SEQUENTIAL TASKS AND MORAL HAZARD

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Sequential Tasks and Moral Hazard

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ABSTRACT

Sequential Tasks and Moral Hazard

A principal owns a two-stage project consisting of two tasks with a finish-to-start dependency where there exists an outcome externality between these tasks. A success in the first task may affect the probability of success of the second task in a positive or negative way, which refers to synergistic and conflicting tasks, respectively. We analyze the optimal incentive schemes where both the number of agents and the order of the tasks are endogenous. We find that, regardless of the optimal set of agents, when both tasks are synergistic, the task with the greater impact is delegated first, while when tasks are conflicting, the one with the lower impact is scheduled first. When one task is synergistic and the other one is conflicting, the principal prefers the synergistic task to be completed first. When the task order is endogenously determined, the principal prefers to employ two identical agents for two different tasks when the tasks are synergistic and a single agent for two tasks when both tasks are conflicting. Since the task allocation is endogenous, we let one task to be synergistic and the other one to be conflicting, in which case it is optimal for the principal to hire two different agents. These results are robust to a possible wage payment after the completion of each task.

ÖZET

Sıralı Görevler ve Ahlaki Tehlike

Bir asil, görevler arasında sonuç dışsallığı olan ve ardışık iki görevden oluşan iki aşamalı bir projenin varlık kısıtı olan ajanlar tarafından tamamlanmasını istemektedir. Birinci görevdeki başarı, ikinci görevin başarı olasılığını olumlu veya olumsuz olarak etkileyebilir. Bu çalışmada, hem vekil sayısının hem de görevlerin içsel olarak belirlendiği durumda optimal teşvik ödemelerini analiz ediyoruz. Bulgularımıza göre, projedeki optimal vekil sayısından bağımsız olarak, her iki görev de sinerjik olduğunda, önce daha büyük etkiye sahip görev devredilirken, görevler çelişkili olduğunda daha düşük etkiye sahip olan görev ilk önce delege edilir. Bir görev sinerjik ve diğeri çelişkili olduğunda, yönetici sinerjik görevin önce tamamlanmasını tercih eder. Görev sırası içsel olarak belirlendiğinde, görevler arasında sinerji var ise iki görevin tamamlanması için iki farklı vekil tayin edilirken bu görevler arasında çelişki olması durumunda ise asil, tek vekilin projeyi tamamlamasını tercih eder. Eğer bir görevin diğerinin başarı olasılığı üzerinde olumlu bir etkisi varken diğerinin olumsuz bir etkisi varsa asil iki farklı vekil tayin eder. Bu sonuçlar her görev tamamlandıktan sonra oluşabilecek olası bir ücret ödemesi için de aynıdır.

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TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Contract theory investigates how players with conflicting interests form agreements based on the optimal design of incentives of the parties. It is divided into two main frameworks: moral hazard and adverse selection, with the latter can take the form of screening or signaling. Moral hazard occurs whenever at least two players reach an agreement in which one party has the ability to undertake additional risks that reduce the other party's utility. In the most basic moral hazard model, the principal (e.g., and employer) delegates a task to the agent (e.g., employee) where the effort exerted by the agent is not observable and not verifiable by the principal. The observable outcome of the task generates revenue to the principal, which is expected to be higher when the effort exerted to the task is higher. This task cannot be carried out by the principal. The agent decides the effort level that he will exert for the task, which generates a disutility for the agent. Both the principal and the agent are utility maximizers and there exists a conflict of interest between them. For that, the principal's objective is to design a contract, that is, an outcome dependent wage scheme, that maximizes her utility while incentivizing the agent to exert effort to perform the task.

Although some agency problems may involve a single agent and a single task, in practice, most agency problems involve multiple tasks and multiple agents. Also, the tasks may have to be conducted in a sequence, which may be endogenous, and these tasks can be delegated to a single agent or a set of agents. Thus, when there are multiple agents available and multiple tasks to be accomplished in an endogenous sequence, the principal may face two decision problems: task scheduling and task assignment. Task scheduling refers to the order in which the tasks are performed whereas task assignment refers to the principal's task bundling decision: assigning different tasks to either a single agent (bundling or integration), or multiple agents (unbundling or separation).

Task Assignment. Task assignment problem is also known as the bundling decision, for which a relevant context is the R&D projects. Companies must decide whether to incorporate R&D processes with operational activities or to create a distinct department for research and development programs. Even though big corporations and banks agree that separating R&D and operational activities is the best approach to optimize profit, numerous firms incorporate R&D activities into related divisions recently. When the tasks are bundled, the rents transferred to the agent are consolidated to motivate the agent to put in high effort. Under limited liability constraint, bundling of the rents may have a spillover effect in the sense that the agent may lose the rent he earns from the first job when the second job fails, encouraging the agent to exert high effort. However, in the context of sequential moral hazard with outcome externality, principal may prefer to hire multiple agents for these tasks when the first stage success increases the success probability of the second stage.

Bundling decision is also commonly addressed in the literature studying public-private partnerships as well.[1] In this context, it is frequently debated whether or not the building of a facility and the supply of services should be combined. Public-private partnership refers to the cooperation between a government entity and a private business where financing, developing and operating a public project (e.g., convention center) are allocated to a single contractor, whereas under traditional procurement, these tasks are separated to different contractors.

Task Scheduling. When a project requires a set of tasks to be completed, the optimal ordering of the tasks may be a decision to be made by the principal. Although in some cases the task order may be exogenously given, in some other cases the order is endogenous. The order decision may not be trivial especially when the tasks are related in terms of the impact of one task's success on the other tasks' success probabilities. For example, in a scientific research project, there may be

[1] See Hart (2003), Bennett and Iossa (2006), Martimort and Pouyet (2008), Chen and Chiu (2010), Chen and Chiu (2012), Iossa and Martimort (2012), Iossa and Martimort (2015), Schmitz (2013b) and Schmitz (2019)

several tasks to be accomplished such as conducting the empirical analysis and solving the theoretical model. These tasks can be taken in any order and several researchers may be in charge of these tasks. Furthermore, an outcome externality between these tasks may exist; empirical model may help build the theoretical model (that is, synergistic tasks) or vice versa.

Another typical example which involves multiple tasks is drug development projects, which consists of identification of disease causal factors, drug discovery, preclinical and clinical research and getting it approved by the FDA. Although, there is usually a given sequence of the main tasks in the drug development, there is some flexibility regarding submission of a drug to the approval of FDA and developing as many candidate drugs as possible: the drug developing team may first try to generate a number of alternative drug options and then start the submission process, or the team could submit the very first drug developed to the FDA, before designing other drug options (Agastya, Bag, & Pepito, 2016).

Combination of task assignment and task scheduling problems are frequently observed in several business projects. For example, the managers of audit departments often face both task assignment and task scheduling decisions. Banking Regulatory and Supervisory Agency (BRSA) in Turkey requires internal audit departments of the banks to complete inspecting the banking processes at the end of each year. These operational processes contain several inspection activities with several tests that are needed to be executed for each of these activities. Each process is a project with multiple tasks, in which each task refers to an inspection activity. These tests can be conducted by a single auditor or the manager can assign these inspection activities to multiple auditors. Moreover, for technical reasons, these inspection activities can only be carried out sequentially. In general, these tests must be performed with personnel from departments that are related to the operational process which is being inspected. Therefore, another test with that department cannot be conducted without finishing the previous test even when other auditors are hired for the project. Furthermore, outcome of one test might have an externality on the other test. For example, one of the inspection activities might be to audit whether or not workers in the Internal Audit Department examines if the check collection processes are working correctly, and the other task might be to check whether a specific process of the check collection is working correctly or not. Then the outcome of the first test would positively affect the effectiveness of the effort exerted to the second task.

In this study, we analyze both task scheduling and task assignment decisions of a principal, where both the task order and the number of agents are endogenously determined in a two-stage/two-task moral hazard setting. We consider outcome externality between the two tasks: the outcome of the first task has an effect (positive or negative) on the success probability of the second task. When the first task is synergistic (conflicting) with the second task, its success increases (decreases) the probability of success of the second task.[2] We consider a set of two identical agents who are risk-neutral and protected by limited liability, and assume that the effort levels of the agents are not observed by the principal and are non-verifiable. We first formulate the principal's problem when she hires only one agent, and characterize the optimal sequence of the two tasks. Then, we consider the principal's problem when she hires two symmetric agents, and solve for the optimal order of tasks. Finally, we analyze the problem of task assignment and find the optimal number of agents the principal hires. We find that, independent from how many agents are responsible for the project, (i) the task with larger (lower) impact is delegated first when both tasks are synergistic (conflicting), (ii) synergistic task is delegated first when one task is synergistic and the other one is conflicting. Moreover, the optimal way to induce effort is to employ two different agents when the tasks are synergistic whereas the principal should hire one agent to carry out two tasks when the tasks are conflicting. Our result regarding task assignment decision of the principal in our model is parallel to the result in Schmitz (2013a), who addresses the task assignment problem with an

[2]However, since the order of tasks is endogenous, we do not fix the nature of outcome externality. We let, for instance, task a (if it is the first task) to be synergistic with task b, while task b (if it is the first task) to be conflicting with task a.

exogenously given task order in the presence of outcome externality, whereas we address both task assignment and task scheduling problems simultaneously, with an endogenous task order.

More specifically, we consider a project involving two tasks to be performed sequentially, which can be completed by a single agent or by assigning two identical agents to two distinct tasks. All parties are risk neutral and the agents are wealth constrained. Outside option is not present, that is the reservation utility is zero for each agent. For each task, an agent must determine whether he will exert costly effort or not, for which effort decision affects the success probability of that task. Effort exerted by the agent is not observable and not verifiable whereas outcome of each task is observable, which can be either success or failure. The principal gets a strictly positive revenue for each successful task, which is assumed to be sufficiently large so that the principal wants to induce high effort at each stage, while gets zero for failed tasks. Furthermore, success in the first stage is assumed to make effort in the subsequent stage either more or less effective, referring to synergistic and conflicting tasks, respectively. Both the order and the agent set to be hired are endogenous.

Several jobs that a principal assigns to an agent are frequently conflicting (Dewatripont & Tirole, 1999). Thus, motivating one agent to work on two competing objectives might be challenging, which may drive the principal to hire two different agents to perform two conflicting tasks (Schmitz, 2013a). When the allocation of the tasks is endogenously determined, we show that the principal hires one agent (two agents) when both tasks are conflicting (synergistic).[3] When one task is synergistic and the other one is conflicting, then the optimal way to induce high effort is to hire two agents to work on the project where the optimal order is to delegate the synergistic task first. When both tasks are synergistic (conflicting), then the optimal allocation of the tasks is to delegate the task with greater (lower) impact first. These results are robust to a possible change in the timing of the transfers. More precisely, the optimal contracts when the wage payments are made after the completion of the

[3]Thus, the results in Schmitz (2013a) still hold.

project are equivalent the optimal contracts with payments made after the completion of each task. Moreover, if the outcome externality is in terms of the effect of a task's success on the marginal cost of the next task, rather than on the marginal success probability, our results indicate that there exists a task order equivalence between outcome externality in terms of marginal cost and marginal probability.[4]

Earlier studies in the moral hazard literature have focused on the models in which the principal delegated a single job to a single agent.[5] The literature regarding multi-task agency problems has been growing as well. The literature on multitask principal-agent problems has begun with the article Holmstrom and Milgrom (1991). They focus on the effort substitution problem of risk averse agents in a simultaneous moral hazard setting. Similarly, Itoh (1994) studies multitask moral hazard problem in which the agents are risk averse, focusing on the trade-off between incentives and insurance.[6]

Although many studies in the multi-task agency literature focus on simultaneous jobs,[7] there are now numerous articles that investigate circumstances where tasks must be delivered sequentially.[8,9] Berkovitch et al. (2010) consider a two-stage moral hazard in which unobservable effort is exerted in the second stage only. Kräkel and Schöttner (2012) and Ohlendorf and Schmitz (2012) depart from this paper in the sense that they do not study the optimal agent set. Related to these articles, Nieken and Schmitz (2012) conduct a laboratory experiment and find out that even if the durations are technologically unrelated, principals can gain by offering long-term contracts with memory because of incentive considerations.

[4] This result is an addition to the equivalence of the optimal agent sets under outcome externality in terms of marginal cost and that in terms of marginal probability (Pi, 2014).

[5] See, for example, Ross (1973), Spence and Zeckhauser (1978), Mirrlees (1976), Holmström (1979), Holmstrom (1981), Grossman and Hart (1983), Harris and Raviv (1979).

[6] See also Baker, Gibbons, and Murphy (1994), Dewatripont, Jewitt, and Tirole (1999) for theoretical studies and Fehr and Schmidt (2004) for experimental study focusing on effort substitution problem.

[7] See Dewatripont, Jewitt, and Tirole (2000), Kragl and Schöttner (2011) and Z. Li, Lu, Ryan, and Sun (2021)

[8] See, for example, Kräkel and Schöttner (2012), Berkovitch, Israel, and Spiegel (2010), Müller (2011) , Ohlendorf and Schmitz (2012), Bessen and Maskin (2009), Cato and Ishihara (2017), Chen and Stephen Chiu (2013), Strausz et al. (1996) and Klor, Kube, Winter, et al. (2014)

[9] Dewatripont et al. (2000), Laffont and Martimort (2009), and Dewatripont, Bolton, et al. (2005) provide comprehensive reviews of multi-task agency models under moral hazard.

Many studies examine multi-task agency problems using a dynamic approach.[10,11] Schöttner (2017) and Kräkel and Schöttner (2016) study dynamic moral hazard model to derive optimal sales and compensation plans, commissions and bonus payments. Özener (2019) studies a dynamic moral hazard with sequential tasks in an infinite horizon dynamic setting, focusing on the most efficient outcome and optimal contract when agents tend to delay the project. None of these studies focus on the task assignment problem of the principal.

There are several dimensions in the literature on task assignment and job design. Gilbert and Riordan (1995), Da Rocha and de Frutos (1999) and Severinov (2008) investigate the advantages of integration under adverse selection and perfect task complementarity, but they ignore task scheduling. Severinov (2008) compares three organizational forms where centralization and decentralization refer to integration and separation respectively, and delegation means that two agents are in charge of two tasks but the principal contracts with one of them only.[12] Kragl and Schöttner (2011) examines a two-stage simultaneous moral hazard problem to find out if the principal should hire one agent to perform both tasks or multiple agents in the presence of wage floors. They find that when there is no wage floor and the agents' reservation utility is not too high, it is best to distribute the jobs to various agents. If the wage floor is high enough, the principal only employs one agent. Dewatripont et al. (2005) investigate this subject in a complete contracting framework, assuming that tasks are completed concurrently and that there is an effort externality between jobs referring to the situation in which the effort exerted to the task in the previous stage increases or reduces the success probability of the subsequent task. They conclude that if the task conflict is severe enough, the principle chooses to employ two distinct agents to work on the two jobs. Otherwise, however,

[10] See, for example Müller and Weinschenk (2015), Schöttner (2017), Kräkel and Schöttner (2010), Kräkel and Schöttner (2016), Paez-Perez and Sanchez-Silva (2016), Asseyer (2018), C. Li and Qiu (2018), Macera (2018), Özener (2019), Rivera (2020) and Szydlowski (2019).

[11] For repeated moral hazard models, see MacLeod and Malcomson (1988), Fuchs (2007) and Müller (2011).

[12] See also Riordan and Sappington (1987), Dana Jr (1993), Lewis and Sappington (1997), Severinov (2003), Lockwood (2000) and Xu, Yin, and Li (2019) investigating the bundling decision of the principal using the adverse selection approach in a two-stage agency model.

only one agent should be in charge of both, as incentivizing one agent is less expensive for the principal.

Economies of scope in the context of multi-task principal-agent problems refers to integration where the rent used to incentivize the agent may be low because of spillover effect.[13] Unlike the present paper, Hirao (1993) does not consider conflicting tasks. Che and Yoo (2001) studies a repeated moral hazard model focusing on team incentives. Tsai and Kung (2011) investigate a two-stage sequential moral hazard problem where the outcome of the first task may not be observable. Kragl and Schöttner (2014) explore economies of scope in which tasks are asymmetric and focus on the effects of wage floors. They show that assigning tasks to distinct agents is efficient due to the cost advantages of specialization.[14]

Several authors, beginning with Hart (2003) and Bennett and Iossa (2006), have focused on the recent debates about public-private partnerships; public-private partnership vs. traditional procurement. They observed that public-private partnerships (PPPs) can provide more incentives to implement cost-reducing investments compared to traditional procurement. Unlike this paper, Hart (2003) considers incomplete contracting. Iossa and Martimort (2012) and Martimort and Straub (2016) use a dynamic multi-task moral hazard problem where renegotiation is possible. Hoppe and Schmitz (2013) conducts a laboratory experiment to find out the optimal decision of the principal for building and operating activities of a public facility. They find support for theoretical prediction. Buso (2019) takes an adverse selection approach to analyze PPPs. The results in Schmitz (2013b) show that it is optimal to employ one agent if the government does not confront any binding budget constraint and separate the tasks otherwise. Hoppe and Schmitz (2021) consider production externality and allow for renegotiation. Buso, Greco, et al. (2021) do not consider any externality and assume the government has a budget constraint.[15]

[13] See Laux (2001), Tamada and Tsai (2007) and Kusterer (2014), for related literature.

[14] See also Hart, Shleifer, and Vishny (1997) and Hoppe and Schmitz (2010) for similar theoretical works on privatization.

[15] For other studies analyzing public-private partnerships by focusing on the bundling decisions, see Martimort and Pouyet (2008), Fang, Liu, Bao, and Fang (2009), Chen and Chiu (2010), Chen and Chiu (2012), Iossa and Martimort (2015) and Greco (2015).

Dewatripont and Tirole (1999) study task assignment problem in an incomplete contracting model and assume that the different tasks that are delegated to the agents can be conflicting. Their results show that when the two tasks are in direct conflict, principal prefers to employ two different agents for two different tasks, which is overturned in the present study. Following Dewatripont and Tirole (1999), Schmitz (2019) analyzes the integration decision of the principal under incomplete contracting framework by focusing on R&D activities. The results in this paper show that joint ownership can be optimal, which is in line with current R&D applications. Close to our setting, Schmitz (2005) studies a sequential moral hazard model under incomplete contracting and considers outcome externality in terms of marginal probability where a success in the first stage increases the effectiveness of the effort exerted in the second stage, i.e., synergistic tasks. However, conflicting tasks and task scheduling are not considered. Hoppe and Kusterer (2011) conduct a laboratory experiment on sequential moral hazard in the presence of outcome externalities and allow for conflicting tasks to exist. Their experimental evidence supports the finding of Dewatripont et al. (2005) that two agents are assigned when the tasks are in conflict whereas the principal prefers to hire one agent when the tasks are synergistic. Grossman and Hart (1986) and Hart and Moore (1990) study principal-agent model under incomplete contracting and focus on vertical integration where owning a firm refers to integration, but contracting for a service from another party who owns this firm is non-integration.[16]

Pi (2014), Pi (2018) and Schmitz (2013a) use two-stage sequential moral hazard models with outcome externalities to find the optimal number of agents to be hired for the project. Schmitz (2013a) considers outcome externality in terms of marginal success probability, where the outcome of the first stage task increases or decreases the effectiveness of effort in the second stage. Related to Schmitz's work, Pi (2018) assumes outcome externality concerning fixed part of the success probability in which the first stage outcome increases or decreases the fixed portion

[16]Similarly, Hemmer (1995), Khalil, Kim, and Shin (2006), Schöttner (2008) and S. Li, Sun, Yan, and Yu (2015) explore the optimal agent set that the principal should assign.

of the success probability of the second stage task. Pi (2014) handles this issue by focusing on the outcome externality regarding the effort cost where marginal cost refers to the situation in which the outcome in the first stage affects the marginal part of the cost of the effort exerted in the second stage and fixed cost refers to case in which first stage outcome affects the fixed term of the second stage effort cost. He concluded that the optimal organizational form, that is, integration vs separation, when there exists outcome externality in terms of marginal cost is equivalent to that of when the outcome externality regarding fixed cost is present. In Section 4, we show that there exists task order equivalence between outcome externality in terms of marginal cost and marginal probability. These three studies are closest to our work in the sense that we built a two-stage sequential moral hazard problem with outcome externality in terms of marginal probability and marginal cost. however, none of these studies consider task ordering problem.

One of the earliest studies that focus on task allocation and stopping decisions is Weitzman (1979), in which an agent, Pandora, opens the boxes in whatever sequence she wants and gets the utility from the greatest prize she discovers net of the cost from opening the boxes. Olszewski and Weber (2015) extends this problem by allowing for the payoffs to depend on all discovered prices. However, these studies do not contain agency considerations. Optimal allocation of tasks in time is also common in queueing literature. Mitra (2001) focuses on first best implementable cost structures, where the principal's problem is to efficiently organize the agents in a queue in order to reduce the total waiting time. Using an empirical approach, Ibanez, Clark, Huckman, and Staats (2018) study the similar problem in radiological services using the perspective of the agents. According to their empirical findings, doctors prioritize similar jobs and those with the shortest estimated processing time.

Mocadlo (2021) examines auditors' ranking and performance of the jobs under time pressure where there exist no agency concerns and no outcome externalities between the tasks. The results show that auditors prefer to work on tasks that have more objective criteria before working on tasks that have more subjective criteria. In Thiele (2010), potential ordering metrics for performance measurements in multi-task agency model are investigated, in which the agent demonstrates task specific skills, that is, the agents are not identical as they are in our model. Winter (2006) studies a sequential moral hazard model where the outcome of the intermediate task is not observable, focusing on the ordering of the tasks in time, which vary in terms of significance, and assigning them to the agents with distinct skills. Different from Winter (2006), in he present study we address the issue of optimal agent set where the agents are identical. Agastya and Birulin (2021) study multi-task agency model focusing on the task ordering problem of the principal with moral hazard and adverse selection, where only specialists can perform these tasks. The closest article investigating the allocation of the tasks in time under agency problem is Agastya et al. (2016), which analyze two-task moral hazard problem with outcome externality. They suggest that even though it is less essential, a work with a significantly bigger impact can be attempted earlier. Yet, none of these studies consider the bundling decision of the principal.

In a very relevant study for our setting, Mylovanov and Schmitz (2008) characterizes the optimal task allocation over time and the optimal task assignment to identical agents in the presence of outcome externality via solving two-stage moral hazard problem with risk neutral and wealth constrained agents. Different from our setting, Mylovanov and Schmitz (2008) considers three identical jobs, and one agent can carry out only two of them in one stage. The scheduling decision that the principal makes is whether to organize all three tasks to be performed in one stage by contracting several agents or to hire one agent to complete all three tasks in two periods. If the project is decided to be spread over two stages, the principal should also decide if the allocation of the tasks over time should be two tasks in the first stage and one task in the second stage or vice versa. Outcome externality which is considered in Mylovanov and Schmitz (2008) is that the assignment of tasks to agents in the second stage may be affected by the outcomes of the first task. For example, a failure in the first stage may lead the principal to alter the agent, i.e., separation of the tasks. The task allocation investigated in the present study, however, is determined by the impact and effectiveness of the tasks where impact refers to the intensity of the outcome externality.

CHAPTER 2

MODEL

Consider a project containing two tasks, $\{a, b\}$, to be performed sequentially. These tasks can be carried out by one agent, or two identical agents can be assigned for two different tasks. For each task, the agent in charge decides whether to exert effort or not. Effort on task $i \in \{a, b\}$ is denoted by $e_i \in \{0, 1\}$ and disutility from exerting effort is denoted by ψe_i. Effort decisions of agents affect the probability of success of the outcome. The outcome of task $i \in \{a, b\}$ is denoted by $q_i \in \{0, 1\}$ where $q_i = 1$ if it is a success and $q_i = 0$ otherwise. The outcome of task i is verifiable, but the effort decision on a task is not observable and not verifiable. The principal gets $R > 0$ for each successful task and zero for each failed task. Throughout this paper, R is assumed to be sufficiently large so that the principal always wants to induce high effort. All parties are risk-neutral, agents are wealth constrained, that is, there is limited liability constraint, and the reservation utility is zero for each agent. The task order is endogenous.

Suppose that task i is delegated first. Then the success probabilities of first and second tasks are

$$\begin{aligned} \Pr\{q_i = 1\} &= \alpha + \rho_i e_i \\ \Pr\{q_{-i} = 1\} &= \alpha + (\rho_{-i} + \gamma_{q_i}) e_{-i} \end{aligned}$$

respectively, where there is an outcome externality of task i on task –i, captured through γ_{q_i}, where

$$\gamma_{q_i} = \begin{cases} \gamma_i & \text{if} \quad q_i = 1 \\ 0 & \text{if} \quad q_i = 0 \end{cases}$$

where $i = a, b$. Note that γ_{q_i} being equal to zero when the first stage task is a failure is for normalization purposes. Here, $1 > \rho_i > 0$ stands for the effectiveness of the effort on the success probability of task i, where higher ρ_i increases the success probability of effort. Also, γ_{q_i} represents the impact of task i on the success probability of task

$-i$, that is, whether or not the first stage task is a success affects the success probability of the second stage task. Specifically, suppose task i is delegated first by the principal. If $\gamma_i > 0$, then task i is synergistic on task $-i$: the first stage success positively affects the second stage success probability. If $\gamma_i < 0$, then task i is conflicting on task $-i$: the first stage success negatively affects the second stage success probability. When $\gamma_i = \gamma_{-i} = 0$, then the tasks are technologically independent.

The parameter α is assumed to be strictly positive so that there is a positive success probability even if the agent does not exert effort. Moreover, it is assumed that $0 < \alpha < 1 - \max\{\rho_a, \rho_b, \rho_a + \gamma_b, \rho_b + \gamma_a\}$ in order to assure that the success probabilities are between zero and one.

Assumption 1: $|\gamma_i| < \rho_{-i}$ whenever $\gamma_i < 0$, for $i = a, b$.

Assumption 1 ensures that $\rho_{-i} + \gamma_{q_i} > 0$. Thus, the monotonicity of the success probability of task $-i$ with respect to effort is preserved, that is, the higher the effort the higher the success probability. The principal offers a wage scheme $\{w_{11}, w_{10}, w_{01}, w_{00}\}$ if one agent is assigned to perform both tasks and $\{(w_{11}^A, w_{10}^A, w_{01}^A, w_{00}^A), (w_{11}^B, w_{10}^B, w_{01}^B, w_{00}^B)\}$ if two agents, A, B, are employed for the two tasks, a, b.

2.1 Principal's problem: One agent

One agent is employed to perform both tasks. Let task a be the first task delegated. Then, the success probabilities are $\Pr\{q_a = 1\} = \alpha + \rho_a e_a$ for the first stage, and $\Pr\{q_b = 1\} = \alpha + (\rho_b + \gamma_{q_a})e_b$ for the second stage. Principal'a problem is to find the cost minimizing wage scheme such that the agent exerts high effort. The agent is willing to participate in the project if the following constraint is satisfied.

$$(\alpha + \rho_a)\left[(\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - \psi\right]$$
$$+(1 - \alpha - \rho_a)\left[(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00} - \psi\right] \geq \psi$$

which is equivalent to

$$\begin{aligned}&(\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11}+(1-\alpha-\rho_b-\gamma_a)w_{10}\right]\\&+(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01}+(1-\alpha-\rho_b)w_{00}\right]\geq 2\psi\end{aligned} \tag{1}$$

The agent is willing to exert high effort in the first stage if the first stage incentive compatibility constraint below is satisfied.

$$\begin{aligned}&(\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11}+(1-\alpha-\rho_b-\gamma_a)w_{10}-\psi\right]\\&+(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01}+(1-\alpha-\rho_b)w_{00}-\psi\right]-\psi\\&\geq \alpha\left[(\alpha+\rho_b+\gamma_a)w_{11}+(1-\alpha-\rho_b-\gamma_a)w_{10}-\psi\right]\\&+(1-\alpha)\left[(\alpha+\rho_b)w_{01}+(1-\alpha-\rho_b)w_{00}-\psi\right]\end{aligned}$$

This is simplified to

$$\rho_a\left[(\alpha+\rho_b+\gamma_a)w_{11}+(1-\alpha-\rho_b-\gamma_a)w_{10}-(\alpha+\rho_b)w_{01}-(1-\alpha-\rho_b)w_{00}\right]\geq\psi \tag{2}$$

The incentive compatibility constraint ensuring that the agent exerts high effort in the second stage, $e_b = 1$, when the first stage is a success, $q_a = 1$, is

$$(\alpha+\rho_b+\gamma_a)w_{11}+(1-\alpha-\rho_b-\gamma_a)w_{10}-\psi\geq\alpha w_{11}+(1-\alpha)w_{10}$$

which can be simplified to

$$(\rho_b+\gamma_a)(w_{11}-w_{10})\geq\psi \tag{3}$$

The incentive compatibility constraint ensuring that the agent exerts high effort in the second stage, $e_b = 1$, when the first stage is a failure, $q_a = 0$, is

$$(\alpha+\rho_b)w_{01}+(1-\alpha-\rho_b)w_{00}-\psi\geq\alpha w_{01}+(1-\alpha)w_{00}$$

which can be rewritten as

$$\rho_b(w_{01} - w_{00}) \geq \psi \tag{4}$$

The principal solves the following cost minimization problem, which we denote by $\mathcal{P}_1$:

$$\min_{\{w_{11},w_{10},w_{01},w_{00}\}} \begin{bmatrix} (\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11} + (1-\alpha-\rho_b-\gamma_a)w_{10}\right] \\ +(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01} + (1-\alpha-\rho_b)w_{00}\right] \end{bmatrix}$$

subject to

$$\begin{aligned}
&(\mathrm{PC}) && (\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11} + (1-\alpha-\rho_b-\gamma_a)w_{10}\right] \\
& && \quad +(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01} + (1-\alpha-\rho_b)w_{00}\right] \geq 2\psi \\
&(\mathrm{IC}_1) && (\alpha+\rho_b+\gamma_a)w_{11} + (1-\alpha-\rho_b-\gamma_a)w_{10} \\
& && -(\alpha+\rho_b)w_{01} - (1-\alpha-\rho_b)w_{00} \geq \frac{\psi}{\rho_a} \\
&(\mathrm{IC}_2^{q_a=1}) && (\rho_b+\gamma_a)(w_{11} - w_{10}) \geq \psi \\
&(\mathrm{IC}_2^{q_a=0}) && \rho_b(w_{01} - w_{00}) \geq \psi \\
&(\mathrm{LL}) && w_{q_a q_b} \geq 0 \quad \text{for each} \quad q_a, q_b \in \{0,1\}
\end{aligned}$$

where the first two constraints are the incentive compatibility constraints in the second stage following the outcome q_a, and the third constraint is the incentive compatibility constraint in the first stage, and the fourth is the participation constraint, and finally the last one is the limited liability constraint.

2.2 Principal's problem: Two agents

Suppose that the principal hires two agents, A and B, for the two tasks, a and b. Now, a wage scheme is given by $\{(w_{11}^A, w_{10}^A, w_{01}^A, w_{00}^A), (w_{11}^B, w_{10}^B, w_{01}^B, w_{00}^B)\}$. Since the agents are identical, which agent performs which task is irrelevant, however, the order of tasks matters. Without loss of generality, let agent A be in charge of task a and agent B be in charge of task b. Moreover, suppose task a is delegated first.

Agent A is willing to participate if the following constraint is satisfied.

$$(\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11}^A+(1-\alpha-\rho_b-\gamma_a)w_{10}^A\right]+(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01}^A+(1-\alpha-\rho_b)w_{00}^A\right] \geq \psi \tag{5}$$

Agent A is willing to exert high effort in the first stage if the first stage incentive compatibility constraint below is satisfied.

$$\begin{gathered}(\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11}^A+(1-\alpha-\rho_b-\gamma_a)w_{10}^A\right]+\\ (1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01}^A+(1-\alpha-\rho_b)w_{00}^A\right]-\psi\\ \geq \alpha[(\alpha+\rho_b+\gamma_a)w_{11}^A+(1-\alpha-\rho_b-\gamma_a)w_{10}^A]+(1-\alpha)[(\alpha+\rho_b)w_{01}^A+(1-\alpha-\rho_b)w_{00}^A]\end{gathered}$$

This is simplified to

$$\rho_a\left[(\alpha+\rho_b+\gamma_a)w_{11}^A+(1-\alpha-\rho_b-\gamma_a)w_{10}^A-(\alpha+\rho_b)w_{01}^A-(1-\alpha-\rho_b)w_{00}^A\right] \geq \psi \tag{6}$$

Agent B is willing to participate in the project if the following two constraints are satisfied.

$$(\alpha+\rho_b+\gamma_a)w_{11}^B+(1-\alpha-\rho_b-\gamma_a)w_{10}^B \geq \psi \tag{7}$$

$$(\alpha+\rho_b)w_{01}^B+(1-\alpha-\rho_b)w_{00}^B \geq \psi \tag{8}$$

The incentive compatibility constraint ensuring that agent B chooses high effort in the second stage, $e_b = 1$, when the first stage is a success, $q_a = 1$, is

$$(\alpha+\rho_b+\gamma_a)w_{11}^B+(1-\alpha-\rho_b-\gamma_a)w_{10}^B-\psi \geq \alpha w_{11}^B+(1-\alpha)w_{10}^B$$

which is rewritten as

$$(\rho_b+\gamma_a)(w_{11}^B-w_{10}^B) \geq \psi \tag{9}$$

The incentive compatibility constraint ensuring that agent B exerts high effort in the second stage, $e_b = 1$, when the first stage task is a failure, $q_a = 0$, is

$$(\alpha+\rho_b)w_{01}^B + (1-\alpha-\rho_b)w_{00}^B - \psi \geq \alpha w_{01}^B + (1-\alpha)w_{00}^B$$

which is simplified to

$$\rho_b(w_{01}^B - w_{00}^B) \geq \psi \tag{10}$$

The principal solves the following cost minimization problem, which we denote by $\mathcal{P}_2$:

$$\min_{\substack{\{(w_{11}^A, w_{10}^A, w_{01}^A w_{00}^A) \\ (w_{11}^B, w_{10}^B, w_{01}^B, w_{00}^B)\}}} \begin{bmatrix} (\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)(w_{11}^A + w_{11}^B) + (1-\alpha-\rho_b-\gamma_a)(w_{10}^A + w_{10}^B)\right] \\ +(1-\alpha-\rho_a)\left[(\alpha+\rho_b)(w_{01}^A + w_{01}^B) + (1-\alpha-\rho_b)(w_{00}^A + w_{00}^B)\right] \end{bmatrix}$$

subject to

$$\begin{aligned}
&(PC_A) && (\alpha+\rho_a)\left[(\alpha+\rho_b+\gamma_a)w_{11}^A + (1-\alpha-\rho_b-\gamma_a)w_{10}^A\right] \\
& && \quad +(1-\alpha-\rho_a)\left[(\alpha+\rho_b)w_{01}^A + (1-\alpha-\rho_b)w_{00}^A\right] \geq \psi \\
&(IC_A) && (\alpha+\rho_b+\gamma_a)w_{11}^A + (1-\alpha-\rho_b-\gamma_a)w_{10}^A \\
& && -(\alpha+\rho_b)w_{01}^A - (1-\alpha-\rho_b)w_{00}^A \geq \frac{\psi}{\rho_a} \\
&(PC_B^{q_a=1}) && (\alpha+\rho_b+\gamma_a)w_{11}^B + (1-\alpha-\rho_b-\gamma_a)w_{10}^B \geq \psi \\
&(PC_B^{q_a=0}) && (\alpha+\rho_b)w_{01}^B + (1-\alpha-\rho_b)w_{00}^B \geq \psi \\
&(IC_B^{q_a=1}) && (\rho_b+\gamma_a)(w_{11}^B - w_{10}^b) \geq \psi \\
&(IC_B^{q_a=0}) && \rho_b(w_{01}^B - w_{00}^B) \geq \psi \\
&(LL) && w_{q_a q_b}^j \geq 0 \quad \text{for each} \quad q_a, q_b \in \{0,1\} \quad \text{and} \quad j \in \{A,B\}
\end{aligned}$$

CHAPTER 3

OPTIMAL CONTRACTS

3.1 Optimal contract with one agent

When there is a single agent, we solve $\mathcal{P}_1$ to find the cost minimizing contract when task i is delegated first.

Lemma 1: Suppose both tasks are delegated to one agent, and that the revenue R is high enough so that the principal induces high effort for each task. Suppose task i is delegated first by the principal.

(i) When $\rho_{-i}^2 + \rho_{-i}\gamma_i + \alpha\gamma_i\rho_i \geq 0$, it is optimal for the principal to offer the contract $\{w_{11}, w_{10}, w_{01}, w_{00}\}$ such that $w_{00} = w_{10} = 0$, $w_{01} = \frac{\psi}{\rho_{-i}}$, and $w_{11} = \frac{\rho_{-i}+\rho_i(\alpha+\rho_{-i})}{\rho_i\rho_{-i}}\frac{\psi}{\alpha+\rho_{-i}+\gamma_i}$. The expected cost of the principal is

$$\psi\left[\frac{\alpha+\rho_i}{\rho_i}+\frac{\alpha+\rho_{-i}}{\rho_{-i}}\right]$$

(ii) When $\rho_{-i}^2 + \rho_{-i}\gamma_i + \alpha\gamma_i\rho_i < 0$, it is optimal for the principal to offer $w_{00} = w_{10} = 0$, $w_{01} = \frac{\psi}{\rho_{-i}}$ and $w_{11} = \frac{\psi}{\rho_{-i}+\gamma_i}$. The expected cost of the principal is

$$\psi\left[\frac{\alpha+\rho_{-i}}{\rho_{-i}}-\frac{\alpha\gamma_i(\alpha+\rho_i)}{\rho_{-i}(\rho_{-i}+\gamma_i)}\right]$$

Proof: See the Appendix.

Suppose that the principal gets a revenue $R > 0$ for each successful task, and 0 for a failed task. Therefore, the possible revenue levels are $\{2R, R, 0\}$; 2R when both tasks are successful, R when one task is successful and the other fails, and 0 when both tasks fail. Then, when high effort is induced for both tasks, and task a is first in order, her expected revenue is

$$\begin{aligned}
ER(e_a = e_b = 1) &= (\alpha+\rho_a)(\alpha+\rho_b+\gamma_a)2R + (\alpha+\rho_a)(1-\alpha-\rho_b-\gamma_a)R \\
&\quad +(1-\alpha-\rho_a)(\alpha+\rho_b)R \\
&= R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]
\end{aligned}$$

Note that the expected revenue is symmetric when the task order is reversed. Given the order of tasks and the effort levels to induce, the optimal wage scheme is determined through the principal's cost minimization problem. To find the optimal task order when both tasks are delegated to one single agent, the principal compares expected total profits from each task order.

Proposition 1: Suppose both tasks are delegated to one single agent, and that the revenue R is high enough so that the principal induces high effort for each task. Then,

(i) If both tasks are synergistic, that is, $\gamma_a > 0$ and $\gamma_b > 0$, then

  (a) When $\rho_a = \rho_b = \rho$, the task with higher impact (higher γ_i) is delegated first.

  (b) When $\gamma_a = \gamma_b = \gamma$, the task with higher effectiveness (higher ρ_i) is delegated first.

(ii) If both tasks are conflicting, that is, $\gamma_a < 0$ and $\gamma_b < 0$, then

  (a) When $\rho_a = \rho_b = \rho$, the task with lower impact (lower $|\gamma_i|$) is delegated first.

  (b) When $\gamma_a = \gamma_b = \gamma$, the task with lower effectiveness (lower ρ_i) is delegated first.

(iii) If one task is synergistic and the other one is conflicting, then the synergistic task is delegated first.

Proof: See the Appendix.

The intuition for the result above is as follows. In the case where both tasks are in synergy with each other, if the first stage task is successful, the success probability of the second task increases marginally with the parameters γ_i (impact) and ρ_{-i} (effectiveness). Since it becomes easier to be successful in the second task as the impact of the first task gets larger, the rent used to motivate the agent to exert high effort becomes lower. If the impact levels are equal, however, the task with larger effectiveness is delegated first as the rent to incentivize the agent to exert effort in the second stage decreases with the level of effectiveness. Similarly, when one task is synergistic and the other one is conflicting, placing the synergistic task first increases the success probability of the second task in case of first stage success, which leads to a lower second stage rent. In contrast, in the case where both tasks are in conflicting with each other, the first stage success decreases the success probability of the second stage, which gets larger as the impact level increases. The higher the impact is, the lower the second stage success probability is, which means the larger the rent is. However, the task with lower effectiveness is delegated first, because as the tasks are conflicting, the probability of success for the first task is then smaller, which in turn decreases the adverse effect on the success probability of the second task.

3.2 Optimal contract with two agents

When there are two symmetric agents, we solve $\mathcal{P}_2$ to find the cost minimizing contract when task i is delegated first.

Lemma 2: Suppose two tasks are delegated to two symmetric agents sequentially, and that the revenue R is high enough so that the principal induces high effort for each task. Suppose task i is delegated first, to agent A. Then, it is optimal for the principal to offer the wage scheme

$$w_{00}^{A} = w_{01}^{A} = 0,\ w_{10}^{A} = w_{11}^{A} = \frac{\psi}{\rho_i}$$

and

$$w_{00}^{B} = w_{10}^{B} = 0,\ w_{01}^{B} = \frac{\psi}{\rho_{-i}},\ w_{11}^{B} = \frac{\psi}{\rho_{-i} + \gamma_i}$$

The expected cost of the principal is

$$\psi\left[(\alpha + \rho_i)\left(\frac{\alpha}{\rho_{-i} + \gamma_i} + \frac{1}{\rho_i}\right) + (1 - \alpha - \rho_i)\frac{\alpha}{\rho_{-i}} + 1\right]$$

Proof: See the Appendix.

Note that unlike the wages for the one-agent case in Lemma 1, here with two agents the outcome of the second task has no effect on the wages of the agent in charge of the first task: $w^A_{i0} = w^A_{i1}$, for i = 0, 1. The principal's expected revenue is

$$\begin{aligned} ER(e_a = e_b = 1) &= (\alpha+\rho_a)(\alpha+\rho_b+\gamma_a)2R + (\alpha+\rho_a)(1-\alpha-\rho_b-\gamma_a)R \\ &\quad +(1-\alpha-\rho_a)(\alpha+\rho_b)R \\ &= R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]. \end{aligned}$$

Therefore, her expected profit is given by

$$EP(e_a = e_b = 1) = R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]$$
$$-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right].$$

Proposition 2: Suppose two tasks are delegated to two symmetric agents sequentially, and that the revenue R is high enough so that the principal induces high effort for each task. Then,

(i) If both tasks are synergistic, that is, $\gamma_a > 0$ and $\gamma_b > 0$, then

 (a) When $\rho_a = \rho_b = \rho$, the task with higher impact (higher γ_i) is delegated first.

 (b) When $\gamma_a = \gamma_b = \gamma$, the task with higher effectiveness (higher ρ_i) is delegated first.

(ii) If both tasks are conflicting, that is, $\gamma_a < 0$ and $\gamma_b < 0$, then

 (a) When $\rho_a = \rho_b = \rho$, the task with lower impact (lower $|\gamma_i|$) is delegated first.

 (b) When $\gamma_a = \gamma_b = \gamma$, the task with lower effectiveness (lower ρ_i) is delegated first.

(iii) If one task is synergistic and the other one is conflicting, then the synergistic task is delegated first.

Proof: See the Appendix.

The order in Proposition 2 for the two-agent case is similar to the one with one agent in Proposition 1. In the one-agent case, the incentives are interrelated, since the second task outcome affects the wages of the agent who is in charge of the first task. However, here with two agents, the outcome of the second task has no effect on the wages of the agent in charge of the first task, as it is seen in Lemma 2. Then, when both tasks are synergistic, the task with higher(lower) impact or higher(lower) effectiveness is delegated first, to make it easier(harder) to give incentives to the agent in charge of the second task. Thus, the result above follows more directly relative to the case with one agent.

3.3 Optimal number of agents

Given the optimal task orders that are described in Propositions 1 and 2, now we analyze the optimal number of agents for the principal to have. To do this we compare the expected profit levels of the principal we derived in Sections 3.1 and 3.2.

Proposition 3: Suppose there are two tasks to be delegated sequentially to either one agent or to two symmetric agents. Suppose the revenue R is high enough so that the principal induces high effort for each task.

(i) If both tasks are synergistic, that is, $\gamma_a > 0$ and $\gamma_b > 0$, then it is optimal for the principal to delegate the two tasks to two agents.

(ii) If both tasks are conflicting, that is, $\gamma_a < 0$ and $\gamma_b < 0$, then it is optimal for the principal to delegate the two tasks to one agent.

(iii) If one task is synergistic and the other one is conflicting, then it is optimal for the principal to delegate the two tasks to two agents.

Proof: See the Appendix.

Note that the optimal task orders, given the number of agents, in Propositions 1 and 2, are parallel. When both tasks are synergistic, the principal hires two agents and in order to benefit from the incentives for the second agent to be relatively easier to give, than the incentives when there is one agent. When the tasks are conflicting, the lower impact/effectiveness task is first in order and in that case, incentives are easier to generate with one agent as opposed to two agents. This is because in the two-agent case, the burden of the reduction in the second task's success probability falls only on the second agent's shoulders. However, with one agent, the principal can generate incentives more easily since now the reduction in the second task's success probability is faced by the agent in charge of the first task as well.

CHAPTER 4

OUTCOME EXTERNALITY ON THE MARGINAL COST

We now investigate an alternative model in which the tasks must still be completed sequentially, but instead of an outcome externality that takes effect on the marginal probability of success on the second task, we assume that there is an outcome externality that takes effect on the marginal cost of the second stage task. Specifically, the probability of success is $\Pr\{q_i = 1\} = \alpha + \rho e_i$ for each $i \in \{a, b\}$, that is, there is no outcome externality in terms of marginal probabilities. However, the effort cost of the first stage task, task i, is ψe_i whereas the effort cost of the second stage task, task –i, is $(\psi + \gamma_{q_i})e_{-i}$, where the outcome externality of task i on task –i is captured through $\gamma_{q_i} e_{-i}$. A similar standardization for $\gamma_{q_i} e_{-i}$ as in Section 2 is employed here. If $\gamma_i > 0$ ($\gamma_i < 0$) then success in the first stage, task i, increases (decreases) the marginal cost of effort in the second stage, task –i, which refers to conflicting (synergistic) tasks. If $\gamma_i = 0$, then the tasks are technologically independent.

Suppose that the principal hires two identical agents, A and B, for the two tasks, a and b. Now, a wage scheme is given by $\{(w^A_{11}, w^A_{10}, w^A_{01}, w^A_{00}), (w^B_{11}, w^B_{10}, w^B_{01}, w^B_{00})\}$. Without loss of generality, suppose agent A is responsible for task a and agent B is responsible for task b. Furthermore, suppose task a is delegated first by the principal.

Participation constraint of the agent A is

$$(\alpha+\rho)\left[(\alpha+\rho)w^A_{11} + (1-\alpha-\rho)w^A_{10}\right] + (1-\alpha-\rho)\left[(\alpha+\rho)w^A_{01} + (1-\alpha-\rho)w^A_{00}\right] \geq \psi \quad (11)$$

First stage incentive compatibility constraint ensuring that agent A exerts high effort, $e_a = 1$, is

$$\begin{aligned}(\alpha+\rho)\left[(\alpha+\rho)w^A_{11} + (1-\alpha-\rho)w^A_{10}\right] + (1-\alpha-\rho)\left[(\alpha+\rho)w^A_{01} + (1-\alpha-\rho)w^A_{00}\right] - \psi \\ \geq \alpha[(\alpha+\rho)w^A_{11} + (1-\alpha-\rho)w^A_{10}] + (1-\alpha)[(\alpha+\rho)w^A_{01} + (1-\alpha-\rho)w^A_{00}]\end{aligned}$$

which is equivalent to

$$\rho[(\alpha+\rho)(w_{11}^{A}-w_{01}^{A})+(1-\alpha-\rho)(w_{10}^{A}-w_{00}^{A})] \geq \psi \tag{12}$$

Agent B is willing to participate in the project if the following constraints are satisfied.

$$(\alpha+\rho)w_{11}^{B}+(1-\alpha-\rho)w_{10}^{B} \geq \psi+\gamma_a \tag{13}$$

$$(\alpha+\rho)w_{01}^{B}+(1-\alpha-\rho)w_{00}^{B} \geq \psi \tag{14}$$

The incentive compatibility constraint ensuring that agent B chooses high effort in the second stage, $e_b = 1$, when the first stage is a success, $q_a = 1$, is

$$(\alpha+\rho)w_{11}^{B}+(1-\alpha-\rho)w_{10}^{B}-(\psi+\gamma_a) \geq \alpha w_{11}^{B}+(1-\alpha)w_{10}^{B}$$

which is rewritten as

$$\rho(w_{11}^{B}-w_{10}^{B}) \geq \psi+\gamma_a \tag{15}$$

The incentive compatibility constraint ensuring that agent B exerts high effort in the second stage, $e_b = 1$, when the first stage task is a failure, $q_a = 0$, is

$$(\alpha+\rho)w_{01}^{B}+(1-\alpha-\rho)w_{00}^{B}-\psi \geq \alpha w_{01}^{B}+(1-\alpha)w_{00}^{B}$$

which is simplified to

$$\rho(w_{01}^{B}-w_{00}^{B}) \geq \psi \tag{16}$$

The principal solves the following cost minimization problem, which we denote by $\mathcal{P}_3$:

$$\min_{\substack{\{(w_{11}^A, w_{10}^A, w_{01}^A w_{00}^A) \\ (w_{11}^B, w_{10}^B, w_{01}^B, w_{00}^B)\}}} \begin{bmatrix} (\alpha+\rho)\left[(\alpha+\rho)(w_{11}^A + w_{11}^B) + (1-\alpha-\rho)(w_{10}^A + w_{10}^B)\right] \\ +(1-\alpha-\rho)\left[(\alpha+\rho)(w_{01}^A + w_{01}^B) + (1-\alpha-\rho)(w_{00}^A + w_{00}^B)\right] \end{bmatrix}$$

subject to

$$\begin{aligned}
(PC_A) \quad & (\alpha+\rho)\left[(\alpha+\rho)w_{11}^A + (1-\alpha-\rho)w_{10}^A\right] \\
& +(1-\alpha-\rho)\left[(\alpha+\rho)w_{01}^A + (1-\alpha-\rho)w_{00}^A\right] \geq \psi \\
(IC_A) \quad & \rho[(\alpha+\rho)(w_{11}^A - w_{01}^A) + (1-\alpha-\rho)(w_{10}^A - w_{00}^A)] \geq \psi \\
(PC_B^{q_a=1}) \quad & (\alpha+\rho)w_{11}^B + (1-\alpha-\rho)w_{10}^B \geq \psi + \gamma_a \\
(PC_B^{q_a=0}) \quad & (\alpha+\rho)w_{01}^B + (1-\alpha-\rho)w_{00}^B \geq \psi \\
(IC_B^{q_a=1}) \quad & \rho(w_{11}^B - w_{10}^B) \geq \psi + \gamma_a \\
(IC_B^{q_a=0}) \quad & \rho(w_{01}^B - w_{00}^B) \geq \psi \\
(LL) \quad & w_{q_a q_b}^j \geq 0 \quad \text{for each} \quad q_a, q_b \in \{0,1\} \quad \text{and} \quad j \in \{A, B\}
\end{aligned}$$

When two identical agents are hired to perform two different tasks. We solve $\mathcal{P}_3$ to find the cost minimizing contract when task i is delegated first.

Lemma 3: Suppose two tasks are delegated to two symmetric agents sequentially, and that the revenue R is high enough so that the principal induces high effort for each task. Suppose task i is delegated first, to agent A.

(i) If $\gamma_a \geq 0$, or $\gamma_a < 0$ and $|\gamma_a| < \psi$, then the principal offers the contract

$$w_{00}^A = w_{01}^A = 0, w_{10}^A = w_{11}^A = \frac{\psi}{\rho}$$

and

$$w_{00}^B = w_{10}^B = 0, w_{01}^B = \frac{\psi}{\rho}, w_{11}^B = \frac{\psi + \gamma_i}{\rho}$$

The expected cost of the principal is

$$\frac{\alpha+\rho}{\rho}[\gamma_i(\alpha+\rho)+2\psi]$$

(ii) If $\gamma_i < 0$ and $|\gamma_i| \geq \psi$, then the optimal contract is

$$w_{00}^A = w_{01}^A = 0,\ w_{10}^A = w_{11}^A = \frac{\psi}{\rho}$$

and

$$w_{00}^B = w_{10}^B = w_{11}^B = 0,\ w_{01}^B = \frac{\psi}{\rho}$$

The expected cost of the principal is

$$(\alpha+\rho)(2-\alpha-\rho)\frac{\psi}{\rho}$$

Proof: See the Appendix.

The principal's expected revenue is

$$ER(e_a = e_b = 1) = (\alpha+\rho)(\alpha+\rho)2R + (\alpha+\rho)(1-\alpha-\rho)R + (1-\alpha-\rho)(\alpha+\rho)R$$
$$= 2R(\alpha+\rho).$$

Proposition 4: Suppose there exist outcome externality on the marginal cost of effort and two identical agents are hired to complete two sequential tasks. Furthermore, suppose the revenue R is high enough such that the principal prefers agent to implement high effort for each task. Then,

(i) If $\gamma_i \geq 0$, or $\gamma_i < 0$ and $|\gamma_i| < \psi$, then

(a) When both tasks are synergistic, that is, $\gamma_a < 0$ and $\gamma_b < 0$, the task with higher impact (higher $|\gamma_i|$, i.e., lower γ_i) is delegated first.

(b) When both tasks are conflicting, that is, $\gamma_a > 0$ and $\gamma_b > 0$, the task with lower impact (lower $|\gamma_i|$, i.e., lower γ_i) is delegated first.

(c) When one task is synergistic and the other one is conflicting, then the synergistic task is delegated first.

(ii) If $\gamma_i < 0$ and $|\gamma_i| \geq \psi$, then the pricipal is indifferent in ordering the tasks.

Proof: See the Appendix.

The intuition for Proposition 4 above is parallel to the one for Proposition 2. A direct implication is an equivalence between the task orders under the two types of outcome externalities: marginal probability and marginal cost.

Corollary 1: When an outcome externality exists in terms of marginal cost, task order is equivalent to the task order when an outcome externality exists in terms of marginal probability.

CHAPTER 5

POSSIBLE WAGE PAYMENT AFTER THE FIRST TASK OUTCOME

The main model focuses on the optimal contracts when the wages are paid once the outcomes are realized after the second task is undertaken, that is, there is no wage payment after the outcome of the first task is realized. More specifically, there might be some payment for the first stage task after the outcome of the first task is obtained. This extension provides a robustness check when there is a possible change in the timing of the wage payments. We alter the timing of the model in Section 2 as follows.

At stage 1, where the first task, task i, is undertaken, after the outcome, $q_i \in \{0, 1\}$, is realized, the wage payment, $w_{q_a} \in \{w_1, w_0\}$, is made to the agent who carries out the first task.

At stage 2, where the second task, task –i, is undertaken, after the outcome, q_i $\in \{0, 1\}$, is realized, the wage payment, $w_{q_a,q_b} \in \{w_{11}, w_{10}, w_{01}, w_{00}\}$, is made to the agent who carries out the second task.

As in Section 2, renegotiation is not possible, and each party is risk neutral, and there is limited liability. Also, the principal wants to implement high effort in each stage. Principal's problem is to find a contract, $\{w_{q_i}, w_{q_i,q_j}\}_{i,j}=$ $\{\{w_1, w_0\}, \{w_{11}, w_{10}, w_{01}, w_{00}\}\}$, that minimizes her expected cost.

Now, suppose principal hires one agent to carry out both tasks and task a is delegated first. Then, principal's expected cost from a contract, $\{\{w_1, w_0\}, \{w_{11}, w_{10}, w_{01}, w_{00}\}\}$, given that it is accepted and the agent exerts e = 1 in both stages, is given by

$$(\alpha + \rho_a)w_1 + (1 - \alpha - \rho_a)w_0 + (\alpha + \rho_a)[(\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10}]$$
$$+(1 - \alpha - \rho_a)[(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00}]$$

The relevant constraints are as follows.

$IC_2^{q_a=1}$: Incentive compatibility constraint ensuring that $e_b = 1$ given $q_a = 1$ is

$$(\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - \psi \geq \alpha w_{11} + (1 - \alpha)w_{10}$$

$IC_2^{q_a=0}$: Incentive compatibility constraint ensuring that $e_b = 1$ given $q_a = 0$ is

$$(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00} - \psi \geq \alpha w_{01} + (1 - \alpha)w_{00}$$

IC_1: First stage incentive compatibility constraint ensuring that $e_a = 1$ is

$$\begin{aligned}(\alpha + \rho_a)w_1 + (1 - \alpha - \rho_a)w_0 + (\alpha + \rho_a)[(\alpha + \rho_b + \gamma_a)w_{11} +\\ (1 - \alpha - \rho_b - \gamma_a)w_{10} - \psi] + (1 - \alpha - \rho_a)[(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00} - \psi] - \psi\\ \geq \alpha w_1 + (1 - \alpha)w_0 + \alpha[(\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - \psi] +\\ (1 - \alpha)[(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00} - \psi]\end{aligned}$$

PC: The participation constraint of the agent is

$$\begin{aligned}(\alpha + \rho_a)w_1 + (1 - \alpha - \rho_a)w_0 + (\alpha + \rho_a)[(\alpha + \rho_b + \gamma_a)w_{11} +\\ (1 - \alpha - \rho_b - \gamma_a)w_{10} - \psi] + (1 - \alpha - \rho_a)[(\alpha + \rho_b)w_{01} + (1 - \alpha - \rho_b)w_{00} - \psi] - \psi\\ \geq 0\end{aligned}$$

Principal's problem is to find the wage scheme that minimizes her expected cost subject to the agent's participation constraint, PC, incentive compatibility constraints, IC_1, $IC_2^{q_a=1}$ and $IC_2^{q_a=0}$, and limited liability constraints. Incentive compatibility constraints and limited liability constraints imply the agent is willing to participate in the project. Furthermore, note that the second stage incentive compatibility constraints are the same as that of the main section. IC_1 is simplified to

$$\rho_a\left[w_1 + (\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - (\alpha + \rho_b)w_{01} - (1 - \alpha - \rho_b)w_{00}\right] \geq \psi$$

where the only difference from inequality 2 is the term $\rho_a w_1$.

(i) Assume $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a \geq 0$. Similar to the main part, ignoring $IC_2^{q_a=1}$ gives a binding IC_1, which gives

$$w_{11} - w_{10} = \frac{\frac{\psi}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\psi - w_{10} - w_1}{\alpha + \rho_b + \gamma_a}$$

The constraint $IC_2^{q_a=1}$ is satisfied whenever;

$$\frac{\psi}{\rho_b + \gamma_a} \leq \frac{\frac{\psi}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\psi - w_{10} - w_1}{\alpha + \rho_b + \gamma_a}$$

Arranging the above inequality gives,

$$w_{10} + w_1 \leq \psi\frac{\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a}{\rho_a\rho_b(\rho_b + \gamma_a)}$$

where the only difference from Inequality 18 is the term w_1 in the left hand side. Therefore, an optimal contract for the principal is to set $w_{10} + w_1 = 0$, where limited liability implies $w_{10} = w_1 = 0$. Moreover, $w_{11} = \frac{\rho_b+\rho_a(\alpha+\rho_b)}{\rho_a\rho_b}\frac{\psi}{\alpha+\rho_b+\gamma_a}$ which implies the optimal contract in this case is the same as that of the problem.

(ii) Assume $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a < 0$. Then IC_1 is non-binding, that is

$$\rho_a\left[w_1 + (\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - (\alpha + \rho_b)\frac{\psi}{\rho_b}\right] > \psi$$

where the only difference from the proof of Lemma 1 is the term $\rho_a w_1$ which makes the following change in Inequality 20

$$w_1 + w_{10} > \frac{\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a}{\rho_a\rho_b(\rho_b + \gamma_a)}$$

Therefore, now the principal sets $w_{10} + w_1 = 0$ which means $w_{10} = w_1 = 0$ because of the limited liability constraints. Moreover, $w_{11} = \frac{\psi}{\rho_b+\gamma_a}$ meaning that the optimal wage scheme when there is a probable wage payment is the same as the optimal wage scheme when the agent receives the payment after the project is completed.

Now suppose there are two agents in charge of two tasks and agent A is in charge of task a and B in charge of task b. Now the wage scheme is $w^j_{q_a,q_b} \in \{\{w^A_0, w^A_1\}, \{w^B_{11}, w^B_{10}, w^B_{01}, w^B_{00}\}\}$. The only difference from the proof of Lemma 2 is IC_A, which is

$$(\alpha + \rho_a)w^A_1 + (1 - \alpha - \rho_a)w^A_0 - \psi \geq \alpha w^A_1 + (1 - \alpha)w^A_0$$

which is equivalent to

$$\rho_a(w^A_1 - w^A_0) \geq \psi$$

In addition to the binding incentive compatibility constraints in proof of Lemma 2, IC_A must bind in the optimum as well. Moreover, the principal sets $w^A_0 = w^B_{00} = w^B_{10} = 0$ in the optimum. Then the binding constraints imply $w^A_1 = \frac{\psi}{\rho_a}$, $w^B_{01} = \frac{\psi}{\rho_b}$ and $w^B_{11} = \frac{\psi}{\rho_b+\gamma_a}$, which indicates that optimal contract when the wages are delivered right upon the completion of the tasks is the same as the optimal contract in Lemma 2, when the wage payments are made after the completion of the second task. Therefore, we conclude that our model is robust to possible transfers after each task.

CHAPTER 6
CONCLUSION

The current study provides a fresh viewpoint on multitask principal-agent problem literature by extending the model in Schmitz (2013a) through allowing for task ordering to be endogenous. A two-stage sequential moral hazard problem with outcome externality is studied. When one task is synergistic, that is success in the previous job increases the marginal success probability of the subsequent job, and the other task is conflicting, that is success in the previous job affects the probability of success of the next job negatively, then the principal prefers the synergistic task to be first in order. The task with higher impact is placed first in order when both tasks are synergistic whereas the task with lower impact is delegated first when two tasks are conflicting. Furthermore, holding the impact levels of the two tasks equal, the task with higher effectiveness is delegated first in the case of synergistic tasks where the one with lower effectiveness is assigned first in the case of conflicting tasks. Moreover, the task order when there is an outcome externality regarding the marginal success probability is equivalent to the task order when there exists an outcome externality regarding marginal cost of effort. The findings regarding the bundling decision of the principal in the present article are parallel to the findings in Schmitz (2013a), where it is optimal for two identical agents to carry out two different tasks when they are synergistic, and a single agent for two tasks when they are both conflicting. When just one job is synergistic, it is best for the principal to contract two distinct agents. These findings are unaffected by a possible wage transfer when each task is completed.

APPENDIX

PROOFS

Proof of Lemma 1: Limited liability and the incentive compatibility constraints together imply that the agent's participation constraint is already satisfied since the cost of no effort to the agent is zero. Moreover, note that $w_{00} = 0$ must be the case. To see this, suppose otherwise, that is, $w_{00} > 0$. Then the principal can increase her profit by reducing w_{00} without violating any other constraints. This contradicts the optimality of the contract. Therefore, the last limited liability constraint must be binding: $w_{00} = 0$. Now, substitute $w_{00} = 0$ into the $IC_2^{q_a=0}$, that is, Equation 4. Thus, agent's incentive compatibility constraint for the second stage when the first stage was a failure becomes $w_{01} \geq \frac{\psi}{\rho_b}$. In the optimum, this constraint must be binding as well. Otherwise, the principal can decrease w_{01} and increase her profit without violating other constraints. Therefore, $w_{01} = \frac{\psi}{\rho_b}$. Then, IC_1, that is, Equation 2 can be rewritten as

$$\rho_a \left[w_{10} + (\alpha + \rho_b + \gamma_a)(w_{11} - w_{10}) - \frac{\alpha + \rho_b}{\rho_b}\psi \right] \geq \psi$$

Without loss of generality, let i = a and –i = b, that is, task a is delegated first.

(i) Assume $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a \geq 0$. For now, ignore the second stage incentive compatibility constraint conditional on a first stage success, $IC_2^{q_a=1}$, that is, Equation 3. Then, the first stage incentive compatibility, IC_1, binds. This is because otherwise, the principal could decrease w_{11} and w_{10}, and increase her profit without violating other constraints. Then,

$$w_{10} + (\alpha + \rho_b + \gamma_a)(w_{11} - w_{10}) - \frac{\alpha + \rho_b}{\rho_b}\psi = \frac{\psi}{\rho_a}$$

which is equivalent to

$$w_{11} - w_{10} = \frac{\frac{\psi}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\psi - w_{10}}{\alpha + \rho_b + \gamma_a} \tag{17}$$

The constraint $IC_2^{q_a=1}$, which is equivalent to $\frac{\psi}{\rho_b+\gamma_a} \leq w_{11} - w_{10}$, is satisfied whenever;

$$\frac{\psi}{\rho_b + \gamma_a} \leq \frac{\frac{\psi}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\psi - w_{10}}{\alpha + \rho_b + \gamma_a}$$

Arranging the above inequality, we get

$$\frac{w_{10}}{\alpha + \rho_b + \gamma_a} \leq \frac{\psi}{\alpha + \rho_b + \gamma_a} \frac{\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a}{\rho_a\rho_b(\rho_b + \gamma_a)}$$

Note that, $\alpha + \rho_b + \gamma_a > 0$, by Assumption 1. Thus, we cancel this term on both sides, and get

$$w_{10} \leq \psi \frac{\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a}{\rho_a\rho_b(\rho_b + \gamma_a)} \tag{18}$$

if $\gamma_a > 0$, then the right hand side of the above inequality is strictly positive. However, if $\gamma_a < 0$, then, by Assumption 1, $|\gamma_a| < \rho_b$, thus, the denominator of the right hand side of above inequality is strictly positive. The nominator can is assumed to be positive since we assumed $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a \geq 0$ in this case. Thus, an optimal contract for the principal is to set $w_{10} = 0$, and

$$w_{11} = \frac{\rho_b + \rho_a(\alpha + \rho_b)}{\rho_a\rho_b} \frac{\psi}{\alpha + \rho_b + \gamma_a}$$

Note that any w_{11} and w_{10} that satisfy Equation 17 and Inequality 18 will constitute an optimal contract together with $w_{01} = \frac{\psi}{\rho_b}$ and $w_{00} = 0$. Each of the above optimal contracts yield the same expected cost to the principal, given by

$$\begin{aligned}
EC(e_a = e_b = 1) &= (\alpha + \rho_a)(\alpha + \rho_b + \gamma_a)\frac{\rho_b + \rho_a(\alpha + \rho_b)}{\rho_a\rho_b} \frac{\psi}{\alpha + \rho_b + \gamma_a} \\
&\quad + (1 - \alpha - \rho_a)(\alpha + \rho_b)\frac{\psi}{\rho_b} \\
&= \psi \left[\frac{\alpha + \rho_a}{\rho_a}(1 + \rho_a \frac{\alpha + \rho_b}{\rho_b}) + (1 - \alpha - \rho_a)\frac{\alpha + \rho_b}{\rho_b} \right] \\
&= \psi \left[\frac{\alpha + \rho_a}{\rho_a} + \frac{\alpha + \rho_b}{\rho_b} \right]
\end{aligned}$$

(ii) Assume $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a < 0$. Now, the first stage incentive compatibility constraint cannot bind since otherwise w_{10} would be less than or equal to a negative number by Equation 18, which contradicts the limited liability constraints. Thus, it must be

$$\rho_a \left[(\alpha + \rho_b + \gamma_a)w_{11} + (1 - \alpha - \rho_b - \gamma_a)w_{10} - (\alpha + \rho_b)\frac{\psi}{\rho_b}\right] > \psi$$

Then, the second stage incentive compatibility constraint conditional on a first stage success, $IC_2^{q_a=1}$, that is, Equation 3, must be binding. To see this, suppose not, that is, $(\rho_b + \gamma_a)(w_{11} - w_{10}) > \psi$. Then, the principal can increase her profit by lowering w_{11} such that the first stage incentive compatibility constraint is not violated. Therefore, the constraint $IC_2^{q_a=1}$ must bind, that is,

$$w_{11} - w_{10} = \frac{\psi}{\rho_b + \gamma_a} \tag{19}$$

Then, the first stage incentive compatibility constraint, IC_1 becomes;

$$(\alpha + \rho_b + \gamma_a)\frac{\psi}{\rho_b + \gamma_a} + w_{10} > \psi[\frac{1}{\rho_a} + (\alpha + \rho_b)\frac{1}{\rho_b}]$$

that is,

$$w_{10} > \psi[\frac{\rho_b + \rho_a(\alpha + \rho_b)}{\rho_a\rho_b} - \frac{\alpha + \rho_b + \gamma_a}{\rho_b + \gamma_a}]$$

which is equivalent to

$$w_{10} > \frac{\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a}{\rho_a\rho_b(\rho_b + \gamma_a)} \tag{20}$$

Since $\rho_b^2 + \rho_b\gamma_a + \alpha\gamma_a\rho_a < 0$ and $\rho_b + \gamma_a > 0$, the right hand side of this inequality is negative. Therefore, the first stage incentive compatibility constraint is always satisfied whenever limited liability constraint $w_{10} \geq 0$ is satisfied. Thus, an optimal contract for the principal is to set $w_{10} = 0$ and $w_{11} = \frac{\psi}{\rho_b+\gamma_a}$. Note that any w_{11} and w_{10} that satisfy Equation 19 and Inequality 20 will constitute an optimal contract together with $w_{01} = \frac{\psi}{\rho_b}$ and $w_{00} = 0$. Each of the above optimal contracts yield the same expected cost to the principal, given by

$$\begin{aligned} EC(e_a = e_b = 1) &= (\alpha+\rho_a)(\alpha+\rho_b+\gamma_a)\frac{\psi}{\rho_b+\gamma_a} + (1-\alpha-\rho_a)(\alpha+\rho_b)\frac{\psi}{\rho_b} \\ &= \psi\left[\frac{\alpha+\rho_b}{\rho_b} - (\alpha+\rho_a)(\frac{\alpha+\rho_b}{\rho_b} - 1 + \frac{\alpha}{\rho_b+\gamma_a})\right] \\ &= \psi\left[\frac{\alpha+\rho_b}{\rho_b} - \frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right] \end{aligned}$$

This finishes the proof

Proof of Proposition 1: Assume that the revenue R is high enough so that the principal induces high effort for each task.

(i) Suppose both tasks are synergistic, that is, $\gamma_a > 0$ and $\gamma_b > 0$. Then, we have $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) > 0$ and $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) > 0$. Then, task a is delegated first by the principal if and only if her expected total profit from delegating task a first is larger than her expected profit from delegating task b first, that is,

$$\begin{aligned} &R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b] - \psi\left[\frac{\alpha+\rho_a}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\right] \\ &\geq R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a] - \psi\left[\frac{\alpha+\rho_a}{\rho_a} + \frac{\alpha+\rho_b}{\rho_b}\right] \end{aligned}$$

that is

$$(\alpha+\rho_a)(\gamma_a+1)+\rho_b \geq (\alpha+\rho_b)(\gamma_b+1)+\rho_a$$

Simplifying, we get

$$\gamma_a(\alpha+\rho_a) \geq \gamma_b(\alpha+\rho_b) \tag{21}$$

(a) Suppose the effectiveness levels of two tasks are the same, $\rho_a = \rho_b = \rho$. Then, Inequality 21 above is equivalent to $\gamma_a \geq \gamma_b$. Thus, when both tasks are synergistic, the task with higher impact is delegated first.

(b) Now, suppose that the effectiveness levels of two tasks are not the same, but the impact levels are the same, that is, $\gamma_a = \gamma_b = \gamma > 0$. Then, by Inequality 21, task a is delegated first if and only if $\rho_a \geq \rho_b$. Thus, the task with higher effectiveness is delegated first.

(ii) Suppose both tasks are conflicting, that is, $\gamma_a < 0$ and $\gamma_b < 0$. There are four cases when both tasks are conflicting.

Case 1: $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) > 0$ and $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) > 0$.

This case induces the same expected profit levels as in part (i) above. Thus, the principal delegates task a first if and only if Inequality 21 holds.

(a) Suppose $\rho_a = \rho_b = \rho$. Then, Inequality 21 reduces to $\gamma_a \geq \gamma_b$. Since $\gamma_a, \gamma_b < 0$, this is equivalent to $|\gamma_a| \leq |\gamma_b|$. Thus, the task with lower impact is delegated first, when both tasks are conflicting.

(b) Now, suppose that the effectiveness levels of two tasks are not the same, but the impact levels are the same, that is, $\gamma_a = \gamma_b = \gamma < 0$. Then, Inequality 21 reduces to $\rho_a \leq \rho_b$. Thus, the task with lower effectiveness is delegated first, when both tasks are conflicting.

Case 2: $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) < 0$ and $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) < 0$.

Then, task a is delegated first by the principal if and only if the inequality below is satisfied.

$$\begin{aligned} &R[(\alpha + \rho_a)(\gamma_a + 1) + \alpha + \rho_b] - \psi\left[\frac{\alpha + \rho_b}{\rho_b} - \frac{\alpha\gamma_a(\alpha + \rho_a)}{\rho_b(\rho_b + \gamma_a)}\right] \\ &\geq R[(\alpha + \rho_b)(\gamma_b + 1) + \alpha + \rho_a] - \psi\left[\frac{\alpha + \rho_a}{\rho_a} - \frac{\alpha\gamma_b(\alpha + \rho_b)}{\rho_a(\rho_a + \gamma_b)}\right] \end{aligned}$$

Simplifying the inequality above gives

$$\begin{aligned} &R[\alpha(\gamma_a - \gamma_b) + \rho_a\gamma_a - \rho_b\gamma_b] \\ &\geq \psi\left[(\alpha + \rho_b)\left(\frac{\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b)}{\rho_a\rho_b(\rho_a + \gamma_b)}\right) - (\alpha + \rho_a)\left(\frac{\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a)}{\rho_a\rho_b(\rho_b + \gamma_a)}\right)\right] \end{aligned} \tag{22}$$

(a) Suppose $\rho_a = \rho_b = \rho$. Then, Inequality 22 reduces to

$$R(\gamma_a - \gamma_b) \geq -\frac{\psi\alpha(\gamma_a - \gamma_b)}{(\rho + \gamma_a)(\rho + \gamma_b)}$$

If $\gamma_a \geq \gamma_b$, that is, $|\gamma_a| \leq |\gamma_b|$, then task a is assigned first if and only if $R \geq -\frac{\alpha\psi}{(\rho+\gamma_a)(\rho+\gamma_b)}$ holds. By Assumption 1, $\rho + \gamma_i > 0$. Thus, the right hand side of this inequality is negative. Since R is strictly positive, this inequality is satisfied, and task a is delegated first. If $\gamma_b \geq \gamma_a$, that is, $|\gamma_a| > |\gamma_b|$, then task a is delegated first if and only if $R \leq -\frac{\alpha\psi}{(\rho+\gamma_a)(\rho+\gamma_b)}$. This cannot hold since R is strictly positive. Therefore, task b is delegated first whenever $|\gamma_a| > |\gamma_b|$.

This concludes that the task with lower impact is delegated first when the effectiveness levels of the tasks are the same.

(b) Now, suppose that the effectiveness levels of two tasks are not the same, but the impact levels are the same, that is, $\gamma_a = \gamma_b = \gamma < 0$. Then, Inequality 22 reduces to

$$R\gamma(\rho_a - \rho_b) \geq \psi(\rho_a - \rho_b)\frac{\alpha}{\rho_a\rho_b}\left[1 - \gamma\frac{\alpha(\rho_a + \rho_b + \gamma) + \rho_a(\rho_a + \gamma) + \rho_b(\rho_b + \gamma) + \rho_a\rho_b}{(\rho_a + \gamma)(\rho_b + \gamma)}\right]$$

If $\rho_a > \rho_b$, the inequality above becomes

$$R\gamma \geq \psi\frac{\alpha}{\rho_a\rho_b}\left[1 - \gamma\frac{\alpha(\rho_a + \rho_b + \gamma) + \rho_a(\rho_a + \gamma) + \rho_a\rho_b + \rho_b(\rho_b + \gamma)}{\rho_a\rho_b(\rho_a + \gamma)(\rho_b + \gamma)}\right]$$

Since $\gamma < 0$, the right hand side is negative. Also, by Assumption 1, the denominator of the fraction in the brackets is positive. Thus, with $\gamma < 0$, the entire right hand side is positive. But then the inequality cannot hold. Thus, task b, the one with lower effectiveness, is delegated first. If $\rho_a < \rho_b$, then task a is delegated first if and only if

$$R\gamma \leq \psi\frac{\alpha}{\rho_a\rho_b}\left[1 - \gamma\frac{\alpha(\rho_a + \rho_b + \gamma) + \rho_a(\rho_a + \gamma) + \rho_a\rho_b + \rho_b(\rho_b + \gamma)}{\rho_a\rho_b(\rho_a + \gamma)(\rho_b + \gamma)}\right]$$

Again, since $\gamma < 0$, the left hand side is negative. Also, the right hand side of this inequality is positive. Thus, this inequality holds. Thus, task a, the one with lower effectiveness, is delegated first.

This concludes that the task with lower effectiveness is delegated first when the impact levels of the tasks are the same.

Case 3: $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) > 0$ and $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) < 0$.

Then task a is delegated first by the principal if and only if the inequality below is satisfied.

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]$$
$$\geq R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]$$

Simplifying, we get

$$R[\alpha(\gamma_a-\gamma_b)+\rho_a\gamma_a-\rho_b\gamma_b]\geq\psi(\alpha+\rho_b)\left[\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_b\rho_a(\rho_a+\gamma_b)}\right] \tag{23}$$

(a) Suppose $\rho_a=\rho_b=\rho$. Then, Inequality 23 reduces to

$$R(\gamma_a-\gamma_b)\geq\frac{\psi}{\rho}\left(\frac{\rho+\gamma_b(1+\alpha)}{\rho+\gamma_b}\right)$$

The condition $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)\geq 0$ implies $|\gamma_a|\leq\frac{\rho}{1+\alpha}$. Likewise, the condition $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)<0$ implies $|\gamma_b|>\frac{\rho}{1+\alpha}$. Thus, $|\gamma_b|>|\gamma_a|$, which implies $\gamma_a-\gamma_b>0$. Thus, the left hand side of the inequality above is positive. Moreover, since $|\gamma_b|>\frac{\rho}{1+\alpha}$, we have $\rho+\gamma_b(1+\alpha)<0$. By Assumption 1, we also have $\rho+\gamma_b>0$. Thus, the right hand side of this inequality is negative. Thus, this inequality holds and task a, the one with lower impact, is delegated first.

(b) Now, suppose that the effectiveness levels of two tasks are not the same, but the impact levels are the same, that is, $\gamma_a=\gamma_b=\gamma<0$. Now, the conditions $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)>0$ and $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)<0$ imply

$$\frac{\rho_a^2}{\rho_a+\alpha\rho_b}<|\gamma|<\frac{\rho_b^2}{\rho_b+\alpha\rho_a}$$

Thus, we have $\rho_a\rho_b(\rho_a-\rho_b)<\alpha(\rho_b-\rho_a)(\rho_a^2+\rho_a\rho_b+\rho_b^2)$. If $\rho_a>\rho_b$, the left hand side of this inequality is positive whereas the right hand side is negative, which is a contradiction. Thus, it must be $\rho_a<\rho_b$. Now, note that task a is delegated first if and only if Inequality 23 holds when $\gamma_a=\gamma_b=\gamma$. That is,

$$R\geq\frac{\psi(\alpha+\rho_b)}{\gamma(\rho_a-\rho_b)}\,\frac{\rho_a^2+\gamma\rho_a+\alpha\gamma\rho_b}{\rho_a\rho_b(\rho_a+\gamma)}$$

Since $\rho_a^2 + \gamma\rho_a + \alpha\gamma\rho_b < 0$, $|\gamma| < \rho_a$, $\gamma < 0$ and $\rho_a < \rho_b$, the right hand side of this inequality is negative. Thus, this inequality is satisfied, and task a, the one with lower effectiveness, is delegated first.

This concludes that the task with lower effectiveness is delegated first when the impact levels of the tasks are the same.

Case 4: $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) < 0$ and $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) > 0$. This case is symmetric with Case 3 above and the result is valid.

(iii) Suppose $\gamma_a > 0$ and $\gamma_b < 0$.[1] Since $\gamma_a > 0$, the condition $\rho_b^2 + \gamma_a(\rho_b + \alpha\rho_a) \geq 0$ is satisfied. However, $\gamma_b < 0$ may induce one of the two cases below.

Case 1: $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) > 0$. Then, task a is delegated first if and only if Inequality 21 is satisfied: $\gamma_a(\alpha + \rho_a) \geq \gamma_b(\alpha + \rho_b)$. Since $\gamma_a > 0$ and $\gamma_b < 0$, this inequality holds and the synergistic task, task a, is delegated first.

Case 2: $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) < 0$. Then, task a is delegated first if and only if Inequality 23 holds, which is given by

$$R[\alpha(\gamma_a - \gamma_b) + \rho_a\gamma_a - \rho_b\gamma_b] \geq \psi(\alpha + \rho_b)\left[\frac{\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b)}{\rho_b\rho_a(\rho_a + \gamma_b)}\right]$$

Since $\gamma_a > 0$ and $\gamma_b < 0$, the left hand side is positive. Also, since $\rho_a^2 + \gamma_b(\rho_a + \alpha\rho_b) < 0$, the right hand side is negative. Thus, the inequality holds in this case and the synergistic task, task a, is delegated first.

Thus, if one task is synergistic and the other is conflicting, then for any effectiveness levels ρ_a and ρ_b, the synergistic task is delegated first.

Proof of Lemma 2: Without loss of generality, suppose first task is a, that is, $i = a$. The limited liability, (LL), and the incentive compatibility (IC_A) constraints for agent A together imply that agent A's participation constraint, (PC_A), is satisfied. Likewise, ($IC_B^{q_a=1}$) and (LL) imply ($PC_B^{q_a=1}$), and ($IC_B^{q_a=0}$) and (LL) imply ($PC_B^{q_a=0}$). Moreover, incentive compatibility constraints, ($IC_B^{q_a=1}$) and ($IC_B^{q_a=0}$) of the second task, task b,

[1] The other case with $\gamma_a < 0$ and $\gamma_b > 0$ is symmetric.

are binding, that is

$$(\rho_b + \gamma_a)(w^B_{11} - w^B_{10}) = \psi$$

$$\rho_b(w^B_{01} - w^B_{00}) = \psi$$

To see this, suppose they do not bind. Then, the principal can increase her expected profit by lowering w^B_{11} and w^B_{01} by a small enough amount without violating other constraints. Moreover, the principal sets w^B_{10} and w^B_{00} as small as possible, that is, $w^B_{10} = w^B_{00} = 0$. Then, the binding constraints imply

$$w^B_{11} = \frac{\psi}{\rho_b + \gamma_a} \quad \text{and} \quad w^B_{01} = \frac{\psi}{\rho_b}$$

The principal also sets w^A_{01} and w^A_{00} to be zero in order to minimize her expected cost. Thus, agent A's incentive compatibility constraint, (IC_A) is simplified to

$$(\alpha + \rho_b + \gamma_a)w^A_{11} + (1 - \alpha - \rho_b - \gamma_a)w^A_{10} \geq \frac{\psi}{\rho_a}$$

This constraint must be binding at the optimal contract, since otherwise the principal would decrease w^A_{11} and w^A_{10} by a sufficiently small amount without violating other constraints. Thus, any combination of w^A_{11} and w^A_{10} satisfying $(\alpha + \rho_b + \gamma_a)w^A_{11} + (1 - \alpha - \rho_b - \gamma_a)w^A_{10} = \frac{\psi}{\rho_a}$ with nonnegative wages minimizes the principal's expected cost. One such contract is where $w^A_{11} = w^A_{10} = \frac{\psi}{\rho_a}$, and the principal's expected cost is then

$$\begin{aligned} EC(e_a = e_b = 1) &= (\alpha + \rho_a)\left[(\alpha + \rho_b + \gamma_a)\left(\frac{\psi}{\rho_a} + \frac{\psi}{\rho_b + \gamma_a}\right) + (1 - \alpha - \rho_b - \gamma_a)\frac{\psi}{\rho_a}\right] \\ &\quad +(1 - \alpha - \rho_a)(\alpha + \rho_b)\frac{\psi}{\rho_b} \\ &= \psi\left[(\alpha + \rho_a)\left(\frac{\alpha}{\rho_b + \gamma_a} + \frac{1}{\rho_a}\right) + (1 - \alpha - \rho_a)\frac{\alpha}{\rho_b} + 1\right] \end{aligned}$$

This finishes the proof.

Proof of Proposition 2: Task a is delegated first by the principal if and only if the expected total profit from delegating task a first is greater than that of b, that is

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]$$
$$\geq R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

which simplifies to

$$R\left[\alpha(\gamma_a-\gamma_b)+\rho_a\gamma_a-\rho_b\gamma_b\right]\geq-\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \tag{24}$$

If $\rho_a=\rho_b=\rho$, then Inequality 24 becomes

$$R(\gamma_a-\gamma_b)\geq-\psi\frac{\alpha(\gamma_a-\gamma_b)}{(\rho+\gamma_a)(\rho+\gamma_b)}$$

Note that by Assumption 1, $\rho+\gamma_a>0$ and $\rho+\gamma_b>0$. Now, if $\gamma_a\geq\gamma_b$, then above inequality becomes $R\geq-\frac{\psi\alpha}{(\rho+\gamma_a)(\rho+\gamma_b)}$, which holds for any α, ψ, ρ and R. If $\gamma_a<\gamma_b$, then above inequality becomes $R\leq-\frac{\psi\alpha}{(\rho+\gamma_a)(\rho+\gamma_b)}$ which never holds, thus task b is delegated first. Thus, if both tasks are synergistic, $\gamma_a>0$ and $\gamma_b>0$, then the the task with higher impact is delegated first. But if both are conflicting, $\gamma_a<0$ and $\gamma_b<0$, then the one with smaller impact is delegated first: for $\gamma_a\geq\gamma_b$, we have $|\gamma_a|<|\gamma_b|$, and task a, the one with lower impact is delegated first; and for $\gamma_a<\gamma_b$, we have $|\gamma_a|>|\gamma_b|$, and task b, again the one with lower impact is delegated first. This proves part (i)(a) and part (ii)(a).

If $\gamma_a=\gamma_b=\gamma$, then Inequality 24 becomes

$$R(\rho_a-\rho_b)\geq-\psi\alpha(\rho_a-\rho_b)\left[\frac{(\alpha+\rho_a+\rho_b)(\rho_a+\gamma)+\rho_b(\alpha+\rho_b)}{\rho_a\rho_b(\rho_a+\gamma)(\rho_b+\gamma)}\right]$$

Suppose the tasks are synergistic, that is, $\gamma>0$. If $\rho_a\geq\rho_b$, then task a is delegated first if and only if

$$R\geq-\psi\alpha\left[\frac{(\alpha+\rho_a+\rho_b)(\rho_a+\gamma)+\rho_b(\alpha+\rho_b)}{\rho_a\rho_b(\rho_a+\gamma)(\rho_b+\gamma)}\right] \tag{25}$$

The right hand side of the inequality above is negative. Therefore, this inequality is satisfied. Thus, task a is delegated first which is the task with higher effectiveness. If $\rho_a < \rho_b$, then task a is delegated first if and only if

$$R \leq -\psi\alpha \left[\frac{(\alpha + \rho_a + \rho_b)(\rho_a + \gamma) + \rho_b(\alpha + \rho_b)}{\rho_a\rho_b(\rho_a + \gamma)(\rho_b + \gamma)} \right] \tag{26}$$

which contradicts with the revenue being strictly positive. Therefore, task b, the task with higher effectiveness, is delegated first. Therefore, we conclude that the task with higher effectiveness is delegated first when the tasks are synergistic. This proves part (i)(b).

Suppose now the tasks are conflicting, that is, $\gamma < 0$. If $\rho_a \geq \rho_b$, then task a is delegated first if and only if Inequality 26 holds. Since right hand side is negative, we conclude that task b is delegated first. If $\rho_a < \rho_b$, then task a is delegated first if and only if Inequality 25 holds. Since this inequality holds, it must be that task a is delegated first. Therefore, we conclude that the task with lower effectiveness is delegated first when the tasks are conflicting. This proves part (ii)(b).

To see part (iii), suppose one task is synergistic and the other is conflicting. Say, without loss of generality, $\gamma_a > 0$ and $\gamma_b < 0$. Then, the left hand side of Inequality 24 is positive. The right hand side of Inequality 24 is given by

$$\begin{aligned} &-\psi\alpha \left[\frac{\gamma_a(\alpha + \rho_a)}{\rho_b(\rho_b + \gamma_a)} - \frac{\gamma_b(\alpha + \rho_b)}{\rho_a(\rho_a + \gamma_b)} \right] \\ &= -\psi\alpha \left[\frac{\gamma_a(\alpha + \rho_a)\rho_a(\rho_a + \gamma_b) - \gamma_b(\alpha + \rho_b)\rho_b(\rho_b + \gamma_a)}{\rho_b(\rho_b + \gamma_a)\rho_a(\rho_a + \gamma_b)} \right] \end{aligned}$$

which is negative since the term in the brackets is positive. This is because $\gamma_a > 0$, $\gamma_b < 0$ and $\rho_b + \gamma_a > 0$ and $\rho_a + \gamma_b > 0$. Thus, task a, the one that is synergistic, is delegated first. This proves part (iii).

Proof of Proposition 3: (i) Suppose $\gamma_a > 0$ and $\gamma_b > 0$. Without loss of generality, suppose $\gamma_a > \gamma_b$. Then, when there is one agent, by Proposition 1, task a is delegated first if and only if $\gamma_a(\alpha + \rho_a) \geq \gamma_b(\alpha + \rho_b)$ holds. If task a is delegated, then the expected profit is given by

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]$$

When there are two agents, if task a is delegated first, then the expected profit is given by

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]$$

Then, the principal delegates to two agents rather than one agent if the following holds

$$\begin{aligned}R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]\\ \geq R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]\end{aligned}$$

that is,

$$(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\leq\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}$$

which is equivalent to $\frac{1}{\rho_b+\gamma_a} \leq \frac{1}{\rho_b}$, which holds since $\gamma_a > 0$. When there are two agents, if task b is delegated first, then the expected profit is given by

$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

Then, the principal delegates to two agents rather than one agent if the following holds

$$\begin{aligned}R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]\\ \geq R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]\end{aligned}$$

that is,

$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]$$
$$\geq \psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha+\rho_b}{\rho_b}\right]$$

which is equivalent to

$$R\left[\gamma_b(\alpha+\rho_b)-\gamma_a(\alpha+\rho_a)\right]\geq \psi\left[\frac{1}{\rho_a+\gamma_b}-\frac{1}{\rho_a}\right]$$

Since both sides are negative, we can rewrite is as

$$R\left[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)\right]\leq \psi\frac{\gamma_b}{\rho_a(\rho_a+\gamma_b)} \tag{27}$$

Moreover, task b is delegated first by the principal if and only if the following holds;

$$R\left[\alpha(\gamma_a-\gamma_b)+\rho_a\gamma_a-\rho_b\gamma_b\right]<-\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \tag{28}$$

Rewriting this inequality gives;

$$R\left[\alpha(\gamma_a-\gamma_b)+\rho_a\gamma_a-\rho_b\gamma_b\right]$$
$$<-\psi\alpha\left[\frac{\gamma_a\gamma_b(\rho_a-\rho_b)(\rho_a+\rho_b+\alpha)+\gamma_a\rho_a^2(\alpha+\rho_a)-\gamma_b\rho_b^2(\alpha+\rho_b)}{\rho_b(\rho_b+\gamma_a)\rho_a(\rho_a+\gamma_b)}\right]$$

Given Inequality 21, the left hand side of this inequality is positive. Moreover, the right hand side of this inequality is negative when $\rho_a \geq \rho_b$. Therefore, when there are two agents in charge of two tasks, task a is delegated first whenever $\rho_a \geq \rho_b$ (and Inequality 21 holds). Now, suppose that $\rho_a < \rho_b$. Then, the right hand side of this inequality is not necessarily negative. To see this, suppose Inequality 21 holds with equality, that is, $\gamma_a = \frac{\gamma_b(\alpha+\rho_b)}{(\alpha+\rho_a)}$. Then, the right hand side is positive and for small enough R, task b is delegated first. Now, suppose that task b is delegated first when there are two agents to carry out the two tasks. Note that the condition for task b to be delegated first implies two different agents are assigned to complete the project, that is, Inequality 28 implies Inequality 27. Since the left hand side of these inequalities are the same, right hand side of Inequality 28 being lower than that of Inequality 27 guarantees that two different agents carry out the two different tasks whenever task b is delegated first by the principal. Therefore, we need to show that

$$-\psi\alpha\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}+\psi\alpha\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}<\psi\frac{\gamma_b}{\rho_a(\rho_a+\gamma_b)}$$

that is,

$$\psi\frac{\gamma_b}{\rho_a(\rho_a+\gamma_b)}\left[1-\alpha(\alpha+\rho_b)\right]+\psi\alpha\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}>0$$

The inequality above is certainly satisfied since $\rho_{-i}+\gamma_{q_i}>0$ by Assumption 1 and $\alpha(\alpha+\rho_b)<1$. Thus, it is optimal for the principal to delegate the two tasks to two agents, when the tasks are synergistic.

(ii) Suppose $\gamma_a<0$ and $\gamma_b<0$. There are four cases when both tasks are conflicting and both tasks are delegated to only one agent.

Case 1: $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)>0$ and $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)>0$. This case induces the same expected profit levels as in part (i). Therefore, with one agent the principal delegates task a first if and only if Inequality 21 holds. Now, assume Inequality 21 holds, that is, task a is delegated first when there is one agent. Then, the condition for two tasks to be assigned to one agent rather than two agents, when task a is also delegated first for two agents case is as follows.

$$\begin{aligned}&R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]\\ \geq\ &R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]\end{aligned}$$

that is,

$$\frac{\alpha(\alpha+\rho_a)}{\rho_b+\gamma_a}\geq\frac{\alpha(\alpha+\rho_a)}{\rho_b}$$

which is equivalent to $\frac{1}{\rho_b+\gamma_a}\geq\frac{1}{\rho_b}$, which is satisfied since $\gamma_a<0$. When there are two agents, if task b is delegated first, then the principal hires one agent rather than two different agents if and only if

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]\geq$$
$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

which is equivalent to,

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)]\geq\psi\frac{\alpha(\alpha+\rho_b)\gamma_b}{\rho_a(\rho_a+\gamma_b)}$$

Following from the Inequality 21, the left hand side of the inequality above is positive. Moreover, right hand side is negative since $\gamma_b < 0$. Thus, the inequality above holds and the principal hires one agent.

Case 2: $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a) < 0$ and $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b) < 0$. There are four combinations when comparing task orders in one-agent and two-agent cases.

(2.1) Suppose task a is delegated first in both cases. Then the principal hires only one agent for the project if and only if

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]$$
$$\geq R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]$$

that is,

$$-\frac{\alpha\rho_a\gamma_a(\alpha+\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\leq(\alpha+\rho_a)\frac{\alpha\rho_a\rho_b+\rho_b(\rho_b+\gamma_a)-\alpha\rho_a(\rho_b+\gamma_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}$$

which is equivalent to $\rho_b(\rho_b+\gamma_a)\geq 0$, which holds since $|\gamma_i| < \rho_{-i}$ for each i by Assumption 1.

(2.2) Now, suppose that task a is delegated first in the one-agent case as in the previous case, and task b is delegated first in the two-agent case. Then, it is optimal to delegate two tasks to one agent if and only if

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]\geq$$
$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

that is,

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)]\geq\psi\left[\frac{\alpha(\alpha+\rho_b)}{\rho_a}-\frac{\alpha(\alpha+\rho_b)}{\rho_a+\gamma_b}-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right] \quad (29)$$

For task a to be first in the order in the one-agent case and for task b to be first in the two-agent case, the following inequalities must hold, respectively.

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)]\geq\psi\left[\frac{\alpha+\rho_b}{\rho_b}+\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right] \quad (30)$$

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)]\leq-\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \quad (31)$$

The two inequalities above necessarily imply the following inequality:

$$\psi\left[\frac{\alpha+\rho_b}{\rho_b}+\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]\leq-\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]$$

which is equivalent to $\alpha(\rho_a-\rho_b)\leq 0$, which holds whenever $\rho_a\leq\rho_b$. Moreover, note that the left hand side of Inequality 29 and 30 are equivalent. Therefore, if the right hand side of Inequality 30 is greater than that of Inequality 29, then we can conclude that the principal hires only one agent whenever task a is delegated first in the one-agent case. Thus, we need to show that,

$$\psi\left[\frac{\alpha+\rho_b}{\rho_b}+\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]$$
$$\geq\psi\left[\frac{\alpha(\alpha+\rho_b)}{\rho_a}-\frac{\alpha(\alpha+\rho_b)}{\rho_a+\gamma_b}-\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]$$

that is,

$$\frac{\alpha(\alpha+\rho_b)(\rho_a+\gamma_b)}{\rho_a(\rho_a+\gamma_b)}+\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha(\alpha+\rho_b)}{\rho_a}\geq 0$$

which is equivalent to $\frac{\alpha+\rho_b}{\rho_b} \geq 0$, which holds since α and ρ_b are strictly positive.

(2.3) Now, suppose that task b is delegated first in the one-agent case, and task a is delegated first in the two-agent case. For task b to be delegated first in the one-agent case, the opposite of Inequality 22 must hold, which can be rewritten as

$$\begin{aligned} &R[\gamma_b(\alpha+\rho_b)-\gamma_a(\alpha+\rho_a)] \\ &\geq -\psi\left[(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)-(\alpha+\rho_a)\left(\frac{\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\right)\right] \end{aligned}$$

Furthermore, task a is delegated first in the two-agent case if and only if the Inequality 24 holds, which can be rewritten as

$$\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \geq R\left[\gamma_b(\alpha+\rho_b)-\gamma_a(\alpha+\rho_a)\right]$$

Then, these inequalities imply

$$\begin{aligned} &\psi\alpha\left[\frac{\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}-\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \\ &> -\psi\left[(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)-(\alpha+\rho_a)\left(\frac{\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\right)\right] \end{aligned}$$

that is $\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha+\rho_a}{\rho_a} > 0$, which is equivalent to $\alpha(\rho_a-\rho_b) > 0$ which is satisfied whenever $\rho_a > \rho_b$. Note that the case here is symmetric to Case (2.2), so two tasks are delegated to one agent if and only if the following (replacing subscripts a with b and b with a in Inequality 29) holds.

$$R[\gamma_b(\alpha+\rho_b)-\gamma_a(\alpha+\rho_a)] \geq \psi\left[\frac{\alpha(\alpha+\rho_a)}{\rho_b}-\frac{\alpha(\alpha+\rho_a)}{\rho_b+\gamma_a}-\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right] \quad (32)$$

To see that this inequality holds it suffices to show the following

$$-\psi\left[(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)-(\alpha+\rho_a)\left(\frac{\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\right)\right]$$

$$\geq \psi\left[\frac{\alpha(\alpha+\rho_a)}{\rho_b}-\frac{\alpha(\alpha+\rho_a)}{\rho_b+\gamma_a}-\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]$$

that is,

$$(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)-(\alpha+\rho_a)\left(\frac{\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\right)$$
$$\leq -\frac{\alpha(\alpha+\rho_a)}{\rho_b}+\frac{\alpha(\alpha+\rho_a)}{\rho_b+\gamma_a}+\frac{\alpha+\rho_b}{\rho_b}+\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}$$

that is,

$$(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)-(\alpha+\rho_a)\left(\frac{\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)}{\rho_a\rho_b(\rho_b+\gamma_a)}\right)$$
$$\leq \frac{-\alpha(\alpha+\rho_a)\rho_a\gamma_a}{\rho_a\rho_b(\rho_b+\gamma_a)}+(\alpha+\rho_b)\left(\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right)$$

which is equivalent to $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)\geq\alpha\rho_a\gamma_a$, that is, $\rho_b(\rho_b+\gamma_a)\geq 0$, which holds by Assumption 1.

(2.4) Suppose task b is delegated first in both cases. Then the principal hires only one agent for the project if and only if

$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]\geq$$
$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

which is simplified to

$$\frac{\alpha(\alpha+\rho_b)}{\rho_a+\gamma_b}+\frac{\alpha+\rho_b}{\rho_b}-\frac{\alpha(\alpha+\rho_b)}{\rho_a}+\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\geq 0$$

which is equivalent to $\frac{\alpha+\rho_b}{\rho_b}\geq 0$ which holds since all the parameters are striclty positive.

Case 3: $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)\geq 0$ and $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)<0$. There are four combinations when comparing task orders in one-agent and two-agent cases.

(3.1) Suppose task a is delegated first in both cases. Then the principal hires only one agent for the project if and only if

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right]$$
$$\geq R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]$$

which is equivalent to $\frac{1}{\rho_b} \leq \frac{1}{\rho_b+\gamma_a}$, which holds since $\gamma_a \leq 0$.

(3.2) Now, suppose that task a is delegated first in the one-agent case, and task b is delegated first in the two-agent case. Then, it is optimal to delegate two tasks to one agent if and only if

$$R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}+\frac{\alpha+\rho_b}{\rho_b}\right] \geq$$
$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

which is simplified to

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)] \geq \psi\alpha\frac{\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)} \tag{33}$$

Task a is delegated first in one-agent case if and only if Inequality 23 holds, which is

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)] \geq \psi(\alpha+\rho_b)\left[\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_b\rho_a(\rho_a+\gamma_b)}\right]$$

To show that Inequality 33 holds it suffices to show the following inequality

$$\psi(\alpha+\rho_b)\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)} \geq \psi\alpha\gamma_b\frac{(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}$$

which can be simplified to $\frac{\rho_a(\rho_a+\gamma_b)}{\rho_b} \geq 0$, which holds by Assumption 1.

(3.3) Now, suppose that task b is delegated first in the one-agent case, and task a is delegated first in the two-agent case. For task b to be delegated first in the one-agent case, the opposite of Inequality 23 must hold, which can be rewritten as

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)] \leq \psi(\alpha+\rho_b)\left[\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right] \tag{34}$$

One agent is assigned to perform both tasks if and only if

$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]$$
$$\geq R[(\alpha+\rho_a)(\gamma_a+1)+\alpha+\rho_b]-\psi\left[(\alpha+\rho_a)\left(\frac{\alpha}{\rho_b+\gamma_a}+\frac{1}{\rho_a}\right)+(1-\alpha-\rho_a)\frac{\alpha}{\rho_b}+1\right]$$

that is,

$$R[\gamma_a(\alpha+\rho_a)-\gamma_b(\alpha+\rho_b)]\leq\psi\left[(\alpha+\rho_b)\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right] \tag{35}$$

To see that Inequality 34 holds, it sufficed to show

$$\psi(\alpha+\rho_b)\left[\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}\right]\leq\psi\left[(\alpha+\rho_b)\frac{\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)}{\rho_a\rho_b(\rho_a+\gamma_b)}-\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\right]$$

which is equivalent to $\frac{\alpha\gamma_a(\alpha+\rho_a)}{\rho_b(\rho_b+\gamma_a)}\leq 0$, which holds by Assumption 1 and $\gamma_a<0$.

(3.4) Now suppose task b is delegated first in both cases. Then, one agent is assigned to perform both tasks if and only if

$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[\frac{\alpha+\rho_a}{\rho_a}-\frac{\alpha\gamma_b(\alpha+\rho_b)}{\rho_a(\rho_a+\gamma_b)}\right]\geq$$
$$R[(\alpha+\rho_b)(\gamma_b+1)+\alpha+\rho_a]-\psi\left[(\alpha+\rho_b)\left(\frac{\alpha}{\rho_a+\gamma_b}+\frac{1}{\rho_b}\right)+(1-\alpha-\rho_b)\frac{\alpha}{\rho_a}+1\right]$$

which is simplified to

$$-\alpha\gamma_b\frac{\alpha+\rho_b}{\rho_a(\rho_a+\gamma_b)}\leq\alpha\frac{\alpha+\rho_b}{\rho_a+\gamma_b}+\frac{\alpha+\rho_b}{\rho_b}-\alpha\frac{\alpha+\rho_b}{\rho_a}$$

that is $\frac{\alpha+\rho_b}{\rho_b}\geq 0$ which holds since the parameters are positive.

Case 4: $\rho_b^2+\gamma_a(\rho_b+\alpha\rho_a)<0$ and $\rho_a^2+\gamma_b(\rho_a+\alpha\rho_b)\geq 0$. This case symmetric to Case 3, thus the same proof applies here as well, which we get by just replacing subscripts a(b) with b(a) in that proof.

(iii) Without loss of generality, suppose $\gamma_a>0$ and $\gamma_b<0$. Propositions 1 and 2 assert that the synergistic task is delegated first. Thus, task a is delegated first in each case. Then, the proof is the same as in part (i) and the principal prefers to hire two identical agents rather than one agent when one task is synergistic and the other one is conflicting.

Proof of Lemma 3: Without loss of generality, suppose first task is a, that is, $i = a$. The limited liability, (LL), and the incentive compatibility (IC_A) constraints for agent A together imply that agent A's participation constraint, (PC_A), is satisfied. Likewise, ($IC_B^{q_a=1}$) and (LL) imply ($PC_B^{q_a=1}$), and ($IC_B^{q_a=0}$) and (LL) imply ($PC_B^{q_a=0}$). Moreover, the limited liability constraints for w_{10}^B and w_{00}^B are binding, since otherwise they can be lowered by the principal so that she maximizes her expected profit. Then, substituting them into the incentive compatibility constraints of agent B gives

$$w_{11}^B \geq \frac{\psi + \gamma_a}{\rho} \quad \text{and} \quad w_{01}^B \geq \frac{\psi}{\rho}$$

Similarly, the principal sets w_{01}^A and w_{00}^A as small as possible, that is $w_{01}^A = w_{00}^A = 0$. Substituting them into IC_A gives

$$(\alpha + \rho)w_{11}^A + (1 - \alpha - \rho)w_{10}^A \geq \frac{\psi}{\rho}$$

Note that $IC_B^{q_a}$ binds, since otherwise the principal would decrease w_{01}^B by a small amount without violating other constraints, which gives $w_{01}^B = \frac{\psi}{\rho}$. Furthermore, IC_A binds at the optimum. Suppose not, i.e., $(\alpha + \rho)w_{11}^A + (1 - \alpha - \rho)w_{10}^A > \frac{\psi}{\rho}$. Then the only constraints that would be affected by lowering w_{11}^A and w_{10}^A are limited liability constraints, $w_{11}^A \geq 0$ and $w_{10}^A \geq 0$, in which at least one of them must be non-binding so that the IC_A holds. Then, if IC_A does not bind, the principal would decrease the non-binding wage in order to maximize her profit without hurting other constraints. Then, any combination of w_{11}^A and w_{10}^A satisfying binding IC_A, $(\alpha + \rho)w_{11}^A + (1 - \alpha - \rho)w_{10}^A = \frac{\psi}{\rho}$, with non-negative wages minimizes the expected cost of the principal. One such contract is $w_{11}^A = w_{10}^A = \frac{\psi}{\rho}$.

Considering $IC_B^{q_a=1}$, if task a is conflicting on task b, i.e., $\gamma_a > 0$, or technologically independent, i.e., $\gamma_a = 0$, or task a is synergistic on task b, i.e., $\gamma_a < 0$, and $|\gamma_a| < \psi$, then this constraint is also binding since the right hand side of this inequality is positive. Then the binding constraint gives

$$w_{11}^B = \frac{\psi + \gamma_a}{\rho}$$

If task a is synergistic on task b, $\gamma_a < 0$, and $|\gamma_a| > \psi$, then the right hand side of $IC_B^{q_a=1}$ is negative and hence non-binding. Then all values of w_{11}^B satisfying limited liability constraint $w_{11}^B \geq 0$ satisfies $IC_B^{q_a=1}$, which implies that the principal sets the minimum value, which is $w_{11}^B = 0$.

Proof of Proposition 4: Task a is delegated first by the principal if and only if the expected total profit from delegating task a first is greater than that of task b, that is

(i) If $\gamma_i \geq 0$, or $\gamma_i < 0$ and $|\gamma_i| < \psi$

$$2R(\alpha + \rho) - \frac{\alpha + \rho}{\rho}[\gamma_a(\alpha + \rho) + 2\psi] \geq 2R(\alpha + \rho) - \frac{\alpha + \rho}{\rho}[\gamma_b(\alpha + \rho) + 2\psi]$$

which is equivalent to $\gamma_a \leq \gamma_b$.

(ii) If $\gamma_i < 0$ and $|\gamma_i| \geq \psi$, then the expected profit from delegating task a first is the same as the expected profit from delegating task b first, which implies that the principal is indifferent in ordering the tasks.

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