ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

DYNAMIC INVESTIGATION OF DIESEL ENGINE GEAR TRAIN SYSTEM UNDER FLUCTUATED TORQUE EXCITATION

M.Sc. THESIS

Osman SUBAŞI

Department of Mechanical Engineering

Automotive Graduate Program

DECEMBER 2017

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<u>ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ</u>

DİZEL MOTOR ZAMANLAMA DİŞLİ SİSTEMİ DİNAMİĞİNİN SALINIMLI TORK ETKİSİ ALTINDA İNCELENMESİ

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Osman SUBAŞI, a M.Sc. student of İTU Graduate School of Science Engineering and Technology student ID 503141714, successfully defended the thesis entitled "DYNAMIC INVESTIGATION OF DIESEL ENGINE GEAR TRAIN SYSTEM UNDER FLUCTUATED TORQUE EXCITATION", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below

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To my family,

FOREWORD

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November 2017

Osman SUBAŞI
Mechanical Engineer

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ABBREVIATIONS

ARCT	**:** Acoustical Relevant Contact Time
CAD	**:** Computer Aided Design
CAE	**:** Computer Aided Engineering
DoF	**:** Degree of Freedom
EOM	**:** Equation of Motion
FIP	**:** Fuel Injection Pump
GUI	**:** Graphical User Interface
IC	**:** Internal Combustion
MBD	**:** Multi-Body Dynamic
NVH	**:** Noise Vibration and Harshness
STE	**:** Static Transmission Error
TD	**:** Timing Drive

SYMBOLS

$\mathbf{F_1,F_2,F_3, F_e}$	**:** Forces
$\mathbf{T_1,T_2,T_3}$	**:** Torques
$\mathbf{I_1, I_2, I_3}$	**:** Inertias
$\mathbf{r_1, r_2, r_3}$	**:** Base circle radius
$\boldsymbol{\Theta_1,\Theta_2,\Theta_3}$	**:** Angular Positions
$\mathbf{k_1,k_2,k_3,k_m}$	**:** Stiffness
$\mathbf{c_1,c_2,c_3,c_m}$	**:** Damping
$\mathbf{m_1,m_2,m_3,m_c}$	**:** Masses
$\mathbf{z_1,z_2,z_3}$	**:** Number of Teeth
$\boldsymbol{\zeta}$	**:** Displacement vector
$\boldsymbol{\delta_m}$	**:** Relative displacement
$\boldsymbol{\lambda}$	**:** Complex Eigenvalues
η	**:** State vector

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DYNAMIC INVESTIGATION OF DIESEL ENGINE GEAR TRAIN SYSTEM UNDER FLUCTUATED TORQUE EXCITATION

SUMMARY

In last decades trend in automotive industry go towards on a new path due to regulations and high demand of costumers. Most automotive companies downsize and boost their internal combustion engine to have a better fuel economy and emissions. Nowadays, all diesel and most of the gasoline engines have direct injection and turbocharger or supercharger system to get more power with high fuel efficiency. However, this brings new challenges especially for noise, vibration and harshness (NVH) because downsizing and boosting means having higher pressure in the combustion chamber and increasing forces that generated by power conversion systems. There are two main sources of noise for an internal combustion engines; combustion noise and mechanical noise. Rotating systems of engines cause the mechanical noise and one of them is timing drive system. There are different type of timing drive system and geared system generally uses for heavy-duty diesel engine due to its higher load and power capability while reducing durability risks. However, gear systems have worst performance when it comes to NVH, mainly two type of gear noise produces by geared timing drive systems; gear whine and gear rattle. Gear rattle, also known gear impact or hammering noise, is a broadband noise and can be seen in wide frequency range. In this study, the sensitivity of gear rattle under fluctuated torque influence is investigated and computer-aided engineering (CAE) based method applied to get a better timing gear system in terms of NVH.

This study can divide into two main chapters; gear dynamic and heavy-duty diesel engine gear system with fluctuated torque source. In the early part of the study, dynamic of a gear pair with different complexity levels is investigated. Firstly, equation of motion of a linear time-invariant gear pair, which has four degrees of freedom governed, and a fluctuated torque applied. Governed equations solved with two different methods, which are analytical and numerical. In addition, a multi-body dynamic (MBD) model built for exactly same system and computationally solved the system. Secondly, same gear pair investigated with backlash. Backlash brings a non-linearity to system. Equation of motion of non-linear system also governed and solved with numerically. Same non-linear system built in computer-based program and results compared with the numeric solution. For all solutions, results are same for different solution for same systems. Thirdly, another gear added to system and degree of freedom increase to six, this system solved only computationally and effect of phasing fluctuated torque presented.

A detailed multi-body dynamics model of a six-cylinder heavy-duty diesel engine's geared timing drive system built to calculate gear mesh forces. There are ten gears in this system and two of them have fluctuated torque characteristics; fuel injection pump (FIP) and camshaft. FIP provides fuel to common rail in diesel engines and has highly fluctuated torque demand from the geared system due to his working principle.

A metric called impact impulse, which calculated from gear mesh force and used for rattle assessment of the system. Impact impulses calculated for each gear mesh and summed individual impact impulse up to get total impact impulse, which correlated with gear rattle noise. Total impact impulse recalculated after phasing torque curve of fuel injection pump to find best working point of the system in terms of gear rattle noise. In addition, a physical test conducted and radiated noise of engine and acceleration from gear housing measured to compare with computational results. Consequently, both computational measurement results show fluctuated torques has a significant effect on geared timing system of a heavy-duty diesel engine and with phasing of fluctuated torque can reduce gear rattle in a geared timing drive system.

DİZEL MOTOR ZAMANLAMA DİŞLİ SİSTEMİNİN DİNAMİĞİNİN SALINIMLI TORK ETKİSİ ALTINDA İNCELENMESİ

ÖZET

Özellikle son on yılda otomotiv sektörü sıkı regulasyonları ve müşteri memnuniyetini karşılayabilmek için yeni bir yola girmiş bulunmaktadır. Bu yolda, otomotiv şirketleri içten yanmalı motor hacimlerinde küçültmeye gidereken daha fazla güç ve daha iyi emisyon değerleri elde etmeye çalışılmaktadır. Günümüzde kullanılan bütün dizel motorlarda ve birçok benzinli motorda direk püskürtmeli yakıt sistemi ve turbo besleme bulunmaktadır. Bu sistemlerin amacı az yakıt ile yüksek verimliliğe ulaşabilmektir. Fakat bu beraberinde titreşim ve gürültü gibi bazı zorluklar getirmektedir. Çünkü turbo beslemeli ve direk yakıt püskürtmeli motorlarda yanma odasındaki basınç daha yüksek değerlere çıkmakta ve bu da beraberinde motorun hareket eden parçalarına daha fazla kuvvet iletmesini getirmektedir. Bu yüksek kuvvetler de titreşim ve gürültü problemlerini arttırmaktadır. İçten yanmalı motorlarda yanma ve mekanik olmak üzere iki temel gürültü ve titreşim kaynağı vardır. İçten yanmalı motorlarda zamanlama sistemi mekanik gürültü kaynaklarından bir tanesidir. üç farklı zamanlama sistemi motorlarda krank milinden eksantrik miline hareket aktarmakta kullanılabilir. Bunlar, dişli kayıs-kasnak sitemi, zincir sistemleri yada dişli sistemleridir. Ağır vasıta dizel motorlarında yüksek güç ve yük iletimi gerektiğinden ve bu koşullara en uygun olan dişli sistemler tercih edilmektedir. Yüksek dayanım performansı olan dişli sistemlerin bazı dezavantajları da vardır, bunların en önemlisi gürültü problemidir. Genelde dişli sistemli kaynaklı iki farklı ses problemi çıkmaktadır bunlar dişli vınlaması ve dişli tıkırtısıdır. Dişli vınlamasının kaynağı dişlilerin elastik yapısından kaynaklanmaktadır ve bu çalışma kapsamında dişli vınlaması çalışılmamış sadece dişli tıkırtısı çalışılmıştır. Dişli tıkırtısı aynı zamanda dişli çarpma sesi ya da dişli çekiç sesi olarakda bilinir ve geniş bandlı bir ses kaynağıdır, dolayısıyla geniş bir frekans aralığında duyulmaktadır. Dişli tıkırtısının ana sebepleri, dişli şaftlarındaki hız dalgalanması, diş boşluğu, sistemde tahrik edilen yağ pompası, yüksek basınçlı yakıt pompası, kompresor gibi yardımcı yan sistemlerin ihtiyaç duyduğu zamana göre değişen tork değerleri(salınımlı torkalar), dişlilerin ve dönen parçaların ataletleridir. Sistemi besleyen ana dişlideki tork dalgalanması içten yanmalı motorun çalışma mantığından kaynaklanmaktadır. Çünkü her parlamada tork değeri maksimuma ulaşırken sonraki patlamaya kadar tork değeri düşmektedir ve motor sabit bir hzındayken zamana bağlı tork değeri sürekli olarak ortalama bir değer etrafında salınım yapmakta bu salınım da dişlilerin açısal hızında bir salınıma sebep olmaktadır, bu salınımı azaltmak mümkündür ancak hem parça dizaynlarında hem sistemin dizaynında büyük değişiklikler yapmak gerekir ki özellikli ileri fazdaki bir proje de bunlar çok fazla mümkün olmayan değişikliklerdir. Dişliler arasında dizayn aşamısında belirlenen ve dişli üretim kalitesine bağlı olan ve ingilizce *backlash* olarak bilinen bir boşluk vardır. Bu boşluk, dişliler arasında olmak zorundadır yoksa dişliler uyum içinde çalışamaz ve kitlenirler ama bu diş boşluğu aynı zamanda sistemdeki tork salınımıyla beraber dişlilerin ön ve arka tarafa çarpmasına izin

vererek yüksek ve anlık dinamik kuvvetlerin oluşmasına, bu yüksek ve anlık dinamik kuvvetlerde tıkırtı sesi oluşumuna sebep verirler. Bu diş boşluğu her zaman belirli bir tolerans aralığında tutlmaya çalışılarak bu gibi sorunların önüne geçilmeye çalışılır. Diğer bir ana sebep olan yardımcı sistemlerin tork değerleri bu diş boşlukları sayesinde sistemde oluşan dinamik kuvvetlerin artmasına sebep olmaktadırlar. Bu çalışmada da dişli tıkırtı sesini salınımlı tork etkisi altından incelenmesi kapsamında sayısal çözümleme yapan bilgisayar destekli simülasyon metodu kullanılması anlatılmıştır, bu salımlı torklar fazlanarak etkileri minumum değerlere ulaştırılması hedeflenmiştir. Daha sonra bu kapsamda dinamometre odasında fiziksel bir test gerçekleştirilmiştir.

Bu çalışma dişli dinamiği ve ağır vasıta dizel motorlarının zamanlama dişli sisteminin salınım torku altında tıkırtı ses probleminin incelenmesi olarak iki ana başlık altında toplanabilir. Bu kapsamda, çalışmanın ilk kısımlarında dişli dinamiği değişik varsıyımlar altında incelenmiştir. İlk olarak basit bir diş çifti- dört serbestlik dereceli- lineer olarak ele alınmış ve bu bağlamda hareket denklemleri çıkarılarak analitik, sayısal ve bilgisayar programı yardımıyla çözülmüştür. Bu sistemde diş yatakları lineer kabul edilmiş ve diş boşlukları olmadığı kabulü yapılarak lineerleştirilmiştir. Bu sistem dış kuvvetlerin etkisi altında olmadığı kabulü yapılmak suretiyle homojenleştirilierek başlangıç koşulu (ilk hız) altında analitik ve sayısal olarak çözülmüş ve her iki çözüm yöntemi de aynı sonuçları vermiştir. Bir sonraki adım da aynı sistem bilgisayar programı ile kurulmuş ve dış kuvvetler altında çözülmüştür. Daha önce sadece başlangıç koşulunda sayısal olarak çözülen sistem de aynı dış kuvvetler uygulanarak bu iki çözüm sistemi de karşılaştırılmıştır ve sonuçlar tamamen aynı çıkmıştır. Daha sonra sisteme dişililer arasında boşluklar hesaba katılar lineer olmayan hale getirilmiş ve yine bu sisteminde hareket denklemleri çıkartılarak nümerik ve bilgisayar programı yardımıyla çözülmüştür. Aynı sistemi aynı parametrelerle çözen tüm bü değişik çözüm yöntemleri beklendiği gibi birbiriyle aynı sonuçlar vermiştir. Sayısal ve analitik çözümleri yapabilmek için çıkarılan hareket denklemlerin çözülmü için Matlab programında kodlar yazılmıştır. Son olarak, lineer olmayan aynı sisteme bir dişli daha eklenerek, daha kompleks hale getirilmiş ve bilgisayar programı yardıyla çözülmüştür. Yine aynı sistemde salınım yapan tork eğrisi belli bir derece fazlanarak bu fazlamanın sistemin dinamiğine özelliklede dişiler arasındaki anlık kuvvete olan etkisi incelenmiştir.

Zamanlama sistemleri içten yanmalı motorlarda hareketi krank milinden eksantrik miline aktarmak için kullanılırlar ve bu sistemler genellikle dişli kayış, zincir ve dişli olarak farklı formlarda kullanılır. Harekete eksantrik miline aktarırken aynı zamanda yardımcı sistemleride tahrik etmektedir. Ağır vasıta motorlarında bu zamanlama sistemi yüksek güç ve yük taşıma kapasitesinden dolayı dişli sistemler olarak seçilirler. Üstünde çalışılınan altı silindirli ağır vasıta dizel motorunun dişli zamanlama sistemi bilgisayar programı yardımıyla detaylıca modellenmiş ve dişliler arasındaki kuvvetler hesaplanmıştır. Bu modelleme sırasında dişlilerin makro değerleri girilmiş ve ISO standartlarına göre dişli çiftlerinin sertlik katsayıları hesaplanarak sabit bir değer olrak modele girilmiştir yine sönüm katsayısı aynı yöntemle hesaplanmıştır. Bu sistemde on tane düz dişli iki düzlemde bulunmaktadır, bu dişlilerden iki tanesi sistemden yüksek salınlı tork çekmektedir, bunlar eksantrik milinin bağlı olduğu dişli ve yüksek basınçlı yakıt sistmim tahrik eden dişlidir. Yüksek basınçlı yakıt pompası tork değeri üreticisinin yaptığı test ile ölçtürülmüş ve modele girdi olarak kullanılmıştır. Yine bir başka salınımlı tork kaynağı olan subab sistemi aynı program yardımıyla noktasal atalet ve kütle noktalarına indirgenerek bu noktalar sertlik ve sönüm katsayısı değerleriyle birbirlerine bağlanarak gerçek hayat

simüle edilmeye çalışılmıştır. Subab mekanizması eksantrik miliyle beraber dişli sistemine simülasyon ortamında monte edilerek bu iki sistem gerçek hayattaki gibi eş zamanlı çalıştırılmıştır böylece subap mekanizmasının dişli sistemine olan etkisi birebir gözlemlenmiştir. Ayrıca bir başka önemli girdi olan olan tahrik dişlisi yani krank miline bağlı olan dişli ye başlangıç koşulu olarak hızdaki dalgalanma başka bir simülasyon programıyla hesaplanarak girilmiştir. Bu simülasyon kapsamında piston, biyel ve krank mekanizması modellenerek ölçüm sonucu olan silindir içi basınç uygulanarak motor hızındaki salınım hesaplanmıştır.
Darbe etkisi (*impact impulse)* olarak adlandırılan bir metrik dişli çiftleri arasındaki kuvvetten hesaplanarak, dişli tırkırtısını değerlendirmek için kullanılmıştır. Darbe etkisi her dişli çifti için ayrı ayrı hesaplanmış ve tüm dişili çftleri için hesaplanan değerler toplanarak toplam darbe etkisi bulunmuştur. Bu toplam değer sistemin dişli tıkırtı sesi hakkında bir değerlendirme imkânı vermiştir. Bu değerin yüksek olduğu sistemler de daha fazla tıkırtı sesi beklenmektedir. Daha sonra yüksek basınçlı pompanın tork değeri farklı faz açılarına fazlanarak –belli bir mantık çerçevesinde- işlem tekrarlanmış ve sistemin bu tork değerine en az hassas olduğu nokta yani sistemin en az tıkırtı sesi vereceği değer bulunmaya çalışılmıştır. Bu kapsamda mümkün olan dokuz farklı faz açısı denenmiştir. Ayrıca dinamometre odasında fiziksel bir test gerçekleştırılmiş ve aynı durum orda da sorgulanmıştır. Bu test sırasında motor etrafına standartlara uygun olarak, motordan birer metre uzakta olucak şekilde dört adet mikrofon yerleştirilmiş, bu mikrofonlara ek olarak bir tane de zamanlama dişli sisteminin etkisini daha net gözlemleyebilmek için motorun arka tarafına (dişli sistemi motorun arka kısmında bulunmaktadır) yerleştirilmiştir. Ayrıca volan ve dişli yuvaları üstüne ivmeölçerler yerleştirilerek bu verilerde toplanmıştır. Motordan toplanan bu ses ve ivme verileri toplanarak bilgisayar programı çıktılarıyla karşılaştırılmıştır. Sonuç olarak hem bilgisayar destekli hesaplamalar hem de fiziksel test sonuçları birbirini doğrular nitelikte olmuştur. Fiziksel test sonuçlarında en iyi durum ile en kötü durum arasında dört desibelden fazla bir fark bulunmuştur. Normalde kullanılcak değer ile en iyi sonucu veren durum arasında ise iki desibelden fazla fark bulunmuştur ve herhangi bir ekstra maliyet getirmeden ve tasarımda değişiklik yapmadan motorun genel ses durumu iki desibelden fazla iyiliştirilmiştir.

1. INTRODUCTION

First geared systems were used hundreds of years ago and nowadays are used in many applications from automotive, aerospace, wind turbines to clocks in different materials form and several varied forms. Usage percentage of these variations are seventy-four percent of gears are spur gears, fifteen percent helical gear, five percent worm, four percent bevel and rest are either epicyclical or internal [1]. There are many several points that need to take attention at the very beginning of a geared system design. One of them is the gear dynamic because gears can produce numerous dynamic loads and many different parameters that have important roles in gear dynamics. Another important point is noise vibration and harshness (NVH) characteristics of the geared system, which is related gear dynamic. With help of modern computers and commercial software, dynamic analysis of gears can be done easily and fast at project design early phase.

Different models have been presented by researchers about gear dynamic in various forms since the 1950s. Simplest one is a gear pair with a single degree of freedom and complexity can be extended to increase degrees of freedom of a gear pair or using multiple gear pairs. Kahraman and Singh state dynamic model of the geared system can be categorized into four main groups, which are linear time-invariant, linear time varying with variable mesh stiffness, time-invariant with including backlash and finally the last group both gear backlash and mesh stiffness variation considered at the same time [2].

This study starts with governing a single gear pair's equation of motion (EOM) with different assumptions and solving them with analytical and numerical methods in Matlab. Also, same gear pair's dynamic model was built in commercial software to benchmark the results and get a confidence level. Those calculation methods applied to entire timing gear train system of a heavy-duty diesel engine. Timing gear train system is used to transmit torque from crankshaft to the accessories drives such as oil pump, air compressor or fuel injection pump and as well as camshaft with respect to valve timing to allow opening and closing valve according to engine strokes. A metric called impact impulse calculated from mesh force between each gear in the system. Impact impulse is a metric that used to evaluate overall rattle performance of a gear train system and allow us to optimize parameters that are important for gear rattle and in this study; we optimized fluctuated torque with phasing to have best rattle performance from the system.

This study can divide into two main chapters. First, one is governing equation of motion of a geared system with two and three gears and analytical, numerical and computational calculation of them under fluctuated torque. The second one is computational calculation and measurement of timing geared system with ten gears in two plain for different fluctuated torque phasing.

1.1 Problem Definition

The sound performance of an internal combustion engine is a key factor evaluating the overall quality of the vehicle. Combustion noise dominates the heavy-duty diesel engine's NVH however in recent year improvements in noise control lead new source of noise that masked previously. Timing gear train system is one of those previously masked systems, its contribution is numerous and cannot neglect, and it is an area that needs many works. Mostly two noise problems can observe in a timing gear system; gear whine and gear rattle. It defined that static transmission error (STE) is the main exciter for gear whine and defined as the deviation of gears angular position due to tooth elasticity and manufacturer errors [3]. If a gear pair has perfectly rigid gears, they would result a constant ratio in angular speed that mean is constant angular speed input will give constant output angular speed [4]. However, in reality, every gear has a flexible tooth and manufacturing errors that cause transmission error and gear whine. Gear whine is a deep subject and there are many researches on it but it does not focus of this study.

Gear rattling is also called gear hammering is the characteristic noise that produces during impacting gears. The main reason for this impacting is clearance type nonlinearities which called backlash. Backlash is clearance between two teeth that are engaged and cause to collapse teeth one side or double side and generate impact noise when one gear try to transfer torque to other. There are several reasons for this collision such as engine torsional vibration and torque characteristic of accessory drive especially highly fluctuated. Crank torsional vibration can reduce but it is not always easy and mostly brings additional cost and significant design changes in the crank train system, on the other hand, fluctuated torques in dynamic system can phase to have a better impact force at the gear meshes that will cause better gear rattle without any additional cost or design changes.

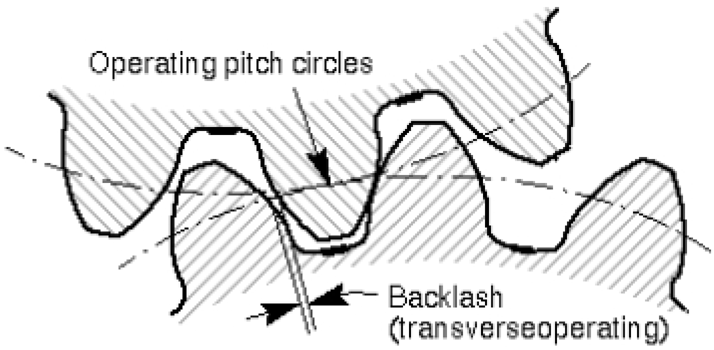

Figure 1.1 : Backlash in a gear pair

R. Singh claim that “the rattle problem is generally a dynamic system problem, i.e, it is not a gear dynamic issue (such as whine)” so in this study first we start with a gear pair system in the second step an idler added to system to see fluctuated torque effects better and finally methodology extended to entire timing gear system [5].

1.2 Literature Review

There are plenty of work on gear dynamics and noise problems. Before starting this study, a literature survey has done. The generic geared system that considered in Kahraman and Singh’s study [6] has applied by real parameter of fluctuated torque gear and its preceding gear. However, they proposed two different systems; the first one is with gear center assumed rigid and has one degree of freedom and the second one has gears with flexible bearing (including shafts) and has three degrees of freedom. First system that will present in this study is same with their second assumption. However, degree of freedom is kept four and did not reduce to three (there is a relationship regarding to reduction ratio between gears and system degrees of freedom can be reduced to three). The reason why we did this is to have more output to compare with the computational results. Also, Kahraman and Singh considered mesh stiffness variation and clearance at the bearing which are not considered in this study. In their study, governed equations of motion solved by both analytical and numerical solution techniques under a different assumption. The frequency response of system obtained and a parametric study conducted to show effect of each parameter such as clearance in gear bearing and backlash at the mesh point.

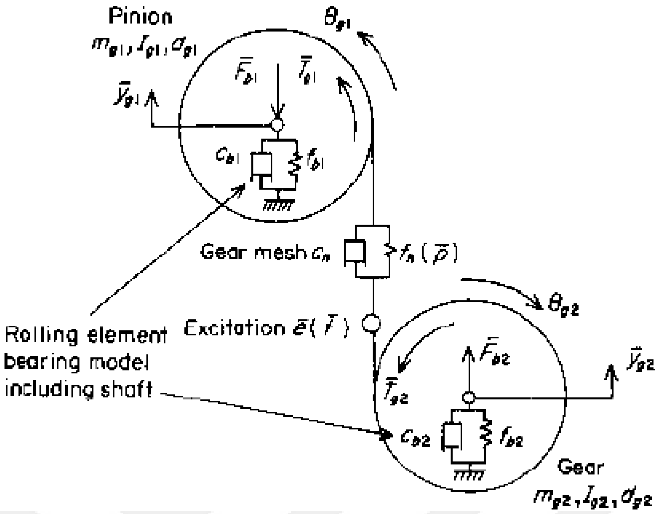

Figure 1.2 : Presented geared system [6]

Another study is published by Özgüven and Houser in 1987[7]. Their model assumes rigid gear bearing, average stiffness at gear mesh and has one degree of freedom. This model is one of the simplest linear models can be build but gives the opportunity to predict dynamic mesh load, dynamic tooth load and dynamic transmission error. The advantage of this method is that it does not need to calculate mesh stiffness function at the mesh point and static transmission error can be used as an input.

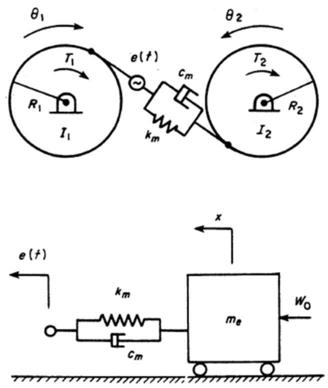

Figure 1.3 : A dynamic model of gear pair and its equivalent single degree of freedom of gear pair [7]

There are many studies about gear rattle in literature, however, most of them about automotive transmission. Although the same rattle phenomenon occurs in transmission rattle and engine timing gear system, there are different points to look in these systems for rattle phenomena. Firstly, main source of gear rattle in an automotive transmission is loose gear because they are coupled but do not transmit any torque and have potential to rattle [8]. Nevertheless, this assumption cannot apply to geared timing system because all gears are transferring torque all the time.
Spressert and Ponsa [9] studied on direct injection diesel engine's timing gears and experimentally showed fuel injection pump has the largest contribution to the overall sound of geared timing drive system because of its highly fluctuated torque characteristic. Another experimental study [10] shows fluctuated torque's effect on gear train's impacts. In this study, each gear's effect has been observed and results clearly show that FIP and camshaft have an incredible effect on gear impact due to their fluctuated torques and gear impact are related with gear rattle noise. However, constant accessory drive torques such as water pump, ac compressor reduce the gear rattle intensity in a geared timing drive. In addition, large changes in inertia of rotating components have little effect on rattle [11].

1.3 Objective

The objective of this thesis is to understand the non-linear gear dynamic under fluctuated external torques and reducing gear noise specifically gear rattle. However, first, a linear system is developed and then extended to non-linear to understand exact differences. Two gear-pairs system is solved with three different ways to compare results. These solution ways are analytical, numerical and computational. After that, it extends to examine Ford Otosan's 12.7 liters inline six cylinders heavy-duty diesel engine's timing gear system in this study. This engine has quite complex gear train system with 13 gears in 3 different plains but only two plains have been considered in computational model because third plain's effect on overall rattle contribution can be negligible compare to rest of the system, all are spur gears and gave an opportunity to reduce gear rattle with phasing highly fluctuated external torque. A fully dynamic analysis has been conducted at all different possible phasing according to torque data and tried to find best for all responses that were analyzed and a decision made on a phasing point which gear train is less sensitive. This does not only minimize the gear rattle since gear rattle purely depends on the mesh force of gear especially at first mating moment but also the overall dynamic response of the entire system.

A detailed dynamic analysis was performed after governing the equation of motions to understand each parameter effect on gear rattle but only fluctuated torques are main interest for this study. There are many accessories that extract torque from timing gear train system but two of them have highly fluctuated torque characteristics; camshaft gear and fuel injection pump (FIP) gear. It is impossible to phasing camshaft gear due to valve timing but FIP can be phased without any problem.

1.4 Limitations

This thesis completed following limitations:

- Stiffness variations are ignored at mesh point and gear bearings so constant stiffness is used for each gear mesh.
- All analysis conducted under steady state condition at constant speed, effects of transient have been ignored due to software limitations.
- A physical test has been conducted but not any of outputs are used directly to correlate between analysis and measurement.

2. ANALYTICAL, NUMERICAL AND COMPUTATIONAL INVESTIGATION OF A LINEAR TIME-INVARIANT SYSTEM

The main objective of this chapter is investigating dynamic behavior of a linear time-invariant gear pair. Focusing on dynamic modeling of a geared system has many reasons such as noise control, durability and stability analysis. Also, gears could generate a great amount of loads and transmit that loads to other machine elements of the system especially shafts and bearings. Gears loads was concern decades ago but the first simple mass-spring model was introduced in the 1950s [12]. In this chapter, governing of linear equation of motion of a gear pair obtained and forced vibration analysis is made and solution of same system has been found by numerical, analytical and computational.

2.1 Four Degrees of Freedom Linear Model – No Backlash

Four degree of freedom (DoF) (it could be reduced to three degree of freedom since there is rotational relationship between two gears) linear model of a geared system with gear inertias I_1 and I_2, gear masses m_1 and m_2 and base circle radius r_1 and r_2 as shown in Figure 2.1 is considered. This system has constant bearing stiffness in the y-direction and constant stiffness between gears –at mesh point- without any backlash so there is nothing that brings non-linearity to system and a good way to start to investigate system dynamic behavior. Gear 1 is driven gear and extracts a fluctuated torque (T_2) from system and gear 2 is driving gear with a constant torque. There is no friction defined at mesh point so that transverse vibrations in the pressure line direction have no effect on the vibrations in the opposite direction that allows us to use one-way stiffness at the gear bearings on the pressure line [6]. The equation of motion obtained on the pressure line and static transmission error does not consider here and ignored in all equations.

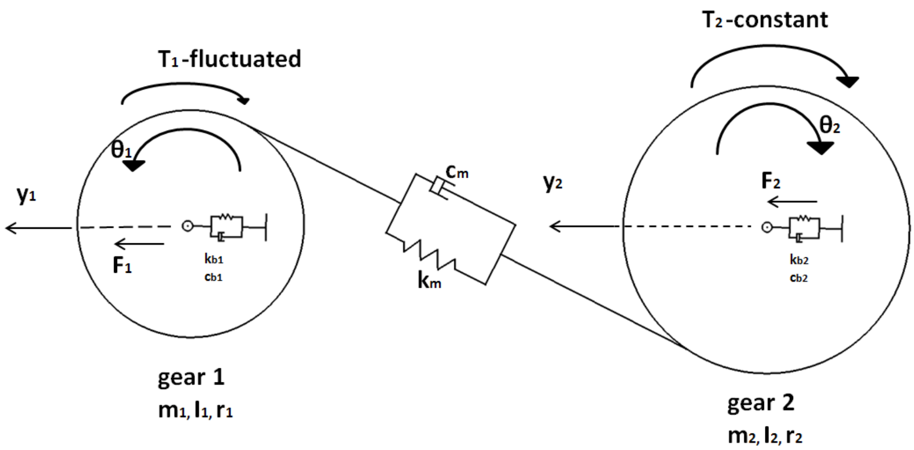

Figure 2.1 : 4 DoF linear geared system

2.2 Governing Equations

Equations are governing under assumptions that mentioned in the previous chapter and figure 2.1 shows the system that equation of motions will govern in this chapter.
Governing equation for the translational motion of Gear 1 (y_1):

$$m_1 y_1'' = F_1 - k_1 y_1 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2) - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \tag{2.1}$$

Governing equation for the translational motion of Gear 2 (y_2):

$$m_2 y_2'' = F_2 - k_2 y_2 + k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2) + c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \tag{2.2}$$

Governing equation for the angular motion of Gear 1 (θ_1):

$$I_1\theta_1'' = -T_1 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2)r_1 - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_1 \tag{2.3}$$

Governing equation for the angular motion of Gear 2 (θ_2):

$$I_2\theta_2'' = T_2 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2)r_1 - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_2 \tag{2.4}$$

These equations can be reduced to three degree of freedom with the following relationships between θ_1 and θ_2

$$x = r_1\theta_1 - r_2\theta_2 \tag{2.5}$$

Let δ_m be relative displacement of gear tooth.

$$\delta_m = y_1 - y_2 + x \tag{2.6}$$

Using (2.6) in equation (2.3) and multiplying by r_2 will give us

$$I_1 r_2\theta_1'' = -T_1 r_2 - k_m r_1 r_2\delta - c_m r_1 r_2\delta_m \tag{2.7}$$

Also, using (2.6) in equation (2.4) and multiplying by r_1 will give us

$$I_2 r_1 \theta_2'' = T_2 r_1 - k_m r_1 r_2 \delta_m - c_m r_1 r_2 \delta_m \tag{2.8}$$

Now subtracting equation (2.8) from equation (2.7) and multiplying with r_2 :

$$I_1 r_2^2 \theta_1'' - I_2 r_1 r_2 \theta_2'' = -T_1 r_2^2 - T_2 r_1 r_2 \tag{2.9}$$

δ_m is previously defined and multiplying second derivative of it with $I_2 r_1$:

$$I_2 r_1^2 \theta_1'' + I_2 r_1 r_2 \theta_2'' = I_2 r_1 \delta'' - I_2 r_1 y_1'' + I_2 r_1 y_2'' \tag{2.10}$$

Now adding up (2.10) and (2.9), also making some arrangement will give us

$$m_c \delta_m'' - m_c y_1'' + m_c y_2'' + k_m \delta_m = \frac{T_1 r_1 I_2 + T_2 r_2 I_1}{I_1 r_2^2 + I_2 r_1^2} \tag{2.11}$$

Where $m_c = \frac{I_1 I_2}{I_1 r_2^2 + I_2 r_1^2}$ and called effective mass.

Finally, three governing equations are (2.1), (2.2) and (2.11), those equations can be combined into matrix formulation:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ -m_c & m_c & m_c \end{bmatrix} \begin{Bmatrix} y_1'' \\ y_2'' \\ \delta_m'' \end{Bmatrix} + \begin{bmatrix} c_1 & 0 & c_m \\ 0 & c_2 & -c_m \\ 0 & 0 & c_m \end{bmatrix} \begin{Bmatrix} y_1' \\ y_2' \\ \delta_m' \end{Bmatrix} + \begin{bmatrix} k_1 & 0 & k_m \\ 0 & k_2 & -k_m \\ 0 & 0 & k_m \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ \delta_m \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_e \end{Bmatrix} \tag{2.12}$$

where F_e is the effective force and can be written as:

$$F_e = \frac{T_1 r_1 I_2 + T_2 r_2 I_1}{I_1 r_2^2 + I_2 r_1^2} \tag{2.13}$$

2.3 Analytical Solution

The four DoF system's linear differential equations of second order with constant coefficients governed in chapter 2.2. However, for analytical solution three DoF equations (2.12) in matrix form will use to simplify solution. If this equation will try to solve as forced vibration, one problem would come across due to fluctuated torque input. Because fluctuated torque input ($T_{1)}$ as can be seen in figure 2.2. is not a function or cannot be written as a function. T_1 input torque that showed in figure 2.2. is a generic measurement result. On the other hand, for analytical solution all external loads should be a function so that external torques removed from system to solve equation of motions analytically. Analytical solution will be exact solution of system mathematically under initial condition and will give a good level of confidence to other solution methods which are numerical and computational.

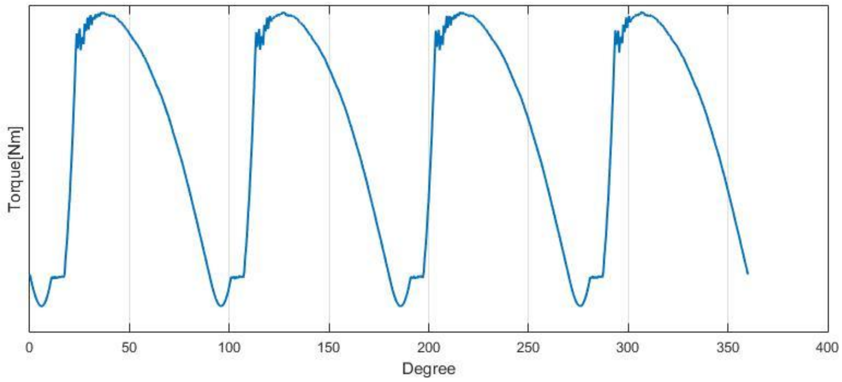

Figure 2.2 : Generic measurement input torque T_1 (x-axis is gear 1 domain)

Therefore, the equation of motion of system reduced to homogeneous to be able to have an exact solution with analytical method. Under this assumption, equation (2.12) becomes:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ -m_c & m_c & m_c \end{bmatrix} \begin{Bmatrix} y_1'' \\ y_2'' \\ \delta_m'' \end{Bmatrix} + \begin{bmatrix} c_1 & 0 & c_m \\ 0 & c_2 & -c_m \\ 0 & 0 & c_m \end{bmatrix} \begin{Bmatrix} y_1' \\ y_2' \\ \delta_m' \end{Bmatrix} + \begin{bmatrix} k_1 & 0 & k_m \\ 0 & k_2 & -k_m \\ 0 & 0 & k_m \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ \delta_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.14}$$

Equation (2.14) can be written in the following form:

$$[\mathbf{M}]\{\zeta''\} + [\mathbf{C}]\{\zeta'\} + [\mathbf{K}]\{\zeta\} = \{0\} \tag{2.15}$$

where

$$[\mathbf{M}] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ -m_c & m_c & m_c \end{bmatrix} \text{ is mass matrix} \tag{2.16a}$$

$$[\mathbf{C}] = \begin{bmatrix} c_1 & 0 & c_m \\ 0 & c_2 & -c_m \\ 0 & 0 & c_m \end{bmatrix} \text{ is damping matrix} \tag{2.16b}$$

$$[\mathbf{K}] = \begin{bmatrix} k_1 & 0 & k_m \\ 0 & k_2 & -k_m \\ 0 & 0 & k_m \end{bmatrix} \text{ is the stiffness matrix} \tag{2.16c}$$

And

$$\{\zeta\} = \begin{Bmatrix} y_1 \\ y_2 \\ \delta_m \end{Bmatrix} \text{ is the displacement vector} \tag{2.17}$$

New vector form needs to be defined to be able to solve this equation in a matrix form and this vector called state vector.

$$\{\eta\} = \begin{Bmatrix} \zeta' \\ \zeta \end{Bmatrix} \tag{2.18}$$

After defining state vector, equation of motion then becomes:

$$\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}\{\eta'\} + \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}\{\eta\} = \{0\} \tag{2.19}$$

the second order matrix equation of motion has been transformed into a first order matrix differential equation.

Let $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}$ be, these two matrixes contain all the mechanical information of system. By multiplying entire expression with the inverse of **A** matrix and re-arranging expression will give:

$$\{\eta'\} + [\mathbf{A}]^{-1}[\mathbf{B}]\{\eta\} = \{0\} \tag{2.20}$$

Now need to assume a solution to equation (2.20) which is

$$\eta(t) = Hexp(\lambda t) \text{ and} \tag{2.21}$$

$$\eta'(t) = H\lambda exp(\lambda t)$$

$\eta(t) = Hexp(\lambda t)$ is well-known guess function for this kind of differential equations.

Final form of equation becomes:

$$\left[\lambda[\mathbf{I}] + [\mathbf{A}]^{-1}[\mathbf{B}]\right]Hexp(\lambda t) = \{0\} \tag{2.22}$$

Equation (2.22) is the classical eigenvalue problem, where λ refers to the complex eigenvalues and H are the corresponding eigenvectors.

$exp(\lambda t) \neq 0\ and\ \{H\} = 0$ give the trivial solution, for non-trivial solution the following determinant has to be zero.

$$|\lambda[I] + [A]^{-1}[B]| = 0 \tag{2.23}$$

Since we do not have any external loads in the system, an initial condition needs to apply to observe system's response during analytical calculation. These initial conditions are y_1 (0) =0, y_2 (0) =0, $\theta_1' = 125.664\ \mathrm{rad/s}$ (1200rpm), $\theta_2' = 62.831\ \mathrm{rad/s}$ (600rpm) and with these initial conditions δ_m can be calculated from equation (2.6) and result will be δ_m: 0.00010362. Parameters that used to calculate results as shown in table 2.1. Those parameters are generic parameters and 2:1 gear ratio was chosen to simplify initial gear angular velocities.

Figure 2.3 and figure 2.3 shows graphs of analytical solution (the general solution of the equation 2.12 with the right side being zero – the homogenous equation) which decays after 0.06 second due to large damping in the system. The homogenous part is also called transient response and if the system is lightly damped, the transient response will linger.

Table 2.1 : Generic parameters of 4 DoF System

Parameter	Unit	Value
m_1	(kg)	0.75
m_2	(kg)	3.5
I_1	(kg.m^2)	0.0009
I_2	(kg.m^2)	0.022
r_1	(m)	0.05181
r_2	(m)	0.10362
T_2	(Nm)	26.72
z_2	-	30
k_2	(N/m)	5000000
k_m	(N/m)	124091320
c_1	(N.s/m)	500
c_2	(N.s/m)	500
c_m	(N.s/m)	1485
T_1	(N.m)	See figure 2.2
z_1	-	60

A Matlab code created to solve the equation (2.23) and this code can be found in Appendix A. Under these initial conditions, displacement and velocity results as shown in Figure 2.3 and Figure 2.4.

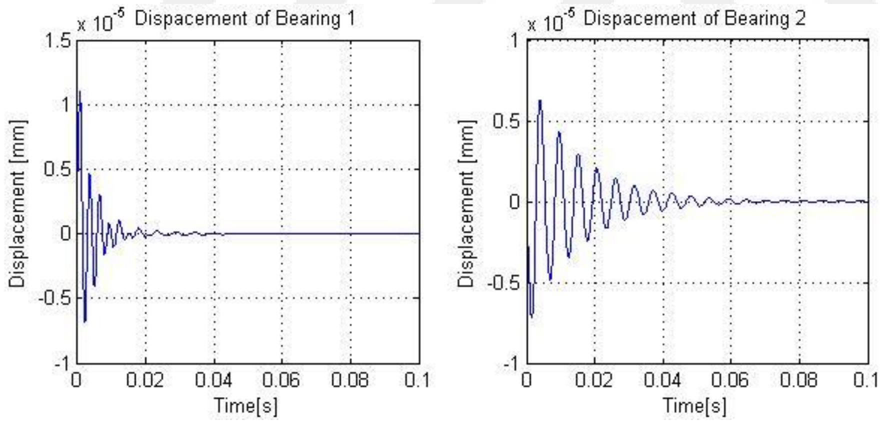

Figure 2.3 : Analytically calculated translational displacements of Gear 1 (y_1) and Gear 2 (y_2) for the homogeneous system (external excitation functions are assumed to be zero)

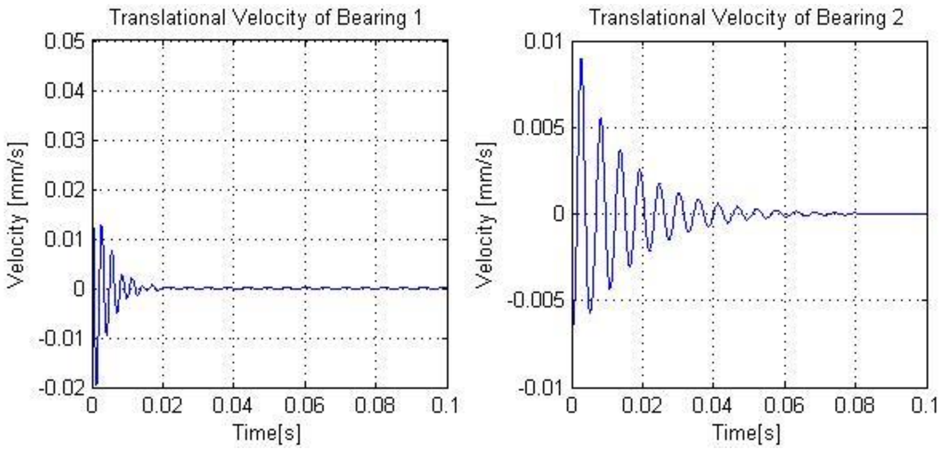

Figure 2.4 : Analytically calculated translational velocities of Gear 1 (y_1') and Gear 2 (y_2') for the homogeneous system (external excitation functions are assumed to be zero)

Those responses are the classical damped free vibration of system's response.

2.4 Numerical Solution

The governed equation of motion solved numerically with help of Matlab. Matlab is numerical computing environment so these equations can solve easily with existed command in Matlab. The command that used to solve equations is based on an explicit Runge-Kutta formula. Created Matlab codes can be found in Appendix B.

Equations 2.1, 2.2, 2.3 and 2.4 are solved with parameters in Table 2.1. Four equations solved to have eight outputs to compare with analytical and computational results. These results are translational displacement of bearing 1 (y_1) and bearing 2 (y_2) and angular displacement of gear 1 (θ_1) and gear 2 (θ_2) are derivative of each of them which will be angular and translational velocities can calculate as an output. T_2 in table 2.1 is calculated from T_1 and relationship between them is that mean value of T_1 multiplied by 2.0065(gear ratio is 2 and 0.0065 is coming from inertial effects, with this value system is running in steady state condition and velocity of each gear does not decrease or increase and always fluctuated around calculated mean value).

Parameters that used to calculate equations of motion can be found in table 2.1.

2.4.1 Numerical results

Gear 1 has 30 teeth and gear 2 has 60 teeth so the ratio between them is 1:2. Numerical calculation done for one revolution (360 degree) of gear 2 that means gear 1 will rotate 720 degree (2 revolutions) and torque in figure2.2. will be applied 2 times in this time of period. Also same an initial conditions that used for an analytical solution have been applied which are initial speed of 600rpm (62.831 rad/s) for gear 2 and 1200rpm (125.664 rad/s) for gear 1 however rest of initial conditions remain zero. Under this initial conditions one revolution of gear 2 will be completed in 0.1 second and all results which will be presented in next chapter are for one cycle of gear 2 (360 degrees) (driving gear) or two cycle of gear 2 (driven gear) .

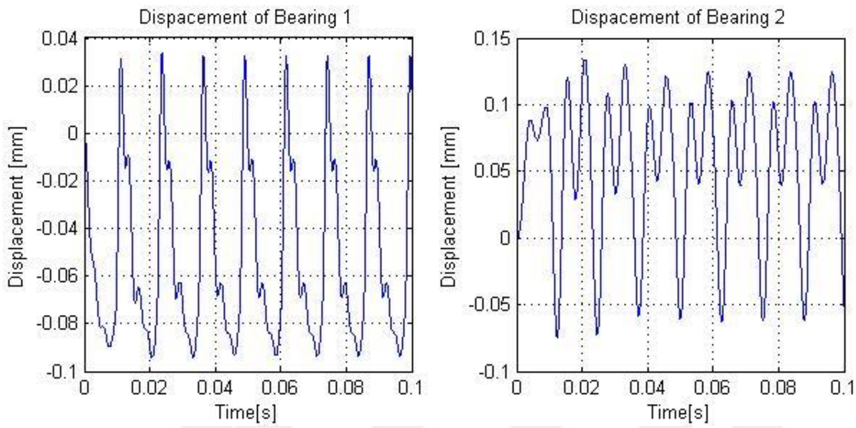

Figure 2.5 : Numerically calculated translational displacements of Gear 1 (y_1) and Gear 2 (y_2) under fluctuating torque excitation

The maximum displacement of bearings is related to generic stiffness value that defined in Matlab. Characteristic of displacement curve related to the applied fluctuated torque. Even displacement curve at bearings can show the effect of highly fluctuated torque effect on the system and when non-linearity come into system, its effect will be greater. On the other hand, these values can multiply by defined stiffness value to have force at bearing.

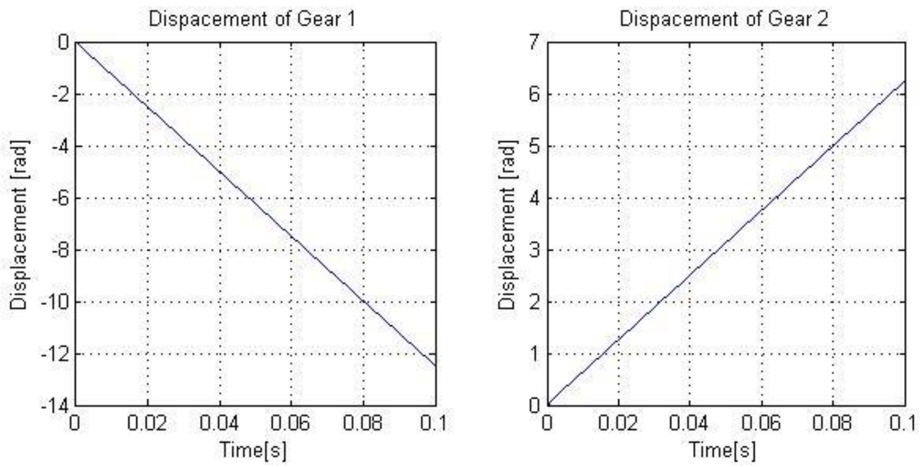

Figure 2.6 : Numerically calculated angular displacements of Gear 1 (θ_1) and Gear 2 (θ_2) under fluctuating torque excitation

These results show all the boundary and initial condition defined correctly in the equation of motions. Gear 2 rotates 6.283 radian which; is 360 degrees in 0.1 second and gear 1 rotates -12.566 radian due to 1:2 ratio in opposite direction in the same time range.

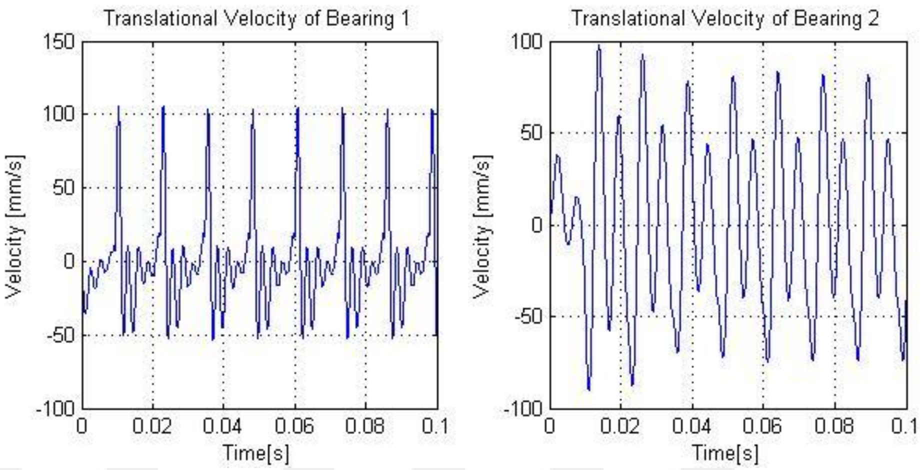

Figure 2.7 : Numerically calculated translational velocities of Gear 1 (y_1') and Gear 2 (y_2') under fluctuating torque excitation

Translational value of bearings obtained simply by derivative of displacement by the governed equation of motions.

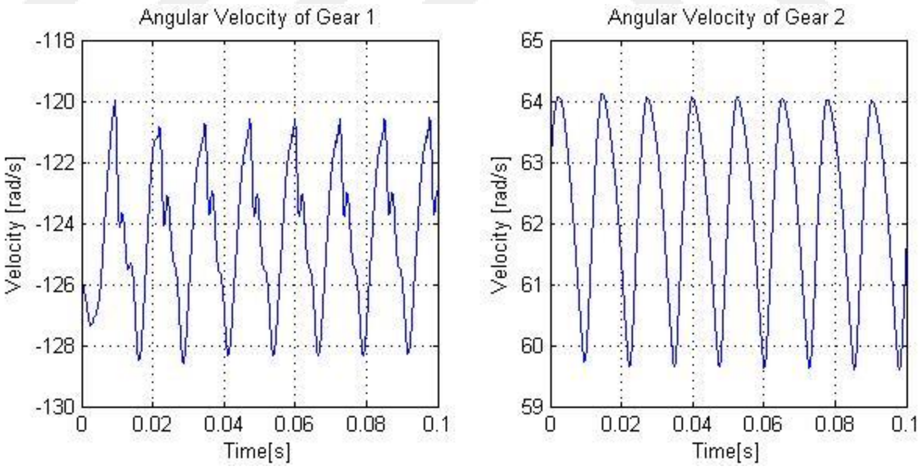

Figure 2.8 : Numerically calculated angular velocity of Gear 1 (θ_1') and Gear 2 (θ_2') under fluctuating torque excitation

Angular velocity of gear 1 shows different characteristic than gear 2 because fluctuated torque is applied to gear 2 and this more complex response is due to applied fluctuated torque. Translational and angular velocities of gears are represented in figure 2.7 and figure 2.8.

Results in figure 2.9 and figure 2.10 show the comparison of numerical and analytical solution approaches.

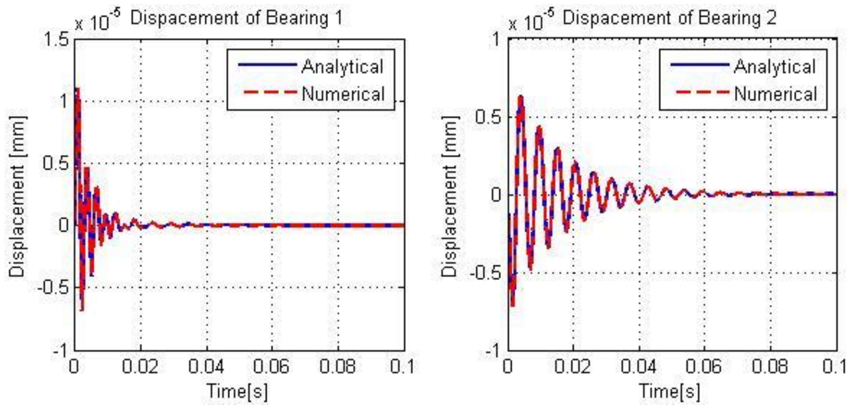

Figure 2.9 : Comparison of analytical and numerical solutions for the translational displacements of Gear 1 (y_1) and Gear 2 (y_2)

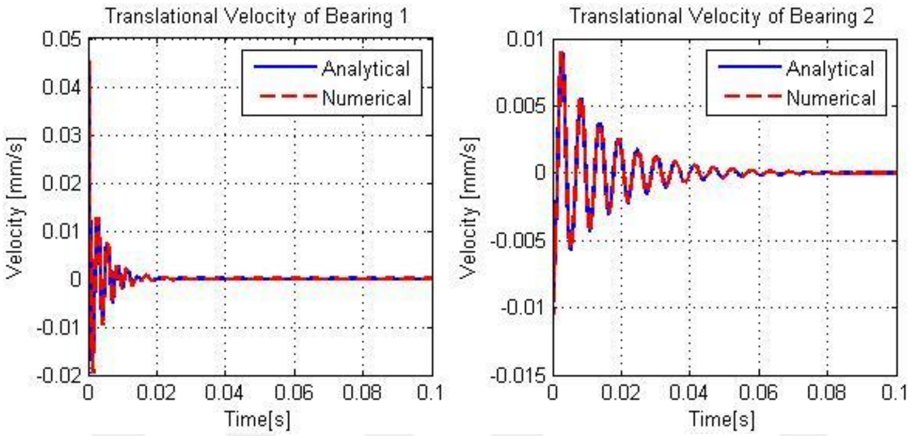

Figure 2.10 : Comparison of analytical and numerical solutions for the translational velocities of Gear 1 (y_1') and Gear 2 (y_2')

Figure 2.9. and Figure 2.10. show good correlation between analytical and numerical solution for both displacement and velocities. Figure 2.11. and Figure 2.12. are zoomed to the very early response of the system where transient effect might be observed.

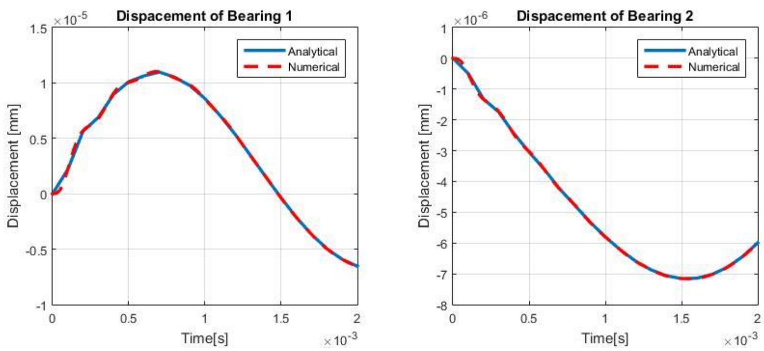

Figure 2.11 : Comparison of analytical and numerical solutions for the translational displacements of Gear 1 (y1) and Gear 2 (y2) for 0 < t < 0.002 sec

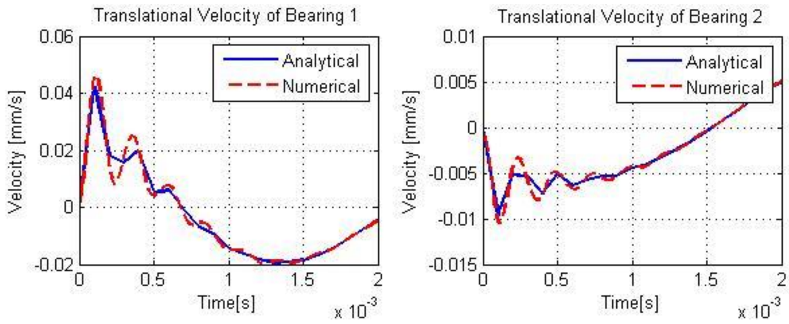

Figure 2.12 : Comparison of analytical and numerical solutions for the translational velocities of Gear 1 (y_1') and Gear 2 (y_2') for 0 < t < 0.002 sec

Even at the transient response of system are quite similar for both calculation method. Results give a good level of confidence about numerical solution. As describes in chapter 2.3 these results do not contain any external force or torque, these are system's response under initial conditions.

2.5 Computational Solution – Ricardo Valdyn

Same system with same parameters in Table 2.1. built in Multi-Body Dynamic software called Ricardo VALDYN. In addition to Valdyn, the same model built in AVL Excite Timing Drive software program as well but results are not presenting in this section. These two software programs are doing pretty much same things and used for modeling timing drive and valve train system of an internal combustion engine. They give the opportunity to model belt driven, chain driven or geared timing drive systems. However for gear modeling, AVL Excite Timing Drive program require many inputs and didn't look feasible to use for this level of system modeling, however, on the other hand, Valdyn gives us the opportunity to model exactly same system with same parameters, initial and boundary conditions for equation of motion that are governed in the previous chapter.

In Valdyn gears are presented as a series of lumped mass/inertia nodes and connected by stiffness and damping. This flexibility enables the user to generate models varying from simple model to highly complex complete system models. Each modeling element has been designed to use the minimum degrees of freedom, whilst retaining sufficient detail to achieve a high level of simulation accuracy.

Valdyn provides a Graphical User Interface (GUI), which is used to generate model-input files. The GUI uses a building block approach whereby icon representations of the various modeling elements can be chosen from a simple toolbox. The individual elements are then connected together in the desired manner using the mouse to build model. The GUI has built-in logic, which helps to ensure that “nonsense” models are not created. Each modeling element has a certain number of required data inputs associated with it (mass, stiffness, etc.). The user is able to supply these values by entering in pre-defined forms within the GUI. The GUI also allows the user to specify the units of each of the required parameters. Figure 2.13. shows the modeled geared system how looks like in Valdyn GUI.

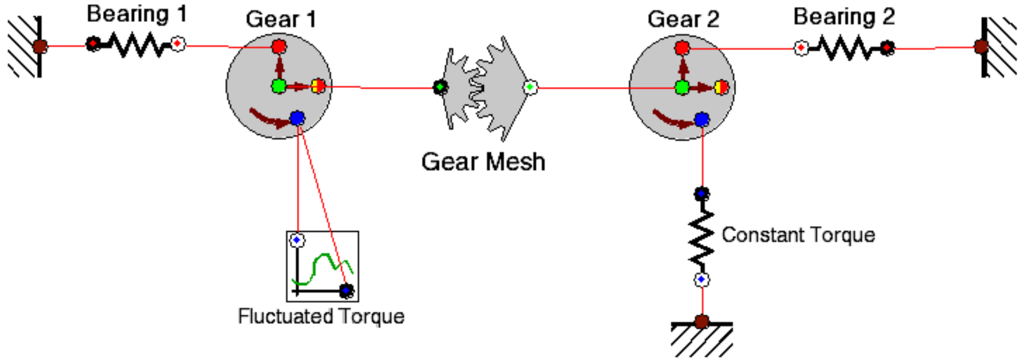

Figure 2.13 : 4 DoF linear geared system in Valdyn GUI

2.5.1 Computational results

Figure 2.8, 2.9, 2.10 and 2.10 shows results of Valdyn with a comparison of the governed equation of motion that solved with help of Matlab.

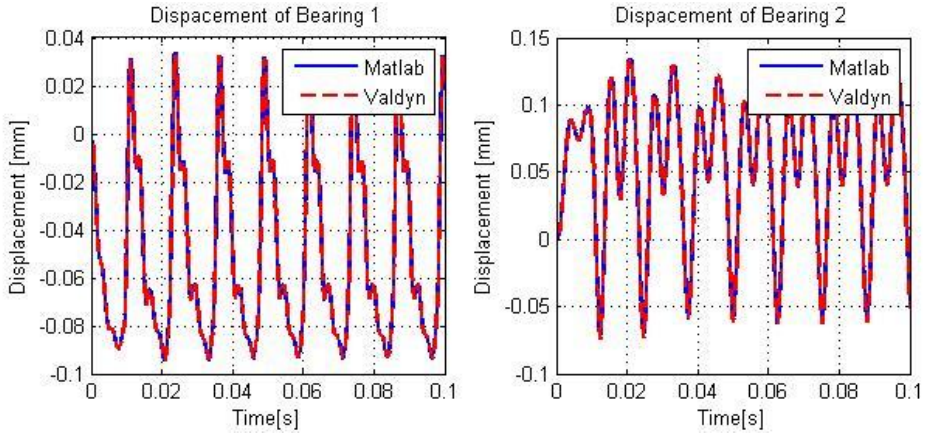

Figure 2.14 : Comparison of numerical and computational predictions for the translational displacements of Gear 1 (y_1) and Gear 2 (y_2)

Figure 2.14 shows translational displacement both numerical and analytical at the gear’s bearing. Both results are identical, no any differences.

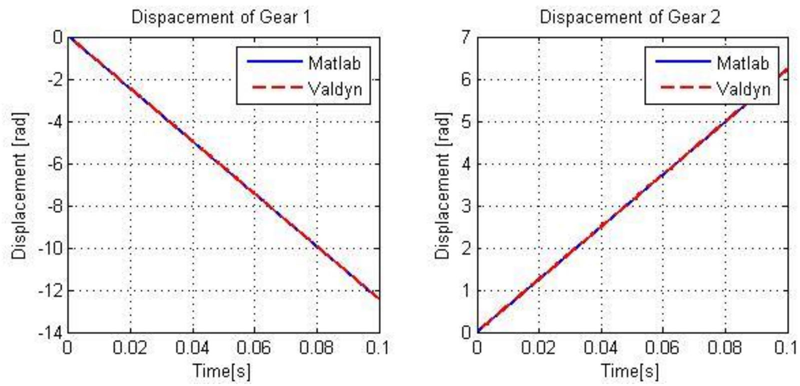

Figure 2.15 : Comparison of numerical and computational predictions for the angular displacements of Gear 1 (θ_1) and Gear 2 (θ_2)

Figure 2.15 shows rotational displacement results of gears for both calculation method and as an expected, they are same.

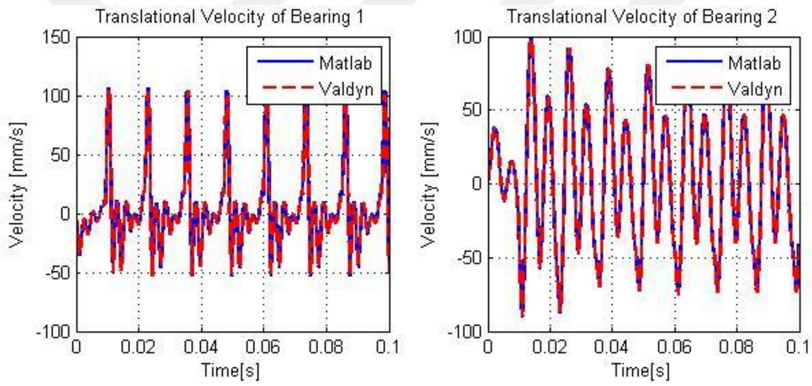

Figure 2.16 : Comparison of numerical and computational predictions for the translational velocities of Gear 1 (y_1') and Gear 2 (y_2')

Figure 2.16 shows the bearing's translational velocity at the gear's bearing.

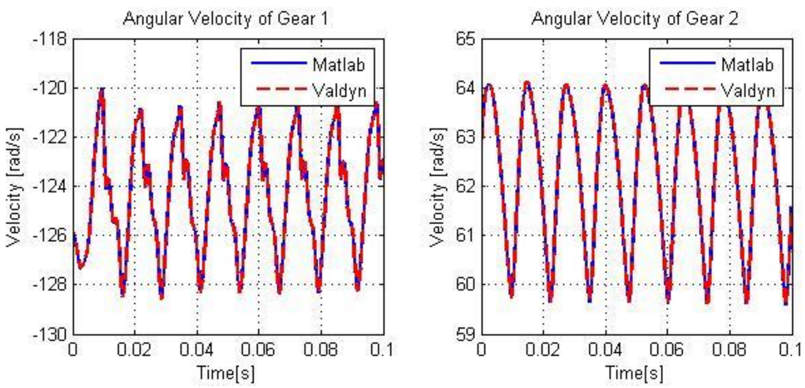

Figure 2.17 : Comparison of numerical and computational predictions for the angular velocities of Gear 1 (θ_1') and Gear 2 (θ_2')

Figure 2.17 shows angular velocity in rad/s of gears. As it can be seen under fluctuated torque, gear shows more uneven regime compare to gear that under the effect of constant torque and both solution method are identical here as well.

As expected both numerical and computational results are exactly same since the model is simple and linear. Results show that Valdyn uses the same equation of motion and solves them with Runge-Kutta iteration method.

3. NUMERICAL AND COMPUTATIONAL INVESTIGATION OF GEARED SYSTEM WITH BACKLASH

As describes in the previous chapter backlash is clearance between gear tooth that comes from design intent or manufacturer errors and it has a critical role in gear design to avoid jamming however, it has to be between design tolerances. Defining backlash lower may result gear jamming, on the other hand, setting backlash too large may raise impact force and that can cause to gear rattle. In this chapter non-linear equation of motion of same-geared system obtained, non-linearity comes with backlash to system. In addition, 4 DoF system will extend to 6 DoF system by adding an idler gear. The main reason for adding another gear is that to show clearly the effect of fluctuated torque's phasing in a non-linear system with more complexity.

3.1 Four Degrees of Freedom Non-Linear System

Same system in chapter 2.2. is considered here but this time backlash at gear mesh point considered and system extended like in Figure 3.1. shows the backlash (b) and it is a function of relative gear position.

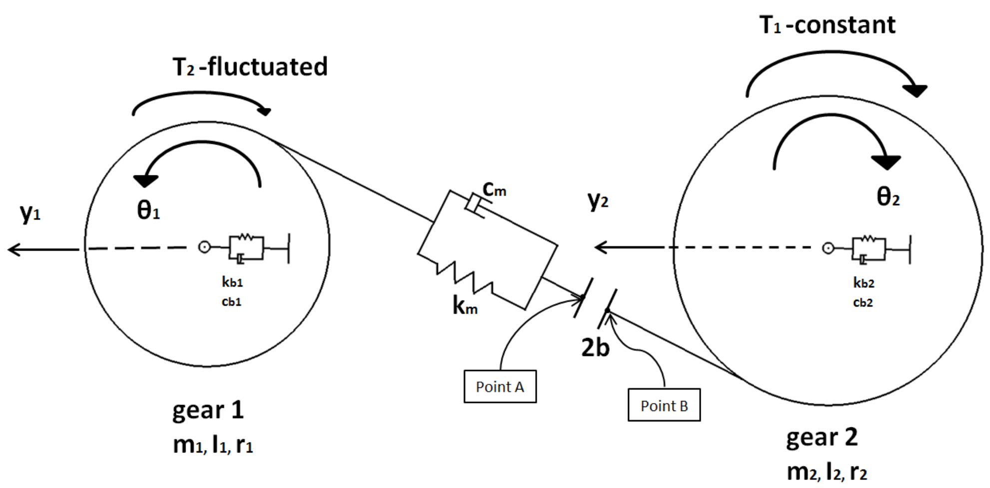

Figure 3.1 : 4 DoF system with backlash

3.1.1 Governing equation

The Same equation of motions will be used here but this time backlash nonlinearity will be considered. It will take account of backlash function that showed Figure 3.2.

Position of tooth 1: $y_1 + r_1\theta_1$

Position of tooth 2: $y_2 + r_2\theta_2$ and distance between tooth 1 and tooth 2 is:

$$\delta_m = y_1 + r_1\theta_1 - y_2 + r_2\theta_2 \tag{3.1}$$

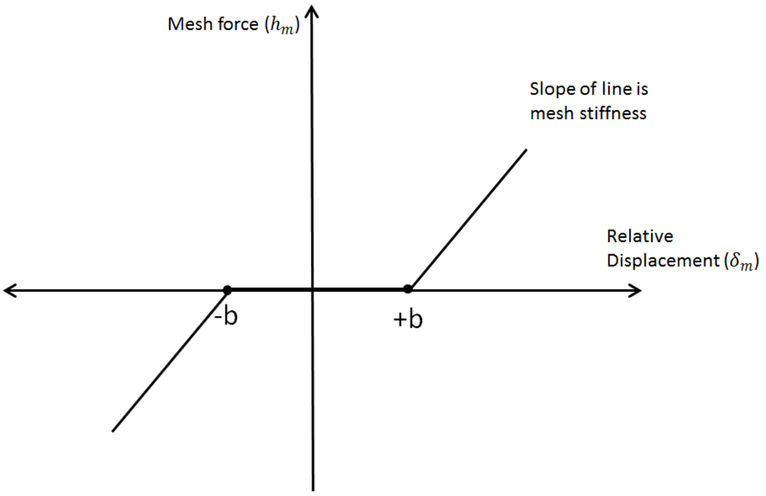

Figure 3.2 : Mesh force function with mesh stiffness and backlash

In backlash situation, there are three different cases, which are:

a) Case I: $\delta_m > b$ Contact at point A, driving contact

b) Case II: $-b \leq \delta_m \geq b$ No contact

c) Case III: $\delta_m < -b$ Contact at point B, backside contact

Tooth positions for these cases can be seen in Figure 3.3.

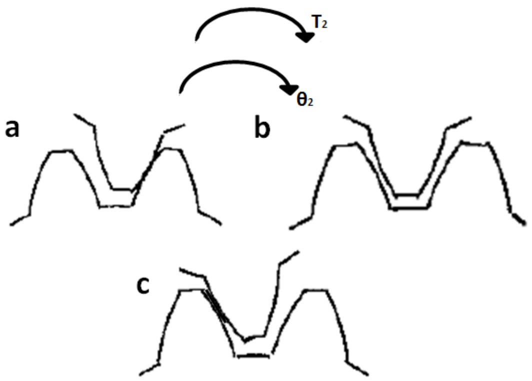

Figure 3.3 : **a)** Case I, Gears contact on driving side

b) Case II, Gears separated, no contact

c) Case III, Gears contact on the driven side

According to these cases equation of motions will be;

For Case I: $\delta_m > b$ Gears contact on driving side

Governing equation for the translational motion of Gear 1 (y_1):

$$m_1 y_1^{"} = F_1 - k_1 y_1 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 - b) - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \quad (3.2)$$

Governing equation for the translational motion of Gear 2 (y_2):

$$m_2 y_2^{"} = F_2 - k_2 y_2 + k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 - b) + c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \quad (3.3)$$

Governing equation for the angular motion of Gear 1 (θ_1):

$$I_1\theta_1^{"} = -T_1 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 - b)r_1 - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_1 \quad (3.4)$$

Governing equation for the angular motion of Gear 2 (θ_2):

$$I_2\theta_2^{"} = T_2 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 - b)r_1 - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_2 \quad (3.5)$$

Case II: $-b \le \delta_m \ge b$ Gears separeted, no contact

Governing equation for the translational motion of Gear 1 (y_1):

$$m_1 y_1^{"} = F_1 - k_1 y_1 \quad (3.6)$$

Governing equation for the translational motion of Gear 2 (y_2):

$$m_2 y_2^{"} = F_2 - k_2 y_2 \quad (3.7)$$

Governing equation for the angular motion of Gear 1 (θ_1):

$$I_1\theta_1^{"} = -T_1 \quad (3.8)$$

Governing equation for the angular motion of Gear 2 (θ_2):

$$I_2\theta_2^{"} = T_2 \quad (3.9)$$

Case III: $\delta_m < -b$ Gear contact on the driven side

Governing equation for the translational motion of Gear 1 (y_1):

$$m_1 y_1^{"} = F_1 - k_1 y_1 - c_1 y_1' - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 + b) - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \quad (3.2)$$

Governing equation for the translational motion of Gear 2 (y_2):

$$m_2 y_2^{"} = F_2 - k_2 y_2 - c_2 y_2' + k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 + b) + c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2') \quad (3.3)$$

Governing equation for the angular motion of Gear 1 (θ_1):

$$I_1\theta_1^{"} = -T_1 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 + b)r_1 - c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_1 \quad (3.4)$$

Governing equation for the angular motion of Gear 2 (θ_2):

$$I_2\theta_2'' = T_2 - k_m(y_1 - y_2 + r_1\theta_1 + r_2\theta_2 + b)r_1 - \ c_m(y_1' - y_2' + r_1\theta_1' + r_2\theta_2')r_2 \tag{3.5}$$

Rewriting equations in terms of δ_m and reducing them into three with following same steps that describes in chapter 2 with having$y_3 = r_1\theta_1 + r_2\theta_2$:

For Case I: $\boldsymbol{\delta_m > b}$

$$m_1y_1'' + k_1y_1 + c_1y_1' + k_m(\delta_m - b) = F_1 \tag{3.6}$$

$$m_2y_2'' + k_2y_2 + c_2y_2' - k_m(\delta_m - b) = F_2 \tag{3.7}$$

$$m_ey_3'' + k_m(\delta_m - b) = F_e \tag{3.8}$$

Case II: $\boldsymbol{-b \leq \delta_m \geq b}$

$$m_1y_1'' + k_1y_1 = 0 \tag{3.6}$$

$$m_2y_2'' + k_2y_2 = 0 \tag{3.7}$$

$$m_ey_3'' = F_e \tag{3.8}$$

Case III: $\boldsymbol{\delta_m < -b}$

$$m_1y_1'' + k_1y_1 + c_1y_1' + k_m(\delta_m + b) = F_1 \tag{3.9}$$

$$m_2y_2'' + k_2y_2 + c_2y_2' - k_m(\delta_m + b) = F_2 \tag{3.10}$$

$$m_ey_3'' + k_m(\delta_m + b) = F_e \tag{3.11}$$

3.1.2 Numerical solution – Matlab

Solving such differential equations, event detection method was used for calculation in Matlab. It divides the task into two phases which are an event detection following by event location phase. First event detection phase occurs and then event location to determine which case will occur next. It starts to numerically solve the equation of motions for case one under initial condition (defined velocity, position etc..) with a time step and when it detects case I satisfied it returns to case II and starts to solve equations for case II until reach to the condition that given. After that it decides either, it is going to be again case I or reach to case III. If the only case I and case II happen, this called single side impact however it might have double side impact when event detects to case III after case II. Schematic view of event detection flowchart is in figure 3.4. Event detection closely related to integration time step and it automatically decided by Matlab by defined tolerance and maximum time step. Matlab codes can be found in Appendix C for four DoF geared system with backlash.

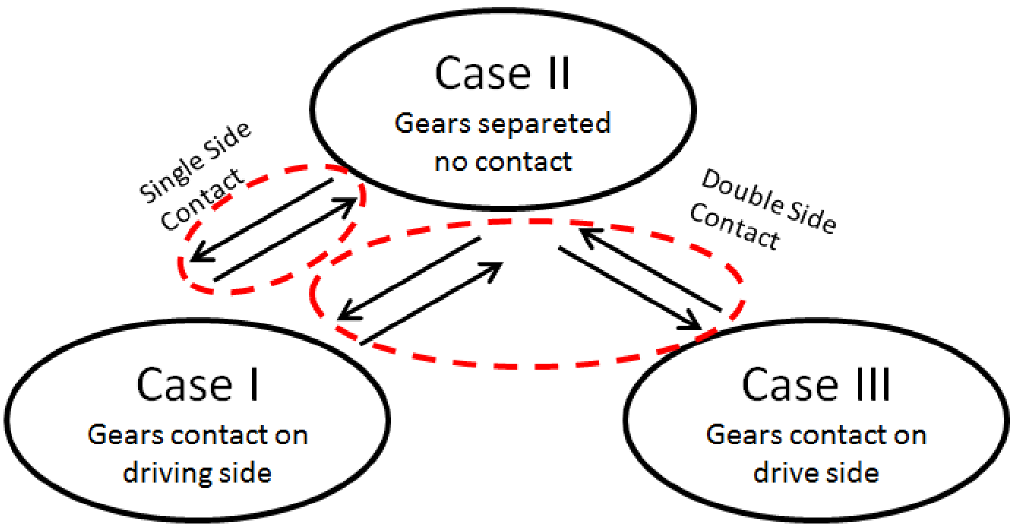

Figure 3.4 : Case detection schematic

In figure 3.4, can be helpful for understanding what these cases represent in real tooth profile. Figure 3.4 a represents case I, Figure 3.4 b represents case II and finally Figure 3.4 c represents case III.

3.1.3 Computational solution – Valdyn

Addition to parameters that defined in Table 2.1 150μm (2b) backlash is defined to a system with backlash. Figure 3.5 shows backlash definition between gears. Results of the geared system with backlash will show next chapter with computational results.

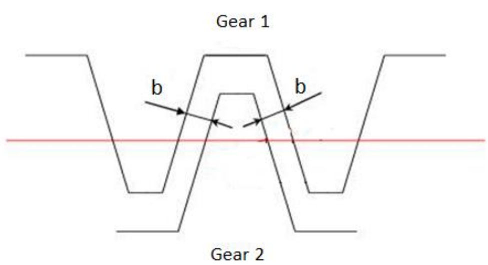

Figure 3.5 : Backlash definition between gears

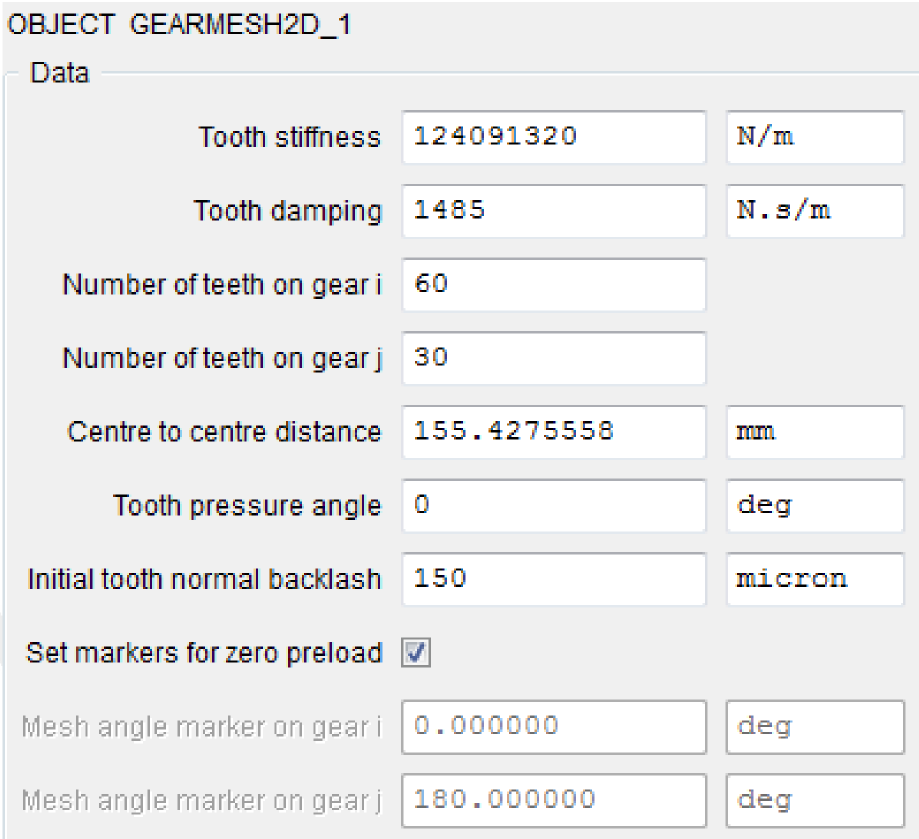

Figure 3.6 : Generic input parameters for gear mesh element in Valdyn

Only differences that made in the model is setting backlash 150 um and it was zero for the linear model. The generic parameters that used for computational calculation represents in figure 3.6.

3.1.4 Numerical and computational solution

Four outputs from 4 DoF system calculated for both numerical and computational, results can be seen in Figure 3.7, 3.8, 3.9 and 3.10.

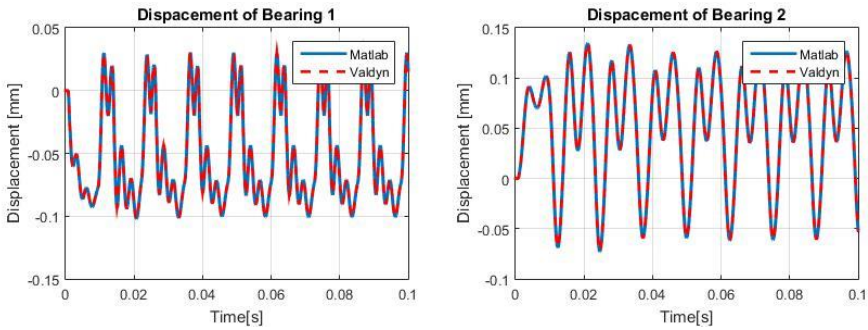

Figure 3.7 : Comparison of numerical and computational predictions for the translational displacements of Gear 1 (y1) and Gear 2 (y2)

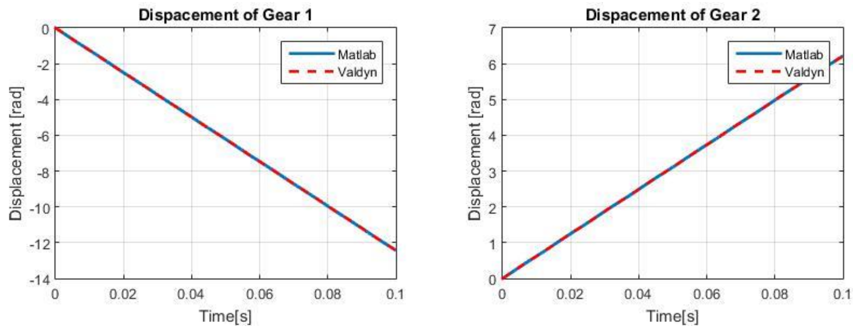

Figure 3.8 : Comparison of numerical and computational predictions for the angular displacements of Gear 1 (θ_1) and Gear 2 (θ_2)

Figure 3.7 shows translational displacement of bearing 1 and bearing 2 (y_1 and y_2) and Figure 3.8 shows gear 1 and gear 2 (θ_1 and θ_2) angular displacement results for both numerical and computational. Both of them are exactly same and have a good correlation each other.

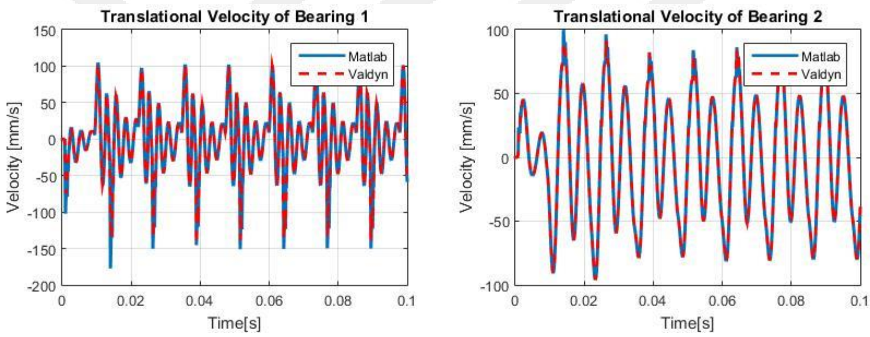

Figure 3.9 : Comparison of numerical and computational predictions for the translational velocities of Gear 1 (y_1') and Gear 2 (y_2')

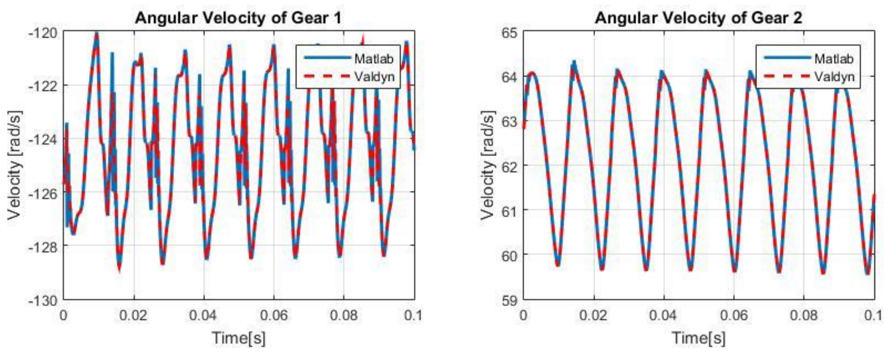

Figure 3.10 : Comparison of numerical and computational predictions for the angular velocities of Gear 1 (θ_1') and Gear 2 (θ_2')

Figure 3.9 and figure 3.10 show translational velocity of bearings (y_1, y_2) and angular velocity of gear 1 and gear 2 (θ_1, θ_2) and both results are good correlated just small differences in sharp peaks due to time step differences because both methods based on explicit Runge-Kutta iteration and time integrator step plays a crucial role here.

3.2 Six Degrees of Freedom Geared System With Backlash

In the previous chapter, four DoF geared system first solved analytically, numerically and computationally without including backlash clearance nonlinearity. Secondly, include the system backlash nonlinearity and solved with numerical and computational. Results are comparable and give a good level of confidence for all possible solution methods.

In this section, another gear added to the system to increase degrees of freedom and non-linearity. Because the focus of study is reducing rattle noise of a heavy-duty diesel engine's gear train system and it has 13 gears with four idler gears. Previous works on this subjects shows that a gear train with more gears – especially idler gears tend to have more gear noises [13]. Figure 3.11 shows a 6DoF system with 3 gears.

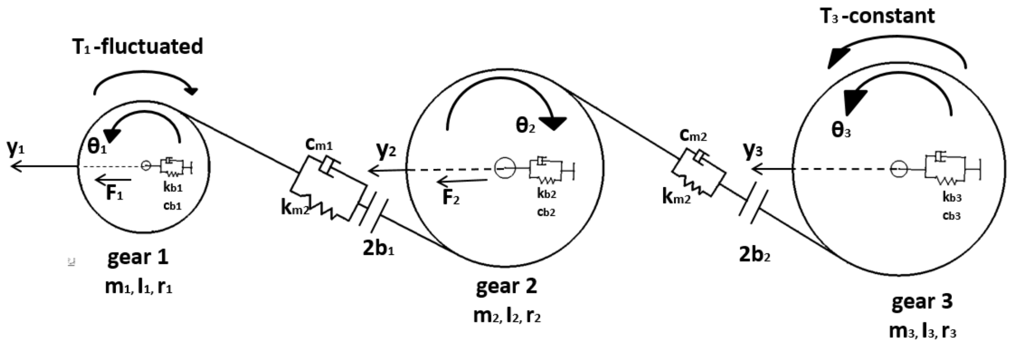

Figure 3.11 : Six DoF geared system with backlash

All the parameters that used during four DoF system that showed in table 2.1 remain same except T_2 because gear two is now an idler and it does not work under an external torque directly, it just transfers the torque from gear 3 to gear 1. However, in this system, there is T_3 and it again adjusted regarding the mean value of T_1 and inertial effect of entire system just like in 4 DoF because it is running gear 1 through gear 2. This 6 DoF system investigated because idler gears are most likely to be reason for high impact force since it is between two torque source. Also increasing DoF of system will show us a better view of phasing of fluctuated torque curve. Also in heavy-duty diesel engine timing gear system includes several idlers. Table 3.1 shows new parameters that are used for mesh two and gear 3.

Table 3.1 : Generic parameters of 6 DoF System

Column A	Column B	Column C
m_3	(kg)	3.133
I_3	(kg.m^2)	0.017880
r_3	(m)	0.077715
T_3	(Nm)	20.0894
z_3	-	45
k_3	(N/m)	5000000
c_3	(N.s/m)	500
k_{m2}	(N/m)	348766540
c_{m2}	(N.s/m)	4536.3086
b_2	µm	75

$$T_3 = mean(T_2) * 1.5 * 1.005667 \quad (3.1)$$

Where 1.5 is coming from gear reduction and 1.005667 is inertial effects and $mean(T_2)$ is mean value of fluctuated T_2 torque. Inertial effects have calculated through Valdyn model, with this value system has a steady state condition, and angular velocity of gears fluctuated around first initial velocities and not decreasing or increasing.

Since computational calculation gave reasonable results in the previous chapter, EOM of 6 degrees of freedom-geared system will not governed here. Because using computer software is much faster than obtaining EOM and solve them with manual codes. Below figure 2.12 is Valdyn model of six DoF system including 150-µm normal backlash (2b) for both mesh points. Figure 3.12 shows how computational model of six DOF systems looks in Valdyn GUI view.

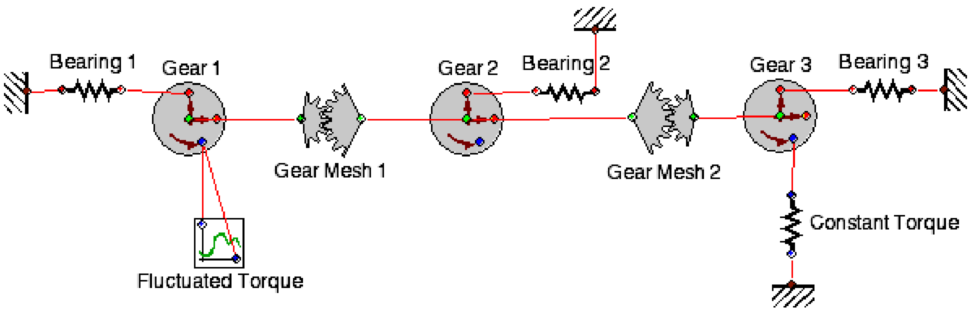

Figure 3.12 : 6 Degree of freedom Valdyn model

Again 600rpm (62.832 rad/s) initial velocity is applied to gear 3 this time and regarding this initial condition of gear 3, gear 2 will have 450rpm (41.888 rad/s) and gear 1 will have 900rpm (94.248 rad/s) initial angular velocity.

3.2.1 Results

Results will represent here are outputs of computational results of 6 DoF system between 0.1 and 0.2 seconds, first cycle which is 0 – 0.1 second didn't consider here because transient effect might be high.

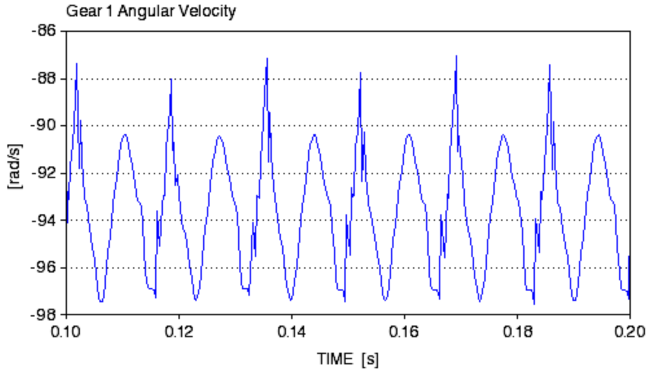

Figure 3.13 : Computationally predicted angular velocity of Gear 1 (θ_1')

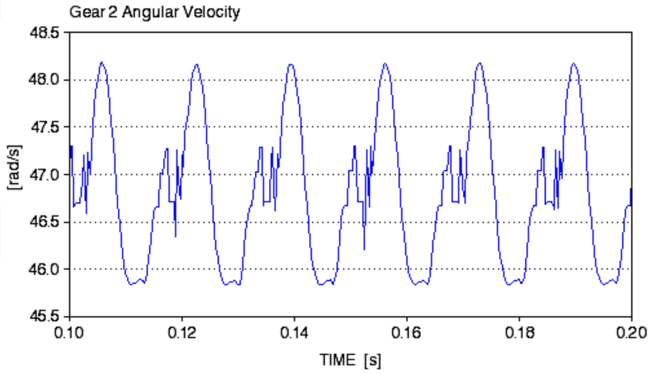

Figure 3.14 : Computationally predicted angular velocity of Gear 2 (θ_2')

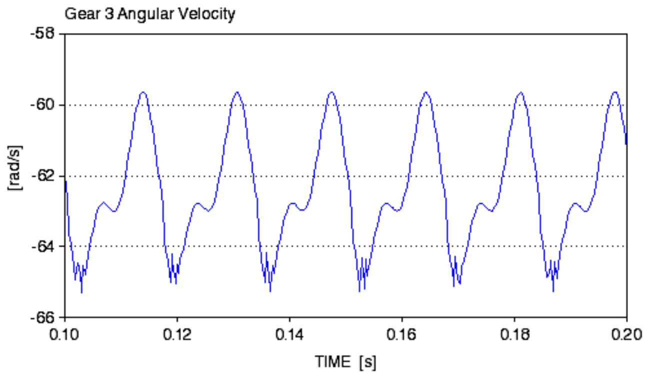

Figure 3.15 : Computationally predicted angular velocity of Gear 3 (θ_3')

Figure 3.13, 3.14 and 3.15 shows the angular fluctuation of gears around initial speeds and there is no decreasing or increasing in mean velocity that shows calculated and applied torques are correct and gives reasonable results. Bearings displacement or velocities are not present here this time because we are not going to compare them with any other calculation method.

3.2.2 Phasing effect on gear dynamic

In this chapter, fluctuated torque will be shifted and rest of the input parameters will remain same and effect of phasing will observe, especially in tooth mesh force because our main interest here reducing gear rattle noise and it is related with the mesh force. As it will explain in detail next chapter acoustical relevant contact time (ARCT) is an important short time duration and will be used to calculate impact impulse which is a metric that correlates with gear rattle also called gear hammering noise is coming from gear mesh force.

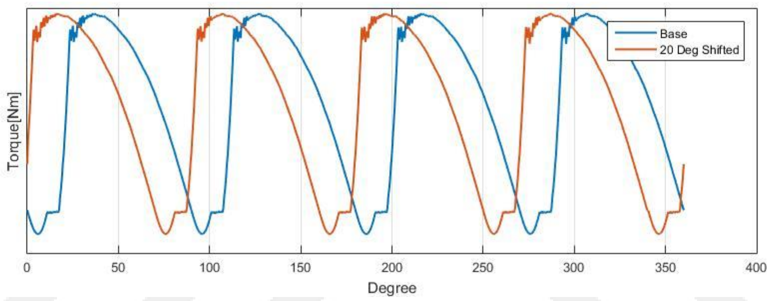

Figure 3.16 : Base and shifted torque curves (generic)

T_1 torque curve is shifted 20 degrees forward, figure 3.16 shows how it looks to compare the base curve when shifted 20 degrees. New shifted curve implemented to model and computational calculation of system under same initial conditions and assumptions applied; only differences in parameters is torque timing of T_1.The important point here is that to show how phasing will effect system response, in other words how a non-linear system will response to a highly fluctuated torque input's phasing. Figure 3.16 and 3.17 shows the effect of phasing, how it effects tangential force at mesh point, these forces will be using for impact impulse calculation in next chapter. As it will explain in detail, impact impulse is a metric that will use for evaluating gear rattle noise and it basis on gear mesh force, especially first engagement force which is mostly very high due to gear impact.

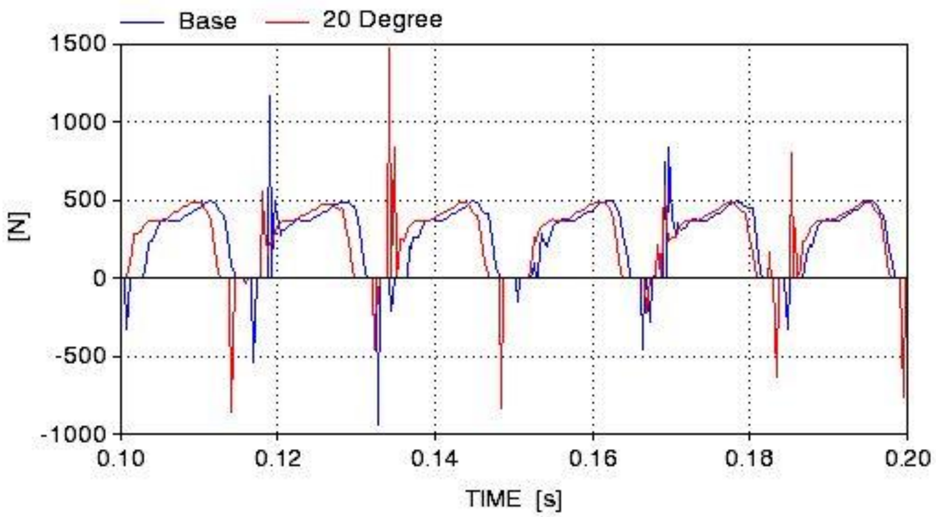

Figure 3.17 : Computationally predicted tangential gear mesh force at mesh 1 under fluctuating torque excitations of Figure 3.15

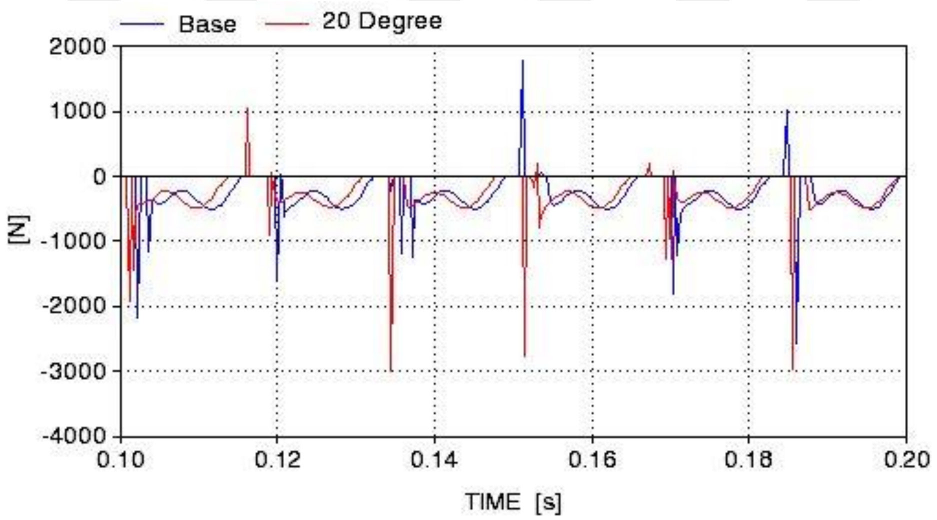

Figure 3.18 : Computationally predicted tangential gear mesh force at mesh 2 under fluctuating torque excitations of Figure 3.15

Figure 3.17 and 3.18 clearly shows phasing has numerous effect on the non-linear geared system. The general trend of blue and red curves are similar in both figures however sharp peaks magnitude are different and the most likely reason for impact noise (rattle) is these sharp peaks. In Figure 3.17 negative values shows double side contact, in figure 3.18 positive values shows double side impacts because gears rotates in different direction and forces are direction sensitive. Bearing forces and gear’s velocity also change significantly by applying external fluctuated torque with different phases, the important thing is to sweep all the possible phasing position and find the best point that entire system’s less sensitive operation point.

4. COMPUTATIONAL INVESTIGATION OF TIMING GEAR TRAIN SYSTEM

Timing drive system is a subsystem of internal combustion (IC) engines that mainly transmit torque from crankshaft to camshaft with a synchronization. Timing drive systems are always used to drive camshaft from crankshaft at half engine speed within an accuracy to allow valves open and close at desired times, in addition to camshaft drives, it also used to drive fuel injection pumps, oil, pumps and water pumps etc. Timing drive systems can be belt drive, chain drive or gear drive. Durability, NVH, system cost, package and friction play roles in deciding system type [14]. Heavy-duty diesel engines have geared system as timing drive system due to durability issues because loads and torque demands of this engine's valve train and auxiliary drives are very high. In this section as mentioned before Ford Otosan's 12.7-liter heavy-duty diesel engine timing gear train system will investigate in terms of rattle noise with respect to FIP phasing. This chapter will cover a computational calculation and psychical test.

4.1 Computational Calculation of Impact Impulse

Gear rattle noise also called gear impact or hammering noise is caused backward and forward relative motion between gear teeth and it is a broadband noise [15]. Crankshaft torsional vibration, oil level and oil viscosity, backlash of the gears, torque characteristic of accessory drives are key factors that affect the level of rattle noise [16].

Impact impulse is a metric that used to evaluate gear rattle noise and gear contact force at gear mesh point used as a basis to calculate it. Acoustical relevant contact time (ARCT) is a very short time period of two gears engagement and rattle noise happens at this time of period. In addition, there are limitations for calculation of impact impulse during ARCT. Firstly, smooth contacts (low contact velocity) give no contribution to impacts; secondly, low forces contribution also does not take into account of impact impulse calculation. Thus, only if force and velocity exceed given limits, impact impulse evaluated [17]. Figure 4.1a and 4.1b explain how impact impulse is calculated from a gear mesh force.

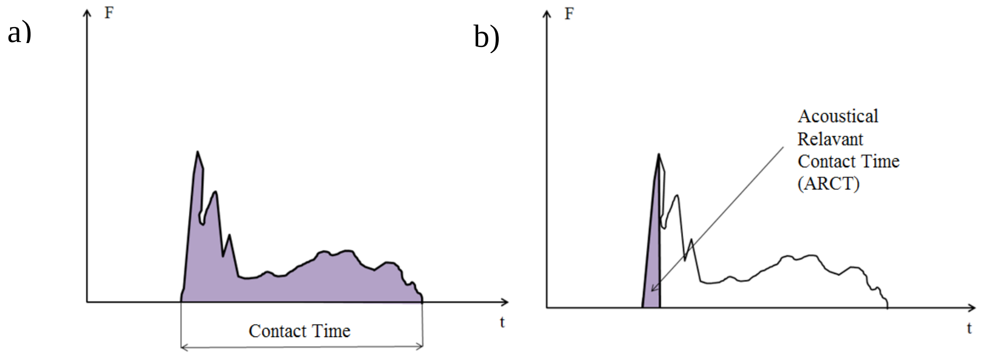

Figure 4.1 : a) Mesh force **b:** Acoustical relevant contact time

As explained before impact impulse calculated from mesh force with respect to ARCT. Equation (4.1) shows the calculation of impact impulse according to figure 4.1b.

$$I = \int_{ARCT} F dt \tag{4.1}$$

Impact impulse can calculate in Multi-Body Dynamic (MBD) computer software. AVL Excite Timing Drive (TD) is one of them and impact impulse is one of its direct output so no need for further calculation after mesh force obtained.

4.2 Timing Gear Train of Heavy-Duty Diesel Engine

Nowadays NVH is coming more and more important for automotive customers and engineer faces more challenges with the demand of high-quality products. Gear train system in heavy-duty diesel engines is one of the main mechanical noise sources. Mainly two different type of gear noise produce by gear train; gear rattle and gear whine. MBD model of Ford Otosan 12.7 liter diesel engine gear train built in AVL Excite TD to evaluate gear rattle with impact impulse and try to find the best performance of it by phasing fluctuated torque curve of FIP. Figure 4.2 shows three-dimensional (3D) view of gear train system. There is another plain below the crank gear but it does not consider in computational calculation because main excitation is coming from crank gear and this third plain does not have any effect to rest of the system that has gears with fluctuated torque characteristics.

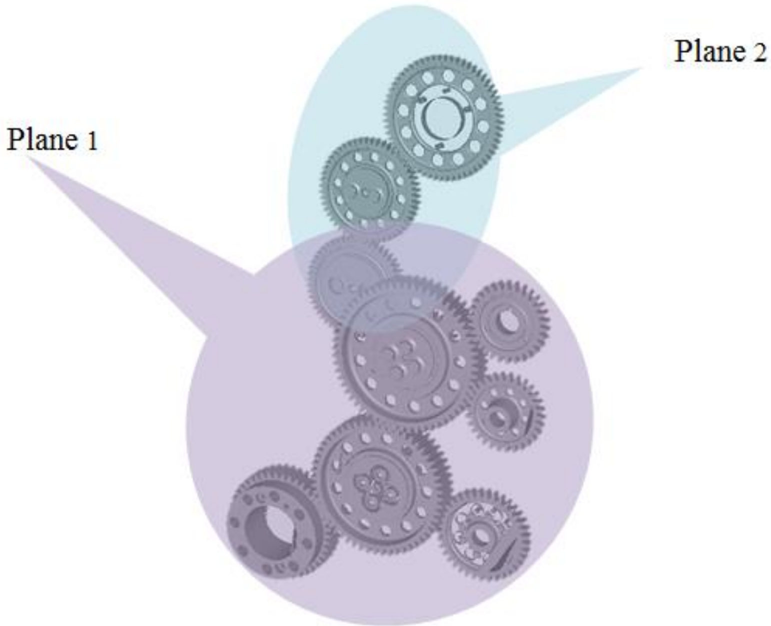

Figure 4.2 : Examined Gear train of heavy duty diesel engine

The system has ten gears in two plains. The gear at the bottom is crank gear and excitation is coming from this gear to entire system. In addition to plains in figure 4.2, there is another plain below the crank gear which is running oil pump however it didn't model in MBD model because crank gear excites the entire system and oil pump doesn't influence systems that include FIP and camshaft.

4.2.1 Gear train MBD model

Simulations of gear train dynamics performed with AVL Excite Timing Drive (TD). Valve and gear train are reduced to a lumped mass system with equivalent dynamic characteristics, which explained in detail during 4 DoF system modeling, however, this time model covers all the parameters of gears such as hydrodynamic radial bearings, pressure line description, gear face width etc. Equations of motion for the equivalent system solved with predictor/corrector method combined with automatic control of integration step width.

In calculations, each gear considered as a mass point, where mass and moment of inertia are located and mass points are connected by spring/damper units describing stiffness and damping of gear meshes. Gear mesh and stiffness that used for the dynamic model calculated with another gear design software called Kissoft and mesh stiffness calculates according to Weber/Banaschek in Kissoft. The method developed by Weber and Banaschek calculates the deformation of a spur gear according to applied load [18]. In gear meshes, backlash between gears and damping within backlash are considered. Damping within backlash depends on relative distance between tooth flanks of mating gears. Damping values are empirical and they are within the software limits. Masses and moments of inertia obtained from computer-aided design (CAD) data. Mass and moment of inertia of auxiliaries added to gear element. Also, each auxiliaries torque data implemented to this inertia points included FIP, same as 4 DoF or 6 DoF systems that showed in previous chapters. Gears supported by one radial and one thrust bearing. Bearings calculated as hydrodynamic and backlashes and bearing clearances considered in the model. Gear train MBD model is showed in figure 4.3

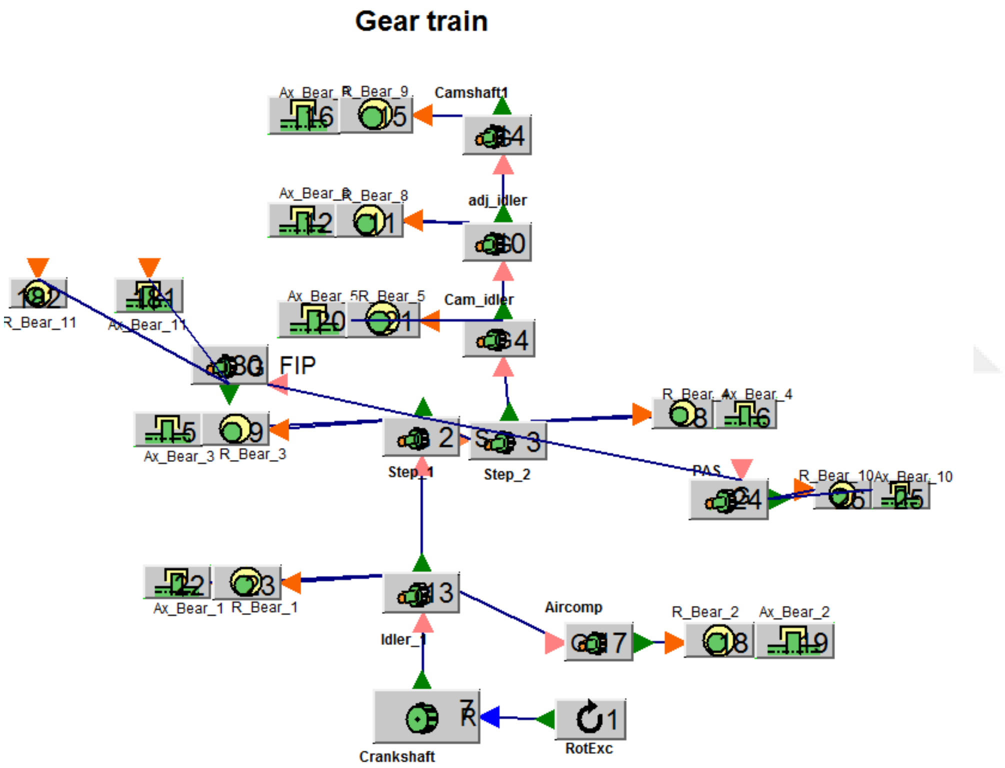

Figure 4.3 : AVL Excite TD Gear train model

One of the fluctuated torque sources of the system is valve train that runs by camshaft gear that can be seen in figure 4.2. Valve train has a significant effect on the system thus needed to model correctly so that detailed model was constructed and implemented to model. A separated valve train model built in Excite TD and results are reasonable and expected such an engine system so valve train system engaged to the system through camshaft gear. The same methodology applied to valve train and system reduced to a lumped mass system with equivalent dynamic characteristics.

Each element replaced by a mass linked to the previous element by a spring and a damper. Furthermore, external forces and friction taken into account at certain elements, like gas forces on valve faces or friction on valve stems. Valve train model is showed in figure 4.4. The engine has a single camshaft that opens and closes both exhaust and intake valves.

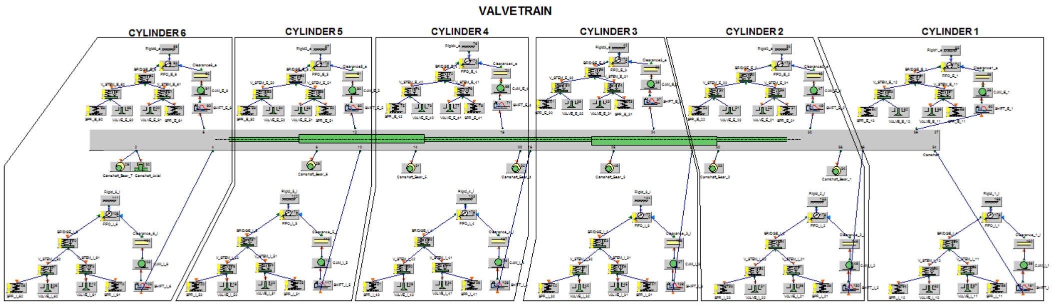

Figure 4.4 : AVL Excite TD valve train model

Another crucial input is crank torsional vibration. It has a direct effect on gear train dynamics as well as one of the man source for gear rattle noise. Torsional vibration is speed / angular displacement fluctuations of rotating engine components around engine nominal speed due to internal combustion engine's nature and become very critical with current trends in powertrain design; downsizing and boosting. The primary source of engine torsional vibration is engine crank train. Measurement results or prediction results could use in Excite TD as speed fluctuation. Since it needs special equipment and time to measure it from experimental set up, therefore, another software called AVL Excite Designer used to predict torsional vibration. A pure torsional model of crank train system has built-in Designer including flywheel, vibration damper, piston and connecting rod mechanical, geometrical details and cylinder pressure used to as an input to excite system. Angular speed fluctuation around nominal crankshaft speed for each speed calculated with help of designer and implemented to gear train model. Figure 4.5 shows the crank train model of engine in Excite Designer GUI.

Torsional vibration is speed / angular displacement fluctuations of rotating engine components around engine nominal speed due to internal combustion engine's nature and become very critical with current trends in powertrain design; downsizing and boosting. Primary source of engine torsional vibration is engine crank train.

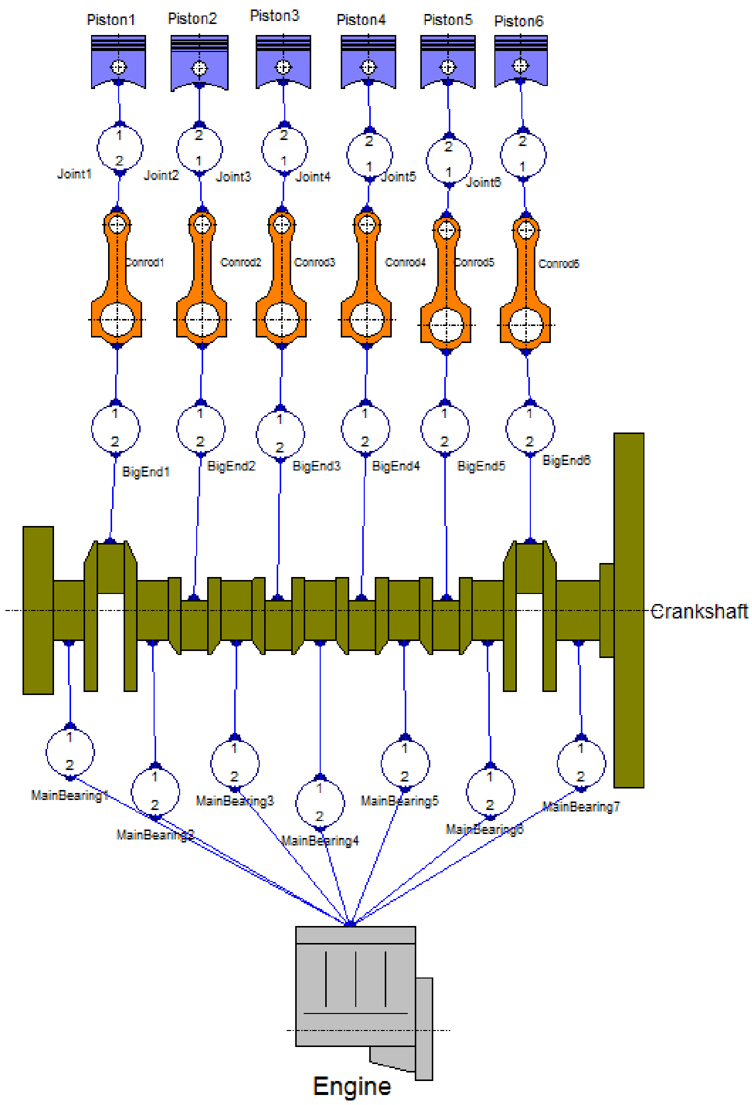

Figure 4.5 : Excite Designer model for Crank train

4.3 Computational Results

Torque data of FIP is provided by its supplier from measurement and implemented to software model. FIP torque data that used for a single constant engine speed can be seen in figure 4.6 and data represents torque demand of FIP for one revolution at this engine speed. For such an engine torque values of FIP can go up to 200-Nm and then drops to minus 20-40 Nm since common rail pressure can go over 2000 bar. Although figure 4.6 shows data for only single speed, the analysis performed from 600-rpm to 2000 rpm with 100-rpm steps for full load condition of engine and for each speed different torque curves provided because pressure in FIP could vary due to engine speed.

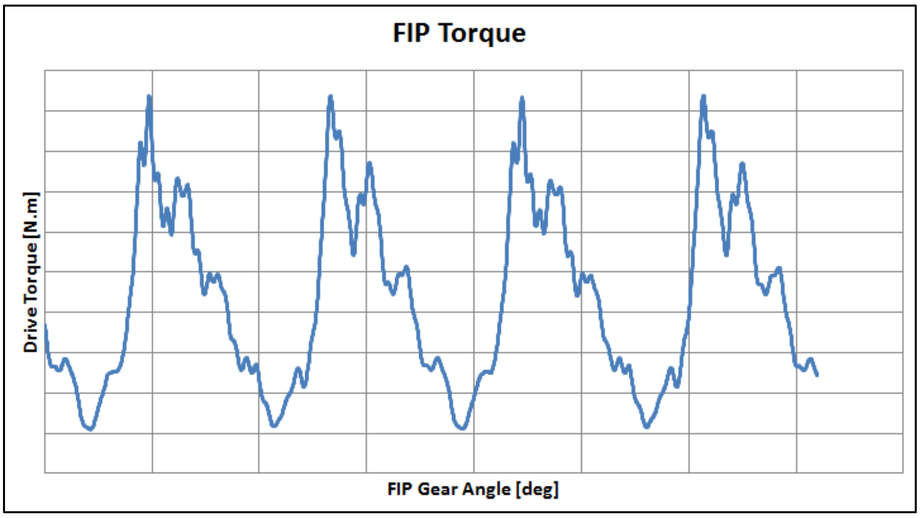

Figure 4.6 : FIP torque for a constant speed

As it can be seen from figure 4.6 data repeat itself every 90° so that it doesn't need to sweep all possible shifting, up to 90° will be sufficient. Gear that runs FIP has 30 tooth and each tooth shifting equal to 12° forwarding data showed in figure 4.6. Shifting done according to one tooth step that is 12° because it is easy to adjust it during physical test. From base to 96° degree with 12°steps total nine different shifting point has been run through MBD model and try to find less sensitive point of gear train for rattle assessment. In each step, only FIP torque shifted and rest of the inputs remain same and impact impulse calculated for every gear mesh and total impact impulse of system determined by summing up individual impact impulse value of gear meshes. In figure 4.7, the total impact impulse results are presented for all possible shifted points.

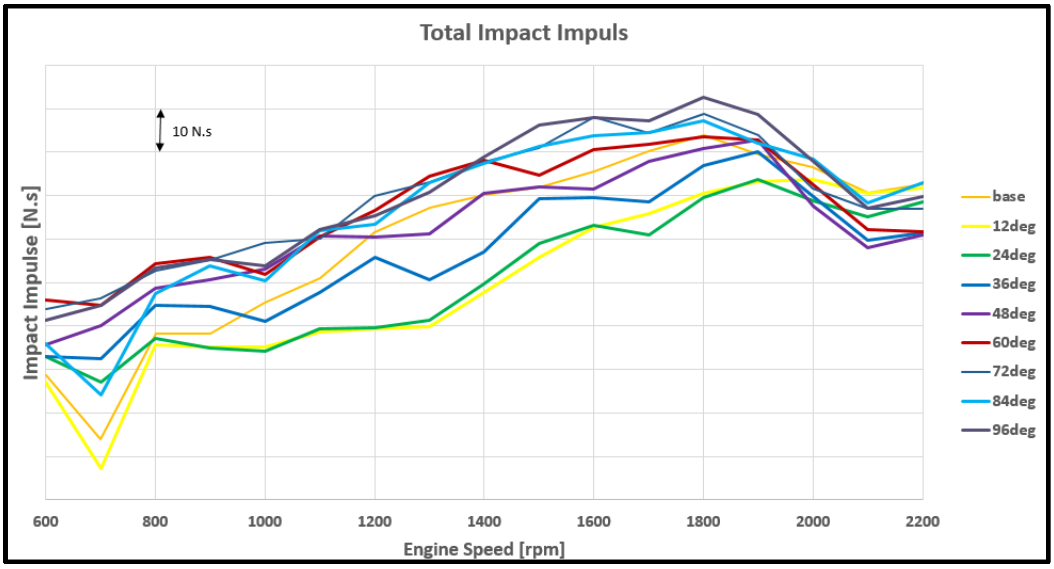

Figure 4.7 : Total Impact impulse for all iterations

Figure 4.7 clearly shows 12° and 24° are best cases and 96°, 84° and 72° are worst cases when compare the total impact impulse.

4.4 Measurement Results

Examined diesel engine without transmission unit was tested in a semi-anechoic chamber and data are collected with acceleration and acoustic transducers. Figure 4.8 shows semi-anechoic chamber .The test is carried out on the engine for full load condition and for nine iterations same as computational calculation. Test carried out from 800rpm to 2100rpm with slow sweep speed to eliminate effect of transient events. The measurement procedure that describes in ISO 3745 is applied [19]. According to ISO 3745, four microphones used for each side of engine; left side, right side, front side and top side to calculate the overall engine noise. These microphones are located one meter away from outer component of engine and surface of engine should be located in center of microphones. ISO 3745 describes how average sound pressure level can be calculated.

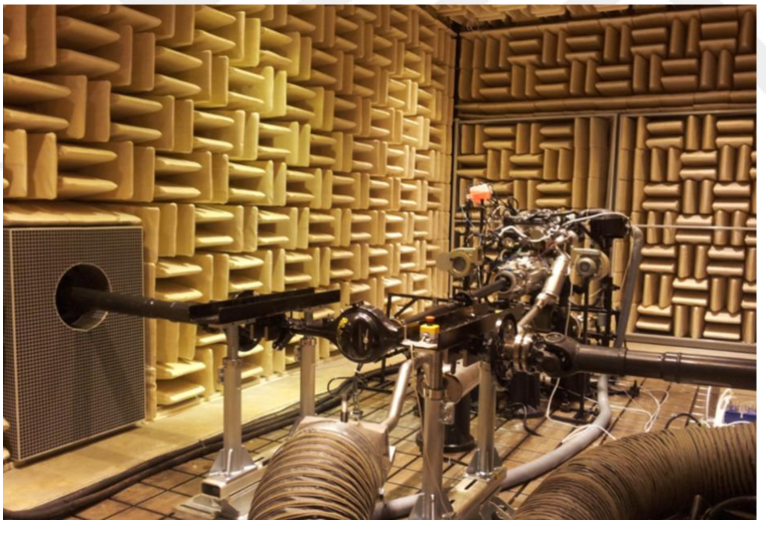

Figure 4.8 : Ford Otosan semi-anechoic test room[20]

In addition to measurement standard another microphone is located at the rear of engine to see direct effect of gear train where located at the rear side of engine. In addition, accelerometers are located at the gear train housing and flywheel housing to see effect of gear impact at these points. Engine torque measured as well, for all iterations to see if FIP phasing has any effect on engine torque. Figure 4.10 and show location of accelerometers in these locations as a zoomed picture and figure 4.11 red points show flywheel top and gear train right acceleration locations on engine.

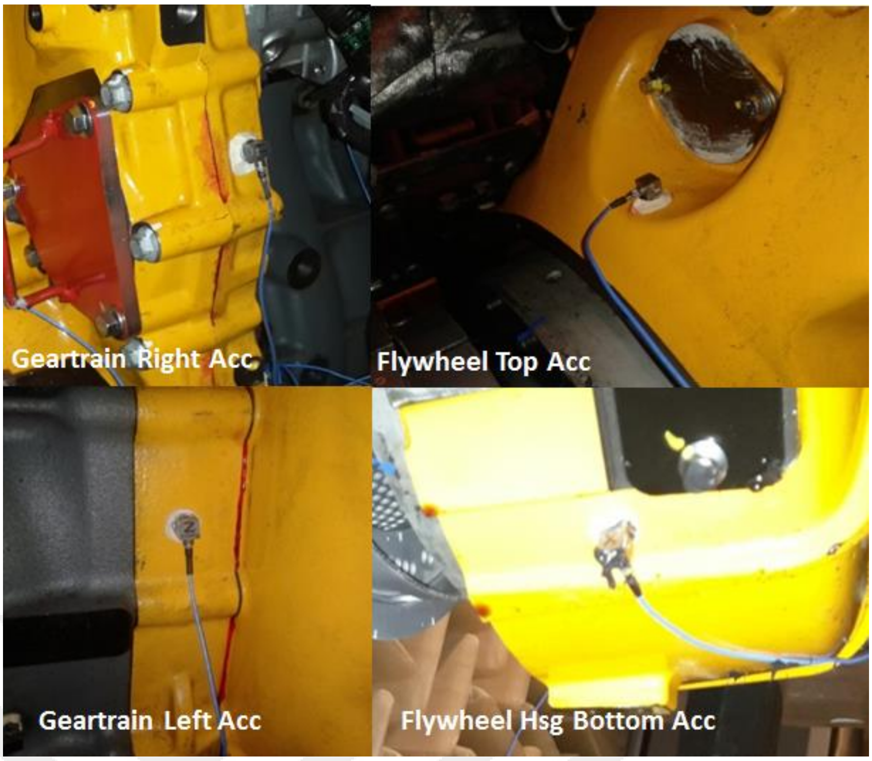

Figure 4.9 : Accelerometer locations [15]

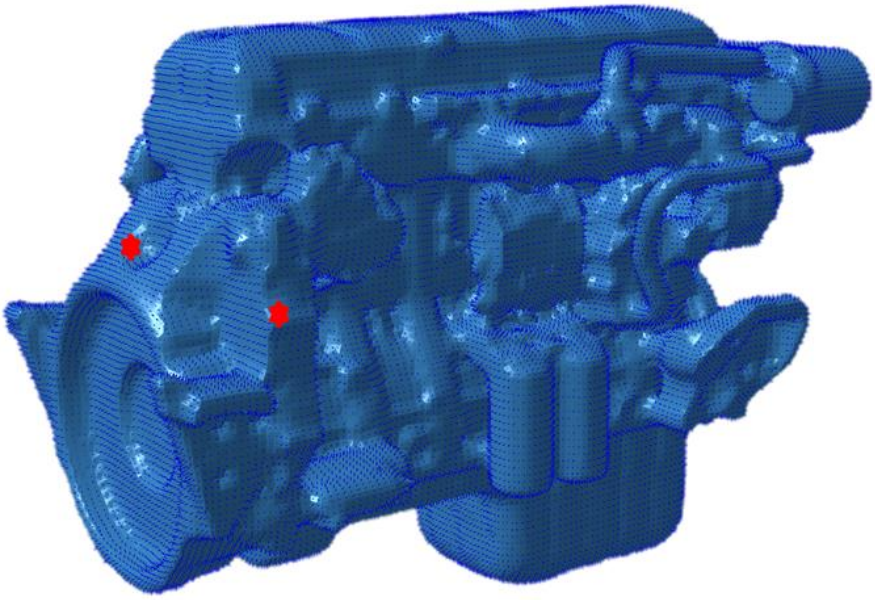

Figure 4.10 : Flywheel top and gear train housing right accelerometer locations

During the experiment, nine different phase angles from baseline to 96 degrees are tested. Each iteration assembly position of FIP gear phased with respect to crankshaft gear in order to applying same differences that have done during computational calculation and observing FIP phase effect.

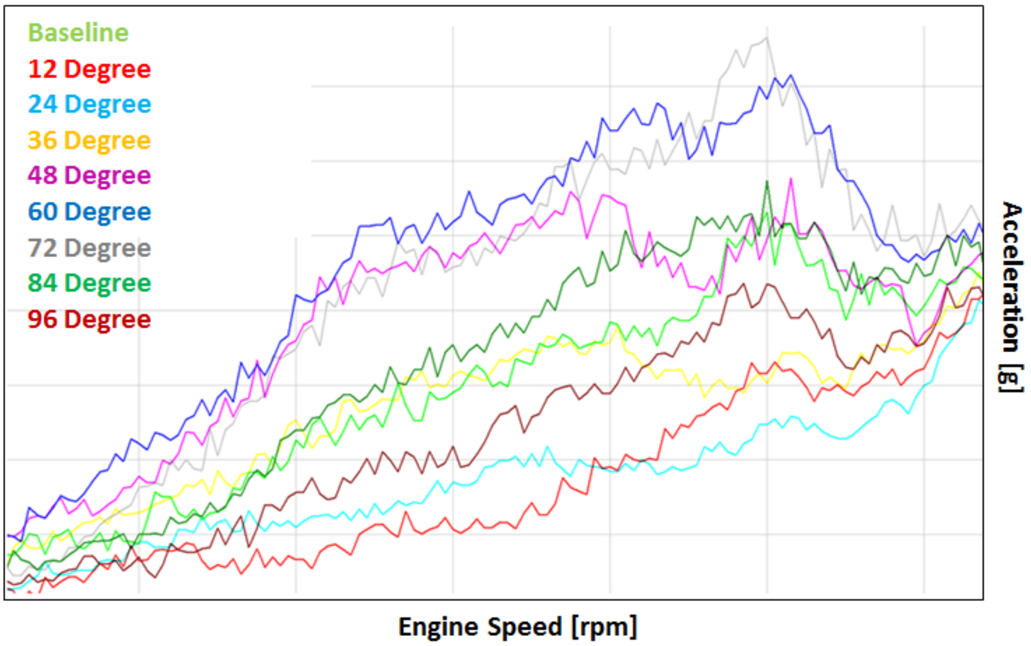

Figure 4.11 : Gear train housing right acceleration results

Figure 4.12 shows acceleration results that measured from gear train right accelerometer. In figure 4.12 indicates 12° and 24° are better than rest of the measurement points as expected from computational results.

Average sound pressure level of four microphones can be seen in figure 4.13 for all phasing points. Figure 4.14 shows rear microphone results, this microphone is the closest one to gear train and gives best opportunity to evaluate phasing effect.

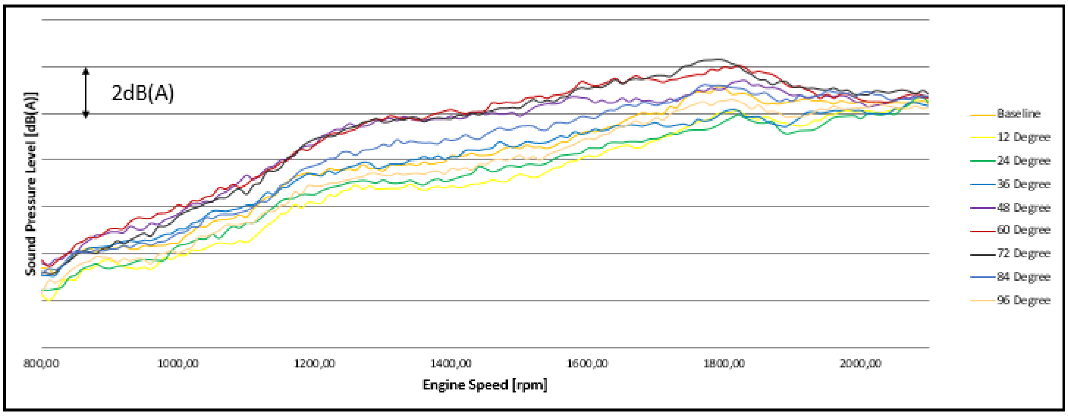

Figure 4.12 : Average sound pressure level of four microphones [15]

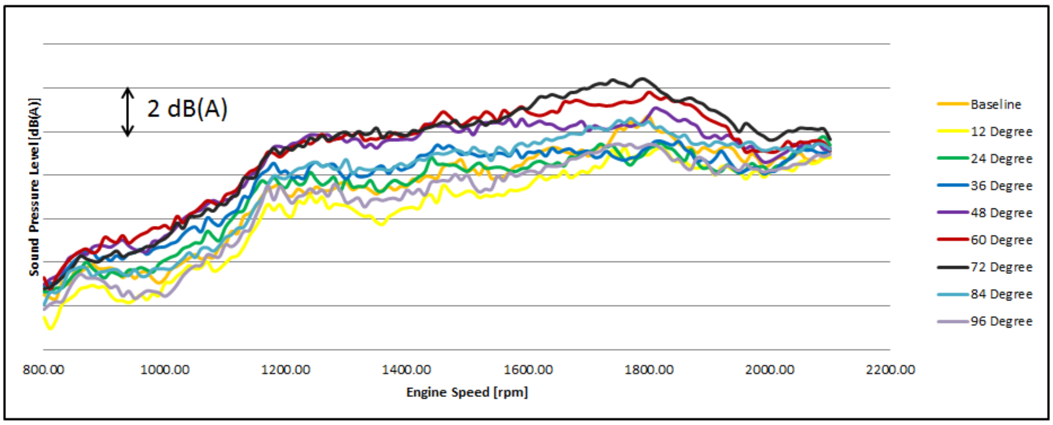

Figure 4.13 : Rear microphone [15]

From figure 4.13 and figure 4.14 show that 12° and 24° gives best results, and 60° and 72° are the worst cases. There are over 4 dB (A) differences between the best and worst cases. The results show fluctuated torque phasing is a very effective way to have a better NVH characteristic for a heavy-duty diesel engine. Because, phasing brings no any additional cost or design changes and almost no effect on other systems of engine.

5. CONCLUSION

Gear train dynamic investigated under fluctuated torque for different type of complexity. Firstly, simple linear gear pair investigated and then extended to non-linear six DoF freedom system. Analytical, numerical and computational solution applied to these different systems to see their dynamic behavior under fluctuated torque excitation. Matlab codes are created to solve linear and non-linear geared system and a MBD software program is used to build same system with same parameters. All different solution methods show a good correlation in each other for same systems. After having a good level of confidence about solutions, study extended to six DoF to calculate gear mesh force. It showed that mesh force between gears are sensitive to phasing of this fluctuated torque due to its non-linear nature. Although general response trend of mesh force remain same, the peak forces are quite different and it showed in literature that these peak forces between gears are main reason for gear rattle noise.

In this study, a metric called impact impulse represented. This metric was calculated from gear mesh forces and correlate with gear rattle noise. It showed that if a system has higher total impact impulse this system would also have higher gear rattle noise.

A detailed gear train and valvetrain system of a heavy-duty diesel engine was built in a commercial MBD software to calculate total impact impulse of system. FIP is one of the component of this system and has highly fluctuated torque characteristics. Know-how from single gear pair system is applied to this complex system and FIP torque curve is shifted with a logical way to reduce impact impulse so do gear rattle.

An experimental study has conducted for examining the same engine in a semi-anechoic chamber. Computational calculations and experimental investigation shows same trend with respect to FIP phasing and showed phasing angle of 12°, 24° are best cases and 72° is worst case. Also, both investigations showed FIP has a strong effect on gear rattle of system. In test results the differences between best and worst case is over 4 dB(A).

Both experimental and computational results show fuel injection pump phasing can have a significant effect on gear rattle noise of a geared timing drive system as well as the overall heavy-duty engine noise level.

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APPENDICES

APPENDIX A

Matlab codes for analytical solution of three degrees of freedom linear system (No backlash).

```
clear
clc
close all
tic

m1 = 0.75;               % Mass of gear #1
m2 = 3.5;                % Mass of gear #2
I1 = 0.0009;             % Inertia of gear #1
I2 = 0.022;              % Inertia of gear #2
r1 = 0.05181;            % Radius of gear #1
r2 = 0.10362;            % Radius of gear #2
mc = (I1*I2)/((I1*r2*r2)+(I2*r1*r1));% Effective mass

k1 = 5e6;               % Stiffness for bearing #1
k2 = 5e6;               % Stiffness for bearing #2
km = 124091320;         % Mesh stiffness

c1 = 500;               % Damping for bearing #1
c2 = 500;               % Damping for bearing #2
cm = 1485;              % Mesh damping

deltam0 = -(125.664*r1)+(62.831*r2)

M = [m1 0 0;0 m2 0;-mc mc mc];
C = [c1 0 cm;0 c2 -cm;0 0 cm];
K = [k1 0 km;0 k2 -km;0 0 km];

A = [zeros(3) M;M C];
B = [-M zeros(3);zeros(3) K];
lambda = eig(-B,A);
[vec,lambda] = eig(-inv(A)*B);
RES = [];
IC = [0;0;deltam0;0;0;0];
COEFS = (vec*diag(exp(diag(lambda)*0)))\IC;
for t = 0:0.0001:.1
    RES = [RES vec*diag(exp(diag(lambda)*t))*COEFS];
e
```

APPENDIX B

Matlab codes for numerical solution of four degrees of freedom linear system (No Backlash)

There are two Matlab files and these two files need to be same folder to be able to run properly.

Script#1: main_GearPair_FlexBrgs_NoBacklash

```
clear
clc
close all

data = load('input_fluctuated_torque');
p.theta1 = data(:,1)*(pi/180);
p.torque1 = data(:,2);

p.b = 0;                % Backlash

m1 = 0.75;              % Mass of gear #1
m2 = 3.5;               % Mass of gear #2
I1 = 0.0009;            % Inertia of gear #1
I2 = 0.022;             % Inertia of gear #2
r1 = 0.05181;           % Radius of gear #1
r2 = 0.10362;           % Radius of gear #2
mc = (I1*I2)/((I1*r2*r2)+(I2*r1*r1));% Effective mass

k1 = 5e6;              % Stiffness for bearing #1
k2 = 5e6;              % Stiffness for bearing #2
km = 124091320;        % Mesh stiffness

c1 = 500;              % Damping for bearing #1
c2 = 500;              % Damping for bearing #2
cm = 1485;             % Mesh damping

p.T2 = 13.3175*2.0065;    % External torque acting on gear #2

tspan = [0 .1];

X = [0 0 0 -125.664 0 0 0 62.831];
options = odeset('RelTol',1e-6,'AbsTol',1e-6*ones(1,8),'MaxStep',1e-4);
[T4dof,X4dof] =
ode45(@ode_GearPair_FlexBrgs_NoBacklash,tspan,X,options,p);
```

Script#2: ode_GearPair_FlexBrgs_NoBacklash

```
function dx = ode_GearPair_FlexBrgs_NoBacklash(t,x,p)

p.T1 = interp1(p.theta1,p.torque1,mod(x(3),(2*pi)),'spline');
p.Fe = ((p.T1*p.r1*p.mc)/p.I1)+((p.T2*p.r2*p.mc)/p.I2);     %
Effective force
dx = zeros(8,1);
dx(1) = x(2);
dx(2) = (-(p.c1*x(2))-(p.cm*(x(2)-x(6)+(p.r1*x(4))+(p.r2*x(8))))...
         -(p.k1*x(1))-(p.km*(x(1)-
x(5)+(p.r1*x(3))+(p.r2*x(7)))))/p.m1;
```

```
dx(3) = x(4);
dx(4) = (p.T1-(p.cm*p.r1*(x(2)-x(6)+(p.r1*x(4))+(p.r2*x(8))))...
            -(p.km*p.r1*(x(1)-x(5)+(p.r1*x(3))+(p.r2*x(7)))))/p.I1;
dx(5) = x(6);
dx(6) = (-(p.c2*x(6))+(p.cm*(x(2)-x(6)+(p.r1*x(4))+(p.r2*x(8))))...
        -(p.k2*x(5))+(p.km*(x(1)-
x(5)+(p.r1*x(3))+(p.r2*x(7)))))/p.m2;
dx(7) = x(8);
dx(8) = (p.T2-(p.cm*p.r2*(x(2)-x(6)+(p.r1*x(4))+(p.r2*x(8))))...
            -(p.km*p.r2*(x(1)-x(5)+(p.r1*x(3))+(p.r2*x(7)))))/p.I2;
```

APPENDIX C

Matlab codes for nonlinear four degree of freedom system (with backlash)

This appendix has seven different scripts and all of them has to be in same folder to be able to run.

Script#1: main_GearPairBrgContactLoss

```
clear
clc
close all

data = load('fluctuated_input_torque');
p.theta1 = data(:,1)*(pi/180);
p.torque1 = data(:,2);

p.b = 75e-6;            % Backlash

p.m1 = 0.7;             % Mass of gear #1
p.m2 = 3.5;             % Mass of gear #2
p.I1 = 0.0009;          % Inertia of gear #1
p.I2 = 0.022;           % Inertia of gear #2
p.r1 = 0.05181;         % Radius of gear #1
p.r2 = 0.10362;         % Radius of gear #2

p.k1 = 5e6;             % Stiffness for bearing #1
p.k2 = 5e6;             % Stiffness for bearing #2
p.km = 124091320;       % Mesh stiffness

p.c1 = 500;             % Damping for bearing #1
p.c2 = 500;             % Damping for bearing #2
p.cm = 1485;            % Mesh damping

% p.T1 = 13.3175;
p.T2 = 13.3175*2.003;   % External torque acting on gear #2

tspan = [0 .1];

y0 = [0 0 0 -125.664 0 0 0 62.831];     %y1,theta1,y2,theta2


tfinal = tspan(1);
yfinal = y0;
if y0(1)-y0(5)+(p.r1*y0(3))+(p.r2*y0(7))>p.b
```

```
    event_no = 1;                % Case1
elseif abs(y0(1)-y0(5)+(p.r1*y0(3))+(p.r2*y0(7)))<=p.b
    event_no = 2;                % Case2
elseif y0(1)-y0(5)+(p.r1*y0(3))+(p.r2*y0(7))<-p.b
    event_no = 3;                % Case3
end
tf = tspan(1);
tol = 1e-9;
while tf < tspan(2);
    if event_no == 1;
        options =
odeset('RelTol',tol,'AbsTol',tol*ones(8,1),'Events',@event_gear_cont
act_case1);
        [t,y] =
ode45(@ode_GearPairBrgContactCase1,tspan,y0,options,p);
        tf = t(end);
        yfinal = [yfinal;y((2:end),:)];
        tfinal = [tfinal;t((2:end),:)];
        event_no = 2
%         tfinal(end)
    elseif event_no == 2;
        options =
odeset('RelTol',tol,'AbsTol',tol*ones(8,1),'Events',@event_gear_cont
act_case2);
        [t,y,te,ye,ie] =
ode45(@ode_GearPairBrgContactCase2,tspan,y0,options,p);
        tf = t(end);
        yfinal = [yfinal;y((2:end),:)];
        tfinal = [tfinal;t((2:end),:)];
        if ie == 1;
            event_no = 1
%             tfinal(end)
        elseif ie == 2;
            event_no = 3
        end
    elseif event_no == 3;
        options =
odeset('RelTol',tol,'AbsTol',tol*ones(8,1),'Events',@event_gear_cont
act_case3);
        [t,y] =
ode45(@ode_GearPairBrgContactCase3,tspan,y0,options,p);
        tf = t(end);
        yfinal = [yfinal;y((2:end),:)];
        tfinal = [tfinal;t((2:end),:)];
        event_no = 2
    end
    if tf < tspan(2);
        tspan = [tf tspan(2)];
        y0 = yfinal(end,:);
    else
        break
    end
end
```

Script#2: ode_GearPairBrgContactCase1

```
function dy = ode_GearPairBrgContactCase1(t,y,p)
p.T1 = interp1(p.theta1,p.torque1,mod(y(3),(2*pi)),'spline');
dy = zeros(8,1);
```

```
dy(1) = y(2);
dy(2) = (-(p.c1*y(2))-(p.cm*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8)))))...
         -(p.k1*y(1))-(p.km*(y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-
p.b)))/p.m1;
dy(3) = y(4);
dy(4) = (p.T1-(p.cm*p.r1*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8))))...
             -(p.km*p.r1*(y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-
p.b)))/p.I1;
dy(5) = y(6);
dy(6) = (-(p.c2*y(6))+(p.cm*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8)))))...
         -(p.k2*y(5))+(p.km*(y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-
p.b)))/p.m2;
dy(7) = y(8);
dy(8) = (p.T2-(p.cm*p.r2*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8))))...
             -(p.km*p.r2*(y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-
p.b)))/p.I2;
```

Script#3: ode_GearPairBrgContactCase2

```
function dy = ode_GearPairBrgContactCase2(t,y,p)

 p.T1 = interp1(p.theta1,p.torque1,mod(y(3),(2*pi)),'spline');
dy = zeros(8,1);
dy(1) = y(2);
dy(2) = (-(p.k1*y(1))-(p.c1*y(2)))/p.m1;
dy(3) = y(4);
dy(4) = p.T1/p.I1;
dy(5) = y(6);
dy(6) = (-(p.k2*y(5))-(p.c2*y(6)))/p.m2;
dy(7) = y(8);
dy(8) = p.T2/p.I2;
```

Script#4: ode_GearPairBrgContactCase1

```
function dy = ode_GearPairBrgContactCase3(t,y,p)
p.T1 = interp1(p.theta1,p.torque1,mod(y(3),(2*pi)),'spline');
dy = zeros(8,1);
dy(1) = y(2);
dy(2) = (-(p.c1*y(2))-(p.cm*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8)))))...
         -(p.k1*y(1))-(p.km*(y(1)-
y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b)))/p.m1;
dy(3) = y(4);
dy(4) = (p.T1-(p.cm*p.r1*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8))))...
             -(p.km*p.r1*(y(1)-
y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b)))/p.I1;
dy(5) = y(6);
dy(6) = (-(p.c2*y(6))+(p.cm*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8)))))...
         -(p.k2*y(5))+(p.km*(y(1)-
y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b)))/p.m2;
dy(7) = y(8);
dy(8) = (p.T2-(p.cm*p.r2*(y(2)-y(6)+(p.r1*y(4))+(p.r2*y(8))))...
             -(p.km*p.r2*(y(1)-
y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b)))/p.I2;
```

Script#5: event_gear_contact_case1

```
function  [value,isterminal,direction] =
event_gear_contact_case1(t,y,p)
value = y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-p.b;
isterminal = 1;
```

```
direction = -1;
```

Script#6: event_gear_contact_case2

```
function  [value,isterminal,direction] =
event_gear_contact_case2(t,y,p)
value = [y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))-p.b y(1)-
y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b];
isterminal = [1 1];
direction = [1 -1];
```

Script#7: event_gear_contact_case3

```
function  [value,isterminal,direction] =
event_gear_contact_case3(t,y,p)
value = y(1)-y(5)+(p.r1*y(3))+(p.r2*y(7))+p.b;
isterminal = 1;
direction = 1;
```

CURRICULUM VITAE

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