Optimisation of cut-off grades for multi-mineral deposits

by

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others

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ABSTRACT

Determination of optimum cut-off grades is a fundamental issue in mineral extraction as it assigns the boundaries between ore and waste over time. As a result, a number of algorithms have been developed to find optimal sequences of cut-off grades. Over the past 30 years the methods have moved from static approaches to dynamic approaches. Although the policy used for cut-off grade determination varies depending on the objectives of the enterprise, the general approach is to extract ore at the highest discounted profit over the life of the mine.

A general approach for optimisation of cut-off grades for multi-mineral deposits is to use parametric cut-off grades. There are, however, problems associated with the use of parametric cut-off grades. For a given parametric cut-off grade there is no unique corresponding grade for each of the minerals/metals that comprise the parametric variable. For the same reason, individually optimising the cut-off grades of the component minerals/metals will not necessarily yield the same answer as optimising the parametric cut-off grades.

There have been some attempts at determining optimal multi-mineral cut-off grades on the basis of the individual constituent minerals/metals. The grid search method developed by Lane, for example, has been applied to two-mineral deposits.

Three different optimisation methods have been used in this research to determine optimum cut-off grades for mineral deposits containing up to three valuable minerals/metals. Lane's grid search method has been extended and adapted for cases of up to three valuable minerals/metals.

Another method that has proved successful in determining optimum cut-off grades and production rates is the dynamic programming method developed by Dowd. Dowd and Xu have extended the method to the optimisation of multi-mineral deposits by means of parametric cut-off grades. In the research described in this thesis, the dynamic programming method has been extended to include multi-mineral deposits by dealing with the minerals separately.

The third method described in this research is based on genetic algorithms. This is the first known application of genetic algorithms to the determination of optimum cut-off grades and the success of the approach is encouraging for wider development and application in this field.

Three computer programs were written to implement the three methods for up to three valuable minerals/metals. The algorithms and computer programs have been applied to three case studies to assess and validate the methods.

For the sake of brevity, in the remainder of this thesis, the valuable constituents of the mined product will be referred to as minerals and this term will be taken to include metals and any other relevant constituent whether or not it satisfies the normal definition of a mineral. For the same reason, the mined product will usually be termed 'ore'.

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CHAPTER 1

INTRODUCTION

1.1 Scope of work

The primary purpose of this research is to develop new methods to optimise sequences of cut-off grades for mineral deposits containing two or more valuable minerals and to generate basic mine production schedules. A complete mine production schedule, including mining and other access constraints, is beyond the scope of this work, although methods for the inclusion of rudimentary access constraints are described. The aim here is to provide indications of optimum cut-off grade policies for a multi-mineral deposit for which all blocks that comprise the orebody model, or a period-limited section of it, are assumed to be immediately accessible. The optimisation criterion used throughout this work is maximum net present value. The scheduling process gives optimum cut-off grades and production rates for multi-mineral deposits and thereby determines the optimal mine life.

All mineralised material with a grade higher than the optimum cut-off grade for a given year would be considered to be ore for that year and be sent to the mineral processing plant. Stockpiling is not considered.

The three methods used in this research (grid search method, dynamic programming and genetic algorithms) require slightly different data and do not necessarily generate the same results.

1.2 General background and literature review

In its broadest definition, a cut-off grade is any grade that is used to separate two courses of action; e.g., to mine or not to mine, to process or to dump, to process or to stockpile. In its most widely used senses, a cut-off grade is a grade that is used to classify mined, or minable, products as ore or waste or to define a production schedule over some period of time. In this thesis cut-off grades are used primarily in the latter sense and, specifically, as sequences of grades over the life of a mine.

The profit from a mining operation is a direct function of the sequences of cut-off grades and associated ore tonnages that define the life-of-mine production schedule. As profit varies with these sequences there will be a sequence, or sequences, that optimise any specified profit criterion. The most widely used cut-off grade optimisation criterion is maximum net present value of profits.

The traditional approach to cut-off grades is to use the break-even grade, at which revenue equals cost. This approach completely ignores the time value of money and usually leads to a sub-optimal valuation of the mineral resource.

Some heuristic approaches have been suggested to modify the traditional point of view. The traditional cut-off grade calculation has been modified to include depreciation and fixed costs for a period of time by applying higher cut-off grades and, consequently, to gain more profit for early years of production.

The time value of money, together with the opportunity cost of taking low grades when higher grades are still available, ensures that there is an advantageous trade-off between mining at higher cut-off grades in the early years and lower cut-off grades in the later years of mine life. The recognition of these factors led a number of researchers to develop algorithmic approaches to the determination of optimal sequences of cut-off grades.

Henning[26] showed for the first time that true maximisation of present value came from a sequence of declining cut-off grades. He was the first to show that, when the discount rate is greater than zero, net present value is maximised only if there is a declining cut-off grade policy. He derived formulas for three cases each with different objectives. He showed analytically that if the objective is to maximise the spread between the present value of annual operating profit and investment cost and if the discount rate is greater than zero, there must be a declining cut-off grade policy. He pointed out that in a free market, this is the most commonly applied objective.

Lane[29, 30] discussed the theoretical background to optimum cut-off grades and developed a comprehensive theory of cut-off grade optimisation. He was the first to distinguish three main stages of a mining operation with their distinctive costs, namely, mining, mineral processing and refinery and/or marketing. Each stage has a certain capacity that may constrain the entire operation. On the basis of these stages, he defined three limiting economic cut-off grades, each being a result of the capacity limitation of one of the three stages. If the mining stage is the constraint, in order to maximise the cash flow, it is necessary to maximise profit per unit of mineralised material. In the same way, if the mineral processing stage is the constraint, in order

to maximise the cash flow, it is necessary to maximise profit per unit of ore mined and if the refinery and/or marketing stage is the constraint, in order to maximise the cash flow, it is necessary to maximise profit per unit of final product. The mining operation may not be constrained by one stage only; it could be simultaneously constrained by two stages. Lane defined three balancing cut-off grades, each resulting from a limitation of a pair of the three mining stages. He proposed that the optimum cut-off grade is one of the six cut-off grades mentioned above. He showed how to find balancing cut-off grades graphically and analytically and demonstrated that a sequence of declining cut-off grades is needed if present value is to be maximised. He introduced a general formula and a solution algorithm for optimisation of cut-off grades. Lane's basic expression for the net present value per unit of resource is:

$$v = (p - k) * x * y * a - x * h - m - (f + F) * t \qquad (1.1)$$

where;

v: net present value,

p: price per unit of marketed product,

k: marketing variable cost,

x: ore/material ratio,

y: yield from treatment (recovery),

a: average grade,

h: mineral processing variable cost,

m: mining variable cost,

f: fixed cost,

F: opportunity cost,

t: time per unit of resource.

The inclusion of the opportunity cost in the formula ensures a sequence of declining cut-off grades. "An opportunity cost is the profit foregone by following one course of action when another would have yielded a higher value. For example, the cost of delaying revenue from available high-grade ore by instead mining lower grade ore. The cut-off grade at any particular time must therefore generate enough revenue to offset the cost of this delay." (Dowd[15]).

Dagdelen[7] and Dowd[15] have itemised the steps involved in implementing Lane's algorithm. Dowd and Xu[16, 18] have adapted and extended Lane's method and incorporated it into an integrated software package for the financial and risk appraisal of mining projects.

Blackwell[6] demonstrated the validity of Lane's approach by applying it to a hypothetical open pit operation. He compared the economic results of the optimum cut-off grades policy with those obtained by applying a range of fixed cut-off grades. He concluded that the net present value obtained by applying the optimum cut-off grades policy has a gross advantage over the highest net present value obtainable by a fixed cut-off grade strategy.

Noren[37] worked on optimum production rates and optimum mining limits. Instead of the cut-off grade, he proposed the use of the average grade as the mining limit, or decision variable of his model. Noren studied optimum production rates and optimum mining limits as separate cases. His approach is to divide the orebody into zones and to use dynamic programming to find optimum production rates. The rate of production may be changed only when mining activity transfers from one zone to

another. Thus, he assumed constant production rates for each zone. His assertion was that changes cause expansion or contraction investments since the rate of production is always equal to the production capacity. For the problem of finding mining limits, the optimum average grades are determined by means of dynamic programming, the average grades being constant for each zone. Noren came to the same conclusion as other authors that, because of the opportunity cost, the optimum average grade decreases as ore extraction proceeds.

Taylor[42, 43] assessed previous work and arrived at a very similar approach to that of Lane. In his papers, he introduced a comprehensive background theory of cut-off grades and showed the importance of determining preliminary cut-off grades for different types of mines. He analysed the effects of financial objectives, a variety of costs and fluctuation of prices on the determination of cut-off grades.

Whittle and Wharton[45] followed Lane's concept but defined opportunity cost in a different way. In addition to normal cash costs, they introduced two further important costs: delay costs and change costs. The delay cost is the cost of delaying the operation or the cost of delaying future profits. The delay cost is essentially what other authors have called the opportunity cost. The change cost, as Whittle and Wharton explained, is the cost of any change in the product price. If the price of the product is expected to be lower next year, it is better to produce and sell this year rather than selling next year at a lower price. On the contrary, if the price is expected to be higher next year, then the change cost becomes negative or a gain. On the other hand, if economic conditions are not expected to change for the future, the change cost is zero by definition. The authors argued that the sum of the delay and change costs is equal to the opportunity cost. The concept of change costs used by these

authors is generally considered and dealt with in sensitivity or risk analysis by other authors (Dowd[14, 15, 21], Smith[41]).

Dowd[14] introduced the use of dynamic and stochastic programming to generate optimal sequences of cut-off grades. He extended Roman[40]'s work of finding optimum production rates by means of dynamic programming to finding optimum cut-off grades and production rates. He showed that the optimum cut-off grade problem could be posed as a sequential decision-making problem in which the orebody could be viewed as a system that could be in any of a number of states at any given stage. Implementing a cut-off grade decision moves the system from a given state in the current stage to a new state in the subsequent stage. Dowd extended the dynamic programming method to include rudimentary access constraints and this is still the only published method to address the problem of access in cut-off grade optimisation. He also used stochastic programming to account for the uncertainties associated with variables such as prices and costs.

Dowd and Xu[16, 18] introduced a computer program capable of optimal production scheduling using Dowd's dynamic programming method and incorporated it into an integrated software package for the financial and risk appraisal of mining projects.

The easiest and most common approach to mineral deposits that contain more than one valuable mineral is to convert all the minerals to their equivalent in terms of one basic mineral. The deposit is then treated as if it is a single mineral deposit. For deposits in which the constituent minerals are highly correlated (e.g., stratigraphic silver/lead/zinc orebodies) this is an intuitively attractive approach as it reflects the evaluation, mining, concentrating and sales components of the operation.

As the correlation among the constituent minerals increases, there is little difference between applying a cut-off grade to one constituent mineral and to the equivalent mineral content. In other types of deposits, however, the validity of the approach is less obvious, especially when the correlation is such that cut-off grades applied independently to each of the constituent minerals identifies mutually exclusive zones within the orebody.

Mine planning and the financial evaluation of mineral deposits that contain more than one valuable mineral are generally done on the basis of parametric cut-off grades or equivalents. Lane[30] explains this technique in detail for mineral deposits that contain two minerals. Dowd and Xu[18] included the equivalents method in their computer program in order to find optimal cut-off grades and production rates for multi-mineral deposits. However, because of problems related to this method, a number of researchers have tried to find an alternative method of optimisation. The main problem arises from the fact that the revenue and the costs must be calculated on the basis of the average grades of the individual minerals from the calculated equivalent grade. If the constituent minerals are highly correlated, the average grades can be estimated by iteration and by defining some additional parameters for the equivalent grade-tonnage data (Dowd and Xu[18]). However, if there is very little correlation among the minerals, the validity of the equivalents method is not obvious.

Lane[30] introduced a number of ideas for cut-off grade optimisation for mineral deposits that contain more than one valuable mineral and suggested a grid search method for optimisation. However such an approach is very likely to find local optima rather than the global optimum. Lane's proposal was that the search area for

the optimum cut-off grade could be approximated by a set of equidistantly spaced grid points. The grid points can then be searched for the highest net present value. The point or points that give the highest value are accepted as the optimum cut-off grade or grades. In fact, the value found is the optimum cut-off grade for the designated grid points rather than for the entire search area. However, if the density of grid points to search for optimum is sufficient, this method can prove quite satisfactory.

Dagdelen and Mohammed[8] implemented the grid search method in a computer program for the optimisation of cut-off grades for two-mineral deposits. They used the same terminology as Lane and the same grid point density.

The dynamic programming method, which has been successfully applied to the optimisation of cut-off grades and production rates for single-mineral deposits (Dowd[14]) and to equivalents for multi-mineral deposits (Dowd and Xu[18]), is another method that has been implemented in this research.

Genetic algorithms comprise another group of search methods used to solve mining industry problems (e.g., Denby and Schofield[9, 10], Denby, Schofield and Hunter[11], Denby, Schofield and Surme[12], Yun et al[47]) that have also been used in this research. Genetic algorithms constitute a class of stochastic algorithms that use a search method based on the laws of biological genetics and natural evolution. Although these algorithms have found applications in many disciplines, they have only recently been applied to the solution of minerals industry problems.

Denby and Schofield[9] used genetic algorithms for optimal open pit design and production scheduling, and underground mine scheduling (Denby and Schofield[10]).

Yun et al[47] applied genetic algorithms to an iron ore mine to find an optimum cut-off grade. They used the algorithms to find a static cut-off grade to be used for the life of the mine, rather than looking for optimal sequences of cut-off grades.

The objectives of the research work reported in this thesis were to develop general methods for determining optimal sequences of cut-off grades for multi-mineral deposits, to implement these methods in computer programs and to compare and assess the performances of the methods. During the course of the research the methods chosen were the grid search method, dynamic programming and genetic algorithms. The computer programs developed for this purpose are capable of determining optimal sequences of cut-off grades for multi-mineral deposits that contain up to three valuable minerals.

CHAPTER 2

GENERAL CUT-OFF GRADE THEORY

2.1 Introduction

Cut-off grades are used in the mineral industry as reference grades that separate two courses of action. A major use is for the classification ore and waste. If the mineral content of the material is above the assigned cut-off grade, the material is classified as ore and could be mined at a profit. If not, the material is classified as waste and, depending on the mining method used, would be sent to waste dumps or left in-situ.

The determination of optimum cut-off grades requires the application of an optimising criterion, usually financial. There are two basic principles involved in cut-of grade optimisation: the "bigger the better" principle (bigger benefits are preferred to smaller ones, *ceteris paribus*) and the "bird in the hand" principle (early benefits are preferred to later ones). Several financial evaluation techniques have been used in cut-off grade optimisation and for the general appraisal of mineral deposits, and there are numerous variations in the application of specific techniques. The two main categories of financial evaluation techniques are undiscounted profit methods and discounted profit methods.

The objective in undiscounted profit methods is to maximise the spread between the investment costs and the sum of the annual profits during the lifetime of the

mine. Two common undiscounted profit methods are the (undiscounted) payback period method and the accounting rate of return.

The payback period method is one of the easiest methods to apply. Payback period is defined as the time required for the cash income from a project to equal the investment in the project. The time for payback can be found by dividing the capital investment at the beginning of mine life by the annual cash flow as in Equation 2.1.

$$n = \frac{I}{c} \tag{2.1}$$

where;

n: number of years or time for payback of the capital investment,

I: capital investment,

c: annual cash flow.

There are several versions of the accounting rate of return method. One version is the average accounting rate of return method. The average accounting rate of return is calculated by dividing the average annual profit by the initial investment. It is shown in Equation 2.2. A more complex method is the average accounting rate of return with depreciation and is calculated by dividing the average annual net profit by the average investment in the project after subtracting the depreciation, that is, the average book value of the investment by a company as shown in the company's books. This is shown in Equation 2.3.

$$aarr = \frac{aanp}{I} \tag{2.2}$$

where;

aarr: average accounting rate of return,

aanp: average annual net profit.

$$\text{aarrd} = \frac{\text{aanp}}{\text{abv}} \tag{2.3}$$

where;

aarrd: average accounting rate of return with depreciation,

abv: average book value.

The weakness of undiscounted profit methods is that they ignore the time value of money and give no weight to annual fluctuations in cash flows.

The purpose of discounted profit methods is to maximise the spread between the present value of annual profits and investment costs. If the time value of money is zero, they are the same as undiscounted profit methods. Discounted cash flow analysis has been a prominent technique for performing valuations for several decades (Atrill and McLaney[1]). The two most widely used discounted methods in the financial analysis of mineral deposits are the net present value method and the internal rate of return method (Johnson[28]).

The net present value is the difference between the total discounted income and the total discounted cost of a project over the lifetime of the mine. That is, all cash flows are discounted back at a discount rate since the time value of money for each year is different. The rationale of this method is that, as long as the net present value is positive, the project is economically justified. The net present value of a mining venture can be found by calculating the sum of the present values of all yearly cash

flows less the initial investment, as shown in Equation 2.4. Maximisation of net present value is the most widely accepted and used method. It takes into account the time value of money and gives a single project value for a given discount rate and cash flows. The larger the net present value, the more profitable the project.

$$NPV = \left(\sum_{t=1}^{n} \frac{c_t}{(1+i)^t} \right) - I \qquad (2.4)$$

where;

NPV: net present value,

c_t: cash flow in year t,

i: discount rate,

n: mine life of the project.

The internal rate of return is defined as the discount rate that equates the total discounted profit to the total discounted cost of a project over the lifetime of the project. The logic behind the internal rate of return method is not very different to that of the net present value method. Basically, the internal rate of return method is the discount rate that makes the net present value zero. The decision criterion is that the internal rate of return must be higher than the discount rate of the project. For a project to be acceptable, it must meet a minimum internal rate of return requirement and this minimum should be the opportunity cost of finance (Atrill and McLaney[1]). One weakness of the internal rate of return method is that it assumes that yearly cash flows of a project are reinvested at an interest rate identical to the internal rate of return of the project. However, such an assumption is not always possible. An expression for internal rate of return is shown in equation 2.5.

$$0 = \left(\sum_{t=1}^{n} \frac{c_t}{(1+IRR)^t} \right) - I \qquad (2.5)$$

where;

IRR: internal rate of return.

There are some other commonly used discounted profit methods such as wealth growth rate, growth rate of return, present value ratio and overall rate of return. These are closely related to the net present value and internal rate of return methods.

The growth rate of return is the rate of return that equates the future value of the capital investment and the future value of total cash flows that are reinvested at an external interest rate. If the reinvestment rate is equal to the internal rate of return for a project, then the internal rate of return and the growth rate of return of that project are identical. The concept of an external rate of return comes from the observation that, if the internal rate of return of a project is very high, then it is unlikely that the interest rate at which the yearly cash flows could be reinvested would be as much as the internal rate of return.

The wealth growth rate is similar to the growth rate of return except that the wealth growth rate is the rate of return at the end of a project. The growth rate of return at the end of a project is equal to the wealth growth rate of the same project. An expression for wealth growth rate or growth rate of return for the end of a project is given in Equation 2.6.

$$0 = \left[\sum_{t=1}^{n} c_t * i' \right] - I * (1+WGR) \qquad (2.6)$$

where;

WGR: wealth growth rate,

i': external interest rate or reinvestment rate.

The present value ratio is a financial evaluation method closely related to the net present value method. It is the ratio of the sum of the present values of all yearly cash flows to the initial investment. The decision criterion is that if the present value ratio is greater than 1, the project is acceptable. An expression for this method is shown in Equation 2.7.

$$PVR = \frac{\left(\sum_{t=1}^{n} \frac{c_t}{(1+i)^t}\right)}{I} \tag{2.7}$$

where;

PVR: present value ratio.

The overall rate of return is related to the present value ratio method. This value is the ratio of the net present value to the initial investment of a project. The decision criterion is to accept the project if the overall rate of return is more than 0. The overall rate of return is given in Equation 2.8.

$$ORR = \frac{\left(\sum_{t=1}^{n} \frac{c_t}{(1+i)^t}\right) - I}{I} \tag{2.8}$$

where;

ORR: overall rate of return.

Among the financial evaluation methods for project appraisal mentioned above, the net present value is the preferred method of most authors (Torries[44]). It is also relatively easy to calculate net present value.

The policy used for mine life cut-off grade determination varies widely, depending on the objectives of the enterprise. The methods used began with what have been termed static approaches (Dowd[13]) and, over the past 50 years, have evolved to dynamic approaches and solutions.

2.2 Static cut-off grades

Traditionally, break-even grades, where revenue balances the cost, have been widely used as cut-off grades. The following formula is used to find a marginal break-even grade:

$$b_m = \frac{v}{(s-r)*y} \tag{2.9}$$

where;

b_m: marginal break-even grade,

v: variable cost,

s: selling price,

r: recovery and/or marketing cost,

y: overall recovery rate.

This is a marginal grade because it ignores all sunk and fixed costs. It is based on the intuitive principle that if all development costs for a given parcel of ore have been incurred (whether or not they should have been) and if all fixed costs must be

borne in the short-term, then they play no part in the economic evaluation of an immediately available parcel of ore. In other words, the only relevant costs are those that have yet to be incurred. Marginal break-even grades are immediate break-even grades applied to decide whether or not an immediate parcel of ore is profitable, rather than for making overall decisions for the deposit (Dowd[15]).

For an overall break-even grade, Equation 2.9 would result in a grade for which revenues would be insufficient to cover all costs, as only variable costs are included. The overall break-even grade is given by:

$$b_o = \frac{v + \frac{f}{a}}{(s - r) * y} \tag{2.10}$$

where;

b_o: overall breakeven grade,

f: fixed costs per time period (usually a year)

a: amount of ore extracted per time period.

Decision criteria for cut-off grades and the methods used to compute them vary for different enterprises depending on their goals.

It is obvious that using the break-even grade as a production cut-off grade would yield a mine life production schedule in which the operating grades were identical in each year of operation. Such a production schedule ignores the time value of money because, in general, higher grades yield greater profits per tonne mined and a higher net present value could be obtained by mining the higher grade tonnages in the earlier years. Thus, constant cut-off grade production schedules usually lead to sub-optimal exploitation of the resource. Such schedules do,

however, enable the enterprise to recover the maximum amount of ore and thereby maximise the life of the mine, which, in some circumstances, may be a legitimate (non-financial and usually political) objective. For various policy reasons (e.g. maximising employment or encouraging ancillary industrial development) or ideological reasons sub-optimal economic exploitation of mineral resources might be preferred to "wasting" or sterilising part of the resource (Dowd[15]).

A constant cut-off grade production schedule is, implicitly or explicitly, based on the objective of maximising the undiscounted profits of the mining operation. When the overall break-even grade, as in Equation 2.9, is used as the cut-off grade throughout the mine life, maximum undiscounted profit is achieved. If, however, the objective is to maximise discounted profit rather than to maximise undiscounted profit, to maximise the life of mine or to recover maximum amount of ore, the technique of assigning a single cut-off grade does not yield a schedule that satisfies the objective. The reason for this is the opportunity cost or the time value of money.

The concept of the time value of money is very important in understanding the discounted profit approach to financial analysis. The concept is based on the fact that money earns interest over time and that there is always an opportunity cost incurred by a financial decision. It is also indirectly based on risk. Recovering the capital cost and getting more benefits from an investment in earlier periods decreases the risk of recovering the investment. For all these reasons a given amount of money is always worth more today than in the future. Since investors would rather receive benefits sooner than later, the value of each yearly cash flow generated over the life of a project can be adjusted for the time value of money (Torries[44]).

There has always been some intuitive recognition of the role that the time value of money plays in determining cut-off grades, as exemplified by wide-spread "high-grading" and "black birding" practices throughout the nineteenth and early twentieth centuries (Dowd[15]). These practices were often guided by the desire for quick profits (and an even quicker withdrawal from the operation) rather than any attempt to maximise net present value. There are equally numerous examples of otherwise astute mining operators who have totally ignored this basic economic principle. South African mining practice, for example, persisted in the use of pay-limit (break-even) grades until the early 1990s (Dowd[15]).

The incorporation of the time value of money into cut-off grade determination began in the 1950s. The general aim of such an approach is to use cut-off grades higher than break-even grades during the early years of a mine and to use the break-even grade for the later years in order to decrease the opportunity cost of having lower grades of ore while higher grades are available.

Berry[4] used present value theory to determine the cut-off grade that gives the maximum profit. But he found the cut-off grade that gives the maximum present value for year zero as the optimum cut-off grade for the life of the mine. By doing so, he justified raising the cut-off grade permanently above the break-even grade and so permanently abandoning poorer ore. Whilst present value maximisation is logical when viewed from year zero, Berry failed to point out that the financial advantage would decline as mining (and, hence, time) progresses. Taylor[42] pointed out that:

> "Any policy that purports to maximize present value at year zero is spurious unless it also maximizes remaining present value at every succeeding stage

of life. Exact maximization may be difficult to calculate. A corollary is that maximum present value and constant cut-off grade are incompatible."

Some authors have applied heuristic approaches to cut-off grade optimisation. The most common of these methods use cut-off grades higher than break-even grade during the early years of an operation and breakeven grade during the later stages of the operation (Halls et al[25]). Some heuristic cut-off grade calculations also include a minimum profit for a period of time to obtain a higher cut-off grade for the early years of production (Ericson[22]). The following equations provide an example of a heuristic approach in which, for the early years, the break-even grade formula is extended to include depreciation and a minimum profit and for the later years includes only depreciation (Dagdelen[7]).

$$c_{mp} = \frac{v + d + mp + \frac{f}{a}}{(s - r) * y} \tag{2.11}$$

where;

c_{mp}: heuristic cut-off grade for early years,

d: depreciation,

mp: minimum profit.

$$c_d = \frac{v + d + \frac{f}{a}}{(s - r) * y} \tag{2.12}$$

where;

c_d: heuristic cut-off grade for later years.

This could be extended to a three-stage sequence by using the overall break-even grade formula for the final years of operation. Such an approach would yield three cut-off grades in decreasing order for the three stages of mine life. In this way, although the undiscounted profit would be less than the constant cut-off grade approach, the net present value generally will be higher.

2.3 Optimal cut-off grades

The objective of maximising the net present value can be achieved by maximising profit per unit of time. This process necessitates applying, in the early years of operation, the highest cut-off grade that can provide sufficient ore to satisfy the requirements of the processing plant. As time passes and the higher-grade is depleted, the cut-off grade must be lowered, thereby lowering the opportunity cost. Hence, the highest net present value is achieved. Although the importance of the time value of money was recognised by other authors (e.g. Henning[26], Noren[37]), it was Lane[29] who first recognised and formulated the principle in this way and laid the foundation for an economic theory of cut-off grades (Dowd[13]).

Lane's maximum net present value production schedule can, however, only be applied when mining can be done in a perfectly selective manner. In reality, access constraints and the geological structure of the orebody inhibit the application of such a schedule. In a perfectly selective operation, the same block of material could be classed as ore or waste depending on the cut-off grade used for the particular stage of operation but if rejected as waste it would always be available at any future stage. In reality, the extent to which a block, once rejected, remains available for future selection depends on the mining method. Underground mining methods are far less

flexible in this respect than open pit methods. The expense of developing ore in underground mining, together with geotechnical and stability constraints, generally mean that mine design is constrained by relatively long-term and rigid cut-off grade policies that make it very difficult to deviate from sequential, "all or nothing" selection. Access and, therefore, selection is much more flexible in surface mining but even here it is very expensive to alter a pit design after it has been established. For both types of mining, stockpiling allows a more flexible approach to achieving a maximum net present value production schedule but the costs of stockpiling must be included in the determination of the cut-off grades and will thus alter the optimal schedule and the net present value.

By using stockpiles material rejected as waste in the early stages of a mining operation, which could be accepted as ore if it was depleted later, can be processed in a later stage of the operation. Dagdelen and Mohammed[8] published an approach to incorporating stockpiles in the determination of optimal cut-off grades. They argued that, the intermediate grade material - mineralised material with grades between the optimum cut-off grades for each year and the break-even cut-off grade - can be stockpiled and used at a later stage to enhance the net present value of the operation. Yi and Sturgul[46] stated, "To achieve a true maximisation in open pit mining, a sub-cut-off grade needs to be defined as a limitation that distinguishes the ore having future potential value and waste that should be discarded permanently. Then, an operating cut-off grade in an open pit has to be redefined as a limitation between the ore having current value and that which might have value in the future." They suggested assigning sub-cut-off grades to distinguish parts of low-grade ores that could be stockpiled and mined later and parts that are waste. However, there is a

cost associated with stockpiling and stockpiles are not applicable for all minerals as some deteriorate over time in open storage.

Lane[29, 30] must be regarded as the pioneer of formal cut-off grade theory. He provided a comprehensive economic theory and an algorithm for determining optimum cut-off grades. The work is based on subdividing a mining operation into three main stages: mining, mineral processing, and refining and/or marketing. Cut-off grades depend on various limitations between pairs of these stages. The quantitative balance between pairs of stages and the capacity constraints of each stage impose different break-even grades. These break-even grades can be used as markers between which the optimal cut-off grade can be found. In addition, because of the time value of money, a declining sequence of cut-off grades is needed to decrease the opportunity cost if the net present value is to be maximised.

The calculation of a single optimum cut-off grade for one particular set of conditions is trivial and the arithmetic is well within the capability of a hand calculator. The calculation of a complete cut-off grade schedule is, however, a different matter. A sequence of cut-off grades for the life of the mine has to be calculated so that the net present value is maximum and the sequence has to be consistent in that the corresponding annual cash flows and present values must relate to each cut-off grade in the sequence.

Complete cut-off policies are not a normal requirement for day-to-day mining operations, but they are important for longer range mine planning. Future cut-off grades affect development requirements, equipment utilisation and output. They also affect future cash flow and the resource, which underlies all mine planning. Besides, the expansion or performance improvement of a project implies a change in cut-off

grade policy and pre-feasibility studies entail the comparison of many alternative plans, each with associated cut-off grade policies, which have to be evaluated for a range of different conditions.

There are three main components of a mining operation related to the throughputs from the three distinct stages mentioned above. They are mineralised material, ore and mineral.

Mineralised material is dealt with in the mining stage. This is generally concerned with making ore available for treatment, i.e. creating access to, and extracting, the ore before the treatment. Costs are incurred per tonne of mineralised material and capacity is the maximum rate at which the mineralised body can be depleted.

The ore is treated in the mineral processing stage. After the mineralised material has been extracted it is moved to the next stage where it is processed. In this stage the costs are incurred per tonne of ore and capacity is the maximum throughput of ore that the processing plant can support.

The mineral, or the product of the mineral processing plant, is moved to the third stage, which involves refining (or further treatment) and/or marketing. Costs are incurred per unit of final product (mineral, metal) and capacity is the maximum refining or marketing rate.

The basic expression for the net present value per unit of resource in a mining operation is given in Equation 1.1.

If the total profit from a finite mineral deposit is to be maximised, it is necessary to maximise the scarcest throughput. Thus, if mining is limited, profit per unit of mineralised material must be maximised. If the limitation comes from a limited

mineral processing capacity, profit per unit of ore must be maximised. If the limitation is from limited refinery or marketing capacity, profit per unit of mineral must be maximised.

Lane introduced three limiting economic cut-off grades based on the mining operation stages. They are mining limited economic cut-off grade, mineral processing limited economic cut-off grade and marketing limited economic cut-off grade. There are, in addition, cut-off grades that balance each pair of stages. These balancing cut-off grades are mining and mineral processing balancing cut-off grade, mining and marketing balancing cut-off grade and mineral processing and marketing balancing cut-off grade. The optimum cut-off grade for each year is calculated from among these six cut-off grades. The result is a sequence of cut-off grades in decreasing order over the life of the mine. But there is a difficult circularity in this formulation. In order to calculate limiting economic cut-off grades, the net present value of the profits for future years must be known and to find this net present value the cut-off grades must be determined. A solution to such an interdependency problem can be obtained by iteration. In the first stage of the iteration process, an estimate of the present value of future profits is required and at the end of each iteration the net present value is improved, until the solution converges to a stable optimum result. There is no guarantee that Lane's algorithm will converge but a solution can usually be found by adopting a suitable tolerance value for convergence. A more detailed description of Lane's method can be found in Chapter 3.

Another method for determining optimal sequences of cut-off grades is dynamic programming. In this method, every possible series of cut-off grade decisions is

enumerated and an optimal solution is found by considering the effects of each decision separately. The main limitation to this approach comes from computer memory limitations. Improvements in computer technology might ease this disadvantage.

Dowd[14] used the dynamic programming technique to determine an optimal sequence of cut-off grades and production rates over the life of a mine. In this method, decision intervals must be set for cut-off grades and production rates and the full range of values is replaced by sets of discrete values defined by the intervals. For each depletion rate, a cut-off grade that gives maximum profit is calculated. The number of depletion rates restricts the number of possible solutions to be searched. The depletion rates are searched over the mine life by means of dynamic programming to find the maximum discounted profit, which gives a decreasing order of cut-off grades.

The search method is very robust in that all possible alternatives are searched to find the global optimum. One way of decreasing computing time is to make use of the fact that an optimal sequence of cut-of grades must be decreasing over time and thus all non-decreasing sequences can be eliminated from the search. This approach, devised and implemented in the research described in this thesis, provides significant improvements in computation time. A more detailed description of the dynamic programming method can be found in Chapter 3.

One way to solve optimisation problems is by simulation. This is a stochastic approach based on random, or pseudo-random, number generation and searching all alternatives to find the optimum. There are a number of different simulation techniques with Monte Carlo simulation being the basic and well-known one. Monte

Carlo simulation is far from a good choice for cut-off grade optimisation since it is based on purely random choices.

Directed search methods, such as hill climbing or simulated annealing, can give better results. Another group of search methods, known collectively as genetic algorithms, was introduced about 25 years ago, and offers the prospect of significant improvements over many other search methods. Genetic algorithms are particularly suited to the solution of large-scale optimisation problems. They belong to the class of probabilistic algorithms but are very different from random algorithms as they combine directed and stochastic searches. Because of this, genetic algorithms are more robust than other directed search methods. Another important property of genetic-based search methods is that they maintain a population of potential solutions – all other methods process a single point of the search space (Yun et al[47]). Genetic algorithms can also easily escape from local optima by using genetic operators, such as mutation.

The search process comes to an end either by the convergence of the solution to a stable optimum result, as in Lane's method, or an assigned number of simulations is reached. A more detailed description of genetic algorithms can be found in Chapter 5.

CHAPTER 3

CUT-OFF GRADES FOR SINGLE-MINERAL DEPOSITS

3.1 Optimisation by Lane's method of limiting and balancing cut-off grades

3.1.1 Introduction

Being natural phenomena, mineralised bodies have unique features that differ from each other. As a result, no two orebodies are identical. However, the basic structure of the mining process is similar for all orebodies and that enables a general approach to optimal exploitation. This process necessitates applying a mining cut-off grade that provides sufficient ore to satisfy the requirements of the mineral processing plant. As time passes, the cut-off grade must be lowered. As a result of this process, the opportunity cost is lowered and hence, the highest net present value is achieved.

3.1.2 Limiting economic cut-off grades

Limiting economic cut-off grades are break-even cut-off grades for the cases in which each mining stage in turn limits the total capacity of the mining operation. They are economic cut-off grades for the cases in which each corresponding stage alone limits

total capacity of a mining operation. They depend directly on mineral price and costs of the operation and indirectly on the grade distribution of the deposit.

There are three limiting economic cut-off grades and each relates to one of the mine operating stages. They are, mining limiting economic cut-off grade, mineral processing limiting economic cut-off grade and marketing limiting economic cut-off grade. The limiting economic cut-off grades are determined by costs and prices. They are grades at which the cost of extracting the mineral is equal to the revenue gained from selling it. If the mining operation is restricted by only one limiting economic cut-off grade then that grade is the optimum cut-off grade.

Lane's basic profit expression for the net present value per unit of resource in a mining operation must be maximised in order to obtain an optimum production schedule. The basic profit expression is:

$$W = (p-k)*x*y*a - x*h - m - f*t \qquad (3.1)$$

where;

W: profit for the next unit of material,

p: price per unit of product,

k: marketing variable cost,

x: ore/material ratio,

y: processing yield (recovery),

a: average grade,

h: variable mineral processing cost,

m: variable mining cost,

f: fixed cost,

t: time unit of resource.

The profit expression is formulated by considering the next unit of material to be mined. The unit mining cost is m. The proportion of the mined material that is sent to the mineral processing plant is determined by the ore/material ratio, x. Thus, for a unit of mined material a quantity, x, of ore is sent to the mineral processing plant and processed at a unit cost of h. After treatment, the mineral might be sold directly or sent to a refinery prior to marketing. In either case, the final product, which is a proportion of the processed product, is sold; the proportion depends on the average grade and recovery rate. A quantity, $x * y * a$, of mineral will be sold at a unit marketing cost of k. If the selling price is p, the total income is $p * x * y * a$.

Each of the stages - mining (depletion) of the orebody, processing the ore and refining and/or marketing – takes a certain time and a proportion of the fixed costs will be allocated to that time period in addition to the variable costs m, h and k. If the fixed cost per unit of time is f and the time period is t, the total fixed cost borne by the stage is $f * t$.

The profit expression calculates the profit obtained by mining an incremental unit of material. However, the objective is to maximise the net present value of the future profits rather than just the total profit. The best way to achieve this objective is to add to the profit function the variable opportunity cost of taking low-grade material when higher-grade material is still available.

Assume that the maximum present value of future profits is V and the maximum present value of future profits after the next incremental unit of material has been mined is A. The maximum present value of future profits is equal to the

profit obtained from mining the next unit of material plus the present value of future profits after the next unit is mined. If d is the discount rate, the present value is:

$$V = \frac{W + A}{(1+d)^t} \tag{3.2}$$

where;

V: net present value of all future cash flows,

A: maximum possible present value of future profits after the next unit of material has been mined,

d: discount rate.

For short intervals of time, $(1 + d)^t$ can be approximated by $1 + d * t$. Then:

$$V - A = W - d * V * t \tag{3.3}$$

V – A is the increase in the present value that accrues from mining the next unit of material. If this increase in the present value is v, then the expression for the net present value per unit of resource in a mining operation is:

$$v = (p-k) * x * y * a - x * h - m - (f + d * V) * t \tag{3.4}$$

where;

v: net present value.

The expression 3.1, which is the basic present value expression, differs from the profit expression by the added term d * V. This can be regarded as the variable opportunity cost and can be denoted by F.

$$v = (p-k) * x * y * a - x * h - m - (f + F) * t \tag{3.5}$$

where;

F: opportunity cost.

By using this expression, the limiting economic cut-off grades can be found by assuming that mining, mineral processing and marketing stages, in turn, limit the operation.

3.1.2.1 Mining limiting economic cut-off grades

If the rate of mining limits the operation and the maximum capacity of the mining stage is M, then the time needed to mine one unit of material is 1/M. Substituting 1/M for t in (3.5) the net present value formula becomes:

$$v_m = (p-k) * x * y * a - x * h - m - \frac{(f+F)}{M} \qquad (3.6)$$

where;

v_m: net present value for a limiting mining rate,

M: maximum mining capacity.

The only difference from the global net present value expression is that the time unit of the operation is determined as the time needed to mine one unit of material.

As mining is the limiting stage, the cut-off grade only affects the mineral processing and marketing stages, and only $(p-k) * x * y * a - x * h$ will vary with the cut-off grade. Therefore, to find the optimum cut-off grade when the mining stage limits the operation, $x * ((p-k) * y * a - h)$, or $(p-k) * y * a - h$ must be maximised.

The average grade can be derived from the cut-off grade. If the cut-off grade to be maximised is g and the mining limiting break-even cut-off grade is g_m, then g can be lowered for as long as $(p - k) * y * g$ is greater than h. Hence, the mining limiting economic break-even cut-off grade is given by $(p - k) * y * g_m = h$. So:

$$g_m = \frac{h}{(p-k) * y} \tag{3.7}$$

where;

g_m: mining limiting economic cut-off grade.

This approximation comes from the same break-even concept that is used for the static cut-off grade approach, i.e., mineralised material should be classified as ore as long as the profit acquired from selling it, $(p - k) * y * g$ is not less than the cost of processing, h.

3.1.2.2 Mineral processing limiting economic cut-off grades

If the rate of mineral processing limits the operation and the maximum capacity of the mineral processing plant is H, then the time needed to process one unit of ore is 1/H. But in this case, one unit of mineralised material yields x units of ore. Therefore, the time needed to process one unit of mineralised material is x/H. Substituting x/H for t in (3.5) the net present value formula becomes:

$$v_h = (p-k) * x * y * a - x * h - m - \frac{(f+F) * x}{H} \tag{3.8}$$

where;

v_h: net present value for a limited mineral processing rate,

H: maximum mineral processing plant capacity.

The difference from the global net present value expression is that the time unit of the operation is determined as the time needed to process the ore supplied by one unit of material.

As cut-off grades affect only the mineral processing and marketing stages, only $(p - k) * x * y * a - x * h - (f + F) * x/H$ will vary with the cut-off grade. Therefore, to find the optimum cut-off grade when the mineral processing stage limits the operation $x * ((p - k) * y * a - h - (f + F) / H)$, or $(p - k) * y * a - h - (f \div F)/H$ must be maximised.

The average grade can be derived from the cut-off grade. If the cut-off grade to be maximised is g and the mineral processing limiting break-even cut-off grade is g_h, then g can be lowered for as long as $(p - k) * y * g$ is greater than $h \div (f + F)/H$. Hence, the mineral processing limiting economic break-even cut-off grade is given by $(p - k) * y * g_h = h + ((f + F) / H)$. So:

$$g_h = \frac{h + \frac{f + F}{H}}{(p - k) * y} \qquad (3.9)$$

where;

g_h: mineral processing limiting economic cut-off grade.

The mineral processing limiting economic cut-off grade formula is very different from the mining limiting economic cut-off grade formula because of the inclusion of the opportunity cost. The opportunity cost measures the net present value

of future cash flows and is a function of time. As time passes the opportunity cost declines. So does the mineral processing limiting economic cut-off grade.

3.1.2.3 Marketing limiting economic cut-off grades

If the rate of refinery or marketing limits the operation and the maximum capacity of the refinery or marketing is K, then the time needed to refine/market one unit of final product is 1/K. But in this case, one unit of mineralised material gives rise to $x * y * a$ units of final product. Therefore, the time needed to refine/market one unit of final product is $x * y * a/K$. Substituting $x * y * a/K$ for t in (3.5), the net present value formula becomes:

$$v_k = (p-k) * x * y * a - x * h - m - \frac{(f+F) * x * y * a}{K} \qquad (3.10)$$

where;

v_k: net present value for a limited refinery or marketing rate,

K: maximum refinery or marketing capacity.

The difference from the global net present value expression is that the time unit of the operation is determined as the time needed to refine or market the final product supplied by one unit of material.

As cut-off grades affect the mineral processing and marketing stages, only $(p - k) * x * y * a - x * h - (f + F) * x * y * a/K$ vary with the cut-off grade. Therefore, to find the optimum cut-off grade when the marketing stage limits the operation, $x * ((p - k) * y * a - h - (f + F) * y * a/K)$, or $(p - k) * y * a - h - (f + F) * y * a/K$ must be maximised.

If the cut-off grade to be maximised is g and mineral processing limiting break-even cut-off grade is g_k, then g can be lowered for as long as $(p - k) * y * g$ is greater than $h + (f + F) * y * g/K$. Hence, the mineral processing limiting economic break-even cut-off grade is given by $(p - k) * y * g_k = h + (f + F) * y * g_k / K)$. So:

$$g_k = \frac{h}{(p - k - \frac{f + F}{K}) * y} \qquad (3.11)$$

where;

g_k: refinery or marketing limiting economic cut-off grade.

The marketing limiting economic cut-off grade formula is also different from the mining limiting economic cut-off grade formula because of the inclusion of the opportunity cost. As the mine ages, the opportunity cost declines. So does the marketing limiting economic cut-off grade.

3.1.3 Balancing cut-off grades

For the limiting economic cut-off grades, one stage of the operation limits the throughput. These cut-off grades are determined by economic factors. However, the optimum cut-off grade is not necessarily a limiting economic cut-off grade. As in any system composed of a number of components, more than one component might limit the operation. For cut-off grade optimisation, a pair of the components can limit the operation. These components are called balancing cut-off grades because they balance the capacities of pairs of stages of the operation. Sometimes all three of these stages limit the operation. If more than one stage limits the mining operation there should be

a balancing point at which related capacities of the mining operation stages are at full capacity.

Balancing cut-off grades are the values at which the capacities of pairs of stages operate at full capacity. Thus, the existing capacities of a mining operation limit the range of possible cut-off grades. As a result, the cut-off grades at the corresponding balancing points are possible candidates for the optimum cut-off grade.

Mining operations with more than one stage of the system working at capacity are preferable to those with only one limiting stage as idle capacities imply wasted fixed costs.

The mining stage normally has a maximum capacity determined by the capacities of the mining machinery and the haulage system (e.g. truck and road capacities in an open pit operation and shaft capacity in an underground operation). Similarly, the mineral processing plant is designed and built to a maximum capacity. Let the maximum depletion (mining) capacity be M units per year and the maximum mineral processing capacity be H units per year. The two stages are said to be in balance when the quantity of ore obtained from a given quantity of mineralised material and the corresponding mineralised material are in the ratio H/M. In other words, the balance occurs when the ore:material ratio is equal to the ratio of maximum processing plant capacity to maximum capacity of depletion. Hence, if we draw a cumulative grade distribution graph, there will be a point on the graph at which mineralised material is equal to the ratio H/M. This point is the grade that balances the mining and mineral processing stages and is denoted g_{mh}. Operating at this cut-off grade ensures that the two stages are working at full capacity. This situation is shown in Figure 3.1.

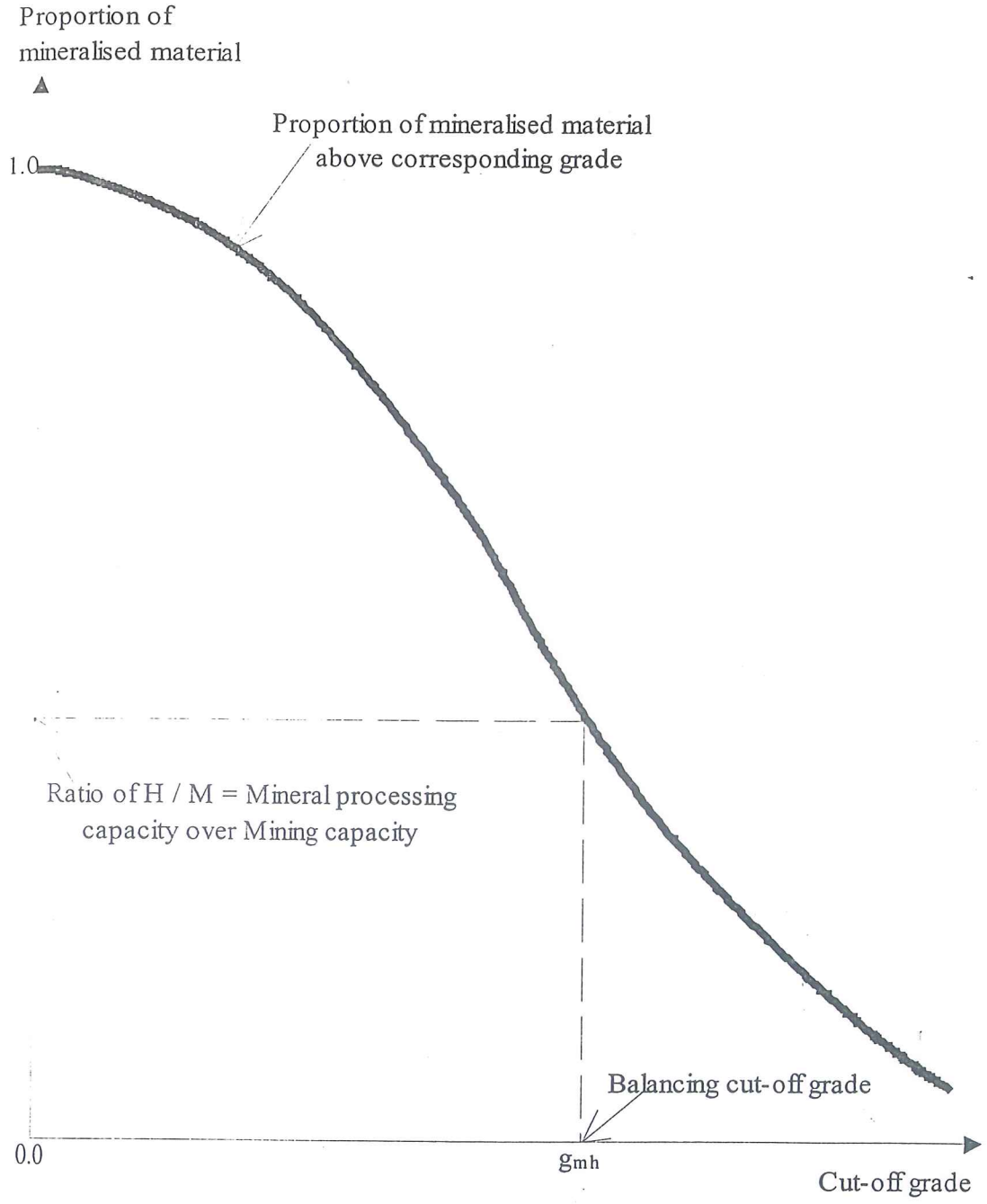

Figure 3.1 Cumulative grade distribution for a mine

The marketing stage also has a maximum capacity depending on the capacities of the refinery and/or the market. Let the maximum marketing capacity be K units per year. The two stages are said to be in balance when the quantity of mineral recovered from a given quantity of mineralised material and the corresponding mineralised material are in the ratio K/M. In other words, the balance occurs when the ratio of recoverable mineral per unit of mineralised material is equal to the ratio of refinery or marketing capacity to depletion capacity. If we draw a graph of recoverable mineral per unit of mineralised material versus cut-off grade, there will be a point on the graph at which the ratio of recoverable mineral to the whole mineralised material is equal to the ratio K/M. The grade at this point is the balancing cut-off grade for the mining and marketing stages and is denoted g_{mh}. Operating at this cut-off grade ensures that the two stages are working at full capacity. This situation is shown in Figure 3.2.

The mineral processing and marketing stages are said to be in balance when the quantity of mineral recovered from a given quantity of ore and the corresponding ore are in the ratio K/H. In other words, the balance occurs when the ratio of recoverable mineral per unit of ore is equal to the ratio of refinery/marketing capacity to mineral processing capacity. If we draw a graph of recoverable mineral per unit of ore versus cut-off grade, then there will be a point on the graph at which the ratio of recoverable mineral to the ore amount is equal to the ratio K/H. This point determines the cut-off grade that balances the mineral processing and marketing stages and is denoted g_{hk}. Operating at this cut-off grade ensures that the two stages are working at full capacities. This situation is shown in Figure 3.3.

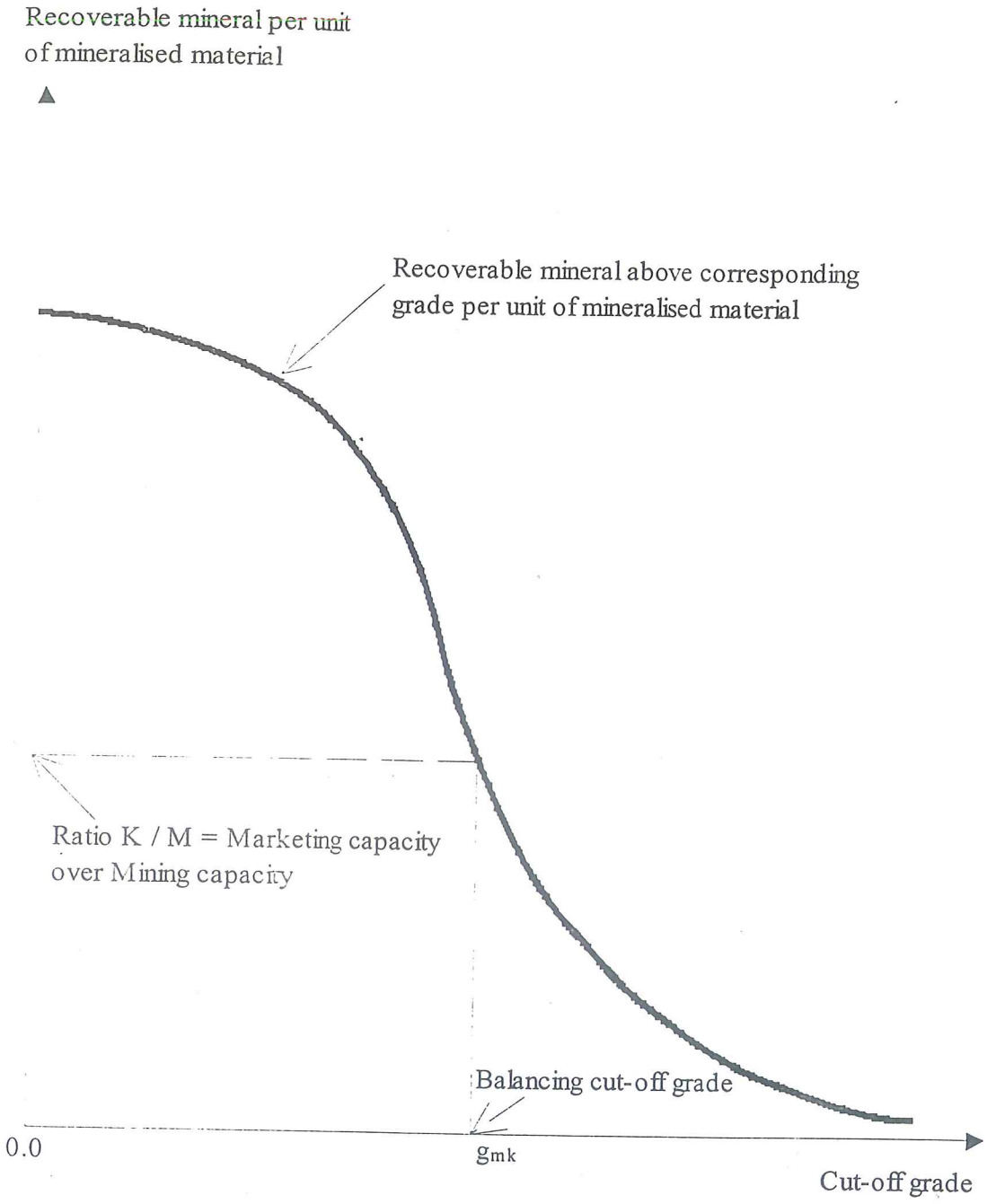

Figure 3.2 Recoverable mineral per unit of mineralised material as a function of grade

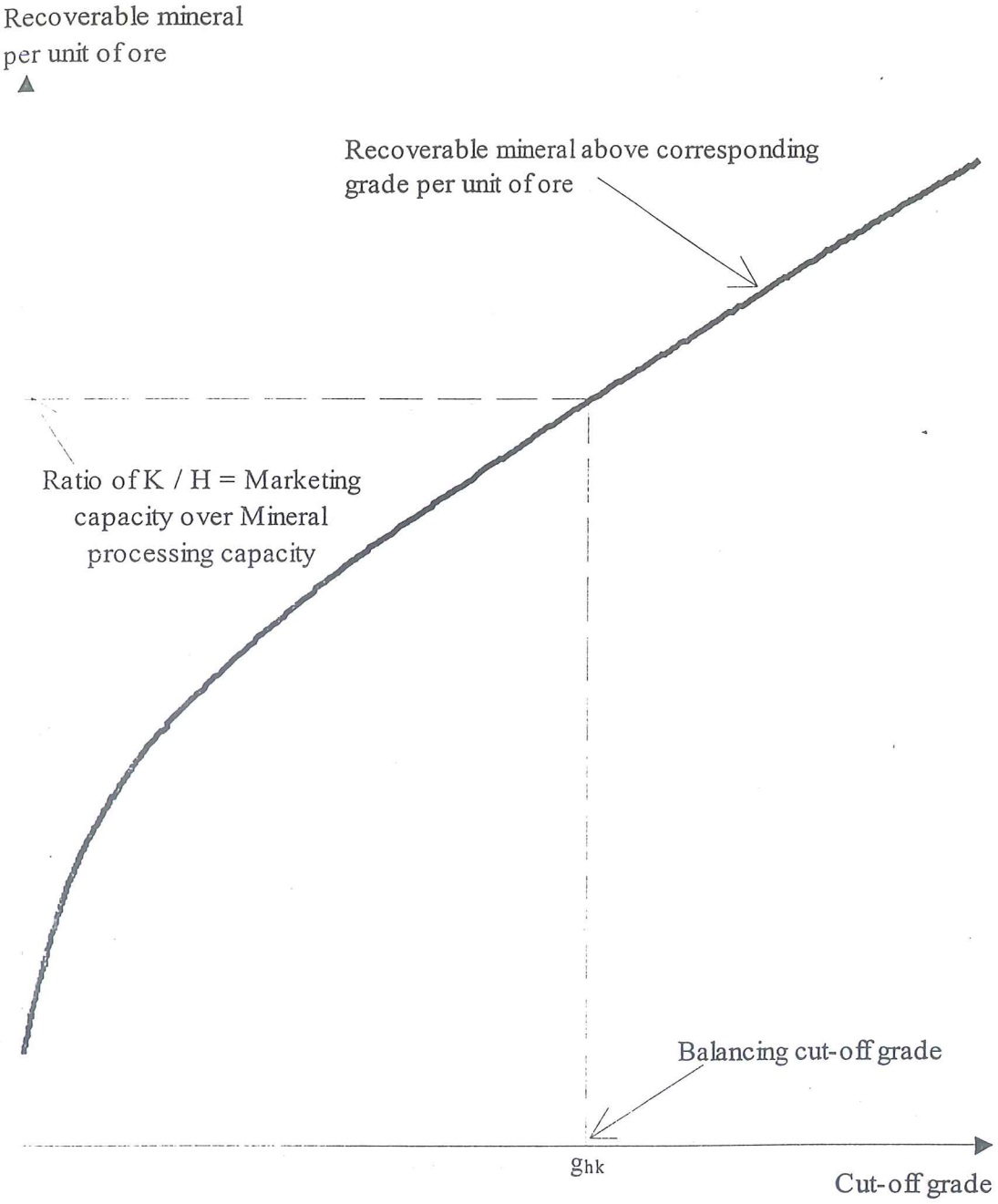

Figure 3.3 Recoverable mineral per unit of ore as a function of grade

The determination of balancing cut-off grades can be achieved graphically using figures 3.1, 3.2 and 3.3. However, the best way to determine them in practice is by calculating them from the grade-tonnage data for the mineral deposit. For each grade category, the ratios of potential ore to total material, the ratio of mineral to total

material and mineral to potential ore can be calculated by using the grade-tonnage data. The balancing cut-off grades are the grade categories for which these ratios are equal to the capacity ratios of the three stages.

If the ratios of the stages are not equal to the ratios H/M, K/M and K/H, the balancing cut-off grades can be found by interpolating between the corresponding ratios.

Mohammed[34] derived a set of formulas to find the balancing cut-off grades analytically. Assume that the grade-tonnage distribution of a mineral deposit consists of Z grade cells. Hence, there would be Z + 1 grade limits. The representation of the corresponding tonnage for different cells would be:

[g(1), g(2)], [g(2), g(3)], ..., [g(Z - 1), g(Z)], [g(Z), g(Z + 1)]

If the lower grade limit g(z) for a given cell [g(z), g(z + 1)] is the cut-off grade representing interval r, then the amount of material above the cut-off grade, amount of material below the cut-off grade and the average grade of ore above the cut-off grade can be found by using the following equations:

$$T_o(r) = \sum_{z=r}^{Z} T_z \tag{3.12}$$

$$T_w(r) = \sum_{z=1}^{r-1} T_z \tag{3.13}$$

$$g_{avg}(r) = \frac{\sum_{z=r}^{Z} T_z * \left[\frac{g(z)+g(z+1)}{2}\right]}{\sum_{z=r}^{Z} T_z} \tag{3.14}$$

where;

r: grade interval,

$To(r)$: amount of material above the cut-off grade for the r^{th} grade interval,

$T_w(r)$: amount of material below the cut-off grade for the r^{th} grade interval,

T_z: amount of material for the given interval,

$g(z)$: lower grade limit of a given cell,

$g(z+1)$: upper grade limit of a given cell,

$g_{avg}(r)$: average grade above the cut-off grade for the r^{th} grade interval.

The ratio of the amount of ore to the total amount of material in the deposit, the ratio of the amount of mineral to the total amount of material in the deposit and the ratio of mineral to the amount of ore for each grade category can be found by using equations 3.15, 3.16 and 3.17.

$$mh(r) = \frac{T_o(r)}{T_o(r) + T_w(r)} \tag{3.15}$$

$$mk(r) = \frac{T_o(r) * g_{avg}(r) * y}{T_o(r) + T_w(r)} \tag{3.16}$$

$$hk(r) = g_{avg}(r) * y \tag{3.17}$$

where;

$mh(r)$: ratio of ore to the total mineralised material for the r^{th} grade interval,

$mk(r)$: ratio of mineral quantity to the total mineralised material for the r^{th} grade interval,

$hk(r)$: ratio of mineral quantity to the ore for the r^{th} grade interval.

The balancing cut-off grades g_{mh}, g_{mk} and g_{hk} can then be found as follows:

For the mining and mineral processing balancing cut-off grade, the ratio of mineral processing plant capacity to depletion capacity is calculated. Then, the ratio (H/M) is compared to the ratios of ore to the total mineralised material (mh). If the ratio H/M is identical to one of the mh ratios, then the corresponding lower grade limit is the mining and mineral processing balancing cut-off grade, g_{mh}. Otherwise, the result is obtained by interpolating between the pair of mh values that contain the value of H/M. The interpolation procedure is:

$$mh(r) \geq \frac{H}{M} \geq mh(r+1) \tag{3.18}$$

$$\Delta_{mh} = \frac{mh(r) - \frac{H}{M}}{mh(r) - mh(r+1)} \tag{3.19}$$

$$g_{mh} = g(r) + \Delta_{mh} * [g(r+1) - g(r)] \tag{3.20}$$

where;

$g(r)$: lower grade limit for the r^{th} grade interval,

g_{mh}: mining and mineral processing balancing cut-off grade.

For the mining and marketing balancing cut-off grade, the ratio of marketing/refinery capacity to depletion capacity is calculated. Then, the ratio K/M is compared to the ratios of mineral quantity to total mineralised material (mk). If the ratio K/M is identical to one of the mk ratios, then the corresponding lower grade limit is the mining and marketing balancing cut-off grade, g_{mk}. Otherwise, the result is obtained by interpolation within the mk interval that contains K/M. The interpolation procedure is:

$$mk(r) \geq (K/M) \geq mk(r+1) \tag{3.21}$$

$$\Delta_{mk} = \frac{mk(r) - (K/M)}{mk(r) - mk(r+1)} \tag{3.22}$$

$$g_{mk} = g(r) + \Delta_{mk} * [g(r+1) - g(r)] \tag{3.23}$$

where;

g_{mk}: mining and marketing balancing cut-off grade.

For the mineral processing and marketing balancing cut-off grade, the ratio of marketing or refinery capacity to mineral processing plant capacity is calculated. Then, the ratio K/H is compared to the ratios of mineral quantity to the ore amount (hk). If the ratio K/H is identical to one of the hk ratios, then the corresponding lower grade limit is the mineral processing and marketing balancing cut-off grade, g_{hk}. Otherwise, the result is obtained by interpolation within the hk interval that contains the K/H value. The interpolation procedure is:

$$hk(r) \geq \frac{H}{M} \geq hk(r+1) \tag{3.24}$$

$$\Delta_{hk} = \frac{hk(r) - \frac{H}{M}}{hk(r) - hk(r+1)} \tag{3.25}$$

$$g_{hk} = g(r) + \Delta_{hk} * [g(r+1) - g(r)] \tag{3.26}$$

where;

g_{hk}: mineral processing and marketing balancing cut-off grade.

3.1.4 Optimum cut-off grades

The optimum cut-off grade is one of the six cut-off grades (three balancing and three limiting). The criterion for choosing the optimum cut-off grade from these six grades

is based on the net present value per unit of resource from mining the next unit of mineralised material. Thus, equation (3.5) must be maximised in order to determine the optimum cut-off grade.

Depending on the time taken to mine the next unit of mineralised material, the equation takes three forms. They are mining limiting net present value increment, mineral processing limiting net present value increment and marketing limiting net present value increment and are illustrated in equations 3.6, 3.8 and 3.10.

The graphs of all the three forms of increments in net present value as a function of the cut-off grades are convex upwards with a single maximum point. This maximum point corresponds to the limiting economic cut-off grade for the related component. Figure 3.4 shows the increment in net present value versus cut-off grade graph when a single stage, mining, limits the process.

If only one stage limits the operation, the maximum point, which gives the highest net present value, is the overall optimum cut-off grade. But if two stages together limit the operation, the possible solution points must be searched for the optimum cut-off grade. To illustrate the case for two limiting stages, the graphs of v_m and v_h, corresponding to the mining and the mineral processing stages, are superimposed and shown in Figure 3.5. The point of intersection of these two graphs is where the corresponding net present values for the two stages have the same value, i.e. $v_m = v_h$. From the equality of equations (3.6) and (3.8), we have $x = H/M$.

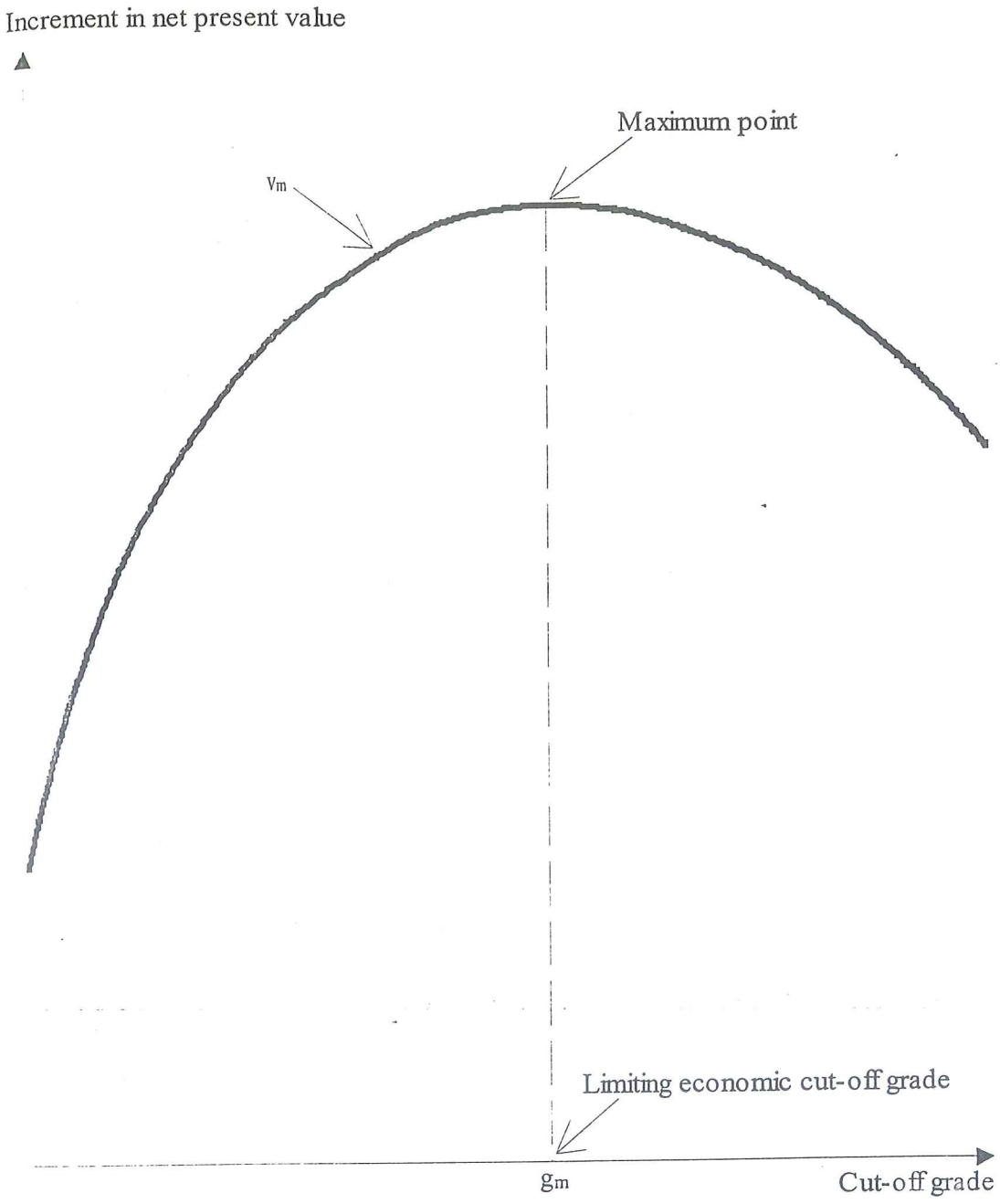

Figure 3.4 Increment in net present value where only mining component is limiting the operation

That is to say that the ore material ratio is equal to the ratio of processing plant capacity to depletion capacity. This means that the point of intersection in the Figure 3.5 corresponds to the balancing cut-off grade g_{mh}. On the other hand, it can easily be seen from Figure 3.5 that for cut-off grades less than the balancing point the mineral

processing plant limits the operation and for values greater than the balancing point the mining stage limits the operation. Therefore, the feasible parts of the net present value increment versus cut-off grade curve are the lower parts of the two curves below the balancing cut-off grade point.

The feasible part can be seen as a thicker line in the Figure. Since the point of intersection gives highest net present value, the balancing cut-off grade, g_{mh}, is the optimum cut-off grade as far as these two stages are concerned. As can easily be seen in the Figure, the curves for mining and mineral processing net present value intersect somewhere in between the two limiting economic cut-off grades. However, this is not always the case. The two other cases in which intersection of the two curves does not occur between the two limiting economic cut-off grades are shown in Figure 3.6 and Figure 3.7.

In Figure 3.6, the curves for v_m and v_h intersect at a point at which the balancing cut-off grade, g_{mh}, is lower than both the mining limiting economic cut-off grade, g_m, and the mineral processing limiting economic cut-off grade, g_h. For cut-off grades less than the balancing point the mineral processing plant limits the operation and for values greater than the balancing point the mining stage limits the operation. Therefore, the feasible part of the net present value increment versus cut-off grade curve is the lower part of the mineral processing net present value increment curve before the balancing cut-off grade point and the upper part of the mining net present value increment curve after the balancing cut-off grade point. In this case, the mining limiting economic cut-off grade, g_m, gives highest net present value. The mining stage is the bottleneck in the operation. Hence, g_m is the optimum cut-off grade as far as the two stages are concerned.

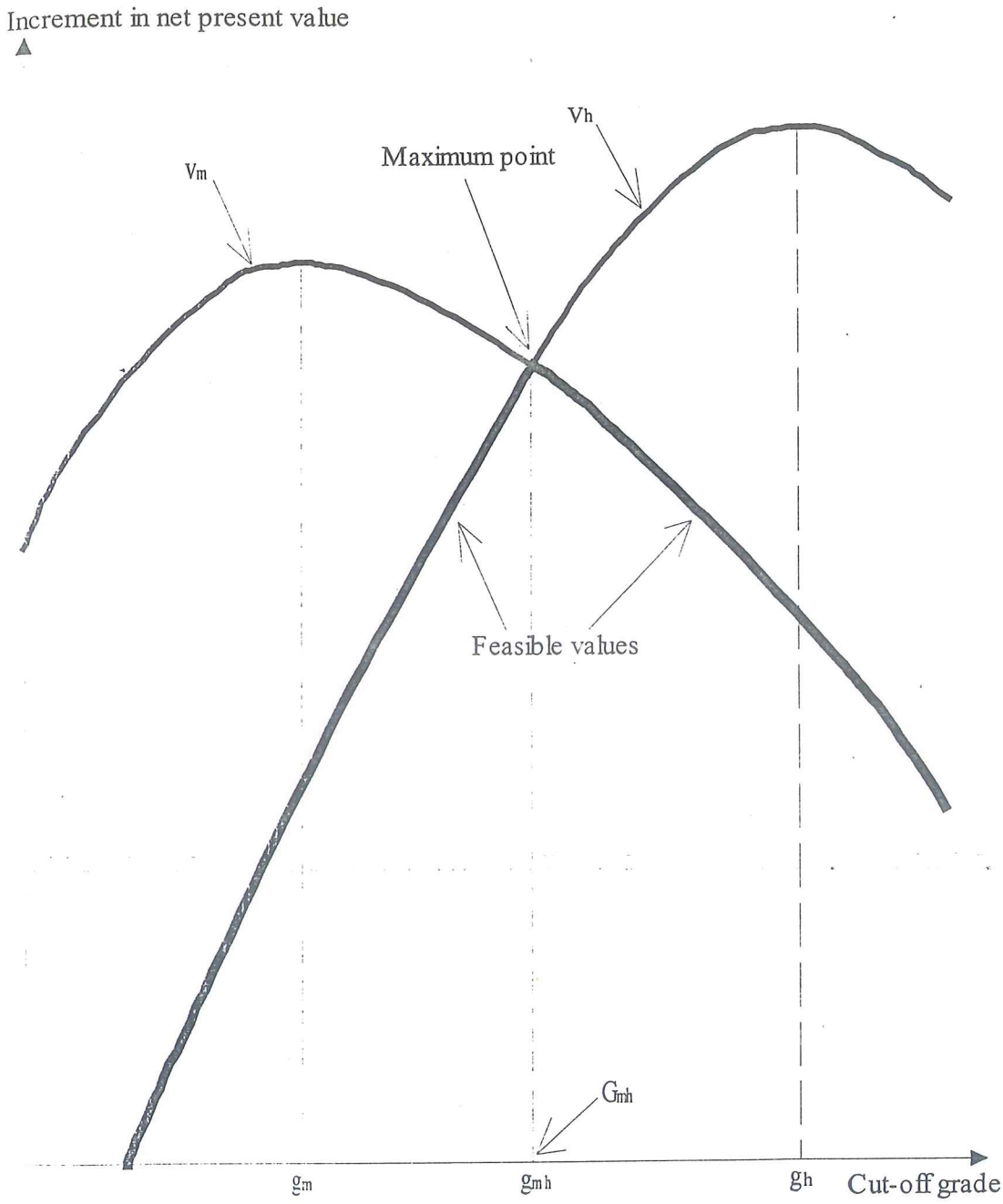

Figure 3.5 Increment in net present value where mining and mineral processing are limiting the operation

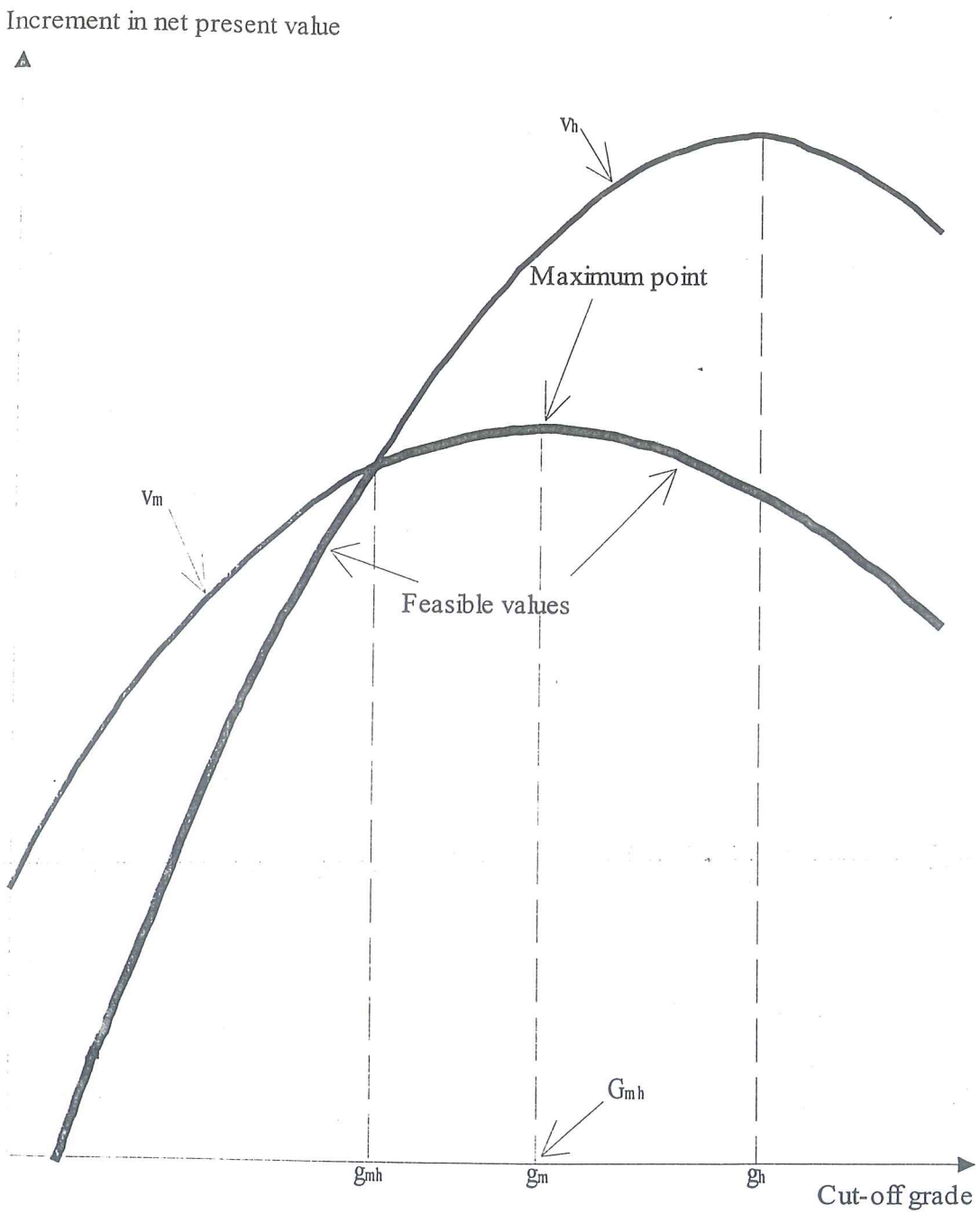

Figure 3.6 Increment in net present value where mining limiting cut-off grade is the optimum

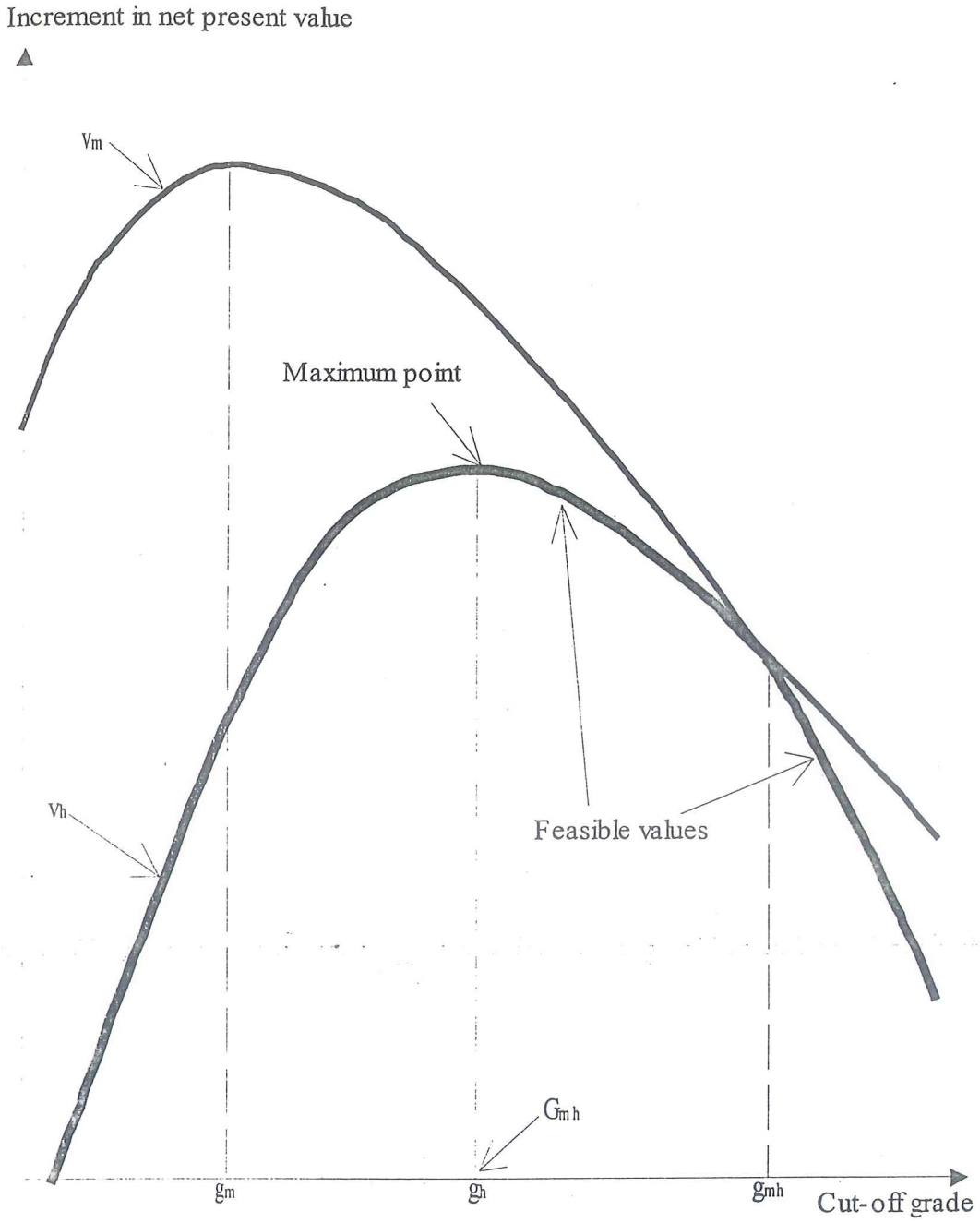

Figure 3.7 Increment in net present value where mineral processing limiting cut-off grade is the optimum

In Figure 3.7, the curves for v_m and v_h intersect at a point for which the balancing cut-off grade is higher than both the mining limiting economic cut-off grade

and the mineral processing limiting economic cut-off grade. For cut-off grades less than the balancing point the mineral processing plant limits the operation and for values greater than the balancing point the mining stage limits the operation. The feasible part of the curve is the upper part of the mineral processing net present value increment curve before the balancing cut-off grade point and the lower part of the mining net present value increment curve after the balancing cut-off grade point. In this case, the mineral processing limiting economic cut-off grade, g_h, gives highest net present value. The mineral processing plant is the bottleneck in the operation. Hence, g_h is the optimum cut-off grade as far as the two stages are concerned.

As a result, a rule can be formulated to find the optimum cut-off grade for a mining operation limited by the mining and mineral processing stages. If the optimum cut-off grade for the mining and mineral processing stages is G_{mh}, then the optimum cut-off grade can be found as follows:

If g_m is less than g_h ($g_m < g_h$),

$G_{mh} = g_m$ if $g_{mh} <= g_m$ (mine is the bottleneck)

$= g_h$ if $g_{mh} >= g_h$ (mineral processing is the bottleneck)

$= g_{mh}$ otherwise (mine and mineral processing are balanced) (3.27)

Else if g_m is greater than g_h ($g_m > g_h$),

$G_{mh} = g_h$ if $g_{mh} <= g_h$ (mineral processing is the bottleneck)

$= g_m$ if $g_{mh} >= g_m$ (mine is the bottleneck)

$= g_{mh}$ otherwise (mine and mineral processing are balanced) (3.28)

Else if g_m is equal to g_h ($g_m = g_h$),

$G_{mh} = g_h = g_m$ (mine and mineral processing are balanced) (3.29)

where;

G_{mh}: optimum cut-off grade for the case in which the mining and the mineral processing stages limit the operation.

There are other pairs of stages that may limit the operation: the mining and marketing stages and the mineral processing and marketing stages. Optimum cut-off grades for these cases can be found in the same way.

If G_{mk} is the optimum cut-off grade for the case in which the mining and marketing stages limit the operation, the optimum cut-off grade can be found as follows:

If g_m is less than g_k ($g_m < g_k$),

$G_{mk} = g_m$ if $g_{mk} <= g_m$ (mine is the bottleneck)

$= g_k$ if $g_{mk} >= g_k$ (marketing is the bottleneck)

$= g_{mk}$ otherwise (mine and marketing are balanced) (3.30)

Else if g_m is greater than g_k ($g_m > g_k$),

$G_{mk} = g_k$ if $g_{mk} <= g_k$ (marketing is the bottleneck)

$= g_m$ if $g_{mk} >= g_m$ (mine is the bottleneck)

$= g_{mk}$ otherwise (mine and marketing are balanced) (3.31)

Else if g_m is equal to g_k ($g_m = g_k$),

$G_{mk} = g_k = g_m$ (mine and marketing are balanced) (3.32)

where;

G_{mk}: optimum cut-off grade for the mine and market limiting case.

If G_{hk} is the optimum cut-off grade for the case in which the mineral processing and marketing stages limit the operation, the optimum cut-off grade is found as follows:

If g_h is less than g_k ($g_h < g_k$),

$G_{hk} = g_h$ if $g_{hk} <= g_h$ (mineral processing is the bottleneck)

$= g_k$ if $g_{hk} >= g_k$ (marketing is the bottleneck)

$= g_{hk}$ otherwise (mineral processing and marketing are balanced) (3.33)

Else if g_h is greater than g_k ($g_h > g_k$),

$G_{hk} = g_k$ if $g_{hk} <= g_k$ (marketing is the bottleneck)

$= g_h$ if $g_{hk} >= g_h$ (mineral processing is the bottleneck)

$= g_{mk}$ otherwise (mineral processing and marketing are balanced) (3.34)

Else if g_h is equal to g_k ($g_h = g_k$),

$G_{hk} = g_k = g_h$ (mineral processing and marketing are balanced) (3.35)

where;

G_{hk}: optimum cut-off grade for mineral processing and marketing limiting case.

The overall optimum cut-off grade, G, is one of the three optimum cut-off grades for the pair-wise stages, G_{mh}, G_{mk} and G_{hk}. This is illustrated graphically in Figure 3.8. The largest increase in net present value that can be achieved at any cut-off grade lies on the boundary defined by the lowest values of the v_m, v_h and v_k curves. This is shown in Figure 3.8 by the three bold curve segments. Hence, the optimum cut-off grade is the highest point on the bold line and it always occurs at the middle value of the optimum cut-off grades for the three pairs of stages, G_{mh}, G_{mk} and G_{hk}. Thus,

G = Middle value (G_{mh}, G_{mk}, G_{hk}) (3.36)

where;

G: overall optimum cut-off grade.

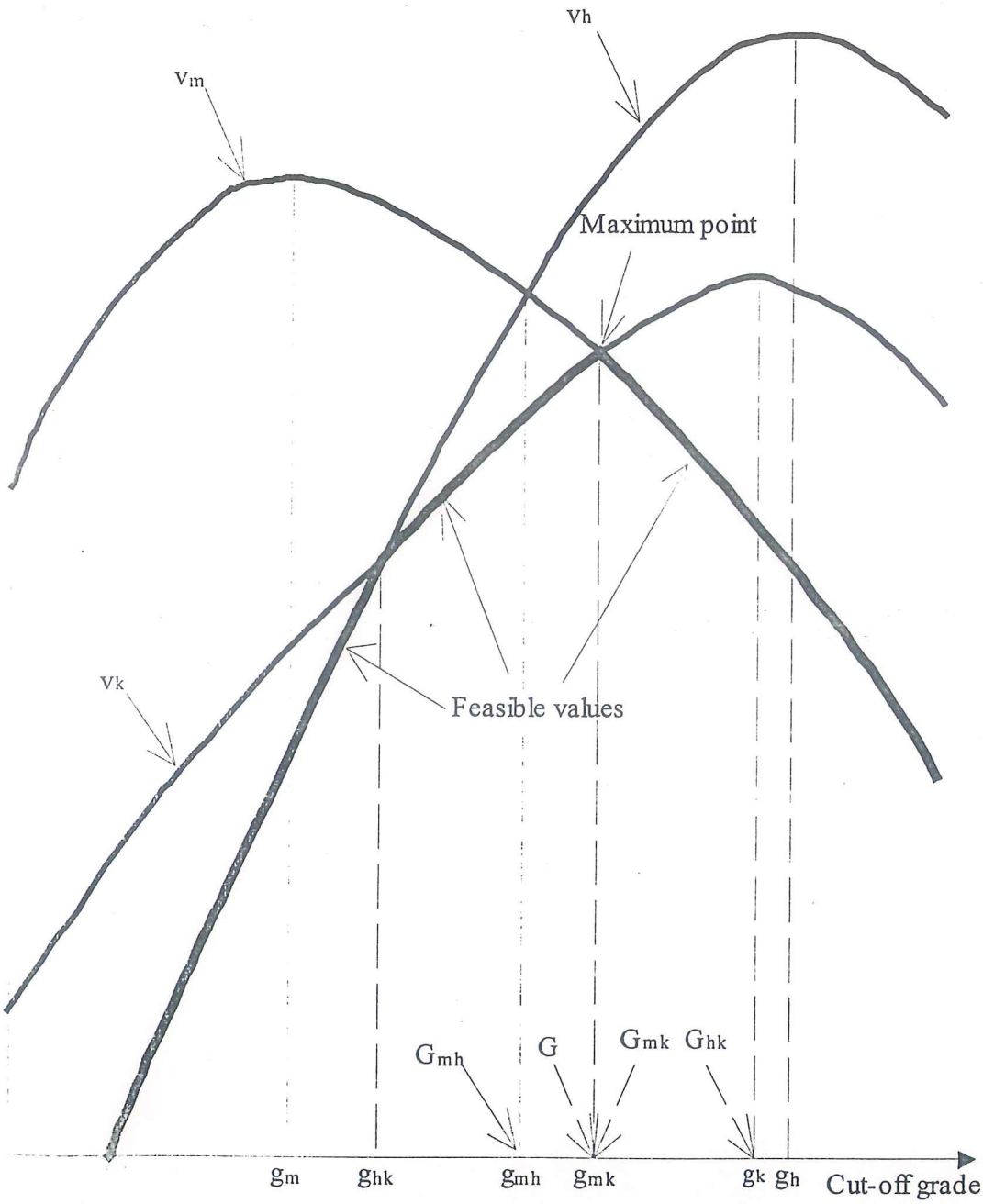

Figure 3.8 Increment in net present value where three stages are limiting the operation

The highest point on the three segments is not necessarily one of the intersection points as in Figure 3.8. Depending on the relative positions of the three incremental present value curves, it may be one of the maximum points g_m, g_h or g_k. In this case, the total capacity is limited by one stage only and two of the associated values of G_{mh}, G_{mk} and G_{hk} coincide. Figure 3.9 illustrates this position. Bold lines show the corresponding net present value increments that are possible, allowing for the capacity restrictions. In this figure G_{mh} and G_{hk} coincide at g_h. Although there aren't three different optimum cut-off grades from which to select the mid value, G_{mh} or G_{hk} can still be considered as the middle value. Hence, the overall cut-off grade is one of the two, G_{mh} or G_{hk}, since, in practice, both are the same, i.e., g_h. This is the effective overall optimum cut-off grade and the mineral processing plant alone limits the operation.

3.1.5 Steps of the algorithm

The algorithm for finding the optimum cut-off grade by Lane's method comprises the following steps:

1 Start with the input data: the selling price, mining cost, mineral processing cost, refinery and/or marketing cost, annual fixed cost, mining capacity, mineral processing capacity, refinery or marketing capacity, recovery, discount rate, grade categories and amount of the mineralised material associated with each of the grade categories.

2. Set the net present value V to zero.

3. Calculate the reserves available in the deposit.

4. Using equations 3.7, 3.9 and 3.11, find the mining limiting, the mineral processing limiting and the marketing limiting cut-off grades.

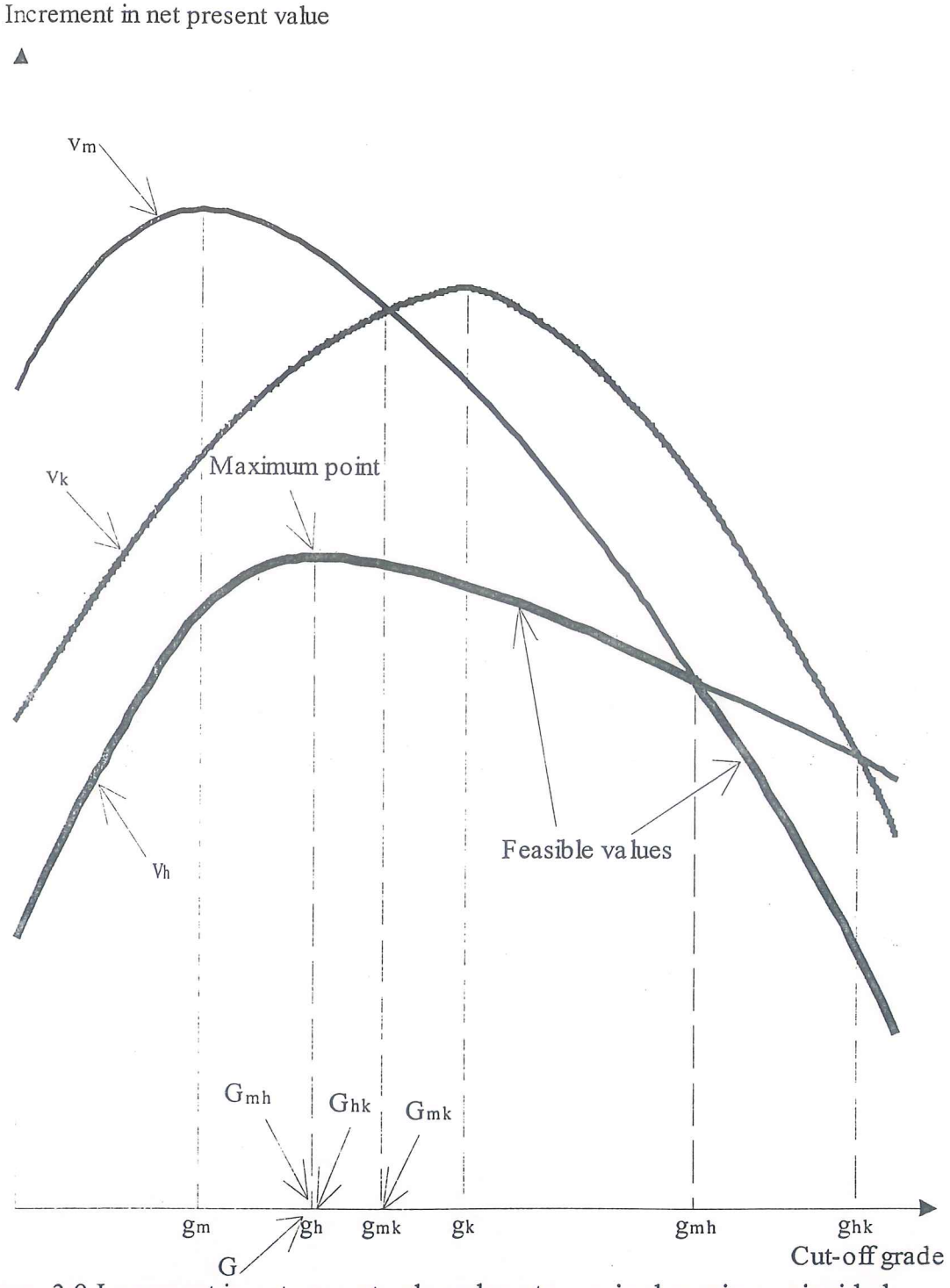

Figure 3.9 Increment in net present value where two paired maxima coincided

5. Calculate the amount of material above the cut-off grade, amount of material below the cut-off grade and the average grade of ore above the cut-off grade for each grade interval by using equations 3.12, 3.13 and 3.14.
6. Find the proportion of ore in the total amount of mineralised material in the deposit, the proportion of mineral in the total amount of mineralised material in the deposit and the proportion of mineral in the ore for each grade interval by using equations 3.15, 3.16 and 3.17.
7. Using expressions 3.18, 3.19, 3.20, 3.21, 3.22, 3.23, 3.24, 3.25 and 3.26 determine the mining and mineral processing balancing cut-off grade, the mining and marketing balancing cut-off grade, and the mineral processing and marketing balancing cut-off grade.
8. Determine the overall optimum cut-off grade by using equations 3.27, 3.28, 3.29, 3.30, 3.31, 3.32, 3.33, 3.34, 3.35, and 3.36.
9. Denoting the depletion rate by Q, use the following profit equation to calculate the annual profit for each year of the mine life when the optimum cut-off grade is used:

$$P = Q*[(p-k)*x*y*a - x*h - m] - f*t \qquad (3.37)$$

where;

P: profit,

Q: depletion rate.

10. Find the life of the mine from:

$$L = \frac{T_o + T_w}{Q} \qquad (3.38)$$

where;

L: mine life,

T_o: amount of ore,

T_w: amount of waste.

11. Find the net present value, V, by discounting profits at the specified discount rate d:

$$V = P * \frac{1 - \frac{1}{(1+d)^L}}{d} \qquad (3.39)$$

12. Using the net present value V and equations 3.7, 3.9 and 3.11, calculate the mining limiting, the mineral processing limiting and the marketing limiting cut-off grades.
13. Calculate the amount of material above the cut-off grade, amount of the material below the cut-off grade and the average grade of ore above the cut-off grade for each grade interval by using equations 3.12, 3.13 and 3.14.
14. Find the proportion of ore in the total amount of mineralised material in the deposit, the proportion of the mineral in the total amount of mineralised material in the deposit and the proportion of mineral in the ore for each grade interval by using equations 3.15, 3.16 and 3.17.
15. By using expressions 3.18, 3.19, 3.20, 3.21, 3.22, 3.23, 3.24, 3.25 and 3.26 determine the mining and mineral processing balancing cut-off grade, mining and marketing balancing cut-off grade, and the mineral processing and marketing balancing cut-off grade.

16. Determine the overall optimum cut-off grade by using equations 3.27, 3.28, 3.29, 3.30, 3.31, 3.32, 3.33, 3.34, 3.35, and 3.36.
17. Using equation 3.37, calculate the annual profit for the life of the mine for the optimum cut-off grade.
18. Find the life of the mine from formula 3.38.
19. Find the net present value for next year from:

$$V_n = V - \frac{P}{(1+d)^L} \qquad (3.40)$$

where;

V_n: net present value of future profits for year after.

20. Set $V = V_n$.
21. Calculate the reserves available in the deposit. If the reserves are greater than zero, go to step 12. Otherwise, go to the next step.
22. Find a new net present value by discounting the profits from each year and summing them. Compare this net present value with the net present value used to find the limiting economic cut-off grades in step 12. If the calculated net present value has converged to the net present value used in the calculations, go to the next step. Otherwise, set the net present value to be used in calculations to the calculated net present and go to step 12.
23. Write out the results.

3.2 Optimisation by dynamic programming

3.2.1 Introduction

Dynamic programming refers to a collection of mathematical tools used to analyse sequential decision processes. The functional equations used in dynamic programming predate the technique but it was Bellman[2] who first formulated the principle of optimality and used it to analyse hundreds of optimisation problems in mathematics, engineering, operational research and other fields. Many of his contributions are compiled in his book on dynamic programming (Bellman[2]) and in his joint work with Dreyfus[3].

Dynamic programming is based on the principle of optimality for sequential decision processes, stated by Bellman[2] as:

> "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Essentially, dynamic programming describes an approach to problem solving in which an instance of a problem can be divided into smaller, related instances. The smaller instances are solved and the results are stored. These results are then retrieved, rather than re-computing them, whenever they are required. The "programming" part of the name refers to the use of a look-up array, or sequence of arrays, in which the solution is constructed. The steps in developing a dynamic programming algorithm are (after Neapolitan and Naimipour[36]):

1. Establish a recursive property that gives the optimal solution to an instance of the problem in terms of sub-instances of the problem.

2. Compute the value of an optimal solution in a bottom-up manner by solving sub-instances first.

3. Construct an optimal solution in a bottom-up manner.

These steps can be implemented to yield an optimal solution only if the principle of optimality applies, i.e. if an optimal solution to an instance of the problem always contains optimal solutions to all sub-instances.

Dynamic programming is a reasonably well-used method in the field of operational research and can be used to find an optimal sequence of decisions for multi-stage decision problems. Dynamic programming is especially suitable to the solution of optimisation problems in which decisions have to be taken sequentially over several time periods. It is assumed that the periods are linked in a way that actions taken in any particular period affect the decision environment in all future periods. The problem is divided into sub-problems and an optimal solution is found for each of the sub-problems. A problem with N decision variables can be transformed into N sub-problems, each containing only one decision variable. The computation time increases exponentially with the number of variables, but only linearly with the number of sub-problems.

It is convenient to consider the whole problem as a system and sub-problems as stages. Stages in dynamic programming generally represent a sequence of time. However, depending on the nature of the problem, some other means may be used instead, e.g. depletion of a mineral deposit.

At each stage assigned to the system, there will be a number of possible states that correspond to the possible conditions, properties or states of the problem. States characterise conditions of incomplete solutions. At each stage the decision-maker

should choose the decision that gives the best return. A decision moves the system from one state to another. A single step in such a process that corresponds to the transition of a system from one state to another is called a stage of the system.

Elementary dynamic programs are often most easily conceptualised as a backward sequence, that is, proceeding from final to initial conditions (Rardin[39]). This backward induction technique is performed by going backwards from the final state to the states of the previous stages.

There have been later contributions to the technique. Blackwell[5] worked on discounted dynamic programming. Rangarajan[38] presented the theoretical background of two different approaches; namely, finite horizon dynamic programming and stationary discounted dynamic programming.

3.2.2 Classification of dynamic programming

There are many different kinds of dynamic programming applications, arising mainly from the nature of the problems.

The classification discrete, as opposed to continuous, refers to the way in which a variable is defined with respect to time. A discrete variable is defined only at a set of discrete points in time. For example, in a machine replacement problem, if the machine age in years is used as a measure of the state of machine wear at each stage, it is a discrete variable. This is, of course, a definition of convenience and, in reality, a machine deteriorates gradually and continuously. Dynamic programming processes in which all the variables are discrete variables are called discrete dynamic programming processes. Otherwise, they are called continuous dynamic programming.

The classification deterministic, as opposed to probabilistic, refers to the way in which the value of a function f(x) is specified by the value of its argument x. The value of a deterministic function is completely specified by the value of its argument. In the machine replacement problem mentioned above, if the return function f(x) is an exact relationship between age of the machine and return function (i.e., cost of replacement) it is assumed to be a deterministic function. In reality, the replacement cost of the machine may also depend on unpredictable factors, such as changes in the selling price of new machinery. Such unpredictable factors can be represented by random variables and the return value may then be described more realistically by a function of the age of the machine and of the random variables. This kind of a function, with a value that depends partly on random variables, is called a probabilistic function. Dynamic programming processes that can be described entirely by deterministic functions are called deterministic dynamic programming. Whereas dynamic programming processes that use a probabilistic benefit function are called probabilistic, or stochastic, dynamic programming.

The classification finite horizon, as opposed to stationary discounted, refers to the overall horizon within which dynamic programming takes place. In finite horizon dynamic programming, the objective is to choose a plan for taking actions at each point in time in order to maximise the sum of the returns over the horizon of the model. The sum to be maximised is (after Rangarajan[38]):

$$\text{maximise} \quad \sum_{t=1}^{T} r_t(s_t, a_t) \tag{3.41}$$

where;

r_t: return function for period (or year or stage) t,

s_t: state of the problem for period t,

a_t: decision to be taken for period t.

However, infinite horizon models lack a last period and this makes it impossible to use backward induction techniques to derive an optimal strategy. The decision-maker is presumed to discount future returns by using a discount factor. Therefore, if an action is taken in a specific period at a specific stage, the return is discounted by multiplying it by the discount factor for the period of time. So, the sum to be maximised is(after Rangarajan[38]):

$$\text{maximise} \quad \sum_{t=0}^{\infty} d^t r(s_t, a_t) \tag{3.42}$$

where;

d: discount factor.

3.2.3 Application of dynamic programming to cut-off grade optimisation

Dynamic programming has been applied to a number of mining problems, such as optimisation of cut-off grades and production rates (Dowd[14,16]), production scheduling (Dowd and Onur[20], Yun and Yegulalp[48]), grade control in sub-level stopping (Dowd and Elvan[19]) and optimal open pit design (Lerchs and Grossmann[31]). Roman[40] introduced dynamic programming to determine production rates and Dowd[14] extended it to optimise both production rates and cut-off grades.

Multistage decision processes and the techniques of dynamic programming are most conveniently introduced by discussing an example of a multistage decision process. The following example is a simplified case inspired by a paper of Dowd[14] to

show how dynamic programming can be applied to cut-off grade optimisation problems.

3.2.3.1 Example: a cut-off grade optimisation problem

Consider a small deposit that can be mined at two different cut-off grades. The maximum production rate of the mine is 500 tonnes a year. Assume that total amount of ore available is 4 000 tonnes if the lower cut-off grade is used. The deposit will be depleted in eight years since the maximum production rate of the mine is 500 tonnes a year. On the other hand, 2 000 tonnes of ore is available at the higher cut-off grade and the deposit would be depleted in four years at this cut-off grade. Suppose that 1 tonne of ore at the lower grade can be sold for £10 and 1 tonne of ore at the higher grade can be sold for £16. The discount rate is 10%.

The problem here is to find the optimal cut-off grade policy that gives the maximum net present value at the end of depletion of the deposit. The possible decisions are illustrated in Figure 3.10. Since the problem is a very small scale and simplified one, it is possible to solve it graphically step by step. It should be understood that this is a very elementary example intended only to demonstrate the basic concepts of dynamic programming.

This is an example of discrete deterministic stationary discounted dynamic programming. Because of the nature of the problem, there is a starting point but there are many end points. In other words, there is a group of possible final points on the upper horizon of the figure, which represent depletion of the deposit.

In the first year, either the deposit will be exploited by applying a lower cut-off grade, indicated by the point A, or the higher cut-off grade leading to the state

indicated by the point B. If the lower cut-off grade is applied, one-eighth of the deposit would be mined and the discounted value of the first year's profit would be £4545. However, if the higher cut-off grade is applied, one-fourth of the deposit would be mined and the discounted value of the first year's profit would be £7273. As a result, we can write the value £4545 next to point A and the value £7273 next to point B, each representing the value of the routes to these points.

In the next stage of the computation, the states of the system are indicated by the points C, D and E. In order to find the return values of these points, one must look for the routes from the point O to C, O to D and O to E. But as a consequence of the principle of optimality, it is only necessary to consider the routes from the points A and B, since they have already been assigned optimal values. For the point C, there is only one route and it is OAC. As a result, one can write £8677 as the value of arriving at point C in an optimal manner; this value is obtained by adding the values of point A to the value of the decision AC. For point E, there is also only one route and it is OBE. Therefore, one can write £13885 as the optimum value of the second year's profit for that point as the sum of the values of state B and the decision BE. However, for D, there are two routes to be considered: OAD and OBD. The value of the route OAD is £11157 and the value of the route OBD is £11405. Route OBD has the higher value and thus the optimum value of the second year's profit for state D is £11405. For the third year, the optimal routes from the starting point O to the points F, G, H and I are searched to find the optimal values of profits of these states. In doing so, one need only consider the values of the points in the previous year, C, D and E, and the routes starting from these points. This process is continued until the upper depletion horizon is reached. At the end of the process, the highest value among

the return values of the points on the upper horizon of the figure gives the maximum profit. That point is Z1. Consequently, the route that leads to the maximum value is the optimal route. The optimal route could be found by tracing back easily. The optimal route is OBDGKRWZ1, which means mining at the higher cut-off grade during the first year and thereafter mining at the lower cut-off grade. The maximum net present value of this policy is £27070.

An expression for this principle of optimisation can be formulated (Dowd[14]) as follows:

Let

$$R(x_1, y_1; x_2, y_2; \ldots\ldots; x_n, y_n) = g_1(x_1, y_1) + g_2(x_2, y_2) + \ldots\ldots + g_n(x_n, y_n) \qquad (3.43)$$

where;

R: total return,

n: period,

x_n: reserve amount in period n,

y_n: grade percentage in period n,

g_n: return in period n.

The objective is to maximise the return *R* subject to the constraints

$x_i \geq 0$ and

$$\sum_{i=1}^{n} xi = x$$

where;

x: total ore reserve.

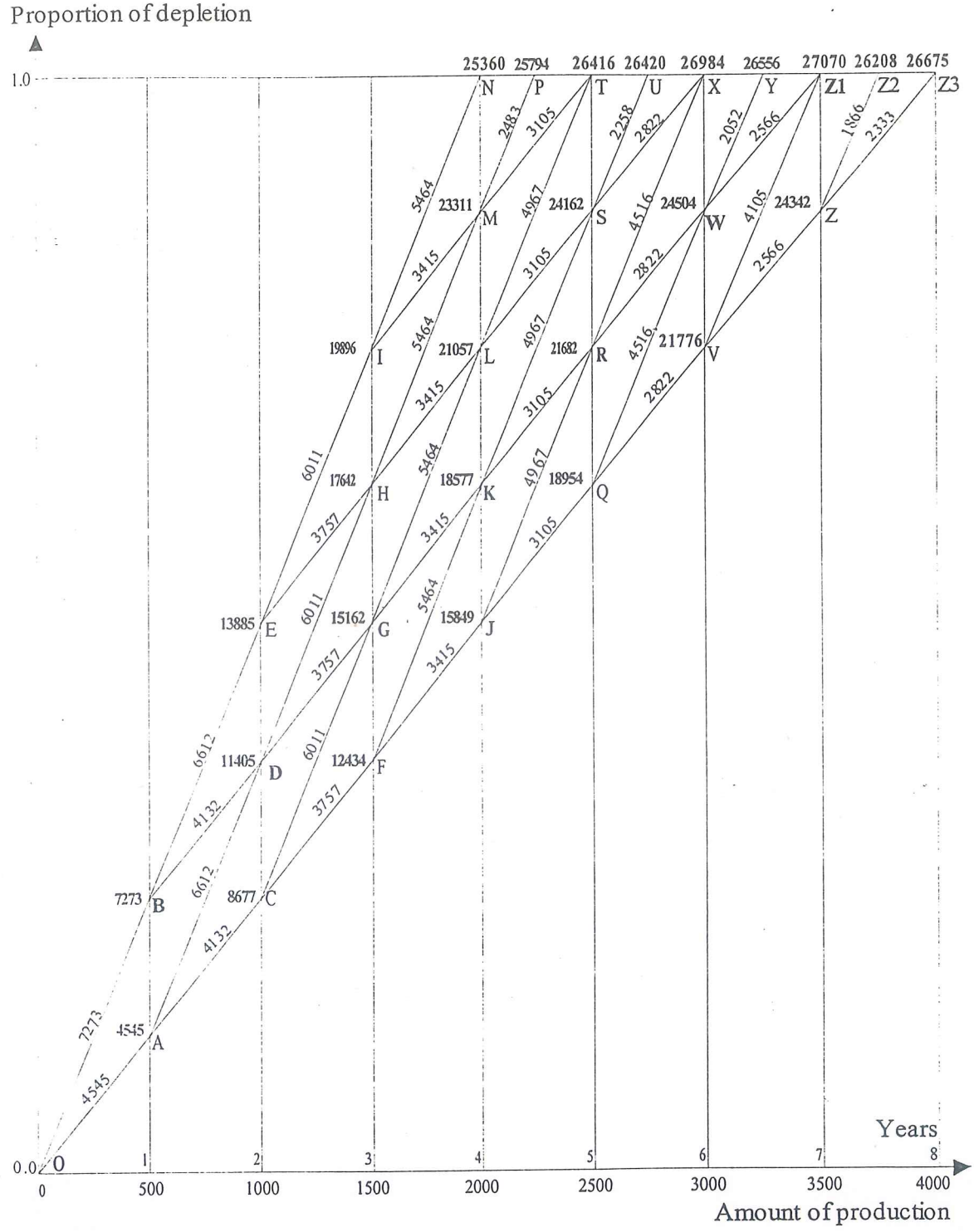

Figure 3.10 Decision graph

The basic functional equation for this maximisation is

$$f_n(x) = \underset{x_n, y_n}{max}[g_n(x_n, y_n) + f_{n-1}(x - x_n)] \quad (3.44)$$

where;

f_n: the return for *n* periods.

3.2.4 Steps of the algorithm

A deterministic dynamic programming algorithm for finding the optimal sequence of cut-off grades and production rates comprises the following steps:

1. Input the data: the selling price, mining cost, mineral processing cost, refinery and/or marketing cost, annual fixed cost, mining capacity, mineral processing capacity, minimum depletion interval, refinery or marketing capacity, recovery, discount rate, grade categories, amount of the mineralised material associated with each of the grade categories, lowest limit of cut-off grade, highest limit of cut-off grades and interval between cut-off grade decisions.
2. Calculate the reserves available in the deposit.
3. Calculate the amount of material above the cut-off grades, amount of material below the cut-off grades and the average grade of ore above the cut-off grades for each grade interval by using equations 3.12, 3.13 and 3.14.
4. Determine the cut-off grades to be searched by using the lower and upper limits for cut-off grade decisions and the interval between the cut-off grade decisions.
5. Calculate the amount of material above each of the cut-off grades, the amount of material below the cut-off grades and the average grade of ore above the cut-off grades for each different cut-off grades to be searched by using interpolation.

6. Calculate the number of different depletions and the different depletions possible according to mining capacity, mineral processing capacity and minimum depletion interval given.
7. Calculate the depletion rates and the production rates associated with each of the cut-off grades, considering maximum capacities of mining, mineral processing and marketing stages.
8. Using profit equation 3.37, calculate the annual profits for each of the cut-off grades.
9. Find the optimal cut-off grades for each of the depletion rates by assigning the cut-off grade that gives the maximum annual profit value.
10. Start with the first year.
11. Assign cut-off grades and production rates based on the different depletion rates found in step 9, with each depletion rate being the state of the year (or stage).
12. Assign the profit values of each state as maximum discounted profit values of that state.
13. Go to the next year (or stage).
14. Start from the first state of the previous year.
15. Start from the first of the different depletion rates found in step 9.
16. Assign a new state for this year (or stage) by applying the different depletion rates to move from the previous year's states. If the cut-off grade of the new state is higher than the previous state, omit it.
17. Calculate the discounted profit from the cut-off grades and production rates assigned for each year, by using the equation:

$$P_n = \frac{P}{(1+d)^n} \tag{3.45}$$

where;

P_n: discounted profit for year n,

P: profit,

d: discount rate,

n: year (or stage).

19. If there is another solution in the same state, i.e. one with the same depletion amount, the one that gives the higher discounted profit should be assigned as the solution of the state.
20. Find the reserve left by subtracting the depletion rate used from the total reserve.
21. Go to the next depletion rate. If this is the final depletion rate go to the next step. Otherwise, go to step 16.
22. Go to the next state of the previous year. If this is the final state go to the next step. Otherwise go to step 15.
23. If the remaining reserve is equal to zero for all states of this stage, go to the next step. Otherwise, go to step 13.
24. Find the state that gives maximum total discounted profit.
25. Find the previous states of previous stages that yield the maximum total discounted profit.
26. Stop and write out the cut-off grades and production rates that give the maximum total discounted profit.

3.3 Optimisation by genetic algorithms

3.3.1 Introduction

There are many optimisation problems for which reasonably fast searching algorithms have not been developed. For some problems we can use probabilistic algorithms, such as simulated annealing or hill climbing, although they do not guarantee the global optimum value. For more than a quarter of a century, there has been a growing interest in solving such problems using evolutionary principles. The common term, *evolution programs,* is used for all evolution-based systems (Michalewicz[33]). The basic principles of an evolution program can be seen in Figure 3.11.

In 1975 Holland[27] proposed the basic principles of genetic algorithms. In this approach, individuals of a population are represented as chromosomes and an expanded set of genetic operations takes place. Genetic algorithms are stochastic algorithms whose search methods are based on the principles of biological genetics and natural evolution. It is presumed that the potential solution of any problem is an individual and can be represented by a set of parameters.

The vocabulary of genetic algorithms is borrowed from genetics science. In nature, each cell of every living organism has a set of chromosomes that make up DNA. Chromosomes are made up of genes, which control different characteristics of an organism. In genetic algorithms, a potential solution to a problem is called an individual or chromosome. Individuals make up a population. Genetic operations, such as crossover, mutation and reproduction, are also used in genetic algorithms.

```
Evolution Program(){
    // start
    t = 0;
    // initialize a population of individuals
    population(t);
    // evaluate the individuals
    evaluate population(t);
    do{
        t = t + 1;
        // select a new population from previous one
        newPopulation(t) = selectedPopulation(t);
        // alter the individuals
        alteredPopulation(t) = alteredPopulation(t);
        // evaluate the individuals
        evaluate alteredPopulation(t);
    }while (program does not converge);
}
```

Figure 3.11 The structure of an evolution program.

Genetic algorithms are used to solve large-scale optimisation problems. They belong to the class of probabilistic algorithms, yet they are very different from the methods based on only random numbers as they combine directed and stochastic searches. Because of this, genetic algorithms are more robust than traditional, directed search methods. Another important property of such genetic-based search methods is

that they maintain a population of potential solutions – all other methods process a single point of the search space (Michalewicz[33]).

A genetic algorithm structure is shown in Figure 3.12 and a flowchart for genetic algorithms is illustrated in Figure 3.13.

The basic principles of genetic algorithms are:

1. A set of strings composed of finite elements, generally a binary code, is assigned. Each string refers to a point in the search space or a solution to the problem among the alternatives. Genetic algorithms work on these strings, called chromosomes or individuals.
2. A first generation, i.e., a population, of individuals is selected. Generally, the selection is done at random.
3. The individuals are evaluated on the basis of their return values. Fitness values are assigned to the individual in order to rank them on the basis of their return values. The ones assigned to better solutions result in higher fitness values.
4. Some of the individuals are selected on the basis of their fitness values. The individuals with lower fitness values lose in competition.
5. Parents are chosen from among the selected individuals. They are crossed over by pairs. The result is two new individuals from each parent.
6. Some chromosomes enter a mutation process. That is, one or more digits of a string are changed at random. A new population is ready.
7. The process is repeated from step 3 until it converges to a stable value or an assigned number of generations is reached.

```
Genetic Algorithm(){
    // initialize the generation counter
    g = 1;
    // start with a random population
    p(g) = RandomPopulation();
    // evaluate fitness values of individuals of the population
    evaluate p(g);
    // generations
    do{
        // increase the generations counter
        g = g + 1;
        // select a new population for reproduction
        p(g) = select p(g);
        // crossover
        crossover p(g);
        // mutation
        mutate p(g);
        // evaluate fitness values of individuals of the population
        evaluate p(g);
    }while (program does not converge);
}
```

Figure 3.12 A genetic algorithms structure.

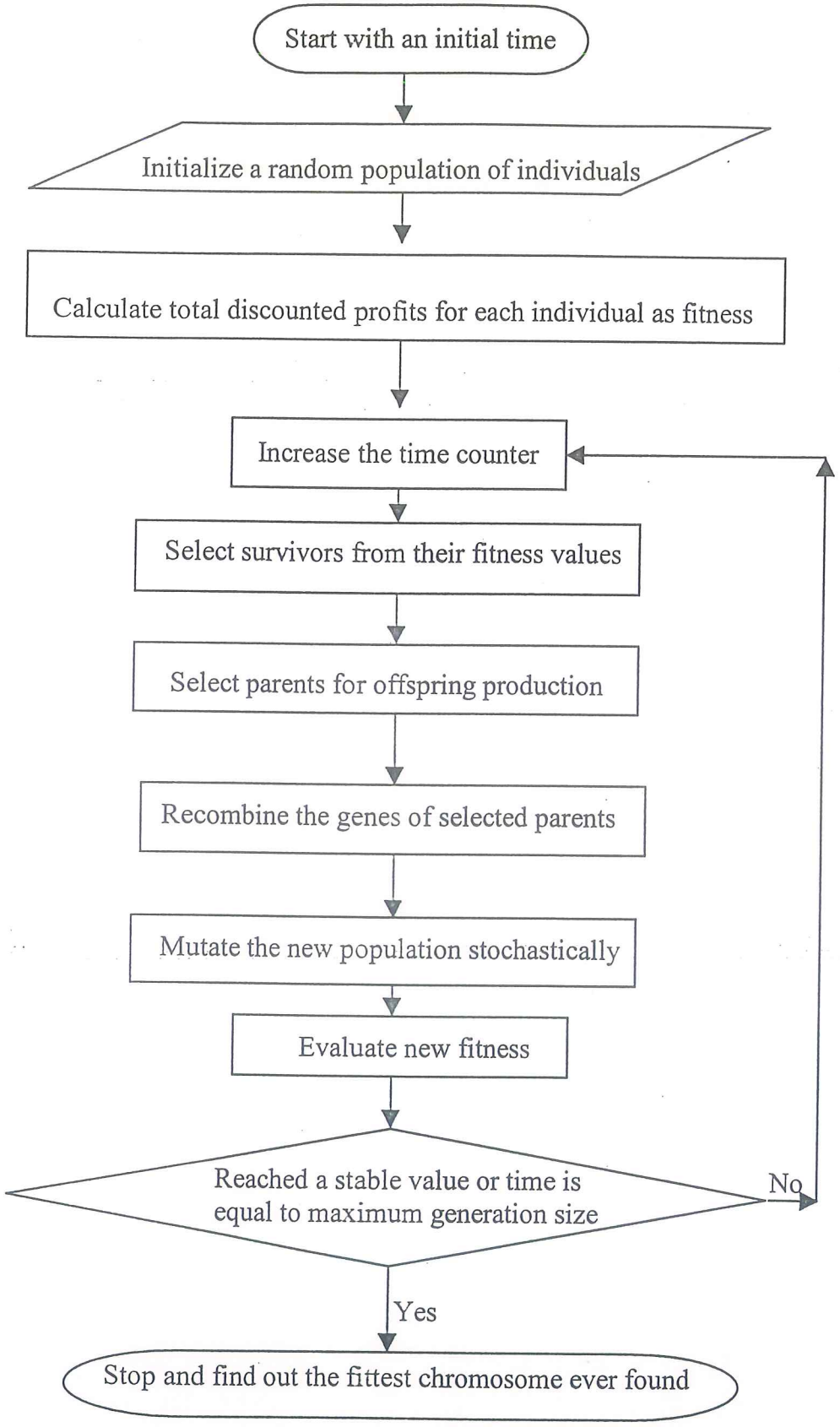

Figure 3.13 A genetic algorithms flowchart

3.3.2 Major Elements of Genetic algorithms

3.3.2.1 Encoding

Genetic algorithms work with representations of solutions rather than the solutions themselves. The representation is a symbol string, which carries all the information about the individual. The string has a fixed length and is called a chromosome or individual. The length of the string of an individual depends on the precision requirements. The string can be composed of real decimal numbers or characters, but the most widely used string representation is binary.

The mapping value of a binary string into a real number is straightforward. The binary string is converted into a real number, which is an integer. Then a corresponding real number, which is the mapping value, is found. The use of binary representation and the evaluation of the mapping value are described in detail by Michalewicz[33].

In order to initialise an individual composed of binary strings, all bits would be randomly initialised.

3.3.2.2 Population

In genetics, individuals make up a population. The bigger the population, the more extended is the search area. But the number of individuals adversely affects the speed of the algorithm. Typically, a population is composed of between 30 and 100 individuals (Yun et al[47], Zalzala and Fleming[49]).

3.3.2.3 Evaluation

After initialisation of a population made up of binary or real number strings, an evaluation process takes place. Each individual is assigned a fitness value, which is calculated on the basis of objective function for the problem.

3.3.2.4 Selection

Good individuals with better fitness values are selected. Each generation produces new individuals out of the current population. Selection is a process of finding how many times each individual from the current population should be copied to generate a new set of solutions or a new population. The process resembles natural selection in that individuals that give better results in the evaluation process have a greater chance of reproducing. The selection process consists of determining the number of times that a particular individual is chosen to have offspring. The selection process can be deterministic or probabilistic.

In deterministic selection, better individuals are determined to have more offspring than poor ones. Individuals with very low fitness values have no chance of survival. This process can be compared with the assignment of members of parliament after an election. Political parties having more votes assign more members to parliament. But there is no unique system of doing so. Rather, there are many different systems, as in the case of the deterministic selection process of genetic algorithms. It might be argued that the selection should be a deterministic one since it helps to get rid of poor individuals and to generate a quick result. But this approach always carries the risk of convergence to a local maximum value, rather than finding the global maximum (Falkenauer[23]).

The roulette wheel selection method is the most widely used selection method in genetic algorithms. It is a probabilistic method in which individuals with better fitness values are more likely to reproduce, although weak individuals still have a chance of survival. Individuals are represented on the wheel as a proportion of their fitness values. There are different kinds of roulette wheel selection methods. The basic roulette wheel selection method is stochastic sampling with replacement (Zalzala and Fleming[49]), where the selection probability of each individual remains the same throughout the selection phase while in other methods the probability could be changed.

There are other parameters that can be applied to the deterministic and probabilistic approaches. Scaling is one of them. When the fitness values of individuals of a population are sufficiently distinct, there is no need for any kind of scaling. But if the fitness values are close to each other, which is generally the case as generations pass and most of the individuals have relatively good fitness values, good individuals will lose competitiveness. Scaling is used to improve the situation. The individuals are scaled in order to improve the competition abilities of the good individuals during the selection process. This is generally achieved by subtracting the same number from all the fitness values of the individuals. Consider a problem with only two individuals. Suppose that their fitness values, based on their performances, are 495 and 497. If one of them is to be selected randomly, the chance of the first individual being selected is 49.9% and of the second is 50.1%. Although one of the individuals is obviously better, the chances of selection are almost the same. However, if the fitness values of the individuals are scaled by subtracting 490 from both, the chances of being selected improve to 41.7% and 58.3% respectively.

Another commonly used method of improving the performance of a genetic algorithms process is elitist selection. In genetic algorithms there is always a risk of losing the best individual when generations pass. To avoid such a problem, the most fit individual, or individuals, after each evaluation phase could be carried to the next generation unchanged (Zalzala and Fleming[49]). This process is called elitist selection.

3.3.2.5 Genetic Operators

As in nature, there are mainly two types of classical genetic operators in genetic algorithms, crossover and mutation.

3.3.2.5.1 Crossover

Crossover is the basic operator for the production of new chromosomes. It mimics the sexual reproduction of living organisms. Two parents come together and they produce two infants whose genes resemble those of the parents. The common forms of crossover are 1-point crossover, 2-point crossover, n-point crossover and uniform crossover (Green[24]).

In 1-point crossover, a crossover point is randomly selected for each couple. Each half of the chromosomes then cross over to find two new offspring that resemble both parents.

The basic difference in 2-point crossover is that there are two crossover points assigned for each couple. The sections between the two crossover points are swapped for the new individuals.

In n-point crossover, there are n crossover points. The parts of the strings of the parents between every two crossover points are swapped for the new infants. As a

result, the infants would get the parts of the string of a parent between every successive crossover point.

In uniform crossover, a number of points are selected at random. Each selected point is swapped over rather than swapping a part of the string.

Parents are chosen from among the selected individuals randomly according to an explicitly assigned crossover probability. Although it depends on the problem to be solved, a crossover probability of around 0.6 generally achieves a fast solution, which is crucial for large-scale problems (Yun et al[47]).

3.3.2.5.2 Mutation

In nature, copying DNA to create offspring can sometimes result in errors. These errors, called mutations, generally do not have a good effect on the fitness of the individual. But sometimes they can result in good features and they can be passed to succeeding generations via reproduction. Mutation is so important for the evolution of living organisms that without it nature would have been a vicious circle rather than an evolutionary process.

Genetic algorithms are very different from other stochastic search methods in the searching methods used. Searching starts as a randomly selected population and future solutions depend on mutual relationships of the individuals. Without mutation, the search area would be so restricted that finding the global maximum point would be almost impossible in large-scale problems. The search area can be widened gradually by mutation and deepened by the crossover process; the features of individuals are improved by selection. Individuals evolve gradually until the solutions converge to a maximum point or a predetermined number of generations is reached.

Like the crossover process, mutation is random. However, the probability of mutation should be comparatively low, since it is not a common process like crossover. There are different types of mutation. In bit-by-bit mutation (Michalewicz[33]), random numbers are generated for each digit of the whole population and, depending on the assigned mutation probability, the digit might be changed. In binary code this is a trivial exercise. If the original digit value were 0, the changed value would be 1, and vice versa. In string mutation (Yun et al[47]), however, the mutation probability value is assigned on the string basis. After random numbers have been generated, if a string is mutated, another random number is generated and assigned to the mutation site for the string.

3.3.3 Application of genetic algorithms to cut-off grade optimisation

Since they were first proposed in 1975, genetic algorithms have been successfully applied to many optimisation problems (Man et al[32], Michalewicz[33]). Many cut-off grade optimisation problems have huge numbers of local optimum values, which are widely separated from the global optimum point and from each other. Stochastic search methods, including genetic algorithms, can fail to reach a solution for such problems. The real challenge in such problems is finding solutions close to the global optimum point for a restricted time. Genetic algorithms are more robust than many other existing search methods.

Yun et al[47] applied genetic algorithms to the Jingtieshan iron ore mine in China in order to optimise cut-off grade and minimum average grade, which is a criterion used in China to define ore for mining purposes. They used net present value as a fitness

value, binary representation, roulette wheel selection and 100 iterations (number of generations).

As mentioned earlier, Genetic Algorithms work on representatives of solutions, which are chromosomes, rather than on solutions directly. Encoding and evaluation processes for the application of genetic algorithms to cut-off grade optimisation are described below.

3.3.3.1 Encoding

Encoding of an individual for the optimisation of a single cut-off grade for an ore deposit with only one valuable mineral is straightforward. The string is composed of only one gene, which represents a cut-off grade. The size of the string depends on the number of cut-off grades to be evaluated (searched). If the binary-coding method is used, the length of a string will be long. A 5-bit string can represent $2^5 = 32$ cut-off grades. To derive real values from the binary code (i.e., mapping) the string the formula is:

$$X = X_{min} + \frac{X_{max} - X_{min}}{2^L - 1} * Y \qquad (3.46)$$

where;

X: The mapping value,

X_{min}: The minimum cut-off grade to be searched for,

X_{max}: The maximum cut-off grade to be searched for,

L: The length of the binary string,

Y: The value of binary representation.

The value of the binary representation for a 5-bit string would be an integer between 0 for string 00000 and 31 for string 11111.

The application of genetic algorithms to the solution of the optimum cut-off grade problem requires a crucial increase in the length of the string. Since in each year in the mine life there might be a different optimum cut-off grade, there should be different genes in the same string. If the mine life is 20 years, the string will be composed of 20 genes, each with a length of 5-bits, making the total length of the chromosome 100. If more than one valuable mineral is to be considered the length of the string will be increased even more.

3.3.3.2 Evaluation

In genetic algorithms, every individual is assigned to a fitness value depending on its performance. In cut-off grade optimisation, the objective function is maximum net present value. The higher the discounted profit the better is the individual.

3.3.4 Steps of the algorithm

The following steps comprise a genetic algorithm approach to determining optimum cut-off grades:

18. Input the data: the selling price, mining cost, mineral processing cost, refinery and/or marketing cost, annual fixed cost, mining capacity, mineral processing capacity, refinery or marketing capacity, recovery, discount rate, grade categories, amount of the mineralised material associated with each of the grade categories, lowest limits of cut-off grades, highest limits of cut-off grades, number of population, number of generations, crossover rate and mutation rate.

19. Calculate the reserves available in the deposit.
20. Calculate amount of the material above the cut-off grades, amount of the material below the cut-off grades and the average grade of ore above the cut-off grades for each grade interval by using equations 3.12, 3.13 and 3.14.
21. Determine the cut-off grades to be searched by using the lower and upper limits and the cut-off grade decision interval.
22. For each cut-off grade, calculate, by interpolation, the amount of material above the cut-off grade, amount of material below the cut-off grade and the average grade of ore above the cut-off grade.
23. Calculate the depletion rate for each cut-off grade considering maximum capacities of mining, mineral processing and marketing stages.
24. Using the following profit equation, calculate the annual profit for the cut-off grades:

$$P = Q*[(p-k)*x*y*a-x*h-m)-f*t \qquad (3.37)$$

25. Set the number of generations to zero.
26. Initialise the number of individuals randomly so that each individual is composed of random numbers that represent different cut-off grade policies.
27. Decode the numbers representing individuals in the population and find the mapping values.
28. Assign the best individual a value of zero.
29. Start with the first individual in the population.
30. Start with the first year.
31. Calculate the discounted profit as a function of the cut-off grades assigned for each year:

$$P_d = \frac{P}{1+d} \qquad (3.47)$$

32. Subtract the depletion rate used from the total reserve.
33. If the total reserve is greater than zero, go to step 14. Otherwise, go to the next step.
34. Find the total discounted profit for the individual.
35. If the total discounted profit of the individual is higher than that of the best individual, assign the individual as the best individual.
36. If all individuals in the population have not been assessed, go to step 13. Otherwise, go to the next step.
37. Assign a fitness value for each individual in the population depending on their total discounted profit values.
38. Generate random variables for each individual for the selection process. Copy the individuals that have high fitness values, for the next generation. Determine a new population of new individuals.
39. Generate random variables for selection of parents for the crossover process. Select the individuals for offspring production stochastically.
40. If the number of parents is an odd number, subtract the last one.
41. Generate random numbers for each chromosome in order to find points where crossover occurs. Recombine the parents and find new individuals carrying their parents' genes.
42. Copy the infants instead of their parents in the population.

43. Generate random numbers for each bit of each chromosome for the mutation process. If the random number is in the range of the mutation probability, mutate the related bit of the related individual.
44. After mutation the new population replaces the previous one.
45. Increase the number of generations by 1.
46. If the maximum number of generations has not been reached, go to the step 10. Otherwise, go to the next step.
47. Stop and write out the results for the best individual, i.e. the chromosome that has the highest fitness value found.

CHAPTER 4

CUT-OFF GRADES FOR MULTI-MINERAL DEPOSITS

4.1 General theory of cut-off grades for multi-mineral deposits

Many orebodies contain more than one mineral of economic value. In these cases, the mining system consists of more than three stages. For example, for a mineral deposit containing two minerals there are four stages: the mining stage, the mineral processing stage, the marketing and/or refining stage for the first mineral and the marketing and/or refining stage for the second mineral.

Mineral deposits that contain more than one mineral are generally evaluated and exploited on the basis of parametric cut-off grades or equivalent cut-off grades. In his book, The Economic Definition of Ore, Lane[30] explains this technique in detail for mineral deposits that contain two minerals. The use of equivalent grades has been a standard practice in the mining industry, especially for base metal deposits, for many years.

4.2 Parametric cut-off grades

In order to avoid the complexities of a two-dimensional grade analysis in a two-mineral deposit, one of the minerals could be converted to its economic equivalent

value in terms of the other mineral. Any analysis can then be conducted as if the orebody contained only one mineral. As a result, the operating cut-off grades do not correspond to the cut-off grades for the grade-tonnage distributions of the individual minerals. Rather, they are parametric cut-off grades based on equivalent values, which provides a way of aggregating two minerals into one. Whilst there is a direct relationship from the individual grades to the equivalent grade there is no unique inverse relationship from the equivalent grade back to the individual grades.

As an example, consider a lead-zinc deposit. The original grade categories are defined in terms of lead and zinc and the equivalent grade is calculated as:

$$Equivalent\ grade\ \% = a\ x\ Pb\% \ +\ b\ x\ Zn\%$$

where a and b are coefficients. Effectively, one of the minerals, preferably the one of secondary importance, is converted into the equivalent of the other mineral. If lead were of secondary importance, the zinc equivalent of the lead in each zinc category would be calculated and added to the zinc content. The equivalence coefficients are usually based on relative prices and costs of production.

The decision of whether to use equivalents is based on a choice between accuracy and practicability. Either the two-dimensional grade distribution should be retained or the mineral deposit should be redefined on the basis of the combination of the two minerals. The difference between the two approaches decreases as the correlation between the two minerals increases. In stratigraphic lead-zinc deposits the correlation between lead and zinc is usually very high, often in excess of 0.8. In stratigraphic silver-lead-zinc deposits correlations between silver and lead usually exceed 0.90. These high correlations mean that the *in situ* concentrations of the minerals are usually coincident and there are thus no adverse implications for mine

planning done on the basis of equivalents rather than on the basis of the individual minerals. In addition, high correlations often reflect strong geochemical relationships with consequent mineral processing linkages (silver, for example, is recovered as a component of the lead concentrate). The use of equivalents for minerals that have little or no correlation, however, will almost certainly lead to problems in mine design and the exploitation of the resource.

After combining the individual mineral values into a single equivalent value, the optimum cut-off grade for the equivalent can be found using any of the methods for single mineral cut-off grade optimisation described in earlier chapters.

The use of equivalents is attractive because it simplifies the determination of cut-off grades. It is acceptable, however, only when the secondary minerals are of minor importance or if all of the minerals are strongly correlated with each other so that the secondary minerals are evenly distributed throughout the grade-tonnage distribution or they are so closely related to the distribution of the primary mineral that they simply amplify it. A detailed discussion and analysis of the definition and use of equivalents can be found in Dowd and Xu[17].

There is a serious problem with the use of equivalents. In the mining process, the equivalent cut-off grades are not based on the grades of an entity but rather they are parametric cut-off grades, which are only indirectly related to the grade distributions. At the end of the mineral processing stage, the amounts of each mineral produced and sent to the refinery/marketing stage are not calculated on the basis of the minerals that compose the deposit but on the basis of equivalents. The procedure is valid if there are no refinery/market limitations. However, if one of the minerals is subject to refinery/market limitation, the process is not valid because

excess production of the particular mineral cannot be sold and, as a result, the mineral cannot be valued on the basis of the contract price. Hence, it is the influence of the capacities of the mineral processing plant and the refinery/market that invalidate the parametric cut-off grade approach and therefore, the minerals must be dealt with separately.

4.3 Optimal cut-off grades

The grade-tonnage distribution for a deposit with n minerals is n-dimensional. In addition, the mining system consists of n+2 stages.

Lane[29] tried to find a cut-off grade optimisation algorithm for mineral deposits containing two minerals. The grade distribution in this case is a two-dimensional surface instead of a curve and may be represented by a series of contours as shown in Figure 4.1.

In a two-mineral deposit, the optimum production schedule comprises pairs of cut-off grades for each production period of the mine life. Cut-off grades are boundaries between ore and waste, and they are represented by a line in Figure 4.1. The simplest way of specifying a line that represents the cut-off grades is by means of its intercepts on the grade axes of the minerals.

In Figure 4.1 the value g_1 is the cut-off grade for mineral 1 in the absence of mineral 2 and g_2 is the cut-off grade for mineral 2 in the absence of mineral 1. The determination of optimum cut-off grades for a two-mineral deposit is, therefore, the problem of determination of pairs of cut-off grades g_1 and g_2, which gives the maximum return value.

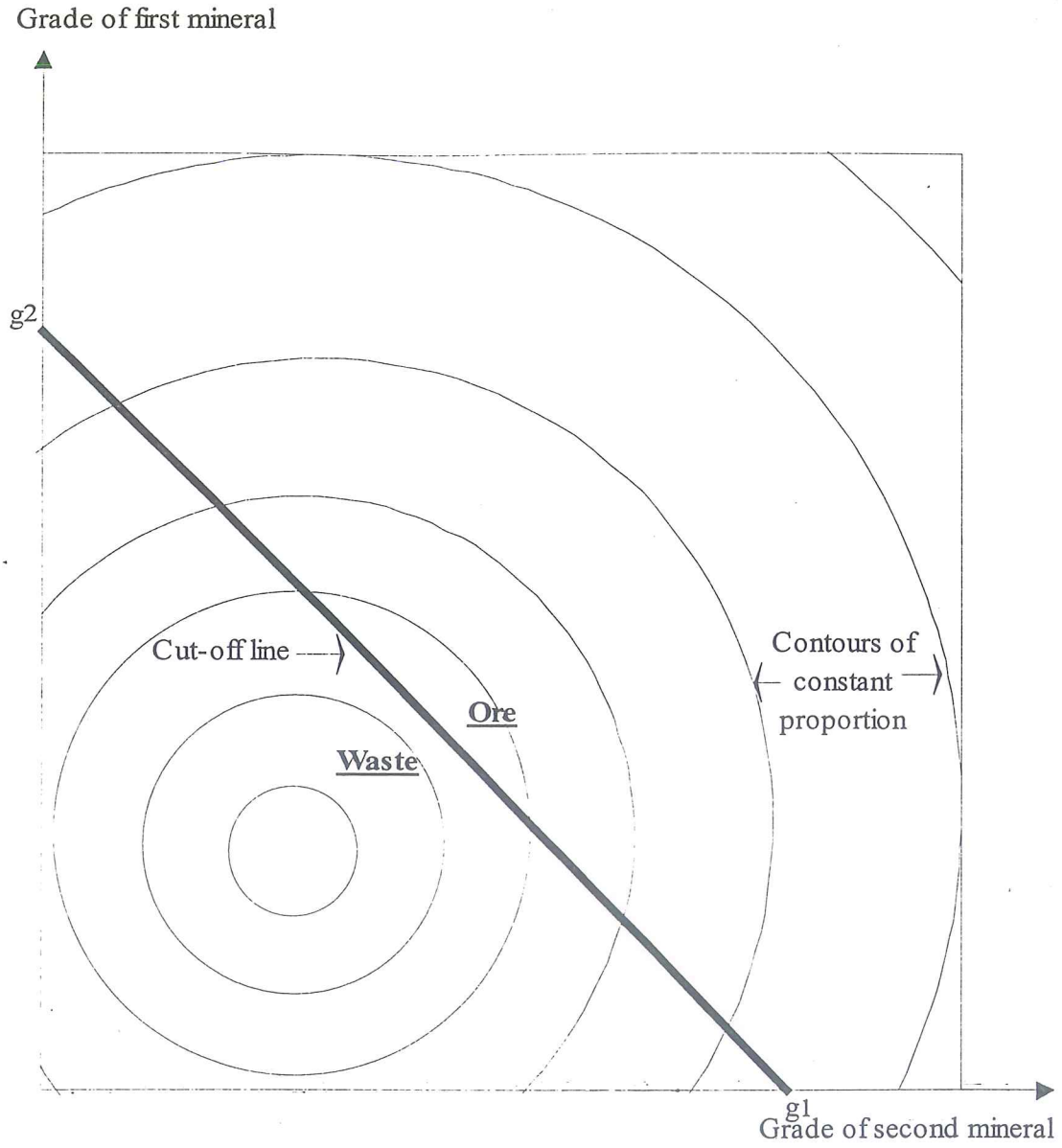

Figure 4.1 Two-dimensional grade distribution

The expression for the net present value is analogous to Equation 2.5 for single mineral deposits given in Chapter 2. The notation is similar with the exception of the addition of price, cost, yield and average grade of the secondary mineral.

The basic expression for the net present value per unit of resource in a mining operation for two-mineral deposits is:

$$v = [(p_1 - k_1) * x * y_1 * a_1 + (p_2 - k_2) * x * y_2 * a_2] - x * h - m - (f + F) * t \qquad (4.1)$$

where:

v: net present value,

p_1: price of the first mineral per unit of product,

p_2: price of the second mineral per unit of product,

k_1: marketing variable cost of the first mineral,

k_2: marketing variable cost of the second mineral,

x: ore / material ratio,

y_1: yield of the first mineral during treatment (recovery),

y_2: yield of the second mineral during treatment (recovery),

a_1: average grade of the first mineral,

a_2: average grade of the second mineral,

h: mineral processing variable cost,

m: mining variable cost,

f: fixed cost,

F: opportunity cost,

t: time unit of resource.

There are four main components of the mining operation related to the throughputs from the four distinct stages: mineralised material, ore, first mineral product and second mineral product.

The first and the second components and the stages are the same as the single mineral case. The mineralised material is dealt with in the mining stage and the ore is dealt with in the mineral processing stage.

The first mineral product is dealt with in the refining and/or marketing stage for the first mineral. Costs are incurred per unit of mineral and capacity is the maximum refining or marketing rate. Analogous descriptions apply for the second mineral.

If the total profit from a finite mineral deposit is to be maximised, it is necessary to maximise the scarcest throughput. Thus, if mining is limited, profit per unit of mineralised material must be maximised. If the limitation comes from a limited mineral processing capacity, profit per unit of ore must be maximised. If the limitation is from limited refinery or marketing capacity of the first mineral, profit per unit of the first mineral must be maximised. If the limitation is from limited refinery or marketing capacity of the second mineral, then, profit per unit of the second mineral must be maximised.

Lane[30] introduced four limiting economic cut-off grades based on the four stages. The formulae he derived for limiting economic cut-off grades are useful when a single stage is limiting the process. However, when the maximum occurs at a balancing point, where more than one capacity restricts the throughput, there is no analytical solution to the problem. In this case, the maximum can only be found by a searching process.

Lane[30] proposed a grid search technique to find optimum cut-off grades for two-mineral deposits. Although the grid search technique does not cover the entire search area, and therefore does not guarantee the global maximum, it can search a wide area.

The grid search method that Lane proposed can be applied to mineral deposits that contain more than two valuable minerals by adding one more stage to the

calculations for each mineral, though it makes the process more time consuming as the number of minerals increases.

Another method for finding optimum cut-off grades is dynamic programming. In this method, every possible sequence of cut-off grade decisions is enumerated and an optimal solution is found by considering the effects of each decision separately.

The determination of cut-off grades and production rates by means of dynamic programming, introduced by Dowd[14], can be used to determine optimum cut-off grades and production rates for multi-mineral deposits. In this method, the number of cut-off grade intervals and the number of depletion rate intervals to be searched for the optimum must be decided. For each depletion rate, optimum cut-off grades for each of the minerals are found. The number of depletion rates restricts the number of possible solutions to be searched. The depletion rates are searched for the life of the mine by means of dynamic programming to find the maximum discounted profit.

The number of different depletion rates to be searched for each year is limited by computer memory. Improvements in computer technology might, in time, ease this disadvantageous situation. The search method is very robust in that all possible alternatives are searched to find the global optimum among them. Bearing in mind the fact that only a series of decreasing cut-off grade points can give the maximum discounted profit, all non-decreasing sequences of cut-off grades can be eliminated from the search. This strategy yields a significant improvement in computation time.

Genetic algorithms can be used as a search method for cut-off grade optimisation for multi-mineral deposits as well as for single-mineral deposits as shown in Chapter 3.

Although genetic algorithm methods provide a very robust means of searching alternatives and in a very short time an acceptable optimum point can be reached, the inclusion of second and third minerals extends the length of the chromosomes used in the algorithms and slows down the process.

The search process comes to an end either by the convergence of the solution to a stable optimum result or by reaching an assigned number of simulations.

4.3.1 Lane's method for multi-mineral deposits

As mentioned above, there is a major problem associated with the use of equivalents in multi-mineral cut-off grade optimisation. The problem lies in the fact that the revenue and the costs must be calculated on the basis of the average grades of the individual minerals (Dowd and Xu[18]). However, at the end of the process, only average equivalent grades above each of the equivalent cut-off grades are available. Dowd and Xu[18] introduced a solution to this problem by defining some additional parameters for the equivalent grade-tonnage data. These parameters are the grade ranges of individual minerals for each of the equivalent grade classes. They assumed that, within each class interval of the equivalent grade, individual grade values follow a uniform distribution and the variability of each of the individual grades follows the same pattern as that of the equivalent grade values. Using these assumptions it is possible to find the average grades for each of the individual minerals above the equivalent grade. However, the grade ranges of the minerals based on the equivalents are not directly related to the grade ranges of the original grade-tonnage distributions.

Lane attempted to find a solution to optimise cut-off grades for a two mineral deposit without using the equivalents approach. He dealt with the minerals separately and developed formulas for two mineral deposits.

Lane's basic profit expression for the net present value per unit of resource must be maximised in order to obtain an optimum exploitation strategy for a mine. This profit expression for mineral deposits that contain two minerals is:

$$W = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - m - f * t \qquad (4.2)$$

where;

W: profit for the next unit of material,

p_1: price per unit of product of the first mineral,

p_2: price per unit of product of the second mineral,

k_1: variable marketing cost of the first mineral,

k_2: variable marketing cost of the second mineral,

x: ore / material ratio,

y_1: yield of the first mineral during treatment (recovery),

y_2: yield of the second mineral during treatment (recovery),

a_1: average grade of the first mineral,

a_2: average grade of the second mineral,

h: variable mineral processing cost,

m: variable mining cost,

f: fixed cost,

t: time unit of resource.

With the addition of the opportunity cost, F, to the profit expression of 4.2 Lane's basic profit expression, Equation 4.1 can be found.

4.3.1.1 Limiting economic cut-off grades

For mineral deposits with two valuable minerals, there are basically four pairs of limiting economic cut-off grades. Each of these arises from limitations on one of the mining operation stages mentioned in the previous chapter. They are mining limiting economic cut-off grade, mineral processing limiting economic cut-off grade, marketing limiting economic cut-off grade of the first mineral and marketing limiting economic cut-off grade of the second mineral.

By using Equation 4.1, the limiting economic cut-off grades can be found by assuming that the mining, mineral processing, marketing of the first mineral and marketing of the second mineral stages in turn limit the operation.

4.3.1.1.1 Mining limiting economic cut-off grades

If the rate of mining limits the operation and the maximum capacity of the mining stage is M, then the time needed to mine one unit of material is 1/M. If 1/M is substituted for t, the net present value formula to be maximised becomes:

$$v_m = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - m - \frac{(f + F)}{M} \qquad (4.3)$$

where;

v_m: net present value for a limited mining rate,

M: maximum mining capacity.

Cut-off grades affect only the mineral processing and marketing stages and so the only part of the expression that varies with the cut-off grade is:

$$((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h \quad (4.4)$$

or

$$x[((p_1 - k_1) * y_1 * a_1) + ((p_2 - k_2) * y_2 * a_2) - h] \quad (4.5)$$

Therefore,

$$((p_1 - k_1) * y_1 * a_1) + ((p_2 - k_2) * y_2 * a_2) - h \quad (4.6)$$

must be maximised in order to find the optimum cut-off grades, if the mining stage is the limiting agent.

If the cut-off grades to be maximised are g_1 and g_2 and the mining limiting break-even cut-off grades are g_{m1} and g_{m2}, then g_1 and g_2 can be lowered for as long as

$$(p_1 - k_1) * y_1 * g_1 + (p_2 - k_2) * y_2 * g_2 \geq h \quad (4.7)$$

Hence, the mining limiting break-even cut-off grades are given by:

$$(p_1 - k_1) * y_1 * g_{m1} + (p_2 - k_2) * y_2 * g_{m2} = h \quad (4.8)$$

So;

$$g_{m1} = h/((p_1 - k_1) * y_1) \quad (4.9)$$

where;

g_{m1}: mining limiting economic cut-off grade for the first mineral in the absence of the second mineral.

and

$$g_{m2} = h/((p_2 - k_2) * y_2) \quad (4.10)$$

where;

g_{m2}: mining limiting economic cut-off grade for the second mineral in the absence of the first mineral.

This approximation comes from the concept that mineralised materials should be classified as ore as long as the profit acquired from selling them is not less than the cost of the mineral processing. That is;

$$(p_1 - k_1) * y_1 * g_1 + (p_2 - k_2) * y_2 * g_2 > h \qquad (4.11)$$

4.3.1.1.2 Mineral processing limiting economic cut-off grades

If the rate of mineral processing limits the operation and the maximum capacity of the mineral processing plant is H, then the time needed to process one unit of ore is 1/H. In this case, one unit of mineralised material yields x units of ore. Therefore, the time needed to process one unit of mineralised material is x/H. Substituting x/H for t, the net present value formula to be maximised becomes:

$$v_h = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - m - \frac{(f + F) * x}{H} \qquad (4.12)$$

where;

v_h: net present value for a limited mineral processing rate,

H: maximum mineral processing plant capacity.

Cut-off grades affect only the mineral processing and marketing stages. So, only part of the expression,

$$((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - \frac{(f + F) * x}{H} \quad (4.13)$$

varies with the cut-off grade. Therefore, to find the optimum cut-off grade when the mineral processing stage is the limiting agent,

$$((p_1 - k_1) * y_1 * a_1) + ((p_2 - k_2) * y_2 * a_2) - h - \frac{(f + F)}{H} \quad (4.14)$$

must be maximised.

As a result, the mineral processing limiting economic break-even cut-off grade is given by

$$((p_1 - k_1) * y_1 * g_{h1}) + ((p_2 - k_2) * y_2 * g_{h2}) = h + \frac{(f + F)}{H} \quad (4.15)$$

So;

$$g_{h1} = \frac{\frac{h + (f + F)}{H}}{(p_1 - k_1) * y_1} \quad (4.16)$$

where;

g_{h1}: mineral processing limiting economic cut-off grade for the first mineral in the absence of the second mineral.

$$g_{h2} = \frac{\frac{h + (f + F)}{H}}{(p_2 - k_2) * y_2} \quad (4.17)$$

where;

g_{h2}: mineral processing limiting economic cut-off grade for the second mineral in the absence of the first mineral.

The mineral processing limiting economic cut-off grade formula is very different to the mining limiting economic cut-off grade formula because of the presence of the opportunity cost.

4.3.1.1.3 Marketing 1 limiting economic cut-off grades

If the refinery or marketing rate of the first mineral limits the operation and the maximum capacity of the refinery or marketing is K_1, then the time needed to process one unit of mineral is $1/K_1$. One unit of mineralised material yields $x * y_1 * a_1$ units of mineral. Therefore, the time needed to process one unit of mineralised material is $x * y_1 * a_1/K_1$. Substituting $x * y_1 * a_1/K_1$ for t, the net present value formula to be maximised becomes:

$$v_{k1} = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - m - \frac{(f + F) * x * y_1 * a_1}{K_1} \quad (4.18)$$

where;

v_{k1}: net present value for a limited refinery 1 or marketing 1 rate for the first mineral,

K_1: maximum refinery or marketing capacity for the first mineral.

Following the same procedure used for the single mineral case, the mineral processing limiting economic break-even cut-off grades are found to be:

$$g_{k11} = \frac{h}{\frac{p_1 - k_1 - (f + F)}{K_1} * y_1} \quad (4.19)$$

where;

g_{k11}: refinery 1 or marketing 1 limiting economic cut-off grade for the first mineral in the absence of the second mineral.

and

$$g_{k12} = \frac{h}{(p_2 - k_2) * y_2} \tag{4.20}$$

where;

g_{k12}: refinery 1 or marketing 1 limiting economic cut-off grade for the second mineral in the absence of the first mineral.

The marketing 1 limiting economic cut-off grade formula is also different from the mining limiting economic cut-off grade formula because of the presence of the opportunity cost. As the mine ages, the opportunity cost declines and so too do the marketing 1 limiting economic cut-off grades.

4.3.1.1.4 Marketing 2 limiting economic cut-off grades

If the refinery or marketing rate of the second mineral limits the operation and the maximum capacity of the refinery or marketing is K_2, then the time needed to process one unit of mineral is $1/K_2$. One unit of mineralised material yields $x * y_2 * a_2$ units of mineral. Therefore, the time needed to process one unit of mineralised material is $x * y_2 * a_2/K_2$. Substituting $x * y_2 * a_2/K_2$ for t, the net present value formula to be maximised becomes:

$$v_{k2} = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) - x * h - m - \frac{(f + F) * x * y_2 * a_2}{K_2} \tag{4.21}$$

where;

v_{k2}: net present value for a limited refinery 2 or marketing 2 rate for the second mineral,

K_2: maximum refinery or marketing capacity for the second mineral.

The mineral processing limiting economic break-even cut-off grades are;

$$g_{k21} = \frac{h}{(p_1 - k_1) * y_1} \tag{4.22}$$

where;

g_{k21}: refinery 2 or marketing 2 limiting economic cut-off grade for the first mineral in the absence of the second mineral.

and

$$g_{k22} = \frac{h}{\frac{p_2 - k_2 - (f + F)}{K_2} * y_2} \tag{4.23}$$

where;

g_{k22}: refinery 2 or marketing 2 limiting economic cut-off grade for the second mineral in the absence of the first mineral.

The marketing 1 limiting economic cut-off grade formula is also different from the mining limiting economic cut-off grade formula because of the presence of the opportunity cost.

The formulae used earlier to find cut-off grades for both of the minerals for four different limiting cases are useful when a single stage is limiting the process.

However, no satisfactory analytical technique has been developed for the case in which more than one capacity restricts the throughput. The problem is geometrically one of four intersecting surfaces that can be envisaged as forming hills and valleys. The maximum point that gives the optimum pair of cut-off grades is best located by using a search process that deals with the constituent minerals separately instead of using parametric cut-off grades.

4.3.2 Optimisation of cut-off grades for multi-mineral deposits by the grid search method

Lane[30] proposed a form of grid search technique that basically involves calculating the net present values for four different limiting cut-off grades. The ore/material ratios and average grades for each pair of cut-off grades can be calculated from the grade distributions. The net present values, namely, v_m, v_h, v_{k1} and v_{k2} can then be calculated. The controlling capacity for each pair of cut-off grades is the one that corresponds to the least of the four net present values calculated. It is this figure that has to be maximised among the different pairs of cut-off grades in order to find the optimum pair.

The main advantage of the grid search method over the equivalents method is that it is more general, i.e. the equivalents method is only applicable when there is a strong correlation among the constituent minerals. The grid search method can be applied even if the secondary minerals are of minor importance.

Lane[30] suggested using a primary grid of 9 * 9 cells, which makes 100 grid points. As a second step, he proposed a secondary grid of 6 * 6 cells, which makes 1 grid points, covering the four original cells that surround the maximum point.

Finally, a third grid of 6 * 6 cells covering the four cells that surround the maximum point in the secondary grid was proposed. He also suggested as a safeguard that, if the maximum occurs on the boundary of the second or the third grid, the grid should be relocated around the near point with no change in scale. This process ensures that the maximum will be found even when it is on a steep ridge running between grid points. The process involves the calculation of only 198 grid points.

Lane claimed that the process of using three subsequent grids gives an accuracy of one in 9 * 3 * 3, in other words 1 in 81, which is near 1%. In fact, there is no any obvious proof that using three subsequent grids, instead of one, does improve the searching process. The optimum point is not necessarily somewhere near the maximum point in the first grid. By discarding the search area outside the four cells around the maximum point in the first grid, there is a danger of missing the target global optimum. Besides, because of the improvements in computer memory capacity and computational speed, it is now possible to search a finer grid that has a better coverage than the method of grid selection suggested by Lane.

Mohammed[34] and Dagdelen and Mohammed[8] applied Lane's grid search method to a copper and gold deposit. Mohammed[34] used the same grid selection method as Lane suggested and developed a computer program for mineral deposits that contain two valuable minerals. He assigned the grade intervals for a given grade-tonnage distribution as grid points for the primary grid. The application of the method to a sample deposit yields pairs of cut-off grades that are in decreasing order.

For orebodies that contain more than two minerals, the grid search method is still useful. In the case of an orebody with three valuable minerals, there will be five

mining operation stages that restrict the throughput, the fifth one being the marketing 3 limiting stage. As a result, there are five different net present values representing each of the stages. The grade-tonnage distribution is three-dimensional. Basically it is not much different than the two minerals case. However, the computing time increases significantly with the inclusion of each mineral.

The following expressions are derived for the calculations to be used in the grid search method for mineral deposits containing three valuable minerals.

Assume that the grade-tonnage distribution of a mineral deposit consists of W grade cells for the mineral 1. Hence, there would be W + 1 grade limits. The representation of the corresponding grade for the different cells would be:

$[g_1(1), g_1(2)], [g_1(2), g_1(3)], ..., [g_1(W - 1), g_1(W)], [g_1(W), g_1(W + 1)]$

As there is more than one mineral in the grade distribution, the deposit will have W number of grade distributions for mineral 2. Assume that each of the grade distributions for mineral 2 have Y individual cells. As a result, there would be Y + 1 grade limits. The representation of the corresponding grade for different cells would be:

$[g_2(1), g_2(2)], [g_2(2), g_2(3)],....., [g_2(Y - 1), g_2(Y)], [g_2(Y), g_2(Y + 1)]$

As there are three minerals in the grade distribution, the deposit will have W*Y number of grade distributions for mineral 3. Assume that each of the grade distributions for mineral 3 have Z individual cells. As a result, there would be Z + 1 grade limits. The representation of the corresponding grade for different cells would be:

$[g_3(1), g_3(2)], [g_3(2), g_3(3)],....., [g_3(Z - 1), g_3(Z)], [g_3(Z), g_3(Z + 1)]$

If the lower grade limit g(w) for a given cell [g(w), g(w + 1)] is the cut-off grade of mineral 1 representing interval p, and if the lower grade limit g(y) for a given cell [g(y), g(y + 1)] is the cut-off grade of mineral 2 representing interval q, and if the lower grade limit g(z) for a given cell [g(z), g(z + 1)] is the cut-off grade of mineral 3 representing interval r, the amount of the material above the cut-off grade, the amount of the material below the cut-off grade, the average grade of mineral 1 above the cut-off grade, the average grade of mineral 2 above the cut-off grade and the average grade of mineral 3 above the cut-off grade can be found by using the following equations:

$$T_{ore}(p,q,r) = \sum_{z=1}^{Z}\sum_{w=1}^{W}\sum_{y=1}^{Y} T_{pqr} \qquad (4.24)$$

where

$$T_{pqr} = \begin{cases} T_{(w,y,z)} & if\ Z \geq z \geq r - \left(\frac{r}{p} * w\right) - \left(\frac{r}{q} * y\right)\ for\ \forall w \in [1,W]\ and\ \forall y \in [1,Y] \\ 0 & otherwise \end{cases}$$

where;

$T_{ore}(p,q,r)$: amount of material above the cut-off grade for the p^{th} and q^{th} and r^{th} grade intervals,

$T_{(w,y,z)}$: amount of material for the given grade limits,

p: grade interval for the first mineral,

q: grade interval for the second mineral,

r: grade interval for the third mineral.

$$T_{waste}(p,q,r) = \sum_{z=1}^{Z}\sum_{w=1}^{W}\sum_{y=1}^{Y} T_{pqr} \qquad (4.25)$$

where

$$T_{pqr} = \begin{cases} T_{(w,y,z)} & \text{if } 0 \le z < r - \left(\frac{r}{p} * w\right) - \left(\frac{r}{q} * y\right) \text{ for } \forall w \in [1, W] \text{ and } \forall y \in [1, Y] \\ 0 & \text{otherwise} \end{cases}$$

where;

$T_{waste}(p,q,r)$: amount of material below the cut-off grade for the p^{th} and q^{th} and r^{th} grade intervals.

$$g_{avg1}(p,q,r) = \frac{\sum_{w=1}^{W}\sum_{y=1}^{Y}\sum_{z=1}^{Z} T_{pqr} * \left(\frac{g_w^1 + g_{w+1}^1}{2}\right)}{\sum_{w=1}^{W}\sum_{y=1}^{Y}\sum_{z=1}^{Z} T_{pqr}} \qquad (4.26)$$

where

$$\begin{bmatrix} T_{pqr} \\ g_w^1 \\ g_{w+1}^1 \end{bmatrix} = \begin{cases} \begin{bmatrix} T_{(w,y,z)} \\ g_1(w) \\ g_1(w+1) \end{bmatrix} & \text{if } W \ge w \ge p - \left(\frac{p}{r} * z\right) - \left(\frac{p}{q} * y\right) \text{ for } \forall z \in [1, Z] \text{ and } \forall y \in [1, Y] \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

where;

$g_{avg1}(p,q,r)$: average grade of the first mineral above the cut-off grade for the p^{th} grade interval of the first mineral and the q^{th} grade interval of the second mineral, and the r^{th} grade interval of the third mineral,

$g_1(w)$: lower grade limit of a given cell for the first mineral,

$g_1(w+1)$: upper grade limit of a given cell for the first mineral.

$$g_{avg1}(p,q,r) = \frac{\sum_{y=1}^{Y}\sum_{w=1}^{W}\sum_{z=1}^{Z} T_{pqr} * \left(\frac{g_y^2 + g_{y+1}^2}{2}\right)}{\sum_{w=1}^{W}\sum_{y=1}^{Y}\sum_{z=1}^{Z} T_{pqr}} \qquad (4.27)$$

where

$$\begin{bmatrix} T_{pqr} \\ g_y^2 \\ g_{y+1}^2 \end{bmatrix} = \begin{cases} \begin{bmatrix} T_{(w,y,z)} \\ g_2(y) \\ g_2(y+1) \end{bmatrix} & if\ Y \geq y \geq q - \left(\frac{q}{r} * z\right) - \left(\frac{q}{p} * w\right)\ for\ \forall z \in [1,Z]\ and\ \forall w \in [1,W] \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & otherwise \end{cases}$$

where;

$g_{avg2}(p,q,r)$: average grade of the second mineral above the cut-off grade for the p^{th} grade interval of the first mineral and the q^{th} grade interval of the second mineral, and the r^{th} grade interval of the third mineral,

$g_2(y)$: lower grade limit of a given cell for the second mineral,

$g_2(y + 1)$: upper grade limit of a given cell for the second mineral.

$$g_{avg1}(p,q,r) = \frac{\sum_{z=1}^{Z}\sum_{y=1}^{Y}\sum_{w=1}^{W} T_{pqr} * \left(\frac{g_z^3 + g_{z+1}^3}{2}\right)}{\sum_{w=1}^{W}\sum_{y=1}^{Y}\sum_{z=1}^{Z} T_{pqr}} \qquad (4.28)$$

where

$$\begin{bmatrix} T_{pqr} \\ g_z^3 \\ g_{z+1}^3 \end{bmatrix} = \begin{cases} \begin{bmatrix} T_{(w,y,z)} \\ g_3(z) \\ g_3(z+1) \end{bmatrix} & if\ Z \geq z \geq r - \left(\frac{r}{p} * w\right) - \left(\frac{r}{q} * y\right) for\ \forall w \in [1,W]\ and\ \forall y \in [1,Y] \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & otherwise \end{cases}$$

where;

$g_{avg3}(p,q,r)$: average grade of the third mineral above the cut-off grade for the p^{th} grade interval of the first mineral and the q^{th} grade interval of the second mineral, and the r^{th} grade interval of the third mineral,

$g_3(z)$: lower grade limit of a given cell for the third mineral,

$g_3(z + 1)$: upper grade limit of a given cell for the third mineral.

The ratio of the amount of ore to the total amount of mineralised material in the deposit, i.e. ore/material ratio can be found by using the Equation 4.29.

$$x(p,q,r) = \frac{T_{ore}(p,q,r)}{T_{ore}(p,q,r) + T_{waste}(p,q,r)} \qquad (4.29)$$

where;

$x(p,q,r)$: ore/material ratio for the p^{th} grade interval of the first mineral and the q^{th} grade interval of the second mineral.

The average grades and the ore/material ratio values that were generated by these equations are for the grade intervals of the specified grade-tonnage distribution. If the grade intervals are chosen as grid points, the net present values

for the five different limiting stages can be calculated by using the equations below. However, if the grid points are assigned explicitly, the corresponding net present values for the grid points, the values of the ore/material ratio, the average grade of mineral 1 above the cut-off grade, the average grade of mineral 2 above the cut-off grade and the average grade of mineral 3 above the cut-off grade for each grid point must be found by interpolation. After finding these values for the grid points representing any pair of cut-off grade points, the net present values for the five different limiting stages can be calculated by using the following equations.

$$v_m = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) + ((p_3 - k_3) * x * y_3 * a_3) - x * h - m - \frac{(f + F)}{M} \quad (4.30)$$

where;

y_1: yield of the first mineral during treatment (recovery),

y_2: yield of the second mineral during treatment (recovery),

y_3: yield of the third mineral during treatment (recovery),

a_1: average grade of the first mineral,

a_2: average grade of the second mineral,

a_3: average grade of the third mineral.

$$v_h = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) + ((p_3 - k_3) * x * y_3 * a_3) - x * h - m - \frac{(f + F) * x}{H} \quad (4.31)$$

$$v_{k1} = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) + ((p_3 - k_3) * x * y_3 * a_3) - x * h - m - \frac{(f + F) * x * y_1 * a_1}{K_1} \quad (4.32)$$

$$v_{k2} = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) + ((p_3 - k_3) * x * y_3 * a_3) - x * h - m - \frac{(f + F) * x * y_2 * a_2}{K_2} \quad (4.33)$$

$$v_{k3} = ((p_1 - k_1) * x * y_1 * a_1) + ((p_2 - k_2) * x * y_2 * a_2) + ((p_3 - k_3) * x * y_3 * a_3) - x * h - m - \frac{(f + F) * x * y_3 * a_3}{K_3} \quad (4.34)$$

where;

v_{k1}: net present value for a limited refinery 1 or marketing 1 rate for the first mineral,

v_{k2}: net present value for a limited refinery 2 or marketing 2 rate for the second mineral,

v_{k3}: net present value for a limited refinery 3 or marketing 3 rate for the third mineral,

K_1: maximum refinery or marketing capacity for the first mineral,

K_2: maximum refinery or marketing capacity for the second mineral,

K_3: maximum refinery or marketing capacity for the third mineral.

After deciding on the net present values for each stage, the highest possible net present value can be found as:

$$v_{(max)} = \text{maximum } \{\text{minimum } (v_m, v_h, v_{k1}, v_{k2}, v_{k3})\} \quad (4.35)$$

As five stages restrict the process, for any grid point, the minimum among the five values gives the maximum possible present value and the maximum among

them gives the optimum grid point. The cut-off grades that lie on the optimum grid point give the optimum cut-off grades for three minerals.

The grid search method used in this research is based on Lane's grid search method but with some significant differences. The main difference is the size and method of selection of the grid: only one grid is assigned instead of three. The selection of grid points in the grid is done explicitly, rather than using the grade intervals for the given grade-tonnage distribution. More grid points have been used in the work described here than were used by Lane.

4.3.2.1 Steps of the algorithm for the grid search method

The following steps comprise the algorithm implemented in the computer program developed for determining the optimum cut-off grades for mineral deposits containing three valuable minerals using the grid search method:

1. Input the data, which include the mining cost, mineral processing cost, annual fixed cost, mining capacity, mineral processing capacity, discount rate. refinery and/or marketing costs, selling prices, refinery or marketing capacities, recoveries and grade categories for each of the minerals, and amount of the mineralised material associated with each of the grade categories.
2. Set the net present value V to zero.
3. Calculate the reserves available in the deposit.
4. Calculate the amount of material above the cut-off grade, the amount of the material below the cut-off grade and the average grade of ore above the cut-off grade of each mineral for each of the grid points by using the equations 4.24, 4.25, 4.26, 4.27 and 4.28 and using the interpolation technique.

5. Calculate the net present values for the five different limiting stages at each point of the grid, v_m, v_h, v_{k1}, v_{k2} and, v_{k3} by using Equations 4.30, 4.31, 4.32, 4.33 and 4.34.
6. Find the minimum values among the four net present values in the step 5 for each grid point.
7. Determine the grid point that gives the maximum value of the minimum values found in the step 6. The cut-off grades corresponding to this grid point are the overall optimum cut-off grades for the first iteration.
8. Using the profit equation below, calculate the annual profit for the life of the mine for the pair of optimum cut-off grades determined:

$$P = Q*[(p_1 - k_1)*x*y_1*a_1 + (p_2 - k_2)*x*y_2*a_2 + (p_3 - k_3)*x*y_3*a_3] - x*h - m - f*t \quad (4.36)$$

where;

P: profit,

Q: depletion rate.

9. Find the life of the mine by using the formula:

$$L = \frac{T_{ore} + T_{waste}}{Q} \quad (4.37)$$

where;

L: mine life.

10. Find the net present value, V, by discounting profits at the specified discount rate d by using the formula:

$$V = P * \frac{1 - \frac{1}{(1+d)^L}}{d} \qquad (4.38)$$

11. Using the net present value V and Equations 4.30, 4.31, 4.32, 4.33 and 4.34, calculate the net present values for the five different limiting stages on each point of the grid.
12. Find the minimum values among the four net present values in the step 11 for each grid point.
13. Determine the grid point that gives the maximum value among the minimum values found in the step 12. The cut-off grades that correspond to this grid point are the overall optimum cut-off grades for the first period of time.
14. Using Equation 4.36, calculate the annual profit for the life of the mine for the optimum cut-off grades determined.
15. Find the life of the mine by using the Equation 4.37.
16. Find the net present value for the next year from the equation:

$$V_n = V - \frac{P}{(1+d)^L} \qquad (4.39)$$

where;

V_n: the net present value of future profits for the following year.

17. Set $V = V_n$.
18. Calculate the remaining reserves available in the deposit. If there are no remaining reserves go to step 22. Otherwise, go to the next step.

19. Using the net present value V and Equations 4.30, 4.31, 4.32, 4.33 and 4.34, calculate the net present values for the five different limiting stages at each point of the grid.
20. Find the minimum values among the four net present values in the step 19 for each grid point.
21. Determine the grid point that gives the maximum value among the minimum values found in the step 20. The cut-off grades corresponding to this grid point are the overall optimum cut-off grades for that particular period of time. Go to step 14.
22. Find a new net present value by discounting profits of each year and summing them. Compare this net present value with the net present value used in step 11. If the calculated net present value has converged to the net present value used in the calculations, go to step 23. Otherwise, update the calculated net present value and go to step 11.
23. Write the results.

4.3.3 Optimisation of cut-off grades for multi-mineral deposits by dynamic programming

Dowd and Xu[18] used dynamic programming for the optimisation of cut-off grades for multi-mineral deposits. Because of the complexities of the process, they used the equivalent grade approach. The difficulty with this approach is that revenue and costs are calculated on the basis of the average grades of the individual minerals and there is no unique way of determining the individual mineral concentrations from values of equivalents.

Chapter 4: Cut-off Grades for Multi-mineral Deposits

The complexities associated with multi-mineral deposits arise from the multi-variate grade-tonnage distribution. For a two-mineral deposit with ten grade intervals assigned for each mineral, ten grade-tonnage curves would be required to provide a complete definition of the deposit. The complexity increases dramatically as the number of minerals increases.

Dynamic programming is a multi-stage decision process that was developed by Bellman[2]. It has been applied to a number of mining problems including optimisation of cut-off grades and production rates. Roman[40] introduced dynamic programming for determination of production rates. Dowd[14] extended it to optimise both production rates and cut-off grades.

The basic characteristic of the approach for the multi-mineral case is similar to the model in the example given in 3.2.3.1. The basic difference in the multi-mineral case is the exponential increase in the number of cut-off grades or the number of pairs of cut-off grades considered. However, as the number of depletion rates is the same for each mineral, the procedure is quite similar to the single mineral case described in Chapter 3.

In the two-mineral case, the grade-tonnage distribution is two-dimensional. Assume that the grade-tonnage distribution of a mineral deposit consists of W grade cells for mineral 1. Hence, there would be W+1 grade limits.

As there are two minerals in the grade distribution, the deposit will have W grade distributions for mineral 2. Assume that each grade distribution for mineral 2 has Y individual cells. As a result, there would be Y+1 grade limits.

In the determination of cut-off grades and production rates by dynamic programming, the rates of depletion are defined by the tonnage interval on which a

decision to mine or not can be made. Thus, after the optimum pairs of cut-off grades have been determined, the process can proceed as for the single mineral case. The optimum for each year can be found from among the alternative depletion rates. Basic equations for this principle of optimisation can be found in equations 3.43 and 3.44.

The dynamic programming used in this research work is based on Dowd's dynamic programming method for optimisation of cut-off grades and production rates. It is the extension of the method into multi-mineral deposits with one exception. In this work, recovery rate is expected as a fixed percentage regardless of the head grade or the amount of throughput as in Lane's method.

The type of dynamic programming technique applied to multi-mineral cut-off grade determination is discrete, deterministic and stationary discounted.

The following expressions specify the numbers of calculations involved in the programs developed in this research work.

The number of different depletion rates is:

$$d = \frac{M}{I} + 1 \qquad (4.40)$$

where;

d: number of possible depletion rates,

M: maximum depletion rate,

I: tonnage interval.

The number of different depletion rates above the lowest depletion rate is:

$$D = d - \frac{H}{I} \qquad (4.41)$$

where;

D: number of different depletion rates above the lowest depletion rate,

H: maximum production rate (or minimum depletion rate).

The number of stages of the system is:

$$n = \frac{T}{H} + 2 \qquad (4.42)$$

where;

n: number of stages,

T: total reserve in the system.

If the stages are 1,2,3,.....,n-2,n-1,n, the number of states for each stage is:

$$s_n = n * (D - 1) + 1 \qquad (4.43)$$

where;

s_n: number of states for each stage

The total number of calculations in searching for all possible combinations is:

$$\text{total number of calculations} = \sum_{m=1}^{n} D * s_m \qquad (4.44)$$

The total number of calculations can be reduced by recognising that only decreasing sequences of cut-off grade points can qualify as candidates for maximum discounted profit. All non-decreasing sequences of cut-off grades can thus be eliminated from the search. The total number of calculations, and consequently the computing time, can be reduced to half that given by Equation 4.44.

4.3.3.1 Steps of the algorithm for dynamic programming

The following steps comprise the dynamic programming algorithm implemented in the computer program developed for determining the optimum cut-off grades for mineral deposits containing three valuable minerals:

1. Input the data: mining cost, mineral processing cost, mining capacity, mineral processing capacity, minimum depletion interval, annual fixed cost, discount rate, refinery and/or marketing costs, selling prices refinery or marketing capacities, recoveries and grade categories for each of the minerals, amount of mineralised material associated with each of the grade categories, lowest limit of cut-off grade, highest limit of cut-off grades and interval between cut-off grade decisions.
2. Calculate the reserves available in the deposit.
3. Calculate the amount of material above the cut-off grades, amount of material below the cut-off grades and the average grades of ore above the cut-off grades for each grade interval by using equations 4.24, 4.25, 4.26, 4.27 and 4.28.
4. Determine the cut-off grades to be searched for the optimum by using the lower and upper cut-off grade limits and the interval between cut-off grade decisions.
5. Find the amount of material above the cut-off grades, amount of material below the cut-off grades and the average grades of ore above the cut-off grades for each of the cut-off grades to be searched for the optimum by using the interpolation technique.

6. Find the number of different depletion rates and the different depletion rates possible according to mining capacity, mineral processing capacity and the specified depletion rate interval.
7. Find the depletion rates and the production rates associated with each pair of cut-off grades, considering the maximum capacities of the mining, mineral processing and marketing stages.
8. Using the profit equation 4.36, calculate the annual profits for each pair of cut-off grades.
9. Find the optimal pair of cut-off grades for each of the depletion rates by assigning the annual profit values of each of the cut-off grades as the objective.
10. Start with the first year.
11. Assign the pairs of cut-off grades and production rates based on different depletion rates found in step 9 as each depletion rate being the state of the year (or stage).
12. Assign the profit values of each state as maximum discounted profit values of that state.
13. Go to the next year (or stage).
14. Start from the first state of the previous year.
15. Start from the first of the different depletion rates found in step 9.
16. Assign a new state for this year (or stage) by applying the different depletion rates to move from previous year's states. If the depletion rate of the new state is higher than the previous state, omit it.
17. Calculate the discounted profit from the cut-off grades and production rates assigned for each year, by using the Equation 3.45.

19. If there is another solution in the same state, i.e. with the same depletion amount, the one with the higher discounted profit should be assigned as the solution for the state.
20. Find the remaining reserve by subtracting the depletion rate used from the total reserve.
21. If this is the final depletion rate go to step 22. Otherwise, choose the next depletion rate and go to step 16.
22. If this is the last state go to step 23. Otherwise, go to the next state of the previous year and go to step 15.
23. If the remaining reserve is zero for all states of the current stage, go to step 24. Otherwise, go to step 13.
24. Find the state that gives the maximum total discounted profit.
25. By moving backwards find the previous states for all previous stages that yield the maximum total discounted profit.
26. Stop and write out the cut-off grades and production rates that give the maximum total discounted profit.

4.3.4 Optimisation of cut-off grades for multi-mineral deposits by genetic algorithms

The optimisation of cut-off grades for multi-mineral deposits is significantly more complex than for single-mineral deposits. A multivariate grade distribution must be used and consequently the dimension of the data increases. This increase in dimension causes an exponential increase in the area to be searched for the optimum.

Genetic algorithms work on representatives of solutions, known as chromosomes. The structures of chromosomes for single-mineral deposits and for multi-mineral deposits differ in that as the number of minerals increases, the length of the related chromosomes increases arithmetically.

In encoding an individual, the size of the string depends on mine life, number of minerals and number of cut-off grades to be searched. For the optimisation of cut-off grades for an ore deposit that contains one valuable mineral with a maximum mine life of 20 years, the string will be composed of 20 genes, representing the cut-off grades for each year. If the binary-coding method is used, the string will be long. A 5-bit string can represent a maximum of $2^5 = 32$ cut-off grades. If a gene of 5-bit length is used, the total string length will be 100. To derive real values from the binary code (i.e., mapping the string) formula 5.1 should be used for a binary string. The value of a binary representation for a 5-bit string would be an integer between 0 and 31 for the mineral as the first five bits of the string change from 00000 to 11111.

The application of genetic algorithms to the optimisation of cut-off grades for multi-mineral deposits brings about a further increase of the length of the string. Since for each year of mine-life there might be a different optimum cut-off grade, there should be different genes in the same string. In the case of a two-mineral deposit, if the mine life is 20 years, the string would be composed of 40 genes, each having 5-bit length, making the total length of the chromosome 200.

The genetic algorithms computer program developed in this research work is capable of optimisation of cut-off grades for mineral deposits that contain up to three minerals. Binary representation is used and if 32 different cut-off grades are to be searched for each mineral, 5-bit genes must be used. Three minerals require a 15-bit

string length. Therefore, if the maximum mine life is 20 years, an ore deposit that contains three minerals requires a string size of 300.

The process of scaling is used in this research work in order to improve the selection process. Fitness values are scaled by subtracting the fitness value of the worst individual from the fitness values of all the individuals of the population.

One safeguard has proved necessary to improve the computation results. Genetic algorithms differ substantially from the other two methods used in this research work in that individuals assigned randomly represent solutions for the entire mine life. They are improved over time on the basis of the principle of survival of the fittest. On the other hand, we know that true maximisation of net present value necessitates a sequence of declining cut-off grades. However, only a very small part of randomly selected populations can have cut-off grades in declining order for the life of the mine. Consequently, the algorithm has been changed in such a way that if the depletion rate for a specific year is more than that of the previous year, the cut-off grade for the specified year is set deterministically to that of the previous year. This policy enables the program to search for the optimum among the alternatives that are limited to sequences of declining cut-off grades and brings about a substantial improvement in the performance of the algorithm.

The genetic algorithms work in this research work was inspired by the work of Yun et al[47]. However, the research work described here is in a different context to that of Yun et al.

With respect to the other two methods used in this research, genetic algorithms use four additional parameters that are not directly related to technical or economic

constraints. Theses parameters are population size, generation size, crossover rate and mutation rate. These parameters have been tested in order to determine an optimum range of control values that will generate the highest discounted profit. The method and test results are included in Appendix B.

4.3.4.1 Steps of the algorithm for genetic algorithms

The following steps comprise the algorithm implemented in the computer program developed for determining optimum cut-off grades for mineral deposits that contain three valuable minerals using genetic algorithms:

1. Input the data: mining cost, mineral processing cost, mining capacity, mineral processing capacity, annual fixed cost, discount rate, selling prices, refinery and/or marketing costs, refinery or marketing capacities, recoveries, grade categories, amount of the mineralised material associated with each of the grade categories, lowest limit of cut-off grades, highest limit of cut-off grades, number of population, number of generations, crossover rate and mutation rate.
2. Calculate the reserves available in the deposit.
3. Calculate the amount of material above the cut-off grades, the amount of material below the cut-off grades and the average grade of ore above the cut-off grades for each of the minerals and for each grade interval by using equations 4.24, 4.25, 4.26, 4.27 and 4.28.
4. Determine the cut-off grades to be searched for the optimum by using the lower and upper cut-off grade limits and the number of different cut-off grades to be searched.

5. Use the interpolation technique to find the amount of material above the cut-off grades, the amount of material below the cut-off grades and the average grade of ore above the cut-off grades for each combination of three cut-off grades representing three minerals.
6. Find the depletion rates for each combination of three cut-off grades considering maximum capacities of mining, mineral processing and marketing stages.
7. Use profit equation 4.36 to calculate the annual profit for the cut-off grade combinations.
8. Set the number of generations as zero.
9. Initialise the number of individuals randomly; each individual is composed of random numbers that represent different cut-off grade policies for different minerals for the life of the mine.
10. Decode the numbers that represent individuals in the population and find the mapping values.
11. Assign the best-ever individual to zero.
12. Start with the first individual in the population.
13. Start with the first year.
14. If this is the first year go to step 16. Otherwise, go to step 15.
15. If the depletion rate is higher than that of the previous year, set the cut-off grade (and depletion rate) equal to that of the previous year.
16. Calculate the discounted profit, depending on the cut-off grades assigned for the year, by using equation 3.47.
17. Subtract the depletion rate used from the total reserve.

18. If the remaining total reserve is greater than zero, go to step 14. Otherwise, go to step 19.
19. Find the total discounted profit for the individual.
20. If the total discounted profit of the individual is higher than that for the best-ever individual, assign the individual as the best-ever individual.
21. If the individual number is less than the total number in the population go to the next individual and go to step 13. Otherwise, go to the next step.
22. Assign a fitness value for each individual in the population depending on their total discounted profit values.
23. Generate random variables for each individual for the selection process. Copy the individuals that have high fitness values, for the next generation. Determine a new population of the same size and of new individuals.
24. Generate random variables for selection of parents for the crossover process. Select the individuals for offspring production.
25. If the number of parents is an odd number, subtract the last one.
26. Generate random numbers for each chromosome in order to find points where crossover occurs. Recombine the parents and find new individuals carrying their parents' genes.
27. Copy the infants instead of their parents in the population.
28. Generate random numbers for each bit of each chromosome for the mutation process. If the random number is in the range of the mutation probability, mutate the related bit of the related individual.
29. Assign the new population after the mutation as the population.
30. Increase the number of generations by 1.

31. If the maximum number of generations has not been reached, go to the step 10. Otherwise, go to the next step.
32. Stop and write the results based on the best-ever individual, as this is the chromosome that has the highest fitness value ever found.

CHAPTER 5

THE SOFTWARE

5.1 Introduction

The software developed for this study includes programs for the determination of optimum cut-off grades for multi-mineral deposits by means of the grid search method, dynamic programming and genetic algorithms. All three programs, namely grid.cpp, dyn.cpp and gen.cpp, were written in C++ code. Although all the programs written are basically for cut-off grade optimisation, they are slightly different in terms of data requirements.

5.2 Grid search method

The software developed for the grid search method is based on the method proposed by Lane[30] for multi-mineral deposits. The steps in the algorithm are shown in Chapter 4 and the program can handle up to three valuable minerals for a maximum mine life of 20 years.

5.2.1 Input requirements

The following is a description of data entry and specifications for a mineral deposit containing three valuable minerals:

1 The number of minerals in the deposit for which cut-off grade optimisation is required. This must be an integer value of 1, 2 or 3. The number entered shows number of minerals to be computed. In this case, 3.

2 The number of grade intervals in the grade-tonnage data for the first mineral.

3 The grade boundaries in the grade-tonnage data for the first mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 2.

4 The number of grade intervals in the grade-tonnage data for the second mineral.

5 The grade boundaries in the grade-tonnage data for the second mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 4.

The number of grade intervals in the grade-tonnage data for the third mineral.

The grade boundaries in the grade-tonnage data for the third mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 6.

The ore quantities in millions of tons for each of the grade intervals specified in steps 2, 4 and 6. The first value entered by the user is the tonnage of ore for the first grade intervals of all minerals. The second value entered is the

tonnage of ore for the first grade intervals of the first and second minerals and the second grade interval of the third mineral, etc.

9 The lower limit for cut-off grades of the first mineral. This is the lowest cut-off grade to be considered.

10 The upper limit for cut-off grades of the first mineral. This is the highest limit of any cut-off grades to be considered.

11 The interval between cut-off grades for the first mineral. The cut-off grade interval determines the cut-off grades and the number of cut-off grades to be considered.

12 The lower limit for cut-off grade of the second mineral.

13 The upper limit for cut-off grade of the second mineral.

14 The interval between cut-off grades for the second mineral.

15 The lower limit for cut-off grade of the third mineral.

16 The upper limit for cut-off grade of the third mineral.

17 The interval between cut-off grades for the third mineral.

18 The maximum depletion rate. This is the annual capacity of the mining stage and must be entered in metric tons.

19 The maximum production rate. This is the annual capacity of the mineral processing plant and must be entered in metric tons.

20 The maximum refining or marketing rate of the first mineral. This is the annual capacity of the refinery or marketing of the first mineral and must be entered in metric tons.

21 The maximum refining or marketing rate of the second mineral.

22 The maximum refining or marketing rate of the third mineral.

23 The market price of the first mineral. This is the selling price of the first mineral.

24 The market price of the second mineral.

25 The market price of the third mineral.

26 The treatment-marketing cost for the first mineral. This is the cost of marketing the concentrate and/or the refinery cost and respective transportation cost.

27 The treatment-marketing cost for the second mineral.

28 The treatment-marketing cost for the third mineral.

29 The recovery rate for the first mineral.

30 The recovery rate for the second mineral.

31 The recovery rate for the third mineral.

32 The variable mining cost. This is the cost of the mining stage per ton of depletion.

33 The variable mineral processing cost. This is the cost of the mineral processing stage per ton of mined product.

34 The fixed costs. This is the annual fixed cost in millions of monetary unit.

35 The discount rate. This is the annual discount rate expressed as a percentage.

5.2.2 Output

The output from the program "grid.cpp" is written to the file "gridresult". The optimum total discounted profit is written first. There are then 14 columns (for the three minerals case) each containing a numerical output:

1 Year.

2 Profit.

3 Discounted profit.

4 Depletion rate.

5 Production rate.

6 Treatment/marketing rate for mineral number 1.

7 Treatment/marketing rate for mineral number 2.

8 Treatment/marketing rate for mineral number 3.

9 Cut-off grade for mineral number 1.

10 Cut-off grade for mineral number 2.

11 Cut-off grade for mineral number 3

12 Average grade for mineral number 1.

13 Average grade for mineral number 2.

14 Average grade for mineral number 3.

5.3 Dynamic programming

The software developed for the dynamic programming approach is based on the dynamic programming method for determination of cut-off grades and production rates by Dowd[14]. It is basically an extension of Dowd's method to the multi-mineral case. The steps of the algorithm are shown in Chapter 4.

5.3.1 Input requirements

The followings are a brief description of data entry and specifications for a mineral deposit that contains three valuable minerals:

1 The number of minerals in the deposit for which cut-off grade optimisation is required. This must be an integer value of 1, 2 or 3. The number entered shows number of minerals to be computed. In this case, 3.

2 The number of grade intervals in the grade-tonnage data for the first mineral.

3 The grade boundaries in the grade-tonnage data for the first mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 2.

4 The number of grade intervals in the grade-tonnage data for the second mineral.

5 The grade boundaries in the grade-tonnage data for the second mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 4.

6 The number of grade intervals in the grade-tonnage data for the third mineral.

7 The grade boundaries in the grade-tonnage data for the third mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 6.

8 The ore quantities in millions of tons for each of the grade intervals specified in steps 2, 4 and 6. The first value entered by the user is the tonnage of ore for the first grade intervals of all minerals. The second value entered is the tonnage of ore for the first grade intervals of the first and minerals and the second grade interval of the third mineral, etc.

9 The lower limit for cut-off grades of the first mineral. This is the lowest cut-off grade to be considered.

10 The upper limit for cut-off grades of the first mineral. This is the highest limit of any cut-off grades to be considered.

11 The interval between cut-off grades for the first mineral. The cut-off grade interval determines the cut-off grades and the number of cut-off grades to be considered.

12 The lower limit for cut-off grade of the second mineral.

13 The upper limit for cut-off grade of the second mineral.

14 The interval between cut-off grades for the second mineral.

15 The lower limit for cut-off grade of the third mineral.

16 The upper limit for cut-off grade of the third mineral.

17 The interval between cut-off grades for the third mineral.

18 The maximum depletion rate. This is the annual capacity of the mining stage and must be entered in millions of tons.

19 The maximum production rate. This is the annual capacity of the mineral processing plant and must be entered in millions of tons.

20 The minimum tonnage interval between a decision to mine or not to mine. This enables to apply different depletion rates to the optimisation process and must be entered in millions of tons.

21 The maximum refining or marketing rate of the first mineral. This is the annual capacity of the refinery or marketing of the first mineral and must be entered in metric tons.

22 The maximum refining or marketing rate of the second mineral.

23 The maximum refining or marketing rate of the third mineral.

24 The market price of the first mineral. This is the selling price of the first mineral.

25 The market price of the second mineral.

26 The market price of the third mineral.

27 The treatment-marketing cost for the first mineral. This is the cost of marketing the concentrate and/or the refinery cost and respective transportation cost.

28 The treatment-marketing cost for the second mineral.

29 The treatment-marketing cost for the third mineral.

30 The recovery rate for the first mineral.

31 The recovery rate for the second mineral.

32 The recovery rate for the third mineral.

33 The variable mining cost. This is the cost of the mining stage per ton of depletion.

34 The variable mineral processing cost. This is the cost of the mineral processing stage per ton of mined product.

35 The fixed costs. This is the annual fixed costs in millions of monetary unit.

36 The discount rate. This is the annual discount rate expressed as a percentage.

5.3.2 Output

The output from the program "dyn.cpp" is written to the file "dynresult". The optimum total discounted profit is written first. There are then 14 columns (for the three minerals case) each containing one item of numerical output. The columns are identical to those in the "gridresult" file.

5.4 Genetic algorithms

The software developed for genetic algorithms is based on the genetic algorithms method explained in Chapter 3. The steps of the algorithm are given in Chapter 6. The program can search for 32 different cut-off grades for each mineral for a maximum mine life of 20 years.

5.4.1 Input requirements

The following is a brief description of data entry and specifications for a mineral deposit that contains three valuable minerals:

1 The number of minerals in the deposit for which cut-off grade optimisation is required. This must be an integer value of 1, 2 or 3. The number entered shows number of minerals to be computed. In this case, 3.

2 The number of grade intervals in the grade-tonnage data for the first mineral.

3 The grade boundaries in the grade-tonnage data for the first mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 2.

4 The number of grade intervals in the grade-tonnage data for the second mineral.

5 The grade boundaries in the grade-tonnage data for the second mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 4.

6 The number of grade intervals in the grade-tonnage data for the third mineral.

7 The grade boundaries in the grade-tonnage data for the third mineral. The number of grade boundaries entered in this step must be one more than the number of grade intervals entered in step 6

8 The ore quantities in millions of tons for each of the grade intervals specified in steps 2, 4and 6. The first value entered by the user is the tonnage of ore for the first grade intervals of all minerals. The second value entered is the tonnage of ore for the first grade intervals of the first and second minerals and the second grade interval of the third mineral, etc.

9 The lower limit for cut-off grades of the first mineral. This is the lowest cut-off grade to be considered.

10 The upper limit for cut-off grades of the first mineral. This is the highest limit of any cut-off grades to be considered.

11 The lower limit for cut-off grade of the second mineral.

12 The upper limit for cut-off grade of the second mineral.

13 The lower limit for cut-off grade of the third mineral.

14 The upper limit for cut-off grade of the third mineral

15 The maximum depletion rate. This is the annual capacity of the mining stage and must be entered in millions of tons.

16 The maximum production rate. This is the annual capacity of the mineral processing plant and must be entered in millions of tons.

17 The maximum refining or marketing rate of the first mineral. This is the annual capacity of the refinery or marketing of the first mineral and must be entered in metric tons.

18 The maximum refining or marketing rate of the second mineral.

19 The maximum refining or marketing rate of the third mineral.

20 The market price of the first mineral. This is the selling price of the first mineral.

21 The market price of the second mineral.

22 The market price of the third mineral.

23 The treatment-marketing cost for the first mineral. This is the cost of marketing the concentrate and/or the refinery cost and respective transportation cost.

24 The treatment-marketing cost for the second mineral.

25 The treatment-marketing cost for the third mineral.

26 The recovery rate for the first mineral.

27 The recovery rate for the second mineral.

28 The recovery rate for the third mineral.

29 The variable mining cost. This is the cost of the mining stage per ton of depletion.

30 The variable mineral processing cost. This is the cost of the mineral processing stage per ton of mined product.

31 The fixed costs. This is the annual fixed costs in millions of monetary unit.

32 The discount rate. This is the annual discount rate expressed as a percentage.

33 The number of population. This is the number of chromosomes or random search points to start with.

34 The number of generations. This is the number of iterations before the program stops.

35 The crossover rate. This is the likelihood percentage of the individuals to crossover in order to produce new individuals.

36 The mutation rate. This is the likelihood percentage of the individuals to mutate.

5.4.2 Output

The output from the program "gen.cpp" is written to the file "genresult". The optimum total discounted profit is written first. There are then 14 columns (for the three minerals case) each representing a numerical output. The columns are identical to those in the "gridresult" file.

CHAPTER 6

CASE STUDIES

6.1 Introduction

Three case studies have been included here to illustrate the application of the software for determining optimal cut-off grades for multi-mineral deposits. The first case study is based on data from a copper deposit, the second is from a copper and gold deposit and the third is from a gold, lead and zinc deposit. This chapter summarises the data and the results obtained by applying the software for the grid search method, dynamic programming and genetic algorithms to each of the case studies.

6.2 Case Study 1

6.2.1 Technical and Economic Data

Case study 1 is a porphyry copper deposit. The grade-tonnage distribution of the deposit is taken from Dowd[14] and is shown in Figure 6.1. The rest of the technical and economic data are taken from MINVEST software package[17] sample files. The technical and economic data for the grid search method, dynamic programming and

for genetic algorithms related to the case study are given in Tables 6.1, 6.2 and 6.3 respectively. The input files are given in Appendix A.

6.2.2 The Results

The results, showing the complete cut-off grade policy, are given in Tables 6.4, 6.5 and 6.6 for the grid search method, dynamic programming and genetic algorithms respectively. The output files follow the order shown in Chapter 5.2.2.

For the grid search method, 32 different cut-off grades were searched by means of Pentium II 300 MHz PC and the optimum was found in less than 1 CPU second. The mining operation terminates in 14.35 years and total production is 143,525,852 tonnes. Total discounted profit is 301,783,000 dollars.

For dynamic programming, 32 different cut-off grades and 7 different production rates were searched by means of Pentium II 300 MHz PC and the solution was found in less than 1 CPU second. The mining operation terminates in 14.60 years and total production is 145,861,749 tonnes. Total discounted profit is 301,145 000,dollars.

For genetic algorithms, 32 different cut-off grades were searched. A total of 500 generations was computed by means of Pentium II 300 MHz PC in 50 CPU seconds. The mining operation terminates in 14.49 years and total production is 144,889,728 tonnes. Total discounted profit is 301,753,000 dollars.

The cut-off grades, and as a result the depletion rates, are lowered progressively throughout the life of the mine. Cut-off grades for each year are similar for all three methods. For all methods, cut-off grades start around 0.96% and lowered to around 0.64% in less than 15 years. The cut-off grades versus years for

each of the methods have been plotted and can be seen in Figure 6.2 in order to trace the decreasing order. The optimum discounted profits for the three methods are very similar. The differences among the discounted profits are negligible, around 0.2%.

The production rates for the grid search method and genetic algorithms are identical and equal to the maximum production rate except for the last year, as shown in Table 8. However, the production rates for dynamic programming differ depending on the cut-off grade, although they are very close to the maximum production rate. This is because, in dynamic programming, pre-determined depletion rates restrict the optimisation process.

The grid search method finds the highest total discounted profit with the shortest mine life.

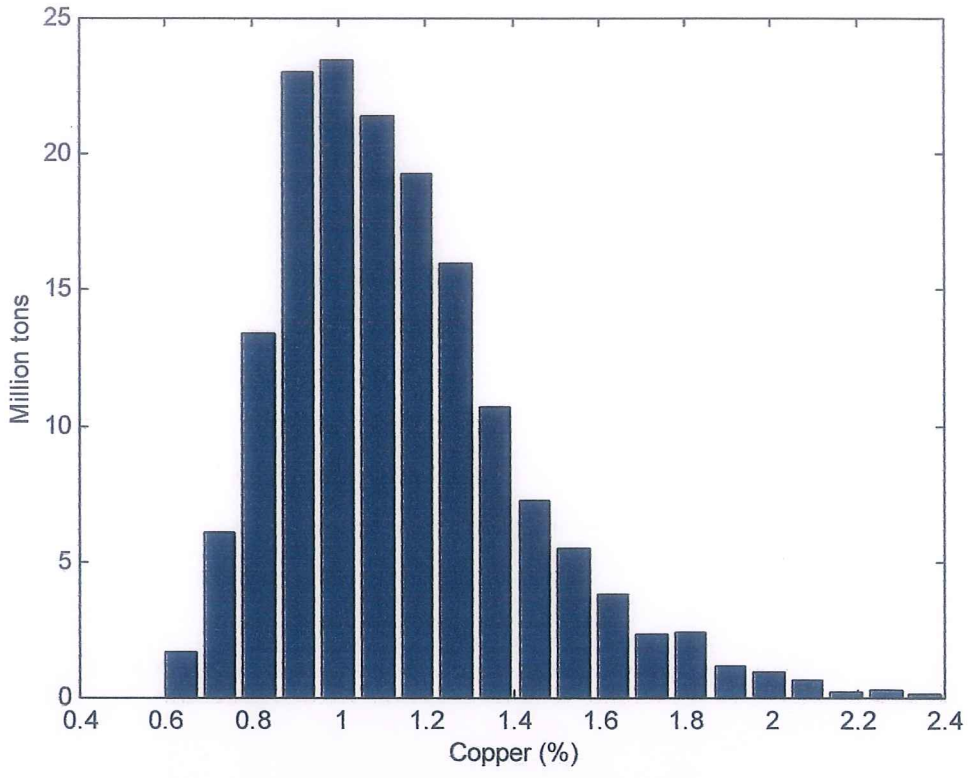

Figure 6.1 Grade-tonnage distribution for the copper deposit.

Description	Value
Lower limit of cut-off grades (%)	0.635
Upper limit of cut-off grades (%)	1.1
Interval between cut-off grade decisions (%)	0.015
Mining Capacity (tons per year)	13 000 000
Mineral processing capacity (tons per year)	10 000 000
Marketing and/or refining capacity (tons per year)	130 000
Selling price (dollars per ton)	1 344
Marketing and/or refining cost (dollars per ton)	506
Recovery rate (%)	92
Variable mining cost of material mined (dollars per ton)	0.8
Variable concentration cost of material processed (dollars per t	4.4
Fixed costs (dollars per year)	1 750 000
Discount rate (%)	10

Table 6.1 Technical and economic data for the grid search method.

Description	Value
Lower limit of cut-off grades (%)	0.635
Upper limit of cut-off grades (%)	1.1
Interval between cut-off grade decisions (%)	0.015
Mining Capacity (tons per year)	13 000 000
Mineral processing capacity (tons per year)	10 000 000
Tonnage interval between decisions to mine or not (tons per yea	500 000
Marketing and/or refining capacity (tons per year)	130 000
Selling price (dollars per ton)	1 344
Marketing and/or refining cost (dollars per ton)	506
Recovery rate (%)	92
Variable mining cost of material mined (dollars per ton)	0.8
Variable concentration cost of material processed (dollars per to	4.4
Fixed costs (dollars per year)	1 750 000
Discount rate (%)	10

Table 6.2 Technical and economic data for dynamic programming.

Description	Value
Lower limit of cut-off grades (%)	0.635
Upper limit of cut-off grades (%)	1.1
Mining Capacity (tons per year)	13 000 000
Mineral processing capacity (tons per year)	10 000 000
Marketing and/or refining capacity (tons per year)	130 000
Selling price (dollars per ton)	1 344
Marketing and/or refining cost (dollars per ton)	506
Recovery rate (%)	92
Variable mining cost of material mined (dollars per ton)	0.8
Variable concentration cost of material processed (dollars per t	4.4
Fixed costs (dollars per year)	1 750 000
Discount rate (%)	10
Population size (number of individuals in the population)	500
Number of generations	500
Crossover rate (%)	50
Mutation rate (%)	60

Table 6.3 Technical and economic data for genetic algorithms.

The optimum total discounted profit is 3.01783e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr	Cutoff	Mean
1	42440442	38582220	12917604	10000000	117571	0.965000	1.277946
2	42440442	35074746	12917604	10000000	117571	0.965000	1.277946
3	42012913	31564923	12533313	10000000	116694	0.950000	1.268413
4	41576677	28397430	12171226	10000000	115828	0.935000	1.258997
5	40693360	25267375	11514989	10000000	114147	0.905000	1.240730
6	40419689	22815861	11332301	10000000	113646	0.890000	1.235285
7	39844250	20446400	10983781	10000000	112627	0.860000	1.224205
8	39543332	18447256	10817438	10000000	112109	0.845000	1.218575
9	38928848	16509632	10505111	10000000	111077	0.815000	1.207364
10	38630067	14893563	10368657	10000000	110591	0.785000	1.202073
11	38309291	13427173	10235702	10000000	110081	0.755000	1.196532
12	37967356	12097570	10106114	10000000	109549	0.725000	1.190752
13	37811360	10952604	10052777	10000000	109312	0.680000	1.188176
14	37698138	9927098	10017531	10000000	109143	0.650000	1.186341
15	13270879	3176943	3525852	3525852	38451	0.635000	1.185390

Table 6.4 Output file for the grid search method.

The optimum discounted profit is 3.01145e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr	Cutoff	Mean
1	41896592	38087811	12500000	9973420	116384	0.950000	1.268413
2	41896592	34625283	12500000	9973420	116384	0.950000	1.268413
3	41896592	31477530	12500000	9973420	116384	0.950000	1.268413
4	40638113	27756378	11500000	9986983	113999	0.905000	1.240730
5	40638113	25233071	11500000	9986983	113999	0.905000	1.240730
6	40638113	22939155	11500000	9986983	113999	0.905000	1.240730
7	38909056	19966498	10500000	9995135	111023	0.815000	1.207364
8	38909056	18151362	10500000	9995135	111023	0.815000	1.207364
9	38909056	16501238	10500000	9995135	111023	0.815000	1.207364
10	38909056	15001125	10500000	9995135	111023	0.815000	1.207364
11	37638789	13192166	10000000	10000000	109056	0.635000	1.185390
12	37638789	11992878	10000000	10000000	109056	0.635000	1.185390
13	37638789	10902616	10000000	10000000	109056	0.635000	1.185390
14	37638789	9911469	10000000	10000000	109056	0.635000	1.185390
15	22583273	5616344	6000000	6000000	65434	0.635000	1.185390

Table 6.5 Output file for dynamic programming.

The optimum total discounted profit is 3.01753e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr	Cutoff	Mean
1	42440442	38582220	12917604	10000000	117571	0.965000	1.277946
2	42440442	35074746	12917604	10000000	117571	0.965000	1.277946
3	42012913	31564923	12533313	10000000	116694	0.950000	1.268413
4	40693360	27794112	11514989	10000000	114147	0.905000	1.240730
5	40693360	25267375	11514989	10000000	114147	0.905000	1.240730
6	40419689	22815861	11332301	10000000	113646	0.890000	1.235285
7	39234140	20133317	10656058	10000000	111586	0.830000	1.212890
8	39234140	18303016	10656058	10000000	111586	0.830000	1.212890
9	39234140	16639105	10656058	10000000	111586	0.830000	1.212890
10	38140918	14704975	10170496	10000000	109818	0.740000	1.193672
11	38140918	13368159	10170496	10000000	109818	0.740000	1.193672
12	37811360	12047865	10052777	10000000	109312	0.680000	1.188176
13	37698138	10919808	10017531	10000000	109143	0.650000	1.186341
14	37638789	9911469	10000000	10000000	109056	0.635000	1.185390
15	18404343	4625757	4889728	4889728	53325	0.635000	1.185390

Table 6.6 Output file for genetic algorithms.

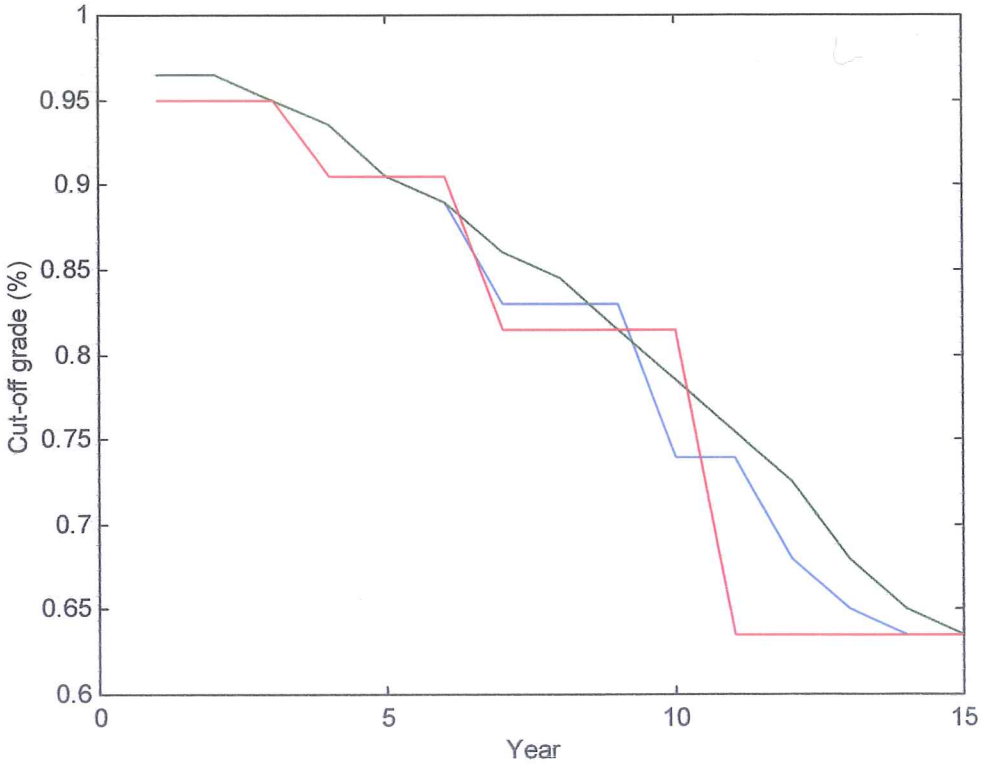

Figure 6.2 Cut-off grades for the three methods. Green line represents the grid search method, red line represents dynamic programming and blue line represents genetic algorithms.

6.3 Case Study 2

6.3.1 Technical and Economic Data

Case study 2 is a copper and gold deposit. The grade-tonnage distribution of the deposit is given in Mohammed[34] and is shown in Figure 6.3. The rest of the technical and economic data is taken from the same source. The technical and economic data for the grid search method , dynamic programming and for genetic

algorithms are given in Tables 6.7, 6.8 and 6.9 respectively. The input files are given in Appendix A.

6.3.2 The Results

The results are given in Tables 6.10, 6.11 and 6.12 for the grid search, dynamic programming and genetic algorithms respectively. The output files follow the order shown in Chapter 5.2.2.

For the grid search method, the solution was found by means of Pentium II 300 MHz PC in 1 CPU second. The mining operation terminates in 12.51 years and total production is 62,548,563 tonnes. Total discounted profit is 410,739,000 dollars.

For dynamic programming, the solution was found by means of Pentium II 300 MHz PC in 1 CPU second. The mining operation terminates in 12.75 years and total production is 63,483,005 tonnes. Total discounted profit is 408,683,000 dollars.

For genetic algorithms, a total of 500 generations was computed by means of Pentium II 300 MHz PC in 65 CPU seconds. The mining operation terminates in 12.69 years and total production is 63,440,968 tonnes. Total discounted profit is 410,211,000 dollars.

The optimum cut-off grades change and the depletion rates decrease throughout the life of the mine. The depletion rates versus years for each of the methods have been plotted and can be seen in Figure 6.4 in order to trace the decreasing order. The differences between the discounted profits are around 0.5%.

The grid search method finds the highest total discounted profit with the shortest mine life.

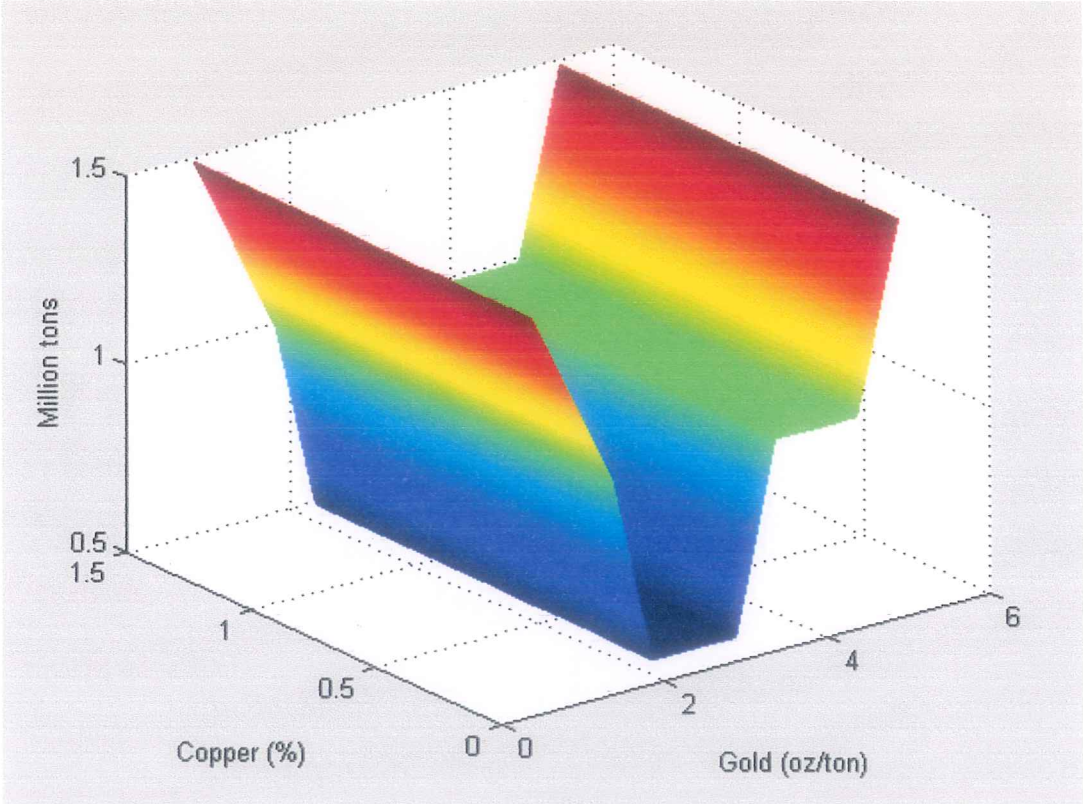

Figure 6.3 Grade-tonnage distribution for the copper and gold deposit.

Description	Value
Lower limit of cut-off grades for copper (%)	0
Upper limit of cut-off grades for copper (%)	0.93
Interval between cut-off grade decisions for copper (%)	0.03
Lower limit of cut-off grades for gold (ounce/ton)	0
Upper limit of cut-off grades for gold (ounce/ton)	0.0465
Interval between cut-off grade decisions for gold (ounce/ton)	0.0015
Mining Capacity (tons per year)	10 000 000
Mineral processing capacity (tons per year)	5 000 000
Marketing and/or refining capacity for copper (tons per year)	50 000
Marketing and/or refining capacity for gold (ounces per year)	150 000
Selling price for copper (dollars per ton)	2 100
Selling price for gold (dollars per ounce)	385
Marketing and/or refining cost for copper (dollars per ton)	100
Marketing and/or refining cost for gold (dollars per ounce)	5
Recovery rate for copper (%)	90
Recovery rate for gold (%)	80
Variable mining cost of material mined (dollars per ton)	2.5
Variable concentration cost of material processed (dollars per ton)	5.5
Fixed costs (million dollars per year)	3 500 000
Discount rate (%)	15

Table 6.7 Technical and economic data for the grid search method for the copper and gold deposit.

Description	Value
Lower limit of cut-off grades for copper (%)	0
Upper limit of cut-off grades for copper (%)	0.93
Interval between cut-off grade decisions for copper (%)	0.03
Lower limit of cut-off grades for gold (ounce/ton)	0
Upper limit of cut-off grades for gold (ounce/ton)	0.0465
Interval between cut-off grade decisions for gold (ounce/ton)	0.0015
Mining Capacity (tons per year)	10 000 000
Mineral processing capacity (tons per year)	5 000 000
Tonnage interval between decisions to mine or not (tons per year)	100 000
Marketing and/or refining capacity for copper (tons per year)	50 000
Marketing and/or refining capacity for gold (ounces per year)	150 000
Selling price for copper (dollars per ton)	2 100
Selling price for gold (dollars per ounce)	385
Marketing and/or refining cost for copper (dollars per ton)	100
Marketing and/or refining cost for gold (dollars per ounce)	5
Recovery rate for copper (%)	90
Recovery rate for gold (%)	80
Variable mining cost of material mined (dollars per ton)	2.5
Variable concentration cost of material processed (dollars per ton)	5.5
Fixed costs (million dollars per year)	3 500 000
Discount rate (%)	15

Table 6.8 Technical and economic data for dynamic programming for the copper and gold deposit.

Description	Value
Lower limit of cut-off grades for copper (%)	0
Upper limit of cut-off grades for copper (%)	0.93
Lower limit of cut-off grades for gold (ounce/ton)	0
Upper limit of cut-off grades for gold (ounce/ton)	0.0465
Mining Capacity (tons per year)	10 000 000
Mineral processing capacity (tons per year)	5 000 000
Marketing and/or refining capacity for copper (tons per year)	50 000
Marketing and/or refining capacity for gold (ounces per year)	150 000
Selling price for copper (dollars per ton)	2 100
Selling price for gold (dollars per ounce)	385
Marketing and/or refining cost for copper (dollars per ton)	100
Marketing and/or refining cost for gold (dollars per ounce)	5
Recovery rate for copper (%)	90
Recovery rate for gold (%)	80
Variable mining cost of material mined (dollars per ton)	2.5
Variable concentration cost of material processed (dollars per ton)	5.5
Fixed costs (million dollars per year)	3 500 000
Discount rate (%)	15
Population size (number of individuals in the population)	500
Number of generations	500
Crossover rate (%)	50
Mutation rate (%)	60

Table 6.9 Technical and economic data for genetic algorithms for the copper and gold deposit.

The optimum total discounted profit is 4.10739e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Cutoff1	Cutoff2	Mean1	Mean2
1	76265217	66317580	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
2	76265217	57667461	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
3	76265217	50145618	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
4	76265217	43604886	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
5	76265217	37917292	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
6	75575424	32673341	6483051	5000000	33407	147288	0.660000	0.046500	0.742373	0.036822
7	74588525	28040589	6270492	5000000	32662	147213	0.660000	0.040500	0.725820	0.036803
8	72028244	23546161	5839695	5000000	31706	142672	0.540000	0.036000	0.704580	0.035667
9	72028244	20474922	5839695	5000000	31706	142672	0.540000	0.036000	0.704580	0.035667
10	69818478	17258069	5543478	5000000	30986	138696	0.450000	0.031500	0.688587	0.034673
11	68427113	14707944	5387324	5000000	30383	137183	0.480000	0.021000	0.675176	0.034295
12	67350690	12588325	5275862	5000000	30127	134966	0.330000	0.025500	0.669483	0.033741
13	33307169	5413346	2599534	2548563	15020	67282	0.150000	0.025500	0.654833	0.033000

Table 6.10 Output file for the grid search method.

The optimum discounted profit is 4.08683e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Cutoff1	Cutoff2	Mean1	Mean2
1	75639608	65773572	6600000	4960784	33349	147961	0.720000	0.046500	0.746957	0.037282
2	75639608	57194410	6600000	4960784	33349	147961	0.720000	0.046500	0.746957	0.037282
3	75342876	49539164	6500000	4970588	33303	146908	0.690000	0.046500	0.744444	0.036944
4	75342876	43077534	6500000	4970588	33303	146908	0.690000	0.046500	0.744444	0.036944
5	74553294	37066163	6300000	4982353	32704	146835	0.690000	0.042000	0.729339	0.036838
6	74553294	32231446	6300000	4982353	32704	146835	0.690000	0.042000	0.729339	0.036838
7	74553294	28027345	6300000	4982353	32704	146835	0.690000	0.042000	0.729339	0.036838
8	72186974	23598050	5900000	4974510	31452	144454	0.690000	0.030000	0.702519	0.036298
9	70909961	20157036	5700000	4992157	31132	141718	0.690000	0.024000	0.692910	0.035485
10	69427124	17161323	5500000	4996732	30784	138399	0.540000	0.030000	0.684532	0.034622
11	67550484	14519519	5300000	4988235	30226	134959	0.420000	0.025500	0.673264	0.033819
12	66472627	12424209	5200000	4996078	29770	133976	0.930000	0.010500	0.662075	0.033520
13	48688431	8200227	3800000	3725490	21956	98353	0.930000	0.009000	0.654833	0.033000

Table 6.11 Output file for dynamic programming.

The optimum discounted profit is 4.10211e+008										
Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Cutoff1	Cutoff2	Mean1	Mean2
1	76265217	66317580	6652174	5000000	33613	149130	0.720000	0.046500	0.746957	0.037282
2	75809402	57322799	6538462	5000000	33500	147778	0.690000	0.046500	0.744444	0.036944
3	75809402	49845912	6538462	5000000	33500	147778	0.690000	0.046500	0.744444	0.036944
4	75063333	42917704	6375000	5000000	32944	147667	0.690000	0.045000	0.732083	0.036916
5	74588525	37083679	6270492	5000000	32662	147213	0.690000	0.040500	0.725820	0.036803
6	74588525	32246678	6270492	5000000	32662	147213	0.690000	0.040500	0.725820	0.036803
7	74588525	28040589	6270492	5000000	32662	147213	0.690000	0.040500	0.725820	0.036803
8	74588525	24383121	6270492	5000000	32662	147213	0.690000	0.040500	0.725820	0.036803
9	72839063	20705408	5976563	5000000	32256	142813	0.540000	0.043500	0.716797	0.035703
10	67350690	16648060	5275862	5000000	30127	134966	0.390000	0.025500	0.669483	0.033741
11	67350690	14476574	5275862	5000000	30127	134966	0.390000	0.025500	0.669483	0.033741
12	67350690	12588325	5275862	5000000	30127	134966	0.390000	0.025500	0.669483	0.033741
13	44970007	7634437	3509787	3440968	20279	90842	0.180000	0.034500	0.654833	0.033000

Table 6.12 Output file for genetic algorithms.

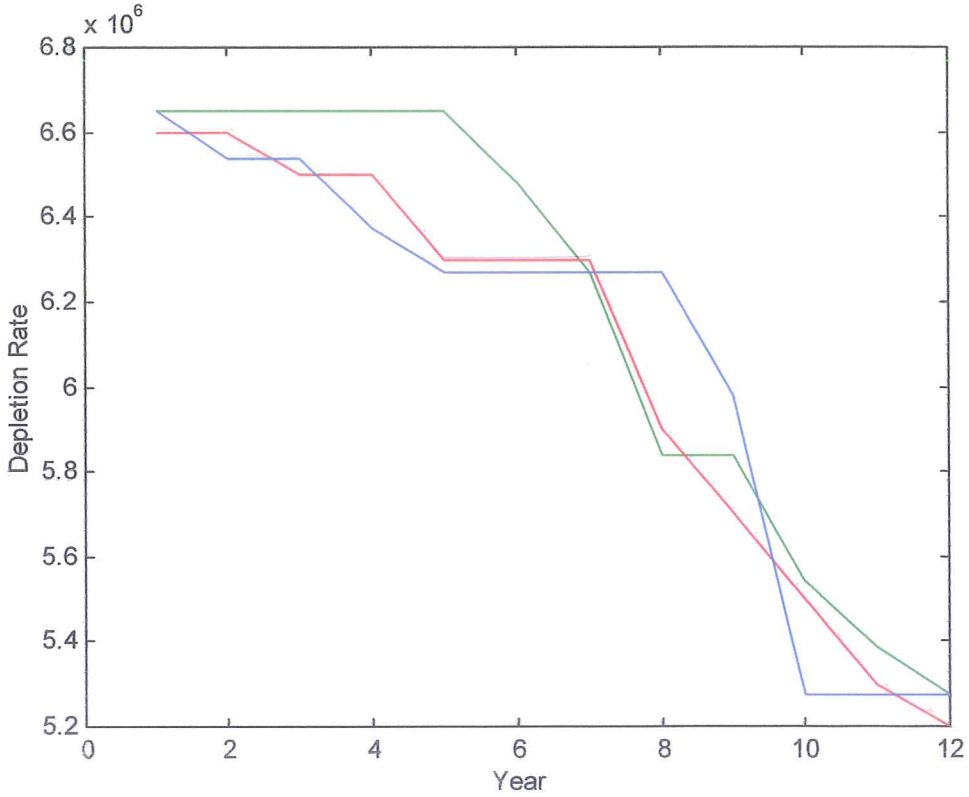

Figure 6.4 Depletion rates for the three methods. Green line represents the grid search method, red line dynamic programming and blue line genetic algorithms.

6.4 Case Study 3

6.4.1 Technical and Economic Data

Case study 3 is from a gold, lead and zinc deposit. The grade-tonnage distribution of the deposit has been created from borehole data and is shown in Figure 6.5. The sections from a to h of the figure shows the grade intervals of lead that are 0.00 0.25 0.50 0.75 1.0 1.25 1.50 and 2.00. For section a the grade of lead is more than 0.00%, for section b the grade of lead is more than 0.25%, etc. The technical and economic data have been approximated from previous works. The technical and economic data for the grid search method, dynamic programming and genetic algorithms are given in Tables 6.13, 6.14 and 6.15 respectively. The input files are shown in Appendix A.

6.4.2 The Results

The results are given in Tables 6.16, 6.17 and 6.18 for the grid search method, dynamic programming and genetic algorithms respectively. The output files follow the order shown in Chapter 5.2.2.

For the grid search method, the solution was found by means of Pentium II 300 MHz PC in approximately 2 CPU seconds. The mining operation terminates in 8.07 years and total production is 8,066,991 tonnes. Total discounted profit is 439,100,000 dollars.

For dynamic programming, 11 different production rates were searched by means of Pentium II 300 MHz PC in approximately 2 CPU seconds. The mining operation terminates in 8.29 years and total production is 8,284,402 tonnes. Total discounted profit is 434,112,000 dollars.

For genetic algorithms, a total of 500 generations was computed by means of Pentium II 300 MHz PC in approximately 25 CPU seconds. The mining operation terminates in 8.26 years and total production is 8,255,730 tonnes. Total discounted profit is 439,854,000 dollars.

The optimum cut-off grades change and the depletion rates decrease throughout the life of the mine. The depletion rates versus years for each of the methods have been plotted and can be seen in Figure 6.6 in order to trace the decreasing order. The differences between the discounted profits are around 1%.

Genetic algorithms find the highest total discounted profit and the grid search method finds the shortest mine life.

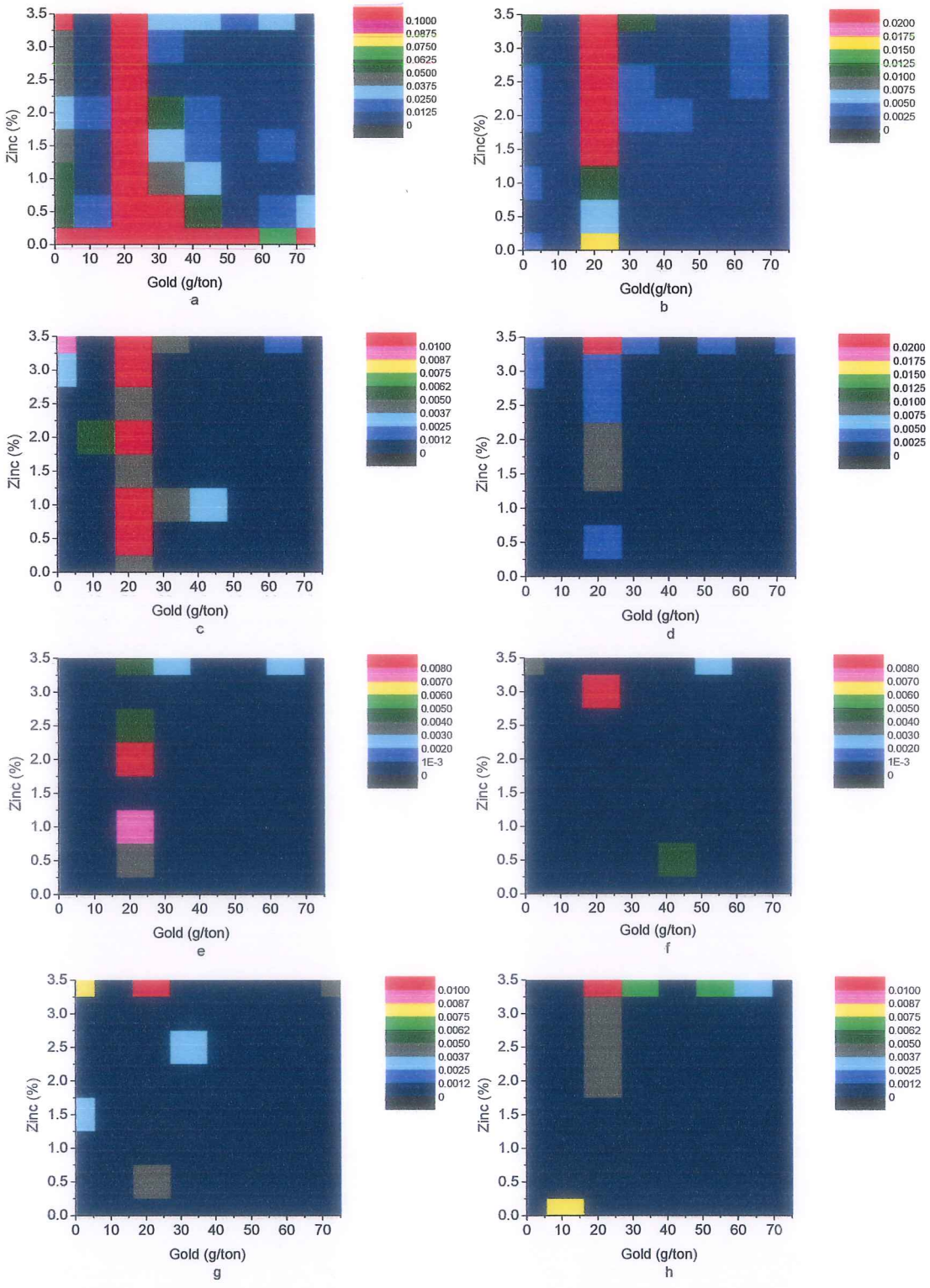

Figure 6.5 Grade-tonnage distribution for the gold, lead and zinc deposit. Distributions from the first to the last lead grade intervals can be seen from sections a to h.

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
Interval between cut-off grade decisions gold (%)	0.0006
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Interval between cut-off grade decisions zinc (%)	0.2
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Interval between cut-off grade decisions for lead (%)	0.1
Mining Capacity (tons per year)	1 200 000
Mineral processing capacity (tons per year)	1 000 000
Marketing and/or refining capacity for gold (tons per year)	10
Marketing and/or refining capacity for zinc (tons per year)	11000
Marketing and/or refining capacity for lead (tons per year)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10

Table 6.13 Technical and economic data for the grid search method for the gold, lead and zinc deposit.

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
Interval between cut-off grade decisions gold (%)	0.0006
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Interval between cut-off grade decisions zinc (%)	0.2
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Interval between cut-off grade decisions for lead (%)	0.1
Mining Capacity (tons per year)	1 200 000
Mineral processing capacity (tons per year)	1 000 000
Tonnage interval between decisions to mine or not (tons per year)	20 000
Marketing and/or refining capacity for gold (tons per year)	10
Marketing and/or refining capacity for zinc (tons per year)	11000
Marketing and/or refining capacity for lead (tons per year)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10

Table 6.14 Technical and economic data for dynamic programming for the gold, lead and zinc deposit.

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Mining Capacity (tons per year)	1 200 000
Mineral processing capacity (tons per year)	1 000 000
Marketing and/or refining capacity for gold (tons per year)	10
Marketing and/or refining capacity for zinc (tons per year)	11000
Marketing and/or refining capacity for lead (tons per year)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10
Population size (number of individuals in the population)	500
Number of generations	500
Crossover rate (%)	50
Mutation rate (%)	60

Table 6.15 Technical and economic data for genetic algorithms for the gold, lead and zinc deposit.

The optimum discounted profit is 4.391e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Tr-mr3	Cutoff1	Cutoff2	Cutoff3	Mean1	Mean2	Mean3
1	82423897	74930816	1167041	1000000	9	10572	1527	0.001200	2.600000	1.500000	0.001956	1.321489	0.179590
2	82423897	68118923	1167041	1000000	9	10572	1527	0.001200	2.600000	1.500000	0.001956	1.321489	0.179590
3	82423897	61926294	1167041	1000000	9	10572	1527	0.001200	2.600000	1.500000	0.001956	1.321489	0.179590
4	82344016	56242071	1165144	1000000	9	10564	1526	0.001200	2.600000	1.300000	0.001954	1.320560	0.179501
5	82344016	51129155	1165144	1000000	9	10564	1526	0.001200	2.600000	1.300000	0.001954	1.320560	0.179501
6	80753096	45583017	1131110	1000000	9	10377	1513	0.000600	2.600000	1.300000	0.001916	1.297138	0.178003
7	80753096	41439107	1131110	1000000	9	10377	1513	0.000600	2.600000	1.300000	0.001916	1.297138	0.178003
8	80301619	37461298	1122803	1000000	9	10374	1510	0.000600	1.600000	1.100000	0.001904	1.296792	0.177614
9	5350339	2269066	74718	66991	1	693	101	0.000600	0.800000	1.400000	0.001893	1.293160	0.177264

Table 6.16 Output file for the grid search method.

The optimum discounted profit is 4.34112e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Tr-mr3	Cutoff1	Cutoff2	Cutoff3	Mean1	Mean2	Mean3
1	81144899	73768090	1160000	999881	9	10584	1525	0.002400	3.000000	0.300000	0.001946	1.323202	0.179468
2	81144899	67061900	1160000	999881	9	10584	1525	0.002400	3.000000	0.300000	0.001946	1.323202	0.179468
3	81144899	60965364	1160000	999881	9	10584	1525	0.002400	3.000000	0.300000	0.001946	1.323202	0.179468
4	81144899	55423058	1160000	999881	9	10584	1525	0.002400	3.000000	0.300000	0.001946	1.323202	0.179468
5	79199360	49176571	1120000	997909	9	10349	1507	0.003000	2.800000	0.200000	0.001903	1.296367	0.177694
6	79199360	44705974	1120000	997909	9	10349	1507	0.003000	2.800000	0.200000	0.001903	1.296367	0.177694
7	79199360	40641795	1120000	997909	9	10349	1507	0.003000	2.800000	0.200000	0.001903	1.296367	0.177694
8	71814509	33501999	1000000	1000000	8	9482	1462	0.009000	3.000000	0.000000	0.001723	1.185280	0.171950
9	20908899	8867414	291151	291151	2	2761	426	0.009000	3.000000	0.000000	0.001723	1.185280	0.171950

Table 6.17 Output file for dynamic programming.

The optimum discounted profit is = 4.39854e+008

Yr	Profit	DisProfit	Depletion	Productn	Tr-mr1	Tr-mr2	Tr-mr3	Cutoff1	Cutoff2	Cutoff3	Mean1	Mean2	Mean3
1	82319950	74836319	1164693	1000000	9	10564	1526	0.001200	3.000000	0.800000	0.001953	1.320533	0.179577
2	82319950	68033017	1164693	1000000	9	10564	1526	0.001200	3.000000	0.800000	0.001953	1.320533	0.179577
3	82020659	61623335	1158748	1000000	9	10582	1524	0.001200	2.600000	0.800000	0.001945	1.322725	0.179298
4	82020659	56021214	1158748	1000000	9	10582	1524	0.001200	2.200000	0.800000	0.001945	1.322725	0.179298
5	82020659	50928376	1158748	1000000	9	10582	1524	0.001200	2.200000	0.700000	0.001945	1.322725	0.179298
6	79839492	45067312	1114896	1000000	9	10342	1507	0.001200	0.800000	0.700000	0.001892	1.292739	0.177344
7	79839492	40970283	1114896	1000000	9	10342	1507	0.001200	0.800000	0.700000	0.001892	1.292739	0.177344
8	72691453	33911099	1000000	1000000	8	9482	1462	0.000000	2.400000	1.100000	0.001723	1.185280	0.171950
9	18589355	8463256	255730	255730	2	2425	374	0.000000	2.200000	0.800000	0.001723	1.185280	0.171950

Table 6.18 Output file for genetic algorithms.

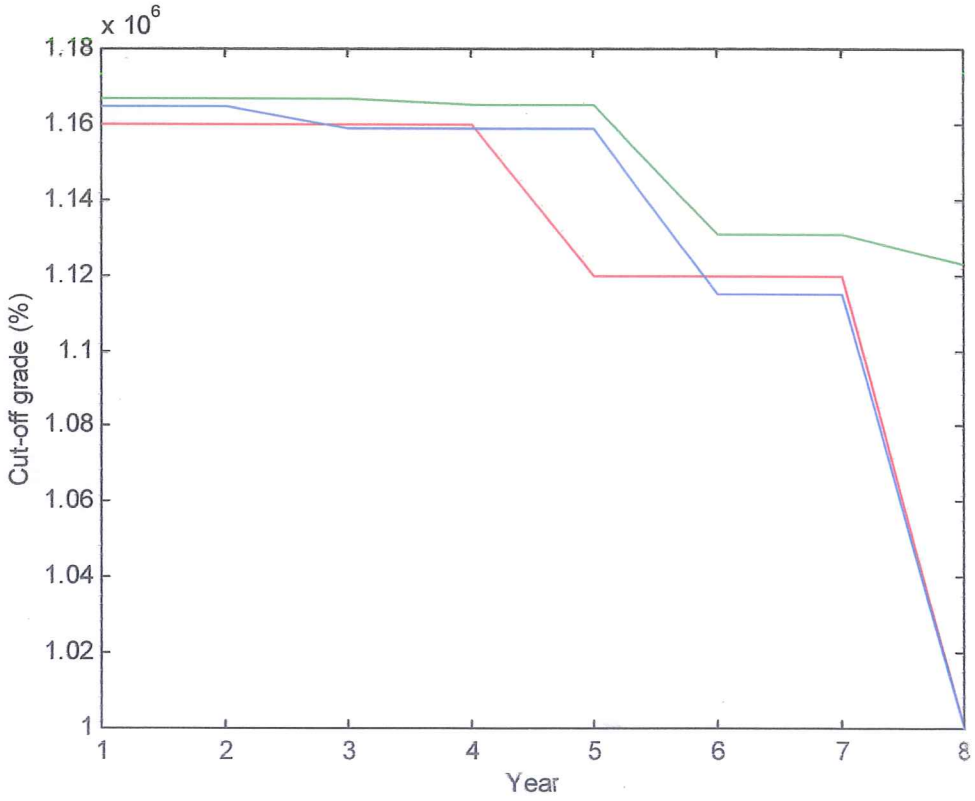

Figure 6.6 Depletion rates for the three methods. Green line represents the grid search method, red line represents dynamic programming and blue line represents genetic algorithms.

6.5 Conclusions

For the three case studies included in this chapter each method gives similar total discounted profits in reasonable computation times. The additional advantage of the dynamic programming method is that it optimises production rates as well as cut-off grades. Pre-determined depletion rates constrain the process as well as cut-off grades. As a result, it is misleading to conduct a detailed comparison of the results generated by the three methods with the expectation that they should yield the same production schedule. However, as it is almost always optimal to produce at

capacity, the optimisation of mining production rates is not usually an important consideration.

The genetic algorithms method requires significantly more CPU time than the other two methods. Genetic algorithms are fast search methods and capable of generating results in a very short time and the results could be improved by assigning a long generation time. The improvements in results are much higher in the early generations and decline rapidly for later generations to such an extent that, in many cases, they can be regarded as negligible. In the case studies above, the intention was to use reasonably long generation times so that high total discounted profits could be achieved.

The cut-off grades and depletion rates are reasonably close for all the methods. The main difference is that there is a greater variation in cut-off grades and depletion rates in the grid search method. There are many reasons for this. First of all, the number of depletion rates in dynamic programming restricts the process further as mentioned above. In addition, as mentioned in Chapter 4, the process of eliminating non-decreasing sequences of cut-off grades for dynamic programming and genetic algorithms generates a solution space that has less variation in cut-off grades and depletion rates over the life of the mine.

All the methods give similar results in terms of total discounted profit. However, there are many different combinations of cut-off grades that give the same, or very similar, maximum discounted profits for multi-mineral deposits. Thus the results generated by the three methods do not necessarily comprise the same combination of cut-off grades for the life of the mine. The schedule with the highest discounted profit must be the best. However, given the uncertainty associated with

all of the technical and economic parameters, it is misleading to place too much emphasis on differences in results unless they are significant.

Decreasing sequences of cut-off grades can be traced easily for single-mineral deposits. But for multi-mineral deposits, a substantial decrease in the cut-off grade of one mineral over a period of time may mean that the cut-off grade for another mineral might increase without increasing the depletion rate. For example, in Table 6.10, the cut-off grade of the second mineral in year 11 is higher than in year 10 but the depletion rate is substantially lower. Because the production rate is generally maximum, the decreasing sequence for multi mineral case must be traced from depletion rates.

Cut-off grades versus years for case study 1 is given in Figure 6.2. It is very clear that for all the methods cut-off grades decrease. However, there is a more gradual decrease in grid search method comparing to dynamic programming and genetic algorithms. The decrease is sharpest in dynamic programming. As a result, grid search method results with the highest total discounted profit and dynamic programming gives the lowest total discounted profit.

For case study 2, depletion rates versus years is shown in Figure 6.4 The order of decrease in depletion rates is obvious from the figure. In the grid search method, there is a more gradual decrease after the first years of production. The decrease is sharpest in dynamic programming.

For case study 3, depletion rates versus years is shown in Figure 6.6. The order of decrease can be seen from the figure. Genetic algorithms result with the highest total discounted.

For the sake of comparison of the methods, the same number of cut-off grades are searched for each of the case studies.

In order to verify the results obtained by means of the software developed, the case study 1 has been implemented to MINVEST software package. It was very encouraging that exactly the same production schedule has been found for dynamic programming. Although the algorithm of Lane's method in MINVEST program is not the same as the grid search method and genetic algorithms used in this research, the data for the grid search method given in Table 6.1 has been applied to by means of Lane's Method. It is encouraging that the results were substantially similar to the production schedule given in tables 6.4 and 6.6. The results obtained by means of MINVEST software package is given in tables 6.19 and 6.20. They can be compared with the tables 6.4, 6.5 and 6.6.

Year	OCO(%)	EQ-EG	CU-EG	QM(Mt)	QC(Mt)	CU-QR	P(m)	PD(m)	PV(m)
1	0.9	1.268	1.268	12.5	10	0.1	41.9	38.1	301.1
2	0.9	1.268	1.268	12.5	10	0.1	41.9	34.6	289.4
3	0.9	1.268	1.268	12.5	10	0.1	41.9	31.5	276.4
4	0.9	1.241	1.241	11.5	10	0.1	40.6	27.8	262.1
5	0.9	1.241	1.241	11.5	10	0.1	40.6	25.2	247.7
6	0.9	1.241	1.241	11.5	10	0.1	40.6	22.9	231.9
7	0.8	1.207	1.207	10.5	10	0.1	38.9	20	214.4
8	0.8	1.207	1.207	10.5	10	0.1	38.9	18.2	196.9
9	0.8	1.207	1.207	10.5	10	0.1	38.9	16.5	177.7
10	0.8	1.207	1.207	10.5	10	0.1	38.9	15	156.6
11	0.6	1.185	1.185	10	10	0.1	37.6	13.2	133.3
12	0.6	1.185	1.185	10	10	0.1	37.6	12	109
13	0.6	1.185	1.185	10	10	0.1	37.6	10.9	82.3
14	0.6	1.185	1.185	10	10	0.1	37.6	9.9	52.9
15	0.6	1.185	1.185	6	6	0.1	22.6	5.4	20.5

Table 6.19 Dowd's Dynamic Programming Method results by means of MINVEST.

Year	OCO(%)	EQ-EG	CU-EG	QM(Mt)	QC(Mt)	CU-QR	P(m)	PD(m)	PV(m)
1	0.968	1.28	1.28	13	10	0.118	42.528	38.662	301.6
2	0.968	1.28	1.28	12.985	10	0.118	42.512	35.134	288.44
3	0.949	1.268	1.268	12.503	10	0.117	41.978	31.539	273.99
4	0.929	1.255	1.255	12.037	10	0.115	41.406	28.281	258.87
5	0.909	1.243	1.243	11.584	10	0.114	40.792	25.329	243.05
6	0.887	1.234	1.234	11.299	10	0.114	40.367	22.786	226.48
7	0.864	1.226	1.226	11.032	10	0.113	39.928	20.489	208.8
8	0.84	1.217	1.217	10.76	10	0.112	39.436	18.397	189.91
9	0.814	1.207	1.207	10.498	10	0.111	38.915	16.504	169.71
10	0.786	1.202	1.202	10.371	10	0.111	38.635	14.896	148.1
11	0.755	1.197	1.197	10.236	10	0.11	38.31	13.428	124.65
12	0.722	1.191	1.191	10.103	10	0.11	37.958	12.095	99.199
13	0.686	1.189	1.189	10.06	10	0.109	37.834	10.959	71.577
14	0.647	1.186	1.186	10.014	10	0.109	37.686	9.924	41.321
15	0.635	1.185	1.185	3.518	3.518	0.038	13.241	3.17	8.189

Table 6.20 Lane's Method results by means of MINVEST.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The ultimate objective of mineral extraction is to generate profits. For some investors, however, the objective may be broader than the normal definition of profits. Governments, for example, may have immediate objectives, such as increasing employment, motivating economic development and/or minimising environmental impacts. Nevertheless all of these objectives can ultimately viewed as profit generation, however indirect that may be. In this research work the objective is to extract ore at the highest possible profit on the basis of using net present value as the financial evaluation technique to achieve that target.

Determination of a complete mine production schedule requires complex modelling of an orebody and the inclusion of access constraints. A realistic optimisation model must include numerous physical, technical and economic constraints. The basis of the research work described in this thesis was to find broad indications of optimum cut-off grades and a mining sequence that gives optimum discounted profits by using technical and economic constraints only. Detailed mine scheduling that includes physical, or access, constraints is beyond the scope of this

research. The orebody is defined by a grade-tonnage distribution, which gives ore tonnage for different grade intervals. Access constraints are not included so that any parcel of orebody is assumed to be immediately accessible. In other words, the grade-tonnage distribution is identical for all parts of the orebody and for all parcels of ore.

Multi-mineral deposits are commonly dealt with by converting them into their equivalent values in terms of one basic mineral in order to ease the difficulties associated with handling multi-mineral grade-tonnage data. The objective in this research was to find optimal sequences of cut-off grades for multi-mineral deposits without using the equivalents, or parametric cut-off grades, approach.

Three computer programs were written for this research based on the three optimisation methods studied. Among them, the grid search method has previously been applied to two-mineral deposits. The work described here has extended the grid search method to mineral deposits that contain up to three valuable minerals. Dynamic programming has also been used previously for cut-off grade optimisation but only for single-mineral deposits and for multi-mineral deposits using equivalents. This research work extends the dynamic programming approach to multi-mineral deposits on the basis of the individual minerals. The final optimisation method used in this research is genetic algorithms, which have not previously been applied to cut-off grade optimisation problems in the same context. The genetic algorithms computer program written during this research is the first known application to the determination of optimum sequences of cut-off grades.

It is very clear from this research, and work done by others, that maximum net present value can only be achieved by a declining cut-off grades policy. That is,

the mining operation should start with a relatively high cut-off grade that declines gradually over the life of the mine. For that reason, and in order to increase the speed of the computations, production schedules that do not have declining cut-off grades are eliminated explicitly in the programs for dynamic programming and genetic algorithms.

As for all the research methods, the three methods used in this research work have finite search space in terms of the lifetime of the deposit. All the computer programs developed for this study are capable of determining optimum cut-off grades for a maximum mine life of 20 years. As the capacity of the programs increases to handle larger mines with longer mine lives, the computing time will increase proportionally.

Although this research is not restricted to open-pit mines, the computer programs developed are more suitable to open-pit mining operations than to underground mines. In underground mines a greater proportion of waste is left *in situ* and access constraints are simultaneously more restrictive and less comprehensive than in open pits. In this study, variable mining costs are applied to a depletion rate which means that all material required to achieve a specific tonnage and grade is effectively excavated regardless of whether it is processed.

The grid search method used in this research is based on Lane's grid search method but with some important differences. The main difference is the size and method of selection of the grid. Lane[30] proposed a primary grid of 9 * 9 cells, a secondary grid of 6 * 6 cells and a third grid of another 6 * 6 cells. The process involves calculation of only 198 grid points. Lane claimed that the process of having three subsequent grids gives an accuracy of 1 in 81. However, there is no obvious

proof that using three subsequent grids improves the search process. Besides, because of the improvements in computer memory capacity and computational speed, it is now possible to search a finer grid, which has a better coverage than the method of grid selection suggested by Lane. As a result, grid points that can be searched for the optimum in this research work are far more than in Lane's method. For the case of two minerals, a grid of 100 * 100 cells, making a total of 10 201 grid points, could be searched and the answer can be obtained in 1 CPU second.

The dynamic programming method used in this research work is based on Dowd's dynamic programming method for optimisation of cut-off grades and production rates. In the work described here the method has been extended to multi-mineral deposits on the basis of the individual minerals. In his work, Dowd[14] expressed concentration recovery as a function of head grade. In this work, the recovery rate is expected as a fixed percentage regardless of the head grade or the amount of throughput.

The genetic algorithms method used in this research work is inspired by the work of Yun et al[47]. These authors did not actually use the method to find a sequence of optimum cut-off grades. Rather, they used it to find a static cut-off grade and a minimum average grade. In addition, in the work described in this thesis the process of scaling is used in order to improve the selection process. As a result, the genetic algorithms method used in this work is not in the same context as the work of Yun et al.

The case studies in Chapter 6 give promising results and they are not expensive in terms of computer memory or CPU time. All programs run efficiently on a reasonably specified PC. However, as the number of minerals in the

computation increases, satisfactory results, especially for genetic algorithms, will require increased computer memory capacity. One possible solution to overcome the computer memory deficiency problem is to implement the algorithms on a workstation.

As a result of this research work, it is now possible to establish a mine production schedule by determining optimum cut-off grades and production rates for multi-mineral deposits by means of the grid search method, dynamic programming and genetic algorithms. However, some improvements can be made in this and related subjects. The followings are some recommendations for future work:

- Determination of optimum cut-off grades with only technical and economic constraints but without access constraints is the main intention in this study. However, for a complete mine schedule physical access constraints should be included. This will, of course, make the optimisation process more complex and more computationally intensive.
- As with all industrial and financial ventures, mining is not free of risk. First of all there are financial risks arising from uncertainties in financial parameters, i.e., prices and costs. Secondly, and peculiar to the mining industry, there are technical risks originating from the uncertainties of the characteristics of mineral deposits such as grades and tonnages. Risk analysis has not been addressed in this research work. However, this has been addressed by others and it could be included with relatively little extra work (Dowd[14]).
- The software could be adapted to make it more suitable for underground operations.

- Detailed cash flow analysis that includes additional factors, such as taxation, could also be included in the process.
- The computer programs developed are DOS programs that read from data files and write to output files. A Windows version of the programs could be developed to increase the flexibility of data handling.

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APPENDIX A

THE INPUT FILES FOR THE CASE STUDIES

The input files for the case study 1

The input file for the grid search method follows the order shown in Chapter 5.2.1. The input file for dynamic programming follows the order shown in Chapter 5.3.1. The input file for genetic algorithms follows the order shown in Chapter 5.4.1.

1 20
0.635 0.725 0.816 0.906 0.997 1.088 1.178 1.269 1.359 1.45 1.54 1.631
1.722 1.812 1.903 1.993 2.084 2.174 2.265 2.355 2.446
13.44 23.04 23.44 21.44 19.28 16 10.72 7.28 5.52 3.84 2.32 2.4 1.2 0.96
0.64 0.24 0.32 0.16
0.635 1.1 0.015 13 10 0.13 1344 506 92 0.8 4.4 1.75 10

Table A.1 Input file for the grid search method for case study 1.

1 20
0.635 0.725 0.816 0.906 0.997 1.088 1.178 1.269 1.359 1.45 1.54 1.631
1.722 1.812 1.903 1.993 2.084 2.174 2.265 2.355 2.446
13.44 23.04 23.44 21.44 19.28 16 10.72 7.28 5.52 3.84 2.32 2.4 1.2 0.96
0.64 0.24 0.32 0.16
0.635 1.1 0.015 13 10 0.5 0.13 1344 506 92 0.8 4.4 1.75 10

Table A.2 Input file for dynamic programming for case study 1.

1 20
0.635 0.725 0.816 0.906 0.997 1.088 1.178 1.269 1.359 1.45 1.54 1.631
1.722 1.812 1.903 1.993 2.084 2.174 2.265 2.355 2.446
13.44 23.04 23.44 21.44 19.28 16 10.72 7.28 5.52 3.84 2.32 2.4 1.2 0.96
0.64 0.24 0.32 0.16
0.635 1.1 13 10 0.13 1344 506 92 0.8 4.4 1.75 10 500 500 50 60

Table A.3 Input file for genetic algorithms for case study 1.

The input files for the case study 2

The input file for the grid search method follows the order shown in Chapter 5.2.1. The input file for dynamic programming follows the order shown in Chapter 5.3.1. The input file for genetic algorithms follows the order shown in Chapter 5.4.1.

```
2 9 0.0 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95 2.0
9 0.00 0.015 0.02 0.025 0.03 0.035 0.04 0.045 0.05 0.055
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
0 0.93 0.03 0 0.0465 0.0015 10 5 0.05 0.15 2100 385 100 5 90 80 2.5 5.5 3.5 15
```

Table A.4 Input file for the grid search method for case study 2.

```
2 9 0.0 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95 2.0
9 0.00 0.015 0.02 0.025 0.03 0.035 0.04 0.045 0.05 0.055
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1.1 1.5
0 0.93 0.03 0 0.0465 0.0015 10 5 0.1 0.05 0.15 2100 385 100 5 90 80 2.5 5.5 3.5 15
```

Table A.5 Input file for dynamic programming for case study 2.

```
2 9 0.0 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95 2.0
9 0.00 0.015 0.02 0.025 0.03 0.035 0.04 0.045 0.05 0.055
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
1.5 1 .5 .5 .5 1 1 1 1.5
0 0.93 0 0.0465 10 5 0.05 0.15 2100 385 100 5 90 80 2.5 5.5 3.5 15 500 500 50 60
```

Table A.6 Input file for genetic algorithms for case study 2.

The input files for the case study 3

The input file for the grid search method follows the order shown in Chapter 7.2.1. The input file for the dynamic programming follows the order shown in Chapter 7.3.1. The input file for the genetic algorithms follows the order shown in Chapter 7.4.1.

```
3 8 0.0 0.0005 0.0010 0.0015 0.002 0.0025 0.005 0.01 0.011
8 0.0 0.50 1.0 1.5 2.0 2.5 3.0 3.5 4.0
8 0.00 0.25 0.50 0.75 1.0 1.25 1.50 2.00 2.60

0.9575 0.00336085 0 0 0 0 0 0
0.0553281 0 0 0 0 0 0 0
0.0607719 0.00309096 0 0 0 0 0 0
0.040923 0 0 0 0 0 0.00279636 0
```

0.0360611 0.00314928 0 0 0 0 0 0
0.0472554 0.00283176 0 0 0 0 0 0
0.0400378 0 0.00314928 0.00297648 0 0 0 0
0.13456 0.0119308 0.00916153 0.0031296 0 0.00336085 0.00790414 0.00770585

0.195926 0 0 0 0 0 0 0
0.0129613 0 0 0 0 0 0 0
0.0052488 0 0 0 0 0 0 0
0.00506218 0 0 0 0 0 0 0
0.0125286 0 0.00508429 0 0 0 0 0
0.0104307 0 0 0 0 0 0 0
0.00292032 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

1.0725 0.015974 0.00423306 0 0 0 0 0
1.07828 0.00719053 0.0162346 0.00426816 0.00399789 0 0.0045684 0
0.481643 0.0116567 0.0213151 0 0.00782347 0 0 0
0.328418 0.0335822 0.00392218 0.00844684 0 0 0 0
0.414266 0.062707 0.0121329 0 0.00832202 0 0 0.00410791
0.23602 0.0333151 0.00427372 0.00444471 0.0042457 0 0 0.00446342
0.141256 0.0202387 0.0164529 0.0041209 0 0.00867839 0 0.00429791
0.423013 0.120201 0.0404464 0.0206626 0.00409147 0 0.0113566 0.0120805

1.22976 0 0 0 0 0 0 0
0.151432 0 0 0 0 0 0 0
0.0381719 0 0.00423014 0 0 0 0 0
0.0322079 0 0 0 0 0 0 0
0.050114 0.0034007 0 0 0 0 0 0
0.00977195 0.003213 0 0 0 0 0.00326514 0
0.0156935 0 0 0 0 0 0 0
0.0332583 0.0103719 0.00399168 0.00381888 0.00289996 0 0 0.00740445

0.402052 0 0 0 0 0 0 0
0.0504829 0 0 0 0 0.0041013 0 0
0.0309103 0 0.00307055 0 0 0 0 0
0.0184902 0 0 0 0 0 0 0
0.0169469 0.00424764 0 0 0 0 0 0
0.00410791 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.0263339 0 0 0 0 0 0 0

0.15576 0 0 0 0 0 0 0
0.00900752 0 0 0 0 0 0 0
0.00859915 0 0 0 0 0 0 0

```
0.00390979 0 0 0 0 0 0 0
0.0110727 0 0 0 0 0 0 0
0.00907694 0 0 0 0 0 0 0
0.00795571 0 0 0 0 0 0 0
0.0180391 0 0 0.0031296 0 0.00293576 0 0.00702065

0.0721093 0 0 0 0 0 0 0
0.0128699 0 0 0 0 0 0 0
0.00152021 0 0 0 0 0 0 0
0.016871 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.00542246 0.00293576 0 0 0 0 0 0
0.00436655 0.00293576 0 0 0 0 0 0
0.0295253 0.00348705 0.00158976 0 0.00295682 0 0 0.00312304

0.269171 0 0 0 0 0 0 0
0.0262184 0 0 0 0 0 0 0
0.00712252 0 0 0 0 0 0 0
0.00125194 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.00128563 0 0 0 0 0 0 0
0.00619769 0.00133753 0 0.0038394 0.00380902 0 0.00455328 0

0 0.009 0.0006 0 3 0.2 0 1.5 0.1 12 10 0.00001 0.011 0.0016 11000000 1400 600
3000000 300 150 46 80 85 0.4 0.4 1 10
```

Table A.7 Input file for the grid search method for case study 3.

```
3 8 0.0 0.0005 0.0010 0.0015 0.002 0.0025 0.005 0.01 0.011
8 0.0 0.50 1.0 1.5 2.0 2.5 3.0 3.5 4.0
8 0.00 0.25 0.50 0.75 1.0 1.25 1.50 2.00 2.60

0.9575 0.00336085 0 0 0 0 0 0
0.0553281 0 0 0 0 0 0 0
0.0607719 0.00309096 0 0 0 0 0 0
0.040923 0 0 0 0 0 0.00279636 0
0.0360611 0.00314928 0 0 0 0 0 0
0.0472554 0.00283176 0 0 0 0 0 0
0.0400378 0 0.00314928 0.00297648 0 0 0 0
0.13456 0.0119308 0.00916153 0.0031296 0 0.00336085 0.00790414 0.00770585
```

0.195926 0 0 0 0 0 0 0
0.0129613 0 0 0 0 0 0 0
0.0052488 0 0 0 0 0 0 0
0.00506218 0 0 0 0 0 0 0
0.0125286 0 0.00508429 0 0 0 0 0
0.0104307 0 0 0 0 0 0 0
0.00292032 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

1.0725 0.015974 0.00423306 0 0 0 0 0
1.07828 0.00719053 0.0162346 0.00426816 0.00399789 0 0.0045684 0
0.481643 0.0116567 0.0213151 0 0.00782347 0 0 0
0.328418 0.0335822 0.00392218 0.00844684 0 0 0 0
0.414266 0.062707 0.0121329 0 0.00832202 0 0 0.00410791
0.23602 0.0333151 0.00427372 0.00444471 0.0042457 0 0 0.00446342
0.141256 0.0202387 0.0164529 0.0041209 0 0.00867839 0 0.00429791
0.423013 0.120201 0.0404464 0.0206626 0.00409147 0 0.0113566 0.0120805

1.22976 0 0 0 0 0 0 0
0.151432 0 0 0 0 0 0 0
0.0381719 0 0.00423014 0 0 0 0 0
0.0322079 0 0 0 0 0 0 0
0.050114 0.0034007 0 0 0 0 0 0
0.00977195 0.003213 0 0 0 0 0.00326514 0
0.0156935 0 0 0 0 0 0 0
0.0332583 0.0103719 0.00399168 0.00381888 0.00289996 0 0 0.00740445

0.402052 0 0 0 0 0 0 0
0.0504829 0 0 0 0 0.0041013 0 0
0.0309103 0 0.00307055 0 0 0 0 0
0.0184902 0 0 0 0 0 0 0
0.0169469 0.00424764 0 0 0 0 0 0
0.00410791 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.0263339 0 0 0 0 0 0 0

0.15576 0 0 0 0 0 0 0
0.00900752 0 0 0 0 0 0 0
0.00859915 0 0 0 0 0 0 0
0.00390979 0 0 0 0 0 0 0
0.0110727 0 0 0 0 0 0 0
0.00907694 0 0 0 0 0 0 0
0.00795571 0 0 0 0 0 0 0
0.0180391 0 0 0.0031296 0 0.00293576 0 0.00702065

```
0.0721093 0 0 0 0 0 0 0
0.0128699 0 0 0 0 0 0 0
0.00152021 0 0 0 0 0 0 0
0.016871 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.00542246 0.00293576 0 0 0 0 0 0
0.00436655 0.00293576 0 0 0 0 0 0
0.0295253 0.00348705 0.00158976 0 0.00295682 0 0 0.00312304

0.269171 0 0 0 0 0 0 0
0.0262184 0 0 0 0 0 0 0
0.00712252 0 0 0 0 0 0 0
0.00125194 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.00128563 0 0 0 0 0 0 0
0.00619769 0.00133753 0 0.0038394 0.00380902 0 0.00455328 0

0 0.009 0.0006 0 3 0.2 0 1.5 0.1 12 10 0.02 0.00001 0.011 0.0016 11000000 1400 600
3000000 300 150 46 80 85 0.4 0.4 1 10
```

Table A.8 Input file for dynamic programming for case study 3.

```
3 8 0.0 0.0005 0.0010 0.0015 0.002 0.0025 0.005 0.01 0.011
8 0.0 0.50 1.0 1.5 2.0 2.5 3.0 3.5 4.0
8 0.00 0.25 0.50 0.75 1.0 1.25 1.50 2.00 2.60

0.9575 0.00336085 0 0 0 0 0 0
0.0553281 0 0 0 0 0 0 0
0.0607719 0.00309096 0 0 0 0 0 0
0.040923 0 0 0 0 0 0.00279636 0
0.0360611 0.00314928 0 0 0 0 0 0
0.0472554 0.00283176 0 0 0 0 0 0
0.0400378 0 0.00314928 0.00297648 0 0 0 0
0.13456 0.0119308 0.00916153 0.0031296 0 0.00336085 0.00790414 0.00770585

0.195926 0 0 0 0 0 0 0
0.0129613 0 0 0 0 0 0 0
0.0052488 0 0 0 0 0 0 0
0.00506218 0 0 0 0 0 0 0
0.0125286 0 0.00508429 0 0 0 0 0
```

0.0104307 0 0 0 0 0 0 0
0.00292032 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

1.0725 0.015974 0.00423306 0 0 0 0 0
1.07828 0.00719053 0.0162346 0.00426816 0.00399789 0 0.0045684 0
0.481643 0.0116567 0.0213151 0 0.00782347 0 0 0
0.328418 0.0335822 0.00392218 0.00844684 0 0 0 0
0.414266 0.062707 0.0121329 0 0.00832202 0 0 0.00410791
0.23602 0.0333151 0.00427372 0.00444471 0.0042457 0 0 0.00446342
0.141256 0.0202387 0.0164529 0.0041209 0 0.00867839 0 0.00429791
0.423013 0.120201 0.0404464 0.0206626 0.00409147 0 0.0113566 0.0120805

1.22976 0 0 0 0 0 0 0
0.151432 0 0 0 0 0 0 0
0.0381719 0 0.00423014 0 0 0 0 0
0.0322079 0 0 0 0 0 0 0
0.050114 0.0034007 0 0 0 0 0 0
0.00977195 0.003213 0 0 0 0 0.00326514 0
0.0156935 0 0 0 0 0 0 0
0.0332583 0.0103719 0.00399168 0.00381888 0.00289996 0 0 0.00740445

0.402052 0 0 0 0 0 0 0
0.0504829 0 0 0 0 0.0041013 0 0
0.0309103 0 0.00307055 0 0 0 0 0
0.0184902 0 0 0 0 0 0 0
0.0169469 0.00424764 0 0 0 0 0 0
0.00410791 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.0263339 0 0 0 0 0 0 0

0.15576 0 0 0 0 0 0 0
0.00900752 0 0 0 0 0 0 0
0.00859915 0 0 0 0 0 0 0
0.00390979 0 0 0 0 0 0 0
0.0110727 0 0 0 0 0 0 0
0.00907694 0 0 0 0 0 0 0
0.00795571 0 0 0 0 0 0 0
0.0180391 0 0 0.0031296 0 0.00293576 0 0.00702065

0.0721093 0 0 0 0 0 0 0
0.0128699 0 0 0 0 0 0 0
0.00152021 0 0 0 0 0 0 0
0.016871 0 0 0 0 0 0 0

```
0 0 0 0 0 0 0 0
0.00542246 0.00293576 0 0 0 0 0 0
0.00436655 0.00293576 0 0 0 0 0 0
0.0295253 0.00348705 0.00158976 0 0.00295682 0 0 0.00312304

0.269171 0 0 0 0 0 0 0
0.0262184 0 0 0 0 0 0 0
0.00712252 0 0 0 0 0 0 0
0.00125194 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0.00128563 0 0 0 0 0 0 0
0.00619769 0.00133753 0 0.0038394 0.00380902 0 0.00455328 0

0 0.009 0 3 0 1.5 12 10 0.00001 0.011 0.0016 11000000 1400 600 3000000 300 150 46
80 85 0.4 0.4 1 10 150 150 50 60
```

Table A.9 Input file for genetic algorithms for case study 3.

APPENDIX B

TESTING FOR CONTROL PARAMETERS OF GENETIC ALGORITHMS

The four control parameters used in genetic algorithms, namely, population size, generation size, crossover rate and mutation rate, are not related to any economic or technical constraints, but are functions of the genetic algorithms process. There is a very wide range of possible scenarios that originate from a set of values for each of the control parameters. Different total discounted profit values can be obtained from many scenarios arising from combinations of different values of the parameters. The control parameters of the genetic algorithms used in this research work were tested in order to find an optimum range of values that give reasonably good results.

There is an obvious difference between the two genetic operators, i.e. crossover rate and mutation rate, and the other two control parameters. The latter parameters define the capacity of the genetic algorithms process, whereas the former parameters define the quality of the process.

The genetic algorithms process shown in case study 1 of Chapter 8 is used for testing the optimum ranges of the control parameters. The total discounted profits obtained are shown in figuress B.1 and B.2. The testing range of crossover

rate and mutation rate is 0% to 100%, of population size is 50 to 500 and of generation size is 100 to 500. A set of different crossover rates and mutation rates were applied while the population size and generation size were set to 100. The basis for comparison is total discounted profit and the results are shown in Figure B.1. In addition, a set of different population sizes and generation sizes have been applied with the crossover rate set to 50% and the mutation rate set to 60%. The results are shown in Figure B.2.

The population size in genetic algorithms is the number of search points assigned for each generation. Assigning a number of solution spaces, by means of population size, is peculiar to genetic algorithms and makes the method unique among search methods. In an ideal situation, a high population rate is likely to yield a better search. However, the genetic algorithms process is based on random values and sometimes it is possible to have a higher discounted profit with a smaller population size than with a larger one as can be seen from Figure B.2. Moreover, as can be seen from the figures, there is a tendency for results to improve as the population size increases. However, as the population size increases there is a proportionate increase in computing time and the population size is also restricted by computer memory capacity. Although larger populations generally give better results, because of computing time and memory constraints, the challenge is to find a reasonable population size that results in a reasonably high discounted profit. Population sizes of 250 to 500 give high performance in a reasonable computing time.

The generation size is the number of iterations used as the control variable for completing searches in genetic algorithms. Computation results improve as the

number of generations increases, as can be seen in Figure B.2. However, these improvements are not proportional to the generation size. In the early generations, or iterations, the level of improvements is higher than in later ones. Besides, computing time increases proportionally with increases in the generation size and population size. The objective, therefore, must be to have a sufficiently high generation size to enable the iterations to continue, for a reasonable computing time, until improvements in computed results are no longer significant. Generation sizes of 400 to 500 give a reasonable performance within a reasonable computing time.

The crossover rate is a genetic operator that makes new individuals by taking parts of two chromosomes or by merging two individuals into one, by means of a crossover point. Although from Figure B.1 it could be said that a crossover range of 20% to 50% is optimum the effect of the crossover rate on the performance of genetic algorithms applied to cut-off grade optimisation is not very obvious.

The mutation rate is the percentage change in one or more bits of the chromosome structure. A high rate of mutation tends to yield better individuals as can be seen in Figure B.1. Mutation rates of 40% to 100% give a reasonable performance. The mutation type used in this research is bit-by-bit mutation and, thus, a 100% mutation rate does not mean that all the chromosomes mutate but rather that it is likely that all the chromosomes mutate. However, for some chromosomes it is usual for more than one bit to mutate while other chromosomes do not mutate.

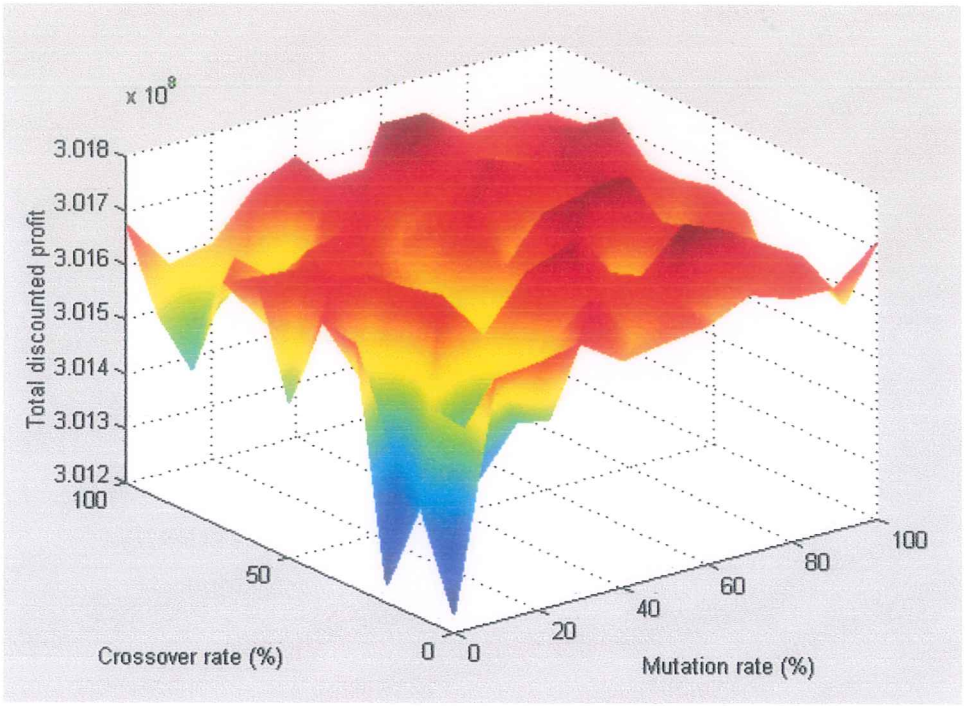

Figure B.1 Total discounted profits when crossover rate and mutation rate are variables. Population size is 100 and generation size is 100.

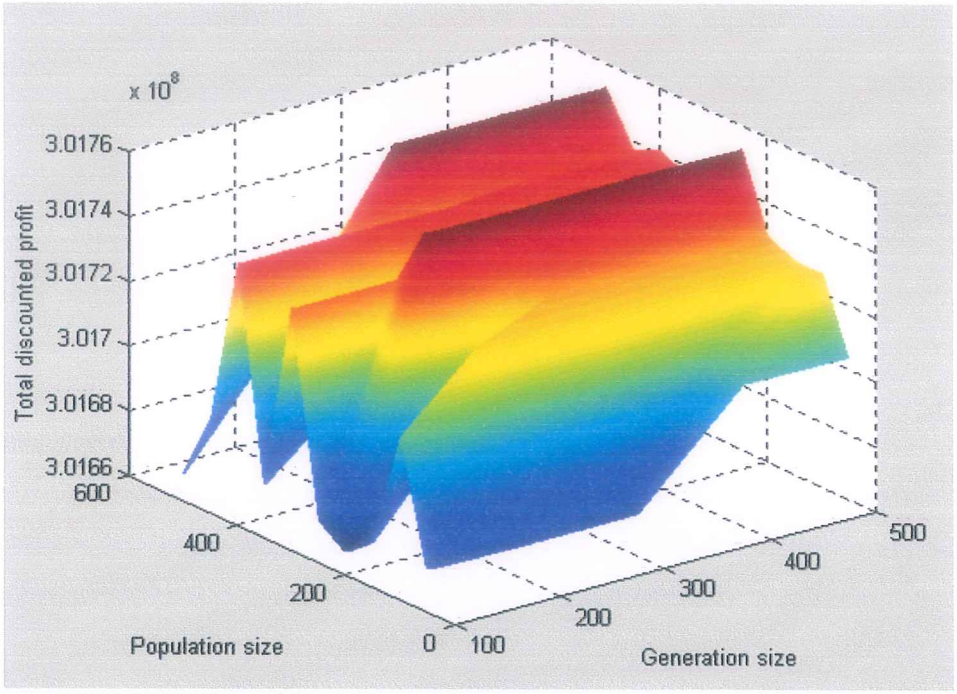

Figure B.2 Total discounted profits when population size and generation size are variables. Crossover rate is 50% and mutation rate is 60%.