Liquid Drop Model and Alpha Decay in Nuclear Structure Study

M.Sc. Thesis

in

Engineering Physics
University of Gaziantep

Supervisor

Assist. Prof. Dr. Mehmet KOÇAK

by

Dashty Taher Jameel AZIZAGHA

September 2016

REPUBLIC OF TURKEY
UNIVERSITY OF GAZİANTEP
GRADUATE SCHOOL OF NATURAL & APPLIED SCIENCES
ENGINEERING PHYSICS DEPARTMENT

Name of the thesis: Liquid Drop Model and Alpha decay in Nuclear Structure Study
Name of the student: Dashty Taher Jameel AZIZAGHA

Exam date: 7th September 2016

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. A. Necmeddin YAZICI
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Ramazan KOÇ
Head of Department

This is to certify that we have read this thesis and that in our consensus opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Mehmet KOÇAK
Supervisor

Examining Committee Members:	Signature
Assoc. Prof. Dr. Fatih HASOĞLU	....
Assist. Prof. Dr. Mehmet KOÇAK	....
Assist. Prof. Dr. M. Yakup HACIİBRAHİMOĞLU	....

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Dashty Taher Jameel AZIZAGHA

ABSTRACT

LIQUID DROP MODEL AND ALPHA DECAY IN NUCLEAR STRUCTURE STUDY

AZIZAGHA, Dashty Taher Jameel
M.Sc. in Engineering Physics
Supervisor: Assist. Prof. Dr. Mehmet KOÇAK
September 2016
110 pages

Six different liquid drop model (LDM) formulas (LDM1, LDM2, LDM3, LDM4, LDM5 and LDM6) were analyzed to describe nuclear binding energy for 3252 nuclei which were classified into four groups; deformation group which contain six sets, decay mode group which contain two sets, spin group which contain six sets and final group which contain all nuclei.

LDM6 was firstly formed and used in this study to represent nuclear binding energy. We determined the coefficients of all LDM formulas by using nonlinear regression method. Moreover we calculated root mean square (RMS) for all nuclei groups by using all LDM formulas. Comparison of experimental data of binding energy with calculations was also studied for all LDM formulas.

It is shown in this study that by adding shell correction and shell effect terms to all LDM formulas RMS is being smaller.

A systematic study on alpha decay half-lives using five different semi-empirical formulas for even-even, even-odd, odd-even and odd-odd nuclei was also done in this thesis. Calculated half-lives were carefully compared with measured ones for 459 different isotopes.

Key words: Liquid drop model, binding energy, shell correction, shell effect, alpha decay, half-life, nonlinear regression.

ÖZET

NÜKLEER YAPI ÇALIŞMASINDA SIVI DAMLA MODELİ VE ALFA BOZUNUMUNUN KULLANIMI

AZIZAGHA, Dashty Taher Jameel
Yüksek Lisans Tezi, Fizik Mühendisliği Bölümü
Tez Yöneticisi: Yard.Doç. Dr. Mehmet KOÇAK
Eylül 2016
110 sayfa

Altı farklı sıvı damla modeli (LDM) formülü (LDM1, LDM2, LDM3, LDM4, LDM5 ve LDM6) 3252 çekirdek izotopunu dört gruba sınıflandırarak: deformasyon grubu ki altı setten oluşur, bozunma modu grubu ki iki setten oluşur, spin grubu ki altı setten oluşur ve son grup ki bütün izotopları gözönünde bulundurur, atom çekirdeğinin bağlanma enerjisini tasvir etmek için incelenmiştir.

LDM6 ilk defa bu çalışmada oluşturulmuş ve çekirdek bağlanma enerjisini hesaplamada kullanılmıştır. Doğrusal olmayan regrasyon metodunu kullanarak bütün LDM formüllerinin katsayıları hesaplanmıştır. Dahası bütün LDM formüllerini kullanarak her grup için kök kare ortalama (RMS) değerlerini hesapladık. Hesaplanan bütün bağlanma enerjisi verileri deneysel ölçüm sonuçlarıyla kıyaslanmıştır.

Bu çalışmada kabuk düzeltmesinin ve kabuk etkisinin bütün LDM formüllerine eklenmesiyle RMS değerlerinin nasıl küçüldüğü gösterilmektedir.

Bu tezde aynı zamanda alfa bozunum yarı ömrü beş farklı yarı ampirik formülle çift-çift, çift-tek, tek-çift ve tek-tek çekirdekler için bir sistematik çalışma yapılmıştır. 459 farklı izotop için hesaplanan yarı ömürler dikkatlice ölçülen değerler ile karşılaştırılmıştır.
Anahtar Kelimeler: Sıvı damla modeli, bağlanma enerjisi, kabuk düzeltmesi, kabuk etkisi, alfa bozunumu, yarı ömür, doğrusal olmayan regrasyon.

Dedication

I dedicate wholeheartedly this thesis to my parents. Without their patience, understanding, support, and most of all love, the completion of this work would not have been possible, to my dear wife which shared their times for me, and to my cute children.

ACKNOWLEDGEMENTS

I am most sincerely thankful to my supervisor Assist. Prof. Dr. Mehmet Koçak for suggesting this project and for his great contribution in supporting my thesis with scientific thoughts and his continuous guiding during the course of this work.

I introduce my high thanks to Assist. Prof. Dr. Ali H. Ahmed for his help in the scientific information during the search period for scientific literature.

I introduce my high thanks to Assist. Prof. Dr. Mumtaz M. Hussain and Dr. Ahmed Sabir for their great assistance during writing of the computer programm. My honest respect to my family specially my wife, my parents, my brother, my sisters and all my relatives.

Last but not least my great thanks to all my colleagues in the department of engineering physics.

TABLE OF CONTENTS

Page

LIST OF TABLES

Page

LIST OF FIGURES

Page

LIST OF SYMBOLS

B.E	Binding energy
E_p	Pairing energy term
G	Gamow factor
T	Penetration probability
$\hbar$	Planks constant
V(r)	Coulomb potential barrier
r	Distance of separation
N_c	Closure magic number
η_c	Semi-degeneracy of shell number
RMS	Root mean square deviation
Q	Reaction energy of α-decay
ℓ	Orbital momentum
μ	Reduced mass
$T_{1/2}$	α-decay half-lives
A	Mass number

CHAPTER 1

INTRODUCTION

1.1 Liquid drop model

A nucleus consists of a number of nucleons, which interact with each other due to the strong forces. The nature of the strong interaction is not known exactly. This makes the study of nuclear structure a complex problem. It is quite difficult to write the Schrodinger equation for a nucleus and solve it. One has to rely on some simple model of the nucleus to understand its properties. The tremendous amount of accumulated experimental data has been used to find the similarities and patterns to develop a variety of models of the nucleus. Niles Bohr who proposed the successful atom model for the atoms having single electrons proposed also the liquid drop model of the nucleus [1].

The binding energies and volumes of the nuclei are proportional to the number of nucleons present in nuclei. The proportionality, however, indicates the short range and saturation characters of the nuclear forces. This follows that there is strong interaction amongst all the neighboring inside the nucleus. These properties of a nucleus are analogous to the properties of the forces which hold a liquid drop together. Hence, a nucleus may be considered to be analogous to a drop of incompressible fluid of very high density (~10^{17} Kg/m^3). This analogy led Niles Bohr and Wheeler and independently Frenkel to propose the liquid drop model of the nucleus. This model is not considered about the individual characteristics of the nucleons and hence this is statistical model. According to this model, the nucleus is regarded as an incompressible and uniformly charged liquid drop [2]

1.1.1 Similarities between a Nucleus and Liquid drop

There are large number of particles in a nucleus as in a drop of liquid, protons and neutrons (except ${}_{1}^{1}H$ which contains one proton only) in the former and molecules or atoms in the latter. For example, considering the volume of a drop of water to be 0.05×10^{-6} m^3, it would contain ≈ 10^{21} molecules, whereas the same volume of nuclear substance would contain ≈ 10^{37} nucleons, the density being 0.165 nucleon per fm^3. Both the liquid drop and nucleus exhibit homogeneity and incompressibility. This implies that the charge density (≈ 10^{17} m^{-3} for nucleus) and all other properties are uniform throughout the drop and nucleus, except at the surface boundary. In both the cases, the density is independent of dimensions.

Each nucleon in a nucleus interacts strongly with a small number of adjacent nucleons just as do the molecules in a liquid drop. This follows that nucleon forces are of short range and of saturation character.

Both the liquid drop and nucleus show surface tension effects, i.e. the surface energy of the nucleus is analogous to the surface tension of liquids.

Apart from the Coulomb repulsion, the forces between the *n-n, n-p, and p-p* are the same in the nucleus, just as the intermolecular forces (between the solute-solute, solute-solvent and solvent-solvent molecules) in an ideal solution.

Because of inter nucleon forces in a nucleus or the intermolecular forces in a liquid drop, any excess energy associated with an individual particles is rapidly shared by all the nucleons in the nucleus or the particles in the drop.

The energy levels of the nucleus are regarded as the quantum states of the nucleus as a whole rather than of a single nucleon like various molecules as in a liquid drop.

Evaporation from a liquid drop is analogous to the loss of nucleons from a nucleus in the nuclear reaction.

The fusion of small drops into a bigger one, and the braking up of large drop into small droplets are quite analogous to the fusion of light nuclei, respectively. The fusion and fission processes in both the cases are exoergic.

The thermal agitation of the molecules in a drop is quit analogous to the kinetic energy of the nucleons [3, 4].

1.1.2 Merits of the Liquid Drop Model

The liquid drop model has been employed with great success in the interpretation of intra-nuclear forces and of nuclear transformations. The merits of this model in the satisfactory explanation of various nuclear phenomena are given below.

Unlike the shell model, the liquid drop model can be applied to the nuclei in the excited states. Thus it provides a mechanism for low energy nuclear reactions, for which Niles Bohr used this model as the basis of the compound nucleus theory.

The phenomenon of nuclear fission has been explained sucesfully with the help of this model. The calculation of the nuclear binding energies and the study of the properties of isobars have been made successfully on the basis of the liquid drop model of the nucleus [5, 6].

1.1.3 Limitations of Liquid Drop Model

In preceding subsection we came across with different important applications of the liquid drop model of the nucleus in connection with the nuclear properties. Nevertheless, this model is associated with the following limitations:

(1) The model dose not considers the independent behavior of the nucleons in the nucleus with respect to their motion, spin, parity, and magnetic moment effects. In other words, the nucleon forces are charge and spin independent.

(2) The binding energy or mass formula ignores the closed shell effects as revealed by the discontinuities of the atomic masses at the neutron or proton magic number (it may be noted that the mass of any isobar at the nucleon magic numbers obtained β-decay energetics lies about 1 or 2 MeV below the value predicted by the smoothly varying mass formula.

(3) The enhanced binding energy of even Z, and N nuclei due to the positive contribution of odd-even effects to the total binding energy is consistent with *p-p* and *n-n* pairing effects.

(4) This model applied well to the medium mass nuclei, as for light nuclei surface effect predominates over the coulomb effect and, for heavy nuclei, the reverse is the case.

The liquid drop model gives a good account of the average behavior of nuclei in regard to mass, or binding energy, i.e. their stability [7, 8].

1.2 Literature review

Royer and Gautier [9] had determined the coefficients of different combinations of terms of the LDM by a least square fitting method to the experimental atomic masses. Also the nuclear masses can be superseded using a Coulomb radius taking into account the increase of the ratio with increasing mass, the fitted coefficient of surface energy term remaining around 18 MeV.

Liu and Zhang [10] had calculated the symmetry energy coefficients for nuclei have the mass numbers A=20–250 which are taken away from nuclear mass measurement. They investigated the density dependence of the symmetry energy terms of nuclear matter at sub normality densities by the semi-empirical connection between the symmetry energy terms coefficients of finite nuclei and the nuclear symmetry energy terms at reference densities.

Basu and Roy [11] had determined the coefficients of volume, surface, asymmetry, Coulomb, and pairing terms of the Bethe-Weizsacker mass formula by least square fitting to the mass excesses. Also they obtained the different sets of the weighting parameters for Bethe-Weizsacker mass formula of LDM from minimizations of chi-square and root mean square deviation. The recent experimental and evaluated mass excesses have been used for the least square fitting method in their study.

Chowdhury and Basu [12] had calculated nuclear masses by using the modified Bethe-Weizsacker mass formula of liquid drop model LDM in which the isotonic shifts have been combined.

Tian and Wang [13] had extracted nuclear symmetry energy coefficients of finite nuclei by using the differences between the isobaric nuclei masses. Based on the masses of nuclei with A = 10–270 which are more than 2400; they explored the model reliance in the extraction of the symmetry energy coefficient. They found that the extraction of the symmetry energy coefficients is strongly get together with the forms of the Coulomb energy coefficients and adopted the mass reliance of the symmetry energy coefficient.

Wang and Wu [14] had proposed a semi-empirical nuclear mass formula which is based on the macroscopic-microscopic method in which the isospin and mass reliance of model parameters are investigated with the Skyrme energy density function. The root mean square deviation with respect to 2149 measured nuclear masses is 0.516 MeV. Also they predicted shell corrections of superheavy nuclei.

Silisteanu [15] had calculated the performed α-decay half-lives in terms of α-cluster and resonance scattering amplitudes specified by self-harmonic nuclear models. His calculations were performed for superheavy nuclei have Z=102-120. He showed the improved agreement with experimental data and basic trends in the systematic of the results were well superseding, by adding even-odd rectification to the calculated α-decay half-lives.

Zdeb and Pomorski [16] had proposed a simple phenomenological model was based on the Gamow theory for the estimation of α-decay half-lives for cluster radioactivity.

Poenaru and Greiner [17] had calculated the α-decay half-lives of transuranium nuclei including superheavies by using different methods such as: a semi-empirical formula taking consideration the magic numbers of nucleons, the analytical universal curves model. The calculations based on kinetic energy of alpha decay (Q_α) values which were determined by using the recently published collection of atomic masses were compared to the experimental results.

Chowdhury and Samanta [18] had calculated the theoretical assessment of α-decay half-lives of several nuclei presented in the decay from element Z= 113-107. By using density dependent M3Y (DDM3Y) interaction in a WKB framework they calculated α-decay half-lives using experimental Q-values. They also found that the calculations on α-decay half-lives to be quite sensitive to the Q-values and angular momentum transfers. They predicted the calculated α-decay half-life decreases, due to more penetrability as well as thinner barrier, as Q-value increases.

Zhang and Li [19] had proposed a new semi-empirical formula for determining the half-lives of radioactive nuclei present cluster radioactivity. The parameters of the formula were procured by using a least-square fit method to the available cluster decay experimental data. The calculated α-decay half-life time was also compared with the values predicted by Viola–Seaborg–Sobiczewski (VSS) formulas and Horoi systematics formula.

Dasgupta-Schubert and Tamez [20] had evaluated three predictabilities α-decay half-life formulae such as the Royer generalized liquid drop model (GLDM), the Viola-Seaborg (VSS) and the Sobiczewski-Parkhomenko (SPM) formulae, by developing a method based on standard experimental benchmarking.

Budaca and Silisteanu [21] had fixed the parameters for a modified of the Brown empirical formula (BEF) via two fitting procedure which enables its rapprochement with Viola–Seaborg (VSS) and Royer formulas, by using the experimental data on the α-decay half-lives and Q_α values for 96 superheavy nuclei.

Zhang and Li [22] had systematically investigated reproducer α particle preformation in heavy nuclei using experimental Q_α value and half-lives. They proposed the formulas for the preformation factors, that can be used to evidence microscopic surveyed on preformation factors, and for perform accurate calculations of the α decay half-lives.

CHAPTER 2

BINDING ENERGY & SEMI-EMPIRICAL MASS FORMULA

The atomic nuclei are made up of two different types of elementary particles, protons and neutrons. The proton is nothing but the nucleus of the hydrogen atom. It carries on electronic unit of positive charge (+e) and has a mass about 1836 times the electronic mass (m_e). The neutron, on the other hand, is electrically neutral and is slightly heavier than the proton.

The protons and the neutrons are held together inside the nucleus by very strong short range attractive force. This force is different from the more commonly known forces like the gravitational or electrical forces and constitutes what may be called the specifically nuclear information. The neutrons and protons are jointly known as the nucleons.

2.1 Nuclear Masses

The nuclei form the core of atoms, with electrons revolving around them. Except for very light nuclei; it is difficult to ionize the atoms completely, so as to 'bare' the nuclei. One, generally, ionizes the atoms by removing only one or two electrons, in various experimental methods used for measuring atomic nuclear masses. It is therefore, more convenient to measure directly the atomic masses including the masses of revolving atomic electrons, rather than nuclear masses. One obtains the nuclear mass, by subtracting from the atomic mass, the mass of the electrons and making correction for the binding energy of electrons, i.e.,

$$M_N = M_A - \left[Zm_e - {(B.E)_e}/{c^2} \right] \qquad 2.1$$

where M_N is the mass of nucleus, M_A is the mass of neutral atom, m_e is the mass of one electron, Z is the atomic number of atoms, and $(B.E)_e$ is the total binding energy of all electrons.

The binding energy of all the electrons $(B.E)_e$ for given atom can be obtained from literature from X-ray spectra or ionization studies. The atomic masses, M_A may be obtained from various methods, expressed in such a way that the atomic mass of carbon-12 atom is defined as 12 atomic mass units (12 A.M.U.) or mass number of carbon-12 is 12. This is the scale used now for atomic or nuclear masses. According to the present scale, the atomic mass unit (A.M.U.) has a value of

$$1 \text{ A.M.U.} = 931.748 \text{ MeV}/c^2$$

$$= 1.6602\times10^{-27} \text{ kg}$$

The mass, referred hereafter, for atomic masses will be based on ^{12}C unless especially mentioned, otherwise [23].

Binding Energy (B.E) is the energy required to break an atom completely into hydrogen atoms and neutrons. This can expressed as:

$$B.E(A,Z) = \left[ZM_H + (A-Z)M_n - M(A,Z)\right]c^2 \qquad 2.2$$

where M_H is the mass of neutral hydrogen atom and M_n is the mass of neutron. It may be noted that the binding energy, as defined above, is the energy required to break an atom (A, Z) into Z hydrogen atoms; and (A-Z) neutrons. This is a bit different from the binding energy of nucleus (A, Z) breaking into Z protons and (A-Z) neutrons; by the difference of electronic binding energies. This difference may be neglected for most of the purposes, but may be taken into account for most precise measurements. This argument applies subsequently also.

(i) Separation Energy: This is the energy required to remove a specific particle from a nucleus, e.g. the last neutron or the last proton or alpha particle, etc. The separation energy is defined as

For a neutron (S_n):

$$S_n = B.E(A,Z) - B.E(A-1,Z)$$

$$= [M(A-1,Z) - M(A,Z) + M_n]c^2 \qquad 2.3$$

For protons (S_p):

$$S_p = B.E(A,Z) - B.E(A-1,Z-1)$$

$$= [M(A-1,Z-1) - M(A,Z) + M_H]c^2 \qquad 2.4$$

And for an α-particle (S_α):

$$S_\alpha = B.E(A,Z) - B.E(A-4,Z-2) - B.E(4,2)$$

$$= [M(A-4,Z-2) - M(A,Z) + M_{He}]c^2 \qquad 2.5$$

and similarly for any particle.

2.2 Bethe -Weizsacker Semi-Empirical Mass Formula

On the basis of the liquid drop model, Weiszacker, and several others, have attempted to express the masses of nuclei in terms of nuclear characteristics in connection with their binding energy and stability. This formula is known as semi-empirical mass formula [24].

Let M(A,Z) be the atomic mass of the isotope of an element X of atomic number Z and mass number A, then

$$M(A,Z) = ZM_H + NM_n - B.E \qquad 2.6$$

where M_H and M_n are the masses of proton and neutron respectively, and $B.E = E_v + E_s + E_c + E_{asym} + E_{pair}$

The Bethe-Weizsacker mass formula is given by

$$M(A,Z) = ZM_H + NM_n - \left(E_v + E_s + E_c + E_{asym} + E_{pair}\right) \qquad 2.7$$

2.2.1 Volume Energy

Nucleons are bound by short-range force, i.e. each nucleon is held strongly by other nucleons in its immediate vicinity. These short range forces are responsible for the binding energy of the nucleus. The binding energy is proportional to the number of nucleons, (i.e. mass number A). Since the nuclear volume is proportional to A, as a first approximation, there is an attraction or volume energy, proportional to A. This volume energy (E_v) is positive and is given by

$$E_v = a_v A \tag{2.8}$$

where the volume term a_v is constant.

2.2.2 Surface Energy

The nucleus is made up of protons and neutrons and the nuclear radius is proportional to the cube root of the nuclear mass number, i.e. $R = r_0 A^{1/3}$. Obviously, the nucleus is assumed to resemble a spherical liquid drop of radius R. As in case of a liquid drop, the nucleons on the surface of the nucleus are not surrounded by as many neighbors as those in the interior. In other words, the nucleons are less strongly bound on the surface, and consequently, the nucleon forces are unsaturated. Since the binding energy is proportional to the number of nucleons (i.e. A), and the number of nucleons is reduced on the surface, the binding energy is, therefore lowered by an amount which various as the surface area of the nucleus. As the nuclear radius is proportional to $A^{1/3}$, the area of the nuclear surface is related to $A^{2/3}$. The energy term corresponding to this surface tension effect is called the surface energy, and is represented by

$$E_s = -a_s A^{2/3} \tag{2.9}$$

where the surface terms a_s is proportionality constant.

2.2.3 Coulomb Energy

The protons present in the nuclear volume experience a coulomb repulsion, (i.e. long range electrostatic force of repulsion) which tends to lower the binding energy. As each proton is repelled by (Z-1) protons, the total Coulomb energy is proportional to

$\frac{Z(Z-1)e^2}{R}$. The effect of this Coulomb repulsion on the binding energy is named as the Coulomb energy and can be calculated as follows:

Let us assume that the total number charge $Q=+Ze$ be distributed uniformly, then the charge density is obtained as

$$\rho_C = \frac{3Ze}{4\pi R^3} = \frac{3Ze}{4\pi r_0^3 A} \qquad 2.10$$

One can calculate the potential energy of the charged sphere of radius R by considering that the sphere is built up layer by layer and making use of the fact that the field outside a spherical distribution of charge is the same as if all the charge is concentrated at the center. Assuming that the sphere has been built up to radius r, one finds the total charge in it as

$$q = \frac{4}{3}\pi r^3 \rho_C \qquad 2.11$$

Now, we add an infinitesimal shell of charge of thickness *dr* to this. The work done in bringing the charge $dq = 4\pi r^2 \rho_C dr$ in the shell from infinity to the radius *r* against the field of charged sphere of radius *r*, as above is

$$dW = \frac{qdq}{4\pi\varepsilon_0 r} = \frac{4}{3}\pi r^3 \rho_C \times \frac{4\pi r^2 \rho_C dr}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{4\pi\varepsilon_0 r} \times \frac{16\pi^2}{3}\rho_C^2 r^4 dr \qquad 2.12$$

($\varepsilon_0 = \frac{10^{-9}}{36\pi}\frac{m^2}{N.c^2}$ is the permittivity of the free space)

Total work done in forming the charged sphere of radius R is obtained by integration over sphere as

$$W = \frac{1}{4\pi\varepsilon_0}\frac{16\pi^2}{3}\rho_C^2\int_0^R r^4 dr = \frac{1}{4\pi\varepsilon_0}\frac{16\pi^2}{3}\rho_C^2\frac{R^5}{5} = \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Q^2}{R} \qquad 2.13$$

We know that the potential energy is the negative of work done and hence the Coulomb energy of the charged sphere is

$$E_C = -\frac{3}{5}\times\frac{Q^2}{4\pi\varepsilon_0 R} = -\frac{3}{5}\times\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

$$= -\frac{3}{5}\left(\frac{(Ze)^2}{4\pi\varepsilon_0 r_0 A^{1/3}}\right) = -a_c\frac{Z^2}{A^{1/3}} \qquad 2.14$$

Where $a_c = \frac{3}{5}\left(\frac{e^2}{4\pi\varepsilon_0 r_0}\right)$ is a constant 2.15

We must note Equation (2.16) for the Coulomb energy is not exact. One has apply corrections due to (i) non uniformity of the nuclear charge distribution; (ii) effect of the uncertainty on the localization of the protons; (iii) the requirement of the discrete arrangement of the charges on the protons; (iv) corrections of the positions of the protons and (v) non-sphericity of the nucleus [25].

2.2.4 Asymmetry Energy

Maximum stability of nuclei occurs when $Z = \frac{A}{2}$ i.e. the number of protons and neutrons are equal in the most stable (light) nuclides but when the number of neutrons exceeds, i.e. (A-Z) > Z, instability of the nuclide appears. This is known as the asymmetry composition effect. Since the excess neutrons occupy the higher quantum states than the other nucleons, they contribute a smaller amount (per neutron) to the total binding energy. Obviously, with the introduction of asymmetry, the binding energy decreases. The lowering of binding energy, called the asymmetry or neutron excess energy is proportional to the square of the neutron excess, $\approx (A-2Z)^2$ and inversely to A, i.e.

$$E_{asym} = -a_{asym} \frac{(A-2Z)^2}{A} \qquad 2.16$$

where a_{asym} is the proportionality constant [26].

2.2.5 Pairing Energy

It has been observed that even-Z, even-N nuclides are the most abundant amongst the stable nuclides. Nuclides with odd-Z, odd-N are the least stable, while nuclides with even-Z, odd-N or vice versa, are of intermediate stability. This may be attributed to the pairing of nucleons of the same type or pairing of the nucleon spins. In case of even-even nuclei, all the spins are paired, and so there is a positive contribution to the binding energy for nuclei with odd-Z, odd-N because of the presence of unpaired proton and unpaired neutron spins. There is no contribution for nuclei with even-Z, odd-N or the reverse. This contribution to the binding energy arising from the spin or odd-even effect is known as the pairing energy. The pairing energy term $E_{pair}(A,Z)$ depends only on A and is taken to be zero for odd A nuclei, positive for even-even nuclei and negative for odd-odd nuclei and given by

$$E_{pair} = \pm a_p A^{-\frac{3}{4}} \qquad 2.17$$

Where a_{pair} the proportionality constant, the positive sign is implies an increase in the binding energy for the even-even nuclei, and the negative sign for the odd-odd nuclei, which results in a decrease in the binding energy. $E_{pair} = 0$ for odd A nuclei. This pairing interaction arises from the short-range and attractive nature of the nuclear force and leads to greater binding between like nucleons if their angular momenta are coupled to zero spin. This pairing interaction between like nucleons is responsible for the last term, the pairing term E_{pair} is generally symbolized by E_p, in the semi-empirical mass formula. The function E_p has been parameterized by

$$E_p = +a_p A^{-\frac{3}{4}} \qquad \text{for even-even nuclei}$$

$$E_p = 0 \qquad \text{for odd A nuclei}$$

$$E_p = -a_p A^{-\frac{3}{4}} \qquad \text{for odd-odd nuclei}$$

and the term accounts for the increased stability observed for nuclei which have an even number of like nucleons. The reason the pairing interaction is not so effective between neutrons and protons in stable nuclei with A > 40 is because such nuclei have N > Z because of the Coulomb repulsion of the protons which displaces the neutron and proton potential wells [27].

The total binding energy of a nuclide of mass number A proton number Z is obtained by combining all the energy terms, i.e.

$$B.E(A,Z) = E_v - E_s - E_c - E_{asym} \pm E_{pair} \qquad 2.18$$

$$= a_v A - a_s A^{\frac{2}{3}} - a_c z^2 A^{-\frac{1}{3}} - a_{asym}(A-2Z)^2 A^{-1} \pm E_b \qquad 2.19$$

The effect of first four terms of binding energy together with the total binding energy per nucleon is given in Figure (2.1).

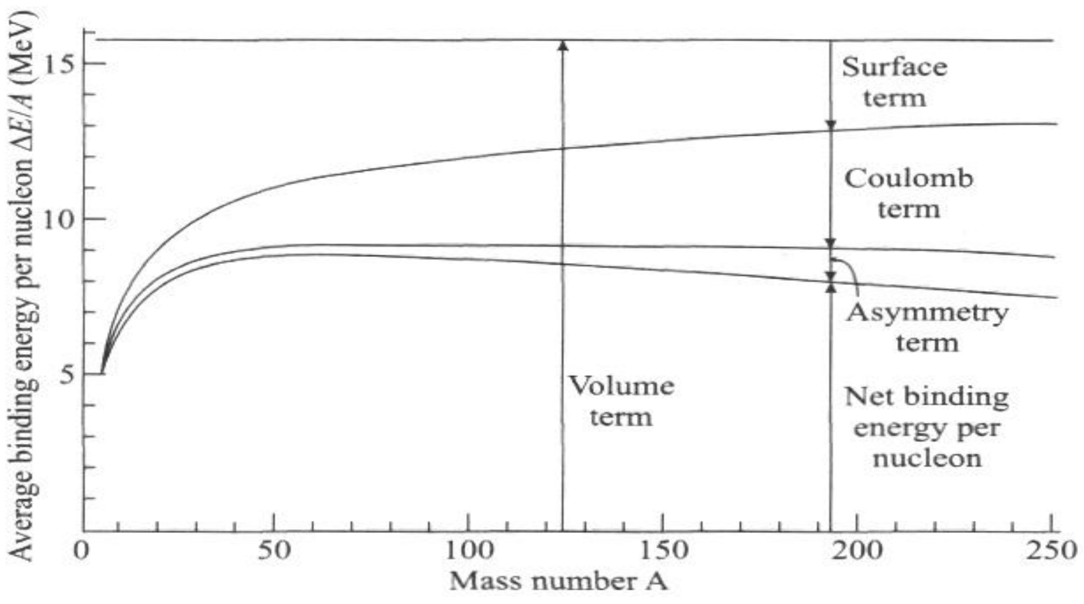

Figure 2.1 illustrating how the volume, surface, Coulomb, and symmetry terms of semi empirical mass formula combine to yield the average binding energy per nucleon.

Equation (2.19) is known as semi-empirical binding energy formula. For the calculation of the binding energy and the atomic mass, the values of constants appearing in (2.19) must be known.

The constants are empirical and can be determined from the experimental values of the masses (or binding energy). The values of the constants in the semi-empirical mass formula have been determined by several workers. But their values are somewhat different. The currently accepted values expressed in unified atomic mass scale are [28]

$a_v = 15.760\ MeV$; $a_s = 17.804\ MeV$; $a_c = 0.71\ MeV$; $a_c = 23.697\ MeV$; $a_c = 33.5\ MeV$

The ratios of the surface, coulomb and asymmetry energies to the volume energy for some typical nuclei in different mass region, i.e. light, intermediate and heavy are shown in Table 2.1.

Table 2.1 Ratios of the surface, Coulomb and Asymmetry energies to the Volume energy [29].

A	Z	$A-2Z$	$E_v(MeV)$	$\frac{E_s}{E_v}(\%)$	$\frac{E_c}{E_v}(\%)$	$\frac{E_a}{E_v}(\%)$	$\frac{B.E}{A}(MeV)$
6	3	0	94.8	62	3.7	0	5.4
20	10	0	316	41	8.2	0	7.95
50	24	2	790	30.5	14	0.24	8.71
160	64	32	2528	20.8	21.2	6	8.22
200	80	40	3160	19.3	24.6	6	7.92
250	100	50	3950	18	28.5	6	7.52

One finds from Table 2.1 that for very light nuclei, the main contributory factor reducing the volume energy is the surface energy term. This is because of the fact that the surface area to volume ratios for these is relativity much larger than for the heavy nuclei. The effect of the surface energy terms diminishes rapidly as A increases in the case of very light nuclei which is responsible for the rapid increases in the value of $\frac{E_v}{A}$ with the increase of A. One also finds from table 2.1 that the Coulomb energy effect $\left(\frac{E_c}{E_v}\right)$, Which is small for lighter nuclei, becomes quite dominant for the heavier nuclei containing a large number of protons. The asymmetry effect is important for heavier nuclei and insignificant at low A values for which N and Z are almost equal.

2.3 Application of semi-empirical mass formula to Alpha Decay

Alpha-decay of ${}_{Z}^{A}X$ nucleus into the nucleus ${}_{Z-2}^{A-4}X$ can be written as

$$ {}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He $$

The α-disintegration energy is

$$ Q_\alpha = M(A,Z) - M(A-4, Z-2) - M\left({}_{2}^{4}He\right) \qquad 2.20 $$

Expressing in terms of binding energies (*B.E*) of the nuclei involved, Equation (2.20) becomes for A and Z large [3]

$$ Q(\alpha, Z) = B.E(A-4, Z-2) + B.E\left({}_{2}^{4}He\right) - B.E(A,Z) \qquad 2.21 $$

$$ = a_v(A-4) - a_s(A-4)^{\frac{2}{3}} - a_c\frac{(Z-2)^2}{(A-4)^{\frac{1}{3}}} - a_{asym}\frac{(A-2Z)^2}{A-4} - a_vA + a_sA^{\frac{2}{3}} + a_cZ^2A^{-\frac{1}{3}} $$

$$ + a_{asym}(A-2Z)^2A^{-1} + B.E\left({}_{2}^{4}He\right) \qquad 2.22 $$

$$ = B.E\left({}_{2}^{4}He\right) - 4a_v + a_s\left[\left\{A^{\frac{2}{3}} - (A-4)^{\frac{2}{3}}\right\}\right] + a_c\left\{\frac{Z^2}{A^{\frac{1}{3}}} - \frac{(Z-2)^2}{(A-4)\frac{1}{3}}\right\} + a_{asym}(A-2Z)^2\left\{\frac{1}{A} - \frac{1}{A-4}\right\} $$

$$ = 28.3 - 4a_v + \frac{8}{2}a_sA^{-\frac{1}{3}} + 4a_cZ\left(1 - \frac{Z}{3A}\right)A^{-\frac{1}{3}} - \frac{4a_{asym}(A-2Z)^2}{A(A-4)} \qquad 2.23 $$

The pairing energy term is neglected because the binding energy of the α-particle has been taken to be 28.3 MeV. Taking the values of constants a_v, a_s, a_c and a_{asym} obtained earlier, one find $Q_\alpha > 0$ for A > 160. Alpha disintegration is actually observed primarily in the region A > 200. For A < 200, i.e. lighter nuclei, the energy release is so small that barrier penetration probability becomes very small.

CHAPTER 3

ALPHA DECAY THEORY

3.1 Introduction

The study of alpha decay started from the year 1896 with the discovery of radioactivity by Becquerel. It was Rutherford who identified the alpha particle with the Helium nucleus in the year 1909. Many heavy nuclei, especially those of the naturally occurring radioactive series, decay through spontaneous emission of alpha particle.

If one shoots an alpha particle on a target nucleus, it experiences Coulomb repulsion $2Ze^2/r$ which increases as *r* decreases and reaches a maximum value $2Ze^2/R$, where R denotes the nuclear radius.

Alpha particles are ${}^4_2He$ nuclei with 2 units of positive charge and mass of 4 amu. Because they are charged particles so they can be deflected by electric and magnetic field. They ionize the medium through which they pass. Their ionizing power is much higher than β^- and γ-rays.

The alpha decay has high ionization power so they can be easily absorbed by few cm of air or fraction of mm thick aluminum. Naturally occurring α-emitters emit α-particles with energy in the range of 5 MeV to 10 MeV [5].

Long exposures to α-emitters produce harmful effects on the human body.

3.2 Q-value for α-decay

An inspection of the naturally occurring radioactive series viz., the Uranium series, the Actinium series, the Thorium series and the Neptunium series reveals that several heavy nuclei are found to decay by emission of α-particles. In α-decay, the nucleus (*A, Z*) transforms into the nucleus (*A-4, Z-2*) by emission of α-particle

$$(A,Z) \rightarrow (A-4, Z-2) + \alpha + Q \qquad 3.1$$

where Q denotes the Q-value of the reaction which is equal to the energy of the missing mass [30].

$$Q = \{M(A,Z) - M(A-4, Z-2) - M(\alpha)\}c^2, \qquad 3.2$$

where M denotes the mass of the respective nucleus. From the table of atomic masses, one can calculate the Q-value, since the binding energy of electrons in corresponding atoms mutually cancels away to a large extent.

Alternatively, one can write Eq.(3.2) in terms of binding energy B of corresponding nuclei.

$$Q = B(^4He) + B(A-4, Z-2) - B(A,Z). \qquad 3.3$$

The binding energy of α-particle is

$$B(^4He) = 28.3 MeV, \qquad 3.4$$

and the binding energy of the daughter and parent nucleus can be obtained from the Weizsäcker mass formula [39],

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} \qquad 3.5$$

Denoting the difference in binding energy of the daughter and the parent nucleus by ΔB,

$$\Delta B = B(A-4, Z-2) - B(A,Z) \qquad 3.6$$

The Q-value of α-decay, given by Eq. (3.3),

$$Q = \Delta B + 28.3 MeV, \qquad 3.7$$

is positive only for heavy nuclei with $A \geq 150$. For nuclei with mass number A lying between 150 and 210, the available decay energy is small and hence the lifetime is large, thereby making the α-decay detection difficult. Beyond A ≈ 210, the decay energy tends to be higher and hence the heavier nuclei decay predominantly by α-emission.

Table 3.1 depicts the range R_α of the α-particle emitted from the radioactive nuclide and the corresponding alpha disintegration energy E_α. It can easily be verified that the alpha disintegration energy is the kinetic energy due to relative motion of the α-particle and the residual daughter nucleus in the center of mass (c.m.) system of the disintegrating nucleus.

Table 3.1 The radioactive thorium series that decay by alpha emission [38].

Nuclei Symbol	R_α (mm)	E_α (MeV)	λ (sec^{-1})	Half-lives ($T_{1/2}$)
$^{232}_{90}Th$	29.0	3.98	1.58×10^{-18}	1.39×10^{10} y
$^{228}_{90}Th$	40.2	5.42	1.16×10^{-8}	1.9 y
$^{224}_{88}Ra$	43.5	5.68	2.20×10^{-6}	3.64 d
$^{220}_{86}Ra$	50.6	6.28	1.27×10^{-2}	54.5 s
$^{216}_{84}Po$	56.8	6.77	4.33	0.16 s
$^{212}_{83}Bi$	47.9	6.05	1.92×10^{-4}	60.5 min
$^{212}_{84}Po$	86.2	8.77	2.31×10^{6}	3×10^{-7} s

3.3 Coulomb potential barrier

For the emission of α-particle from the nucleus, the α-particle has to overcome a Coulomb repulsive potential barrier. To estimate the height of the Coulomb potential barrier, consider the α-particle and the residual nucleus to be just at the point of separation [31].

If V is the height of the Coulomb potential barrier, then

$$V = \frac{(Z-2)2e^2}{R} \qquad 3.8$$

where e is the unit of electron charge and R is the distance of separation. Assuming the $A^{\frac{1}{3}}$ as law for the nuclear radius,

$$R = r_1 + r_2 = r_0\left\{(A-4)^{\frac{1}{3}} + 4^{\frac{1}{3}}\right\} \qquad 3.9$$

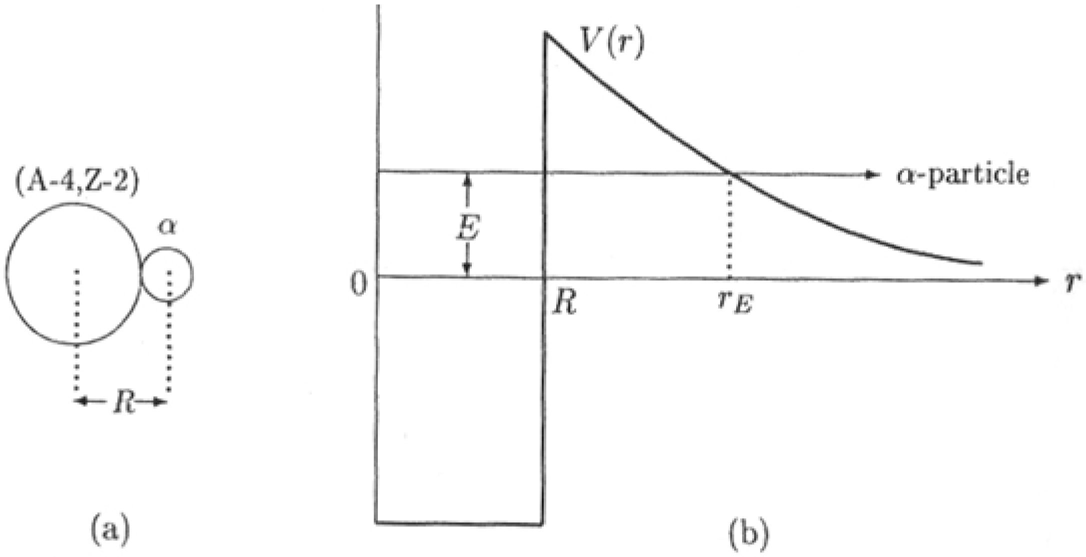

Figure 3.1 (a) Alpha particle and the daughter nucleus (A-4, Z-2) at the point of separation, (b) Coulomb potential barrier V(r) that a particle experiences in α-decay [38].

where r_0 is approximately equal to 1.3 fm. The Coulomb repulsive potential that the α-particle experiences is maximum when it is just at the point of separation and thereafter decreases steadily with the separation distance *r* as shown in figure (3.1b).

3.4 Gamow's Theory of alpha decay

Gamow's theory of α-decay is based on the following two postulates [32]:

(1) Alpha particles are preformed inside the nucleus. It is a very reasonable assumption since the α-particle has the lowest energy (i.e. maximum binding energy) among the possible few nucleon clusters.

(2) Alpha particles tunnel through the Coulomb potential barrier which is much higher than the kinetic energy of the emitted α -particle. This is a quantum mechanical effect.

The penetrability for α-particles has been deduced and given by

$$T = \exp(-2G) \qquad 3.10$$

where T is generally called as penetration probability and the quantity G is known as the Gamow factor which can be evaluated as shown below [30].

$$G = \frac{\sqrt{2\mu}}{\hbar} \int_R^{rE} (V(r) - E)^{\frac{1}{2}} dr \tag{3.11}$$

Where μ denotes the reduced mass of the α-particle. The Coulomb potential barrier *V(r)* is

$$V(r) = \frac{2Z_d e^2}{r} \tag{3.12}$$

where r is the distance of separation and Z_d = Z - 2 is the charge number of the daughter nucleus and Z the charge number of the radioactive parent nucleus. The potential barrier *V(r)* is maximum at the point of separation of the α-particle from the daughter nucleus (*i.e.*, when $r = R$) [33]

$$V(R) = \frac{2Z_d e^2}{R} \tag{3.13}$$

and decreases as the distance of separation increases. When $r = r_E$, *V(r)* becomes equal to the energy of the emitted α-particle.

$$V(r_E) = E = \frac{2Z_d e^2}{r_E} \tag{3.14}$$

Substituting these values in equation (3.17), we obtain [30]

$$G = \frac{\sqrt{4\mu Z_d e^2}}{\hbar} \int_R^{r_E} \left(\frac{1}{r} - \frac{1}{r_E} \right)^{\frac{1}{2}} dr \tag{3.15}$$

To evaluate the integral

$$I = \int_R^{r_E} \left(\frac{1}{r} - \frac{1}{r_E} \right)^{\frac{1}{2}} dr \tag{3.16}$$

make the substitution

$$r = r_E \cos^2 \theta \tag{3.17}$$

Then

$$dr = -2r_E \cos\theta \sin\theta . d\theta \tag{3.18}$$

and

$$I = -2\sqrt{r_E}\int \sin^2\theta d\theta$$

$$= -\sqrt{r_E}\int (1-\cos 2\theta)\, d\theta$$

$$= -\sqrt{r_E}\left(\theta - \frac{1}{2}\sin 2\theta\right)$$

$$= -\sqrt{r_E}\,(\theta - \sin\theta\cos\theta) \qquad 3.19$$

From Equation (3.11), it follows that [34]

$$\cos\theta = \sqrt{\frac{r}{r_E}}\ ; \qquad \theta = \cos^{-1}\sqrt{\frac{r}{r_E}}\ ; \qquad \text{and} \qquad \sin\theta = \sqrt{\left(1-\frac{r}{r_E}\right)}$$

Now the integral *l* becomes

$$I = -\sqrt{r_E}\left[\cos^{-1}\sqrt{\frac{r}{r_E}} - \sqrt{\left(1-\frac{r}{r_E}\right)\left(\frac{r}{r_E}\right)}\right]_R^{r_E}$$

$$= -\sqrt{r_E}\left[\cos^{-1}\sqrt{\frac{R}{r_E}} - \sqrt{\left(1-\frac{R}{r_E}\right)\left(\frac{R}{r_E}\right)}\right] \qquad 3.20$$

Let

$$x = \frac{R}{r_E} \qquad 3.21$$

From equations (3.13) and (3.14), it follows that

$$x = \frac{R}{r_E} = \frac{V(r_E)}{V(R)} = \frac{E}{V_0} \qquad 3.22$$

Then

$$G = \frac{\left(4\mu Z_d e^2 r_E\right)^{\frac{1}{2}}}{\hbar}\left(\cos^{-1}\sqrt{x} - \sqrt{(1-x)\,x}\right)$$

$$= \frac{4Z_d e^2}{\hbar v}\left(\cos^{-1}\sqrt{x} - \sqrt{(1-x)x}\right) \qquad 3.23$$

where v is the velocity of the α-particle. The last equation is obtained by using the relations [35]

$$r_E = \frac{2Z_d e^2}{E} \quad \text{and} \quad v = \sqrt{\frac{E}{\mu}} \tag{3.24}$$

Normally, $x << 1$ since $V_0 >> E$. In such cases, we can further simplify the expression for G. Let

$$\cos^{-1}\sqrt{x} = \theta \tag{3.25}$$

Then

$$\sqrt{x} = \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \approx \frac{\pi}{2} - \theta \tag{3.26}$$

and equation (3.23) can be rewritten using equations (3.25) and (3.26)

$$G = \left(\frac{4Z_d e^2}{\hbar v}\right)\left[\frac{\pi}{2} - \sqrt{x} - \sqrt{x(1-x)}\right]$$

$$= \left(\frac{4Z_d e^2}{\hbar v}\right)\left[\frac{\pi}{2} - 2\sqrt{x}\right] \quad \text{(neglecting higher powers of } x\text{)}$$

$$= \left(\frac{2\pi Z_d e^2}{\hbar v}\right) - \left(\frac{8Z_d e^2}{\hbar v}\right)\sqrt{x}$$

$$= \left(\frac{2\pi Z_d e^2}{\hbar v}\right) - \left(\frac{4e}{\hbar}\right)\sqrt{Z_d \mu R} \tag{3.27}$$

To obtain the last step, we have used the relations.

$$v = \left(\frac{2E}{\mu}\right)^{\frac{1}{2}} \quad \text{and} \quad x = \frac{E}{V_0} = \frac{ER}{2Z_d e^2} \tag{3.28}$$

by making a slight manipulation, an alternative expression for G can be obtained.

$$G = \left(\frac{e}{\hbar}\right)(Z_d \mu R)^{\frac{1}{2}}\left[\pi\left(\frac{V_0}{E}\right)^{\frac{1}{2}} - 4\right] \tag{3.29}$$

3.5 The decay constant and the half-life

The decay constant λ in α-decay is the probability of emission of α-particle per second from the nucleus and it is equal to the penetrability T (probability of emission of α-particle per collision) and the number of collisions it makes per second $\left(\frac{v}{R}\right)$.

$$\lambda = \left(\frac{v}{R}\right)T = \left(\frac{v}{R}\right)\exp(-2G) \qquad 3.30$$

From the decay constant λ, otherwise known as disintegration constant, one can obtain the mean life or average life τ and half-life $T_{1/2}$ of the nucleus [30].

$$\tau = \frac{1}{\lambda} \text{ and } T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} = 0.693\tau \qquad 3.31$$

from (3.30), we obtain

$$\ln\lambda = \ln\left(\frac{v}{R}\right) - 2G \;; \qquad 3.32$$

$$\log\lambda = \log\left(\frac{v}{R}\right) - \frac{2G}{2.303} \qquad 3.33$$

The factor 2.303 occurs when we use logarithm to the base 10.

CHAPTER 4

METHODS OF CALCULATIONS

4.1 Liquid Drop Model (LDM) & Semi-empirical Mass Formula

In this this part we studied six liquid drop model formulas (LDM1, LDM2, LDM3, LDM4, LDM5, LDM6) to calculate binding energies of 3252 different isotopes which were classified into four groups: i) deformation group which contain six sets, ii) decay mode group which contain two sets, iii) spin group which contain six sets and finally iv) all nuclei group which contains 3252 nuclei. We firstly used Nonlinear Regression Method to determine the coefficients of all formulas then calculated binding energies for each of them. One of the formulas, LDM6, which is formed by selecting suitable terms in the mass formulas given in literature, has just been used to calculate binding energies in this study.

4.1.1 Liquid drop model mass formula 1: LDM1

The first one is an improved version of the Liquid drop model formula with modified to be a volume term (with isospin dependence), surface term, coulomb term, the third term representing correction to Coulomb energy due to surface diffuseness. The first liquid drop model formula can be assumed to be [36-38]

$$LDM1 = a_v[1+\frac{4k_v}{A^2}T_z\left(T_z+1\right)]A + a_s[1+\frac{4k_s}{A^2}T_z\left(T_z+1\right)]A^{2/3} + \frac{3Z^2e^2}{5r_0A^{1/3}} + \frac{C_4Z^2}{A} \qquad 4.1$$

where, $T_z=(N-Z)/2$ is the third component of isospin and e is the electrostatic charge. The coefficients a_v is volume energy term, k_v is isospin dependence of volume energy term, a_s is surface energy term, k_s is isospin dependence of surface energy term, r_o is coulomb radius, C_4 is the correction to coulomb energy term due to surface diffuseness are treated as free parameters [39,40].

4.1.2 Liquid drop model mass formula 2: LDM2

The second formula is obtained from a phenomenological search and more generalized Bethe-Weizsacker mass formula [41].

$$LDM2 = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{\left(1+e^{-A/k}\right)A} + \delta_{new} \qquad 4.2$$

where the asymmetry energy term and the pairing energy term, δ_{new} , had been modified to obtain great improvements to the fits of the binding energy. Fitted value of the asymmetry energy term modified constant k = 17, while the modified pairing energy term δ_{new} can be expressed as [42, 43]

$$\delta_{new} = \left(1-e^{-A/c}\right)E_p$$

where c = 30, 4.3

and δ being the old pairing energy term is given as

$$E_p = a_p A^{-1/2} \quad \text{for even N even Z}$$

$$= -a_p A^{-1/2} \quad \text{for odd N odd Z}$$

$$= 0 \quad \text{for odd A}$$

4.1.3 Liquid drop model mass formula 3: LDM3

The third formula is obtained from the modification of the Bethe-Weizsacker mass formula which is based on the macroscopic-microscopic method [44, 45], which incorporates explicitly isospin effect [46]

$$LDM3 = a_v A + a_s A^{2/3} + E_C + a_{sym} I^2 A + a_{pair} A^{-1/3} \delta_{np} \qquad 4.4$$

with isospin asymmetry $I = (N-Z)/A$. The paring energy term is taken from [44]

$$\delta_{np} = \begin{cases} 2-|I| & \text{N and Z even} \\ |I| & \text{N and Z odd} \\ 1-|I| & \text{N even, Z odd and N > Z} \\ 1-|I| & \text{N odd, Z even and N < Z} \\ 1 & \text{N even Z odd and N<Z} \\ 1 & \text{N odd Z even and N>Z} \end{cases} \quad 4.5$$

and the symmetry energy coefficient, including an I correction on the conventional surface symmetry energy term of Liquid drop model to approximately describe the Wigner effect for heavy nuclei, is written as

$$a_{sym} = c_{sym}\left[1-\frac{k}{A^{1/3}}+\frac{2-|I|}{2+|I|A}\right] \quad 4.6$$

The Coulomb energy exchange slightly $Z(Z-1)$ is replaced by Z^2,

$$E_C = a_c\frac{Z^2}{A^{1/3}}\left[1-Z^{-2/3}\right] \quad 4.7$$

4.1.4 Liquid drop model mass formula 4: LDM4

The fourth formula is obtained from the liquid drop model with pairing energies term of the Thomas-Fermi model [47-50]

$$LDM4 = a_v\left(1-k_vI^2\right)A - a_s\left(1-k_sI^2\right)A^{2/3} - \frac{3}{5}\frac{e^2Z^2}{R_0} + E_{pair} - a_kA^{1/3}$$
$$-a_0A^0 - f_p\frac{Z^2}{A} - W|I| \quad 4.7$$

The first term is the volume energy corresponding to the saturation exchange force and infinite nuclear matter. In this form it includes the asymmetry energy of the Bethe-Weizsacker mass formula via the relative neutron excess $I=(N-Z)/A$. The second term is the surface energy term. It takes into account the deficit of binding energy of the nucleons at the nuclear surface and corresponds to semi-infinite nuclear matter. The dependence on I is not considered in the Bethe-Weizsacker mass formula. Third term is the Coulomb energy term where $R_0 = r_0A^{1/3}$. It gives the loss of binding energy due to the repulsion between the protons. In the Bethe-Weizsacker mass formula the proportionality to $Z(Z-1)$ is assumed. The fourth term is pairing energy same as defined in LDM2.

The fifth term is curvature energy is a repairing to the surface energy appearing when the surface energy is considered as a function of local properties of the surface consequently depends on the mean local curvature [51]. The sixth term term appears when the surface term of the LDM is extended to contain higher order terms in $A^{-1/3}$ and I. The seventh term performs a diffuseness correction to the sharp radius Coulomb energy. The eighth term is represented the Wigner energy term [45, 51] which appears in the counting of identical pairs in a nucleus.

4.1.5 Liquid drop model mass formula 5: LDM5

The macroscopic mass formula for liquid drop model is expressed as [52, 53]

$$LDM5 = a_v\left(1+k_v I^2\right)A + a_s\left(1+k_s I^2\right)A^{2/3} + C_1 \frac{2-|I|}{2+|I|A} I^2 A + a_c \frac{Z^2}{A^{1/3}}\left(1-0.76Z^{-2/3}\right) \quad 4.8$$

Considering the saturation property of nuclear force, first term represented the volume energy term, with I^2A being the symmetry energy of the Beth-Weizsacker mass formula [54]. The second term is represented the surface energy term, which accounts for the deficit of binding energy of nucleons at the surface. The third term combined with I^2A dependence of the first two terms consists of the symmetry energy coefficient $a_v k_v + a_s k_s / A^{1/3} + C_1 \frac{2-|I|}{2+|I|A}$. The fourth term is included the Coulomb repulsion between protons and diffuseness correction to the sharp radius Coulomb energy [55].

4.1.6 Liquid drop model mass formula 6: LDM6

We modified liquid drop model of Bethe-Weizsacker mass formula, the first term is volume energy, the second term is surface energy, third terms is coulomb repulsion energy, fourth term is asymmetry term, and the final two terms is pairing energy term and Wigner energy term respectively [37, 56-57].

$$LDM6 = a_v[1+\frac{4k_v}{A^2}T_z(T_z+1)]A - a_s(1-k_sI^2)A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}}(1-bZ^{-2/3}) - a_{sym}\frac{(N-Z)^2}{A}$$
$$+E_p + E_W \qquad 4.9$$

Wigner term has been calculated using:

$$E_W = C_oA\left(\frac{2-|I|}{2+|I|A}\right)$$

4.2 Liquid Drop Model plus Shell corrections

The binding energy is maximal for those nuclei with N or Z equal to 2, 8, 20, 28, 50, 82 and 126. The Temis et al. [58] suggest to use 14 instead of 20 as a magic number.

To improve the predictive of LDM’S formulas with the inclusion of two extra term. To keep their parameters as close as possible to its original values, the new terms should have a null average contribution. To this goal their mean is removed by defining

$$F = \frac{n_v+n_\pi}{2} - \left\langle\frac{n_v+n_\pi}{2}\right\rangle \qquad 4.10$$

and

$$FF = \left(\frac{n_v+n_\pi}{2}\right)^2 - \left\langle\left(\frac{n_v+n_\pi}{2}\right)^2\right\rangle \qquad 4.11$$

where n_v (n_π) is the number of valance neutron (proton) particles or holes counted from the nearest closed shell. The notation comes from the counting of bosons adopted in the neutron-proton interaction boson model [59,60].

The number of valence neutron n_v is defined by [58]:

$$n_v = N - N_c \quad \text{if } N \le N_{med} \qquad 4.12$$

$$n_v = N_{c+1} - N \quad \text{if } N\rangle N_{med} \qquad 4.13$$

where we have introduced the closure magic numbers N_c :

$$N_c = 8, 14, 28, 50, 82, 126, 184, 258, \quad \text{with } c = 1, 2, \ldots, 8 \tag{4.14}$$

and their mid closures:

$$N_{med} = 11, 21, 39, 66, 104, 155, 221. \tag{4.15}$$

Similar expression hold for the number of valence proton n_π .

Introducing the semi-degeneracy η_c of the shell number c as:

$$\eta_c = \frac{N_{c+1} - N_c}{2} \tag{4.16}$$

The mean of the valence nucleons can be expressed as:

$$\langle n_v \rangle = \langle n_\pi \rangle = \frac{\eta_c}{2} \tag{4.17}$$

$$\left\langle \left(\frac{n_v + n_\pi}{2} \right)^2 \right\rangle = \frac{\langle n_v^2 \rangle + \langle n_\pi^2 \rangle + 2\langle n_v \rangle \langle n_\pi \rangle}{4} \tag{4.18}$$

where

$$\langle n_v^2 \rangle = \frac{2\eta_c^2 + 1}{6} \tag{4.19}$$

$$\langle n_\pi^2 \rangle = \frac{2\eta_c^2 + 1}{6} \tag{4.20}$$

While the removal of their mean values guarantees that the microscopic terms will have no average contribution when all nuclei between closed shells are included, when the analysis is restricted to nuclei with measured masses the average value of these two terms is not zero. To compensate for this effect a constant term is added to the complete all masses formulas, such as LDM2 which reads

$$LDM2 + shell = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{(1+e^{-A/k})A} + \delta_{new} + a_f F + a_{ff} FF \quad 4.21$$

In this thesis work the shell correction terms $a_f F + a_{ff} FF$ were applied to all LDM formulas to calculate the binding energies.

4.3 Liquid Drop Model plus Shell effect

The shell effect to the Liquid drop model can be found, in general these method are rather laborious [61]. A simple method was proposed in ref. [59,62] based on counting the number of valence nucleons, as suggested by the interaction boson model (IBM) [63]. This shell effect is linear and quadratic in the total number of valence nucleons, as [61]:

$$E_{shell} = b_1(n_\nu + n_\pi) + b_2(n_\nu + n_\pi)^2 \qquad 4.22$$

where n_ν and n_π is the number of valance neutron and proton (particles or holes) and b_1 and b_2 are parameters.

4.4 Root mean square deviation (RMS) of liquid drop model

The RMS deviation was calculated using this relation: [58].

$$RMS = \sqrt{\frac{\sum\left[BE_{\exp} - BE_{the}\right]^2}{N_{nucl}}} \qquad 4.23$$

where BE_{exp} and BE_{the} are experimental and theoretical binding energies, respectively and N_{nucl} is the total number of nuclei.

4.5 Calculations of α-decay half-lives

We studied five different formulas for α-decay to calculate half-lives of α-particles. The coefficient of those formulas was determined.

4.5.1 Viola-Seaborg-Sobiczewski (VSS) formula

The Geiger-Nuttall law was generalized and proposed by Viola-Seaborg formula as [64, 65]:

$$log_{10}T_\alpha = \left(aZ + b\right)Q_\alpha^{1/2} + \left(cZ + d\right) \qquad 4.24$$

where Z is the atomic number of parent nuclei and Q is the reaction energy, the *a, b, c* and *d* are free parameters obtained by Nonlinear Regression fit to nucleus [66]. In this formula the value of half-lives is in *seconds*. The calculation of α-decay half-lives of even-even, even-odd, odd-even, odd-odd and all nuclei by this formula agree well with the experimental data.

4.5.2 Guy Royer formula

From the generalized liquid drop model included the proximity potential effects between the α-particle and the daughter nucleus and adjusted to reproduce the Q value [67], Guy Royer deduced the α-emission half-lives from WKB barrier penetration probability [68]. A fitting procedure leaded to the following empirical formula:

$$log_{10}\left[T_\alpha(s)\right] = -a - bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} \qquad 4.25$$

For even-even, even-odd, odd-even, odd-odd and all nuclei, where *Z* and *A* are the atomic number and mass number of parent nucleus, respectively and *Q* is the reaction energy and *a*, *b* and *c* are free parameters obtained by Nonlinear Regression to nucleus.

4.5.3 Denisov & Khudenko (DEKH) formula

From the Guy Royer empirical formula, Denisov & Khudenko developed empirical expressions for α-decay half-lives between ground-states-to-ground-states α- transition of parent and daughter nuclei. They obtained the following expression for the half-lives as [69]:

$$log_{10}\left(T_{\alpha}(s)\right) = -a - \frac{bA^{1/6}\sqrt{Z}}{\mu} + \frac{cZ}{\sqrt{Q_{\alpha}}} + \frac{d\sqrt{\ell\left(\ell+1\right)}}{QA^{-1/6}} - e\left((-1)^{\ell} - 1\right) \qquad 4.26$$

Here A and Z are the mass number and atomic number of parent nucleus, respectively, ℓ is the orbital moment of emitted α particle, and $\mu = \left(A/\left(A-4\right)\right)^{1/6}$. Q is the reaction energy value. The α-particle emission from nuclei obeys the spin-parity selection rule [70, 71]

$$\ell_{\min} = \begin{cases} \Delta j & \text{for even } \Delta j \text{ and } \pi_p = \pi_d \\ \Delta j + 1 & \text{for odd } \Delta j \text{ and } \pi_p = \pi_d \\ \Delta j & \text{for odd } \Delta j \text{ and } \pi_p \neq \pi_d \\ \Delta j + 1 & \text{for even } \Delta j \text{ and } \pi_p \neq \pi_d \end{cases} \qquad 4.27$$

where $\Delta j = \left|j_p - j_d\right|$, j_p, π_p, j_d, π_d are spin and parity values of the parent and daughter nuclei, respectively, and $\ell = \ell_{\min}$, the *a*, *b*, *c*, *d*, and *e* are free parameters obtained by nonlinear regression to nucleus.

4.5.4 The Universal Decay Law (UDL) formula

Starting from α-like (extension to the heavier cluster of α-decay theory) R-matrix theory and the microscopic mechanism of the charged particle emission, a new universal decay law (UDL) for α-decay and cluster decay model was introduced by Qi- et al., [72,73]. The model was presented in an interesting way, which made it possible to represent, on the same plot with a single straight line, the logarithm of the half-lives minus some quantity versus one of the two parameters (χ' and ρ') that depend on the atomic and mass numbers of the daughter and emitter particles as well as the Q value. UDL related the half-lives of monopole radioactive decay with the Q values of the outgoing particles as well as the masses and charges of the nuclei involved in the decay and can be written in the logarithmic form as [74],

$$\log_{10}\left(T_{1/2}\right) = aZ_cZ_d\sqrt{\frac{A}{Q}} + b\sqrt{AZ_cZ_d\left(A_d^{1/3}+A_c^{1/3}\right)} + c \qquad 4.28$$

$$= a\chi' + b\rho' + c \qquad 4.29$$

where the quantity $A = \frac{A_dA_c}{A_d + A_c}$ and *a, b* ,and *c*, are free parameters coefficient sets of equation (4.29) obtained by Nonlinear Regression to nucleus of both α and cluster decays with A_d and A_c are the daughter and cluster mass numbers, respectively.

4.5.5 Unified α-decay and cluster radioactivity (NRDX) formula

The equation of half-lives and decay energies for α decay and cluster radioactivity can be written as [75-78]

$$\log_{10} T_{1/2} = a\sqrt{\mu}Z_cZ_dQ^{-1/2} + b\sqrt{\mu}(Z_cZ_d)^{1/2} + c \qquad 4.30$$

where Z_c and Z_d are the atomic number of the cluster and daughter nuclei, respectively, and *a, b,* and *c* free parameters coefficient sets obtained by Nonlinear Regression to nucleus of both α and cluster decays. μ is the reduced mass of the cluster –daughter where $\mu = A_cA_d/(A_c + A_d)$.

4.6 Root mean square deviation of alpha decay

The root mean square deviation for alpha decay was calculated using this expression [79]:

$$RMS = \sqrt{\frac{\sum\left(\log_{10} T_{\exp} - \log_{10} T_{the}\right)^2}{N}} \qquad 4.31$$

where T_{exp} and T_{the} are experimental and theoretical alpha decay half-lives, respectively and N is the number of nuclei.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Liquid Drop Model

In this part we investigated the binding energy calculated from six different liquid drop model formulas with presenting a suggested model (sixth formula) which has been modified depending on the Bethe-Weizsacker model.

Firstly a total of 3252 nuclei with N, $Z \geq 8$ [80] were classified into four different groups: deformation group, decay mode group, spin group and a group of all nuclei together. Each group has been subdivided into the following sets:

1- Deformation group: This group is classified into six sets according to the quadrupole deformations e_2 [81] in the ranges as shown in Table 5.1.

Table 5.1 The quadrupole deformations group.

Sets	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
e_2 **min**	-0.65	-0.11	0.04	0.12	0.18	0.23
e_2 **max**	-0.11	0.04	0.12	0.18	0.23	0.65
N_{nucl}	364	781	415	412	669	604

N_{nucl} is the number of nuclei in each group, listed in fourth row of Table 5.1. We remark that regions more nuclei are belong to sets 2, 5 and 6. Notice that group1 contain most of oblate nuclei, that the more spherical nuclei belong to groups 2, 3, and that the more prolate deformation nuclei are included in groups 4, 5 and 6.

2- Decay mode group: This group is classified into two sets according to the (β+, and β-) nuclei which is taken from reference [90] as shown in Table 5.2.

Table 5.2 The Decay mode group.

Sets	Set β+	Set β-
N_{nucl}	1117	1277

It is obvious that although number of nuclei decayed by negatron (β^-) is larger than the number of nuclei decayed by positron emission (β+) in table 5.2 there is no big difference between these two.

3- Spin group: This group is classified into six sets according to the spin of nuclei as shown in Table 5.3.

Table 5.3 The Spin group.

Sets	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
Spin	0	1/2	3/2	5/2	7/2	9/2
N_{nucl}	850	458	508	497	323	245

4- All Nuclei group: This group contain all Nuclei having N, $Z \geq 8$.

For the four mentioned groups, the Nonlinear Regression numerical analysis method has been tried to calculate the coefficient of LDM's formulae which are multi-variable nonlinear equations. The MATLAB 2014 software code has been used in the determination of the coefficients and presented in Appendix A.

By using equation (4.23), we determined the Root Mean Square (RMS) for each sixth formula as illustrated in Table A.7, A21 and A25, which shows the consistency of presented models to the experimental results.

From Table A.7 the RMS of Deformation group, it turned out that set 5 has smallest value for all six formulas which are in order of 11.591, 23.048, 8.939, 7.956, 11.535 and 7.958, keV and the largest RMS was for set 2 for all six formulas which are in order of 39.005, 42.805,31.946, 28.373, 40.137 and 31.411, keV. According to these results it is obvious that nuclei having prolate deformation give better description of the measured binding energies of nuclei.

For decay mode group, it turned out that set 1 which says nuclei having β^+ decay has the smallest value for all six formula which are in order of 25.746, 25.480, 22.576, 21.737, 26.797 and 21.452 keV and the largest RMS was for set 2, which says nuclei having β^- decay, for all six formula which are in order of 30.861, 32.207, 27.511, 26.809, 30.759 and 26.815 keV.

For spin group, it turned out that set 6 which refer to spin 9/2 has smallest value for all six formula which are in order of 20.418, 26.551, 20.708, 18.949, 20.368 and 19.004 keV and the largest RMS was for set 3 for all six formula which are in order of 28.331, 39.321, 26.721, 25.486, 29.461 and 25.754 keV. Here calculation for spin 9/2 best describes the measured binding energy of nuclei.

For all nuclei group, it turned out that the RMS value for all six formulas which are in order of 28.619, 36.546, 24.599, 23.733, 29.654 and 24.015, keV. According to these result it is said that LDM4 gives better description of the measured binding energies of nuclei. LDM6 which was used to calculate binding energy firstly in this study has a relatively comparable RMS value with LDM4.

In this thesis work, the shell correction has been considered by adding equations (4.10) and (4.11) to all the liquid drop mass formulae, and then the resulted RMS values are shown in Table A.14.

It seems in Table A.14 when calculated data are fitted to measured binding energies for deformation group set 5 has smallest RMS value among all six formula which are in order of 10.140, 22.288, 6.872, 6.411, 10.461 and 6.552, keV and the largest RMS was for set 2 for all six formula which are in order of 23.671, 34.479, 17.821, 16.192, 24.335 and 16.806 keV. These results shows us that nuclei having prolate deformation gives better description of the measured binding energies of nuclei as we proved in a previous calculation.

For decay mode group, it turned out that set 1 which says nuclei having β^+ decay has the smallest value for all six formula which are in order of 16.910, 16.512, 11.982, 10.635, 18.152 and 10.497 keV and the largest RMS was for set 2, which says nuclei having β^- decay, for all six formula which are in order of 21.103, 24.772, 17.606, 16.501, 21.092 and 16.651 keV.

For spin group, it turned out that set 6 which refer to spin 9/2 has smallest value for all six formula which are in order of 8.532, 15.943, 8.628, 8.271, 9.337 and 8.149 keV and the largest RMS was for set 3 for all six formula which are in order of 20.027, 33.133, 18.550, 17.452, 20.729 and 17.988 keV. Here calculation for spin 9/2 best describes the measured binding energy of nuclei.

For all nuclei group, it turned out that the RMS value for all six formula which are in order of 20.279, 31.408, 15.806, 14.635, 21.508 and 14.978 keV. By the inclusion of shell correction also we get the similar behaviour of the previous calculations. Again LDM4 gives best description of the measured data but LDM6 which is a new formula gives RMS very close to that formula.

The Shell effect was inserted by adding Equation (4.22) to all six liquid drop model formulas in this study.

RMS values was determined and listed in Table A.21. It is seen that for the Deformation group, set 5 has smallest RMS value among all six formula which are in order of 10.224, 21.500, 6.936, 6.252, 10.535 and 6.929 keV and the largest RMS was for set 2 for all six formula which are in order of 23.912, 37.193, 18.899, 16.792, 24.836 and 17.073 keV. These results shows us that nuclei having prolate deformation gives better description of the measured binding energies of nuclei as we proved in a previous calculations.

For decay mode group, it appears that set 1 which says nuclei having β^+ decay has the smallest value for all six formula which are in order of 17.122, 17.638, 13.081, 10.156, 19.349 and 10.083 keV and the largest RMS was for set 2, which says nuclei having β^- decay, for all six formula which are in order of 22.175, 26.145, 18.641, 17.632, 22.212 and 17.836 keV.

For spin group, it turned out that set 6 which refer to spin 9/2 has smallest value for all six formula which are in order of 8.261, 15.735, 8.704, 7.538, 10.125 and 7.149 keV and the largest RMS was for set 3 for all six formula which are in order of 21.570, 35.756, 19.966, 18.518, 22.108 and 18.961 keV. Here calculation for spin 9/2 best describes the measured binding energy of nuclei.

For all nuclei group, it turned out that the RMS value for all six formula which are in order of 20.760, 32.521, 16.884, 15.166, 22.422 and 15.315 keV. By the inclusion of shell effect also we get the similar behaviour of the previous calculations. Again LDM4 gives best description of the measured data but LDM6 which is a new formula gives RMS very close to that formula.

Hence, it is noted that although we have the three different calculations, just LDM, adding shell correction to LDM and adding shell effect to LDM formulas, in each case we get same results with different numbers of RMS. It means for deformation group set 5 has smallest but set 2 has largest RMS values, for decay mode group set 1 has smallest but set 2 has largest RMS, for spin group set 6 has smallest but set 3 has larget RMS values and in all nuclei group LDM4 and LDM6 very close to each other and better describe the measured binding energies.

So far we did the numerical analysis of the calculation and presented the calculated results in Tables A.7, A14 and A21 in Appendix part. In the following subsection we discuss the results with figures.

5.1.1 First Liquid Drop Model (LDM1)

Figure 5.1 shows the difference between the theoretical binding energy, which was determined by liquid drop model LDM1, and measured binding energy vs mass number A for the all nuclei group.

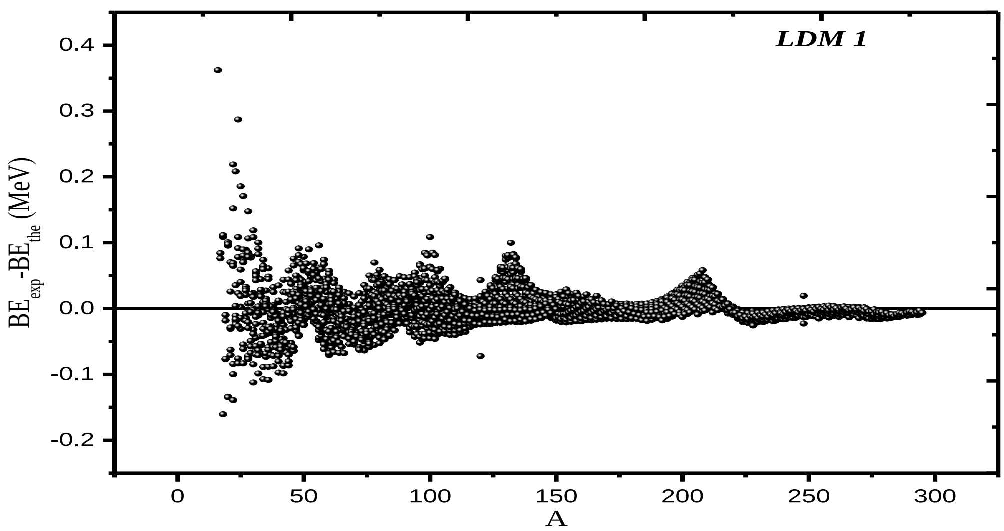

Figure 5.1 The difference between theoretical binding energy from LDM1 and experimental value vs. mass number for all nuclei group.

Figure 5.2 shows the difference between the theoretical binding energy, which is determined by Liquid drop model LDM1+shell correction, and the experimental binding energy vs. mass numbers of all nuclei group.

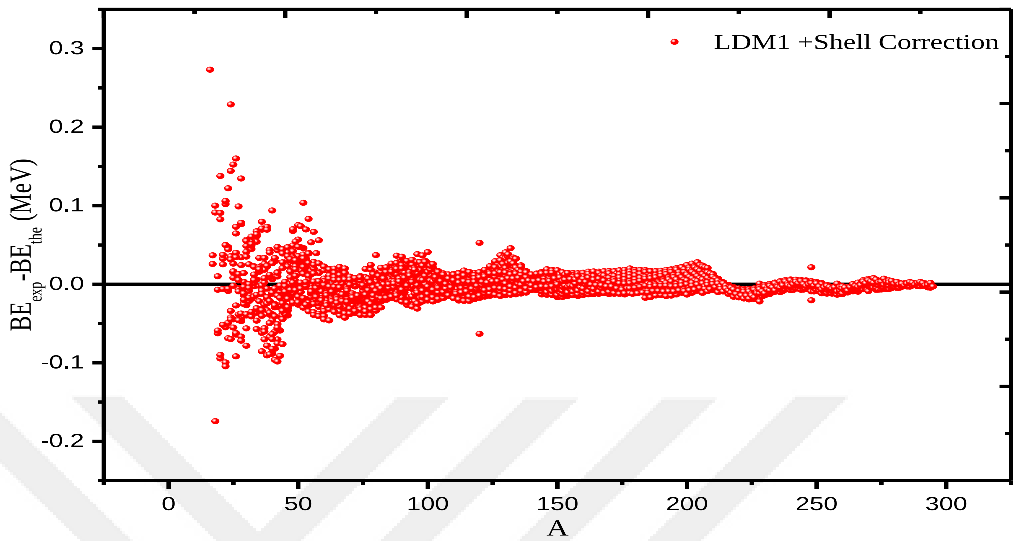

Figure 5.2 The difference between theoretical binding energy from LDM1+shell correction and experimental value vs. mass number for all nuclei group.

Figure 5.3 shows the difference between theoretical binding energy, which is determined by formula 1 of Liquid drop mode LDM1+shell effect, and the experimental binding energy vs. mass number A of all nuclei group.

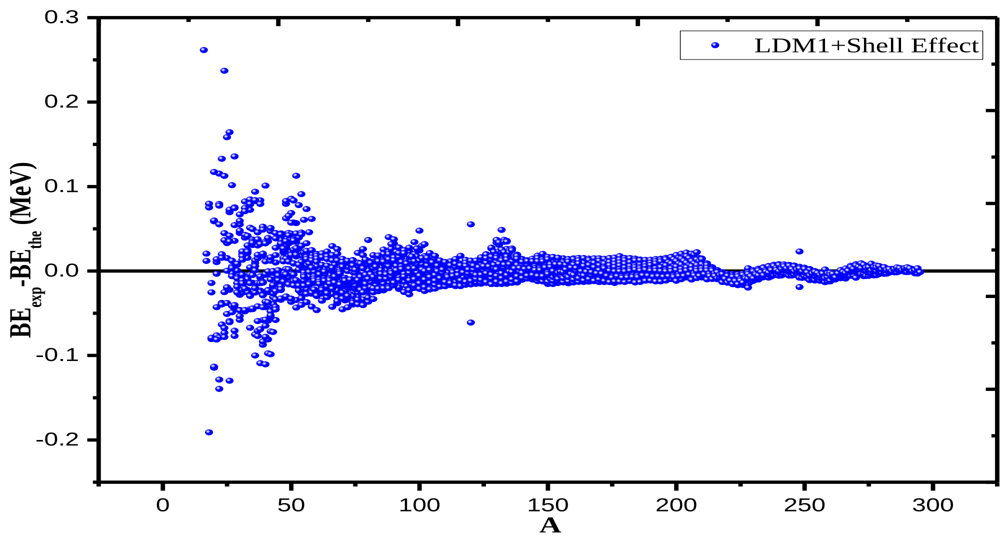

Figure 5.3 The difference between theoretical binding energy from LDM1+shell effect and experimental value vs. mass number for all nuclei group.

A comparison between the difference of experimental binding energies and those resulted from LDM1, LDM1+Shell Correction, and LDM1+Shell Effect formulae have been shown in Figure 4-5, in which the last formula record more reasonable agreements than the other two ones

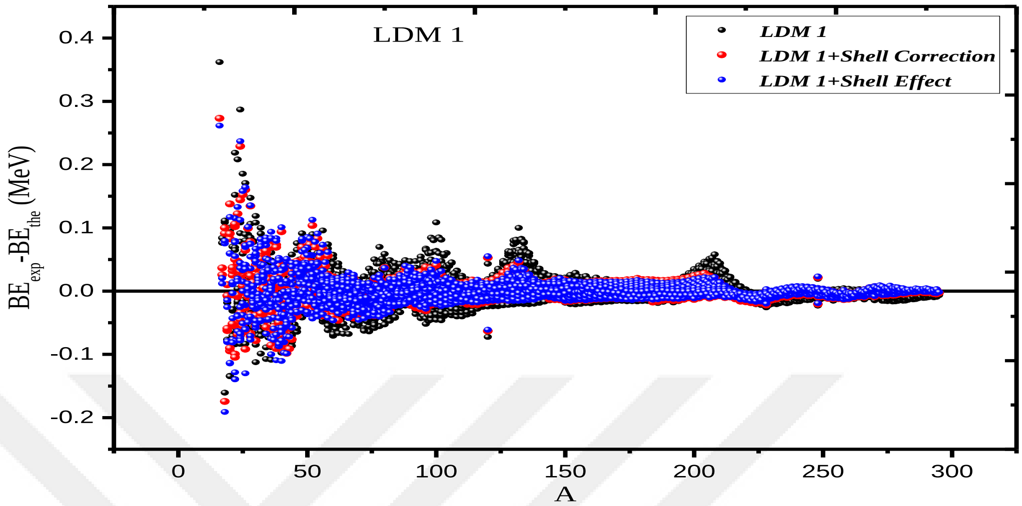

Figure 5.4 The difference between theoretical binding energy from (LDM1, LDM1+shell correction, and LDM1+shell effect) and experimental value vs. mass number for all nuclei group.

5.1.2 Second Liquid Drop Model LDM2

Figure 5.5 shows the difference between theoretical binding energy, which is determined by Liquid drop mode LDM2, and experimental binding energy vs. mass number A of all nuclei group.

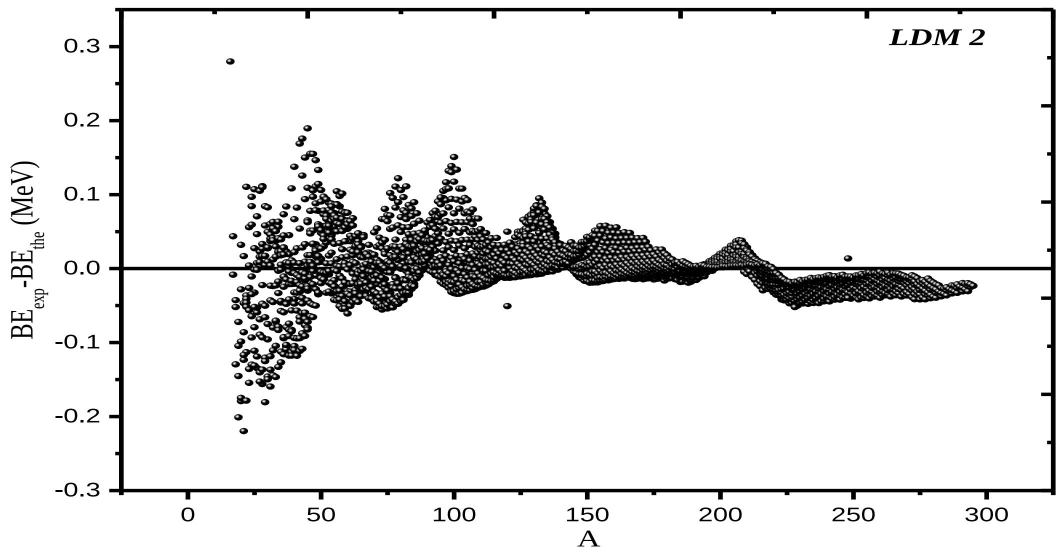

Figure 5.5 The difference between theoretical binding energy from LDM2 and experimental value vs. mass number for all nuclei group.

The Figure 5.6 shows the difference between theoretical binding energy, which is determined by Liquid drop mode LDM2, and experimental binding energy vs. mass number of all nuclei group.

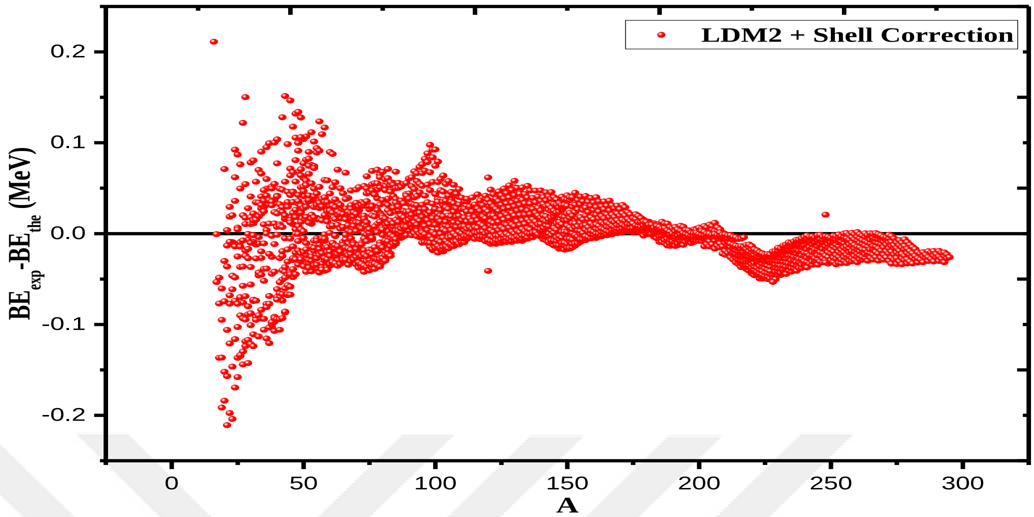

Figure 5.6 The difference between theoretical binding energy from LDM2+shell correction and experimental value vs. mass number for all nuclei group.

The figure (5.7) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM2+shell effect- to experimental binding energy vs to mass number of all nuclei group.

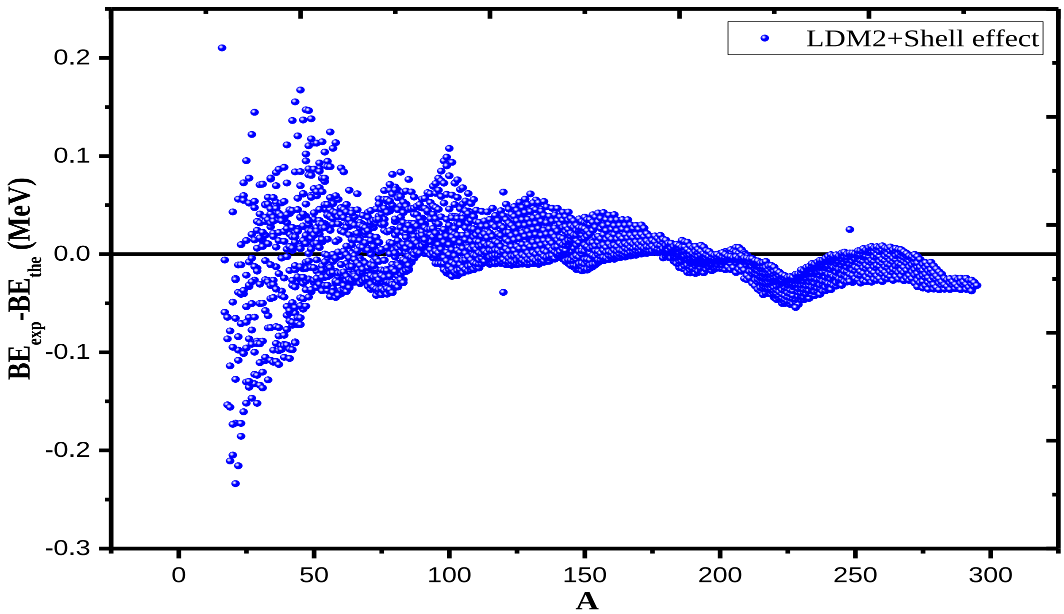

Figure 5.7 The difference between theoretical binding energy from LDM2+shell effect and experimental value vs. mass number for all nuclei group.

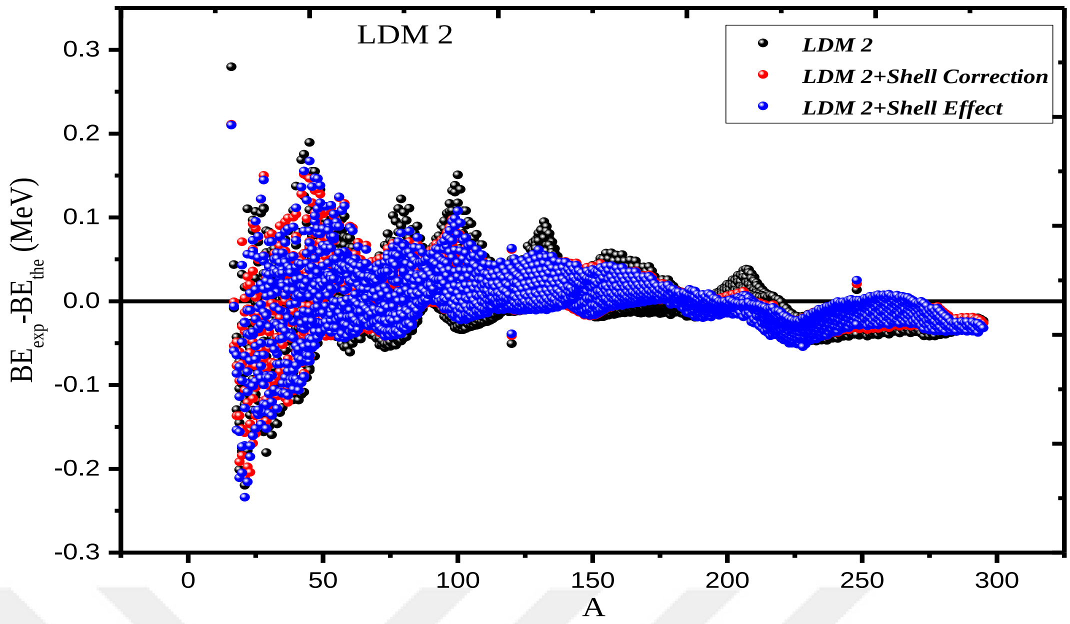

Figure 5.8 The difference between theoretical binding energy from (LDM2, LDM2+shell correction, and LDM2+shell effect) and experimental value vs. mass number for all nuclei group

5.1.3 Third Liquid Drop Model LDM3

The figure (5.9) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM3- to experimental binding energy vs to mass number of all nuclei group.

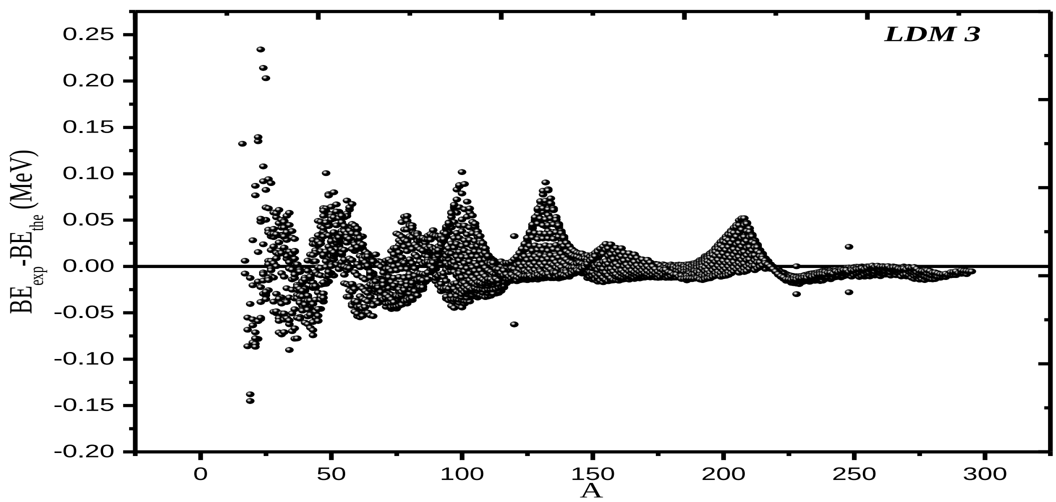

Figure 5.9 The difference between theoretical binding energy from LDM3 and experimental value vs. mass number for all nuclei group

The figure (5.10) show the agreement between theoretical binding energy- which is determined by of Liquid drop mode LDM3+shell correction- to experimental binding energy vs to mass number of all nuclei group.

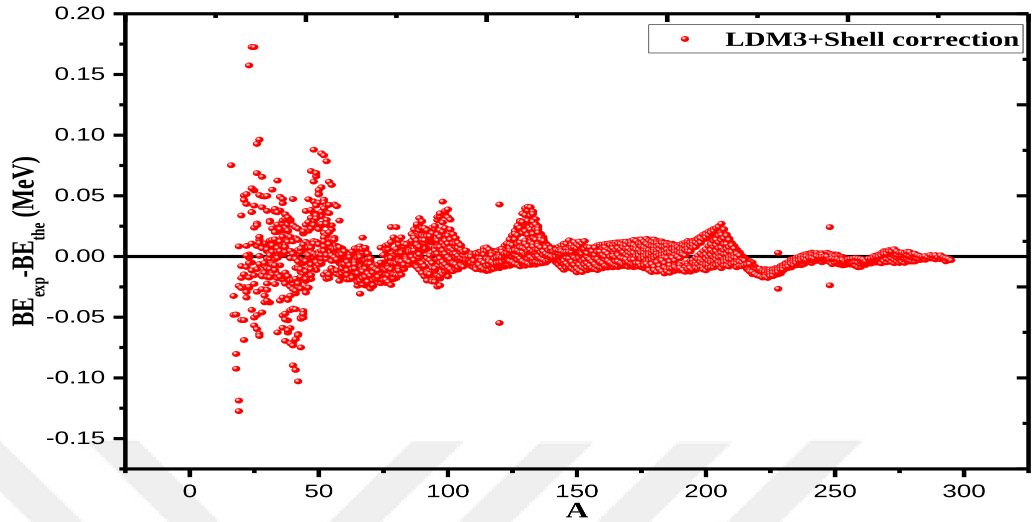

Figure 5.10 The difference between theoretical binding energy from LDM3+shell correction and experimental value vs. mass number for all nuclei group.

The figure (5.11) show the agreement between theoretical binding energy- which is determined by formula 3 of Liquid drop mode LDM3+shell effect- to experimental binding energy vs to mass number of all nuclei group.

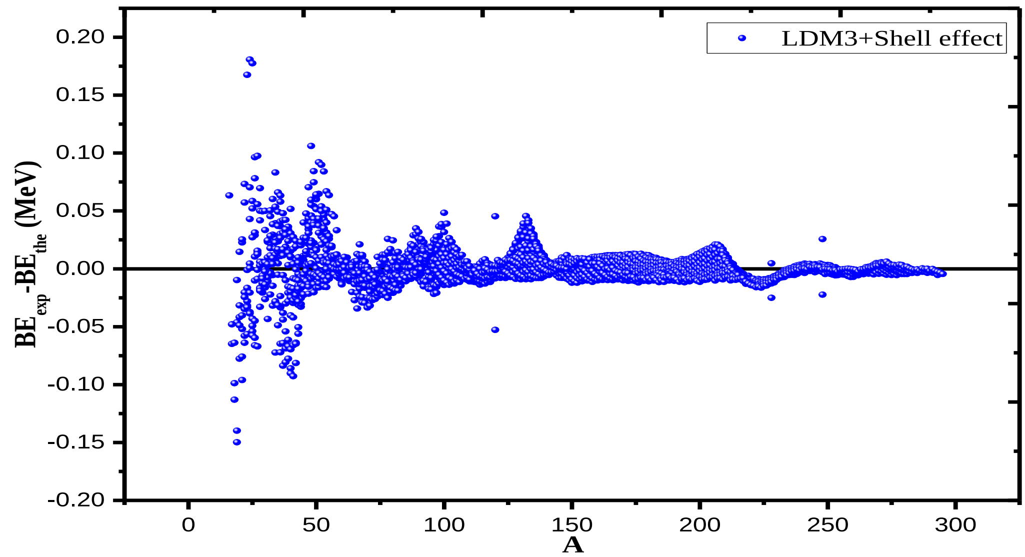

Figure 5.11 The difference between theoretical binding energy from LDM3+shell effect and experimental value vs. mass number for all nuclei group.

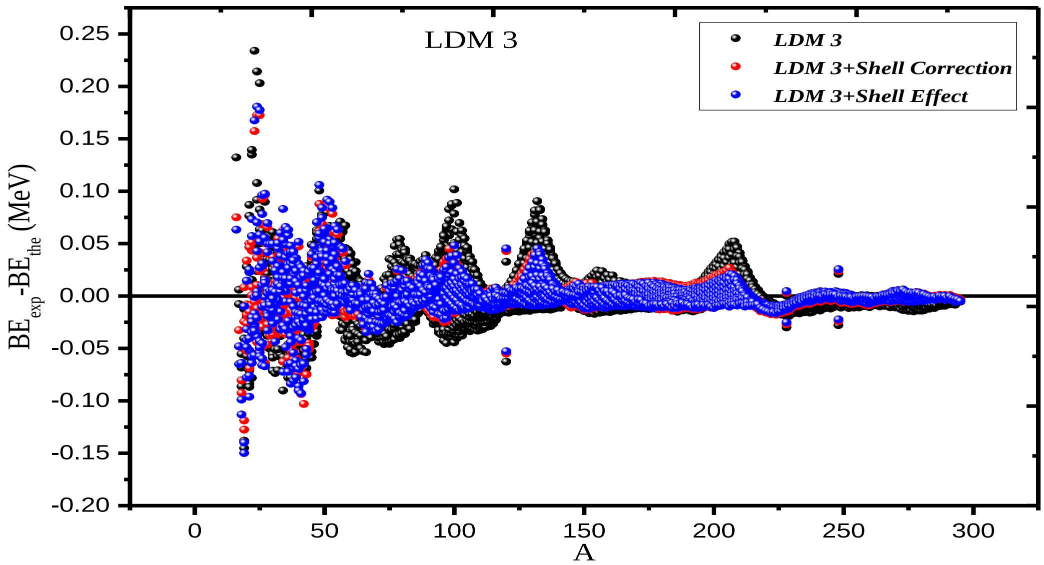

Figure 5.12 The difference between theoretical binding energy from (LDM3, LDM3+shell correction, and LDM3+shell effect) and experimental value vs. mass number for all nuclei group

5.1.4 Fourth Liquid Drop Model LDM4

The figure (5.13) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM4- to experimental binding energy vs to mass number of all nuclei group.

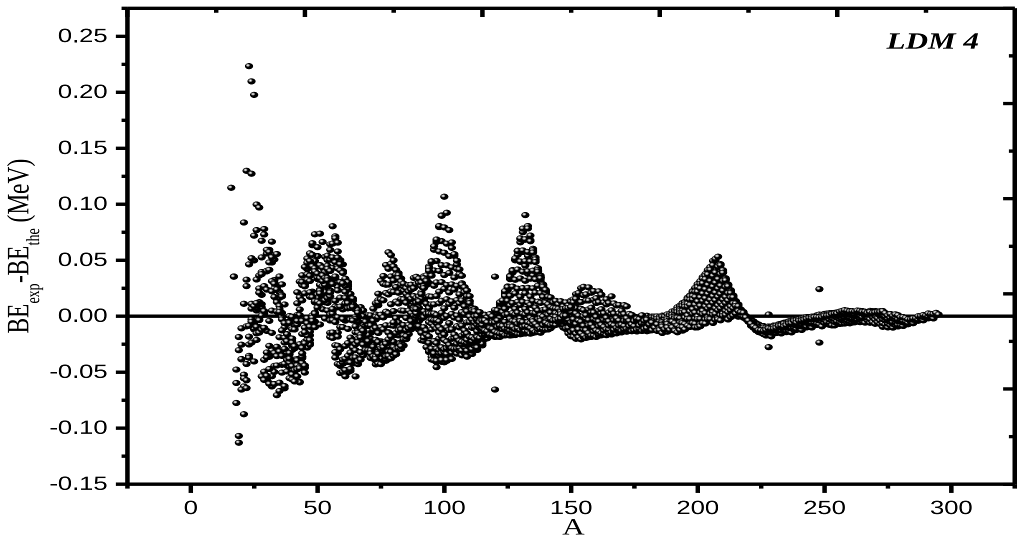

Figure 5.13 The difference between theoretical binding energy from LDM4 and experimental value vs. mass number for all nuclei group.

The figure (5.14) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM4+shell correction formula- to experimental binding energy vs to mass number of all nuclei group.

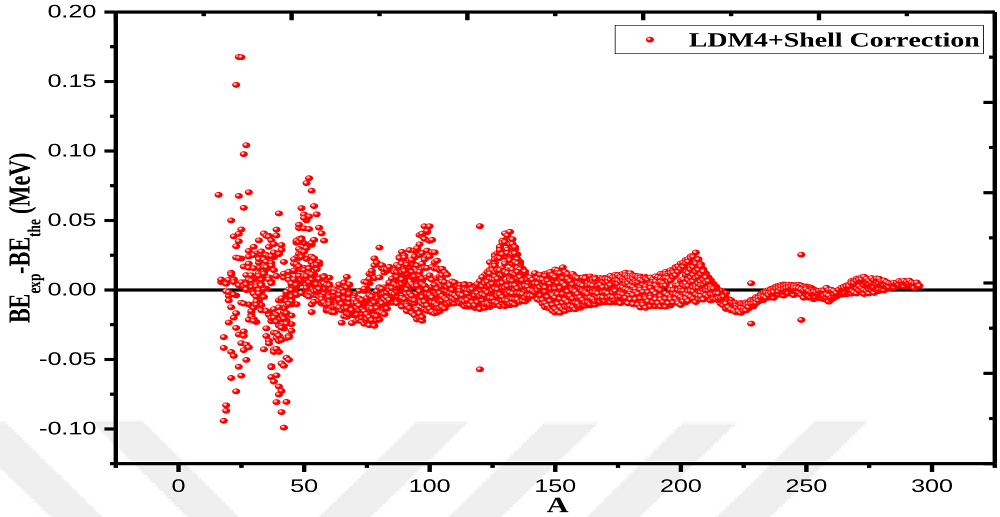

Figure 5.14 The difference between theoretical binding energy from LDM4+shell correction and experimental value vs. mass number for all nuclei group.

The figure (5.15) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM4+shell effect formula- to experimental binding energy vs to mass number of all nuclei group.

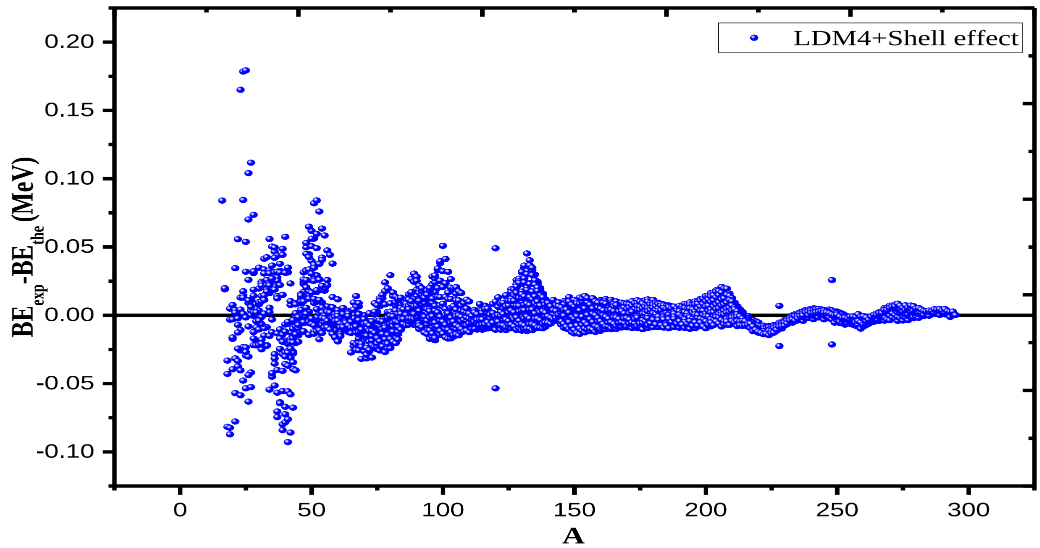

Figure 5.15 The difference between theoretical binding energy from LDM4+shell effect and experimental value vs. mass number for all nuclei group.

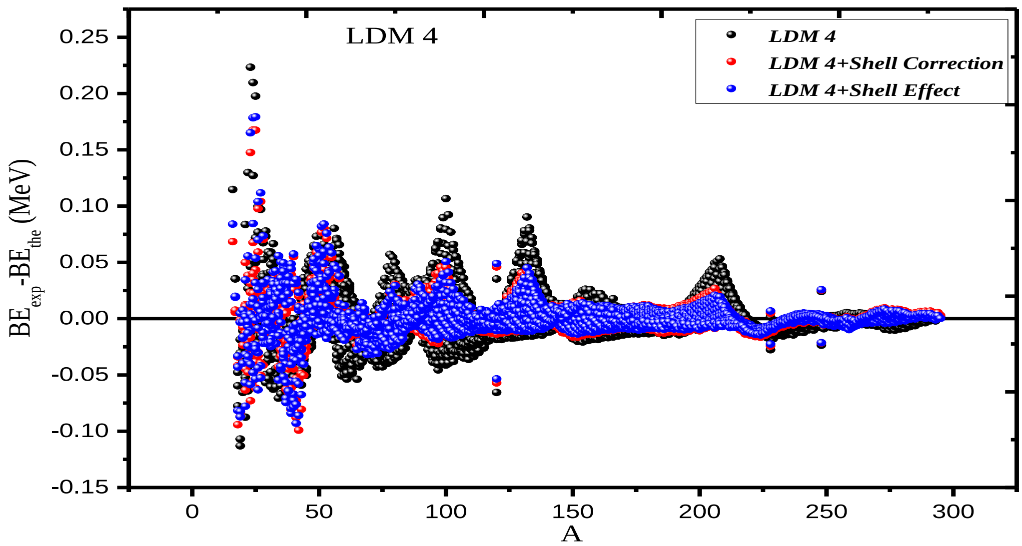

Figure 5.16 The difference between theoretical binding energy from (LDM4, LDM4+shell correction, and LDM4+shell effect) and experimental value vs. mass number for all nuclei group

5.1.5 Fifth Liquid Drop Model LDM5

The figure (5.17) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM5 formula- to experimental binding energy vs to mass number of all nuclei group.

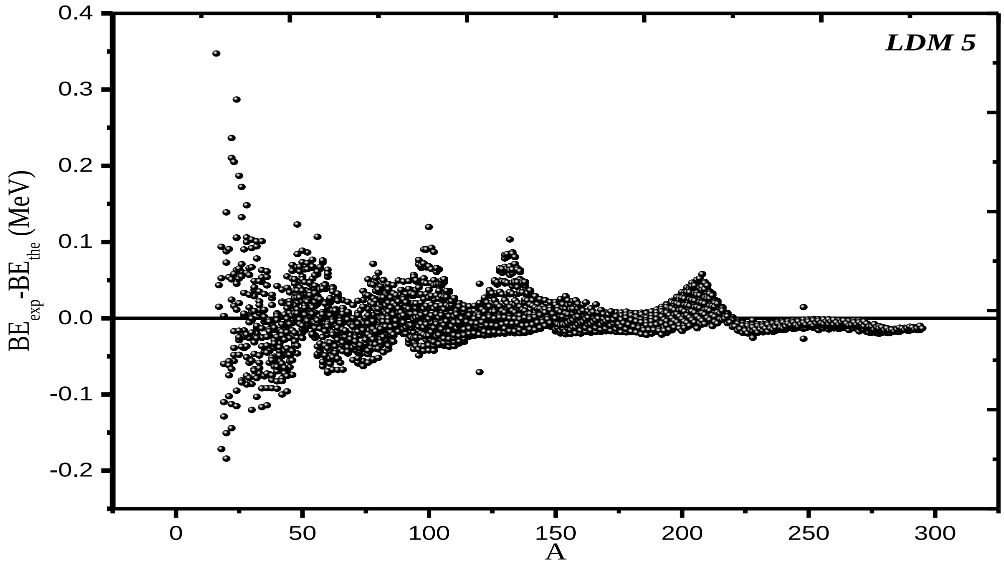

Figure 5.17 The difference between theoretical binding energy from LDM5 and experimental value vs. mass number for all nuclei group.

The figure (5.18) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM5+shell correction formula- to experimental binding energy vs to mass number of all nuclei group.

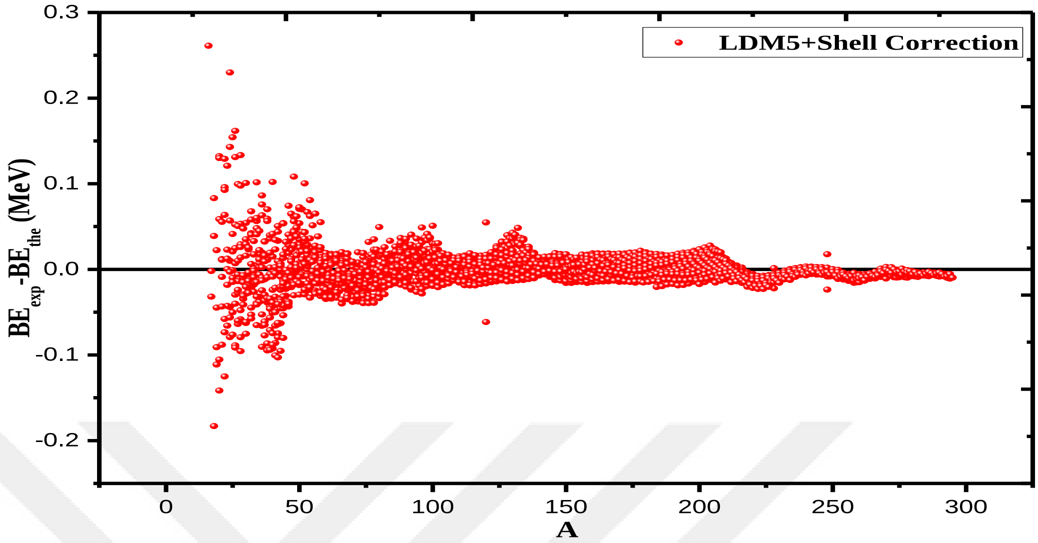

Figure 5.18 The difference between theoretical binding energy from LDM5+shell correction and experimental value vs. mass number for all nuclei group.

The figure (5.19) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM5+shell effect formula- to experimental binding energy vs to mass number of all nuclei group.

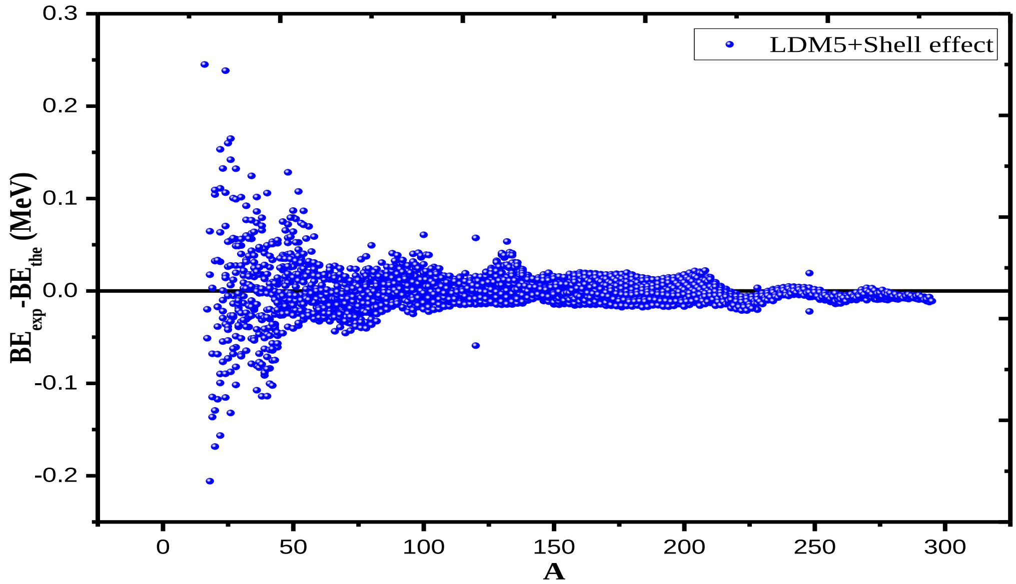

Figure 5.19 The difference between theoretical binding energy from LDM5+shell effect and experimental value vs. mass number for all nuclei group.

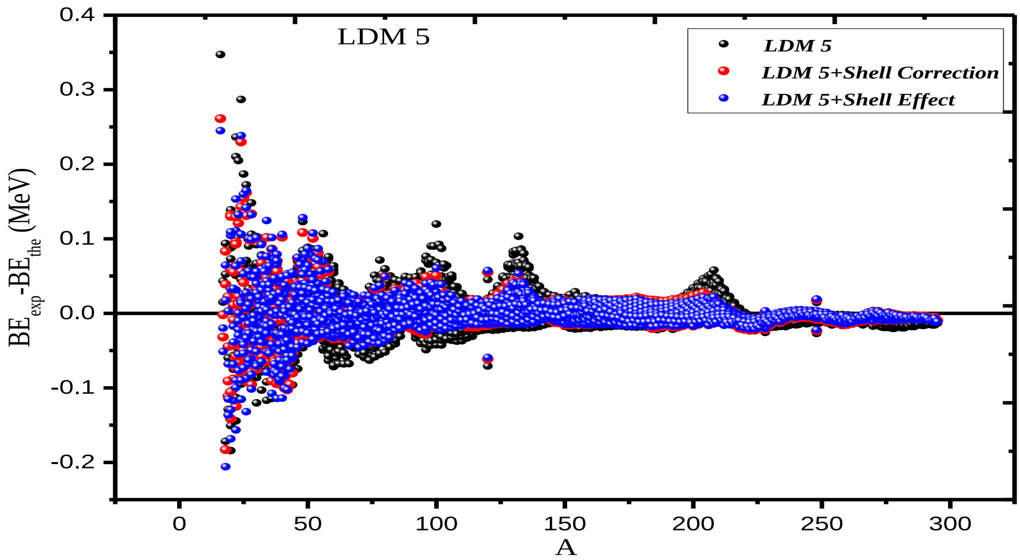

Figure 5.20 The difference between theoretical binding energy from (LDM5, LDM5+shell correction, and LDM5+shell effect) and experimental value vs. mass number for all nuclei group

5.1.6 Sixth Liquid Drop Model LDM6

The figure (5.21) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM6 formula- to experimental binding energy vs to mass number of all nuclei group.

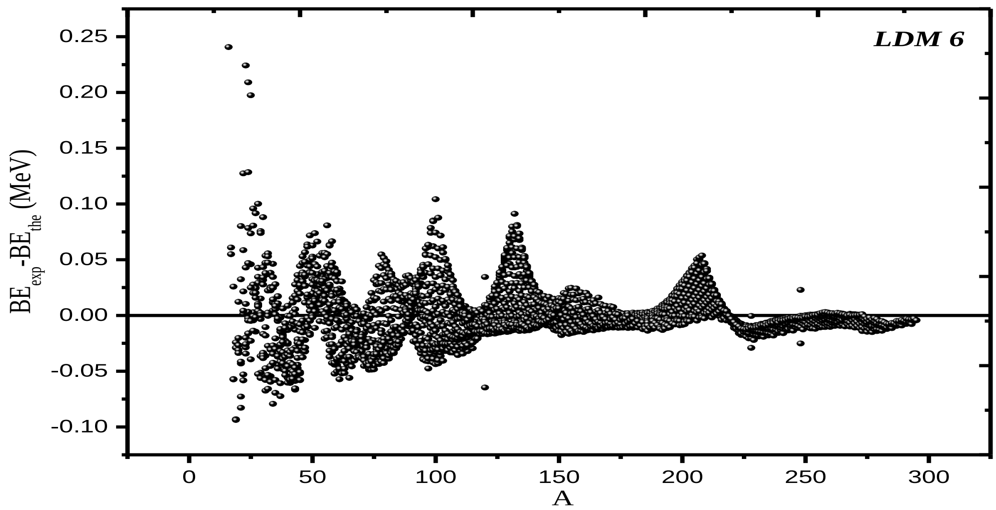

Figure 5.21 The difference between theoretical binding energy from LDM6 and experimental value vs. mass number for all nuclei group.

The figure (5.22) show the agreement between theoretical binding energy- which is determined by Liquid drop mode LDM6+shell correction formula- to experimental binding energy vs to mass number of all nuclei group.

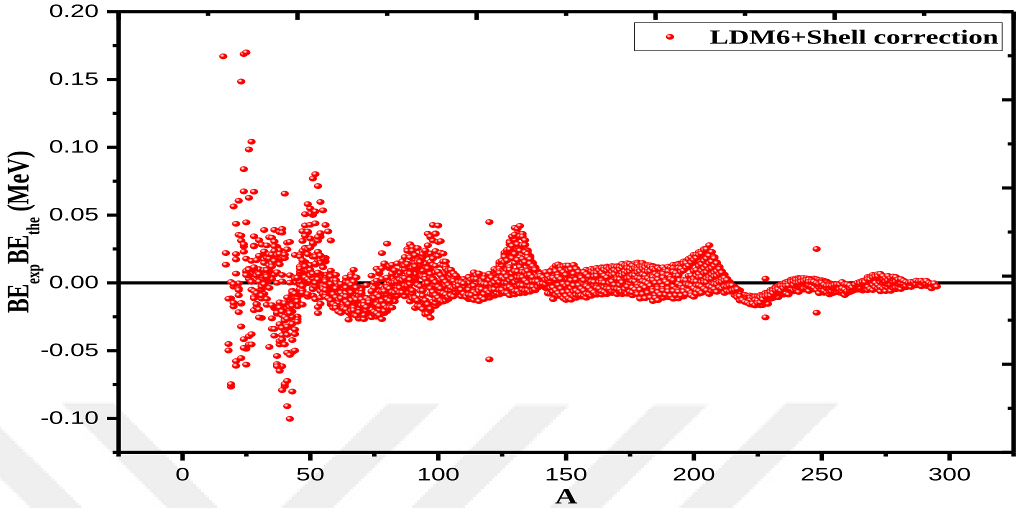

Figure 5.22 The difference between theoretical binding energy from LDM6+shell correction and experimental value vs. mass number for all nuclei group.

The figure (5.23) show the agreement between theoretical binding energy- which is determined by of Liquid drop mode LDM6+shell effect formula- to experimental binding energy vs to mass number of all nuclei group.

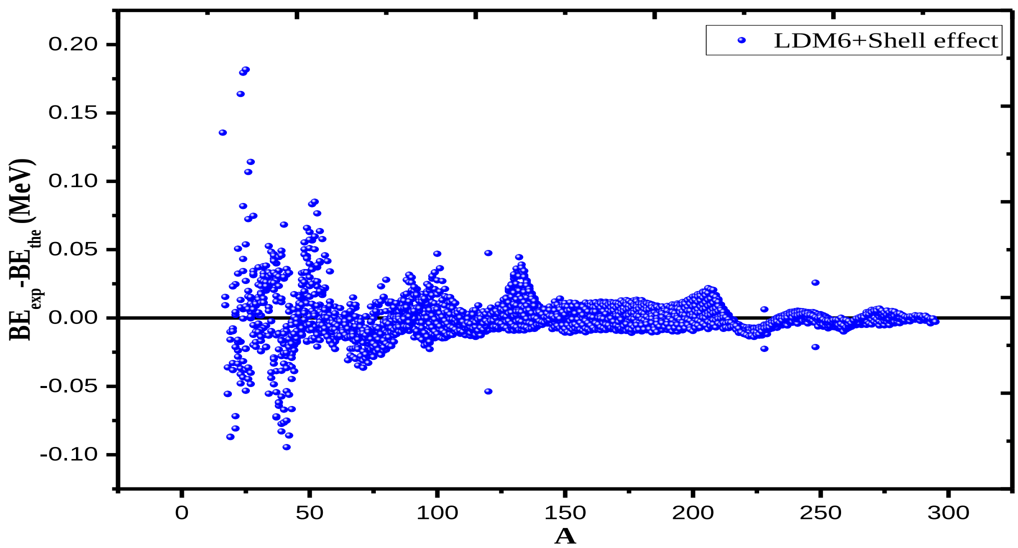

Figure 5.23 The difference between theoretical binding energy from LDM6+shell effect and experimental value vs. mass number for all nuclei group.

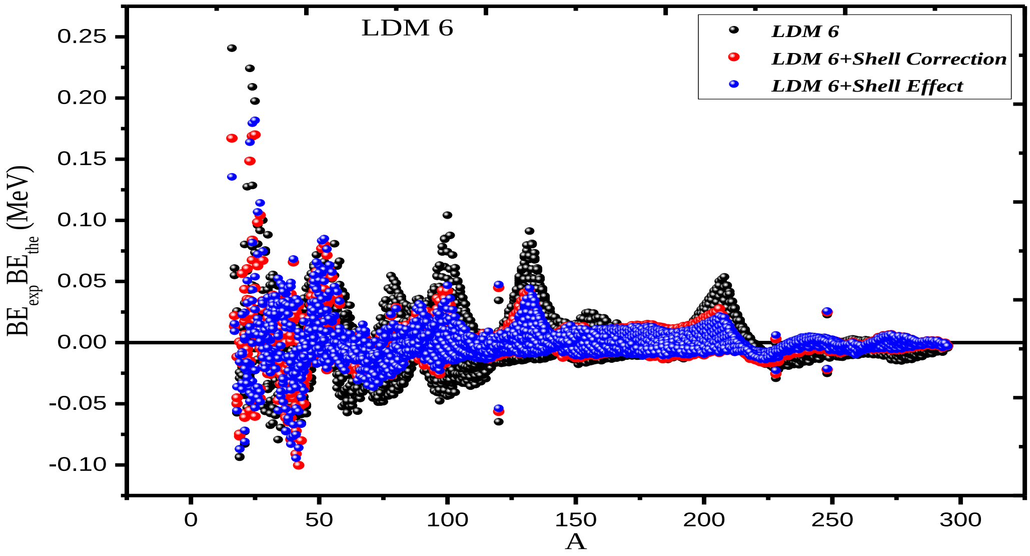

Figure 5.24 The difference between theoretical binding energy from (LDM6, LDM6+shell correction, and LDM6+shell effect) and experimental value vs. mass number for all nuclei group

In this subsection we present 24 figures comparing calculated and measured binding energies when just considering pure LDM formulas and taking shell correction and shell effect into consideration. It is remarkable that for heavier nuclei calculated results are very close to measured ones but for lighter nuclei this is not the case. Also in each figure there are sharp peaks at different points. This effect is caused from magic numbers that they have large binding energies.

5.2 Alpha Decay half-lives

A second important issue that we stand on is the analysis of α-decay half-lives. To do this we have used five different empirical formulas described in Chapter 4.

In this part we used the nonlinear regression method to calculate the parameters given in Equations (4.24), (4.25), (4.26), (4.28), and (4.30). This method of calculation was done by MINITAB 17 software program. The obtained parameters for all formula were listed in Table A.22-A.26 in Appendix A.

The Root Mean Square (RMS) was estimated by using Equation (4.31) and presented in Table (5.4). For the 459 studied nuclei the cases (even(Z)-even(N), even(Z)-odd(N), odd(Z)-even(N), and odd(Z)-odd(N)) are classified and listed in this table.

Table 5.4 The Calculated RMS values of α-decay half-lives for all formula

Case	**No. of Nuclei**	**VSS**	**Royer**	**DEKH**	**UDL**	**NRDX**
even-even	133	0.5211	0.5920	0.5433	0.5556	0.5677
odd-even	119	0.9417	0.9881	0.8518	0.9716	0.9453
even-odd	109	0.7453	0.7612	0.7260	0.7467	0.7749
odd-odd	98	0.8667	0.8601	0.8055	0.8605	0.8681

It is possible to obtain the range of compatibility between the theoretical and experimental values of half-lives of α-decay for the studied nuclei. If the result is less than one, it is possible to rely on the model for explaining α-particle theory, and also to determine α-decay half-lives. In addition, this method (nonlinear regression method) can be considered as accurate statistical method explaining much of the changes that need to be processed.

Table 5.4 illustrates that for even-even nuclei, the smallest value of RMS was 0.5211 for VSS formula and the largest value was 0.592 for Royer formula. For odd-even nuclei the smallest value was 0.8518 for DEKH formula whereas the largest value was 0.9881 for Royer formula. For even-odd nuclei the smallest value of RMS was 0.7260 for DEKH formula and the largest value was 0.7749 for NRDX formula. For odd-odd nuclei the smallest value of RMS was 0.8055 for DEKH formula and largest value was 0.8681 for NRDX formula.

After adding the angular momentum parameter and μ term which represents $\mu = \left(A / \left(A - 4 \right) \right)^{1/6}$ to Royer formula we predict the decrease of RMS as shown in DEKH formula.

The difference between experimental and theoretical values of α-decay half-lives for some special cases of nuclei are presented in Figures (5.25), (5.26), (5.27) and (5.28).

The relation between α-decay energy (Q_α) and α-decay half-lives for all semi-empirical formulae are studied and the results are shown in Figures (5.29), (5.30), (5.31) and (5.32).

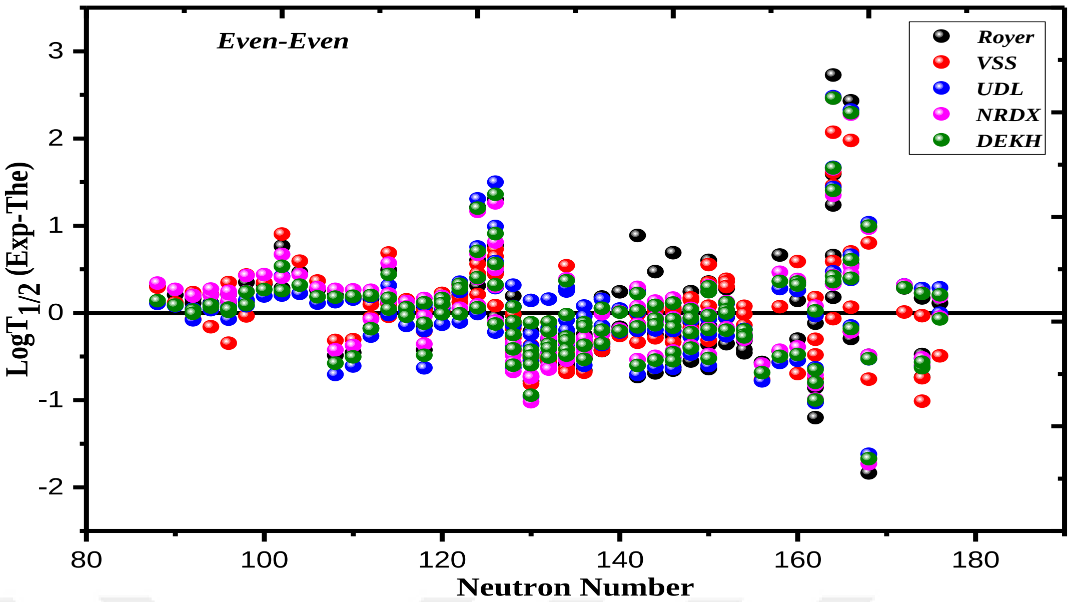

Figure 5.25 The difference between experimental and theoretical α-decay half-lives of even-even nuclei for all formulae.

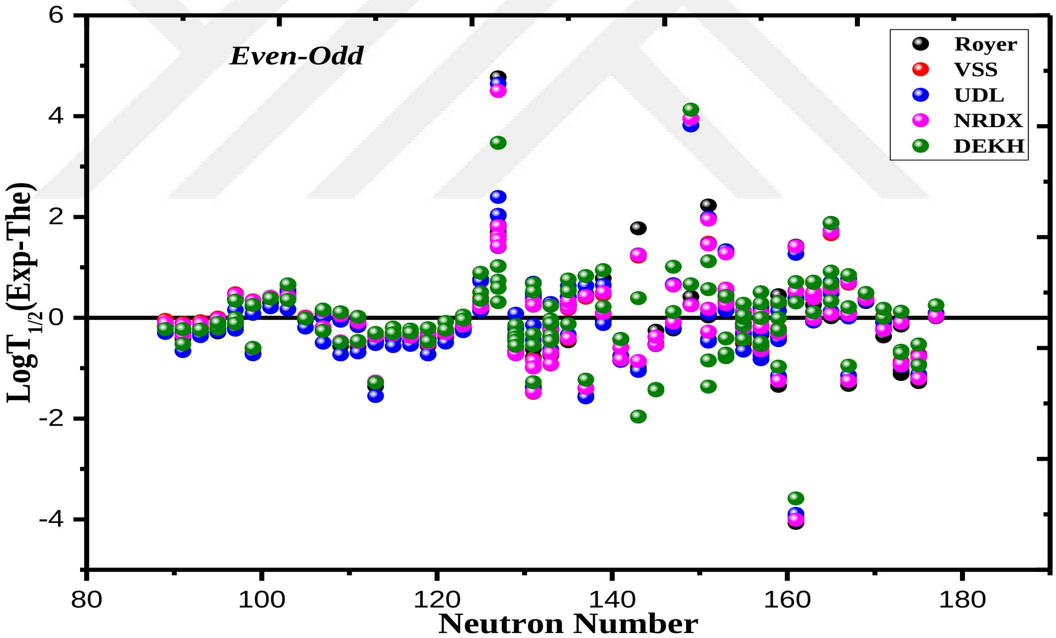

Figure 5.26 The difference between experimental and theoretical α-decay half-lives of even-odd nuclei for all formulae.

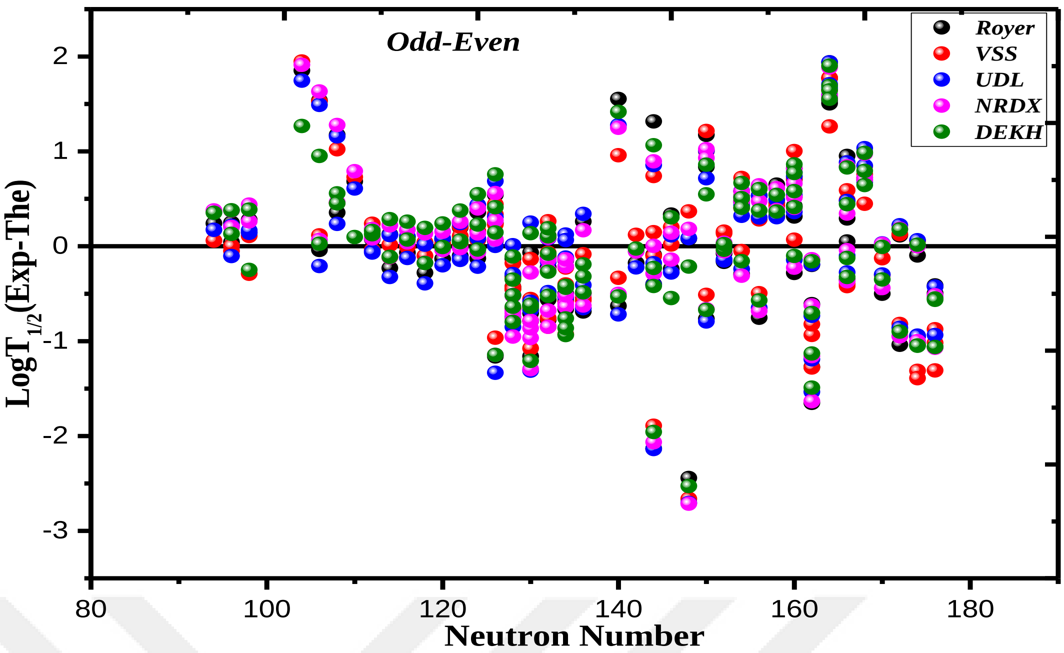

Figure 5.27 The difference between experimental and theoretical α-decay half-lives of odd-even nuclei for all formulae..

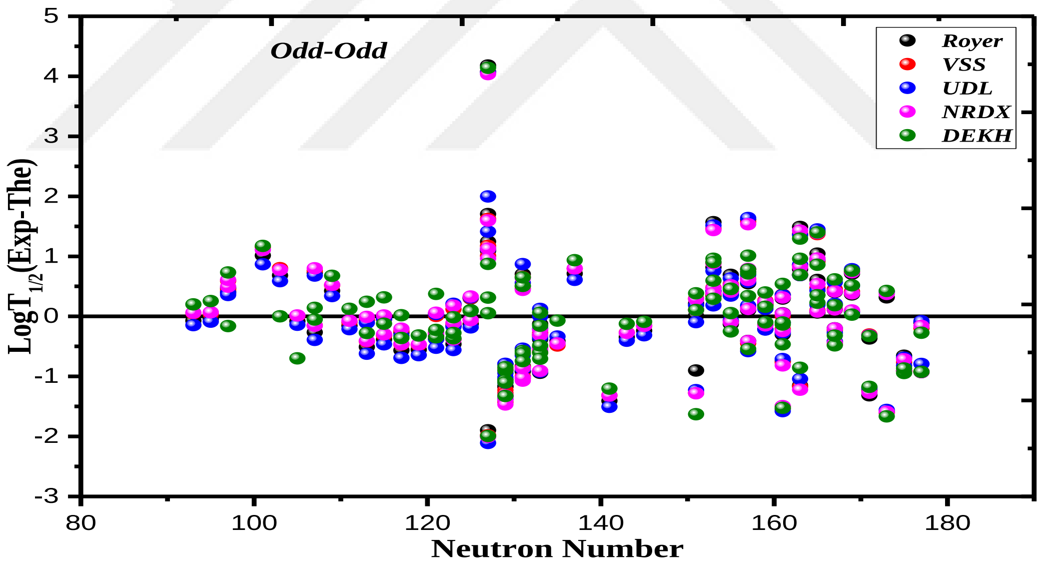

Figure 5.28 The difference between experimental and theoretical α-decay half-lives of odd-odd nuclei for all formulae.

It has been observed that the deviations from the experimental data of the α-decay half-lives are acceptable excepting the magic and near magic numbers.

Within all the semi-empirical formulae, the predicted α-decay half-lives of the even-even nuclei were best among those of the other nuclei.

Accordingly, the difference between experimental data and the calculated α-decay half-lives from the formulae are small for heavy nuclei, but it becomes large for the super heavy nuclei.

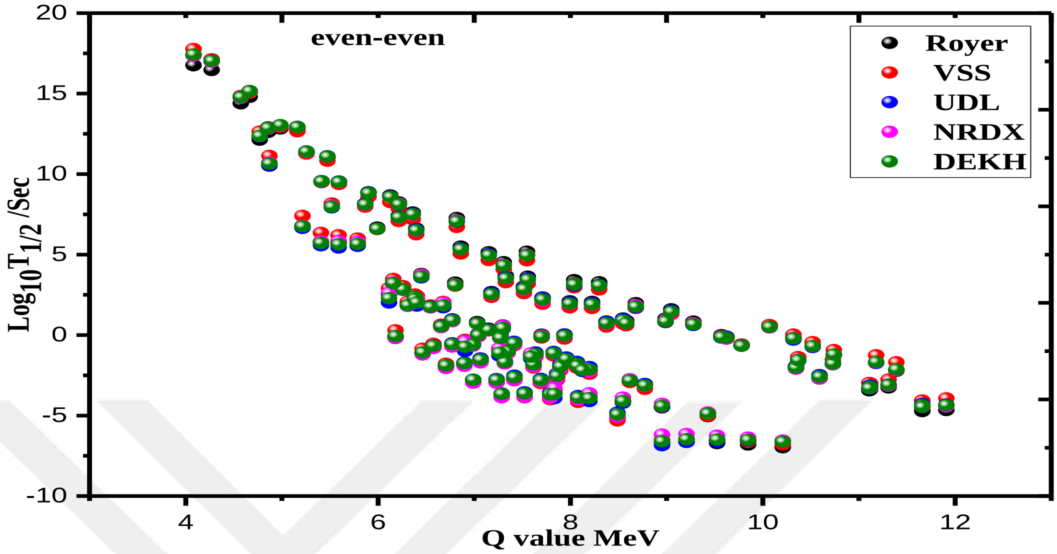

Figure 5.29 Comparison of theoretical α-decay half-lives with Q_α of even-even nuclei for all formula.

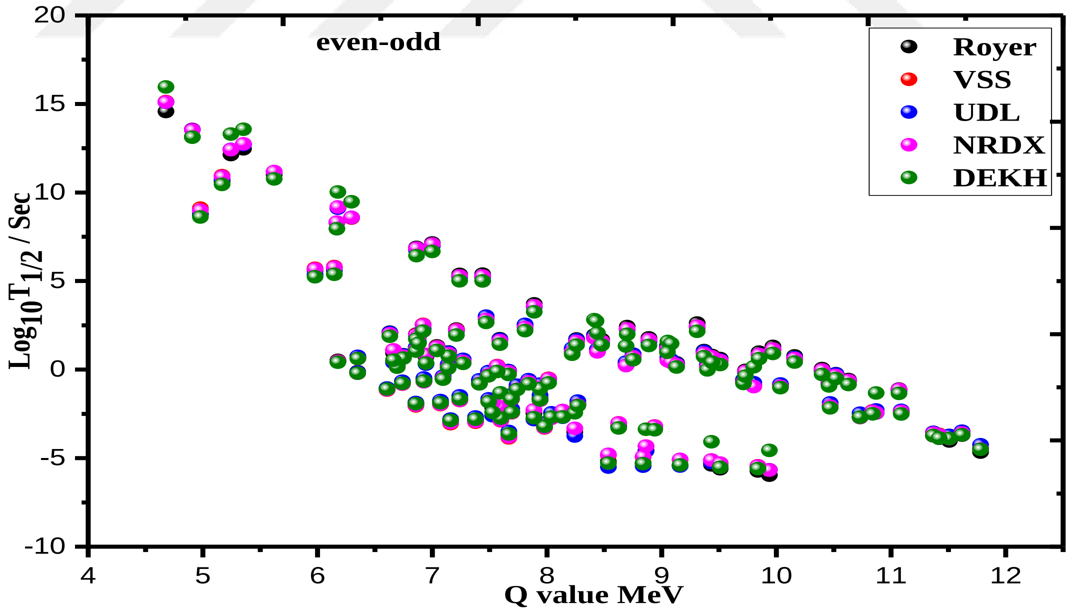

Figure 5.30 Comparison of theoretical α-decay half-lives with Q_α of even-odd nuclei for all formula.

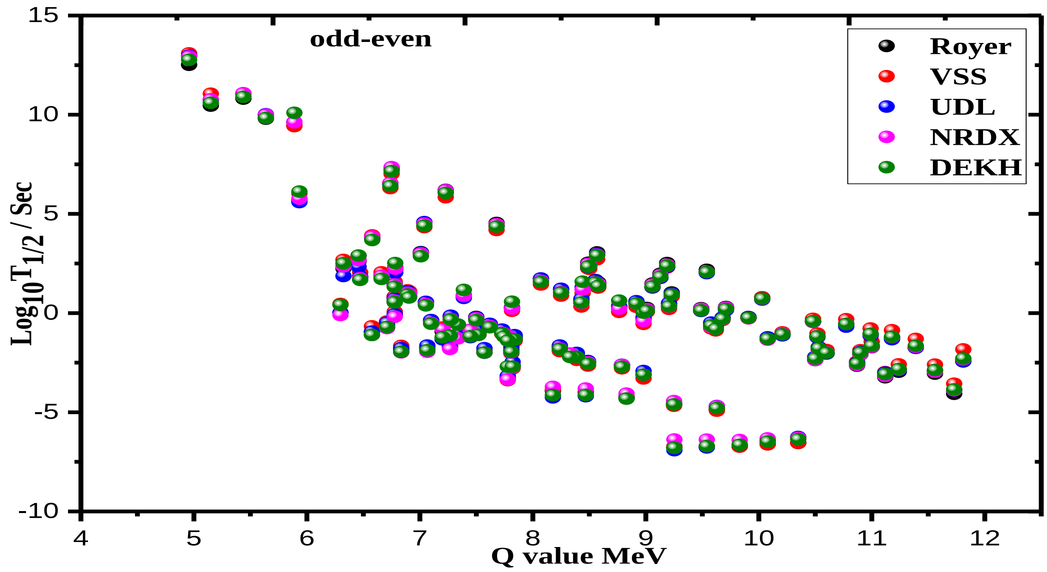

Figure 5.31 Comparison of theoretical α-decay half-lives with Q_α of odd-even nuclei for all formula.

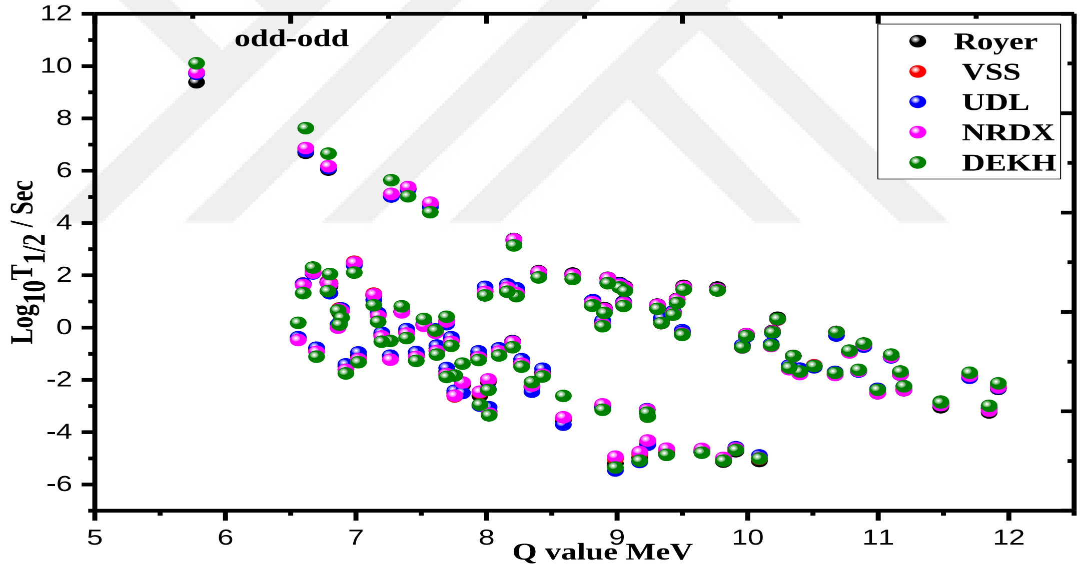

Figure 5.32 Comparison of theoretical α-decay half-lives with Q_α of odd-odd nuclei for all formula.

Through Figures 5.29 - 5.32, it is clear that the Q_α values have affects the α-decay half-lives; the increase of Q_α leads to the decrease of α-decay half-lives in a manner reflecting the major role of Q_α in the isotopes stability as well as in the barrier penetration probability of α-decay.

CHAPTER 6

CONCLUSION

We studied the ability of six liquid drop models to describe binding energies in deformation group, decay mode group, spin group and all nuclei group for nuclei have N, Z $\geq$ 8.

Also we analyzed the effects of shell correction and shell effect to all liquid drop models formula. The shell correction and shell effect decreased the height of sharp peaks in binding energy which they occur around magic numbers.

Classification of nuclei was wider (four groups) as compared to published studies on liquid drop mass formulas. It is the first time that nuclei is classified according to beta (β^+ and β^-) decays and nuclear spin to study on binding energies by using semi empirical mass formulas. Classification of nuclei according to nuclear deformation was studied by so many physicists but the number of isotopes that they study is not more than two thousand. However in this work it was studied for more than three thousand and two hundred different isotopes that this is almost all of the isotopes which have measured binding energies. Number of isotopes studied is important for some reasons such as it is statistically desired to study with enough numbers of quantities to get more accurate results.

All calculations on binding energies were done for five different well-known mass formulas in addition to LDM6 which was formed and used for the first time in this thesis. RMS calculations reveals the fact that LDM4 and LDM6 which have very close results best describe the measured binding energies even we add or not the shell correction and shell effects to LDM formulas.

We have shown in this study that the nuclei having prolate deformation gives better description of the measured binding energies of nuclei. Moreover, nuclei having β^- decay describe better the data as compared to β^+. Also calculations on spin have shown that nuclei having spin 9/2 gives best description of measured binding energies among all spins.

As a second application on nuclear structure we did a systematic study on alpha decay half-lives for 459 different isotopes using five different semi-empirical formulas such as Royer, VSS, UDL, NRDX and DEKH. Calculations with all these formulas are in good agreement with measured α-decay half-lives.

Isotopes for alpha decay classified into four groups: even-even, even-odd, odd-even and odd-odd. RMS calculations on these groups have shown that while for the even-even nuclei VSS formula best describes the measured half-lives for other cases DEKH formula gives best description of the measured data. So we can rely on these formulas to determine α-decay half-lives for new isotopes.

As we discussed in the previous chapter Q-value has a major role that affect alpha decay half-lives. So Q-values required in all steps of calculating alpha decay half-lives computed carefully.

We studied in this thesis work on some different applications of nuclear structure studies successfully. Our aim hereafter is to concentrate on some special topics of the thesis comprehensively.

REFERENCES

[1] Jagdish, V., & Bhandari. R. (2013). Fundamental of Nuclear Physics. CBS Publishers.

[2] Perston, M. A., & Bhaduri, R. K. (1993). Structure of the nucleus. Westview Press.

[3] Kakani. S. L., & Kakani. Sh. (2011). Nuclear and Particle Physics. Viva Book Press.

[4] Ghoshal, S. N. (2011). Nuclear physics. S. Chand..

[5] Mittal, V. K., & Verma, R. C. (2013). Introduction to Nuclear and Particle Physics. PHI learning press.

[6] Mayer, M. G.& Jensn, J. H. (2013). Elementary Theory Of Nuclear Shell Structure, Literary Licensing .

[7] Basdevant, J. L. & Rich R. (2005). Fundamentals of Nuclear Physics. Springer

[8] Bohr, A. & Mottelson, B.. (1999). Nuclear Structure.Volume I & II. World Scientific Pub.

[9] Royer, G., & Gautier, C. (2006). Coefficients and terms of the liquid drop model and mass formula. *Physical Review C*, 73(6), 067302.

[10] Liu, M., Wang, N., Li, Z. X., & Zhang, F. S. (2010). Nuclear symmetry energy at subnormal densities from measured nuclear masses. *Physical Review C*, 82(6), 064306.

[11] D.N. Basu and P. Roy Chowdhury, *arXiv*:nucl-th/0408013.

[12] Chowdhury, P. R., Samanta, C., & Basu, D. N. (2005). Modified bethe–weizsäcker mass formula with isotonic shift and new driplines. *Modern Physics Letters A*, 20(21), 1605-1618.

[13] Tian, J., Cui, H., Zheng, K., & Wang, N. (2014). Effect of Coulomb energy on the symmetry energy coefficients of finite nuclei. *Physical Review C*, 90(2), 024313.

[14] Wang, N., Liu, M., & Wu, X. (2010). Modification of nuclear mass formula by considering isospin effects. *Physical Review C*, 81(4), 044322.

[15] Silisteanu, I. (2011). Simple systematics for α-decay half-lives of superheavy nuclei. *Journal of Advanced Research in Physics*, 2(1).

[16] Zdeb, A., Warda, M., & Pomorski, K. (2013). Half-lives for α and cluster radioactivity within a Gamow-like model. *Physical Review C*, 87(2), 024308.

[17] Poenaru, D. N., Plonski, I. H., & Greiner, W. (2006). α-decay half-lives of superheavy nuclei. *Physical Review C*, 74(1), 014312.

[18] Chowdhury, P. R., Basu, D. N., & Samanta, C. (2007). α-decay chains from element 113. *Physical Review C*, 75(4), 047306.

[19] Zhang, H. F., Royer, G., Wang, Y. J., Dong, J. M., Zuo, W., & Li, J. Q. (2009). Analytic expressions for α particle preformation in heavy nuclei. *Physical Review C*, 80(5), 057301.

[20] Dasgupta-Schubert, N., Reyes, M. A., & Tamez, V. A. (2009). The analysis of predictability of alpha-decay half-life formulae and the alpha partial half-lives of some exotic nuclei. *The European Physical Journal A*, 42(1), 121-133.

[21] Budaca, A. I., Budaca, R., & Silisteanu, I. (2016). Extended systematics of alpha decay half lives for exotic superheavy nuclei. *Nuclear Physics A*, 951, 60-74.

[22] Zhang, H. F., Royer, G., Wang, Y. J., Dong, J. M., Zuo, W., & Li, J. Q. (2009). Analytic expressions for α particle preformation in heavy nuclei. *Physical Review C*, 80(5), 057301.

[23] Hans, H. S. (2008). Nuclear Physics: experimental and theoretical. New Age International.

[24] Meyerhof, W. E. (1967). Elements of nuclear physics. McGraw-Hill Book.

[25] Lilley, J. (2013). Nuclear physics: principles and applications. John Wiley & Sons.

[26] Cohen, B. L. (2007). Concept of Nuclear Physics. McGraw-Hill Book.

[27] Beth, H. A. & Morrison, P. (2006). Elementary Nuclear Theory. Courier Corporation.

[28] Roy, R. R., & Nigam, B. P. (2015). Nuclear physics: Theory and experiment. New Age International.

[29] Jana, Y. (2015). An Introduction to Nuclear Physics. Narosa Publishing House.

[30] Devanathan, V. (2013). Nuclear Physics. Narosa Publishing House.

[31] Kaplan, I. (1998). Nuclear physics. Narosa Publishing House.

[32] Serge, E. (1977). Nuclei and particles. An introduction to nuclear and subnuclear physics.

[33] Gamow, G. (1928). Zur quantentheorie des atomkernes. Zeitschrift für Physik,51(3-4), 204-212.

[34] Gamow, G. (1930). Fine Structure of alpha-Rays. Nature, 126, 397.

[35] Geiger, H., & Nuttall, J. M. (1911). LVII. The ranges of the α particles from various radioactive substances and a relation between range and period of transformation. The London, Edinburgh, and Dublin Philosophical Magazine and

[36] Bhagwat, A., Viñas, X., Centelles, M., Wyss, R., & Schuck, P. (2011). Microscopic–Macroscopic Mass Calculations with Wigner–Kirkwood expansion. *In Journal of Physics: Conference Series* (Vol. 321, No. 1, p. 012053). IOP Publishing.

[37] Bhagwat, A., Vinas, X., Centelles, M., Schuck, P., & Wyss, R. (2010). Microscopic-macroscopic approach for binding energies with the Wigner-Kirkwood method. *Physical Review C*, 81(4), 044321.

[38] Bhagwat, A., Viñas, X., Centelles, M., Schuck, P., & Wyss, R. (2012). Microscopic-macroscopic approach for binding energies with the Wigner-Kirkwood method. II. Deformed nuclei. *Physical Review C*, 86(4), 044316.

[39] Bhagwat, A. (2014). Simple nuclear mass formula. *Physical Review C*, 90(6), 064306.

[40] Pomorski, K., & Dudek, J. (2003). Nuclear liquid-drop model and surface-curvature effects. *Physical Review C*, 67(4), 044316.

[41] Samanta, C., & Adhikari, S. (2002). Extension of the Bethe-Weizsäcker mass formula to light nuclei and some new shell closures. *Physical Review C*, 65(3), 037301.

[42] Basu, D. N. (2003). On the extension of the Bethe-Weizsacker mass formula to light nuclei. *arXiv* preprint nucl-th/0309045.

[43] Samanta, C., Chowdhury, P. R., & Basu, D. N. (2005, November). Modified Bethe-Weizsaecker Mass Formula with Isotonic Shift, New Driplines and Hypernuclei. *In AIP Conference Proceedings* (Vol. 802, No. 1).

[44] Wang, N., Liu, M., & Wu, X. (2010). Modification of nuclear mass formula by considering isospin effects. *Physical Review C*, 81(4), 044322.

[45] Moller, P., Nix, J. R., Myers, W. D., & Swiatecki, W. J. (1995). Nuclear ground-state masses and deformations. *Atomic Data and Nuclear Data Tables*, 59(2), 185-381.

[46] Wang, N., Liang, Z., Liu, M., & Wu, X. (2010). Mirror nuclei constraint in nuclear mass formula. *Physical Review C*, 82(4), 044304.

[47] Royer, G., & Gautier, C. (2006). Coefficients and terms of the liquid drop model and mass formula. *Physical Review C*, 73(6), 067302.

[48] Royer, G., Guilbaud, M., & Onillon, A. (2010). Macro-microscopic mass formulae and nuclear mass predictions. *Nuclear Physics A*, 847(1), 24-41.

[49] Myers, W. D., & Swiatecki, W. J. (1996). Nuclear properties according to the Thomas-Fermi model. *Nuclear Physics A*, 601(2), 141-167.

[50] Myers, W. D., & Swiatecki, W. J. (1994). LBL. Lawrence Berkeley Laboratory Report No 36803.

[51] Barbero, C., Hirsch, J. G., & Mariano, A. E. (2012). Deformation and shell effects in nuclear mass formulas. *Nuclear Physics A*, 874, 81-97.

[52] Zhang, H., Dong, J., Ma, N., Royer, G., Li, J., & Zhang, H. (2014). An improved nuclear mass formula with a unified prescription for the shell and pairing corrections. *Nuclear Physics A*, 929, 38-53.

[53] Royer, G. (2008). On the coefficients of the liquid drop model mass formulae and nuclear radii. *Nuclear Physics A*, 807(3), 105-118.

[54] Myers, W. D., & Swiatecki, W. J. (1974). The nuclear droplet model for arbitrary shapes. *Annals of Phys*ics, 84(1-2), 186-210.

[55] Tajima, N. (2001). Hartree-Fock+ BCS approach to unstable nuclei with the Skyrme force. *Progress of Theoretical Physics Supplement*, 142, 265-296.

[56] Royer, G., & Subercaze, A. (2013). Coefficients of different macro–microscopic mass formulae from the AME2012 atomic mass evaluation. *Nuclear Physics A*, 917, 1-14.

[57] Viñas, X., Bhagwat, A., Centelles, M., Schuck, P., & Wyss, R. (2015). Applications to nuclear properties of the microscopic–macroscopic model based on the semiclassical Wigner–Kirkwood method. *Physica Scripta*,90(11), 114001.

[58] Mendoza-Temis, J., Frank, A., Hirsch, J. G., Vieyra, J. L., Morales, I., Barea, J., & Velázquez, V. (2008). Nuclear masses and the number of valence nucleons. *Nuclear Physics A*, 799(1), 84-93.

[59] Van Isacker, P. (2006, March). Symmetry and wigner energy in nuclear matter. In Capture Gamma-Ray Spectroscopy and Related Topics: *12th International Symposium* (Vol. 819, No. 1, pp. 57-64). AIP Publishing.

[60] Arima, A., Ohtsuka, T., Iachello, F., & Talmi, I. (1977). Collective nuclear states as symmetric couplings of proton and neutron excitations. *Physics Letters B*, 66(3), 205-208.

[61] Dieperink, A. E. L., & Van Isacker, P. (2009). Shell corrections to a liquid-drop description of nuclear masses and radii. *The European Physical Journal A*,42(2), 269-279.

[62] Dieperink, A. E. L., & Van Isacker, P. (2007). The symmetry energy in nuclei and in nuclear matter. The European Physical Journal A, 32(1), 11-18.

[63] Iachello, F., & Arima, A. (1987). The interacting boson model. Cambridge University Press.

[64] Viola, V. E., & Seaborg, G. T. (1966). Nuclear systematics of the heavy elements—II Lifetimes for alpha, beta and spontaneous fission decay. *Journal of Inorganic and Nuclear Chemistry*, 28(3), 741-761.

[65] Sobiczewski, A., Patyk, Z., & Ćwiok, S. (1989). Deformed superheavy nuclei.*Physics Letters B*, 224(1), 1-4.

[66] Dong, T., & Ren, Z. (2005). New calculations of α-decay half-lives by the Viola-Seaborg formula. *The European Physical Journal A-Hadrons and Nuclei*, 26(1), 69-72.

[67] Royer, G. (2000). Alpha emission and spontaneous fission through quasi-molecular shapes. *Journal of Physics G: Nuclear and Particle Physics*, 26(8), 1149.

[68] Royer, G., & Zhang, H. F. (2008). Recent α decay half-lives and analytic expression predictions including superheavy nuclei. *Physical Review C*, 77(3), 037602.

[69] Denisov, V. Y., & Khudenko, A. A. (2009). α-decay half-lives: Empirical relations. *Physical Review C*, 79(5), 054614.

[70] Denisov, V. Y., & Khudenko, A. A. (2009). α-Decay half-lives, α-capture, and α-nucleus potential. *Atomic Data and Nuclear Data Tables*, 95(6), 815-835.

[71] Denisov, V. Y., Davidovskaya, O. I., & Sedykh, I. Y. (2015). Improved parametrization of the unified model for α-decay and α capture. *Physical Review C*, 92(1), 014602.

[72] Qi, C., Xu, F. R., Liotta, R. J., & Wyss, R. (2009). Universal decay law in charged-particle emission and exotic cluster radioactivity. *Physical review letters*, 103(7), 072501.

[73] Qi, C., Xu, F. R., Liotta, R. J., Wyss, R., Zhang, M. Y., Asawatangtrakuldee, C., & Hu, D. (2009). Microscopic mechanism of charged-particle radioactivity and generalization of the Geiger-Nuttall law. *Physical Review C*, 80(4), 044326.

[74] Santhosh, K. P., & Priyanka, B. (2014). Heavy particle radioactivity from superheavy nuclei leading to 298 114 daughter nuclei. *Nuclear Physics A*, 929, 20-37.

[75] Ni, D., Ren, Z., Dong, T., & Xu, C. (2008). Unified formula of half-lives for α decay and cluster radioactivity. *Physical Review C*, 78(4), 044310.

[76] Wang, Y. Z., Wang, S. J., Hou, Z. Y., & Gu, J. Z. (2015). Systematic study of α-decay energies and half-lives of superheavy nuclei. *Physical Review C*,92(6), 064301.

[77] Santhosh, K. P., Sukumaran, I., & Priyanka, B. (2015). Theoretical studies on the alpha decay of 178–220 Pb isotopes. *Nuclear Physics A*, 935, 28-42.

[78] Ismail, M., Seif, W. M., Ellithi, A. Y., & Abdurrahman, A. (2015). Single universal curve for α decay derived from semi-microscopic calculations. *Physical Review C*, 92(1), 014311.

[79] Jian-Min, D., Hong-Fei, Z., Wei, Z., & Jun-Qing, L. (2008). Half-lives of superheavy nuclei in Z= 113 alpha decay chain. *Chinese Physics Letters*,25(12), 4230.

[80] Audi, G., A. H. Wapstra, and C. Thibault. (2003) .The AME2003 atomic mass evaluation :(II). Tables, graphs and references. *Nuclear Physics A*, 729.1, 337-676.

[81] Audi, G., Kondev, F. G., Wang, M., Pfeiffer, B., Sun, X., Blachot, J., & MacCormick, M. (2012). The Nubase2012 evaluation of nuclear properties. *Chinese Physics C*, 36(12), 1157.

APPENDICES

Table A.1 Coefficients of liquid drop model of (LDM1) formula (MeV)

LDM 1						
Sets	a_v	k_v	a_s	k_s	r_o	C_4
All Nuclei	15.479	-1.821	-17.617	-2.167	-1.231	0.962
DGS 1	15.075	-1.700	-16.189	-1.813	-1.297	0.500
DGS 2	15.535	-1.807	-17.723	-2.165	-1.225	0.923
DGS 3	15.433	-1.787	-17.580	-2.072	-1.226	1.232
DGS 4	15.698	-1.837	-18.678	-2.227	-1.189	1.553
DGS 5	15.464	-1.817	-17.670	-2.151	-1.231	1.052
DGS 6	15.303	-1.758	-17.148	-1.953	-1.269	0.730
DMGSB 1	15.535	-1.769	-18.153	-2.010	-1.214	1.431
DMGSB 2	15.735	-1.904	-17.732	-2.339	-1.232	-0.198

Table A.1 Coefficients of liquid drop model of (LDM1) formula (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	r_o	C_4
SGS 1	15.264	-1.807	-16.707	-2.167	-1.259	0.739
SGS 2	15.423	-1.838	-17.433	-2.239	-1.249	0.763
SGS 3	15.582	-1.833	-17.990	-2.203	-1.219	0.987
SGS 4	15.464	-1.815	-17.568	-2.120	-1.235	0.900
SGS 5	15.826	-1.829	-19.051	-2.163	-1.177	1.507
SGS 6	15.698	-1.858	-19.034	-2.432	-1.187	1.928

Table A.2 Coefficients of liquid drop model of (LDM2) formula (MeV)

LDM 2					
Sets	a_v	a_s	a_c	a_{sym}	a_p
All Nuclei	15.708	17.949	0.711	22.824	15.307
DGS 1	15.427	17.333	0.684	21.581	8.787
DGS 2	15.698	18.007	0.705	22.605	17.212
DGS 3	15.486	17.141	0.697	22.577	5.745
DGS 4	15.717	17.897	0.715	22.959	13.939
DGS 5	15.549	17.395	0.703	22.777	14.698
DGS 6	15.515	17.687	0.686	22.176	18.636
DMGSB 1	15.600	17.695	0.695	23.936	15.605
DMGSB 2	16.198	19.507	0.761	22.455	12.237

Table A.2 Coefficients of liquid drop model of (LDM2) formula (MeV) (cont.)

Sets	a_v	a_s	a_c	a_{sym}	a_p
SGS 1	15.524	17.302	0.699	22.506	18.760
SGS 2	15.613	17.750	0.700	22.889	13.273
SGS 3	15.668	17.874	0.707	22.436	23.985
SGS 4	15.828	18.347	0.718	23.123	10.483
SGS 5	15.708	17.965	0.709	22.841	29.681
SGS 6	15.243	16.298	0.682	22.243	35.011

Table A.3 Coefficients of liquid drop model of (LDM3) formula (MeV)

LDM 3						
Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}
All Nuclei	15.567	-18.170	-0.707	-29.659	1.516	6.651
DGS 1	15.284	-17.375	-0.686	-27.322	1.361	5.613
DGS 2	15.572	-18.207	-0.706	-29.066	1.508	7.399
DGS 3	15.516	-17.878	-0.705	-29.794	1.521	3.704
DGS 4	15.595	-18.192	-0.712	-29.733	1.498	7.225
DGS 5	15.452	-17.780	-0.701	-28.888	1.437	5.648
DGS 6	15.437	-17.966	-0.692	-28.489	1.436	5.895
DMGSB 1	15.543	-18.113	-0.704	-30.842	1.684	5.862
DMGSB 2	15.617	-18.326	-0.710	-29.774	1.507	7.422

Table A.3 Coefficients of liquid drop model of (LDM3) formula (MeV) (cont.)

Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}
SGS 1	15.495	-17.935	-0.702	-29.434	1.512	6.087
SGS 2	15.415	-17.747	-0.694	-29.589	1.572	4.856
SGS 3	15.602	-18.304	-0.709	-30.075	1.564	7.048
SGS 4	15.504	-17.963	-0.703	-29.302	1.481	5.316
SGS 5	15.681	-18.478	-0.717	-29.580	1.476	5.865
SGS 6	15.503	-17.932	-0.704	-29.806	1.589	10.954

Table A.4 Coefficients of liquid drop model of (LDM4) formula (MeV)

LDM 4										
Sets	a_v	k_v	a_s	k_s	r_o	a_p	a_k	a_o	f_p	W
All Nuclei	15.853	1.876	19.334	2.253	1.195	-10.734	1.225	-8.696	-1.238	33.862
DGS 1	16.290	1.790	25.029	1.700	1.210	-8.294	-20.984	16.252	-1.003	40.244
DGS 2	15.903	1.953	15.246	3.207	1.141	-9.844	28.437	-52.712	-1.571	37.207
DGS 3	14.026	1.930	20.027	1.905	1.260	-7.655	59.249	-75.607	-0.975	53.577
DGS 4	15.769	1.848	19.912	2.151	1.200	-11.633	-4.876	3.035	-1.397	39.021
DGS 5	14.912	1.871	12.743	3.040	1.269	-10.709	15.885	-21.167	-0.671	45.033
DGS 6	16.546	1.690	30.053	1.174	1.227	-9.379	-45.509	54.305	-1.064	27.276
DMGSB 1	15.466	1.817	16.479	2.411	1.206	-10.498	9.243	-15.473	-1.452	40.989
DMGSB 2	16.844	1.745	27.493	1.457	1.116	-9.785	-18.752	9.757	-2.371	15.588

Table A.4 Coefficients of liquid drop model of (LDM4) formula (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	r_o	a_p	a_k	a_o	f_p	W
SGS 1	15.722	1.909	17.721	2.569	1.198	-10.395	7.821	-17.838	-1.218	37.641
SGS 2	15.628	1.849	18.569	1.345	1.116	-9.817	-7.771	-18.235	-1.698	35.814
SGS 3	17.024	1.874	29.373	1.753	1.134	-10.311	-28.866	23.928	-1.722	34.521
SGS 4	15.368	1.850	15.108	2.448	1.220	-8.154	14.144	-22.351	-1.052	26.730
SGS 5	16.521	1.925	17.625	2.446	1.232	-9.987	15.236	-19.652	-1.452	32.532
SGS 6	15.832	1.825	18.526	2.144	1.216	-10.235	-22.632	-20.453	-1.525	35.752

Table A.5 Coefficients of liquid drop model of (LDM5) formula (MeV)

LDM 5						
Sets	a_v	k_v	a_s	k_s	a_c	C_1
All Nuclei	15.385	-1.856	-17.298	-2.248	-0.693	-23.808
DGS 1	15.084	-1.703	-16.501	-1.732	-0.670	-18.937
DGS 2	15.327	-1.824	-17.100	-2.284	-0.688	-26.660
DGS 3	15.405	-1.930	-17.153	-2.805	-0.698	-42.337
DGS 4	15.469	-1.875	-17.557	-2.249	-0.699	-21.474
DGS 5	15.333	-1.878	-17.065	-2.463	-0.691	-33.030
DGS 6	15.285	-1.793	-17.222	-2.030	-0.679	-22.913
DMGSB 1	15.349	-2.142	-17.222	-3.868	-0.687	-38.013
DMGSB 2	15.660	-1.811	-18.381	-1.840	-0.709	-8.195

Table A.5 Coefficients of liquid drop model of (LDM5) formula (MeV) (cont.)

Set	a_v	k_v	a_s	k_s	a_c	C_1
SGS 1	15.191	-1.820	-16.602	-2.146	-0.681	-21.378
SGS 2	15.351	-1.820	-17.363	-2.004	-0.686	-16.037
SGS 3	15.443	-1.872	-17.554	-2.306	-0.695	-23.296
SGS 4	15.417	-1.838	-17.463	-2.094	-0.694	-20.542
SGS 5	15.582	-1.897	-17.851	-2.498	-0.708	-34.183
SGS 6	15.709	-2.184	-18.143	-4.149	-0.715	-65.447

Table A.6 Coefficients of liquid drop model of (LDM6) formula (MeV)

LDM 6									
Sets	**av**	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p
All Nuclei	15.486	0.436	18.128	2.200	0.706	1.033	-35.363	31.660	10.922
DGS 1	15.042	-1.326	16.393	1.770	0.663	0.441	-5.819	-7.153	8.502
DGS 2	15.685	1.766	18.848	2.210	0.722	1.185	-56.480	61.584	12.014
DGS 3	15.573	1.176	18.489	2.672	0.719	1.318	-48.079	62.697	7.059
DGS 4	15.549	1.188	18.378	2.603	0.718	1.278	-48.149	60.098	10.146
DGS 5	15.258	-1.274	17.281	2.316	0.689	0.919	-8.812	1.490	10.644
DGS 6	15.134	-1.147	17.056	1.725	0.669	0.750	-8.900	-8.954	9.513
DMGSB 1	15.432	0.260	18.109	3.361	0.705	1.252	-35.155	47.133	9.897
DMGSB 2	15.799	1.906	19.335	1.843	0.730	1.163	-58.582	51.454	9.618

Table A.6 Coefficients of liquid drop model of (LDM6) formula (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p
SGS 1	15.431	-0.480	17.937	2.342	0.700	1.004	-21.472	15.578	14.217
SGS 2	15.517	1.583	18.203	2.126	0.701	0.850	-53.124	51.812	6.397
SGS 3	15.580	0.791	18.536	2.206	0.714	1.138	-41.258	37.209	10.775
SGS 4	15.560	1.082	18.401	2.057	0.711	1.039	-45.281	43.160	6.693
SGS 5	15.738	1.069	19.421	2.117	0.735	1.628	-45.790	43.006	6.292
SGS 6	15.626	3.591	18.516	3.703	0.714	1.063	-88.860	125.077	7.313

Table A.7 RMS for All Groups & All Formulas (keV)

Group Nuclei		**LDM 1**	**LDM 2**	**LDM 3**	**LDM 4**	**LDM 5**	**LDM 6**
All Nuclei Group		28.619	36.546	24.599	23.733	29.654	24.015
Deformation Group	**DGS 1**	21.332	30.899	17.116	15.995	21.456	16.676
	DGS 2	39.005	42.805	31.946	28.373	40.137	31.411
	DGS 3	22.780	34.447	21.442	18.961	22.486	20.227
	DGS 4	16.506	29.460	14.403	12.977	20.732	11.530
	DGS 5	11.591	23.048	8.939	7.956	11.535	7.958
	DGS 6	18.760	24.293	12.384	8.946	19.521	9.942
Decay Mod Group	**DMGSB 1**	25.746	25.480	22.576	21.737	26.797	21.452
	DMGSB 2	30.861	32.207	27.511	26.809	30.759	26.815

Table A.7 RMS for All Groups & All Formulas (keV) (cont.)

Group Nuclei		LDM 1	LDM 2	LDM 3	LDM 4	LDM 5	LDM 6
Spin Group	SGS 1	28.331	37.410	26.138	24.859	28.855	25.750
	SGS 2	24.651	28.917	24.152	22.116	24.641	22.582
	SGS 3	27.946	39.321	26.721	25.486	29.461	25.389
	SGS 4	22.654	34.539	21.736	20.694	23.187	20.614
	SGS 5	22.180	38.542	24.198	21.513	26.233	22.009
	SGS 6	20.418	26.551	20.708	18.949	20.368	19.004

Table A.8 Coefficients of liquid drop model of (LDM1+Shell Correction formula) (MeV)

LDM 1 +Shell Correction								
Sets	a_v	k_v	a_s	k_s	r_o	C_4	*af*	*aff*
All Nuclei	15.525	-1.847	-17.744	-2.230	-1.224	0.934	-1.209	0.041
DGS 1	15.364	-1.769	-17.198	-1.993	-1.249	0.783	-0.588	-0.002
DGS 2	15.496	-1.839	-17.576	-2.250	-1.227	0.857	-1.726	0.083
DGS 3	15.483	-1.841	-17.579	-2.168	-1.229	0.924	-1.149	0.034
DGS 4	15.726	-1.860	-18.705	-2.267	-1.191	1.376	-1.023	0.046
DGS 5	15.581	-1.855	-18.100	-2.258	-1.217	1.099	-0.831	0.028
DGS 6	15.352	-1.776	-17.265	-2.003	-1.258	0.769	-0.526	0.022
DMGSB 1	15.533	-1.788	-18.081	-2.033	-1.217	1.311	-1.064	0.030
DMGSB 2	15.775	-1.931	-17.954	-2.410	-1.222	-0.025	-1.236	0.042

Table A.8 Coefficients of liquid drop model of (LDM1+Shell Correction formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	r_o	C_4	a_f	a_{ff}
SGS 1	15.337	-1.835	-16.936	-2.226	-1.249	0.735	-1.099	0.034
SGS 2	15.643	-1.886	-18.221	-2.354	-1.213	0.961	-1.398	0.052
SGS 3	15.510	-1.832	-17.621	-2.176	-1.231	0.761	-1.034	0.030
SGS 4	15.558	-1.855	-17.901	-2.253	-1.222	0.939	-1.302	0.051
SGS 5	15.691	-1.828	-18.523	-2.159	-1.195	1.324	-0.997	0.033
SGS 6	15.938	-1.869	-19.345	-2.176	-1.166	1.381	-1.895	0.076

Table A.9 Coefficients of liquid drop model of (LDM2+Shell Correction formula) (MeV)

LDM 2 + Shell Correction							
Sets	a_v	a_s	a_c	a_{sym}	a_p	a_f	a_{ff}
All Nuclei	15.726	18.009	0.713	22.910	13.824	-0.979	0.025
DGS 1	15.527	17.651	0.692	21.808	7.766	-0.912	0.057
DGS 2	15.659	17.965	0.702	22.501	13.067	-2.195	0.186
DGS 3	15.611	17.604	0.706	23.050	7.976	-1.776	0.074
DGS 4	15.821	18.318	0.721	23.276	15.656	-2.131	0.093
DGS 5	15.565	17.522	0.701	22.821	14.788	-1.189	0.026
DGS 6	15.491	17.677	0.682	22.056	18.541	0.717	-0.030
DMGSB 1	15.624	17.790	0.697	24.126	14.650	-1.102	0.034
DMGSB 2	16.228	19.582	0.765	22.610	10.861	-1.163	0.042

Table A.9 Coefficients of liquid drop model of (LDM2+Shell Correction formula) (MeV) (cont.)

Sets	a_v	a_s	a_c	a_{sym}	a_p	a_f	a_{ff}
SGS 1	15.554	17.390	0.702	22.633	19.125	-0.930	0.022
SGS 2	15.722	18.067	0.709	23.164	11.770	-0.905	0.028
SGS 3	15.677	17.906	0.709	22.544	20.172	-1.067	0.028
SGS 4	15.859	18.448	0.721	23.130	8.928	-0.853	0.029
SGS 5	15.572	17.620	0.697	22.559	26.040	-0.872	0.007
SGS 6	15.678	17.724	0.713	23.661	21.782	-2.135	0.084

Table A.10 Coefficients of liquid drop model of (LDM3+Shell Correction formula) (MeV)

LDM 3 + Shell Correction								
Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}	a_f	a_{ff}
All Nuclei	15.584	-18.213	-0.709	-30.028	1.539	5.905	-1.106	0.036
DGS 1	15.444	-17.802	-0.699	-28.401	1.420	5.360	-0.286	0.022
DGS 2	15.504	-17.944	-0.705	-29.621	1.551	5.116	-1.430	0.059
DGS 3	15.612	-18.243	-0.712	-30.256	1.521	4.337	-1.198	0.039
DGS 4	15.668	-18.492	-0.717	-29.931	1.490	8.038	-1.289	0.057
DGS 5	15.550	-18.129	-0.707	-29.645	1.494	6.059	-0.843	0.027
DGS 6	15.498	-18.083	-0.699	-29.110	1.473	5.912	-0.679	0.029
DMGSB 1	15.551	-18.150	-0.705	-30.790	1.652	5.533	-1.067	0.032
DMGSB 2	15.648	-18.402	-0.713	-30.450	1.548	6.525	-1.139	0.038

Table A.10 Coefficients of liquid drop model of (LDM3+Shell Correction formula) (MeV) (cont.)

Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}	a_f	a_{ff}
SGS 1	15.517	-17.984	-0.705	-29.817	1.531	5.274	-1.047	0.032
SGS 2	15.564	-18.191	-0.707	-30.715	1.639	4.008	-1.274	0.045
SGS 3	15.553	-18.128	-0.707	-29.799	1.534	5.753	-0.995	0.028
SGS 4	15.540	-18.073	-0.707	-30.017	1.554	3.907	-1.184	0.043
SGS 5	15.556	-18.110	-0.708	-29.103	1.448	5.969	-0.986	0.027
SGS 6	15.795	-18.797	-0.728	-30.310	1.460	5.372	-1.884	0.075

Table A.11 Coefficients of liquid drop model of (LDM4+Shell Correction formula) (MeV)

LDM 4 + Shell Correction												
Sets	a_v	k_v	a_s	k_s	r_o	a_k	a_o	f_p	w	a_p	a_f	a_{ff}
All Nuclei	16.095	1.892	21.618	2.118	1.185	-6.925	1.610	-1.245	34.551	-9.593	-1.119	0.038
DGS 1	15.543	1.834	17.212	2.239	1.223	5.539	-12.020	-0.939	33.368	-7.936	-0.394	-0.018
DGS 2	15.676	1.937	17.681	2.710	1.201	5.743	-13.320	-1.093	40.715	-8.550	-1.268	0.041
DGS 3	15.052	1.873	12.244	3.031	1.227	22.660	-30.121	-1.084	34.346	-8.544	-1.301	0.054
DGS 4	15.401	1.826	15.350	2.221	1.212	12.300	-17.341	-1.048	22.012	-10.747	-1.459	0.067
DGS 5	15.579	1.901	17.443	2.543	1.219	5.342	-12.743	-1.005	43.468	-10.533	-0.781	0.027
DGS 6	16.489	1.721	29.125	1.274	1.223	-41.659	49.231	-1.073	29.684	-9.279	-0.376	0.018
DMGSB 1	15.727	1.823	19.054	2.090	1.202	-0.095	-4.115	-1.372	38.857	-9.896	-1.043	0.030
DMGSB 2	17.675	1.775	34.517	1.354	1.082	-41.479	35.663	-2.353	20.890	-8.600	-1.164	0.040

Table A.11 Coefficients of liquid drop model of (LDM4+Shell Correction formula) (MeV) (cont.)

Sets	a_v	*kv*	a_s	*ks*	*ro*	*ak*	*ao*	*fp*	*w*	a_p	*af*	*aff*
SGS 1	16.011	1.921	20.535	2.321	1.187	-2.513	-4.489	-1.227	38.285	-9.595	-1.059	0.035
SGS 2	16.096	1.845	22.116	1.797	1.192	-10.437	9.080	-1.163	17.809	-7.377	-1.380	0.051
SGS 3	17.095	1.892	29.929	1.770	1.136	-30.772	24.934	-1.542	39.104	-7.862	-1.017	0.031
SGS 4	15.753	1.898	18.676	2.348	1.203	2.035	-7.791	-1.119	32.798	-6.606	-1.209	0.046
SGS 5	15.562	1.894	15.766	2.749	1.192	16.062	-29.090	-1.316	39.356	-7.816	-1.039	0.035
SGS 6	14.872	1.864	10.377	3.227	1.233	24.633	-25.444	-0.639	42.077	-4.854	-1.838	0.073

Table A.12 Coefficients of liquid drop model of (LDM5+Shell Correction formula) (MeV)

LDM 5 + Shell Correction								
Sets	a_v	k_v	a_s	k_s	a_c	C_1	a_f	a_{ff}
All Nuclei	15.433	-1.883	-17.454	-2.315	-0.698	-23.803	-1.209	0.041
DGS 1	15.282	-1.745	-17.089	-1.770	-0.685	-16.915	-0.624	0.009
DGS 2	15.312	-1.890	-17.051	-2.546	-0.691	-29.656	-1.776	0.085
DGS 3	15.447	-1.899	-17.437	-2.356	-0.699	-26.881	-1.109	0.029
DGS 4	15.602	-1.806	-18.166	-1.664	-0.707	-5.209	-1.578	0.059
DGS 5	15.415	-1.899	-17.376	-2.468	-0.696	-30.612	-0.742	0.023
DGS 6	15.327	-1.829	-17.271	-2.189	-0.685	-25.982	-0.566	0.025
DMGSB 1	15.377	-2.061	-17.331	-3.296	-0.690	-31.650	-1.121	0.035
DMGSB 2	15.707	-1.860	-18.509	-1.999	-0.714	-10.234	-1.232	0.043

Table A.12 Coefficients of liquid drop model of (LDM5+Shell Correction formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	a_c	C_1	a_f	a_{ff}
SGS 1	15.261	-1.853	-16.828	-2.234	-0.687	-22.064	-1.095	0.034
SGS 2	15.535	-1.857	-17.943	-2.020	-0.701	-13.175	-1.356	0.050
SGS 3	15.425	-1.870	-17.488	-2.285	-0.696	-23.713	-1.115	0.035
SGS 4	15.481	-1.899	-17.671	-2.368	-0.699	-23.652	-1.265	0.050
SGS 5	15.456	-1.871	-17.499	-2.391	-0.699	-31.578	-0.978	0.026
SGS 6	15.882	-2.009	-18.754	-2.837	-0.732	-42.381	-1.882	0.075

Table A.13 Coefficients of liquid drop model of (LDM6+Shell Correction formula) (MeV)

LDM 6 + Shell Correction											
Sets	a_v	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p	a_f	a_{ff}
All Nuclei	15.503	0.195	18.147	2.261	0.707	0.978	-32.034	26.505	9.728	-1.109	0.037
DGS 1	15.275	-1.220	17.328	1.862	0.687	0.789	-8.358	-7.504	8.042	-0.528	-0.005
DGS 2	15.418	-0.506	17.772	2.548	0.702	0.902	-21.634	16.482	9.032	-1.401	0.050
DGS 3	15.543	0.390	18.430	2.226	0.714	1.198	-35.134	31.486	8.177	-1.292	0.049
DGS 4	15.593	0.550	18.490	2.121	0.716	1.056	-37.427	33.160	10.104	-1.267	0.058
DGS 5	15.447	-0.223	17.991	2.303	0.703	0.985	-25.501	20.204	10.374	-0.749	0.026
DGS 6	15.162	-1.164	17.090	1.851	0.674	0.757	-9.100	-7.140	9.389	-0.440	0.021
DMGSB 1	15.401	-0.322	17.909	2.888	0.701	1.092	-25.140	27.476	9.574	-1.008	0.028
DMGSB 2	15.761	1.433	19.258	1.946	0.729	1.221	-51.536	41.806	8.470	-1.138	0.038

Table A.13 Coefficients of liquid drop model of (LDM6+Shell Correction formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p	a_f	a_{ff}
SGS 1	15.458	-0.623	17.977	2.395	0.703	0.949	-19.700	12.304	12.460	-1.066	0.034
SGS 2	15.662	1.134	18.818	2.109	0.717	1.046	-46.992	38.955	5.518	-1.381	0.052
SGS 3	15.542	0.618	18.199	2.219	0.707	0.856	-38.537	34.882	8.360	-0.932	0.025
SGS 4	15.492	0.057	18.142	2.269	0.706	0.970	-29.926	23.000	5.959	-1.184	0.044
SGS 5	15.624	1.025	18.872	2.108	0.723	1.394	-44.710	42.135	6.812	-1.003	0.033
SGS 6	15.798	1.167	19.128	2.543	0.731	1.089	-49.149	56.801	3.956	-1.802	0.071

Table A.14 RMS for All Groups & All Formulas + Shell Correction (keV)

Group Nuclei		LDM 1	LDM 2	LDM 3	LDM 4	LDM 5	LDM 6
All Nuclei Group		20.279	31.408	15.806	14.635	21.508	14.978
Deformation Group	**DGS 1**	17.670	28.934	13.529	12.164	18.113	12.391
	DGS 2	23.671	34.479	17.821	16.192	24.335	16.806
	DGS 3	13.278	24.098	10.413	9.312	13.694	9.736
	DGS 4	14.119	22.690	8.967	7.083	15.759	7.004
	DGS 5	10.140	22.288	6.872	6.411	10.461	6.552
	DGS 6	18.309	22.425	11.270	8.557	19.110	9.446
Decay Mod Group	**DMGSB 1**	16.910	16.512	11.982	10.635	18.152	10.497
	DMGSB 2	21.103	24.772	17.606	16.501	21.092	16.651

Table A.14 RMS for All Groups & All Formulas + Shell Correction (keV) (cont.)

Group Nuclei		LDM 1	LDM 2	LDM 3	LDM 4	LDM 5	LDM 6
Spin Group	SGS 1	19.352	31.790	17.169	15.597	20.069	16.523
	SGS 2	15.546	25.312	16.110	13.250	15.997	12.637
	SGS 3	20.027	33.133	18.550	17.452	20.729	17.988
	SGS 4	12.480	31.669	12.570	10.953	14.389	11.437
	SGS 5	12.949	29.989	14.041	12.040	16.850	12.191
	SGS 6	8.532	15.943	8.628	8.271	9.337	8.149

Table A.15 Coefficients of liquid drop model of (LDM1+ Shell Effect formula) (MeV)

LDM 1 + Shell Effect								
Sets	a_v	k_v	a_s	k_s	r_o	C_4	***b1***	***b2***
All Nuclei	15.504	-0.187	17.877	2.329	0.710	1.058	-26.306	18.252
DGS 1	15.266	-1.669	17.084	1.985	0.687	0.774	-1.899	-15.453
DGS 2	15.409	-0.870	17.338	2.700	0.704	0.951	-16.427	9.877
DGS 3	15.575	0.132	18.243	2.264	0.719	1.278	-31.370	23.893
DGS 4	15.604	0.109	18.177	2.201	0.720	1.096	-30.801	24.768
DGS 5	15.463	-0.456	17.788	2.405	0.706	1.022	-22.173	16.724
DGS 6	15.198	-1.171	17.142	1.882	0.679	0.835	-9.172	-7.770
DMGSB 1	15.411	-0.735	17.780	2.636	0.708	1.309	-18.020	16.199
DMGSB 2	15.552	0.820	18.800	1.827	0.728	1.895	-40.652	24.547

Table A.15 Coefficients of liquid drop model of (LDM1+ Shell Effect formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	r_o	C_4	***b1***	***b2***
SGS 1	15.464	-0.835	17.696	2.456	0.706	1.017	-16.599	7.067
SGS 2	15.689	-1.896	-17.969	-2.421	-1.201	1.061	-0.792	0.015
SGS 3	15.517	0.113	17.884	2.310	0.708	0.936	-30.870	25.029
SGS 4	15.535	-0.277	17.997	2.408	0.712	1.075	-25.285	16.377
SGS 5	15.616	0.719	18.591	2.088	0.724	1.410	-39.828	34.461
SGS 6	15.897	3.692	19.170	2.437	0.739	1.198	-89.450	98.789

Table A.16 Coefficients of liquid drop model of (LDM2+ Shell Effect formula) (MeV)

LDM 2 + Shell Effect							
Sets	a_v	a_s	a_c	a_{sym}	a_p	***b1***	***b2***
All Nuclei	15.749	17.851	0.715	22.948	14.445	-0.404	0.004
DGS 1	15.489	17.397	0.691	21.763	8.387	-0.212	0.005
DGS 2	15.634	17.393	0.708	22.750	13.392	-0.929	0.037
DGS 3	15.653	17.357	0.709	23.098	7.879	-0.585	0.000
DGS 4	15.876	18.000	0.727	23.453	15.254	-0.839	0.011
DGS 5	15.575	17.276	0.703	22.828	15.214	-0.456	0.004
DGS 6	15.436	17.745	0.677	21.902	18.320	-0.447	0.009
DMGSB 1	15.668	17.614	0.701	24.315	15.618	-0.541	0.008
DMGSB 2	16.272	19.407	0.771	22.696	11.448	-0.546	0.009

Table A.16 Coefficients of liquid drop model of (LDM2+ Shell Effect formula) (MeV) (cont.)

Sets	a_v	a_s	a_c	a_{sym}	a_p	***b1***	***b2***
SGS 1	15.573	17.180	0.704	22.680	-0.541	-0.447	0.004
SGS 2	15.718	17.832	0.709	23.171	12.295	-0.381	0.005
SGS 3	15.714	17.807	0.711	22.590	22.905	-0.328	0.001
SGS 4	15.889	18.404	0.723	23.172	9.414	-0.222	0.001
SGS 5	15.604	17.488	0.698	22.614	28.033	-0.407	0.000
SGS 6	15.750	17.402	0.720	23.895	25.619	-1.045	0.020

Table A.17 Coefficients of liquid drop model of (LDM1+ Shell Effect formula) (MeV)

LDM 3 + Shell Effect								
Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}	$b1$	$b2$
All Nuclei	15.622	-18.017	-0.714	-30.300	1.551	5.857	-0.561	0.009
DGS 1	15.485	-17.813	-0.701	-28.777	1.450	5.405	-0.126	0.008
DGS 2	15.520	-17.575	-0.709	-29.942	1.559	4.894	-0.692	0.011
DGS 3	15.669	-18.103	-0.717	-30.660	1.551	4.188	-0.582	0.007
DGS 4	15.721	-18.296	-0.722	-30.280	1.503	7.681	-0.650	0.012
DGS 5	15.595	-17.966	-0.711	-29.921	1.508	6.076	-0.547	0.008
DGS 6	15.529	-18.064	-0.702	-29.264	1.482	5.863	-0.250	0.005
DMGSB 1	15.597	-17.986	-0.710	-30.975	1.642	5.694	-0.542	0.008
DMGSB 2	15.702	-18.240	-0.719	-30.682	1.556	6.646	-0.588	0.010

Table A.17 Coefficients of liquid drop model of (LDM1+ Shell Effect formula) (MeV) (cont.)

Sets	a_v	a_s	a_c	C_{sym}	k	a_{pair}	***b1***	***b2***
SGS 1	15.511	-17.602	-0.707	-29.929	1.533	4.401	-0.577	0.009
SGS 2	15.574	-17.827	-0.710	-30.828	1.635	3.568	-0.675	0.012
SGS 3	15.609	-17.996	-0.713	-30.156	1.547	5.322	-0.496	0.007
SGS 4	15.601	-17.905	-0.713	-30.524	1.583	2.886	-0.623	0.011
SGS 5	15.593	-17.991	-0.711	-29.142	1.441	7.128	-0.465	0.006
SGS 6	15.876	-18.627	-0.734	-31.294	1.555	6.866	-0.921	0.018

Table A.18 Coefficients of liquid drop model of (LDM4+ Shell Effect formula) (MeV)

LDM 4 + Shell Effect												
Sets	a_v	k_v	a_s	k_s	r_o	a_k	a_o	f_p	w	a_p	$b1$	$b2$
All Nuclei	16.225	1.884	22.975	2.003	1.180	-14.379	12.719	-1.285	34.210	-10.017	-0.602	0.010
DGS 1	15.660	1.842	18.402	2.174	1.222	0.052	-4.480	-0.933	35.174	-8.220	-0.187	-0.006
DGS 2	15.949	1.939	20.327	2.460	1.191	-6.138	2.506	-1.160	42.346	-8.631	-0.693	0.010
DGS 3	15.158	1.882	12.795	2.942	1.217	19.486	-25.349	-1.136	30.938	-8.500	-0.718	0.014
DGS 4	15.201	1.820	13.757	2.325	1.222	13.411	-15.200	-0.954	21.853	-10.870	-0.762	0.016
DGS 5	15.434	1.911	15.958	2.761	1.224	8.447	-14.859	-0.956	44.170	-10.883	-0.496	0.008
DGS 6	16.425	1.719	28.525	1.275	1.224	-40.390	48.558	-1.067	28.537	-9.341	-0.178	0.004
DMGSB 1	16.132	1.820	22.766	1.843	1.183	-14.357	13.975	-1.535	39.753	-10.291	-0.593	0.009
DMGSB 2	17.807	1.743	36.255	1.246	1.076	-50.561	49.315	-2.556	17.437	-9.026	-0.614	0.011

Table A.18 Coefficients of liquid drop model of (LDM4+ Shell Effect formula) (MeV) (cont)

Sets	a_v	k_v	a_s	k_s	r_o	a_k	a_o	f_p	w	a_p	$b1$	$b2$
SGS 1	16.180	1.915	22.184	2.177	1.179	-10.690	7.141	-1.277	38.067	-9.562	-0.600	0.010
SGS 2	16.573	1.836	27.056	1.575	1.175	-31.317	36.417	-1.283	20.838	-7.583	-0.786	0.015
SGS 3	17.169	1.882	30.926	1.704	1.134	-37.252	35.280	-1.590	38.295	-8.439	-0.531	0.008
SGS 4	15.489	1.911	16.245	2.628	1.213	7.066	-10.718	-1.058	31.547	-5.888	-0.671	0.013
SGS 5	15.709	1.873	17.157	2.462	1.186	9.254	-19.062	-1.363	35.949	-8.982	-0.512	0.008
SGS 6	14.350	1.798	5.839	3.852	1.262	34.262	-31.001	-0.386	24.399	-4.632	-0.947	0.018

Table A.19 Coefficients of liquid drop model of (LDM5+ Shell Effect formula) (MeV)

LDM 5 +Shell Effect								
Sets	a_v	k_v	a_s	k_s	a_c	C_1	***b1***	***b2***
All Nuclei	15.474	-1.899	-17.237	-2.405	-0.702	-24.435	-0.615	0.010
DGS 1	15.303	-1.775	-16.962	-1.926	-0.687	-19.181	-0.250	0.001
DGS 2	15.320	-1.919	-16.551	-2.717	-0.695	-31.307	-0.902	0.019
DGS 3	15.515	-1.915	-17.347	-2.426	-0.704	-25.793	-0.590	0.007
DGS 4	15.667	-1.836	-17.910	-1.836	-0.714	-8.617	-0.786	0.012
DGS 5	15.445	-1.914	-17.200	-2.574	-0.699	-32.518	-0.460	0.007
DGS 6	15.348	-1.828	-17.270	-2.168	-0.686	-24.802	-0.187	0.004
DMGSB 1	15.415	-2.076	-17.136	-3.392	-0.694	-32.594	-0.548	0.008
DMGSB 2	15.757	-1.871	-18.303	-2.066	-0.720	-10.938	-0.631	0.011

Table A.19 Coefficients of liquid drop model of (LDM5+ Shell Effect formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	a_c	C_1	*b1*	*b2*
SGS 1	15.309	-1.873	-16.619	-2.335	-0.692	-22.529	-0.626	0.010
SGS 2	15.549	-1.871	-17.577	-2.108	-0.705	-14.167	-0.709	0.013
SGS 3	15.483	-1.902	-17.309	-2.466	-0.702	-26.987	-0.566	0.009
SGS 4	15.565	-1.933	-17.574	-2.525	-0.707	-25.086	-0.651	0.012
SGS 5	15.480	-1.863	-17.344	-2.365	-0.700	-30.817	-0.433	0.005
SGS 6	15.945	-2.019	-18.529	-2.839	-0.736	-36.788	-0.931	0.018

Table A.20 Coefficients of liquid drop model of (LDM6+ Shell Effect formula) (MeV)

LDM 6 + Shell effect											
Sets	a_v	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p	$b1$	$b2$
All Nuclei	15.504	-0.187	17.877	2.329	0.710	1.058	-26.306	18.252	10.038	-0.600	0.010
DGS 1	15.266	-1.669	17.084	1.985	0.687	0.774	-1.899	-15.453	8.311	-0.270	-0.002
DGS 2	15.409	-0.870	17.338	2.700	0.704	0.951	-16.427	9.877	8.796	-0.731	0.011
DGS 3	15.575	0.132	18.243	2.264	0.719	1.278	-31.370	23.893	8.255	-0.721	0.013
DGS 4	15.604	0.109	18.177	2.201	0.720	1.096	-30.801	24.768	10.668	-0.687	0.014
DGS 5	15.463	-0.456	17.788	2.405	0.706	1.022	-22.173	16.724	10.735	-0.499	0.008
DGS 6	15.198	-1.171	17.142	1.882	0.679	0.835	-9.172	-7.770	9.451	-0.225	0.005
DMGSB 1	15.411	-0.735	17.780	2.636	0.708	1.309	-18.020	16.199	10.126	-0.574	0.009
DMGSB 2	15.552	0.820	18.800	1.827	0.728	1.895	-40.652	24.547	8.943	-0.595	0.010

Table A.20 Coefficients of liquid drop model of (LDM6+ Shell Effect formula) (MeV) (cont.)

Sets	a_v	k_v	a_s	k_s	a_c	b	a_{sym}	C_0	a_p	$b1$	$b2$
SGS 1	15.464	-0.835	17.696	2.456	0.706	1.017	-16.599	7.067	11.305	-0.612	0.010
SGS 2	15.670	1.001	18.484	2.197	0.723	1.166	-45.143	36.667	6.197	-0.788	0.015
SGS 3	15.517	0.113	17.884	2.310	0.708	0.936	-30.870	25.029	8.892	-0.490	0.007
SGS 4	15.535	-0.277	17.997	2.408	0.712	1.075	-25.285	16.377	5.489	-0.670	0.013
SGS 5	15.616	0.719	18.591	2.088	0.724	1.410	-39.828	34.461	8.299	-0.498	0.008
SGS 6	15.897	3.692	19.170	2.437	0.739	1.198	-89.450	98.789	4.434	-0.934	0.018

Table A.21 RMS for All Groups & All Formulas + Shell effect (keV)

Group Nuclei		LDM 1	LDM 2	LDM 3	LDM 4	LDM 5	LDM 6
All Nuclei Group		20.760	32.521	16.884	15.166	22.422	15.315
Deformation Group	**DGS 1**	18.001	30.420	13.663	12.449	18.660	12.435
	DGS 2	23.912	37.193	18.899	16.792	24.836	17.073
	DGS 3	13.206	24.913	10.852	9.461	13.885	9.791
	DGS 4	14.093	24.651	9.754	7.429	16.262	7.199
	DGS 5	10.224	23.440	6.936	6.252	10.535	6.292
	DGS 6	18.369	21.500	11.751	8.590	19.321	9.393
Decay Mod Group	**DMGSB 1**	17.122	17.638	13.081	10.156	19.349	10.083
	DMGSB 2	22.175	26.145	18.641	17.632	22.212	17.836

Table A.21 RMS for All Groups & All Formulas + Shell effect (keV) (cont.)

Group Nuclei		LDM 1	LDM 2	LDM 3	LDM 4	LDM 5	LDM 6
Spin Group	SGS 1	18.662	32.440	17.633	15.924	19.749	16.449
	SGS 2	16.446	26.349	17.489	13.921	17.561	13.576
	SGS 3	21.570	35.756	19.966	18.518	22.108	18.961
	SGS 4	11.797	32.825	13.148	11.025	14.865	11.178
	SGS 5	13.906	29.981	15.079	12.676	18.300	12.773
	SGS 6	8.261	15.735	8.704	7.538	10.125	7.149

Table A.22 Coefficient for VSS formula

VSS Formula					
Nuclei State	***a***	***b***	***c***	***d***	***e***
Even-Even	0.71594	74.02256	0.09743	-59.52929	0
Even-Odd	1.43019	11.67919	-0.14241	-37.99915	0
Odd-Even	0.70280	76.65339	0.10910	-60.83229	0
Odd-Odd	1.21434	17.09616	-0.09934	-36.71378	0

Table A.23 Coefficient for Royer formula

Royer Formula			
Nuclei State	***a***	***b***	***c***
Even-Even	24.46249	1.98223	1.14814
Even-Odd	23.66439	2.10361	1.16470
Odd-Even	24.93498	1.84830	1.14181
Odd-Odd	21.06454	1.78591	1.02132

Table A.24 Coefficient for DEKH formula

DEKH Formula					
Nuclei State	***a***	***b***	***c***	***d***	***e***
Even-Even	27.78775	0.96474	1.52261	0	0
Even-Odd	27.40198	0.17109	1.55400	0.20969	0.57396
Odd-Even	29.17532	0.87983	1.50882	1.01027	0.01369
Odd-Odd	25.65088	0.80972	1.35849	0.85511	0.44309

Table A.25 Coefficient for UDL formula

UDL Formula			
Nuclei State	***a***	***b***	***c***
Even-Even	0.39681	-0.06690	-35.65815
Even-Odd	0.41161	-0.07140	-36.01793
Odd-Even	0.39548	-0.06415	-35.87090
Odd-Odd	0.36605	-0.06244	-32.15804

Table A.26 Coefficient for NRDX formula

NRDX formula			
Nuclei State	***a***	***b***	***c***
Even-Even	0.38538	-1.26983	-15.81005
Even-Odd	0.40095	-1.36740	-14.68439
Odd-Even	0.38518	-1.20084	-17.36897
Odd-Odd	0.35790	-1.18383	-13.98005