REPUBLIC OF TURKEY
CUKUROVA UNIVERSITY
INSTITUTE OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS

BOOTSTRAPPING METHODS AND APPLICATIONS ON TURKISH MACROECONOMIC VARIABLES

Mehmet Fatih TRAŞ

DOCTORATE OF PHILOSOPHY

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DOCTORATE OF PHILOSOPHY

ADANA / 2016

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MEHMET FATİH TRAŞ

ABSTRACT

BOOTSTRAPPING METHODS AND APPLICATIONS ON TURKISH MACROECONOMIC VARIABLES

Mehmet Fatih TRAŞ

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This paper mainly has two aims. In the first place, an extensive discussion on bootstrap methods used in statistical estimation and inference with cross section and time series data sets is presented. In particularly, dynamics through which bootstrap refines the statistics of interest and outperforms classical statistical approach are discussed in detail. In addition, a special attention is paid to the application of bootstrap with different data sets such as cross section and time series and how the procedure must be conducted are explained comprehensively.

As the second aim of this study, three bootstrap applications with Turkish macroeconomic time series are performed. Because stationarity tests are quite common among empirical practitioners and because they have explicit implications in terms of long term cointegration relationships between time series, the first two applications of this study focus on bootstrapped unit root tests under the presence of small samples where bootstrap approach is known to be superior to conventional unit root tests suggested in literature. Besides, bootstrap unit root tests results and unit root tests allowing for single and multiple endogenous structural breaks are compared. The third application of the study is devoted to obtain bootstrap estimation of the Keynesian consumption function for the Turkish economy, by employing an auxiliary method used in determining the number of structural breaks occurring in the relationship between the series involved in the model.

Results show that conventional and bootstrapped unit root test results show substantial differences on the presence of unit root. Furthermore, bootstrapped unit root test results are highly in line with the results suggested by the unit root tests allowing for single and multiple structural breaks. Furthermore, the Keynesian consumption function estimated by Ordinary Least Square and bootstrap method differ significantly in terms of the magnitude of the model coefficients estimated across three regimes determined endogenously.

Keywords: Bootstrap, unit root tests, small sample, Keynesian consumption function

ÖZET

BOOTSTRAPPING YÖNTEMLERİ VE TÜRKİYE'NİN TEMEL MAKROEKONOMİK DEĞİŞKENLERİ İLE UYGULAMALARI

Mehmet Fatih TRAŞ

Doktora Tezi, İktisat
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En genel hali ile bu çalışmanın iki amacı vardır. İlk olarak, yatay kesit verisi ve zaman serileri ile gerçekleştirilen ve istatistiksel tahmin ile çıkarım süreçlerinde kullanılan bootstrap yöntemlerinin özetlenmesi hedeflenmiştir. Özellikle bootstrap yönteminin ilgilenilen istatistiğin niteliğini iyileştirdiği ve klasik istatistiksel yaklaşım ile elde edilmiş varyasyonlarına karşı üstün kıldığı dinamikler ayrıntıları ile tartışılmıştır. Ek olarak, bootstrap yaklaşımının yatay kesit verisi ve zaman serileri gibi farklı veri tiplerindeki kullanımlarına özel bir ilgi atfedilmiştir.

Çalışma ikinci olarak Türkiye'nin temel makroekonomik değişkenleri ile yapılmış üç uygulamayı sunmaktadır. Durağanlık testleri uygulamalı çalışmalar yapan bilim insanları arasında oldukça yaygın olduğundan ve zaman serileri arasındaki uzun dönemli eşbütünleşik ilişkilere dair önemli ipuçları verdiğinden, bu çalışmadaki ilk iki uygulama bootstrap yönteminin geleneksel birim kök testlerine karşı üstünlüğü olduğu bilinen küçük örneklem koşulları altında birim kök testlerine odaklanmaktadır. Geleneksel birim kök testlerine ek olarak çalışmada, tek ve çoklu içsel kırılmalara izin veren birim kök testleri de kullanılmış ve bu testlerin sonuçları da bootstrap birim kök testleri ile karşılaştırılmıştır. Çalışmaya dair üçüncü uygulama yapısal kırılmaların sayısı ve tarihini içsel olarak belirlemeye izin veren yardımcı bir yöntemi de kullanarak Türkiye ekonomisi için Keynezyen Tüketim fonksiyonunun tahminini kapsamaktadır.

Çalışmanın bulguları, geleneksel ve bootstrap birim kök testleri sonuçları arasında serilerin durağanlığına ilişkin çarpıcı farklılıklar olduğunu göstermektedir. Buna ek olarak, bootstrap birim kök test sonuçlarının tek ve çoklu içsel kırılmalara izin veren birim kök test sonuçları ile önemli derece paralellik göstermesi de çalışmanın bulguları arasındadır. Bu bulgulara ek olarak, içsel olarak belirlenmiş üç rejim dönemi

için Sıradan En Küçük Kareler metodu ile tahmin edilmiş Keynezyen tüketim fonksiyonu, aynı dönemler için bootstrap yöntemi ile tahmin edilmiş Keynezyen tüketim fonksiyonundan model katsayılarının büyüklüğü bakımından önemli farklılıklar göstermektedir.

Anahtar Kelimeler: Bootstrap, birim kök testleri, küçük örneklem, Keynezyen tüketim fonksiyonu

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LIST OF ABBREVIATONS

ADF	**:** Augmented Dickey Fuller
AR	**:** Autoregressive
BIC	**:** Bayesian Information Criteria
Cons	**:** Aggregated Domestic Consumption
DF	**:** Dickey Fuller
er	**:** Exchange Rate
ex	**:** Export
GDP	**:** Gross Domestic Product
i	**:** Interest rate
im	**:** Import
KPSS	**:** Kwiatkowski Phillips Schmidt Shin
LP	**:** Lumsdaine Papell
LS	**:** Lee Strazicich
m1	**:** Money Supply in m1 term
m2	**:** Money Supply in m2 term
OLS	**:** Ordinary Least Square
P	**:** Consumer Price Index
PP	**:** Phillips Perron
ZA	**:** Zivot Andrews

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CHAPTER I

INTRODUCTION

1.1. Statement of the Problem

A vast body of literature has been built around statistical inference methods which are of vital importance in obtaining the closest approximation of the population distribution and its corresponding parameters, such as the mean, the variance and the standard deviation and in assessing the performance of the estimated versions of these parameters. A conventional way of doing this for hypothesis testing, for example, is to assume unknown population distribution to exhibit certain and known characteristics, through which rejection probabilities and the critical values corresponding to these probabilities can be derived. In other words, classical statistical approach allows one to examine population characteristics in terms of a statistics of interest assuming a well behaved sampling distribution.

An alternative approach to this is bootstrap. Bootstrap is a resampling method which intends to maximize the information extracted from population sample. The key point bootstrap outperforms classical approach in estimation and hypothesis testing is its ability in constructing empirical distribution of estimation or a statistics, rather than assuming the presence and correspondence of well-behaving sampling distribution. In other words, bootstrap derives empirical distribution of a statistics by which inferences can be made without any assumption regarding the population distribution. In this way, bootstrap method has the capability of obtaining estimated parameters, distribution of test statistics and critical values which can be benefited from in hypothesis testing with certain level of significance levels.

The research questions of this study are based on the advantage bootstrap has over classical statistical approach and hypothesis testing in time series applications with small sample. Therefore, this study focuses on

- How do conventional unit root tests perform under the presence of small sample?
- Through which dynamics does bootstrap approach provide refinements to the drawbacks observed in the use of classical statistical approach and hypothesis testing under the presence of small sample?

- To what extent are the ordinary least square and bootstrap estimation methods in agreement on the magnitude and sign of the estimated coefficients of a structural regression model?

1.2. The Aim of the Research

This study intends to present the principles of bootstrapping methods and to discuss the refinements bootstrap may provide to applied econometrics researchers. In particular, a clear explanation on how bootstrap betters off the desired properties of estimates and test statistics is given with an emphasis on the different areas in statistics and econometrics where bootstrap method can be employed. For this purpose a brief summary of the concepts and definitions used in the assessment of asymptotic properties of test statistics is provided in Appendix A. This chapter provides a description of the tools benefited to explore how a test statistics behaves in asymptotical sense under the presence of independent and identically distributed (i.i.d.) random variables and heterogeneously distributed variables with serial dependence. Performing bootstrap schemes with cross-section data is relatively simpler and straight forward. The use of bootstrap with time series, however, requires special attention due to the possible dependence between consecutive observations. Accordingly, this paper summarizes methods suggested for the application of bootstrap with different data types.

Although unit root tests are one of the most commonly used econometric tools among practitioners in economics, they do suffer from certain conditions frequently encountered in empirical applications. Major problem unit roots tests have is their poor performance in small samples. In particular poor performance of unit root tests in small samples is of particular interest because of its implications on statistical inference procedures. With the advances in technology and computation abilities, simulation methods and the related synthetic data generating approaches including bootstrap have gained prominence in applied econometric research, not only because of relatively less calculation burden, but also because of empirical refinements these approaches have to provide. Therefore, as the second purpose of this study, time series properties of the selected Turkish macroeconomic variables are explored using bootstrapped unit root tests. The major focus of this investigation is on the presence of small samples. To this end, bootstrapped unit root tests suggested by Psaradakis (2001) and Harris (1992) are selected to perform. Besides, for comparison purposes, some well-known and conventional unit root tests and tests allowing for single or multiple structural breaks are employed. Furthermore, bootstrap method will be used to investigate the existence of possible structural relationships between macroeconomic variables. For this purpose, Keynesian consumption function for the Turkish economy will be estimated depending on a bootstrap approached designated for stationary time series along with an auxiliary method which allows for inspection of the number and position of structural breaks.

The structure of this study is as follows. In the second chapter bootstrap methods are introduced, and the dynamics by which this method outperforms the products of classical approach in some particular applications are discussed. In Appendix A, a summary of definitions and concepts needed to characterize asymptotic behaviors of estimates and test statistics is given. The third chapter focuses on time series and how bootstrap can be used with serially dependent data structures. Besides, in the same chapter, an extensive summary of unit root tests with or without structural breaks is provided. In the fourth chapter, two applications of bootstrap unit root test with Turkish macroeconomic time series is performed and the results are compared with the conventional unit root tests with or without structural breaks. In addition to that, bootstrap estimation of the Keynesian consumption function for the Turkish economy, by employing an auxiliary method used in determining the number of structural breaks occurring in the relationship between the series involved in the model is also presented in Chapter four. Appendix B, presents the distribution of data specific critical values with 1%, 5% and 10% rejection probabilities for each series used in the analysis. Besides, Appendix C presents empirical distribution of the parameters estimated by bootstrapped version of Ordinary Least Squares. Furthermore, selected R codes used in different steps of the analysis are given in Appendix D. The final chapter concludes the main findings of the study.

CHAPTER II

PRINCIPLES OF BOOTSRAP

2.1. Understanding Principles of Bootstrap

The major concern in statistics science has been inferring about the true value of a population parameter, depending on a sample size of n. More precisely, statistical theory attempts to answer three major questions;

1. How should I collect my data?
2. How should I analyze and summarize the data that I have collected?
3. How accurate are my data summarizes? (Efron and Tibshirani, 1994, p.1)

Statistical inference on which a vast body of literature has been devoted to the ways of obtaining the closest approximation of the population distribution and its corresponding parameters, such as the mean, the variance and the standard deviation is included in the third question. A classical approach suggested for hypothesis testing, for instance, is making assumptions about the unknown population distribution. In this framework once assumptions on unknown population distribution are made, characteristics of the sampling distribution, by which hypothesis test on the value of the true population parameter can be carried out, are determined. In other words, classical probability theory departs from the composition of a population and ends up with deducing the properties of a random sample and of statistics calculated from this sample (Efron and Tibshirani, 1994, p.20). A common example of hypothesis test of this sort is tests of the mean of a normal distribution, in which sample mean can be standardized as a result of the assumptions on parameters and normality of the parent population distribution (Newbold, 1991, p.247).

An alternative approach that does not require any pre-specified assumption on population distribution and its corresponding parameters in statistical inference is bootstrap, suggested by Efron in his pioneering work in 1979. To illustrate the main points of this resampling method assume F_n is the empirical distribution of independent identically distributed random vectors $X_1, X_2, \dots, X_n$ and $\hat{\theta}$ is an estimate obtained using the random vectors $X_1, X_2, \dots, X_n$, while F is the unknown population distribution. The principle behind this approach is that, given sufficiently large sample size n, law of large numbers for independent identically distributed random variables assures that empirical distribution F_n, that has probability $\frac{1}{n}$ assigned to each sample value, converges to the unknown population distribution F (Chernick, 2007, p.11). In other words, empirical distribution F_n substitutes the fitted model when no mathematical form such as normal or exponential is assumed for F (Davison and Hinkley, 1997, p.12).

In Efron (1979)'s perspective statistical inference problem is defined as estimating parameters of a sampling distribution, depending on a set of observations (p.2). For this purpose, he suggests the use of observed empirical distribution F_n, rather than pre-specified properties of population distribution F, by applying the following steps,

1. Construct the empirical probability distribution F_n, putting mass $1/n$ at each point of the observations.
2. With F_n fixed, draw a random sample of size n from F_n and call this the bootstrap sample.
3. Approximate the sampling distribution using the bootstrap sample (p. 3).

A clearer explanation of the steps of the basic bootstrap and a better understanding of the resampling procedure Efron (1979) implicitly stresses, can be found in Chernick (2007),

1. Generate a sample with replacement from the empirical distribution (a bootstrap sample),
2. Compute θ^*, the value of $\hat{\theta}$ obtained by using the bootstrap sample in the place of the original sample,
3. Repeat steps 1 and 2 k times (p.9).

After this clarification, Chernick (2007) explains the essence of resorting to empirical distribution;

"The basic idea behind the bootstrap is the variability of θ^*(based on F_n) around $\hat{\theta}$ will be similar to (or mimic) the variability of $\hat{\theta}$ (based on the true population distribution F) around the true parameter value, θ. There is good reason to believe that this will be true for large sample sizes, since as n gets larger and larger, F_n comes closer and closer to F and so sampling with replacements from F_n is almost like random sampling from F" (p.11).

An alternative explanation of the steps of the basic bootstrap can also be stated as,

1. Define a matrix X^* of $n \times B$, where n is the number of sample and B is the number of bootstrap sample series to be drawn from the original sample X.
1. 2. Draw bootstrap samples $x_i^*,\ \ i = 1,2,3, \dots , B$ B times each of size n from the original series X with replacement and assign each to the columns of $X^*_{n \times B}$.
2. Calculate the statistics of interest for each column of $X^*_{n \times B}$ and obtain the corresponding sample statistics distribution.

2.2. Literature Review

Bootstrap was introduced by Efron (1979) in his pioneering work. In his study Efron (1979) empirically shows satisfactory performance of bootstrap technique by focusing on several estimation problems such as variance of sample mean, error rates in a linear discriminant analysis, ratio estimation, and estimation regression parameters. Focusing on the exact convergence rate of the bootstrap approximation of the distributions of sample quintiles, Singh (1981) and Bickel and Freedman (1981) independently investigate the theoretical proofs of the statistical refinements that bootstrap provides. Singh (1981) demonstrates consistency of the bootstrap estimation and the advantage bootstrap method has over approximations performed by the limiting normal distribution. Bickel and Freedman (1981), on the other hand, prove that bootstrap method is particularly useful for the estimation of means and for pivotal quantities of the familiar t-statistics sort, by expressing the reservation that bootstrap may fail in certain cases where a statistic is attempted to estimate using uniformly distributed variables. In addition to that Gine and Zihn (1989) define the necessity conditions for bootstrap to have refinement in the estimation of population mean.

In the literature cases where bootstrap applications fail are also discussed from a variety of aspects. In one of these studies, Athreya (1987) point outs that bootstrap methods aiming to estimate population mean may fail under the presence of the heavy tailed case. Besides, depending on an empirical study, Knight (1989) verifies Athreya

(1987)'s theoretical discussions. Likewise, Angus (1992) evaluates the performance of bootstrap distributions in the case of an IID sequence with outliers and draws the conclusion that almost sure convergence is not necessarily attainable with the sequences involving extremes. Besides, Hall, Hardle and Simar(1993) considers bootstrapped estimators and the necessity conditions under which estimators can be consistent.

The discussions on the use of bootstrap in time series are primarily centered around stationary ARIMA models, introduced by Box and Jenkins (1976). Efron (1982) develops bootstrapping residuals within the scope of regression analysis. In a further study, Freedman and Peters (1985) extend the use of bootstrap in time series to the estimation of forecasting error and the model selection. Referring to Freedman and Peters (1985)'s finds, Findley (1986) emphasizes that the usefulness of bootstrap procedure for time series may be ambiguous for estimating mean square forecast errors. Giving a broad perspective on the importance of autoregressive structure present in the data, Efron and Tibshirani (1986) show that bootstrapped standard errors for an AR(1) process outperform conventional standard errors. Besides, Stine (1987) demonstrates the refinement of bootstrapped confidence intervals under autoregressive data generating processes.

Dependent data characteristic of time series has also led to discussions which propose different bootstrapping procedures. In this respect, Carlstein (1986) proposes replacement of bootstrap re-sampling by re-sampling non-overlapping subseries. In a more extensive study, Künsch (1989) develops the idea of adopting overlapping blocks in re-sampling procedure. Besides, block size selection and consequences of choosing different block length are addressed by Hall, Horowitz and Jing (1995). Another bootstrap method considered to be more appropriate for dependent data is sieve bootstrap, suggested by Bühlmann (1997).

Unit root concept in time series is developed by Dickey and Fuller (1979). Proposing a unit root test for restricted residuals, the first attempt for testing the presence of unit root using the bootstrap is made by Basawa, Mallik, McCormick, Reeves and Taylor (1991). In another study, studying on an AR(1) process, Ferretti and Romo (1996) propose a unit root test on unrestricted residuals and prove the asymptotic validity of the test statistic in the case of independent and autoregressive errors. In addition, Moreno and Romo (2000) develop a different approach to test unit root with bootstrap by employing the least absolute deviation estimators and establish that their test has better power behavior compared to other proposed tests up to then. In another study, Psaradakis (2000) proposes a unit root test with bootstrap for seasonal autoregressive model. Furthermore, Psaradakis (2001) employs sieve bootstrap to test the presence of unit root for the models that include an intercept and/or a linear time trend. Procidano and Luchini (2001) use the external bootstrap method to test the unit root hypothesis in various forms of heteroskedasticity and establish that their method indicates more accurate results than Dickey-Fuller unit root test does.

In a later study, Paparoditis and Politis (2003) propose a block-bootstrap based unit root test method that generates unit root integrated pseudo-series upon which a test statistics can be structured and examine its finite sample performance. Park (2003) examines the asymptotic performance of bootstrapped unit root tests and shows that bootstrap provides asymptotic refinements in terms of validity of the unit root hypothesis testing. Swensen (2003) suggests two bootstrap algorithms that can be used for unit root hypothesis testing, the first of which is creating a re-sampled data through fitting an autoregressive model to the first-differenced observations and the second of which is bootstrapping directly from the first-differenced observations. Paparoditis and Politis (2005), investigate behavior of different bootstrap alternatives and propose a new approach based on unrestricted residuals.

2.3. Some Bootstrap Applications in Statistics

Bootstrap has a widespread level of applicability in a number of fields in statistical estimation and inference processes. In particular, bootstrap is especially useful in the cases where making assumptions on the population distribution of a test statistics is not feasible. Besides, bootstrap method is quite favorable in the estimation of important parameters and accuracy statements such as population mean and its standard error, both of which are consulted in further investigations in statistical inference. For this reason bootstrap is involved as a tool providing desired refinements in a variety of applications some of which are explained below.

2.3.1. Bootstrapped Standard Errors

An important statistic that can be used to evaluate the precision and the reliability of an estimate is standard error (Newbold, 1991, p.246). Standard error of an estimate can be regarded as an accuracy measure indicating to what extent an estimate is far away from its expected value (Efron and Tibshirani, 1994, p.11). As an example, estimated standard error for sample mean with independent identically distributed data points $x_1, x_2, \dots, x_n$ with $\bar{x} = \sum_{i=1}^{n} {x_i}/{n}$ is,

$$\widehat{se} = \sqrt{\frac{s^2}{n}} \tag{1}$$

where $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$ (Efron and Tibshirani, 1994, p.11).

Distribution of sample mean $\bar{x}$ is shown to be $\bar{x} \sim (\mu, \sigma^2/n)$, where μ and σ^2 are population mean and variance, respectively. Furthermore, as n gets large, this distribution converges to a normal distribution with the same parameters;

$$\bar{x} \sim N(\mu, \sigma^2/n) \tag{2}$$

which implies that $\bar{x}$ can be expected to be less than one standard error away from μ about 68% of the time, and less than two standard errors away from μ about 95% of the time (Newbold, 1991, p. 248).

All these results discussed above are derived from the central limit theorem. Contrary to this conventional setting, bootstrap offers an alternative way for estimating standard error of an estimate. One of the advantages of bootstrap is that it does not necessarily require the use of the central limit theorem and offers alternative accuracy statements, such as bootstrapped standard error (Efron and Tibshirani, 1994, p.40). Figure 1 illustrates the suggested bootstrap process for estimating standard error of an estimate.

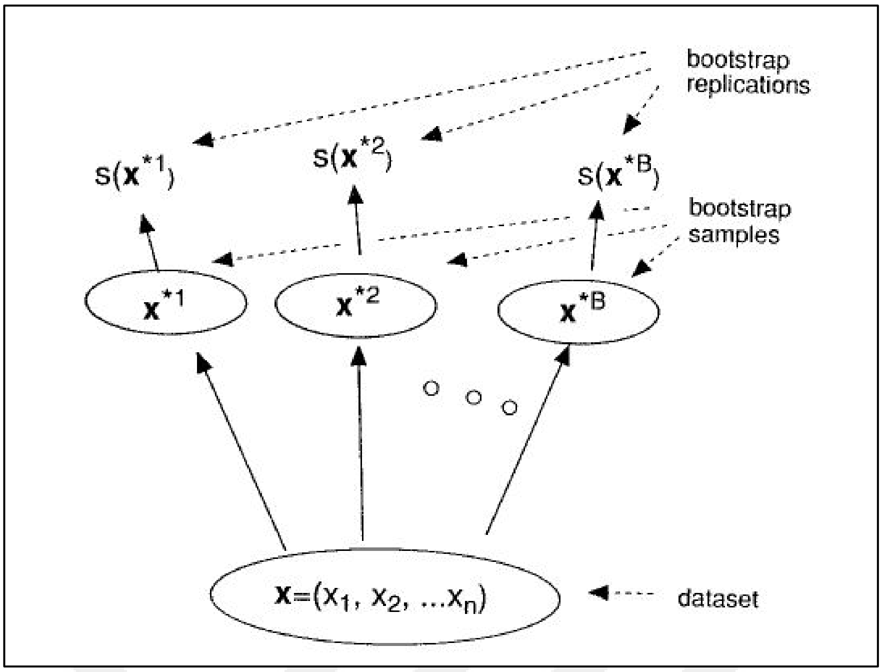


Figure 1. Schematic process of the bootstrap process for estimating standard error of an estimate.
Source: Efron and Tibshirani, 1994.

In this procedure, Efron and Tibshirani (1994) suggest an algorithm that starts with drawing samples with replacement from observations $\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}$ to obtain a set of samples $\mathbf{x}^{*\mathbf{1}}, \mathbf{x}^{*\mathbf{2}}, \ldots, \mathbf{x}^{*\boldsymbol{B}}$, each of size $\boldsymbol{n}$ vectors. Here $\boldsymbol{B}$ indicates the number of bootstrap samples. In the next stage an estimate of interest is calculated for each bootstrap sample $\mathbf{x}^{*\mathbf{1}}, \mathbf{x}^{*\mathbf{2}}, \ldots, \mathbf{x}^{*\boldsymbol{B}}$ that corresponds to bootstrap replications $\boldsymbol{s}(\boldsymbol{x}^{*\mathbf{1}}), \boldsymbol{s}(\boldsymbol{x}^{*\mathbf{2}}), \ldots \boldsymbol{s}(\boldsymbol{x}^{*\boldsymbol{B}})$, forming a distribution, by which inferences about the standard error of the estimate can be made. In this setting, Efron and Tibshirani (1994) show that the bootstrap estimate of standard error is the standard deviation of the bootstrap replications (p.13);

$$\widehat{\boldsymbol{se}}_{\boldsymbol{boot}} = \left\{\sum_{\boldsymbol{b}=\mathbf{1}}^{\boldsymbol{B}} [\boldsymbol{s}(\boldsymbol{x}^{*\boldsymbol{b}}) - \boldsymbol{s}(.)]^2 / (\boldsymbol{B}-\mathbf{1})\right\}^{\frac{1}{2}} \tag{3}$$

where $s(.) = \sum_{b=1}^{B} s(x^{*b}) / B.$ (4)

Continuing with the sample mean case, in the presence of sufficiently large B, bootstrapped standard error converges to the standard error estimate of sample mean given under standard probability theory,

$$\{\sum_{i=1}^{n}(x_i - \bar{x})^2/n^2\}^{\frac{1}{2}} \tag{5}$$

Needless to say, a big importance is attached to the number of bootstrap samples B, for the fact that desired approximation in the distribution of the relevant statistic occurs only under sufficiently large value of B. Efron and Tibshirani (1994) suggest that ideal value of B for estimating standard error of an estimate is between 50 and 200 (p.13). In a more formal way, Efron and Tibshirani (1994) give the following relationship between the size of B and the approximation expected;

$$\lim_{B\to\infty} \widehat{se}_{boot} = se_{F_n} = se_{F_n}(\theta^*). \tag{6}$$

In other words, Efron and Tibshirani (1994) evaluate the convergence in standard error on the basis that $\widehat{se}_{boot}$ approaches se_{F_n} as B goes to infinity and reach the conclusion that an empirical standard deviation approaches the population standard deviation as the number of replications gets large (p.47). In consequence, Efron and Tibshirani (1994) state two rules of thumb that can be helpful in the selection of the number of bootstrap replications for standard error estimate of $\hat{\theta}$;

“ 1) Even a small number of bootstrap replications, say $B = 25$, is usually informative. $B = 50$ is often enough to give a good standard error estimate of $\hat{\theta}$.

2) Very seldom are more than $B = 200$ replications needed for estimating a standard error “ (pg. 52).

2.3.2. Bootstrapped Confidence Intervals

An important tool that can be used in statistical inference for describing an interval in which population parameter θ might be contained with a certain probability is confidence intervals. To keep up with the frame given above, in the first place classical probability theory approach in structuring confidence intervals will be summarized for a better illustration of how bootstrap approach differs from it. Secondly, dynamics through which bootstrap provides improvements over the accuracy of the interval estimate is explained.

To explain classical probability approach more explicitly, following Efron and Tibshirani (1994), assume an estimate $\hat{\theta}$ has the distribution $\hat{\theta} \sim N(\theta, se^2)$. This distribution can also be written as;

$$Z = \frac{\hat{\theta}-\theta}{\widehat{se}} \sim N(0,1) \tag{7}$$

Moreover, let $z^{(\alpha)}$ indicate the 100. αth percentile point of a $N(0,1)$ distribution, which can be found in a standard normal distribution table. Under these circumstances, confidence interval for θ, depending on $\hat{\theta}$, $\widehat{se}$ and $z^{(\alpha)}$ value corresponding to the 100. αth percentile point of a $N(0,1)$ distribution can be defined as;

$$Prob\left\{z^{(\alpha)} \leq \frac{\hat{\theta}-\theta}{\widehat{se}} \leq z^{(1-\alpha)}\right\} = 1 - 2\alpha \tag{8}$$

or equivalently

$$Prob\left\{\theta \in [\hat{\theta} - z^{(1-\alpha)}.\widehat{se}, \hat{\theta} - z^{(\alpha)}.\widehat{se}]\right\} = 1 - 2\alpha \tag{9}$$

(Efron and Tibshirani, 1994, p.154).

By computing this interval for θ, it is possible to reach the implication that $1 - 2\alpha$ of the time, the true value of θ will be contained by a random interval constructed in this way (Efron and Tibshirani, 1994, p.155). Classical probability theory offers a better approximation for the distribution of Z where only small number of sample, say 20, is available. In this case, a better approximation, called Student's t distribution with $n-1$ degree of freedom, is given as;

$$Z = \frac{\hat{\theta}-\theta}{\widehat{se}} \sim t_{n-1} \tag{10}$$

which transforms the confidence interval to be;

$$Prob\left\{\theta \in [\hat{\theta} - t_{n-1}^{(1-\alpha)}.\widehat{se}, \hat{\theta} - t_{n-1}^{(\alpha)}.\widehat{se}]\right\} = 1 - 2\alpha \tag{11}$$

(Efron and Tibshirani, 1994, p.159).

Contrary to classical probability approach that assumes $Z \sim N(0,1)$ or $\sim t_{n-1}$, bootstrap suggests a more straightforward way for computing confidence intervals without making theoretical assumptions. Bootstrap approach in confidence intervals relies upon building up a Z table specific to the data at hand, rather than using an assumed one (Efron and Tibshirani, 1994, p.160). In this approach bootstrapped Z values $Z^*(b)$ is calculated as follow,

$$Z^*(b) = \frac{\theta^*(b) - \hat{\theta}}{\widehat{se}(b)} \tag{12}$$

where $\theta^*(b)$ is the bootstrapped statistics calculated from bootstrap samples $\mathbf{x}^{*\mathbf{1}}, \mathbf{x}^{*\mathbf{2}}, \ldots, \mathbf{x}^{*\boldsymbol{B}}$ and $\widehat{se}(b)$ is the estimated standard error of $\theta^*(b)$. After obtaining distribution of $Z^*(b)$, interested percentiles can be calculated from

$$\#\{Z^*(b) \leq \hat{t}^{(\alpha)}\}/B = \alpha \tag{13}$$

where # is used to indicate the number of cases described in $\{.\}$. Bootstrap confidence interval using bootstrapped $\hat{t}$ statistic instead of the assumed t can eventually be written as

$$(\hat{\theta} - \hat{t}^{(1-\alpha)}.\widehat{se}, \hat{\theta} - \hat{t}^{(\alpha)}.\widehat{se}) \tag{14}$$

(Efron and Tibshirani, 1994, p.160)

As shown by Efron and Tibrishani (1994), in large samples the coverage of the bootstrap t intervals are found to be closer to the desired levels such as 90% or 95%, compared to the coverage of the standardized t table used in classical approach (p.323).

2.3.3. Bootstrap in Regression Models

Regression models are commonly used tools in statistics and applied economics. Inspired by Keynes (1936)'s interpretation of econometrics, regression can be defined

as investigating consistency between observed facts and a theorized phenomena. In Gujarati (….)'s words, this "bread-and-butter tool" of econometrics
"… is concerned with the study of the dependence of one variable, the dependent variable, on one or more other variables, the explanatory variables, with a view to estimation and/or predicting the (population) mean or average value of the former in terms of the known or fixed (in repeated sampling) values of the latter" (P.18).

To present the role bootstrap plays in regression models, primary focus will firstly be on the conventional setting in linear regression analysis and the ordinary least squares method. Then, how bootstrap contributes to linear regression models and to what extend this contribution refines the estimates is explained.

To begin with, following Efron and Tibshirani (1994), assume the data set $\mathbf{x}$ consists of $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$, where each $\mathbf{x_i}$ corresponds to $\mathbf{c_i}$ and y_i pairs. Here $\mathbf{c_i}$ is a $p \times 1$ vector called covariate vector or predictor, while y_i is a real number called response (p.105). Furthermore, assume μ_i indicates the conditional expectation of ith response y_i given the covariate vector $\mathbf{c_i}$,

$$\mu_i = \mathrm{E}(y_i \setminus \mathbf{c_i}) \qquad (i = 1,2, \ldots, n) \tag{15}$$

Efron and Tibshirani (1994) point out that critical point in linear regression model is assuming μ_i as a linear function of covariates $\mathbf{c_i}$,

$$\mu_i = \mathbf{c_i}\boldsymbol{\beta} = \sum_{j=1}^{p} c_{ij}\beta_j \tag{16}$$

where $\boldsymbol{\beta}$ is the parameter vector about which regression analysis attempts to infer, relying on the observed data $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n})$ (p.106). The stochastic form of the linear relationship defined by this regression model can be written as;

$$y_i = \mathbf{c_i}\boldsymbol{\beta} + \epsilon_i \tag{17}$$

where ϵ_i is a random component with the distribution F having the following properties;

$$F \rightarrow (\epsilon_1, \epsilon_2, \ldots, \epsilon_n) \qquad \text{and} \qquad [\mathrm{E}_F(\epsilon) = 0]$$

The major aim of the ordinary least squares method is to minimize residual squared error (RSE). To see this let **b** indicate a trial value of $\boldsymbol{\beta}$, while $\widehat{\boldsymbol{\beta}}$ indicates the ordinary least square estimate minimizing RSE among the set of estimates **b**,

$$\mathrm{RSE}(\mathbf{b}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{y_i} - \mathbf{c_i b})^2 \tag{18}$$

$$\mathrm{RSE}(\widehat{\boldsymbol{\beta}}) = \min_{\mathbf{b}}[\mathrm{RSE}(\mathbf{b})] \tag{19}$$

Then the normal equations and their solution that gives the ordinary least squares estimate $\widehat{\boldsymbol{\beta}}$ can be stated respectively as;

$$\mathbf{C^T C}\widehat{\boldsymbol{\beta}} = \mathbf{C^T y} \tag{20}$$

$$\widehat{\boldsymbol{\beta}} = (\mathbf{C^T C})^{-\mathbf{1}}\mathbf{C^T}\boldsymbol{y} \tag{21}$$

where $\mathbf{C}$ is a $n \times p$ matrix with ith row $\mathbf{c_i}$ and **y** is the vector $(y_1, y_2, \dots, y_n)^T$ (Efron and Tibshirani, 1994, p.107) .

Regarding the accuracy of the ordinary least squares estimate classical probability approach provides standard error of the jth component of $\widehat{\boldsymbol{\beta}}$ as,

$$\mathrm{se}(\hat{\beta}_j) = \sigma_F \sqrt{G^{jj}} \tag{22}$$

where σ_F indicates square root of $\sigma_F^2 = \mathrm{var}_F(\epsilon)$ which can be estimated by

$$\hat{\sigma}_F = \{\textstyle\sum_{i=1}^{n} (\mathrm{y_i} - \mathbf{c_i b})^2/n\}^{1/2} = \left\{\mathrm{RSE}(\widehat{\boldsymbol{\beta}})/n\right\}^{1/2} \tag{23}$$

or by the bias corrected version

$$\bar{\sigma}_F = \left\{\mathrm{RSE}(\widehat{\boldsymbol{\beta}})/n - p\right\}^{1/2} \tag{24}$$

and G^{jj} is the jth diagonal element of the inverse of a matrix **G,**
defined as a $p \times p$ inner product matrix $\mathbf{G} = \mathbf{C^T C}$ (25)

(Efron and Tibshirani, 1994, p.109).

Before discussing how bootstrap approach works in regression and in the ordinary least squares method, it is essential to introduce "two sample problem" in statistics. So far we assumed that observation $\mathbf{x}$ is drawn from an unknown population probability distribution F. However, there are other cases where independent set of observations, say $\mathbf{z}$ and $\mathbf{y}$, come from different unknown probability distributions F and G. To discuss two sample case in more detail, following Efron and Tibshirani (1994), assume $\mathbf{x}$ is composed of the pairs $(\mathbf{z}, \mathbf{y})$ and P is an unknown probability model defined as $P = (F, G)$ which can be estimated to be $\hat{P} = (F_n, G_n)$ (p.88). Illustration of how $\hat{P}$ can be estimated using bootstrap algorithm is given in Figure 2.

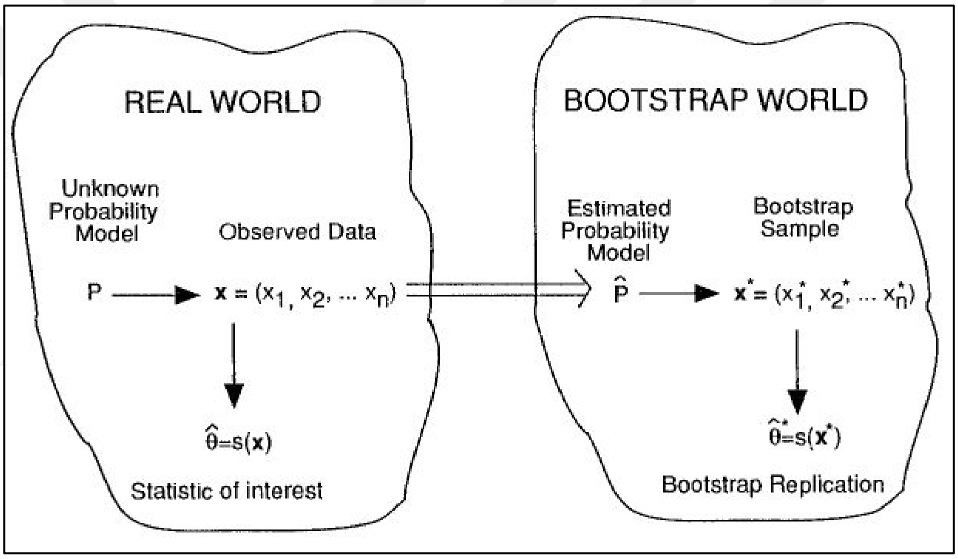


Figure 2. Bootstrap approach to the estimation of P^

Source: Efron and Tibshirani, 1994.

In Figure 2 bootstrap process begins with evaluation of observations $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ to estimate a statistic of interest $\hat{\theta}$ from classical probability perspective. The right panel of the Figure 2, on the other hand, shows how bootstrap is put into use in the estimation of the probability model $\hat{P}$. In principal, the same algorithm by which F_n is estimated is followed to get the estimated probability model $\hat{P}$. However, it is important to note that, unlike the single sample case, bootstrap samples $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$ of size n drawn with replacement consist of the pairs $x_i = (z_i, y_i)$. Using this way, statistic of interest θ^* and its distribution can be derived.

In order to associate the two sample concept with regression models and the ordinary least squares approach, following Efron and Tibshirani (1994), assume that unknown probability model P is defined as

$$P = (\boldsymbol{\beta}, F) \tag{26}$$

where $\boldsymbol{\beta}$ is the parameter vector of regression coefficients and F is the probability distribution of the error terms. In order to obtain the estimation of the probability model $\hat{P}$, estimation of $\widehat{\boldsymbol{\beta}}$ and F_n, first of which is already available from the ordinary least squares estimate, is required, (Efron and Tibshirani, 1994, p.111). In order to obtain F_n approximate errors can be deployed. By the words of Efron and Tibshirani (1994);

“How can we estimate F_n ? If $\boldsymbol{\beta}$ were known we could calculate the errors $\epsilon_i = y_i - \mathbf{c_i}\boldsymbol{\beta}$ for $i = 1,2, \dots, n$ and estimate F by their empirical distribution. We don’t know $\boldsymbol{\beta}$, but we can use $\widehat{\boldsymbol{\beta}}$ to calculate approximate errors

$$\hat{\epsilon}_i = y_i - \mathbf{c_i}\widehat{\boldsymbol{\beta}} \qquad \text{for i} = 1,2, \dots, \text{n}. \tag{27}$$

… The obvious estimate of F is the empirical distribution of the $\hat{\epsilon}_i$, which can be stated as $F_n\!: probability\ \ 1/n\ \ on\ \hat{\epsilon}_i \qquad for\ i = 1,2, \dots, n$" (p.111).

2.3.3.1. Regression with Bootstrapped Pairs

One way of applying bootstrap in linear regression models is performing resampling process within the pairs $\mathbf{x_i} = (z_i, y_i\ )$ in a straightforward fashion. Accordingly, the algorithm of bootstrapping pairs is,

1. Sample from $\mathbf{x}$ with replacement to get bootstrap pairs $\mathbf{x}^* = (\mathbf{x_1^*}, \mathbf{x_2^*}, \dots, \mathbf{x_n^*})$, where $\mathbf{x}_i^* = (z_i^*, y_i^*)$
2. Set $\mathbf{x} = \mathbf{x}^*$
3. Fit ordinary least squares regression to $\mathbf{x}$ giving bootstrapped coefficient estimate $\boldsymbol{\beta}^*$.
4. Repeat the process for B times (Davison and Hinkley, 1997, p.264).

It should be noted, however, that this type of bootstrapping may not be appropriate for all data types where the sequences are not exchangeable, as discussed above. Besides, it should be kept in mind that bootstrapping pairs to estimate regression parameters is a clear violation in Ordinary Least Square framework, where covariates are assumed to be fixed and non-stochastic.

2.3.3.2. Regression with Bootstrapped Residuals

A different approach to bootstrap in linear regression is resampling estimated error terms $\hat{\epsilon}_i$'s of a regression model. More precisely, following Efron and Tibshirani (1994), assume an ordinary least squares regression is fitted using the observation set $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_n}) = ((\mathbf{c_1}, y_1), (\mathbf{c_2}, y_2, \dots, (\mathbf{c_n}, y_n))$ where y_i indicates the response variable, while $\mathbf{c_i}$ is the covariate vector (P.111). This approach comprises of generating $\mathbf{x}^*$ by resampling residuals $\hat{\epsilon}_i$ to get bootstrapped residuals $\epsilon^* = (\epsilon_1^*, \epsilon_2^*, \dots, \epsilon_n^*)$ that are employed in the construction of bootstrapped response variable y^*,

$$y_i^* = \mathbf{c_i}\widehat{\boldsymbol{\beta}} + \epsilon_i^* \tag{28}$$

(Efron and Tibshirani, 1994, p.111).

With the y_i^* at hand, now it is possible to estimate the bootstrap least squares estimate and its variance by using the same set of covariate vector $\mathbf{c_i}$,

$$\boldsymbol{\beta}^* = (\mathbf{C^T C})^{-1}\mathbf{C^T}\mathbf{y}^* \tag{29}$$

$$\mathrm{var}(\boldsymbol{\beta}^*) = (\mathbf{C^T C})^{-1}\mathbf{C^T}\mathrm{var}(\mathbf{y}^*)\mathbf{C}(\mathbf{C^T C})^{-1} \tag{30}$$

$$= \sigma_{\mathrm{F}}^2(\mathbf{C^T C})^{-1} \tag{31}$$

(Efron and Tibshirani, 1994, p.112).

This method is questioned by Chernick (2007) since fitting an incorrect model to the original sample would lead to distorted bootstrap samples (p.103).

CHAPTER III

BOOTSTRAPPING WITH DEPENDENT DATA STRUCTURES

3.1. Bootstrap in Time Series

Bootstrap method and resampling procedures in time series must be applied appropriately because of the autocorrelation involved in this data type. So far, two approaches, bootstrapping the original sample and bootstrapping the residuals, have been introduced. Among these approaches bootstrapping the original sample is not suitable for time series to draw repeated samples since it would deteriorate the autocorrelation among observations and would construct an irrelevant bootstrap sample in each repetition. Because of this, reason major focus in studies concerning the use of bootstrap with time series is on bootstrapping residuals transformed in an appropriate way.

In the literature two ways of bootstrapping in time series are suggested. First approach suggested is bootstrapping residuals as Efron and Tibshirani (1994) point out. Second method that can be employed in time series is moving block bootstrap proposed by Künsch (1989).

3.1.1. Bootstrap in Regression with Time Series

3.1.1.1. Bootstrapping residuals in regression with time series

To clarify how bootstrapping residuals in time series works, following Chernick (2007), assume y_t generates from a first order autoregressive process;

$$y_t = \beta_1 y_{t-1} + e_t, \tag{32}$$

where y_t is stationary and e_t is random innovations with a Gaussian distribution around zero.

The algorithm to bootstrap residuals in time series is as follows,

1. Estimate $y_t = \beta_1 y_{t-1} + e_t$ to get $\hat{\beta}_1$ and corresponding $\hat{e}_t$'s.
2. Sample with replacement from $\hat{e}_t$ to get $e_2^*, e_3^*, \dots e_t^*$.
3. Let $y_1^* = y_1$ and get $y_2^*, y_3^*, \dots, y_t^*$ using

$$y_2^* = \hat{\beta}_1 y_1^* + e_2^* , \quad y_3^* = \hat{\beta}_1 y_2^* + e_3^* , \quad \dots \quad , y_t^* = \hat{\beta}_1 y_{t-1}^* + e_t^* \tag{33}$$

4. Estimate the model $y_t^* = \beta_1^* y_{t-1}^* + v_t$ to obtain the bootstrapped statistic β_1^* .
5. Repeat the first four steps B times (Chernick, 2007, p.100).

3.1.1.2. Moving Block Bootstrap

An alternative approach that can be used in bootstrapping time series is moving block bootstrap. In this method, original time series at hand is bootstrapped on the basis of a predetermined block size of consecutive observations. As an example, basic method of bootstrapping pairs discussed above is a special case of moving block bootstrap where block size is one.

In his pioneering study Künsch (1989) states that ignoring the dependence in the data and bootstrapping time series observations result in misleading statistics and suggests choosing n/l blocks of length l with replacement from the $n - l + 1$ blocks of observed data (p. 1218). In this method blocks of consecutive observations are resampled, rather than resampling a single observation in the construction of bootstrap sample, so that the dependence structure of the original observations is sustained within the each block (Lahiri, 2003, p.25). In other words fundamental idea of using moving blocks to bootstrap the original observations is that blocks of length l of observations can be expected to be uncorrelated, provided that the series is stationary and block size l is sufficiently large to separate the series far enough in time in such a way that series can be treated as exchangeable (Chenick, 2007, p.104). Figure 3 illustrates the background of moving blocks bootstrap.

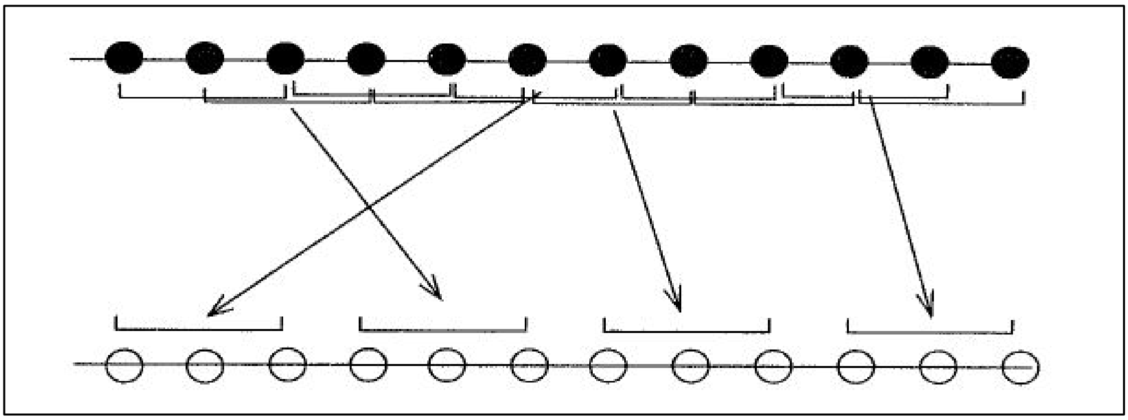


Figure 3. An illustration of the moving blocks bootstrap. The black circles in the figure represent the original sample. In this diagram block size is chosen to be three.

Source: Efron and Tibshirani, 1994

An advantage of the moving blocks bootstrap in linear regression is that, contrary to bootstrapping estimated residuals imposing the estimation of population parameter β, it offers a bootstrapping mechanism which is not dependent on a fitted model (Efron and Tibshirani, 1994, p.102).

Although moving blocks bootstrap is regarded more appropriate for dependent data structures like time series, it has disadvantages. First and foremost, theoretical requisite of block bootstrap that series must be stationary does not allow application of this method to time series. As an example, stationary condition of time series brings severe limitations on applicability of block bootstrap to construct a procedure testing the presence of unit root. Besides, two other highlighted disadvantages of block bootstrap are,

1. Resampled blocks may not exhibit exactly the same nature of the original time series observations.
2. Resampled blocks may tend to break up the dependence in the series (Chernick, 2008, p.15).

3.1.2.1.1. Choosing Optimal Length of Blocks

A crucial point that determines the accuracy of a statistics in moving blocks bootstrap is the chosen size of block length (Lahiri, 2003, p. 173). The principal logic in the selection of the block size lies in the tradeoff between choosing a long block size, leading to a well preserved dependence structure during the resampling process and choosing a short block size, rendering distinct values of statistic to be as many as possible so that resampling process yields a good distributional approximation of the statistic of interest (Davison and Hinkley, 1997, p.397).

To see how to set the optimal size of block length, following Davison and Hinkley (1997), let $\mathcal{K}$ indicate a statistic of interest based on a sample size of n, and l is the block size chosen. As mentioned above it is known that different block sizes generate different moving blocks estimates $\widehat{\mathcal{K}}(n, l)$ (Davison and Hinkley, 1997, p.401). The mean squared error of the moving blocks estimate is given as

$$\frac{1}{n^d}\left(\frac{C_1}{l^2}+\frac{C_2 l^c}{n}\right) \tag{34}$$

where C_1 and C_2 rely on $\mathcal{K}$ and the dependence structure of the series (Davison and Hinkley, 1997, p.402). In this expression d and c take the following values;

1. $d = 2$ and $c = 1$ if $\mathcal{K}$ is bias or variance,
2. $d = 1$ and $c = 2$ if $\mathcal{K}$ is a one-sided significance probability,
3. $d = 2$ and $c = 3$ if $\mathcal{K}$ is a two-sided significance probability (Davison and Hinkley, 1997, p.402).

Under this setting the algorithm to estimate the optimal block length is as follows;

1. Choose an arbitrary block size of l and get $\widehat{\mathcal{K}}(n, l)$,
2. Choose a value for m and k such that $m < n$ and $k < l$ and estimate $\widehat{\mathcal{K}}(m, k)$. Mean square error estimate of $\widehat{\mathcal{K}}(m, k)$ is given as

$$MSE(m,k) = \frac{1}{n-m+1}\sum_{j=1}^{n-m+1}\left\{\widehat{\mathcal{K}_j}(m,k) - \widehat{\mathcal{K}}(n,l)\right\}^2 \tag{35}$$

3. Repeat this procedure for different values of k but with the same m, through which value of $\hat{k}$ minimizing $MSE(m, k)$ can be found.
4. Find the optimum block size $\hat{l}$ using
$$\hat{l} = \hat{k} \times (\mathrm{n}/\mathrm{m})^{1/(\mathrm{c}+2)} \tag{36}$$

(Davison and Hinkley, 1997, p.402).

3.1.1.3. Sieve Bootstrap

Sieve bootstrap is another approach that can be used in time series applications. Following Chernick and LaBudde (2011) let P denote the unknown joint probability distribution for an infinitely long time series with the infinite set of observations $[X_1, X_2, X_3, \ldots, X_n, \ldots,]$. On the other hand, further assume that F_n is the empirical distribution of the first n observations. In the case where i.i.d. property for X holds with population distribution of F, the joint distribution of the first n observations is derived by F^n, characterizing that a logical estimate of the joint distribution is $(F_n)^n$ (Chernick and LaBudde, 2011, p.128). On the other hand, as Chernick and LaBudde (2011) noted, if i.i.d. property for X does not hold, joint distribution cannot be derived through one-dimensional joint distribution, giving bootstrap a ground to approximate this joint distribution P as n increases (p.128).

There are different views in literature on the application of sieve bootstrap time series. Bühlmann (1997) suggests employing the joint distribution of an autoregressive regression to approximate P, by resampling from the residuals of AR(p) process fitted (p. 123). Another empirical strategy is to benefit from variable-length Markow chains and approximate the joint distribution P by means of simulation techniques (Bühlman, 2002, p.10).

3.1.1.4. Wild Bootstrap

Wild bootstrap proposed by Liu (1988) is another method applied to time series. Basic idea of this method relies on modifying the residuals of a suitably specified model in such as way that wild bootstrapped residuals are derived by $u^* = h_i(\hat{u}_i)\varepsilon_i$, where $\varepsilon_i$displays the property of having 0 mean and a variance of 1 and h_i is a transformation, in order to investigate the appropriate model parameter estimates and their standard errors under the presence of heteroscedasticity (Chernick and LaBudde, 2011, p. 141). In a further study Shao (2010) proposes the concept of dependent bootstrap. In this approach the main focus is on obtaining a procedure superior to block bootstrap with a broader applicability to time series (Chernick and LaBudde, 2011, p.126). The major difference between the algorithms of wild and dependent wild bootstrap is that the latter allows the auxiliary variable ε_i to be dependent so that the dependence structure in the data is reflected in the bootstrap samples (Shao, 2010, p.218).

3.2. Stationarity, Unit Root and Unit Root Tests

Concept of unit root has serious implications both in statistical and economical sense. In particular, most of econometric studies of empirical world attempting to reveal a casual long-run relationship between macroeconomic variables in the framework of ordinary least square regression model rely heavily on cointegration approach, which can only be employed after a complete investigation of stationarity of variables involved. Besides, stationary is an important concept in statistical inference in regressions with time series. In particular, a time series trending upwards over time may invalidate all asymptotic properties and convergences defined and utilized in statistical decisions (Baltagi,2008, p.355).

Broadly speaking stationarity of time series is defined in terms of the mean and variance of time series (Goldberger, 1991, p.281). In more detail, a stationary time series is

1. expected to exhibit a tendency towards returning a constant mean around which it fluctuates,
2. inclined to have a time-independent variance,
3. observed to have a correlogram that dies away as lag length increases (Enders, 2008, p. 2012).

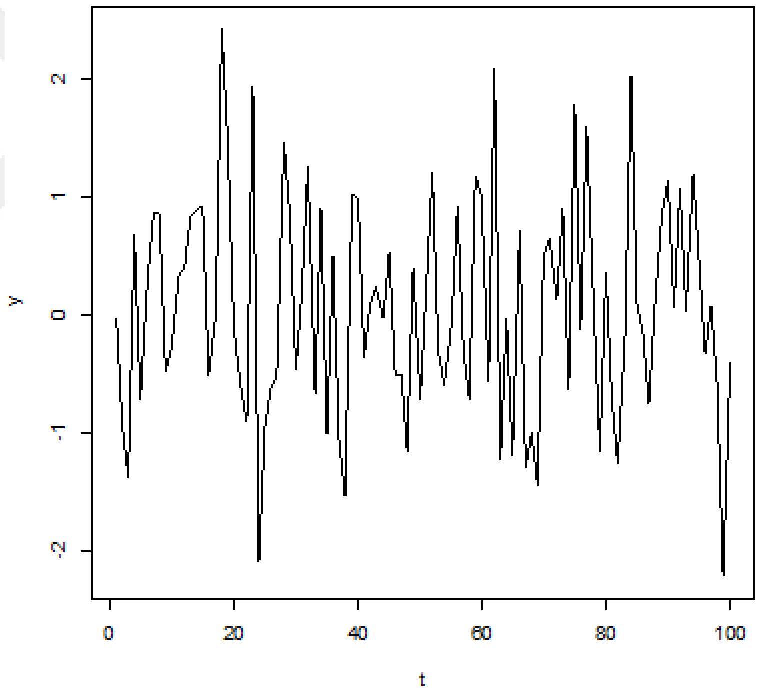


Figure 4. A realization of stationary process with mean zero and variance 1

Figure 4 presents a stationary process of 100 independent samples drawn from the population $\boldsymbol{N}(\mathbf{0},\mathbf{1})$. As can be expected, the series moves around a constant mean zero with a constant variance of 0.886. On the other hand, properties of a nonstationary series are as follows;

1. A long-run mean constant mean series returns is not observed,

2. Variance of the series changes across time and converges to infinity as time dimension expands,
3. Autocorrelations do not diminish, but in finite sample case, correlograms die away gradually (Enders, 2008, p. 2012).

Figure 5, shows realization of two nonstationary time series, namely random walk and random walk with drift. In the upper panel of Figure 5, a sample of 100 drawn from a population with the data generating process;

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{37}$$

where $\rho = 1$ and ε_t is an independent and identically distributed error term with $N(0,1)$. In order to simulate Y_t recursively, initial value Y_0 is set to be equal to zero, that is, Y_1 is equal to ε_1. The estimated value $\hat{\rho}$ that is obtained from ordinary least square estimation of the model $Y_t = \rho Y_{t-1} + \varepsilon_t$ for the series drawn is 0.98. As can be expected random walk process series do exhibit the properties characterizing nonstationary time series with a time variant pattern and increasing variance as time approaches infinity. In the lower panel of Figure 5, a random walk with drift process is seen. This 100 observations are drawn from the population defined in the form of

$$Y_t = \phi_1 + \rho Y_{t-1} + \varepsilon_t \tag{38}$$

where $\phi_1 = 1.2$ and $\rho = 1$ and ε_t is an independent and identically distributed error term with $N(0,1)$. In simulating the data generating process of random walk with drift, same algorithm used in random walk is followed, together with a deterministic intercept of 1.2, creating a "drift" leading to a stochastic trend.

Stationarity of a time series is directly related to the presence of unit root in time series. Connection between stationarity and unit root stems from the fact that the first and the second moments of an autoregressive form upon which the stationarity concept is defined are strongly influenced by the value of ρ. To be more precise, following Maddala and Kim (1998), assume time series x_t and y_t satisfy the relationship

$$x_t = \rho x_{t-1} + u_t \qquad |\rho| < 1 \tag{39}$$

$$y_t = y_{t-1} + v_t \tag{40}$$

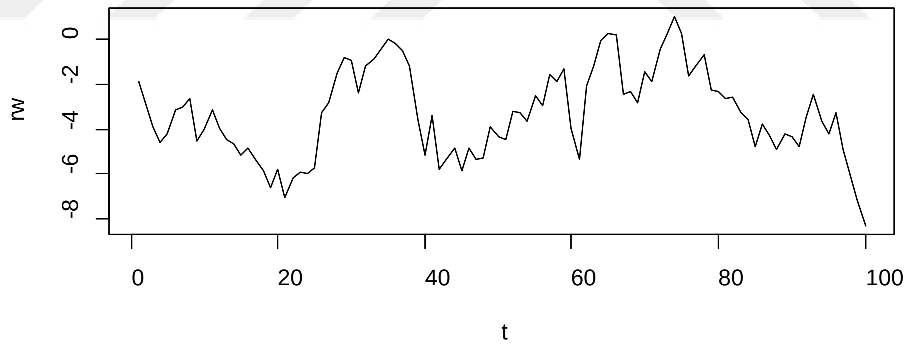
where u_t and v_t indicate white noise error terms with independently identically distributed with $N(0,1)$. Investigation of stationariy of the series x_t and y_t requires inspection of the mean (the first moment), the variance and covariance (the second moment) of the series (Maddala and Kim, 1998, p.21). To that end, let both stochastic processes x_t and y_t be rearranged by successive substitution as,

$$x_t = \rho x_{t-1} + u_t = \rho(\rho x_{t-2} + u_{t-1}) + u_t = \cdots \tag{41}$$

$$= \rho^t x_0 + u_t + \rho u_{t-1} + \cdots + \rho^{t-1} u_1 \tag{42}$$

$$= \rho^t x_0 + \sum_{i=0}^{t-1} \rho^i u_{t-i} \tag{43}$$

$$y_t = y_0 + \sum_{i=0}^{t-1} v_{t-i} \tag{44}$$



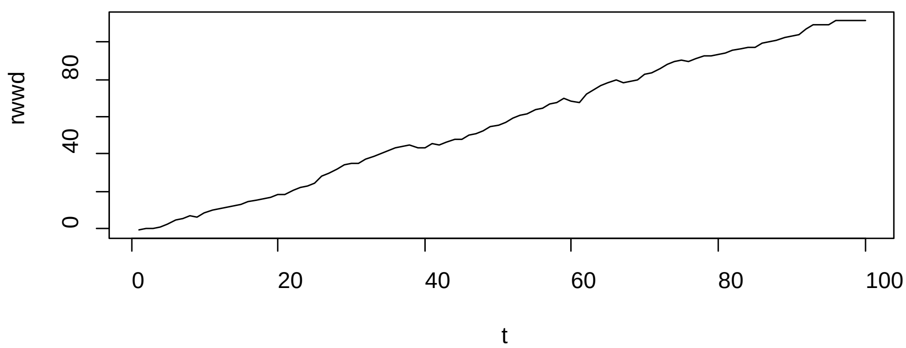


Figure 5. Realizations of random walk and random walk with drift processes.

Assuming the initial conditions are $x_0 = 0$ and $y_t = 0$ yields $E(x_t) = 0$ and $E(y_t) = 0$. On the other hand, the variances of these two stochastic processes are[1],

$$var(x_t) = \sum_{i=0}^{t-1} \rho^{2i} var(u_{t-i}) \rightarrow \frac{1}{1-\rho^2} \tag{45}$$

$$var(y_t) = \sum_{i=0}^{t-1} var(v_{t-i}) = t \tag{46}$$

where → indicates to where the expression on the left hand side converges as time approaches infinity.

Not least, autocovariances of two processes are given as,

$$\gamma_r^x = E(x_t x_{t+r}) = E[(\sum_{i=0}^{t-1} \rho^i u_{t-i})(\sum_{i=0}^{t+r-1} \rho^i u_{t+r-i})] \tag{47}$$

$$= \sum_{i=0}^{t+r-1} \rho^i \rho^{r+i} \tag{48}$$

And

$$\gamma_r^y = E(y_t y_{t+r}) = E[(\sum_{i=0}^{t-1} v_{t-i})(\sum_{i=0}^{t+r-1} v_{t+r-i}) = (t - r) \tag{49}$$

What these moments of the processes x_t and y_t tell us is that they give a detailed exposition of the characteristics of the mean, the variance and the autocovariances of the series. First and foremost, the value of the mean for both series is not influenced whether $|\rho| = 1$ or $|\rho| < 1$ in the autoregression form with no intercept. However, the form of variance and covariances are highly dependent on the value of ρ, as can be observed from y_t's variance and autocovariance functions, both of which are a function of time. What's more, for the stochastic process x_t where $|\rho| < 1$, it is observed that both variance and covariance converge to a constant as time approaches infinity. Therefore, as time increases, the variance of y_t increases whereas that of x_t converges to a constant (Maddala and Kim, 1998, p. 22).

Same investigation can also be extended to the case where a constant term is involved in the data generating process of the series x_t and y_t. For this case, assume the series have the following processes,

[1] Derivaiton of the first and the second moments of the processes is based on Maddala and Kim (1998).

$$x_t = \phi_1 + \rho x_{t-1} + u_t \qquad |\rho| < 1 \tag{50}$$

$$y_t = \phi_1 + y_{t-1} + v_t \tag{51}$$

Using successive substitution we get,

$$x_t = \rho^t x_0 + \phi_1 \sum_{i=0}^{t} \rho^i + \sum_{i=0}^{t} \rho^i u_{t-i} \tag{52}$$

And

$$y_t = y_0 + \phi_1 t + \sum_{i=0}^{t-1} v_{t-i} \tag{53}$$

The variance and the autocovariance of x_t and y_t under the data generation process of (1) is the same with random walk process. However, in this case where data generating process involves a constant ofϕ_1 first moment of the stochastic processes also show important differences. More formally, the means of these processes are,

$$E(x_t) \to \frac{\phi_1}{1-\rho} \tag{54}$$

$$E(y_t) \to \phi_1 t \tag{55}$$

In this case, not only the variance and the covariance of the stochastic process y_t defined under the condition that $|\rho| = 1$ is a function of time, but also its mean changes across time.

Consequently, investigating whether a time series under consideration has unit root or not is equivalent to investigating the stationarity condition of time series. The stationarity condition of time series can be examined by several ways;

1. Visual examination of time series; A mean exhibiting substantial differences across time signals nonstationarity in time series
2. An inspection of the autocorrelation functions of the series; rapidly decaying autocorrelation functions may be a sign of stationarity.

3. Hypothesis testing of the presence of unit root; statistical decision that null of the presence of unit root (i.e. $\rho = 1$) is not rejected can be interpreted as series is nonstationariy (Stundenmund, 2000, p. 426).

There are two ways to transform a nonstationary series into stationary one. First one is detreding the series and therefore removing deterministic trend. Nonstationary series that can be transformed to be stationary in this way are called trend stationary time series. Second solution to the problem of nonstationary is differencing the series. This kind of series is called difference stationary.

Needless to say, the most common approach of such investigation is to perform unit root tests and during this subsection, principal attention will be focused on unit root tests conventionally used in empirical studies.

3.2.1. Economic Implications of Unit Root

Unit root is a concept commonly discussed in economics and mostly considered from a mathematical perspective. The intuitive idea behind this phenomenon, however, is a set of common characteristics peculiar to most of macroeconomic variables.

First of all, presence of unit root in most macroeconomic variables implies that random shocks have an enduring effect on the subsequent values of the variables, rendering fluctuations of greater persistence. In Perron (1994)'s own words;

*"As far as macroeconomic theories are concerned the most important implication of the unit root revolution is that under this hypothesis random shocks have a permanent effect on the system. Fluctuations are too transitory. This implication, as forcefully argued by Nelson and Plosser, has profound consequences for business cycle theories. It runs counter to the prevailing view that business cycles are transitory fluctuations around a more or less stable trend path. It is therefore important to assess carefully the reliability of the unit root hypothesis as an empirical fact" (*p.113).

This implication of unit root is in particularly informative for policy makers and empirical researchers, since information regarding the presence of unit root would allow one to assess whether one time shocks observed in important time series of an economy have permanent effect in the future.

3.2.2. Conventional Unit Root Tests

3.2.1.1. Unit Root Tests without Structural Breaks

3.2.1.1.1. Augmented Dickey Fuller Unit Root Test

In Dickey Fuller (DF) unit root test Dickey and Fuller (1979) depart from a first order autoregressive model as the data generating process;

$$Y_t = \rho Y_{t-1} + \epsilon_t \quad (56)$$

or equivalently;

$$\Delta Y_t = \delta Y_{t-1} + \epsilon_t \quad (57)$$

where ϵ_t is a sequence of independent normal random variables with mean zero and variance σ^2 and $\delta = (1-\rho)$. Given this data generating process, Dickey and Fuller (1979) attempt to determine whether coefficient ρ is statistically equal to 1. To this end, they employ three models with different deterministic components under the alternative hypothesis;

$$\Delta Y_t = \delta Y_{t-1} + \epsilon_t \quad (58)$$

$$\Delta Y_t = \mu + \delta Y_{t-1} + \epsilon_t \quad (59)$$

$$\Delta Y_t = \mu + \beta t + \delta Y_{t-1} + \epsilon_t \quad (60)$$

The first model here is a pure random walk, the second is a random walk with a drift component μ and the third is random walk with drift and a deterministic trend. DF test is performed under the null hypothesis that $\rho = 1$ (or $\delta = (1-\rho) = 0$). Hypothesis-testing regarding the statistical value of δ requires a special attention. As Dickey and Fuller (1979) show, with $\rho = 1$ under the null hypothesis, test statistics for $\delta = 0$ does not converge to Student's t distribution. Therefore they construct a new distribution to extract critical values to be used in testing $\delta = 0$. This new statistics is called τ (tau). Dickey and Fuller (1979) report the critical values for the test statistics τ

for all the three models and different sample sizes, for the test statistics τ that can be used to test whether $\delta = 0$ is highly dependent on the inclusion of drift and trend terms and sample size. In a further study, these critical values are expanded and reported more accurately by MacKinnon (2010).

Strong assumption regarding the nature of the error term ϵ_t caused some revisions in DF unit root test. In DF unit root test Dickey and Fuller (1979) assumed that ϵ_t is a sequence of independent normal random variables with mean zero and variance σ^2. In Augmented Dickey Fuller (ADF) unit root test this assumption was replaced with a weaker assumption stating that error term u_t is uncorrelated in the following equations;

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t \tag{61}$$

$$\Delta Y_t = \mu + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t \tag{62}$$

$$\Delta Y_t = \mu + \beta t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t \tag{63}$$

This form of DF test is known as Augmented Dickey Fuller (ADF) unit root test. MacKinnon (2010) states that asymptotical distribution of τ is not affected from inclusion of lagged dependent variables $\sum_{i=1}^{m} \alpha_i \Delta Y_{t-i}$ in these models (p. 4). Therefore, critical values simulated for DF test can also be used for testing the hypothesis that $\delta = 0$ in ADF unit root test.

3.2.2.1.2. Phillips – Perron Unit Root Test

In ADF unit root test the problem of autocorrelation in error terms is solved by adding lagged dependent variables into the regression equation through which the validity of the null hypothesis is tested. This parametric approach of ADF test is alternated by Phillips and Perron (1988)'s approach. In Phillips-Perron (PP) test, Phillips and Perron (1988) utilize from a nonparametric methodology allowing for the investigation of unit root in weakly dependent and heterogeneous time series, under the null hypothesis that series has unit root. To this end, Phillips and Perron (1988) assume error term u_t to have the following conditions;

1. $E(u_t) = 0$ for all t
2. $sup_t E|u_t|^{\beta+\varepsilon} < \infty$ for some $\beta > 2$ and $\varepsilon > 0$
3. as $T \to \infty$, $\sigma^2 = limE(T^{-1}S_T^2)$exists and $\sigma^2 > 0$, where $S_t = u_1 + \cdots + u_t$
4. $\{u_t\}$ is strong mixing with mixing coefficients $\alpha_m$that satisfy $\sum \alpha_m^{1-2/\beta} < \infty$, where the sum is over $m = 1, \dots, \infty$.

Under this specification of error terms, Phillips and Perron (1988) state that 2nd and 4th conditions control the permissible heterogeneity and the allowable temporal dependence of the process (p. 336). In unit test procedure, Phillips and Perron (1988) focus in particularly on the following models' estimated coefficients and distribution of their test statistics;

$$y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + \hat{u}_t \tag{64}$$

$$y_t = \tilde{\mu} + \tilde{\beta}\left(t - \frac{1}{2}T\right) + \tilde{\alpha} y_{t-1} + \tilde{u}_t \tag{65}$$

Introducing a weakly dependent and heterogeneous process, Phillips and Perron (1988) state that test statistics' distribution under this setting has the same form with that of Dickey and Fuller (1979) (p.339). Furthermore, authors discuss under which circumstances their approach is more beneficial. In this regard they state that their approach is particularly advantageous for time series with moving average components (p. 336). In addition to that, this approach can also be applicable to the processes with independent and identically distributed errors (Phillips and Perron, 1988, p. 345).

3.2.2.1.3. Kwiatkowski–Phillips–Schmidt–Shin (KPSS) Unit Test

Another test proposed in literature in an attempt to investigate whether series contain a unit root or not is Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. In this approach Kwiatkowski et al. (1992) propose a test whose null hypothesis is the series is stationary around a deterministic trend, employing the Lagrange Multiplier (LM) test. Kwiatkowski et al. (1992) use a decomposer approach in time series and focus on three major components. These components are a deterministic trend, a random walk, and a stationary error. According to KPSS test, the null of trend stationarity condition is equivalent to the hypothesis that the variance of the random walk is equal to zero (Kwiatkowski et al., 1992, p. 161). Kwiatkowski et al. (1992) state that with the introduction of some additional assumptions combined with normal error terms, LM test statistics to be calculated for the hypothesis would converge to locally best invariant (LBI) test statistics (p. 161). However, since, as most of empirical studies show, these assumptions are not so realistic, Kwiatkowski et al. (1992) derive asymptotical distribution of the test statistics, by providing a modified version of the one-sided LM statistics.

Kwiatkowski et al. (1992) start with decomposing series into three components;

$$y_t = \xi t + r_t + \varepsilon_t \tag{66}$$

where t, r_t and ε_t show deterministic trend, random walk characterized by $r_t = r_{t-1} + u_t$ with $u_t \sim iid(0, \sigma_u^2)$ and stationary error, respectively. In KPSS unit root test stationarity of the series is examined by testing the hypothesis that $\sigma_u^2 = 0$. In this test if ε_t is assumed to be stationary, then null hypothesis turns out to be y_t is a trend stationary process, whereas if $\xi t = 0$ is assumed then the null hypothesis is y_t is stationary around a level (Kwiatkowski et al., 1992, p. 162).

KPSS unit root test relies on the residuals obtained from fitted autoregressive models with constant and with constant trend. In this sense assume $e_t, t = 1,2,\dots,T$ be the residuals obtained from the model

$$y_t = \alpha + \xi t + \varepsilon_t \tag{67}$$

and assume $\hat{\sigma}_\varepsilon^2$ is the standard error of the regression above. Then Kwiatkowski et al (1992) give the LM test statistics as

$$LM = \sum_{t=1}^{T} S_t^2 / \hat{\sigma}_\varepsilon^2 \tag{68}$$

where $S_t = \sum_{i=1}^{t} e_i$, t=1,2,…,T. When a deterministic trend is involved in the regression model, corresponding test statistics can only be used to test the null that y_t is a trend stationary process, while residuals obtained from a regression with a constant only can be used to test the null of level stationarity (Kwiatkowski et al., 1992, p.163). Kwiatkowski et al. (1992) derive asymptotical distribution of the test statistics and provide a modified version of the one-sided LM statistics which can be used to test the presence of unit root in series.

3.2.2.2. Unit Root Test with Structural Breaks

With the introduction of high computing capabilities and importance of stationarity for analyses time series used as the variables several unit root tests taking structural breaks into account have been suggested. These tests can be divided into two groups. The first group is the unit root tests in which the break date is considered as exogenous. In this approach a-prior knowledge regarding the date of the break or the regime shift is incorporated without a formal practice. Second group of unit root tests with structural breaks is those in which break date is allowed to be determined within the model. Unit root tests with endogenous structural break can also be classified depending on the number of breaks, as some of these tests allow for a single break only, while some tests take more than one breaks into consideration.

3.2.2.2.1. Unit Root Tests with Exogenous Structural Break

3.2.2.2.1.1. Perron Unit Root Test

First unit root test allowing for structural breaks in the series is Perron unit root test. Perron (1989) highlights that unit root presence in the economic time series and therefore persistence of the economic shocks may be caused by structural changes which deeply affect economies (p. 1362). Perron (1989) defines structural break on three bases. In the first place, a structural break may occur in intercept as a result of a crash (one time increase or a decrease) change in the level of the series. Secondly, a smooth slope change such as a productivity slow-down may be observed. Finally, a change both in level and slope may be seen. Accordingly, in his path breaking study, Perron (1989) investigates the presence of unit root in Nelson and Plosser data, by choosing the events of Great Depression in 1929 and oil crisis in 1979 as the structural break dates in level (crash) and trend (a change in slope), respectively. Perron (1989) starts with a data generating process $y_t$which is characterized by random walk with drift. Under this process Perron (1989) constructs three hypothesis based on three models;

Model (A) $y_t = \mu + dD(TB)_t + y_{t-1} + e_t,$

Model (B) $y_t = \mu_1 + y_{t-1} + (\mu_2 - \mu_1)DU_t + e_t,$

Model (C) $y_t = \mu_1 + y_{t-1} + dD(TB)_t + (\mu_2 - \mu_1)DU_t + e_t.$

Where;

$$D(TB)_t = 1 \quad if \quad t = T_B + 1, \quad 0\ otherwise \tag{69}$$

$$DU_t = 1 \quad if \quad t > T_B, \quad 0\ otherwise \tag{70}$$

$$A(L)e_t = B(L)v_t, \tag{71}$$

In this expression, $v_t \sim i.i.d.(0, \sigma^2)$ and $A(L)$ and $B(L)$ are pth and qth order polynomials, respectively (p. 1364). On the other hand, alternative hypothesis against the nulls with structural breaks are given as,

Model (A) $y_t = \mu_1 + \beta t + (\mu_2 - \mu_1)DU_t + e_t,$

Model (B) $y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + e_t,$

Model (C) $y_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1)DU_t + (\beta_2 - \beta_1)DT_t + e_t$

Here, T_B indicates the time of break. In this setting, Model A corresponds to crash change, while Model B refers to "changing growth" model. Model C, on the other hand incorporates a change both in level and trend. (Perron, 1989, p. 1364). Perron (1989) starts with regressing the series y_t on appropriate model among Model (A), Model (B) and Model (C) to obtain the residuals $\tilde{y}_t^i$ which will be used in $\tilde{y}_t^i = \tilde{\alpha}^i \tilde{y}_{t-1}^i + \tilde{e}_t$, where i:A for the model with a constant, a time trend and DU_t, i: B for the models with a constant, a time trend and DT_t^* and i: C for the model with a constant, a time trend, DU_t and DT_t. Perron (1989) notes that to derive asymptotical values of the statistics of interest requires some additional effort compared to ADF test because of two reasons,

1. there are unusual regressors under the null hypothesis, which in turn influences the distribution of t_{α^i},
2. existence of sample's divided nature, resulting from break dummies involved under the null hypothesis (p. 1373)

To tackle these additional characteristics, Perron (1989) computes asymptotical distribution and corresponding critical values of t_{α^i}, depending on the parameter λ, the ratio of the number of observations after break date to the total number of observations (p. 1375).

Perron (1994) admits that a single break in economic series may be translated into being too optimistic on the stability of these series. However, he still advocates that this form of pattern is still able to reflect most of economic variables' nature (p.114). Furthermore, Perron (1994) is aware that introducing structural breaks into time series methodology implies adding randomness dimension to intercept and trend coefficients, albeit this randomness is rarely observed over quite long span of time. Perron (1994) claims that the intuitive behind a modeling in which structural break dates and kinds are incorporated through observed infrequent randomness in intercept and trend of economic time series still provides opportunity to benefit from exogenous break dates in the investigation of unit root in the series (p. 114).

3.2.2.2.2. Unit Root Tests with Endogenous Structural Break

3.2.2.2.2.1. Zivot Andrews Unit Root Test

A new chapter in the investigation of unit root presence in economic time series begun with the seminal study of Zivot and Andrews (1992). In this study authors suggest estimation of break dates within the testing procedure, rather than externally imposing them as Perron (1989) proposes. More specifically, Zivot and Andrews (1992) criticize Perron (1989)'s exogenous approach on the ground that the period perceived as a structural break in Perron (1989)'s view may belong to either tail of the distribution of the underlying data generating process (p. 252). In this regard, Zivot and Andrews (1992) offer a more formal way utilizing a data dependent algorithm to inspect whether a realization can be regarded as a structural break.
Zivot and Andrews (1992) state their null hypothesis as

$$y_t = \mu + y_{t-1} + e_t \tag{72}$$

To determine the break dates endogenously Zivot and Andrews (1992) assume that y_t can be modeled as a trend stationary process with a one time break in the trend whose date is unknown (p. 254). In so doing, the authors aim to determine the point in which the testing process is the closest to accept the trend stationary alternative hypothesis. In Perron (1989)'s terms, Zivot and Andrews (1992) state that λ is chosen as the break point where the test statistics α^i $(i = A, B, C)$ is minimized. In light of this discussion, Zivot and Andrews (1992) suggest the following models to test the null that stipulates series is a random walk with drift;

$$y_t = \hat{\mu}^A + \hat{\theta}^A DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^A \Delta y_{t-j} + \hat{e}_t \quad (73)$$

$$y_t = \hat{\mu}^B + \hat{\beta}^B t + \hat{\gamma}^B DT_t^*(\hat{\lambda}) + \hat{\alpha}^B y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^B \Delta y_{t-j} + \hat{e}_t \quad (74)$$

$$y_t = \hat{\mu}^C + \hat{\theta}^C DU_t(\hat{\lambda}) + \hat{\beta}^C t + \hat{\gamma}^C DT_t^*(\hat{\lambda}) + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^C \Delta y_{t-j} + \hat{e}_t \quad (75)$$

With this parameterization and modeling, Zivot Andrews unit root test's critical values differ substantially from that of Perron (1992). Accordingly, Zivot and Andrews (1992) provide asymptotical and small sample critical values corresponding to different λ ratios for the null hypothesis that $\alpha^i = 1$ $(i = A, B, C)$.

3.2.2.2.2.2. Lumsdaine and Papell Unit Root Test

Departing from the idea that one time structural break in economic time series is not realistic, Lumsdaine and Papell (1997) suggest a new unit root test in which two endogenous breaks are included. To this end, Lumsdaine and Papell (1997) proposes the following models to test the presence of unit root with two structural break,

$$y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_y \quad (76)$$

$$y_t = \mu + \beta t + \theta DU1_t + \omega DU2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_y \quad (77)$$

$$y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t$$

$$+ \psi DT2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_y \quad (78)$$

where $DU1_t$and $DU2_t$are dummy variables for changes in level at time $TB1$ and $TB2$ while $DT1_t$ and $DT2_t$ are dummies indicating changes in trend (p. 213). To test the null hypothesis that series are random walk without drift, Lumsdaine and Papell (1997) focus on the test statistics associated with α in these models, and derive corresponding critical values through Monte Carlo simulations.

3.2.2.2.2.3. Lee and Strazicich Unit Root Test

So far all the unit root tests discussed above involve hypothesis testing procedures defined under the null hypothesis that series is random walk with or without drift. Lee and Strazicich (2003)'s seminal work, however, furthers the work of Lumsdaine and Papell (1997) by allowing two endogenous structural breaks not only under the alternative hypothesis but also under the null hypothesis. Lee and Strazicich (2003) claims that rejecting unit root null with a test without structural breaks under the null implies rejection of unit root without breaks and this result cannot necessarily be interpreted as rejection of a unit root with break (p. 1082). To overcome this uncertainty and ambiguous results, Lee and Strazicich (2003) propose an endogenous two-break Lagrange multiplier unit root test which permits structural breaks under both the null and alternative hypothesis. Lee and Strazicich (2003) state that, with a testing procedure as such, rejection of unit root null now means the series is trend stationary.

Using a different notation, Lee and Strazicich (2003) consider the following data generating process,

$$y_t = \delta' Z_t + e_t, \qquad e_t = \beta e_{t-1} + \varepsilon_t \tag{79}$$

where Z_t stands for a vector of exogenous variables and $\varepsilon_t \sim iidN(0, \sigma^2)$. To incorporate the two structural breaks under the null Lee and Strazicich (2003) specify that Model A allows for two changes in level by defining Z_t as

$$Z_t = [1, t, D_{1t}, D_{2t}]' \tag{80}$$

where $D_{ji} = 1$ for $t \geq T_{Bj} + 1, \quad j = 1,2,$ and 0 otherwise and by the same token, Model C has the vector of Z_t in such a way that

$$Z_t = [1, t, D_{1t}, D_{2t}, DT_{1t}, DT_{2t}]' \tag{81}$$

where $DT_{jt} = t - T_{Bj}$ for $t \geq T_{Bj} + 1, \quad j = 1,2$, and 0 otherwise (p. 1083).

Accordingly, Lee and Strazicich (2003) define alternative and corresponding null hypothesis of the Model A and C as follows

Model A

Null $\quad y_t = \mu_0 + d_1 B_{1t} + d_2 B_{2t} + y_{t-1} + v_1$

Alternative $\quad y_t = \mu_1 + \gamma t + d_1 D_{1t} + d_2 D_{2t} + v_{2t}$

Model C

Null $\quad y_t = \mu_0 + d_1 B_{1t} + d_2 B_{2t} + d_3 D_{1t} + d_4 D_{2t} + y_{t-1} + v_1$

Alternative $\quad y_t = \mu_1 + \gamma t + d_1 D_{1t} + d_2 D_{2t} + d_3 DT_{1t} + d_4 DT_{2t} + v_{2t}$

Using LM (score) principle, Lee and Strazicich (2003) use the following regression to perform the unit root test,

$$\Delta y_t = \delta' \Delta Z_t + \phi \tilde{S}_{t-1} + u_t \tag{82}$$

where $\tilde{S}_t = y_t - \widetilde{\Psi}_x - Z_t \tilde{\delta}, \quad t = 2, \dots, T$ and the unit root null hypothesis is expressed by $\phi = 0$ (p. 1083).Therefore, Lee and Strazicich (2003) define LM test statistics as

$$\tilde{\rho} \to -\frac{1}{2}\frac{\sigma_\varepsilon^2}{\sigma^2}\left(\int_0^1 \underline{V}_B^{(m)}(r)^2 dr\right)^{-1} \tag{83}$$

$$\tilde{\tau} \to -\frac{1}{2}\frac{\sigma_\varepsilon}{\sigma}\left(\int_0^1 \underline{V}_B^{(m)}(r)^2 dr\right)^{-\frac{1}{2}} \tag{84}$$

where

$$\sigma_\varepsilon^2 = \lim_{T\to\infty} T^{-1} E(\varepsilon_1^2 + \dots + \varepsilon_T^2)^2 \tag{85}$$

$$\sigma^2 = \lim_{T\to\infty} T^{-1} E(\varepsilon_1 + \dots + \varepsilon_T)^2 \tag{86}$$

and $\underline{V}_B^{(m)}(r)$ is defined for $m = A$ and C, respectively (p. 1083). Lee and Strazicich (2003) show that $\underline{V}(r) = V(r) - \int_0^1 V(r)dr$ which leads the authors to the conclusion that the asymptotic null distribution of the test statistics for Model A is independent of the break dates' location λ and magnitude of breaks. On the other hand, Lee and Strazicich (2003) note that the invariance of the test statistics to the location and magnitude of breaks does not hold for Model C.

3.2.3. Unit Root Tests with Bootstrap

Time series econometrics attributes a special importance to the presence of unit root in time series for the presence of unit root may have serious consequences on the results of applications. Because of this importance, a vast literature has been formed on the investigation of unit root, utilizing some assumed distributions of test statistics.

The principal behind hypothesis testing in time series including unit root tests is to assume a theoretical data generating process, of which we can observe only a single realization. In this framework, parameters in the relevant data generating process are attempted to be derived and their estimations are statistically tested. In other words, statistical inference with time series requires the presence of a population in which alternative realizations of the same series over the same course of time exist. While classical statistical inference approach in time series only assumes this population to exist, bootstrap approach claims to obtain this population.

The main reason why bootstrapped unit root tests are used is the consistencies between the exact and assumed distributions of a test statistics. This problem may be in particularly the case where only a limited a number of observations is available (Harris, 1992, 619).

3.2.3.1. Harris Unit Root Test Procedure

To test the presence of unit root in small sample case, Harris (1992) proposes a bootstrap approach by which exact empirical distribution of the test statistics is used rather than assumed one. Harris (1992) considers the most extensive ADF model;

$$\Delta y_t = \alpha y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{t-j} + \delta + \gamma t + v_t \tag{87}$$

Harris (1992) focuses on the t statistics of the parameter α by applying the following bootstrap algorithm;

1. Estimate the ADF equation with restriction that $\alpha = 0$, in order to obtain the residuals $\{\tilde{v}_t\}$.
2. draw 5000 random samples (with replacement) $\{v_t^*\}_1^n$ from the centered normalized residuals $\left\{\tilde{v}_t/(1-kn^{-1})^{1/2}\right\}_1^n$ where k and n denote number of regressors and sample size respectively. Then, generate y_t^* recursively under condition that $\alpha = 0$.
3. use y_t^* to estimate unrestricted ADF equation and obtain the distribution of α's t values.
4. Find the critical values corresponding to %1, %5 and %10 quintiles of the ordered t statistics (p. 619).

Presenting Monte Carlo simulation results regarding power and size properties of the approach he proposes, Harris (1992) shows that his method provides a considerable refinement on the performance of ADF test.

3.2.3.2. Psaradakis' Unit Root Test Procedure

Another approach suggested to test the presence of unit root with bootstrap is Psaradakis (1998)'s methodology. This approach depends on constructing a sieve so as to transform residuals of an unknown model to create bootstrap replicates. Psaradakis (1998) proposes the following steps for generating the test statistics of interest;

1. With the realization$\mathbf{X}_n$, obtain parameter estimates $(\hat{\phi}_{1,n}, \hat{\phi}_{2,n}, \dots, \hat{\phi}_{p,n})$ from the AR(p) model $\sum_{j=0}^{p} \phi_{j,n}\hat{u}_{t-j} = \varepsilon_{t,n}$ $(\phi_{0,n} \coloneqq 1)$, where $\hat{u}_t = y_t = x_t - x_{t-1}$, if $d_t = 0$ or $d_t = \beta_0$, and $\hat{u}_t = y_t - \bar{y}_n$ as $d_t = \beta_0 + \beta_1 t$, with $\bar{y}_n = n^{-1}\sum_{t=1}^{n} y_t$.
2. Construct a set of residuals $\left\{\hat{\varepsilon}_{t,n}\right\}_{t=p+1}^{n}$ using $(\hat{\phi}_{1,n}, \hat{\phi}_{2,n}, \dots, \hat{\phi}_{p,n})$ as

$$\hat{\varepsilon}_{t,n} = \sum_{j=0}^{p} \hat{\phi}_{j,n}\hat{u}_{t-j} \quad \text{for } t = p+1, \dots, n \tag{88}$$

where $\hat{\phi}_{0,n} = 1$.

3. Sample an i.i.d. set $\left\{\varepsilon_{t,n}^*\right\}_{t=1}^{n}$ from the empirical distribution function

$$\hat{F}_n(w) = (n-p)^{-1} \sum_{t=p+1}^{n} \mathbf{1}_{(-\infty,w]}(\hat{\varepsilon}_{t,n} - \bar{\hat{\varepsilon}}_n) \text{ for } w \in \mathbb{R} \quad (89)$$

where $\bar{\hat{\varepsilon}}_n = (n-p)^{-1} \sum_{t=p+1}^{n} \hat{\varepsilon}_{t,n}$

5. Obtain bootstrap errors $\{u_{t,n}^*\}_{t=1}^{n}$, using $\{\varepsilon_{t,n}^*\}_{t=1}^{n}$ with recursion,

$$\sum_{j=0}^{p} \hat{\phi}_{j,n} u_{t-j,n}^* = \varepsilon_{t,n}^* \text{ for } t = 1, \dots, n \quad (90)$$

where $(u_{1-p,n}^*, \dots, u_{0,n}^*) = (0, \dots, 0)$.

6. By means of $\{u_{t,n}^*\}_{t=1}^{n}$, obtain bootstrap replicates $\{x_{t,n}^*\}_{t=1}^{n}$ according to

$$x_{t,n}^* = x_{t-1,n}^* + u_{t,n}^* \text{ for } t = 1, \dots, n \quad (91)$$

If $d_t = 0$ or $d_t = \beta_0$, or,

$$x_{t,n}^* = \bar{y}_n + x_{t-1,n}^* + u_{t,n}^* \text{ for } t = 1, \dots, n \quad (92)$$

If $d_t = \beta_0 + \beta_1 t$, with $x_{0,n}^* = x_0$.

7. Bootstrap test statistics then can be calculated from,

$$B_{m,n}^* = n(\hat{\alpha}_{m,n}^* - 1) \text{ for } m = 0,1,2 \quad (93)$$

and

$$T_{m,n}^* = \frac{\hat{\alpha}_{m,n-1}^*}{\tau_{m,n}^*} \text{ for } m = 0,1,2 \quad (94)$$

where $\hat{\alpha}_{m,n}^*$ and $\tau_{m,n}^*$ are, respectively, the least square estimate of α^* and its standard error in the regression

$$x_{t,n}^* = \alpha^* x_{t-1,n}^* + d_t^* + u_{t,n}^* \quad (95)$$

where $d_t^* = 0$, $d_t^* = \delta_0^*$, or $d_t^* = \delta_0^* + \delta_1^* t$ as $m = 0, m = 1$, or $m = 2$

8 Repeat step 3 to 6 a large number of times (p. 582).

Psaradakis (1998) notes that specification of the AR structure in step 1 is an important issue and requires a special attention. Accordingly, Psaradakis (1998) states that AR(p) model in step 1 can be specified by using a complexity-penalized likelihood criterion such as Akaike information criterion (p. 582).

3.2.3.3. Smeekes and Taylor's Unit Root Test Procedure

Smeekes and Taylor (2012) propose a wild bootstrap scheme to test the unity of the roots of time series. In this framework, Smeekes and Taylor (2012), consider the data generating process,

$$y_t = x_t + \mu + \beta_T t, \quad t = 0,1,\dots,T \tag{96}$$

$$x_t = \rho_T x_{t-1} + u_t, \quad t = 0,1,\dots,T \quad \rho_T = 1 - c/T \tag{97}$$

$$u_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} = \psi(L)\varepsilon_t \quad (\psi_0 = 1) \tag{98}$$

by focusing on the hypothesis $H_0: c = 0$ against $H_1: c > 0$. Smeekes and Taylor (2012) propose the following algorithm to calculate bootstrap analogue of the unit root test statistics;

1. Detrend y_t using $\hat{x}^{\tau}_{t,\tilde{\gamma}} = y_t - \hat{\theta}'_{\tilde{\gamma}} z^{\tau}_t$
2. Estimate an augmented version of DF regression of order by ordinary least square and obtain the residuals

$$\hat{\varepsilon}^{\tau}_{q,t} = \Delta\hat{x}^{\tau}_{t,\tilde{\gamma}} - \hat{\lambda}\hat{x}^{\tau}_{t-1,\tilde{\gamma}} - \sum_{j=1}^{q} \hat{\phi}_{q,j} \Delta\hat{x}^{\tau}_{t-j,\tilde{\gamma},} \quad t = q+1,\dots,T \tag{99}$$

3. Replicate bootstrap errors ε^*_t using one of the following ways;

i) Resample with replacement from the centered residuals $(\hat{\varepsilon}^{\tau}_{q,t} - \bar{\hat{\varepsilon}}^{\tau}_{q,t})$

ii) Sample from $\varepsilon^*_t = \xi^*_t \varepsilon^{\tau}_{q,t}$ by using the function ξ^*_t where satisfies $\mathrm{E}^*\xi^*_t = 0$ and $\mathrm{E}^*{\xi^*_t}^2 = 1$

4. By recursive calculation, obtain $u_t^* = \sum_{j=1}^{q} \hat{\phi}_{q,j} u_{t-j}^* + \varepsilon_t^*$, using the estimated parameters $\hat{\phi}_{q,j}$ in step 2 and build x_t^* as $x_t^* = x_{t-1}^* + u_t^*$ with $x_0^* = 0$ and get $y_t^* = x_t^* + \theta^{*\prime} z_t$, t=0,1,…,T
5. Follow the same detrending procedure using the bootstrap sample y_t^* to obtain $\hat{x}_{t,\gamma}^{\delta *} = y_t^* - \hat{\theta}_\gamma^* z_t^\delta$ where $\hat{\theta}_\gamma^*$ is specified with the bootstrap replications. Calculate the bootstrapped augmented DF test statistics from

$$\Delta\hat{x}_{t,\gamma}^{\delta *} = \lambda^* \hat{x}_{t-1,\gamma}^{\delta *} + \sum_{j=1}^{p^*} \phi_{p^*,j} \Delta\, \hat{x}_{t-j,\gamma}^{\delta *} + \varepsilon_{p^*,t}^* y = p^* + 1, \dots, T \tag{100}$$

6. Repeat steps 3 to 5 N times to obtain bootstrap test statistics $DF - \gamma_b^{\delta *}$, $b = 1, \dots, N$ and corresponding critical values $cv_{\gamma,\tilde{\gamma}}^{\delta *}(\pi)$ as

$$cv_{\gamma,\tilde{\gamma}}^{\delta *}(\pi) = max\{x : N^{-1} \sum_{b=1}^{N} I(DF - \gamma_{\tilde{\gamma},b}^{\delta *} < x) \leq \pi\} \tag{101}$$

or, equivalently sort the statistics of interest in an ascending order and extract 1%, 5% and 10% critical values.

CHAPTER IV

BOOTSTRAP APPLICATIONS WITH TURKISH MACROECONOMIC VARIABLES

4.1. Bootstrapped Unit Root Test Applications with Turkish Macroeconomic Variables

In this chapter, principal focus is on identifying the characteristics of Turkish macroeconomic variables. For this purpose, firstly unit root presence in the series will be comprehensively investigated. Secondly, aggregate consumption function for the Turkish economy will be estimated.

To give a broad picture on the characteristics of Turkish macroeconomic time series, three sets of empirical tools will be used. In the first place, some conventional unit root tests such as ADF, PP and KPSS will be employed. Furthermore, unit root tests allowing for endogenous single and multiple structural breaks, namely Zivot Andrews (ZA), Lumsdaine Papell (LP) and Lee Strazicich (LS) unit root tests will be performed. The last but not the least, two parametric bootstrap unit root tests will be employed, in an attempt to conclude better on the presence of unit root in the series and to prevent inconsistencies between the exact and apparent significance levels in hypothesis testing, resulting mainly from size distortions observed in the cases where relatively small number of sample is present. These approaches are Harris' and Psaradakis' unit root testing methodologies. Additionally, results of these tests will be compared and characteristics of Turkish macroeconomic time series will be enlightened by taking the different aspects of the structure of the series into account.

4.1.2. Data

To explore the characteristics of the selected Turkish macroeconomic variables is the main focus of this study. To this end, Gross Domestic Product (GDP) and aggregated consumption (cons) in 1998 prices, Consumer Price Index (p) with 2003 base year, US Dollar-Turkish Lira nominal exchange rate (er), import (im) and exports (ex) in current US dollars, annual deposit interest rate (i) and money supply in m1 and m2 measures are among the variables focused. All series are expressed in logarithms, except for annual deposit interest rate (i) and extracted from Turkish Central Bank's Electronic Data Delivery System. The number of yearly observations ranges from 30 to 24. The series and the corresponding ranges are presented in Table 1.

Table 1.

List of Series and the Number of Observations Used in Analysis

Series	**Date Start-Date End**	**Number of Observation**
lnGDP	1985-2014	30
lncons	1987-2014	28
lnp	1985-2014	30
lner	1985-2014	30
lnex	1985-2014	30
lnim	1985-2014	30
i	1985-2014	30
lnm1	1991-2014	24
lnm2	1991-2014	24

Graph of the series in non-logarithmic and logarithmic terms are given in Figure 6 and Figure 7, respectively. Shaded areas in Figure 6 and Figure 7 correspond to the main financial and economic crises in the world, namely, 1997 Asia Financial Crisis, 2001 Turkish Economic Crisis and 2008 Housing Bubble and Credit Crisis.

4.1.3. Conventional Unit Root Tests

Primarily, stationarity of major Turkish macroeconomic variables is tested using some conventional unit root tests such as Augmented Dickey Fuller (1979) (ADF), Phillips Perron (PP) and Kwiatkowski–Phillips–Schmidt–Shin (1992) (KPSS). In ADF unit root test a methodology allowing for testing the presence of deterministic trend and constant components is followed based on the ADF specifications;

Model A: $\Delta Y_t = \phi_1 + \delta Y_{t-1} + \phi_3 t + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t$

Model B: $\Delta Y_t = \phi_1 + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t$

Model C: $\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t$

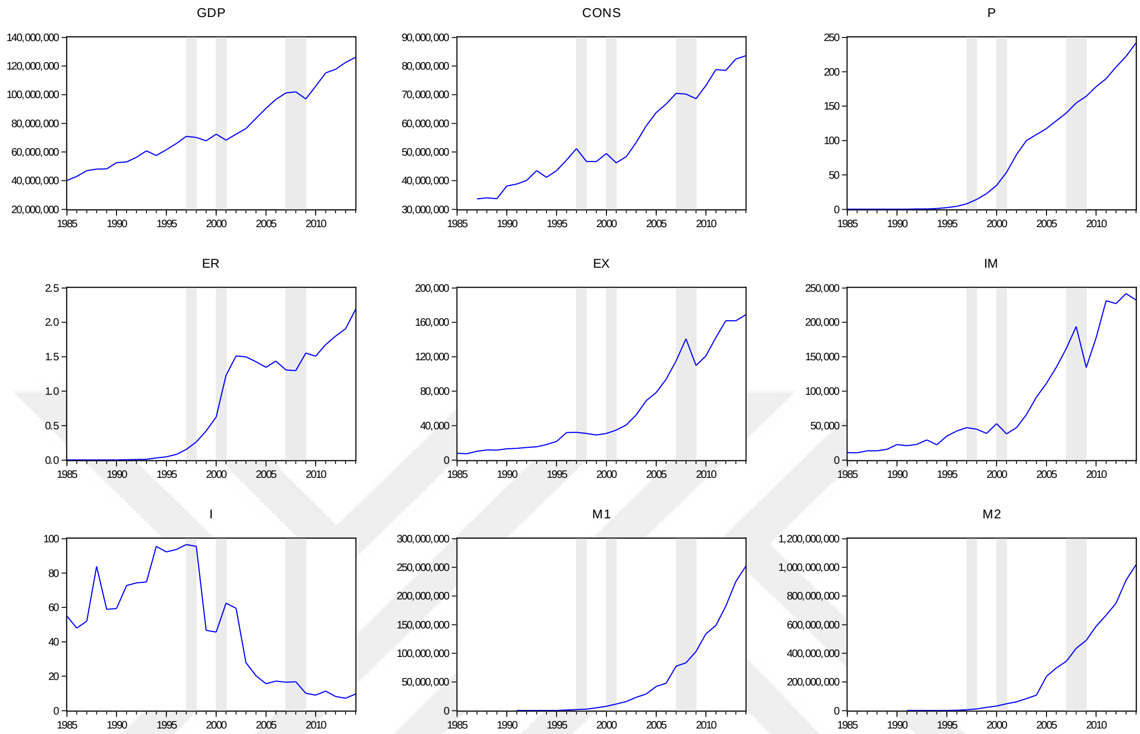


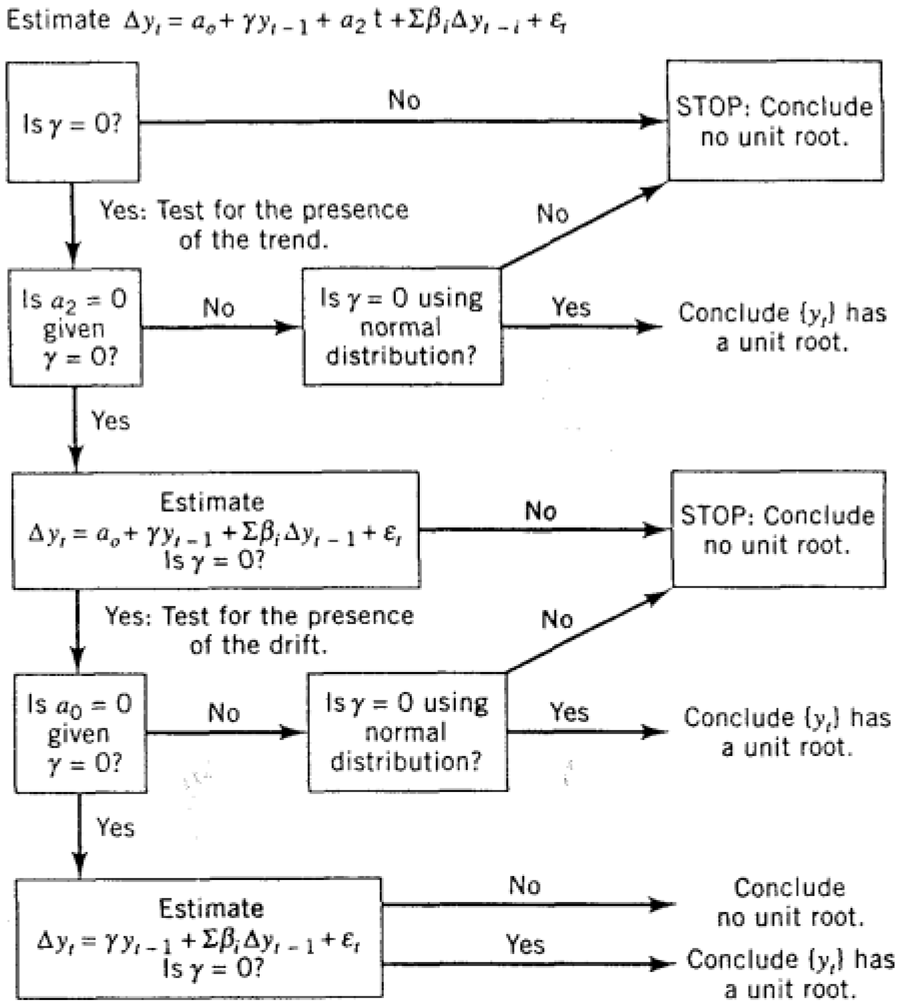
Figure 6. Major Turkish macroeconomic variables between 1985-2014

To test the stationarity of the series methodology outlined in Walter and Enders (2008) is followed. The steps of this methodology are presented in Table 2.

Table 2.

Methodology Followed in Testing Stationarity of the Series.

Source: Walter and Enders (2008), p.264.



Accordingly, in the first place the most extensive model (Model A) with constant and trend is estimated. If the null hypothesis of unit root is rejected, it is concluded series is stationary. However, if the hypothesis $\delta = 0$ is not rejected, our focus is on the significance of the trend term, given that series has unit root. For this purpose, we test the joint hypothesis that $\phi_3 = 0$ under the maintained hypothesis that $\delta = 0$. If the trend term is statistically different from zero, then we retest $\delta = 0$ using the standardized normal distribution. Critical values used in retesting the presence of unit root are -2.33, -1.64 and -1.28, for 1%, 5% and 10% significance levels, respectively. On the other hand, if the trend term is statistically equal to zero provided that series has unit root, we estimate the Model B and repeat the same procedure for the constant term. In the light of this methodology results of ADF, PP, and KPSS unit root tests are given in Table 3 and Table 4.

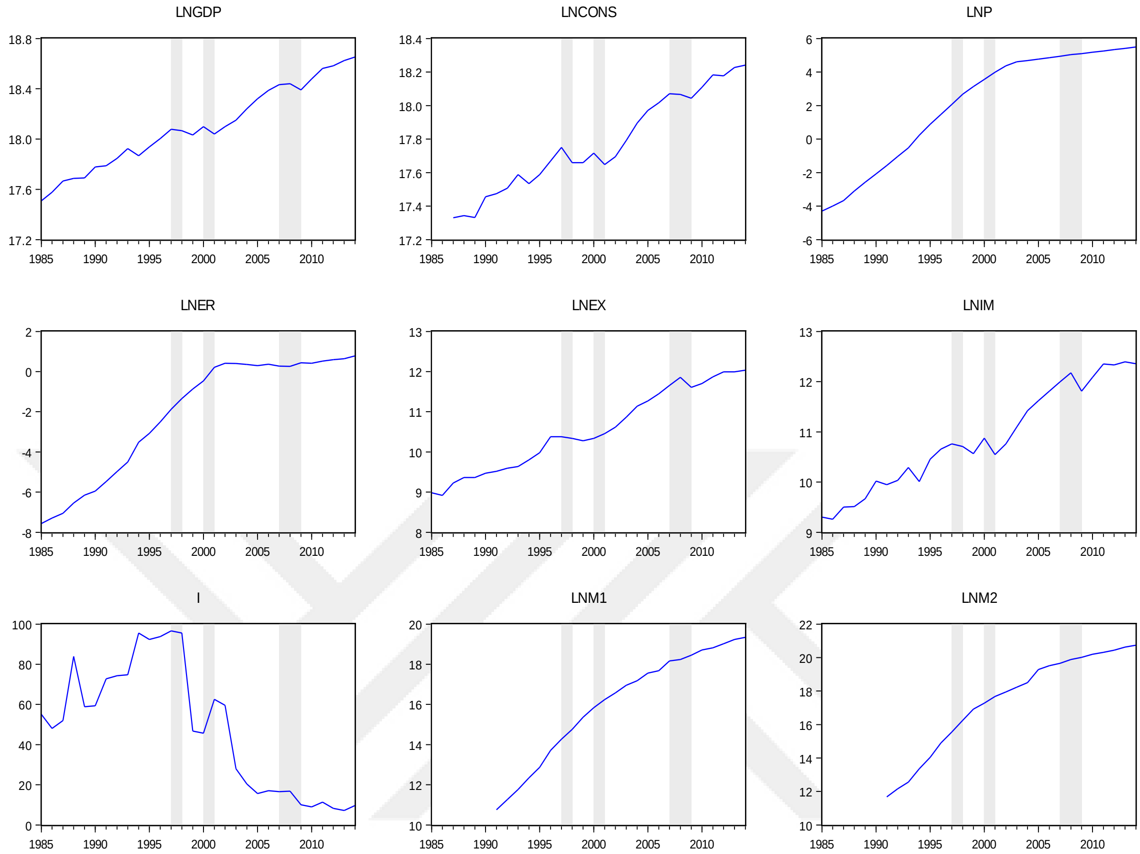


Figure 7. Major Turkish macroeconomic variables in logarithms between 1985-2014

In the light of the methodological guidance followed in ADF test, firstly Model A is considered. As can be seen from Table 3, there is no evidence that GDP series is stationary as the presence of unit root cannot be rejected. In the next step we test the presence of a deterministic trend. For this purpose, F test is benefited under the joint restriction that δ=0 and $\phi_3 = 0$. The results for GDP series show that this null hypothesis cannot be rejected. In other words, trend coefficient under the presence of unit root is not statistically different from zero; leading to the conclusion that trend variable can be omitted from the specification. This decision seems to be plausible considering the fact that the exact value of trend coefficient in Model A is equal to 0.02 with a standard error of 0.007. Since Model A does not seem to be appropriate to characterize the GDP series at hand, the testing procedure continues with the evaluation of Model B. Model B also suggests that GDP series is nonstationary, where as null hypothesis that δ=0 and $\phi_2 = 0$ is rejected.

Table 3.

ADF Unit Root Test Results of the Major Turkish Macroeconomic Variables

	ADF							
	Model A			Model B			Model C	
Series	k	t	ϕ_3	k	t	ϕ_1	k	t
lnGDP	1	-2.903242	4.218758	1	-0.424468	6.619193**	1	3.656305
lncons	1	-2.569690	3.301706	1	-0.439126	4.166330*	1	2.891007
lnp	1	-1,58	5,67	1	-3,38**	5,80**	1	-1,70*
lner	1	-0.685604	2.453885	1	-2.253756	3.941653	1	-2.470015**
lnex	1	-2,66	3,76	1	-0.924554	5.604339**	1	3.047671
lnim	1	-2.540373	3.318506	1	-0.776435	5.592015**	1	3.148563
İ	6	-2,95	4,4	1	-0.832194	0.476330	1	-0.878994
lnm1	2	-1,46*	6,75*	2	-3,78**	7,22**	3	-0,45
lnm2	1	-0.789334	5.642877	1	-3.419425**	7.485752**	1	0,91

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

Since the results show that a significant constant term is involved in the model, we retest the significance of t statistics depending on the standardized normal distribution and find that the test statistics t is not negative enough to reject the null of unit root. This result indicates that Turkish GDP series in logarithmic terms can be characterized as random walk with drift with an integration degree of 1. As can be expected, Turkish aggregated consumption exhibits a similar characteristic to that of GDP, as the findings of ADF test results are in line with a random walk with drift process. ADF test results in Model A for Consumer Price Index show that series is not stationary, while trend term turns out to be statistically equal to zero. Omitting trend term and proceeding with the Model B, Consumer Price Index is found to be stationary. Furthermore, USD-TRY nominal exchange rate (lnER) in logarithmic terms is not found to be stationary in Model A and Model C, with insignificant trend and constant terms, respectively. Interestingly, Model C presents the evidence that USD-TRY nominal exchange rate (lnER) is integrated of order 0 with a random walk without drift nature. Needless to say, this finding seems suspicious since lnER series contains a visible trend with an apparent onetime break and at least a drift term that would generate a trend would be expected. This interesting result indicates the necessity for further investigation on the characteristics of lnER through alternative unit root tests which will be discussed below.

Moreover, in the ADF test results, lnex and lnim series are found to be nonstationary with a significant constant term, whereas annual deposit interest rate (i) is nonstationary with a random walk without drift feature. Furthermore, according to the ADF test results, there is no evidence that money supply in m1 measure (lnm1) does not contain a unit root at 10% significance level, as the findings suggest that series has a significant deterministic trend with a test statistics t more negative than -1.28, the standardized normal distribution's critical value corresponding to 10% significance level. Besides, unit root hypothesis for money supply in m2 measure (lnm2) is rejected with 95% confidence in Model B.

Table 4.

PP and KPSS Unit Root Test Results of the Major Turkish Macroeconomic Variables

	PP						KPSS	
	Model A		Model B		Model C			
Series	Bandwith	t	Bandwith	t	Bandwith	t	η_t	η_μ
lnGDP	0	-3,03	4	0,84	5	6,8	0,08	0,713**
lncons	1	-2,65	6	-0,11	6	5,34	0,08	0,66**
lnp	4	0,42	4	-2,85*	4	-0,29	0,18**	0,66**
lner	4	-0,23	4	0,20	4	-3,3***	0,17**	0,64**
lnex	3	-2,48	6	-0,41	6	5,59	0,07	0,7**
lnim	2	-3,2*	0	-0,68	1	3,13	0,07	0,69**
İ	1	-2,23	1	-0,93	3	-0,95	0,13*	0,49**
lnm1	1	-0,16	2	-6,3***	3	3,78	0,18**	0,68**
lnm2	0	-0,12	1	-4,6***	3	3,24	0,18**	0,67**

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

Broadly speaking, the results of PP test results given in Table 4 are in line with ADP test results. KPSS test results, on the other hand, present evidences on the ambiguity of the integration degree of the series Consumer Price Index (lnP), USD-TRY nominal exchange rate (lnER), lnM1 and lnM2. First of all it should be noted that in KPSS test the null hypothesis is series is stationary. According to the KPSS test results in Table 4, stationarity hypothesis for Consumer Price index, USD-TRY nominal exchange rate and money supplies M1 and M2 cannot be accepted in the model $\eta_t$where both a constant and linear trend are involved.

Table 5.

ADF, PP and KPSS Unit Root Tests for Turkish Macroeconomic Variables in First Differences

Series	ADF		PP		**KPSS**	
	Model C		Model C			
	k	t	Bandwith	t	η_t	η_μ
D(lnGDP)	1	-2.554391***	3	-3.748814***	0.105746	0.100249
D(lncons)	1	-2.695665***	1	-3.908802***	0.118427	0.112308
D(lnP)	1	-0.813300	2	-0.761848	0.500986**	0.130526*
D(lnER)	1	-0.998224	3	-1.324439	0.407084*	0.111474
D(lnEX)	1	-2.440210**	1	-3.098515***	0.114999	0.114612
D(lnIM)	1	-3.043679***	4	-5.342296***	0.050824	0.050121
D(i)	1	-5.046256***	2	-5.582889***	0.185800	0.115424
D(lnM1)	3	-1.500656	1	-1.258404	0.603565**	0.093427
D(lnM2)	1	-0.908650	2	-1.156424	0.543635**	0.076454

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

On the other hand results of the same tests performed with first differenced series are given in Table 5. First of all, it is confirmed that lnGDP, lncons, lnex, lnim and i series are integrated of degree 1, as their first differences yield stationary series. On the other hand, KPSS test results show that if the model specified in the ADF testing procedure is adopted, first differenced lnM1 and lnM2 series are found to be integrated of degree 1. Besides, lnER is also found to be stationary in the first differences in the model η_μ where a constant term is involved. However, there still is inconclusiveness on the integration level of lnP since the KPSS test statistics calculated with the first differenced lnP fails to accept the null of stationarity in both models, giving rise to the conclusion that Consumer Price Index (lnP) is integrated of degree 2. In fact, these results regarding the integration order of Consumer Price Index (lnP) can actually be interpreted as a possible evidence of over-differencing of this series in the procedures suggested by the unit root tests without breaks. In other words, if a stationary series is mistakenly found to be integrated of degree 1, and differenced accordingly, then the differenced series turns out to be nonstationary, misleadingly implying that the original series is I(2).

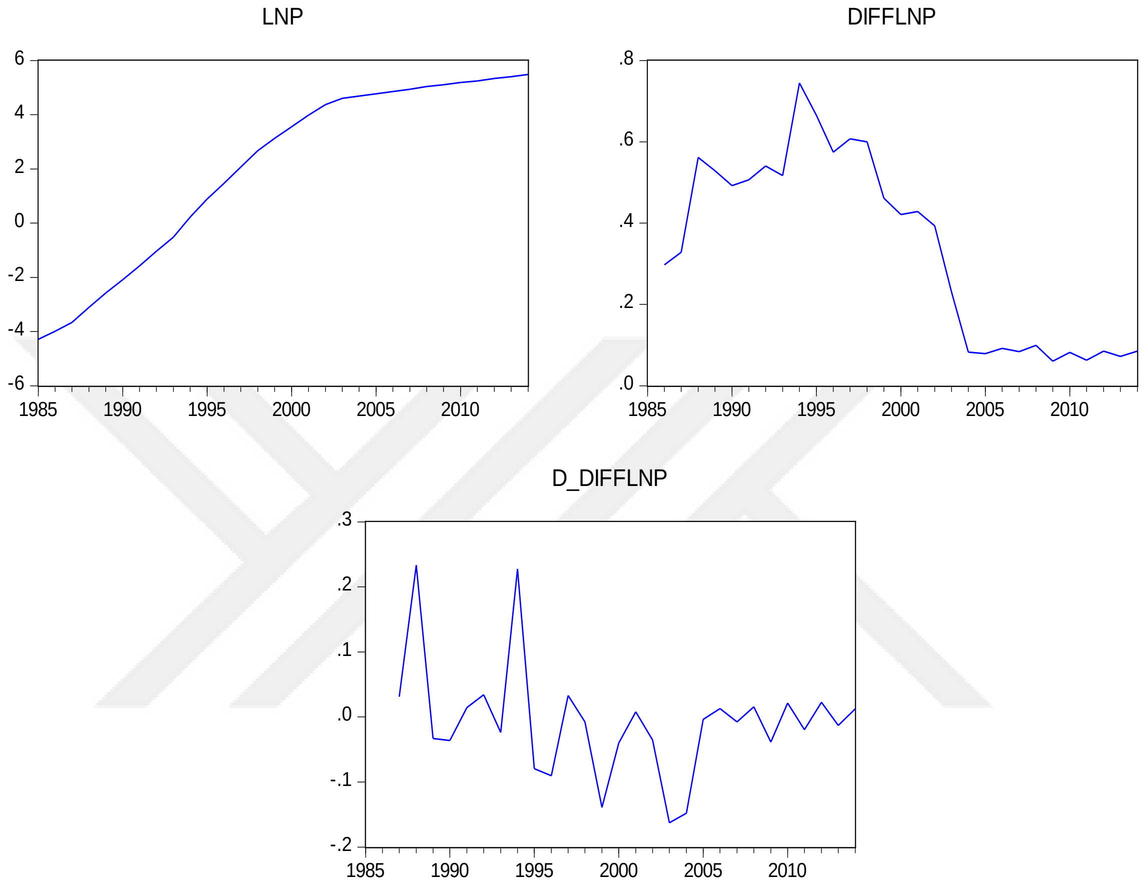


Figure 8. Graphs of lnP, d(lnP) and d(d(lnP))

To investigate the presence of such possibility further, Figure 8 presents the graph of lnP, D(lnP) and D(D(lnP)). In Figure 8, lnP in the upper left panel of the graph appears to be generated by a trend stationary process with a change in slope. However, firs differenced lnP series in the upper right panel of the graph seems to contain a notable stochastic trend. Eventually, second differenced lnP series resembles to be generated from a white noise data generating process. These visual examinations amplify the suspicions of possible over-differencing in these series.

Series and their integration degrees corresponding to ADF, PP and KPSS unit root tests are given in Table 6. Table 6 shows that the degree of integration of Consumer Price Index (lnP), USD-TRY nominal exchange rate (lnER) and M1 and M2 money supplies are ambivalent as the different unit root tests suggest different integration degrees. This uncertainity is in particularly present between ADF and KPSS test results.

Table 6.

Integration Degrees of the Series with respect to Conventional Unit Root Tests

Series	**ADF**	**PP**	**KPSS**
lnGDP	I(1)	I(1)	I(1)
lncons	I(1)	I(1)	I(1)
lnP	I(0)	I(0)	I(2)
lnER	I(0)?	I(0)?	I(1)
lnEX	I(1)	I(1)	I(1)
lnIM	I(1)	I(1)	I(1)
i	I(1)	I(1)	I(1)
lnM1	I(0)	I(0)	I(1)
lnM2	I(0)	I(0)	I(1)

4.1.4. Unit Root Tests with Structural Breaks

4.1.4.1. Zivot Andrews Unit Root Test

Ambiguity on the integration degree of the Turkish macroeconomic time series implies that alternative unit root tests must be employed to scrutinize the behavior of these series. Considering the fact that the period between 1985 and 2014 witnessed a serious number of domestic and international economic crises and structural changes such as 2001 economic crisis in Turkey, 2008 Housing Bubble and Credit Crisis in the United States, and the following Federal Reserve's Quantitative Easing Policy between November 2008 and June 2013, as the second step of the empirical strategy Zivot Andrews (ZA) unit root test allowing for a single endogenous break is used to investigate the stationarity of Turkish macroeconomic time series. To account for structural breaks in the series, all three models associating a break in level, trend or both in level and trend are employed, because the period 1985-2014 may involve characteristics of a break in level (crash) like 2001 economic crisis in Turkey, a change in trend like Kemal Derviş's Transition to Strong Economy Program in 2001 or in both level and trend like the period of Adalet ve Kalkınma Partisi's (AKP) 14 year long single party government. Results of ZA unit root test are given in Table 6.

Table 7.

Zivot Andrews Unit Root Test Results

	Null: Series has unit root..					
	Zivot Andrews					
Model	Level		Trend		Both	
Series	Statistics	Time of Break	Statistics	Time of Break	Statistics	Time of Break
lnGDP	*-4.09240*	1999	-3.48369	2003	-3.48369	2003
lncons	*-3.92355*	1998	-3.09084	2002	-3.82892	1998
lnP	*-3.26953*	1994	-4.57631**	1999	-4.46572	1994
lnER	*-2.54600*	1994	-5.42117***	2002	-4.53807	2001
lnEX	*-3.94936*	2004	-3.02272	1992	-3.70811	2004
lnIM	*-3.72882*	2004	-2.73631	2002	-3.44311	2005
i	*-3.73112*	2003	-3.34194	1995	-4.92351	1999
lnM1	*-4.15640*	1996	-4.79716**	1998	-5.00935	1996
lnM2	*-1.88577*	1996	-2.85655	1997	-2.52682	1996

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

In Table 7 we see a completely different picture regarding the behavior of Turkish macroeconomic variables. First and foremost, according to ZA test results, lnGDP and domestic consumption (lncons) series are still found to be integrated of order 1. The most striking results in Table 6, however, are about the Consumer Price Index (P) and USD-TRY nominal exchange rate (ER). Results show that if an endogenous break is allowed, Consumer Price Index (P) is found to be stationary in both three models. This case is also valid for USD-TRY nominal exchange rate (ER), controlling for a break in trend and in level and trend. In other words, justifying suspicions previously discussed, the ZA unit root test model allowing for a structural break in trend cannot reject the alternative hypothesis that lnP and lnER series are stationary with a onetime break in trend in 1999 and 2002, respectively. Possible events that the break dates in lnP and lnER series correspond to IMF stand-by arrangement in 1999 and the commencement of the AKP governance. Regarding the lnEX and lnIM series, ZA test results indicate that unit root null cannot be rejected. Besides, money supply in M1 measure is also found to stationary with the model allowing for a break in trend. The ZA test results performed with the first differences of the series are provided in Table 8.

Table 8.

Zivot Andrews Unit Root Test Results in First Differences

	Null: Series has unit root.					
	Zivot Andrews					
Model	Level		Trend		Both	
Series	Statistics	Time of Break	Statistics	Time of Break	Statistics	Time of Break
d(lnGDP)	*-5.041***	2003	-4.362	2000	-4.959	2004
d(lncons)	*-6.218****	2003	-4.523**	1999	-6.058***	2003
d(lnP)	*-4.883***	2003	-3.092	2010	-3.750	2003
d(lnER)	*-4.899***	2002	-2.599	1995	-4.135	2002
d(lnEX)	*-5.357****	2009	-4.860**	2007	-5.354**	2009
d(lnIM)	*-5.673****	2003	-4.637**	2007	-5.806***	2003
d(i)	*-7.361****	1999	-6.415***	2004	-8.070***	1999
d(lnM1)	*-4.30617*	2002	-3.690	2010	-3.886	1998
d(lnM2)	*-4.810***	2000	-3.998	2011	-4.281	2005

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

4.1.4.2. Lumsdaine Papell Unit Root Test

As noted by Lumsdaine and Papell (1997), onetime structural break in economic time series may not be so realistic, and more than one break in different deterministic components of the unit root test equations must be incorporated. Considering the fact that the span of 1985-2014 can be regarded as a turbulent period, we utilize Lusdaine Papell (LP) unit root test allowing for two structural breaks in the series, under the null hypothesis that the series has unit root. Results of Lumsdaine and Papell unit root test are given in Table 9.

Table 9.

Lumsdaine Papell Unit Root Test Results

	Lumsdaine Papell					
Model	Level		Trend		Both	
Series	Statistics	Time of Break	Statistics	Time of Break	Statistics	Time of Break
lnGDP	*-5.8570*	2000 and 2004	-4.5048	1998 and 2002	-6.2749	2000 and 2008
lncons	*-4.8170*	1997 and 2003	-3.6648	2004 and 2007	-5.3570	1998 and 2003
lnP	*-3.6289*	1993 and 2003	-5.2231	1999 and 2003	-6.0126	1993 and 2000
lnER	*-3.6212*	1993 and 2003	-7.1343**	1991 and 2002	-6.3823	1993 and 2000
lnEX	*-4.5476*	1995 and 2003	-4.8152	2003 and 2008	-4.4656	1995 and 2003
lnIM	*-4.4194*	1998 and 2003	-3.9068	2004 and 2008	-5.6047	2000 and 2008
İ	*-5.2509*	1998 and 2002	-7.6273***	1998 and 2007	-7.9766***	1998 and 2004
lnM1	*-1.2807*	1996 and 2007	-4.7262	2001 and 2008	-4.7751	2001 and 2006
lnM2	*-1.7601*	2005 and 2010	-7.1322**	2000 and 2008	-12.143***	2000 and 2004

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

Consistent with the result of aforementioned unit root tests, the results show that we cannot reject the null of unit root for the series lnGDP and lnCONS. Besides, LP unit root test results also confirm that lnER is stationary in level with two breaks in trend in 1991 and 2002. However, compared to ZA unit root test, LP test gives mixed results regarding the nature of the series lnP, i, lnM1 and lnM2. It can be seen in Table 9 that, the null of unit root is not rejected at any conventional significance level for lnP and lnM1. Besides, interest rate i is found to be stationary in models with breaks in trend and both in level and trend.

4.1.4.3. Lee Strazicich Unit Root Test

So far we have discussed three groups of the pioneering unit root tests. These are conventional unit roots tests (ADF, PP and KPSS), unit root test with a onetime endogenous break (ZA) and unit root test with multiple breaks (LP). In neither of these tests no structural breaks are involved under the null hypothesis and rather fundamental data generating processes such as random walk with or without drift are assumed. Different from these unit root tests, however, Lee Strazicich (LS) unit root test incorporates the structural breaks under the null hypothesis and generates critical values accordingly. As shown by Lee and Strazicich (2003), the critical values used for the model allowing for breaks in level do not change with respect to the break dates' location and magnitude. On the other hand, this invariance of the critical values is not valid for the model embodying breaks in both level and trend. In other words, critical values to be used in the model for breaks both in level and trend must be generated by depending on the positions and the magnitudes of the break dates. Because of this difficulty, in this study, along with the global critical values independent of the position and magnitude of the break dates for model with breaks in level, we use critical values provided by Lee and Strazicich (2003) under the null without structural breaks for the model with breaks in both level and trend. Therefore, critical values used in the hypothesis testing of unit root are extracted from Lee and Strazicich (2003). The result of the LS unit root test is presented in Table 10.

Table 10.

Lee Strazicich Unit Root Test Results

	Lee Strazicich			
Model	Level		Level and Trend	
Series	Statistics	Time of Break	Statistics	Time of Break
lnGDP	*-3.8028**	*2000 and 2003*	*-5.9741****	1999 and 2004
lncons	*-3.3666*	1997 and 2002	*-5.6187***	1996 and 2003
lnP	*-3.9558***	*1988 and 2004*	*-3.9887*	1992 and 2001
lnER	*-2.1722*	*1988 and 1993*	*-4.1243*	1992 and 2002
lnEX	*-3.4970*	*2003 and 2007*	*-4.3807*	1998 and 2005
lnIM	*-3.5099**	*2001 and 2007*	*-4.9897**	1997 and 2003
i	*-2.2841*	*1990 and 2002*	*-8.0727****	1997 and 2002
lnM1	*-1.3665*	*2004 and 2006*	*-3.6245*	1997 and 2005
lnM2	*-1.9630*	*2002 and 2004*	*-4.8355*	1997 and 2007

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

Results show that lnGDP is level stationary with two structural breaks in both models. In particularly, the LS model taking two breaks in level and trend into consideration points out the years 1999 and 2004 as the break dates. These dates are in particularly notable, as the former signaling a break in level corresponds to 1999 Gölcük earthquake which heavily destroyed the industrialized urban areas in Turkey, the latter implying a break in trend coincides with the commencement of the AKP governance. Besides, the results show that lnCONS also exhibits a level stationary feature, with approximately the same break dates compared to that of lnGDP. Besides, lnIM and i series are found to stationary with structural breaks clustering around 1997 and 2002.

Table 11.

Lee Strazicich Unit Root Test Results in First Differences

	Lee Strazicich			
Model	Level		Level and Trend	
Series	Statistics	Time of Break	Statistics	Time of Break
d(lnGDP)	*-5.1586****	*1996 and 2003*	*-5.7823***	2000 and 2006
d(lncons)	*-5.6854****	2001 and 2012	*-6.7869****	2001 and 2008
d(lnP)	*-1.4678*	*1993 and 1998*	*-7.5110****	1996 and 2003
d(lnER)	*-3.4316*	*1990 and 2001*	*-9.8436****	1992 and 2002
d(lnEX)	*-4.1466***	*1996 and 2008*	*-5.0127**	1995 and 2001
d(lnIM)	*-5.2061****	*1998 and 2003*	*-6.4320****	1998 and 2001
d(i)	*-5.7205****	*1994 and 1997*	*-7.4128*	1997 and 2001
d(lnM1)	*-4.0809*	*1999 and 2003*	*-6.5586****	1995 and 2005
d(lnM2)	*-3.4251*	*1995 and 1999*	*-4.9099*	2000 and 2005

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

It can be seen from Table 10 that lnP is level stationary in the model allowing for two structural breaks which correspond to 1988 and 2004, while the null of unit root is not rejected for lnER, lnEX, lnM1 and lnM2 at any conventional significance level. LS test performed with the first differenced series (see Table 11) shows that lnP, lnER, lnEX and lnM1 are integrated order of 1. Contrary to these series, lnM2 is has integration order of 2.

4.1.5. Unit Root Tests with Bootstrap

Although unit root tests are one of the most commonly used econometric tools among practitioners in economics, they do suffer from certain conditions frequently encountered in empirical applications. Two major problems unit root tests have are their poor performance in small samples and high dependency to nuisance parameters such as trend and intercept (Angelis, Fachin and Young., 1997, p. 1155). In particular poor performance of unit root tests in small samples is of particular interest because of its implications on statistical inference procedures. More specifically, for most of the popularly used unit root tests, discrepancies between the actual and nominal level of rejection of probabilities are evident (Park, 2003, p. 1845).

With the advances in technology and computation abilities, simulation methods and the related synthetic data generating approaches including bootstrap have gained prominence in applied econometric research, not only because of relatively less calculation burden, but also because of empirical refinements these approaches have to provide. Among these approaches, bootstrap is shown to be in particularly useful in achieving the actual and the desired rejection probabilities (Ferretti and Romo, 1996, Nankervis and Savin (1996)). Therefore, in this section we investigate the time series properties of Turkish macroeconomic variables using two bootstrap unit root test, namely Psaradakis (2001) and Harris (1992).

4.1.5.1. Harris' Unit Root Test Procedure

In his seminal paper Harris (1992) criticizes the weak performance of the conventional unit root tests in the case of small samples and proposes a new methodology in which a parametric bootstrap scheme is integrated to derive the characteristics of the series. In this approach we use two models, namely model with both constant and a deterministic trend (Model A) and model with a constant only (Model B). More formally these models can be written as follows,

Model A : $\Delta y_t = \alpha y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{t-j} + \delta + \gamma t + v_t$

Model B: $\Delta y_t = \alpha y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{t-j} + v_t$

where δ in Model A and Model B and t in Model A are constant and deterministic trend, respectively. The lag degree of the series Δy_t is determined based on the lag structure suggested by Schwarz Information Criterion in conventional ADF model estimation. Our principal concern is to obtain empirical distribution of the statistics corresponding to $t_i^* = \frac{\hat{\alpha}_i^*}{se(\hat{\alpha}_i^*)}$, where $\hat{\alpha}_i^*$ stands for bootstrapped version of the parameter estimate $\hat{\alpha}$. As suggested by Harris (1992) number of bootstrap replications is chosen as 5000, which implies that $i = 1, 2, \ldots., 5000$.

In application of Harris' (1992) bootstrap unit test approach, we start with estimating a restricted version of Model A and Model B imposing that $\alpha = 0$ and obtain residuals $\hat{v}_t$ for each model. Secondly, attaining $\frac{1}{n}$ probability to each observation, we resample from centralized residuals in order to generate bootstrap replications of the error terms v_i^* , which is used to derive the bootstrapped replicates of the series y_t^* by recursion with the initial condition that $y_0^* = y_1$. The recursions used to obtain bootstrapped version of y_t in Model A for 1 (d=1), 2 (d=2) and 6 (d=6) lags in the models are as follows;

For d=1:

$$\Delta y_t = \alpha y_{t-1} + \hat{\beta}_1 \Delta y_{t-1} + \hat{\delta} + \hat{\gamma} t + v_t^* \tag{102}$$

$$y_1^* = \hat{\beta}_1 \Delta y_0 + \hat{\delta} + \hat{\gamma} t + v_1^* \tag{103}$$

$$y_2^* = \hat{\beta}_1 \Delta y_1^* + \hat{\delta} + \hat{\gamma} t + v_2^* \tag{104}$$

$$y_3^* = \hat{\beta}_1 \Delta y_2^* + \hat{\delta} + \hat{\gamma} t + v_3^* \tag{105}$$

For d=2:

$$\Delta y_t = \alpha y_{t-1} + \hat{\beta}_1 \Delta y_{t-1} + \hat{\beta}_2 \Delta y_{t-2} + \hat{\delta} + \hat{\gamma} t + v_t^* \tag{106}$$

$$y_1^* = y_0 + \hat{\beta}_1 \Delta y_0 + \hat{\beta}_2 \Delta y_{-1} + \hat{\delta} + \hat{\gamma} t + v_1^* \tag{107}$$

$$y_2^* = y_1^* + \hat{\beta}_1 \Delta y_1^* + \hat{\beta}_2 \Delta y_0 + \hat{\delta} + \hat{\gamma} t + v_2^* \tag{108}$$

$$y_3^* = y_2^* + \hat{\beta}_1 \Delta y_2^* + \hat{\beta}_2 \Delta y_1^* + \hat{\delta} + \hat{\gamma} t + v_3^* \tag{109}$$

$$y_4^* = y_3^* + \hat{\beta}_1 \Delta y_3^* + \hat{\beta}_2 \Delta y_2^* + \hat{\delta} + \hat{\gamma} t + v_4^* \tag{110}$$

For d=6:

$$\Delta y_t = \alpha y_{t-1} + \hat{\beta}_1 \Delta y_{t-1} + \hat{\beta}_2 \Delta y_{t-2} + \hat{\beta}_3 \Delta y_{t-3} + \hat{\beta}_4 \Delta y_{t-4} + \hat{\beta}_5 \Delta y_{t-5} + \hat{\beta}_6 \Delta y_{t-6} + \hat{\delta} + \hat{\gamma} t + v_t^* \tag{111}$$

$$y_1^* = y_0 + \hat{\beta}_1 \Delta y_0 + \hat{\beta}_2 \Delta y_{-1} + \hat{\beta}_3 \Delta y_{-2} + \hat{\beta}_4 \Delta y_{-3} + \hat{\beta}_5 \Delta y_{-4} + \hat{\beta}_6 \Delta y_{-5} + \hat{\delta} + \hat{\gamma} t + v_1^* \tag{112}$$

$$y_2^* = y_1^* + \hat{\beta}_1 \Delta y_1^* + \hat{\beta}_2 \Delta y_0 + \hat{\beta}_3 \Delta y_{-1} + \hat{\beta}_4 \Delta y_{-2} + \hat{\beta}_5 \Delta y_{-3} + \hat{\beta}_6 \Delta y_{-4} + \hat{\delta} + \hat{\gamma} t + v_2^* \tag{113}$$

$$y_3^* = y_2^* + \hat{\beta}_1 \Delta y_2^* + \hat{\beta}_2 \Delta y_1^* + \hat{\beta}_3 \Delta y_0 + \hat{\beta}_4 \Delta y_{-1} + \hat{\beta}_5 \Delta y_{-2} + \hat{\beta}_6 \Delta y_{-3} + \hat{\delta} + \hat{\gamma} t + v_3^* \quad (114)$$

$$y_4^* = y_3^* + \hat{\beta}_1 \Delta y_3^* + \hat{\beta}_2 \Delta y_2^* + \hat{\beta}_3 \Delta y_1^* + \hat{\beta}_4 \Delta y_0 + \hat{\beta}_5 \Delta y_{-1} + \hat{\beta}_6 \Delta y_{-2} + \hat{\delta} + \hat{\gamma} t + v_4^* \quad (115)$$

$$y_5^* = y_4^* + \hat{\beta}_1 \Delta y_4^* + \hat{\beta}_2 \Delta y_3^* + \hat{\beta}_3 \Delta y_2^* + \hat{\beta}_4 \Delta y_1^* + \hat{\beta}_5 \Delta y_0 + \hat{\beta}_6 \Delta y_{-1} + \hat{\delta} + \hat{\gamma} t + v_5^* \quad (116)$$

$$y_6^* = y_5^* + \hat{\beta}_1 \Delta y_5^* + \hat{\beta}_2 \Delta y_4^* + \hat{\beta}_3 \Delta y_3^* + \hat{\beta}_4 \Delta y_2^* + \hat{\beta}_5 \Delta y_1^* + \hat{\beta}_6 \Delta y_0 + \hat{\delta} + \hat{\gamma} t + v_6^* \quad (117)$$

$$y_7^* = y_6^* + \hat{\beta}_1 \Delta y_6^* + \hat{\beta}_2 \Delta y_5^* + \hat{\beta}_3 \Delta y_4^* + \hat{\beta}_4 \Delta y_3^* + \hat{\beta}_5 \Delta y_2^* + \hat{\beta}_6 \Delta y_1^* + \hat{\delta} + \hat{\gamma} t + v_7^* \quad (118)$$

where v_t^* shows bootstrapped residuals extracted from Model A. In addition, the algorithm needed to perform Harris' unit root test approach was coded in R (programming language) and for the sake of replication of the same results, seed was set to 100.

Table 12.

Harris' Unit Root Test Results

	Harris					
	Model A			Model B		
Series	k	t	Critical Values	k	t	Critical Values
lnGDP	1	-2.903242*	-3.774; -3.165; -2.830	1	-0.424468	-1.881; -1.309; -1.027
lncons	1	-2.569690	-4.336; -3.582; -3.231	1	-0.439126	-2.847; -1.990; -1.662
lnP	1	-1,58	-6.664; -5.397; -4.710	1	-3,38*	-5.242; -3.685; -2.969
lnER	1	-0.685604	-5.933; -4.269; -3.599	1	-2.253756	-6.877; -5.356; -4.615
lnEX	1	-2,66	-4.458; -3.833; -3.522	1	-0.924554	-2.010; -1.590; -1.381
lnIM	1	-2.540373	-3.444; -2.859; -2.624	1	-0.776435	-2.314; -1.806; -1.550
İ	6	-2,95**	-3.236; -1.983; -1.554	1	-0.832194	-4.406; -3.093; -2.582
lnM1	2	-1,46	-5.594; -4.311; -3.771	2	-3,78*	-5.569; -4.287; -3.653
lnM2	1	-0.789334	-5.954; -4.177; -3.497	1	-3.419425*	-4.626; -3.501; -3.079

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

The results of Harris' unit root test approach are presented in Table 12. In Table 12, critical values given correspond to 1%, 5% and 10% quantiles of the empirical distribution obtained through bootstrapping residuals, respectively. In deciding whether the series has unit root or not, calculated t statistics for the autoregressive coefficient α is compared with the corresponding critical value. If the calculated t statistic is more negative than the value derived from the desired quantile of the empirical distribution, a.k.a. significance level, then it is concluded that the null of unit root can be rejected with a certain confidence level.

In the light of the discussion above, from Table 12 it can be seen that null of unit root for lnGDP series can be rejected with 90% confidence, implying that the series is stationary with a constant and deterministic trend. On the contrary, lncons is not found to be stationary at any conventionally used significance level. On the other hand, Harris' unit root test results show that the series lnP does not contain a unit root at 10% significance level. Furthermore, while the series lnER, lnEX, lnIM and i are found to have unit root, lnM1 and lnM2 are stationary at 10% significance level in Model B where a sole constant is involved. These results highly contradict with the results suggested by conventional unit root tests such as ADF, PP and KPSS. On the other hand, there are notable similarities between Harris' bootstrapped unit root test and the test with single and multiple endogenous breaks. These similarities are particularly evident for the series lnGDP and lnP, both are found to be stationary by LS and Harris' unit root tests.

4.1.5.2. Psaradakis' Unit Root Test Procedure

To give an extensive picture on the characteristics of Turkish macroeconomic time series, besides using unit root tests with break, we also focus on Psaradakis' unit root approach outlined in 6.3.2. In this approach we start with first differencing the series. Secondly, appropriate AR(p) specification is determined using Box and Jenkins (1976)'s (BJ) methodology. As stated in Psaradakis (2001), selection of the AR(p) model fitting the series best can be performed using the two information criteria, namely, Akaike Information Criteria (AIC) and Schwarz Information Criteria (SIC) (p.582). In Table 13 AIC and SIC values corresponding to different AR(p) p=1,2,3,4 values are provided.

Table 13.

Determination of AR(p) Structure of the Series Using BJ Methodology

Variable/ AR(p)	Criteria	p=1	p=2	p=3	p=4	Decision
D(lnGDP)	AIC	-2.853620	-2.857779	-2.819037	-2.711719	AR(2)
	SIC	-2.806041	-2.761791	-2.673872	-2.516699	AR(1)
D(lncons)	AIC	-2.659920	-2.546101	-2.728912	-2.590666	AR(3)
	SIC	-2.611532	-2.448591	-2.581656	-2.393189	AR(1)
D(lnp)	AIC	-2.045909	-1.947351	-2.211865	-2.138766	AR(3)
	SIC	-1.998330	-1.851363	-2.066700	-1.943746	AR(3)
D(lner)	AIC	-0.224086	-0.274559	-0.306914	-0.215492	AR(3)
	SIC	-0.176507	-0.178571	-0.161749	-0.020472	AR(2)
D(lnex)	AIC	-0.918709	-1.033881	-0.931926	-0.879713	AR(2)
	SIC	-0.871130	-0.937893	-0.786761	-0.684693	AR(2)
D(lnim)	AIC	0.003230	0.047614	0.056795	0.170274	AR(1)
	SIC	0.050809	0.143602	0.201960	0.365294	AR(1)
D(i)	AIC	8.292865	8.297067	8.222625	8.143786	AR(4)
	SIC	8.340444	8.393054	8.367790	8.338807	AR(4)
D(lnm1)	AIC	-0.553879	-1.051343	-1.087808	-0.934592	AR(3)
	SIC	-0.504286	-0.951865	-0.938448	-0.735763	AR(2)
D(lnm2)	AIC	-0.248293	-0.296009	-0.407881	-0.282306	AR(3)
	SIC	-0.198700	-0.196531	-0.258521	-0.083477	AR(3)

BJ results in Table 13 show that, depending on the two information criterion, AR(p) structure differs slightly for the lnGDP, lnCONS, lnER and lnM1 series. On the other hand for lnP, lnEX, lnIM, i and lnM2, AIC and SIC measures yield the same AR(p) specification. Psaradakis' unit root test results performed according to AR(p) structures specified by AIC and SIC are given in Table 13. Once AR(p) structure is determined and its residuals are obtained, resampling procedure is performed attaining $1/n$ probability to each observation with replacement. In this step, the number of bootstrapped residuals through which the bootstrapped versions of the error terms are generated by the recursion is chosen as 10000. The recursions used to obtain bootstrapped version of the error terms for AR(1), AR(2), AR(3) and AR(4) specifications are as follows;

For AR(1):

$$\varepsilon_0^* = \phi_0 u_0^* = \phi_1 u_{-1}^* \quad \rightarrow u_0^* = 0 \tag{119}$$

$$\varepsilon_1^* - \phi_0 u_1^* = u_0^* \qquad \rightarrow u_1^* = \varepsilon_1^* \tag{120}$$

$$\varepsilon_2^* - \phi_0 u_2^* = \phi_1 u_1^* \quad \rightarrow u_2^* = \frac{\varepsilon_2^* - \phi_1 u_1^*}{\phi_0} \tag{121}$$

$$\varepsilon_3^* - \phi_0 u_3^* = \phi_1 u_2^* \quad \rightarrow u_3^* = \frac{\varepsilon_3^* - \phi_1 u_2^*}{\phi_0} \tag{122}$$

For AR(2):

$$\varepsilon_0^* = \phi_0 u_0^* = \phi_1 u_{-1}^* + \phi_2 u_{-2}^* \quad \rightarrow u_0^* = 0 \tag{123}$$

$$\varepsilon_1^* - \phi_0 u_1^* = \phi_1 u_0^* + \phi_2 u_{-1}^* \quad \rightarrow u_1^* = \varepsilon_1^* \tag{124}$$

$$\varepsilon_2^* - \phi_0 u_2^* = \phi_1 u_1^* + \phi_2 u_0^* \quad \rightarrow u_2^* = \frac{\varepsilon_2^* - \phi_1 u_1^*}{\phi_0} \tag{125}$$

$$\varepsilon_3^* - \phi_0 u_3^* = \phi_1 u_2^* + \phi_2 u_1^* \quad \rightarrow u_3^* = \frac{\varepsilon_3^* - \phi_1 u_2^* - \phi_2 u_1^*}{\phi_0} \tag{126}$$

For AR(3):

$$\varepsilon_0^* = \phi_0 u_0^* = \phi_1 u_{-1}^* + \phi_2 u_{-2}^* + \phi_3 u_{-3}^* \quad \rightarrow u_0^* = 0 \tag{127}$$

$$\varepsilon_1^* - \phi_0 u_1^* = \phi_1 u_0^* + \phi_2 u_{-1}^* + \phi_3 u_{-2}^* \quad \rightarrow u_1^* = \varepsilon_1^* \tag{128}$$

$$\varepsilon_2^* - \phi_0 u_2^* = \phi_1 u_1^* + \phi_2 u_0^* + \phi_3 u_{-1}^* \quad \rightarrow u_2^* = \frac{\varepsilon_2^* - \phi_1 u_1^*}{\phi_0} \tag{129}$$

$$\varepsilon_3^* - \phi_0 u_3^* = \phi_1 u_2^* + \phi_2 u_1^* + \phi_3 u_0^* \quad \rightarrow u_3^* = \frac{\varepsilon_3^* - \phi_1 u_2^* - \phi_2 u_1^*}{\phi_0} \tag{130}$$

$$\varepsilon_4^* - \phi_0 u_4^* = \phi_1 u_3^* + \phi_2 u_2^* + \phi_3 u_1^* \quad \rightarrow u_4^* = \frac{\varepsilon_4^* - \phi_1 u_3^* - \phi_2 u_2^* - \phi_3 u_1^*}{\phi_0} \tag{131}$$

.

For AR(4)

$$\varepsilon_0^* = \phi_0 u_0^* = \phi_1 u_{-1}^* + \phi_2 u_{-2}^* + \phi_3 u_{-3}^* + \phi_4 u_{-4}^* \quad \rightarrow u_0^* = 0 \tag{132}$$

$$\varepsilon_1^* - \phi_0 u_1^* = \phi_1 u_0^* + \phi_2 u_{-1}^* + \phi_3 u_{-2}^* + \phi_4 u_{-3}^* \quad \rightarrow u_1^* = \varepsilon_1^* \tag{133}$$

$$\varepsilon_2^* - \phi_0 u_2^* = \phi_1 u_1^* + \phi_2 u_0^* + \phi_3 u_{-1}^* + \phi_4 u_{-2}^* \quad \rightarrow u_2^* = \frac{\varepsilon_2^* - \phi_1 u_1^*}{\phi_0} \tag{134}$$

$$\varepsilon_3^* - \phi_0 u_3^* = \phi_1 u_2^* + \phi_2 u_1^* + \phi_3 u_0^* + \phi_4 u_{-1}^* \quad \rightarrow u_3^* = \frac{\varepsilon_3^* - \phi_1 u_2^* - \phi_2 u_1^*}{\phi_0} \tag{135}$$

$$\varepsilon_4^* - \phi_0 u_4^* = \phi_1 u_3^* + \phi_2 u_2^* + \phi_3 u_1^* + \phi_4 u_0^* \quad \rightarrow u_4^* = \frac{\varepsilon_4^* - \phi_1 u_3^* - \phi_2 u_2^* - \phi_3 u_1^*}{\phi_0} \tag{136}$$

$$\varepsilon_5^* - \phi_0 u_5^* = \phi_1 u_4^* + \phi_2 u_3^* + \phi_3 u_2^* + \phi_4 u_1^* \quad \rightarrow u_5^* = \frac{\varepsilon_5^* - \phi_1 u_4^* - \phi_2 u_3^* - \phi_3 u_2^* - \phi_4 u_1^*}{\phi_0} \tag{137}$$

where ε_t^* is resampled standardized residuals from AR(p) specification, u_t^* is the error term by which bootstrapped series is generated and ϕ_i is the coefficients of the corresponding AR(p) model estimated by OLS. The backwards iteration scheme utilizing u_t^* to generate bootstrapped series x_t^* is

$$x_t^* = x_{t-1}^* + u_t^* \tag{138}$$

If $d_t = 0$ or $d_t = \beta_0$, or,

$$x_t^* = \bar{y}_n + x_{t-1}^* + u_t^* \tag{139}$$

If $d_t = \beta_0 + \beta_1 t$, with $x_{0,n}^* = x_0$.

$$x_{t,n}^* = x_{t-1,n}^* + u_{t,n}^* \quad for\ t = 1, \dots, n \tag{140}$$

If $d_t = 0$ or $d_t = \beta_0$, or,

$$x^*_{t,n} = \bar{y}_n + x^*_{t-1,n} + u^*_{t,n} \qquad for \quad t = 1, \dots, n \tag{141}$$

If $d_t = \beta_0 + \beta_1 t$, with $x^*_{0,n} = x_0$.

where $\bar{y}$ stands for the original series' mean. In addition, the algorithm needed to perform the Psaradakis' unit root test approach was coded in R (programming language) and for the sake of replication of the same results, seed was set to 100.

Table 14.

Psaradakis Unit Root Test Results

Series	Specification	**Psaradakis (2001)**								
		Model A			Model B			Model C		
		k	t	Ciritical values	k	t	Critical Values	k	T	Critical Values
lnGDP	*AR(2)* *AR(1)*	1	-2.903242**	-2.946; -2.411; -2.152 -3.272; -2.821; -2.577	1	-0.424468	-2.504; -2.055; -1.815 -3.599; -2.929; -2.596	1	3.656305	-3.139; -2.336; -1.945 -1.982; -1.459; -1.169
lncons	*AR(3)* *AR(1)*	1	-2.57*	-3.682; -3.107; -2.807 -3.411; -2.852; -2.573	1	-0.439126	-2.828; -2.455; -2.247 -3.483; -2.755; -2.470	1	2.891007	-2.403; -1.634; -1.272 -1.949; -1.359; -1.072
lnP	*AR(3)*	1	-1,58	-16.091; -13.772; -13.032	1	-3,38	-24.062; -20.311; -18.658	1	-1,70***	-0.452; -0.408; -0.384
lnER	*AR(3)* *AR(2)*	1	-0.685604	-3.238; -2.137; -1.560 -2.808; -1.966; -1.516	1	-2.253756***	-1.923; -1.324; -1.040 -2.379; -1.619; -1.298	1	-2.470015***	-2.340; -1.236; -0.617 -2.418; -1.535; -1.017
lnEX	*AR(2)*	1	-2,66	-4.470; -3.90; -3.534	1	-0.924554	-2.419; -1.969; -1.756	1	3.047671	-3.408; -2.600; -2.206
lnIM	*AR(1)*	1	-2.540373*	-3.299; -2.776; -2.482	1	-0.776435	-2.868; -2.367; -2.110	1	3.148563	-2.993; -1.913; -1.428
i	*AR(4)*	6	-2,95	-4.050; -3.502; -3.151	1	-0.832194	-4.399; -3.521; -3.064	1	-0.878994*	-1.660; -1.118; -0.836
lnM1	*AR(3)* *AR(2)*	2	-1,46	-6.441; -4.733; -3.838 -5.437; -3.994; -3.382	2	-3,78***	-1.506; -0.647; 0.001 -2.423; -1.761; -1.424	3	-0,45	-6.325; -4.862; -4.056 -5.873; -4.383; -3.566
lnM2	AR(3)	1	-0.789334	-5.128; -3.575; -2.978	1	-3.419425***	-2.525; -1.779; -1.148	1	0,91	-6.554; -4.632; -3.849

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

In Table 14 Model A refers to the most extensive model where both a deterministic constant and trend term is involved, Model B stands for the model with a constant only and Model C represents the model without constant, under the null that series has unit root. First, second and the third critical values in Table 14 correspond to 1%, 5% and 10% rejection probabilities, or equivalently, 100th, 500^{th} and 1000^{th} test statistics values, respectively. Distributions of all data specific test statistics are provided in Appendix B, with the borders of rejection areas marked with dashed lines.

Hypothesis testing procedure is straight forward; if the calculated t statistics of the autoregressive term is more negative than the critical value at the desired level, we can reject the null of unit root. Calculated t statistics of the autoregressive term in the relevant model for each series is compared with AR(p) structures suggested by AIC and SIC. Needless to say, if AIC and SIC measures yield the same AR(p) specification, then the statistical inference on the value of the autoregressive term is performed depending on one set of critical values.

The results of Psaradakis' unit root approach show that unit root hypothesis for lnGDP can be rejected at 5% significant level in Model A where both constant and a linear trend are involved. This means that lnGDP is stationary with an integration degree of zero. This find contradicts with the conventional unit root tests which are known to perform poorly under small sample presence in term of disparities between actual and nominal significance level. On the other hand, it is noteworthy that the findings of Psaradakis unit root approach on the integration degree of lnGDP are highly parallel with Lee Strazicich (LS) unit root test results, where two structural breaks are maintained under the null of unit root. Furthermore, it is in particularly meaningful that LS test rejects the null of unit root at 1% significance level in the model where Psaradakis' unit root approach also rejects, giving rise to the argument that bootstrap approach conveys all the information involved in the sample including the breaks to the statistical inference process regarding the unity of time series' roots. Distribution of the test statistics derived for all three models with different AR(p) structures are provided in Appendix B (see Table A1, for example, for Model A with AR(1) specification).

Regarding the result of Psaradakis unit root test for lncons, a similar pattern can be seen. According to the results, null hypothesis of unit root can be rejected with 90% confidence in the model where two deterministic components, a constant and a linear trend, are involved. This result also implies a high disagreement between conventional and bootstrap unit root tests. On the other hand, the model statistically rejecting the presence of unit root in bootstrapped test also coincides with that of unit root tests with structural breaks.

With 99% confidence, Psaradakis test results show that lnP does not contain a unit root in the model where no intercept and trend are involved. Besides, as the thresholds of 1%, 5% and 10% probabilities in Model A and Model B suggest, the test strongly rejects the presence of a constant and a linear trend. Besides, lnER is strongly found to be stationary in level, as the t statistics calculated for the autoregressive terms are more negative than 1% significance threshold in Model B and Model C. Furthermore, given the Turkish macroeconomic time series focused in this study and the results of Psaradakis unit root test, it can be seen in Table 13 that lnEX is the only macroeconomic time series which has a unit root. lnEX series is consistently found to have I(1) in all the conventional unit root tests and in the tests with onetime or multiple structural breaks. On the other hand, consistent with LS and LP test results, null of unit root hypothesis cannot be accepted for Model A in Psaradakis unit root test. Among others, this finding is also interesting and in line with the intuition that bootstrap method provides good refinements regarding identification of time series characteristics, by incorporating structural break information into the empirical distribution of test statistics through bootstrapped series. Furthermore, results show that null of unit root can be rejected for the series i, with 10% significance level in Model C. Besides, both lnM1 and lnM2 show level stationary characteristics, as the hypothesis testing the unity of the roots can be rejected with 99% confidence in Model B. Psaradakis' unit root test performed with the first differences of the series for Model C can be seen in Table 15.

Table 15.

Psaradakis' Unit Root Test Result with the first differenced series

Series	Specification	Model C		
		k	t	Critical Values
DlnGDP	*AR(2)*	1		-1.045; -0.500; -0.190
	AR(1)		-3.710**	-3.906; -2.799; -2.221
Dlncons	*AR(1)*	1	-3.917***	-3.454; -2.465; -2.014
DlnP	*AR(3)*	1	-0.813	-2.614; -1.708; -1.279
DlnER	*AR(3)*	1	-0.998**	-1.520; -0.841; -0.503
	AR(2)			-3.753; -3.120; -2.788
DlnEX	*AR(2)*	1	-2.440*	-3.103; -2.503; -2.226
DlnIM	*AR(1)*	1	-3.043***	-2.0504; -1.463; -1.174
Di	*AR(1)*	1	-5.046***	-3.779; -2.627; -1.997
DlnM1	*AR(1)*	1	-1.176	-5.632; -3.930; -3.0916
DlnM2	AR(3)	1	-0.908	-2.678; -2.088; -1.805

****, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10% significance levels, respectively*

4.2. An Investigation on the Stability of Turkish Consumption Function with Block Bootstrap

The main purpose of this chapter is to investigate the stability of Keynesian consumption function for the Turkish economy with endogenously determined regimes between 1987 and 2014 by employing block bootstrap procedure. As the empirical strategy we follow a methodology working in four avenues. Firstly, we perform unit root test for Turkish Gross Domestic Product (gdp) and consumption (cons) series by taking two endogenous breaks into account. Once the integration degree of the series is determined, we proceed to detect endogenous breaks using Bai and Perron (1998 and 2003)'s approach. The reason why this methodology is chosen is its ability in considering structural breaks not only from a single-series based perspective, but also from a perspective in which the relationship between different series are considered. Thirdly, we obtain an OLS estimation of the Keynesian consumption function by taking the structural breaks occurred in terms both of individual series and their relationship into consideration. Therefore, we compare the magnitude of income's coefficient across different regimes and discuss how it evolved during endogenously selected regime terms. Lastly, in an attempt to verify the findings regarding the stability of income's coefficient for the Turkish economy, we obtain bootstrap estimation of the Keynesian consumption function corresponding to each regime.

4.2.1. Data and Methodology

To investigate the stability of consumption for the Turkish economy, we focus on the Keynesian consumption function;

$$C = a + bY \qquad 0 < b < 1 \qquad (142)$$

where C, Y are consumption and income, respectively. In the empirical application we estimate the model

$$cons_t = a + b \times gdp_{\ t} + u_t \qquad 0 < b < 1 \qquad (143)$$

where cons and gdp stand for final consumption expenditure of resident households and Gross Domestic Product in 1998 prices for the Turkish economy, respectively and u_t is an error term with IID property . Both series used in the analysis extracted from Electronic Data Delivery System of Central Bank of Republic of Turkey. Graphs of the series over the period 1987-2014 is are given in Figure 9. Besides, the scatter plot of the corresponding series is provided in Figure 10. Shaded areas in Figure 10 show the periods in which the relationship between consumption and GDP series appears to breakdown with the respective year intervals.

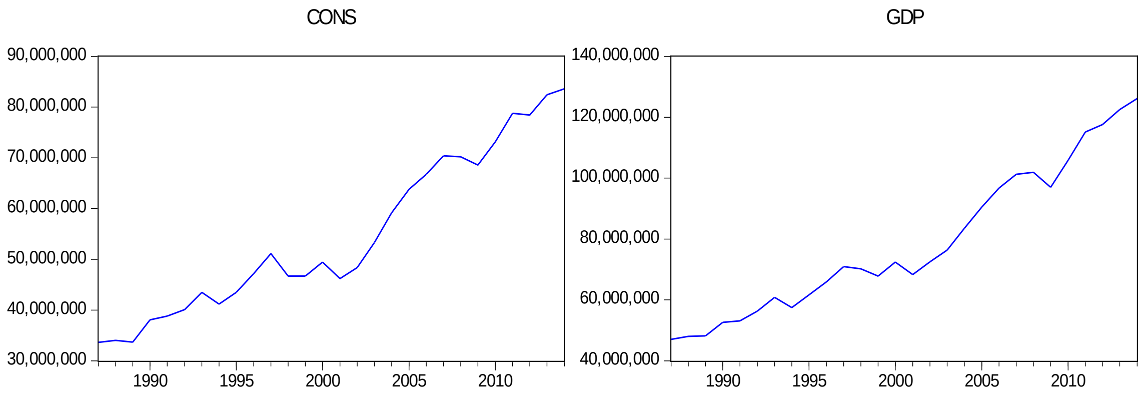


Figure 9. Turkish aggregate consumption and gross domestic product between 1987 and 2014.

As the first part of the empirical strategy, we test stationarity condition of the series cons and gdp using Lumsdaine-Papell unit root test outlined in chapter 2. Next, we endogenously determine structural breaks in the series and in the relationship between the series. For this purpose, we employ Bai and Perron (1998 and 2003)'s methodology.

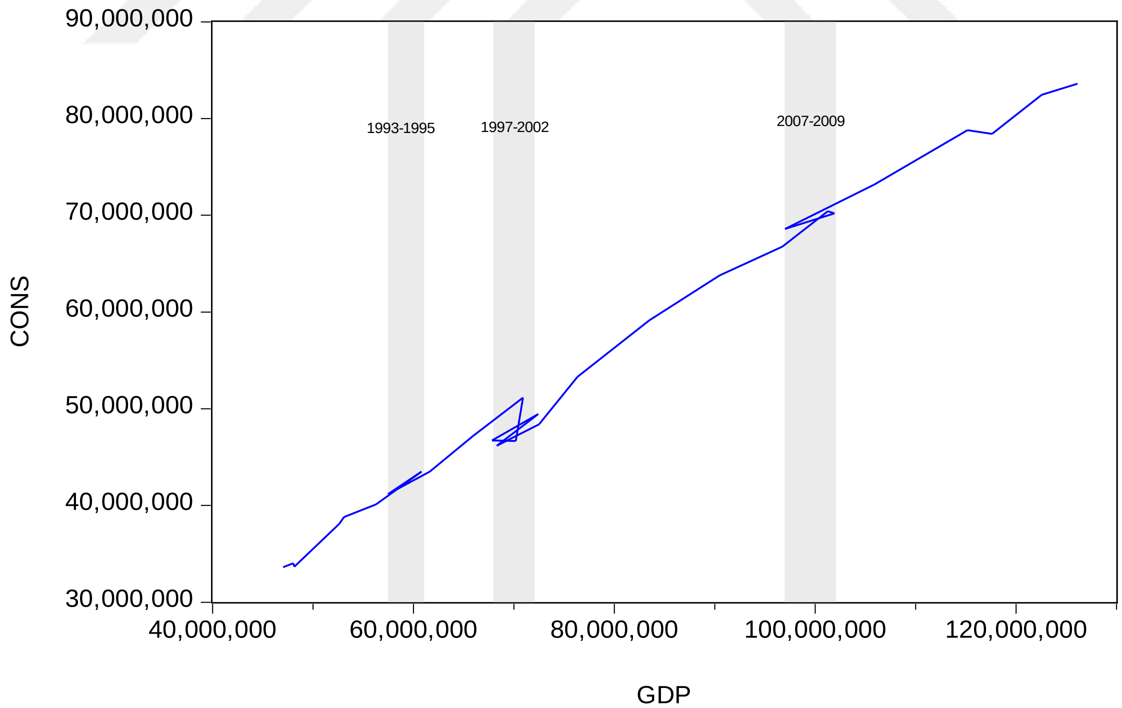


Figure 10. Scatter of aggregate consumption and gross domestic product for the period between 1987-2014

Bai and Perron (1998) suggest an OLS-based methodology to date and monitor structural breaks without predetermining the number of break dates. Bai and Perron (1998) depart from a multiple linear regression model

$$y_t = x_t'\beta + z_t'\delta_j + u_t \qquad (t = T_{j-1}+, \dots, T) \tag{144}$$

where y_t is the independent variable, while x_t' and z_t^t are independent variable vectors with $(p \times 1)$ and $(q \times 1)$ dimensions, respectively. Bai and Perron (1998) use the break points indices $(T_1, \dots, T_m)$ as unknown (p. 49). In matrix notation Bai and Perron (1998) state the data generating process as follows

$$Y = X\beta^0 + \bar{Z}^0\delta^0 + U \tag{145}$$

Bai and Perron (1998) aim to estimate unknown coefficients β^0, $\delta^0, \dots, \delta_{m+1}^0$ and $T_1^0, \dots, T_m^0$ by depending on a least square principle in which for each m-partition $(T_1, \dots, T_m)$, shown as $\{T_j\}$, sum of squared residuals $\sum_{i=1}^{m+1}\sum_{t=T_{i-1}+1}^{T}[y_t - x_t'\beta - z_t'\delta_i]^2$ is minimized (p. 50).

In the next step we obtain OLS estimate of Keynesian consumption function with the corresponding break dates and compare the magnitude of the income's coefficients. As the last tool of our empirical strategy, we employ block bootstrap to verify the magnitude differences.

4.2.2. Empirical Findings

As the first step of our empirical strategy we check the stationary condition of the series cons and gdp, by employing Lee-Strazich unit root test allowing for two endogeneous structural breaks. The results of Lee-Strazich unit root test are given in Table 16.

Table 16.

Lee-Strazich Unit Root Test Results for the Series Cons and Gdp

	Lee-Strazicich					
	Level			Both level and trend		
	Lag	Test Statistic	Break Dates	Lag	Test Statistic	Break Dates
gdp	1	-2.3771	1993; 2002	3	-5.1759*	1993; 1999
cons	3	-3.3341	1993; 2010	1	-5.7531**	1999; 2004

Critical values for 1%(***), 5%(**) and 10%(*) significance levels are -4.54; -3.84; and -3.50 for the model with two breaks in level, and -5.823, -5.286 and -4.989 for the model with two breaks in both trend and constant, respectively. Critical values are derived from Lee and Strazich (2003) and Altınay (2005).

Table 16 shows that under the model where two breaks are associated with both intercept and slope, the series cons and gdp are integrated degree of zero at 10% and 5% significance levels, respectively. Besides, the corresponding break dates for the series cons is 1999 and 2004. On the other hand, Lee-Strazich test results show that break points for the series gdp are 1999 and 2004. As a result, we conclude that stationary condition for the series holds and further estimations taking the structural changes in the series into consideration can be trusted.

For a more comprehensive evaluation of the series and the nature of relationship between them, we also investigate the structural breaks occurring in terms of the relationship between cons and gdp. For this purpose, we utilize from Bai and Perron (1998)'s proposed methodology. This methodology is in particularly superior to other methods associating structural breaks, because the former determines the number of break dates endogenously, as well as corresponding break dates. To this end, we start with evaluating each series independently. In the analysis, we use trimming value as four and determine the number of break dates depending on Bayesian Information Criteria (BIC). The number of break dates chosen based on BIC is given in Figure 11.

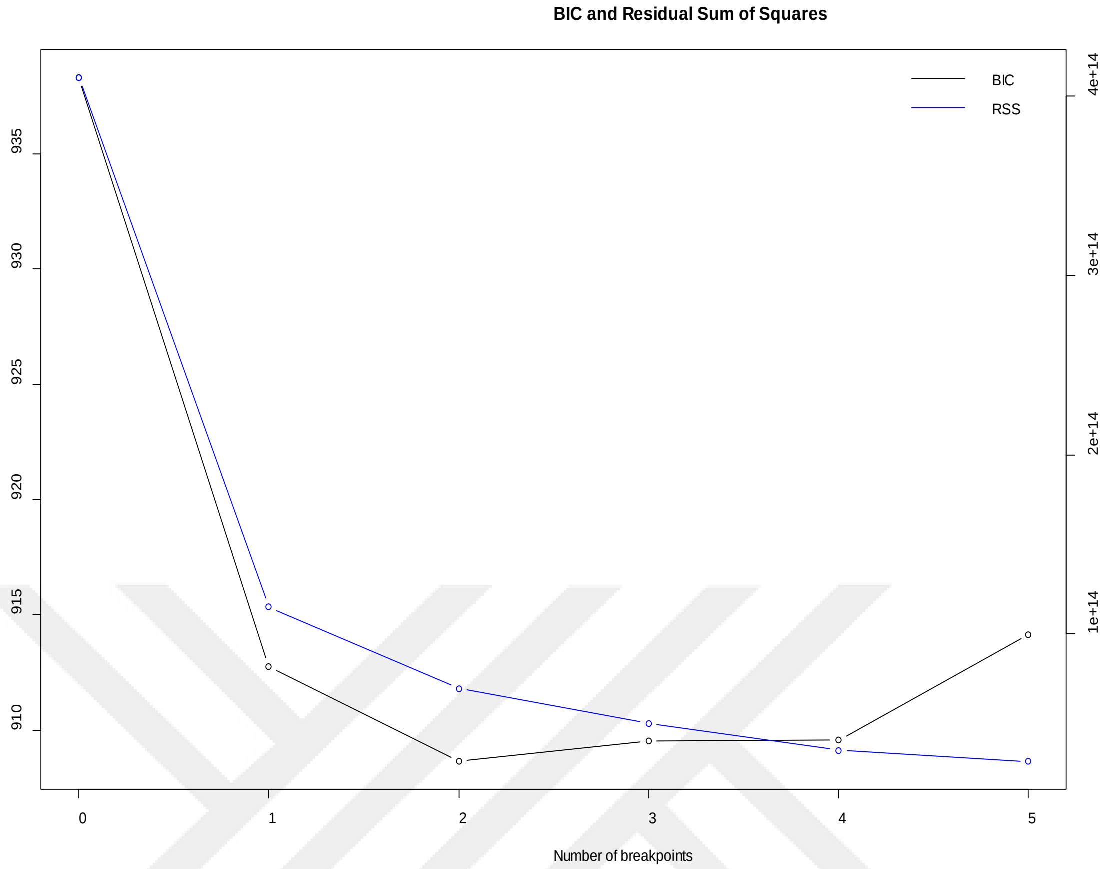


Figure 11. Number of break date selection for cons

In Figure 11 the number of break dates is determined where the BIC is minimized. Accordingly, Figure 11 shows that BIC suggests two break dates for the series cons. In other words, Bai and Perron (1998)'s methodology anticipates three regimes that are segregated by the corresponding break dates of 2000 and 2007. The break dates and the respective confidence intervals are presented in Figure 12. In Figure 12, the solid line shows the average consumption for each regime, while the dashed lines represent structural breaks determined. In addition, 95% confidence intervals for the break dates 2000 and 2007 are given as 1999-2001 and 2006-2008, respectively.

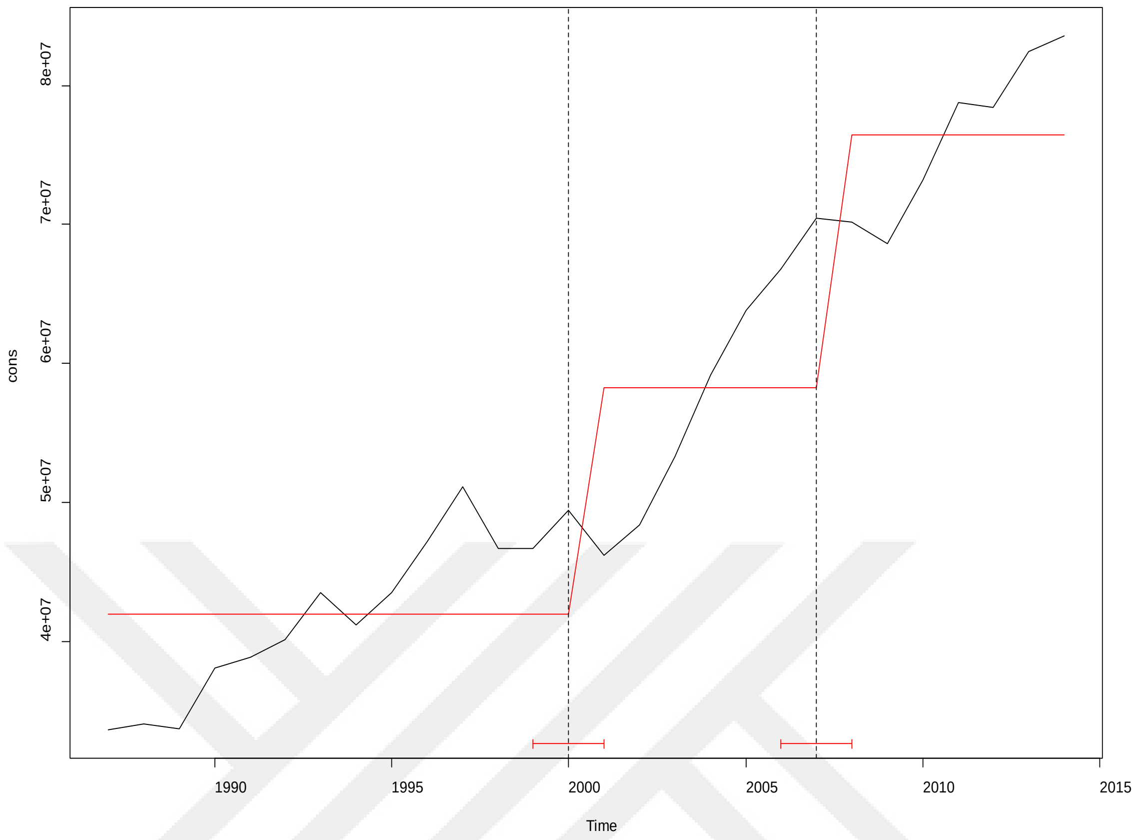


Figure 12. Structural breaks, regime averages and 95% confidence intervals for cons

Next, we investigate the presence of structural changes for the series gdp. Using BIC as the decision criteria with the trimming condition of four, we find that gdp series is also characterized by two structural breaks. The value of BIC with respect to different regime shifts can be found in Figure 13, where BIC is minimized at two structural breaks.

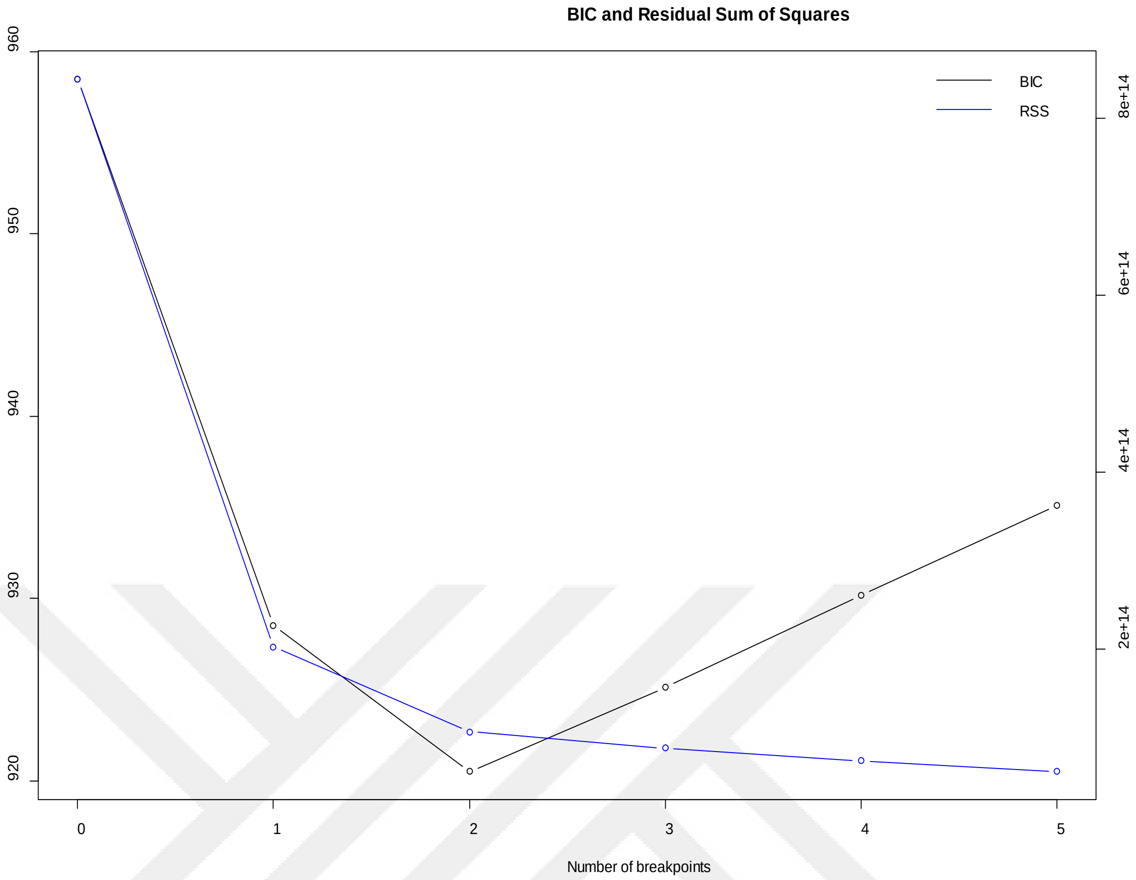


Figure 13. Number of break date selection for gdp

The break dates for gdp chosen by Bai and Perron (1998)'s approach are 2000 and 2008. Besides, %95 confidence intervals regarding these break dates chosen are 1999-2001 for 2000 and 2007-2009 for 2008. Regime averages and break dates combined with 95% confidence intervals for gdp are presented in Figure 14.

An important distinction of the methodology proposed by Bai and Perron (1998) is that it allows investigating the relationship between two series in a structural break framework. In other words, use of Bai and Perron (1998)'s methodology allows not only evaluating the structural changes reflected in individual series, but also the changes occurring in the co-movement of different series. To perform this, we consider the cons and gdp series simultaneously in a multiple structural break framework.

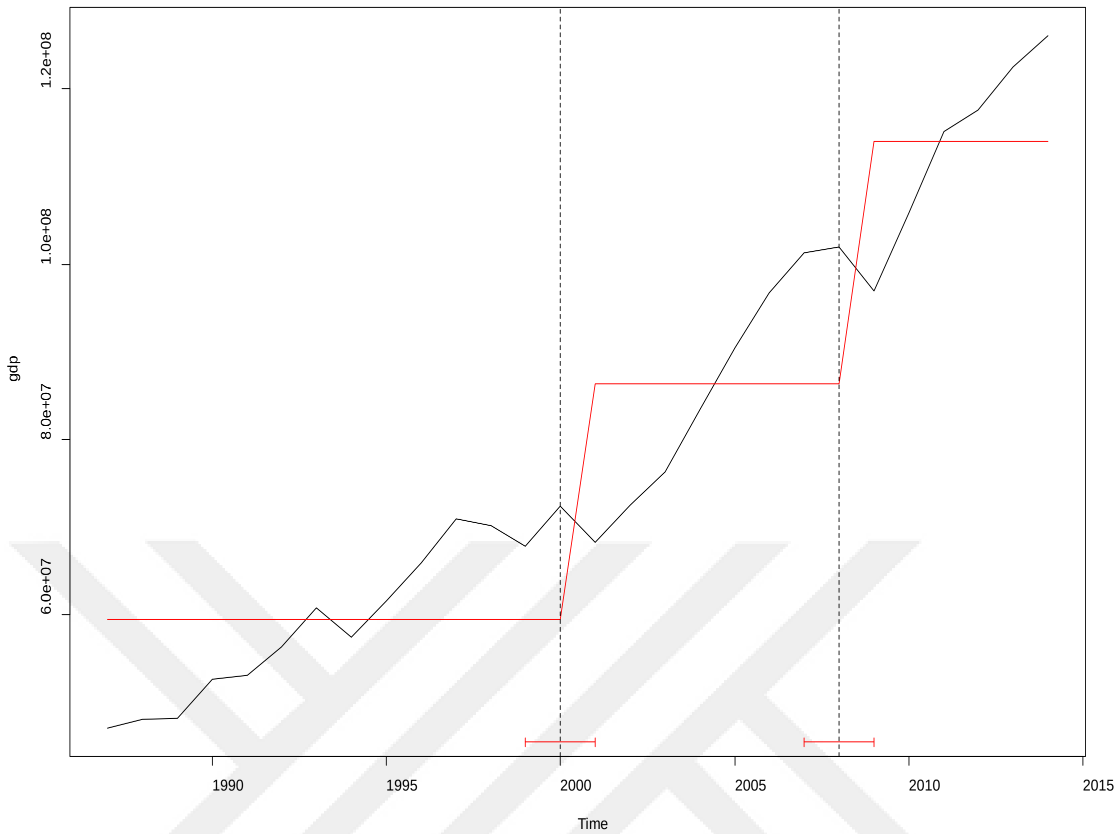


Figure 14. Structural breaks, regime averages and 95% confidence intervals for gdp

In the analysis of multiple structural breaks between cons and gdp, we choose trimming value as four and use the Bayesian Information Criteria to determine the number of breaks between the series. Again we start with determining the number of breaks occurring in the relationship between cons and gdp. As can be seen in Figure 15, we find that there are two structural breaks in the relationship between cons and gdp. These break dates correspond to 1997 and 2004. Furthermore, 95% confidence intervals for the break dates of 1997 and 2004 are 1995-1998 and 2003-2005, respectively.

Gure

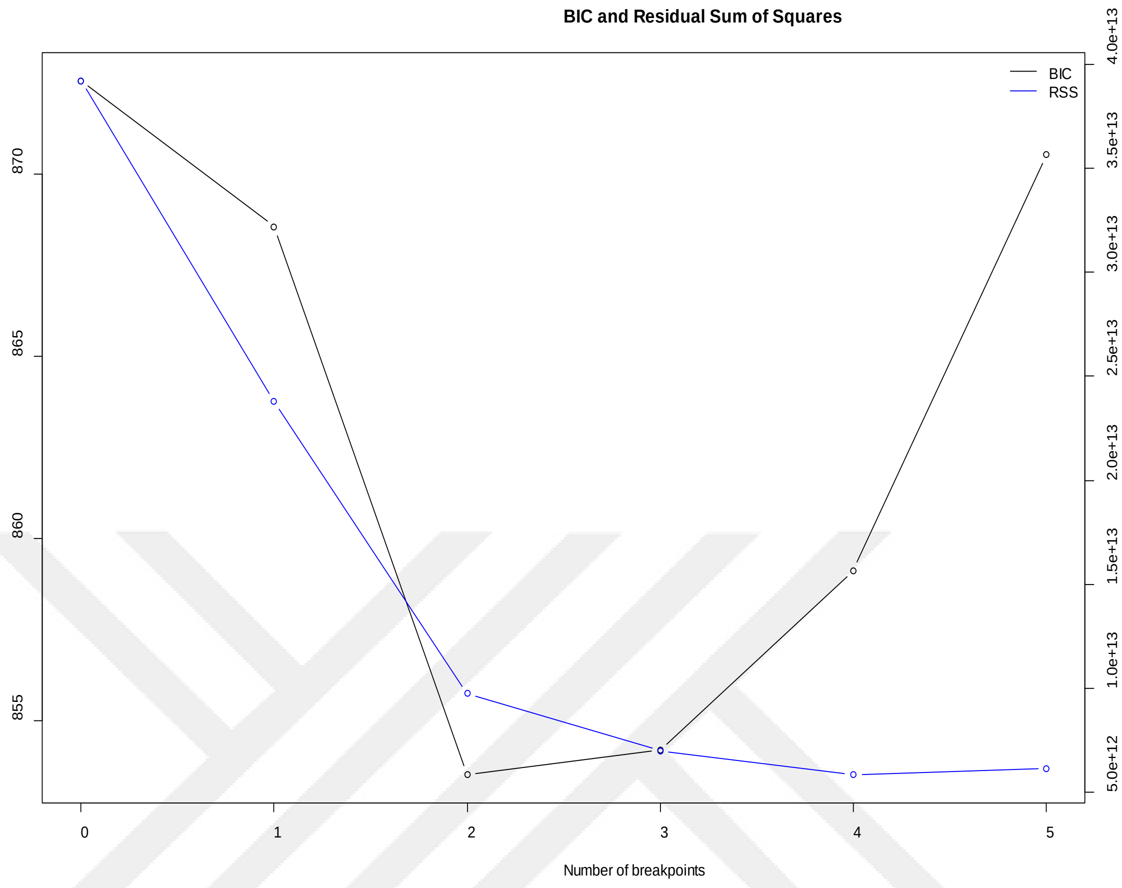


Figure 15. Number of break date selection for cons and gdp

Break dates of the co-movement of cons and gdp and the respective 95% confidence intervals are given in Figure 16. As can be seen from Figure 16, Bai and Perron (1998)'s methodology suggests three regimes for Keynesian consumption function of the Turkish economy for the period 1987-2014. These regimes are 1987-1997, 1998-2004 and 2005-2014.

Armed with the information regarding the structural breaks, it is now possible to investigate the stability of Keynesian consumption function for the Turkish economy for the period 1987-2014. For this purpose, we firstly employ ordinary least squares to estimate the consumption function for each regimes determined by Bai and Perron (1998)'s methodology, using the model

$$cons_t = c_1 \times DUMMY_{1987-1997} + \alpha_1 \times (DUMMY_{1987-1997} \times gdp_t) + c_2 \times DUMMY_{1998-2004} + \alpha_2 \times (DUMMY_{1998-2004} \times gdp_t) + c_3 \times DUMMY_{2005-2014} + \alpha_3 \times (DUMMY_{2005-2014} + \times gdp_t)$$

where $DUMMY_{1987-1997}$, $DUMMY_{1998-2004}$ and $DUMMY_{2005-2014}$ indicate the dummy variables constructed for the regimes, 1987-1997, 1998-2004 and 2005-2014, respectively.

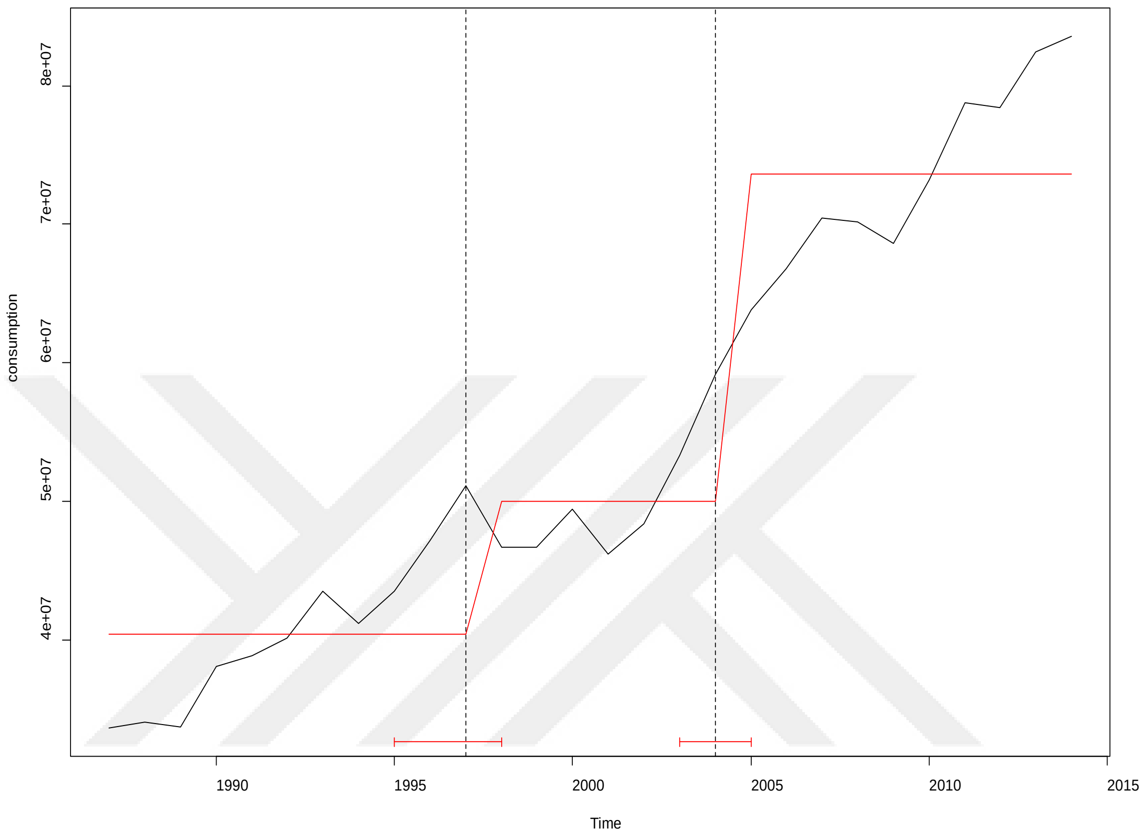


Figure 16. Structural breaks, regime averages and 95% confidence intervals for co-movement between cons and gdp

OLS estimates of the Keynesian consumption function for the Turkish economy for the period 1987-2014 can be seen in Table 17. The models estimated for the respective regime periods can be written as

$$cons_t = -801600 \times DUMMY_{1987-1997} + 0.7292 \times (DUMMY_{1987-1997} \times gdp_t) + -12430000 \times DUMMY_{1998-2004} + 0.8546 \times \ (DUMMY_{1998-2004} \times gdp_t) + 13430000 \times DUMMY_{2005-2014} + 0,5600 \times (DUMMY_{2005-2014} + \times\, gdp_t)$$

From the models estimated it can be seen that Keynesian consumption function for the Turkish economy shows substantial differences in the period between 1987 and 2014. These differences are in particularly evident regarding the magnitude of the income’s coefficient, as it considerably varies between 0.73 and 0.56. This coefficient is estimated as 0.73 is for the first regime determined as the period between 1987 and

1997. Furthermore, the highest value of the same coefficient is estimated as 0.85 for the period 1998-2004. Last but not least, effect of one unit increase in income on consumption is estimated as 0.56 for the period 2005-2014.

Table 17.

OLS estimates of the Keynesian Consumption Function for the Turkish Economy for the Period 1987-2014

cons={gdp}	c_1	c_2	c_3	β_1	β_2	β_3
$cons = \{gdp, DUMMY_{1987-1997}$	-801600	-	-	0.73	-	-
	(-0.518)	-	-	(26.8***)	-	-
$cons = \{gdp, DUMMY_{1988-2004}$	-	-12430000	-	-	0.85	-
	-	(-3.408 **)	-	-	(17.1 ***)	-
$cons = \{gdp, DUMMY_{2005-2014}$	-	-	13430000	-	-	0.56
	-	-	(6.797 ***)	-	-	(30.6 ***)

t values are in paranthesis. ***,** and * indicate significnce at 1%, 5% and 10% levels, respectively.

To scrutinize the stability of the Keynesian consumption function for the period 1987-2014 further, as the last step of our empirical strategy, we utilize from bootstrap in the estimation of the consumption function. Since cons and gdp series are found to be stationary, moving block bootstrap approach outlined in the subsection 1.1.2. becomes particularly functional in investigating the characteristics of Keynesian consumption function across the regimes identified. In our view, block bootstrap estimates of the consumption function are of particular interest since this approach takes not only the small sample presence into consideration, but also accounts for possible autocorrelations between successive observations of the series. Needless to say, such an approach requires one to determine to the autocorrelation presence and degree in the series, and at the end of the day, the size of the blocks used in resampling the series with repetition. To perform this, we use autocorrelation function of the series cons, which indicates significant autocorrelations up to 5th lag. Therefore, we determine the block size as 5 and resample cons and gdp in a pairwise setting with 1000 replications. In this way, we construct 1000 times cons and gdp series and obtain the respective parameter estimates for each repetition. The results of block bootstrap estimation of the Keynesian consumption function of Turkey for the period 1987-2014 are given in Table 18. To investigate the significance of the respective parameters estimates, 95% confidence intervals are used under the hypothesis asserting that the corresponding coefficient is equal to zero. Therefore, if the confidence interval estimated for a parameter estimate involves zero, then we conclude that the null hypothesis cannot be rejected.

Table 18.

Block Bootstrap OLS Estimates of the Keynesian Consumption Function for the Turkish Economy for the Period 1987-2014 with the Block Size of 5.

cons={gdp}	c_1^*	c_2^*	c_3^*	β_1^*	β_2^*	β_3^*
$cons$ $= \{gdp, DUMMY_{198}$	97575,7	-	-	0,725	-	-
$cons$ $= \{gdp, DUMMY_{198}$	-	-4258054	-	-	0,787	-
$cons$ $= \{gdp, DUMMY_{200}$	-	-	15045971	-	-	0,558
95% Confidence Interval	-25839143, 5598968	-72692214, -4154540	-15956720, 18536110	0.6008, 1.0492	0.7495, 1.2307	0.5096, 0.6153

Table 18 shows that there are notable differences between OLS estimates presented in Table 17 and the bootstrapped OLS estimates of the consumption function. These differences are in particularly evident in the magnitude of the constant terms of the models. More specifically, the constant term of the OLS regression for the period 1987-1997 is estimated as -801600, while that of bootstrapped OLS model is 97575.7. On the other hand, this coefficient is found to be statistically insignificant in both models. Regarding the constant term of the OLS and block bootstrapped OLS regressions estimated for the period 1998-2004, a substantial difference in the magnitude of the coefficient estimates is also observed, as the former estimates the constant terms as -12430000, the latter indicates a value of -4258054. In both models the null hypothesis asserting that the coefficients equal to zero can be rejected with 95% confidence. Furthermore, the constant term that corresponds to the regression estimated by OLS is 13430000, while the same parameter is estimated as 15045971 in bootstrapped OLS model. Besides, we observe a dramatic difference in the decision regarding the statistical value of this parameter. In the hypothesis testing regarding population value of the parameter, OLS model suggests with 95% confidence that the parameter is significantly different from zero. Conversely, the constant term for the period 1998-2004 is not found to be statistically different from zero at any conventional significance level. A regular pattern emerged in the magnitude of the constant terms of the OLS and the bootstrapped OLS regression models is that the former underestimates the constant term in each regime.

The distribution of the constant term for the period 1987-1997 estimated by the block bootstrapped OLS is given in Figure 17. The dashed line on the left panel of Figure 17 shows the value of the constant term estimated by OLS method. Besides, on the right panel, Normal Q-Q plot of the distribution of the parameter estimated is presented to evaluate the consistency of the distribution with the normal distribution. Normal Q-Q plots are used to visually assess the normality of the distributions by comparing normal theoretical quantiles shown as the dashed lines in the figure with the quantiles obtained from a given distribution. For a distribution to be normal, the quantiles obtained from a distribution must be close to the line making an angle of 45 degree, implying a one-to-one accordance in quantiles. The right panel of Figure 17 shows that the distribution of the constant term form the period 1987-1997 is far from being normal. Besides, from the left panel of Figure 17 it can be seen that the quantiles calculated for the distribution are considerably different from the quantiles suggested by standard normal distribution. The distributions of the parameters estimated by bootstrapping OLS are provided in Appendix C.

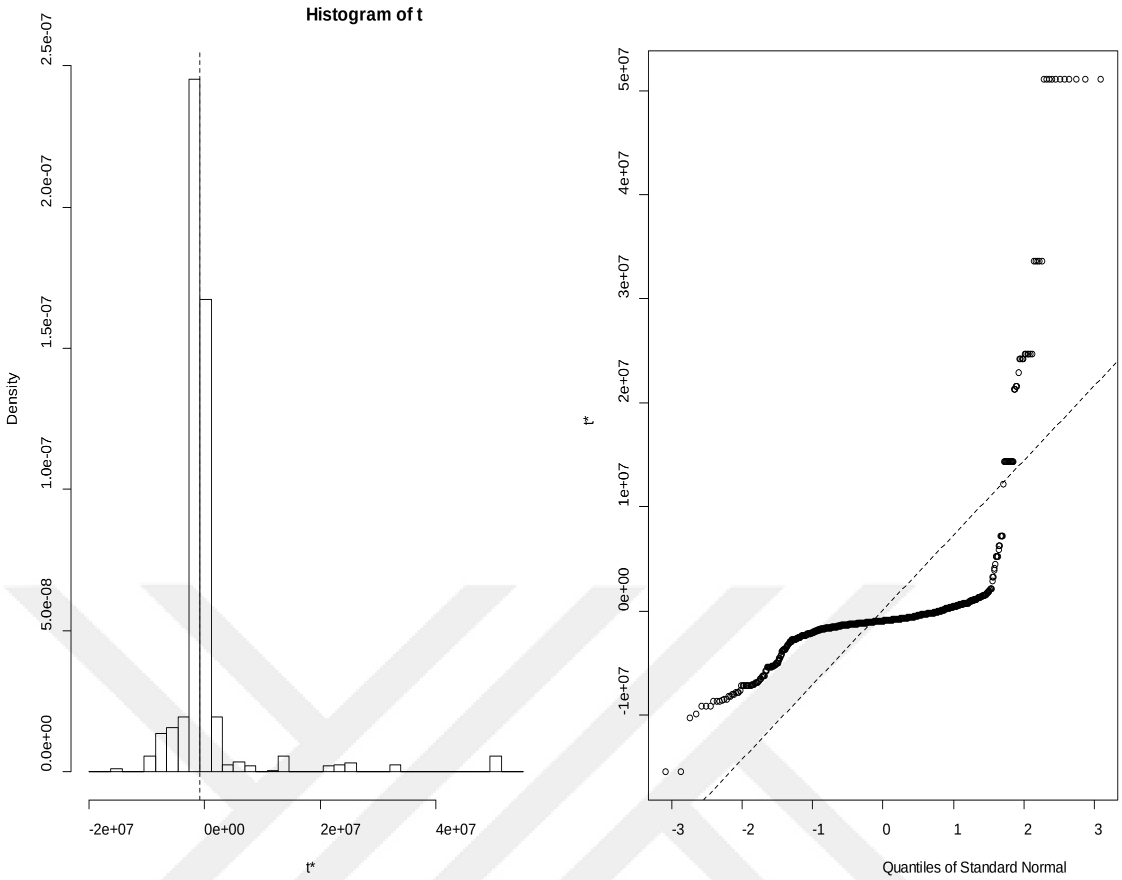


*Figure 17. T*he distribution of the constant term for the period 1987-1997 estimated by the block bootstrapped OLS

Considering the magnitude of the income's coefficient estimated by bootstrapping OLS method, in Table 18 we see an unstable characteristic in the Keynesian consumption function of the Turkish economy for the period 1987-2014, as suggested by the OLS estimation results of the model. Broadly speaking, the effect of an increase in income across regimes can be divided into two groups. The regimes corresponding to the periods 1987-1997 and 1998-2004 constitutes the first group exhibiting similar consumption patterns with the estimated parameter values of 0.725 and 0.787, respectively. Based on this information, it can be inferred that the consumption in the Turkish economy exhibited relatively stronger pattern for these two periods. On the other hand, our results show that income's effect on consumption dropped considerably from 0.787 in the period 1998-2004 to 0.558 in the period 2005-2014.

These results show that the consumption function of the Turkish economy exhibited an unstable nature between 1987 and 2014, in terms particularly of the effect of income on consumption. This result is also confirmed by bootstrapped version of the OLS method, which has known theoretical and empirical advantages in the case where the overall period focused is quite small and theoretical distributions used in statistical inference process cannot be trusted.

CHAPTER V

CONCLUSIONS

5.1. Conclusions

This study intends to explain bootstrap methods and its applications in applied economics by highlighting the dynamics through which bootstrap provides refinements by means of constructing empirical distribution of an estimation or a statistics, rather than assuming a well-behaving distribution with known characteristics. In particular, a clear explanation on how bootstrap improves the desired properties of estimates and test statistics is given with an emphasis on the different areas in statistics and econometrics where bootstrap method can be employed.

Type of data is an important issue in choosing the appropriate bootstrap method. In this sense, it can be stated that performing bootstrap schemes with cross-section data is relatively easy and straight forward. Bootstrapping with time series, however, raises a number of questions, stemming mainly from the autocorrelation between consecutive observations which invalidates the application of two main bootstrap approaches, bootstrapping the original sample and bootstrapping the residuals. Among these approaches bootstrapping the original sample is in particularly not suitable for time series to draw repeated samples since it would deteriorate the autocorrelation among observations and would construct an irrelevant bootstrap sample in each repetition. Because of this nature of time series, major aim of the studies intending to take the advantage of bootstrap is to transform residuals obtained from an idealized model fitted so as to perform a resampling procedure.

Unit root tests are important tools in econometrics, since they allow researchers to scrutinize properties of time series and determine suitability for further analysis. Because of this importance, as the first empirical application, this study focuses on the use of bootstrap in unit root tests. Firstly unit root presence in the series is comprehensively investigated. Especially, for comparison purposes, conventional unit root tests, as well as tests allowing for single or multiple endogenous breaks are performed with selected Turkish macroeconomic time series. To give a broad picture on the characteristics of Turkish macroeconomic time series, three sets of empirical tools are used. In the first place, some conventional unit root tests such as ADF, PP and KPSS are employed. Furthermore, unit root tests allowing for endogenous single and multiple structural breaks, namely Zivot Andrews (ZA), Lumsdaine Papell (LP) and Lee Strazicich (LS) unit root tests are implemented. The last but not the least, two parametric bootstrap unit root tests are performed, in an attempt to conclude better on the presence of unit root in the series and to prevent inconsistencies between the exact and apparent significance levels in hypothesis testing, resulting mainly from size distortions observed in cases where relatively small number of sample is present.

The main findings show that conventional unit root tests pose serious risks of over-differencing in small samples with apparent structural breaks. More specifically, over-differencing occurs when a stationary series is found to be integrated order of 1, and differenced accordingly, then the differenced series turns out to be nonstationary, misleadingly implying that the original series is I(2). This case is in particularly evident for Consumer Price Index (lnP) for the period 1985-2014 as the conventional unit root tests overwhelmingly indicate that the first differenced series is integrated order of 1. Besides, conventional unit root tests which do not take one time shocks in level and/or smooth structural changes in trend into account exhibit serious discrepancies regarding integration degrees of the time series considered.

Another finding of this study is the notable similarities between the results of unit root tests allowing for single and multiple endogenous breaks and that bootstrapped unit root tests. Most particularly, Turkish Gross Domestic Product (lnGDP), aggregate consumption (lncons), and annual deposit interest rate (i) series found to be stationary in their level in bootstrapped unit root test and unit root tests with structural breaks. These findings give rise to the argument that bootstrap approach conveys all the information involved in the sample including the breaks to the statistical inference process carried out to conclude about the unity of time series' roots. Furthermore, the results are in line with the intuition that bootstrap method provides good refinements regarding identification of time series characteristics, by incorporating structural break information into the empirical distribution of test statistics through bootstrapped series.

As the second application of bootstrap to time series, we investigated stability of Keynesian consumption function for the Turkish economy for the period 1987-2014, by employing moving block bootstrap which can be used in stationary time series. Our results show that consumption function for the Turkish economy exhibits dramatic differences across three regime periods determined endogenously as 1987-1997, 1998-2004 and 2005-2014. Our findings show that the effect of income on consumption for Turkish consumers dropped dramatically from 0.7 in the periods 1987-1997 and 1998-

2004 to 0.5 in 2005-2014. Furthermore, we find substantial differences in terms of statistical significance of the parameters estimated by OLS and block bootstrapped version of OLS. In our view, parameter estimates of the consumption function estimated by moving block bootstrap are more robust in both magnitude and statistical inference, considering the fact that the overall sample selected to work with is considerably small.

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APPENDIX A

ASYMTOTIC STATISTICS

1. Some Concepts of Asymptotic Investigation

The main purpose of this chapter is to give a brief insight into the concepts that will be used and referred to in the further chapters of the study. This chapter intends to provide a unified summary of several concepts essential for describing how a test statistics behaves in asymptotical sense under the presence of independent and identically distributed (i.i.d.) random variables and heterogeneously distributed variables with serial dependence.

1.1. Independent and Identically Distributed Variables Case

1.1.1. Convergence

Following Hamilton (1994), let $\{c_t\}_{T=1}^{\infty}$ be a sequence of deterministic numbers. This sequence is said to converge to c if for any $\varepsilon > 0$, there exists an N such that $|c_T - c| < \varepsilon$ whenever $T \geq N$ and can be denoted as

$$\lim_{T\to\infty} c_T = c$$

In other words, the value by which estimator c_T diverges from its actual value c diminishes, as the sample size increases.

1.1.2. Convergence in Probability

Limiting behavior of a statistics can also be discussed based on convergence in probability, rather than convergence to an actual value. To show this let $\{X_T\}_{T=1}^{\infty}$ be a sequence of scalar random variables. Given every $\varepsilon > 0$ and every $\delta > 0$, the condition for this sequence to converge to c in probability is

$$P\{|X_T - c| > \delta\} < \varepsilon$$

Alternatively, this can be shown as

$$plim X_T = c$$

Introducing convergence of the corresponding elements of X_T and c, this definition can be transformed so as to cover a sequence of random vectors or random matrices (Hayashi, 2000).

1.1.3. Convergence in Mean Square

Following Karr (1993), convergence in a power of expected value can generally be given as

$$\lim_{T\to\infty} E(X_T - c)^{\pi}$$

where π denotes the convergence degree. Thus, convergence between X_T and c in mean and in mean square occur for $\pi = 1$ and $\pi = 2$, respectively.

As shown in Rao (2002), a sequence of random variables is said to convergence in mean square if,

$$\lim_{T\to\infty} E(X_T - c)^2 = 0$$

In other words, square of the difference between observed and actual value of a random variable can be expected to be zero, provided that sample size in sequence increases sufficiently. An implicit subtraction that may follow this argument is that as the sample size increases, the difference between two components dies away and eventually ends up with zero on average.

This condition of convergence is stronger than convergence in probability, implying that a sequence of random variables converging to zero in mean square also converges to c in probability (Rao, 2002).

A following result closely related with convergence in mean square is Chebyshev's inequality. As reviewed in Hamilton (1994), Chebyshev's inequality defines the following condition,

$$P\{|X - c| > \delta\} \leq \frac{E|X - c|^{\pi}}{\delta^{\pi}}$$

In convergence in mean square case, we get;

$$P\{|X - c| > \delta\} \leq \frac{E|X - c|^{2}}{\delta^{2}}$$

To see the basic intuition behind this, it is useful to focus more on the left hand side of the inequality. Implicitly, the corresponding interval for $|X - c| > \delta$ is

$$X - c < -\delta \qquad X - c > \delta$$

$$X < c - \delta \qquad X > c + \delta$$

$$(-\infty, c - \delta) \cup (c + \delta, +\infty).$$

$$(-\infty, c - \delta) \bigcup (c + \delta, +\infty)$$

On the other hand, as shown by Rao (2002), Chebyshev's inequality can also be written as follows;

Let X be a random variable with mean c and varianceσ^2. Using variance definition,

$$\sigma^2 = \int (X-c)^2 f(x)dx$$

Employing the relevant interval;

$$\sigma^2 = \int_{-\infty}^{c-\lambda\sigma} (X-c)^2 f(x)dx + \int_{c-\lambda\sigma}^{c+\lambda\sigma} (X-c)^2 f(x)dx + \int_{c+\lambda\sigma}^{+\infty} (X-c)^2 f(x)dx$$

Omitting the second term with which random variable X does not comply;

$$= \int_{-\infty}^{c-\lambda\sigma} (X-c)^2 f(x)dx + \int_{c+\lambda\sigma}^{+\infty} (X-c)^2 f(x)dx$$

Replacing $(X-c)^2$ by the smallest value in the terms;

$$\sigma^2 \geq \lambda^2\sigma^2 \int_{-\infty}^{c-\lambda\sigma} f(x)dx + \lambda^2\sigma^2 \int_{c+\lambda\sigma}^{+\infty} f(x)dx$$

$$\sigma^2 \geq \lambda^2\sigma^2 P(|X-c| \geq \lambda\sigma$$

$$P\{|X-c| > \lambda\sigma\} \leq \frac{1}{\lambda^2} \qquad \lambda > 0$$

where σ denotes standard error of X. Replacing δ by $\lambda\sigma$, relevant interval is turns out to be,

$$(-\infty, c-\lambda\sigma) \bigcup (c+\lambda\sigma, +\infty)$$

With the introduction of standard error and use of a given of λ, it is now possible to give a probability for X to be within or without this interval. If, for example, $\lambda = \sqrt{2}$, 50% of random variable X lies within the interval$(c-2\sigma, c+2\sigma)$, that is, 50% of X falls between the area bounded by twice the standard error.

Ultimately, Chebishev's inequality is of interest because of its capability in establishing an interval within which a random variable lies by a certain probability.

1.1.4. Law of Large Numbers

So far the point of interest has been confined to the behavior of a single estimate obtained through a single sample selection procedure. For a more informative view on limiting behavior of statistics, however, distributional nature of an estimate put forth by a repeated sampling perspective must be taken into consideration. In the words of Rao (2002), *"In practice, estimates are made of an unknown quantity (parameter) by taking the average of a number of repeated measurements of the quantity, each of which may be in error. It is, therefore, of interest to study the properties of such an estimate. An initial enquiry is made concerning its behavior as the number of measurements increases. Does the estimate converge in some sense to the true value of the parameter under study?"*

To open up this understanding, following (Hamilton, 1994), let $\overline{Y_T}$ denote sample mean defined by $\bar{Y}_T = (\frac{1}{T})\sum_{t=1}^{T} Y_t$, where Y_t is identically and independently distributed random variable with mean μ and variance σ^2. The expected value and variance of $\overline{Y_T}$ are μ and σ^2/T, respectively. Variance of $\overline{Y_T}$, in particular, has important implications on the performance of the estimator. Needless to say, as sample size increases, $\overline{Y_T}$ turns out to be a consistent estimator of population parameter, for its variance converges to zero and becomes degenerated in the form of a spike centered at the population parameter (Hamilton, 1994). In other words, since variance of $\overline{Y_T}$ is inversely related with sample size T, variance of $\overline{Y_T}$ is equal to zero in limit, implying that as sample size rises variation in the value of estimator diminishes. This consistent nature of sample mean is said to be law of large numbers.

1.1.5. Convergence in Distribution

Following Hayashi (2002), let $\{X_T\}_{T=1}^{\infty}$ and $F_{X_T}(x)$ be a sequence of random scalars and corresponding cumulative distribution function, respectively. Convergence in distribution occurs when cumulative distribution function of X_T converges to that of X. Formally,

$$\lim_{T\to\infty} F_{X_T}(x) = F_X(x)$$

However, as highlighted by Karr (1993), this form of convergence is difficult to adopt since continuity of the relevant cumulative distribution function is required.

1.1.6. Central Limit Theorem

By the law of large numbers discussed above important distributional parameters of random variables and their conventional response to limiting sample size conditions are well defined. However, no particular distribution type a random variable exhibits has yet been introduced.

As discussed by (Hogg, 1978), if $X_1, X_2, \dots, X_T$ is a random sample size of T from a population with variance σ^2, then the asymptotic distribution of the random variable $\sqrt{T}(\bar{X}_T - \mu)/\sigma$ exhibits a random distribution with zero mean and unit variance. This implies that if the condition that $\sqrt{T}(\bar{X}_T - \mu)/\sigma$ is normally distributed with mean zero and unit variance is met, then, using these properties, it will be possible to draw an inference on probabilities of $\bar{X}$ (Hogg, 1978).

1.2. Heterogeneously Distributed Variables with Serial Dependence Case

In this section, the focus of attention is turned on definition of heterogeneously distributed variables with serial dependence and the concepts benefited from to derive asymptotic properties of these variables.

To start with, let $(Y_1, Y_2, \dots, Y_T)$ be a sample size T from a covariance stationary data generating process with the following properties,

$E(Y_T) = \mu \qquad for\ all\ t$

$E(Y_t - \mu)\big(Y_{t-j} - \mu\big) = \gamma_j \qquad for\ all\ t$

$$\sum_{j=0}^{\infty} |\gamma_j| < \infty$$

Following Hamilton (1994), the expectation of sample mean, $(\hat{Y}_T)$, generated from a process described above is μ, whereas its variance is given as,

$$\mathrm{E}(\overline{\mathrm{Y}}_{\mathrm{T}} - \mu)^2 = \mathrm{E}[(\frac{1}{\mathrm{T}}) \sum_{\mathrm{t}=1}^{\mathrm{T}} (\mathrm{Y}_{\mathrm{t}} - \mu)]^2$$

$$= \left(\frac{1}{T^2}\right) E\{[(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)] \times [(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)]\}$$

$$=(1/T^2)\, E\{(Y_1 - \mu)[(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)]$$
$$+(Y_2 - \mu)[(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)]$$
$$+(Y_3 - \mu)[(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)]$$
$$+ \cdots + (Y_T - \mu)[(Y_1 - \mu) + (Y_2 - \mu) + \cdots + (Y_T - \mu)]\}$$

Substituting γ_j for $E(Y_t - \mu)\big(Y_{t-j} - \mu\big)$,

$$= \left(\frac{1}{T^2}\right)\{[\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \cdots + \gamma_{T-1}]$$

$$+[\gamma_1 + \gamma_0 + \gamma_1 + \gamma_2 + \cdots + \gamma_{T-2}]$$
$$+[\gamma_2 + \gamma_1 + \gamma_0 + \gamma_1 + \cdots + \gamma_{T-3}]$$
$$+ \cdots + [\gamma_{T-1} + \gamma_{T-2} + \gamma_{T-3} + \cdots + \gamma_0]\}$$

Rearranging,

$$E(\bar{Y}_T - \mu)^2 = \left(\frac{1}{T^2}\right)\{T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + 2(T-3)\gamma_3 + \cdots + 2\gamma_{T-1}\}$$

or

$$E(\bar{Y}_T - \mu)^2 = \left(\frac{1}{T}\right)\{\gamma_0 + \left[\frac{T-1}{T}\right](2\gamma_1) + \left[\frac{T-2}{T}\right](2\gamma_2) + \left[\frac{T-3}{T}\right](2\gamma_3) + \cdots + \left[\frac{1}{T}\right](2\gamma_{T-1})\}$$

By expanding both sides by sample size T, to check the limiting value of this expression, we get,

$$T \cdot E(\bar{Y}_T - \mu)^2 = \left|\gamma_0 + \left[\frac{T-1}{T}\right](2\gamma_1) + \left[\frac{T-2}{T}\right](2\gamma_2) + \left[\frac{T-3}{T}\right](2\gamma_3) + \cdots + \left[\frac{1}{T}\right](2\gamma_{T-1})\right|$$

This expanded form of variance which appears to be a weighted average of the covariances can be interpreted based on three, namely, large, modest and small, sample size cases. In the presence of large sample size, for instance, $T \cdot E(\bar{Y}_T - \mu)^2$ converges to γ_0 , for all other terms sum to zero. In the modest sample case, each term related to sample size T does affect the sum, albeit their contribution approaches unity. In the final, small sample size, case, the sum is affected all the way through the weighted average of the covariances (Hamilton, 1994).

1.2.2. Martingale Difference Sequence

Martingale difference sequence is used in order to examine whether a random variable is dependent on or independent of its past values. For this purpose, let $\{Y_t\}_{t=1}^{\infty}$ be a random variable with $E(Y_t) = 0$. On the other hand, assume Ω_t indicates information available at date t, through current and lagged values of Y . If the conditional expectation that,

$$E(Y_t \setminus \Omega_{t-1}) = 0 \qquad for\ t = 2,3, \ldots$$

is met, then $\{Y_t\}$ is said to be a martingale difference sequence with respect to Ω_t. This condition can equivalently be shown as,

$$E(Y_t \setminus Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0$$

As can be seen in this expression, Martingale Difference Sequences are highly associated with the presence of serially uncorrelated series, regardless from the type of relation such as linear or nonlinear (Hamilton, 1994).

APPENDIX B

DISTRIBUTIONS of PSARADAKIS' UNIT ROOT TEST STATISTICS

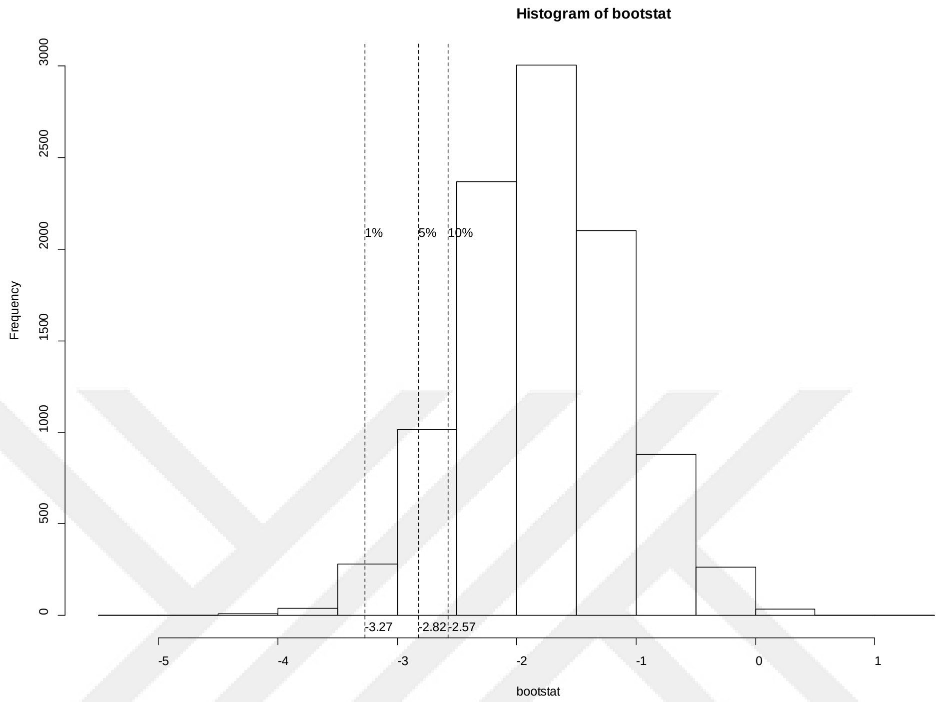

Figure A1, lngdp, Model A, AR(1)

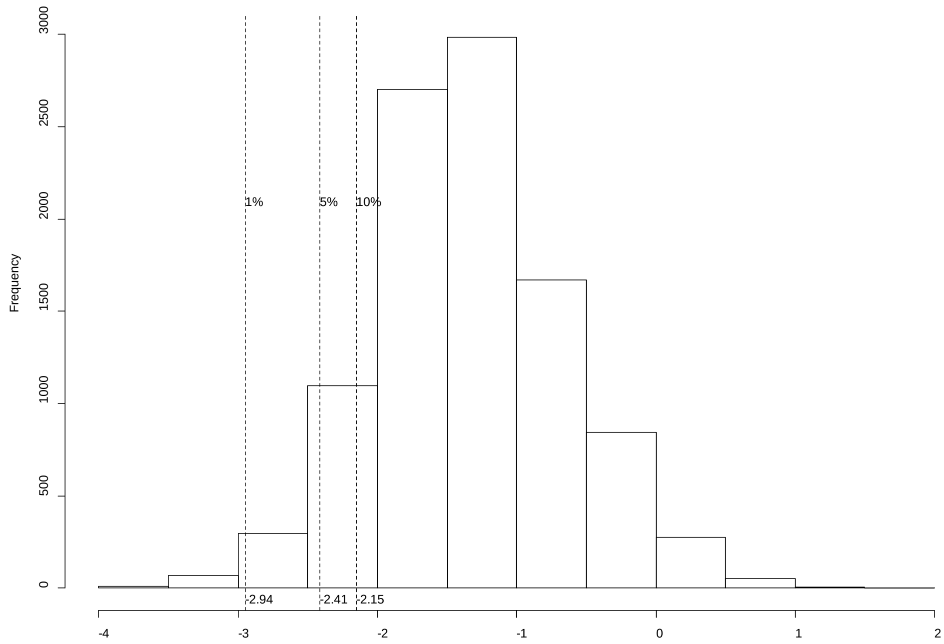

Figure A2, lngdp, Model A, AR(2)

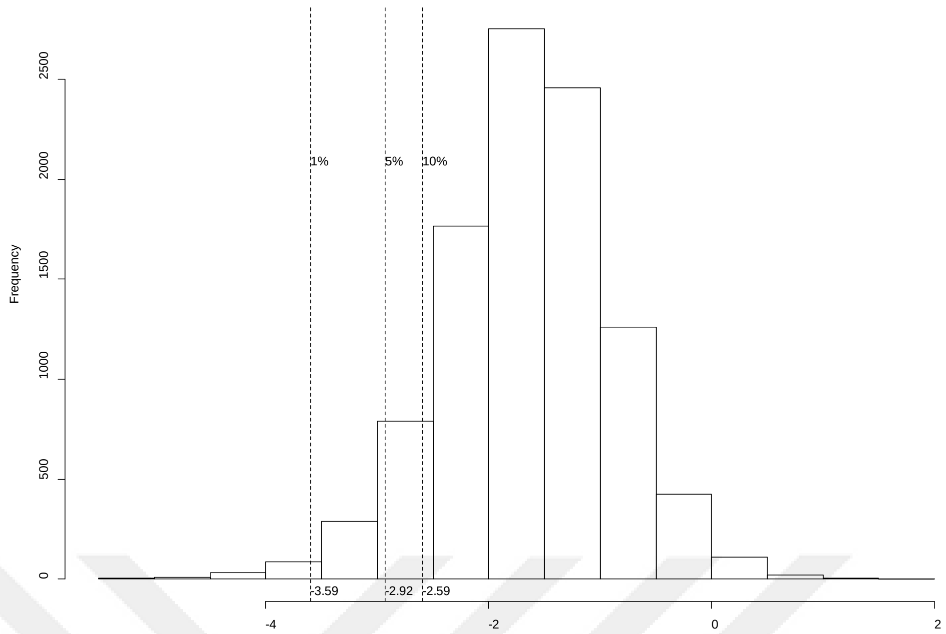

Figure A3, lngdp, Model B, AR(1)

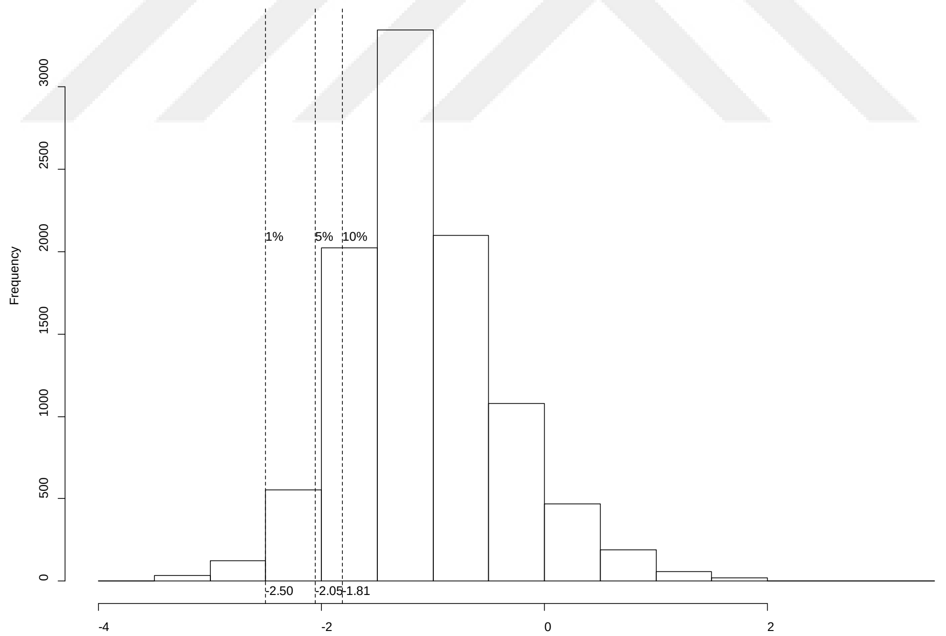

Figure A4, lngdp, Model B, AR(2)

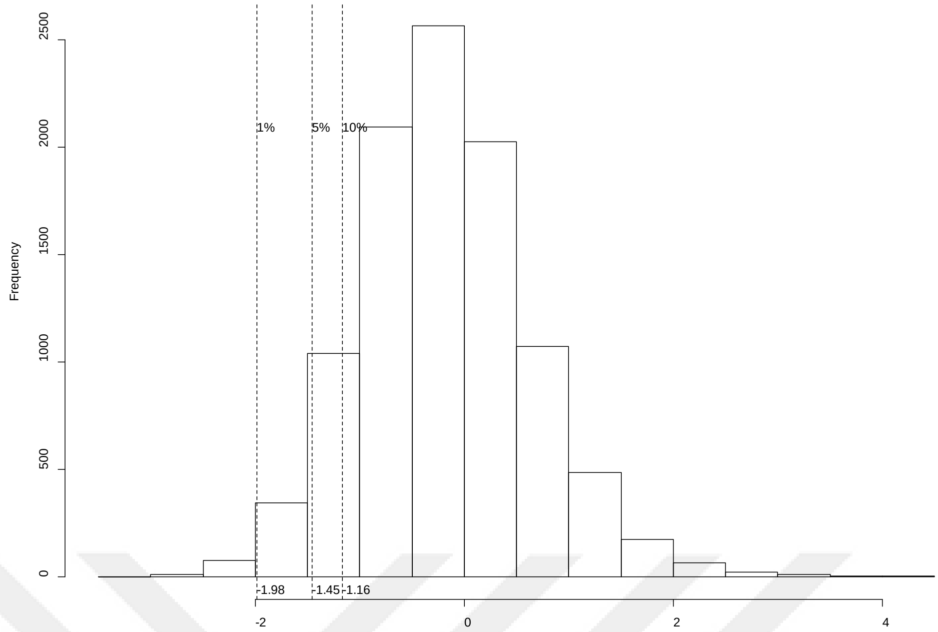

Figure A5, lngdp, Model C, AR(1)

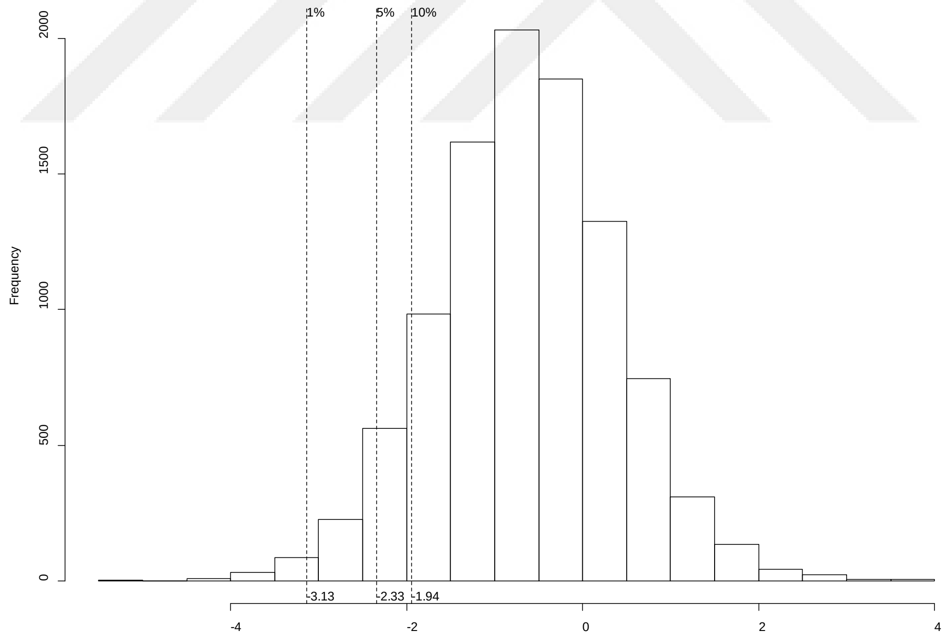

Figure A6, lngdp, Model C, AR(2)

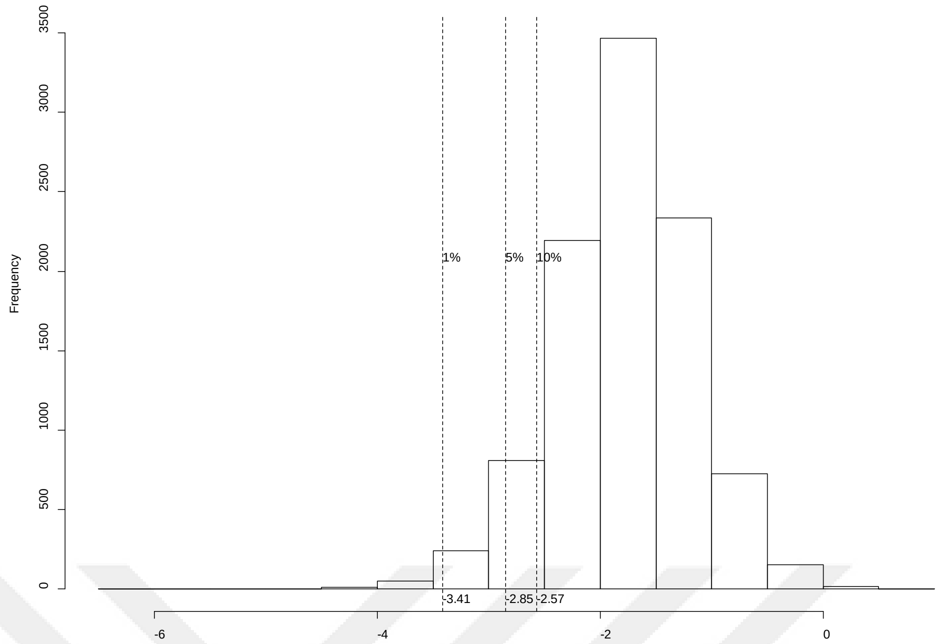

Figure A7, lncons, Model A, AR(1)

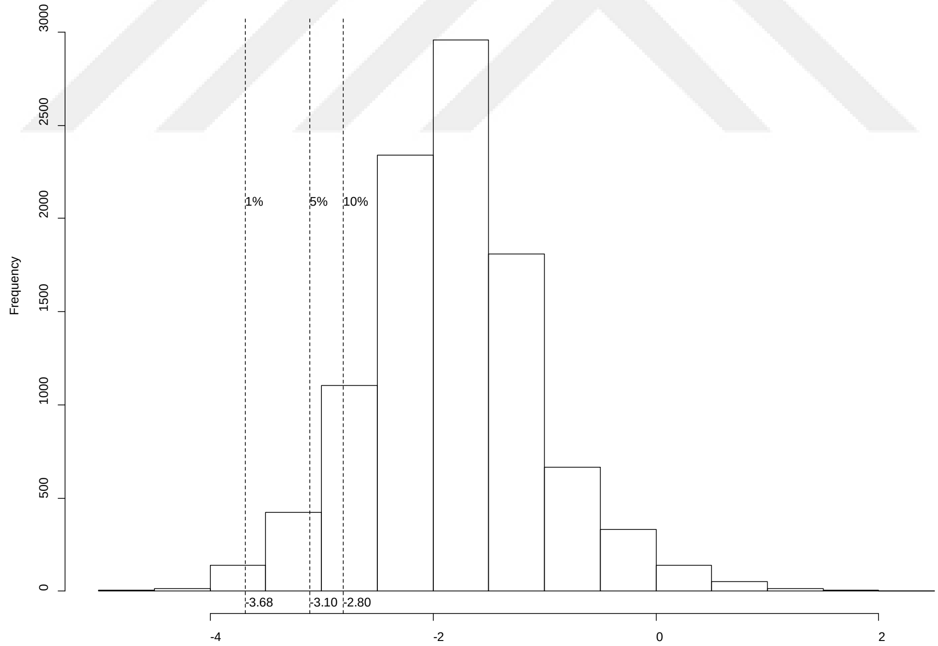

Figure A8, lncons, Model A, AR(3)

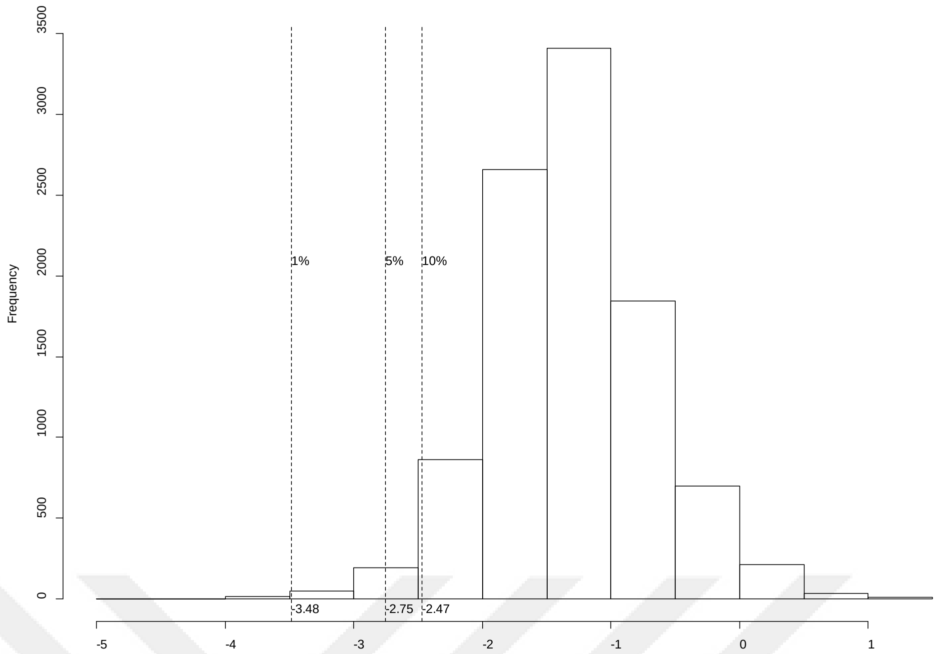

Figure A9, lncons, Model B, AR(1)

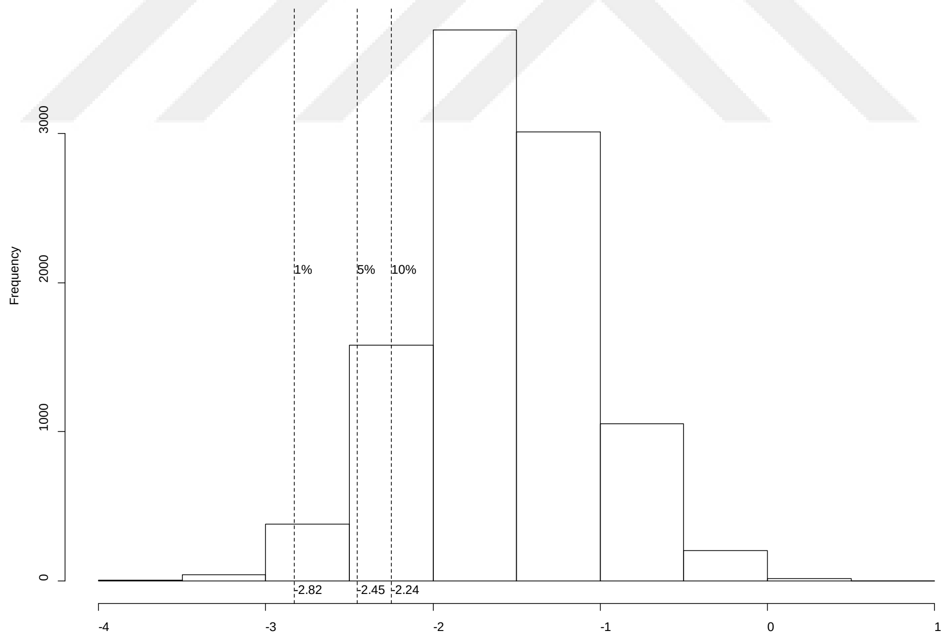

Figure A10, lncons, Model B, AR(3)

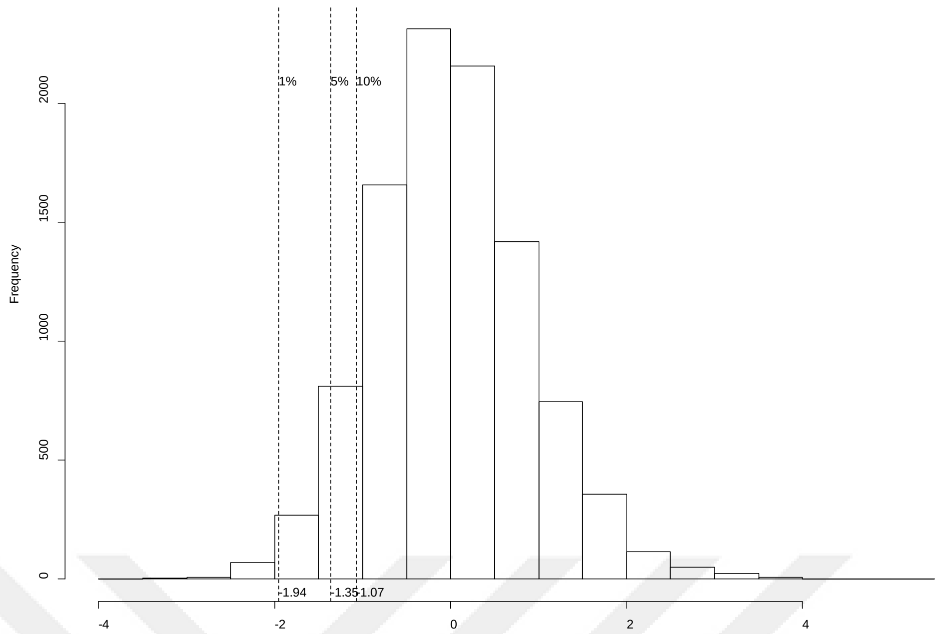

Figure A11, lncons, Model C, AR(1)

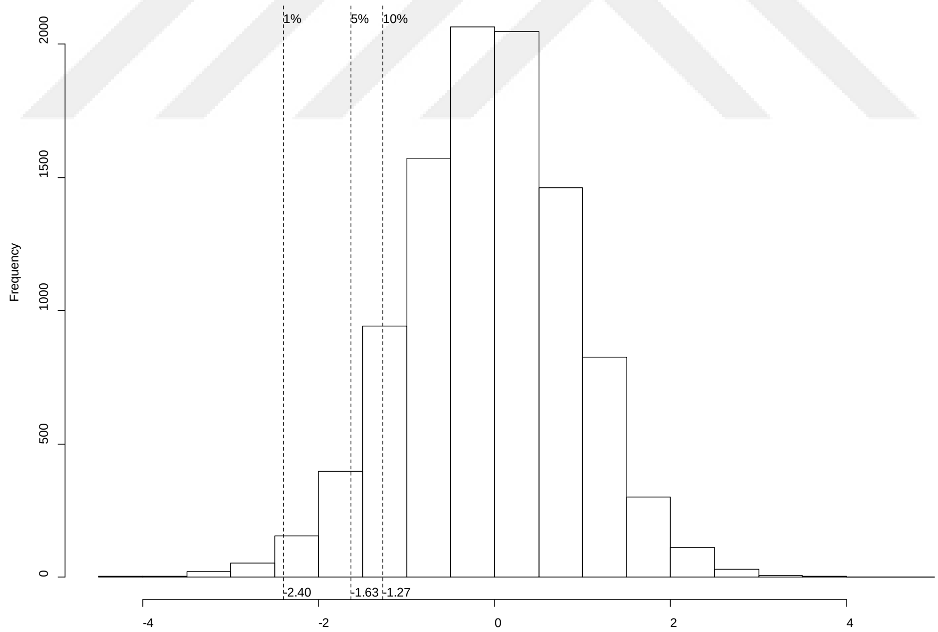

Figure A12, lncons, Model C, AR(3)

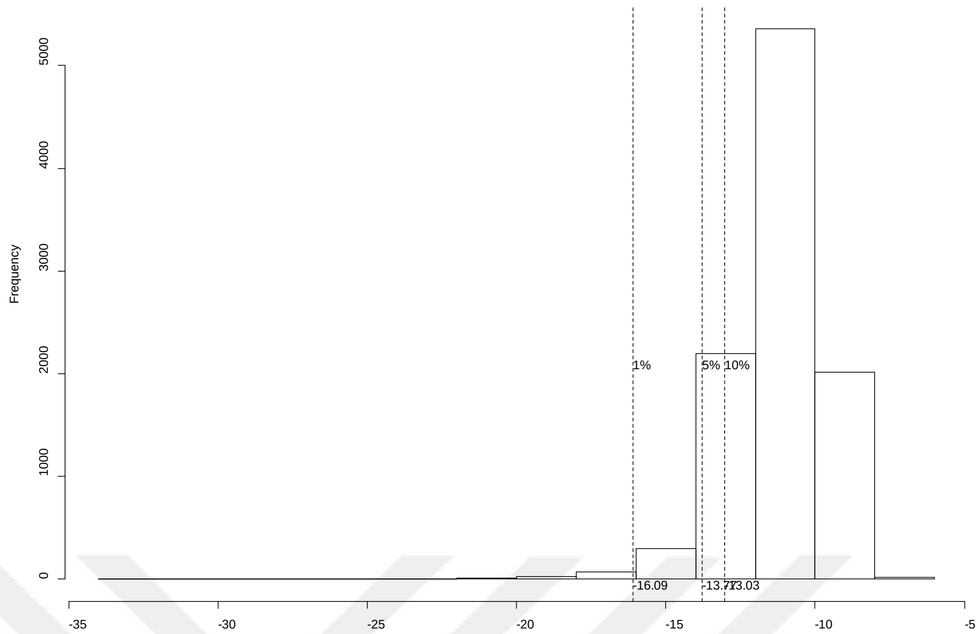

Figure A13, lnp, Model A, AR(3)

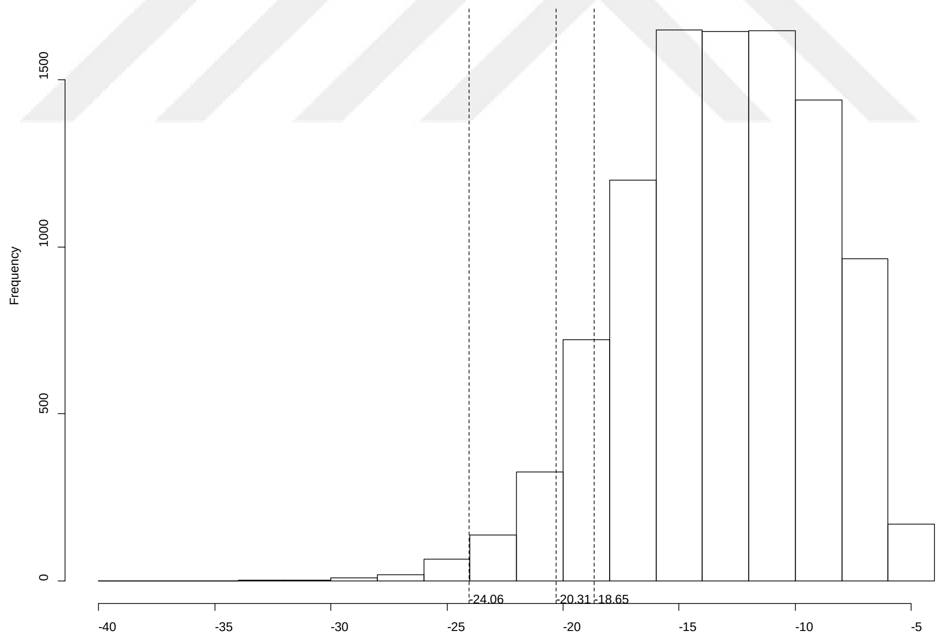

Figure A14, lnp, Model B, AR(3)

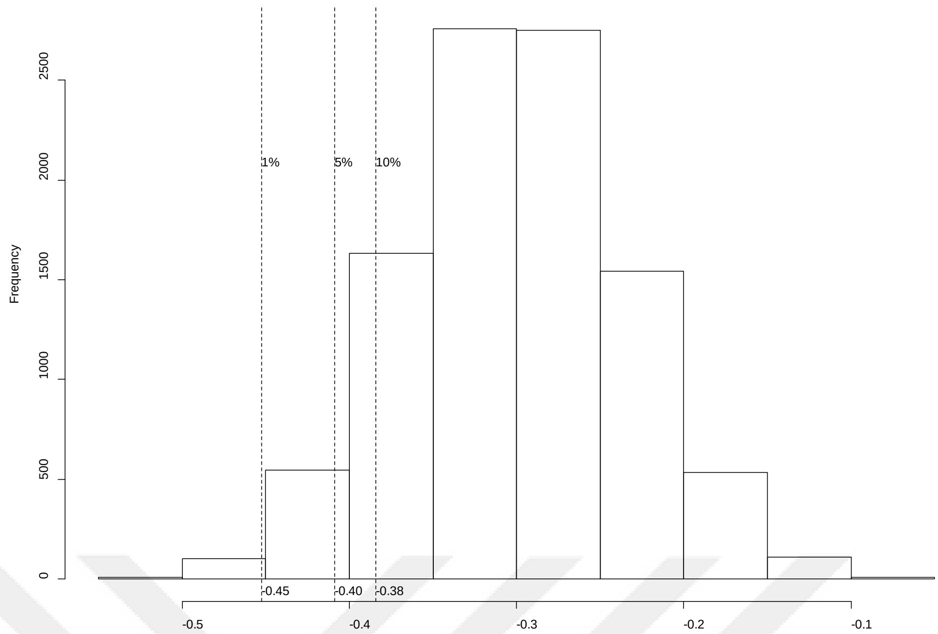

Figure A15, lnp, Model C, AR(3)

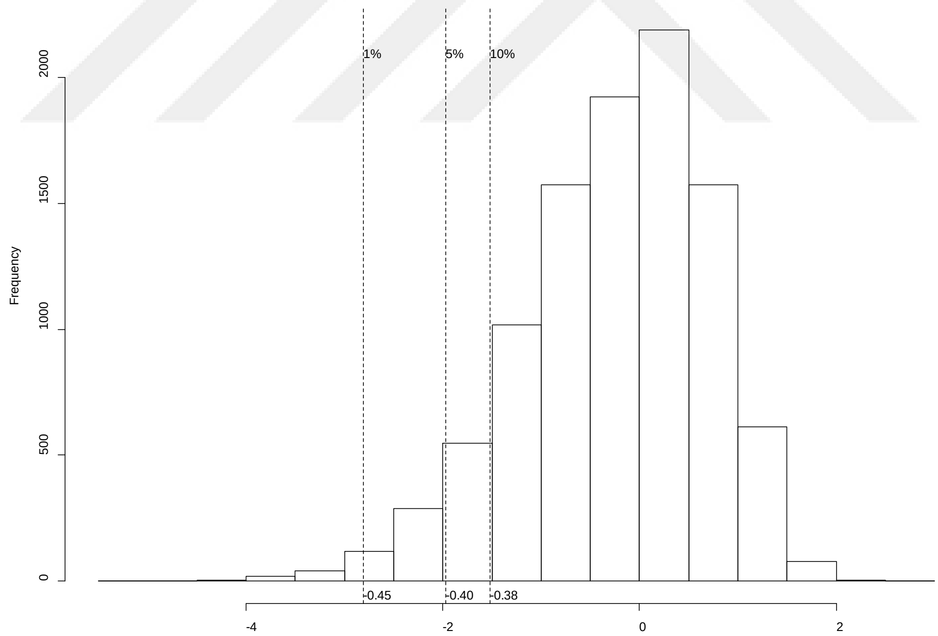

Figure A16, lner, Model A, AR(2)

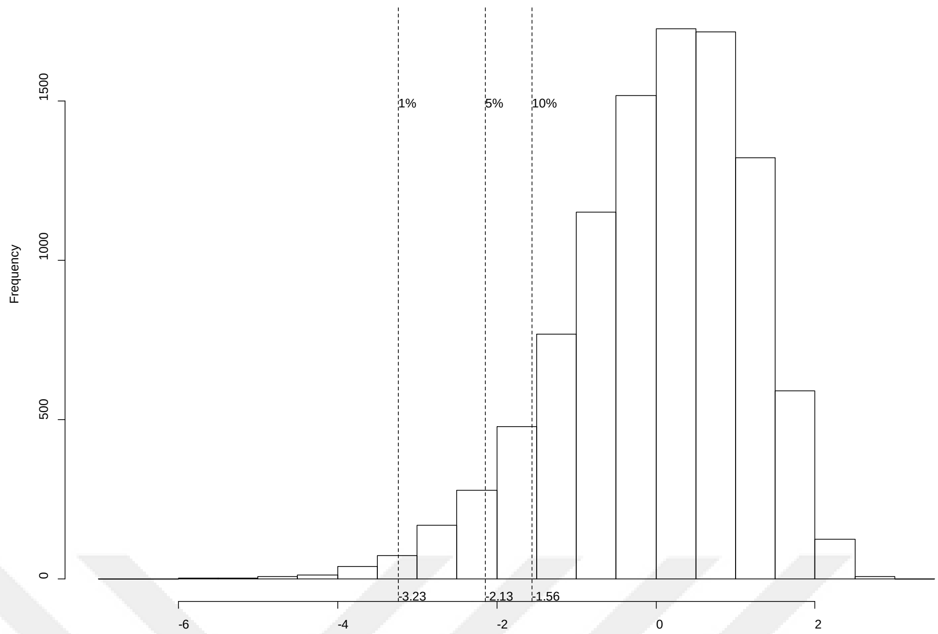

Figure A17, lner, Model A, AR(3)

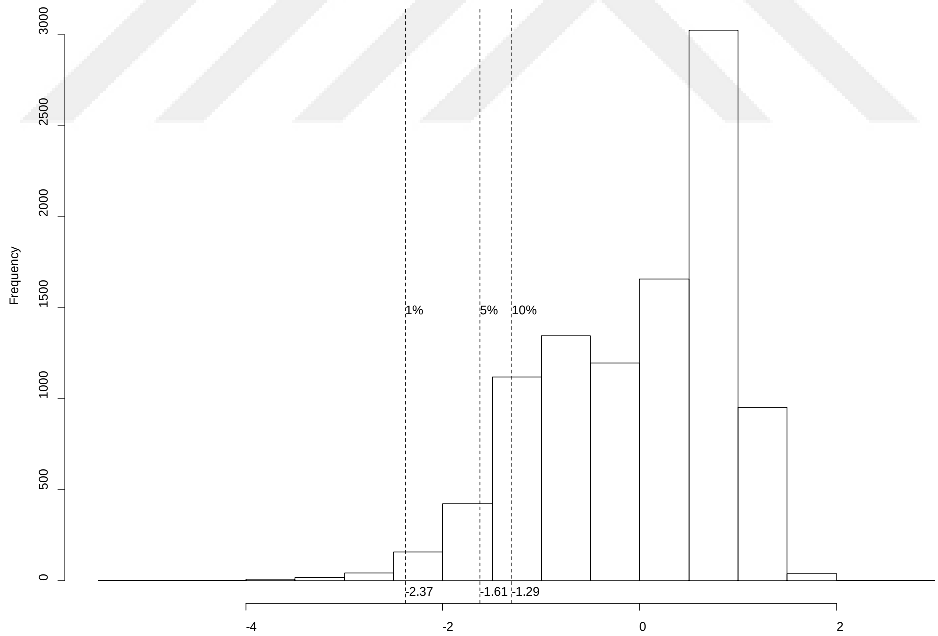

Figure A18, lner, Model B, AR(2)

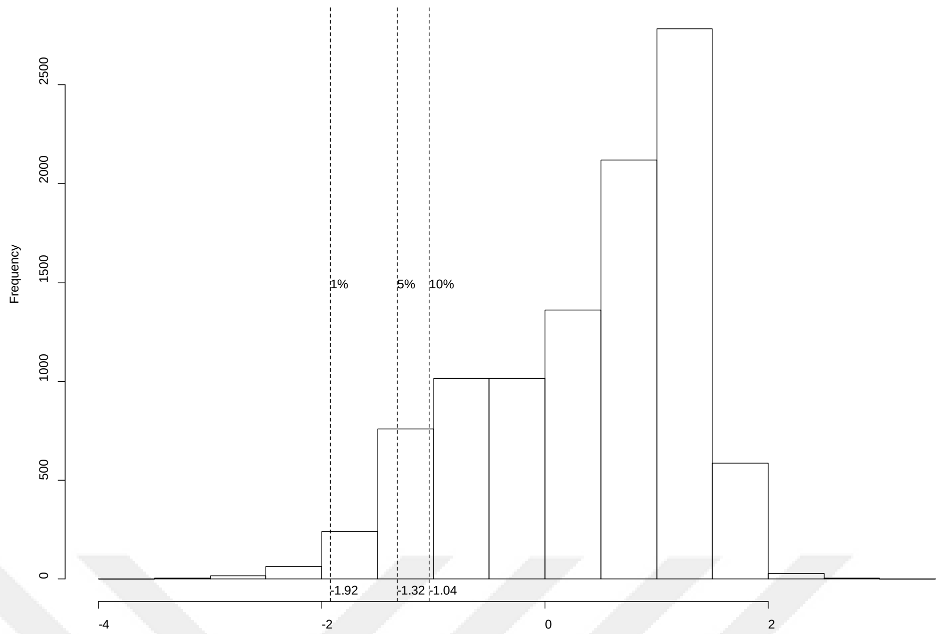

Figure A19, lner, Model B, AR(3)

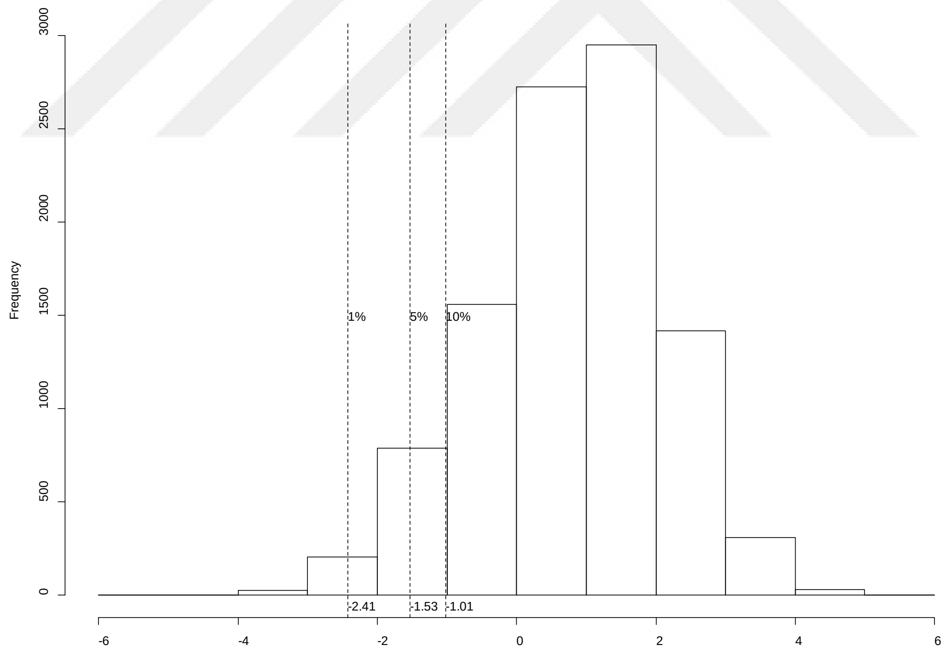

Figure A20, lner, Model C, AR(2)

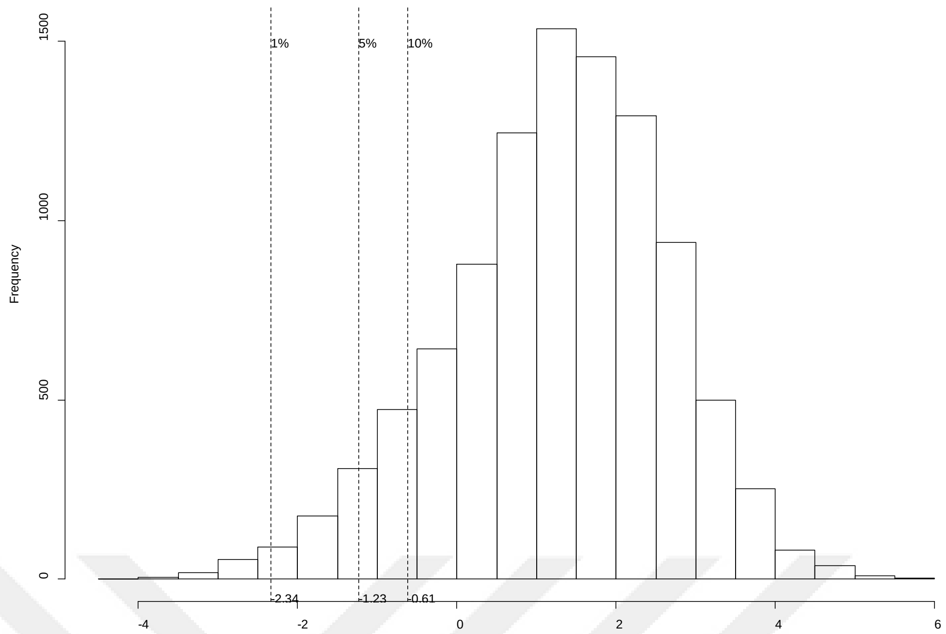

Figure A21, lner, Model C, AR(3)

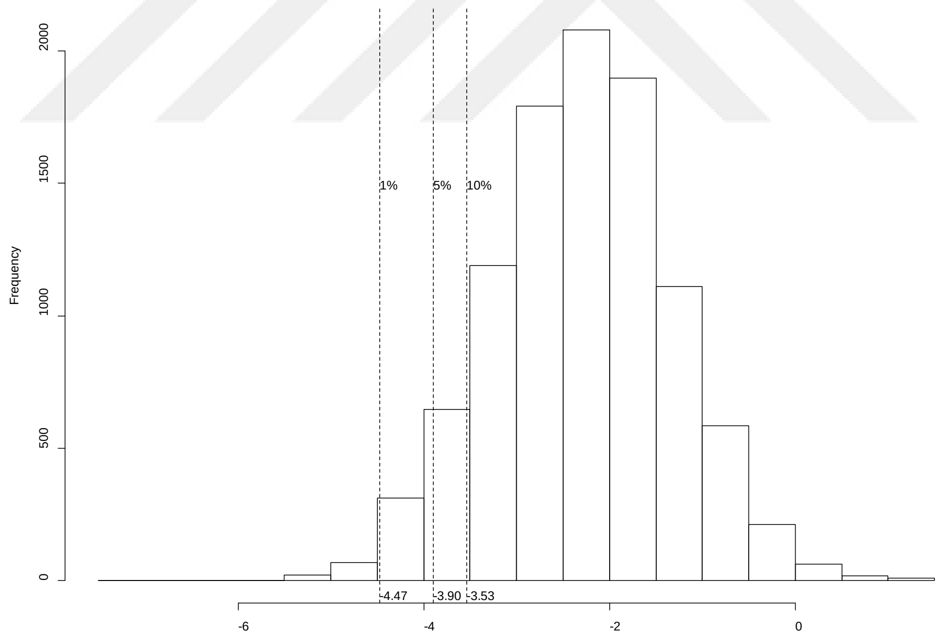

Figure A22, lnex, Model A, AR(2)

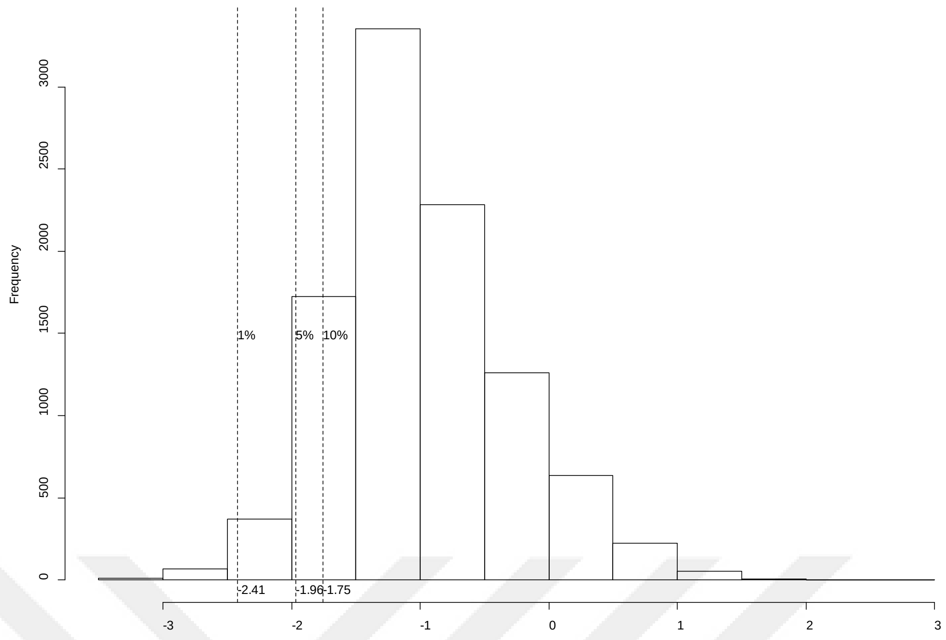

Figure A23, lnex, Model B, AR(2)

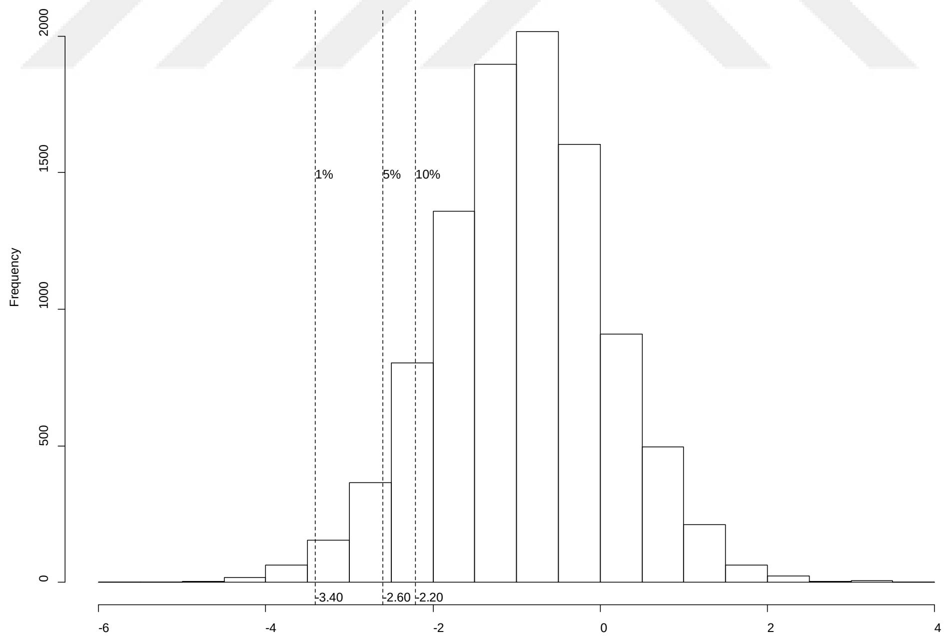

Figure A24, lnex, Model C, AR(2)

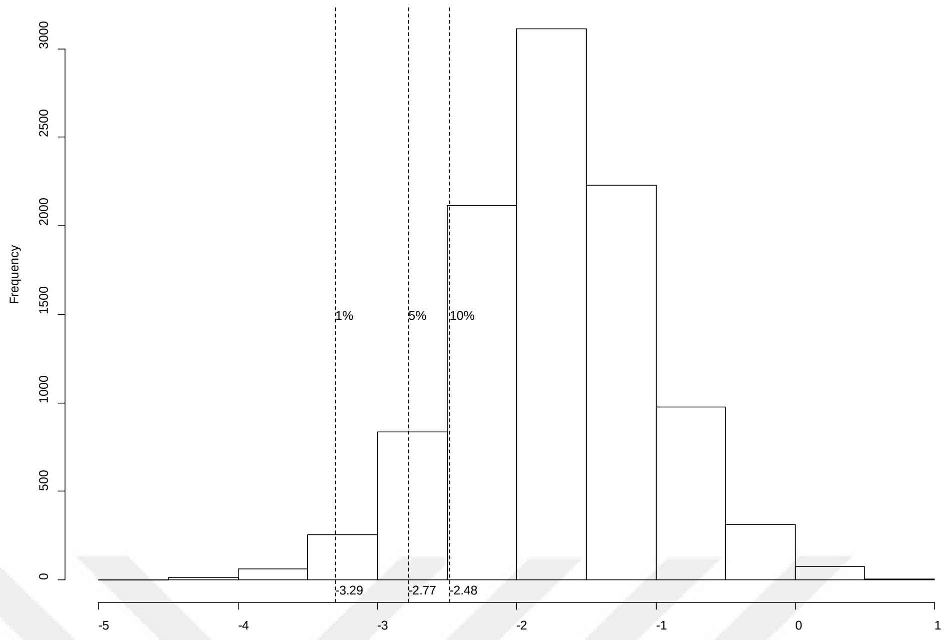

Figure A25, lnim, Model A, AR(1)

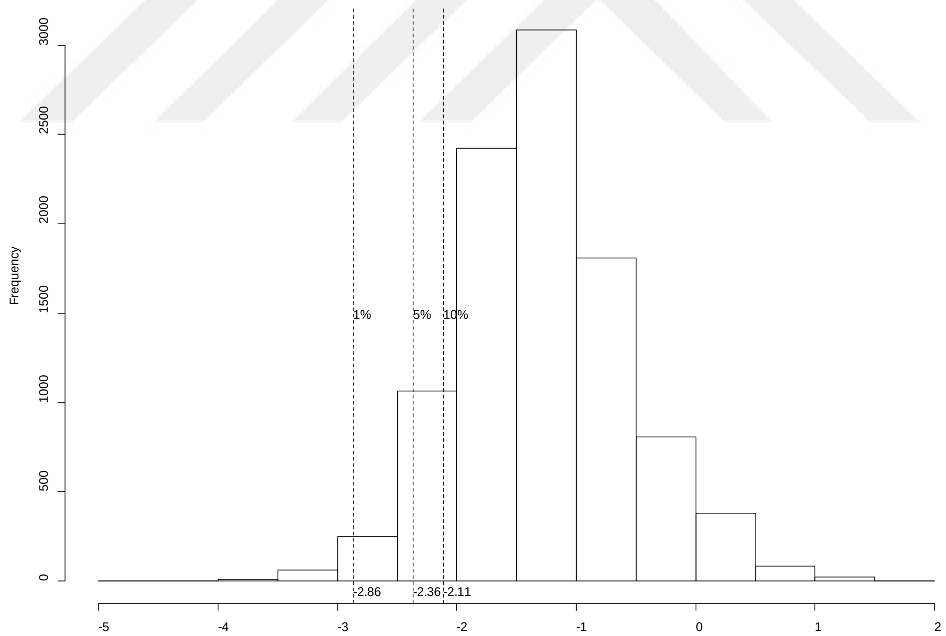

Figure A26, lnim, Model B, AR(1)

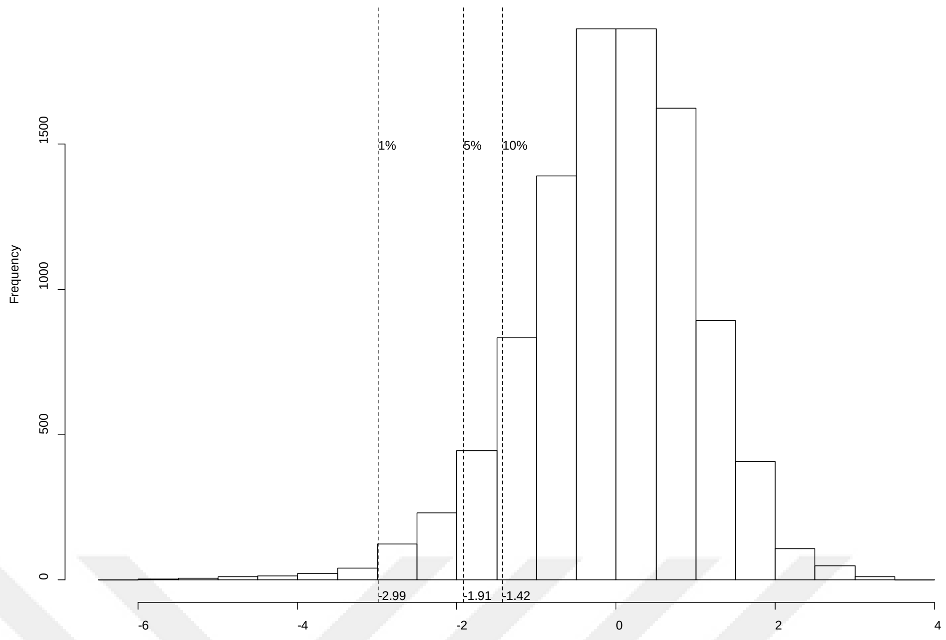

Figure A27, lnim, Model C, AR(1)

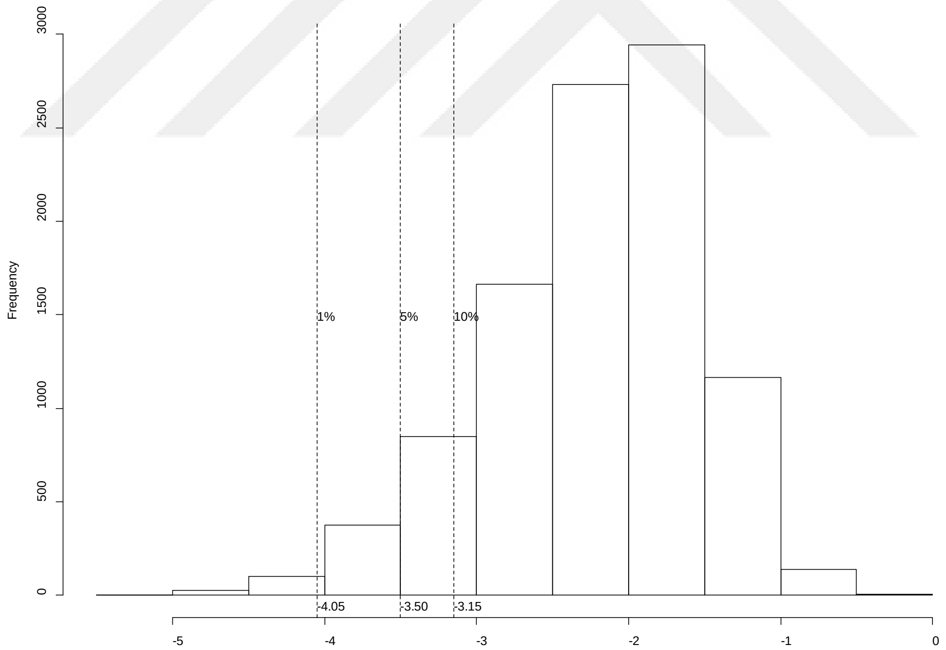

Figure A28, i, Model A, AR(4)

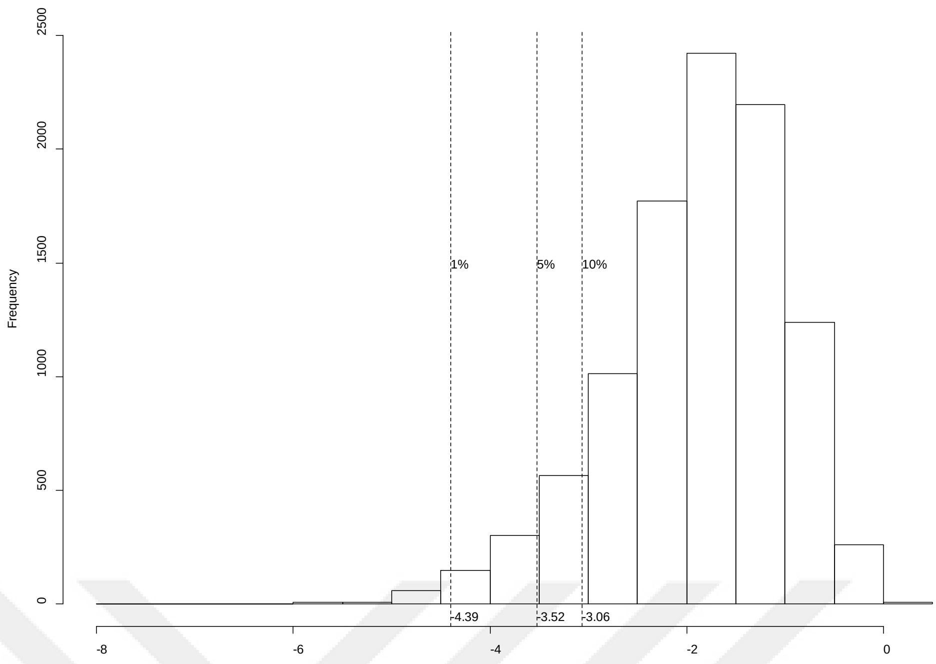

Figure A29, i, Model B, AR(4)

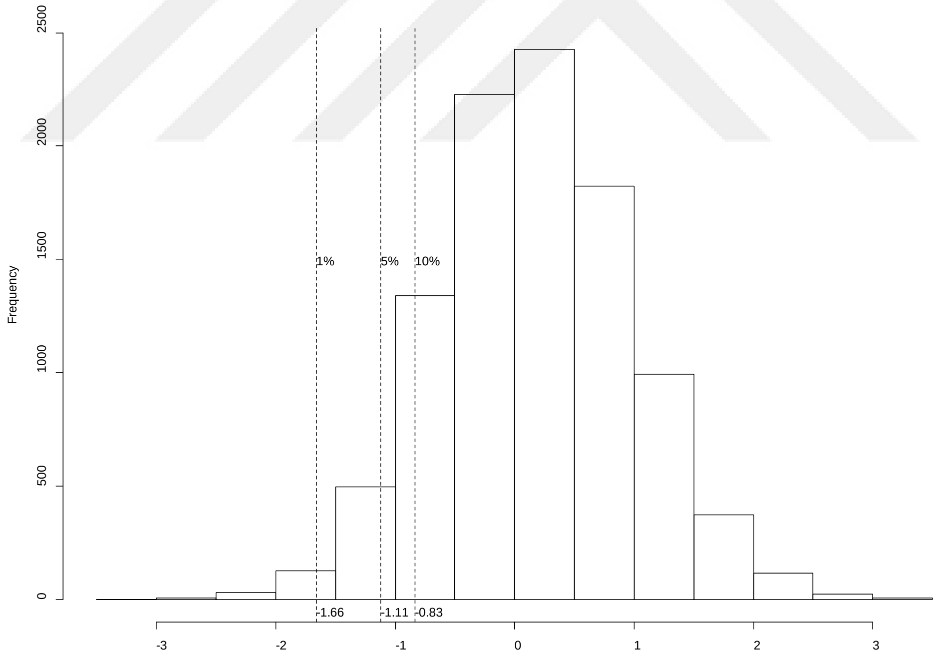

Figure A30, i, Model C, AR(4)

APPENDIX C

EMPIRICAL DISTRIBUTIONS of PARAMETER ESTIMATES of KEYNESIAN CONSUMPTION FUNCTION

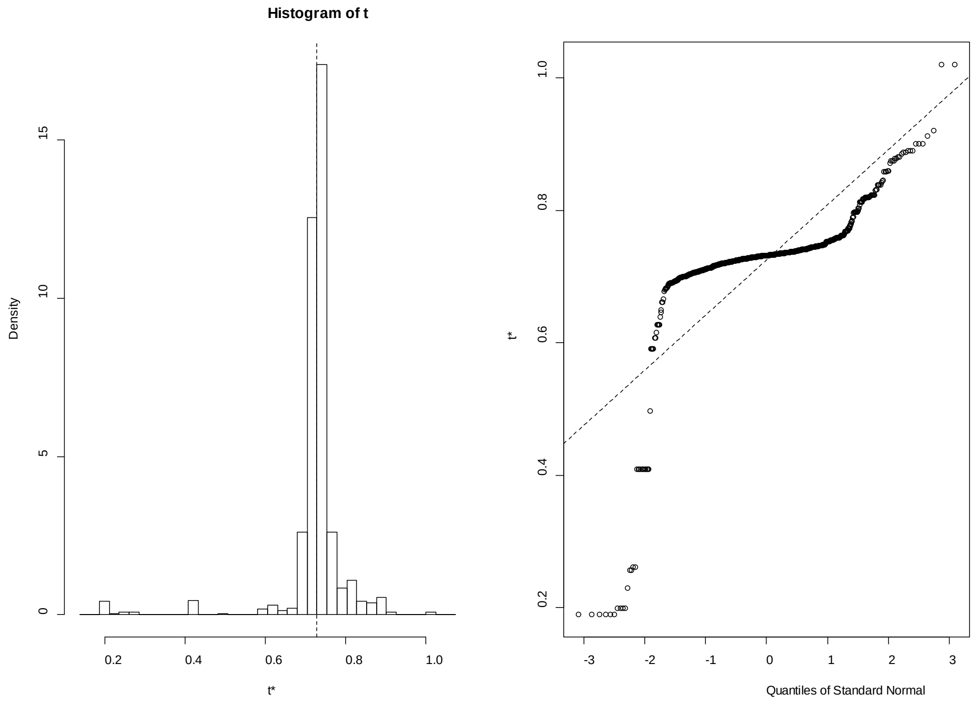

Figure C1, Distribution and Q-Q plot of $\hat{\alpha}_1$

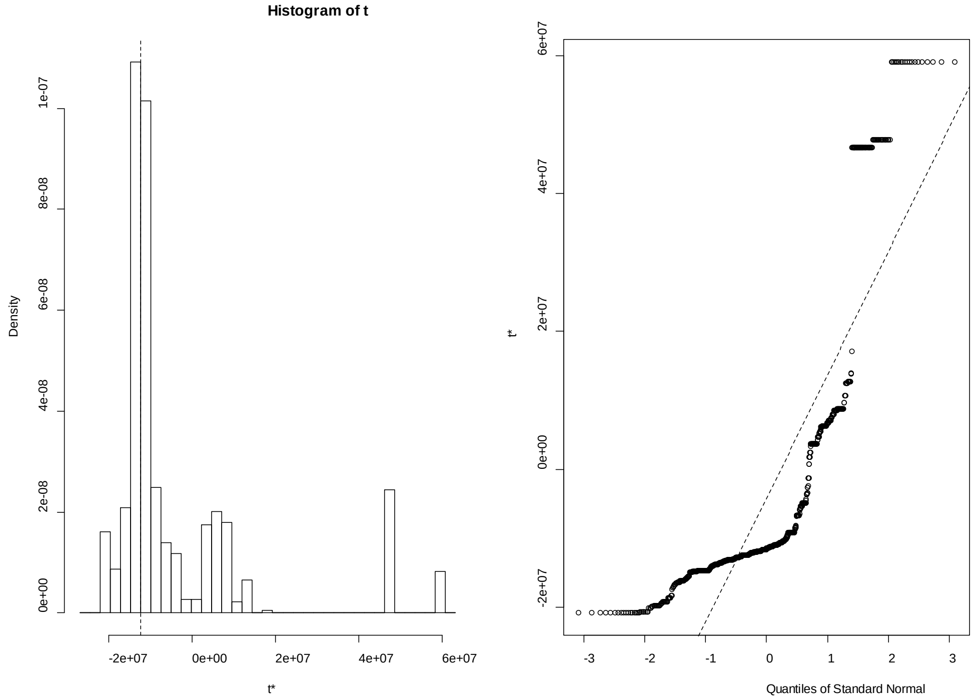

Figure C2, Distribution and Q-Q plot of $\hat{c}_2$

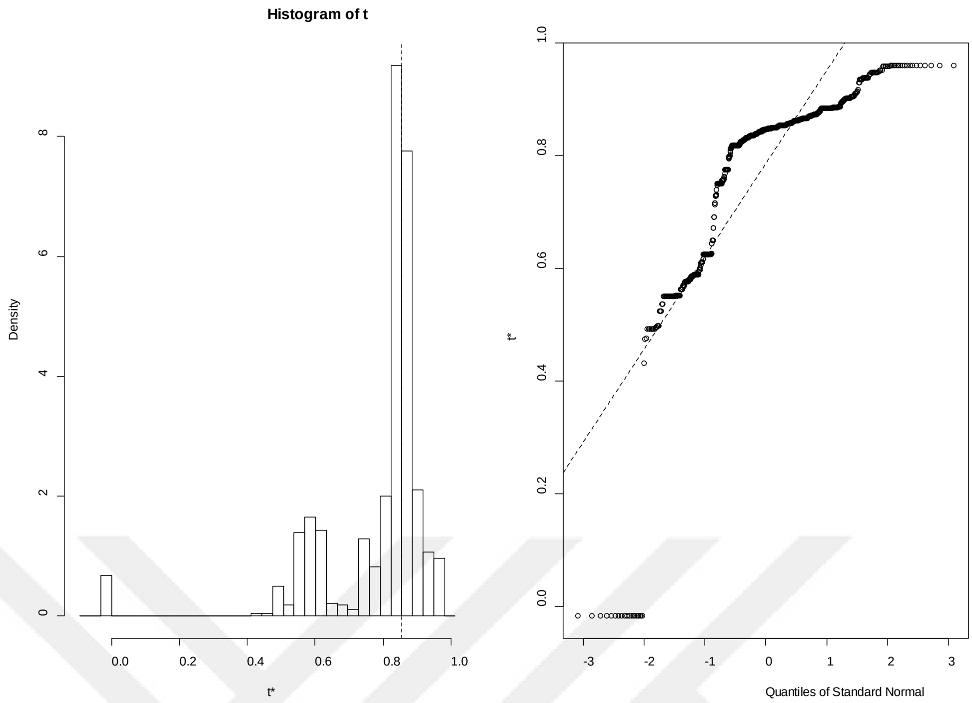

Figure C3, Distribution and Q-Q plot of $\hat{\alpha}_2$

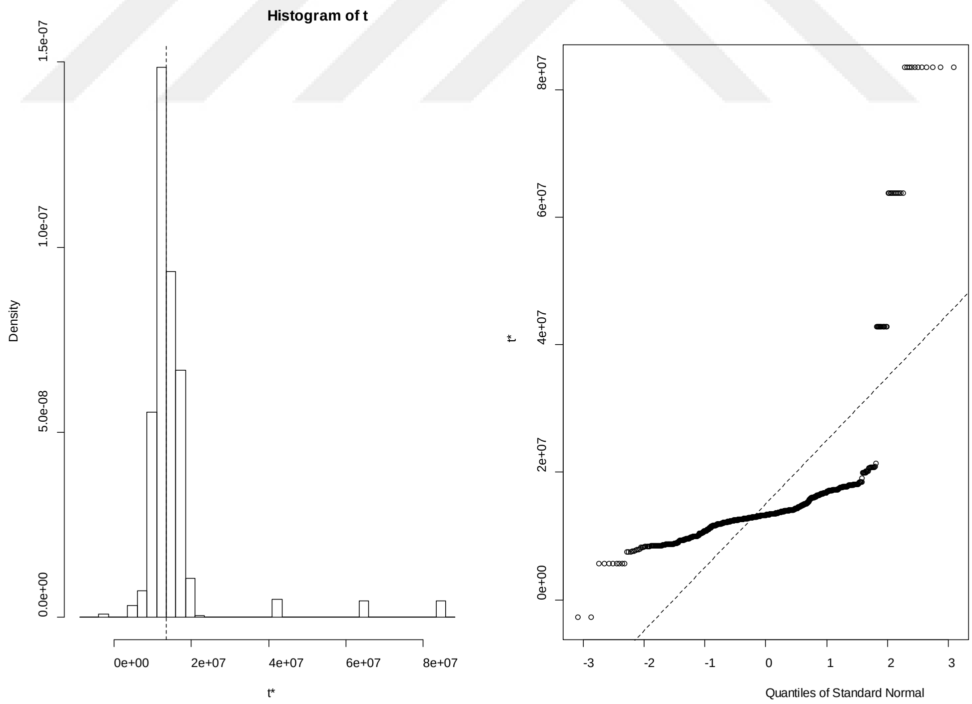

Figure C4, Distribution and Q-Q plot of $\hat{c}_3$

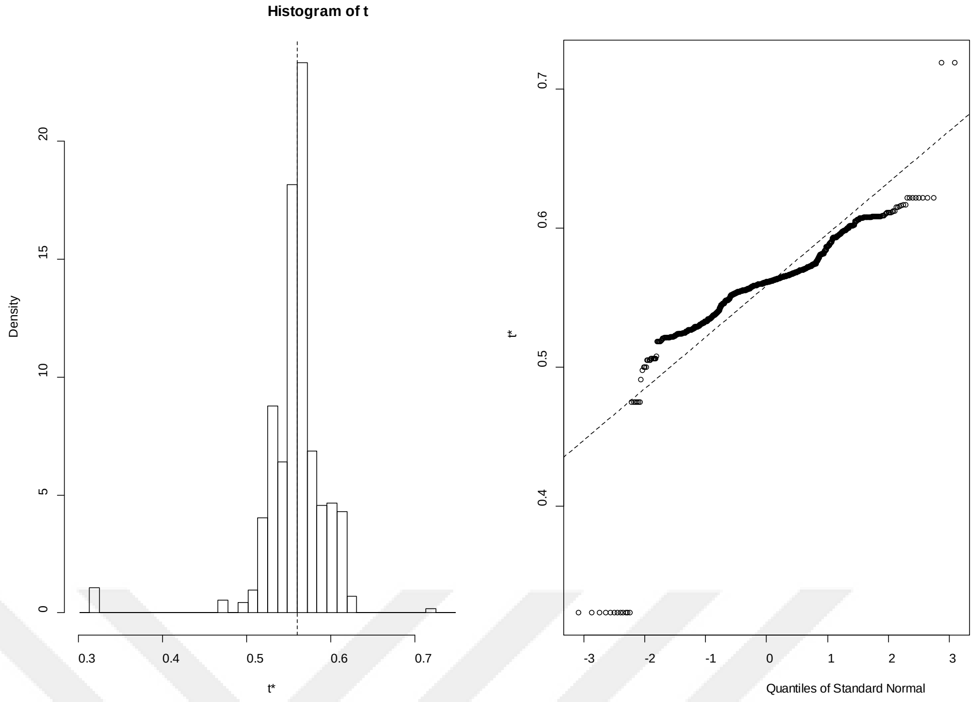

Figure C5, Distribution and Q-Q plot of$\hat{\alpha}_3$

APPENDIX D

CODES

1. R Codes Used in the Analysis

1.2. Psaradakis' Unit Root Approach

1.2.1. Model without Trend and Constant

Lngdp with AR(2) structure;

```
set.seed(100)
library(zoo)
uhat=zoo(diff(lngdp, differences=1))
laguhat1=lag(uhat, -1, na.pad=TRUE)
laguhat2=lag(uhat, -2, na.pad=TRUE)
uhat=uhat[-c(1,2)]
laguhat1=laguhat1[-c(1,2)]
laguhat2=laguhat2[-c(1,2)]
otoreg1=lm(uhat~laguhat1 + laguhat2-1)
phi=matrix(1:2, nrow=1)
phi[,1]=summary(otoreg1)$coefficients[1,1]
phi[,2]=summary(otoreg1)$coefficients[2,1]
epsilon=fitted(otoreg1)
meanepsilon=sum(epsilon)/(27-2)
sepsilon=(epsilon-meanepsilon)/(27-2)
xboot=zoo(matrix(1:280000, nrow=28))
uboot=matrix(1:280000, nrow=28)
sepsilonboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
sepsilonboot[,i]=sample(sepsilon, 28, replace=TRUE )}
uboot[1,]=sepsilonboot[1,]
uboot[2,]=(sepsilonboot[2,]-phi[,1]*uboot[1,])
for (i in 3:28){
uboot[i,]=(sepsilonboot[i,]-phi[,1]*uboot[i-1]-phi[,2]*uboot[i-2])}
```

```
xboot[1,]=lngdp[1]
for (i in 2:28){
xboot[i,]= xboot[i-1,]+uboot[i,]
}
lagxboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
 lagxboot[,i]=lag(xboot[,i], -1, na.pad=TRUE )
 }
 lagxboot=lagxboot[-1,]
 xboot=xboot[-1,]
rho=seq(10000)
sdrho=seq(10000)
for (i in 1:10000){
rhoreg=lm(xboot[,i]~lagxboot[,i]-1)
rho[i]=summary(rhoreg)$coefficients[1,1]
sdrho[i]= summary(rhoreg)$coefficients[1,2] }
hist(rho)
bootstat=seq(10000)
for (i in 1:10000){
bootstat[i]=(rho[i]-1)/sdrho[i]
}
sorted.bootstat= sort(bootstat, decreasing=FALSE)
print(sorted.bootstat[100])
print(sorted.bootstat[500])
print(sorted.bootstat[1000])
```

1.2.2. Model with Constant

Lngdp with AR(2) structure;

```
otoreg1=lm(uhat~laguhat1 + laguhat2-1)
phi=matrix(1:2, nrow=1)
phi[,1]=summary(otoreg1)$coefficients[1,1]
phi[,2]=summary(otoreg1)$coefficients[2,1]
```

```
epsilon=fitted(otoreg1)
meanepsilon=sum(epsilon)/(27-2)
sepsilon=(epsilon-meanepsilon)/(27-2)
xboot=zoo(matrix(1:280000, nrow=28))
uboot=matrix(1:280000, nrow=28)
sepsilonboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
sepsilonboot[,i]=sample(sepsilon, 28, replace=TRUE )}
uboot[1,]=sepsilonboot[1,]
uboot[2,]=(sepsilonboot[2,]-phi[,1]*uboot[1,])
for (i in 3:28){
uboot[i,]=(sepsilonboot[i,]-phi[,1]*uboot[i-1]-phi[,2]*uboot[i-2])}
xboot[1,]=lngdp[1]
for (i in 2:28){
xboot[i,]= xboot[i-1,]+uboot[i,]
}
lagxboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
 lagxboot[,i]=lag(xboot[,i], -1, na.pad=TRUE )
 }
 lagxboot=lagxboot[-1,]
 xboot=xboot[-1,]
rho=seq(10000)
sdrho=seq(10000)
for (i in 1:10000){
rhoreg=lm(xboot[,i]~lagxboot[,i])
rho[i]=summary(rhoreg)$coefficients[2,1]
sdrho[i]= summary(rhoreg)$coefficients[2,2] }
bootstat=seq(10000)
for (i in 1:10000){
bootstat[i]=(rho[i]-1)/sdrho[i]
}
sorted.bootstat= sort(bootstat, decreasing=FALSE)
print(sorted.bootstat[100])
```

```
print(sorted.bootstat[500])
print(sorted.bootstat[1000])
```

1.2.3. Model with Constant and Trend

Lngdp with AR(2) structure;

```
otoreg1=lm(uhat~laguhat1 + laguhat2-1)
phi=matrix(1:2, nrow=1)
phi[,1]=summary(otoreg1)$coefficients[1,1]
phi[,2]=summary(otoreg1)$coefficients[2,1]
epsilon=fitted(otoreg1)
meanepsilon=sum(epsilon)/(27-2)
sepsilon=(epsilon-meanepsilon)/(27-2)
xboot=zoo(matrix(1:280000, nrow=28))
uboot=matrix(1:280000, nrow=28)
sepsilonboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
sepsilonboot[,i]=sample(sepsilon, 28, replace=TRUE )}
uboot[1,]=sepsilonboot[1,]
uboot[2,]=sepsilonboot[2,]-phi[,1]*uboot[1,]
for (i in 3:28){
uboot[i,]=sepsilonboot[i,]-phi[,1]*uboot[i-1,]-phi[,2]*uboot[i-2] }
xboot[1,]=lngdp[1]
for (i in 2:28){
xboot[i,]= ymean+xboot[i-1,]+uboot[i,]
}
lagxboot=matrix(1:280000, nrow=28)
for (i in 1:10000){
 lagxboot[,i]=lag(xboot[,i], -1, na.pad=TRUE )
 }
 lagxboot=lagxboot[-1,]
 xboot=xboot[-1,]
 trend=seq(27)
```

```
  for (i in 1:27) {
  trend[i]=i }
rho=seq(10000)
sdrho=seq(10000)
for (i in 1:10000){
rhoreg=lm(xboot[,i]~lagxboot[,i]+trend)
rho[i]=summary(rhoreg)$coefficients[2,1]
sdrho[i]= summary(rhoreg)$coefficients[2,2] }
bootstat=seq(10000)
for (i in 1:10000){
bootstat[i]=(rho[i]-1)/sdrho[i]
}
sorted.bootstat= sort(bootstat, decreasing=FALSE)
print(sorted.bootstat[100])
print(sorted.bootstat[500])
print(sorted.bootstat[1000])
```

1.2. Harris' Unit Root Approach

1.2.1. Model with Constant

```
set.seed(100)
library(zoo)
delta_y=zoo(diff(lngdp, differences=1))
lag_delta_y=lag(delta_y, -1, na.pad=TRUE)
delta_y=delta_y[-c(1)]
 lag_delta_y=lag_delta_y[-c(1)]
 RADF=lm(delta_y~ lag_delta_y)
 v=residuals(RADF)
 cenv=v/sqrt(1-3*28^-1)
bootv=matrix(1:140000,nrow=28)
 for (i in 1:5000) {
bootv[,i]=sample(v, 28, replace=TRUE )}
```

```
 bootyt=matrix(1:140000, nrow=28)
bootyt[1,]=lngdp[1]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficient
s[2,1]*lngdp[1]+bootv[1,]
bootyt[2,]=bootyt[1,]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficien
ts[2,1]*(bootyt[1,]-lngdp[1])+bootv[2,]
for (i in 3:28){
bootyt[i,]=bootyt[i-
1,]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficients[2,1]*(bootyt[i-
1,]-bootyt[i-2,])+bootv[i,] }
bootyt=zoo(bootyt)
difyt=zoo(matrix(1:135000, nrow=27))
for (i in 1:5000){
difyt[,i]=zoo(diff(bootyt[,i], differences=1))}
lagyt=zoo(matrix(1:140000, nrow=28))
 for (i in 1:5000){
lagyt[,i]=lag(bootyt[,i],-1, na.pad=TRUE)}
lag_difyt=zoo(matrix(1:135000, nrow=27))
 for (i in 1:5000){
lag_difyt[,i]=lag(difyt[,i], -1, na.pad=TRUE)
}
difyt=difyt[-1,]
lagyt=lagyt[-1,]
lagyt=lagyt[-1,]
lag_difyt=lag_difyt[-1,]
tau=seq(5000)
 for (i in 1:5000){
 BADF=lm(difyt[,i]~lagyt[,i]+lag_difyt[,i])
  tau[i]= summary(BADF)$coefficients[2,3]}
    hist(tau)
 sorted.tau= sort(tau, decreasing=FALSE)
 print(sorted.tau[50])
 print(sorted.tau[250])
  print(sorted.tau[500])
```

1.2.1. Model with Constant and Trend

```
set.seed(100)
library(zoo)
delta_y=zoo(diff(lngdp, differences=1))
lag_delta_y=lag(delta_y, -1, na.pad=TRUE)
delta_y=delta_y[-c(1)]
 lag_delta_y=lag_delta_y[-c(1)]
  trend=seq(28)
 for (i in 1:28){
 trend[i]=i}
RADF=lm(delta_y~trend+ lag_delta_y)
 v=residuals(RADF)
 cenv=v/sqrt(1-3*28^-1)
 bootv=matrix(1:140000,nrow=28)
 for (i in 1:5000) {
bootv[,i]=sample(v, 28, replace=TRUE )}
 bootyt=matrix(1:140000, nrow=28)
bootyt[1,]=lngdp[1]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficient
s[2,1]*trend[1]+summary(RADF)$coefficients[3,1]*lngdp[1]+bootv[1,]
bootyt[2,]=bootyt[1,]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficien
ts[2,1]*trend[2]+summary(RADF)$coefficients[3,1]*(bootyt[1,]-lngdp[1])+bootv[2,]
for (i in 3:28){
bootyt[i,]=bootyt[i-
1,]+summary(RADF)$coefficients[1,1]+summary(RADF)$coefficients[2,1]*trend[i]+su
mmary(RADF)$coefficients[3,1]*(bootyt[i-1,]-bootyt[i-2,])+bootv[i,] }
bootyt=zoo(bootyt)
difyt=zoo(matrix(1:135000, nrow=27))
for (i in 1:5000){
difyt[,i]=zoo(diff(bootyt[,i], differences=1))}
lagyt=zoo(matrix(1:140000, nrow=28))
 for (i in 1:5000){
lagyt[,i]=lag(bootyt[,i],-1, na.pad=TRUE)}
lag_difyt=zoo(matrix(1:135000, nrow=27))
```

```
 for (i in 1:5000){
lag_difyt[,i]=lag(difyt[,i], -1, na.pad=TRUE)
}
difyt=difyt[-1,]
lagyt=lagyt[-1,]
lagyt=lagyt[-1,]
lag_difyt=lag_difyt[-1,]
trend=seq(26)
 for (i in 1:26){
 trend[i]=i}


tau=seq(5000)
 for (i in 1:5000){
 BADF=lm(difyt[,i]~lagyt[,i]+lag_difyt[,i]+trend)
  tau[i]= summary(BADF)$coefficients[2,3]}
     hist(tau)
 sorted.tau= sort(tau, decreasing=FALSE)
 print(sorted.tau[50])
 print(sorted.tau[250])
  print(sorted.tau[500])
```

1.3. Bai and Perron’s Endogenous Multiple Break Test

1.3.1. Multiple Breaks in Individual Series

```
library(strucchange)
plot(gdp, ylab="GDP")
trend=ts(1:28)
bp.gdp <- breakpoints(gdp ~ 1 + trend, h = 4)
plot(bp.gdp)
summary(bp.gdp)
b=2
fac.gdp<- breakfactor(bp.gdp, breaks = b, label = "seg")
fm.gdp<- lm(gdp ~ 0 + fac.gdp)
```

```
summary(fm.gdp)
coef(bp.gdp, breaks = b)
sapply(vcov(bp.gdp, breaks = b, ), sqrt)
confint(bp.gdp, breaks = b, )
plot(gdp)
lines(ts(fitted(fm.gdp), start = 1987, freq = 1), col = 2)
lines(confint(bp.gdp, breaks = b, ))
```

1.3.2. Multiple Breaks in the Relationship between the Series

```
bp.consumption <- breakpoints(consumption ~ 1 + gdp, h = 4)
plot(bp.consumption)
summary(bp.consumption)
b=2
fac.consumption<- breakfactor(bp.consumption, breaks = b, label = "seg")
fm.consumption<- lm(consumption ~ 0+ fac.consumption)
summary(fm.consumption)
coef(bp.consumption, breaks = b)
sapply(vcov(bp.consumption, breaks = b, ), sqrt)
confint(bp.consumption, breaks = b, )
plot(consumption)
lines(ts(fitted(fm.consumption), start = 1987, freq = 1), col = 2)
lines(confint(bp.consumption, breaks = b, ))
z=lm(consumption~gdp)
summary(z)
zz= lm(consumption~d1+d1gdp+d2+d2gdp+d3+d3gdp-1)
summary(zz)
 lines(ts(fitted(z), start = 1987, freq = 1), col = 2)
 lines(ts(fitted(zz), start = 1987, freq = 1), col = 3)
```

1.4.Moving Block Bootstrap for Keynesian Consumption Function

```
varmat=ts(matrix(1:196, nrow=28))
 varmat[,1]=consumption
  varmat[,2]=d1
```

```
  varmat[,3]=d1gdp
  varmat[,4]=d2
  varmat[,5]=d2gdp
  varmat[,6]=d3
  varmat[,7]=d3gdp
set.seed(100)
library(boot)
blockboot <- function(varmat)
{
z=lm(varmat[,1]~varmat[,2]+varmat[,3]+varmat[,4]+varmat[,5]+varmat[,6]+varmat[,7]
-1)
 b1=coef(z)
}
boot.1 <- tsboot(varmat, blockboot, R = 1000, l = 5, sim = "fixed")
boot.1
plot(boot.1, index=1)
plot(boot.1, index=2)
plot(boot.1, index=3)
plot(boot.1, index=4)
plot(boot.1, index=5)
plot(boot.1, index=6)
```

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