

TRANSFORMATION INDUCED CRACK CLOSURE AND FATIGUE LIFE
ANALYSIS OF SHAPE MEMORY ALLOYS USING FINITE ELEMENTS

by

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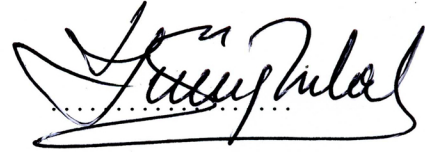
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ABSTRACT

TRANSFORMATION INDUCED CRACK CLOSURE AND FATIGUE LIFE ANALYSIS OF SHAPE MEMORY ALLOYS USING FINITE ELEMENTS

Fatigue crack propagation behavior of shape memory alloys is investigated using finite elements. Fatigue crack growth rate is numerically determined in terms of effective stress intensity factor range and maximum stress intensity factor. Finite element model of crack closure used is similar to that used by Kibey et al. [68] on plasticity induced crack closure in elastic-plastic materials with hardening. Before studying fatigue crack growth, fracture behavior of the material (such as size of phase transformation zone in cracked specimen, J-integral, stress intensity factor, etc.) is discussed. Stress distribution in transformation zone around the crack tip is solved analytically using HRR singularity field. A correlation between J-integral and stress intensity factor is checked.

No plastic strain accumulation during cyclic loading is allowed in the material model used for the superelastic alloy, therefore the phase transformation is considered to be the only mechanism triggering crack closure. A detailed analysis is made to see effects of load ratio, initial crack length and crack propagation history on elastic and transformation strain energies during crack propagation. To estimate the fatigue life of a typical shape memory alloy, stress intensity factor in terms of final crack length and applied load is estimated by using an empirical formulation. Results obtained from this method are compared to analytical results in literature. Finally, service life of the material is calculated regarding all aspects of its fatigue behavior.

ÖZET

ŞEKİL HAFIZALI ALAŞIMLARDA FAZ DÖNÜŞÜMÜ VE YORULMA ÇATLAĞI İLERLEMESİ İLİŞKİSİNİN SONLU ELEMENLAR YÖNTEMİYLE ANALİZİ

Sonlu elemanlar yazılımı kullanılarak, şekil hafızalı alaşımların yorulma çatlakları ilerleme davranışı incelenmiştir. Etkin gerilim şiddet çarpan aralığı ve azami gerilim şiddet çarpanı ile doğru orantılı çatlak ilerleme hızı sayısal olarak belirlenmiştir. Kullanılan sonlu eleman modeli; Kibey ve diğerleri [68] tarafından, gerilim sertleşmesi yaşayan malzemelerde görülen plastisiteye bağlı çatlak kapanması ile ilgili analizlerde kullanılan modelle benzeştirilmiştir. Yorulma çatlak ilerlemesi ile ilgili çalışmalardan önce, malzemenin kırılma mekaniği (çatlaklı numunelerde faz dönüşüm bölgesinin büyüklüğü, J-integrali, gerilim şiddet çarpanı, vb.) incelenmiştir. HRR tekil alan çözümü kullanılarak, çatlak çevresindeki gerilim dağılımı analitik olarak çözülmüştür. J-integrali ile gerilim şiddet çarpanı arasında bir bağıntı olup olmadığı sınanmıştır.

Kullanılan ticari yazılım ile tümleşik süperelastik malzeme modelinde, çevrimsel yükleme esnasında görülen kalıcı gerilim birikimi tanımlanmamıştır; bu yüzden çatlak kapanma sadece faz değişimi olgusu tarafından tetiklenmektedir. Çatlak ilerlemesi sırasında; gerilim oranını, çatlak ilk oluştuğu noktanın yeri ve ilerleme hikayesinin enerji düzeylerine etkisi detaylı bir biçimde analiz edilmiştir. Tipik bir şekil hafızalı alaşımın yorulma ömrünü tahmin edebilmek için sonlu elemanlar yöntemi kullanılarak çatlak uzunluğuna ve uygulanan yüke bağlı bir gerilim şiddeti çarpanı hesaplanmaya çalışılmıştır. Bulunan sonuçlar literatürdeki analitik çözümler ile karşılaştırılmıştır. Çalışmanın sonunda, malzemenin yorulma ile ilgili tüm davranışları göz önünde bulundurularak hizmet ömrü saptanmıştır.

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LIST OF SYMBOLS/ABBREVIATIONS

a	Crack length
$\{a\}$	Flow vector
A	Austenite phase
A_f	Austenite finish temperature
A_s	Austenite start temperature
C	Constant slope
C_1, C_2, C_3	Arbitrary constants
$[D]$	Usual matrix of elastic constants
d_m	Constant depending on type of material plasticity
E	Modulus of elasticity
E_{ij}	Transformation strains
e, el	Elastic
ep	Elastic-plastic
F	Yield function of associated flow
G	Shear modulus
H	Hardening modulus
I_1, J_2, J_3	Invariants of stress tensor
I_m	Integration constant
in	Inelastic part
J	J-Integral
K_I	Mode I stress intensity factor
K_{II}	Mode II stress intensity factor
K_{eff}	Effective stress intensity
K_{th}	Threshold stress intensity
K_{tr}	Stress intensity contribution due phase transformation
m	Power-law hardening parameter
M	Martensite phase
M_d	Martensite desist temperature
M_f	Martensite finish temperature

M_s	Martensite start temperature
N	Number of cycles
p, p^l	Plastic
P	Pressure
Q	Plastic potential function
R	Load ratio
$\Re$	Special defined ratio
r, θ	Polar coordinates at the crack tip
S	Single variant martensite
s	HRR field singularity constant
T	T-stress
T_0	Reference temperature
T_i	Traction vector
t	Deviatoric part of Kirchoff stress
th	Thermoelastic part
U	Complementary strain energy
u_{ij}	Displacement field
$\tilde{u}_i(\theta)$	HRR singular field displacement quantities
W	Plate width
α	Drucker-Prager flow constant
δ	Measured value of crack opening displacement
δ_c	Critical value of crack opening displacement
δ_t	Crack tip opening displacement
$\tilde{\varepsilon}_{ij}(\theta, m)$	HRR singular field strain quantities
$\varepsilon_t, \varepsilon^L$	Transformation strain
ε_y	Yield strain
ζ	Crack opening displacement criterion
κ	Amplitude of stress function
$d\lambda$	Plastic multiplier
μ	Friction

ν	Poisson's ratio
ξ	Phase fraction
σ_0, σ_{app}	Applied remote stress
σ_{cl}	Crack closing stress
σ_{eq}	Equivalent stress
σ_f	Finishing stress of detwinning
$\tilde{\sigma}_{ij}(\theta, m)$	HRR singular field stress quantities
σ_{op}^I	Mode I opening stress
σ_{op}^{II}	Mode II opening stress
σ_{rr}	Normal stress along radial direction
$\sigma_{r\theta}$	Shear stress on r-plane along θ direction
σ_s	Starting stress of detwinning
σ_y	Yield stress
σ_{yy}	Normal stress along y-direction
$\sigma_{\theta\theta}$	Normal stress along tangential direction
Φ	Stress function
$\tilde{\Phi}(\theta)$	Dimensionless function of the angle
AS	Conversion of austenite into single-variant martensite
COD	Crack opening displacement
CTOD	Crack tip opening displacement
FEA	Finite element analysis
HRR	Hutchinson-Rice-Rosengren
PICC	Plasticity induced crack closure
RICC	Roughness induced crack closure
SA	Conversion of single-variant martensite into austenite
SMA	Shape memory alloy
SMCC	Sliding mode crack closure
SS	Reorientation of the single-variant martensite
TICC	Transformation induced crack closure

1. INTRODUCTION

As time goes by, the demand of special materials is increasing parallel to developments in technology, which require precision, reliability and durability. Due to their unique characteristics, shape memory alloys (SMA) can be classified as smart materials and they have been supplied more and more, recently. The name “shape memory” originate from their ability to regain the original shape after a thermal cycle. Beside the shape memory effect, at sufficiently high temperatures, the material behaves superelastically, meaning that large deformations can be recovered during loading-unloading cycles at constant temperature. By undergoing a hysteretic shape change, SMAs can absorb and dissipate energy when they are subjected to mechanical cyclic loading [1]. Due to these characteristic properties, investigation of micromechanical and mechanical behavior (fracture, fatigue, creep, etc.) of SMAs are increasing.

During the early 1900s, martensitic transformation was observed in Fe-C alloys, however these transformations were considered as an irreversible process. Later in CuZn and in CuAl alloys, thermally reversible martensitic structures are observed. In 1962, Buehler and Wiley [2] developed a series of nickel-titanium alloys with a composition of 53 to 57 % nickel by weight. It was noticed that in addition to its good mechanical properties, comparable to some common engineering metals, the material also possessed a shape recovery capability. Severely deformed specimens with residual strains of 8-15 % regained their original shape when temperature was increased. The Ni-Ti compound was named as ” NiTiNOL ”, in honor of its discovery at the Naval Ordnance Laboratory (NOL).

Shape memory alloys have been used in many fields and commercial applications last fifty years. Some alloys showing shape memory effect are listed in Table 1.1 [3]. The first major industrial application of a shape memory alloy was a cryogenic pipe fitting device developed by Raychem Corporation in 1969. In 1970s, a prototype heat engine with an efficiency estimated about 4-5 % was developed. Because of its high corrosion resistance and bio-compatibility, shape memory alloys have become popular

Table 1.1. Composition and transformation temperature range of some shape memory alloys

Alloy	Composition	Transformation Temperature Range [°C]
AgCd	44-49 at. % Cd	(-190) - (-50)
AuCd	46.5-50 at. % Cd	(30) - (100)
CuAlNi	14 wt. % Al , 4 wt. % Ni	(-140) - (100)
CuSn	15 at. % Sn	(-120) - (30)
CuZn	38.5-41.5 wt. % Zn	(-180) - (-10)
NiAl	36-38 at. % Al	(60) - (100)
NiTi	46.2-51 at. % Ti	(-50) - (110)
TiNiCu	15 at. % Cu	(-150) - (100)
TiNiNb	15 at. % Nb	(-200) - (50)
TiNiAu	50 at. % Ni + Au	(20) - (610)
MnCu	5-35 at. % Cu	(-250) - (180)
FeMnSi	32 wt. % Mn , 6 wt. % Si	(-200) - (150)

in medical field recently. They have been used in production of medical guidewires used for insertion of devices [4], orthodontic wires used in dental applications, artificial joints used in bone surgery. Perhaps the most attractive application of shape memory alloys are stents. The superelastic behavior of the material makes the insertion of initially stretched stents easier. After being released from the delivery system, the stent self-expands to over twice of its compressed diameter and exerts a nearly constant radial force on vessel wall [1, 5].

Airplane-pipe couplings, electrical connectors, thermal actuators are some other applications that use the advantage of large deformation recovery property of shape memory alloys. Thermal sensors and actuators based on phase transformations with small hysteresis are getting more reliable recently [6]. Other commercial fields where SMAs are used are production of eyeglass frames, golf clubs and antennas.

1.1. Phase Transformation in Shape Memory Alloys

The martensitic phase transformation induced by stress or heat is a solid to solid transformation, where no diffusion and no change in relative positions of atoms occur. Instead, the transformation from one structure to another depends on shear lattice distortion [7]. A material, that can undergo a martensitic phase transformation, will have two different crystalline structures : Austenite phase that is stable at high temperatures and low stress levels; Martensite phase which is less-ordered and stable at low temperatures. Each martensitic crystal has a different orientation direction, called “variants”. The number of martensite variants depends on the change of the symmetry during transformation. Austenitic crystals have no variants.

When a shape memory alloy is transformed from austenite to martensite by cooling, the phenomenon called “twinning” happens. The name comes from the twinned structure of martensite variants and an optical illustration is given in Figure 1.1. Twinning occurs due to many different martensitic phases generated during cooling process having different orientations and equal energy. In the case of a detwinned martensite, a single variant dominates all assembly. The formation is isothermal and triggered by mechanical loads.

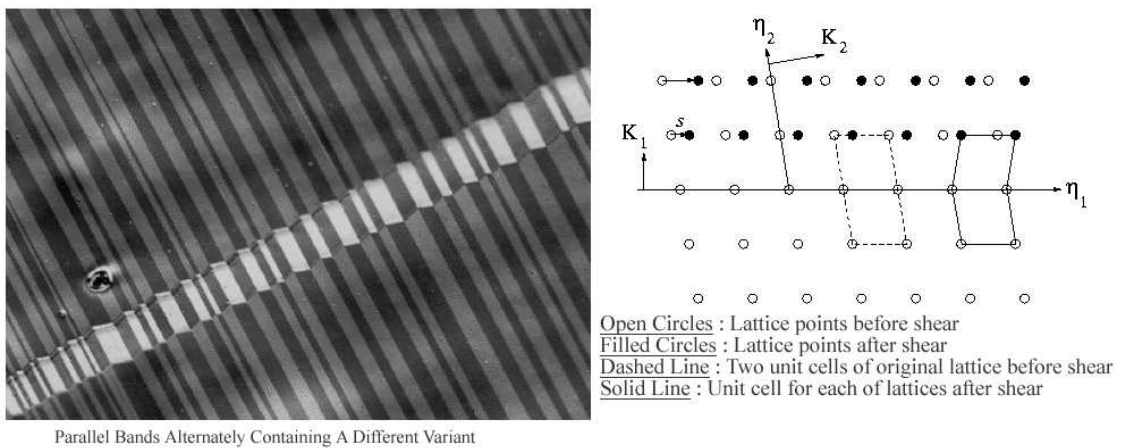


Figure 1.1. Optical micrograph of the microstructure in CuAlNi alloy showing crossing twins [8, 9]

In Table 1.1, in addition to composition of different shape memory alloys, the transformation temperature ranges are also given. The phase transition tempera-

tures are characteristic for each material and under stress-free conditions, they are denoted as M_f (Martensite Finish Temperature), M_s (Martensite Start Temperature), A_f (Austenite Finish Temperature) and A_s (Austenite Start Temperature).

These transformation starting and finishing temperatures depend on stress state and their values are changing linearly with increasing stress [1]. Figure 1.2 shows the dependency of martensitic phase transformation on stress and temperature.

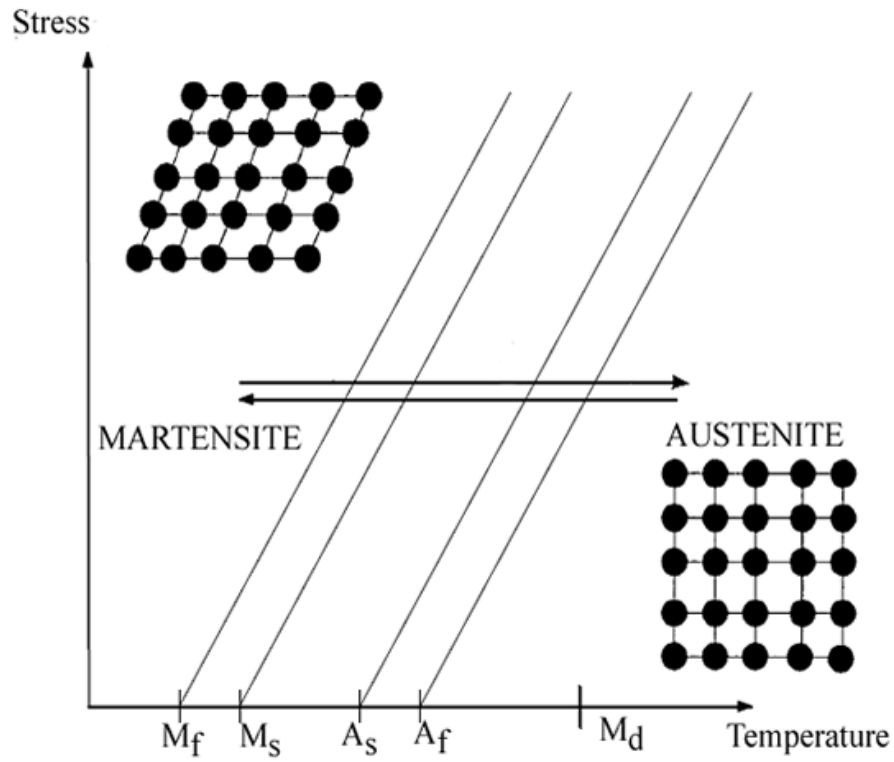


Figure 1.2. Stress-temperature phase diagram for SMAs

For isothermal, stress-induced transformations, the martensite formation begins only above the austenite finish temperature A_f . Another limiting variable M_d is given for isothermal cases to show the maximum temperature over which no transformation will occur.

1.2. Shape Memory Effect

To illustrate the shape memory effect, a stress-strain-temperature diagram is needed. By following the thermomechanical load path, the formation of different crystalline structures can be better understood. In Figure 1.3, the stress-strain-temperature data of a NiTi specimen is displayed. The initial temperature is over the austenite finish temperature A_f , so the material is in austenite phase. Without exerting any mechanical loads, cooling below martensite finish temperature M_f results in the formation of twinned martensite (path A-B). The twinned martensite is subjected isothermally to an applied stress that exceeds the critical stress value σ_s , so that the reorientation process starts. The detwinning process continues until the stress level approaches to σ_f (path B-C). Then, the specimen is elastically unloaded (path C-E). The material is still in detwinned martensite phase. After removing the load completely, the specimen is reheated in order to complete the thermal cycle (path E-F). Upon increasing the temperature, the detwinned martensite phase transforms into the original austenite phase and the original shape is regained. The described ability of these special materials is called one-way shape memory effect. As seen in Figure 1.3, the recovered strain is greater than four percent. The strain recovered from detwinned martensite to austenite is termed as transformation strain and denoted by ε_t .

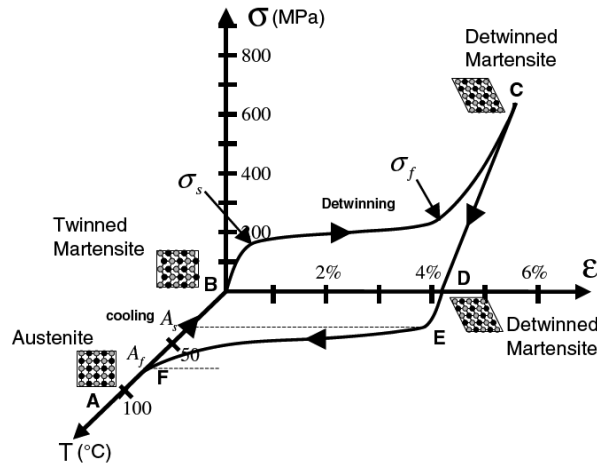
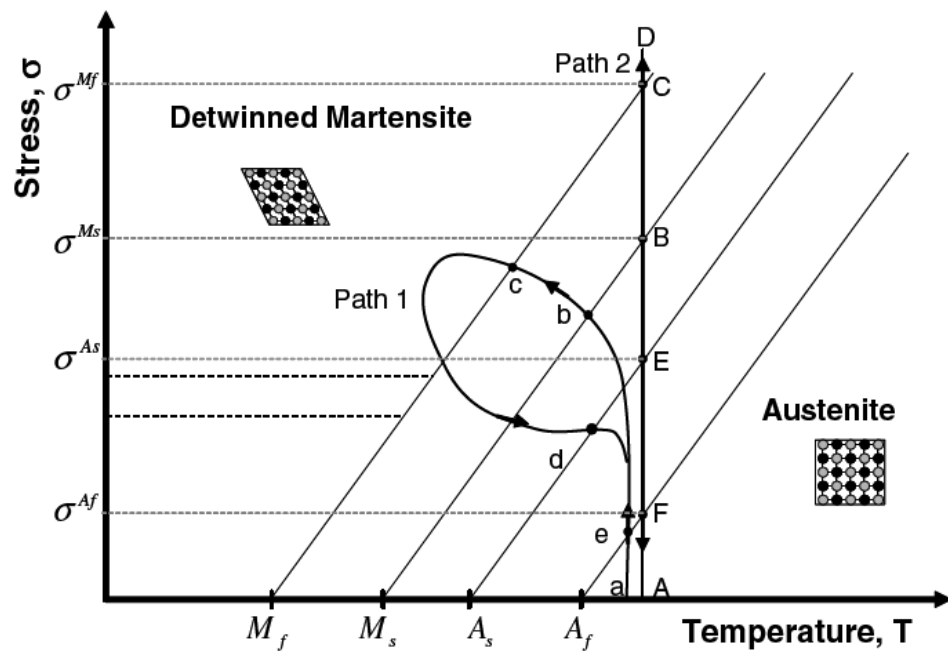


Figure 1.3. Stress-strain-temperature plot exhibiting the shape memory effect [1]

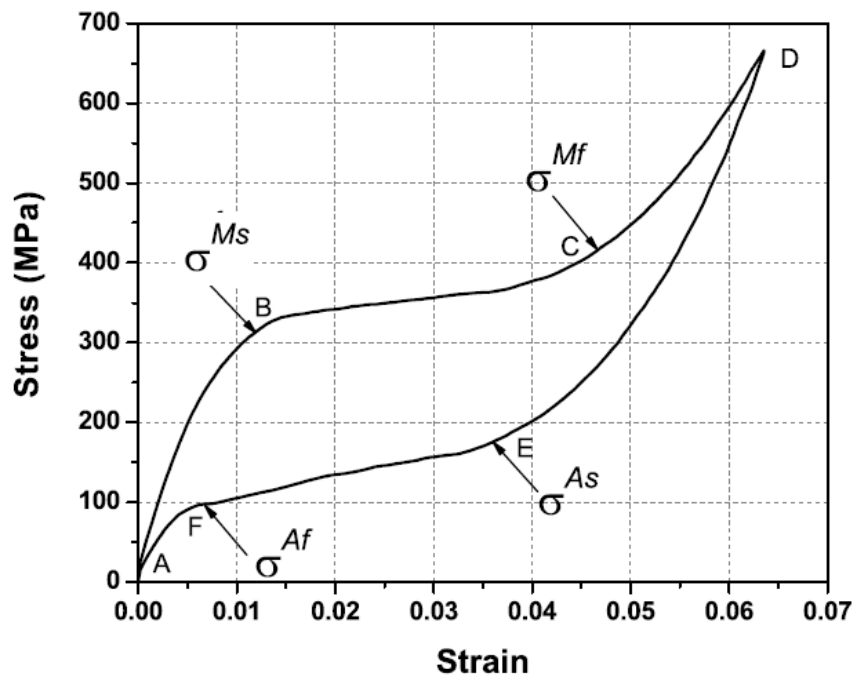
1.3. Pseudoelasticity

Another attribute making the shape memory alloys superior to other commercial materials is pseudoelasticity or superelasticity associated with stress-induced transformation. In Figure 1.4(a), two distinct loading-unloading paths are illustrated. Path 1 is a thermomechanical, pseudoelastical loading path; however by applying path 2, the temperature is kept constant above the critical austenite finish temperature at zero-stress state. For formation of stress-induced detwinned martensite phase, the temperature should remain above the austenite finish temperature A_f and below the martensite desist temperature M_d .

Considering only the mechanical loading path 2 in Figure 1.4(a) and its associated stress-strain curve displayed in Figure 1.4(b), superelasticity of shape memory alloys will be explained. Initially the specimen is hold stress-free and the surrounding temperature is above A_f . When a mechanical load is applied, the material deforms elastically until the critical stress level σ_s^M , which is the beginning of detwinned martensite formation (path A-B). Stress-induced martensite formation generates large inelastic strains nearly 7% - 12%, depending on the composition and initial conditions. Therefore, in forward transformation, the stress-strain curve of SMAs behaves like the curves of perfectly-plastic materials (path B-C). The transformation proceeds until the applied stress approaches the final stress level σ_f^M , where the loading path intersects with the stress-dependent martensite finish temperature M_f line. At this stage, the slope of stress-strain curve changes and the material continues deforming elastically with the modulus of martensitic phase (path C-D). When the stress is released gradually by unloading, the elastic deformation of martensite is recovered until another critical stress level σ_s^A , which indicates the beginning of the reverse transformation process (path D-E). Similar to forward transformation, the reverse transformation happens between two limiting stresses, σ_s^A and σ_f^A , where the strain due to phase transformation is recovered (path E-F). Indeed, loading curve and unloading curve don't coincide, which is a signal of energy dissipation during transformation and existence of hysteresis.



(a) Stress-Temperature diagram



(b) Stress-Strain diagram

Figure 1.4. Superelastic behavior of a NiTi specimen [1]

1.4. Cyclic Behavior of SMAs

The fatigue behavior of SMAs depends on several internal (defects on grain boundaries, microcracks, impurities, etc.) and external (applied stress, strain, temperature gradients, corrosive medium, etc.) conditions. Experimental observations state that by repeatedly loading the material following a certain mechanical or thermal loading path, the hysteretic response of shape memory alloys stabilizes and the inelastic strain saturates [10, 11]. During the first thermal or mechanical cycle, only a part of transformation strain developed in cooling or pressurizing the material, can be recovered; some permanent (mostly plastic) strain remains after each cycle is completed. The additional permanent strains accumulate in a slow fashion and a saturation takes place in the pseudoelastic regime after some cycles. This response of a nitinol wire is drawn in Figure 1.5.

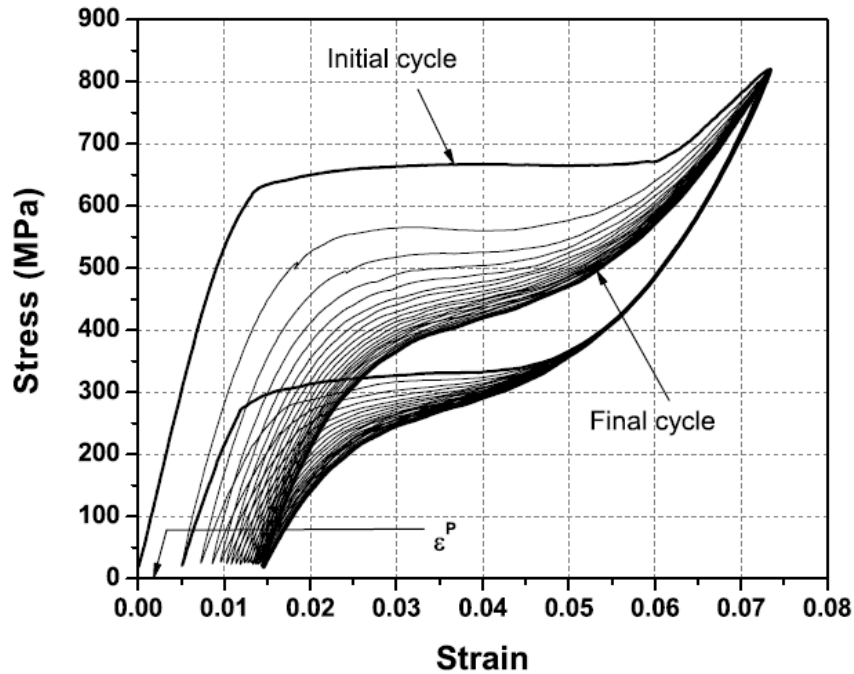


Figure 1.5. Pseudoelastic response of an as-received NiTi wire with $A_f = 65^\circ\text{C}$, tested at a temperature of 70°C [12]

The accumulation of permanent strains cause perturbation of general mechanical behavior of materials and failures may occur. If the deformation or stress level applied to the SMA specimen remains within the elastic regime, this can lead to fatigue life as high as nearly 10^7 cycles [13].

1.5. Overview of Constitutive Models for SMAs

In Sections 1.2 and 1.3, it has been stated that shape memory alloys are metallic alloys being able to recover from significant deformation, when subjected to some mechanical or thermal loading conditions. This ability is gained by using two major properties of shape memory alloys; the shape memory effect and superelasticity. Besides, the martensitic phase transformations occurring in SMAs are diffusionless, solid-solid phase transformations, where the austenite and martensite phases can co-exist during the phase transformation. These transformations are dependent on stress state and temperature. To review constitutive thermomechanical models, describing these fundamental properties of SMAs, by resembling experimental studies carried out before, will be a great guideline for the future work on fracture mechanics study of this material.

1.5.1. Choice of Internal State Variables

To represent the SMA thermomechanical response being related to changes in microstructure, two internal state variables are defined : Martensitic volume fraction ξ_m and the transformation strain ε_t . The martensitic volume fraction is a scalar quantity and it counts the contribution of all different martensitic variants in the material. As stated before, the martensitic phase can exist in different forms; either the twinned martensite induced by cooling or the detwinned martensite induced by stress. In the case of detwinned martensite formation, the presence of stress leads to a macroscopic shape change. This deformation is associated with the transformation strain, used as an internal state variable by formulating constitutive models.

1.5.2. Kinematic Assumptions

The choice of a reference configuration for SMAs, with respect to which deformations can be described under applied loads, is very complicated due to existence of two material reference configurations; the austenitic and martensitic parent phases. Conventionally, the austenitic parent phase is selected as the reference configuration and the transition from austenite to martensite is described by internal state variables. When the deformation gradient tensor approaches to identity, the difference between reference and deformed configurations becomes negligible and small-strain assumption formulation can be used. Experimental observations show that polycrystalline SMAs are able to recover strains of about 6%, therefore small-strain formulation will be valid for describing their deformations [1]. With this assumption, there will be no distinction between the reference and current coordinates and the total strain tensor will be given in the form of :

$$\varepsilon = \frac{1}{2}[(\nabla u) + (\nabla u)^T] \quad (1.1)$$

The total strain is decomposed of a thermoelastic part and an inelastic part, which is further decomposed of transformation strain and formation of plastic strains. For simplicity, the inelastic part of strain is considered to be only associated with the transformation strain and total strain can be written as the sum of thermoelastic strain and transformation strain :

$$\varepsilon = \varepsilon^{th} + \varepsilon^{in} \quad (1.2)$$

1.6. Thermomechanical Constitutive Models for SMAs

Aiming of predicting the material behavior, many researches have been done and several different constitutive models have been proposed since 1970s. Describing the phase transition characteristics of SMA has been the main concern of researchers, which includes modeling the hardening during phase transformation, the different response that SMAs exhibit in tension and compression, modeling of detwinning of martensite and the accumulation of plastic strains during cyclic loading [14, 15, 16].

Considering the methodology used to describe the material behavior, there exists two major groups. First group of researchers is dealing with micromechanical engineering features to calculate macro-level quantities and their constitutive models are microplane-based. The other solution is building the constitutive model, formulated by phase transformation kinetics or Gibbs free energy and/or Helmholtz free energy concepts.

1.6.1. Micromechanical Constitutive Models

1D mathematical models, which include martensitic transformation phenomenon, are firstly formulated by Delaey, Krishnan, Tas and Warlimont [17, 18, 19] and Achenbach and Müller [20]. Fischer and Tanaka [21] studied thermomechanical and transformation behaviors of polycrystalline shape memory alloys in the process of stress-induced martensitic transformation from the micromechanical point of view. Patoor et al. [22] addressed transformation of single crystals with multiple martensitic variants and extended the method to polycrystalline materials. Sun and Hwang [23, 24] proposed a micromechanics constitutive model to describe the pseudoelastic and shape memory behavior of polycrystalline shape memory alloys under various temperatures and studied the constitutive relation for various phenomena of SMA (superelasticity, rubber-like elasticity, shape memory effect). Huang and Brinson [25] presented a general 3D multivariant model based on thermodynamics and micromechanics for single crystal shape memory alloy based on the habit plane and transformation directions for the variants of martensite in a given material. Cherkaoui et al. [26] derived the

behavior for an austenitic single crystal from which the overall behavior of polycrystalline TRIP steels can be deduced using classical scale transition method. Gao et al. [27, 28] developed a general 3-D multivariant model for shape memory alloy constitutive behavior based on the habit planes and transformation directions for variants of martensite and used a thermodynamic and micromechanics approach to develop the governing equations for thermomechanical response. Guquan et al. [29, 30] established a microstructure-based constitutive model and investigated numerically on the role of each microstructure parameter in the kinematical and kinetic evolution of polycrystalline SMA under axisymmetrical tension loading. Gao and Brinson [31] improved their multivariant model describing SMA constitutive behavior for improved prediction of thermomechanical response. Zhu et al. [32] simulated interior hysteresis loops during loading/unloading by switching the critical driving forces when an opposite transition takes place. Sadjadpour and Bhattacharya [33] built a general 3D microstructure-based constitutive model with the key idea of an effective transformation strain of martensitic microstructure. Peng et al. [34] proposed a model to describe the main characteristics of pseudoelastic behavior of NiTi shape memory alloys under pure tension and pure torsion. Levitas and Ozsoy [35, 36] derived the universal thermodynamic driving force for coherent interface reorientation during first-order transformations in solids for small and finite strains and applied the theory to the numerical modeling of the evolution of martensitic microstructure under three-dimensional thermomechanical loading during cubic-tetragonal and tetragonal-orthorhombic phase transformations.

1.6.2. Constitutive Models Based on Phase Diagram

Beside the researches done for formulating a model at macroscale by using micromechanics, other models based on different engineering concepts (phase transformation kinetics, theory of plasticity and hardening rules, Gibbs free energy, Helmholtz free energy, etc.) are also constructed throughout the history of shape memory alloy investigation. Tanaka et al. [37, 38, 39] used an exponential hardening rule for the phase transformation, during which material properties remain constant. Later, Liang and Rogers [40] used a cosine hardening rule for the phase transformation. In their model, again, material properties remain constant during phase transformation. Brin-

son [41] and later, Bekker and Brinson [42, 43] formulated their mathematical model based on the concept of Helmholtz free energy, where two internal variables allowing modeling of both detwinned and self accommodated martensite are introduced and a cosine hardening law and variable elastic stiffness during phase transformation are used. Ivshin and Pence [44] used a hyperbolic tangent function to model the transformation hysteresis. Phase fraction of austenite is directly postulated to have a given functional form where cases of hysteretic and non-hysteretic phase transformations are considered. Boyd and Lagoudas [45] used the volume fraction of martensite as internal variable and a transformation evolution equation to connect it with the transformation strain. A polynomial hardening function is introduced and during reverse transformation, non-proportional loading paths are accounted for. Auricchio, Taylor and Lubliner [46, 47, 48] worked on the isothermal pseudoelasticity and dealt with the 3D response using Drucker-Prager-type surface to describe the critical stresses for pseudoelastic transformation. Hardening law is exponential and the model has been implemented on finite elements. Lagoudas, Bo and Qidwai [49] unified the earlier models of Tanaka, Liang and Rogers, and, Boyd and Lagoudas. Auricchio and Sacco [50] worked on a 1D mathematical model concerning isothermal pseudoelasticity. In their model, a linear hardening law is used to determine critical transformation stresses. Govindjee and Kasper's model [51] includes kinematic hardening plasticity and a distinction between martensitic variants in tension and compression is made. Helm and Haupt [52] considered both temperature-induced and stress-induced martensite, where the forward and reverse transformation is modeled by single function with kinematic hardening. Anand and Gurtin [53] decomposed the deformation gradient into an elastic term and a term associated with the martensitic transformation. Reese and Christ [54] used large deformation finite element analysis and worked on isothermal pseudoelasticity. Kinematic hardening is used to describe the critical stresses for transformation.

1.7. Auricchio - Taylor - Lubliner Constitutive Model

1.7.1. Drucker - Prager Flow Potential

Since the models for shape memory alloys proposed by Auricchio, Taylor and Lubliner are within the generalized plasticity framework, there is a need for explaining some basic concepts of plasticity. Drucker - Prager flow potential is used to characterize phase transition of SMAs, so it is convenient to give a brief description of this function.

The post-yielding, loading-and-unloading behavior of materials are related to the stress-strain relation for plastically deformed solids. In the plastic region, the total strain increment in uniaxial stress state is the sum of elastic and plastic strain increment :

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \quad (1.3)$$

The total strain increments in the three-dimensional case can be generalized as

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (1.4)$$

The decomposed increments are elastic and plastic strain increments. The decomposition is correct only for infinitesimal strain. In elastic theory the strain tensor was related to the stress tensor through an elastic potential function, the complementary strain energy U , such that :

$$\varepsilon_{ij} = \frac{\partial U}{\partial \sigma_{ij}} \quad (1.5)$$

By extending this idea, a plastic potential function $Q(\sigma_{ij})$ was proposed and the

plastic strain increments can be derived similarly :

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \quad (1.6)$$

where $d\lambda$ is a scalar. The constant plastic potential Q represents a surface in the six-dimensional stress space, and the plastic strain can be represented to a vector perpendicular to the surface $Q(\sigma_{ij})$. The common approach is to assume the plastic potential function same as the yield function, this assumption is called “associated flow rule”. According to Drucker’s stability postulate, associated flow rule is proved mathematically [55]. The complete incremental relationship between stress and strain for elasto-plastic deformation is found to be :

$$d\varepsilon_{ij} = \frac{1}{E}[(1 + \nu)\delta_{ik}\delta_{jl} - \nu\delta_{ik}\delta_{kl}]d\sigma_{kl} + d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \quad (1.7)$$

The incremental constitutive relation can also be expressed in matrix formulation. The yield function is given as :

$$F(\sigma_{ij}, k) = 0 \quad (1.8)$$

where k is a material parameter determined experimentally. Differentiating the hardening yield function 1.8, the following expression is obtained :

$$dF = \frac{\partial F}{\partial \sigma_{ij}}d\sigma_{ij} + \frac{\partial F}{\partial k}dk = \frac{\partial F}{\partial \sigma_1}d\sigma_1 + \frac{\partial F}{\partial \sigma_2}d\sigma_2 + \frac{\partial F}{\partial \sigma_3}d\sigma_3 + \frac{\partial F}{\partial k}dk = 0 \quad (1.9)$$

Introducing a parameter $A = -\frac{1}{d\lambda} \frac{\partial F}{\partial k} dk$, Equation 1.9 can be rewritten to a matrix form

$$\frac{\partial F}{\partial \sigma_{ij}}d\sigma_{ij} - Ad\lambda = 0 \quad (1.10)$$

Introducing the flow vector $\{a\} = \frac{\partial F}{\partial \sigma_{ij}}$, the expression 1.10 becomes :

$$\{a\}^T d\{\sigma\} - A d\lambda = 0 \quad (1.11)$$

The strain increments can be derived as follows

$$d\{\varepsilon\} = [D]^{-1} d\{\sigma\} + \{a\} d\lambda \quad (1.12)$$

where $[D]$ is the usual matrix of elastic constants. Multiplying both sides of Equation 1.12 and combining with Equation 1.11, one can obtain :

$$\{a\}^T [D] d\{\varepsilon\} = A d\lambda + d\lambda \{a\}^T [D] \{a\} \quad (1.13)$$

The plastic multiplier is found :

$$d\lambda = \frac{\{a\}^T [D] d\{\varepsilon\}}{A + \{a\}^T [D] \{a\}} \quad (1.14)$$

By substituting the plastic mulplier $d\lambda$ into Equation 1.12, the complete stress-strain relation can be derived to be

$$d\{\sigma\} = [D]_{ep} d\{\varepsilon\} = \left[[D] - \frac{[D] \{a\} \{a\}^T [D]}{A + \{a\}^T [D] \{a\}} \right] d\{\varepsilon\} \quad (1.15)$$

According to Owen and Hinton [56], the generic flow vector $\{a\}$ can be expressed as

$$\{a\}^T = C_1 \{a_1\} + C_2 \{a_2\} + C_3 \{a_3\} \quad (1.16)$$

where

$$\{a_1\}^T = \frac{\partial I_1}{\partial \{\sigma\}} = \{1, 1, 1, 0, 0, 0\} \quad (1.17)$$

$$\{a_2\}^T = \frac{\partial \sqrt{J_2}}{\partial \{\sigma\}} = \frac{1}{2\sqrt{J_2}} \{\sigma'_{xx}, \sigma'_{yy}, \sigma'_{zz}, 2\tau_{yz}, 2\tau_{zx}, 2\tau_{xy}\} \quad (1.18)$$

$$\{a_3\}^T = \frac{\partial J_3}{\partial \{\sigma\}} \quad (1.19)$$

and I_1 is the first invariant of the stress tensor, J_2 and J_3 are the second and third invariants of the deviatoric stress tensor, respectively. C_1, C_2, C_3 are constants required to determine the flow vector and their values depend on the type of the yield function. For Drucker-Prager type flow potentials, these constants are $C_1 = \alpha$, $C_2 = 1$ and $C_3 = 0$.

1.7.2. Macromodeling of Superelastic Behavior

In traditional materials, both shape memory effect and superelasticity are not present, hence conventional inelastic theories don't satisfy the requirements in formulating a constitutive model for SMAs. Auricchio et al. [48] have proposed a small-deformation three-dimensional model for the superelastic behavior of shape-memory material. Then using the same framework, Auricchio and Taylor [57] extended the superelastic constitutive model to the case of finite strains. The main goal of these papers are constructing algorithms which can be used in computational tools to help the design process of SMA devices. Experimental evidence shows that :

- The austenite is stable at high temperatures and high values of stress. For a stress-free state, T_f^{SA} is the temperature above which only the austenite is stable and T_f^{AM} is the temperature below which only martensite is stable.
- At the crystallographic level, if there is no preferred direction of occurrence of transformation, multiple-variant martensite is produced, meaning that the

martensite takes advantage of the existence of different habit planes. Habit plane is the contact plane between the austenite and the martensite during a single-crystal transformation [58, 59]; and if the martensite crystals tend to be formed on the most favorable habit plane, single-variant martensite is produced. Multiple-variant martensite is characterized by a twinned structure, whereas the single-variant martensite by a detwinned one.

- The martensitic transformation is pressure dependent [60].
- Phase transformations are insensitive to temperature and stress rates [61].

In the model, the temperature T and the Kirchhoff stress τ are selected as control variables. One-type of martensite is considered so that only two phases, the austenite (A) and the single-variant martensite (S), are worked with. As internal variables the fraction ξ of austenite and single-variant martensite are given. The conditions

$$\xi_S + \xi_A = 1 \quad (1.20)$$

$$\dot{\xi}_S + \dot{\xi}_A = 0 \quad (1.21)$$

are always satisfied. Three phase transformation are stated :

- (i) AS : The conversion of austenite into single-variant martensite ($A \Rightarrow S$)
- (ii) SA : The conversion of single-variant martensite into austenite ($S \Rightarrow A$)
- (iii) SS : The reorientation of the single-variant martensite ($S \Rightarrow S$)

AS, SA, SS will be used as superscripts in following equations in order to differentiate phase transformations. Since there is no change in the martensitic fraction during reorientation, the change in the martensite fraction can be written as :

$$\dot{\xi}_S = \dot{\xi}_S^{AS} + \dot{\xi}_S^{SA} \quad (1.22)$$

To fully understand the transformation mechanism, an uniaxial model is better suited to explain. In order to describe the single-variant martensite or the austenite production, some material parameters should be defined (see Figure 1.6). T_s and T_f are the starting and final temperatures at which the transformation may occur at zero stress. These temperatures are dependent on the stress level, therefore a constant slope C is given as a material parameter.

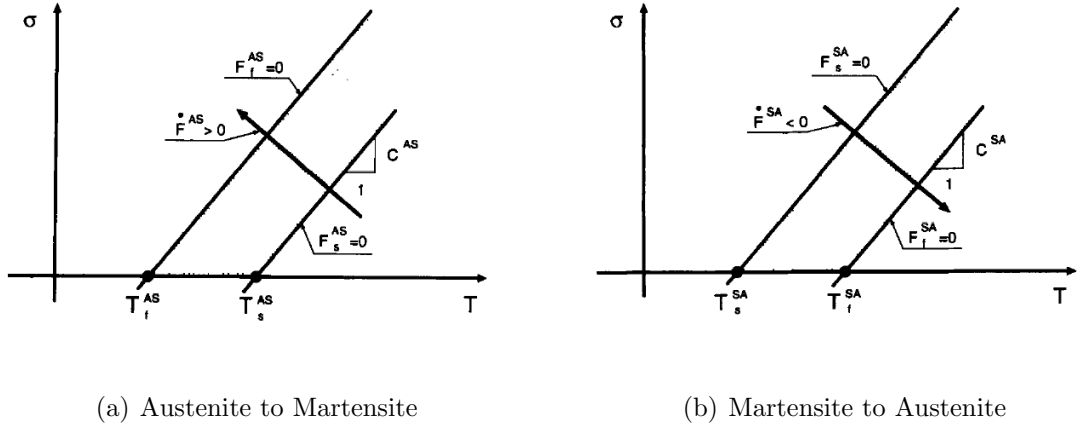


Figure 1.6. Production and reproduction of phases in 1D [48]

The first case is the forward transformation from austenite to single-variant martensite. According to Figure 1.6(a), the transformation functions for martensite production are defined as:

$$F^{AS} = \sigma - C^{AS}T \quad (1.23)$$

$$F_s^{AS} = F^{AS} - R_s^{AS} \quad (1.24)$$

$$F_f^{AS} = F^{AS} - R_f^{AS} \quad (1.25)$$

where

$$R_s^{AS} = C^{AS}T_s^{AS} \quad (1.26)$$

$$R_f^{AS} = C^{AS}T_f^{AS} \quad (1.27)$$

The conditions for transformation to martensite are given as :

$$F_s^{AS} > 0$$

$$F_f^{AS} < 0$$

Moreover, to activate the production of single-variant martensite, a stress increase, a temperature decrease or a combination of both activations should occur. Therefore, the condition

$$\dot{F}^{AS} > 0$$

should be satisfied.

The transformation functions for austenite reproduction are defined in the same fashion (Figure 1.6(b)) such as

$$F^{SA} = \sigma - C^{SA}T \quad (1.28)$$

$$F_s^{SA} = F^{SA} - R_s^{SA} \quad (1.29)$$

$$F_f^{SA} = F^{SA} - R_f^{SA} \quad (1.30)$$

with

$$R_s^{SA} = C^{SA}T_s^{SA} \quad (1.31)$$

$$R_f^{SA} = C^{SA}T_f^{SA} \quad (1.32)$$

The conditions for transformation to austenite are given as :

$$F_s^{SA} < 0$$

$$F_f^{SA} > 0$$

$$\dot{F}^{SA} < 0$$

The uniaxial model is extended to three-dimensional case for superelasticity, in which Drucker-Prager type loading functions are introduced to describe the three different phase transformations mentioned. The function, which is in the form of Equation 1.8 and depends on control variables Kirchoff stress τ and temperature T , is :

$$F^{AS}(\tau, T) = \|t\| + 3\alpha p - C^{AS}T \quad (1.33)$$

where t is the deviatoric part of Kirchoff stress, p is pressure, α is the Drucker-Prager flow constant (see Equation 1.16) and $\|X\|$ is the Euclidean norm of X . In both conversions the same transformation functions are used as in the uniaxial case, where

$$R_s^{AS} = \left[\sigma_s^{AS} \left(\sqrt{\frac{2}{3}} + \alpha \right) - C^{AS}T_s^{AS} \right] \quad (1.34)$$

$$R_f^{AS} = \left[\sigma_f^{AS} \left(\sqrt{\frac{2}{3}} + \alpha \right) - C^{AS}T_f^{AS} \right] \quad (1.35)$$

$$R_s^{SA} = \left[\sigma_s^{SA} \left(\sqrt{\frac{2}{3}} + \alpha \right) - C^{SA}T_s^{SA} \right] \quad (1.36)$$

$$R_f^{SA} = \left[\sigma_f^{SA} \left(\sqrt{\frac{2}{3}} + \alpha \right) - C^{SA}T_f^{SA} \right] \quad (1.37)$$

and, where σ_s and σ_f are starting and finishing values of stresses in each phase transformation processes.

To relate the formation rate of each phase with the loading functions, scalar parameters of state variables (internal and control) H^{AS} and H^{SA} are formulated :

$$H^{AS} = \begin{cases} 1 & \text{if } F_s^{AS} > 0, \quad F_f^{AS} < 0, \quad \dot{F}^{AS} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$H^{SA} = \begin{cases} 1 & \text{if } F_s^{SA} < 0, \quad F_f^{SA} > 0, \quad \dot{F}^{SA} < 0 \\ 0 & \text{otherwise} \end{cases}$$

By using these parameters, the evolution of single-variant martensite fraction and austenite fraction can be assumed in either exponential form

$$\dot{\xi}_s^{AS} = H^{AS} \beta^{AS} (1 - \xi_s) \frac{\dot{F}^{AS}}{(F_f^{AS})^2} \quad (1.38)$$

$$\dot{\xi}_s^{SA} = H^{SA} \beta^{SA} \xi_s \frac{\dot{F}^{SA}}{(F_f^{SA})^2} \quad (1.39)$$

where β^{AS} and β^{SA} are material parameters, or a linear form:

$$\dot{\xi}_s^{AS} = -H^{AS} (1 - \xi_s) \frac{\dot{F}^{AS}}{F_f^{AS}} \quad (1.40)$$

$$\dot{\xi}_s^{SA} = H^{SA} \xi_s \frac{\dot{F}^{SA}}{F_f^{SA}} \quad (1.41)$$

1.8. Objective of the Study

The mechanical properties of shape memory alloys have been studied experimentally by many researchers. However, it has been recognized that the number and results of finite element simulations for their fracture behavior are not consummating. Especially, there is a lack in FE modeling of fatigue crack propagation and in analysis of phase transformation induced crack closure. Considering this fact, the contribution of

this study to literature will be a detailed and comparative analysis of SMA's fatigue behavior. The first major goal of the thesis is observing effects of phase transformation on mechanical properties after applying cyclic loads. A parametric analysis will be carried out in order to find out the importance of load ratio, initial and cumulative crack length. Last, the fatigue life of a typical SMA specimen will be predicted by using finite elements.

Chapter 2 is reproduction chapter of the analysis on plasticity induced crack closure. Modeling techniques introduced here will be further used in SMA fatigue crack closure analysis. In Chapter 3, fracture mechanics behavior of shape memory alloys are investigated in detail. Required parameters like size of phase transformation zone, stress intensity factor and energy release rate are calculated and the validity of estimation procedures are checked by comparing the FE results to analytical results available in literature for other similar materials. The information obtained from quasi-static analysis in Chapter 3 is used in the Chapter 4, where parametric analysis on phase transformation induced fatigue crack closure is carried out. A stress intensity factor description for single-edge cracked SMA plate is going to be made, which can also be used in calculation of crack growth rate and fatigue life. The study is concluded in Chapter 5, where also future work suggestions are given.

2. MODELING OF FATIGUE CRACK CLOSURE USING FINITE ELEMENTS

Over past 35-40 years, several finite element models have been proposed to study crack closure which has an important effect on fatigue crack growth. For an arbitrary material, the general crack growth equation (Paris-Erdogan equation) is written as follows [62] :

$$\frac{da}{dN} = C(\Delta K_{eff})^m \quad (2.1)$$

where a is the crack length and N is the number of fatigue cycles. C and m are empirical constants describing the materials' fatigue behavior. Ideally, if a crack is perfectly sharp and behaves completely elastically, then this crack closes when unloading from tension is complete ($\sigma = 0$). For real materials, especially in which the plastic zone ahead of the crack tip advances during crack propagation, permanent deformation in the wake affects the interaction of crack faces during unloading. Under plane-stress conditions, stretching in the plastic zone at the crack tip leads to local necking of the component [63]. During crack propagation, this process results in formation of a wedge and material flow to crack flanks (see Figure 2.1). Consequently, crack stretched crack faces come into contact before complete unloading from tension. This phenomenon is called “crack closure” and the extent of closure greatly affects the crack propagation rate.

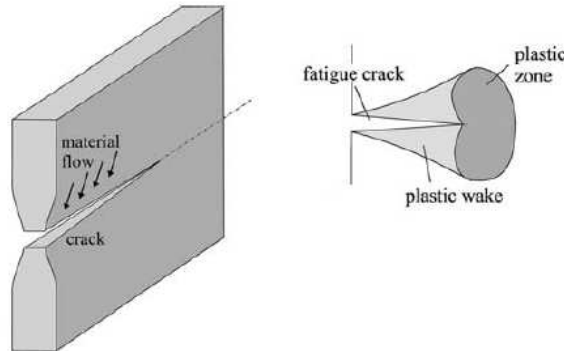


Figure 2.1. Crack closure and mechanism of material flow

Observations state that, fatigue cracks can grow only when they are completely open and only part of the fatigue cycles, during which the crack is open, contributes to crack propagation [64]. If the closure effect is ignored, the stress intensity range is calculated from :

$$\Delta K = \begin{cases} K_{\max} - K_{\min} & \text{if } \sigma_{\min} > 0 \\ K_{\max} & \text{if } \sigma_{\min} < 0 \end{cases} \quad (2.2)$$

If there exists a premature contact of crack faces before complete removal of load, the stress intensity range ΔK needs to be reduced by the part of stress intensity range, where the crack is closed (see Figure 2.2). Thus, a new parameter, ΔK_{eff} , should be introduced, that drives stable propagation of fatigue cracks and is used in Equation 2.1. The effective stress intensity range is calculated by :

$$\Delta K_{eff} = K_{\max} - K_{cl/op} = f(\sigma_{\max}, \sigma_{cl/op}) \quad (2.3)$$

where K_{cl} and K_{op} are stress intensity factors corresponding to crack closure and crack opening stress levels, respectively.

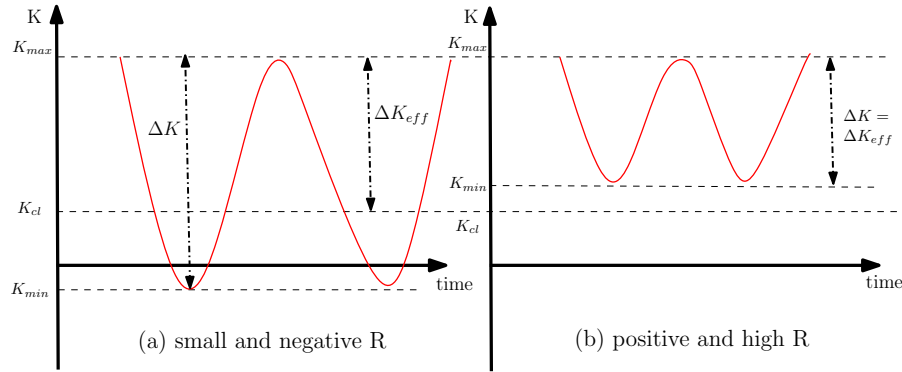


Figure 2.2. Significance of the effective stress intensity factor

The effect of fatigue crack closure can be quantified with ΔK_{eff} , which depends on various factors like maximum applied stress $\sigma_{\max}$, yield stress σ_y , load ratio $R = \sigma_{\min}/\sigma_{\max}$, hardening modulus H , elasticity modulus E , friction μ between crack faces, crack length a , geometry and microstructure of specimen (centercrack, edge crack,

etc.), crack orientation, plane stress or strain constraints, etc. Shortly,

$$\Delta K_{eff} = f \left(\frac{\sigma_{max}}{\sigma_y}, R, \frac{H}{E}, \mu, a, Dimensions, \dots \right) \quad (2.4)$$

In Figure 2.3, a typical plot of da/dN versus stress intensity range ΔK , which depends basically on the stress distribution and geometry, is given. The curve is divided into three regions: Possible Threshold Region, Simple Linear Relationship, Static Tearing. The threshold range, below which no crack growth can be observed, is denoted by ΔK_{th} . The expression 2.1 describes the linear behavior in region II and is convenient using this equation for a wide range of Mode-I crack growth rates, between 10^{-9} and 10^{-6} m/cycle [62]. To describe the complete sigmoidal curve, the following equation can be used :

$$\frac{da}{dN} = C(\Delta K)^m \left[\frac{1 - \left(\frac{\Delta K_{th}}{\Delta K} \right)^{n_1}}{1 - \left(\frac{K_{max}}{K_c} \right)^{n_2}} \right]^{n_3} \quad (2.5)$$

where n_1, n_2, n_3 are empirical constants and K_{IC} is the fracture toughness of the material above which the crack growth will be unstable.

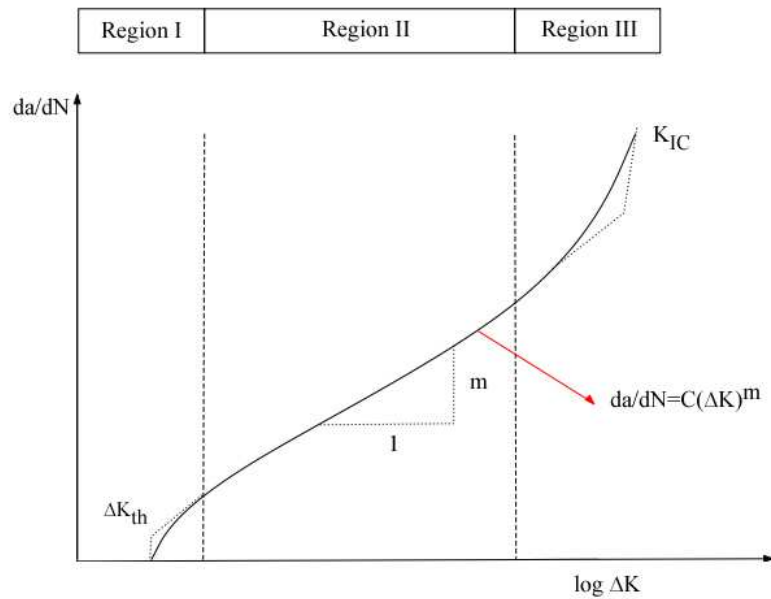


Figure 2.3. Representative plot of crack growth rate vs. stress intensity range

The threshold stress intensity range ΔK_{th} was assumed to be a material constant, however it is dependent on several external parameters like load ratio, overloading, test frequency etc., and internal parameters like grain size, multiphases, texture etc. [65]. Introducing the concept of crack closure, the threshold stress intensity needs to be replaced by an effective fatigue threshold stress intensity, $\Delta K_{th,eff}$, which is considered to be a true material constant [66]. The closure behavior of a real material is shown in Figure 2.4. By reducing the external load from K_{max} , the crack opening is reduced proportionally along the straight line down to K_{cl} , where crack faces first come into contact. On further unloading until K_{closed} , where COD is equal to zero, the linear relationship between K and COD vanishes due to interaction of several asperities. On reloading, the sequence is inverse, however K_{op} will generally slightly higher than K_{cl} due to frictional forces at the fracture surface [67]. For the case $K_{closed} < K_{min} < K_{cl}$, the modified effective stress intensity threshold is given as :

$$\Delta K_{th,eff} = \Delta K_{th} - \Delta K_{shielding} \quad (2.6)$$

where $\Delta K_{shielding}$ takes into account the difference between the closure-free and the closure-affected crack.

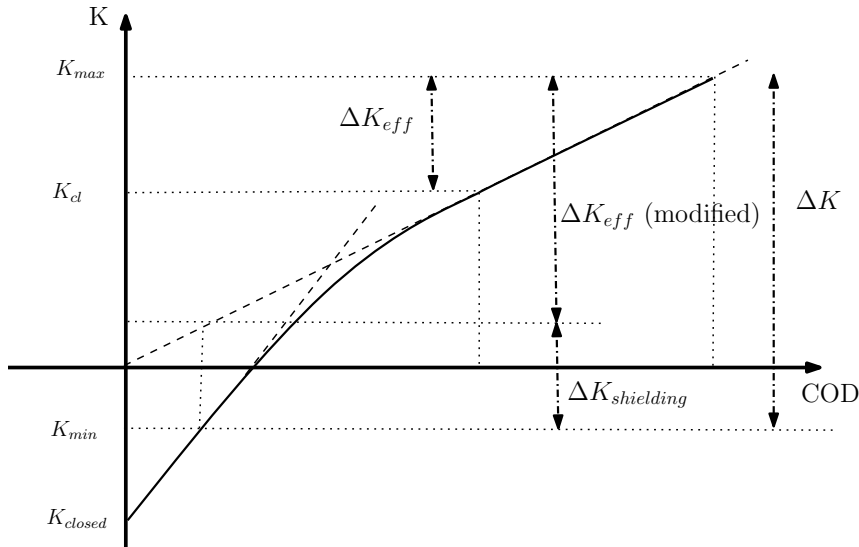


Figure 2.4. Effect of crack closure on threshold stress intensity range

The concept of crack closure has been important in the explanation of experimental results. Elber [64] was the first researcher, who found that fatigue cracks are closed for a significant portion of a tensile load cycle, probably owing to residual plastic deformation left in the wake of a growing crack. The mechanism described by him is plasticity induced crack closure (PICC) and for a long period, it was assumed to be the only mechanism for closure. Besides PICC, there are further mechanisms causing premature contact of crack faces [63]:

- Roughness induced crack closure (RICC) deals with reduction of nominal stress intensity factor due to friction between crack faces.
- Oxidation induced crack closure is due to oxidation of new surfaces generated by crack advance.
- Fluid induced crack closure is due to penetration of a highly viscous fluid into the crack.
- Transformation induced crack closure is due to volume change in the vicinity of the crack tip.

The objective of this chapter is to determine modeling parameters and techniques for an arbitrary material to investigate its fatigue crack closure behavior. The data obtained from finite element analysis will be compared with the results available in literature. The knowledge gained in this particular stage will be further implemented to shape memory alloys to study their fatigue response, in next chapters.

2.1. 2-D Edge Crack in a Homogenous Elastic-Plastic Plate

For the verification of fatigue crack closure analysis used here, a two dimensional model has been developed. Material data and specimen geometries are extracted from the paper of Kibey et al. [68]. All finite element analyses are carried out by using ABAQUS/Standard.

2.1.1. Modeling Parameters and Geometry

The stress-strain relationship of the test material is given in Figure 2.5. The material follows a bilinear path and the Young's modulus and the yield stress are 200 GPa and 480 MPa, respectively. The Poisson's ratio is assumed to be 0.3. To capture Baushinger effect associated with reversed yielding, a Mises yield criterion with a constant kinematic hardening modulus $H = 0.01 E$ is set. This hardening modulus is typical for low hardening steels and aluminium alloys.

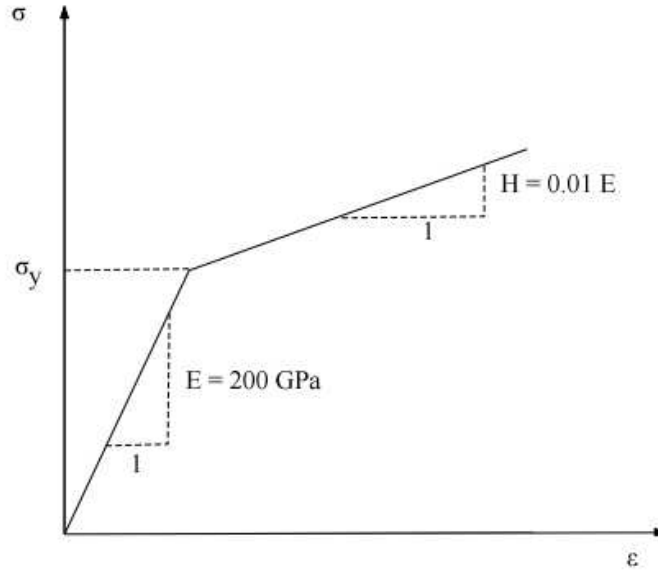


Figure 2.5. Stress-strain curve of elastic-plastic material with kinematic hardening

The thickness of the plate is 1 mm and plane stress conditions are applied. The geometry of edge-cracked specimens are illustrated in Figure 2.6. The height H and the width W are same in all models and their values are 70 mm and 50 mm, respectively. L_o and L_f are the initial and final crack lengths, which are different in each model to examine the effect of crack orientation and crack path on plasticity induced crack closure. Four different geometries are considered, which are consistent with geometries in literature [68]:

- (i) MODEL 1 : Specimen having single edge straight crack with 0.3 mm initial size and 5 mm final length.

- (ii) MODEL 2 : Specimen having single edge slanted crack with 0.3 mm initial size and inclined at an angle of 45 to the direction of applied loading in model I. The fatigue crack is allowed to propagate to a final length of 5 mm without deviating from its path.
- (iii) MODEL 3 : Specimen having single edge slanted crack with 1.1 mm initial and 3.1 mm final size; subsequently deflected and growing perpendicular to the mode I loading. The deflected portion has a length of 5 mm.
- (iv) MODEL 4 : Specimen having single edge slanted crack with 1.1 mm initial size and 8.1 mm final length.

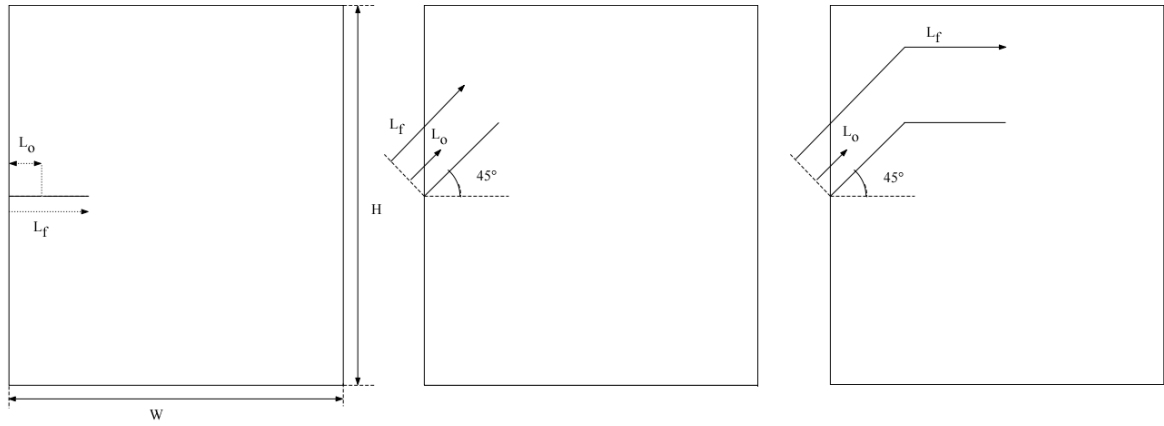


Figure 2.6. 2-D geometry of specimens with (a) straight (b) slanted (c) deflected single-edge cracks

2.1.2. Crack Propagation Analysis and Debond Option in ABAQUS

The software ABAQUS provides the opportunity of modeling quasi-static crack growth in two dimensions (planar or axisymmetric). Two distinct initially bonded surfaces are required to model the potential crack surfaces, between which the crack will propagate. Like in all surface based contact models, a master and a slave surface are determined and their interaction properties are proposed. By defining a specific fracture criteria, initially bonded surfaces are allowed to be separated [69].

In Figure 2.7, a schematic representation of crack tip region is given. The first step is identifying the initially bonded crack part, so that the nodes in the crack propagation path can be recognized by the program. The slave and master surfaces are defined; any

contact formulation except finite-sliding, surface-to-surface formulation can be used. In the present analysis, the sliding formulation is small-sliding and the discretization method is node-to-surface. The unbonded portion of the slave surface will behave as a regular contact surface, which is very important attribute, especially in research of fatigue crack closure effect. The initially bonded portion is also defined along the slave surface; this node set represents the location of current crack tip and crack extension direction. Bonding is allowed only in normal direction and typically used to model bonded contact conditions in Mode I crack problems.

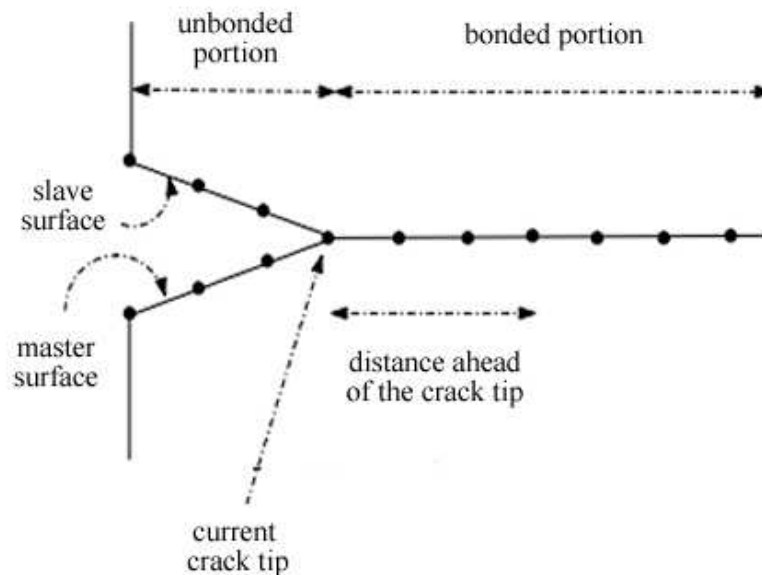


Figure 2.7. Illustration of unbonded and bonded regions around crack tip

The crack propagation from the crack tip is activated through special commands. If it is not activated, the master and slave surfaces will not separate and the contact conditions will have the same effect as that provided by the tied contact properties. After debonding, the traction between two surfaces is initially carried as equal and opposite force at the slave and its corresponding master node. Once a crack-tip node debonds, this traction should be ramped down to zero according a predefined amplitude curve given to the code. This reduction process must be completed in step time of debonding and the tendency may change for different materials. For rate-independent materials, a linear amplitude curve is adequate; whereas for large strain problems and

rate-dependent materials, the stresses are recommended to be decreased slowly. The convergence of the solution is also affected by reversals in plastic flow due to crack propagation; hence very small time increments are required to continue the analysis.

The most important part of the debonding process is specifying a fracture criterion. For 2D models, ABAQUS provides three different criteria :

- (i) Critical Stress Criterion
- (ii) Crack Length versus Time Criterion
- (iii) Critical Crack Opening Displacement Criterion

In critical stress criterion, the normal and shear failure stresses at a specific location must be specified to allow debonding. The other crack propagation model - prescribed crack length versus time - is usually used to verify the results from experiments. For stable crack growth in ductile materials, experimental evidence indicates that the value of the crack opening displacement at a specified distance behind crack tip is usually constant, therefore warning the software about the critical crack opening displacement is another sensible method. In this analysis, COD (Critical Opening Displacement) criterion will be used.

The crack opening displacement criterion is defined as

$$\zeta = \frac{\delta}{\delta_c} \quad (2.7)$$

where δ is the measured value of crack opening displacement and δ_c is the critical value of the crack opening displacement (user-specified). The crack-tip node debonds when the fracture criterion reaches the value 1.0.

The required dimensions are given in Figure 2.8. A distance value equal to the distance behind the crack tip along the slave surface, at which the crack opening displacement is measured, should be determined. The cumulative crack length has to

be also proposed, which is the distance between the initial crack tip and the current crack tip measured along the slave surface in current configuration.

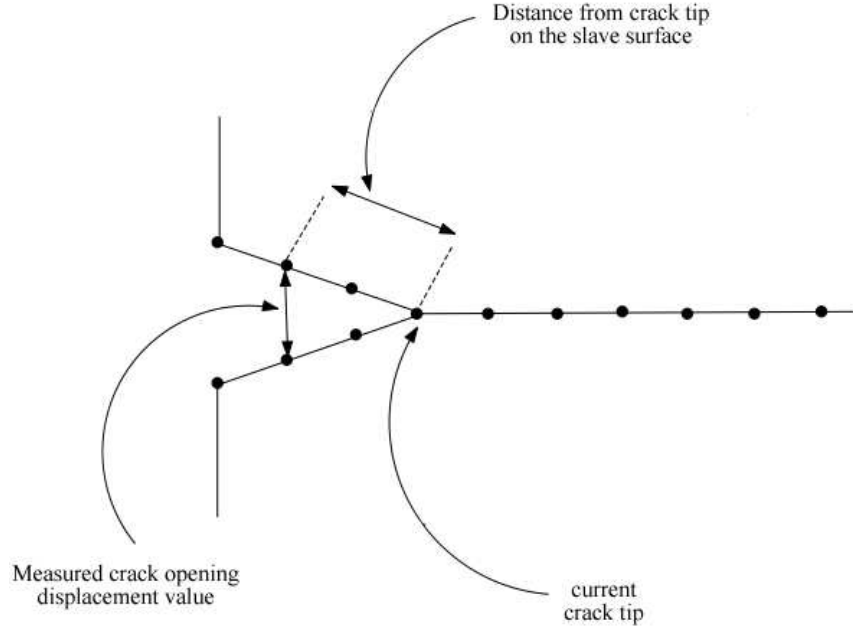


Figure 2.8. Representation of critical crack tip opening displacement criterion

For crack propagation analysis, ABAQUS recommends the usage of first order elements (CPE4, CAX4, CPS4, etc.) As output parameters; the time when bond failure occurred, fraction of stress at bond failure that still remains, all components of stress remained in the failed bond can be demanded. Contour integrals can be requested for two-dimensional crack propagation analysis, too.

2.1.3. Finite Element Mesh

Figure 2.9 shows the geometry, loading and boundary conditions of MODEL 1. The symmetry does not exist in other models, hence entire plate needs to be discretized. Besides, the crack surfaces are not allowed to penetrate through each other during unloading and to accomplish the contact behavior they should be characterized as free surfaces. To use the crack propagation property of ABAQUS, a master and a slave surface are needed to be defined that refer to two distinct parts. These two parts named in Figure plate 1 and 2 are assembled to complete the whole geometry in each

case, where released nodes during fatigue crack propagation are initially bonded.

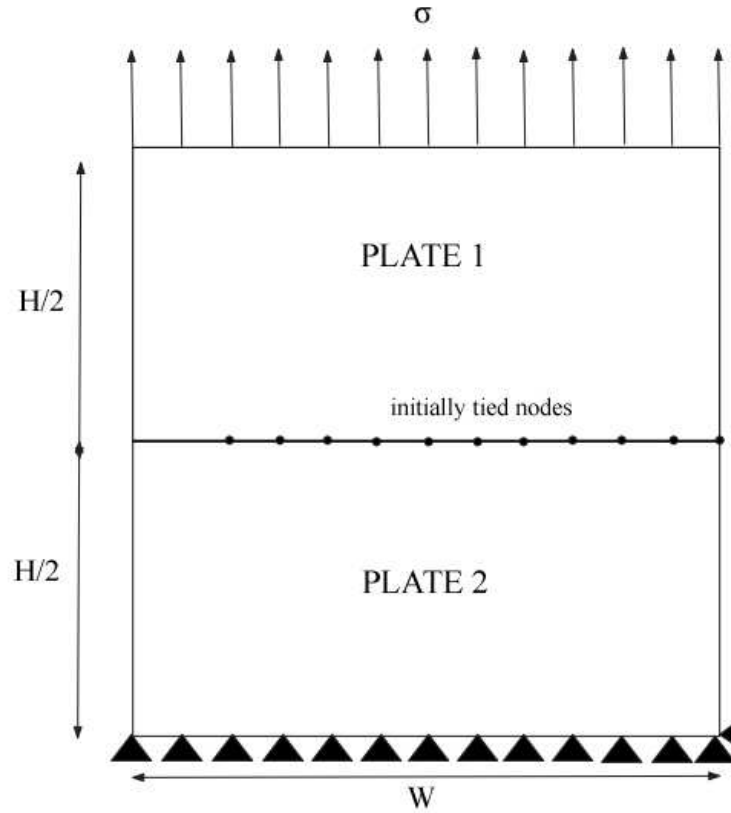


Figure 2.9. Loading and boundary conditions of MODEL 1

Modeling contact interactions between crack faces becomes very important when the mechanical behavior of two distinct parts affects each other. As mentioned in Section 2.1.2, surfacebased contact elements are used to define the deformable contacting surfaces and small sliding is allowed to capture the motion between crack faces. Friction coefficient μ of 0.1 is selected for mixed mode cracks; for MODEL 1 frictionless tangential properties are defined. In addition, hard normal contact between contact surfaces is recommended.

Displacement boundary conditions, that restrict the vertical displacement of bottom surface nodes and rigid body motion of specimens, are set in each case (see Figure 2.9).

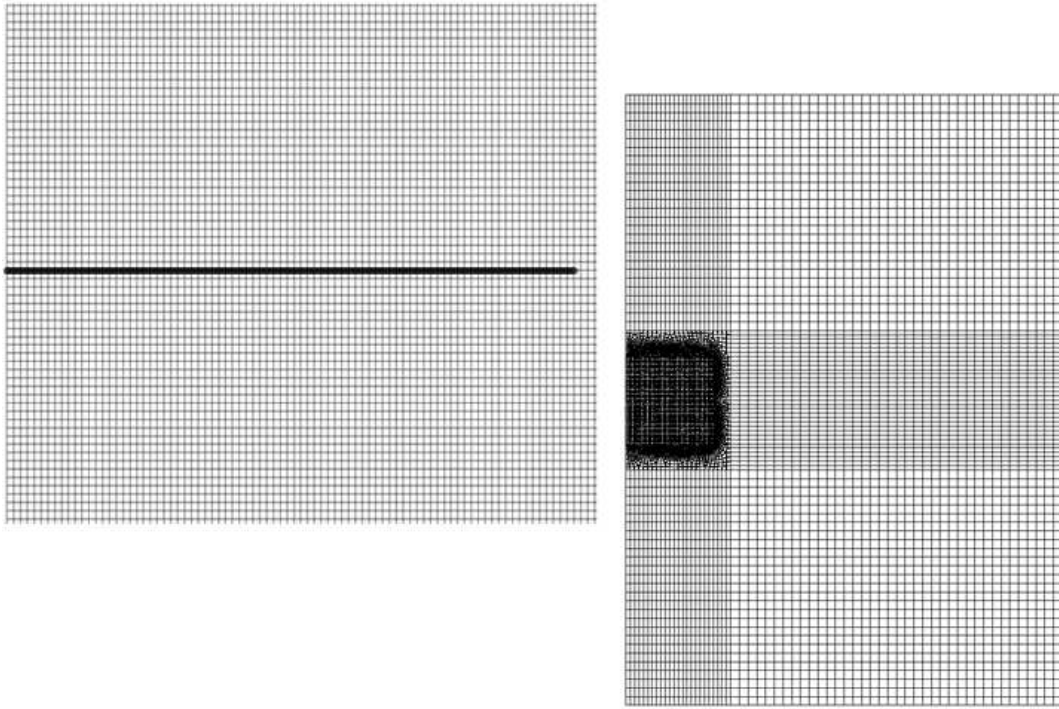
In Figures 2.10 and 2.11, the meshed plate and the near tip finite element meshes for four models are shown. Each model is divided into regions and all regions have different mesh density. For abundant usage of software's debonding ability, four-node quadratic plane stress quadrilateral elements (CPS4) have been used to discretize the geometries and uniform structural meshes are defined for assembly. Since reversed plasticity during unloading affects the fatigue crack closure considerably, the mesh is refined in crack tip region to capture plastic deformations and both forward and reversed plastic zones. Regarding only the plasticity doesn't meet the requirements for mesh sufficiency, the mesh should be refined also to cover the contact mechanics correctly. However, there is no quantitative criteria for determining required mesh size for contact modeling, therefore the crack tip region has been discretized according to mesh sufficiency criteria of plasticity and then validated for interaction properties of crack faces through several simulations. The mesh sufficiency criteria adapted from literature [70] is :

$$\Delta a/r_p \leq 0.10 \quad \text{for} \quad R = 0 \quad (2.8)$$

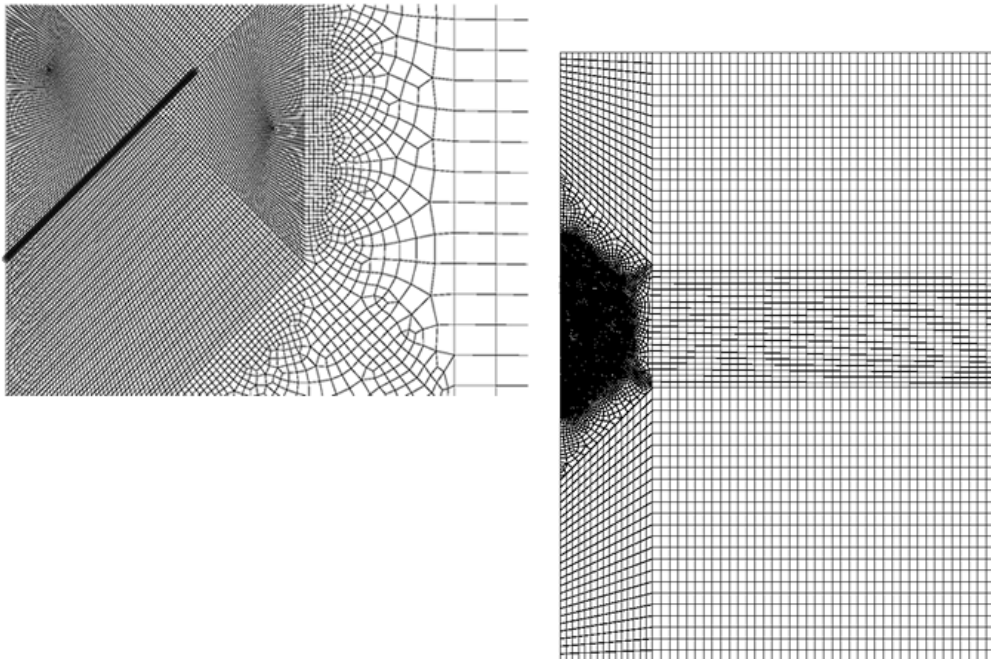
$$\Delta a/r_p \leq 0.15 \quad \text{for} \quad R = -1 \quad (2.9)$$

if four-node quadrilateral elements have been used. R is the load ratio of minimum and maximum applied stresses, r_p is the radius of the plastic zone around crack tip and Δa is recommended minimum element length. Another difficulty faced in this stage is determination of the plastic zone size. The results for plastic zone sizes are given in Table 2.1, in which the values are calculated according to three different approaches (Irwin, Modified Irwin and Strip-Yield Model). Selecting an element size of ~ 0.1 mm around crack tip meets the requirements of the plasticity mesh-sufficiency criterion for all models.

In addition to mesh sufficiency around crack tip, the whole discretization of all models are tested. For this purpose, stationary models of four geometries are prepared. The final crack lengths of fatigue crack propagation are assumed to be the crack lengths of these stationary models and same mesh densities are implemented. Since analyt-

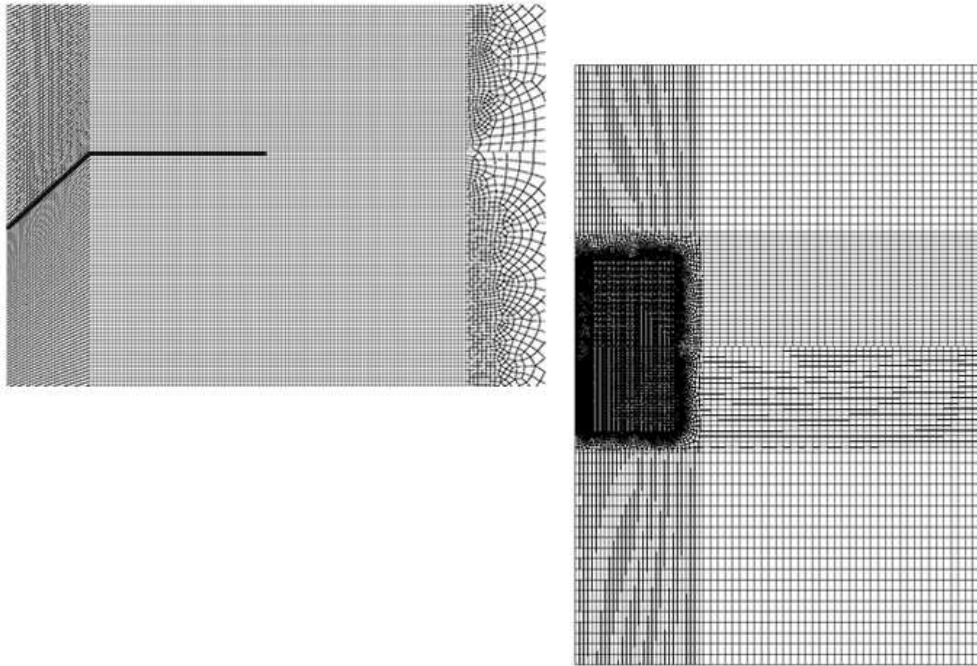


(a) MODEL 1

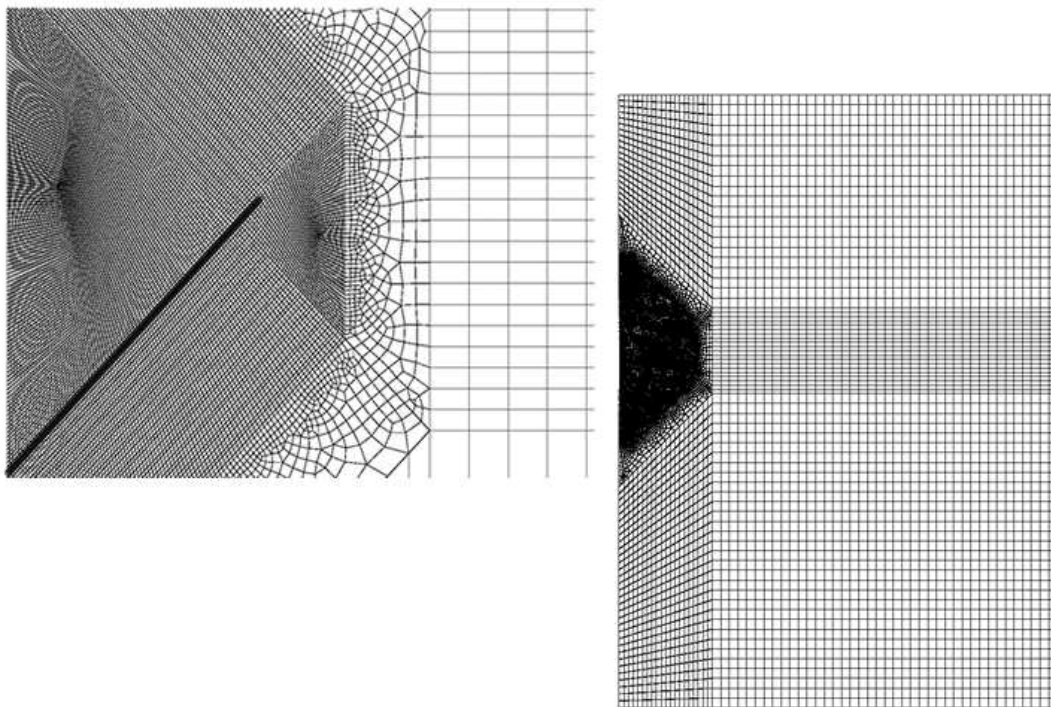


(b) MODEL 2

Figure 2.10. Descritization of plates and near crack tip region mesh densities



(a) MODEL 3



(b) MODEL 4

Figure 2.11. Descritization of plates and near crack tip region mesh densities

Table 2.1. Plastic zone sizes and recommended minimum element lengths

	r_p [mm]	Δa [mm]	
		$R = -1$	$R = 0$
Irwin Approach	2.259357	0.338904	0.225935
Modified Irwin	4.518714	0.677807	0.451871
Strip-Yield Model	4.759498	0.713925	0.475950

ical solutions for stress intensities of elastic materials are available in literature, the selected material is pure elastic with an elastic modulus E of 200 GPa and μ of 0.33. The contour independent J-integral and stress intensities K_I and K_{II} are requested and compared with analytical results. These results are listed in Table 2.2. The analytical solution methods can be arranged in three different groups :

(i) Pure Mode I cracks : For single-edge cracked specimen with finite dimensions, K_{II} is assumed to be zero and K_I is given as [62]:

$$K_I = f(a/W)\sigma_{app}\sqrt{W} \quad (2.10)$$

$$f(a/W) = \sqrt{\frac{2 \tan\left(\frac{\pi a}{2W}\right)}{\cos\left(\frac{\pi a}{2W}\right)}} \left(0.752 + 2.02\frac{a}{W} + 0.37 \left(1 - \sin\left(\frac{\pi a}{2W}\right)\right)^3\right) \quad (2.11)$$

(ii) Slanted cracks : According to Fett [71], the mixed-mode stress intensity factors for a single edged obliqued crack in tension can be calculated as :

$$K_I = \int_0^a \sigma_n(x) h_I(x, a) dx \quad (2.12)$$

$$K_{II} = \int_0^a \tau(x) h_{II}(x, a) dx \quad (2.13)$$

where σ_n and τ are normal and shear component in the plane of crack and

$$h_I = \sqrt{\frac{2}{\pi a}} \left(\frac{1}{\sqrt{1-x/a}} + A\sqrt{1-x/A} \right) \quad (2.14)$$

$$h_{II} = \sqrt{\frac{2}{\pi a}} \left(\frac{1}{\sqrt{1-x/a}} + B\sqrt{1-x/A} \right) \quad (2.15)$$

$$A = 0.3515 + 0.2851(a/W) + 0.1723(a/W)^2 + 1.212(a/W)^3 \quad (2.16)$$

$$B = 0.04362 - 0.0883(a/W) - 0.8302(a/W)^2 + 0.19617(a/W)^3 \quad (2.17)$$

(iii) Deflected cracks : There is no analytical solution for deflected cracks, instead the displacement field around crack tip is calculated numerically and fitted to a curve according to formulation

$$\begin{aligned} u_y = & \frac{K_I}{2G} \left(\frac{r}{2\pi} \right)^{1/2} \sin \frac{\theta}{2} \left(\frac{3-v}{1+v} - \cos \theta \right) - \frac{Tv}{2\mu(1+v)} r \sin \theta \\ & + \frac{K_{II}}{4G} \left(\frac{r}{2\pi} \right)^{1/2} \left(\frac{5v-3}{1+v} \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right) + A_1 r \cos \theta + u_{0y} \end{aligned} \quad (2.18)$$

by using least-square method [72]. The stress intensities for elastic materials can be derived with this method.

By comparing the analytical and numerical results, it is decided that the mesh densities are sufficient for investigation of fatigue crack propagation. Furthermore, by adding kinematic hardening to the elastic material model and carrying out the same analysis with same geometries and meshes, the difference between contour dependent J-integrals of plastically deformed material and in J-integrals of pure elastic material can be recognized (see Figure 2.12).

Table 2.2. Verification of meshes by comparison of numerical and analytical stress intensity results

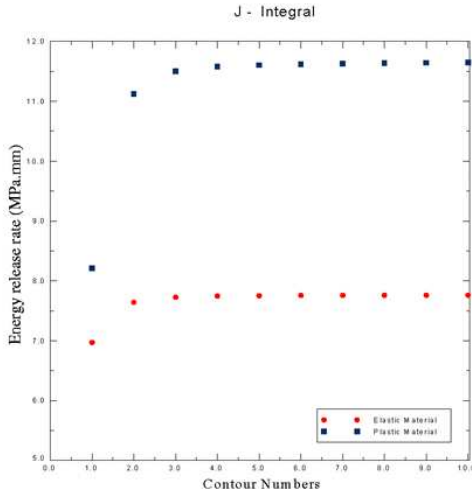
CONTOUR	MODEL 1		MODEL 2		MODEL 3		MODEL 4	
	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}
1	1738.13	-1.5262	1044.96	561.643	2217.28	-18.153	1435.84	732.371
2	1806.56	-1.3936	1101.03	571.652	2297.13	-6.6074	1488.30	759.696
3	1810.14	-1.3993	1105.69	572.826	2301.81	-6.7762	1491.06	761.482
4	1810.85	-1.4000	1106.59	572.834	2302.70	-6.7642	1491.57	761.765
5	1811.11	-1.4003	1106.90	572.846	2303.02	-6.7642	1491.74	761.885
6	1811.24	-1.4004	1107.06	572.864	2303.18	-6.7643	1491.82	761.944
7	1811.31	-1.4004	1107.15	572.896	2303.27	-6.7643	1491.86	761.979
8	1811.35	-1.4005	1107.21	572.961	2303.32	-6.7645	1491.88	762.000
9	1811.38	-1.4005	1107.25	573.133	2303.36	-6.7651	1491.90	762.015
10	1811.40	-1.4005	1107.27	573.710	2303.38	-6.7656	1491.91	762.026
FEA [MPa $\sqrt{mm}$]	1810.14	-1.3993	1105.69	572.826	2301.81	-6.7762	1491.06	761.482
ANALYTICAL	1808.52	0.0000	1103.39	575.285	2324.10	-11.400	1491.55	763.211
ERROR [%]	0.0895	100	0.2084	0.4274	0.9591	40.56	0.0332	0.2266

2.1.4. Crack Advance Scheme and Crack Face Interaction in a Fatigue Cycle

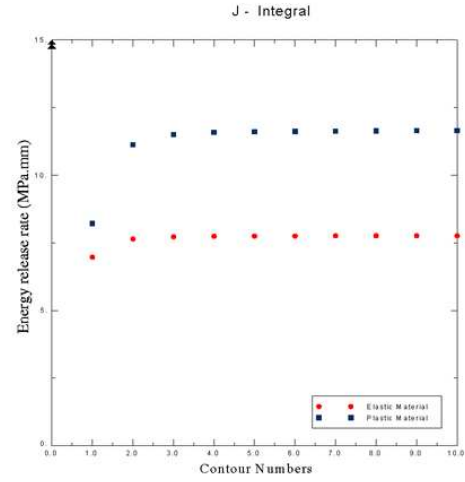
In the physical situation, the crack advances in continuous series of infinitesimally small length increments, which can be coarsely simulated. The accuracy of crack propagation is limited to the minimum element length around crack tip. In each cycle crack tip node has been released and the crack has been extended by one element size. To model the fatigue crack closure, several node release methods are tested :

- Node release at maximum load and redistribution of the loads before beginning the unloading [73, 74, 75, 76]
- Node release at minimum load [77, 78]
- Node release immediately after the maximum load during the first increment of unloading [79]

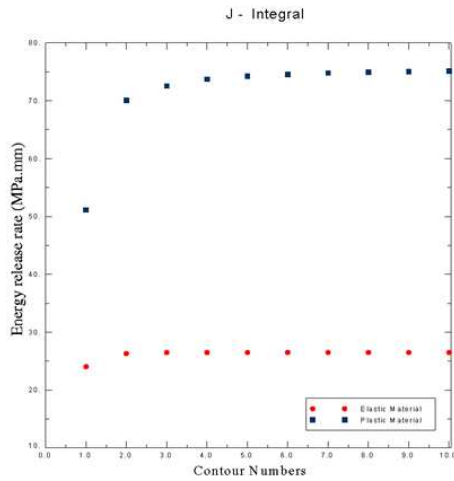
According McClung and Sehitoglu [70], all possible schemes give similar results; however due to its general efficiency and consistent performance the nodes are debonded



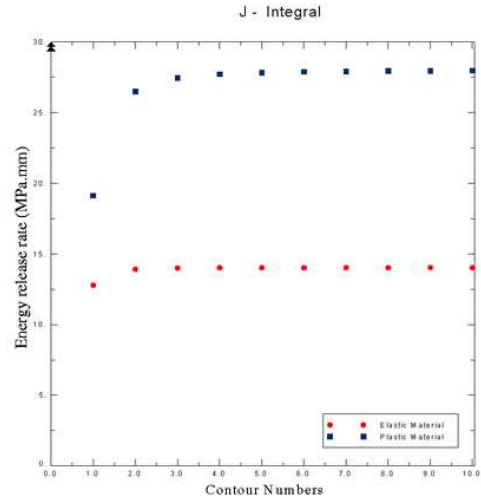
(a) MODEL 1



(b) MODEL 2



(c) MODEL 3



(d) MODEL 4

Figure 2.12. J-Integral results for pure elastic and elastic-plastic materials

at a separate step between loading and unloading steps in our case.

Another important issue in the analysis is the interaction of crack faces during unloading and reloading. In Figure 2.13, the relative behavior of crack faces in a fatigue cycle is drawn, where beside mode I fracture (opening), mode II fracture (shear offset) is also considered. After the specimen is loaded with maximum stress σ_{max} and maximum crack opening displacement is measured, the crack tip node will be released and the plate begins to be unloaded. σ_{cl} is a critical stress during unloading, below which

the crack closure begins since the crack openings at first and second nodes behind the crack tip become zero. By further unloading the specimen, the crack gradually starts to close and the upper crack face slides relative to the lower crack face until the maximum compressive load σ_{min} . When the loading conditions change from compression to tension, first the mode II displacement ΔU_{II} is observed to be zero and complete sticking condition occurs until a specific load. By further loading the specimen, the upper crack slides relative to the lower crack again, but in the opposite direction. σ_{op}^{II} is a mode II opening level, where complete slip initiates. From this point on, the crack starts to open in mode I until the mode I opening level σ_{op}^I , where the compressive stresses behind the crack tip become tensile.

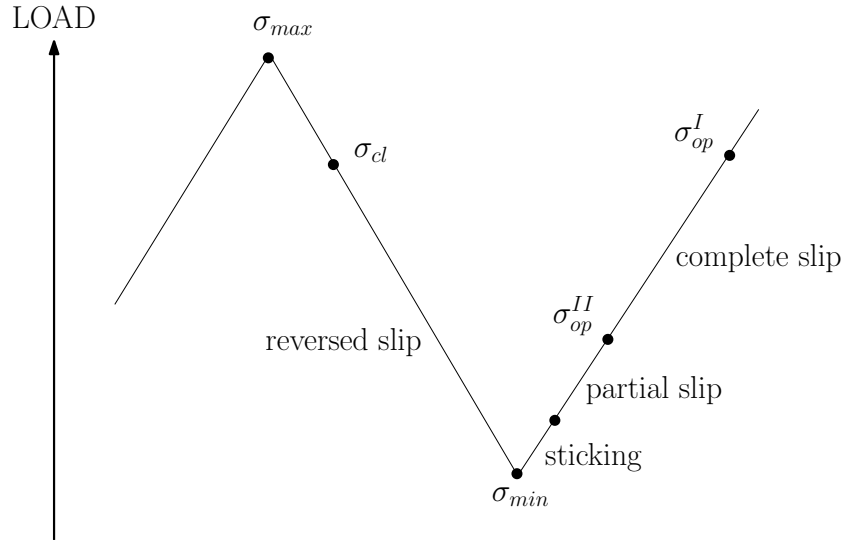


Figure 2.13. Crack face interaction in a fatigue cycle

2.1.5. Finite Elements Results

In this study, the fatigue fracture behavior of single-edge cracked elastic-plastic specimens are examined. In order to be consistent in comparing cracks with different orientation angles, initial and final lengths, all plates are subjected to the same maximum tensile load of $0.8\sigma_y = 384$ MPa and load ratio $R = -1$. The loading path and critical stresses on this path are already given in Figure 2.13. Since geometries with slanted cracks describe the crack face interaction during a fatigue cycle more generally, the crack region displacements of MODEL 2 on different applied stresses and related

crack opening levels are demonstrated in Figure 2.15. The corresponding mode I and mode II opening levels ($\sigma_{op}^I, \sigma_{op}^{II}$) and beginning stress of crack closure σ_{cl} are plotted in Figure 2.14, where stress levels are normalized by maximum applied stress in the cycle and given as a function of fatigue crack distances normalized by initial crack length L_0 . As the crack length increases, mode I opening levels will increase and mode II opening levels will decrease for a 45° slanted crack. There is no significant change in values of σ_{cl} . By carefully examining the Figure, it can be noted that crack opens in mode I before slip reaches the crack tip during initial few cycles ($\sigma_{op}^{II} > \sigma_{op}^I$) and monotonic rise in mode II opening levels are observed. This is due to fact that the crack lengths are too small to cause substantial forward plastic deformation and mode II opening levels are primarily affected by mode I opening levels [68]. For larger cracks, mode I and mode II opening levels are no longer equal; mode II levels continuously drop until reaching a steady state.

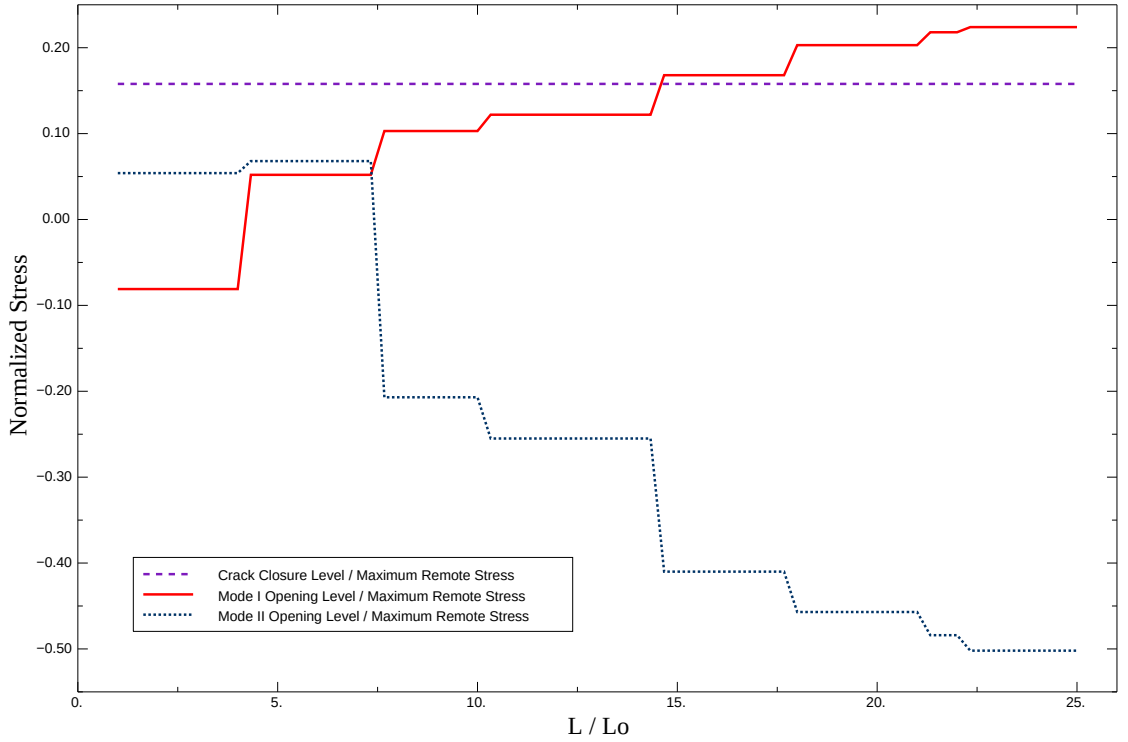


Figure 2.14. Normalized mode I and mode II opening levels for 45° slanted crack (MODEL 2) for $R = -1$ and $\sigma_{max} = 384$ MPa

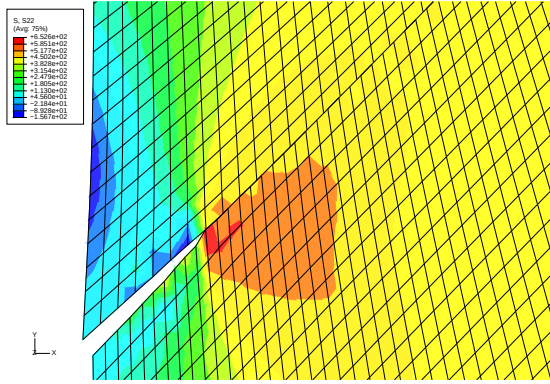
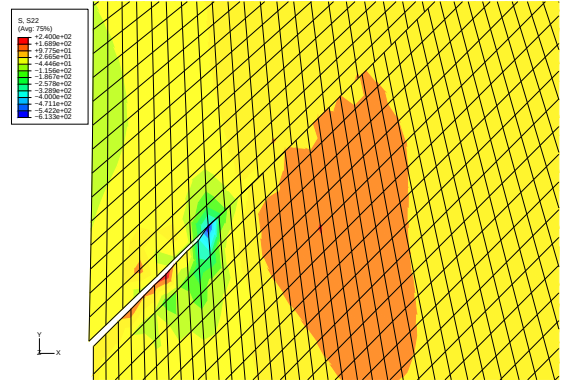
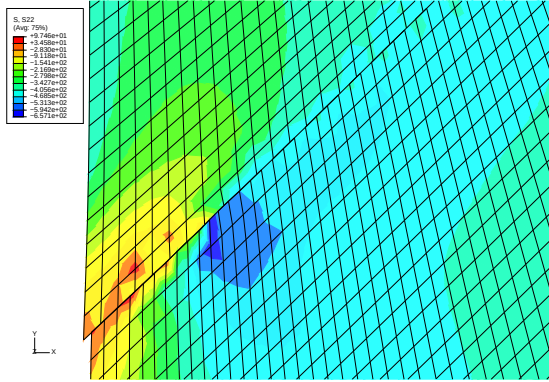
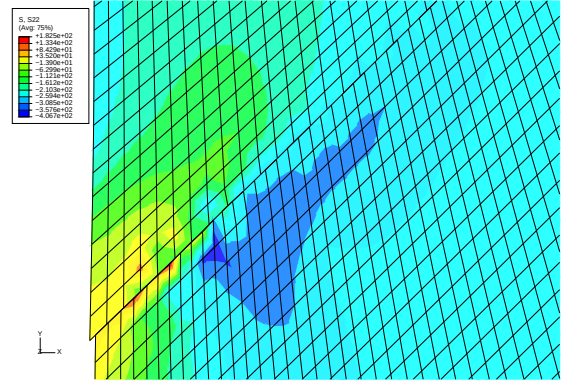
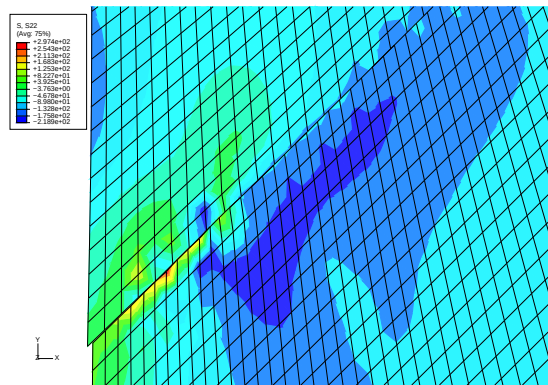
(a) at $\sigma_{app} = \sigma_{max}$ (b) at $\sigma_{app} = \sigma_{cl}$ (c) at $\sigma_{app} = \sigma_{min}$ (d) at $\sigma_{app} = \sigma_{op}^{II}$ (e) at $\sigma_{app} = \sigma_{op}^I$

Figure 2.15. Crack face interaction of MODEL 2 at different stress levels during fatigue cycle ($R = -1$, $\sigma_{max} = 384$ MPa)

The crack orientation has a crucial effect in crack opening levels. It is expected that the mode I crack opening level of a slanted crack is higher than the level of a straight crack with the same length. To understand this phenomenon, the mode I crack opening displacements of four models are calculated when the applied tensile load is maximum and cracks have reached at their final lengths (see Figure 2.16). The crack opening displacements are plotted with respect to the distance behind the crack tip, which is parallel to the crack growth direction. It can be deduced that the crack opening displacements of straight fatigue cracks are greater than the COD's of inclined fatigue cracks of the same length; since the applied load only contributes mode I opening behavior in MODEL 1 and MODEL 3 while a fraction of load is used in mode II shear offset of slanted cracks. To ensure the correctness of FEA solution, the results for COD's in literature are also put in another figure and compared with recent ones (see Figure 2.17).

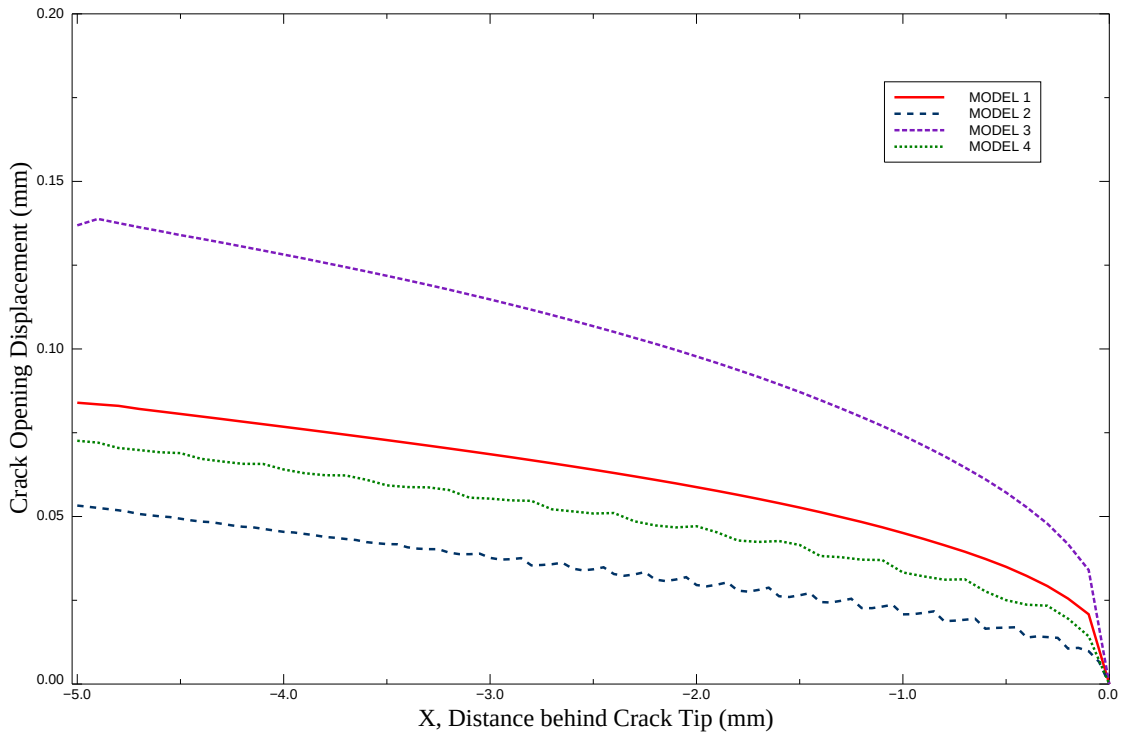


Figure 2.16. COD's of all models under maximum load at final fatigue crack length

The effect of plasticity induced fatigue crack closure (PICC) can be seen in Figures 2.18 and 2.19, where the COD's of fatigue cracks and stationary cracks without any residual deformations in their wakes are compared. Crack opening event will occur only

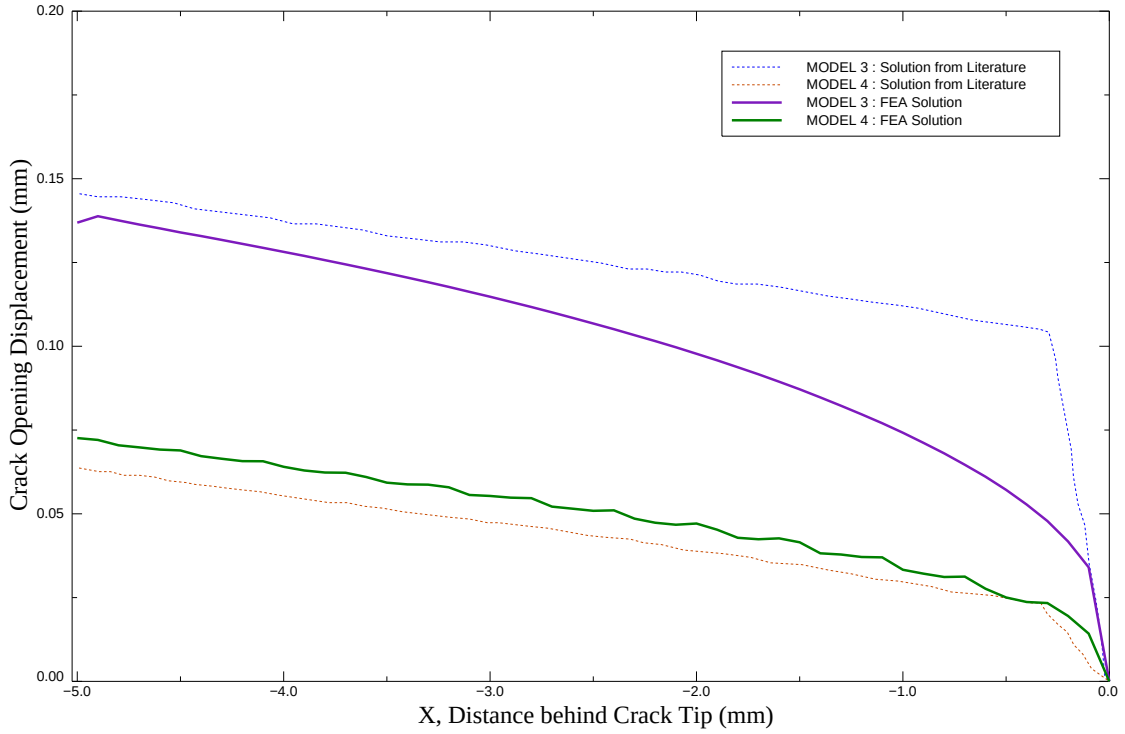


Figure 2.17. COD's of MODEL 3 and MODEL 4

if crack opening displacement near the crack tip exceeds the residual deformation, which is the difference in displacements between fatigue and stationary crack. To visualize this situation, equivalent plastic strain at integration points are drawn for both stationary and fatigue cases of MODEL 1 and 3 in Figure 2.20. First impression got from the figure is that the plastic strains are adding up during fatigue crack propagation and maximum equivalent plastic strain in the fatigue crack zone is more than twice that of the stationary crack, in both models. Besides, the size and shape of plastically deformed zones are completely different. This difference affects the maximum crack opening distances, so that COD of a stationary crack will be greater than that in a fatigue crack propagation process under the same maximum loading conditions. Another important issue related with the crack opening levels is the ratio of residual deformation and crack opening displacement of a stationary crack. This ratio is around 0.29 for MODEL 3, which is smaller than the fraction of MODEL 4 having the value of 0.32, meaning that the opening levels for straight crack will be lower as expected.

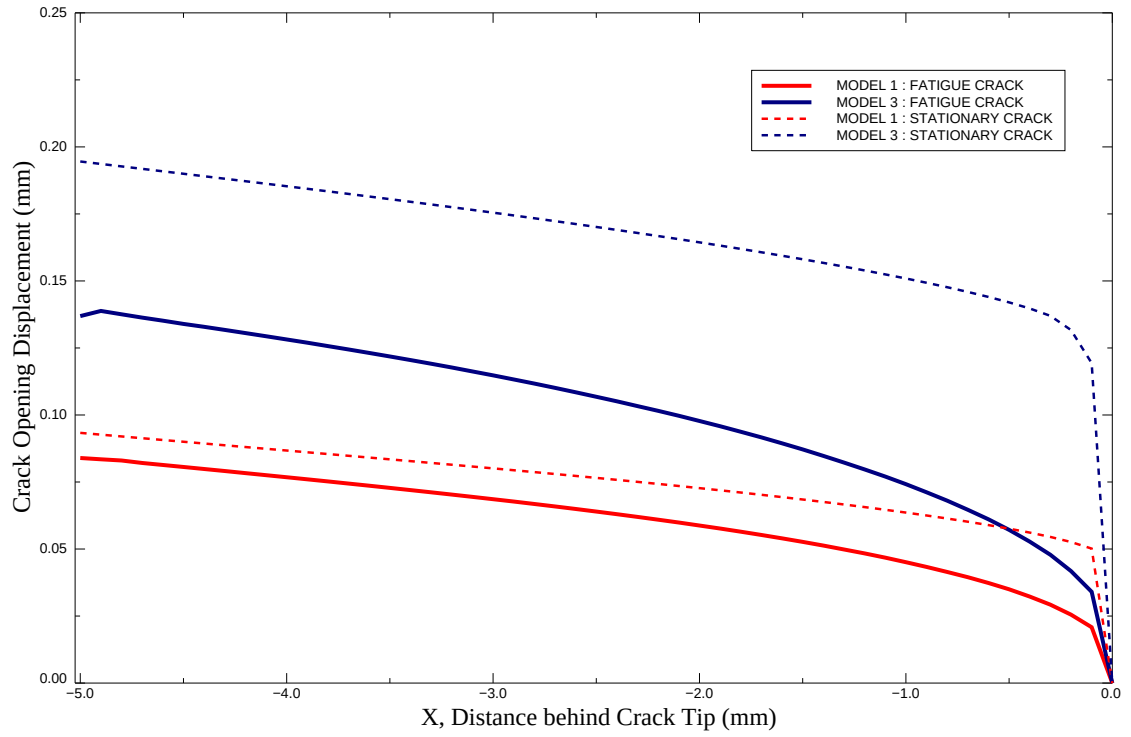


Figure 2.18. COD of fatigue crack versus COD of stationary crack (MODEL 1 and MODEL 3)

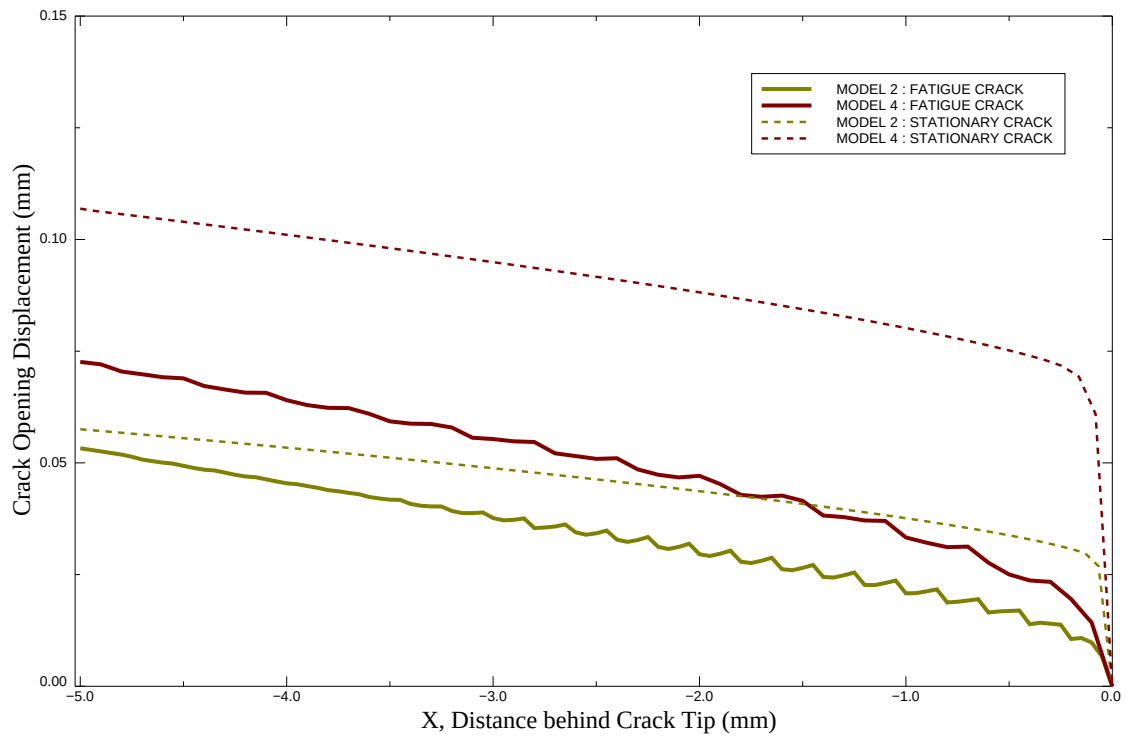


Figure 2.19. COD of fatigue crack versus COD of stationary crack (MODEL 2 and MODEL 4)

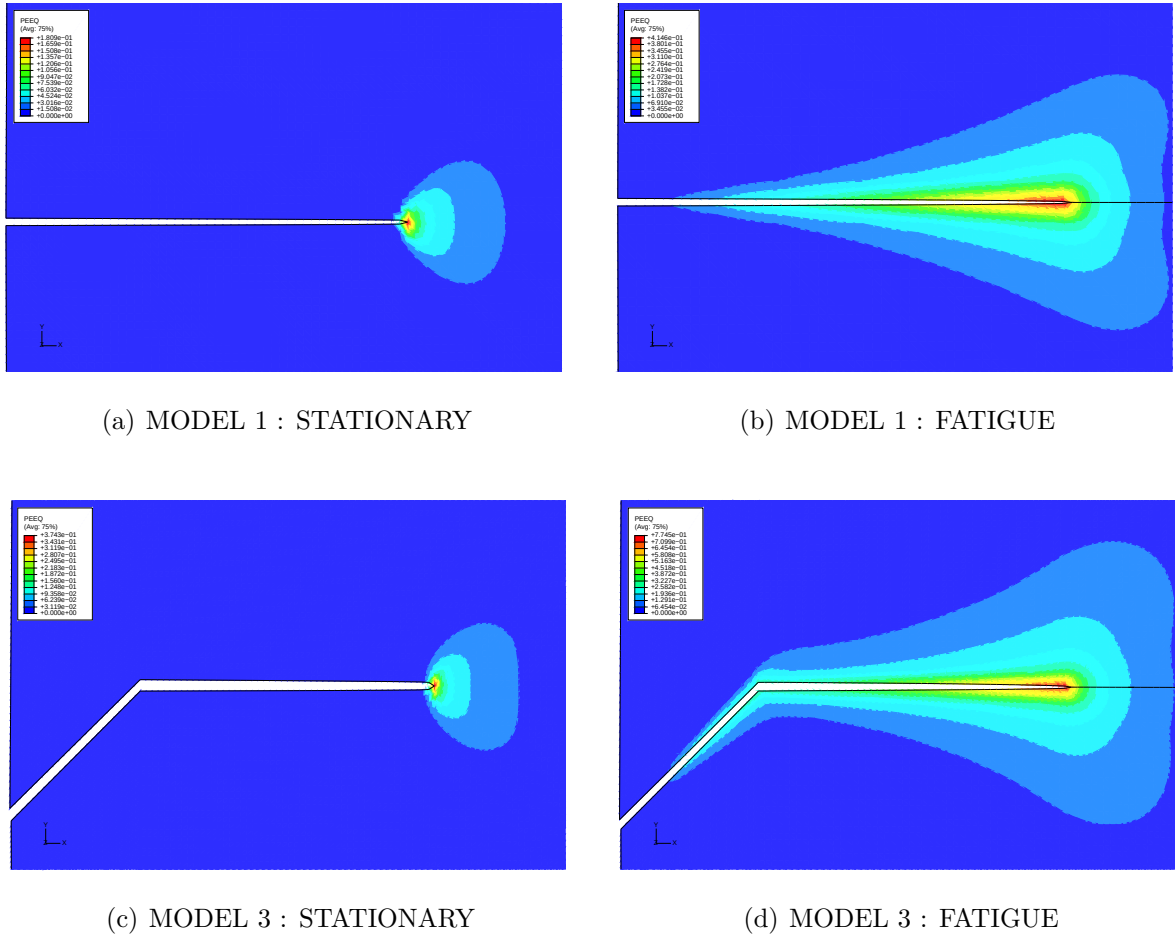


Figure 2.20. Equivalent plastic strain at integration points at final crack length and under maximum load

2.2. Crack Tip Opening Displacement Analysis of Elastic-Plastic Plate

There are lots of node release schemes suggested in fatigue crack propagation studies. As discussed before, one of them was picked up in order to interpret the physical situation numerically. The chosen fracture criterion was the excess of critical opening displacement and the relation between selected node release scheme and fracture criterion has not yet been discussed.

In analysis, MODEL 1 will be used. The maximum vertical displacements between crack faces at a distance of 0.1 and 0.2 mm from crack tip are measured and they are plotted in terms of crack length a . As a control parameter the maximum COD at the plate edge is also added to the figure. Results are demonstrated in Figure 2.21.

From this figure it can be deduced that the crack opening will increase linearly, when the crack propagates. Moving away from crack tip and raising the parameter a are disturbing the linear relationship between crack opening displacement and crack length. Therefore, the linear behavior of critical crack opening displacement is allowed to be used as a fracture function submitted to the finite element code, only if the vertical displacements are measured close to the crack tip. Besides, there exists a requirement of an analytical solution for CTOD, before running the code.

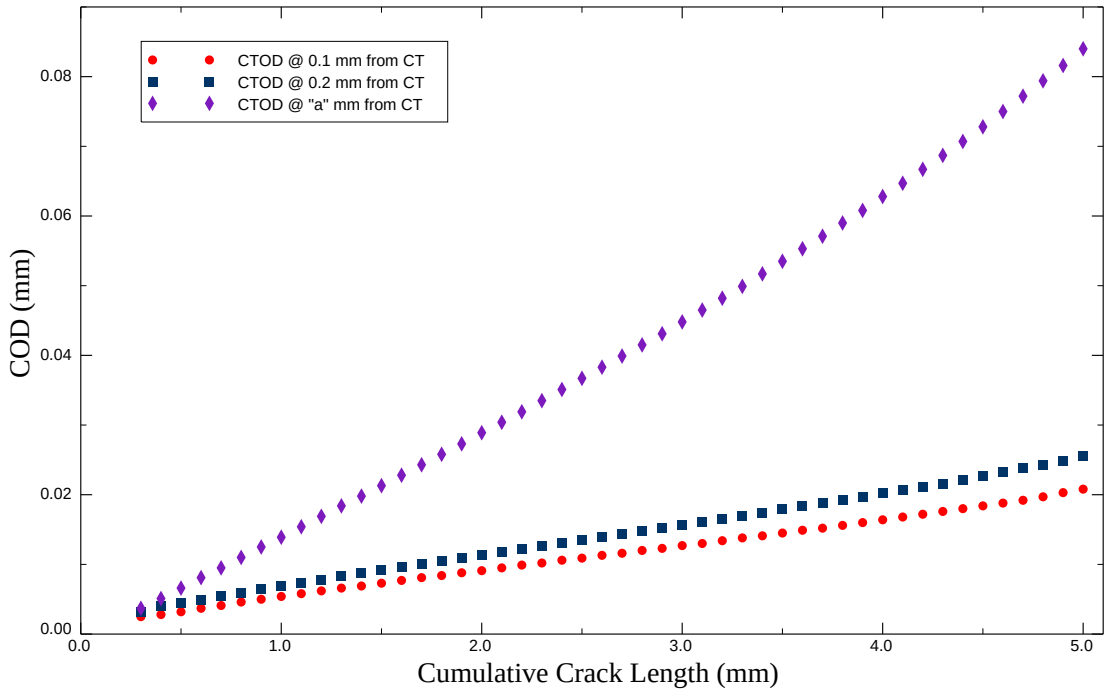


Figure 2.21. Crack openings during fatigue crack propagation

In analytical solution of CTOD for elastic-plastic material, results from EPFM approach will be utilized. CTOD can be defined as the crack opening displacement at the end of the strip-yield zone and according to this definition it is given in terms of Mode I stress intensity K_I as [62] :

$$\delta_t = \frac{8\sigma_y a}{\pi E} \ln \sec \left(\frac{\pi \sigma}{2\sigma_y} \right) \cong \frac{K_I^2}{E\sigma_y} \quad (2.19)$$

The ratio of numerically determined COD's over results calculated by Equation 2.19 is plotted in terms of crack length (see Figure 2.22). The curves emphasize the fact that the ratios converge to a certain value during fatigue crack propagation. Thus, CTOD

may be calculated by small modification of EPFM formulation, if initial crack length is larger than a threshold value. The modified version of the equation is :

$$\delta_t = d_m \frac{K_I^2}{E\sigma_y} \quad (2.20)$$

where d_m is constant depending on the type of material plasticity.

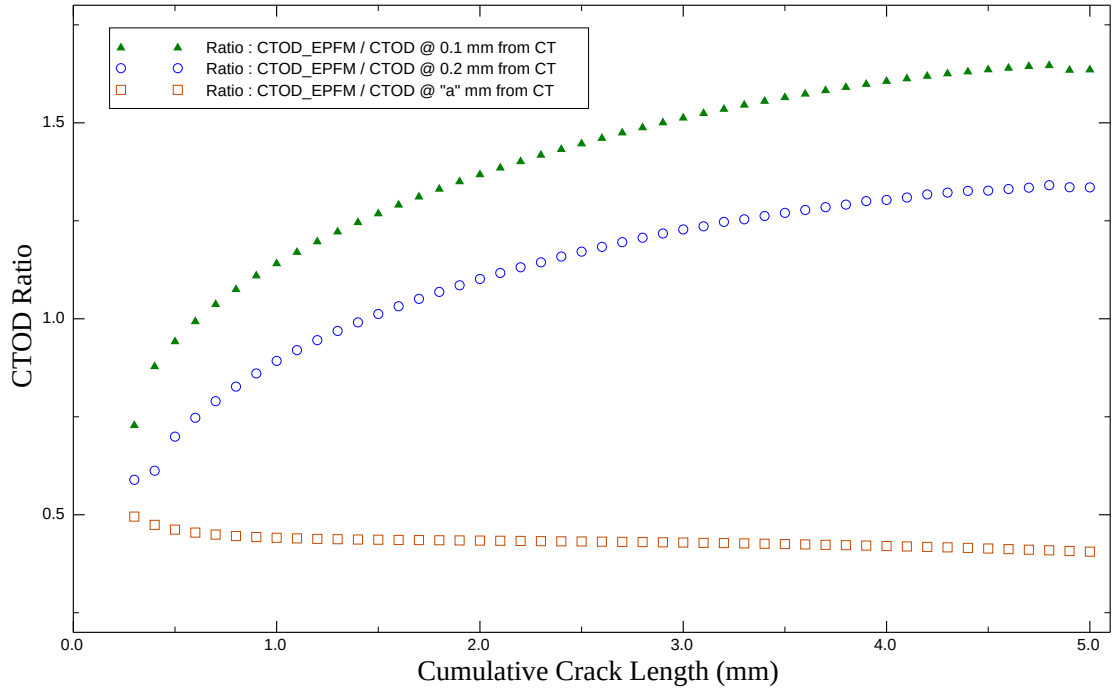


Figure 2.22. CTOD : Analytical solution vs. FEA results

3. FRACTURE MECHANICS OF SHAPE MEMORY ALLOYS

One of the main objectives of this study is to analyze phase-transformation induced crack closure of shape memory alloys, which is directly related with the service life of these materials. Shape memory alloys are manufactured through powder metallurgy and sintering processes, where it is inevitable to completely remove residual porosity and this porosity leads to microcracks in alloys. The main reason of material failure is low strength and fracture resistance due to these microcracks available in shape memory alloys. Hence it will be useful to analyze first the fracture behavior of a single-edge cracked shape memory alloy plate under static loading before working on fatigue crack propagation schemes.

3.1. Abaqus/Umat Subroutine for Superelasticity of Shape Memory Alloys

For solving problems using finite elements, the required material model should be introduced to the software. Auricchio-Taylor-Lubliner constitutive model for shape memory alloys described in Section 1.7 has been implemented to ABAQUS by Hibbitt, Karlsson and Sorensen through an user defined material subroutine [80]. This subroutine describes the superelastic behavior of the shape memory alloy. The material is assumed to be isotropic [81] and only transformations between austenite and single-variant martensite phases are considered. As stated in Section 1.5.2, the total strain is taken as the sum of elastic and transformation strains. Allowed transformation strain is in the order of 6-7 %, however the elastic strain is limited to a maximum of 2 % [80].

Uniaxial superelastic behavior as described by ABAQUS is shown in Figure 3.1. Because start and finish stresses of forward and reverse transformations are temperature-dependent, a stress-temperature diagram is also added to the figure. The subroutine requires 15 input parameters and the definition of these parameters are tabulated in Table 3.1. Most of these parameters were described in previous chapters in

detail, other important parameters are the reference temperature T_0 , at which uniaxial tensile test is carried out, and number of annealing. Since the constitutive behavior in tension and compression may not be symmetric, start of transformation stress during loading in compression can be given as a separate value. Linear elastic strains, transformation strains, equivalent transformation strain, volumetric transformation strain, fraction of martensite, linear elastic strain energy values and transformation strain energy values can be requested as output parameters.

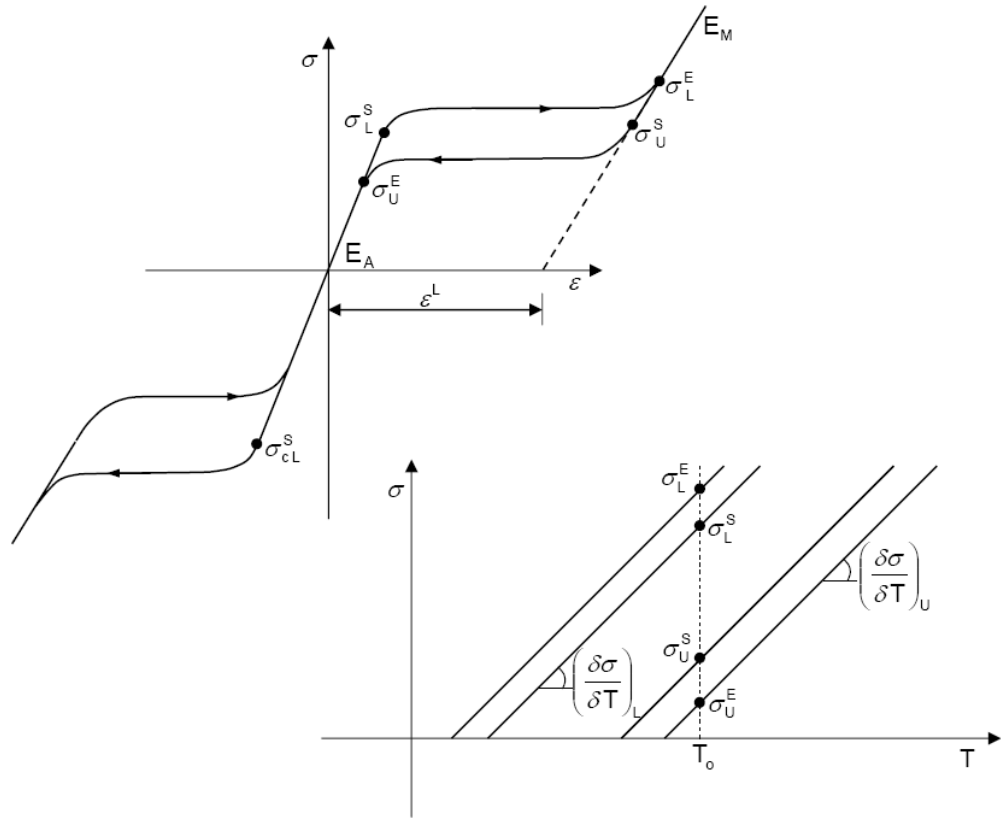


Figure 3.1. ABAQUS UMAT/VUMAT uniaxial stress-strain and stress-temperature relationships [80]

Table 3.1. Definition of input parameters used by superelastic material model

E_A	Austenite Elastic Modulus
E_M	Martensite Elastic Modulus
ν_A	Austenite Poissons ratio
ν_M	Martensite Poissons ratio
σ_L^S	Start of transformation (loading)
σ_L^E	End of transformation (loading)
σ_U^S	Start of transformation (unloading)
σ_U^F	End of transformation (unloading)
σ_{CL}^S	Start of transformation stress during loading in compression
T_0	Reference temperature
$(\frac{\delta\sigma}{\delta T})_L$	$\delta\sigma/\delta T$ loading
$(\frac{\delta\sigma}{\delta T})_U$	$\delta\sigma/\delta T$ unloading
ε^L	Transformation strain
ε_v^L	Volumetric transformation strain
N_A	Number of annealing to be performed during analysis

3.2. Previous Studies on Fracture and Fatigue Failure of Shape Memory Alloys

There are some experimental, analytical and numerical studies related with fracture of shape memory alloys in literature. Since Nitinol is favorable for the production of endovascular stents, majority of prior researches is focused on determination of fatigue properties of wire and stent-like samples. However, fatigue crack propagation data for shape memory alloys adopted from these studies are limited. In 1999, Holtz et al. [82] worked on fatigue threshold behavior of nickel-titanium shape memory alloy and stated that the presence of crack closure in stress induced martensite regime cannot be ruled out. McKelvey and Ritchie [83] studied fatigue crack propagation behavior of Nitinol and investigated the effect of stress induced martensite transformation on crack growth resistance. Fatigue crack growth was found to increase with decreasing temperature, so that fatigue thresholds are estimated higher in martensite when compared to stable austenite. Hornbogen [84] worked on effects of martensitic transformation on fatigue resistance and claimed that fatigue induced defects will modify the temperature dependence of the transformation cycles as well as the hysteresis

loop of the stress-strain curves. Chen et al. [85] stated that crack propagates always in the direction of the maximum normal stress after performing mechanical tests on Nitinol specimens. Robertson and Ritchie [13] focused on characterizing subcritical and critical crack growth behavior in thin-walled nitinol tubing and found the fracture toughness and crack-resistance properties with a dependence on crack-growth angle. The toughness of tubes is found to be lower than that of bulk Nitinol.

The fracture toughening mechanism of shape memory alloys are studied analytically and numerically by several researchers. Stam and Giessen [86] explored that shear strains play an important role in transformation processes and the toughness of shape memory alloys will increase if the phase transformation is irreversible. Yi and Gao [87] stated that the martensite transformation reduces the crack tip stress intensity factor and increases the toughness. They analyzed the shape and size of the transformation zones for stationary and steady advanced cracks under mode I loading and found the great effect of temperature rise in reduction of crack tip stress intensity factor. In 2007, Wang [88, 89] compared a Ni-Ti alloy, undergoing a stress-induced martensite transformation, to a fully transformed martensite NiTi alloy. He stated that as the applied load increases, first partially and fully martensitic zones appear around the crack tip and a permanent deformation occurs in the fully martensitic zone afterwards. This plastic deformation due to phase transformation increases the fracture load and improves the toughness. Not all researchers have compromised in the claim that fracture toughness increases by phase transformation. Yan et al. [90] concluded that transformation volume contraction observed in SMAs increases the effective stress intensity near an advancing crack tip to over 10 % and reduces the toughness. Xiong and Liu [91] gave an analytical solution for increase in crack tip stress intensity factor of Ni-Mn-Ga shape memory alloys and stated that volume change during martensitic transformation is composition dependent, and it can be positive or negative. Freed et al. [92] used a cohesive zone model to simulate crack growth using finite elements and found that the reversibility of the transformation can reduce toughness of the alloy. In literature, a dearth exists in experimentally measurement of the fracture toughness K_{Ic} . Few details about material and testing procedures are given in the study of He et al. [93]. Wang et. al [94] obtained the fracture toughness by doing experiments on

Table 3.2. Material properties of heat treated 50Ni-50Ti (at.pct.)

E_A	62000 MPa
E_M	22000 MPa
ν_A	0.33
ν_M	0.33
σ_L^S	400 MPa
σ_L^E	410 MPa
σ_U^S	200 MPa
σ_U^F	190 MPa
σ_{CL}^S	400 MPa
T_0	37°C
$(\frac{\delta\sigma}{\delta T})_L$	5.71 MPa/°C
$(\frac{\delta\sigma}{\delta T})_U$	5.71 MPa/°C
ε^L	0.0264
ε_v^L	0.0264
N_A	0

a compact tension (CT) specimens, that were pre-cracked at a low fatigue load. The fracture toughness of the studied material was found to be $39.4 \text{ MPa}\sqrt{m}$.

3.3. Phase Transformation Zone around Crack Tip

In this section, the shape and the size of phase transformation zone in single-edge cracked plate is analyzed in detail. Material properties, which are extracted from the paper of McKelvey and Ritchie [83], are tabulated in Table 3.2. These properties are obtained from mechanical testing of 50Ni-50Ti (at.pct) shape memory alloy, which was heat treated in air at 500°C for 35 minutes and then ice water quenched to induce an austenite finish temperature slightly lower than 37°C. The transformation temperatures M_s , M_f , A_s , A_f , M_d are -26.9°C, -54.8°C, 13.6°C, 31.3°C, 120°C respectively. The reference temperature T_0 for complete uniaxial constitutive behavior is fixed at 37°C.

The determined geometry for static analysis is a rectangular plate with $W = 100 \text{ mm}$ and $H = 200 \text{ mm}$. Because Mode I edge cracks are investigated, only upper half of the plate is modeled. Four different crack lengths are set in order to observe

the effect of crack tip location on phase transformation. To avoid boundary effects, a/W ratio range is taken between 0.25 and 0.40. The crack tip is perfectly sharp and crack faces are traction-free. The geometry is divided into partitions and each partition has a different mesh density. The mesh density around crack tip is higher for more accurate results, where the minimum element length is 0.1 mm. Besides, a circular sweep mesh descritization at the crack tip is recommended to obtain accurate results for some fracture parameters like J-integral and strain energy density. Plane stress conditions are assumed and the thickness of the section is taken as 1 mm. In order to compare sizes of transformation zones, remote tensile stresses of 80, 100 and 120 MPa are applied.

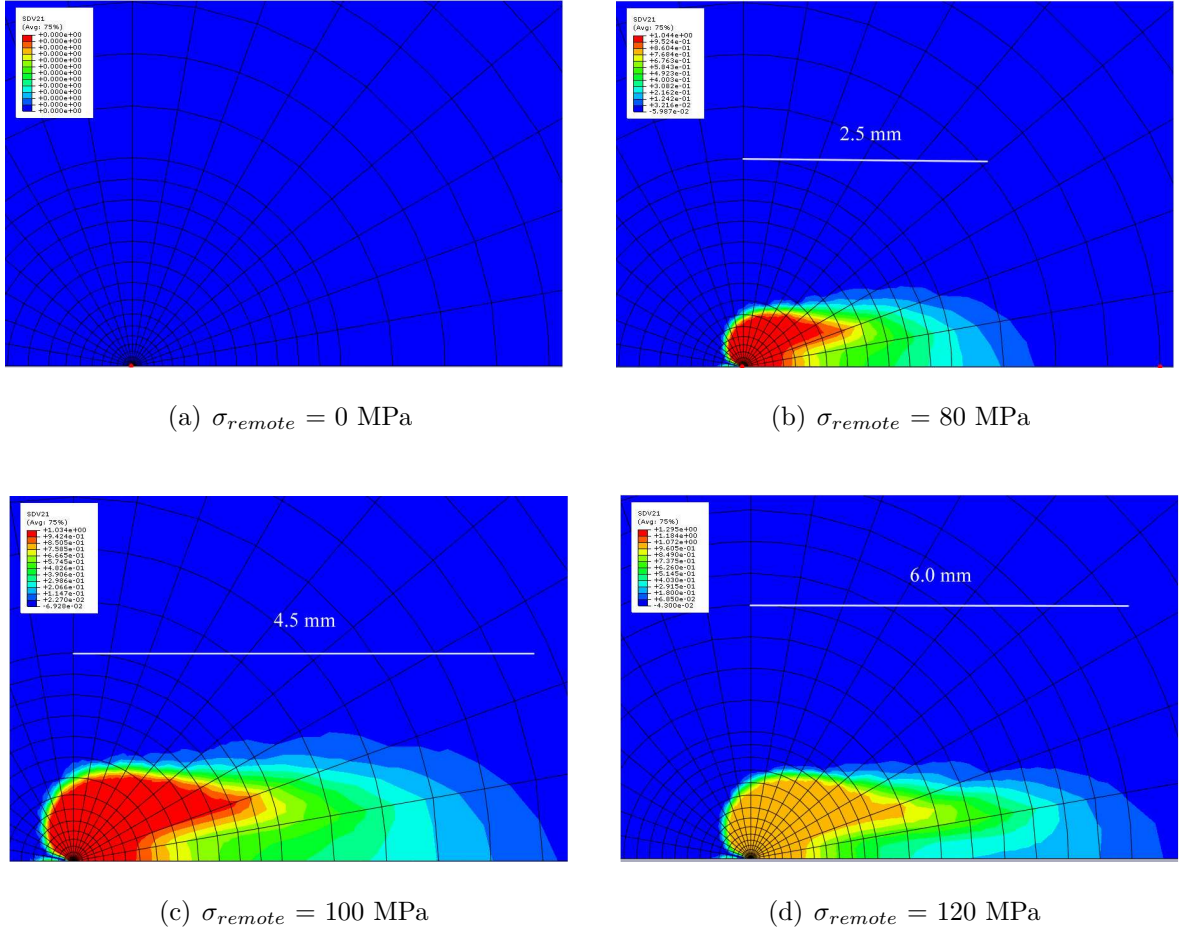


Figure 3.2. Shape and size of transformation zone under different applied loads

The transformation region is plotted using martensite fractions calculated at integration points, that take values between 0 and 1. The size of the region is directly related to the mode I stress intensity K_I . As K_I increases, there is a quick initiation

and progression of phase transformation region outwards from the crack tip. Stress intensity is directly proportional to applied load and a/W ratio, hence a circular shaped zone around crack tip becomes fully martensite when one continues to increase stress intensity. Following this saturated region of martensite, a transition region and a fully austenite region can be seen by moving outwards from the crack tip. In Figures 3.2 and 3.3, results of the effect of loading and crack length on transformation zones in the single-edge cracked plate are plotted. In all contour plots, kidney-shaped red areas are representing saturated martensite region, while blues zones stand for non-transformed austenite. Between fully transformed and non-transformed regions, there is a non-homogenous transformation zone having different martensite fraction in its different locations. The greater the applied load and/or initial crack length is, the longer is the horizontal and vertical dimensions of the fully transformed and transforming zones. If the initial crack length remains 30 mm and the applied stress is increased from 80 MPa to 120 MPa, the horizontal length of transformation zone will become bigger than twice its initial length. Under the same applied load of 100 MPa, the transformation region around crack tip enlarges threefold when the initial crack propagates from 25 mm to 40 mm. Due to plane stress conditions there will be no significant change in vertical dimensions as compared with horizontal dimensions, when the stress intensity increases.

3.4. J-Integral

For homogenous elastic materials, one of the ways used to calculate stress intensity factor is J-integral, which represents energy release rate around a closed path and indicates deformation at a crack tip. In Figure 3.4, J-integral results versus contour numbers at the same applied load of 100 MPa and for different a/W ratios are plotted. For homogenous materials at constant temperature, J-integral is contour-independent; meaning that J-integrals evaluated at an arbitrary closed path must always give the same value. In transformation zone of shape memory alloys, this is not the case. Although the material remains elastic, the phase transformation around crack tip region cancels the homogeneity of the material, so that the initial material properties like elastic modulus E_A should be modified in fully martensite and mixed zones. As a re-

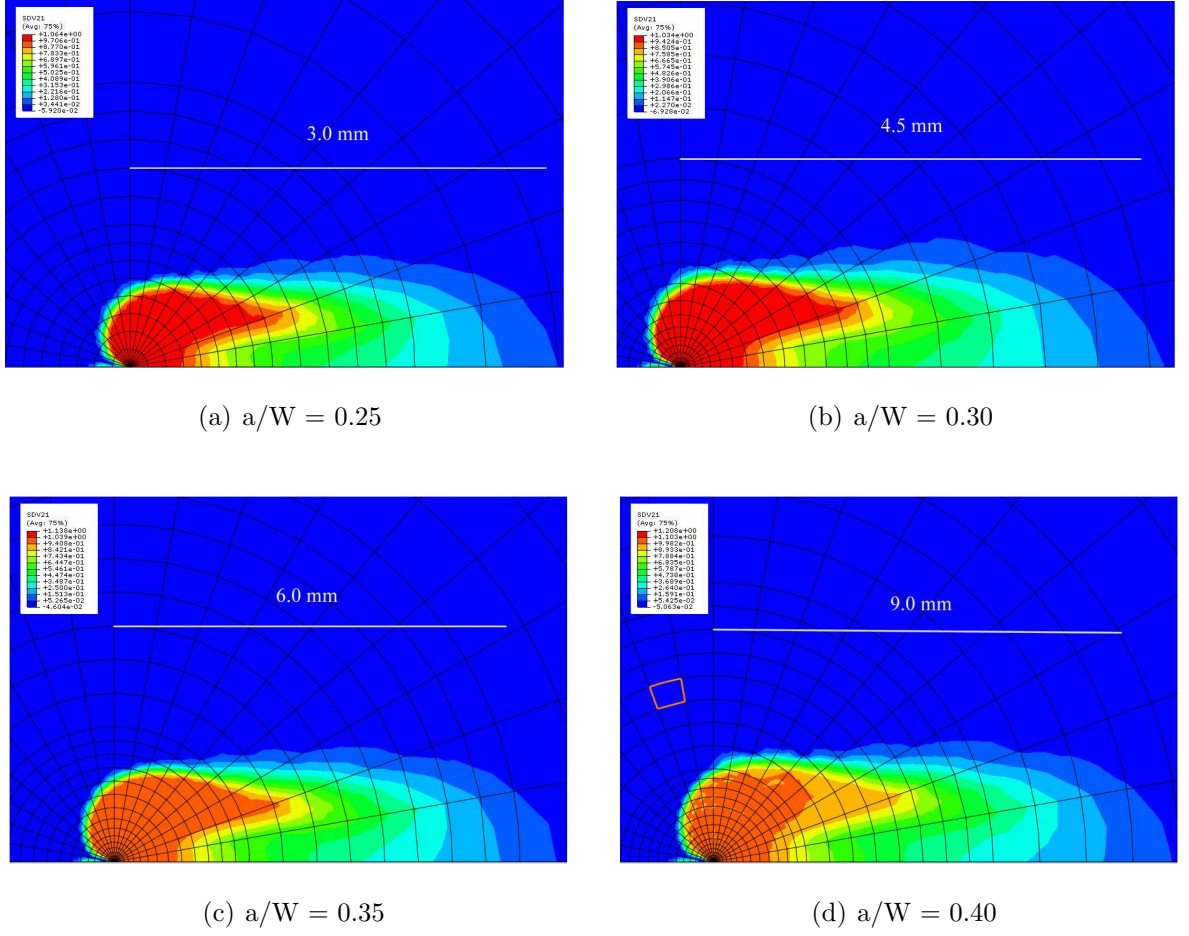
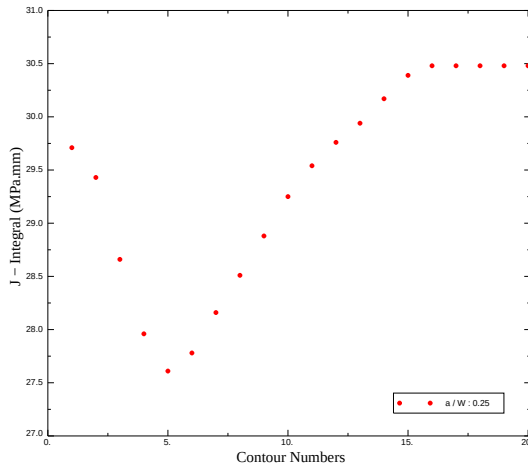


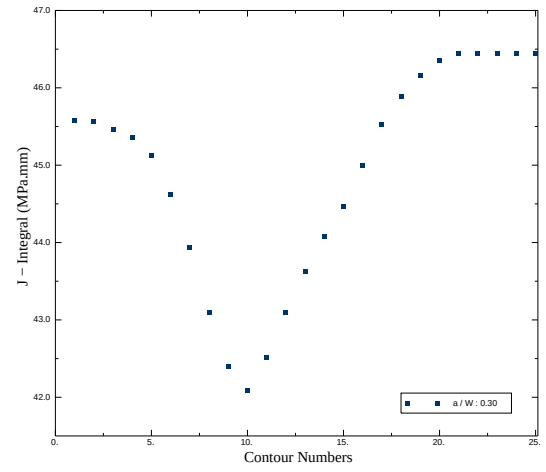
Figure 3.3. Shape and size of transformation zone of single-edge cracked plates having different initial crack lengths

sult, the determination of mode I stress intensity factor K_I is not straightforward like in pure elastic models.

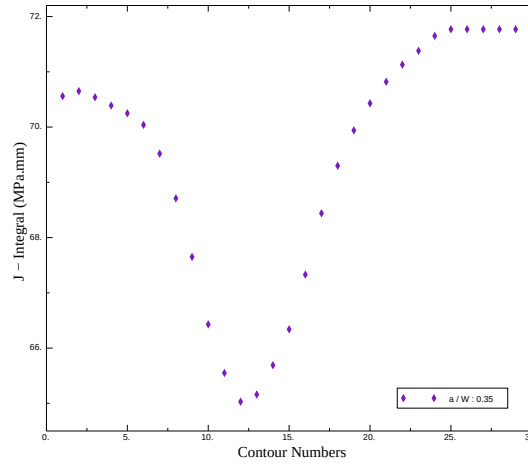
When results plotted in Figure 3.4 are studied in detail, the first impression is that the J-integral results remain constant only in fully austenite region. For instance, the J-integral for $a/W = 0.25$ converges to a value of 30.48 MPa.m on the 17th contour, which has a radius of 3.0 mm. The contour plots in Figure 3.3 reveals that this radius indicates the starting position of fully austenite (homogenous) region. In transformed regions on the plate, J-integral results are contour-dependent. Another interesting common trait of these plots is that J-integral values decrease with increasing contour numbers until approaching a minimum value and then increase again. To interpret this, a geometry and mesh configuration are selected ($a/W=0.3$ and $\sigma_{app} = 100$ MPa) and



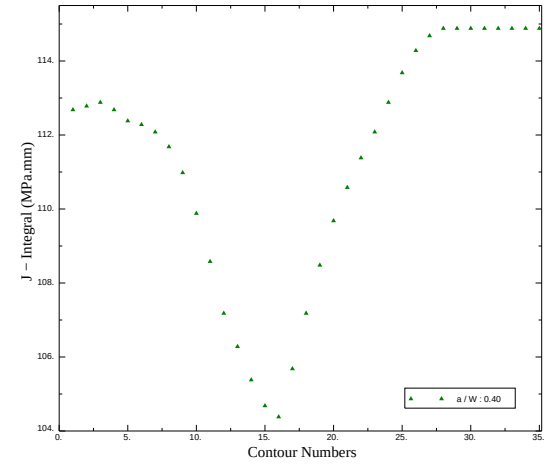
(a)



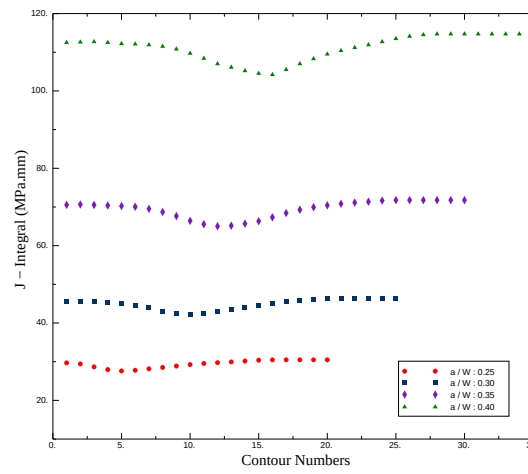
(b)



(c)



(d)



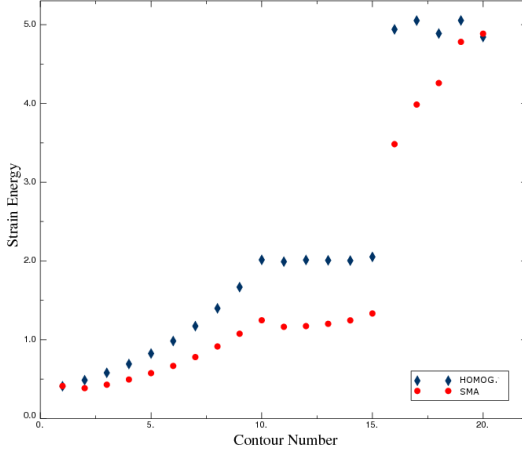
(e)

Figure 3.4. SMA : J-Integral vs. contour number ($\sigma_{app} = 100$ MPa)

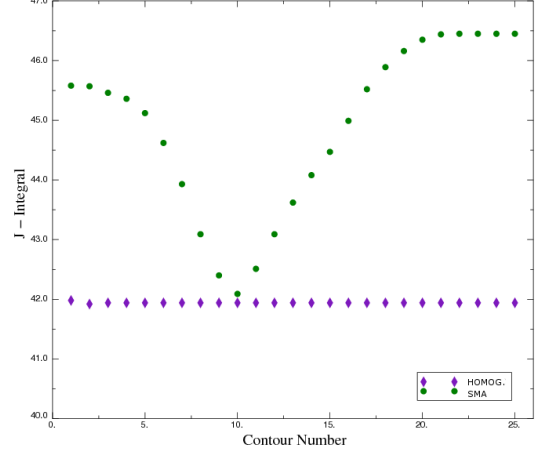
Table 3.3. Comparison between strain energy densities of SMA and elastic material models having same $a = 30$ mm and $\sigma_{app} = 100$ MPa

CONTOUR	STRAIN ENERGY SUM $\int_{\Gamma} w dy$	
	SMA ($E_A = 62$ GPa , $E_M = 22$ GPa)	ELASTIC (E = 62 GPa)
1	4,13E-01	4,10E-01
2	3,86E-01	4,87E-01
3	4,30E-01	5,81E-01
4	4,96E-01	6,92E-01
5	5,77E-01	8,25E-01
6	6,68E-01	9,84E-01
7	7,80E-01	1,17E+00
8	9,15E-01	1,40E+00
9	1,08E+00	1,67E+00
10	1,25E+00	2,01E+00
11	1,16E+00	1,99E+00
12	1,17E+00	2,01E+00
13	1,20E+00	2,01E+00
14	1,25E+00	2,00E+00
15	1,33E+00	2,05E+00
16	3,48E+00	4,94E+00
17	3,98E+00	5,05E+00
18	4,26E+00	4,89E+00
19	4,78E+00	5,05E+00
20	4,88E+00	4,84E+00

the material used is changed to an elastic one with an elastic modulus of $E = E_A = 62$ GPa. Because strain energy density is an important parameter in J-integral evolution, the sum of strain energy in each element on each contour is imported for both SMA and pure elastic cases. The results are tabulated and plotted (see Table 3.3 and Figure 3.5). One can conclude that the phase transformation and relevant nonhomogeneity greatly affect strain energy densities and energy release rates. The drop in J-integral values are directly related to the difference between strain energy densities of homogenous and nonhomogenous material. The most striking result obtained from these graphs is that the local minimum of both strain energy and J-integral of SMA's is seen at the 11th contour, while the values of strain energy and J-intergral of the elastic material remain almost constant between 10th and 15th contours. Moreover, although fully austenite region has the same material properties with homogenous elastic material, J-integral in austenite region of SMA is affected by phase transformation and its value is different from that of pure homogenous material.



(a) Strain Energy vs. Contour Number



(b) J-Integral vs. Contour Number

Figure 3.5. Effect of phase transformation on strain energy and J-Integral

3.5. Calculation of Stress Intensity Factor

The stress intensity factor K_I is the key parameter in analysis of crack propagation, therefore an accurate solution method has to be proposed. Unfortunately, there are few studies in literature focusing on stress intensity determination in the case of shape memory alloys. As mentioned in Section 3.2, there are discussions on the effect of phase transformation on toughness and different proposals for determination of stress intensity factors have been made. Hence, all suggested analytical and numerical methods will be studied in this section. The most recent closed form solution is given by Xiong and Liu [91] in 2007, who state that the phase transformation of SMAs leads to a volume contraction at the crack tip and the stress intensity increases.

In their study, Xiong and Liu 3.2 work on ferromagnetic shape memory alloys such as Ni-Mn-Ga, which has poor fatigue properties compared to Nitinol, and expresses their crack tip intensity factor K_I^* as :

$$K_I^* = K_I + K_{tr} \quad (3.1)$$

where K_I is the classical stress intensity factor as a result of loading and K_{tr} is due to localized phase transformation around the crack tip. Their theory is based on the existence of a small scale plastically deformed region at the crack tip and dimensions of plastic zone (r_p), fully transformed zone (r_m), whole transformation zone (r_{tr}) and the modified sizes of plastic zone and fully transformed zone after stress field redistribution around the crack tip (r_p^* , r_m^*) are predicted by using linear elastic fracture mechanics (see Figure 3.6). For a center cracked plate with the half-crack length a , the corrected K_I and K_I^* are given as follows :

$$K_I = \sigma_0 \sqrt{\frac{2\pi \cdot a \cdot \sigma_y^2}{2\sigma_y^2 - \sigma_0^2}} \quad (3.2)$$

$$K_I^* = \sqrt{-\frac{2(C^{AS})^2(M_f - T)^2 \{4\sigma_s^M \sigma_y Z - \sigma_0^2 \pi a [\sigma_y^2 + (\sigma_s^M)^2]\}}{\sigma_s^M (\sigma_s^M - \sigma_y) [3(C^{AS})^2 (M_f - T)^2 + \sigma_y]}} \quad (3.3)$$

$$Z = K_I^2 + \sigma_0 \sigma_y \left[\sqrt{\sigma_0^2 \pi a \left(\frac{1}{\sigma_y^2} + \frac{1}{(\sigma_s^M)^2} \right)} - \frac{K_I^2}{\sigma_y^2} - \frac{K_I}{\sigma_y} \right] K_I \quad (3.4)$$

where Z is an operation constant. The other required terms are yield stress σ_y , remote stress σ_0 , loading at which martensitic transformation starts σ_s^M , slope of critical stress for formation versus temperature C^{AS} and martensite finish temperature M_f .

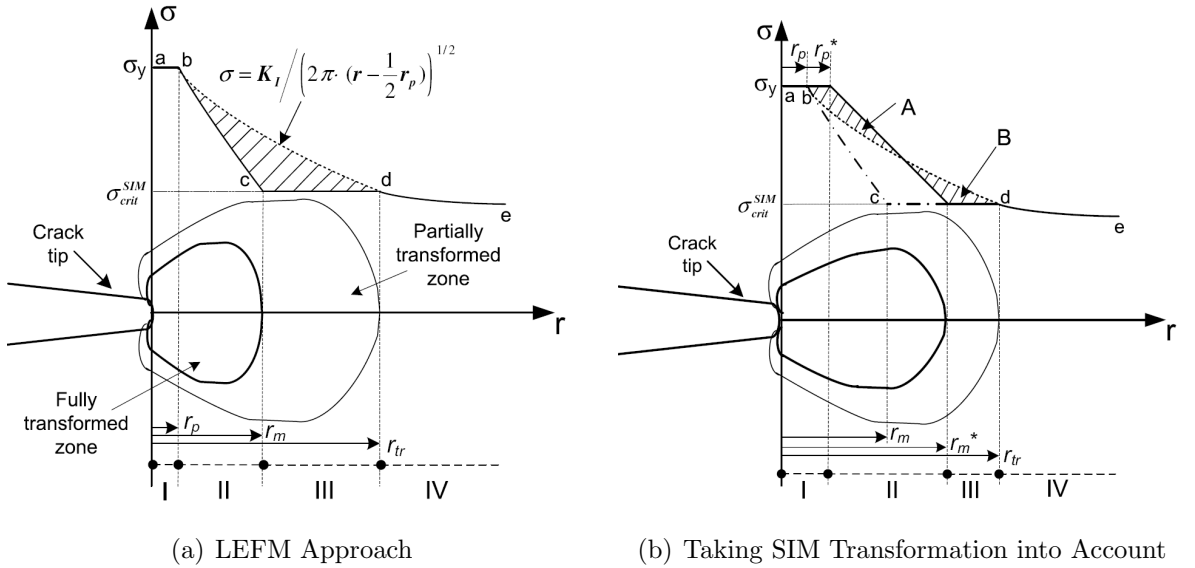


Figure 3.6. Stress distribution in transformation zone [91]

Aside from shape memory alloys, there are several materials like ceramics, that undergo stress induced phase transformation. Vena et al. [95] worked on fatigue crack growth rate of transformation toughening in zirconia tetragonal polycrystal ceramics and proposed a closed form solution for Mode I stress intensity factor. Equation 3.1 is valid for their case, in which K_{tr} is evaluated by means of following domain integral as suggested by Budiansky et al. [96] :

$$K_{tr} = \int_{\Omega} dK_{tr} dA \quad (3.5)$$

$$dK_{tr} = \frac{1}{\sqrt{8\pi}} \frac{E}{(1 - \nu^2)} r^{-\frac{3}{2}} M(E_{ij}^{tr}, \beta) \quad (3.6)$$

where

$$M = (E_{11}^{tr} + E_{22}^{tr}) \cos\left(\frac{3}{2}\beta\right) + \sin(\beta) \left[3E_{12}^{tr} \cos\left(\frac{5}{2}\beta\right) + \frac{3}{2}(E_{11}^{tr} - E_{22}^{tr}) \sin\left(\frac{5}{2}\beta\right) \right] \quad (3.7)$$

and Ω is the transformed region. β and r are polar coordinates of points where transformation strains E_{ij}^{tr} occur, measured from the tip of the crack.

In Table 3.4, SIFs that are calculated using Xiong and Liu's suggestion formulation are summarized for various crack lengths and applied loads. Because their formulation is for a center-cracked plate, the contribution from applied load K_I is modified by a correction factor (CF), which is taken from handbooks for each H/W and a/W ratio of a single-edge cracked specimen [97]. Both dimensional and nondimensional values of K_{tr} and K_I^* are tabulated. It is seen that the contribution from stress induced martensite transformation K_{tr} is a function of temperature and applied stress intensity factor K_I (see Equation 3.3), but independent of crack size. Besides, the output values indicate that the toughness is decreasing due to transformation around the crack tip. On the other hand, when the formulation of Vena et al. is applied, the solution will be different. The comparison of stress intensity factors calculated by both formulations are given in Table 3.5. In this case, the remote stress is 100 MPa and the

Table 3.4. K_I^* analytical results for $a / W = 0.25-0.40$ and $\sigma = 80, 100, 120$ MPa

			$\sigma_0 = 80$ MPa					
a [mm]	a/W	CF	K_I [MPa $\sqrt{mm}$]	K_I^{mod}	K_{tr}	K_I^*	ND K_{tr}	ND K_I^*
25	0.25	1.09	709.65	770.27	16.51	786.78	0.04128	1.9669
30	0.30	1.12	777.39	873.41	18.09	891.50	0.04128	2.0345
35	0.35	1.17	839.67	979.00	19.54	998.54	0.04128	2.1097
40	0.40	1.21	897.65	1089.33	20.89	1110.22	0.04128	2.1942
			$\sigma_0 = 100$ MPa					
a [mm]	a/W	CF	K_I [MPa $\sqrt{mm}$]	K_I^{mod}	K_{tr}	K_I^*	ND K_{tr}	ND K_I^*
25	0.25	1.09	887.54	963.35	20.65	984.01	0.04130	1.9680
30	0.30	1.12	972.25	1092.34	22.62	1114.96	0.04130	2.0356
35	0.35	1.17	1050.15	1224.40	24.44	1248.84	0.04130	2.1109
40	0.40	1.21	1122.66	1362.39	26.12	1388.51	0.04130	2.1954
			$\sigma_0 = 120$ MPa					
a [mm]	a/W	CF	K_I [MPa $\sqrt{mm}$]	K_I^{mod}	K_{tr}	K_I^*	ND K_{tr}	ND K_I^*
25	0.25	1.09	1065.75	1156.78	24.80	1181.58	0.04132	1.9692
30	0.30	1.12	1167.47	1311.66	27.16	1338.83	0.04132	2.0369
35	0.35	1.17	1261.01	1470.24	29.34	1499.58	0.04132	2.1122
40	0.40	1.21	1348.07	1635.93	31.37	1667.30	0.04132	2.1968

crack length is fixed at 30 mm. In determination of crack tip stress intensity, transformation strains E_{ij}^{tr} are extracted from ABAQUS and used in Equation 3.7. The elastic modulus E is the crack tip modulus and equal to E_M . By using this methodology, it is seen that the toughness will increase after phase transformation, since K_{tr} is negative.

Table 3.5. Comparison of K_{tr} solved by different approaches

$\sigma_0 = 100$ MPa and $a / W = 0.30$						
Solution Method	K_I [MPa $\sqrt{mm}$]	K_I^{mod}	K_{tr}	K_I^*	ND K_{tr}	ND K_I^*
XIONG	887.54	963.35	20.652	984.01	0.04130	1.96801
VENA	887.54	963.35	-54.01	909.34	-0.10802	1.81869

Phase transformation in SMAs leads to nonhomogeneity, so that the equality between J and energy release rate G for linear elastic materials is inadequate in calculation of mode I stress intensity factor. Still, it has shown that for general nonhomogenous materials, the crack tip fields are identical to those in homogenous materials if the material properties are continuous and piecewise continuously differentiable [98]. Using this idea, the stress intensity K_I can be determined by following equations :

$$K_I = \sqrt{G \cdot E_{tip}} = \sqrt{\frac{\partial U}{\partial a} \cdot E_M} \quad \text{for plane stress} \quad (3.8)$$

$$K_I = \sqrt{\frac{G \cdot E_{tip}}{1 - \nu^2}} = \sqrt{\frac{\partial U}{\partial a} \cdot \frac{E_M}{1 - \nu^2}} \quad \text{for plane strain} \quad (3.9)$$

Table 3.6. K_I solution of Nitinol : 1) FEA Results K_I^{FEA} 2) Closed Form Formulation Results K_I^*

[MPa]		[MPa.mm]		[MPa. $\sqrt{mm}$]		%		
σ_0	a/W	J^A	G^{FEA}	K_I^{FEA}	K_I^*	Difference	ND K_I^{FEA}	ND K_I^*
80	0.25	19.00	19.62	656.93	786.78	16.50	1.6423	1.9670
	0.30	28.59	29.63	807.41	891.50	9.43	1.8426	2.0346
	0.35	43.27	45.43	999.75	998.54	-0.12	2.1124	2.1098
	0.40	66.92	62.60	1173.54	1110.22	-5.70	2.3194	2.1943
100	0.25	30.48	31.63	834.16	984.01	15.23	1.6683	1.9680
	0.30	46.45	48.97	1037.93	1114.96	6.91	1.8950	2.0356
	0.35	71.77	76.07	1293.67	1248.84	-3.59	2.1867	2.1109
	0.40	114.9	123.20	1646.33	1388.51	-18.57	2.6031	2.1954
120	0.25	45.41	47.68	1024.19	1181.58	13.32	1.7070	1.9693
	0.30	70.24	74.56	1280.75	1338.83	4.34	1.9486	2.0370
	0.35	112.10	120.52	1628.32	1499.58	-8.59	2.2936	2.1123

In Table 3.6, FEA results for energy release rate and corresponding stress intensity factor are given. Furthermore, J-integral outputs in fully austenite region and analytical results for stress intensity factor, which is calculated by Equation 3.3, are also tabulated. It is observed that the J^A values are close to energy release rate outputs, however the equality $J = G$ for linear elastic materials doesn't hold. Still, they may be used in pre-analysis stage of shape memory alloys' fracture studies. By looking at differences between analytical and finite element results of K, it will be very difficult to judge, which result is closer to the real value of K. Depending on a/W ratio

and remote stress σ_0 , these differences are becoming larger or smaller; only at special conditions (e.g. $a/W = 0.35$ and $\sigma_0 = 80$ MPa), the analytical and numerical results are very close to each other. It can be concluded that, there is no consistency between ABAQUS outputs and closed form solutions, one can't be sure about the validity of calculated stress intensity factor.

3.6. A Proposal to Calculate Relationship between J, Crack Tip Stress Fields and Crack Tip Opening Displacement

In the previous section, the requirement of an accurate stress intensity calculation couldn't be satisfied. Actually, stress intensity factor is an amplitude used in description of stress and strain fields around crack tip of linear elastic materials. Following a similar pattern, stress distribution in transformation zone should be identified and a stress intensity factor definition for Nitinol might be made. In the introductory part of the study, it is emphasized that in construction of SMA's constitutive models, some basic concepts of plasticity are followed up. Hence, it will be a good idea to dissect crack tip stress fields of elastic-plastic materials in first place. Analogous properties of shape memory alloys and plastic materials will be further discussed.

3.6.1. HRR (Hutchinson-Rice-Rosengren) Singularity Field

Hutchinson, Rice and Rosengren independently evaluated the character of stress tip fields in the case of power-law hardening materials, which follow the nonlinear stress-strain relationship :

$$\frac{\varepsilon}{\varepsilon_y} = \alpha \left(\frac{\sigma}{\sigma_y} \right)^m \quad (3.10)$$

where α is a proportionality factor and m is the hardening exponent. Hutchinson [99] defined a stress function, Φ , for the problem. The governing differential equation for deformation plasticity theory for a plane problem in power-law hardening material is

given as :

$$\nabla^4 \Phi + \gamma(\Phi, \sigma_{eff}, r, m, \alpha) = 0 \quad (3.11)$$

where the function γ differs for plane stress and plane strain. For mode I cracks, the stress function is expressed in terms of an asymptotic expansion,

$$\Phi = C_1(\theta)r^s + C_2(\theta)r^t + \text{H.O.T.} \quad \text{and} \quad s < t \quad (3.12)$$

where C_1 and C_2 are constants, that depend on the angle θ from the crack plane in polar coordinates. As approaching to the crack tip ($r \rightarrow 0$), the first term dominates the equation and the stress function can be expressed as follows :

$$\Phi = \kappa \sigma_y r^s \tilde{\Phi}(\theta) \quad (3.13)$$

where κ is the amplitude of the stress function and $\tilde{\Phi}(\theta)$ is a dimensionless function of the angle θ . Thus, the stresses in polar coordinates are given by

$$\sigma_{rr} = \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} = \kappa \sigma_y r^{s-2} \tilde{\sigma}_{rr}(\theta) = \kappa \sigma_y r^{s-2} (s \tilde{\Phi} + \tilde{\Phi}'') \quad (3.14)$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \Phi}{\partial r^2} = \kappa \sigma_y r^{s-2} \tilde{\sigma}_{\theta\theta}(\theta) = \kappa \sigma_y r^{s-2} s(s-1) \tilde{\Phi} \quad (3.15)$$

$$\sigma_{r\theta} = -\frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} = \kappa \sigma_y r^{s-2} \tilde{\sigma}_{r\theta}(\theta) = \kappa \sigma_y r^{s-2} (1-s) \tilde{\Phi}' \quad (3.16)$$

$$\sigma_{eq} = \kappa \sigma_y r^{s-2} \left(\tilde{\sigma}_{rr}^2 + \tilde{\sigma}_{\theta\theta}^2 - \tilde{\sigma}_{rr} \tilde{\sigma}_{\theta\theta} + 3 \tilde{\sigma}_{r\theta}^2 \right)^{0.5} \quad (3.17)$$

When the boundary conditions for the crack problem, $\tilde{\Phi}(\pm\pi) = \tilde{\Phi}'(\pm\pi) = 0$, are substituted into Equation 3.11, a nonlinear eigenvalues equation for s is obtained, which can be solved numerically for a range of m values. According to Hutchinson,

the numerical analysis indicates that s could be described quite accurately by a simple formula :

$$s = \frac{2m + 1}{m + 1} \quad (3.18)$$

which can be used in definition of stress function Φ and angular functions $\tilde{\sigma}_{ij}$. Also it implies that the strain energy density varies as $1/r$ near the crack tip.

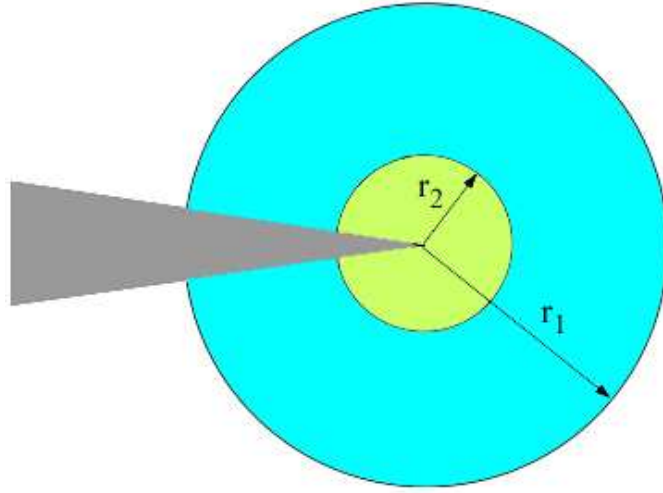


Figure 3.7. Two circular contours around the crack tip

In Figure 3.7, two circular contours of radius r_1 and r_2 are plotted around the crack tip in a small scale yielding region; r_1 is in the zone dominated by the elastic singularity, while arbitrary defined r_2 is in plastic zone, where the leading term of the Hutchinson asymptotic expansion dominates. If J is evaluated along r_1 , the equality $J = K_I^2/E$ is found as expected. Solving J -integral along the plastic contour leads to

$$J = r \int_{-\pi}^{+\pi} \left[w(r, \theta) \cos \theta - T_i \frac{\partial u_i}{\partial x} \right] \quad (3.19)$$

$$= \alpha \varepsilon_y \sigma_y \kappa^{n+1} r_2^{(m+1)(s-2)+1} I_m \quad (3.20)$$

where the integration constant I_m is given by

$$I_m = \int_{-\pi}^{+\pi} \left\{ \frac{m}{m+1} \tilde{\sigma}_{eff}^{m+1} \cos \theta - \left[\sin \theta \{ \tilde{\sigma}_{rr}(\tilde{u}_\theta - \tilde{u}'_r) - \tilde{\sigma}_{r\theta}(\tilde{u}_r - \tilde{u}'_\theta) \} + \cos \theta (m(s-2) + 1)(\tilde{\sigma}_{rr}\tilde{u}_r + \tilde{\sigma}_{\theta\theta}\tilde{u}_\theta) \right] \right\} d\theta \quad (3.21)$$

and

$$u_r = \alpha \varepsilon_y \kappa^m r^{m(s-2)+1} \tilde{u}_r(\theta) \quad (3.22)$$

$$u_\theta = \alpha \varepsilon_y \kappa^m r^{m(s-2)+1} \tilde{u}_\theta(\theta) \quad (3.23)$$

In order for J to be path independent, Equation 3.18 must hold. After doing the required arrangements, the singular stress and strain fields as a function of J are found to be :

$$\sigma_{ij} = \sigma_y \left(\frac{J}{\alpha \sigma_y \varepsilon_y I_m r} \right)^{\frac{1}{1+m}} \tilde{\sigma}_{ij}(\theta, m) \quad (3.24)$$

$$\varepsilon_{ij} = \alpha \varepsilon_y \left(\frac{J}{\alpha \sigma_y \varepsilon_y I_m r} \right)^{\frac{m}{1+m}} \tilde{\varepsilon}_{ij}(\theta, m) \quad (3.25)$$

where $\tilde{\sigma}_{ij}$ and $\tilde{\varepsilon}_{ij}$ values are tabulated for various θ and m 's in many resources [100]. Besides, for plane stress conditions, the integration constant I_m can be calculated as a polynomial function of hardening coefficient by the following equation :

$$I_m = 4.546 - 0.2827m + 0.0175m^2 - 0.45816 \times 10^{-4}m^3 \quad (3.26)$$

The results are important for establishing J as a stress amplitude parameter within the plastic zone, where the linear elastic solution is invalid. Because elastic strains are assumed to be negligible, the solution is only valid near the crack tip within

the plastic zone. Moreover, the solution is also invalid for very small values of r due to finite geometry changes at the crack tip.

3.6.2. Stress Field around Crack Tip in Power-Law Hardening Material

To better understand the solution methodology described in previous section, a plastic material, that undergoes power-law kinematic hardening after yielding, is tested. The stress field is determined by ABAQUS and compared to solution obtained from HRR singularity field quantities. The plasticity exponent m and the constant α are selected arbitrarily. To find analytical results, J 's on every circular contour at a distance r from crack tip are calculated using finite elements and then used in Equation 3.24. The coefficients $\tilde{\sigma}_{ij}$, which is a function of angle θ and m , are calculated by interpolation of tabulated values [100].

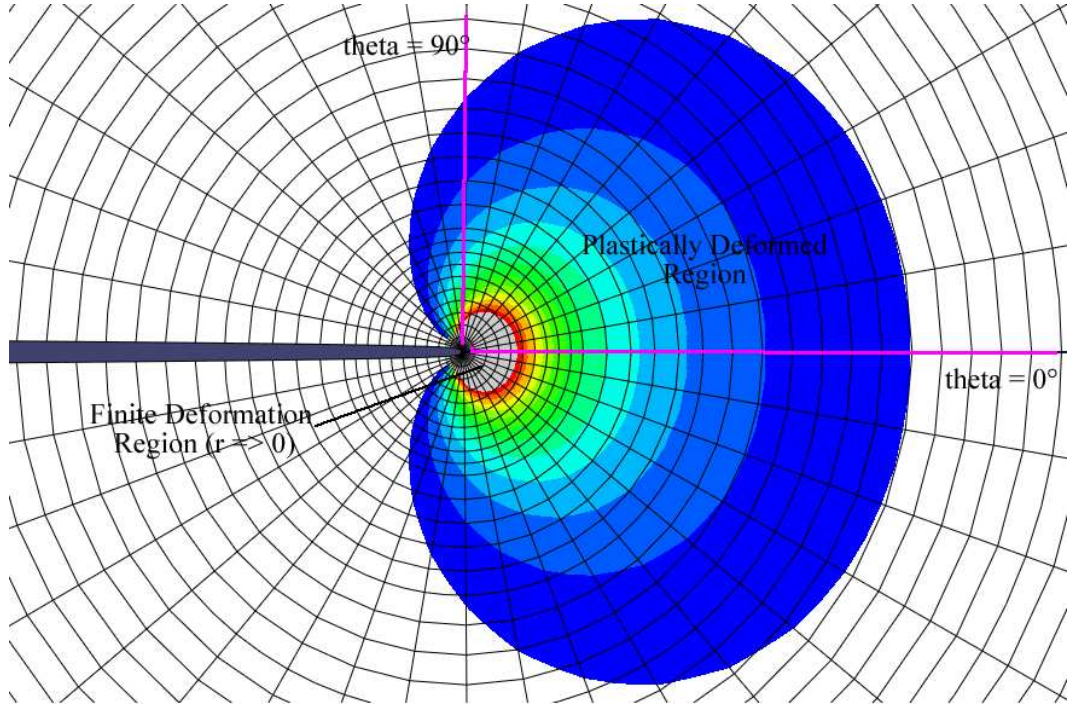


Figure 3.8. Contour plots of equivalent plastic strains around crack tip

In Figure 3.8, the circular mesh around the crack tip is shown. The single-edge crack is located at the distance $a = 40$ mm on a 100 mm wide plate, which has a thickness of 1 mm. Plane stress and symmetry conditions are applied, in which the remote tensile load σ_0 is chosen to be $0.31\sigma_y = 150$ MPa. Material properties α , m , E

are 0.8152, 3.11 and 200 GPa, respectively.

Figure 3.8 shows contour plots of equivalent plastic strains around the crack tip with three distinct zones. The closest zone to crack tip is the finite deformation zone, where plastic strains and equivalent stresses are very large. In this zone, the HRR solution is expected to be invalid. The white region far from the crack tip represents elastically deformed region, where plastic strains are zero or negligibly small and HRR method is not allowed to be used either. Hence, only stresses at integration points in plastically deformed region are extracted and analyzed. To simplify the task, FE results of normal stresses σ_{yy} at $\theta = 0^\circ$ and 90° are compared to analytical ones (HRR). The prescribed conditions are $\sigma_{yy} = \sigma_{\theta\theta}$ at $\theta = 0^\circ$ and $\sigma_{yy} = \sigma_{rr}$ at $\theta = 90^\circ$. The results are tabulated below (see Table 3.7).

Table 3.7. Comparison of FE and analytical results of HRR singular field quantities

r [mm]	PLASTIC MATERIAL ($\sigma_y = 480$ MPa . $\sigma_0 = 150$ MPa . $m = 3.11$. $\alpha = 0.8152$)							
	$\theta = 0^\circ$ ($\sigma_{yy} = \sigma_{\theta\theta}$, $\bar{\sigma}_{\theta\theta} = 1.109114$)				$\theta = 90^\circ$ ($\sigma_{yy} = \sigma_{rr}$, $\bar{\sigma}_{rr} = 0.660167$)			
	J[MPa.mm]	$\sigma_{\theta\theta}^{HRR}$ [MPa]	$\sigma_{\theta\theta}^{FEA}$	Difference [%]	J[MPa.mm]	σ_{rr}^{HRR} [MPa]	σ_{rr}^{FEA}	Difference [%]
0.10	82.94	1868.97	2323.13	19.55	82.94	1267.69	1097.20	15.54
0.20	88.78	1605.27	1814.13	11.51	88.78	1088.83	995.29	9.40
0.30	90.05	1459.53	1611.32	9.42	90.05	989.97	924.26	7.11
0.40	90.66	1363.11	1487.51	8.36	90.66	924.57	875.65	5.59
0.50	91.06	1292.46	1399.12	7.62	91.06	876.65	837.95	4.62
0.60	91.37	1237.38	1331.28	7.05	91.37	839.30	807.97	3.88
0.70	91.61	1192.59	1276.64	6.58	91.61	808.91	783.07	3.30
0.80	91.76	1154.93	1231.30	6.20	91.76	783.37	762.00	2.81
0.90	91.89	1122.69	1192.70	5.87	91.89	761.50	743.73	2.39
1.00	92.01	1094.62	1159.12	5.56	92.01	742.46	726.60	2.18
1.20	92.17	1047.56	1105.44	5.24	92.17	710.54	698.57	1.71
1.40	92.36	1009.50	1059.98	4.76	92.36	684.73	676.32	1.24
1.60	92.51	977.61	1021.88	4.33	92.51	663.10	657.82	0.80
1.80	92.55	950.10	989.29	3.96	92.55	644.43	641.95	0.39
2.00	92.58	926.14	958.84	3.41	92.58	628.18	627.42	0.12
2.25	92.63	900.09	929.72	3.19	92.63	610.52	611.18	0.11
2.50	92.70	877.46	902.50	2.77	92.70	595.17	596.67	0.25
2.75	92.77	857.51	878.37	2.37	92.77	581.64	583.99	0.40
3.00	92.85	839.73	854.74	1.76	92.85	569.58	572.01	0.43
3.25	92.94	823.72	836.78	1.56	92.94	558.71	561.15	0.43
3.50	93.01	809.14	818.61	1.16	93.01	548.83	550.68	0.34
3.75	92.95	795.56	801.94	0.80	92.95	539.61	541.03	0.26
4.00	92.88	783.02	786.50	0.44	92.88	531.11	532.01	0.17
4.25	92.83	771.45	772.13	0.09	92.83	523.26	523.50	0.05
4.50	92.80	760.74	758.68	0.27	92.80	515.99	515.52	0.09

Table 3.7 and Figure 3.9 reveal that minor differences exist between analytical and FE results except at finite deformation zone very close to crack tip ($0.1 \text{ mm} \leq r \leq 1.0 \text{ mm}$). That proves the validity of the method for power-law hardening materials.

The differences may arise due to interaction of neighboring large plasticity zone and HRR singular field during finite elements implementation and/or due to interpolation errors during analytical determination of some parameters. Still, an error below 1% looks acceptable, which is seen at several points on the stress field.

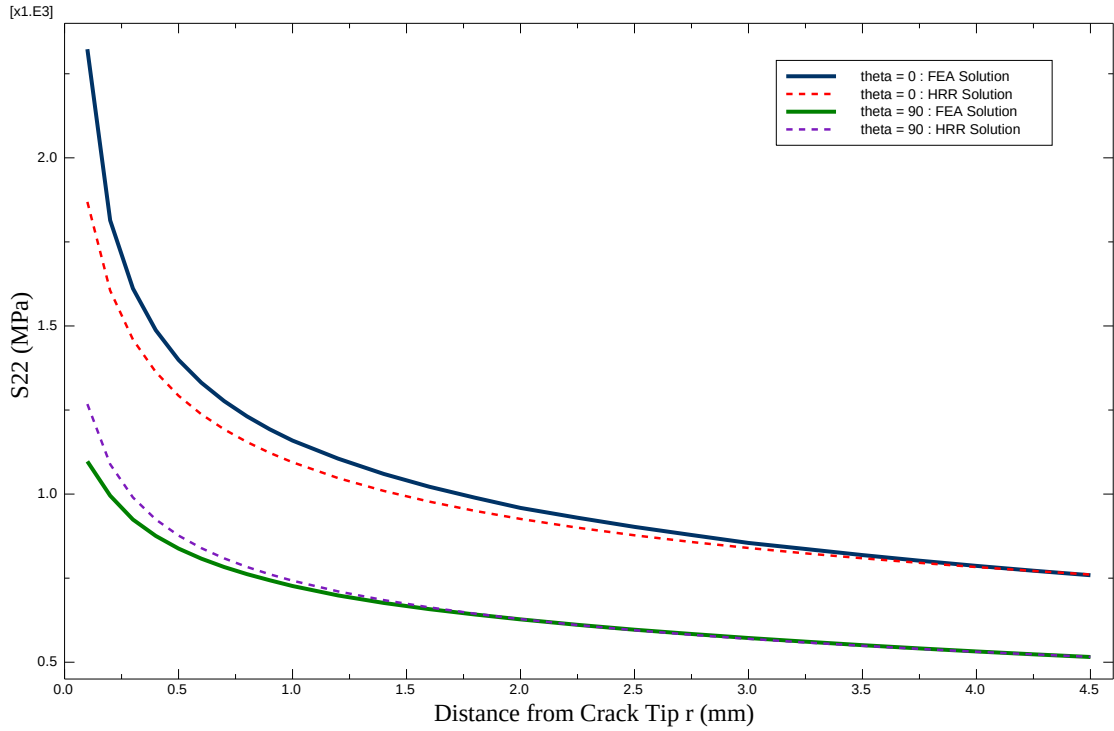


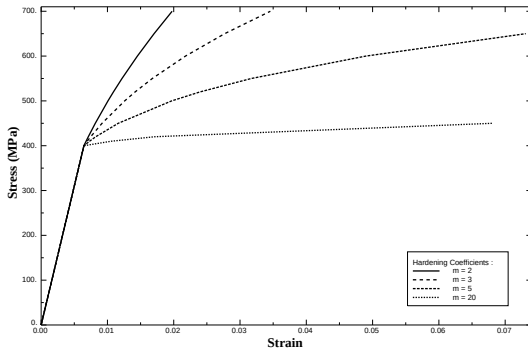
Figure 3.9. σ_{yy} vs. r : FE and analytical results

3.6.3. Stress Field around Crack Tip in Shape Memory Alloy

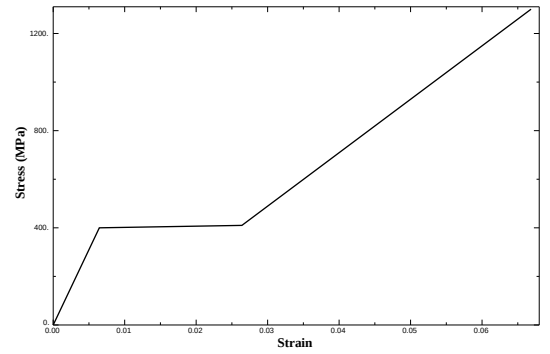
Using a similar approach, stress distribution in phase transformation region is going to be investigated. Same geometry, crack tip location, mesh densities around crack tip, initial and boundary conditions of the study in previous subsection are used in analysis. Material properties were already given in Table 3.2. Two remote tensile loads of 40 and 80 MPa will be applied. The FE outputs of stress in transformation zone are extracted and tabulated (see Table 3.9, 3.10 and 3.11).

Unfortunately, the stress field in cracked shape memory alloys has not yet been clarified analytically. The linear elastic solution holds as long as there is no martensitic phase transformation. If the plastically deformed zone of materials with power-law

hardening and phase transformation zone of shape memory alloys are compared, one will see that the size and shape of these zones resemble to each other (see Figures 3.2 and 3.8). Hence, we are curious in the fact that $(1/r)^{\frac{1}{1+m}}$ singularity of plastic materials may hold in transformation zone of shape memory alloys. For this purpose, the existence of HRR singularity field is checked in the transformation zone of SMAs. HRR solution is an accurate method, however the first assumption made in formulation was that the material should be plastic with power-law hardening, which follows a nonlinear stress-strain relationship. To use this formulation, similarities between uniaxial stress-strain behaviors of SMAs and power-law hardening materials must be found (see Figure 3.10). Moreover, the constitutive law during uniaxial tensile loading of Nitinol should be represented by a using a similar form of Equation 3.10. To overcome this hurdle, series of numerical paths are followed. First, stress and strain data on every integration point of the loaded plate are demanded from ABAQUS and the distribution of normalized strain in terms normalized stress is drawn. Several curves are fitted on different data sets to find out, which m and α values represent tensile behavior of shape memory alloys the best. By using these constants, analytical results for stress field could be obtained.



(a) Plastic Material with Power-Law Hardening



(b) Nitinol

Figure 3.10. Uniaxial stress-strain plots

As discussed before, Nitinol behaves like a perfectly plastic material in the phase transformation region ($\sigma_s^M < \sigma < \sigma_f^M$) and regains its elastic behavior after complete martensitic transformation ($\sigma > \sigma_f^M$, $E = E_M$). Using only single data set would be not enough to describe complete stress-strain relationship. At least two curves in the power curve form of $y = ax^b$ are needed. Thus, following data sets are created :

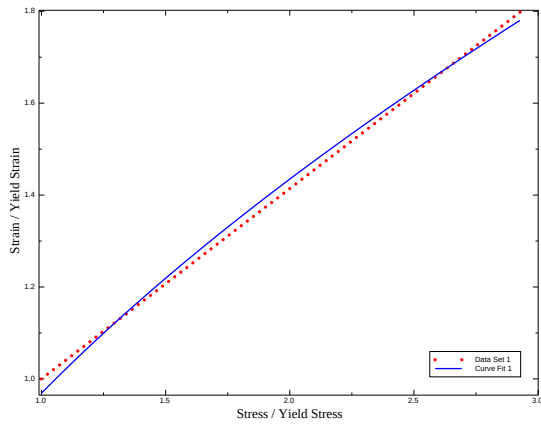
- DATA SET 1 : $410 \text{ MPa} < \sigma < 1200 \text{ MPa}$
- DATA SET 2 : $400 \text{ MPa} < \sigma < 1300 \text{ MPa}$
- DATA SET 3, DATA SET 4, DATA SET 5 : $400 \text{ MPa} < \sigma < 800 \text{ MPa}$
- DATA SET 6, DATA SET 7 : $400 \text{ MPa} < \sigma < 410 \text{ MPa}$
- DATA SET 8 : $400 \text{ MPa} < \sigma < 1200 \text{ MPa}$

Plots in Figure 3.11 are showing normalized strain-stress distribution of each data set and fitted power curves on these sets. The thumb of rule by curve fitting is that adjusted R-square values should be near unity. However, due to unexpected inaccurate FE results on the stress-strain domain, making R equal to 1 doesn't help to show general tendency of stress-strain relationship after yielding. In order to fit the best curve on data sets, these sets are smoothed by using methods recommended by MATLAB. For example, DATA SET 7 and DATA SET 8 are smoothen versions of DATA SET 6 and 2. The calculated m , α and HRR singular field quantities for every power curve fit are tabulated in Table 3.8. Eight different stress fields are determined analytically by using these parameters and contour-dependent J-integrals are drawn using ABAQUS.

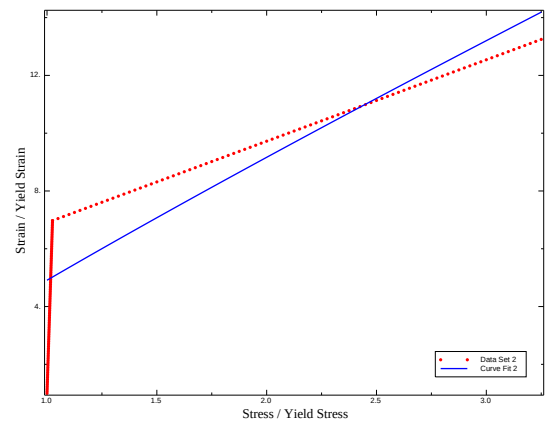
Table 3.8. Curve fitting results

Fitting Number	σ_y [MPa]	ε_y	α	m	$\tilde{\sigma}_{\theta\theta}$	$\tilde{\sigma}_{rr}$	I_m
1	410	0.045	0.9688	0.5665	0.084	1.276	4.391
2	400	0.006452	4.911	0.9001	0.706	1.051	4.306
3	400	0.006452	4.561	1.230	0.905	0.927	4.224
4	400	0.006452	6.153	0.651	0.380	1.203	4.369
5	410	0.045	6.507	0.593	0.171	1.252	4.385
6	400	0.006452	1.767	58.17	1.152	0.328	2.580
7	400	0.006452	1.219	37.16	1.151	0.352	2.670
8	400	0.006452	4.429	0.919	0.723	1.042	4.301

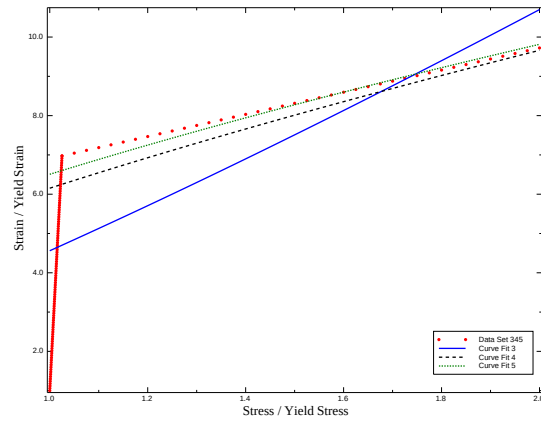
In Tables 3.9, 3.10 and 3.11, the analytical HRR singular field solution and finite elements solution of SMA's stress distribution around crack tip are compared and the parameter $\Re$, that represents the ratio of $\frac{\sigma_{yy}^{HRR}}{\sigma_{yy}^{FEA}}$, is found in terms of distance r



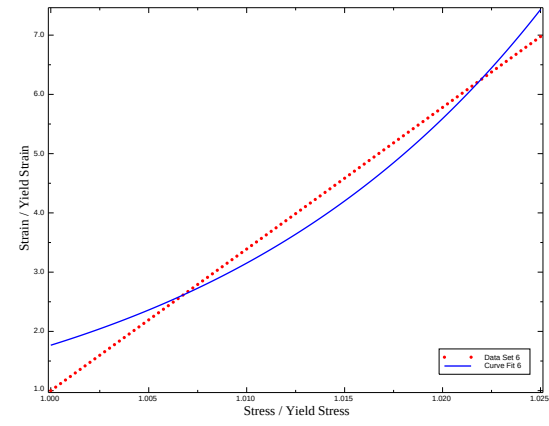
(a) DATA SET 1



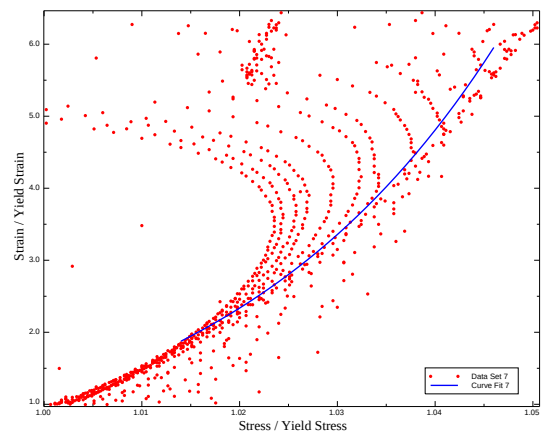
(b) DATA SET 2



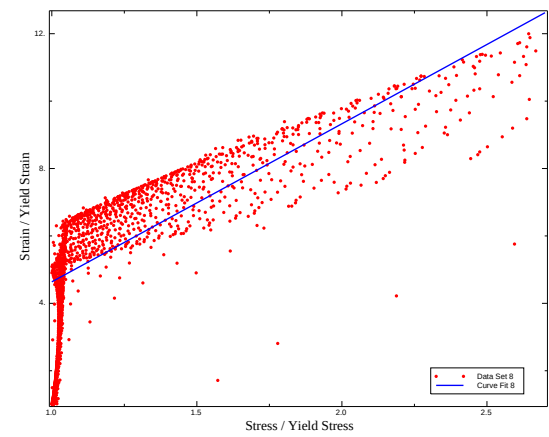
(c) DATA SET 345



(d) DATA SET 6



(e) DATA SET 7



(f) DATA SET 8

Figure 3.11. Power curve fitting on $\frac{\varepsilon}{\varepsilon_y}$ vs. $\frac{\sigma}{\sigma_y}$

Table 3.10. Stress field results using HRR and FE ($\theta = 90^\circ$, $\sigma_0 = 80$ MPa)

$\theta = 90^\circ$, $\sigma_0 = 80$ MPa			Fit 1		Fit 2		Fit 3		Fit 4	
r	J	σ_{yy}^{FEA}	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$
0.10	80.19	2501.18	2306.86	0.922	1729.36	0.691	1290.43	0.516	2119.68	0.847
0.20	80.12	1112.58	1481.17	1.331	1200.20	1.079	945.30	0.850	1392.06	1.251
0.30	79.94	892.17	1141.81	1.280	968.46	1.086	787.38	0.883	1087.43	1.219
0.40	79.75	761.12	948.77	1.247	831.32	1.092	691.32	0.908	912.15	1.198
0.50	79.38	664.24	820.35	1.235	737.39	1.110	624.19	0.940	794.56	1.196
0.60	78.80	604.49	726.85	1.202	667.38	1.104	573.32	0.948	708.35	1.172
0.70	78.21	561.31	655.58	1.168	612.94	1.092	533.22	0.950	642.26	1.144
0.80	77.54	526.46	598.69	1.137	568.74	1.080	500.28	0.950	589.25	1.119
0.90	76.81	505.33	551.97	1.092	531.89	1.053	472.53	0.935	545.52	1.080
1.00	76.06	482.55	512.88	1.063	500.64	1.037	448.77	0.930	508.78	1.054
1.20	75.15	466.88	453.02	0.970	451.95	0.968	411.30	0.881	452.26	0.969
$\theta = 90^\circ$, $\sigma_0 = 80$ MPa			Fit 5		Fit 6		Fit 7		Fit 8	
r	J	σ_{yy}^{FEA}	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$
0.10	80.19	2501.18	668.60	0.267	140.92	0.056	158.67	0.063	1786.00	0.714
0.20	80.12	1112.58	432.40	0.389	139.27	0.125	155.81	0.140	1243.92	1.118
0.30	79.94	892.17	334.75	0.375	138.32	0.155	154.15	0.173	1005.84	1.127
0.40	79.75	761.12	279.00	0.367	137.64	0.181	152.99	0.201	864.70	1.136
0.50	79.38	664.24	241.81	0.364	137.11	0.206	152.07	0.229	767.90	1.156
0.60	78.80	604.49	214.67	0.355	136.67	0.226	151.32	0.250	695.66	1.151
0.70	78.21	561.31	193.95	0.346	136.30	0.243	150.68	0.268	639.45	1.139
0.80	77.54	526.46	177.38	0.337	135.97	0.258	150.12	0.285	593.77	1.128
0.90	76.81	505.33	163.76	0.324	135.68	0.269	149.62	0.296	555.66	1.100
1.00	76.06	482.55	152.34	0.316	135.42	0.281	149.17	0.309	523.32	1.084
1.20	75.15	466.88	134.83	0.289	134.97	0.289	148.41	0.318	472.90	1.013

Table 3.11. Stress field results using HRR and FE ($\theta = 0^\circ$, $\sigma_0 = 40$ MPa)

$\theta = 0^\circ$, $\sigma_0 = 40$ MPa			Fit 7		Fit 8	
r	J	σ_{yy}^{FEA}	σ_{yy}^{HRR}	$\Re$	σ_{yy}^{HRR}	$\Re$
0.10	17.37	880.314	498.69	0.567	905.47	1.029
0.20	17.16	456.043	489.56	1.074	636.97	1.397
0.30	16.57	456.847	483.94	1.059	511.36	1.119
0.40	16.45	459.908	480.22	1.044	441.41	0.960
0.50	16.77	460.942	477.66	1.036	398.74	0.865
0.60	17.06	461.038	475.59	1.032	367.21	0.796
0.70	17.27	460.651	473.83	1.029	342.12	0.743
0.80	17.43	459.977	472.29	1.027	321.55	0.699
0.90	17.57	459.125	470.93	1.026	304.42	0.663
1.00	17.67	458.203	469.70	1.025	289.66	0.632
1.20	17.81	456.306	467.56	1.025	265.53	0.582
1.40	17.95	454.161	465.77	1.026	246.84	0.544
1.60	18.05	451.580	464.21	1.028	231.58	0.513

from crack tip. As mentioned earlier, the HRR results are invalid at the crack tip and points too close to the tip, so deep investigation in these fields is out of scope. Besides, stresses are not calculated in regions, where martensite fraction ξ is zero or negligibly small, because the material in fully austenite region is linearly elastic.

First impression from the tables is that there is no consistency in $\mathfrak{R}$ results for the whole domain of r and θ . For example, analytical solution by using Fit 1 parameters gives reasonable results for θ of 90° , whereas for θ of 0° the error between numerical and analytical result becomes giant. An explanation for this situation may be that a circular shape assumption of plastic zone is made in the formulation, whereas the shape memory alloy has a kidney-shaped transformation zone. By looking at results in Table 3.10 and Figure 3.13, the ratio $\mathfrak{R}$ remains in a fixed range of $[0.8:1.2]$ for curves describing the whole transformation zone and fully martensite phase (fit 1,2,3,4,8), thus we deduce that HRR solution might be a good approximation method, if stresses on $\theta = 90^\circ$ path are searched for.

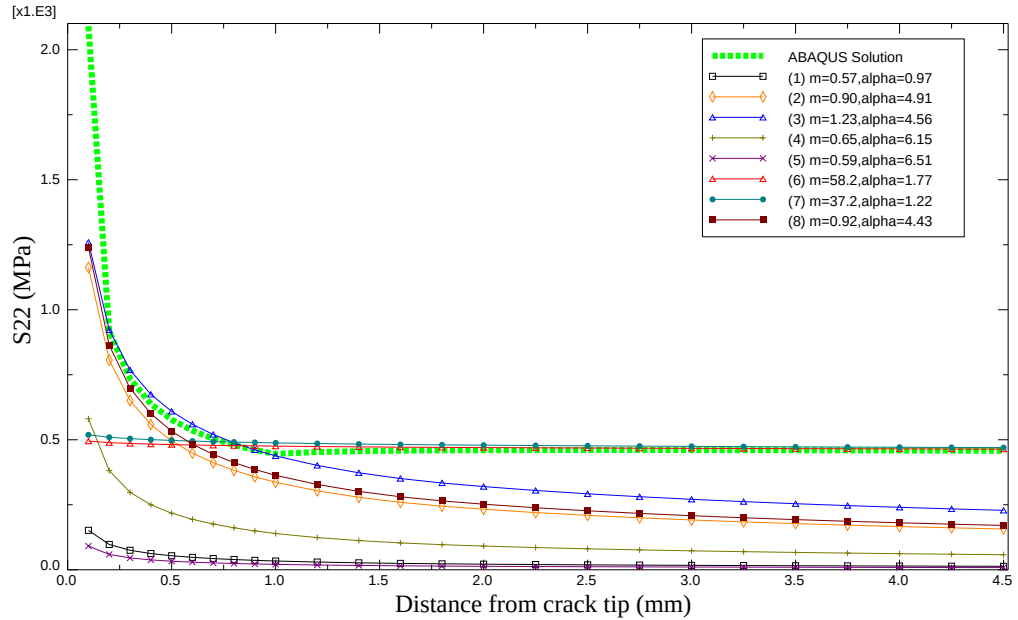


Figure 3.12. FEA vs. curve fitting results : Stress distribution ($\theta = 0^\circ$, $\sigma_0 = 80$ MPa)

Next, we concentrate on results of HRR solution for θ of 0° and two different loads. To prove the validity of the method, keeping the stress ratio $\mathfrak{R}$ in the range of $[0.95:1.05]$ looks acceptable. Within these limits, following curve fits are chosen as the

best ones :

- Curve Fit 3 : $\Re=[0.98:1.05]$ in $r=[0.20:1.00]$
- Curve Fit 6 : $\Re=[0.94:1.06]$ in $r=[0.70:3.75]$
- Curve Fit 7 : $\Re=[0.98:1.05]$ in $r=[0.70:3.75]$
- Curve Fit 8 : $\Re=[0.90:0.95]$ in $r=[0.20:0.60]$

Noting that the fits 7 and 8 are related with smoothed data sets, only results of them are focused under the applied remote load of 80 MPa and 40 MPa (see Figures 3.14 and 3.15).

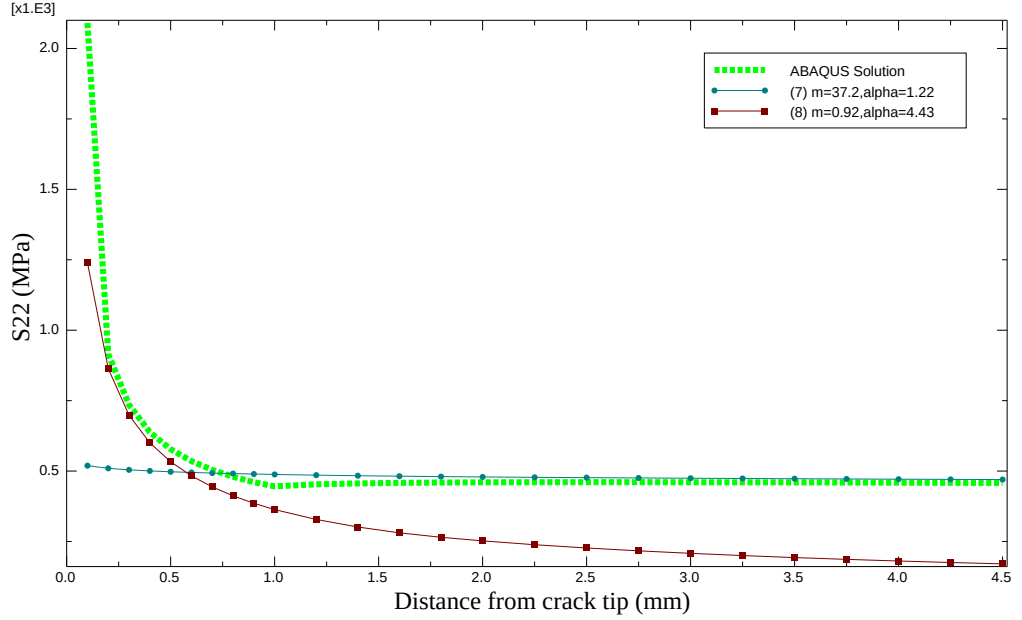


Figure 3.13. FEA vs. curve fitting results : Stress distribution ($\theta = 90^\circ$, $\sigma_0 = 80$ MPa)

The main difference between the last two curve fits is the normalized strain-stress range, that they are representing. The first one is limited between martensite transformation start and finish stresses, therefore it is expected that HRR solution by using this curve is suitable for finding stresses at relative distant points from crack tip in the transformation zone. The latter curve tries to pass on possibly many data points between martensite formation starting stress and actual yield stress of the material (in this case 1200 MPa). The fitted curve contains information from both transformation

and fully martensite zone, however phase transformation zone constitutes only $\sim 1.2\%$ of the curve. Hence, it is expected that inserting constants of fit 8 into the analytical solution should give more accurate results in the homogenous martensite zone close to the crack tip.

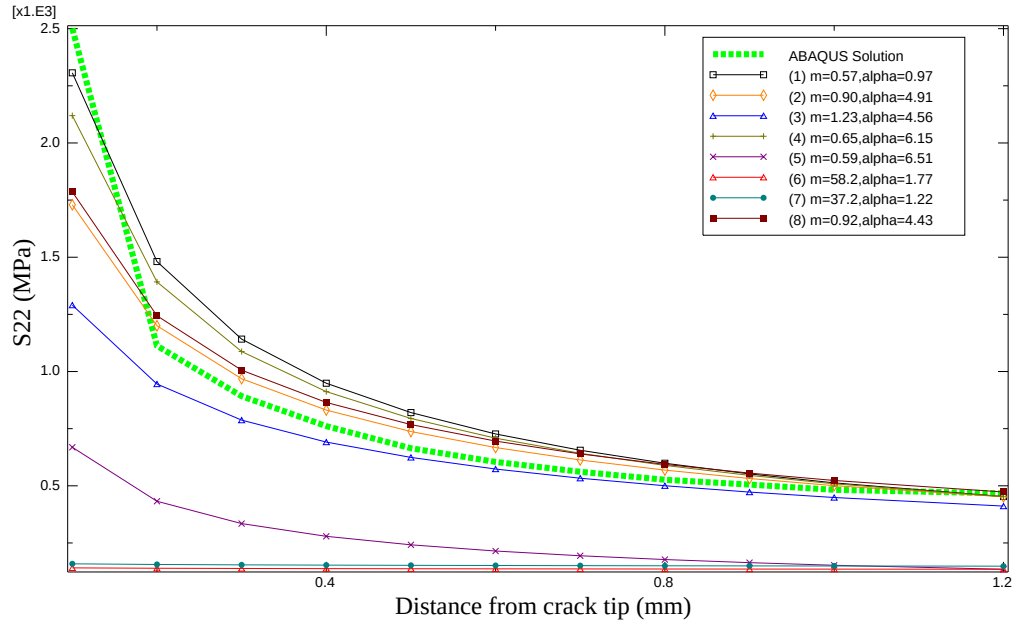


Figure 3.14. FEA vs. curve fitting 7-8 : Stress distribution ($\theta = 0^\circ$, $\sigma_0 = 80$ MPa)

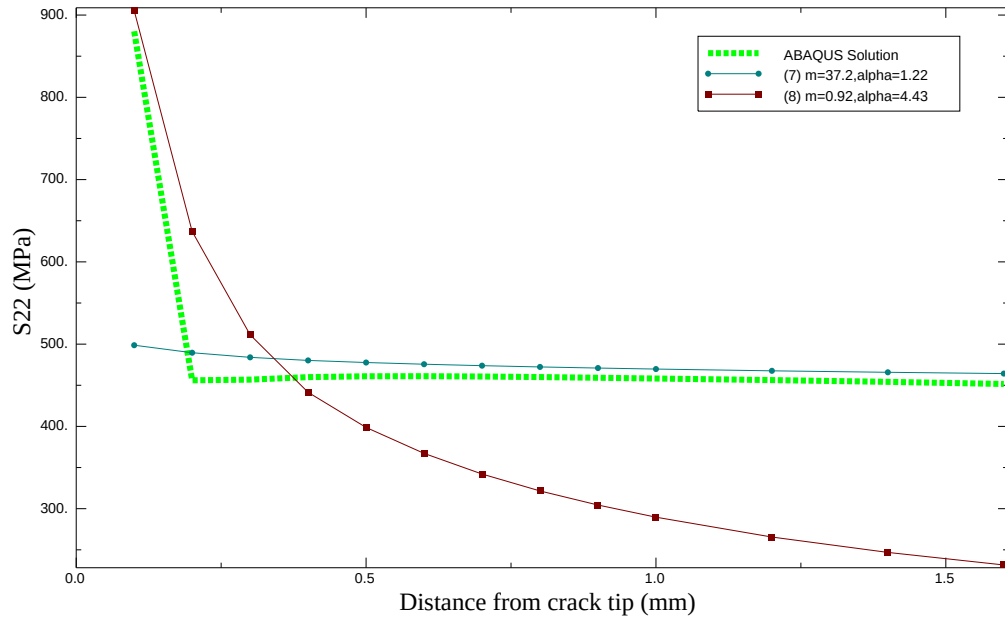


Figure 3.15. FEA vs. curve fitting 7-8 : Stress distribution ($\theta = 0^\circ$, $\sigma_0 = 40$ MPa)

The Figures 3.14 and 3.15, which compare HRR solution method with ABAQUS outputs, justify our predictions. It can be easily seen that the analytical solutions are tangent to the finite elements solution at previously defined locations for both loading conditions. Curve 7 coincides with ABAQUS solution as r increases until the end of transformation zone. This is logical because the condition $m \rightarrow \infty$ leads to perfect plasticity, which suits in description of uniaxial stress-strain behavior during phase transformation. On the other hand, Curve 8 will become tangent as r goes to 0, where fully martensite zone appears. If m is equal to unity, the stress field equation (see Equation 3.24) satisfies the $1/\sqrt{r}$ singularity, which can be used to calculate the stress distribution in both fully austenite and martensite regions. The hardening coefficient of curve 8 is 0.919, which is close to unity, therefore stress results of FE solution and analytical solution match in homogenous region around the crack tip.

In order to better illustrate this phenomenon, stress ratios $\mathfrak{R}$ in terms of distance r are plotted in Figures 3.16 and 3.17. The ratio of curve fit 7 equals to unity if $r > 1.0$ and $r > 0.25$ under loads of 80 and 40 MPa, respectively. On the other hand, in the remaining section of plots, curve fit 8 seems to be more successful. Therefore, the combination of both curve fits may provide a constant $\mathfrak{R}$ and the results of FE and analytical solution proposed in this section would get closer.

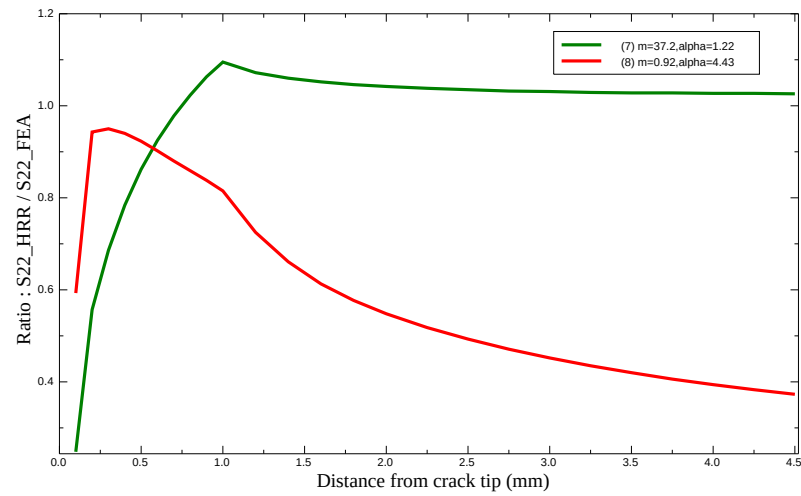


Figure 3.16. $\mathfrak{R} = \frac{\sigma_{yy}^{HRR}}{\sigma_{yy}^{FEA}}$ in terms of r for $\theta = 0^\circ$ and $\sigma_0 = 80$ MPa

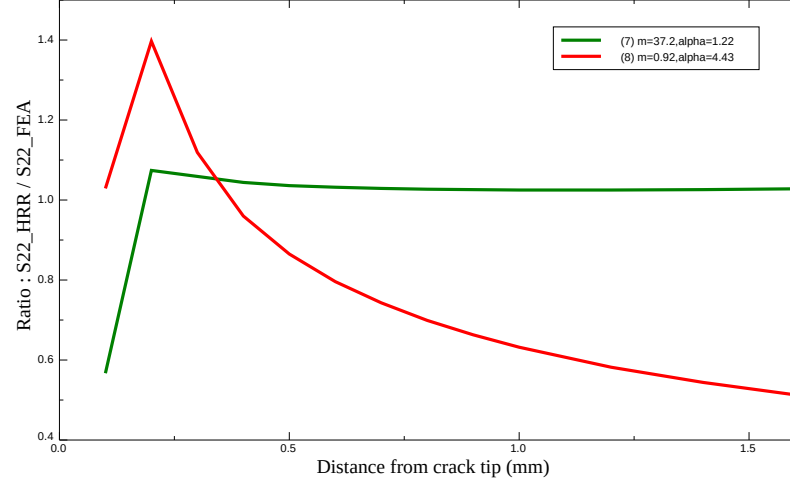


Figure 3.17. $\mathfrak{R} = \frac{\sigma_{yy}^{HRR}}{\sigma_{yy}^{FEA}}$ in terms of r for $\theta = 0^\circ$ and $\sigma_0 = 40$ MPa

In short, one can conclude that HRR singular field solution is an established analytical method, that can be used in determination of stress and strain fields in phase transformation zone of shape memory alloys. If there is an opportunity to describe the SMA's stress-strain by using a single power curve, the results in terms of the distance from crack tip will be correct for whole domain. However due to different uniaxial behavior at different locations on the stress-strain curve, at least two distinct power functions with different hardening parameter m are needed to determine these important mechanical properties analytically. Nevertheless, the validity of HRR method is an important sign of nonlinear elastic behavior of shape memory alloys. Furthermore, the plasticity-based solution techniques used in determination some fracture parameters like stress intensity factor may be implemented and their results will be compared with that of conventional linear elastic approaches.

3.6.4. Relationship between J-integral and COD

Based on the power-law description; Hutchinson, Rice and Rosengren showed a stress and strain field formulation in the vicinity of a crack. Similarly, the crack opening profile $\delta(x)$ can also be expressed. Shih [101] tried to find a coefficient d_m , which will relate J to crack tip opening displacement. This coefficient depends on material deformation properties, on the other hand it should be independent of crack

configuration under small-scale yielding conditions. In his study, the crack face vertical displacement is formulated as :

$$\delta = \frac{2\alpha\sigma_y}{E} \left(\frac{EJ}{\alpha\sigma_y^2 I_m} \right)^{\frac{m}{m+1}} r^{\frac{1}{1+m}} \tilde{u}_y(m) \quad (3.27)$$

where $\tilde{u}_y$ is a displacement quantity depending on hardening parameter. The crack tip opening displacement is defined as the opening distance between the intercept of two 45° lines, so that equation 3.27 takes the form of :

$$\delta_t = d_m \frac{J}{\sigma_y} \quad (3.28)$$

and

$$d_m = \left(\frac{\alpha\sigma_y}{E} \right)^{\frac{1}{m}} (\tilde{u}_x + \tilde{u}_y)^{\frac{1}{m}} \frac{2\tilde{u}_y}{I_m} \quad (3.29)$$

First, Shih's suggestion for crack opening profile determination will be implemented in elastic-plastic material. Remember that, in Section 2.2 crack tip opening displacement was determined for bilinear case. Here, finite element results of the power-law hardening material will be used to check the accuracy of the relationship. The displacement u_y along the crack edge is plotted in Figure 3.18 and it is observed that finite element results and Shih's equation results are very close. Thus, the validity of Equation 3.27 is proven for power-law hardening materials.

Next, crack tip opening δ_t will be calculated for shape memory alloys assuming that SMA's global behavior is like power-law hardening materials' under uniaxial tensile loading. Especially, assuming perfect-plasticity under small scale yielding conditions give rewarding results in stress distribution determination of transformation zone. For this particular reason, Equation 3.28, which was formulated in basis of HRR singular field quantities, is used to find crack openings. Required parameters are the yield stress σ_y , J and the coefficient d_m . The yield stress will be equal to the equivalent transformation starting stress of 400 MPa. Variation of d_m with hardening parameter

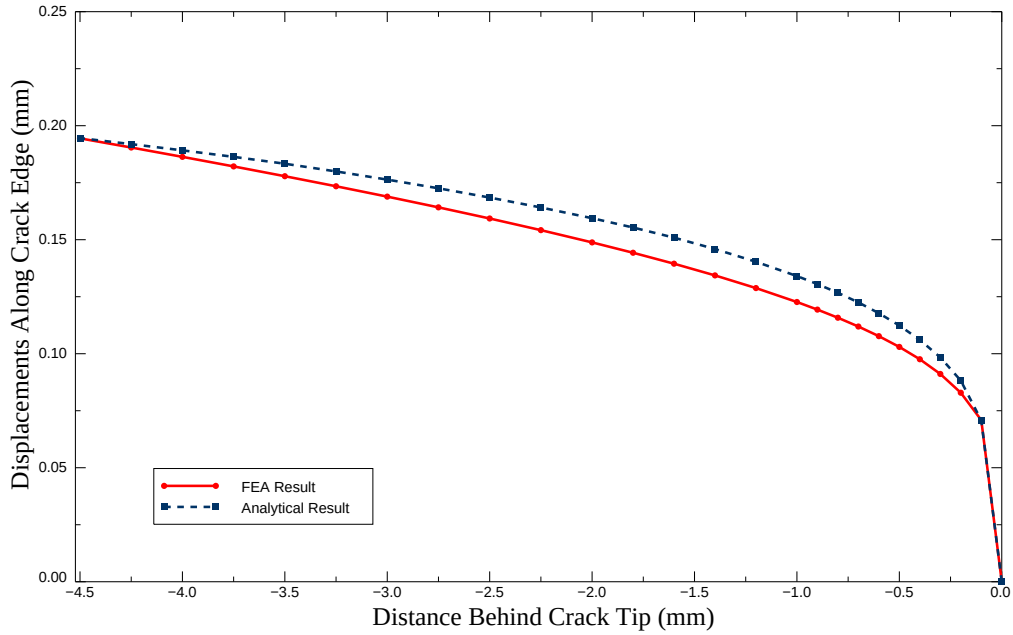


Figure 3.18. Deformed crack profile for power-law hardening material

m is plotted in Shih's study [101] and it is 0.98 for our case. Since J is contour dependent in shape memory alloys, the value of second contour, which is located in fully martensite region, is employed in calculations. Not only crack tip opening displacements are found. It is known that, at low stress levels Dugdale strip-yield model gives the relation $\delta_t = K^2 / (E\sigma_y)$, so that there is an opportunity to find a stress intensity factor K_I by implementation of HRR method. Results are summarized in Table 3.12. The selected geometry and material properties are same as that in Section 3.5.

In the previous chapter, it had been stressed that ΔK results are crucial in numerical prediction of fatigue crack growth. After finding K_I by using HRR approach, they should be compared with results of closed form analytical solution (Xiong et al.'s proposal) and strain energy difference method (named as FEA method in this study) given in Section 3.5. It is seen that at smaller initial crack lengths, Xiong's approach gives the relatively biggest stress intensity, whereas the other two FEA based approaches are more conservative at greater loads and a/W ratios. Focusing on outputs of last methodology, it can be deduced that the approach is consistent in itself, however its usability in stress intensity determination has not yet been proven. After a quick look on the comparison Table 3.12, this method seems to undertake a conciliatory

Table 3.12. Parametric study on crack tip opening displacement and stress intensity factor based on HRR method

[MPa] σ_0	a/W	[MPa.mm] J^M	[mm] δ_t	[MPa. $\sqrt{mm}$] K_I^{HRR}	ND K_I^{HRR}	Comparison of Results
80	0.25	18.28	0.04479	627.79	1.56947	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.30	28.00	0.06860	776.97	1.77318	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.35	43.43	0.10640	967.65	2.04454	$K_I^{FEA} > K_I^* > K_I^{HRR}$
	0.40	65.74	0.16106	1190.53	2.35299	$K_I^{HRR} > K_I^{FEA} > K_I^*$
100	0.25	29.71	0.07279	800.34	1.60068	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.30	45.57	0.11165	991.21	1.80969	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.35	70.65	0.17309	1234.19	2.08615	$K_I^{FEA} > K_I^* > K_I^{HRR}$
	0.40	112.80	0.27636	1559.48	2.46575	$K_I^{FEA} > K_I^{HRR} > K_I^*$
120	0.25	44.46	0.10893	979.06	1.63177	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.30	69.36	0.16993	1222.87	1.86053	$K_I^* > K_I^{FEA} > K_I^{HRR}$
	0.35	110.10	0.26975	1540.70	2.17022	$K_I^{FEA} > K_I^{HRR} > K_I^*$

mission between suggestions discussed in previous chapters. Nevertheless, it is a simple applicable, conservative approximation method and will be utilized in next chapter.

3.7. Elastic Strain Energy and Transformation Strain Energy

In every analysis, it has been assumed that the process is adiabatic and the initial temperature remains constant. Since there is no energy loss to heat, the external work leading to deformation is stored as internal energy in the medium. Generally, internal energy is composed of three measures and written as :

$$E_I = E_S + E_P + E_C \quad (3.30)$$

where E_S is elastic strain energy, E_P is the energy dissipated by plasticity and E_C is the energy dissipated by time-dependent deformation such as creep, swelling or viscoelasticity. There will be no time-dependent deformation in the model, so the last

term is zero. The other energy measures are calculated by following equations :

$$E_S = \int_0^t \left(\int_V \sigma^c : \dot{\varepsilon}^{el} dV \right) d\tau \quad (3.31)$$

$$E_P = \int_0^t \left(\int_V \sigma^c : \dot{\varepsilon}^{pl} dV \right) d\tau \quad (3.32)$$

where σ^c is the stress derived from the user-specified constitutive equation. Remembering that no plasticity is defined in construction of SMA finite elements model, the second term in the equation should also vanish. Considering the plasticity based transformation models of shape memory alloys, the energy term E_P indicates transformation strain energy and ε^{pl} in Equation 3.32 will be replaced by transformation strain ε^{tr} . ABAQUS supplies elastic (ALLSE) and transformation (ALLPD) strain energies for every single element and for the whole model. These energy measures are plotted in terms of a/W range between 0.1 and 0.2 under an applied load of 100 MPa in Figures 3.19 and 3.20.

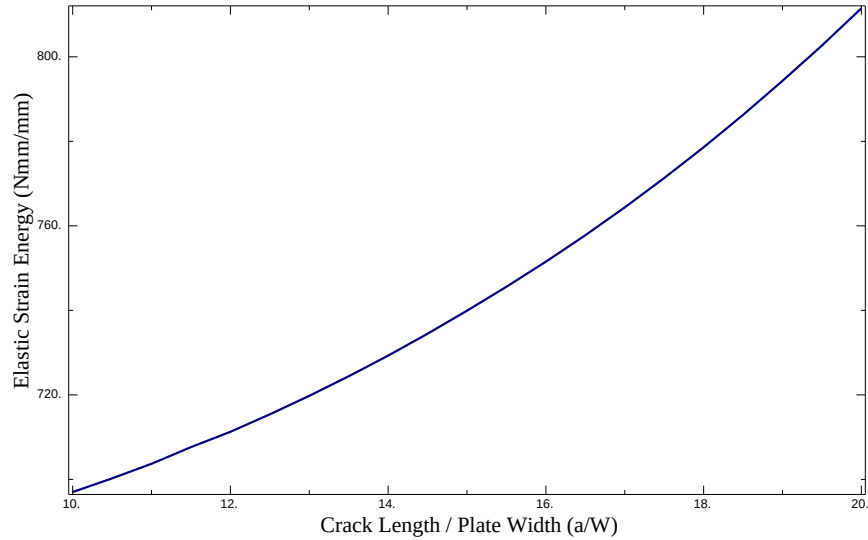


Figure 3.19. Elastic strain energy in terms of crack length

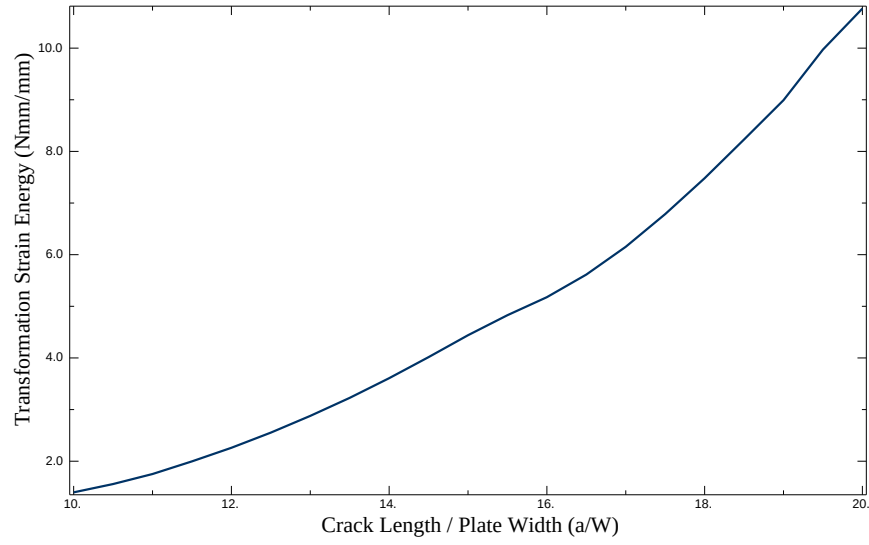


Figure 3.20. Transformation strain energy in terms of crack length

Table 3.13. ALLSE, ALLPD and total internal energy in terms of crack length a
under $\sigma_0 = 100$ MPa

[mm]	[Nmm/mm]			[mm]	[Nmm/mm]		
a	E_S	E_P	E_I	a	E_S	E_P	E_I
10.0	697.005	1.397	698.402	15.5	745.591	4.828	750.419
10.5	700.241	1.558	701.799	16.0	751.521	5.177	756.698
11.0	703.692	1.752	705.444	16.5	757.776	5.615	763.391
11.5	707.634	1.997	709.631	17.0	764.365	6.153	770.518
12.0	711.263	2.260	713.523	17.5	771.308	6.786	778.094
12.5	715.396	2.554	717.950	18.0	778.596	7.481	786.077
13.0	719.772	2.878	722.650	18.5	786.259	8.231	794.490
13.5	724.403	3.227	727.630	19.0	794.317	8.992	803.309
14.0	729.289	3.608	732.897	19.5	802.671	9.972	812.643
14.5	734.475	4.016	738.491	20.0	811.485	10.766	822.251
15.0	739.925	4.440	744.365				

4. FATIGUE CRACK CLOSURE OF SHAPE MEMORY ALLOYS

Crack closure concept has been used in correlation of crack-growth rate data and for predicting crack growth under variable amplitude loading. In Chapter 2, the effect of plasticity on fatigue crack closure was revealed. In the FE material model used by ABAQUS, shape memory alloys have been considered as elastic and all strain is expected to be recovered during unloading. Unless defining a plastic region around crack tip, plasticity induced fatigue crack closure will not appear in SMA's for variable amplitude loading. Another process must be significant in development of closure. The only triggering mechanism in the model is phase transformation from austenite to martensite and vice versa. Hence, the concept of transformation-induced crack closure (TICC) will be first discussed. Since we will consider only Mode I cracks and no friction between crack faces will be defined, investigation of roughness-induced closure development will be out of scope.

4.1. Transformation Induced Crack Closure

In cracked plates made of certain metastable alloys, stresses at crack tip can lead to phase transformation. If there is a volume increase as a result of transformation, closure can develop as the crack penetrates the transformed region, and there can be retardation in fatigue crack growth rate [102]. Limited number of researches have been carried out to investigate this phenomenon. Mayer et al. [103] studied fatigue crack growth under constant and random loading for a metastable austenitic-bainitic steel and compared to a ferritic chromium steel at very low crack growth rates. They observed that high loads in a random loading-sequence cause large plastic deformation around crack tip, which results in phase transformation and increase of material volume around crack tip. This increase leads to a residual compressive stress field and crack closure, so that the measured crack growth rates are much lower than the results from linear Miner calculations. Launey et al. [104] worked on effects of volume changes and residual

stresses on fatigue behavior of Zr-Ti-Ni-Cu-Be bulk metallic glasses. They deduced that both plane strain and fatigue fracture toughness are degraded by a reduction in free volume, but fatigue growth rates are insensitive to free volume variations.

In SMAs, the effect of phase transformation on fatigue crack growth rate has not yet been discussed in depth in literature. The most detailed study was carried out by Robertson and Ritchie [13], who investigated fatigue crack growth in and toughness behavior of thin-walled Nitinol endovascular stents . With 50 Hz test frequency and for three different load ratios of $R=0.1$, 0.5 and 0.7 , fatigue crack growth in Nitinol tubes were measured and plotted in terms of ΔK and K_{max} (see Figure 4.1). Robertson and Ritchie demonstrated the dependency of crack growth rate on both ΔK and K_{max} and quantified this dependence. In addition to that, they observed a decrease in fatigue thresholds with increasing load ratio and they associated occurrence of such thresholds with crack closure mechanisms.

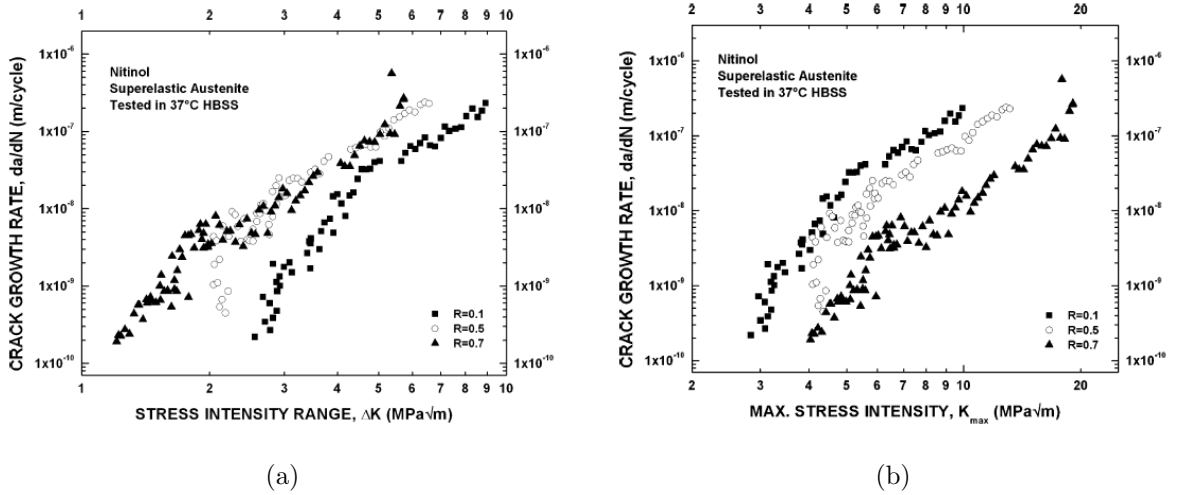


Figure 4.1. Variation in fatigue crack growth rates of Nitinol [13]

For making complete description of linear behavior shown in Figure 2.3, the fatigue growth rate equation (see Equation 2.1) is modified, where the rates are quantified in terms of maximum stress intensity and stress intensity range as shown below :

$$\frac{da}{dN} = C(1 - R)^n (K_{max})^n \Delta K^p \quad (4.1)$$

where C , n and p are Paris-law constants. In their study, Robertson and Ritchie tabulated these constants in terms of load ratio as shown in Table 4.1.

Table 4.1. Fatigue crack growth constants, n and p , and stress intensity thresholds, ΔK_{th} , determined experimentally

R	n	p	$C(1 - R)^n$ [m/cycle/(MPa $\sqrt{m}$) ^{m}]	ΔK_{th} [MPa $\sqrt{m}$]
0.1	1.87 / 1.53	2.56 / 2.68	1.52×10^{-11} / 1.60×10^{-11}	2.48 / 2.48
0.5	1.25 / 0.75	2.56 / 2.68	6.55×10^{-11} / 1.76×10^{-10}	2.02 / 1.86
0.7	1.06 / 1.03	2.56 / 2.68	6.87×10^{-11} / 7.21×10^{-11}	1.15 / 1.44

4.2. Cyclic Behavior of Nitinol

In Section 1.4, general cyclic behavior of shape memory alloys was described and experimental studies were presented. Observations state that some permanent strain remains after each cycle and they strains accumulate slowly (see Figure 1.5). According to Auricchio-Taylor-Lubliner constitutive theory on the other hand, which has been implemented in ABAQUS's superelastic material model, no detectable permanent strain should remain after complete unloading of the specimen and the transformation should be geometrically reversible. Unfortunately, this material model isn't able to fully describe uniaxial stress-strain behavior of Nitinol. The uniaxial tensile test results of 50Ni-50Ti given in Table 3.2, which have been used in this study, the 0.2 % offset plastic-yield point from the second elastic linear region (behavior of martensite) can be noticed at a strain of 8 % and the corresponding yield stress is around 1300 MPa [83]. Since any kind of crack in a medium is a stress-raiser, it is inevitable to have a plastic zone around crack tip of shape memory alloys. Therefore, the contribution of permanent plastic strain on fatigue crack growth should also be discussed.

To see the effect of plasticity on cyclic behavior, two separate computational models will be studied using ABAQUS. In both models, an edge cracked plate with both height and width equal to 100 mm and thickness equal to 1 mm is used. The crack length is fixed at 20 mm. Although symmetry exists in both cases, two separate plates

like the ones given in Figure 2.9 are used to establish interaction between crack faces. Four-node quadratic plane stress quadrilateral elements (CPS4) in a uniform mesh are used to discretize the geometry. Minimum element length around crack tip is taken as 0.1 mm, which obeys the mesh sufficiency criterion discussed in Chapter 2. Apart from fatigue studies carried out before, no node will be released in this section; only cyclic behavior of some mechanical parameters at a fixed crack length will be investigated. Maximum remote loads are 100 and 150 MPa and the load ratio is equal to -1. Both plates are kept at a constant temperature of 37°C.

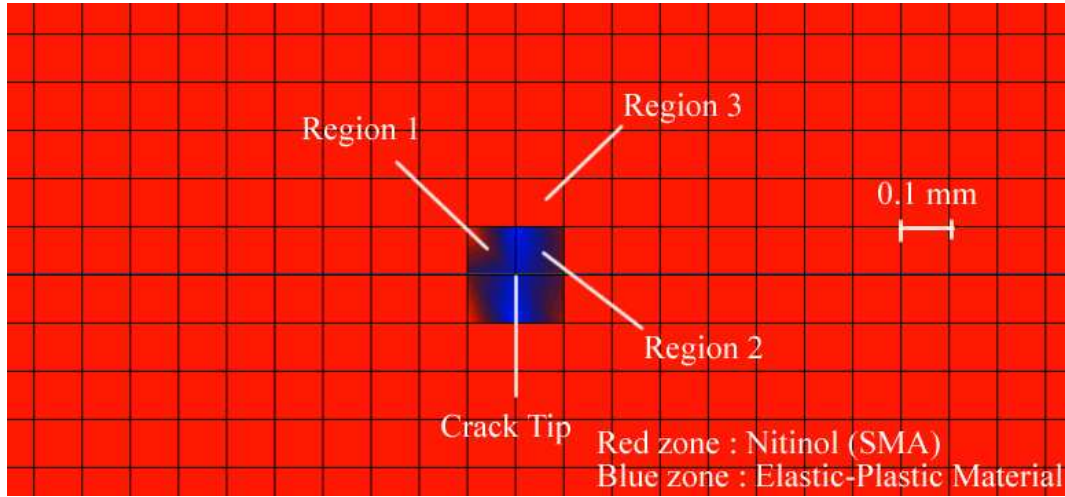


Figure 4.2. Material definition and mesh density around crack tip

The only difference between two mesh configurations is the material definition at the crack tip elements. In Model I, user-defined plastic zone is inserted around the crack tip as shown in Figure 4.2, where the material of four elements is changed from Nitinol to an elastic-plastic material with isotropic strain hardening. The yield stress of elastic-plastic material is 1300 MPa and the corresponding yield strain is 0.08. Model II doesn't contain any plastic region and cyclic response of Model I and Model II under same applied load will be compared. Finite element results are extracted from region 1, region 2 and region 3 on contour plot, which are demonstrated in Figure 4.2.

Stress and strain values, that are measured on the same integration point (IP3), are plotted in Figures 4.3 - 4.11 on each loading-unloading cycle **N**. Independent of the type of material and the remote load, tensile stresses and strains in region 2 should be

greater than those in regions 1 and 3. Changing material properties at the crack tip has a great influence on stress levels and saturation time of permanent strain accumulation. By definition of elastic-plastic material properties in four crack tip elements, a drop in stress values is observable in all three regions (see Figures 4.12 - 4.15). Related to the stress decrease, the size of the transformation zone becomes smaller and the material properties at some locations on the plate will change. The most remarkable result obtained from this computation is the uniaxial stress-strain behavior on each cycle in different regions. Whether it is minor or major, a shift due to strain accumulation is observed on every nine plots. For pure superelastic case, the development of residual stresses and considerable small permanent transformation strains can easily be noticed. Formations of these residual mechanical terms and their effects on fatigue crack closure will be investigated later. Similar results of stress-strain history in Figure 1.5 are obtained after entering plasticity in the model and they are demonstrated in Figures 4.6 - 4.8. It is obvious that, only a part of the transformation strain could be recovered during the first mechanical cycle and some permanent plastic strain would remain after each cycle. Equivalent plastic strain is about 2% at integration point 3 in region 2, after only initial cycle is completed. As proposed before, the accumulation of permanent strains is very slow.

Another important aspect is energy levels during cyclic loading. By looking at Figures 4.16 and 4.17, one can deduce that a hysteresis exists during unloading of Nitinol plate. The difference between elastic strain energy levels at maximum and minimum remote stresses is non-zero, therefore some energy is dissipated during transformation. Besides, at the end of the initial cycle the transformation strain energy doesn't return to its starting value of zero and a transformation strain energy accumulation appears after several cycles are completed.

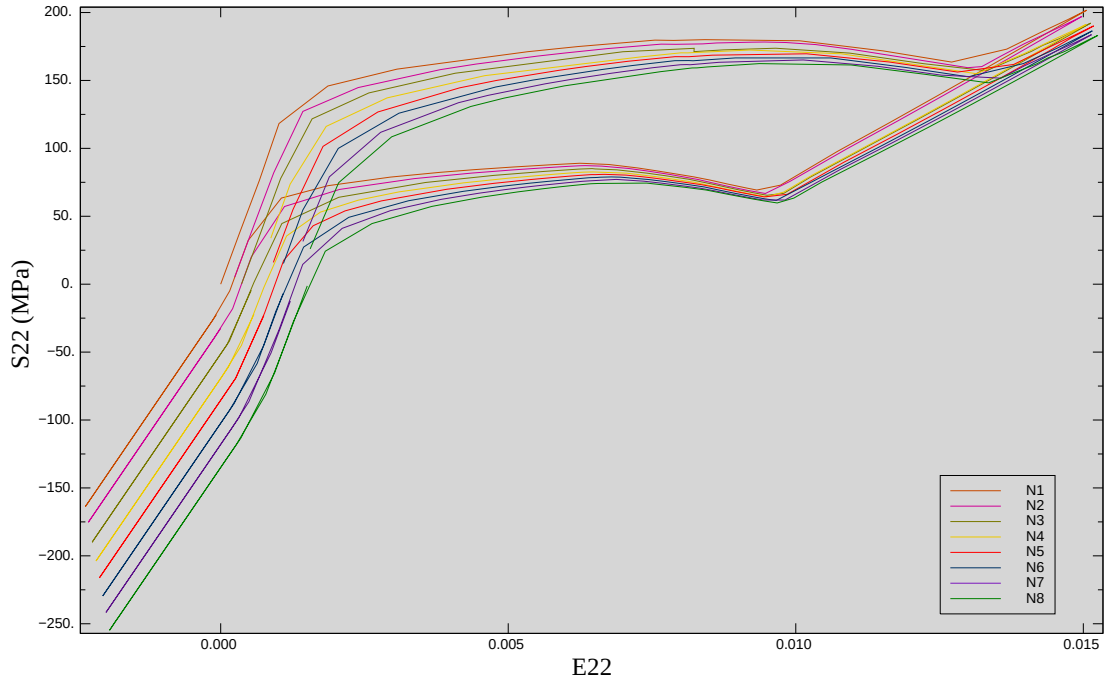


Figure 4.3. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 1 at integration point 3 (Model II, no plastic zone, $\sigma_0 = 150$ MPa)

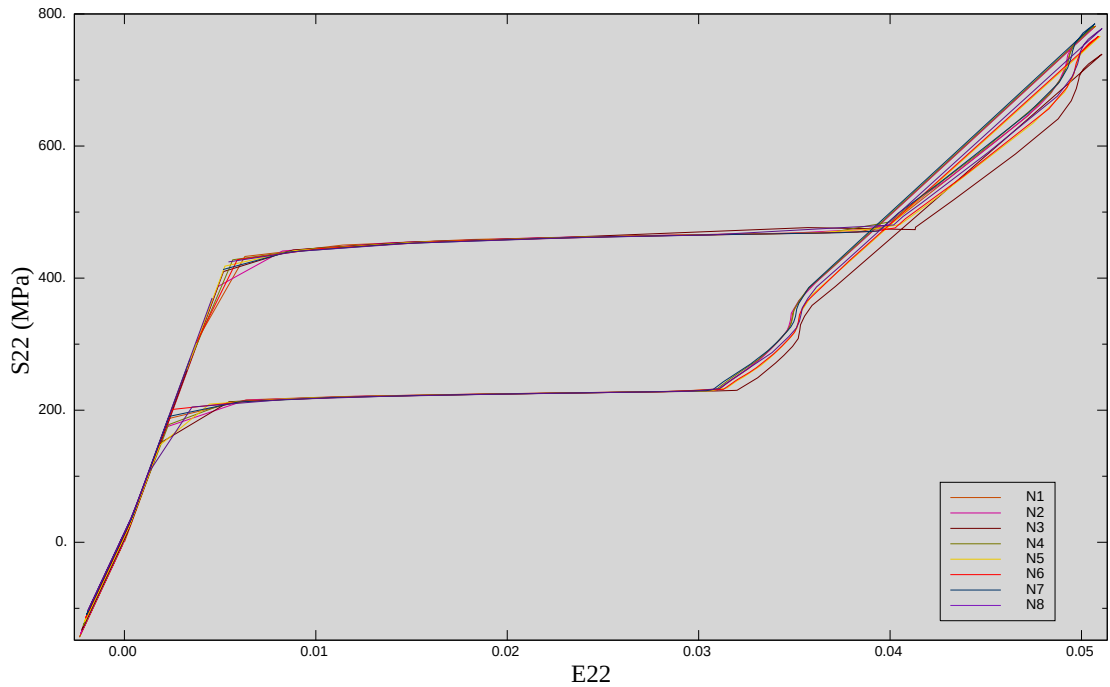


Figure 4.4. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 2 at integration point 3 (Model II, no plastic zone, $\sigma_0 = 150$ MPa)

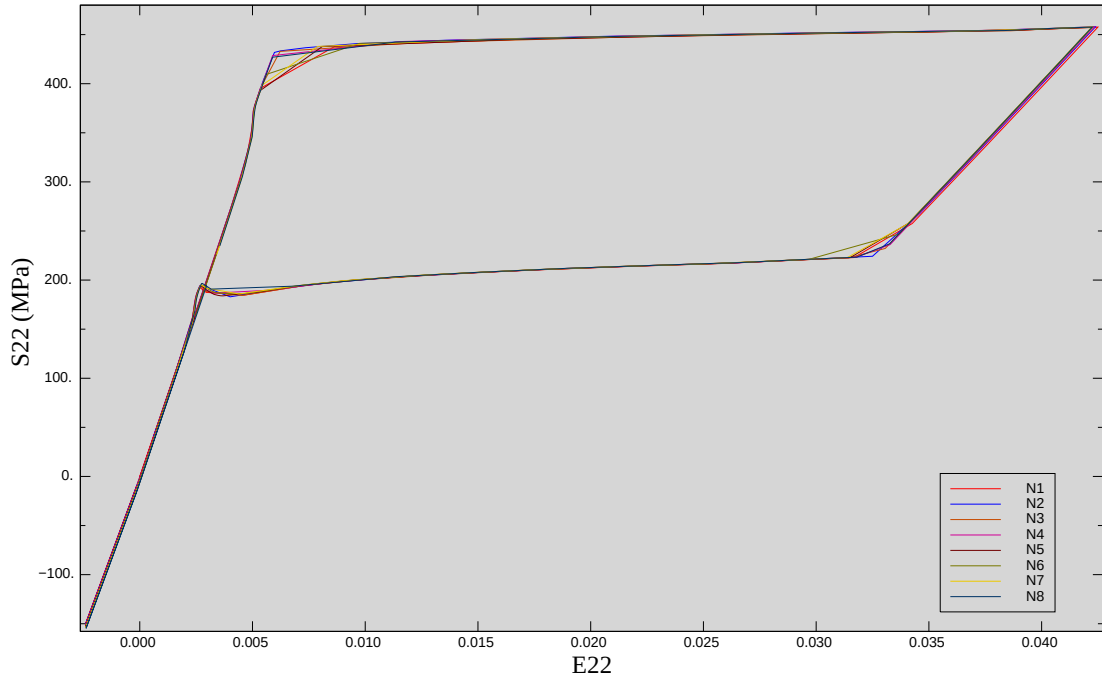


Figure 4.5. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 3 at integration point 3
(Model II, no plastic zone, $\sigma_0 = 150$ MPa)

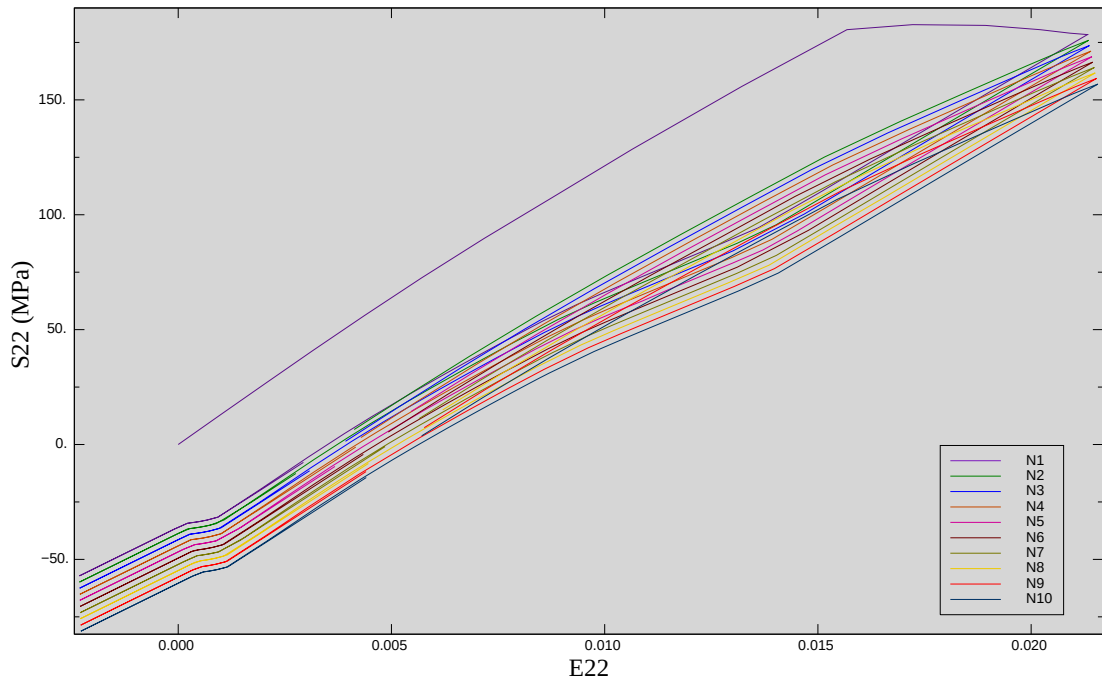


Figure 4.6. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 1 at integration point 3
(Model I, plastic zone inserted, $\sigma_0 = 150$ MPa)

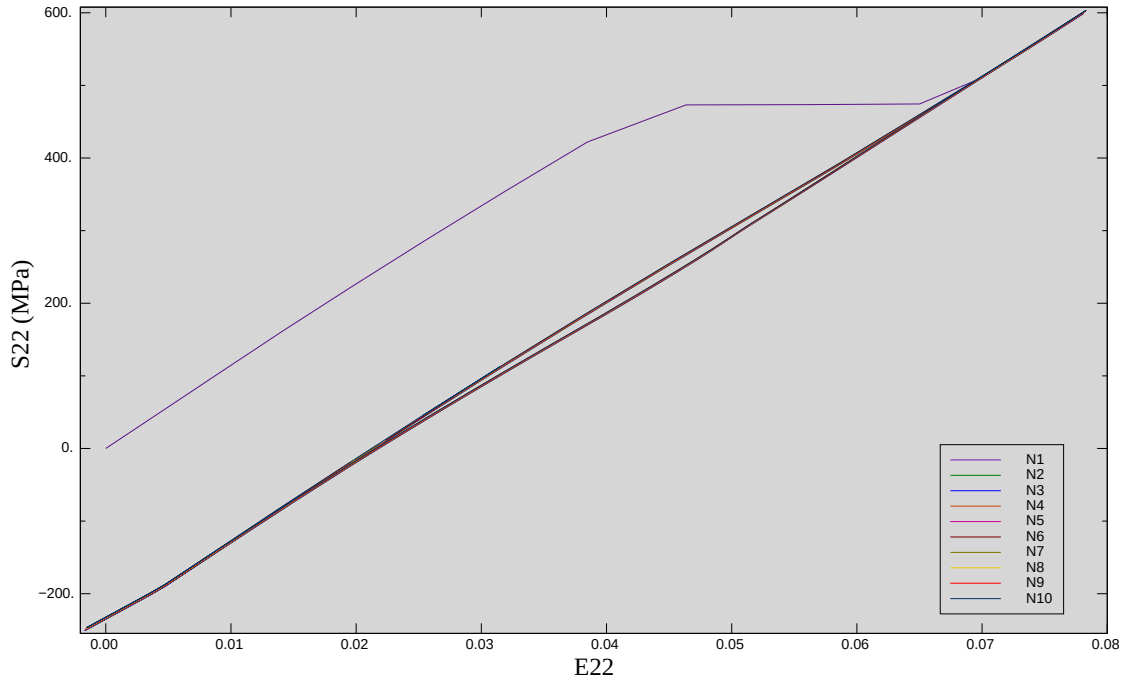


Figure 4.7. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 2 at integration point 3
(Model I, plastic zone inserted, $\sigma_0 = 150$ MPa)

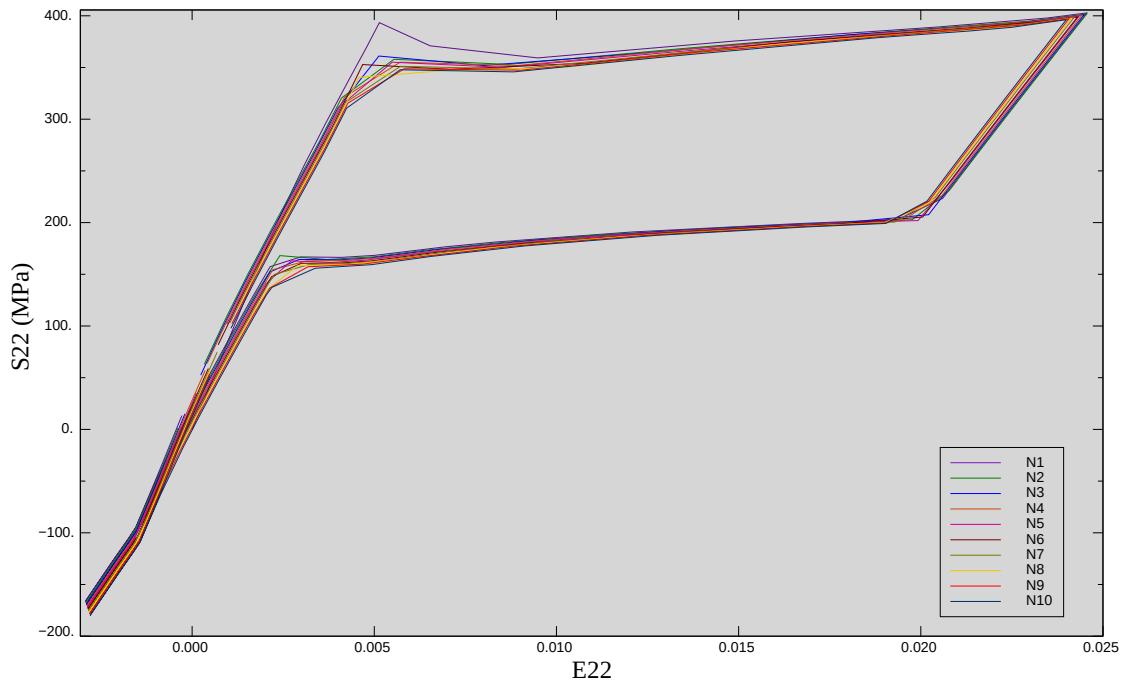


Figure 4.8. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 3 at integration point 3
(Model I, plastic zone inserted, $\sigma_0 = 150$ MPa)

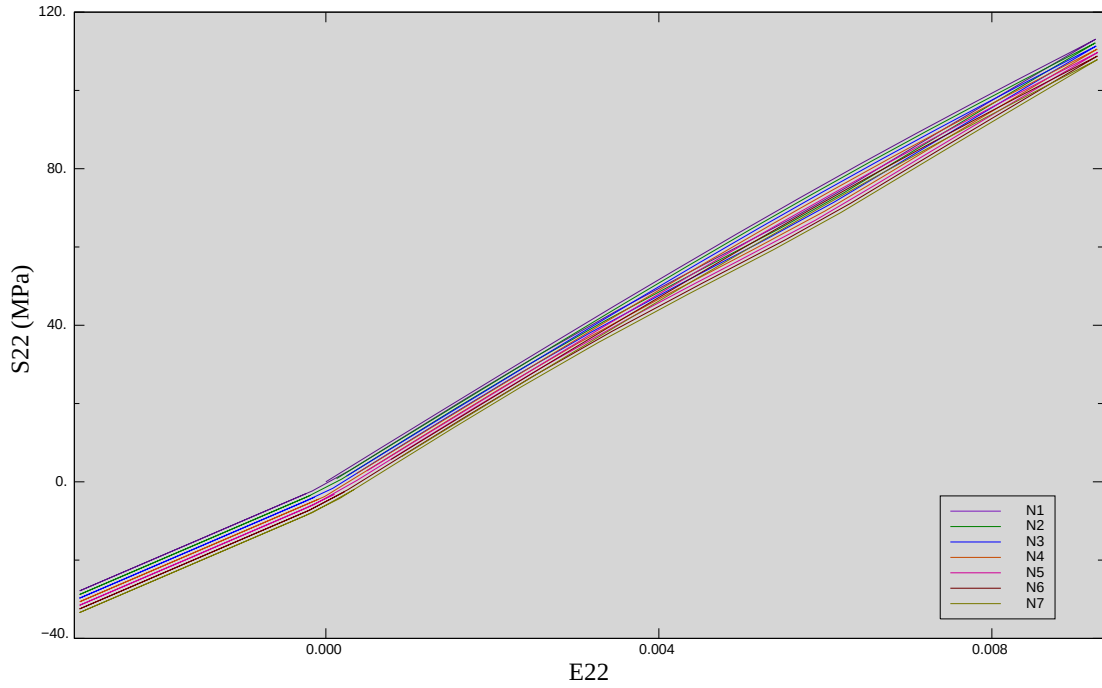


Figure 4.9. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 1 at integration point 3
(Model I, plastic zone inserted, $\sigma_0 = 100$ MPa)

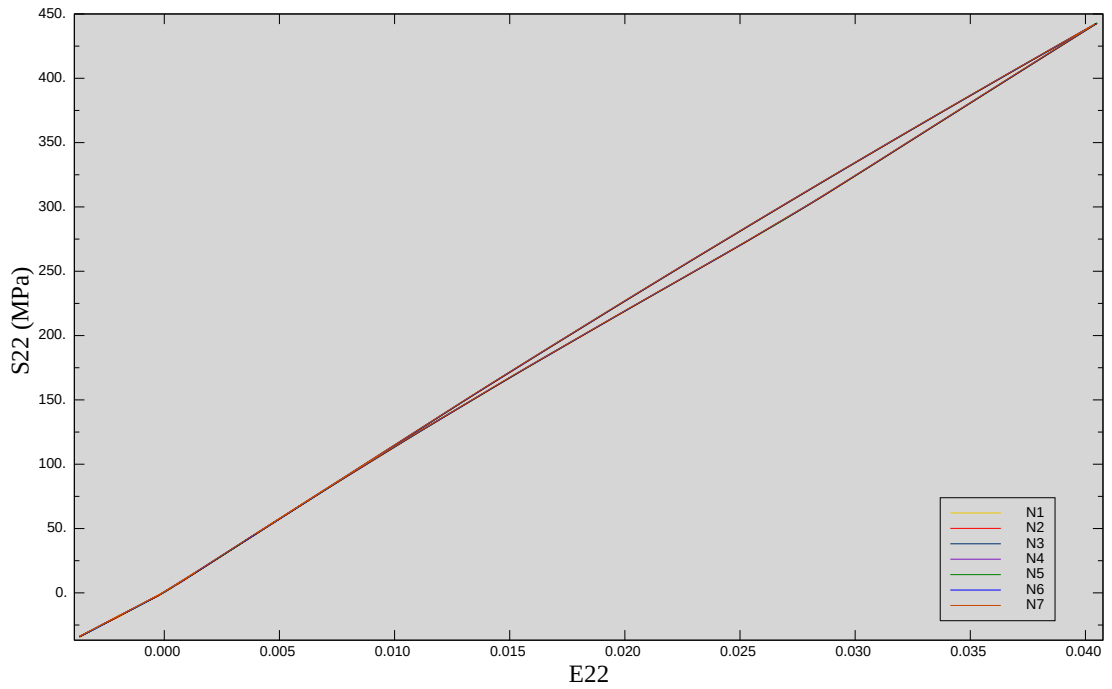


Figure 4.10. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 2 at integration point 3
(Model I, plastic zone inserted, $\sigma_0 = 100$ MPa)

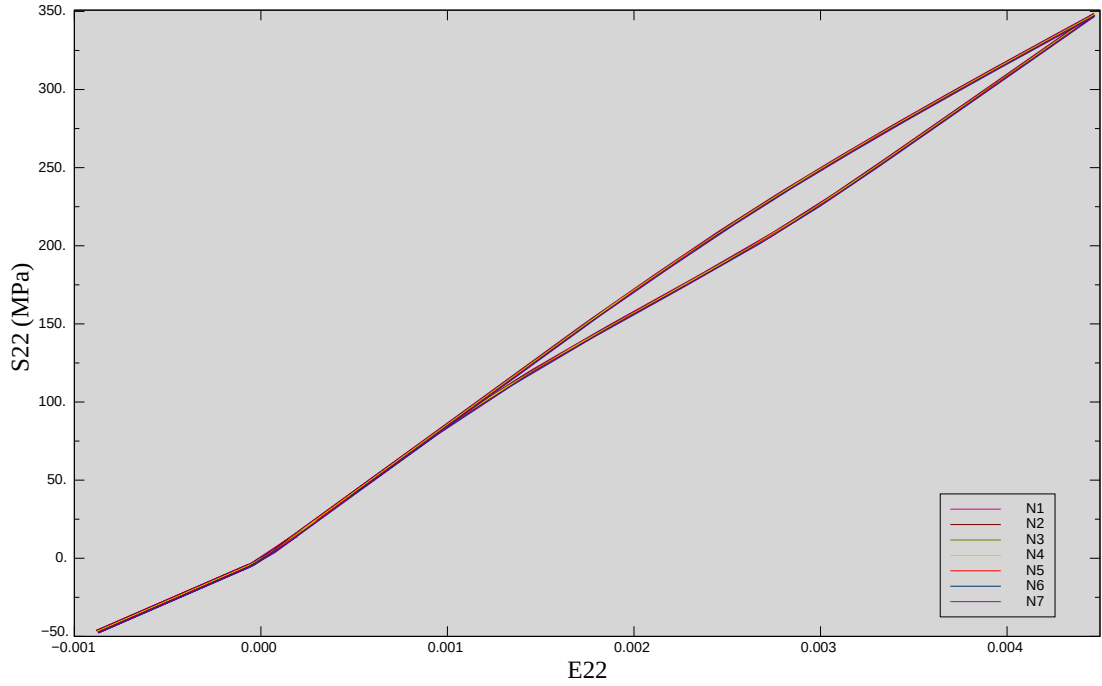


Figure 4.11. Cyclic behavior of σ_{yy} vs. total ε_{yy} in region 3 at integration point 3 (Model I, plastic zone inserted, $\sigma_0 = 100$ MPa)

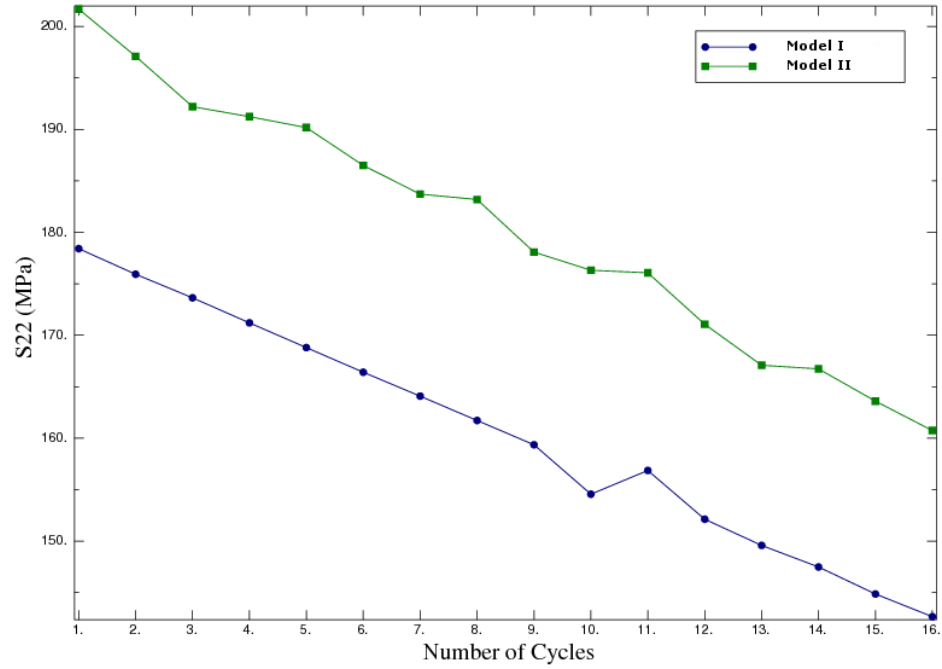


Figure 4.12. Comparison of maximum stress in the loading-unloading cycle between Model I and Model II (Region 1, IP3, $\sigma_0 = 150$ MPa)

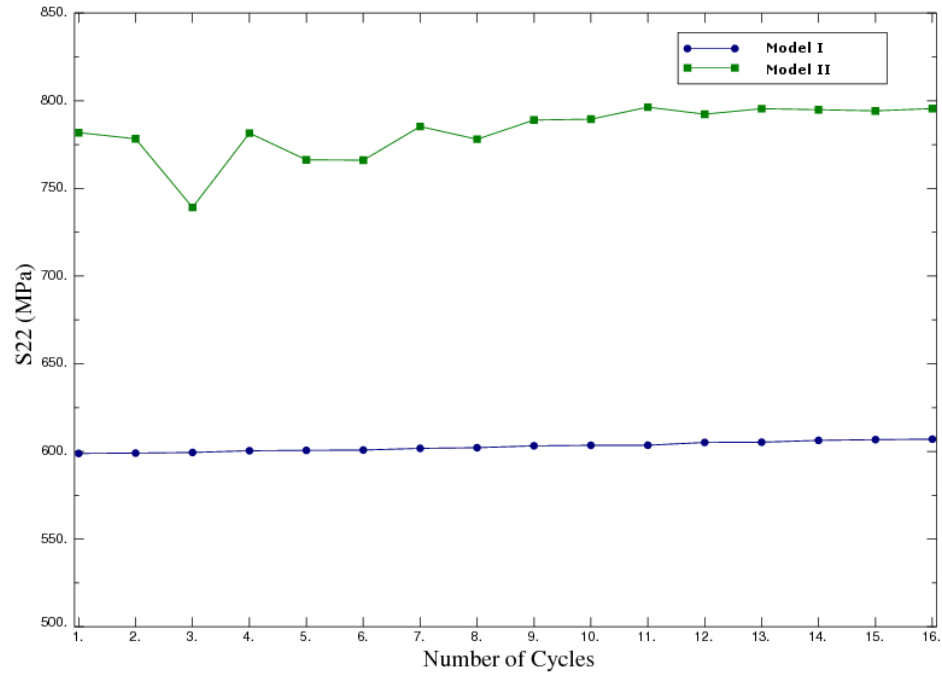


Figure 4.13. Comparison of maximum stress in the loading-unloading cycle between Model I and Model II (Region 2, IP3, $\sigma_0 = 150$ MPa)

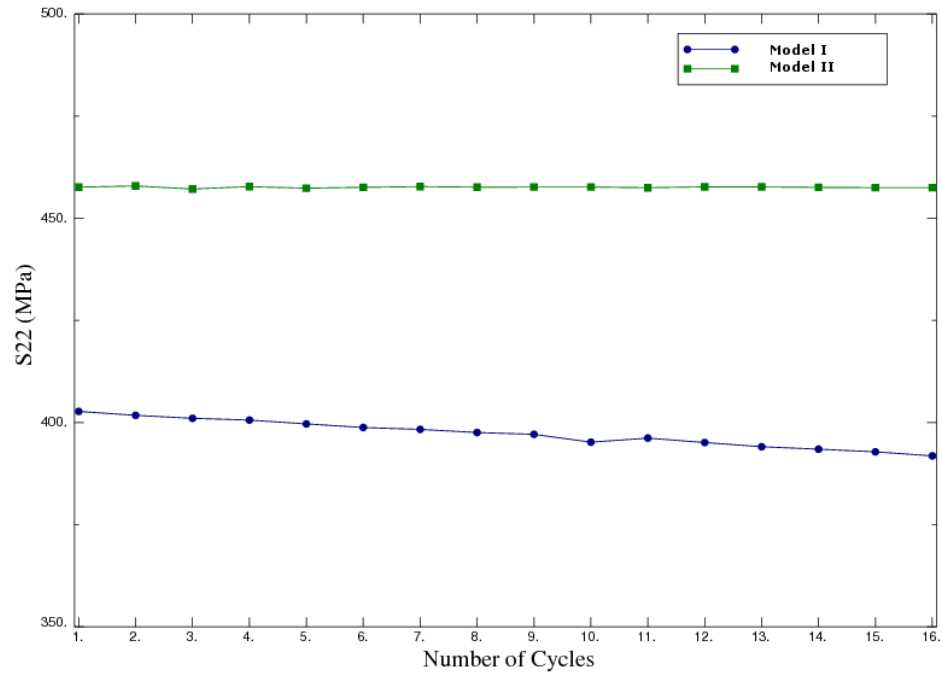


Figure 4.14. Comparison of maximum stress in the loading-unloading cycle between Model I and Model II (Region 3, IP3, $\sigma_0 = 150$ MPa)

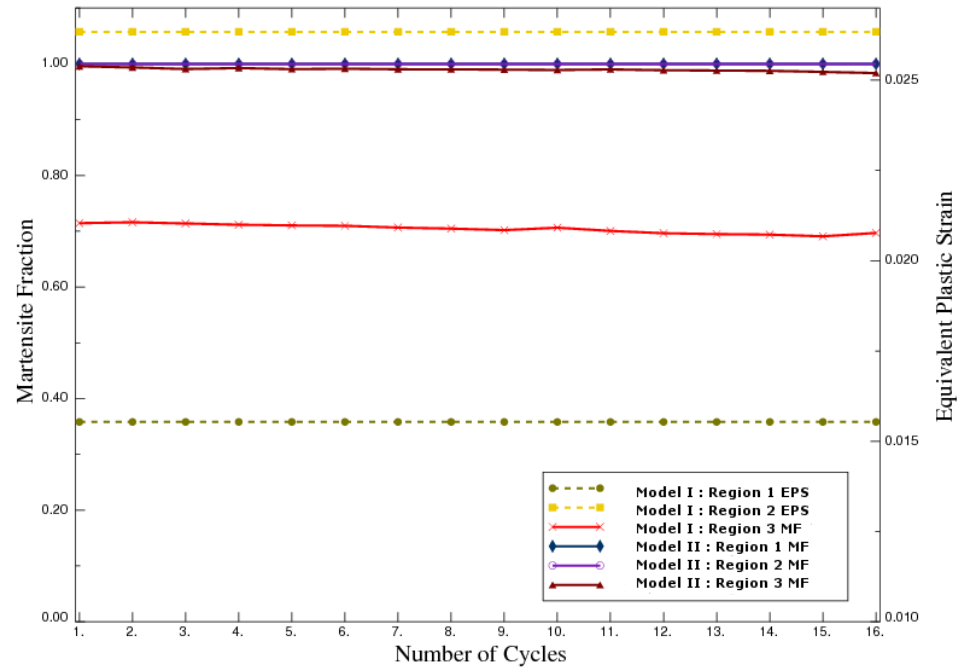


Figure 4.15. Comparison of martensite fraction and/or equivalent plastic strain at maximum load in a cycle between Model I and Model II (All three regions, $\sigma_0 = 150$ MPa)

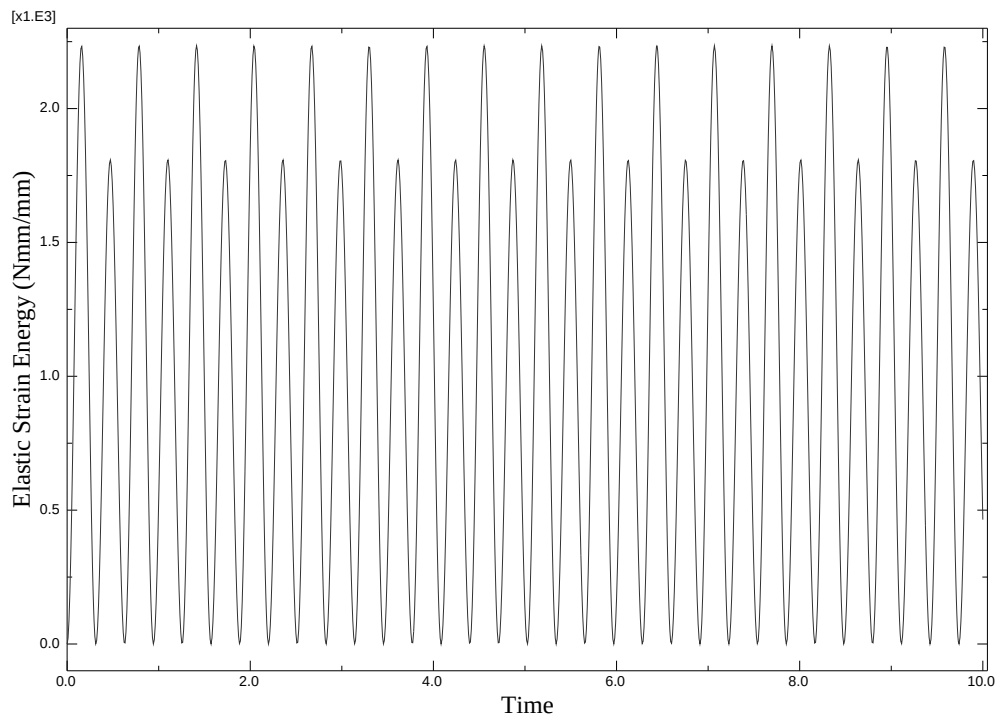


Figure 4.16. SMA elastic strain energy during cyclic analysis

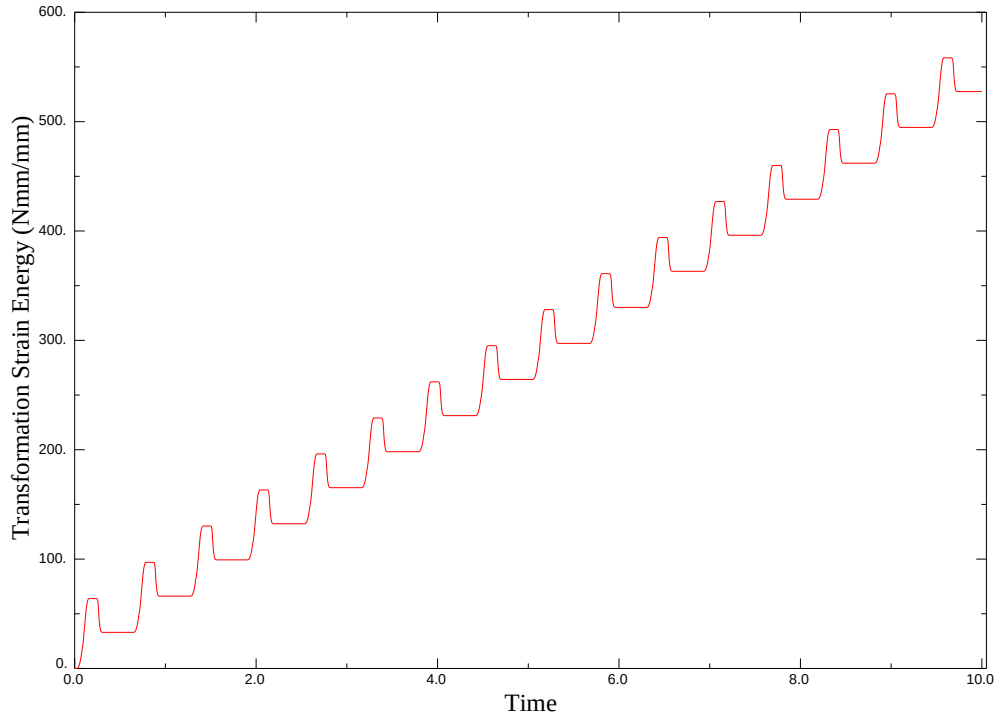


Figure 4.17. SMA transformation strain energy during cyclic analysis

Briefly, the shortage of ABAQUS superelastic model in analysis of crack tip plasticity has come up during investigation of SMA's cyclic behavior. The only way to neglect the effect of plasticity on fatigue crack closure is to confine the yielded portion of the specimen in a small scale zone around crack tip. Hence, the crack tip stress intensities will be kept as low as possible in the next major analysis.

4.3. Fatigue Crack Propagation Analysis of an Edge-Cracked Nitinol Plate

4.3.1. Modeling Parameters, Geometry and Finite Element Mesh

Modeling parameters selected for fatigue analysis of Nitinol plate are similar to those used in Chapter 2 for the elastic-plastic material. Material data have already been tabulated (see Table 3.2). The specimen has a single straight crack with an initial size of a_0 and a final length of a_f , so that only mode I crack problem will be solved. Plane stress conditions are imposed to the model and the thickness of the plate is given as 1 mm. As discussed in previous chapters, a full square plate with dimensions $W = H = 100$ mm is going to be meshed in order to take into account of crack face

interactions during fatigue loading. Frictionless tangential properties and hard normal contact are defined between contact surfaces. Vertical displacement of the plate is restricted and a cyclic load is applied at the top surface. The only predefined field in the model is temperature, which is held constant at 37°C throughout the analysis. The geometry of the plate, boundary and loading conditions and mesh densities in different locations are demonstrated in Figure 4.18.

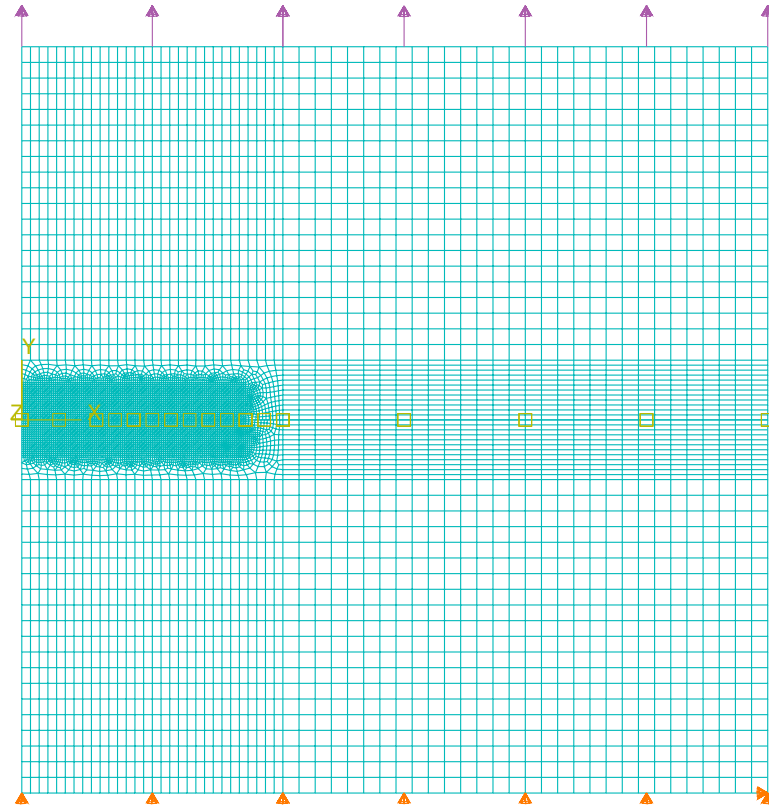


Figure 4.18. FE discretization of Nitinol plate, boundary conditions and crack propagation path

Since there isn't any suitable mesh sufficiency criterion for materials undergoing phase transformation, the minimum element length in crack tip region is calculated from a modified version of criteria adapted from literature [70]. The effect of reverse transformation on crack closure during unloading will be investigated, therefore the software has to capture forward and reversed transformation zones. So, the mesh

sufficiency criteria after required modifications become :

$$\Delta a/r_{tr} \leq 0.10 \quad \text{for} \quad R = 0 \quad (4.2)$$

$$\Delta a/r_{tr} \leq 0.15 \quad \text{for} \quad R = -1 \quad (4.3)$$

if four-node quadrilateral elements are used. As stated before, R is the load ratio of minimum and maximum applied stresses, r_{tr} is the radius of the transformation zone around crack tip and Δa is the recommended minimum element length. Next, the size of the transformation zone should be found analytically. According to Xiong and Liu [91], whose formulation was referred to stress intensity determination in the previous chapter, the radius of the transformation zone is given as :

$$r_{tr} \simeq \frac{1}{2} \left\{ \frac{1}{(\sigma_s^M)^2} + \frac{1}{\sigma_y^2} \right\} \sigma_0^2 a \quad (4.4)$$

if the applied stress σ_0 is relatively small compared with the yield stress of Nitinol. The results for r_{tr} and corresponding element sizes are given in Table 4.2. Although required element length must be smaller, an element size of ~ 0.05 mm will be selected in order to prevent an excessive calculation effort.

Table 4.2. Mesh sufficiency criteria in terms of σ_0 , R and a

	$\sigma_0 = 80$	Δa [mm]		$\sigma_0 = 100$	Δa [mm]		$\sigma_0 = 120$	Δa [mm]	
a [mm]	r_{tr} [mm]	$R = 0$	$R = -1$	r_{tr} [mm]	$R = 0$	$R = -1$	r_{tr} [mm]	$R = 0$	$R = -1$
10	0.2189	0.0219	0.0328	0.3421	0.0342	0.0513	0.4926	0.0493	0.0739
15	0.3284	0.0328	0.0493	0.5131	0.0513	0.0770	0.7389	0.0739	0.1108
20	0.4379	0.0438	0.0657	0.6842	0.0684	0.1026	0.9852	0.0985	0.1478
25	0.5473	0.0547	0.0821	0.8552	0.0855	0.1283	1.2315	0.1232	0.1847
30	0.6568	0.0657	0.0985	1.0263	0.1026	0.1539	1.4778	0.1478	0.2217

4.3.2. Finite Element Results

In this section, the fatigue fracture behavior of a single-edge cracked Nitinol specimens are examined using FE. All models are subjected to the same maximum tensile load of $0.25\sigma_s^M = 100$ MPa, which is enough to trigger a martensitic phase transfor-

mation around the crack tip and is expected to obey small scale yielding criterion discussed in Section 4.2. Here, the effect of two parameters, the initial crack length a_0 and the load ratio R , on finite element results are going to be discussed. Besides, fatigue fracture of the specimen will be compared with the stationary fracture behavior at the final distance a_f to see the effect of forward and reverse phase transformations on crack closure.

The effect of transformation induced fatigue crack closure (TICC) can be seen in Figures 4.19 and 4.20, where vertical displacements between faces of fatigue crack and stationary cracks at final crack distance of 18.75 mm are demonstrated. The fatigue crack propagation begins at $a/W = 0.1$ and crack tip node is released at the end of every load cycle with $R=0.1$, whereas the model of stationary crack doesn't contain any residual deformation in its wake. As mentioned in the results section of Chapter 2, crack opening will occur only if crack opening displacement near the tip exceeds the residual deformation. In fatigue analysis of elastic-plastic materials, a blatant difference between COD of fatigue and stationary cracks are seen. In this case, there also exists a difference, but this minor difference cannot be recognized by simply looking at the Figure 4.19. Thus, a zoomed version is supplied, where it can be noticed that crack opening displacement of stationary crack is greater than that of fatigue crack. To see the details, the finite element results are tabulated in Table 4.3. By keeping the minimum element length in the finite element mesh as small as possible, a difference in the range of 0.07-0.18 % can be captured. Although this gap seems to be negligible, it may be seen as an indication of transformation induced fatigue crack closure.

In the study of fatigue crack closure, some modeling parameters are changed to view the response under different conditions. To see the importance of initial crack length, the crack tip is moved from a_0 of 10 mm and 15 mm to a_f of 18.5 mm separately. Some mechanical properties are calculated at the end of each loading cycle with $R=0.1$ under a maximum tensile load of 100 MPa. The maximum crack opening displacement and the elastic strain energy of the whole plate are plotted in terms of the propagated crack distance in Figures 4.21 and 4.22. Due to the very small variation between results, the effect of initial crack length can be disregarded. The most significant difference

exists in transformation strain energy, that is plotted in Figure 4.23. Regarding the energy outputs at the crack location of 17 mm, one will see that the elastic strain energy has the value around 920 Nmm/mm for both models. On the other hand, the model with an initial crack length of 10 mm reaches a transformation strain energy level of 80 Nmm/mm, which is twice of that of the model with an initial crack length of 15 mm. Calculations are repeated for ξ , e^{tr} and e^{el} at constant distances of $\Delta x = \Delta y = 0.5$ mm from the recent crack tip during crack propagation and these results are tabulated in Table 4.4. As expected, there are no significant changes in stress, strain, martensite fraction and elastic strain energy density values. However, the model with older crack propagation history has a larger transformation strain energy density. For example, transformation energy density in the model denoted with letter A is three times larger the energy of the model B at $a_f = 15$ mm, which corresponds to the initial loading cycle of the second model as well. On further steps, the energy gap between these two models is closed and approaches ~ 0.5 % to the energy value at the final distance of 18.5 mm.

Another very important parameter in fatigue crack closure is the load ratio R , whose effects have been taken into account by some researchers in fatigue life predictions [63]. For this purpose, five models with similar geometry are pulled and compressed under the same maximum tensile load of 100 MPa, but with different load ratios. The applied load ratios are $R = -1, 0, 0.1, 0.25$ and 0.5 . The effect of R on energy levels and crack openings are plotted in Figures 4.24-4.28 and the data in terms of final crack lengths are tabulated in Tables 4.5 and 4.6. No significant change in elastic strain energy levels E_s are observed, when minimum load in the model is modified. Influence of stress ratio is significant in transformation strain energy E_{tr} and crack opening displacement calculations. When R is more positive, E_{tr} is generally lower and crack opening displacements between crack faces are generally greater for the same final crack length. Thus, a higher R value implies a higher mean load resulting crack flanks will be relatively further apart and there will be less crack closure.

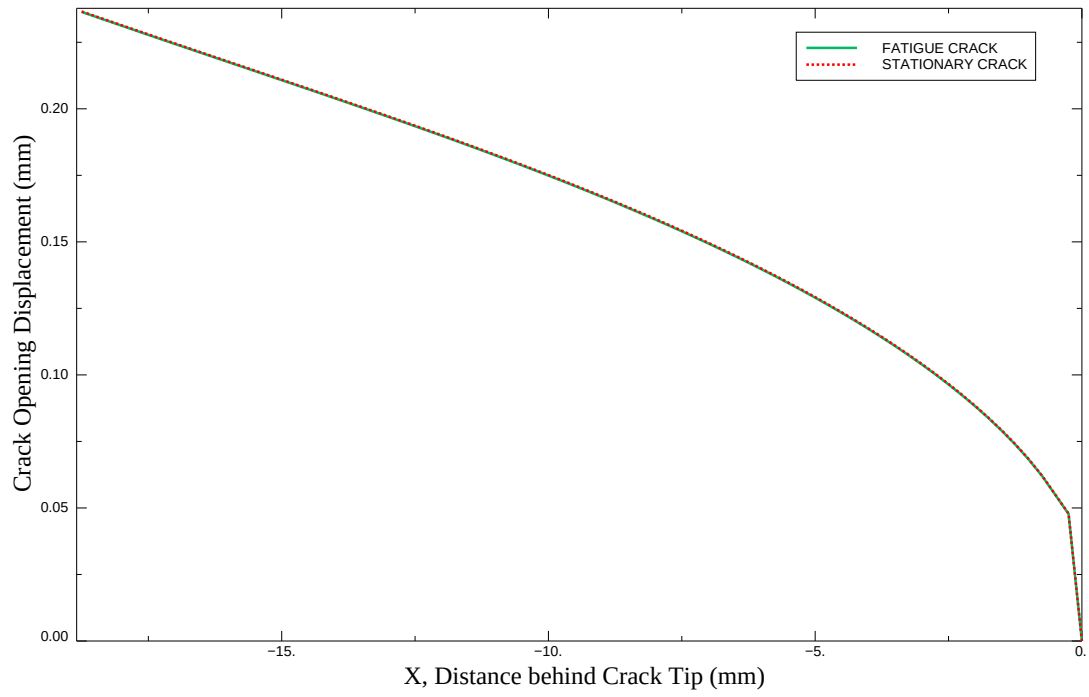


Figure 4.19. Crack opening displacement for fatigue and stationary cracks at a_f
 $(a_0=10 \text{ mm}, a_f=18.75 \text{ mm}, \sigma_0=100 \text{ MPa}, R=0.1)$

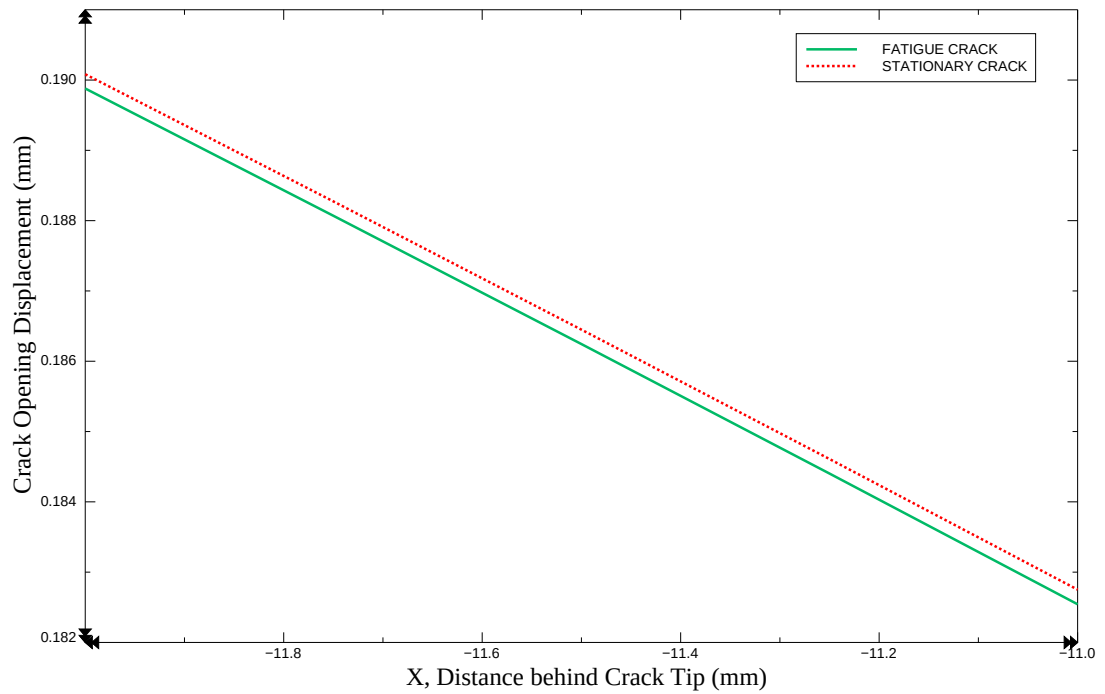


Figure 4.20. Zoomed version of COD's of fatigue and stationary cracks ($a_0=10 \text{ mm}$,
 $a_f=18.75 \text{ mm}, \sigma_0=100 \text{ MPa}, R=0.1$)

Table 4.3. Crack opening displacement comparison between fatigue and stationary crack ($a_0=10$ mm, $a_f=18.75$ mm, $\sigma_0=100$ MPa, $R=0.1$)

Distance Behind Crack Tip	Crack Opening Displacement [mm]		
[mm]	STATIONARY	FATIGUE	% Difference
0.00	0	0	0
2.25	0.0925829	0.092417	0.179190758
7.50	0.1542105	0.1539703	0.155761119
9.00	0.173159	0.1729412	0.125780352
10.75	0.1808745	0.1806647	0.115992028
15.00	0.2109545	0.2107625	0.091014887
17.00	0.2245524	0.2243647	0.083588508
18.75	0.2365875	0.2364025	0.078195171

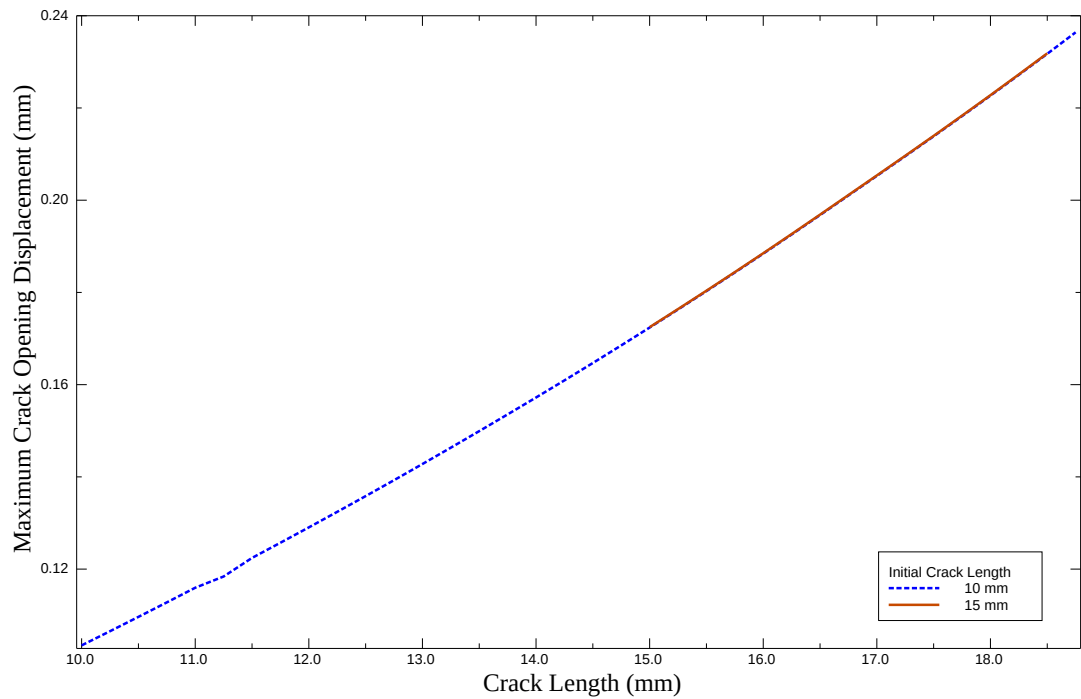


Figure 4.21. Effect of initial crack length a_0 on crack opening displacement ($a_f=18.5$ mm, $\sigma_0=100$ MPa, $R=0.1$)

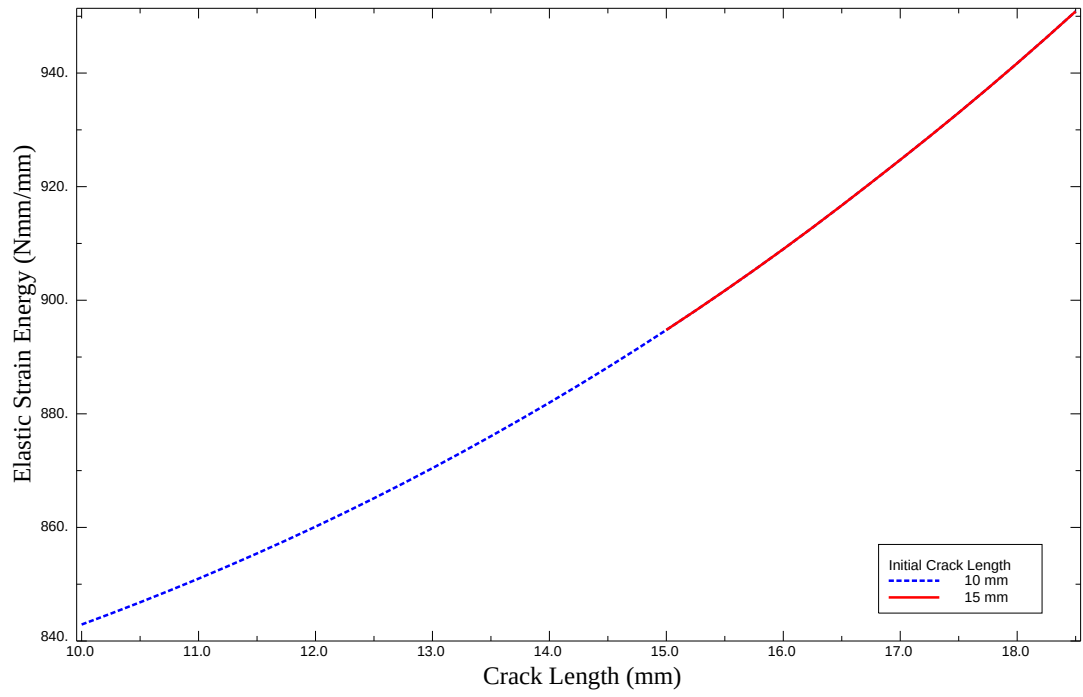


Figure 4.22. Effect of initial crack length a_0 on elastic strain energy ($a_f=18.5$ mm, $\sigma_0=100$ MPa, $R=0.1$)

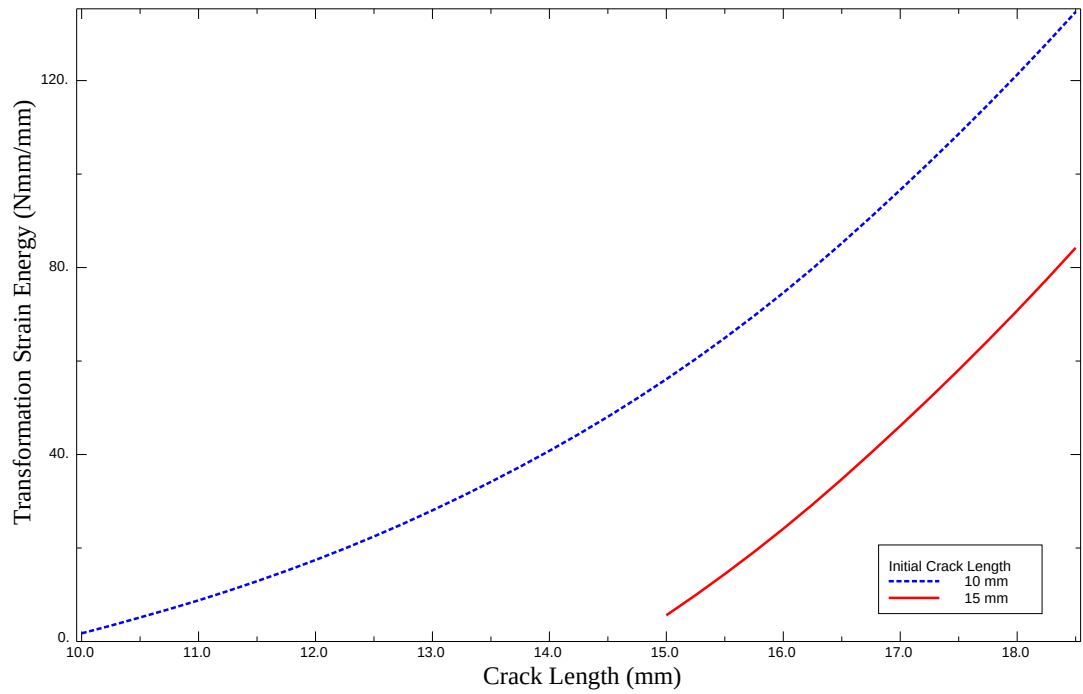


Figure 4.23. Effect of initial crack length a_0 on transformation strain energy ($a_f=18.5$ mm, $\sigma_0=100$ MPa, $R=0.1$)

Table 4.4. Effect of initial crack length on various terms measured at $\Delta x = \Delta y = 0.5$ mm from crack tip of each cycle ($a_f = 18.5$ mm, $\sigma_0 = 100$ MPa, $R = 0.1$, $A \Rightarrow a_0 = 10$ mm, $B \Rightarrow a_0 = 15$ mm)

	σ_{eq}		σ_{yy}		ξ		e^{tr}		e^{el}	
a_f	A	B	A	B	A	B	A	B	A	B
15.00	398.644	398.604	453.21	453.283	0.006796	0.006680	0.23162	0.07157	1.56719	1.56555
15.25	398.617	398.608	452.877	452.99	0.006949	0.006840	0.24075	0.13014	1.5699	1.5684
15.50	398.511	398.509	452.475	452.587	0.007115	0.007022	0.27541	0.20837	1.5716	1.57033
15.75	398.395	398.394	452.067	452.169	0.007303	0.007220	0.31898	0.29129	1.57386	1.57272
16.00	398.44	398.363	451.829	451.836	0.007442	0.007402	0.35204	0.34514	1.57921	1.57674
16.25	398.362	398.357	451.484	451.558	0.007656	0.007595	0.41156	0.40282	1.58266	1.58145
16.50	398.452	398.46	451.309	451.387	0.007803	0.007743	0.49020	0.48041	1.58794	1.58691
16.75	398.459	398.459	451.111	451.176	0.007986	0.007938	0.58915	0.57917	1.59374	1.59279
17.00	398.483	398.506	451.152	451.235	0.008173	0.008128	0.70971	0.70018	1.59883	1.598
17.25	399.235	399.201	452.065	452.085	0.008355	0.008315	0.84563	0.83567	1.60736	1.60664
17.50	400.101	400.093	452.955	453.011	0.008841	0.008756	1.01725	1.00622	1.61637	1.61571
17.75	400.179	400.175	452.939	452.991	0.009674	0.009589	1.22564	1.21429	1.6237	1.62306
18.00	400.242	400.239	452.879	452.928	0.010641	0.010561	1.45386	1.44247	1.6316	1.63101
18.25	400.299	400.298	452.799	452.833	0.011726	0.011715	1.70534	1.69544	1.63927	1.63868
18.50	400.467	400.461	452.867	452.913	0.015568	0.015490	2.05376	2.04348	1.64707	1.64686
[mm]	[MPa]						[Energy Density]			

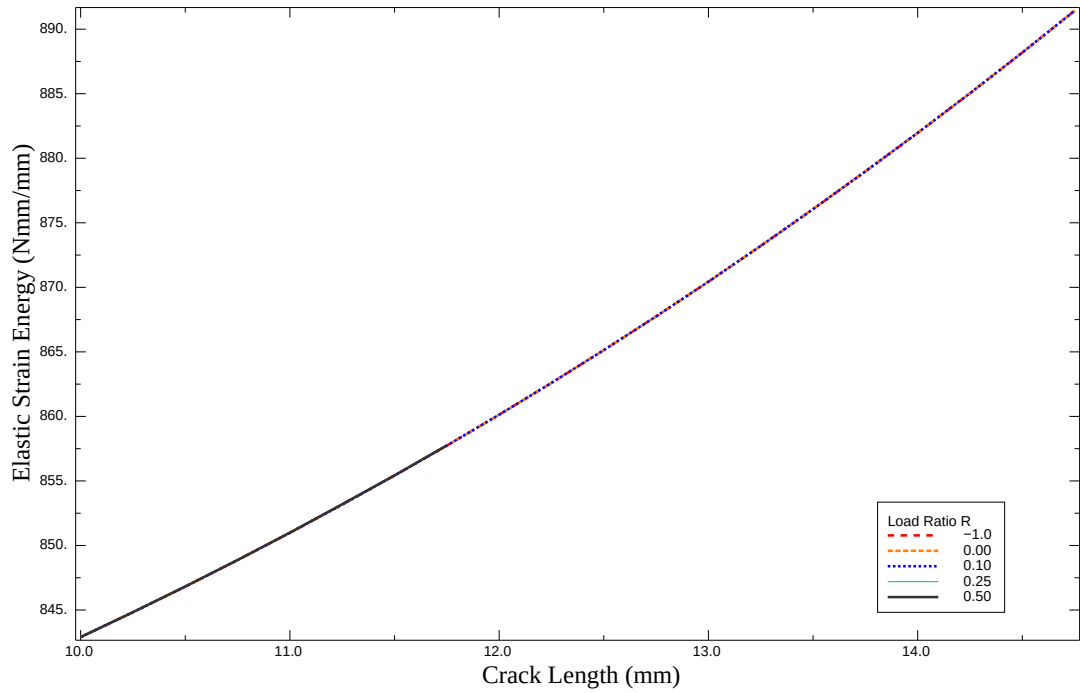


Figure 4.24. Effect of load ratio R on elastic strain energy ($\sigma_0 = 100$ MPa)

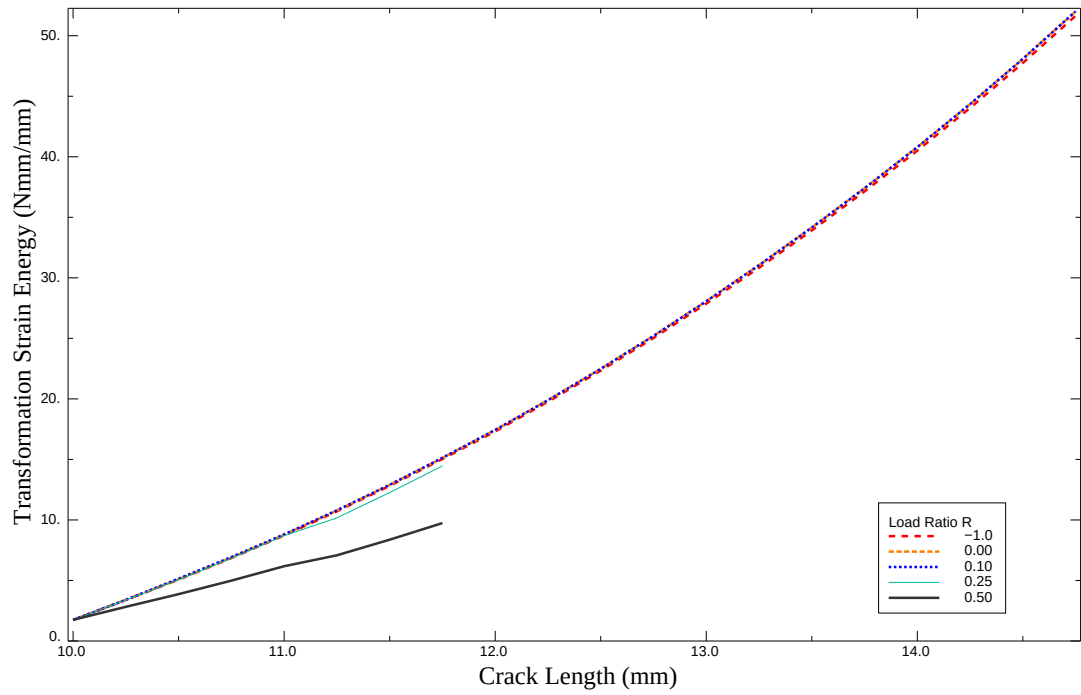


Figure 4.25. Effect of load ratio R on transformation strain energy ($\sigma_0=100$ MPa)

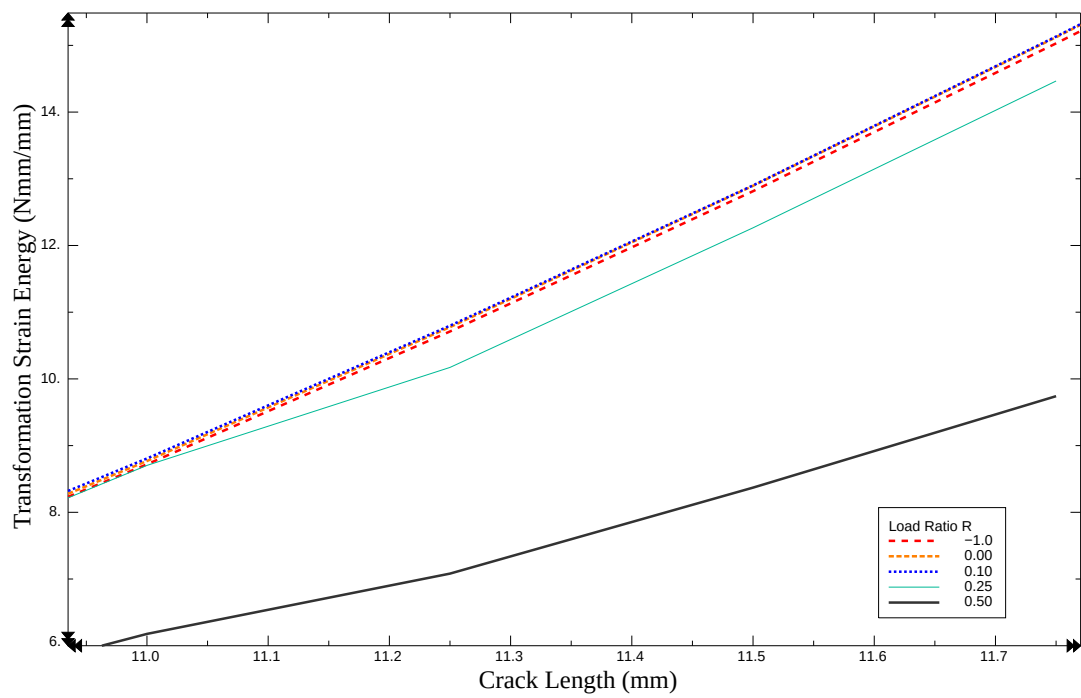


Figure 4.26. Zoomed version : Effect of load ratio R on transformation strain energy
($\sigma_0=100$ MPa)

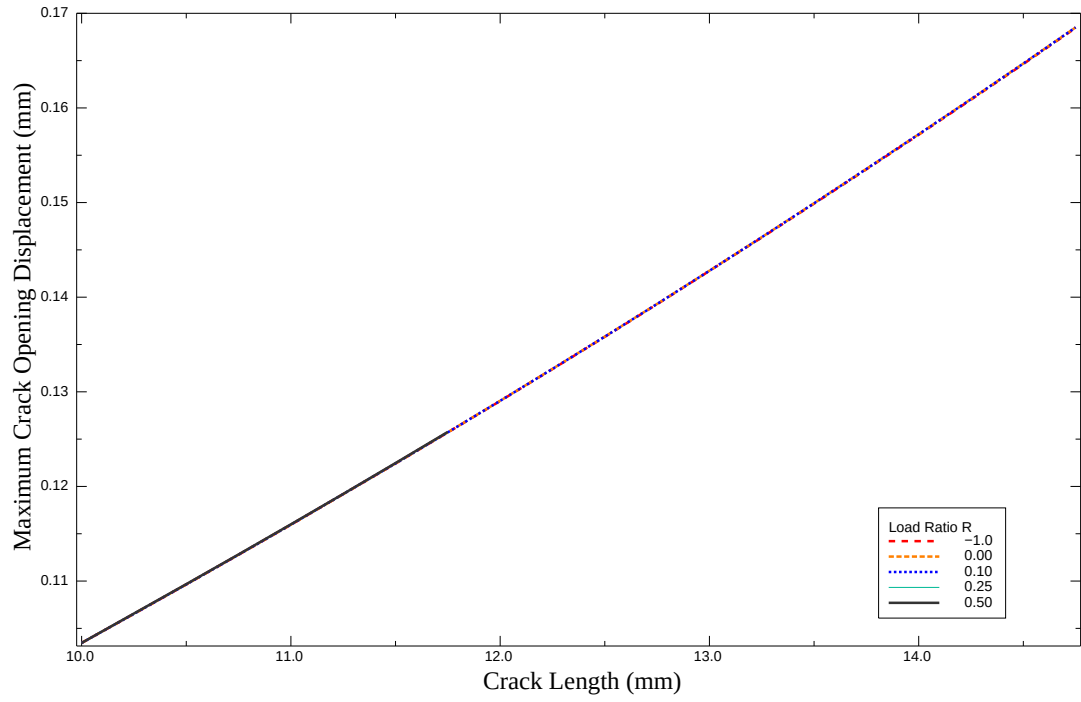


Figure 4.27. Effect of load ratio R on maximum crack opening distance ($\sigma_0=100$ MPa)

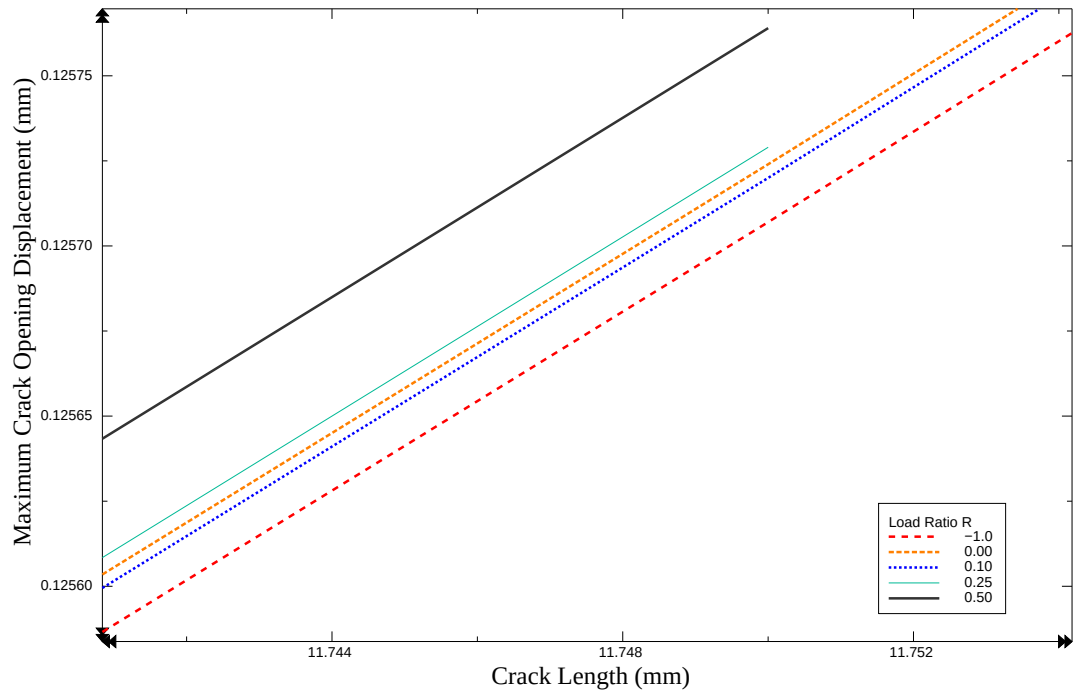


Figure 4.28. Zoomed version : Effect of load ratio R on maximum crack opening distance ($\sigma_0=100$ MPa)

Table 4.5. Comparison of energy measures of five models with different load ratios

	E_S [Nmm/mm]					E_{tr} [Nmm/mm]				
a [mm]	R=0.1	R=-1	R=0	R=0.25	R=0.50	R=0.1	R=-1	R=0	R=0.25	R=0.50
10.00	842.901	842.901	842.901	842.901	842.901	1.746	1.746	1.746	1.746	1.746
10.25	844.826	844.826	844.826	844.826	844.835	3.399	3.389	3.399	3.387	2.824
10.50	846.814	846.814	846.814	846.815	846.823	5.153	5.071	5.093	5.066	3.872
10.75	848.867	848.866	848.866	848.867	848.875	6.935	6.847	6.881	6.836	4.988
11.00	850.983	850.983	850.983	850.983	850.993	8.807	8.719	8.769	8.703	6.177
11.25	853.168	853.167	853.167	853.167	853.199	10.798	10.711	10.776	10.172	7.081
11.50	855.420	855.419	855.419	855.420	855.428	12.901	12.814	12.895	12.265	8.370
11.75	857.741	857.739	857.741	857.741	857.753	15.133	15.030	15.122	14.466	9.741
12.00	860.132	860.131	860.132			17.438	17.314	17.425		
12.25	862.597	862.596	862.597			19.892	19.766	19.898		
12.50	865.136	865.134	865.135			22.481	22.336	22.488		
12.75	867.750	867.747	867.749			25.197	25.030	25.203		
13.00	870.439	870.44	870.44			28.062	27.870	28.064		
13.25	873.212	873.21	873.211			31.045	30.832	31.048		
13.50	876.061	876.059	876.061			34.152	33.912	34.154		
13.75	878.982	878.98	878.982			37.401	37.139	37.401		
14.00	881.969	881.966	881.968			40.797	40.507	40.794		
14.25	885.038	885.035	885.037			44.35	44.035	44.344		
14.50	888.192	888.189	888.191			48.089	47.746	48.081		

Table 4.6. Comparison of maximum COD of five models with different load ratios

	Maximum Crack Opening Displacement [mm]				
a [mm]	R=0.1	R=-1	R=0	R=0.25	R=0.5
10.00	0.103462	0.103462	0.103462	0.103462	0.103462
10.25	0.106536	0.106534	0.106536	0.106537	0.106571
10.50	0.109644	0.109641	0.109645	0.109646	0.109674
10.75	0.112784	0.112781	0.112787	0.112790	0.112815
11.00	0.115959	0.115951	0.115960	0.115964	0.115996
11.25	0.118399	0.118390	0.119179	0.119182	0.119207
11.50	0.122431	0.122420	0.122434	0.122438	0.122471
11.75	0.125720	0.125707	0.125724	0.125729	0.125764
12.00	0.129047	0.129030	0.129051		
12.25	0.132420	0.132401	0.132425		
12.50	0.135831	0.135809	0.135836		
12.75	0.139285	0.139258	0.139289		
13.00	0.142790	0.142760	0.142794		
13.25	0.146337	0.146302	0.146340		
13.50	0.149922	0.149884	0.149925		
13.75	0.153551	0.153511	0.153554		
14.00	0.157215	0.157169	0.157217		
14.25	0.160926	0.160875	0.160927		

Another important parameter to understand the existence of crack closure is mode I opening levels, σ_{op}^I . As mentioned before, for an ideal elastic crack, σ_{op} and σ_{cl} are equal to zero. For a plastic material with bilinear hardening, these stress levels are found to be different than zero and the existence of plasticity induced crack closure is proven. Same procedure is implemented to investigation of transformation crack closure in Nitinol. To examine the effect of maximum stress and stress ratio on mode I opening level, the specimen is subjected to constant amplitude fatigue loading with five different stress levels σ_{max}/σ_s^L : 0.125, 0.2, 0.4, 0.5 and 0.625. The dependence of opening levels on load ratio is examined by considering two different values of R : 0 and -1. Figures 4.29(a) and 4.29(b) show crack opening levels as a function of crack length and these stress levels are normalized by maximum applied stress in the cycle. First impression got from figures is that, crack opening stresses become greater than zero after the initial cycle for all cases. Note that, normalized opening stress significantly decreases with increasing maximum stress for both $R = 0$ and $R = -1$. Same trend was seen in plastic material and according to Kibey et al. [68], in all numerical and analytical models this phenomenon was confirmed. However the dependency of opening stress level on maximum stress has not yet been clarified. Normally, a higher applied stress σ_0 leads to a greater inelastic deformation in the wake of the crack and should result in increased closure. The results deny this argument. An explanation for this anomaly is given by McClung and Sehitoglu [70] and they explained the results by comparing COD of fatigue and stationary crack of same length subjected to the same maximum stress, like we did in this study and plotted results in Figure 4.20. According to them [68], “the fatigue crack opening is seen as a competition between residual deformation and the **ideal** crack opening displacement at maximum load”.

Crack opening levels are also affected by different load ratios. Figures 4.29(a) and 4.29(b) show that normalized opening level will be lower when the load ratio is more negative, when the specimen is subjected to the same maximum stress σ_0 . This phenomenon was explained by Ritchie et al. [105], who observed that crack faces are flattened by compressive overload leading to a change of opening stress level.

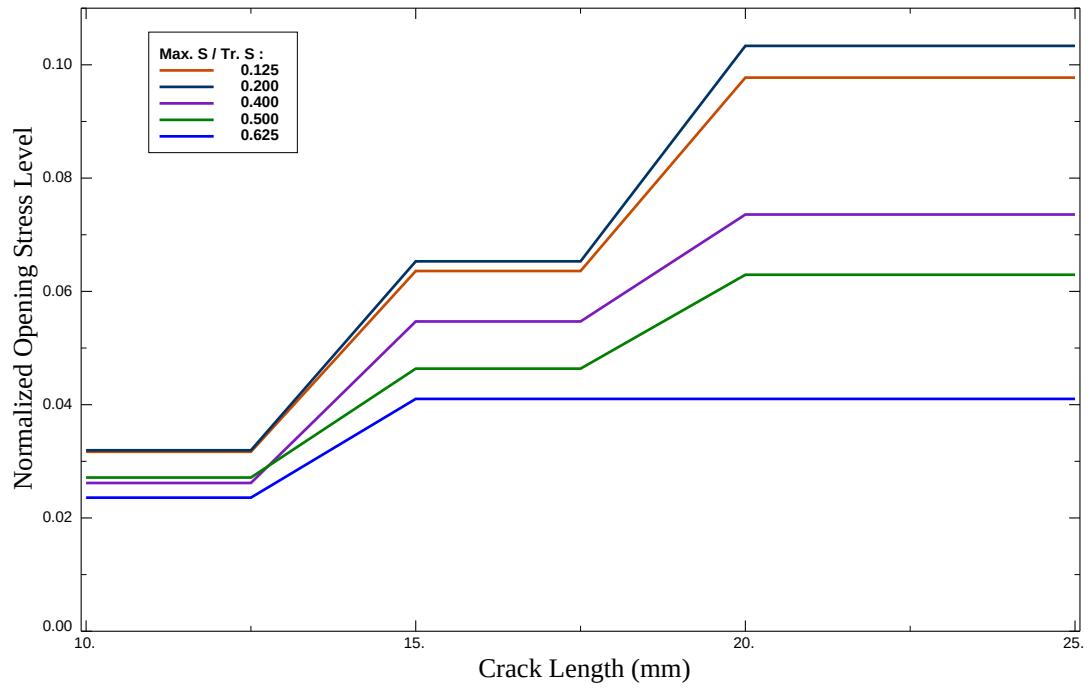
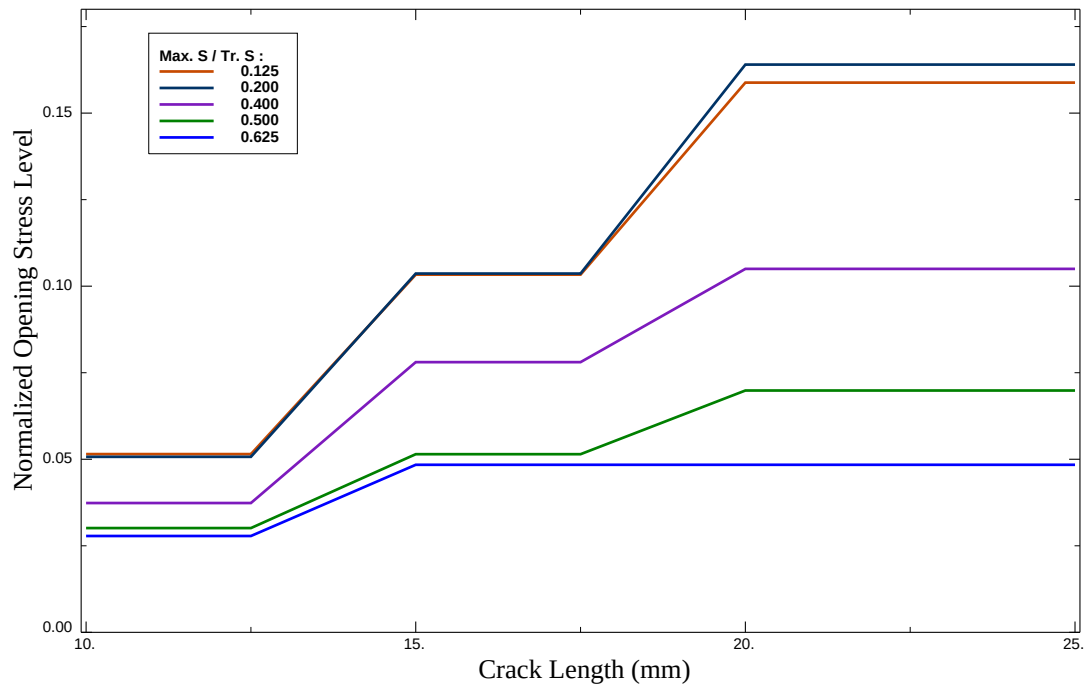
(a) $R = -1$ (b) $R = 0$

Figure 4.29. Normalized mode I opening levels for two load ratios

For materials with only limited closure effects, ΔK_{th} is more and less independent of R . On the other hand, the effect of stress ratio on the threshold stress intensity range has been proven in some materials like aluminium alloys [63]. By looking at the threshold values given in Table 4.1, it can be observed that they are decreasing with increasing stress ratio. As mentioned in Chapter 2, ΔK_{th} is not a material constant and it is dependent on several internal and external parameters. In this study, load ratio R is the dominant external parameter and it greatly affects the threshold stress intensity factor. To obtain a true material constant, ΔK_{th} should be converted into $\Delta K_{th,eff}$, which is out of the scope of this study. We can only say that, no crack closure will occur at R values above a critical value R_c .

As the applied stress decreases from σ_{max} , the crack tip opening angle decreases owing to elastic relaxation (martensite to austenite phase transformation) of the cracked body. However, the crack surfaces are prevented from becoming parallel, because the material along the flanks causes closure before minimum load depending on R is reached. This closure results in the flanks exerting reaction forces onto each other. This phenomenon was recognized in plastic materials with isotropic hardening. Same situation is happening during crack propagation of shape memory alloys, but in smaller scale compared with the plastic case. By looking at results discussed in this section for Nitinol, it can still be expected that mode I stress intensity will develop, which increases as the applied stress decreases. This claim generates the interest about the value of stress intensity K and fatigue crack growth length da/dN under constant amplitude loading.

4.3.3. Determination of K_{max} and ΔK

Unlike linear elastic materials and some elastic-plastic materials, that follow certain hardening rules, shape memory alloys have no clear closed form analytical solutions for stress intensity factors. In Section 3.4, some numerical methods were reviewed to maintain an opinion, in which range K takes values in a single-edged cracked specimen of Nitinol. These methods are strain energy difference method and a sort of J-K relationship, where Hutchinson-Rice-Rosengren singularity solutions for power-law

hardening materials are implemented in stress field determination of SMA's transformation region. In order to find proper K_{max} and ΔK results in a crack propagating model, which are going to be used in prediction of fatigue crack growth under constant amplitude loading, a reasonable and consistent J-K relationship should be constructed.

The mechanism in J-integral evaluation is simple. Since crack is propagating, there will be number of crack tips during the analysis. Therefore, at the end of every loading cycle, J is calculated around several nodes, which will sooner or later become the new crack tip during the crack propagation analysis. The mechanism is demonstrated in Figure 4.30, where the J-integral progress around some locations along the centerline of the plate are plotted. If there is no singularity, J at that location will be zero. The time step, where J reaches to a global maximum, indicates that the new crack tip is the node, around which contour integral is evaluated. Before that, there may appear other local maximums caused by intersection of contour radii with crack flanks.

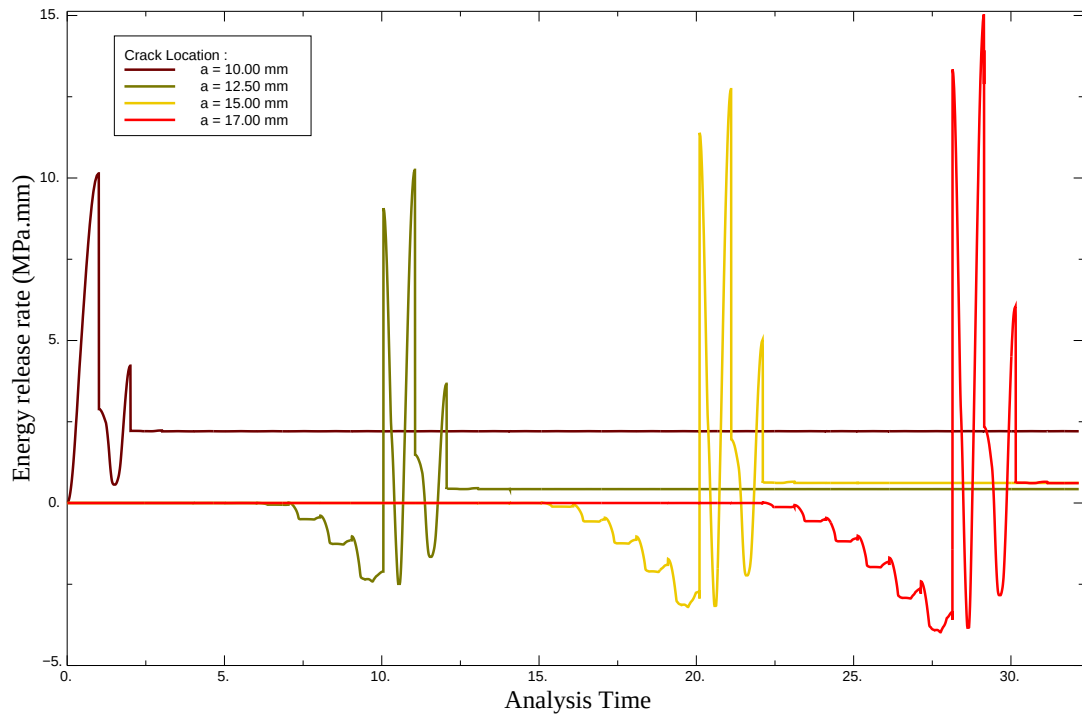


Figure 4.30. The progression of J during the fatigue crack growth analysis

In order to figure out the effect of phase transformation on energy release rate,

J's are computed at contours with different radius and at different crack locations. The first contour around each crack tip has a radius of 0.25 mm and it is increasing by $\Delta r=0.25$ mm on the next contours. In Figure 4.31, the contour dependency of J-integral in shape memory alloys is shown. In the analysis, the maximum applied load is again 100 MPa and the load ratio R is 0.1. As mentioned in Section 3.3, a drop in J values to a local minimum is observed with increasing contour radius and this drop will be prominent with larger stress intensity. The common trait of seven curves in the figure is that they converge to a J-value, when the contour radii are bigger and J's are calculated on these contours outside the transformation zone. The drop in J results could be differently captured by changing the element sizes, so that no mesh-independent, consistent J's can be calculated on contours in transformation zones. The only mesh configuration and contour independent results for J-integral are obtained outside the transformation zone in shape memory alloys and they will increase with greater stress intensity, as shown in Figure 4.32. Although no phase transformation is happening in these calculation zones, the energy release rate is definitely affected by martensite formation near the crack tip, so that J-values on far contours in Nitinol will be higher than that of the linear elastic material with elastic modulus of E_A .

In linear elastic materials, it is known that the equations $G=J$ and $J=K^2/E$ hold for plane stress conditions. Assuming that the region very close to the crack tip is homogenous and fully martensite, K_I^{HRR} described in Chapter 3 was calculated in the same fashion, in which $E_{tip} = E_M$ and $J = J^M$. In this case, however, the mesh dependency of J^M has been proven, so that using this parameter in K-determination will be unrealistic. On the other side, the consistency of J^{far} was ensured and the effect of transformation on this property has been shown. Even though using the converged J's in stress intensity determination might give results above the real stress intensity, it will help to stay on the conservative side. Hence, the stress intensity for Nitinol will be found by using following equation :

$$\bar{J} = \frac{K^2}{E_M} \quad \text{where} \quad \bar{J} = J^{far} \quad (4.5)$$

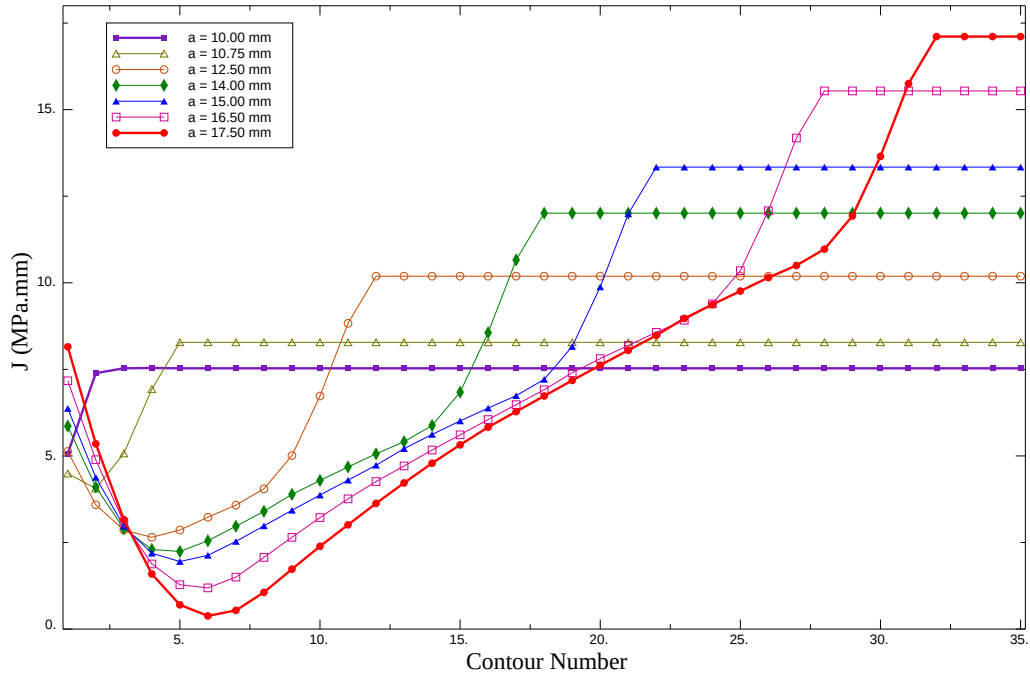


Figure 4.31. Contour dependent J values at different crack lengths ($R=0.1$, $\sigma_{max}=100$ MPa)

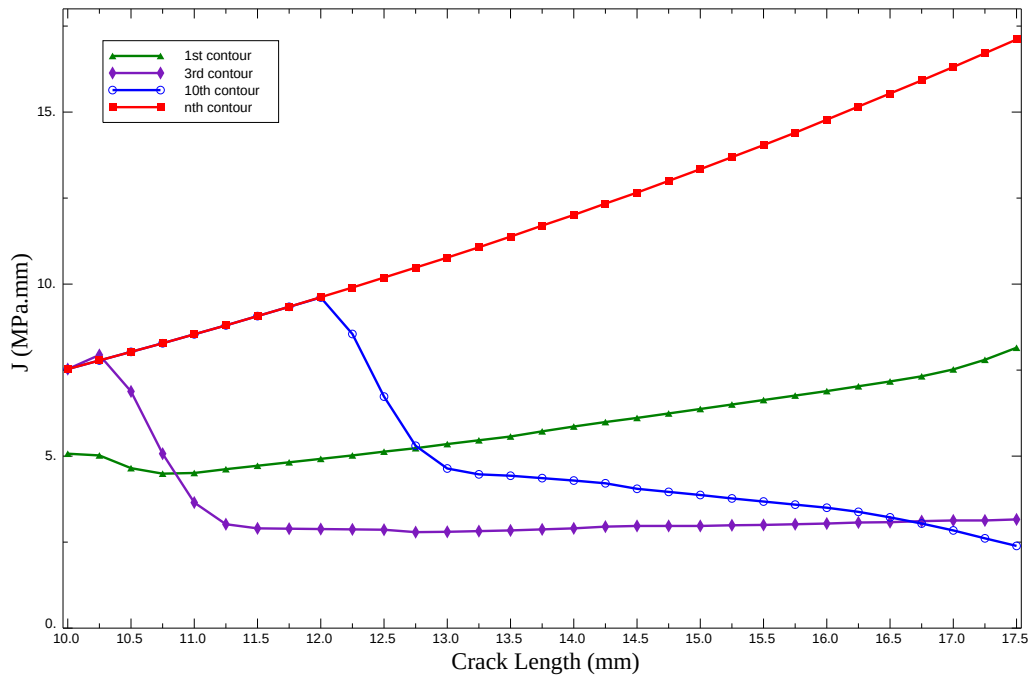


Figure 4.32. J in terms of propagated crack final length ($R=0.1$, $\sigma_{max}=100$ MPa, $\Delta x=\Delta y=0.25$ mm for each contour)

The $\bar{J}$ outputs are extracted from ABAQUS for different maximum tensile loads of σ_0 and they are converted to K by using Equation 4.5. The results are summarized in Table 4.7, where K/σ_0 values are also submitted.

Table 4.7. $\bar{J}$, K and K/σ_0 results in terms of the crack length and maximum applied tensile load (R=0.1)

		Crack Length [mm]												
σ_0 [MPa]	J [MPa.mm]	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.25	12.50	12.75	
	60	2.652	2.737	2.824	2.912	3.002	3.092	3.185	3.278	3.374	3.471	3.569	3.669	
	70	3.626	3.743	3.862	3.979	4.097	4.215	4.333	4.451	4.569	4.687	4.805	4.923	
	80	4.759	4.913	5.071	5.228	5.389	5.553	5.721	5.889	6.062	6.237	6.415	6.596	
	90	6.060	6.264	6.455	6.655	6.853	7.050	7.248	7.446	7.644	7.841	8.039	8.237	
	100	7.533	7.779	8.029	8.282	8.539	8.802	9.069	9.340	9.619	9.900	10.186	10.478	
	120	11.050	11.400	11.770	12.127	12.487	12.847	13.207	13.567	13.927	14.287	14.647	15.007	
	150	17.755	18.325	18.910	19.485	20.063	20.640	21.218	21.795	22.373	22.950	23.528	24.105	
	180	26.480	27.395	28.310	29.225	30.140	31.055	31.970	32.885	33.800	34.715	35.630	36.545	
200	33.770	34.860	35.980	37.080	38.185	39.290	40.395	41.500	42.605	43.710	44.815	45.920		
σ_0 [MPa]	K [MPa.mm ^{1/2}]	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.25	12.50	12.75	
	60	241.545	245.385	249.233	253.087	256.969	260.814	264.687	268.544	272.428	276.317	280.191	284.109	
	70	282.420	286.941	291.467	295.876	300.231	304.523	308.756	312.932	317.053	321.121	325.138	329.105	
	80	323.571	328.764	333.993	339.124	344.323	349.507	354.755	359.942	365.175	370.424	375.673	380.936	
	90	365.115	371.225	376.842	382.634	388.277	393.840	399.325	404.736	410.075	415.346	420.550	425.691	
	100	407.081	413.675	420.270	426.854	433.426	440.037	446.674	453.299	460.020	466.690	473.384	480.109	
	120	493.052	500.799	508.861	516.515	524.125	531.627	539.024	546.322	553.523	560.631	567.651	574.585	
	150	624.988	634.941	644.996	654.729	664.361	673.855	683.217	692.452	701.566	710.563	719.448	728.224	
	180	763.256	776.331	789.189	801.842	814.297	826.565	838.654	850.570	862.322	873.916	885.359	896.655	
200	861.940	875.740	889.697	903.194	916.553	929.720	942.704	955.510	968.148	980.622	992.940	1005.107		
σ_0 [MPa]	K / σ_0	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.25	12.50	12.75	
	60	4.026	4.090	4.154	4.218	4.283	4.347	4.411	4.476	4.540	4.605	4.670	4.735	
	70	4.035	4.099	4.164	4.227	4.289	4.350	4.411	4.470	4.529	4.587	4.645	4.702	
	80	4.045	4.110	4.175	4.239	4.304	4.369	4.434	4.499	4.565	4.630	4.696	4.762	
	90	4.057	4.125	4.187	4.251	4.314	4.376	4.437	4.497	4.556	4.615	4.673	4.730	
	100	4.071	4.137	4.203	4.269	4.334	4.400	4.467	4.533	4.600	4.667	4.734	4.801	
	120	4.109	4.173	4.241	4.304	4.368	4.430	4.492	4.553	4.613	4.672	4.730	4.788	
	150	4.167	4.233	4.300	4.365	4.429	4.492	4.555	4.616	4.677	4.737	4.796	4.855	
	180	4.240	4.313	4.384	4.455	4.524	4.592	4.659	4.725	4.791	4.855	4.919	4.981	
200	4.310	4.379	4.448	4.516	4.583	4.649	4.714	4.778	4.841	4.903	4.965	5.026		

		Crack Length [mm]												
σ_0 [MPa]	J [MPa.mm]	13.00	13.25	13.50	13.75	14.00	14.25	14.50	14.75	15.00	17.50	20.00		
	60	3.771	3.874	3.979	4.086	4.195	4.305	4.417	4.532	4.650	5.591	6.587		
	70	5.041	5.159	5.277	5.395	5.513	5.631	5.749	5.867	5.985	7.165	8.345		
	80	6.781	6.969	7.160	7.354	7.551	7.752	7.956	8.164	8.376	10.080	11.885		
	90	8.435	8.632	8.830	9.028	9.226	9.423	9.621	9.819	10.017	11.994	13.972		
	100	10.775	11.075	11.380	11.695	12.010	12.335	12.660	12.995	13.335	16.070	18.967		
	120	15.367	15.727	16.087	16.447	16.807	17.167	17.527	17.887	18.247	21.847	25.447		
	150	24.683	25.260	25.838	26.415	26.993	27.570	28.148	28.725	29.303	35.078	40.853		
	180	37.460	38.375	39.290	40.205	41.120	42.035	42.950	43.865	44.780	53.930	63.080		
200	47.025	48.130	49.235	50.340	51.445	52.550	53.655	54.760	55.865	66.915	77.965			
σ_0 [MPa]	K [MPa.mm ^{1/2}]	13.00	13.25	13.50	13.75	14.00	14.25	14.50	14.75	15.00	17.50	20.00		
	60	288.012	291.938	295.868	299.820	303.775	307.750	311.727	315.759	319.827	350.702	380.670		
	70	333.026	336.901	340.732	344.521	348.268	351.975	355.644	359.275	362.870	397.032	428.479		
	80	386.227	391.544	396.874	402.215	407.567	412.956	418.368	423.802	429.269	470.914	511.331		
	90	430.771	435.791	440.754	445.662	450.517	455.320	460.072	464.776	469.433	513.685	554.416		
	100	486.867	493.598	500.360	507.238	514.023	520.932	527.750	534.687	541.636	594.594	645.961		
	120	581.436	588.207	594.901	601.521	608.069	614.546	620.957	627.302	633.583	693.273	748.216		
	150	736.896	745.466	753.940	762.319	770.607	778.807	786.921	794.953	802.904	878.467	948.027		
	180	907.811	918.831	929.720	940.484	951.126	961.650	972.060	982.359	992.552	1089.247	1178.032		
200	1017.128	1029.009	1040.755	1052.369	1063.856	1075.221	1086.467	1097.597	1108.616	1213.314	1309.668			
σ_0 [MPa]	K / σ_0	13.00	13.25	13.50	13.75	14.00	14.25	14.50	14.75	15.00	17.50	20.00		
	60	4.800	4.866	4.931	4.997	5.063	5.129	5.195	5.263	5.330	5.845	6.344		
	70	4.758	4.813	4.868	4.922	4.975	5.028	5.081	5.132	5.184	5.672	6.121		
	80	4.828	4.894	4.961	5.028	5.095	5.162	5.230	5.298	5.366	5.866	6.392		
	90	4.786	4.842	4.897	4.952	5.006	5.059	5.112	5.164	5.216	5.708	6.160		
	100	4.869	4.936	5.004	5.072	5.140	5.209	5.277	5.347	5.416	5.946	6.460		
	120	4.845	4.902	4.958	5.013	5.067	5.121	5.175	5.228	5.280	5.777	6.235		
	150	4.913	4.970	5.026	5.082	5.137	5.192	5.246	5.300	5.353	5.856	6.320		
	180	5.043	5.105	5.165	5.225	5.284	5.342	5.400	5.458	5.514	6.051	6.545		
200	5.086	5.145	5.204	5.262	5.319	5.376	5.432	5.488	5.543	6.067	6.548			

Following observations are made after calculation of stress intensity factors and checking crack opening displacements at maximum loads :

- 1) For linear elastic materials, the ratio K/σ_0 depends only on plate geometry and crack length. This ratio is not constant at a fixed crack distance in Nitinol; besides, it might increase or decrease with increasing load depending on crack size.
- 2) The non-dimensional stress intensity factor, $K/\sigma_0\sqrt{a}$, is a function of a/W in linear elastic materials. For Nitinol, this ratio depends on both crack length and applied remote load.
- 3) As expected, the ratio COD^{max}/a at an arbitrary location on crack flanks is affected by applied maximum stress.
- 4) In order to find a fracture criterion in debonding process, a mathematical relation between K and COD would like to be constructed. The non-dimensional ratio between these parameters will become :

$$\mathfrak{U} = \frac{K}{\sigma_0\sqrt{a}} \frac{a}{COD} = \frac{K\sqrt{a}}{\sigma_0 COD} \quad (4.6)$$

After the analysis, it is deduced that the ratio $\mathfrak{U}$ depends also on both crack length and applied load (see Table 4.8). Therefore, it is not possible to build a linear relationship between stress intensity and crack opening.

Table 4.8. Relationship between K and COD (Note that maximum COD 's are read)

$\sigma_0 = 80 \text{ MPa}$										
a [mm]	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5
$\frac{K}{\sigma_0\sqrt{a}}$	1.27903	1.28840	1.29771	1.30764	1.31771	1.32820	1.33900	1.35019	1.36159	1.37336
$\frac{COD}{a}$	0.00818	0.00825	0.00833	0.00841	0.00849	0.00857	0.00866	0.00875	0.00884	0.00894
$\mathfrak{U}$	156.357	156.122	155.843	155.576	155.269	154.976	154.649	154.317	153.970	153.603
$\sigma_0 = 100 \text{ MPa}$										
a	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5
$\frac{K}{\sigma_0\sqrt{a}}$	1.28730	1.29698	1.30683	1.31717	1.32796	1.33893	1.35032	1.36181	1.37379	1.38594
$\frac{COD}{a}$	0.01035	0.01044	0.01054	0.01065	0.01075	0.01087	0.01098	0.01111	0.01123	0.01136
$\mathfrak{U}$	124.423	124.205	123.967	123.720	123.484	123.215	122.936	122.623	122.333	122.019

In fatigue crack growth analysis of this study, one or several nodes are released at the end of every loading cycle, whatever the maximum applied tensile load is. In reality, failure at crack tip is decided by looking at the fracture toughness of material, which can be determined experimentally. Hence, the actual remote loads, which cause failure at crack tip, should be determined by checking the condition of $K_I > K_{IC}$. Since several results for stress intensity factor in terms of crack length and applied load have been obtained, a suitable extrapolation is needed in order to determine the critical remote load σ_0^{IC} for crack propagation. After couple of trial and errors, the following equation is judged to be the most accurate one, which is :

$$K_{IC} = C_1(\sigma_0^{IC})^{C_2} \quad \text{where} \quad C_1, C_2 = f(a) \quad (4.7)$$

By using the results in Table 4.7 and Equation 4.7, constants C_1, C_2 and the critical load σ_0^{IC} are determined for $K_{IC}=39.4 \text{ MPa}\sqrt{m}$ and they are tabulated in Table 4.9.

Table 4.9. Results for C_1, C_2 and extrapolation results for σ_0^{IC} in terms crack length of Nitinol plate with $K_{IC}=39.4 \text{ MPa}\sqrt{m}$

a [mm]	10.00	10.25	10.50	10.75	11.00	11.50	11.75	12.00	12.25	12.50	12.75
C_1	3.01	3.054	3.095	3.141	3.188	3.289	3.341	3.396	3.453	3.511	3.571
C_2	1.066	1.067	1.067	1.067	1.067	1.067	1.067	1.066	1.065	1.064	1.064
σ_0^{IC} [MPa]	285.04	279.70	276.23	272.44	268.67	260.93	257.12	254.54	251.90	249.27	245.34
a [mm]	13.00	13.25	13.50	13.75	14.00	14.50	14.75	15.00	17.50	20.00	25.00
C_1	3.633	3.697	3.762	3.831	3.899	4.044	4.12	4.198	4.606	5.051	5.845
C_2	1.063	1.062	1.06	1.059	1.058	1.055	1.053	1.052	1.051	1.048	1.044
σ_0^{IC} [MPa]	242.65	239.93	238.47	235.63	232.94	228.53	226.85	224.00	206.13	191.67	170.05

In Figures 4.33 and 4.34, results for stress intensity factor behavior with increasing remote loads during fatigue crack propagation and critical stresses in terms of crack locations, which lead to fracture failure of Nitinol plate, are demonstrated. There is an asymptotic decrease in critical stress, when crack is growing, which agrees with the stress intensity increase by incremental application of tensile load at a fixed crack tip location.

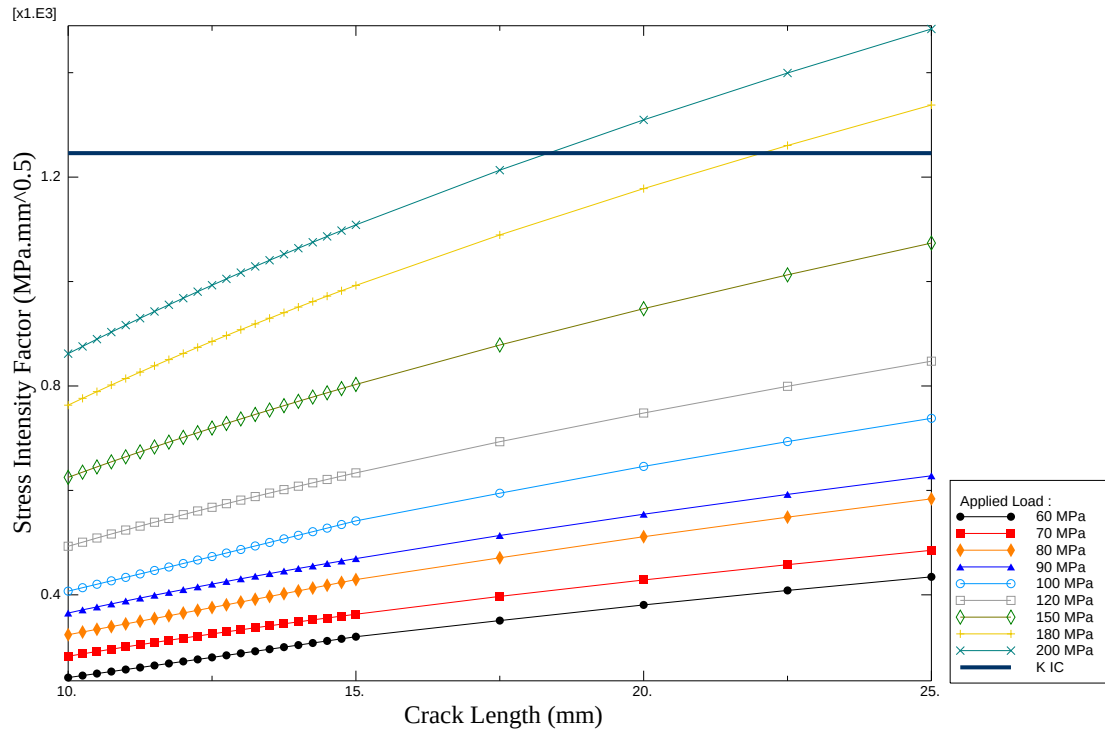


Figure 4.33. K in terms of crack length and applied maximum stress

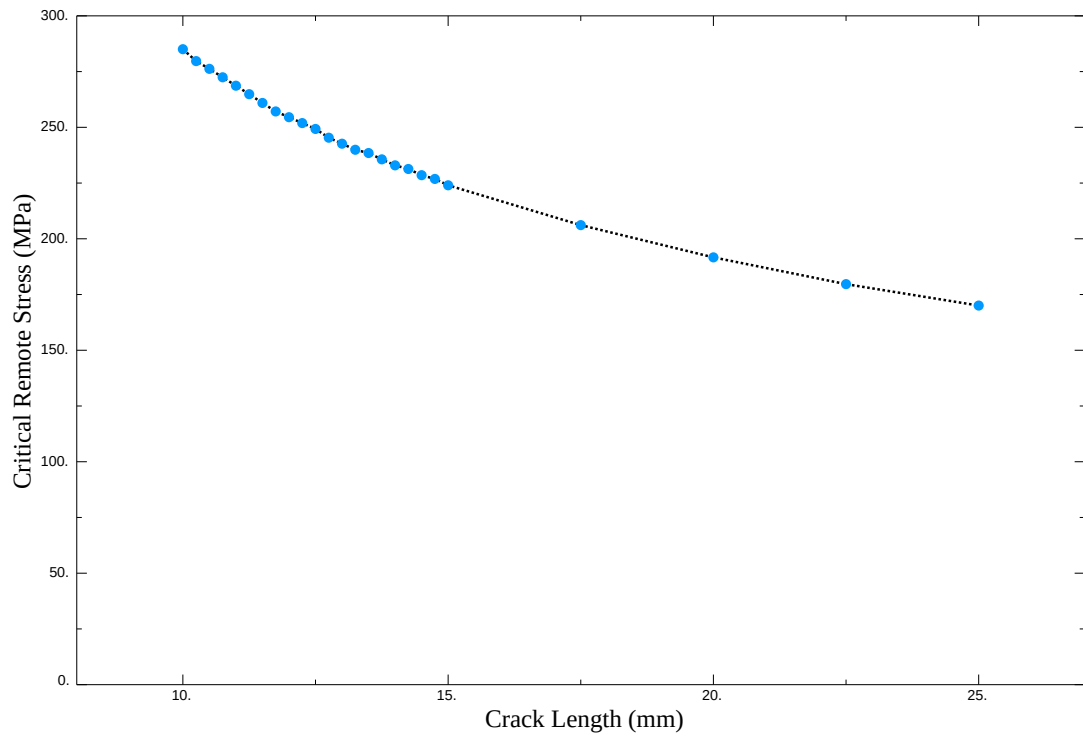


Figure 4.34. σ_0^{IC} in terms of crack length

4.3.4. Calculation of Crack Growth Rate and Fatigue Life

In Section 4.1, a fatigue growth rate equation for Nitinol was given, which was a generalized version of Paris-Erdogan law for crack propagation. The equation is quantified in terms of load ratio, maximum stress intensity and stress intensity range. Load ratio in the formulation helps for including the effect of crack closure, so that ΔK can be transformed into ΔK_{eff} . Thus, crack growth rate corresponding the same ΔK will be lower, when load ratio is greater. The stress intensity factor K for single-edged cracked specimen, which is the most crucial parameter in crack growth rate equation, was determined using a proper methodology in the previous subsection. Applying the same idea, da/dN is going to be formulated. Note that, stress intensity factor is given as :

$$K = C_1(\sigma_0)^{C_2} \quad \text{where} \quad C_1, C_2 = f(a) \quad (4.8)$$

where the values of C_1 and C_2 in an a/W range of $[0.1:0.25]$ are obtained from FE results. By using these results, the behavior of C_1 and C_2 can be described in terms of the crack length. After appropriate curve fittings, they are formulated for the whole a/W domain, assuming that no major changes occur in their tendencies beyond $a/W=0.25$. The results are :

$$C_1 = 2.715a^{0.3939} - 3.785 \quad (4.9)$$

$$C_2 = 1.136a^{-0.02683} \quad (4.10)$$

which are put into Equation 4.8 and stress intensity factor is given as :

$$K(a, \sigma_0) = (2.715a^{0.3939} - 3.785)(\sigma_0)^{1.136a^{-0.02683}} \quad (4.11)$$

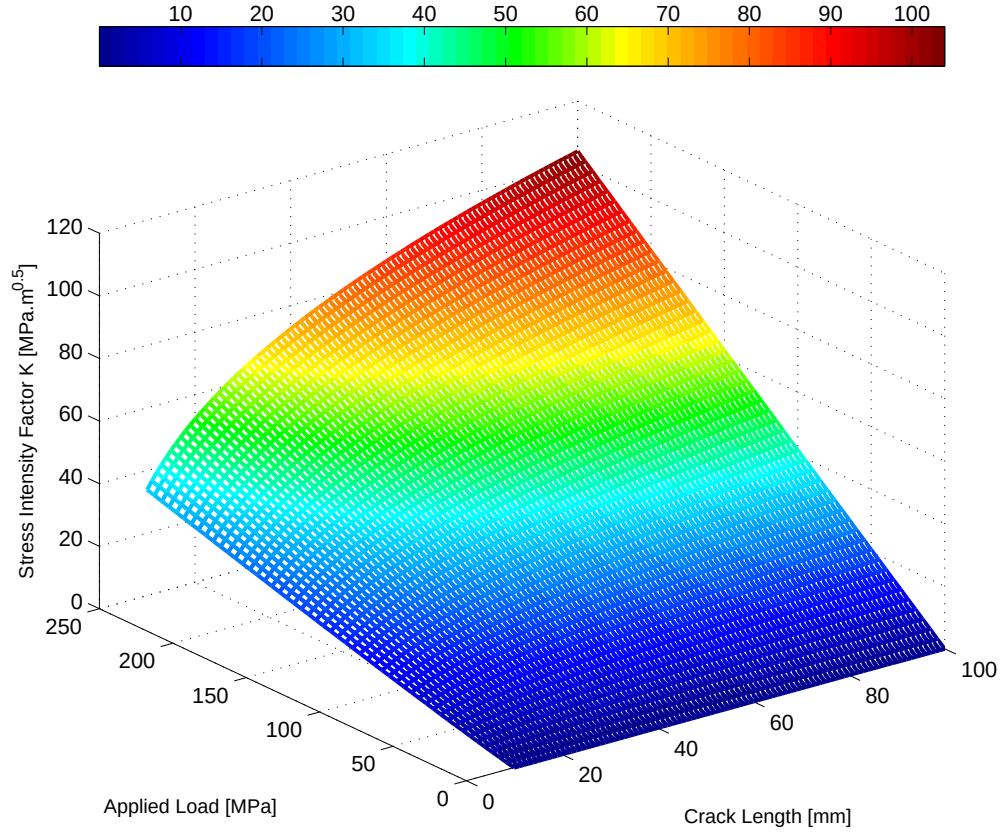


Figure 4.35. Surface grid of K in terms of applied load and crack length

A 3D surface grid of K in terms of σ_0 and a is given in Figure 4.35. Putting the term K into Equation 4.1, the crack growth rate of Nitinol takes the form of :

$$\frac{da}{dN} = C(1 - R)^n (C_1 \sigma_{max}^{C_2} - C_1 \sigma_{min}^{C_2})^p (C_1 \sigma_{max}^{C_2})^n \quad (4.12)$$

where subscripts max and min stand for maximum and minimum stress in the loading cycle, respectively. Hence, total number of loading cycles required to make a crack propagate from a_i to a_f is calculated from the following formulation :

$$N = \int_{a_i}^{a_f} \frac{1}{C(1 - R)^n [C_1 \sigma_0^{C_2} - C_1 (R \sigma_0)^{C_2}]^p (C_1 \sigma_0^{C_2})^n} da \quad (4.13)$$

The integral above is evaluated by using two different numerical methods. The first method is 8-point Gauss-Legendre formulas to estimate fatigue life. The original equa-

tion takes the form of :

$$N = \int_{-1}^1 \frac{a_f - a_i}{2C(1 - R)^n [C_1 \sigma_0^{C_2} - C_1 (R \sigma_0)^{C_2}]^p (C_1 \sigma_0^{C_2})^n} dx_d \quad (4.14)$$

and

$$x_d = \frac{2a}{a_i + a_f} \quad (4.15)$$

The integral is evaluated from initial crack length of 10 mm to $a_f=W$ for different maximum loads and results seem to be unacceptable due to large errors and inconsistency. The numerical errors and inconsistency arise from the characteristics of crack growth rate equation, hence another numerical integration method, the trapezoidal rule, is used to calculate the fatigue life of SMAs. In this numerical approach, a suitable increment of crack growth, $\Delta a_i = a_{i+1} - a_i$, should be chosen, but it must be not too small to cause an excessive calculation effort. Then, ΔK and K_{max} are calculated for the crack length corresponding to the mean of the crack growth increment and da/dN is determined for these values of ΔK and K_{max} . The number of cycles required to make a crack propagate as Δa_i , is calculated from :

$$\Delta N_i = \frac{\Delta a_i}{\left(\frac{da}{dN}\right)_i} \quad (4.16)$$

To find $N_i = \sum_{i=1}^n N_i$, the previous steps over the required range of the crack growth should be repeated and the values of ΔN_i are summed. To use the trapezoidal rule, two important aspects of Paris-Erdogan equation are remembered :

1. In order to use Paris-law constants given in Table 4.1, the minimum ΔK should be greater than the ΔK_{th} . Therefore, depending on load ratio R, there exist threshold maximum stresses, σ_0^{th} , under which no crack propagation occurs. These threshold stresses are calculated and plotted in terms of crack lengths for three different load ratios (see Figure 4.36). The minimum σ_0 used in this study is 60 MPa and it is above the threshold stresses in whole a/W domain. Thus, ΔK is

always greater than ΔK_{th} and linear relationship between da/dN and $\log(\Delta K)$ is valid.

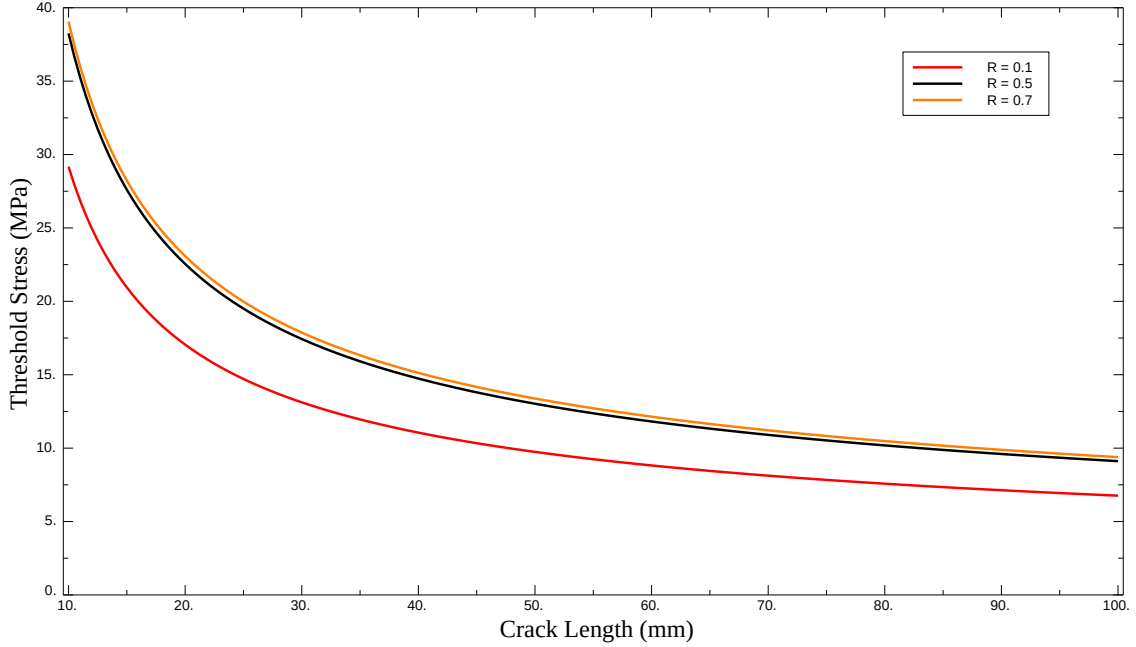


Figure 4.36. Threshold stresses in terms of crack length for different R's

2. The condition $K_{max} > K_{IC}$ leads to unstable crack growth and sudden failure. Similar to σ_0^{IC} in terms crack length, a definition for a^{IC} is needed in terms of applied maximum load σ_0 . Assuming that, $\bar{J}$ results in Table 4.7 are increasing linearly beyond $a/W=0.25$, some a_f^{IC} values are calculated and tabulated below. No propagation is allowed beyond critical crack length and the specimen is counted as failed.

Table 4.10. Critical crack lengths in terms of applied maximum loads (R=0.1)

σ_0 [MPa]	60	70	80	90	100	120	150	180	200
a_f^{IC} [mm]	$> W$	$> W$	$> W$	91.5	64.5	51.3	32.8	22.1	18.3

Taking the initial crack length as 10 mm, the service life of Nitinol working under different loading conditions (R=0.1/0.5/0.7) is determined and tabulated (see Table 4.11). To see the effect of load ratio on crack propagation rate, a σ_0 vs. N and a $\Delta\sigma$

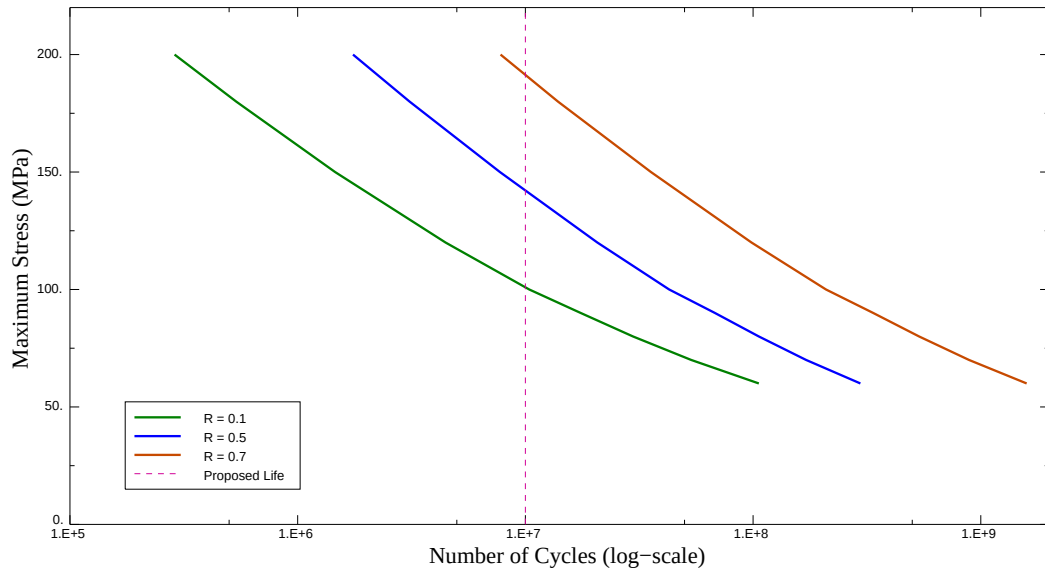
vs. N diagram are plotted (see Figures 4.37(a) and 4.37(b)), where $\Delta\sigma = (1 - R)\sigma_0$. As the load ratio increases under the same σ_0 , the crack growth rate will diminish and the crack reaches at its critical length later. Furthermore, if ΔK remains constant and load ratio increases, the effect of crack closure will be reduced. In this condition, the fatigue life of Nitinol is going to be shorter.

Table 4.11. Fatigue life of Nitinol under various cyclic loads at R=0.1,0.5 and 0.7

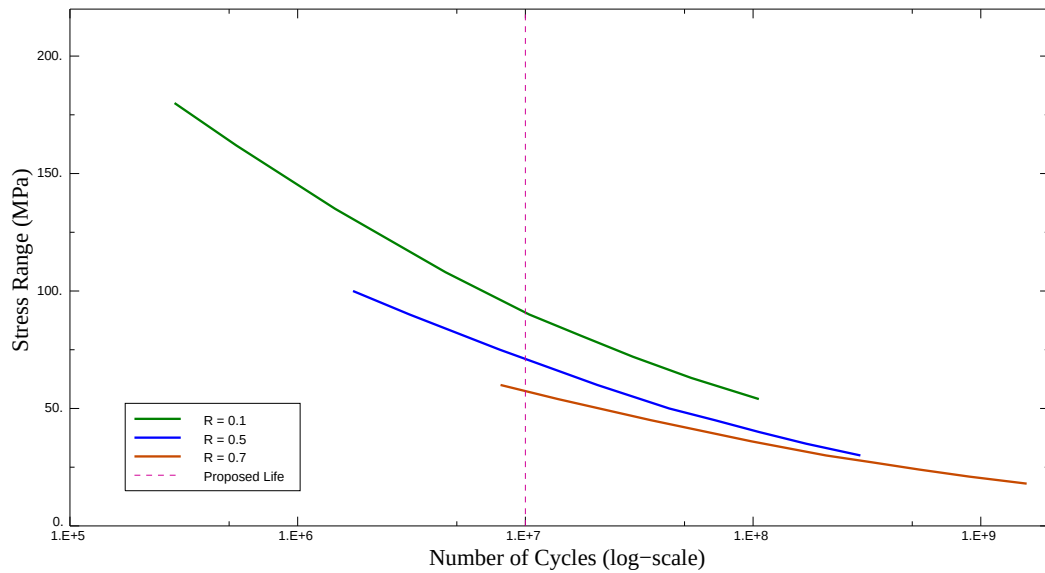
R	$C(1 - R)^n$	n	p		ΔK_{th}	
0.1	1.60 x 10 ⁻¹¹	1.53	2.68		2.48	
0.5	1.76 x 10 ⁻¹⁰	0.75	2.68		1.86	
0.7	6.87 x 10 ⁻¹¹	1.06	2.56		1.15	
	R=0.1		R=0.5		R=0.7	
σ_0 [MPa]	$\Delta\sigma$ [MPa]	NC	$\Delta\sigma$ [MPa]	NC	$\Delta\sigma$ [MPa]	NC
60	54	1.06E+08	48	2.96E+08	18	1.59E+09
70	63	5.35E+07	56	1.71E+08	21	8.88E+08
80	72	2.97E+07	64	1.06E+08	24	5.36E+08
90	81	1.75E+07	72	6.80E+07	27	3.37E+08
100	90	1.04E+07	80	4.28E+07	30	2.09E+08
120	108	4.46E+06	96	2.07E+07	36	9.84E+07
150	135	1.46E+06	120	7.73E+06	45	3.56E+07
180	162	5.37E+05	144	3.10E+06	54	1.39E+07
200	180	2.88E+05	160	1.75E+06	60	7.78E+06

Results obtained from the second method match with the life predictions of Nitinol in literature [13, 91, 94], which stress that Nitinol-like SMAs have a fatigue life near 10^7 cycles. Thus, the actual fatigue life of Nitinol is thought to be close to that calculated by using finite elements and numerical integration methods. There are two concerns related with life calculations. According to Robertson and Ritchie [13], Mode I critical fracture toughness values for Nitinol, which were reported by He et al. [93] and have been used throughout this study, is questionable due to non-standard fracture mechanics specimen geometry of the test material. In addition to that, any change in

this toughness value will result in retardation or acceleration of fatigue crack growth rate. Whether fracture toughness remains constant or it is affected by phase transformation, should also be discussed to obtain more accurate results.



(a) σ_0 vs. N



(b) $\Delta\sigma$ vs. N

Figure 4.37. Fatigue life prediction for different R's

5. CONCLUSION

The aim of this thesis was to study the effect of phase transformation on fracture and fatigue crack growth in SMAs. Crack closure analysis of a plastic material available in literature [68] is used as a starting point to investigate fatigue behavior and service life of Nitinol. Following observations are made :

- Crack closure phenomenon should be included to investigate fatigue crack growth and to predict service life of shape memory alloys. The only mechanism, which may cause crack closure, is the phase forward- and back-transformation. The existence of closure is shown in Section 4.3.2, where crack opening stresses calculated are greater than zero during fatigue crack propagation.
- Uniaxial tensile tests prove that the material used has a yield stress. Because no plasticity is allowed by the constitutive model for the superelastic shape memory alloy, the effect of plasticity on crack closure is kept negligible by application of small loads.
- Crack opening occurs only if crack opening displacement near the tip overcomes residual deformation. Every loading-unloading cycle causes formation of residual stresses in crack wake. That's why, a blatant difference between COD of stationary and fatigue cracks is seen in fatigue analysis of plastic materials. This difference is minor in shape memory alloys, because residual deformation after unloading isn't as high as that in plastic materials with hardening.
- The phase transformation from austenite to martensite is not a geometrically reversible process and some energy is dissipated during unloading. Hence, the choice of initial crack length changes transformation strain energy levels considerably. On the other hand, elastic strain energy is not affected by crack propagation history, because it is a measure of elastic strains only.
- Influence of stress ratio R is significant in transformation strain energy and crack opening displacement calculations. A higher stress ratio implies a higher mean load resulting less crack closure. Moreover, ΔK_{th} will decrease by increasing R . Considering the effects of load ratio on crack closure levels, the fatigue crack

growth equation takes the form of :

$$\frac{da}{dN} = f(\Delta K_{eff}, K_{max}) \quad (5.1)$$

where

$$\Delta K_{eff} = f(R)\Delta K \quad (5.2)$$

- In order to determine stresses and strains in the transformation zone, HRR singular field solution is used. The uniaxial behavior of stress-strain curve of SMA resembles that of a plastic material with power-law hardening. At least two distinct power-law stress-strain relations are needed to fully describe stress field around the crack tip. Based on this assimilation, analytic stress intensity factor calculation methods of plastic materials following certain hardening rules are employed for SMAs in Section 3.6.
- J-integral is contour-dependent in the transformation zone of SMAs. A drop in J is observed in fully martensitic region and the extent of this drop can only be captured by keeping the element length close to the crack tip very small. The only mesh-independent parameter in the analysis is far-field J, which is also affected by forward phase transformation. The results for mesh-independent J at a given crack length is close to the ratio of $\Delta U/\Delta A$, therefore it is used in the approximate determination of stress intensity factor.
- A closed form for stress intensity factor, $K_I = C_1(\sigma_0)^{C_2}$, is built to determine ΔK and K_{max} . Crack length dependent C_1 and C_2 are evaluated using FE results. Regarding the condition of unstable crack propagation, critical stresses in terms of crack length, and critical crack lengths in terms of applied maximum loads are calculated. An asymptotic decrease in critical stress levels is observed during crack propagation.
- An average service life of 10^7 cycles is found for Nitinol, which matches experimental observations in literature [13, 91, 94].

The drawback in this work is restrictions brought in by the constitutive model used by ABAQUS. The material model should be modified or replaced with suitable ones to obtain better results. For future studies, additional experimental data (like fracture toughness change during fatigue crack propagation, actual stress intensity, etc.) are needed to compare them with results obtained from finite elements.

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