

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**STRESS DISTRIBUTION OF CRYOGENIC PRESSURE VESSELS UNDER
VARIOUS BOUNDARY CONDITIONS**



M.Sc. THESIS

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Department of Aeronautics and Astronautics Engineering

Aeronautics and Astronautics Engineering Programme

MAY 2020

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MAY 2020

İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

**FARKLI SINIR KOŞULLARI ALTINDAKİ KRİYOJENİK BASINÇ
TANKLARININ GERİLME DAĞILIMLARI**

YÜKSEK LİSANS TEZİ

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MAYIS 2020

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Date of Submission : 04 May 2020
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To my family,



FOREWORD

This is the M.Sc. graduation project completed as a requirement of Aeronautical and Astronautical Engineering Program of Graduate School of Science Engineering and Technology at Istanbul Technical University. In this project, stress distributions of cryogenic pressure vessels are analyzed.

I am very grateful to my advisor Prof. Dr. İbrahim ÖZKOL for being so patient and tolerant to me during thesis period. I owe a debt of gratitude to Recep UFUK and my other coworkers at DeltaV Uzay Teknolojileri A.Ş. for their continuous support. Finally, I thank to my beloved wife and daughter for always being there for me whenever I needed.

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SYMBOLS

A	: area, m ²
a	: linear dimension; hot spot radius or plate radius, m
B	: bulk modulus, Pa
C	: specific heat, J/kg-K
C_v	: specific heat at constant volume, J/kg-K
$C_1, C_2, \dots$	: constants of integration
D_i	: pipe inside diameter, m
D_0	: pipe outside diameter, m
$D_m=D_0-t$	: pipe mean diameter, m
e_m	: elongation due to mechanical loads, m
e_t	: thermal strain parameter, dimensionless
E	: young's modulus, Pa
f_s	: factor of safety, dimensionless
F	: force, N
F_T	: thermal force
G	: shear modulus, Pa
h	: bar or beam dimension, plate thickness, m
S	: area moment of inertia, m ⁴
T	: polar moment of inertia, m ⁴
K_{sp}	: spring constant, N/m
K_t	: thermal conductivity, W/m-K
K_a, K_b, K_c	: factors in, dimensionless
K_y, K_z	: factors, N-m
L	: length, m
ΔL	: change in length, m
M_T	: thermal bending moment, N-m
p	: pressure, Pa
P	: external force, N
r_0	: equilibrium atomic spacing, m
S_u	: ultimate strength, Pa
S_y	: yield strength, Pa
T	: temperature, K or °C
T_0	: temperature in the stress-free condition, K or °C
$\Delta T = T - T_0$	: temperature change, K or °C
$\Delta T_1 = T_1 - T_0$	: temperature change, K or °C
TSP	: thermal shock parameter, K-m/s ^{1/2}
TSR	: thermal stress ratio, K
U	: potential energy; internal energy, J/kg
V	: volume, m ³
V_T	: thermal shear force, N
ΔV	: volume change, m ³
M_r	: bending stress resultant (moment per unit length) in the radial direction, N

M_x	: bending stress resultant (moment per unit length) in the x-direction, N	
M_y	: bending stress resultant (moment per unit length) in the y-direction, N	
M_θ	: bending stress resultant (moment per unit length) in the polar direction, N	
M_ϕ	: bending stress resultant (moment per unit length) in the azimuth direction, N	
N_θ	: mechanical edge force per unit length, N/m	
N_r	: radial membrane stress resultant, N/m	
N_T	: thermal stress resultant, N/m	
N_x	: x-membrane stress resultant, N/m	
N_ϕ	: polar membrane stress resultant, N/m	
Q_x	: shear stress resultant (force per unit length) in the x-direction, N/m	x-



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STRESS DISTRIBUTION OF CRYOGENIC PRESSURE VESSELS UNDER VARIOUS BOUNDARY CONDITIONS

SUMMARY

In this thesis, stress distribution of the systems which works in cryogenic temperatures are analyzed specifically for pressure vessels. In the first chapter of this thesis, concept of thermal stress has been described. In this chapter, a guideline for designing a system at which the temperature changes are critical has been emphasized and some important parameters which are utilized during design process have been mentioned. The terms which indicates mechanical and thermal properties of the material which is going to be selected for the design have been explained. It is also depicted how to obtain these terms both analytically and numerically.

In the second chapter, thermal stress for structures which are simpler than pressure tanks has been introduced. In this chapter, stress-strain formulas for the parts which are exposed to an uniform temperature distribution have been derived. Also, both analytical and numerical design methods have been presented for the situations which have a nonuniform temperature variation across the cross section or axially nonuniform temperature variation. Thermal force and thermal moment have been introduced in this chapter.

In the third chapter, thermal moment concept has been explained. Assumptions and limitations of the analysis has been indicated. Deformation, stress, strain, etc. formulas have been derived for the systems that are exposed to thermal bending loading.

In the fourth chapter, there are calculations and derivations for cylindrical pressure vessels. All the necessary derivations have been made for the systems which are exposed to axisymmetric loading or axial temperature variation.

In the fifth chapter, a practical application of this thesis's topic has been studied. In this chapter, the liquid oxygen tank of TESOR Sounding Rocket which has been developed by Delta V Uzay Teknolojileri A.Ş., has been analyzed under various boundary conditions and loadings and all the necessary calculations have been made. It has been indicated that the results which are obtained from the analytical calculations will be compared with the results which are obtained from the ANSYS package program at the conclusion part. In the fifth chapter, firstly, the layout of the TESOR Sounding Rocket has been depicted via solid model. After that, the dimensions and the geometry of the liquid oxygen tank have been illustrated. Information about the liquid oxygen tank's material selection and its manufacturing methods have been shared. Liquid oxygen tank's loading condition during its operation which are LOX filling, pressurization and flight, respectively, has been explained. The analysis has been accomplished for all three stages of operation separately. Accordingly, at the first step, after the LOX filling the tank is cooled until it reaches 90 Kelvin and only thermal loading are run for this stage. All the calculations have been made until all the necessary inputs are obtained. After this point, a MATLAB code which is provided in APPENDIX of this thesis is executed in order to get the results. At the second step,

the tank is pressurized to 70 bar in addition to thermal loadings. At the third step, the thrust loading which is occurred during the rocket's flight is added to step 1 and 2 conditions. In the same manner as the first step, the calculations have been made until all the necessary inputs are obtained. After this point, a MATLAB code which is provided in APPENDIX of this thesis is executed in order to get the results for second and third steps.

The sixth chapter is the conclusions and recommendations part. All the results of the analysis which are achieved in the fifth chapter have been shared in this chapter with the results which are achieved from ANSYS package program. Also, details of the generated model for ANSYS and its boundary conditions have been explained in this chapter. The prepared MATLAB codes provide the listed parameter's distributions below:

- Axial bending stress resultant distribution
- Axial stress distribution
- Deformation distribution
- Hoop stress distribution
- Polar bending stress resultant distribution
- Polar membrane stress resultant distribution
- Shear stress resultant distribution
- Thermal stress resultant distribution
- Transverse shear stress distribution

All the distribution graphics which are listed above have been provided for all three stages separately. After that, selected parameters have been compared with the ANSYS results and error percentages with respect to each other have been given. Finally, the important points to consider while working with the cylindrical pressure vessels which is operated with cryogenic fluids under various types of loadings have been mentioned and recommendations for designer/manufacture have been made.

FARKLI SINIR KOŞULLARI ALTINDAKİ KRIYOJENİK BASINÇ TANKLARININ GERİLME DAĞILIMLARI

ÖZET

Bu tez çalışmasında kriyojenik sıcaklıklarda çalışılan sistemlerin gerilme dağılımları, basınçlı tanklar özelinde incelenmiştir. Kriyojenik sıcaklık aralığı -150 santigrat derece ile mutlak sıfır (-273 santigrat derece) arası olarak tanımlanan bir sıcaklık aralığıdır. Sıcaklığın düşük olduğu durumlarda maddelerin birtakım özelliklerinde önemli farklılıklar oluşur. Maddenin bu özelliklerine esneklik, elektriksel direnç, dayanım ve iletkenlik örnek verilebilir. Kriyojenik malzemeler ilk olarak 1877’de oksijenin -183 santigrat dereceye kadar soğutulup sıvı faza geçirilmesiyle elde edilmiştir. Kriyojenik malzemeler, hali hazırda soluduğumuz hava içerisinde bulunan gazların basınçlandırılarak soğutulması ve daha sonra aniden genişlemelerine izin verilerek çok çok düşük sıcaklıklara ulaştırılmasıyla elde edilir. Başlıca kriyojenik malzemeler sıvı azot, sıvı oksijen, sıvı argon, sıvı propan ve sıvı helyumdur.

Dünya üzerinde tarih boyunca ölçülen en düşük hava sıcaklığı -89 santigrat derece civarındadır. Dünya üzerinde ölçülen en düşük hava sıcaklığı ile kriyojenik sıcaklıklar arasındaki fark bile oldukça fazladır. Malzeme üzerinde kısıtlama olduğu durumlarda, bir sıcaklık değişimine maruz kalındığında malzeme üzerinde ısı gerilmeleri oluşmaktadır. Böyle dramatik sıcaklık farklılıklarının olduğu sistemlerde ise ısı gerilmeleri oldukça önem arz etmektedir.

Isıl gerilmelerin dahil olduğu durumlar hava-uzay yapılarından zirkonyum geçirilmiş nükleer yakıt çubuklarına kadar birçok mühendislik alanında artış göstermektedir. Bir mühendis için ısı gerilmelerin hafifletilmesinin öneminin anlaşılması çok önemlidir. Örneğin, kriyojenik akışkanları aktarma sistemlerinin tasarımı, vakum gömlekli transfer hatları, ortam sıcaklığından kriyojenik sıcaklıklara soğutulduğunda birkaç santimetreye kadar diferansiyel kontraksiyonlara müsaade edecek seviyede olmalıdır. Herhangi bir sistemin komponentleri, sistem mekanik olarak kısıtlandığında ve sıcaklık değişimine bağlı olarak genişleme veya büzülme serbest olmadığında, sıcaklık değişikliğine maruz kaldığı her durumda ısı gerilmeleri oluşacaktır.

Çoğunlukla ısı gerilmeleri parçayı daha büyük yaparak değiştirilemez. Isıl gerilmeler mekanik kısıtlamalar sebebiyle oluşur ve bu kısıtlamaların kapsamı azaltılarak kontrol altına alınabilir.

Bu tez çalışmasının birinci bölümünde ısı gerilme konsepti anlatılmıştır. Öncelikle ısı gerilmelerin oluşmasının ön şartı olan kısıtlamaların türlerinden bahsedilmiş ve örnekler verilerek konu açıklanmıştır. Ayrıca bu bölümde sıcaklık değişimlerinin kritik olduğu sistemlerin tasarımında izlenecek yol haritası belirtilmiş ve tasarım aşamasında kullanılan önemli parametrelerin üzerinden geçilmiştir. Tasarımda kullanılacak olan malzemenin mekanik ve ısı özelliklerini belirten terimlerin açıklamaları ve analitik veya numerik olarak bu değerlerin nasıl elde edildiği bu bölümde anlatılmıştır.

İkinci bölümde basınç tanklarından daha basit yapılar üzerinde ısı gerilim konseptine giriş yapılmıştır. Bu bölümde düzgün sıcaklık dağılımına maruz kalan parçalar için gerilim-gerinim formül derivasyonları yapılmıştır. Yine ikinci bölümde, eksenel ve kesit alanı boyunca düzgün olmayan sıcaklık değişimine maruz kalındığı durumlar için hem analitik hem de numerik tasarım yöntemleri sunulmuştur. Isıl kuvvet ve ısı moment kavramlarına giriş yapılmıştır.

Üçüncü bölümde ısı moment kavramı açıklanmıştır. Yapılan analizlerdeki kısıtlamalar belirtilerek, ısı moment yüklemesine maruz kalan sistemler için deformasyon, gerilim, gerinim vb. formül derivasyonları yapılmıştır.

Dördüncü bölümde silindirik tanklar için hesaplamalar ve derivasyonlar bulunmaktadır. Hem eksenel-simetrik yükleme için hem de eksenel sıcaklık değişimine maruz kalan silindirik tanklar için gerekli derivasyonlar yapılmıştır.

Beşinci bölümde bu tezde anlatılan konunun pratik bir uygulaması işlenmektedir. Bu bölümde Delta V Uzay Teknolojileri A.Ş. şirketinin geliştirmiş olduğu TESOR Sonda Roketi'nin sıvı oksijen tankı, farklı sınır koşullar ve farklı yüklemeler altında analiz edilmiş ve gerekli hesaplamalar yapılmıştır. Yapılan analitik çözümlemelerin, sonuç bölümünde ANSYS paket programı ile edilen sonlu elemanlar yöntemi sonuçlarıyla karşılaştırılacağı belirtilmiştir. Bu bölümde ilk olarak TESOR Sonda Roketi'nin yerleştirme düzeni katı model üzerinde gösterilmiştir. TESOR Sonda Roketi, yaklaşık yedi metre yüksekliğindedir ve 280 mm çapa sahiptir. Her parçası ve alt komponenti Delta V Uzay Teknolojileri A.Ş. bünyesinde tasarlanmış, üretilmiş, test edilmiş ve uçuşa hazır hale getirilmiştir. Burun konisi LD haack serisi geometriye sahiptir ve uzunluk/çap oranı 3 olarak belirlenmiştir. Burun konisi fiberglass kompozit malzemeden üretilmiştir ve uç kısmında titanyum parça bulunmaktadır. Aerodinamik sürtünme kaynaklı ısınma ve yüksek hızlarda maruz kalınan dinamik basınç faktörlerine dayanım gösterebilmesi için burun konisi ucunda titanyum malzemesi seçimi yapılmıştır. Burun konisi altında gövde parçası bulunmaktadır. Gövde, burun konisinde olduğu gibi fiberglass kompozit malzemeden üretilmiştir. Gövde içerisinde aviyonik kompartmanı bulunduğu için sinyal iletiminde kesinti olmaması adına fiberglass malzemesi seçilmiştir. Gövde üzerinde telemetri antenleri, görüntü aktarma antenleri, GPS (Global Positioning System – Küresel Konumlandırma Sistemi) antenleri ve yerden güç besleme portu için pencereler açılmıştır. Gövde parçası burun konisi ile gövde-burun arayüzü vasıtasıyla bağlanır. Bu arayüz alüminyum 7075 malzemeden üretilmiştir. Gövde içerisinde aviyonik kompartmanının yanı sıra, ayrılma sistemi ve kurtarma sistemi bulunmaktadır. Ayrılma sistemi roketi istenilen irtifaya geldiğinde iki parçaya ayırıp, paraşütlerin dışarıya salınımını sağlayacak olan sistemdir. Kurtarma sisteminde bir adet drogue paraşütü ve bir adet ana paraşüt bulunmaktadır. Gövdenin altında basınçlandırma tankı bulunmaktadır. Basınçlandırma tankı ile gövde birbirlerine basınçlandırma tankı-gövde arayüzü denilen ve alüminyum 7075 malzemeden üretilen parça ile bağlanır. TESOR Sonda Roketi basınç beslemeli bir sonda roketidir. Yani sıvı oksijenin istenilen basınçta kalması ve motor içerisine istenilen debide akıtılmasını sağlamak için basınçlandırma tankı ile sıvı oksijen tankının içerisine basınçlı gaz, regülatörler aracılığıyla aktarılmaktadır. Basınçlandırma tankının altında sıvı oksijen tankı bulunmaktadır. Bu bölümde, sıvı oksijen tankının geometrisi ve boyutları hakkında kısaca bilgi verilmiştir. Sıvı oksijen tankının malzeme seçimi anlatılmış ve tankın üretim metodu hakkında bilgilendirme yapılmıştır. Sıvı oksijen tankının operasyon koşullarındaki yükleme durumu, maruz kalınan yükler açıklanmıştır. Sıvı oksijen tankı 3 farklı aşama

için analiz edilmiştir. Tankın operasyonu sırasıyla sıvı oksijen dolumu, tankın basınçlandırılması ve roketin uçuşu olarak gerçekleştirilmektedir. Buna paralel olarak birinci aşamada, sıvı oksijen dolumu yapılırken tankın sıvı seviyesine kadar olan kısmın 90 Kelvin (-183 santigrat derece) dereceye kadar soğutulması esnasında sadece ısı yükleri uygulanarak analiz yapılmıştır. Hesaplamalar bütün girdiler elde edilinceye kadar yapılmış ve sonuçlar EK te verilen MATLAB koduyla elde edilmiştir. İkinci aşamada ısı yüklemeye ek olarak tank basıncı da dahil edilmiştir. Tank, motorda istenilen stokiometrik karışımı elde edebilmek için ihtiyaç duyulan sıvı oksijen debisini sağlayacak basınca ulaştırılır (70 Bar veya 7 Megapascal). Tank üzerinde hem ısı yüklemesi hem de iç basınç varken analizler yapılmıştır. Aynı şekilde, hesaplamalar bütün girdiler elde edilinceye kadar yapılmış ve sonuçlar EK te verilen MATLAB koduyla hesaplanmıştır. Üçüncü aşamada roketin uçuşu esnasında tank üzerine binen itki yüklemesi de denkleme dahil edilmiştir. TESOR Sonda Roketi KÖRTEM hibrit roket motoru ile güçlendirilmektedir. KÖRTEM hibrit roket motoru, Delta V Uzay Teknolojileri tarafından geliştirilmiştir. Bu motorun itkisi yaklaşık 22000 newton değerindedir. Bu itki değerinin kriyojenik sıvı oksijen tankı üzerindeki yüklemesi simüle edilmiştir. Üçüncü aşamada ısı yüklemesi, tank iç basıncı ve itki yüklemesi mevcut iken bütün analizler ve hesaplamalar yapılmıştır. Birinci ve ikinci aşamada olduğu gibi, hesaplamalar bütün girdiler elde edilinceye kadar yapılmış ve sonuçlar EK te verilen MATLAB koduyla hesaplanmıştır.

Altıncı bölüm sonuçlar ve tavsiyeler bölümüdür. Beşinci bölümde yapılan tüm analizlerin ve ANSYS paket programında yapılan analizlerin sonuçları bu bölümde paylaşılmıştır. Ayrıca ANSYS analizlerinde oluşturan modelin detayları, sınır koşulları bu bölümde açıklanmıştır. Analiz yöntemi ve analiz yönteminin seçilme nedenleri belirtilmiştir. MATLAB kodu aşağıda listelenen parametrelerin tank üzerindeki dağılımının grafiklerini vermektedir.

- Eksenel bükülme gerilim bileşenleri dağılımı
- Eksenel gerilim dağılımı
- Deformasyon dağılımı
- Çevresel gerilim dağılımı
- Polar bükülme gerilim bileşenleri dağılımı
- Polar membran gerilim bileşenleri dağılımı
- Kesme gerilim bileşenleri dağılımı
- Isıl gerilim bileşenleri dağılımı
- Enine kesme gerilim dağılımı

Yukarıda listelenen bütün dağılım grafikleri her üç aşama için ayrı ayrı yazdırılmıştır. Daha sonra seçilen parametreler, ANSYS sonuçlarıyla karşılaştırılmış ve birbirleri arasındaki hata payı paylaşılmıştır. Son olarak, kriyojenik akışkanlarla çalışılan silindirik basınçlı tankların farklı yüklemeler durumunda dikkat edilmesi gereken noktalar üzerinde durulmuş ve tasarımcı/üretimci için tavsiyelerde bulunulmuştur.



1. INTRODUCTION

1.1 Thermal Stress

Thermal stresses are generated when the material is exposed to a temperature variation while there are constraints. These stresses are mechanical stresses as a result of the forces caused by a component makes an attempt to contract or grow in the presence of constraints.

Thermal stresses do not exist without any constraints. As an illustration, if a bar which can be seen from the Figure 1.1 is considered, no stress would be observed with unconstrained ends of the bar while the bar was exposed to a temperature variation $\Delta T = 25^\circ\text{C}$. In addition, in the presence of the same bar with the identical temperature variation while the edges of the bar were firmly connected this time, stresses would be arised in the bar resulting from the force at the bar's edges. This stress is named thermal stress.

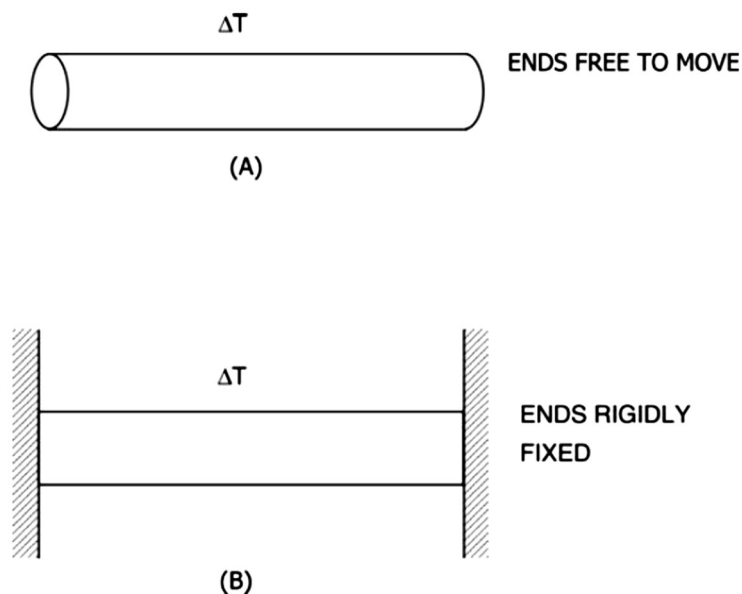


Figure 1.1 : Depiction of external constraints.

For the thermal stress point of view, inner and outer constraints are the two types of constraints. In the presence of temperature variation, outer constraints put limitations

all over the system which inhibit contradiction or expansion of the whole body. For instance, if a steel bar which is fixed at two edges, this constraint would be an external one.

On the other hand, inner constraints are limitations that exist interior of the material since the expansion or contradiction of the material is not the same all around the part, however the material must remain continuous. If it was supposed that a pipe were unconstrained, and sufficient amount of hot liquid were poured into the pipe that provide 10 °C temperature increase at the inner surface of the pipe while the outer surface of the pipe remain at the initial temperature (Figure 1.2). For this type of case, inner layers of the pipe would tend to expand because of the temperature change and outer layers would tend to remain undeformed. Because of the inner and outer layers of the pipe cannot move independently, thermal stresses would be generated for this circumstance. This type of constraints are inner constraints.

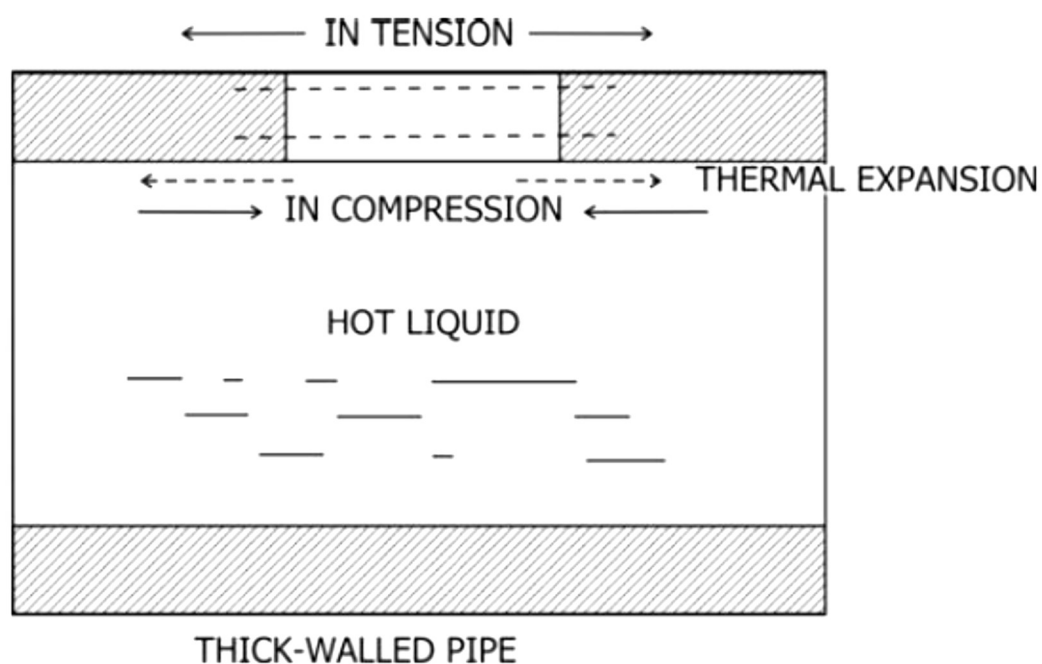


Figure 1.2 : Depiction of internal constraints.

1.2 Thermal Design

“The design process involves more than “solving the problem” in a mathematical manner” (Shigley and Mischke, 1989). If at all possible, other than safety, there would be no design limitations. Nevertheless, various factors have to be considered usually

when a product is being designed. A conventional design flowchart can be seen in Figure 1.3.

In the point of question of thermal stress for the period of design, the component's

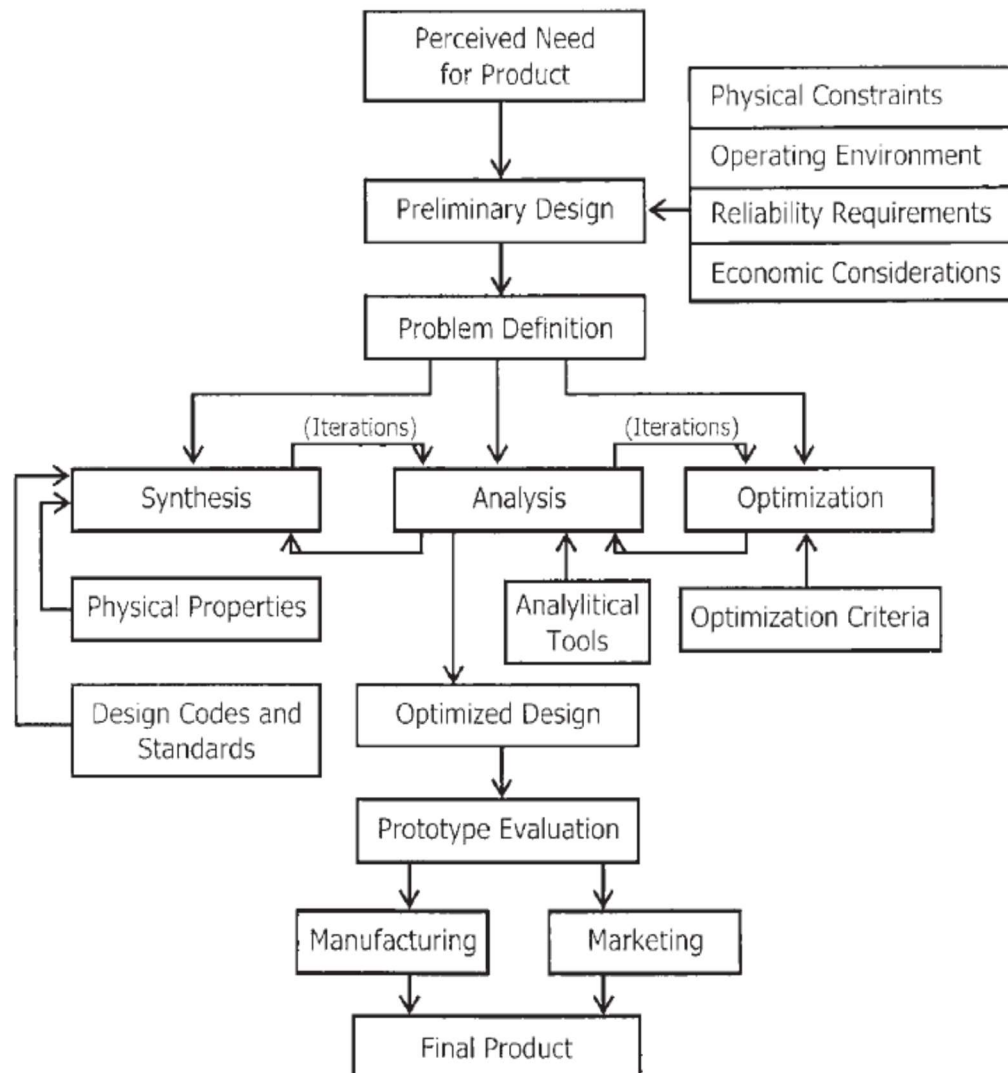


Figure 1.3 : Flowchart of design process.

dimensions are out of topic or poorly related. In these circumstances, at least three options can be considered: choice of the materials, restraint of temperature variation, and constraints relief.

Separate materials would be exposed unequal thermal stresses if the same loading and environmental conditions are applied. For instance, a stainless-steel bar and an aluminum bar, both have the same conditions and same boundary conditions, steel bar

would be exposed to a thermal stress which is nearly double of the amount of aluminum's level.

Thermal stress generally decreases because of the temperature variation's decrement. If the temperature variation is reduced to 25 °C from 50 °C of a bar which is cantilevered from both edges, the thermal stress decreases fifty percent with respect to the amount of thermal stress for the 50 °C temperature variation. Thermal insulation can be used for some steady-state thermal condition in order to reduce the temperature change of the part.

Decreasing or relaxing the constraints on the system while the design process is carried on is the most effective method in many cases in order to restrict the stresses which is caused by temperature variation. By using more flexible elements, the system can be made less constrained.

1.3 Safety Factor

Generally, a designer works on a part so that the failure does not occur, if it is not desired specifically. For instance, when a critical electric current amount is exceeded, fuses have to fail in order to protect the electrical system. Conversely, for a liquid oxygen transfer line, the wall thickness of the pipes must be determined in a suitable way in order to prevent the failure of the pipes while the system is operating.

Another parameter is the safety factor for designing concerns. Safety factor value is obtained by dividing the parameter of failure by the same parameter which will be used in the design. What failure criteria must be considered for the part or the system is the decision that the designer should make firstly. There are various failure criteria such as;

- Rupture of the part
- Yielding of the part
- Fatigue
- Buckling
- Extreme displacement or vibration
- Extreme noise generation

In order to evaluate and plan for safety, determination of the appropriate failure criteria is mostly the key factor.

If breaking of the part would be considered as failure point then the ultimate strength of the material will be the parameter of failure when the part is exposed to a stress and the creep effects are negligible which means temperature is not high enough. Nevertheless, the yield strength would be selected as failure parameter when the failure criterion is yielding. For both cases, the highest stress value which is acted on the part is the design parameter. The safety factor can be depicted as below.

$$f_s = \frac{S_u}{\sigma} \text{ or } f_s = \frac{S_y}{\sigma} \quad (1.1)$$

For any other failure criterion, safety factor can be determined in a similar way.

The safety factor can be determined for such cases from the codes such as ASME (American Society of Mechanical Engineers) codes. For example, ASME Code for Unfired Pressure Vessels states that the safety factor is maintained at 3.5 in order to design a cylindrical pressure vessel. The designer has to determine the factor of safety during the early periods of the design activity if it is not prescribed. Designing a part or a system with a factor of safety such that it is assured that it does not fail under the worst possible combinations of conditions, is not generally cost effective. Consequently, the designer often determines the factor of safety in accordance with the personal knowledge from in similar design activities. Generally, the uncertainties in many factors involved in the design are depicted by the f_s . Some of these ambiguities are as follows:

- Material property data uncertainties
- Maximum loading uncertainties
- Assumptions uncertainties
- Environmental effects (corrosive, etc.)

The uncertainty in the design process is attempted to reduce by the stochastic or trustworthiness -based design approach. Nevertheless, because ambiguity exist in the “uncertainty” information and the information is not comprehensive, the method has its disadvantages based on this fact.

With the help of the clarifying the origins of the scatter in the info for the ultimate or yield strength, the uncertainty of the strength parameter may be mollified. The material properties such as ultimate and yield strengths are generally typical amount or average values in the literature. In this circumstance, this means while the 50% of the datum stays higher from the average value and the other 50% of the datum stays lower from the listed amount. From the tables which illustrates normal probability distributions, the strength value that the presumptions of experiencing a strength which is smaller than this amount can be calculated, if the standard deviation is received. For this circumstance, S_u is depicted below:

$$S_u = k_S \bar{S}_u \quad (1.2)$$

S_u : ultimate strength

k_S : factor which is described as below:

$$k_S = 1 - F_P(\hat{\sigma}_S/\bar{S}_u) \quad (1.3)$$

Values of the F_P (probability factor) may be found from the Table 1-1. Other strength parameter may use the same expression.

There are no ready-to-use suitable information for the strength datum $\hat{\sigma}_S$ for all materials. The designer is responsible to decide what amount of risk is tolerable for the lowest strength which is applied in design. If there is no standard deviation information about the material, an approximation may be used for the $(\hat{\sigma}_S/\bar{S}_u)$ ratio such that it is equal to 0.05 for S_u ; 0.075 for S_y and 0.1 for fatigue strength.

The value of the safety factor which is selected is affected by the reliability of the anticipated maximum mechanical or thermal loading which is utilized in the design. If some safeguards such as pressure relief valves etc. are utilized in the design which inhibit surpassing a regulated amount for loading, then the safety factor can be determined lower with respect to the circumstances that more ambiguous loading is applied.

The mathematical model is generated based on assumptions or simplifications and the validity of the model used in the design has a certain effect on the selected safety

Table 1.1 : Examples of probability factor.

Survival Rate	Failure Rate	Probability Factor
0.9	0.1	1.282
0.95	0.05	1.645
0.975	0.025	1.96
0.99	0.01	2.33
0.999	0.001	3.09
0.9999	0.0001	3.72

factor. A very complicated numerical analysis is not correct exactly. In order to determine the safety factor, the anticipated ambiguity in the analysis can be utilized.

1.4 Thermal Expansion Coefficient

Thermal expansion coefficient is a significant material property regarding to the thermal stress. The linear thermal expansion coefficient (α) and the volumetric thermal expansion coefficient (β_v) are the two thermal expansion coefficients which are going to be generally considered.

During the stress on a body is maintained as steady, the fractional variation in length, width or depth per unit temperature variation is the definition of the linear thermal expansion coefficient. The mathematical expression of the linear thermal expansion coefficient is stated below:

$$\alpha = \frac{1}{L} \left(\frac{\partial L}{\partial T} \right)_{\sigma} \quad (1.4)$$

Usually under the stress-free circumstance, the linear thermal expansion coefficient is measured.

While the pressure is maintained as constant, the fractional change in volume per unit change in temperature is the definition of the volumetric thermal expansion coefficient. The mathematical expression of the volumetric thermal expansion coefficient is stated below:

$$\beta_t = \frac{1}{L} \left(\frac{\partial V}{\partial T} \right)_p \quad (1.5)$$

If the material's properties are same in all directions, the material is called isotropic. The linear thermal expansion coefficient and the volumetric thermal expansion coefficient are related by the expression as stated below for an isotropic material:

$$\beta_t = 3\alpha \quad (1.6)$$

If the material's intermolecular forces are considered, the change of the thermal expansion coefficient regarding to temperature can be explained. In Figure 1.4, it may be seen that an atom couple's intermolecular potential energy graphic is asymmetrical. While the temperature increases or the atom acquires more energy, the average space between the two atoms increases which means that expansion occurs.

If the potential energy graphic would not be asymmetric, the average of the r_1 and r_2 positions would be constant ($r_{(ave)}=r_0$). The equilibrium spacing at $T=0$ is r_0 .

Even though the atoms tend to drift apart while the temperature is raised, the mean space between the atoms stay constant so that the thermal expansion will not be generated acted on the material due to the equilibrium spacing between the atoms remains constant with a symmetrical energy curve, independent from the energy level.

While the material's temperature is raised, atom couple's equilibrium spacing becomes larger because the true potential energy curve is not symmetrical around the equilibrium spacing at 0 °K. As the temperature or the energy is raised, mean distance between the atoms change rate increases. As a consequence of this, the thermal expansion coefficient increases while the temperature is raised. If the material temperature approaches to 0 °K, the thermal expansion coefficient value converges to 0 as stated by the thermodynamics' 3rd law.

Grüneisen relationship depicts that the relation between the thermal expansion coefficient and the interatomic vibrational energy since that the material's C_v rely on the atoms' vibrational energy for crystalline solids.

$$\beta_t = \frac{\gamma_G C_v \rho}{B} \quad (1.7)$$

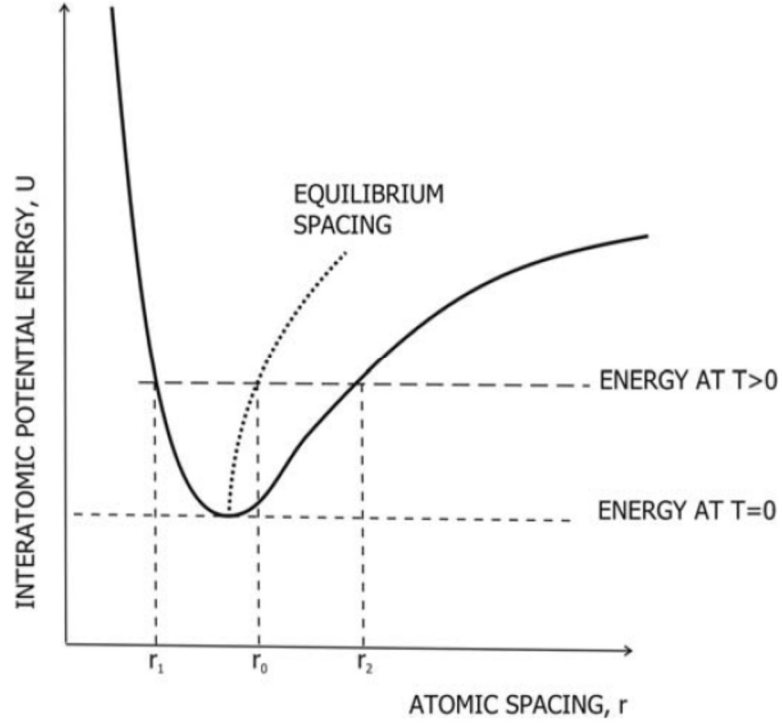


Figure 1.4 : Interatomic potential energy vs atomic spacing graph.

or

$$\alpha = \frac{\gamma_G C_v \rho (1 - 2\nu)}{E} \quad (1.8)$$

C_v : specific heat at constant volume

ρ : density

E : Modulus of elasticity

γ_G : Grüneisen Constant

B : isothermal bulk modulus

γ_G values for various materials are depicted in Table 1.2.

For a metal, the density and the bulk modulus are not largely dependent on temperature. For particular ranges of temperature, Grüneisen constant is dependent of temperature. If it was actually not dependent on temperature, then β_t equation above depicts that the thermal expansion coefficient will change in the similar way with temperature as the C_v does. Thermal expansion coefficient is proportionate to T^3 for a

Table 1.2 : Grüneisen constant examples.

Material	Lattice Structure	γ_G
Al	FCC	2.17
Cu	FCC	1.96
Au	FCC	2.4
Pb	FCC	2.73
Ni	FCC	1.88
Pa	FCC	2.23

genuine crystalline solid at minor temperatures. The thermal expansion coefficient is proportionate to temperature at about medium temperature and higher than medium temperature, and it is far fewer reliant on temperature that is the circumstance at temperatures which are pretty low.

Specifically, for cryogenic temperatures, within satisfactory certainty, thermal expansion coefficient may not be assumed constant. For cryogenic temperatures, the thermal strain parameter e_t may be used. Thermal strain parameter is depicted as below:

$$e_t(T) = \int_0^T \alpha \, dT \quad (1.9)$$

Between T_1 and T_2 , the mean thermal expansion coefficient is given as below

$$\bar{\alpha} = \frac{e_t(T_2) - e_t(T_1)}{T_2 - T_1} \quad (1.10)$$

1.5 Modulus of Elasticity

Modulus of elasticity is another material property which gives the material's amount of elasticity that is significant in determining thermal stresses. Under constant temperature which means isothermal condition, the Young's modulus is usually measured. Young's modulus' mathematical expression is as stated below:

$$E = \left(\frac{\partial \sigma}{\partial \epsilon} \right)_T \quad (1.11)$$

For the material, the stress level (σ) is below the relational limit. Mechanical strain is denoted by “ ϵ ” and it is caused by the stress (σ).

A characteristic interatomic force graphic can be seen in Figure 1.5. Modulus of elasticity is firstly the cause of the forces between the atoms in a material. The curve's slope at the balance space of the atoms which can be seen in Figure 1.5 defines the young's modulus' value. Theoretical expression of the young's modulus is as stated below:

$$E = \frac{9Z^2e^2}{16\pi\epsilon_0r_0^4} \quad (1.12)$$

Z : atomic ion's valence

e : charge of an electron

ϵ_0 : permittivity of free space

r_0 : equilibrium spacing of the atoms

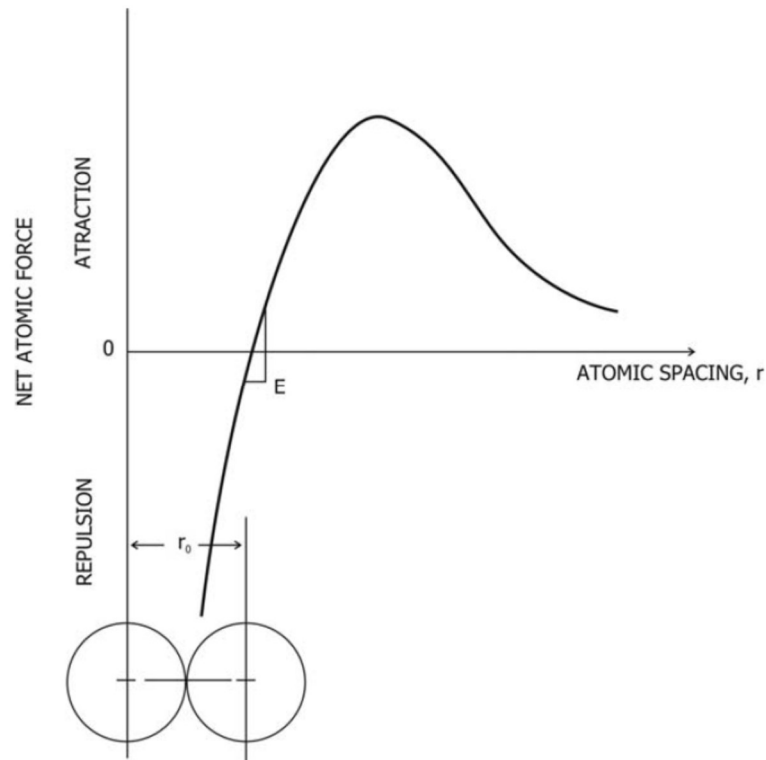


Figure 1.5 : Net atomic force vs atomic spacing graph.

1.6 Poisson's Ratio

When a force is applied to a material in a certain direction, the atoms of the material are pulled apart and a corresponding contraction which is perpendicular to the applied force is generated. The ratio of the lateral strain to the strain is called Poisson's ratio (μ). Poisson's ratio can be stated as below, assuming that the force is acted on the x-coordinate:

$$\mu = -\frac{\epsilon_y}{\epsilon_x} \quad (1.13)$$

ϵ_y : mechanical strain in the y-direction

ϵ_x : mechanical strain in the x-direction

Because there is a contradiction in the transverse direction, negative strain value in the y-direction is introduced. If the applied force in the x direction causes negative strain, then the lateral strain would have positive value.

If it is assumed that a tensile force is applied to a bar, the effect of this force on the volume of the material can be analyzed. If the bar has a length of L and a cross section area of $a \times b$. Stress-free volume of the bar would be:

$$V_0 = Lab \quad (1.14)$$

After the bar were exposed to a tensile load, the final volume of the bar (V_1) is connected to the initial measures and Poisson's ratio:

$$V_1 = (L + \epsilon_x L)(a + \epsilon_y a)(b + \epsilon_z b) \quad (1.15)$$

If the Poisson's ration is introduced to this equation, we get:

$$\begin{aligned} V_1 &= Lab(1 + \epsilon_x)(1 - \mu\epsilon_x)(1 - \mu\epsilon_x) \\ &= V_0(1 + \epsilon_x)(1 - 2\mu\epsilon_x + \mu^2\epsilon_x^2) \end{aligned} \quad (1.16)$$

ϵ_x^2 and ϵ_x^3 are negligible because their small values, the fractional change of volume may be expressed as below:

$$\frac{\Delta V}{V} = \frac{V_1 - V_0}{V} = \epsilon_x(1 - 2\mu) \quad (1.17)$$

The Poisson's ratio's value for a homogeneous isotropic material is between 0 and 0.5. A material which is exposed a pressure that would increase the volume of the material if the Poisson's ratio is greater than 0.5 and for engineering materials this behavior is not observed. The volume of a material remains constant while a force (tensile or compressive) is applied if the Poisson's ratio of the material is 0.5 ($\mu = 0.5$).

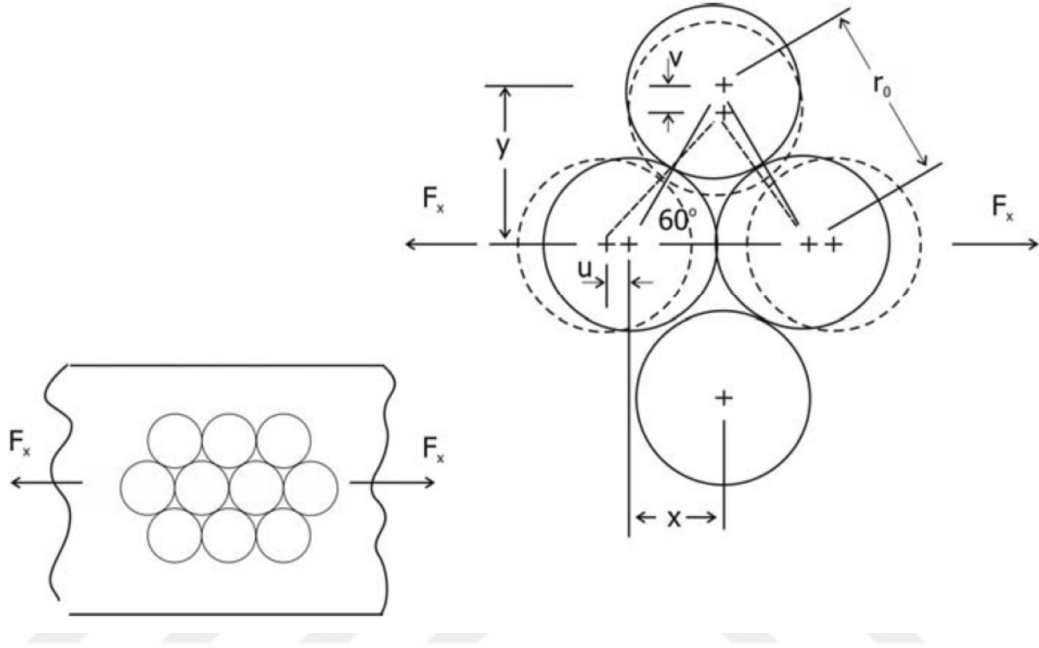


Figure 1.6 : Poisson's ratio depiction.

Poisson's ratio is almost independent from the temperature because the it is a feature which is relied on the geometry or the atoms' pattern in the material. For a metal which has a face centered cubic lattice pattern or hexagonal close pack lattice pattern, the Poisson's ratio should be equal to 1/3. From the Figure 1.6:

$$x^2 + y^2 = r_0^2 = \text{constant}$$

The displacements can be calculated for u , v as below:

$$2xu + 2yv \approx 2xdx + 2ydy = 0$$

Or

$$\frac{u}{v} = \frac{y\varepsilon_y}{x\varepsilon_x} = \frac{-x}{y}$$

With the Poisson's ratio in the equation:

$$\mu = \frac{-\varepsilon_y}{\varepsilon_x} = \left(\frac{x}{y}\right)^2 = \tan^2 30 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

1.7 Other Elastic Moduli

Several other elastic moduli have been described beside from the modulus of elasticity and Poisson's ratio. Only two of the elastic moduli are independent for an isotropic material.

Shear modulus of elasticity is denoted as 'G' and it is obtained for a material below the elastic limit by dividing shearing stress (τ) by shear strain (γ). Elastic region means that stresses are less than the proportional limit. The shear modulus of elasticity is also referred as the shear modulus and the modulus of rigidity.

$$G = \frac{\tau}{\gamma} \quad (1.18)$$

If the material is isotropic, the shear modulus is related to Young's modulus and Poisson's ratio by the equation below:

$$G = \frac{E}{2(1 + \mu)} \quad \text{eq. (1.19)}$$

Modulus of rigidity would be equal to (3/8)E if the Poisson's ratio of a material is equal to 1/3.

Under steady temperature conditions, the pressure change per unit volumetric strain (volumetric strain = $\Delta V/V$) of a material is called the isothermal bulk modulus (B). It is also denoted as the volume modulus of elasticity.

$$B = V \left(\frac{\partial p}{\partial V} \right)_T \quad (1.20)$$

If the material is isotropic, the bulk modulus is connected to modulus of elasticity and Poisson's ratio by the relationship below:

$$B = \frac{E}{3(1 - 2\mu)} \quad (1.21)$$

If a material has a Poisson's ratio value of 1/3 ($\mu = 1/3$), bulk modulus of elasticity would be equal to Young's modulus of elasticity ($B=E$). As stated before, if a material

has a Poisson's ratio value of 0.5, there are no volume change when it is exposed to a pressure, therefore if $\mu = 0.5$, the bulk modulus of elasticity is infinite.

In order to relate bulk modulus of elasticity (B), modulus of rigidity (G) and Young's modulus (E):

$$\frac{1}{B} + \frac{3}{G} = \frac{3(1 - 2\mu)}{E} + \frac{(3)(2)(1 + \mu)}{E} = \frac{9}{E} \quad (1.22)$$

λ_L is written as below:

λ_L : Lamé elastic constant

$$\lambda_L = \frac{\mu E}{(1 + \mu)(1 - 2\mu)} = \frac{3\mu B}{1 + \mu} \quad (1.23)$$

The Lamé constant is equal to 0.75E if the Poisson's ratio of the material is equal to 1/3. Lamé constant can be rewritten by using the modulus of rigidity and the bulk modulus for any Poisson's ratio as below:

$$\lambda_L + \frac{2}{3}G = B \quad (1.24)$$

1.8 Thermal Diffusivity

There exist transient or time dependent temperature distributions in many circumstances including thermal stresses. The thermal stress distribution and the temperature distribution for these situations are dependent on the thermal diffusivity (κ). By using the density, thermal conductivity, and specific heat, the thermal diffusivity may be described:

$$\kappa = \frac{k_t}{\rho c} \quad (m^2/s) \quad (1.25)$$

Thermal diffusivity's unit is square meter per second (m^2/s) in the SI system.

The thermal diffusivity value helps us to understand how quickly energy can be absorbed by a solid body. Energy can diffuse fast into the material which has a large amount of thermal diffusivity and that means for short distance on the material, large temperature changes cannot be observed. Therefore, thermal stress would be lower with respect to the material which has a smaller thermal diffusivity.

Thermal diffusivity for a solid body may be expressed as stated below:

$$\kappa = \frac{1}{3} \bar{v} \lambda_m \quad (1.26)$$

$\bar{v}$: velocity of the energy carriers

λ_m : mean free path

At medium temperature and above, the thermal diffusivity is relatively constant with temperature variations for metals. The thermal diffusivity of metals is strictly dependent on temperature, at very low temperatures and it changes as T^{-3} to T^{-4} .

1.9 Thermal Shock Parameters

If a material is exposed to a sudden temperature change in the medium around the material, thermal shock is generated. Reentry of a spacecraft into the atmosphere or a cold internal combustion engine's start-up period are examples of sudden temperature change in the environment. Some materials are less effected to thermal shock than other under identical environmental conditions. Thermal shock may be a significant difficulty for materials which are brittle, because they show small mechanical strains before rupture. Although the materials which are ductile can endure more mechanical strains before failure, yielding of the material may be occurred by thermal shock. Also, thermal fatigue failure can be observed in the presence of repeated thermal shock for ductile materials.

In order to determine the materials in order to endure a specific tensile load for least weight of the body, the strength-to-weight ratio (S_y / ρ) is a substantial parameter. Likewise, for a given temperature variation, thermal shock parameter is a suitable material feature in order to determine the material which would endure the thermal shock. Thermal shock parameter was introduced by Schott and Winkelmann in 1894 as below:

$$TSP = \frac{S_u \sqrt{\kappa}}{\alpha E} \quad (1.27)$$

S_u : Ultimate tensile strength

κ : Thermal diffusivity

A higher level of stress may be endured if a material has a large value of S_u with respect to a material with a low ultimate tensile strength. Lower thermal strains and smaller

thermal stresses correspondingly would be generated if a material has a low thermal expansion coefficient with respect to a material which grows bigger when the temperature of the material is varied. Thermal strain would be accommodated better by a material which has a small Young's modulus value with respect to a material with larger modulus of elasticity because it is more elastic. Lastly, less temperature gradient would be able to be generated by a material which has a large thermal diffusivity with respect to a material with a lower thermal diffusivity, because thermal energy may be extend throughout the material with the high thermal diffusivity value rapidly.

Thermal shock parameter brings together the ultimate tensile strength, thermal expansion coefficient, Young's modulus and thermal diffusivity which means that a material which has a large value of thermal shock parameter should have a large S_u , a small Young's modulus, a large thermal diffusivity and a small thermal expansion coefficient in order to have an acceptable thermal shock resistance. The thermal shock parameter values of various materials may be seen in Table 1.3.

Table 1.3 : TSP and TSR values for some materials.

Material	αE	TSR	TSP
Aluminum, 2024-T3	1.652	280	1.94
Aluminum, 3003-H12	1.553	83.7	0.68
Aluminum, 6061-T6	1.615	192	1.57
Beryllium copper	2.208	521	2.59
Brass, 70/30	1.21	291	1.7
Bronze, UNS-22000	1.833	202	0.6

The time-dependent thermal features do not affect the thermal stresses under steady state conditions. For such circumstances, the thermal stress ratio is a significant property of material in order to use in evaluating the resistance of the material to the thermal stresses.

$$TSR = \frac{S_u}{\alpha E} \quad (1.28)$$

In general, a material can endure better to thermal stresses if it has a large thermal stress ratio. In Table 1.3, various materials' thermal stress ratio TSR values may be seen.



2. THERMAL STRESS IN BARS

2.1 Stress and Strain

Stress can be described as a force per unit area that the force is acted on, from a basic viewpoint. Based on the engineering's viewpoint, direct or extensional stresses and shear stresses are the two types of stresses. For extensional or direct stresses, the force is applied to the perpendicular or normal to the surface. For shear stresses, the force is applied parallel or tangential to the surface. Direct stresses are denoted with the symbol σ while shear stresses are denoted with the symbol τ . Tensile stresses are assumed to have a positive sign while compressive stresses are assumed to have a negative sign.

Strain can be described as a expansion or contraction per unit length from a basic viewpoint. Based on the engineering's viewpoint, direct or extensional strains and shearing strains are the two types of strains. For directional or extensional strains, deformation is a variation in material's length. For shear strains, deformation is a distortion of the material's shape. Direct strains are denoted with the symbol ε while the shear strains are denoted with the symbol γ . Positive strains will be caused by the material's elongation or stretch. On the other hand, negative strains will be caused by the contraction of the material.

If temperature change occurs, the material's strain is a temperature and stress' function. If the chain rule is applied, differential direct strain can be defined as below:

$$d\varepsilon = \left(\frac{\partial \varepsilon}{\partial \sigma} \right)_T d\sigma + \left(\frac{\partial \varepsilon}{\partial T} \right)_\sigma dT \quad (2.1)$$

The first term is related to the modulus of elasticity. The physical quantity in the subscript on the partial derivatives states that they are held constant for evaluating of the derivative. If the stress is below the elastic limit for the material,

$$E = \left(\frac{\partial \sigma}{\partial \varepsilon} \right)_T = \frac{\sigma}{\varepsilon_m} \quad (2.2)$$

ε_m : Mechanical strain due to applied force

In order to evaluate the second term in the $d\varepsilon$ equation above:

$$\alpha = \frac{1}{L} \left(\frac{\partial L}{\partial T} \right)_{\sigma} = \left(\frac{\partial \varepsilon}{\partial T} \right)_{\sigma} \quad (2.3)$$

Direct strain's differential may be expressed as below:

$$d\varepsilon = \frac{d\sigma}{E} + \alpha dT \quad (2.4)$$

If this equation is integrated from initial temperature to any temperature value, stress-strain relationship is obtained for 1-D systems:

$$\varepsilon = \frac{\sigma}{E} + \int_{T_0}^T \alpha dT = \frac{\sigma}{E} + [e_t(T) - e_t(T_0)] \quad (2.5)$$

e_t : Thermal strain parameter

This equation can be rewritten if the thermal expansion coefficient is assumed as constant:

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T \quad (2.6)$$

$$\Delta T = T - T_0$$

And,

$\frac{\sigma}{E}$: Mechanical strain

$\alpha \Delta T$: Thermal strain

In order to get the substitute expression of the stress-strain-temperature relationship, this equation can be solved for the stress

$$\sigma = \varepsilon E - \alpha E \Delta T \quad (2.7)$$

2.2 Nonuniform Temperature Distribution

Thus far to this point, temperature changes that cause the thermal stress have been considered uniform throughout the material's length and across the material's cross section. At this point, the temperature variation's effects lengthwise and through the transection within the material will be analyzed.

2.2.1 Temperature variation in the lengthwise direction

If it is considered that the case that the temperature changes throughout the bar's length yet the temperature across the cross-section remain constant. The temperature variation is

$$\Delta T = T(x) - T_0 \quad (2.8)$$

T_0 is the material's temperature under the circumstance of zero stress and the x-coordinate is evaluated from the bar's one edge.

The displacement of one side of a differential section of the bar at any location along the bar is denoted by u . The displacement of the other side of the differential element at $x+dx$ is $[u+(du/dx)dx]$ if the displacement is not constant. From the definition of the strain, which is the variation in length per unit length, the strain of the differential element may be calculated as:

$$\varepsilon = \frac{\left(u + \frac{du}{dx} dx\right) - u}{dx} = \frac{du}{dx} \quad (2.9)$$

For a particular position, if the strain is uniform or not, the stress-strain temperature relation is as below:

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T = \frac{du}{dx} \quad (2.10)$$

If a force balance to the element is applied, we get the equation below if traction or shear forces do not exist:

$$\sigma A - \left(\sigma + \frac{d\sigma}{dx} dx\right) A = 0 \quad (2.11)$$

Thus

$$\frac{d\sigma}{dx} = 0 \text{ or } \sigma = \text{constant} \quad (2.12)$$

In order to get the displacement throughout the bar at any position, the strain equation can be integrated with the circumstance that the no dislocation is occurred at $x=0$:

$$u = \frac{\sigma}{E} x + \int_0^x \alpha \Delta T(x) dx \quad (2.13)$$

This equation may be rewritten if α is assumed constant:

$$u(x) = \frac{\sigma}{E}x + \alpha \int_0^x \Delta T(x)dx \quad (2.14)$$

The displacement at $x=L$ can be stated as below:

$$u(L) = \frac{\sigma}{E}L + \alpha L(\Delta T)_{ave} \quad (2.15)$$

$(\Delta T)_{ave}$: average temperature change along the bar

$$(\Delta T)_{ave} = \frac{1}{L} \int_0^L \Delta T(x)dx \quad (2.16)$$

Overall strain ε_0 is described as:

$$\varepsilon_0 = \frac{u(L)}{L} = \frac{\sigma}{E} + \alpha(\Delta T)_{ave} \quad (2.17)$$

This equation is the same in form as the uniform temperature change situation's expression which is previously stated. This means if the average temperature variation is utilized like for this situation, in order to find out the thermal stress for a non-uniform temperature change along the length, uniform temperature change expressions may be utilized.

2.2.2 Temperature variation across the cross section

If it is considered that the situation the temperature can change through the transection, yet the temperature is uniform throughout the length of the bar.

Furthermore, it is considered which the situation that the distribution of the temperature is symmetrical about the centroid axis of the cross section or the situation that the bar cannot bend because it is constrained externally. Symmetrical temperature distribution through the transection is as below:

$$\int_A \alpha E \Delta T(y) y dA \equiv 0 \quad (2.18)$$

From the centroid axis of the cross-section y-coordinate is measured. Transverse deformation and bending stresses will be generated if the bar is not constrained in bending or the temperature distribution is not symmetrical.

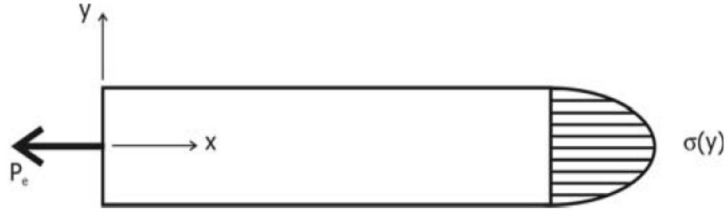


Figure 2.1 : Temperature variation across the thickness.

For Figure 2.1, the equation below is obtained if a force balance is applied:

$$P_e = \int_A \sigma dA \equiv 0 \quad (2.19)$$

Throughout the bar's length, temperature does not vary. In this circumstance, total strain is constant. This equation may be expanded as below by introducing stress:

$$P_e = \int_A E \varepsilon_0 dA - \int_A \alpha E \Delta T dA \quad (2.20)$$

And the thermal force is:

$$F_T = \int_A \alpha E \Delta T dA \quad (2.21)$$

If it is noted that the total strain is not a function of the cross-sectional area, if the thermal force is introduced to the equation above:

$$P_e + F_T = \varepsilon_0 \int_A E dA \quad (2.22)$$

For the circumstance that the thermal expansion coefficient and Young's modulus are not functions of temperature, thermal force and total strain may be rewritten as below

$$F_T = \alpha E \int_A \Delta T dA = \alpha E A \Delta T_m \quad (2.23)$$

$$\varepsilon_0 = \frac{P_e + F_T}{EA} \quad (2.24)$$

ΔT_m : average temperature variation through the transection

$$\Delta T_m = \frac{1}{A} \int_A \Delta T dA \quad (2.25)$$

In order to calculate the stress at any location which is far enough from the ends of the bar by using $\sigma = -\alpha E \Delta T$:

$$\sigma(y) = E \varepsilon_0 - \alpha E \Delta T(y) = \frac{P_e}{A} + \alpha E [\Delta T_m - \Delta T(y)] \quad (2.26)$$

At the edges of the bar, the loading would not be uniform and at the ends of the bar the stress distribution cannot be calculated particularly by the equation above. This equation can be used for the stress at locations away from the ends, in accordance with the Saint Venant's principle. The loading's effect is not limited to the area near the applied load's point for some thin walled structures such as monocoque cylinders under concentrated loads and various sheet and stringer geometries.

Because in the vicinity of the regions near the boundary, the deviation's magnitude between the true and anticipated stresses and dislocations is needed to state approximations, a comprehensively common and brief declaration of Saint Venant's principle cannot be made easily (Boresi and Chong, 1987). From the engineering design view, Saint Venant's principles can be summed up as follows. Almost the identical displacements and stress distribution in the areas which are far removed enough from the area where the force is applied, are generated by the two distinctive yet statically same force systems that is applied on the small part of the external of a solid.

It can be secured that the stresses in the bar at distances around two or three times the thickness of the bar may be defined from the equation above for any symmetrical temperature distribution regarding the Saint Venant's principle. For this circumstance, on the ends of the bar, the resultant force is P_e and the resultant moment is equal to zero.

If it is considered that the bar is not constrained at the ends or $P_e=0$. By using thermal force and total strain formulas, the overall strain may be found as below

$$\varepsilon_0 = \frac{F_T}{EA} = \alpha \Delta T_m \quad (2.27)$$

The stress distribution is found as:

$$\sigma = -\alpha E [\Delta T(y) - \Delta T_m] \quad (2.28)$$

It is observed that, because the external surface and internal areas tend to grow or shrink at distinctive rates, there are internal constraints, although external constraints do not exist. However, the bar must remain one piece. In these cases, thermal stresses are generated because of the internal constraints even though there are no external forces acting on the bar.





3. THERMAL BENDING

3.1 Limitations for The Analysis

For the case of a temperature change through the transection of a beam exists, the relationships between the stress and the strain will be examined in this chapter. the strength of materials approach will be the base of these analysis, so the following restrictions will be introduced.

- All planes of the cross section stay plane
- Effects of Poisson's ratio are negligible
- The lengthwise axis is straight at first
- Torsion forces do not exist
- The stresses are in elastic region
- Stresses are in the elastic region

In order to be exactly accurate, an ordinary moment which is acted on the part's edges is the exclusive condition that after bending, the transection of the part would stay plane. Shear stresses are the cause of the distortion of the transection when the part is exposed to a load and no outer crosswise forces will produce shearing stresses in the part (Timoshenko and Goodier,1970). The shear stress effect's significance depends on the loading condition. Nevertheless, where the part's length is denoted by L , if the part's depth is less than $L/10$, the error between the strength of materials treatment and a more accurate elastic theory treatment does not exceed 5% generally.

The sidewise expansion's or contraction's effect on the stress is significant if the part is very wide. Where μ is the Poisson's ratio, for a part with a width (b) which does not exceed the $h/20\mu$, the effect of lateral contraction does not exceed 5% if $b/h < 1$. For $0.25 < \mu < 0.33$, where the part's depth is denoted by h , the part's width should be smaller than $0.15h$ to $0.20h$ in order to be able to neglect Poisson's ratio effects.

The stress distribution would not be symmetrical around the neutral axis if the part's lengthwise axis is not initially straight. Furthermore, the part's transection's centroid

and neutral axis would not be identical. In a bended part, the bending stress is not proportionate to the distance from the centroid axis or the neutral axis. Radius of curvature is denoted by R , if the depth of a part is smaller than $0.05R$, the stress which is evaluated by the straight beam treatment is smaller than 5% different from the stress which is determined with beam curvature's results treated in general.

3.2 Stress Relationships

If it is considered that the situation of thermal stresses which are generated in a part that the temperature changes in the crosswise direction through the transection exclusively. If it is assumed that bending of the beam may exist, total strain can be expressed as the sum of two component

- Strain throughout the centroid axis which is a uniform extensional strain ε_0 .
- Bending strain ε_b , which changes linearly through the transection

For temperature dependent properties, or for constant material properties strain may be expressed. If the material properties is considered constant, the strain is:

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T = \varepsilon_0 + \varepsilon_b \quad (3.1)$$

In order to evaluate the strain component, it is considered that the beam, which is depicted in Figure 3.1, for little amount of rotation, the tangent of the ψ is almost equivalent to the ψ itself, defined as below:

$$\tan \psi \approx \psi = \frac{\varepsilon_b L}{y} \quad (3.2)$$

The angle of rotation may be written by using the length of the part and the radius of curvature from the geometry of the part's deflection

$$\frac{L}{2R} = \tan \frac{\psi}{2} \approx \frac{\psi}{2} \quad (3.3)$$

For the angle ψ , by introducing two expression above, the following equation for the bending strain component may be obtained:

$$\varepsilon_b = \frac{y}{R} \quad (3.4)$$

By combining the equations, the stress may be expressed as:

$$\sigma = E\varepsilon_0 + \frac{E y}{R} - \alpha E \Delta T \quad (3.5)$$

If a force equilibrium is made between the part's ends and any transection of the part then utilize the equation above, the following expression is obtained.

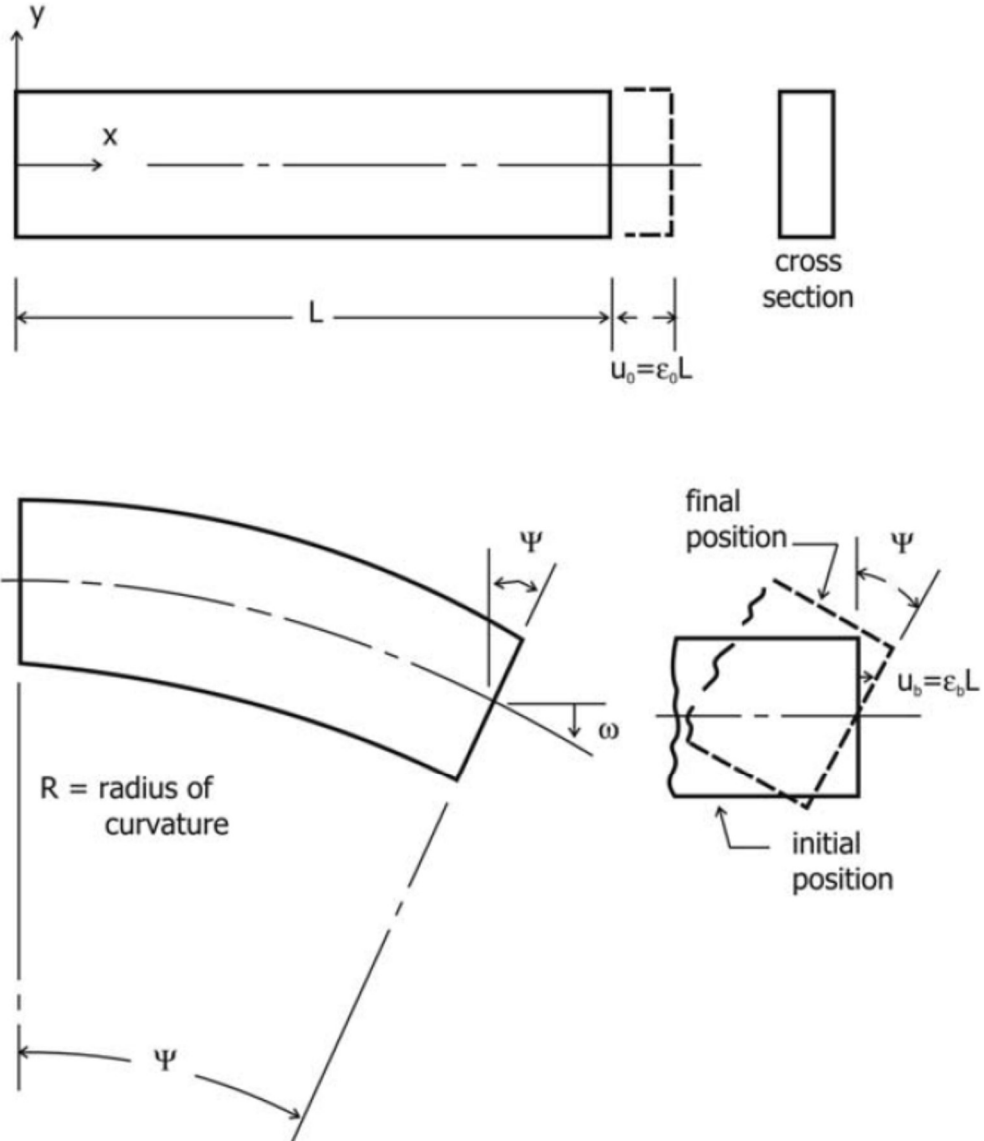


Figure 3.1 : Beam's deflection during bending.

$$P_e = \int \sigma dA = EA\varepsilon_0 + 0 - \alpha E \int \Delta T dA \quad (3.6)$$

Because the centroid of the transection is where the origin is positioned, the second term on the right-hand side is equal to zero. Last term is the thermal force. For constant material properties:

$$F_T = \alpha E \int \Delta T dA \quad (3.7)$$

By combining two of the previous equations the strain at the centroid axis may be solved:

$$\varepsilon_0 = \frac{\varepsilon_0 + F_T}{EA} \quad (3.8)$$

By using equation (3.5), the moment of mechanical bending at any point throughout the part's length can be calculated:

$$M = \int \sigma y dA = 0 + \frac{E}{R} \int y^2 dA - \alpha E \int \Delta T y dA \quad (3.9)$$

Because the centroid of the transection is where the origin is positioned, first term on the right is equal to zero. The last term is called thermal moment and it can be rewritten as below:

$$M_T = \alpha E \int \Delta T y dA \quad (3.10)$$

The thermal moment may be described as below if the features of the material changes with temperature:

$$M_T = \int \alpha E \Delta T y dA \quad (3.11)$$

Around the z-axis of the part, the area moment of inertia of the cross section is described by:

$$I = \int y^2 dA \quad (3.12)$$

If the substitutions from the equations of M_T and I into equation M above are made, the part's radius of curvature or its reciprocal can be solved:

$$\frac{1}{R} = \frac{M + M_T}{EI} \quad (3.13)$$

If the definition for the strain at the centroid axis and the expression of the radius of curvature are introduced to the stress relationship, common illustration for evaluating stress in a part which is exposed to mechanical and thermal loads is obtained:

$$\sigma = \frac{P_e + F_T}{A} + \left(\frac{M + M_T}{I} \right) y - \alpha E \Delta T \quad (3.14)$$

A sign convention is needed in order to guarantee that the mechanical and the thermal moments are described in a coherent way. According to sign convention for the moment, a moment which would generate a tensile stress or an extension in the positive y region of the transection is positive bending moment.

If the transection of the part is asymmetric or the temperature changes in both lateral and longitudinal directions, as can be seen in Figure 3.2, then the beam's stress must be calculated by using the equation below:

$$\sigma = \frac{P_e + F_T}{A} + \frac{M_y + K_y y + K_z z}{I_z} - \alpha E \Delta T \quad (3.15)$$

K_y and K_z are described below:

$$K_y = \frac{I_y M_T - I_{yz} M_{Tz}}{I_y - (I_{yz}^2 / I_z)} \quad (3.16)$$

$$K_z = \frac{I_z M_{Tz} - I_{yz} M_T}{I_z - (I_{yz}^2 / I_y)} \quad (3.17)$$

Thermal moment in the spanwise direction is:

$$M_{Tz} = \alpha E \int \Delta T z dA \quad (3.18)$$

The area moment of inertia I_y and the product of inertia I_{yz} are:

$$I_y = \int z^2 dA \quad (3.19)$$

$$I_{yz} = \int yz dA \quad (3.20)$$

3.3 Displacement Relations

From the Figure 3.2, if the beam element in bending is considered, the strain for bending can be introduced from the common strain description:

$$\varepsilon_b = \frac{\text{change in length}}{\text{initial length}} = \frac{-y d\omega}{ds} \approx \frac{-y d\omega}{dx} \quad (3.21)$$

The negative sign is caused by the revolution of the part which is described as below:

$$\omega = \frac{dv}{dx} \quad (3.22)$$

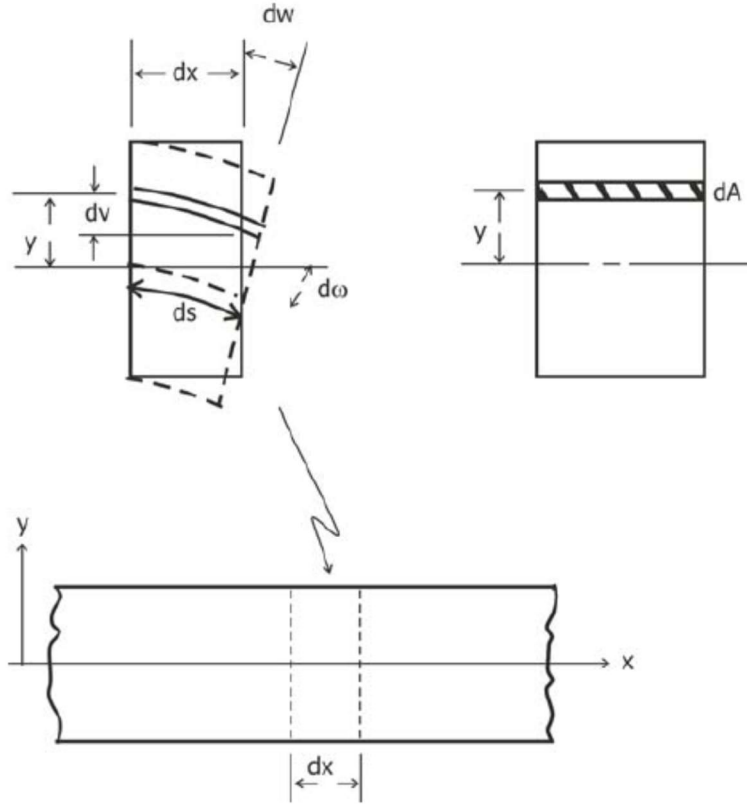


Figure 3.2 : Beam's differatial element.

When the displacement is in the positive y direction, the lateral displacement v is considered as positive. The analysis for the sidewise dislocation v is small with respect to the part's depth is limited by the approximation of $ds \approx dx$.

The following equation which is for the bending strain in terms of the crosswise dislocation is obtained by using the ω equation in the ϵ_b equation above:

$$\epsilon_b = -y \frac{d^2 v}{dx^2} \quad (3.23)$$

The equation for the part's radius of the curvature is found as:

$$\frac{1}{R} = -\frac{d^2 v}{dx^2} \quad (3.24)$$

Furthermore, the significant relationship below is found by using the radius of curvature:

$$\frac{d^2 v}{dx^2} = -\frac{M + M_T}{EI} \quad (3.25)$$

In order to get the definition for the part's crosswise dislocations v , the equation above may be integrated from both sides, if the thermal moment and mechanical bending moment are expressed as a function of the location x throughout the part's length.

3.4 General Thermal Bending Relations

If the part's differential element which can be seen in Figure 3.3 is considered, the quantity q is the applied transverse load per unit length. This quantity is a known quantity in the design of the beam, and it can change with the longitudinal position x . If the force's direction is towards to the positive y direction, the transverse load is considered positive also.

V : total shear force (N or lbf) through a transection

If a force equilibrium is made in the y -direction for the part, the following expression is obtained:

$$\sum F = 0 = -V + (V + dV) + qdx$$

$$q = -\frac{dV}{dx} \quad (3.26)$$

If the mechanical bending moments for the part about an axis at the left end of the element are added, it is obtained that:

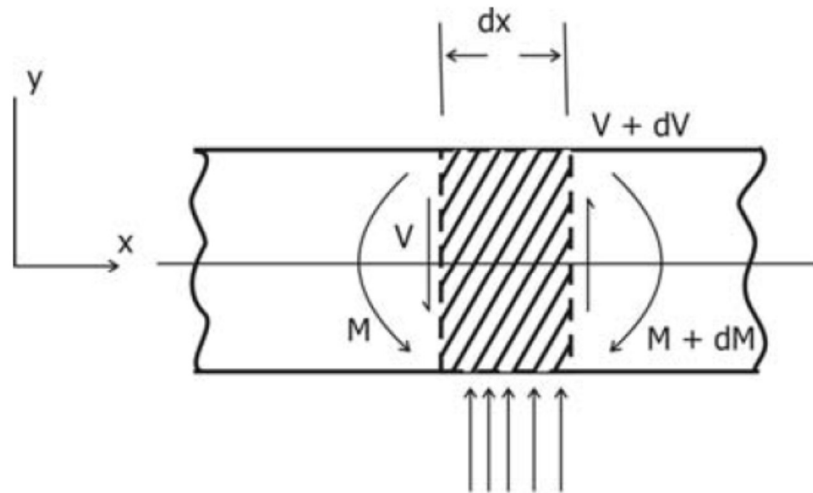


Figure 3.3 : Loading depiction.

$$\sum M = 0 = -M + (M + dM) - (V + dV)dx + \frac{1}{2}q(dx)^2 \quad (3.27)$$

If the higher order differentials are neglected, the relationship between the crosswise shear force and the mechanical bending moment is obtained:

$$V = \frac{dM}{dx} \quad (3.28)$$

In order to assign the mechanical bending moment and the used crosswise load:

$$q = -\frac{dV}{dx} = -\frac{d^2M}{dx^2} \quad (3.29)$$

The mechanical moment can be solved by using the crosswise dislocation

$$M = -EI \frac{d^2v}{dx^2} - M_T \quad (3.30)$$

The expression below is obtained by utilizing this equation in the crosswise shear definition:

$$V = \frac{dM}{dx} = -EI \frac{d^3v}{dx^3} - \frac{dM_T}{dx} \quad \text{eq. (3.31)}$$

The crosswise load, q, and the crosswise dislocations, v, may be related each other by combining the equations:

$$q = -\frac{dV}{dx} = EI \frac{d^4v}{dx^4} + \frac{d^2M_T}{dx^2} \quad (3.32)$$

Or

$$\frac{d^4v}{dx^4} = \frac{1}{EI} \left(q - \frac{d^2M_T}{dx^2} \right) \quad (3.33)$$

3.5 Shear Stresses

If the longitudinal stress is not steady or changes with respect to longitudinal distance linearly throughout the part, shear stresses will be generated by thermal loads. If $\Delta T = \Delta T(y)$, which means that the temperature variation is a function of y, no shear stresses will be generated by the thermal loads in statically stable parts.

If a part which is depicted in Figure 3.4 is considered, the part's transection's width is a function of the coordinate y. A part which has a symmetrical transection will be

assumed in this section. Generally, the temperature variation is considered that is a function of both x and y . Integration of the temperature change through the transection is involved by the thermal force F_T and the thermal moment M_T , so these quantities are functions of the longitudinal coordinates only.

If a force equilibrium for the part's shaded region is made:

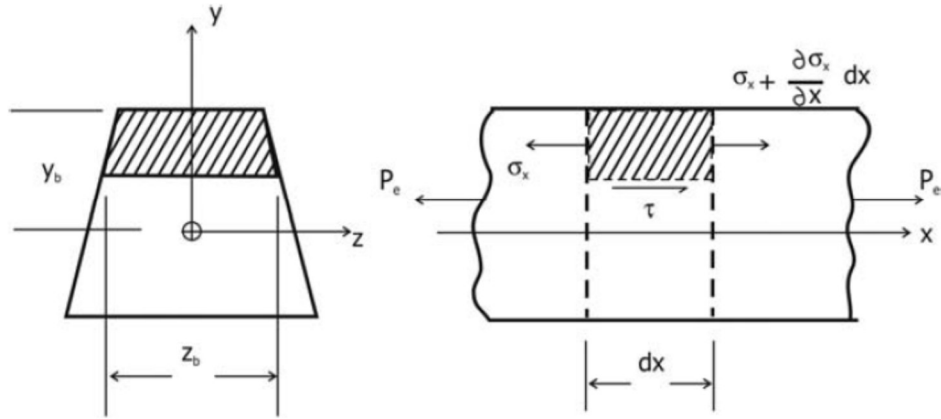


Figure 3.4 : Shear stress depiction.

$$\tau z_b dx + \int_y^{y_b} \frac{\partial \sigma}{\partial x} dx dA = 0 \quad (3.34)$$

As illustrated in Figure 3.4, the quantity y_b is the measure of the coordinate y at top of the part transection. The shear stress expression can be stated as below:

$$\tau = -\frac{1}{z_b} \int_y^{y_b} \frac{\partial \sigma}{\partial x} dA = -\frac{1}{z_b} \frac{\partial}{\partial x} \left(\int_y^{y_b} \sigma dA \right) \quad (3.35)$$

Direct stress may be found as:

$$\sigma = \frac{P_e + F_T}{A} + \left(\frac{M + M_y}{I} \right) y - \alpha E \Delta T \quad (3.36)$$

It is obtained by substituting into the equation

$$\tau = -\frac{1}{z_b} \frac{\partial}{\partial x} \left[\frac{P_e + F_T}{A} \int_y^{y_b} dA + \frac{M + M_y}{I} \int_y^{y_b} y dA - \alpha E \int_y^{y_b} \Delta T dA \right] \quad (3.37)$$

The external axial force P_e is constant since the shear forces on the part's external does not exist. Following quantities are defined:

$$\Gamma_y = \int_y^{y_b} y dA \quad (3.38)$$

$$A_y = \int_y^{y_b} dA \quad (3.39)$$

$$A_y \Delta T_y = \int_y^{y_b} \Delta T dA \quad (3.40)$$

If these equations are substituted into previous equation:

$$\tau = -\frac{1}{z_b} \frac{\partial}{\partial x} \left[\frac{A_y F_T}{A} + \frac{(M + M_T) \Gamma_y}{I} - \alpha E A_y \Delta T_y \right] \quad (3.41)$$

If it is differentiated:

$$\tau = -\frac{1}{z_b} \left[\frac{A_y}{A} \frac{dF_T}{dx} + \frac{\Gamma_y}{I} \left(\frac{dM}{dx} + \frac{dM_T}{dx} \right) - \alpha E A_y \frac{d(\Delta T_y)}{dx} \right] \quad (3.42)$$

And the thermal shear force V_T is defined as:

$$V_T = \frac{dM_T}{dx} \quad (3.43)$$

The last definition for the shear stress for the part is obtained by substituting the mechanical shear force:

$$\tau = -\frac{\Gamma_y}{z_b r_g^2} \left(\frac{V}{A} + \frac{V_T}{A} \right) - \frac{A_y}{z_b A} \frac{dF_T}{dx} + \frac{\alpha E A_y}{z_b} \frac{d(\Delta T_y)}{dx} \quad (3.44)$$

For the cross sectional area, the quantity r_g is the radius of gyration which is defined by:

$$r_g^2 = \frac{I}{A} \quad (3.45)$$

4. THERMAL STRESSES IN CYLINDRICAL SHELLS

Membrane and general analysis which bending is comprised are the two extensive analysis for shells. Shear forces and bending are neglected for the analysis of membrane. In-plane forces resist to the loads which acted on the membrane's surface. Static equilibrium concerns provide the solutions of the membrane. Beside the areas which is near to the constraint or discontinuity, membrane stresses are the essential ones for pressure vessels.

Alternatively, the shell's general analysis which bending is comprised is statically indeterminate. For this kind of analysis, in order to be able to answer the stress problem, stress strain relationship is needed. Discontinuity stresses are the stresses which are caused by shell's bending.

There exist numerous complexity levels when the general analysis which bending is contained is considered:

- i- an axisymmetric loading is exposed to a cylindrical shell,
- ii- an axisymmetric loading is exposed to a revolution's shells
- iii- a nonsymmetric loading is exposed to a cylindrical shell,
- iv- a nonsymmetric loading is exposed to a revolution's shells

4.1 Axisymmetric Loading for Cylindrical Shells

It may be seen from the Figure 4.1 that the moments and forces which is exposed to a shell element in the absence of symmetrical mechanical and symmetrical thermal loading around the cylinder axis.

$$(Q_x + dQ_x)a d\theta - Q_x a d\theta + p_r a d\theta dx - 2N_\theta \sin\left(\frac{1}{2}d\theta\right) dx = 0 \quad (4.1)$$

a : cylindrical shell's midplane radius

Also,

$$\sin\left(\frac{1}{2}d\theta\right) \approx \frac{1}{2}d\theta \text{ for small angles ;}$$

Then,

$$\frac{dQ_x}{dx} - \frac{N_\theta}{a} + p_r = 0 \quad (4.2)$$

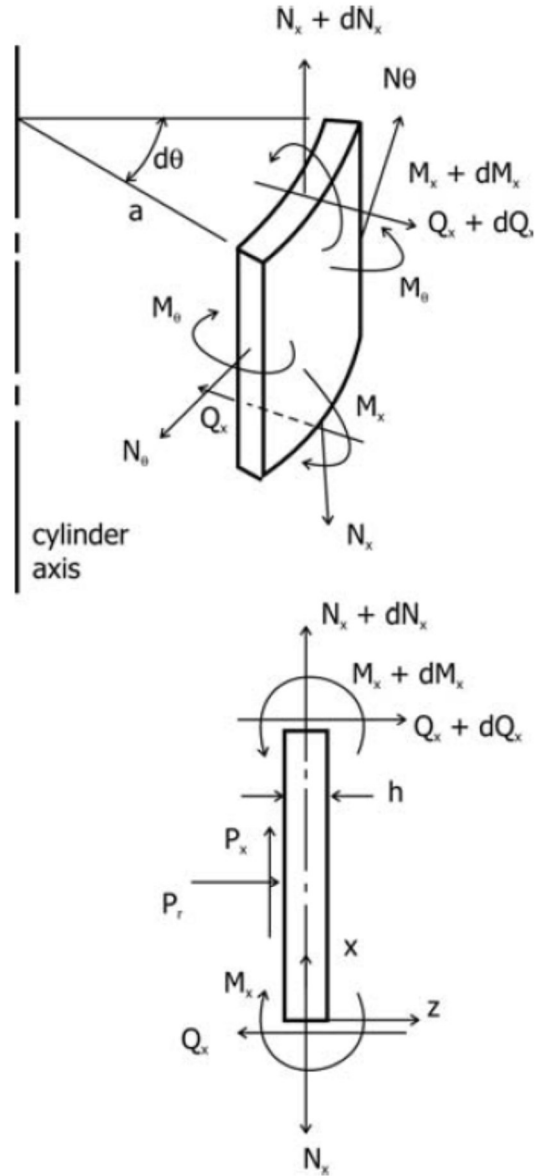


Figure 4.1 : Axisymmetric loading depiction.

It is obtained that;

$$(N_x + dN_x)a \, d\theta - N_x a \, d\theta + p_x a \, d\theta dx = 0 \quad (4.3)$$

If we make simpler the equation above;

$$a[(N_x + dN_x) \, d\theta - N_x \, d\theta + p_x \, d\theta dx] = 0 \quad \text{and} \quad a \neq 0$$

$$\rightarrow N_x d\theta + dN_x d\theta - N_x d\theta + p_x d\theta dx = 0$$

$$\rightarrow dN_x d\theta + p_x d\theta dx = 0$$

$$\rightarrow \frac{dN_x d\theta}{d\theta dx} + \frac{p_x d\theta dx}{d\theta dx} = 0$$

$$\boxed{\frac{dN_x}{dx} + p_x = 0} \quad (4.4)$$

If the equation (4.4) is integrated for N_x :

$$N_x = - \int p_x dx + C \quad (4.5)$$

If balance of moments is applied;

$$(M_x + dM_x)a d\theta - M_x a d\theta - Q_x a d\theta dx = 0 \quad (4.6)$$

From this balance equation:

$$a[M_x d\theta + dM_x d\theta - M_x d\theta - Q_x d\theta dx] = 0$$

$$a \neq 0$$

$$dM_x d\theta - Q_x d\theta dx = 0$$

$$\frac{dM_x d\theta}{d\theta dx} - \frac{Q_x d\theta dx}{d\theta dx} = \frac{0}{d\theta dx}$$

$$\boxed{\frac{dM_x}{dx} - Q_x = 0} \quad (4.7)$$

4.1.1 Stress-strain relations

In the absence of symmetry, displacement is equal to zero, and relations for the displacement and strain for the midplane are:

$$\bar{\varepsilon}_x = \frac{(du + dx) - dx}{dx} = \frac{du}{dx} \quad (4.8)$$

$$\bar{\varepsilon}_\theta = \frac{(a + w)d\theta - ad\theta}{ad\theta} = \frac{w}{a} \quad (4.9)$$

For the mid-plane, $\bar{\varepsilon}$ and N relations:

$$\bar{\varepsilon}_x = \frac{1}{Eh}(N_x - \mu N_\theta) + \frac{N_T}{Eh} \quad (4.10)$$

$$\bar{\varepsilon}_\theta = \frac{1}{Eh}(N_\theta - \mu N_x) + \frac{N_T}{Eh} \quad (4.11)$$

N_T is known as :

$$N_T = \int_{-h/2}^{h/2} \alpha E (T - T_0) dz = \int_{-h/2}^{h/2} \alpha E \Delta T dz \quad (4.12)$$

The N_x and N_θ equations are acquired if equation (4.10) and equation (4.11) are solved for the stress resultants:

$$N_x = \frac{Eh}{1-\mu^2} (\bar{\varepsilon}_x + \mu \bar{\varepsilon}_\theta) - \frac{N_T}{1-\mu} = \frac{Eh}{1-\mu^2} \left(\frac{du}{dx} + \mu \frac{w}{a} \right) - \frac{N_T}{1-\mu} \quad (4.13)$$

$$N_\theta = \frac{Eh}{1-\mu^2} (\bar{\varepsilon}_\theta + \mu \bar{\varepsilon}_x) - \frac{N_T}{1-\mu} = \frac{Ehw}{a} + \mu N_x - N_T \quad (4.14)$$

From equation (4.13), $\bar{\varepsilon}_x$ can be found as:

$$\bar{\varepsilon}_x = \frac{du}{dx} = \frac{(1-\mu^2)N_x}{Eh} - \mu \frac{w}{a} + (1+\mu)N_T \quad (4.15)$$

If w is obtained, $\bar{\varepsilon}_x$ can be calculated. In the circumferential direction, element's rotation cannot be observed, and axial direction's rotation can be calculated by using:

$$\omega_x = \frac{dw}{dx} \quad (4.16)$$

The resultants of bending moments are:

$$M_x = -D \frac{d\omega_x}{dx} - \frac{M_T}{1-\mu} = -D \frac{d^2w}{dx^2} - \frac{M_T}{1-\mu} \quad (4.17)$$

$$M_\theta = -\mu D \frac{d\omega_x}{dx} - \frac{M_T}{1-\mu} = -\mu D \frac{d^2w}{dx^2} - \frac{M_T}{1-\mu} \quad (4.18)$$

D : shell's flexural rigidity

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (4.19)$$

if equation (4.7) and equation (4.17) are combined:

$$Q_x = -D \frac{d^3 w}{dx^3} - \frac{1}{1-\mu} \frac{dM_T}{dx} \quad (4.20)$$

Q_x : transverse shear resultant

4.1.2 Displacement formulation

In order to remove the Q_x term, the derivative with respect to x is taken for the both sides of the equation (4.20) and equation (4.2) is utilized:

$$\frac{d^4 w}{dx^4} + \frac{Eh}{Da^2} w = -\frac{\mu N_x}{Da} - \frac{1}{(1-\mu)D} \frac{d^2 M_T}{dx^2} + \frac{N_T}{Da} + \frac{p_r}{D} \quad (4.21)$$

If the equation (4.19) is inserted to the equation (4.21), 2nd term of the equation (4.21) is going to be:

$$\frac{Eh}{Da^2} = \frac{12(1-\mu^2)}{a^2 h^2} \quad (4.22)$$

Following the transverse displacement's solution, values of the other parameters can be found as:

$$M_x = -D \frac{d^2 w}{dx^2} - \frac{M_T}{1-\mu} \quad (4.23)$$

$$M_\theta = -\mu D \frac{d^2 w}{dx^2} - \frac{M_T}{1-\mu} \quad (4.24)$$

$$Q_x = -D \frac{d^3 w}{dx^3} - \frac{1}{1-\mu} \frac{dM_T}{dx} \quad (4.25)$$

$$N_x = -\int p_x dx + C \quad (4.26)$$

$$N_\theta = \frac{Ehw}{a} + \mu N_x - N_T \quad (4.27)$$

σ_x and σ_θ can be found at the shell's lower or upper surface as:

$$\sigma_x = \frac{N_x}{h} \pm \frac{6M_x}{h^2} \quad (4.28)$$

$$\sigma_\theta = \frac{N_\theta}{h} \pm \frac{6M_\theta}{h^2} \quad (4.29)$$

Shear stress' peak value takes place at the midplane and it can be found as follows:

$$\tau_{xz,max} = \frac{3Q_x}{2h} \quad (4.30)$$

Before the equation (4.21), if the displacement is removed instead of Q_x term and the equation is solved for Q_x , the formulation of stress can be achieved. If equation (4.9) is utilized with the rotation formula we get:

$$\omega_x = \frac{dw}{dx} = a \frac{d\bar{\varepsilon}_\theta}{dx} \quad (4.31)$$

If equation (4.11) is inserted to the rotation formula:

$$\omega_x = \frac{a}{Eh} \left(\frac{dN_\theta}{dx} - \mu \frac{dN_x}{dx} + \frac{dN_T}{dx} \right) \quad (4.32)$$

If equation (4.2) and equation (4.4) are utilized, stress resultant of the membrane can be removed from the equation:

$$\omega_x = \frac{a^2}{Eh} \left(\frac{d^2 Q_x}{dx^2} + \frac{dp_r}{dx} + \mu \frac{p_x}{a} + \frac{1}{a} \frac{dN_T}{dx} \right) \quad (4.33)$$

By using the equation (4.23) and equation (4.33), M_x can be calculated in terms of Q_x :

$$M_x = - \left(\frac{Da^2}{Eh} \right) \left(\frac{d^3 Q_x}{dx^3} + \frac{d^2 p_r}{dx^2} + \frac{1}{a} \frac{d^2 N_T}{dx^2} + \frac{\mu}{a} \frac{dp_x}{dx} \right) - \frac{M_T}{1-\mu} \quad (4.34)$$

If equation (4.7) and equation (4.34) are combined:

$$\frac{d^4 Q_x}{dx^4} + \left(\frac{Eh}{Da^2} \right) Q_x + \frac{d^3 p_r}{dx^3} + \frac{1}{a} \frac{d^3 N_T}{dx^3} + \frac{\mu}{a} \frac{d^2 p_x}{dx^2} + \frac{Eh}{(1-\mu)Da^2} \frac{dM_T}{dx} = 0 \quad (4.35)$$

Following the transverse shear stress' solution, values of the other parameters can be found as:

$$M_x = - \left(\frac{Da^2}{Eh} \right) \left(\frac{d^3 Q_x}{dx^3} + \frac{d^2 p_r}{dx^2} + \frac{1}{a} \frac{d^2 N_T}{dx^2} + \frac{\mu}{a} \frac{dp_x}{dx} \right) - \frac{M_T}{1-\mu} \quad (4.36)$$

$$M_\theta = - \left(\frac{\mu Da^2}{Eh} \right) \left(\frac{d^3 Q_x}{dx^3} + \frac{d^2 p_r}{dx^2} + \frac{1}{a} \frac{d^2 N_T}{dx^2} + \frac{\mu}{a} \frac{dp_x}{dx} \right) - \frac{M_T}{1-\mu} \quad (4.37)$$

$$N_x = - \int p_x dx + C \quad (4.38)$$

$$N_\theta = a \frac{dQ_x}{dx} + p_r a \quad (4.39)$$

$$\omega_x = \frac{a^2}{Eh} \left(\frac{d^2 Q_x}{dx^2} + \frac{dp_r}{dx} + \mu \frac{p_x}{a} + \frac{1}{a} \frac{dN_T}{dx} \right) \quad (4.40)$$

$$w = \left(\frac{a^2}{Eh} \right) \left(\frac{dQ_x}{dx} - \frac{\mu}{a} N_x + \frac{N_T}{a} + p_r \right) \quad (4.41)$$

4.2 Axial Temperature Variation for Cylindrical Shells

In some cases, a pressure vessel may develop thermal stresses which include change of temperature in the axial direction. It may be caused by the vertically oriented cylindrical pressure vessel as indicated in Figure 4.2, which is filled partially with a liquid and the vapor of the liquid has non-uniform temperature gradient.

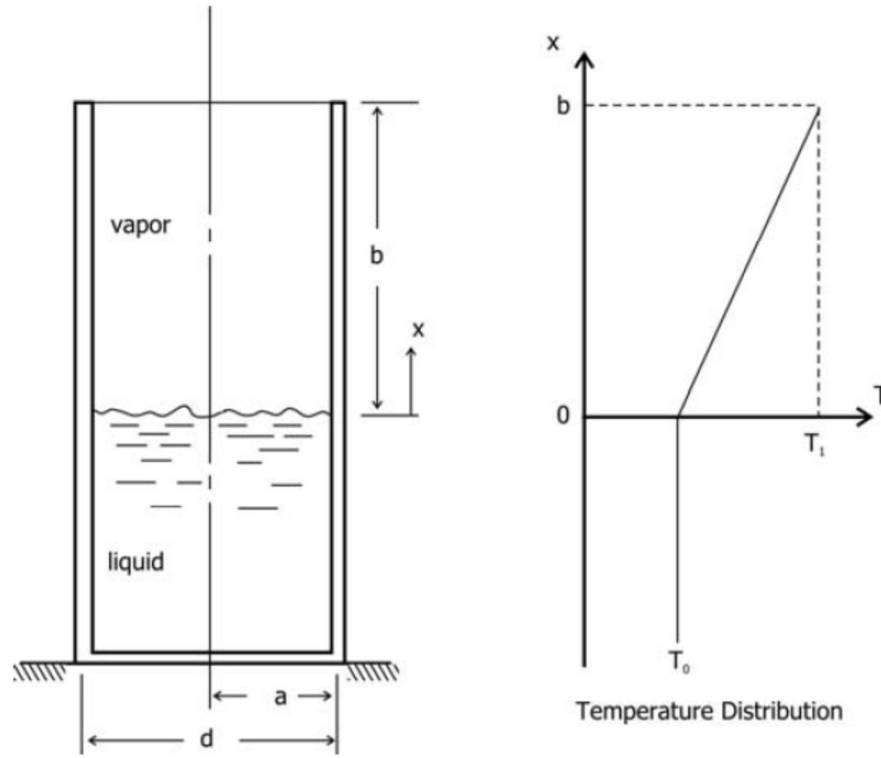


Figure 4.2 : Partially filled vertical cylindrical vessel depiction.

If it is assumed that the temperature is horizontally uniform and vertically non uniform at where the liquid vapor exists:

$$\Delta T(x) = T(x) - T_0 = (T_1 - T_0)(x/b) = \Delta T_1(x/b) \quad (4.42)$$

T_0 : Constant temperature of the shell at where the liquid contacts

Resultant thermal stress of the membrane and the bending stress are:

$$N_T = \alpha E \int_{-h/2}^{h/2} \Delta T_1(x/b) dz = \alpha E \Delta T_1 h(x/b) = N_0(x/b) \quad (4.43)$$

$$N_0 = \alpha E \Delta T_1 h \quad (4.44)$$

$$M_T = \alpha E \int_{-h/2}^{h/2} \Delta T_1(x/b) z dz = 0 \quad (4.45)$$

If the formulation of displacement is utilized, in the situation of mechanical loads are equal to zero so that the stress resultant N_x is equal to zero in accordance with the equation (4.5). If these quantities are combined with the equation (4.21), for the part of the cylindrical pressure vessel at which the vapor is exposed, the following equation may be achieved.

$$\frac{d^4 w}{dx^4} + \frac{Eh}{Da^2} w = \left(\frac{N_0}{Da} \right) \left(\frac{x}{b} \right) \quad (4.46)$$

Solution:

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + \frac{N_0}{4\beta^4 Da} \left(\frac{x}{b} \right) \quad (4.47)$$

and β ,

$$4\beta^4 \equiv \frac{Eh}{Da^2} \quad \text{or} \quad \beta = \frac{[3(1 - \mu^2)]^{\frac{1}{4}}}{\sqrt{ah}} \quad (4.48)$$

If $\beta b > 9$, utilizing the formulation of β in order to simplify $\frac{N_0}{4\beta^4 Da} \left(\frac{x}{b} \right)$ term from equation (4.48) yields the governing equation as follows:

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + \frac{N_0 a}{Eh} \left(\frac{x}{b} \right) \quad (4.49)$$

From Figure 4.2, at $x=0$ where the vapor and liquid border exist, the shell's deflection in the radial direction is:

$$\omega_x(0) = w_0 = C_1 \quad (4.50)$$

The shell's rotation:

$$\omega_x(x) = \frac{dw}{dx} = -\beta e^{-\beta x} [(C_1 - C_2) \cos \beta x + (C_1 + C_2) \sin \beta x] + \frac{N_0 a}{Ehb} \quad (4.51)$$

At $x=0$:

$$\omega_x(0) = -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb} \quad (4.52)$$

If the solution of deflection is utilized in order to calculate the stress resultant, by using equation (4.17) with $M_T = 0$, the resultant of stress of the axial bending can be achieved:

$$M_x = -D \frac{d^2 w}{dx^2} = -2D\beta^2 e^{-\beta x} (C_1 \sin \beta x - C_2 \cos \beta x) \quad (4.53)$$

At $x=0$:

$$(M_x)_{x=0} = 2D\beta^2 C_2 = M_0 \quad (4.54)$$

with $M_T = 0$, Q_x can be calculated by using equation (4.20):

$$Q_x = -D \frac{d^3 w}{dx^3} = -2D\beta^3 e^{-\beta x} [(C_1 + C_2) \cos \beta x - (C_1 - C_2) \sin \beta x] \quad (4.55)$$

At $x=0$:

$$(Q_x)_x = -2D\beta^3 (C_1 + C_2) = Q_0 \quad (4.56)$$

At Figure 4.3, the moments and forces are depicted. For $x \leq 0$, to have M_0 and $-Q_0$, rotation and displacement are:

$$w(x) = \frac{e^{+\beta x}}{2D\beta^3} [\beta M_0 (\sin \beta x - \cos \beta x) + Q_0 \cos \beta x] \quad (4.57)$$

At $x=0$:

$$w(0) = -\frac{1}{2D\beta^3} (\beta M_0 - Q_0) \quad (4.58)$$

At $x=0$, the slope is:

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (2\beta M_0 - Q_0) \quad (4.59)$$

If equation (4.50) and equation 4.57 are merged with the same deflection at both $x < 0$ and $x > 0$:

$$w(0) = C_1 = -\frac{1}{2D\beta^3} [2D\beta^3 C_2 + 2D\beta^3 (C_1 + C_2)] = -C_1 - 2C_2 \quad (4.60)$$

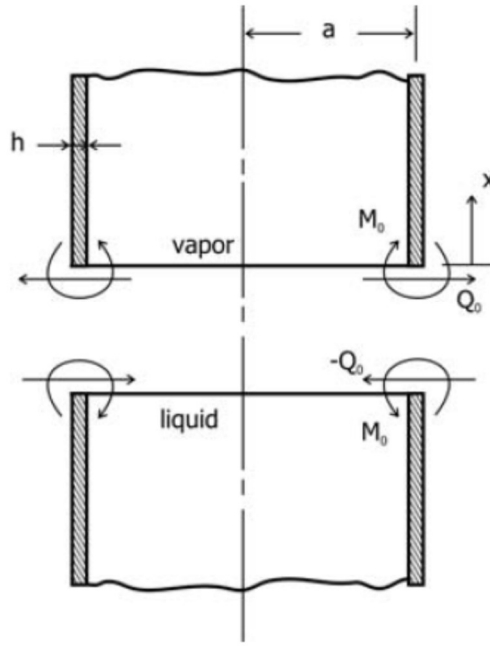


Figure 4.3 : Illustration of moments and forces at $x=0$.

Or

$$C_1 = -C_2 \quad (4.61)$$

If equation (4.52) and equation (4.59) are merged with the same axial rotation at both $x < 0$ and $x > 0$:

$$(\omega_x)_{x=0} = -\beta(C_1 - C_2) + \frac{N_0 a}{E h \beta} = -\frac{1}{2D\beta^2} [4D\beta^3 C_2 - 2D\beta^3 (C_1 + C_2)] \quad (4.62)$$

And C_1 and C_2 can be calculated as follows:

$$C_1 = -C_2 = \frac{N_0 a}{4E h \beta b} = \frac{\alpha \Delta T_1 a}{4\beta b} \quad (4.63)$$

If equation (4.63) is added into the equation (4.49) and equation (4.53):

$$w(x) = \alpha \Delta T_1 a \left[\left(\frac{x}{b} \right) + \frac{e^{-\beta x}}{4\beta b} (\cos \beta x - \sin \beta x) \right] \quad (4.64)$$

$$M_x(x) = -\frac{\alpha \Delta T_1 D a \beta}{2b} e^{-\beta x} (\cos \beta x + \sin \beta x) \quad (4.65)$$

Bending stress in the axial direction:

$$\sigma_x(x) = \pm \frac{6M_x}{h^2} = \mp \frac{3\alpha\Delta T_1 Da\beta}{bh^2} e^{-\beta x} (\cos \beta x + \sin \beta x) \quad (4.66)$$

σ_x reaches the peak value at $x=0$:

$$(\sigma_x)_{max} = \frac{3\alpha\Delta T_1 Da\beta}{bh^2} = \frac{3\alpha E\Delta T_1 d}{2[12(1-\mu^2)]^{3/4} b} \sqrt{\frac{h}{d}} \quad (4.67)$$

Figure 4.4 indicates the distribution of stress axially.

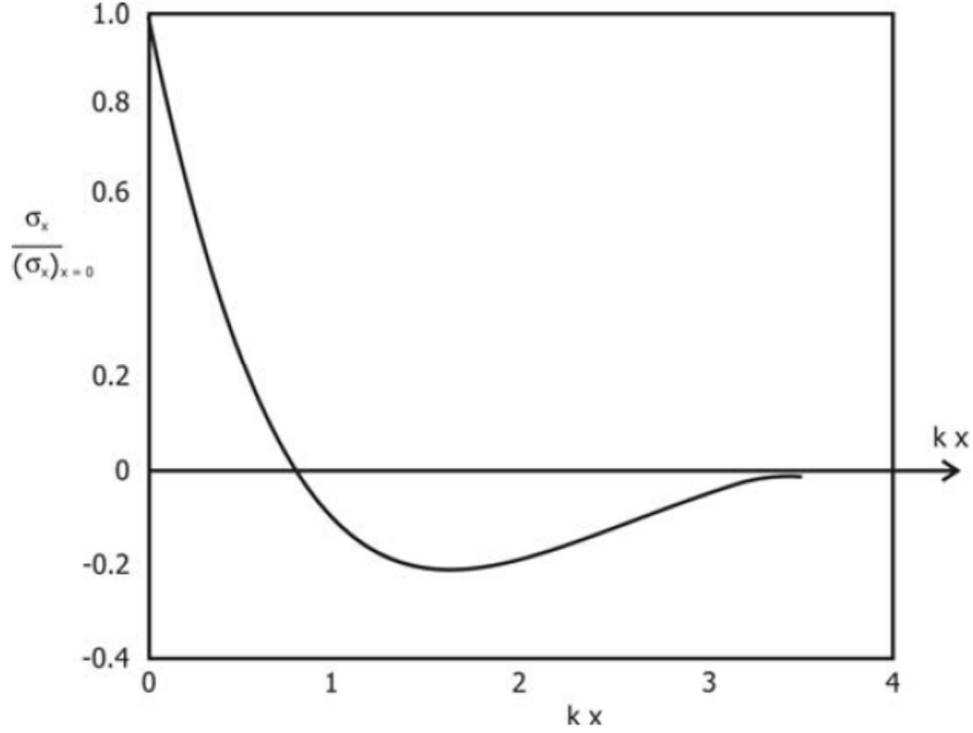


Figure 4.4 : Axial stress distribution.

Equation (4.14) gives the formulation of N_θ , if $N_x = 0$ is applied to this formula:

$$N_\theta = \frac{Ehw}{a} - N_0 \left(\frac{x}{b} \right) + \mu N_x = \frac{\alpha E\Delta T_1 h}{4\beta b} e^{-\beta x} (\cos \beta x - \sin \beta x) \quad (4.68)$$

If $M_x = 0$ is added to the M_θ formula from the equation 8-14b:

$$M_\theta = -\mu D \frac{d^2 w}{dx^2} - \frac{M_T}{1-\mu} = -\mu D \frac{d^2 w}{dx^2} = \mu M_x \quad (4.69)$$

$\sigma_\theta(x)$ is:

$$\sigma_\theta(x) = \frac{\alpha E\Delta T_1}{4\beta b} e^{-\beta x} \left[\left(1 \mp \frac{12\mu Da\beta^2}{Eh^2} \right) \cos \beta x - \left(1 \pm \frac{12\mu Da\beta^2}{Eh^2} \right) \sin \beta x \right] \quad (4.70)$$

If β and D definitions are added to the formula in order to simplify the equation:

$$\frac{12\mu D\alpha\beta^2}{Eh^2} = \frac{\mu}{\sqrt{\frac{1}{3}(1-\mu^2)}}$$

The stress reaches the peak value at $x=0$:

$$(\sigma_\theta)_{max} = \frac{\alpha E \Delta T_1}{4\beta b} \left[1 + \frac{\mu}{\sqrt{\frac{1}{3}(1-\mu^2)}} \right] = \frac{\alpha E \Delta T_1 d}{4[12(1-\mu^2)]^{1/4} b} \sqrt{\frac{h}{d}} \left[1 + \frac{\mu}{\sqrt{\frac{1}{3}(1-\mu^2)}} \right] \quad (4.71)$$

5. PRACTICAL APPLICATION OF THIS STUDY

In this chapter, a cryogenic pressure vessel which is utilized as an oxidizer tank of a sounding rocket which is powered by a hybrid rocket engine is going to be analyzed. First of all, all the calculations is going to be made with all the conditions and the results are going to be compared with the results which are obtained from a FEM (Finite Elements Method) package programme (ANSYS).

This pressure vessel was used as an oxidizer tank onboard TESOR Sounding Rocket which is developed by DeltaV Uzay Teknolojileri A.Ş. (hereinafter referred as DeltaV). DeltaV is a subsidiary company of Presidency of Defence Industries of the Presidency of the Republic of Turkey (T.C. Cumhurbaşkanlığı Savunma Sanayii Başkanlığı).

TESOR Sounding Rocket is equipped and powered by a flight-proven KÖRTEM hybrid rocket engine which is also developed, manufactured and tested by DeltaV. KÖRTEM provides 22000 Newton thrust to TESOR Sounding Rocket. Hybrid rocket engines are consist of solid fuel and liquid oxidizer. In this case, liquid oxygen(LOX) is used as oxidizer. LOX is a cryogenic liquid which has a temperature around 90 °K (-183 °C). Layout of the TESOR Sounding Rocket is illustrated in Figure 5.2.

Liquid oxygen tank is made from Aluminum 6061-T6. It is selected due to its strength and weldability. It is a cylindrical pressure vessel with semi-elliptical heads at both sides. It is manufactured by welding three parts which are two semi elliptic heads and one cylindrical body. Cylindrical body is manufactured by machining both inside and outside of an aluminum rod. The tank's wall thickness is 5 mm. However, due to the decrease of the strength of aluminum which is caused by welding, the upper and lower sections of the welding areas are left 9 mm thickness. The cross section of the LOX tank may be seen at Figure 5.3 with the major dimensions.

TESOR Sounding Rocket has a pressure-fed system which means required pressure of the LOX is maintained by a pressurized gas via pressure regulators. Because LOX is extremely reactive, Helium is used in order to pressurize oxidizer tank. Tank is filled to a certain level. Due to vaporization, above this level is filled with gas

oxygen(GOX). Gaseous oxygen has a temperature around $-70\text{ }^{\circ}\text{C}$ ($\sim 203\text{ }^{\circ}\text{K}$) in this state. In Figure 5.1, the layout of the GOX and LOX is illustrated. For this case, in order to obtain KÖRTEM's required combustion stoichiometry, LOX is filled upto 221 mm below the semi-elliptic region.



Figure 5.1 : GOX and LOX illustration.

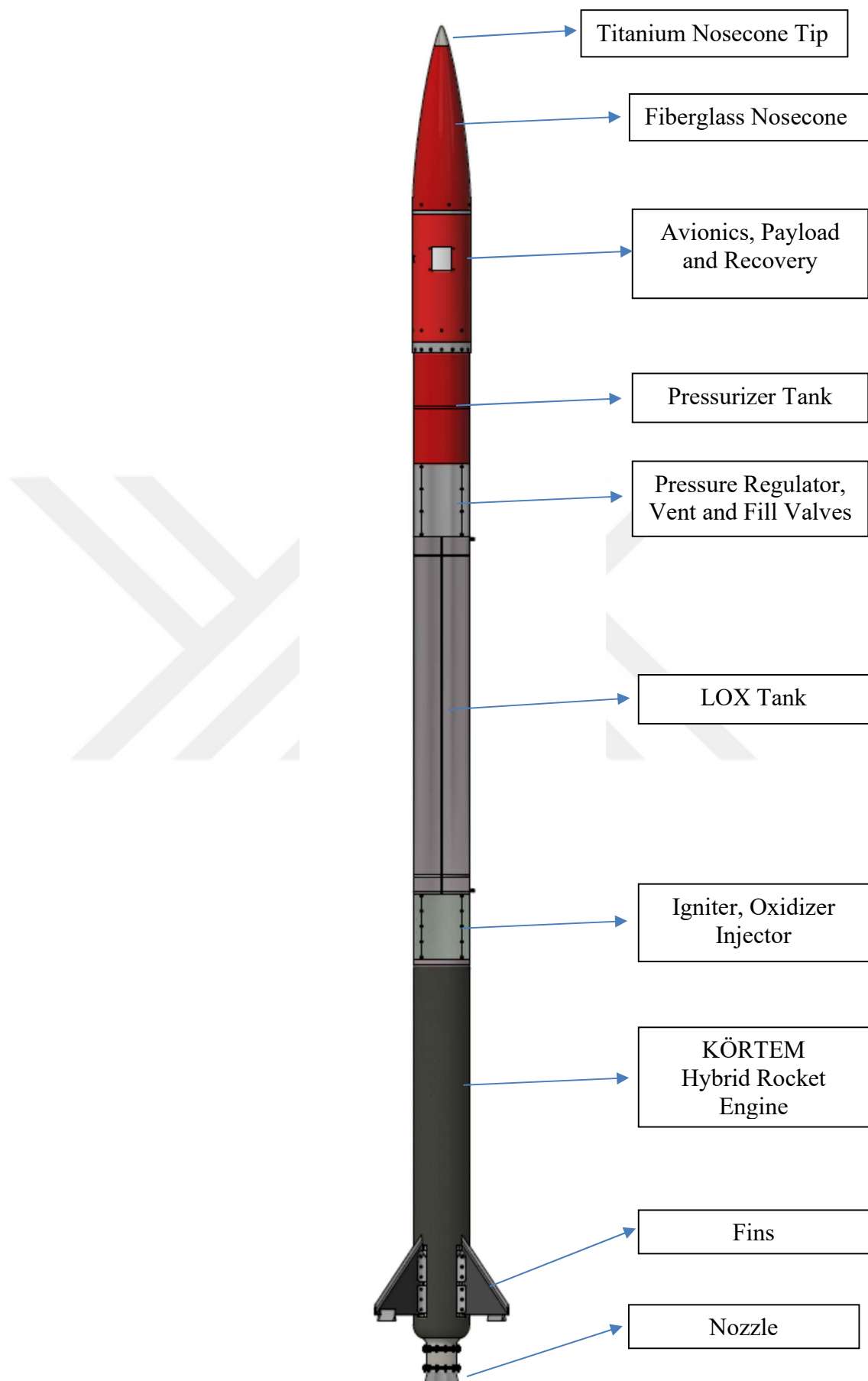


Figure 5.2 : TESOR layout.

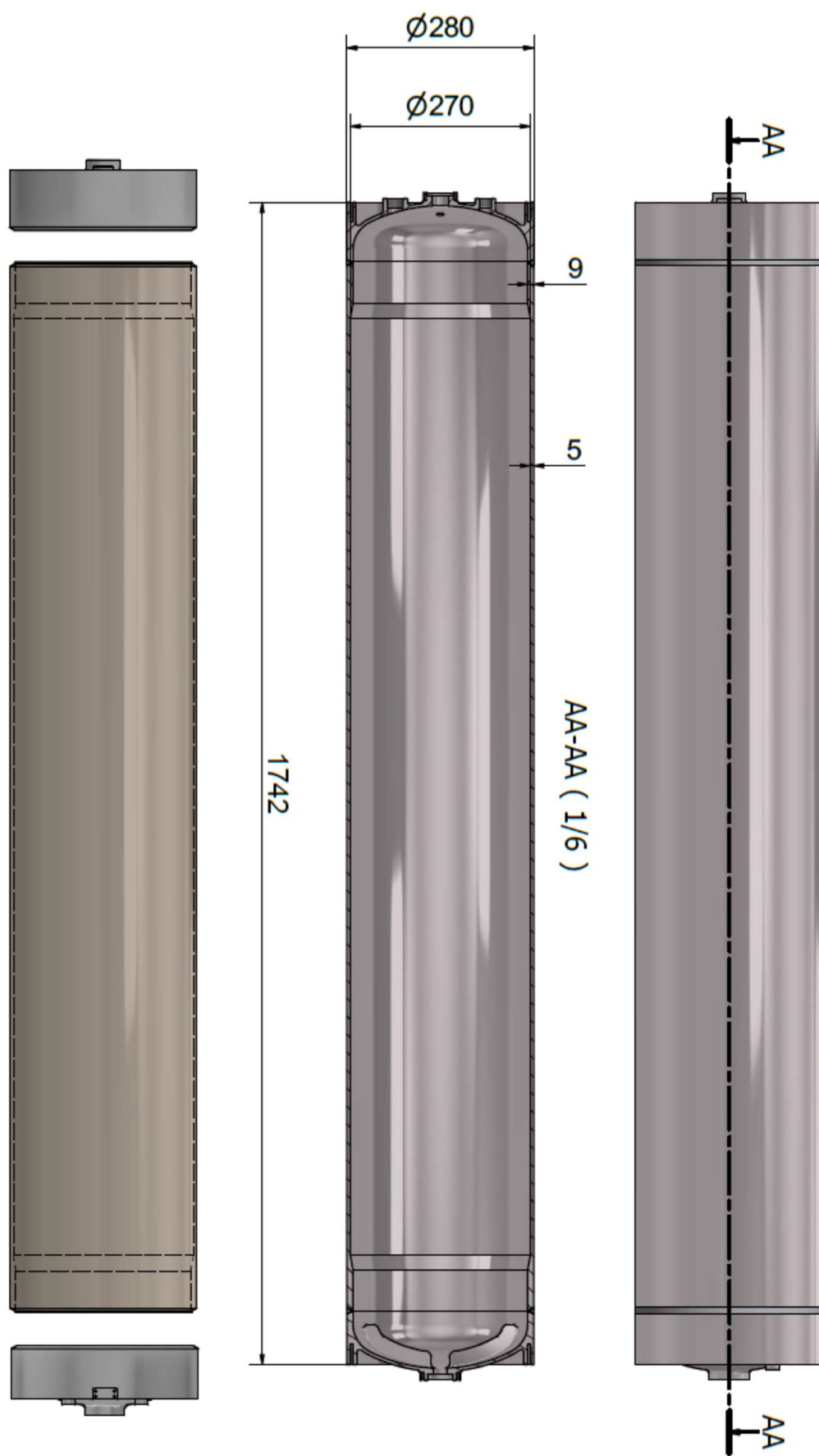


Figure 5.3 : Liquid oxygen pressure vessel.

All the calculations are made for three steps with respect to its operations which are LOX filling, pressurizing and flight conditions, respectively:

Step 1:

At first step, only thermal effects are considered. At this stage, tank is not pressurized, and the rocket is stagnant.

Step 2:

At second step, pressurization is taken into consideration in addition to thermal effects. Tank pressure is maintained as 70 bar (7 MPa). The rocket is still stagnant.

Step 3:

At third step, thrust effect is taken into consideration while the other parameters are still present. This simulates the rocket's flight.

For all three steps, convection from the ambient temperature is neglected because a thermal blanket is used around the tank till the flight. During the flight, the ambient temperature decreases dramatically in a short period of time and the liquid oxygen is completely consumed in 20 seconds.

Steps of analyses are illustrated in Table 5.1.

Table 5.1 Steps of Analysis

Loading	Step 1	Step 2	Step 3
Thermal Loading	Present	Present	Present
Pressurization	-	70 Bar	70 Bar
Thrust Loading	-	-	22000 N

For all three steps, differential equations are going to be calculated in detail until the variables and differential constants are obtained. After this point, MATLAB is utilized for solutions for each millimeter of the tank's section. MATLAB codes are attached for each steps to APPENDIX.

5.1 Calculation of TESOR Sounding Rocket's Liquid Oxygen Tank

If $\beta b = \frac{[3(1-\mu^2)]^{0.25}}{\sqrt{h/a}} \left(\frac{b}{a}\right) \geq 9.21$, the cylindrical shell can be assumed as an infinite cylinder. TESOR Sounding Rocket's Liquid Oxygen Tank satisfies infinite cylinder assumption requirement ($\beta b = 10.78 \geq 9.21$). It means, effects of moments and forces at the cylinder's both ends have damped out in the middle portion of the cylinder. Therefore, our calculations will be focused on the $0 \leq x \leq b$ portion of the shell since the maximum values are going to be obtained from this section of the cylindrical shell. If the tank is safe for

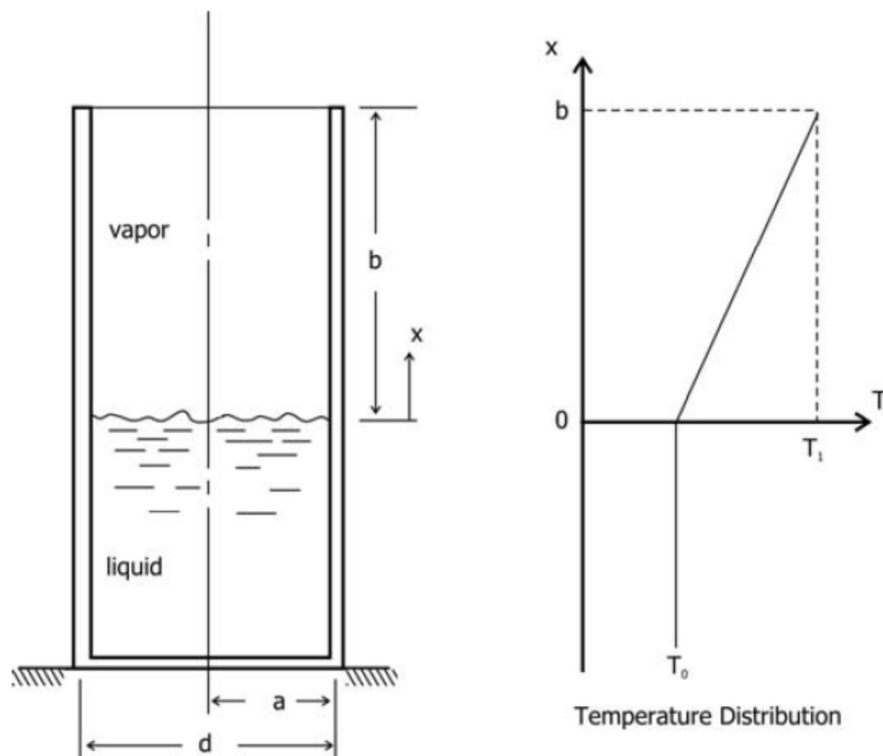


Figure 4.2 : Partially filled vertical cylindrical vessel depiction.

5.1.1 Step 1

Initial conditions:

$$T_0 = -183 \text{ }^{\circ}\text{C}$$

$$T_1 = -70 \text{ }^{\circ}\text{C}$$

$$b = 221 \text{ mm}$$

$$a = 137.5 \text{ mm}$$

$$h = 5 \text{ mm}$$

$$\mu = 0.33$$

$$p_r = 0$$

$$N_x = 0$$

$$M_T = 0$$

$$N_T = \alpha E \int_{-h/2}^{h/2} \Delta T_1(x/b) dz = \alpha E \Delta T_1 h(x/b) = N_0(x/b)$$

$$N_0 = \alpha E \Delta T_1 h$$

$$\Delta T(x) = T(x) - T_0 = (T_1 - T_0)(x/b) = \Delta T_1(x/b)$$

From displacement equation:

$$\frac{d^4 w}{dx^4} + \frac{Eh}{Da^2} w = -\frac{\mu N_x}{Da} - \frac{1}{(1-\mu)D} \frac{d^2 M_T}{dx^2} + \frac{N_T}{Da} + \frac{p_r}{D}$$

If the initial conditions are added to the equation:

$$\frac{Eh}{Da^2} = 4\beta^4$$

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{N_0}{Da} \left(\frac{x}{b}\right)$$

Solution for the homogeneous part:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = 0$$

Let $w = e^{mx}$;

$$\frac{de^{mx}}{dx} = me^{mx}$$

and

$$\frac{d^2 e^{mx}}{dx^2} = m^2 e^{mx}$$

and

$$\frac{d^3 e^{mx}}{dx^3} = m^3 e^{mx}$$

and

$$\frac{d^4 e^{mx}}{dx^4} = m^4 e^{mx}$$

If these are put into equation:

$$e^{mx}(m^4 + 4\beta^4) = 0$$

For every value of m $e^{mx} \neq 0$;

$$m^4 + 4\beta^4 = 0$$

$$(m^2 + 2\beta^2)^2 - 4m^2\beta^2 = 0$$

$$(m^2 + 2\beta^2)^2 = 4m^2\beta^2$$

$$(m^2 + 2\beta^2) = \pm 2m\beta$$

Therefore:

$$m^2 + 2m\beta + 2\beta^2 = 0 \text{ and } m^2 - 2m\beta + 2\beta^2 = 0$$

If these equations are solved for m;

$$m_1 = -\beta - \beta i$$

$$m_2 = -\beta + \beta i$$

$$m_3 = +\beta - \beta i$$

$$m_4 = +\beta + \beta i$$

Therefore:

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + \frac{N_0}{4\beta^4 D a} \left(\frac{x}{b}\right)$$

And

$$4\beta^4 \equiv \frac{Eh}{Da^2}$$

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + \frac{N_0 a}{Eh} \left(\frac{x}{b}\right)$$

If $\beta b = \frac{[3(1-\mu^2)]^{0.25}}{\sqrt{h/a}} \left(\frac{b}{a}\right) \geq 9.21$, the cylindrical shell can be assumed as an infinite cylinder.

$$\beta b = \frac{[3(1 - 0.33^2)]^{0.25}}{\sqrt{5/137.5}} \left(\frac{221}{137.5}\right) = 10.78 \geq 9.21$$

If the cylinder is assumed as infinite, $e^{\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) = 0$. Therefore;

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + \frac{N_0 a}{Eh} \left(\frac{x}{b}\right)$$

First, Second and third derivatives of the displacement equation are:

$$\omega_x(x) = \frac{dw}{dx} = -\beta e^{-\beta x} [(C_1 - C_2) \cos \beta x + (C_1 + C_2) \sin \beta x] + \frac{N_0 a}{Ehb}$$

$$\frac{d^2 w}{dx^2} = -2\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$\frac{d^3 w}{dx^3} = 2\beta^3 e^{-\beta x} [(C_1 + C_2) \cos \beta x + (C_2 - C_1) \sin \beta x]$$

If second derivative of displacement equation is utilized:

$$M_x = -D \frac{d^2 w}{dx^2} - \frac{M_T}{1 - \mu} = -D \frac{d^2 w}{dx^2} = 2D\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$M_{x(x=0)} = 2D\beta^2 C_2 = M_0$$

If third derivative of displacement equation is utilized:

$$\begin{aligned}
Q_x &= -D \frac{d^3 w}{dx^3} - \frac{1}{1-\mu} \frac{dM_T}{dx} = -D \frac{d^3 w}{dx^3} \\
&= -2D\beta^3 e^{-\beta} [(C_1 + C_2)\cos \beta x + (C_2 - C_1)\sin \beta x] \\
Q_{x(x=0)} &= -2D\beta^3 (C_1 + C_2) = Q_0
\end{aligned}$$

For the lower portion of the liquid-vapor interface, $x \leq 0$:

$$w(0) = -\frac{1}{2D\beta^3} (\beta M_0 - Q_0) \quad (4.58)$$

$$w(0) = -\frac{1}{2D\beta^3} (2D\beta^3 C_2 + 2D\beta^3 (C_1 + C_2)) = -C_1 - 2C_2$$

and

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (2\beta M_0 - Q_0) \quad (4.59)$$

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (4D\beta^3 C_2 + -2D\beta^3 (C_1 + C_2)) = -\beta(C_1 + 3C_2)$$

At $t = 0$, lower and upper values are equal, therefore:

$$\begin{aligned}
w(0) &= C_1 = -C_1 - 2C_2 \\
C_1 &= -C_2
\end{aligned}$$

And

$$(\omega_x)_{x=0} = -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb} = -\beta(C_1 + 3C_2)$$

$$\beta(-4C_2) = \frac{N_0 a}{Ehb}$$

$$-C_2 = \frac{N_0 a}{4\beta Ehb} = C_1$$

After this point, solutions are calculated by using MATLAB iteratively for each millimeter cross section of the LOX tank by utilizing equations (4.23) thru (4.30). MATLAB codes are provided in APPENDIX. Aluminum 6061-T6 material properties are inputs in the code's first lines in accordance with its cryogenic behavior.

5.1.2 Step 2

Initial conditions:

$$T_0 = -183 \text{ }^\circ\text{C}$$

$$T_1 = -70 \text{ }^\circ\text{C}$$

$$b = 221 \text{ mm}$$

$$a = 137.5 \text{ mm}$$

$$h = 5 \text{ mm}$$

$$\mu = 0.33$$

$$p_r = 70 \text{ bar}$$

$$N_x = 0$$

$$M_T = 0$$

From displacement equation:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = -\frac{\mu N_x}{Da} - \frac{1}{(1-\mu)D} \frac{d^2 M_T}{dx^2} + \frac{N_T}{Da} + \frac{p_r}{D}$$

It becomes:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{N_T + p_r a}{Da}$$

If this differential equation is solved as it is solved at Step 1:

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + \frac{N_0 a}{Eh} \left(\frac{x}{b} \right) + \frac{p_r a^2}{Eh}$$

First, second and third derivatives of the $w(x)$:

$$\omega_x(x) = \frac{dw}{dx} = -\beta e^{-\beta x} [(C_1 - C_2) \cos \beta x + (C_1 + C_2) \sin \beta x] + \frac{N_0 a}{Ehb}$$

$$\frac{d^2 w}{dx^2} = -2\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$\frac{d^3 w}{dx^3} = 2\beta^3 e^{-\beta} [(C_1 + C_2) \cos \beta x + (C_2 - C_1) \sin \beta x]$$

If second derivative of displacement equation is utilized:

$$M_x = -D \frac{d^2 w}{dx^2} - \frac{M_T}{1-\mu} = -D \frac{d^2 w}{dx^2} = 2D\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$M_{x(x=0)} = 2D\beta^2 C_2 = M_0$$

If third derivative of displacement equation is utilized:

$$\begin{aligned} Q_x &= -D \frac{d^3 w}{dx^3} - \frac{1}{1-\mu} \frac{dM_T}{dx} = -D \frac{d^3 w}{dx^3} \\ &= -2D\beta^3 e^{-\beta} [(C_1 + C_2) \cos \beta x + (C_2 - C_1) \sin \beta x] \end{aligned}$$

$$Q_{x(x=0)} = -2D\beta^3 (C_1 + C_2) = Q_0$$

For the lower portion of the liquid-vapor interface, $x \leq 0$:

$$w(0) = -\frac{1}{2D\beta^3} (\beta M_0 - Q_0) \quad (4.58)$$

$$w(0) = -\frac{1}{2D\beta^3} (2D\beta^3 C_2 + 2D\beta^3 (C_1 + C_2)) = -C_1 - 2C_2$$

and

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (2\beta M_0 - Q_0) \quad (4.59)$$

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (4D\beta^3 C_2 + -2D\beta^3 (C_1 + C_2)) = -\beta(C_1 + 3C_2)$$

At $x = 0$, lower and upper values are equal, therefore:

$$w(0) = e^0 (C_1 \cos 0 + C_2 \sin 0) + \frac{N_0 a}{Eh} \left(\frac{0}{b}\right) + \frac{p_r a^2}{Eh} = C_1 + \frac{p_r a^2}{Eh}$$

$$C_1 + \frac{p_r a^2}{Eh} = -C_1 - 2C_2$$

Therefore:

$$C_1 = -C_2 - \frac{p_r a^2}{2Eh}$$

also

$$\begin{aligned} \omega_x(0) &= \frac{dw}{dx} = -\beta e^0 [(C_1 - C_2) \cos 0 + (C_1 + C_2) \sin 0] + \frac{N_0 a}{Ehb} \\ &= -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb} \end{aligned}$$

and

$$\begin{aligned} -\beta(C_1 + 3C_2) &= -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb} \\ -\beta(C_1 + 3C_2 - C_1 + C_2) &= \frac{N_0 a}{Ehb} \end{aligned}$$

$$C_2 = \frac{-N_0 a}{4Eh\beta b}$$

$$C_1 = -C_2 - \frac{p_r a^2}{2Eh}$$

After this point, solutions are calculated by using MATLAB iteratively for each millimeter cross section of the LOX tank by utilizing equations (4.23) thru (4.30).

MATLAB codes are provided in APPENDIX. Aluminum 6061-T6 material properties are inputs in the code's first lines in accordance with its cryogenic behavior.

5.1.3 Step 3

Initial conditions:

$$T_0 = -183 \text{ }^\circ\text{C}$$

$$T_1 = -70 \text{ }^\circ\text{C}$$

$$b = 221 \text{ mm}$$

$$a = 137.5 \text{ mm}$$

$$h = 5 \text{ mm}$$

$$\mu = 0.33$$

$$p_r = 70 \text{ bar}$$

$$N_x = \sim 1.0088 \times 10^4 \text{ N.m}^{-1}$$

$$M_T = 0$$

From displacement equation:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = -\frac{\mu N_x}{Da} - \frac{1}{(1-\mu)D} \frac{d^2 M_T}{dx^2} + \frac{N_T}{Da} + \frac{p_r}{D}$$

It becomes:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = -\frac{\mu N_x}{Da} + \frac{N_T}{Da} + \frac{p_r}{D}$$

If this differential equation is solved as it is solved at Step 1:

$$w(x) = e^{-\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + \frac{N_0 a}{Eh} \left(\frac{x}{b} \right) + \frac{p_r a^2}{Eh} - \frac{\mu N_x a}{Eh}$$

First, second and third derivatives of the $w(x)$:

$$\omega_x(x) = \frac{dw}{dx} = -\beta e^{-\beta x} [(C_1 - C_2) \cos \beta x + (C_1 + C_2) \sin \beta x] + \frac{N_0 a}{Ehb}$$

$$\frac{d^2 w}{dx^2} = -2\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$\frac{d^3 w}{dx^3} = 2\beta^3 e^{-\beta x} [(C_1 + C_2) \cos \beta x + (C_2 - C_1) \sin \beta x]$$

If second derivative of displacement equation is utilized:

$$M_x = -D \frac{d^2 w}{dx^2} - \frac{M_T}{1-\mu} = -D \frac{d^2 w}{dx^2} = 2D\beta^2 e^{-\beta x} [C_2 \cos \beta x - C_1 \sin \beta x]$$

$$M_{x(x=0)} = 2D\beta^2 C_2 = M_0$$

If third derivative of displacement equation is utilized:

$$Q_x = -D \frac{d^3 w}{dx^3} - \frac{1}{1-\mu} \frac{dM_T}{dx} = -D \frac{d^3 w}{dx^3} \\ = -2D\beta^3 e^{-\beta x} [(C_1 + C_2) \cos \beta x + (C_2 - C_1) \sin \beta x]$$

$$Q_{x(x=0)} = -2D\beta^3 (C_1 + C_2) = Q_0$$

For the lower portion of the liquid-vapor interface, $x \leq 0$:

$$w(0) = -\frac{1}{2D\beta^3} (\beta M_0 - Q_0) \quad (4.58)$$

$$w(0) = -\frac{1}{2D\beta^3} (2D\beta^3 C_2 + 2D\beta^3 (C_1 + C_2)) = -C_1 - 2C_2$$

and

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (2\beta M_0 - Q_0) \quad (4.59)$$

$$(\omega_x)_{x=0} = -\frac{1}{2D\beta^2} (4D\beta^3 C_2 + -2D\beta^3 (C_1 + C_2)) = -\beta(C_1 + 3C_2)$$

For the upper portion of the liquid-vapor interface, $x \geq 0$:

$$w(0) = e^0 (C_1 \cos 0 + C_2 \sin 0) + \frac{N_0 a}{Eh} \left(\frac{0}{b}\right) + \frac{p_r a^2}{Eh} - \frac{\mu N_x a}{Eh} = C_1 + \frac{p_r a^2}{Eh} - \frac{\mu N_x a}{Eh}$$

$$\omega_x(0) = \frac{dw}{dx} = -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb}$$

At $x = 0$, lower and upper values are equal, therefore:

$$w(0) = -C_1 - 2C_2 = C_1 + \frac{p_r a^2}{Eh} - \frac{\mu N_x a}{Eh}$$

$$C_1 = \frac{\mu N_x a}{2Eh} - \frac{p_r a^2}{2Eh} - C_2$$

and

$$\omega_x(0) = -\beta(C_1 + 3C_2) = -\beta(C_1 - C_2) + \frac{N_0 a}{Ehb}$$

$$\beta(C_1 - C_2 - C_1 - 3C_2) = \frac{N_0 a}{Ehb}$$

$$C_2 = \frac{-N_0 a}{4Eh\beta b}$$

and

$$C_1 = \frac{\mu N_x a}{2Eh} - \frac{p_r a^2}{2Eh} + \frac{N_0 a}{4Eh\beta b}$$

After this point, solutions are calculated by using MATLAB iteratively for each millimeter cross section of the LOX tank by utilizing equations (4.23) thru (4.30).

MATLAB codes are provided in APPENDIX. Aluminum 6061-T6 material properties are inputs in the code's first lines in accordance with its cryogenic behavior.



6. CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the results of the calculations from the previous chapter are going to be illustrated for each step of solutions. Then the results are going to be compared with the results which are obtained from ANSYS.

6.1 ANSYS FEM Model

Two-dimensional axisymmetrical model is generated. This model provides less CPU power consumption with less number of elements and also prevents waste of time.

There are 11040 second order elements and 36791 nodes in this model. Mesh detail of the model may seen in Figure 6.1.

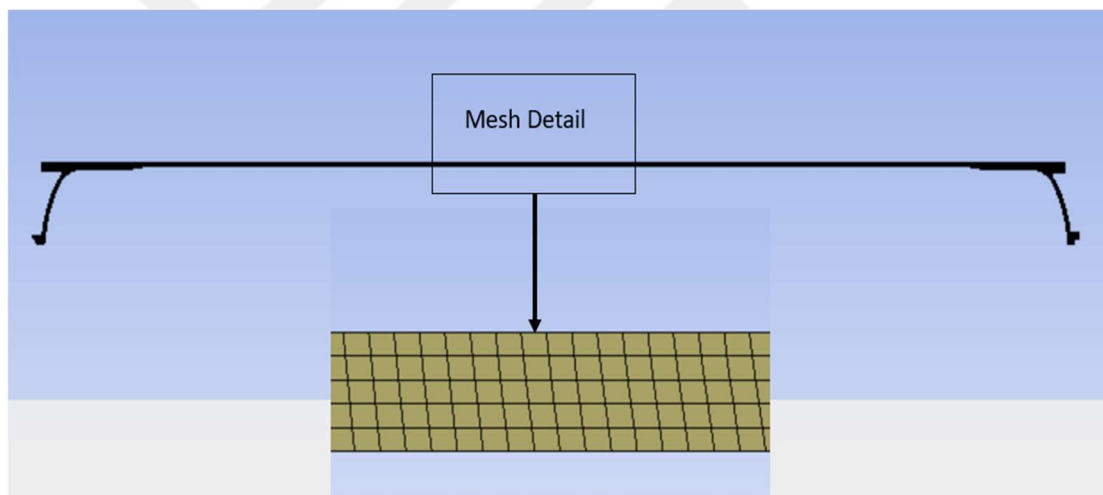


Figure 6.1 : Mesh detail.

Thermomechanical analysis model boundary conditions may be seen in Figure 6.2 and Figure 6.3:

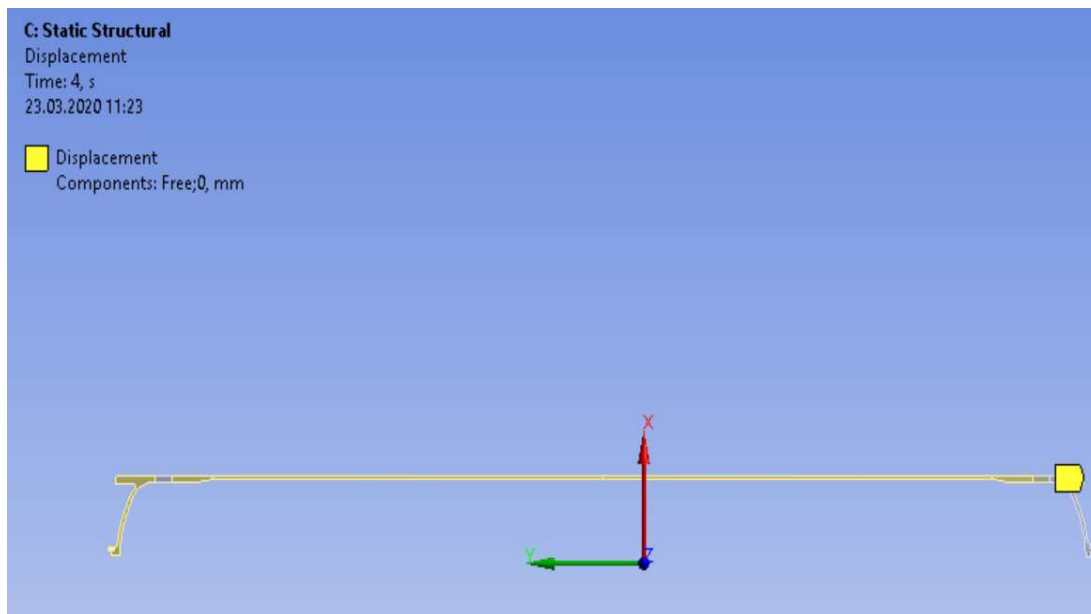


Figure 6.2 : Boundary condition 1 - free in x-dir. and $y=0$.

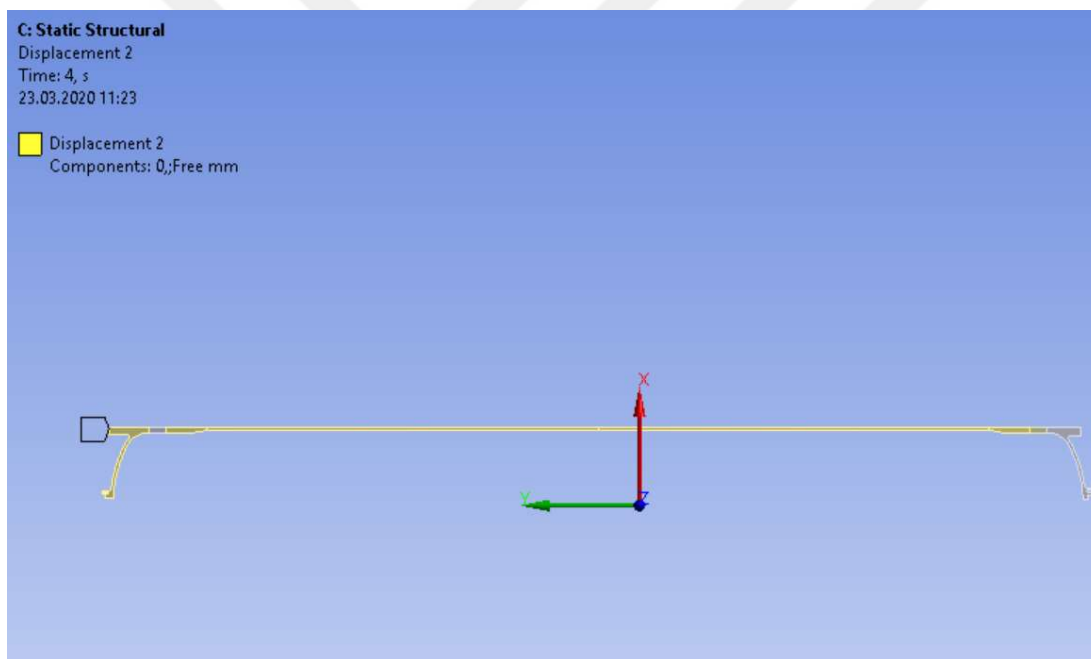


Figure 6.3 : Boundary condition 1 - free in y-dir. and $x=0$.

Boundary condition 1 and boundary condition 2 are executed simultaneously.

Three steps of solutions are illustrated in Figure 6.4.

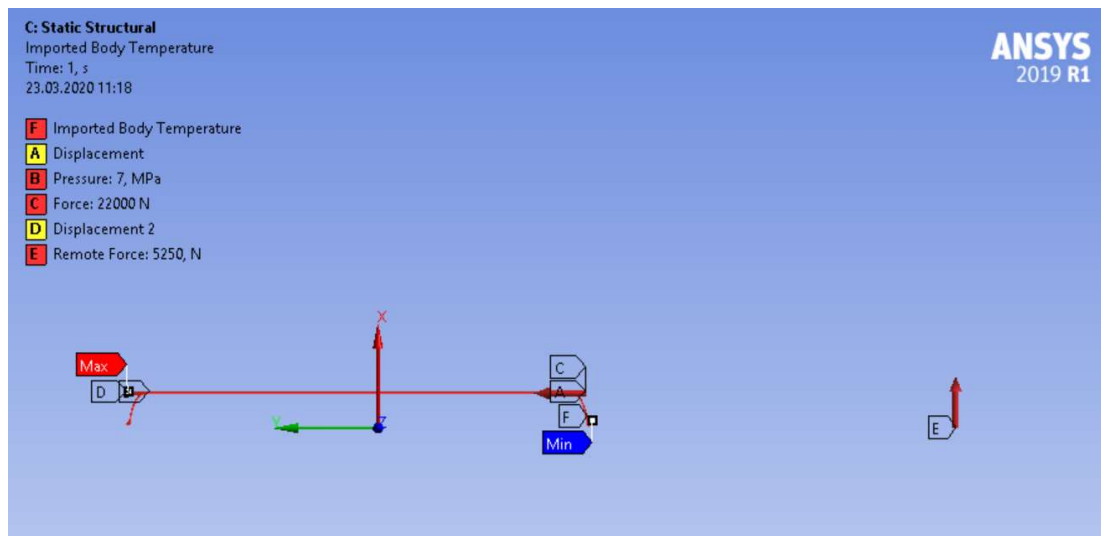


Figure 6.4 : Solution steps.

6.1.1 Step 1 solutions:

While the thermal effects are applied only, total deformation results are as in Figure 6.5.

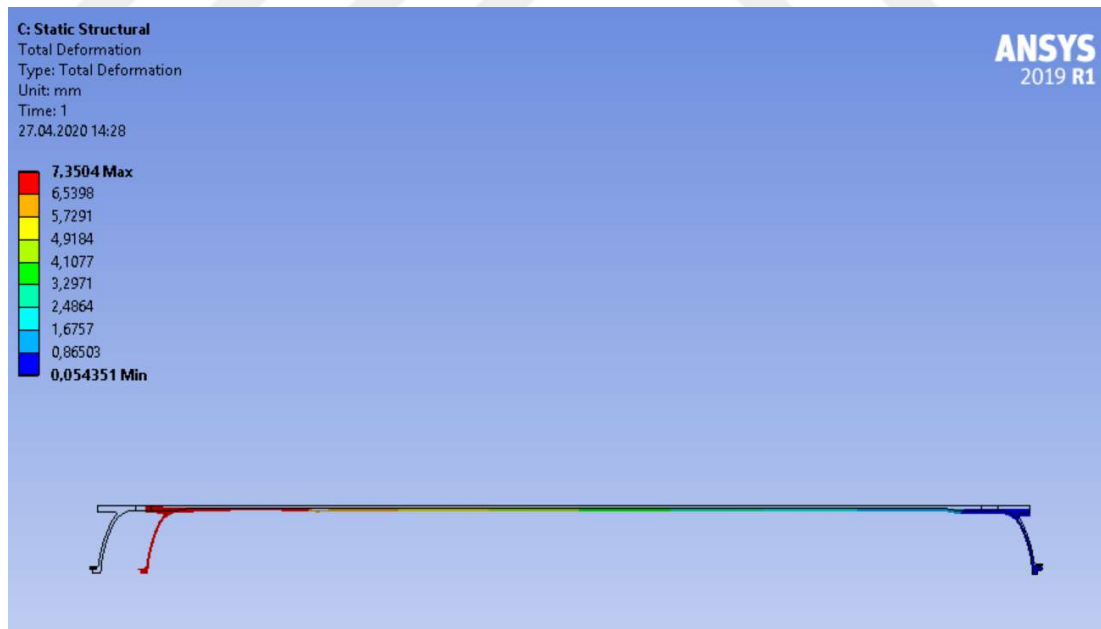


Figure 6.5 : Deformation for step 1.

Maximum principle stresses, hoop stresses are as in Figure 6.6.

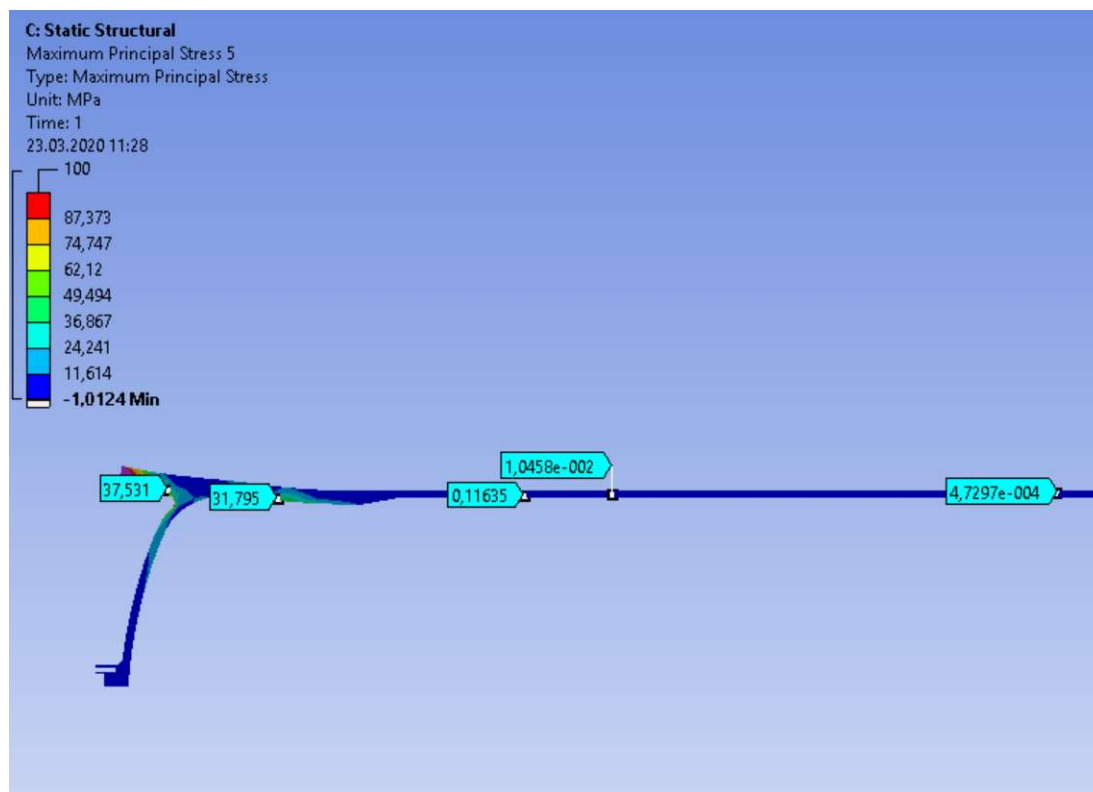


Figure 6.6 : Hoop stresses for step 1.

6.1.2 Step 2 solutions:

At step 2, beside thermal loading, tank is pressurized to 70 bar. For this condition total deformation is as in Figure 6.7.

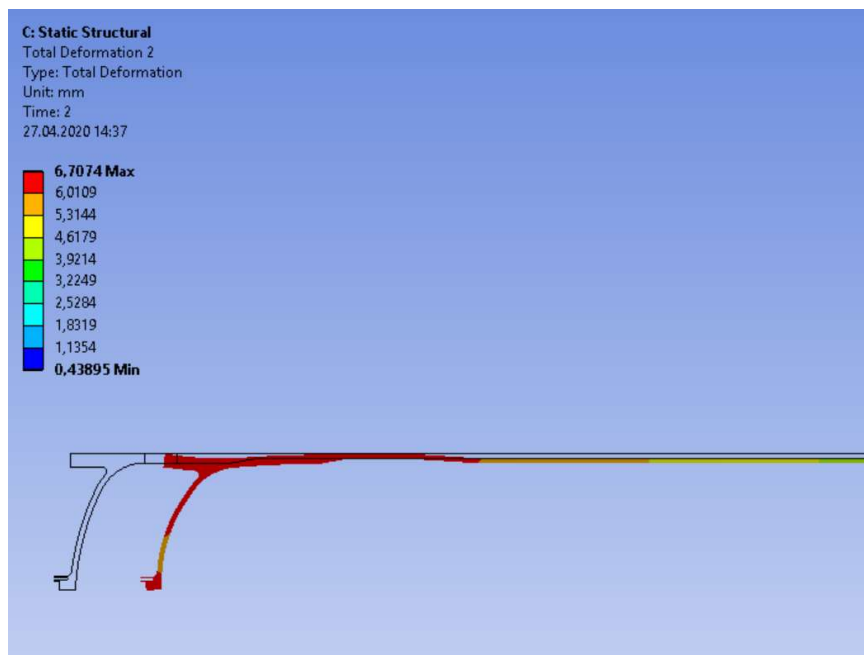


Figure 6.7 : Deformation for step 2.

Maximum principle stresses, hoop stresses are as in Figure 6.8.

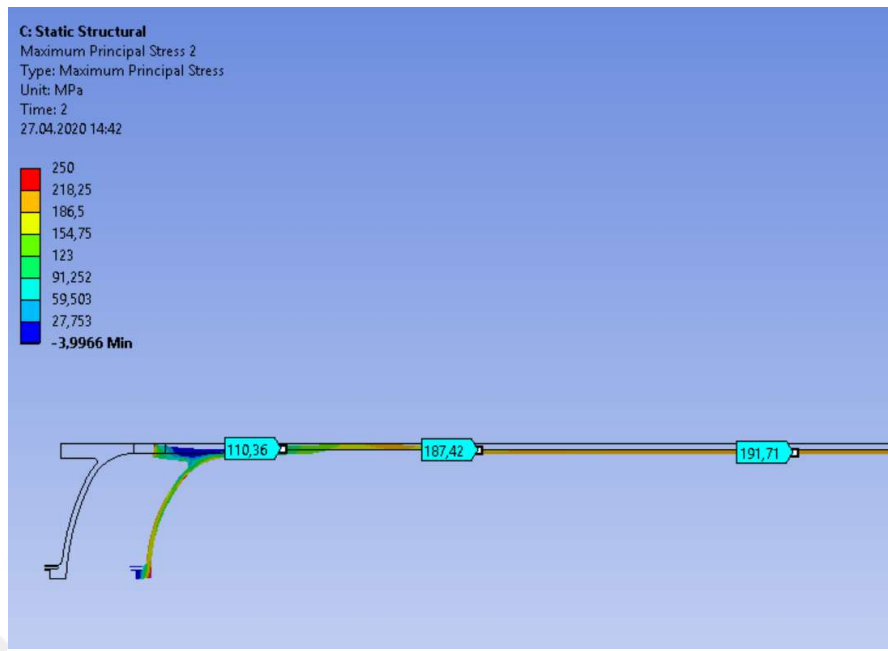


Figure 6.8 : Hoop stresses for step 2.

6.1.3 Step 3 solutions:

At step 3, beside thermal loading and 70 bar tank pressure, 22000 N thrust load is added.. For this condition total deformation is as in Figure 6.9.

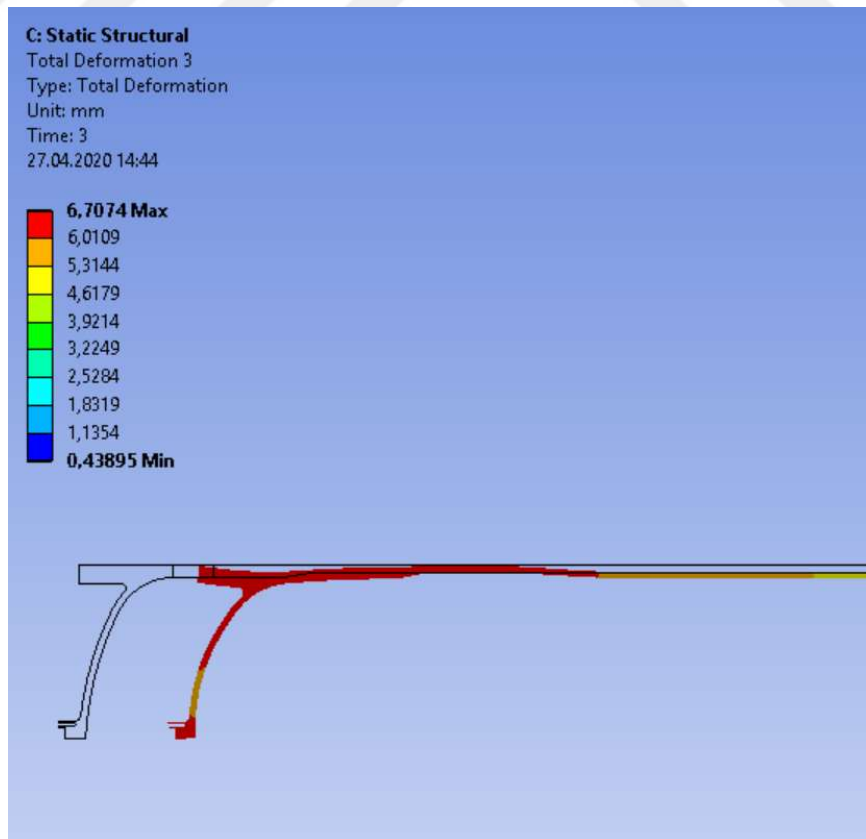


Figure 6.9 : Deformation for step 3.

Maximum principle stresses, hoop stresses are as in Figure 6.10.

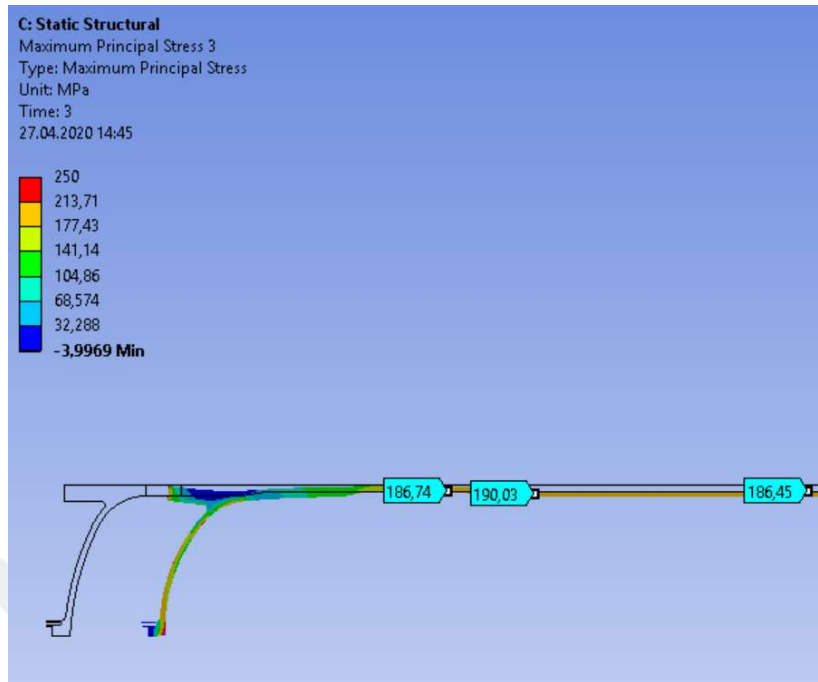


Figure 6.10 : Hoop stresses for step 3.

6.2 Analytical Calculation Results

In the light of the study of this thesis, prepared MATLAB codes provides the listed parameters below:

- Displacement/Deformation (w)
- Axial Stress (σ_x)
- Polar Stress/Hoop Stress (σ_θ)
- Axial Bending Stress Resultant (M_x)
- Polar Bending Stress Resultant (M_θ)
- Polar Membrane Stress Resultant (N_θ)
- Shear Stress Resultant (Q_x)
- Thermal Stress Resultant (N_T)
- Transverse Shear Stress (τ_{xz})

6.2.1 Step 1 solutions:

Deformation of the shell is illustrated in Figure 6.11.

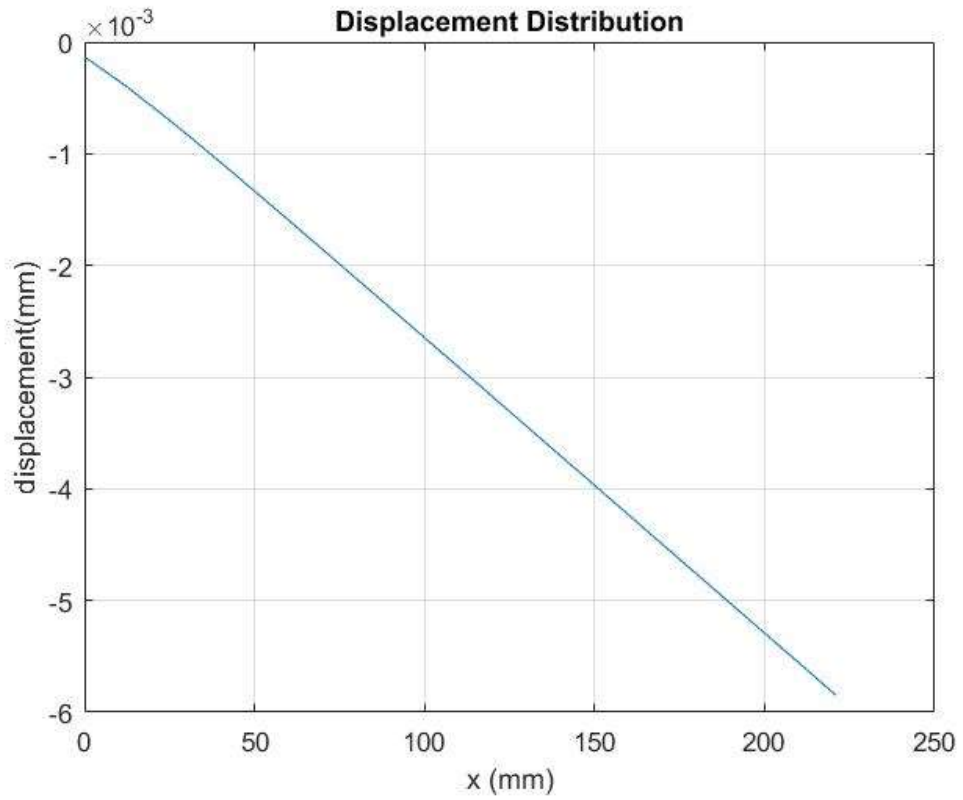


Figure 6.11 : Displacement distribution for step 1.

Axial Stress(σ_x) of the shell is illustrated in Figure 6.12.

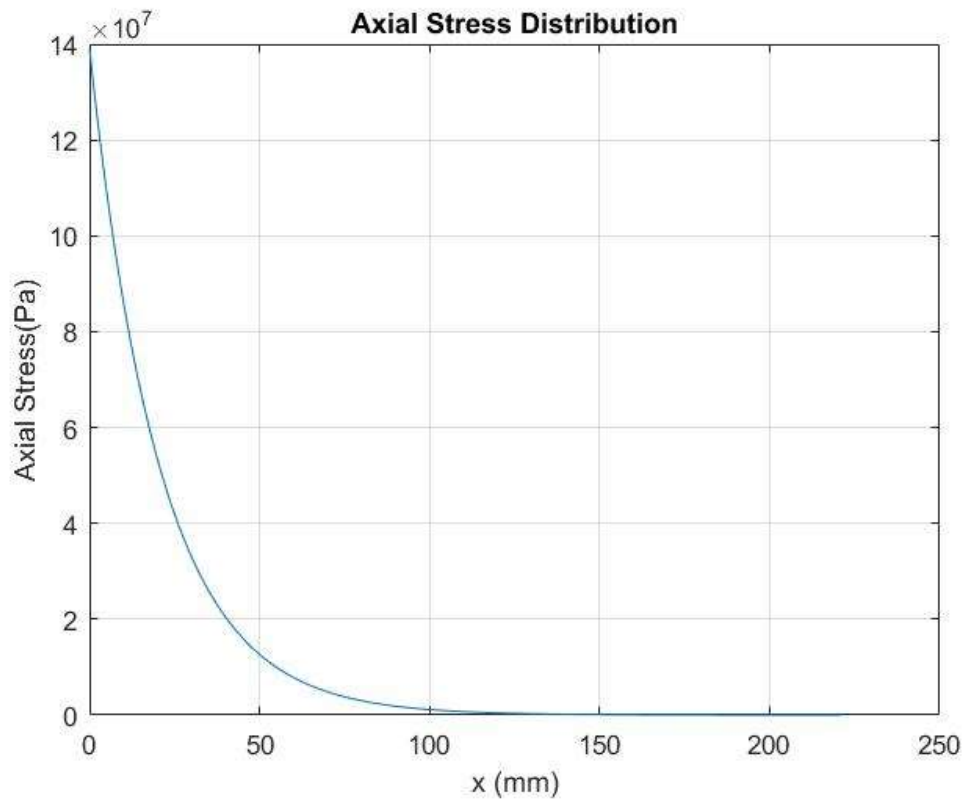


Figure 6.12 : Axial stress distribution for step 1.

Hoop Stress(σ_θ) of the shell is illustrated in Figure 6.13.

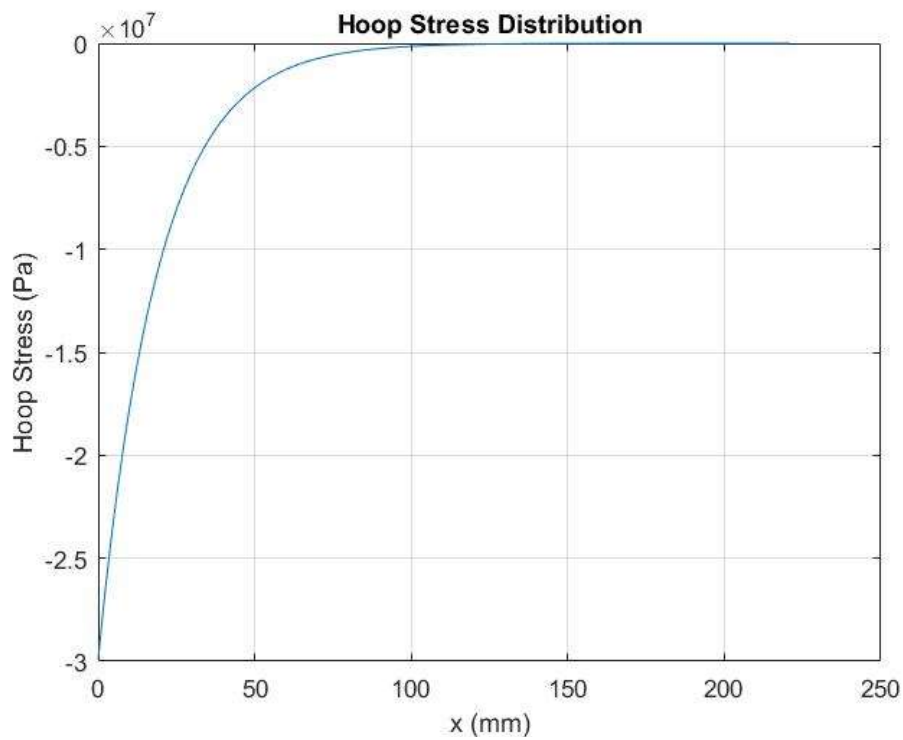


Figure 6.13 : Hoop stress distribution for step 1.

Axial Bending Stress Resultant(M_x) of the shell is illustrated in Figure 6.14.

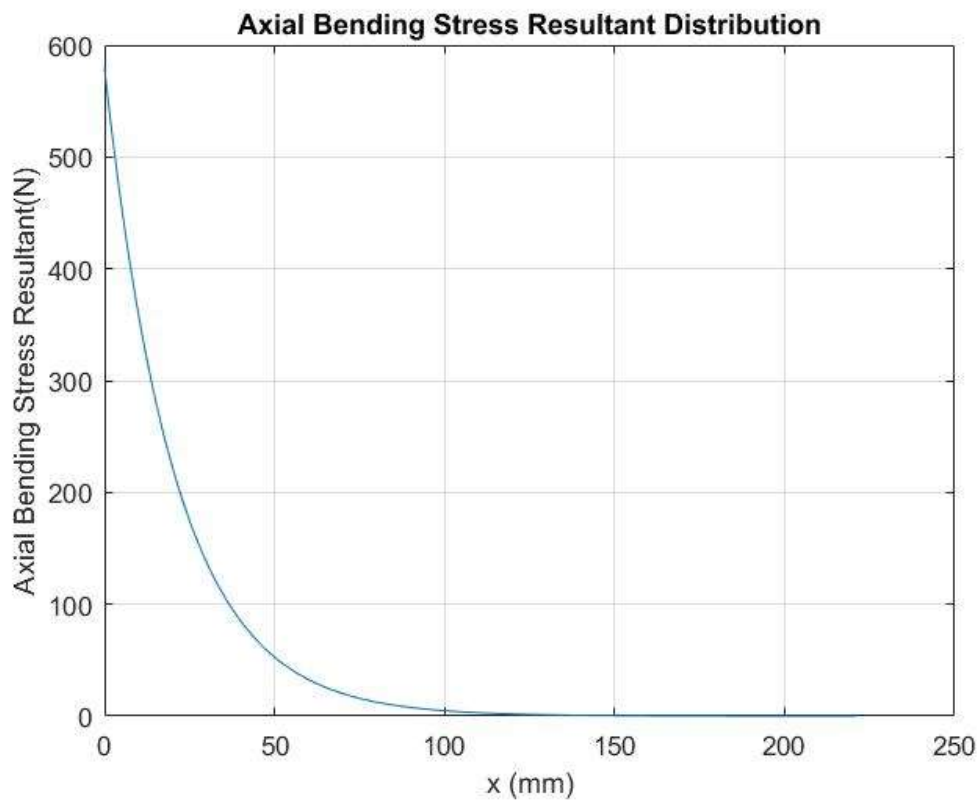


Figure 6.14 : Axial bending stress resultant distribution for step 1.

Polar Bending Stress Resultant(M_θ) of the shell is illustrated in Figure 6.15.

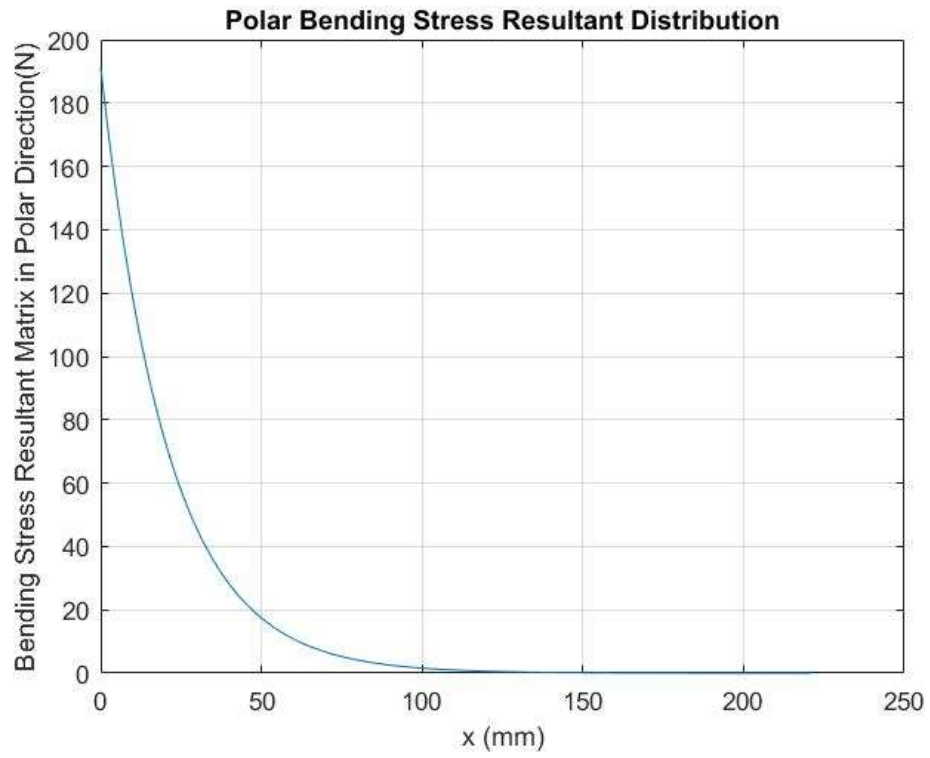


Figure 6.15 : Polar bending stress resultant distribution for step 1.

Polar membrane stress resultant(N_θ) of the shell is illustrated in Figure 6.16.

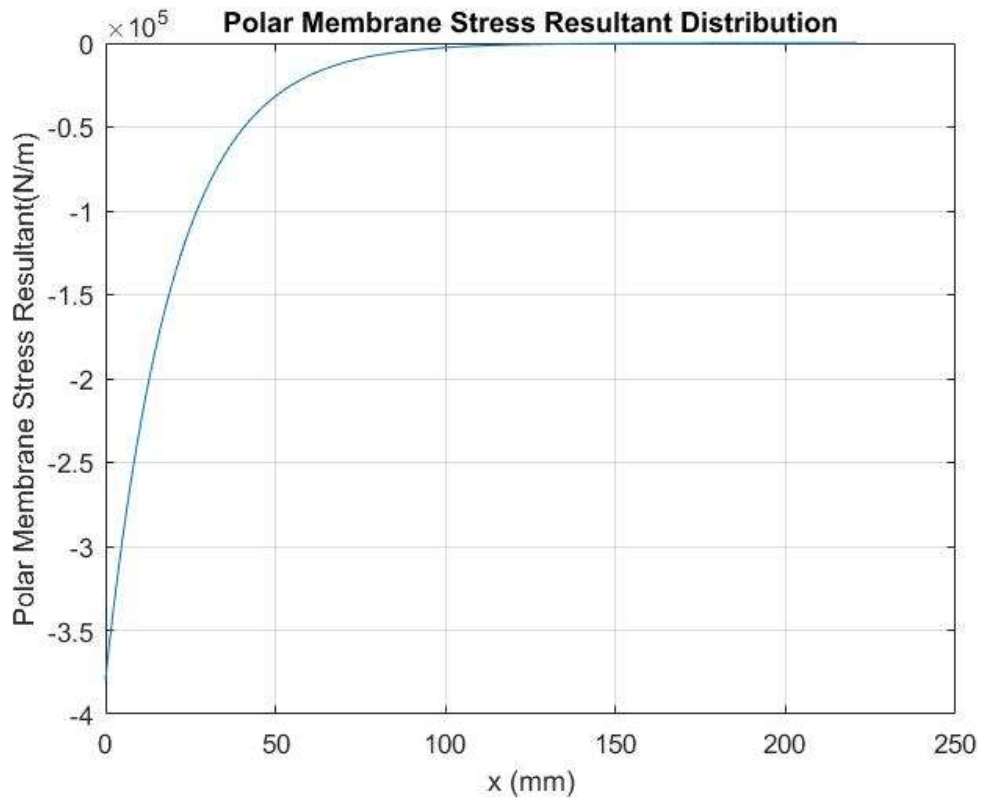


Figure 6.16 : Polar membrane stress resultant distribution for step 1.

Shear Stress Resultant (Q_x) of the shell is illustrated in Figure 6.17.

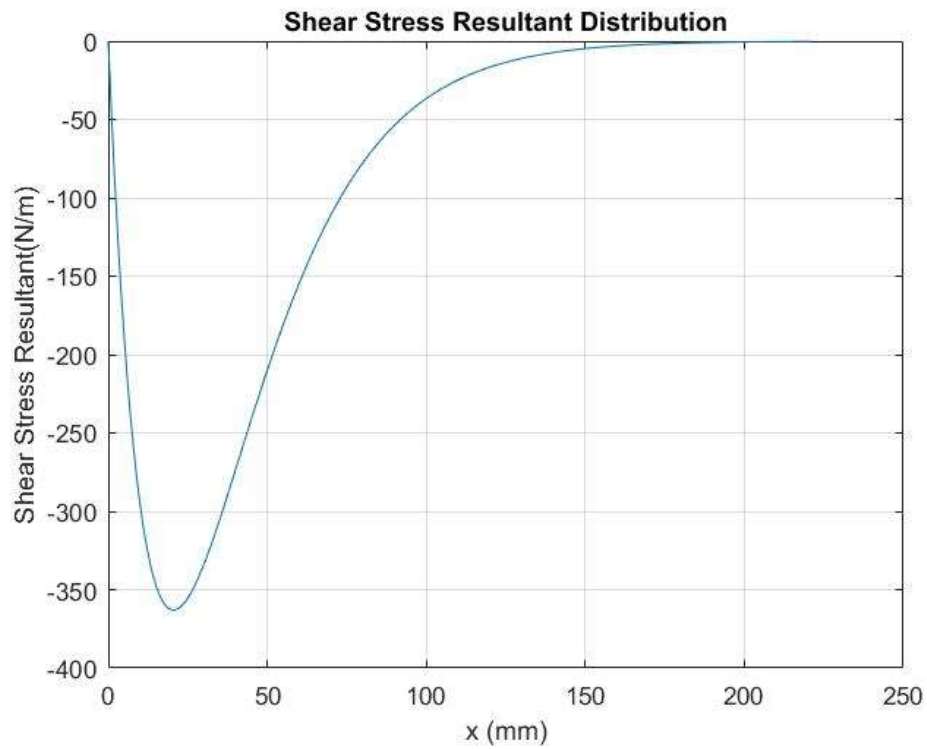


Figure 6.17 : Shear stress resultant distribution for step 1.

Thermal Stress Resultant (N_T) of the shell is illustrated in Figure 6.18.

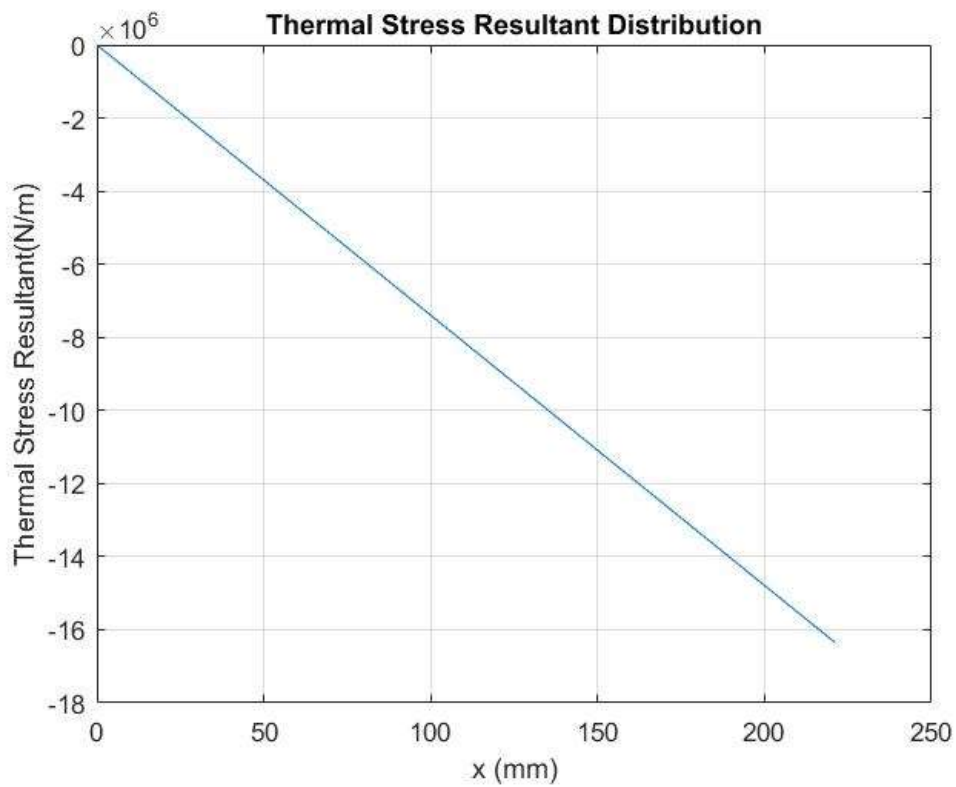


Figure 6.18 : Thermal stress resultant distribution for step 1.

Transverse Shear Stress (τ_{xz}) of the shell is illustrated in Figure 6.19.

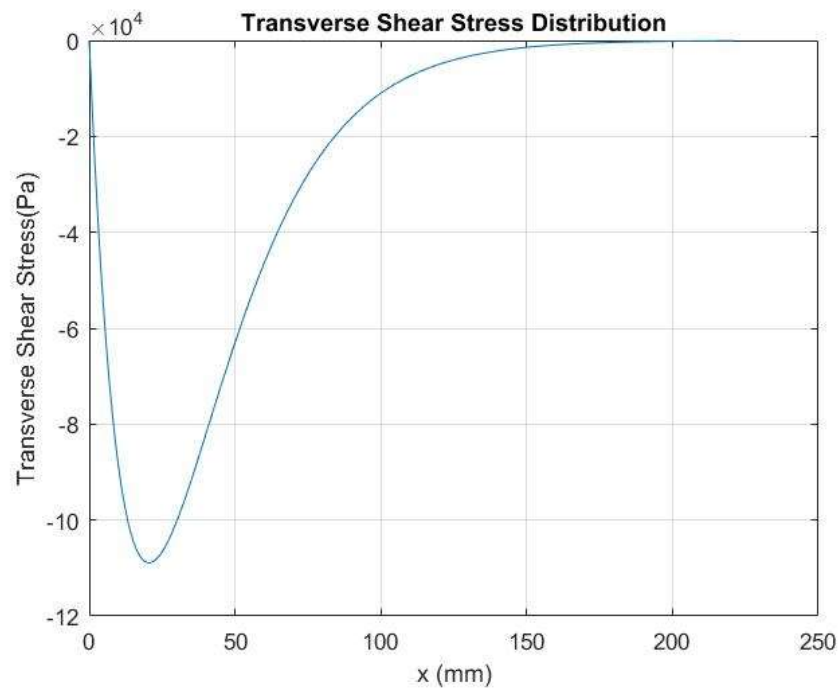


Figure 6.19 : Transverse shear stress distribution for step 1.

6.2.2 Step 2 solutions:

Deformation of the shell is illustrated in Figure 6.20.

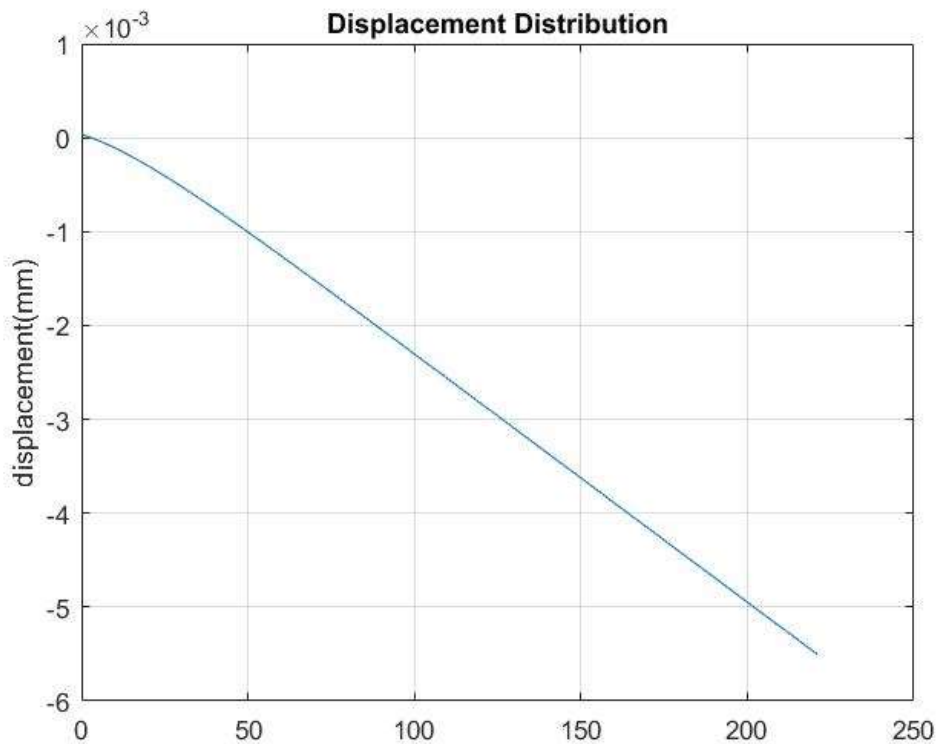


Figure 6.20 : Displacement distribution for step 2.

Axial Stress(σ_x) of the shell is illustrated in Figure 6.21.

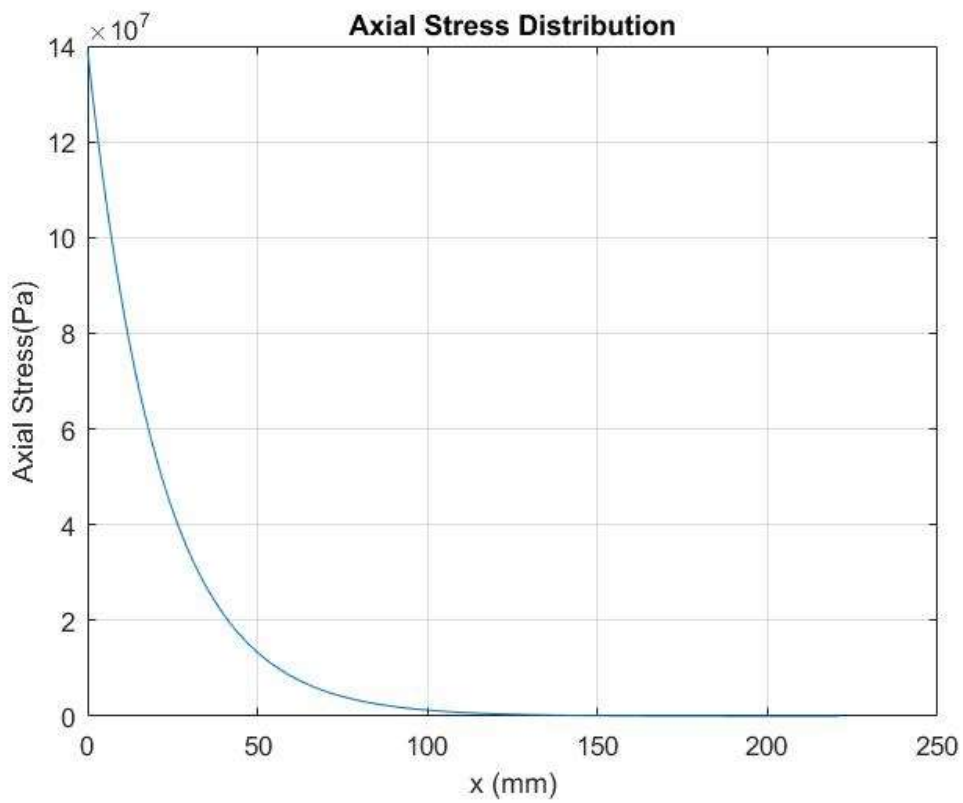


Figure 6.21 : Axial stress distribution for step 2.

Hoop Stress(σ_θ) of the shell is illustrated in Figure 6.22.

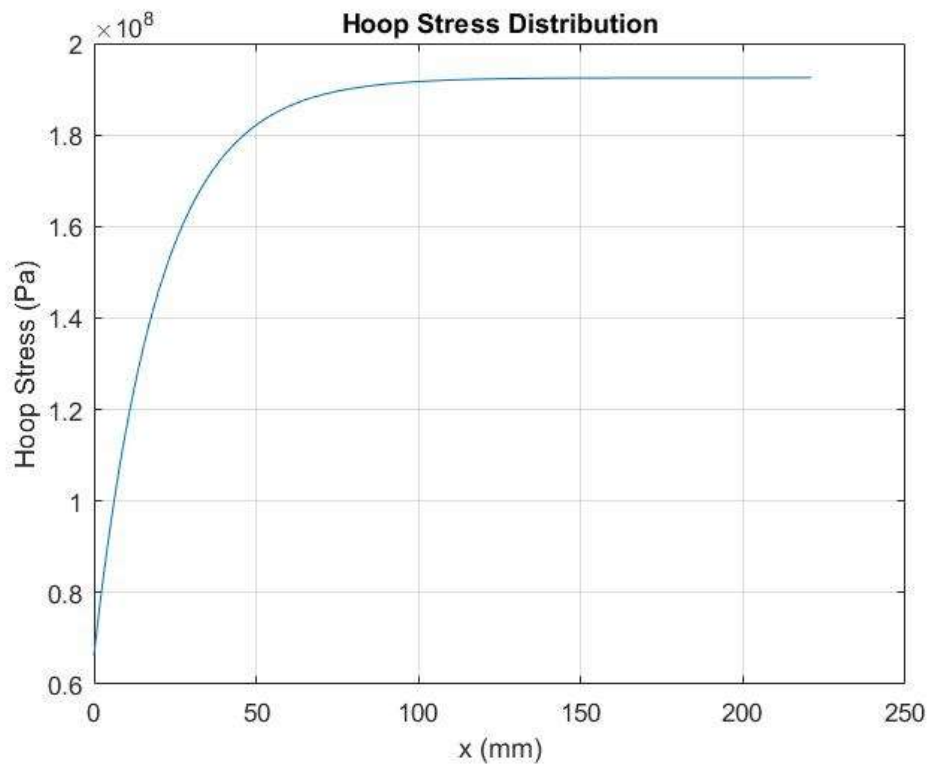


Figure 6.22 : Hoop stress distribution for step 2.

Axial Bending Stress Resultant(M_x) of the shell is illustrated in Figure 6.23.

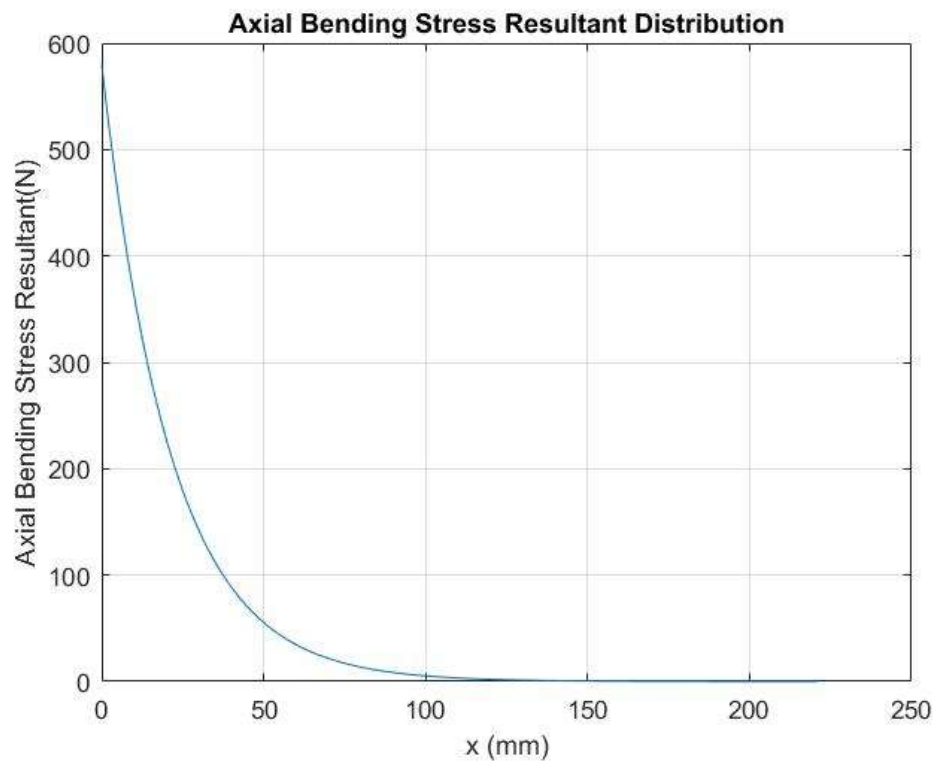


Figure 6.23 : Axial bending stress resultant distribution for step 2.

Polar Bending Stress Resultant(M_θ) of the shell is illustrated in Figure 6.24.

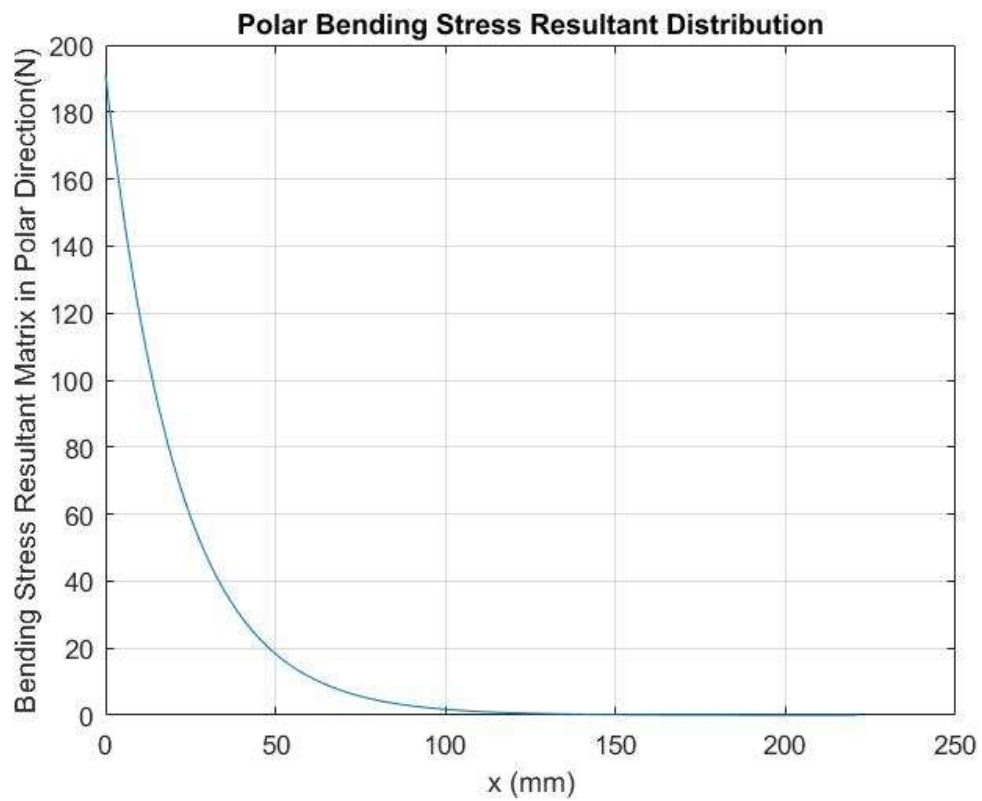


Figure 6.24 : Polar bending stress resultant distribution for step 2.

Polar membrane stress resultant(N_θ) of the shell is illustrated in Figure 6.25.

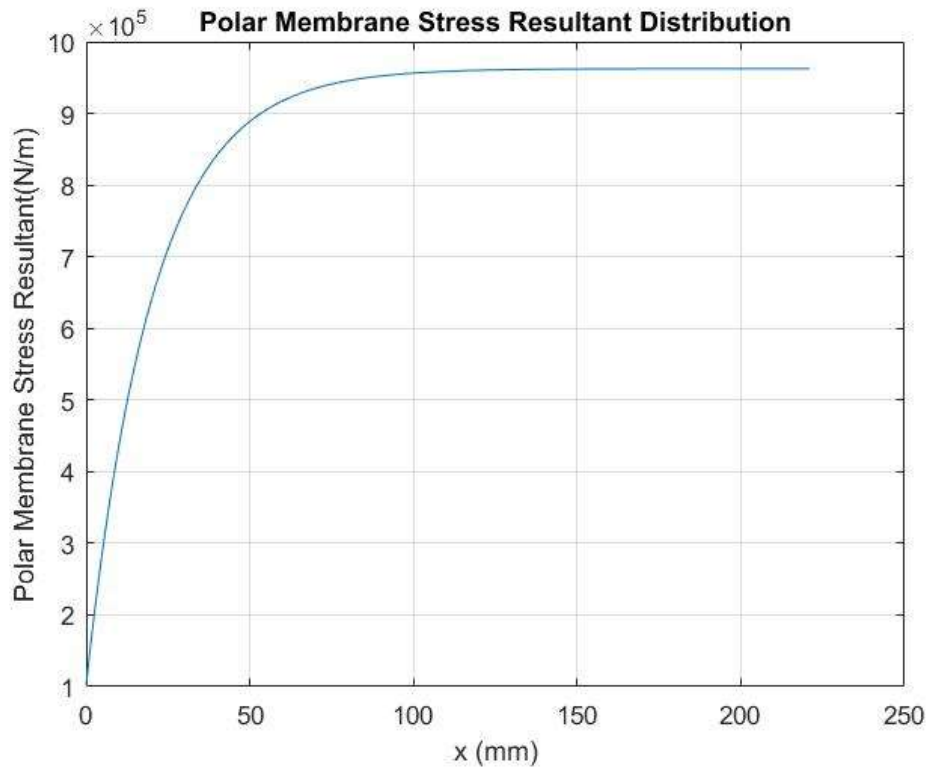


Figure 6.25 : Polar membrane stress resultant distribution for step 2.

Shear Stress Resultant (Q_x) of the shell is illustrated in Figure 6.26.

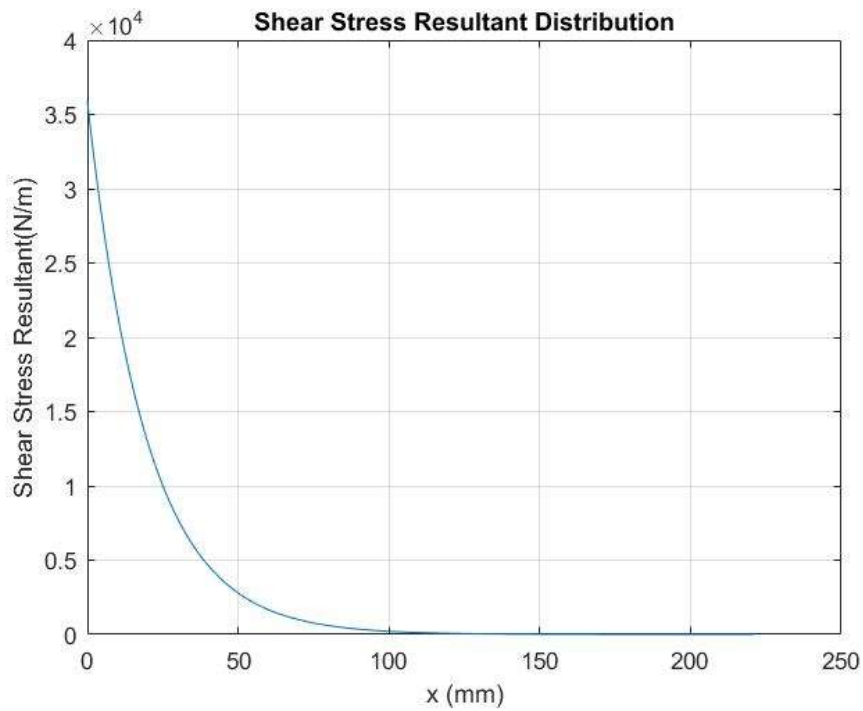


Figure 6.26 : Shear stress resultant distribution for step 2.

Thermal Stress Resultant (N_T) of the shell is illustrated in Figure 6.27.

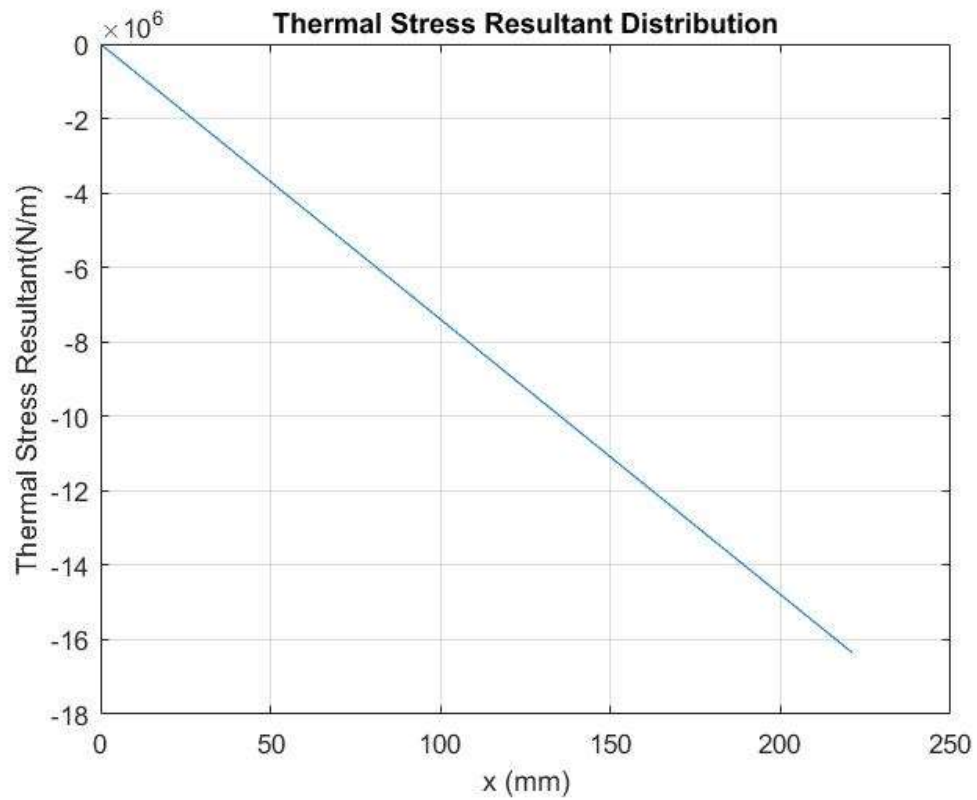


Figure 6.27 : Thermal stress resultant distribution for step 2.

Transverse Shear Stress (τ_{xz}) of the shell is illustrated in Figure 6.28.

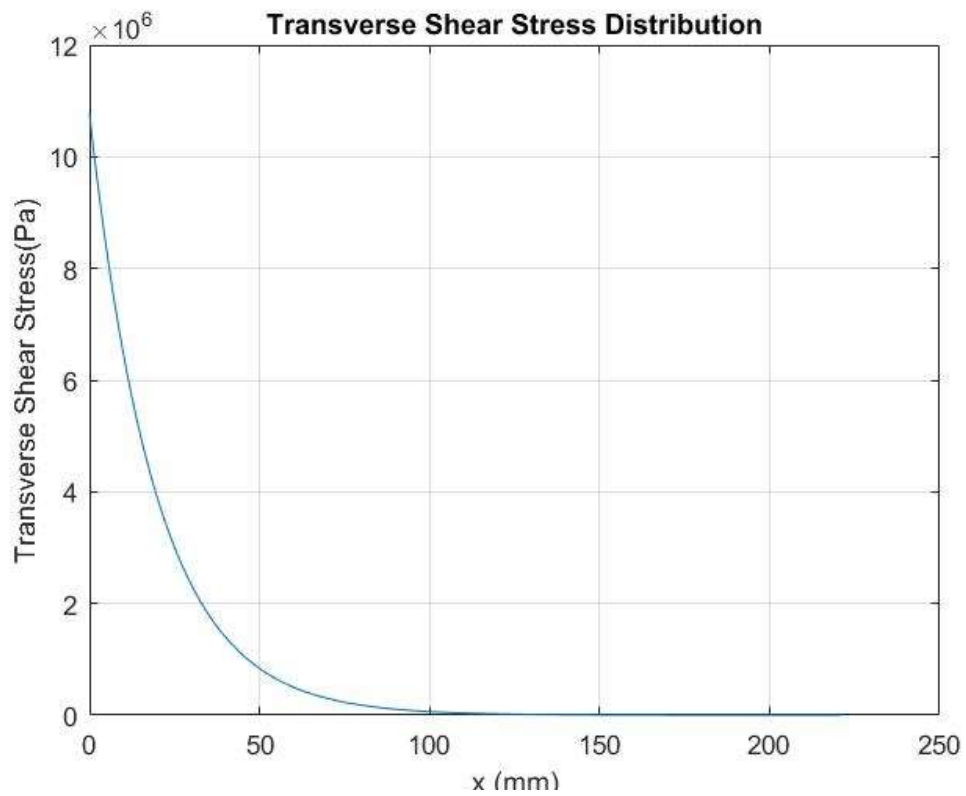


Figure 6.28 : Transverse shear stress distribution for step 2.

6.2.3 Step 3 solutions:

Deformation of the shell is illustrated in Figure 6.29.

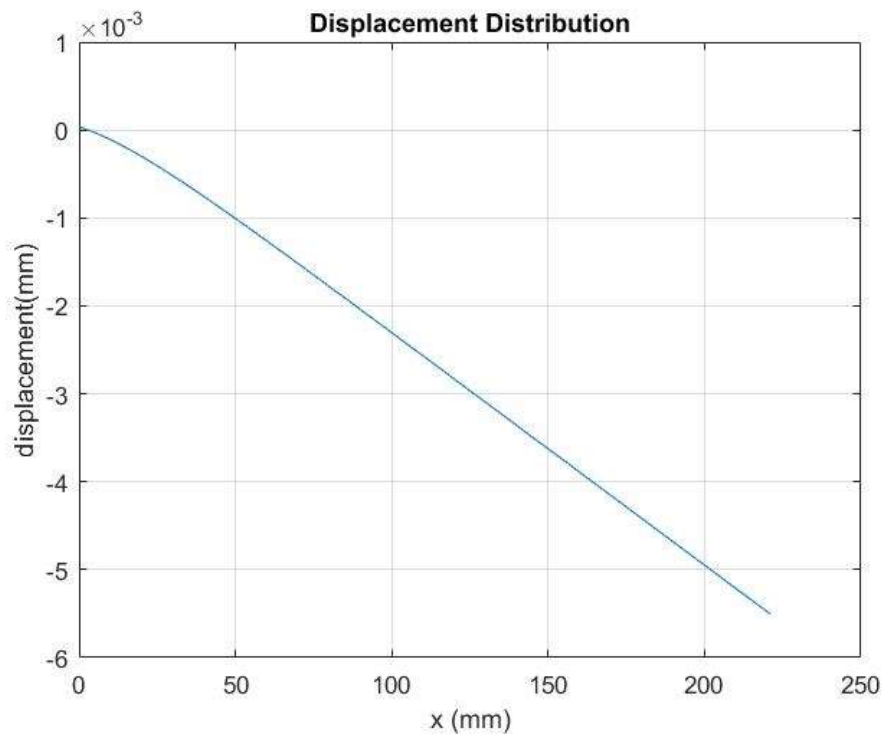


Figure 6.29 : Displacement distribution for step 3.

Axial Stress(σ_x) of the shell is illustrated in Figure 6.30.

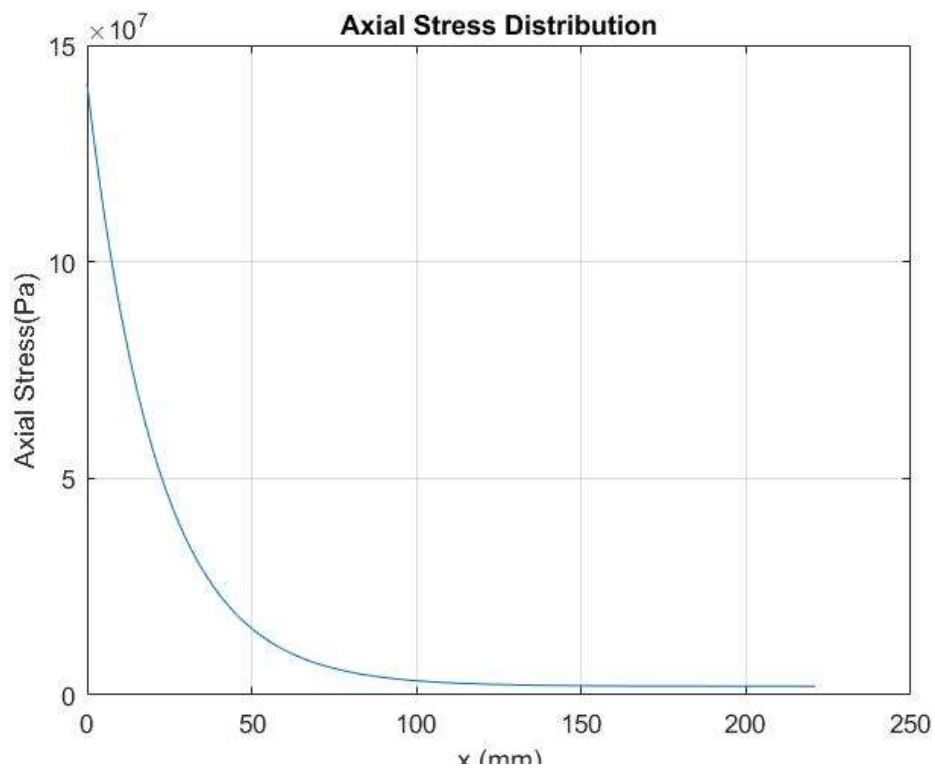


Figure 6.30 : Axial stress distribution for step 3.

Hoop Stress(σ_θ) of the shell is illustrated in Figure 6.31.

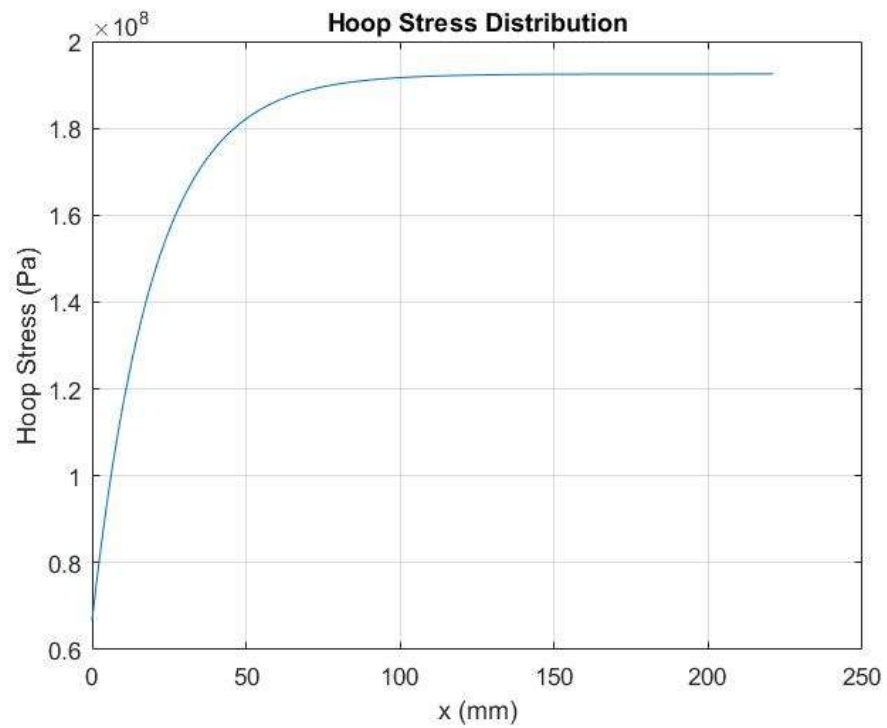


Figure 6.31 : Hoop stress distribution for step 3.

Axial Bending Stress Resultant(M_x) of the shell is illustrated in Figure 6.32.

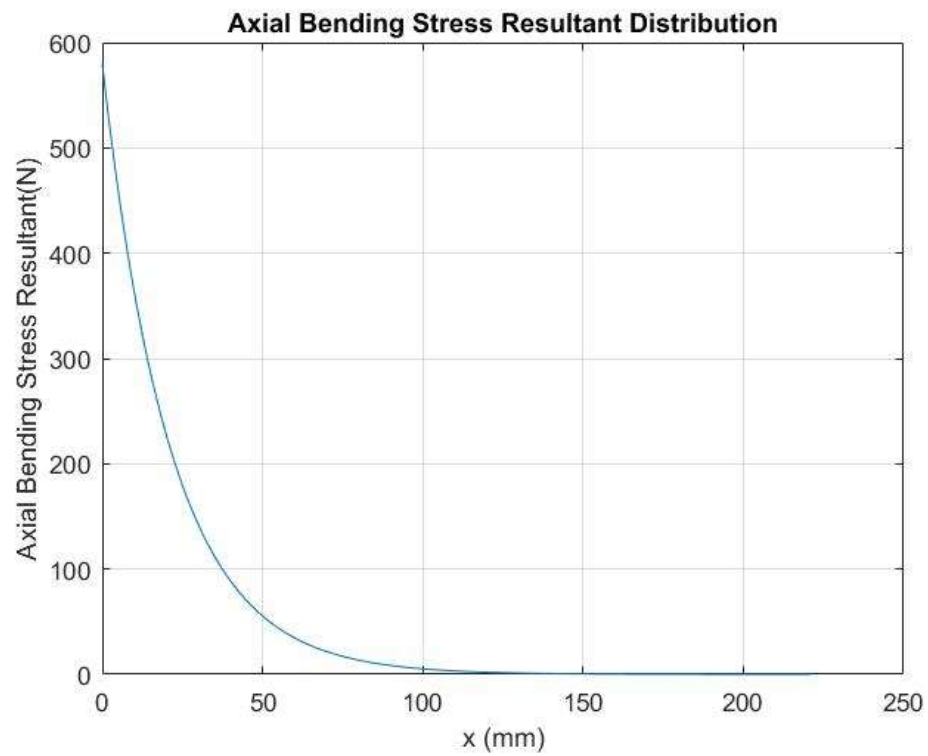


Figure 6.32 : Axial bending stress resultant distribution for step 3.

Polar Bending Stress Resultant(M_{θ}) of the shell is illustrated in Figure 6.33.

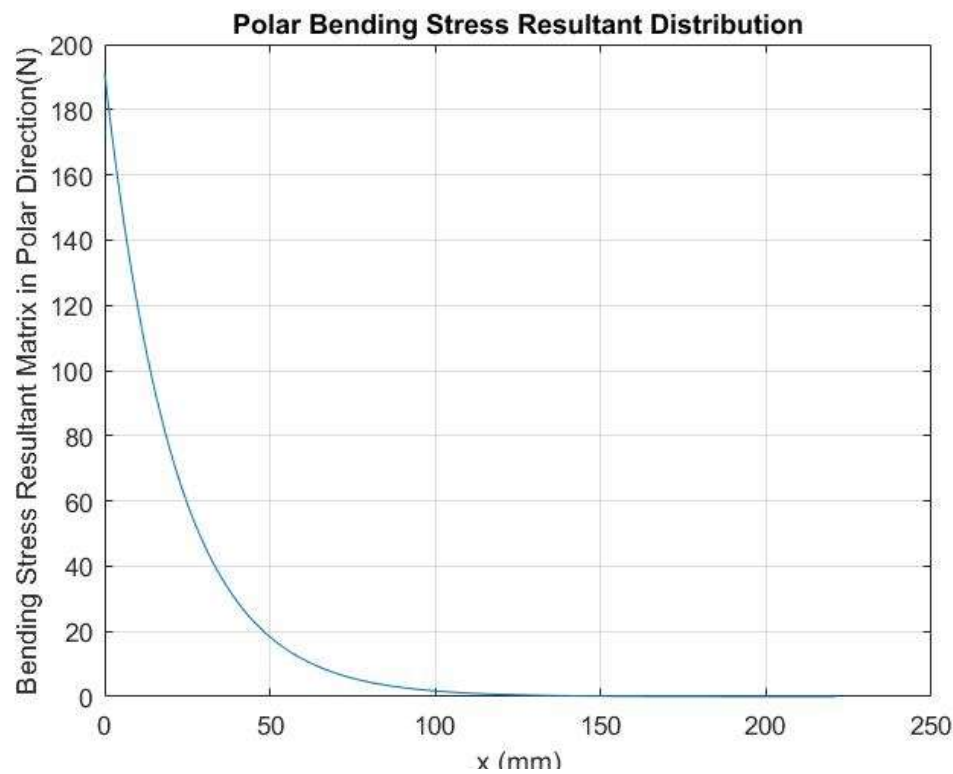


Figure 6.33 : Polar bending stress resultant distribution for step 3.

Polar membrane stress resultant(N_{θ}) of the shell is illustrated in Figure 6.34.

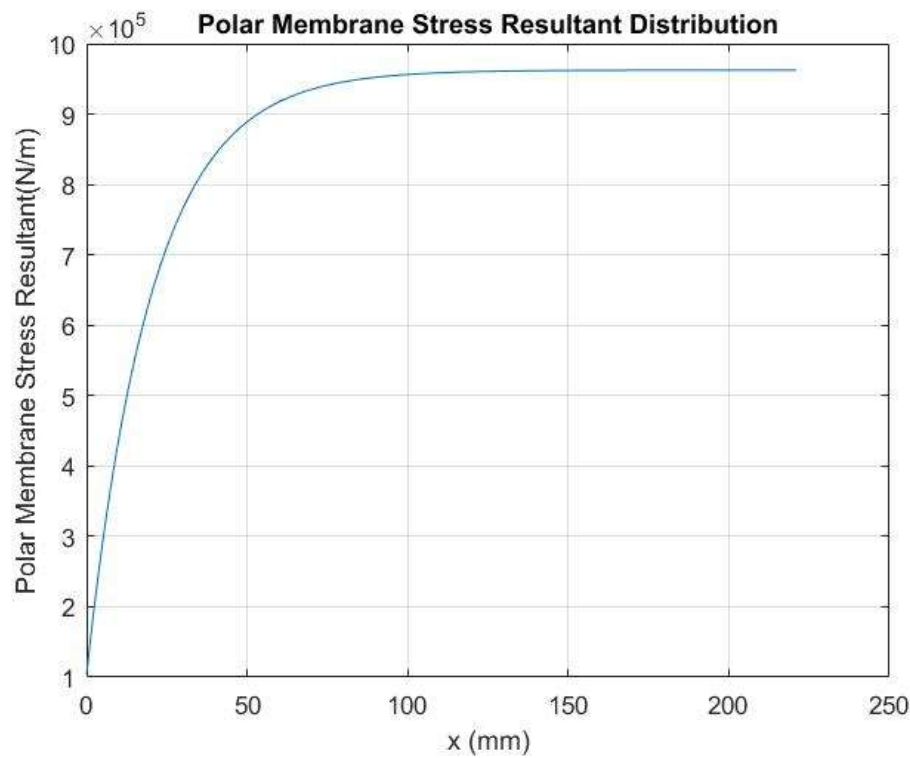


Figure 6.34 : Polar membrane stress resultant distribution for step 3.

Shear Stress Resultant (Q_x) of the shell is illustrated in Figure 6.35.

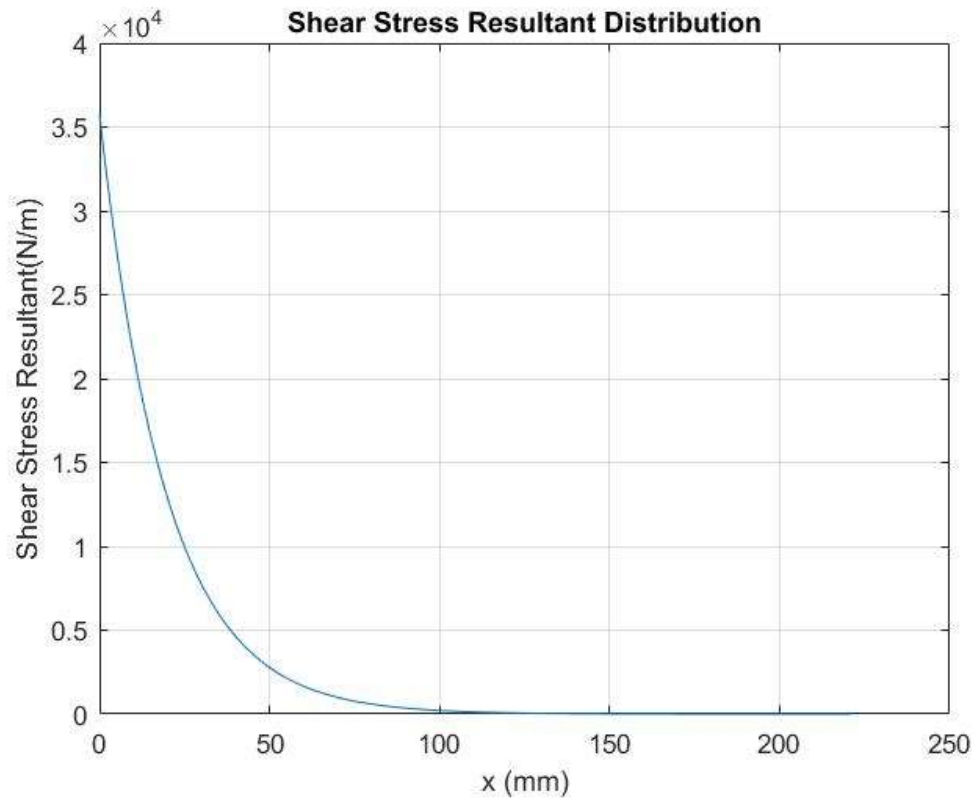


Figure 6.35 : Shear stress resultant for step 3.

Thermal Stress Resultant (N_T) of the shell is illustrated in Figure 6.36.

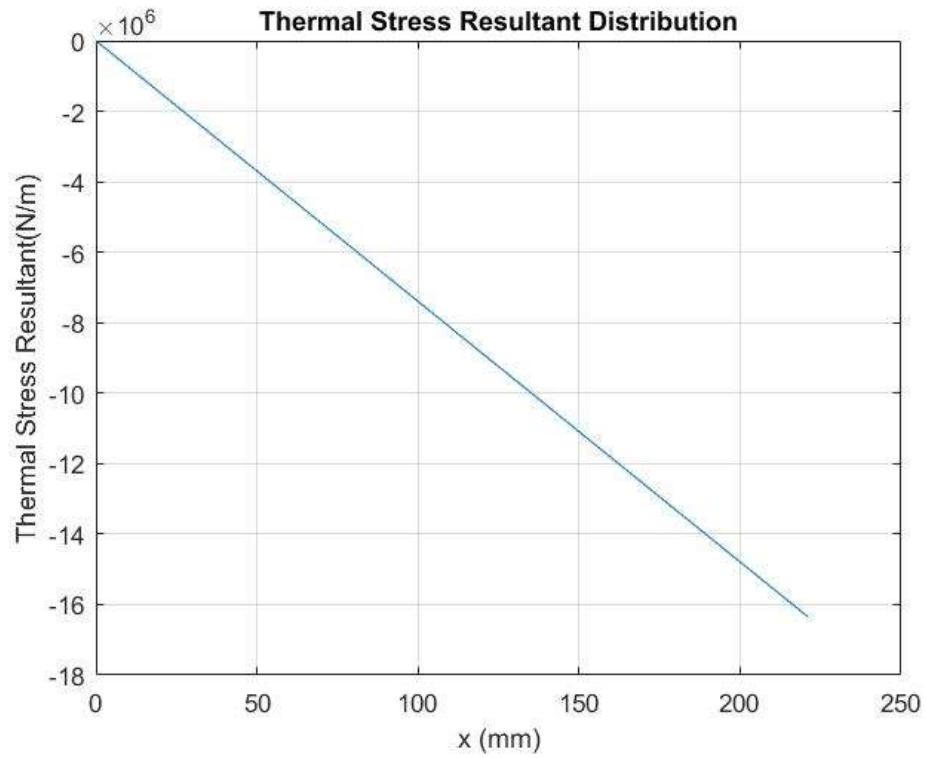


Figure 6.36 : Thermal stress resultant distribution for step 3.

Transverse Shear Stress (τ_{xz}) of the shell is illustrated in Figure 6.37.

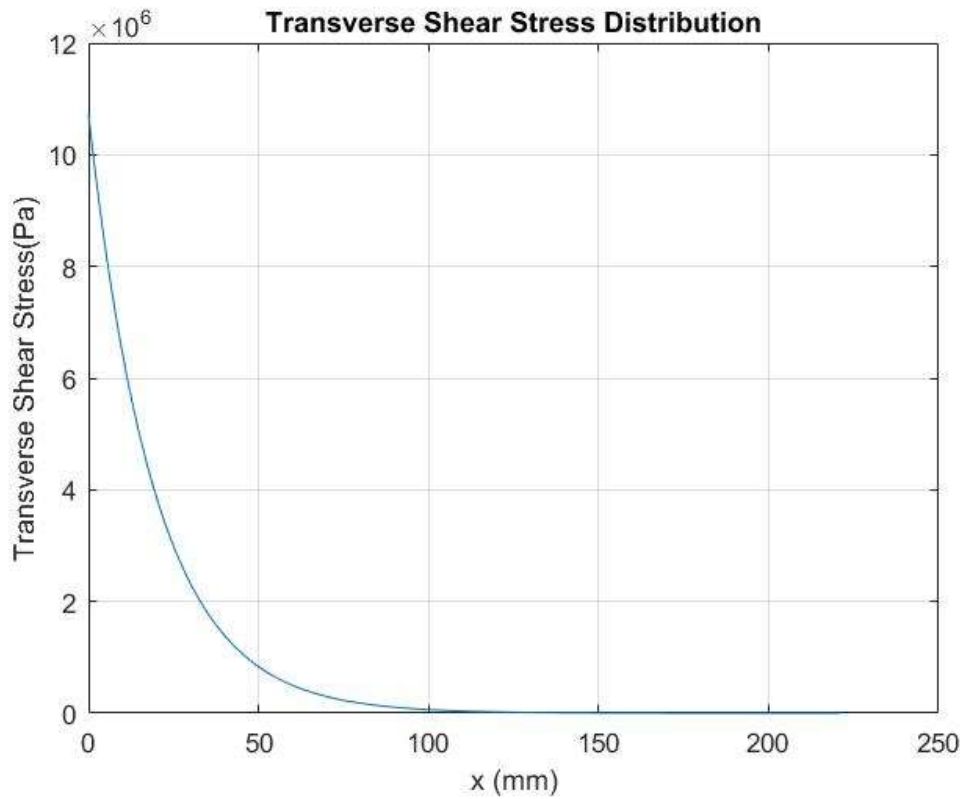


Figure 6.37 : Transverse shear stress distribution for step 3.

6.3 Comparison of Results and Recommendations

Maximum deformation and maximum hoop stress values which are obtained from analytical calculations and ANSYS FEM model for three different conditions are compared each other and it may be seen in Table 6.1.

Table 6.1 : Comparison of results.

	Max. Value	Analytical Solution	ANSYS	Error%
Step 1	Deformation	5.85 mm	6.54 mm	~10.5%
	Hoop Stress	29.91 MPa	31.795 MPa	~5.9%
Step 2	Deformation	5.51 mm	6.1 mm	~9.7%
	Hoop Stress	192.5 MPa	191.71 MPa	~0.4%
Step 3	Deformation	5.51 mm	6.1 mm	~9.7%
	Hoop Stress	192.5 MPa	190.03	~1.3%

Results are mostly similar and according to tensile and shear yield strength of LOX Tank's material which is Aluminum 6061-T6 at 90 °K ($S_y = 324 \text{ MPa}$ at 90 °K), it can be said that the tank is safe and fully operational.

If the displacement distributions for Step 1, Step 2 and Step 3 are examined (Figure 6.11, 6.20 and 6.29 respectively), it may be seen that the maximum deformation is caused by the thermal effects. Because of the material is cooled to cryogenic temperatures, the tank tends to contract. Therefore, when an inner pressure is added to the system, the materials deformation decreases slightly. Also, it may be interpreted that the thrust loading on the tank caused by the flight operation of the rocket has no effect on deformation. An engineer/designer should consider the thermal effects as first priority if the pressure vessel is going to be filled with cryogenic liquids when the deformation of the tank is more critical.

If the axial stress distributions for Step 1, Step 2 and Step 3 are examined (Figure 6.12, 6.21 and 6.30 respectively), it may be seen that in the situation of only thermal effects is effective on the tank, the maximum axial stress is around 139.1 MPa. In the same manner, for Step 2, when tank pressure is added to thermal effects, the maximum axial stress remains the same, 139.1 MPa. However, when thrust loading is taken into account, the maximum axial stress reaches to 141.2 MPa. This indicates that the thrust of the rocket's engine increases the liquid oxygen tank's axial stress. For all of the three steps, maximum axial stress occurs LOX-GOX interface. An engineer/designer should make sure that the amount of thrust which causes an increment of the axial stress of the tank must be in the limit of the pressure vessel's tensile yielding stress in accordance with the selected safety factor.

If the hoop stress distributions for Step 1, Step 2 and Step 3 are examined (Figure 6.13, 6.22 and 6.31 respectively), it may be seen that for Step 1 at where the only thermal loads are in action, maximum hoop stress is observed around LOX-GOX interface (-29.91 MPa). However, after the tank is pressurized hoop stress increases dramatically. Since the cooling of the tank provides contraction and pressurization works the opposite direction, hoop stress value reaches its maximum (+192.5 MPa) at the upper level of the GOX section. Thrust loading caused by the rocket's engine has no effect on hoop stress. An engineer/designer should be careful when designing a cryogenic pressure vessel since the cryogenic liquid level is crucial when it is pressurized.

Axial bending stress resultant distributions for Step 1, Step 2 and Step 3 (Figure 6.14, 6.23 and 6.32 respectively) and polar bending stress resultant distribution for Step 1, Step 2 and Step 3 (Figure 6.15, 6.24 and 6.33 respectively) are the same for all the cases since the moment loads does not change and the bending stress resultants represent the moment per unit length.

The polar membrane stress resultant distributions for Step 1, Step 2 and Step 3 (Figure 6.16, 6.25 and 6.34 respectively) is related with the hoop stress distribution and the polar bending stress resultant distribution. Since the bending stress resultant distributions in polar direction are same for all three steps, the hoop stress distribution's interpretation is valid for the polar membrane stress resultant distribution. An engineer/designer should clarify if the moment loads have a variation through the pressure vessel, if not, hoop stress interpretation might be adequate.

If the shear stress resultant distributions for Step 1, Step 2 and Step 3 (Figure 6.17, 6.26 and 6.35 respectively) and transverse shear stress distribution for Step 1, Step 2 and Step 3 (Figure 6.19, 6.28 and 6.37 respectively) are examined, it may be seen that when the only thermal loads are in action, shear stresses increase for a short period of time first then it decreases again. However, when the tank is pressurized, shear stress reaches its maximum value at LOX-GOX interface and it is equal to 10.73 MPa. Shear stress decreases down to 111.8 Pa through the edge of the cylindrical shell. During flight, thrust loading has a negligible amount of increment on the shear stress. While at the interface shear stress is 10.77 MPa and through the edge of the cylindrical shell it reaches 112.5 Pa. For all three stages of the analysis for a cylindrical pressure vessel, compared to hoop and axial stress, shear stress does not have a considerable amount of significance.

If the thermal stress resultant distribution for Step 1, Step 2, and Step 3 (Figure 6.18, 6.27 and 6.36 respectively) are examined, it may be seen that distributions are the same for all three analysis. Since the thermal stress is based on the temperature variation only and the thermal stress resultant represents the thermal stress per unit length, the same behavior is observed for all three cases. Temperature variation of the tank is given in Figure 4.2. It may also be observed from Figure 4.2 that the thermal stress resultant increases directly proportional with the temperature change.

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APPENDIX

MATLAB Code for Step 1:

```
clear all;
clc ;

r_in = 135e-3 ; %inner radius (m)
d_in = r_in*2; %inner diameter (m)
r_out = 140e-3 ; %outer radius (m)
d_out = r_out*2 ; %outer diameter (m)
a = 0.5*(r_in + r_out); %mean radius (m)
h = 5e-3; %thickness of the shell (m)
b = 221e-3; % GOX level (m)

T0 = 90; % initial temperature in K
T1 = 203; % final temperature in K
delta_T = T1-T0; % change of temperature (K)

%Modulus of Elasticity equation of Al6061-T6 as a function of temperature
k = 7.771221e1; % modulus of elasticity constant
l = 1.030646e-2; % modulus of elasticity constant
m = -2.924100e-4; % modulus of elasticity constant
n = 8.993600e-7; % modulus of elasticity constant
o = -1.070900e-9; % modulus of elasticity constant
% Modulus of elasticity (Pa)
E = (k + l*T0 + m*T0^2 + n*T0^3 + o*T0^4)*1e9;

Mu = 0.33; %Poisson's Ratio of Al6061-T6
D = (E*h^3)/(12*(1-Mu^2)); % flexural rigidity (N.m)
%Linear Thermal Expansion Coefficient of Al6061-T6 at 90 K (1/K)
alfa = -376.4e-6 ;
beta = ((3*(1-Mu^2))^0.25)/sqrt(a*h); % m^-1
N_0 = alfa*E*delta_T*h;
P_r = 0;
thrust = 0; % N
thrust_area = pi*((140e-3)^2-(127e-3)^2); %m^2
N_x = (thrust/thrust_area)*h;
M_T = 0;
derivative_of_M_T = 0;

%Initial solution matrices are generated
displacement = zeros(222,1); %Displacement Matrix
```

```

derivative_of_displacement = zeros(222,1);      %Derivative of Displacement
Matrix
second_derivative_of_displacement = zeros(222,1); %Second Derivative of
Displacement Matrix
third_derivative_of_displacement = zeros(222,1); %Third Derivative of
Displacement Matrix
M_x = zeros(222,1);
%Bending Stress Resultant Matrix
N_T = zeros(222,1);
%Thermal Stress Resultant Matrix in x-direction
sigma_x_upper = zeros(222,1);
%Axial Stress Matrix
sigma_x_lower = zeros(222,1);
%Axial Stress Matrix
N_theta = zeros(222,1);
%Polar Membrane Stress Resultant Matrix
M_theta = zeros(222,1);
%Bending Stress Resultant Matrix in Polar Direction
sigma_theta_upper = zeros(222,1);
%Hoop Stress Matrix
sigma_theta_lower = zeros(222,1);
%Hoop Stress Matrix
Q_x = zeros(222,1);
%Shear Stress Resultant Matrix
tau_xz = zeros(222,1); %Transverse Shear Stress Matrix
length = zeros(222,1);
% x=0 to x=221mm

%Integration Constants
C1=N_0*a/(4*E*h*beta*b);
C2= -C1;
C3=0;
C4=0;

for i=0:221

    length(i+1,1) = i;

end

for x=0:221

displacement(x+1,1) = exp(-beta*x*1e-3)*(C1*cos((beta*x*1e-3)*(pi/180)) +
C2*sin((beta*x*1e-3)*(pi/180))) + (N_0*a*x*1e-3)/(E*h*b);
derivative_of_displacement(x+1,1) = (-beta*exp(-beta*x*1e-3))*((C1-
C2)*cos((beta*x*1e-3)*(pi/180)) + (C1+C2)*sin((beta*x*1e-3)*(pi/180))) +
((N_0*a)/(E*h*b));
second_derivative_of_displacement(x+1,1) = 2*(beta^2)*(exp(-beta*x*1e-
3))*(C1*sin((beta*x*1e-3)*(pi/180))-C2*cos((beta*x*1e-3)*(pi/180)));

```

```
third_derivative_of_displacement(x+1,1) = 2*(beta^3)*exp(-beta*x*1e-3)*((C1+C2)*cos((beta*x*1e-3)*(pi/180)) + (C2-C1)*sin((beta*x*1e-3)*(pi/180)));
```

```
N_T(x+1,1) = N_0*(x*1e-3/b);
```

```
N_theta(x+1,1) = ((E*h*displacement(x+1,1))/a) - (N_0*(x*1e-3)/b) + (Mu* N_x);
```

```
M_x(x+1,1) = -D*second_derivative_of_displacement(x+1,1)-(M_T/(1-Mu));
```

```
M_theta(x+1,1) = Mu*M_x(x+1,1)-((M_T)/(1-Mu));
```

```
sigma_x_upper(x+1,1) = N_x/h + 6*M_x(x+1,1)/h^2;
```

```
sigma_x_lower(x+1,1) = N_x/h - 6*M_x(x+1,1)/h^2;
```

```
sigma_theta_upper(x+1,1) = (N_theta(x+1,1))/h + (6* M_theta(x+1,1))/h^2;
```

```
sigma_theta_lower(x+1,1) = (N_theta(x+1,1))/h - (6* M_theta(x+1,1))/h^2;
```

```
Q_x(x+1,1) = -D*third_derivative_of_displacement(x+1,1) - (1/(1-Mu))*derivative_of_M_T;
```

```
tau_xz(x+1,1) = (3*Q_x(x+1,1))/(2*h);
```

```
end
```

```
figure(1)
```

```
plot(length,displacement);
```

```
xlabel('x (mm)');
```

```
ylabel('displacement(mm)');
```

```
title('Displacement Distribution');
```

```
grid on
```

```
figure(2)
```

```
plot(length,sigma_theta_upper);
```

```
xlabel('x (mm)');
```

```
ylabel('Hoop Stress (Pa)');
```

```
title('Hoop Stress Distribution');
```

```
grid on
```

```
figure(3)
```

```
plot(length,sigma_theta_lower);
```

```
xlabel('x (mm)');
```

```
ylabel('Hoop Stress (Pa)');
```

```
title('Hoop Stress Distribution')
```

```
grid on
```

```
figure(5)
```

```
plot(length,sigma_x_upper);
```

```
xlabel('x (mm)');
```

```
ylabel('Axial Stress(Pa)')
```

```
title('Axial Stress Distribution');
```

```
grid on
```

```
figure(6)
```

```

plot(length,sigma_x_lower);
xlabel('x (mm)');
ylabel('Axial Stress(Pa)');
title('Axial Stress Distribution');
grid on

```

```

figure(7)
plot(length,N_T);
xlabel('x (mm)');
ylabel('Thermal Stress Resultant(N/m)');
title('Thermal Stress Resultant Distribution');
grid on

```

```

figure(8)
plot(length,N_theta);
xlabel('x (mm)');
ylabel('Polar Membrane Stress Resultant(N/m)');
title('Polar Membrane Stress Resultant Distribution');
grid on

```

```

figure(9)
plot(length,M_x);
xlabel('x (mm)');
ylabel('Axial Bending Stress Resultant(N)');
title('Axial Bending Stress Resultant Distribution');
grid on

```

```

figure(10)
plot(length,M_theta);
xlabel('x (mm)');
ylabel('Bending Stress Resultant Matrix in Polar Direction(N)');
title('Polar Bending Stress Resultant Distribution');
grid on

```

```

figure(11)
plot(length,Q_x);
xlabel('x (mm)');
ylabel('Shear Stress Resultant(N/m)');
title('Shear Stress Resultant Distribution');
grid on

```

```

figure(12)
plot(length,tau_xz);
xlabel('x (mm)');
ylabel('Transverse Shear Stress(Pa)');
title('Transverse Shear Stress Distribution');
grid on

```

MATLAB Code for Step 2:

```
clear all;
clc ;

r_in = 135e-3 ; %inner radius (m)
d_in = r_in*2; %inner diameter (m)
r_out = 140e-3 ; %outer radius (m)
d_out = r_out*2 ; %outer diameter (m)
a = 0.5*(r_in + r_out); %mean radius (m)
h = 5e-3; %thickness of the shell (m)
b = 221e-3; % GOX level (m)

T0 = 90; % initial temperature in K
T1 = 203; % final temperature in K
delta_T = T1-T0; % change of temperature (K)

%Modulus of Elasticity equation of Al6061-T6 as a function of temperature
k = 7.771221e1; % modulus of elasticity constant
l = 1.030646e-2; % modulus of elasticity constant
m = -2.924100e-4; % modulus of elasticity constant
n = 8.993600e-7; % modulus of elasticity constant
o = -1.070900e-9; % modulus of elasticity constant
E = (k + l*T0 + m*T0^2 + n*T0^3 + o*T0^4)*1e9;
% Modulus of elasticity (Pa)

Mu = 0.33; %Poisson's Ratio (unitless)
D = (E*h^3)/(12*(1-Mu^2));
alfa = -376.4e-6 ;
%Linear Thermal Expansion Coefficient (K^-1)
beta = ((3*(1-Mu^2))^0.25)/sqrt(a*h); % m^-1
N_0 = alfa*E*delta_T*h;
P_r = 7e6; %tank pressure in Pa
thrust = 0; % N
thrust_area = pi*((140e-3)^2-(127e-3)^2); %m^2
N_x = (thrust/thrust_area)*h;
M_T = 0;
derivative_of_M_T = 0;

%Initial solution matrices are generated
displacement = zeros(222,1); %Displacement Matrix
derivative_of_displacement = zeros(222,1); %Derivative of Displacement Matrix
second_derivative_of_displacement = zeros(222,1); %Second Derivative of Displacement Matrix
third_derivative_of_displacement = zeros(222,1); %Third Derivative of Displacement Matrix
M_x = zeros(222,1);
%Bending Stress Resultant Matrix
N_T = zeros(222,1);
```

```

%Thermal Stress Resultant Matrix in x-direction
sigma_x_upper = zeros(222,1);
%Axial Stress Matrix
sigma_x_lower = zeros(222,1);
%Axial Stress Matrix
N_theta = zeros(222,1);
%Polar Membrane Stress Resultant Matrix
M_theta = zeros(222,1);
%Bending Stress Resultant Matrix in Polar Direction
sigma_theta_upper = zeros(222,1);
%Hoop Stress Matrix
sigma_theta_lower = zeros(222,1);
%Hoop Stress Matrix
Q_x = zeros(222,1);
%Shear Stress Resultant Matrix
tau_xz = zeros(222,1); %Transverse Shear Stress Matrix
length = zeros(222,1);
% x=0 to x=221mm m

%Integration Constants
C1 = ((N_0*a)/(4*E*h*beta*b))-((P_r*a^2)/(2*E*h));
C2 = -C1-((P_r*a^2)/(2*E*h));
C3 = 0;
C4 = 0;

for i=0:221

    length(i+1,1) = i;

end

for x = 0:221

displacement(x+1,1) = exp(-beta*x*1e-3)*(C1*cos((beta*x*1e-3)*(pi/180))+C2*sin((beta*x*1e-3)*(pi/180)))+((N_0*a*x*1e-3)/(E*h*b))+
((P_r*a^2)/(E*h)) ;
derivative_of_displacement(x+1,1) = -beta*exp(-beta*x*1e-3)* ((C1-
C2)*cos((beta*x*1e-3)*(pi/180))+(C1+C2)*sin((beta*x*1e-3)*(pi/180)))+(N_0*a)/(E*h*b);
second_derivative_of_displacement(x+1,1) = -2*(beta^2)*exp(-beta*x*1e-3)*(C2*cos((beta*x*1e-3)*(pi/180))-C1*sin((beta*x*1e-3)*(pi/180)));
third_derivative_of_displacement(x+1,1)= 2*(beta^3)*exp(-beta*x*1e-3)*((C1+C2)*cos((beta*x*1e-3)*(pi/180)) + (C2-C1)*sin((beta*x*1e-3)*(pi/180)));

N_T(x+1,1) = N_0*(x*1e-3/b);
N_theta(x+1,1) = ((E*h*displacement(x+1,1))/a) - (N_0*(x*1e-3)/b) + (Mu* N_x);

M_x(x+1,1) = -D*second_derivative_of_displacement(x+1,1)-(M_T/(1-Mu));
M_theta(x+1,1) = Mu*M_x(x+1,1)-(M_T/(1-Mu));

```

```

sigma_x_upper(x+1,1) = N_x/h + 6*M_x(x+1,1)/h^2;
sigma_x_lower(x+1,1) = N_x/h - 6*M_x(x+1,1)/h^2;

sigma_theta_upper(x+1,1) = (N_theta(x+1,1))/h + (6* M_theta(x+1,1))/h^2;
sigma_theta_lower(x+1,1) = (N_theta(x+1,1))/h - (6* M_theta(x+1,1))/h^2;

Q_x(x+1,1) = -D*third_derivative_of_displacement(x+1,1) - (1/(1-
Mu))*derivative_of_M_T;
tau_xz(x+1,1) = (3*Q_x(x+1,1))/(2*h);

```

end

```

figure(1)
plot(length,displacement);
xlabel('x (mm)');
ylabel('displacement(mm)');
title('Displacement Distribution');
grid on

```

```

figure(2)
plot(length,sigma_theta_upper);
xlabel('x (mm)');
ylabel('Hoop Stress (Pa)');
title('Hoop Stress Distribution');
grid on

```

```

figure(3)
plot(length,sigma_theta_lower);
xlabel('x (mm)');
ylabel('Hoop Stress (Pa)');
title('Hoop Stress Distribution')
grid on

```

```

figure(5)
plot(length,sigma_x_upper);
xlabel('x (mm)');
ylabel('Axial Stress(Pa)')
title('Axial Stress Distribution');
grid on

```

```

figure(6)
plot(length,sigma_x_lower);
xlabel('x (mm)');
ylabel('Axial Stress(Pa)');
title('Axial Stress Distribution');
grid on

```

```

figure(7)
plot(length,N_T);
xlabel('x (mm)');

```

```
ylabel('Thermal Stress Resultant(N/m)');
title('Thermal Stress Resultant Distribution');
grid on
```

```
figure(8)
plot(length,N_theta);
xlabel('x (mm)');
ylabel('Polar Membrane Stress Resultant(N/m)');
title('Polar Membrane Stress Resultant Distribution');
grid on
```

```
figure(9)
plot(length,M_x);
xlabel('x (mm)');
ylabel('Axial Bending Stress Resultant(N)');
title('Axial Bending Stress Resultant Distribution');
grid on
```

```
figure(10)
plot(length,M_theta);
xlabel('x (mm)');
ylabel('Bending Stress Resultant Matrix in Polar Direction(N)');
title('Polar Bending Stress Resultant Distribution');
grid on
```

```
figure(11)
plot(length,Q_x);
xlabel('x (mm)');
ylabel('Shear Stress Resultant(N/m)');
title('Shear Stress Resultant Distribution');
grid on
```

```
figure(12)
plot(length,tau_xz);
xlabel('x (mm)');
ylabel('Transverse Shear Stress(Pa)');
title('Transverse Shear Stress Distribution');
grid on
```

MATLAB Code for Step 3:

```
clear all;
clc ;

r_in = 135e-3 ; %inner radius (m)
d_in = r_in*2; %inner diameter (m)
r_out = 140e-3 ; %outer radius (m)
d_out = r_out*2 ; %outer diameter (m)
a = 0.5*(r_in + r_out); %mean radius (m)
h = 5e-3; %thickness of the shell (m)
b = 221e-3; % GOX level (m)

T0 = 90; % initial temperature in K
T1 = 203; % final temperature in K
delta_T = T1-T0; % change of temperature (K)

%Modulus of Elasticity equation of Al6061-T6 as a function of temperature
k = 7.771221e1; % modulus of elasticity constant
l = 1.030646e-2; % modulus of elasticity constant
m = -2.924100e-4; % modulus of elasticity constant
n = 8.993600e-7; % modulus of elasticity constant
o = -1.070900e-9; % modulus of elasticity constant
E = (k + l*T0 + m*T0^2 + n*T0^3 + o*T0^4)*1e9;
% Modulus of elasticity (Pa)

Mu = 0.33; %Poisson's Ratio (unitless)
D = (E*h^3)/(12*(1-Mu^2));
alfa = -376.4e-6 ;
%Linear Thermal Expansion Coefficient (K^-1)
beta = ((3*(1-Mu^2))^0.25)/sqrt(a*h); % m^-1
N_0 = alfa*E*delta_T*h;
P_r = 7e6; %tank pressure in Pa
thrust = 22e3; % N
thrust_area = pi*((140e-3)^2-(127e-3)^2); %m^2
N_x = (thrust/thrust_area)*h;
M_T = 0;
derivative_of_M_T = 0;

%Initial solution matrices are generated
displacement = zeros(222,1); %Displacement Matrix
derivative_of_displacement = zeros(222,1); %Derivative of Displacement Matrix
second_derivative_of_displacement = zeros(222,1); %Second Derivative of Displacement Matrix
third_derivative_of_displacement = zeros(222,1); %Third Derivative of Displacement Matrix
M_x = zeros(222,1);
%Bending Stress Resultant Matrix
```

```

N_T = zeros(222,1);
%Thermal Stress Resultant Matrix in x-direction
sigma_x_upper = zeros(222,1);
%Axial Stress Matrix
sigma_x_lower = zeros(222,1);
%Axial Stress Matrix
N_theta = zeros(222,1);
%Polar Membrane Stress Resultant Matrix
M_theta = zeros(222,1);
%Bending Stress Resultant Matrix in Polar Direction
sigma_theta_upper = zeros(222,1);
%Hoop Stress Matrix
sigma_theta_lower = zeros(222,1);
%Hoop Stress Matrix
Q_x = zeros(222,1);
%Shear Stress Resultant Matrix
tau_xz = zeros(222,1); %Transverse Shear Stress Matrix
length = zeros(222,1);
% x=0 to x=221mm

%Integration Constants
C1 = (((Mu*N_x*a)-(P_r*a^2))/(2*E*h))+((N_0*a)/(4*E*h*beta*b));
C2 = -(N_0*a)/(4*E*h*beta*b);
C3 = 0;
C4 = 0;

for i=0:221

    length(i+1,1) = i;

end

for x = 0:221

    displacement(x+1,1) = exp(-beta*x*1e-3)*(C1*cos((beta*x*1e-3)*(pi/180))+C2*sin((beta*x*1e-3)*(pi/180)))+((N_0*a*x*1e-3)/(E*h*b))+((P_r*a^2)/(E*h))-((Mu*N_x*a)/(E*h));
    derivative_of_displacement(x+1,1) = -beta*exp(-beta*x*1e-3)*((C1-C2)*cos((beta*x*1e-3)*(pi/180))+(C1+C2)*sin((beta*x*1e-3)*(pi/180)))+(N_0*a)/(E*h*b);
    second_derivative_of_displacement(x+1,1) = -2*(beta^2)*exp(-beta*x*1e-3)*(C2*cos((beta*x*1e-3)*(pi/180))-C1*sin((beta*x*1e-3)*(pi/180)));
    third_derivative_of_displacement(x+1,1) = 2*(beta^3)*exp(-beta*x*1e-3)*((C1+C2)*cos((beta*x*1e-3)*(pi/180)) + (C2-C1)*sin((beta*x*1e-3)*(pi/180)));

    N_T(x+1,1) = N_0*(x*1e-3/b);
    N_theta(x+1,1) = ((E*h*displacement(x+1,1))/a) - (N_0*(x*1e-3)/b) + (Mu*N_x);

    M_x(x+1,1) = -D*second_derivative_of_displacement(x+1,1)-(M_T/(1-Mu));

```

```

M_theta(x+1,1) = Mu*M_x(x+1,1)-(M_T/(1-Mu));

sigma_x_upper(x+1,1) = N_x/h + 6*M_x(x+1,1)/h^2;
sigma_x_lower(x+1,1) = N_x/h - 6*M_x(x+1,1)/h^2;

sigma_theta_upper(x+1,1) = (N_theta(x+1,1))/h + (6* M_theta(x+1,1))/h^2;
sigma_theta_lower(x+1,1) = (N_theta(x+1,1))/h - (6* M_theta(x+1,1))/h^2;

Q_x(x+1,1) = -D*third_derivative_of_displacement(x+1,1) - (1/(1-
Mu))*derivative_of_M_T;
tau_xz(x+1,1) = (3*Q_x(x+1,1))/(2*h);

```

end

```

figure(1)
plot(length,displacement);
xlabel('x (mm)');
ylabel('displacement(mm)');
title('Displacement Distribution');
grid on

```

```

figure(2)
plot(length,sigma_theta_upper);
xlabel('x (mm)');
ylabel('Hoop Stress (Pa)');
title('Hoop Stress Distribution');
grid on

```

```

figure(3)
plot(length,sigma_theta_lower);
xlabel('x (mm)');
ylabel('Hoop Stress (Pa)');
title('Hoop Stress Distribution')
grid on

```

```

figure(5)
plot(length,sigma_x_upper);
xlabel('x (mm)');
ylabel('Axial Stress(Pa)');
title('Axial Stress Distribution');
grid on

```

```

figure(6)
plot(length,sigma_x_lower);
xlabel('x (mm)');
ylabel('Axial Stress(Pa)');
title('Axial Stress Distribution');
grid on

```

```

figure(7)

```

```

plot(length,N_T);
xlabel('x (mm)');
ylabel('Thermal Stress Resultant(N/m)');
title('Thermal Stress Resultant Distribution');
grid on

```

```

figure(8)
plot(length,N_theta);
xlabel('x (mm)');
ylabel('Polar Membrane Stress Resultant(N/m)');
title('Polar Membrane Stress Resultant Distribution');
grid on

```

```

figure(9)
plot(length,M_x);
xlabel('x (mm)');
ylabel('Axial Bending Stress Resultant(N)');
title('Axial Bending Stress Resultant Distribution');
grid on

```

```

figure(10)
plot(length,M_theta);
xlabel('x (mm)');
ylabel('Bending Stress Resultant Matrix in Polar Direction(N)');
title('Polar Bending Stress Resultant Distribution');
grid on

```

```

figure(11)
plot(length,Q_x);
xlabel('x (mm)');
ylabel('Shear Stress Resultant(N/m)');
title('Shear Stress Resultant Distribution');
grid on

```

```

figure(12)
plot(length,tau_xz);
xlabel('x (mm)');
ylabel('Transverse Shear Stress(Pa)');
title('Transverse Shear Stress Distribution');
grid on

```

CURRICULUM VITAE



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- April 2016 – April 2017 Aircraft System Engineer at Genel Havacılık A.Ş.