

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL

CONSTRAINT CONTROL OF ROBOTIC MANIPULATORS



SAMET GÜL

A THESIS OF DOCTORATE
DEPARTMENT OF COMPUTER ENGINEERING

ADVISOR: PROF. ERKAN ZERGEROĞLU

NOVEMBER 2024

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GEBZE TEKNİK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

ROBOT MANİPÜLATÖRLERİN KISIT KONTROLÜ

SAMET GÜL

DOKTORA TEZİ
BİLGİSAYAR MÜHENDİSLİĞİ ANABİLİM DALI

DANIŞMAN: PROF. DR. ERKAN ZERGEROĞLU

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JURY

MEMBER

(THESIS ADVISOR) : Prof. Erkan ZERGEROĞLU

MEMBER

: Assoc. Prof. Janset DAŞDEMİR

MEMBER

: Assoc. Prof. Habil KALKAN

MEMBER

: Prof. Şeref Naci ENGİN

MEMBER

: Asst. Prof. Serhat OBUZ

APPROVAL

Gebze Technical University Graduate Administration Board

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ABSTRACT

Constraints play a crucial role in the development of physical systems, being fundamental to their functionality and design. Especially in cases where direct manipulation in the states of a system is not possible, at least keeping these system states within safe and functional limits is necessary to prevent potential loss of life and property. Integrating barrier Lyapunov functions (BLF) into controller design for nonlinear systems is a significant development addressing this need. This thesis focuses on integrating BLF in nonlinear controller designs for robotic manipulators. The general objective of the studies in the thesis is to effectively perform trajectory tracking of robot manipulators under various operational conditions while applying constraints on certain system states. In line with this purpose, the thesis proposes three different control strategies to enhance robot manipulators' operational efficiency and safety. Our first study introduces two innovative full-state feedback desired model-based, joint position-constrained robot controller designs developed using BLFs. These constraint controllers allow effective tracking control despite dynamic uncertainties in robot dynamics while reliably limiting position-tracking errors within predetermined bounds. Our second study presents output feedback controller designs with joint position tracking error constraints for robot manipulators with uncertain dynamic parameters. The proposed controllers maintain the joint position tracking error within predetermined bounds and ensure its convergence to zero even without velocity measurements. Our last study presents a tracking control strategy with task space tracking error constraints for kinematically redundant robot manipulators. This controller design has a robust component to deal with parametric uncertainties and external disturbances, as well as a BLF component to limit tracking errors. The constraint imposed on the task space tracking error term ensures the manipulator's end effector has a limited behavior and predictable transient performance. Throughout the thesis, extensive simulation studies and experimental validations are presented to demonstrate the effectiveness and performance of each proposed feedback controller design.

Keywords: Robotic Manipulator, Constraint Control, Lyapunov-Based Approach, Barrier Lyapunov Function

ÖZET

Kısıtlar, fiziksel sistemlerin geliştirilmeleri ve etkili bir şekilde çalışmaları için hayati bir öneme sahiptir. Özellikle bir sistemin durumlarına doğrudan müdahale edilmesinin mümkün olmadığı durumlarda en azından bu sistem durumlarının güvenli ve işlevsel sınırlar içinde tutulması can ve mal kaybının önüne geçmek için bir gerekliliktir. Bu ihtiyacı ele alan önemli bir gelişme, doğrusal olmayan sistemler için kontrolcü tasarımına bariyer Lyapunov fonksiyonlarının (BLF) entegre edilmesidir. Bu tez de, özellikle robot manipülatörler için doğrusal olmayan kontrolcü tasarımlarında BLF'nın entegrasyonunu ele almaktadır. Tezdeki çalışmaların genel hedefi, BLF'nı kullanarak geliştirilen kontrolcülerle çeşitli operasyonel koşullar altında robot manipülatörlerin etkili bir şekilde yörünge takibi gerçekleştirmesidir. Bu amaç doğrultusunda, robot manipülatörlerin operasyonel etkinliğini ve güvenliğini artırmayı hedefleyen üç farklı kontrol stratejisi önerilmektedir. İlk çalışmamızda, BLF kullanılarak geliştirilen iki özgün tam durum geri beslemeli, istenen model tabanlı ve eklem pozisyonu kısıtlı robot kontrolcü tasarımları tanıtılmaktadır. Bu kısıt kontrolcüler, robot dinamiğindeki belirsizliklere rağmen, etkili bir takip kontrolüne izin verirken aynı zamanda pozisyon takip hatalarını önceden belirlenmiş sınırlar içinde güvenilir bir şekilde sınırlandırmaktadır. İkinci çalışmamız, belirsiz dinamik parametrelere sahip robot manipülatörler için eklem pozisyonu takip hatası kısıtlı çıkış geri beslemeli kontrolcü tasarımları sunmaktadır. Önerilen kontrolcüler, eklem pozisyonu takip hatasını önceden belirlenmiş sınırlar içinde tutmayı ve hız ölçümleri olmamasına rağmen sifıra yakınsamasını sağlamaktadır. Son çalışmamız ise, kinematik olarak fazlalıklı robot manipülatörler için görev alanı takip hatası kısıtlı bir takip kontrol stratejisi sunmaktadır. Bu kontrolcü tasarımı parametrik belirsizlikleri ve dış bozucu etkileri dengelemek için bir gürbüz kontrolcü bileşeni ile takip hatasını kısıtlamak için bir BLF bileşeni bulunmaktadır. Kontrolcü sayesinde görev alanı takip hatası terimine uygulanan kısıt, manipülatörün uç efektörünün sınırlandırılmış bir davranışa ve tahmin edilebilir bir geçici performansa sahip olmasına imkan vermektedir. Tez boyunca, önerilen her bir geri besleme kontrolcü tasarımcısının etkinliğini ve performansını göstermek için kapsamlı simülasyon çalışmaları ve deneysel doğrulamalar sunulmuştur.

Anahtar Kelimeler: Robot Manipülatör, Kısıt Kontrolü, Lyapunov Tabanlı Yaklaşım, Bariyer Lyapunov Fonksiyonu

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LIST of ABBREVIATIONS and ACRONYMS

GTU	: Gebze Technical University
BLF	: Barrier Lyapunov Function
DOF	: Degrees of Freedom
n	: Joint Space Dimension
m	: Task Space Dimension
q	: Joint Position
q_d	: Desired Joint Position
$M(q)$	: Inertia Matrix
$C(q, \dot{q})$	: Centripetal-Coriolis Matrix
$G(q)$	: Gravitational Forces
F_d	: Frictional Effects
$\tau(t)$	: Torque Input
$Y(q, \dot{q}, \ddot{q})$	: Regression Matrix of Known Variables
θ	: Uncertain Constant System Parameters
$\xi_d(t)$	: Unknown Bounded Additive Disturbances
$W(q, \dot{q}, \ddot{q})$	: Regression Matrix of Known Variables
$x(t)$	: Task Space Position of the End Effector
$f(q)$	: Forward Kinematics
$J(q)$	: Jacobian of the Manipulator
J^+	: Pseudo-Inverse of the Jacobian
$e(t)$	: Position Tracking Error
Δ	: Predetermined bound on Position Error
$r(t)$	: Filtered Tracking Error
α	: Positive Definite Diagonal Control Gain Matrix
χ	: Uncertain Vector
K_e	: Error Dependent Control Gain Matrix
$\hat{\theta}(t)$	: Parameter Estimation Vector
Γ	: Adaptation Gain Matrix
$\tilde{\theta}(t)$	: Parameter Estimation Error
V_l	: Logarithmic Barrier Lyapunov Function
V_t	: Trigonometric Barrier Lyapunov Function
$e_f(t)$	: Surrogate Velocity Signal
$\eta(t)$	: Auxiliary Variable
$V(\cdot)$	: Barrier Lyapunov Function
$e_N(t)$	: Sub-Task Tracking Error
$g(t)$	: Auxiliar Joint Velocity Vector
$\rho(\cdot)$	: A Positive Function
ε	: Small Positive Scalar Constant
ζ_θ	: A Known Positive Constant

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1. INTRODUCTION

1.1. Motivation

A fundamental aspect of control theory involves analyzing dynamic systems and developing control algorithms. This research field can be broadly categorized into two branches: linear control theory, which applies to systems represented by linear differential equations, and nonlinear control theory. Although linear control theory has a rich history and an array of advanced methods, it faces limitations when dealing with many physical systems that exhibit nonlinear characteristics. These systems often require linearization around a specific operating point before applying linear control methods. In current industrial practices, linear control techniques are widely employed and generally sufficient. However, the linearization process may not be suitable for specific applications that demand wide operating ranges or higher performance, as it can result in the loss of some system information and behavior. In such instances, nonlinear control theory, which forms the other major branch of control theory, is utilized to achieve the desired outcomes and acceptable performance levels. Over time, there have been significant developments in nonlinear control theory, leading to the proposal of various controllers for systems with nonlinear behaviors, such as high-order nonlinearities or discontinuities.

The mathematical representation of systems, typically through differential equations, is fundamental for analysis and controller design. These physics-based models describe the relationship between the system states, their derivatives, and the control inputs. Although these models are valuable tools based on a solid foundation of physics and reasoning, developing a sufficiently realistic model has its challenges. These models often rely on assumptions that may not always hold in real-world scenarios, such as frictionless joints, negligible forces at low velocities, dead zones, or external disturbances. These complexities that are hard to precisely model are known as non-parametric uncertainties. Additionally, system parameters may change depending on operating conditions and may not be known precisely or differ over time in some systems. Such uncertainties are called parametric uncertainties. Various nonlinear control strategies have been researched and developed in response to these challenges over the past four decades. Among these

strategies, Lyapunov's method stands out as a particularly effective technique. It is widely used for analyzing the stability of systems with uncertain nonlinear dynamics and designing controllers that can maintain stability under these conditions. Due to their nonlinear nature and the presence of uncertain terms in their dynamics, robotic manipulators have attracted many researchers, and controlling them has been a yielding research field.

Constraints play a critical role in the functionality and design of practical systems, and they can take different forms, such as physical limits, saturations, and specific performance and safety requirements. Robotic manipulators, widely used physical systems in various industries, highlight the importance of effective human-robot collaboration. Safety concerns, such as ensuring robots maintain their intended trajectory and maintaining errors in acceptable margins while performing tasks are essential when people are working near these machines. In cases where direct manipulation of system states is not feasible, enforcing appropriate constraints is critical to avoiding operational problems, safety hazards, or damage. Additionally, a significant theoretical problem in controller designs employing Lyapunov-type analysis is the lack of detailed understanding of the system states' transient behavior. While Lyapunov-based analysis can confirm the system's overall stability, ensuring that states remain within bounds and eventually reach the desired states, it falls short of providing a detailed view of the behavior of the system states during the transient response. Given these practical necessities and theoretical gaps, focusing on managing position constraints in the design of controllers has emerged as an essential area of research. Incorporating barrier Lyapunov functions (BLF) into the design of controllers for nonlinear systems represents a notable advancement in meeting this condition.

The main theme of this dissertation is the incorporation of BLF into the design of nonlinear controllers for robotic manipulators. The proposed controller designs aim to achieve precise trajectory tracking for robotic manipulators under a variety of operational conditions while also imposing specific constraints on certain states of the system. To this end, the dissertation introduces three distinct control strategies to improve robotic manipulators' efficiency and safety. The first part of the research introduces novel constraint controller designs for robotic manipulators in full-state feedback form. These designs enable precise control over the robot's trajectory, effectively managing dynamic

uncertainties in robot behavior and maintaining position-tracking errors within specified limits. The second study focuses on output feedback controller designs for robotic manipulators with dynamically uncertain parameters. These proposed controllers are designed to keep the joint position tracking error within predefined limits, ensuring that it converges to zero, even in the absence of velocity measurements. The final study presents a tracking control strategy with constraints on task space tracking errors, particularly for kinematically redundant robot manipulators. This strategy includes a robust component to address parametric uncertainties and external disturbances alongside a BLF component for limiting tracking errors. The constraint on the task space tracking error is key, ensuring limited and predictable transient behavior of the manipulator's end effector.

1.2. Literature Review

Numerous control strategies for systems featuring nonlinear components are discussed in the literature [1, 2]. Dynamic systems often contains uncertainties, as acquiring complete system knowledge is typically unfeasible. Systems having uncertainties have garnered considerable attention, leading to various control schemes based on uncertainty types. The adaptive control method is often preferable for systems with structured, parametric uncertainties [3, 4]. However, this approach has a downside: the requirement to tune more gains increases with the quantity of unknown parameters. For systems with unstructured uncertainties, robust control methods are often favored [5, 6]. Nevertheless, these methods may induce undesired effects like chattering in practical applications. To address these issues in robust and adaptive control methods, hybrid schemes known as robust adaptive control have been developed, combining the strengths of both approaches [7, 8]. Additionally, learning control methods are useful for dynamically uncertain systems with periodic characteristics [9, 10], and neural network-based controllers have been designed to handle uncertainties [11].

As we mentioned in the previous part, one approach to address constraint control challenges is the use of BLF in controller design for nonlinear systems, which is particularly useful for managing constraints. BLFs are characterized by their infinite growth near certain boundaries, ensuring that these boundaries are not crossed by keeping the BLF bounded within a closed-loop system. Controllers incorporating BLFs have been developed for various types of constraints: output constraint [12], full-state constraint [13],

and partial-state constraint [14]. In [15], an adaptive neural controller was introduced for output feedback nonlinear systems with unknown functions. The incorporation of integral BLFs in adaptive control methods was introduced for switched nonlinear systems in [16], and an asymmetric BLF-based controller was developed for output tracking control in constrained nonlinear switched systems in [17]. Uncertain parametric nonlinear systems with full-state constraints and unknown control direction were addressed using adaptive control techniques in [18]. Additionally, BLF-based adaptive asymptotic tracking control schemes were proposed for strict-feedback nonlinear systems with unknown virtual control coefficients and full-state constraints in [19]. Adaptive control for uncertain stochastic nonlinear systems with full-state constraints was explored in [20]. Command-filtered output feedback control using BLFs for nonlinear systems with full-state constraints and uncertain time-varying disturbances was presented in [21]. High-order tangent function type BLFs were employed for adaptive output tracking controller design in high-order uncertain nonlinear systems with full-state constraints in [22]. A BLF-based systematic motion controller for servo systems [23] and a neural network-based controller using BLFs for robotic manipulators with model uncertainties [24]. Fixed-time control for biped robots using tangent BLF was introduced in [25]. Neural network-based adaptive methods coupled with BLFs were applied to Euler Lagrange systems [26], and marine vessels [27], [28], [29]. A BLF-based design for distributed state constraint non-singular terminal sliding mode in robot manipulators was proposed in [30]. Additionally, a unified neural control scheme combining neural networks and tangent function type BLF through a backstepping control technique for trajectory tracking of an unknown space manipulator under output constraints was presented in [31]. A BLF-based approach for the joint limit avoidance sub-task of redundant robot manipulator control problem was introduced in [32].

Research in controlling robotic manipulators has been a significant area of study. A key area of focus has been the output feedback control problem, where control design relies solely on joint-level position measurements. This problem is noteworthy for two reasons: theoretically, it presents the challenge of designing controllers without joint-level velocity data, and practically, it enables the elimination of costly sensors from each joint, reducing manufacturing costs. Several output feedback controllers for robotic manipulators have been introduced by researchers. For instance, Nicosia and

Tomei suggested a model-based observer to address velocity measurement issues in [33]. A robust controller approach was introduced in [6], while a passivity-based controller-observer formulation was presented in [34]. Additionally, a model-independent observer-based output feedback tracking controller designed for dynamically uncertain robotic manipulators was developed in [35]. Repetitive learning controllers, suitable for periodic trajectories, have been adapted for output feedback tracking in robotic devices [36]. Furthermore, filtered-based methods, using a velocity surrogate signal from a filter, have been applied to robotic manipulator control [37]. A composite adaptive scheme combining gradient-based and least-squares-based estimators with a filtered-based velocity surrogate was presented in [38]. Most output feedback controllers are semi-global, but there have been developments in global output feedback controllers for uncertain dynamics, as seen in [39] and [40]. A globally exponentially stable controller relying only on position measurements was introduced in [41], although it requires precise knowledge of dynamical parameters. More recently, an adaptive output feedback controller with velocity observers was proposed in [42].

Robotic manipulators with kinematic redundancy possess more degrees of freedom (DOF) than are required for specific tasks in their operational space. This redundancy, characterized as having an excess of $n - m$ DOF where n is the joint space dimension and m is the task space dimension, allows the use of additional independent parameters to define the robot's output configuration in the task space. Robots can thus perform sub-tasks like improving manipulability, avoiding singularities, and circumventing obstacles by adjusting their joints within the null space of the Jacobian matrix [43], [44]. Although task space is often used for task definition and execution, much of the past research has concentrated on developing tracking controllers for robotic manipulators in joint space [45]. One approach to address this issue is to transform the desired task into joint space, drawing on the extensive literature on joint tracking control. However, this necessitates solving the inverse kinematics problem, which can have multiple joint configuration solutions for a specific end-effector position. An alternative strategy is to formulate the task space tracking error while designing the joint space's control torque [46]. This avoids the inverse kinematics calculation but requires using the inverse Jacobian matrix. Several control approaches have been proposed in this context. A feedback linearization controller for end-effector tracking and redundant joint velocity control was introduced

in [47]. An augmented inverse Jacobian solution for kinematic redundancy was proposed in [48]. A robust adaptive controller was presented for redundant manipulator motion control in [49]. Compliant motion control using an extended task space formulation for redundant manipulators was suggested in [50]. Hierarchical multi-objective controllers employing null space projections were developed in [51]. A model-based robust control scheme for redundant robotic manipulators, utilizing self-motion for multiple sub-tasks under parametric uncertainties, was proposed in [52].

In our first study, as elaborated in [53], we introduced and examined two innovative full-state feedback, desired model-based controllers for robots with joint position constraints using BLFs. These controllers are designed to keep position-tracking errors within specified limits despite dynamic uncertainties in robotic systems. Our approach, which blends BLF design with desired model-based compensation, was novel and substantiated through comprehensive simulations and experimental tests on a two-link direct-drive robot arm. Furthermore, in [54], we developed a distinctive output feedback controller for robot manipulators with uncertain dynamics, focusing on joint position tracking error constraints. This controller ensures that the joint position tracking error remains within predetermined boundaries and achieves zero convergence, even without velocity measurements. Extensive simulation studies and experimental validations have proven its efficacy and performance. More recently, as described in [55], our research shifted towards kinematically redundant robotic manipulators. We formulated a specialized model-based controller for these systems, targeting the constraint of task space tracking errors. This controller guarantees exponential tracking of link positions and subtasks, provided the initial tracking error of the end-effector begins within a certain range. By applying a constraint to the task space tracking error, we effectively managed the overshoot of the manipulator's end-effector and secured predictable transient behavior. In our most recent endeavors, we have investigated and proposed a robust tracking control technique for kinematically redundant robotic manipulators, focusing on task space position constraints. This approach, integrating a pseudo-inverse-based method with a robust control component, handles parametric uncertainties and external disturbances, representing another notable advancement in the field.

2. MATHEMATICAL PRELIMINARIES AND MODELS

This section furnishes the essential mathematical foundation required to comprehend the studies presented in this dissertation, thereby rendering this document a comprehensive source of information.

2.1. Dynamic Model of Robot Manipulator

Dynamic Model: The equation representing the dynamic model for a robot manipulator with n revolute joints is described as follows [2]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} = \tau, \quad (2.1)$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ respectively denote the joint positions, velocities, and accelerations. $M(q) \in \mathbb{R}^{n \times n}$ represents the inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ denotes the Coriolis matrix, $G(q) \in \mathbb{R}^n$ corresponds to gravitational forces, $F_d \in \mathbb{R}^{n \times n}$ indicates the frictional effects, and $\tau(t) \in \mathbb{R}^n$ is the torque input. The dynamic model in (2.1) is known to satisfy the following properties.

Property 1: The inertia matrix $M(q)$ is symmetric and positive definite, adhering to the inequalities [2]

$$m_1\|\delta\|^2 \leq \delta^T M(q)\delta \leq m_2\|\delta\|^2 \quad \forall \delta \in \mathbb{R}^n \quad (2.2)$$

where m_1, m_2 are positive constants, and $\|\cdot\|$ represents Euclidean norm.

Property 2: The inertia matrix $M(q)$ and the Coriolis matrix $C(q, \dot{q})$ satisfy the skew-symmetry condition [2]

$$\xi^T (\dot{M} - 2C)\xi = 0 \quad \forall \xi \in \mathbb{R}^n. \quad (2.3)$$

Property 3: The Coriolis matrix $C(q, \dot{q})$ adheres to the following relation [2]

$$C(\xi, v)\eta = C(\xi, \eta)v \quad \forall \xi, v, \eta \in \mathbb{R}^n. \quad (2.4)$$

Property 4: The components of the dynamic model in (2.1) can be bound by the

following inequalities [2]

$$\|M(\xi) - M(v)\|_{i\infty} \leq \zeta_{m1} \|\xi - v\| \quad (2.5)$$

$$\|C(\xi, v)\|_{i\infty} \leq \zeta_{c1} \|v\| \quad (2.6)$$

$$\|C(\xi, v) - C(\eta, v)\|_{i\infty} \leq \zeta_{c2} \|\xi - \eta\| \quad (2.7)$$

$$\|G(\xi) - G(v)\| \leq \zeta_g \|\xi - v\| \quad (2.8)$$

for all $\xi, v, \eta \in \mathbb{R}^n$, where $\zeta_{m1}, \zeta_{c1}, \zeta_{c2}, \zeta_g$ are positive constants.

Property 5: The robot dynamics in (2.1) can be expressed using the following representation

$$Y(q, \dot{q}, \ddot{q})\theta = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_d\dot{q}, \quad (2.9)$$

where $Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times p}$ serves as a regression matrix consisting of known variables, and $\theta \in \mathbb{R}^p$ comprises uncertain constant system parameters. This regression matrix is also represented for a desired trajectory as

$$Y(q_d, \dot{q}_d, \ddot{q}_d)\theta = M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d, \quad (2.10)$$

with $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^n$ represent the desired positions, velocities, and accelerations, respectively.

Dynamic Model Including Additive Disturbances: The robot's dynamics, as outlined in (2.1), can also be presented in the form

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} + \xi_d = \tau, \quad (2.11)$$

where $\xi_d(t) \in \mathbb{R}^n$ represents unknown bounded additive disturbances, and the other terms have already been described in (2.1). All properties applicable to model (2.1) equally apply to (2.11).

Property 6: The robot dynamics expressed in (2.11) can be redefined as

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} + \xi_d = W(q, \dot{q}, \ddot{q})\theta + \zeta, \quad (2.12)$$

where $W(q, \dot{q}, \ddot{q})\theta$ includes the components of the dynamics from (2.1) that can be linearly parameterized. The term $\zeta \in \mathbb{R}^n$ represents the remaining elements that are not linearly parameterizable. In (2.11), $\theta \in \mathbb{R}^p$ denotes the constant system parameters, which include factors like the mass and center of mass of the robot links. $W(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times p}$ denotes the regression matrix, which is composed of known functions that depend on $q(t), \dot{q}(t)$, and $\ddot{q}(t)$.

2.2. Kinematic Model

The task-space position and orientation of the end-effector, symbolized by $x(t) \in \mathbb{R}^m$, is expressed as

$$x = f(q), \quad (2.13)$$

wherein $f(q) \in \mathbb{R}^m$ is the representation of forward kinematics, and $q(t) \in \mathbb{R}^n$ represents the positions of the links. From this expression, we deduce the correlation between the end-effector's position and the positions of the links as

$$\begin{aligned} \dot{x} &= J(q)\dot{q}, \\ \ddot{x} &= \dot{J}(q)\dot{q} + J(q)\ddot{q}, \end{aligned} \quad (2.14)$$

where $\dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ signify the velocities and accelerations of the links, respectively, and $J(q) \in \mathbb{R}^{m \times n}$ stands for the Jacobian of the manipulator, defined as

$$J(q) = \frac{\partial f(q)}{\partial q}. \quad (2.15)$$

The pseudo-inverse of the Jacobian, $J^+(q) \in \mathbb{R}^{n \times m}$, is defined by

$$J^+ = J^T (JJ^T)^{-1}, \quad (2.16)$$

and it satisfies the property

$$JJ^+ = I_m, \quad (2.17)$$

with $I_m \in \mathbb{R}^{m \times m}$ being the identity matrix of size m . The pseudo-inverse obeys the Moore-Penrose Conditions, which are

$$\begin{aligned} JJ^+J &= J, \quad J^+JJ^+ = J^+, \\ (J^+J)^T &= J^+J, \quad (JJ^+)^T = JJ^+. \end{aligned} \quad (2.18)$$

Additionally, the matrix $(I_n - J^+J)$ possesses the following attributes

$$\begin{aligned} (I_n - J^+J)(I_n - J^+J) &= (I_n - J^+J), \quad J(I_n - J^+J) = 0, \\ (I_n - J^+J)^T &= (I_n - J^+J), \quad (I_n - J^+J)J^+ = 0. \end{aligned} \quad (2.19)$$

Remark 1: For the feasibility of the control design, it is crucial to have access to the pseudo-inverse $J^+(q)$ for every joint configuration $q(t)$. This can be ensured by avoiding kinematic singularities in the robot's range of motion.

Remark 2: All dynamic and kinematic elements of the robot model are kept within bounded limits for all viable $q(t)$ values. This boundedness is ensured because their dependency on $q(t)$ is mediated through trigonometric functions.

3. DESIRED MODEL COMPENSATION BASED POSITION CONSTRAINED CONTROL OF ROBOTIC MANIPULATORS

3.1. Problem Definition and Error System Development

The main goal of the control strategy is to design the control input torque $\tau(t)$ in a way that ensures the joint position vector $q(t)$ progressively aligns with the desired joint position vector $q_d(t)$, fulfilling the tracking control objective. An additional control aim is to guarantee that the position tracking error components, represented as $e(t) \in \mathbb{R}^n$, stay within a predetermined bound $\Delta_i > 0$ for each joint i , ensuring that ¹

$$|e_i(t)| < \Delta_i \quad \forall t > 0, \quad (3.1)$$

when the each initial tracking error term $e_i(0)$ starts within the predetermined boundary defined by (3.1). The control problem considers joint position and joint velocity measurements as available. Still, it is complicated by parametric uncertainties in the robot dynamics (*i.e.*, the θ vector in (2.10) is uncertain). The desired joint position trajectory is assumed to be sufficiently smooth, meaning it and its first two derivatives are bounded time functions.

To articulate the primary control objective, the joint position tracking error $e(t) \in \mathbb{R}^n$ is defined as

$$e \triangleq q_d - q, \quad (3.2)$$

and to facilitate the subsequent design and analysis using only first-time derivatives, a filtered error $r(t) \in \mathbb{R}^n$ is introduced as

$$r \triangleq \dot{e} + \alpha e, \quad (3.3)$$

where $\alpha \in \mathbb{R}^{n \times n}$ is a positive definite diagonal control gain matrix. By taking the time derivative of $r(t)$ and pre-multiplying it by the inertia matrix, we obtain

$$M(q)\dot{r} = M(q)(\ddot{q}_d + \alpha\dot{e}) + C(q, \dot{q})(\dot{q}_d + \alpha e) - C(q, \dot{q})r + G(q) + F_d\dot{q} - \tau, \quad (3.4)$$

¹Here, the subscript i in a diagonal matrix or a column vector indicates the i th diagonal entry of the matrix or the i th entry of the vector, respectively.

where (2.1) is substituted and (3.3) is utilized along with the derivative of (3.2). Adding and subtracting the desired robot dynamics from (2.10) to the right-hand side of (3.4) results in

$$M(q)\dot{r} = -C(q, \dot{q})r + \chi + Y_d\theta - \tau, \quad (3.5)$$

where $\chi(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) \in \mathbb{R}^n$ is an uncertain vector defined by

$$\chi \triangleq M(q)(\ddot{q}_d + \alpha\dot{e}) + C(q, \dot{q})(\dot{q}_d + \alpha e) + G(q) + F_d\dot{q} - Y_d\theta. \quad (3.6)$$

Utilizing the dynamic model properties 1, 3, and 4, it can be demonstrated that χ is upper bounded as [2]

$$\|\chi\| \leq \rho_1\|e\| + \rho_2\|r\|, \quad (3.7)$$

with $\rho_1(\|e\|)$ and $\rho_2(\|e\|)$ being known, non-negative, and non-decreasing functions (see Appendix-A for detailed explanation).

3.2. Controller Design

The control input torque $\tau(t)$, derived from the forthcoming stability analysis, is designed as

$$\tau = Y_d\hat{\theta} + K_r r + K_e e + v_R \quad (3.8)$$

where $K_r \in \mathbb{R}^{n \times n}$ is a diagonal, positive definite and constant control gain matrix. $K_e(e) \in \mathbb{R}^{n \times n}$ is an error dependent control gain matrix that derived from the stability analysis, and it is also positive definite and diagonal. Additionally, $v_R(e, r) \in \mathbb{R}^n$ is introduced to mitigate the adverse impact of χ , and is formulated as

$$v_R = (k_n \rho_1^2 + \rho_2)r \quad (3.9)$$

where $k_n \in \mathbb{R}$ is a positive damping constant. The parameter estimation vector $\hat{\theta}(t) \in \mathbb{R}^p$ is adaptively modified according to

$$\dot{\hat{\theta}} = \Gamma Y_d^T r \quad (3.10)$$

in which $\Gamma \in \mathbb{R}^{p \times p}$ is a constant, positive definite, diagonal adaptation gain matrix.

Inserting the tailored control input torque from (3.8) and (3.9) into the open loop error system given in (3.5) leads to the subsequent closed loop error system

$$M(q)\dot{r} = -C(q, \dot{q})r - K_r r - K_e e + \chi - (k_n \rho_1^2 + \rho_2)r + Y_d\tilde{\theta} \quad (3.11)$$

where $\tilde{\theta}(t) \in \mathbb{R}^p$ represents the parameter estimation error, defined as

$$\tilde{\theta} \triangleq \theta - \hat{\theta}. \quad (3.12)$$

The introduction of the tracking error-dependent control gain matrix K_e distinguishes this work from prior research. The design of K_e facilitates a novel stability analysis, ensuring the *a priori* boundedness of tracking error entries within predetermined upper limits. To achieve this, two different designs for K_e are proposed. The first design is

$$K_e = \text{diag} \left\{ \frac{k_i}{\Delta_i^2 - e_i^2} \right\} \quad (3.13)$$

where k_i , for $i \in \{1, \dots, n\}$, are constant gains. The second design is

$$K_e = \text{diag} \left\{ 1 + \tan^2 \left(\frac{\pi}{2} \frac{e_i^2}{\Delta_i^2} \right) \right\} \quad (3.14)$$

with Δ_i previously defined in (3.1).

3.3. Stability Analysis

Although the two designs of the error dependent control gain matrices are fundamentally different, the initial stages of the stability analysis are the same. The stability analysis for both designs is established based on the following theorem.

Theorem 3.1: *For the mathematical model of the robot manipulator given in (2.1), employing the controller described in (3.8) and (3.9), along with the adaptive update law in (3.10) and utilizing the tracking error dependent control gain matrix design from either (3.13) or (3.14), ensures global asymptotic convergence of the tracking error and the filtered error to the origin. It also guarantees that the entries of the joint tracking error remain within a predefined bound, while demonstrating closed-loop stability by ensuring the boundedness of all system trajectories, provided that the damping gain k_n introduced in (3.9) is chosen to be sufficiently high.*

Proof. For the diagonal controller gain matrix design in (3.13) dependent on tracking error, we initiate the analysis by introducing the barrier Lyapunov function $V_l(r, e, \tilde{\theta}) \in \mathbb{R}$ defined as

$$V_l \triangleq \frac{1}{2} r^T M(q) r + \sum_{i=1}^n \frac{k_i}{2} \ln \left(\frac{\Delta_i^2}{\Delta_i^2 - e_i^2} \right) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \quad (3.15)$$

given that the initial values of the joint tracking error entries satisfy $|e_i(0)| < \Delta_i$ for all $i \in \{1, \dots, n\}$, thus, $V_l(\cdot) : \mathbb{R}^n \times \{e_i \in \mathbb{R} : |e_i| < \Delta_i\} \times \mathbb{R}^p \rightarrow \mathbb{R}_+$ is positive definite and radially unbounded within its respective domain.

The time derivative of (3.15) is expressed as

$$\dot{V}_l = r^T M(q) \dot{r} + \frac{1}{2} r^T \dot{M}(q) r + \sum_{i=1}^n k_i \frac{e_i \dot{e}_i}{\Delta_i^2 - e_i^2} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \quad (3.16)$$

where

$$\sum_{i=1}^n k_i \frac{e_i \dot{e}_i}{\Delta_i^2 - e_i^2} = e^T K_e \dot{e} \quad (3.17)$$

with the diagonal structure of K_e in (3.13). Substituting the closed-loop error system in (3.11) for r dynamics, (3.17), (3.3) for e dynamics, and the time derivative of (3.12) along with (3.10) for $\tilde{\theta}$ dynamics into (3.16) yields

$$\begin{aligned} \dot{V}_l = & r^T \left[-C(q, \dot{q}) r - K_r r - K_e e + \chi - (k_n \rho_1^2 + \rho_2) r + Y_d \tilde{\theta} \right] \\ & + \frac{1}{2} r^T \dot{M}(q) r + e^T K_e (-\alpha e + r) - \tilde{\theta}^T Y_d^T r. \end{aligned} \quad (3.18)$$

Utilizing the skew-symmetry relationship in (2.3), bounding χ with (3.7), and canceling common terms, we simplify (3.18) to

$$\dot{V}_l \leq -r^T K_r r - e^T K_e \alpha e + [\rho_1 \|e\| \|r\| - k_n \rho_1^2 \|r\|^2] \quad (3.19)$$

where the term in square brackets, as shown in [56], can be further bounded as

$$\rho_1 \|e\| \|r\| - k_n \rho_1^2 \|r\|^2 \leq \frac{1}{4k_n} \|e\|^2, \quad (3.20)$$

and this allows us to establish an upper bound for the right-hand side of (3.19) as

$$\dot{V}_l \leq -\lambda_{\min}\{K_r\} \|r\|^2 - \left(\lambda_{\min}\{K_e \alpha\} - \frac{1}{4k_n} \right) \|e\|^2. \quad (3.21)$$

Defining the combined error vector $x \triangleq [r^T \ e^T]^T \in \mathbb{R}^{2n}$ and a positive constant $\beta \in \mathbb{R}$ as

$$\beta \triangleq \min \left\{ \lambda_{\min}\{K_r\}, \lambda_{\min}\{K_e \alpha\} - \frac{1}{4k_n} \right\} \quad (3.22)$$

we derive the following upper bound for the right-hand side of (3.21)

$$\dot{V}_l \leq -\beta \|x\|^2 \quad (3.23)$$

as long as k_n is chosen to be sufficiently high.

From the structures of (3.15) and (3.23), the boundedness of V_l is established, implying that $r(t), e(t), \tilde{\theta}(t) \in \mathcal{L}_\infty$. By leveraging the boundedness of these terms, along with the boundedness of the desired trajectory and its time derivatives, $\dot{e}(t), \dot{r}(t) \in \mathcal{L}_\infty$ can be proven using (3.3) and (3.11), respectively. It is straightforward to show that the remaining terms are also bounded. Integrating (3.23) over time from the initial time to infinity establishes that $x(t)$ is square integrable, and consequently, $r(t), e(t) \in \mathcal{L}_2$. Since $x(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{x}(t) \in \mathcal{L}_\infty$, Barbalat's Lemma [57] guarantees $x(t) \rightarrow 0$ as $t \rightarrow \infty$.

It should be noted that due to the formulation of $V_l(t)$ as shown in (3.15) and the negativeness of $\dot{V}_l(t)$ from equation (3.23), the positive function V_l decreases over time. Given that the first and third terms of $V_l(t)$ are quadratic in terms of $r(t)$ and $\tilde{\theta}$ respectively, the middle term, as illustrated in Figure 3.1, is also observed to decline with time. If the initial error signal, $e_i(0)$, is set within the bounds $(-\Delta_i, +\Delta_i)$, the tracking error signal will consistently remain within this range and will asymptotically converge to zero [12], thereby consistently avoiding the singularities in equation (3.13).

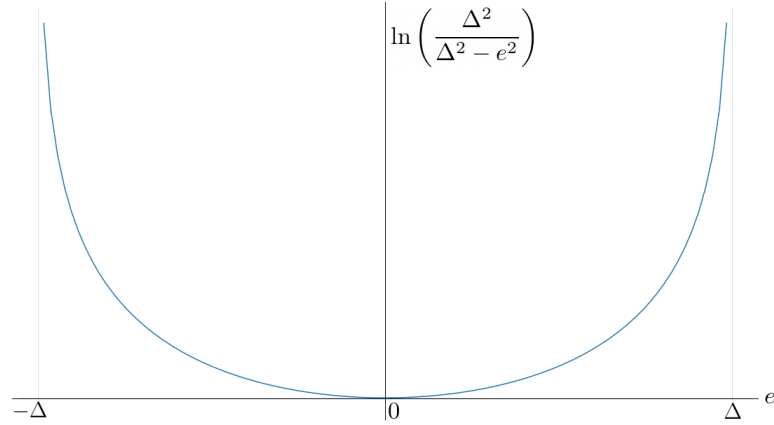


Figure 3.1: Illustration showing how the logarithmic component $V(\cdot)$ relates to the corresponding argument $e(t)$

For designing the tracking error dependent diagonal controller gain matrix in (3.14), a similar analysis is conducted by establishing the barrier Lyapunov function $V_t(r, e, \tilde{\theta}) \in$

$\mathbb{R}$ as follows

$$V_t \triangleq \frac{1}{2} r^T M(q) r + \sum_{i=1}^n \frac{\Delta_i^2}{\pi} \tan\left(\frac{\pi}{2} \frac{e_i^2}{\Delta_i^2}\right) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (3.24)$$

where the only difference from the term in (3.15) is in the second term. It is noted that $V_t(r, e, \tilde{\theta})$ is positive definite and radially unbounded, assuming that the initial values of the joint tracking error entries fulfill the condition $|e_i(0)| < \Delta_i$ for each $i \in \{1, \dots, n\}$.

The time derivative of (3.24) is obtained as

$$\dot{V}_t = r^T M(q) \dot{r} + \frac{1}{2} r^T \dot{M}(q) r + \sum_{i=1}^n e_i \dot{e}_i \left(1 + \tan^2\left(\frac{\pi}{2} \frac{e_i^2}{\Delta_i^2}\right)\right) + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \quad (3.25)$$

where, with the diagonal structure of the K_e design in (3.14), the third term is reformulated as

$$\sum_{i=1}^n e_i \dot{e}_i \left(1 + \tan^2\left(\frac{\pi}{2} \frac{e_i^2}{\Delta_i^2}\right)\right) = e^T K_e \dot{e}. \quad (3.26)$$

A closer examination of the structure of (3.25), in light of (3.26), reveals that it is the same as (3.16) used with (3.17). Consequently, the remainder of the analysis proceeds identically to the previous part. $\square$

Remark 3: *Despite achieving the same outcome for both selections of tracking error-dependent control gain matrices, the resulting β values may differ due to the distinctions in their designs in (3.13) and (3.14). For the design in (3.13), $\lambda_{\min}\{K_e \alpha\}$ is equivalent to $\min\{\frac{k_i \alpha_i}{\Delta_i^2 - e_i^2}\}$ over $i \in \{1, \dots, n\}$, which can conservatively be obtained as $\frac{\min\{k_i \alpha_i\}}{\max\{\Delta_i^2\}}$. On the other hand, for the design in (3.14), $\lambda_{\min}\{K_e \alpha\}$ is equivalent to $\min\{\alpha_i \left(1 + \tan^2\left(\frac{\pi}{2} \frac{e_i^2}{\Delta_i^2}\right)\right)\}$ over $i \in \{1, \dots, n\}$. Noting that the $\tan^2(\cdot)$ term is nonnegative, this can conservatively be obtained as $\min\{\alpha_i\}$.*

3.4. Simulation Studies

Numerical simulations were conducted on a two-link planar robot manipulator model to showcase the effectiveness of the proposed adaptive control strategy. The robot

manipulator model is described by the following equation

$$\begin{aligned} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} &= \begin{bmatrix} \theta_1 + 2\theta_3 \cos(q_2) & \theta_2 + \theta_3 \cos(q_2) \\ \theta_2 + \theta_3 \cos(q_2) & \theta_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} -\theta_3 \dot{q}_2 \sin(q_2) & -\theta_3 (\dot{q}_1 + \dot{q}_2) \sin(q_2) \\ \theta_3 \dot{q}_1 \sin(q_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} \theta_4 & 0 \\ 0 & \theta_5 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}. \end{aligned} \quad (3.27)$$

In this model, $\theta_1, \theta_2, \theta_3$ are uncertain model parameters related to the masses and lengths of the robot manipulator links, while θ_4 and θ_5 represent viscous friction parameters. The actual values of these parameters are set as follows in the dynamical model (3.27)

$$\theta = [3.473 \ 0.193 \ 0.242 \ 5.3 \ 1.1]^T. \quad (3.28)$$

Following the analysis above, these model parameters are treated as uncertainties not available for control design. The control input torque values were realistically saturated at $\pm 10 \text{ Nm}$.

For the all simulations, the desired joint trajectory below was used (see Figure 3.2)

$$q_d(t) = \begin{bmatrix} 0.7 \sin(t) (1 - \exp(-0.3t^3)) \\ 1.2 \sin(t) (1 - \exp(-0.3t^3)) \end{bmatrix} [\text{rad}] \quad (3.29)$$

where the exponential term in the above equation is used to provide a smooth start to the system. The initial joint positions of the robot manipulator were chosen to yield an initial tracking error of 2.9° for both joints. The initial values of the parameter estimation vector, $\hat{\theta}(0)$, were set to zero.

Three sets of simulations were conducted with Δ_i chosen as $3^\circ, 5^\circ$, and 10° , respectively, for each joint. Control and adaptation gain matrices were tuned through trial and error, with the following selected gains for all three simulations:

$$\alpha = 2I_2, \quad K_r = 50I_2, \quad k_1 = k_2 = 2 \quad (3.30)$$

$$\Gamma = \text{diag}\{50, 0.8, 1.255, 100.6, 40.2\}. \quad (3.31)$$

The simulation results, including joint tracking errors, parameter estimate updates, and control input torques, are presented in Figures 3.4, 3.5, and 3.6 for different values of

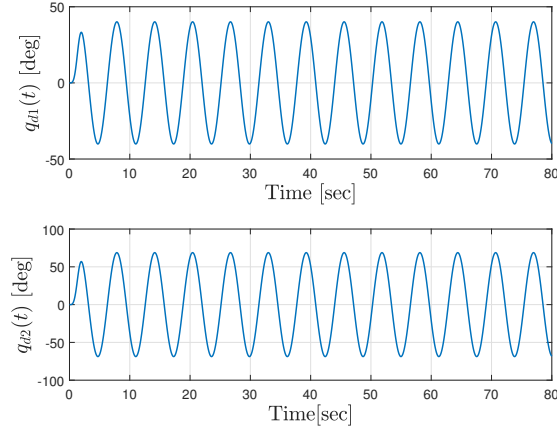


Figure 3.2: Desired joint position trajectory used in simulation studies

position constraints. The $\mathcal{L}_2$ norms of the controller efforts for each simulation's initial state are presented in Table 3.1, showing a decrease in control effort as the position constraint increases from 3° to 10° . The $\mathcal{L}_2$ norms of the controller efforts at steady state are presented in Table 3.2, indicating no significant change in the amount of control effort when the system reaches steady state. Additionally, Figures 3.4, 3.5, and 3.6 show that increasing the position constraint on the joint trajectories prolongs the convergence time of the parameter estimates.

Table 3.1: $\mathcal{L}_2$ norm of the torque inputs for $0 \leq t \leq 20$ [sec]

	$\Delta_i = 3$ [deg]	$\Delta_i = 5$ [deg]	$\Delta_i = 10$ [deg]
$\{\int_0^{20} \tau_1(t)dt\}^{1/2}$	540.7435	523.7784	514.0927
$\{\int_0^{20} \tau_2(t)dt\}^{1/2}$	150.1854	143.568	141.5573

Table 3.2: $\mathcal{L}_2$ norm of the torque inputs for $70 \leq t \leq 80$ [sec]

	$\Delta_i = 3$ [deg]	$\Delta_i = 5$ [deg]	$\Delta_i = 10$ [deg]
$\{\int_{70}^{80} \tau_1(t)dt\}^{1/2}$	338.3291	338.3548	338.3563
$\{\int_{70}^{80} \tau_2(t)dt\}^{1/2}$	95.3373	95.3455	95.3514

3.5. Experimental Validation

To demonstrate the feasibility and performance of the proposed position-constrained adaptive controller, experimental studies were conducted on a two-link, direct-drive, planar robot manipulator assembled in Gebze Technical University, Control Applications, and Robotics Laboratory. The dynamical model of the robot is similar to (3.27) with the values of $\theta_i, i = 1 \dots 5$, being uncertain. The links of the manipulator are constructed from aluminum with link lengths of 16 cm for the first link and 6.5 cm for the second link. Link 1 is actuated via a SanyoDenko dc motor equipped with a 4096 counts/revolution encoder, and the second link is actuated via an Escap dc motor with an encoder having 2048 counts/revolution. Each motor is actuated via an in-house-built linear amplifier with unity gain. A schematic diagram of the experimental setup is illustrated in Figure 3.3.

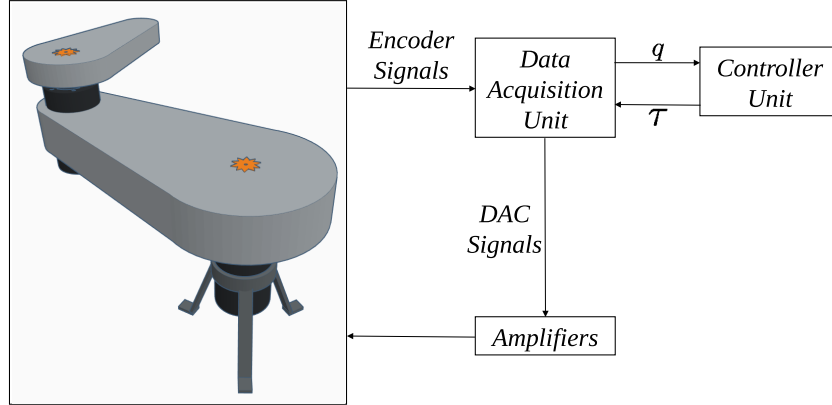


Figure 3.3: Block representation of the experimental setup

For the experimental studies, the position tracking error constraint was set to 10° . The same desired trajectory used in the simulation studies was applied (i.e., equation (3.29)). The initial values of the parameter estimates were set to zero, and the controller and adaptation gains were tuned until the best error tracking and parameter adaptations were

obtained. The selected control and adaptation gains were as follows

$$\alpha = \text{diag}\{ 10, 20 \}, \quad k_e = \text{diag}\{ 12, 24 \},$$

$$K_r = \text{diag}\{ 180, 50 \}$$

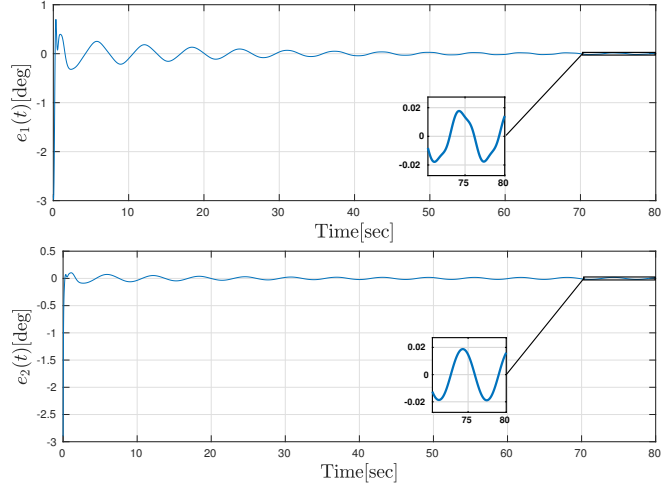
and

$$\Gamma = \text{diag}\{ 6, 0.5, 0.4, 24, 4.5 \}$$

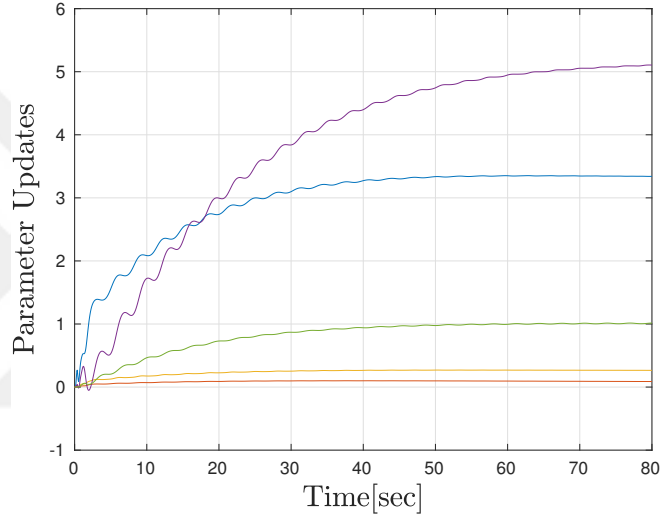
The results of the experiments are presented in Figure 3.7, where the top two sub-figures show the tracking error performances, the middle sub-figure displays the parameter estimations, and the bottom two sub-figures depict the corresponding voltage outputs of the current amplifiers (representing the control torque inputs). As seen from Figure 3.7, after around 20 seconds, the parameter estimates converge to some values, and the tracking error performances for links 1 and 2 converge to values below $\pm 0.5^\circ$.

3.6. Conclusion

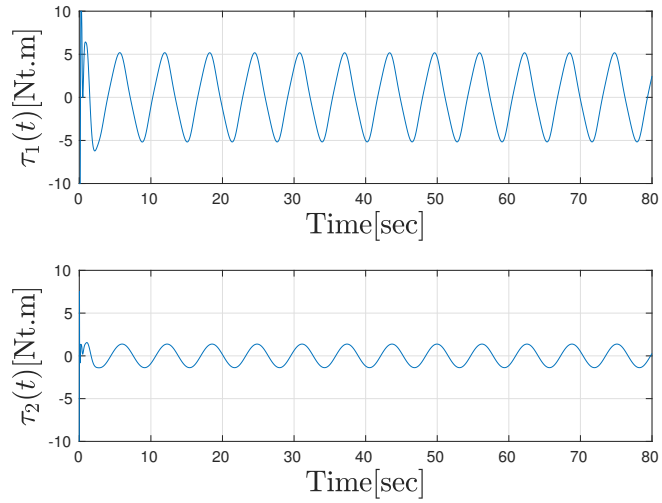
This study have introduced the design and analysis of two distinct types of full-state feedback controllers for robot manipulators. These controllers utilize BLFs and are designed to enforce joint position constraints, ensuring that the position tracking error remains within a predefined bound and converges to zero when the initial tracking error is within this region. Notably, the proposed controllers incorporate BLF design with a desired model-based compensation approach, representing a novel fusion in controller methodology. The utilization of the desired trajectory signal, instead of actual state variables, enhances controller smoothness and reduces sensitivity to sensory noise. Extensive numerical simulations demonstrate the effectiveness of the proposed methods under varying joint position constraints. Experimental studies conducted on a two-linked direct-drive manipulator further validate the feasibility of the proposed controllers.



(a) The tracking error performances

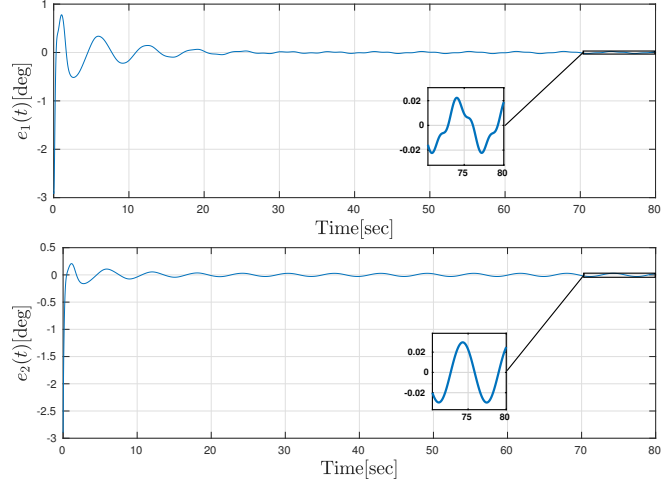


(b) Parameter estimates

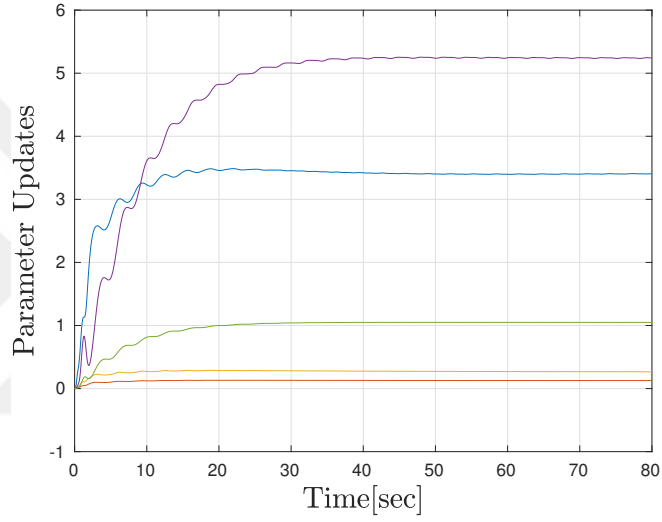


(c) Control torque input signals

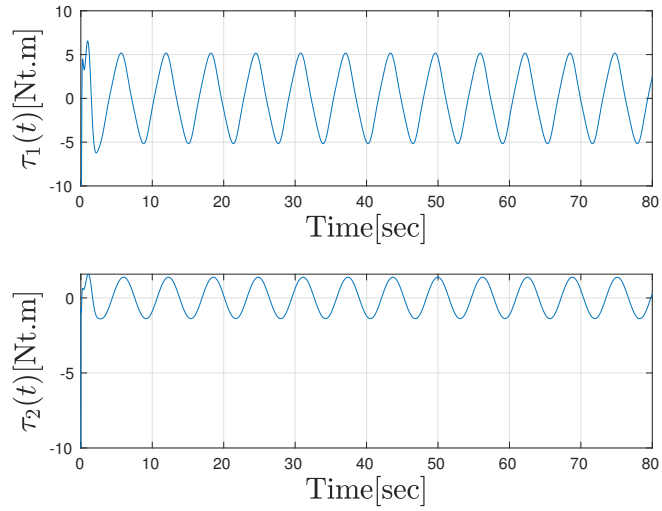
Figure 3.4: Simulation results with the position constraint Δ set at 3°



(a) The tracking error performances

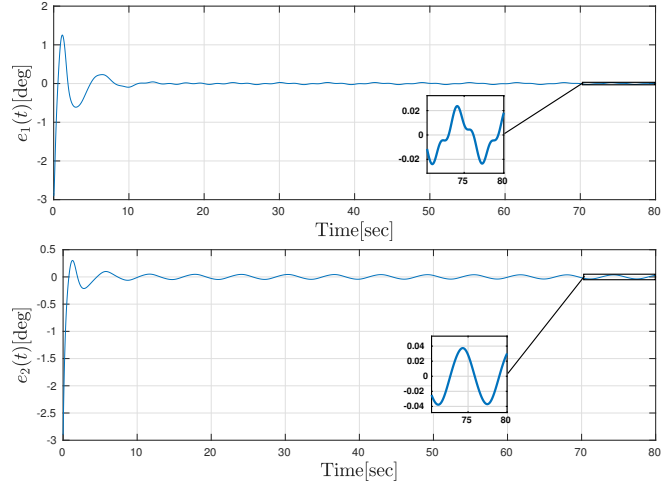


(b) Parameter estimates

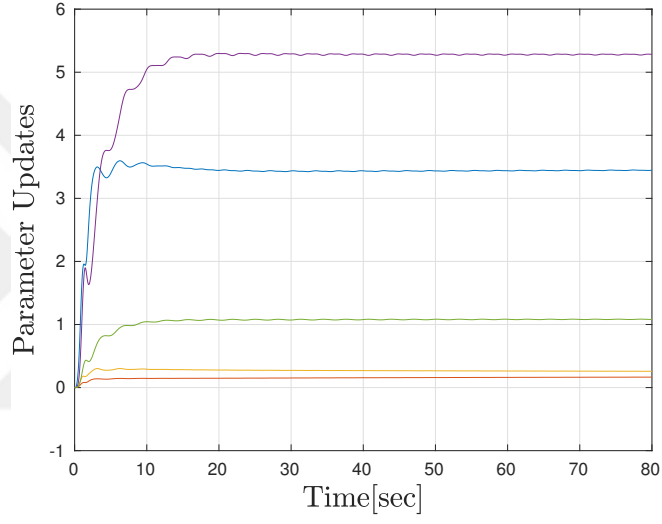


(c) Control torque input signals

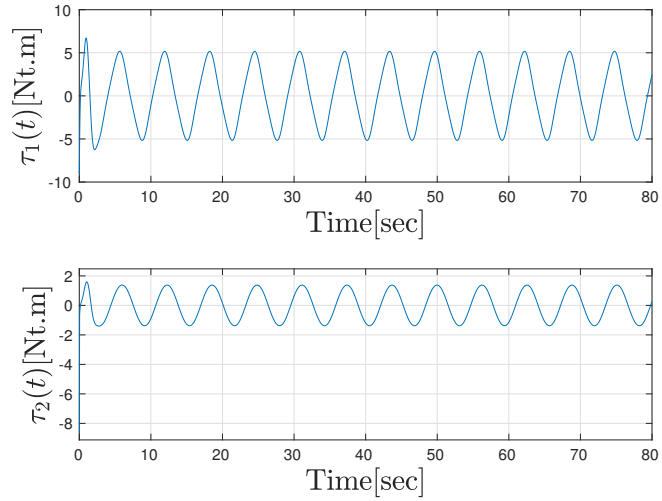
Figure 3.5: Simulation results with the position constraint Δ set at 5°



(a) The tracking error performances

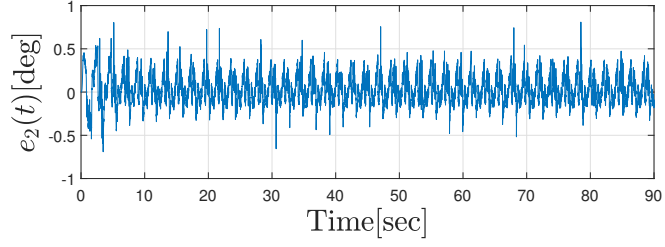
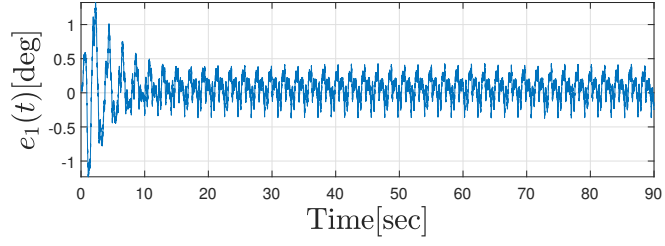


(b) Parameter estimates

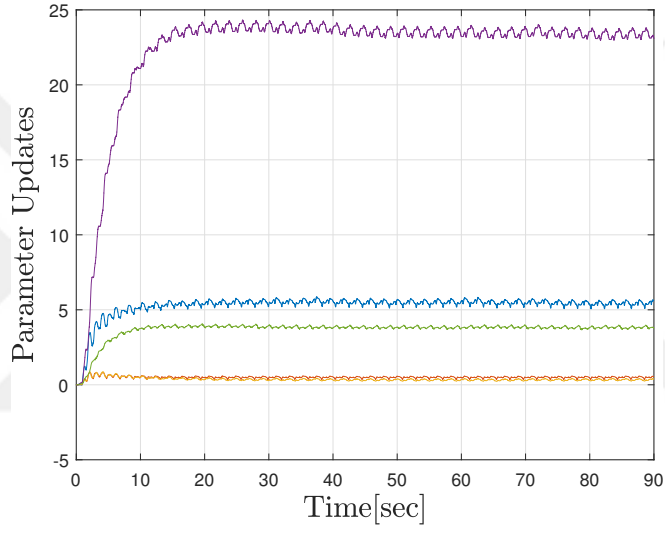


(c) Control torque input signals

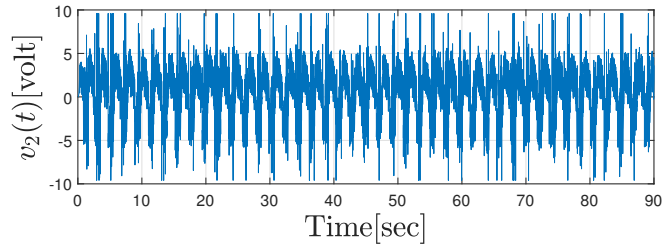
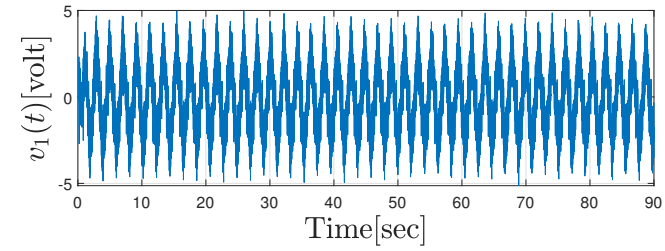
Figure 3.6: Simulation results with the position constraint Δ set at 10°



(a) The tracking error performances



(b) Parameter estimates



(c) Control outputs applied to the amplifiers

Figure 3.7: Experimental results when the position constraint Δ set at 10°

4. POSITION CONSTRAINED, ADAPTIVE CONTROL OF ROBOTIC MANIPULATORS WITHOUT VELOCITY MEASUREMENTS

Similar to the first study, this research also addresses the constrained tracking control problem; however, in this case, it focuses on a robot manipulator in an output feedback format. Additionally, the dynamics in this study involve parametric uncertainties.

4.1. Problem Definition and Error System Development

To define the main control objective, we introduce the joint position tracking error $e(t) \in \mathbb{R}^n$ as

$$e = q_d - q. \quad (4.1)$$

We also establish an upper bound for the tracking error term

$$|e_i(t)| < \Delta_i \quad \forall t > 0, i \in \{1, \dots, n\}. \quad (4.2)$$

The main control objective is to design the control input signal $\tau(t)$ such that when the initial tracking error term $e(0)$ starts within the boundary defined by (4.2), it not only remains within this boundary but also ultimately converges to zero. Given the absence of joint velocity measurements, we propose a velocity filter-based approach. Specifically, a surrogate velocity signal $e_f(t) \in \mathbb{R}^n$ is utilized in place of the actual velocity tracking error. The velocity surrogate signal is obtained from the output of the following filter

$$e_f = -ke + w \quad (4.3)$$

where k is a positive gain and $w(t) \in \mathbb{R}^n$ is updated according to the differential relationship

$$\dot{w} = -(k+1)e_f - ke + K_e e, \quad w(0) = ke(0). \quad (4.4)$$

Here, $K_e(e) \in \mathbb{R}^{n \times n}$ is an error-dependent gain matrix that is designed as follows

$$K_e = \text{diag} \left\{ \frac{k_{ei}}{\Delta_i^2 - e_i^2} \right\} \quad (4.5)$$

where k_{ei} , for each $i \in \{1, \dots, n\}$, are positive constant gain values. To derive the dynamics of $e_f(t)$, we start by taking the time derivative of (4.3) and substituting (4.4),

resulting in the following expression

$$\dot{e}_f = -e_f - k\eta + K_e e. \quad (4.6)$$

Here, $\eta(t) \in \mathbb{R}^n$ serves as an auxiliary variable and is defined by

$$\eta \triangleq \dot{e} + e + e_f. \quad (4.7)$$

Since the definition of $\eta(t)$ includes the time derivative of the tracking error, it cannot be used for designing controls or estimators. Nonetheless, it will be utilized in discussing the stability and convergence analysis. The equation for the position tracking error dynamics is derived by rearranging equation (4.7) to get

$$\dot{e} = -e - e_f + \eta. \quad (4.8)$$

To derive the dynamics of $\eta(t)$, the time derivative of (4.7) is multiplied by $M(q)$ from the left, yielding

$$\begin{aligned} M(q) \dot{\eta} &= M(q) (\ddot{q}_d + \dot{e} + \dot{e}_f) \\ &+ V_m(q, \dot{q}) \dot{q} + G(q) + F_d \dot{q} - \tau \end{aligned}$$

where (2.1) is used to replace the $M(q)\ddot{q}$ term. Adding and subtracting the desired dynamics in (2.10) to the right-hand side of the above expression and manipulating the terms using (4.6) and (4.8), the dynamics of η is reformulated as

$$M(q) \dot{\eta} = -V_m(q, \dot{q}) \eta - kM(q) \eta + Y_d \theta + \chi - \tau \quad (4.9)$$

where $\chi(e, e_f, \eta, t) \in \mathbb{R}^n$ is structured as follows

$$\begin{aligned} \chi &\triangleq M(q) (\ddot{q}_d + \eta - 2e_f + K_e e - e) \\ &+ V_m(q, \dot{q}) (\dot{q}_d + e_f + e) \\ &+ G(q) + F_d \dot{q} - Y_d \theta. \end{aligned} \quad (4.10)$$

It's important to highlight, as detailed in Appendix B of [45] and also included in Appendix-B of the dissertation for thoroughness, that an upper bound for the term χ can be established as follows

$$\|\chi\| \leq \zeta_1 \|x\| + \zeta_2 \|x\|^2. \quad (4.11)$$

In this equation, ζ_1 and ζ_2 represent known positive constants, and the vector $x(t) \in \mathbb{R}^{3n}$, which is defined as the combined error vector, is given by

$$x \triangleq [\eta^T \quad e_f^T \quad e^T]^T. \quad (4.12)$$

Now that we have derived the dynamics of η , we can proceed to design the control and adaptive estimation terms.

4.2. Controller Design

Given the error dynamics structure in (4.8) and the derived dynamical relations in (4.6) and (4.9), and following subsequent stability analysis, the control input torque $\tau(t)$ is formulated as follows

$$\tau = Y_d \hat{\theta} + K_e e - k e_f. \quad (4.13)$$

Here, $\hat{\theta}(t) \in \mathbb{R}^p$ represents the adaptive estimate of the uncertain parameter vector θ , introduced in (2.10). The adaptive estimate is designed to have the following structure:

$$\begin{aligned} \hat{\theta} &= \Gamma \int_0^t Y_d^T(\sigma) (e_f(\sigma) + e(\sigma)) d\sigma \\ &+ \Gamma Y_d^T e - \Gamma \int_0^t \frac{d\{Y_d^T(\sigma)\}}{d\sigma} e(\sigma) d\sigma \end{aligned} \quad (4.14)$$

where $\Gamma \in \mathbb{R}^{p \times p}$ represents a constant, diagonal, positive definite matrix known as the adaptation gain. Considering the structure of η as specified in (4.7), the time derivative of the adaptive estimate $\hat{\theta}$ given in (4.14) can be expressed as

$$\dot{\hat{\theta}} = \Gamma Y_d^T \eta. \quad (4.15)$$

Substituting the control input torque τ from (4.13) into (4.9), we arrive at the closed-loop equation

$$\begin{aligned} M(q) \dot{\eta} &= -V_m(q, \dot{q}) \eta - k M(q) \eta \\ &- K_e e + k e_f + Y_d \tilde{\theta} + \chi. \end{aligned} \quad (4.16)$$

Here, $\tilde{\theta}(t) \in \mathbb{R}^p$ represents the parameter estimation error and is represented by the equation

$$\tilde{\theta} \triangleq \theta - \hat{\theta}. \quad (4.17)$$

4.3. Stability Analysis

Theorem 4.1: *The control input torque described in (4.13), along with the parameter update mechanism in (4.14), and the filter design specified in (4.3) and (4.4), guarantees the boundedness of all signals within the closed-loop system and the convergence of the tracking error term. This assurance is contingent upon the initial value of the tracing error term beginning within a predetermined region and the fulfillment of the gain condition $\min\{k_{ei}\} \geq \max\{\Delta_i^2\}$. The control gain k is formulated as*

$$k = \frac{1}{m_1} (1 + \zeta_1^2 k_n + \zeta_2^2 k_n). \quad (4.18)$$

The damping gain k_n , which is positively valued and defined in (4.18), is specifically designed to meet the following condition

$$k_n > 1 + \frac{\lambda_2}{\lambda_1} \|z(0)\|^2 \quad (4.19)$$

where $z(t) \triangleq \begin{bmatrix} x^T & \tilde{\theta}^T \end{bmatrix}^T \in \mathbb{R}^{3n+p}$, with $x \in \mathbb{R}^{3n}$ defined as per (4.12). The constants m_1, m_2 represent the minimum and maximum bounding constants for the inertia matrix.

The constants λ_1 and λ_2 are positive constants determined as

$$\lambda_1 \triangleq \frac{1}{2} \min\{m_1, 1, \frac{\min\{k_{ei}\}}{\max\{\Delta_i^2\}}, \lambda_{\min}\{\Gamma^{-1}\}\}, \quad (4.20)$$

$$\lambda_2 \triangleq \frac{1}{2} \max\{m_2, 1, \max\{k_{ei}\}, \lambda_{\max}\{\Gamma^{-1}\}\}. \quad (4.21)$$

Here, the notations $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ denote the minimum and maximum eigenvalues of a matrix, respectively.

Proof. We initiate the proof by introducing a non-negative function denoted as $V(\eta, e_f, e, \tilde{\theta})$ defined by

$$\begin{aligned} V &\triangleq \frac{1}{2} \eta^T M(q) \eta + \frac{1}{2} e_f^T e_f \\ &+ \sum_{i=1}^n \frac{k_{ei}}{2} \ln \left(\frac{\Delta_i^2}{\Delta_i^2 - e_i^2} \right) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \end{aligned} \quad (4.22)$$

This function is both lower and upper-bounded, as expressed by

$$\lambda_1 \|x\|^2 \leq \lambda_1 \|z\|^2 \leq V \leq \lambda_2 \|z\|^2 \quad (4.23)$$

where λ_1 and λ_2 are constants defined in the Theorem. Notably, the third term in $V(\eta, e_f, e, \tilde{\theta})$ is always positive given that the initial error values of all joints satisfy $|e_i(0)| < \Delta_i$, thereby ensuring the positive definiteness of $V(\eta, e_f, e, \tilde{\theta})$.

Upon deriving the time derivative of $V(t)$, we employ the relationship

$$\sum_{i=1}^n k_{ei} \frac{e_i \dot{e}_i}{\Delta_i^2 - e_i^2} = e^T K_e \dot{e}, \quad (4.24)$$

next, we incorporate expressions from (4.6), (4.8), (4.16), (4.15), considering the time derivative of (4.17). After eliminating common term, leveraging the skew-symmetry property (2.3) of the model, the resulting expression for the time derivative of V becomes

$$\dot{V} = -e_f^T e_f - e^T K_e e - k\eta^T M(q)\eta + \eta^T \chi. \quad (4.25)$$

Upon substituting the expression for k from (4.18), and utilizing the upper bound for the χ term (4.11), the upper bound of $\dot{V}$ takes the form

$$\begin{aligned} \dot{V} \leq & -\|x\|^2 + [\xi_1 \|\eta\| \|x\| - \xi_1^2 k_n \|\eta\|^2] \\ & + [\xi_2 \|\eta\| \|x\|^2 - \xi_2^2 k_n \|\eta\|^2]. \end{aligned} \quad (4.26)$$

The reformulation of the upper bound for $\dot{V}(t)$ can be expressed as

$$\begin{aligned} \dot{V} \leq & -\|x\|^2 \\ & - \left[\xi_1^2 k_n \|\eta\|^2 - \xi_1 \|\eta\| \|x\| + \frac{1}{4k_n} \|x\|^2 \right] + \frac{1}{4k_n} \|x\|^2 \\ & - \left[\xi_2^2 k_n \|\eta\|^2 - \xi_2 \|\eta\| \|x\|^2 + \frac{1}{4k_n} \|x\|^4 \right] + \frac{1}{4k_n} \|x\|^4. \end{aligned} \quad (4.27)$$

Here, terms $\frac{1}{4k_n} \|x\|^2$ and $\frac{1}{4k_n} \|x\|^4$ have been strategically added and subtracted to the bracketed terms of (4.26) respectively. The bracketed terms in equation (4.27) can be expressed as the squares $\left(\xi_1 \sqrt{k_n} \|\eta\| - \frac{1}{2\sqrt{k_n}} \|x\| \right)^2$ and $\left(\xi_2 \sqrt{k_n} \|\eta\| - \frac{1}{2\sqrt{k_n}} \|x\|^2 \right)^2$, which are inherently non-negative. Given the negative sign preceding these terms, their presence leads to a non-positive contribution in the inequality. Consequently, removing these non-positive terms from the right-hand side of equation (4.27) does not affect the validity of the inequality, enabling a simplification of the upper bound to

$$\dot{V} \leq -\|x\|^2 + \frac{1}{4k_n} \|x\|^2 + \frac{1}{4k_n} \|x\|^4 \quad (4.28)$$

which can further be simplified to

$$\dot{V} \leq - \left[1 - \frac{1}{4k_n} (1 + \|x\|^2) \right] \|x\|^2. \quad (4.29)$$

Utilizing (4.23), the upper bound for $\dot{V}(t)$ can be expressed as

$$\dot{V} \leq - \left[1 - \frac{1}{4k_n} \left(1 + \frac{V}{\lambda_1} \right) \right] \|x\|^2. \quad (4.30)$$

From the relationship in (4.30), we understand that $\dot{V}(t)$ remains negative or zero when the term inside the brackets is positive. Consequently, to ensure that $\dot{V}(t)$ does not increase, it is critical to choose the control gain k_n such that

$$1 - \frac{1}{4k_n} \left(1 + \frac{V(t)}{\lambda_1} \right) > 0. \quad (4.31)$$

This condition guarantees that $\dot{V}(t)$ is either negative or zero, thereby preserving the non-increasing nature of the function $V(t)$, which is inherently positive. Given that $V(t)$ is non-increasing function, it will never exceed its initial value $V(0)$. This insight is instrumental in selecting an appropriate control gain k_n that aligns with specified stability conditions. By setting k_n according to the criteria in equation (4.19), it ensures that

$$\dot{V} \leq -\gamma \|x\|^2 \quad (4.32)$$

for some $0 < \gamma < 1$. Based on the structures of (4.22) and (4.32), we can conclude that $V(t) \in \mathcal{L}_\infty$, meaning it remains bounded over time. This boundedness ensures that the components of $V(t)$ are also bounded, $e(t), e_f(t), \eta(t), \tilde{\theta}(t) \in \mathcal{L}_\infty$. Given that $e(t)$ is bounded and the desired joint trajectory vector is also bounded, it follows that the joint trajectory is $q(t) \in \mathcal{L}_\infty$. Utilizing these boundedness statements, along with equations (4.6) and (4.7), it can be shown that $\dot{e}_f(t), \dot{e}(t) \in \mathcal{L}_\infty$. This is further reinforced by the standard control theory argument of signal chasing which asserts that all signals in the closed-loop system, including the control torque input signal $\tau(t)$, specified in (4.13), are bounded. The boundedness of $\dot{e}(t)$ enables the conclusion that the tracking error signal is uniformly continuous. Furthermore, the structure of (4.32) allows us to establish that $x(t)$ and, consequently, $e(t)$ are square integrable, i.e., $x(t), e(t) \in \mathcal{L}_2$. Therefore, employing Barbalat's Lemma [57], it can be stated that $\lim_{t \rightarrow \infty} e(t) = 0$, concluding the proof. $\square$

Remark 4: Examining the structure of (4.19), one might infer that the proposed method necessitates high gain values for implementation. It is essential to note that, due to the initial constraint on $e(0)$, the initial values of $z(0)$ are also constrained. Consequently, the gain condition on the damping gain, k_n , compared to its adaptive counterparts with global stability or high-gain robust controllers, does not necessarily demand high values. From an implementation standpoint, as long as the initial tracking error value can be measured or is available, choosing the appropriate damping gain should be

straightforward.

4.4. Simulation Studies

Simulations were performed using the dynamic model of a two-link planar robot manipulator that was outlined in (3.27), using identical model parameters from (3.28). The desired trajectory from the first study in (3.29) was replicated in these simulations. Control torque during the simulations was capped at $\pm 10 \text{ Nm}$ to mimic saturation effects found in real systems. Each simulation began with the joint tracking error initialized at 3° and the parameter estimation vector, $\hat{\theta}(0)$, starting at zero.

Three simulation sets were carried out with position constraints of 4° , 8° , and 10° , respectively. In each simulation, the same controller and parameter estimation gains were used, with only the tracking constraint Δ_i being altered. The controller and estimation gains were approximately adjusted through a trial and error process

$$k = \text{diag}\{ 100 \quad 20 \}, \quad k_e = 2I_2$$

and

$$\Gamma = \text{diag}\{ 50 \quad 0.5 \quad 1 \quad 80 \quad 2.5 \}.$$

The results of the simulation studies are presented in Figures 4.1, 4.2, and 4.3. Each figure includes two sub-figures at the top showing tracking error performances, a middle sub-figure displaying parameter estimations, and two sub-figures at the bottom illustrating the control torque inputs. The control torque efforts for varying tracking constraints are analyzed in Tables 4.1 and 4.2. Table 4.1 indicates that the $\mathcal{L}_2$ norms of the initial control efforts decrease as the position constraint widens from 4° to 10° in the transient phase. Furthermore, Table 4.2 reveals that the controller effort remains relatively stable when the system achieves a steady state. Regarding the dynamic parameter estimates, the figures show that a larger position constraint on the joint errors prolongs the convergence time for the parameter estimates. Despite the controller not guaranteeing perfect estimation theoretically, the simulation outcomes demonstrate that the parameter estimation values ultimately align with the exact numerical values specified in (3.28).

In order to provide a comparative analysis in our numerical studies, we also developed

Table 4.1: $\mathcal{L}_2$ norm of the torque inputs for $0 \leq t \leq 10$ [sec]

	$\Delta_i = 4$ [deg]	$\Delta_i = 8$ [deg]	$\Delta_i = 10$ [deg]
$\{\int_0^{10} \tau_1(t)dt\}^{1/2}$	395.7179	393.3586	391.6419
$\{\int_0^{10} \tau_2(t)dt\}^{1/2}$	130.7111	123.5858	122.0114

Table 4.2: $\mathcal{L}_2$ norm of the torque inputs for $70 \leq t \leq 80$ [sec]

	$\Delta_i = 4$ [deg]	$\Delta_i = 8$ [deg]	$\Delta_i = 10$ [deg]
$\{\int_{70}^{80} \tau_1(t)dt\}^{1/2}$	338.2321	338.2384	338.0253
$\{\int_{70}^{80} \tau_2(t)dt\}^{1/2}$	95.3125	95.3121	95.3085

and simulated a similar output feedback controller without constraints, defined as

$$\tau = Y_d \hat{\theta} + k_\alpha e - k e_f. \quad (4.33)$$

Here, the controller gain matrix k_α is a diagonal matrix composed of positive constants and belongs to $\mathbb{R}^{n \times n}$. Similar to the filter design of the proposed controller in (4.3) and (4.4), the velocity surrogate signal $e_f(t)$ for the unconstrained controller is structured as follows

$$\begin{aligned} e_f &= -k e + w \\ \dot{w} &= -(k+1) e_f - k e + k_\alpha e. \end{aligned} \quad (4.34)$$

The adaptation and controller gains were set to match those of the constrained version for the simulations of the unconstrained output feedback controller in (4.33). However, the value of k_α was chosen as the mean value of the time-varying controller gain K_e from the 10° tracking constrained version in Figure 4.3. The outcomes of the unconstrained simulation are displayed in Figure 4.4. While comparing the outputs of the unconstrained and the 10° tracking error-constrained versions, no significant differences were noted. Nevertheless, as illustrated in Figures 4.1, 4.2, and 4.3, it is evident that reducing the constraint enhances control effort (as shown in Table 4.1) and accelerates the convergence of parameter estimates.

4.5. Experimental Validation

Experiments were conducted using a two-link, direct-drive, planar robot manipulator, detailed in Section 3.5 of Chapter 3, to validate the feasibility and effectiveness of the proposed position constraint adaptive output feedback controller. The schematic of the experimental setup can be seen in Figure 3.3.

In the experimental studies, a slightly faster version of the desired joint trajectory from the numerical studies was implemented

$$q_d(t) = \begin{bmatrix} 0.7 \sin(1.7t) (1 - \exp(-0.3t^3)) \\ 1.2 \sin(1.7t) (1 - \exp(-0.3t^3)) \end{bmatrix} [\text{rad}] \quad (4.35)$$

and, three sets of experiments were carried out with the position constraints for tracking errors set at 8° , 10° , and 12° . In all the experiments conducted, the same adaptation and controller gains were used. These gains, along with the parameter estimate gains, were tuned through trial and error until satisfactory results were achieved in terms of parameter adaptations and tracking error performance. Initial parameter estimates were initialized to zero, and the corresponding controller and adaptation gains were chosen as follows

$$k = \text{diag}\{ 380 \quad 120 \}, \quad k_e = 50I_2$$

and

$$\Gamma = \text{diag}\{ 9 \quad 1 \quad 2.5 \quad 20 \quad 5 \}.$$

The experimental results for 8° , 10° , and 12° constraints are depicted in Figures 4.5, 4.6, and 4.7, respectively. Each figure includes two sub-figures at the top showcasing tracking error performances, a middle sub-figure for parameter estimations, and two sub-figures at the bottom displaying the voltage outputs from the current amplifiers as control torque inputs. Similarly to the numerical studies, experiments were also conducted for the controller without joint tracking constraints as described in (4.33) and (4.34), using the same control and adaptation gains in the constrained experiments. The results of the unconstrained experiment are presented in Figure 4.8. The experimental data show that after an initial period, the tracking error performance consistently falls below $\pm 0.5^\circ$. Given the resolution of the encoders, this indicates that the tracking performance of the proposed controller is effective. Surprisingly, the parameter estimations converged to comparable values (detailed in Table 4.3). However, due to unknown dynamics,

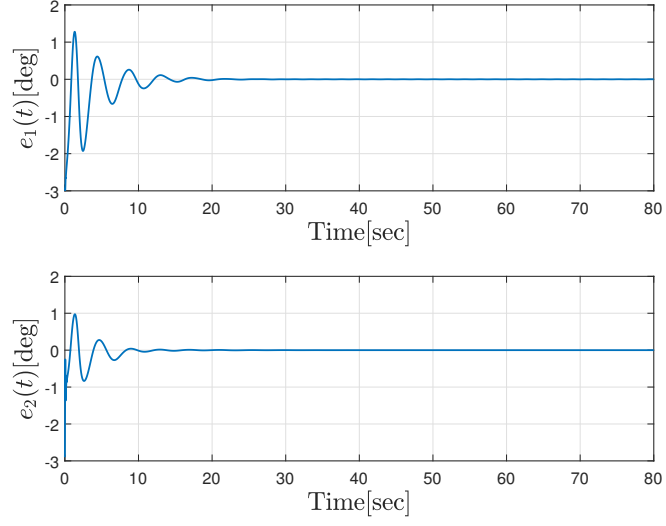
such as friction coefficients in the experimental setup, it cannot be confirmed that these estimations are centered around the actual values.

Table 4.3: Parameter Estimation Convergence in Experimental Studies

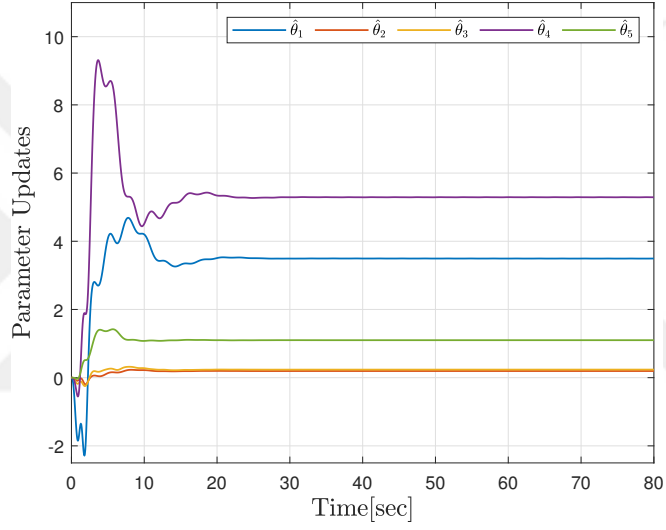
	$\Delta_i = 8^\circ$	$\Delta_i = 10^\circ$	$\Delta_i = 12^\circ$	Unc ($K_e = 1$)
θ_1	2.2 ~ 2.65	2.7 ~ 3.3	2.7 ~ 2.9	1.9 ~ 2.15
θ_2	-0.4 ~ 0.5	-0.2 ~ 0.25	0 ~ 0.3	-0.4 ~ 0
θ_3	-0.3 ~ 0.3	-0.45 ~ 0	-0.2 ~ 0	0.15 ~ 0.45
θ_4	71.5 ~ 72	68.6 ~ 69.2	68.5 ~ 68.7	71 ~ 71.5
θ_5	8 ~ 9.5	9.4 ~ 9.9	9.2 ~ 9.8	8.6 ~ 9.1

4.6. Conclusion

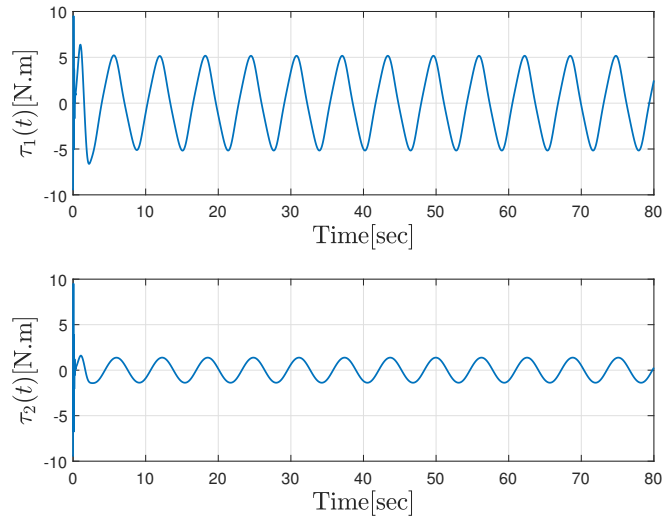
This study introduces the design of an adaptive output feedback controller for robotic manipulators with uncertain dynamics, focused on constrained joint position tracking errors. The controller guarantees that, provided the initial joint position error is kept within a predetermined limit, the tracking errors for each joint will remain within this limit and ultimately converge to zero, despite the absence of velocity measurements and the presence of dynamic uncertainties. To the best of our knowledge, this is the first instance of an adaptive output controller for robotic manipulators that enforces constrained position tracking. The imposed constraint on joint positions helps to minimize overshoot and, when applied rigorously, could be beneficial for achieving specific transient performances. The effectiveness and performance of the proposed controller are demonstrated through comprehensive simulation studies and experimental validations.



(a) The tracking error performances

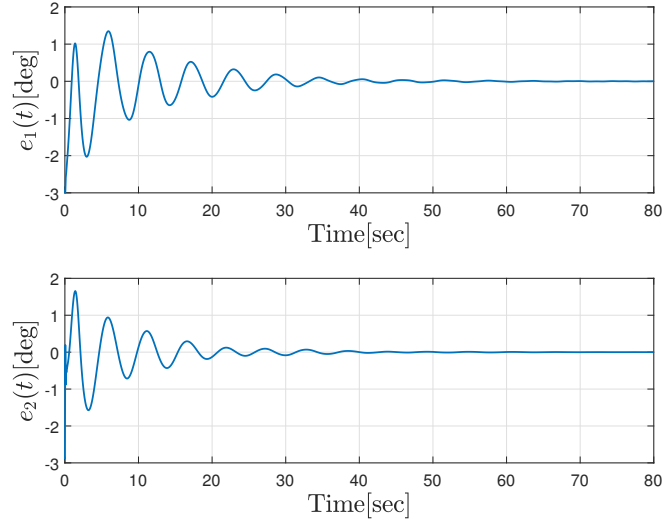


(b) Parameter estimates

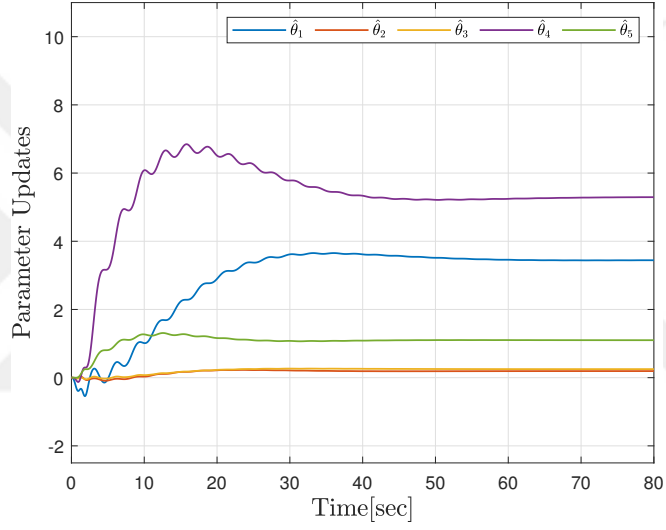


(c) Control torque input signals

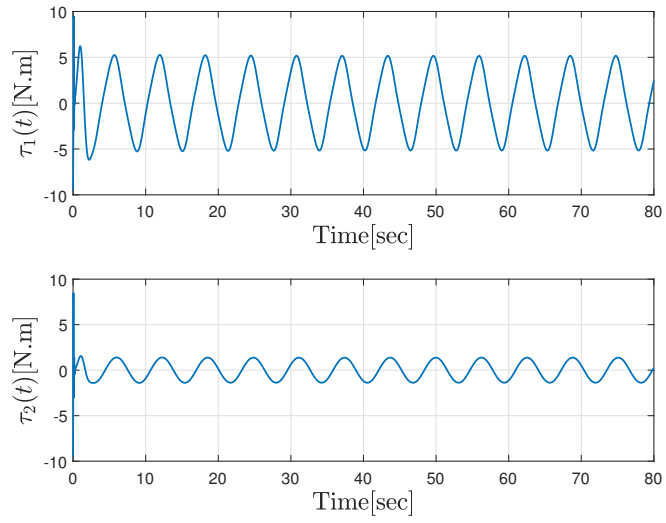
Figure 4.1: Simulation results when the position constraint on the tracking error is 4°



(a) The tracking error performances

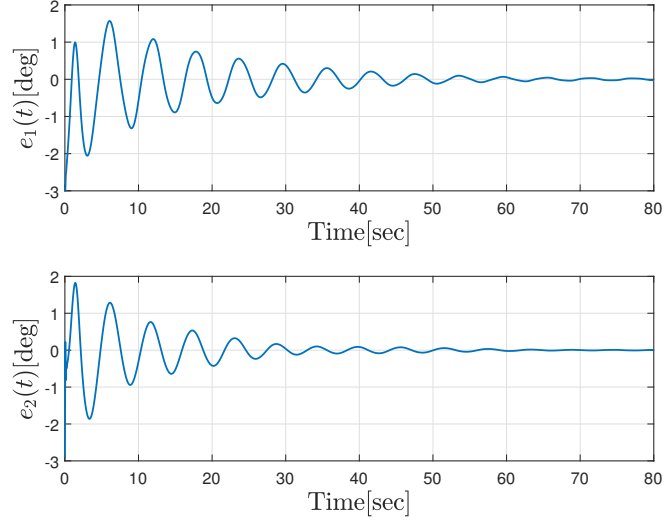


(b) Parameter estimates

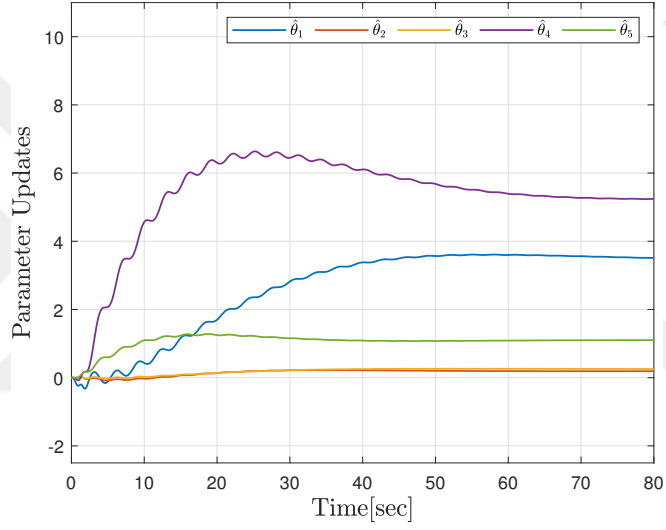


(c) Control torque input signals

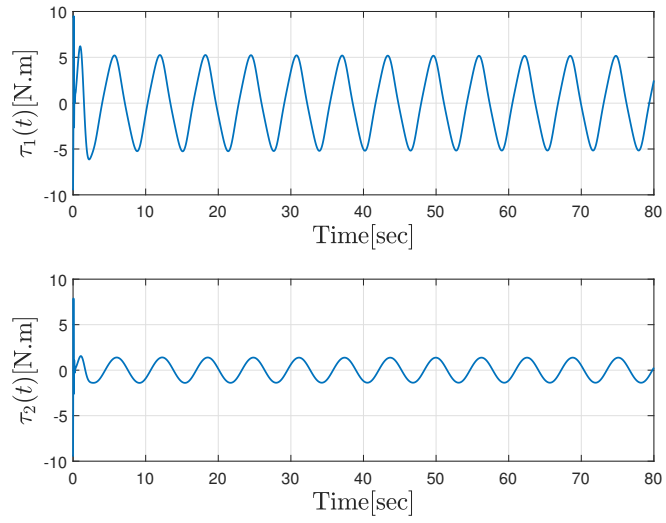
Figure 4.2: Simulation results when the position constraint on the tracking error is 8°



(a) The tracking error performances

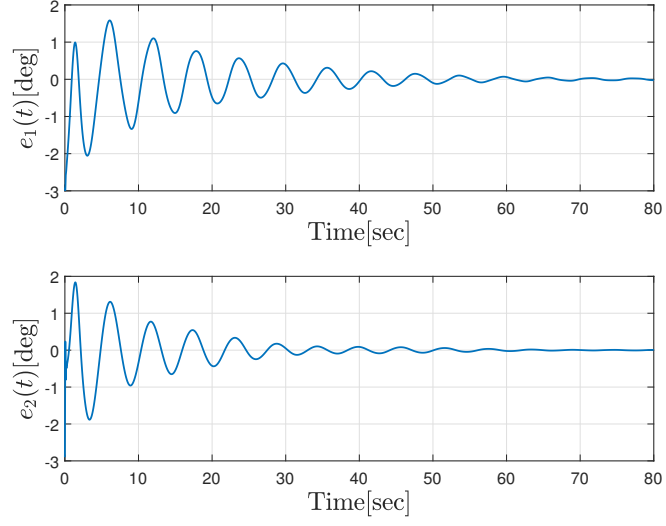


(b) Parameter estimates

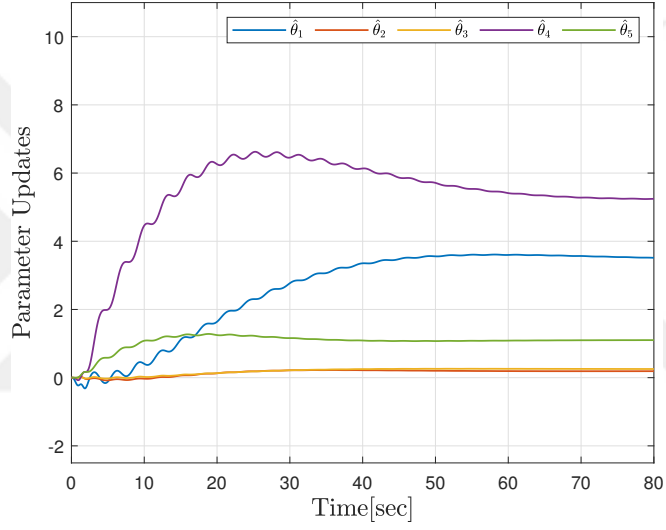


(c) Control torque input signals

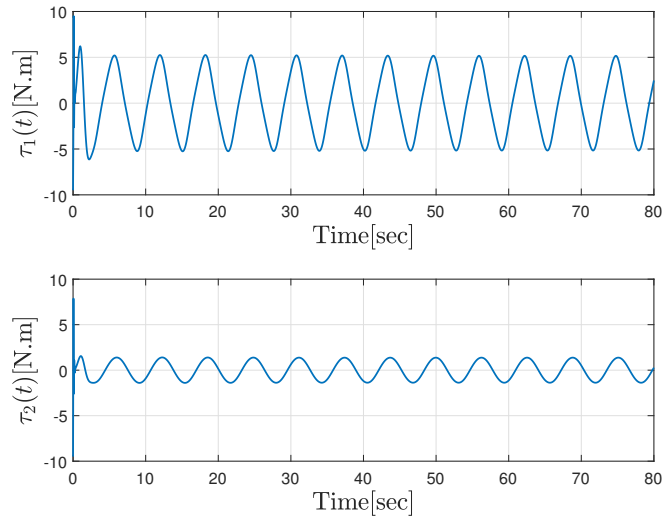
Figure 4.3: Simulation results when the position constraint on the tracking error is 10°



(a) The tracking error performances

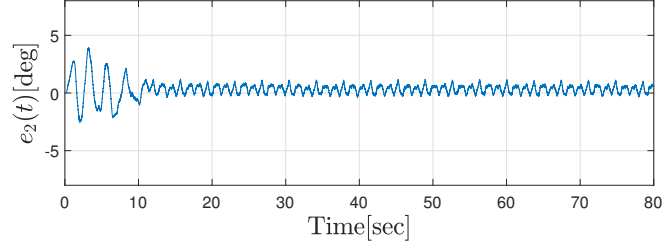
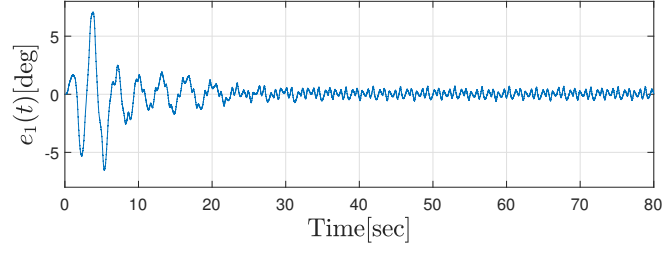


(b) Parameter estimates

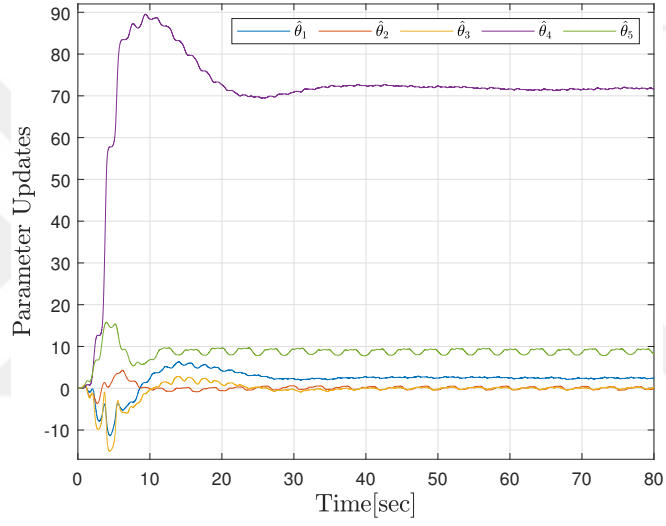


(c) Control torque input signals

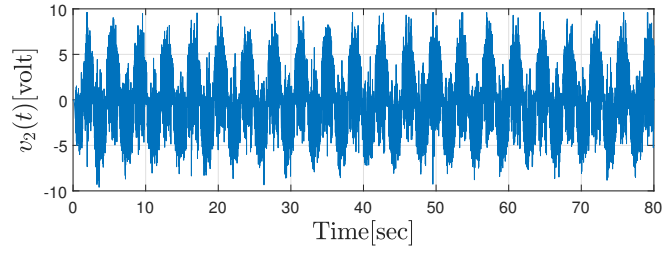
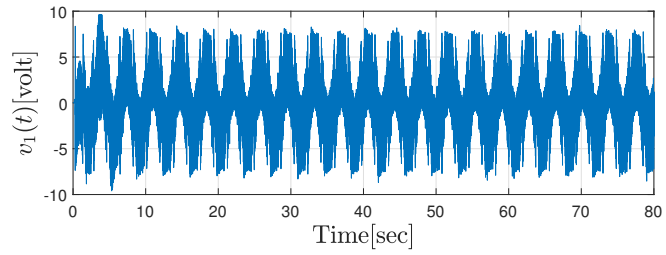
Figure 4.4: Simulation results with no position constraints



(a) The tracking error performances

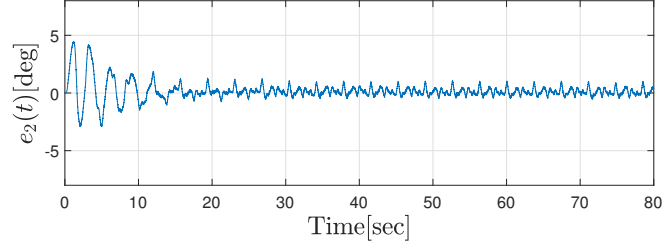
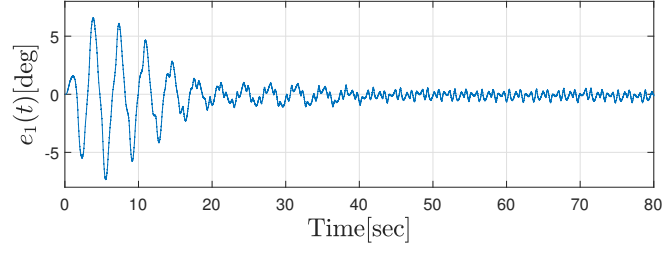


(b) Parameter estimates

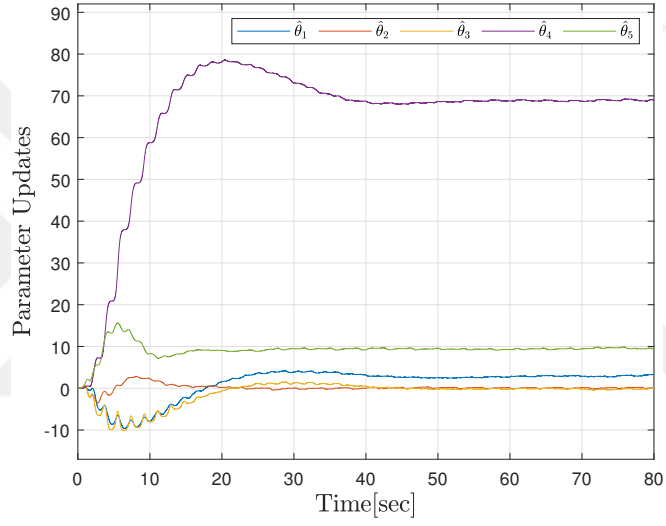


(c) Control outputs applied to the amplifiers

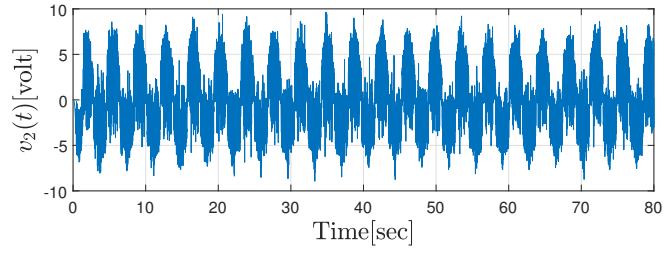
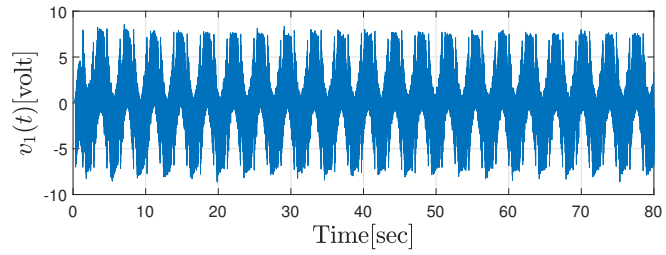
Figure 4.5: Experimental results for $\Delta_i = 8^\circ$ position error constraints



(a) The tracking error performances

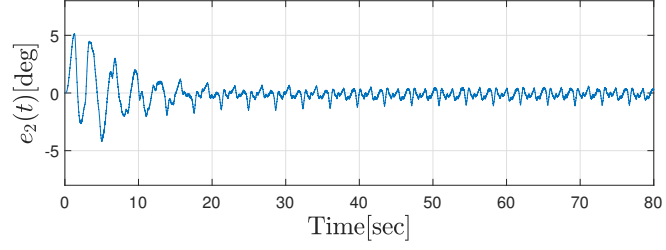
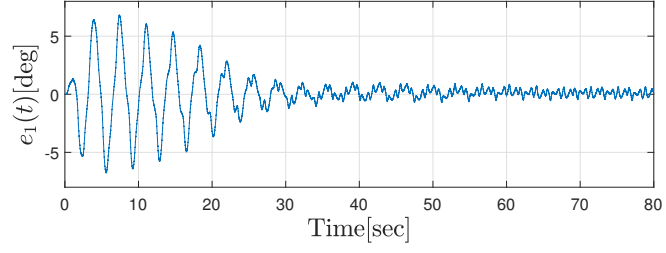


(b) Parameter estimates

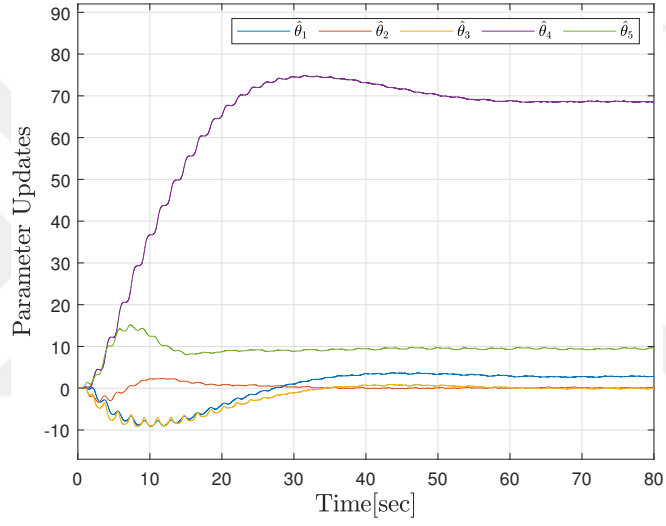


(c) Control outputs applied to the amplifiers

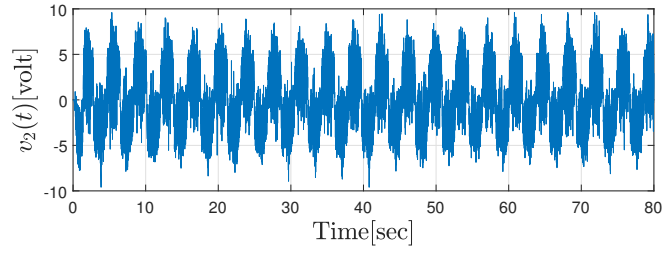
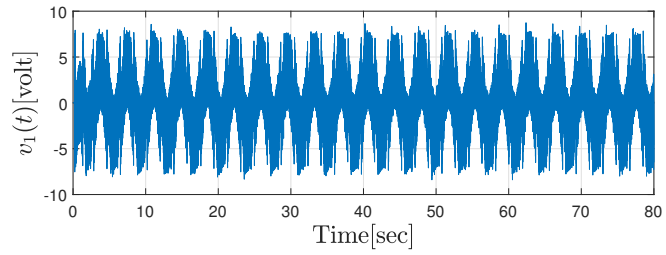
Figure 4.6: Experimental results for $\Delta_i = 10^\circ$ position error constraints



(a) The tracking error performances

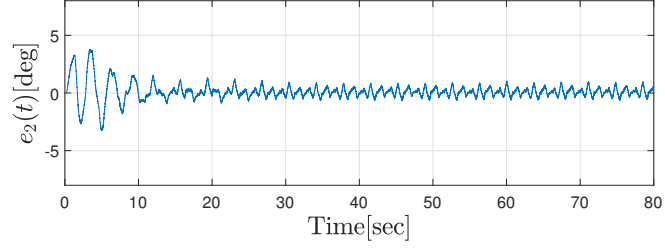
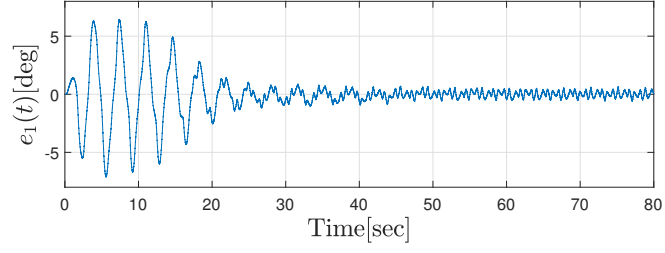


(b) Parameter estimates

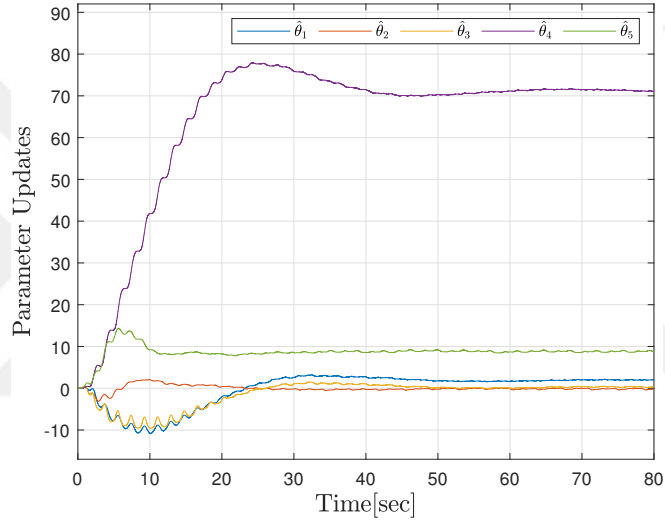


(c) Control outputs applied to the amplifiers

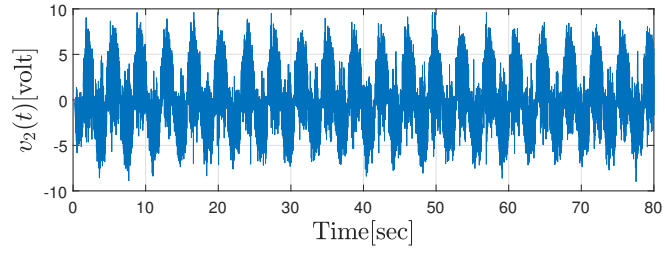
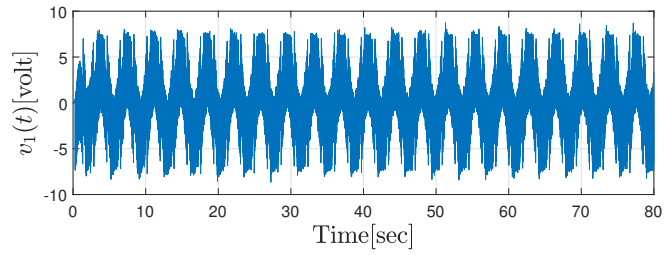
Figure 4.7: Experimental results for $\Delta_i = 12^\circ$ position error constraints



(a) The tracking error performances



(b) Parameter estimates



(c) Control outputs applied to the amplifiers

Figure 4.8: Experimental results without position error constraints ($K_e = 1$)

5. ROBUST TASK SPACE POSITION CONSTRAINT CONTROL OF KINEMATICALLY REDUNDANT MANIPULATORS

In the initial two studies, the desired trajectories were specified in joint space. In this study, however, we explore desired trajectories in task space. Moreover, this dynamic model includes the challenge of additional disturbances and non-parametric uncertainties, which complicate the design of our task space constraint tracking controller.

5.1. Problem Definition and Error System Development

The primary goal is to design the control input torque signal $\tau(t)$ so that the robot's end-effector in task space $x(t)$ accurately tracks a desired end-effector position and orientation signal $x_d(t)$. A secondary goal is to ensure that the tracking error $e_i(t)$ for each dimension i in the m -dimensional task space stays within a specified limit Δ_i at all times, described as

$$|e_i| < \Delta_i \quad \text{for all } t > 0, \quad (5.1)$$

provided that the initial tracking error $e_i(0)$ lies within the predetermined boundary defined by (5.1). Given the availability of joint position and velocity measurements, we define the task space position tracking error $e(t) \in \mathbb{R}^m$ using the equation

$$e = x_d - x, \quad (5.2)$$

assuming that the desired task space trajectory and its first two derivatives are bounded functions.

Additionally, the designed control signal should leverage the manipulator's redundancy to perform subtasks defined by at least one motion optimization measure, such as avoiding joint limits and navigating around obstacles. Adopting the method outlined in [47], we define the sub-task tracking error $e_N(t) \in \mathbb{R}^n$ as follows

$$e_N = (I_n - J^+ J)(g - \dot{q}), \quad (5.3)$$

where $g(t) \in \mathbb{R}^n$ represents an auxiliary joint velocity vector designed to enhance the manipulator's functionality, aligning with sub-task control objectives like mechanical

limit avoidance or obstacle navigation. To justify the definition of the sub-task control objective in (5.3), we begin by differentiating (5.2) with respect to time, substituting in the expression from (2.14) for $\dot{x}(t)$. We then apply the properties of the Jacobian pseudo-inverse outlined in (2.17), as well as the characteristics of the null space projection matrix $(I_n - J^+J)$ detailed in (2.19). As a result, we derive $\dot{e}$ expressed as

$$\dot{e} = -\alpha e + J \left(J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g - \dot{q} \right), \quad (5.4)$$

where the term $\pm\alpha e$ is incorporated to aid in the formulation of the control strategy, with $\alpha \in \mathbb{R}^{m \times m}$ being a diagonal, positive definite gain matrix. Building on (5.4) and the following analysis, we define the filtered tracking error, represented by $r(t) \in \mathbb{R}^n$, using the equation

$$r = J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g - \dot{q}. \quad (5.5)$$

The final expression for the closed-loop task space position tracking error system is given by

$$\dot{e} = -\alpha e + Jr. \quad (5.6)$$

The equation in (5.6) demonstrates that controlling the tracking error $e(t)$ requires regulating the filtered tracking error $r(t)$.

To establish the open-loop dynamics for $r(t)$, we begin by differentiating (5.5) with respect to time and then pre-multiplying by the inertia matrix $M(q)$. Substituting the dynamic model from (2.11) and considering the terms $\pm V_m(q, \dot{q})r$, we derive the following equation

$$M(q)\dot{r} = -V_m(q, \dot{q})r + \omega_r - \tau \quad (5.7)$$

where $\omega_r(t)$ is defined by the expression

$$\begin{aligned} \omega_r \triangleq & M(q) \frac{d}{dt} \left\{ J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g \right\} \\ & + V_m(q, \dot{q}) \left\{ J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g \right\} \\ & + G(q) + F_d \dot{q} + \xi_d. \end{aligned} \quad (5.8)$$

Using equation (2.12), it's important to note that $\omega_r(t)$ can be separated into two distinct components as shown below

$$\omega_r = Y\theta + \xi_r. \quad (5.9)$$

Here, $Y(\ddot{x}_d, \dot{x}_d, x, q, \dot{q}, \dot{g}, g) \in \mathbb{R}^{n \times p}$ represents a regression matrix, $\theta \in \mathbb{R}^p$ represents the constant system parameters, and $\xi_r(t) \in \mathbb{R}^n$ comprises the residual elements of ω_r that are not linearly parameterizable.

Additionally, we can show that the sub-task tracking error (5.3) can be effectively controlled by managing the filtered tracking error. By pre-multiplying (5.5) with the null space projection matrix $(I_n - J^+ J)$ and leveraging its properties as outlined in (2.19), we arrive at the following equation

$$e_N = (I_n - J^+ J)r. \quad (5.10)$$

From (5.10), it becomes clear that by regulating the filtered tracking error $r(t)$, control over the sub-task tracking error $e_N(t)$ is also achieved.

5.2. Controller Design

Expanding upon the error system and the subsequent stability analysis, the control input $\tau(t)$ is designed as follows

$$\tau = Y\hat{\theta} + K_r r + v_R + J^T K_e e, \quad (5.11)$$

where $\hat{\theta} \in \mathbb{R}^p$ is the approximate estimates the constant parameter vector θ as detailed in (5.9), $K_r \in \mathbb{R}^{n \times n}$ represents a constant diagonal controller gain matrix, and $v_R(t) \in \mathbb{R}^n$ is a defined auxiliary robust control term

$$v_R = \frac{r\rho^2}{\|r\|\rho + \varepsilon}. \quad (5.12)$$

Here, $\rho(\cdot)$ is a positive function crafted to ensure

$$\rho \geq \|Y\tilde{\theta}\| + \|\xi_r\|, \quad (5.13)$$

and ε is a small positive scalar constant. In this context, $\tilde{\theta} \in \mathbb{R}^p$ as mentioned in (5.13) represents the parameter estimation error, which is defined as the difference between the actual parameters and their estimates

$$\tilde{\theta} = \theta - \hat{\theta}. \quad (5.14)$$

In the analysis that follows, we assume that the norm of the parameter estimation error, $\|\tilde{\theta}\|$, is bounded by ζ_θ , where $\zeta_\theta \in \mathbb{R}$ is a known positive constant. Furthermore,

$K_e(e) \in \mathbb{R}^{m \times m}$ denotes a control gain matrix that is dependent on the tracking error, positive definite, and diagonal. This study introduces a control gain matrix K_e that varies with the tracking error, setting it apart from similar research in the field. This matrix enables a novel stability analysis, ensuring that the tracking error remains within predefined bounds. The formulation of K_e is given by

$$K_e = \text{diag} \left\{ \frac{k_{ei}}{\Delta_i^2 - e_i^2} \right\} \quad (5.15)$$

where k_{ei} are constant controller gains for each dimension i of the task space, ranging from 1 to m . Δ_i represents the predefined limits for the tracking error in each dimension, as detailed in (5.1). This configuration ensures that the gain increases as the error e_i approaches its limit Δ_i , intensifying the controller response to keep the error within the specified bounds.

5.3. Stability Analysis

Theorem 5.1: The robust control strategy introduced in (5.11) guarantees that both the task-space position error $e(t)$ and the sub-task tracking error $e_N(t)$ are uniformly ultimately bounded. Moreover, the controller design in (5.11), when used in conjunction with the error-dependent control gain matrix in (5.15), ensures that task-space tracking errors are kept within established boundaries, assuming the initial errors in task space begin within these specified limits.

Proof. The analysis starts with the following introduction of BLF $V(r, e) \in \mathbb{R}$

$$V \triangleq \frac{1}{2} r^T M(q) r + \sum_{i=1}^n \frac{k_{ei}}{2} \ln \left(\frac{\Delta_i^2}{\Delta_i^2 - e_i^2} \right). \quad (5.16)$$

This function is positive definite and radially unbounded provided that the initial values of the joint tracking error entries satisfy $|e_i(0)| < \Delta_i$ for all $i \in 1, \dots, m$. By applying the inequalities in (2.2), the bounds for (5.16) are as follows

$$\frac{1}{2} a \|z\|^2 \leq V(t) \leq \frac{1}{2} b \|z\|^2 \quad (5.17)$$

where $z \triangleq [r^T \ e^T]^T$, $a \triangleq \min(m_1, 1)$, and $b \triangleq \max(m_2, 1)$. Here, m_1 and m_2 refer to the constants previously defined in (2.2). By taking the time derivative of V and incorporating the open-loop dynamics of $r(t)$ from (5.7), the time derivative of V is

given by

$$\dot{V} = \frac{1}{2} r^T \dot{M}(q) r + r^T \{-V_m(q, \dot{q}) r + \omega_r - \tau\} + e^T K_e \dot{e}, \quad (5.18)$$

where the term $e^T K_e \dot{e}$ can be expanded by utilizing the diagonal structure of K_e in (5.15) as follows

$$\sum_{i=1}^m k_{ei} \frac{e_i \dot{e}_i}{\Delta_i^2 - e_i^2} = e^T K_e \dot{e}. \quad (5.19)$$

By leveraging the skew symmetry property of the robot dynamics in (2.3), incorporating the controller defined in (5.11), cancelling common terms, and substituting $\dot{e}(t)$ from (5.6), the derivative of the Lyapunov function simplifies to

$$\dot{V} = r^T \left(Y \tilde{\theta} + \xi_r - \frac{r \rho^2}{\|r\| \rho + \varepsilon} - K_r r \right) - e^T K_e \alpha e. \quad (5.20)$$

Using the inequality from (5.13), an upper bound for $\dot{V}$ can be determined as follows

$$\dot{V} \leq \left[\rho \|r\| - \frac{\|r\|^2 \rho^2}{\|r\| \rho + \varepsilon} \right] - K_r \|r\|^2 - \sum_{i=1}^m \alpha_i \|e_i\|^2 \frac{k_{ei}}{\Delta_i^2 - e_i^2}. \quad (5.21)$$

The bracketed term in (5.21) can be reformulated as

$$\rho \|r\| - \frac{\|r\|^2 \rho^2}{\|r\| \rho + \varepsilon} = \rho \|r\| \left(1 - \frac{\|r\| \rho}{\|r\| \rho + \varepsilon} \right) = \varepsilon \frac{\|r\| \rho}{\|r\| \rho + \varepsilon} \leq \varepsilon. \quad (5.22)$$

Utilizing this redefined expression, the upper bound for (5.21) can be further refined to

$$\dot{V} \leq -\beta \|z\|^2 + \varepsilon \quad (5.23)$$

where

$$\beta \triangleq \min \left\{ \lambda_{\min}(K_r), \min \left\{ \alpha_i \frac{k_{ei}}{\Delta_i^2 - e_i^2} \right\} \middle|_{i \in \{1, 2, \dots, m\}} \right\},$$

with $\lambda_{\min}$ denoting the smallest eigenvalue of a matrix. Using the upper bound of $V(t)$ in (5.17), (5.23) can be further refined to

$$\dot{V} \leq -\gamma V + \varepsilon \quad (5.24)$$

where $\gamma \triangleq \frac{2\beta}{b}$. By solving the differential inequality in (5.24), the solution for $V(t)$ is found to be

$$V(t) \leq V(0) \exp(-\gamma t) + \frac{\varepsilon}{\gamma} (1 - \exp(-\gamma t)). \quad (5.25)$$

Applying the bounds provided by (5.17) to the solution in (5.25), an upper bound for $\|z(t)\|$ is obtained as follows

$$\|z(t)\| \leq \sqrt{\frac{b}{a} \|z(0)\|^2 \exp(-\gamma t) + \frac{2\varepsilon}{a\gamma} (1 - \exp(-\gamma t))}. \quad (5.26)$$

Here, a , b are constants defined in (5.17). Given the upper bounds derived from (5.26) and considering the composition of $z(t)$, which includes both the filtered tracking error $r(t)$ and the position tracking error $e(t)$, we can establish bounds for $e(t)$ as follows

$$\|e(t)\| \leq \sqrt{\frac{b}{a}\|z(0)\|^2 \exp(-\gamma t) + \frac{2\varepsilon}{a\gamma}(1 - \exp(-\gamma t))}. \quad (5.27)$$

Given the established boundedness of $J(q)$ and $J^+(q)$ as noted in (5.10), and leveraging the same bound for the overall system state vector $z(t)$, we can deduce a bound for the sub-task tracking error $e_N(t)$ as follows

$$\|e_N(t)\| \leq \|I_n - J^+ J\|_{i\infty} \sqrt{\frac{b}{a}\|z(0)\|^2 \exp(-\gamma t) + \frac{2\varepsilon}{a\gamma}(1 - \exp(-\gamma t))}. \quad (5.28)$$

Based on the bounds provided in (5.27) and (5.28), it is clear that the conditions outlined in the theorem are met. These bounds demonstrate that both the task-space position error $e(t)$ and the sub-task tracking error $e_N(t)$ are uniformly ultimately bounded. Given that the trajectory signals $x_d(t)$, $\dot{x}_d(t)$, $\ddot{x}_d(t)$, as well as $g(t)$ and $\dot{g}(t)$ are all bounded, and considering that the kinematic and dynamic terms such as $M(q)$, $V_m(q, \dot{q})$, $G(q)$, $J(q)$, and $J^+(q)$ are bounded for all configurations of $q(t)$, it is feasible to demonstrate using standard signal analysis techniques that all system signals remain bounded. While the boundedness of $q(t)$ itself cannot be assured due to the self-motion characteristic of redundant manipulators, it is crucial to note that the boundedness of all other signals associated with the manipulator's kinematics, dynamics, and control processes is guaranteed, irrespective of the specific behavior of $q(t)$, as $q(t)$ only appears within trigonometric functions.

The second component of the Lyapunov function introduced in (5.16) is crafted to enforce specific constraints on the position tracking error $e(t)$. As illustrated in Figure 3.1, this logarithmic component of the function experiences infinite growth as its variable $e(t)$ nears the Δ boundaries. Given the proof that the positive-valued function $V(\cdot)$ specified in (5.16) and its components $r(t)$ and $e(t)$ are bounded, and considering that the initial error signals $e_i(0)$ begin within the predetermined range $(-\Delta_i, +\Delta_i)$, it follows that the tracking error signals $e_i(t)$ will remain within these limits [12]. This relationship underscores the effectiveness of the control design in maintaining the tracking errors within safe operational boundaries. $\square$

5.4. Simulation Studies

Simulation studies were conducted on a planar robotic manipulator featuring three DOF and revolute joints. The dynamics of the robot are encapsulated by the following matrices

$$M(q) = \begin{bmatrix} \beta_1 + 2p_1c_2 + p_2c_{23} + p_3c_3 & \beta_2 + p_1c_2 + p_2c_{23} & \beta_3 + p_2c_{23} + p_3c_3 \\ \beta_2 + p_1c_2 + p_2c_{23} & \beta_2 + 2p_3c_3 & \beta_3 + p_3c_3 \\ \beta_3 + p_2c_{23} + p_3c_3 & \beta_3 + p_3c_3 & \beta_3 \end{bmatrix},$$

$$V_m(q, \dot{q}) = \begin{bmatrix} V_{m11} & V_{m12} & V_{m13} \\ V_{m21} & V_{m22} & V_{m23} \\ V_{m31} & V_{m32} & V_{m33} \end{bmatrix},$$

$$F(\dot{q}) = \begin{bmatrix} f_{d1} & 0 & 0 \\ 0 & f_{d2} & 0 \\ 0 & 0 & f_{d3} \end{bmatrix}.$$

In this model, the parameters β_1 , β_2 , β_3 , p_1 , p_2 , and p_3 represent mass parameters, and the trigonometric functions c_i , s_i , c_{ij} , and s_{ij} denote $\cos(q_i)$, $\sin(q_i)$, $\cos(q_i + q_j)$, and $\sin(q_i + q_j)$, respectively. The elements of the centripetal-Coriolis matrix V_{mij} , ($i, j \in 1, 2, 3$) maintain a skew-symmetric relationship with the inertia matrix, previously described in (2.3), as follows

$$\begin{aligned} V_{m11} &= -(p_1s_2 + p_2s_{23})\dot{q}_2 - (p_2s_{23} + p_3s_3)\dot{q}_3, \\ V_{m12} &= -(p_1s_2 + p_2s_{23})(\dot{q}_1 + \dot{q}_2) - (p_2s_{23} + p_3s_3)\dot{q}_3, \\ V_{m13} &= (p_2s_{23} + p_3s_3)(-\dot{q}_1 + \dot{q}_2 + \dot{q}_3), \\ V_{m21} &= (p_1s_2 + p_2s_{23})\dot{q}_1 + (p_3s_3)\dot{q}_3, \\ V_{m22} &= -(p_2s_{23} + p_3s_3)\dot{q}_3, \\ V_{m23} &= -p_3s_3(3\dot{q}_1 + \dot{q}_2 + \dot{q}_3), \\ V_{m31} &= (p_1s_2 + p_2s_{23})\dot{q}_1 - (p_3s_3)\dot{q}_2, \\ V_{m32} &= p_3s_3(3\dot{q}_1 + \dot{q}_2), \\ V_{m33} &= 0. \end{aligned} \tag{5.29}$$

The dynamic friction coefficients for the joints are represented by f_{d1} , f_{d2} and f_{d3} . The masses of the manipulator's links were set as follows: link 1 weighs 3.6 kg, link 2

weighs 2.6 *kg*, and link 3 weighs 2 *kg*. The lengths of these links were also specifically chosen: 0.4 *m* for link 1, 0.36 *m* for link 2, and 0.3 *m* for link 3. With these numerical values, the system parameters were assigned the following numerical values

$$\begin{aligned}\beta_1 &= 1.1956 \text{ kg.m}^2 & \beta_2 &= 0.3946 \text{ kg.m}^2 & \beta_3 &= 0.0512 \text{ kg.m}^2 \\ p_1 &= 0.4752 \text{ kg.m}^2 & p_2 &= 0.1280 \text{ kg.m}^2 & p_3 &= 0.1152 \text{ kg.m}^2 \\ f_{d1} &= 5.3 \text{ Nm.s} & f_{d2} &= 2.4 \text{ Nm.s} & f_{d3} &= 1.1 \text{ Nm.s} .\end{aligned}\quad (5.30)$$

Subsequently, these parameters are compiled into the constant system parameter vector θ defined in (5.7) for use in the control and simulation models, as described by

$$\theta = [\beta_1 \quad \beta_2 \quad \beta_3 \quad p_1 \quad p_2 \quad p_3 \quad f_{d1} \quad f_{d2} \quad f_{d3}]^T .$$

In every simulation scenario, the manipulator was initialized with the following joint positions

$$q_1 = -0.23 \text{ rad}, \quad q_2 = 1.36 \text{ rad}, \quad q_3 = 1.38 \text{ rad}. \quad (5.31)$$

The trajectory for the desired task space was set as

$$x_d(t) = \begin{bmatrix} 0.3 + 0.2\cos(t)(1 - \exp(-0.3t^3)) \\ 0.4 + 0.1\sin(t)(1 - \exp(-0.3t^3)) \end{bmatrix}. \quad (5.32)$$

Additionally, the auxiliary signal $g(t)$, as referenced in [47], was defined as

$$g = -2(q_3 - q_2 + 0.5q_1)[1 \quad -1 \quad 1]^T. \quad (5.33)$$

It should be noted that the auxiliary signal $g(t)$ is actually the negative gradient of the function

$$f = ((q_3 - 0.5q_2) - 0.5(q_2 - q_1))^2. \quad (5.34)$$

This setup indicates that the control strategy is designed to optimize the joint configuration so that $(q_3 - 0.5q_2) = 0.5(q_2 - q_1)$. The control and adaptation gain matrices were tuned to achieve satisfactory task space position tracking performance using the following values

$$\begin{aligned}\alpha &= 5I_2, \quad K_r = \text{diag}\{15 \quad 10 \quad 5\}, \quad k_{e1} = k_{e2} = 1, \\ \rho &= 1, \quad \varepsilon = 0.05 .\end{aligned}\quad (5.35)$$

In this configuration, the values for the system parameter estimates were intentionally set at half of their actual values.

Four different simulation runs were conducted to assess the robustness and effectiveness of the control strategy, with consistent settings for the control gains (5.35), the trajectory profile (5.32), and the initial joint positions (5.31) across all trials. These initial joint positions were chosen to ensure that the tracking errors began within the predetermined regions, accommodating even the tightest Δ constraints for task space position error.

The primary variable in each simulation was the choice of the Δ constraints applied to the task space position error, as specified in (5.15). These constraints directly influenced the error-dependent control gain K_e , which adjusts dynamically to keep the position errors within the set bounds. For example, a constraint setting of $\Delta = \text{diag}\{0.3 \ 1.2\} \text{ cm}$ means that the error e_1 would consistently be contained within $\pm 0.3 \text{ cm}$ and e_2 within $\pm 1.2 \text{ cm}$.

Tables 5.1 and 5.2 provide a comprehensive overview of the control efforts exerted under various constraint settings. The first column in each table is dedicated to the unconstrained simulation scenario where the error-dependent gain matrix, ($K_e = 1$), remains constant regardless of changes in position errors. In contrast, the subsequent columns in these tables showcase the results under different Δ constraint configurations, affecting how the error-dependent control gain K_e adapts to maintain the position errors within specified limits.

Table 5.1 focuses on illustrating the $\mathcal{L}_2$ norms of the controller efforts during the initial phase of each simulation. This data clearly demonstrates that the total control effort increases as the position constraints on the task space error become more restrictive. This increase in control effort reflects the controller's response to more stringent requirements for maintaining task space accuracy.

Table 5.2 details the $\mathcal{L}_2$ norms of the controller efforts once the system has reached a steady state. Interestingly, the data show only minor variations in control effort across different simulations at this stage. This uniformity suggests that, once stabilized, the system's errors are small enough relative to the set constraints, leading to a similar magnitude of control effort irrespective of the constraint Δ settings.

The torque input signal plots presented in Figure 5.1 further support these findings, providing visual confirmation of the varying control efforts required during the initial phases versus the steady state, and underscoring the consistency of the control strategy's

effectiveness once the system stabilizes.

Table 5.1: $\mathcal{L}_2$ norm of the torque inputs for $0 \leq t \leq 5$ [sec]

	$Unc (K_e = 1)$	$diag\{0.9 \ 3.6\}$	$diag\{0.6 \ 2.4\}$	$diag\{0.3 \ 1.2\}$
$\{\int_0^5 \tau_1(t)dt\}^{1/2}$	76.1233	78.0169	87.5897	113.0088
$\{\int_0^5 \tau_2(t)dt\}^{1/2}$	24.0600	33.6861	53.2038	93.0462
$\{\int_0^5 \tau_3(t)dt\}^{1/2}$	10.2997	12.8117	18.0882	48.6135

Table 5.2: $\mathcal{L}_2$ norm of the torque inputs for $55 \leq t \leq 60$ [sec]

	$Unc (K_e = 1)$	$diag\{0.9 \ 3.6\}$	$diag\{0.6 \ 2.4\}$	$diag\{0.3 \ 1.2\}$
$\{\int_{55}^{60} \tau_1(t)dt\}^{1/2}$	57.3895	56.7613	56.4942	56.3340
$\{\int_{55}^{60} \tau_2(t)dt\}^{1/2}$	25.8498	26.7254	27.2466	27.9030
$\{\int_{55}^{60} \tau_3(t)dt\}^{1/2}$	11.9488	12.7754	13.1375	13.5435

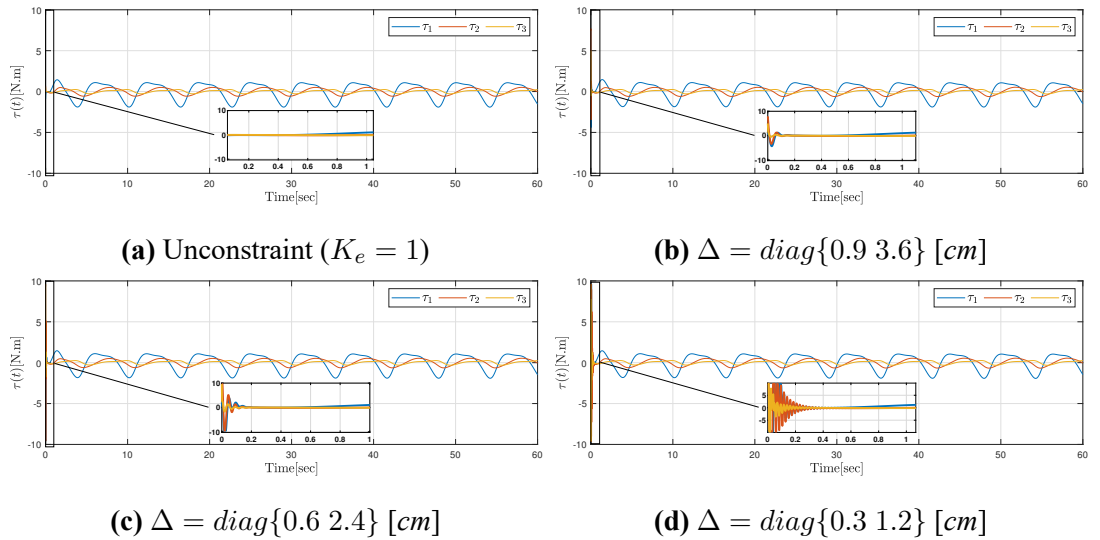


Figure 5.1: Simulation results of the torque input signals under various Δ constraints

The impact of varying Δ constraint choices on the trajectory tracking performance of

the end-effector is illustrated in Figure 5.2. These plots, which compare the desired and actual trajectories, demonstrate that tighter constraints on the position errors lead to quicker convergence and enhanced overall tracking accuracy. This result is a direct outcome of the more stringent control efforts enforced by tighter Δ values, which compel the controller to more aggressively correct deviations from the desired trajectory.

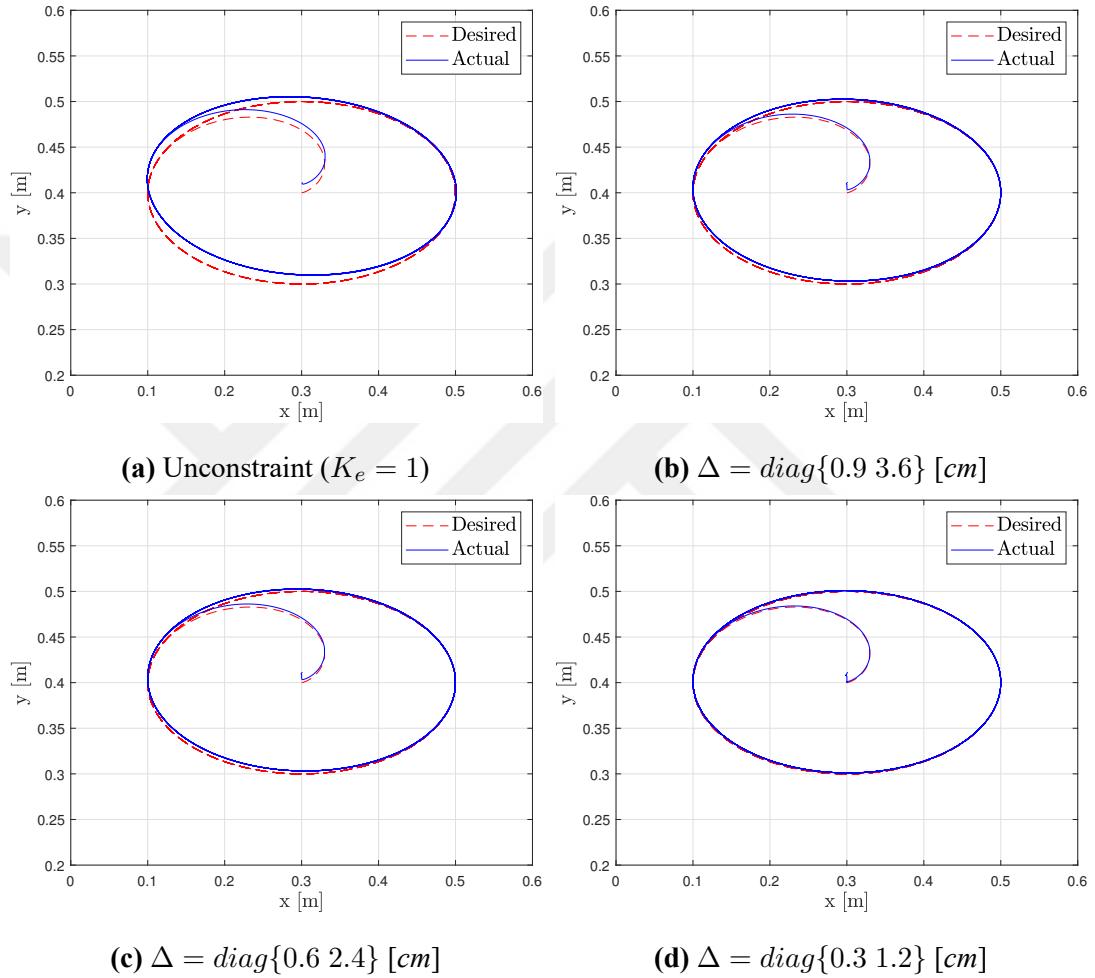


Figure 5.2: Simulation results showing constraint choices' effect on the actual trajectory

Additionally, the task space position tracking errors depicted in Figure 5.3 indicate that tighter constraints not only improve trajectory adherence but also effectively minimize the task space tracking errors. This enhancement occurs despite the presence of uncertainties within the manipulator's dynamics, underscoring the robustness of the control approach under stringent conditions.

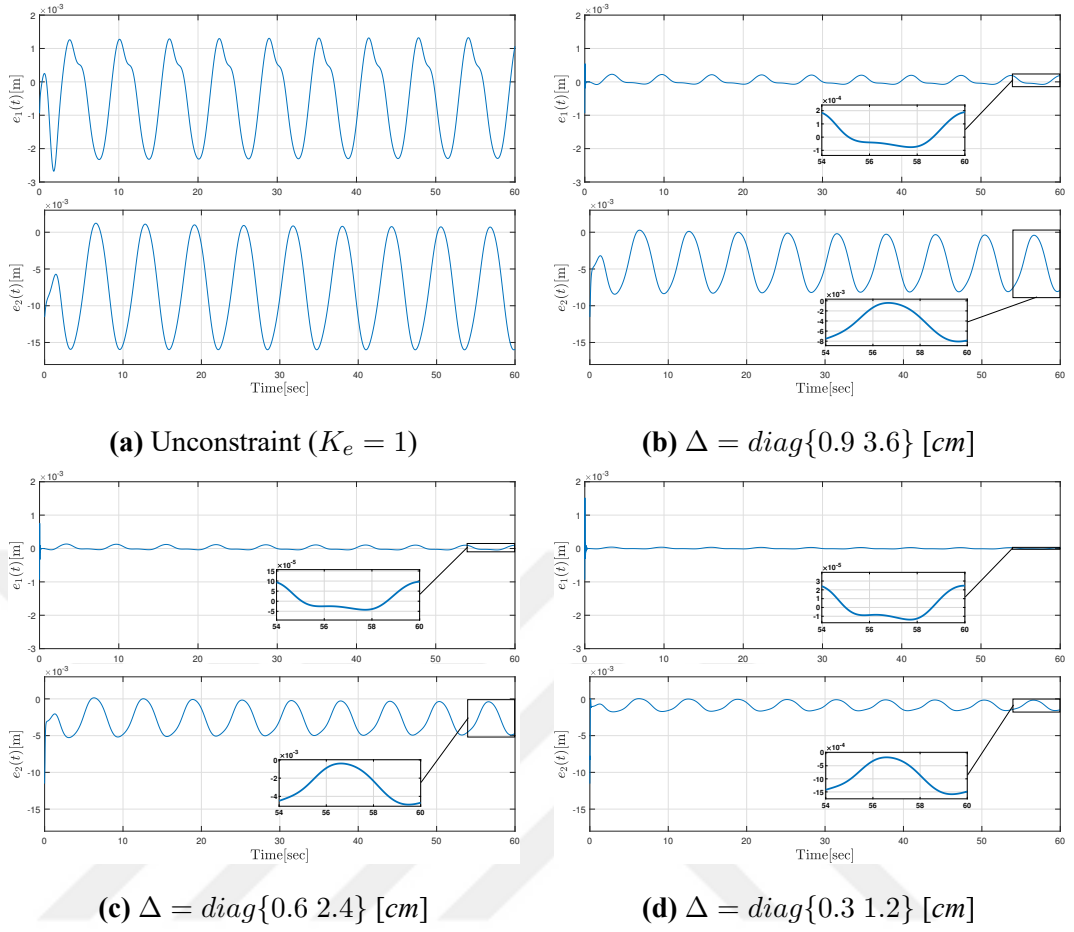


Figure 5.3: Simulation results of the task space position errors for different constraints

A comparative analysis of the tracking error results from two specific simulations is presented in Figure 5.4. This comparison is crucial for understanding the system's behavior under different control settings, particularly with the goal of ensuring that the error e_2 does not exceed $\pm 1.2 \text{ cm}$. In the scenario where the tracking error is unconstrained ($K_e = 1$), it is observed that the errors frequently breach the predefined limits. Conversely, when the constraints are set to $\Delta = \text{diag}\{0.3 \ 1.2\} \text{ [cm]}$, the error-dependent gain matrix K_e in (5.11) dynamically adjusts in response to the error magnitudes, effectively maintaining the errors within the designated boundaries.

5.5. Experimental Validation

Although our simulation results for the proposed controller were encouraging, practical applications often require robustness to handle uncertainties. To this end, we under-

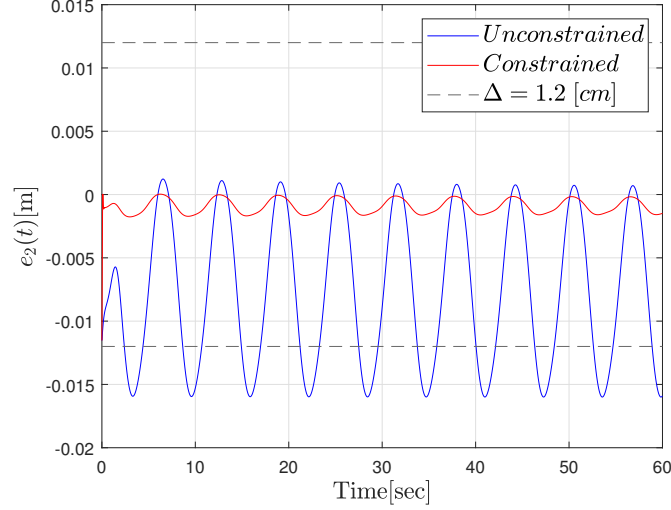


Figure 5.4: Comparative analysis of e_2 signals in simulation studies: unconstrained $K_e = 1$ versus constrained $\Delta = \text{diag}\{0.3 \ 1.2\}$ [cm]

took experimental studies using a three-link, direct-drive, planar robot manipulator constructed in the Control Applications and Robotics Laboratory at Gebze Technical University. This robot features aluminum links measuring 15 cm, 10 cm, and 6.5 cm in length for the first, second, and third links, respectively. The weights of these links are 1.65 kg, 0.86 kg, and 0.13 kg. Each link is driven by a Maxon DC motor equipped with an encoder capable of delivering 5000 counts per revolution. The motors are controlled using custom-built linear amplifiers set to unity gain. To prepare for the experiments, we conducted three separate test runs where a basic linear controller initially positioned the robot at

$$q = [0 \ 80 \ 110]^T [\text{deg}]. \quad (5.36)$$

The chosen initial position is strategically important for two reasons: first, it places the robot away from singular points, and second, it is crucial for the success of our BLF strategy, ensuring that error signals start within predetermined regions in all experimental runs. The desired task space trajectory was selected as follows

$$x_d(t) = \begin{bmatrix} 15 + 5 \cos(t) \\ 10 + 4 \sin(t) \end{bmatrix} [\text{cm}], \quad (5.37)$$

with the same auxiliary signal $g(t)$ used in the simulation studies. The controller gains were meticulously adjusted to achieve a satisfactory task space position tracking and transient performance across all experimental setups. The controller gains for the

experiments were set as follows

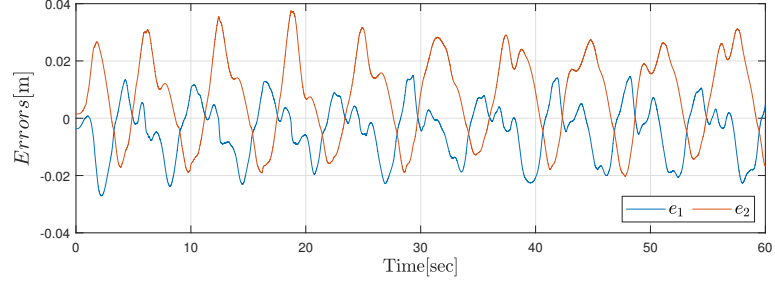
$$\begin{aligned}\alpha &= 10I_2, \quad K_r = \text{diag}\{40 \quad 30 \quad 10\}, \quad k_{e1} = k_{e2} = 1, \\ \rho &= 2, \quad \varepsilon = 0.08.\end{aligned}\tag{5.38}$$

Since we constructed the robot ourselves, we have approximate measurements of the lengths and weights of its links. The first six dynamic parameters are related to this information, allowing us to calculate estimated values for the first six θ values. As an added challenge, we decided to use half of these calculated values as the first six dynamic parameters in our model. The remaining dynamic elements, including those due to friction and gravity, were treated as entirely unknown and not linearly separable, represented as the ζ term in (2.12).

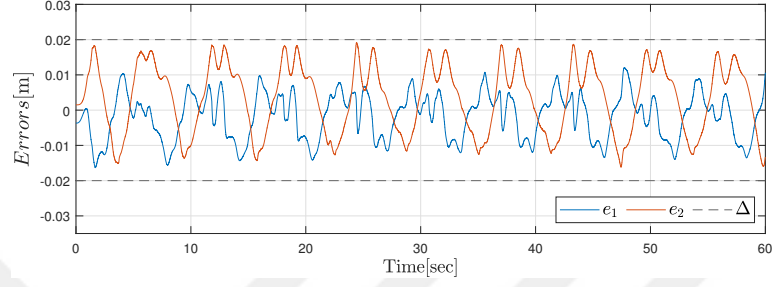
The distinction among these three experiments lies in the selection of the constraint. In the initial experiment, a constant gain of $K_e = 1$ was used instead of the error-dependent K_e gain defined in (5.15). Consequently, no constraints were imposed on errors, allowing them to reside in a predetermined region. For the second and third experiments, constraints of $\Delta_1 = \Delta_2 = \pm 2$ cm and $\Delta_1 = \Delta_2 = \pm 0.5$ cm were chosen, respectively. The position error signal and torque input signals for these experiments are depicted in Figures 5.5 and 5.6. As evident in these figures, the constraints were consistently adhered to. However, it is noteworthy that as the constraints became more stringent, the control effort required to maintain them increased.

5.6. Conclusion

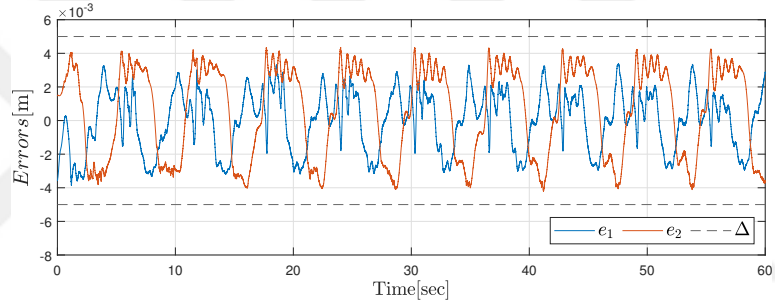
This study introduces an innovative model-based nonlinear robust control approach tailored for kinematically redundant robot manipulators with task space position tracking constraints. The central emphasis lies in restricting the task space position tracking errors while ensuring the uniform ultimate boundedness of both end-effector position and sub-task tracking. The proposed control methodology guarantees that, when the initial end-effector position tracking error commences within a predefined region, it remains within this region, even in the presence of uncertainties in the robot dynamics parameters. The imposition of constraints on the position tracking error term ensures a predictable overshoot for the end-effector, facilitating predetermined transient performance. The stability and convergence of the closed-loop system are established through a BLF-based



(a) Unconstraint ($K_e = 1$)



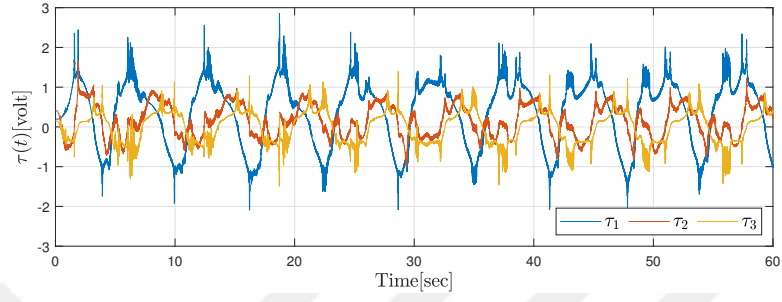
(b) $\Delta = 2I_2$ [cm]



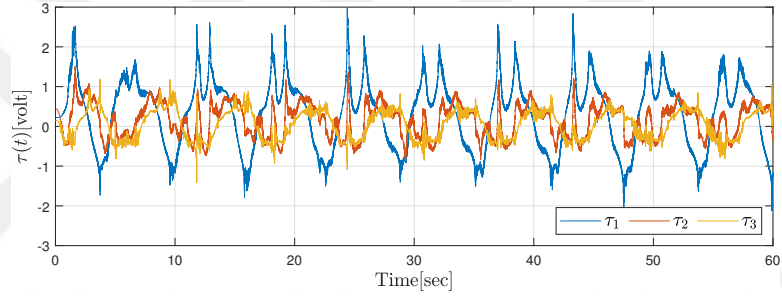
(c) $\Delta = 0.5I_2$ [cm]

Figure 5.5: Experimental results of task space position errors for various Δ constraints

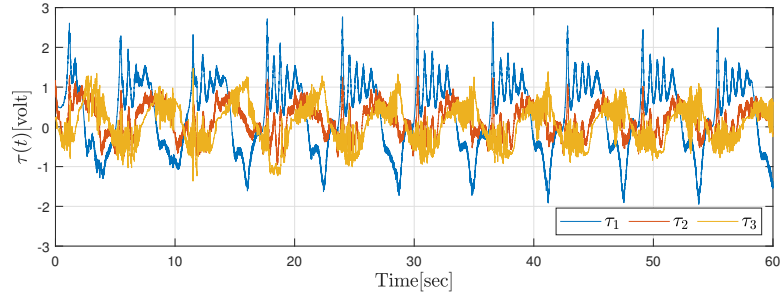
argument. Moreover, the designed control strategy utilizes the pseudo-inverse of the manipulator Jacobian and eliminates the need for computing the inverse kinematics at the position level. Extensive simulation and experimental studies affirm the validity and robustness of the proposed control methodology, underscoring its potential to enhance accuracy and safety in robotic manipulator applications.



a) Unconstraint ($K_e = 1$)



b) $\Delta = 2I_2$ [cm]



c) $\Delta = 0.5I_2$ [cm]

Figure 5.6: Experimental results of torque input signals for different Δ constraints

6. CONCLUSION

In this dissertation, we have developed constraint control strategies for tracking control problem of robotic manipulators with uncertainties, ensuring that position errors remain within predetermined limits. The application of constraints on system states has been an absolute necessity in control systems because of increased performance requirements and security. Additionally, the aforementioned drawback of using Lyapunov's method to design a controller, which is the lack of information on the transient performance must be handled. The BLF approach which is a novel concept in the nonlinear control field gives a potential solution to these concerns.

Our initial research introduced and analyzed two novel full state feedback, joint position constrained robot controllers using BLFs. These controllers consistently kept position tracking error within predetermined limits despite dynamic uncertainties in the robot dynamics. Our approach, combining BLF design with desired model-based compensation, was unprecedented and validated through extensive simulations and experimental studies on a two-linked direct-drive robotic manipulator.

Expanding on our initial study, our subsequent research concentrated on developing adaptive output feedback controllers for robotic manipulators, a notably challenging task when relying solely on joint position measurements for several reasons. First, the absence of joint velocity data complicates the theoretical design of the controller. Second, using output feedback controllers offers a practical advantage by potentially eliminating the need for costly sensors in each joint, thereby reducing manufacturing costs. Provided that the initial joint position error stays within certain boundaries, the adaptive controller we designed ensures that the tracking errors for each joint remain within these boundaries and eventually converge to zero, despite the absence of velocity data and the presence of dynamic uncertainties. The constraints applied to joint positions are instrumental in reducing overshoot and, if strictly implemented, can improve the prescribed transient performance. Supported by comprehensive simulations and experimental tests, these results pave the way for further investigations into output feedback controllers that integrate learning capabilities and task space constraints.

The first two studies emphasized establishing predefined limits on joint position errors

to address safety concerns, even though it is the end-effector in the task space that performs the essential functions of the robot. Typically, the dynamic model of a robot manipulator is formulated in joint-space, but the intended movements are described in task-space, where the end-effector executes the actual tasks. Building on this, our last study introduced a robust control strategy for robot manipulators with kinematic redundancy, focusing on constraining task space position errors. This control strategy utilizes a logarithmic BLF-based Lyapunov analysis to design the constraint controller. It ensures that both task space position error and sub-task error remain bounded, enhancing the predictability of transient performance. Additionally, the strategy simplifies the control process by using the pseudo-inverse of the manipulator's Jacobian, thus avoiding the extensive computations involved in inverse kinematics. The stability and effectiveness of this approach were rigorously validated through simulations and experiments, highlighting its potential to improve both accuracy and safety in robotic applications.

For future studies, we have consistently utilized symmetrical Barrier Lyapunov Functions (BLFs) throughout our research. However, there exists the potential to explore asymmetrical versions, which allow for the application of asymmetric constraints. While our work has predominantly involved logarithmic and tangent types of BLFs, other varieties are tailored for specific purposes and could deepen our understanding of constrained control systems. Additionally, robot manipulators are commonly employed in real-world scenarios to perform repetitive tasks. Learning controllers, which can adapt to dynamic model uncertainties through updating algorithms, present a valuable approach for managing such periodic activities. Building on our foundation of designing constraint controls in task space across various feedback mechanisms, our next step could involve developing position-constrained learning-type controller algorithms utilizing BLFs.

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BIOGRAPHY

Samet GÜL received his B.Sc. in Electrical-Electronics Engineering from Erciyes University in 2015, followed by an M.Sc. in 2018 from the same department. Over the years, he has gained diverse experience as an entrepreneur, founding a startup company, and working as a software engineer and research assistant. He is currently a research assistant in the Department of Computer Engineering at Gebze Technical University. His research interests include control systems, robotics, and applications of artificial intelligence, and he is eager to apply his expertise in dynamic roles in both academia and industry that involve innovative projects and professional growth opportunities.



PUBLICATIONS WITHIN the SCOPE of THESIS STUDY

Gul, S., Zergeroglu, E., Tatlicioglu, E., & Kilinc, M. V. (2022). Desired model compensation-based position constrained control of robotic manipulators. *Robotica*, 40(2), 279-293.

Gul, S., Zergeroglu, E., & Tatlicioglu, E. (2022). Position tracking constrained adaptive output feedback control of robotic manipulators. *Journal of Dynamic Systems, Measurement, and Control*, 144(11), 111006.

Gul, S. & Zergeroglu, E. (2023). Kinematik Fazlalıklı Robot Manipulatorların Görev Alanı Kısıt Kontrolü. 24. Otomatik Kontrol Ulusal Toplantısı (TOK'2023), pp. 13-18. Istanbul, Turkey.

APPENDICES

APPENDIX-A: Boundedness of χ for the Full-State Feedback Design

The auxiliary term χ given in (3.7) can be explicitly expressed as follows

$$\begin{aligned} \chi \triangleq & M(q)(\ddot{q}_d + \alpha \dot{e}) + C(q, \dot{q})(\dot{q}_d + \alpha e) + G(q) + F_d \dot{q} \\ & - M(q_d)\ddot{q}_d - C(q_d, \dot{q}_d)\dot{q}_d - G(q_d) - F_d \dot{q}_d \end{aligned} \quad (\text{A.1})$$

where (2.10) has been utilized. In a manner similar to that outlined in [2] (Chapter 6 Eq: 6.2-9), the term χ can be upper-bounded as follows

$$\|\chi\| \leq \zeta_1 \|e\| + \zeta_2 \|e\|^2 + \zeta_3 \|r\| + \zeta_4 \|r\| \|e\| \quad (\text{A.2})$$

where $\zeta_i, i = 1, 2, 3, 4$, are positive bounding constants that depend on the desired trajectory and the physical parameters such as link mass, link length, and friction coefficients. Following some mathematical manipulations of (A.2), the upper bound can be revised as

$$\|\chi\| \leq (\zeta_1 + \zeta_2 \|e\|) \|e\| + (\zeta_3 + \zeta_4 \|e\|) \|r\|. \quad (\text{A.3})$$

The bounding functions $\rho_1(e)$ and $\rho_2(e)$ can be straightforwardly selected as follows to satisfy the bound given in (3.7)

$$\begin{aligned} \rho_1(e) &= \zeta_1 + \zeta_2 \|e\| \\ \rho_2(e) &= \zeta_3 + \zeta_4 \|e\|. \end{aligned} \quad (\text{A.4})$$

APPENDIX-B: The Bound on χ term for the Output Feedback Design

We begin by redefining (4.10) as

$$\chi = \tilde{Y} + M(q)\eta - 2M(q)e_f + V_m(q, \dot{q})(e_f + e) \quad (\text{B.1})$$

where the auxiliary variable $\tilde{Y}(e, e_f, \eta, t) \in \mathbb{R}^p$ is introduced and defined by

$$\tilde{Y} = M(q)\ddot{q}_d + V_m(q, \dot{q})\dot{q}_d + F_d \dot{q} + G(q) - Y_d \theta.$$

Based on (B.1) and applying the triangular inequality, an upper bound for $\|\chi\|$ is derived as follows

$$\|\chi\| \leq \|\tilde{Y}\| + \|M(q)\eta\| + 2\|M(q)e_f\| + \|V_m(q, \dot{q})e_f\| + \|V_m(q, \dot{q})e\| \quad (\text{B.2})$$

According to [2], the term $\|\tilde{Y}\|$ can be upper bounded by

$$\|\tilde{Y}\| \leq \alpha_0 \|e\| + \alpha_2 \|e\|^2 + \alpha_3 \|\dot{e}\| + \alpha_1 \|\dot{e}\|^2. \quad (\text{B.3})$$

Here, $\alpha_i, i = 0, 1, 2, 3$ are positive constants that depend on the physical parameters of the mechanical system (such as link lengths, the center of gravity of a link, the total mass of a link, etc.) and the bounds on the desired motion trajectory. The upper bound of $\tilde{Y}$, along with the induced matrix norms for the other terms on the right-hand side of (B.2), facilitates establishing the following bound for $\|\chi\|$

$$\begin{aligned} \|\chi\| &\leq \alpha_0 \|e\| + \alpha_2 \|e\|^2 + \alpha_3 \|\dot{e}\| + \alpha_1 \|\dot{e}\|^2 \\ &\quad + \|M(q)\|_{i_2} \|\eta\| + 2\|M(q)\|_{i_2} \|e_f\| \\ &\quad + \|V_m(q, \dot{q})\|_{i_\infty} \|e_f\| + \|V_m(q, \dot{q})\|_{i_\infty} \|e\|. \end{aligned} \quad (\text{B.4})$$

Here, $\|\cdot\|_{i_2}$ and $\|\cdot\|_{i_\infty}$ denote the induced 2-norm and induced ∞ -norm of a matrix, respectively. By applying the properties $\|M(q)\|_{i_2} \leq m_2$ and $\|V_m(q, \dot{q})\|_{i_\infty} \leq \zeta_{c1} \|\dot{q}\|$ to the respective matrix norms in (B.4), we can refine the upper bound on $\|\chi\|$ as follows

$$\begin{aligned} \|\chi\| &\leq \alpha_0 \|e\| + \alpha_2 \|e\|^2 + \alpha_3 \|\dot{e}\| + \alpha_1 \|\dot{e}\|^2 \\ &\quad + m_2 \|\eta\| + 2m_2 \|e_f\| + \zeta_{c1} \|\dot{q}\| \|e_f\| + \zeta_{c1} \|\dot{q}\| \|e\|. \end{aligned} \quad (\text{B.5})$$

By substituting $\dot{q} = \dot{q}_d - \dot{e}$ and using $\|\dot{e}\| \leq \|\eta\| + \|e_f\| + \|e\|$ in (B.5), an upper bound containing known constants and the variables $\eta(t)$, $e_f(t)$, and $e(t)$ is obtained

$$\begin{aligned} \|\chi\| &\leq \alpha_0 \|e\| + \alpha_2 \|e\|^2 + \alpha_3 (\|e\| + \|e_f\| + \|\eta\|) \\ &\quad + \alpha_1 (\|e\| + \|e_f\| + \|\eta\|)^2 + m_2 \|\eta\| + 2m_2 \|e_f\| \\ &\quad + \zeta_{c1} \zeta_{d2} \|e_f\| + \zeta_{c1} (\|e\| + \|e_f\| + \|\eta\|) \|e_f\| \\ &\quad + \zeta_{c1} \zeta_{d2} \|e\| + \zeta_{c1} (\|e\| + \|e_f\| + \|\eta\|) \|e\|. \end{aligned} \quad (\text{B.6})$$

Using the fact that $\|x\| \geq \|\eta\|, \|e_f\|, \|e\|$ and collecting common terms yields the final upper bound placed on (B.6) as shown below

$$\|\chi\| \leq (\alpha_0 + 2\zeta_{c1} \zeta_{d2} + 3\alpha_3 + 3m_2) \|x\| + (9\alpha_1 + \alpha_2 + 6\zeta_{c1}) \|x\|^2 \quad (\text{B.7})$$

It is evident that when the bounding constants ξ_1 and ξ_2 are defined as

$$\begin{aligned} \xi_1 &= \alpha_0 + 2\zeta_{c1} \zeta_{d2} + 3\alpha_3 + 3m_2, \\ \xi_2 &= 9\alpha_1 + \alpha_2 + 6\zeta_{c1} \end{aligned} \quad (\text{B.8})$$

the bound given in (4.11) is obtained.