

FLIGHT SIMULATION AND CONTROL OF FIXED-WING AIRCRAFT

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FOREWORD

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ABBREVIATIONS

RK4	: the 4 rd order Runge Kutta
App	: Appendix
Fig.	: Figure
Figs.	: Figures
Eq .	: Equation
Eqs.	: Equations
CE	: Characteristic Equation
SP	: Short Period
PH	: Phugoid
DoF	: Degree of Freedom

SYMBOLS

U, V, W	components of aircraft velocity, m/s
u, v, w	perturbed values of U, V, W , m/s
P, Q, R	angular velocity components, rad/s
p, q, r	perturbated values of P, Q, R , rad/s
$\bar{q}$	dynamic pressure
Ψ, Θ, Φ	Euler angles
ψ, θ, ϕ	perturbated values of Euler angles
Ψ_T	thrust line inclination angle w.r.t XZ-plane
ϕ_T	thrust line inclination angle with respect to YX-plane
β	angle of sideslip, deg or rad
α	angle of attack (AoA), deg or rad
$\dot{\alpha}$	rate of change of AoA, rad/s
γ	flight path angle, deg or rad
g	acceleration of gravity, m/s ²
L, D, T	lift, drag and thrust, N
U_1, W_1	steady-state velocity components along X and Z direction
x_E, y_E, z_E	aircraft coordinates with respect to earth fixed axis system
I_{xx}, I_{yy}, I_{zz}	moments of inertia kg.m ²
I_{xy}, I_{yz}, I_{xz}	products of inertia kg.m ²
F_{Ax}, F_{Ay}, F_{Az}	aerodynamic force components
F_{Tx}, F_{Ty}, F_{Tz}	thrust force components
S	wing area, m ²
$\bar{c}$	mean aerodynamic chord, m
b	span length, m
L_A, L_T	aerodynamic and thrust induced rolling moment, N.m
M_A, M_T	aerodynamic and thrust induced pitching moment, N.m
N_A, N_T	aerodynamic and thrust induced yawing moment, N.m
δ_e	elevator deflection
δ_a	aileron deflection
δ_r	rudder deflection

δ_T	thrust ratio, ratio of actual thrust to maximum thrust available
i_h	stabilizer incidence angle
AR	aspect ratio
e	span efficiency factor
d_T	distance from the thrust line to the center of gravity
$T_{\max_{SL}}$	maximum sea level thrust force
σ	density ratio
ζ	damping ratio
ω_N	natural frequency

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FLIGHT SIMULATION AND CONTROL OF FIXED-WING AIRCRAFT

SUMMARY

In this study, the dynamics of fixed-wing aircraft is investigated. The historical development of flight control systems is explained. A transport aircraft is studied. Aircraft stability and control derivatives, which are used in equations of motion of aircraft, are obtained by modelling the aircraft geometry and the given flight condition with respect to the body-axis system at AAA software. Linear and nonlinear dynamic model of the aircraft is obtained with respect to body-axis system. Nonlinear equations are solved by using the 4th order Runge Kutta method numerically and linear equations are solved by using both the 4th order Runge Kutta method numerically and Laplace Transformation method exactly. The aircraft response to elevator deflection is compared by three different approaches. Firstly, the stability analysis is performed by using transfer functions obtained from longitudinal equations of aircraft motion for a given flight condition. The aircraft modes are calculated from characteristic equation. Moreover, longitudinal motion of aircraft is investigated by Bode Plot. The uncertainty of stability and control derivatives is explained. Then, two stability derivatives are determined as uncertain parameters. Afterwards, the Edge and Bialas theorems, which are used for robust flight stability analyses, are introduced. Stability analysis for all possible conditions in the flight envelope is reduced to the stability analysis of four polynomials using these two theorems. Thus, instead of analysing the system for all possible conditions, the stability analysis of aircraft is performed only by investigating the stability of these four polynomials which are obtained easily. Three and six degree of freedom trim analyses are performed by using the Gauss-Seidel and the Newton-Raphson methods for aircraft in different conditions. These methods are compared.

SABİT KANATLI UÇAKLARIN UÇUŞ BENZETİMİ VE KONTROLU

ÖZET

Bu çalışmada, sabit kanatlı uçakların dinamiği araştırılmıştır. Uçuş kontrol sistemlerinin gelişimi tarihsel olarak anlatılmıştır. Bir taşıma uçağı model olarak seçilmiştir. Uçağın hareket denklemlerinde kullanılan uçak kararlık ve kontrol türevleri, gövde eksen takımına göre uçak geometrisi ve uçuş koşulu AAA yazılımında modellenerek elde edilmiştir. Uçağın doğrusal ve doğrusal olmayan dinamik modeli gövde eksen takımına göre oluşturulmuştur. Doğrusal olmayan denklemler sayısal olarak 4. mertebe Runge Kutta yöntemi kullanılarak, doğrusal denklemler ise hem sayısal olarak 4. mertebe Runge Kutta hem de kesin olarak Laplace dönüşüm yöntemleri kullanılarak çözülmüştür. Uçağın elevator kontrol yüzeyinin değişimlerine verdiği yanıt 3 farklı yöntemle çözümlenerek kıyaslanmıştır. Kararlılık analizi ilk olarak belli bir uçuş koşulunda uçağın boylamsal hareketi için elde edilen tranfer fonksiyonlarından yararlanılarak yapılmıştır. Uçağın karakteristik denkleminde kısa ve uzun modları hesaplanmıştır. Ayrıca uçağın boylamsal hareketi, Bode çizimi yardımıyla da incelenmiştir. Bir taşıma uçağının uçuş zarfında uçuş kararlık ve kontrol türevlerindeki belirsizlik anlatılmıştır. İki kararlık türevi belirsiz parametre olarak saptanmıştır. Daha sonra dayanıklı uçuş kararlılık analizinde kullanılacak olan Kenar ve Bialas yöntemleri tanıtılmıştır. Tüm olası uçuş koşulları için kararlılık analizi bu iki teorem kullanılarak elde edilen dört polinomun kararlılık analizine indirgenmiştir. Dolayısıyla uçuş kararlılık analizi, bu iki belirsiz kararlık türevinin olası her değişimi için kolaylıkla yapılmıştır. Farklı koşullardaki sabit kanatlı uçakların üç ve altı serbestlik dereceli denge analizleri çalışılmıştır. Üç serbestlik dereceli denge analizi, yatay bir düzlemdeki uçağın hareketine uygulanmıştır. Altı serbestlik dereceli denge analizi, sabit bir irtifada sabit bir yarıçapla dönüş yapan bir uçak için uygulanmıştır. Denge analizleri, Gauss-Seidel ve Newton Raphson yöntemleri kullanılarak yapılmıştır. Bu yöntemler kıyaslamıştır.

1. INTRODUCTION

1.1 Literature Survey

In this section, the historical development of flight control systems has been investigated from the early studies of aircraft design at the end of 1980's till now. The inherent stability becoming a starting point for these studies and the factors leading the Wright Brothers to achieve the first manned powered flight have been explained. The necessity of an autopilot and the first successful application of an autopilot have been explained. Projects of various countries, in that area, during World War I have also been mentioned. Delayed application of the first commercial autopilot has been explained with reasons. Theoretical studies till World War II have been mentioned. Moreover, the gap between theory and application has been stated. Developments in the Second World War have been explained, and improvements after World War II have been summarized chronologically. Finally, the future of flight control systems has been discussed.

Flying has been a very long standing desire for people. Actually, the first manned flight had been achieved before Wright Brothers' first flight. Balloons and gliders can be given as examples. In this paper, the historical development of flight control systems is handled with its relationship to the other improvements in aeronautics chronologically.

This section is arranged as follows:

The inherent stability problem, which was the first study area in this subject, and the first manned-powered flight attempts are explained. The contribution of the Wright Brothers to flight control and how they were able to achieve the first manned-powered flight are explained. Then, another milestone in the history of aeronautics is handled. Just 11 years after the Wright Brothers' flight, Lawrence Sperry made a demonstration flight in Paris with an autopilot that set into "Curtis Flying Boat". The effects of World War I are explained. Afterwards, first commercial autopilot efforts are discussed. The theoretical studies are discussed until World War II. World War II

period is handled. The improvements after World War II are explained. Finally, the future of flight control systems is discussed.

1.1.1 The Inherent Stability Problem

Early aeronautical scientists such as Otto Lilienthal from Germany, Lanchester and Pilcher from England, Octave Chanute and Samuel Pierpont Langley from the U.S. tried to construct an aircraft which has inherent stability without feedback [1, 2]. In 1893, Albert Zahm from the US was the first person who stated that the center of gravity has to be located in front of the aerodynamic center for the static stability to be correct [3].

Afterwards, they discovered how to determine the tail incidence angle and use wing dihedral [2]. In these designs, the pilot's duty was to direct the aircraft. However, afterwards, it was understood that designs, based only on inherent stability, were not safe in case of sudden and strong winds and affected by atmospherical disturbance easily, and that aircraft needed to be more maneuverable [2, 3].

People such as Otto Lilienthal, Octave Chanute and Samuel Langley had influence on Wright Brothers. These researchers aimed to achieve powered flight of an aircraft under human control. Lilienthal did experiments to determine the characteristics of wings and saved data on more than 2000 glider flight in detail. They gave inspiration to many aviation pioneers [3].

Lilienthal's designs had inherent stability, but their controllability was very limited. Therefore, despite his all success in inherent stability, he lost his control in 1896 and died by falling from a height of 50 ft [1, 3].



Figure 1.1 : ¹Lilienthal's Glider Flight on June 29th 1895



Figure 1.2 : Otto Lilienthal

¹ All figures in this section were taken from www.wikipedia.org

Samuel Pierpont Langley did experiments and collected aerodynamic data; then, he concluded that the powered flight was possible with vehicles heavier than air. He achieved an unmanned powered model flown 3 quarters of a mile in 1.5 seconds on May 6th 1896. After his successful flight he won a \$ 50,000 project to develop a manned powered aircraft. In 1899, he started his work and 4 years after he held trials with his assistant Charles Manley on September 7th 1903 and December 8th 1903 in the middle of the Potomac River, Virginia. In both trials, he failed owing to the problems in the launch system. He was badly influenced by criticism that was received from media. However, 20 years later his aircraft could fly with its new engine successfully [3].

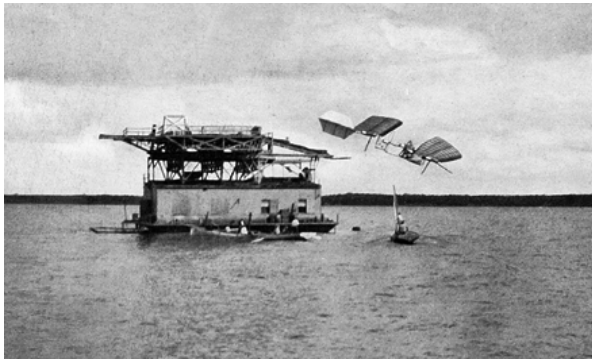


Figure 1.3 : First failure of the manned Aerodome, Potomac

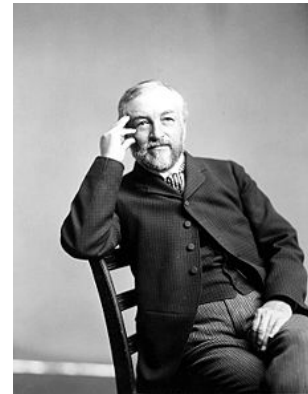


Figure 1.4 : Samuel Pierpont Langley

1.1.2 The Contribution of the Wright Brothers

Contrary to the studies based on inherent stability, Wright Brothers focused on the necessity of having more controllability [1, 2].

In a speech, Wilbur Wright stated that “Men already know how to construct wings or aeroplanes, which when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine, and of the engineer as well. Men also know how to build engines and screws of sufficient lightness and power to drive these planes at sustaining speed... Inability to balance and steer still confronts students of the flying problem. When this one feature has been worked out, the age of flying machines will have arrived, for all other difficulties are of minor importance.” [2].

He believed that the main problem was in control, while the others focused on works based on inherent stability, which had not been successfully finalized. He defined the

necessity of strong control for a pilot to provide equilibrium and steering. This idea can be assumed as the main contribution of Wilbur and Orville Wright. Eventually, pilot was able to achieve feedback to stabilize an aircraft that has no inherent stability [2, 4].



Figure 1.5 : The Wright Brothers' First Flight on December 17th 1903

On December 17th 1903, Wright Brothers performed their historical flight with the Kitty Hawk for 12 seconds about 125 ft. in Kitty Hawk, North Carolina. They did 3 more trials. Their final flight was 59 seconds and 852 ft. Wright Brothers determined that their aircraft gave much more response compared to their previous glider flight. Therefore they had difficulty in controlling it [3].

1.1.3 First Autopilot Usage

Although Wright Brothers' aircraft was not statically stable, its maneuverability was rather high. Owing to their experience gained in glider experiments, they trained themselves to use an unstable aircraft [3]. Flights that were based on a pilot's feeling of aircraft required good weather conditions and eye contact between pilot and ground.

Therefore, this approach was insufficient in case of a cloudy sky and during the night when eye contact between the pilot and the ground was impossible. There were 2 solutions: First, providing pilots with instruments that show aircraft's behavior. The other was the usage of an autopilot [1]. In 1909, Elmer Sperry suggested using a passive gyroscope and tried to sell his idea to Wright Brothers. Between 1912 and 1914 Elmer Sperry and his son Lawrence made important contributions. The Sperry

Gyroscope Company that was under the management of Dr. E. A. Sperry developed an autopilot called “Sperry Stabilizer”. It included gyros that were sensitive to the change of altitude and servomotors that made elevators and ailerons activated [5]. Sperrys used four gyroscopes for this system [1]. On June 18th 1914, Lawrence Sperry performed a demonstration flight with Curtis Flying Boat equipped with this autopilot in Paris. In this demonstration, pilot Lawrence Sperry standing in the cockpit with both hands over his head, while the mechanic was standing and walking back and forth on the wing. Therefore, he won 50,000 franc award which was given to the aircraft having the best stability [4, 5]. This success was 11 years after the Wright Brothers’ Flight [29].

1.1.4 World War I and Its Effects

World War I had started before autopilots were introduced commercially. War led to new necessities. In these terms, work was on indicators that assist pilots rather than autopilots, for instance turn indicator, artificial horizons and slip indicators [1]. While World War I (1914-1918) caused important developments in aircraft design, it did not lead to such developments in the autopilot area [6]. In 1915, Lawrence Sperry recommended that his father had worked on an azimuth stabilizer which would be able to provide flight without pilot when it was used with normal Sperry stabilizer. After 1918, work on autopilots was done by military authorities such as (RAE) Royal Aircraft Establishment in England, the Naval Research Laboratory in USA. After 1915, research on autopilots was hidden, owing to military security, and thus, knowledge was limited. Meredith and Cooke from RAE were allowed to develop various autopilots in 1926 and patent them [1]. In addition, since 1928, Siemens Company, under the management of Eduard Fischel, had been developing an electro-hydraulic system which provided control of heading, control of pitch and roll as well [7].

1.1.5 First Commercial Autopilots

In the 1930s, autopilots started to be developed for commercial purposes. In 1933, Wiley Post insisted on using a Sperry Autopilot prototype for his round-the-world flight. Therefore, the monopoly of military in that area ended [1, 5].

In July 1933, Wiley Post achieved his round-the world flight in less than eight days with his Lockheed Vega aircraft called “Winnie Mae”, which was equipped with the

Sperry A2 autopilot. He was able to perform this flight owing to the autopilot, which allowed the pilot to sleep for short periods [5]. According to a story, a monkey wrench in his hand had fallen when he was asleep, then he was able to understand that he needed to switch on the autopilot [5, 8].



Figure 1.6 : Wiley Post's Lockheed Vega Aircraft "Winnie Mae"

The Smith Instrument Company in England introduced a commercial version of RAE autopilot. In addition, Siemens, Askania, Sperry Gyroscope, General Electric, Honeywell Company had been working in this area [1]. In 1934, United Airlines equipped Boeing 247 aircraft with an A2 autopilot [8].



Figure 1.7 : Boeing 247, in United Air Lines markings at Paine Field

1.1.6 Theoretical Studies until World War II

Flight stability was first studied analytically by F.W. Lanchester and the solution of simplified equations of aircraft was represented [2, 9]. In 1904, Bryan and Williams presented linearized equations of motion which would be the basics of dynamic stability and control studies [2, 10]. In 1911, both longitudinal and lateral motions were investigated by Bryan.

He recommended determining “stability derivatives”, which are coefficients of equations, experimentally [2, 11]. In 1912, Bairstow and Melvill Jones took up Bryan’s suggestion and developed wind tunnel techniques at the National Physical Laboratory in Great Britain to measure stability derivatives. They reported the results the following year. Between 1915 and 1919, studies concerning the determination of stability derivatives were continued by a lot of scientists [12-16]. Between 1920 and 1930, works on auto heading mechanism based on gyroscopic compass gathered interest [8]. In 1922, Minorsky suggested a method which is known as PID control design today [17]. Application tests were performed until the end of 1920s. However, they were ended owing to the fact that the USA army stopped its support [8]. The main provider of the commercial autopilots, Sperry Company, explained and analyzed the Minorsky system [18]. At the end of the 1930s, Nyquist frequency domain stability criteria and Bode-complex-frequency domain theory to explain relationship between gain-phase started to develop. In the 1930s, despite all experimental and theoretical effort to flight stability and control subject, aircraft designers benefitted in a very limited manner. There was a gap between the theory and application [6]. Designs were intuitive rather than theoretical. This situation did not change until the end of 1940s [1, 2, 8]. Mevil Jones thought this resulted from the difficulty of calculation [2, 19]. However, in the following decade, the gap between theory and application continued in spite of invention of methods that decreased the complexity of computations [20, 21], simple and general methods that used dimensionless representations [22], and charts that provided summary for stability factors [23, 24].

All improvement did not motivate aircraft designers to benefit from theoretical studies [2]. Moreover, autopilot designers were aware of the theoretical developments. Meredith and Cooke who worked at Royal Aircraft Establishment explained theoretical and practical improvement in 1937. The lack of application of theoretical studies was obvious [8]. With the advent of the war, studies concerning this subject accelerated and some issues such as wing-fuselage interaction, motor effects on stability derivatives, aircraft response to control surfaces such as spoilers, and its influence on lift were understood better [2].

1.1.7 The Period of World War II

Military and commercial security hindered sharing knowledge for this subject. However, the slowness in theoretical developments cannot relate only to military and commercial security. Another reason was the fact that at that time even the most difficult problems could be solved by trial and error, intuition and experience without making any dynamical stability analysis. This situation continued until the years of 1947-1948 which McRuer and Graham saw as a milestone [8]. Contrary to World War I, World War II (1939-1945) caused important developments in control theory [6]. The advent of war revealed a lot of large and small problems. Some of these problems: voltage stabilization for aircraft generator (1941) [25], theoretical investigation of aircraft stability, development of aircraft flight instruments, management of power station. In addition, finding a target, tracking, the estimation of the next position of moving target, the pointing and firing of guns can be counted among interesting problems that needed to be answered during this era [8]. Powered control surfaces in large aircraft led to the development of the hydraulic servomechanism; necessity of flying at night and bad weather conditions caused radio-navigation instruments. On the other hand, in World War II, the speed-altitude envelope expanded. Therefore, necessities of transporting and dropping heavy loads required better understanding of aircraft dynamics [6].

1.1.8 Developments after World War II

After the Wiley Post's successful flight in 1933, the subject of "Navigational Autopilots" became the center of attention and afterwards in 1947, the Airforce C47 aircraft achieved a transatlantic flight. It was performed by autopilot without any human intervention including take-off and landing. In the US, the speed-altitude envelope expanded with the "X" series first jet aircraft [6]. The Bell X-1 rocket aircraft, which performed its first flight in January 1946, exceeded the speed of sound that had been assumed impossible. Moreover in August 1949, a flight at 72000 ft was achieved [26].



Figure 1.8 : Bell X-1 rocket aircraft (NASA)

In 1948, W.R. Evan's Root-Locus methods became an important improvement to analyze and design control systems [6]. Bigger and bigger motors were installed in aircrafts with shorter and thinner wings to achieve higher performance. This approach caused important changes in the moment of inertia [5]. Inertia cross-coupling had been ignored safely in the past. Charles Yeager, the first person who was able to exceed the speed of sound, had almost died, when he made a flight with X-1A aircraft at 75,000 ft and, at Mach 2.44, because the aircraft started to turn around the 3 axes [5, 6].



Figure 1.9 : The Pilot Charles Yeager

The fact that the speed-altitude envelope expanded caused very big changes in aircraft dynamics. Moreover, the fact that the mass characteristics of aircraft changed and the aerodynamic surface decreased led to the inertia coupling problem which required more analytical approaches. The modes of aircraft changed depending on the design and mass change. Therefore, aircraft became more difficult for pilots to

control [6]. On 27th July 1949, the English de Havilland Comet, which was known as first turbojet passenger aircraft, performed its first flight with four engines that were located on the bottom of the wings [27].



Figure 1.10 : Royal Air Force Comet C.2

The Lockheed X-7 was built and used for the 5-year test program that started in 1951. In these periods, significant experience was gained on high velocity aerodynamics, aerothermodynamics, and special fuel and materials. Afterwards, in the designs of F-104 and SR-71, these experiences were used. X-15 rocket aircraft expanded the speed-altitude envelope to 6 Mach- 30000 ft. It had been equipped with an adaptive stability augmentation system which was designed by the Honeywell and provided the stability in 3 axes [6].



Figure 1.11 : Rocket-powered aircraft X-15

The program of the X-15 test aircraft, which was the fastest manned aircraft in that time, was terminated on October 3rd 1967. This program contributed to subjects

such as control and stability at hypersonic velocity, and control in case of entrance to the atmosphere [26]. At the beginning of the 1960s, fighter aircrafts were approaching the speed level of Mach 2 [6]. At the end of the 1960s, Mach 2.6 speed was reached with the McDonnell F-4 aircraft [26]. The improvement of digital computers in the engineering area contributed to the development of the modern control theory at the middle of 1960s. In 1970s, by the help of digital computers, the simulation of complex systems and real time application of guidance-control theories developed rather quickly [6]. The F-16 aircraft with its single motor was accepted as an important aircraft owing to its contribution to velocity performance and fight ability [26].

The F-16 was the first aircraft that used “fly-by-wire” systems consisting of electronic circuits and servo-actuators. The system was based on electrical signals fully [28]. The electrical system had previously been used with mechanical backup. At present, the analog system is still more commonplace. The First American Space Shuttle performed its first flight in 1981.

The control of hypersonic aircraft was another problem. The large uncertainties of aerodynamic coefficients in hypersonic flights made the design difficult and restricted the design process. A lot of experience was gained in that subject. Gain-scheduled or self adaptive control methods were used to overcome this problem. Thus, the changes in aircraft dynamics would be allowed in a wide flight envelope. However, when the fixed scheduling method was used, some problems in sensing the flight conditions were faced. Since external probes could be used in low Mach numbers, air data such as dynamic pressure and Mach number were obtained from the navigation system and the stored atmospherical models. However, since real atmosphere has unpredictable large changes, the control systems had to be designed to allow these changes [6].

Historically, linear control design has more usage and has been prevalent in flight control applications. Hence it is established well. At present, linearized dynamics of aircraft is still used successfully to develop control system designs [29, 30]. However, linear controller is designed to reach stability and performance criterion for a linearized model of the system dynamics at particular operating points. Today’s high performance aircrafts are expected to be more maneuverable and they deal with complex nonlinear dynamics [31] .

Khan and Lu represent a new technique for nonlinear control of aircraft and state that modern high-performance aircrafts operate frequently in regimes where angle of attack is considerably large and angular rate is immense. Under these circumstances, nonlinearities become more important for the aircraft dynamics [32].

The nonlinear dynamics of aircraft is described in many text books [2, 3, 5, 6, 33, 34]. A lot of researches on nonlinear dynamics have been done in so many years and various results have been demonstrated in the literature [31, 32, 35, 36].

Duties that were expected from an aircraft revealed new designs and more difficult control problems that had to be solved. In the past, flight control systems benefitted the areas of stability, velocity, fuel, performance and maneuverability in aeronautics. However, today's unconventional aircrafts and UAVs require better developed flight control systems. On the other hand, the developments in control theories encouraged aircraft designers to improve unconventional designs. Therefore, control system design has been required to be done with aircraft design simultaneously. Control of aircrafts revealed a lot of problems. When the goals are reached, the duties expected from flight control systems increase and become more difficult. Today's advanced aircraft requires more sophisticated methods such as nonlinear control, numerical optimization, sensitivity and robustness analyses, adaptive methods, multi-variable controls. Work in this area has been going on in various countries all over world.

1.2 Research Objectives

Nonlinear dynamics has been gaining more importance. Failures in control surfaces have caused many aviation accidents. The dynamics of the seriously damaged aircraft could meander into highly nonlinear regimes; depending on the sharpness of the imperfection and states before failure. Therefore the use of nonlinear control theory is stimulated to satisfy the stability and performance criterions in the existence of highly nonlinear dynamics. In this study both linear and nonlinear equations of motion are aimed to investigate. The nonlinear equations are solved by using the Runge Kutta Method. Linear equations are solved numerically by using the Runge Kutta method and they are also solved exactly by using the Laplace Transformation method. All of these solutions are obtained by using the body axis system. The results of the simulations are plotted for different control deflections.

Many classical approaches use stability axis system for the solution of linear equations. However, in this thesis Transfer Functions of the aircraft are redefined in the body axis system, because stability axes change with angle of attack and some of the stability derivatives need to be re-evaluated for each angle of attack, when stability axes are used. Moreover, in addition to classical text book approach, linear equations are also solved by using the 4th order Runge Kutta method.

A transport aircraft, which can be a representation of Boeing 737-400, is selected for simulations. The stability derivatives of this aircraft are elaborated and obtained by using AAA[®]. All stability derivatives are aimed to calculate with respect to the body-axis system.

Longitudinal analysis of the aircraft is performed in frequency and time domain for a state of equilibrium. A longitudinal stability analysis of a transport aircraft has been done considering the change of two stability derivatives within a range in whole flight envelope. Stability of the aircraft is aimed to be determined by investigating eigenvalues of the matrix which is obtained by this four polynomials.

The three and six degree of freedom trim analyses of fixed wing aircraft are aimed to study. Trim analysis is the solution of aircraft force and moment equations at a given instant, to obtain unknown parameters. This solution requires an iterative procedure. Two different iteration methods are tested. These are Gauss Seidel and Newton Raphson iterative methods. Three degree of freedom trim analysis is applied for the motion of a transport aircraft in a vertical plane. Six degree of freedom trim analysis is applied for the steady, constant altitude, constant radius turning motion of a business jet in a horizontal plane.

1.3 Thesis Outline

This dissertation is outlined as follow:

Chapter 1 is devoted to presenting literature survey and research objectives. Calculation of flight stability and control derivatives are described in Chapter 2. Both linear and nonlinear equations of motion of the aircraft are presented and solved in Chapter 3. In Chapter 4, firstly, the stability analysis is done with respect to any flight condition, then it is extended to whole flight envelope. The three and six degree of freedom trim analyses of fixed wing aircraft are studied by using different

methods in Chapter 5. Finally, in Chapter 6, the conclusion is reached from the results of each Chapter.

2. CALCULATION OF FLIGHT STABILITY AND CONTROL DERIVATIVES

2.1 Objectives

In this chapter, the longitudinal stability derivatives of a transport aircraft have been calculated by modeling the aircraft configuration and flight condition. The applying method is able to use for all sizes of civil and military aircrafts.

During the dynamic modeling of aircrafts, stability derivatives are generally obtained from the experimental or previous studies about the related aircraft. Therefore, obtaining new derivatives of an aircraft whose derivatives have not been calculated yet is difficult for researchers. The Advanced Aircraft Analysis (AAA) program applies to most fixed wing configurations (civil or military) and permit engineers to fast calculate stability derivatives straightforwardly [37-40]. In this study, a longitudinal motion of a transport aircraft has been studied. This aircraft model approximately represents the characteristics of an aircraft similar to a Boeing 737-400.

All stability derivatives are calculated with respect to the body-axis system for two main reasons. 1) Some stability derivatives are functions of characteristic distances of aircraft such as x and z coordinates of the vertical tail aerodynamic center etc. as seen in Fig. 2.1. In body axis system these distances are fixed but in stability axes they differ with angle of attack. Therefore body axis system is more convenient to evaluate the stability derivatives. 2) In nonlinear simulations, both equations of motion and also stability derivatives are derived with respect to the body-axis system. Therefore, it is appropriate to use the same derivatives in the linearized solutions.

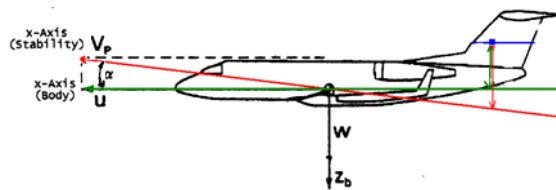


Figure 2.1 : Stability and Body Axes Systems

2.2 Calculation of Flight Stability and Control Derivatives by Using AAA[®]

Calculation of flight stability and control derivatives of a transport aircraft includes two parts: 1. Modelling of Configuration 2. Modeling of Flight Condition

2.2.1 Modelling of Aircraft Configuration

Configuration of the aircraft has been set in Geometry Module of AAA. The aim of the Geometry module assist the user to characterize the geometry of the fuselage, wing, horizontal tail, vertical tail and work out related parameters [37-40]. After the parameters of the aircraft are set, corresponding plots have been obtained.

Configuration modeling parameters of the aircraft: fuselage, wing, horizontal and vertical tails, aileron, elevator control surfaces are determined and shown in Fig. 2.2-2.7 respectively.

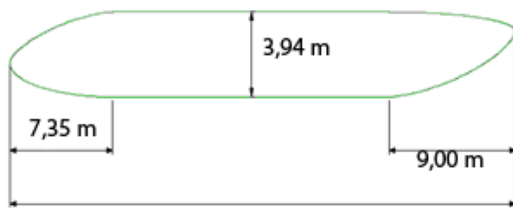


Figure 2.2 : Fuselage Geometry

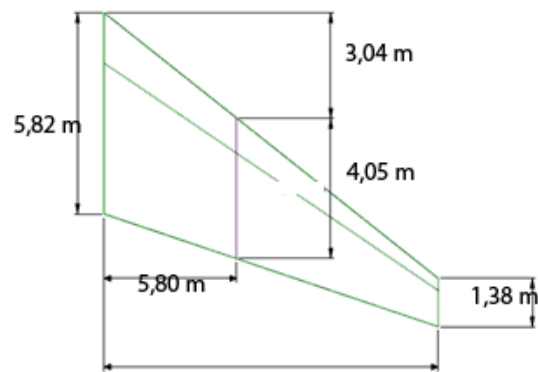


Figure 2.3 : Wing Geometry

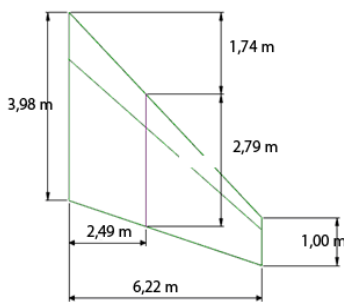


Figure 2.4 : Horizontal Tail

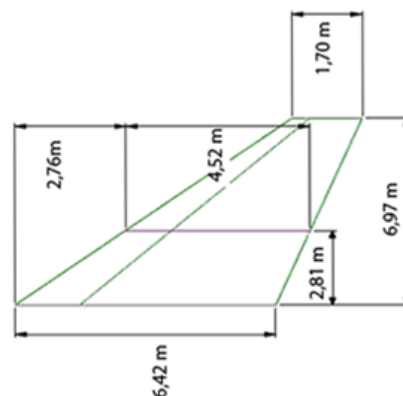


Figure 2.5 : Vertical Tail

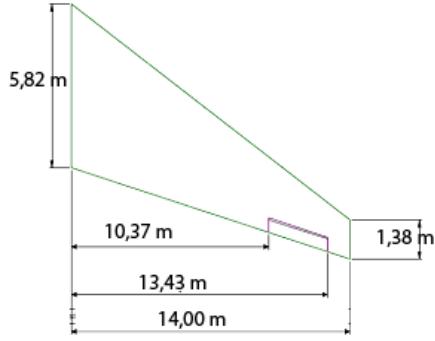


Figure 2.6 : Aileron Geometry

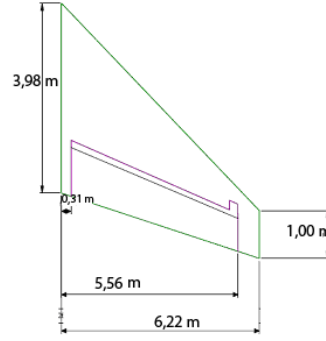


Figure 2.7 : Elevator Geometry

2.2.2 Modelling of Flight Condition

Flight condition for the test problem as described in Table 2.1.

Table 2.1: Flight Condition

Altitude	$h = 3896 \text{ m}$
Steady State Flight Speed:	$U_1 = 537,0 \text{ km/hr}$
Airplane Current Weight:	$W_{current} = 500000,0 \text{ N}$
Steady State Pitch Attitude:	$\theta_1 = 3,0 \text{ deg}$
Dynamic Pressure in Steady State:	$q_1 = 9151,60 \text{ N/m}^2$
Wing Loading at Current Flight Condition:	$W/S = 4761,90 \text{ N/m}^2$

2.2.3 Program Results

According to the aircraft configuration and flight condition, stability derivatives of the aircraft are obtained by AAA[®] program and presented in Table 2.2.

Table 2.2: Stability and control derivatives obtained from AAA[®] program:

$C_{L_0} = 0.2144$	$C_{m_0} = 0.1167$	$Z_u = -0,1476 \text{ s}^{-1}$	$M_u = 0,0023$
$C_{L_\alpha} = 5.4039 \text{ 1/rad}$	$C_{m_\alpha} = -1.7938 \text{ 1/rad}$	$Z_\alpha = -102,0394 \text{ m/s}^2$	$M_{Tu} = 0,0000 \text{ 1/m.s}$
$C_{L_{i_h}} = 1.1835 \text{ 1/rad}$	$C_{m_{i_h}} = -4.2496 \text{ 1/rad}$	$Z_{\dot{\alpha}} = -0,6563 \text{ m/s}$	$M_\alpha = -2,7367 \text{ s}^{-2}$
$C_{L_{\dot{\alpha}}} = 0.5534 \text{ 1/rad}$	$C_{m_{\dot{\alpha}}} = -1.9872 \text{ 1/rad}$	$Z_q = -2,0160 \text{ m/s}$	$M_{\dot{\alpha}} = -0,1915 \text{ s}^{-1}$
$C_{L_{\ddot{\alpha}}} = 2.860 \text{ 1/rad}$	$C_{m_{\ddot{\alpha}}} = -10.2692 \text{ 1/rad}$	$X_u = -0,0082 \text{ s}^{-1}$	$M_q = -0,6106 \text{ s}^{-1}$
$C_{L_q} = 8.7571 \text{ 1/rad}$	$C_{m_q} = -32.5170 \text{ 1/rad}$	$X_{Tu} = -0,0000 \text{ s}^{-1}$	$X_{\dot{\alpha}} = -0,1921 \text{ m/s}^2$
$\bar{C}_{D_0} = 0.0203$		$X_\alpha = 5,2770 \text{ m/s}^2$	$Z_{\dot{\alpha}} = -9,3490 \text{ m/s}^2$
			$M_{\ddot{\alpha}} = -2,7284 \text{ s}^{-2}$

3. EQUATIONS OF MOTION OF FIXED-WING AIRCRAFT

3.1 Nonlinear Equations of Motion of Aircraft

Aircraft force equations can be written in the body axis x, y, z directions as follows [33, 37, 41]: Eqs. (3.1 – 3.2)

$$m(\dot{U} + QW - RV) = -mg \sin\Theta + (-D \cos\alpha + L \sin\alpha) + T \cos\Psi_T \quad (3.1)$$

$$m(\dot{V} + RU - PW) = mg \sin\Phi \cos\Theta + F_{A_y} + F_{T_y} \quad (3.2)$$

$$m(\dot{W} + PV - QU) = mg \cos\Phi \cos\Theta + (-D \sin\alpha - L \cos\alpha) - T \sin\Phi_T \quad (3.3)$$

Linear and Angular (Rotational Velocities) can be seen in Fig. 3.1.

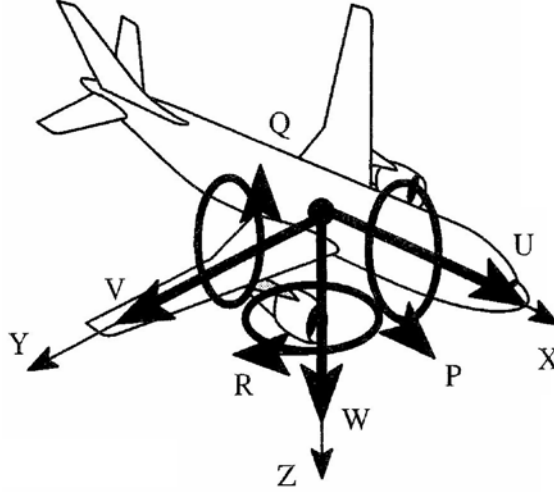


Figure 3.1 : Linear and Angular (Rotational) Velocities [37]

Besides, the aircraft moment equations about the x, y, z body axes, with their origin at the center of gravity point, are given in Eqs. 3.4-3.6 [33, 37, 41].

$$\dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} = L_A + L_T \quad (3.4)$$

$$\dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} = M_A + M_T \quad (3.5)$$

$$\dot{R}I_{zz} + PQ(I_{yy} - I_{xx}) + (QR^2 - \dot{P})I_{xz} = N_A + N_T \quad (3.6)$$

Eqs. (3.4-3.6) are related to rolling moment, pitch moment, and yaw moment respectively.

Following equations give relations between angular velocities and Euler angles [33, 37, 41].

$$P = -\sin(\Theta)\dot{\Psi} + \dot{\Phi} \quad (3.7)$$

$$Q = \sin(\Phi)\cos(\Theta)\dot{\Psi} + \cos(\Phi)\dot{\Theta} \quad (3.8)$$

$$R = \cos(\Phi)\cos(\Theta)\dot{\Psi} - \sin(\Phi)\dot{\Theta} \quad (3.9)$$

The following three differential equations are used to determine the coordinates of the flight path with respect to earth axis system [34].

$$\begin{aligned} \dot{x}_E = & U \cos\Theta \cos\Psi + V (\sin\Phi \sin\Theta \cos\Psi - \cos\Phi \sin\Psi) \\ & + W (\cos\Phi \sin\Theta \cos\Psi + \sin\Phi \sin\Psi) \end{aligned} \quad (3.10)$$

$$\begin{aligned} \dot{y}_E = & U \cos\Theta \sin\Psi + V (\sin\Phi \sin\Theta \sin\Psi + \cos\Phi \cos\Psi) \\ & + W (\cos\Phi \sin\Theta \sin\Psi - \sin\Phi \cos\Psi) \end{aligned} \quad (3.11)$$

$$\dot{z}_E = -U \sin\Theta \cos\Psi + V \sin\Phi \cos\Theta + W \cos\Phi \cos\Theta \quad (3.12)$$

In summary, there are 12 equations which have to be solved to perform simulations of the aircraft: 3 force equations, 3 moment equations, 3 kinematic equations, and 3 coordinate equations.

These 12 equations are rearranged in a form suitable for solution by the RK4 Method. In each equation the derivative term, such as $\dot{U}$, $\dot{V}$, $\dot{Q}$, etc. is placed to the left hand side. $\dot{R}$ and $\dot{P}$ are solved from equations (3.4) and (3.6). $\dot{\Psi}$ and $\dot{\Theta}$ are solved from equations (3.8) and (3.9).

The variables R4, R6, and R9 are defined for convenience [41, 42]

$$R4 = \frac{1}{I_{xx}} \left[-QR(I_{zz} - I_{yy}) + PQI_{xz} + L_A + L_T \right] \quad (3.13)$$

$$R6 = \frac{1}{I_{zz}} \left[-PQ(I_{yy} - I_{xx}) - QRI_{xz} + N_A + N_T \right] \quad (3.14)$$

$$R9 = \frac{R \cos \Phi + Q \sin \Phi}{\cos \Theta} \quad (3.15)$$

Following 3 equations are obtained from longitudinal forces and moments.

y'_i means the time derivative of the i^{th} variable ($i=1, 2, \dots, 12$).

Force equations are arranged as:

$$y'_1 = \dot{U} = RHS1 = -QW + RV - g \sin \Theta + \frac{1}{m} (F_{Ax} + F_{Tx}) \quad (3.16)$$

$$y'_2 = \dot{V} = RHS2 = -RU + PW + g \sin \Phi \cos \Theta + \frac{1}{m} (F_{Ay} + F_{Ty}) \quad (3.17)$$

$$y'_3 = \dot{W} = RHS3 = -PV + QU + g \cos \Phi \cos \Theta + \frac{1}{m} (F_{Az} + F_{Tz}) \quad (3.18)$$

The 5th variable is the pitch rate, Q , $\dot{Q}$ can be solved from the pitching moment equation. The time derivatives of roll rate and yaw rate, $\dot{P}$ and $\dot{R}$, exist both in rolling moment equation and yawing moment equation. Two new equations can be obtained for $\dot{P}$ and $\dot{R}$ by solving them from the rolling moment and yawing moment equations using the Cramer rule.

$$y'_4 = \dot{P} = RHS4 = \frac{R4 + R6 \frac{I_{xz}}{I_{xx}}}{1 - \frac{I_{xz}^2}{I_{xx} I_{zz}}} \quad (3.19)$$

$$y'_5 = \dot{Q} = RHS5 = \frac{1}{I_{yy}} \left[PR(I_{zz} - I_{xx}) - (P^2 - R^2)I_{xz} + M_A + M_T \right] \quad (3.20)$$

$$y'_6 = \dot{R} = RHS6 = \frac{R6 + R4 \frac{I_{xz}}{I_{zz}}}{1 - \frac{I_{xz}^2}{I_{xx} I_{zz}}} \quad (3.21)$$

The next three equations are obtained from kinematic equations.

$$y'_7 = \dot{\Phi} = RHS7 = P + \sin \Theta R9 \quad (3.22)$$

$$y'_8 = \dot{\Theta} = RHS8 = Q \cos \Phi - R \sin \Phi \quad (3.23)$$

$$y'_9 = \dot{\Psi} = RHS9 = R9 \quad (3.24)$$

Finally, the three coordinate equations are arranged as follows,

$$y'_{10} = \dot{x}_E = RHS10 = U \cos \Theta \cos \Psi + V (\sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi) + W (\cos \Phi \sin \Theta \cos \Psi + \sin \Phi \sin \Psi) \quad (3.25)$$

$$y'_{11} = \dot{y}_E = RHS11 = U \cos \Theta \sin \Psi + V (\sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi) + W (\cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi) \quad (3.26)$$

$$y'_{12} = \dot{z}_E = RHS12 = -U \sin \Theta \cos \Psi + V \sin \Phi \cos \Theta + W \cos \Phi \cos \Theta \quad (3.27)$$

Hence, the 12 equations are in a suitable form for the solution by the RK4 method.

3.1.1 Aerodynamic Forces and Moments

The 12 differential equations include aerodynamics forces and moments. During the nonlinear solutions, every force and moment are recalculated at each step.

These coefficients include both steady and non-steady terms.

Lift coefficient is written as follows (3.28) [37].

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{i_h}} i_h + C_{L_{\delta_e}} \delta_e + C_{L_{\dot{\alpha}}} \frac{\dot{\alpha} \bar{c}}{2U_1} + C_{L_q} \frac{q \bar{c}}{2U_1} \quad (3.28)$$

where:

C_{L_0} is the value of C_L for: $\alpha = i_h = \delta_e = 0$

$C_{L\alpha}$ is the change in aircraft lift due to a change in α

C_{Li_h} is the change in aircraft lift due to a change in stabilizer incidence angle, i_h , for:

$$\alpha = \delta_e = 0$$

$C_{L_{\delta_e}}$ is the change in aircraft lift due to a change in elevator angle, i_h , for: $\alpha = \delta_e = 0$

$C_{L_{\dot{\alpha}}}$ is variation of aircraft lift coefficient with dimensionless rate of change of angle of attack

C_{Lq} is variation of aircraft lift coefficient with dimensionless pitch rate

Aircraft drag force is described as [37];

$$D = C_D \bar{q} S$$

$$C_D = C_{D0} + C_{D\alpha} \alpha \quad (3.29)$$

$$C_{D\alpha} = \frac{2C_L C_{L\alpha}}{\pi A R e}$$

where:

C_D is the total aircraft drag coefficient

C_{D0} is the value of C_D for $\alpha = i_h = \delta_e = 0$ when using locally linearized drag equation ($C_D - \alpha$).

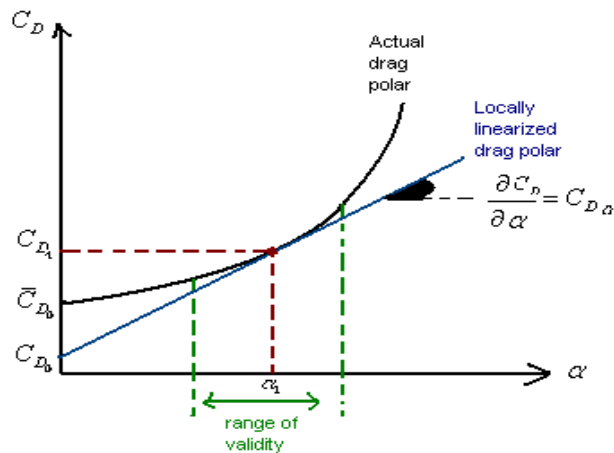


Figure 3.2 : Drag coefficient versus alpha

$C_{D\alpha}$ is the slope of the locally linearized drag equation

Besides drag coefficient can be expressed as:

$$C_D = \bar{C}_{D0} + \frac{C_L^2}{\pi \cdot AR \cdot e} \quad (3.30)$$

where:

$\bar{C}_{D0}$ is the value of aircraft drag coefficient at zero lift coefficient.

From equations (3.29, 3.30), C_{D0} is defined as:

$$C_{D0} = \bar{C}_{D0} + \frac{C_L^2}{\pi \cdot AR \cdot e} - C_{D\alpha} \alpha \quad (3.31)$$

Aircraft pitching moment is formulated as follows [37];

$$M_A = C_m \bar{q} S \bar{c} \quad (3.32)$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e + C_{m_{\dot{\alpha}}} \frac{\dot{\alpha} \bar{c}}{2U_1} + C_{m_q} \frac{q \bar{c}}{2U_1}$$

where:

C_{m_0} is the value of C_L for: $\alpha = i_h = \delta_e = 0$

C_{m_α} is the change in aircraft lift due to a change in α

$C_{m_{i_h}}$ is the change in aircraft lift due to a change in stabilizer incidence angle, i_h ,

for: $\alpha = \delta_e = 0$

$C_{m_{\delta_e}}$ is the change in aircraft lift due to a change in elevator angle, i_h , for: $\alpha = \delta_e = 0$

$C_{m_{\dot{\alpha}}}$ is variation of aircraft pitching moment coefficient with dimensionless rate of change of AoA

C_{m_q} is variation of aircraft pitching moment coefficient with dimensionless pitch rate

Rolling moment is written as follows [37]:

$$L_A = C_l \bar{q} S b \quad (3.33)$$

$$C_l = C_{l_0} + C_{l\beta} \beta + C_{l\delta_a} \delta_a + C_{l\delta_r} \delta_r + C_{lp} \frac{pb}{2U_1} + C_{lr} \frac{rb}{2U_1}$$

where:

C_l is the aircraft aerodynamic rolling moment coefficient

C_{l_0} is the value of C_l for $\beta = \delta_a = \delta_r = 0$

$C_{l\beta}$ is the change in aircraft rolling moment coefficient due to a change in aircraft sideslip angle, β

$C_{l\delta_a}$ is the change in aircraft rolling moment coefficient due to a change in aileron deflection δ_a

$C_{l\delta_r}$ is the change in aircraft rolling moment coefficient due to a change in rudder deflection δ_r

C_{lp} is the variation of aircraft rolling moment coefficient with dimensionless rate of change of roll rate

C_{lr} is the variation of aircraft rolling moment coefficient with dimensionless rate of change of yaw rate

Side-Force is expressed as [37]

$$F_{A_y} = C_y \bar{q} S \quad (3.34)$$

$$C_y = C_{y_0} + C_{y\beta} \beta + C_{y\delta_a} \delta_a + C_{y\delta_r} \delta_r + C_{yp} \frac{pb}{2U_1} + C_{yr} \frac{rb}{2U_1}$$

where:

C_y is the aircraft aerodynamic side-force coefficient.

C_{y_0} is the value of C_y for $\alpha = \beta = \delta_a = \delta_r = 0$.

C_{y_β} is the change in aircraft side-force coefficient due to a change in aircraft sideslip angle.

$C_{y_{\delta_a}}$ is the change in aircraft side-force coefficient due to a change in aileron deflection δ_a

$C_{y_{\delta_r}}$ is the change in aircraft side-force coefficient due to a change in rudder deflection δ_r

C_{y_p} is the variation of aircraft side-force coefficient with dimensionless rate of change of roll rate

C_{y_r} is the variation of aircraft side-force coefficient with dimensionless rate of change of yaw rate

Finally yawing moment is obtained as follows [37];

$$N_A = C_n \bar{q} S b \quad (3.35)$$

$$C_n = C_{n_0} + C_{n_\beta} \beta + C_{n_{\delta_a}} \delta_a + C_{n_{\delta_r}} \delta_r + C_{n_p} \frac{pb}{2U1} + C_{n_r} \frac{rb}{2U1}$$

where:

C_n is the aircraft aerodynamic yawing moment coefficient

C_{n_0} is the value of C_n for $\beta = \delta_a = \delta_r = 0$

C_{n_β} is the change in aircraft yawing moment coefficient due to a change in aircraft sideslip angle, β

$C_{n_{\delta_a}}$ is the change in aircraft yawing moment coefficient due to a change in aileron deflection δ_a

$C_{n_{\delta_r}}$ is the change in aircraft yawing moment coefficient due to a change in rudder deflection δ_r

C_{n_p} is the variation of aircraft yawing moment coefficient with dimensionless rate of change of roll rate

C_{n_r} is the variation of aircraft yawing moment coefficient with dimensionless rate of change of yaw rate.

3.1.2 Thrust Forces and Moments

Thrust forces and moments are expressed as follows [41, 42]:

$$F_{T_x} = T_{\max_{SL}} \sigma \delta_T \cos \phi_T \quad (3.36)$$

$$F_{T_z} = -T_{\max_{SL}} \sigma \delta_T \sin \phi_T \quad (3.37)$$

$$M_T = -T_{\max_{SL}} \sigma \delta_T d_T \quad (3.38)$$

Fig. 3.3 shows steady state thrust forces and pitching moment in stability axis.

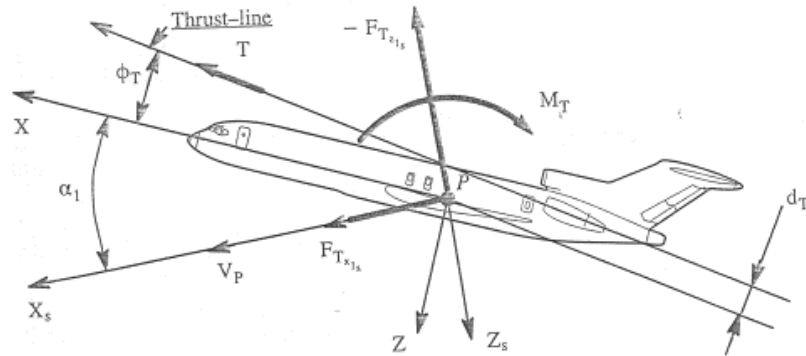


Figure 3.3 : Steady State Thrust Forces and Pitching Moment in Stability Axes

3.2 Linear Equations of Motion of Aircraft

In this section, the linearized equations of motion are obtained. Traditionally, Transfer Functions (TFs) of aircrafts are obtained with respect to stability axes [33, 37, 43]. However, in this paper TFs of the aircraft are redefined in the body axis system. Therefore, some additional terms are added to classical TFs to solve linear equations exactly.

Classically, in text books, linearized equations are solved by using TF methods [33, 37, 43]. Alternatively, this paper includes numerical solutions of these equations as well. This approach provides more understanding to aeronautical engineering students and demonstrates the efficiency of RK4 method in solving aircraft equations of motion.

3.2.1 Longitudinal Linear Equations of Motion

Longitudinal motion includes two translations and one rotation. Translations are in x direction (Eq. 3.1) and z direction (Eq. 3.3) and rotation is about the y axis (Eq. 3.5). Therefore following three equations are necessary to investigate longitudinal motion [33, 37, 41].

Longitudinal nonlinear equations are linearized by using small perturbation theory for a trim point, as discussed in [33] in depth. Eqs. (3.1-3.3) are reduced into Eqs. (3.51-3.53).

$$\dot{u} + W_1 q = -g\theta \cos \theta_1 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e \quad (3.51)$$

$$\dot{w} - U_1 q = -g\theta \sin \theta_1 + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q + Z_{\delta_e} \delta_e \quad (3.52)$$

$$\dot{q} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_{\delta_e} \delta_e \quad (3.53)$$

$X_u, X_{T_u}, X_\alpha, X_{\delta_e}, Z_u, Z_\alpha, Z_{\dot{\alpha}}, Z_q, Z_{\delta_e}, M_u, M_{T_u}, M_\alpha, M_{T_\alpha}, M_{\dot{\alpha}}, M_q, M_{\delta_e}$ are dimensional linearized stability derivatives of the aircraft. The descriptions of them are demonstrated and they are deeply investigated in [37-39]. In this study, they are obtained by using AAA program.

Note that, if stability axis system was chosen, W_1 would be equal to zero. Due to the choice of the body axis system, we have to use two velocity components, U_1 and w_1 . Therefore, following equations related to linearized theory are derived without the simplification of $U_1=0$ differently from classical approach.

Defining $q = \dot{\theta}$, $\dot{q} = \ddot{\theta}$, and taken into account the following assumptions, for small perturbations;

$$\alpha \approx \frac{w}{U_1} \Rightarrow w = \alpha U_1 \quad \text{and} \quad \dot{w} = \dot{\alpha} U_1, \text{ we get:}$$

$$\dot{u} + W_1 \dot{\theta} = -g \theta \cos \theta_1 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e \quad (3.54)$$

$$\dot{\alpha} U_1 - U_1 \dot{\theta} = -g \theta \sin \theta_1 + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q \dot{\theta} + Z_{\delta_e} \delta_e \quad (3.55)$$

$$\ddot{\theta} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q \dot{\theta} + M_{\delta_e} \delta_e \quad (3.56)$$

Rearranging Eqs. (3.54-3.56), following equations are obtained:

$$\dot{u} = -W_1 \dot{\theta} - g \theta \cos \theta_1 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e \quad (3.57)$$

$$\dot{\alpha} = \frac{1}{U_1 - Z_{\dot{\alpha}}} \{ (U_1 + Z_q) \dot{\theta} - g \theta \sin \theta_1 + Z_u u + Z_\alpha \alpha + Z_{\delta_e} \delta_e \} \quad (3.58)$$

$$\ddot{\theta} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q \dot{\theta} + M_{\delta_e} \delta_e \quad (3.59)$$

In Test Problem section, Eqs.(3.57-3.59) are solved numerically by using the 4th order Runge Kutta method and they are also solved exactly by using the Laplace Transformation method. Therefore, Eqs. (3.57-3.59) are also represented in Laplace transform with zero initial as follows:

$$\begin{bmatrix} (s - X_u - X_{T_u}) & -X_\alpha & g \cos \theta_1 + W_1 s \\ -Z_u & [s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha] & [-(Z_q + U_1)s - g \sin \theta_1] \\ -(M_u + M_{T_u}) & -[M_{\dot{\alpha}} s + M_\alpha + M_{T_\alpha}] & (s^2 - M_q s) \end{bmatrix} \begin{bmatrix} \frac{u(s)}{\delta_e(s)} \\ \frac{\alpha(s)}{\delta_e(s)} \\ \frac{\theta(s)}{\delta_e(s)} \end{bmatrix} = \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \quad (3.60)$$

Each of the three longitudinal transfer functions are determined by using Cramer's rule as shown Eqs(3.61-3.63).

The speed-to-elevator transfer function $u(s)/\delta_e(s)$ is written as follows:

$$\frac{u(s)}{\delta_e(s)} = \frac{N_u}{D_1} = \frac{\begin{vmatrix} X_{\delta_e} & -X_\alpha & g \cos \theta_1 + W_1 s \\ Z_{\delta_e} & [s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha] & [-(Z_q + U_1)s - g \sin \theta_1] \\ M_{\delta_e} & -[M_{\dot{\alpha}} s + M_\alpha + M_{T_\alpha}] & (s^2 - M_q s) \end{vmatrix}}{\begin{vmatrix} (s - X_u - X_{T_u}) & -X_\alpha & g \cos \theta_1 + W_1 s \\ -Z_u & [s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha] & [-(Z_q + U_1)s - g \sin \theta_1] \\ -(M_u + M_{T_u}) & -[M_{\dot{\alpha}} s + M_\alpha + M_{T_\alpha}] & (s^2 - M_q s) \end{vmatrix}} \quad (3.61)$$

In similar way, the angle of attack-to-elevator transfer function $\alpha(s)/\delta_e(s)$ is obtained as follows:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{N_\alpha}{D_1} = \frac{\begin{vmatrix} (s - X_u - X_{T_u}) & X_{\delta_e} & g \cos \theta_1 + W_1 s \\ -Z_u & Z_{\delta_e} & \{-(Z_q + U_1)s + g \sin \theta_1\} \\ -(M_u + M_{T_u}) & M_{\delta_e} & (s^2 - M_q s) \end{vmatrix}}{|\overline{D_1}|} \quad (3.62)$$

Finally, the pitch-attitude-to-elevator transfer function $\theta(s)/\delta_e(s)$ is expressed as:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{N_\theta}{D_1} = \frac{\begin{vmatrix} (s - X_u - X_{T_u}) & -X_\alpha & X_{\delta_e} \\ -Z_u & \{s(U_1 - Z_\alpha) - Z_\alpha\} & Z_{\delta_e} \\ -(M_u + M_{T_u}) & -\{M_\alpha s + M_\alpha + M_{T_\alpha}\} & M_{\delta_e} \end{vmatrix}}{|\overline{D_1}|} \quad (3.63)$$

3.2.2 Lateral-Directional Linear Equations of Motion

Lateral-directional Equations of Motion are related to rolling moment (Eq. 3.4), side force (Eq. 3.2), and yaw moment (Eq. 3.6) respectively.

Similarly, lateral nonlinear equations are linearized by using small perturbation theory for a trim point, as discussed in [33].

$$\dot{v} + U_1 r - W_1 p = g \phi \cos \Theta_1 + Y_\beta \beta + Y_p p + Y_r r + Y_{\delta_\alpha} \delta_\alpha + Y_{\delta_r} \delta_r \quad (3.64)$$

$$\dot{p} + \overline{A_1} \dot{r} = L_\beta \beta + L_p p + L_r r + L_{\delta_\alpha} \delta_\alpha + L_{\delta_r} \delta_r \quad (3.65)$$

$$\dot{r} + \overline{B_1} \dot{p} = N_\beta \beta + N_{T_\beta} \beta + N_p p + N_r r + N_{\delta_\alpha} \delta_\alpha + N_{\delta_r} \delta_r \quad (3.66)$$

where

$$A_1 = \frac{I_{xz}}{I_{xx}} \text{ and } B_1 = \frac{I_{xz}}{I_{zz}}$$

For trim condition and small perturbations, following assumptions are made.

$$p = \dot{\phi} \text{ and } r = \dot{\psi}$$

$$\beta \approx \frac{v}{U_1} \Rightarrow v \approx \beta U_1 \text{ and } \dot{v} \approx \dot{\beta} U_1$$

These assumptions are introduced to Eqs. (3.63-3.66).

$$U_1 \dot{\beta} + U_1 \dot{\psi} - W_1 \dot{\phi} = g \phi \cos \Theta_1 + Y_\beta \beta + Y_p \dot{\phi} + Y_r \dot{\psi} + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r \quad (3.67)$$

$$\ddot{\phi} + \bar{A}_1 \ddot{\psi} = L_\beta \beta + L_p \dot{\phi} + L_r \dot{\psi} + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \quad (3.68)$$

$$\ddot{\psi} + \bar{B}_1 \ddot{\phi} = N_\beta \beta + N_{T_\beta} \beta + N_p \dot{\phi} + N_r \dot{\psi} + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r \quad (3.69)$$

And then Eqs(3.67-3.69) are rearranged as:

$$\dot{\beta} = \frac{1}{U_1} \{ g \phi \cos \Theta_1 + Y_\beta \beta + (Y_p + W_1) \dot{\phi} + (Y_r - U_1) \dot{\psi} + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r \} \quad (3.70)$$

$$\ddot{\phi} = L_\beta \beta + L_p \dot{\phi} - \bar{A}_1 \ddot{\psi} + L_r \dot{\psi} + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \quad (3.71)$$

$$\ddot{\psi} = N_\beta \beta + N_{T_\beta} \beta - \bar{B}_1 \ddot{\phi} + N_p \dot{\phi} + N_r \dot{\psi} + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r \quad (3.72)$$

Eqs.(3.70-3.72) are solved numerically by using the 4th order Runge Kutta method and they are also solved exactly by using the Laplace Transformation method . Therefore, Eqs. (3.73-3.76) are also represented in Laplace transform with zero initial as follows:

$$\begin{bmatrix} (sU_1 - Y_\beta) & -(sY_p + g \cos \Theta_1 - sW_1) & s(U_1 - Y_1) \\ -L_\beta & (s^2 - L_p s) & -(s^2 \bar{A}_1 + sL_1) \\ -N_\beta - N_{T_\beta} & -(s\bar{B}_1 + N_p s) & (s^2 + sN_r) \end{bmatrix} \begin{bmatrix} \beta(s) \\ \phi(s) \\ \psi(s) \end{bmatrix} = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a(s) \\ \delta_r(s) \end{bmatrix} \quad (3.73)$$

Each of the lateral-directional transfer functions are determined by using Cramer's rule as it was done for longitudinal transfer functions:

sideslip-to-aileron (or-rudder) transfer function $\beta(s)/\delta(s)$

$$\frac{\beta(s)}{\delta(s)} = \frac{\begin{vmatrix} Y_\delta & -(sY_p + g \cos \Theta_1 - sW_1) & s(U_1 - Y_1) \\ L_\delta & (s^2 - L_p s) & -(s^2 \bar{A}_1 + sL_1) \\ N_\delta & -(s\bar{B}_1 + N_p s) & (s^2 + sN_r) \end{vmatrix}}{\begin{vmatrix} (sU_1 - Y_\beta) & -(sY_p + g \cos \Theta_1 - sW_1) & s(U_1 - Y_1) \\ -L_\beta & (s^2 - L_p s) & -(s^2 \bar{A}_1 + sL_1) \\ -N_\beta - N_{T_\beta} & -(s\bar{B}_1 + N_p s) & (s^2 + sN_r) \end{vmatrix}} = \frac{N_\beta}{\bar{D}_2} \quad (3.74)$$

bank angle-to-aileron (or-rudder) transfer function $\phi(s)/\delta(s)$

$$\frac{\phi(s)}{\delta(s)} = \frac{\begin{vmatrix} (sU_1 - Y_\beta) & Y_\delta & s(U_1 - Y_1) \\ -L_\beta & L_\delta & -(s^2 \bar{A}_1 + sL_1) \\ -N_\beta - N_{T_\beta} & N_\delta & (s^2 + sN_r) \end{vmatrix}}{\bar{D}_2} = \frac{N_\phi}{\bar{D}_2} \quad (3.75)$$

heading-to-aileron (or-rudder) transfer function $\psi(s)/\delta(s)$

$$\frac{\psi(s)}{\delta(s)} = \frac{\begin{vmatrix} (sU_1 - Y_\beta) & -(sY_p + g \cos \Theta_1 - sW_1) & Y_\delta \\ -L_\beta & (s^2 - L_p s) & L_\delta \\ -N_\beta - N_{T_\beta} & -(s\bar{B}_1 + N_p s) & N_\delta \end{vmatrix}}{\bar{D}_2} = \frac{N_\psi}{\bar{D}_2} \quad (3.76)$$

In summary, in this section, Eqs. (3.70-3.72) and Eqs. (3.74-3.76) gives us opportunity to solve linear equations numerically by RK4 method and exactly by using the Laplace Transformation method respectively.

3.3 Test Problems and Results

Stability derivatives in Table 2 are substituted into Eqs. (3.64-3.66) to obtain Eqs. (3.77-3.79).

The speed-to-elevator transfer function, $u(s)/\delta_e(s)$:

$$\frac{u(s)}{\delta_e(s)} = \frac{N_u}{\bar{D}_1} = \frac{-28.94s^3 + 2977.40s^2 + 3654.97s - 2434.42}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (3.77)$$

the angle of attack-to-elevator transfer function $\alpha(s)/\delta_e(s)$:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{N_\alpha}{\bar{D}_1} = \frac{-9.35s^3 - 409.52s^2 - 5.19s - 4.14}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (3.78)$$

the pitch-attitude-to-elevator transfer function $\theta(s)/\delta_e(s)$:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{N_\theta}{\bar{D}_1} = \frac{-409.52s^2 - 256.25s + 4.43}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (3.79)$$

After then, nonlinear equations of motions and linear equations of motions are modeled in Matlab Simulink® and solved at the same time to obtain same RK4 step size for linearized and nonlinear solution.

In the simulation, a step elevator deflection was given between the 0th and 5th seconds as shown Fig.3.4.

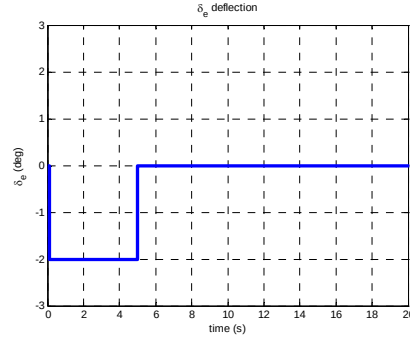


Figure 3.4 : Elevator Deflection

Simulations are obtained for 10 different elevator deflections ranging from -1° to -10° . Responses to elevator input are investigated. As mentioned before, same test cases are solved by three different ways and compared with each other.

Figure 2-4 are plotted to show the velocity responses to both $\delta_e = -1\text{deg}$ and -3deg elevator inputs. As seen in the figures, when input amplitude increase, responses increase, as well.

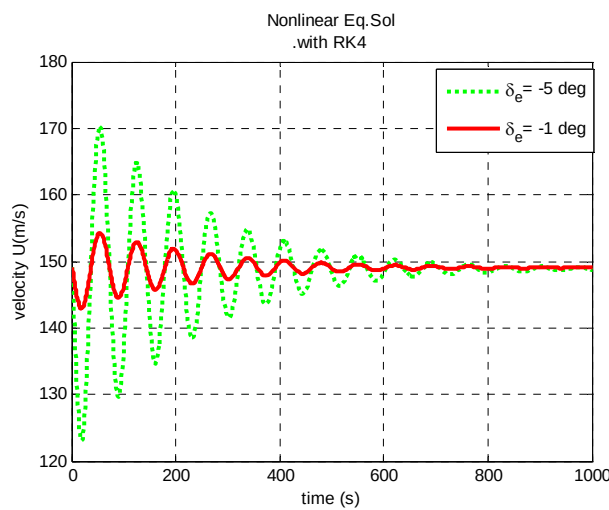


Figure 3.5 : Velocity response to elevator input by solving nonlinear equations with RK4 method

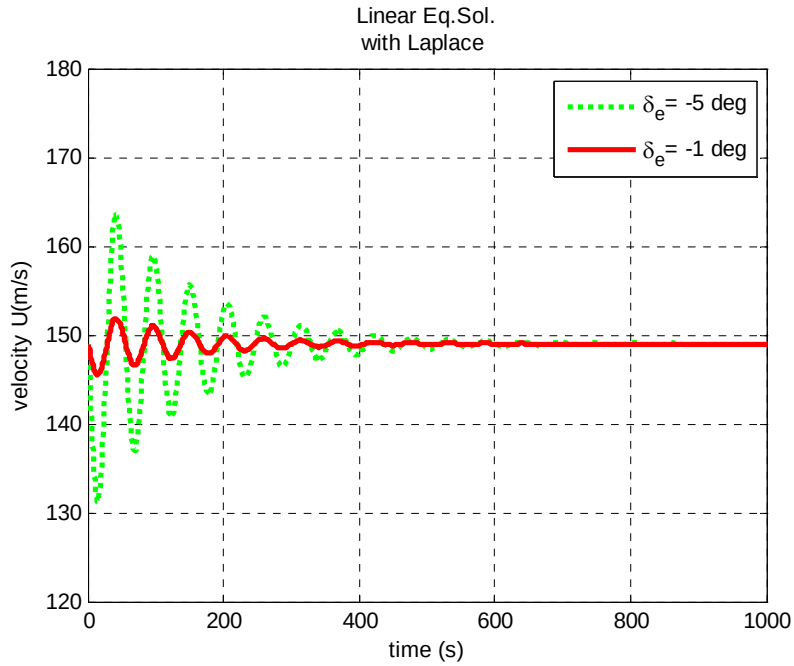


Figure 3.6 : Velocity response to elevator input by solving linear equations with Laplace Transform

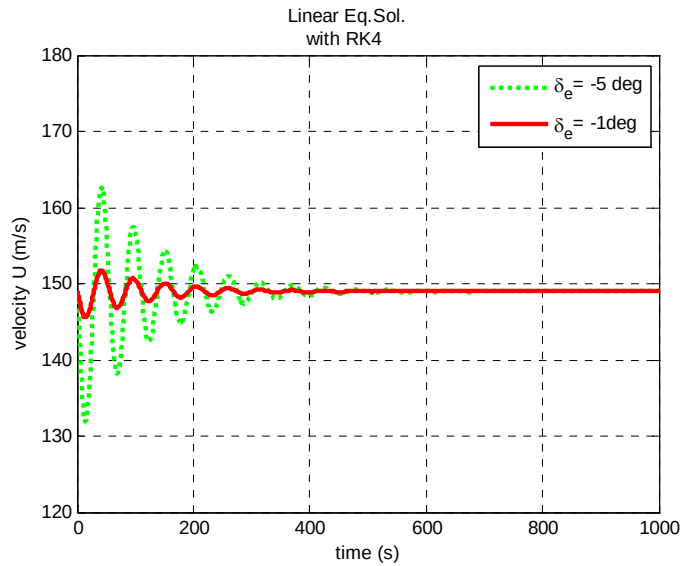


Figure 3.7 : Velocity response to elevator input by solving linear equations with RK4 method

Figures 3.5-3.7 indicate that behaviors of systems are similar.

However, the response magnitudes of the nonlinear system are more than the linear system. In addition, the difference of Laplace Transform and the RK4 method to

solve linearized equations is rather small for both elevator deflections, in comparison to the difference of Laplace Transform method and RK4 method to solve nonlinear equations. This results show RK4 method's efficiency to solve this kind of problems with a suitable step size.

Figures 3.8 and 3.9 present comparisons with results.

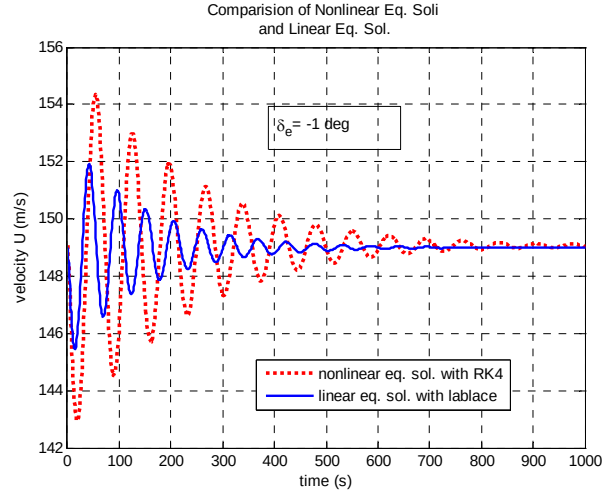


Figure 3.8 : Comparison of Nonlinear Equations Solution with RK4 and Linear Equations Solution with Laplace at $\delta_e = -1$ deg.

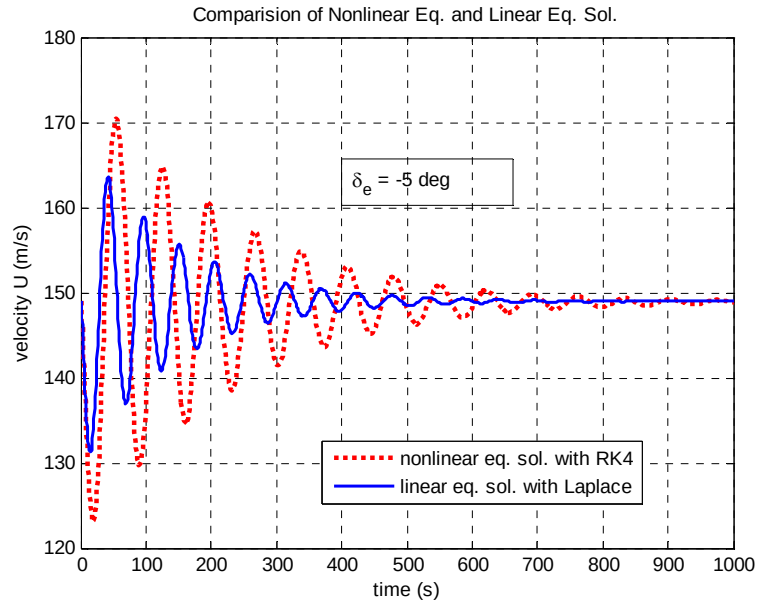


Figure 3.9 : Comparison of Nonlinear Equations Solution with RK4 and Linear Equations Solution with Laplace at $\delta_e = -5$ deg.

Figures 3.8 and 3.9 demonstrate that, the differences increase with ascending elevator inputs. Therefore following analyses are done in Fig. 10.

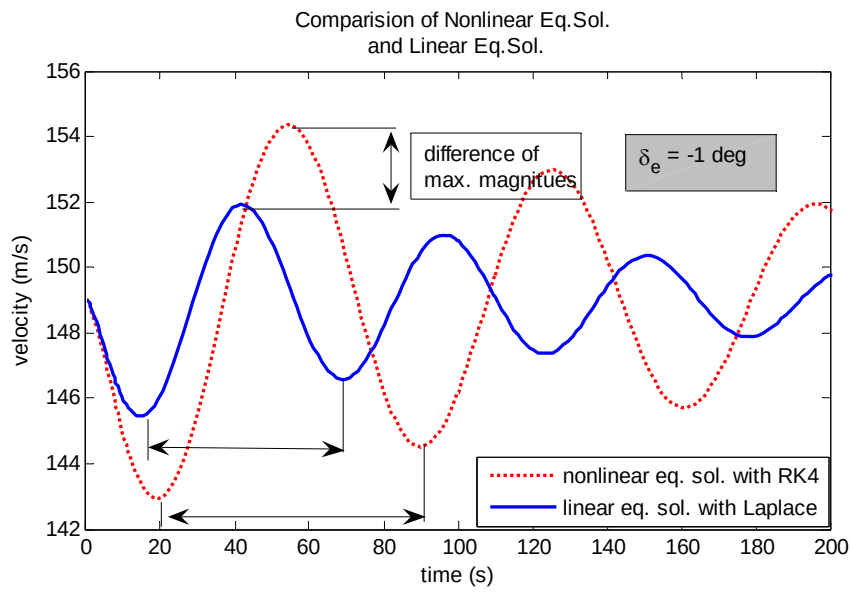


Figure 3.10 : Differences of Magnitudes and Periods

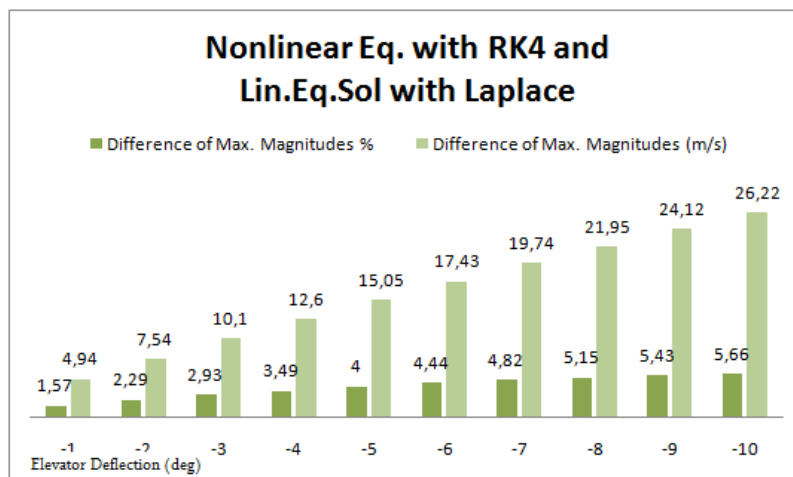


Figure 3.11 : Difference of Max. Magnitudes

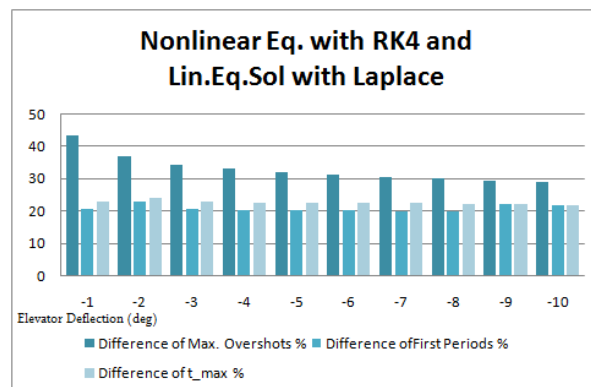


Figure 3.12 : Difference of Max. Overshoot, Periods and tmax

We can plot figures like Fig. 3.10 also for each elevator deflections. From these figures we can obtain Fig. 3.11 in which the differences in maximum magnitudes are compared.

The difference of maximum magnitudes % is defined as:

$$\frac{\text{Max magnitudes of Nonlinear Solution} - \text{Max magnitudes of Linear Solution}}{\text{Max magnitudes of Nonlinear Solution}} \times 100 \quad (3.80)$$

The difference of maximum magnitudes is defined as:

$$\text{Max magnitudes of Nonlinear Solution} - \text{Max magnitudes of Linear Solution} \quad (3.81)$$

The difference of maximum overshoots % is defined as:

$$\frac{\text{Max overshoot of Nonlinear Solution} - \text{Max overshoot of Linear Solution}}{\text{Max overshoot of Nonlinear Solution}} \times 100 \quad (3.82)$$

The difference of periods % is defined as:

$$\frac{\text{First Period of Nonlinear Solution} - \text{First Period of Linear Solution}}{\text{First Period of Nonlinear Solution}} \times 100 \quad (3.83)$$

The difference of tmax is defined as:

$$\frac{t_{\max} \text{ of Nonlinear Solution} - t_{\max} \text{ of Linear Solution}}{t_{\max} \text{ of Nonlinear Solution}} \times 100 \quad (3.84)$$

where tmax is the time of the first maximum

As seen in Figure 3.11, the differences increase with increasing magnitude elevator inputs. The difference is only 1.57 % at $\delta_e = -1$ deg., but it increases to 5.66% at $\delta_e = -10$ deg deflection.

Figure 3.12 indicated that period differences do not depend on elevator deflection and is always about 23%.

In Figure 3.13, angle of attack deflection response to elevator is demonstrated by solving both nonlinear and linear equations separately for -3 degree elevator deflection.

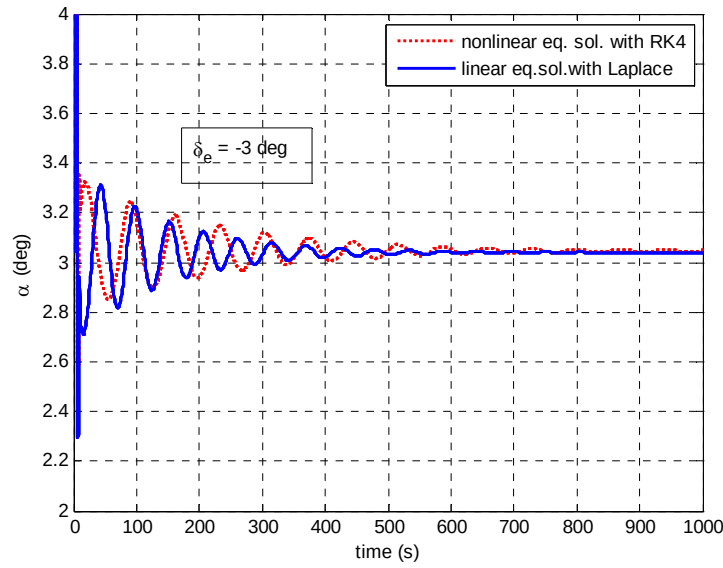


Figure 3.13 : Angle of attack response to elevator deflection

Short period mode can be seen straightforwardly in Figure 3.14, which is a scaled up plot of Figure 3.13.

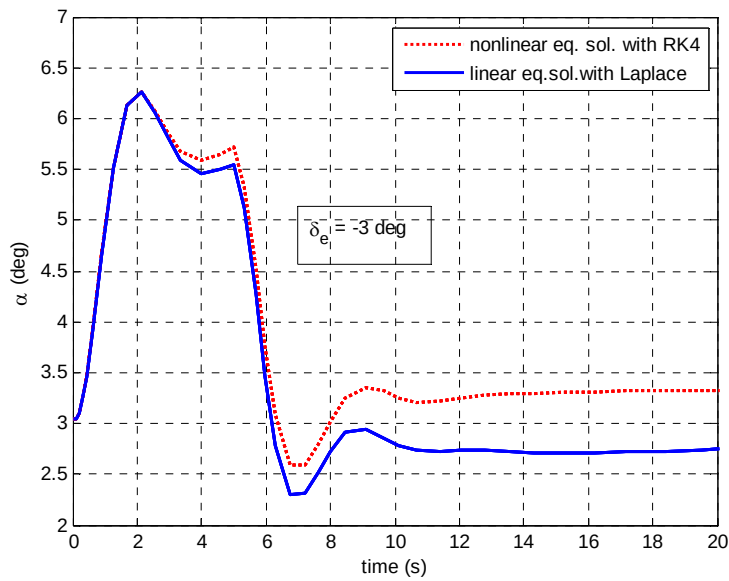


Figure 3.14 : Scaled up plot of angle of attack response to elevator deflection (short period)

Differences of magnitudes and periods for -7 deg. elevator deflection can be seen in Figure 3.15. Similarly this can be shown for different elevator deflections in Figure 3.16.

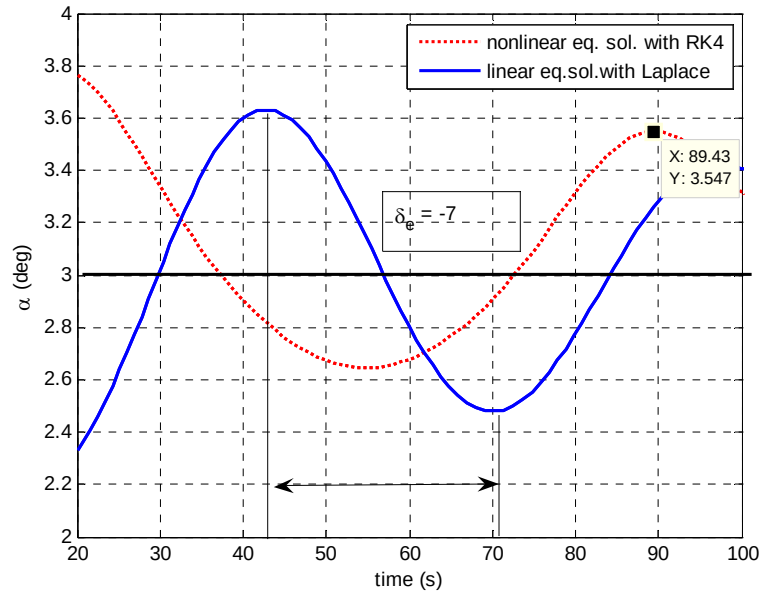


Figure 3.15 : Differences of Magnitudes and Periods

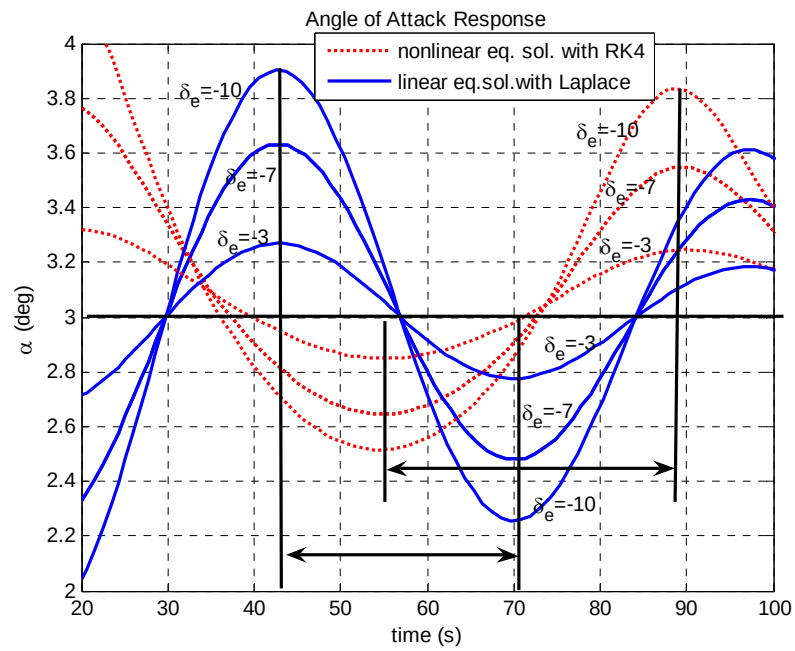


Figure 3.16 : Detailed Plot to analyze periods and time delay

4. FLIGHT STABILITY ANALYSIS

In this chapter, stability analysis is performed for both any flight condition and whole flight envelope, respectively.

4.1 Flight Stability Analysis for any Flight Condition

In Eqs. (3.77-3.79) aircraft Transfer Function is obtained for the flight condition which is stated in Table 2.1 .

The speed-to-elevator transfer function, $u(s)/\delta_e(s)$:

$$\frac{u(s)}{\delta_e(s)} = \frac{N_u}{D_1} = \frac{-28.94s^3 + 2977.40s^2 + 3654.97s - 2434.42}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (4.1)$$

the angle of attack-to-elevator transfer function $\alpha(s)/\delta_e(s)$:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{N_\alpha}{D_1} = \frac{-9.35s^3 - 409.52s^2 - 5.19s - 4.14}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (4.2)$$

the pitch-attitude-to-elevator transfer function $\theta(s)/\delta_e(s)$:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{N_\theta}{D_1} = \frac{-409.52s^2 - 256.25s + 4.43}{150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25} \quad (4.3)$$

For the three TFs, the characteristic equation of the aircraft which determines the the dynamic behavior of the aircraft is the same:

$$CE = 150.66s^4 + 223.60s^3 + 472.61s^2 + 9.57s + 6.25 \quad (4.4)$$

The fourth order characteristic equation for longitudinal motion can be written as the product of two second-order polynomials [33].

$$(s^2 + 2\zeta_{SP}\omega_{N_{SP}}s + \omega_{N_{SP}}^2)(s^2 + 2\zeta_{PH}\omega_{N_{PH}}s + \omega_{N_{PH}}^2) = 0 \quad (4.4)$$

The aircraft CE can be written in this form as Eq. (4.5).

$$CE = 150.66 (s^2 + 1.47 s + 3.102278) (s^2 + 0.0141379 s + 0.01337) \quad (4.5)$$

The time required for the oscillation to damp to one-half amplitude is defined as follows:

$$T_{1/2} = \frac{0.693}{\xi \omega_n} \quad (4.6)$$

Then from CE we can obtain the phugoid and short periods modes parameters: the roots, natural frequencies, damping ratios and the time required for the oscillation to damp to one-half amplitude as follows:

Table 4.1: The Aircraft's Longitudinal Modes

Phugoid	Short Period
$\xi_{PH} = 0.061$ $\omega_{n_{PH}} = 0.1156 \frac{rad}{s}$ $T_{1/2_{PH}} = 98.28 \text{ sec}$	$\xi_{SP} = 0.417$ $\omega_{n_{SP}} = 1.1761 \frac{rad}{s}$ $T_{1/2_{SP}} = 1.026 \text{ sec}$

These values are quite typical for this kind of aircraft.

4.1.1 Frequency Domain Analysis

The frequency response shows the magnitude and phase relationship between input and output [3]. In linear systems, the total output can be evaluated as the sum of system response to each individual input. For this reason, the response behaviour of the output signal in response to an input signal of constant amplitude, but changing frequency is interested. Bode plot demonstrates a method for estimating the frequency response of the airplane [37]. If the response of a linear system to either a step or sinusoidal input can be, then the system's response to an arbitrary input can also be estimated owing to the principle of superposition. In Bode plot, we can define the response of the system by the ratio of the output to input amplitude and the phase difference as the frequency goes from zero to infinity.

Following bode plots Figs.4.1-4.3 were drawn by using Matlab for the TFs. in Eqs. 4.1-4.3 to investigate aircraft longitudinal stability in depth as discussed in ref. [5, 43].

The Transfer Functions of longitudinal motion, $(\frac{u(s)}{\delta_e(s)}, \frac{\alpha(s)}{\delta_e(s)}, \frac{\theta(s)}{\delta_e(s)})$, are investigated. Let say $G(s)$ is one of the longitudinal motion TFs.

$$M(\omega) = |G(j\omega)| \quad \text{and} \quad \Phi = \angle G(j\omega)$$

$M(\omega)$ and Φ demonstrate the very useful fact that the steady state amplitude ratio and phase shift of a linear system can be obtained directly from the system transfer function. The procedure is simply to substitute $s = j\omega$ into TF and evaluate the absolute magnitude and phase angle of the result [43].

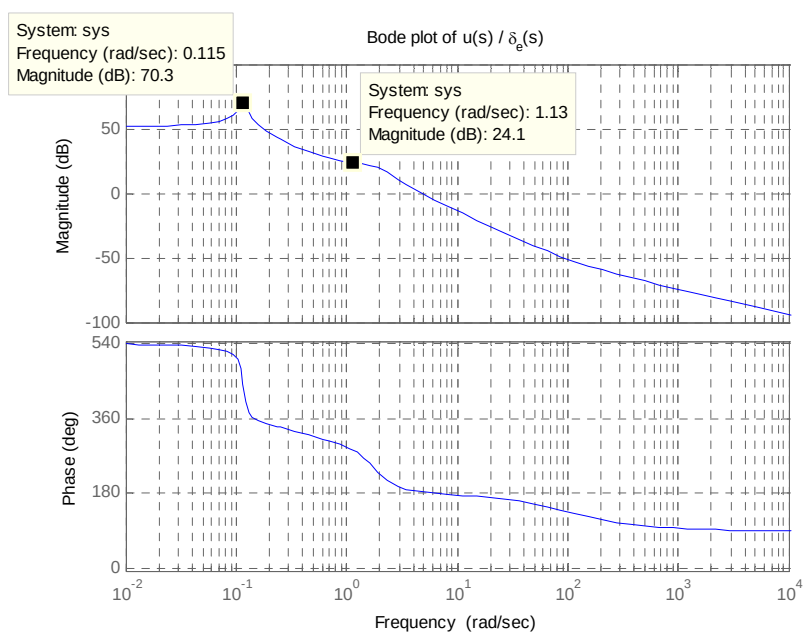


Figure 4.1 : Bode Plot of $\frac{u(s)}{\delta_e(s)}$

In Fig. 4.1 there are two graphs. First one is the magnitude versus the frequency on a logarithmic scale, as the other phase angle versus the frequency on a logarithmic scale.

This plot can be obtained by replacing the Laplace variable s by $j\omega$ in TF.

$$\text{Magnitude (dB)} = 20 \log_{10} \frac{\text{the output amplitude}}{\text{the input amplitude}}$$

The maximum occurs exactly at a frequency of:

$$\omega_{\text{peak magnitude}} = \omega_n \sqrt{1 - 2\zeta^2}$$

Fig. 4.2 indicates that the amplitude of the $\frac{u(s)}{\delta_e(s)}$ response is very small at the natural frequency of the short period oscillation. It means that there is very little change in the forward velocity during the short period oscillation.

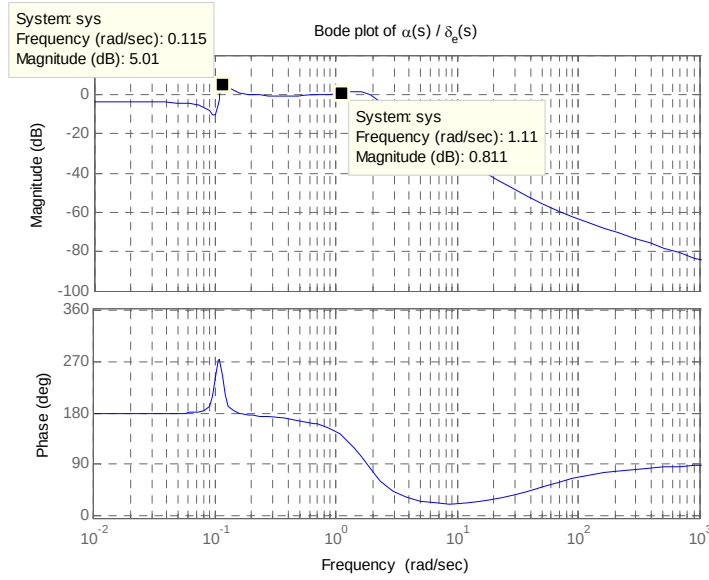


Figure 4.2 : Bode Plot of $\frac{\alpha(s)}{\delta_e(s)}$

Fig.4.2 is compatible to the idea that the phugoid oscillation takes place at almost constant angle of attack.

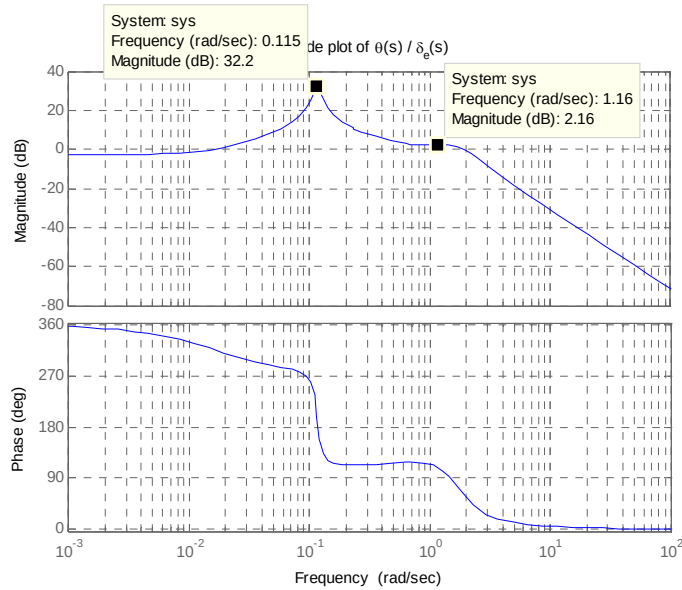


Figure 4.3 : Bode Plot of $\frac{\theta(s)}{\delta_e(s)}$

It can be seen from the Bode plots, the longitudinal flight is obviously imposed in phugoid mode owing to a given elevator deflection as anticipated from the natural frequencies and damping ratios of the aircraft.

4.2 Uncertainty in Flight Stability Derivatives

A transport aircraft required to fly in a flight envelope similar to that shown in Fig. 4.4.

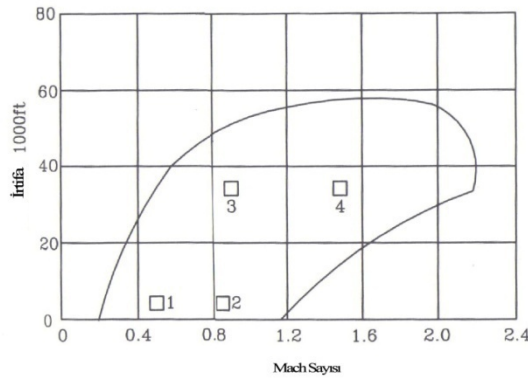


Figure 4.4 : Flight Envelope

Since aircraft stability derivatives have different values for changing flight conditions, the stability analysis needs to be repeated for all possible conditions. However, since an infinite number of flight conditions are possible, making this analysis directly is not feasible. According to experience in stability derivatives, it can be said that the uncertainty of stability derivatives ranges from 20% to 60% [44]. In this study, it is assumed that the uncertainty of two stability derivatives (X_α, Z_α) is 50% in order to determine the range of their variation. As a result, for the particular plane considered here the following inequalities are used:

$$2.6 < X_\alpha < 7.8 \quad (4.7)$$

$$-153.04 < Z_\alpha < -51.01 \quad (4.8)$$

4.3 Robustness Analysis Methods

In this section, a longitudinal stability analysis of a transport aircraft has been done considering the change of two stability derivatives within a given range of whole

flight envelope. Longitudinal stability analysis for all condition in the flight envelope is reduced the stability analysis of four polinomials by using Edge and Bialas theorems [45, 47, 48]. Whether or not the aircraft is stable can be determined by investigating eigenvalues of the matrix which is obtained by this four polinomials straightforwardly.

4.3.1 Parameter Box

The set of all values that can be assumed by a parameter vector formed by interval parameters is called the parameter box.

$$Q = \{ q | q_i \in [q_i^-, q_i^+] \} \quad i = 1, 2, \dots, l \quad (4.9)$$

The parameter box is a hyper-rectangle in the parameter space as seen in Fig. 4.5.

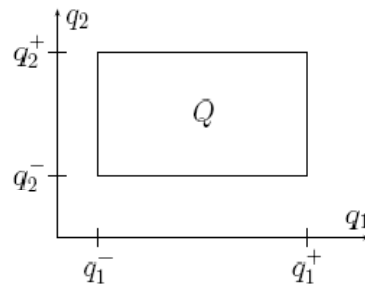


Figure 4.5: The plant parameter vector is bounded by an operating domain, $q \in Q$ [45]

An uncertain system is said to be stable if and only if, it is stable for all possible uncertainties [45].

$$G(s, Q) \text{ is stable} \Leftrightarrow \nexists q \in Q | G(s, Q) \text{ is unstable}$$

In general, there is no quick way to test the stability of an uncertain system. However, there are numerous methods and theorems for practical special cases. Some of these tools, which are relevant to our system, are introduced next.

4.3.2 Pole Spread

As shown in Fig.4.7, pole spread is obtained for every point in Fig. 4.6. by gridding the uncertain paremeters. In this example, No poles of the system is on the right hand plane (RHP). Therefore, it is possible to assert that the system is stable for the given range of uncertain parameters.

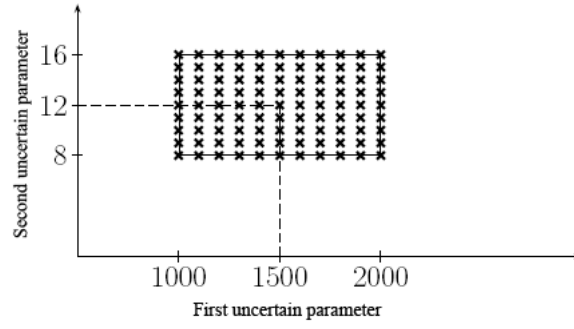


Figure 4.6: An 11 x 9 grid over the operating domain Q [45]

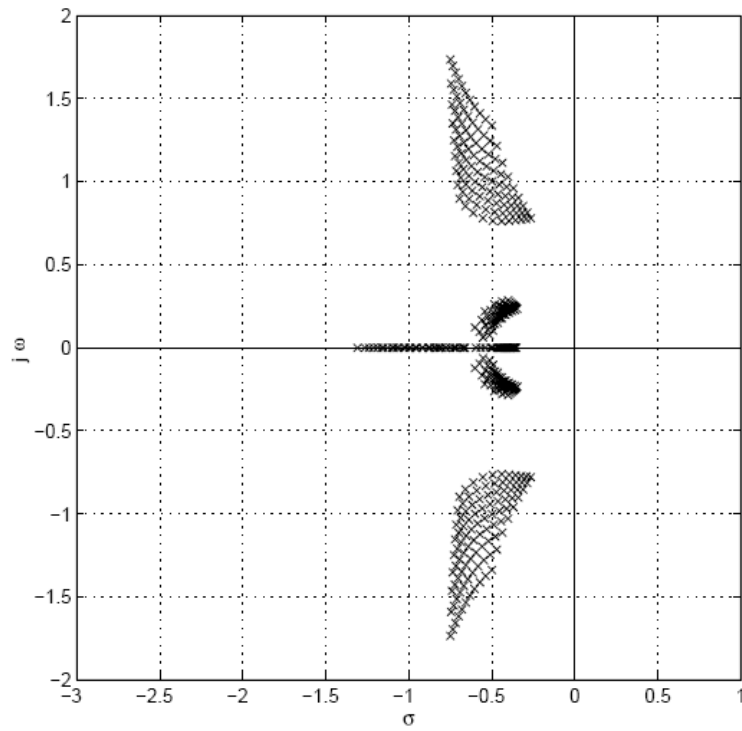


Figure 4.7: Root set for the grid points of Fig. 4.6 [45]

It is important to remark that the pole spread method may require a lot of computation power. In addition, it does not guarantee the stability theoretically no matter how small the grid size is, since it is not possible to cover all possible points. The characteristic polynomial derived from the aircraft body axis system has some interesting properties. As result of these, the stability of the system under the possible parameter changes can be easily analyzed.

4.3.3 Parametric Polynomials with Affine Linear Coefficients and Value Set

The characteristic polynomial of an aircraft's longitudinal motion, which includes two uncertain stability derivatives is an affine linear polynomial. Therefore, in this section, affine linear polynomials and the edge theorem, which is used for the stability analysis of these kind of polynomials are introduced.

Polynomials with affine-linear coefficients can be shown as follows:

$$p(s, \mathbf{q}) = p_0(s) + q_1 p_1(s) + \dots + q_l p_l(s) \quad (4.10)$$

$$\mathbf{q} = [q_0 \quad q_1 \quad \dots \quad q_l]^T$$

$$q_i \in [q_i^-, q_i^+] \quad i = 1, 2, 3, \dots, l$$

$q_0 \quad q_1 \quad \dots \quad q_l$: uncertain parameters

q_i^- ve q_i^+ : lower and upper limits of the i th uncertain parameter

For instance:

$$p(s, \mathbf{q}) = s^4 + q_1(s^3 + s^2 - s) + q_2(s^2 + s) + q_3(3s^2 + 1) \quad q_0, q_2 \in [0 \quad 1]$$

This polynomial is a polynomial with affine-linear coefficients.

The value set of an affine-linear polynomial family is as shown in Fig. 4.8.

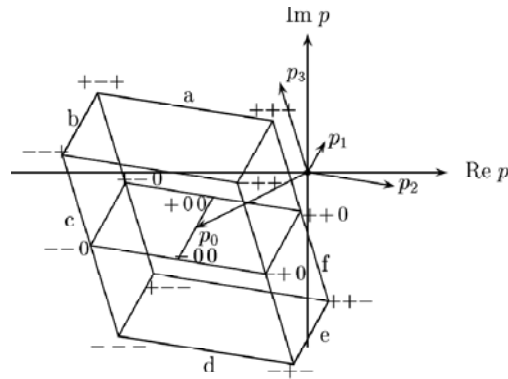


Figure 4.8: The value set for an affine linear polynomial at a fixed frequency [45]

In order to polynomial family is robust stable, its value set has to exclude the origin for all $\omega > 0$ as shown Fig. 4.9.

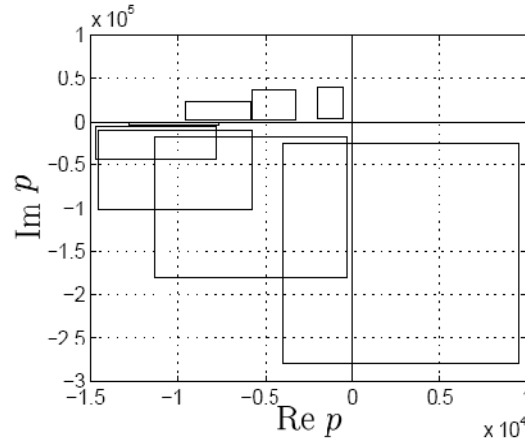


Figure 4.9: Value sets for several frequencies [45]

Boundary crossing theorem states that, A polynomial family is robustly stable if, and only if, its value set excludes the origin for all $\omega > 0$.

4.3.4 Edge Theorem

In the above case, the Edge Theorem eases the analysis process by stating that the investigation of stability of the edge polynomials is enough.

For a given polynomial family, the polynomials corresponding to vertices of the parameter box are called as vertex polynomials. Note that the uncertain parameters assume their minimum and/or maximum values at the vertex polynomials.

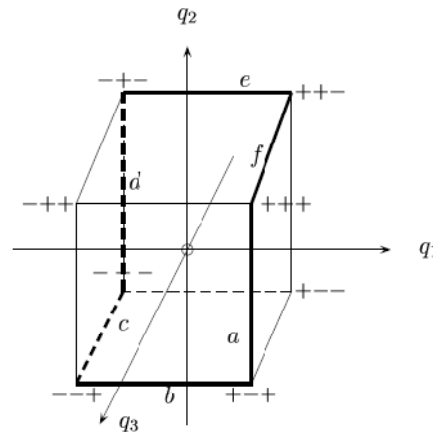


Figure 4.10 : The edges of a value set [45]

Usually it is not sufficient to test the stability of vertex polynomials to determine robust stability of an affine linear polynomial family.

If exactly one of the uncertain parameters varies between its minimum and maximum values while the remaining $l-1$ parameters stick either to their minimum or their maximum values, the resulting polynomial family is called as an edge polynomial.

if $l=3 \Rightarrow 3 \times 2^2$ edges

$$p^{--o}(s, q) = p(s, q_1^-, q_2^-, q_3) \quad (4.11)$$

$$p^{-+o}(s, q) = p(s, q_1^-, q_2^+, q_3) \quad (4.12)$$

$$p^{-o-}(s, q) = p(s, q_1^-, q_2, q_3^-) \quad (4.12)$$

Edge Theorem:

An affine linear polynomial family is robustly stable if, and only if, the edge polynomials are robustly stable [46].

An edge polynomial can be obtained from two vertex polynomials.

$$P(s, Q) = \{(1 - \lambda) p_b(s) + \lambda p_c(s) \mid \lambda \in [0;1]\} \quad (4.13)$$

where:

$p_b(s)$: 1st vertex polynomial

$p_c(s)$: 2nd vertex polynomial

λ : uncertain parameter

After vertex polynomials are obtained, there exists only one uncertain parameter in edge polynomials irrespective of the number of uncertain parameters.

In following step, the Bialas Theorem [47, 48], which is very suitable to investigate a polynomial with one uncertain parameter, is given.

4.3.5 The Bialas Theorem

Consider the characteristic equation given by

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$$

Hurwithz matrix of this polynomial is defined as follows [45]:

$$H_n = \begin{bmatrix} a_1 & a_3 & a_5 & \dots & 0 & 0 & 0 \\ a_0 & a_2 & a_4 & \dots & \cdot & \cdot & \cdot \\ \cdot & a_1 & a_3 & \dots & a_n & 0 & 0 \\ \cdot & a_0 & a_2 & \dots & a_{n-1} & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & a_{n-2} & a_n & 0 \\ \cdot & \cdot & \cdot & \dots & a_{n-3} & a_{n-1} & 0 \\ 0 & 0 & 0 & \dots & a_{n-4} & a_{n-2} & a_n \end{bmatrix}$$

Assume that H_n^b and H_n^c are the Hurwitz matrices of $P_b(s)$ and $P_c(s)$ respectively [45].

$$P_b(s) = b_0 + b_1s + b_2s^2 + \dots + b_ns^n, \quad b_n > 0$$

$$P_c(s) = c_0 + c_1s + c_2s^2 + \dots + c_ns^n, \quad c_n > 0$$

The polynomial family $P(s, Q) = \{(1 - q)P_b(s) + qP_c(s) | q \in [0; 1]\}$

is stable if and only if

- 1) $P_b(s)$ is stable
- 2) the matrix $(H_n^b)^{-1} H_n^c$ has no nonpositive real eigenvalues

Stability of edge polynomials which include one uncertain parameter can be investigated straightforwardly owing to the Bialas Theorem.

4.4 Robust Flight Stability Analysis

The differences between characteristics of aircraft results from variations of aircraft geometries and stability derivatives. As stability derivatives differs from one aircraft to another, they also changes for the same aircraft for different flight conditions. Therefore, a stability analysis for a flight condition is valid only for this condition. Hence, stability analysis of longitudinal motion of aircraft should be investigated for all flight conditions by determining two uncertain stability derivatives. In this section, the stability analysis of aircraft longitudinal motion for all flight envelope are reduced to the analysis of four polynomials which are obtained by using Edge and Bialas Theorems.

The characteristic equation of aircraft longitudinal motion can be represented as seen in Eq. 4.14.

$$CE = \begin{vmatrix} (s - X_u - X_{T_u}) & -X_\alpha & g \cos \theta_1 + W_1 s \\ -Z_u & \{s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha\} & \{-(Z_q + U_1)s - g \sin \theta_1\} \\ -(M_u + M_{T_u}) & -\{M_{\dot{\alpha}}s + M_\alpha + M_{T_\alpha}\} & (s^2 - M_q s) \end{vmatrix} \quad (4.14)$$

After the determinant operation is applied, following expression (Eq. 4.15) can be obtained.

$$\begin{aligned} CE = & -(-M_{T_\alpha} - M_\alpha - s M_{\dot{\alpha}}) ((s - X_{T_u} - X_u) (g \sin[\theta_1] + s(-U_1 - Z_q)) + (g \cos[\theta_1] + s W_1) Z_u) + \\ & (-M_{T_u} - M_u) (-X_\alpha (g \sin[\theta_1] + s(-U_1 - Z_q)) - (g \cos[\theta_1] + s W_1) (-Z_\alpha + s(U_1 - Z_{\dot{\alpha}}))) + \\ & (s^2 - s M_q) (-X_\alpha Z_u + (s - X_{T_u} - X_u) (-Z_\alpha + s(U_1 - Z_{\dot{\alpha}}))) \end{aligned} \quad (4.15)$$

When all the values of stability derivatives except X_α and Z_α in Table 2 are substituted to Eq. 4.5, the characteristic equation reduce to Eq. 4.6.

$$CE = 3.94568 + 204735.s + 1.41501 \times 10^8 s^2 + 281886.s^3 + 149.823 s^4 + 0.00118026 X_\alpha - 0.248322 s X_\alpha + 0.1476 s^2 X_\alpha - 0.0225321 Z_\alpha - 1150.01 s Z_\alpha - 0.6188 s^2 Z_\alpha - s^3 Z_\alpha \quad (4.16)$$

It can be seen that the charecteristic equation is an affine-linear polynomial family as mentioned before in Eq.4.13.

where:

$$CE = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4$$

$$\begin{aligned} a_0 &= 3.945 + 0.001180 X_\alpha - 0.02253 Z_\alpha; \\ a_1 &= (204734.7151 - 0.248322 X_\alpha - 1150.0050 Z_\alpha); \\ a_2 &= (1.41501 \times 10^8 + 0.1476 X_\alpha - 0.6188 Z_\alpha); \\ a_3 &= 281886.237 - Z_\alpha; \\ a_4 &= 149.822; \end{aligned}$$

The minimum and/or maximum values of uncertain parameters, which are given as $(2.6 < X_\alpha < 7.8)$ and $(-153.04 < Z_\alpha < -51.01)$, are substituted into Eq. 4.16, and the following vertex polinomials are obtained (Eq. 4.17-4.20).

$$p^{--}(s) = 7.39706 + 380731.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4 \quad (4.17)$$

$$p^{-+}(s) = 5.09811 + 263396.s + 1.41501 \times 10^8 s^2 + 281037.s^3 + 149.823 s^4 \quad (4.18)$$

$$p^{++}(s) = 5.10425 + 263395.s + 1.41501 \times 10^8 s^2 + 281037.s^3 + 149.823 s^4 \quad (4.19)$$

$$p^{+-}(s) = 7.4032 + 380730.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4 \quad (4.20)$$

Since the characteristic equation has affine-linear coefficients, edge polynomials needs to be obtained from the vertex polynomials.

Edge polynomials can be obtained as follows (Eqs. 4.21-4.24).

$$edge01 = (1 - \lambda) p^{--}(s) + \lambda p^{+-}(s) \quad (4.21)$$

$$edge02 = (1 - \lambda) p^{+-}(s) + \lambda p^{++}(s) \quad (4.22)$$

$$edge03 = (1 - \lambda) p^{++}(s) + \lambda p^{-+}(s) \quad (4.23)$$

$$edge04 = (1 - \lambda) p^{-+}(s) + \lambda p^{--}(s) \quad (4.24)$$

If Eqs 7-10 are substituted into Eqs. 4.21-4.24, then Eqs. 4.25- 4.28 are obtained.

$$edge01 = (1 - \lambda)(7.39706 + 380731.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4) + \lambda(7.4032 + 380730.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4) \quad (4.25)$$

$$edge02 = (1 - \lambda)(7.4032 + 380730.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4) + \lambda(7.4032 + 380730.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4) \quad (4.26)$$

$$edge03 = (1 - \lambda)(5.10425 + 263395.s + 1.41501 \times 10^8 s^2 + 281037.s^3 + 149.823 s^4) + \lambda(5.09811 + 263396.s + 1.41501 \times 10^8 s^2 + 281037.s^3 + 149.823 s^4) \quad (4.27)$$

$$edge04 = (1 - \lambda)(5.09811 + 263396.s + 1.41501 \times 10^8 s^2 + 281037.s^3 + 149.823 s^4) + \lambda(7.39706 + 380731.s + 1.41501 \times 10^8 s^2 + 282039.s^3 + 149.823 s^4) \quad (4.28)$$

Edge polynomials include only one uncertain parameter as seen in Eqs. 4.25-4.28.

Therefore, Bialas Theorem, which provide an efficient and easy way to test the stability of polynomials with one uncertain parameters is benefitted.

When the Bialas Theorem is applied to edge polynomials, it can be seen in Table 4.2 that the hurwitz derminant of the first polinomials are postive and the matrix of $(H_n^b)^{-1} H_n^c$ has no nonpositive real eigenvalues. Therefore, it can be said that all edge polynomials are stable.

In conclusion, it can be said that the system is stable for the parameter ranges determined at the beginning.

Table 4.2 : Robust Stability Test

	Hurwitz determinants of $p_b(s)$	Eigenvalues of $(H_n^b)^{-1} H_n^c$ matrices	
Edge 1	282039., 3.99088x10 ¹³ , 1.51945 x10 ¹⁹	1., 0.999997, 0.999993	stable
Edge 2	282039., 3.99085x10 ¹³ , 1.51943x10 ¹⁹	1., 0.999639, 0.691815	stable
Edge 3	281937., 3.98941x10 ¹³ , 1.05079x10 ¹⁹	1.,1.,1.	stable
Edge 4	281937., 3.98941 x10 ¹³ , 1.05079x10 ¹⁹	1.44547, 1.00036, 1.	stable

5. THREE AND SIX DEGREE OF FREEDOM TRIM ANALYSES

In this section, three and six degree of freedom trim analyses of fixed wing aircraft are studied. Trim analysis is the solution of aircraft force and moment equations at a given instant, to obtain unknown parameters. This solution requires an iterative procedure. Two different iteration methods are tested. These are Gauss Seidel and Newton Raphson iterative methods. Three degree of freedom trim analysis is applied for the motion of a transport aircraft in a vertical plane. Six degree of freedom trim analysis is applied for the steady, constant altitude, constant radius turning motion of a business jet in a horizontal plane.

5.1 Longitudinal Trim Analysis

Longitudinal motion includes two translations and one rotation. Translations are in x direction (Eq. 3.1) and z direction (Eq. 3.3) and rotation is about the y axis (Eq. 3.5). Therefore these three equations are necessary to investigate longitudinal motion. All the terms of the x force equation (Eq. 3.1) are collected to the one side of the equation and named as the x force function (XF). Similarly the z force function and pitching moment functions can be obtained. For steady flow in a vertical plane these functions are as follows:

$$\begin{aligned} XF = & -mg \sin(\gamma + \alpha) - (C_{D_0} + C_{D_\alpha} \alpha) q S_w \cos \alpha \\ & + (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h + C_{L_{\delta_e}} \delta_e) q S_w \sin \alpha \\ & + T_{\max,SL} \sigma \cos \phi_T \delta_T = 0 \end{aligned} \quad (5.1)$$

$$\begin{aligned} ZF = & mg \cos(\gamma + \alpha) - (C_{D_0} + C_{D_\alpha} \alpha) q S_w \sin \alpha \\ & - (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h + C_{L_{\delta_e}} \delta_e) q S_w \cos \alpha \\ & - T_{\max,SL} \sigma \sin \phi_T \delta_T = 0 \end{aligned} \quad (5.2)$$

$$YM = (C_{m_0} + C_{m_\alpha} \alpha + C_{m_{ih}} i_h + C_{m_{\delta_e}} \delta_e) q S_w c_w + T_{\max,SL} \sigma \sin \phi_T \delta_T z_t = 0 \quad (5.3)$$

There are three equations (Eqs. 5.1–5.3) for the longitudinal motion. Therefore three unknown can be solved from these equations. For instance, in a vertical plane the unknowns may be selected as α (angle of attack), δ_T (throttle setting) and one of i_h (horizontal stabilizer angle) or δ_e (elevator deflection). These problems are solved by two different methods in next section respectively: 1) The Gauss-Seidel Method 2) The Newton-Raphson Methods.

5.1.1 Three DoF Trim Analysis by Using the Gauss-Seidel Method

If δ_e is known, δ_T can be solved from the x force equation, α can be solved from the z force equation and i_h can be solved from the pitching moment equation as follows.

The equations 5.1 -5.2 are arranged to get following equations respectively [41, 42]:

x force equation is from Eq. 5.1:

$$\dot{U} = -QW + RV - g \sin \Theta + \frac{1}{m}(-D \cos \alpha + L \sin \alpha + T \cos \phi_T) \quad (5.4)$$

z force equation is from Eq. 5.3:

$$\dot{W} = -PV + QU + g \cos \Phi \cos \Theta + \frac{1}{m}(-D \sin \alpha - L \cos \alpha - T \sin \phi_T) \quad (5.5)$$

pitching moment equation is from Eq. 5.2:

$$\dot{Q} = \frac{1}{I_{YY}}[PR(I_{zz} - I_{xx}) - (P^2 - R^2)I_{xz} + M_A - M_T] \quad (5.6)$$

Following assumptions are made for steady horizontal flight.

$$\gamma = 0 \Rightarrow \Theta = \alpha$$

$$\dot{U}, \dot{W}, \dot{Q}, P, R, \Phi = 0$$

Substituting these assumptions into Eqs. (5.4-5.6) we can obtain Eqs. (5.7-5.9).

x force equation:

$$0 = -mg \sin \alpha - D \cos \alpha + L \sin \alpha + T \cos \phi_T \quad (5.7)$$

z force equation:

$$0 = mg \cos \alpha - D \sin \alpha - L \cos \alpha - T \sin \phi_T \quad (5.8)$$

pitch moment equation:

$$\begin{aligned} 0 &= M_A - M_T \\ 0 &= M_A - T d_T \end{aligned} \quad (5.9)$$

Aerodynamic forces and moments that appear in equations (5.7-5.9) are calculated by using Eqs(3.28-3.38). As expressed in Ref. [41, 42], substituting these aerodynamic force and moment equations into Eqs. (5.7-5.9), we get:

x force equation :

$$\begin{aligned} 0 &= W \sin \alpha - (C_{D_0} + C_{D_\alpha} \alpha) \bar{q} S_w \cos \alpha \\ &\quad + (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h C_{L_{\delta e}} \delta_e) \bar{q} S_w \sin \alpha + T_{\max, SL} \cdot \sigma \cdot \delta_T \cos \phi_T \end{aligned} \quad (5.10)$$

z force equation:

$$\begin{aligned} 0 &= W \cos \alpha - (C_{D_0} + C_{D_\alpha} \alpha) \bar{q} S_w \sin \alpha \\ &\quad - (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h C_{L_{\delta e}} \delta_e) \bar{q} S_w \cos \alpha - T_{\max, SL} \cdot \sigma \cdot \delta_T \cos \phi_T \end{aligned} \quad (5.11)$$

pitch moment equation:

$$0 = (C_{m_0} + C_{m_\alpha} \alpha + C_{m_{ih}} i_h + C_{m_{\delta e}} \delta_e) \bar{q} S_w \bar{c}_w - T_{\max, SL} \cdot \sigma \cdot \delta_T \cdot d_T \quad (5.12)$$

Rearranging Eq. (5.10), δ_T is solved as:

$$\delta_T = \frac{W \sin \alpha - (C_{D_0} + C_{D_\alpha} \alpha) \bar{q} S_w \cos \alpha + (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h C_{L_{\delta e}} \delta_e) \bar{q} S_w \sin \alpha}{T_{\max, SL} \cdot \sigma \cdot \cos \phi_T} \quad (5.13)$$

α is solved from z-force equation (5.11):

$$\alpha = \frac{W \cos \alpha - (C_{D_0} + C_{D_\alpha} \alpha) \bar{q} S_w \sin \alpha - (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{ih}} i_h C_{L_{\delta e}} \delta_e) \bar{q} S_w \cos \alpha - T_{\max, SL} \cdot \sigma \cdot \delta_T \sin \phi_T}{(C_{D_\alpha} \sin \alpha + C_{L_\alpha} \cos \alpha) \bar{q} S_w} \quad (5.14)$$

i_h is solved from pitch moment equation (5.12):

$$i_h = \frac{(C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e) \bar{q} S_w \bar{c}_w - T_{\max, SL} \cdot \sigma \cdot \delta_T \cdot d_T}{C_{M_{i_h}} \bar{q} S_w \bar{c}_w} \quad (5.15)$$

Hence, it is straightforward to solve these three equations by the Gauss-Seidel method iteratively.

5.1.2 Three DoF Trim Analysis by Using the Newton-Raphson Method

The following procedure is used to obtain the converged solution. First the initial values of the unknowns (δ_T, α, i_h) are guessed and initial values of the XF , ZF and YM functions are calculated. The values of these functions are equal to zero for the converged solution. For the iteration level $i+1$, the new guesses of the unknowns are written as,

$$\begin{aligned} \delta_{T,i+1} &= \delta_{T,i} + d\delta_T \\ \alpha_{i+1} &= \alpha_i + d\alpha \\ i_{h,i+1} &= i_{h,i} + di_h \end{aligned} \quad (5.16)$$

The new values of the functions are,

$$\begin{aligned} XF_{i+1} &= XF_i + \frac{\partial XF}{\partial \delta_T} d\delta_T + \frac{\partial XF}{\partial \alpha} d\alpha + \frac{\partial XF}{\partial i_h} di_h = 0 \\ ZF_{i+1} &= ZF_i + \frac{\partial ZF}{\partial \delta_T} d\delta_T + \frac{\partial ZF}{\partial \alpha} d\alpha + \frac{\partial ZF}{\partial i_h} di_h = 0 \\ YM_{i+1} &= YM_i + \frac{\partial YM}{\partial \delta_T} d\delta_T + \frac{\partial YM}{\partial \alpha} d\alpha + \frac{\partial YM}{\partial i_h} di_h = 0 \end{aligned} \quad (5.17)$$

$d\delta_T, d\alpha, di_h$ are solved from the following linear system of equations.

$$\begin{bmatrix} \frac{\partial XF}{\partial \delta_T} & \frac{\partial XF}{\partial \alpha} & \frac{\partial XF}{\partial i_h} \\ \frac{\partial ZF}{\partial \delta_T} & \frac{\partial ZF}{\partial \alpha} & \frac{\partial ZF}{\partial i_h} \\ \frac{\partial YM}{\partial \delta_T} & \frac{\partial YM}{\partial \alpha} & \frac{\partial YM}{\partial i_h} \end{bmatrix} \begin{bmatrix} d\delta_T \\ d\alpha \\ di_h \end{bmatrix} = - \begin{bmatrix} XF_i \\ ZF_i \\ YM_i \end{bmatrix} \quad (5.18)$$

where:

$$\begin{bmatrix} \frac{\partial XF}{\partial \delta_T} & \frac{\partial XF}{\partial \alpha} & \frac{\partial XF}{\partial i_h} \\ \frac{\partial ZF}{\partial \delta_T} & \frac{\partial ZF}{\partial \alpha} & \frac{\partial ZF}{\partial i_h} \\ \frac{\partial YM}{\partial \delta_T} & \frac{\partial YM}{\partial \alpha} & \frac{\partial YM}{\partial i_h} \end{bmatrix} : \text{Jacobian Matrix}; \quad \begin{bmatrix} d\delta_T \\ d\alpha \\ di_h \end{bmatrix} : \text{force vector}; \quad \begin{bmatrix} d\delta_T \\ d\alpha \\ di_h \end{bmatrix} : \text{step size.}$$

Iterations continue until the converged solution is obtained.

If the values of variables at the Table 2.2 to the equation 5.1-5.3, we can get following equations :

$$XF = 142821. dT - 974648. (CD0 + CDa \alpha) \cos \alpha - 500000 \sin \alpha + 974648. (0.2165 + 1.0652 i_h + 5.4039 \alpha) \sin \alpha \quad (5.19)$$

$$ZF = 500000 \cos \alpha - 974648. (0.2165 + 1.0652 i_h + 5.4039 \alpha) \cos \alpha - 974648. (CD0 + CDa \alpha) \sin \alpha \quad (5.20)$$

$$YM = 228514. dT + 3.94733 \times 10^6 (0.1093 - 3.8246 i_h - 1.7936 \alpha) \quad (5.21)$$

To obtain Jacobian Matrix of the force equations we need partial derivatives of these equations with respect to unknown trim variables.

$$\partial_{dT} XF = 142821. \quad (5.22)$$

$$\partial_{\alpha} XF = -500000 \cos[\alpha] - 974648. CDa \cos[\alpha] + 974648. (0.2165 + 1.0652 i_h + 5.4039 \alpha) \cos[\alpha] + 5.2669 \times 10^6 \sin[\alpha] + 974648. (CD0 + CDa \alpha) \sin[\alpha] \quad (5.23)$$

$$\partial_{i_h} XF = 1.0382 \times 10^6 \sin[\alpha] \quad (5.24)$$

$$\partial_{dT} ZF = 0 \quad (5.25)$$

$$\partial_{\alpha} ZF = -5.2669 \times 10^6 \cos[\alpha] - 974648. (CD0 + CDa \alpha) \cos[\alpha] - 500000 \sin[\alpha] - 974648. CDa \sin[\alpha] + 974648. (0.2165 + 1.0652 i_h + 5.4039 \alpha) \sin[\alpha] \quad (5.26)$$

$$\partial_{i_h} ZF = -1.0382 \times 10^6 \cos[\alpha] \quad (5.27)$$

$$\partial_{dT} YM = 228514. \quad (5.28)$$

$$\partial_{\alpha} YM = -7.07992 \times 10^6 \quad (5.29)$$

$$\partial_{i_h} YM = -1.50969 \times 10^7 \quad (5.30)$$

Then the step size is obtained from following equation.

$$dx = -\text{inv}[\text{Jacobian Matrix}] * [\text{Force Vector}] \quad (5.31)$$

In the next step the values of unknowns is calculated as follow:

$$x_{i+1} = x_i + dx \quad (5.32)$$

This procedure is continued until the sufficient convergence is obtained.

5.1.3 The Comparison of the Gauss-Seidel and Newton- Raphson Methods

For this problem with the same initial guesses, the Newton-Raphson Method converges faster than the Gauss-Seidel Method as seen Table 5.1.

Table 5.1: Comparison of Newton-Raphson and Gauss-Seidel Methods

----- INITIAL GUESSES FOR ALPHA, IH AND DELTA THRUST ----- ALPHA= 0 radian IH =0.000000 radian DTHRUST =0.750000			
-----START NEWTON-RAPHSON -----			
iteration	alpha	ih	dthrust
1,	0.05302 radian,	0.00825 radian,	0.29962
1,	3.03798 degree,	0.47256 degree,	0.29962
2,	0.05317 radian,	0.00700 radian,	0.22155
2,	3.04644 degree,	0.40087 degree,	0.22155
3,	0.05317 radian,	0.00699 radian,	0.22112
3,	3.04656 degree,	0.40045 degree,	0.22112
4,	0.05317 radian,	0.00699 radian,	0.22112
4,	3.04656 degree,	0.40045 degree,	0.22112
5,	0.05317 radian,	0.00699 radian,	0.22112
5,	3.04656 degree,	0.40045 degree,	0.22112
6,	0.05317 radian,	0.00699 radian,	0.22112
6,	3.04656 degree,	0.40045 degree,	0.22112
7,	0.05317 radian,	0.00699 radian,	0.22112
7,	3.04656 degree,	0.40045 degree,	0.22112
8,	0.05317 radian,	0.00699 radian,	0.22112
8,	3.04656 degree,	0.40045 degree,	0.22112
9,	0.05317 radian,	0.00699 radian,	0.22112
9,	3.04656 degree,	0.40045 degree,	0.22112
10,	0.05317 radian,	0.00699 radian,	0.22112
10,	3.04656 degree,	0.40045 degree,	0.22112
-----START GAUSS SEIDEL ITERATION-----			
iteration	alpha	ih	dthrust
1,	0.05487 radian,	0.00517 radian,	0.15329
1,	3.14375 degree,	0.29604 degree,	0.15329
2,	0.05352 radian,	0.00682 radian,	0.22076
2,	3.06656 degree,	0.39075 degree,	0.22076
3,	0.05320 radian,	0.00697 radian,	0.22105
3,	3.04835 degree,	0.39954 degree,	0.22105
4,	0.05318 radian,	0.00699 radian,	0.22112
4,	3.04673 degree,	0.40036 degree,	0.22112
5,	0.05317 radian,	0.00699 radian,	0.22112
5,	3.04657 degree,	0.40044 degree,	0.22112
6,	0.05317 radian,	0.00699 radian,	0.22112
6,	3.04656 degree,	0.40045 degree,	0.22112
7,	0.05317 radian,	0.00699 radian,	0.22112
7,	3.04656 degree,	0.40045 degree,	0.22112
8,	0.05317 radian,	0.00699 radian,	0.22112
8,	3.04656 degree,	0.40045 degree,	0.22112
9,	0.05317 radian,	0.00699 radian,	0.22112
9,	3.04656 degree,	0.40045 degree,	0.22112
10,	0.05317 radian,	0.00699 radian,	0.22112
10,	3.04656 degree,	0.40045 degree,	0.22112

The same results are obtained for each methods. The Newton-Raphson Method finds the results at the 3rd step as The Gauss-Seidel Method finds them at the 6th step.

5.2 Six DoF Trim Analysis by Using the Newton-Raphson Method

Similar to 3 DoF Trim Analysis, the force and moment equations can be arranged as the force and moment functions as follows.

$$XF = m(\dot{U} + QW - RV) - F_X = 0 \quad (5.33)$$

$$YF = m(\dot{V} + RU - PW) - F_Y = 0 \quad (5.34)$$

$$ZF = m(\dot{W} + PV - QU) - F_Z = 0 \quad (5.35)$$

$$XM = \dot{P} I_{XX} + QR(I_{ZZ} - I_{YY}) - (R + PQ)I_{XZ} - L = 0 \quad (5.36)$$

$$YM = \dot{Q} I_{YY} - PR(I_{ZZ} - I_{XX}) + (P^2 - R^2)I_{XZ} - M = 0 \quad (5.37)$$

$$ZM = \dot{R} I_{ZZ} + PQ(I_{YY} - I_{XX}) + (QR - \dot{P})I_{XZ} - N \quad (5.38)$$

As an example, constant altitude, steady turning flight of a business jet is studied. The aircraft is a business jet similar to Learjet 35A or 36A which is described in Table B7 of Jan Roskam's "Airplane Flight Dynamics and Automatic Flight Controls Part 1" book [37]. An earth fixed observer observes this aircraft during a constant altitude, constant radius, and constant speed turning flight as seen in Figure 5.1. At the instant of $t=0$ the nose of the aircraft is headed to the North and the aircraft is observed as making a left turn.

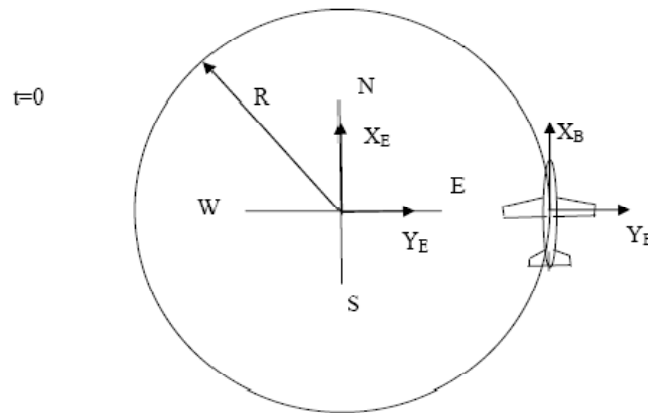


Figure 5.1: Aircraft position at $t=0$

The aircraft has two jet engines with,

$$T_{\max,SL} = 2 \times 15.6 \text{ kN}$$

Other parameters about this flight are:

$$h = 2000 \text{ m}$$

$$\text{Mach} = 0.2$$

$$R = 500 \text{ m}$$

$$\vec{V}_{WIND} = 0$$

The unknowns of this 6 DoF trim analysis are $\theta, \phi, \delta_T, \delta_a, \delta_r, \delta_e$.

Let $X_1 = \theta; X_2 = \phi; X_3 = \delta_T; X_4 = \delta_a; X_5 = \delta_r; X_6 = \delta_e$

and $F_1 = XF; F_2 = YF; F_3 = ZF; F_4 = XM; F_5 = YM; F_6 = ZM$

The Newton Raphson Method is defined as follows,

$$\begin{aligned} \frac{\partial F_1}{\partial x_1}.dx_1 + \frac{\partial F_1}{\partial x_2}.dx_2 + \dots + \frac{\partial F_1}{\partial x_6}.dx_6 &= -F_{1,i} \\ \frac{\partial F_2}{\partial x_1}.dx_1 + \frac{\partial F_2}{\partial x_2}.dx_2 + \dots + \frac{\partial F_2}{\partial x_6}.dx_6 &= -F_{2,i} \\ &\vdots \\ \frac{\partial F_6}{\partial x_1}.dx_1 + \frac{\partial F_6}{\partial x_2}.dx_2 + \dots + \frac{\partial F_6}{\partial x_6}.dx_6 &= -F_6 \end{aligned} \quad (5.39)$$

or in matrix form.

$$\begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \frac{\partial F_1}{\partial X_2} & \cdot & \cdot & \cdot & \frac{\partial F_1}{\partial X_6} \\ \frac{\partial F_2}{\partial X_1} & \frac{\partial F_2}{\partial X_2} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial F_6}{\partial X_1} & \frac{\partial F_6}{\partial X_2} & \cdot & \cdot & \cdot & \frac{\partial F_6}{\partial X_6} \end{bmatrix} \begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \\ dx_4 \\ dx_5 \\ dx_6 \end{Bmatrix} = \begin{Bmatrix} -F_1 \\ -F_2 \\ -F_3 \\ -F_4 \\ -F_5 \\ -F_6 \end{Bmatrix} \quad (5.40)$$

The partial derivatives are calculated numerically,

$$\frac{\partial F_1}{\partial X_1} \cong \frac{F_1(X_{1_{i+1}}, X_{2i}, X_{3i}, X_{4i}, X_{5i}, X_{6i}) - F_1(X_{1i}, X_{2i}, X_{3i}, X_{4i}, X_{5i}, X_{6i})}{X_{1_{i+1}} - X_{1i}} \quad (5.41)$$

The same process, which is expressed before at 3 DoF Trim Analysis, is performed. Following results are obtained Figs. 5.2 – 5.3.

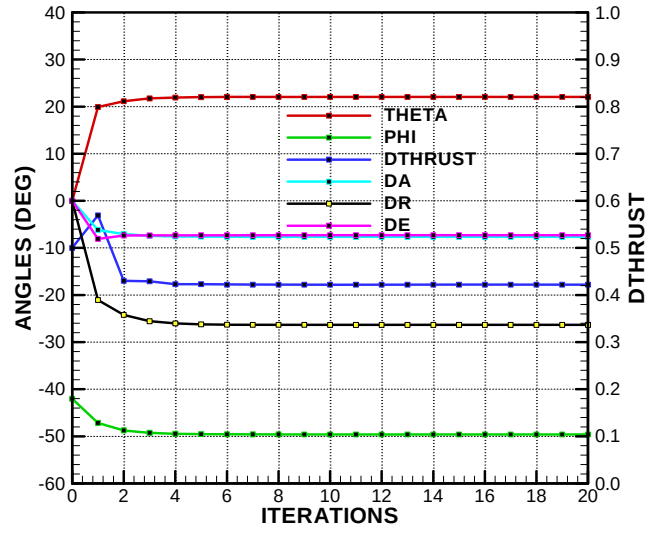


Figure 5.2 : Convergence of the trim variables

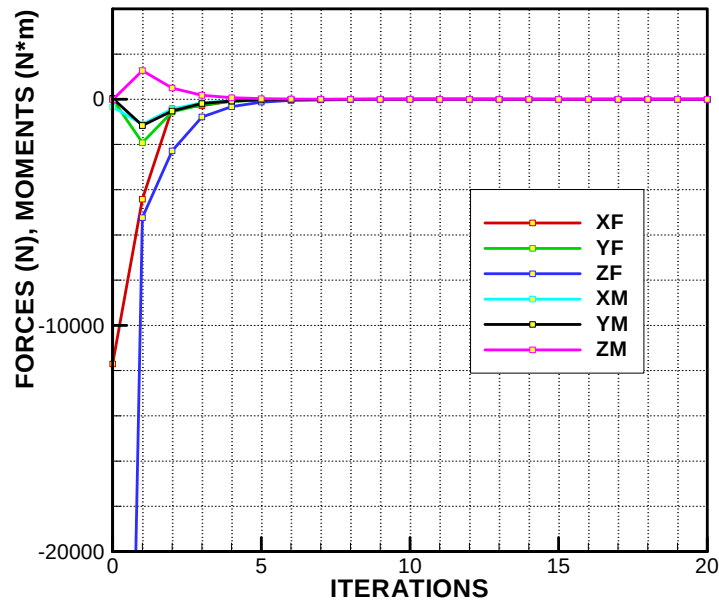


Figure 5.3: Convergence of the force and moment equations

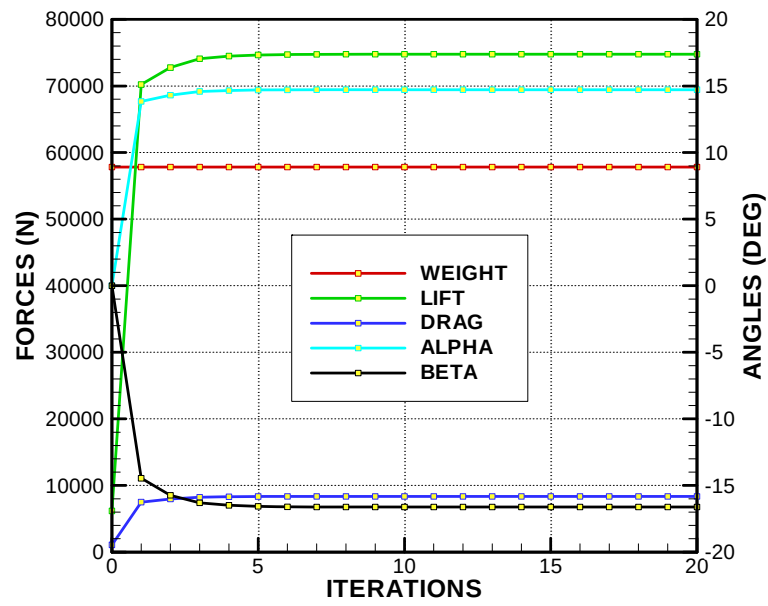


Figure 5.4 : Convergence of weight, lift, drag,alpha, and beta

6. CONCLUSION AND RECOMMENDATIONS

In this study non-linear and linear aircraft equations of motion are modeled and solved by using the body axis system. Many classical approaches use stability axis system for the solution of linear equations. However, stability axes change with angle of attack and some of the stability derivatives need to be re-evaluated for each angle of attack. Therefore body axes which are fixed to the aircraft are more convenient for the solution of flight dynamic problems. Nonlinear equations are solved by using the 4th order Runge Kutta method. Linear Equations are solved by using the exact Laplace transformation method and also by using the 4th order Runge Kutta method. Solution of linear equations by both methods gave the same results as expected. There were important differences in amplitude and frequency of oscillations which are obtained by the solution of non-linear and linear equations. These differences grow with growing input control deflection. Therefore it is appropriate to prefer nonlinear approach to obtain more satisfactory results.

Robust stability methods are introduced. The stability analysis of longitudinal aircraft motion is reduced to the stability investigation of four polynomials which are obtained by the help of the Edge Theorem. The stability analysis of these four polynomials is then performed using Bialas Theorem which allows to investigate stability of polynomials with one uncertain parameter. Thus, instead of analysing the system for all possible conditions, the stability analysis of aircraft is performed only by investigating the stability of these four polynomials which are obtained easily. As a result of the analysis, the longitudinal aircraft motion is stable for the given range of two stability derivatives.

REFERENCES

- [1] **Bennett, S.**, 1986: A history of control engineering 1800-1930, IEE Control Engineering Series.
- [2] **McRuer, D., Ashkenas, I., and Graham, D.**, 1973: Aircraft Dynamics and Automatic Control, Princeton University Press.
- [3] **Nelson, R. C.**, 1989: Flight Stability and Automatic Control, McGraw-Hill, New York.
- [4] **Draper, C. S.**, 1955: Flight Control. *Journal of the Royal Aeronautical Society* Vol. **59**, pp. 451-477.
- [5] **Blakelock, J. H.**, 1965: Automatic Control of Aircraft and Missiles, Aircraft Dynamics and Automatic Control, Wiley, New York.
- [6] **Stevens, B. L. and Lewis, L. F.**, 2003: Aircraft Control and Simulation, John Wiley, Hoboken NJ.
- [4] **Oppelt, W.**, 1976: A historical review autopilot development, research, and theory in Germany. *J. Dynamic Systems, Measurements, and Control, Trans. ASME, Series G*, Vol. **98**, pp. 215-223.
- [8] **Bennett, S.**, 1993: A history of control engineering 1930-1955, IEE Control Engineering Series.
- [9] **Lanchester, F. W.**, 1908: Aerodenetics, Archibald Constable, London.
- [10] **Bryan, G. H. and Williams, W. E.**, 1904: The Longitudinal Stability of Aerial Gliders. *Proc. Roy. Soc. Series A*, Vol. 73, No.489, pp. 100-116.
- [11] **Bryan, G. H.**, 1911: Stability in Aviation, Macmillan, London.
- [12] **Hunsaker, J. C.**, 1915: Experimental Analysis of Inherent Longitudinal Stability of an Airplane, *NACA TR 1, Pt. I*.
- [13] **Glauert, H.**, 1919: The Stability Derivatives of an Airscrew, *ARC R & M 642*.
- [14] **Bryant, L. W. and Irwing, H. B.**, 1919: Apparatus for the Measurement of M_q on a Complete Model Airplane, *ARC R & M 616*.
- [15] **Cowley, W. L.**, 1918: The Effect of the Lag of Downwash on the Longitudinal Stability of an Aeroplane, *ARC R & M 718*.
- [16] **Glauert, H.**, 1919: Analysis of Phugoids Obtained by a Recording Airspeed Indicator, *NACA TR 70*.
- [17] **Minorsky, N.**, 1922: Directional stability of automatically steered bodies. *J. American Society of Naval Engineers*, Vol. **34**, pp. 280-309.
- [18] **Minorsky, N.**, 1937: The principles and practice of automatic control. *The Engineer*, pp. 322-3.

- [19] **Jones, B. M.**, 1943: Dynamics of the Aeroplane. *Aerodynamic Theory*, Vol. **5**, Durand Reprinting Committee, Pasadena, Calif.
- [20] **Bryant, L. W. and Williams, D. H.**, 1930: The Application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane, *ARC R & M 1346*.
- [21] **Jones, R. T.**, 1936: A simplified application of the Method of Operators to the Calculation of the Disturbed Motion of an Airplane, *NACA TR-560*.
- [22] **Glaubert, H.**, 1927: A Non-dimensional Form of the Stability Equations of an Aeroplane, *ARC R & M 1093*.
- [23] **Koppen, O. C.**, 1934: Happier Landings. *Aviation*, Vol. **33**, pp. 275-7.
- [24] **Koppen, O. C.**, 1940: Airplane Stability and Control from the Designer's Point of View. *J. Aeron. Sci.*, Vol. **7**, No. 4, pp. 135-140.
- [25] **Holiday, T. B.**, 1941: Application of electric power in aircraft. *Trans. American of Electrical Engineers*, Vol. **60**, pp. 218-225.
- [26] **Thurston, D. B.**, 2000: The World's Most Significant and Magnificent Aircraft: Evolution of the Modern Airplane, SAE International, Warrendale, PA.
- [27] **Winter, H. W.**, 2003: 100 Years of Flight: A Chronicle of Aerospace History 1903-2003, AIAA, Reston, Virginia.
- [28] **Kinney, R. K.**, 2006: Airplanes: The Life Story of a Technology, Greenwood Press, Westport.
- [29] **Aykan, R. and Hajiyeve C., and Caliskan, F.**, 2005: Kalman filter and neural network-based icing identification applied to A340 aircraft dynamics. *Aircraft Engineering and Aerospace Technology: An International Journal*, Vol. 77, Num. 1, pp. 23-33.
- [30] **Hajiyeve C., Caliskan, F.**, 2005: Sensor and control surface/actuator failure detection and isolation applied to F-16 flight. *Aircraft Engineering and Aerospace Technology: An International Journal*, Vol. 77, Num. 2, pp. 152-160.
- [31] **Kim, N.**, 2003: Improved Methods in Neural Network-Based Adaptive Output Feedback Control with Applications to Flight Control, *PhD Thesis*, School of Aerospace Engineering, Georgia Institute of Technology.
- [32] **Khan, M. A. And Lu, P.**, 1994: New Technique for Nonlinear Control of Aircraft. *Journal of Guidance, Control, and Dynamics*, Vol. **17**, No:5, p. 1055.
- [33] **Yechout, T. R., Morris, L. R, Bossert, D. E. and Hallgren, W. F.**, 2003: Introduction to aircraft flight mechanics: performance, static stability, dynamic stability, and classical feedback control, AIAA, Reston, VA.
- [34] **Etkin, B. and Reid, L. D.**, 1995: Dynamics of Flight Stability and Control, John Wiley & Sons.

- [35] **Shaheed, M. H.**, 2005: Feedforward neural network based non-linear dynamic modelling of a TRMS using RPROP algorithm. *Aircraft Engineering and Aerospace Technology: An International Journal*, Vol. **77**, Num. 1, pp. 13-22.
- [36] **Cao, Y., Chen, Y., Yuan, K. and Jin, Ch.**, 2004: Nonlinear inverse dynamics control of the aircraft in the presence of windshear. *Aircraft Engineering and Aerospace Technology: An International Journal*, Vol. **76**, Num. 6, pp. 562-599.
- [37] **Roskam, J.**, 2001: Airplane Flight Dynamics and Automatic Flight Controls, Part I and II, DAR Corporation.
- [38] **Lan, C. E and Roskam, J.**, 1980: Airplane Aerodynamics and Performance, Roskam Aviation and Engineering.
- [39] **Roskam, J.**, 1989: Airplane Design Parts I-VIII, DAR Corporation.
- [40] **Ozdemir, U., Jafarov E. M., and Kavsaoglu, M. S.**, 2007: Calculation of the Longitudinal Stability Derivatives of a Transport Aircraft and Analysis of Longitudinal Modes, *The 9th WSEAS International Conferences on Automatic Control, Modelling and Simulation (ACMOS07)*, Istanbul, Turkey, May 27-29.
- [41] **Ozdemir, U., and Kavsaoglu, M. S.**, 2008: Linear and Nonlinear Simulations of Aircraft Dynamics Using Body Axis System. *Aircraft Engineering and Aerospace Technology: An International Journal*, Vol. **80**, No:6, pp. 638-648.
- [42] **Ozdemir, U., and Kavsaoglu, M. S.**, 2007: Non-linear and Linear Simulations of Aircraft Dynamics”, *4th Ankara International Aerospace Conference*), Ankara, Turkey, September 10-12.
- [43] **McLean, D.**, 1990: Automatic Flight Control Systems, Prentice Hall, Englewood Cliffs, NJ.
- [44] **Sadraey, M. H.**, 1995: Design of Nonlinear Robust Controller for a Complete Unmanned Aerial Vehicle Mission, *PhD Thesis*, University of Kansas.
- [45] **Ackerman, J.**, 2002: Robust Control: the parametric space approach, Springer, New York, London.
- [46] **Hollot, C. V. And Lin, H.**, 1998: Root locations of an entire polytope of polynomials: it suffices to check the edges. *Mathematics of Control Signals and Systems*, Vol. **1**, No:1, pp. 61-71.
- [47] **Bialas, S.**, 1985: A necessary and sufficient condition for the stability of convex combinations of stable polynomials or matrices. *Bull. Polish Academy of Sciences (Technical Sciences)*, Vol. **33**, No:9-10, pp. 473-480.
- [48] **Bialas, S. and Garloff, J.**, 1985: Convex combination of stable polynomials. *J. of the Franklin Inst.*, Vol. **319**, No: 3, pp. 373-377.

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- Turkoglu, K., **Ozdemir, U.**, Nikbay, M., Jafarov, E.M., “PID Parameter Optimization of an UAV Longitudinal Flight Control System”, International Conference on Control, Automation, Robotics and Vision (ICCARV 2008), Paris, 21-23 Nov 2008.
- **Ozdemir, U.**, Kavsaoglu, M.S. , Soylemez, M.T. “Stability Analysis of an Transport Aircraft including Uncertain Stability Derivatives”, in Turkish, 2st National Aerospace Conference (UHUK 2008), Istanbul, 15-17 October 2008.
- **Ozdemir, U.**, Kavsaoglu, M.S., *Non-linear and Linear Simulations of Aircraft Dynamics*, 4th Ankara International Aerospace Conference (AIAC 2007), Ankara, September 10-12, 2007.
- **Ozdemir, U.**, Jafarov, E.M., Kavsaoglu, M.S., *Calculation of the Longitudinal Stability Derivatives of a Transport Aircraft and Analysis of Longitudinal Modes*, 9th WSEAS International Conference on Automatic Control, Modeling and Simulation (ACMOS 07), Istanbul, pp. 105-110 , 27-29 May 2007.