

TOOLS AND TECHNIQUES FOR REAL TIME MODAL IDENTIFICATION

by

Yavuz Kaya

B.S., Civil Engineering, Yıldız Technical University, 1998

M.S., Earthquake Engineering, Boğaziçi University, 2002

Submitted to the Kandilli Observatory and Earthquake Research Institute

in partial fulfillment of

the requirements for the degree of

Doctor of Philosophy

Graduate Program in Earthquake Engineering

Boğaziçi University

2009

TOOLS AND TECHNIQUES FOR REAL TIME MODAL IDENTIFICATION

APPROVED BY:

Prof. Dr. Erdal Şafak
(Thesis Supervisor)

Prof. Dr. Mustafa Ö. Erdik

Prof. Dr. M. Nuray Aydınoğlu

Prof. Dr. Eser Durukal

Assoc. Prof. Dr. Kemal Beyen

DATE OF APPROVAL: 09.November.2009

To my father, with all my love

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude and appreciation to my adviser Professor Erdal Şafak for his invaluable guidance and help during the preparation of this thesis, who has been always a wise mentor. It is very hard to describe the honor of working with such a great engineer and a valuable scientist. I would like to thank for his valuable contributions to my experience in engineering practice, for his patience and understanding for the times I had to split focus to my job as a practicing engineer.

I also would like to express my appreciation to the faculty of Earthquake Engineering during my PhD education with whom I had the opportunity to work with and take courses, especially to Prof. M. Nuray Aydınoğlu and Assoc. Prof. Kemal Beyen, additionally to Prof. Mustafa Erdik, Prof. Özal Yüzügüllü, Prof. Atilla Ansal, and Prof. Eser Durukal. Their kind concern and support are greatly acknowledged.

I also would like to express my special thanks to my close friend and colleague Habib Cem Yenidoğan for his kind concern and support in all aspects.

ABSTRACT

TOOLS AND TECHNIQUES FOR REAL TIME MODAL IDENTIFICATION

Framework of the structural health monitoring (SHM) is to detect and locate the damage in the structure. Real-time identification of modal properties of a structure is very important in SHM because the changes in modal properties during its service life may be related to the damage on the structure. Therefore, it is obvious that modal properties should be monitored and identified accurately in real-time in order to have a good estimate in SHM. A new real-time software together with new real-time tools/techniques have been developed in this dissertation to be used in SHM to identify the modal properties of the structure in real-time. In the new proposed tool the real-time modal frequencies are estimated by using basic signal processing tools such as baseline correction, band-pass filtering, windowing, FFT, and smoothing while the real-time damping is estimated with half-power bandwidth technique and/or Logarithmic decrement technique. The developed real-time software, KOERIMIDS, has been tested with the ambient vibration data set recorded from Hagia Sophia Museum. Modal properties of the structure have been identified successfully in real-time. Results of the Hagia Sophia test have been compared with the previous studies conducted by different researchers. Comparison shows that the results of the KOERI-MIDS are in good agreement with that of the previous studies.

ÖZET

MODAL PARAMETRELERİN GERÇEK ZAMANLI BELİRLENMESİ İÇİN TEKNİKLER VE YÖNTEMLER

Yapı sağlığı izleme (YSİ) sistemlerinin amacı yapıda meydana gelecek hasarın tespiti ve bu hasarların yapı üzerinde yerlerinin belirlenmesidir. Yapının modal özelliklerinin gerçek zamanlı olarak gözlemlenmesi ve belirlenebilmesi YSİ sistemlerinde çok önemlidir çünkü yapının modal özelliklerindeki değişimler yapıdaki muhtemel hasar ile ilişkilendirilir. Dolayısı ile YSİ sistemlerinde iyi bir öngörü elde edebilmek için yapıların modal özelliklerinin gerçek zamanlı olarak izlenmesi gerekir. Bu tez kapsamında YSİ sistemlerinde kullanılacak, yapıların gerçek zamanlı modal özelliklerini yeni önerilen teknikleri ve yöntemleri kullanarak hesaplayabilen yeni bir yazılım geliştirilmiştir. Geliştirilen yazılımda gerçek zamanlı olarak değişen modal frekansların hesabında temel sinyal işleme teknikleri kullanılmıştır, bunların başlıcaları sıfır eksen düzeltmesi (baseline correction), bant geçiren filtre, pencereleme, FFT ve yumuşatma (smoothing). Gerçek zamanlı modal sönüm oranları Half-power bandwidth ve/veya Logarithmic decrement teknikleri ile hesaplanmıştır. Geliştirilen yazılım, *KOERI-MIDS*, Aya Sofya Müzesi'nden alınan çevresel titreşim kayıtları kullanılarak test edilmiştir. Yapının gerçek zamanlı modal özellikleri başarılı bir şekilde hesaplanmıştır. Elde edilen sonuçlar daha önceden farklı araştırmacılar tarafından yapılan çalışmalar ile kıyaslanmıştır. Bu kıyaslama sonucunda *KOERI-MIDS* sonuçlarının daha önceden farklı araştırmacılar tarafından yapılan çalışma sonuçları ile çok iyi bir uyum içerisinde olduğu görülmüştür.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iv
ABSTRACT	v
ÖZET	vi
TABLE OF CONTENTS	vii
LIST OF FIGURES	x
LIST OF TABLES	xii
LIST OF SYMBOLS / ABBREVIATIONS	xiii
1. INTRODUCTION	1
1.1. Main Objective	2
1.2. Overview	3
2. SEISMIC INSTRUMENTATION AND DATA PROCESSING	6
2.1. Seismic Instrumentation	6
2.2. Data Processing	9
2.2.1. Baseline Correction and Detrending	9
2.2.2. Filtering	10
2.2.3. Windowing	11
2.2.4. Spectral Leakage	12
2.2.5. Smoothing	15
3. MODAL IDENTIFICATION	17
3.1. Identification of Modal Frequencies	19
3.1.1. Parametric Methods for Power Spectrum Estimation	19
3.1.1.1. The Relationship Between the Autocorrelation and the Model Parameters	20
3.1.1.2. The Yule-Walker Method for AR Model Parameters	22
3.1.1.3. The Burg Method for the AR Model Parameters	23
3.1.1.4. Unconstrained Least-Square Method for the AR Model Parameters	26
3.1.1.5. Selection of AR Model Order.	27
3.1.2. Eigenanalysis Algorithms for Spectrum Estimation	28

3.1.2.1. MUSIC Algorithm.	30
3.1.2.2. ESPRIT Algorithm.	31
3.1.2.3. Order Selection Criteria.	34
3.2. Identification of Modal Damping.....	35
3.2.1. Logarithmic Decrement Technique	35
3.2.1.1. Damped Free Vibration.....	35
3.2.1.2. Decay of Motion.	37
3.2.2. Half-Power Bandwidth Technique	39
3.2.2.1. Harmonic Vibration with Viscous Damping.....	39
3.2.2.2. Sharpness of Resonance.	43
3.2.3. Energy Dissipated by Viscous Damping	46
3.2.4. Equivalent Viscous Damping	48
3.2.5. Structural Damping.....	49
3.3. Identification of Mode Shape.....	51
3.4. Other Identification Techniques.....	53
3.4.1. Stochastic Subspace Identification	53
3.4.2. Random Decrement Technique	56
3.4.3. Frequency Domain Decomposition	59
4. REAL TIME DATA PROCESSING ALGORITHMS	64
4.1. Real-Time Data Analysis Techniques.....	65
4.1.1. Analysis of Ambient Vibration.....	65
4.1.2. Analysis of Other Vibrations	67
4.2. Real-Time Modal Frequency Identification Algorithms	68
4.3. Real-Time Damping Identification Algorithms	72
4.4. Real-Time Mode Shape Identification Algorithm	74
5. DESCRIPTION OF KOERI-MIDS	76
5.1. General Properties of the KOERI-MIDS	77
5.2. Structural Geometry and Measurement Stations.....	80
5.3. Selection of Modal Property Calculation Techniques	81
5.3.1. Spectrum Estimation Technique Panel	83
5.3.2. Damping Estimation Technique Panel	83
5.3.3. Mode Shape Estimation Technique Panel	83
5.3.4. Real-Time Modal Properties (Output File).....	83

5.4. Modal Frequency Band Selection	84
5.5. Real-Time Data Processing Settings	88
5.5.1. Signal Properties Panel	89
5.5.2. Decimation Panel	89
5.5.3. Filter Properties Panel	90
5.5.4. Window Parameters Panel	91
5.5.5. Spectrum Smoothing Panel	92
5.5.6. Screen Settings Panel	93
5.6. Real-Time Mode Shape Estimation	94
6. AMBIENT VIBRATION TEST APPLICATIONS	96
6.1. Hagia Sophia Museum	96
6.1.1. History of Hagia Sophia	96
6.1.2. Past Earthquakes	97
6.1.3. Related Previous Studies	98
6.1.4. Description of the Structure	99
6.1.5. Instrumentation of the Structure	100
6.1.6. Parameter Selection for KOERI-MIDS	101
6.1.6.1. Selection of Parameters for Real-Time Data Processing Settings.	101
6.1.6.2. Selection of Modal Property Calculation Types.	105
6.1.6.3. Selection of Frequency Bands.....	106
6.1.7. Results of Real-Time Modal Identification of Hagia Sophia	107
7. CONCLUSION.....	114
REFERENCES	119

LIST OF FIGURES

Figure 2.1.	A sensor network installed in a structure	8
Figure 2.2.	Windowing functions and their frequency domain representations.....	12
Figure 2.3.	Spectral leakage caused by windowing function	14
Figure 2.4.	Reducing the spectral leakage effects by windowing	15
Figure 2.5.	Selection of optimum windowing length	16
Figure 2.6.	Smoothing a fourier amplitude spectrum of a recorded earthquake data.....	16
Figure 3.1.	Damped free vibration.....	36
Figure 3.2.	Effects of damping on the natural vibration frequency (Anil K.Chopra, 2001)	37
Figure 3.3.	Free decay curve (Anil K.Chopra, 2001)	38
Figure 3.4.	Exact and approximate relations between logarithmic decrement and damping ratio (Anil K.Chopra, 2001)	39
Figure 3.5.	Response of SDOF to harmonic vibration. (Anil K.Chopra, 2001)	40
Figure 3.6.	Deformation response factor and phase angle for a damped system excited by harmonic motion. (Anil K.Chopra, 2001).....	42
Figure 3.7.	Halfpower bandwidth technique	43
Figure 3.8.	Halfpower bandwidth technique with 2 closely spaced peaks	45
Figure 3.9.	Hysteresis loop of a SDOF for spring and viscous damper	47
Figure 3.10.	Free vibration of an undamped system in its first natural mode of vibration (Anil K.Chopra, 2001)	51
Figure 3.11.	Schematic view of random decrement technique.....	58
Figure 4.1.	Schematic view for truncated data segment and the running window	66
Figure 4.2.	Flow chart of real-time algorithm of smoothed FAS for the determination of modal frequency	70
Figure 4.3.	Flow chart of real-time algorithms of BURG, Yule-Walker and MUSIC for the determination of modal frequency.....	71
Figure 4.4.	Flow chart of real-time algorithm of half-power bandwidth for the determination of modal damping ratio.....	73
Figure 4.5.	Flow chart of real-time algorithm for the determination of mode shapes.....	75
Figure 5.1.	General view (main window) of <i>KOERI-MIDS</i>	79

Figure 5.2.	Modal property calculation window.....	82
Figure 5.3.	Modal frequency band selection.....	86
Figure 5.4.	Real-time data processing settings	89
Figure 5.5.	Filter response	90
Figure 5.6.	Smoothing control	93
Figure 5.7.	Real-time mode shape	95
Figure 6.1.	General view of Hagia Sophia.....	97
Figure 6.2.	Locations of strong ground motion instrumentation in Hagia Sophia	101
Figure 6.3.	Real-time data processing settings for Hagia Sophia museum	102
Figure 6.4.	Filter response of the forth order band-pass filter	103
Figure 6.5.	Optimum smoothing length for station 1 in east-west direction	104
Figure 6.6.	Optimum smoothing length for station 4 in east-west direction	104
Figure 6.7.	Modal poroperty calculation	105
Figure 6.8.	Frequency band selection	107
Figure 6.9.	Ambient vibration of KOERI-MIDS.....	108
Figure 6.10.	One hour real-time ambient vibration data and instrument locations in Hagia Sophia, accelerations are amplified by a factor of two in both East-West and North-South directions.....	109
Figure 6.11.	Time variations of the first two predominant modal frequency and damping ratios of the Hagia Sophia for twenty three hours (Joint Number 1)	110
Figure 6.12.	Time variations of the first two predominant modal frequency and damping ratios of the Hagia Sophia for twenty three hours (Joint Number 2)	110
Figure 6.13.	First mode shape in east-west direction and particle motion for joint number 1.....	112
Figure 6.14.	First mode shape in east-west direction and partial motion for joint number 1.....	113

LIST OF TABLES

Table 6.1.	Change of modal frequencies of Hagia Sophia	111
------------	---	-----

LIST OF SYMBOLS / ABBREVIATIONS

$x(t)$	Input data sequence
$y(t)$	Output data sequence (measurements)
a, b	Coefficients of best straight line in least square error function
a_k, b_k	Filter coefficients
$a_m(m)$	Prediction coefficients
$h(k)$	Windowing function
$bw(n)$	Boxcar windowing function
$f_m(n)$	Forward prediction error for Burg method
$f_p(n)$	Forward prediction error for Unconstrained Least-Square Method
$g_m(n)$	Backward prediction error for Unconstrained Least-Square Method
$g_p(n)$	Backward prediction error for Burg method
n_w	Optimum smoothing window length
$r(m)$	Biased autocorrelation matrix
$s(f)$	Complex sinusoidal vector
u_i	Displacement response of a SDOF
w_i	i^{th} mode Eigen-frequency
w_D	Damped modal frequency
$z(n)$	Time displayed vector
$E[.]$	Expected value operator
E_s	Potential energy
E_K	Kinetic energy
E_I	Input energy to the system
E_D	Energy dissipated by viscous damping
E_{ST}	Energy dissipated by structural damping
w_p	Process noise due to disturbances and modeling inaccuracies
v_p	Measurement noise due to sensor inaccuracy

A	State matrix in state-space representation of a system
C	Output matrix in state-space representation of a system
$D_{xx}(\tau)$	Random decrement function
$\hat{D}_{xx}(\tau)$	Estimated random decrement function
$T_{X(t)}$	Triggering condition
N	Total number of points in data sequence
N_r	Total number of triggering points
G	"next state-output" covariance matrix
$G_{yy}(w)$	Output power spectral density matrix
$G_{xx}(w)$	Input power spectral density matrix
H	Hankel matrix
$H(w), H(f)$	Frequency response function (FRF)
$H(z)$	Rational system function
K_m	Complex-valued m^{th} reflection coefficient in the lattice filter realization
P_i	Orthogonal projection of the row space of the future outputs
$P^{YW}(f)$	Power spectrum estimate for Yule-Walker method
$P^{BU}(f)$	Power spectrum estimate for Burg method
$P^{LS}(f)$	Power spectrum estimate for unconstrained-Least square method
$P_{MUSIC}(f)$	Power spectrum estimate for MUSIC method
Q	Quality factor
R_m	Residue Matrix
R_d	Deformation response function
$\mathcal{E}$	Least square error
$\mathcal{E}_m$	Forward and backward prediction error for least square estimation
τ	Time lag in random decrement function
φ_m	m^{th} mode shape vector
$\Gamma(f)$	Power spectral density of the data sequence
γ_m	m^{th} mode participation vector

γ	Structural damping factor
$\gamma(m)$	Autocorrelation of the data sequence
σ_x^2	Variance of the data sequence
χ_m	Operational vector
Y^t	Observed data vector
Y_f	Future outputs defined in Hankel matrix
Y_p	Past outputs defined in Hankel matrix
W^t	Noise vector
O_i	Observability matrix
x_k	State vector
$\hat{X}_i$	Kalman state sequence
y_k	Output measurements at discrete time instance k
δ_{pq}	Kronecker delta function
δ	Logarithmic decrement
Λ_i	Output Covariance Matrix
Λ	Diagonal complex eigenvalue matrix
λ_i^c	Continuous time eigenvalues
λ_i	Discrete time eigenvalues
λ_m	Function of poles
ζ	modal damping ratio
AIC	Akaike Information Criterion
ADSL	Asymmetric Digital Subscriber Line
AR	Autoregressive
ASCII	American Standard Code for Information Interchange
CAT	Criterion Autoregressive Transfer
ERA	Eigensystem Realization Algorithm
ESPRIT	Estimation of signal parameters via rotational invariance techniques
FAS	Fourier Amplitude Spectrum
FDD	Frequency Domain Decomposition

FFT	Fast Fourier Transform
FRF	Frequency Response Function
FPE	Final Prediction Error
HDD	Hard Disk Drive
ITD	Ibrahim Time Domain Technique
KOERI-MIDS	Kandilli Observatory and Earthquake Research Institute Modal Identification Software
MDL	Minimizes the Description Length
MUSIC	Multiple Signal Classification
PSD	Power Spectral Density
RDT	Random Decrement Technique
SDOF	Single Degree of Freedom System
SFAS	Smoothed Fourier Amplitude Spectrum
SHM	Structural Health Monitoring
SSI	Stochastic Subspace Identification
SVD	Singular Value Decomposition

1. INTRODUCTION

Suspension bridges, high rise buildings, offshore structures, and ports are some of the important civil engineering structures that have been built during the last decade in many developing countries. Due to their importance in emerging economies, a special attention should be given to their state of health during their service life. These structures experience different level of external loads during their service life such as wind loads, live loads, earthquake loads, and temperature loads, etc. These different levels of loads may result in some undesired consequences such as structural and non-structural damages in structure.

Some of these damages in a structure may be caused by extreme loads such as earthquakes, hurricanes, and blast loads, while other damages are the result of the structure's normal usage during their service life. For structural safety, it is an important issue to detect and locate the damage as soon as they occur in the structure. In this concept a structural monitoring system (SHM) can be used to detect and locate the damage in structures. It is well known that the damages in structure results in the change in dynamic properties of the structure. Although the reverse may not be true, the main intention in SHM is to monitor the vibration of the structure and analyze them in order to detect the changes in dynamic properties of the structure in real-time. These changes in dynamic properties may indicate a possibility for damage in the structure as soon as they occur. Based on the SHM system used, a decision should be made automatically whether there is damage and if it is dangerous for the structure.

In recent years, SHM has emerged as a new research area in earthquake and civil engineering. SHM is an important tool to reduce the maintenance cost of civil structures. Visual inspection and evaluation of civil structures are usually done by means of site inspections, which sometimes cannot provide enough information for the maintenance due to the some undetectable structural damage that can be hidden in the structure. With the help of rapid developments in digital data acquisition and processing speed, and the availability of minicomputers and microprocessors, many different intelligent SHM methods, using only response data, to identify the structures' modal characteristics have

been developed. The accurate estimation of dynamic modal properties of structures plays an important role to determine and locate the damage in SHM because they describe the inherent dynamic properties of the structure. Since these dynamic properties are directly related to the mass and the stiffness properties of the structural system, experimentally obtained modal parameters provide the latest value of these two physical properties of a structure. Once the latest stage of modal characteristics of the structures, such as the modal frequencies, the modal damping ratio and the mode shapes are identified, real-time changes in these modal properties can be related to detect damage and also to locate it in the structure.

1.1. Main Objective

In general, the different source of dynamic excitation for civil engineering structures can be categorized as: ambient vibrations, experimental force vibration, earthquakes, and underground explosions. These dynamic excitations are usually results in change in modal properties of the structure. The real-time change of structure's modal properties with respect to different environmental conditions can be monitored by means of experimental tests such as impulse response and force vibration test. Although these tests have been used as reliable method to obtain modal properties of a structure, they are not always practical in earthquake engineering applications due to their cost. As an alternative, among the different methods of full-scale testing, ambient vibration records can be used to identify the structure's modal properties in real-time. The superiority of ambient testing is due to the light-weight equipment requirement, easy of experimentation, and limited and minimum disturbance to the normal operation of the structure.

As a result it is obvious that the availability of a real-time modal identification tools is crucial for on-line control and monitoring of structures under ambient loads. Therefore the main intension of this dissertation is to develop real-time modal identification software to be used in SHM. The software developed in this dissertation is for real-time modal identification, which allows constructing a mathematical model of a dynamic system that is based only on the measured response.

The software developed has some important properties such as;

1. It is a real-time modal identification software but it can also be used for off-line identification.
2. The software is developed in a modular way that it can easily be improved.
3. Modal frequencies, damping ratios, and corresponding mode shapes are estimated using different estimation methods in time or frequency domains. Estimated modal properties can be compared against each other.
4. Other modal property estimation methods that are not included in the current code can easily be added when needed.
5. The software is coded in MATLAB.

1.2. Overview

In Chapter 2, seismic instrumentation and data processing tools that are used in the real-time data processing algorithms are explained. The issues of long-term seismic monitoring (instrumentation layout, selection of appropriate sensors and their locations and other hardware, and data processing) are discussed. The optimal sensor placement is explained briefly using the limited number of sensors. The minimum hardware requirements for a recording in a SHM are also explained. The minimum requirements for a data acquisition system are discussed in terms of data security and maintenance.

The basic data processing tools used in the real-time data processing algorithms are explained in detail. In general raw data do not provide enough information about the structure's dynamic properties due to environmental conditions, instrument noise and etc. To minimize the effects of such factors in the structures dynamic properties, the recorded raw data should be processed first before using it. Processing data include the base line correction of the measured data, band pass filtering, windowing to prevent the spectral leakage and to track the time variations of data, and smoothing of Fourier Amplitude Spectrum (FAS).

In Chapter 3, different modal identification algorithms in time and frequency domains are discussed. Among the time domain techniques, Stochastic Subspace Identification (SSI), and Random Decrement Technique (RDT), and the Ibrahim Time Domain Technique (ITD) are explained in detail with their advantages and disadvantages

in general usage. On the other hand for frequency domain technique, Frequency Domain Decomposition (FDD) method is explained. The modal frequencies in the developed software are estimated by peak-picking technique within a pre-defined frequency bands using either the estimated power spectrum or the smoothed FAS. Different power spectrum estimation techniques are explained such as the Yule-Walker method, the Burg method, the Multiple Signal Classification (MUSIC) method and the Rotational Invariance Technique (ESPRIT). The modal damping ratios are estimated by Half-Power bandwidth technique by using either smoothed FAS or power spectrum. The logarithmic decrement technique, which is given as a second option in the software to estimate the modal damping ratios, is also explained in detail. Mode shapes, corresponding to each identified modal frequency, are estimated using two different techniques. First technique, which is the default option in the software, is the double integration of the band-pass filtered acceleration data that is measured at different location of the structure. This yields the displacement time history for all measurement locations. Amplitudes of these displacement histories correspond to the amplitudes of the mode shapes at a given modal frequency. Second technique is based on the FAS and phase spectrum of measured acceleration data. The amplitude of FAS of measured data gives the amplitude of mode shape at the corresponding measurement location in a given modal frequency while the phase information is used to determine the relative mode shape amplitude directions.

In Chapter 4, the importance of time-synchronization in the estimation of modal properties is explained in detail. The concept of short and long duration windows is discussed in detail for the analysis of real-time data processing. The real-time data analysis techniques are explained for ambient induced vibrations and other source of vibrations. Real-time modal frequency identification algorithm is explained for four different methods, such as BURG, Yule-Walker, MUSIC, and SFAS (Smoothed Fourier Amplitude Spectrum). The real-time modal damping and mode-shape estimation algorithms are explained in detail.

In Chapter 5, the developed software, *KOERI-MIDS* is explained in detail. The software basically identifies the real-time modal properties of a structure under ambient vibration. The general properties of the program are given with the explanation of several user interfaces windows and dialog boxes. The input and output file formats for the

program are explained and discussed in detail. The geometric dimensions of the structure should be provided by user as a separate file in ASCII format. The software can read the response of the structure for different file formats, such as ASCII, SAC, and GCF. However, the output files are only in ASCII file format. Real-time modal frequency, damping and mode-shape estimation techniques can be selected with user selectable parameters using the *Selection of Modal Property Calculation* window. The analyses of the real-time measurements are carried out by means of parameters defined in the *Real-Time Data Processing Settings* window.

Application of the developed software is given in Chapter 6. The related application test is performed for Hagia Sophia museum. The brief historical background of the structure is given and the past earthquake that caused damaged to structure is discussed in detail in chapter 6. The Hagia Sophia first built in 360 AD and several renovations have been done until now. The structure has suffered several big earthquakes during its lifetime. Many of them caused significant damages to the structure. Several previous studies about the Hagia Sophia are presented briefly. Two of these studies are used to validate the results of the KOERI-MIDS. The geometric description of the structure and the instrumentation details are explained in detail. Nine accelerometers are installed in the structure and the current instrumentation has been in operation since 1991. The selection of the parameters for KOERI-MIDS is given in detail. Real-Time data processing settings, modal property calculation settings, and frequency band limits for Hagia Sophia museum are explained. Results of KOERI-MIDS are compared with the previous studies. Results shows that the there is an enough consistency with the previous studies and the developed software is reliable enough to be used for real-time modal identification.

2. SEISMIC INSTRUMENTATION AND DATA PROCESSING

2.1. Seismic Instrumentation

It is obvious that the response of a structure should be monitored in real-time in order to identify the dynamic characteristics of a structure with continuously varying environmental conditions. The need for structural seismic instrumentation has been constantly recognized by earthquake engineering research centers and organizations. There are three main issues related to long-term seismic instrumentation of structures. These issues are; instrumentation layout, selection of appropriate sensors and other hardware, and data processing tools to be used. The basic requirements for practical and economical feasibility of seismic instrumentation hardware are reliability, easy of operation (including synchronizing), data communication, instrumentation maintenance, and cost effectiveness over the life of the instruments.

It is well known that ambient loads (wind, temperature, live loads, and traffic loads, etc.) acting on the structure are usually remains unknown during an ambient vibration. Furthermore the amplitude of ambient vibrations is relatively smaller than that of earthquake induced vibrations. As a result only a limited number of modes of the structure are expected to be excited in an ambient vibration test. On the other hand it is know that only the first few modes of vibration are the significant participants in the structural response except some complicated and unsymmetrical structures. Therefore identifying the first few modes of the structure is usually enough to understand the general dynamic behavior of the structure.

There are usually a limited number of sensors to be used in the structure for real-time modal identification. Therefore, the measurement locations in the structure should be selected carefully in order to obtain maximum modal information from the structure. To improve the signal-to-noise ratio while recording the structural response, the sensors need to be located as close as possible the antinodes of the lower frequency mode shape. More

details about the optimum sensor placement can be found in Udwadai (1984), and Rojahn and Matthiessen (1977).

The optimal instrumentation can be found by satisfying the following steps;

- 1) Modal frequencies and mode shapes of the structure should be first calculated based on an analytical model, such as finite element model
- 2) An ambient test is performed to confirm the results of the analytical results
- 3) If necessary, modifications in the analytical model should be made to match the experimentally identified structural parameters
- 4) Based on the final analytical results, the optimal locations of the sensors are selected

The number of measurement locations in a structure depends on the expected response and also the size of the structure. For instance, if only horizontal vibrations are expected in the structure three sensors measuring the horizontal vibrations for each floor are enough. On the other hand, if there is rocking motion expected in the structure additional three or four sensors measuring the vertical vibrations are required in the foundation level. The locations of the sensor in a structure, for instance high rise building, should satisfy the following conditions.

- a) First set of sensor should be placed at the top of the structure
- b) Second set of sensor should be placed at the foundation level
- c) Third set of sensor should be located where the typical floor mass and rigidity changed throughout the height of the building.
- d) The last set of the sensor must be placed where amplitude of free vibration modes are expected to be maximum.

As for the hardware of a recording instrument, force-balanced servo accelerometers are nowadays the most commonly used type of sensors in structural health monitoring. This type of accelerometers has reliable performance. They have high sensitivity in both high-amplitude seismic and low-level ambient vibrations.

A sensor network installed in a typical structure is shown in Figure 2.1. It monitors the response of a structure under ambient loads in real-time. In general the measurements from different locations of the structure are usually first stored in a central computer, *PC1*, which is generally placed in the structure monitored. There are different techniques available for data acquisition between sensors and the central computer. For instance in cases of Hagia Sophia museum, measurements are transmitted to the central computer through ADSL modem. In practice the data acquisition systems are not perfect. A single value or portions of the data can be lost due to malfunctions in the sensors or errors in communication links. It means that the data acquisition system may stop at any time. In such case the measurements are stored in the ring-buffer of the instrument. Later the missing measured data are streamed to the central computer when the connection is established again. This type of data acquisition prevents data loss and usually maintains the data transmission speed in real-time monitoring.

During an ambient vibration, the dynamic response of the structure is measured at different locations of the structure in the form of total accelerations (Sotoudeh and Brady, 1985). Once the measurements are saved in *PC1*, they can be transmitted to other PCs via ADSL modem or other transmission lines. The measured data should be processed in real-time as explained in Chapter 4. This data processing can be performed in *PC1* or in *PC2*. More detail about the real-time data processing is given in Chapter 4.

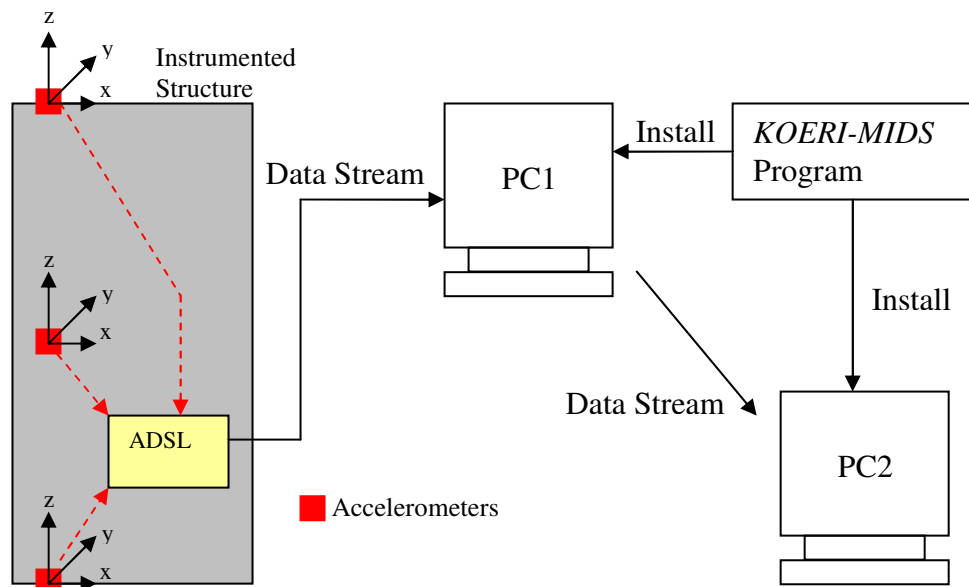


Figure 2.1. A sensor network installed in a structure

2.2. Data Processing

In general, recorded data in their original form (i.e., raw data) do not usually provide sufficient information about the modal identification of the structure. Some of the main reasons for this can be the environmental conditions, instrument noise in the recorded signal, possible low-frequency drifts in the data, outliers (i.e., erroneous isolated peaks), and other flaws in the sensors, installations and recordings. To minimize the undesirable effects of such factors on the modal identification, the recorded data should be processed first before using it for identification. Data processing involves the removal of mean, windowing, removal of linear trends, low-frequency drifts, outliers, and other unwanted frequencies by filtering, instrument corrections, and decimation if needed.

In case of filtering, band-pass filters are preferred in data processing. Band-pass filters remove mean value from recorded data and eliminate the components dominated by noise. Typically, a low-frequency corner is selected to remove the mean from the record (i.e., DC or static component) and other low frequency errors. On the other hand, a high-frequency corner is defined to remove the high frequencies that are usually dominated by noise. The filter bandwidth should be selected such that all significant modes of the considered structure are within that bandwidth. It should be noted that some of the digital filters distort the data by changing the phase of the signal. To overcome this problem, either zero phase band-pass filters (e.g. Ormsby filter) should be used, or the filter should be applied twice (e.g. Butterworth filters), first in forward and then in backward directions.

It should be kept in mind that the band-pass filters do not remove the noise from the entire frequency range, but actually eliminates the components of the record that are dominated by the noise. Therefore, the noise is still present within the bandwidth of the filter, although its amplitudes are low when compared to those of the signal.

2.2.1. Baseline Correction and Detrending

The mean value of a vibration record represents the static component of the structure's response. Normally, the mean value of vibration records should be zero. In real-

time monitoring, the mean value may fluctuate around zero due to various factors, such as imperfections in sensors, environmental factors (e.g., wind, rain, etc.), natural changes in structure (e.g., structural modifications, changes in load, aging, etc.), or permanent damage after an extreme event (e.g., structural inelastic deformations during an earthquake). Therefore, it is important that the changes in mean value are tracked accurately.

Detrending, also known as baseline correction, removes a linear trend from records. Again, for real-time data it should be applied by utilizing running time windows, so that the trend that is being removed is current one. The equations for detrending have been well developed by determining the best straight line fit, (i.e., $y(t) = ax(t) + b$), to the recorded data by minimizing the error defined in Equation (2.1) in the least-square sense.

$$\begin{aligned}
 \varepsilon(t) &= \sum_{i=1}^t [x(i) - ax(i) - b]^2 \\
 a &= \frac{N \sum_{k=1}^N k \cdot x_k - \sum_{k=1}^N k \cdot \sum_{k=1}^N x_k}{N \sum_{k=1}^N k^2 - \left(\sum_{k=1}^N k \right)^2} \\
 b &= \frac{\sum_{k=1}^N k^2 \cdot \sum_{k=1}^N x_k - \sum_{k=1}^N k \cdot \sum_{k=1}^N k \cdot x_k}{N \sum_{k=1}^N k^2 - \left(\sum_{k=1}^N k \right)^2}
 \end{aligned} \tag{2.1}$$

Where N is the total number of points in the data. The coefficients a and b of the straight line are determined by minimizing the error function in Equation (2.1) with respect to a and b . Because of the term b in the equation, detrending also removes the mean value from the data. Baseline correction and detrending are two basic signal processing tools that must be applied to any raw data.

2.2.2. Filtering

Filtering is required to remove the unwanted frequency components in data. The desired filter characteristics are determined in the frequency domain in terms of the desired

magnitude and phase response of the filter. There are mainly two types of filters, FIR (Finite Impulse Response) and IIR (Infinite Impulse Response) filters. Different applications require different filter types.

FIR filters have linear phase characteristics within their pass band. Either an IIR or an FIR filter may be employed unless there is a requirement for a linear phase characteristics. However in general, an IIR filter has lower sidelobes in their stopband than an FIR filter having the same number of parameters. Therefore, if some phase distortion is either tolerable or unimportant, an IIR filter is always preferable. Furthermore implementation of IIR filters involves fewer parameters, requires less memory and has lower computational complexity. It is obvious that using either FIR or IIR filter generate some phase shift (phase distortion) in data. To overcome this problem, the recorded data is filtered in both forward and reverse directions. The resulting filtered data have zero-phase distortion.

In general, to reduce the noise in recorded data, the standard procedure is to use bandpass filters. Bandpass filters remove the frequency components that are believed to be dominated by noise (typically the very low and very high ends of the frequency band). Although the noise is still present in the remaining frequency band, it is assumed that the signal-to-noise ratio is high enough so that the noise can be ignored in the pass-band. All types of filtering (bandpass, lowpass, and highpass) can be designed by using recursive time domain filters as shown in Equation (2.2).

$$y(t) = - \sum_{k=1}^m a_k y(t-k) + \sum_{l=1}^n b_l x(t-l) \quad (2.2)$$

where $y(t)$ is filtered zero-phase distortion data, and $x(t)$ is the original data to be filtered, m and n are filter order depends on filter corner frequencies, a_k and b_k are the filter coefficients depends on the filter type and desired filter characteristics.

2.2.3. Windowing

Windowing functions required to reduce the spectral leakage in FAS, to track time variations in measured data, and to smooth the FAS. There are several types of windowing

functions available in the literature, as shown in Figure 2.2. Three different windowing functions are used in this dissertation, a Hamming, Hanning and a Triangular window. For instance, mathematical formulation of the Hamming window is given in Equation (2.3).

$$h(k+1) = 0.54 - 0.46 \cos\left(2\pi \frac{k}{n-1}\right) \quad k = 0, \dots, n-1 \quad (2.3)$$

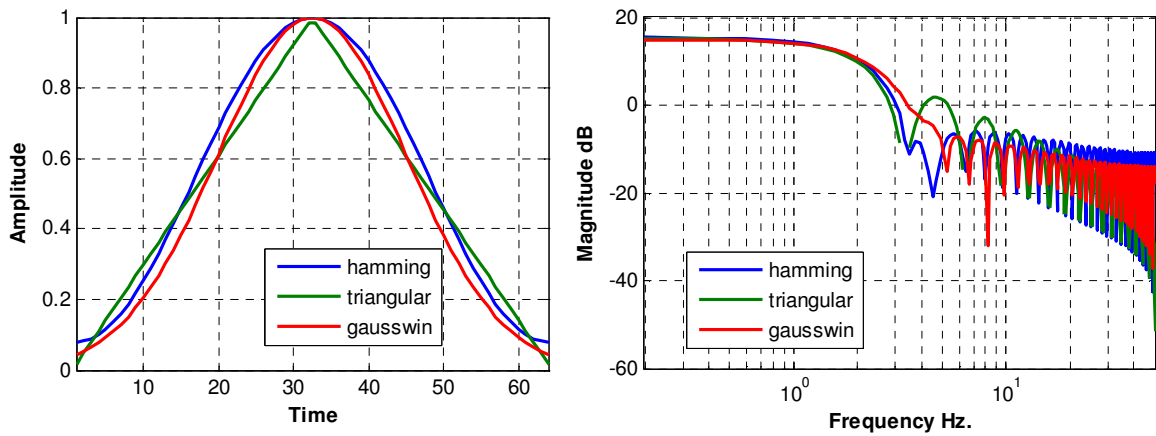


Figure 2.2. Windowing functions and their frequency domain representations

All window function's FAS look like a series of bumps, or lobes. The tallest, and usually the widest, bump is at DC (an impulse at zero Hz.) and is called the Main Lobe. All other bumps at higher frequencies are called Side Lobes. Application of a windowing function tends to smear an impulse over a range of frequencies close to the true frequency. This distortion will make it difficult if a bump in the spectrum was generated by a single sine wave (single impulse) or a series of two or more closely spaced impulses (sinusoids at closely spaced frequencies). To measure the severity of this problem, attention must be paid on the width of the window main lobe. Smaller main lobe is always preferred.

2.2.4. Spectral Leakage

By definition, Fourier transform is defined for signals with infinite length. In other words, to compute the FAS of a signal, the values of the signal for all times are required. However, in practice, we observe signals for only a finite duration. The finite-duration record has different spectral characteristics from the infinite-duration record.

Discontinuities at the beginning and at the end of the finite duration record produce leakage in spectral information, which results in a discrete-time spectrum that is a smeared version of the infinite duration records. Consequently, FAS of an infinite duration records can never be calculated but can only be approximated from a finite-duration record. The reason is that infinitely long measurements are impossible to obtain. Therefore in practice, measurement with finite time period can be modeled mathematically by multiplying the infinitely long measurement by a windowing function. The simplest windowing function is the Boxcar window as defined in Equation (2.4).

$$bw(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases} \quad (2.4)$$

where $bw(n)$ is Boxcar window function, and L is the number of samples.

Application of a Boxcar window function to an infinite time series yields a finite time data segment. This process results in spectral leakage. If we consider an infinite length of a sine wave having the frequency of $f=10 \text{ Hz}$, its Fourier Amplitude Spectrum will give a peak at 10 Hz as seen in Figure 2.3. It should be noted that windowed FAS is not localized to a single frequency, but instead it is spread out over the whole frequency range. Consequently the power of the original signal sequence that was concentrated at single frequency has been spread by the window function into the entire frequency range. In other words, the energy in a specific frequency is shifted or leaked out into the entire frequency range. This phenomenon, which is a characteristic of windowing the signal, is called spectral leakage. The severity of this problem is measured by the difference between the amplitudes of the window's main lobe and the highest side lobes, which is called the side lobe attenuation. Lower side lobes are always preferred.

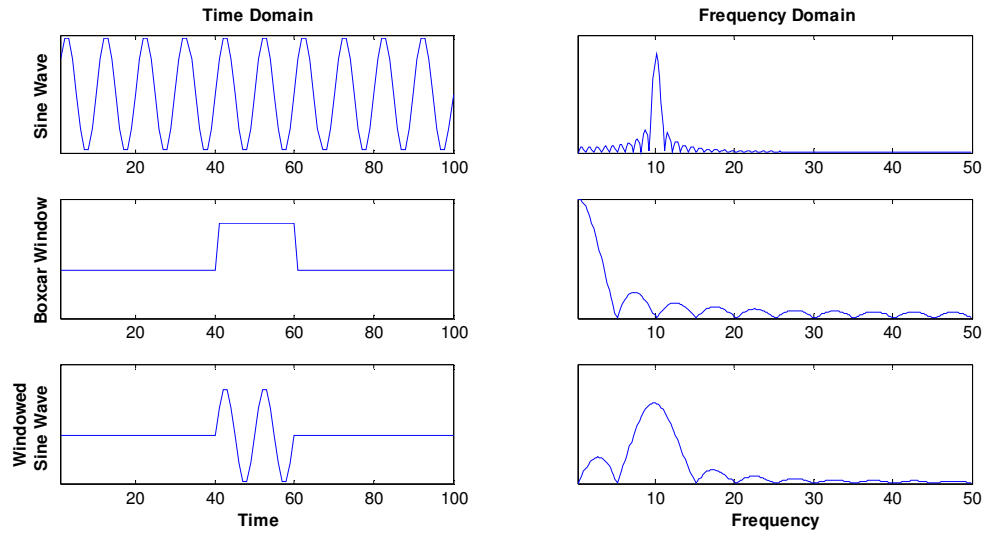


Figure 2.3. Spectral leakage caused by windowing function

To reduce the leakage effect on FAS, we can select a windowing function $W(f)$ that has lower side lobes, and smaller main lobe width in the frequency domain compared with the Boxcar window. However, a reduction of the side lobes in a window function is obtained at the expense of an increase in the width of the main lobe of the windowing function and hence a loss in spectral resolution.

There are no straightforward rules to select the best windowing function, $W(f)$ for a specific record to reduce the spectral leakage effect. Consider a truncated sine wave with predominant frequencies of $f_1=10\text{ Hz}$ and $f_2=30\text{ Hz}$ with a duration of 100 seconds and time interval of 0.01 second. As a result of finite duration record, spectral leakage over the entire frequency range can be seen in Figure 2.4. Finite duration record is multiplied by the Hamming window to avoid the discontinuities as much as possible at the beginning and at the end of the finite duration record. The resulting FAS of windowed data are almost free of spectral leakage as seen in Figure 2.4. It is impossible to avoid spectral leakage effects in FAS, but it can be reduced by means of windowing functions. The amplitude of the resulting FAS is different than that of finite duration record while the predominant frequencies appear at the same frequencies.

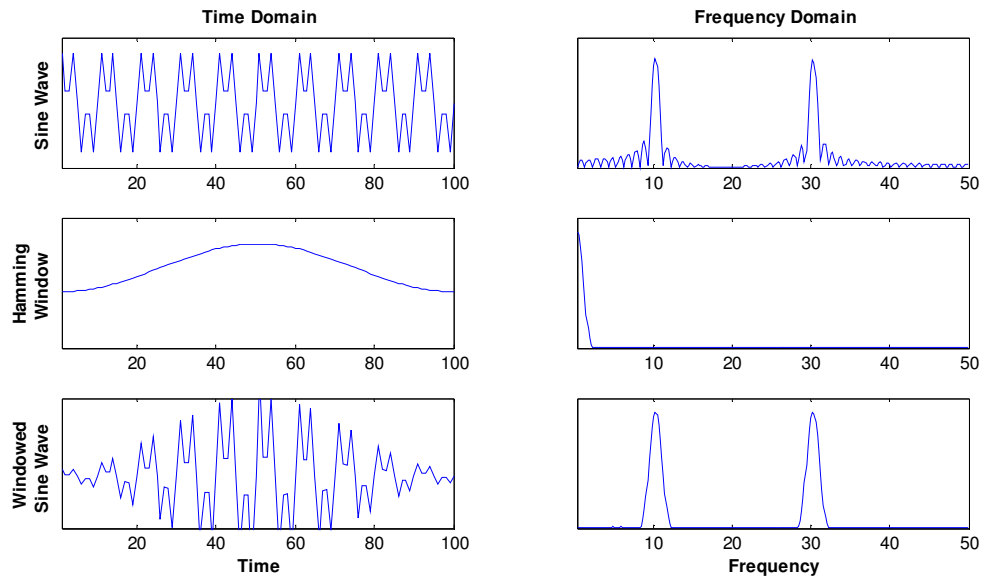


Figure 2.4. Reducing the spectral leakage effects by windowing

2.2.5. Smoothing

Experimental data are usually contaminated with high amplitude noise due to environmental conditions and mechanical imperfections in the instruments. As a result, high amplitude noise is often observed in ambient vibration records, which results in low signal-to-noise ratios and causes distortions in FAS. Therefore some of the peaks in FAS may not be observed clearly. It can make calculations of other signal features, such as slopes, areas, peak widths and so forth, difficult. To overcome this problem, smoothing windows are used to reduce the effect of noise in FAS.

Smoothing windows are a simple means of reducing the influence of noise in Fourier Amplitude Spectra. There are different types of smoothing windows such as triangle, and Hamming windows. The length of the smoothing window should be selected carefully. Too short smoothing window may not provide sufficient noise reduction, whereas too long smoothing may eliminate some of the real peaks in FAS. A simple technique suggested by Safak, 2006 can be used for the selection of optimum window length. The proposed technique uses the area under squared FAS with increasing window length to determine the optimum window length. Figure 2.5 shows a continuously decaying curve with increasing window length. The decay is very fast at the beginning, but becomes slower as the window

length increases. If the noise-free FAS are assumed to be the smooth function of frequency, it can be shown that the window length where the rate of decay in the curve changes from fast to slow corresponds to the optimal window length.

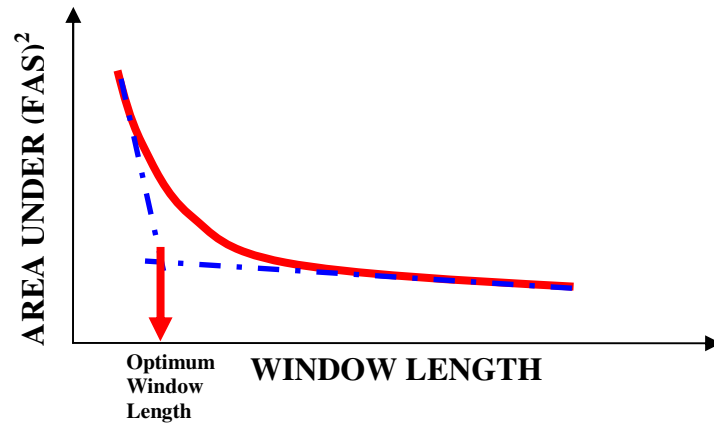


Figure 2.5. Selection of optimum windowing length

A simple example for smoothing FAS is given in Figure 2.6 for the time history data recorded at Hagia Sophia at February 17, 2009. The FAS of the measured data contains high frequency noise amplitudes. There are also closely spaced frequencies. These make it difficult to pick the dominant frequencies in FAS. In order to overcome this problem, a Hamming type of smoothing window function with a length of 53 samples is used for smoothing the FAS. It is clearly noticeable that the resulting smoothed FAS enables dominant peak frequencies to be selected more easily when compared to that of raw data.

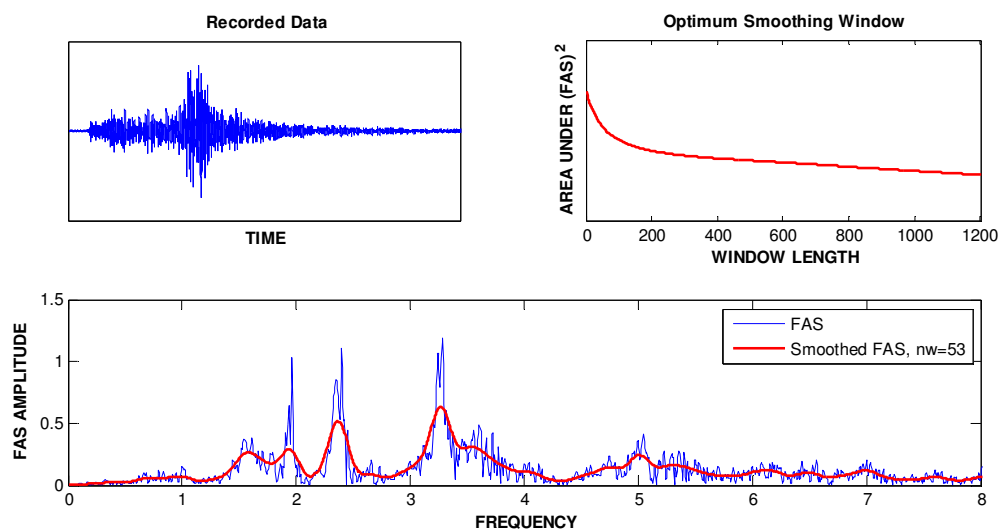


Figure 2.6. Smoothing a fourier amplitude spectrum of a recorded earthquake data

3. MODAL IDENTIFICATION

Modal identification is the process of developing or improving a mathematical representation of a structure using experimental data. In other words it refers to estimate the dynamic properties of a structure (the natural frequencies, damping ratios and mode shapes) under different load conditions. The civil engineering structures have been subjected to various environmental loads that usually cannot be measured accurately such as wind loads, temperature loads, and live loads. The real-time change in these modal properties with respect to different environmental loads may be identified by means of experimental tests. Among these are the impulse responses and forced vibrations tests. Although these tests have been used as reliable method to obtain modal properties of a structure, they are not always practical in earthquake engineering applications due to their cost. Therefore, as an alternative method, the use of ambient vibration of structures for the identification of modal properties is seemed to be more feasible when compared to forced vibration tests. On the other hand, there are some disadvantages of this method. First of all the ambient loads to structure remains unknown therefore the modal identification must be performed base only on the output-only measurements. Secondly the method does not allow identifying all modes of the structure because significant number of modes may not be excited due to low level of amplitudes of ambient vibration. Beside that there is usually limited number of sensors available to be used for structural modal identification. Therefore, the measurement locations in the structure should be selected carefully to obtain maximum modal information from the limited number of measurement locations.

As mentioned in Chapter 1, SHM has emerged as a new research area in earthquake and civil engineering community. In this context, the accurate estimation of dynamic modal properties of structures plays an important role to determine and locate the damage in SHM. There are several methods that have been developed for the last decays to estimate the modal properties of a structure using ambient responses.

Power spectrum estimation techniques can be used in order to estimate the modal frequencies of the structure. Two different power spectrum estimation methods are

explained in this section. First one is the parametric methods, such as Yule-Walker, Burg, and Unconstrained Least-square method. Second one is the eigen-analysis algorithms such as multiple signal classification (MUSIC) method, estimation of signal parameters via rotational invariance techniques (ESPRIT) method. Such methods are usually called nonparametric methods since they do not (explicitly) employ a finite-dimensional parameter vector in the search of best description. All methods have its advantageous and disadvantageous. Even though one method may give better results compared to others for a specific problem, the selection of which method to use is still depends on the nature of the problem.

As for the identification of modal damping, among many different estimation techniques, logarithmic decrement technique and half-power bandwidth techniques are explained in detail in this section. Logarithmic decrement technique is based on the damped free vibration of the structure. Decay of motion gives information about the damping ratio. On the other hand, half-power band width technique is based on the response of a structure under harmonic excitation. It is shown that the total response of harmonically excited system converges to the steady-state response in some time. Making use of the deformation response function that is calculated based on the Steady-state response, the half-power points are calculated that yields to damping ratio of the system later. Equivalent viscous damping and structural damping concepts are explained in detail.

Two different mode shape identification technique is explained in this section. First method is the integration method. It estimates the mode shapes based on the double integration of the measured data that is band-pass filtered around the identified modal frequency. Second method is the FAS method. In this method, first the FAS and the phase spectrum of the measurements are calculated. The amplitudes of FAS are related to the amplitudes of the mode shapes at a given modal frequency. Phase spectrum of the recorded signal gives information about the sign of the amplitude of the mode shapes.

3.1. Identification of Modal Frequencies

3.1.1. Parametric Methods for Power Spectrum Estimation

There are mainly two types of power spectrum estimation methods; nonparametric methods and parametric methods. The nonparametric estimation methods are relatively simpler than the parametric methods. However nonparametric methods require the availability of long data records in order to obtain the necessary frequency resolution. Furthermore, these methods suffer from spectral leakage effects, due to windowing, that are inherent in finite length data records. On the other hand parametric models are based on modeling the data sequence $y(n)$ as the output of a linear system. It is characterized by a rational system function in frequency domain as can be seen in Equation (3.1).

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^q b_k z^{-k}}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (3.1)$$

$$y(n) = -\sum_{k=1}^p a_k y(n-k) + \sum_{k=0}^q b_k x(n-k)$$

where $y(n)$ is the observed output sequence data and $x(n)$ is the input sequence to the system, which is usually not observable. On the other hand, if the observed output sequence, $y(n)$ is characterized as a stationary random process, then the input sequence, $x(n)$ is also assumed to be a stationary random process. In such case the power density spectrum (PSD) of the output sequence, $\Gamma_{yy}(f)$ can be calculated as given in Equation (3.2).

$$\Gamma_{yy}(f) = |H(f)|^2 \Gamma_{xx}(f) \quad (3.2)$$

where $\Gamma_{xx}(f)$ is the PSD of the input sequence and $H(f)$ is the frequency response of the model. If it is assume that the input sequence, $x(n)$ is zero mean white noise sequence with autocorrelation $\gamma_{xx}(m) = \sigma_x^2 \delta(m)$, then the PSD of the observed data is can be calculated as given in Equation (3.3).

$$\Gamma_{yy}(f) = \sigma_x^2 |H(f)|^2 = \sigma_x^2 \frac{|B(f)|^2}{|A(f)|^2} \quad (3.3)$$

where σ_x^2 is the variance (i.e., $\sigma_x^2 = E[|w(n)|^2]$).

In the model-based PSD estimation approach, the spectrum estimation procedure consists of two steps.

1. Estimation of the model parameters, $\{a_k\}$, and $\{b_k\}$ based on the given data sequence $y(n)$.
2. Estimation of the PSD based on the model chosen (AR, MA, ARMA).

Among these three models the AR model is by far the most widely used model because the AR model is suitable for representing spectra with narrow peaks (resonances), and the AR model results in very simple linear equations for the AR parameters. On the other hand, MA model is seldom used by itself as model because the MA model requires more coefficients to represent a narrow spectrum when compared to AR model. The ARMA model, however, requires less number of model parameters and provides a more efficient representation of the spectrum of a random process.

3.1.1.1. The Relationship Between the Autocorrelation and the Model Parameters. Let us consider a wide-sense stationary process $\{y(n)\}$ with autocorrelation sequence $\{\gamma_{yy}(m)\}$, and power spectral density $\Gamma_{yy}(f)$. Assuming that $\Gamma_{yy}(f)$ is real and continuous for all $|f| < 0.5$, then the basic relationship between the autocorrelation $\{\gamma_{yy}(m)\}$ and the model parameters $\{a_k\}$, and $\{b_k\}$ can be established for the ARMA(p, q) as given in Equation (3.4).

$$\gamma_{xx}(m) = \begin{cases} -\sum_{k=1}^p a_k \gamma_{yy}(m-k), & m > q \\ -\sum_{k=1}^p a_k \gamma_{yy}(m-k) + \sigma_y^2 \sum_{k=m}^{p-m} h(k) b_{k+m}, & 0 \leq m \leq q \\ \gamma_{xx}^*(-m) & m < 0 \end{cases} \quad (3.4)$$

This relationship provide a formula for determining the model parameters $\{a_k\}$ by restricting our attention to the case $m > q$. The set of linear equations as given in Equation (3.5) can be used to solve for model parameters $\{a_k\}$ by using the estimates of the autocorrelation sequence in place of $\{\gamma_{yy}(m)\}$ for $m > q$.

$$\begin{bmatrix} \gamma_{yy}(q) & \gamma_{yy}(q-1) & \dots & \gamma_{yy}(q-p+1) \\ \gamma_{yy}(q+1) & \gamma_{yy}(q) & \dots & \gamma_{yy}(q+p+2) \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{yy}(q+p-1) & \gamma_{yy}(q+p-2) & \dots & \gamma_{yy}(q) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} \gamma_{yy}(q+1) \\ \gamma_{yy}(q+2) \\ \vdots \\ \gamma_{yy}(q+p) \end{bmatrix} \quad (3.5)$$

If the AR(p) model is adopted, then the relationship between the AR parameters and the autocorrelation sequence is obtained by setting $q=0$ in Equation (3.6).

$$\gamma_{xx}(m) = \begin{cases} -\sum_{k=1}^p a_k \gamma_{yy}(m-k), & m > 0 \\ -\sum_{k=1}^p a_k \gamma_{yy}(m-k) + \sigma_y^2, & m = 0 \\ \gamma_{xx}^*(-m) & m < 0 \end{cases} \quad (3.6)$$

In this case, the AR parameters $\{a_k\}$ are obtained from the solution of the Yule-Walker or normal equations as given in Equation (3.7).

$$\begin{bmatrix} \gamma_{yy}(0) & \gamma_{yy}(-1) & \dots & \gamma_{yy}(p+1) \\ \gamma_{yy}(1) & \gamma_{yy}(0) & \dots & \gamma_{yy}(-p+2) \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{yy}(p-1) & \gamma_{yy}(p-2) & \dots & \gamma_{yy}(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} \gamma_{yy}(1) \\ \gamma_{yy}(2) \\ \vdots \\ \gamma_{yy}(p) \end{bmatrix} \quad (3.7)$$

where the variance σ_x^2 can be obtained from the Equation (3.8).

$$\sigma_x^2 = \gamma_{xx}(0) + \sum_{k=1}^p a_k \gamma_{xx}(-k) \quad (3.8)$$

The Equations (3.7) and (3.8) can be combined into a single matrix equation as given in Equation (3.9). Since the resulting matrix is Toeplitz, it can be efficiently inverted by use of the Levinson-Durbin algorithm.

$$\begin{bmatrix} \gamma_{yy}(0) & \gamma_{yy}(-1) & \dots & \gamma_{yy}(-p) \\ \gamma_{yy}(1) & \gamma_{yy}(0) & \dots & \gamma_{yy}(-p+1) \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{yy}(p) & \gamma_{yy}(p-1) & \dots & \gamma_{yy}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_x^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3.9)$$

Therefore, all the system parameters in the AR(p) model are easily determined from the autocorrelation sequence $\{\gamma_{yy}(m)\}$ for $0 \leq m \leq p$.

Lacoss (1971) showed that in estimating the power spectrum of sinusoidal signals via AR models, spectral peaks in an AR spectrum estimate are proportional to the square of the power of the sinusoidal signal. On the other hand, the area under the peak in the power density spectrum is linearly proportional to the power of the sinusoid. This characteristic behavior holds for all AR model-based estimation methods.

3.1.1.2. The Yule-Walker Method for AR Model Parameters. In the Yule-Walker method AR parameters are estimated using the Equation (3.7). The biased form of the autocorrelation of the measured data given in Equation (3.10) will be used to ensure that the autocorrelation matrix is positive and semidefinite.

$$r_{yy}(m) = \frac{1}{N} \sum_{n=0}^{N-m-1} y^*(n) y(n+m) \quad m \geq 0 \quad (3.10)$$

where $r_{yy}(m)$ is the biased autocorrelation matrix. If $r_{yy}(m)$ is substituted for $\gamma_{yy}(m)$ in Levinson-Durbin algorithm, the result is a stable AR parameters. The corresponding power spectrum estimate is given in Equation (3.11).

$$P_{yy}^{YW}(f) = \frac{\hat{\sigma}_{xp}^2}{\left| 1 + \sum_{k=1}^p \hat{a}_p(k) e^{-j2\pi f k} \right|^2} \quad (3.11)$$

$$\hat{\sigma}_{xp}^2 = \hat{E}_p^f = r_{xx}(0) \prod_{k=1}^p (1 - |\hat{a}_k(k)|^2)$$

where $\hat{a}(k)$ are estimates of the AR parameters obtained from the Levinson-Durbin recursions, $\hat{\sigma}_{xp}^2$ is the estimated minimum mean-square value for p^{th} -order predictor.

3.1.1.3. The Burg Method for the AR Model Parameters. This method is introduced by Burg (1968). The method is basically a least-square lattice algorithm, which estimates the AR parameters based on the minimization of the forward and backward prediction errors in linear predictors. Forward and backward linear predictions can be defined as given in Equation (3.12) for a given data sequence of $y(n)$, $n=0,1,\dots,N-1$. The order of the linear prediction is m .

$$\hat{y}(n) = -\sum_{k=1}^m a_m(k) x(n-k)$$

$$\hat{y}(n-m) = -\sum_{k=1}^m a_m^*(k) x(n+k-m) \quad (3.12)$$

where $a_m(k)$, $0 \leq k \leq m-1$, $m=1,2,\dots,p$ are the prediction coefficients, which should also satisfy the Levinson-Durbin recursion. The forward and backward errors for the linear predictors are minimized by means of least-square error as given in Equation (3.13).

$$\begin{aligned}
f_m(n) &= x(n) - \hat{x}(n) \\
g_m(n) &= x(n-m) - \hat{x}(n-m) \\
\mathcal{E}_m &= \sum_{n=m}^{N-1} \left[|f_m(n)|^2 + |g_m(n)|^2 \right]
\end{aligned} \tag{3.13}$$

where $f_m(n)$, and $g_m(n)$ are the forward and the backward prediction errors. The prediction coefficients, $a_m(k)$ are subjected to constrain that they satisfy the Levinson-Durbin recursion as given in Equation (3.14).

$$a_m(k) = a_{m-1}(k) + K_m a_{m-1}^*(m-k), \quad 1 \leq k \leq m-1, \quad 1 \leq m \leq p \tag{3.14}$$

where $K_m = a_m(m)$ is the m^{th} reflection coefficient in the lattice filter realization of the predictor. When Equation (3.14) is substituted into the Equation (3.13), the result is the pair of order-recursive equations for the forward and backward prediction errors. If we substitute these prediction errors into Equation (3.14) and perform the minimization of $\mathcal{E}_m$ with respect to the complex-valued reflection coefficients K_m , the result is the estimated reflection coefficients as can be seen in Equation (3.15).

$$\hat{K}_m = \frac{-\sum_{n=m}^{N-1} f_{m-1}(n) g_{m-1}^*(n-1)}{\frac{1}{2} \sum_{n=m}^{N-1} \left[|f_{m-1}(n)|^2 + |g_{m-1}(n-1)|^2 \right]} \quad m = 1, 2, \dots, p \tag{3.15}$$

The denominator in Equation (3.15) is simply the least-square estimate of the forward and backward errors, E_{m-1}^f and E_{m-1}^b , respectively. On the other hand the numerator is an estimate of the crosscorrelation between the forward and backward prediction errors.

$$\hat{K}_m = \frac{-\sum_{n=m}^{N-1} f_{m-1}(n) g_{m-1}^*(n-1)}{\frac{1}{2} \sum_{n=m}^{N-1} \left[\hat{E}_{m-1}^f + \hat{E}_{m-1}^b \right]} \quad m = 1, 2, \dots, p \tag{3.16}$$

where $\hat{E}_{m-1}^f + \hat{E}_{m-1}^b$ is an estimate of the total squared error, E_m . The Burg algorithm estimates the reflection coefficients by means of least-square error in the equivalent lattice structure while Levinson-Durbin algorithm is used to obtain the AR model parameters. Estimated AR parameters are used to form the power spectrum estimate as given in Equation (3.17).

$$P_{yy}^{BU} = \frac{\hat{E}_p}{\left| 1 + \sum_{k=1}^p \hat{a}_p(k) e^{-j2\pi k} \right|^2} \quad (3.17)$$

The Burg method has several advantages such as high frequency resolution, yielding a stable AR model, and computationally efficient. However there are also some major disadvantages of the Burg method. First, it exhibits spectral line splitting at high signal-to-noise ratio, which means that the spectrum of $y(n)$ may have a single sharp peak, but the Burg method may result in two or more closely spaced peaks. Second, for higher order models, the Burg method also results in false peaks. Third, for short data records of sinusoidal signals in noise, the Burg method reveals sensitivity to the initial phase of the sinusoid, which results in a frequency shift from the true frequency that is called phase dependent frequency bias.

Introduction of weighting (window) sequence in the least-square error function as seen in Equation (3.18) may overcome the limitations of the Burg method. There are different weighting windows such as hamming, quadratic or parabolic window, and delta-adaptive energy weighting windows.

$$\begin{aligned} \mathcal{E}_m^{WB} &= \sum_{n=m}^{N-1} w_m(n) \left[|f_m(n)|^2 + |g_m(n)|^2 \right] \\ \hat{K}_m &= \frac{- \sum_{n=m}^{N-1} w_{m-1}(n) (f_{m-1}(n) g_{m-1}^*(n-1))}{\frac{1}{2} \sum_{n=m}^{N-1} w_{m-1}(n) \left[|f_{m-1}(n)|^2 + |g_{m-1}(n-1)|^2 \right]} \end{aligned} \quad (3.18)$$

Application of such windowing function and energy weighting methods are considerably effective in reducing the occurrence of line splitting, spurious peaks and frequency bias.

3.1.1.4. Unconstrained Least-Square Method for the AR Model Parameters. The basic idea of this method is same as the Burg method except that the unconstrained least-square algorithm is used for determining the AR parameters. As defined in the Burg method, the forward and backward prediction errors are minimized by means of least-square error as given in the Equation (3.19).

$$\begin{aligned}\varepsilon_p &= \sum_{n=m}^{N-1} \left[|f_p(n)|^2 + |g_p(n)|^2 \right] \\ &= \sum_{n=p}^{N-1} \left[\left| y(n) + \sum_{k=1}^p a_p(k) y(n-k) \right|^2 + \left| y(n-p) + \sum_{k=1}^p a_p^*(k) y(n+k-p) \right|^2 \right]\end{aligned}\quad (3.19)$$

where $a_p(k)$ are the prediction coefficients. The unconstrained minimization of ε_p with respect to the prediction coefficients yield the set of linear equations with a function of the autocorrelation, $r_{xx}(l, k)$ of the sequence $y(n)$.

$$\begin{aligned}\sum_{k=1}^p a_p(k) r_{yy}(l, k) &= -r_{yy}(l, 0), \quad l = 1, 2, \dots, p \\ r_{yy}(l, k) &= \sum_{n=p}^{N-1} \left[x(n-k) x^*(n-l) + x(n-p+l) x^*(n-p+k) \right]\end{aligned}\quad (3.20)$$

The resulting residual least-square error and the unconstrained least-square power spectrum estimation are given in the Equation (3.21).

$$\varepsilon_p^{LS} = r_{xx}(0, 0) + \sum_{k=1}^p \hat{a}_p(k) r_{xx}(0, k) \quad (3.21)$$

$$P_{xx}^{LS} = \frac{\mathcal{E}_p^{LS}}{\left| 1 + \sum_{k=1}^p \hat{a}_p(k) e^{-j2\pi k} \right|^2}$$

The unconstrained least-square method has been proposed for spectrum estimation and its performance characteristics are superior to the Burg method, in the sense that unconstrained least-square methods does not exhibit the same sensitivity to such problems as line splitting, frequency bias and false peaks. In view of computational efficiency, unconstrained least-square method is very attractive when compared to the Levinson-Durbin algorithm. However, there is no guarantee that the estimated AR parameters yield a stable AR model, which is not considered to be a problem in spectrum estimation.

3.1.1.5. Selection of AR Model Order. The selection of model order, p is one of the most important aspect when using AR model. Wrong selection of model order may cause undesired results. If a low order of AR the model is selected, a highly smoothed spectrum may be resulted in. On the other hand if p is selected too high, low-level false peaks may appear in the resulting spectrum. Mean-square value of the residual error controls the AR model performance. As the residual error decreases, the order of the AR model increases. The rate of the decrease can be monitored to find the optimum AR model order.

However, there are several methods that can be used to calculate the optimum model order such as *Final Prediction Error (FPE) criterion*, *Akaike Information Criterion (AIC)*, *Minimizes the Description Length (MDL)*, and *Criterion Autoregressive Transfer (CAT)*.

$$FPE(p) = \hat{\sigma}_{wp} \left(\frac{N + p + 1}{N - p - 1} \right) \quad (3.22)$$

Proposed by Akaike (1969-1974), FPE is based on the minimizing the mean-square error for one-step predictor as seen in Equation (3.22) while AIC is based on selecting the order that minimizes the Equation (3.23). It should be noted that the term $\hat{\sigma}_{wp}$ in AIC decreases and therefore $\ln \hat{\sigma}_{wp}$ also decreases as the order of the AR model increases.

However, $2p/N$ increases with an increase in p . Hence a minimum value is obtained for some p .

$$AIC(p) = \ln \hat{\sigma}_{wp}^2 + 2p / N \quad (3.23)$$

An alternative information criterion, MDL, proposed by Rissanen (1983), is based on the selecting the order that minimize the Equation (3.24).

$$MDL(p) = N \ln \hat{\sigma}_{wp}^2 + p \ln N \quad (3.24)$$

The last criterion, CAT that is proposed by Parzen (1974), is based on selecting the model order, p that to minimize the Equation (3.25). The mean should be removed from the data. Since $\hat{\sigma}_{wk}^2$ depends on the of spectrum estimate, the model order is also a function of criterion.

$$CAT(p) = \left(\frac{1}{N} \sum_{k=1}^p \frac{1}{\bar{\sigma}_{wk}^2} \right) - \frac{1}{\hat{\sigma}_{wp}^2} \quad (3.25)$$

$$\bar{\sigma}_{wk}^2 = \frac{N}{N-k} \hat{\sigma}_{wk}^2$$

3.1.2. Eigenanalysis Algorithms for Spectrum Estimation

In this section, sinusoid signals that are corrupted by additive white noise are considered. The algorithm used is based on an eigen-decomposition of the correlation matrix of the noise-corrupted signal. In general a signal consisting of p sinusoids components should satisfies the difference equation and system function given in Equation (3.26).

$$x(n) = -\sum_{m=1}^{2p} a_m x(n-m) \quad (3.26)$$

$$H(z) = \frac{1}{1 + \sum_{m=1}^{2p} a_m z^{-m}}$$

If the sinusoids are corrupted by a white noise sequence $w(n)$ with $E[|w(n)|^2]$. Then the noise-corrupted signal can be written as seen in Equation (3.27).

$$y(n) = x(n) + w(n) \quad (3.27)$$

If Equation (3.27) is substituted in Equation (3.26), we obtain the difference equation for ARMA(p, p) process in which the AR and MA parameters are identical as seen in Equation (3.28).

$$\begin{aligned} y(n) - w(n) &= - \sum_{m=1}^{2p} [y(n-m) - w(n-m)] a_m \\ \sum_{m=0}^{2p} a_m y(n-m) &= \sum_{m=0}^{2p} a_m w(n-m) \end{aligned} \quad (3.28)$$

where, by definition, $a_0 = I$. This difference equation can be expressed in matrix form as

$$\mathbf{Y}^t \mathbf{a} = \mathbf{W}^t \mathbf{a} \quad (3.29)$$

where $\mathbf{Y}^t = [y(n) \ y(n-1) \cdots y(n-2p)]$ is the observed data vector of dimension $(2p+1)$, $\mathbf{W}^t = [w(n) \ w(n-1) \cdots w(n-2p)]$ is the noise vector, and $\mathbf{a} = [1 \ a_1 \cdots a_{2p}]$ is the coefficient vector. If Equation (3.29) is pre-multiplied by $\mathbf{Y}$ and take the expected value, we will obtain an eigenvalue problem as shown in the Equation (3.30).

$$\begin{aligned} E(\mathbf{Y}\mathbf{Y}^t) \mathbf{a} &= E(\mathbf{Y}\mathbf{W}^t) \mathbf{a} = E[(\mathbf{X} + \mathbf{W})\mathbf{W}^t] \mathbf{a} \\ (\Gamma_{yy} - \sigma_w^2 \mathbf{I}) \mathbf{a} &= 0 \end{aligned} \quad (3.30)$$

where $w(n)$ is assumed to be zero mean and white, σ_w^2 is an eigenvalue of the autocorrelation matrix $\mathbf{\Gamma}_{yy}$, and $\mathbf{X}$ is deterministic signal. The eigenvalue problem in Equation (3.30) forms the basis for the Pisarenko harmonic decomposition method.

3.1.2.1. MUSIC Algorithm. Multiple signal classification (MUSIC) method proposed by Schmidt (1981), is a noise subspace frequency estimator. Let's consider a "weighted" spectral estimate as seen in Equation (3.31).

$$P(f) = \sum_{k=p+1}^M w_k \left| s^H(f) v_k \right|^2 \quad (3.31)$$

where $\{v_k, k = p+1, \dots, M\}$ is the eigenvectors in the noise subspace, $\{w_k\}$ is a set of positive weights, and $s(f) = [1, e^{j2\pi f}, e^{j4\pi f}, \dots, e^{j2(M-1)\pi f}]$ is the complex sinusoidal vector. It should be noted that at $f = f_i$, $s(f_i) \equiv s_i$, so that at any one of the p sinusoidal frequency components of the signal, $P(f_i) = 0$, $i = 1, 2, \dots, p$. Therefore, the reciprocal of $P(f)$ is a sharply peaked function of frequency and provides a method for estimating the frequencies of the sinusoidal components.

$$\frac{1}{P(f)} = \frac{1}{\sum_{k=p+1}^M w_k \left| s^H(f) v_k \right|^2} \quad (3.32)$$

Although theoretically $1/P(f)$ is infinite at $f = f_i$, in practice the estimation errors result in finite values for $1/P(f)$ at all frequencies.

The MUSIC sinusoidal frequency estimator is a special case of Equation (3.32) in which the weights $w_k = 1$ for all k .

$$P_{MUSIC}(f) = \frac{1}{\sum_{k=p+1}^M \left| s^H(f) v_k \right|^2} \quad (3.33)$$

The estimate of the sinusoidal frequencies are the peaks of $P_{\text{MUSIC}}(f)$. Once the sinusoidal frequencies are estimated, the power of each of the sinusoids can be obtained by solving Pisarenkon Harmonic Decomposition Method.

3.1.2.2. ESPRIT Algorithm. Estimation of signal parameters via rotational invariance techniques (ESPRIT) is a method for estimating frequencies of a sum of sinusoids by use of an eigen-decomposition approach. ESPRIT exploits an underlying rotational invariance of signal subspace spanned by two temporally displaced data vectors. If p complex-valued sinusoids in additive white noise are considered, then the output sequence will be a vector as seen in Equation (3.34).

$$y(n) = [y(n), y(n+1), \dots, y(n+M-1)]^t = x(n) + w(n) \quad (3.34)$$

where $x(n)$ is the signal vector and $w(n)$ is the noise vector. To identify the deterministic character of the sinusoids, a time-displaced vector $z(n)=y(n+1)$ should be defined as given in Equation (3.35).

$$\begin{aligned} z(n) &= [z(n), z(n+1), \dots, z(n+M-1)]^t = [y(n+1), y(n+2), \dots, y(n+M)]^t \\ y(n) &= Sa + w(n) \\ z(n) &= S\Phi a + w(n) \end{aligned} \quad (3.35)$$

where $a = [a_1, a_2, \dots, a_p]^t$, $a_i = A_i e^{j\phi_i}$, and Φ is a diagonal matrix consisting of the relative phase between adjacent time samples of each of the complex sinusoids as seen in Equation (3.36). It should be noted that the rotational operator matrix, Φ relates the time-displaced vectors $y(n)$ and $z(n)$. The matrix S is the Vandermonde matrix specified as given in Equation (3.36).

$$\begin{aligned} \Phi &= \text{diag}[e^{j2\pi f_1}, e^{j2\pi f_2}, \dots, e^{j2\pi f_p}] \\ s_i &= [1, e^{j2\pi f_i}, e^{j4\pi f_i}, \dots, e^{j2\pi(M-1)f_i}] \quad i = 1, 2, \dots, p \end{aligned} \quad (3.36)$$

Auto-covariance matrix for the data vector, $y(n)$ is can be estimated as given in Equation (3.37).

$$\begin{aligned}
 \Gamma_{yy} &= E[y(n)y^H(n)] \\
 &= SP S^H + \sigma_w^2 \mathbf{I} \\
 P &= \text{diag}[|a_1|^2, |a_2|^2, \dots, |a_p|^2] \\
 &= \text{diag}[P_1, P_2, \dots, P_p]
 \end{aligned} \tag{3.37}$$

where $\mathbf{P}$ is a diagonal matrix consisting of the powers of the complex sinusoids. $\mathbf{P}$ must be a diagonal matrix because complex sinusoids of different frequencies are orthogonal over the infinite interval. However, ESPRIT algorithm does not require $\mathbf{P}$ to be diagonal. Therefore, it is applicable to the case in which the covariance matrix is estimated from finite data records.

The cross-covariance matrix of the signal vectors $y(n)$ and $z(n)$ can be calculated as seen in Equation (3.38).

$$\begin{aligned}
 \Gamma_{yz} &= E[y(n)z^H(n)] = SP\Phi^H S^H + \Gamma_w \\
 \Gamma_w &= E[w(n)w^H(n+1)] = \sigma_w^2 \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \equiv \sigma_w^2 Q
 \end{aligned} \tag{3.38}$$

The auto and cross-covariance matrices Γ_{yy} and Γ_{yz} are given as seen in Equation (3.39) and (3.40). It should be noted that these Γ_{yy} and Γ_{yz} are Teoplitz matrices.

$$\Gamma_{yy} = \begin{bmatrix} \gamma_{yy}(0) & \gamma_{yy}(1) & \dots & \gamma_{yy}(M-1) \\ \gamma_{yy}^*(1) & \gamma_{yy}(0) & \dots & \gamma_{yy}(M-2) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{yy}^*(M-1) & \gamma_{yy}(M-2) & \dots & \gamma_{yy}(0) \end{bmatrix} \tag{3.39}$$

$$\Gamma_{yz} = \begin{bmatrix} \gamma_{yy}(1) & \gamma_{yy}(2) & \cdots & \gamma_{yy}(M) \\ \gamma_{yy}(0) & \gamma_{yy}(1) & \cdots & \gamma_{yy}(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{yy}^*(M-2) & \gamma_{yy}^*(M-3) & \cdots & \gamma_{yy}(1) \end{bmatrix} \quad (3.40)$$

where $\gamma_{yy}(m) = E[y^*(n)y(n+m)]$. Based on this formulation the problem reduces to determine the frequencies $\{f_i\}$ and their powers $\{P_i\}$ from the autocorrelation sequence $\{\gamma_{yy}(m)\}$. It is clear that the matrix $\mathbf{SPS}^H$ has rank p . Therefore, Γ_{yy} has $(M-p)$ identical eigenvalues equal to σ_w^2 .

$$\begin{aligned} \Gamma_{yy} - \sigma_w^2 \mathbf{I} &= \mathbf{SPS}^H \equiv C_{yy} \\ \Gamma_{yz} - \sigma_w^2 \Gamma_w &= \mathbf{SP}\Phi^H S^H \equiv C_{yz} \end{aligned} \quad (3.41)$$

The matrix $C_{yy} - \lambda C_{yz}$ can be written as seen in Equation (3.42).

$$C_{yy} - \lambda C_{yz} = \mathbf{SP}(I - \lambda\Phi^H)S^H \quad (3.42)$$

The column space of $\mathbf{SPS}^H$ is identical to the column space $\mathbf{SP}\Phi^H S^H$. Consequently, the rank of $C_{yy} - \lambda C_{yz}$ is equal to p . Based on these mathematical relationship we can formulate an algorithm (ESPRIT) for estimating the frequencies $\{f_i\}$.

1. The autocorrelation values $r_{yy}(m)$, $m=1,2,\dots,M$ are computed using the data available. $\mathbf{R}_{yy}$, and $\mathbf{R}_{yz}$ matrices are formed using the corresponding estimates of Γ_{yy} and Γ_{yz} .
2. Eigenvalues of $\mathbf{R}_{yy}$ are computed. For $M > p$, the minimum eigenvalues is an estimate of σ_w^2 .
3. Compute $\hat{C}_{yy} = \mathbf{R}_{yy} - \hat{\sigma}_w^2 \mathbf{I}$ and $\hat{C}_{yz} = \mathbf{R}_{yz} - \hat{\sigma}_w^2 \mathbf{Q}$
4. Compute the generalized eigenvalues of the matrix pair $\{\hat{C}_{yy}, \hat{C}_{yz}\}$. The p generalized eigenvalues of these matrices lie on the unit circle. Then using these eigenvalues, elements of Φ are determined, and hence the sinusoidal frequencies. The remaining $M-p$ eigenvalues will lie at the origin.

There are two methods to compute the power of sinusoidal components. First, Pisarenko Harmonic Decomposition method can be used with $r_{yy}(m)$ substituted for $\gamma_{yy}(m)$. Second method is based on the computation of the generalized eigenvectors $\{v_i\}$ corresponding to the generalized eigenvalues $\{\lambda_i\}$ as given in Equation (3.43).

$$(C_{yy} - \lambda_i C_{yz})v_i = SP(I - \lambda_i \Phi^H)S^H v_i = 0 \quad (3.43)$$

It should be noted that generalized eigenvector v_i is orthogonal to s_j , $j \neq i$. Then the power of the signal can be calculated as given in Equation (3.44).

$$P_i = \frac{v_i^H C_{yy} v_i}{|v_i^H s_i|^2} \quad i = 1, 2, \dots, p \quad (3.44)$$

3.1.2.3. Order Selection Criteria. The MUSIC and ESPRIT methods are used to estimate the frequencies and the powers of the sinusoids. They also provide information about the number of sinusoidal components in the recorded measurements. If there are p sinusoids, the eigenvalues associated with the signal subspace are $\{\lambda_i + \sigma_w^2, \quad i = 1, 2, \dots, p\}$ while the remaining $(M-p)$ eigenvalues are all equal to σ_w^2 . AIC criteria can be extended to the eigen-decomposition method to determine the number of sinusoids, p in the signal by selecting the minimum value of $MDL(p)$ as given in Equation (3.45).

$$MDL(p) = -\log \left[\frac{G(p)}{A(p)} \right]^N + E(p) \quad (3.45)$$

where

$$G(p) = \prod_{i=p+1}^M \lambda_i \quad p = 0, 1, \dots, M-1$$

$$A(p) = \left[\frac{1}{M-p} \sum_{i=p+1}^M \lambda_i \right]^{M-p}$$

$$E(p) = \frac{1}{2} p(2M-p) \log N$$

N: number of samples used to estimate the M autocorrelation lags.

3.2. Identification of Modal Damping

Modal damping can be calculated using the measured data. Modal damping is a highly nonlinear dynamic parameter of a structure. It varies with the frequency content of the earthquake load acting on the structure and the displacement amplitude of the structure. Instead of raw data, the autocorrelation functions may give better results when dealing with small earthquake records with high noise contamination, wind or ambient vibrations.

3.2.1. Logarithmic Decrement Technique

3.2.1.1. Damped Free Vibration. Classical formulation for viscously damped single degree of freedom (SDOF) system is given in Equation (3.46).

$$a(t) + 2\zeta w_n v(t) + w_n^2 u(t) = 0$$

$$\zeta = \frac{c}{2mw_n} = \frac{c}{c_{cr}} \quad (3.46)$$

where $a(t)$ is the acceleration, $v(t)$ is the velocity, $u(t)$ is the displacement, ζ is damping ratio or fraction of critical damping, w_n is natural circular frequency, and c_{cr} is critical damping coefficient that is the smallest value of c that inhibits oscillation completely.

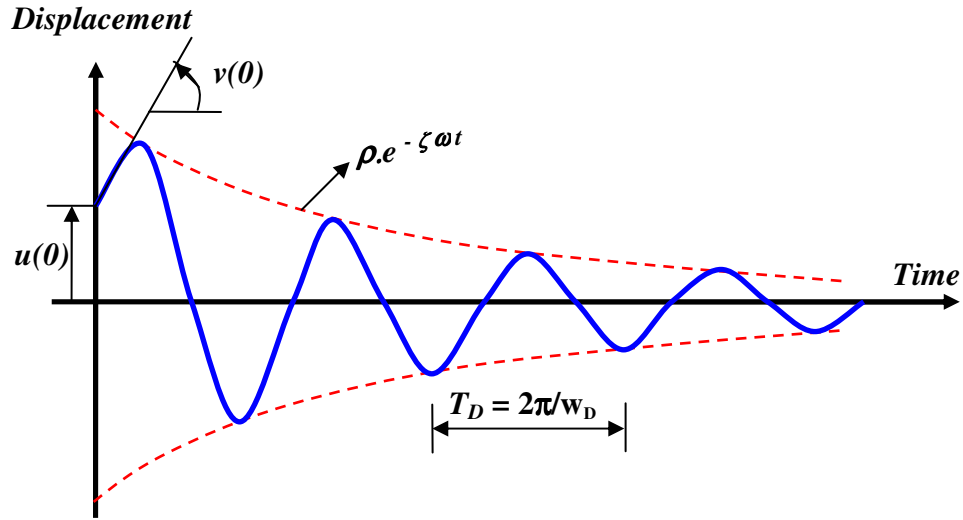


Figure 3.1. Damped free vibration

The displacement response of viscously damped SDOF is given in Figure 3.1. The solution for the SDOF subjected to initial conditions (initial displacement $u(0)$, and initial velocity $v(0)$) for systems with $c < c_{cr}$ is given in Equation (3.47). It should also be noted that the displacement amplitude is decreasing exponentially in time with every cycle of vibration due to damping effect. The envelope of this decay is given in Equation (3.48).

$$u(t) = e^{-\zeta \omega_n t} \left[u(0) \cos(\omega_D t) + \frac{v(0) + \zeta \omega_n u(0)}{\omega_D} \sin(\omega_D t) \right] \quad (3.47)$$

$$\omega_D = \omega_n \sqrt{1 - \zeta^2}$$

$$\rho = \sqrt{u(0)^2 + \left[\frac{u(0) + \zeta \omega_n u(0)}{\omega_D} \right]^2} \quad (3.48)$$

The damping in structures is an important modal property that should be studied carefully. Damping has the effect of lowering the natural frequency from ω_n to ω_D . On the other hand this effect is negligible for structures having damping ratio below 20% as can be seen in Figure 3.2.

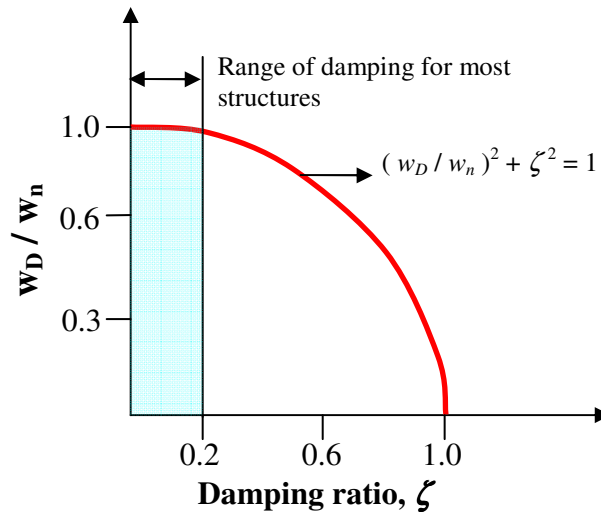


Figure 3.2. Effects of damping on the natural vibration frequency (Chopra, 2001)

3.2.1.2. Decay of Motion. Logarithmic decrement method is used to measure the damping ratio for viscously damped vibrating systems. In this method, the free vibration displacement amplitude history of a system to an initial conditions (initial displacement and velocity) is measured and recorded. A typical free decay curve is shown in Figure 3.1. Logarithmic decrement is the natural logarithmic value of the ratio of two adjacent peak values of displacement in free decay vibration.

$$\frac{u(t)}{u(t + T_D)} = \frac{\exp(-\zeta w_n t)}{\exp(-\zeta w_n (t + T_D))} = \exp(\zeta w_n T_D) = \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad (3.49)$$

$$\frac{u_i}{u_{i+1}} = \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad (3.50)$$

The natural logarithm of this ratio is called the *logarithmic decrement*, denoted by δ as can be seen in Equation (3.51). If ζ is small enough, the term $\sqrt{1-\zeta^2}$ converges to 1 and the damping ratio leads to an approximate formulation.

$$\delta = \ln \left(\frac{u_i}{u_{i+1}} \right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad (3.51)$$

$$\delta \cong 2\pi\zeta$$

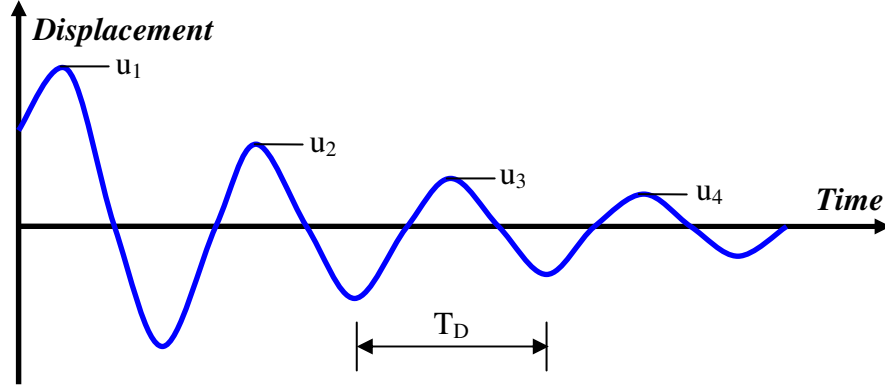


Figure 3.3. Free decay curve (Chopra, 2001)

If the decay of motion is slow, as is the case for lightly damped systems, it is desirable to relate the ratio of two amplitudes several cycles apart, instead of successive amplitudes, to the damping ratio. Over j cycles the motion decreases from u_1 to u_{j+1} . Figure 3.4 shows a plot of the exact and approximate relations between logarithmic decrement, δ and damping ratio, ζ . It should be noted that Equation (3.52) is valid for $\zeta < 0.2$, which covers most of the structures.

$$\frac{u_1}{u_{j+1}} = \frac{u_1}{u_2} \frac{u_2}{u_3} \frac{u_3}{u_4} \dots \frac{u_j}{u_{j+1}} = e^{j\delta} \quad (3.52)$$

$$\delta = \frac{1}{j} \ln \left(\frac{u_1}{u_{j+1}} \right) \cong \frac{1}{j} 2\pi\zeta$$

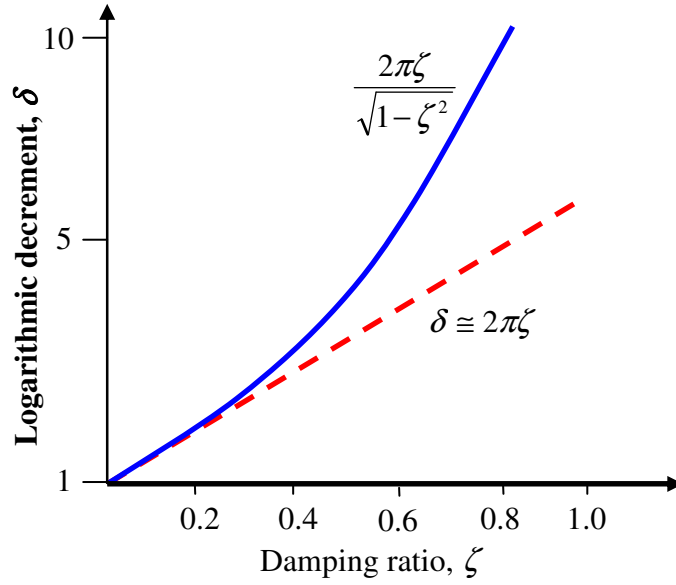


Figure 3.4. Exact and approximate relations between logarithmic decrement and damping ratio (Chopra, 2001)

3.2.2. Half-Power Bandwidth Technique

3.2.2.1. Harmonic Vibration with Viscous Damping. Classical formulation for viscously damped single degree of freedom (SDOF) system to a harmonic motion is given in Equation (3.53).

$$ma(t) + cv(t) + ku(t) = P_o \sin(\omega t) \quad (3.53)$$

where $a(t)$ is the acceleration, $v(t)$ is the velocity, $u(t)$ is the displacement, P_o is the amplitude of the harmonic motion, ω is the frequency of forcing function, and m , c , and k are the mass, the damping coefficient, and the stiffness of SDOF system, relatively.

The total solution of Equation (3.53) includes complimentary solution and particular solution. The complimentary solution is the damped free vibration response explained in section 3.2.1.1. The particular solution of SDOF system to a harmonic motion is the steady-state response as given in Equation (3.54). The total response and the steady-state response of the SDOF can be seen in Figure 3.5. It is noticeable that the difference between total response and the steady-state response decays exponentially with time depending on

the (w/w_n) and ζ . After a certain time, the total response converges to the steady-state response of the SDOF.

$$u_p(t) = C \sin(wt) + D \cos(wt)$$

$$C = \frac{P_0}{k} \frac{1 - (w/w_n)^2}{[1 - (w/w_n)^2]^2 + [2\zeta(w/w_n)]^2} \quad (3.54)$$

$$D = \frac{P_0}{k} \frac{-2\zeta(w/w_n)}{[1 - (w/w_n)^2]^2 + [2\zeta(w/w_n)]^2}$$

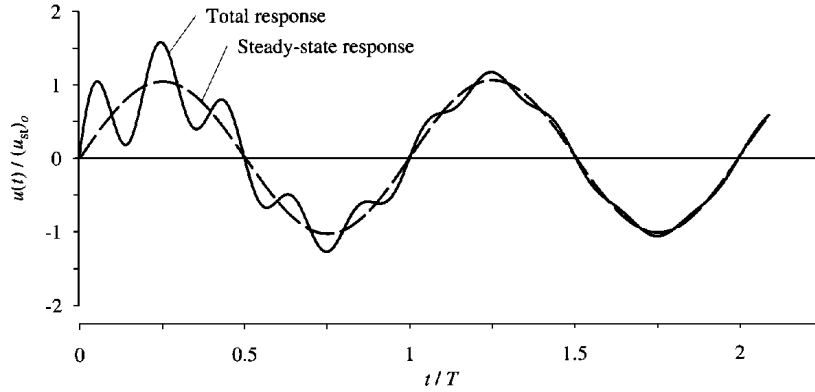


Figure 3.5. Response of SDOF to harmonic vibration. (Chopra, 2001)

The steady-state response of SDOF system excited by harmonic force, given in Equation (3.54), can be rewritten as seen in Equation (3.55).

$$u(t) = u_0 \sin(wt - \phi) = (u_{st})_0 R_d \sin(wt - \phi) \quad (3.55)$$

where $u_0 = \sqrt{C^2 + D^2}$ and $\phi = \tan^{-1}(-D/C)$. If D and C is substituted, then the deformation response factor, R_d and phase angle ϕ can be written as given in Equation (3.56).

$$R_d = \frac{u_0}{(u_{st})_0} = \frac{1}{\sqrt{[1 - (w/w_n)^2]^2 + [2\zeta(w/w_n)]^2}} \quad (3.56)$$

$$\phi = \tan^{-1} \left(\frac{2\zeta(w/w_n)}{1 - (w/w_n)^2} \right)$$

where $(u_{st})_0$ is the maximum value of the static deformation.

Figure 3.6 shows the change of deformation response function R_d , with respect to (w/w_n) for different damping ratios. It is obvious that the R_d decreases with increasing damping ratio for the entire excitation frequencies.

1. The steady-state motion described by Equation (3.55) is harmonic and is of the same frequency as the excitation frequency. The amplitude of the steady-state response is a function of the amplitude and frequency of the excitation as well as of the natural frequency and damping ratio of the SDOF system.
2. Steady-state response and the harmonic excitation force are not in phase. They do not attain their maximum values at the same time. The response lags the harmonic excitation by the phase angle ϕ as shown in Equation (3.55).
3. If $(w/w_n) \ll 1$ or $(w/w_n) \cong 0$, R_d is close to 1 and becomes almost independent of damping. In this case the amplitude of dynamic response is the same as the static deformation and controlled by the stiffness of the system

$$u_0 \cong (u_{st})_0 = \frac{P_0}{k} \quad (3.57)$$

4. If $(w/w_n) \gg 1$ or $(w/w_n) \rightarrow \infty$, then the deformation response factor, R_d tends to zero and unaffected by damping. In this case the dynamic response is controlled by the mass of the system.

$$u_0 \cong (u_{st})_0 \frac{w_n^2}{w^2} = \frac{P_0}{mw^2} \quad (3.58)$$

5. If $(w/w_n) \cong 1$, R_d becomes very sensitive to the damping ratio and R_d can be several times larger than one. In this case the forcing frequency is very close to the frequency of SDOF system. If $(w/w_n) = 1$, the dynamic deformation becomes as given in Equation (3.59). In this specific case, the response is controlled by the damping ratio of the SDOF system. The deformation response factor, R_d becomes a function of damping ratio.

$$u_0 = \frac{(u_{st})_0}{2\zeta} = \frac{P_0}{cw_n}$$

$$R_d = \frac{1}{2\zeta}$$
(3.59)

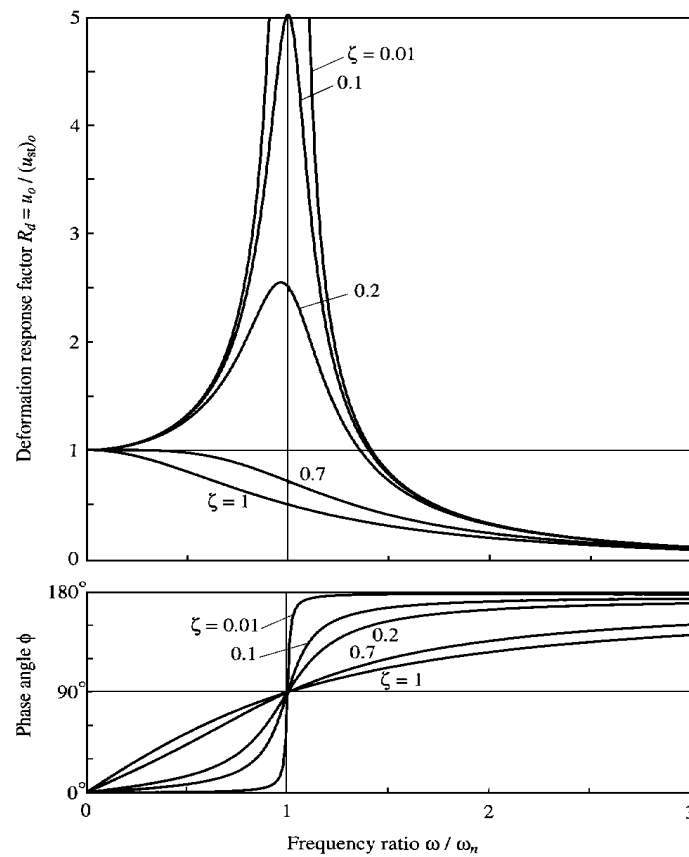


Figure 3.6. Deformation response factor and phase angle for a damped system excited by harmonic motion. (Chopra, 2001)

3.2.2.2. Sharpness of Resonance. In harmonic excitation, there is a quantity Q related to damping that is a measure of the sharpness of resonance. In the case of resonance, when $w/w_n=1$, the deformation response factor, R_d is $1/2\zeta$ as shown in Equation (3.59). We seek the two frequencies on either side of resonance where the amplitude is $1/\sqrt{2}$ times the resonant amplitude. These frequency points are also referred to as the half-power points and are shown in Figure 3.7. Let their frequencies be called w_1 and w_2 with corresponding frequency ratios w_1/w_n and w_2/w_n .

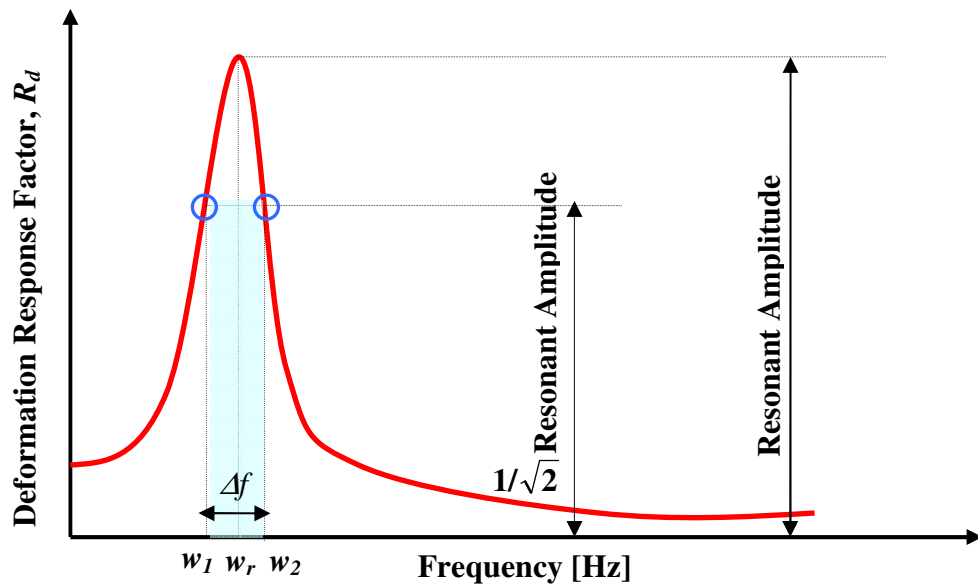


Figure 3.7. Halfpower bandwidth technique

The halfpower frequency points can be obtained by using the deformation response factors, R_d ;

At resonance frequency $w=w_r$; $R_d = 1/2\zeta$

$$R_d = \frac{1}{2\zeta} = \frac{1}{\sqrt{[1 - (w/w_n)^2]^2 + [2\zeta(w/w_n)]^2}}$$

At half-power points $w=w_1$ and/or $w=w_2$, $R_d = (1/\sqrt{2}) \cdot (1/2\zeta)$

$$R_d = \frac{1}{\sqrt{2}} \frac{1}{2\zeta} = \frac{1}{\sqrt{[1 - (w/w_n)^2]^2 + [2\zeta(w/w_n)]^2}}$$

$$(w/w_n)^4 - 2(1 - 2\zeta^2)(w/w_n)^2 + (1 - 8\zeta^2) = 0 \quad (3.60)$$

Solving for $(w/w_n)^2$, we have two roots given by Equation (3.61).

$$(w/w_n)^2 = (1 - 2\zeta^2) \pm 2\zeta\sqrt{1 - \zeta^2} \quad (3.61)$$

Assuming $\zeta \ll 1$ and neglecting higher order terms of ζ , and using binomial expansion;

$$\begin{aligned} (w/w_n)^2 &= (1 \pm 2\zeta) \\ w_2/w_n &= (1 + 2\zeta)^{1/2} = 1 + \frac{1}{2}(2\zeta) + \dots \\ w_1/w_n &= (1 - 2\zeta)^{1/2} = 1 - \frac{1}{2}(2\zeta) + \dots \end{aligned} \quad (3.62)$$

$$\begin{aligned} 4\zeta &= \frac{w_2^2 - w_1^2}{w_n^2} \cong 2 \left(\frac{w_2 - w_1}{w_n} \right) \\ \zeta &\cong \frac{w_2 - w_1}{2w_n} \end{aligned} \quad (3.63)$$

The quantity factor Q is then defined as

$$Q = \frac{w_n}{w_2 - w_1} = \frac{f_n}{f_2 - f_1} = \frac{1}{2\zeta} \quad (3.64)$$

This common method is very simple and quick. It has been already practiced in many applications. The method is simply based on the fact that the width of the displacement response amplitude of a SDOF system is proportional to the systems damping ratio. The more the damping, the more the frequency ranges between this two frequencies, f_1 and f_2 . Half-power bandwidth is defined as the ratio of the frequency range between the two half-power points, f_1 and f_2 to the natural frequency at this mode.

A possible problem is, for example, when the spectral peak is missing. In fact, there is a good chance not to have an exact spectral peak. If the peak is missing, the corresponding band width tends to be greater than the real value and the damping is likely to be overestimated. Another possible problem is the closely spaced peaks as shown in Figure 3.8. In this case the, halfpower points are located at the amplitude of $(1/n)$ times the resonant amplitude. Therefore, the above formulation should be rederived for $(1/n)$ instead of $1/\sqrt{2}$.

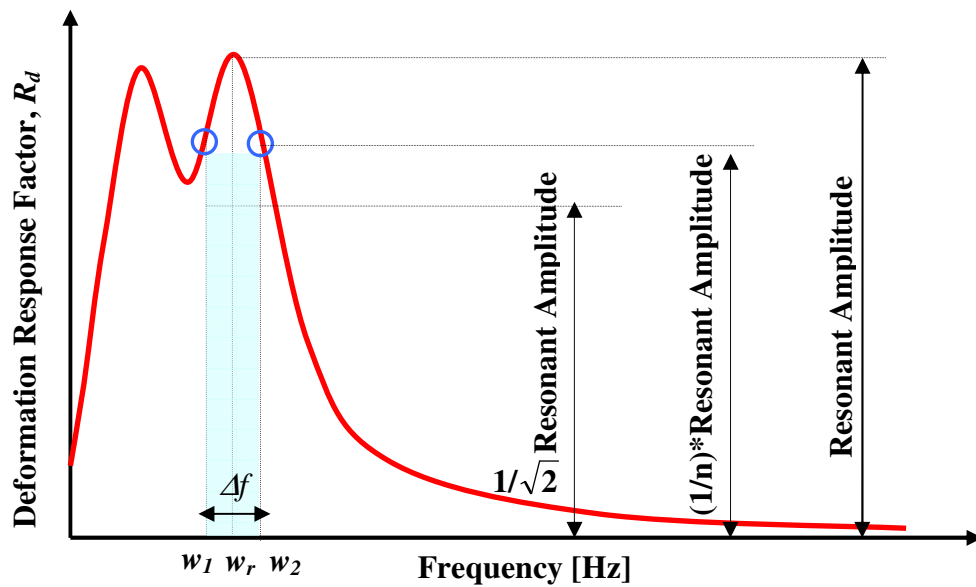


Figure 3.8. Halfpower bandwidth technique with 2 closely spaced peaks

The accuracy of the halfpower bandwidth method is mostly depends on the frequency resolution of spectral response of the structure.

A parameter, w , proposed by Brownjohn, 1994 can be used for estimating the bias of the damping values, $W = \Delta f / df$. The single parameter W appears to be the major parameter in determining the bias of the damping ratio. Bias can be characterized by the increase of measured W from the true W arising from the combination of frequency, damping and resolution.

3.2.3. Energy Dissipated by Viscous Damping

Consider the response of steady-state motion of an SDOF system to a harmonic motion $p(t) = p_o \sin(\omega t)$ as given in Equation (3.53). The potential energy, E_s and the kinetic energy, E_K dissipated in one cycle of harmonic motion are zero as shown in Equation (3.65).

$$\begin{aligned} E_s &= \int f_s du = \int_0^{2\pi/\omega} (ku) v dt = \int_0^{2\pi/\omega} k [u_0 \sin(\omega t - \phi)] [\omega u_0 \cos(\omega t - \phi)] dt = 0 \\ E_K &= \int f_I du = \int_0^{2\pi/\omega} (m\ddot{u}) v dt = \int_0^{2\pi/\omega} m [-\omega^2 u_0 \sin(\omega t - \phi)] [\omega u_0 \cos(\omega t - \phi)] dt = 0 \end{aligned} \quad (3.65)$$

The E_s and E_K is zero for one cycle of harmonically excited SDOF system, therefore all the energy should be dissipated by the viscous damping. The energy dissipated in one cycle of harmonic motion by viscous damping, E_D is defined in Equation (3.66). The dissipated energy is proportional to the square of the amplitude of the motion, damping ratio of the SDOF system and the frequency of excitation force.

$$E_D = \int f_D du = \int_0^{T=2\pi/\omega} c v^2 dt = c \int_0^{2\pi/\omega} [\omega u_0 \cos(\omega t - \phi)]^2 dt = \pi c \omega u_0^2 = 2\pi \zeta \frac{\omega}{\omega_n} k u_0^2 \quad (3.66)$$

Figure 3.9 shows the total force $f_s + f_D$ acting on SDOF system where the system is executing steady-state motion. The lower portion of the curve is for motion with $v > 0$ whereas the upper of the curve is for the portion of the cycle when $v < 0$. The triangular area is the peak potential energy, E_{s0} stored elastically. The area inside the elliptical curve is the energy dissipated per cycle by the damping force, E_D as given in Equation (3.66).

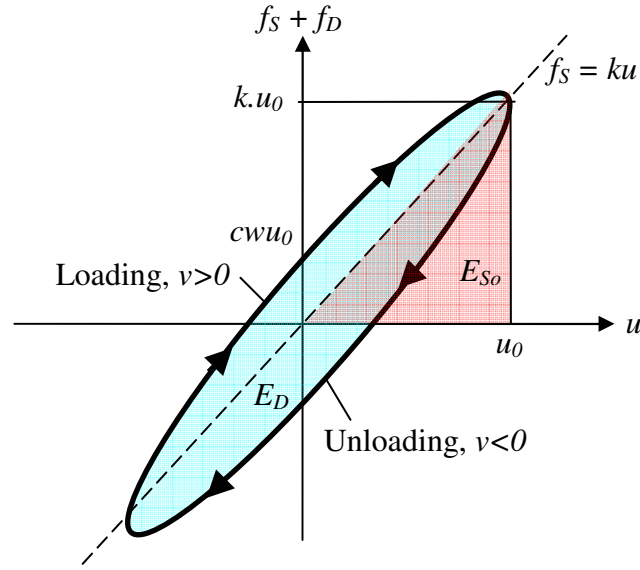


Figure 3.9. Hysteresis loop of a SDOF for spring and viscous damper

In steady-state vibration, the energy input to the system due to the applied force is dissipated in viscous damping. The external force $p(t)$ inputs to the SDOF system as given in Equation (3.67).

$$E_I = \int p(t) du = \int_0^{T=2\pi/w} p(t) v dt = \int_0^{2\pi/w} [p_0 \sin(wt)] [wu_0 \cos(wt - \phi)] dt = \pi p_0 u_0 \sin(\phi) \quad (3.67)$$

At resonance frequency, where $w=w_n$ and $\phi=90^\circ$, the input energy, E_I will be

$$E_I = \pi p_0 u_0 \quad (3.68)$$

Input energy to the SDOF system at resonance frequency will be dissipated by the viscous damping. In other word, in steady-state force vibration the loss of energy is balanced by the energy that is supplied by the excitation. Therefore Equation (3.68) should be equal to Equation (3.66). The result is consistent with Equation (3.59).

$$\pi p_0 u_0 = \pi c w_n u_0^2$$

$$u_0 = \frac{P_o}{c w_n} \quad (3.69)$$

3.2.4. Equivalent Viscous Damping

Damping is present in all oscillatory systems. The process by which free vibration steadily diminishes in amplitude is called damping. The primary effect of the damping is to remove energy from the system. The loss of energy from the damped system results in the decay of amplitude of free vibration as seen in Figure 3.5. Unfortunately the damping coefficient, c , as seen in Equation (3.53), cannot be calculated from the dimensions of the structure. Therefore the vibration experiments on actual structures are usually carried out in order to calculate the damping coefficient.

There are various damping mechanisms exists in structures which dissipates energy in a vibrating system. It is usual that there is more than one energy dissipating mechanism in the structure at the same time such as friction at steel connections, opening and closing of microcracks in concrete, and friction between the structure itself, and nonstructural elements such as partition walls. It is almost impossible to measure or to estimate the contribution of all these energy dissipating mechanism to global energy dissipation in a real structure. Therefore, an idealization for the damping in structures is needed to represent all the damping mechanisms in structure. An equivalent viscous damper can be used to idealize the actual damping in SDOF system.

The viscous damping coefficient, c is selected so that the vibration energy dissipated by viscous damper is equal to the energy dissipated by all the damping mechanisms exist in the actual structure. The most common method to calculate the equivalent viscous damping is to equate the energy dissipated by the actual structure, E_I and an equivalent viscous system, E_D .

$$E_I = 4\pi\zeta_{eq} \frac{w}{w_n} E_{So}$$

$$\zeta_{eq} = \frac{1}{4\pi} \frac{1}{w/w_n} \frac{E_D}{E_{So}} \quad (3.70)$$

where the strain energy, $E_{So} = ku_o^2 / 2$ should be calculated at resonance $w=w_n$ where the response of the system is most sensitive to the damping. The damping ratio ζ_{eq} determined for resonance $w=w_n$ would not be correct for other exciting frequency.

It should be noted that the equivalent viscous damper idealization is valid for deformations within the linear elastic limit of the structure. The damping coefficient, c may vary nonlinearly beyond the linear elastic limit.

3.2.5. Structural Damping

When materials are cyclically stressed, energy is dissipated internally within the material itself. For most structural metals, such as steel or aluminum, the energy dissipated per cycle is independent of the frequency over a wide frequency range and proportional to the square of the amplitude of vibration. Internal damping fitting this classification is called *solid damping* or *structural damping*. With the energy dissipation per cycle proportional to the square of the vibration amplitude, the loss coefficient is a constant and the shape of the hysteresis curve remains unchanged with amplitude and independent of the strain rate.

Energy dissipated by structural damping can be written as given in Equation (3.71).

$$E_{ST} = \alpha U^2 \quad (3.71)$$

where α is constant with units of force/displacement. By using the concept of equivalent viscous damping, energy dissipated by viscous damping should be equal to the energy dissipated by structural damping as given in Equation (3.72).

$$\pi c_{eq} \omega X^2 = \alpha X^2$$

$$c_{eq} = \frac{\alpha}{\pi \omega} \quad (3.72)$$

By substituting c_{eq} for c in Equation (3.53), the differential equation of motion for the system with structural damping can be written as given in Equation

$$x\ddot{a}(t) + \left(\frac{\alpha}{\pi \omega} \right) \dot{v}(t) + ku(t) = P_0 \sin(\omega t) \quad (3.73)$$

If it is assumed that the oscillation is harmonic, then the Equation (3.73) can be rewritten as;

$$x\ddot{a}(t) + \left(k + i \frac{\alpha}{\pi} \right) u(t) = P_0 e^{i\omega t} \quad (3.74)$$

By factoring out the stiffness k and letting $\gamma = \alpha/\pi k$, the preceding equation becomes

$$x\ddot{a}(t) + k(1 + i\gamma)u(t) = P_0 \sin(\omega t) \quad (3.75)$$

The quantity $k(1+i\gamma)$ is called *complex stiffness* and γ is the structural damping factor. Using the concept of complex stiffness for problems in structural vibrations is advantageous in that one needs only to multiply the stiffness term in the system by $(1+i\gamma)$. The method is justified, however, only for harmonic oscillations. With the solution $u \equiv \bar{U}e^{i\omega t}$, the steady-state amplitude from Equation (3.75) becomes

$$\bar{U} = \frac{P_0}{(k - m\omega^2) + i\gamma k} \quad (3.76)$$

The amplitude at resonance frequency is then

$$|U| = \frac{P_o}{\gamma k} \quad (3.77)$$

Comparing this with the resonance response of a system with viscous damping as given in Equation (3.59), we can conclude that with equal amplitudes at resonance, the structural damping factor is equal to twice the viscous damping factor.

$$\gamma = 2\zeta \quad (3.78)$$

3.3. Identification of Mode Shape

If free vibration is initiated by appropriate distribution of displacement in the various DOFs in an undamped structure, the response will be the simple harmonic motion without change of deflected shape. In other word, if the structure is displaced in one of its mode shape and released, it will vibrate in simple harmonic motion, maintaining the initial deflected shape. All DOFs on structure reach their extreme displacements at the same time and they also pass through the equilibrium position at the same time as shown in Figure 3.10. This characteristic property of deflected shape of structure is called *the natural mode of vibration* of the structure. A natural period of vibration, T_1 is the time required to complete one cycle of simple harmonic motion in one of natural modes.

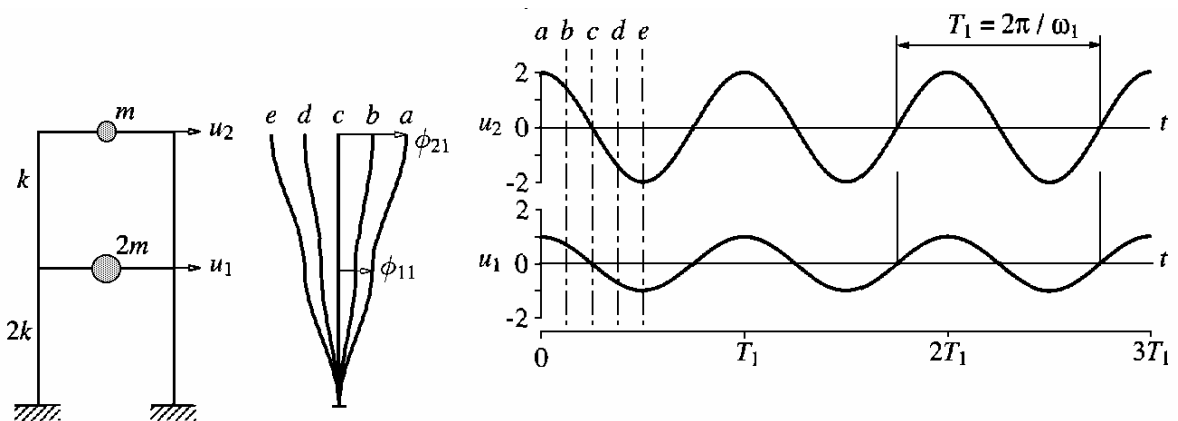


Figure 3.10. Free vibration of an undamped system in its first natural mode of vibration
(Chopra, 2001)

Dynamic motion of a multi degree of freedom (MDOF) system can be defined as the sum of the dynamic response of SDOF systems. The number of the SDOF system must be equal to the degree of freedom of the MDOF system. Such SDOF systems are called the free vibration modes of the structure and they can be calculated by free vibration modal analysis. Predominant frequencies calculated by means of FAS of the measured data are the modal frequencies of the structure (MDOF).

If the damping in the structure is linearly proportional with the mass and the stiffness of the structure, the modal frequencies and the corresponding mode shapes can be calculated as real values. Otherwise, the modal frequency and the mode shape can be defined as complex values.

Two methods are explained in this section for the estimation of real-time mode shape of a vibrating structure. First one is the double integration of the measurements, which is called *the integration method*. Second method is based on the FAS and phase spectrum of measurements, which is called *FAS amplitude method*.

In the integration method, a specified duration of raw data, for instance 1 minute, from a measurement location is used for the mode shape identification. The selected data segment (raw data) is first baseline corrected and detrended. Second the data segment is band-pass filtered around the identified modal frequency using methods explained in section 3.1. In doing so, the phase content of the data should be preserved after band-pass filtering by either using zero-phase filters or by filtering in both forward and reverse directions. The resultant data is a sine wave, which has a frequency equal to the identified modal frequency and has varying amplitude in time. Taking double integration of the band-pass filtered data will yield the displacement history of this measurement location. The amplitudes of the displacement history at the considered measurement location correspond to the amplitude of the mode shape at the identified modal frequency. If this procedure is followed for all the measurement locations in the structure, the displacement history of the structure at all measurement locations is obtained for any given estimated modal frequency. Therefore, mode shapes at any estimated easily modal frequency can be estimated using this method.

In the FAS amplitude method, a specified duration of raw data, for instance 1 minute, from a measurement location is selected for the mode shape identification as in the first method. The selected data segment (raw data) is first baseline corrected and detrended. Second the FAS and phase spectrum of the data segment are calculated. The amplitude of FAS at the identified modal frequency corresponds to the amplitude of the mode shape at this modal frequency for the considered measurement location. The sign of the amplitude of the mode shape is determined using phase spectrum. Therefore if this procedure is followed for all measurement locations in the structure, the amplitudes of the mode shape for all measurement locations are estimated for the identified modal frequency. The above procedure can be repeated for any identified modal frequency.

3.4. Other Identification Techniques

There are some other identification methods that estimate the modal properties of a structure. Even though the classification of these methods is still an arguable matter in the literature, they are simply classified as time domain methods and frequency domain methods in this dissertation. Stochastic Subspace Identification (SSI), Eigen system Realization Algorithm (ERA), and Random Decrement Technique (RDT) are some of the time domain modal identification techniques while the Frequency Domain Decomposition (FDD) method is defined in frequency domain. All of these methods can estimate the modal frequencies, modal damping ratio, and mode shapes all together at the same time. Even though these methods are not coded in the developed software, it is the author's intention that they will be included in the software later by means of real-time algorithm.

3.4.1. Stochastic Subspace Identification

SSI, as presented by Van *et al.* (1996), is just a method. This method defines a stochastic state model based on the output-only measurements. SSI does not involve any nonlinear computation and therefore it is much faster than other methods. A system excited by white noise can be modeled by the following stochastic state space model in Equation (3.79).

$$\begin{aligned} x_{k+1} &= A x_k + w_k, \\ y_k &= C x_k + v_k, \end{aligned} \quad E \left[\begin{pmatrix} w_p \\ v_p \end{pmatrix} \begin{pmatrix} w_q^T & v_q^T \end{pmatrix} \right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{pq} \quad (3.79)$$

where $y_k \in \Re^{l \times 1}$ are the measurements of l outputs at discrete time instant k ($t=k\Delta t$, $k \in N$ with Δt , the sampling interval); $x_k \in \Re^{n \times 1}$ is the state vector. The vector $w_k \in \Re^{n \times 1}$ is the process noise due to disturbances and modeling inaccuracies and models also the white noise input; $v_k \in \Re^{l \times 1}$ is the measurement noise due to sensor inaccuracy. They are both unmeasurable vector signals assumed to be zero mean, white and with covariance matrices given in Equation (3.79), where E is the expected value operator; δ_{pq} is the Kronecker delta; p, q are two arbitrary time instance. $A \in \Re^{n \times n}$ is the state matrix describing the dynamics of the system while $C \in \Re^{l \times n}$ is the output matrix that describes how the internal states are transferred to the outside world.

The output covariance matrix of lag i , Λ_i , and the "next state-output" covariance matrix G are defined in Equation (3.80).

$$\Lambda_i = E[y_{k+1} y_k^T], \quad G = E[x_{k+1} x_k^T], \quad \Lambda_i = C A^{i-1} G \quad (3.80)$$

The relationship between Λ_i and G means that output covariance can be considered as impulse response of the deterministic linear time invariant system. Therefore the classical realization theory of Ho and Kalman, (1996) can be used to solve the Equation (3.80). The output measurements are gathered in a block Hankel Matrix $H \in \Re^{2li \times j}$ with $2i$ block rows and j columns as given in Equation (3.81). For statistical reasons it is assumed that $j \rightarrow \infty$.

$$H = \frac{1}{\sqrt{j}} \begin{pmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \cdots & \cdots & \cdots & \cdots \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{pmatrix} = \begin{pmatrix} Y_{0li-1} \\ Y_{i2i-1} \end{pmatrix} = \begin{pmatrix} Y_p \\ Y_f \end{pmatrix} \quad (3.81)$$

If P_i is the orthogonal projection of the row space of the future outputs into the row space of the past outputs, it can be factorized as the product of the observability matrix and the Kalman filter state sequence as defined in Equation (3.82). This projection enables to hold all the information in the past that is useful to predict the future.

$$P_i = \frac{Y_f}{Y_p} = Y_f Y_p^T (Y_p Y_p^T)^\diamond Y_p = \begin{pmatrix} C \\ CA \\ CA^2 \\ \cdots \\ CA^{i-1} \end{pmatrix} (\hat{x}_i \quad \hat{x}_{i+1} \quad \cdots \quad \hat{x}_{i+j-1}) = O_i \hat{X}_i \quad (3.82)$$

where $(\bullet)^\diamond$ represents the pseudo-inverse of a matrix, O_i is the observability matrix and $\hat{X}_i$ is the Kalman state sequence. Using the Singular Value Decomposition (SVD), the Kalman state sequence can be calculated using only the output data that is $\hat{X}_i = O_i^\diamond P_i$.

A different projection can be defined by shifting the past and future outputs in Hankel matrix one block down as given in Equation (3.83).

$$P_{i-1} = Y_f^- / Y_p^+ = Y_{i+12i-1} / Y_{0i} = O_{i-1} \hat{X}_{i+1} \quad (3.83)$$

Then the system matrices in Equation (3.79) can be determined using the over-determined set of linear equations given in Equation (3.84).

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} \hat{X}_i + \begin{pmatrix} W_i \\ V_i \end{pmatrix} \quad (3.84)$$

where $Y_{i|i}$ is a Hankel matrix with only one block row, W_i , and V_i are the least square residuals. Kalman state sequences and the outputs are known and the residuals $(W_i^T \ V_i^T)$ are uncorrected with $\hat{X}_i$. Therefore the set of equations in Equation (3.84) can be solved for A , C in a least square sense.

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} (\hat{X}_i)^\diamond \quad (3.85)$$

The eigenvalue decomposition of matrix A yields the modal properties of the system.

$$A = \Psi \Lambda \Psi^{-1} \quad (3.86)$$

where $\Lambda = \text{diag}(\lambda_i), i = 1, 2, \dots, n$ is a diagonal matrix containing the discrete time complex eigenvalues and Ψ contains the eigenvectors as columns. The continuous time eigenvalues λ_i^c can be obtained from the discrete ones as given in Equation (3.86). They occur in complex conjugate pairs.

$$\lambda_i^c = \frac{\ln \lambda_i}{\Delta t}, \quad \lambda_i^c, \lambda_i^{c*} = -\zeta_i w_i \pm j w_i \sqrt{1 - \zeta_i^2} \quad (3.87)$$

where ζ_i is the modal damping ratio of the mode i , and w_i is the eigenfrequency of the mode i (rad/sec). The mode shapes at the sensor locations can be obtained by pre-multiplying Ψ with the output matrix C that is $\Phi = C\Psi$.

3.4.2. Random Decrement Technique

The Random Decrement (RDT) technique was first introduced by Cole (1971a – 1971b- 1968 - 1973) at NASA. Cole was looking for "a simple and direct method for

translating the time history into a form meaningful to observer". RDT is basically a method to transform measured ambient time history response (a Stochastic Response), $X(t)$, of linear structure into Random Decrement functions, which are equivalent to free decay of the structure. Free decays contains only information about the structure itself and do not contain information about the random load applied to the structure. Due to its simple formulation in time domain, it is now accepted by many authors that the RDT is a fast method to estimate the free response curve of the structures.

The Random Decrement function of a stationary process, $X(t)$ are defined as conditional mean values as given in Equation (3.88).

$$D_{xx}(\tau) = E[X(t + \tau) | T_{X(t)}] \quad (3.88)$$

where $T_{X(t)}$ is the triggering condition.

The basic idea behind RDT is that, when driven by a stationary input, the dynamic response of a structure at an arbitrary time instant $t_o + t$ is composed of three parts, namely;

1. Deterministic part of step response due to initial displacement at time t_o .
2. Deterministic part of impulse response due to initial velocity at time t_o .
3. A random part, which is due to random excitation applied to the structure in the interval time t_o to $t_o + t$.

Let us assume that a certain length of time segment of random response is picked out whenever it reaches an initial displacement, $x(t)=a$, usually called triggering point as seen in Figure 3.11. If N_r numbers of these time segments are averaged, the result is the estimated Random Decrement signature. Therefore, the RD functions are estimated as the empirical mean values as given in Equation (3.89). When the number of averaged time segments increases, the random part due to random excitation decreases and eventually cancels out itself. Since the velocity at each triggering point is a random variable, the resulting initial velocity after averaging will be very close to zero. Therefore, the result is a free decay response due to initial displacement, $x(t)=a$.

$$\hat{D}_{XX}(\tau) = \frac{1}{N_r} \sum_{i=1}^N x(t_i + \tau) | x(t_i) = a \quad (3.89)$$

where the stochastic process, $X(t)$ is assumed to be the ergodic, $\hat{D}_{XX}$ is estimated Random Decrement (RD) function, τ is the time variable in $\hat{D}_{XX}$ and N_r is the total number of triggering point. As seen in Equation (3.89) it is very easy to program the estimated RD function in MatLAB. Unlike FFT algorithm, the algorithm contains only additions with respect to a triggering condition therefore the algorithm is very fast when compared to other estimation techniques.

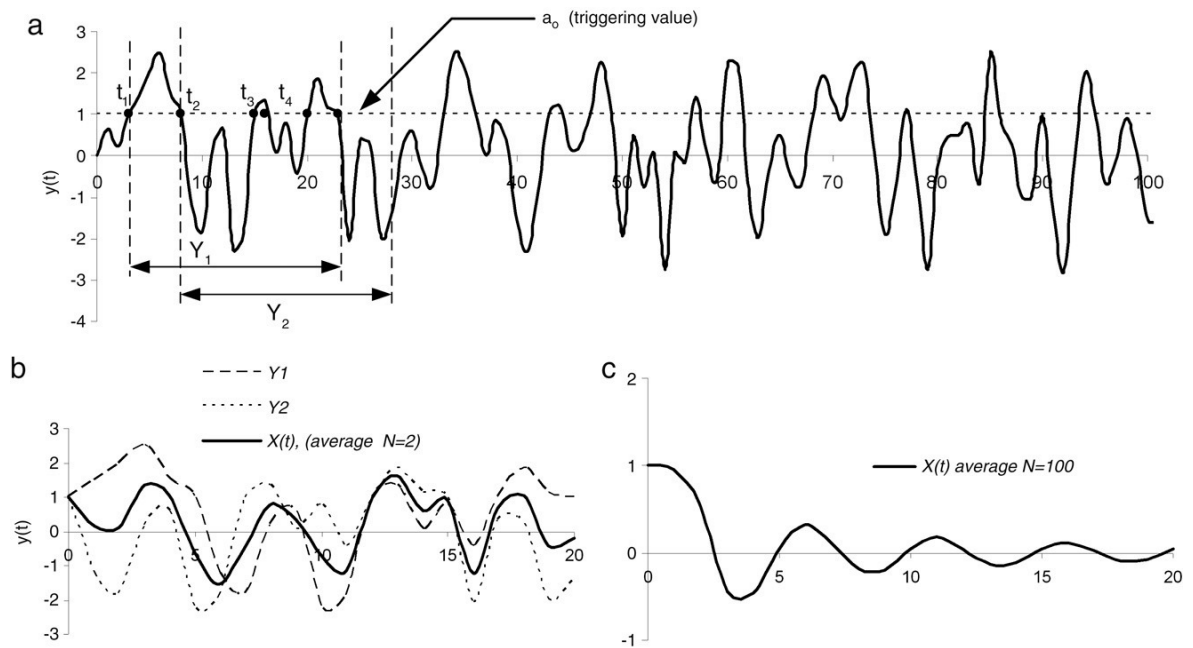


Figure 3.11. Schematic view of random decrement technique

There are two main issues in real application of RDT. First one is the selection of the reference station from which the triggering points obtained. Second one is the determination of the triggering level. Incorrect selection this parameters may result in high signal-to-noise ratio in the estimated RD function at the end of the decay that causes the difficulty of estimating modal properties though RD function (Free response of structure). On the other hand, since the measurements, x_i , are discrete in time, fulfilling the triggering condition $X(t)=a$ is almost impossible. Therefore, four different triggering conditions were introduced to overcome this problem by Asmussen (1997); a) Level Crossing Triggering

Condition, b) Local Extremum Triggering Condition, c) Positive Point Triggering Condition, d) Zero Crossing Triggering Condition. In application of the ambient testing, or in case of real-time ambient data processing, the positive point triggering condition is usually applied.

Ibrahim (1977a) has introduced the auto and cross-RD functions using a reference station among multichannel network, and then the mode shapes are first derived using the RDT. Vandiver (1982) has proved that the conditional expectation of RD functions is proportional to the auto and –cross correlation function of the random response provided that they are stationary and Gaussian distributed. Bedewi (1986) has showed the relation between the conditional expectation of RD functions and auto and cross-correlation functions together with triggering level and the standard deviation of the random response.

Once the free decay (RD function) of a structure is estimated, the modal parameters of the structure can be extracted by using well-known methods like Polyreference Time Domain technique, Ibrahim Time Domain Technique, and Auto-Regressive Vector models, etc. These methods are developed to extract the modal parameters from the free response of linear lumped mass parameter systems.

Ibrahim (1977b) developed a technique (Ibrahim Time Domain Technique, ITD) that estimates the modal properties of a lumped mass linear systems through the free response curve (RD function) using the eigenvalue decomposition.

3.4.3. Frequency Domain Decomposition

The Frequency Domain Decomposition (FDD) was first introduced by Brincker *et al.* (2000) as technique for spectral decomposition based on SVD. The modal coordinates are identified by sorting the set of singular values according to singular vectors. Once a modal peak has been identified, the first singular value at this peak was chosen as the mode shape estimate.

Zhang *et al.* (2005) introduced another way of estimating the modal coordinate instead of sorting existing singular values, the single-degree-of- freedom (SDOF) system

response was isolated by using a projection technique, called spatial decomposition. This provided better estimates of the SDOF system and thus made the subsequent modal identification easier.

The FDD technique is based on the relationship between input and output PSD for stochastic process.

$$G_{yy}(w) = H(w)G_{xx}(w)H(w)^H$$

$$H(w) = \sum_{m=1}^M \left(\frac{R_m}{iw - \lambda_m} + \frac{R_m^*}{iw - \lambda_m^*} \right) \quad (3.90)$$

where $G_{xx}(w)$, $G_{yy}(w)$ are input and output PSD matrices, respectively, $H(w)$ is the Frequency Response Function (FRF) matrix, superscript H denotes the complex conjugate and transpose. FRF can be express as function of poles, λ_m and residuals, R_m in a partial fraction form where $\lambda_m = -\sigma_m + iw_m$ is the m^{th} pole, and $R_m = \varphi_m \gamma_m^T$ is the corresponding residue matrix, $\varphi_m = [\varphi_{1m} \ \varphi_{2m} \ \dots \ \varphi_{Nm}]^H$, $\gamma_m = [\gamma_{1m} \ \gamma_{2m} \ \dots \ \gamma_{Rm}]^H$ are the m^{th} mode shape and modal participation factor vector, respectively. Assuming that the excitation is white noise (has flat spectrum), then its PSD is a constant matrix, $G_{xx}(w) = G_{xx}$.

$$G_{yy}(w) = \sum_{m=1}^M \sum_{s=1}^M \left(\frac{R_m}{iw - \lambda_m} + \frac{R_m^*}{iw - \lambda_m^*} \right) G_{xx} \left(\frac{R_s}{iw - \lambda_s} + \frac{R_s^*}{iw - \lambda_s^*} \right)^H \quad (3.91)$$

Multiplying the two partial fraction factors and making use of the Heaviside partial theorem, the output PSD can be reduced to a pole-residue form as seen in Equation (3.92).

$$G_{yy}(w) = \sum_{m=1}^M \left(\frac{A_m}{iw - \lambda_m} + \frac{A_m^H}{-iw - \lambda_m^*} + \frac{A_m^*}{iw - \lambda_m^*} + \frac{A_m^T}{-iw - \lambda_m} \right)$$

$$A_m = R_m G_{xx} \sum_{s=1}^M \left(\frac{R_s^H}{-\lambda_m - \lambda_s^*} + \frac{R_s^T}{-\lambda_m - \lambda_s} \right) \quad (3.92)$$

where A_m is the residue matrix of the output PSD corresponding to the m^{th} pole λ_m when all the measurement points are taken as reference point. Taking Equation (3.92) into account the output PSD can be modally decomposed as given in Equation;

$$G_{yy}(w) = \sum_{m=1}^M \left(\frac{\varphi_m \chi_m^T}{iw - \lambda_m} + \frac{\varphi_m^* \chi_m^H}{iw - \lambda_m^*} + \frac{\chi_m \varphi_m^T}{-iw - \lambda_m} + \frac{\chi_m^* \varphi_m^H}{-iw - \lambda_m^*} \right) \quad (3.93)$$

where φ_m and χ_m are the m^{th} mode shape and operational reference vector, respectively. The reference vector is a function of the modal parameters and the PSD matrix of the unknown random input forces. The time domain representation of the PSD is the correlation function matrix given in Equation (3.94).

$$C_{yy}(\tau) = \sum_{m=1}^M \left(A_m e^{\lambda_m \tau} + A_m^* e^{\lambda_m^* \tau} - A_m^T e^{-\lambda_m |\tau|} - A_m^H e^{-\lambda_m^* |\tau|} \right) \quad (3.94)$$

where positive poles corresponds to positive time lags (first two term), and the negative poles corresponds to negative time lags (last two terms). Since the PSD is estimated from a limited amount of data, the spectral density matrix is typically characterized by high noise levels compared to FRF's. The noise problem can be overcome by using time domain identification algorithm starting from the first N_w positive lags of the correlation function. The Fourier transform of the corresponding correlation function yields the PSD of output matrix as given in Equation (3.95).

$$\begin{aligned} C_{yy}(\tau) &= \sum_{m=1}^M \left(A_m e^{\lambda_m \tau} + A_m^* e^{\lambda_m^* \tau} \right) \\ G_{yy}(w) &= \sum_{m=1}^M \left(\frac{\varphi_m \chi_m^T}{iw - \lambda_m} + \frac{\varphi_m^* \chi_m^H}{iw - \lambda_m^*} \right) \end{aligned} \quad (3.95)$$

The output PSD matrix can be re-written in matrix form as given in Equation (3.96).

$$G_{yy}(w) = \Phi [iwI - \Lambda]^{-1} X^T + \Phi^* [iwI - \Lambda^*]^{-1} X^H = \Phi_{2M} [iwI - \Lambda_{2M}]^{-1} X_{2M} \quad (3.96)$$

The decomposition of the spectral matrix can also be seen as results of uncorrelated complex modal coordinates as seen in Equation (3.97).

$$y(t) = \tilde{\Phi} \tilde{q}(t) \quad \tilde{\Phi} = [\Phi \quad \Phi^*] \quad \tilde{q}(t) = \begin{bmatrix} q(t) \\ \tilde{q}(t)^* \end{bmatrix} \quad (3.97)$$

The covariance matrix of the response will be obtained as given in Equation (3.98).

$$C_{yy}(\tau) = E\{y(t+\tau) y(t)^H\} \quad (3.98)$$

$$C_{yy}(\tau) = E\{\tilde{\Phi} \tilde{q}(t+\tau) \tilde{q}(t)^H \tilde{\Phi}^H\} = \tilde{\Phi} E\{\tilde{q}(t+\tau) \tilde{q}(t)^H\} \tilde{\Phi}^H = \tilde{\Phi} \tilde{C}_{qq}(\tau) \tilde{\Phi}^H$$

where $E\{\bullet\}$ denotes the expectation operator, $\tilde{C}_{qq}(\tau)$ is the covariance matrix of the extended modal coordinates. If we assume that the modal coordinates are uncorrelated, and then all off-diagonal elements are zero in $\tilde{C}_{qq}(\tau)$. Therefore, $\tilde{C}_{qq}(\tau)$ is diagonal matrix $\tilde{C}_{qq} = [c_m]$, which contains two identical contributions from each mode, $c_m = c_{M+m} = C_m(\tau)$, $m = 1 \dots M$, while $C_m(\tau)$ is the auto covariance function of the modal coordinate of mode m . Fourier transform of the covariance matrix of the response will yield the response PDS matrix in frequency domain as given in Equation (3.99).

$$G_{yy}(w) = \tilde{\Phi} \tilde{G}_{qq}(w) \tilde{\Phi}^H \quad (3.99)$$

where $G_{yy}(w)$ is the power spectral density matrix of the extended modal coordinates, which is diagonal with positive real components. The response power spectral matrix decomposes into modal contributions as given in Equation (3.100), which shows that the influence of mode shape complexity vanishes on the response PSD matrix.

$$G_{yy}(w) = G_1(w) (\varphi_1 \varphi_1^H + (\varphi_1 \varphi_1^H)^*) + G_2(w) (\varphi_2 \varphi_2^H + (\varphi_2 \varphi_2^H)^*) + \dots \quad (3.100)$$

In the FDD identification, the first step is to estimate the PSD matrix. The estimate of the output PSD, $G_{yy}(w)$ is then decomposed by taking the SVD of the matrix as shown in

Equation (3.101). All spectral density matrices describes energy, thus they are not only Hermitian, but also positive definite. Therefore, for a SVD decomposition of a square PSD matrix at a certain frequency line given in Equation (3.101) always holds true for SVD. If all eigenvalues are positive, then G_{yy} is positive definite, and $U=V$.

$$\begin{aligned} G_{yy} &= U S V^H \\ G_{yy} &= U S U^H \end{aligned} \quad (3.101)$$

The key of the FDD technique is to conduct a factorization of the output PSD estimated at discrete frequencies $w=w_k$, therefore eigenvalues and eigenvectors are a function of frequency as given in Equation (3.102).

$$\begin{aligned} S(w_k) &= \text{diag}(s_{k1}, s_{k1}, \dots, s_{kR}) \\ V_k &= V(w_k) = [v_{k1}, v_{k2}, \dots, v_{kR}] \\ U_k &= U(w_k) = [u_{k1}, u_{k2}, \dots, u_{kM}] \end{aligned} \quad (3.102)$$

When the discrete frequency considered approaches to a certain dominating modal frequency w_{md} , the rank of the PSD matrix yields one because only the first eigenvector will contribute to the spectral matrix as given in Equation (3.103). This contribution is an estimate of the contributions from the dominant mode. The first eigenvalue that reaches maximum at the natural frequency $w_k=w_{dm}$ describes the modal coordinate and the corresponding eigenvector $u_1(w_{md})$ is an estimate of the m^{th} modeshape, therefore $\hat{\phi}_m \approx u_1(w_{md})$ with unitary normalization.

$$\Delta G_{yy}(w_k) \approx s_1(w_k) u_1(w_k) v_1(w_k)^H \quad (3.103)$$

4. REAL TIME DATA PROCESSING ALGORITHMS

It is known that the modal properties of a structure may change with varying environmental conditions. To measure the change in these modal properties, the response of the structure is monitored and analyzed in real-time. The real-time data processing algorithm enables to analyze the response of the structure in real-time to identify the modal properties (modal frequency, damping ratio and mode shape) under different ambient loads. Quite often, the initial finite element model of a structure may be found to be incorrect by as much as 10% in the lowest frequencies. These inaccuracies are due to a combination of factors, approximations in the finite element derivations, mismodeling of structural elements, differences of actual material properties and dimensions from those assumed in the model, and lack of convergence of numerical model. Therefore, the accurate calculation of modal frequencies becomes very important by means of experimental data. In this point, the real-time data processing algorithms are very important because they enable to monitor and detect the changes in the modal properties of a structure with respect to different environmental conditions using experimental data (measured real-time data from structure). The accurate estimation of the dynamic modal properties of a structure plays an important role in a real-time modal identification. Therefore, the real-time data processing algorithms for modal identification are an important tool for SHM.

Time synchronization of sensors installed in the structure is another important issue in SHM. It is important when estimating the dynamic properties of the structure in real-time. Hence, it is also important in determining and locating the damage in the structure in SHM. In case of non-synchronization, the real-time variation of modal properties of the structure may not be detected accurately that may result in that the potential damage in the structure may not be detected and located accurately in real-time. Time synchronization becomes most important when estimating the real-time mode shapes of the structure. All sensors in the structure should be time-synchronized and should have a GPS connection because the displacement history of the measurements stations at a given modal frequency corresponds to the amplitudes of the mode shape. Therefore, time synchronization enables the amplitudes of the mode shape to be correct in real-time.

It is obvious that a real-time modal identification program should be used in order to monitor the changes in modal properties of a structure. To do this, a new real-time modal identification software, called ***KOERI-MIDS (KOERI-Modal Identification Software)***, is developed in this dissertation. The developed software basically calculates the dynamic modal properties of the structure in real-time based on the structure's response under various ambient loads. It also enables to monitor the real-time change of modal properties of the structure. The developed program uses the real-time algorithms explained in this chapter. More details about this software can be found in Chapter 5.

Structures experience different level of loadings during their service life such as ambient loads, wind loads, live loads, earthquake loads, blast loads, and temperature loads, etc. In order to analyze the real-time response of the structure under these loads, the developed software has mainly two different real-time analysis techniques working simultaneously. First analysis technique is used for analyzing the response of the structure under ambient vibrations whereas the second analysis technique is used for analyzing the response of the structure for the remaining external loads. The first analysis technique enables to monitor the variation of the real-time modal properties by means of truncated data segment. This requires that the structure should remain elastic within the truncated window duration as in the case of ambient vibration. The second analysis technique enables to monitor the variation of the real-time modal properties within short window duration. In case of earthquake, blast and even wind loads, the structure may not remain elastic; therefore the sudden change of real-time modal properties can be detected by short windows. More details about these analysis techniques are given in the following sections.

4.1. Real-Time Data Analysis Techniques

4.1.1. Analysis of Ambient Vibration

In practical applications the response of a structure is measured continuously in real-time. Even though the measured data sequence in SHM is infinitely long in duration it should be, somehow, processed in real-time to calculate the real-time modal properties of the structure. Such a long duration of data cannot be analyzed one at a time due to the limited memory of digital computers. In such cases, the structural vibration data must be

truncated into equal size of data segments in order to analyze it in real-time as seen in Figure 4.1. It is also known that the ambient vibration data is usually contaminated with high amplitude noise due to environmental conditions and mechanical imperfections in the instruments. To overcome this problem, the truncated data can be segmented into fixed-size of running windows and each window is processed via the data processing tools defined in Chapter 2.1. Since all the data processing tools defined in Chapter 2.1 are linear, successive running windows can be analyzed one at a time and the output of each running window are averaged to form the overall output for the truncated data segment. The running window moves through the truncated data segment by predefined overlap ratio. The effect of noise in the truncated data segment is reduced by averaging the outputs of running windows. Since the nature of the noise is random, it should appear randomly in the outputs of each running window. Therefore, it is obvious that averaging the outputs of each running window will eventually decrease the effect of the noise in the overall output of truncated data segment. Increasing the overlap ratio also increases the number of running windows in the truncated data segment. Therefore, if the number of running window is increased, the effect of noise in the overall output of the truncated data segment will decrease.

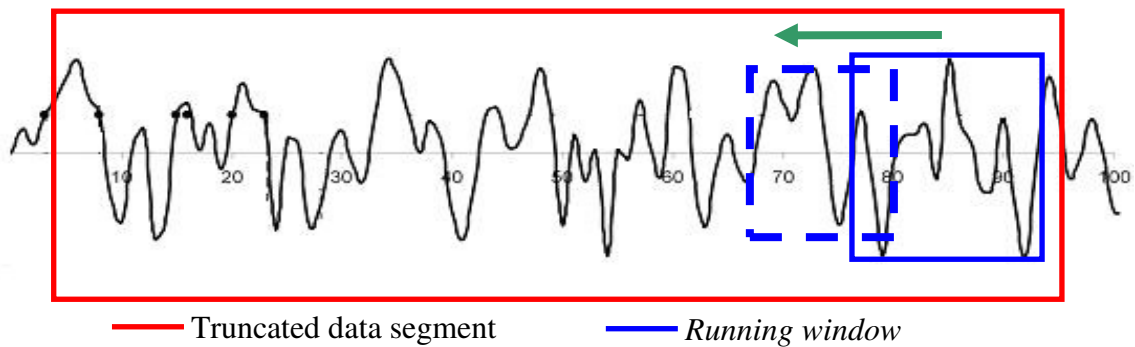


Figure 4.1. Schematic view for truncated data segment and the running window

It is assumed that the structure remains elastic within the length of truncated data segment. Even though this assumption does not hold true for earthquake induced vibrations, it is acceptable for most of the structures under ambient loads. Therefore, averaging the outputs of running windows will result in an overall output for the truncated data segment that is almost identical to the output obtained if the truncated data had been processed one at a time under ambient vibrations.

The length of running window depends on one parameter. It is the smallest modal frequency of the structure in consideration. The length of running window should be at least 10 times great than the maximum modal period of the structure. The length of truncated data segment depends on the number of running windows. By keeping the length of truncated data segment constant while increasing the number of running window will decrease the effect of noise in the truncated data segment but it also decrease the length of running window. In order to get better results from the modal identification of a structure, the number of running windows should be increased in the truncated data segment. On the other hand, increasing the length of truncated data segment increases the number of running windows but it will also increase the calculation time. Too long truncated data segment may not provide sufficient information about the real-time changes in the modal properties of the structure due to sudden change of ambient loads whereas too short truncation data segment may not provide sufficient noise reduction due to decrease in the number of running windows. There is no unique solution to determine the optimum length for the truncated data segment for a specific structure. It is suggested that several set of experimental data is collected and analyzed with varying lengths of truncated data segments. The length of truncated data segment and running window should be determined based on the results of this analysis.

To form the overall output for a truncated data segment requires a calculation time. It depends on the length of the truncated data segment and the number of running windows. Total calculation time is the duration to form the overall outputs for the truncated data segments for all measurement stations and for all directions. Once the outputs for all truncated data segments are calculated in real-time, all truncated data segments are shifted by predefined streaming time and the above defined analysis technique (procedure) is repeated. The streaming time must be bigger than the total calculation time. Otherwise, the streaming time lags the calculation time, therefore the calculated modal properties of the structure lag in time.

4.1.2. Analysis of Other Vibrations

In an ambient vibration analysis, the response of the structure is divided into equal size of truncated data segments. Then later these data blocks are further divided into

running data segments and analyzed separately. The overall output of the truncated data segment is calculated based on the averaging of the results of running data segments. Application of such analysis technique requires that the structure remains elastic during the truncated data segment.

However this is not always the case. Structures may experience different level of loadings during their service life such as wind loads, earthquake loads, blast loads, and temperature loads, etc. Some of the structures may not remain elastic under these loads and residual damages in the structure can be observed. This may results in that the real-time dynamic properties of the structure may change immediately in a short time. Therefore, such a sudden change of modal properties of the structure cannot be caught by ambient vibration analysis technique that is based on the averaging of the results of the running windows. The sudden change of modal property in a short time may lose its contribution in averaging process. Therefore, a short duration window should be used in order to catch the sudden variations of the modal properties. The length of the short window should be at least ten times greater than the maximum modal period of the structure. To do so, the definitions of the truncated data segments and the running windows in the ambient vibration analysis will be used for the analysis of other vibrations. The first running window inside the truncated data segment as seen in Figure 4.1 corresponds to the short window length. The analysis of the first running window will result in the real-time modal properties of the structure.

4.2. Real-Time Modal Frequency Identification Algorithms

As mentioned before, the real-time measurements are infinitely long data sequences and they cannot be processed one at a time. Therefore, they should be truncated into equal length of data segments in order to analyze them and identify the modal frequencies of the structure. In order to minimize the effects of noise, the truncated data segment is divided into equal size of running windows. Each running window is analyzed separately and the outputs of all running windows are averaged to form the overall output of the truncated data segment. The length of the truncation and the running window size depends on the structure's modal characteristics and the noise contamination in the measurements.

In order to identify the real-time modal frequencies of a structure, four different real-time algorithms are defined in this section based on the modal frequency identification techniques explained in Chapter 3.1. They are namely;

- 1) SFAS (Smoothed Fourier Amplitude Spectrum)
- 2) BURG
- 3) Yule-Walker
- 4) MUSIC

The flow chart for the Smoothed Fourier Amplitude Spectrum (SFAS) algorithm is given in Figure 4.2. The procedure starts with segmenting the measurements into equal size of truncated data segments. In the following step, the truncated data segment is analyzed to calculate the modal frequencies of the structure. To do so, it is divided into equal size of running windows as seen in Figure 4.1. The length of the truncated data segment and the running window is determined according to Chapter 4.1.1.

The analysis of each running window starts with the base line correction to remove the static component then the possible trend is removed from the running window. Next step is the filtering. Standard butter-worth band-pass filter is applied with corner frequencies of f_1 and f_2 . The lower corner frequency should be less than the minimum modal frequency of the structure whereas the upper corner frequency should be greater than the maximum modal frequency of the structure in consideration. Following step is the windowing process. Three different windowing functions (hamming, hanning, and triangular) can be used for windowing to reduce the spectral leakage. The length of the windowing function is equal to the length of running window. The last step is the calculation of FAS of the running window and smoothing the FAS. Three different smoothing windows (hamming, hanning, and triangular) can be used for smoothing the FAS with varying smoothing window lengths of n_w . The optimum smoothing window length should be calculated in advance and it will be kept constant for all running windows in the truncated data segment. The smoothed FAS is stored in the memory for each running window. Smoothed FAS of the truncated data segment is calculated by averaging the smoothed FASs in memory. The resultant smoothed FAS (SFAS) can be used for the real-time modal frequency identification.

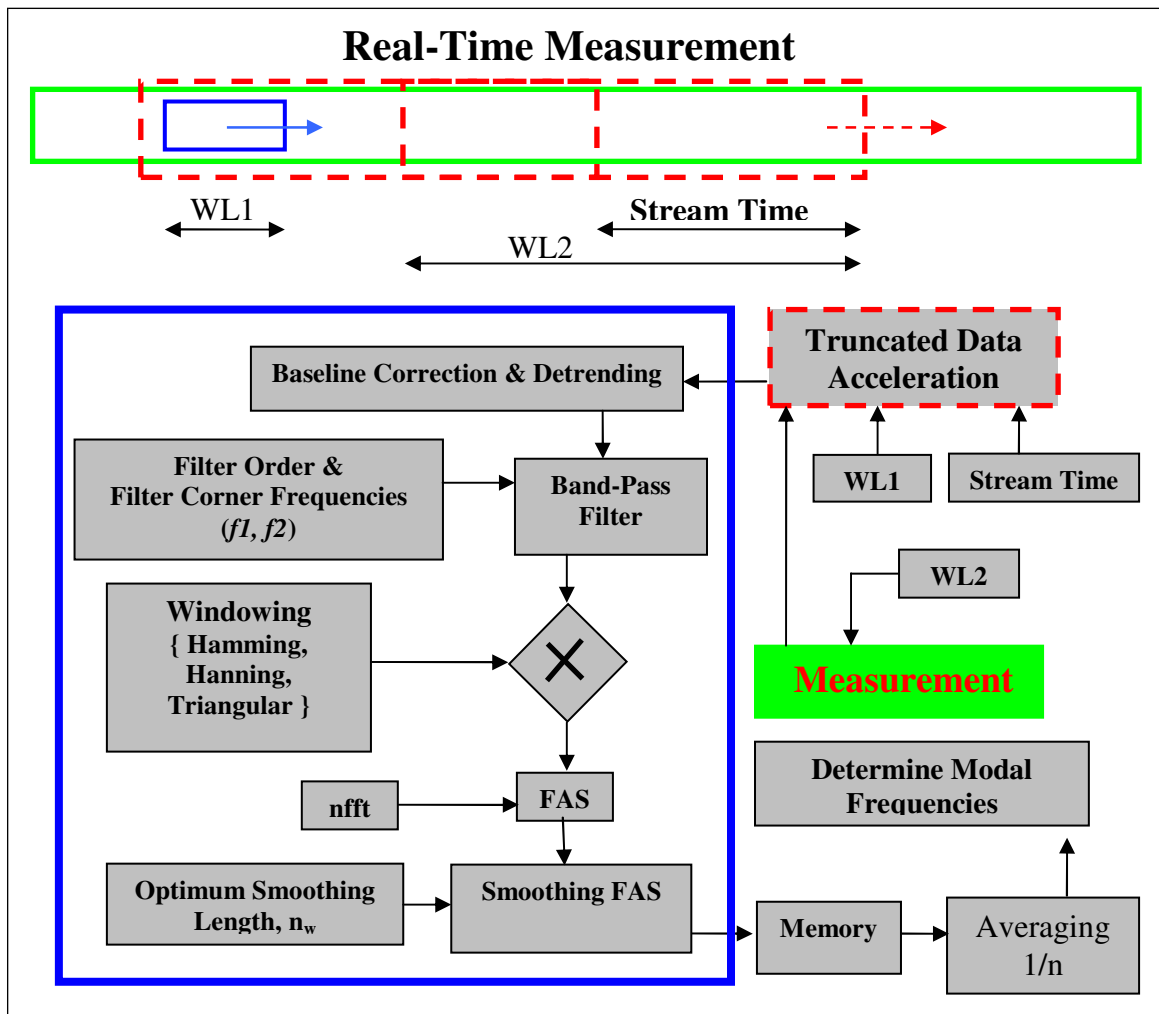


Figure 4.2. Flow chart of real-time algorithm of smoothed FAS for the determination of modal frequency

The flow chart for BURG, Yule-Walker and MUSIC algorithms are given in Figure 4.3. The analysis procedure is very similar to the SFAS algorithm. The truncated data segment is divided into equal length of running windows that is analyzed separately and averaged to form the overall output for truncated data segment. The analysis of each running window starts with baseline correction and detrending to remove the static component and possible trend exist in each running window. Next step is the band-pass filtering of the running window with butter-worth filter. In the following step, the power spectrum of the running window is estimated by one of the estimation techniques, BURG, Yule-Walker and MUSIC as explained in detail in Chapter 3.1.2. All the calculated power spectrums are stored in memory. The Power spectrum for the truncated data segment is calculated by averaging the power spectrums in memory. The resulting power spectrum

can be used to identify the modal frequencies of the structure considered. Peaks of power spectrum correspond to the modal frequencies of the structure.

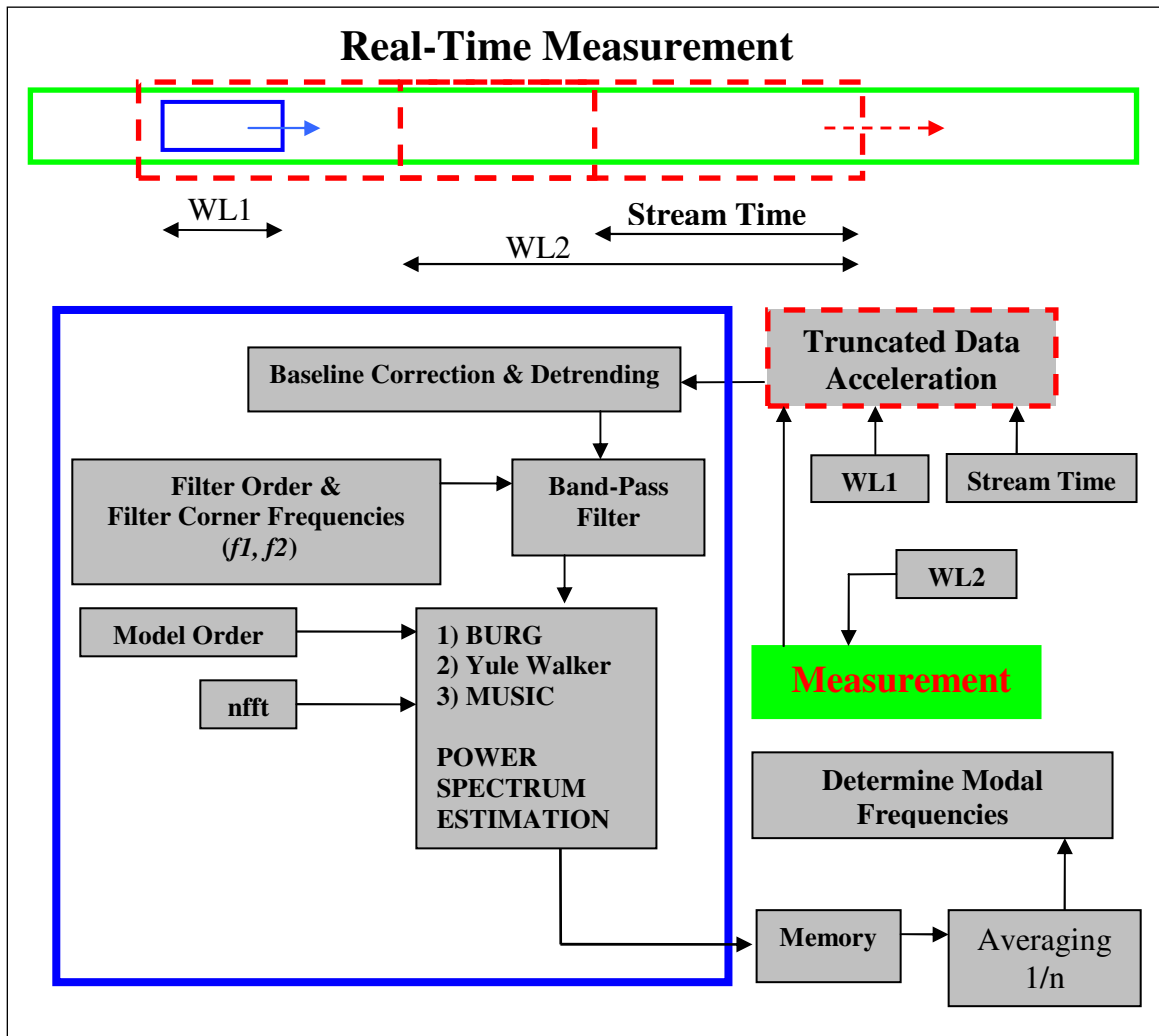


Figure 4.3. Flow chart of real-time algorithms of BURG, Yule-Walker and MUSIC for the determination of modal frequency

Peaks of SFAS usually correspond to the modal frequencies of the structure. If there is a clear peak in SFAS in a single frequency for all measurement stations, it is easy to identify it as a modal frequency. However this is not always the case. In real-time data processing, it is common to observe that the peaks of SFAS do not coincide in a single frequency but in a frequency range (frequency band). If there is a clear peak in SFAS of all measurement stations within the specified frequency band, it can be said that there is a modal frequency within this frequency band. Therefore, a series of frequency bands are

defined in advance for the identification of real-time modal frequencies. All the real-time modal frequencies are searched within this frequency bands defined in advance. No modal frequency is searched outside of these frequency bands. Therefore, the determination of frequency bands is an important part of the real-time modal frequency identification algorithm. Some pre-knowledge about the dynamic properties of the structure will help to locate the frequency bands accurately. More details about specifying the frequency bands are given in Chapter 5.4.

Once the modal frequencies of the structure are determined for the truncated data segment considered, it is shifted by predefined stream time. Then the above defined procedure is repeated to identify the modal frequencies of the structure for the shifted truncated data segment. Therefore, the real-time modal frequencies of the structure are calculated by means of truncated data segments, running windows and averaging.

4.3. Real-Time Damping Identification Algorithms

As mentioned before due to the limited memory of digital computers, the real-time measured data is truncated into equal length of data segments. The real-time damping ratio is calculated for each identified modal frequency and for each truncated data segments. The modal frequencies must be identified first for each truncated data segment before the real-time modal damping estimation. In order to identify the modal damping ratio, two different real-time algorithms are explained in this section, half-power bandwidth and logarithmic decrement.

Flow chart for half-power bandwidth algorithm for the identification of modal damping ratio is given in Figure 4.4. The calculation of the real-time modal damping ratio depends on the FAS of the truncated data segment. Therefore, the algorithm should start with calculating the FAS of truncated data segment considered. To do this the truncated data segment is divided into equal length of running windows and each window is analyzed separately to calculate the FAS. In doing so, the each running window is baseline corrected and detrended. Then they are band-pass filtered and it is multiplied with a windowing function (hamming, hanning, and triangular) to reduce the spectral leakage. The length of the windowing function is equal to the length of running window. In the next

step the FAS of each running window is calculated and stored in the memory. The real-time damping estimation algorithm up to this point is same as the one that is used for the real-time modal frequency identification algorithm as explained in Chapter 4.2. The average of FAS in memory will yield the FAS of truncated data segment considered. Using the half-power bandwidth technique, the damping ratio for the truncated data segment can be calculated for each identified modal frequency. In doing so, first the half-power points are determined based on the structure's identified modal frequency considered. Then the modal damping ratio is calculated based on the half-power points. If this procedure is followed for other identified modal frequencies, the modal damping ratio for each identified modal frequencies can be calculated for the truncated data considered.

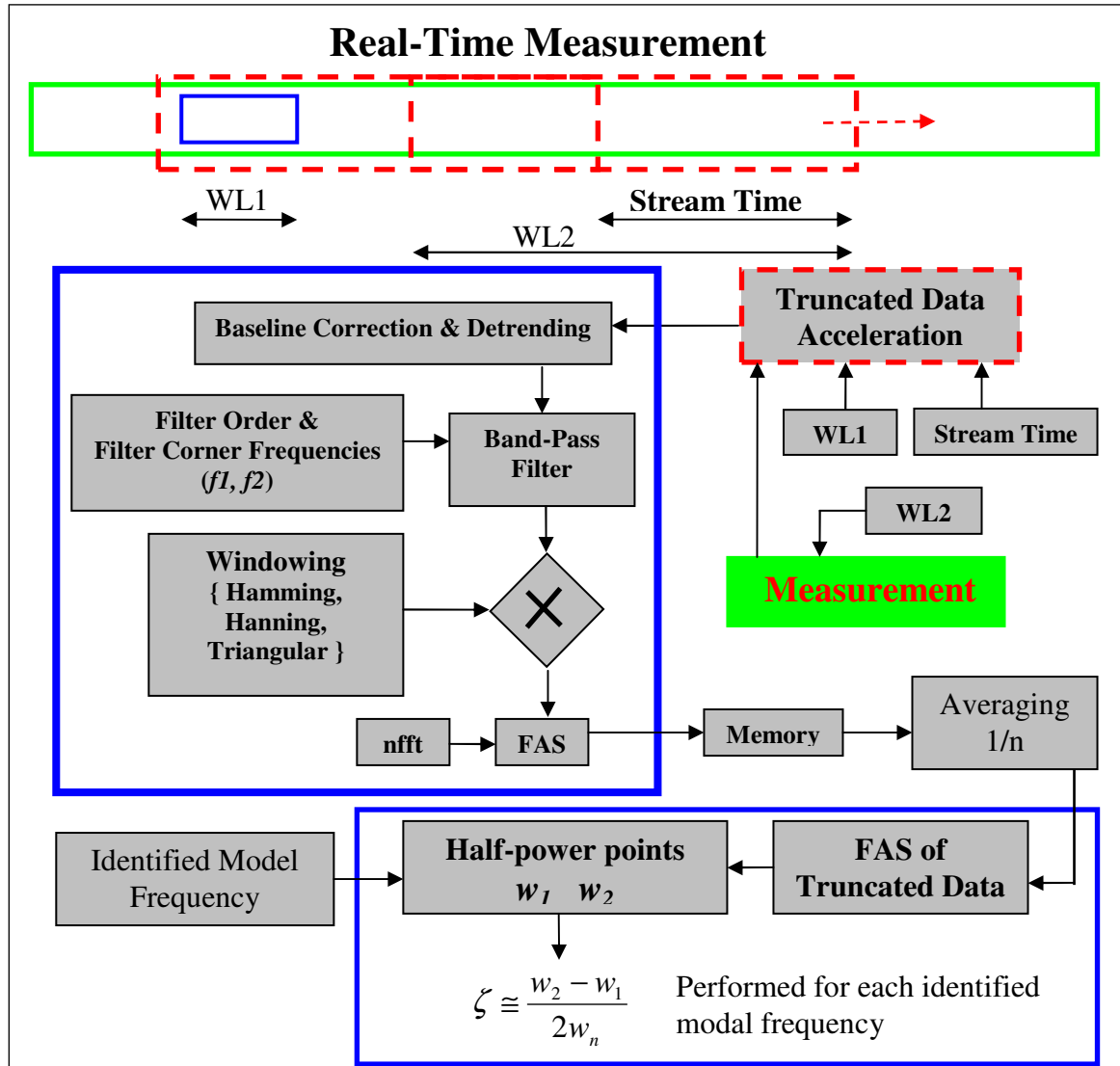


Figure 4.4. Flow chart of real-time algorithm of half-power bandwidth for the determination of modal damping ratio

Once the modal damping ratios of the structure are determined for the truncated data segment considered, the truncated data segment is shifted by predefined stream time. The above defined procedure is repeated to identify the modal damping ratio of the structure for the shifted truncated data segment. Therefore, the real-time damping ratio can be analyzed with varying ambient loads (e.g., wind, traffic load, and live load) by means of half-power bandwidth technique.

4.4. Real-Time Mode Shape Identification Algorithm

The flow chart for real-time mode shape identification is given in Figure 4.5. As mentioned before, the real-time measurements are truncated into equal size of data segments so that they can be analyzed in real-time. The real-time mode shape is calculated for each identified modal frequency and for each truncated data segments. The calculation of real-time mode shape requires the identification of the real-time modal frequencies. Therefore, the real-time modal frequencies must be calculated before the mode shape identification procedure starts. To do that the entire truncated data segment is first baseline corrected and detrended. In the next step the truncated data segment is band-pass filtered around the identified modal frequency considered. The filter corner frequencies should be located around the identified modal frequency considered. The distance between the filter corner frequencies and the structure's modal frequency should be at least three times the smallest frequency that can be observed in the measured data segment. Double numeric integration of the filtered truncated data segment with respect to time will yield the displacement time history of the truncated data segment for the modal frequency considered. The resulting displacement history corresponds to the amplitude of the mode shape at the modal frequency considered in the band-pass filtering. If this procedure is repeated for all measurement stations in the structure, the displacement history of the structure at all measurement locations is obtained for the modal frequency considered. This corresponds to the real-time mode shape history of the structure at the identified modal frequency considered.

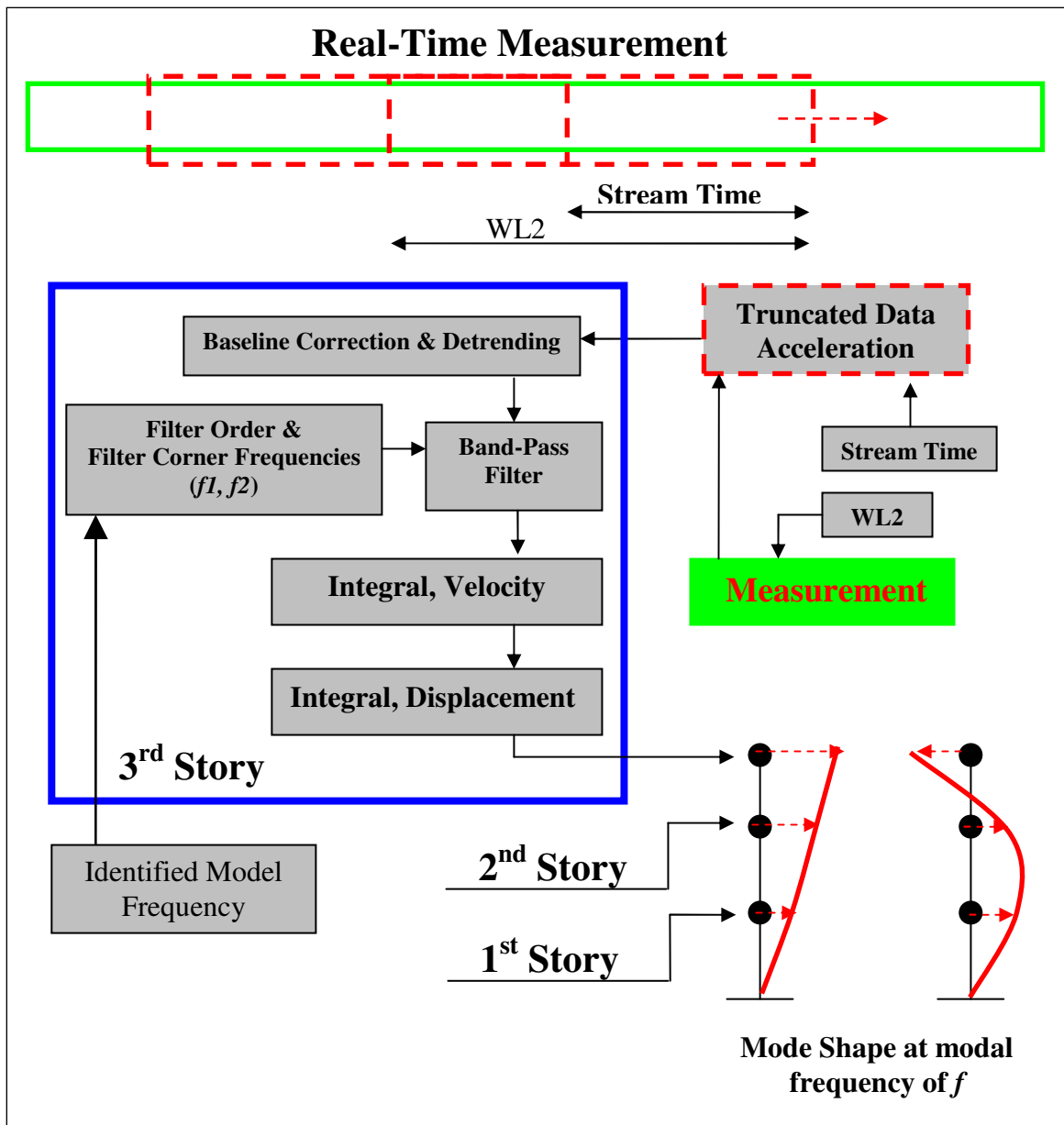


Figure 4.5. Flow chart of real-time algorithm for the determination of mode shapes

5. DESCRIPTION OF KOERI-MIDS

Structural health monitoring (SHM) is a growing field of engineering that improves our knowledge of the safety and maintainability of civil structures. SHM combines various physical sensing and measurement techniques with continuous, remote processing to capture, log, and continuously analyze real-time data. The size and complexity of the structures being monitored require engineers to be well versed in disciplines ranging from sensing techniques, to multisystem synchronization, to structural dynamics, to data management and analysis, and more.

Visual inspections and evaluation of civil structures are usually difficult and very expensive. In addition to that they are usually done by means of site inspections, which sometimes cannot provide enough information for the maintenance due to the some undetectable structural damage that can be hidden in the structure. Accumulation of such hidden structural damage may result in serious problems in the structure and even may result in local or global structural failures. In recent years many catastrophic structural failures have occurred around the globe resulting in the need for structural health monitoring. Engineers in both academia and industry have recognized the importance of implementing damage detection and monitoring the health state of structures, such as bridges, buildings, dams, tunnels and even pipelines. In order to overcome such catastrophic structural failures, many civil engineering structures have been instrumented and have being monitored for the last decay. On the other hand, there is a tendency that instrumentation and monitoring of structures will be compulsory with the new design codes. Therefore, there will be a clear need for establishing SHM systems not only for new structures but also for existing structures. The main purpose of monitoring is not only to build more intelligent civil infrastructures but also to control the health state of structures in real-time. Through the real-time monitoring and the real-time data analysis techniques that have been developed for the last decay, the engineers are now almost capable of reacting the potential failures in the structure before they even occurred.

As a result it is obvious that the availability of a real-time modal identification programs is crucial for on-line control and monitoring of structures under ambient loads. Therefore the main intension of this dissertation is to develop real-time modal identification software to be used in SHM. The software developed in this dissertation is for real-time modal identification, which allows constructing a mathematical model of a dynamic system that is based on the measured response of the structure. The name of the software is *KOERI-MIDS* (Kandilli Observatory and Earthquake Research Institute - Modal Identification Software). All the tools and techniques developed in *KOERI-MIDS* are coded in MATLAB. Many modal identification and signal processing tools have already been developed and available in MATLAB. Some of these tools have been used for developing the *KOERI-MIDS*. On the other hand some of the real-time modal identification tools are newly developed in this dissertation as explained in Chapter4.

5.1. General Properties of the KOERI-MIDS

KOERI-MIDS is a single program that is fully integrated with Microsoft Windows. It features graphical user interface that is matched in terms of easy-of-use. Creation of the general dimensions of the structure, execution of real-time analysis and production of outputs are all accomplished using several user interfaces. A single structure can be used for a variety of different types of real-time analysis.

The program, *KOERI-MIDS* feature some complicated capabilities, such as monitoring the real-time measured responses, truncation of real-time measurements, calculation of power spectrums and smoothed FAS, performing complicated signal processing tools, calculating the real-time modal properties (modal frequency, damping ratio, and mode shape), production of outputs, and calculation of many strong motion parameters in real-time. The entire above defined features are performed in the same run based on the real-time response of the structure. The developed program can be used for different type of structure, such as concrete, steel, and masonry.

The general view of *KOERI-MIDS* can be seen in Figure 5.1. The real-time measured responses of the structure are monitored in the main window of the *KOERI-MIDS* by means of truncated data segments. The real-time measurements are continuously streamed

in the program. The truncation length and the streaming duration can be adjusted manually by the user in the *Settings* pushbuttons. The streaming time is shown in the *Stream Time* edit box. The general dimensions of the structure are shown with the seismic instrumentation installed in the structure. Either the smoothed FASs or the power spectrums for each truncated data segment are calculated and plotted in a single graph in the main interface of the program. A common peak in the graph may indicate a possible modal frequency of the structure. Based on the frequency bands defined by the *Frequency Band Selection* pushbutton, the modal frequencies of the structure are determined in real-time. Each frequency band defined correspond a single modal frequency of the structure. Therefore, the number of *Frequency Band* defined in the software must be equal to the number of mode to be identified by the software. The modal damping ratio is calculated based on Half-Power Bandwidth technique. The real-time modal frequencies and damping ratios are calculated for each truncated data segments and they are plotted in different graphs in the main window of the program. The parameters used in the calculation of modal properties can be adjusted by the *Setting* pushbutton. The *Mode Shape* pushbutton enables to calculate the real-time mode shapes of the structure by opening a new user interface window. There are different types of real-time modal property calculation techniques as explained in Chapter 4. Which method to use can be determined by *Calculation Type* pushbutton. The three pop-up menus in the main interface of program enable to select the measurement direction (*East-West*, *North-South*, and *Vertical*), measurement station, and the frequency band in consideration, respectively. The value of pop-up menus can be changed at any time while the program running in real-time. All the plots and graphs will be updated automatically according to the value of pop-up menus in real-time. The above defined calculation procedure requires a calculation time. This is indicated in the *Elapsed Time* edit box in the main window. The elapsed time strongly depends on the software settings defined for the specific structure.

The data acquisition system installed in the structure collects the structural response data in the static hard disc (HDD) of the central computer as seen in Figure 2.1. The collected data can be stored in different file formats, such as ASCII, SAC, or GCF. The developed program is capable of reading these file formats. *KOERI-MIDS* can be installed in the central computer in the structure or in a different computer in an office. If installed in a different computer, the real-time collected data in the central computer must be

transferred into the computer where the *KOERI-MIDS* is installed via ADSL modem or other transmission lines. The program reads the response of the structure from the related file in HDD. The file is not read once at a time. On the contrary it is read by means of data blocks, therefore there is no limitation on the length of the file. The length of the data blocks depends on the length of the truncation and the streaming time defined in the *Setting* pushbutton in the main window of the program.

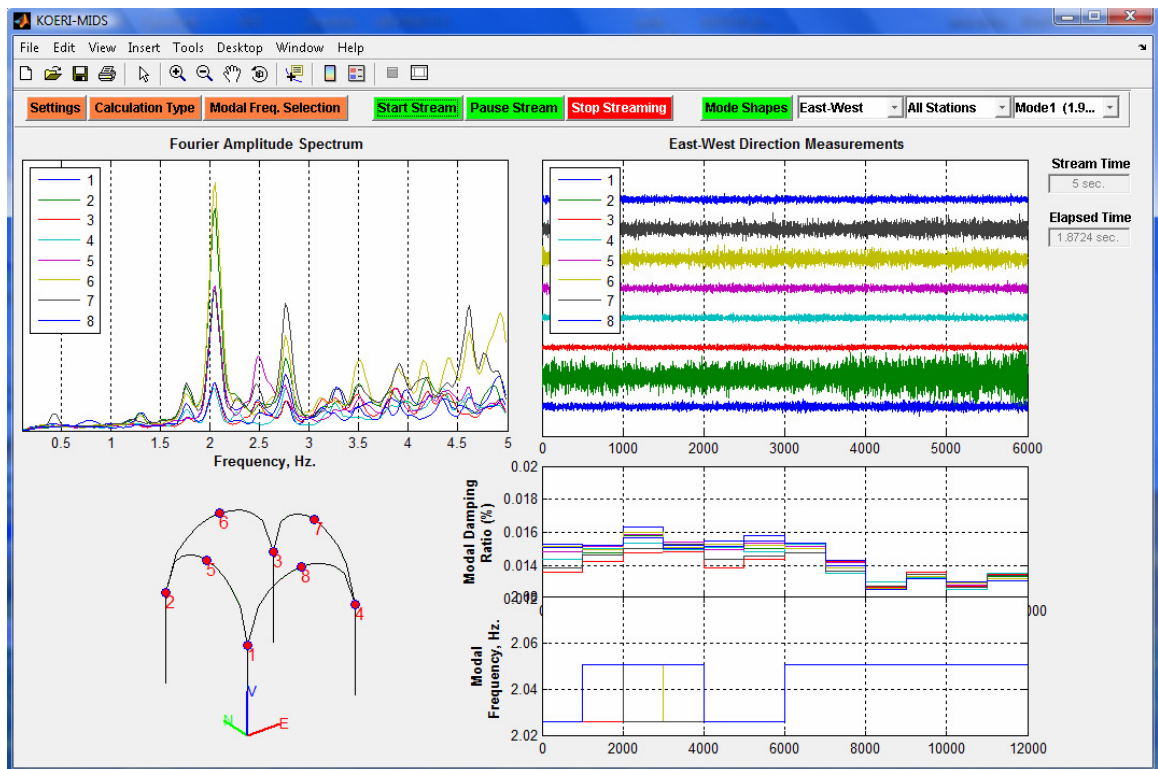


Figure 5.1. General view (main window) of *KOERI-MIDS*

The developed program, *KOERI-MIDS* starts real-time calculations by clicking on the *Start Stream* pushbutton. It first reads a specific file where the general dimensions of the structure and the location of measurement stations are given. The name of the file is *GeneralInfo.txt*. The general dimension of the structure together with the measurement locations are plotted in the main window of the program. Later, the program starts reading the response of the structure block by block from the related file(s). Each data block is analyzed separately and the results are plotted in the corresponding graphs in the main interface of the software. The elapsed time for the real-time calculation can be seen in the *Elapsed Time* edit box. The real-time calculation can be paused by *Pause Stream* pushbutton. The name of the pushbutton will be changed to *Resume Stream* immediately

when the software is paused. Clicking on the *Resume Stream* pushbutton resumes the real-time calculations. *Stop Stream* pushbutton stops the real-time calculations and closes all the files that the program is using currently. These files are basically the files that the response of the structure is stored or the files that the output reports are stored.

5.2. Structural Geometry and Measurement Stations

The geometric dimensions of the structure and the locations of the measurement stations should be given in a separate text file in ASCII format. The name of the text file should be *GeneralInfo.txt*. It is read by the software as soon as the software starts real-time calculation. The general dimensions of the structure will be plotted in a graph in the main window of the *KOERI-MIDS*.

The first line in the text file is reserved for user to enter the project name or the structure name. The number of joints to be defined in the general dimensions of the structure should be given in the second line. Following lines (equals to the number of joints defined) consist of the four columns. The first column is reserved for the joint number and the remaining three columns are for the joint's global coordinates in X, Y, and Z directions, respectively. The joint numbers should be given in ascending order. All joint coordinates must be in unit of meter. The next line after the definitions of joint coordinates is reserved for the number of frame elements defined in the general dimensions of the structure. Following lines (equals to the number of frame elements defined) consist of five columns. First column is for the frame number whereas the remaining four columns are for the joint numbers forming the frame corners. The frame element numbers should be given in ascending order. The next line after the definition of the frame elements is reserved for the number of measurement stations in the structure. Following lines (equals to the number of measurement stations defined) consist of two columns. The first column is for the measurement station number and the next column is for joint number where the measurement station is located. The measurement station number should be given in ascending order. An example of *GeneralInfo.txt* text file is given below;

<i>HAGIA SOPHIA</i>				% Structure Name or Project Name
44				% Number of Joints Defined
1	0	0	0	% Joint-ID X-Coordinate Y-Coordinate Z-Coordinate
2	0	31	0	
3	31	31	0	
4	31	0	0	
5	0	0	24	
6	0	31	24	
7	31	31	24	
8	31	0	24	
9	0	15.5	39.5	
.	.	.	.	
.	.	.	.	
.	.	.	.	
44				% Number of Frame
1	1	5	5	1
				% FrameNumber 1 st Joint 2 nd Joint 3 rd Joint 4 th Joint
2	4	8	8	4
3	2	6	6	2
4	3	7	7	3
5	5	17	17	5
6	17	18	18	17
7	18	19	19	18
8	19	20	20	19
.	.	.	.	
.	.	.	.	
.	.	.	.	
8				% Number of Measurement Station
1	5			% Measurement Number - Joint Number
2	6			
3	7			
4	8			
5	9			
6	10			
7	11			
.	.			
.	.			

5.3. Selection of Modal Property Calculation Techniques

The developed software, *KOERI-MIDS* has mainly four successive steps to perform in real-time modal identification. These steps are namely the identification of real-time modal frequencies, damping ratios, mode shapes and production of reports, respectively. The modal frequencies of the structure are estimated in frequency domain by means of power spectrum or smoothed FAS while the modal damping ratio is calculated for each identified modal frequency identified using either Half-Power bandwidth technique or Logarithmic decrement technique. On the other hand, mode shapes are calculated for each

identified modal frequency using the band-pass filters and numeric integration. The results of the calculated real-time modal properties of the structure depend on the calculation techniques used in the analysis. The calculated real-time modal properties of the structure can be stored in an output file when needed. The name of the output file should be given in the *Output File Name* edit box. The user can determine the number of modes to be identified automatically by the program. It should be given in the *Number of Modes to be Identified* edit box. Since just a limited number of modes of the structure are usually excited under ambient vibrations, there is a limitation put in the number of modes to be identified in order to decrease the total calculation time for each steamed truncated data segment. By default the minimum number of modes to be identified is one whereas the maximum is five.

If the *Calculation Type* pushbutton is clicked from the main window of program, a new window called *Selection of Modal Property Calculation* is opened as shown in Figure 5.2 where the above defined modal property calculation techniques and other parameters can be selected. Clicking the *Update* pushbutton will update all the selections in the *Selection of Modal Property Calculation* window. They will be updated automatically in real-time.

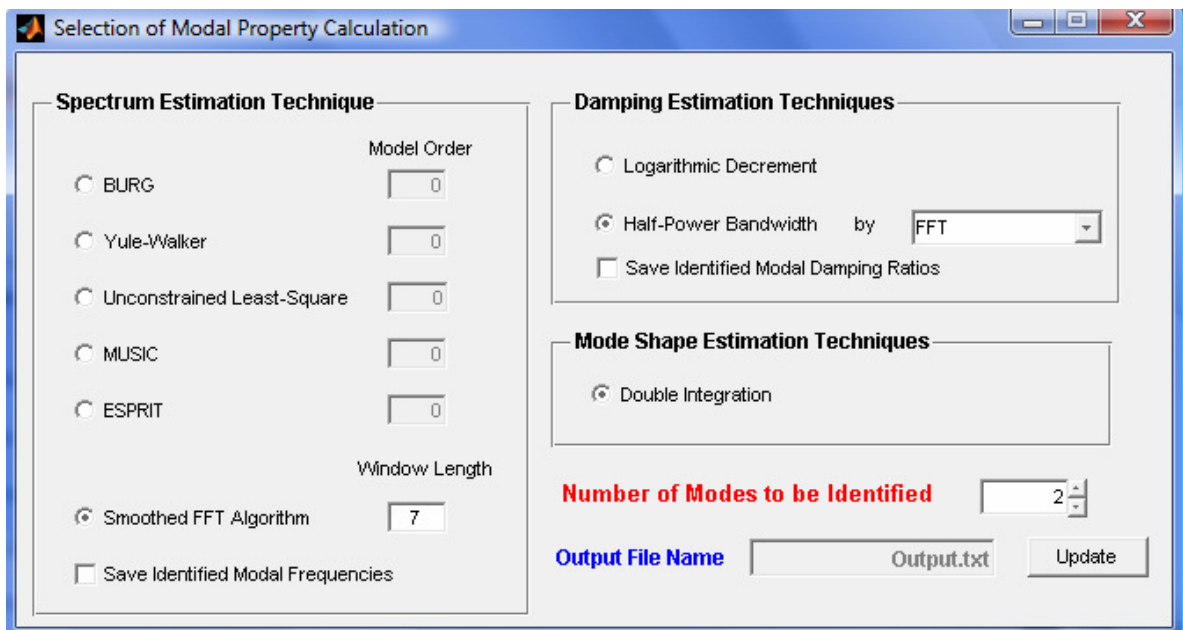


Figure 5.2. Modal property calculation window

5.3.1. Spectrum Estimation Technique Panel

There are five different power spectrum estimation techniques (BURG, Yule-Walker, Unconstrained Least-Square, MUSIC, and ESPRIT) and one smoothed FAS calculation technique available for the calculation of real-time modal frequencies of the structure. The smoothed FAS algorithm is the default spectrum estimation technique in the software. The window length should be an odd number and it defines the length of smoothing window. If a different spectrum estimation technique is selected, the model order should be specified for the selected spectrum estimation method. If the *Save Identified Modal Frequencies* check-box is selected all the identified modal frequencies will be saved in an output text file in ASCII format.

5.3.2. Damping Estimation Technique Panel

As for the calculation of real-time modal damping ratio, there are two different estimation techniques available, Logarithmic decrement and Half-power bandwidth techniques as seen in Figure 5.2. Using half-power bandwidth technique, the damping ratio can be calculated based on two options, FAS or the estimated power spectrum. The half-power bandwidth technique with FAS option is the default technique in the software for the real-time modal damping estimation. If the *Save Modal Damping Ratios* check-box is selected, all the identified real-time modal damping ratios will be saved in an output text file in ASCII format.

5.3.3. Mode Shape Estimation Technique Panel

There is only one method for the calculation of real-time mode shape that is the double integration method.

5.3.4. Real-Time Modal Properties (Output File)

Real-Time modal properties of the structure are stored in an output file in ASCII format when necessary. The format of the output file is given below. It consists of five

header lines; name of the structure, sampling rate, number of stations, and start time of the output file. The identified modal frequencies and the modal damping ratios for all measurement locations are stored in a single file. The number of the columns in the file depends on the number of identified modes of the structure. The first two columns indicate the first modal frequency and the related damping ratio of the structure, respectively. The third and the forth columns indicated the second modal frequency and the related damping ratio, respectively and so on. Therefore the number of columns in the file is equal to twice the number of modes indentified by the software. The first row after the header corresponds to the first measurement location, second row for second measurement location and so on. An example of output file is given below;

```
% KOERI-MIDS for HAGIA SOPHIA
% Sampling Rate: 200.000000
% Number of Station: 8.000000
% Start Date (YYYY-MM-DD-HH): 2009-3-17-11
% 1st mode 2nd mode 3rd mode 4th mode
2.050781 0.023027 1.757813 0.017809 % 1st measurement location
2.050781 0.021983 1.757813 0.017899 % 2nd measurement location
2.050781 0.023062 1.757813 0.017871 % 3rd measurement location
2.050781 0.023903 1.757813 0.017956 % 4th measurement location
2.050781 0.021251 1.757813 0.017897 % 5th measurement location
2.050781 0.022242 1.757813 0.017845 % 6th measurement location
2.050781 0.020906 1.757813 0.017951 % 7th measurement location
2.075195 0.023923 1.757813 0.017790 % 8th measurement location
. . . .
. . . .
. . . .

2.050781 0.024235 1.757813 0.017723
2.050781 0.022885 1.757813 0.017610
2.050781 0.020781 1.757813 0.017213
2.050781 0.028656 1.757813 0.017415
2.050781 0.021826 1.757813 0.017234
2.050781 0.023463 1.757813 0.017588
2.050781 0.022222 1.757813 0.017371
2.050781 0.027073 1.757813 0.017578
```

5.4. Modal Frequency Band Selection

Based on the settings defined in the *Selection of Modal Property Calculation* window, either the real-time power spectrum or the smoothed FAS are calculated for all truncated data segments and are plotted in a single graph in the main window of the program in real-time. The calculation of the real-time modal properties is all based on

these calculated spectrums. The peaks of the estimated spectrums usually correspond to the modal frequencies of the structure.

In order to identify the modal frequencies of the structure, the real-time power spectrums of all measuring stations are calculated and plotted in one graph simultaneously in the main window of the *KOERI-MIDS*. If there is a clear peak in a specific frequency in all spectrums, this frequency corresponds to a modal frequency of the structure. However, this is not always the case. It is common to observe that the peaks of the spectrum sometimes do not coincide in a single frequency but in a frequency band. They may be very near to each other but do not meet in a single frequency. Therefore, identification of a modal frequency is not always easy. In order to overcome this problem, frequency band concept is utilized in the developed software. Using frequency band has mainly two major advantages. It increases the calculation time and it is much easier to code it in MATLAB.

If a series of frequency bands are defined in advance in the software, the modal frequencies of the structure can be searched within these frequency bands. If there is a clear peak within a frequency band in all calculated spectrums, it can be easily said that there is a modal frequency exist in this frequency band as seen in Figure 5.3. Therefore, specifying the frequency bands is an important part of *KOERI-MIDS* because the program automatically searches the modal frequencies of the structure only within these frequency bands. Therefore, wrong selection of frequency bands may even result in no modal frequency to be identified. This is because of that no modal frequencies are searched outside of frequency bands defined. Some pre-knowledge about the dynamic properties of the structure will help to define the frequency band limits accurately. Therefore, it is strongly suggested that the user should collect a certain length of ambient vibration test data from the structure and should analyze them to have a general idea about the predominant modal frequencies of the structure. If possible finite element model of the structure would give valuable information about the modal frequencies of the structure. It is obvious that only a few predominant frequencies of the structure will be excited under ambient vibration. Therefore, the number of frequency bands defined in the program must be equal to the number of modes that has been identified using ambient vibration test data.

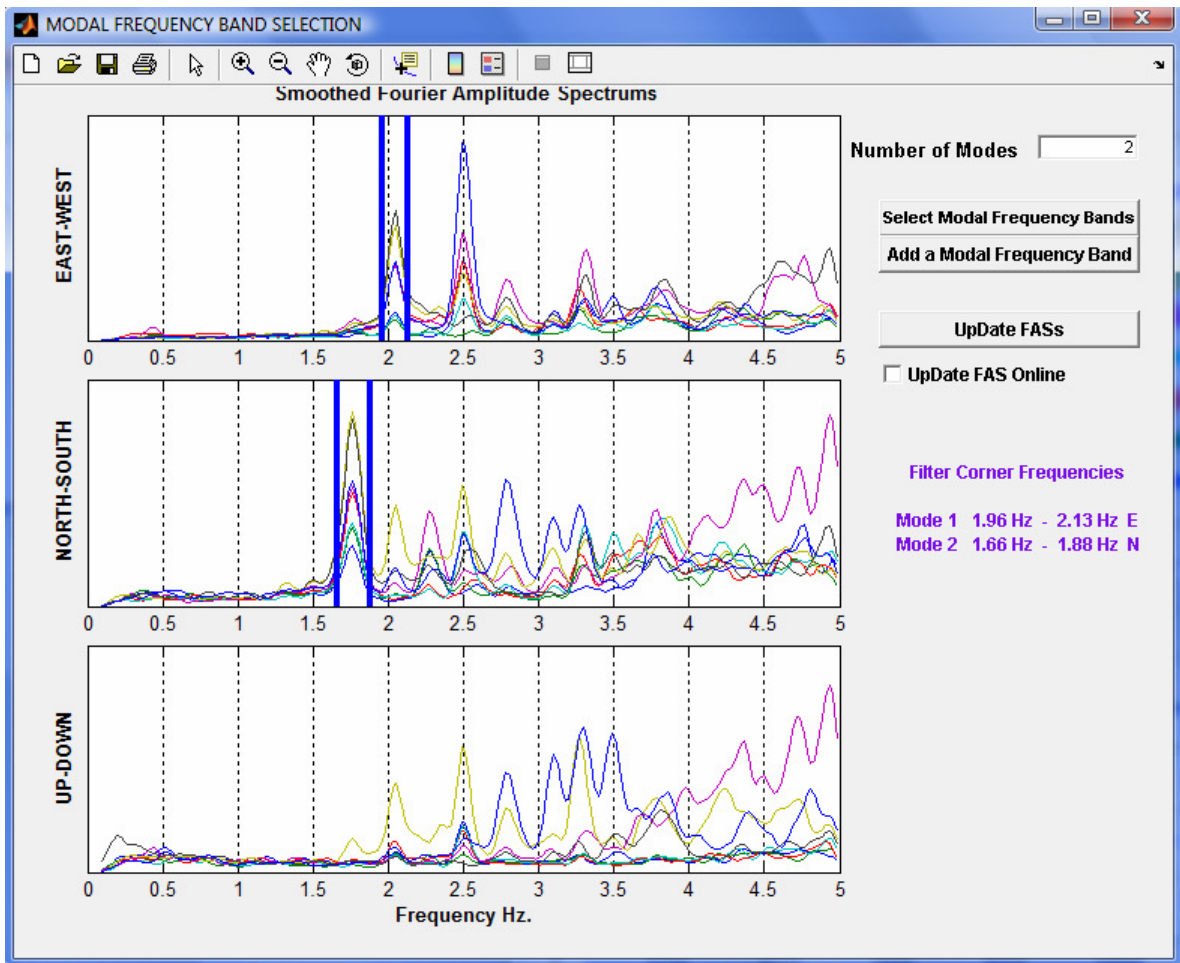


Figure 5.3. Modal frequency band selection

If the *Frequency Band Selection* pushbutton is clicked from the main window of the program, the *Modal Frequency Band Selection* window is opened as seen in Figure 5.3. There are three pushbuttons, three plot boxes one edit box and one check-box in the *Modal Frequency Band Selection* window. The real-time smoothed FAS or the estimated power spectrums are plotted automatically in the three plot boxes as soon as the *Modal Frequency Band Selection* window is opened. In the first plot, spectrums of all measuring stations in East-West direction are plotted whereas spectrums of all measuring stations in the North-South and Vertical directions are plotted in the second and the third plots, respectively. In addition to this, the pre-defined frequency bands are also plotted in the corresponding plot boxes as seen in Figure 5.3.

The *Number of Modes* edit box shows the number of modes to be identified by the *KOERI-MIDS* program. The first pushbutton, *Select Modal Frequency Bands* enables to

define a series of modal frequency bands. The number of frequency band that will be defined by the user should be entered into the *Number of Modes* edit box. The related pushbutton deletes the existing frequency band information from memory and from the corresponding plot boxes in the figure. It waits for user to define the first frequency band. It is defined by clicking on a plot box in the figure two times. First click defines the first corner and the second click defines the second corner of the frequency band. After the definition of first frequency band, the program automatically waits for user to define the next frequency band. This procedure is repeated for the *Number of Mode* defined in the program. The program does not enable to select a single frequency band by clicking on the two different plot boxes. Each frequency band should be defined in a single plot box. All the corner frequencies of the selected frequency bands will be summarized in the figure while they are being defined. There is no limitation for the width of the frequency band in the *KOERI-MIDS*. The user can define any width for any frequency band. However, it is strongly suggested that the width of the frequency band should be defined such that the limits of any frequency band must be placed around the modal frequency where the amplitude of the modal frequency attains its minimum for all stations installed in the structure.

The second pushbutton, *Add a Modal Frequency Band* enables to define an additional frequency band while keeping the existing frequency band information in the memory. The related pushbutton waits for user to click on a plot box in the figure to specify the first corner of the frequency band. Second click on the same plot box defines the second corner of the frequency band. This option automatically increases the *Number of Modes* by one. This procedure can be repeated until the maximum number of modes defined in the program is reached. The corner frequencies of the selected frequency bands will be summarized in the figure.

Since the *KOERI-MIDS* analysis the structural response in real-time, the power spectrum of all measurement stations are calculated in real-time. The third pushbutton, *UpDate FASs* enables to update the current calculated power spectrum of all measurement stations so that the modal frequency bands can be selected using the most recently calculated power spectrums. The calculated power spectrums in all directions will be updated automatically in the corresponding plot boxes. The *UpDate FASs* pushbutton

manually updates the calculated spectrums. On the other hand, the check box, *UpDate FAS Online* also enables to update the recent calculated power spectrums but in real-time.

5.5. Real-Time Data Processing Settings

If the *Settings* pushbutton is clicked from the main window of the *KOERI-MIDS*, the *Real-Time Data Processing Settings* window is opened as seen in Figure 5.4. This window controls the calculation parameters of the real-time modal frequency identification by means of dialog boxes; Signal Properties, Filter Properties, Decimation, Spectrum smoothing, Window Parameters, and Screen Settings. As mentioned before, the real-time measurements are usually infinite in duration. Therefore, they have to be divided into equal length of data segments and should be analyzed separately. Several signal processing tools (windowing, filtering, decimation, and smoothing) are applied to each data block to calculate the real-time modal properties. All these signal processing tools are optional in the *KOERI-MIDS*. Each of options might be selected or deselected by clicking on the related *Enable* check-box as seen in Figure 5.4. By default, all the *Enable* check boxes are selected in the program.

Real-Time Data Processing Settings

Signal Properties

Number of Station: 8

Sampling Frequency, Hz.: 200

Decimation

☒ Enable

Decimation Order, r : 2

New Samp. Frequency: 100 Hz

Filter properties

☒ Enable ☐ Filter Response

Filter Freq. (f_1) (Hz.): 0.2

Filter Freq. (f_2) (Hz.): 20

Filter Order: 4

Butterworth Filter is applied to all channels

Window Parameters

☒ Short Window Length [sec.]: 0.4

Total Window Length [min.]: 1

Running Window Length [min.]: 0.4

OverLap Ratio (%): 0.6

nfft: 4096

Number of Averaging window: 4

Spectrum Smoothing

☒ Enable ☐ Smoothing Response

Window Type: Hamming

Window Length, n_{ww} : 7

Smoothing is performed for FFT of Acceleration

Screen Settings

Streaming Time [Sec.]: 5

Fmin [Hz.]: 0.1

Fmax [Hz.]: 5

Update

Figure 5.4. Real-time data processing settings

5.5.1. Signal Properties Panel

The number of measurement stations can be determined from the *Number of Station* edit box. The number of measurement stations cannot be changed while the program is running in real-time. It should be determined before the program starts running in real-time. The developed program assumes that all measurement stations in the structure have the same sampling frequency. Therefore, a unique sampling frequency should be set for all measurement stations in the structure.

5.5.2. Decimation Panel

If the *Enable* check-box is clicked for the *Decimation*, decimation process is applied to all running windows for all measurement stations. *Decimation order*, r must be an integer number. The *New Sampling Frequency* will be calculated automatically based on

the decimation order specified. Decimation decreases the sampling frequency by *Decimation Order*, r . Therefore, decimation acts like a low-pass filter. The contribution of higher frequency modes to the general response of the structure is usually smaller than that of lower frequency modes. Since the decimation is carried out for the response of the structure, some of the higher frequency components of the response will be lost in the resulting decimated data. Therefore, decimation does not change the structural response significantly. Another benefit of the decimation is that it decreases the calculation time considerably by decreasing the sampling frequency.

5.5.3. Filter Properties Panel

If the *Enable* check-box is clicked for the *Filter Properties*, a standard butterworth band-pass filter is applied to all running windows in the truncated data segment for all measurement stations. The user can adjust the filter's lower corner frequency, f_1 upper corner frequency, f_2 and the filter-order manually in real-time. If the *Filter Response* check-box is selected, a new window of *Filter Response* is opened as seen in Figure 5.5.

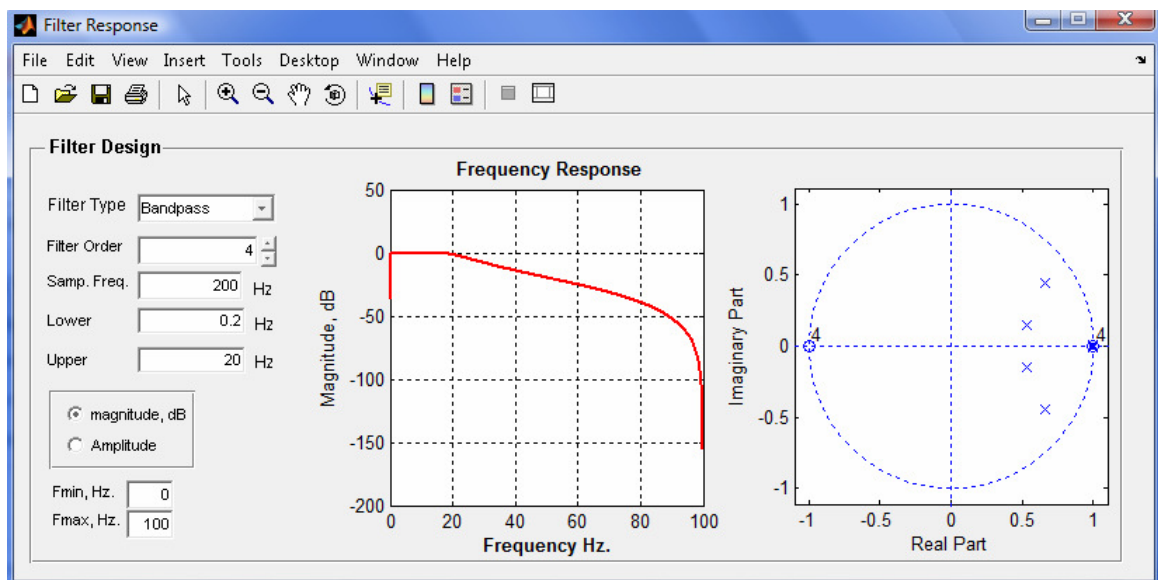


Figure 5.5. Filter response

The *Filter Response* window enables to check the response of the filter and to re-design it if necessary. There are three filter types available, Band-pass, High pass, or Low-pass filter. Band-pass filter type with filter order of 4 is the default option in the program.

A correct and stable butterworth filter requires several conditions. Among them, the two most important conditions are to have minimum ripples in the pass-band of the filter and no poles located outside of the unite circle. These conditions can be checked out by the two plot boxes in the *Filter Response* window. First plot box shows the filter response in frequency domain with two option, decibel and amplitude. Second plot box shows the filter's pole-zero maps.

There are four edit boxes in the *Filter Response* window to define the filter order, sampling frequency, and filter lower and upper corner frequencies, respectively. The user can design a digital filter by means of these parameters. All the options in the *Filter Response* window can be changed while the program is running in real time. All the parameters defined in this window will be automatically updated by closing the window.

5.5.4. Window Parameters Panel

Analysis of ambient vibration records are carried out according to the techniques defined in Chapter 4.1.1. The real-time measurements are usually infinitely long sequences. Therefore, they are divided into equal length of data segments in order to analyze them in real-time. The modal identification is carried out for these truncated data segments. The *Total Window Length* in Figure 5.4 defines the length of the truncated data segment. In order to minimize the effects of noise in the truncated data segments, they are further divided into equal length of running windows. The length of each running window should be given in the *Running Window Length* edit box. The length of the truncation and the running window size depends on the modal characteristics of the structure and the noise contamination in the measurements. Each running window is overlapped with an overlap ratio defined in the *Overlapped Ratio* edit box. The outputs of all running windows are averaged to form the overall output of the truncated data segment. The parameter *nfft* is the number of points in the smoothed FAS or power spectrum. It is automatically calculated as the smallest power of two that is greater than or equal to the length of the running window. The parameter *Number of Averaging Window* is the number of running windows in the truncated data segment.

In order to catch the sudden change of modal properties of the structure, an additional short window is defined in *KOERI-MIDS*. The length of short window is usually equal to the first running window inside the truncated data segment as explained in Chapter 4.1.2. Therefore, the analysis of first running window will result in the real-time variation of modal properties of the structure. This procedure can be activated by clicking on the *Short Window Length* check box in the Figure 5.4. Even though the length of the short window is equalized to the length of running window automatically, it can be changed by the user. In such case, it must be bigger than the length of *Running Window*.

5.5.5. Spectrum Smoothing Panel

If the *Enable* check-box is clicked for the *Spectrum Smoothing*, smoothing process is applied to the FAS of each running windows in the truncated data segments for all measurement station. There are three smoothing window type available, Hamming, hanning, and triangular. Smoothing window length must be an odd number. Smoothing will reduce the noise in the FAS. Different smoothing window types and length may result in different noise reduction ratios on FAS. The optimum smoothing window length can be determined by clicking the *Smoothing Response* check-box. This will open a new window of *Smoothing Control* as seen in Figure 5.6.

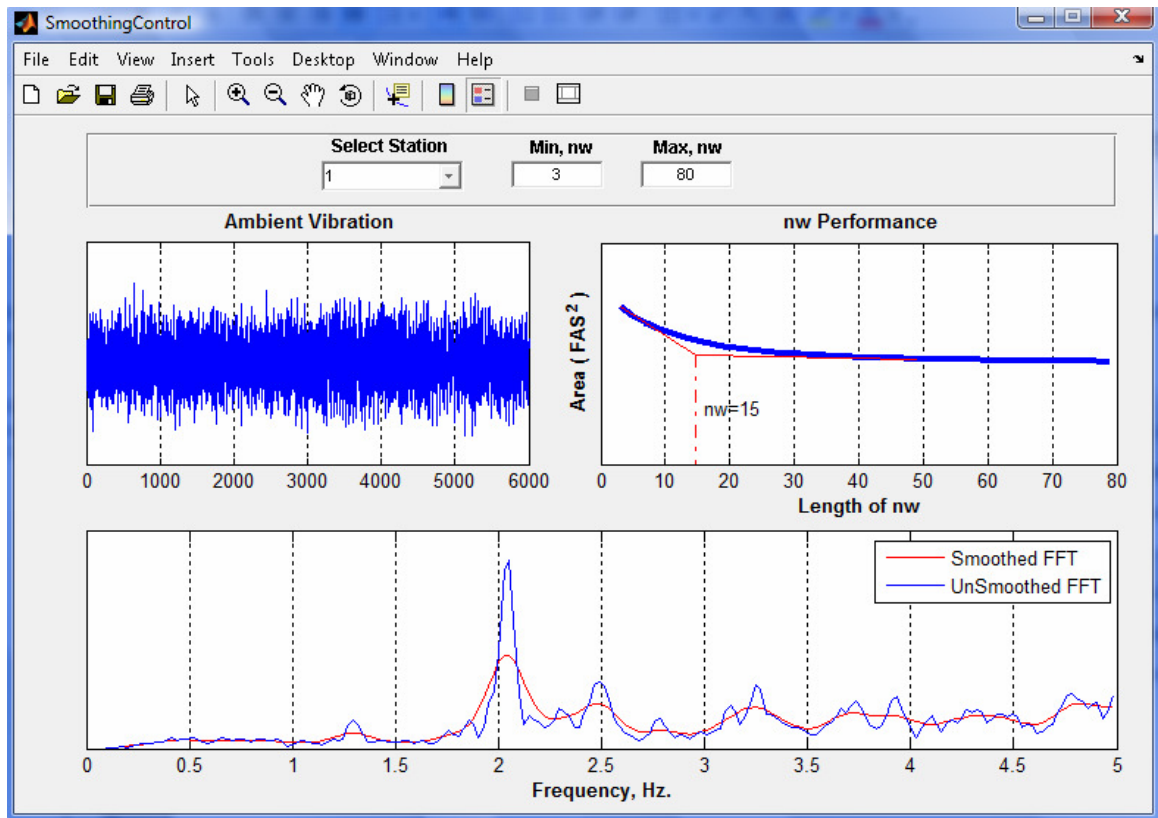


Figure 5.6. Smoothing control

In the Smoothing Control window, there are two edit boxes, one pop-up menu, and three plot boxes. The first plot box shows the ambient vibration response of the structure. The pop-up menu enables to select the measurement station. The program automatically calculates the area under the squared-FAS with respect to varying smoothing window lengths. The user can determine the minimum and the maximum smoothing window lengths by the two edit boxes. The calculated curve is plotted in the second plot box. The optimum length for the smoothing window is automatically calculated and plotted in the second plot box. The calculated smoothed and unsmoothed FAS are plotted in the third plot box using the optimum smoothing window length.

5.5.6. Screen Settings Panel

The real-time smoothed FAS or power spectrum is calculated and plotted in the main window of the program. The axis limits of this plot can be adjusted by the user. The *Fmin* and *Fmax* edit boxes enables to determine the minimum and maximum frequency limits in Hz in the main window of the KOERI-MIDS.

After the calculation of modal properties for the truncated data segment, it is shifted by the *Streaming Time* in order to calculate the power spectrum for the shifted data block. This *Streaming Time* can be adjusted by the user. The default streaming time is set to five seconds.

5.6. Real-Time Mode Shape Estimation

Clicking on the *Mode Shape* pushbutton in the main window of the *KOERI-MIDS* will open *Mode Shape Animation* window. There are three pop-up menus, three pushbuttons, one check box, two slider bars, and four plot boxes in the *Mode Shape Animation* window. The real-time raw data, calculated modal frequencies and damping ratios are plotted in three graphs as soon as the *Mode Shape Animation* window is opened. The *Direction* pop-up menu enables to select the measurement direction whereas the *StationID* pop-up menu enables to select the measurement station number in the structure. The *ModeID* pop-up menu enables to animate the different mode shapes of the structure.

The raw data will be automatically filtered around the modal frequency bands defined in the *ModeID* pop-up menu. The filter response can be checked by *Check Filter Response* check-box.

The *Animation* pushbutton starts to animate of the mode shape where as the *Pause* and *Stop* pushbuttons are pause and stops the mode shape animation, respectively. The mode shape animation is plotted in three different camera positions, such as East-Vertical, North-Vertical, and East-North directions, respectively. All camera positions are animated synchronously. The *Scale* slider bar enables to adjust the amplitudes of the mode shape while the *Speed* slider bar controls the animation speed.

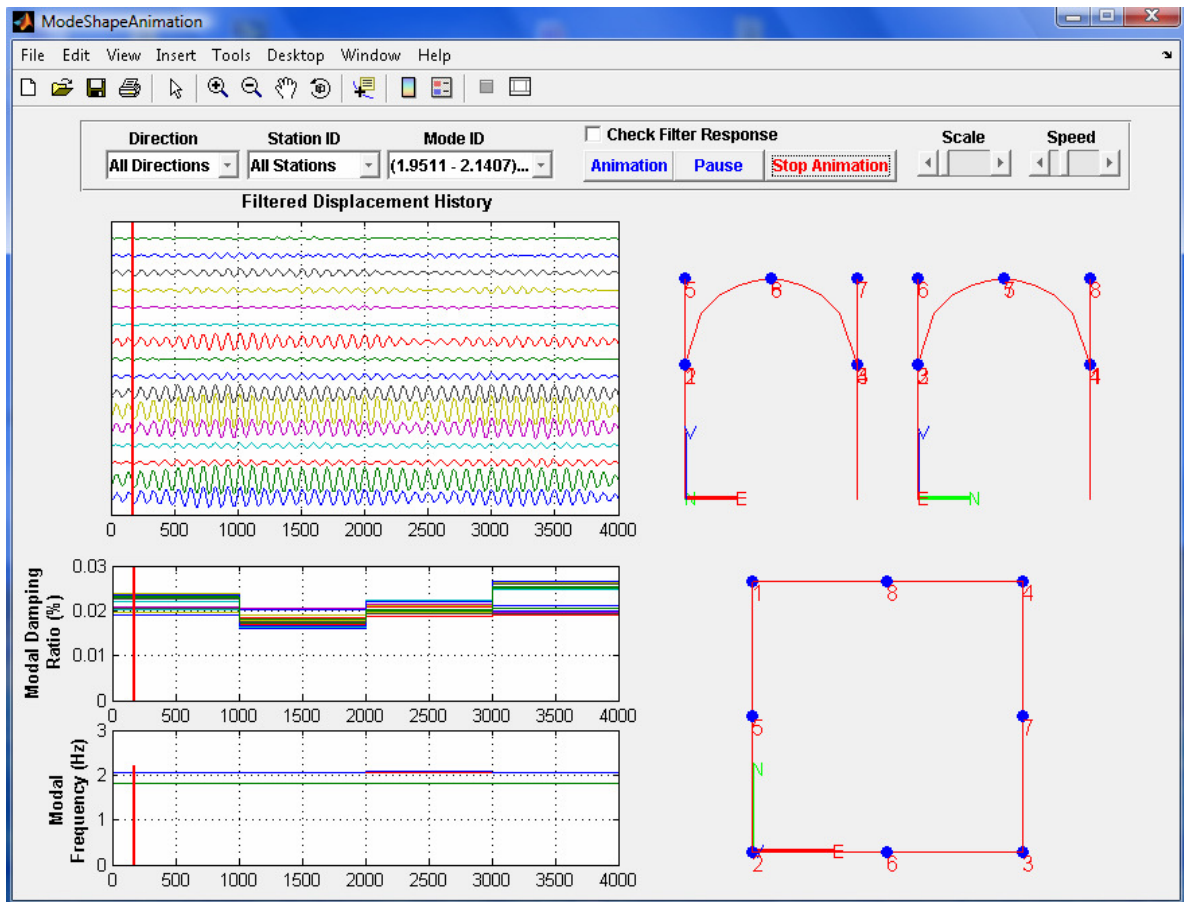


Figure 5.7. Real-time mode shape

6. AMBIENT VIBRATION TEST APPLICATIONS

6.1. Hagia Sophia Museum

6.1.1. History of Hagia Sophia

Hagia Sophia (in Turkish Ayasofya) is one of the most magnificent buildings in the history of architecture. It was the cathedral of Byzantine Constantinople for more than a thousand years, and then for nearly five centuries after the Ottoman conquest it was among the most important mosques of Istanbul. The edifice was restored and opened as a museum in 1935.

The present edifice of Hagia Sophia as seen in Figure 6.1 is the third church of that name to stand upon this site. The first church was built in around 360 AD by emperor Constantinus (son of Constantine the great), and dedicated to hagia Sophia, the divine wisdom, one of the three attributes of Christ according to Orthodox Christian faith. The other two are Irene (peace) and dynamos (strength). Three churches were built and dedicated to each of these attributes; only two churches are surviving; Hagia Sophia and Hagia Irene.

The church was burnt down on 9 June 404 during a riot by the supporters of John Chrysostom, the patriarch, who had been removed from his see by empress Eudoxia, wife of emperor Arcadius. A new church was later built on the same site by Theodosius II and opened to service on 10 October 415. This structure known as Theodosian church was destroyed by fire in 532 in the very first day of Nika Revolt. The present church was built by the order of Emperor Justinian who initiated the work and projects just a month after the end of rebellion. The emperor appointed two important architect of the age; Anthemius of Tralles and Isidorus of Miletus, as Anthemius died during the first year of the construction, Isidorus carried the project through the completion in 537 A.D and it was consecrate on 26 December of that year.

When the Ottoman Turks conquered Constantinople on 29th may 1453, the church was converted into a mosque, a place of islamic worship as Ottoman Sultan Mehmet II "the Conqueror" ordered after inspecting the building. The following friday he attended the first muslim service to be held in Aya Sofya Camii, the Mosque of Hagia Sophia.



Figure 6.1. General view of Hagia Sophia

6.1.2. Past Earthquakes

The church suffered and partially destroyed due to several earthquakes that shook the city between 553 and 557. On 7 may 558, the eastern part of the dome collapsed along with the arch and semidome on the side of the church. The original architects were no longer alive, so Justinian entrusted the task of rebuilding the church to a nephew of Isodorus, who decided to change the design of the dome, making it less shallow so as to reduce the lateral stresses. The reconstruction was completed late in 563.

On February 9th 869, during the reign of Emperor Basil I, an earthquake damaged the western side of the building and it was repaired in 879. On october 25th 968, a violent

earthquake resulted in collapse of the western apse and caused partial damage to the dome. The church had to be closed until architect Tridat finished repairing it in 994.

The church interior suffered very much in the latin sack of Constantinople in 1204, when it was stripped of all its sacred relics and other precious objects. During the Latin occupation, Hagia Sophia served as the Roman Catholic cathedral of the city. Following the recapture of Constantinople by the Byzantines in 1261, Hagia Sophia was re consecrated as a Greek Orthodox sanctuary. In 1348, the eastern half of the dome collapsed, and was afterwards repaired. In the 15th century, travelers described Hagia Sophia as being in a state of disrepair.

6.1.3. Related Previous Studies

The dynamic behavior of Hagia Sophia has been analyzed by many researchers in the last decades. Selected studies are briefly explained in chronological order in the following;

Durukal (1992) has prepared a thesis studying the structural identification and seismic vulnerability assessment of Hagia Sophia. The model frequencies obtained from ambient vibration survey are matched with ones obtained from finite element model. The dynamic characteristics of the structure are investigated using the calibrated finite element model. According to the results of the dynamic analysis, stress concentrations are observed in the top of the four main arches and at their springing points. The main dome is in good condition, but in the main semidomes, the stresses intensify in areas close to the east and west main arches.

Mark *et al.* (1993) created a numerical model of the Hagia Sophia. They have investigated the short and long term linear and non-linear material behavior. SAP90 and FENDAC structural analysis programs are used for linear and non-linear analysis, respectively.

Erdik *et al.* (1994) analyzed the earthquake response of the Hagia Sophia on the basis of the strong ground motion data recorded by accelerometer network installed in the structure. They compared the modal vibration frequencies and the mode shapes obtained

through the empirical transfer function to those based on the ambient vibration survey and analytical finite element studies.

Cakmak *et al.* (1993) applied an interdisciplinary approach to the study of dynamic behavior and earthquake response of Hagia Sophia. Material properties of the structure have been adjusted for the numerical model to match the modal properties of the structure that has been identified from the measured response of a low-intensity earthquake. The numerical analysis of the structure is performed using the calibrated model.

Durukal *et al.* (1998) carried out non-destructive test namely Ultrasonic Pulse Velocity and Schmidt Hammer Test to investigate the material properties of Hagia Sophia. The compressive strengths of the main structural materials and modulus of elasticity have been investigated.

Durukal, *et al.* (2000) has carried out a study utilizing the records of 1999 Kocaeli and Düzce Earthquakes to investigate the earthquake specific modal frequencies using time and frequency domain analysis. The results indicate that a drop up to 25% and 27% was observed in the first and second modal frequencies, respectively in Hagia Sophia as compared to the values determined based on pre-earthquake ambient vibration survey.

Keypour (2001) prepared a thesis about the non-linear dynamic analysis of historical structures and application to Hagia Sophia Mosque. Results showed that the first mode (1.845 Hz.) is complete motion of structure in east-west direction, which is weaker than the other direction. The second mode (2.148 Hz.) is the same as first mode but the motion is in north-south direction. Due to the slight difference of the east and west of the structure by the small mihrab semidome, a very small rotation was observed in conjunction with this mode. The results of the modal analysis have been found in good agreement with the ambient vibration tests.

6.1.4. Description of the Structure

The imperial gate leads to the central nave of Hagia Sophia. The dome is approximately 55 meters high and 32 meters wide. Due to repairs and earthquakes over the

centuries, the dome lost its original shape and not completely circular anymore. The ceiling is completely covered with mosaics. The dome rests on four large arches which are supported by four pillars. Weight of the dome is transmitted to semi-domes in the north and south, and to the lower sections.

The interior contains 107 columns, 40 of these are found on the ground floor and the rest are up in the gallery. Over the years, buttresses have been built outside, against almost every wall, to lessen the stress of the building and to counteract the damage caused by earthquakes.

Hagia Sophia was built on a domed basilica plan; therefore it has a central nave, northern and southern side naves, and two narthexes in the west. Only in the central nave is it possible to see all the way up to the dome. A second store, the gallery, was built over the two side naves and the inner narthex. The distance from the imperial gate to the apse is 79.30 meters; the length of the edifice is approximately 100 meters. Hagia Sophia occupies an area of 7,500 square meters and is the fourth largest, as well as being the oldest church in the world after St. Peter's in Rome, Sevilla Cathedral and Milan Cathedral.

6.1.5. Instrumentation of the Structure

Big attentions should be given to such cultural heritage structures in order to preserve them against natural disasters. Such preservation in seismic areas is one of the important issues in earthquake engineering. As a result strong motion network (SMN) was installed in Haiga Sophia to monitor the vibration characteristics of the structure.

The SMN has been operational in the structure since August 1991. Many changes have been made since the first installation. The latest instrument setup that is present at the museum right now can be seen in Figure 6.2. The instruments basically cover the three different levels of the structure. One instrument is at the ground level while four of them are located at cornice level next to the springing points of the four main arches. The last four instruments are located at dome base level on the crowns of the main arch.

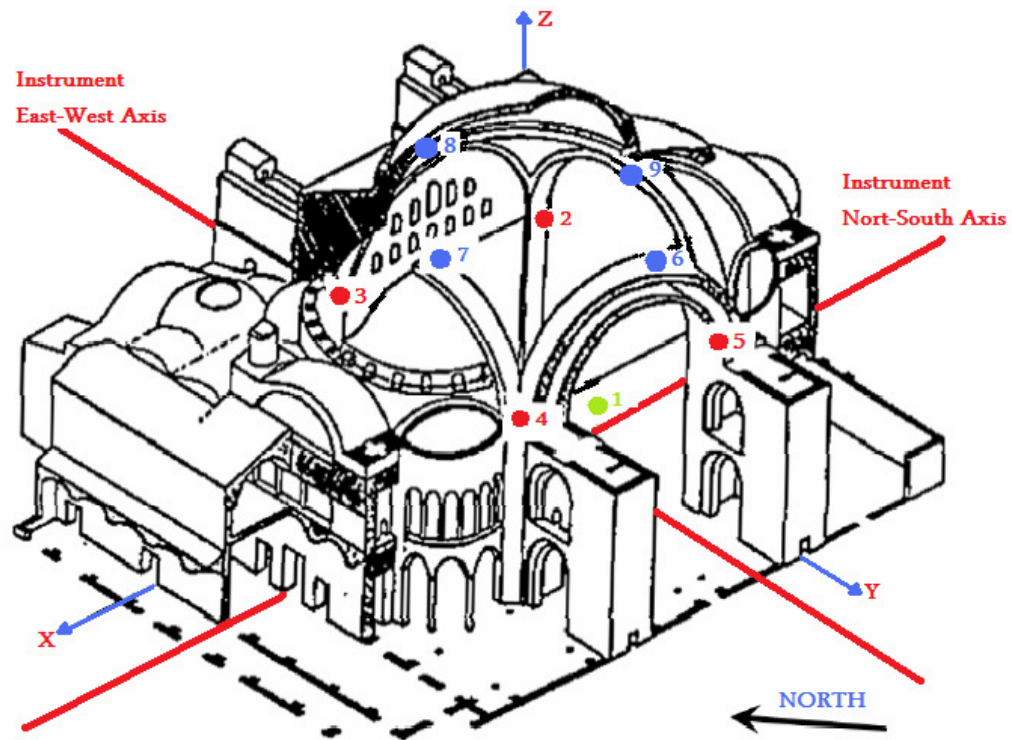


Figure 6.2. Locations of strong ground motion instrumentation in Hagia Sophia

6.1.6. Parameter Selection for KOERI-MIDS

KOERI-MIDS software is used for the real-time modal identification of the Hagia Sophia museum. There are nine accelerometers installed in the Hagia Sophia. However, only eight of them are used in the real-time modal identification analysis. The first accelerometer installed on the ground level is not used in the analysis.

6.1.6.1. Selection of Parameters for Real-Time Data Processing Settings. Since the real-time response of the structure is recorded continuously in real-time, they are usually infinitely long data sequences. Therefore, they should be truncated into equal size of data segments in order to analyze them in real-time. The total length of each truncated data segment is fixed to one minute for the Hagia Sophia Museum. Each truncated data segment is then further divided into equal length of running windows and each running window is analyzed separately. The length of each running window is set to 0.4 minutes (24 seconds), which should be at least ten times greater than the first fundamental modal period of the structure. Each running window is overlapped with the previous running window by an overlap ratio. The overlap ratio is fixed to 0.6 as seen in Figure 6.3. The result of all

running windows is averaged to form the overall output of the truncated data segment. The structural response is recorded with a sampling frequency of 200 Hz. Each running window is decimated by a decimation order of two. Therefore, the sampling frequency of all records in the structure is reduced from 200 Hz to 100 Hz.

Real-Time Data Processing Settings

Signal Properties

Number of Station: 8

Sampling Frequency, Hz.: 200

Decimation

☒ Enable

Decimation Order, r: 2

New Samp. Frequency: 100 Hz

Filter properties

☒ Enable ☐ Filter Response

Filter Freq. (f1) (Hz.): 0.2

Filter Freq. (f2) (Hz.): 20

Filter Order: 4

Butterworth Filter is applied to all channels

Window Parameters

☒ Short Window Length [sec.]: 5

Total Window Length [min.]: 1

Running Window Length [min.]: 0.4

OverLap Ratio (%): 0.6

nfft: 4096

Number of Averaging window: 4

Spectrum Smoothing

☒ Enable ☐ Smoothing Response

Window Type: Hamming

Window Length, nww: 7

Smoothing is performed for FFT of Acceleration

Screen Settings

Streaming Time [Sec.]: 5

Fmin [Hz.]: 0.1

Fmax [Hz.]: 5

Update

Figure 6.3. Real-time data processing settings for Hagia Sophia Museum

In real-time modal analysis, each running window is baseline corrected and then band-pass filtered. The forth order butterworth band-pass filter has been used with corner frequencies of 0.2 Hz and 20 Hz. The response of butterworth filter can be seen in Figure 6.4. The pole zero map shows that all poles are located inside the unite circle while all zeros are located on the unite circle. The filter parameters, the filter frequency response and pole-zero-map show that the forth order butterworth filter used is a stable filter response.

The Fourier Amplitude Spectrum (FAS) of each running window is smoothed in order to reduce the noise effect from FAS. There are three different smoothing windows available namely the hamming, hanning and triangular windows. Among them the hamming type of smoothing window is used as default with an optimum length of seven. The optimum length for the selected smoothing window can be determined for each measurement station as seen in Figure 6.5 and Figure 6.6.

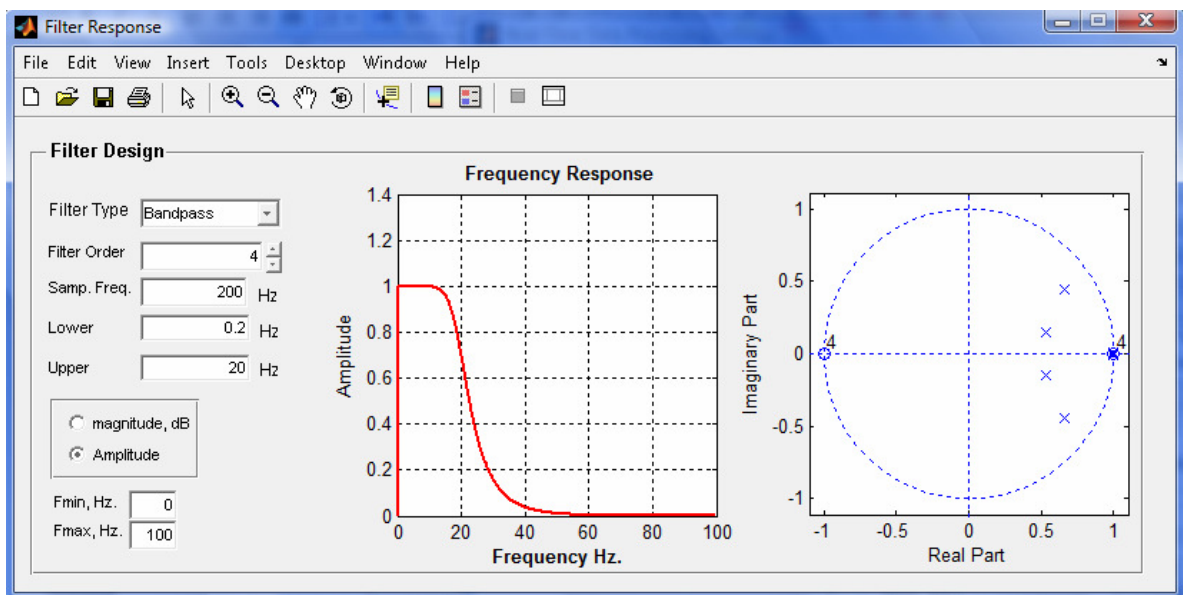


Figure 6.4. Filter response of the forth order band-pass filter

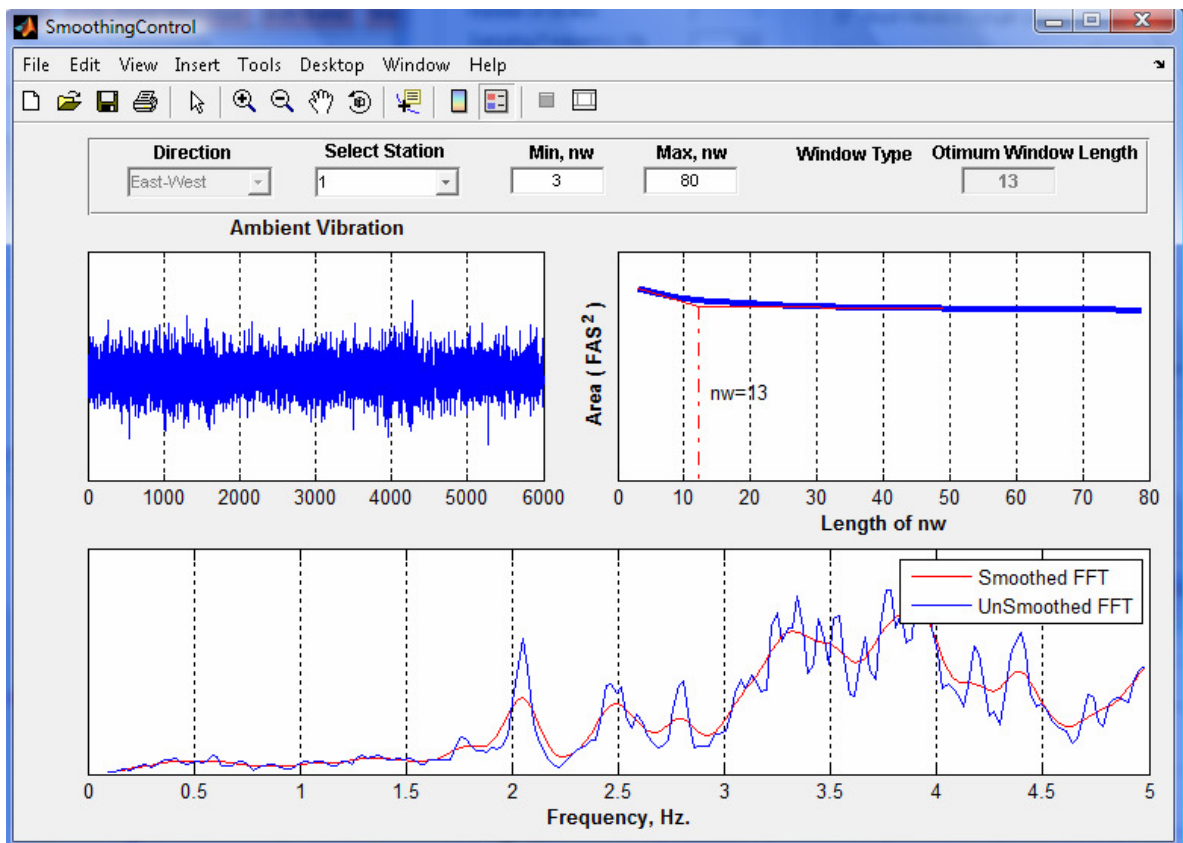


Figure 6.5. Optimum smoothing length for station 1 in east-west direction

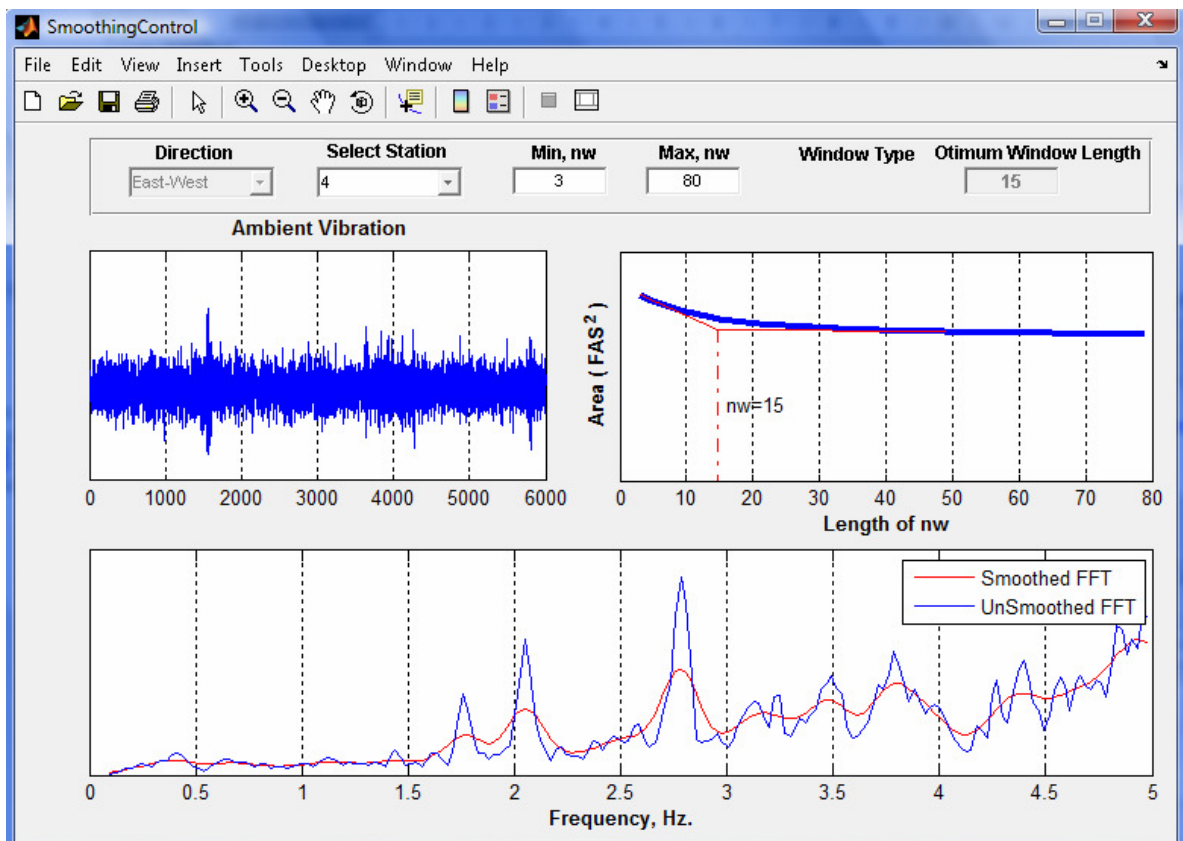


Figure 6.6. Optimum smoothing length for station 4 in east-west direction

6.1.6.2. Selection of Modal Property Calculation Types. There are four different spectrum estimation techniques available in KOERI-MIDS. The details of these techniques can be found in Chapter 3.1. Three of which estimates the power spectrum of the signal while the last one calculates the smoothed FAS of the signal. Among different estimation techniques, smoothed FAS algorithm is selected as the default estimation technique to calculate the modal frequencies of the structure. However, different estimation techniques may result in different modal frequency estimation. The difference may vary with the estimation technique (model) used and the order of the model chosen.

There are two different modal damping estimation techniques namely the Half-Power bandwidth and Logarithmic decrement as explained in Chapter 4.3. Half-Power bandwidth technique with FFT option is selected for the estimation of real-time modal damping ratio. As for real-time mode-shape, there is only one option available that is the Double Integration method. Since the only a few number of modes are excited during ambient vibration, the number of modes to be identified is set to two for Hagia Sophia museum as seen in Figure 6.7. It is intended to identify only the first two fundamental modes of the structure in north-south and east-west directions, respectively.

Selection of Modal Property Calculation

Spectrum Estimation Technique

Model Order

☐ BURG

☐ Yule-Walker

☐ Unconstrained Least-Square

☐ MUSIC

☐ ESPRIT

Window Length

☒ Smoothed FFT Algorithm

☐ Save Identified Modal Frequencies

Damping Estimation Techniques

☐ Logarithmic Decrement

☒ Half-Power Bandwidth by

☐ Save Identified Modal Damping Ratios

Mode Shape Estimation Techniques

☒ Double Integration

Number of Modes to be Identified

Output File Name

Figure 6.7. Modal poroperty calculation

6.1.6.3. Selection of Frequency Bands. Fifteen minutes of ambient vibration data are collected from all the stations installed on the structure in order to determine the frequency band limits of the Hagia Sophia Museum as suggested in Chapter 5.4. The real-time data analysis technique that is explained in Chapter 4.1 is used for the determination of fundamental modal frequencies of the structure. Since the collected data set is an ambient vibration data, only a few modes of the structure is excited. Therefore, only first two fundamental modal frequencies are identified at frequencies of 1.758 Hz and 2.051 Hz in north-south and east-west directions, respectively. Therefore, two frequency bands are defined around these identified modal frequencies for the real-time modal identification of the structure. The first frequency band is defined at frequencies of 1.96 Hz and 2.13 Hz in north-south direction while the second frequency band is defined at frequencies of 1.66 Hz and 1.88 Hz in east-west direction. The frequency band limits are defined at frequencies where the amplitude of the modal frequency attains its minimum. However, the minimum amplitudes do not always coincide in a single frequency for each measurement station in the structure. Therefore a common average is selected among the minimum amplitudes as the limits of the frequency bands as seen in Figure 6.8.

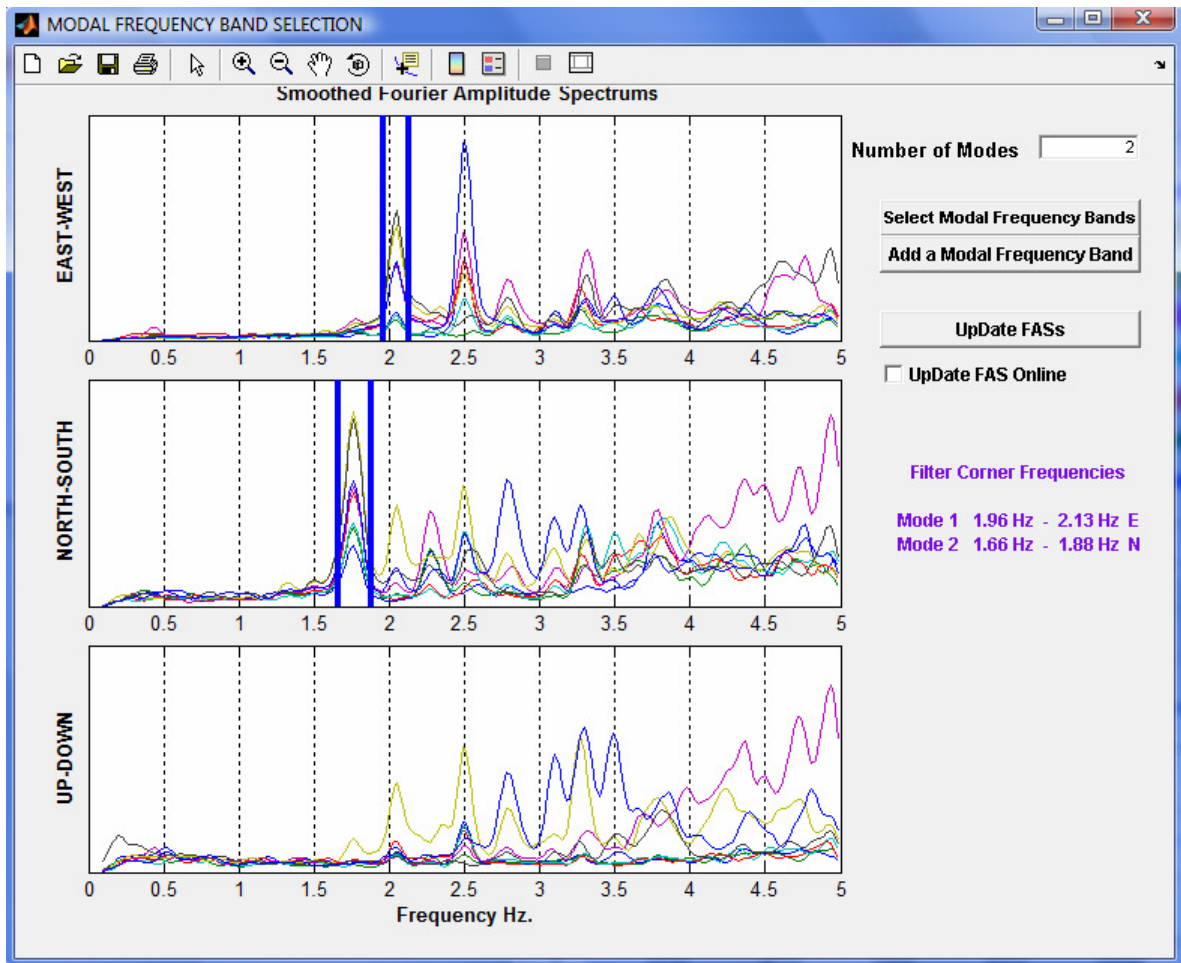


Figure 6.8. Frequency band selection

6.1.7. Results of Real-Time Modal Identification of Hagia Sophia

Twenty three hour length of real-time ambient vibration data was collected from Hagia Sophia and was analyzed using *KOERI-MIDS* as seen in Figure 6.9. The starting time of the real-time vibration data is 08:00 am, 17th of March, 2009 and the end time is 06:00 am, 18th of March, 2009. The first and the second fundamental modes of the structure are identified based on the settings defined in Chapter 6.1.6. Even though the total length of the real-time data is twenty three hours, there are some gaps in the data due to several reasons such as malfunctioning of ADSL modem, problems in software or hardware and so on. In such cases, modal identification is not performed for these data segments.

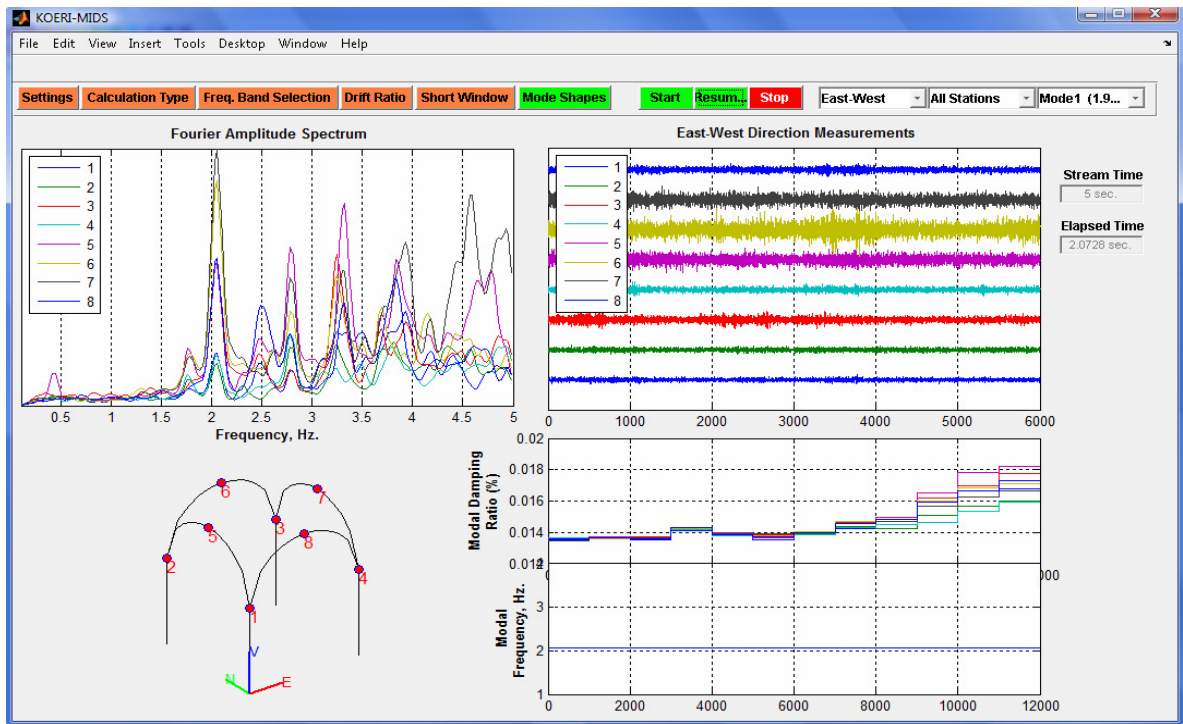


Figure 6.9. Ambient vibration of KOERI-MIDS

One hour duration of real-time raw data can be seen in Figure 6.10 for all stations installed in Hagia Sophia. It is observed that the maximum mean acceleration amplitudes at the top of main arches are amplified by a factor of at least two in both East-West and in North-South directions when compared to that of the amplitudes at the top of main columns as seen in Figure 6.10.

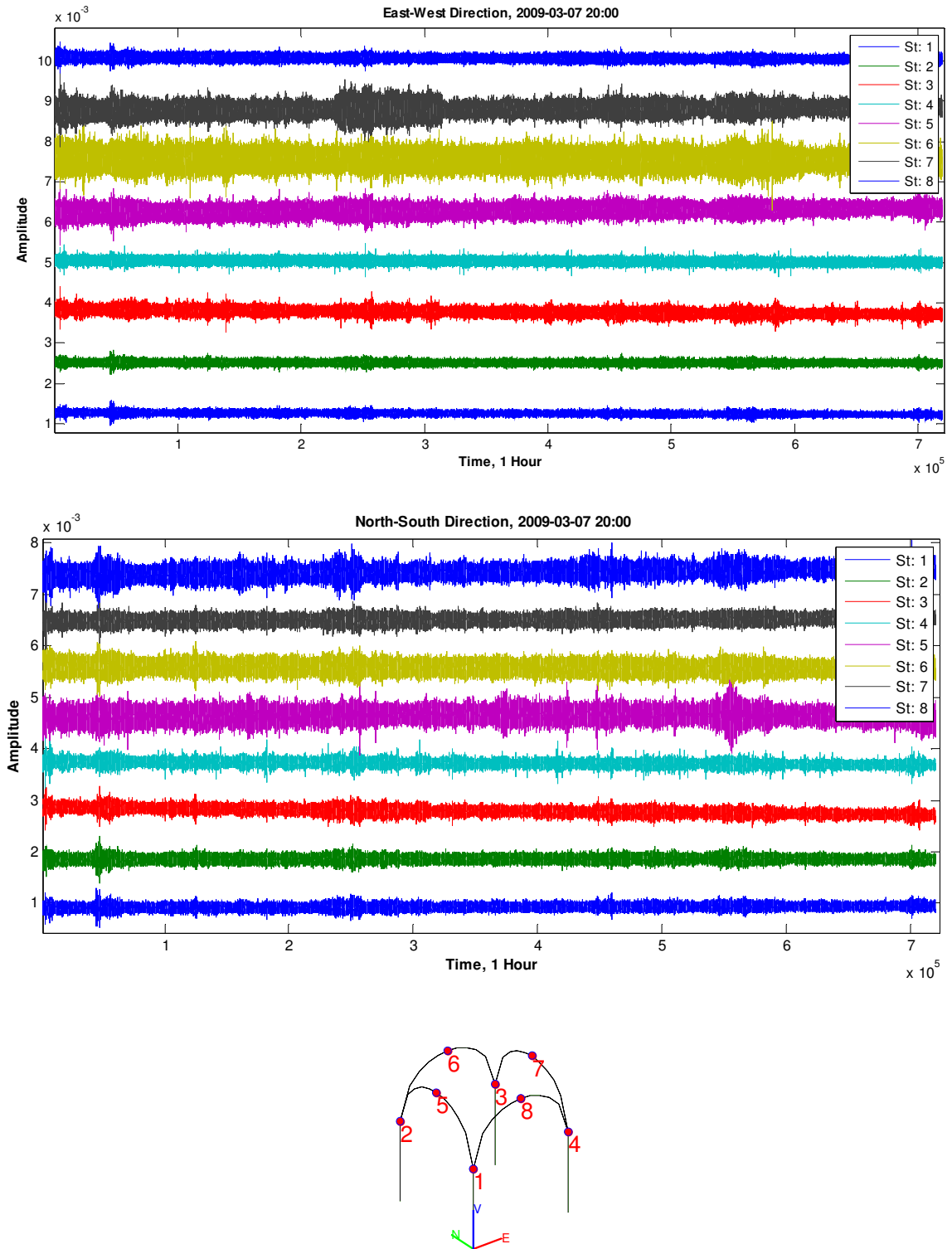


Figure 6.10. One hour real-time ambient vibration data and instrument locations in Hagia Sophia, accelerations are amplified by a factor of two in both East-West and North-South directions

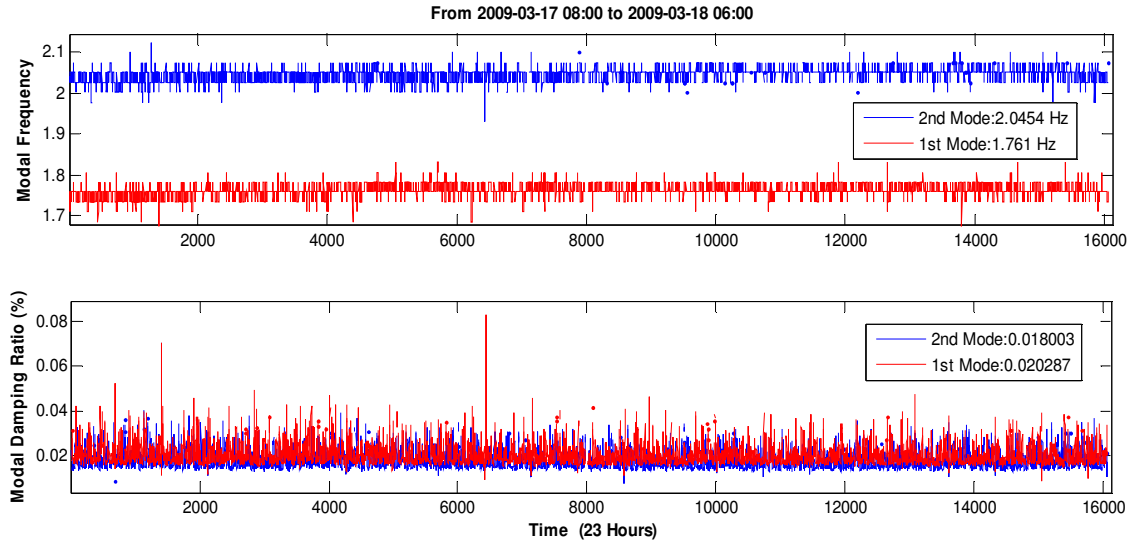


Figure 6.11. Time variations of the first two predominant modal frequency and damping ratios of the Hagia Sophia for twenty three hours (Joint Number 1)

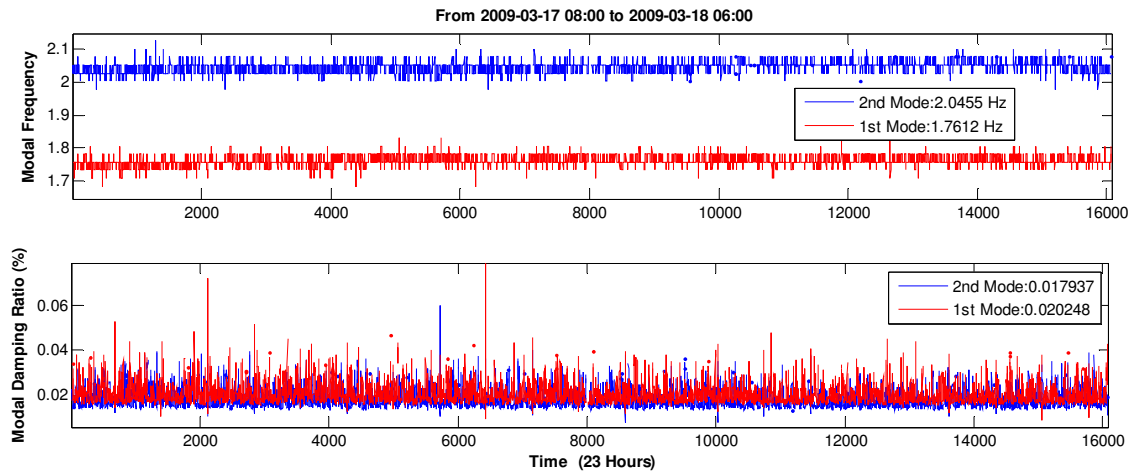


Figure 6.12. Time variations of the first two predominant modal frequency and damping ratios of the Hagia Sophia for twenty three hours (Joint Number 2)

Ambient vibration data is analyzed for twenty three hours duration and the results are depicted in Figure 6.11 and Figure 6.12 for joint number 1 and 2, respectively. The result shows that mean value of the first two predominant modal frequencies for joint number 1 during twenty three hours duration are 1.761 Hz and 2.05 Hz in East-West and North-South directions, respectively. Time variations of the first two predominant modal frequencies do not show any significant variations for the joint number 2. Similar results were obtained for the rest of instrument locations. Nevertheless, these results could not be

correlated with respect to temperature and wind velocity because those data were not measured during the ambient vibration. The modal damping ratios of the first two modes are very small. It is well known that the damping is proportional to the amplitude of the vibration. Since the amplitudes of the vibrations (real-time acceleration data) during the ambient vibration are not large enough, the damping in the structure is not considerably large.

Durukal *et al.* (2001) have conducted a research study about the changes in modal frequencies of Hagia Sophia during an ambient vibration and during different earthquakes in August, 1999. Ambient vibration test gives first natural frequency at 1.85 Hz in North-South direction while the second natural frequency is at 2.10 Hz in East-West direction. It is observed that the modal frequencies change with respect to the ambient results based on the considered ground shaking as seen in Table 6.1. It is seen that there is significant drop in both the first modal frequency and the second modal frequency.

Table 6.1. Change of modal frequencies of Hagia Sophia

Event	Hagia Sophia	
	1st Modal Frequency	2nd Modal Frequency
Ambient	1.85 Hz	2.10 Hz
17.08.1999	1.41 Hz	1.59 Hz
13.09.1999	1.43 Hz	1.61 Hz
21.09.1999	1.51 Hz	1.71 Hz
29.09.1999	1.80 Hz	1.74 Hz
21.10.1999	1.42 Hz	1.71 Hz
07.11.1999	1.61 Hz	1.79 Hz
11.11.1999	1.45 Hz	1.65 Hz

Results of KOERI-MIDS are compared with respect to that of study of Durukal. The difference in the identified modal frequencies is negligible for ambient vibration event, 8% and 2% in North-South and East-West directions, respectively.

Keypour, 2001 prepared a thesis about the non-linear dynamic analysis of historical structures and application to Hagia Sophia Mosque. According to the finite element model prepared in LUSAS (finite element analysis software), the first two modal frequencies of the structure are identified as 1.845 Hz and 2.148 Hz in east-west and north-south

directions, respectively. The results of the modal analysis in LUSAS have been found in good agreement with the ambient vibration tests results in KOERI-MIDS.

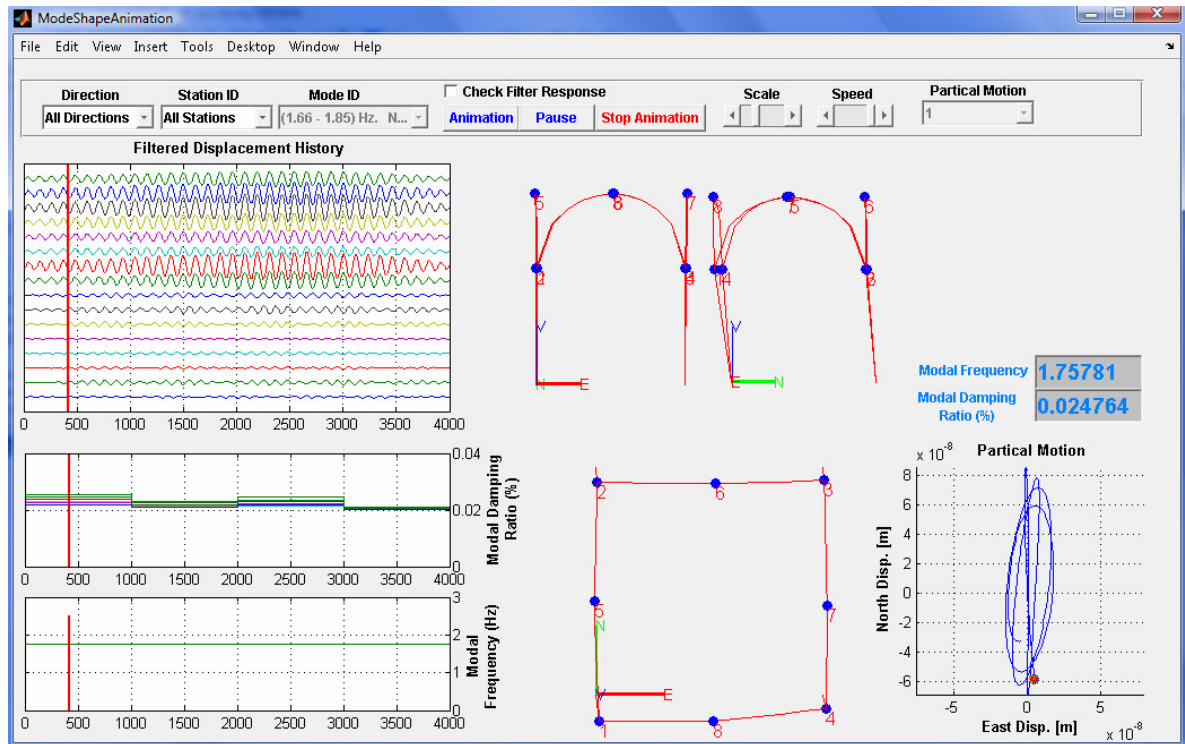


Figure 6.13. First mode shape in east-west direction and particle motion for joint number 1

The first two mode shapes of the structure are identified using KOERI-MIDS as seen in Figure 6.13 and Figure 6.14. The first two modal frequencies as seen in the *Modal Frequency* graph indicate a unique frequency for all instrument locations. The first two mode shapes are identified at 1.76 Hz and 2.02 Hz in north-south and east-west directions, respectively. The particle motion of joint number 1 for the first mode shape shows that the motion is strongly dominated in north-south direction while the same joint motion for the second mode shows that the mode is dominated in two directions.

The raw acceleration data is band-pass filtered around the identified modal frequencies; $f_1=1.66$ Hz, $f_2=1.85$ Hz for north-south direction and $f_1=1.97$ Hz, $f_2=2.12$ Hz for east-west direction. The filtered data is then integrated twice to calculate the displacement time history of all instrument locations. The result is the displacement history of the instrument locations in sinusoidal wave form as seen in *Filtered Displacement History* graph. The amplitudes of this wave form correspond to the amplitudes of the mode

shape. The mode shapes are presented in three different directions; one plan view and two vertical views. The mode shape animations of three different views are all synchronized.

The identified modal damping ratios are shown in the mode shape animation window. The modal damping ratios are very small (around 0.02%) due to the low level of ambient vibration amplitudes. Each instrument installed in the structure results in different modal damping ratio for a specific frequency.

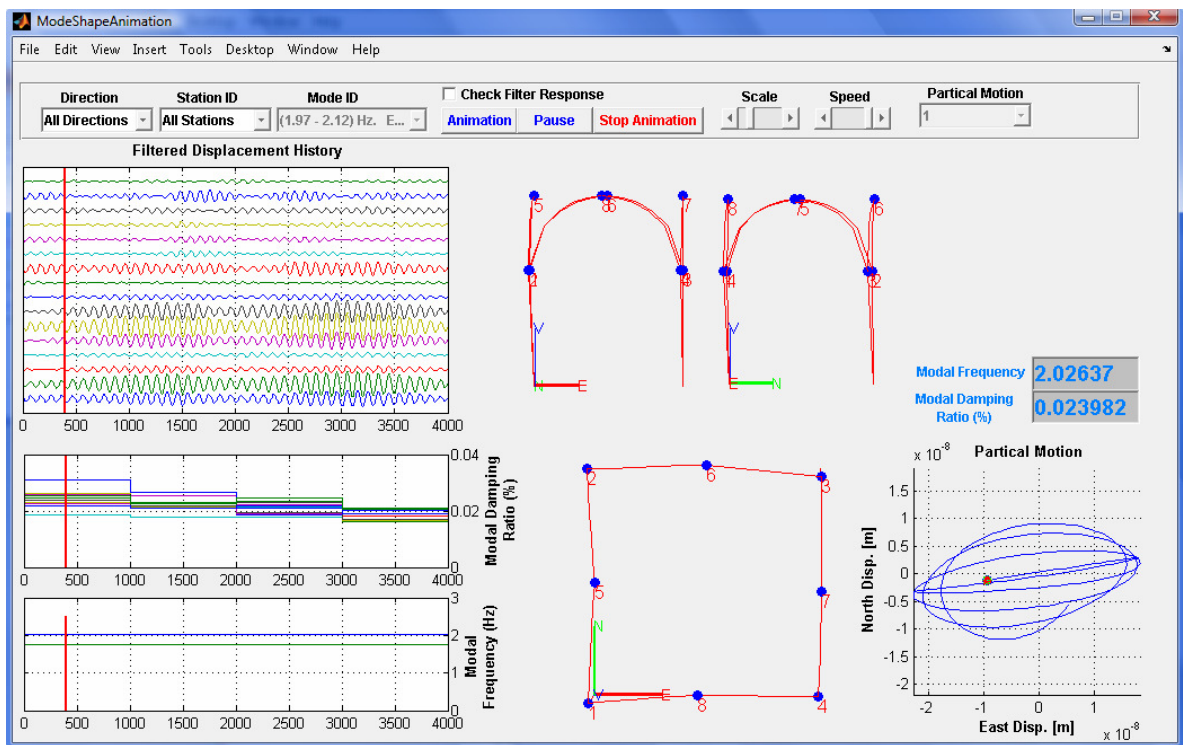


Figure 6.14. First mode shape in east-west direction and partial motion for joint number 1

7. CONCLUSION

Many important civil engineering structures have been built during the last decade in many developing countries. Due to their importance in emerging economies, a special attention is given to their state of health during their service life. These structures experience different level of external loads during their service life such as wind loads, live loads, earthquake loads, and temperature loads, etc. These different levels of loads may result in damages in structure. Some of the damages in a structure may be caused by extreme loads such as earthquakes, hurricanes, and blast loads, while other damages are the result of the structure's normal usage during its service life. For structural safety, it is an important issue to detect and locate the damage as soon as they occur. In this concept a structural health monitoring system (SHM) can be useful tool to detect and locate damage in structures. It is well known that the damages in structure results in the change in dynamic properties of the structure that may indicate a possibility for damage in the structure. Based on the SHM system used, a decision should be made automatically whether there is damage and if it is dangerous for the structure.

SHM has been emerged as a new research area in earthquake and civil engineering community. Visual inspection and evaluation of civil structures are usually done by means of site inspections. There are some undetectable structural damage that can be hidden in the structure. SHM usually provide enough information for the maintenance. There are many different intelligent SHM methods that have been developed for last decay. Rapid developments in the digital acquisition and processing speed enable fast and reliable damage detections using change in dynamic modal properties of the structures. In this concept the accurate estimation of dynamic modal properties of structures plays an important in SHM. Once the modal characteristics of the structures, such as the modal frequencies, the modal damping ratio and the mode shapes are identified, real-time changes in these modal properties can be related to detect damage and also to locate it in the structure.

There are different source of dynamic excitation for structures such as ambient vibration, experimental force vibration, earthquakes, strong winds, and underground explosions and so on. These dynamic excitations may cause damage in the structure which results in change in modal properties of the structure. These changes of modal properties with respect to different environmental conditions can be monitored by means of experimental tests such as impulse response and force vibration test. Among the different methods of full-scale testing, ambient vibration records were used to identify the structure's modal properties in real-time. The superiority of ambient vibration testing is due to the light-weight equipment requirement, easy of experimentation, and limited and minimum disturbance to the normal operation of the structure.

There are three main issues related to long-term seismic instrumentation of structures. These issues are; instrumentation layout, selection of appropriate sensors and other hardware, and data processing tools to be used. The basic requirements for practical and economical feasibility of seismic instrumentation hardware are reliability, easy of operation (including synchronizing), data communication, instrumentation maintenance, and cost effectiveness over the life of the instruments.

Ambient loads acting on the structure are usually remains unknown during an ambient vibration. Furthermore the amplitude of ambient vibrations is relatively smaller than that of earthquake induced vibrations. Therefore only a limited number of modes of the structure are excited. On the other hand it is know that only the first few modes of vibration are the significant participants in the structural response except some complicated and unsymmetrical structures. Therefore identifying the first few modes of the structure is usually enough to understand the general dynamic behavior of the structure.

Recorded raw data do not usually provide enough information about the modal identification of the structure due to the environmental conditions, instrument noise in the recorded signal, possible low-frequency drifts in the data, and other flaws in the sensors, installations and recordings. Therefore, the recorded raw data should be processed first before using it for identification. Data processing involves the removal of mean, windowing, removal

of linear trends, low-frequency drifts, outliers, and other unwanted frequencies by filtering, instrument corrections, and decimation if needed.

The accurate estimation of dynamic modal properties of structures plays an important role to determine and locate the damage in SHM. There are several methods that have been developed for the last decades to estimate the modal properties of a structure using ambient responses.

Power spectrum estimation techniques were used in order to estimate the modal frequencies of the structure. Two different power spectrum estimation methods are used; the parametric methods (Yule-Walker, Burg, and Unconstrained Least-square) and the eigen-analysis algorithms (multiple signal classification (MUSIC), estimation of signal parameters via rotational invariance techniques (ESPRIT)). All methods have their own advantageous and disadvantageous. Even though one method may give better results compared to others for a specific problem, the selection of which method to use is still depends on the nature of the problem.

As for the identification of modal damping, the half-power bandwidth technique is used. The half-power band width technique is based on the response of a structure under harmonic excitation. It is shown that the total response of harmonically excited system converges to the steady-state response in some time. Making use of the deformation response function that is calculated based on the Steady-state response, the half-power points are calculated that yields to damping ratio of the system later. The real-time mode shape identification is performed by the integration method. It estimates the mode shapes based on the double integration of the measured data that is band-pass filtered around the identified modal frequency.

The response of a structure is measured continuously in real-time. Therefore, they are infinitely long data sequences. Such a long duration of data cannot be analyzed one at a time due to the limited memory of digital computers. Hence the structural vibration data must be truncated into equal size of data segments in order to analyze them in real-time. It is also known that the ambient vibration data is usually contaminated with high amplitude noise due to

environmental conditions and mechanical imperfections in the instruments. To overcome this problem, the truncated data are segmented into fixed-size of running windows and each window is processed separately. Successive running windows were analyzed one at a time and the output of each running window were averaged to form the overall output for the truncated data segment. The effect of noise in the truncated data segment is reduced by averaging.

It is, of course, assumed that the structure remains elastic within the length of truncated data segment. Even though this assumption does not hold true for earthquake induced vibrations, it is acceptable for most of the structures under ambient loads. Therefore, averaging the outputs of running windows will results in an overall output for the truncated data segment that is almost identical to the output obtained if the truncated data had been processed one at a time under ambient vibrations.

The main purpose of monitoring of structures under ambient vibrations is not only to build more intelligent civil infrastructures but also to control the health state of structures in real-time. It is obvious that the availability of a real-time modal identification programs is crucial for on-line control and monitoring of structures under ambient loads. Therefore the real-time modal identification software to be used in SHM was developed in this dissertation. The software, coded in MATLAB, allows constructing a mathematical model of a dynamic system that is based on the measured response of the structure.

The name of the software is KOERI-MIDS. It features graphical user interface that is matched in terms of easy-of-use. Creation of the general dimensions of the structure, execution of real-time analysis and production of outputs are all accomplished using several user interfaces. A single structure can be used for a variety of different types of real-time analysis. The software feature some complicated capabilities, such as monitoring the real-time measured responses, truncation of real-time measurements, calculation of power spectrums and smoothed FAS, performing complicated signal processing tools, calculating the real-time modal properties (modal frequency, damping ratio, and mode shape), production of outputs, and calculation of many strong motion parameters in real-time. The entire above defined features are performed in the same run based on the real-time response of the structure under

ambient vibration. The developed program can be used for different type of structure, such as concrete, steel, and masonry.

REFERENCES

- Akaike, H., 1969, "Power Spectrum Estimation Through Autoregression Model Fitting", *Annals of the Institute of Statistical Mathematics*, Vol. 21, pp. 407-149, July.
- Akaike, H., 1974, "A New Look at the Statistical Model Identification", *IEEE Trans. Automatic Control*, Vol. AC-19, pp. 716-723, December.
- Asmussen, J.C., 1997, *Modal Analysis Based on the Random Decrement Technique*, Ph.D. Dissertation, University of Aalborg, Denmark.
- Bedewi, N.E., 1986, *The Mathematical Foundation of the Auto and Cross Random Decrement Technique and the Development of a System Identification Technique for Detection of Structural Deterioration*. Ph.D. Dissertations, University of Maryland.
- Brincker, R., L.M. Zhang, and P. Anderson, 2000, "Modal Identification from Ambient Response using Frequency Domain Decomposition" *Proceedings of 18th IMAC*, San Antonio, TX, USA.
- Brownjohn, M.W., 1995, "Estimation of Damping in Suspension Bridges", *Proceedings of Institution of Civil Engineers Structures and Buildings*, Vol. 104, pp. 401-415, November.
- Brownjohn, M.W., and A.A. Dumanoglu, and R.T. Severn, 1992, "Ambient Vibration Survey of the Fatih Sultan Mehmet (Second Bosphorus) Suspension Bridge", *Earthquake Engineering and Structural Dynamics*, Vol. 21, pp. 881-906, April.

- Burg, J.P., 1968, A New Analysis Technique for Time Series Data, *NATO Advanced Study Institute on Signal Processing with Emphasis on Underwater Acoustics*, August 12-23. Reprinted in *Modern Spectrum Analysis*, D. G. Childers, ed., IEEE Press, New York.
- Cakmak, A.S., R. Davidson, C.L. Mullen, and M. Erdik, 1993, *Dynamic Analysis and Earthquake Response of Hagia Sophia*, Structural Repair and Maintenance of Historical Buildings III, Computational Mechanics Publications, pp. 67-84.
- Chopra, A.K., 2001, *Dynamics of Structures: Second Edition*, Prentice Hall, New Jersey.
- Durukal, E., 1992, *Structural Identification and Seismic Vulnerability Assessment of Hagia Sophia*, Ph.D. Dissertation, Bogazici University.
- Durukal, E., Ö. Yüzügüllü, and K. Beyen, 1998 "Non-destructive testing techniques of Structural Materials in Historical Structures.", *Compatible Materials Recommendations for the Preservation of European Cultural Heritage PACT 55*, pp. 221-230.
- Durukal, E., S. Cimilli, and M. Erdik, 2000, "Dynamic Response of Hagia Sophia and Süleymaniye in Istanbul Inferred from the Recordings of 1999 Kocaeli and Düzce Earthquakes", *Compatible Materials Recommendations for the Preservation of European Cultural Heritage PACT 59*, pp. 19-28.
- Durukal, E., M. Erdik, and S. Cimilli, 2001 "Strong Motion Networks: A Tool for the Assessment of Earthquake Response of Historical Monuments", *Strong Motion Instrumentation for Civil Engineering Structures*, pp. 209-228.
- Erdik, M., E. Durukal, Ö. Yüzügüllü, K. Beyen and U. Kadakal, 1993, *Strong-motion instrumentation of Hagia Sophia and the Analysis of Response to an Earthquake of 4.8 magnitude*, Structural Repair and Maintenance of Historical Buildings III, Computational Mechanics Publications, pp. 99-114.

- Ho, B.L., and R.E. Kalman, 1996, "Effective Construction of Linear State-Variable Models from Input/Output Data.", *Regelungstechnik*, Vol. 14, pp. 545-548.
- Cole, H.A., 1971a, *Failure Detection of a Space Shuttle Wing Flutter Model by Random Decrement*. NASA TMX-62, 041, May.
- Cole, H.A., 1971b, *Method and Apparatus for Measuring the Damping Characteristics of a Structure*, United States Patent, No. 3, 620.069, November.
- Cole, H.A., 1973, "On-Line Failure Detection and Damping Measurement of Aerospace Structures by Random Decrement Signatures.", NASA CR-2205, March.
- Cole H.A., 1968, "On-the-Line Analysis of Random Vibrations.", AIAA, No. 68-288, AIAA/ASME Ninth Structures, Structural Dynamics and Materials Conference, Palm Springs, California.
- Ibrahim, S.R., 1977a "Random Decrement Technique for Modal Identification of Structures" *Journal of Spacecraft and Rockets*, Vol. 14, No. 11., Nov. pp. 696-700.
- Ibrahim, S.R., 1977b "The Use of Random Decrement Technique for Identification of Structural Modes of Vibration" *AIAA*, Vol. 77, pp. 1-9.
- Keypour, H., 2001, *Nonlinear Dynamic Analysis of Historical Structures and Application to Ayasofya Mosque*, Ph.D. Dissertation, Istanbul, Boğaziçi University.
- Lacoss, R.T., 1971, "Data Adaptive Spectral Analysis Methods", *Geophysics*, Vol. 36, pp. 661-675, August.
- Mark R., A.S. Çakmak, K. Hill and R. Davidson, 1993, *Structural Analysis of Hagia Sophia: a Historical Perspective*, Structural Repair and Maintenance of Historical Buildings III, Computational Mechanics Publications, pp. 33-46.

- Mathworks, 2007a, *System Identification toolbox for MATLAB*, Mathworks, Inc., Nattick, MA.
- Mathworks, 2007b, *Signal Processing toolbox for MATLAB*, Mathworks, Inc., Nattick, MA.
- Parzen, E., 1974, "Some Recent Advances in Time Series Modeling", *IEEE Trans. Automatic Control*, Vol. AC-19, pp. 723-730, December.
- Proakis, J.G., and D.G. Manolakis, 1996, "*Digital Signal Processing, Principles, Algorithms, and Applications*", Prentice Hall, New Jersey.
- Riassanen, J., 1983 "A Universal Prior for the Integers and Estimation by Minimum Description Length", *Annals of Statistics*, Vol. 11, pp. 416-431, November.
- Rojahn, C., and R.B. Matthiesen, 1977, "Earthquake Response and Instrumentation of Buildings." *Journal of the Technical Concils of ASCE*, Vol. 103, No. TC1, pp. 1-12, December.
- Schmidt, R.D., 1981, *A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation*, Ph.D. Dissertation, Department of Electrical Engineering, Stanford University, Stanford, California, November.
- Şafak, E., 1992, "Response of a 42-Storey Steel-Frame Building to the $M_s=7.1$ Loma Prieta Earthquake", *Engineering Structures*, Vol. 15, pp. 403-421, December.
- Şafak, E., and K. Hudnut, 2006, "Real-time Structural Monitoring and Damage Detection by Acceleration and GPS Sensors", *8th US National Conference on Earthquake Engineering*, San Francisco, California, pp. 18-22, April.

- Şafak, E., 2006, "On Estimation of Site Amplification from Ambient Ground Noise", *Proceedings of 1st European Conference on Earthquake Engineering and Seismology*, paper number, 713, September, Geneva, Switzerland.
- Şafak, E., 2001, "Analysis of Earthquake Records from Structures: An Overview", *Luwer Academic Publishers*, Printed in the Netherlands.
- Sotoudeh, V., and A.G. Brady, 1985, *Dynamic Study of Great Western Building, Berkeley, California*, USGS open file report in press, February.
- Udwadia, F.E., and N. Jerath, 1984, "Time Variations of Structural Properties During Strong Ground Shaking.", *Journal of the eighth World Conference of Earthquake Engineering*, San Francisco, Vol. 106, No. 1, pp. 111-121, February.
- Van, O.P, and B. De Moor, 1996, *Subspace Identification for Linear Systems: Theory – Implementation – Applications*. Kluwer Academic Publishers, Dordrecht, Netherlands.
- Zhang, L., T. Wang, and Y. Tamura., 2005, "Frequency-spatial Domain Decomposition Technique with Application to Operational Modal Analysis of Civil Engineering Structures." *Proceedings of IOMAC*, Copenhagen, Denmark, April.