INCORPORATION OF FOREIGN EXCHANGE RISK TO FAMA-FRENCH FACTOR MODEL: A STUDY ON BORSA İSTANBUL

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF APPLIED MATHEMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

FURKAN HÖÇÜK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
FINANCIAL MATHEMATICS

FEBRUARY 2022

Approval of the thesis:

INCORPORATION OF FOREIGN EXCHANGE RISK TO FAMA-FRENCH FACTOR MODEL: A STUDY ON BORSA İSTANBUL

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ABSTRACT

INCORPORATION OF FOREIGN EXCHANGE RISK TO FAMA-FRENCH FACTOR MODEL: A STUDY ON BORSA İSTANBUL

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February 2022, 172 pages

This empirical study compares the relative performances of the Fama-French five-factor model without foreign exchange risk and the five-factor model incorporating foreign exchange risk on capturing portfolio returns in Borsa İstanbul. The main contribution of our study to the asset pricing literature is the incorporation of FX risk to the Fama-French five-factor model. We propose an additional factor as a proxy for FX risk.

Another contribution of this study is implementing a machine learning technique, support vector regression (SVR), to estimate portfolio returns through the FF5F model without FX risk and FF5F model incorporating FX risk for Borsa İstanbul stocks. Although there are numerous researches investigated on Borsa İstanbul, any other study did not implement SVR via CAPM or Fama French multi-factor models to the best of our knowledge.

There are empirical studies that confirm the efficiency of SVR. Some studies also compare the performance of the linear factor regression method with alternative statistical tools, including machine learning methods. Our study stands out in combining predictions of simple linear regression and SVR methods. Optimal weights obtained from linear combinations imply more precise estimations through

SVR. In 28 out of 36 combinations, we observed that optimal weights assigned to SVR estimations were greater than those assigned to SLR estimations. Linear regression methods may be too restrictive to reflect the non-linearity of factor exposures under the Fama-French multi-factor model scheme. Asset pricing models, which take nonlinear aspects of the stock markets into consideration, might generate more precise estimations.

Keywords: Fama-French**,** Foreign Exchange Risk, Multi-Factor Models, Borsa İstanbul, Support Vector Regression, Forecast Combinations

ÖZ

DÖVİZ KURU RİSKİNİ İÇEREN FAMA-FRENCH FAKTÖR MODELİ:
BORSA İSTANBUL ÜZERİNE BİR ÇALIŞMA

Höçük, Furkan
Yüksek Lisans, Finansal Matematik Bölümü
Tez Yöneticisi: Doç. Dr. Esma Gaygısız

Şubat 2022, 172 sayfa

Bu ampirik çalışmada, Fama-French Beş Faktör modeli ile söz konusu modelin, yabancı para risk faktörünü içeren versiyonun, Borsa İstanbul'da işlem gören hisse senetlerinin getiri tahminlerine ilişkin performansları karşılaştırılmaktadır. Bu çalışmanın varlık fiyatlaması literatürüne temel katkısı, Fama-French Beş Faktör modeline döviz kuru riskinin eklenmesidir. Döviz kuru riski göstergesi olarak ilave bir faktör tanımlanmıştır.

Bu çalışmanın diğer katkısı, bir makine öğrenme tekniği olan, destek vektör regresyon (DVR) yöntemiyle döviz kuru riskini içermeyen FF5F modeli ile döviz kuru riskini içeren FF5F modeli kullanılarak Borsa İstanbul hisse senetleri için portföy getirilerinin tahmin edilmesidir. Borsa İstanbul üzerine yapılan çok sayıda ampirik çalışma bulunmasına karşın, bildiğimiz kadarıyla DVR yöntemiyle SVFM veya Fama-French çoklu faktör modelleri kullanılmak suretiyle Borsa İstanbul üzerine uygulanan başka bir çalışma bulunmamaktadır.

DVR yönteminin etkliğini gösteren ampirik çalışmalar bulunmaktadır. Ayrıca bazı çalışmar doğrusal faktör regresyon yöntemi ile makine öğrenme tekniği gibi alternatif yöntemlerin performanslarını karşılaştırmaktadır. Çalışmamız, basit doğrusal regresyon ve DVR tahminlerinin birleştirilmesi suretiyle önce çıkmaktadır. Doğrusal birleşimler sonucunda hesaplanan optimum ağırlık değerleri DVR tahminlerinin daha güçlü olduğunu işaret etmektedir. 36 kombinasyonun 28'inde, DVR tahminlerine ait optimum ağırlık değerlerinin, basit doğrusal regresyon tahminlerine ait ağırlıklardan daha yüksek olduğu gözlenmektedir. Doğrusal regresyon yöntemi, Fama-French çoklu faktör modeli bağlamında, faktörlerin doğrusal olmayan etkilerini yansıtamamaktadır. Hisse senedi piyasalarının doğrusal olmayan boyutunu göz önünde bulunduran varlık fiyatlama modelleri daha sağlıklı tahminler üretebilirler.

Anahtar Kelimeler: Fama-French, Döviz Kuru Riski, Çoklu Faktör Modelleri, Borsa İstanbul, Destek Vektör Regresyonu, Tahmin Kombinasyonu

To My Mother

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Assoc. Prof. Dr. Esma Gaygısız for her patience and overwhelming support throughout the research process. I consider myself extremely lucky to have the opportunity to perform this study under her supervision. I could not even imagine what this thesis be like without her guidance. I also thank thesis committee members, Assoc. Prof. Dr. Gül İpek Tunç and Assoc. Prof. Dr. Didem Pekkurnaz, for their insightful comments.

There is no way to express my appreciation to my mother, Nurhan, my father, Hüseyin, and my brothers, Burhan & Burak. They were always there for me during my MSc studies. Their encouragement inspired me with confidence to pursue my research.

I would also like to thank my friends Cem Toker and Gürcan Arıcı for their moral support and valuable feedbacks throughout my studies. I always felt their intimacy.

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LIST OF ABBREVIATIONS

AAV	Average absolute value
ADF	Augmented Dickey-Fuller
B/M	Book-to-market
BA	Intersection of portfolios composed of big size and aggressive investment stocks
BC	Intersection of portfolios composed of big size and conservative investment stocks
BH	Intersection of portfolios consisting of big size and high market-to-book ratio stocks
BI	Intersection of portfolios composed of big size and intermediate investment stocks
BIST100	Borsa İstanbul-100 Index
BL	Intersection of portfolios consisting of big size and low market-to-book ratio stocks
BLUE	Best Linear Unbiased Estimator
BM	Intersection of portfolios consisting of big size and middle profitability stocks
BN	Intersection of portfolios consisting of big size and neutral market-to-book ratio stocks
BOP	Intersection of portfolios consisting of big size and open FX position stocks
BPOZ	Intersection of portfolios consisting of big size and FX surplus stocks
BR	Intersection of portfolios consisting of big size and robust profitability stocks
BW	Intersection of portfolios consisting of big size and weak profitability stocks
CAPM	Capital Asset Pricing Model

CMA	Conservative minus aggressive
CPI	Consumer Price Index
E/P	Earnings-to-price
FF3F	Fama-French Three-Factor
FF5F	Fama-French Five-Factor
FX	Foreign Exhange
GLS	Generalized least squares
GRS	Gibbon, Ross and Shanken
HML	High minus low
JSE	Johanessburg Stock Exchange
LHS	Left hand side
ME	Market Equity
OLS	Ordinary least squares
PP	Phillips Perron
RMW	Robust minus weak
SA	Intersection of portfolios composed of small size and aggressive investment stocks
SC	Intersection of portfolios composed of small size and conservative investment stocks
SH	Intersection of portfolios consisting of small size and high market-to-book ratio stocks
SI	Intersection of portfolios composed of small size and intermediate investment stocks
SL	Intersection of portfolios consisting of small size and low market-to-book ratio stocks
SLB	Sharpe-Lintner-Black
SM	Intersection of portfolios consisting of small size and middle profitability stocks
SMB	Small minus big
SN	Intersection of portfolios consisting of small size and neutral market-to-book ratio stocks

SOP	Intersection of portfolios consisting of small size and open FX position stocks
SPOZ	Intersection of portfolios consisting of small size and FX surplus stocks
SR	Intersection of portfolios consisting of small size and robust profitability stocks
SVR	Support vector regression
SW	Intersection of portfolios consisting of small size and weak profitability stocks
VIF	Variance inflation factor

CHAPTER 1

INTRODUCTION

In the finance literature, asset pricing has always been an attention-grabbing subject that had never been out of date for decades since the first studies towards understanding the nature of the relation between risk and return. Researchers put endless efforts into exploring the dynamics of stock return movements and underlying risk factors. Academics conducted countless studies to define proxies for average stock returns and develop various solutions to best estimate stock prices under different settings. Sharpe [1], Lintner [2], and Black [3] introduced the asset pricing model (Sharpe-Lintner-Black, SLB model) to capture the relation between average returns and deviations. The model predicted that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz [4].

Fama & French [5] proved that size and book-to-market equity factors also affect the average stock returns as the market factor does. When Fama and French [5] developed the three-factor model, size and B/M were two well-known proxies for the average stock returns, which were left unexplained by the CAPM. The findings of Novy and Marx [6], Titman, Wei, and Xie [7], and others towards the three-factor model is incomplete for expected returns, led Fama and French to augment the three-factor model by adding investment and profitability factors. Hence, Fama and French [8] developed the five-factor model with investment and profitability exposures to the regression equation.

We evaluated the performance of the FF5F model without FX risk and the FF5F model incorporating FX risk to determine whether one outperforms the other. We also predicted excess returns of intersection portfolios using support vector regression method. Subsequently, we combined estimations of simple linear regression and support vector regression. We found that SVR outperforms SLR for both versions of the Fama-French five-factor with and without FX risk.

1.1 Foreign Exchange Exposure

The core motivation behind this study is to examine the performance of the Fama-French five-factor model incorporating FX risk in explaining deviations in expected stock returns. The FF5F model is extended by incorporating a 6^{th}-factor variable as a proxy for FX risk.

One factor for incorporating FX exposure to the Fama-French five-factor is the net FX position of non-financial companies in Turkey. Figure 1.1 depicts the evolution of real sector firms' FX position between 2002-21. There is an increasing trend in the net FX position of the non-financial companies for the corresponding period. In December 2002, real sector companies had a net FX position of USD 4.4 billion. In March 2018, the net FX position rose to USD 196.7 billion at its peak. Most recent data indicate that the net FX position is USD 115,4 billion as of November 2021. Companies[1] with short FX positions have been found to possess lower efficiency and profitability than those with long FX positions [9].

[11] The sample consisted of 30 firms among the companies included in BİST100 index, over Q3 2012-Q2 2015, 20 of which were manufacturing and 10 were service firms.

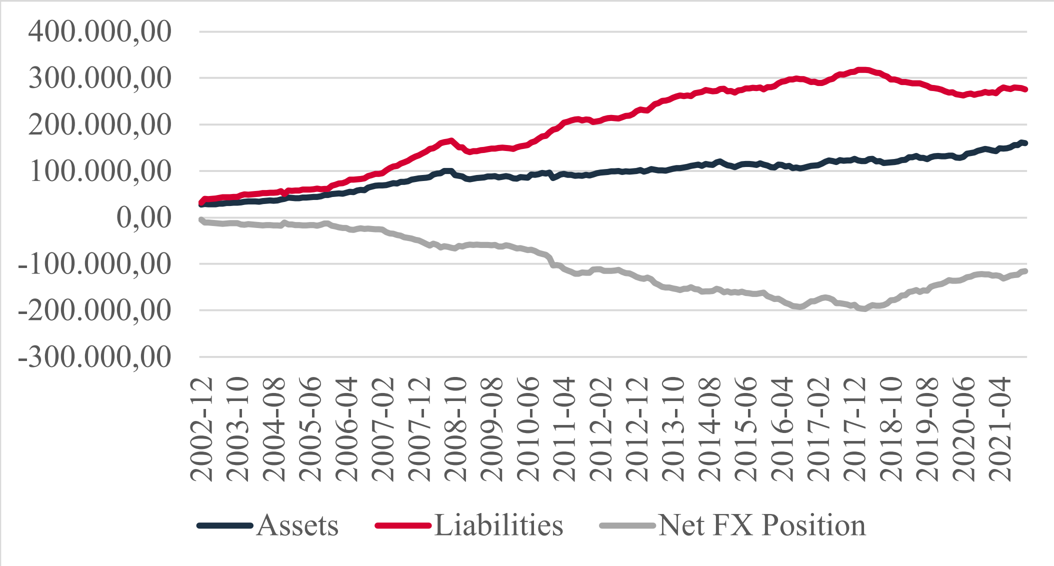

Figure 1.1: Foreign Exchange Assets and Liabilities of Non-Financial Companies (million US Dollars)

Source: The Central Bank of the Republic of Turkey, EVDS

Figure 1.2 portrays the comparison between the net FX position of non-financial companies and the aggregated total equity data of all companies.[2] We normalized these variables from -1 to +1. A decrease in net FX position indicates a worsening in the net FX positions of non-financial companies and vice-versa. Likewise, a decrease in total equity refers to a decrease in total equity values and vice-versa. Net FX position recorded a net decrease between 2009 and 2020, while the total equity index realized a net increase. Figure 1.2 points out that the spread between liabilities and assets denominated in foreign currencies recorded an increase as the companies grew until 2017. We observed improvements in the net FX position of non-financial companies in line with the increase in the total equity after 2017.

[2] We used aggregated total equity values of all companies because of lack of necessary data for non-financial companies.

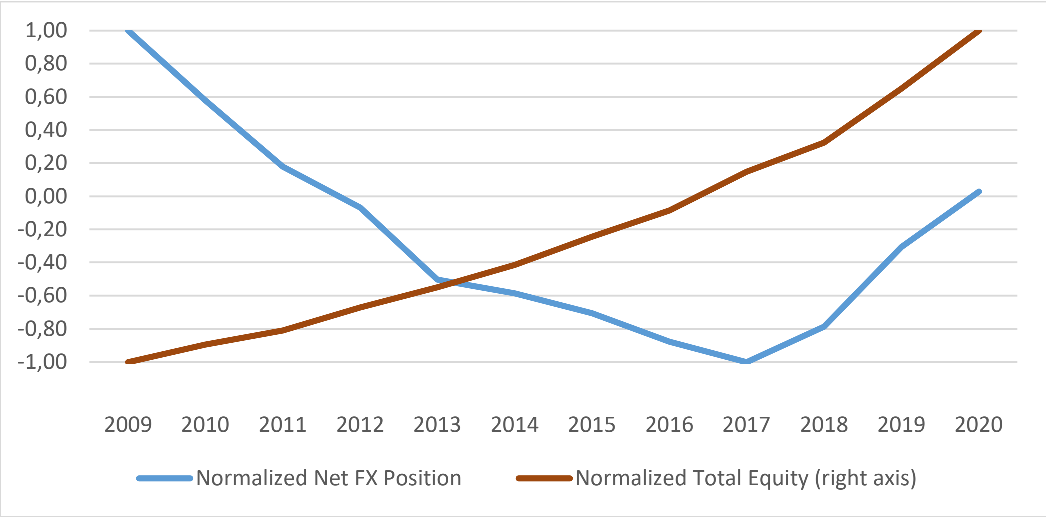

Figure 1.2: Net FX position of non-financial companies and aggregated total equity values of all Turkish companies (normalized)
Source: The Central Bank of the Republic of Turkey

Another factor for considering FX risk is the level of external loans. Figure 1.3 shows the developments in the external loans of non-financial companies in Turkey. We observed a significant jump in external loans between November 2005 and August 2008, the high level thenceforth until November 2021 had been sustained. External loans of non-financial companies realized as USD 102,8 billion in the same month of 2021.

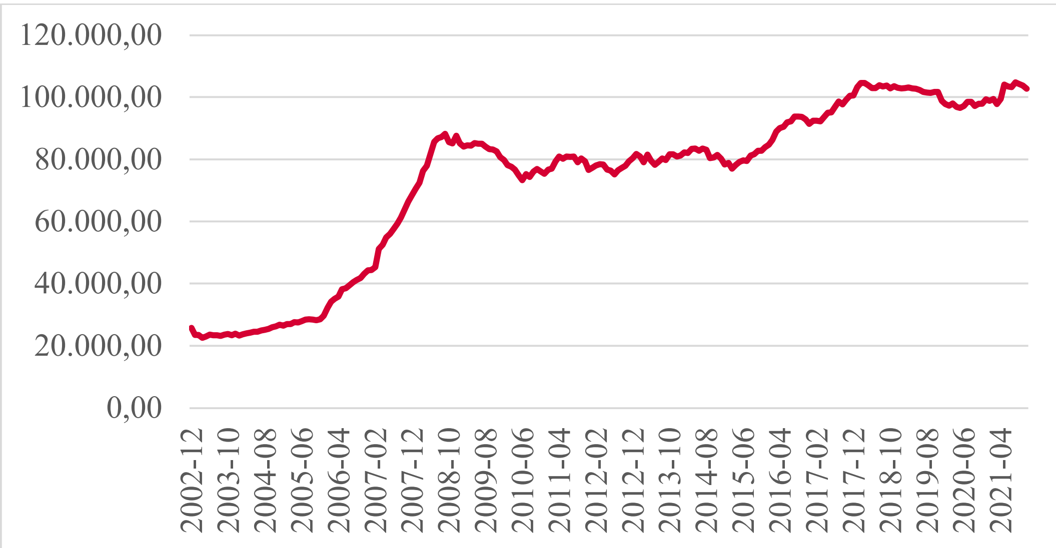

Figure 1.3: External Loans of Non-Financial Companies (million US Dollars)
Source: The Central Bank of the Republic of Turkey, EVDS

Ruch [10] argues that emerging countries' rising external, corporate sector, and sovereign weaknesses (between 2007 & 2018) make them more fragile against an adverse shock. He adds that the impact of financial stress on the growth of emerging economies would depend on their weaknesses and the degree policymakers would react.

We calculated the ratio of the sum of cash and non-cash loans denominated in foreign currencies to total cash and non-cash loans (Figure 1.4). The ratio has never fallen below 0.5 throughout 2009 and 2020. Turkish companies tend to use foreign currency loans overwhelmingly for the corresponding period.

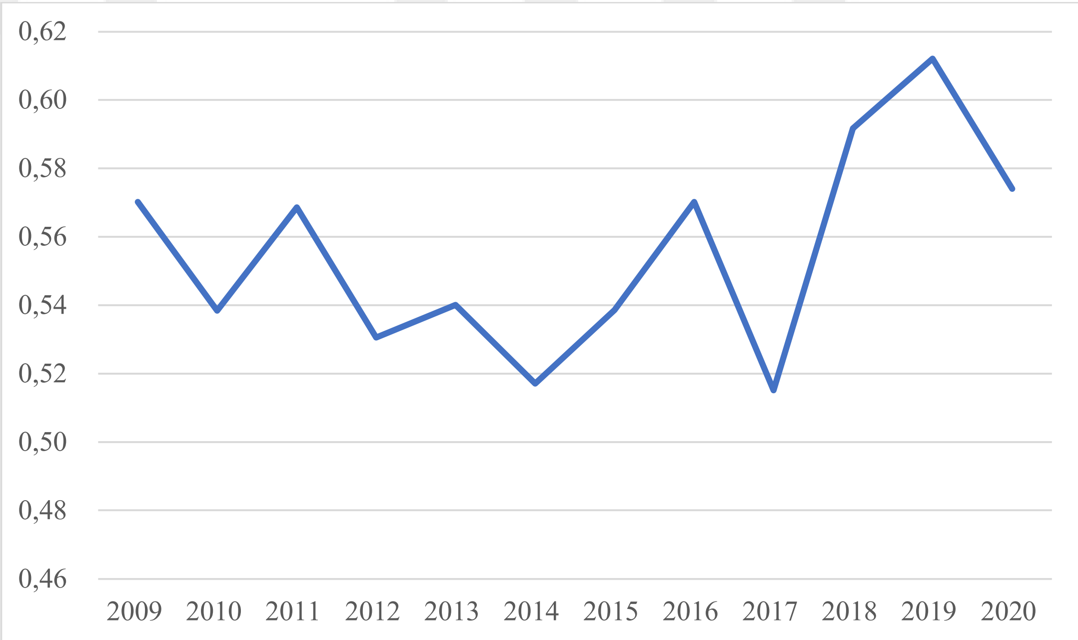

Figure 1.4: Ratio of Loans Denominated in Foreign Currencies to Total Loans
Source: The Central Bank of the Republic of Turkey

Another argument behind incorporating FX risk is the balance of payments developments. Figure 1.5 and Figure 1.6 portray annual and monthly current account statistics. The current account, excluding 2001 and 2019, recorded deficits between 1999-2021. The imbalances in the current account imply FX exposures to companies that have intensive use of imported inputs.

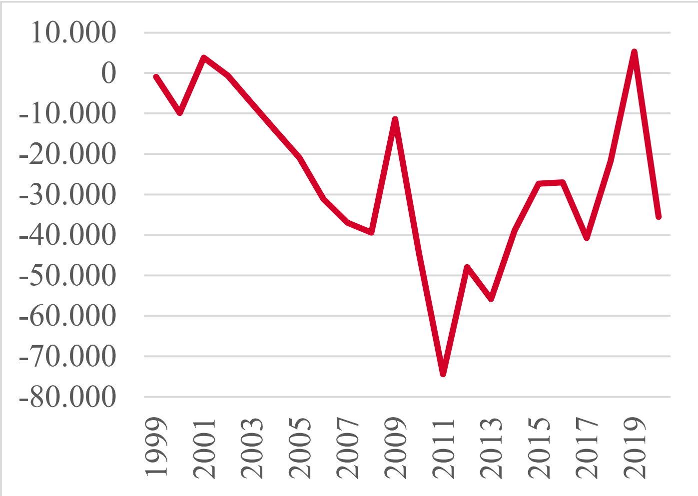

Figure 1.5: Current Account (million US Dollars, annual data) as of November 2021

Source: The Central Bank of the Republic of Turkey, EVDS

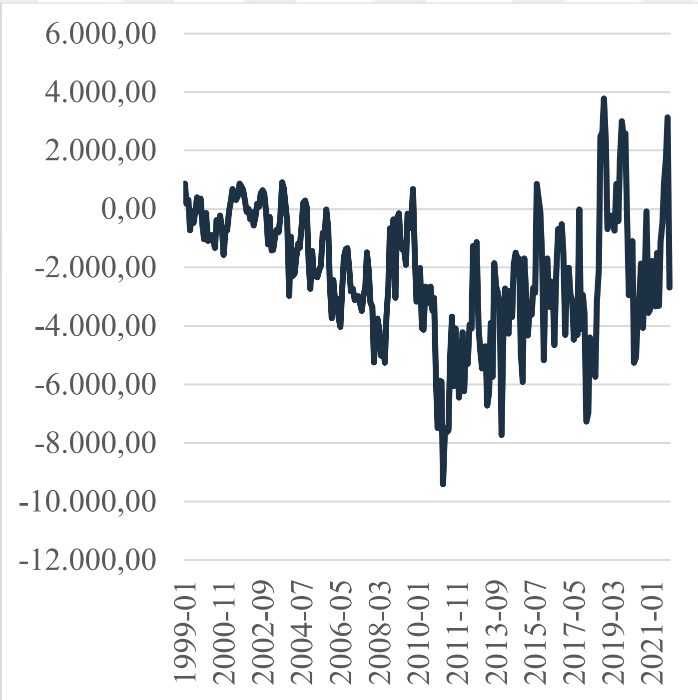

Figure 1.5: Current Account (million US Dollars, monthly data) as of November 2021

Source: The Central Bank of the Republic of Turkey, EVDS

Figure 1.7 depicts the ratio of the current account balance to GDP in US dollars between 1999 and 2020. In 2011, the ratio recorded the smallest value with -8.8%.

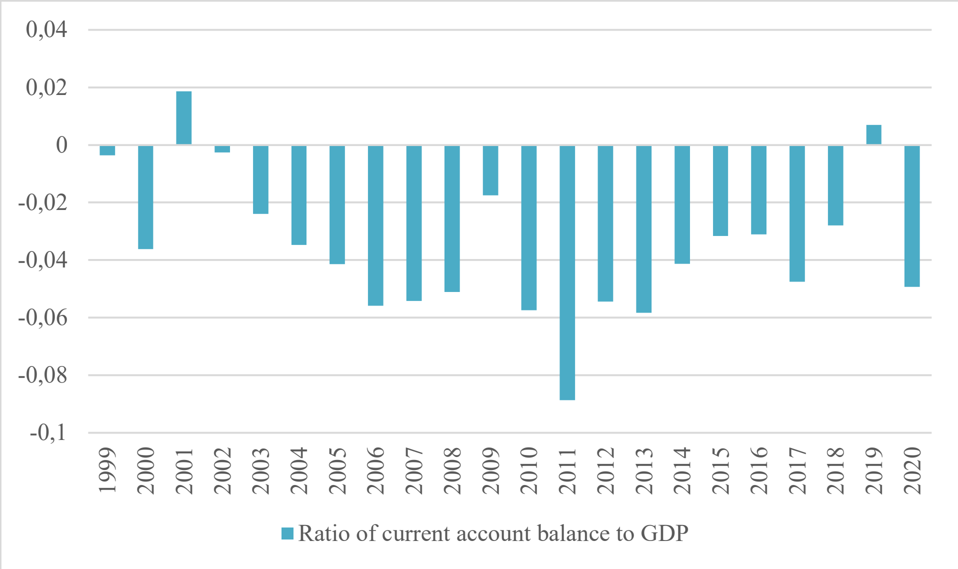

Figure 1.7: Ratio of Current Account Balance to GDP

Source: The Central Bank of the Republic of Turkey, World Bank

Figure 1.8 reveals the US Dollar-Turkish Lira and US Dollar-South African Rand[3] GARCH volatility predictions between 2001-2022. Volatility predictions indicate similar movements in the variability of two currencies against the US Dollar between 2002-2018. Nevertheless, US Dollar to Turkish Lira volatility predictions signify larger movements in the exchange rate in 2018 and subsequent periods.

[3] We added US Dollar-South African Rand GARCH volatility predictions to Figure 5, because South Africa is amongs BRICS (Brazil, Russia, India, China and South Africa). We avoided using volatility predictions of other BRICS currencies for simplicity issues.

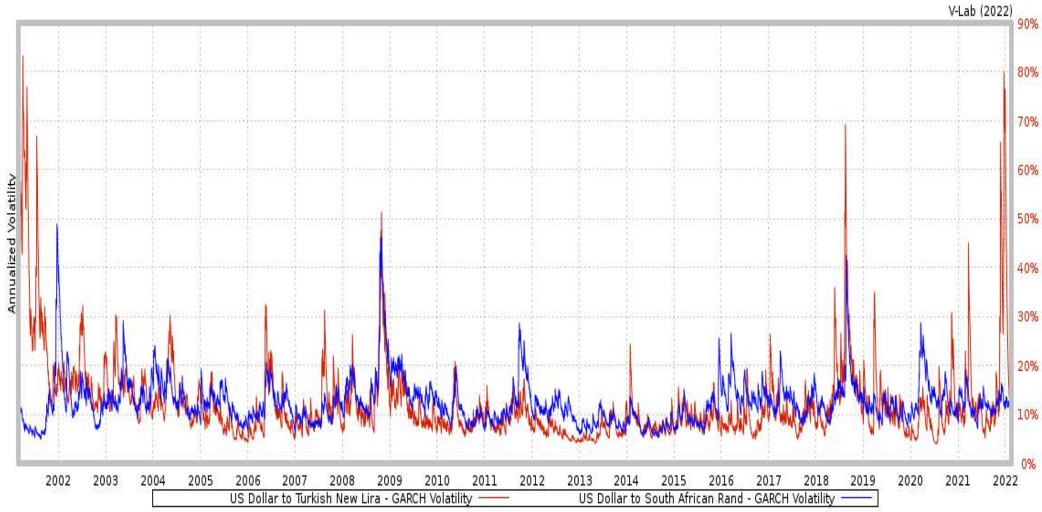

Figure 1.8: US Dollar-Turkish Lira & US Dollar-South African Rand GARCH Volatility Predictions

Source: The Volatility Laboratory (V-Lab)

Hajilee and Nasser [11] refer to the presence of theoretical and empirical studies that document the impact of exchange rate volatility on stock market performance. Their findings suggest a relationship between exchange rate volatility and stock returns.

Exporters' reliance on imported goods is another indicator that might impact FX exposure. Akgündüz & Fendoğlu [12] estimate the ratio of imported inputs in exports is 24%. Once bringing exporters' suppliers into the picture, the degree of reliance on imports is estimated at 45% for exporters in Turkey. They also found that exporters with high levels of reliance on imported goods or those working with suppliers with high levels of reliance are more likely to increase producer-currency export prices and avoid increasing export volumes in case of domestic currency depreciation.

Figure 1.9 shows exports, imports, and CPI-based real effective exchange rates. The real exchange rate depreciated by 36,7% between November 2017 and November 2021. However, the imports increased by 41,45% while the level of exports recorded an increase of only 26,5% for the corresponding period.

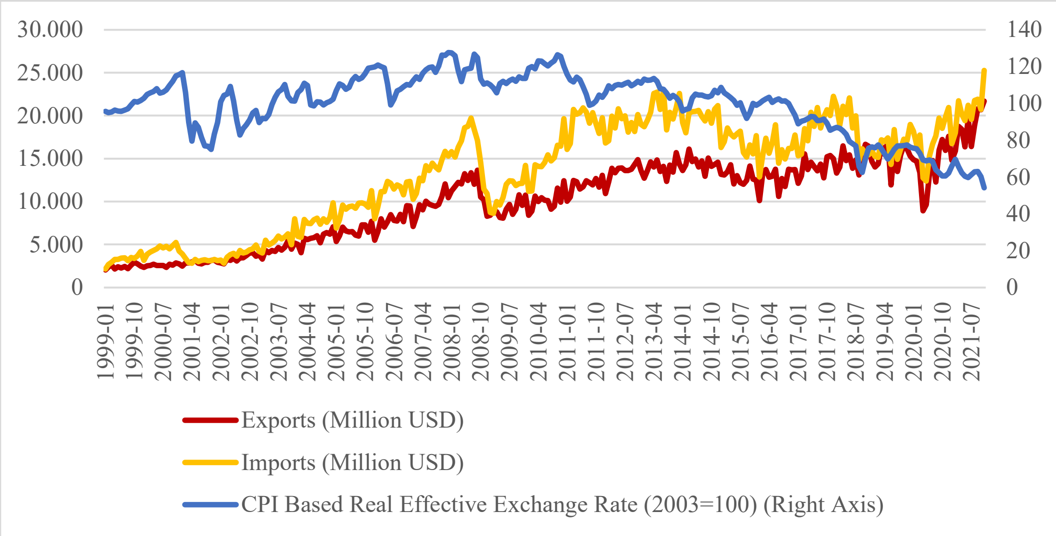

Figure 1.9: Exports, Imports and CPI Based Real Effective Exchange Rate Developments

Source: The Central Bank of the Republic of Turkey, EVDS

Toraganlı & Yalçın [13] argue that depreciation in the real exchange rate has a limited positive effect for Turkish exporters with a high level of reliance on imported inputs. Toraganlı & Yalçın also discuss that movements in the real exchange rates have a less substantial impact on exporters which have a balanced or low ratio of debt-to-exports, as they define such firms as "*naturally-hedge*".

Figure 1.10 shows the innovations in ratio of imports to exports between 1999-2021. Import-to-export ratios are above 100% for the majority of the period.

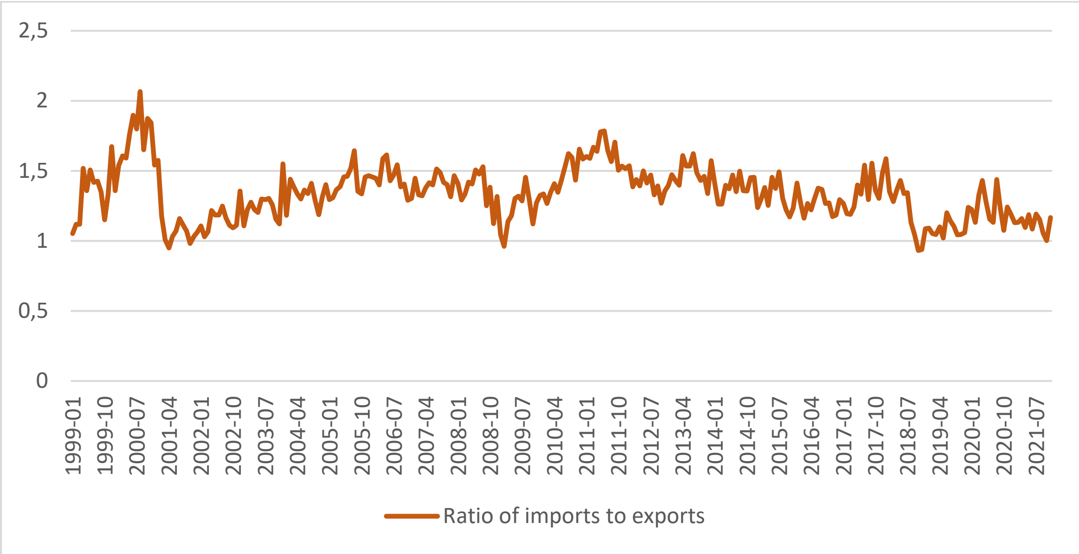

Figure 1.10: Ratio of Imports to Exports

Source: Author's calculations

To summarize, we picked FX risk as an additional factor because of the following structural weaknesses which make companies more vulnerable to external shocks:

- Prolonging net FX open position of non-financial companies,
- The tendency of Turkish companies to borrow from external markets,
- Volatile exchange rates,
- Exporters' intensive use of imported inputs,
- Capital account imbalances

In chapter 2, the theoretical background of Fama-French multi-factor models, from the introduction of the asset pricing model of Sharpe [1], Lintner [2], and Black [3] to CAPM and the contemporary Fama-French factor models and empirical studies in the asset pricing field are depicted. Chapter 3 focuses on methodology and data. We elaborate on the formation of factor variables and intersection portfolios. Chapter 4 is about time-series properties and prediction results of the FF5F model with and without FX risk. We will also compare model performances. In Chapter 5, we predict excess portfolio returns by using a machine learning method, support vector regression. Chapter 6 contains the combined predictions of simple linear and support vector regression methods. We will use *forecast combination* method to combine estimations. The final chapter discusses our main findings and contributions to Fama-French factor modeling literature.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

Asset pricing has been one of the major fields of financial studies. Researchers put endless efforts into exploring the dynamics of stock return movements and underlying risk factors. Countless studies were conducted to define proxies for average stock returns and develop models to estimate stock prices under different settings accurately.

This study is the first one that considers FX risk as an additional risk factor incorporated into Fama-French five-factor model on Borsa İstanbul. Incorporating FX risk into the five-factor model improves the prediction performance for most portfolios tested[4]. On top of that, we applied a machine learning algorithm; support vector regression to discover the non-linear relationship between risk and return. Moreover, there is no need to satisfy Gauss-Markov assumptions [14] to implement support vector regression. Support vector regression method proved its usefulness in terms of root MSEs. We obtained lower average root MSEs for either Fama-French five-factor model with or without FX risk. Finally, we combined the predictions obtained from simple linear regression and support vector regression methods and verified that the support vector regression method outperforms.

In subsection 2.2, we will summarize the process of introduction CAPM, the development of the Fama-French factor models, and studies in between and

[4] Portfolio construction procedure will be explanied in detail in Chapter 3.

thenceforth. In 2.3, we will investigate the Fama-French three-factor and five-factor models and their components. Subsection 2.4 will report major empirical studies on the Fama-Frech factor models literature. In 2.5, we will present empirical studies on the Fama-French factor models investigated on Borsa İstanbul.

2.2 Introduction of CAPM and Evolution of Fama-French Factor Models

Sharpe [1], Lintner [2], and Black [3] introduced the asset pricing model (Sharpe-Lintner-Black, SLB model) to capture the relation between average returns and standard deviations. The model predicted that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz[4]. Sharpe [1] asserts that returns of individual assets or portfolios formed through combinations of risky assets are positively correlated with the market return.

Na, Green, and Maggioni [15] summarizes Sharpe, Lintner, and Black's assumptions for the asset pricing model as:

- Investors are mean-variance optimizers only for a single period.
- Investors' assessments are the same for the first two moments of asset returns.
- There are perfect markets where securities are freely traded without restrictions and transaction costs.

Fama-French [16] defines the relation between expected return and market beta, as they describe "Sharpe-Lintner CAPM" as follows:

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_{im}, \quad i = 1,\ldots,N. \tag{2.1}$$

where:

$E(R_i)$: expected return on asset i

R_f : the risk-free rate

R_m : expected market return

$E(R_m) - R_f$: risk premium

β_{im} (market beta of asset i) $= \dfrac{\text{cov}(R_i, R_m)}{\sigma^2(R_m)}$ where $\sigma^2(R_m)$: variance of market return

Figure 2.1 pictures the risk and return relationship implied by CAPM. The curve *abc* represents the portfolios of risky assets that minimize the risk; $\sigma^2(R)$ for different levels of expected return.[5] Along *abc* curve, portfolios above the point *b* are "*mean-variance efficient portfolios*" as any point above *b* also correspond to portfolios with maximum expected return for a given level of risk.

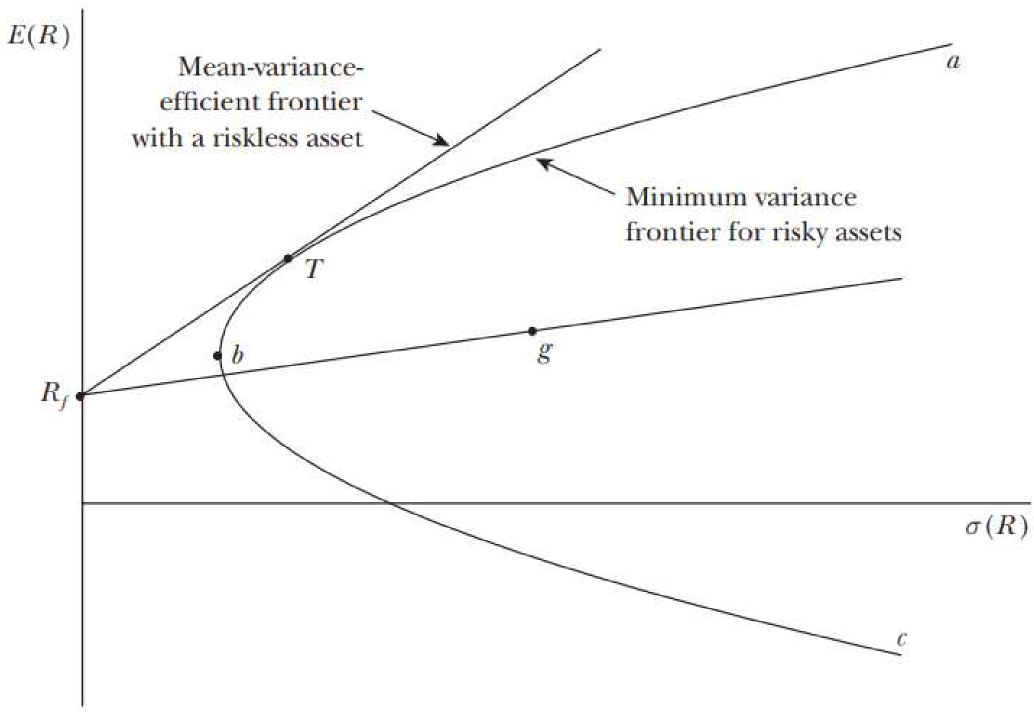

Figure 2.1: Investment Opportunities and Efficient Frontier
Source: Fama & French (2004)

[5] Fama & French [16] underline that risk-free borrowing and lending are not included in these portfolios.

Combinations of risk-free lending or borrowing with some risky portfolio *g* form a straight line from R_f through *g*. Mean-variance efficient portfolio when risk-free lending or borrowing is available is obtained at the point where a single risky portfolio *T* is tangent to the *abc* curve. The straight line representing efficient frontier when risk-free lending or borrowing is available contains all combinations of risky portfolio T and risk-free borrowing or lending (see [16]).

According to Fama & French [16], Black considered Sharpe and Lintner's assumption for risk-free borrowing and lending without any limit unrealistic. He develops a version of CAPM without borrowing and lending at a risk-free rate. Black [3] demonstrated that mean-variance efficient portfolios could be achieved through unrestricted short sales of risky assets. Sharpe-Lintner CAPM indicates that $E(R_{Zm})$ must be equal to the risk-free rate. Whereas Black's version of the CAPM indicates that expected returns on assets with "0" market betas, $E(R_{Zm})$ must be less than $E(R_m)$.

Fama-French [16] consider Sharpe [1] and Lintner's [2] assumption of unrestricted risk-free borrowing and lending and Black's [3] assumption that short selling risky assets is unrestricted unrealistic. Fama-French [5] also stated that the SLB model has various empirical inconsistencies. The most prominent drawback of the SLB model is the size effect of Banz [17]. Banz captured the impact of market equity (ME) on predicting the average stock returns and found that stocks with lower market capitalization tend to have higher average returns (see Fama-French [5],[18]). In addition to the market's β and ME effect, Bhandari [19] finds that the leverage helps explain the cross-section of average stock returns. Stattman [20] and Rosenberg, Reid, and Lanstein [21] define the relation between average returns on US stocks and the ratio of a firm's book value of common equity. Book-to-market equity plays a vital role in explaining the cross-section of average returns on Japanese stocks, as Chan, Hamao, and Lakonishok [22] demonstrated.

Basu [23] illustrates that alongside size factor and market β, earning-price ratios

(E/P) also help explain returns on US stocks. Ball [24] argues that the E/P ratio is a "catch-all proxy" for unknown factors in expected returns. He puts forward that E/P is higher for stocks with higher risks and higher expected returns no matter the unnamed sources of risk are. According to Fama-French [5], Ball's proxy argument for E/P ratios might be extended to cover size, leverage, and book-to-market equity. According to Keim [25], size, leverage, and book-to-market equity parameters suggest different ways to predict stock prices, to obtain the information in prices about risk and expected returns. (see [5]). As E/P, ME, leverage, and BE/ME factors are scaled versions of price, Fama-French presumed that some of them might be redundant to explain average returns. According to Fama & French [5], for the period 1941-1990, unlike the simple relation between average return and market β, the univariate links between average return, size, leverage, E/P, and book-to-market equity are strong. The model performed by Fama & French [5] reveals that size and book-to-market equity factors describe the average return for 1941-1990, while the relation between market β and average return is weak. However, investment and profitability factors explaining part of the average return are left unexplained in Fama-French 3 Factor model. (see [5]).

Novy & Marx [6], Titman, Wie, and Xie [7], and others argue three-factor model of Fama-French is an incomplete model to capture variations in expected returns. To them, Fama-French's three factors are inefficient in explaining the variations in average expected returns related to profitability and investment.

There are empirical studies to improve the three-factor model of Fama and French with additional factors like a momentum factor of Carhart [26] and liquidity factor of Amihud [27], Pástor and Stambaugh [28], Acharya and Pedersen [29], and coskewness factor of Harvey and Siddique [30].

Campbell and Shiller [31] argue that the dividend discount model (2.2) is a tautology that defines the internal rate of return, r of a stock:

$$P_t = \sum_{\tau=1}^{\infty} \frac{E(d_{t+\tau})}{(1+r)^{\tau}} \tag{2.2}$$

where:

P_t : share price at time t

$E(d_{t+\tau})$: expected dividend per share for period τ

r : internal rate of return on expected dividends

Similarly, as equation 2.3 derives directly from equation 2.2, equation 2.3 is also a tautology (see [32]).

$$\frac{P_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau}}{B_t} \tag{2.3}$$

where:

P_t : share price at time t

B_t : book equity at time t

$Y_{t+\tau}$: total equity earning for the period $t+\tau$

$dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$: the change in total book equity

Fama-French [33],[8], Xing [34], Hou, Xue, and Zhang [35] augment the three-factor model by incorporating additional factors for investment and profitability exposures. Fama and French [33],[8] compare the three-factor model [5] with the five-factor model to determine which model more efficiently explains the average returns related to proxies left out by the three-factor model. Hence, they documented that the five-factor model is superior in explaining average stock returns. On the other hand, with the addition of profitability and investment factors, the value factor in the three-factor model becomes obsolete in explaining average returns in their sample (see [33]).

There is also evidence that stocks with high ratios of a fundamental like book value or cash flow to price have higher average returns than stocks with low ratios of fundamentals to price (see [36], [5],[18],[37]).

Novy & Marks [6] identify a proxy for expected profitability with high exposure on the average return (see [18]). There is also proof for a weaker but statistically reliable relation between investment and average return (see also [38], [39], [40], [7], [32], [41], and [42]).

2.3 Fama-French Three-Factor & Five-Factor Models

Fama and French [5] developed a three-factor model to portray the relation between market value (size factor) and average return and; a price ratio like B/M and average return alongside market β. When the three-factor model of Fama and French was developed, size and B/M were two well-known proxies for the average stock returns, which were left unexplained by the CAPM.

The three-factor model of Fama and French [5]is as follows:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it} \tag{2.4}$$

where:

R_{it} : return on security or portfolio i at time t

R_{ft} : return on a risk-free asset at time t

R_{mt} : return of value-weighted market portfolio at time t

SMB_t : the difference between the returns on a diversified portfolio of small and big stocks

HML_t : the difference between the returns on diversified portfolios of high and low B/M stock

ε_t : disturbance term with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}, \varepsilon_{is}) = 0$; $t \neq s$ and $\varepsilon_{it} \sim iid(0, \sigma^2)$

The findings of Novy and Marx [6], Titman, Wei, and Xie [7], and others towards the three-factor model is incomplete for expected returns, led Fama and French to augment the three-factor model by adding investment and profitability factors.

Fama and French's three-factor model with the addition of investment and profitability exposures (five-factor model) is as:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \quad (2.5)$$

where:

RMW_t : the difference between the returns on a diversified portfolio of stocks with robust and weak profitability

CMA_t : the difference between the returns on a diversified portfolio of stocks of low and high investments firms

r_i, c_i : factor exposures for profitability and investment, respectively

ε_{it} : disturbance term with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}, \varepsilon_{is}) = 0$; $t \neq s$ and $\varepsilon_{it} \sim iid(0, \sigma^2)$

Fama and French [33] highlight that the mean-variance efficient tangency portfolio, which prices all assets, combines the risk-free asset, the market portfolio, SMB, HML, RMW, and CMA in the sense of Huberman and Kandel [43]. In addition, neither size, B/M, profitability, and investment nor the factor portfolios formed through combinations of risky assets are not themselves state variables and state variables mimicking portfolios in the sense of Merton's [44] model. SMB, HML, RMW, and CMA factors are just diversified portfolios that offer combinations of exposures to unknown state variables (see [45]).

2.4 Empirical Studies on Fama-French Models

Connor and Sehgal [46] examined the market, size, and book-to-market factor exposures on returns of stocks in the Indian Market over June 1989-March 1999. They found that Fama-French's three factors predict the cross-section of average stocks returns.

Ajili [47] applied the Fama-French three-factor model and the CAPM on the French Market stocks over July 1976 – June 2001. His research indicated that the Fama-French three-factor model performs better in explaining stock return deviations than the CAPM. His research also suggested that both the three-factor model and the CAPM successfully predicted the cross-section of stock returns.

Drew, Naughton, and Veeraraghavan [48] verified the effect of Fama-French's three-factor on the cross-section of the average stock returns in the Shanghai Stock Exchange. They also reported that small firms with low B/M ratios generate higher returns than big firms with high B/M ratios. Their finding is contradictory to Fama-French [45].

Dirkx and Peter [49] implemented Fama-French five-factor model and an augmented version with momentum factor on the German stock market over 2002-2019. For the country-specific case of the German stock market, their results indicated that profitability and investment factors, in addition to the momentum, did not contribute to the explanatory power of the Fama-French three-factor model.

López-García et al. [50] proposed a new factor exposure with long-term memory to augment the Fama-French model. The authors encountered that the long-term memory factor stands significant in a sample of 2500 stocks with the highest liquidity in the US stock market when the market factor in the model is an equally weighted portfolio of stocks. The long-term memory factor becomes insignificant when the market factor is calculated as a capitalization-weighted portfolio.

Li et al. [51] augment the Fama-French five-factor model by adding long memory and memory factors. The authors investigate the augmented version of the model on A-share market in the Chinese stock market from January 2010 to July 2020. The results expose that long-term memory, and momentum factors improved the model's explanatory power for the Chinese stock market.

Foye [52] compares the performances of the Fama-French five-factor and three-factor models for emerging market stock returns in a sample consisting of 18 countries. The five-factor model was found superior in Eastern Europe and Latin America. In contrast, the five-factor model fails to contribute explanatory power of the three-factor model in Asian countries.

Leite, Klotzle, Pinto, and Barbedo [53] argued that shocks to aggregate dividend yield and term spread, default spread, and one-month T-bill rates are proxies for size and value factors previous literature documented. However, they fail to explain the profitability factor. Leite et al. included the innovations in CPI in the set of state variables and verified that portfolio returns are highly correlated with innovations in CPI and the slope of the term structure of interest rates. A model where factor variables are excess market returns, and unexpected changes in CPI and term structure of interest rates, explain common time-varying behaviors in portfolio returns more efficiently than five-factor and three-factor models.

Cox and Britten [54] examined the effectiveness of the Fama-French five-factor model in predicting returns on the Johanessburg Securities Exchange over 1991-2017. The profitability and investment factor are significant in explaining the returns on the JSE. Their research suggested that the profitability factor is more consistent than the investment factor for the JSE.

Faff, Gharghori, and Nguyen [55] amplify Vassalou's [56] GDP growth factor by conditioning the Fama-French five-factor model through the same macroeconomic variables used to build the GDP growth factor of Vassalou. Faff et al. [55] evaluated the performances of an extended version of the Fama-French model with GDP and conditional Fama-French model using non-nested techniques on the Australian Securities Exchange over 1990-2010. Empirical results suggested that the conditional Fama-French model outperforms the GDP-augmented version of the Fama-French model.

Dhaoui and Bensalah [57] compare the performance of the Fama-French five-factor model and Fama-French's momentum factor, and the augmented version by incorporating an additional factor of an investor sentiment index on the NYSE over July 1965-September 2015. Evaluations indicate that the augmented version of the model is more successful in explaining stock return deviations.

2.5 Recent Empirical Studies on Fama-French Models Investigated on Borsa İstanbul

Bereket [58] implements Fama-French four-factor model and researches its validity on İstanbul Stocks Exchange[6] over July 2004-June 2013. Despite the validity of the four-factor model, it does not outperform the CAPM and three-factor model.

Ceylan, Dogan, and Berument [59] incorporate an additional factor for the excess holding of foreign investors to the Fama-French three-factor model. They find a statistically significant and adverse relationship between the alternative factor variable and deviations in portfolio returns.

Erdinç [60] compares CAPM, Fama-French three- and five-factor models for the Turkish stock market over June 2000-May 2017. He shows that the five-factor model successfully explains variations in stock returns.

Acaravcı and Karaomer [61],[62] examine the performance of the CAPM and Fama-French factor models on BİST over July 2005-June 2016. The GRS-F test indicates pricing error for CAPM, while the Fama-French models do not possess price error and are found valid in the Turkish stock market. Among two versions of Fama-French models, the five-factor model outperforms the three-factor model.

Aras et al. [63] investigate the validity of the FF5F model on the Turkish stock market and compare the model's performance with its predecessors FF3F model and CAPM over January 2005-June 2017. The authors use excess returns of 18 intersection portfolios as response variables. In consideration of statistical

[6] İstanbul Stock Exchange is re-branded as "Borsa İstanbul" on April, 2013. https://www.borsaistanbul.com/en/sayfa/3753/legal-framework

indicators, the authors find that the performance of the FF5F model is superior to the FF3F model and the CAPM for the Turkish stock market.

Tan & Taş [64] analyze the relationship between investor attention and stock return movements using CAPM, FF3F, and Carhart's four-factor models on Borsa İstanbul stocks over April 2013-September 2017. The authors used abnormal Google search volume index (ASVI) as a proxy for investor attention. Their findings suggest that firms attracting high attention tend to benefit from higher stock prices.

Zeren et al. [65] test the validity of the FF5F model on 18 of Borsa İstanbul Sustainability Index stocks over Q1 1995-Q3 2017. The authors did not find sufficient evidence towards the validity of the FF5F model over Sustainability Index stocks.

Table 2.1 summarizes the recent empirical studies investigated on Borsa İstanbul.

Table 2.1: Empirical Studies Investigated on Borsa İstanbul

Author	**Publication Year**	**Title of the Study**	**Major Findings**
Bereket	2014	The Validity of Fama-French Four Factor Model in Istanbul Stocks Exchange	Despite the validity of the four-factor model, it does not outperform the CAPM and three-factor model.
Ceylan, Dogan and Berument	2015	Three-Factor Asset Pricing Model and Portfolio Holdings of Foreign Investors: Evidence from an Emerging Market – Borsa Istanbul	The relationship between a proxy for the excess holding of foreign investors and portfolio returns is significant and negative.

Table 2.1: (continued)

Erdinç	2017	Comparison of CAPM, Three-Factor Fama-French Model and Five-Factor Fama-French Model for the Turkish Stock Market	Compared to CAPM and the Fama-French three-factor model, the five-factor model is better at explaining stock return deviations.
Acaravcı and Karaomer	2018	The Comparative Performance Evaluation of the Fama-French Five-Factor Model in Turkey	The Fama-French five-factor model outperforms the Fama-French three-factor model. The GRS-F test indicates a pricing error for CAPM.
Aras, Çam, Zavalsız and Kekin	2018	A Comparison of the Performance of Fama-French Multifactor Asset Pricing Models: An Application on Borsa İstanbul	The performance of the FF5F model is superior to the FF3F model and the CAPM for the Turkish stock market.
Tan and Taş	2019	Investor Attention and Stock Returns: Evidence from Borsa Istanbul	There is a positive relationship between high investor attention and stock returns.
Zeren, Yılmaz and Belke	2019	Testing the Validity of Fama French Five Factor Asset Pricing Model: Evidence From Turkey	The authors did not find sufficient evidence that the FF5 model is valid for İstanbul Stock Market Sustainability Index stocks

2.6 Conclusion

Various empirical researches have been carried out since the introduction of CAPM. Researchers conducted studies that consider different stock markets and emphasize factors like momentum and memory factors, innovations in CPI, GPD growth, investment sentiment index associated with different risks contributed to the assets pricing literature.

Looking into empirical studies investigated on Borsa İstanbul, we noticed that the main focus is on exploring the validity of Fama-French factor models and CAPM for the Turkish stock market.

Our study is the first to incorporate FX risk to the Fama-French five-factor model in the literature. Moreover, we did not detect any prior study that applies the support vector regression method to estimate the average stock returns on Borsa İstanbul. Ahead of all, this thesis documents the superiority of support vector regression through combinations of predictions.

CHAPTER 3

METHODOLOGY AND DATA

3.1 Introduction

In this chapter, our emphasis is to put forward the methodology and data retrieved to construct the Fama-French five-factor without FX risk and the model with FX risk. In 3.2, the formation of intersection portfolios similar to Fama-French factor models' procedures is clarified. In 3.3, we elaborate on the factor variables, i.e., what they refer to and how they are constructed. Factor formation is highly crucial because factor variables are exogenous variables of the Fama-French multi-factor models. In 3.4, we will present some information about Borsa İstanbul. Finally, in 3.5, we will present sample-related data.

3.2 Portfolio Formation

3.2.1 Size – M/B Portfolios

Stocks included in the sample are divided into two groups based on the market values. As of June on year t, market values of stocks are ranked from the largest to the smallest. The cluster of stocks whose market values are above the sample median value forms *big-size portfolios*. On the other hand, the set of stocks whose market values are below or equal to the sample median value forms *small-size portfolios*. The exact process is repeated over 2009-2019 to obtain *big and small-size portfolios*. Consequently, big and small-size portfolios are formed through market values as of June on year t, spanning the 12 months between July on year t and June on year t+1.

In the next step, stocks included in the sample are divided into three categories based on the market-to-book ratios (MV/BV). MV/BV ratios of stocks as of June on year t are ranked from the largest to the smallest. The cluster of stocks with MV/BV ratios at the highest %30 among the sample companies is categorized as *high market-to-book ratio portfolios*. On the contrary, the cluster of stocks with MV/BV ratios in the lowest %30 among the sample companies is classified as *low market-to-book ratio portfolios*. Among the sample companies, the cluster of stocks whose MV/BV ratios are in the middle %40 between high and low market-to-book ratio portfolios are categorized as *neutral market-to-book ratio portfolios.* For each year, the exact process is repeated over 2009-2019 to obtain *high, neutral, and low market-to-book ratio portfolios*. Consequently, portfolios are formed through market-to-book ratios as of month June on year t, high, neutral, and low market-to-book ratio portfolios, spanning the 12 months between July on year t and June on year t+1.

Table 3.1 presents six intersection portfolios through combinations of size and market-to-book ratio factors.

Table 3.1: Intersection of size-M/B portfolios

		Market-to-book (M/B)		
		High	Neutral	Low
Size (Market Value)	Big	BH	BN	BL
	Small	SH	SN	SL

The intersection of size-M/B portfolios are defined as follows:

- BH: Intersection of portfolios consisting of big size and high market-to-book ratio stocks
- BN: Intersection of portfolios consisting of big size and neutral market-to-book ratio stocks
- BL: Intersection of portfolios consisting of big size and low market-to-book ratio stocks
- SH: Intersection of portfolios consisting of small size and high market-to-book ratio stocks
- SN: Intersection of portfolios consisting of small size and neutral market-to-book ratio stocks
- SL: Intersection of portfolios consisting of small size and low market-to-book ratio stocks

Returns on the portfolios, as mentioned above, are value-weighted averages of individual stocks. Market values of stocks as of June on year t are used to calculate the returns of intersection portfolios for the period July on year t – June on year t+1. We will calculate the returns of other intersection portfolios similarly.

3.2.2 Size – Profitability Portfolios

Stocks included in the sample are divided into three groups based on the profitability ratios. We used net operating profit divided by book equity as a proxy for profitability. Profitability ratios of stocks as of month June on year t are ranked from the largest to the smallest. The cluster of stocks whose profitability ratios are at the highest %30 among the sample companies are classified as *robust profitability portfolios*. On the contrary, the cluster of stocks whose profitability ratios are in the lowest %30 among the sample companies is classified as *weak profitability portfolios*. The cluster of stocks whose profitability ratios are in the middle %40 between *robust* and *weak profitability portfolios* are classified as *middle profitability portfolios.* Each year, the exact process is repeated over 2009-2019 to obtain *robust, middle, and weak profitability portfolios*. Consequently, through profitability ratios as of month June on year t, *robust, middle, and weak profitability portfolios* are formed, spanning the 12 months between July on year t and June on year t+1.

The six intersection portfolios through combinations of size and profitability factors are represented in Table 3.2.

Table 3.2: Intersection of size-profitability portfolios

		Profitability (Net operating income/book equity)		
		Robust	Middle	Weak
Size (Market Value)	Big	BR	BM	BW
	Small	SR	SM	SW

Intersections of size-profitability portfolios are defined as follows:

- BR: Intersection of portfolios consisting of big size and robust profitability stocks
- BM: Intersection of portfolios consisting of big size and middle profitability stocks
- BW: Intersection of portfolios consisting of big size and weak profitability stocks
- SR: Intersection of portfolios consisting of small size and robust profitability stocks
- SM: Intersection of portfolios consisting of small size and middle profitability stocks
- SW: Intersection of portfolios consisting of small size and weak profitability stocks

3.2.3 Size – Investment Portfolios

We split stocks into three groups based on the investment ratios. We used the total asset growth ratio as a proxy for investment.

The total asset growth ratio formula is as follows:

$$Total\ asset\ growth\ ratio\ (t) = \frac{\text{total asset (t)} - \text{tota\ \ assets (t} - 1)}{\text{total assets (t} - 1)} \qquad (3.1)$$

where:
total assets (t): total asset as of December on year t
total assets (t-1): total asset as of December on year t-1

Total asset growth ratios of stocks as of December on year t-1 are ranked from the largest to the smallest. The cluster of stocks whose investment ratios take part at the highest %30 among the sample companies is classified as *aggressive investment portfolios*. On the contrary, the cluster of stocks whose profitability ratios take part in the lowest %30 among the sample companies is classified as *conservative investment portfolios*. The cluster of stocks whose profitability ratios take part in the middle %40 between *aggressive* and *conservative profitability portfolios* among the sample companies are called *intermediate investment portfolios.* Each year, the exact process is repeated over 2009-2019 to obtain *aggressive, intermediate, and conservative investment portfolios*. Consequently, through total asset growth ratios as of month December on year t-1, *aggressive, intermediate, and conservative investment portfolios* are constructed, spanning the 12 months between July on year t and June on year t+1.

The six intersection portfolios using combinations of size and investment factors are shown in Table 3.3.

Table 3.3:Intersection of size-investment portfolios

		Investment (Total asset growth ratio)		
		Aggresive	Intermediate	Conservativ e
Size (Market Value)	Big	BA	BI	BC
	Small	SA	SI	SC

Intersections of size-investment portfolios are described as follows:

- BA: Intersection of portfolios composed of big size and aggressive investment stocks
- BI: Intersection of portfolios composed of big size and intermediate investment stocks
- BC: Intersection of portfolios composed of big size and conservative investment stocks
- SA: Intersection of portfolios composed of small size and aggressive investment stocks
- SI: Intersection of portfolios composed of small size and intermediate investment stocks
- SC: Intersection of portfolios composed of small size and conservative investment stocks

3.2.4 Size – FX Risk Portfolios

Stocks included in the sample are divided into two groups based on the FX positions. We used the net FX position divided by book equity as a proxy fx position. FX position ratios of stocks as of month December on year t-1 are ranked from the largest to the smallest. The stocks of companies whose assets in FX exceed FX liabilities form *FX surplus portfolios*. On the other hand, the group of stocks whose FX position proxies are negative form *open position portfolios*. We did not include stocks with zero FX assets and liabilities in the sample. Each year, the exact process is repeated over 2009-2019 to obtain *FX surplus and open position portfolios*. Consequently, FX surplus and open position portfolios are formed through FX position proxies as of December on year t-1, spanning the 12 months between July on year t and June on year t+1.

The four intersection portfolios using combinations of size and FX position factors are defined in Table 3.4.

Table 3.4:Intersection of size-FX position portfolios

		FX position	
		Surplus	Open
Size (Market Value)	Big	BPOZ	BOP
	Small	SPOZ	SOP

Intersections of size-FX portfolios are described as follows:

- BPOZ: Intersection of portfolios consisting of big size and fx surplus stocks
- BOP: Intersection of portfolios consisting of big size and open fx position stocks
- SPOZ: Intersection of portfolios consisting of small size and fx surplus stocks
- SOP: Intersection of portfolios consisting of small size and open fx position stocks

3.3 Factor Definitions and Calculations

3.3.1 Factor Definitions

Fama and French [33],[8] defined five factors as proxies for risks associated with deviations in stock returns. These five factors are summarized as follows:

- *Market factor*: the difference between the market return and the risk-free return. In this study, we used the Treasury's average cost of borrowing as a proxy for risk-free returns.
- *Size factor (SMB)* is the difference between the average returns of big companies and small companies in terms of market value.
- *Value factor (HML)*: is the difference between the average returns of companies with high and low market-to-book ratios.
- *Profitability factor (RMW)* is the difference between the average returns of companies that have robust and weak profitability ratios. In this thesis, we used *return on equity* (ROE) as an indicator of profitability.
- *Investment factor (CMA)* is the difference between the average returns of companies with conservative and aggressive investment strategies. In this thesis, we used *total asset growth ratio* as an indicator of investment strategy.

We incorporated an additional risk factor to the Fama-French five-factor model. The factor which is a proxy for FX risk is summarized as follows:

- *FX position factor* is the difference between the average returns of companies whose foreign currency assets exceed foreign currency liabilities and companies which have open FX positions. In this thesis, we propose:

$$\frac{Net\ FX\ position}{Total\ Equity} \tag{3.2}$$

as an indicator of the FX risk of a company.

3.3.2 Fama-French's Five Factors Without FX risk

Table 3.5 puts together size-M/B, size-profitability, and size-investment portfolios.

Table 3.5: Intersection portfolios in the sense of Fama and French Five-Factor without FX Risk

		Size (Market Value)	
		Big	Small
Market-to-book	High	BH	SH
	Neutral	BN	SN
	Low	BL	SL
Profitability	Robust	BR	SR
	Middle	BM	SM
	Weak	BW	SW
Investment	Aggresive	BA	SA
	Intermediate	BI	SI
	Conservative	BC	SC

3.3.2.1 Factor Calculations

Market, *size*, *value*, *profitability,* and *investment* factors for the FF5F model without FX risk are calculated as follows:

- **Market Factor:**

$$R_m - R_f$$

- **Size Factor**

$$SMB_{M/B} = \frac{SH + SN + SL}{3} - \frac{BH + BN + BL}{3}$$

$$SMB_{\Pr ofitability} = \frac{SR + SM + SW}{3} - \frac{BR + BM + BW}{3}$$

$$SMB_{Investment} = \frac{SA + SI + SC}{3} - \frac{BA + BI + BC}{3}$$

$$SMB = \frac{SMB_{M/B} + SMB_{\Pr ofitability} + SMB_{Investment}}{3} \quad (3.3)$$

- **Value Factor**

$$HML = \frac{SH - SL}{2} + \frac{BH - BL}{2} \quad (3.4)$$

- **Profitability Factor**

$$RMW = \frac{SR - SW}{2} + \frac{BR - BW}{2} \quad (3.5)$$

- **Investment Factor**

$$CMA = \frac{SC - SA}{2} + \frac{BC - BA}{2} \quad (3.6)$$

3.3.3 Fama-French's Five Factors Incorporating FX Risk

Intersection portfolios, including size-FX position portfolios, are gathered in Table 3.6.

Table 3.6: Intersection portfolios including FX position portfolios

		Size (Market Value)	
		Big	Small
Market-to-book	High	BH	SH
	Neutral	BN	SN
	Low	BL	SL
Profitability	Robust	BR	SR
	Middle	BM	SM
	Weak	BW	SW
Investment	Aggresive	BA	SA
	Intermediate	BI	SI
	Conservative	BC	SC
FX position	Long FX position	BPOZ	SPOZ
	Short FX position	BOP	SOP

Returns of all intersection portfolios are value-weighted averages of returns of individual stocks in the sample.

3.3.3.1 Factor Calculations

Market, *value*, *profitability,* and *investment* factor calculations are the same as shown in section 3.3.2.1. New factor exposure and the modified version of size factor due to the incorporation of new factor exposure are calculated as follows:

- **Size Factor (modified)**

$$SMB_{M/B} = \frac{SH + SN + SL}{3} - \frac{BH + BN + BL}{3}$$

$$SMB_{\text{Profitability}} = \frac{SR + SM + SW}{3} - \frac{BR + BM + BW}{3}$$

$$SMB_{Investment} = \frac{SA + SI + SC}{3} - \frac{BA + BI + BC}{3}$$

$$SMB_{FXposition} = \frac{SOP + SPOZ}{2} - \frac{BOP + BPOZ}{2}$$

$$SMB = \frac{SMB_{M/B} + SMB_{\Pr ofitability} + SMB_{Investment} + SMB_{FXposition}}{4} \tag{3.7}$$

- **FX Risk Factor**

$$FX = \frac{SOP - SPOZ}{2} + \frac{BOP - BPOZ}{2} \tag{3.8}$$

3.4 Data

Borsa İstanbul was founded as a securities exchange on December 30, 2012. Borsa İstanbul combines all the exchanges for capital market instruments, foreign currencies, precious metals and gems, and other contracts, documents, documents, and assets approved by the Capital Markets Board of Turkey under a single roof. The evolution of total market capitalization and the number of companies listed in Borsa İstanbul can be seen in Figure 3.1.

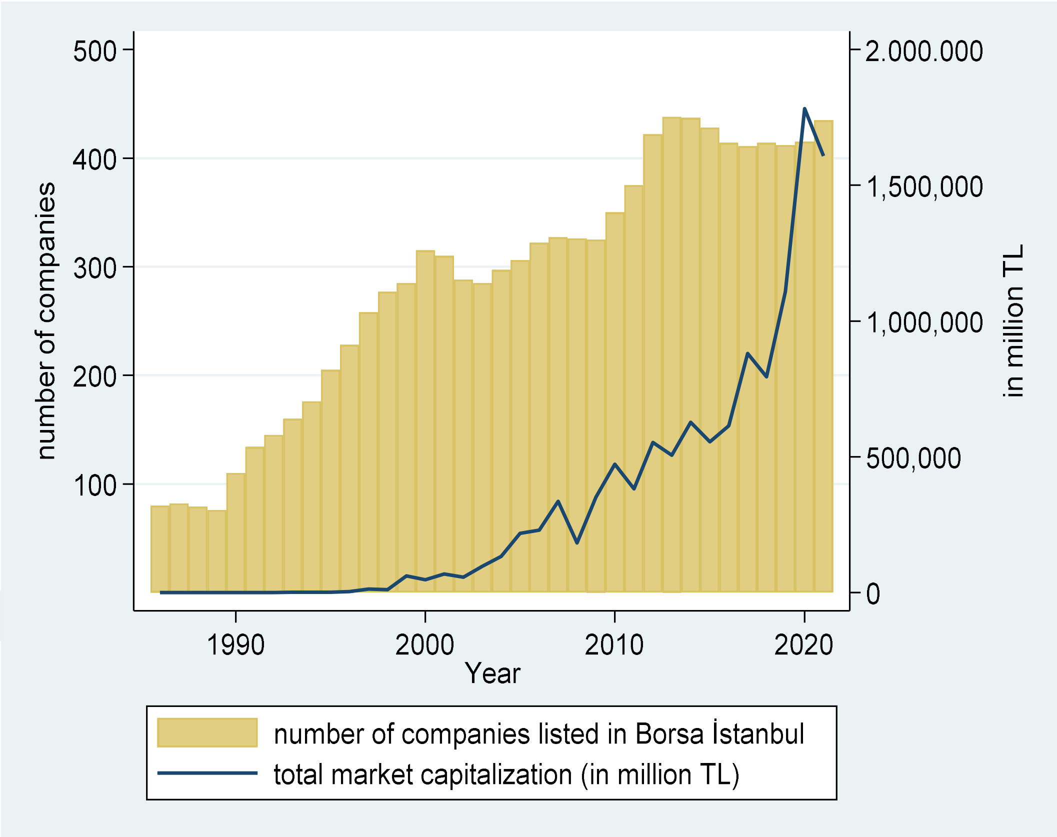

Figure 3.1: Evolution of market capitalization and number of companies listed in Borsa İstanbul (1986-2021 June)

Source: Borsa İstanbul

The market capitalization of the sample companies and the number of companies included in the sample can be seen in Figure 3.2 and Table 3.7. The sample consists of companies listed in Borsa İstanbul between 2009 July and 2019 June. We avoided using 2008 and pre-2008 data because of the possible effect of global financial crises in 2008 on stock prices.[7] Returns of stocks of financial companies and banks are excluded from the sample. Balance sheet and income statement data, market values, share prices, and BİST100 index are downloaded from the Finnet Electronic database.

[7] In the presence of 2008 and pre-2008 data, GRS-F test results might indicate pricing errors.

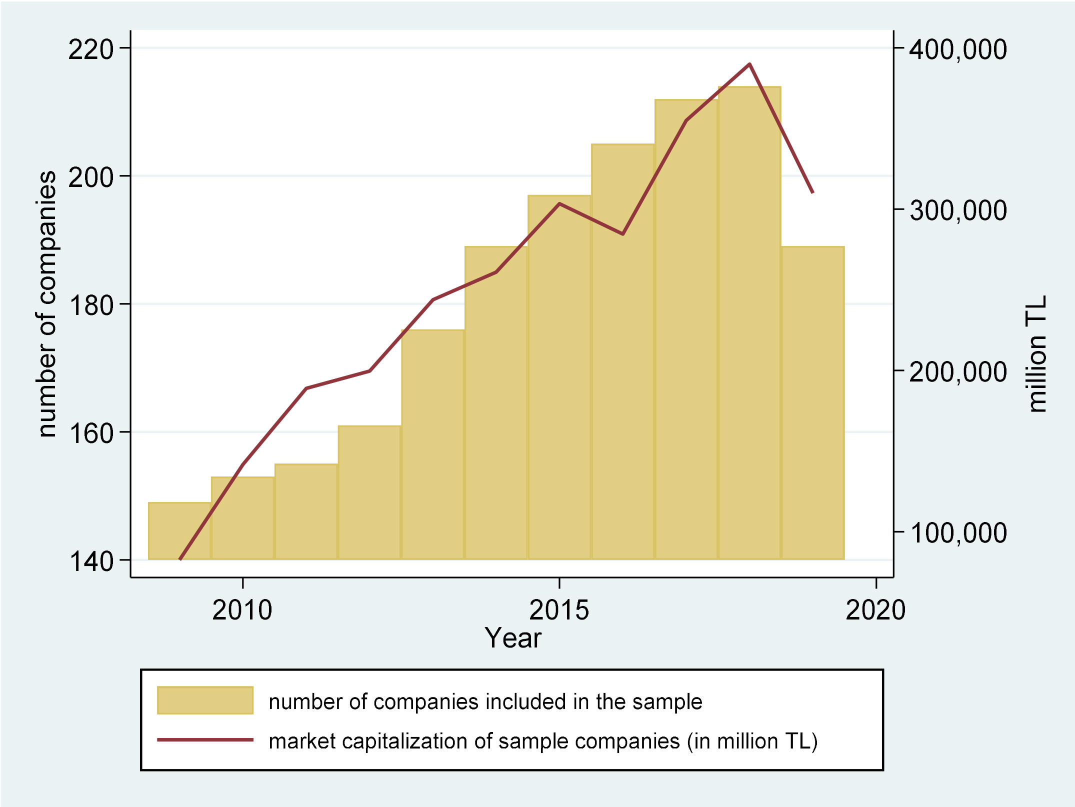

Figure 3.2: Market capitalization and number of companies included in the sample (2009 June – 2019 June)

Source: Author's calculations

Table 3.7: Market capitalization of sample companies and number of companies included in the sample

Year	Market Capitalization of sample companies (million TL.)	Number of Companies Listed in sample
2009	82,621.84	149
2010	141,798.26	153
2011	188,890.07	155
2012	199,704.04	161
2013	243,881.02	176
2014	260,991.09	189
2015	303,405.19	197
2016	284,493.64	205
2017	355,061.03	212
2018	389,875.12	214
2019	310,013.49	189

Source: Author's calculations

A similar methodology to Fama and French [33],[8] is applied in this study. The sample companies' equity and market value, net profit, and asset growth data are used to construct size, market-to-equity, profitability and investment factors, and intersection portfolios. In addition, the sample companies' foreign exchange assets and liabilities data are used to construct the FX risk factor. We incorporate FX risk in Fama-French five-factor model to capture the effect of the FX position of companies on average stock returns.

A firm with a negative equity value is excluded from the sample. If a firm has a positive equity value in year t+1, it is included in the sample even though it possesses a negative equity value in year t. In other words, for each year in the sample, the equity values of sample companies are assessed, and firms with positive equity values are added to the model.

For a company to be included in the sample, there should be market value data available as of June on year t. There should also be equity value, net profit, total assets, foreign exchange assets, and foreign exchange liabilities data available as of December on year t-1. Companies with a "0" value for the FX position are excluded from the sample. A firm with a positive or negative FX position value in year t+1 is included in the sample, even if it has a "0" value in year t.

Monthly returns of individual stocks are calculated as the percentage change between closing prices at the last days of two consecutive months. For instance, return for month t is calculated as the percentage change between the closing price quoted on the last day of month t and the closing price quoted on the last day of month t-1. For a stock to be included in the sample for July on year t-1 to June on year t, there should be closing price data available at least for the last 36 consecutive months until June on year t. The stocks that miss necessary share price data are excluded from the sample for July on year t-1 to June on year t. We obtained closing prices of stocks from the Finnet portal.

The average cost of domestic borrowing of the Ministry of Treasury and Finance is used as the risk-free rate. Simple monthly rates are calculated from annual compounded borrowing rates issued each month. The average cost of domestic borrowing is obtained from the public finance statistics of the Ministry of Treasury and Finance.[8]

BIST100 index is used as a proxy for the market portfolio. Monthly percentage changes are calculated BIST100 ındex values are obtained from the Finnet portal.

3.5. Sample Characteristics and Descriptive Statistics

Table 3.8 shows the number of stocks included in intersection portfolios formed through market value (size) and market-to-book ratios.

Table 3.8: Number of companies in size-M/B portfolios

	Size - M/B Portfolios						
Time Period	**BH**	**BN**	**BL**	**SH**	**SN**	**SL**	**Total Number of Companies**
2009 July - 2010 June	27	30	17	17	31	27	149
2010 July - 2011 June	30	33	13	15	30	32	153
2011 July - 2012 June	28	33	16	18	29	31	155
2012 July - 2013 June	29	34	17	19	31	31	161
2013 July - 2014 June	34	39	15	18	33	37	176
2014 July - 2015 June	38	41	15	18	36	41	189
2015 July - 2016 June	44	42	12	15	37	47	197
2016 July - 2017 June	40	47	15	21	36	46	205
2017 July - 2018 June	42	47	17	21	39	46	212
2018 July - 2019 June	40	51	16	24	35	48	214

[8]https://en.hmb.gov.tr/public-finance

Table 3.8: (continued)

2019 July - 2020 June	39	36	19	17	41	37	189
Average Number of Companies	**35.55**	**39.36**	**15.64**	**18.45**	**34.36**	**38.45**	**181.82**

Notes: B/S: Big/Small in size, H/N/L: High/Neutral/Low in market-to-book ratios
Source: Author's calculations

BN cluster has the highest average number of stocks, whereas BL cluster has the lowest one.

Table 3.9 shows the number of stocks included in intersection portfolios formed using market values and profitability ratios.

Table 3.9: Number of companies in size-profitability portfolios

	Size - Profitability Portfolios						
Time Period	**BR**	**BM**	**BW**	**SR**	**SM**	**SW**	**Total Number of Companies**
2009 July - 2010 June	32	25	17	12	36	27	149
2010 July - 2011 June	28	33	15	17	30	30	153
2011 July - 2012 June	34	30	13	12	33	33	155
2012 July - 2013 June	32	32	16	16	33	32	161
2013 July - 2014 June	36	39	13	16	33	39	176
2014 July - 2015 June	42	36	16	14	41	40	189
2015 July - 2016 June	40	41	17	19	38	42	197
2016 July - 2017 June	42	39	21	19	44	40	205
2017 July - 2018 June	46	39	21	17	47	42	212
2018 July - 2019 June	43	48	16	21	38	48	214
2019 July - 2020 June	34	39	21	22	38	35	189
Average Number of Companies	**37.18**	**36.45**	**16.91**	**16.82**	**37.36**	**37.09**	**181.82**

Notes: B/S: Big/Small in size, R/M/W: Robust/Middle/Weak profitability
Source: Author's calculations

SM cluster has the highest average number of stocks, whereas SR cluster has the lowest one.

Table 3.10 demonstrates the number of stocks included in intersection portfolios formed using market values and investment rations.

Table 3.10: Number of companies in size-investment portfolios

	Size - Investment Portfolios						
Time Period	**BA**	**BI**	**BC**	**SA**	**SI**	**SC**	**Total Number of Companies**
2009 July - 2010 June	28	27	19	16	34	25	149
2010 July - 2011 June	27	29	20	18	34	25	153
2011 July - 2012 June	21	36	20	25	27	26	155
2012 July - 2013 June	23	38	19	25	27	29	161
2013 July - 2014 June	30	36	22	22	36	30	176
2014 July - 2015 June	27	44	23	29	33	33	189
2015 July - 2016 June	25	48	25	34	31	34	197
2016 July - 2017 June	31	45	26	30	38	35	205
2017 July - 2018 June	31	45	30	32	41	33	212
2018 July - 2019 June	36	47	24	28	39	40	214
2019 July - 2020 June	36	40	18	20	37	38	189
Average Number of Companies	**28.64**	**39.55**	**22.36**	**25.36**	**34.27**	**31.64**	**181.82**

Notes: B/S: Big/Small in size, A/I/C: Aggressive/Intermediate/Conservative investment strategy
Source: Author's calculations

BI cluster has the highest average number of stocks, whereas BC cluster has the lowest one.

Table 3.11 depicts the number of stocks included in intersection portfolios formed using market values and FX positions

Table 3.11: Number of companies in size-FX position portfolios

	Size - FX Position Portfolios				
Time Period	**BPOZ**	**BOP**	**SPOZ**	**SOP**	**Total Number of Companies**
2009 July - 2010 June	21	53	23	52	149
2010 July - 2011 June	27	49	24	53	153
2011 July - 2012 June	24	53	29	49	155
2012 July - 2013 June	26	54	25	56	161
2013 July - 2014 June	25	63	27	61	176
2014 July - 2015 June	27	67	32	63	189
2015 July - 2016 June	33	65	32	67	197
2016 July - 2017 June	37	65	38	65	205
2017 July - 2018 June	37	69	38	68	212
2018 July - 2019 June	33	74	41	66	214
2019 July - 2020 June	30	64	44	51	189
Average Number of Companies	**29.09**	**61.45**	**32.09**	**59.18**	**181.82**

Notes: B/S: Big/Small in size, POZ/OP: Pozitif FX position/Open FX position
Source: Author's calculations

BOP cluster has the highest average number of stocks, whereas BPOZ cluster has the lowest one.

Table 3.12 displays the summary statistics of excess returns of eighteen intersection portfolios over the risk-free rates. Portfolios constructed through the cluster of size and FX positions are also included in the table below.

Table 3.12: Summary statistics of intersection portfolios

R_{it}-R_{ft}	Mean	Median	Max	Min	Std. Dev.	Variance	Skewness	Kurtosis	Obs
EBL	0.0208	0.0234	0.2449	-0.1726	0.0774	0.0060	-0.0646	3.1829	132
ESL	0.0212	0.0163	0.4232	-0.1705	0.0855	0.0073	0.8799	6.0553	132
EBN	0.0103	0.0078	0.1690	-0.1758	0.0698	0.0049	-0.0736	2.9454	132
ESN	0.0174	0.0175	0.2568	-0.1780	0.0790	0.0062	0.2534	3.4988	132
EBH	0.0064	0.0084	0.2133	-0.1735	0.0573	0.0033	0.0974	3.9423	132
ESH	0.0211	-0.0059	1.3310	-0.1871	0.1504	0.0226	5.2641	45.1782	132
EBW	0.0079	0.0069	0.1757	-0.1914	0.0775	0.0060	-0.1253	2.5446	132
ESW	0.0157	0.0073	0.2939	-0.1605	0.0869	0.0076	0.3405	3.2327	132
EBR	0.0066	0.0048	0.2114	-0.1889	0.0605	0.0037	-0.0257	3.7621	132
ESR	0.0172	0.0187	0.2573	-0.1662	0.0782	0.0061	0.2708	3.3467	132
EBC	0.0091	0.0108	0.2265	-0.2251	0.0764	0.0058	-0.1978	3.1313	132
ESC	0.0210	0.0134	0.3465	-0.1609	0.0836	0.0070	0.6541	4.4518	132
EBA	0.0082	0.0101	0.1568	-0.1659	0.0598	0.0036	-0.2519	3.0848	132
ESA	0.0123	0.0071	0.2973	-0.2180	0.0845	0.0071	0.4197	4.0947	132
EBOP	0.0093	0.0089	0.1955	-0.2015	0.0611	0.0037	-0.0892	3.6711	132
ESOP	0.0166	0.0174	0.3629	-0.1806	0.0830	0.0069	0.4000	4.6526	132
EBPOZ	0.0079	0.0102	0.1432	-0.1528	0.0614	0.0038	-0.2381	2.8169	132
ESPOZ	0.0243	0.0101	0.8418	-0.1539	0.1069	0.0114	3.6538	27.7080	132

Notes: In the left column, the letter E indicates excess return
Source: Author's calculations

Weighted averages of excess returns of eighteen intersection portfolios can be sorted as:

ESPOZ > ESL > ESH > ESC > EBL > ESN > ESR > ESOP > ESW > ESA > EBN > EBOP > EBC > EBA > EBW > EBPOZ > EBR > EBH.

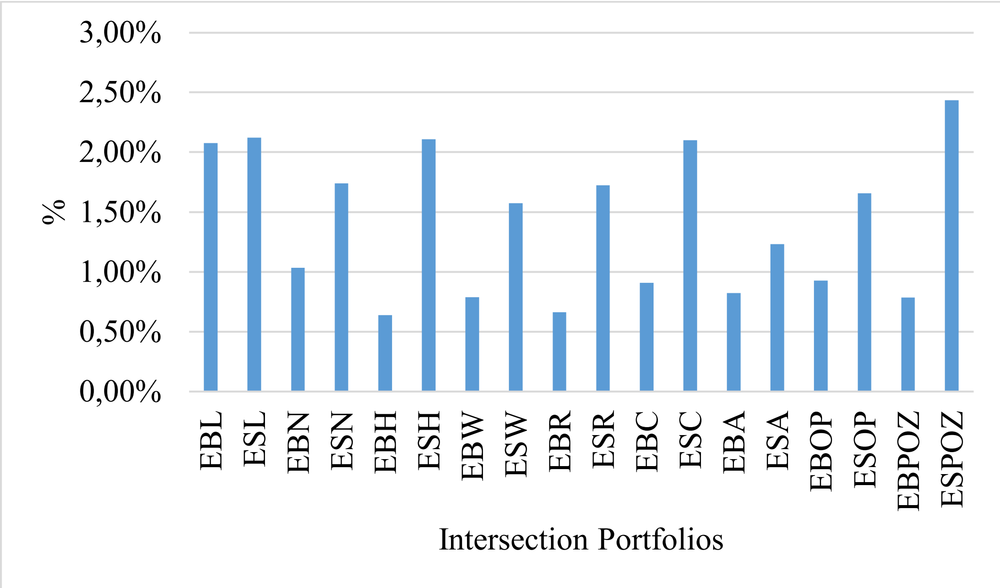

Figure 3.3: Average Excess Returns of Intersection Portfolios
Source: Author's calculations

Table 3.13 shows the summary statistics of Fama-French's five factors without FX risk.

Table 3.13: Summary statistics of factor variables

Factors	**Mean**	**Median**	**Max**	**Min**	**Std. Dev.**	**Variance**	**Skewness**	**Kurtosis**	**Obs**
R_m-R_f	0.0026	0.0017	0.1576	-0.1608	0.0676	0.0046	-0.0156	2.3979	132
SMB	0.0090	0.0032	0.3138	-0.1044	0.0508	0.0026	2.0772	12.2642	132
HML	-0.0073	-0.0129	0.5995	-0.1242	0.0719	0.0052	4.7831	40.1682	132
RMW	0.0001	0.0040	0.0852	-0.1461	0.0368	0.0014	-0.4584	3.8294	132
CMA	0.0048	0.0052	0.0797	-0.0779	0.0288	0.0008	0.0941	2.8336	132

Source: Author's calculations

Average values of the factor variables are sorted as:

$$SMB > CMA > R_m - R_f > RMW > HML$$

The correlation matrix of Fama-French's five factors is presented in Table 3.14.

Table 3.14: Correlation matrix of factor variables in the sense of FF5F model without FX risk

	SMB	**HML**	**RMW**	**CMA**	**R_m-R_f**
SMB	1.0000				
HML	0.4605	1.0000			
RMW	-0.3312	0.0553	1.0000		
CMA	0.2190	0.0861	-0.3407	1.0000	
R_m-R_f	0.0738	-0.0193	-0.1849	0.1457	1.0000

Source: Author's calculation

The most significant correlation coefficient value with 0.4605 is between size and value factor variables. We obtained the weakest correlation coefficient among five factors between market and value portfolios. All the correlation coefficients are between -0.5 and 0.5.

Tables 3.15 & 3.16 demonstrate the summary statistics and the correlation matrix of the Fama-French five-factor model with FX risk. Summary statistics for market, value, profitability, and investment factors are identical for both versions of the FF5F model with and without FX risk.

The equation for Fama-French five-factor model incorporating FX risk is as follows:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t^{'} + h_i HML_t + r_i RMW_t + c_i CMA_t + f_i FX_t + \varepsilon_{it} \quad (3.9)$$

We will compare the performance of the traditional FF5F model with the performance of the FF5F model incorporating FX risk in explaining deviations in portfolio returns in the following chapters.

Table 3.15: Summary statistics of factor variables including FX risk

Factors	Mean	Median	Max	Min	Std. Dev.	Variance	Skewness	Kurtosis	Obs
R_m-R_f	0.0026	0.0017	0.1576	-0.1608	0.0676	0.0046	-0.0156	2.3979	132
SMB	0.0097	0.0036	0.3452	-0.1068	0.0524	0.0027	2.3974	15.0643	132
HML	-0.0073	-0.0129	0.5995	-0.1242	0.0719	0.0052	4.7831	40.1682	132
RMW	0.0001	0.0040	0.0852	-0.1461	0.0368	0.0014	-0.4584	3.8294	132
CMA	0.0048	0.0052	0.0797	-0.0779	0.0288	0.0008	0.0941	2.8336	132
FX risk	-0.0032	0.0018	0.0567	-0.3573	0.0402	0.0016	-5.2951	47.0602	132

Source: Author's calculations

Average values of the factor values, including the FX risk, are sorted as:

$$SMB > CMA > R_m - R_f > RMW > FX\_Risk > HML$$

Size and investment factors generate higher returns than the market portfolio. However, returns on value, FX position, and profitability factors fall behind the market portfolio. Value and FX position factors yield negative average returns (see Figure 3.4 & 3.5)

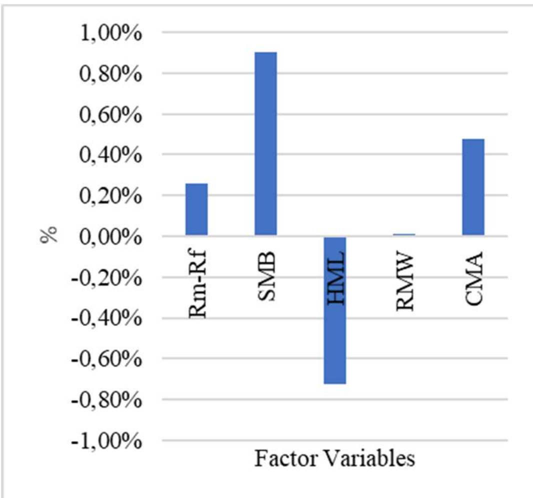

Figure 3.4: Average returns on factor variables in FF5F model without FX risk

Source: Author's calculations

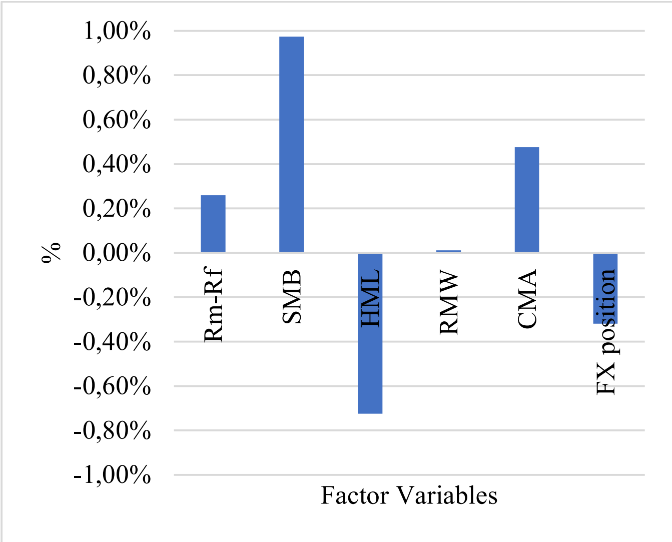

Figure 3.5: Average returns on factor variables in FF5F model incorporating FX risk

Source: Author's calculations

Table 3.16: Correlation matrix of factor variables including FX position factor

	SMB	HML	RMW	CMA	FXposition	R_m-R_f
SMB	1.0000					
HML	0.4658	1.0000				
RMW	-0.3463	0,0553	1.0000			
CMA	0.2305	0.0861	-0.3407	1.0000		
FXposition	-0.4240	-0.4580	0.0264	-0.1018	1.0000	
R_m-R_f	0.0855	-0.0193	-0.1849	0.1457	0.0683	1.0000

Source: Author's calculations

The correlation coefficient between size and value factors is the strongest, just as the correlation coefficient in the traditional FF5F model without FX risk. Likewise, the weakest correlation coefficient among factors in the FF5F model incorporating FX risk is between market and value factors. Similarly, all the correlation coefficients reside between -0.5 and 0.5 once the FX risk is incorporated in the FF5F.

Despite Parlak & İlhan's [9] findings towards a company's FX position, asset efficiency, and profitability, the correlation coefficient between CMA (profitability) factor and FX risk is weak.

Table 3.17 shows the variance inflation factor (VIF) values for both versions of the FF5F without FX risk and the FF5F model incorporating FX risk.

Table 3.17:Estimated VIF Values

Variance Inflation Factor (VIF)	**R_m-R_f**	**SMB**	**HML**	**RMW**	**CMA**	**FX position**	**Mean VIF**
FF5F model without FX risk	1.04	1.52	1.36	1.33	1.16	-	1.28
FF5F model incorporating FX risk	1.05	1.69	1.51	1.36	1.16	1.38	1.36

Source: Author's calculations

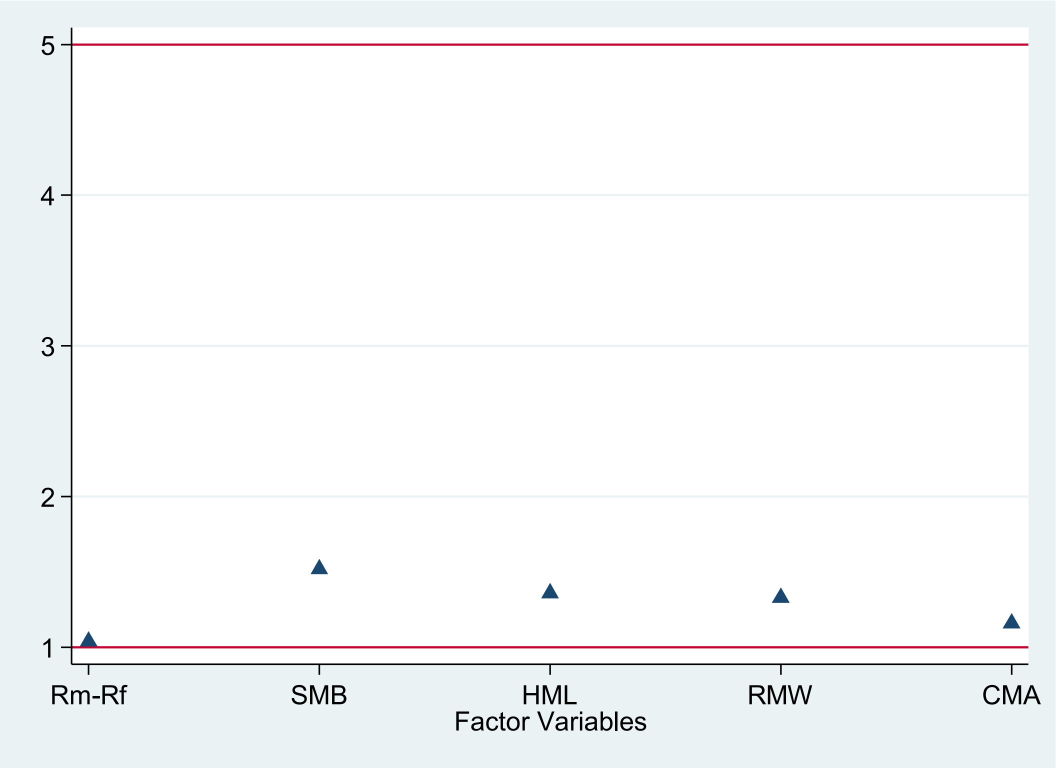

Figure 3.6: Estimated VIF values for the FF5F model without FX risk

Source: Author's calculations

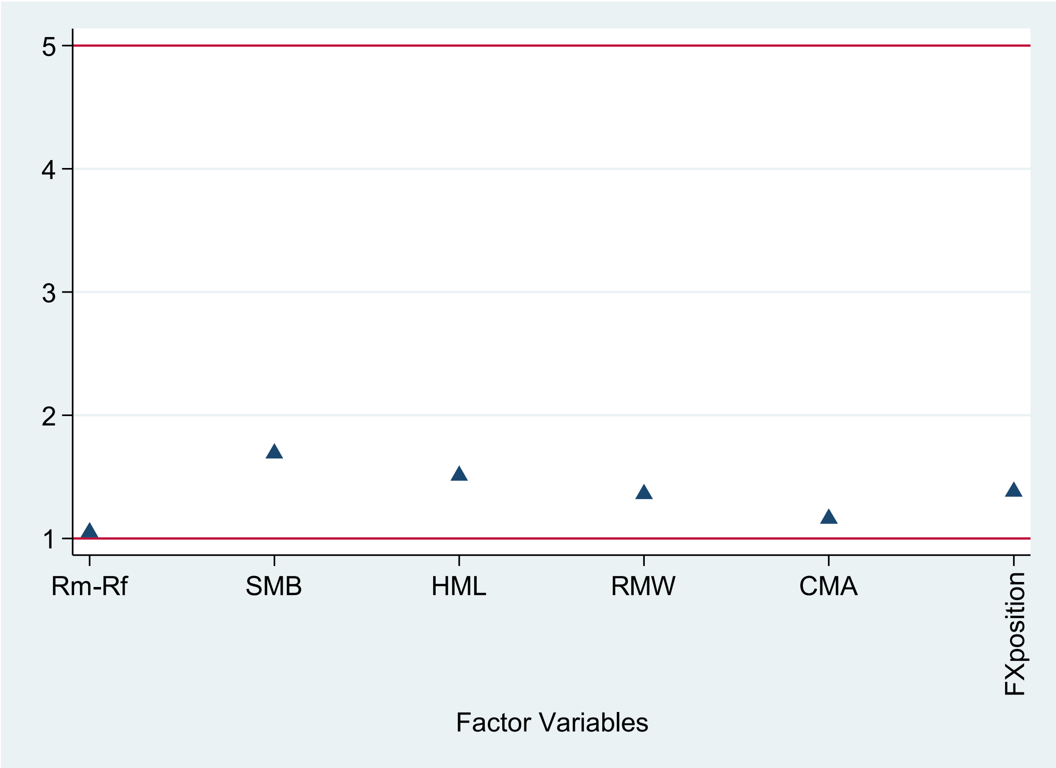

Figure 3.7: Estimated VIF values for the FF5F model incorporating FX risk

Source: Author’s calculations

VIF values for Fama-French's *market*, *size*, *value*, *profitability*, and *investment* factors are between 1.04-1.52. Similarly, the six factors of the FF5F model incorporating FX risk are between 1.05 and 1.69. So, VIF values do not indicate an explicit correlation between any pairs of factor variables for both versions of the model.

Regarding correlation coefficients and VIF values, we conclude that multicollinearity does not exist in the original and modified version of FF5F.

3.6 Conclusion

In this chapter, we described the methodology and data. We followed a similar methodology to Fama-French to construct a five-factor model without FX risk. We proposed a new factor to measure the effect of FX risk and incorporated it in the Fama-French five-factor model. Chapter 4 will focus on time series and other statistical properties of factor variables. Moreover, we will obtain estimation results through linear regression. Finally, we will compare the Fama-French five-factor model incorporating FX risk and the FF5 model without FX risk based on several statistical indicators such as the adjusted R^2 values, Gibbons, Ross, and Shanken's [66] GRS-F test, and average absolute values (AAV) of the intercept terms of the regressions. All of these indicators will be approached in detail in the next chapter.

CHAPTER 4

TIME-SERIES PROPERTIES AND COMPARISON OF MODEL PERFORMANCES

4.1 Introduction

This chapter presents the estimation results for Fama-French five-factor model without FX risk and the five-factor model incorporating FX risk. The excess returns of eighteen intersection portfolios[9] are estimated through the factor variables. Estimations are obtained following Fama & French's [33][8] five-factor methodology and a similar methodology incorporating FX risk in the five-factor model. We adopted a time series approach encompassing 132 months between July 2009 and June 2020. Intercept values, slopes, t statistics, p values, R^2 values, and F statistics are evaluated and interpreted.

In Chapter 3, correlation coefficients and variance inflations factor (VIF) values indicated that factor variables do not suffer multicollinearity. Before obtaining the estimation results, we will apply diagnostic tests to have a broader perspective on our data. We will examine the stationarity of excess returns of the market and intersection portfolios and factor variables. We will apply Augmented Dickey-Fuller and Phillips Perron tests. Test procedures, test results, and what the results indicate will be disclosed in 4.2 and 4.3, respectively. In 4.4, we will apply Breusch-Pagan/Cook Weisberg Test for heteroskedasticity. If the Breusch-Pagan/Cook Weisberg test result indicates heteroscedasticity for a particular portfolio, we apply

[9] Including portfolios formed based on size and B/M cluster

the Generalized Least Squares (GLS) method for the corresponding regression. The application of GLS will be discussed in subsection 4.5.

In 4.6, we will present prediction results with the FF5F model without FX risk and the FF5F model incorporating FX risk. We will also report our findings in that section. In the final subsection, we will compare the model performances and explain our findings.

4.2 Augmented Dickey-Fuller Test

In the OLS estimation of the AR(1) process with Gaussian errors;

$$y_t = \rho * y_{t-1} + \varepsilon_t$$

where:

ε_t : error term iid $\sim N(0,\sigma^2)$

y_0 : 0

ρ: autocorrelation parameter estimator, given by $\hat{\rho}_n = \dfrac{\sum_{t=1}^{n} y_t y_{t-1}}{\sum_{t=1}^{n} y_t^2}$

If $|\rho| < 1$, then

$$\sqrt{n}(\hat{\rho}_n - \rho) \rightarrow N(0, 1-\rho^2)$$

The equation for ADF is as:

$$\Delta y_t = a + \beta y_{t-1} + \sum_{j=1}^{k} \varsigma_i \Delta y_{t-j} + \varepsilon_t \qquad (4.1)$$

The ADF equation includes extra lagged terms of the dependent variable, Δy_t to eliminate the autocorrelation. The ADF test has the following hypothesis:

H_0: Unit root exists (the serials are not stationary),

H_1: Unit root does not exist (the serials are stationary).

The ADF test statistics and p-values for the Fama-French Five-Factor model's excess returns of intersection portfolios and factor variables are shown in Table 4.1.

Table 4.1: Augmented Dickey-Fuller test statistics and p values for the Fama French Five-Factor Model without FX risk

Intersection Portfolios and Factor Variables	**t statistics**	**p-values**
EBL	-10.462	0.0000
ESL	-9.850	0.0000
EBN	-11.919	0.0000
ESN	-10.461	0.0000
EBH	-12.141	0.0000
ESH	-11.291	0.0000
EBW	-10.420	0.0000
ESW	-10.641	0.0000
EBR	-12.175	0.0000
ESR	-10.416	0.0000
EBC	-12.923	0.0000
ESC	-9.540	0.0000
EBA	-11.402	0.0000
ESA	-10.217	0.0000
EBOP	-11.597	0.0000
ESOP	-10.910	0.0000
EBPOZ	-12.697	0.0000
ESPOZ	-10.262	0.0000
SMB	-9.509	0.0000
HML	-10.920	0.0000
RMW	-10.222	0.0000
CMA	-12.035	0.0000
Rm-Rf	-12.021	0.0000

Note: "E" refers to excess returns.
Source: Author's calculations

P-values of "0" indicate that t statistics of the serials are above the MacKinnon critical values at all significance levels. As Table 19 demonstrates, p values of excess returns of intersection portfolios and factor serials are equal to 0. Hence, we rejected the null hypothesis, H_0, and concluded that the serials are stationary.

The ADF test statistics and p-values of additional factor (FX risk) and recalculated size factor formed due to the incorporation of FX risk in the FF5F model are presented in Table 4.2.

Table 4.2: Augmented Dickey-Fuller test statistics and p values for FX risk and recalculated size factor formed due to the incorporation of FX risk in the FF5F model

Factor Variables	t statistics	p-value
SMB’	-9.552	0.0000
FX risk	-12.718	0.0000

Source: Author’s calculations

The ADF test statistics of alternative size and FX risk factors constructed through the FF5F model incorporating FX risk are above the MacKinnon critical values. Therefore, we rejected the null hypothesis, H_0, and confirmed that alternative size and FX risk factors are also stationary.

4.3 Phillips Perron Test

The equation for the Phillips Perron test is:

$$\Delta y_t = \beta' D_t + \pi y_t + \mu_t \tag{4.2}$$

where:

$\mu_t : I(0)$

There are two statistics, Z_p and Z_τ, calculated as:

$$Z_p = n(\hat{\rho}_n - 1) - \frac{1}{2}\frac{n^2\hat{\sigma}^2}{s_n^2}(\hat{\lambda}_n^2 - \hat{\gamma}_{0,n})$$

$$Z_t = \sqrt{\frac{\hat{\gamma}_{0,n}}{\hat{\lambda}_n^2}} \frac{\hat{\rho}_n - 1}{\hat{\sigma}} - \frac{1}{2}(\hat{\lambda}_n^2 - \hat{\gamma}_{0,n}) \frac{1}{\hat{\lambda}_n} \frac{n\hat{\sigma}}{s_n}$$

$$\hat{\gamma}_{j,n} = \frac{1}{n} \sum_{i=j+1}^{n} \hat{u}_i \hat{u}_{i-j}$$

$$\hat{\lambda}_n^2 = \hat{\gamma}_{0,n} + 2\sum_{j=1}^{q} (1 - \frac{j}{q+1}) \hat{\gamma}_{j,n}$$

$$s_n^2 = \frac{1}{n-k} \sum_{i=1}^{n} \hat{u}_i^2$$

where:

u_i : OLS residual

k : the number of covariates in the regression

q : the number of Newey-West lags in calculating $\hat{\lambda}_n^2$

$\hat{\sigma}$: the OLS standard error of $\hat{\rho}$

The PP test has the following hypothesis for unit root testing:

H_0: Unit root exists (the serials are not stationary),

H_1: Unit root does not exist (the serials are stationary).

The PP test statistics and p-values for the Fama-French Five-Factor model's excess returns of intersection portfolios and factor variables are shown in Table 4.3.

Table 4.3: Phillips Perron test statistics and p values for the Fama French Five-Factor Model without FX risk

Intersection Portfolios and Factor Variables	t statistics	p-values
EBL	-10.462	0.0000
ESL	-9.857	0.0000
EBN	-12.126	0.0000
ESN	-10.410	0.0000
EBH	-12.433	0.0000

Table 4.3: (continued)

ESH	-11.290	0.0000
EBW	-10.395	0.0000
ESW	-10.605	0.0000
EBR	-12.588	0.0000
ESR	-10.369	0.0000
EBC	-13.436	0.0000
ESC	-9.516	0.0000
EBA	-11.525	0.0000
ESA	-10.149	0.0000
EBOP	-11.734	0.0000
ESOP	-10.897	0.0000
EBPOZ	-13.089	0.0000
ESPOZ	-10.263	0.0000
SMB	-9.641	0.0000
HML	-10.909	0.0000
RMW	-10.266	0.0000
CMA	-12.158	0.0000
Rm-Rf	-12.297	0.0000

Note: "E" refers to excess returns.
Source: Author's calculations

Like the ADF test, PP statistics of the serials are well above the MacKinnon critical values at all significance levels p values are equal to 0. Therefore, we rejected the null hypothesis, H_0, and established that the serials are stationary.

In Table 4.4, the PP test statistics and p-values for the FX risk and size factors for the alternative model are presented.

Table 4.4: Phillips Perron test statistics and p values for additional and recalculated factor variables formed according to the FF5F model incorporating FX risk

Factor Variables	**t statistics**	**p-values**
SMB'	-9.683	0.0000
FX risk	-12.806	0.0000

Source: Author's calculations

The PP test statistic values for the additional serials are also above the MacKinnon critical values. As Table 4.4 shows, p values of excess returns two-factor serials are 0. Hence, we rejected the null hypothesis, H_0, and concluded that the serials are stationary.

All in all, Augmented Dickey-Fuller and Phillips Perron tests indicate that all of the input and output serials are stationary.

4.4 Breusch-Pagan/Cook Weisberg Test for Heteroskedasticity

Consider a regression model with k independent variables:

$$Y_t = \beta_0 + \sum_{i=1}^{k} \beta_i X_{ti} + u_t$$

After we estimated the model, we obtained:

$$\tilde{\sigma} = \frac{\sum \hat{u}_t^2}{T}$$

where $\tilde{\sigma}$ is the maximum-likelihood estimator of σ.

Let $P_t = \frac{\hat{u}_t^2}{\tilde{\sigma}^2}$, then the initial regression model is transformed to:

$$P_t = a_0 + \sum_{i=1}^{m} a_i X_{ti} + v_t$$

Let $\hat{P}_t$ be the estimated values of P_t according to OLS regressions.

Breusch Pagan test statistic is calculated as:

$$\delta = \frac{\sum_{t=1}^{T} \hat{p}_t}{2} \tag{4.3}$$

where:

$\hat{p}_t = \hat{P}_t - \bar{\hat{P}}_t$ and $\delta \underset{asy}{\sim} \chi^2_m$

The Breusch-Pagan/Cook Weisberg test has the following hypothesis for heteroskedasticity:

H_0: Constant variance exists (homoskedasticity),

H_1: Constant variance does not exist (heteroskedasticity).

If δ>0 critical chi-square value at the chosen level of significances, we reject the null hypothesis H_0 and conclude that heteroskedasticity exists.

The Breusch-Pagan/Cook Weisberg test statistics and p-values for the traditional FF5F model without FX risk and the five-factor model incorporating FX risk are shown in Table 4.5.

Table 4.5: Breusch-Pagan/Cook Weisberg Test Statistics and p-values for the FF5F Model without FX Risk and the FF5F Model Incorporating FX Risk

	FF5F model without FX risk		**FF5F model incorporating FX risk**	
R_{it}-R_{ft}	**Chi-Square test statistics (5), δ**	**p-value**	**Chi-Square test statistics (6), δ**	**p-value**
EBL	27.86	0.0000	7.84	0.2501
ESL	40.65	0.0000	45.82	0.0000
EBN	2.88	0.7181	7.07	0.3141
ESN	4.90	0.4282	4.80	0.5697
EBH	34.07	0.0000	21.56	0.0015
ESH	60.03	0.0000	14.50	0.0245
EBW	12.60	0.0274	12.65	0.0489
ESW	24.07	0.0002	6.70	0.3491
EBR	16.59	0.0054	14.79	0.0219
ESR	8.99	0.1095	8.16	0.2270

Table 4.5: (continued)

EBC	4.94	0.4227	9.38	0.1531
ESC	24.41	0.0002	24.34	0.0005
EBA	2.34	0.8003	3.86	0.6957
ESA	2.04	0.8431	4.96	0.5495
EBOP	1.65	0.8947	3.26	0.7760
ESOP	47.71	0.0000	4.49	0.6111
EBPOZ	3.17	0.6739	6.10	0.4124
ESPOZ	371,39	0,0000	8,04	0,2353

Note: "E" refers to excess returns.
Source: Author's calculations

Breusch-Pagan/Cook Weisberg test statistics for estimations of EBL, ESL, EBH, ESH, EBW, ESW, EBR, ESC, ESOP, and ESPOZ among 18 equations in the FF5F model without FX risk, are above the critical chi-square values at 0.05 significance level. Hence, we rejected the null hypothesis of constant variance for those equations (Table 4.5).

Considering the Fama-French five-factor model incorporating FX risk, chi-square test statistics for equations to predict ESL, EBH, ESH, EBW, EBR, and ESC are above the critical values at 0.05 level of significance. Hence, we rejected the null hypothesis of constant variance and concluded that residuals of those six equations are not distributed with constant variance.

Although an OLS estimator is unbiased with heteroskedasticity conditions, it would be inefficient to explain deviations in the dependent variable. An estimator obtained through the *Generalized Least Squares* (GLS)[10] method is the efficient unbiased estimator when the variance of residuals is not constant. Therefore, we will apply the GLS method to estimate both versions of FF5F model equations, whose residuals have inconstant variances.

[10] GLS is an application of OLS method which satisfies the Gauss-Markov conditions for a BLUE estimator.

4.5 Estimations with Generalized Least Squares (GLS) Method

We can express the Fama-French five-factor model without FX risk and the five-factor model incorporating FX risk in the form of (see Gujarati & Porter [14]):

$$Y_t = \beta_0 + \sum_{i=1}^{k} \beta_i X_{ti} + \varepsilon_t$$

where:

β_0 : intercept term

β_i : coefficients of factor variables, X_{ti}

k =5 for Fama-French five-factor model without FX risk and k =6 for the five-factor model incorporating FX risk

ε_t : disturbances and $Var(\varepsilon_t) = \sigma_t^2 = \sigma^2 \lambda_t$

Dividing both sides by λ_t, we obtain

$$Var(\varepsilon_t)\frac{1}{\lambda_t} = \sigma^2 \cancel{\lambda_t} \frac{1}{\cancel{\lambda_t}}$$

$$Var(\varepsilon_t)\frac{1}{\lambda_t} = \sigma^2$$

$$Var(\frac{1}{\sqrt{\lambda_t}}\varepsilon_t) = \sigma^2$$

So, once we multiply each observation by $\frac{1}{\sqrt{\lambda_t}}$, we will obtain homoskedastic residuals.

Let $w_t = \frac{1}{\lambda_t}$; regression equations for Fama-French five-factor model without FX risk and five-factor model incorporating FX risk turn out to be:

$$Y_t w_t = \beta_0 w_t + \sum_{i=1}^{k} \beta_i X_{ti} w_t + \varepsilon_t w_t$$

Let $Y_t^* = Y_t w_t$, $X_{to}^* = w_t$, $X_{ti}^* = X_{ti} w_t$, and $\varepsilon_t^* = \varepsilon_t w_t$

Then:

$$Y_t^* = \beta_0 X_{t0}^* + \sum_{i=1}^{k} \beta_i X_{ti}^* + \varepsilon_t^*$$

Similar to the OLS method, we calculate the GLS estimators by minimizing the residual sum of squares as follows:

$$\sum_{t=1}^{132} Y_t w_t = \hat{\beta}_0 \sum_{t=1}^{132} w_t + \sum_{t=1}^{132} \sum_{i=1}^{k} \hat{\beta}_i X_{ti} w_t + \sum_{t=1}^{132} \hat{\varepsilon}_t w_t$$

$$\sum_{t=1}^{132} \hat{\varepsilon}_t^* = \sum_{t=1}^{132} Y_t w_t - \hat{\beta}_0 \sum_{t=1}^{132} w_t - \sum_{t=1}^{132} \sum_{i=1}^{k} \hat{\beta}_i X_{ti} w_t$$

$$\sum_{t=1}^{132} \hat{\varepsilon}_t^{*2} = (\sum_{t=1}^{132} Y_t w_t - \hat{\beta}_0 \sum_{t=1}^{132} w_t - \sum_{t=1}^{132} \sum_{i=1}^{k} \hat{\beta}_i X_{ti} w_t)^2$$

$$\sum_{t=1}^{132} \hat{\varepsilon}_t^{*2} = \sum_{t=1}^{132} (Y_t w_t - \hat{\beta}_0 w_t - \sum_{i=1}^{k} \hat{\beta}_i X_{ti} w_t)^2$$

Note that:

$$\sum_{t=1}^{132} (Y_t w_t - \hat{\beta}_0 w_t - \sum_{i=1}^{k} \hat{\beta}_i X_{ti} w_t)^2 = \sum_{t=1}^{132} w_t^2 (Y_t - \hat{\beta}_0 - \sum_{i=1}^{k} \hat{\beta}_i X_{ti})^2$$

And

$$Y_t - \hat{\beta}_0 - \sum_{i=1}^{k} \hat{\beta}_i X_{ti} = \hat{\varepsilon}_t$$

Hence:

$$\sum_{t=1}^{132} \hat{\varepsilon}_t^{*2} = \sum_{t=1}^{132} \hat{\varepsilon}_t^2 w_t^2$$

GLS estimators for the intercept term and factor variables are calculated as [14]:

$$\hat{\beta}_0 = \frac{\sum_{t=1}^{T} w_t^2 Y_t}{\sum_{t=1}^{T} w_t^2} - \sum_{i=1}^{k} \hat{\beta}_i \left[\frac{\sum_{t=1}^{T} w_t^2 X_{ti}}{\sum_{t=1}^{T} w_t^2} \right]$$

$$\hat{\beta}_0^{GLS} = \overline{Y}_w - \sum_{i=1}^{k} \hat{\beta}_i \overline{X}_{wi}$$

$$\hat{\beta}_i^{GLS} = \frac{\sum_{t=1}^{T} w_t^2 \hat{v}_{tiw} y_{tw}}{\sum_{t=1}^{T} w_t^2 v_{tiw}^1}$$

where**:**

$\overline{Y}_w$: weighted averages of the observations on portfolio returns, Y in the sample

$\overline{X}_w$: weighted averages of the observations on factor variables, X in the sample

$w_t^2 = \frac{1}{\lambda_t}$

$y_{tw} = Y_t - \overline{Y}_w$

$w_t^2 \hat{v}_{tiw} : t^{th}$ residual from the regression of X_{ti}^* on the remaining transformed factor variables

4.6. Estimation Results with FF5 Model without FX Risk and FF5 Model Incorporating FX Risk

The excess returns of intersection portfolios are used as output variables. The time series regressions are conducted for the Fama-French five-factor model without FX risk and the alternative model incorporating FX risk.

4.6.1. Estimation Results with Fama-French Five-Factor Model without FX risk

The traditional Fama-French five-factor model without FX risk is defined as follows:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \qquad (4.4)$$

where:

R_{it} : return on a security or a portfolio *i* at time t

R_{ft} : return on a risk-free asset at time t

R_{mt} : return of value-weighted market portfolio at time t

SMB_t : the difference between the returns on a diversified portfolio of small and big stocks

HML_t : the difference between the returns on diversified portfolios of high and low B/M stocks

RMW_t : the difference between the returns on a diversified portfolio of stocks with robust and weak profitability

CMA_t : the difference between the returns on a diversified portfolio of stocks of low and high investment firms

ε_{it} : disturbance term with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}, \varepsilon_{is}) = 0$; $t \neq s$ and $\varepsilon_{it} \sim iid(0, \sigma^2)$

FF5F model without FX risk has five independent variables which imitate underlying risk factors behind the deviations on portfolio returns. Excess market return over the risk-free rate, SMB_t (size factor), HML_t (value factor), RMW_t (profitability factor), and CMA_t (investment factor) are factor variables that are not state variables as Fama & French [8] define. However, they are proxies that help explain stock return movements.

LHS variables consist of excess returns of intersection portfolios over the risk-free rates. We constructed 18 intersection portfolios: EBL, ESL, EBN, ESN, EBH, ESH, EBW, ESW, EBR, ESR, EBC, ESC, EBA, ESA EBOP, ESOP, EBPOZ, and ESPOZ[11] for the Fama-French five-factor model without FX risk. Excess returns of intersection portfolios are estimated using five-factor variables. Table 4.6 shows intercepts, coefficient of factor variables, t values, F statistics, and adj. R^2 values.

Among 18 estimations, eight intercept terms are not statistically significantly different from 0 as t values signal. All of the intercept values are positive.
All of the β's are statistically significant and positive.

In 7 out of 18 estimations, size coefficients (s) are not statistically significant. Those seven coefficients belong to excess returns of big-size portfolios. Excess returns of all small-size portfolios have statistically significant size coefficients. Signs of the 16 out of 18 size coefficients are positive.

In 7 out of 18 estimations, value coefficients (h) are not statistically significantly different from 0. Six belong to big-size portfolios among seven insignificant size coefficients. 11 out of 18 value coefficients are negative.

Only six of the profitability factor coefficients (r) are statistically significant Investment factor coefficients (c) are statistically significant only for the excess returns of size-investment portfolios and ESH.

Adjusted R^2 values range between 0,7302 to 0,9592. The average Adj R^2 value is 0,8452. Considering the F-statistic test values for each regression, we can conclude that factor variables can jointly estimate excess returns of intersection portfolios reliably.

[11] The letter E indicates excess returns. B/S: Big/Small, L/N/H: Low/Neutral/High in size, W/R: Weak/Robust profitability, C/A: Conservative/Aggresive investment strategy, OP/POZ: Open/Pozitif FX position

Table 4.6: Estimation Results of the Fama French Five Factor Model without FX Risk

Excess Returns	a	t_a	β	$t_β$	s	t_s	h	t_h	r	t_r	c	t_c	Adj R^2	F-stat
EBL	0.012***	2.76	0.809***	14.42	0.035	0.34	-0.851***	-11.32	0.002	0.02	-0.056	-0.39	0.7383	74.93
ESL	0.008***	3.12	0.837***	23.88	0.911***	12.28	-0.396***	-7.81	-0.211***	-2.72	0.078	0.70	0.8667	159.58
EBN	0.007***	2.65	0.928***	23.05	0.095	1.46	0.027	0.62	0.008	0.10	-0.036	-0.36	0.8095	112.30
ESN	0.005**	2.04	0.854***	25.48	0.927***	17.23	-0.255***	-7.10	-0.022	-0.32	0.042	0.50	0.8969	228.90
EBH	0.006**	2.59	0.813***	30.92	-0.072	-1.25	0.117***	2.66	0.098*	1.70	0.075	1.06	0.8901	208.44
ESH	0.012**	2.17	0.925***	15.48	1.219***	11.32	0.409***	3.59	0.105	1.10	-0.441***	-2.98	0.7383	71.54
EBW	0.003	0.89	0.951***	26.52	0.177**	2.49	-0.042	-0.93	-0.569***	-7.28	0.089	0.85	0.8688	173.15
ESW	0.005	1.59	0.878***	22.22	0.766***	10.57	-0.200***	-3.31	-0.569***	-6.07	0.061	0.57	0.8614	161.41
EBR	0.005***	2.66	0.835***	30.17	-0.052	-0.99	0.071**	2.07	0.147**	2.44	0.078	1.06	0.8789	189.66
ESR	0.004	1.19	0.816***	18.65	1.042***	14.84	-0.287***	-6.11	0.429***	4.73	-0.011	-0.10	0.8210	121.21
EBC	0.003	1.00	0.914***	21.90	0.078	1.16	0.014	0.30	-0.151*	-1.75	0.677***	6.55	0.8292	128.17
ESC	0.007***	2.67	0.821***	27.22	0.954***	13.44	-0.271***	-6.00	-0.075	-1.07	0.392***	4.72	0.9044	245.05
EBA	0.007***	3.14	0.801***	25.39	0.098*	1.93	-0.050	-1.48	-0.119*	-1.81	-0.415***	-5.31	0.8402	138.80
ESA	0.001	0.30	0.882***	22.17	1.066***	16.68	-0.266***	-6.24	-0.092	-1.11	-0.491***	-4.99	0.8729	181.00
EBOP	0.007***	3.24	0.842***	27.51	0.044	0.90	0.023	0.69	0.058	0.91	-0.013	-0.18	0.8562	156.98
ESOP	0.003	1.06	0.767***	37.54	1.055***	17.31	-0.247***	-6.36	-0.260***	-4.95	0.014	0.21	0.9592	602.95
EBPOZ	0.004	1.31	0.758***	17.98	0.146**	2.15	-0.077*	-1.71	-0.035	-0.40	0.039	0.37	0.7302	71.91
ESPOZ	0.003	0.79	0.821***	17.34	1.288***	15.03	0.159	1.59	0.091	1.39	0.145	1.56	0.8522	109.38

Notes: 1. In the "Excess Returns" column, **red colors** indicate GLS estimations; **black colors** indicate OLS estimations
2. *,**,*** indicate 0.10 , 0.05 , 0.01 significance levels respectively
Source: Author's calculations

4.6.2. Estimation Results with Fama-French Five-Factor Model Incorporating FX Risk

We incorporated FX risk in the Fama-French five-factor model and obtained a version as:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t^* + h_i HML_t + r_i RMW_t + c_i CMA_t + f_i FX_t + \varepsilon_{it}$$

(4.5)

where:

FX_t : the difference between the returns on a diversified portfolio of stocks of firms with positive and open FX position

SMB_t^* : Transformed version of size factor exposure, namely SMB_t in original FF5

ε_{it} : disturbance term with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}, \varepsilon_{is}) = 0$; $t \neq s$ and $\varepsilon_{it} \sim iid(0, \sigma^2)$

While the FF5F model has five factor variables, the five-factor model incorporating FX risk has six. We created the 6th-factor variable as a proxy for the FX risk of a firm to examine the effect of the new factor variable on the model performance. The size factor (*SMB*) is re-calculated, and the FX risk factor (*FX*) is generated under the *Factor Calculations* section in Chapter 3. Table 4.7 shows intercepts, coefficients of factor variables, t values, F statistics, and adj. R^2 values.

Among 18 regression results, 7 of the intercept terms are not significantly different from 0. All of the intercept terms are positive, as illustrated in Table 4.7.

Like the Fama-French five-factor model without FX risk, estimation results indicate that all of the β's are statistically significant and positive for the FF5F model incorporating FX risk.

In 6 of the estimations out of 18, size coefficients (s) are not statistically significant. All of the insignificant size coefficients consist of excess returns of big-size portfolios. The sign of the 16 out of 18 coefficients is positive.

Six among 18 value coefficients (h) are not statistically significantly different from 0. Once the value factor coefficient (h) obtained through estimation of EBPOZ[12] is not statistically significantly different from 0 in the FF5F model without FX risk, it becomes significant after applying the FF5F model incorporating FX risk. In 12 out of 18 estimations, value coefficients are negative.

Only 6 of the profitability factor coefficients (r) are statistically significant among 18 prediction results. Only the size-investment portfolios and EHS's investment factor coefficients (c) are statistically significant.

12 out of 18 FX risk factor coefficients are statistically significant. Six of the estimation results FX risk factor slopes are negative. The signs of the FX factor coefficients have negative values for the models where EBPOZ[13] and ESPOZ[14] are dependent variables.

Adjusted R^2 values range between 0.6622 and 0.9718. Considering the F-statistics values for each estimation, we can conclude that factor variables can reliably estimate excess returns of intersection portfolios.

[12] Excess returns of intersection portfolio of big size stocks and stocks whose FX assets exceed FX liabilities

[13] Excess returns of intersection portfolio of companies whose FX assets exceed FX liabilities and whose market value is big.

[14] Excess returns of intersection portfolio of companies whose FX assets exceed FX liabilities and whose market value is small.

Table 4.7: Estimation Results of the Fama-French Five-Factor Model Incorporating FX Risk

Excess Returns	a	t_a	β	t_β	s	t_s	h	t_h	r	t_r	c	t_c	f	t_f	Adj R^2	F-stat
EBL	0.012***	2.97	0.799***	13.39	0.186*	1.91	-0.494***	-7.35	-0.157	-1.26	-0.041	-0.28	-0.356***	-3.10	0.6622	43.81
ESL	0.005**	2.27	0.818***	27.56	1.155***	18.63	-0.459***	-11.36	-0.126	-1.53	-0.161*	-1.84	0.034	0.43	0.9718	689.69
EBN	0.007**	2.52	0.936***	23.45	0.060	0.92	-0.004	-0.08	0.002	0.02	-0.046	-0.47	-0.163**	-2.11	0.8148	97.06
ESN	0.005*	1.95	0.842***	24.54	0.930***	16.60	-0.236***	-6.13	0.001	0.02	0.039	0.46	0.129*	1.96	0.8930	183.26
EBH	0.005***	2.65	0.736***	24.89	-0.031	-0.58	0.110***	3.12	-0.022	-0.37	0.179**	2.45	0.139**	2.14	0.8494	121.31
ESH	0.012**	2.52	0.830***	14.40	1.178***	12.93	0.724***	9.76	-0.105	-0.95	0.024	0.17	-0.350**	-2.55	0.8371	113.21
EBW	0.003	0.84	0.908***	24.41	0.184***	2.65	-0.055	-1.21	-0.660***	-8.13	0.059	0.55	-0.066	-0.94	0.8712	147.52
ESW	0.004	1.62	0.791***	21.68	0.910***	15.28	-0.165***	-4.03	-0.579***	-7.61	0.115	1.28	0.300***	4.27	0.9000	197.59
EBR	0.005***	2.62	0.835***	29.87	-0.036	-0.71	0.089***	2.76	0.129**	2.14	0.073	0.94	0.091	1.55	0.8782	155.99
ESR	0.004	1.16	0.799***	18.27	1.062***	14.87	-0.260***	-5.29	0.462***	5.06	-0.012	-0.11	0.189**	2.24	0.8226	102.25
EBC	0.003	1.06	0.908***	21.76	0.107	1.57	0.033	0.70	-0.140	-1.62	0.682***	6.63	0.122	1.51	0.8310	108.39
ESC	0.008***	3.56	0.794***	27.95	0.867***	15.23	-0.186***	-5.53	-0.144**	-2.48	0.423***	5.41	0.139**	2.36	0.9131	226.99
EBA	0.007***	3.15	0.798***	25.13	0.107**	2.07	-0.041	-1.14	-0.114*	-1.72	-0.413***	-5.28	0.056	0.92	0.8399	115.54
ESA	0.001	0.34	0.863***	21.90	1.096***	17.04	-0.225***	-5.09	-0.058	-0.70	-0.488***	-5.03	0.261***	3.43	0.8768	156.38
EBOP	0.007***	3.54	0.831***	28.41	0.099**	2.07	0.056*	1.70	0.076	1.24	-0.004	-0.05	0.209***	3.71	0.8698	146.79
ESOP	0.005**	2.01	0.803***	25.06	1.064***	20.35	-0.137***	-3.79	-0.138**	-2.07	0.047	0.59	0.494***	8.01	0.9154	237.25
EBPOZ	0.003	1.19	0.777***	20.05	0.030	0.47	-0.138***	-3.17	-0.068	-0.84	0.020	0.21	-0.391***	-5.23	0.7744	75.93
ESPOZ	0.009***	3.39	0.857***	23.78	1.133***	19.26	0.057	1.42	0.006	0.08	0.024	0.27	-0.906***	-13.04	0.9355	317.58

Notes: 1. In the "Excess Returns" column, **red colors** indicate GLS estimations; **black colors** indicate OLS estimations.
2. *,**,*** indicate 0.10 , 0.05 , 0.01 significance levels respectively.
Source: Author's calculations

4.7 Comparison of the Model Performances

Table 4.8 demonstrates the average adj R^2 values, the minimum and maximum of adj R^2 values, GRS F-test statistics and GRS p-values, and averages of absolute values of alphas of both versions of the FF5F model.

Table 4.8: Performance Indicators of the FF5F Model with and without FX Risk

Model	Min-Max of Adj R^2 Values	Avg Adj R^2 Values	GRS F-test statistics	GRS p-value	AAV
FF5F without FX risk	0.7302-0.9592	0.8452	1.154141	0.312349	0.0056
FF5F incorporating FX risk	0.6622-0.9718	0.8588	1.072721	0.389156	0.0058

Notes: AAV indicates the average of absolute values of the intercept terms
Source: Author's calculations

Comparing the adj R^2 values of the Fama-French five-factor model without FX risk and the five-factor model incorporating FX risk, we observed a small jump from 0.8452 to 0.8588 on avg adj R^2 values. Encompassing FX risk to the FF5 model leads to a marginal increase in the explanatory power concerning avg adj R^2 values. Estimation results indicate adj R^2 values scattered between 0.7302 and 0.9592 for the Fama-French five-factor model without FX risk. Estimation results obtained from Fama-French five-factor model incorporating FX risk indicate that adj R^2 values are spread out between 0.6622-0.9718. Table 4.9 shows comparisons in adj R^2 values.

Table 4.9: The Comparative Adj. R^2 Values

Number of predictions	Intersection portfolios	FF5F model without FX risk	FF5F model incorporating FX risk	Change in adj R^2 values due to incorporation of FX risk
1	EBL	0.7383	0.6622	-
2	ESL	0.8667	0.9718	+
3	EBN	0.8095	0.8148	+
4	ESN	0.8969	0.8930	-
5	EBH	0.8901	0.8494	-

Table 4.9: (continued)

6	ESH	0.7383	0.8371	+
7	EBW	0.8688	0.8712	+
8	ESW	0.8614	0.9000	+
9	EBR	0.8789	0.8782	-
10	ESR	0.8210	0.8226	+
11	EBC	0.8292	0.8310	+
12	ESC	0.9044	0.9131	+
13	EBA	0.8402	0.8399	-
14	ESA	0.8729	0.8768	+
15	EBOP	0.8562	0.8698	+
16	ESOP	0.9592	0.9154	-
17	EBPOZ	0.7302	0.7744	+
18	ESPOZ	0.8522	0.9355	+

Notes:1. "E" indicates excess return over the risk-free return for portfolio i at time t.
2. B/S: Big/Small, L/N/H: Low/Neutral/High market-to-book ratios, W/R: Weak/Robust profitability, C/A: Conservative/Aggressive investment strategy, OP/POZ: open FX position/FX surplus.
3. In the right column, "+" indicates improvements in adj R^2 values by incorporating FX risk to the Fama-French five-factor model, and "-" indicates diminishing values.
Source: Author's calculations

In 12 estimation results out of 18, for the portfolio excess returns, namely, ESL, EBN, ESH, EBW, ESW, ESR, EBC, ESC, ESA, EBOP, EBPOZ, and ESPOZ, the five-factor model incorporating FX risk has higher adj R^2 values compared to the Fama-French five-factor model without FX risk[15].

In 6 out of 18 estimations where dependent variables are EBL, ESN, EBH, EBR, EBA, and ESOP, the five-factor model incorporating FX risk has lower adj R^2 values than the FF5F model without FX risk[16].

[15] Differences between values of adj R^2 values of predictions for EBN, EBW, ESR, EBC and ESA negligible.
[16] Differences between values of adj R^2 values of predictions for ESN, EBR and EBA are negligible.

More specifically, Fama-French five-factor model incorporating FX risk is highly efficient at predicting excess returns of portfolios defined as SL[17], SH[18], SW[19] , and SPOZ[20].

Another statistical indicator is the GRS-F statistic of Gibbons et al. [66]. GRS-F statistic indicates whether the intercept terms (alphas) are jointly zero in regressions where excess returns of portfolios or assets are dependent variables and where factor exposures are independent variables.

Jobson & Korkie [67] adapts GRS F-test statistic of Gibbons, Ross, and Shanken [66] to multi-factor models as follows (see also Gökgöz [68]):

$$J = \frac{(T-N-K)}{N} * (1 + \mu_k^1 \Omega^{-1} \mu_k^{-1})^{-1} \hat{a}^1 \hat{\Sigma}^1 \hat{a} \tag{4.7}$$

where:
J: GRS-F test statistic value
T: number of observations (132)
N: number of assets or portfolios (18 intersection portfolios for both versions of the FF5F model with and without FX risk)
K: number of factor variables in the model (5 for the FF5F without FX risk, 6 for the FF5F incorporating FX risk)
μ_k: is a k-vector of factor means
Ω: k x k covariance matrix of factor returns
$\hat{a}$: estimated intercept values
$\sum$: variance-covariance matrix of estimated error terms

Rejection of the GRS-F test indicates pricing error, and therefore factor variables can not correctly explain deviations in returns.

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[17] Small size companies and companies which have low market-to-book value
[18] Small size companies and companies which have high market-to-book value
[19] Small size companies and companies which have weak profitability
[20] Small size companies and companies whose assets in foreign curreny exceed liabilities in foreign currency

The GRS-F test has the following hypothesis for testing pricing errors:

H_0 : intercept terms are jointly 0 (pricing error does not exist),

H_1 : intercept terms are not indistinguishable from 0 (pricing error exists).

The GRS-F test statistics and p-values for the Fama-French five-factor model without FX risk and the five-factor model incorporating are shown in Table 26. GRS-F test statistic for the FF5F model without FX risk is 1.154141, and the p-value is 0.312349. Since GRS_p-value is well above 0.05, we accept the H_0 and conclude that there is no pricing error for the FF5F model without risk. GRS-F test statistic value of the Fama-French five-factor model incorporating FX risk is calculated as 1.072721, and its GRS_p-value is calculated as 0.389156. Hence, we accepted H_0 for the FF5F model incorporating FX risk and verified that there is no pricing error. Both versions of FF5F with and without FX risk do not have pricing errors, and consequently, both models are valid for Borsa İstanbul over the period July 2009-June 2020.

A smaller GRS-F value is desirable, for it leads to higher p-values. Hence, the Fama-French five-factor model incorporating FX risk is superior in GRS-test statistics and p-values.

According to Fama-French, the average absolute value of intercept is another indicator to assess models' relative performances. A model with a smaller AAV is considered superior and vice-versa. (see [33])

The formula for AAV is:

$$AAV = \frac{\sum_{i=1}^{n} |a_i|}{n} \quad (4.8)$$

where:

a_i : intercept term of estimation i; i=1,2,…17,18

n: number of estimations; 18

Table 4.10 lists the intercept values and AAVs for the FF5F model without FX risk and the FF5F model incorporating FX risk.

Table 4.10: Intercept Terms and AAVs of Fama-French Five-Factor without FX risk and Five-Factor Model Incorporating FX Risk

Excess Portfolio Returns	**FF5 model without FX risk**	**FF5 model incorporating FX risk**
EBL	0.0124	0.0124
ESL	0.0081	0.0052
EBN	0.0074	0.0070
ESN	0.0048	0.0047
EBH	0.0055	0.0054
ESH	0.0124	0.0115
EBW	0.0028	0.0026
ESW	0.0051	0.0041
EBR	0.0051	0.0051
ESR	0.0036	0.0035
EBC	0.0029	0.0031
ESC	0.0066	0.0078
EBA	0.0069	0.0070
ESA	0.0008	0.0009
EBOP	0.0069	0.0072
ESOP	0.0029	0.0045
EBPOZ	0.0038	0.0032
ESPOZ	0.0028	0.0085
Average Absolute Value (AAV)	**0.0056**	**0.0058**

Source: Author's calculations

We obtained close results of AAVs for the FF5F model without FX risk and the FF5F model incorporating FX risk. While AAV for the FF5F model without FX risk is assessed as 0.0056, AAV for the FF5 incorporating FX risk to the model is 0.0058. Although the FF5F model without FX risk seemingly slightly performs better than the five-factor model incorporating FX risk, the difference is trivial. Hence, the performances of both models are similar in the sense of AAV.

4.8 Findings

In chapter 3, we have observed moderate correlation coefficients and variance inflation factor (VIF) values among factor variables. Hence, there is no evidence of multicollinearity in our data. We also conducted Augmented Dickey-Fuller and Phillips Perron tests which suggest that neither factor variables nor intersection portfolios imply serial correlation. Breusch-Pagan/Cook Weisberg test results indicate the presence of heteroskedasticity problems for some of the intersection portfolios. We applied the Generalized Least Squares (GLS) method for portfolios that demonstrate heteroskedasticity issues. For remaining, we estimated the portfolio excess returns via the Ordinary Least Squares (OLS) method[21].

To estimate 10 out of 18 excess portfolio returns with the Fama-French five-factor methodology without FX risk, we implemented the GLS method. On the other hand, in 6 out of 18 estimations of excess portfolio returns, we used the GLS method with the five-factor model incorporating FX risk.

The average adj R^2 value estimated out of the Fama-French five-factor model incorporating FX risk is slightly higher than the average adj R^2 value obtained from Fama-French five-factor model without FX risk. Assessing portfolio return estimations one by one, 12 out 18 estimations indicate improvement in adj R^2 due to incorporation of FX risk to the five-factor model. Explicitly, Fama-French five-factor model incorporating FX risk is exceptionally efficient at predicting excess returns of portfolios defined as SL[22], SH[23], SW[24] , and SPOZ[25].

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[21] Henceforth, we will use the SLR and OLS/GLS interchangebly
[22] Small size companies and companies which have low market-to-book value
[23] Small size companies and companies which have high market-to-book value
[24] Small size companies and companies which have weak profitability
[25] Small size companies and companies whose assets in foreign curreny exceed liabilities in foreign currency

Despite the popularity of the simple linear regression method in the Fama-French factor model literature, there are significant drawbacks retaining researchers from understanding the complex nature of the relation between risk and return. Most prominently, due to its nature, the simple linear regression method cannot capture non-linear relationships between factor variables and excess portfolio returns. On top of that, the simple linear regression method is not flexible for parameters that should satisfy Gauss-Markov assumptions to implement the model correctly. Parameters may not always satisfy these conditions.

In Chapter 5, we will apply a machine learning technique, support vector regression method, to predict portfolio returns. Consequently, we will compare the simple linear regression method and support vector regression method in the sense of Fama-French five-factor without FX risk and Fama-French five-factor incorporating FX risk.

CHAPTER 5

ESTIMATING PORTFOLIO RETURNS BY USING SUPPORT VECTOR REGRESSION METHOD

5.1 Introduction

In a substantial part of the empirical studies in the asset pricing literature, it is preferable to implement simple linear estimation methods because the relationship between risk factors and asset returns is considered linear (see Fame & French[8] and others). Moreover, it looks as though SLR is easier to perform. Nevertheless, neither the relationship between risk and stock returns might be linear as traditional Fama-French multi-factor models imply, nor is it easy to implement the SLR technique as it seems. According to Fang and Taylor [69], although the linear factor model provides simple specifications and fitting characteristics, it has limitations to capture the genuine relationship between expected returns and risk factors. Nakagawa et al. [70] criticize the linear factor models for their accuracy is limited because of the non-linear dynamics in the financial markets.

Some recent studies apply alternative methods such as machine learning, deep learning, deep neural network, and others. Dittman [71] examines the impact of non-linear pricing kernels on estimating the cross-section of stock returns. He found that non-linear pricing kernels outperform Fama-French three-factor model in explaining the deviations in stocks returns. Bagudu et al. [72] combine Bayesian optimization and the SVR method to estimate industries' portfolio returns in the United States with the Fama-French three-factor and five-factor models. The results of their study indicated the superiority of BSVR over the traditional methods in the asset pricing literature.

Chen et al. [73] apply deep neural network technique to predict US stock returns over 1967-2016. Sharpe ratios, the amount of explained variations, and pricing errors indicate better results for the deep neural network method than benchmark techniques such as deep learning and the traditional Fama-French five-factor model. Nakagawa et al. [70] develop a deep recurrent multi-factor model and examine its performance for the Japanese stock market over December 1990-March 2015. Annualized return, volatility, Sharpe ratio, MAE, and RMSE statistics suggest that the deep recurrent multi-factor model obtained the best performance among other techniques such as deep factor, linear regression, SVR, and random forest.

Gogas et al. [74] test CAPM, FF3F & FF5F, and arbitrage pricing theory (APT) model by implementing OLS and SVR methods. They also used three different kernel functions: linear, radial basis, and polynomial. Adj R^2 and Mean Absolute Percentage Error statistics suggest that the SVR method with RBF and polynomial kernels produce better results. Estimation results of the APT model via SVR tool with RBF kernel indicated the best overall performance among the combinations of four models and two statistical techniques[26].

In this chapter, we will apply a machine learning tool, the *support vector regression* method, which is becoming popular in asset pricing literature. The tool was introduced by Cortes & Vapnik [75]. SVR is nothing but support vector machine (SVM) when the type of dependent variable is numerical[27] instead of categorical.

In the simple linear regression (SLR) method, the parameters to be estimated must satisfy the Gauss-Markov assumptions. Unlike SLR, distributions in the SVR technique depend on kernel functions to be specified; therefore, parameters do not have to satisfy Gauss-Markov conditions to implement SVR. According to Bagudu et al. [72], SVR is an effective tool, especially when the number of observations and independent variables is small. Another advantage of SVR is considered as its

[26] Including SVR with three different kernel functions (see Gogas et al. [74]).
[27] In SVM, dependent variable is categorical by definition.

ability to capture non-linear relationships as well as linear relationships. SVR is also flexible for forming non-linear models without changing independent variables. Bagudu et al. [72] argue that another advantage of SVR is that through non-linear transformation with the kernel function, it permits the mapping of the data into a higher-dimensional space.

In the following subsection, we will touch upon the general methodology of the support vector regression. In 5.3, we will report prediction results and compare the performances of two techniques for the Fama-French five-factor without FX risk and the five-factor model incorporating FX risk. Subsection 5.4 discusses our main findings.

5.2 Support Vector Regression (SVR) Methodology

Similar to OLS and GLS methods, the objective of SVR is to find the best fit line or hyper-plane. However, instead of focusing on residuals of each observation, the SVR method sets a threshold value and ignores observation points that remain within threshold values. Threshold value is the distance between the best fit line (hyper-plane) and the boundary values around the best fit line (hyper-plane). In other words, the best fit line (hyper-plane) and upper and lower boundaries around the line form *ε-insensitive tube* (see [76],[77] & Figure 5.1).

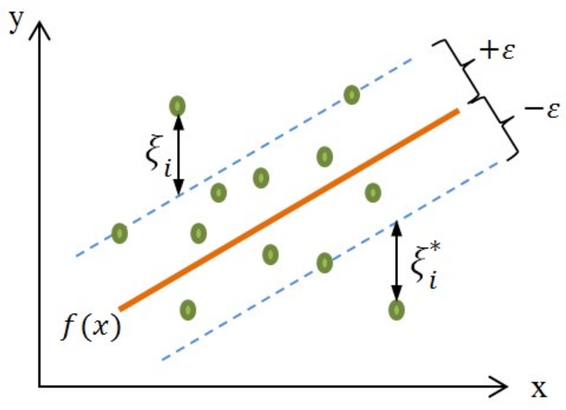

Figure 5.1: Illustration of Linear Support Vector Regression Model
Source: Chanklan et al. (2018) (see [77])

As Figure 5.1 indicates, observation points that reside outside the ε-insensitive tube are called *slack variables* (ξ_i and ξ_i^*). SVR aims to minimize the distance of slack variables from the ε-intensive tube. Slack variables and maximum allowable error terms are defined such that:

ε : the distance between best fit line (hyper-plane) and boundary values, the threshold value

ξ_i : the distance between observation i and the upper boundary values of ε-insensitive tube

ξ_i^* : the distance between observation i and the lower boundary values of ε-insensitive tube

Bagudu et al. [72] express the linear regression model as follows:

$$f(x_i) = \omega^T \phi(x_i) + b \tag{5.1}$$

where:

$f(x_i)$: output function

ω: weight vector

$\phi(x_i)$: non-linear mapping function

b : intercept vector

To determine ε-insensitive tube with minimum distance ε , we must obtain the lowest ω by minimizing the Euclidian norm $\|\omega\|^2$. Bagudu et al. [72] transform the problem as follows:

$$\min \frac{1}{2}\|\omega\|^2 \tag{5.2}$$

s.t.

$$y_i - \omega^T \phi(x_i) - b \le \varepsilon$$
$$\omega^T \phi(x_i) + b - y_i \le \varepsilon$$

Equation 5.2 assumes that there are no slack variables outside the ε-insensitive tube. To allow for the slack variables ξ_i and ξ_i^* to exist, we add slack variables with a cost (regularization) parameter to equation 5.3. As a result, Bagudu et al. [72] define the extended version of the objective function and the corresponding constraints as shown:

$$\min \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \tag{5.3}$$

s.t.

$$y_i - \omega^T \phi(x_i) - b \le \varepsilon + \xi_i$$
$$\omega^T \phi(x_i) + b - y_i \le \varepsilon + \xi_i^*$$
$$\xi_i, \xi_i^* \ge 0, i = 1, 2 \ldots n$$

where:

C : cost or regularization parameter for penalizing over-fitting problem

The existence of slack variables can be formulated as ([72]):

$$|\xi|_\varepsilon = \begin{cases} 0, if\ |\xi| \le \varepsilon \\ |\xi| - \varepsilon, otherwise \end{cases}$$

Bagudu et al. [72] solve equation (5.3) using lagrangian multiplier such that:

$$L = \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$$

$$-=\sum_{i=1}^{l} a_i(\varepsilon+\xi_i-y_i+(\omega,x_i)+b)$$

$$-=\sum_{i=1}^{l} a_i^*(\varepsilon+\xi_i^*+y_i-(\omega,x_i)-b)$$

$$-=\sum_{i=1}^{l}(\eta_i\xi_i+\eta_i^*\xi_i^*)$$

where:

a_i, a_i^*, η_i and η_i^* are Lagrangian multipliers and $a_i, a_i^*, \eta_i, \eta_i^* > 0$.

Solving for weight vector Bagudu et al. [72] obtain:

$$\omega=\sum_{i=1}^{l}(a_i^*-a_i)\phi(x_i) \tag{5.4}$$

Hence objective function can be expressed as:

$$f(x)=\sum_{i=1}^{l}(a_i^*-a_i)K(x_i,x)+b \tag{5.5}$$

where:

$K(x_i, x)$: kernel function, which transforms non-linear function to higher-dimensional space.

Some of the common kernel functions that the SVM algorithm uses are summarized below (see[72],[78], [79] & [80]):

1. Linear kernel: $K(x_i, x_j) = (x_i, x_j)$,

2. Gaussian kernel or radial basis function (RBF) kernel:

$K(x_i, x_j) = \exp\left(\frac{-\|x_i - x\|^2}{2\sigma^2}\right)$ or $K(x_i, x_j) = \exp\left(-\gamma \|x_i - x\|^2\right)$, where

$\gamma = \frac{1}{2\sigma^2}$

3. polynomial kernel: $K(x_i, x_j) = (\langle x_i, x_j \rangle + 1)^d$

4. sigmoid kernel: $K(x_i, x_j) = \tanh(a x_i^T x_j + c)$

5. hyperbolic tangent kernel: $K(x_i, x_j) = \tanh(\kappa x_i x_j + c)$ for some κ >0 and c <0.

We used the radial basis kernel function (RBF) since it is convenient for the general case and is the most appropriate one when there is no prior information regarding the data set[28].

The implementation process consists of testing and training parts. We implemented the SVR with specific cost parameters and maximum allowed errors in the testing phase. In the training phase, we performed iterations with different values of cost parameters and maximum allowed errors to obtain a better model.

5.3 Estimation Results

Fama-French five-factor model without FX risk is defined as:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}$$

and the five-factor model incorporating FX risk is defined as:

$$R_{it} - R_{ft} = a_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t^{'} + h_i HML_t + r_i RMW_t + c_i CMA_t + f_i FX_t + \varepsilon_{it}$$

[28] Although selection of appropriate kernel function under SVR algorithm may require optimization techniques, we applied radial basis function.

We estimated portfolio excess returns by implementing the SVR algorithm. Table 5.1 reports the best performance, kernel type, and other features related to the output model for the Fama-French five-factor without FX risk.

Table 5.1: SVR Estimations of Fama-French Five-Factor Model without FX Risk

Excess Returns	**Cost** (C)	**Max allowable error, threshold** (ε)	**Best performance (tuned)** $\min \frac{1}{2}\lVert\omega\rVert^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$	**Number of support vectors**	**Kernel**
EBL	1	0.1	0.002140105	109	RBF
ESL	2	0.1	0.002137511	102	RBF
EBM	2	0.1	0.001749613	113	RBF
ESM	3	0.1	0.001488924	102	RBF
EBH	2	0.1	0.001069025	101	RBF
ESH	4	0.1	0.01536202	104	RBF
EBW	2	0.1	0.001904504	112	RBF
ESW	1	0.1	0.001826025	101	RBF
EBR	1	0.1	0.001124394	107	RBF
ESR	2	0.1	0.002075445	113	RBF
EBC	17	0.1	0.001595393	110	RBF
ESC	5	0.1	0.002097765	101	RBF
EBA	1	0.1	0.0009510101*	112	RBF
ESA	1	0.1	0.002191365	98	RBF
EBOP	2	0.1	0.001023667	108	RBF
ESOP	1	0.1	0.001787266	97	RBF
EBPOZ	2	0.1	0.001524481	117	RBF
ESPOZ	4	0.1	0.006304148	99	RBF

Notes: 1. The letter E indicates excess returns. B/S: Big/Small size, L/N/H: Low/Neutral/High market-to-book ratio, W/R: Weak/Robust profitability, C/A: Conservative/Aggressive investment strategy, OP/POZ: Open/Pozitif FX position.
2. *: indicates the smallest value among 18 objective functions.
3. $C = 1:10$; $\Delta C = 1$ and $\varepsilon = 0:1$; $\Delta\varepsilon = 0.01$, Hence $10 \times 101 = 1010$ iterations are performed in the tuning phase. For the excess returns of the SR portfolio (small size-robust profitability), we carried out $20 \times 101 = 2020$ iterations by increasing the upper *bound value of the cost parameter*[29], C *to 20 to augment the best performance result out of tuning operation*.
Source: Author's calculations

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[29] Increments for cost parameter and maximum allowable error term for the excess returns of small size-robust profitability (SR) portfolios are as: $C = 1:20$; $\Delta C = 1$ *and* $\varepsilon = 0:1$; $\Delta\varepsilon = 0.01$

Table 5.2 shows the best performance, kernel type, and other features related to the output model for the Fama-French five-factor model incorporating FX risk.

Table 5.2: SVR Estimations of Fama-French Five-Factor Model Incorporating FX Risk

Excess Returns	Cost (C)	Max allowable error, threshold (ε)	Best performance (tuned) $\min \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$	Number of support vectors	Kernel
EBL	3	0.1	0.001932289	111	RBF
ESL	2	0.1	0.002271685	105	RBF
EBM	1	0.1	0.001773139	113	RBF
ESM	4	0.1	0.001610444	99	RBF
EBH	1	0.1	0.001257049	113	RBF
ESH	5	0.1	0.01556683	95	RBF
EBW	1	0.1	0.00203358	113	RBF
ESW	2	0.1	0.001795211	110	RBF
EBR	1	0.1	0.001331536	113	RBF
ESR	1	0.1	0.002226208	104	RBF
EBC	2	0.1	0.001938286	106	RBF
ESC	3	0.1	0.001960812	99	RBF
EBA	2	0.1	0.0009822933*	113	RBF
ESA	1	0.1	0.002181529	96	RBF
EBOP	2	0.1	0.001044133	102	RBF
ESOP	2	0.1	0.001932986	107	RBF
EBPOZ	1	0.1	0.001391832	109	RBF
ESPOZ	4	0.1	0.006307358	94	RBF

Notes: 1. The letter E indicates excess returns. B/S: Big/Small size, L/N/H: Low/Neutral/High market-to-book value, W/R: Weak/Robust profitability, C/A: Conservative/Aggressive investment strategy, OP/POZ: Open/Pozitif FX position.
2. *: indicates the smallest value among 18 objective functions.
3. $C = 1:10; \Delta C = 1$, $\varepsilon = 0:1; \Delta\varepsilon = 0.01$, $10 \times 101 = 1010$ iterations are performed in the tuning phase.
Source: Author's calculations

SVR output regressions for both the FF5F model without FX risk and the FF5F model incorporating FX risk signify that best performance in terms of the value of the objective function is attained when the excess returns of intersection portfolios of big size and aggressive investment stocks (EBA).

Table 5.3 & Table 5.4 demonstrate the comparative performances of SLR and SVR in the sense of Fama-French five-factor without FX risk and five-factor model incorporating FX risk.

Table 5.3: Comparisons of SLR and SVR methods for FF5F model without FX risk

	SLR			**SVR**	
Excess Returns	**Root MSE**	**Adj R^2**	**Estimation Technique**	**Root MSE**	**Adj R^2**
EBL	0.0419	0.7383	GLS	0.0341	0.6890
ESL	0.0258	0.8667	GLS	0.0249	0.8829
EBN	0.0305	0.8095	OLS	0.0240	0.8482
ESN	0.0254	0.8969	OLS	0.0168	0.9477
EBH	0.0196	0.8901	GLS	0.0173	0.8765
ESH	0.0398	0.7383	GLS	0.0563	0.7325
EBW	0.0273	0.8688	GLS	0.0241	0.8795
ESW	0.0303	0.8614	GLS	0.0252	0.8820
EBR	0.0213	0.8789	GLS	0.0231	0.7821
ESR	0.0331	0.8210	OLS	0.0252	0.8698
EBC	0.0316	0.8292	OLS	0.0135	0.9658
ESC	0.0232	0.9044	GLS	0.0141	0.9674
EBA	0.0239	0.8402	OLS	0.0214	0.8169
ESA	0.0301	0.8729	OLS	0.0300	0.8045
EBOP	0.0232	0.8562	OLS	0.0183	0.8765
ESOP	0.0124	0.9592	GLS	0.0290	0.8087
EBPOZ	0.0319	0.7302	OLS	0.0257	0.7656
ESPOZ	0.0240	0.8522	GLS	0.0330	0.8515
Avg.	**0.0275**	**0.8452**	**-**	**0.0253**	**0.8471**

Source: Author's calculations

In 14 out of 18 regressions, the SVR method exhibits superior performance in terms of root MSE (RMSE). Average RMSE for the FF5F model without FX risk dropped from 0,0275 to 0,0253 due to SVR. Nevertheless, there is no significant change in the average adj R^2 values between SLR and SVR methods.

With SVR, the highest increase in adj R^2 values and the best progress in root MSE are achieved in excess returns of intersection portfolio of big size and conservative stocks (EBC).

SLR estimates the highest adj R^2 (0.9592) and the lowest root MSE (0.01236) for the excess returns of intersection portfolio of big size stocks and stocks whose FX liabilities exceed FX assets (ESOP). In contrast, the SVR method verifies the highest adj R^2 for excess returns of intersection portfolio of small size and conservative stocks (ESC) and the lowest root MSE for excess returns of intersection portfolio of big size and conservative stocks (EBC).

Table 5.4: Comparisons of SLR and SVR methods for FF5F model incorporating FX Risk

	SLR			SVR	
Excess Returns	**Root MSE**	**Adj R^2**	**Estimation Technique**	**Root MSE**	**Adj R^2**
EBL	0.0450	0.6622	OLS	0.0246	0.8695
ESL	0.0169	0.9718	GLS	0.0249	0.8809
EBN	0.0301	0.8148	OLS	0.0276	0.7662
ESN	0.0258	0.8930	OLS	0.0150	0.9588
EBH	0.0208	0.8494	GLS	0.0234	0.7105
ESH	0.0423	0.8371	GLS	0.0418	0.8708
EBW	0.0292	0.8712	GLS	0.0277	0.8301
ESW	0.0275	0.9000	OLS	0.0192	0.9415
EBR	0.0214	0.8782	GLS	0.0239	0.7568
ESR	0.0330	0.8226	OLS	0.0301	0.7709
EBC	0.0314	0.8310	OLS	0.0236	0.8750

Table 5.4: (continued)

ESC	0.0202	0.9131	GLS	0.0170	0.9510
EBA	0.0239	0.8399	OLS	0.0178	0.8870
ESA	0.0297	0.8768	OLS	0.0305	0.7896
EBOP	0.0220	0.8698	OLS	0.0170	0.8970
ESOP	0.0241	0.9154	OLS	0.0213	0.9123
EBPOZ	0.0292	0.7744	OLS	0.0260	0.7140
ESPOZ	0.0271	0.9355	OLS	0.0308	0.8697
Avg.	**0.0278**	**0.8587**	**-**	**0.0246**	**0.8473**

Source: Author's calculations

Like SVR predictions for the FF5 model without FX risk, 13 out of 18 regressions, the SVR method exhibits superior performance in root MSE for the FF5 model incorporating FX risk. The average root MSE for the augmented FF5 factors dropped from 0,0278 to 0,246 due to SVR. On the contrary, the average adj R^2 dropped from 0.8587 to 0.8473 through the utilization of SVR. While the adj R^2 values are between 0.6622 and 0.9718 in the regression outputs of SLR, adj R^2 oscillates between 0.7105 and 0.9588 with SVR.

We observed the most significant improvement in adj R^2 values and root MSE in excess returns of intersection portfolio of big size and low market-to-book ratio stocks (EBL).

For the FF5 model, including FX risk, SLR estimates the highest adj R^2 (0.9718) and the lowest root MSE (0.0169) for the excess returns of intersection portfolio of small size and low market-to-book ratio stocks (ESL). In contrast, the SVR method predicts the highest adj R^2 and the lowest root MSE for excess returns of intersection portfolio of small size and neutral market-to-book ratio (ESN).

5.4 Findings

In Chapter 4, we applied OLS and GLS methods to predict excess returns of intersection portfolios. Afterward, we discussed the prediction results of the Fama-French five-factor model without FX risk and the Fama-French five-factor model incorporating FX risk. Results indicated certain advantages of the five-factor model incorporating FX risk.

Unlike OLS, instead of focusing on minimizing the sum of residuals, the basic logic behind SVR is to determine threshold values around the predictions and to minimize the distance between residuals and threshold values for residuals that exceed a threshold value.

In this chapter, portfolio returns are predicted by support vector regression. SVR exhibits more remarkable performance in average adj R^2 and RMSE values for the Fama-French five-factor model without FX risk. Furthermore, in 14 out of18 predictions, the SVR method generates smaller RMSEs.

Similarly, the SVR exhibits more remarkable performance in terms of average root MSEs for the Fama-French five-factor model incorporating FX risk. However, SVR produces slightly smaller average adj R^2 values. In 13 out of 18 predictions with FX risk, SVR yields smaller RMSEs.

So far, we predicted excess portfolio returns using two different models and two different methods. The fundamental problem is how these results should be interpreted. For instance, we reported the efficiency of the SVR method over SLR in terms of average adj R^2 and RMSE values for Fama-French five-factor model without FX risk. On the other hand, the SVR method produced a smaller average root MSE value and a smaller value of average adj R^2 for the five-factor model incorporating FX risk. At this point, results obtained from SLR and SVR methods for both versions of the FF5F model with and without FX risk indicate an inconclusive argument. The problem appears to be selecting the best method for two different models. The motivation to obtain the best prediction results encouraged us to apply the forecast combination method.

Forecast combination (it would be more accurate to express it as *prediction combination* for this study)[30] is a technique to create more precise forecasts out of individual forecasts generated from different forecasting methods. In chapter 6, we shall combine the predictions of SLR and SVR for Fama-French five-factor model with and without FX risk. Timmermann [81] argues that among the individual forecasts, the one which possesses greater weight has a superior performance. In addition, we will draw a more precise comparison between SLR and SVR for Fama-French five-factor model with and without FX risk and obtain more accurate predictions for excess returns of portfolios.

In chapter 6, we will clarify the theoretical background of the linear combination technique and will calculate the optimal combinations.

[30] Note that the term, "forecast" implies estimation of future values of a varibale prior to past values. However, the term, "prediction" has a broader concept of understanding the nature of relations of events through given data.

CHAPTER 6

COMBINATIONS OF PREDICTIONS

6.1 Introduction

According to Timmermann [81], empirical studies have shown the superiority of the forecast combinations, generating more precise forecasts than individual predictions. He suggests that simple forecast combinations (i.e., linear combination of forecasts) produce better estimations than sophisticated combination methods. This chapter will construct linear combinations of SVR and OLS/GLS estimations to minimize MSE (mean squared error) and determine optimal weights accordingly for both versions of the Fama-French five-factor model with and without FX risk.

In the following subsection, referring to Timmermann's methodology [81], we will elaborate on the theoretical background to clarify why combining estimations is efficient. In 6.3, we will make linear combinations of SLR and SVR estimations. In the final part, we will discuss our findings

6.2 Linear Combinations of Predictions

Timmermann [81] defines combination problem when there are two estimation results and under MSE loss of the form:

$$min\ (Y_t - w_1\hat{Y}_{t1} - w_2\hat{Y}_{t2})^2 \tag{6.1}$$

s.t.

$$w_1 + w_2 = 1$$

where:

Y_t **:** excess returns of intersection portfolios

$\hat{Y}_{t1}$ **:** estimations via SLR method

$\hat{Y}_{t2}$ **:** estimations via SVR method

w_1 **:** weights assigned to estimations via SLR

w_2 **:** weights assigned to estimations via SVR

and t=1,2,3…132

Let e_1 and e_2 denote the individual estimation errors attributed to SLR and SVR, respectively**.** Therefore, assuming $e_1 \sim (0, \sigma_1^2)$ and $e_2 \sim (0, \sigma_2^2)$

$$e_1 = Y - \hat{Y}_{t1}$$

$$e_2 = Y - \hat{Y}_{t2}$$

where:

$\sigma_1^2 = \text{var}(e_1)$, $\sigma_2^2 = \text{var}(e_2)$ and $\sigma_{12} = \text{cov}(e_1 e_2) = \rho_{12}\sigma_1\sigma_2$, ρ_{12} is the correlation coefficient between estimation errors attributed to SLR and SVR.

After rearranging the unity condition, $w_1 + w_2 = 1$; we obtain $w_2 = 1 - w_1$**.** Hence error term of the combined estimations, e^c is calculated as follows:

$$e^c = w_1 e_1 + (1 - w_1) e_2 \tag{6.2}$$

The variance of the error term of the combined estimations is obtained through:

$$\sigma_c^2(w_1) = w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2 + 2w_1(1 - w_1)\sigma_{12} \tag{6.3}$$

Taking the derivative of equation 6.3 with respect to w_1 :

$$\frac{d\sigma_c^2(w_1)}{dw_1} = 2w_1\sigma_1^2 - 2(1-w_1)\sigma_2^2 + (2-4w_1)\sigma_{12}$$

Solving for the first-order condition:

$$2w_1\sigma_1^2 - 2\sigma_2^2 + 2w_1\sigma_2^2 + 2\sigma_{12} - 4w_1\sigma_{12} = 0$$

$$2w_1\sigma_1^2 + 2w_1\sigma_2^2 - 4w_1\sigma_{12} = 2\sigma_2^2 - 2\sigma_{12}$$

$$\not{2}w_1(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) = \not{2}(\sigma_2^2 - \sigma_{12})$$

we have

$$w_1^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \tag{6.4}$$

and

$$w_2^* = 1 - w_1^* = 1 - \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

$$w_2^* = \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} - \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

$$w_2^* = \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} - \sigma_2^2 + \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

$$w_2^* = \frac{\sigma_1^2 + \not{\sigma}_2^2 - 2\sigma_{12} - \not{\sigma}_2^2 + \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \tag{6.5}$$

where w_1^* and w_2^* are optimal weights assigned to SLR and SVR estimations, respectively.

The logical statement, $\sigma_2^2 - \sigma_{12} > \sigma_1^2 - \sigma_{12}$ is equivalent to $\sigma_2^2 > \sigma_1^2$. In addition, when $\sigma_2^2 > \sigma_1^2$ we obtain $w_1^* > w_2^*$ and vice-versa. Timmermann [81] points out that a precise estimation (with a smaller variance of error term) possesses a higher weight.

We know that $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$, and we showed $w_1^* = \dfrac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$ in equation 6.5.

If $\sigma_2^2 - \sigma_{12} < 0$;

$$\sigma_2^2 - \rho_{12}\sigma_1\sigma_2 < 0$$

$$\sigma_2^{\cancel{2}} < \rho_{12}\sigma_1\cancel{\sigma_2}$$

$$\rho_{12} > \frac{\sigma_2}{\sigma_1}$$

Once the condition, $\rho_{12} > \dfrac{\sigma_2}{\sigma_1}$ holds, one of the weights will have a negative value, and the other will be larger than unity. Some of the combinations of SLR and SVR estimations verified this condition with empirical evidence. We will document the empirical results in the next section.

Inserting w_1^* in $\sigma_c^2(w_1)$ in equation 6.3, we obtain:

$$\sigma_c^2(w_1^*) = \left(\frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\right)^2 \sigma_1^2 + \left(\frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\right)^2 \sigma_2^2 + 2\left(\frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\right)\left(\frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\right)\sigma_{12}$$

after some algebra, we obtain:

$$\sigma_c^2(w_1^*) = \frac{\sigma_1^2\sigma_2^2(1-\rho_{12}^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \qquad (6.6)$$

Using equation 6.6, we can verify that $\sigma_c^2(w_1^*) \le \min(\sigma_1^2, \sigma_2^2)$.

6.3 Specification of Optimal Weights

In this section, we will calculate the optimal weights of SLR and SVR predictions to obtain $\sigma_c^2(w_1^*)$ so that the MSE of the combined predictions is minimized.

Tables 6.1 & 6.2 depict the sum of squared residuals (SSR) and MSEs of individual and combined predictions, correlation coefficients, $\frac{\sigma_{SVM}}{\sigma_{OLS/GLS}}$ ratios, and optimal weights for the linear combination scheme for the FF5F model without FX risk and the FF5F model incorporating FX risk, respectively.

Table 6.1: Combination of Predictions for Fama-French Five-Factor Model without FX Risk

Excess Returns	**SSR_SLR**	**MSE_SLR**	**SSR_SVM**	**MSE_SVM**	p_{12}	$\frac{\sigma_{SVM}}{\sigma_{OLS/GLS}}$	w_1^*	w_2^*	**SSR_combined**	**MSE_combined**
EBL**	0.4651	0.0037	0.1537	0.001220	0.4935	0.5748	0.0612	0.9388	0.1523	0.001209
ESL**	0.1145	0.0009	0.0819	0.000650	0.8301	0.8458	0.0428	0.9572	0.0818	0.000649
EBN*	0.1171	0.0009	0.0758	0.000602	0.8770	0.8043	-0.2478	1.2478	0.0741	0.000588
ESN*	0.0811	0.0006	0.0372	0.000295	0.7858	0.6773	-0.1865	1.1865	0.0361	0.000287
EBH**	0.0770	0.0006	0.0396	0.000315	0.7722	0.7174	-0.0967	1.0967	0.0393	0.000312
ESH**	0.7796	0.0062	0.4180	0.003317	0.8662	0.7283	-0.3737	1.3737	0.3924	0.003114
EBW**	0.1481	0.0012	0.0766	0.000608	0.7750	0.7190	-0.0999	1.0999	0.0760	0.000603
ESW**	0.1059	0.0008	0.0841	0.000667	0.6935	0.8913	0.3158	0.6842	0.0782	0.000621
EBR**	0.0668	0.0005	0.0702	0.000557	0.8676	1.0253	0.5941	0.4059	0.0638	0.000506
ESR*	0.1381	0.0011	0.0840	0.000667	0.8031	0.7796	-0.0514	1.0514	0.0839	0.000666
EBC*	0.1258	0.0010	0.0239	0.000190	0.6082	0.4352	-0.1141	1.1141	0.0228	0.000181
ESC**	0.0968	0.0008	0.0262	0.000208	0.6735	0.5180	-0.1412	1.1412	0.0252	0.000200
EBA*	0.0719	0.0006	0.0603	0.000478	0.8627	0.9153	0.1864	0.8136	0.0596	0.000473
ESA*	0.1144	0.0009	0.1189	0.000944	0.7666	1.0176	0.5373	0.4627	0.1028	0.000816
EBOP*	0.0675	0.0005	0.0442	0.000351	0.8363	0.8090	-0.0734	1.0734	0.0441	0.000350
ESOP**	0.1043	0.0008	0.1112	0.000883	0.7109	1.0304	0.5517	0.4483	0.0919	0.000729
EBPOZ*	0.1283	0.0010	0.0873	0.000693	0.8736	0.8187	-0.1874	1.1874	0.0868	0.000689
ESPOZ**	0.2703	0.0021	0.1439	0.001142	0.7805	0.7402	-0.0762	1.0762	0.1430	0.001135

Notes: 1. The letter E indicates excess returns. B/S: Big/Small size, L/N/H: Low/Neutral/High market-to-book ratio, W/R: Weak/Robust profitability, C/A: Conservative/Aggressive investment strategy, OP/POZ: Open/Pozitif FX position
2. *: OLS estimation, **: GLS estimation for SLR
Source: Author's calculations

Table 6.2: Combinations of Predictions for Fama-French Five-Factor Model Incorporating FX Risk

Excess Returns	SSR_SLR	MSE_SLR	SSR_SVM	MSE_SVM	p_{12}	$\frac{\sigma_{SVM}}{\sigma_{OLS/GLS}}$	w_1^*	w_2^*	SSR_combined	MSE_combined
EBL**	0.2527	0.002022	0.0797	0.000638	0.6461	0.5616	-0.0805	1.0805	0.0787	0.000630
ESL**	0.1057	0.000845	0.0816	0.000653	0.8048	0.8788	0.1818	0.8182	0.0804	0.000643
EBN*	0.1129	0.000903	0.1003	0.000802	0.8588	0.9422	0.2918	0.7082	0.0977	0.000781
ESN*	0.0835	0.000668	0.0298	0.000239	0.7072	0.5975	-0.1280	1.1280	0.0291	0.000233
EBH**	0.0701	0.000561	0.0724	0.000579	0.8285	1.0161	0.5465	0.4535	0.0651	0.000520
ESH**	0.3542	0.002834	0.2302	0.001841	0.6964	0.8053	0.1665	0.8335	0.2249	0.001799
EBW**	0.1393	0.001114	0.1016	0.000812	0.8659	0.8539	-0.0410	1.0410	0.1015	0.000812
ESW**	0.0945	0.000756	0.0488	0.000391	0.7398	0.7174	-0.0355	1.0355	0.0488	0.000390
EBR**	0.0654	0.000523	0.0756	0.000605	0.8329	1.0754	0.7141	0.2859	0.0634	0.000508
ESR*	0.1358	0.001086	0.2002	0.001602	0.7428	1.2082	0.8457	0.1543	0.1337	0.001069
EBC*	0.1234	0.000987	0.0737	0.000590	0.8252	0.7721	-0.1273	1.1273	0.0731	0.000585
ESC**	0.1070	0.000856	0.0380	0.000304	0.6924	0.5938	-0.1105	1.1105	0.0374	0.000299
EBA*	0.0715	0.000572	0.0420	0.000336	0.8002	0.7664	-0.0717	1.0717	0.0419	0.000335
ESA*	0.1100	0.000880	0.1229	0.000983	0.7183	1.0552	0.5949	0.4051	0.0993	0.000794
EBOP*	0.0607	0.000485	0.0379	0.000304	0.7931	0.7899	-0.0068	1.0068	0.0379	0.000304
ESOP**	0.0728	0.000582	0.0600	0.000480	0.7315	0.9059	0.3190	0.6810	0.0562	0.000450
EBPOZ*	0.1064	0.000851	0.0893	0.000715	0.8381	0.9150	0.2319	0.7681	0.0875	0.000700
ESPOZ**	0.0921	0.000737	0.1254	0.001003	0.4060	1.1640	0.6259	0.3741	0.0740	0.000592

Notes: 1. The letter E indicates excess returns. B/S: Big/Small size, L/N/H: Low/Neutral/High market-to-book ratio, W/R: Weak/Robust profitability, C/A: Conservative/Aggressive investment strategy, OP/POZ: Open/Pozitif FX position
2. *: OLS estimation, **: GLS estimation for SLR
Source: Author’s calculations

According to Table 6.1, w_2^* (the optimal weight assigned to the SVR predictions) has greater values in 15 out of 18 combinations. Hence, SVR predictions are more precise for corresponding portfolio returns. On the other hand, w_1^* (optimal weights assigned to the OLS/GLS method) has larger values in only 3 out of 18 combinations. All the MSE values of combined predictions satisfy the condition; $\sigma_c^2(w_1^*) \leq \min(\sigma_1^2, \sigma_2^2)$ where $\sigma_c^2(w_1^*)$ is the variance of the combined predictions.

As Table 6.2 illustrates, w_2^* has greater values in 13 out of 18 combinations and, therefore, SVR outperforms SLR at predicting corresponding portfolio returns. In remaining combinations, w_1^* has larger values. All MSE values of combined predictions of the FF5F model incorporating FX risk also satisfy the condition $\sigma_c^2(w_1^*) \leq \min(\sigma_1^2, \sigma_2^2)$.

Either Table 6.1 or Table 6.2 verify that when the condition; $\rho_{12} > \dfrac{\sigma_2}{\sigma_1}$ is present, one of the weights possesses a value greater than 1. Interestingly, out of 36 combinations, the optimal weights that exceed unity are assigned to the predictions of support vector regression. The total number of optimal weights that exceed 1 is 19.

We draw two main conclusions based on the combined OLS/GLS and SVR predictions. One of them is the superiority of combined predictions over individual ones. Out of all 36 combinations, the MSEs of the combined predictions are smaller than the MSEs of individual predictions. The other finding is the superior performance of support vector regression forecasts. SVR forecasts are assigned with higher weights in 28 out of 36 combinations.

6.4 Findings

In this chapter, we made linear combinations of SLR and SVR predictions for the

Fama-French five-factor model without FX risk and the five-factor model incorporation FX risk. Our objective is to minimize Mean Squared Error, and the sum of the weights is restricted to 1.

We combined the SLR and SVR predictions for Fama-French five-factor model without FX risk. Optimal weights assigned to SVR predictions are greater than weights assigned to SLR predictions in 15 out of 18 combinations.

We also combined the SLR and SVR predictions for the Fama-French five-factor model incorporating FX risk. Results indicate that weights assigned to SVR predictions are greater in 13 out of 18 combinations. Hence, SVR outperforms SLR at predicting excess returns of the intersection portfolios in total, 28 out of 36 combinations. In addition to the superiority of SVR, we also verified that linear combinations yield more precise predictions. We obtained less volatile error terms and lower values of MSEs accordingly.

In the next chapter, we will bring together puzzle pieces. We will clarify significant findings and the contributions of this study to the Fama-French factor modeling literature.

CHAPTER 7

CONCLUSION

7.1 Introduction

The primary focus of this study is to investigate the performance of the Fama-French five-factor model encompassing FX risk in explaining deviations in excess portfolio returns. We modified the former Fama-French five-factor model by incorporating a new factor variable, a proxy for exposure to foreign exchange risk of a firm. We analyzed the performances and compared the Fama-French five-factor model without FX risk and the five-factor model incorporating FX risk in light of several statistical indicators.

In 7.2, we will discuss significant findings within the scope of this study. In 7.3, we will assess the contributions of this thesis to the literature of Fama-French five-factor modeling.

7.2 Main Findings

In Chapter 4, we estimated excess returns of intersection portfolios using the Fama-French five-factor model with and without FX risk and by applying OLS/GLS methods. We reported some similarities and differences in terms of performances based on several indicators.

To estimate 10 out of 18 excess portfolio returns with the Fama-French five-factor methodology without FX risk, we applied the GLS method. On the other hand, for

6 out of 18 excess portfolio returns with a five-factor model incorporating FX risk, we used the GLS method.

Average adj R^2 values for Fama-French five-factor model incorporating FX risk is slightly higher than average adj R^2 values for Fama-French five-factor model without FX risk. Assessing portfolio return predictions individually, 12 out 18 predictions indicate improvement in adj R^2 due to incorporation of FX risk to the five-factor model. In particular, the Fama-French five-factor model incorporating FX risk is highly efficient at predicting excess returns of portfolios defined as SL[31], SH[32], SW[33], and SPOZ[34].

While the GRS_F test statistics do not indicate pricing error for both models, Fama-French five-factor model incorporating FX risk is superior in terms of GRS_F test statistics. Both models have small and close average absolute values (AAV) of intercepts. Fama-French argued that the smaller the AAV, the better the model's performance is. So, both models are successful in terms of AAV values.

In Chapter 5, we investigated the performance of both models through a machine learning technique, support vector regression. The SVR method exhibits excessive performance in average adj R^2 and root MSE values for the Fama-French five-factor model without FX risk. Furthermore, in 14 out of 18 predictions, the SVR method generates smaller root MSEs.

Similarly, SVR exhibits smaller average root MSEs for the Fama-French five-factor model incorporating FX risk. However, SVR produces slightly smaller average adj R^2 values. In 13 out of 18 estimations with FX risk, SVR yields smaller values of root MSEs.

[31] Small size companies and companies which have low market-to-book value
[32] Small size companies and companies which have high market-to-book value
[33] Small size companies and companies which have weak profitability
[34] Small size companies and companies whose assets in foreign curreny exceed liabilities in foreign currency

Our curiosity to obtain the most efficient predictions encouraged us to apply the forecast combination method.
In Chapter 6, we linearly combined estimations of SLR and SVR methods for the Fama-French five-factor model without FX risk and the five-factor model incorporation FX risk.

We combined the SLR and SVR estimations for Fama-French five-factor model without FX risk. In 15 out of 18 combinations, optimal weights assigned to SVR predictions are greater than weights assigned to SLR predictions.

We also combined the SLR and SVR model predictions for the Fama-French five-factor model incorporating FX risk. Results indicate that in 13 out of 18 combinations, weights assigned to SVR predictions are greater.

Hence, in 28 out of 36 combinations, SVR outperforms SLR at predicting excess returns of the intersection portfolios. In addition to the superiority of SVR, we also verified that linear combinations yield better predictions. We obtained less volatile error terms and lower values of MSEs accordingly.

7.3 Contributions to Asset Pricing Literature

This study contributes to the literature on asset pricing through several dimensions. It is known that the tendency of Turkish companies to borrow from external markets at advantageous rates [9], prolonging the net FX position of non-financial companies, volatile foreign exchange rates, intensive use of imported inputs, and other similar factors increase their vulnerabilities to an external shock. Accordingly, we considered FX risk as potentially a significant determinant of portfolio returns. The main contribution of our study to the existing literature on asset pricing is the incorporation of FX risk to Fama-French five-factor models.

We also applied a machine learning method, *support vector regression* (SVR), to estimate excess returns of intersection portfolios and compared the performance of the SVR method with the linear regression method. Although there are various empirical studies that investigated Borsa İstanbul, any other study did not apply the SVR tool to estimate portfolio returns via CAPM or Fama French multi-factor models. Another contribution of this study is implementing the SVR technique to estimate portfolio returns through the FF5F model without FX risk and the FF5F model incorporating FX risk for Borsa İstanbul stocks.

Even though there are empirical studies that confirm the efficiency of SVR and studies that compare the performance of the linear factor regression method with alternative statistical tools, our study is unique in terms of combining estimations out of SLR and SVR methods via Timmermann's methodology [81]. Optimal weights obtained out of combinations imply more precise estimations through SVR. In 28 out of 36 combinations, optimal weights assigned to SVR estimations are larger than those assigned to SLR estimations. For each binary combination, we reported improvements in overall performance.

Linear regression methods may be too restrictive to reflect the nonlinearity of factor exposures under the Fama-French multi-factor model scheme. Models, which take nonlinear dynamics of Fama-French factors into consideration, might generate more precise estimations.

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APPENDIX A

TUNED SVR PREDICTIONS OF FF5F MODEL WITHOUT FX RISK

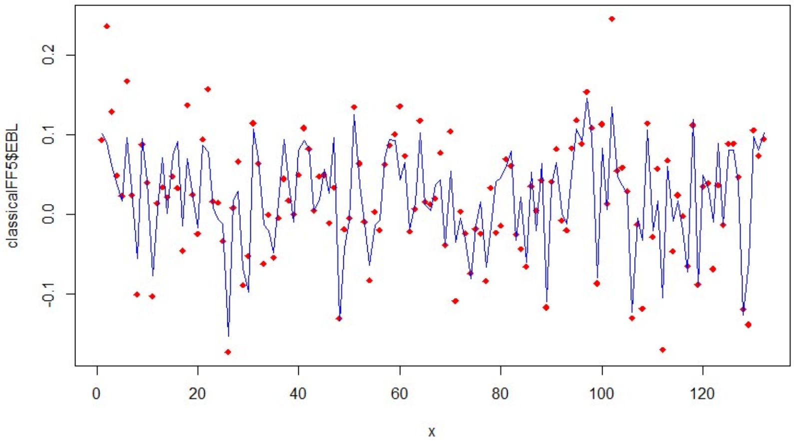

Figure A.1: EBL predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

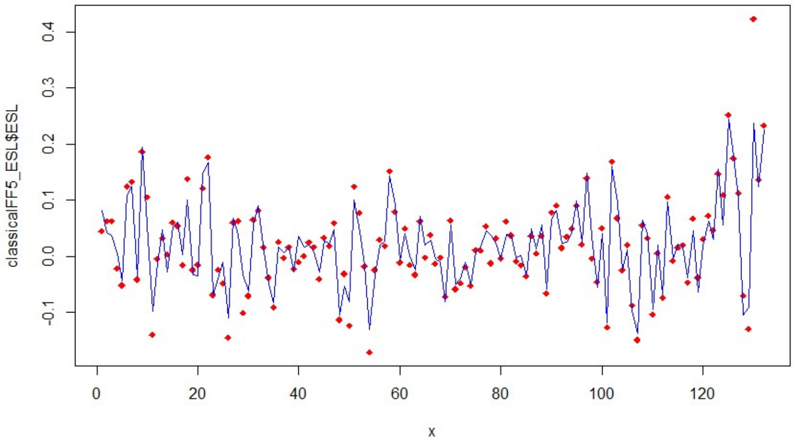

Figure A.2: ESL predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

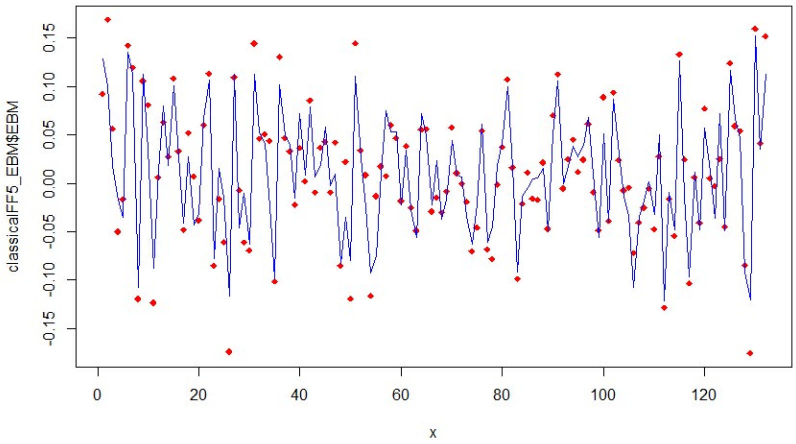

Figure A.3: EBM predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

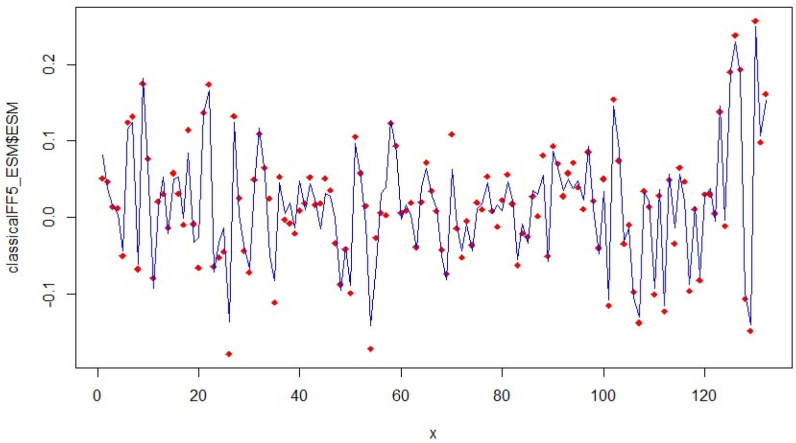

Figure A.4: ESM predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

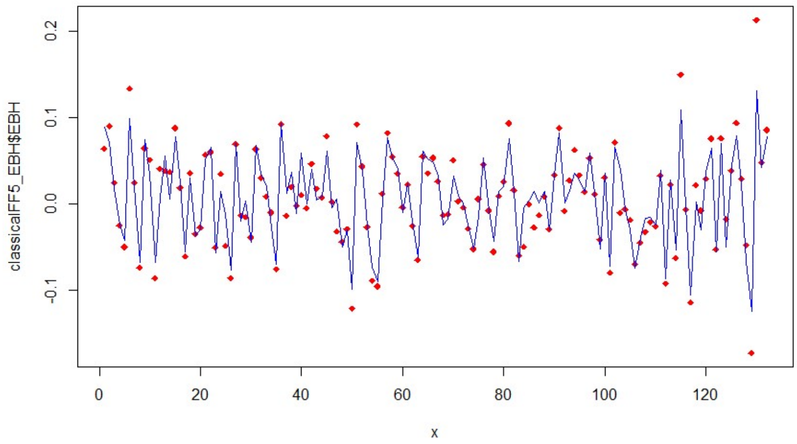

Figure A.5: EBH predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

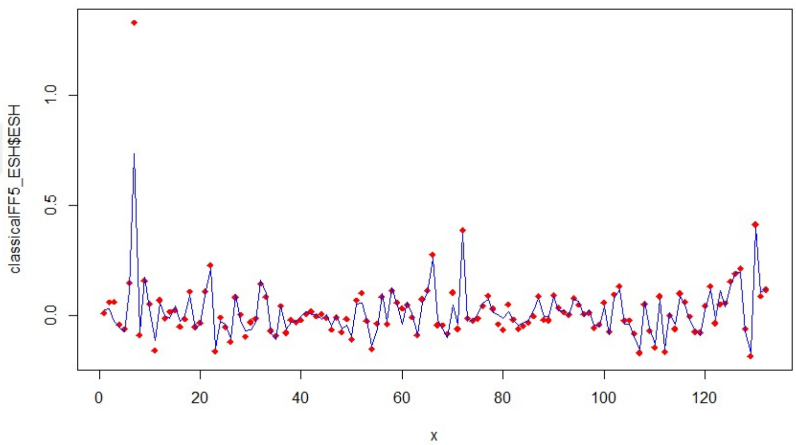

Figure A.6: ESH predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

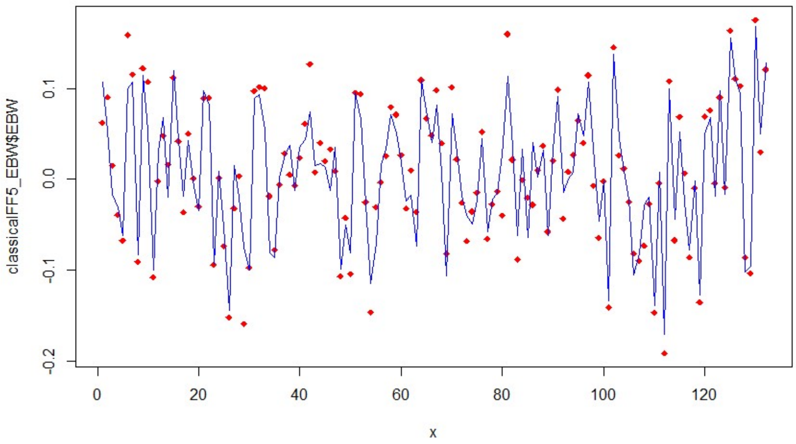

Figure A.7: EBW predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

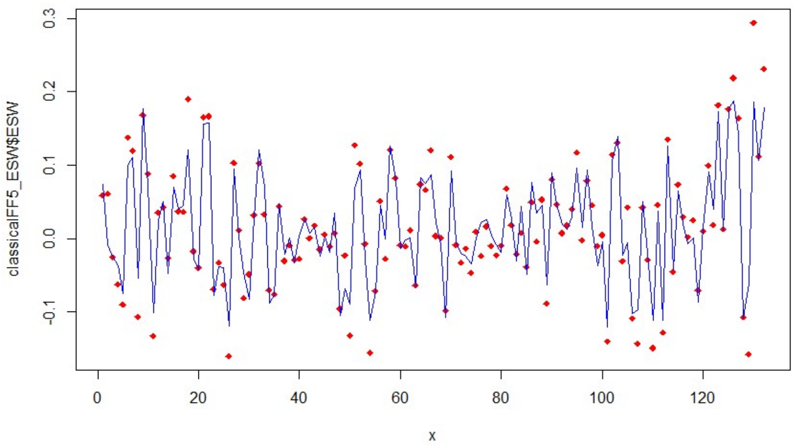

Figure A.8: ESW predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

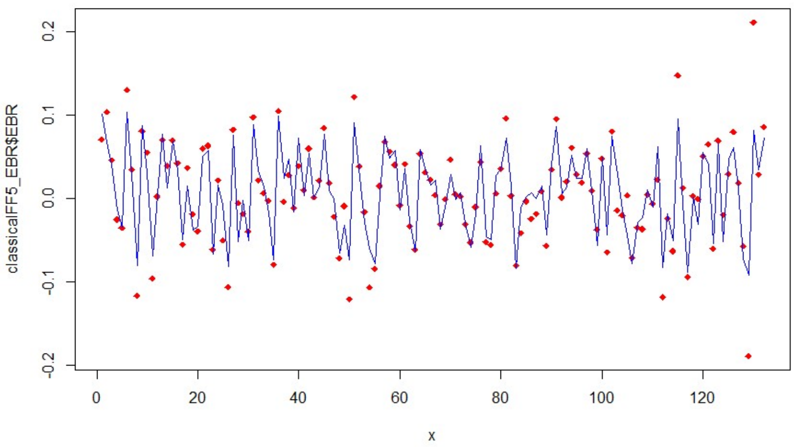

Figure A.9: EBR predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

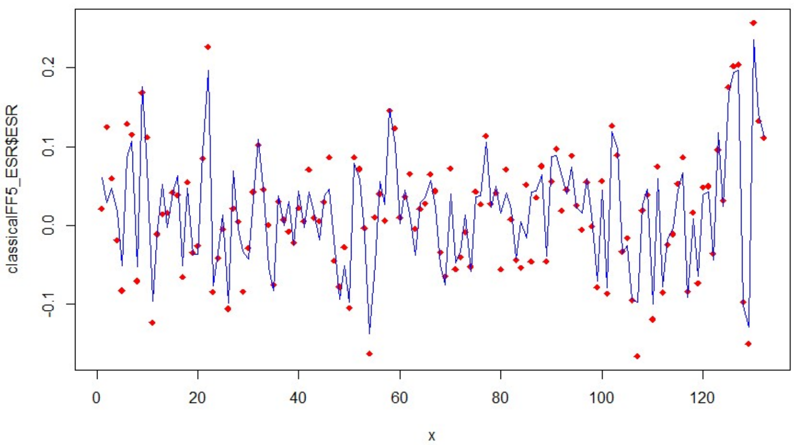

Figure A.10: ESR predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

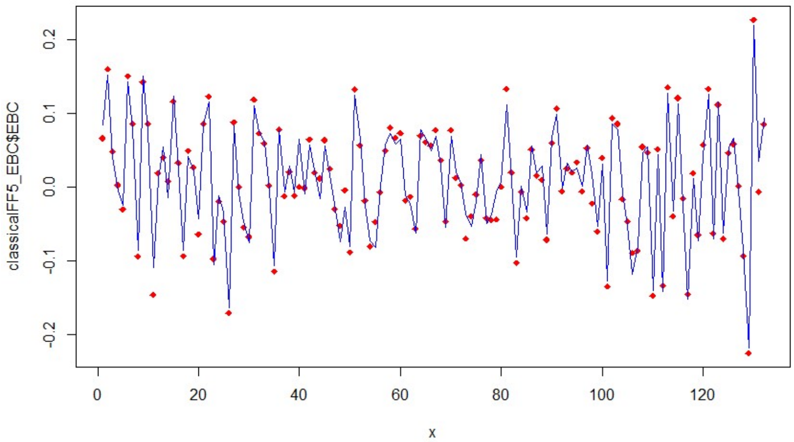

Figure A.11: EBC predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

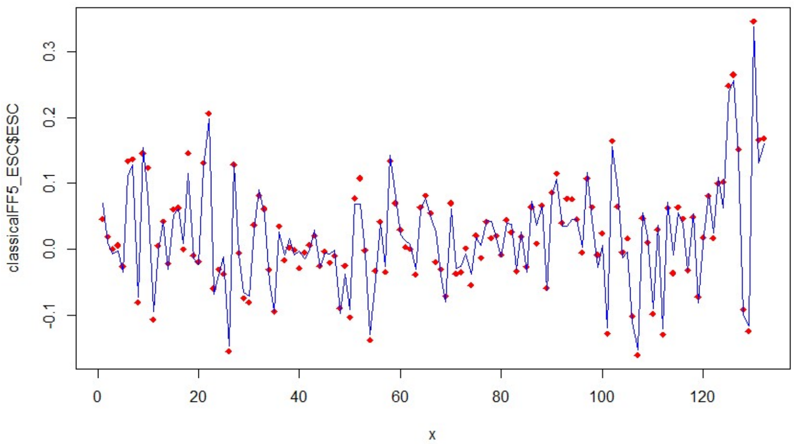

Figure A.12: ESC predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

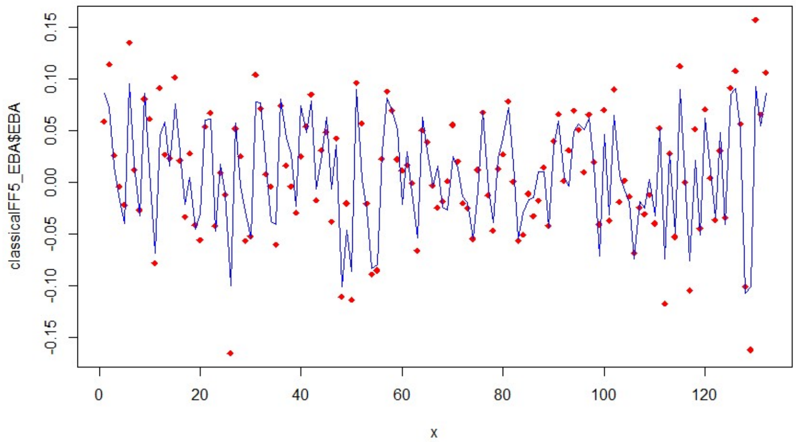

Figure A.13: EBA predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

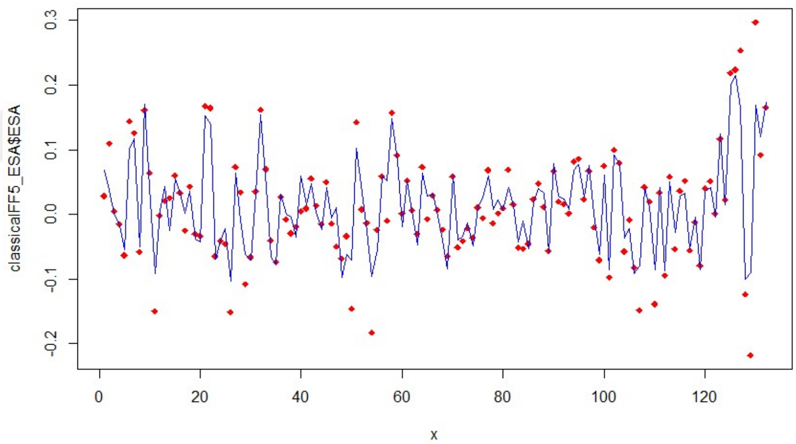

Figure A.14: ESA predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

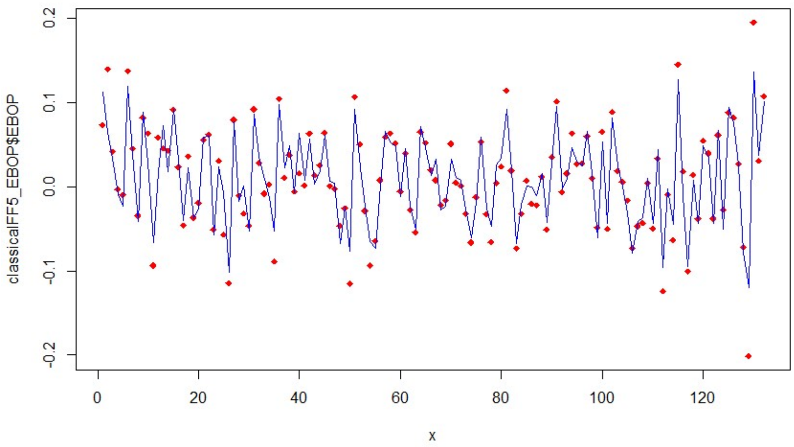

Figure A.15: EBOP predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

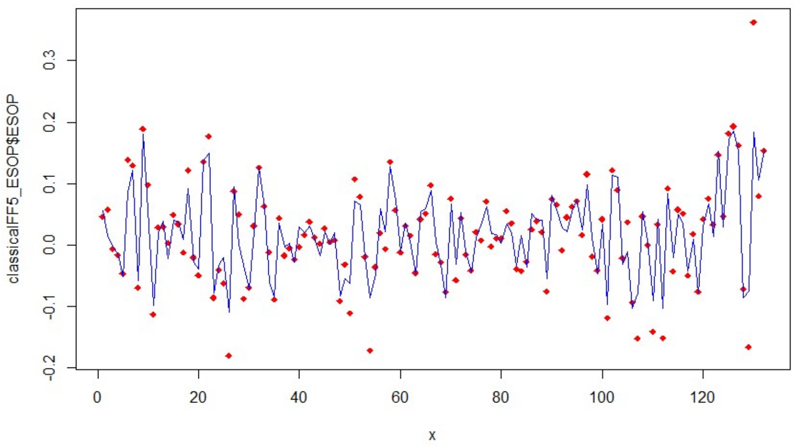

Figure A.16: ESOP predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

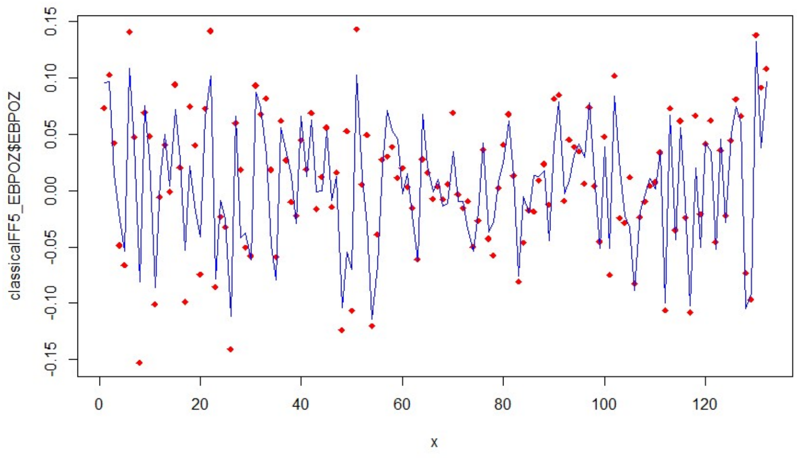

Figure A.17: EBPOZ predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

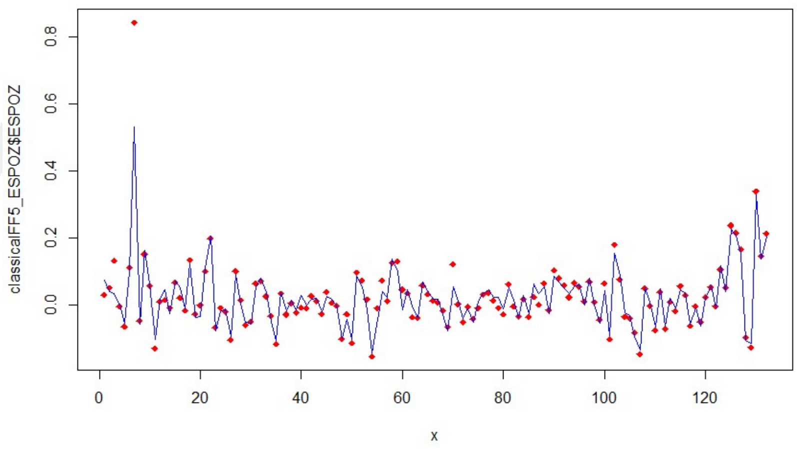

Figure A.18: ESPOZ predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

APPENDIX B

TUNED SVR PREDICTIONS OF FF5F MODEL INCORPORATING FX RISK

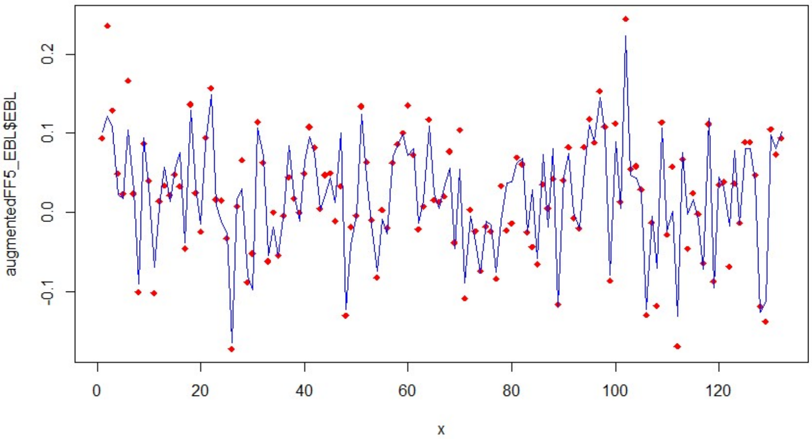

Figure B.1: EBL predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

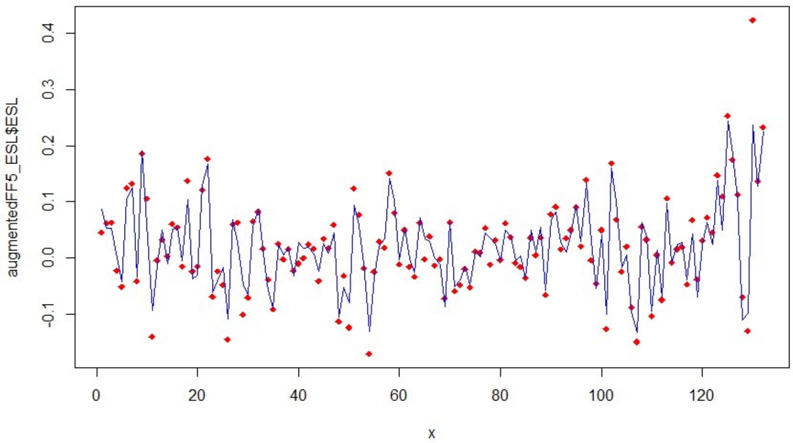

Figure B.2: ESL predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

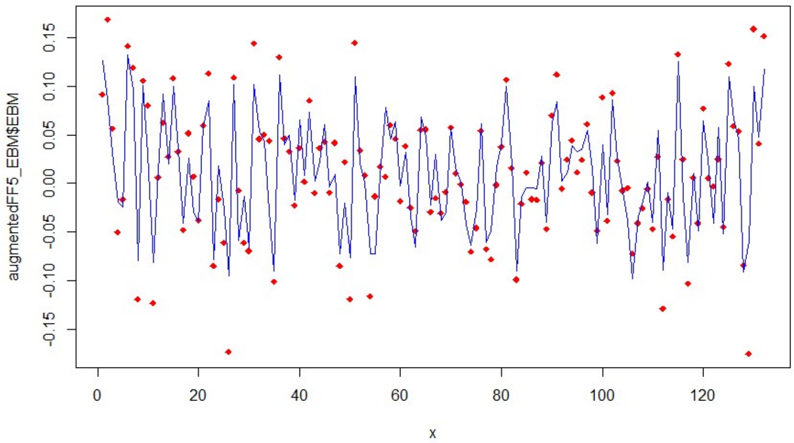

Figure B.3: EBM predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

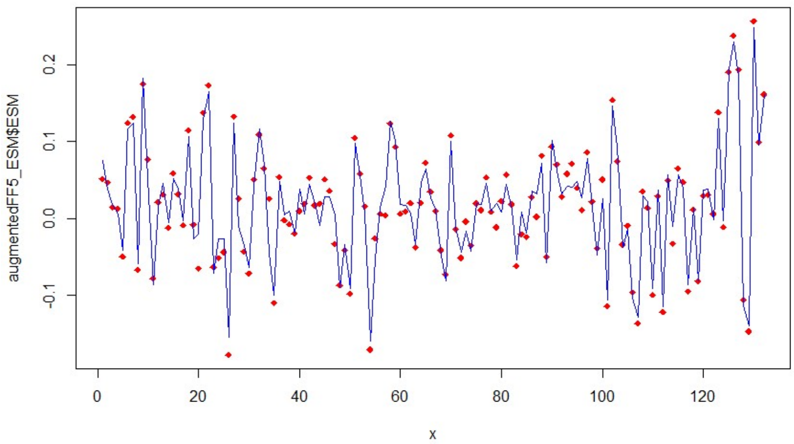

Figure B.4: ESM predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

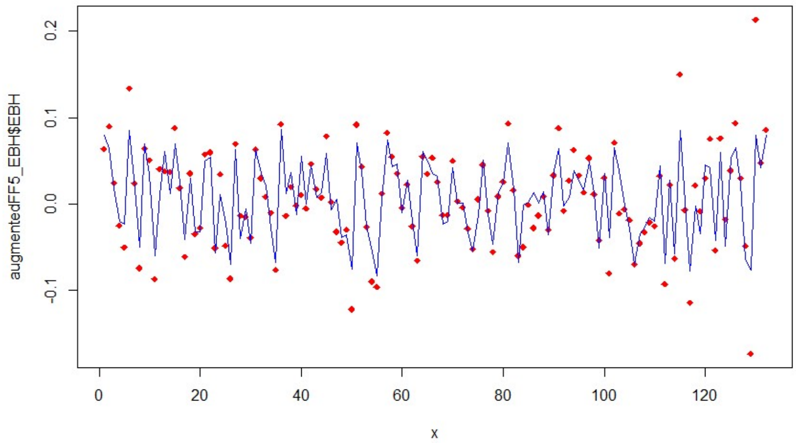

Figure B.5: EBH predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

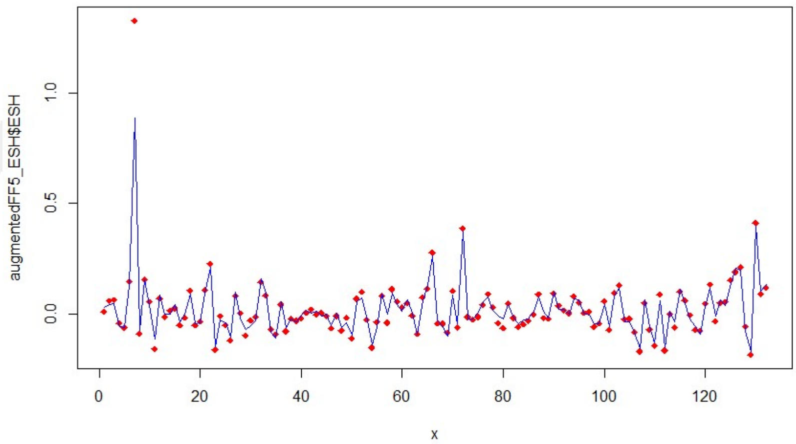

Figure B.6: ESH predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

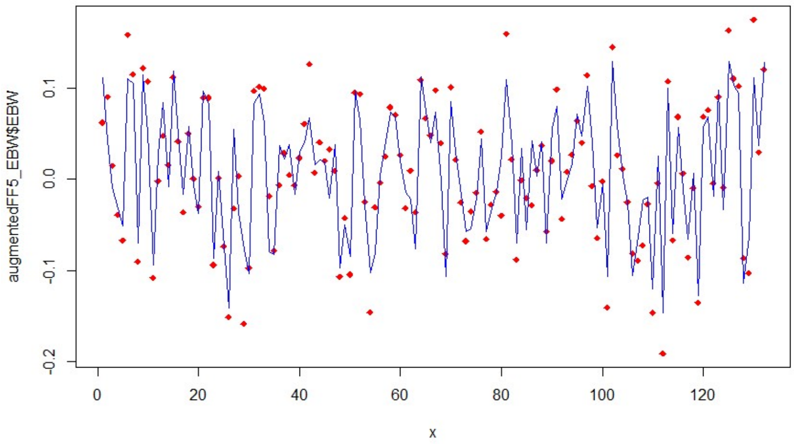

Figure B.7: EBW predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

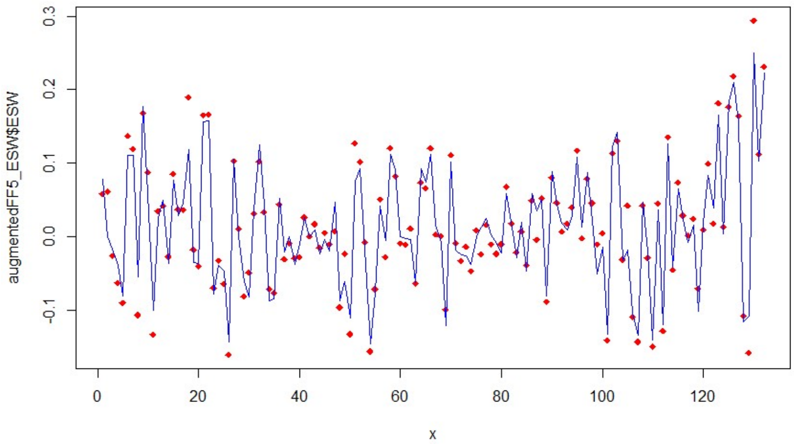

Figure B.8: ESW predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

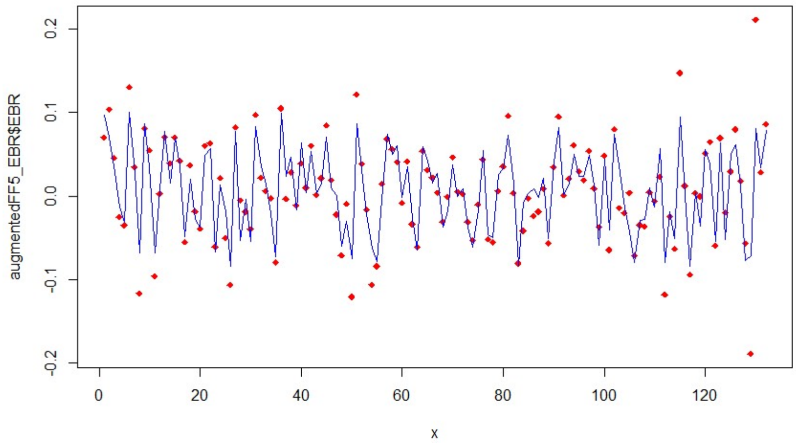

Figure B.9: EBR predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

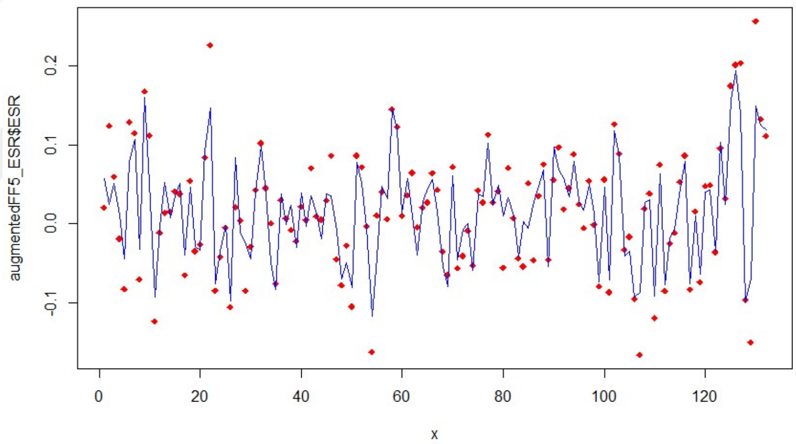

Figure B.10: ESR predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

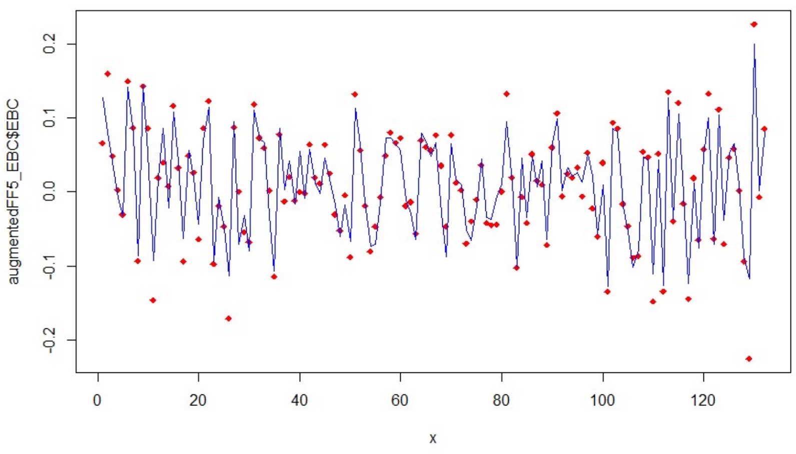

Figure B.11: EBC predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

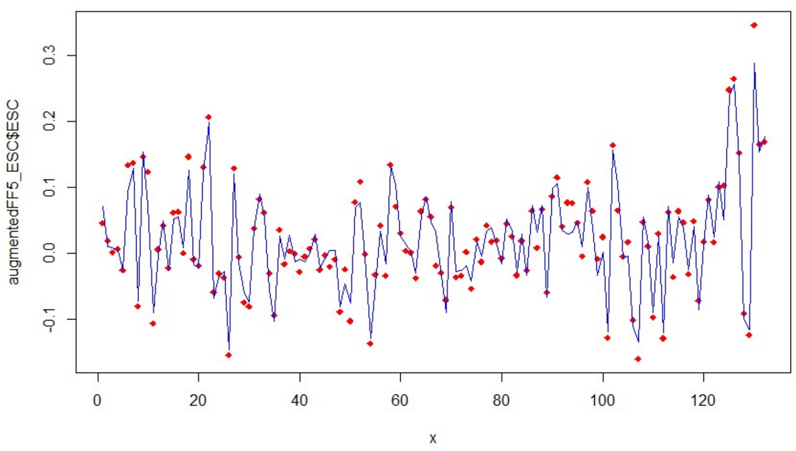

Figure B.12: ESC predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

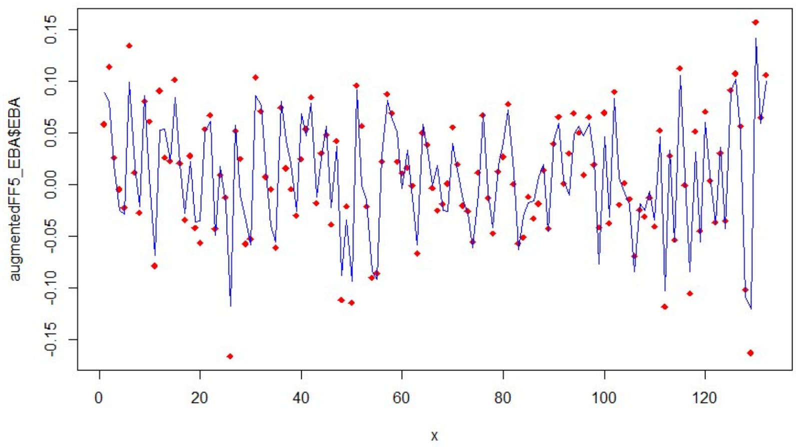

Figure B.13: EBA predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

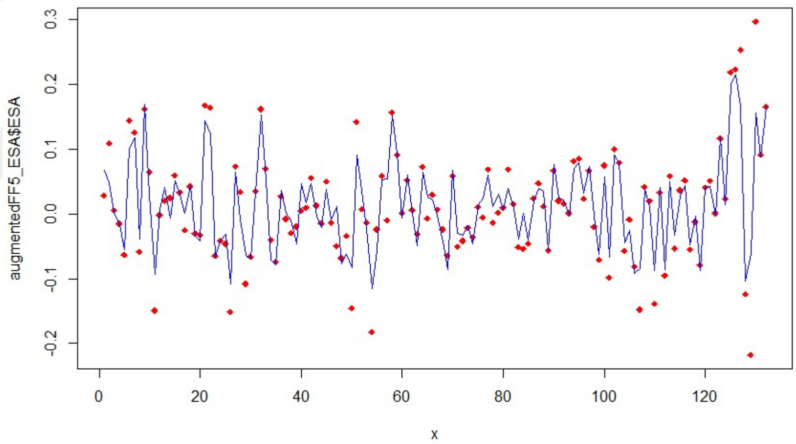

Figure B.14: ESA predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author's calculations

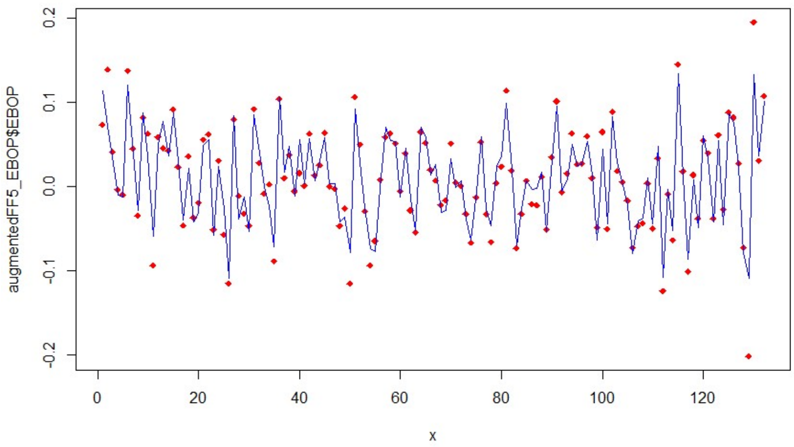

Figure B.15: EBOP predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

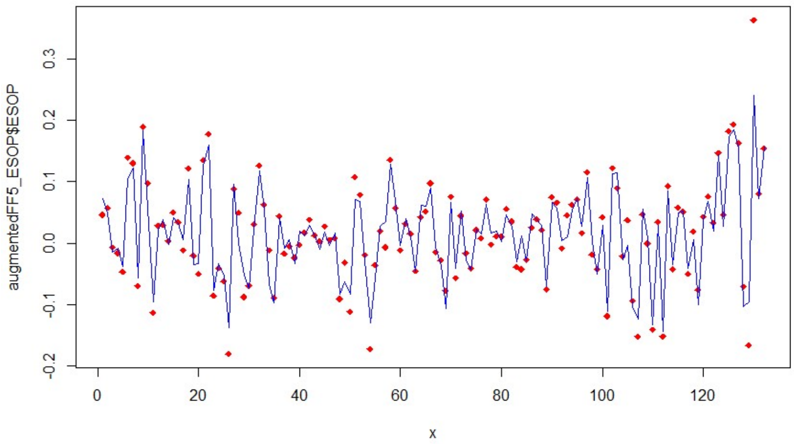

Figure B.16: ESOP predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

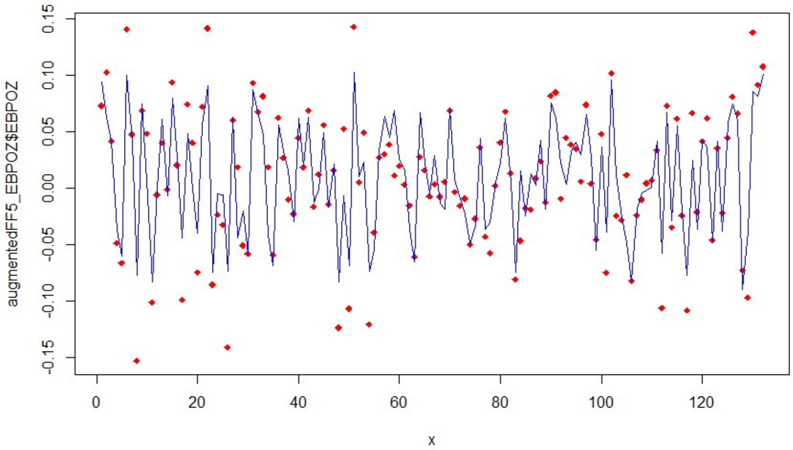

Figure B.17: EBPOZ predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

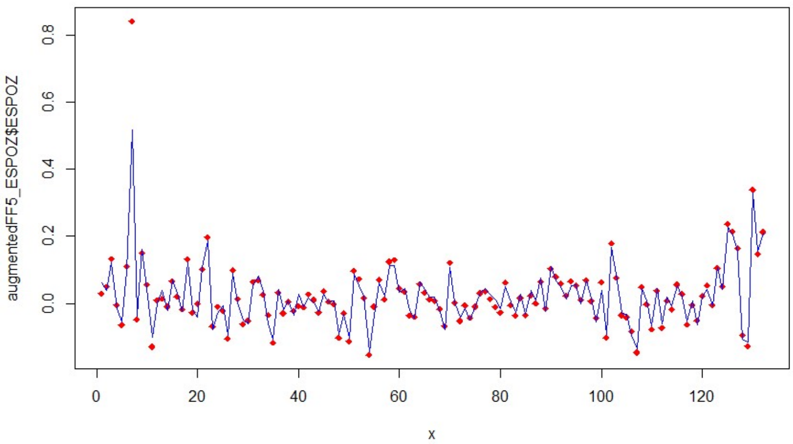

Figure B.18: ESPOZ predictions

Notes: Blue line indicates SVR predictions, and red dots denote actual data.
Source: Author’s calculations

APPENDIX C

PREDICTION ERRORS OBTAINED OUT OF FF5F MODEL WITHOUT FX RISK

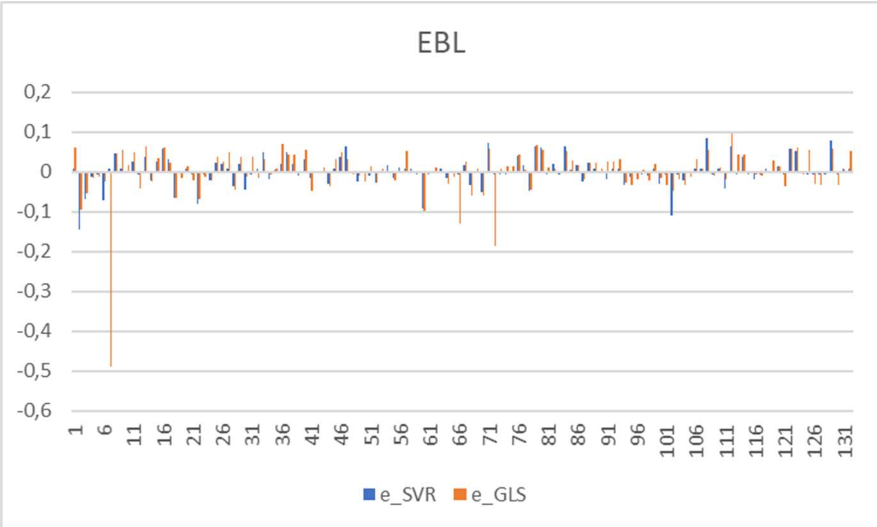

Figure C.1: EBL prediction errors obtained from GLS and SVR

Source: Author's calculations

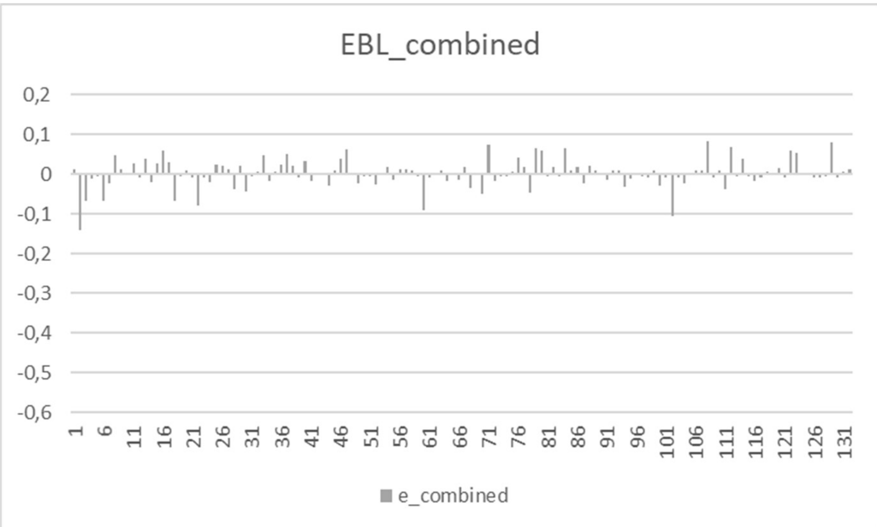

Figure C.2: Errors of combined predictions for EBL

Source: Author's calculations

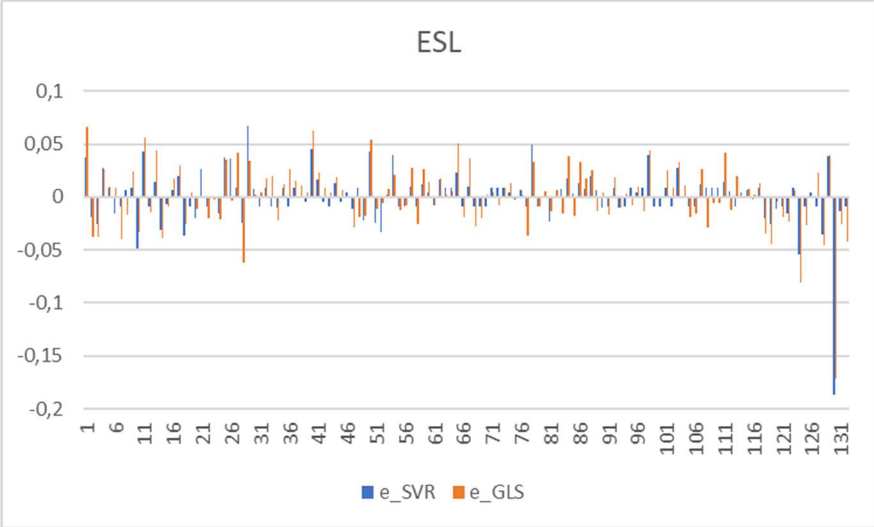

Figure C.3: ESL prediction errors obtained from GLS and SVR

Source: Author's calculations

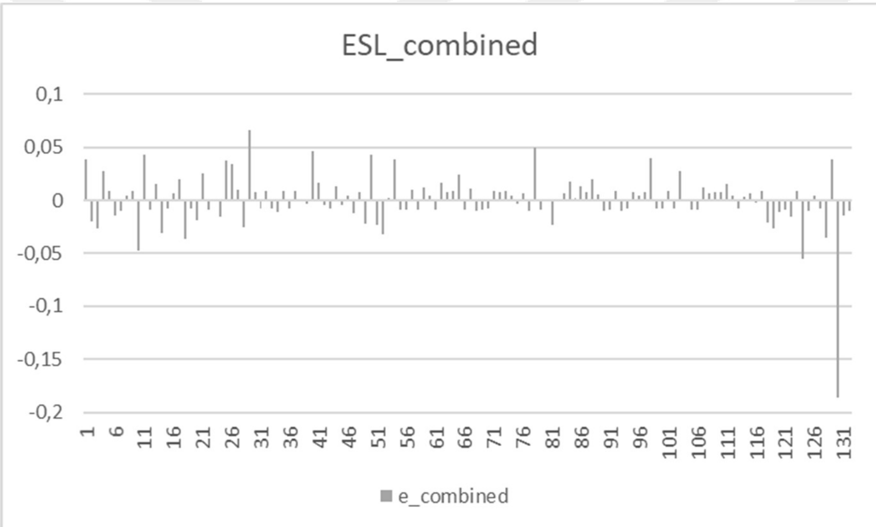

Figure C.4: Errors of combined predictions for ESL

Source: Author's calculations

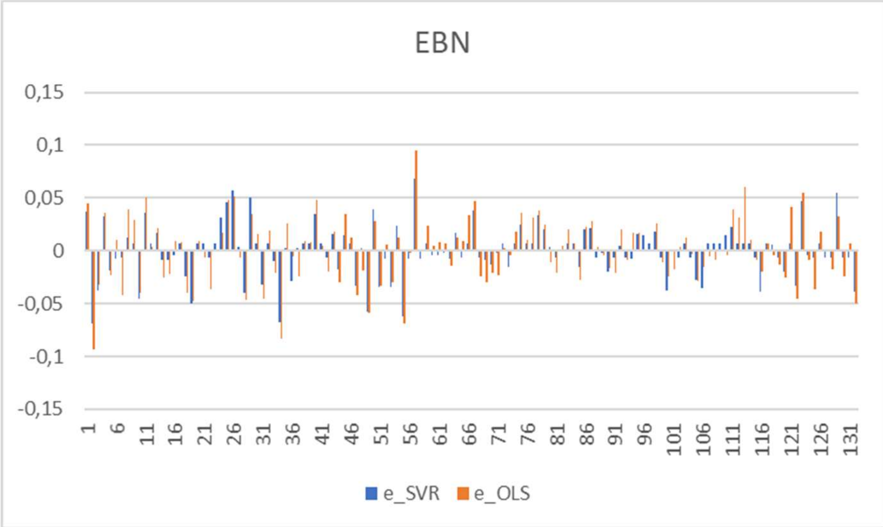

Figure C.5: EBN prediction errors obtained from OLS and SVR

Source: Author’s calculations

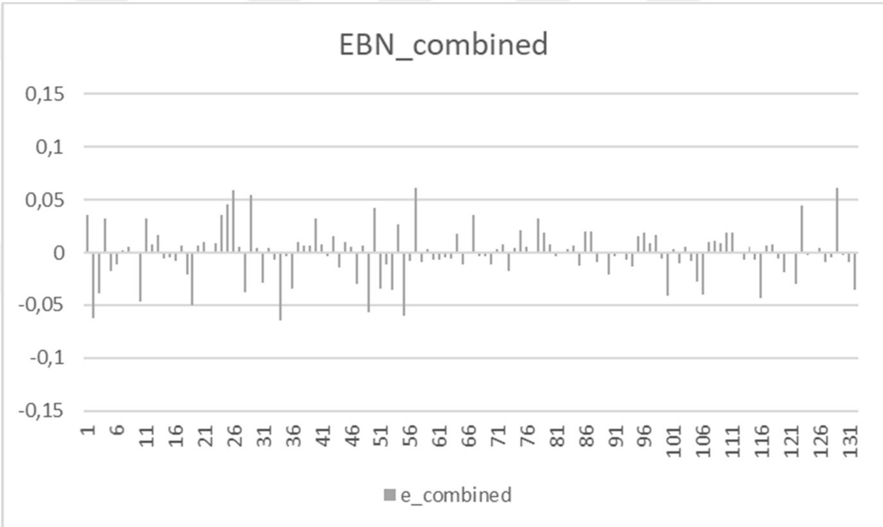

Figure C.6: Errors of combined predictions for EBN

Source: Author’s calculations

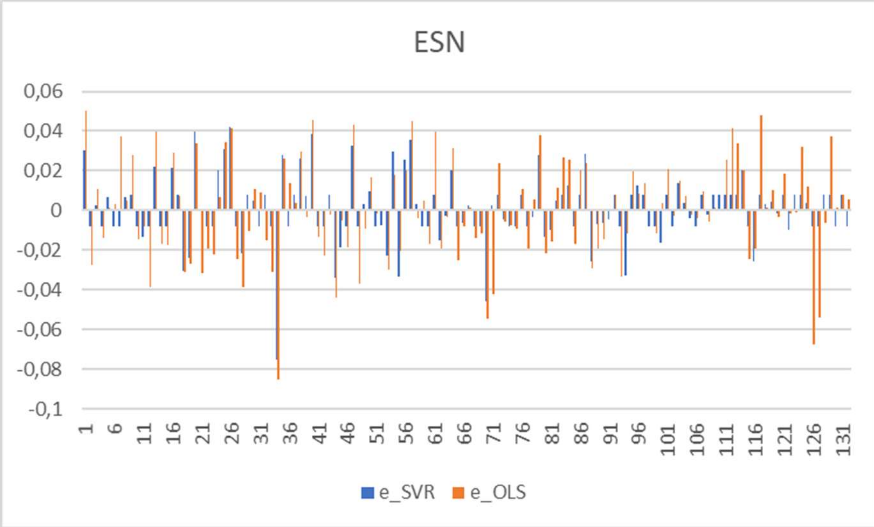

Figure C.7: ESN prediction errors obtained from OLS and SVR
Source: Author's calculations

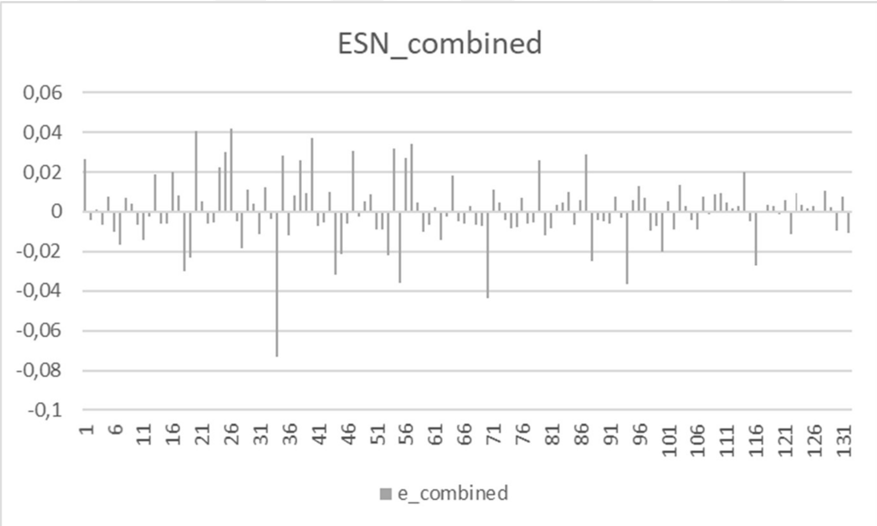

Figure C.8: Errors of combined predictions for ESN
Source: Author's calculations

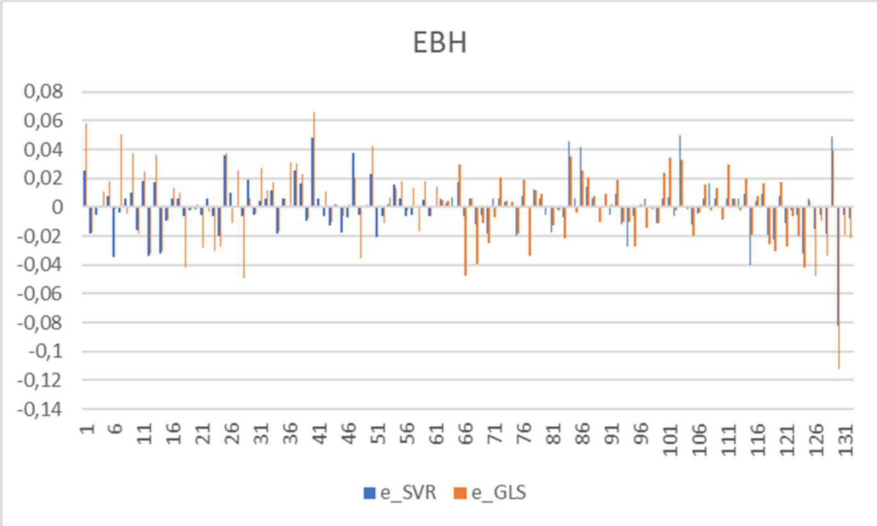

Figure C.9: EBH prediction errors obtained from GLS and SVR

Source: Author's calculations

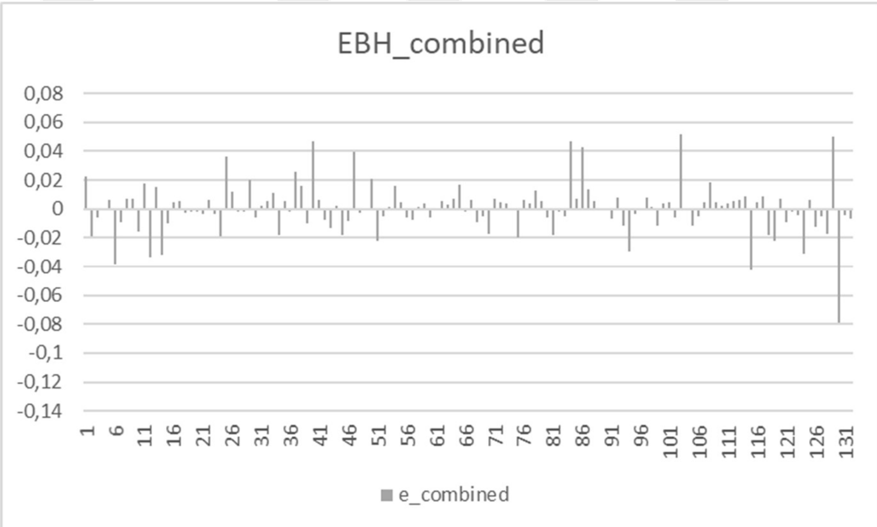

Figure C.10: Errors of combined predictions for EBH

Source: Author's calculations

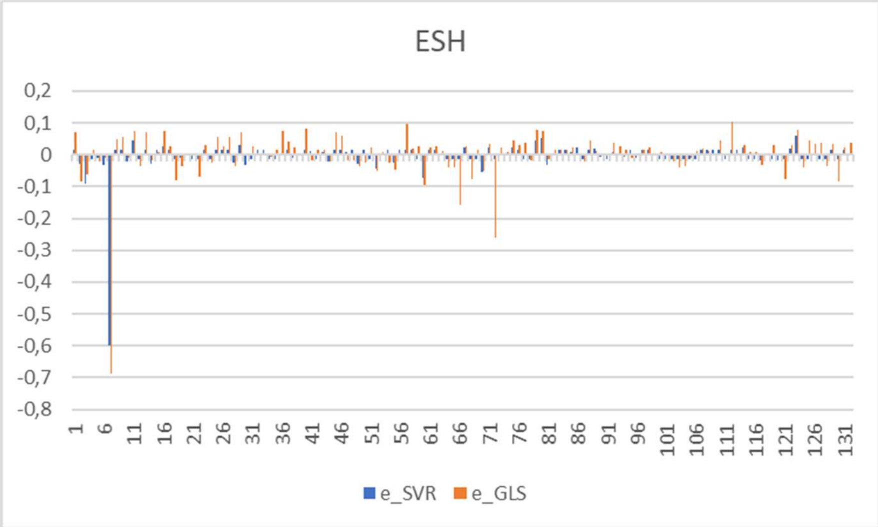

Figure C.11: ESH prediction errors obtained from GLS and SVR
Source: Author's calculations

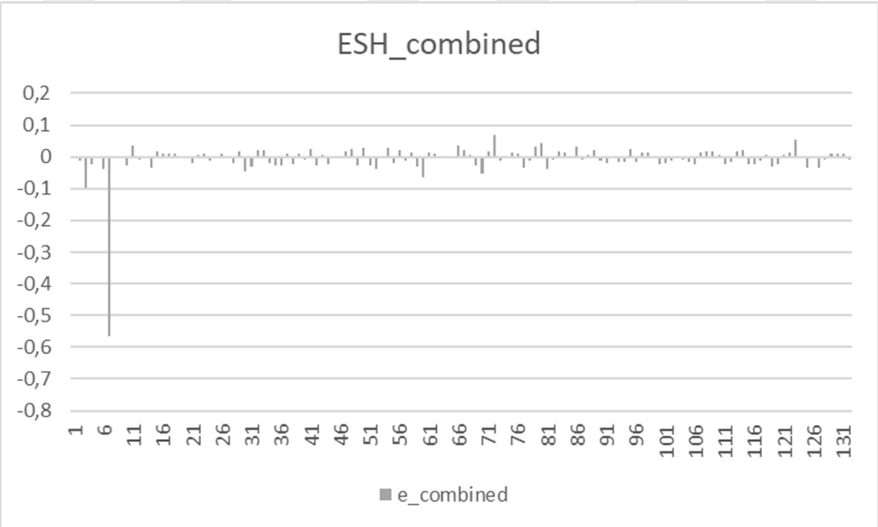

Figure C.12: Errors of combined predictions for ESH
Source: Author's calculations

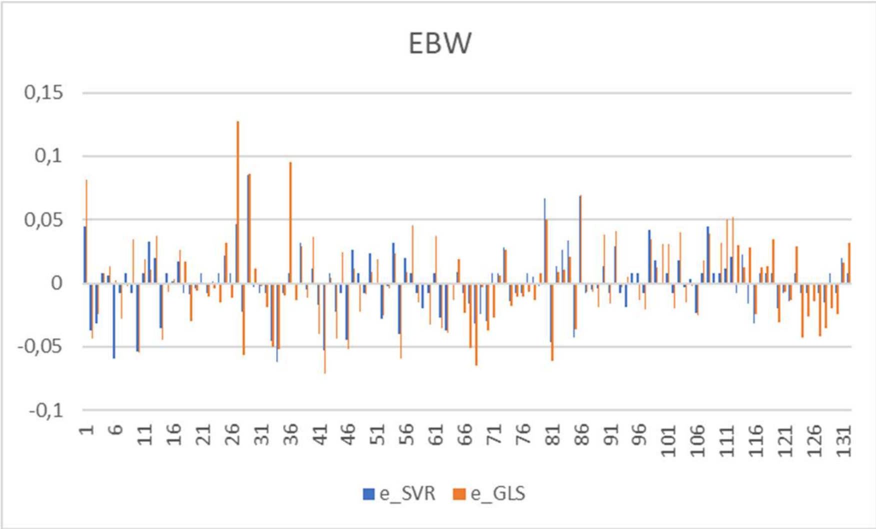

Figure C.13: EBW prediction errors obtained from GLS and SVR

Source: Author's calculations

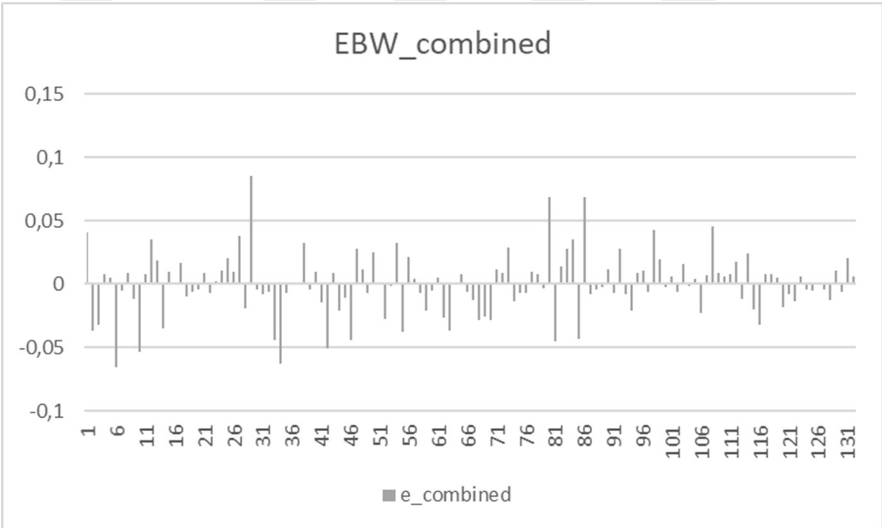

Figure C.14: Errors of combined predictions for EBW

Source: Author's calculations

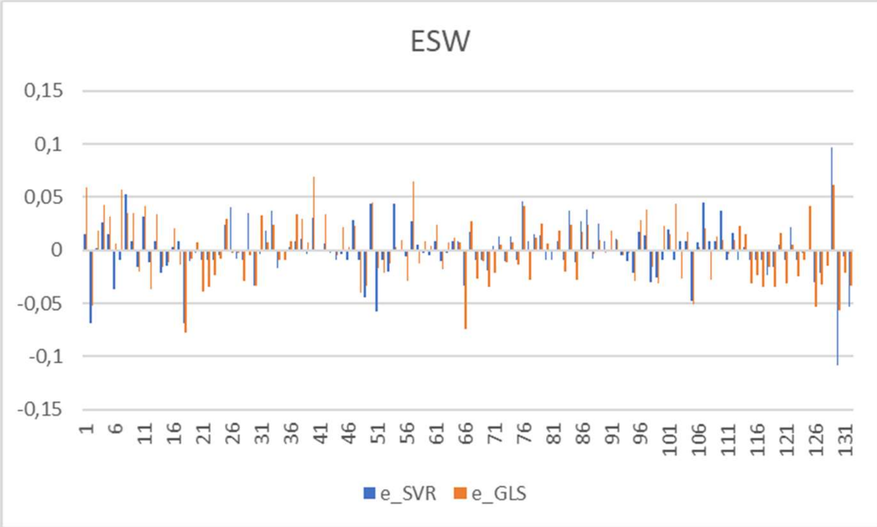

Figure C.15: ESW prediction errors obtained from GLS and SVR

Source: Author's calculations

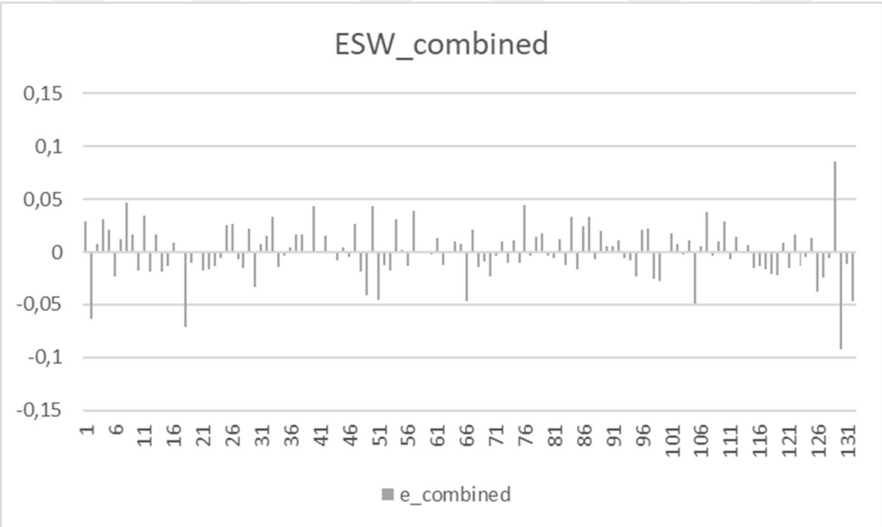

Figure C.16: Errors of combined predictions for ESW

Source: Author's calculations

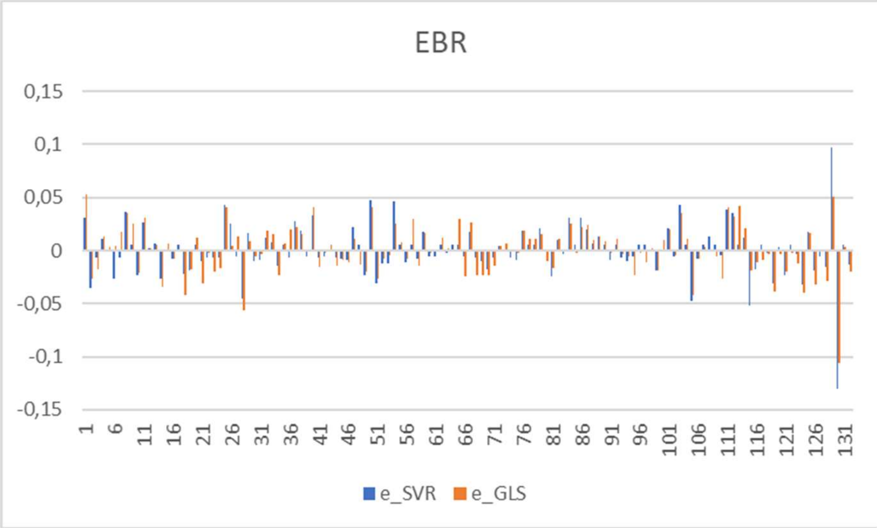

Figure C.17: EBR prediction errors obtained from GLS and SVR

Source: Author's calculations

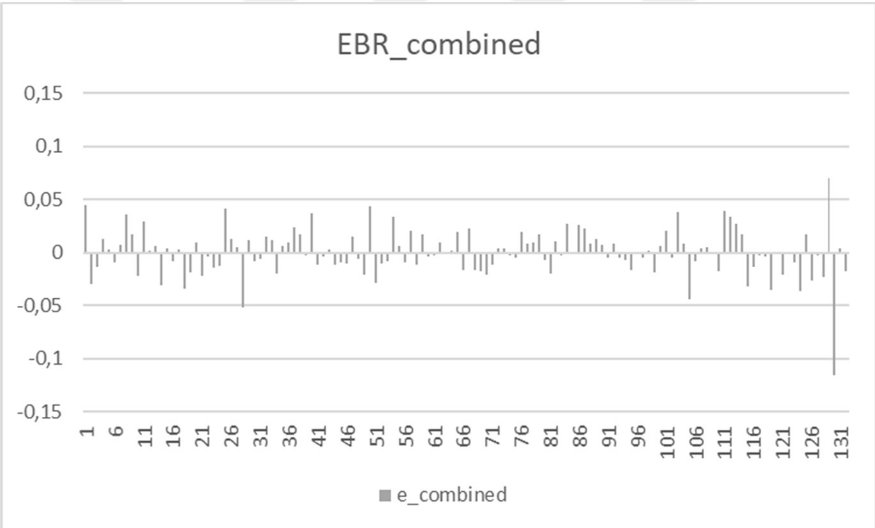

Figure C.18: Errors of combined predictions for EBR

Source: Author's calculations

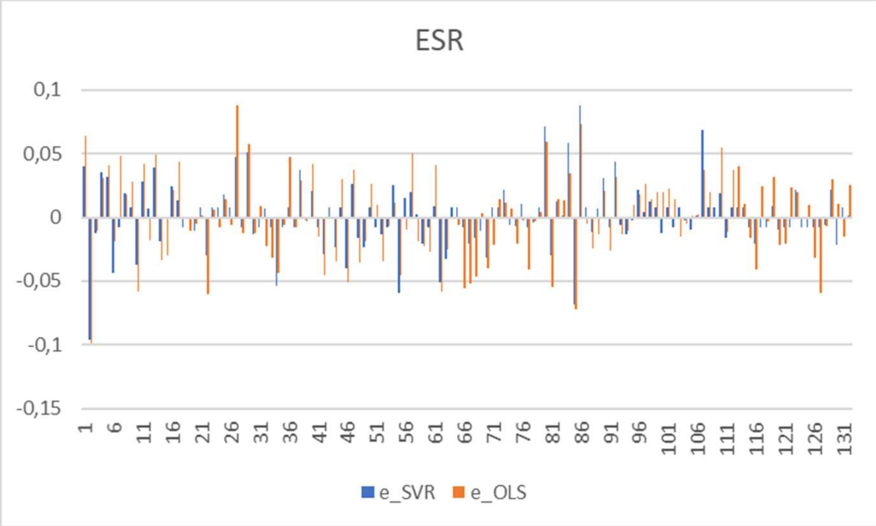

Figure C.19: ESR prediction errors obtained from OLS and SVR

Source: Author's calculations

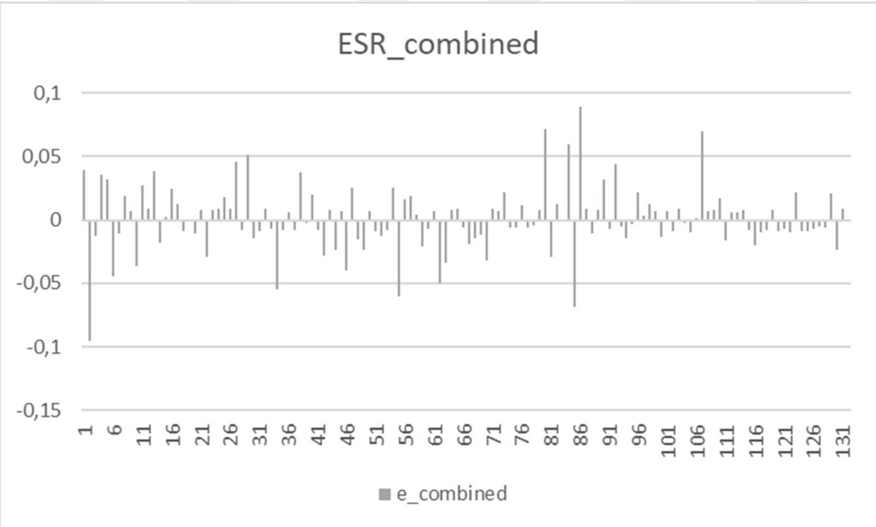

Figure C.20: Errors of combined predictions for ESR

Source: Author's calculations

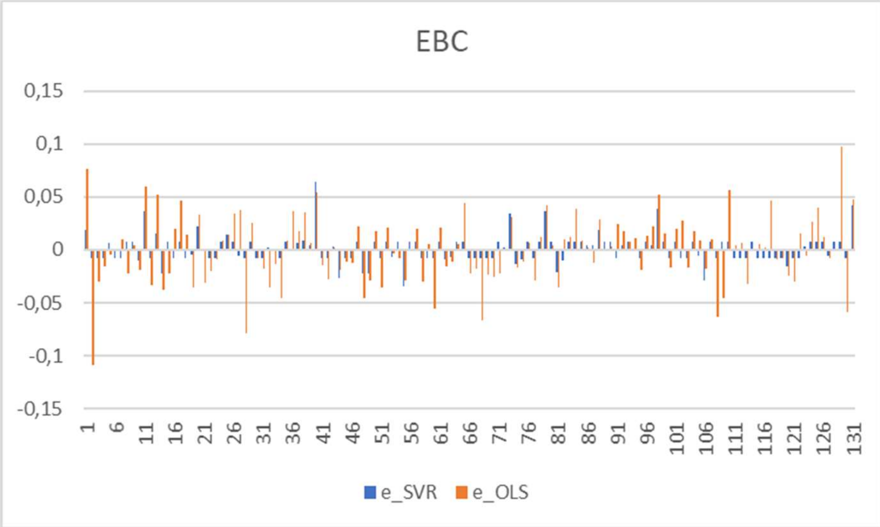

Figure C.21: EBC prediction errors obtained from OLS and SVR
Source: Author's calculations

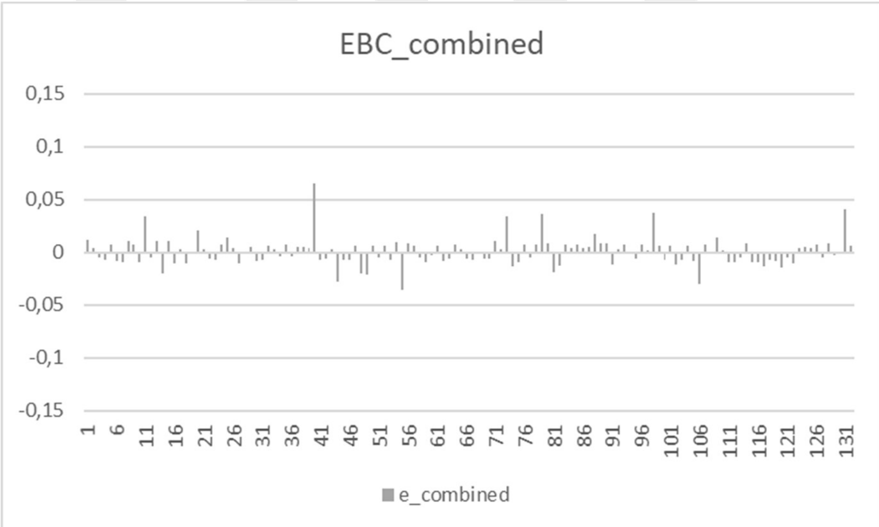

Figure C.22: Errors of combined predictions for EBC
Source: Author's calculations

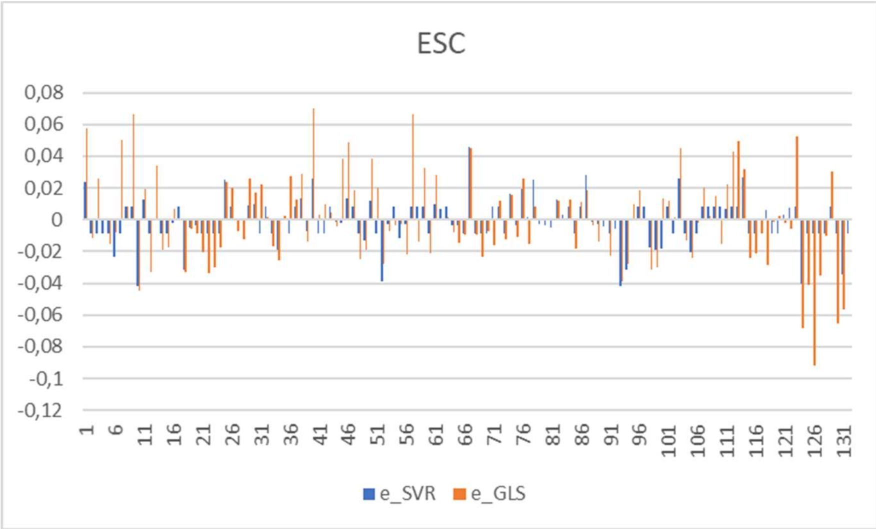

Figure C.23: ESC prediction errors obtained from GLS and SVR

Source: Author's calculations

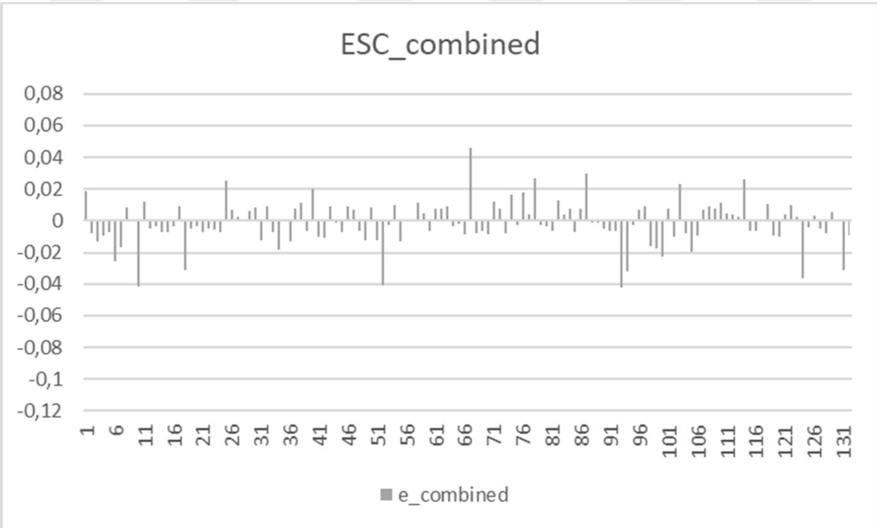

Figure C.24: Errors of combined predictions for EBC

Source: Author's calculations

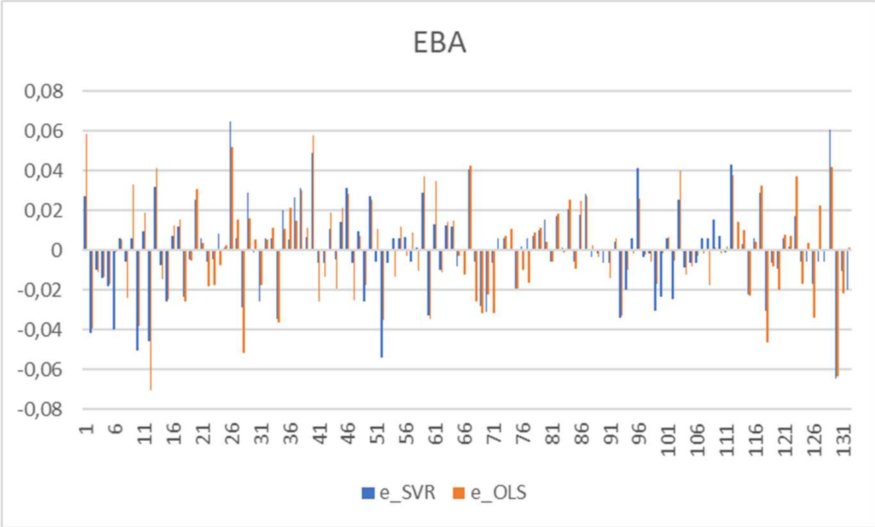

Figure C.25: EBA prediction errors obtained from OLS and SVR

Source: Author's calculations

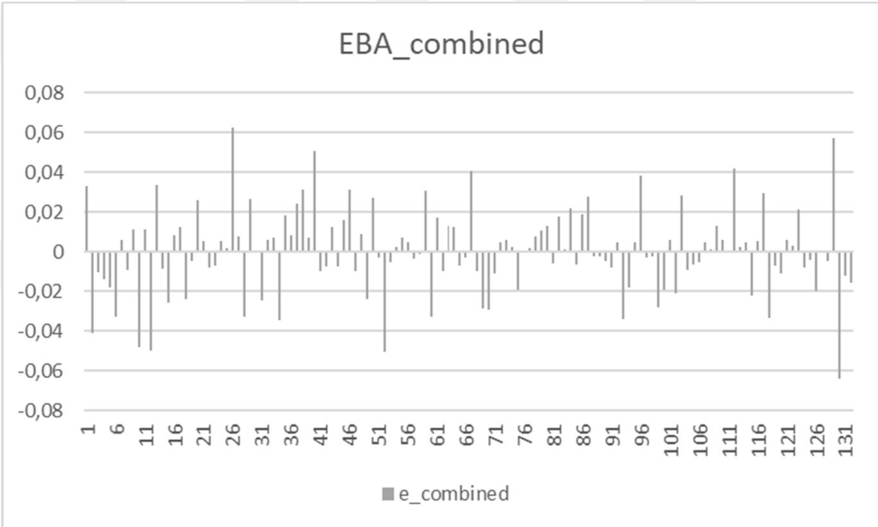

Figure C.26: Errors of combined predictions for EBA

Source: Author's calculations

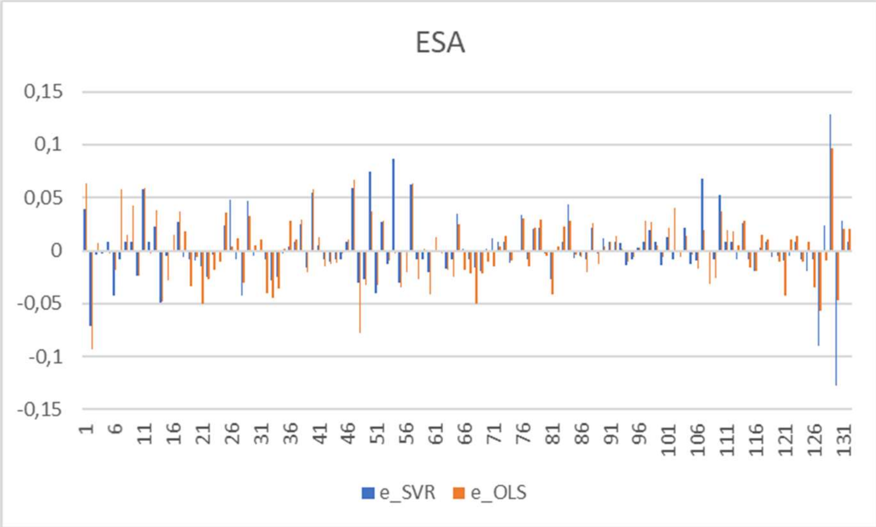

Figure C.27: ESA prediction errors obtained from OLS and SVR

Source: Author's calculations

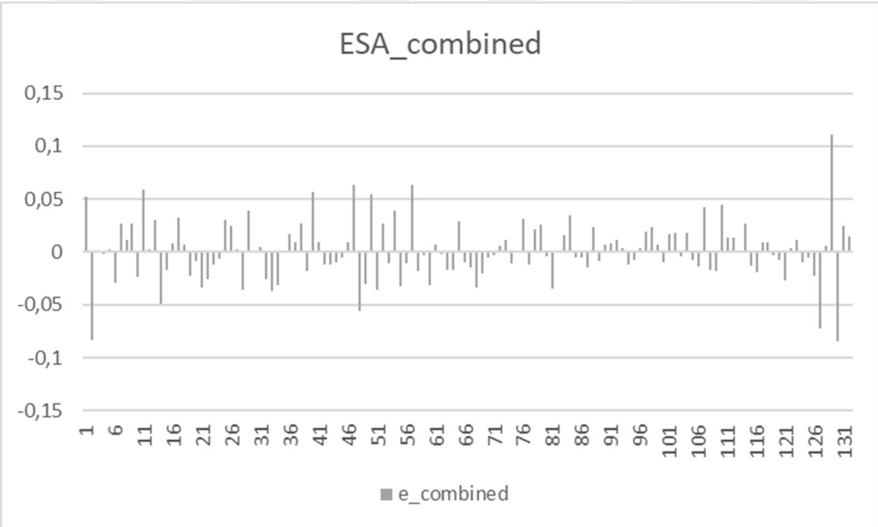

Figure C.28: Errors of combined predictions for ESA

Source: Author's calculations

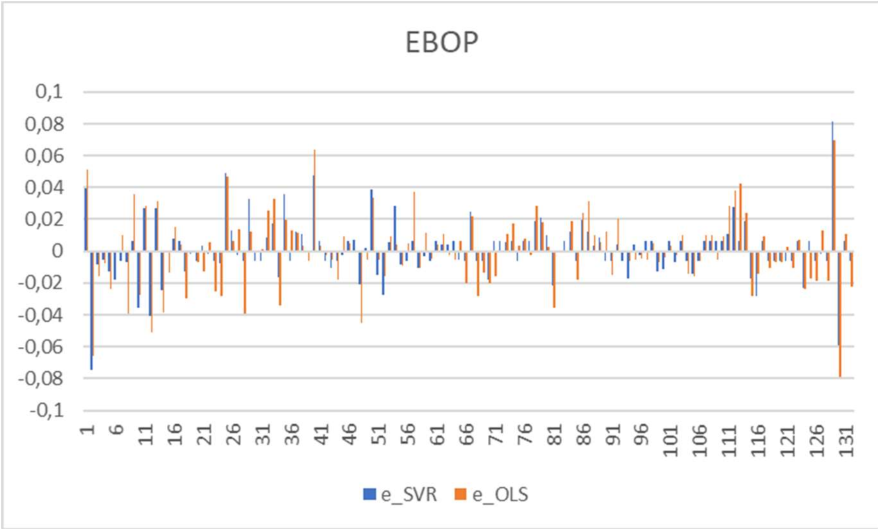

Figure C.29: EBOP prediction errors obtained from OLS and SVR

Source: Author's calculations

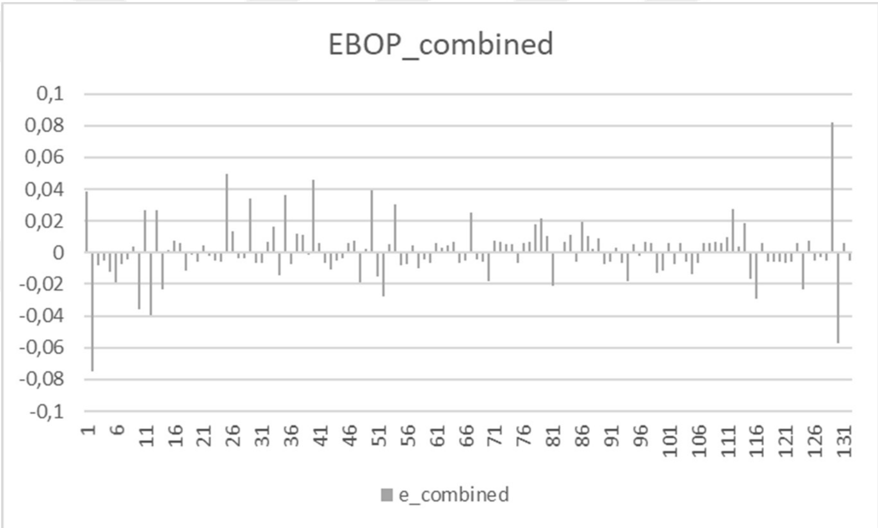

Figure C.30: Errors of combined predictions for EBOP

Source: Author's calculations

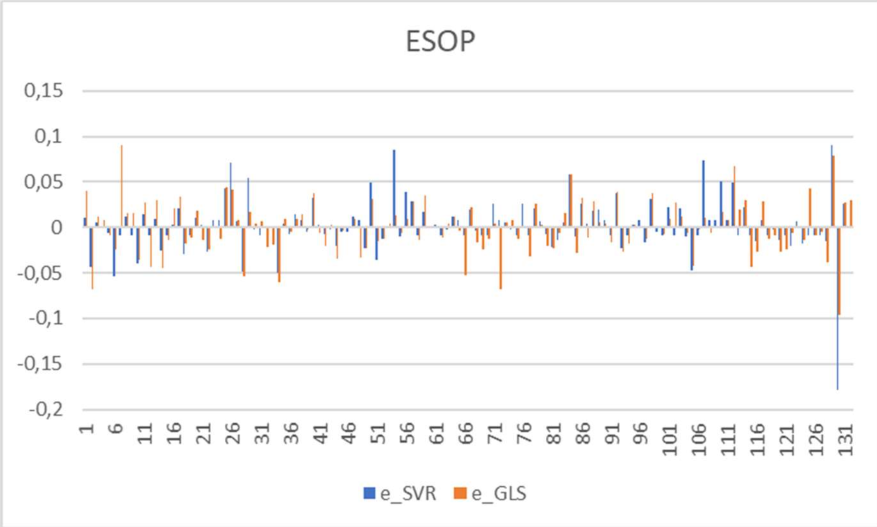

Figure C.31: ESOP prediction errors obtained from GLS and SVR

Source: Author's calculations

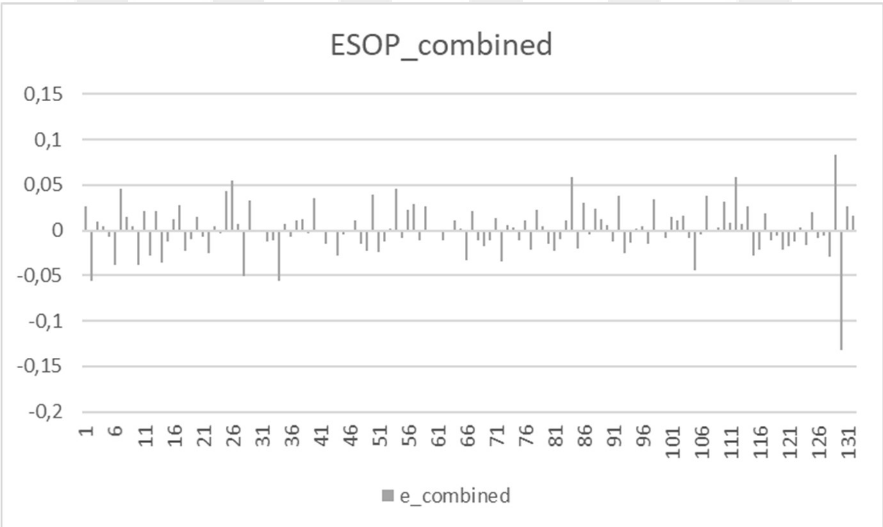

Figure C.32: Errors of combined predictions for ESOP

Source: Author's calculations

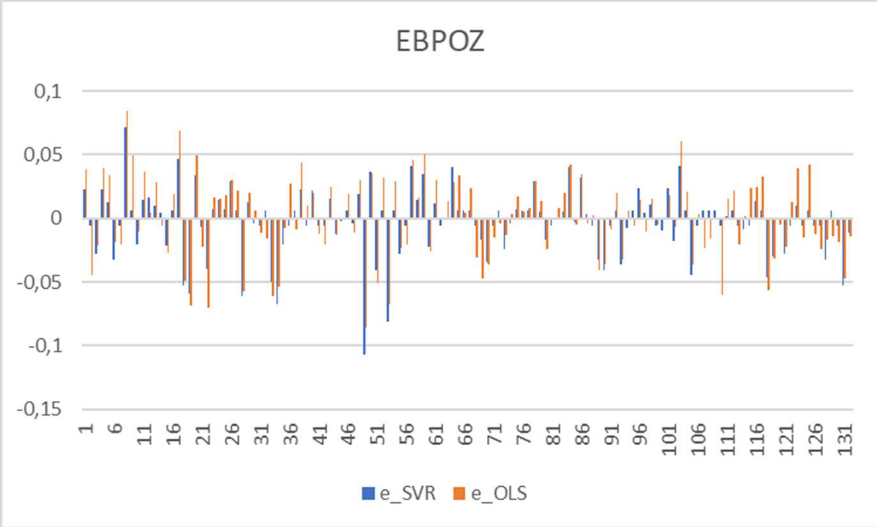

Figure C.33: EBPOZ prediction errors obtained from OLS and SVR

Source: Author's calculations

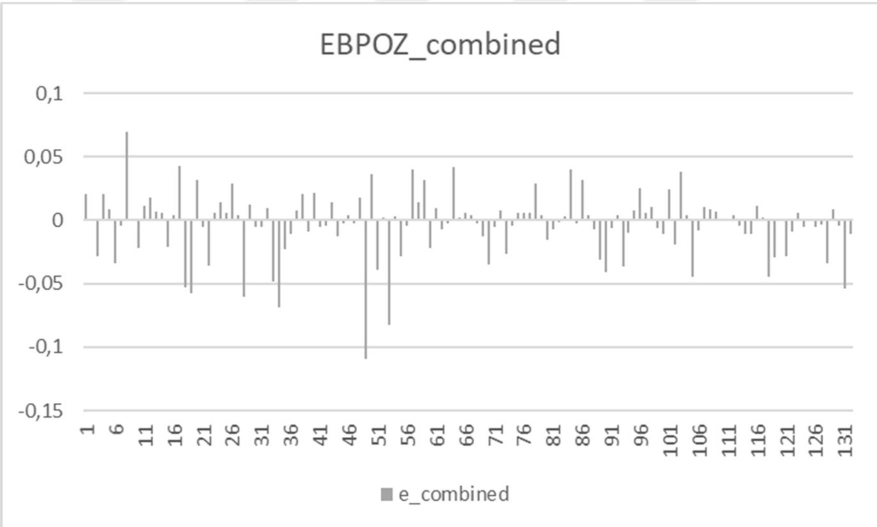

Figure C.34: Errors of combined predictions for EBPOZ

Source: Author's calculations

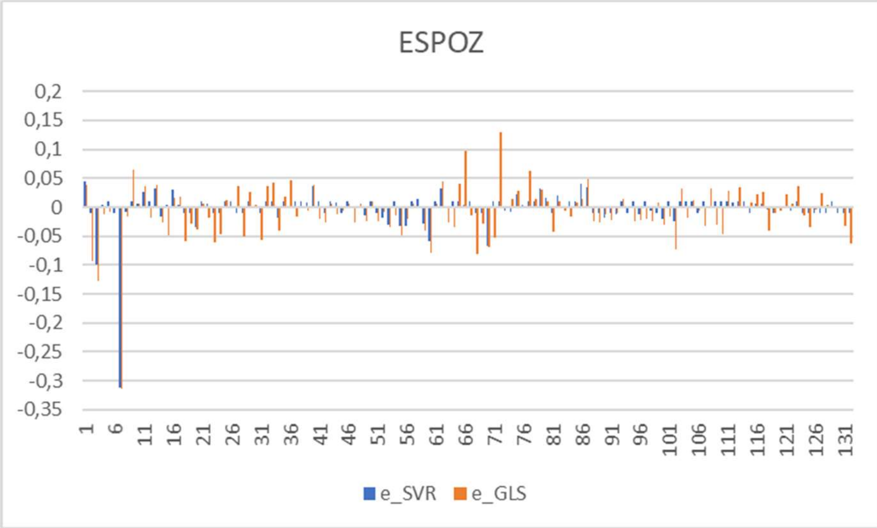

Figure C.35: ESPOZ prediction errors obtained from GLS and SVR

Source: Author's calculations

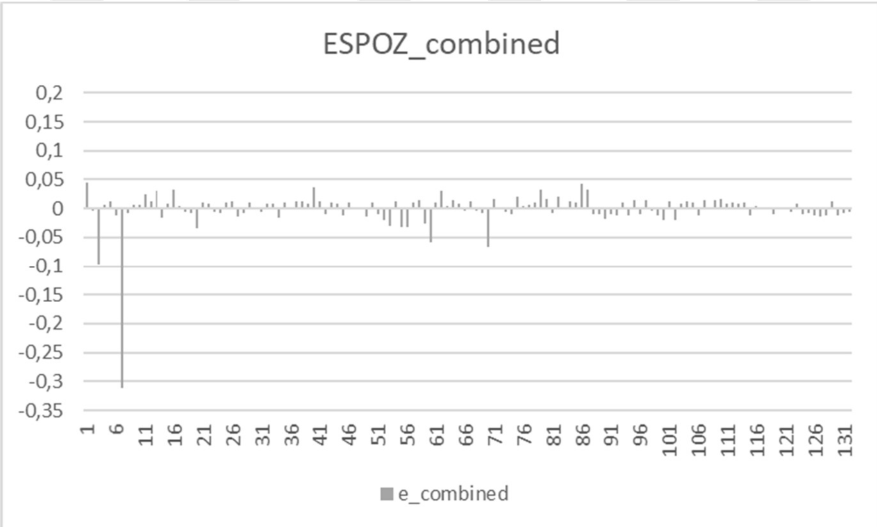

Figure C.36: Errors of combined predictions for ESPOZ

Source: Author's calculations

APPENDIX D

PREDICTION ERRORS OBTAINED OUT OF FF5F MODEL INCORPORATING FX RISK

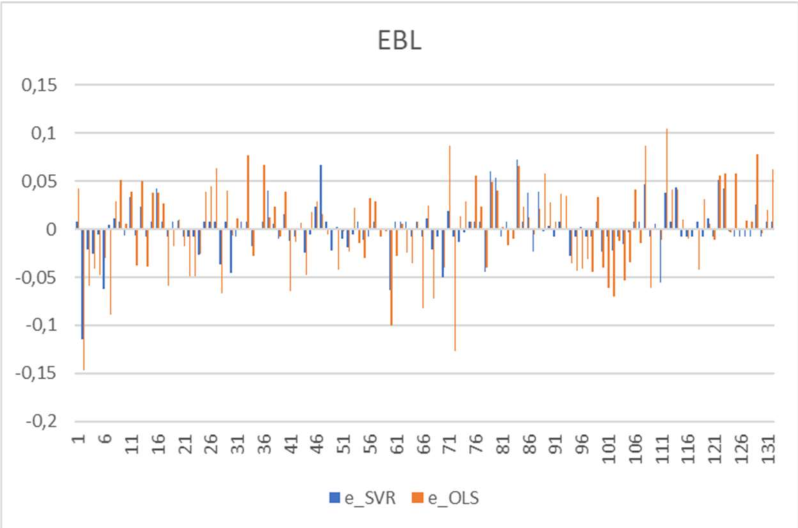

Figure D.1: EBL prediction errors obtained from OLS and SVR

Source: Author's calculations

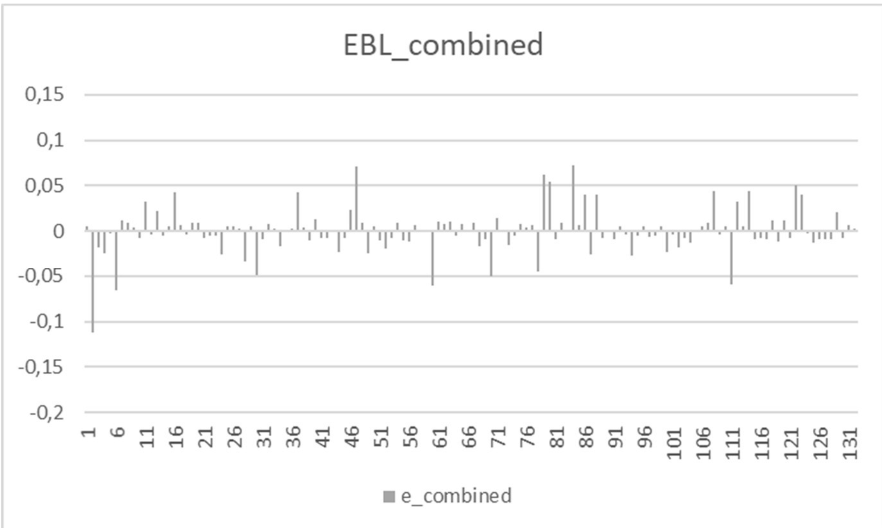

Figure D.2: Errors of combined predictions for EBL

Source: Author's calculations

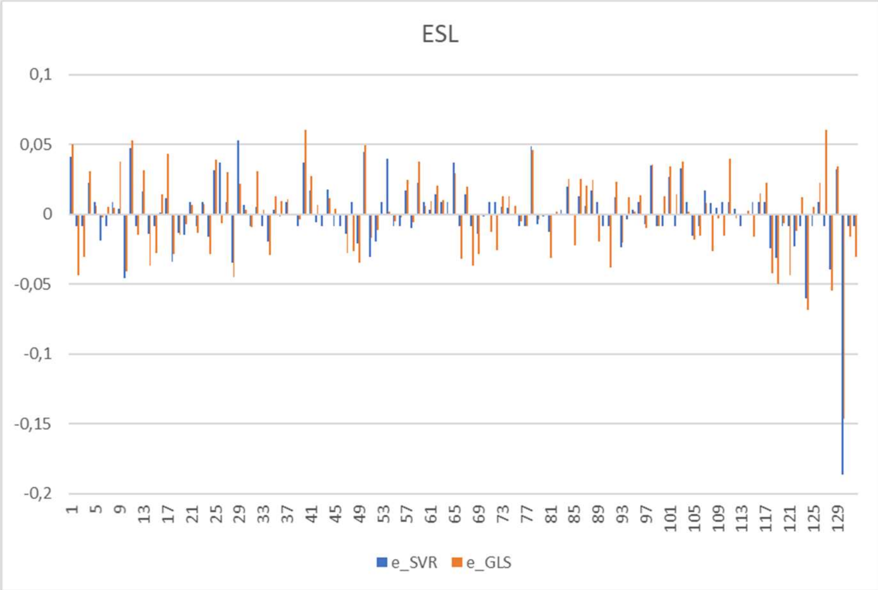

Figure D.3: ESL prediction errors obtained from GLS and SVR
Source: Author’s calculations

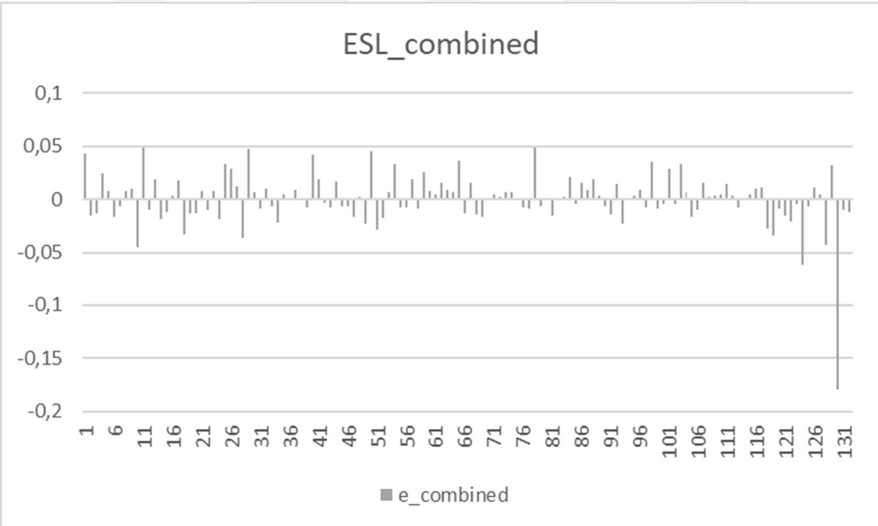

Figure D.4: Errors of combined predictions for ESL
Source: Author’s calculations

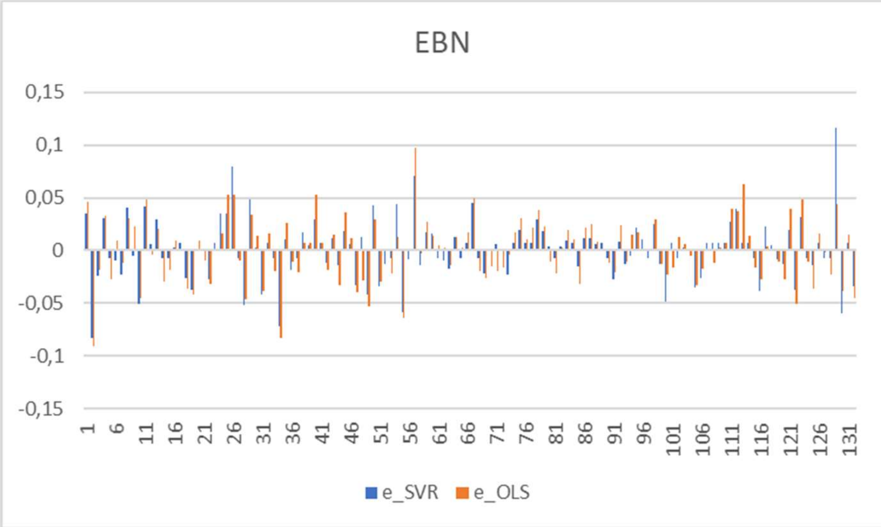

Figure D.5: EBN prediction errors obtained from OLS and SVR

Source: Author's calculations

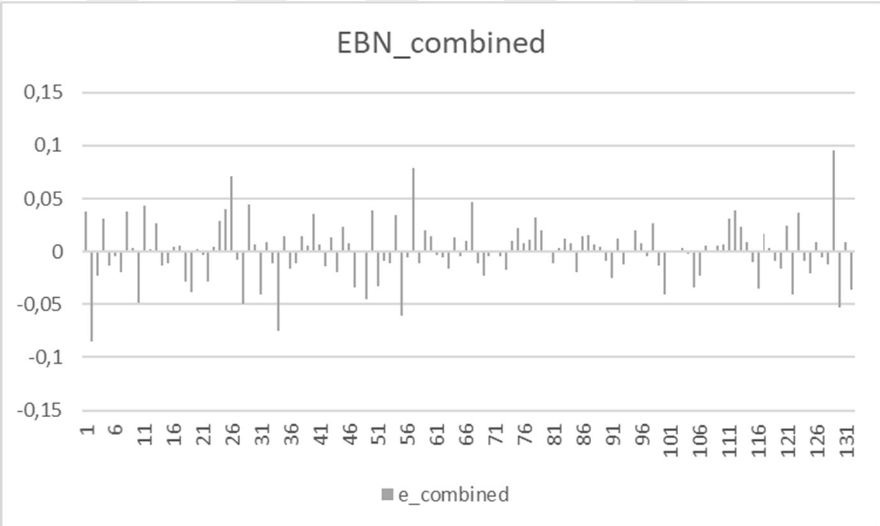

Figure D.6: Errors of combined predictions for ESL

Source: Author's calculations

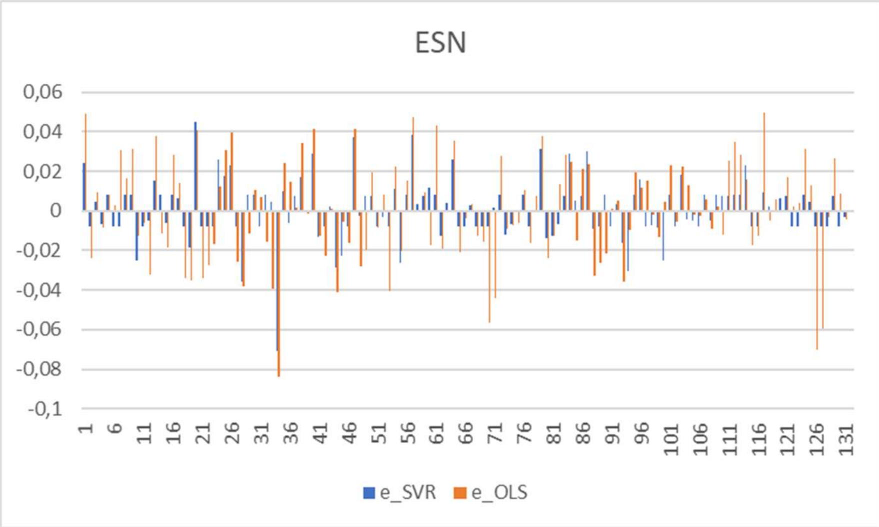

Figure D.7: ESN prediction errors obtained from OLS and SVR

Source: Author's calculations

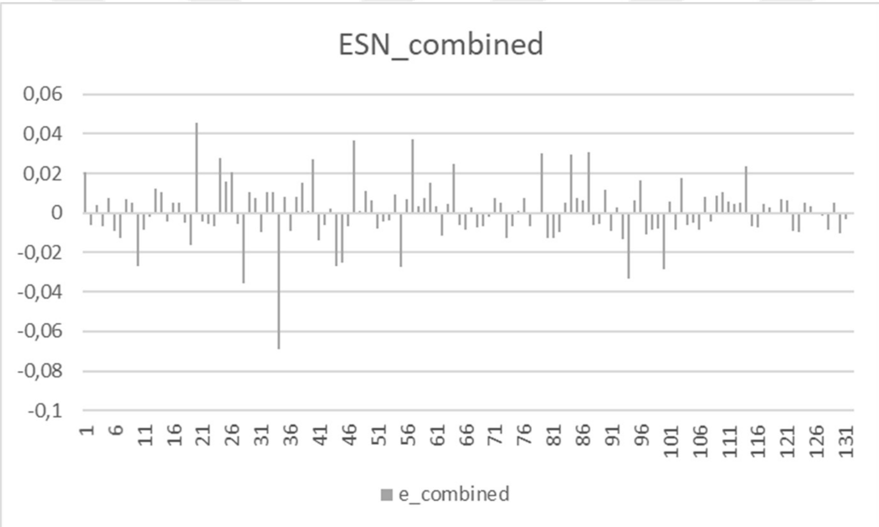

Figure D.8: Errors of combined predictions for ESN

Source: Author's calculations

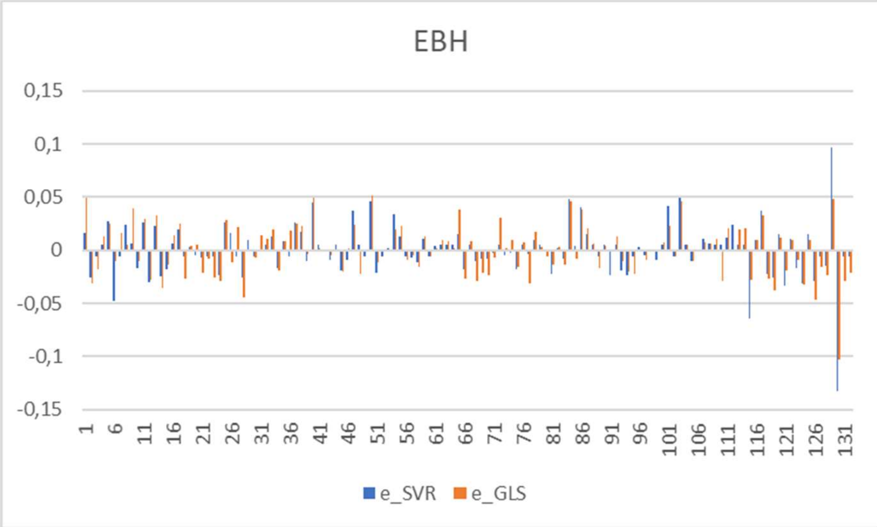

Figure D.9: EBH prediction errors obtained from GLS and SVR

Source: Author's calculations

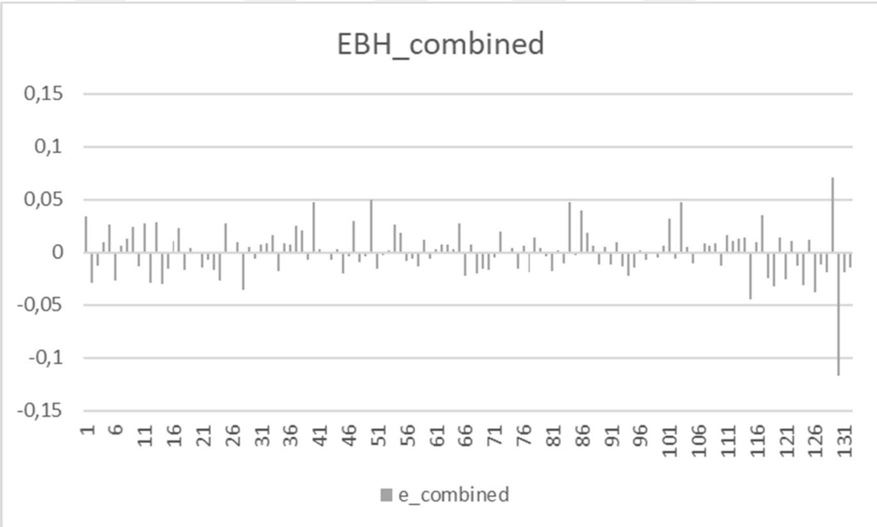

Figure D.10: Errors of combined predictions for EBH

Source: Author's calculations

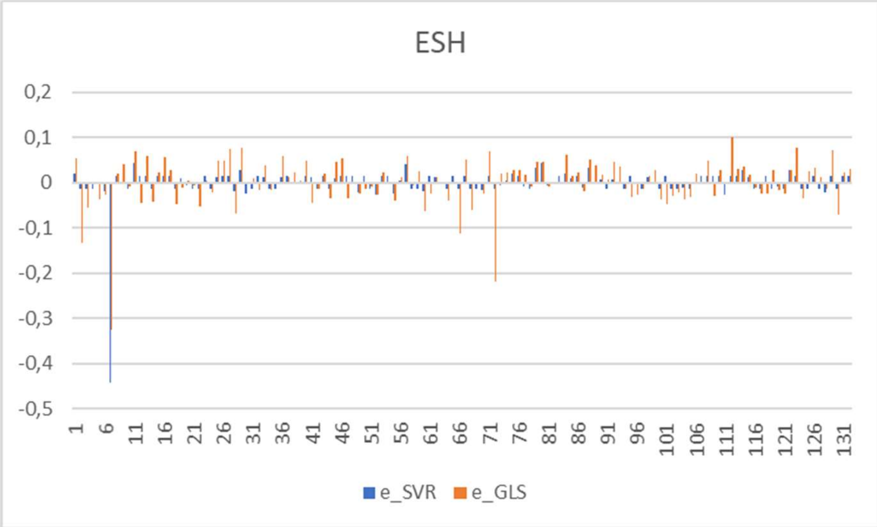

Figure D.11: ESH prediction errors obtained from GLS and SVR

Source: Author's calculations

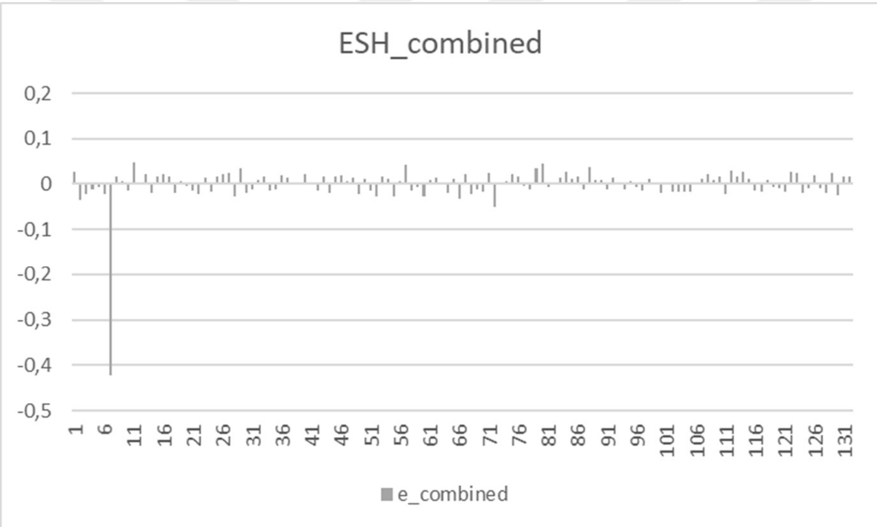

Figure D.12: Errors of combined predictions for ESH

Source: Author's calculations

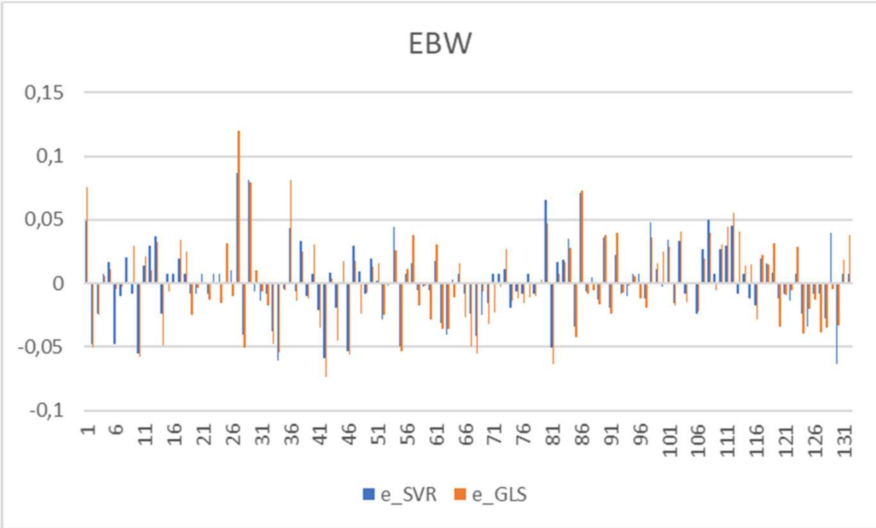

Figure D.13: EBW prediction errors obtained from GLS and SVR

Source: Author's calculations

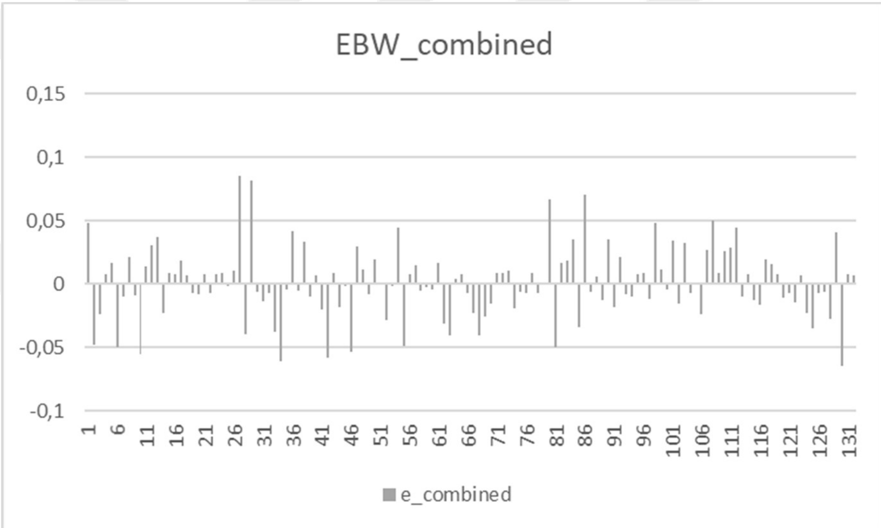

Figure D.14: Errors of combined predictions for EBW

Source: Author's calculations

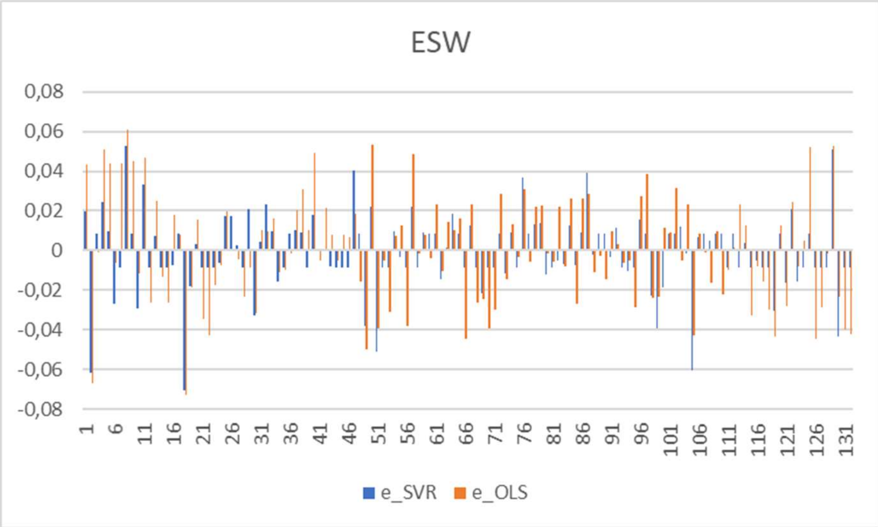

Figure D.15: ESW prediction errors obtained from OLS and SVR

Source: Author's calculations

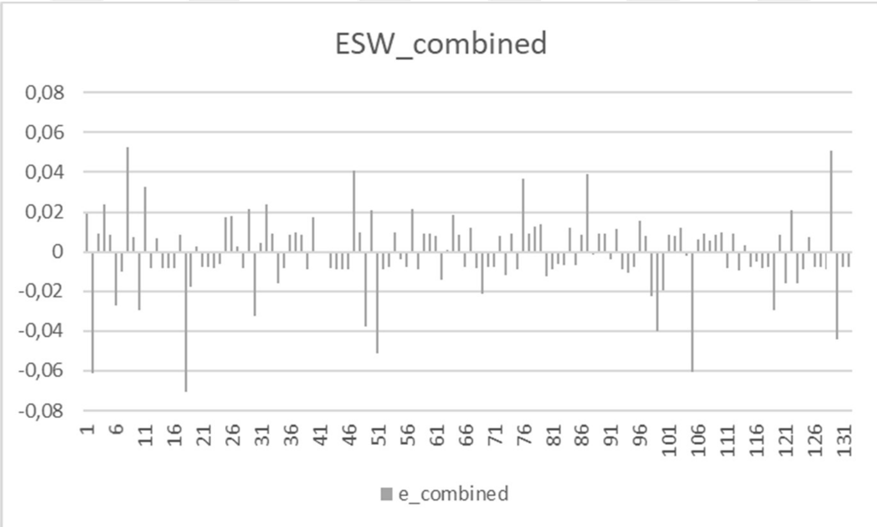

Figure D.16: Errors of combined predictions for ESW

Source: Author's calculations

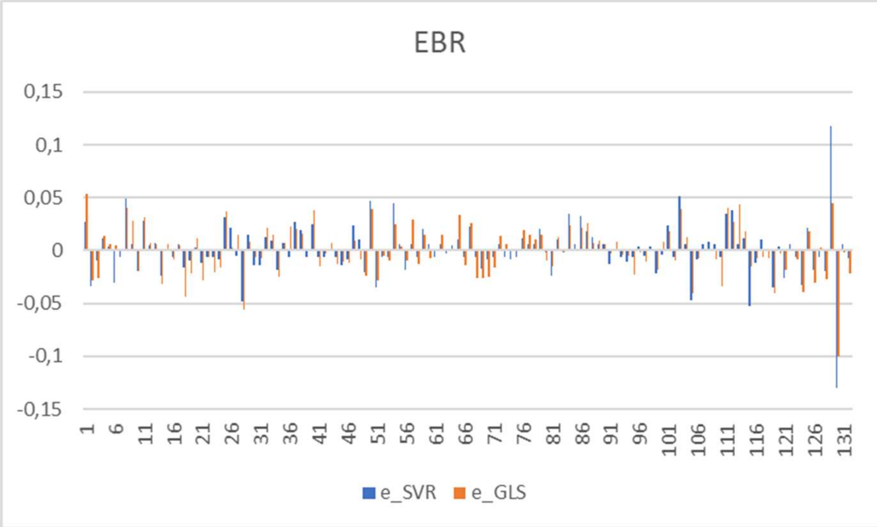

Figure D.17: EBR prediction errors obtained from GLS and SVR

Source: Author’s calculations

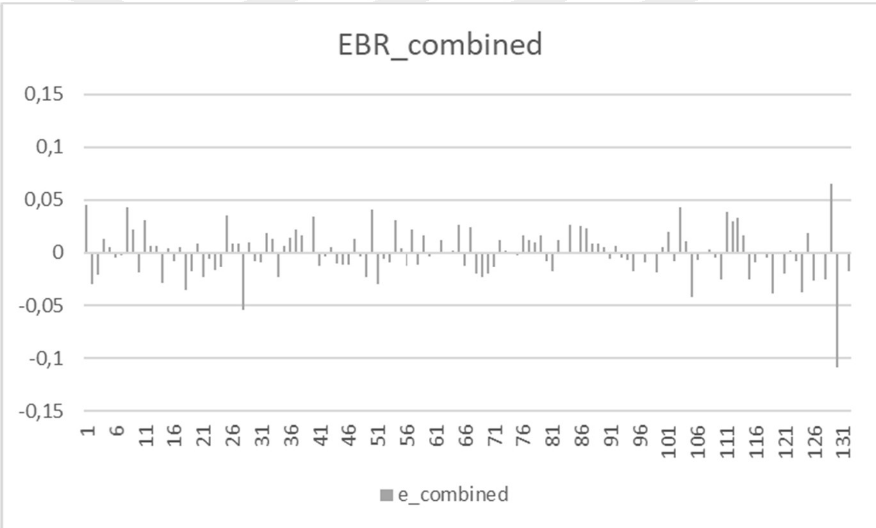

Figure D.18: Errors of combined predictions for EBR

Source: Author’s calculations

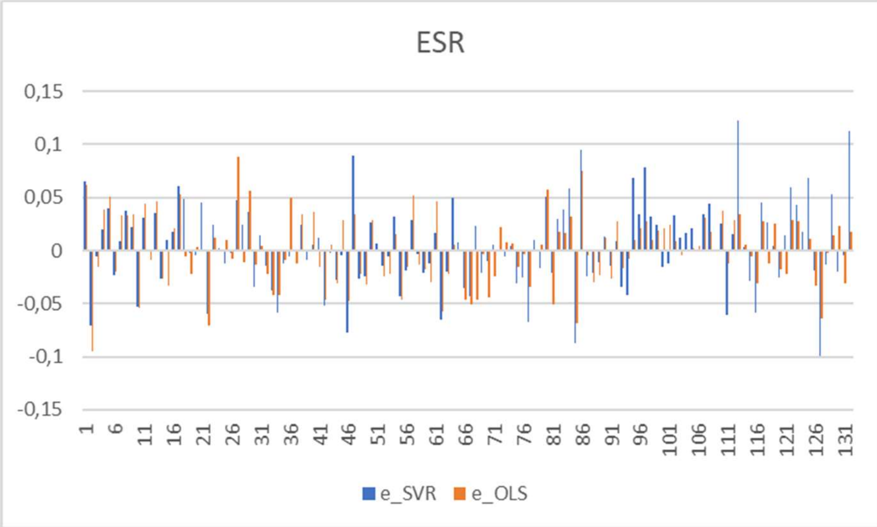

Figure D.19: ESR prediction errors obtained from OLS and SVR
Source: Author’s calculations

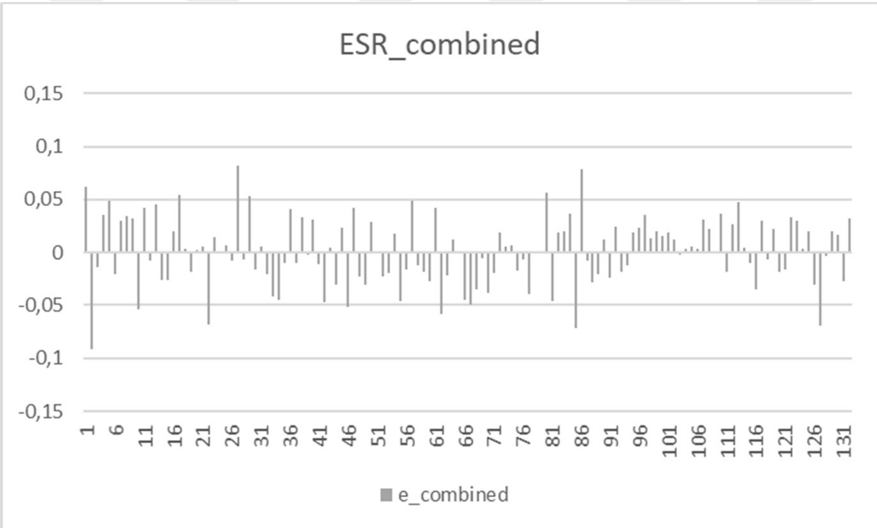

Figure D.20: Errors of combined predictions for ESR
Source: Author’s calculations

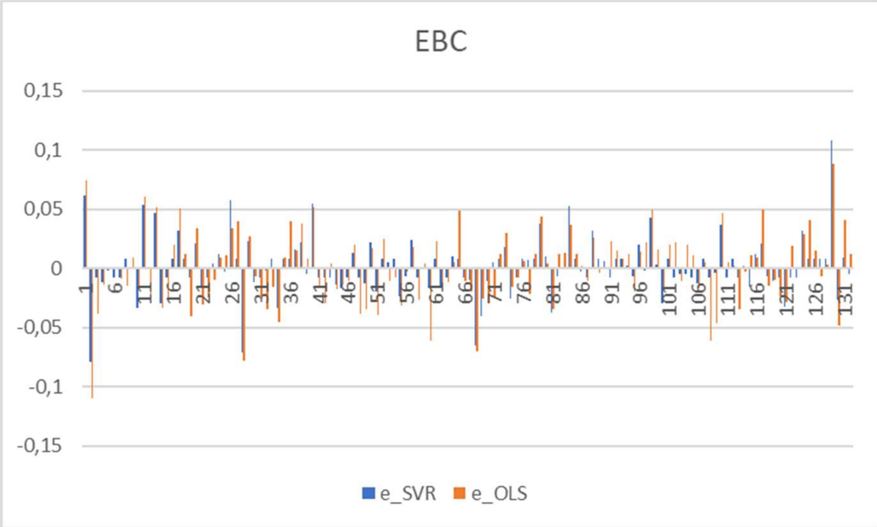

Figure D.21: EBC prediction errors obtained from OLS and SVR

Source: Author's calculations

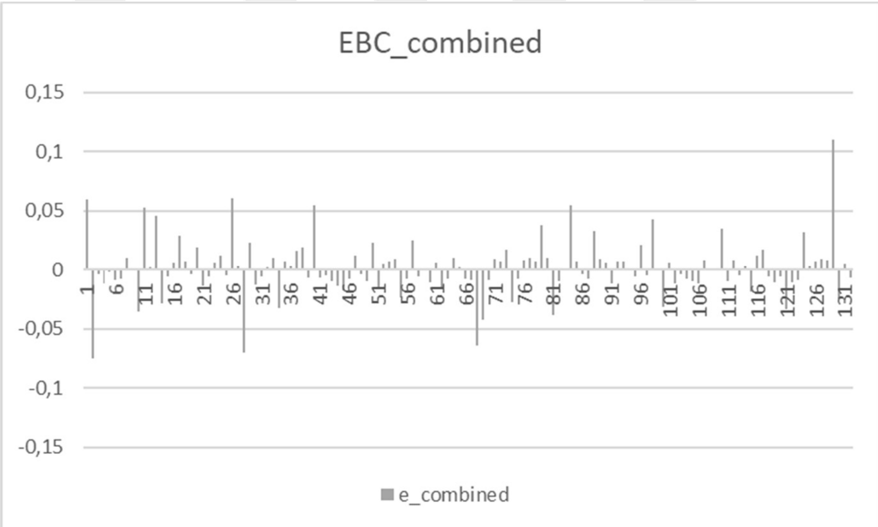

Figure D.22: Errors of combined predictions for EBC

Source: Author's calculations

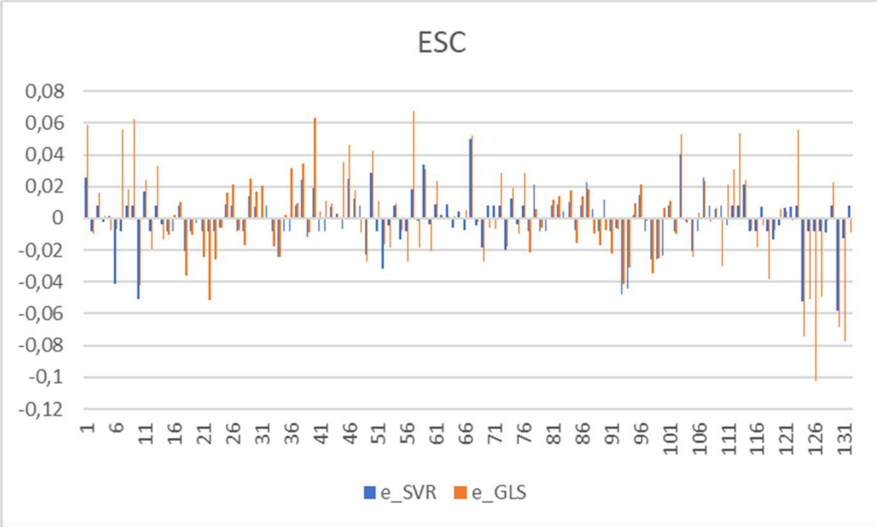

Figure D.23: ESC prediction errors obtained from GLS and SVR

Source: Author's calculations

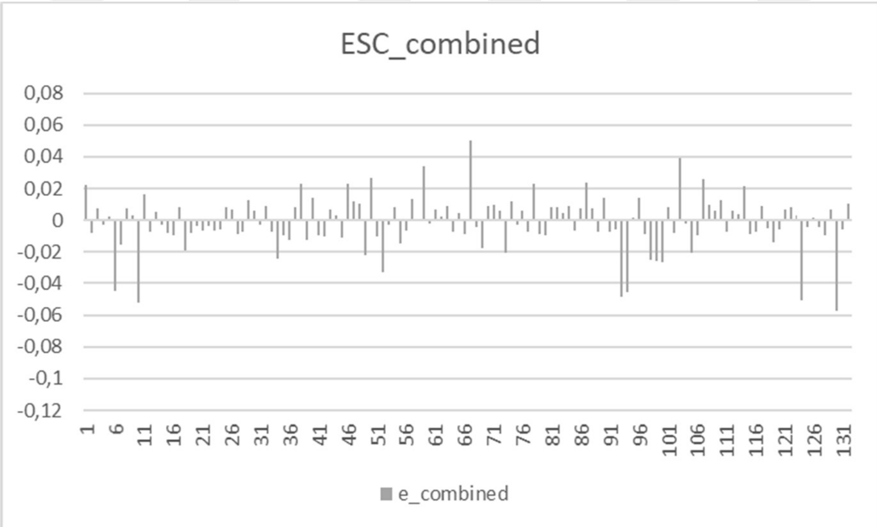

Figure D.24: Errors of combined predictions for ESC

Source: Author's calculations

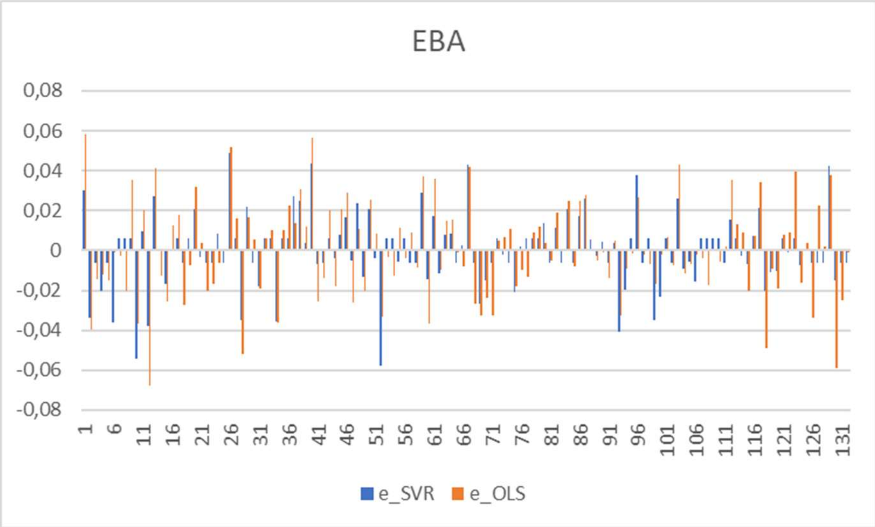

Figure D.25: EBA prediction errors obtained from OLS and SVR

Source: Author’s calculations

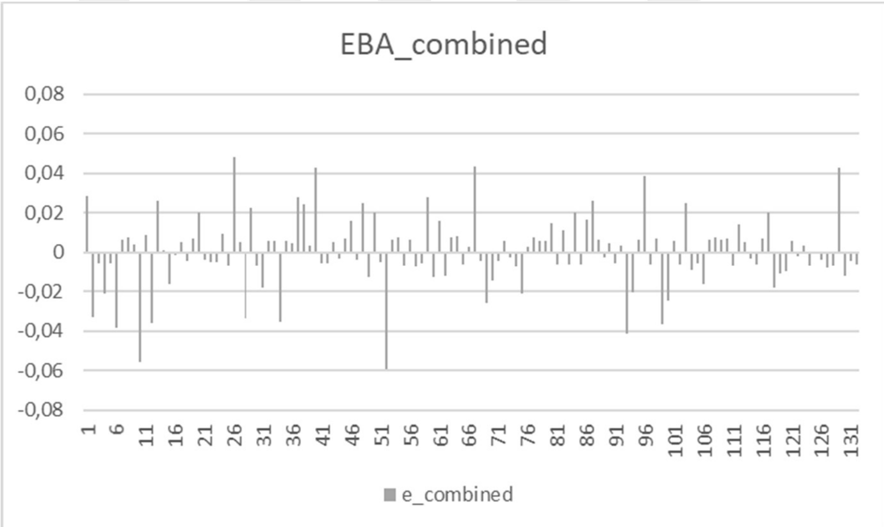

Figure D.26: Errors of combined predictions for EBA

Source: Author’s calculations

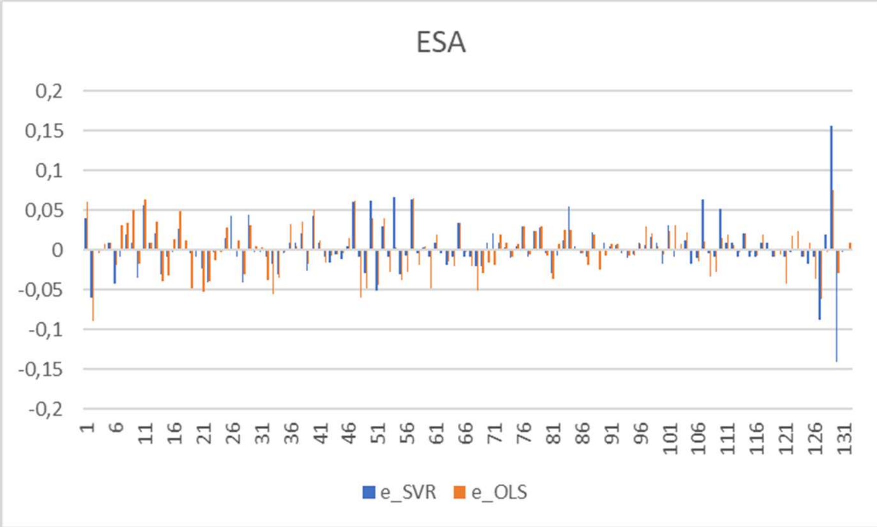

Figure D.27: ESA prediction errors obtained from OLS and SVR
Source: Author's calculations

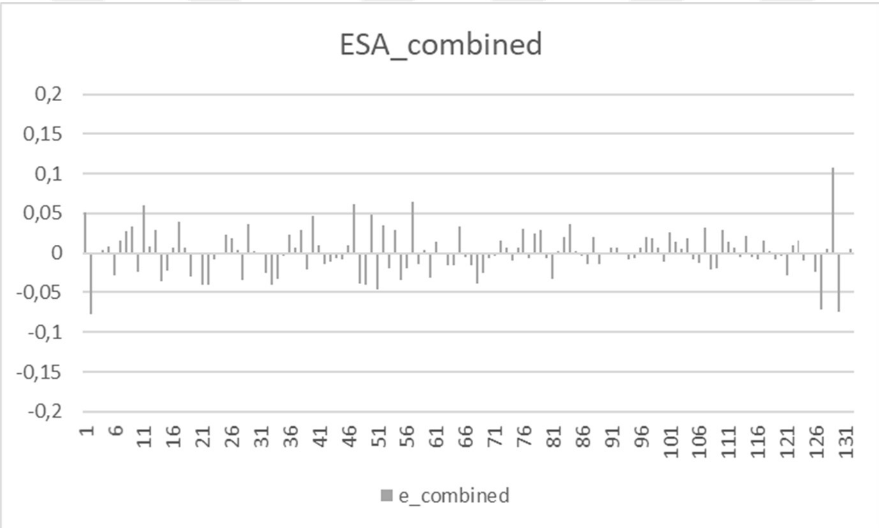

Figure D.28: Errors of combined predictions for ESA
Source: Author's calculations

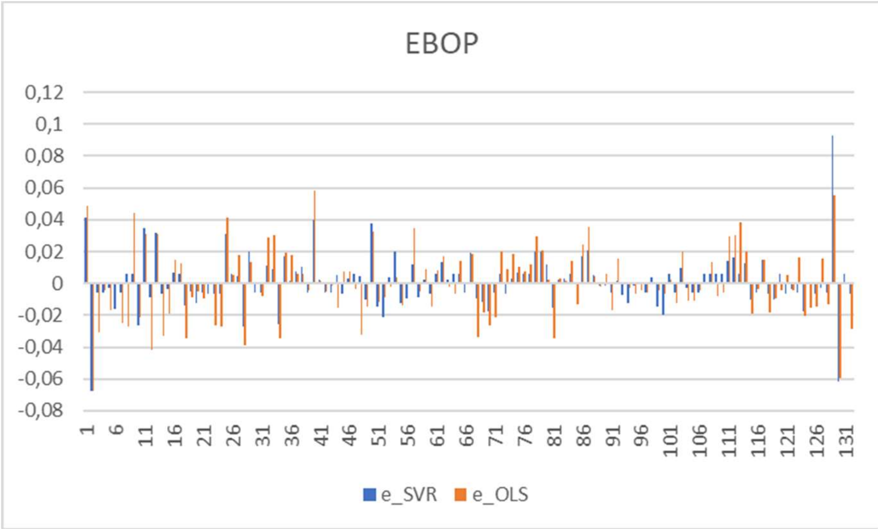

Figure D.29: EBOP prediction errors obtained from OLS and SVR

Source: Author's calculations

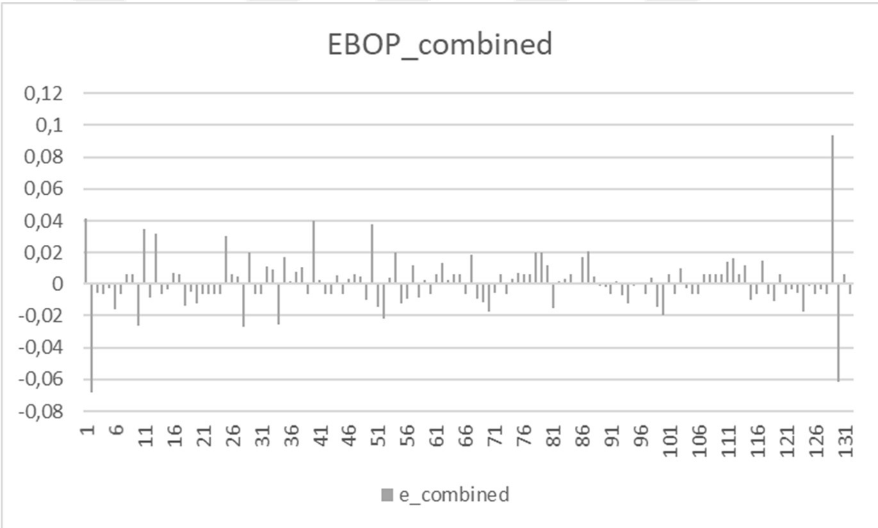

Figure D.30: Errors of combined predictions for EBOP

Source: Author's calculations

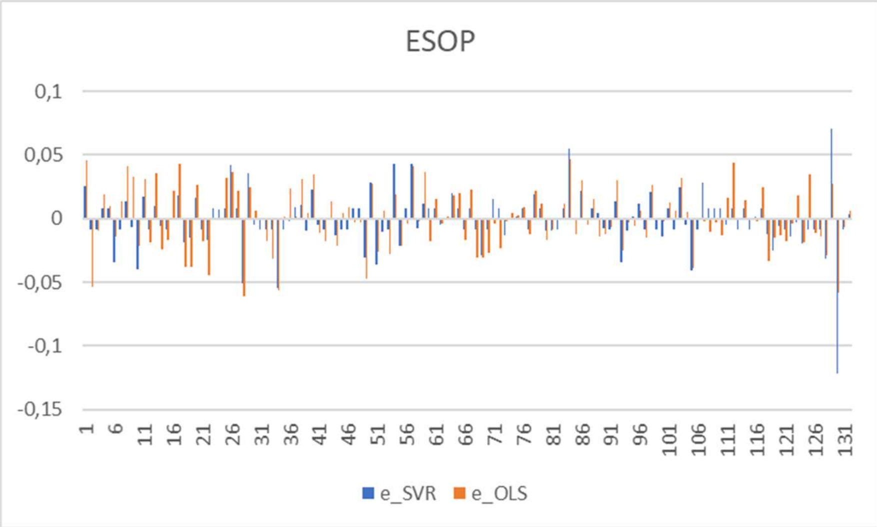

Figure D.31: ESOP prediction errors obtained from OLS and SVR

Source: Author’s calculations

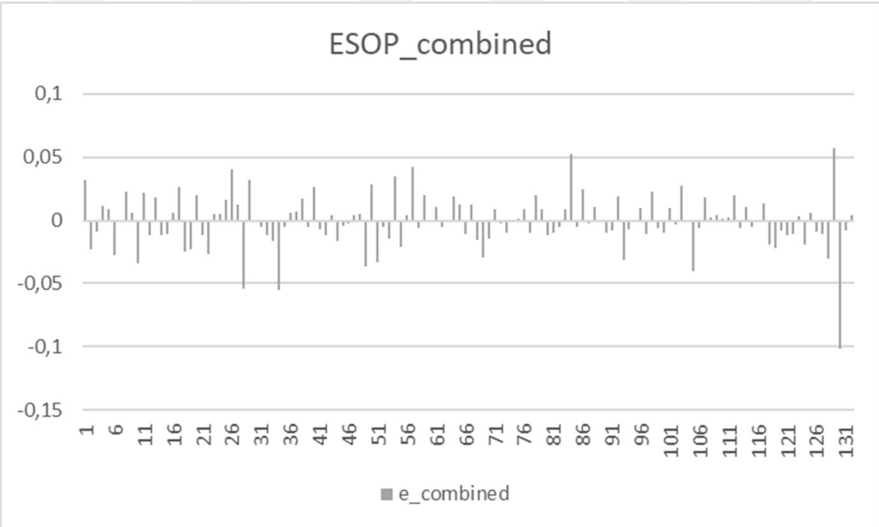

Figure D.32: Errors of combined predictions for ESOP

Source: Author’s calculations

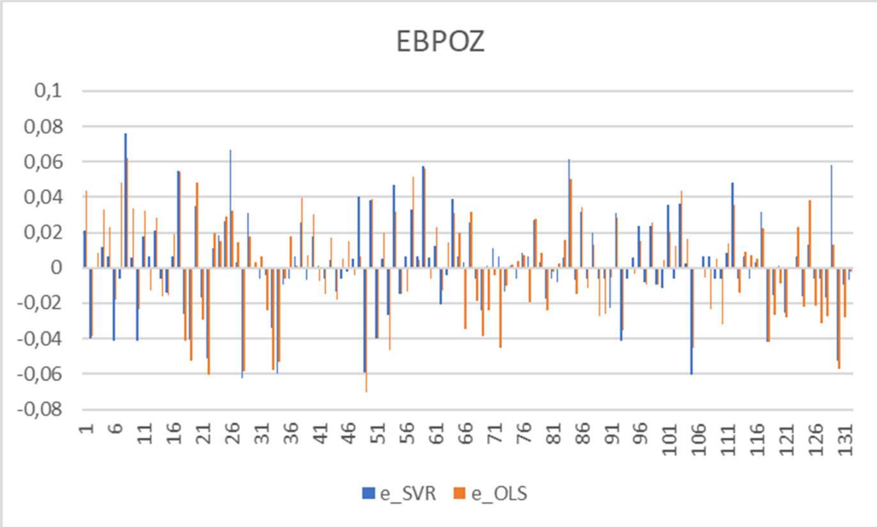

Figure D.33: EBPOZ prediction errors obtained from OLS and SVR
Source: Author's calculations

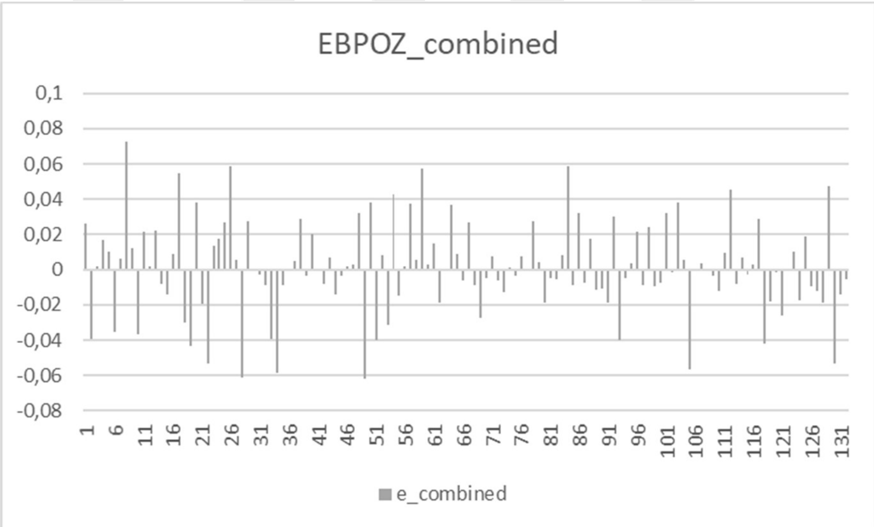

Figure D.34: Errors of combined predictions for EBPOZ
Source: Author's calculations

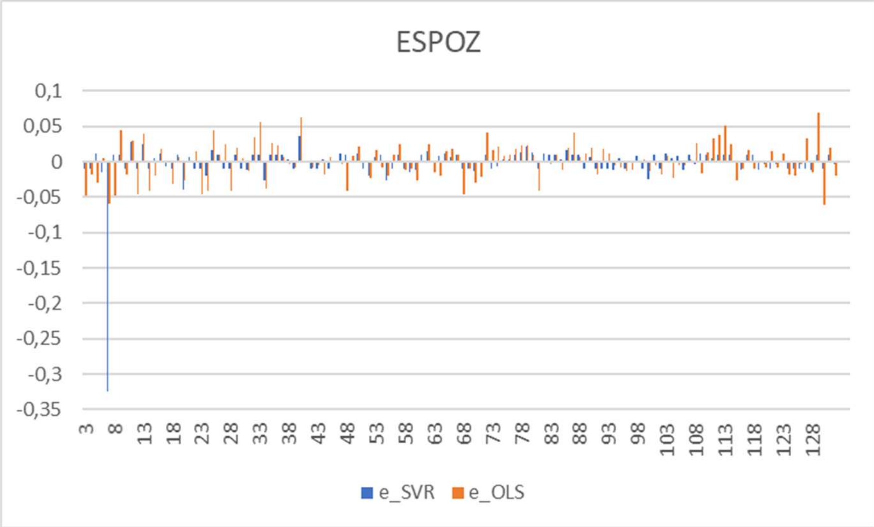

Figure D.35: ESPOZ prediction errors obtained from OLS and SVR
Source: Author’s calculations

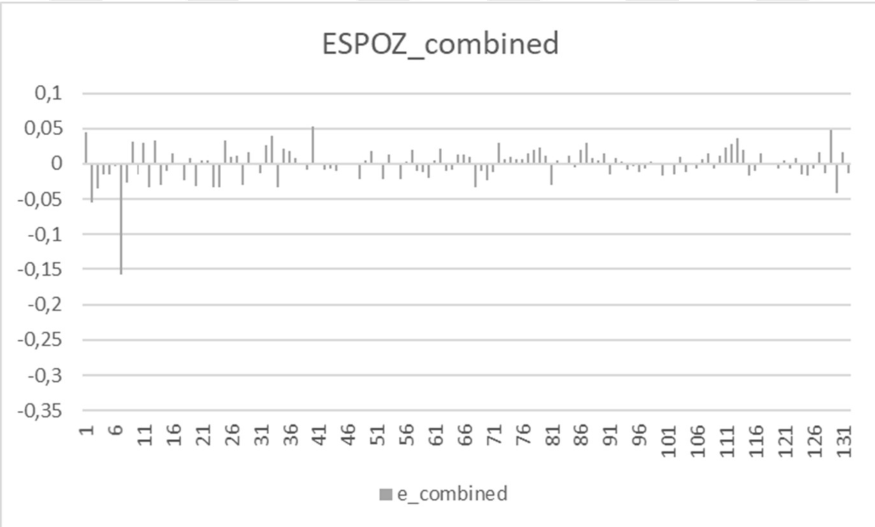

Figure D.36: Errors of combined predictions for ESPOZ
Source: Author’s calculations