



FATİH UNIVERSITY

The Graduate School of Sciences and Engineering

**Master of Science in
Physics**

**DEVELOPMENT OF A MODULAR VERDET
CONSTANT MEASUREMENT SYSTEM FOR NOVEL
MATERIALS**

by

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**M.S.
2015**

October 2015



**DEVELOPMENT OF A MODULAR VERDET CONSTANT
MEASUREMENT SYSTEM FOR NOVEL MATERIALS**

by

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A thesis submitted to

The Graduate School of Sciences and Engineering

of

Fatih University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Physics

October 2015

Istanbul, Turkey

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M.S. Thesis – Physics

October 2015

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ABSTRACT

In this thesis our major aim was to build a modular system where we can measure the Verdet constant for a transparent liquid for some certain laser wavelengths. Verdet constant is well known to be wavelength dependent and temperature dependent. Detecting or obtaining high Verdet constant materials are important for some possible applications in optical electronics. Having materials with large Verdet constants may lead us to have more sensitive detectors of magnetic fields and currents as well. We design and implement a device to do the necessary measurements in a practical way.

Keywords: Optical Communication Systems, Detectors, Lasers, Faraday Effect, Polarimetry

BAZI MATERYALLER İÇİN MODÜLER VERDET SABİTİ ÖLÇÜM SİSTEMİ GELİŞTİRİLMESİ

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Yüksek Lisans Tezi – Fizik

Ekim 2015

Tez Danışmanı: Yard. Doç. Dr. Osman Çağlar AKIN

ÖZ

Bu tezde temel amacımız bazı belirli lazer dalga boylarında saydam herhangi bir sıvı için Verdet sabitlerini belirlememizi sağlayacak modüler bir sistem kurmaktır. Verdet sabitinin sıcaklık ve dalgaboyuna bağlı olduğu iyi bilinmektedir. Yüksek Verdet sabitine sahip materyalleri tespit etme ya da elde etmek opto-elektronikte bazı olası uygulamalar bakımından önem taşımaktadır. Yüksek Verdet sabitine sahip materyaller elde etmek daha hassas manyetik alan ve akım sensörlerinin geliştirilmesine de yol açabilir. Gerekli ölçümleri pratik bir şekilde almamızı sağlayacak bir aygıt tasarladık ve gerçekleştirdik.

Anahtar Kelimeler: Optik haberleşme sistemler, Dedektörler, Lazerler, Faraday etkisi, Polarimetri

To my wife and parents

ACKNOWLEDGEMENT

I express sincere appreciation to Assist. Prof. Osman Çağlar AKIN for his time, guidance and insight throughout the research.

Thanks go to the other faculty members, Prof. Mustafa KUMRU and Assoc. Prof. Mehmet Burak YILMAZ, for their valuable suggestions and comments. The technical assistance of Erdin SARP KAYA and Hasan SİL are gratefully acknowledged.

I express my sincere gratitude to Assoc. Prof. Ramazan ÖZTÜRK and İrfan İLHAN, for lending some of their instruments for my measurements.

I express my thanks and appreciation to my family for their understanding, motivation and patience. Lastly, but in no sense the least, I am thankful to all colleagues and friends who made my stay at the university memorable with valuable experiences.

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL/ABBREVIATION

$T \cdot m/A$	Tesla meter per ampere
ϵ_0	Permittivity of free space
μ_0	Permeability of free space
f_{BW}	Bandwidth
C	Capacitance
R	Reflectance
T	Transmittance
B	Magnetic field
I	Current
$\vec{L}$	Interaction length vector
θ	Rotation angle
$\vec{S}$	Poynting vector
c	Speed of light
n	Number of windings
P	Power
u	Energy density
V	Voltage

CHAPTER 1

LITARATURE SURVEY ON FARADAY EFFECT

1.1 INTRODUCTION

In this thesis our aim is to construct a set up with which we can measure the Verdet constants of various transparent liquids regardless of being pure, colloidal or a mixture.

The interaction is based on Faraday effect, which provides an interplay of an applied external magnetic field with the polarization properties of propagating light.

Faraday Effect, as we will cover in the next chapter, is well known to exist for a very long time. Some materials reveal an interesting property when subject to magnetic fields parallel to the propagating electromagnetic fields within them, they rotate the direction of polarization of the incoming polarized light. The effect has been observed in nature in further galaxies Faraday effect is also used in one of the most abundant molecules on the surface of the earth, water, and a cell used as a modulator as well with relatively high voltages driving the liquid Faraday modulator cell [1].

The Verdet constant of water is relatively very small like most dielectric materials and the effect is small, so the measurement system had to use multiple passes to obtain a larger interaction length and very high currents and voltages to obtain significant magnetic fields. It is also possible to use more advanced electro-optical equipment for the measurement of Verdet constant for materials of smaller quantity, and hence a much small interaction length, but then since the effect is very small, the applications are promising only for those materials with extreme Verdet constants.

Shakir and coworkers [2] (2013) discusses the case where they set up an experiment to measure the Verdet constant of virgin olive oil where they came up with

98.68 rad/T.m for 650 nm, and for this wavelength the Verdet constant for commercially available materials although more expensive- would be like 119 rad/T.m. is also noteworthy that for virgin olive oil the Verdet constant for HeNe laser at 632.8 nm is measured as 33.37 rad/T.m. They detected the constant for 804.3 nm to be pretty small and for 780 nm it is missing. So we aim to find that value too for virgin olive oil.

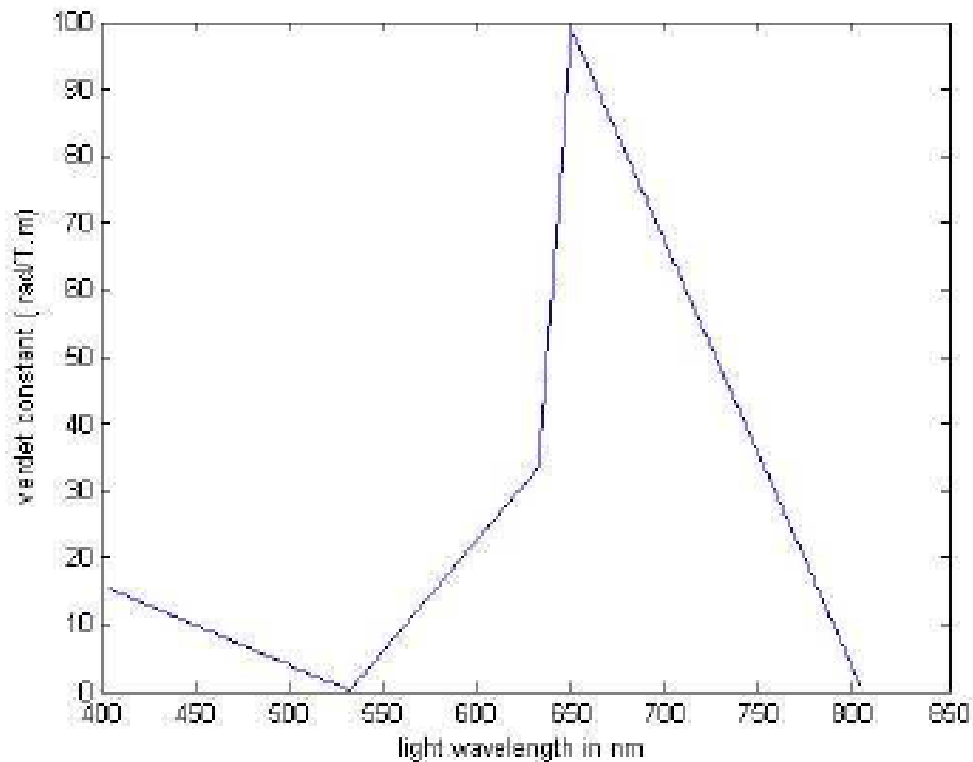


Figure 1.1 Verdet constants of virgin olive oil in different wavelengths [2].

It is seen from Figure. the Verdet constant of the virgin olive oil reaches the maximum value of 650 nm wavelength. According to the result Verdet constant of the virgin olive oil at 650 nm wavelength close to the Verdet constant of Terbium Gallium Garnet (TGG) crystal.

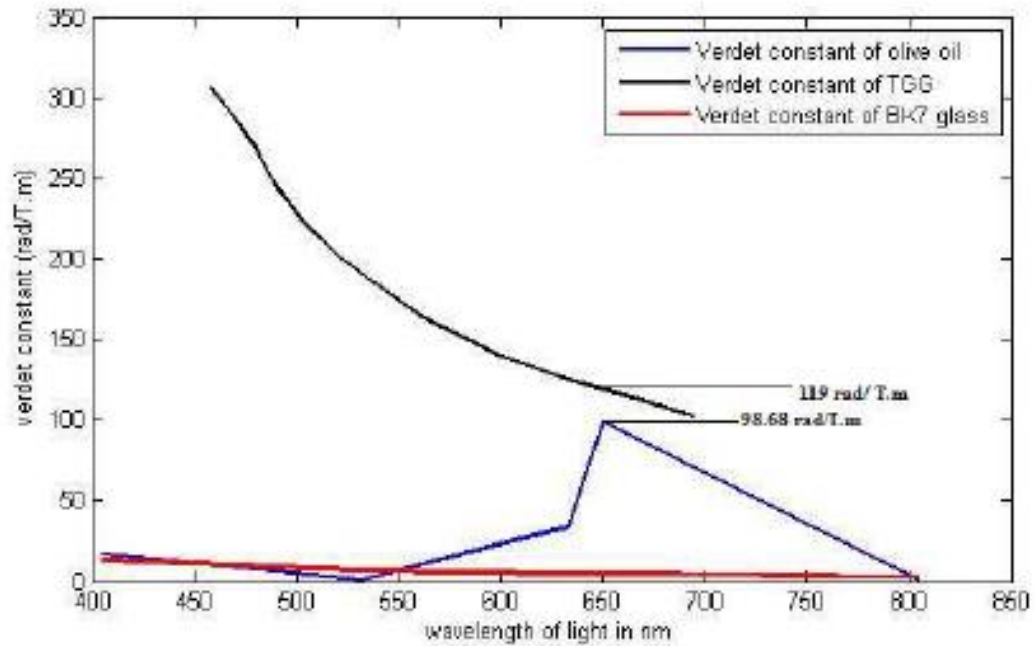


Figure 1.2 Verdet constant for the virgin olive oil in comparison with TGG and BK7 glass [3].

In 2010 Iris Crassee et.al. [3] widely used the Faraday effect and the magneto-optical Kerr effect in data storage, optical communication, and computing. They suggested that the use graphene for fast tunable ultrathin magneto-optical devices should be explored. Indeed, the possibility of easy and fast ambipolar doping together with a strong Faraday effect in a wide frequency range is a unique combination not present in other known materials.

Abu-Taha et.al. [4] (2013) attempted at measuring the Verdet constants of various vegetable oils. They made the measurements with a light source at 525 nm, and they claim relying on their results, they could distinguish between oils with additives, the amount of additives, their geographical origin and their life time on the bench. We want to extend these measurements for various regions of the spectrum.

When it comes to high Verdet constant materials it is noticed that mesogenic organic molecules have pretty high Verdet constants by Vardendriessche et.al. (2013) [5]. Also note that this measurement is done by the experimental set up of Thierry

Verbiest, which we aim at the construction of one at our University as a continuation of this thesis.

It is also noteworthy that Faraday effect is used as a modulator for the technology of superconductivity in the work of Sobolewski et.al. (2011) [6]

Some researchers like Weller (2012) [7] proposed using TGG (Terbium Galium Garnett), YIG (Yttrium Indium Garnett) ve Rubidium gas for the development of optical isolators. It is important to have a practical active material to use as an optical isolator, one of our aims is to develop such materials during the course of this thesis.

Bochun Wu [8] (2012) investigates a measurement system with city electricity and notices that second harmonics inevitably show up at the FFT spectrum with one single detector scheme, which is the common method. We propose a double detector scheme will solve this problem.

It is common practice to use Faraday effect for stand off measurement of currents with non-interfering techniques using fibers and such apparatus Barszack et.al. (2011) [9]

CHAPTER 2

FARADAY EFFECT AND VERDET CONSTANT

2.1 INTRODUCTION

In 1845 Michael Faraday discovered that an interaction between magnetic field and light. According to this discovery a magnetic field in the direction of propagation of a light beam in a transparent medium produces a circular birefringence. So for the first time he established that there is a relationship between optics and electromagnetism. As a result of his many years of research he has discovered the Faraday Effect. [10]

With the discovery of new mathematical methods in the field by William Thomson (Lord Kelvin), Faraday attempted to research how light passes through an electrolyte was affected. He repeated the experiments but he could not get result. Then, on September 13, 1845, he put a piece of glass in front of an electromagnet and he observed that a polarized light through the glass. Thus, he realized that the electromagnet played a vital role in the polarization of light. When he changed the direction of electromagnet the polarization of light had changed. According to this experiment light was affected by applied magnetic forces. This “magneto-optical effect” was later named the Faraday Effect. [11]

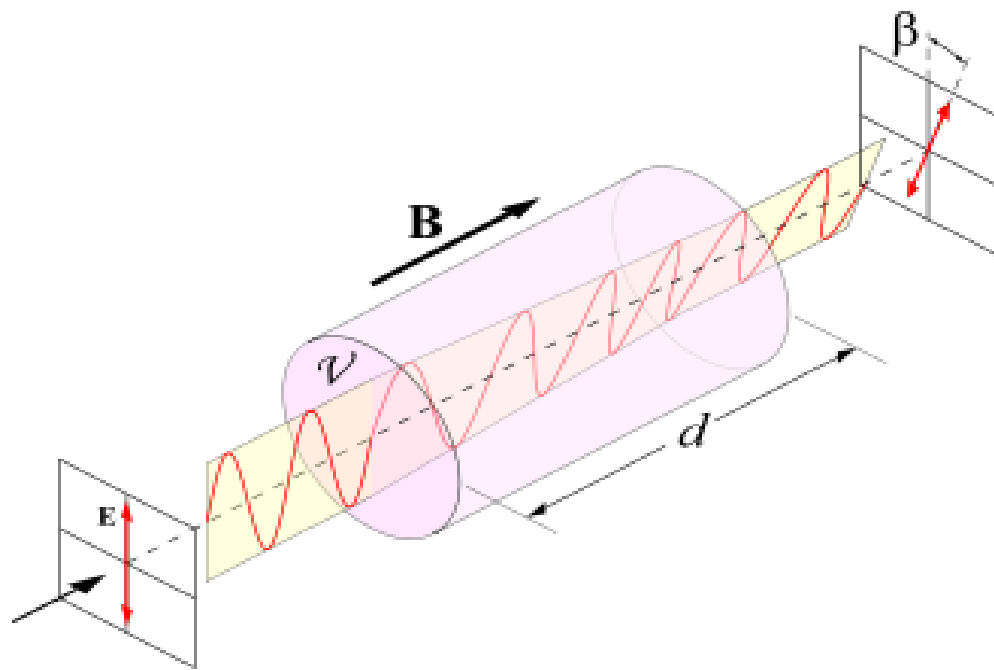


Figure 2.1 Rotation of the direction of Polarization due to Faraday effect.

2.2 BACKGROUND

BC in 800 years the Greeks were aware of magnetism by observing that the iron particles attracted some stone. In 1269 Fierre de Maricourt has achieved a map of the direction taken by the needle by placing needles at various points of natural magnet surface. He saw that these directions create lines across the diameter of the sphere. He gave the name of the magnet poles on this point. In subsequent experiments, whatever the shape of each magnet has been seen having two poles as north and south. These poles are checked impact force upon each other like electric charges. In other words, similar poles repel each other and opposites attract each other. In 1600 William Gilbert also made using various materials for the experiment. He suggested the idea that the world is constantly a huge magnet by using the fact that the needle of a compass directed towards a certain goal. In 1750, John Michell (1724-1793) found that magnetic poles create an attractive or repulsive force on each other and used a torsion balance to indicate that the forces are inversely proportional to the square of distance between the poles. A similar load with a force between two electric force and the two magnetic poles but there is a difference between them. There is a single electric charge (electron or proton) but not a single magnetic pole. That is, the magnetic poles always exist in pairs.

Luigi Galvani discovered the Animal electricity theory. [12] This theory is still applicable today for the patients in the hospitals. In 1786, Luigi Galvani, when a metal blades touch the frog legs found dead leg twitched violently. He thought that it must include an electrical current in the frog muscle. The date of 1792 another Italian scientist Alessandro Volta proved the existence of electricity in animals and concluded that the same idea. [13]

Volta showed that electricity occurs when two different metal from moisture. This is took him to invention of the first electric battery made of damp cardboard, copper and zinc. [14] In this way, he made an important contribution to the electrical science. He showed that electricity can be carried through from one place to another place by using wires.

The relationship between electricity and magnetism were discovered in 1819 by Danish scientist Hans Oersted. [15] During a demonstration of Oersted current passing through a wire near the plant he has seen it deflects compass needles. Shortly after, Andre Ampere (1775-1836) achieved quantitatively laws of magnetic force between the

current carrying conductor. He also suggested that due to the current loops in the molecular size of all magnetic phenomena. This idea forms the basis of the modern theory of magnetism. Today, still known by the name of Oersted's Law. 1820 s, Michael Faraday and Joseph Henry independently of him (1797-1878), also showed another relationship between electric current and magnetism. They showed that can create an electrical current in a circuit by moving a magnet near the circuit. These observations suggest that a changing magnetic field induces an electric field. Faraday effect of light is discovered in 1845 by Michael Faraday. Linear polarization with the same intensity and the same frequency can be regarded as the overlap of two circular polarized light waves. Years later, Maxwell's theoretical work showed that the changing magnetic field creates an electric field. A short time later, Heinrich Hertz confirmed the predictions of Maxwell by creating electromagnetic waves in laboratory. [16]

Étienne-Louis Malus (1775–1812) was a French mathematician and physicist who worked about the light. He explored geometrical systems called ray systems, depending on the geometry of Julius Plucker. Malus tried to describe the relationship between reflection and polarization angle. Although he found the right relationship for water, but it was unable to find the correct refractive index between the surface and the glass. Malus is remembered for Malus' law.

$$I = I_0 \cdot \cos^2(\theta_i) \quad (2.1)$$

where, I_0 is the initial intensity, and θ_i is the angle between the axis of the polarizer and the light's initial polarization direction.

By 1845, of people who use polarized light to investigate the properties of the transparent material was known as Fresnel [17], Malus and the others. Michael Faraday believed that light is an electromagnetic wave, and it is affected by magnetic forces. He spent his years looking for an evidence of electric forces affecting the polarization of light. However, Faraday's experimental methods was not precise enough and he did not reach. After thirty years John Kerr was measured the effect known as 'Kerr Effect'.

Faraday, tried to look the light through the various substances at the effects of magnetic force. According to Faraday, the applied magnetic force moving towards the light beam passes through the glass at an angle rotates proportionally. [18]

Unpolarized beam of light at all angles considered possible comprising a uniform blend of linear polarization. Since the θ – average value of $\cos^2\theta$ is $1/2$ the I_0 becomes

$$\frac{I}{I_0} = \frac{1}{2} \quad (2.2)$$

2.3 FARADAY ROTATION AND VERDET CONSTANT

Verdet constant is a materials property and a quantitative measurement of the Faraday rotation ability of the material. The angle of rotation θ is expressed as $\theta = VBL = \pi(\Delta n/\lambda)L$ where Δn is the magnitude of circular birefringence. (the difference between the refractive indices for left and right circularly polarized light in the medium) [19]

Generally, Faraday rotation is at its strongest in inorganic substances containing paramagnetic ions or in magnetic and superparamagnetic materials. Unfortunately, these materials are often very expensive and difficult to process. Nevertheless, a significant advantage of organic materials would be their ease of processing, limited weight. Organic diamagnetic materials in contrast saturate at for larger magnetic fields and additionally are relatively to temperature changes which is beneficial for sensing applications. An additional advantage of organic materials over inorganic garnets for Faraday rotation is their processability and flexibility. Aliphatic organic compounds are also exhibiting Faraday rotation. In that Pioneer report Stefan Vandendriessche [20] have found Faraday rotation and its dispersion. [21] They measured and calculated the Faraday rotation and its dispersion in the 400-800 nm wavelength range for organic liquids.

Faraday rotation is important for optical isolators, current sensors, integrated communication optics, and detection of biomagnetic fields. Isolators are invariably used to improve the frequency stability of MW generators such as klystron and magnetron.

Verdet constant depends on the magnetic field and its proportional to the magnetic field. In an experiment conducted in 2003, Thomas G. Spears measured the Verdet

constant of the light flint glass and reached similar conclusions. His in this experiment, the angle of rotation and the magnetic field shows a proportional. [22]

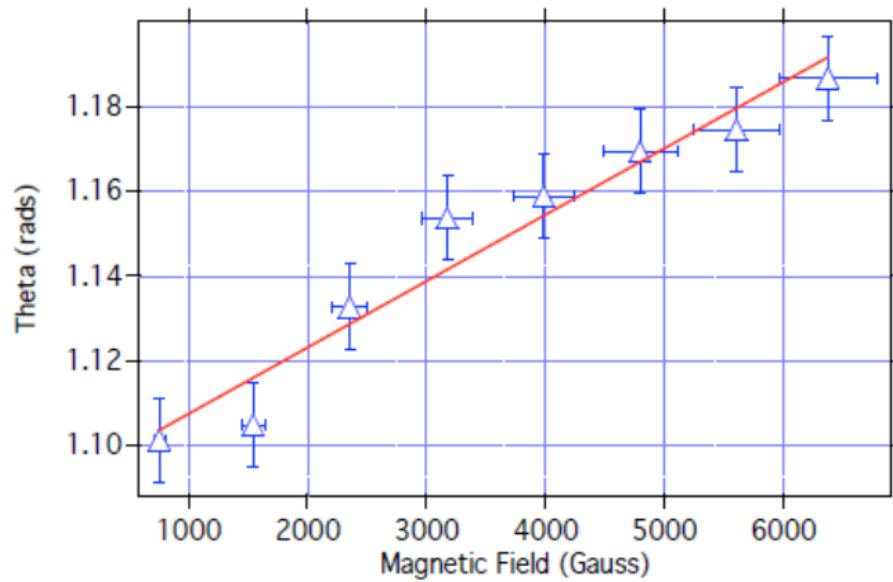


Figure 2.2 A plot of the angle of rotation of the plane of polarization vs. the magnetic field with an associated linear fit [22].

Verdet constant depends on the wavelength. In visible light wavelengths, a graph showing the wavelength is inversely proportional to the Verdet constant. If wave length decreases gradually increase the Verdet constant that we can see in the chart below.

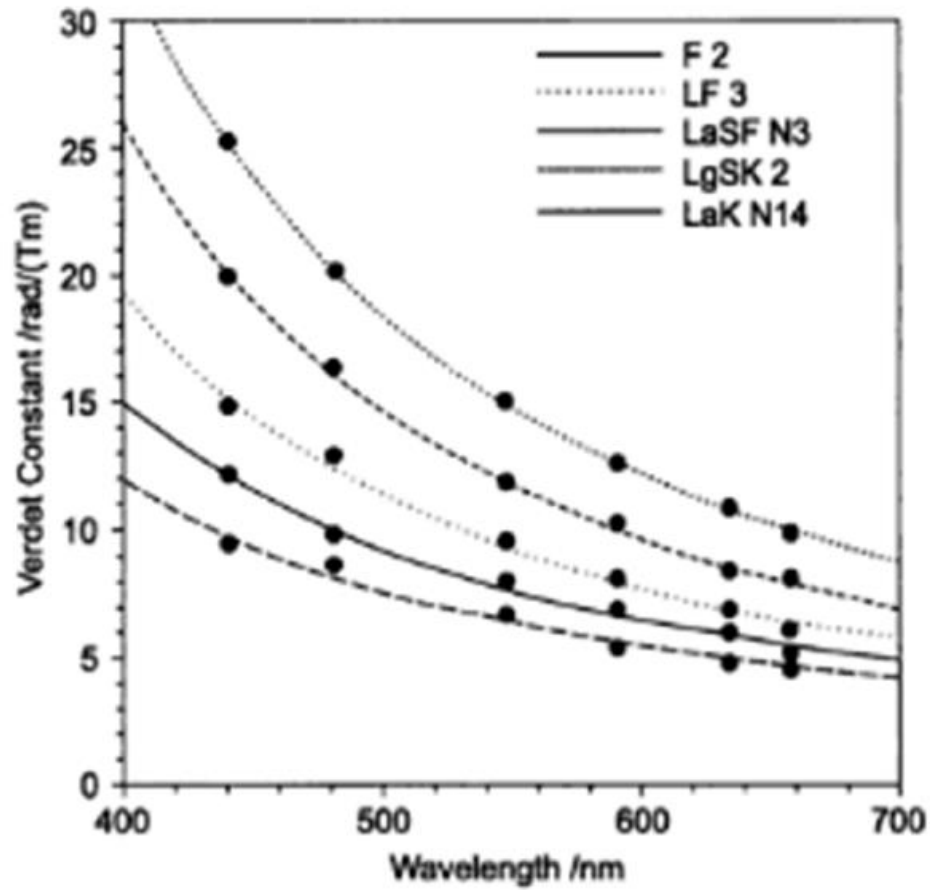


Figure 2.3 Verdet constant due to the visible wavelengths [22].

Typically Verdet constant depends on the wavelength of the light and has a value of a few $\text{rad}/\text{T} \cdot \text{m}$. For the solenoid we used in this lab, the magnetic field at the center is approximately 5 mT, and our material, a full of virgin olive oil tube, is 40 cm long. For a current of 0-1,5 A, we expect a rotation a few $\times 10^{-4}$ radians. This is a pretty small angle and it is detected by using the optical power meter.

2.4 EXPECTATION OF ROTATION ANGLE

$$\theta[rad] = V \cdot B \cdot L \quad (2.3)$$

V is the Verdet constant, B is magnetic field and L is interaction length of the solenoid.

By assuming the following values for the wavelength and the verdet constant

$$\lambda = 633 \text{ nm}$$

$$V = 33.37 \frac{rad}{T.m}$$

$$B_{max} \sim 5500 \mu T = 0,0055 T = 5.5 \times 10^{-3} T \quad (2.4)$$

$$L = 40 \text{ cm} = 0.4 \text{ meters}$$

$$\theta[rad] = 33.37 \frac{rad}{T.m} \times 5.5 \times 10^{-3} T \times 0.4 m \quad (2.5)$$

$$\theta = 33.37 \times 5.5 \times 4 \times 10^{-4} rad = 33.37 \times 22 \times 10^{-4} rad \quad (2.6)$$

$$1 rad = 57.3^\circ \quad (2.7)$$

$$\theta = (33.37 \times 22 \times 57.3 \times 10^{-4})^\circ = 4.2066222^\circ \quad (2.8)$$

Assuming Olive oil $V = 33.37 \frac{rad}{T.m}$ with HeNe laser this value reads $\theta = 4.2066222^\circ$ (see Table 2.1 below)

Table 2.1 The Verdet constants of some materials for $\lambda=632.8$ nm [23].

Medium	V [rad/T.m]
Olive oil	33.37
Bromid of potassium	12.37
Cooking salt	10.04
Calcite	5.52
Quarz	4.84
Water	3.78
Acetone (20°C)	3.25
Ethyl alcohol (25°C)	3.23

The essential feature of the Faraday effect is its irreversibility, i.e. the rotation of light wave polarization plane propagating in a medium being subjected to a magnetic field with a consistent and opposite propagation direction with respect to the field will not get compensated but it will be doubled. So reverse the direction of propagation of light, but the rotation of the polarization is still in the same sense.

2.5 OPTICAL PHENOMENA

2.5.1 Polarization by reflection

We noticed that total internal reflection of polarized light, or reflection of scattered light from dielectric surfaces may indeed cause a rotation of the polarization axis. For clarification of why this is an important phenomenon, we will overview the subject of polarization by reflection, and then we will discuss where it becomes important to avoid it in the experimental procedures.

Augustin Jean Fresnel is a scientist who derived a set of expressions to calculate the amount of light reflected and transmitted at an interface. [24] Considering the electric field vector of the incident light, we can express the electric fields of the

incident, reflected and transmitted waves (in other words $E_i(r, t)$, $E_r(r, t)$ and $E_t(r, t)$). These expressions are the laws of reflection and refraction. Amplitudes of the incident, reflected and transmitted waves, shown as follows : E_{0i} , E_{0r} and E_{0t} , we find using the boundary conditions of the electric field vectors that

$$r_{perpendicular} \equiv \left(\frac{E_{0r}}{E_{0i}} \right) = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \quad (2.9)$$

$$t_{perpendicular} \equiv \left(\frac{E_{0t}}{E_{0i}} \right) = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} \quad (2.10)$$

As the expressions for the amplitude coefficients of reflection and transmission, respectively. When the electric field is parallel to the plane of incidence, we find

$$r_{parallel} \equiv \left(\frac{E_{0r}}{E_{0i}} \right) = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t - n_t \cos \theta_i} \quad (2.11)$$

$$t_{parallel} \equiv \left(\frac{E_{0t}}{E_{0i}} \right) = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i} \quad (2.12)$$

In addition to these field amplitude ratios, one can define the ratio of the reflected (reflectance, R) and the transmitted (transmittance, T) to the incident flux (power). In other words,

$$R = \left(\frac{E_{0r}}{E_{0i}} \right)^2 = r^2 \quad (2.13)$$

and

$$T = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} \left(\frac{E_{0t}}{E_{0i}} \right)^2 = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i} \right) t^2 \quad (2.14)$$

are the reflected and transmitted power coefficients.

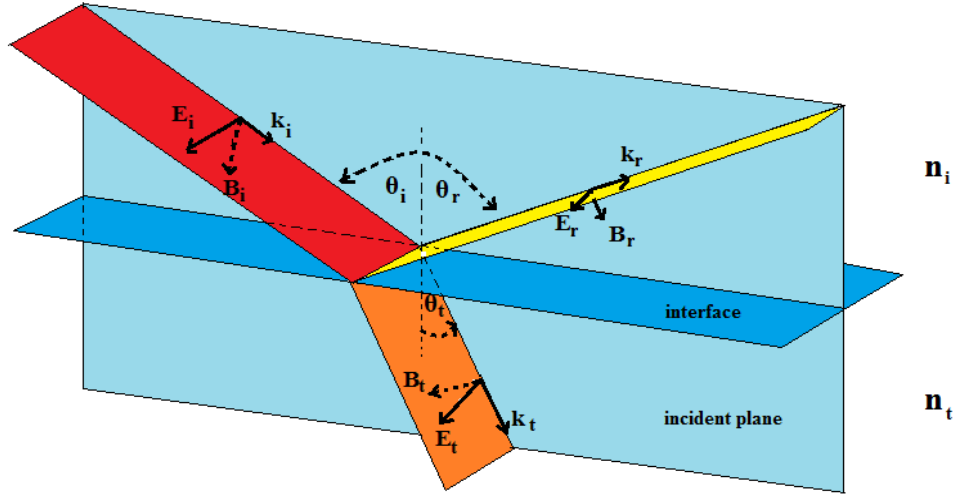


Figure 2.4 Reflected and transmitted light at a dielectric interface.

For the amplitude coefficients of reflection, perpendicular and parallel to the plane of incidence, the reflectance is $R = r^2$.

$$R_{parallel} = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \quad (2.15)$$

$$R_{perpendicular} = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \quad (2.16)$$

These reflectances for external reflection at different values of the incident angle θ_i for an air glass interface are plotted on the below graph in Figure. By using these equations we found:

$$R_n = \frac{1}{2} (R_{parallel} + R_{perpendicular}) \quad (2.17)$$

Observe that parallel reflectance is equal 0 when its denominator equals ∞ ; that is, when $\theta_i + \theta_t = 90^\circ$.

We can get a rigorous expression since Snells Law is valid at the interface as

$$\theta_t = \arcsin\left(\frac{n_i}{n_t} \sin(\theta_i)\right) \quad (2.18)$$

therefore

$$R_{parallel} = \frac{\tan^2(\theta_i - \arcsin(\frac{n_i}{n_t} \sin(\theta_i)))}{\tan^2(\theta_i + \arcsin(\frac{n_i}{n_t} \sin(\theta_i)))} \quad (2.19)$$

$$R_{perpendicular} = \frac{\sin^2(\theta_i - \arcsin(\frac{n_i}{n_t} \sin(\theta_i)))}{\sin^2(\theta_i + \arcsin(\frac{n_i}{n_t} \sin(\theta_i)))} \quad (2.20)$$

Once the refractive indices at the interface is know, we can plot the Reflectances as a function of the incident angle θ_i . The refractive index of Acrylic is known to be 1.49, that of Olive oil is also pretty close to 1.49. For clarification we can plot these functions for $n_i = 1$ and $n_t = 1.5$

```
In[35]:= ni = 1
Out[35]= 1

In[38]:= nt = 1.5
          y[x] = ArcSin[ $\frac{ni}{nt} \star \sin[x]$ ]
Out[38]= 1.5

          ArcSin[0.6666666666666666 Sin[x]]

In[40]:= ArcSin[0.6666666666666666 Sin[x]]
Out[40]= ArcSin[0.666667 Sin[x]]

          Plot[y[x], {x, 0,  $\pi/2$ }]
```

Figure 2.5 Mathematica Plot code for θ_t as a function of θ_i [rad].

We run a Mathematica code to plot the figures, where the horizontal axis is in terms of radian

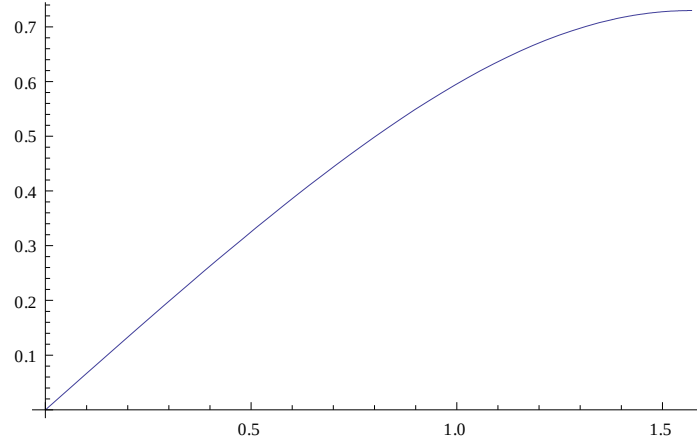


Figure 2.6 Plot of θ_t as a function of θ_i [rad], where $n_i = 1$ and $n_t = 1.5$.

As shown in the Figure 2.7 below for Acrylic-air interface, reflectance coefficients for parallel electric field components to the plane of incidence and $R_{||}$ and perpendicular electric field components $R_{\perp}$ read different values for the same incidence angle, θ_i . This tells us that reflection at dielectric surfaces may themselves be an effective cause of rotation of the plane of polarization, so, one must be very carefull with the reflections, including, total internal reflactions.

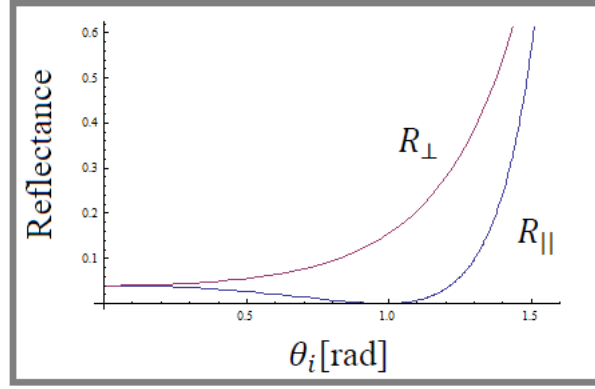


Figure 2.7 Reflectance as a function of the incidence angle for parallel and perpendicular electric field components to the plane of incidence.

2.5.2 Birefringence

We use birefringence as a template to understand what happens with the Faraday effect. Classically, Faraday effect is modelled as the case of circular birefringence, where a linearly polarized light is considered as a superposition of two oppositely rotating circularly polarized light waves. If their indices of refraction were to be different for some reason, we would have a rotation of the polarization axis. We make an overview of a rigorous derivation of the Faraday effect elsewhere in the thesis.

The refractive index is a property of the optical material and the phase velocity of a wavelength is the same in all directions. This can be obtained in the cubic crystals as well as for non crystalline substances such as plastic, unstressed glass, water and air.

Usually, crystals are anisotropic. The atomic binding forces on the electron clouds are different in various directions and as a result the refractive indices are different. We considered here only with uniaxial birefringent crystal such contains a single symmetry axis (actually a direction) known as the optic axis and displays two distinct principal indices of refraction. Calcite crystal, CaCO_3 , is a good example. Because the distribution of atoms is clearly anisotropic as its response to the light. For this reason we chose the Glan-Thompson calcite polarizer to get the ratio of 100 000:1 polarized light. We see in below Figure 2.8 how the light propagates to the uniaxial birefringent material.

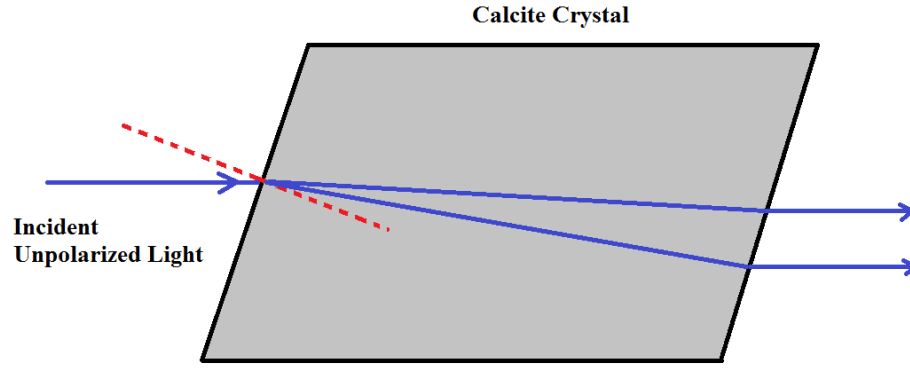


Figure 2.8 Linear birefringence of Calcite Crystal.

2.5.3 Energy and Momentum in Electromagnetic Waves

In this section we make an overview of the wave theory of light concerning the relationship of electric field strength with the Poynting vector and the radiated power, as well as the intensity of light. We want to establish relevant equations between the Intensity and the electric field amplitude, since, with faraday effect what is rotated under an applied magnetic field, is the optical component (the direction of the electric field of the light), but what is measured is the incident power of the light at the detector. [25]

For an electromagnetic wave, the total energy density u is given by

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \quad (2.21)$$

where ϵ_0 and μ_0 are, respectively, the permittivity and permeability of free space. For electromagnetic waves in vacuum, the magnitudes E and B are related by

$$B = \frac{E}{c} = \sqrt{\epsilon_0 \mu_0} E \quad (2.22)$$

We can also express the energy density u in a simple electromagnetic wave in vacuum as

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} (\sqrt{\epsilon_0 \mu_0} E)^2 = \epsilon_0 E^2 \quad (2.23)$$

This shows us the energy density associated with the $\vec{E}$ field in our simple wave is equal to the energy density of the $\vec{B}$ field. In general, magnitude of the electric field is a function of position and time, as for the sinusoidal wave described. Thus the energy density u of the electromagnetic wave depends on position and time.

Considering an area A on a stationary plane, the energy passes through the area A in time dt . The energy flow per unit time per unit area, which we called, S , is

$$S = \frac{1}{A} \frac{dU}{dt} = \epsilon_0 c E^2 \quad (\text{in vacuum}) \quad (2.24)$$

Using the equations $c^2 = \frac{1}{\epsilon_0 \mu_0}$ and $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

$$S = \frac{\epsilon_0}{\sqrt{\epsilon_0 \mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \frac{EB}{\mu_0} \quad (\text{in vacuum}) \quad (2.25)$$

The units of S are energy per unit time per unit area, or power per unit area. The SI unit of S is $1 \text{ J/s} \cdot \text{m}^2$ or 1 W/m^2 .

We can define a vector quantity that describes both the magnitude and direction of the energy flow rate:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad (\text{in vacuum}) \quad (2.26)$$

The vector $\vec{S}$ is called the Poynting vector; it was introduced by the British physicist John Poynting (1852-1914).

2.5.4 Relationship between Poynting vector and Intensity

We can describe electromagnetic waves by means of wave functions

$$y(x, t) = A \cos(kt - wt) \quad (2.27)$$

where $y(x, t)$ is the transverse displacement from its equilibrium position at time t of a point with coordinate x on the string. The quantity A is the amplitude of the wave, w is its angular frequency, equal to 2π times frequency f ; and k is the wave number, equal to

$2\pi/\lambda$ is the wavelength. For an electromagnetic wave the wave functions are $E_y(x, t)$ and $B_z(x, t)$.

$$E_y(x, t) = E_0 \cos(kx - wt) \quad (2.28)$$

$$B_z(x, t) = B_0 \cos(kx - wt) \quad (2.29)$$

(sinusoidal electromagnetic plane wave, propagating in +x direction)

When time $t=0$, the electric and magnetic fields as functions of x are, $\vec{E}(x, t = 0)$ and $\vec{B}(x, t = 0)$. As time goes by, the wave travels with speed c . Electromagnetic wave in vacuum is:

$$E_0 = cB_0 \quad (2.30)$$

The magnetic field has the form $B = B_0 \cos(kx - wt)$ and therefore

$$S = c^2 \epsilon_0 E_0 \times B_0 \cos^2(kx - wt) \quad (2.31)$$

$$\langle S \rangle = c^2 \epsilon_0 |E_0 \times B_0| \langle \cos^2(kx - wt) \rangle \quad (2.32)$$

Calculating the average over a time interval of length T , we find:

$$\langle \cos^2(kx - wt) \rangle = \frac{1}{T} \int_t^{t+T} \cos^2(kx - wt') dt' \quad (2.33)$$

$$= \frac{1}{2} - \frac{1}{4wt} \{ \sin[2kx - 2w(t + T)] - \sin 2(kx - wt) \}$$

When $wT \gg 1$,

$$\langle \cos^2(kx - wt) \rangle = 1/2 \quad (2.34)$$

Consequently, since $E_0 = cB_0$ intensity of light is calculated

$$I = \frac{c\epsilon_0}{2} E_0^2 \quad (2.35)$$

or

$$I = c\epsilon_0 \langle E^2 \rangle \quad (2.36)$$

This proves our point that the intensity, as well as the detected power at the detector is proportional to the square of the strength of the incident electric field component of the light.

2.6 POLARIZATION

2.6.1 Polarization

Polarization is a property of waves that can oscillate along more than one orientation. Electromagnetic waves (such as light) and gravity waves can be polarized. We need a polarizer to obtain polarized light. The polarizer is an optical filter or device that passes light of a specific polarization waves of other polarizations.

2.6.2 Polarizing Filter

A polarizing filter is often placed in front of the lens to darken skies, manage reflections, or suppress glare from the surface of water. Since reflections tend to be at least partially linearly polarized, a linear polarizer can be used to change the balance of the light. The direction of rotation of the filter is set in an artistic manner. [26]

2.6.3 Polarization by scattering

When viewed with a rotated polaroid to the sky, the low and high density portion of blue color can be seen. Molecules in Earth's atmosphere changes the direction (scattering) of sunlight. The incident sunlight is unpolarized but Earth's atmosphere ensures that light is polarized.

2.6.4 Types of Polarization

In the leftmost figure below, x and y components of the electric field are exactly in phase. Consequently, the polarization along a specific direction in the x-y plane. A single line in the vector plane progresses as different from the other two, in this

particular case this is called linear polarization. Most of the polarizing filters produce linear polarization from unpolarized light.

In the middle figure, the x and y components still have the same amplitude but they are 90 degrees out of phase. In this case the electric vector traces out a circle in the plane, this is circular polarization.

In the elliptical polarization there is more general case with the two components out of phase by a different amount and with different amplitudes. This is called elliptical polarization because the electric vector traces out an ellipse. This is shown in the Figure on the right below.

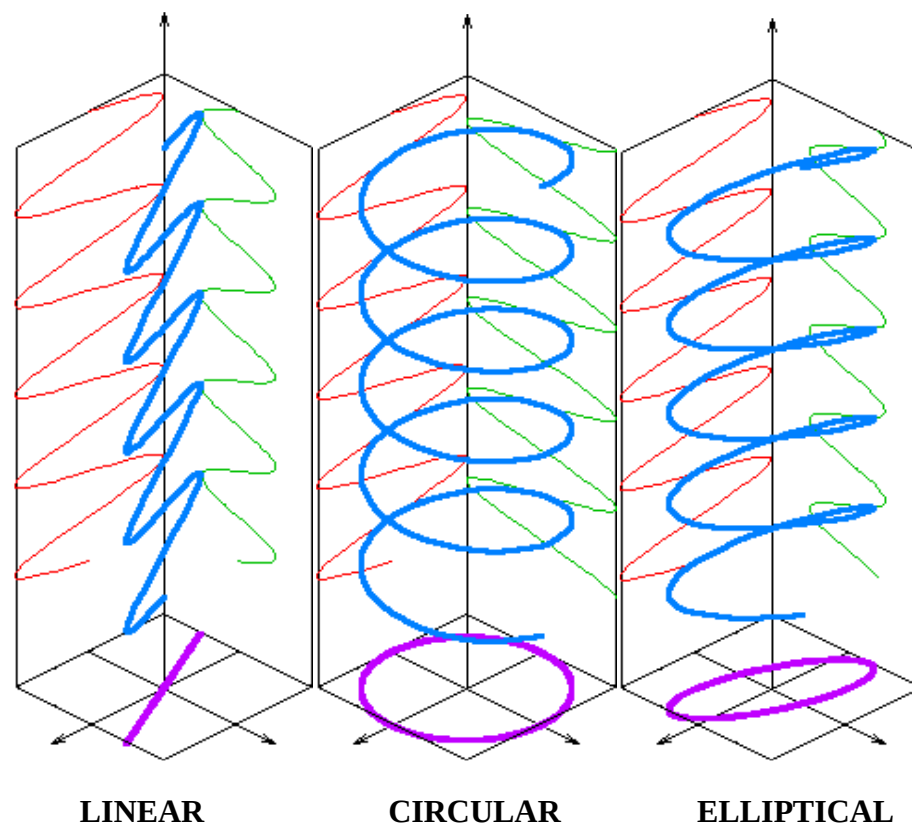


Figure 2.9 Types of Polarizations [27].

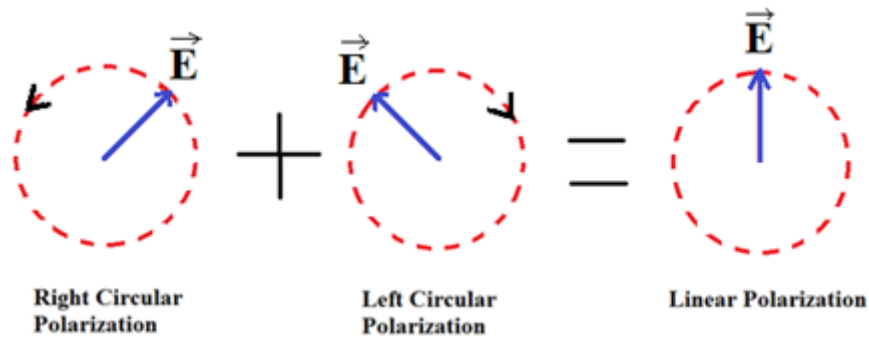


Figure 2.10 Linear Polarization.

2.7 EXPLANATION OF FARADAY EFFECT

2.7.1 The Classical Explanation of Faraday Effect

In this section we give a brief explanation standing for the reason for Faraday Effect within the realm of classical physics. We start with considering an electron rotating on a radius r .

2.7.2 Larmor Frequency

Suppose an electron is rotating counter-clockwise as shown in the Figure 2.11 below.

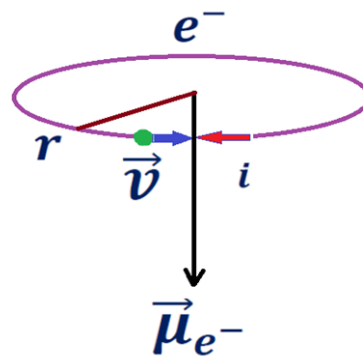


Figure 2.11 Electron e^- rotating counterclockwise about a vertical axis on a radius r , its current i and magnetic moment vector $\vec{\mu}_{e^-}$.

The area of the circle it circulates is $A = \pi r^2$ and since it is an electron with a negative charge, the current is in the opposite direction to the direction of the velocity. Righthand rule gives the magnetic moment vector to be downwards.

$$|\vec{\mu}_{e^-}| = i A \quad (2.37)$$

$$|\vec{\mu}_{e^-}| = \pi r^2 i \quad (2.38)$$

However, the angular momentum vector is in the exact opposite direction for the case of an electron.

$$\vec{L} = \vec{r} \times \vec{p} \quad (2.39)$$

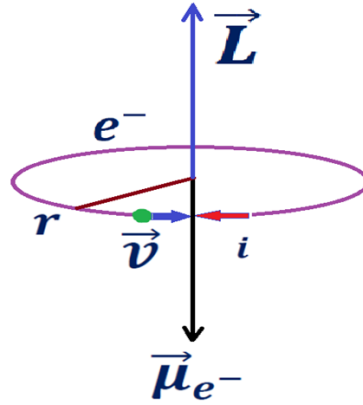


Figure 2.12 Angular momentum vector and magnetic moment of an electron.

$$\vec{L} = m_{e^-} \vec{r} \times \vec{v} \quad (2.40)$$

$$|\vec{L}| = m_{e^-} r v \quad (2.41)$$

$$|\vec{L}| = m_{e^-} r^2 \omega \quad (2.42)$$

By definition the current is

$$i = \frac{dQ}{dt} = e^- v = \frac{e^- \omega}{2\pi} \quad (2.43)$$

If the magnetic moment is placed inside a uniform magnetic field as shown in the Figure 2.13, there will be a torque on the magnetic moment due to the applied magnetic field.

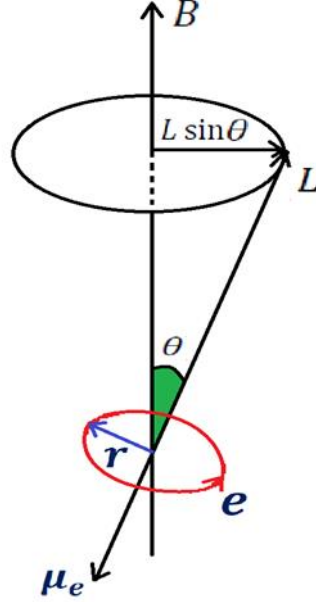


Figure 2.13 Precession of the magnetic moment of an electron in an applied magnetic field [28].

Torque on the dipole by the $\vec{B}$ field is

$$\vec{\tau} = \vec{\mu}_e \times \vec{B} \quad (2.44)$$

which implies

$$|\vec{\tau}| = |\vec{\mu}_e| |\vec{B}| \sin(\theta) \quad (2.45)$$

The angular impulse by the torque is

$$\overline{dL} = \vec{\tau} dt \quad (2.46)$$

The angle of rotation

$$d\phi = \frac{dL}{L'} \quad (2.47)$$

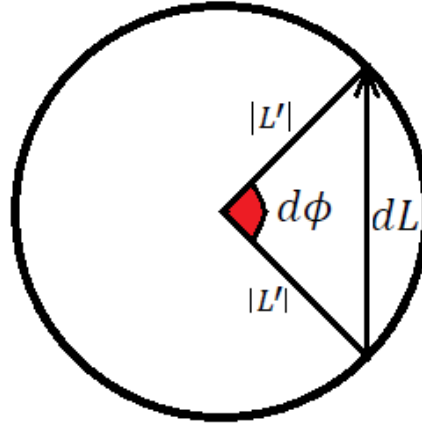


Figure 2.14 Rotation of the magnetic dipole moment.

$$d\phi = \frac{\tau dt}{L \sin(\theta)} \quad (2.48)$$

The precession angular frequency is

$$\omega_p = \frac{d\phi}{dt} \quad (2.49)$$

$$\omega_p = \frac{\tau}{L \sin(\theta)} \quad (2.50)$$

$$\omega_p = \frac{|\vec{\mu}_{e-}| |\vec{B}| \sin(\theta)}{L \sin(\theta)} \quad (2.51)$$

$$\omega_p = \frac{|\vec{\mu}_{e-}| |\vec{B}|}{L} \quad (2.52)$$

substituting all the parameters

$$\omega_p = \frac{\frac{e^- \omega}{2\pi} \cdot \pi r^2 |\vec{B}|}{m_{e^-} r^2 \omega} \quad (2.53)$$

$$\omega_p = \frac{e^- |\vec{B}|}{2 m_{e^-}} \quad (2.54)$$

Then the precession frequency is

$$\nu_p = \frac{e^- |\vec{B}|}{4 \pi m_{e^-}} \quad (2.55)$$

This is the Larmor frequency ν_L which is independent of the particular orientation of the magnetic dipole moment. The effect of the applied magnetic field overall is to rotate the electronic structure which consists of many circulating currents about the direction of the applied magnetic field.

2.7.3 Right and Left Circular polarization components of Light

Let us suppose light with a frequency ν is traversing the material where there is an applied magnetic field along the direction of propagation. The Larmor frequency expression of the precessing magnetic dipole moments is already derived above.

Relative to the electronic structure, which is bound by precession due to the applied external magnetic field, the *l-component* of the light appears to rotate with a frequency $\nu + \nu_L$ and the *r-component* with a frequency of $\nu - \nu_L$.

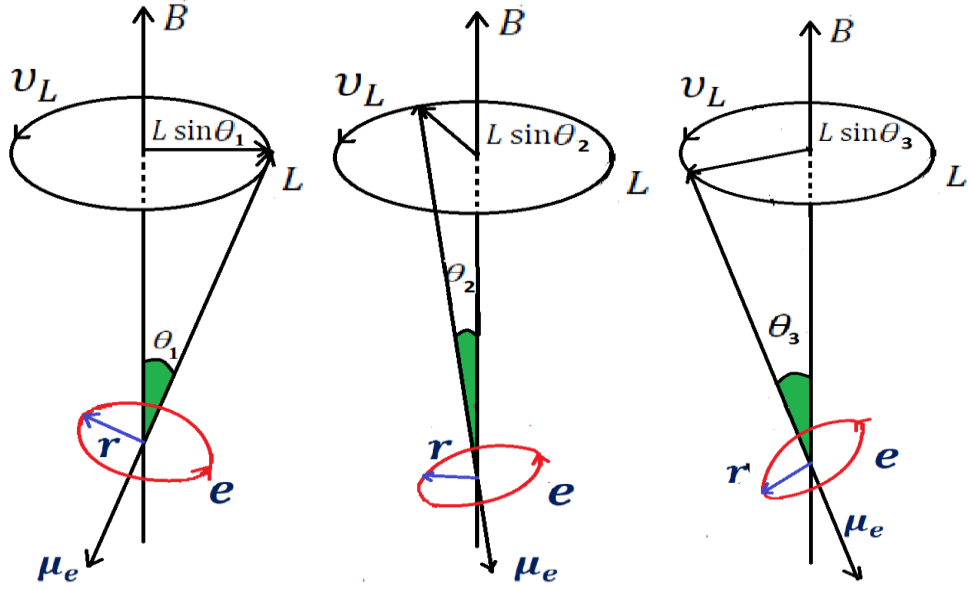


Figure 2.15 Regardless of the angle between the magnetic field and the angular momentum vector of the dipole, the magnetic moments will have the same Larmor frequency of precession about the applied magnetic field.

Considering that this is a dispersive medium, the refractive index is a function of frequency, so for left handed circular polarization and left handed circular polarization components we have a slightly different refractive index assuming that the medium is a dispersive medium

$$n_l = n(\nu - \nu_L) \quad (2.56)$$

and

$$n_r = n(\nu + \nu_L) \quad (2.57)$$

Basically the situation is like that of a dispersive medium with zero applied magnetic field but the left handed and right handed circularly polarized light have different index of refractions. Which in a medium of length d leads to an optical path difference of:

$$(n_r - n_l)d$$

$$(n_r - n_l) = n(\nu + \nu_L) - n(\nu - \nu_L) \quad (2.58)$$

$$(n_r - n_l) = \left(n(\nu) + \frac{dn}{d\nu} \nu_L \right) - \left(n(\nu) - \frac{dn}{d\nu} \nu_L \right) \quad (2.59)$$

$$(n_r - n_l) = 2\nu_L \frac{dn}{d\nu} \quad (2.60)$$

since

$$\nu = \frac{c}{\lambda} \quad (2.61)$$

$$d\nu = \frac{-c}{\lambda^2} d\lambda \quad (2.62)$$

$$\left| \frac{d\lambda}{d\nu} \right| = \frac{\lambda^2}{c} \quad (2.63)$$

since

$$\frac{dn}{d\nu} = \frac{dn}{d\lambda} \frac{d\lambda}{d\nu} = \frac{\lambda^2}{c} \frac{dn}{d\lambda} \quad (2.64)$$

$$(n_r - n_l) = 2 \frac{e^- |\vec{B}|}{4 \pi m_{e^-}} \frac{\lambda^2}{c} \frac{dn}{d\lambda} \quad (2.65)$$

2.7.4 Phase difference due to different refractive indices

The wavelength of light in a medium with a refractive index n_r reads $\lambda' = \frac{\lambda}{n_r}$. Or in other words, the phase change in an optical path of $n_r d$ is

$$\phi_r = \frac{n_r d}{\lambda} 2\pi \quad (2.66)$$

for left handed light, the phase change is

$$\phi_l = \frac{n_l d}{\lambda} 2\pi \quad (2.67)$$

Note that the two phases for left and right circularly polarized lights are identical as they enter the sample, so we can set the initial phase for both to be at zero.

2.7.5 Superposition of left and right circularly polarized optical components

The total electric field is the vector sum of the components of the left and right circularly polarized lights, taking into account their phase changes as well

$$\vec{E} = \vec{E}_r + \vec{E}_l \quad (2.68)$$

From the representative Figure 2.16 below one can write:

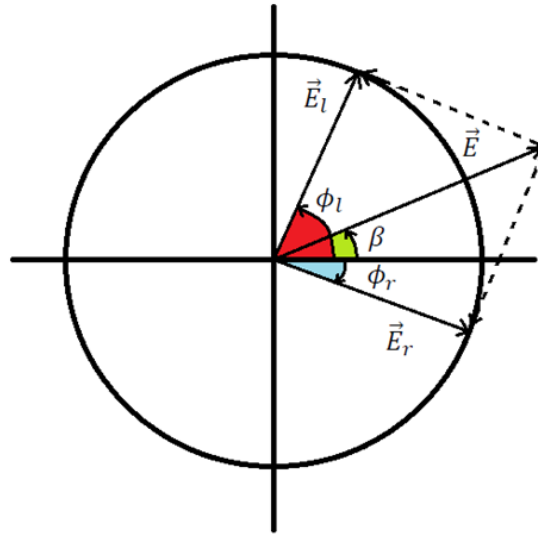


Figure 2.16 Left and right circular polarized light electric field components and their phase.

From the figure

$$\phi_l - \beta = \phi_r + \beta \quad (2.69)$$

$$\beta = \frac{1}{2} (\phi_l - \phi_r) \quad (2.70)$$

$$\beta = \frac{1}{2} \left(\frac{2\pi d}{\lambda} \right) \cdot (n_l - n_r) \quad (2.71)$$

$$\beta = \frac{\pi d}{\lambda} \cdot \frac{e^- |\vec{B}|}{2 \pi m_{e^-}} \cdot \frac{\lambda^2}{c} \frac{dn}{d\lambda} \quad (2.72)$$

$$\beta = \frac{e^-}{2 m_{e^-}} \cdot \frac{\lambda}{c} \frac{dn}{d\lambda} \cdot |\vec{B}| \cdot d \quad (2.73)$$

since

$$\beta = V B d \quad (2.74)$$

It means that the Verdet constant is given as

$$V = \frac{e^-}{2 m_{e^-}} \cdot \frac{\lambda}{c} \frac{dn}{d\lambda} \quad (2.75)$$

At a first glance it may lead to a thought that for longer wavelengths the Verdet constant might be larger, yet, the formula also includes the dispersion term, for which, usually at longer wavelegths is pretty smaller.

2.8 VISUALIZATION OF LINEAR BIREFRINGENCE

As we have just demonstrated, the Faraday effect is caused a circular birefringence. For some materials, if a linearly polarized light passes through a media, the polarization direction of light will be rotate because of the magnetic field. This is called as circular birefringence or optical rotation.



Figure 2.17 Demonstration of birefringence in Calcite Crystal.

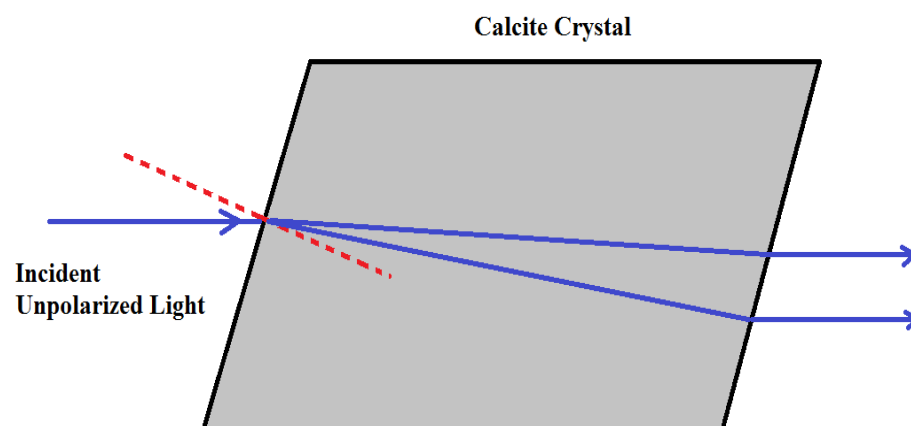


Figure 2.18 Linear Birefringe occurs in some materials where the refractive index.

Circular birefringence causes a rotation of the plane of polarization that merely adds to that caused by Faraday effect. The circular birefringence is a property that right and left circularly polarized waves propagating at different speeds. If the circular birefringence is sufficiently large, linear birefringence arising from bending or other effects has little effect. For some active materials, the circular birefringence can be changed by applying a magnetic field. This effect can vary for different materials depending on the nature of the material

used. Current sensors based on annealed coils offer better temperature stability than has been achieved with either of the other approaches, a further advantage is inexpensively produced. [29]

For a certain material, the strength of the Faraday Effect is determined by an optical constant, which is called as the 'Verdet constant'. This opinion was discovered by the French Physicist Emile Verdet (1824-1866). Emile Verdet studied in optics and magnetism. Also he edited the works of Fresnel about the compact lens.



Figure 2.19 Verdet constant is named after Emile Verdet.

2.9 PURPOSE OF STUDY

The interaction of light and matter in an external magnetic field is an important topic in many respects. Magnetic field measurements in itself is an important topic for systems where the current in a system has to be watched carefully and the measurements has to be taken instantly, yet the measurement system should not or can not interfere with the current. In such systems, like measurement systems for high power currents, measurement of the interaction of light and magnetic field plays a vital role.

As a phenomena of interaction of light and magnetic fields parallel to the direction of propagation of light in a material media, Faraday effect has been known for a long time. It is well known that for materials with so called larger Verdet constants, it is possible to detect and determine how much the direction of polarization of polarized light will change under the effect of an applied magnetic field

Most materials have small Verdet constants of the order of 10^{-4} rad/Tm, but some materials have larger values of the order of 10^2 or more. Such materials yield rotations of the direction of polarization which are easily measurable. Using uniform magnetic fields of known magnitudes over a distance in the direction of propagation of light, one can cause rotation of polarization direction, which is measurable by an adequate experimental set up.

In this thesis our aim is to set up an experimental apparatus which is appropriate for the measurement of Verdet constant of different materials. Our system is considered to be a modular system for which we can start with the measurement of Verdet constants for known materials for a calibration, and then we can change the sample materials.

Development of Novel materials with high Verdet constants is important for several aspects of applied science. The first is a high Verdet constant material can be used as an optical isolator, which is an important component in applications of optics such as fiber lasers and fiber optics. Moreover, we also think that it might act as a modulator for a communication system, and we think we can find an opportunity to test our theory.

We want to set up an experimental set up for Verdet constant measurement delicately. The thesis will outline the experimental details and the measurement techniques.

2.10 PROPOSED EXPERIMENTAL SET UPS

In this experimental set up the magnetic field strength applied by properly assesses the relationship between falling on the photo detector. The set up can be used in determining the Verdet constant. If the coil is connected to an oscillating current, the fundamental frequency and the spectrum is detected and the second harmonic of the detector may be determined experimentally. The purpose of Malus's Law polariser, the light intensity against the possibility of exceeding the threshold value of the laser detector to be able to adjust if necessary.

Analyzer is an optical apparatus whose intensity transmission is proportional to a specific polarization state in the incident beam. In polarimeters analyzers are placed before the detector. The transmitted polarization state emerging from an analyzer can be same or different as the state which is being analyzed. [30]

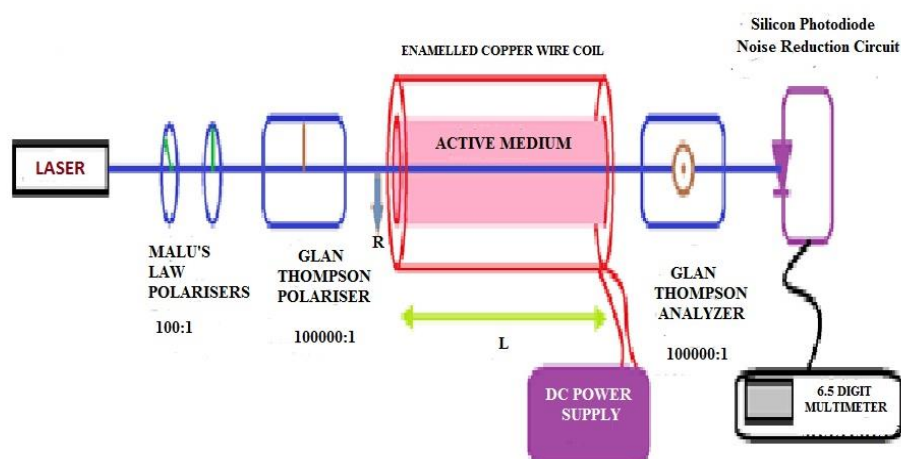


Figure 2.20 One Silicon Photo Detector Test Device.

Another detector system would be a two photodiode system detector. Wollaston prism splits the incident beam into two beams which has been polarised before at an angle of 45° with the horizontal and vertical polarization and separates them from each other at an angle about 20° . In this position, the light falling on silicon detectors is calibrated to be in balance. When the polarization direction of polarized light returned

in a time-dependent manner, the difference of power falling on the detector varies with. In this work we will utilize a one detector system.

2.11 FORMULATION

The propagation direction of light and the direction of the externally applied magnetic field $\vec{B}$ are the same, and that the path of light in the active medium which receives is L , also the Verdet constant of the active medium given by V , the rotation angle of polarization can be found following formula.

$$\theta = V \cdot B \cdot L \quad (2.76)$$

According to radius size, a narrow and long solenoid's uniform magnetic field is

$$B = \mu_0 \cdot n \cdot I \quad (2.77)$$

$$\mu_0 = 1.25663706 \times 10^{-6} \text{ m kg s}^{-2} \text{ A}^{-2}$$

μ_0 is the vacuum permeability of free space

Table 2.2 Relationship between Diameter of Copper Wire and Mass.

Diameter of Copper Wire [mm]	Mass [kg]
0.3	1.32
0.5	2.2
0.8	3.52
1.0	4.4

In the ideal case a small magnetic field to a material having a very high Verdet constant is sufficient. Already, our ultimate goal is to develop such a substance. For this reason, we want to make a good measurement and characterization, and we start with this work as a first step.

Table 2.3 Relationship between the Diameter of Copper Wire and produced Magnetic Field for a constant current of 0.1A.

Diameter of Copper Wire [mm]	Magnetic Field Produced by 0.1 A [μ T]
0.3	4188
0.5	2510
0.8	1570
1.0	1256

Let us suppose a tube of Radius $R=1\text{cm}$ and length $L=1$ meter. Let us also assume that there will be one layer of winding there. How many windings will fit in depends on the Radius of the cross-section of the wire. For a thinner wire, the solenoid holds more number of windings for a single layer of the windings on the solenoid.

For instance, for a wire with a diameter of 1mm, not more than 1000 windings will fit on a single layer assuming of course that there are no gaps between successive windings, and also note that, for a wire with a diameter of 0.3 mm, the upper limit for the number of windings is 3333 per meter. One can take the inner core radius to be very close to the outer Radius for a crude estimate at this stage, but in reality, there is a small cladding of the wire. With this kind of a calculation, if we have ten layers on the solenoid and let a current of 0.1 Amper pass through the wire, we can calculate how much magnetic field we can obtain. We can also get an estimate of how long and how much in terms of mass of the wire is necessary for such a solenoid. We can also have an estimate of how much DC voltage would be necessary to drive the solenoid.

Another thing we can calculate is, having the above mentioned parameters, how much of a polarization rotation in units of degrees could be expected say if we use a HeNe laser of wavelength 632.8 nm in an active medium of a known material. For the calculations we have used the known values for the resistivity and the density of copper. The results here do not contain the facts that at every layer the diameter of the solenoid is increased slightly, but they yield good first degree estimates at this point.

Based on these calculations, the Voltage necessary to produce 0.1 Amper of current goes as the inverse of the cube of the Radius of the cross section of the wire used. And the angle of rotation is inversely proportional to the Radius of the cross section. Accordingly, it seems possible to make an optimization.

Approximate values to 10 layers of wire wound on 1 cm radius and 1 m length of the tube.

Table 2.4 Relationship between the Diameter of Copper wire and the necessary length of the wire to maintain 10 layers of windings with a cylinder template radius of 0.5 cm.

Diameter of Copper Wire [mm]	Minimum Length of Wire [m]
0.3	2093
0.5	1256
0.8	784
1.0	628

Table 2.5 Relationship between Diameter of Copper wire and Voltage necessary to produce 0.1Amps of current in the above mentioned conditions.

Diameter of Copper Wire [mm]	Voltage Required to Produce 0.1 A [V]
0.3	49.748
0.5	10.746
0.8	2.62
1.0	1.34

When the above-mentioned approximate values are taken into account we believe that a little more increased as a result of the length and projected voltage. Further calculation will consider again at the design stage.

Values shown in the table will be increased approximately 3-times for the HeNe laser ($\lambda=650$ nm) and extra virgin olive oil. Now we can look at what it means.

In the experimental set up when we used the Glan Thompson polarizers with 100 000:1 extraction ratio, we anticipate polarization be perfect.

Table 2.6 Relationship between Diameter of Copper wire and the resulting rotation angle, while the current is maintained at 0.1A.

Diameter of Copper Wire [mm]	Rotation Angle for the HeNe Laser and Active Medium of Extra Virgin Olive Oil in 1 m of active material length [°]
0.3	8.4
0.5	5.04
0.8	3.15
1.0	2.52

Polarizer and analyzer in the full upright position as soon as they have each other we can find the amount of brightness falling on a single detector as a function of angle

$$I = I_0 \sin^2 \theta \quad (2.78)$$

When considered in terms of the power of the laser falling on the detector

$$P = P_0 \sin^2 \theta \quad (2.79)$$

Besides the LED characteristics of the laser we use really we would like to get as well as laser properties, so we wanted to 1nm FWHM for laser diodes or get less. Accordingly, the power of diode laser is 4.5mW, (one out of 0.9 mW), the power of the HeNe laser is such that at least 15 mW. Accordingly, even a small deviation angles, we think that detector can detect laser radiation falling on it. In the dual-detector system this deviation angle can be detected even more easily.

CHAPTER 3

INSTRUMENTATION

3.1 INTRODUCTION

In this section we inform a brief review of the instruments we used in our experimental set up. We use Glan-Thompson polarizers since the optical set up is essentially a very much polarization dependent set up. Glan Thompson polarizers are well known to have a very high extraction ratio. They are made of Calcite. Calcite has a small thermal conductivity. Therefore the material is sensitive to temperature gradients. We also use Silicone photodiodes as detectors. We use FDS100 from Thorlabs, although it is not the fastest Si-detector on the market (FDS100 rise time is of the order of ns but FDS100 has a rise time of the order of 10 ns), the detector response is fast, more than one would need in this experiment. We also review opto-mechanics which holds one of the main costs of this project. We did not use an optical table, not only because it would be a great load on the budget but also on purpose, we did not want some metal around that could be permanently magnetized. In the ideal case, we want the magnetic field to go to zero when we switch off the current. Therefore we use a Dekota table. Dekota is a material which we can punch holes easily and use it as a cheap and non-ferrous made optical table. For any material it is hard to speak of as being non-magnetic. Most materials are either paramagnetic or diamagnetic if not ferro-magnetic to a degree. Therefore we use Dekota as the optical table with caution but at least we are sure that it will not be a ferro-magnet as the usual optical tables.

3.2 INSTRUMENTATION

3.2.1 Glan Thompson Polarizer-GTH10M-A

Glan-Thompson polarizers are made of calcite mineral. It consists of two prisms which have high optical grade. Already polarized light from our HeNe laser with a polarization ratio of 500:1 enters the polarizer and is divided into two at the interface between the two crystals. So we want to get rid of that one part too and make this ratio as 100 000:1, that is why we use this particular polarizer. Also note that the analyzing polarizer also consists of the same polarizer, therefore we are pretty sure that there is not a significant leak from the polarizers.

3.2.2 Features of the Glan Thompson Polarizers

Wide Field of View: 40° Typically. High Extinction Ratio of the order of 100,000:1. This is an important parameter. Usual polarizers on the market provide a polarization ratio of 100:1. We have chosen a laser that already has a polarized output on the order of 500:1, much better polarization ratio than a non-polarized laser light passing through a usual polarizer. And then of course we increase this ratio by passing through Glan Thompson polarizer even further to as large as 100 000:1.

350 nm to 2.3 μm Operation for Uncoated Prisms. This is important since we want to use the same polarizers and the same set up if in case necessary for other wavelengths in the visible spectrum too. AR-Coated Versions for 350 - 700 nm or 650 - 1050 nm range are available. We use 350-700 nm version which covers the entire visible spectrum. So the polarizer does not reflect much on the front or back surface of the polarizer. It utilizes the highest grade optical Calcite cemented prisms and have a Damage Threshold of 1 W/cm². This is also important since we do not want to damage the crystal with the laser power. The laser power we use is high for a HeNe laser. It must be held below the damage threshold when we consider the power per unit area.

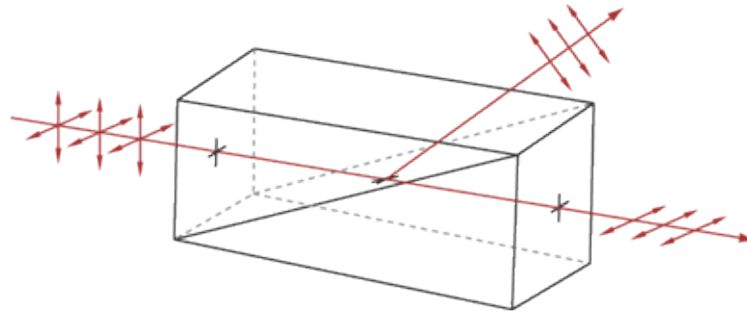


Figure 3.1 Polarization-Dependent Refraction - Glan-Thompson Calcite Polarizer.

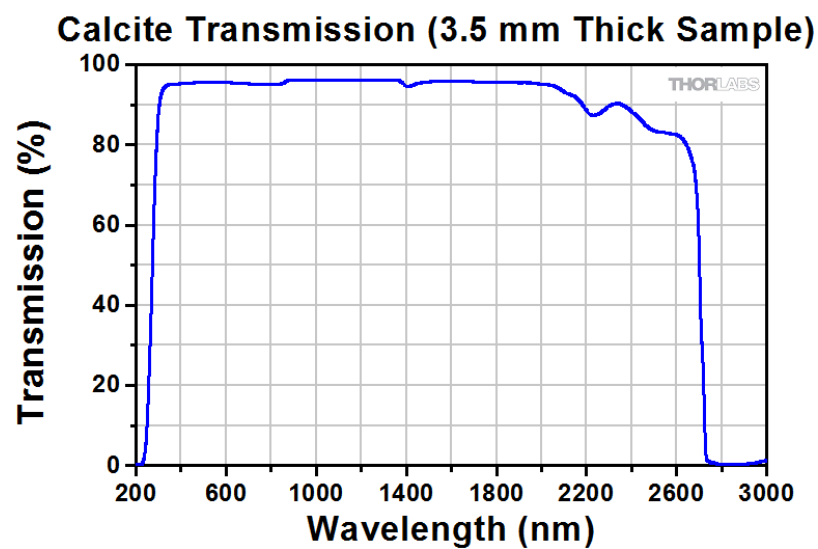


Figure 3.2 Typical Transmission of the calcite Glan-Thompson polarizer.

The above graph shows us the transmission of Glan-Thompson polarizers. Calcite is an inartificial material, and thus transmission is changeable, especially in the UV and IR. [28]. We use type-A for the polarizer so the dimensions are also tabulated.

Table 3.1 Dimensions of the polarizer.

Dimensions, Diameter and Length		
A	21,6 mm	36,6 mm
B	12,7 mm	25,4 mm

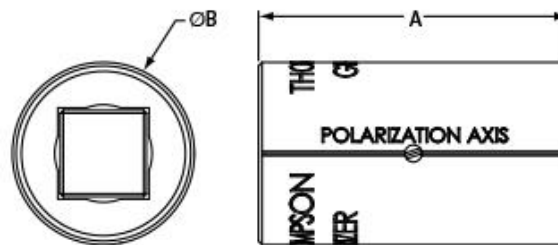


Figure 3.3 Shape of the Glan Thompson polarizer.

The laser light is passed through the above mentioned square. The polarization axis being the same as the laser polarization axis would be important, since we do not want to lose intensity at this level.

3.2.3 Lens Tube SM1L20

Lens tubes are used to create optical infrastructures. The lens is positioned at a desired depth in the tube using an optical spanner.



Figure 3.4 Lens tube SM1L20.

We use the lens tube to position the Glan-Thompson polarizers. We also use rotation mounts which we will discuss next to have a proper measurement of the relative angular positioning of the system.

3.2.4 FDS100 Si Photodiode

The FDS100 is a high-speed silicon photodiode with a spectral response from 350nm to over 1100nm. The detector is modeled with the circuit depicted below. The amount of photocurrent generated depends on the incident light and wavelength and it can be viewed by using an oscilloscope or a multimeter. The task of the RC filter is to filter any high frequency noise from the input source. [31]

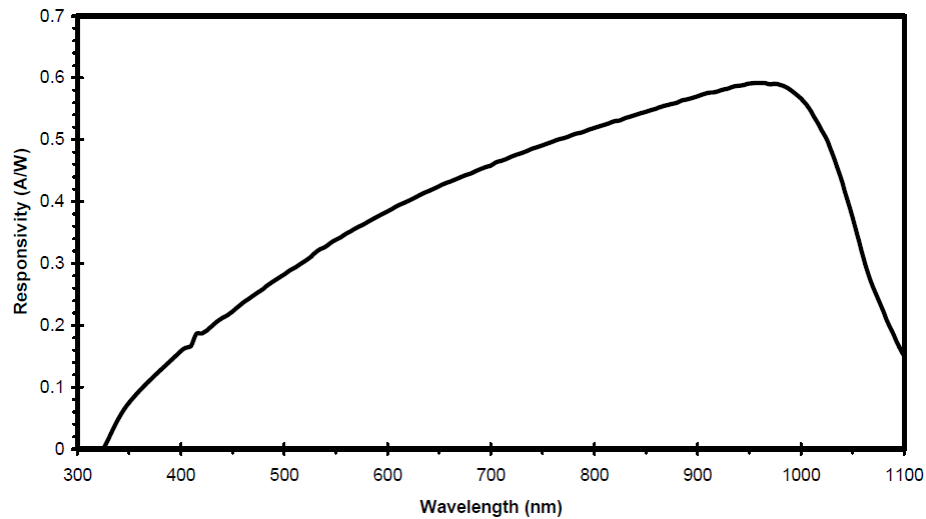


Figure 3.5 Spectral Responsivity graph of FDS100 Photodiode.

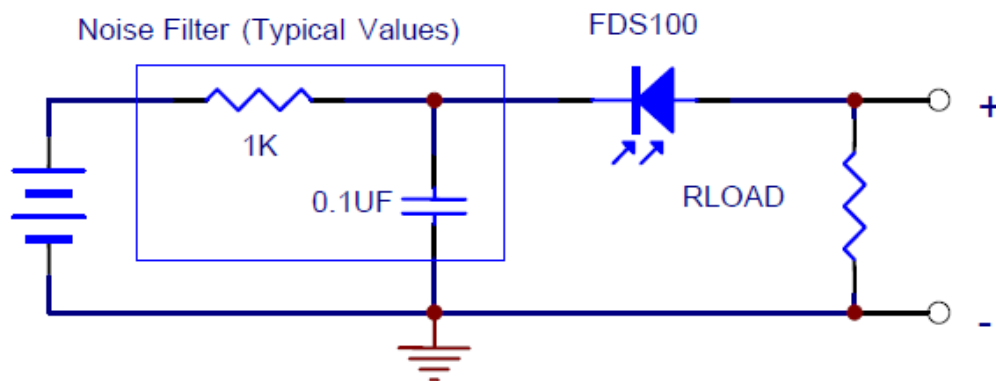


Figure 3.6 Typical circuit diagram of the FDS100 Photodiode.

Damage threshold of the photodiode is 100 mW/cm^2 , for this reason to avoid damaging the photodiode we need to fine-tuning the intensity of the light falling on the detector. The photodiode anode produces a current, which is a function of the incident light power and the wavelength. The responsivity $R(\lambda)$ to estimate the amount of photocurrent to expect. This can be converted to a voltage by placing a load resistor

(R_{LOAD}) from the photodiode anode to the circuit ground. The output voltage is derived as:

$$V_0 = P \cdot R(\lambda) \cdot R_{LOAD} \quad (3.1)$$

The bandwidth, f_{BW} , and the rise time response, t_R , are determined from the diode capacitance, C , and the load resistance, R_{LOAD} , as shown below. Placing a bias voltage from the photo diode cathode to the circuit ground can lower the photo diode capacitance.

$$f_{BW} = \frac{1}{(2\pi R_{LOAD} C)}, t_R = \frac{0.35}{f_{BW}} \quad (3.2)$$

The responsivity of the detector for a single detector system only determines the sensitivity of the detector system. Because the angle is measured in terms of intensity ratios, the absolute value of the responsitiy does not matter for the angle value, however, it is important for the precision of the measurement. [32]

3.2.5 Meniscus Lens LE1156-A

We use this lens to focus the beam that already passed through the active medium on the detector. These lenses are designed to minimize spherical aberrations. One surface of the lens is concave, while the other surface is convex. We used the lens to focus and increase the brightness of the light falling on the photodetector. Positive meniscus lenses are preferred on purpose. Also note that they are anti-reflective coated, which adds to the transmitted intensity as well as the cost, but the measurements then turn out to be much more reliable.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 DISCUSSION ABOUT THE MAGNETIC FIELDS

4.1.1 Basic Properties of the Magnetic Fields

There are various things we better remember about the magnetic fields. First thing is the density of the magnetic field lines, which boils down to the flux passing through a designated area and that is about the strength of the magnetic field in that region. Secondly, magnetic fields are subject to an important limitation, which is one of Maxwell's equations written as

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (4.1)$$

That is the divergence of the magnetic fields must vanish, something known as the lack of magnetic monopoles. The implications are obvious, the influx into a control volume must be equal to the out flux on the same control volume. In other words, it means also that the magnetic field lines will close on to themselves, they will eventually form curves at the same point, infinitely and unbound straight lines of magnetic fields is not possible by definition. So, eventually, magnetic field lines are basically curved in space.

4.1.2 Tesla as a Unit of the Magnetic Field Strength

Another thing to keep in mind is about the strength of the magnetic fields. Although in nature it is recently observed that immense magnetic fields of the order of 10^{12} Tesla is possible on the surface of some neutron stars which are known as magnetars.[32] A magnetar is a kind of neutron star and it has extremely powerful magnetic fields. Actually, on the Earth in usual daily life even 1 T is a very big measure for a magnetic field and can actually be obtained by rather expensive and special instruments. If a Scientific Facility must use magnetic fields say of the order of 10^2 Tesla for instance, even though one uses superconductors, the water cooling system may require water flux to be of the order of the Mississippi River.

So now we know that actually obtaining a magnetic field of the order of 1 T is not really something that we will usually notice on a spring day in the fields.

4.1.3 Polarizer-Analyzer Pair

Recall the equation concerning the Faraday effect rotation for a material of Verdet constant

$$\theta[\text{rad}] = V \cdot B \cdot L \quad (4.2)$$

Say for glass V is about 4 rad/T.m

So for 1 mm = 10^{-3} m of thick glass within a magnetic field of the order of 1 Tesla would lead to

$$\theta[\text{rad}] = 4 \cdot 10^{-3} \text{rad} \quad (4.3)$$

$$\theta[\text{degrees}] \approx 4 \cdot 57,3^\circ \cdot 10^{-3} \text{degrees} \quad (4.4)$$

$$\theta \cong 0,23 \text{degrees} \quad (4.5)$$

a very small rotation angle.

But if we had a material of the Verdet constant $V \sim 100 = 10^2$ rad/T.m then we obtain by the same parameters.

$$\theta[\text{rad}] = 10^2 \left[\frac{\text{rad}}{\text{T}} \cdot \text{m} \right] \cdot 1 \text{ T} \cdot 10^{-3} \text{ m} \quad (4.6)$$

$$\theta[\text{rad}] = 10^{-1} \text{ rad} \sim 5.73^\circ \quad (4.7)$$

The intensity measured through a set up of polarizer-analyzer pair directed with 90° of difference of the polarization directions would yield an intensity ratio to the incoming intensity of

$$\frac{I}{I_0} = \sin^2(\theta) \quad (4.8)$$

And if the incident radiation, say the irradiation from a laser light is powerful enough, this radiation is detectable.

4.1.4 The Interplay of the Magnetic Field and the Interaction length

Obtaining magnetic fields of the order of 1 Tesla requires high currents and voltages. But the equation about the rotation of the polarization of light suggests us one thing interesting. Keeping the product of the parallel magnetic field strength and the interaction length a constant leads to the same degree of rotation. In other words $B = 1$ Tesla with $L = 1$ mm leads to the same degree of rotation with $B = 10^{-3}$ Tesla and $L = 1$ meter.

There are a few things to note at this stage. First is that more interaction length in an active medium also means more absorption and scattering of the light passing through, which we may describe as

$$I_{\text{Transmitted}} = I_{\text{Incident}} \cdot e^{-\alpha x} \quad (4.9)$$

Where α is the absorption coefficient of the optically active medium and $I_{\text{Transmitted}}(x)$ is the intensity remaining at a depth of x within the medium. For an interaction length of L this would lead to

$$I(L) = I_{\text{Transmitted}} \cdot e^{-\alpha L} \quad (4.10)$$

Taking this into account we use a rather powerful He-Ne laser, where this type of laser is known for its stability and HNL150-L is a He-Ne laser with a minimum of 15 mW output power and with already a linearly polarized output of ratio 500:1. Considering that polarizers used in Student Optics labs only produce extraction ratios of 100:1, this is pretty good degree of polarization. An already polarized laser output has an obvious advantage over a non-polarized laser output here that, half of the intensity of the laser source will not be wasted at the input polarizer of our experimental set up.

The second thing to take note of is that, although the strength of the magnetic field is reduced once we decide to use a much longer interaction length, this time, due to the above mentioned Maxwell's equation $\vec{\nabla} \cdot \vec{B} = 0$. It is now a primary concern to have a uniform magnetic field throughout the interaction length.

4.1.5 Obtaining a Uniform Magnetic Field within an External Region

Magnetic fields are capricious. To obtain a uniform magnetic field in a certain space, one needs to have the magnetic field lines to be (1) In the same direction and (2) Have the same flux over that region. One way to obtain this for extended lengths is to use a solenoid. The use of Faraday Integral Law that

$$\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot I \quad (4.11)$$

suggests a uniform magnetic field within a solenoid. With a field strength of

$$B[\text{Tesla}] = \mu_0 \left[\text{T} \cdot \frac{\text{m}}{\text{A}} \right] \quad (4.12)$$

where $\mu_0 = 1,2566370614 \cdot 10^{-6} \text{ T} \cdot \text{m}/\text{A}$ is the magnetic permeability of vacuum, N is the total number of windings and L is the length of the solenoid. So now we can define a new parameter, that is the number of windings per unit length of the solenoid.

$$B[\text{Tesla}] = \mu_0 \left[\text{T} \cdot \frac{\text{m}}{\text{A}} \right] \cdot n \left[\frac{1}{\text{m}} \right] \cdot I[\text{Ampere}] \quad (4.13)$$

$$B[\text{Tesla}] = 1,2566370614 \cdot 10^{-6} \cdot n[\text{length of wire/meter}] \cdot I[\text{A}] \quad (4.14)$$

There are few things to remember here. Ampere is rather large unit for current.

$$B[\text{Tesla}] = 1,2566370614 \cdot 10^{-6} \cdot n \cdot I \quad (4.15)$$

Where $n[1/m]$ and $I[A]$

For practical reasons related to the heating of the coil, the current in wire must be kept at moderate values since the power due to joule heating goes as the square of the following current

$$P = I^2 \cdot R \quad (4.16)$$

Although in a winding coil, the uniform flow of the electric current through the cross-section of the wire is same what loose assumption since all the solenoid is under the effect of magnetic fields due to current else where in the solenoid.

For a moderate electrical current of the order of 10^{-1} Ampere to obtain desired values of the magnetic field B to be order of 10^{-3} Tesla, the required number of windings per meter must be of the order of 10^4 . Basically to have $B = 10^{-3}$ Tesla over an interaction length of about 1 meter, one needs 10^4 windings per meter with a relatively low current of 10^{-1} Ampere, or alternatively 10^3 windings with a current of 1 Ampere.

The length of the solenoid with 10^4 windings/meter in compared to that of a solenoid with 10^3 windings/meter is like 10 times more, which implies that its resistance is about 10 times more, or even more than that because the diameter of the wire adds to the diameter of the coil at every layer. But now simplicity, let us assume the resistance of an order of magnitude larger. To have the desired magnetic field, one tenth of the current would be sufficient though, so the joule-heating formula leads to 10 times more for the solenoid with less number of windings per meter. We can then conclude that heating power, which is a crucial parameter goes as the reciprocal of the number of windings for the same wire and the same initial solenoid diameter, namely

$$P \propto \frac{1}{n} \quad (4.17)$$

But note also that there is mechanical limiting factor on the solenoid template too, the wire also has a weight and the template can hold a certain weight before it cracks.

4.2 ESTIMATION OF THE MAGNETIC FIELD IN A SOLENOID

4.2.1 Coil Winding

It is well known from Freshman Physics classes as an application of Faraday Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot I \quad (4.18)$$

which is just the line integral form of Maxwell's equation $\vec{\nabla} \cdot \vec{B} = 0$ that, a good way to obtain a uniform magnetic field is to wind a long solenoid. A long solenoid is one in which the length of the solenoid is large in compared to the diameter of the solenoid.

Here we review the application of Faraday's Law to a solenoid and remember its' prerequisites. Let us consider a solenoid with one layer of windings only. For starters let L be the length of the solenoid and N_1 be the number of windings that cover the length of the solenoid which would lead to n_1 , the number of windings per unit length in one layer is

$$n_1 = \frac{N_1}{L} \quad (4.19)$$

For the sake of simplicity I will also assign a variable to the number of layers that cover the solenoid. I can say one layer of windings will contain N_1 turns, two layers will contain $2 N_1$ turns, and m layers will contain $m \cdot N_1$ turns. And since all of this is uniformly distributed throughout the length L of the solenoid is

$$n = \frac{m \cdot N_1}{L} \quad (4.20)$$

L is determined by our choice of the template instrument for the solenoid, m is of our choice, but the maximum value of N_1 is determined by the outer diameter of the wire of our choice including the cladding. (See Figure 4.1)

An isolated copper wire has its core diameter and cladding diameter. Core diameter plays role in the total resistance, while the maximum number of windings are determined by the cladding diameter.

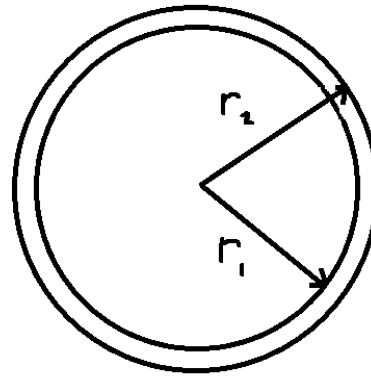


Figure 4.1 Core and Cladding Diameters of the Cross-section of a wire r_1 and r_2 .

We can estimate a maximum number of loops for a certain wire in one layer as

$$N_{1,\max} = \frac{L}{r_2} \quad (4.21)$$

This of course assumes that there is no gap between successive windings, which would be the ideal situation, but is also unrealistic.

For a general discussion we can take the case of a copper wire with cladding diameter of 0.65 mm. We may assume that the core diameter will be 0.60 mm relying on the producer suggestion. Wire may have slightly different cladding thickness depending on the producer. But it seems to be safe to assume from the catalogue values of wires that a 0.65 mm wire will have a core diameter and a wire with a cladding diameter of 0.45 mm will have a core diameter of about 0.4 mm.

Based on these values we can obtain realistic values of the number of windings in one layer. For a 0.65 mm wire

$$N_{1,\max}(0.65) = \frac{1100 \text{ mm}}{0.65 \text{ mm}} \quad (4.22)$$

$$N_{1,\max}(0.65) \sim 1692 \text{ windings} \quad (4.23)$$

The same parameter for a wire with cladding diameter $r_2 = 0.45$ mm

$$N_{1,\max}(0.45) = \frac{1100 \text{ mm}}{0.45 \text{ mm}} \quad (4.24)$$

$$N_{1,\max}(0.45) \sim 2444 \text{ windings} \quad (4.25)$$

A more realistic assumption would be

$$N_{1,\max}(0.65) \sim 1500 \quad (4.26)$$

and

$$N_{1,\max}(0.45) \sim 2200 \quad (4.27)$$

But what matters for the calculation of the magnetic field strength would be the number of windings per unit length of the solenoid and the maximum values considered for one meter of solenoid then reads

$$n_{\max}(0.65) \sim 1363 \quad (4.28)$$

$$n_{\max}(0.45) \sim 2000 \quad (4.29)$$

For 3 layers of windings on the solenoid, the number of windings per unit length would be

$$n_{\max}(0.65 \text{ mm}, m = 3) = 4090 \text{ windings per meter} \quad (4.30)$$

$$n_{\max}(0.45 \text{ mm}, m = 3) = 6000 \text{ windings per meter} \quad (4.31)$$

Although we are planning to we samples with tube lengths not more than 1 meter, we use a solenoid of length 110 cm to account for the possible fringing effects at the tips of the solenoid. So we can safely consider that within the sample we have a truly uniform magnetic field.

For 3 layers of windings it appears that while we have (for a wire of diameter 0.65 mm) a total number of windings of 4500, the number windings per unit length would be like 4090. If we use a wire with diameter 0.45 mm, the total number of windings along the solenoid for 3 layers would be like 6600 and, the number of windings per unit length would be like 6000.

4.2.2 Derivation of the Magnetic Field Strength for a Solenoid

We have Maxwell's equation in integral form which is known the Faraday's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{\text{total}} \quad (4.32)$$

And we may apply this equation to the case of a long solenoid we need to choose an adequate path to apply the equation in confidence.

This solution is a standart solution in Freshman Physics classes as the application of Faraday Law. We choose a path that goes through the solenoid as shown in the Figure 4.2.

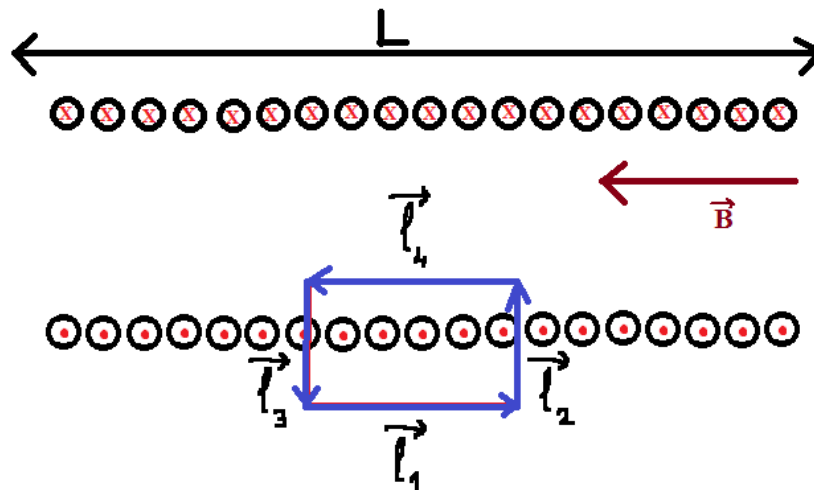


Figure 4.2 Application of Faraday Law to a long solenoid.

If the path is chosen as follows $\vec{B} \cdot d\vec{l}$ vanishes for l_4 and l_2 since $\vec{B} \cdot d\vec{l}$ are perpendicular. And for l_1 , $\vec{B} \cdot d\vec{l}$ vanishes again assuming outside the solenoid the field is rather small assuming l_1 is located rather far from the solenoid. So only l_3 remains as the sole contributor.

$$\int_{l_3} \vec{B} \cdot d\vec{l} = \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{\text{total}} \quad (4.33)$$

If L is the total length of the solenoid and N is the total number of windings, the number of windings that pass through the selected contour reads

$$\frac{l_3}{L} \cdot N$$

Where, the number of windings per unit length is

$$n = \frac{N}{L} \quad (4.34)$$

This means that the total current flowing through the selected contour is

$$I_{\text{total}} = I \cdot l_3 \cdot n \quad (4.35)$$

where I is the current passing through the wire.

Then Faraday equation is reduced to

$$B \cdot l_3 = \mu_0 \cdot I \cdot l_3 \cdot n \quad (4.36)$$

the magnetic field expression within a solenoid is

$$B = \mu_0 \cdot I \cdot n \quad (4.37)$$

Which we will make use of quite frequently.

4.2.3 Calculation of the Expected Magnetic Field Strengths

Now we can obtain a realistic estimate for the expected magnetic field strengths. The number of windings per unit lengths we estimated as in the following table.

Table 4.1 Estimated number of windings per unit length for various wire options with 3 layers of windings.

	Windings / meter
$n_{\max}$ (0,65 mm, $m = 3$)	4090
$n_{\max}$ (0,45 mm, $m = 3$)	6000

Taking into account that magnetic permeability of free space is defined as

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{N}}{\text{A}^2} \quad (4.38)$$

or

$$\mu_0 = 1,2566370614 \cdot 10^{-6} \frac{\text{T} \cdot \text{m}}{\text{A}} \quad (4.39)$$

We can calculate the maximum order of expected magnetic field strengths for given currents.

The above mentioned values yields the maximum expected magnetic field strengths of the order of the variables enlisted in the following table.

Table 4.2 Maximum estimated B-field strength in terms of current for m=3 layers.

	B field [T] in terms of current I [A]
$B_{\max} (0,65 \text{ mm}, m = 3)$	$5139,645581 * 10^{-6} * I[A]$
$B_{\max} (0,45 \text{ mm}, m = 3)$	$7539,82368 * 10^{-6} * I[A]$

For a current of $I=1\text{A}$ the magnetic field estimated will be about $5140 \mu\text{T}$ for $r_2 = 0,65 \text{ mm}$ and $7540 \mu\text{T}$ for $r_2 = 0,45 \text{ mm}$ wired.

For a current of $I= 1.25 \text{ A}$ on the otherhand, the magnetic field strength is estimated to be of the order of $6425 \mu\text{T}$ for $r_2 = 0,65 \text{ mm}$ and $9425 \mu\text{T}$ for $r_2 = 0,45 \text{ mm}$ diameter wires.

Of course we need to remember that these are estimates and at some point they contain some assumptions too. That's why experimental values need to be taken carefully.

4.2.4 Estimation of DC Resistance and Current

We can estimate the resistance of the coils since we know the order of magnitude of the core diameter of the wire used for the solenoid. For core diameter of $0,60 \text{ mm}$ we will have a $0,30 \text{ mm}$ radius and for a core diameter of $0,40$ we will have a $0,20 \text{ mm}$ radius.

At this point it might be necessary to refer to the solenoid we construct. Taking into account the divergence of the laser light, we estimated that for a sample size of length 1 meter , it would be less practical to have a width of the sample less than 1.6 centimeters . But we are also compelled to have a sample size as small as possible in width since the solenoid to be wound will be even larger than that and more radius means more wiring, more material and hence more mass. And the materials have a limit to handle that kind of mass. Therefore we had to choose sample widths, which means the sample tube diameter to be as small as possible, but not smaller. Also we had to go with

what we have on the market. The materials on the market also set a limit. We use acrylic tubes for the sample and the solenoid. The sample tube diameters we use are 21 mm at the inner and 25 mm at the outer. The solenoid has to hold it so the acrylic tube for the solenoid has 26 mm at the inner and 30 mm at the outer. So we have to take into account the the solenoid will have a radius of 15 mm to start with, but as we wind the solenoid it is obvious that the wire diameter also adds to this radius. So we need to take into account that too while we are to calculate the total length of the wire necessary to make our solenoid.

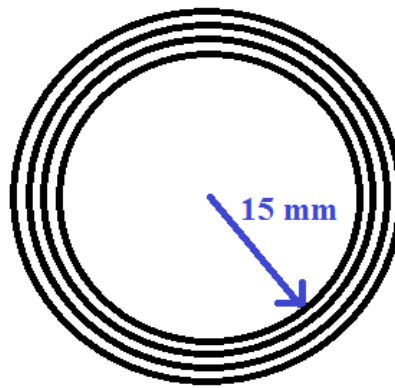


Figure 4.3 Crossection of the solenoid wire windings, three layers.

The first layer of coils will have $15\text{mm} + r_1$, the second layer will have $15\text{mm} + 3r_1$ and the third layer will have $15\text{mm} + 5r_1$. Since all have equal number of windings we can take an average and $\langle R \rangle = 15,60\text{mm}$ for $0,45\text{ mm}$ cladding radius wires. So an average loop will have a length of

$$2\pi \langle R \rangle = 98,0177\text{ mm} \quad (4.40)$$

for $0,45\text{ mm}$ wire.

For 110 cm the number of windings for 3 layers of windings will be 4499 windings and 6600 windings for $0,65\text{ mm}$ and $0,45\text{ mm}$ windings.

So the total length of the solenoid will be number of windings multiplied by the average length of one loop.

Table 4.3 Length of the wire for three layers of the solenoid.

	Length of the wire for three layers of solenoid
$r_2 = 0,65 \text{ mm}$	$l = 449462,02235 \text{ mm} \sim 449,46 \text{ m}$
$r_2 = 0,45 \text{ mm}$	$l = 646916,82 \text{ mm} \sim 646,92 \text{ m}$

We can proceed with the estimated cross section of wires.

$$\pi \cdot r_1^2 = 0,28274 \text{ mm}^2 \text{ for } r_2 = 0,65 \text{ mm} \quad (4.41)$$

and

$$\pi \cdot r_1^2 = 0,12566 \text{ mm}^2 \text{ for } r_2 = 0,45 \text{ mm} \quad (4.42)$$

The cross-sectional area of the wires are tabulated below.

Table 4.4 Estimated cross-sectional area of the wires.

	A (Cross-sectional Area)
$r_2 = 0,65 \text{ mm}, r_1 = 0,30 \text{ mm}$	$2,8274 * 10^{-7} \text{ m}^2$
$r_2 = 0,45 \text{ mm}, r_1 = 0,20 \text{ mm}$	$1,2566 * 10^{-7} \text{ m}^2$

At this point we are ready to get an estimate for the resistance of the coil for a DC current. Note that the resistivity of Copper is $\rho_{\text{Cu}} = 1,68 * 10^{-8} \Omega \cdot \text{m}$

The resistance is

$$R = \rho \cdot \frac{l}{A} \quad (4.43)$$

Table 4.5 Estimated resistances for three layers of windings.

	$l[m]$	$A[m^2]$	$R[\Omega]$
$r_2 = 0,65 \text{ mm}$	449,46	$2,8274 \cdot 10^{-7}$	26,70626
$r_2 = 0,45 \text{ mm}$	646,92	$1,2566 \cdot 10^{-7}$	86,48938

Regulated DC power supplies are usually, 30 V maximum and 5 A maximum type of 60 V maximum 5 A maximum, so we can calculate the expected currents for the above mentioned resistances.

Table 4.6 Estimated currents using estimated resistances.

	$I(30V) [A]$	$I(60V) [A]$
$r_2 = 0,65 \text{ mm}$	1,1233 A	2,2466 A
$r_2 = 0,45 \text{ mm}$	0,3468 A	0,6937 A

4.2.5 Winding the Solenoid

Since we need at least several thousand windings for the solenoid, we needed to construct a machine for this purpose. We need a solenoid of length 110 cm, but no one would do this for us with a meticulous care it needed to be done, therefore we did that ourselves. We needed a motor to rotate the solenoid. A frame to hold the rotation axis, and the axis should have as little resistance as possible while being able to hold the weight of the solenoid and the wire. The solenoid main frame is made of acrylic, therefore we also don't want it to break under the weight of the wire or the torque of the electrical motor. We also need a mechanism to couple the motor to the acrylic main

frame on both ends. We needed a high torque low rotation per minute rate motor. We bought two motors of low rpm and high torque and used one of them we considered to be adequate for assembly. We attached the motor to a steel holder and the holder to a clamp. The cramps we used have the ability to rotate in all three dimensions therefore we had an adequate mechanism to ensure that both ends of the solenoid will be at the same height and on the same rotation axis. This saves us a lot of designing and construction, as well as it provides us an ability to change the solenoid length if in case we want to construct yet another one afterwards.



Figure 4.4 Electrical motor and Winding the solenoid.

The parts necessary to construct the mills were designed and fabricated using a 3D printer. The solenoid is had to be varnished every now and then and air circulation was used to dry the varnish, to hold the wires as well as to provide a more reliable isolation system for the wires. For safety reason after varnishing, the coil is left to dry out.

4.2.6 Estimation of the Uniform Magnetic Field Strength

Now that we have an estimate of the currents flowing through the wires for both solenoid options, we can have a plausible calculation for the magnetic fields using

$$B = \mu_0 \cdot n \cdot I \quad (4.44)$$

where μ_0 is the permeability of free space and n is the number of windings per unit length.

Table 4.7(a) Estimated magnetic fields that could be obtained using three layers of windings.

30V	$n_{\max} \left[\frac{\text{windings}}{\text{meter}} \right]$	$I[\text{A}]$	$B_{\max}[\mu\text{T}]$
$r_2(0,65 \text{ mm})$	4090	1,1233 A	5773,37 μT
$r_2(0,45 \text{ mm})$	6000	0,3468 A	2614,81 μT

Table 4.7(b) Estimated magnetic fields that could be obtained using three layers of windings.

60V	$n_{\max} \left[\frac{\text{windings}}{\text{meter}} \right]$	$I[\text{A}]$	$B_{\max}[\mu\text{T}]$
$r_2(0,65 \text{ mm})$	4090	2,2466 A	11546,74 μT
$r_2(0,45 \text{ mm})$	6000	0,6937 A	5229,62 μT

We can put the above estimates in a single equation for these wires. For three layers of windings we have

$$B (r_2 = 0,65 \text{ mm})[\mu\text{T}] = 5139,6456 \cdot I[\text{A}] \quad (4.45)$$

and

$$B (r_2 = 0,45 \text{ mm})[\mu\text{T}] = 7539,8224 \cdot I[\text{A}] \quad (4.46)$$

We tabulate and plot the data in units of milli-amperes and micro-Tesla therefore we can rewrite these same equations in units of mA and μT as follows.

$$B (r_2 = 0,65 \text{ mm})[\mu\text{T}] = 5.1396456 \cdot I[\text{mA}] \quad (4.47)$$

and

$$B (r_2 = 0,45 \text{ mm})[\mu\text{T}] = 7.5398224 \cdot I[\text{mA}] \quad (4.48)$$

It is obvious that since all the above mentioned calculations are theoretical and based on some assumptions, we did not include any error calculations and the resolution appears to be pretty high. When we take data, we can compare the results with these predictions and speculate on the physical parameters.

4.2.7 Experimental Results

4.2.7.1 Questions concerning the magnetic field vector

In this section we take the data for the solenoid. Our idea was that, what really matters for the Faraday Effect is the total interaction length and the magnetic field strength in terms of the following equation,

$$\int_0^L \vec{B}(\mathbf{x}) \cdot d\vec{x} \quad (4.49)$$

This equation would reduce to

$$\vec{B} \cdot \vec{L} \quad (4.50)$$

If the magnetic field vector is a constant, i.e., the magnetic field is uniform throughout the interaction length. A solenoid in the ideal case will provide for us a uniform magnetic field. However, since the derivation of the solenoid equation includes

the assumption that the solenoid will be infinitely long, at the ends of the solenoid this assumption seems to fail. Therefore, we thought that the interaction length if considered to be like 1 meter, to get rid of these fringing effects as far as possible, we need to make the solenoid longer. We have wind a solenoid of length 110 cm, where we have access of 5 cm of windings even for the longest interaction lengths of 1 meter.

In short, what we actually do is to take the average magnetic field strength and multiply it with the interaction length in case where the two vectors are parallel.

$$\langle \vec{B} \rangle \cdot \vec{L} \quad 4.51$$

Where $\langle \vec{B} \rangle$ denotes the average magnetic field. We assume that the average magnetic field seems to be constant throughout the solenoid, but this assumption needs to be checked and we will check if the magnetic field changes significantly in the solenoid. Another question of course is whether the active medium effects the magnetic field strength or not. It could be that the active medium could be diamagnetic, or paramagnetic, that we do not know yet if we don't check it. A third question to ask would be whether or not the active medium allows the light to penetrate through, since 1 m of interaction length is a long one, and absorption of light could consume the light in great proportions. In the following experiments we will check all these questions as far as the instruments allow us to do.

4.2.7.2 Absorption of the Laser Light

An important question is whether or not the laser light will pass through the virgin olive oil as an active medium. This question can only be answered if we now the absorption coefficient of the olive oil or if we observe that the laser light passes through the medium with detectable intensity.

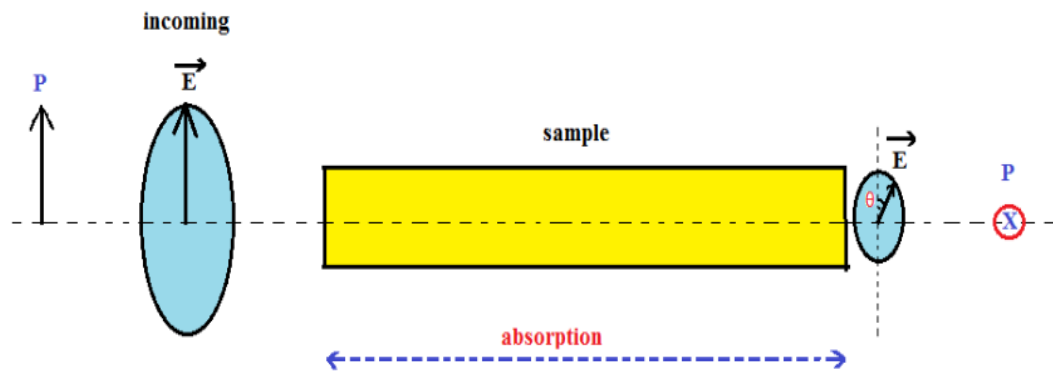


Figure 4.5 Decreasing the Electric Field in the Active Medium.

In the following we let the light from HeNe laser to pass through a 0.4 meter long tube full of virgin olive oil. We can directly see the light being absorbed as it propagates in the active medium.

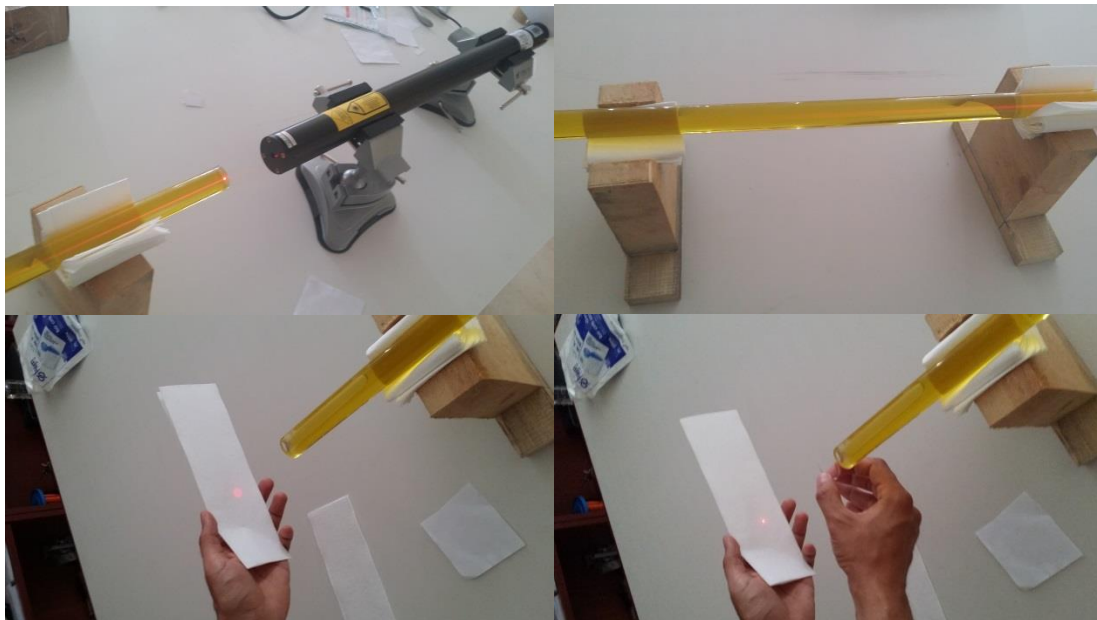


Figure 4.6 Exponential die out and residual laser light after passing through 0.4 meter virgin olive oil.

We can also easily notice that since the windows on the tips of the tube are not anti-reflective coated windows but rather simple clear laser cut acrylic material, we lose some of the intensity on the reflections at the edges of the tube. Which seems to be something inevitable at this stage. The intensity at the focus determines what remains as the maximum power and the analyzer does not let almost any of this intensity to pass through but only a 10^{-5} of it will pass through, since the Glan Thompson calcite polarizers have an extraction ratio of 100 000:1. It adds to the background noise and any rotation that causes less intensity change is not detectable since it will be immersed in noise. But considering that regular polarizers have an extraction ratio of only 100:1, this extraction ratio yields a great advantage in the precision measurements.

It is also noteworthy that the acrylic windows have a certain reflection ratio inherent on the incident laser light at the front and back surfaces. Since the refractive coefficient of virgin olive oil is about 1.46-1.49, and that of acrylic is almost the same, at the olive-oil-acrylic interfaces we don't expect large reflection ratios, but acrylic-air interface goes as 1.5 to 1 as refractive indices respectively, so the reflection is more pronounced. So a certain cause of intensity loss also stems from this reason. But our experiment says the following:

- 1) Even if the active medium is 0.4 meter long, from a Laser light of 15mW, still a considerable intensity remains to pass through completely.
- 2) Making the interaction length longer is questionable from an experimental point of view. The intensity decay is exponential, and if less than half of the intensity is to remain when we double the interaction length, it is an act not to be taken.
- 3) In an ideal case, one needs windows with anti-reflective coatings to avoid intensity loss at the window surfaces of the sample tube. Maybe next time.

4.2.7.3 Magnetic Field versus the current through the Solenoid

Several questions are pending concerning the structure of the magnetic field. Now that we have constructed active medium tubes of length 1 m and 40 cm, several questions arise and the only way it seems to answer these questions properly is to take experimental data. The questions are as follows:

(1) Is the magnetic field dependent on the position within the solenoid? We ask this question since the 1m tube ends like 5 meters deep into the solenoid and 40 cm tube ends at 35 cm deep within the tube. Does the magnetic field change appreciably within the solenoid or does the uniform field assumption apply safely? To answer this question we will take data in a depth of 5 cm and 25 cm of the solenoid, and plots of magnetic field versus current will emerge to answer this question.

(2) Is the magnetic field altered with the existence of the tube and the active medium? This is a legitimate question since many materials we think to be non-magnetic are actually not really non-magnetic but diamagnetic. We have experienced one case painfully. We have used polyethylene holders for the solenoid at the tips. And for some not-so-obvious reason we saw that the magnetic field turns out smaller than we thought it would be. It turned out that the material itself, polyethylene is a diamagnetic material, and it was the reason for smaller field strengths. We had to disassemble the polyethylene holders from the tips but cutting them out. Eventually we have observed larger magnetic fields. So, now we know that one has to be careful about those kind of assumptions. Therefore we will take the same exact data for the magnetic field when there is the active medium in the solenoid. We want to see if the active medium affects the magnetic field in any way.

Therefore we took data with varying the Applied Voltage with a regulated DC Power Supply. We tabulated the Voltage, Current and Measured Magnetic Field Data in detail. We plot the Magnetic Field versus Current data for each and every case.

We have used PHYWE Teslameter for taking the magnetic field. The Teslameter has a resolution of 10 microTesla. If we consider even the Teslameter application on iPads to have 1 microTesla resolution, this resolution is not so good. Therefore we take lots of data and also show the error bars on the plots for the magnetic field data to be ± 10 microTesla.

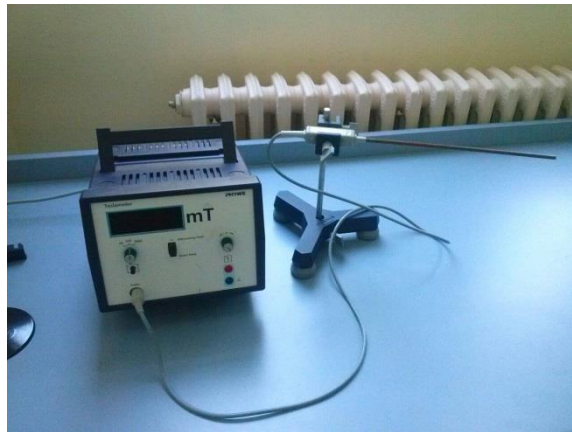


Figure 4.7 PHYWE Teslameter and probe with 10 microTesla resolution.

Also we have used handheld multimeters to take the Voltage and current data, so for the current data, we plot the error bars of 1mA, which is the resolution of the multimeters. Actually with so many data points it seems that we did not have a real problem with resolution for the data set to plot a graph that makes sense.



Figure 4.8 Taking magnetic field data from a depth within the solenoid.

We have also taken the Temperature data at the time of recording the data. It so appears that the solenoid heat up by almost 5°C during the process of taking data. Although it does not change the plots in its essence, it is obvious that Temperature is also a factor in magnetism. We will come back to discuss this point when it is due.



Figure 4.9 B-field, Temperature, Current and Voltage data taking set.

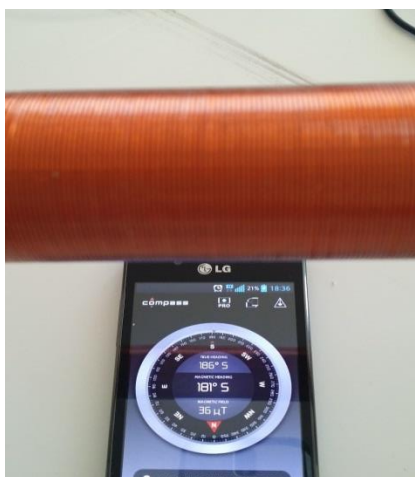


Figure 4.10 The solenoid is positioned along the East-West direction not to interfere with the Earth magnetic field.

4.3 MAGNETIC FIELD MEASUREMENTS

In this section we plot the data and discuss the implications. First question is what is the relationship between the applied current and the magnetic field strength? Do we meet the results of our theoretical calculations? And if there is something wrong, where did we go wrong?

We first check to see the data where we have no material in the solenoid other than air, which is pretty close to the case of vacuum for a magnetic field.

In this form these results tell us that the relationship between the magnetic field and current is a linear one as expected. Taking into account a few very tiny deviations from the linear relationship, we can also draw some conclusions regarding the Physics of the materials used in the set up. So we better compare the results with theoretical predictions and each other.

From the above comparison we can see that the depth and fringing effect of the magnetic field is negligibly small, although we can clearly see that there is a small difference. It seems that the magnetic field deep into the solenoid is very slightly stronger, and we attribute this difference to the fringing effect closer to the tip of the solenoid.

Another important question to ask were whether or not the existence of virgin olive oil as the active medium in the solenoid affects the magnetic field strength or not. For that we need to compare the data for both cases and see if there is a significant effect.

The jump at about 20 Volts DC regulated power (which is about 850 mA) is obvious. The only thing that was significant in the data taking process is that the solenoid was left too cool off for a while at that interval. So it means that, the active medium temperature is also an ingredient. The solenoid warms by several degrees Centigrade during the data taking process. For instance the insides of the solenoid seems to increase from 28°C to 33.5°C. This slight difference in temperature seems to cause a slight difference in the magnetic permeability of the active medium, and hence cause a slight change in the magnetic field around it. Magnetic permeability of materials being Temperature dependent is also a phenomenon the Scientific community is aware of, but we came to observe it here unintentionally. But overall, the observed magnetic field does not seem to be affected significantly by the presence of virgin olive oil as the active medium in the solenoid.

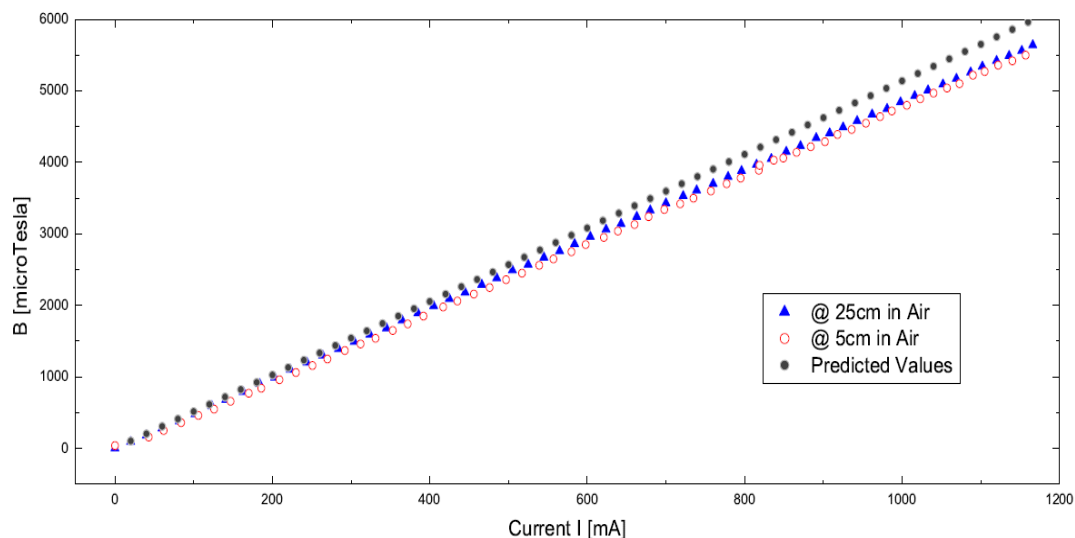


Figure 4.11 Comparison of in air B-field data with estimated values.

Since we have inferred a formula for the predicted values of the B-field as a function of the current in the solenoid, we can also compare our obtained data with the theoretical estimates. The Figure deals with that. Obviously the estimates are not bad, but the data are somewhat smaller than the estimates. The reason it appears to us may be either our estimate of the length of the solenoid is a little bit off, or the inner diameter estimate of the wire may be a little more than what it really is. We are more inclined to think it is the inner core radius estimate that is a little off that caused this tiny shoot off. We also mention that the little systematic difference for 5cm depth in compared to 25cm is considered to be the result of fringing effect at the tip of the selenoid. We also took data from other depths inside the selenoid, from 10cm, 20 cm and 30 cm, and the results were consistent. So we think that the magnetic field is essentially not a function of depth within the selenoid, since the selenoid is a long one, but there is a tiny fringing effect at the tips of the selenoid.

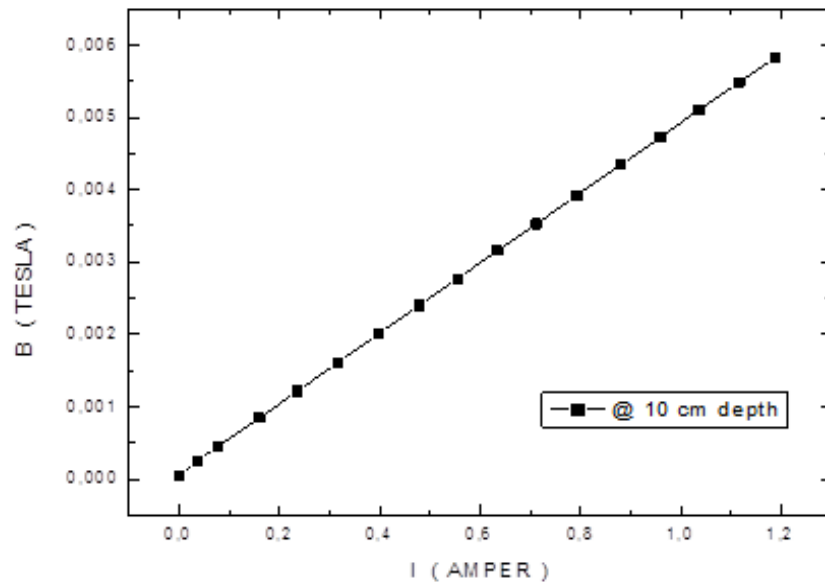


Figure 4.12 B vs I when the Gaussmeter probe is in a depth of 10 *cm*.

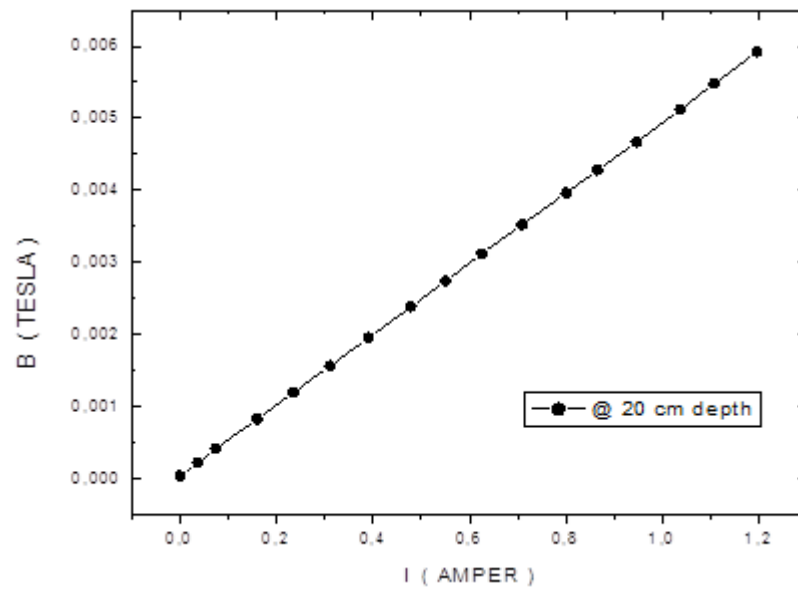


Figure 4.13 B vs I when the Gaussmeter probe is at a depth of 20 *cm*.

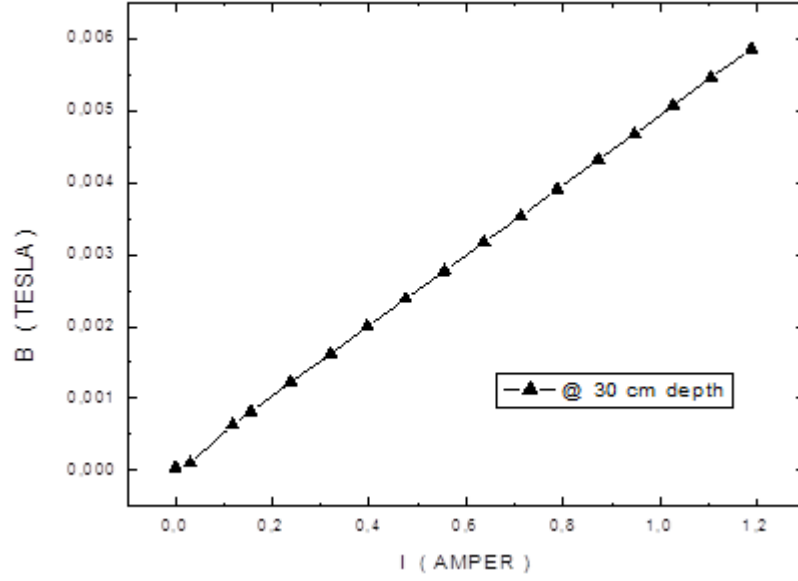


Figure 4.14 B vs I when the Gaussmeter probe is at depth of 30 *cm*.

According to measurements taken at three different depths there has not been much change in the magnetic field.

Other people found depth dependence when their solenoid has a large diameter and a short length. Their ratio is important. [33]

We put the olive oil-filled tube into the solenoid and then we made our depth-dependent measurements. There is not a very big difference compared to the measurement results and graphics. As we expect a linear graph occurred. We make a fit to the data.

$$B = 6.10723 \times 10^{-5} + 0.00491 \cdot I - 4.2876 \times 10^{-5} \cdot I^2 \quad (4.52)$$

The fit is a nice one, but we already know that it should be a linear one. Luckily, the non-linear component is so small and the current we used is also small, so that non-linear term is virtually non-existent, as it should be. Therefore, from now on I assume for the rest of the experiment comfortably that

$$B(I) = 6.10723 \times 10^{-5} + 0.00491 \cdot I \quad (4.53)$$

I did not set the first term to zero, since this time the set up is not along the East-West direction, therefore, with accumulated experimental errors, one could have such a constant magnetic field on the axis when $I = 0$.

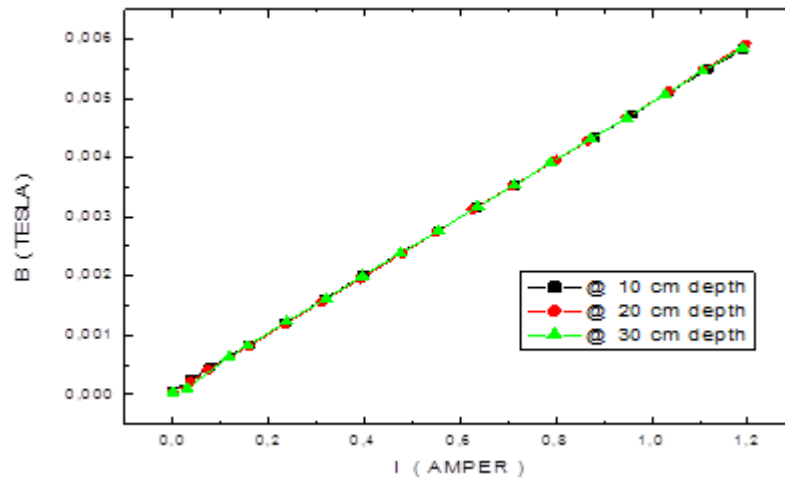


Figure 4.15 All $B(I)$ data @ different depths and their fit.

4.4 OPTICAL DETECTOR

In this short section we will discuss how one can obtain and interpret data with a single photo-detector system.

The incident light on the active medium is polarized, on the order of 100 000:1. Some of this light is reflected back on the front window of the tube, some is reflected back at the back surface, and some is absorbed and scattered by the active medium itself. But none of these acts change the polarization direction of the residual light by any bit. Therefore the light that passed through the active medium and went out, is still very well polarized. Now if the analyzer (which itself is another Glan Thompson polarizer) is exactly in the same direction as the already polarized residual laser light, some tiny bit will also be cut out by the analyzer itself. Since, remember that, the polarizers are very good transparent media, but not perfect. So, after all these processes,

for which we don't have to know what really happens in detail, namely, how much is transmitted, how much intensity is reflected back, or whatever. The transmitted radiation from the analyzer defines the maximum available intensity for the detector. So this Power we refer to as the maximum possible incident power on the photodiode and the notation would be P_0 .

So the next step, we consider that the Analyzer is at exactly 90° angle with the polarizer, in which case the transmitted radiation is nominally zero. But reality, the detector may measure that one out of a hundred thousand light that escapes through the analyzer, as well as some other light in the room, or its own noise that may still remain after filtering out the noise, but that only defines a tiny background, which we know would be zero in an ideal case, therefore, even if we can measure something small, we know that the data that comes out from the detector when the analyzer is perpendicular to the polarizer is only a value to be reset.

If at this stage we apply the magnetic field, we know now that the polarization direction changes slightly, and hence the incident light on the detector will be increased as a function of the rotation angle. Remember that the rotation angle itself is a function of the magnetic field, which we measure simultaneously, the interaction length, which we already know, and the Verdet constant, which may be an unknown. So let us denote the power incident on the detector as a function of the rotation angle as $P(\theta)$.

How much of this Power will be transformed into a current is determined by the Responsivity of the detector, which is a function of the wavelength of the incident light. Let us denote the responsivity as a function of wavelength as $R(\lambda)$ where λ denotes the wavelength of the incident light on the detector.

Since the wavelength is the same, we can write the following equation

$$P(\theta)R(\lambda) = P_0 \cdot R(\lambda) \cdot \sin^2(\theta) \quad (4.54)$$

When it translates into the current the photo-diode detects

$$\frac{I(\theta)}{I(0)} = \sin^2(\theta) \quad (4.55)$$

And the angle of rotation is

$$\theta = \arcsin\left(\sqrt{\frac{I(\theta)}{I(0)}}\right) \quad (4.56)$$

Since

$$\theta = B \cdot V \cdot L = \arcsin\left(\sqrt{\frac{I(\theta)}{I(0)}}\right) \quad (4.57)$$

means that the Verdet constant of the material would be

$$V = \frac{\arcsin\left(\sqrt{\frac{I(\theta)}{I(0)}}\right)}{B \cdot L} \quad (4.58)$$

This formula suggests that two measurements practically could yield the Verdet constant of a material. The maximum intensity measurement with zero applied magnetic field $I(0)$ and the measurement with applied magnetic field, which would be minimum (ideally zero) when no field is applied. Only two measurements can give us the Verdet constant.

4.4.1 Power Measurements

The change of power of the light measured by two methods.

- (1) The increase of power from a minimum by applying $\vec{B}$.
- (2) The decrease of power from a maximum by applying $\vec{B}$.

4.4.1.1 Power increase from a minimum

First, the brightness of the light would be at its minimum (the ideal case is zero) by adjusting the orientation of the polarizer. But in practice there are two reasons for it not being exactly zero. First one is the ambient light, another reason is that the incident light can not be fully polarized (500:1). As we have noticed during the experiment there is also the effect of non-polarized light due to polarization by reflection within the system. As a result of light reflections from the surfaces such a situation can also occur. The polarization of light is affected by these reflections, therefore, if the alignment is

not perfect, the power of the light does not go to zero on the detector where it should, instead it reaches a relative minimum. We expect an increase in brightness by applying a magnetic field when this minimum value is reached. Increasing the brightness of the light means the rotation of the polarized light. The reason for this rotation is because of the applied magnetic field along the axis of propagation, that is, the Faraday effect.

4.4.1.2 Power decrease from a maximum

Second method is as follows. The polarizer is placed at a special angle where the light would reach its maximum brightness. At this point, we measure the power of the light without any applied magnetic field. Then we applied the external magnetic field and observe and measure a little decrease in brightness due to the rotation of the polarization. Because of the existence of the ambient light in any case, we also measure and remove the contribution of the ambient light value from both measurements.

We measured power of the polarized light by using two different devices. Our first device was an optical power meter called ‘Ophir Nova’. This device was very easily affected by environmental factors so to take measurements was not easy. For this reason, the light falling on the detector side, we prepared a box to prevent the ambient light from reaching the detector easily. In the case where the magnetic field is applied, since the polarized light rotates with an angle of θ degrees, the intensity of light is electric field component of the light falling down to $E_0 \cos \theta$. Remember that the intensity goes as the square of the electric field.

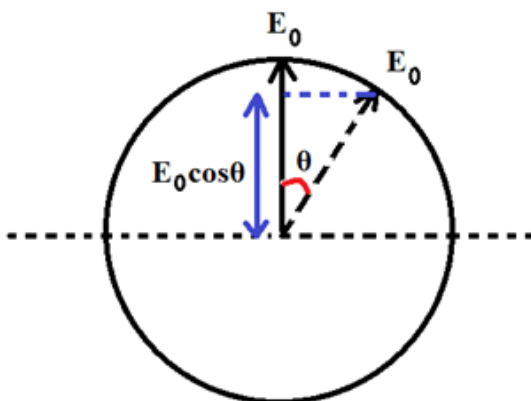


Figure 4.16 Rotation of the polarization angle causes a decrease in the Electric field amplitude if we start with a maximum.

We know that the intensity of light is proportional to the square of the electric field.

$$P \propto E^2 \quad 4.59$$

$$P_0 \propto E_0^2 \quad 4.60$$

where E_0 is the electric field amplitude for the maximum power P_0 on the detector and P_θ is the power on the detector when the polarization direction rotated by an angle of θ

$$P_\theta = E_0^2 \cdot \cos^2 \theta \quad 4.61$$

$$\frac{P_0 - P_\theta}{P_0} = \frac{E_0^2 - E_0^2 \cdot \cos^2 \theta}{E_0^2} \quad 4.62$$

$$\frac{E_0^2(1 - \cos^2 \theta)}{E_0^2} = \sin^2 \theta \quad 4.63$$

so

$$\frac{P_0 - P_\theta}{P_0} = \sin^2 \theta \quad 4.64$$

$$\theta = \sin^{-1} \sqrt{\left(\frac{P_0 - P_\theta}{P_0}\right)} \quad 4.65$$

So the measurement of the maximum power with no field applied, and the power on the detector with applied magnetic field will be sufficient to yield the rotation angle.

4.4.1.3 Measurements with Ophir Nova Powermeter

According to our measurements and calculations, the results are as follows. Firstly, we note that the ambient light is about 54×10^{-11} units because the device is very sensitive to light. Then we measured the intensity of the light by applying the magnetic field by increasing the voltage (30V, minimum). When we applied the

magnetic field the intensity was $13\,800 \times 10^{-11}$ *units*. In cases where no magnetic field is applied (Voltage is 0, maximum) the intensity was $14\,000 \times 10^{-11}$ *units*. If we calculate the results by using the above formula,

$$\sqrt{\frac{(14\,000 - 54) - (13\,800 - 54)}{(14\,000 - 54)}} = \sin \theta \quad 4.66$$

$$0.119754 = \sin \theta \quad 4.67$$

$$\sin^{-1}(0.119754) = 6.878^\circ \quad 4.68$$

$$\theta = 6.878^\circ \quad 4.69$$

This result is a little off, since we expect values about 4° for these parameters, wavelength, active medium length etc.

4.4.1.4. Measurements with Industrial Fiber Optic Photometer

Our second device is a borrowed photometer from Dr. Öztürk. Similar measurements were made with this photometer as the measurements with Ophir Nova, and we reached the following results.

Without covering the detector from ambient light and then off setting the ambient light the ambient light is 4.13 *units*. When no magnetic field is applied (Voltage is 0V, maximum) and the intensity is 11.04 *units*.

When we applied the magnetic field (Voltage is 30V, minimum) and the intensity is 10.86 *units*.

So the rotational angle using the second method explained above is,

$$\sqrt{\frac{(11.04 - 4.13) - (10.86 - 4.13)}{(11.04 - 4.13)}} = \sin \theta \quad 4.70$$

$$0.107598 = \sin \theta \quad 4.71$$

$$\sin^{-1}(0.107598) = 6.177^\circ \quad 4.72$$

4.4.1.5 Measurements with Photometer device by preventing ambient light

Then we covered the detector path such that we try to protect the active region from the ambient light by covering the detector and letting only directional light from the system as much as possible.

The ambient light in this case is only 0.14 *units*.

When no magnetic field is applied (Voltage is 0V, maximum laser light intensity position for GTP) and the total maximum intensity is 11.10 *units*, note that this result includes the effect of tiny ambient light.

When we applied the magnetic field (Voltage is 30V, power on the detector decreases) and the intensity is 11.06 *units*.

So the rotational angle is,

$$\sqrt{\frac{(11.10 - 0.14) - (11.06 - 0.14)}{(11.10 - 0.14)}} = \sin \theta \quad 4.73$$

$$0.060412 = \sin \theta \quad 4.74$$

$$\sin^{-1}(0.060412) = 3.463^\circ \quad 4.75$$

This last measurement result is a more realistic one.

In this case the mechanism is running, and it is seen that occurrence of a rotation amount of polarized light. The result is a little different out of the calculated value of the measurement results is the one of the detectors (Ophir Nova) can be very sensitive. It is an old but versatile device. May be the other reason is that measurements were made in daytime, although sunlight was blocked, the room is not a totally dark room. Unfortunately we could not create an environment full of darkness, whereas a fully dark room would be ideal for such precision measurements.

According to measurement results, the system we have established is working. The result of using a solenoid of 1 meter length and a tube 40 cm in length filled with olive oil should not be overlooked that some of the losses.

4.4.2 Problems in Optical Alignment: Polarization by Reflection

A major problem we noticed with alignment is preventing unpolarized light from the systems internal reflections. Although the incoming laser is very well polarized, we noticed that if we can not align the system properly, reflections at the boundaries, or reflections of scattered light from the interfaces may disturb our measurements.

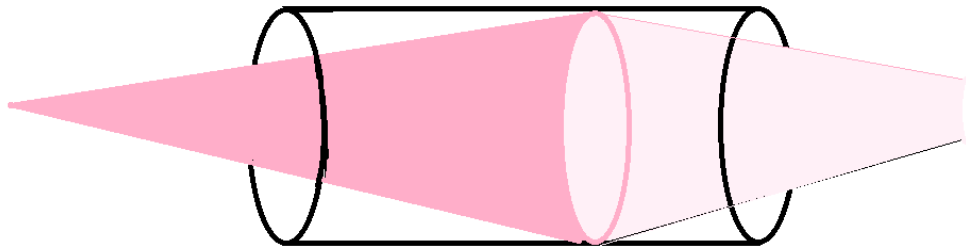


Figure 4.17 Total internal reflection of a strongly divergent laser beam at the interface of a dielectric cylinder.

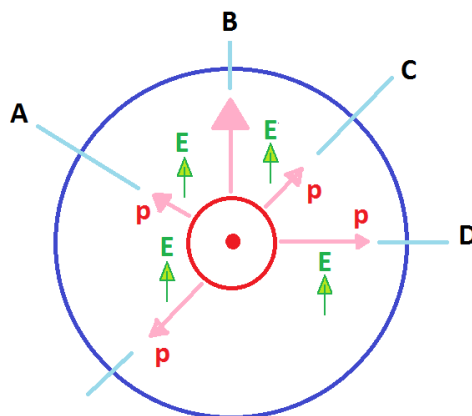


Figure 4.18 Cross sectional representation of total internal reflection for the case of a diverging laser beam into a cylindrical dielectric interface.

Here we refer to the reflection coefficients derived in previous chapters. Remember that, a diverging laser light (where the laser light itself is polarized) the wavefront propagates in all directions into the cylinder, but in any case the electric field direction is in the same direction as it propagates into the cylinder. E in the figure denotes the Electric field direction, which, in this case is upwards. In this scheme p represents the propagation wavefront of the laser light into the cylinder. For the case of point B for instance, the Electric field might fall into the plane of incidence. For the case of point D however, the plane of incidence has nothing to do with the direction of polarization of the electric field, the electric field is totally parallel to the surface. So, we can say that, since the reflectance coefficients are different for parallel and perpendicular polarized components with respect to the plane of incidence, the reflected waves will have their axis of polarizations rotated. Which means that one will have all kinds of polarizations after a total internal reflection on a dielectric surface, even though the source itself is polarized.

4.4.3 Scattered light as a source of noise

The same applies to the case of reflected wave on the interface of scattered light, which is a good deal of a fraction of the incoming laser light. Scattered light may come back into play with reflections at the boundaries as unpolarized light. That kind of light may prevent the Glan Thompson polarizer to act properly when the laser light is supposed to go to zero assuming the light is properly polarized. This phenomena makes the alignment of the optics much more important. We think that we can prevent this by using larger and shorter selenoids. We used several iris in the set up to clean up stray light from the system. Such unpolarized light is a major source of noise in the system.

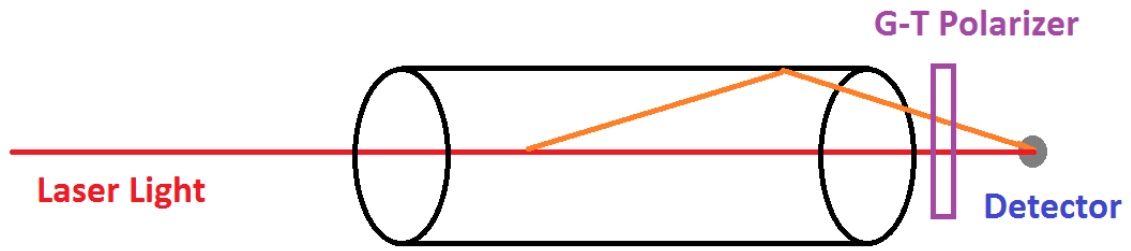


Figure 4.19 Total internal reflection of scattered light from the active medium may fall on the detector.

Scattered light goes in all directions. Some of this light may fall on the dielectric air interface, may be reflected back and fall on the detector. During all these optical processes the polarization direction of this light may have changed. When we align the GT polarizer in such a way that the polarization direction of the incoming laser light is perpendicular to that of the GT polarizer, we expect no transmission. Or negligible amounts of transmission. But the scattered and reflected light coming into play as noise may have all kinds of polarizations. Therefore, we noticed that, during the experiment, we have to use several irie, to prevent the scattered light from falling on the detector.

4.4.4 The Calculation of Verdet Constant of Our Olive Oil

We obtained that Verdet constant of the olive oil which we used by using formulas we found in the previous chapters. According to the calculations, the angle of rotation, θ , was found 3.463 degrees. As we know the rotation angle value we can calculate the Verdet constant of the olive oil. From the equation 4.56

$$\theta = VBL \quad 4.76$$

If Verdet constant, V , is left alone

$$V = \frac{\theta}{BL} \quad 4.77$$

Since the angle of rotation, θ , is in degrees unit, we have to convert it to radians unit. Because θ is given in terms of radiant in the formula.

Also, since the applied magnetic field we found the plot of the graph:

$$B = 6.10723 \times 10^{-5} + 0.00491 \cdot I \quad 4.78$$

The current is 1.12 Ampere

$$B = 0.00497214 \text{ T} \quad 4.79$$

Interaction length is 0.4 meter, so the Verdet constant is

$$V = \frac{\frac{3.463 \times \pi}{180}}{0.00497214 \times 0.4} \quad 4.80$$

$$V = 30.39 \frac{\text{rad}}{\text{T} \cdot \text{m}} \quad 4.81$$

Thus we measured the Verdet constant of the olive oil we used. Our result is close to the findings of Shakir et al. [2]. Composition of olive oil from different sources may vary due to the source and region it is grown. So we constructed a Verdet constant measurement system that works. Now we can work on the details of the system as a next step.

CHAPTER 5

CONCLUSION

In this thesis, we developed a modular Verdet constant measurement system for essentially liquid transparent or translucent materials. Each material has a characteristic Verdet constant with different values. Since the Verdet constant of virgin olive oil is observed to be relatively high recently we chose the olive oil as active material. Because if we use a material which has a high Verdet constant, we will have a larger angle of rotation and we presumed the detection will be simpler. Another reason for choosing the olive oil that this material is both relatively cheap and easy to find in compared to other materials and crystals with high Verdet constants. Also, the absorption coefficient seems to be relatively small that lets us use the active media with longer interaction lengths.

There were several alternatives for the construction of a Verdet constant measurement system. We could use two or one detector system. We noticed that first we had to go through a single detector system. We could use a DC system with a Multimeter or an AC system with a Lock-in amplifier. We also noticed that it would be a leap step if we did not go through the DC system first. And hence we could use a long interaction length system or a short one. Since we did not utilize large magnetic fields that require heavy instrumentation, we have produced our own solenoid and went through the measurements with a regulated DC Power supply.

The main difficulty in the experimentation has been by putting the virgin olive oil into the tubes length of 40 *cm* and 1 *meter*. Making a tube with virgin olive oil in it that would not leak or have air in it has been extensively difficult and we had to go through different methods repeatedly. One conclusion we could draw at this stage

would be to use a more practical and simple method to locate the samples. It is a good idea to go with the good old quartz sample holders with larger magnetic fields and may be larger currents in practice.

We put the tubes into the solenoid which is 110 *cm* in length and prepared with about 4000 windings per meter with copper wire. Then we applied different voltages between 0 *V*-30 *V* (which is the maximum available from the regulated DC Power supply) and we got our magnetic field results depending on the current. We used two different media for the measurements. Firstly, the olive oil-filled tubes were not put into the solenoid and the magnetic field measurements were taken in open air. Secondly, the olive oil-filled tubes were placed into the solenoid and the magnetic field measurements were taken to see if the virgin olive oil itself has some significant effects on the magnetic field. We see that the presence of olive oil does not effect the field.

The system for a Verdet constant measurement system has been constructed. We also analyze the physical sources for error. We notice that a very long interaction length has its disadvantages as well as its advantages. The scattered light or off-axis misaligned light from the olive oil is reflected back into the system. During the reflection, the polarization state of the light is altered. It contributes to the system as a background noise. To prevent this effect we think that it will be a better idea to use a shorter but a wider solenoid. This requires the use of larger magnetic fields. An improvement on the system will be to build a new system with a wider solenoid which can give larger magnetic fields.

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