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ESTIMATION OF STATIC AND DYNAMIC OPTIMAL HEDGE RATIOS: AN APPLICATION TO THE BIST30 INDEX FUTURES

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STATİK VE DİNAMİK OPTİMAL KORUNMA ORANLARININ TAHMİNİ: BIST30 ENDEKS VADELİ SÖZLEŞMELERİ ÜZERİNE BİR UYGULAMA

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ABSTRACT

Estimation of Static and Dynamic Optimal Hedge Ratios: An Application to the BIST30 Index Futures

Especially in recent years, due to fluctuations in both domestic and international financial market, investors pay more attention to control the risks exposed due to their positions in the markets. Various econometric methods have been developed to estimate the optimal hedge ratio. In the literature, some of researchers studied with the static models, while others used dynamics models for estimation of the optimal hedge ratio. The aim of this paper is determining the optimal hedge ratio by providing a comparision of various econometric models. The daily closing prices of BIST 30 Index and BIST30 Index Futures are used for forecasting and the optimal hedge ratio are estimated by employing static models such as the Ordinary Least Squared (OLS) model, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models , the Error Correction Model (ECM) and the Vector Error Correction Model (VECM). Also we employ the bivariate VECM-Diag-BEKK-GARCH model to estimate dynamic hedge ratio. The hedging performance is measured in terms of the variance reduction provided by each models. According to the findings, the VECM and the GARCH models provide only slightly better performance than the traditional linear regression model and the bivariate VECM-Diag-BEKK-GARCH model fails to outperformed the static models.

KEYWORDS: *Optimal hedge ratio; Hedging effectiveness BIST30 futures index; GARCH Models;VECM-Diag-BEKK-GARCH Model*

ÖZET

Statik ve Dinamik Optimal Korunma Oranlarının Tahmini: BIST30 Endeks Vadeli Sözleşmeleri Üzerine Bir Uygulama

Özellikle son yıllarda hem yurt içi hem de uluslararası finansal piyasalardaki dalgalanmalar nedeniyle yatırımcılar piyasalardaki pozisyonlarından dolayı maruz kaldıkları riskleri kontrol etmeye daha fazla önem vermektedirler. Optimal korunma oranını tahmin etmek için çeşitli ekonometrik modeller geliştirilmiştir. Literatürde, optimal korunma oranını tahmin etmek için hem statik hem de dinamik modellerin kullanıldığı birçok çalışma bulunmaktadır. Bu çalışmanın amacı, belirli ekonometrik modellerin karşılaştırmasını sağlayarak optimal korunma oranını belirlemektir. Çalışmada, analizler BIST30 hisse senedi endeks kapanış fiyatları ile BIST30 endeks vadeli sözleşmelerinin kapanış fiyatları üzerinden gerçekleştirilmiş ve optimal korunma oranının tahmini, En küçük kareler (OLS), otoregresif koşullu değişen varyans (GARCH), basit hata düzeltme modeli (ECM) ve vektör hata düzeltme modeli (VECM) gibi statik modeller kullanılarak gerçekleştirilmiştir . Ayrıca dinamik optimal korunma oranını tahmin etmek için iki değişkenli VECM-Diag-BEKK-GARCH modeli kullanılmıştır. Modellerin riskten koruma performansı varyans değişiminde meydana gelen azalma olarak ölçülmüştür. Çalışma sonuçları, VECM ve GARCH modellerinin geleneksel doğrusal regresyon modeline göre sınırlı olarak daha iyi performans gösterdiğini ve dinamik VECM-Diag-BEKK-GARCH modelinin statik modellerden daha iyi performans sergileme konusunda yetersiz kaldığını göstermiştir.

Anahtar Kelimeler: *Optimal korunma oranı; Korunma etkinliği, BIST30 endeks vadeli sözleşmesi; GARCH Modelleri; VECM-Diag-BEKK-GARCH Modeli*

INTRODUCTION

It is quite obvious that investors face a trade off between expected return and risk. There will always be certain risk associated with investments and realized returns will always be different from the expected returns. Invertors almost seek the highest return and the lowest risk, however, it is impossible due to uncertainty in values of financial variables.

The unpredictable events, accordingly, fluctuations in exchange rates and interest rates always have existed in financial marketplaces and these factors bring about the risks which investors and economic units are exposed to, and risk management has become an important phenomenon.

Hedging is very important tool in where economic fluctuation and financial risk exist, especially in volatile economies. In countries under volatile economic conditions, using derivatives instruments is play important role to hedge investors' portfolio. In simple term, hedging is a techique that oftsets certain sources of risk and the hedge ratio is the number of hedging instruments such as derivatives required to offset the risk of the portfolio.

Investors are only concerned with historical return of portfolio in order to predict future return. If there is a particular risk investors faced, they want to construct a hedge strategy to close to perfect as possible as. It is exactly known that perfect hedge is not possible, but they try to reach out the best hedging ratio they can. When constructing a model in order to optimal hedge ratio, behaviours of the variables used in the model are crucial to attain the correct results. Whether the variance is constant or not, and presence of autocorrelation and cointegration are important for the effectiveness of the model to be used in predicting future returns. If series have normal distribution, variance is a used for measurement of risk. However, several studies showed that financial series have not normal distribution and measuring risk with only variance assumption would not give accurate result. Nevertheless, models based on this assumption are still used for comparison purposes.

As for portfolio, risk is calculated based on covariance matrices containing the variance and covariance of the return of financial assets included in porfolio. Although classical variance and covariance approaches ignore the autoregressive relationship of squared returns, they are still used for estimates. Varios univariate and multivariate models have been developed by researchers. The models with autoregressive conditions , which was firslty presented by Engle(1982), have been used in finance literature, however, some of researchers showed that the model may not be exactly appropriate in some instance.

Many studies have performed based on OLS and GARCH models to calculate the optimal hedge ratio and several studies show that heteroskedasticity in spot and futures return have an impact on the optimal hedge ratio and its effectiveness. The GARCH family models take into account the time varying covariance futures and spot return. At the same time, even though unrealistic assumptions, minimum variance hedge ratio estimated by OLS still provides a clear benchmark to evaluate hedging performance.

In this study, a term of minimum variance hedge ratio is deemed as a number of futures contracts that should be purchased with regards to spot position held which is minimizes the variance of return of the hedged portfolio. We construct several models in order to calculate the optimal hedge ratio by using a portfolio of spot instrument to be hedged with the use of BIST30 index futures contracts and then seek to determine which model is the best to estimate the hedge ratio.

This study consists of three chapters. In the first chapter, we will explain fundamental components of risk and risk management. In chapter two, we will provide backround and research developments related to hedge ratio and then, explain the methologdy used in the study. Both the static and dynamic models are performed in order to estimate the optimal hedge ratio. In the last chapter, empirical results of the models will be demonstrated and their performances will be compared according to both obtained variance reduction and statistical consistency of the model.

CHAPTER 1

1.1 Definition of Volatility

Value of portfolio depends on the market variables such as interest rates, exchange rates, equity price, etc. and it is very important for an investor to monitor volatilities of these market variables to protect yourselves from any financial lost. Even though volatility is not completely predictable, the use of several econometric measures help to construct occuring volatility proxies.

In the simplest term, volatility of a variable is defined as the standard deviation of the return. When volatility is used for risk management, it is the standard deviation of the continuously compounded return per day. Volatility and correlation may be relatively high during some periods, whereas during other periods it may be relatively low. Hedging instrument is crucial to prevent financial risk arising from the volatility.

For times series, a common assumption is that the change in assets returns is normally distributed, however, in practice, most financial variables such as exchange rates, interest rates, and equity prices, are more likely to experience larger or smaller changes than the normal distribution. It means that the returns are not identically distributed and have heavy tails. As a consequence, the volatility changes over time and is in tendency to cluster in the periods of low volatility and periods of large volatility. It is called autoregressive conditional heteroskadasticty. Also, today's volatility of a variable may depends on the past volatility of the variable which shows us the autocorrelation.

When it is assumed that the returns on each day are independent with a constant variance, the volatility of the return over N days is equal to the standart deviation of the return times $\sqrt{N}$. If the volatility per day of a return on day n is defined as σ_n

and the the continuously compounded return at the end of day i described as R_n, the volatility can be defined by following formula:

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} \alpha_i R_{n-i}^2, \qquad \left(R_i = ln\frac{S_i}{S_{i-1}}\right) \tag{1}$$

where the variable m is the most recent observation and the variable S_i indicates the price of the variable.

In equation (1), it is assumed that each of R_{n-i}^2 has the same weight. If the weight is taken into account consedering recency of the data, the equation is formed as:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i R_{n-i}^2 \tag{2}$$

Where α is the weight of return and γ is the weight of the long run variance (V_L). The sum of the weights must be equals one. This term was first proposed by Engle (1982) and is named as ARCH (m). And also, if the most recent forecast of the variance is taken into account, the equation (2) will be expressed the following form:

$$\sigma_n^2 = \gamma V_L + \alpha R_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{3}$$

Where β inidicates the weight of σ_{n-1}^2, and again the sum of all weight terms is equals to one (α + γ + β =1). The β is know as GARCH (1) and is first suggested by Bollerslev(1986).

The autoregressive conditional heterorscedasticity (ARCH) and generalized autoregressive conditional heterorscedasticity (GARCH) models are widely used approaches to estimate financial volatility through non-linear pattern.

1.2 Definition of Correlation

Estimation of correlation between variables is very important to financial markets participant in order to correctly assess the risk exposure arising from the market volatility. If the two assests are correlated with each other, it will cause higher effect on the portfolio as compared with the zero correlation. When the two assets have positive correlation and the price of assets are down as a result of negative shock in the market, the decrease in value of portfolio including these assets will be huge. Conversely, if the assets have negative correlation, the decrease in portfolio value will be less. The correlation coefficient of assets returns is defined as:

$$\rho = \frac{E(r_1 r_2) - E(r_1)E(r_2)}{\sigma_{r1}\sigma_{r2}} = \frac{cov(r_1 r_2)}{\sigma_{r1}\sigma_{r2}} \tag{4}$$

Where $cov(r_1 r_2)$ indicates the covariance between r_1 and r_2 and is a indicator of relationship between the variables. A variance - covarince matrix is derived for the variance and covariance rates of the variables.

1.3 Hedging Instruments

The general view about risk is that it is beyond our control. However risk is unpredictable, it can be managed and minimized by using correct financial strategy. If companies or individuals, or any actor of financial markets, do not want to bear the risk arising from fluctuation in prices, exchange rate, interest rates or any other unexpected market conditions, hedge the risk by using specialized financial products such as futures, options, forward, currency swap, etc. These products are collectively called as derivatives instrument and used for managing risk as well as speculating. Moreover, derivatives are used to predict the price of underlying assets. For example, the spot price of the futures can serve as estimation of price of underlying commodity. Derivative products are derived from another asset and their values are based on that underlying assets.

Hedgers purchase derivatives insturements to offet the sensivity of their positions against the market risk and consider the hedge ratio, which is the number of hedging vehicles required to eliminate the risk of defenceless position. The most common types of derivatives are futures, options, forward and swaps.

1.3.1 Futures Contracts

Futures contract is an agreement which entitle the transaction parties to buy or sell an underlying asset of standardized quantitity and quality in the future at an agreed price. Futures contracts are traded on organized market with standardized features such as contract sizes, mature and underlying asset and they are not designed according to specific needs of investors. The contracts standardization and the organized market provide more liquidation rather than creating of new forward contract. The profit is defined as the difference between price of futures contract paid on deal date and the price of underlying asset at the maturity. The futures contract are generally traded on commodity, foreign currency, exchange rates, stocks and index. Trading participants have counterparty risk which is a probability of failure to comply with the contracts obligations. For this reason, creditworthiness of the traders is a vital component of financial markets. In the futures markets, standardized arragements and market practices allow transactions to be more realibale without default and counterparty risk is less of a concern.

Futures contracts are most often used by hedgers to protect themselves against price movements. On the other side, for the spreculators, the futures contracts require less margin and provide more leverage than trading of underlying asset directly.

1.3.2 Options

Options give holders the right to buy or sell an asset. There are two types of options. A call option entitles the holder to buy an underlying asset or financial indicator for a certain price and amount on or before the maturity date. A put option gives the right to sell an underlying asset or financial indicator for a certain price and certain amount on or before the maturity date. The contract price is known as the strike price and the maturity date is known as the expire date. The profit of option is the difference between value of option and the purchase price of the option. Options are traded both on organized market and in the OTC (over the counter market) and are traded on currencies, interest rates, commodities and any other products.

Generally, a positive correlation exist between price of a call option and its underlying assets, whereas, negative correlation does exist price of a put option and the underlying assests. The value of call option increases if price of the underlying asset goes up, and the value of put option increases if the underlying goes down. The value of call option cannot exceed the underlying assets. However, there is no upper or lower boundary for put options.

Option and futures contract should not be confused with each other. Option holder has a exercise right to buy or sell the asset, where futures contract gives a binding obligation to deal parties. There are unlimited variety of option strategies which include combination of calls and puts option and provide different payoff patterns such as straddle, collars and spreads etc. Options strategies can be designed according to specific needs of the investor.

1.3.3 Forward Contracts

Forward contracts are the customized futures contracts and traded in the OTC (over the counter market). There is no standardized maturity or underlying asset, and any forward contract can be written with any quantity of asset on any maturity date. When compared with the futures contracts, forward contracts allow more flexibility in managing the risk with adjustable maturity date and contract size. However, they are commonly traded on non organized markets and this limits to reach transaction parties to each other. Generally, seller of a forward contract requests specific requirements and high collateral to execute the transaction and the buyer bear additional cost. In financial market, investors, firms as well as financial institutions are commonly used to forward contracts based on currency pairs. They have short or long position according to expectations on economic indicator. For example, when the investors expect to asset price goes up, they will have long position in forwards, or otherwise, asset price goes down, short position will be taken.

1.3.4 Swap

Swap transactions allow investors to exchange the financial instruments or cash flows for a certain time. Interest rate swap and cross currency swap are generally used for managing the risk. Currency rate swaps allow agreement parties to exchange a series of payments in one currency for a series of payments in another currency. Interest rate swaps allow to exchange of series of interest payments based on a given interest rate for series of interest payments based on floating interest rate. Generally, investor who has a currency position with future dated, in order to hedge the currency position, buy a interest swap contract where paying a fixed rate and receiving on floating rate, such as EURIBOR or other rates. In this way, investor protect herself against fluctuations in the currency.

1.4. Optimal Hedge Ratio

The optimal hedge ratio, or minimum variance hedge ratio, is an important factor in hedging process and determines the optimal number of contracts to purchase to hedge market exposure. Hedging is considered as a position taken to prevent potential losses arisen from market fluctuations and is not considered as a trade for profit. There have been many studies on hedging strategies in the literature and various techiniques have been employed to estimate constant and time varying hedges ratio.

Hedging strategies can be classified into three type: Naive or one-to-one hedge, the beta hedge and minimum variance hedge. Naive or one to one hedge assumes that futures and cash prices do move exactly together and a perfect futures contract exist in the market. In the naive hedging, spot and futures positions have the same magnitude but in opposite direction, which means that hedge ratio is equal to -1. If the change in prices of spot asset and futures are the same, the change in the value of the spot position will be fully compensated by the reverse change in the futures position and a perfect hedge takes place. But the prices do not completely move together.

Imperfect correlation between spot and futures prices was challenged and the first emprical model was employed by Ederington (1979). In his study, the minimum variance hedge ratio is expressed as the ratio of the covariance of the spot and forward portfolio $\sigma_{s,f,t}$ over the conditional variance of the futures contract σ_f^2:

$$h_{t-1} = -\frac{cov(S,F)_t}{var(F)_t} = -\frac{\sigma_{s,f,t}}{\sigma_f^2} \tag{5}$$

When spot and futures prices are not co-integrated and conditional variance-covariance matrix is time invariant, simple Ordinary Least Square (OLS) approach can be used to calculate to optimal hedge ration. The slope coeffient from a simple regression model can be used to obtain a constant hedge ratio:

$$\Delta S_t = \alpha + \beta^* \Delta F_t + e \tag{6}$$

where β^* is the optimal hedge ratio. Time varying conditional variance and the imperfect correlation between the series have not been taken into account by OLS. To deal with this problem, hedging strategies based on ARCH model families have been used and allowed the hedge ratio to be time varying. Also, some researchers pointed out that cointegration between spot and futures markets should be taken into consideration and an error correction term should be included in the model.

In order to estimate the minimum variance hedge ratio, we will construct OLS, ECM, VECM and GARCH models and then compare the hedging effectiveness of these models by comparing them with the unhedged portfolio in terms of variance minimization. We will generate the models where the hedge ratio will assumed to remain constant over time and also perform bivariate error correction GARCH model in order to allow time varying hedge ratios.

CHAPTER 2

2.1 Literature Review

Various studies have been performed to determine the optimal hedge ratio. Some of the applied studies have shown that the OLS method is sufficient in estimating the optimal hedge ratio and the others have used GARCH models with the concern that the OLS method is not an accurate approximation of the hedge ratio.

In the traditional hedging theory, the position in the futures market is equal to position taking in the spot market with same magnitude but opposite sign in order to hedge the spot position. Working (1953) challenged the traditional theory and argued that the hedging was not only minimize the risk but rather to be a form of arbitrage.
Ederington (1979) evaluated the Government National Mortgage Market as a cash instruments and T-Bills futures markets in order to estimate the hedge ratio. He argues that, the hedge ratio is less than one and the main motivation of hedgers is to the risk minimization. He defines the return of portfolio which consist of futures and spot market holdings and he proves that "cash and futures market holding may even the same sign" (page 161). In that study, the slope of the ordinary least squares (OLS) is defined as a minimum variance hedge ratio, which is also expressed as the the covariance between the cash and futures price to variance of the future price. The hedging effectiveness is measured by percent reduction in variance when compared with the unhedged position. Later on, in order to estimate the hedge ratio, the conventional OLS model have been carried out by several researchers such as Figlewski (1984).

The least squares model (OLS) assumes that the expected value of all error terms are the same at any given point when squared and are normally distributed. This assumption is called homoskedasticity. Another assumption of OLS is that dependent and independent variables are not cointegrated and the conditional variance is time invariant.

When the finance literature is examined, there are numerous evidences that financial time series do not meet the conditions of OLS. The spot and futures markets have not the same volatility and have long run relationship. It means that the conditional variance can be changed over the time. Ignoring of the long-term relationships between the series in the OLS model has been criticized by other researches and in order to overcome this problem, alternative models, which allow variance and covariance to change over time, have been proposed.

ARCH model was first introduced by Engle (1982) and its extension to GARCH by Bollerslev (1986). Engle (1982) developed a new stochastic process and called autoregressive conditional heteroskedastic (ARCH) process. In order to take into consideration of time dependent conditional variance, the ARCH process has been included in the model to strengthen the optimal hedge ratio.

At the same time, the error correction model was developed by Granger (1981) and then extended by Engle and Granger (1987). The paper suggests that an error correction factor of series must be included when series are correlated.They find that short run and long run interest rate are cointegrated while wages and prices are not.

The GARCH model assumes that the effects of positive shocks and negative shocks on the volatility are symmetrical. However, several studies show that the negative and positive shocks may asymmetrically affect the volatility. The EGARCH model developed by Nelson (1991) to solve this problem and the asymmetric factor in the volatility distribution is captured in the model.

Autoregressive Conditional Variable Variance on Mean (ARCH-M) was studied by Engle, Lilien, and Robins (1987) where conditional variance was considered as an explanatory variable in the mean equation of ARCH model. They determine that the conditional variance as a indicator of risk premium is time dependent and changes systematically with the perception of undelying uncertainty. The ARCH-

M model is important in testing the relationship between uncertainty and return, which has an important place in financial theory.

Although estimation of optimal hedge ratios developed with GARCH models provide consistent results, they assume that the correlation between the variables remains constant over time. However, the correlation between the variables changes over time and the hedge ratio varies over time as well. Also the conventional regression model disregards both error correction term and the lagged changes in spot and futures prices. To solve this problem, dynamic hedging strategies based on GARCH models have been developed.

The constant variance assumption was challenged by the Baillie and Myers (1991). They studied spot and futures prices of six different commodities and commodity price were modelled by using bivariate GARCH model to estimate the optimal hedge ratio. They underlines that the optimal hedge ratio is changed over time and the assumption of constant optimal rates is not appropriate. Bivariate GARCH models seem to fit data well and give significant time varying estimates of the conditional covariance matrix. Besides, Myers (1991) developed a dynamic model based on GARCH to estimate time varying optimal hedge ratios. He points out that the hedge ratio estimated by a bivariate GARCH model provides better hedging performance compared with the constant hedge ratio. However, he underlines that the performance superiority is very slight and the constant hedge ratio estimated by linear regression model may give better performance.

Lien and Luo (1993) and then Lien (1996) studied an error correction model in order to reach to optimal hedge ratio. In ECM model, delayed values of both spot and forward returns and one delayed value of the error term obtained from the regression equation are added to the model in order to overcome the autocorrelation. Ghosh (1993) and Lien (1996) show that optimal hedge ratios are downwardly biased due to misspecification, if spot and futures are cointegrated and the error correction term (ECT) is not included in the regression.

Kroner and Sultan (1993) used a error correction terms and performed the bivariate error correction model with GARCH structure to consider time varying structure of returns and cointegrated assets prices. They used spot and futures data for six exchange rates: the British Pound, the Canadian Dollar, the German Mark, the Japanese Yen, and Swiss Franc for the period between 1985 and 1990. They show that new information causes the changes in the return distribution of the assets and it become grounds for time varying hedge ratios. Therefore, if the minimum variance hedge ratios is calculated by using conventinal models, it may provide the incorrect estimation. According to both in sample and out of sample results, they conclude that the bivariate GARCH model provides a superior result in comparision with the other methods, even when the transaction cost is taken into account.

Consolidated models were employed such as the the Constant Correlation (CCC-GARCH) models and the later the Dynamic Conditional Correlation (DCC GARH) models. The first model of this type is the Constant Conditional Correlation (CCC) model employed by Bollerslev (1990). In this model, it is supposed that the conditional correlation is constant over time, and the conditional standard deviation is time-varying. Later, the DCC-GARCH model is examined by Engle and Sheppard (2001) which is an extension of the CCC-GARCH model and the conditional correlation matrix is allowed to change over the time.

In a nutshell, some researchers have performed the OLS model to optimal hedge ratio estimation, others have offered GARCH family models by allowing time varying variances and considering other factors disregarded by the OLS assumptions. Later, in the alternatives of the conventional GARCH model have been suggested in recent years. The error correction term has been used to take into account the presence of cointegration and bivariate GARCH models are performed. In spite of the fact that the better hedging performance have been provided by alternatives model, some doubts still exist about the effectiveness of these models. The numerous studies support that there is no standard hedge ratios in the financial literature and the major superior results are not provided as compared with the OLS model.

2.2 Methodology

In this study, we perform static and dynamic models to find the optimal hedge ratios. The hedge ratios derived from the models are compared with unhedged portfolio in terms of reduction in variance ratio. The variance of hedged portfolio is defined by Kroner and Sultan (1993) as:

$$Var\ (H) = \sigma_s^2 + h^2\sigma_f^2 - 2h^*\sigma_{sf} \qquad (7)$$

Where σ_s and σ_f indicate the standard deviation of the spot and futures prices respectively and h^* is the optimal hedge ratio calculated from the models. We use the equation (7) to calculate the variance of the models and then evaluate the hedging effectiveness by using as the ratio of the variance of the unhedged portfolio minus the variance of the hedged protfolio, over the variance of the unhedged portfolio:

$$\text{Hedge effectiveness} = \frac{\text{Var(Unhedged)} - \text{Var(Hedged)}}{\text{Var(Unhedged)}} \qquad (8)$$

Percentage reduction in the variance of the hedged portfolio shows the hedging effectiveness. The hedge ratio of the conventional models will be obtained as the coefficient of the futures return in the model. While in the dynamic model, the ratio between the variance and the covariance of the residuals is instead used in order to obtain the hedge ratio.

2.2.1 Unhedged Portfolio

Unhedged potfolio has very straightforward concept; we purely assume that β is equal to zero in unhedged portfolio in following equation:

$$R_{t=} \Delta S_t - \beta^* \Delta F_t \quad (9)$$

Unhedged portfolio will be used as a benchmark to calculate the variance reduction.

2.2.2 The conventional OLS approach

The simplest way of approximating the minimum variance hedge ratio is by using OLS regression.

The OLS regression is a statistical linear model for estimating the relationship between dependent and independent variables and has been used to calculate risk. The relationship between dependent and independent variables can be represented in the equation (6) and the β is the minimum variance hedge ratio.

The OLS model has several assumptions. One of them is, especially concerned in our study, the OLS assumes that expected value of all error terms are normallly distributed and has a constant variance which is called homoskedasticiy. Also, it assumes that all independent variables are uncorrelated and the observation of error terms are not correlated, it means that there is no seraial correlation.

In despite of some shortcomings, the OLS model is more simple and less complex model when comparing with the dynamics model.

2.2.3 ARCH and GARCH Models

Many studies show that changes in prices of financial assets can be more volatile in financial crises and rather less volatile in steady economic environment. The magnitude of volatility can be different in negative and positive economic shocks. GARCH class of models allow time varying conditional variance and covariance. A conventional regression model such as the OLS model, does not take into account this volatility and the variation of returns is not completely reflected in the model. GARCH family models shed light on describing volatility in financial markets. On the contrary of classic OLS assumption, which supposes the expected value of residuals is the same at any given point when they are squared, GARCH family models pay attention to heteroskedastic process. The GARCH model, or Generalized Autoregressive Conditionally Heteroskedasticity is an extension of ARCH model and is conditionally heteroskedastic with a constant unconditional variance.

The investigation of ARCH effects is very important to consider the situation "error term and recent error terms are more related to each other than the error terms of previous periods", which is observed in many time series and causes the estimation to be less effective if neglected. The ARCH LM Test, which was developed by Engle (1982) to determine whether ARCH effect exists in time series.

In the implementation of the ARCH model, some restrictions have been placed on the parameters in the conditional variance equation due to the use of relatively long delays and the suggestion of a fixed lag structure. Generalized ARCH (GARCH) model based on more historical information and has a more flexible lag structure in order to eliminate the drawback of not meeting these restrictions and reaching parameter estimates with negative variances (Bollerslev, 1986). In this type of model, conditional volatility is defined as a function of q delays of past squared yields as ARCH (q) and in this way, time varying variance covariance structure is allowed. In addition to the ARCH (q) term, GARCH (p) term is added to model which represents the delays of the conditional volatility within itself. GARCH family models have been extensively studied and particularly employed in financial and econometric modelling and analysis. The most important benefit of GARCH framework is that it allows time dependent variance in the model.

The GARCH method is similar to the ARMA model which is a model that includes the autoregressive process (AR) of the error term, as well as the lagged values of the error term variance. The first order autoregression is indicated as:

$$y_t = \gamma y_{t-1} + \varepsilon_t \tag{10}$$

where ε is indicated as white noise with $V(\in) = \sigma^2$. The parameter σ^2 represents the conditional variance of y_t while $\sigma^2/_{1-\gamma^2}$ represents the unconditional variance. Engle (1982) suggests that the forecast intervals will be better if the past information is taken into consideration to prediction of variance and defines the conditional variance as a linear function of the previous period error term squares:

$$R_t = X_t\delta + \varepsilon_t \tag{11}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{12}$$

The equation (11) is the mean equation and the equation (12) is the variance equation of the model. In order to the conditional variance of the model to be positive, the parameters in the model must be positive; ω >0 and α_1≥0,..., α_p≥0. Since $v_t = \varepsilon_t^2 - \sigma_t^2$, the equation can be written as the following form:

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + v_t \tag{13}$$

Bollerslev (1986) models the error terms obtained from the regression model of in the GARCH (p, q) process. The conditional mean equation is defined as:

$$Y_t = a + b'X_t + \varepsilon_t \,,\ \varepsilon_t / \Psi_{t-1} \sim N(0, \sigma_t^2) \tag{14}$$

Conditionally varying variance of error terms according to the GARCH (p, q) model, which includes p number of delayed autoregressive squared error terms and number of delayed conditional variances is defined as (Bollerslev, 1986) :

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-q}^2 \tag{15}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$$

$$\omega > 0, \; \alpha_1, \alpha_2, \ldots, \alpha_p, \; \beta_1, \beta_2, \ldots, \beta_q \geq 0$$

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$$

$$\varepsilon_t \,|\, I_{t-1} \sim N\,(0, \; \sigma_t^2)$$

In the equation (14) and (15), Y_t and σ^2 represent the conditional mean and conditional variance respectively; ε_t is the error term with zero mean and constant variance, q is the lag lenth of the conditional variance and p is the lag lenth of error terms, X_t is the vector of independent variable, α_i and β_j represent respectively the ARCH and GACRH effect on the conditional variance.

For the conditional variance estimation at time t, the number of parameters is reduced by taking the error term one period before and the variance equation at t-1 as a less current error terms. In order to secure the validity of the model, the constant parameters must be greater than zero and the α_i and β_j must be equal to or greater than zero. The p must be equal to or greater than zero and also q must be greater than zero. At the same time, the sum of the α_i and β_j parameters is lower than one to ensure the stationary.

GARCH (p,q) allows that volatility changes with time. For some periods, volatility is relatively high and for other periods, it is relatively low. Also, more recent data have greater weight than older ones. The GARCH model allows weights assigned to variables decrease exponentially as the data become older. If the GARCH model is working well, the autocorrelation should be removed in the model. GARCH models is helpful to evaluate expected return and risk when the returns have clustering volatility. If series have residuals with serially autocorrelated variance following an ARMA (autoregressive moving average) process, GARCH is designed to this time series. The current variance is estimated by considering of past squared observation and past variance in GARCH models.

The equation (16) is reached by modeling both the square of the delayed values of the error and the conditional variance with their own delayed values. Obtaining the OHR with the conventional GARCH (p,q) model is calculated by estimating the mean and variance equations where b indicates the optimal hedge ratio.

$$\Delta S_t = a + b\Delta F_t + u_t \quad u_t \backsim N(0, \sigma_t^2) \tag{16}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

The conventional GARCH model does not consider the asymmetric effects of negative and positive shocks on the volatility. In order to consider this restriction, EGARCH model has been performed in this paper. The EGARCH model (Nelson,1991) can be shown as:

$$h_t = \log(\sigma_t^2) = \omega_t + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{q} \alpha_i \frac{|z_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} \gamma_i \frac{z_{t-i}}{\sigma_{t-i}} \tag{17}$$

In the equation (17), γ is a factor measuring the asymmetric effect. If the γ has negative sign, bad news have a larger impact on the volatility than good news. If the γ is zero, it means that the model is symmetrical and positive and negative shocks have the same leverage effect on the model. β is the measurement of the GARCH effect and α is the measurement of the ARCH effect. A significant positive α signalize the volatility clustering and thus, using of EGARH model may give better results in this kind of situation. When the sum of the α and β is lower than one, it can be interpreted as variance stationary.

Additionally, we perform the GARCH model with Generalised Error Distribution (GED) in order to consider the state of fat tails and non normal distribution in the series. The density function of GED (Nelson,1991) is given by:

$$f(z) = \frac{k * exp\left[-\frac{1}{2}\left|\frac{z}{\lambda}\right|^{k}\right]}{\lambda 2^{(1+\frac{1}{k})}\Gamma\frac{1}{k}} \tag{18}$$

Where 0<k<∞ and k is a measurement of the thickness of tail and is also called as degree of freedom. When k is equal to 2, GED returns have normal distribution. If k is lower than 2, it has heavier tail than the normal distribution; if k is greater than 2, it has thinner tail than the normal distribution.

2.2.4 ECM and VECM

To consider the cointegration and short run and long run relationship between the series, we use the error correction model and the vector error correction model. The ECM directly calculates adjustment speed of dependent variable towards to long run equilibrium arising from a change in independent variables and examines both the short run and long run dynamics of the series. It is defined as following formula:

$$\Delta y_t = \beta_0 + \sum_{i=1}^{n} \beta_i \, \Delta y_{t-i} + \sum_{i=0}^{n} \delta_i \, \Delta x_{t-i} + \varphi z_{t-1} + u_t \tag{19}$$

Where z is the Error Correction term and is defined as:

$$\Delta z_{t-1} = ECT = y_{t-1} - \beta_0 - \beta_1 x_{t-1} \tag{20}$$

The coefficient of ECT, φ, is measurement of the speed at which spot returns to equilibrium after a change in futures.

Before performing the ECM and VECM, short run and long run causality should be examined. If there is no long run relationship despite the short run relationship, the ECM may not be suitable in terms of significancy of the model.

Error correction model can be estimated by Engle and Granger two steps approach. In this context, the series should be non-stationary originally and then become stationary after taking first differences. If the series are cointegrated at the same level and then we can run the error correction model.

In order to run ECM, the lag values of the spot and the futures return and also lag values of error terms derived from the regression will be added to the OLS model (Lien, 1996).

$$\Delta S_t = c + b\Delta F_t + \theta_1 \Delta F_{t-i} + \phi_1 \Delta S_{t-i} + \varphi z_{t-1} + u_t \qquad (21)$$

We also run the vector error correction model to take into account the cointegration. The VECM model is a special condition of the Vector Autoregressive Models (VAR) and all variables are included as endogenous variables. Since the long-term relationship is thought to be lost in the VAR model, the VECM model has been developed.

In VECM, each variables is linear function of past lags of the other variables and past lags of itself. Also error correction term, which represents the residuals from long run cointegrating regression, is included in the model.

$$\Delta S_t = \alpha_1 + \sum_{i=1}^{m} \theta_{1i}\, \Delta F_{t-i} + \sum_{i=1}^{n} \phi_{1i}\, \Delta S_{t-i} + \sum_{i=1}^{r} \varphi_{1i} z_{t-1} + u_{s,t} \qquad (22)$$

$$\Delta F_t = \alpha_2 + \sum_{i=1}^{m} \theta_{2i}\, \Delta S_{t-i} + \sum_{i=1}^{n} \phi_{2i}\, \Delta F_{t-i} + \sum_{i=1}^{r} \varphi_{2i} z_{t-1} + u_{f,t} \qquad (23)$$

Where $u_{s,t}$ and $u_{f,t}$ represent the error terms derived from the VECM. The hedge ratio can be formulated as following:

$$Hedge\ Ratio = HR_{VECM} = -\frac{cov(S,F)}{\sigma_f^2} \tag{24}$$

When we used the OLS, ECM and VECM in order to estimate the hedge ratio, it was assumed that the variance of residuals remained constant over time. However, if the variances of the residuals are varying over time, ECM-GARCH model can give better results. In order to consider the time varying variance, the following model can be used where the residuals derived from the ECM will be added in the variance equation of GARCH model.

$$\Delta S_t = c + b\Delta F_t + \theta_1 \Delta F_{t-i} + \phi_1 \Delta S_{t-i} + \varphi z_{t-1} + u_t \tag{25}$$

$$u_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \varphi_0 + \varphi_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Estimation of the optimal hedge ratio by performed with both traditional GARCH family models and ECM method is static and calculate a constant hedge ratio. The VECM-Diag-BEKK-GARCH model will be used to calculate dynamic hedge ratio, taking into account the time-dependent correlation between variables.

In order to generate VECM-Diag-BEKK-GARCH model, firstly the VAR specification with Vector Error Correction is estimated and residuals from the VECM specification are saved and then draw on for modelling of conditional variance covariance matrix.

The variance equation of GARCH model with BEKK parameterization, which is modeled by Engle and Kroner (1995), is formulated as the following form:

$$H_t = C_0'^* C_0^* + A_{11}'^* \varepsilon_{t-1}\varepsilon_{t-1}' A_{11}^* + \beta_{11}'^* H_{t-1}\beta_{11}^* \tag{26}$$

$$C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix} \quad A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{21}^* \end{bmatrix} \quad B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{21}^* & \beta_{22}^* \end{bmatrix}$$

By combining the VECM and the GARCH model with BEKK parameterization, we generate the following formula in order to estimate dynamic hedge ratio.

$$\Delta S_t = \alpha_1 + \sum_{i=1}^{m} \theta_{1i}\, \Delta F_{t-i} + \sum_{i=1}^{n} \phi_{1i}\, \Delta S_{t-i} + \sum_{i=1}^{r} \varphi_{1i} z_{t-1} + u_{s,t} \tag{27}$$

$$\Delta F_t = \alpha_2 + \sum_{i=1}^{m} \theta_{2i}\, \Delta S_{t-i} + \sum_{i=1}^{n} \phi_{2i}\, \Delta F_{t-i} + \sum_{i=1}^{r} \varphi_{2i} z_{t-1} + u_{f,t} \tag{28}$$

$$\sigma_{s,t}^2 = \alpha_{0,1,1} + \left(\alpha_{1,1,1}\right)^2 u_{s,t-1}^2 + \left(\beta_{1,1,1}\right)^2 \sigma_{s,t-1}^2 \tag{29}$$

$$\sigma_{f,t}^2 = \alpha_{0,2,2} + \left(\alpha_{1,2,2}\right)^2 u_{f,t-1}^2 + \left(\beta_{1,2,2}\right)^2 \sigma_{f,t-1}^2 \tag{30}$$

$$\sigma_{s,f,t}^2 = \alpha_{0,1,2} + \alpha_{1,1,1} u_{s,t-1} \alpha_{1,2,2} + \beta_{1,1,1} \sigma_{s,f,t-1} + \beta_{1,2,2} \tag{31}$$

2.3 Data

2.3.1 Derivatives Market in Turkey

In Turkey, future and option contracts are traded in Derivatives Market (Vadeli İşlem ve Opsiyon Piyasası-VIOP) within Borsa Istanbul. Investors who want to be protected from risk, can manage risk by trading on VIOP. VIOP also provides an opportunity to investors who want to make a profit by investing in the same amount of underlying assets with a lower collateral than spot markets.

Derivative products are derived from another asset based on a specific underlying asset and whose value is based on that underlying asset. There are various derivative contract traded in VIOP. Currently, future contracts based on stock, index, foreign currency, energy, commodities, precious metals, foreign index, metal and TLREF are actively traded in VIOP while option contracts based on stock, index and foreign currency are traded in VIOP.

Transactions in VIOP are carried out through VIOP member and investor is not allowed to trade directly in VIOP; they have to open an account at intermediary institutions and banks which are the member of VIOP. Both for futures and options trading, an initial margin is required in order to trade and a margin call will be demanded to the Investor when the collateral falls below the initial margin amount. Clearing and exchange fees are charged when investor buys and sells transactions of futures and options contracts at VIOP. For futures contracts, fees are calculated based on the traded value and for options, fees are calculated based on premium value. Also, investor who are residents in Turkey are subject to tax on their income earned from future and option contracts. Besides that, investors generally pay fees to the institution where their accounts are located. These are referred to transaction costs which are paid by investor when trading in VIOP. Investors take into account these costs when making investment decisions.

2.3.2 BIST30 Index

Indices have been created to provide an assessment and evaluation on the price and the return performances of group of stocks. Index is reffered to a relative indicator of the change in the average price levels of stocks included in the indices.

The composition of BIST 30 index consists of the 30 stocks of companies traded on the BIST Stars Market and selected according to free float market value and the daily average traded value in the review period. In order to be included in the BIST 30 indices, it is required to be traded in Borsa Istanbul for at least 60 days at the end of the valuation period. On 27 July 2020, after omitting two zeros from the Turkish stock exchange, including BIST30, the index value is expressed with 1,000 point instead of 100,000 point and contract size is decreased from 100 to 10. The price of stocks did not change by removing of zeros.

2.3.3 Data Selection

We use BIST30 Index and BIST30 Index Futures between 2010 and 2017 and they are obtained from Borsa İstanbul Database. Non-trading days of the series are removed and the data between 2010 and 2017 are used to estimate the models. The data between 2017-2019 are used to evaluate the performance of the models as an out ot sample.

In order to avoid spurious regression problem, the return series are obtained by taking the logarithmic price change. We have transformed each series into a series of logarithmic returns according to the following function:

$$\Delta S_t = log\left(\frac{S}{S_{t-1}}\right), \qquad \Delta F_t = log\left(\frac{F}{F_{t-1}}\right) \tag{32}$$

Where ΔSt and ΔFt represent the daily log returns of respectively BIST30 and BIST30 Index Futures. The daily prices are represented in Figure 1 and Figure 2. As we can clearly see from the figures, the series move together and have similar pattern between 2010 and 2019.

Figure 1: Daily Prices of te BIST30 Index in the period from 2010 to 2019

Figure 2: Daily Prices of the BIST30 Index Futures in the period from 2010 to 2019

CHAPTER 3

EMPRICAL RESULTS

In this section, we will evaluate the minimum variance hedge ratios which are estimated from the OLS, GARCH families, ECM, VECM and lastly bivariate VECM-Diag-BEKK- GARCH model. First, we perform certain tests to determine the characteristics of the series. Table 1 exhibits the descriptive statistics of the daily log return of BIST30 Index (BIST30) and BIST30 Index Futures (XU030). Descriptive statistics give a strong evidence that the series that are being studied during the sample period may have time varying variance.

Table 1: Descriptive Statistics of BIST30 Index and BIST30 Index Futures

Descriptive Statitics	BIST30	XU030
Mean	0.0002	0.0002
Standard Deviation	0.015	0.015
Skewness	-0.383	-0.206
Kurtosis	5.870	5.820
Jarque-Bera	924.035***	850.851***
Prabability	0.000	0.000

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

The table 1 shows that H_0 normality hypothesis cannot be accepted as the calculated Jarque Bera test statistics are larger than chi square table value with two degree of freedom and statistically significant at %1 percent level; the series are not normally distributed. The Histogram graphs and stats show that both series are leptokurtic and have fat tails as expected in financial series.

Figure 3: Histogram Graph of BIST30 Index

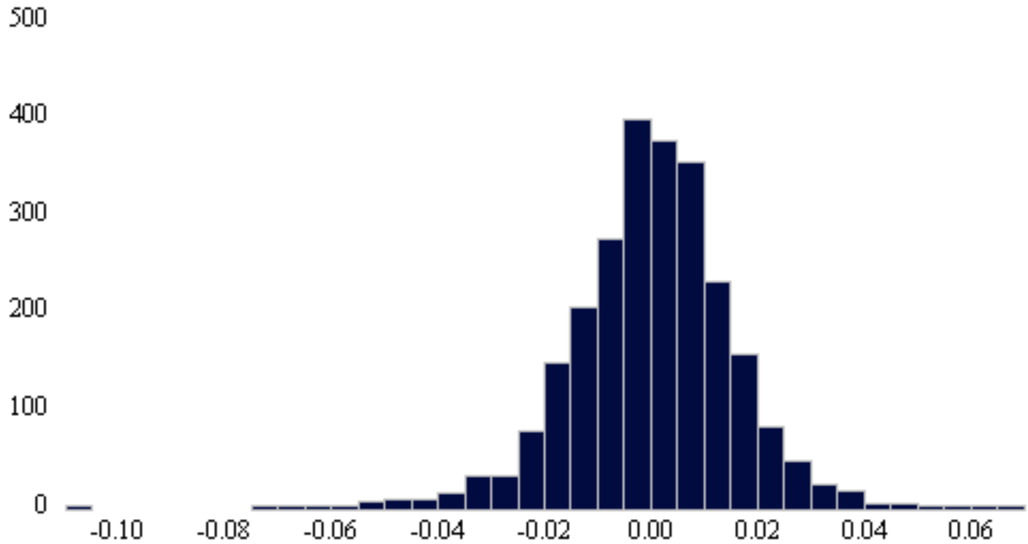
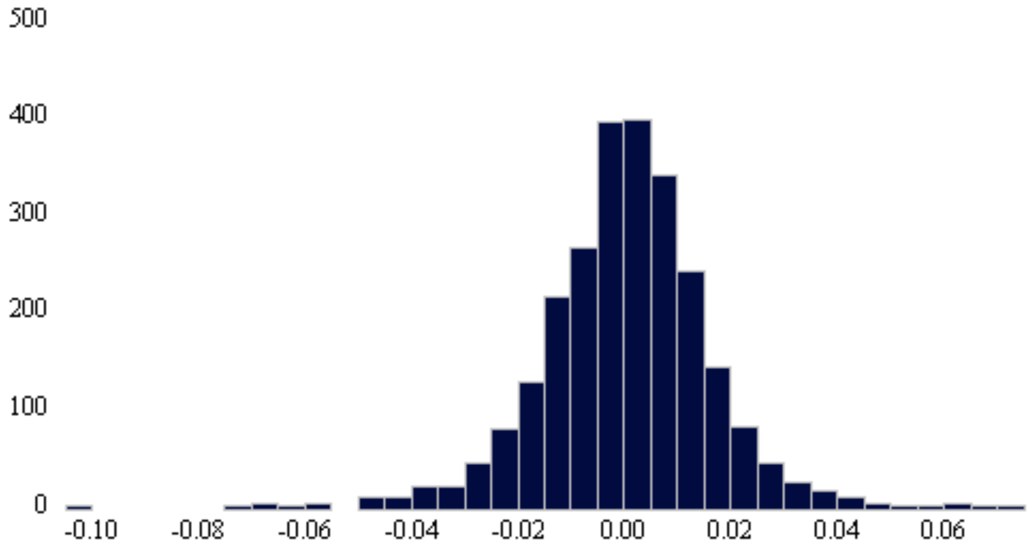


Figure 4: Histogram Graph of BIST30 Index Futures



To check the presence of heteroskedasticty, we model the logarithmic returns by using following GARCH(1,1) formula for both series separately:

$$\Delta Y_i = \mu + \varepsilon_i \qquad (33)$$

$$\varepsilon_i | \Omega_{i-1} \sim td(0, \sigma^2)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Estimation results are shown in table 2. All coefficient of variance equations are positive and significant, the sum of the ARCH and GARCH parameters is less than one for both series. This situation can be accepted as considering the heteroskadasticy gives significant results to optimal hedge ratio estimation.

Table 2: Estimates of Simple GARCH Model for BIST30 Index and BIST30 Futures Index

Coefficients	BIST30	XU030
μ	0.0007 0.880	0.0006 0.900
ω	0.072*** 0.000	0.000*** 0.000
α	0.082*** 0.000	0.063*** 0.000
β	0.857*** 0.000	0.894*** 0.000

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Also in the following sections, we are going to test the presence of autocorrelation. The existence of such relationship might undermine the validity of the OLS approach. Besides, we check whether there exists a long run cointegration relationship between spot and futures prices of currencies.

3.1 Stationarity Of Series

Generally, most economic series are non-stationary. It is important for the research to test of stationarity before generalizing any relationship. Therefore we are starting to test for the presence of unit roots using the Augmented Dickey-Fuller (ADF) test. The delayed differences of the series is taken as the explanatory variable, the following forms can be applied:

$$\Delta y = a_0 y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t \qquad (34)$$

We run ADF test with trend and intercept for each series. If the ADF test statistic value is bigger than the critical values of the MacKinnon test statistic, we can reject the null hypothesis of unit root.

Table 3 shows that the ADF statistic value for the series respectively are -51.004 and -51.499 and the associated p value is 0.000. Notice here that the ADF statistics are less than the critical values at the 1% percent levels so that we can reject the null hypothesis that there is a unit root.

Table 3: The ADF Test Statistics for BIST30 Index and BIST30 Futures Index

	T-Statistic	**Prob.**
BIST30		
Augmented Dickey-Fuller test statistic	-51.004***	0.000
Test critical values: 1% level	-3.43	
5% level	-2.86	
10% level	-2.56	
XU030		
Augmented Dickey-Fuller test statistic	-51.499***	0.000
Test critical values: 1% level	-3.43	
5% level	-2.86	
10% level	-2.56	

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Also, logarithmic returns of BIST30 index and BIST30 index futures are shown in Figure 5 and Figure 6 respectively.

Figure 5: Daily Log Returns of BIST30 Index in the period from 2010 to 2016

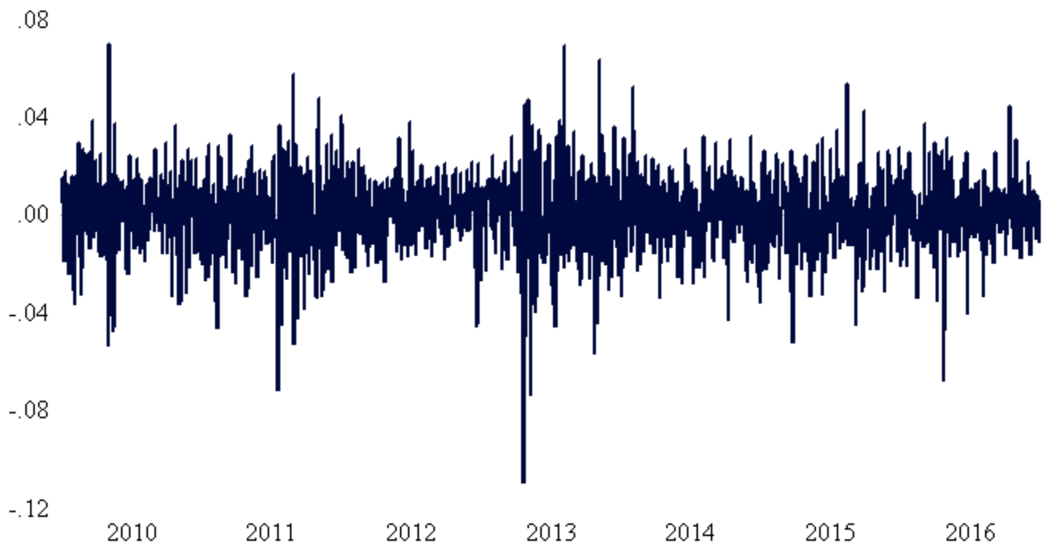
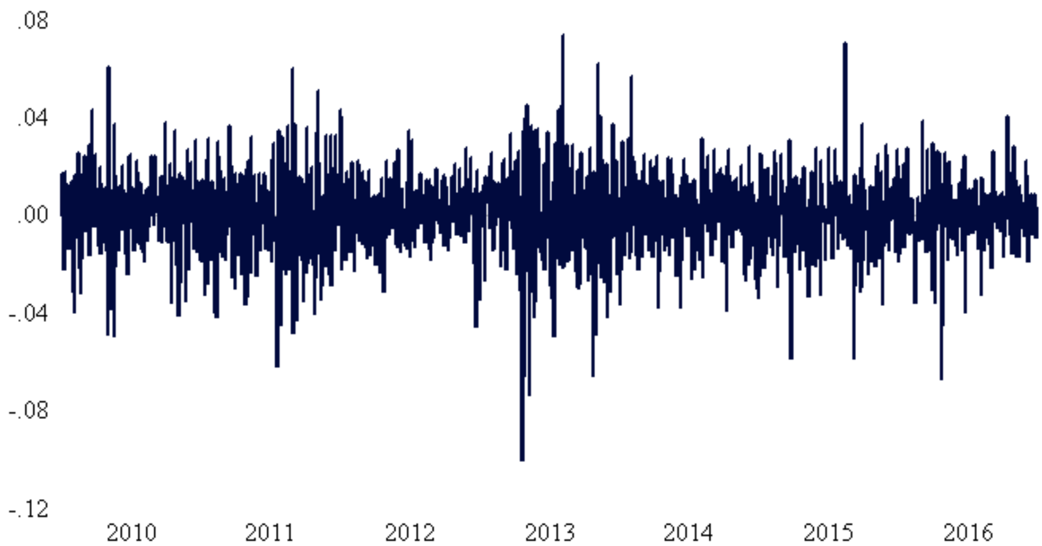


Figure 6: Daily Log Returns of BIST30 Index Futures in the period from 2010 to 2016



Figures 5 and 6 show that the general distribution of the logaritmic return series are around the zero average, suggesting that the return series are stationary and both series do not contains unit root. Also, we clearly see that trend of both series have volatility clustering, periods of large changes being followed by periods of large changes and periods of small changes being followed by small changes.

3.2 OLS Model

The minimum variance hedge ratio estimated by using the OLS are static. This means that once estimated, the hedger uses this ratio of futures to spot during the entire hedging period. The estimates of the OLS model are given in table 4. As we can see the OLS minimum variance hedge ratio is 0.955 and it is less than the traditional hedge stratedy where hedge ratio is -1.

Table 4: Estimates of the OLS Model

Variables	Coefficients	Standard Error	T-Statistic	P-Value
α	1.734	8.862	0.195	0.845
β	0.955***	0.005	169.267	0.000

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

In table 4, the results show that the coefficient is statistically significant, however, the intercept is not statisticaly significant at level %10. It means that there is no linear trend in the data generation process and the estimated coefficients should be biased and inconsistent. This indicates that the hedge ratio estimated from OLS model may be biased and some diagnostic test should be performed in order to evaluate the performance of the regression model.

The results of diagnostic tests are shown in table 5. We start with performing Breusch-Godfrey Serial Correlation test and Ljung-Box Q-statistics test in order to check whether there is serial correlation or not. Breusch-Godfrey Serial Correlation test is performed for 16 lags in order to check the presence of serial autocorrelation.

According to test results, we can reject the null hypothesis as P-value is less than 5, it means that, there is serial correlation at up to 16 lags. Also, we perform to Ljung-Box to check whether the residuals have autocorrelation. The correlogram results are indicated in table 5 and the test statistic shows that there is serial correlation in the residuals. Both the Q-statistic and the Breusch-Godfrey test indicate that the residuals are serially correlated.

Table 5: Results of Diagnotistic Tests on OLS Model

Test	Test Statistic	P-value	Conclusion
Breusch-Godfrey	448.97***	0.000	Reject
Ljung-Box			
Q(5)	338.35***	0.000	Reject
Q(10)	341.70***	0.000	Reject
Q(15)	356.09***	0.000	Reject
Jargue-Bera	385.53***	0.000	Reject
White's Test	67.26***	0.000	Reject
ARCH LM Test	73,546***	0.000	Reject

Note: ‘***’, ‘**’, ‘*’ indicate significance at 1%, 5% and 10% respectively.

Then, we perform normality test for the residuals with using the Jargue-Bera (the JB) test. JB statistic measures that the series have skewness and kurtosis of normal distribution. Under the null hypothesis of normality, the JB is distributed as chi squared with two degrees of freedom. If the Jargue-Bera reported probability exceeds the 5% siginificance level, the hypothesis of normal distribution is accepted. The JB test results is indicated in table 5 and implies that residuals are not normally distributed.

The test results sign that there may be heteroskedasticity and residuals do not conform with a linear pattern, they may be disposed to cluster. The White’s Test is carried out to check whether there is presence of heteroskedasticity or not. White’s test is a test of the null hypothesis of homoskedasticity and demonstrates whether there is no heteroskedasticity. White's test statistic is asymptotically distributed as a chi squared with degrees of freedom equal to the number of slope coefficients with excluding the constant in the regression. The test statistic is shared in the table 5 and shows that there is heteroskedasticity which means that the variance of the residuals in the regression model is not constant. Finally, we run the Lagrange multiplier (LM) test for testing autoregressive conditional heteroskedasticity (ARCH) effects in the residuals. The ARCH test, which is a Lagrange multiplier (LM) test was developed by Engle (1982), in order to detect the presence of heteroskedasticity in the data. The null hypothesis represents that there is no ARCH up to order q in the residuals and based on following formula:

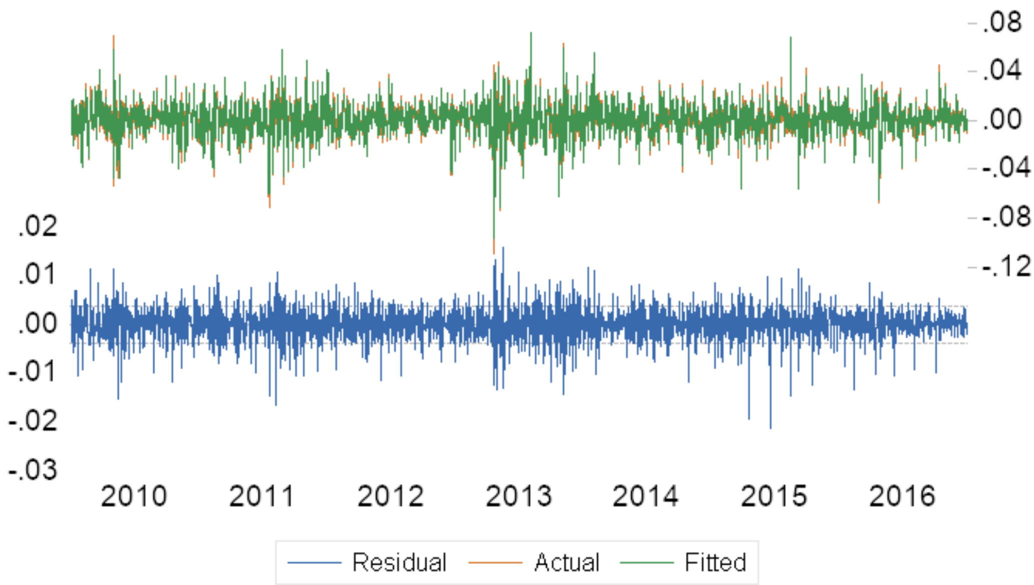
$$e_t^2 = \beta_0 + \left(\sum_{s=1}^{q} \beta_s e_{t-s}^2 \right) + v_t \tag{35}$$

where e indicates the residuals. The ARCH LM test results is indicated in table 5. The P value of Chi squared (q) is less than %5 level which implies there is ARCH effect for up to three order in the residuals and this prove the presence of heteroskadasticity. Proving of presence of autocorrelation and heteroskedasticty bring into doubt the effectivity of using the OLS model to reach the optimal hedge ratio.

3.3 ARCH and GARCH Models

The statistic tests conducted for examining heteroskedasticity in the OLS regression model point out that GARCH approach may give better results for estimating the optimal hedge ratio. GARCH family models incorporate three steps: estimation of a best fitting model, computation the autocorrelation and lastly, testing significance of the model. Firstly, we will check whether there is clustering volatility and the ARCH effect. If there is clustering volatility, periods when large changes are followed by further large changes and periods when small changes are followed by further small changes.

Figure 7: Residual Graph of the OLS



When we look at the graph of the error terms in figure 7, it is observed that the volatility of the series show similarities and error terms may have conditionally varying variances. As it can be clearly observed from figure 7, the return series exhibit volatility clustering, which is similar with the studies in the literature. Also,we have already test the ARCH effect by using The ARCH LM test. After proving of clustering volatility and also ARCH effect, so we can develop an GARCH family model.

We perform several GARCH models and we will compare all these model to find out which one is the best to estimate optimal hedge ratio. Once the models are performed, diagnostic checking will be examined by using some procedures. The preferred model(s) should have no heteroskedasticity and no serial correlations, as well as, have the lowest SIC and AIC value. The normality condition is ignored, because GARCH models by nature have fat tails and are skewed. While model performing, it was noticed that the GARCH(1,1) models underperformed compared to other GARCH (p,q) models, and therefore these models are excluded for comparision of hedge ratios. The test results and the coefficient statistics are displayed on table 6.

According to table 6, the sums of coefficients of GARCH and ARCH effects are lower than one except for GED-GARCH, it can be considered proof of variance stationary. Also, in the EGARCH model, we clearly see that the coefficient of asymmetric effect (γ) has negative sign, it means that there is a asymmetric effect and bad news has a larger impact on the volatility than good news.

Table 6: Estimates of the GARCH Models

	GARCH (1,1)	**GARCH (1,2)**	**EGARCH (1,2,2)**	**GED-GARCH (1,2)**
c	1.548 [0.212] (0.831)	1.372 [0.188] (0.850)	0.0003*** [3.920] (0.000)	9.485 [1.312] (0.189)
h	0.962*** [260.04] (0.000)	0.963*** [266.95] (0.000)	0.974*** [280.75] (0.000)	0.959*** [227.01] (0.000)
α_0	9.229*** [11.999] (0.000)	5.912 [1.596] (0.110)	-0.195*** [-3.382] (0.000)	5.098 [1.022] (0.306)
α_1	0.311*** [8.473] (0.000)	0.284*** [8.223] (0.000)	0.429*** [9.710] (0.000)	0.304*** [6.0193] (0.000)
α_2		-0.266*** [-7.642] (0.000)	-0.382*** [-8.525] (0.000)	0.285*** [-5.627] (0.000)
γ_1			0.192*** [6.658] (0.000)	

γ_2			-0.051* [-1.618] (0.105)	
β_1	0.067 [1.154] (0.248)	0.978*** [177.99] (0.000)	0.985*** [195.69] (0.000)	0.978*** [122.32] (0.000)
k				1.443*** [26.389] (0.000)
AIC	-8.374	-8.391	-8.401	-8.423
SIC	-8.359	-8.373	-8.377	-8.402
R^2	0.940	0.940	0.939	0.940

Notes: (a) T-statistic indicated in [ ] and related p-value in ( ).
(b) '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Ljung-Box Q statistic is performed for testing the autocorrelations and partial autocorrelations of the squared residuals. When the p value of Q-statistic is higher than 5% percent level, the null hypothesis of no serial correlation is accepted, otherwise, the presence of autocorrelation is proven. Ljung-Box Q-statistics and their p-values are reported in table 7 and indicate that The Q-statistics are significant at all lags, which means that there is no serial correlation.

Table 7: Results of the Diagnotistic Tests Conducted on GARCH Models

Model	**Test**	**Test Statistic**	**P-value**	**Result**
GARCH(1,1)	Ljung Box			
	Q(5)	5.062	0.408	Accept
	Q(10)	11.07	0.352	Accept
	Q(15)	17.33	0.299	Accept

GARCH(1,2)	Ljung Box			
	Q(5)	1.927	0.859	Accept
	Q(10)	8.450	0.585	Accept
	Q(15)	9.669	0.840	Accept
EGARCH(1,2,2)	Ljung Box			
	Q(5)	4.134	0.530	Accept
	Q(10)	9.046	0.528	Accept
	Q(15)	11.34	0.728	Accept
GED-GARCH(1,2)	Ljung Box			
	Q(5)	2.633	0.756	Accept
	Q(10)	9.600	0.476	Accept
	Q(15)	10.639	0.778	Accept

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Finally, to test if there is still heteroskedasticity, the Langrange Multiplier test for ARCH effects is conducted. The test results are shown in table 8. The null hypothesis of ARCH-LM Test is that there is no ARCH effect up to selected order in the residuals. According to ARCH LM test results, there is no ARCH up to 36 order in the residuals, it means that ARCH effects are removed by using GARCH models.

Table 8: Results of the ARCH-LM Test Conducted on GARCH Models

Model	ARCH Test		
	Tes Statistic	**P-value**	**Result**
GARCH(1,1)	0.278014	0.598	Accept
GARCH(1,2)	0.704853	0.703	Accept
EGARCH(1,2,2)	3.477868	0.175	Accept
GED-GARCH(1,2)	1.353034	0.508	Accept

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

3.4 ECM and ECM-GARCH Models

Herein, we generate the error correction model and the ECM-GARCH model to estimate the hedge ratio. First, we run the Granger Causality Test. The precondition of the test is that series must be non-stationary but when the series are converted into first difference level, then they should be stationary. We carried out the Augmented Dickey- Fuller Test to test existence of unit roots. The results are shown in table 9. According to test results, the ADF statistic values of BIST30 at level is -1.707 and the associated one-sided p-value is 0.427. The ADF statistic values of XU030 at level is -1.73 and the associated one-sided p-value is 0.415. For both series, the test statistics values are greater than the critical values so we do not reject the null; both series contain unit roots. When first level differences are taken, both series become stationary as the test statistics are less than %5 and reject the null hypothesis.

Table 9: Results of the ADF Unit Root test

	BIST30	**ΔBIST30**	**XU030**	**ΔXU030**
Critical Value (at 1%)	-3.432	-3.432	-3.432	-3.432
Test statistics	-1.707	-20.452***	-1.730	-51.499***
P-Value	0.427	0.000	0.415	0.000
Result	Accept	Reject	Accept	Reject

Notes: (a)The tests are performed based on the logarithmic prices of series.
(b)The null hypothesis is represented as series contains a unit root.
(c) '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Next, we will check if the series are cointegrated. The Granger causality test shows evidence of causality from BIST30 to XU030. After proving the cointegration, we run the model by using the OLS. All parameters of the error correction model are statistically significant.

$$\Delta S_t = 1.84934 + 0.968914\Delta F_t + 0.601801\Delta F_{t-1} + 0.345239\Delta F_{t-2} + 0.173562\Delta F_{t-3} - 0.60365\Delta S_{t-1} - 0.345771\Delta S_{t-2} - 0.180361\Delta S_{t-3} + u_t$$

Table 10: Estimates of the ECM

Variables	Coefficients	T-statistic	P-Value
α_1	1.849	0.278	0.780
b	0.969***	228.18	0.000
θ_1	0.601***	30.44	0.000
θ_2	0.345***	15.629	0.000
θ_3	0.173***	8.750	0.000
ϕ_1	-0.603***	-30.034	0.000
ϕ_2	-0.345***	-15.425	0.000
ϕ_3	-0.180***	-9.015	0.000
R-squared:0.9667 Log Likelihood: 8110.325 AIC: -8.892 SIC: -8.865			

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

Lastly, selected diagnotistic test are performed. According to White'e test result, the statistic values of observation*R^2 is 242.50 and the associated one-sided p-value is 0.000. The value of N*R^2 is greater than the χ^2 table value at %5 percent level, the null hypothesis of homoskedasticity is rejected. The Breusch-Godfrey Serial Correlation test is performed in order to check the presence of serial autocorrelation. According to test results, the N*R^2 is 1506.09 and is greater than the χ^2 table value, the hypothesis of no serial correlation up to order three is rejected. It means that the residuals are serially correlated.

After performing residuals diagnotistic test, the presence of autocorrelation and also heteroskadasticity motivate us to respecified the model for more effective result in estimating the optimal hedge ratio. For that purposed, we performed the ECM-GARCH model.

$$\Delta S_t = 0.0002 + 0.984634\Delta F_t + 0.78154\Delta F_{t-1} + 0.470538\Delta F_{t-2} + 0.202563\Delta F_{t-3} - 0.78777\Delta S_{t-1} - 0.476158\Delta S_{t-2} + 0.902063\Delta S_{t-3} + u_t$$

$$\sigma_t^2 = 1.10371 + 0.975869u_{t-1}^2 + 0.001106\sigma_{t-1}^2$$

Table 11: Estimates of the ECM – GARCH Model

Mean Equation			
Variables	**Coefficients**	**T-statistic**	**P-Value**
α_1	0.0002***	5.8105	0.000
b	0.985***	198.265	0.000
θ_1	0.781***	74.242	0.000
θ_2	0.470***	42.512	0.000
θ_3	0.202***	20.682	0.000
ϕ_1	-0.787***	-71.743	0.000
ϕ_2	-0.476***	-41.306	0.000
ϕ_3	0.902***	99.042	0.000
Variance Equation			
Variables	**Coefficients**	**T-statistic**	**P-Value**
φ_0	1.103***	8.498	0.000
φ_1	0.975***	10.385	0.000
β_1	0.001	0.037	0.970
R-squared. 0.938 AIC -9.022 SIC -8.986			

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

In table 11, all parameters of ECM- GARCH (1,1) models are statistically significant except for α_1 and b. The Ljung-Box Q-statistics are not significant with near 0.9 p-values. The ARCH LM test indicates that we can not reject the null hypothesis that there is no ARCH up to 36 order in the residuals. As this stand, estimation of ECM with the GARCH model eliminates the ARCH effect and time varying variance.

Table 12: ARCH LM Test Results

Heteroskedasticity Test: ARCH			
F-statistic	0.720	Prob. F(36,1749)	0.890
Obs*R-squared	26.108	Prob. Chi-Square(36)	0.887

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

3.5 VECM and VECM-GARCH Models

In order to take into consideration the cointegration between the series, we will model the mean equation using VECM model and then we generate the VECM with the bivariate GARCH model in order to obtain dynamic minimum hedge ration. If the series are cointegrated or have long run relationship, then we can run restirected VAR, that is VECM model model. However, if the variables are not cointagrated, we can not run VEC model, rather unrestiricted VAR should be effective.

Three steps are performed to develop a VECM model. First, we decide how many lag should be choosen. To determine the optimal lag length, we compare the lag length selection criterias. According to VAR Lag Order Selection Criteria, three lag order is selected. Then, the Johansen Cointegration Test is performed to determine the existence of cointegration. The test results are indicated in table 13: there is at most one cointegration between series and they move together in the log run.

Table 13: Unrestricted Cointegration Rank Test (Trace)

Number of Cointegration	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.038	101.946	15.494	0.000
At most 1	0.001	2.907	3.841	0.088

Notes: (a) Trace test indicates 1 cointegrating at the 0.05 level.
(b) * denotes rejection of the hypothesis at the 0.05 level.
(c) **MacKinnon-Haug-Michelis (1999) p-values.

After the preconditions of Johansen Cointegration Test is ensured, we can run the test with linear deterministic trend. If the presence of cointegration between the series and heteroskedasticty are proven, we expect that the estimation of time varying hedge ratios with using time-varying variances and covariances provides better results. In this case, the residuals obtained from VECM will be saved and multivariate GARCH model will be estimated.

The estimates of VECM are indicated in table 14. The coefficient of error correction term represents the speed of adjustment towards to equilibrium. If it has negative sign and is significant, then we can say that there is a long run causality from BIST30 Future Index to BIST30. Estimated equations and the cointegration equation are displayed as:

$$\begin{aligned}\Delta BIST30_t = &-0.03045ect_{t-1} - 0.28276BIST30_{t-1} + 0.16367BIST30_{t-2} \\ &+ 0.07240BIST30_{t-3} + 0.265927XU030_{t-1} \\ &- 0.14411XU030_{t-2} - 0.04566XU030_{t-3} + 0.000239\end{aligned}$$

$$\begin{aligned}\Delta XU030_t = &\ 0.13699ect_{t-1} + 0.14830BIST30_{t-1} + 0.35119BIST30_{t-2} \\ &+ 0.145687BIST30_{t-3} - 0.170688XU030_{t-1} \\ &- 0.34004XU030_{t-2} - 0.11544XU030_{t-3} + 0.000232\end{aligned}$$

$$ect_{t-1} = 1.0000 - 0.99072XU030_t - 6.94528$$

Table 14: Estimates of the VECM

Variables	Coefficients	Std. Error	T-statistic	P-Value
α_1	0.0002	0.000	0.661	0.477
α_2	0.0002	0.000	0.630	0.246
ϕ_{11}	-0.282	-0.125	-2.254	0.403
ϕ_{12}	0.163*	-0.128	1.275	0.087
ϕ_{13}	0.072	-0.111	0.650	0.369
ϕ_{21}	0.148***	-0.127	1.165	0.004
ϕ_{22}	0.351***	-0.130	2.696	0.006
ϕ_{23}	0.145	-0.112	1.290	0.225
θ_{11}	0.265	-0.123	2.153	0.510
θ_{12}	-0.144	-0.126	-1.139	0.130
θ_{13}	-0.045	-0.110	-0.414	0.609
θ_{21}	-0.170***	-0.125	-1.362	0.001
θ_{22}	-0.340***	-0.128	-2.649	0.009
θ_{23}	-0.115	-0.111	-1.032	0.431
φ_1	-0.030	-0.087	-0.348	0.381
φ_2	0.136*	-0.088	1.544	0.102

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

The coefficient of ECT is negative and it is significant with test statistic-0.348 under one-tailed test at %1 percent level. It proves that the long-run causality exist between the series. The optimal hedge ratio is calculated by the following formula:

$$Hedge\ Ratio = \frac{cov(S,F)}{\sigma_f^2} = \frac{0.000236}{0.000245} = 0.0632$$

In order to test the validity of the model, selected residuals diagnotistic tests are performed. First, Portmanteau Tests for Autocorrelations is run to check if there is autocorrelation. The test result is showed in table 15. According to test result, there is serial correlation up to four lags. Next, the VEC Residual Heteroskedasticity test is applied and the null hypothesis of homoskedasticty is rejected with zero p-value.

Table 15: System Residual Portmanteau Tests for Autocorrelations

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
4	15.284***	0.018	15.317	0.017	6

Notes: (a) '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.
(b) The test is valid only for lags larger than the System lag order.
(c) Null Hypothesis: No residual autocorrelations up to lag h.

Table 16: VEC Residual Heteroskedasticity Test

Chi-sq	df	Prob.
292.235***	42	0.000

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

When we consider the cointegration as well as the time varying variance, employing a multivariate GARCH model such as VECM-Diag-BEKK-GARCH model, may produce efficient result to estimate the optimal hedge ratio.

Herein, we employ the bivariate Diag-BEKK-GARCH model with error correction term. We run the model with the diagonal BEKK via software package and the coeficients estimates are reported in table 17:

Table 17: Estimates of the VEC-Diag BEKK-GARCH

Variables	Coefficients	Std. Error	T-statistic	P-Value
α_1	0.0005*	0.000	1.630	0.103
α_2	0.0005*	0.000	1.615	0.106
ϕ_{11}	-0.398***	0.117	-3.393	0.000
ϕ_{12}	0.073	0.121	0.608	0.542
ϕ_{13}	0.134	0.103	1.306	0.191

ϕ_{21}	-0.04	0.117	-0.343	0.731
ϕ_{22}	-0.282**	0.125	-2.259	0.023
ϕ_{23}	-0.185**	0.104	-1.768	0.077
θ_{11}	0.415***	0.114	3.639	0.000
θ_{12}	-0.073	0.122	-0.602	0.547
θ_{13}	-0.101	0.102	-0.990	0.322
θ_{21}	0.047	0.120	0.395	0.692
θ_{22}	0.273**	0.122	2.227	0.025
θ_{23}	0.217**	0.104	2.072	0.038
φ_1	0.021	0.078	0.273	0.784
φ_2	0.186**	0.081	2.279	0.022

Note: '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.

In order to check the presence of autocorrelation, the Portmanteau Autocorrelation Test is performed. Table 18 shows that the bivariate GARCH model with diagonal BEKK parameterization removes the serial correlation previously detected in the VECM.

Table 18: Estimates of the Portmanteau Autocorrelation Test for VECM-Diag-BEKK-GARCH

Lags	**Q-Stat**	**Prob.**	**Adj Q-Stat**	**Prob.**	**df**
1	0.370	0.984	0.370	0.984	4
2	1.158	0.997	1.159	0.997	8
3	6.101	0.910	6.111	0.910	12
4	10.763	0.823	10.782	0.822	16
5	13.641	0.848	13.668	0.846	20

Note: (a) '***', '**', '*' indicate significance at 1%, 5% and 10% respectively.
(b) Null Hypothesis: no residual autocorrelations up to lag h.

Fgures 8 and 9 display residuals for BIST30 and XU030 respectively.

Figure 8: Residual Graph of BIST30 estimated form VEC-Diag BEKK-GARCH Model

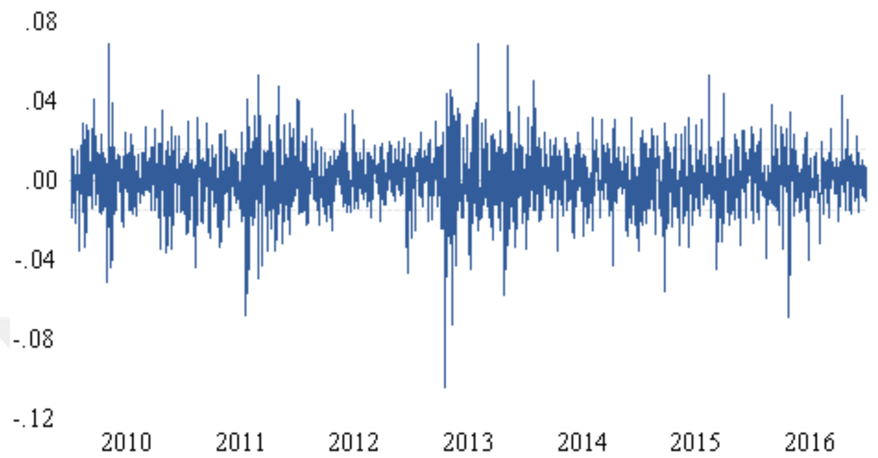
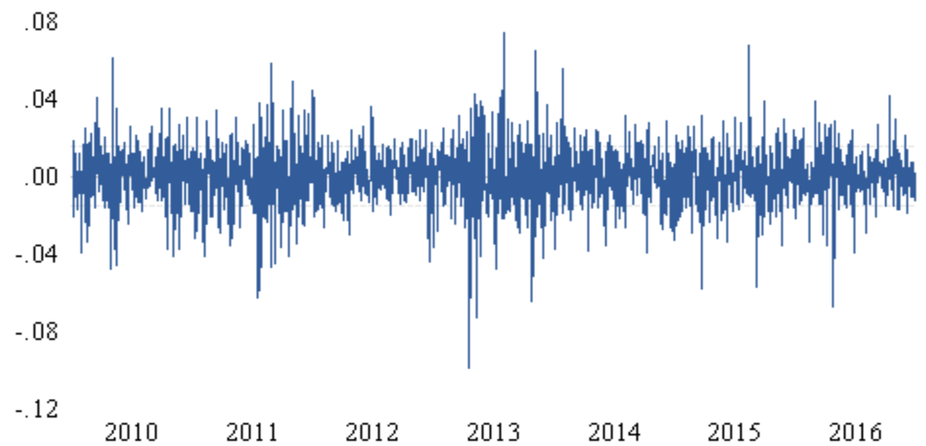
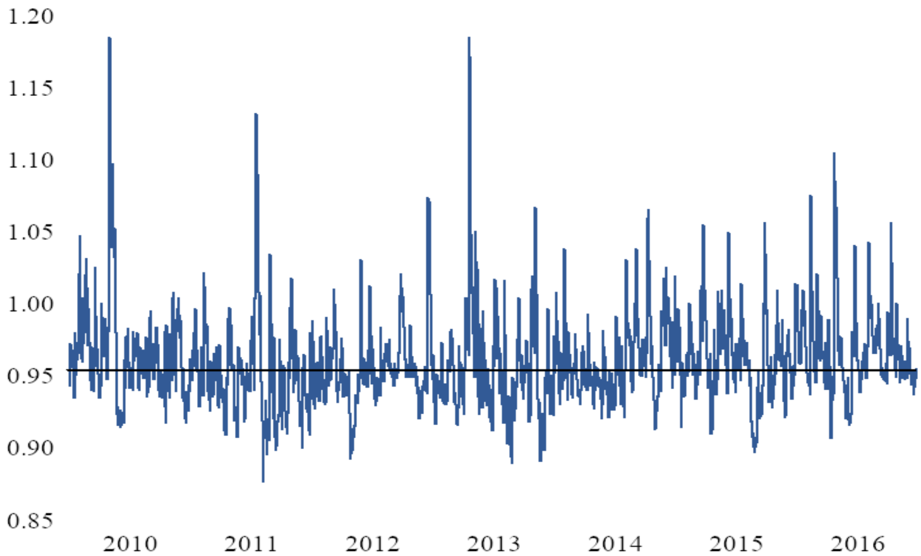


Figure 9: Residual Graph of XU030 Estimated From VEC-Diag BEKK-GARCH Model



After extracting the conditional variance and covariance matrix from the model, we estimate the dynamic hedge ratio via software package. Figure 10 displays the dynamic hedge ratio calculated with the bivariate GARCH(1,1) model. We also add the hedge ratio obtained from the OLS model and show it in black line in figure 10. The sample mean of the dynamic hedge ratio is 0.96 and the ratio range from 0.87 to 1.18 as clearly observed from the Figure 10.

Figure 10: The Plot of the Dynamic Hedge Ratio



3.6 Hedging Performance

In this section, we evaluate and compare the hedging effectiveness of the models. The evaluation of the performance is done for both in sample and out of sample analyses. The out of sample analysis is much more important as it indicates the real market situation. The hedge ratios and the variance reduction rates for in sample and out of sample analyses are show in table 19 and table 20 respectively.

Table 19: Hedging Performances for In Sample

Model	Hedge Ratio	Variance	Variance Reduction %
Unhedged	0.000	0.000244	-
OLS	0.955	0.0000147	0.940
GARCH(1,2)	0.963	0.0000147	0.940
EGARCH(1,2,2)	0.975	0.0000148	0.939
GED-GARCH(1,2)	0.960	0.0000147	0.940
ECM	0.969	0.0000147	0.940

ECM-GARCH	0.985	0.0000149	0.939
VECM	0.963	0.0000147	0.940
VEC-Diag BEKK GARCH	0.960	0.0000147	0.940

Table 20: Hedging Performances for Out of Sample

Model	Hedge Ratio	Variance	Variance Reduction %
Unhedged	0.000	0.0001709	-
OLS	0.884	0.0000219	0.872
GARCH(1,2)	0.908	0.0000219	0.872
EGARCH(1,2,2)	0.937	0.0000223	0.870
GED-GARCH(1,2)	0.976	0.0000233	0.864
ECM	0.907	0.0000219	0.872
ECM-GARCH	0.916	0.0000220	0.871
VECM	0.901	0.0000219	0.872
VEC-Diag BEKK GARCH	0.889	0.0000219	0.872

The hedging performance results show that in sample and out of sample analysis provide a smiliar picture. All models provide a significant reduction in the variance of portfolio compared to the unhedged portfolio. Also, it can be see that the hedge ratios obtained from the GARCH models as well as the ECM and VECM, are greater than the OLS model. Similar to other studies, it can be inferred that the hedge ratio can be biased downward when the autocorrelation and the cointegration relationships are ignored.

In sample analysis, the variance reduction associated with the all model is equal to the %94 except for the EGARCH and ECM-GARCH. The EGARCH and ECM-GARCH provide the variance reduction by %93.9. In out of sample analysis, the variance reduction associated with the OLS model, the GARCH model, the ECM, the VECM and VEC- Diag BEKK- GARCH model is %87.2 while the other models offering a poorer performance. Also, the VEC-Diag BEKK- GARCH model come second after the OLS for providing the lowest hedge ratio.

All models provide a substantial risk reductions compared to the unhedged portfolio and it proves that the hedging with futures contracts allows for risk protection. However, their hedging performance are only slightly better than the OLS. It is cleary observed that the OLS does not underperform the other models, although it has some methodological drawbacks.

CONCLUSION

This study has provided empirically assessment to both static and dynamic hedge ratios estimation by using hedging models with BIST 30 futures contract of Borsa Istanbul and underlines the static and dynamic methods for estimating optimal hedge ratios. The static hedges ratios are derived from the OLS method, GARCH models, ECM and VECM methods and also the dynamic hedge ratios derived from bivariate VECM-Diag-BEKK-GARCH model. The observation between 2010 and 2017 are used to estimate the models and observation between 2017 and 2019 are saved in order to perform out of sample evalution. The model performances have been compared according to variance reduction as a measure.

The hedging performances of the models studied in this paper are very close to each other and this result is in accordance with the findings of other studies. Eventhough, the conventional GARCH models as well as the dynamic GARCH model appear to fit the data well, there is no clear evidence found that they provide better hedging performance for BIST30 Index futures. It seems that the OLS provides the same hedging performance with simple practice, but still the reliability of the model has always been open to discussion due to the misspecification problems. On the other hand, although the time varying hedging ratio seems more realistic, the VECM-Diag-BEKK-GARCH model does not provide a superior result in comparision with the other models despite its complexity. It should always be taken into account that the hedging performance of models vary from asset to asset and it can be complicated to reach the unique optimal hedge ratio. For further studies, it is suggested to carry on more deeply study to analyse whether the GARCH models reflect the behaviours of the returns and the market fluctuations properly.

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