

REWEIGHTED ROBUST DISPERSION ESTIMATION METHODS FOR  
UNIVARIATE S-CHARTS

by

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UNIVARIATE S-CHARTS

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## ABSTRACT

# REWEIGHTED ROBUST DISPERSION ESTIMATION METHODS FOR UNIVARIATE S-CHARTS

Maintaining the quality of manufactured products at a desired level depends on the stability of process dispersion and location parameters and detection of perturbations in these parameters as promptly as possible. In the application of S-Charts, which are one of the most widely used techniques to monitor process variability in statistical process monitoring, sample standard deviation and sample mean are known to be the most efficient traditional estimators in determining process parameters, based on the assumption of independent and normally distributed datasets. In the cases of estimated process parameters from Phase I data clouded with outliers, efficiency of traditional estimators is significantly reduced, and performance of S-Charts are undesirably low. The aim of this thesis is to propose various robust estimators and reweighting procedures to increase the performance of S-Charts in Phase II monitoring. Three dispersion estimators: sample standard deviation, median absolute deviation and scale M-estimator, and three location estimators: sample mean, Harrell-Davis  $q^{th}$  quantile estimator and location M-estimator, are employed to directly construct the Phase II control limits of S-Charts, and also reweighted via different methods. Phase I efficiency of the proposed estimators and Phase II performance of S-Charts constructed from these estimators are assessed both under normality and against diffuse-localized and symmetric-asymmetric contaminations at different contamination density and magnitudes using 50,000-100,000 Monte Carlo simulations. As a result, scale M-estimator combined with Harrell-Davis  $0.5^{th}$  quantile estimators yield parameter estimates with the highest efficiency, and reweighting at skipping level 2-4% using a common location estimate in individuals charts to screen outlier subgroups, and individual observations are found to improve the Phase II performance of the S-Charts.

## ÖZET

# TEK DEĞİŞKENLİ S-GRAFİKLERİ İÇİN YENİDEN AĞIRLIKLANDIRMALI SAĞLAM YAYILIM TAHMİN YÖNTEMLERİ

Elde edilen ürünlerin kalitesinin istenilen seviyede olması ve bu seviyenin korunması, üretim sürecinin konum ve yayılım parametrelerinin kararlılığına ve bu parametrelerdeki değişimlerin mümkün olabildiğince hızlı tespit edilmesine bağlıdır. Sürecin yayılımındaki değişimleri denetleyebilmek için, istatistiksel süreç kontrolünün en yaygın araçlarından biri olan S kontrol grafikleri kullanılmaktadır. Örneklenmiş evrenlerin normal dağılıma uygunluğu varsayıldığında, örneklem ortalaması ve standart sapması, konum ve yayılım tahmin etmek için kullanılan en verimli istatistiklerdir. Öte yandan süreç parametrelerinin tahmin edildiği durumlarda ve örneklemelerde uçdeğer verilerin varlığında, geleneksel örneklem ortalaması önemli ölçüde değişmekte ve örneklem standart sapmasının ani yükselişi kalitesiz ürünlerin tespitini geciktirmektedir. Bu çalışmada, süreç yayılımını tahmin etmek için örneklem standart sapması, medyan mutlak sapma ve M-ölçek tahmin ediciler; ve sürecin konumunu tahmin etmek için ise örneklem ortalaması, Harrell-Davis ve M-konum tahmin ediciler kullanılmış, ve elde edilen istatistikler 2. Aşama kontrol limitlerinin oluşturulmasında doğrudan ve yeniden ağırlıklandırılarak kullanılmıştır. Önerilen tahmin edicilerin 1. Aşama verimlilikleri ve 2. Aşama güçleri, normal dağılımda ve dağılmış-bölgesel, simetrik-asimetrik bozulmalara karşı farklı bozulma miktar ve yoğunluklarında 50,000-100,000 mertebesinde sayılarında yapılan Monte Carlo benzetimleri ile kıyaslanmıştır. Sonuç olarak, %2-4 kırpma miktarı ile yeniden ağırlıklandırılan M-ölçek ve Harrell-Davis konum tahmin edicilerinin beraber kullanımı ve ortak konum parametre tahmini ile çizilen tekil gözlem grafikleri yardımıyla uç değerlerden ayıklanmış yayılım tahmini kullanılarak oluşturulan S kontrol grafiklerinin 2. Aşama performansının yükseldiği belirlenmiştir.

## TABLE OF CONTENTS

|                                                                                                 |        |
|-------------------------------------------------------------------------------------------------|--------|
| ACKNOWLEDGEMENTS . . . . .                                                                      | iii    |
| ABSTRACT . . . . .                                                                              | iv     |
| ÖZET . . . . .                                                                                  | v      |
| LIST OF FIGURES . . . . .                                                                       | ix     |
| LIST OF TABLES . . . . .                                                                        | xiv    |
| LIST OF SYMBOLS . . . . .                                                                       | xxv    |
| LIST OF ACRONYMS/ABBREVIATIONS . . . . .                                                        | xxviii |
| 1. INTRODUCTION . . . . .                                                                       | 1      |
| 2. STATISTICAL PROCESS MONITORING . . . . .                                                     | 4      |
| 2.1. Shewhart Control Charts . . . . .                                                          | 7      |
| 2.2. Two Phases of Data Collection: Phase I and Phase II . . . . .                              | 12     |
| 2.3. Rational Subgrouping . . . . .                                                             | 13     |
| 2.4. Construction of Shewhart Control Charts for Processes with Unknown<br>Parameters . . . . . | 14     |
| 2.5. Performance Measures of Control Charts . . . . .                                           | 17     |
| 2.6. Robust Statistics . . . . .                                                                | 19     |
| 3. METHODOLOGY . . . . .                                                                        | 22     |
| 3.1. Proposed Phase I Estimators . . . . .                                                      | 22     |
| 3.1.1. Dispersion Estimators . . . . .                                                          | 23     |
| 3.1.2. Location Estimators . . . . .                                                            | 25     |
| 3.2. Algorithms of Phase I Estimators . . . . .                                                 | 26     |
| 3.2.1. Method I . . . . .                                                                       | 28     |
| 3.2.2. Method II . . . . .                                                                      | 30     |
| 3.2.3. Method III . . . . .                                                                     | 32     |
| 3.2.4. Method IV . . . . .                                                                      | 34     |
| 3.2.5. Method V . . . . .                                                                       | 35     |
| 3.3. Contamination Models . . . . .                                                             | 39     |
| 3.3.1. Main Contamination Models . . . . .                                                      | 40     |
| 3.3.1.1. CM 1: Diffuse Symmetric Variance Contamination Model                                   | 41     |

|          |                                                                           |    |
|----------|---------------------------------------------------------------------------|----|
| 3.3.1.2. | CM 2: Diffuse Asymmetric Variance Contamination Model . . . . .           | 42 |
| 3.3.1.3. | CM 3: Localized Variance Contamination Model . . . . .                    | 42 |
| 3.3.1.4. | CM 4: Diffuse Mean Contamination Model . . . . .                          | 43 |
| 3.3.1.5. | CM 5: Localized Mean Contamination Model . . . . .                        | 44 |
| 3.3.2.   | Superimposed Contamination Models . . . . .                               | 45 |
| 3.3.2.1. | SCM 1: Diffuse + Localized Variance Contamination Model . . . . .         | 45 |
| 3.3.2.2. | SCM 2: Diffuse + Localized Mean Contamination Model . . . . .             | 46 |
| 3.3.2.3. | SCM 3: Diffuse Variance + Mean Contamination Model . . . . .              | 46 |
| 3.3.2.4. | SCM 4: Localized Variance + Mean Contamination Model . . . . .            | 46 |
| 3.3.2.5. | SCM 5: Diffuse Variance + Localized Mean Contamination Model . . . . .    | 46 |
| 3.3.2.6. | SCM 6: Localized Variance + Diffuse Mean Contamination Model . . . . .    | 46 |
| 3.3.3.   | Autoregressive Contamination Model . . . . .                              | 46 |
| 3.3.3.1. | ACM 1: Autoregressive Process with Normal Data . . . . .                  | 47 |
| 3.3.3.2. | ACM 2: Autoregressive Process with CM 1 . . . . .                         | 47 |
| 3.3.3.3. | ACM 3: Autoregressive Process with CM 4 . . . . .                         | 47 |
| 4.       | RESULTS . . . . .                                                         | 48 |
| 4.1.     | Evaluation of Phase I Efficiency of Proposed Estimation Methods . . . . . | 49 |
| 4.2.     | Derivation of Phase II Control Limits . . . . .                           | 58 |
| 4.3.     | Comparison of Phase II Performances of Proposed Estimators . . . . .      | 59 |
| 4.4.     | Performance of the Estimators on an out-of-Control Process . . . . .      | 74 |
| 4.5.     | Real Data Example . . . . .                                               | 76 |
| 4.5.1.   | Hard-bake Flow Width Example . . . . .                                    | 76 |
| 4.5.2.   | Syringe Length Example . . . . .                                          | 81 |
| 5.       | CONCLUSIONS AND RECOMMENDATIONS . . . . .                                 | 86 |
|          | REFERENCES . . . . .                                                      | 89 |
|          | APPENDIX A: BIAS CORRECTION FACTORS AND COEFFICIENTS . . . . .            | 97 |

|                                                                |     |
|----------------------------------------------------------------|-----|
| APPENDIX B: PHASE I EFFICIENCIES OF PROPOSED ESTIMATORS . . .  | 99  |
| APPENDIX C: PHASE II PERFORMANCES OF S-CHARTS FOR $m=50$ . . . | 109 |
| APPENDIX D: PHASE II PERFORMANCES OF S-CHARTS FOR $m=20$ . . . | 151 |



## LIST OF FIGURES

|              |                                                                         |    |
|--------------|-------------------------------------------------------------------------|----|
| Figure 2.1.  | Representation of process variation via a) histogram b) control charts. | 5  |
| Figure 2.2.  | A typical Shewhart $\bar{X}$ control chart. . . . .                     | 8  |
| Figure 2.3.  | Types of control charts. . . . .                                        | 9  |
| Figure 3.1.  | Steps in computing $\hat{\sigma}^I$ and $\hat{\mu}^I$ . . . . .         | 23 |
| Figure 3.2.  | Algorithm of Method I.A. . . . .                                        | 28 |
| Figure 3.3.  | Algorithm of Method I.B. . . . .                                        | 29 |
| Figure 3.4.  | Algorithm of Method II.A. . . . .                                       | 30 |
| Figure 3.5.  | Algorithm of Method II.B. . . . .                                       | 31 |
| Figure 3.6.  | Algorithm of Method III. . . . .                                        | 33 |
| Figure 3.7.  | Algorithm of Method IV.A. . . . .                                       | 36 |
| Figure 3.8.  | Algorithm of Method IV.B. . . . .                                       | 37 |
| Figure 3.9.  | Algorithm of Method V. . . . .                                          | 38 |
| Figure 3.10. | An example of normally distributed Phase I data. . . . .                | 40 |
| Figure 3.11. | Phase I sampling subjected to CM 1 (K=20%, d=3). . . . .                | 41 |

|              |                                                                                                        |    |
|--------------|--------------------------------------------------------------------------------------------------------|----|
| Figure 3.12. | Phase I sampling subjected to CM 2 ( $K=20\%$ , $d=1$ ). . . . .                                       | 42 |
| Figure 3.13. | Phase I sampling subjected to CM 3 ( $K=20\%$ , $d=3$ ). . . . .                                       | 43 |
| Figure 3.14. | Phase I sampling subjected to CM 4 ( $K=20\%$ , $d=3$ ). . . . .                                       | 44 |
| Figure 3.15. | Phase I sampling subjected to CM 5 ( $K=20\%$ , $d=3$ ). . . . .                                       | 45 |
| Figure 4.1.  | Phase I efficiency of the proposed estimators for $m = 50$ under normal distribution. . . . .          | 51 |
| Figure 4.2.  | Phase I efficiency of the proposed estimators for $m = 20$ under normal distribution. . . . .          | 51 |
| Figure 4.3.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 1 with $K = 0.15$ . . . . .        | 52 |
| Figure 4.4.  | Phase I efficiency of the Method IV and V estimators for $m = 50$ under CM 1 with $K = 0.15$ . . . . . | 53 |
| Figure 4.5.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 2 with $K = 0.15$ . . . . .        | 53 |
| Figure 4.6.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 3 with $K = 0.15$ . . . . .        | 54 |
| Figure 4.7.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 4 with $K = 0.15$ . . . . .        | 55 |
| Figure 4.8.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 5 with $K = 0.15$ . . . . .        | 56 |

|              |                                                                                                                                                                                                                                                                                             |    |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure 4.9.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM 5 with a. $K = 0.10$ and b. $K = 0.20$ . . . . .                                                                                                                                                                        | 56 |
| Figure 4.10. | Phase I efficiency of the proposed estimators for $m = 50$ under a. SCM 1, b. SCM 2, c. SCM 3, d. SCM 4, e. SCM 5, f. SCM 6. . .                                                                                                                                                            | 57 |
| Figure 4.11. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using normally distributed Phase I data. . . . .                                                                                                                                                   | 60 |
| Figure 4.12. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 1 ( $d=3, K=0.15$ ). . . . .                                                                                                                                           | 68 |
| Figure 4.13. | Phase II Performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 2 ( $d=1, K=0.15$ ). . . . .                                                                                                                                           | 69 |
| Figure 4.14. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 3 ( $d=3, K=0.15$ ). . . . .                                                                                                                                           | 70 |
| Figure 4.15. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under CM 4 ( $d=3, K=0.15$ ). . . . .                                                                                                                                           | 71 |
| Figure 4.16. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under a. SCM 1 ( $\gamma = 1$ ) b. SCM 1 ( $\gamma = 1.5$ ) c. SCM 2 ( $\gamma = 1$ ) d. SCM 2( $\gamma = 1.5$ ) e. SCM 3 ( $\gamma = 1$ ) f. SCM 3 ( $\gamma = 1.5$ ). . . . . | 72 |
| Figure 4.17. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and b. $\gamma = 1.5$ using Phase I data under a. SCM 4 ( $\gamma = 1$ ) b. SCM 4 ( $\gamma = 1.5$ ) c. SCM 5 ( $\gamma = 1$ ) d. SCM 5( $\gamma = 1.5$ ) e. SCM 6 ( $\gamma = 1$ ) f. SCM 6 ( $\gamma = 1.5$ ). . . . . | 73 |

|              |                                                                                                                                                                                                                                                                                                                                  |     |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4.18. | Phase II performance of S-Charts for $m = 50$ when a. $\gamma = 1$ and<br>b. $\gamma = 1.5$ using Phase I data under CM 5 ( $d=3, K=0.15$ ). . . . .                                                                                                                                                                             | 74  |
| Figure 4.19. | Box plots of dispersion estimates via various estimators for $m = 50$<br>in the presence of ACM 1 with a. $\phi = 0, \sigma_e^2 =$ b. $\phi = 0.4, \sigma_e^2 = 0.5$<br>c. $\phi = 0.4, \sigma_e^2 = 1$ d. $\phi = 0.8, \sigma_e^2 = 0.5$ e. $\phi = 0.4, \sigma_e^2 = 1$ , and<br>in the presence of f. ACM 2 g. ACM 3. . . . . | 75  |
| Figure 4.20. | S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and<br>V.3 estimators. . . . .                                                                                                                                                                                                                                        | 78  |
| Figure 4.21. | S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and<br>V.3 estimators. . . . .                                                                                                                                                                                                                                        | 85  |
| Figure B.1.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM<br>1 at a. $K = 0.10$ b. $K = 0.20$ . . . . .                                                                                                                                                                                                                | 99  |
| Figure B.2.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM<br>2 at a. $K = 0.10$ b. $K = 0.20$ . . . . .                                                                                                                                                                                                                | 100 |
| Figure B.3.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM<br>3 at a. $K = 0.10$ b. $K = 0.20$ . . . . .                                                                                                                                                                                                                | 101 |
| Figure B.4.  | Phase I efficiency of the proposed estimators for $m = 50$ under CM<br>4 at a. $K = 0.10$ b. $K = 0.20$ . . . . .                                                                                                                                                                                                                | 102 |
| Figure B.5.  | Phase I efficiency of the proposed estimators for $m = 20$ under CM<br>1 at a. $K = 0.10$ b. $K = 0.15$ c. $K = 20$ . . . . .                                                                                                                                                                                                    | 103 |
| Figure B.6.  | Phase I efficiency of the proposed estimators for $m = 20$ under CM<br>2 at a. $K = 0.10$ b. $K = 0.15$ c. $K = 20$ . . . . .                                                                                                                                                                                                    | 104 |

|              |                                                                                                                                 |     |
|--------------|---------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure B.7.  | Phase I efficiency of the proposed estimators for $m = 20$ under CM 3 at a. $K = 0.10$ b. $K = 0.15$ c. $K = 20$ . . . . .      | 105 |
| Figure B.8.  | Phase I efficiency of the proposed estimators for $m = 20$ under CM 4 at a. $K = 0.10$ b. $K = 0.15$ c. $K = 20$ . . . . .      | 106 |
| Figure B.9.  | Phase I efficiency of the proposed estimators for $m = 20$ under CM 5 at a. $K = 0.10$ b. $K = 0.15$ c. $K = 20$ . . . . .      | 107 |
| Figure B.10. | Phase I efficiency of the proposed estimators for $m = 20$ under a. SCM 1 b. SCM 2 c. SCM 3 d. SCM 4 e. SCM 5 f. SCM 6. . . . . | 108 |

## LIST OF TABLES

|            |                                                                                                                                                                              |    |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Table 3.1. | Phase I CLs and bias correction factors of Method I for $m = 50$ .                                                                                                           | 29 |
| Table 3.2. | Phase I CLs and bias correction factors of Method II for $m = 50$ .                                                                                                          | 31 |
| Table 3.3. | Phase I CLs and bias correction factors of Method III for $m = 50$ .                                                                                                         | 34 |
| Table 3.4. | Phase I CLs and bias correction factors of Method IV for $m = 50$ .                                                                                                          | 35 |
| Table 3.5. | Phase I CLs and bias correction factors of Method V for $m = 50$ .                                                                                                           | 39 |
| Table 4.1. | Summary of proposed estimators.                                                                                                                                              | 50 |
| Table 4.2. | Multipliers $U_n$ and $L_n$ for Phase II control limits.                                                                                                                     | 58 |
| Table 4.3. | Multipliers $U_n$ and $L_n$ to determine Phase II control limits cont.                                                                                                       | 59 |
| Table 4.4. | Phase II performance of S-Charts constructed from Method I estimators for $m = 50$ under CM 1-CM 4 ( $d=3$ , $K=0.15$ for CM 1,3-4 and $d=1$ , $K = 0.15$ for CM 2).         | 62 |
| Table 4.5. | Phase II performance of S-Charts constructed from Method II estimators for $m = 50$ under CM 1-CM 4 ( $d=3$ , $K=0.15$ for CM 1,CM 3-CM 4 and $d=1$ , $K = 0.15$ for CM 2).  | 63 |
| Table 4.6. | Phase II performance of S-Charts constructed from Method III estimators for $m = 50$ under CM 1-CM 4 ( $d=3$ , $K=0.15$ for CM 1,CM 3-CM 4 and $d=1$ , $K = 0.15$ for CM 2). | 65 |

|             |                                                                                                                                                                                              |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Table 4.7.  | Phase II performance of S-Charts constructed from Method IV.A estimators for $m = 50$ under CM 1-CM 4 ( $d=3$ , $K=0.15$ for CM 1, CM 3-CM 4 and $d=1$ , $K = 0.15$ for CM 2). . . . .       | 66  |
| Table 4.8.  | Phase II performance of S-Charts constructed from Method IV.B and V estimators for $m = 50$ under CM 1-CM 4 ( $d=3$ , $K=0.15$ for CM 1, CM 3-CM 4 and $d=1$ , $K = 0.15$ for CM 2). . . . . | 67  |
| Table 4.9.  | Flow width measurements (microns) for the hard-bake process. . .                                                                                                                             | 77  |
| Table 4.10. | Process dispersion estimates of hard-bake process and CLs of S-Charts. . . . .                                                                                                               | 79  |
| Table 4.11. | Additional flow width measurements (micron) for monitoring. . . .                                                                                                                            | 80  |
| Table 4.12. | Syringe length (inch) samples. . . . .                                                                                                                                                       | 81  |
| Table 4.13. | Process dispersion estimates of syringe manufacturing process and CLs of S-Charts. . . . .                                                                                                   | 83  |
| Table 4.14. | Additional syringe length measurements (inch) for monitoring. . .                                                                                                                            | 84  |
| Table A.1.  | Bias correction factors are used in the calculation of process dispersion estimate. . . . .                                                                                                  | 97  |
| Table A.2.  | Phase I CLs and bias correction factors for $m = 20$ . . . . .                                                                                                                               | 98  |
| Table C.1.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using normally distributed Phase I data. . . . .                                                                             | 110 |

|             |                                                                                                                                  |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------|-----|
| Table C.2.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using normally distributed Phase I data. . . . .               | 111 |
| Table C.3.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=2$ , $K=0.10$ ). . . . .      | 112 |
| Table C.4.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 ( $d=2$ , $K=0.10$ ). . . . .    | 113 |
| Table C.5.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=3$ , $K=0.15$ ). . . . .      | 114 |
| Table C.6.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 ( $d=3$ , $K=0.15$ ). . . . .    | 115 |
| Table C.7.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=3$ , $K=0.20$ ). . . . .      | 116 |
| Table C.8.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 1 ( $d=3$ , $K=0.20$ ). . . . .    | 117 |
| Table C.9.  | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 2 ( $d=0.75$ , $K=0.10$ ). . . . .   | 118 |
| Table C.10. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under CM 2 ( $d=0.75$ , $K=0.10$ ). . . . . | 119 |
| Table C.11. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under CM 2 ( $d=1$ , $K=0.15$ ). . . . .      | 120 |

|             |                                                                                                                                  |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------|-----|
| Table C.12. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.15$ ). . . . . | 121 |
| Table C.13. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.20$ ). . . . .   | 122 |
| Table C.14. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.20$ ). . . . . | 123 |
| Table C.15. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=2$ , $K=0.10$ ). . . . .   | 124 |
| Table C.16. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=2$ , $K=0.10$ ). . . . . | 125 |
| Table C.17. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.15$ ). . . . .   | 126 |
| Table C.18. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.15$ ). . . . . | 127 |
| Table C.19. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.20$ ). . . . .   | 128 |
| Table C.20. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.20$ ). . . . . | 129 |
| Table C.21. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=2$ , $K=0.10$ ). . . . .   | 130 |

|             |                                                                                                                                  |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------|-----|
| Table C.22. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=2$ , $K=0.10$ ). . . . . | 131 |
| Table C.23. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.15$ ). . . . .   | 132 |
| Table C.24. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.15$ ). . . . . | 133 |
| Table C.25. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.20$ ). . . . .   | 134 |
| Table C.26. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.20$ ). . . . . | 135 |
| Table C.27. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=2$ , $K=0.10$ ). . . . .   | 136 |
| Table C.28. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=2$ , $K=0.10$ ). . . . . | 136 |
| Table C.29. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.15$ ). . . . .   | 137 |
| Table C.30. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.15$ ). . . . . | 137 |
| Table C.31. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.20$ ). . . . .   | 138 |

|             |                                                                                                                                  |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------|-----|
| Table C.32. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.20$ ). . . . . | 138 |
| Table C.33. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under SCM 1. . . . .                       | 139 |
| Table C.34. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under SCM 1. . . . .                     | 140 |
| Table C.35. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under SCM 2. . . . .                       | 141 |
| Table C.36. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under SCM 2. . . . .                     | 142 |
| Table C.37. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under SCM 3. . . . .                       | 143 |
| Table C.38. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under SCM 3. . . . .                     | 144 |
| Table C.39. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under SCM 4. . . . .                       | 145 |
| Table C.40. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using<br>Phase I data under SCM 4. . . . .                     | 146 |
| Table C.41. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using<br>Phase I data under SCM 5. . . . .                       | 147 |

|             |                                                                                                                               |     |
|-------------|-------------------------------------------------------------------------------------------------------------------------------|-----|
| Table C.42. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 5. . . . .                     | 148 |
| Table C.43. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1$ using Phase I data under SCM 6. . . . .                       | 149 |
| Table C.44. | Phase II performance of S-Charts for $m = 50$ when $\gamma = 1.5$ using Phase I data under SCM 6. . . . .                     | 150 |
| Table D.1.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using normally distributed Phase I data. . . . .              | 152 |
| Table D.2.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using normally distributed Phase I data. . . . .            | 153 |
| Table D.3.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=2$ , $K=0.10$ ). . . . .   | 154 |
| Table D.4.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 1 ( $d=2$ , $K=0.10$ ). . . . . | 155 |
| Table D.5.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=3$ , $K=0.15$ ). . . . .   | 156 |
| Table D.6.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using Phase I data under CM 1 ( $d=3$ , $K=0.15$ ). . . . . | 157 |
| Table D.7.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using Phase I data under CM 1 ( $d=3$ , $K=0.20$ ). . . . .   | 158 |

|             |                                                                                                                                     |     |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------|-----|
| Table D.8.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 1 ( $d=3$ , $K=0.20$ ). . . . .    | 159 |
| Table D.9.  | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 2 ( $d=0.75$ , $K=0.10$ ). . . . .   | 160 |
| Table D.10. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 2 ( $d=0.75$ , $K=0.10$ ). . . . . | 161 |
| Table D.11. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.15$ ). . . . .      | 162 |
| Table D.12. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.15$ ). . . . .    | 163 |
| Table D.13. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.20$ ). . . . .      | 164 |
| Table D.14. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 2 ( $d=1$ , $K=0.20$ ). . . . .    | 165 |
| Table D.15. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=2$ , $K=0.10$ ). . . . .      | 166 |
| Table D.16. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=2$ , $K=0.10$ ). . . . .    | 167 |
| Table D.17. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.15$ ). . . . .      | 168 |

|             |                                                                                                                                  |     |
|-------------|----------------------------------------------------------------------------------------------------------------------------------|-----|
| Table D.18. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.15$ ). . . . . | 169 |
| Table D.19. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.20$ ). . . . .   | 170 |
| Table D.20. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 3 ( $d=3$ , $K=0.20$ ). . . . . | 171 |
| Table D.21. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=2$ , $K=0.10$ ). . . . .   | 172 |
| Table D.22. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=2$ , $K=0.10$ ). . . . . | 173 |
| Table D.23. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.15$ ). . . . .   | 174 |
| Table D.24. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.15$ ). . . . . | 175 |
| Table D.25. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.20$ ). . . . .   | 176 |
| Table D.26. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 4 ( $d=3$ , $K=0.20$ ). . . . . | 177 |
| Table D.27. | Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=2$ , $K=0.10$ ). . . . .   | 178 |

|                                                                                                                                              |     |
|----------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Table D.28. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=2$ , $K=0.10$ ). . . . . | 178 |
| Table D.29. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.15$ ). . . . .   | 179 |
| Table D.30. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.15$ ). . . . . | 179 |
| Table D.31. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.20$ ). . . . .   | 180 |
| Table D.32. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under CM 5 ( $d=3$ , $K=0.20$ ). . . . . | 180 |
| Table D.33. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 1. . . . .                       | 181 |
| Table D.34. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 1. . . . .                     | 182 |
| Table D.35. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 2. . . . .                       | 183 |
| Table D.36. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 2. . . . .                     | 184 |
| Table D.37. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 3. . . . .                       | 185 |

|                                                                                                                          |     |
|--------------------------------------------------------------------------------------------------------------------------|-----|
| Table D.38. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 3. . . . . | 186 |
| Table D.39. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 4. . . . .   | 187 |
| Table D.40. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 4. . . . . | 188 |
| Table D.41. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 5. . . . .   | 189 |
| Table D.42. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 5. . . . . | 190 |
| Table D.43. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1$ using<br>Phase I data under SCM 6. . . . .   | 191 |
| Table D.44. Phase II performance of S-Charts for $m = 20$ when $\gamma = 1.5$ using<br>Phase I data under SCM 6. . . . . | 192 |

## LIST OF SYMBOLS

|             |                                                                                  |
|-------------|----------------------------------------------------------------------------------|
| $ARL$       | Average run length                                                               |
| $ARFS$      | Average rate of false signal                                                     |
| $ARS$       | Average rate of signal                                                           |
| $ave$       | Sample average                                                                   |
| $b_n$       | Unbiasing coefficient of $MAD_n$                                                 |
| $c$         | Tuning constant of $ML$ estimator                                                |
| $C$         | Constant to set allowable distance from the central line                         |
| $cARL$      | Conditional average run length                                                   |
| $cARL_{.1}$ | $cARL$ for 10% quantiles of dispersion estimate distributions                    |
| $cARL_{.5}$ | $cARL$ for 50% quantiles of dispersion estimate distributions                    |
| $cARL_{.9}$ | $cARL$ for 90% quantiles of dispersion estimate distributions                    |
| $cov$       | Covariance                                                                       |
| $c_4$       | Unbiasing coefficient of $S_n$                                                   |
| $d$         | Contamination magnitude                                                          |
| $d_2$       | Unbiasing coefficient of $R$                                                     |
| $d_3$       | Coefficient in defining relationship between $\hat{\sigma}_R$ and $\hat{\sigma}$ |
| $D_3$       | Upper control limit coefficient of R-chart                                       |
| $D_4$       | Upper control limit coefficient of R-chart                                       |
| $F_i$       | Event that $\hat{\sigma}_i^{II}$ falls beyond CLs                                |
| $G_i$       | $i^{th}$ subgroup of Phase I sample                                              |
| $H_i$       | Logical value of screened outliers in individuals chart                          |
| $HD_q$      | Harrell-Davis $q^{th}$ quantile estimator                                        |
| $HD_{.5}$   | Harrell-Davis median estimator                                                   |
| $I_i^L$     | Logical value of screened samples due to location estimates                      |
| $I_i^S$     | Logical value of screened samples due to dispersion estimates                    |
| $K$         | Contamination density                                                            |
| $L_n$       | Lower control limit coefficient                                                  |
| $L_n^I$     | Lower control limit coefficient estimate                                         |

|             |                                                                              |
|-------------|------------------------------------------------------------------------------|
| $L_n^*$     | Known lower control limit coefficient                                        |
| $LCL^*$     | Lower control limit                                                          |
| $m$         | Subgroup (sample) size                                                       |
| $MAD_n$     | Median absolute deviation                                                    |
| $mARL$      | Marginal (unconditional) average run length                                  |
| $mARL_0$    | Marginal average run length when process is in-control                       |
| $ML$        | Location M-estimator                                                         |
| $MS_n$      | Scale M-estimator                                                            |
| $MSE$       | Mean square error                                                            |
| $n$         | Number of individual observations in a subgroup                              |
| $N_{run}$   | Number of Monte Carlo simulation                                             |
| $O^L$       | Outlier subgroups which have different location estimate                     |
| $O^S$       | Outlier subgroups which have different dispersion estimate                   |
| $p$         | Probability of giving false signal                                           |
| $R_i$       | Range of $i^{th}$ sample                                                     |
| $R_i$       | Individual outliers in $i^{th}$ sample                                       |
| $RL$        | Run length                                                                   |
| $\bar{S}$   | Grand mean of standard deviations of subgroup                                |
| $S_i$       | Standard deviation of $i^{th}$ sample                                        |
| $S_n$       | Sample standard deviation                                                    |
| $SDRL$      | Standard deviation of run length                                             |
| $U_{ij}$    | Logical value to indicate screened individual outliers                       |
| $U_n$       | Upper control limit coefficient                                              |
| $U_n^I$     | Upper control limit coefficient estimate                                     |
| $U_n^*$     | Known upper control limit coefficient                                        |
| $UCL^*$     | Upper control limit                                                          |
| $W_i$       | Weights of observations in $HD_q$                                            |
| $\bar{X}_i$ | Mean of $i^{th}$ sample                                                      |
| $Y_{ij}$    | Phase II data                                                                |
| $Z_{ij}$    | Phase I data                                                                 |
| $Z_k^*$     | Pooled observations after screening outlier subgroup and individual outliers |

|                       |                                                                |
|-----------------------|----------------------------------------------------------------|
| $\alpha$              | Type I error rate                                              |
| $\alpha_1$            | Skipping level of subgroup screening                           |
| $\alpha_2$            | Skipping level of after screening individual outliers          |
| $\gamma$              | Standard deviation of Phase I sample                           |
| $\Gamma$              | Gamma function                                                 |
| $\Theta_i$            | Bias correction term                                           |
| $\mu$                 | Process mean                                                   |
| $\hat{\mu}$           | Process mean estimate                                          |
| $\hat{\mu}^I$         | Process location estimate of Phase I sample                    |
| $\hat{\mu}_i^I$       | Location estimate of $i^{th}$ Phase I sample                   |
| $\mu_0$               | In-control process mean                                        |
| $\sigma$              | Process standard deviation                                     |
| $\hat{\sigma}$        | Process standard deviation estimate                            |
| $\hat{\sigma}_R$      | Process standard deviation estimate using range                |
| $\hat{\sigma}^I$      | Process standard deviation estimate of Phase I sample          |
| $\hat{\sigma}_i^I$    | Sample standard deviation estimate of $i^{th}$ Phase I sample  |
| $\hat{\sigma}_i^{II}$ | Sample standard deviation estimate of $i^{th}$ Phase II sample |
| $\psi$                | Psi function                                                   |
| $\omega$              | Measurement related to quality characteristics                 |

## LIST OF ACRONYMS/ABBREVIATIONS

|       |                                                            |
|-------|------------------------------------------------------------|
| ACM 1 | Autoregressive process with normal data                    |
| ACM 2 | Autoregressive process with diffuse variance contamination |
| ACM 3 | Autoregressive process with diffuse mean contamination     |
| AR    | Autoregressive process                                     |
| BDP   | Breakdown point                                            |
| CM 1  | Diffuse symmetric variance contamination model             |
| CM 2  | Diffuse asymmetric variance contamination model            |
| CM 3  | Localized variance contamination model                     |
| CM 4  | Diffuse mean contamination model                           |
| CM 5  | Localized mean contamination model                         |
| CL    | Control limit                                              |
| IF    | Influence function                                         |
| i.i.d | Independently and identically distributed                  |
| LCL   | Lower control limit                                        |
| MC    | Monte Carlo                                                |
| SCM 1 | Diffuse and localized variance contamination model         |
| SCM 2 | Diffuse and localized mean contamination model             |
| SCM 3 | Diffuse variance and mean contamination model              |
| SCM 4 | Localized variance and mean contamination model            |
| SCM 5 | Diffuse variance and localized mean contamination model    |
| SCM 6 | Localized variance and diffuse mean contamination model    |
| SPC   | Statistical process control                                |
| SPM   | Statistical process monitoring                             |
| SQM   | Statistical quality control                                |
| UCL   | Upper control limit                                        |

## 1. INTRODUCTION

Competition in consumer market has affected manufacturers to move towards three major interrelated arenas: quality, cost, and responsiveness. The concepts of cost and responsiveness have been defined so as to encompass financial and non-financial aspects of the process requirements and management, respectively. The word of quality came into existence many years ago with different definitions. Fitness for the purpose, degree of customer satisfaction, accuracy in meeting the specification or design, meeting the standards or norms, and degree of excellence are some of these definitions [1]. Although its meaning changes from time to time, it is generally used to symbolize excellence of products or services referring to certain physical characteristics defined by performance, serviceability, durability, reliability, aesthetics, features, and/or conformance to standards of these products and services. First and foremost, customer needs must be recognized to define and manage quality [2]. While subjectivity and polysemy of quality makes the concept difficult to comprehend, quality should be attained and kept at a satisfactory level as much as possible, regardless of its definition [3].

Rendering consistent customer satisfaction through high quality products and services makes it possible to survive in market, therefore manufacturers need quality control techniques to monitor their processes and improve product quality by reducing process variability. A variety of tools are used in many disciplines, such as chemistry, medicine, environment [4], industry, economics [5, 6], informatics, even in traffic management [7], for employing quality control. Effective control techniques reduce scrap and rework, increase production capacity and decrease in production cost via keeping the process under control.

Statistical process monitoring (SPM) can be defined as the collection of tools and techniques to reduce variation of quality characteristics of target product by recognizing the causes behind variation, and bringing the process into a state of control. The most common and valuable tool in SPM is control charts, devised by Dr. Shewhart in 1920s [8]. A control chart is basically a graphical representation of the variation of

quality characteristics over time; extensive use and popularity of control charts are due to their effectiveness and simplicity. Different control charts, such as S charts, X-bar charts, EWMA and CUSUM charts, and methods are used by practitioners in the cases of univariate and multivariate processes to monitor the change in process mean and variance.

Efficiency of a process is related with its stability, commonly measured by its dispersion and location parameters, and prompt detection of changes in these parameters is crucial in SPM applications. In many cases, in-control process parameters would be unknown, therefore control charts are to be constructed using estimated control limits using Phase I data. In SPM, sample mean and standard deviation statistics are known to be the most efficient traditional estimators to determine the process location and dispersion parameters, respectively. The use of traditional estimators rely on the assumption of independent and normally distributed data sets. On the other hand, real-world process data may not be normally distributed in the presence of outliers, which contaminate samples. In such cases, efficiency of conventional estimators may be unacceptably dampened. Phase I efficiency of the proposed estimators are commonly evaluated using mean square error for both normal and non-normal processes. In the literature, outliers within in-control data are usually modeled via contaminated normal distributions; Phase I data are dominated by clean observations, which come from an in-control process, while contaminated by outliers. Both parameter estimation and existence of outliers in Phase I data deteriorate the performance of control charts, so it is an important task in SPM literature to propose estimators with high robustness and efficiency. Phase II performance of control charts constructed various estimators are generally evaluated using a large number of Monte Carlo simulations and metrics, such as average run length, conditional run length, average rate of signals.

This study aims to introduce novel robust approaches to construct univariate S-Charts, which have higher power compared to traditional dispersion estimation methods, and assess the Phase II performance of S-Charts based on the proposed estimators using computer simulations and a wide palette of statistical measures. The outline of the thesis is the following. In the current thesis, historical development, theoretical

concepts and practical applications of control charts are discussed in Chapter 2. Chapter 3 consists of an overall discussion of dispersion estimators, i.e. sample standard deviation, median absolute deviation and scale M-estimator, and location estimators, i.e. sample average, Harrell-Davis  $q^{th}$  quantile estimator and M-Huber location estimator. Step-by-step explanation of the proposed Phase I estimators and the process contamination models, utilized in the current study, are also given in same chapter. Efficiency and performance of S-Charts, application of the proposed methods on two real life examples, flow width measurements of hard bake process and syringe lengths, are in Chapter 4. Finally, conclusions and recommendations for future researchers are stated.

## 2. STATISTICAL PROCESS MONITORING

Quality of products and services plays an important role in decision making processes of different customer segments. Maintaining quality at the desired level, though may be challenging, is imperative in achieving high level of customer satisfaction, as well as, maximizing revenue and market share, rendering elimination of waste product for companies and prolongation of product loyalty for the customers. The second law of thermodynamics states that all systems move towards a state of greater entropy (or disorderliness). This phenomenon is observed also in manufacturing processes: operators may make a mistake, machines may break down, etc., when equipments are not controlled or regulated. Therefore, quality of a process is strongly affected by its stability and repeatability. Although it is impossible to prevent the inherent tendency towards disorderliness of processes, various techniques may reduce the process variability sufficiently. This is why quality control and improvement is an important concern to industries.

Reduction in variability of quality characteristics is achieved via online statistical process monitoring (SPM) techniques. SPM may be defined as the engineering tools and statistical techniques applied to measure and observe the quality characteristics of a process for monitoring and quality improvement. Process monitoring may be achieved via seven quality tools, which are histogram plots, pareto charts, cause and effect diagrams, defect-concentration diagrams, scatter diagrams, sampling inspections and check sheets, which yield limited discrete knowledge about the process rather than distinguishing the variation over a period (Figure 2.1a). Control charts, on the other hand, offer continuous information about the process, and provide an opportunity to apply systematic approaches to decrease the variability in quality characteristics (Figure 2.1b) [9].

The concept of quality control was introduced by Dr. Walter Andrew Shewhart, a physicist, during his service in Bell Telephone Laboratories in 1920s. He needed to decrease the frequency of failures and repairs of transmission systems, so he employed

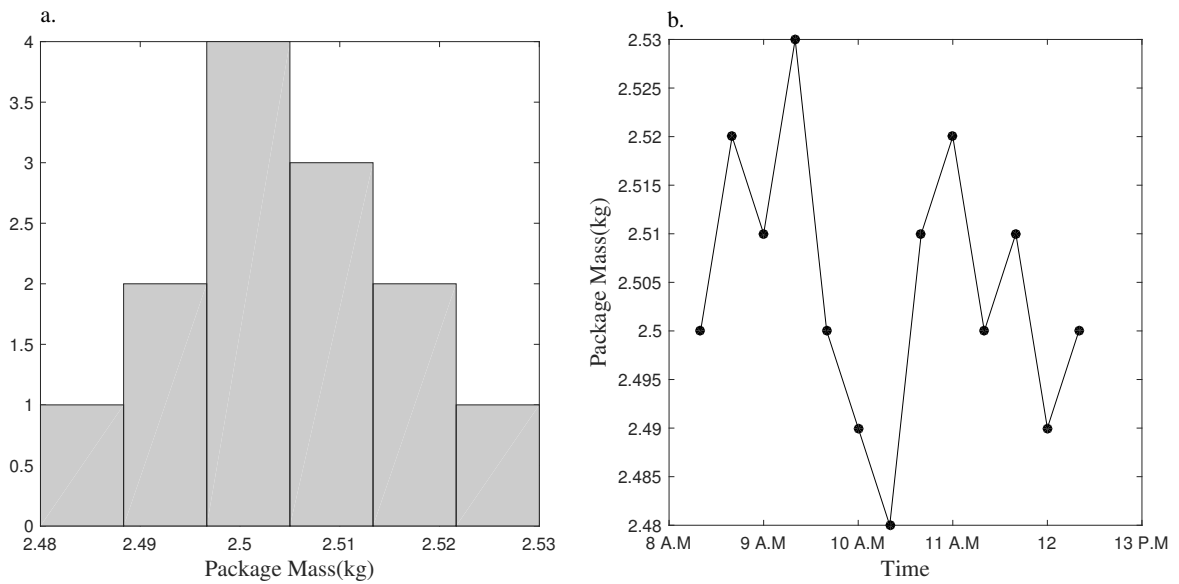


Figure 2.1. Representation of process variation via a) histogram b) control charts.

control charts. In the following years, quality control drew the attention of researchers and practitioners. Deming, who is another widely known proponent of statistical quality control (SQC), brought a new perspective to the application of control charts by emphasizing the responsibilities of the top management. He developed his complete philosophy of management and popularized the Deming PDCA cycle based on the idea to “plan the necessities”, “do what is needed”, “check whether it works or not”, and “act to prevent abnormalities”. Juran defined the quality as “fitness for use” to emphasize that achievement of desired quality cannot happen accidentally, it should be planned and controlled. He advocated a trilogy for quality management as planning, improvement and control. Usage of pareto charts to prioritize variables was his suggestion, as well. Ishikawa ideology, on the other hand, focused on the importance of total quality control of an organization, rather than just focusing on products and/or services. He urged managers to resist becoming content with merely improving a product’s quality, insisting that quality improvement can always go one step further. He emphasized the importance of “internal customers”; customer would continue receiving service even after receiving the product. This service would extend across the company in all levels of management, and even beyond the company to everyday lives of those in-

volved. Importance of quality control, however, was not widely recognized particularly by the U.S. chemical industries. After World War II, due to necessity of improving the manufacturing efficiency, application areas of SQC were highly extended in chemical industries. Numerous studies demonstrated the positive relationship between quality control and manufacturing efficiency. In 1980s, an ingenious perception of Taguchi showed the importance of design of experiments in SQC. In the following years, further research was conducted in statistical techniques more efficiently. [8, 10, 11].

Aim of online application of control charts is not to decrease inherent variability in a process regardless of design and maintenance, but to detect unusual changes. Shewhart, Deming and Juran view the unusual changes as a result of local faults/special causes and sporadic problems, while they see inherent variation as a consequence of chance causes, system faults/common causes and chronic problems. Process is defined as set of causes and conditions, which come together in different combinations to transform inputs (chemical compounds, information, etc.) into outputs (chemical product, service, behaviour, etc.). If one samples products in equal time intervals, these measurements are subjected to variation which may stem from common and/or special causes. When variation is only caused by common causes, the process is stable and statistically in-control, since variability of common causes generates a fluctuation within an acceptable and predictable bandwidth. Common causes do not have a prominent effect on equilibrium point, because the underlying probability distributions of mean and standard deviation estimates of a process result remain stable over time [12]. Special causes, on the other hand, are the causes that stem from assignable sources and indicators of problems, which need to be solved. Special causes may be due to extraordinary conditions, incidents or lack of attention, or a new supplier for raw material, etc. A process, in which outputs are affected by both common and special causes, is called an unstable, or statistically out-of-control process. The magnitude of the variation and change in the quality characteristics are unpredictable. Consequently, special causes should be identified and removed for maintaining a stable process. Deming [13] illustrated the necessity of a stable process as follows.

- A stable process has an identity, and its performance is foreseeable; therefore, it allows rational planning.
  - Maximum productivity and minimum costs are obtained in the existence of stability in a process.
  - Effect of changes for a stable process can be identified in a more reliable way.
- In an unstable process, it is difficult to distinguish special causes from common causes. If the inherent variability of a process is not identified separately, it would be more challenging to know when a change results in improvement.

As importance of quality management has been accepted over years, application areas of control charts have been widened in manufacturing processes [14–16]. The earliest modifications of control charts, e.g. cumulative sum, acceptance control, and multivariate characteristic charts, were first published in the studies by Gibra [17], Vance [18] and Vardeman and Cornell [19], respectively. Saniga and Shirland [20] and Faltin et al. [21] discussed industrial applications of control charts in their studies. Incorporation of statistical process methods into industrial practice, relationship between hypothesis testing and control charting, and modelling of performance were discussed by Woodall, Montgomery and their co-workers [9, 12, 22–25].

### 2.1. Shewhart Control Charts

Typical control charts consist of central line, which is the expected value of the corresponding statistics representing the quality characteristics of the in-control process, and two horizontal lines, named upper control limit (UCL) and lower control limit (LCL). These control limits (CLs) represent the acceptance boundaries of natural variability of the in-control process. If the measured or computed characteristics falls outside these limits, then the process is deemed to be out-of-control, and one should try to define the source of the problem. To illustrate the connection of hypothesis testing and control charts, suppose that the central line in Figure 2.2 is the process mean  $\mu = \mu_0$ , and the current sample average ( $\bar{x}_i$ ) is within the CLs, i.e. process mean is in-control. When  $\bar{x}_i$  exceeds the CLs and gives false signal,  $\mu$  is deemed to

be not equal to  $\mu_0$  anymore, i.e. process is out-of-control. Consequently, being within CLs is equivalent to failing to reject the null hypothesis, i.e. process is assumed to be in-control [9].

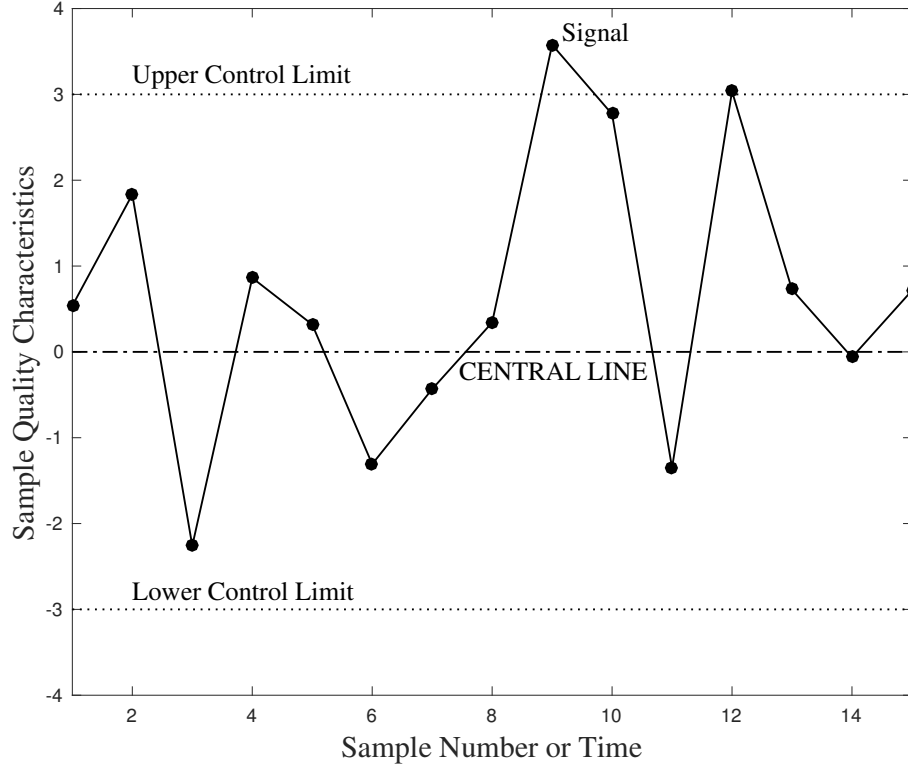


Figure 2.2. A typical Shewhart  $\bar{X}$  control chart.

A process may be subjected to different causes which may perturb mean, or variability, or both parameters of the process. For example, process mean and variation may shift instantaneously, then remain stable at that level, which is called “sustained shift”, or process parameters may shift abruptly for a short time, and turn back to the previous in-control value, or there may be a systematic or gradual increasing/decreasing change in the process mean or variation, or they may vary from time to time in erratic fashion. Usual hypothesis testing framework can deal especially with sustainable shifts, therefore the use of hypothesis testing in inference of control chart performance is more reasonable [8]. In the analysis of the performance of a control chart, hypothesis testing perspective moreover gives information about the probability of type I and type II errors, i.e. the false positive and false negative signals, respectively, for a control chart.

Control charts are classified with respect to the measurability of the quality characteristics. If quality characteristics can be expressed and measured quantitatively, such as temperature, length, concentration, etc., it is common practice to use variables control chart, in which estimates of location and dispersion parameters of the quality characteristics are plotted; otherwise, attributes control charts, in which the number of nonconformities, are used to make qualitative judgements, such as pass/fail, good/bad, yes/no, etc (Figure 2.3). Main purpose of both control charts is to make inferences from the sampled data about the performance of the process, or quality of a product. Variables control charts are widely used in industrial applications because quality characteristics of the processes are generally identified with numerical measurements.

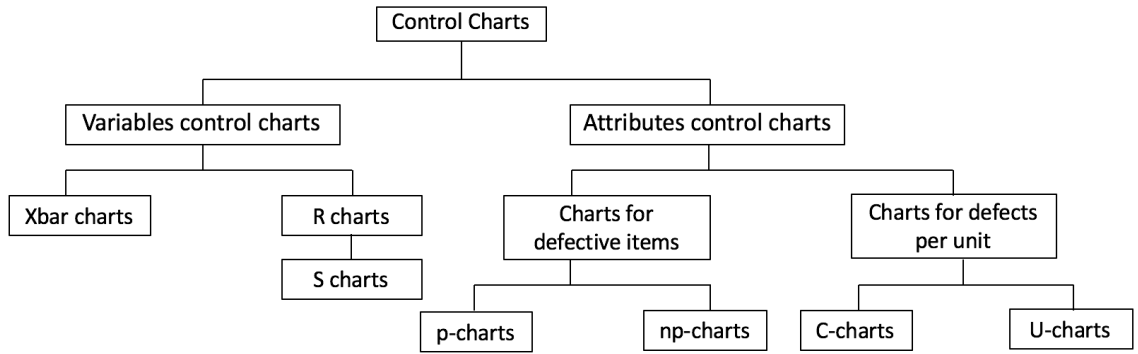


Figure 2.3. Types of control charts.

A prevailing technique used to monitor the quality of products is to take measurements generally as about 20-50 samples of 1-9 units, and analyze the process using control charts such as  $\bar{X}$  and S, or  $\bar{X}$  and R charts. For a general model, let  $\omega$  be a measurement related quality characteristics. Then central line, UCL and LCL become as follows:

$$UCL = \mu_{\omega} + C\sigma_{\omega} \quad (2.1)$$

$$CL = \mu_{\omega} \quad (2.2)$$

$$LCL = \mu_{\omega} - C\sigma_{\omega} \quad (2.3)$$

where  $C$  is the allowable distance from the central line. From this point on, the subscript  $\omega$  will not be used, and  $\mu$  and  $\sigma$  will represent the population parameters of the measurement related with the quality characteristic.

This approach may be applied to all  $\bar{X}$ , S/R control charts to check whether there is a significant shift in the level of process mean and variation, respectively. For a normally distributed process with mean  $\mu$  and standard deviation  $\sigma$ , the CL, LCL and UCL of  $\bar{X}$  chart may be shown as follows:

$$UCL = \mu + 3\sigma/\sqrt{n} \quad (2.4)$$

$$CL = \mu \quad (2.5)$$

$$LCL = \mu - 3\sigma/\sqrt{n} \quad (2.6)$$

here,  $n$  denotes the sample size. Sample averages of new measurements are computed and checked whether they fall within CLs, or not. Central line of S-Charts is taken to be equal to  $c_4\sigma$ , with a constant  $c_4$ , rather than  $\sigma$  itself, because  $S$  is a biased estimator of  $\sigma$ . Furthermore, standard deviation of  $S$  is equal to  $\sigma\sqrt{1-c_4^2}$ , so 3-sigma limits of S-chart are approximated as in Equation 2.7-9. In prospective analysis, sample standard deviation of new measurements are found, and checked whether they fall within these CLs, or not.

$$UCL = c_4\sigma + 3\sigma\sqrt{1-c_4^2} \quad (2.7)$$

$$CL = c_4\sigma \quad (2.8)$$

$$LCL = c_4\sigma - 3\sigma\sqrt{1-c_4^2} \quad (2.9)$$

here, a numerical example is given to illustrate how  $\bar{X}$  and S-Charts are constructed. Hardness measurement from a manufacturing process of allow is assumed to represent the quality characteristics, and a sample consists of five hardness measurements ( $n = 5$ ). Furthermore, mean and standard deviation of the hardness measurements are known to be equal to 45 and 0.1 mm, respectively. Standard deviation of the  $\bar{X}$  is

determined as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.1}{\sqrt{5}} = 0.0447 \quad (2.10)$$

Based on the assumption of normally distributed in-control process,  $100(1 - \alpha)\%$  of sample mean ( $\bar{X}$ ) is expected to fall within  $45 - z_{\alpha/2}(0.0447)$  and  $45 + z_{\alpha/2}(0.0447)$  mm. It is common practice to use the constant C as 3, therefore  $z_{\alpha/2}$  is customarily chosen as 3. Then, UCL and LCL become as:

$$UCL = 45 + 3(0.0447) = 45.134 \quad (2.11)$$

$$LCL = 45 - 3(0.0447) = 44.866 \quad (2.12)$$

If a future sample mean falls in (44.866, 45.134) interval, process is deemed to be in-control, otherwise process is assumed to be out-of-control. It should be noted that, 3-sigma approach is approximately equivalent to that  $\alpha = 0.0027$ ; that is, the probability of exceeding CLs is equal to 0.0027, when process is in-control. After determining CLs, a hypothesis testing procedure is applied for each future measurement sequentially over time.

$$H_0 : \mu = 45 \quad (2.13)$$

$$H_1 : \mu \neq 45 \quad (2.14)$$

Similarly for S-Charts, the constant  $c_4$  is found to be equal 0.94 (the values for different sample sizes are given in Table A.1), therefore 3-sigma control limits of S-chart determined as follows:

$$UCL = 0.94(0.1) + 3(0.1)\sqrt{1 - 0.94^2} = 0.196 \quad (2.15)$$

$$LCL = 0.94(0.1) - 3(0.1)\sqrt{1 - 0.94^2} \approx 0 \quad (2.16)$$

## 2.2. Two Phases of Data Collection: Phase I and Phase II

It is straightforward to apply control charts when process parameters are known, as discussed in the preceding section. However, when the process parameters are unknown, construction of control charts may be challenging. The parameters would be accurately determined if it were possible to collect a large volume of data during in-control operation [26], but it is difficult to take in-control measurements for a long period of time. Therefore, one may assume that parameters of most processes are practically unknown, and should be determined empirically. To solve this problem, process monitoring is predominantly handled in two consecutive phases: Phase I and Phase II. In Phase I, which is known as the retrospective phase, samples are collected in rational subgroups. Stability of the process should be validated prior to parameter estimation using Phase I samples. In Phase II, which is known as the prospective phase, control limits are determined using parameter estimates from Phase I, and hypothesis tests are continuously performed for new observations. Here, the aim is to detect occurrence of a disturbance on the process promptly, using subgroup statistics and Phase II CLs. [24, 27]. Additional investigation may be applied to understand whether the alarm is false or not. In the presence of a valid alarm, changes in process operating conditions may be required to bring the process in-control [25].

Phase I control charts are used to check the process stability and ensure that collected samples represent the variability of the process: quality characteristics are decided, rational subgrouping strategy is determined, long-term/short-term stability is assessed, and nature of the process variation is understood. When the process is established as not stable, one should first stabilize the process by analyzing historical data and eliminating the variation stemming from special causes. Once the process is deemed to be in-control, a control chart is constructed based on Phase I data. If any sample among Phase I samples exceeds the CLs, then potential special causes are identified and eliminated, and revised CLs are determined. It should be noted that a more efficient Phase I sampling yields a more powerful Phase II monitoring [25].

### 2.3. Rational Subgrouping

Subgroups (samples) should be collected in a way to yield observations within subgroups being subjected to only common causes, and differing subgroups if any special cause exists. In other words, subgroups should be constructed so as to minimize variability that stems from common (natural) causes within subgroups and maximize the variability that stems from special (assignable) causes between subgroups. This procedure is called rational subgrouping. When rational subgrouping is employed conveniently, CLs will show the acceptable borders within which all common cause variability is included, and any abnormalities due to special causes will tend to give a signal [9, 10, 28]. Sample size and sampling frequency are important parameters in a rational sampling scheme, and these parameters should be determined carefully. Sample size is generally taken to be equal to, 4 to 8 [26, 29–31]. The sample size should be small enough to be economically appealing to be collected. On the other hand, subgroups should also be large enough for the sample mean to be normally distributed, particularly for non-normal distributed observations. Additionally, subgroups should have high sensitivity in prompt detection of special causes, and sensitivity is increased with larger sample size. It is stated that longer sampling periods may yield delays in detection of an out-of-control status of a process [32]. Furthermore, outliers in a process, which has small sample size, have a relatively higher effect on process monitoring than that in a process having larger sample size [32–34].

The homogeneity of rational subgroups is commonly guaranteed via two different methods. In the first one, measurements in a single subgroup are collected approximately at the same time, thus common cause variability can be included via consecutive sampling. The level of process mean and variation, however, may abruptly change for a short time and turn back to the in-control state between samples, and samples include outlier(s). In such a case, this method would not be feasible. The second approach states that the measurements should be collected randomly instead of simultaneous sampling, thus possibility of occurrence of abrupt shifts in level of process mean and variation stem from special causes can be minimized [9, 10].

In addition to sample size, sampling frequency should also be carefully considered. If a process has not been monitored using control charts and shows somewhat erratic behaviour, samples should be collected as frequently as possible to detect abnormalities and develop a strategy quickly. When process does not allow quality characteristic to be collected frequently, individual measurements could be grouped and sampling frequency may be decreased as the process shows less erratic behaviour. Another important point is to detect the events which may create special causes in a process: sampling interval may be readjusted with respect to the probability of occurrence of special causes, i.e. adjustment in process equipments, change in operators or raw materials may result the need to increase sampling frequency. In all these considerations, cost of sampling should also be taken into account [10,35].

Rational subgrouping is an essential concept in monitoring a process, and its importance is generally underestimated. In many applications, it is appealing to combine the data from different machines and different raw materials, which are assumed to be nearly identical and to use a single control chart to monitor these samples. The variations of stratification and mixing are problematic. In stratification of subgroups and mixing observations, each of operators, machines and raw materials, and so on, yields an observation in a subgroup. In this case, each sample represents different machines, raw materials, etc, equally. This unavoidably leads to an increase of variance within subgroups, and reduces the power of control charts [3,10]. Therefore, sampling should be carefully designed to obtain useful information from the control charts [3,9,28,35].

#### **2.4. Construction of Shewhart Control Charts for Processes with Unknown Parameters**

Construction of control limits and control charts for known process parameters has been discussed in Section 2.1. In real world, however, it is almost impossible to know the true parameters of a process, so process monitoring is handled in two consecutive phases, as discussed in Section 2.2. Process parameters  $(\mu, \sigma)$  should be estimated reliably from Phase I sampling, and known process parameters are replaced

with their estimates  $(\hat{\mu}, \hat{\sigma})$  for computing central line, UCL and LCL for  $\bar{X}$  chart as:

$$UCL = \hat{\mu} + C\hat{\sigma} \quad (2.17)$$

$$CL = \hat{\mu} \quad (2.18)$$

$$LCL = \hat{\mu} - C\hat{\sigma} \quad (2.19)$$

and for  $S$  chart as

$$UCL = U_n\hat{\sigma} \quad (2.20)$$

$$CL = \hat{\sigma} \quad (2.21)$$

$$LCL = L_n\hat{\sigma} \quad (2.22)$$

Based on the assumption of normally distributed statistic, CLs of  $\bar{X}$  charts are determined according to 3-sigma approach using the location estimate ( $\hat{\mu}$ ) and dispersion estimate ( $\hat{\sigma}$ ) from range or standard deviation. Let  $\bar{X}_i$  be the mean of  $i^{th}$  sample,  $\hat{\mu}$  can be determined by the grand mean of  $m$  available samples, and is equal to  $\hat{\mu} = \bar{\bar{X}} = \frac{1}{m} \sum_{i=1}^m \bar{X}_i$ . Dispersion parameter  $\sigma$  can be estimated using range or standard deviation of the samples. Taking  $R_i$  as the range of  $i^{th}$  sample,  $\bar{R}$  is the average sample range and equals  $\bar{R} = \frac{1}{m} \sum_{i=1}^m R_i$ . Since the unbiased estimate of standard deviation is  $\hat{\sigma} = \bar{R}/d_2 = \bar{S}/c_4$ . Central line, UCL and LCL are computed as follows:

$$UCL = \bar{\bar{X}} + \frac{3}{d_2\sqrt{n}}\bar{R} \quad (2.23)$$

$$= \bar{\bar{X}} + \frac{3}{c_4\sqrt{n}}\bar{S} \quad (2.24)$$

$$CL = \bar{\bar{X}} \quad (2.25)$$

$$LCL = \bar{\bar{X}} - \frac{3}{d_2\sqrt{n}}\bar{R} \quad (2.26)$$

$$= \bar{\bar{X}} - \frac{3}{c_4\sqrt{n}}\bar{S} \quad (2.27)$$

Variations in process location and dispersion parameters are monitored simultaneously in real world applications of control charts. Standard deviation of sample range is proportional to the process standard deviation, and computed with the Equation 2.28.

$$\hat{\sigma}_R = d_3 \hat{\sigma} = d_3 \frac{\bar{R}}{d_2} \quad (2.28)$$

Hence, central line, UCL and LCL of R-chart are computed as follows:

$$UCL = \bar{R} + \frac{3d_3}{d_2} \bar{R} \quad (2.29)$$

$$CL = \bar{R} \quad (2.30)$$

$$LCL = \bar{R} - \frac{3d_3}{d_2} \bar{R} \quad (2.31)$$

If constants are defined as  $D_3 = 1 - 3d_3/d_2$  and  $D_4 = 1 + 3d_3/d_2$  (the constant  $D_3$  and  $D_4$  are tabulated for different sample sizes in Table A.1), then CLs of R-chart can be expressed as follows:

$$UCL = D_4 \bar{R} \quad (2.32)$$

$$CL = \bar{R} \quad (2.33)$$

$$LCL = D_3 \bar{R} \quad (2.34)$$

Instead of sample range, standard deviation can also be used as a dispersion estimator. Taking  $S_i$  to be equal to standard deviation of the  $i^{th}$  sample, the unbiased standard deviation estimate ( $\hat{\sigma}$ ) is equal to  $\hat{\sigma} = \bar{S}/c_4$ , where  $\bar{S}$  is the grand mean of standard deviations of subgroups, and equals  $\bar{S} = \frac{1}{m} \sum_{i=1}^m S_i$ . If constants are defined as  $L_n = 1 - \sqrt{1 - c_4^2}/c_4$  and  $U_n = 1 + \sqrt{1 + c_4^2}/c_4$ , the central line and control limits

of S-chart can be now expressed as follows:

$$UCL = \bar{S} + 3 \frac{\bar{S}}{c_4} \sqrt{1 - c_4^2} \quad (2.35)$$

$$= U_n \bar{S} \quad (2.36)$$

$$CL = \bar{S} \quad (2.37)$$

$$LCL = \bar{S} - 3 \frac{\bar{S}}{c_4} \sqrt{1 - c_4^2} \quad (2.38)$$

$$= L_n \bar{S} \quad (2.39)$$

Two different approaches, namely 3-sigma limits and probability limits, are used in practice to determine the constants  $C$ ,  $U_n$  and  $L_n$ . Although 3-sigma approach ( $C = 3$ ) is commonly used in industrial applications, the probability limits approach is preferred particularly. In probability limits approach, control limit coefficients are adjusted to obtain a desired type I error probability. Taking  $\hat{\sigma}^I = \bar{S}/c_4(n)$  as the standard deviation estimate of the process using Phase I samples, and  $\hat{\sigma}_i^{II}$  is the sample standard deviation of the  $i^{th}$  Phase II sample,  $F_i$  is defined as the event that  $\hat{\sigma}_i^{II}$  falls beyond the control limits, and  $P(F_i) = p$  is the probability of occurrence of  $F_i$  (false alarm probability).  $U_n$  and  $L_n$  are adjusted to yield a type I error rate  $\alpha$ , generally taken to be equal to 0.0027.

$$P(F_i|\hat{\sigma}^I) = P(L_n \hat{\sigma}^I \leq \hat{\sigma}_i^{II} \leq U_n \hat{\sigma}^I) = 1 - \alpha \quad (2.40)$$

## 2.5. Performance Measures of Control Charts

Suppose that  $Z_{ij}$  ( $i=1,2,\dots,m$  and  $j=1,2,\dots,n$ ) represents the  $m$  subgroups of  $n$  individual measurements, assumed to be sampled from a stable in-control process with  $N(\mu_0, \sigma_0^2)$  distribution. Phase II data, on the other hand, consist of an undefined number of subgroups with the same sample size ( $n$ ), denoted by  $Y_{ij}$ , ( $i=1,2,\dots$  and  $j=1,2,\dots,n$ ).  $Y_{ij}$  is similarly assumed to be independent, but  $N(\mu_0, (\gamma\sigma_0)^2)$  distributed,

where  $\gamma$  is a constant. When  $\gamma=1$ , process is in-control; otherwise it is exposed to disturbances.

Run length ( $RL$ ) is defined as the number of successive subgroups, which have not fallen beyond the limits until the first signal. If the plotted statistics are independent and identically distributed random variables, the events  $F_i$  are independent and thus,  $RL$  has a geometric distribution with parameter  $p = P(F_i) = \alpha$ . The performance of control charts are evaluated using the average  $RL$  ( $ARL$ ), and the standard deviation of  $RL$  ( $SDRL$ ). When the process parameters are known,  $ARL$  is given by  $1/p$  and  $SDRL$  is by  $\sqrt{1-p}/p$ .

$$ARL = E(RL) = \frac{1}{P(F_i)} = \frac{1}{p} = \frac{1}{\alpha} \quad (2.41)$$

In the case of unknown process parameters, control limits are determined using Equation 2.35-2.40. For a given  $\hat{\sigma}^I$ , the distribution of  $RL$  is geometric with parameter  $P(F_i|\hat{\sigma}^I)$  [36,37]. Consequently, the conditional  $ARL$  ( $cARL$ ) is given by

$$cARL = E(RL|\hat{\sigma}^I) = \frac{1}{P(F_i|\hat{\sigma}^I)} \quad (2.42)$$

Contrary to conditional  $RL$  distribution, unconditional  $RL$  distribution is not geometric with parameter  $P(F_i) = p$ , since events  $F_i$  are not independent [38,39]; they will be independent only when conditioned on a specific value of  $\hat{\sigma}^I$ . Consequently, the marginal (unconditional)  $mARL$  may be obtained by averaging a large number of  $cARL$  over all possible values of  $\hat{\sigma}_i^I$ .

$$mARL = E\{E(RL|\hat{\sigma}^I)\} = E\left\{\frac{1}{P(F_i|\hat{\sigma}^I)}\right\} \quad (2.43)$$

and  $SDRL$  is given by [40]

$$SDRL = \sqrt{\text{var}(RL)} = \sqrt{E(\text{var}(RL|\hat{\sigma}^I) + \text{var}(E(RL|\hat{\sigma}^I)))} \quad (2.44)$$

$$= \sqrt{2E\left(\frac{1}{p(F_i|\hat{\sigma}^I)}\right)^2 - \left(E\frac{1}{p(F_i|\hat{\sigma}^I)}\right)^2 - E\left(\frac{1}{p(F_i|\hat{\sigma}^I)}\right)} \quad (2.45)$$

Although  $mARL$  is an accepted performance measure of control charts due to its simple interpretation for researchers, it is peculiar for practitioners since it could be realized only by extremely long runs [33, 37]. It is suggested to use a more intuitive metric, which is called average rate of false signal ( $ARFS$ ), since control charts are constructed to be used for a certain period of time. In order to clarify the difference between these performance measures, the nature of  $RL$  distribution is elaborated as follows.  $SDRL$  exceeds  $ARL$ , when the parameters are unknown [25, 26, 32, 37, 39, 41–45], i.e. large number of quite short runs are balanced by a very few number of long runs. If  $ARFS$  is taken to be equal to 0.0027 for determining CLs, then all these runs will be equally weighted, whereas short runs have higher importance (weight) when  $mARL$  is taken to be  $\approx 370$  to determine CLs. This is why  $ARFS$  is higher than 0.0027, when CLs are determined with respect to intended  $mARL$  value [32, 33].

## 2.6. Robust Statistics

Statistical inferences and estimation methods are based on samples from population, and certain assumptions are required for their underlying distributions. i.e. independently and identically distributed (i.i.d), or normally distributed observations. There is no guarantee that these assumptions will be actually realized, however their use may be justified with the help of continuity and stability principle: a minor error in the model is likely to yield small errors in conclusions. Unfortunately, this is not always true; sensitivity of traditional methods to minor deviations from assumed models sometimes necessitates use of alternative robust procedures [43, 46]. There is a subtle difference between robustness and outlier resistance. The term outlier resistant is used to imply that appropriate statistics is minimally affected in the existence of

outliers, e.g. median is one of the commonly used outlier resistant estimators. Robustness, on the other hand, indicates insensitivity of a procedure despite the deviations in underlying assumptions [46, 47].

Concept of robustness may be clarified emphasizing its difference from non-parametric methods. In non-parametric methods, no assumption is made about the underlying joint distribution. Robustness, on the other hand, is related with the strength against the deviations from the assumptions [46]. Models are based on assumptions, which hold for the majority of observations, however a fraction of observations may have a different pattern than the majority, or may have no pattern. These observations are called as outliers [48]. Deviations may be handled via nonparametric methods, but it is possible to use parametrized set of underlying distribution for the majority of the observations and arbitrary distribution for the outliers via robust statistics [49].

In the presence of outliers, traditional methods are not efficient in determining process parameters, due to increase in bias and variance, therefore outlier resistant statistics are employed to remove outliers in Phase I sampling before estimating CLs [9, 43, 50]. Howell proposed to use midrange to estimate location, and range to estimate dispersion, and determine the Phase II control limits, accordingly [51]. Ferrell proposed to use median of median [52], while Howell proposed to use mean of median as the measurement of central line. Moreover, Clifford proposed to bare CLs are based on the median of subgroup range [53]. Langenberg and Iglewicz suggested trimmed mean and trimmed range to compute central line and Phase II CLs [54]. Tatum showed that efficiency of reweighted estimators is higher than that of traditional methods in process parameter estimation against diffuse and localized disturbances, in which outliers are equally likely to affect the observations in all subgroup, and are allowed only to effect specific subgroups, respectively [39]. Schoonhoven, Does and Riaz, furthermore, improved the performance of control charts with the use of adaptive skipping and screening of individual observations using outlier resistant estimators [37, 40, 44, 45]. Although, these methods are relatively more resistant against outliers, the problem of skewed process distribution has not been thoroughly studied. Robust control charts using median [55, 56], sample range [57] and median absolute

deviations from the median [34, 58] also showed higher efficiency and higher power, mainly due to insensitivity to deviations from normality assumption rather than traditional estimators. Robust estimators are found to be more efficient both against diffuse-localized and symmetric-asymmetric contaminations, and have higher power in detecting disturbances, compared to non-reweighted and reweighted traditional estimators [26, 43, 50, 55–57, 59–63].



### 3. METHODOLOGY

In SPM, identical statistics (generally sample mean and sample standard deviation/range) are generally used to estimate the dispersion parameter in both Phase I ( $\sigma^I$ ) and Phase II ( $\sigma^{II}$ ). In the current study, shifts in level of process mean and variation (outliers) are assumed to contaminate only Phase I samples, so Phase I estimators should be robust against contaminations and efficient under normal situations, while Phase II estimator should be sensitive to process disturbances. Hence, robust estimator and/or reweighting procedures are better suited to be employed in determining process parameters using Phase I data [24, 37]. Robustness of an estimator can be measured by the local stability assessed via influence function (IF), and global reliability via breakdown point (BDP). Asymptotic BDP measures the fraction of contamination, which is capable to make the estimate useless [64], while IF measures effect of a single observation, varying from  $-\infty$  to  $+\infty$ , on the estimate [65]. A robust estimator is expected to have a bounded and small IF and high BDP to resist against outliers in a dataset [64, 66].

Robustness properties of three dispersion estimators: sample standard deviation ( $S_n$ ), median absolute deviation ( $MAD_n$ ) and scale M-estimator ( $MS_n$ ), and three location estimators: sample mean ( $ave$ ), Harrell-Davis  $q^{th}$  quantile estimator ( $HD_q$ ) and location M-estimator ( $ML$ ) are investigated in Section 3.1. Reweighting procedures employed on these estimators; the algorithms of different reweighting procedures to construct CLs of S-Charts, used to monitor Phase II data are discussed in Section 3.2. Robustness of the non-reweighted and reweighted estimators are elucidated against various Phase I contaminations in Section 3.3.

#### 3.1. Proposed Phase I Estimators

In the following discussion, it is assumed that  $Z_{ij}$  ( $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ ) denotes the  $j^{th}$  individual observation of the  $i^{th}$  subgroup (sample), while  $G_i$  ( $i = 1, 2, \dots, m$ ) represents the  $i^{th}$  subgroup comprising a set of  $n$  individual ob-

servations in Phase I sampling. In the design of  $S$  control charts for an in-control process, standard deviation estimates of rational subgroups ( $\hat{\sigma}_i^I$ ) are determined using dispersion estimators, then process standard deviation is estimated ( $\hat{\sigma}^I$ ) via location estimation of  $\hat{\sigma}_i^I$ s of  $m$  subgroups. Although process location estimation is not needed in the simple application of  $S$  control charts, employing individuals charts for reweighting procedure requires determining location parameters of rational subgroups ( $\hat{\mu}_i^I$ ) and the process location estimate ( $\hat{\mu}^I$ ) (Figure 3.1).

|            |          |                       |         |                                               |                                                       |
|------------|----------|-----------------------|---------|-----------------------------------------------|-------------------------------------------------------|
|            | $Z_{11}$ | $Z_{21} \dots Z_{n1}$ | $= G_1$ | $\Rightarrow \hat{\sigma}_1^I, \hat{\mu}_1^I$ |                                                       |
| $Z_{ij} =$ | $Z_{12}$ | $Z_{22} \dots Z_{n2}$ | $= G_2$ | $\Rightarrow \hat{\sigma}_2^I, \hat{\mu}_2^I$ |                                                       |
|            | $\vdots$ |                       |         |                                               | $\Rightarrow \hat{\sigma}^I \text{ and } \hat{\mu}^I$ |
|            | $Z_{1m}$ | $Z_{2m} \dots Z_{nm}$ | $= G_m$ | $\Rightarrow \hat{\sigma}_m^I, \hat{\mu}_m^I$ |                                                       |

Figure 3.1. Steps in computing  $\hat{\sigma}^I$  and  $\hat{\mu}^I$ .

### 3.1.1. Dispersion Estimators

Sample standard deviation  $S_n$  is known to be most efficient traditional dispersion estimator for normally distributed datasets.

$$S_n(G_i) = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (Z_{ij} - \text{ave}_{j=1}^n Z_{ij})^2} \quad (3.1)$$

here,  $\text{ave}_{j=1}^n Z_{ij}$  represents the sample mean of  $i^{\text{th}}$  subgroup and  $n$  denotes the number of observations in a subgroup. It should be noted that  $S_n$  is biased estimator of  $\sigma^I$ , thus a correction factor is applied ( $\sigma = E(S)/c_4(n)$ ) [9], that is  $\hat{\sigma}_i^I = S_n(G_i)/c_4(n)$ , where  $c_4(n)$  is a constant (Table A.1). Besides having 0% BDP,  $S_n$  has unbounded IF, i.e. when a single observation is replaced with an arbitrary number, dispersion estimate is affected. The next estimator is a well known robust estimator, median absolute deviation [67]:

$$MAD_n(G_i) = MAD_{j=1}^n Z_{ij} = \frac{1}{\phi^{-1}(3/4)} \text{med}_{j=1}^n |Z_{ij} - \text{med}_{j=1}^n Z_{ij}| \quad (3.2)$$

here,  $med_{j=1}^n Z_{ij}$  represents the median of  $i^{th}$  subgroup, and  $\phi$  is the cumulative normal distribution function. In case of normality,  $\phi^{-1}(3/4) = 1.4826$  [46]. Since  $MAD_n$  is a biased estimator of  $\sigma^I$ , a correction factor is required to obtain an unbiased dispersion parameter estimate,  $E(MAD_n(G_i)) = \hat{\sigma}_i^I/b_n$ , where  $b_n$  is a constant (computed via simulations). Although  $MAD_n$  has bounded IF and 50% BDP, having low asymptotic efficiency (37%) and discontinuities in IF make the estimator less favourable especially in small samples [66]. The next robust dispersion estimator is scale M-estimator with auxiliary location.  $MS_n$  is a maximum likelihood-like estimator, which minimizes  $\sum_{i=1}^n \rho(x_i, \theta)$ , where  $\theta$  is the corresponding statistics. Having bounded and smooth IF, and high BDP ( $= \kappa/\rho(\infty)$ ),  $MS_n$  is minimized using the following constraint:

$$\frac{1}{n} \sum_{j=1}^n \rho\left(\frac{Z_{ij} - \hat{\mu}_i}{MS_n(G_i)}\right) = \kappa \quad (3.3)$$

$$\kappa = E_{\phi}\{\rho\} \quad (3.4)$$

here,  $med_{j=1}^n Z_{ij}$  and  $MAD_n$  estimates are used for  $\hat{\mu}_i$  and initialization of  $MS_n(G_i)$ , respectively. The fully iterated M-estimator with logistic  $\phi$  function ( $\rho$  function is shown below), which yields  $\kappa = 0.5$  and  $\rho(\infty) = 1$ , is used in this study, to give a BDP of 50% [66].

$$\rho(x) = \left( \frac{2}{1 + e^{\frac{-x}{0.3739}}} - 1 \right)^2 \quad (3.5)$$

$MS_n(G_i)$  is a biased estimate of  $\sigma^I$ , and its correction factor is determined via computer simulations.

After subgroup standard deviation estimates ( $\hat{\sigma}_i^I$ ) are determined via using dispersion estimators discussed above, location estimators discussed in following subsection are used to determine the process dispersion estimate ( $\hat{\sigma}^I$ ). Location estimators are additionally used to determine location estimates of rational subgroups and the process.

### 3.1.2. Location Estimators

Sample mean (*ave*), which is the average value of observations, is the most widely used location estimator. Process dispersion and location estimates are calculated using Equation 3.6 and 3.7, respectively:

$$\hat{\sigma}^I = ave(\hat{\sigma}_i^I) = \frac{1}{m} \sum_{i=1}^m \hat{\sigma}_i^I \quad (3.6)$$

$$\hat{\mu}^I = ave(\hat{\mu}_i^I) = \frac{1}{m} \sum_{i=1}^m \left( \frac{1}{n} \sum_{j=1}^n Z_{ij} \right) \quad (3.7)$$

Similar to  $S_n$ , *ave* is sensitive to outliers in a dataset, and has asymptotic BDP of 0% (its finite sample BDP is  $1/k$ ), and unbounded IF. The next estimator is distribution-free Harrell-Davis quantile estimator ( $HD_q$ ), which uses the weighted average of all order statistics [68, 69]:

$$\hat{\sigma}^I = HD_q(\hat{\sigma}_i^I) = \sum_{i=1}^m W_i \hat{\sigma}_i^I \quad (3.8)$$

$$\hat{\mu}^I = HD_q(\hat{\mu}_i^I) = \sum_{i=1}^m W_i \hat{\mu}_i^I \quad (3.9)$$

where weights ( $W_i, i = 1, 2, \dots, m$ ) are obtained via computing  $P(g)$  as:

$$W_i = P\left(\frac{i-1}{m} \leq G_i \leq \frac{i}{m}\right) \quad (3.10)$$

here, probability density function  $G_i$ , a beta random variable with parameters  $\alpha = (m+1)q$  and  $\beta = (m+1)(1-q)$ , where  $q$  is the quantile ( $q = 0.5$  is used in this study). Weights ( $W_i, i = 1, 2, \dots, m$ ) are obtained via computing  $P(g)$  as

$$P(g) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) + \Gamma(\beta)} g^{\alpha-1} (1-g)^{\beta-1} \quad (3.11)$$

where  $\Gamma$  represents gamma function. Using all observations in a dataset rather than the order statistics makes  $HD_q$  highly efficient, however its efficiency is lowered in the existence of relatively extreme quantiles. It was recommended to be used in the estimation of sample median due to its high asymptotic efficiency, when sample size is small [70], therefore  $HD_{.5}$  is used in this study.

Another robust location estimator is location M-estimator ( $ML$ ), which has been first proposed by Huber [46]. Huber  $\psi$  function is used instead of logistic  $\psi$  function due to its superiority against asymmetrically located (one-sided) outliers. Denoting  $X_i$  as the representation of  $\hat{\sigma}_i^I$  ( $i = 1, 2, \dots, m$ ), solution of the equation yields the unbiased  $ML$  estimate of  $\hat{\sigma}^I$

$$\sum_{i=1}^m \psi\left(\frac{X_i - ML}{\hat{\sigma}_X}\right) = 0 \quad (3.12)$$

here, initial estimate of  $ML$  ( $= \hat{\sigma}^I$ ) is obtained using  $med_{j=1}^n Z_{ij}$ , and  $\hat{\sigma}_X$  is replaced by  $MAD_n$  estimator for  $\hat{\sigma}_X$ .  $ML$  estimator has BDP of 50%, and bounded and linear IF with  $\psi(x) = \min(c, \max(x - c))$  and the tuning constant  $c = 1.5$ .

In order to obtain process location parameter,  $\hat{\mu}_i^I$  and  $\hat{\mu}^I$  are determined. Assuming that  $Y_i$  denotes  $\hat{\mu}_i^I$  ( $i = 1, 2, \dots, m$ ), which is estimated using the following equation, then  $\hat{\mu}^I$  is determined using the following equations respectively:

$$\sum_{j=1}^n \psi\left(\frac{Z_{ij} - \hat{\mu}_{ij}^I}{\hat{\sigma}_i^I}\right) = 0 \quad (3.13)$$

$$\sum_{i=1}^m \psi\left(\frac{Y_i - \hat{\mu}^I}{\hat{\sigma}_Y}\right) = 0 \quad (3.14)$$

### 3.2. Algorithms of Phase I Estimators

Traditional estimators,  $S_n$ ,  $ave$ , and robust estimators  $MAD_n$ ,  $MS_n$ , and  $HD_{.5}$ ,  $ML$  are employed both directly, and in a reweighted manner using skipping/screening

procedures to screen outliers in Phase I sampling to determine final dispersion estimates. Although asymptotic properties of reweighted estimators were found to be questionable, finite sample properties of reweighted estimators were found to be desirable [71]. In reweighted estimators, subgroups may be screened to eliminate localized variance contaminations at a desired type I probability  $\alpha_1^I$  using  $P(F_i) = P(L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I) = 1 - \alpha_1$ , and in some of methods, to eliminate localized mean contaminations with desired type I probability ( $\alpha_2$ ) using  $P(F_i) = P(\hat{\mu}^I - C_n^I \hat{\sigma}^I / \sqrt{n} \leq \hat{\mu}_i^I \leq \hat{\mu}^I + C_n^I \hat{\sigma}^I / \sqrt{n}) = 1 - \alpha_2$ . The main purpose here is to detect the subgroups which are subjected to localized contaminations. Furthermore, individuals charts may be constructed to eliminate diffuse variance and mean contaminations with the criteria  $P(F_i) = P(\hat{\mu}^I - C_n^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n^I \hat{\sigma}^I) = 1 - \alpha_2$ . All upper ( $U_n^I$ ) and lower ( $L_n^I$ ) Phase I CLs,  $C_n^I$  coefficients, and required bias correction terms  $\Theta_i$  ( $i = 1, 2, \dots, 5$ ) are determined via 100,000 computer simulations. The coefficients and multipliers for  $m = 50$  are shown below, while those for  $m = 20$  can be found in Table A.2.

Five main reweighting methods, shown with Roman numerals, are studied at different trimming amounts. These methods are denoted with upper case letters to represent the subtle differences between sub-methods, and with numbers to represent different skipping levels. Traditional estimators are used in Method I.A, while reweighting with same estimators is employed in Method I.B. Skipping level increases from Method I.B1 to I.B4. Robust estimators and their reweighted versions are used in Method II.A and II.B, respectively. Reweighting procedure is extended in Methods III-V, and individuals charts are constructed to screen individual outlier observations in addition to outlier subgroups. These three methods differ among themselves in application of individuals charts. In Method III, each subgroup location parameter  $\hat{\mu}_i^I$  is estimated and placed as the central line of individuals charts, separately. In Method IV.A, on the other hand, a single robust location estimate of the process  $\hat{\mu}^I$  is used, and a single individuals chart is employed for all subgroups. In Method IV.B, process location estimates are reweighted and a robustified  $\hat{\mu}^I$  is used. In Method V, after screening subgroups via robust dispersion and location estimates, individual observations are pooled without their subgroup information and robust  $\hat{\sigma}^I$  and  $\hat{\mu}^I$  are used in the individuals charts.

In the following subsections, detailed algorithms of five main methods are discussed in detail to explain underlying mechanisms of the reweighting procedures. Here,  $\alpha_1^I$  and  $\alpha_2^I$  represent the skipping level when subgroups and (additionally) individual observations are screened, respectively, while  $\alpha^I$  shows the total skipping level.

### 3.2.1. Method I

Traditional estimators, and common procedure of estimation methods are applied in Method I.A (Figure 3.2). First, subgroup standard deviations  $\hat{\sigma}_i^I$  are estimated from  $S_n$  using Equation 3.1, then  $\hat{\sigma}_i^I$ s are multiplied with bias correction factor  $\Theta_1$ ; since  $n$  equals 5,  $\Theta_1 = 1/c_4(n) = 1.064$ . Process dispersion parameter estimate  $\hat{\sigma}^I$  is obtained by taking the average of dispersion estimates using Equation 3.6. Bias correction is not needed ( $\Theta_2 = 1$ ), since *ave* is an unbiased estimator of location.

- i. Compute  $\mathbf{S}_n(\mathbf{G}_i)$  as defined in Equation 3.1,
- ii. Employ bias correction:  $\hat{\sigma}_i^I = \Theta_1 \times \mathbf{S}_n(\mathbf{G}_i)$ ,
- iii. Compute  $\hat{\sigma}^I$  as defined in Equation 3.6.

Figure 3.2. Algorithm of Method I.A.

In Method I.B,  $S_n$  and *ave* are used in an adaptive skipping scheme (Figure 3.3). Similar to that performed in Method I.A,  $\hat{\sigma}_i^I$ s are estimated using  $S_n$ , and multiplied with  $\Theta_1$  to obtain unbiased standard deviation estimates of rational subgroups. The difference from Method I.A lies in the next step; outlier subgroups are eliminated until all the remaining subgroups fall within the computed Phase I CLs.  $U_n^I$  and  $L_n^I$  are determined with respect to desired skipping level  $\alpha_1$ , which is taken to be equal to 0.0027, 0.010, 0.025, 0.050. used in Method I.B1-B4, respectively. As seen in Figure 3.3, outlier subgroups set  $O^S$  is initialized as a null set, then subgroups are added to  $O^S$  if they exceed  $L_n^I \hat{\sigma}^I$ , or  $U_n^I \hat{\sigma}^I$ . In each iteration, new  $\hat{\sigma}^I$ ,  $L_n^I$  and  $U_n^I$  are computed, and the average of the remaining subgroups is multiplied with  $\Theta_3$  to obtain

an unbiased  $\hat{\sigma}^I$ . The Phase I CLs ( $L_n^I$  and  $U_n^I$ ), and bias correction terms ( $\Theta_1$ ,  $\Theta_2$  and  $\Theta_3$ ) for  $m = 50$  are shown in Table 3.1. While  $\Theta_2$  and  $\Theta_3$  are available in literature, other parameters are determined via computer simulations. As skipping level increases, higher  $L_n^I$  and lower  $U_n^I$  are employed to render tighter CLs.

**i.** Employ Method I.A. Set  $\mathbf{O}^S = \{\}$ ,  
**ii.** Compute the following for  $i^{\text{th}}$  subgroup, which satisfy ( $i \in M$  to  $i \notin O^S$ ),  
 using  $\alpha_1 = (0.0027, 0.010, 0.025, 0.050)$ ,  

$$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$

$$O_{\text{new}} \leftarrow O^S \cup \{i | I_i^S = 0\}$$

$$\hat{\sigma}^I = \frac{1}{m - |O_{\text{new}}|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
**if**  $O_{\text{new}} \neq O^S$  **then**  
      $O^S = O_{\text{new}}$  and go to Step **ii**,  
**else**  
     Continue.  
**end if**  
**iii.** Employ bias correction as  $\hat{\sigma}^I = \Theta^{\text{III}} \times \hat{\sigma}^I$ .

Figure 3.3. Algorithm of Method I.B.

Table 3.1. Phase I CLs and bias correction factors of Method I for  $m = 50$ .

| Method      | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ |
|-------------|------------|------------|---------|---------|------------|
| <b>I.A</b>  | 1.064      | 1.000      | -       | -       | -          |
| <b>I.B1</b> | 1.064      | 1.000      | 0.175   | 2.236   | 1.001      |
| <b>I.B2</b> | 1.064      | 1.000      | 0.244   | 2.048   | 1.002      |
| <b>I.B3</b> | 1.064      | 1.000      | 0.309   | 1.907   | 1.003      |
| <b>I.B4</b> | 1.064      | 1.000      | 0.371   | 1.789   | 1.004      |

### 3.2.2. Method II

Instead of traditional estimators, robust estimators are used without reweighting in Method II.A. Three different combinations of dispersion and location estimators are employed, which are  $MS_n + ML$ ,  $MS_n + HD_{.5}$  and  $MAD_n + HD_{.5}$  in Method II.A1-II.A3, respectively (Figure 3.4). Biased  $\hat{\sigma}_i^I$ 's are calculated using  $MS_n$  in Method II.A1 and II.A2 and using  $MAD_n$  in Method II.A3. The unbiased  $\sigma_i^I$ 's are estimated via multiplying with bias correction term  $\Theta_1$ . Unbiased process dispersion estimate  $\hat{\sigma}^I$  is further determined by calculating the location of  $\hat{\sigma}_i^I$ 's using  $ML$  for Method II.A1, and using  $HD_{.5}$  for Method II.A2 and II.A3, followed by multiplication with bias correction term  $\Theta_2$ .

Use one of the estimators below, and compute  $\hat{\sigma}_i$  and  $\hat{\sigma}^I$  using

- i. Eq. 3.3-3.5 and Eq. 3.12 for Method II.A1,
- ii. Eq. 3.3-3.5 and Eq. 3.8, 3.10-3.11 for Method II.A2,
- iii. Eq. 3.2 and Eq. 3.8, 3.10-3.11 for Method II.A3.
- iv. Employ bias correction:  $\hat{\sigma}_i = \Theta^I \hat{\sigma}_i$  and  $\hat{\sigma}^I = \Theta^{II} \hat{\sigma}^I$ .

Figure 3.4. Algorithm of Method II.A.

In Method II.B, one-step reweighting is performed on the same combinations of estimators employed in Method II.A1-II.A3, respectively (Figure 3.5). The main difference in reweighting method employed in Method II.B from one in Method I.B is that skipping is employed only once (one-step skipping) rather than adaptive skipping. Additionally, skipping level  $\alpha_1$  is constant at 0.025 for Method II.B1-3, while it varies in Method I.B. First, Method II.A procedure is performed, then outlier subgroups  $O^S$ , which fall beyond the  $L_n^I \hat{\sigma}$  and  $U_n^I \hat{\sigma}$ , are determined and eliminated. Averaging the remaining subgroup dispersion estimates and bias correction yield  $\hat{\sigma}^I$ .

- i. Use Method II.A1, II.A2, or II.A3, corresponding to Methods II.B1, II.B2, or II.B3, respectively,
- ii. Set  $\mathbf{O}^S = \{\}$ ,
- iii. Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup, using  $\alpha_1 = 0.025$ ,
- $$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S = \{i | I_i^S = 0\}$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
- iv. Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\mathbf{III}} \hat{\sigma}^{\mathbf{I}}$ .

Figure 3.5. Algorithm of Method II.B.

The Phase I CLs and bias correction terms of Method II estimators for  $m = 50$  are given in Table 3.2. Since reweighting is not employed in Method II.A algorithm,  $L_n^I$ ,  $U_n^I$  and  $\Theta_3$  terms correspond to empty cells.  $\Theta_1$  and  $\Theta_2$  are identical to those for Methods II.A1 and II.B1, II.A2 and II.B2, and II.A3 and II.B3, since identical combinations of estimators are used.

Table 3.2. Phase I CLs and bias correction factors of Method II for  $m = 50$ .

| Method       | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ |
|--------------|------------|------------|---------|---------|------------|
| <b>II.A1</b> | 1.319      | 1.024      | -       | -       | -          |
| <b>II.A2</b> | 1.319      | 1.055      | -       | -       | -          |
| <b>II.A3</b> | 1.803      | 1.094      | -       | -       | -          |
| <b>II.B1</b> | 1.319      | 1.024      | 0.137   | 2.325   | 1.008      |
| <b>II.B2</b> | 1.319      | 1.055      | 0.136   | 2.369   | 1.008      |
| <b>II.B3</b> | 1.803      | 1.094      | 0.106   | 2.656   | 1.013      |

### 3.2.3. Method III

Outliers, which are localized in a subgroup, may be, to a certain extent, eliminated via subgroup screening used in Method I.B and II.B, but diffuse outliers cannot be detected by this method since robust estimators used on subgroups dampen the effect of individual outliers. Reweighting procedure is extended in Method III using individuals charts to screen individual observations in the remaining subgroups.  $MS_n + ML$  estimators are used in Method III.1-III.3, while  $MS_n + HD_{.5}$  are used for Method III.4-III.6. Skipping level in the first subgroup screening stage ( $\alpha_1$ ) are taken to be equal to 0.0027, 0.010 and 0.025, and total skipping level after individuals chart ( $\alpha$ ) is adjusted to be equal to 0.005, 0.020 and 0.040 for Method III.1 and III.4, III.2 and III.5, and III.3 and III.6, respectively. First, Method II.B algorithm is performed using different  $\alpha_1$ , and intermediate  $\hat{\sigma}^I$ s are computed. Then,  $ML$  is used for Method III.1-III.3, while  $HD_{.5}$  is used for Method III.4-III.6 as location estimators. For each remaining subgroup  $m$  different individuals charts, with a common dispersion estimate and different location estimates are constructed, i.e. each individuals chart has different central line ( $\hat{\mu}_i^I$ ), and identical acceptance interval ( $\hat{\sigma}^I$ ). Then  $n$  observations are placed on corresponding individuals chart and observation(s), which are deemed to be outlier(s), are determined ( $R_i$ ). Since even robust estimation methods may fail in the presence of high fraction of outliers in determining all the outliers, if the number of outlier observations in a subgroup is found to be higher than  $n/2$ , then that subgroup is deemed to be subjected to localized contamination, and eliminated, otherwise only individual outlier(s) is/are screened. After all outlier subgroups and individual observations are eliminated, unbiased  $\hat{\sigma}^I$ s are computed from the remaining subgroups via first computing  $S_n$  using the remaining individual observations for each subgroup, then by taking their average (Figure 3.6). The Phase I CLs and bias correction terms used in Method III estimators for  $m = 50$  are given in Table 3.3. Since the same procedure is repeated in subgroup screening,  $\Theta_1$  and  $\Theta_2$  are similar to those in Method II.  $L_n^I$ ,  $U_n^I$  and  $\Theta_3$  terms are computed via computer simulations. Columns of  $C_n^I$  and  $\Theta_4$  represent the coefficients used in individuals chart and bias correction terms after screening individual observations.

- i. Use Method II.A1, or Method II.A2 corresponding to Method III.1-III.3, or Method III.4-III.6, respectively,
- ii. Set  $\mathbf{O}^S = \{\}$ ,
- iii. Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup, using  $\alpha_1 = 0.0027, 0.010, 0.020$ ,
- $$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S = \{i | I_i^S = 0\}$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
- iv. Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\mathbf{III}} \hat{\sigma}^{\mathbf{I}}$ ,
- v. Compute  $\hat{\mu}_i^{\mathbf{I}}$  using Eq. 3.13-3.14, or Eq. 3.9-3.11, ( $i \in M, i \notin O^S$ ), for Method III.1-3, or Method III.4-6, respectively,
- vi. Compute the following for the individual observations using  $\alpha = 0.005, 0.020, 0.040$ ,
- $$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}_i^I - C_n^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}_i^I + C_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- vii. Remove the individual outliers, and use Eq. 3.1 on the remaining data,
- $$R_i = \{j | U_{ij} = 0\}$$
- $$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$
- $$\hat{\mu}_i = \frac{1}{n - |R_i|} \sum_{j=1}^n Z_{ij} U_{ij}, i \notin O^S$$
- $$\hat{\sigma}_i^I = \sqrt{\frac{\sum_{j=1}^n (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|), i \notin O^S$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^m \hat{\sigma}_i^I$$
- viii. Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\mathbf{IV}} \hat{\sigma}^{\mathbf{I}}$ .

Figure 3.6. Algorithm of Method III.

Table 3.3. Phase I CLs and bias correction factors of Method III for  $m = 50$ .

| Method       | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ | $C_n^I$ | $\Theta_4$ |
|--------------|------------|------------|---------|---------|------------|---------|------------|
| <b>III.1</b> | 1.319      | 1.024      | 0.045   | 2.893   | 1.002      | 2.991   | 1.015      |
| <b>III.2</b> | 1.319      | 1.024      | 0.087   | 2.570   | 1.005      | 2.462   | 1.049      |
| <b>III.3</b> | 1.319      | 1.024      | 0.137   | 2.325   | 1.008      | 2.298   | 1.070      |
| <b>III.4</b> | 1.319      | 1.055      | 0.045   | 2.953   | 1.002      | 3.040   | 1.013      |
| <b>III.5</b> | 1.319      | 1.055      | 0.087   | 2.622   | 1.004      | 2.481   | 1.050      |
| <b>III.6</b> | 1.319      | 1.055      | 0.137   | 2.361   | 1.008      | 2.353   | 1.069      |

### 3.2.4. Method IV

In Method III, individuals charts are employed to eliminate diffuse outliers in Phase I sampling by estimating subgroup location estimates separately ( $\hat{\mu}_i^I$ ), and constructed using the limits  $\hat{\mu}_i^I \pm C_n^I \hat{\sigma}^I$ , however  $\hat{\mu}_i^I$  is not an efficient location estimator for processes with small sample size ( $n$ ). Use of single intermediate process location estimate ( $\hat{\mu}^I$ ) is proposed to remedy this problem, and identical CLs ( $\hat{\mu}^I \pm C_n^I \hat{\sigma}^I$ ) are used for all individuals charts in Method IV.A. First, subgroups which are deemed to have different standard deviations are eliminated, and  $\hat{\sigma}^I$  is computed as an intermediate process dispersion estimate (Figure 3.7). Robust location estimators are then employed to calculate  $\hat{\mu}_i^I$  from the remaining subgroups, and followed by estimating a single  $\mu^I$  as an intermediate process location estimate.  $MS_n + ML$  dispersion and location estimators are used in Method IV.A1-IV.A3, and  $MS_n + HD_{.5}$  are used for Method IV.A4-IV.A6. Skipping level  $\alpha_1$  takes the values 0.0027, 0.010 and 0.025, and  $\alpha$  is taken to be equal to 0.005, 0.020 and 0.040 for Method IV.A1, IV.A4, IV.A2, IV.A5 and IV.A3, IV.A6, respectively.

The difference of Method IV.B from Method IV.A lies in reweighting the process location estimate and screening subgroups with different locations. Here,  $MS_n + HD_{.5}$  dispersion and location estimators are employed. After intermediate  $\hat{\mu}^I$  and  $\hat{\sigma}^I$  are calculated, subgroups with process location estimates, which fall beyond the CLs de-

finer by  $\hat{\mu}^I \pm C_n^I \hat{\sigma}^I / \sqrt{n}$ , are eliminated. Then individuals charts are constructed for the remaining subgroups using the CLs  $\hat{\mu}^I \pm C_n^I \hat{\sigma}^I$ . It should be noted that the same coefficient  $C_n$  is used for both subgroup screening charts and individuals charts to yield  $\alpha$  to be equal to 0.005, 0.020 and 0.040 for Method IV.B1-IV.B3, respectively. The Phase I CLs and bias correction terms of Method IV.A and IV.B estimators for  $m = 50$  are given in Table 3.4.

Table 3.4. Phase I CLs and bias correction factors of Method IV for  $m = 50$ .

| Method | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ | $C_n^I$ | $\Theta_4$ |
|--------|------------|------------|---------|---------|------------|---------|------------|
| IV.A1  | 1.319      | 1.024      | 0.045   | 2.893   | 1.002      | 3.070   | 1.011      |
| IV.A2  | 1.319      | 1.024      | 0.087   | 2.570   | 1.005      | 2.578   | 1.038      |
| IV.A3  | 1.319      | 1.024      | 0.137   | 2.325   | 1.008      | 2.411   | 1.057      |
| IV.A4  | 1.319      | 1.055      | 0.045   | 2.953   | 1.002      | 3.092   | 1.011      |
| IV.A5  | 1.319      | 1.055      | 0.087   | 2.622   | 1.004      | 2.577   | 1.039      |
| IV.A6  | 1.319      | 1.055      | 0.137   | 2.361   | 1.008      | 2.420   | 1.056      |
| IV.B1  | 1.319      | 1.055      | 0.045   | 2.953   | 1.002      | 3.030   | 1.013      |
| IV.B2  | 1.319      | 1.055      | 0.087   | 2.622   | 1.004      | 2.913   | 1.019      |
| IV.B3  | 1.319      | 1.055      | 0.137   | 2.361   | 1.008      | 2.762   | 1.030      |

### 3.2.5. Method V

$MS_n + HD_{.5}$  dispersion and location estimators are employed, and skipping level  $\alpha_1$  takes the values 0.0027, 0.010 and 0.025, and  $\alpha$  is taken to be equal to 0.005, 0.020 and 0.040 for Method V.1-V.3, respectively. After skipping subgroups deemed to be outliers, as in Method IV.B, intermediate  $\hat{\mu}^I$  and  $\hat{\sigma}^I$  are determined via pooling of remaining observations in subgroups without their subgroup information. Corresponding robust dispersion and location estimators are employed to determine  $\hat{\mu}^I$  and  $\hat{\sigma}^I$  from the pooled observations in the remaining subgroups and individuals chart is constructed with the CLs defined by  $\hat{\mu}^I \pm C_n^I \hat{\sigma}^I$ . The Phase I CLs and bias correction terms of Method V estimators for  $m = 50$  are given in Table 3.5.

- i.** Use Method II.A1 and II.A2 corresponding to Method IV.A1-IV.A3, or Method IV.A4-IV.A6, respectively,
- ii.** Set  $\mathbf{O}^S = \{\}$ ,
- iii.** Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup, using  $\alpha_1 = 0.0027, 0.010, 0.020$ ,
- $$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S = \{i | I_i^S = 0\}$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
- iv.** Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{III}} \hat{\sigma}^I$ ,
- v.** Compute  $\hat{\mu}_i^{\mathbf{I}}$  and  $\hat{\mu}^{\mathbf{I}}$  using Eq. 3.13-3.14, or Eq. 3.9-3.11, ( $i \in M, i \notin O^S$ ), for Method IV.A1-IV.A3, or Method IV.A4-IV.A6, respectively,
- vi.** Compute the following for the individual observations, ( $i \in M, i \notin O^S$ ), using  $\alpha = 0.005, 0.020, 0.040$ . Note that, contrary to step vi of Method III, a single location estimate  $\hat{\mu}^{\mathbf{I}}$  is used,
- $$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^{\mathbf{I}} - C_n^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^{\mathbf{I}} + C_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- vii.** Remove the individual outliers, and use Eq. 3.1 on the remaining data,
- $$R_i = \{j | U_{ij} = 0\}$$
- $$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2; \\ 0 & \text{if otherwise.} \end{cases}$$
- $$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$
- $$\hat{\mu}_i = \frac{1}{n - |R_i|} \sum_{j=1}^n Z_{ij} U_{ij}, i \notin O^S$$
- $$\hat{\sigma}_i^I = \sqrt{\frac{\sum_{j=1}^n (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|), i \notin O^S$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^m \hat{\sigma}_i^I$$
- viii.** Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{IV}} \hat{\sigma}^I$ .

Figure 3.7. Algorithm of Method IV.A.

- i. Repeat Method II.A2, and set  $\mathbf{O}^S = \{\}$ ,
- ii. Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup, using  $\alpha_1 = 0.0027, 0.010, 0.020$ ,
- $$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S = \{i | I_i^S = 0\}$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
- iii. Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{III}} \hat{\sigma}^{\mathbf{I}}$ ,
- iv. Compute  $\hat{\mu}_i^{\mathbf{I}}$  and  $\hat{\mu}^{\mathbf{I}}$  using Eq. 3.9-3.11, ( $i \in M, i \notin O^S$ ), and set  $\mathbf{O}^L = \{\}$ ,
- v. Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup ( $i \in M, i \notin O^S$ ),
- $$I_i^L = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n^I \hat{\sigma}^L / \sqrt{n} \leq \hat{\mu}_i^I \leq \hat{\mu}^I + C_n^I \hat{\sigma}^L / \sqrt{n} \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^L = \{i | I_i^L = 0\} \text{ and } O^S \leftarrow O^S \cup O^L$$
- vi. Re-estimate the process location parameter using the following reweighted estimator,
- $$\hat{\mu} = \frac{\sum_{i=1}^m \hat{\mu}_i I_i^L I_i^S}{m - |O^L| - |O^S|},$$
- vi. Compute the following for the individual observations, ( $i \in M, i \notin O^S$ ), using  $\alpha = 0.005, 0.020, 0.040$ ,
- $$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- viii. Remove the individual outliers and use Eq. 3.1 on the remaining data,
- $$R_i = \{j | U_{ij} = 0\}$$
- $$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$
- $$\hat{\mu}_i = \frac{1}{n - |R_i|} \sum_{j=1}^n Z_{ij} U_{ij}, i \notin O^S$$
- $$\hat{\sigma}_i^I = \sqrt{\frac{\sum_{j=1}^n (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|), i \notin O^S$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^m \hat{\sigma}_i^I$$
- ix. Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{IV}} \hat{\sigma}^{\mathbf{I}}$ .

Figure 3.8. Algorithm of Method IV.B.

- i.** Repeat Method II.A2, and set  $\mathbf{O}^S = \{\}$ ,
- ii.** Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup, using  $\alpha_1 = 0.0027, 0.010, 0.020$ ,
- $$I_i^S = \begin{cases} 1 & \text{if } L_n^I \hat{\sigma}^I \leq \hat{\sigma}_i^I \leq U_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S = \{i | I_i^S = 0\}$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i=1}^m \hat{\sigma}_i^I I_i^S$$
- iii.** Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{III}} \hat{\sigma}^{\mathbf{I}}$ ,
- iv.** Compute  $\hat{\mu}_i^{\mathbf{I}}$  and  $\hat{\mu}^{\mathbf{I}}$  using Eq. 3.9-3.11, ( $i \in M, i \notin O^S$ ), and set  $\mathbf{O}^L = \{\}$ ,
- v.** Compute the following for  $\mathbf{i}^{\text{th}}$  subgroup ( $i \in M, i \notin O^S$ ),
- $$I_i^L = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n^I \hat{\sigma}^L / \sqrt{n} \leq \hat{\mu}_i^I \leq \hat{\mu}^I + C_n^I \hat{\sigma}^L / \sqrt{n} \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^L = \{i | I_i^L = 0\} \text{ and } O^S \leftarrow O^S \cup O^L$$
- vi.** Pool remaining observations and compute process parameter estimates,
- $$Z_k^* = \{Z_{ij} | i \in M \setminus O^S, j = 1, 2, \dots, n\}$$
- $$\hat{\mu}^I = HD_{.5}(Z_k^*) \text{ and } \hat{\sigma}^I = MS_n(Z_k^*)$$
- vii.** Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\mathbf{V}} \hat{\sigma}^{\mathbf{I}}$ ,
- viii.** Compute the following for the individual observations, ( $i \in M, i \notin O^S$ ), using  $\alpha = 0.005, 0.020, 0.040$ ,
- $$U_{ij} = \begin{cases} 1 & \text{if } \hat{\mu}^I - C_n^I \hat{\sigma}^I \leq Z_{ij} \leq \hat{\mu}^I + C_n^I \hat{\sigma}^I \\ 0 & \text{if otherwise} \end{cases}$$
- ix.** Remove the individual outliers, and use Eq. 3.1 on the remaining data,
- $$R_i = \{j | U_{ij} = 0\}$$
- $$H_i = \begin{cases} 1 & \text{if } |R_i| \geq n/2 \\ 0 & \text{if otherwise} \end{cases}$$
- $$O^S \leftarrow O^S \cup \{i | H_i = 0\}$$
- $$\hat{\mu}_i = \frac{1}{n - |R_i|} \sum_{j=1}^n Z_{ij} U_{ij}, i \notin O^S$$
- $$\hat{\sigma}_i^I = \sqrt{\frac{\sum_{j=1}^n (Z_{ij} - \hat{\mu}_i)^2 U_{ij}}{(n - |R_i| - 1)}} / c_4(n - |R_i|), i \notin O^S$$
- $$\hat{\sigma}^I = \frac{1}{m - |O^S|} \sum_{i \notin O^S}^m \hat{\sigma}_i^I$$
- x.** Employ bias correction as  $\hat{\sigma}^{\mathbf{I}} = \Theta^{\text{IV}} \hat{\sigma}^{\mathbf{I}}$ .

Figure 3.9. Algorithm of Method V.

Table 3.5. Phase I CLs and bias correction factors of Method V for  $m = 50$ .

| Method     | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ | $C_n^I$ | $\Theta_4$ | $\Theta_5$ |
|------------|------------|------------|---------|---------|------------|---------|------------|------------|
| <b>V.1</b> | 1.319      | 1.055      | 0.045   | 2.953   | 1.002      | 3.030   | 1.013      | 1.0009     |
| <b>V.2</b> | 1.319      | 1.055      | 0.087   | 2.622   | 1.004      | 2.913   | 1.019      | 1.019      |
| <b>V.3</b> | 1.319      | 1.055      | 0.137   | 2.361   | 1.008      | 2.762   | 1.030      | 1.017      |

### 3.3. Contamination Models

Phase I data ( $Z_{ij}$ ) is composed of  $m = 20$  or  $m = 50$  subgroups (samples) with  $n = 5$  observations in each sample.  $G_i$  denotes the  $i^{th}$  subgroup of Phase I data. Samples consist of normally distributed data clouded with outliers described with various models in this section. Five main contamination models (CM 1-5), suggested by Tatum [39], are used, and outlier density ( $K$ ) and magnitude of contamination ( $d$ ) are adjusted to devise more realistic scenarios similar. Furthermore, these main contamination models are superimposed (SCM 1-6) to test the efficiency and performance of the proposed estimators more rigorously. In each case, in-control phase I data are assumed to be consist of standardized Gaussian variable  $Z_G \sim N(0, 1)$ , while outliers are represented with another random variable  $Z_C$ , describing contamination models, which correspond to temporary out-of-control states of the out-of-control process. In CM 1-5 and SCM 1-6 models, observations are assumed to be i.i.d ( $corr(Z_{ij}, Z_{ik}) = 0, j \neq k$ ), whereas autoregressive contamination models (ACM 1-3) exclude the independence assumption and assumes an unstable process.

Outliers in Phase I samples are drawn either from another normal distribution, which has a mean different than the main normal distribution to represent shifts in process mean, or from a distribution, which has a different dispersion to correspond to the changes in process variation. In some cases, outliers are spread in all subgroups, i.e. observations in subgroups have equal probability of coming from  $Z_C$  (diffuse contamination models, mainly CM 1, 2 and 4), while, in other case, outliers are concentrated in certain subgroups (localized contamination models, mainly CM 3 and 5), i.e. observations in subgroups come from  $Z_C$ , and the remaining subgroups are clean.

Denoting the Bernoulli distributed indicator with  $I_{ij}$  ( $I_{ij} \sim B(1, K)$ ) to represent the contaminated observations in diffuse contamination models, each  $I_{ij}$  ( $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ ) has two outcomes: success (outlier) with probability of  $K$  and failure (clean) with probability of  $1 - K$ . The indicator  $I_i$ , ( $i = 1, 2, \dots, m$ ) is used to represent contaminated subgroups in localized contamination models.

An example of simulated normally distributed ( $Z_{ij} \sim N(0, 1)$ ) Phase I data is shown in Figure 3.10. Here, there are neither individual outliers within subgroups, nor outlier subgroups.

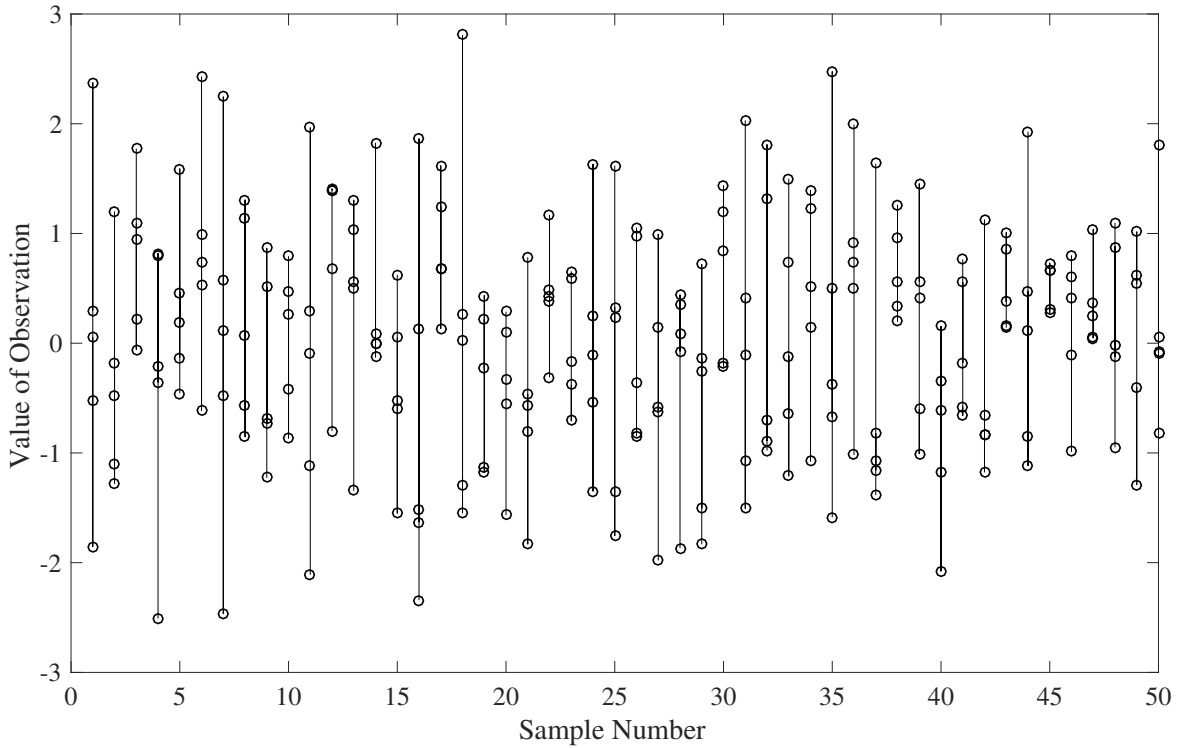


Figure 3.10. An example of normally distributed Phase I data.

### 3.3.1. Main Contamination Models

Five different contamination models are discussed in this section. In CM 1, 3-5, parameters are taken to be equal to  $K = (0.10, 0.15, \text{ and } 0.20)$  and  $d = (0.5, 1, \dots, 6)$  for Phase I efficiency analysis, and  $(K, d) = (0.10, 2; 0.15, 3; 0.20, 3)$  for Phase II performance analysis. Only in CM 2,  $K = (0.10, 0.15, \text{ and } 0.20)$  and  $d = (0.25, 0.50, \dots, 3)$

for Phase I efficiency, and  $(K, d) = (0.10, 0.75; 0.15, 1; 0.20, 1)$  for Phase II performance are studied.

3.3.1.1. CM 1: Diffuse Symmetric Variance Contamination Model. Each observation in each subgroups has  $K$  probability of being drawn the population of  $Z_C \sim N(0, d^2)$  and  $1 - K$  probability of coming from the normal distribution  $Z_G \sim N(0, 1)$ . Therefore, observations may be described by the following probability distribution:

$$Z_{ij} = \begin{cases} \sim N(0, 1) & \text{if } I_{ij} = 0 \\ \sim N(0, d^2) & \text{if } I_{ij} = 1 \end{cases} \quad (3.15)$$

A simulated example of Phase I sampling under CM 1 is shown in Figure 3.11, in which observations denoted with empty circles are clean observations, i.e.  $Z_G \sim N(0, 1)$  ( $I_{ij} = 0$ ), while ones with black filled circles are outliers, i.e.  $Z_C \sim N(0, 3^2)$  ( $I_{ij} = 1$ ).

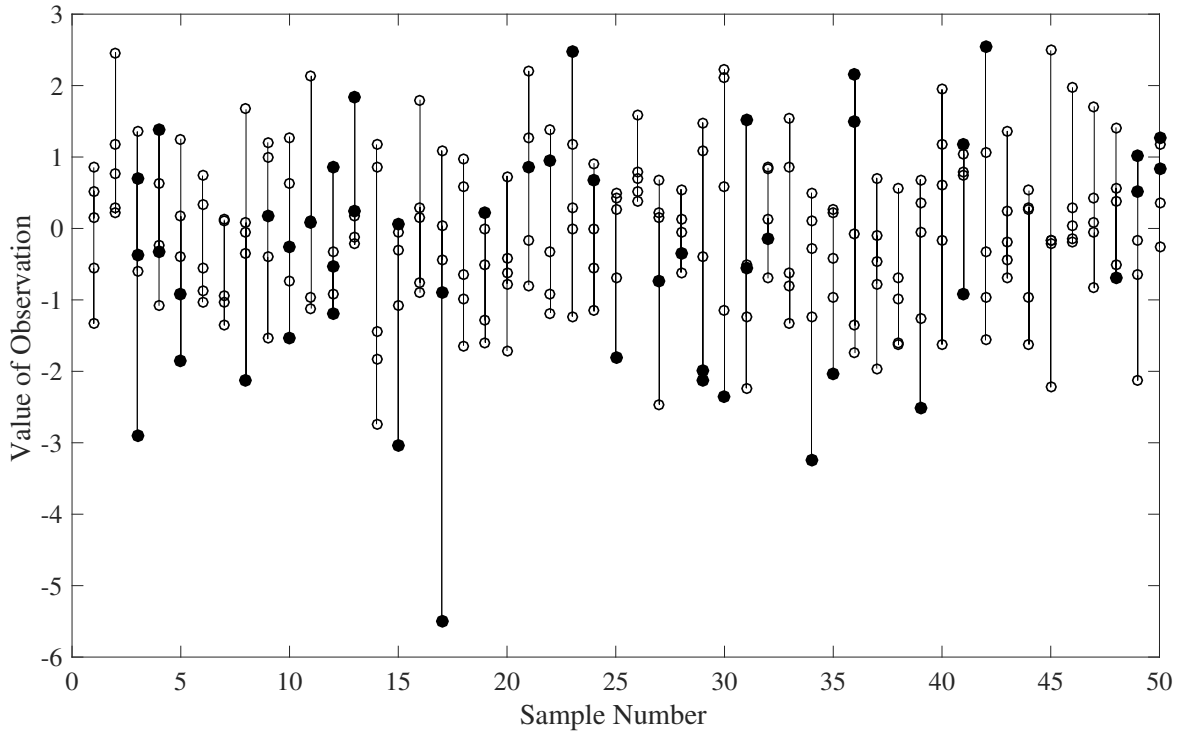


Figure 3.11. Phase I sampling subjected to CM 1 ( $K=20\%$ ,  $d=3$ ).

3.3.1.2. CM 2: Diffuse Asymmetric Variance Contamination Model. Each observation in each subgroup has  $K$  probability of being drawn from a chi-square distribution with one degree of freedom,  $Z_C \sim d\chi_1^2$ , and  $1 - K$  probability of being drawn from the normal distribution  $Z_G \sim N(0, 1)$ .

$$Z_{ij} = \begin{cases} \sim N(0, 1) & \text{if } I_{ij} = 0 \\ \sim d\chi_1^2 & \text{if } I_{ij} = 1 \end{cases} \quad (3.16)$$

A simulated example of Phase I sampling under CM 2 is shown in Figure 3.11, in which observations denoted with empty circles are clean observations, i.e.  $Z_G \sim N(0, 1)$  ( $I_{ij} = 0$ ), while ones with black filled circles are outliers, i.e.  $Z_C \sim 1\chi_1^2$  ( $I_{ij} = 1$ ). Compared to Figure 3.11, one can clearly see the asymmetry of distribution.

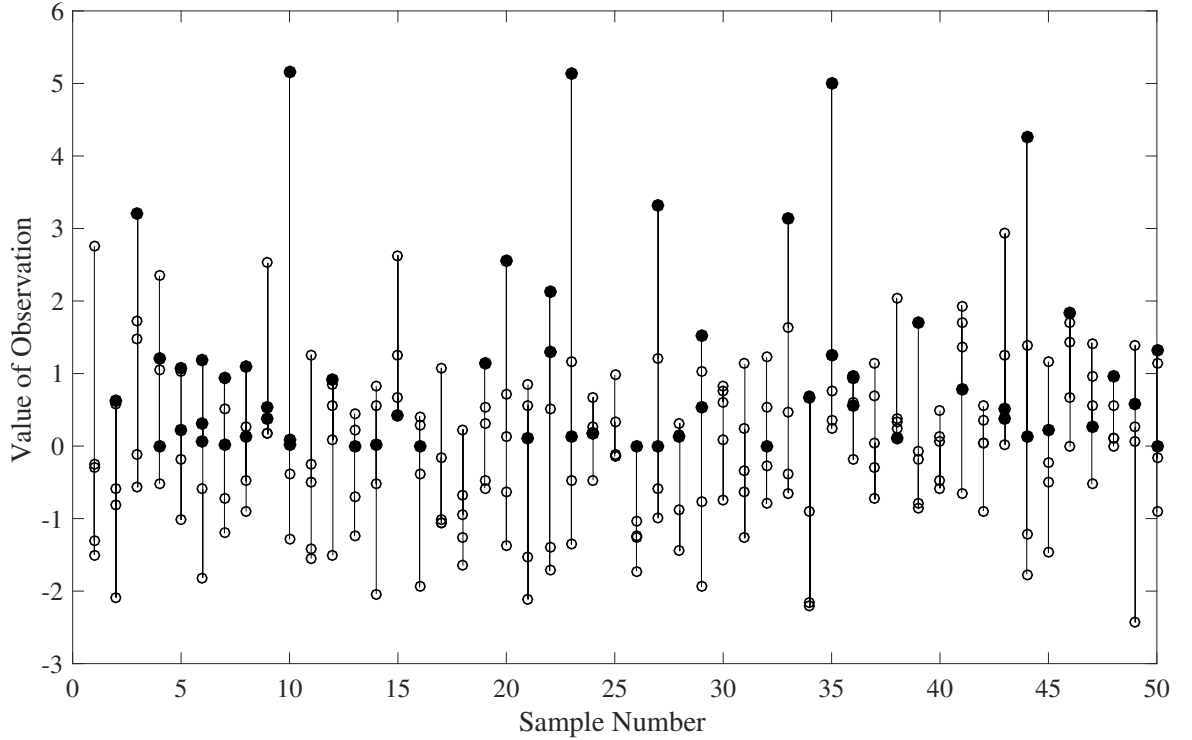


Figure 3.12. Phase I sampling subjected to CM 2 ( $K=20\%$ ,  $d=1$ ).

3.3.1.3. CM 3: Localized Variance Contamination Model. All observations in the first  $K \times m$  subgroups are sampled from  $Z_C \sim N(0, d^2)$ , while the rest of the subgroups are

free from the outliers.

$$Z_{ij} = \begin{cases} \sim N(0, d^2) & \text{if } 1 \leq i \leq \text{round}(K \times m) \\ \sim N(0, 1) & \text{if } \text{round}(K \times m) + 1 \leq i \leq m \end{cases} \quad (3.17)$$

A simulated example of Phase I sampling under CM 3 is shown in Figure 3.13. While observations in the first 10 subgroups, shown with black filled circles are outliers, i.e.  $Z_C \sim N(0, 3^2)$ , the rest of the subgroups, denoted with empty circles are clean, i.e.  $Z_G \sim N(0, 1)$ .

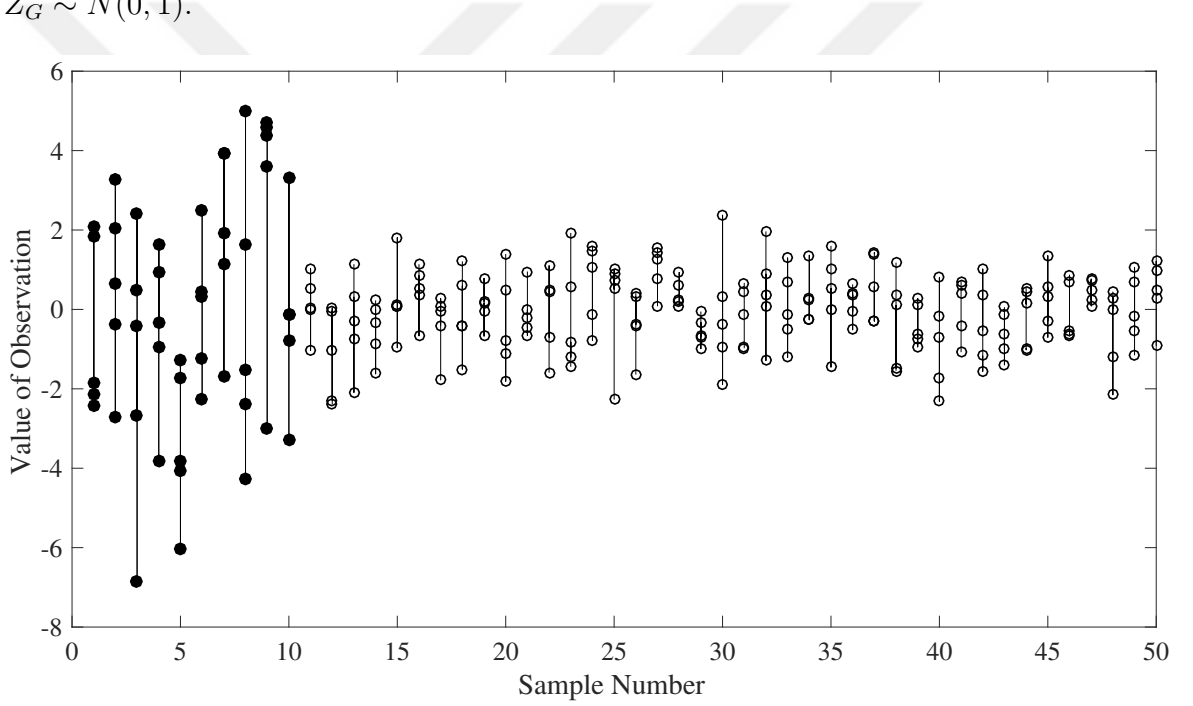


Figure 3.13. Phase I sampling subjected to CM 3 ( $K=20\%$ ,  $d=3$ ).

3.3.1.4. CM 4: Diffuse Mean Contamination Model. In this case, each observation in each subgroup has  $K$  probability of being drawn from the population of  $Z_C \sim N(d, 1)$ , and  $1 - K$  probability of from normal distribution  $Z_G \sim N(0, 1)$ .

$$Z_{ij} = \begin{cases} \sim N(0, 1) & \text{if } I_{ij} = 0 \\ \sim N(d, 1) & \text{if } I_{ij} = 1 \end{cases} \quad (3.18)$$

A simulated example of Phase I sampling under CM 4 is shown in Figure 3.14, in which observations denoted with empty circles are clean observations, i.e.  $Z_G \sim N(0, 1)$  ( $I_{ij} = 0$ ), while ones with black filled squares are outliers, i.e.  $Z_G \sim N(3, 1)$  ( $I_{ij} = 1$ ). Asymmetric distribution of observations is similar to that seen under CM 2, but outliers are less obvious, since they are concentrated at smaller positive values.

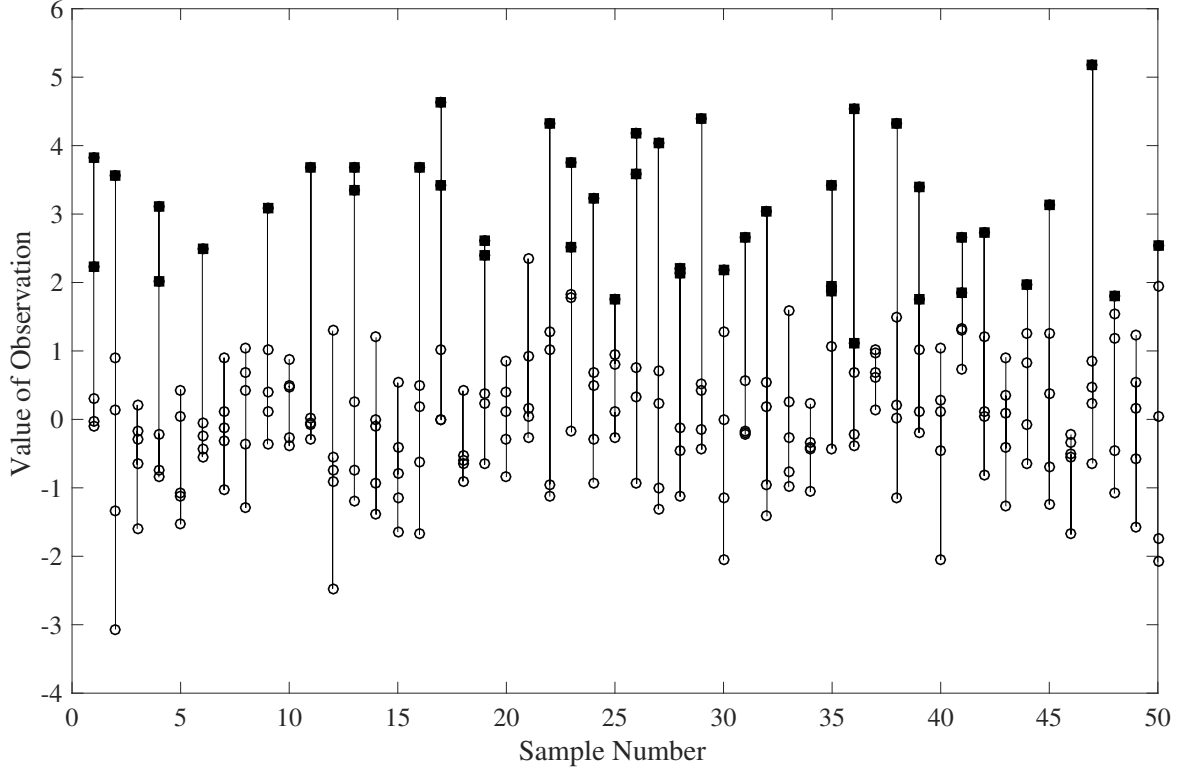


Figure 3.14. Phase I sampling subjected to CM 4 ( $K=20\%$ ,  $d=3$ ).

**3.3.1.5. CM 5: Localized Mean Contamination Model.** This contamination model is similar to CM 3; the only difference is the shift in process mean instead of that in process variation. In this case, first  $K \times m$  subgroups are sampled from  $Z_G \sim N(d, 1)$ , while the rest of the subgroups are free from the outliers.

$$Z_{ij} = \begin{cases} \sim N(d, 1) & \text{if } 1 \leq i \leq \text{round}(K \times m) \\ \sim N(0, 1) & \text{if } \text{round}(K \times m) + 1 \leq i \leq m \end{cases} \quad (3.19)$$

A simulated example of Phase I sampling under CM 5 is shown in Figure 3.15, in which observations denoted with empty circles are clean observations, i.e.  $Z_G \sim N(0, 1)$ , while ones with black filled squares are outliers, i.e.  $Z_C \sim N(3, 1)$ .

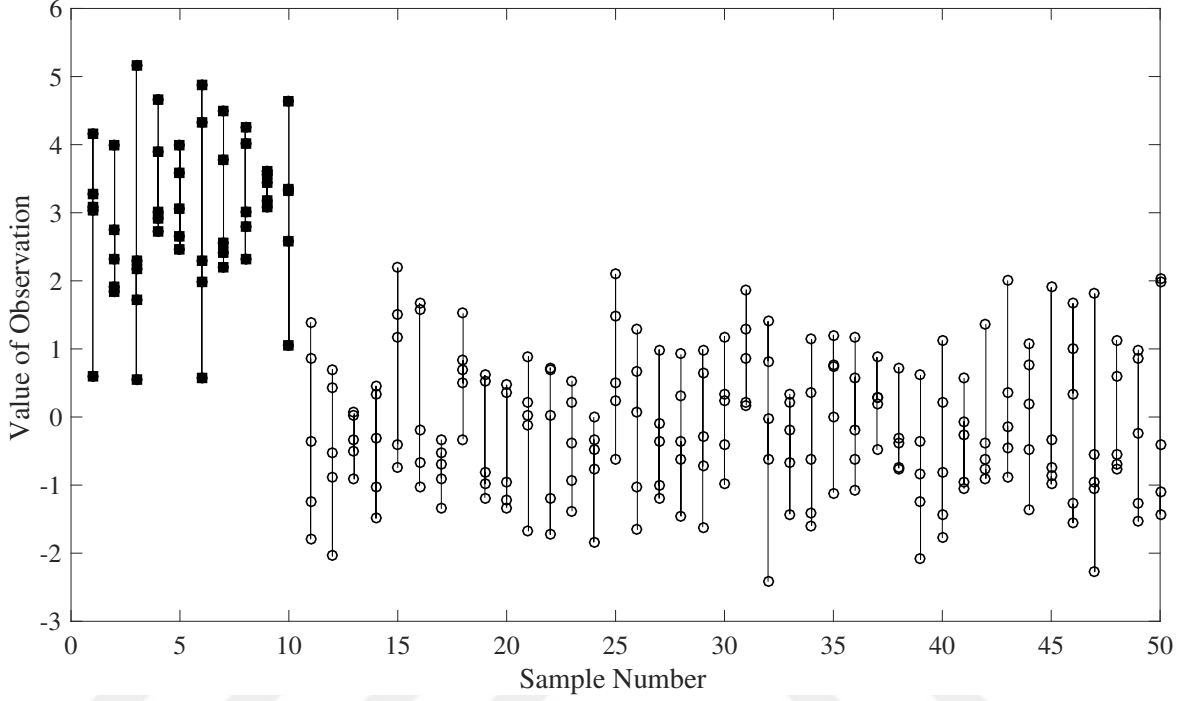


Figure 3.15. Phase I sampling subjected to CM 5 ( $K=20\%$ ,  $d=3$ ).

### 3.3.2. Superimposed Contamination Models

Six superimposed CMs (SCM) are discussed here to examine the effect of simultaneous perturbation in process mean and variation, and/or local and diffuse contaminations. In SCM 1-6, single contamination magnitude is taken to be equal to ( $d = 3$ ), while contamination density  $K$  takes the values  $K = 0.08$  and  $0.10$  for  $m = 50$  and  $m = 20$  to yield overall outlier densities to be equal to  $0.16$  and  $0.20$  for  $m = 50$  and  $m = 20$ , respectively.

**3.3.2.1. SCM 1: Diffuse + Localized Variance Contamination Model.** The first  $K \times m$  subgroups are sampled from normal distribution  $Z_C \sim N(0, 3^2)$ , and each observations in the rest of subgroups has  $K$  probability of being drawn from the same  $Z_C$ .

3.3.2.2. SCM 2: Diffuse + Localized Mean Contamination Model.  $K \times m$  subgroups are sampled from normal distribution  $Z_C \sim N(3, 1)$ , and each observations in the rest of subgroups has  $K$  probability of being drawn from the same population  $Z_C$ .

3.3.2.3. SCM 3: Diffuse Variance + Mean Contamination Model. Each observations in each subgroups has  $K$  probability of being drawn the population of  $Z_C \sim N(3, 1)$ , and  $K$  probability of being drawn the population of  $Z_C \sim N(0, 3^2)$  simultaneously.

3.3.2.4. SCM 4: Localized Variance + Mean Contamination Model.  $K \times m$  subgroups are sampled from normal distribution  $Z_C \sim N(3, 1)$ , and another  $K \times m$  subgroups from another normal distribution  $Z_C \sim N(0, 3^2)$ , while the rest of the subgroups are normally distributed.

3.3.2.5. SCM 5: Diffuse Variance + Localized Mean Contamination Model. The first  $K \times m$  subgroups are sampled from normal distribution  $Z_C \sim N(3, 1)$ , and observations in the rest of the subgroups have a probability of  $K$  to be drawn from another normal distribution  $Z_C \sim N(0, 3^2)$ .

3.3.2.6. SCM 6: Localized Variance + Diffuse Mean Contamination Model. The first  $K \times m$  subgroups are sampled from normal distribution  $Z_C \sim N(0, 3^2)$ , and observations in the rest of the subgroups have a probability of  $K$  to be drawn from another normal distribution  $Z_C \sim N(3, 1)$ .

### **3.3.3. Autoregressive Contamination Model**

Phase I estimators are used to detect shifts in process mean and variation, and determine whether process is in statistically control or not. Autoregressive contamination models (ACM) assumes that the process is out-of-control and clouded with outliers. Phase I sampling under ACM will be discussed only to examine the efficiency of estimators in detecting of out-of-control status of the process, and not further be

examined in comparison of Phase II performance. A first order autoregressive (AR) process is employed to model process mean, as follows.

$$\mu_i = \phi\mu_{i-1} + e_i \quad (3.20)$$

in which  $e_i$  is normally distributed ( $e_i \sim N(0, \sigma_e^2)$ ); the location of the  $i^{th}$  subgroup is correlated with the location parameter of the  $(i - 1)^{th}$  subgroup, Phase I samples are modeled as:

$$Z_{ij} = \mu_i + \epsilon_{ij} \quad (3.21)$$

3.3.3.1. ACM 1: Autoregressive Process with Normal Data. In this model, the process does not contain outliers, and two different parameter value sets  $\phi = (.4, .8)$  and  $\sigma_e^2 = (.5, 1)$  are used to simulate out-of-control processes.

3.3.3.2. ACM 2: Autoregressive Process with CM 1. Out-of-control process is described with the parameter values  $\phi = .8$  and  $\sigma_e^2 = 1$ .  $\epsilon_{ij}$  in Equation 3.21 is modeled by a contaminated normal distribution, corresponding to diffuse variance contamination in Equation 3.15 with  $K = .15$  and  $d = 3$ .

3.3.3.3. ACM 3: Autoregressive Process with CM 4. Same parameters of  $\phi = .8$  and  $\sigma_e^2 = 1$  are used, but  $\epsilon_{ij}$  in Equation 3.21 is modeled by a contaminated normal distribution, corresponding to diffuse mean contamination in Equation 3.18 with  $K = .15$  and  $d = 3$ .

## 4. RESULTS

In this chapter we investigate the Phase I efficiency of the proposed estimation methods and Phase II performance of S-Charts constructed from these methods. Phase I data, which are denoted with  $Z_{ij}$  ( $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ ), consist of  $m$  subgroups with  $n$  individual observations, and Phase II data, which are denoted with  $Y_{ij}$  ( $i = 1, 2, \dots$ , and  $j = 1, 2, \dots, n$ ), consist of an identical number of individual observations ( $n$ ), while  $k$  depends on the length of time that the control chart is used. In robust application of control charts, Phase I estimator should be efficient under normal distribution and robust against contaminations, while Phase II estimator should be sensitive to disturbances [24]. Therefore, the proposed Phase I estimators are used to determine  $\hat{\sigma}^I$ , and the dispersion estimate of the  $i^{th}$  sample of Phase II data is computed by  $\hat{\sigma}_k^{II} = S_n(Y_{ij})/c_4(n)$ , and checked using Phase II CLs.

This chapter is structured as follows. Phase I performance of estimators is evaluated in Section 4.1, when uncontaminated normal and contaminated situations are encountered. The results for  $m = 50$  are given at moderate contamination density ( $K = 0.15$ ) in the same section (Figure 4.3-4.10), while those for low ( $K = 0.10$ ) and high contamination density ( $K = 0.20$ ) are given in Appendix B (Figure B.1-B.4). The results for  $m = 20$  under all CMs are given in Appendix B (Figure B.5-B.10). In Section 4.2, derivation of Phase II CLs for S-Charts using the proposed estimation methods is presented. Next, Phase II performances of S-Charts using the proposed estimators for  $m = 50$  are shown in Section 4.3 under normal distribution and in the presence of contaminations with various outlier densities. For easier inspection, visual presentations of the results for  $K = 0.15$  are presented in Figure 4.11-4.18. Phase II results for  $m = 50$  are given in (Table C.1-C.44), while those for  $m = 20$  are tabulated in Appendix D (Table D.1-D.44). Phase II performance metrics are obtained via 50,000 computer simulations. In Section 4.4, Phase I performance of the proposed estimators is studied on out-of-control processes. Finally, control charts are tested on real-data examples in Section 4.5.

#### 4.1. Evaluation of Phase I Efficiency of Proposed Estimation Methods

Phase I efficiency of dispersion estimators is assessed by mean square error ( $MSE$ ), which is equal to the expected value of the squared difference between the unbiased estimate and the true value:

$$MSE = \frac{1}{N_{run}} \sum_{q=1}^{N_{run}} (\hat{\sigma}_{\{q\}}^I - \sigma_0)^2 \quad (4.1)$$

here,  $\hat{\sigma}_{\{q\}}^I$  represents the standard deviation estimate of  $q^{th}$  Monte Carlo (MC) simulation, and  $N_{run}$  is the number of MC simulations, and is equal to 100,000.

$MSE$  values under normality are shown for  $m = 50$  and  $m = 20$  in Figure 4.1 and 4.2, respectively. In both figures, x axes denote the estimator number, and dashed lines separate proposed estimation methods from Methods I to V, for which subgroup statistics and trimming amounts are given in Table 4.1. In the absence of outliers, traditional sample standard deviation ( $S_n$ ) is the most efficient estimator, and its efficiency slightly deteriorates as trimming amount increases (Method I). Gaussian efficiency of non-reweighted and reweighted robust estimators (Method II) is far from desirable, especially when  $MAD_n$  is used as dispersion estimator (Estimator No. 8 and 11). Reweighting procedure increases the efficiency of Method II.A2 and II.A3 estimators, in which  $HD_{.5}$  is used as the location estimator, but decreases the efficiency of Method II.A1 estimator, in which  $ML$  is utilized. Employing individuals charts within subgroups in Method III-V improves the Phase I efficiency of robust estimators under normality. Method V yields the minimum MSEs, followed by Method IV.A, IV.B and III, respectively. While the increase in skipping level ( $\alpha$ ) deteriorates the Gaussian efficiency of all estimators, it should be noted that the efficiency loss is minimum in Method V and Method IV.B (Figure 4.1). Similar trends are observed when the sample size is relatively low ( $m = 20$ ), the only difference is that  $MSE$  values are relatively higher, because low sample size inflates the variance of estimators (Figure 4.2).

Table 4.1. Summary of proposed estimators.

| Est. No. | Method | Subgroup Statistics |           | $\alpha_1$ | $\alpha$ |
|----------|--------|---------------------|-----------|------------|----------|
| 1        | I.A    | $S_n$               | $ave$     | -          | -        |
| 2        | I.B1   | $S_n$               | $ave$     | 0.0027     | -        |
| 3        | I.B2   | $S_n$               | $ave$     | 0.010      | -        |
| 4        | I.B3   | $S_n$               | $ave$     | 0.025      | -        |
| 5        | I.B4   | $S_n$               | $ave$     | 0.050      | -        |
| 6        | II.A1  | $MS_n$              | $ML$      | -          | -        |
| 7        | II.A2  | $MS_n$              | $HD_{.5}$ | -          | -        |
| 8        | II.A3  | $MAD_n$             | $HD_{.5}$ | -          | -        |
| 9        | II.B1  | $MS_n$              | $ML$      | 0.025      | -        |
| 10       | II.B2  | $MS_n$              | $HD_{.5}$ | 0.025      | -        |
| 11       | II.B3  | $MAD_n$             | $HD_{.5}$ | 0.025      | -        |
| 12       | III.1  | $MS_n$              | $ML$      | 0.0027     | 0.005    |
| 13       | III.2  | $MS_n$              | $ML$      | 0.010      | 0.020    |
| 14       | III.3  | $MS_n$              | $ML$      | 0.025      | 0.040    |
| 15       | III.4  | $MS_n$              | $HD_{.5}$ | 0.0027     | 0.005    |
| 16       | III.5  | $MS_n$              | $HD_{.5}$ | 0.010      | 0.020    |
| 17       | III.6  | $MS_n$              | $HD_{.5}$ | 0.025      | 0.040    |
| 18       | IV.A1  | $MS_n$              | $ML$      | 0.0027     | 0.005    |
| 19       | IV.A2  | $MS_n$              | $ML$      | 0.010      | 0.020    |
| 20       | IV.A3  | $MS_n$              | $ML$      | 0.025      | 0.040    |
| 21       | IV.A4  | $MS_n$              | $HD_{.5}$ | 0.0027     | 0.005    |
| 22       | IV.A5  | $MS_n$              | $HD_{.5}$ | 0.010      | 0.020    |
| 23       | IV.A6  | $MS_n$              | $HD_{.5}$ | 0.025      | 0.040    |
| 24       | IV.B1  | $MS_n$              | $HD_{.5}$ | 0.0027     | 0.005    |
| 25       | IV.B2  | $MS_n$              | $HD_{.5}$ | 0.010      | 0.020    |
| 26       | IV.B3  | $MS_n$              | $HD_{.5}$ | 0.025      | 0.040    |
| 27       | V.1    | $MS_n$              | $HD_{.5}$ | 0.0027     | 0.005    |
| 28       | V.2    | $MS_n$              | $HD_{.5}$ | 0.010      | 0.020    |
| 29       | V.3    | $MS_n$              | $HD_{.5}$ | 0.025      | 0.040    |

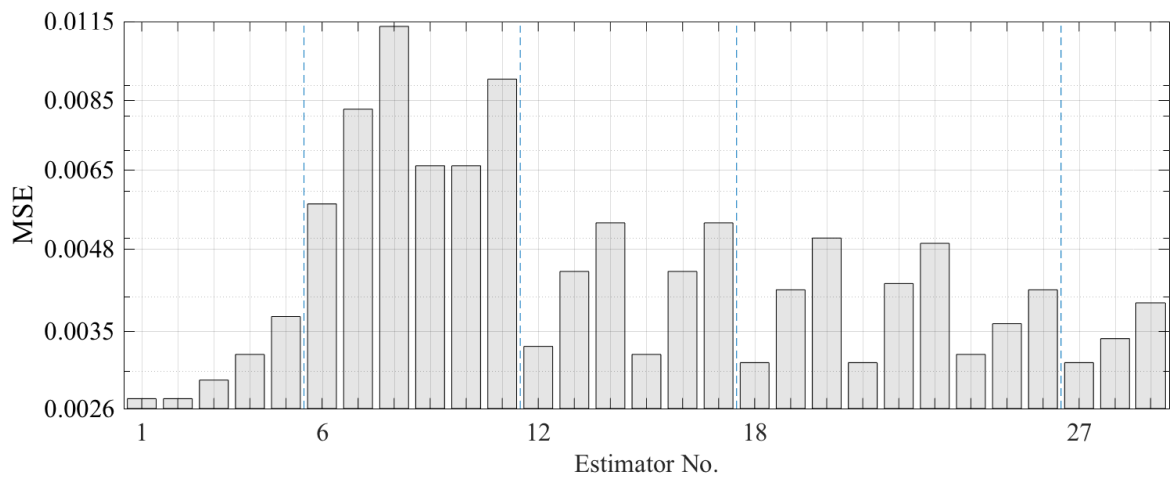


Figure 4.1. Phase I efficiency of the proposed estimators for  $m = 50$  under normal distribution.

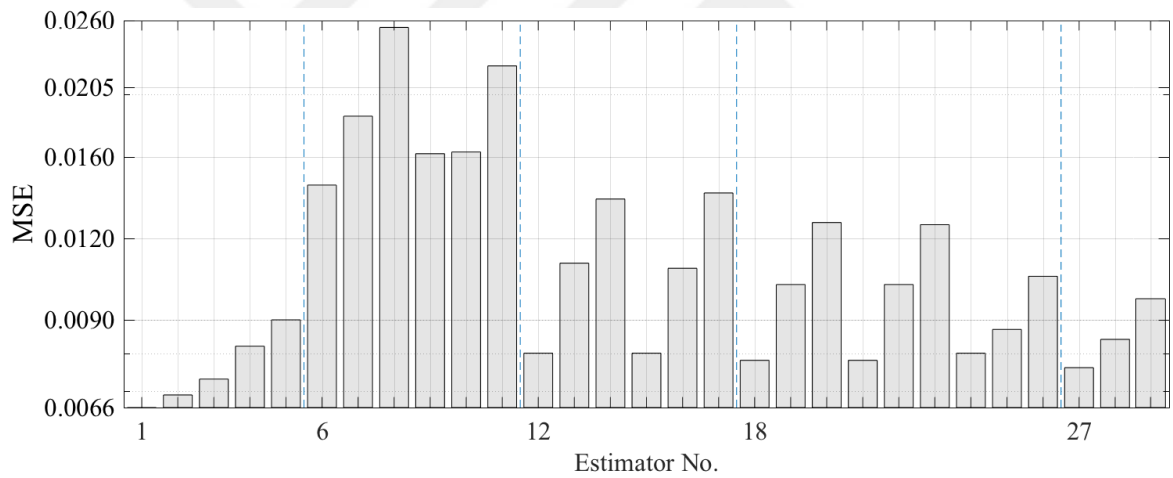


Figure 4.2. Phase I efficiency of the proposed estimators for  $m = 20$  under normal distribution.

Figure 4.3 shows the  $MSE$  values when Phase I samples is subjected to CM 1. Method I estimator, having the highest Gaussian efficiency, promptly loses its superiority to robust estimators as contamination magnitude ( $d$ ) increases. Although Method II estimators are more robust compared to Method I estimators against CM 1, their performances are worse than Method III-V estimators. It is not expected from Method I and II estimators to have high efficiencies, since these methods ignore individual outliers, and are based on subgroup screening only. Among the estimation methods

in which individuals charts are employed, maximum efficiency is observed in Method V, closely followed by Method IV.A6, IV.B3 and III.6, sorted with respect to the decreasing efficiencies. It should be noted that Method III estimator falls short slightly at low and moderate contamination densities, and its inferiority is more apparent at higher contamination density (Figure B.1b). This can be explained on the basis of differences in reweighting algorithms: individuals chart are constructed using location estimates of each subgroup, which are estimated separately in Method III, and can still be influenced by outliers, while a single location estimate is used in Method IV-V. It should further be noted that Method IV.A estimator has higher efficiency than Method IV.B, possibly because partitioning the total skipping level into screening subgroups and individual observations in Method IV.B yields wider CLs, which seems to decrease its efficiency against diffuse outliers.

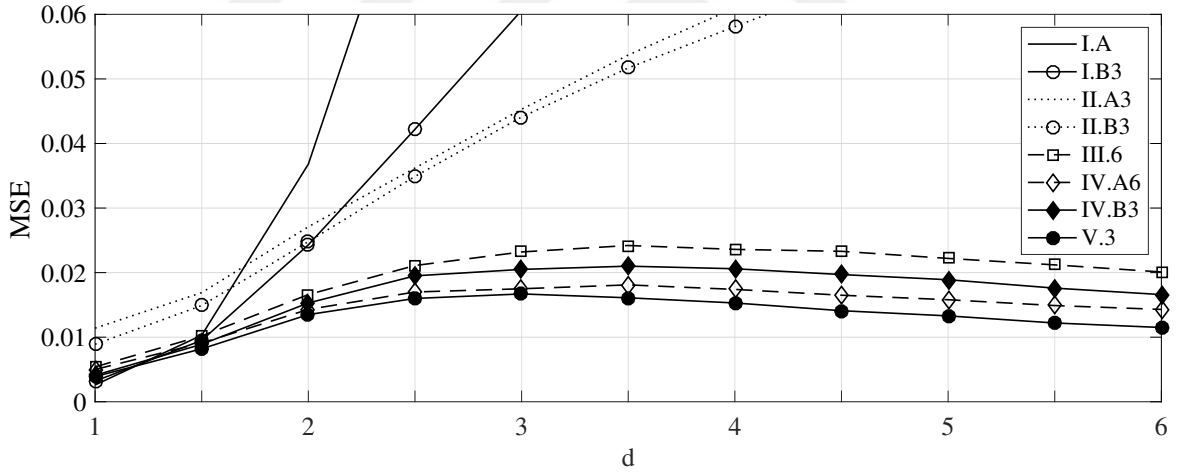


Figure 4.3. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 1 with  $K = 0.15$ .

The effect of skipping level on Phase I efficiency of Method IV and V estimators can be seen in Figure 4.4. Although lower skipping level yields higher Gaussian efficiency for both methods (Figure 4.1), efficiency of the estimators under contamination is increased when skipping level is inflated, recognized even at low contamination magnitude ( $d \approx 2$ ). Method V estimator is affected less by skipping level, rendering this estimator more favorable.

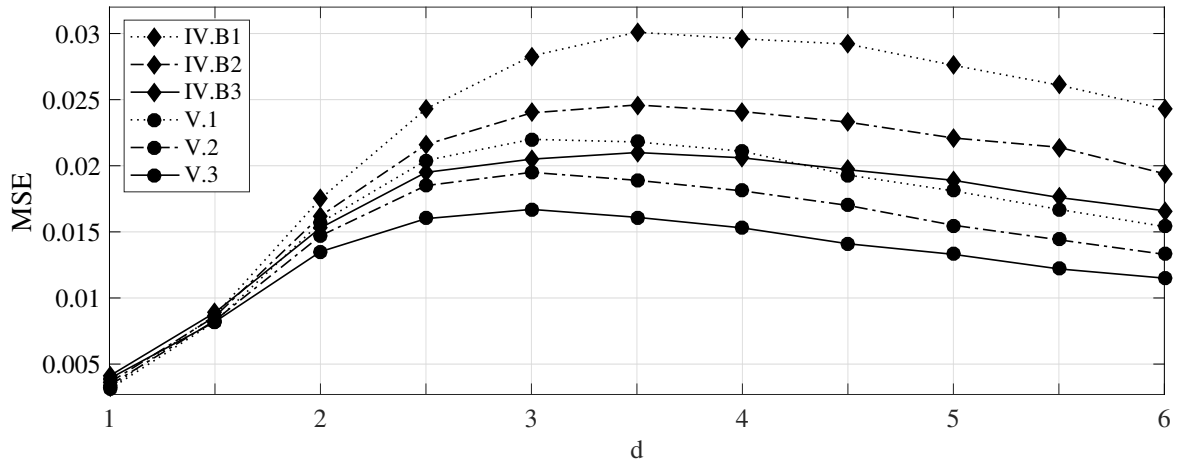


Figure 4.4. Phase I efficiency of the Method IV and V estimators for  $m = 50$  under CM 1 with  $K = 0.15$ .

When Phase I sampling is under CM 2, results are similar to those under CM 1. The main difference is seen in the marginally superior efficiency of Method IV.B compared to that of Method IV.A, particularly for  $d \lesssim 1.5$ , which is a realistic range for the outliers in practice. (Figure 4.5 and Figure B.2). A similar picture emerges also for  $m = 20$ ; efficiency of Method IV.A is higher than that of Method IV.B only at the highest contamination level (Figure B.6).

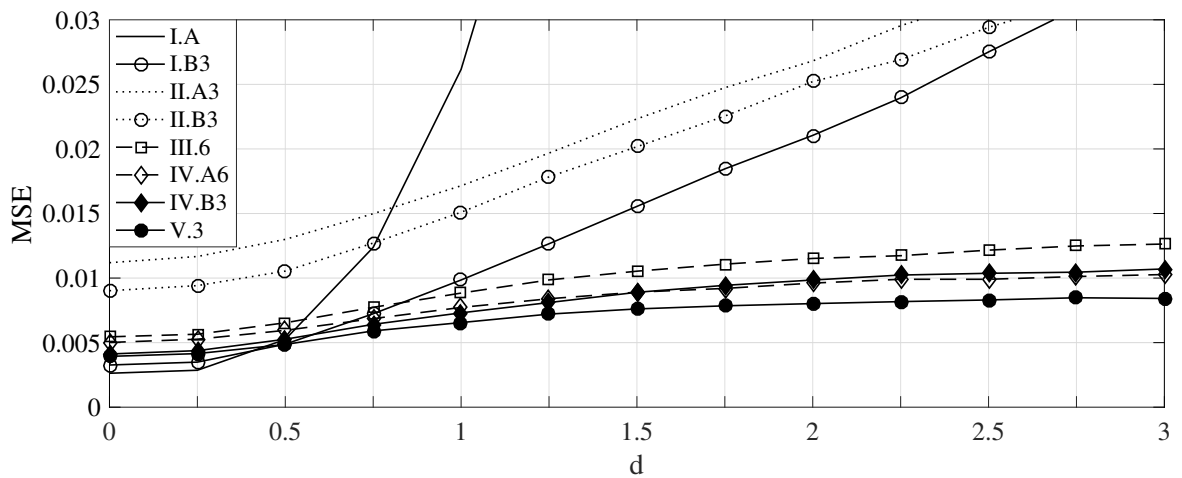


Figure 4.5. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 2 with  $K = 0.15$ .

Figure 4.6 shows the efficiency of estimators when outliers are allowed only to effect specific subgroups in Phase I samples (CM 3). Contrary to that seen in the presence of diffuse outliers, reweighting significantly improves the efficiency of traditional estimators, since Method I.B estimators are mainly based on screening the subgroups which have different dispersion estimates. Method V and IV.B estimators, on the other hand, yield very close  $MSE$  values to those of Method I.B estimator against CM 3 until contamination magnitude  $d \lesssim 4$ , and will be more efficient than that of Method I.B as contamination magnitude is further increased. Similar behaviour is observed at different contamination densities (Figure B.3), and for smaller number of subgroups (Figure B.7).

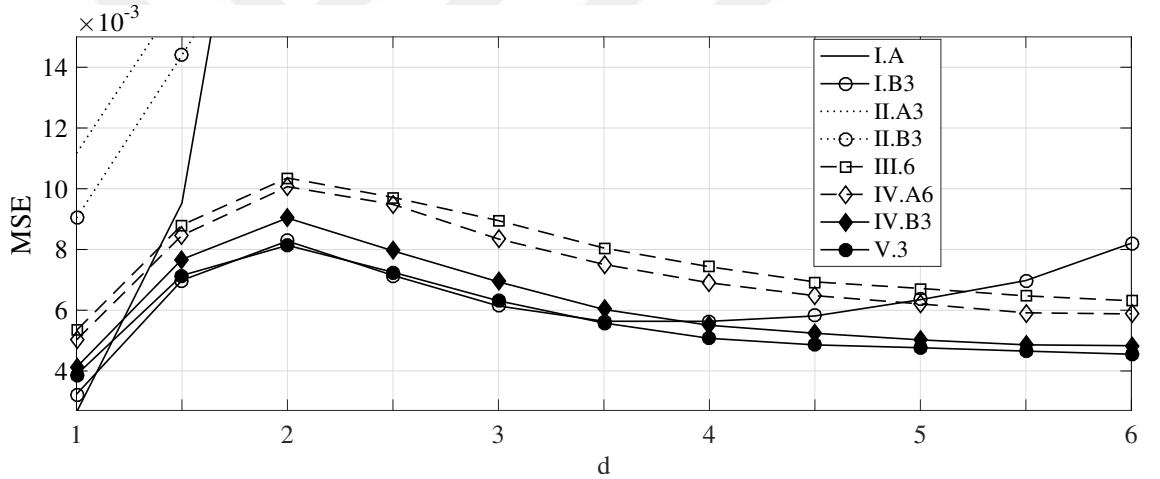


Figure 4.6. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 3 with  $K = 0.15$ .

When diffuse mean contaminations (CM 4) are present in Phase I samples (Figure 4.7), Method I estimators perform far worse than the other estimators. Method II.A and II.B estimators perform significantly better, but not as well as the estimators using individuals chart. Even though all robust estimators have close  $MSE$  values until  $d \approx 2$  at moderate contamination density, Method III-V estimators will be more resistant to outliers in contrast with Method II as  $d$  increases, i.e.  $MSE$  values of Method II.A and II.B remain approximately the same when contamination magnitude is further increased  $d \gtrsim 3.5$ , but those of Method III-V decrease with increasing contamination

magnitude. Against CM 4, Phase I efficiency of Method IV.B estimator is slightly lower for  $m = 50$  compared to other estimators, which utilize individuals charts, while being superior to both Method III and IV.A when  $m$  equals 20 (Figure B.8).

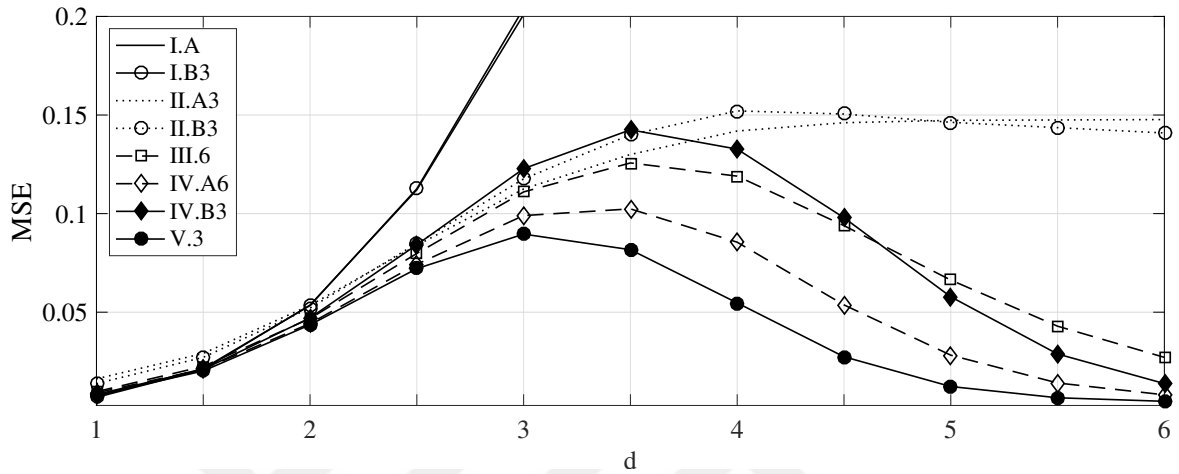


Figure 4.7. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 4 with  $K = 0.15$ .

Method I-III estimators are not affected by localized mean contaminations, since reweighting steps are based on only subgroup statistics in these estimators. When individuals charts are constructed using a single location estimate, on the other hand, between-subgroup variation may affect the final process dispersion estimate, hence efficiency of only Method IV.A, IV.B and V estimators against CM 5 are examined at  $K = 0.15$  (Figure 4.8),  $K = 0.10$  and  $K = 0.20$  (Figure 4.9a and 4.9b, respectively). While Method IV.B and V estimators are not perturbed significantly, there seems to be a peak around ( $d \approx 2$ ) in the  $MSE$  profile of Method IV.A estimator; weakness of Method IV.A estimator lies in including subgroups with different location estimates in individuals charts. It should, further, be noted that  $MSE$  of Method III estimators are higher than  $5.0 \times 10^{-3}$  under normal distribution (Figure 4.1), hence efficiencies of Method IV.B and V estimators are still higher than that of Method III estimator under CM 5.

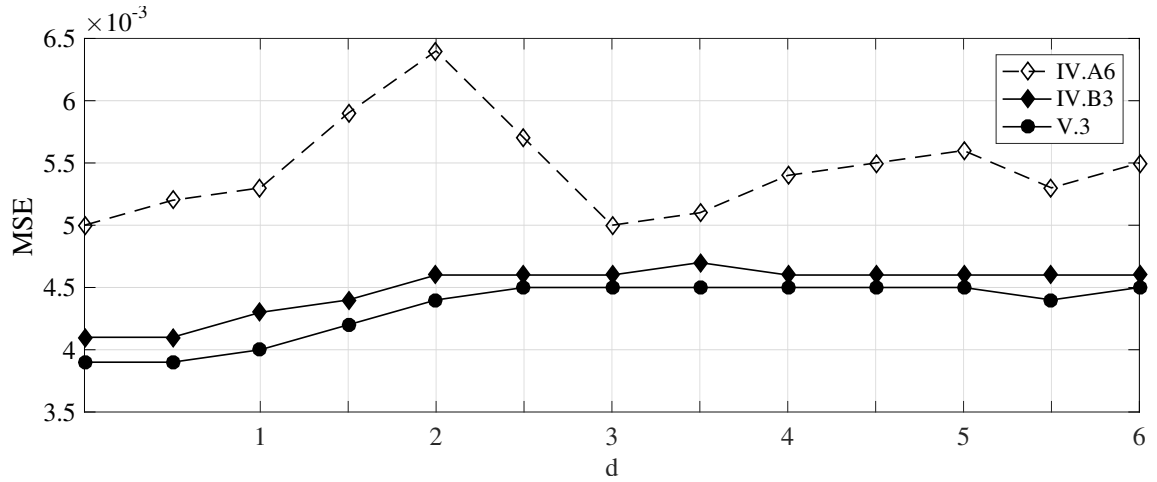


Figure 4.8. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 5 with  $K = 0.15$ .

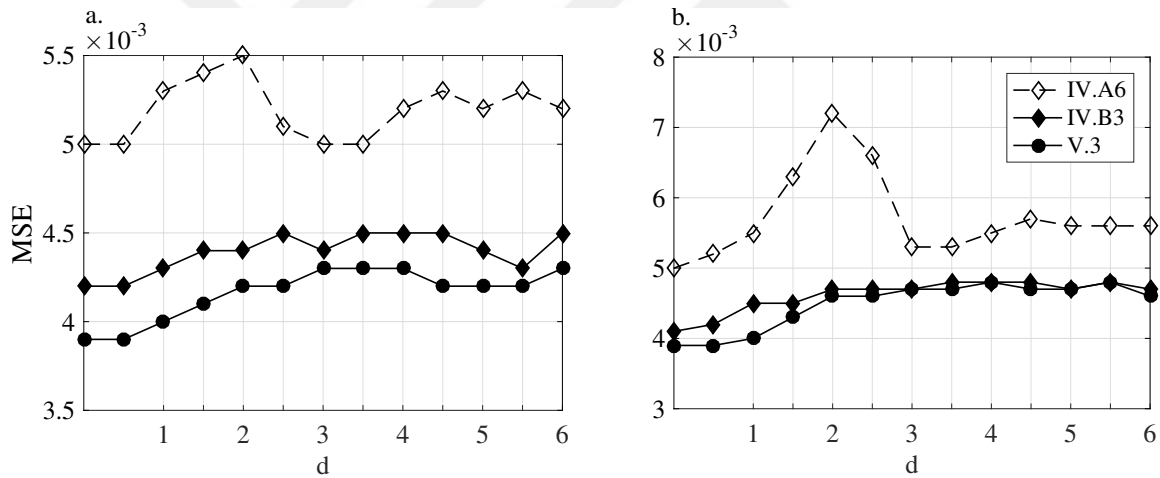


Figure 4.9. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 5 with a.  $K = 0.10$  and b.  $K = 0.20$ .

Figure 4.10 shows the efficiency of estimators when Phase I samples are subjected to superimposed contamination models (SCM 1-6). Under all SCMs, Method I estimators has the lowest efficiency with the highest  $MSE$ . Efficiencies of Method I.B and II estimators vary with respect to contaminated model. While, Method I.B estimators are more efficient when localized variance disturbance is present in SCM (Figure 1a, 1d, 1e), Method II estimators give lower  $MSE$  in the presence of individual outliers in subgroups (Figure 1b, 1c, 1f). Except in the presence of SCM 4, efficiency of Method

III-V estimators are higher than those of Method I and II estimators. Comparing estimators using individuals charts, Method V estimator yields the lowest  $MSE$  against SCM 1, 3 and 6, while Method IV.A has the highest efficiency against SCM 2 and SCM 5. It should be noted that increase in skipping level improves the robustness of the estimators, and maximum efficiency is observed at  $\alpha = 0.04$  for  $m = 50$ , and frequently shifts to  $\alpha = 0.02$  when  $m$  equals 20 (Figure B.10).

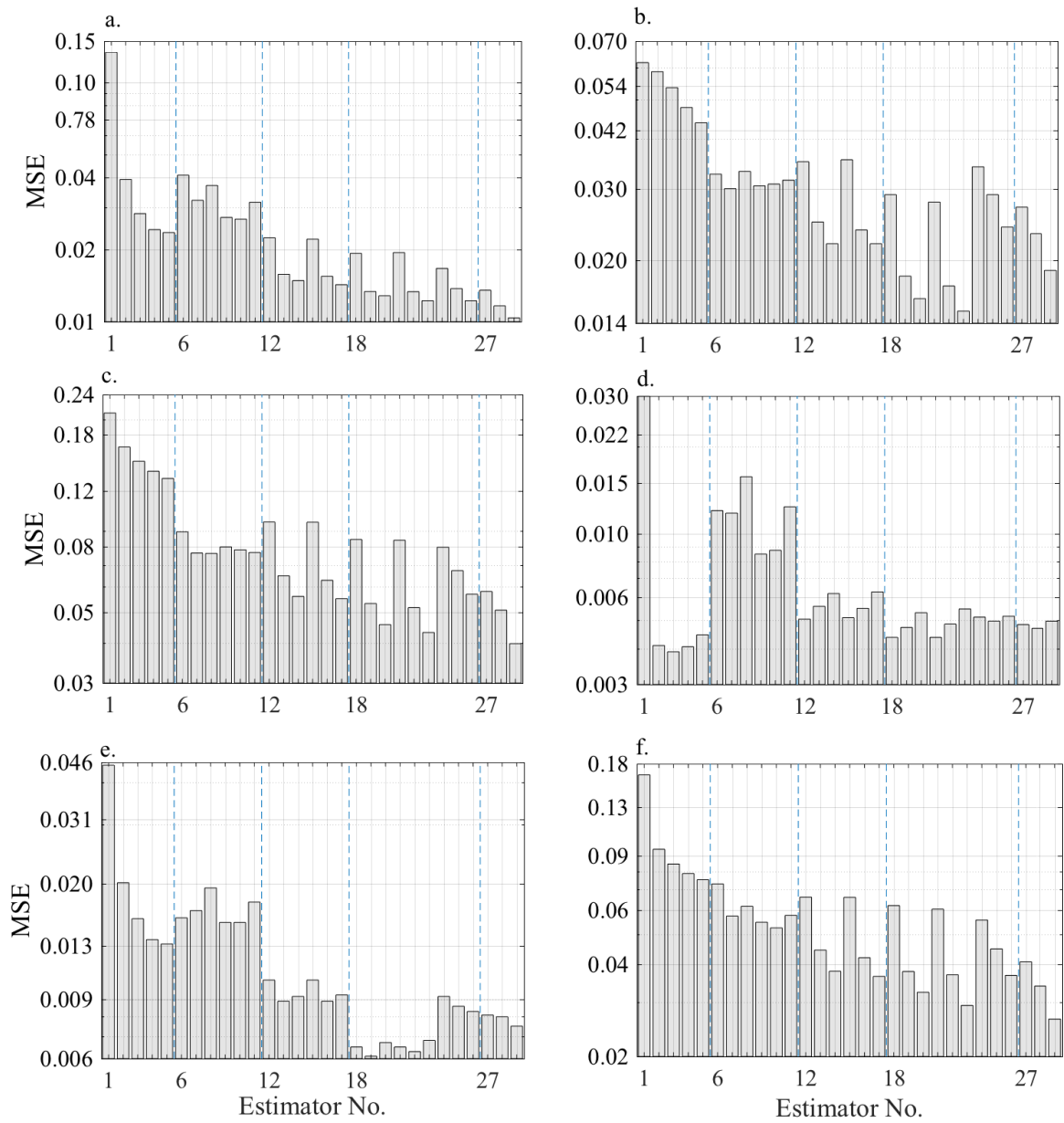


Figure 4.10. Phase I efficiency of the proposed estimators for  $m = 50$  under a. SCM 1, b. SCM 2, c. SCM 3, d. SCM 4, e. SCM 5, f. SCM 6.

## 4.2. Derivation of Phase II Control Limits

Phase II CLs for the proposed Phase I estimators are determined using Equation 2.40. Process dispersion parameter in Phase II samples is estimated by  $S_n/c_4(n)$  for all charts.  $U_n$  and  $L_n$  adjusted so as to make  $ARFS$  equal to 0.0027 with equal probability of exceeding the upper and lower control limits. 50,000 computer simulations are used to determine multipliers. Tables 4.2 and 4.3 show  $U_n$  and  $L_n$  for  $m = 50$  and  $m = 20$ , respectively.

Table 4.2. Multipliers  $U_n$  and  $L_n$  to determine Phase II control limits.

|              | $m = 50$ |       | $m = 20$ |       |
|--------------|----------|-------|----------|-------|
| Method       | $L_n$    | $U_n$ | $L_n$    | $U_n$ |
| <b>I.A</b>   | 0.172    | 2.285 | 0.171    | 2.356 |
| <b>I.B1</b>  | 0.172    | 2.289 | 0.171    | 2.362 |
| <b>I.B2</b>  | 0.172    | 2.295 | 0.171    | 2.372 |
| <b>I.B3</b>  | 0.172    | 2.300 | 0.171    | 2.398 |
| <b>I.B4</b>  | 0.172    | 2.310 | 0.170    | 2.415 |
| <b>II.A1</b> | 0.171    | 2.344 | 0.169    | 2.507 |
| <b>II.A2</b> | 0.171    | 2.384 | 0.169    | 2.585 |
| <b>II.A3</b> | 0.171    | 2.441 | 0.168    | 2.709 |
| <b>II.B1</b> | 0.172    | 2.358 | 0.169    | 2.559 |
| <b>II.B2</b> | 0.171    | 2.358 | 0.169    | 2.581 |
| <b>II.B3</b> | 0.170    | 2.407 | 0.168    | 2.703 |
| <b>III.1</b> | 0.172    | 2.302 | 0.171    | 2.398 |
| <b>III.2</b> | 0.172    | 2.302 | 0.170    | 2.467 |
| <b>III.3</b> | 0.171    | 2.342 | 0.170    | 2.550 |
| <b>III.4</b> | 0.172    | 2.296 | 0.171    | 2.396 |
| <b>III.5</b> | 0.172    | 2.325 | 0.170    | 2.472 |
| <b>III.6</b> | 0.172    | 2.345 | 0.170    | 2.546 |

Table 4.3. Multipliers  $U_n$  and  $L_n$  to determine Phase II control limits cont.

|              | $m = 50$ |       | $m = 20$ |       |
|--------------|----------|-------|----------|-------|
| Method       | $L_n$    | $U_n$ | $L_n$    | $U_n$ |
| <b>IV.A1</b> | 0.172    | 2.296 | 0.171    | 2.391 |
| <b>IV.A2</b> | 0.172    | 2.317 | 0.171    | 2.451 |
| <b>IV.A3</b> | 0.172    | 2.332 | 0.171    | 2.504 |
| <b>IV.A4</b> | 0.172    | 2.300 | 0.171    | 2.391 |
| <b>IV.A5</b> | 0.172    | 2.317 | 0.170    | 2.453 |
| <b>IV.A6</b> | 0.172    | 2.338 | 0.170    | 2.515 |
| <b>IV.B1</b> | 0.172    | 2.295 | 0.171    | 2.413 |
| <b>IV.B2</b> | 0.172    | 2.306 | 0.171    | 2.413 |
| <b>IV.B3</b> | 0.172    | 2.321 | 0.171    | 2.467 |
| <b>V.1</b>   | 0.172    | 2.298 | 0.171    | 2.383 |
| <b>V.2</b>   | 0.172    | 2.302 | 0.171    | 2.403 |
| <b>V.3</b>   | 0.172    | 2.318 | 0.171    | 2.442 |

#### 4.3. Comparison of Phase II Performances of Proposed Estimators

Phase II performances of the estimators are assessed by  $ARFS$  (marginal probability of a false signal from an in-control process),  $mARL_0$  (marginal average run length of an in-control process),  $ARS$  (average rate of signal under a disturbance in Phase II), and  $mARL$  (marginal average run length under a disturbance in Phase II). When the process is in-control, i.e  $\gamma = 1$ , it is desired to obtain  $mARL_0$  equal to or higher than  $\sim 370$ , and  $ARFS$  equal to or lower than 0.0027. Furthermore, it is desired that  $SDRL$  to be close to its intended value  $\sim 370$  for an in-control process. In the out-of-control situation, i.e.  $\gamma = 1.5$ , the change in the process variation is desired to be detected promptly, so  $mARL$  should be as low as possible with high  $ARS$ . In addition to marginal performance metrics,  $cARL$  is computed for 0.10, 0.50 and 0.90 quantiles of the  $\hat{\sigma}^I$  distributions, denoted with  $cARL_{.1}$ ,  $cARL_{.5}$  and  $cARL_{.9}$ , respectively, to examine the best, moderate and worst performance scenarios. Visual representation

of the performance measures for  $K = 0.15$  and  $m = 50$  are given in Figure 4.11-4.18, in which bar plots indicate the  $mARL_0/mARL$  values, and lines denote  $ARFS/ARS$  values. Phase II performances of S-Charts using all estimation methods are shown for  $m = 50$  in Appendix C (Table C.1-C.44), for  $m = 20$  in Appendix D (Table D.1-D.44)

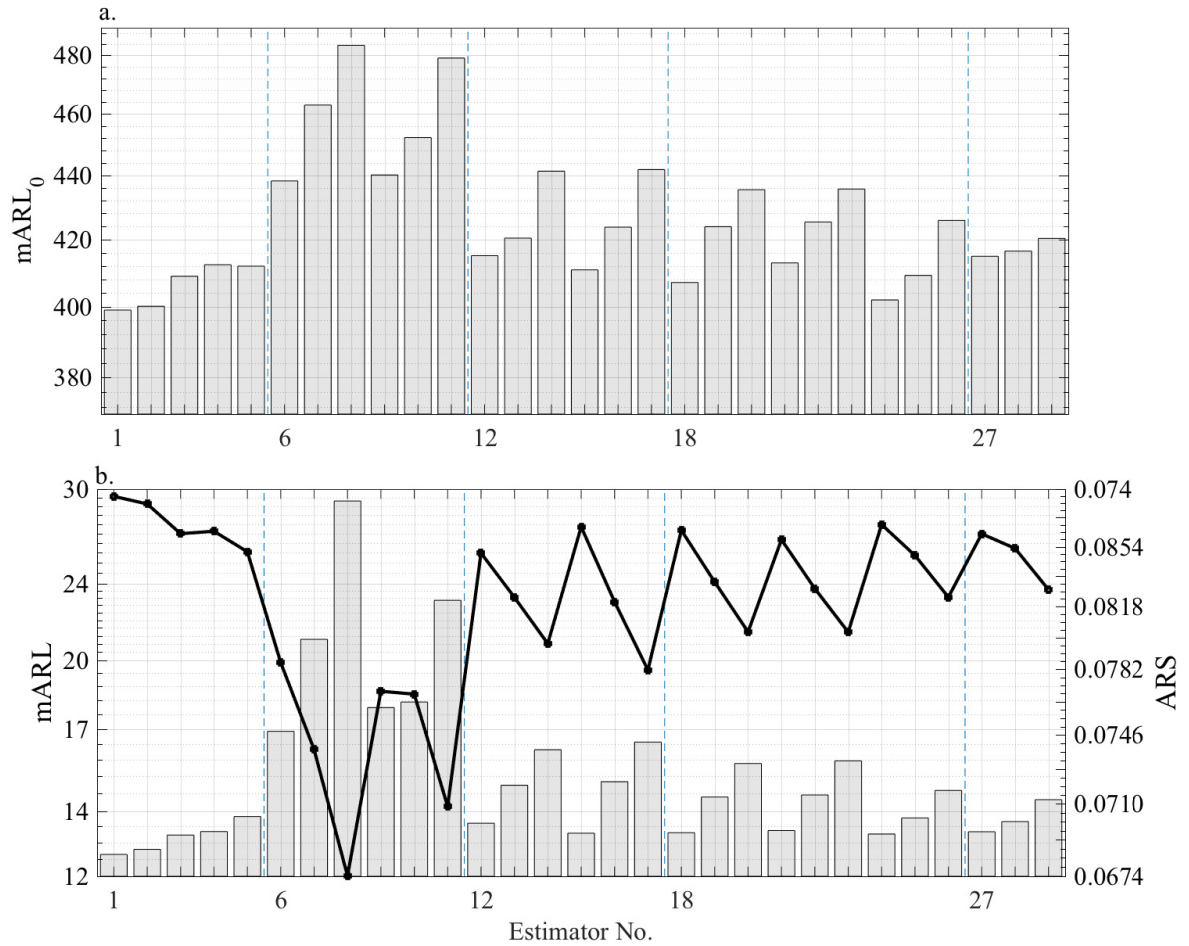


Figure 4.11. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using normally distributed Phase I data.

First, the case, in which the process follows a normal distribution and the Phase I samples are not contaminated, is considered. Regardless of the sample size,  $mARL_0$  values of all estimators are higher than 370 (Figure 4.11.a). When the process is out-of-control in Phase II, power of  $S_n$  estimator is slightly decreased via reweighting. It is seen that reweighting procedure improves the power of S-Charts using Method II estimators with  $HD_{.5}$  estimator, but not with  $ML$  estimator. S-Charts using  $MAD_n$

yield low power and high  $SDRL$ , hence  $MAD_n$  estimator is not found to be convenient, particularly for low subgroup size (Table D.2). Utilizing individuals charts in Method III-V estimators gives sensitivity (Figure 4.11a, Table C.1) and specificity (Figure 4.11b, Table C.2) values comparable to those by Method I.A, while Method II estimators are remarkably less powerful. When  $m$  is equal to 20, higher  $mARL_0$  and  $SDRL$  values are obtained, since variance of the average run lengths inflates due to low sample size (Table D.1, D.2).

In the presence of outliers in Phase I samples, S-Charts constructed using standard deviation give  $mARL_0$  higher than 370 only when contamination density and magnitude are low, i.e.  $K = 0.10$  and  $d = 2$  for CM 1, 3-5 and  $d = 0.75$  for CM 2 (Table C.3, C.9, C.15, C.27), but  $mARL_0$  decreases to values lower than 370, when contamination is increased (except under CM 2). Reweighting traditional estimators, on the other hand, has a significantly positive effect on the sensitivity of S-Charts, when outliers come from an asymmetric long-tailed distribution (CM 2), or specific subgroups are affected by outliers (CM 3), e.g.  $mARL_0$  values are equal to 225.2, 418.4, 259.4 and 187.3 when Method I.A estimator is employed, and equal to 368.5, 452.4, 452.4 and 195.8, when Method I.B4 estimator is used in S-Charts using Phase I data under CM 1-4, respectively, at moderate contamination density (Table 4.4). Increasing skipping level in Method I.B generally yields higher  $mARL_0$  and lower  $ARFS$  (except under CM 4), and the best performance is attained at  $\alpha = 0.025 - 0.050$  for both  $m = 50$  and  $m = 20$ . Reweighting has a substantial effect also on the power of S-Charts constructed by  $S_n$  in detecting shifts in the process variation (except under CM 4), particularly against CM 1 and CM 3, e.g.  $mARL$  of S-Charts using Phase I data under CM1 and CM 3 with moderate contamination is equal to 246.5 and 159.1 when Method I.A estimator is used, but decreases to 136.0 and 20.0 even with the low skipping amount ( $\alpha = 0.0027$ , Table 4.4). It should be noted that S-Charts constructed using Method I estimators may become ARL-biased when Phase data is under CM 4 for the given contamination level, i.e.  $mARL_0$  may be lower than  $mARL$  [24]. Reweighting  $S_n$  estimator has no curative effect on the status of ARL-biased S-Charts, and increase in power is negligibly small in the presence of CM 4 in Phase I sampling (Table 4.4).

Table 4.4. Phase II performance of S-Charts constructed from Method I estimators for  $m = 50$  under CM 1- CM 4 ( $d=3$ ,  $K=0.15$  for CM 1,CM 3-CM 4 and  $d=1$ ,  $K = 0.15$  for CM 2).

| CM   | Method | $ARFS$ | $mARL_0$ | $SDRL$ | $ARS$  | $mARL$ | $SDRL$ |
|------|--------|--------|----------|--------|--------|--------|--------|
| CM 1 | I.A    | 0.0048 | 225.2    | 244.2  | 0.0063 | 246.5  | 312.7  |
|      | I.B1   | 0.0037 | 301.8    | 328.6  | 0.0150 | 136.0  | 211.3  |
|      | I.B2   | 0.0033 | 331.8    | 357.6  | 0.0193 | 109.1  | 180.5  |
|      | I.B3   | 0.0031 | 354.0    | 378.3  | 0.0226 | 93.1   | 160.8  |
|      | I.B4   | 0.0030 | 368.5    | 393.6  | 0.0232 | 90.4   | 157.8  |
| CM 2 | I.A    | 0.0025 | 418.4    | 428.4  | 0.0357 | 40.4   | 59.9   |
|      | I.B1   | 0.0023 | 451.0    | 458.0  | 0.0508 | 25.5   | 33.5   |
|      | I.B2   | 0.0023 | 456.7    | 464.4  | 0.0533 | 24.3   | 31.6   |
|      | I.B3   | 0.0023 | 451.2    | 460.1  | 0.0551 | 23.7   | 31.0   |
|      | I.B4   | 0.0023 | 452.4    | 461.9  | 0.0549 | 24.3   | 32.3   |
| CM 3 | I.A    | 0.0041 | 259.4    | 271.8  | 0.0089 | 159.1  | 204.9  |
|      | I.B1   | 0.0025 | 430.5    | 443.7  | 0.0660 | 20.0   | 27.1   |
|      | I.B2   | 0.0024 | 440.2    | 455.7  | 0.0697 | 18.2   | 23.2   |
|      | I.B3   | 0.0024 | 444.8    | 458.9  | 0.0679 | 18.5   | 23.2   |
|      | I.B4   | 0.0023 | 452.4    | 463.6  | 0.0624 | 20.4   | 25.8   |
| CM 4 | I.A    | 0.0056 | 187.3    | 196.4  | 0.0038 | 323.7  | 364.8  |
|      | I.B1   | 0.0056 | 188.8    | 199.3  | 0.0038 | 329.7  | 372.7  |
|      | I.B2   | 0.0056 | 189.4    | 202.1  | 0.0040 | 328.7  | 374.4  |
|      | I.B3   | 0.0056 | 193.9    | 210.0  | 0.0041 | 337.9  | 387.8  |
|      | I.B4   | 0.0056 | 195.8    | 216.1  | 0.0044 | 338.1  | 394.2  |

When S-Charts are based on Method II estimators,  $ARFS$  values do not exceed 0.0027 at low contamination level (Table C.3, C.9, C.15, C.21), while they may be higher or lower than 0.0027, depending on the underlying distribution of contamination in Phase I samples, at higher outlier densities. When outliers come from an asymmetric long-tailed distribution (CM 2) as seen in Table C.11 and C.13, or specific subgroups are affected by outliers (CM 3) as seen in Table C.17 and C.19, at relatively higher contamination densities,  $ARFS$  is generally lower than 0.0027. While S-Charts with higher  $mARL_0$  values than the nominal value do not present a problem in application,

power of these charts are undesirably low compared to those constructed by Method III-V estimators. S-Charts based on Method II estimators yield higher power than those based on Method I.B estimators in presence of CM 1 and CM 4 in Phase I samples (Table 4.5).

Table 4.5. Phase II performance of S-Charts constructed from Method II estimators for  $m = 50$  under CM 1- CM 4 ( $d=3$ ,  $K=0.15$  for CM 1,CM 3-CM 4 and  $d=1$ ,  $K = 0.15$  for CM 2).

| CM   | Method | $ARFS$ | $mARL_0$ | $SDRL$ | $ARS$  | $mARL$ | $SDRL$ |
|------|--------|--------|----------|--------|--------|--------|--------|
| CM 1 | II.A1  | 0.0028 | 382.0    | 405.4  | 0.0209 | 90.1   | 146.0  |
|      | II.A2  | 0.0026 | 412.8    | 441.2  | 0.0228 | 97.6   | 170.2  |
|      | II.A3  | 0.0027 | 421.3    | 460.0  | 0.0213 | 131.4  | 238.6  |
|      | II.B1  | 0.0028 | 389.6    | 414.5  | 0.0238 | 87.2   | 147.8  |
|      | II.B2  | 0.0027 | 398.8    | 424.0  | 0.0245 | 87.2   | 150.2  |
|      | II.B3  | 0.0026 | 421.5    | 454.4  | 0.0231 | 111.7  | 201.2  |
| CM 2 | II.A1  | 0.0022 | 475.1    | 488.5  | 0.0490 | 30.7   | 46.4   |
|      | II.A2  | 0.0023 | 470.9    | 490.6  | 0.0480 | 37.3   | 65.1   |
|      | II.A3  | 0.0023 | 486.2    | 516.1  | 0.0440 | 52.8   | 107.4  |
|      | II.B1  | 0.0023 | 461.3    | 477.8  | 0.0502 | 31.6   | 49.3   |
|      | II.B2  | 0.0023 | 467.0    | 483.9  | 0.0504 | 32.2   | 51.0   |
|      | II.B3  | 0.0023 | 489.9    | 515.4  | 0.0464 | 42.3   | 76.7   |
| CM 3 | II.A1  | 0.0025 | 422.1    | 440.8  | 0.0262 | 64.9   | 101.5  |
|      | II.A2  | 0.0023 | 461.8    | 482.4  | 0.0341 | 56.6   | 97.8   |
|      | II.A3  | 0.0024 | 456.4    | 488.4  | 0.0283 | 90.6   | 172.0  |
|      | II.B1  | 0.0024 | 456.6    | 475.3  | 0.0477 | 36.9   | 63.2   |
|      | II.B2  | 0.0024 | 462.8    | 484.4  | 0.0510 | 34.9   | 60.8   |
|      | II.B3  | 0.0024 | 468.9    | 498.7  | 0.0411 | 59.3   | 119.9  |
| CM 4 | II.A1  | 0.0042 | 263.7    | 291.6  | 0.0073 | 246.2  | 329.3  |
|      | II.A2  | 0.0039 | 293.6    | 331.1  | 0.0088 | 248.0  | 350.2  |
|      | II.A3  | 0.0039 | 303.9    | 349.9  | 0.0089 | 285.4  | 403.4  |
|      | II.B1  | 0.0042 | 267.7    | 299.7  | 0.0083 | 245.8  | 336.7  |
|      | II.B2  | 0.0041 | 278.1    | 313.0  | 0.0091 | 236.4  | 330.1  |
|      | II.B3  | 0.0039 | 301.6    | 346.8  | 0.0091 | 273.2  | 385.9  |

Reweighting Method II.A estimators at  $\alpha = 0.025$  yields relatively higher  $ARS$  values, but improvement is not remarkable (maximum relative percentage improvement  $\approx 8.3\%$  except against CM 3). Performances of robust estimators can be ordered as  $MS_n + HD_{.5}$ ,  $MS_n + ML$ ,  $MAD_n + HD_{.5}$ , from best to the worse.  $MAD_n$  is neglected for further study due to its low power and high  $SDRL$ . It should be noted that, even though S-Charts are very efficient under CM 4 at low level of contamination, their efficiency are lost when contamination magnitude and density are increased. S-Charts, which are constructed using Method II estimators, are not ARL-biased at moderate contamination density, but not satisfactory, because the power of S-Charts and efficiency of estimators against CM 4 are very low (Table 4.5).

Utilizing individuals chart to screen individual outliers in subgroups generally yields higher  $mARL_0$  compared to Method I.B estimators, and slightly lower compared to Method II estimators, but significantly improves the power of S-Charts compared to those constructed using Method I and II estimators (Table 4.6). There is no remarkable difference between the methods which use  $HD_{.5}$  and  $ML$  estimators, but  $HD_{.5}$  and  $MS_n$  location and dispersion estimators are used in S-Charts based on Method IV.B and V estimators due to having relatively higher Phase I efficiency (Table 4.6, 4.7). Using single location estimate in the construction of individuals chart in Method IV-V, rather than using subgroup location estimates in Method III, yields higher  $mARL_0$  and lower  $mARL$  against all CMs. In all CMs, the highest power is attained by Method V estimator, at generally high skipping level, except against CM 3. Method III estimator, on the other hand, always falls behind the Method IV and V estimators. Superiority of Method IV.B over IV.A, on the other hand, varies with respect to contamination model, i.e. Method IV.A estimator attains higher power when Phase I sampling is subjected to CM 1 and CM 4; however, Method IV.B estimator is more powerful against CM 2 and CM 3. Phase II performances of Method III-V estimators are comparable to the results for  $m = 20$ . Although the highest power is again ranked from best the worst as Method V, IV and III,  $\alpha = 0.020$  is more favorable for  $m = 20$ . Consequently, higher skipping level is suggested when subgroup size is high, i.e.  $m = 50$ , however skipping level should be decreased for datasets having smaller subgroup size, i.e  $m = 20$ , yet still an  $\alpha$  value much higher than .0027 is recommended (Table 4.8).

Table 4.6. Phase II performance of S-Charts constructed from Method III estimators for  $m = 50$  under CM 1- CM 4 ( $d=3$ ,  $K=0.15$  for CM 1,CM 3-CM 4 and  $d=1$ ,  $K = 0.15$  for CM 2).

| CM   | Method | <i>ARFS</i> | <i>mARL</i> <sub>0</sub> | <i>SDRL</i> | <i>ARS</i> | <i>mARL</i> | <i>SDRL</i> |
|------|--------|-------------|--------------------------|-------------|------------|-------------|-------------|
| CM 1 | I.B4   | 0.0030      | 368.5                    | 393.6       | 0.0232     | 90.4        | 157.8       |
|      | II.B2  | 0.0027      | 398.8                    | 424.0       | 0.0245     | 87.2        | 150.2       |
|      | III.1  | 0.0027      | 392.0                    | 408.4       | 0.0288     | 59.4        | 96.8        |
|      | III.2  | 0.0025      | 418.4                    | 432.9       | 0.0360     | 47.2        | 77.1        |
|      | III.3  | 0.0024      | 442.6                    | 459.9       | 0.0368     | 47.8        | 78.7        |
|      | III.4  | 0.0027      | 387.9                    | 404.1       | 0.0288     | 59.0        | 95.2        |
|      | III.5  | 0.0025      | 423.7                    | 438.2       | 0.0365     | 46.6        | 75.6        |
|      | III.6  | 0.0024      | 434.7                    | 451.1       | 0.0372     | 47.8        | 79.1        |
| CM 2 | I.B4   | 0.0023      | 452.4                    | 461.9       | 0.0549     | 24.3        | 32.3        |
|      | II.B2  | 0.0023      | 467.0                    | 483.9       | 0.0504     | 32.2        | 51.0        |
|      | III.1  | 0.0023      | 457.4                    | 465.6       | 0.0575     | 22.1        | 27.9        |
|      | III.2  | 0.0023      | 458.8                    | 472.2       | 0.0598     | 22.4        | 29.3        |
|      | III.3  | 0.0023      | 469.5                    | 486.5       | 0.0588     | 24.0        | 33.1        |
|      | III.4  | 0.0023      | 449.9                    | 458.5       | 0.0587     | 21.5        | 27.0        |
|      | III.5  | 0.0023      | 454.4                    | 467.6       | 0.0592     | 22.6        | 29.5        |
|      | III.6  | 0.0023      | 465.9                    | 482.3       | 0.0576     | 24.6        | 34.0        |
| CM 3 | I.B4   | 0.0023      | 452.4                    | 463.6       | 0.0624     | 20.4        | 25.8        |
|      | II.B2  | 0.0024      | 462.8                    | 484.4       | 0.0510     | 34.9        | 60.8        |
|      | III.1  | 0.0023      | 450.7                    | 462.5       | 0.0552     | 25.3        | 35.2        |
|      | III.2  | 0.0024      | 450.9                    | 467.7       | 0.0606     | 23.3        | 32.7        |
|      | III.3  | 0.0023      | 469.5                    | 486.5       | 0.0588     | 24.0        | 33.1        |
|      | III.4  | 0.0024      | 444.0                    | 456.5       | 0.0578     | 24.0        | 33.5        |
|      | III.5  | 0.0024      | 452.0                    | 469.2       | 0.0619     | 23.0        | 32.5        |
|      | III.6  | 0.0024      | 457.0                    | 476.5       | 0.0614     | 23.8        | 34.7        |
| CM 4 | I.B4   | 0.0056      | 195.8                    | 216.1       | 0.0044     | 338.1       | 394.2       |
|      | II.B2  | 0.0041      | 278.1                    | 313.0       | 0.0091     | 236.4       | 330.1       |
|      | III.1  | 0.0049      | 221.8                    | 242.9       | 0.0060     | 272.1       | 340.0       |
|      | III.2  | 0.0043      | 264.8                    | 295.8       | 0.0094     | 220.6       | 307.0       |
|      | III.3  | 0.0040      | 289.4                    | 325.8       | 0.0111     | 209.1       | 305.9       |
|      | III.4  | 0.0050      | 220.8                    | 242.0       | 0.0062     | 265.0       | 331.3       |
|      | III.5  | 0.0043      | 266.1                    | 298.3       | 0.0098     | 218.7       | 307.6       |
|      | III.6  | 0.0040      | 287.0                    | 323.3       | 0.0116     | 210.0       | 310.1       |

Table 4.7. Phase II performance of S-Charts constructed from Method IV.A estimators for  $m = 50$  under CM 1- CM 4 ( $d=3$ ,  $K=0.15$  for CM 1,CM 3-CM 4 and  $d=1$ ,  $K = 0.15$  for CM 2).

| CM   | Method | $ARFS$ | $mARL_0$ | $SDRL$ | $ARS$  | $mARL$ | $SDRL$ |
|------|--------|--------|----------|--------|--------|--------|--------|
| CM 1 | III.6  | 0.0024 | 434.7    | 451.1  | 0.0372 | 47.8   | 79.1   |
|      | IV.A1  | 0.0025 | 414.8    | 428.7  | 0.0346 | 46.5   | 73.7   |
|      | IV.A2  | 0.0024 | 432.4    | 444.3  | 0.0413 | 37.8   | 58.8   |
|      | IV.A3  | 0.0024 | 441.7    | 454.8  | 0.0421 | 37.7   | 58.9   |
|      | IV.A4  | 0.0024 | 441.7    | 454.8  | 0.0421 | 37.7   | 58.9   |
|      | IV.A5  | 0.0024 | 436.1    | 448.3  | 0.0413 | 37.9   | 58.6   |
|      | IV.A6  | 0.0023 | 453.2    | 468.2  | 0.0431 | 37.6   | 59.3   |
| CM 2 | III.6  | 0.0023 | 465.9    | 482.3  | 0.0576 | 24.6   | 34.0   |
|      | IV.A1  | 0.0023 | 453.3    | 462.4  | 0.0613 | 20.3   | 25.1   |
|      | IV.A2  | 0.0023 | 458.2    | 472.0  | 0.0621 | 20.9   | 26.6   |
|      | IV.A3  | 0.0024 | 458.1    | 473.3  | 0.0616 | 22.1   | 29.8   |
|      | IV.A4  | 0.0023 | 451.9    | 460.8  | 0.0608 | 20.4   | 25.3   |
|      | IV.A5  | 0.0023 | 463.5    | 477.2  | 0.0617 | 21.2   | 27.2   |
|      | IV.A6  | 0.0024 | 462.3    | 478.6  | 0.0610 | 22.4   | 29.7   |
| CM 3 | III.6  | 0.0024 | 457.0    | 476.5  | 0.0614 | 23.8   | 34.7   |
|      | IV.A1  | 0.0023 | 446.0    | 457.6  | 0.0560 | 24.9   | 35.0   |
|      | IV.A2  | 0.0024 | 442.3    | 456.8  | 0.0610 | 22.9   | 31.7   |
|      | IV.A3  | 0.0024 | 453.2    | 470.3  | 0.0612 | 23.1   | 32.5   |
|      | IV.A4  | 0.0024 | 439.4    | 451.1  | 0.0564 | 24.9   | 35.2   |
|      | IV.A5  | 0.0025 | 441.8    | 458.1  | 0.0628 | 22.4   | 31.4   |
|      | IV.A6  | 0.0024 | 452.3    | 471.1  | 0.0627 | 23.0   | 33.1   |
| CM 4 | III.6  | 0.0040 | 287.0    | 323.3  | 0.0116 | 210.0  | 310.1  |
|      | IV.A1  | 0.0049 | 223.7    | 246.4  | 0.0064 | 262.7  | 329.9  |
|      | IV.A2  | 0.0042 | 268.4    | 300.5  | 0.0106 | 212.3  | 303.4  |
|      | IV.A3  | 0.0039 | 292.8    | 328.3  | 0.0130 | 192.0  | 290.0  |
|      | IV.A4  | 0.0048 | 227.1    | 249.4  | 0.0066 | 260.8  | 330.6  |
|      | IV.A5  | 0.0041 | 276.2    | 309.9  | 0.0115 | 200.8  | 293.0  |
|      | IV.A6  | 0.0037 | 308.3    | 346.7  | 0.0146 | 179.1  | 278.3  |

Table 4.8. Phase II performance of S-Charts constructed from Method IV.B and V estimators for  $m = 50$  under CM 1- CM 4 ( $d=3$ ,  $K=0.15$  for CM 1,CM 3-CM 4 and  $d=1$ ,  $K = 0.15$  for CM 2).

| CM   | Method | $ARFS$ | $mARL_0$ | $SDRL$ | $ARS$  | $mARL$ | $SDRL$ |
|------|--------|--------|----------|--------|--------|--------|--------|
| CM 1 | III.6  | 0.0024 | 434.7    | 451.1  | 0.0372 | 47.8   | 79.1   |
|      | IV.A6  | 0.0023 | 453.2    | 468.2  | 0.0431 | 37.6   | 59.3   |
|      | IV.B3  | 0.0024 | 435.0    | 448.4  | 0.0410 | 39.7   | 63.8   |
|      | V.3    | 0.0023 | 445.2    | 456.8  | 0.0450 | 33.5   | 50.3   |
| CM 2 | III.6  | 0.0023 | 465.9    | 482.3  | 0.0576 | 24.6   | 34.0   |
|      | IV.A6  | 0.0024 | 462.3    | 478.6  | 0.0610 | 22.4   | 29.7   |
|      | IV.B3  | 0.0023 | 460.5    | 474.2  | 0.0613 | 21.3   | 27.3   |
|      | V.3    | 0.0023 | 453.9    | 467.1  | 0.0637 | 20.1   | 25.4   |
| CM 3 | III.6  | 0.0024 | 457.0    | 476.5  | 0.0614 | 23.8   | 34.7   |
|      | IV.A6  | 0.0024 | 452.3    | 471.1  | 0.0627 | 23.0   | 33.1   |
|      | IV.B3  | 0.0025 | 443.5    | 461.5  | 0.0666 | 20.1   | 26.9   |
|      | V.3    | 0.0025 | 440.6    | 459.0  | 0.0695 | 18.9   | 24.9   |
| CM 4 | III.6  | 0.0040 | 287.0    | 323.3  | 0.0116 | 210.0  | 310.1  |
|      | IV.A6  | 0.0037 | 308.3    | 346.7  | 0.0146 | 179.1  | 278.3  |
|      | IV.B3  | 0.0042 | 271.0    | 306.5  | 0.0113 | 210.9  | 303.5  |
|      | V.3    | 0.0036 | 313.3    | 346.6  | 0.0157 | 153.5  | 242.8  |

Visual representations of the Phase II performances of the proposed estimators are given in Figure 4.12-4.15 for Phase I samples under main CMs, and in Figure 4.16 and 4.17 for Phase I samples under SCMs. While S-Charts based on Method I.A estimator yield  $mARL_0$  values lower than the nominal value, reweighting used in Method I.B increases its specificity. Power of S-Charts constructed using Method I.B estimators is considerably low when Phase I samples are subjected to CM 4 (Figure 4.15b, 4.16d, 4.16f, 4.17f), and expectedly high in the presence of CM 3 (Figure 4.14b, 4.16b, 4.17b).

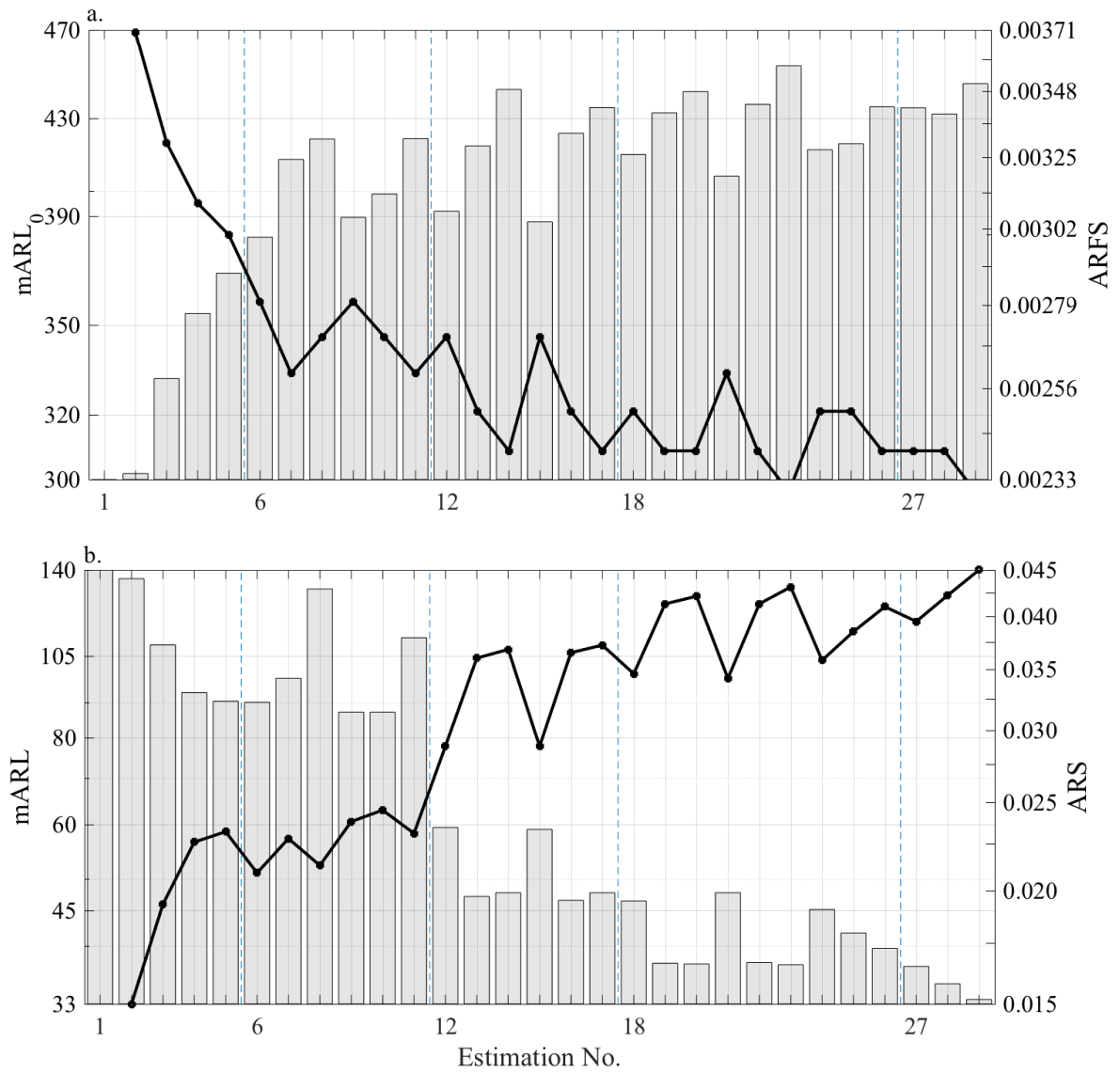


Figure 4.12. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.15$ ).

It should be noted that, S-Charts constructed using Method I.B estimators are not efficient in detecting perturbations in variance using Phase I data under diffuse mean contamination at even very low contamination density; for 92% of observations in the subgroups equally likely (at a probability of 0.08) to come from a distribution, which have higher mean than zero in SCM 6, power of Method I.B estimators is unacceptably low, rendering application of reweighted traditional estimators in constructing S-Charts useless.

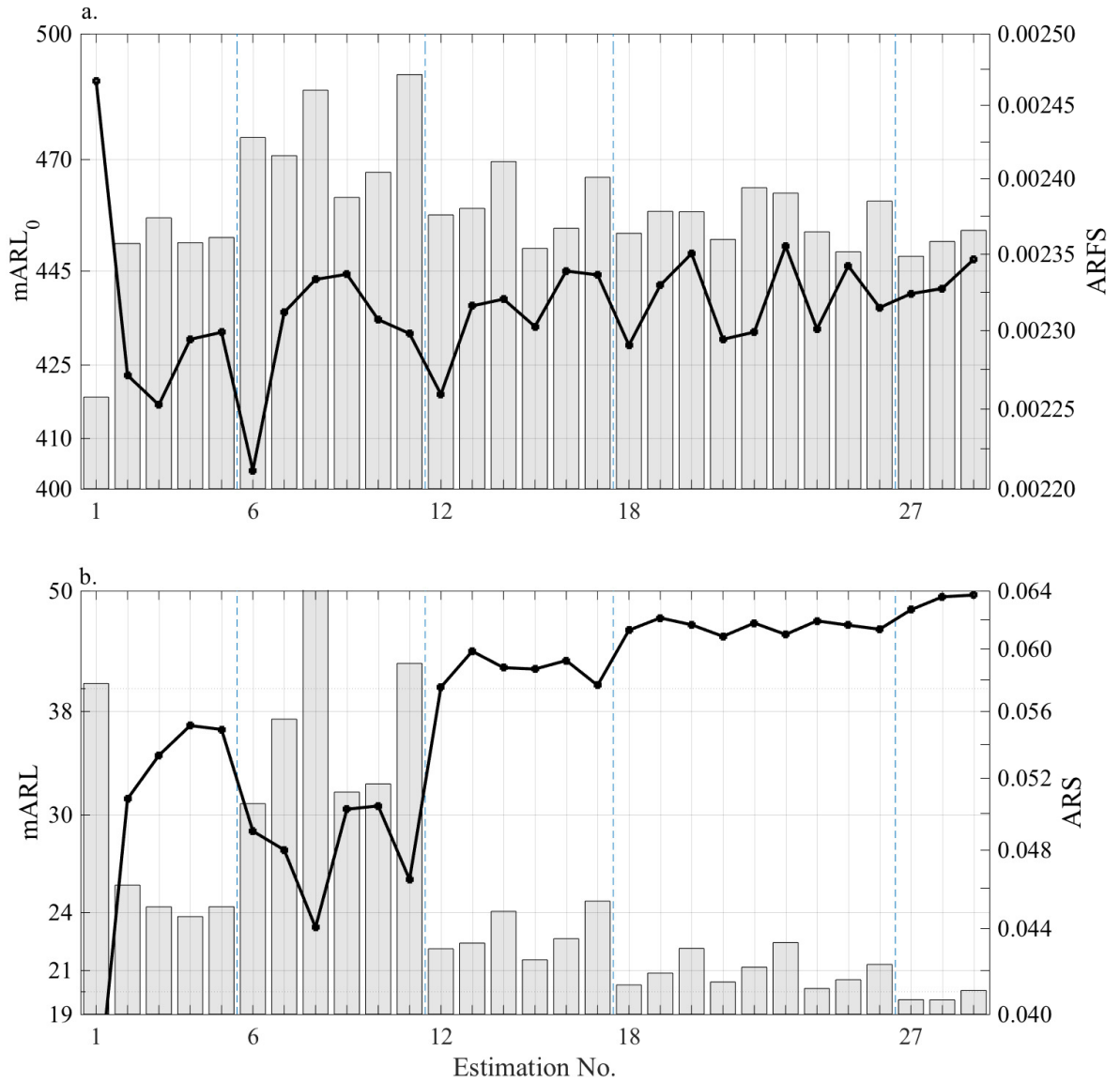


Figure 4.13. Phase II Performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=1, K=0.15$ )

S-Charts based on Method II estimators yield lower  $mARL$  values than those based on Method I.B estimators, when Phase I samples are under only diffuse symmetric (Figure 4.12), diffuse mean contaminations (Figure 4.15), their combinations (Figure 4.16f), but higher  $mARL$  values higher than those based on Method I.B estimators in the presence of CM 2 (Figure 4.13). Variation in the power of S-Charts based on Method II and Method I.B estimators with respect to different contaminations is possibly due to difference in tail length of the asymmetric distributions; distribution of

Phase I samples under CM 2 and CM 4 are long- and short-tailed, respectively, showing that reweighted traditional estimators may be more effective compared to robust estimators in long tailed asymmetric error distributions.

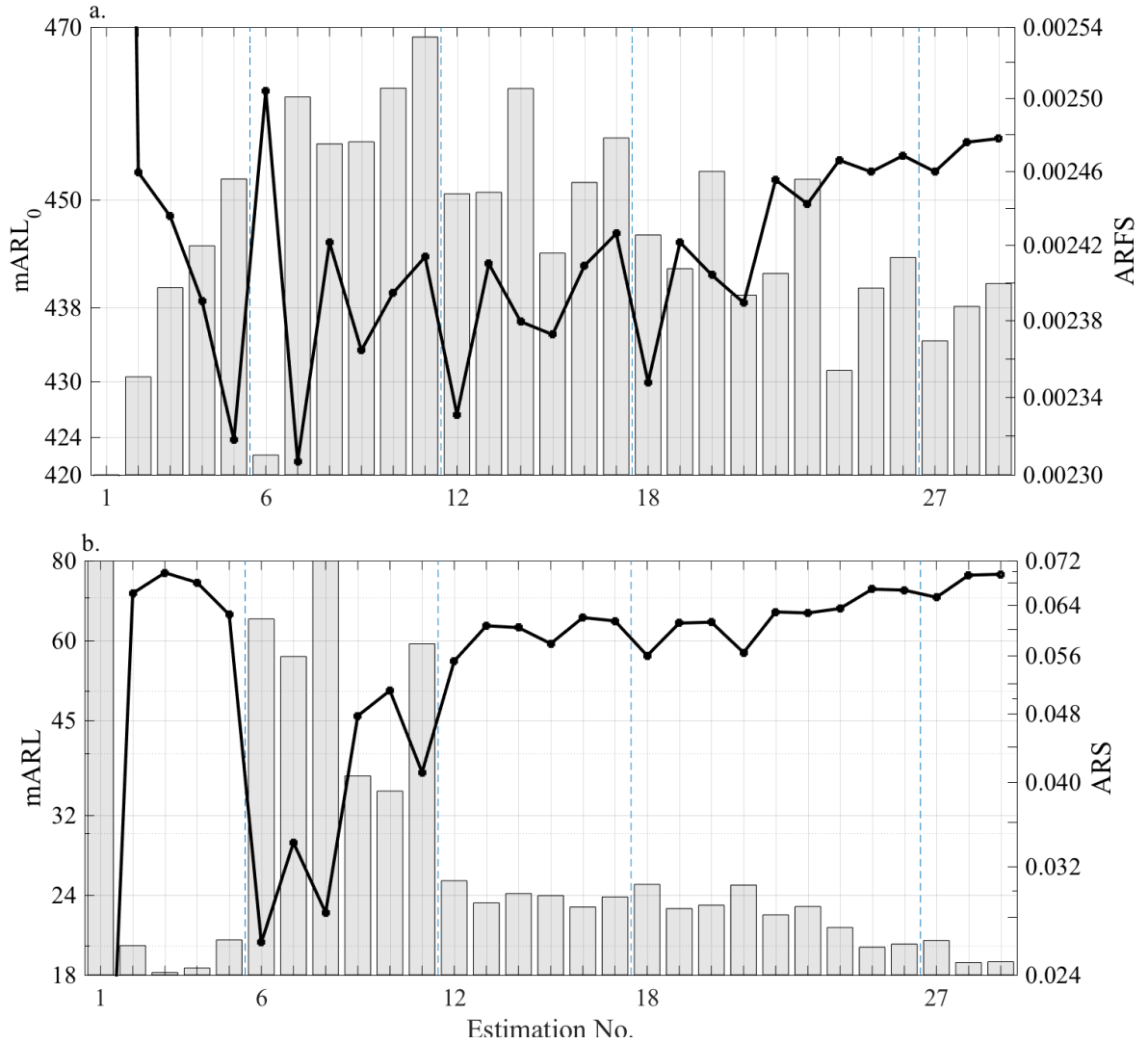


Figure 4.14. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.15$ ).

S-Charts based on individuals charts yield  $mARL_0$  values, which are always higher than the nominal value, at high skipping level in the presence of SCM 1-6 in Phase I samples. It is seen that use of single process location estimate instead of subgroup location estimates in individuals charts increases the power of S-Charts

against all SCMs. On the other hand, superiority of Method IV.B over IV.A varies with respect to contamination model: Method IV.A yields lower  $mARL$  in the existence of light localized mean contamination. (Figure 4.16d, 4.17b, 4.17d).

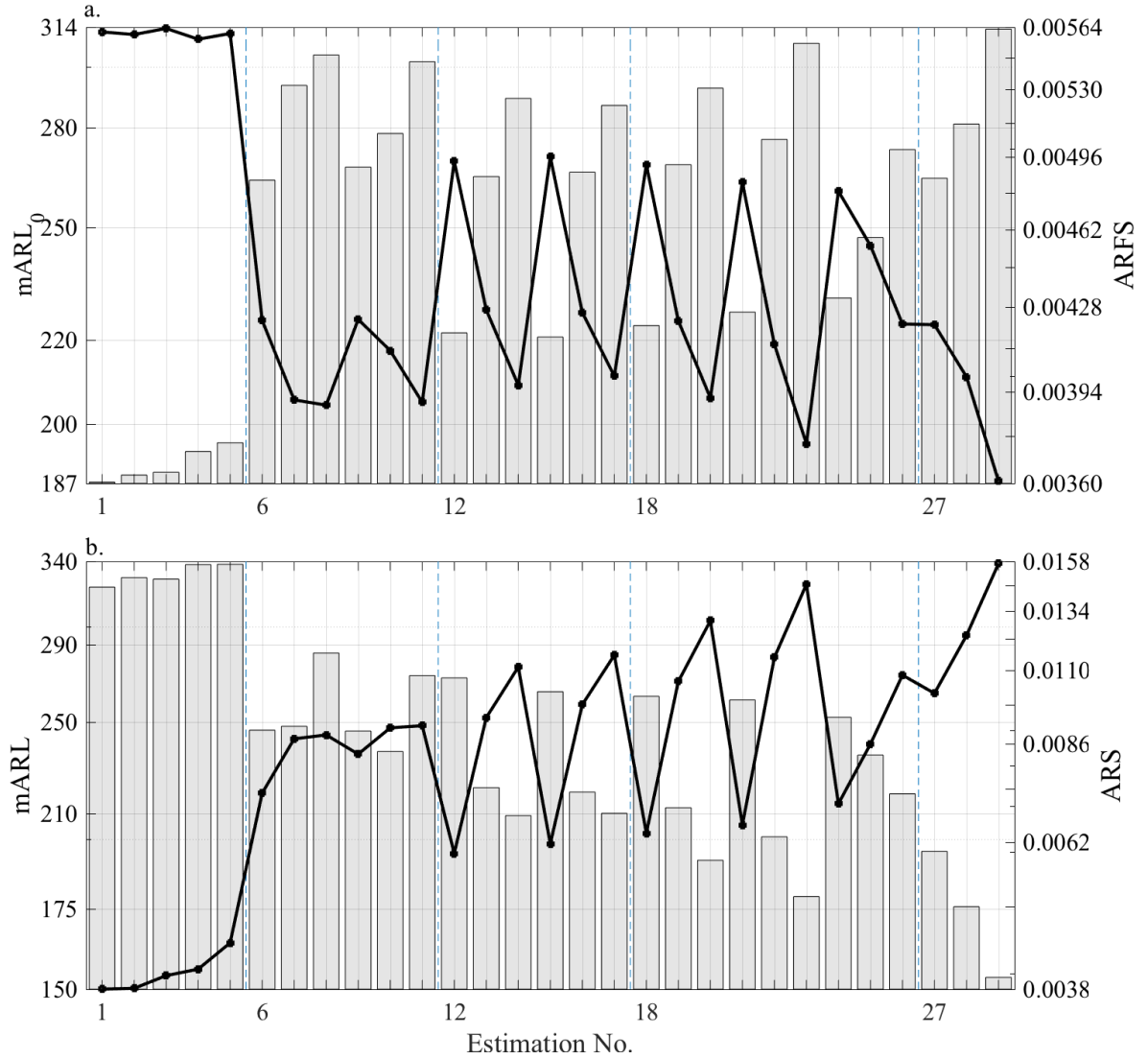


Figure 4.15. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.15$ ).

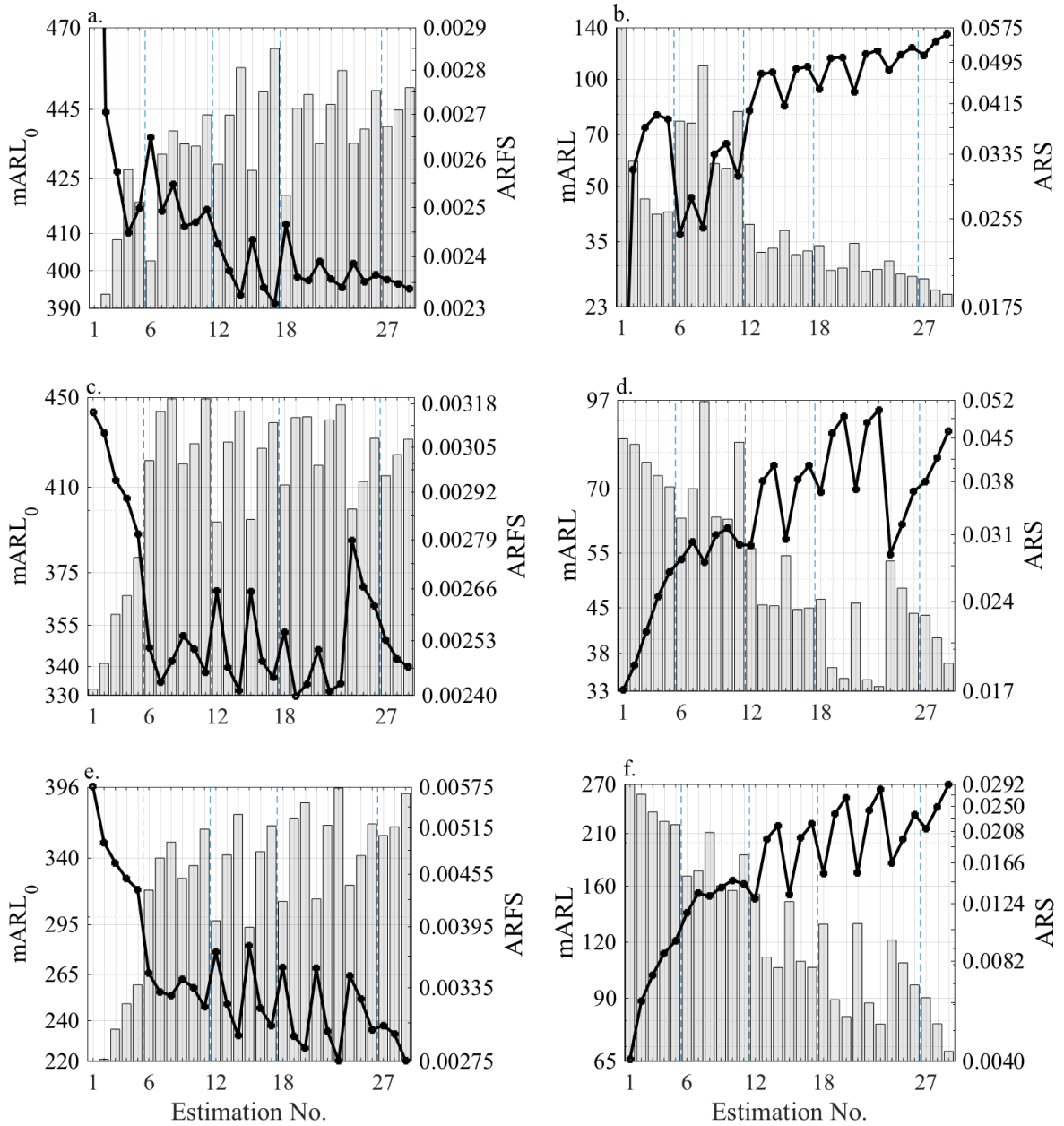


Figure 4.16. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under a. SCM 1 ( $\gamma = 1$ ) b. SCM 1 ( $\gamma = 1.5$ ) c. SCM 2 ( $\gamma = 1$ ) d. SCM 2 ( $\gamma = 1.5$ ) e. SCM 3 ( $\gamma = 1$ ) f. SCM 3 ( $\gamma = 1.5$ ).

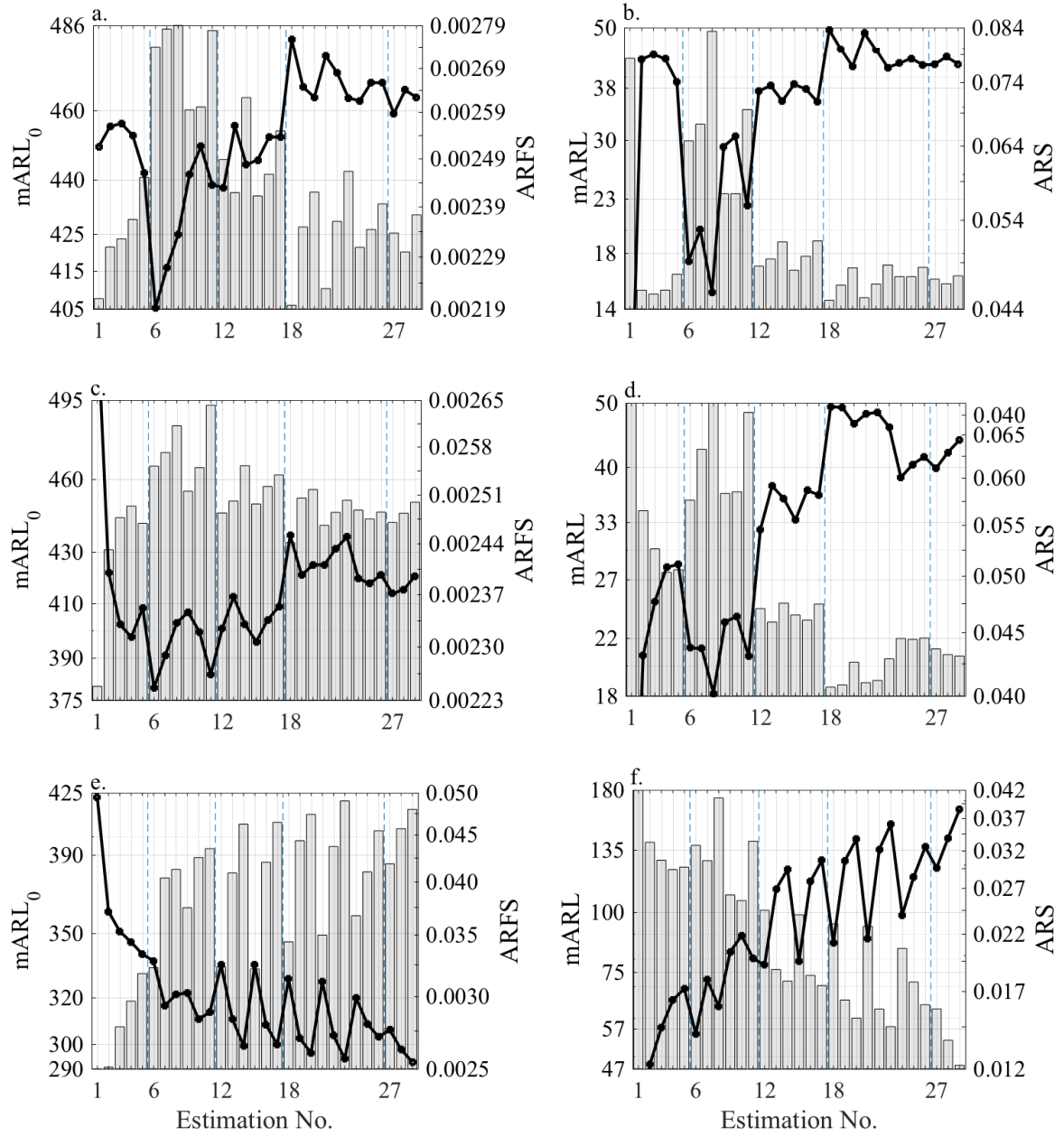


Figure 4.17. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under a. SCM 4 ( $\gamma = 1$ ) b. SCM 4 ( $\gamma = 1.5$ ) c. SCM 5 ( $\gamma = 1$ ) d. SCM 5 ( $\gamma = 1.5$ ) e. SCM 6 ( $\gamma = 1$ ) f. SCM 6 ( $\gamma = 1.5$ ).

Effect of between-subgroup variation on the Phase II performance of S-Charts based on Method IV and V estimators can be seen in Figure 4.18 and Table C.27-C.32 for  $m = 50$ . Performance of S-Charts using Method I-III estimators against CM 5 is identical to those under normally distributed data (Figure 4.11), since reweighting steps are based on only subgroup statistics in these estimators.  $ARFS$  values obtained from Method IV.B and V estimators stay almost constant at 0.0027-0.0028, while those from Method IV.A exceed 0.0030. The results show that, while S-Charts constructed using Method IV.A estimators have higher power for some cases Method IV.B and V estimators are negligibly affected from between-subgroup variation, and less likely to give false signals.

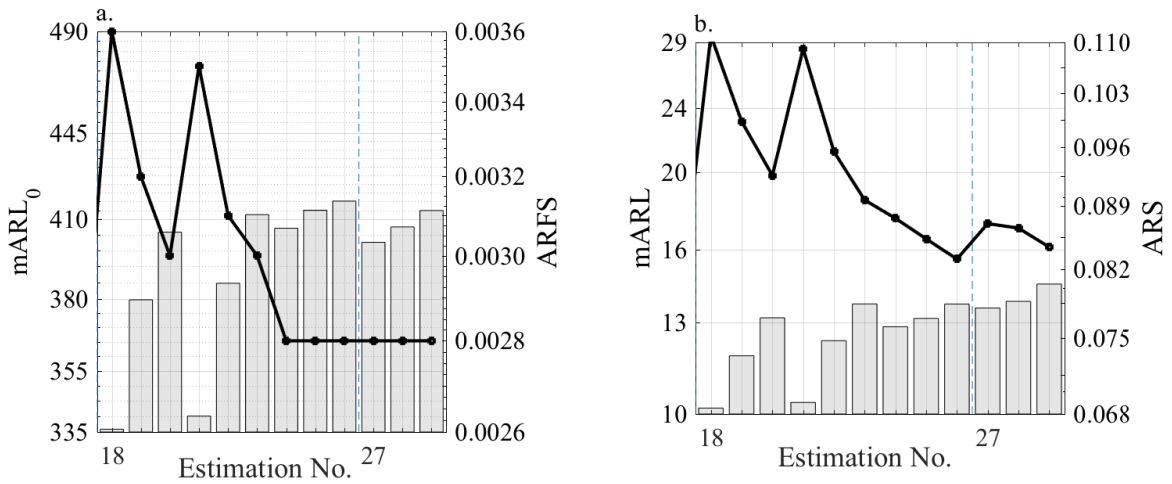


Figure 4.18. Phase II performance of S-Charts for  $m = 50$  when a.  $\gamma = 1$  and b.  $\gamma = 1.5$  using Phase I data under CM 5 ( $d=3$ ,  $K=0.15$ ).

#### 4.4. Performance of the Estimators on an out-of-Control Process

The current study focuses on designing robust Phase I estimators used to compute CLs of S-Charts for efficient Phase II monitoring, hence S-Charts have been evaluated for independent observations in Section 4.3. Autocorrelated observations, on the other hand, are prevalent in industrial applications. Some studies have found that such autocorrelation renders S-Charts misleading if dependency of observations is ignored

[72], and application of S-Charts to the correlated data affects the *ARFS*, and decreases its performance undesirably [73]. In this section, autoregressive models of order one are studied to determine the effect of between-subgroup variation on the proposed estimators and to investigate the capability of the estimators in determining whether process is in-control using correlated Phase I data.

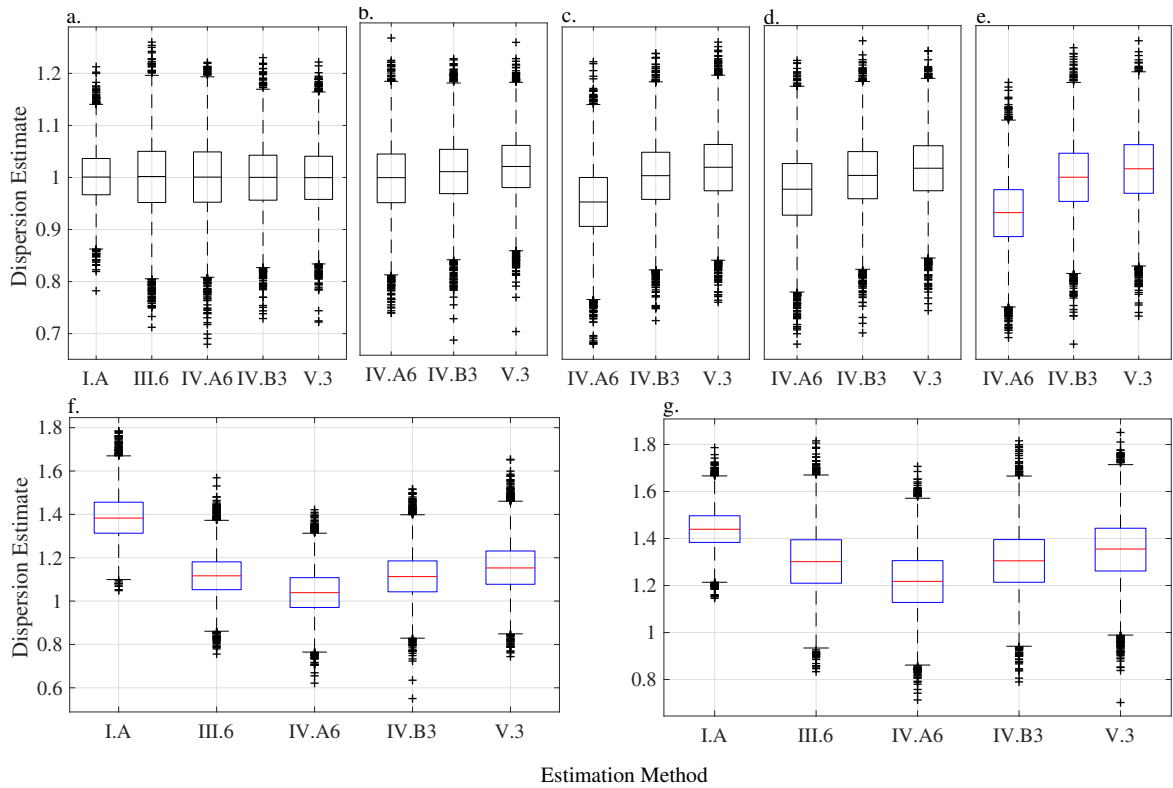


Figure 4.19. Box plots of dispersion estimates via various estimators for  $m = 50$  in the presence of ACM 1 with a.  $\phi = 0, \sigma_e^2 =$  b.  $\phi = 0.4, \sigma_e^2 = 0.5$  c.  $\phi = 0.4, \sigma_e^2 = 1$  d.  $\phi = 0.8, \sigma_e^2 = 0.5$  e.  $\phi = 0.4, \sigma_e^2 = 1$ , and in the presence of f. ACM 2 g. ACM 3.

In Figure 4.19a, box plots of the most promising dispersion estimates (along with the traditional standard deviation estimator) are shown, when process is in-control. As  $\phi$  and  $\sigma_e$  are changed (Figure 4.19b, 4.19c, 4.19d, 4.19e), Method I-III estimators are not affected. Increase in ACM 1 parameters yields a negative bias by Method IV.A estimator, since individual observations, which are distant from bulk of the data, are likely to be screened due to tight CLs of individuals chart in that method. In Method

IV.B and V, on the other hand, this effect is mostly eliminated, and nearly unbiased dispersion estimates are obtained, since most of the subgroups which have substantially different location estimates are already screened in these methods. When diffuse outliers exist in autocorrelated Phase I sampling (Figure 4.19f, 4.19g), Method I.A yields highly positively biased dispersion estimates, thus wider CLs would be constructed, and it would be difficult to detect that the process is out-of-control. Method V estimator yields dispersion estimates smaller than that of Method I.A, yet still having a high positive bias. This shows that shifts in the process mean slightly decreases the efficiency of the screening methods. Method IV.B estimator seems convenient in detecting out-of-control status of the process, since estimates are not negatively biased in the absence of outliers, and not highly positively biased in the presence of outliers.

## 4.5. Real Data Example

In this section, implementation of the control charts constructed using the proposed estimators to real-world process data is studied. Two different examples are discussed: hard-bake [8] and syringe [74] production process. For both examples, dispersion estimates are computed from the first 20 subgroups using algorithms given in Section 3.2;  $\sigma^I$ , upper and lower CLs are estimated using  $U_n^I$  and  $L_n^I$  shown in Table 4.2, and remaining subgroups are monitored. It is not possible to determine the best dispersion estimate from such a limited dataset, but comparing the dispersion estimates and CLs of S-Charts constructed from proposed estimators gives an overall idea about the performance and potential of the estimation methods.

### 4.5.1. Hard-bake Flow Width Example

A hard-bake process is used in conjunction with photolithography in semiconductor manufacturing. First twenty samples of flow width measurements (in microns), which are assumed to be collected when process is in-control, are shown in Table 4.9. Each sample consists five observations, which are sampled every hour. Dispersion estimates of hard-bake process using proposed Phase I estimators are shown in Table 4.10, and they are used to determine Phase II upper and lower CLs, which are given

in the same table, e.g.  $\hat{\sigma}^I$  based on  $S_n$  is equal to 0.1511, and the corresponding lower and upper CL coefficients are 0.171 and 2.356, the Phase II CLs are 0.0258 and 0.3559. Observations and dispersion estimates of the last twenty-five samples and dispersion parameters which are obtained using  $S_n$  are shown in Table 4.11.  $\hat{\sigma}_i^{II}$  values are used to check whether they fall between the pre-determined CLs, deemed to represent the variability of the in-control process.

Table 4.9. Flow width measurements (microns) for the hard-bake process.

| Sample No. | Observations |        |        |        |        |
|------------|--------------|--------|--------|--------|--------|
|            | 1            | 2      | 3      | 4      | 5      |
| 1          | 1.3235       | 1.4128 | 1.6744 | 1.4573 | 1.6914 |
| 2          | 1.4314       | 1.3592 | 1.6075 | 1.4666 | 1.6109 |
| 3          | 1.4284       | 1.4871 | 1.4932 | 1.4324 | 1.5674 |
| 4          | 1.5028       | 1.6352 | 1.3841 | 1.2831 | 1.5507 |
| 5          | 1.5604       | 1.2735 | 1.5265 | 1.4363 | 1.6441 |
| 6          | 1.5955       | 1.5451 | 1.3574 | 1.3281 | 1.4198 |
| 7          | 1.6274       | 1.5064 | 1.8366 | 1.4177 | 1.5144 |
| 8          | 1.4190       | 1.4303 | 1.6637 | 1.6067 | 1.5519 |
| 9          | 1.3884       | 1.7277 | 1.5355 | 1.5176 | 1.3688 |
| 10         | 1.4039       | 1.6697 | 1.5089 | 1.4627 | 1.5220 |
| 11         | 1.4158       | 1.7667 | 1.4278 | 1.5928 | 1.4181 |
| 12         | 1.5821       | 1.3355 | 1.5777 | 1.3908 | 1.7559 |
| 9          | 1.3884       | 1.7277 | 1.5355 | 1.5176 | 1.3688 |
| 10         | 1.4039       | 1.6697 | 1.5089 | 1.4627 | 1.522  |
| 11         | 1.4158       | 1.7667 | 1.4278 | 1.5928 | 1.4181 |
| 12         | 1.5821       | 1.3355 | 1.5777 | 1.3908 | 1.7559 |
| 13         | 1.2856       | 1.4106 | 1.4447 | 1.6398 | 1.1928 |
| 14         | 1.4951       | 1.4036 | 1.5893 | 1.6458 | 1.4969 |
| 15         | 1.3589       | 1.2863 | 1.5996 | 1.2497 | 1.5471 |
| 16         | 1.5747       | 1.5301 | 1.5171 | 1.1839 | 1.8662 |
| 17         | 1.368        | 1.7269 | 1.3957 | 1.5014 | 1.4449 |
| 18         | 1.4163       | 1.3864 | 1.3057 | 1.621  | 1.5573 |
| 19         | 1.5796       | 1.4185 | 1.6541 | 1.5116 | 1.7247 |
| 20         | 1.7106       | 1.4412 | 1.2361 | 1.382  | 1.7601 |

In the original reference, from which this example is taken [8] (pp. 232-234), no out-of-control signal is obtained in  $\bar{R}$  and  $\bar{X}$  charts for any of the subgroups, hence one may assume that the in-control process data is free from outliers. In this case, mean of the standard deviations of the subgroups is likely to yield the best dispersion estimate.

Figure 4.20 shows the S-Chart, in which CLs are computed via Method I.A and methods using individuals charts. All of the Phase II dispersion parameter estimates fall between CLs, showing that the proposed estimators yield similar results to the original source.

Since process is in-control and Phase I samples are not contaminated, it is expected from S-Charts based on Method I.A to have maximum power with tighter CLs. Method I.B estimators yield close dispersion estimates and upper and lower CLs, followed by Method V, IV.A, IV.B and III. Method II estimators, on the other hand, yield dispersion parameter estimates with low efficiency, and wider CLs are observed in S-Charts constructed from these estimators.

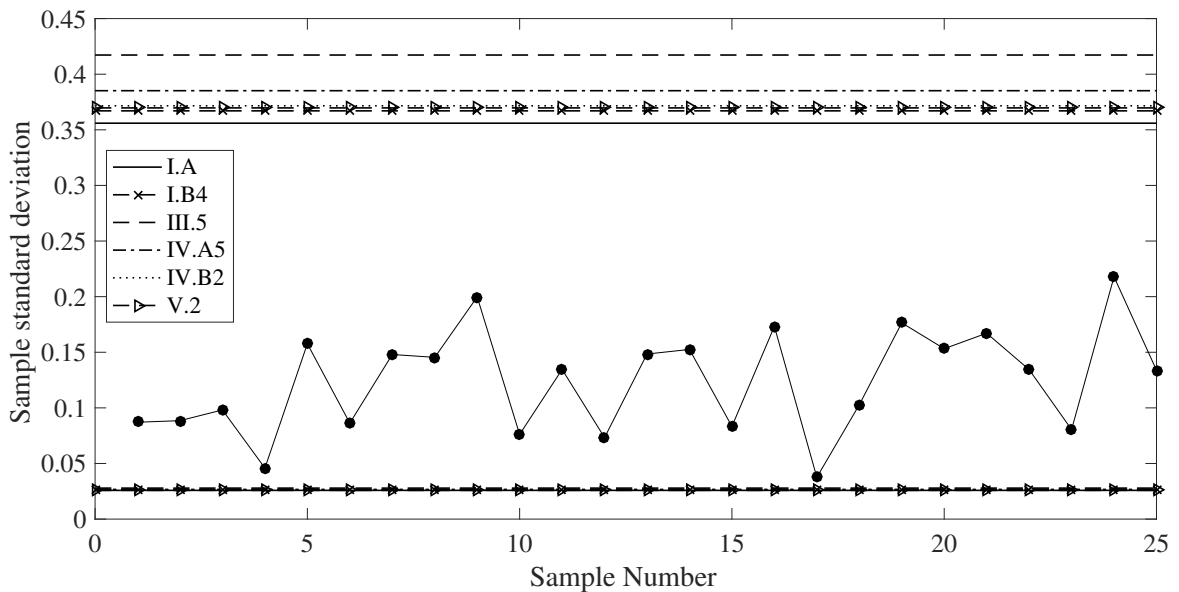


Figure 4.20. S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and V.3 estimators.

Table 4.10. Process dispersion estimates of hard-bake process and CLs of S-Charts.

| Method       | $\hat{\sigma}^I$ | $UCL$  | $LCL$  |
|--------------|------------------|--------|--------|
| <b>I.A</b>   | 0.1511           | 0.3559 | 0.0258 |
| <b>I.B1</b>  | 0.1512           | 0.3572 | 0.0259 |
| <b>I.B2</b>  | 0.1514           | 0.3591 | 0.0259 |
| <b>I.B3</b>  | 0.1517           | 0.3637 | 0.0259 |
| <b>I.B4</b>  | 0.1520           | 0.3671 | 0.0258 |
| <b>II.A1</b> | 0.1636           | 0.4100 | 0.0276 |
| <b>II.A2</b> | 0.1724           | 0.4456 | 0.0291 |
| <b>II.A3</b> | 0.1973           | 0.5344 | 0.0331 |
| <b>II.B1</b> | 0.1625           | 0.4163 | 0.0275 |
| <b>II.B2</b> | 0.1625           | 0.4195 | 0.0275 |
| <b>II.B3</b> | 0.1815           | 0.4905 | 0.0305 |
| <b>III.1</b> | 0.1531           | 0.3670 | 0.0262 |
| <b>III.2</b> | 0.1581           | 0.3901 | 0.0269 |
| <b>III.3</b> | 0.1637           | 0.4175 | 0.0278 |
| <b>III.4</b> | 0.1531           | 0.3668 | 0.0262 |
| <b>III.5</b> | 0.1581           | 0.3909 | 0.0269 |
| <b>III.6</b> | 0.1639           | 0.4172 | 0.0279 |
| <b>IV.A1</b> | 0.1528           | 0.3653 | 0.0261 |
| <b>IV.A2</b> | 0.1568           | 0.3844 | 0.0268 |
| <b>IV.A3</b> | 0.1615           | 0.4043 | 0.0276 |
| <b>IV.A4</b> | 0.1529           | 0.3656 | 0.0262 |
| <b>IV.A5</b> | 0.1570           | 0.3852 | 0.0267 |
| <b>IV.A6</b> | 0.1609           | 0.4047 | 0.0274 |
| <b>IV.B1</b> | 0.1531           | 0.3662 | 0.0261 |
| <b>IV.B2</b> | 0.1540           | 0.3716 | 0.0263 |
| <b>IV.B3</b> | 0.1562           | 0.3853 | 0.0267 |
| <b>V.1</b>   | 0.1530           | 0.3646 | 0.0262 |
| <b>V.2</b>   | 0.1538           | 0.3697 | 0.0263 |
| <b>V.3</b>   | 0.1559           | 0.3808 | 0.0267 |

Table 4.11. Additional flow width measurements (micron) for monitoring.

|            | Observations |        |        |        |        |                       |
|------------|--------------|--------|--------|--------|--------|-----------------------|
| Sample No. | 1            | 2      | 3      | 4      | 5      | $\hat{\sigma}_i^{II}$ |
| 21         | 1.4371       | 1.5051 | 1.3485 | 1.5670 | 1.4880 | 0.0871                |
| 22         | 1.4738       | 1.5936 | 1.6583 | 1.4973 | 1.4720 | 0.0885                |
| 23         | 1.5917       | 1.4333 | 1.5551 | 1.5295 | 1.6866 | 0.0981                |
| 24         | 1.6399       | 1.5243 | 1.5705 | 1.5563 | 1.5530 | 0.0459                |
| 25         | 1.5797       | 1.3663 | 1.6240 | 1.3732 | 1.6887 | 0.1576                |
| 26         | 1.4483       | 1.5458 | 1.4538 | 1.4303 | 1.6206 | 0.0863                |
| 27         | 1.5435       | 1.6899 | 1.5830 | 1.3358 | 1.4187 | 0.1478                |
| 28         | 1.5175       | 1.3446 | 1.4723 | 1.6657 | 1.6661 | 0.1454                |
| 29         | 1.5454       | 1.0931 | 1.4072 | 1.5039 | 1.5264 | 0.1997                |
| 30         | 1.4418       | 1.5059 | 1.5124 | 1.4620 | 1.6263 | 0.0762                |
| 31         | 1.4301       | 1.2725 | 1.5945 | 1.5397 | 1.5252 | 0.1345                |
| 32         | 1.4981       | 1.4506 | 1.6174 | 1.5837 | 1.4962 | 0.0733                |
| 33         | 1.3009       | 1.5060 | 1.6231 | 1.5831 | 1.6454 | 0.1484                |
| 34         | 1.4132       | 1.4603 | 1.5808 | 1.7111 | 1.7313 | 0.1525                |
| 35         | 1.3817       | 1.3135 | 1.4953 | 1.4894 | 1.4596 | 0.0833                |
| 36         | 1.5765       | 1.7014 | 1.4026 | 1.2773 | 1.4541 | 0.1732                |
| 37         | 1.4936       | 1.4373 | 1.5139 | 1.4808 | 1.5293 | 0.0376                |
| 38         | 1.5729       | 1.6738 | 1.5048 | 1.5651 | 1.7473 | 0.1028                |
| 39         | 1.8089       | 1.5513 | 1.8250 | 1.4389 | 1.6558 | 0.1765                |
| 40         | 1.6236       | 1.5393 | 1.6738 | 1.8698 | 1.5036 | 0.1532                |
| 41         | 1.4120       | 1.7931 | 1.7345 | 1.6391 | 1.7791 | 0.1671                |
| 42         | 1.7372       | 1.5663 | 1.4910 | 1.7809 | 1.5504 | 0.1344                |
| 43         | 1.5971       | 1.7394 | 1.6832 | 1.6677 | 1.7974 | 0.0805                |
| 44         | 1.4295       | 1.6536 | 1.9134 | 1.7272 | 1.4370 | 0.2179                |
| 45         | 1.6217       | 1.8220 | 1.7915 | 1.6744 | 1.9404 | 0.1339                |

#### 4.5.2. Syringe Length Example

In this example, syringe length samples manufactured by a midwest pharmaceutical company, are used for both determining CLs in Phase I and monitoring in Phase II. First twenty samples of syringe length measurements (in inch), which consist five observations, are sampled at every 15 minutes, and the measurements are shown in Table 4.12.

Table 4.12. Syringe length (inch) samples.

|                   | <b>Observations</b> |       |       |       |       |
|-------------------|---------------------|-------|-------|-------|-------|
| <b>Sample No.</b> | 1                   | 2     | 3     | 4     | 5     |
| <b>1</b>          | 4.960               | 4.946 | 4.950 | 4.956 | 4.958 |
| <b>2</b>          | 4.958               | 4.927 | 4.935 | 4.940 | 4.95  |
| <b>3</b>          | 4.971               | 4.929 | 4.965 | 4.952 | 4.938 |
| <b>4</b>          | 4.94                | 4.982 | 4.97  | 4.953 | 4.96  |
| <b>5</b>          | 4.964               | 4.95  | 4.953 | 4.962 | 4.956 |
| <b>6</b>          | 4.969               | 4.951 | 4.955 | 4.966 | 4.954 |
| <b>7</b>          | 4.96                | 4.944 | 4.957 | 4.948 | 4.951 |
| <b>8</b>          | 4.969               | 4.949 | 4.963 | 4.952 | 4.962 |
| <b>9</b>          | 4.984               | 4.928 | 4.96  | 4.943 | 4.955 |
| <b>10</b>         | 4.97                | 4.934 | 4.961 | 4.94  | 4.965 |
| <b>11</b>         | 4.975               | 4.959 | 4.962 | 4.971 | 4.968 |
| <b>12</b>         | 4.945               | 4.977 | 4.95  | 4.969 | 4.954 |
| <b>13</b>         | 4.976               | 4.964 | 4.97  | 4.968 | 4.972 |
| <b>14</b>         | 4.97                | 4.954 | 4.964 | 4.959 | 4.968 |
| <b>15</b>         | 4.982               | 4.962 | 4.968 | 4.975 | 4.963 |
| <b>16</b>         | 4.961               | 4.943 | 4.95  | 4.949 | 4.957 |
| <b>17</b>         | 4.98                | 4.97  | 4.975 | 4.978 | 4.977 |
| <b>18</b>         | 4.975               | 4.968 | 4.971 | 4.969 | 4.972 |
| <b>19</b>         | 4.977               | 4.966 | 4.969 | 4.973 | 4.97  |
| <b>20</b>         | 4.975               | 4.967 | 4.969 | 4.972 | 4.972 |

In the original study, it was stated that the first 15 subgroups were sampled from the first shift, while the rest of the subgroups were sampled from the new shift, which started at 4:00 p.m. of the same day. UCL and LCL are computed for  $\bar{X}$  and R charts. When the corresponding statistics of first 15 samples are placed on the  $\bar{X}$  and R control charts, these charts show that process is already out-of-control in both center and variation. The operators from the first shift continued to collect 15 samples, each of size 5 and plotted the sample means and ranges in  $\bar{X}$  and R charts using the CLs obtained from the first set of 15 samples. These charts also showed clearly the center of the process is to be out-of-control.

In the current study, process dispersion estimates are computed from the first 20 samples to determine the CLs of the corresponding S-Charts (Table 4.13), and the remaining 27 samples (Table 4.14) are monitored. Methods using individual charts are found to give lower  $\hat{\sigma}^I$  at higher skipping levels. In the cited reference, the first 15 subgroups were said to be collected from an already out-of-control process; hence monitoring the rest of the observations using CLs obtained from an out-of-control process is not very meaningful. Nevertheless, it should be expected for some of the future samples collected from an in-control process to be below the dispersion LCL. In the cited reference, the lower limit of R-chart was taken to be equal to zero, hence it was not possible to determine future samples, with a smaller dispersion parameter estimate. Tighter CLs are obtained in S-Charts constructed using Method IV and V estimators (Figure 4.21a). Here, sample standard deviation of the 23<sup>th</sup> sample (the third sample in Figure 4.21) is found to be equal to 0.0013, which is lower than the LCL of the S-charts, constructed via all estimators excepted reweighted standard deviation estimator. Also considering that Method II estimators give remarkably high UCLs, it may suggested that Method III, IV, and V estimators are capable of determining a future sample, which has a significantly lower dispersion than that seen in Phase I data. (Figure 4.21b).

Table 4.13. Process dispersion estimates of syringe manufacturing process and CLs of S-Charts.

| Method       | $\hat{\sigma}^I$ | $UCL$  | $LCL$  |
|--------------|------------------|--------|--------|
| <b>I.A</b>   | 0.0094           | 0.0222 | 0.0016 |
| <b>I.B1</b>  | 0.0088           | 0.0208 | 0.0015 |
| <b>I.B2</b>  | 0.0071           | 0.0169 | 0.0012 |
| <b>I.B3</b>  | 0.0069           | 0.0166 | 0.0012 |
| <b>I.B4</b>  | 0.0067           | 0.0163 | 0.0012 |
| <b>II.A1</b> | 0.0114           | 0.0285 | 0.0019 |
| <b>II.A2</b> | 0.0102           | 0.0263 | 0.0017 |
| <b>II.A3</b> | 0.0117           | 0.0316 | 0.0020 |
| <b>II.B1</b> | 0.0108           | 0.0277 | 0.0018 |
| <b>II.B2</b> | 0.0108           | 0.0280 | 0.0018 |
| <b>II.B3</b> | 0.0116           | 0.0313 | 0.0020 |
| <b>III.1</b> | 0.0096           | 0.0230 | 0.0016 |
| <b>III.2</b> | 0.0095           | 0.0235 | 0.0016 |
| <b>III.3</b> | 0.0090           | 0.0220 | 0.0015 |
| <b>III.4</b> | 0.0096           | 0.0229 | 0.0016 |
| <b>III.5</b> | 0.0090           | 0.0223 | 0.0015 |
| <b>III.6</b> | 0.0090           | 0.0229 | 0.0015 |
| <b>IV.A1</b> | 0.0096           | 0.0228 | 0.0016 |
| <b>IV.A2</b> | 0.0094           | 0.0230 | 0.0016 |
| <b>IV.A3</b> | 0.0091           | 0.0227 | 0.0016 |
| <b>IV.A4</b> | 0.0096           | 0.0229 | 0.0016 |
| <b>IV.A5</b> | 0.0090           | 0.0221 | 0.0015 |
| <b>IV.A6</b> | 0.0091           | 0.0229 | 0.0015 |
| <b>IV.B1</b> | 0.0097           | 0.0232 | 0.0017 |
| <b>IV.B2</b> | 0.0090           | 0.0216 | 0.0015 |
| <b>IV.B3</b> | 0.0091           | 0.0225 | 0.0016 |
| <b>V.1</b>   | 0.0097           | 0.0231 | 0.0017 |
| <b>V.2</b>   | 0.0092           | 0.0220 | 0.0017 |
| <b>V.3</b>   | 0.0091           | 0.0222 | 0.0016 |

Table 4.14. Additional syringe length measurements (inch) for monitoring.

|            | Observations |       |       |       |       |                       |
|------------|--------------|-------|-------|-------|-------|-----------------------|
| Sample No. | 1            | 2     | 3     | 4     | 5     | $\hat{\sigma}_i^{II}$ |
| 21         | 4.980        | 4.960 | 4.974 | 4.965 | 4.971 | 0.0083                |
| 22         | 4.979        | 4.967 | 4.976 | 4.970 | 4.978 | 0.0056                |
| 23         | 4.973        | 4.976 | 4.974 | 4.974 | 4.973 | 0.0013                |
| 24         | 4.978        | 4.970 | 4.976 | 4.975 | 4.971 | 0.0036                |
| 25         | 4.979        | 4.967 | 4.972 | 4.977 | 4.970 | 0.0053                |
| 26         | 4.987        | 4.963 | 4.979 | 4.974 | 4.977 | 0.0093                |
| 27         | 4.979        | 4.971 | 4.975 | 4.977 | 4.978 | 0.0034                |
| 28         | 4.974        | 4.970 | 4.973 | 4.972 | 4.971 | 0.0017                |
| 29         | 4.993        | 4.974 | 4.980 | 4.988 | 4.980 | 0.0080                |
| 30         | 4.992        | 4.961 | 4.971 | 4.986 | 4.970 | 0.0135                |
| 31         | 4.988        | 4.976 | 4.981 | 4.986 | 4.979 | 0.0053                |
| 32         | 4.987        | 4.975 | 4.980 | 4.984 | 4.979 | 0.0049                |
| 33         | 4.975        | 4.945 | 4.952 | 4.968 | 4.960 | 0.0128                |
| 34         | 4.981        | 4.956 | 4.966 | 4.974 | 4.968 | 0.0099                |
| 35         | 4.980        | 4.956 | 4.971 | 4.966 | 4.962 | 0.0097                |
| 36         | 4.965        | 4.958 | 4.960 | 4.962 | 4.960 | 0.0028                |
| 37         | 4.974        | 4.960 | 4.966 | 4.971 | 4.964 | 0.0059                |
| 38         | 4.973        | 4.962 | 4.969 | 4.964 | 4.967 | 0.0046                |
| 39         | 4.960        | 4.974 | 4.968 | 4.965 | 4.968 | 0.0054                |
| 40         | 4.967        | 4.956 | 4.962 | 4.959 | 4.961 | 0.0043                |
| 41         | 4.964        | 4.968 | 4.967 | 4.966 | 4.965 | 0.0017                |
| 42         | 4.971        | 4.963 | 4.966 | 4.969 | 4.966 | 0.0033                |
| 43         | 4.966        | 4.959 | 4.963 | 4.961 | 4.961 | 0.0028                |
| 44         | 4.953        | 4.967 | 4.960 | 4.958 | 4.957 | 0.0055                |
| 45         | 4.969        | 4.951 | 4.958 | 4.963 | 4.959 | 0.0071                |
| 46         | 4.970        | 4.950 | 4.962 | 4.957 | 4.966 | 0.0083                |
| 47         | 4.965        | 4.957 | 4.958 | 4.963 | 4.962 | 0.0036                |

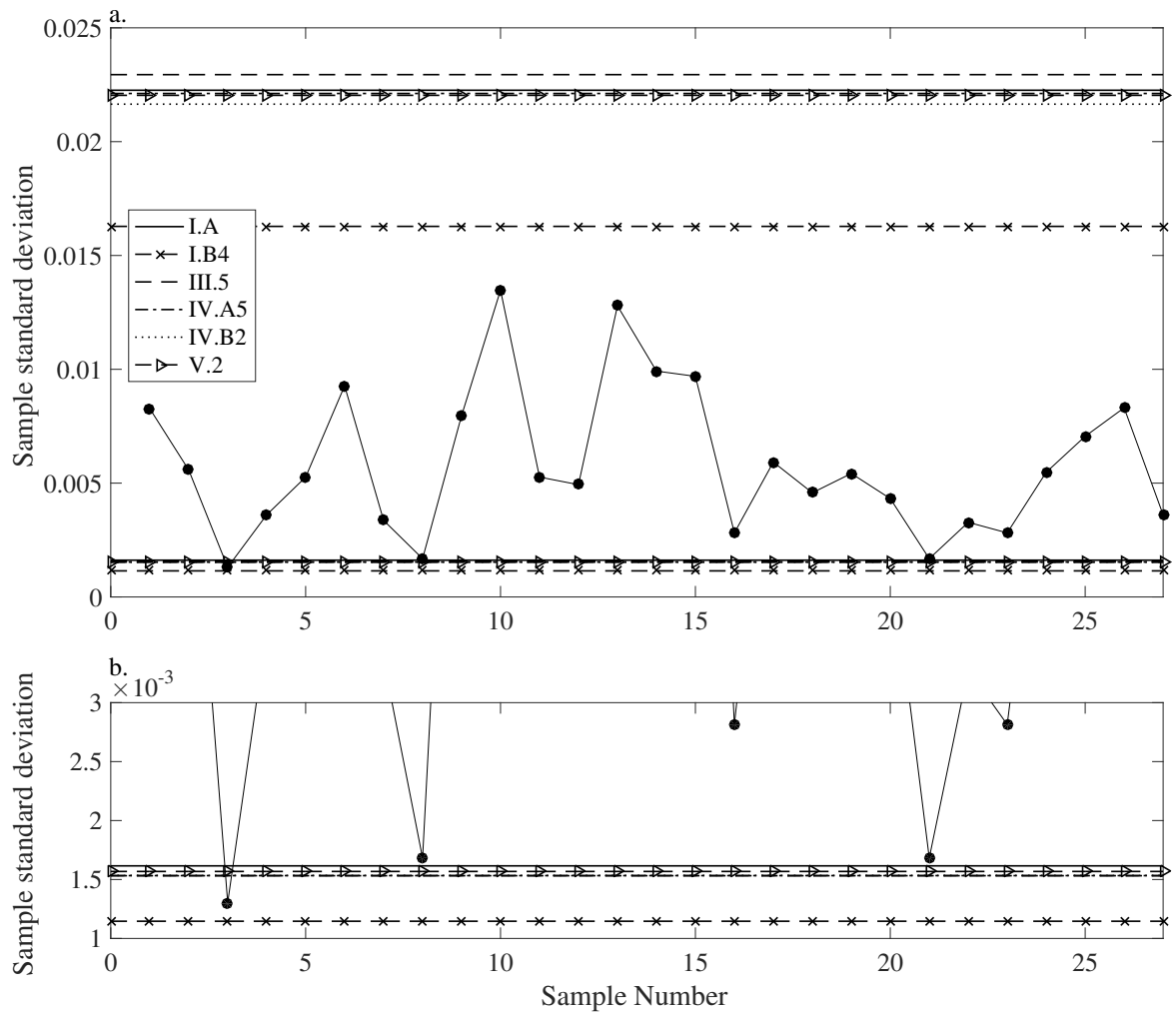


Figure 4.21. S-Charts constructed using Method I.A, III.6, IV.A6, IV.B3 and V.3 estimators.

## 5. CONCLUSIONS AND RECOMMENDATIONS

Based on the assumption of independent and normally distributed dataset, sample standard deviation and sample mean are known to be most efficient traditional estimators to determine process parameters, and S-Charts based on these estimators yield high power under normality. However, the efficiency of traditional estimators is significantly diminished, and power of S-Charts is undesirably low when process parameters are estimated, and outliers are present in sampling [25, 37, 40, 43–45, 50]. Inability to promptly detect disturbances in a process causes an inefficient process monitoring and a serious delay in detection of inferior products. One way of overcoming this problem is reweighting traditional estimators, based on screening the outlier subgroups. While reweighted traditional estimators are highly efficient against outliers that affect only certain subgroups, their efficiency is significantly low against outliers that come from short-tailed asymmetric distributions. An improved method is to use robust statistics for parameter estimation in Phase I. Application of robust estimators may be valuable in the increasing performance of control charts, but robust estimators have low efficiency under normality. Furthermore, screening outlier subgroups does not significantly increase the efficiency and performance of robust estimators. It should be noted that power of S-Charts based on reweighted traditional estimators (Method I.B), non-reweighted (Method II.A) / reweighted robust estimators (Method II.B) are higher than those based on traditional estimators (Method I.A) when outliers exist in samples, but the performance against diffuse mean contamination (CM 4) may be undesirably low; S-Charts may become ARL-biased. To address this problem, the current study focuses on using individuals charts in reweighting robust estimators, being a method recommended by Does and co-workers [37, 40].

In the current study, a number of improvements over the currently existing reweighting methods are proposed to increase Phase II performance of S-Charts in the presence of outliers, keeping high power and efficiency under normality. First of all, good efficiency and performance of scale M-estimator with logistic influence function  $MS_n$  is to be noted. Average distance to the median ( $ADM_n$ ) [37], interquartile

range ( $IQR$ ) [43, 50], Rousseeuw's and Croux'  $Q_n$  [29] estimator are also frequently used in the literature for SPM applications. Although performance of  $Q_n$  was found to be superior to  $ADM_n$  and  $IQR$  [75],  $MAD_n$  and  $MS_n$  showed more satisfactory performance in our preliminary studies [76]. For these reasons,  $S_n$ ,  $MAD_n$  and  $MS_n$  are the dispersion estimators focused in this study. In conclusion,  $MS_n$  is shown to be a convenient robust dispersion estimator, while  $MAD_n$  yields low power due to inflated variance of parameter estimation. Robust location estimators  $HD_{.5}$  and  $ML$  do not show significant difference in parameter estimates and performance of S-Charts.

Utilizing individuals chart to screen outlier observations in subgroups (Method III-V) increases the efficiency of estimators, and S-Charts based on these estimators are found to be more powerful against all CMs. Difference of reweighting procedures in Methods III-V lies in obtaining the initial and intermediate parameter estimates of the subgroups. In Method III, suggested by Does and co-workers, each subgroup location parameter ( $\hat{\mu}_i^{II}$ ) is estimated separately, and individual charts are constructed using subgroup location estimates [37]. It should be noted that, ( $\hat{\mu}_i^{II}$ ) is not likely to be an efficient estimator of process location, since it can still be affected by outliers in a sample with a small size of  $n$  observations. To deal with this problem, a single robust location estimate based on subgroup statistics (Method IV.A, IV.B and V) is proposed. Efficiency of individual charts in detecting outliers is significantly increased using single location estimate. In Method IV.A, subgroups, which have different location estimates than the others, are still included in determining the CLs of individual charts, hence the final standard deviation estimates of subgroups with a different location parameter are likely to be lower than the in-control process standard deviation. To remedy this problem, subgroups, which have different location estimates, are screened before constructing individuals chart in Method IV.B and V. In Method V, intermediate location ( $\hat{\mu}^{II}$ ) and dispersion ( $\hat{\sigma}^{II}$ ) estimates are determined via pooling all the individual observations, regardless of their subgroups in Method V. The results show that Method III estimator falls short at low and moderate contamination densities, and its inferiority is more apparent at higher contamination density, compared to Method IV-V estimators. It should further be noted that Method V estimators yield the best parameter estimation against all CMs, and generally followed by Method IV.B and

IV.A. Method IV.A estimator has higher efficiency than Method IV.B against CM 1 and CM 4, possibly because partitioning the total skipping level into screening subgroups and individual observations in Method IV.B yields wider CLs, decreasing its efficiency against diffuse outliers, but Method IV.B is more efficient against CM 2 and CM 3.

Another important point in the current study is the effect of skipping level on the S-chart performance using various estimators. Higher skipping levels have been shown to increase the performance of S-Charts against contaminations in Phase I samples, and maximum power is observed at  $\alpha = 0.04 - 0.02$  for  $m = 50$ , and  $\alpha = 0.02 - 0.005$  for  $m = 20$ , both being much higher than 0.0027. The loss of efficiency and power due to increasing skipping level under normality is found to be acceptably low, when S-Charts are constructed from Method IV.B and V estimators.

In this study, novel reweighted robust dispersion estimation methods are proposed, and their Phase I efficiency and Phase II performance are assessed both under normality and against various CMs at different contamination magnitude and densities, which are adjusted to better represent the real-world data. It is found that Method IV.B and V estimators are the most promising dispersion estimators, and found to be efficient at high skipping levels. Estimation methods may be extended performing different combination of dispersion and location estimators on subgroups and individual observations, i.e. while  $HD_{.5}$  estimators is used to determine initial process location estimate in subgroup screening,  $ML$  may be used to determine intermediate process location estimate in screening individual outliers. Phase II power analysis of Method IV.B and V estimators may further be studied using  $ML$  as a location estimator, since it has high BDP. A previous study suggested to use  $MS_n$  estimator with bisquare  $\rho$  function [77], further study may be performed to compare the power and efficiency of  $MS_n$  with different  $\rho$  functions using proposed reweighting procedures. Higher skipping levels can also be further studied, and may be optimized for estimation methods with respect to sample size. Proposed reweighting procedures can be studied in  $\bar{X}$  control charts and multivariate processes as well.

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## APPENDIX A: BIAS CORRECTION FACTORS AND COEFFICIENTS

Table A.1. Bias correction factors are used in the calculation of process dispersion estimate.

| $n$ | $c_4$  | $d_2$ | $d_3$ | $D_3$ | $D_4$ |
|-----|--------|-------|-------|-------|-------|
| 1   | -      | -     | 1     | 0.820 | 1     |
| 2   | 2.205  | 0.603 | 1.128 | 0.853 | 0.954 |
| 3   | 2.016  | 0.463 | 1.693 | 0.888 | 1.588 |
| 4   | 0.921  | 0.389 | 2.059 | 0.879 | 1.978 |
| 5   | 0.940  | 0.341 | 2.326 | 0.864 | 2.257 |
| 6   | 0.952  | 0.308 | 2.534 | 0.848 | 2.472 |
| 7   | 0.959  | 0.282 | 2.704 | 0.833 | 2.645 |
| 8   | 0.965  | 0.262 | 2.847 | 0.820 | 2.791 |
| 9   | 0.969  | 0.246 | 2.970 | 0.808 | 2.915 |
| 10  | 0.973  | 0.232 | 3.078 | 0.797 | 3.024 |
| 11  | 0.975  | 0.221 | 3.173 | 0.787 | 3.121 |
| 12  | 0.978  | 0.211 | 3.258 | 0.779 | 3.207 |
| 13  | 0.979  | 0.202 | 3.336 | 0.770 | 3.285 |
| 14  | 0.981  | 0.194 | 3.407 | 0.763 | 3.356 |
| 15  | 0.982  | 0.187 | 3.472 | 0.756 | 3.422 |
| 16  | 0.983  | 0.181 | 3.532 | 0.750 | 3.482 |
| 17  | 0.985  | 0.175 | 3.588 | 0.744 | 3.538 |
| 18  | 0.985  | 0.170 | 3.640 | 0.739 | 3.591 |
| 19  | 0.986  | 0.165 | 3.689 | 0.734 | 3.640 |
| 20  | 0.987  | 0.161 | 3.735 | 0.729 | 3.686 |
| 21  | 0.988  | 0.157 | 3.778 | 0.724 | 3.73  |
| 22  | 0.988  | 0.153 | 3.819 | 0.720 | 3.771 |
| 23  | 0.989  | 0.150 | 3.858 | 0.716 | 3.811 |
| 24  | 0.989  | 0.147 | 3.895 | 0.712 | 3.847 |
| 25  | 0.9904 | 0.144 | 3.931 | 0.708 | 3.883 |

Table A.2. Phase I CLs and bias correction factors for  $m = 20$ .

| Method       | $\Theta_1$ | $\Theta_2$ | $L_n^I$ | $U_n^I$ | $\Theta_3$ | $C_n^I$ | $\Theta_4$ | $\Theta_5$ |
|--------------|------------|------------|---------|---------|------------|---------|------------|------------|
| <b>I.A</b>   | 1.064      | 1          | -       | -       | -          | -       | -          | -          |
| <b>I.B1</b>  | 1.064      | 1          | 0.179   | 2.216   | 1.001      | -       | -          | -          |
| <b>I.B2</b>  | 1.064      | 1          | 0.247   | 2.032   | 1.002      | -       | -          | -          |
| <b>I.B3</b>  | 1.064      | 1          | 0.314   | 1.892   | 1.004      | -       | -          | -          |
| <b>I.B4</b>  | 1.064      | 1          | 0.376   | 0.775   | 1.006      | -       | -          | -          |
| <b>II.A1</b> | 1.319      | 1.022      | -       | -       | -          | -       | -          | -          |
| <b>II.A2</b> | 1.319      | 1.055      | -       | -       | -          | -       | -          | -          |
| <b>II.A3</b> | 1.803      | 1.083      | -       | -       | -          | -       | -          | -          |
| <b>II.B1</b> | 1.319      | 1.022      | 0.14    | 2.335   | 1.008      | -       | -          | -          |
| <b>II.B2</b> | 1.319      | 1.055      | 0.138   | 2.406   | 1.007      | -       | -          | -          |
| <b>II.B3</b> | 1.803      | 1.083      | 0.108   | 2.724   | 1.012      | -       | -          | -          |
| <b>III.1</b> | 1.319      | 1.022      | 0.047   | 2.978   | 1.002      | 3.090   | 1.013      | -          |
| <b>III.2</b> | 1.319      | 1.022      | 0.088   | 2.602   | 1.005      | 2.511   | 1.047      | -          |
| <b>III.3</b> | 1.319      | 1.022      | 0.127   | 2.393   | 1.008      | 2.221   | 1.084      | -          |
| <b>III.4</b> | 1.319      | 1.055      | 0.046   | 3.063   | 1.002      | 3.089   | 1.013      | -          |
| <b>III.5</b> | 1.319      | 1.055      | 0.087   | 2.668   | 1.004      | 2.548   | 1.047      | -          |
| <b>III.6</b> | 1.319      | 1.055      | 0.127   | 2.446   | 1.008      | 2.261   | 1.085      | -          |
| <b>IV.A1</b> | 1.319      | 1.022      | 0.047   | 2.978   | 1.002      | 3.142   | 1.011      | -          |
| <b>IV.A2</b> | 1.319      | 1.022      | 0.088   | 2.602   | 1.005      | 2.620   | 1.038      | -          |
| <b>IV.A3</b> | 1.319      | 1.022      | 0.127   | 2.393   | 1.008      | 2.350   | 1.069      | -          |
| <b>IV.A4</b> | 1.319      | 1.055      | 0.046   | 3.063   | 1.0016     | 3.151   | 1.012      | -          |
| <b>IV.A5</b> | 1.319      | 1.055      | 0.087   | 2.668   | 1.004      | 2.628   | 1.039      | -          |
| <b>IV.A6</b> | 1.319      | 1.055      | 0.127   | 2.446   | 1.008      | 2.371   | 1.065      | -          |
| <b>IV.B1</b> | 1.319      | 1.055      | 0.046   | 3.063   | 1.002      | 3.110   | 1.013      | -          |
| <b>IV.B2</b> | 1.319      | 1.055      | 0.087   | 2.668   | 1.004      | 3.010   | 1.019      | -          |
| <b>IV.B3</b> | 1.319      | 1.055      | 0.127   | 2.446   | 1.008      | 2.743   | 1.034      | -          |
| <b>V.1</b>   | 1.319      | 1.055      | 0.046   | 3.063   | 1.002      | 3.111   | 1.013      | 1.016      |
| <b>V.2</b>   | 1.319      | 1.055      | 0.087   | 2.668   | 1.004      | 3.001   | 1.018      | 1.019      |
| <b>V.3</b>   | 1.319      | 1.055      | 0.127   | 2.446   | 1.008      | 2.734   | 1.024      | 1.024      |

## APPENDIX B: PHASE I EFFICIENCIES OF PROPOSED ESTIMATORS

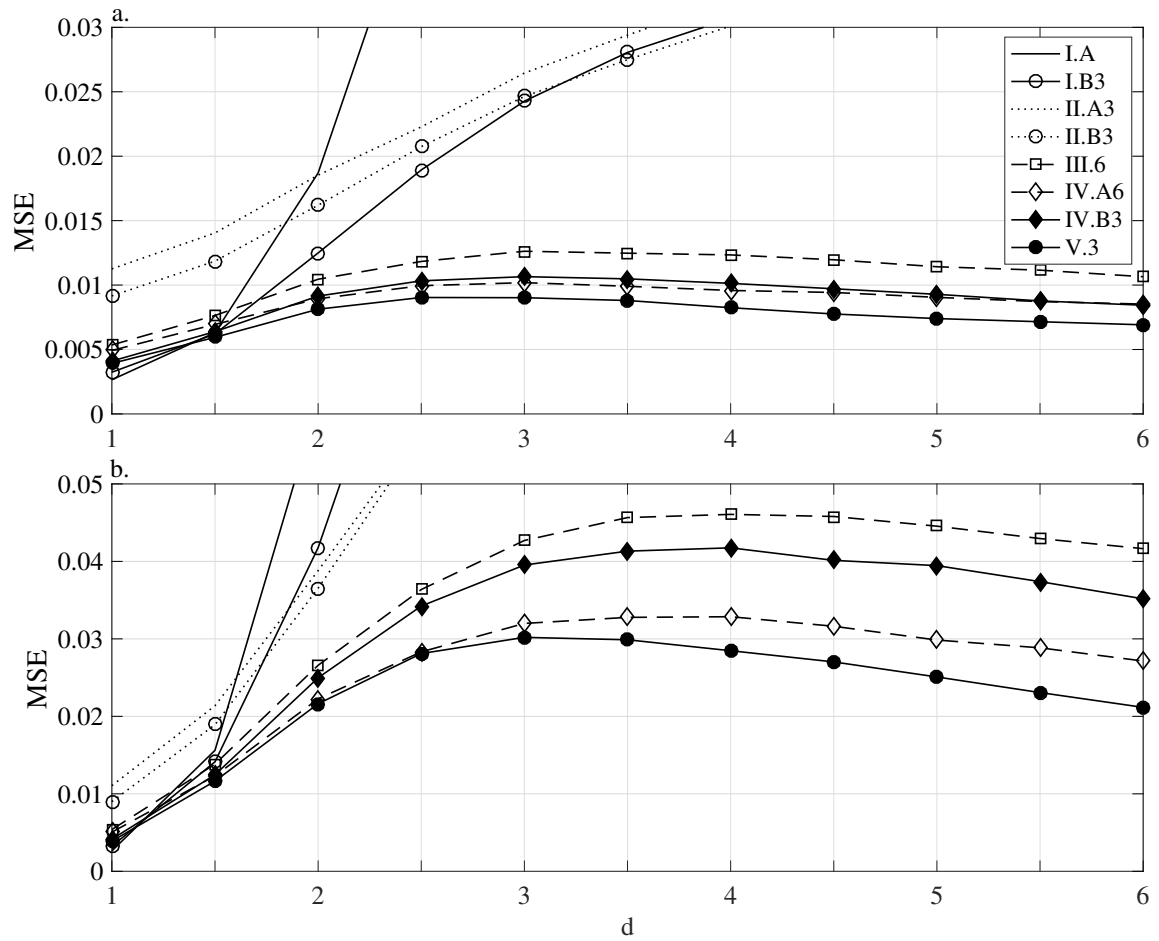


Figure B.1. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 1 at  
a.  $K = 0.10$  b.  $K = 0.20$ .

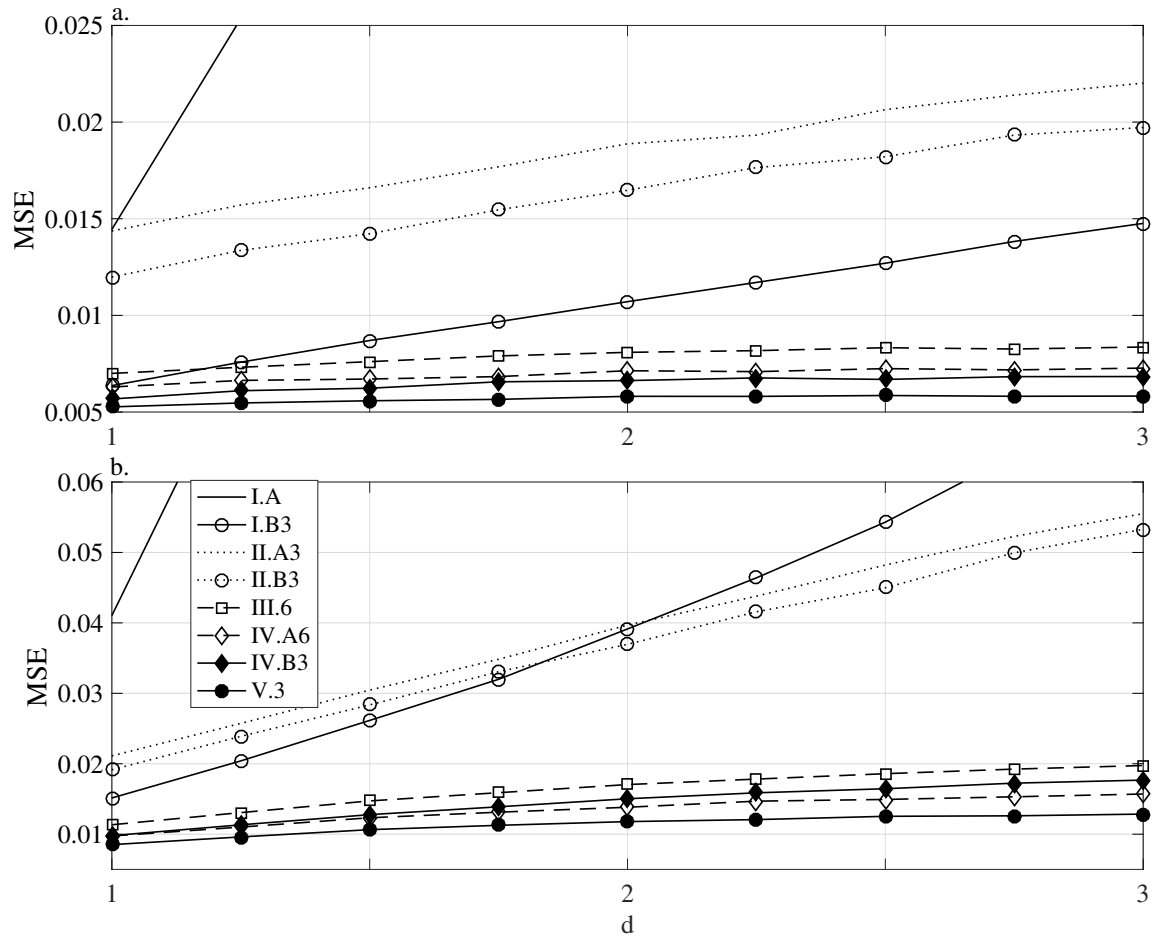


Figure B.2. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 2 at  
a.  $K = 0.10$  b.  $K = 0.20$ .

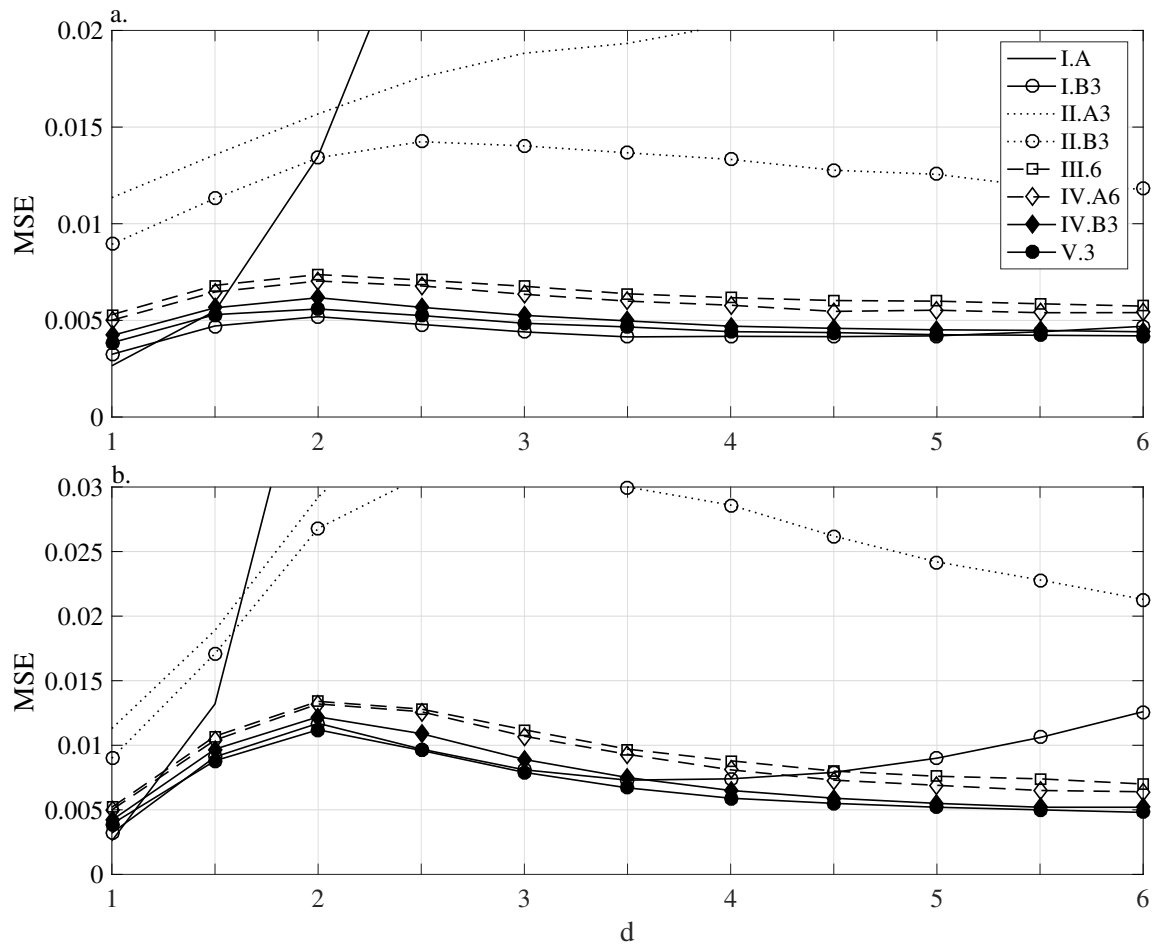


Figure B.3. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 3 at  
a.  $K = 0.10$  b.  $K = 0.20$ .

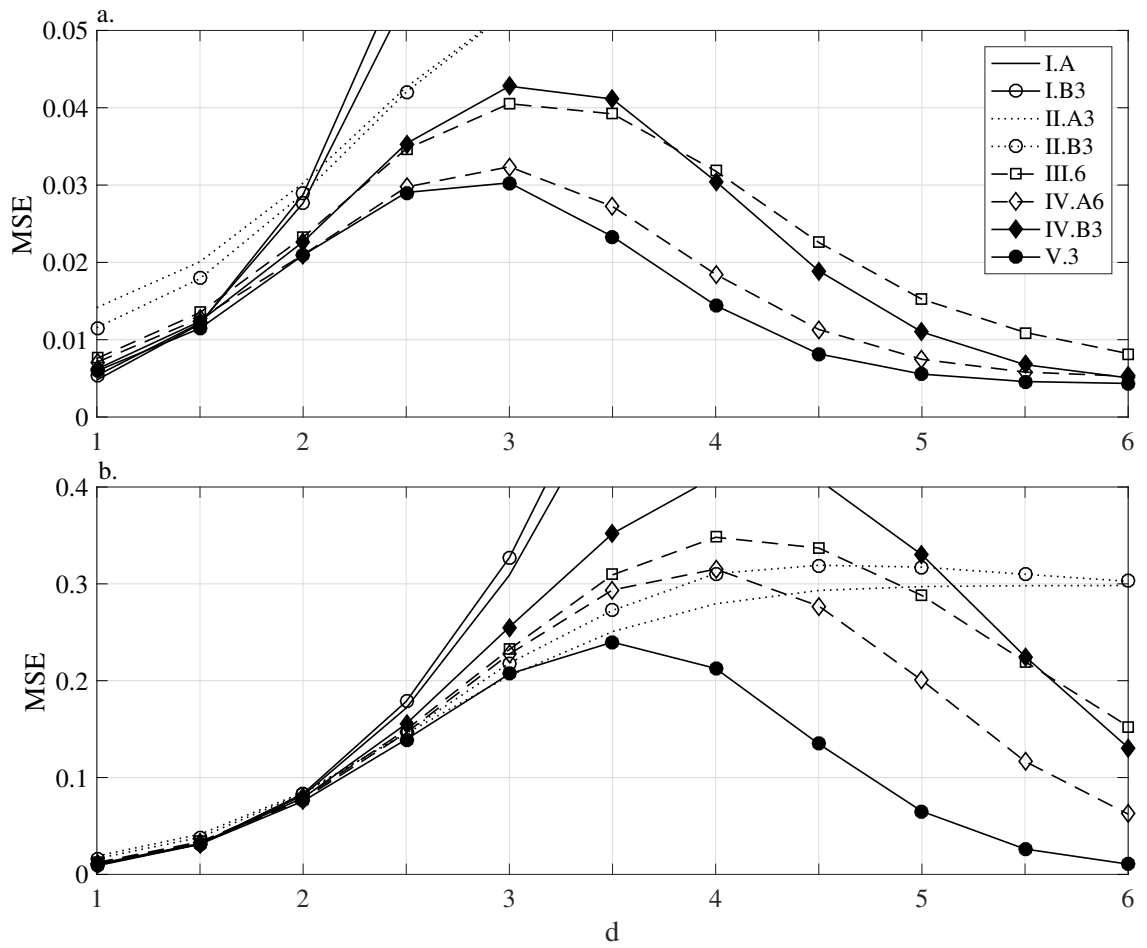


Figure B.4. Phase I efficiency of the proposed estimators for  $m = 50$  under CM 4 at  
a.  $K = 0.10$  b.  $K = 0.20$ .

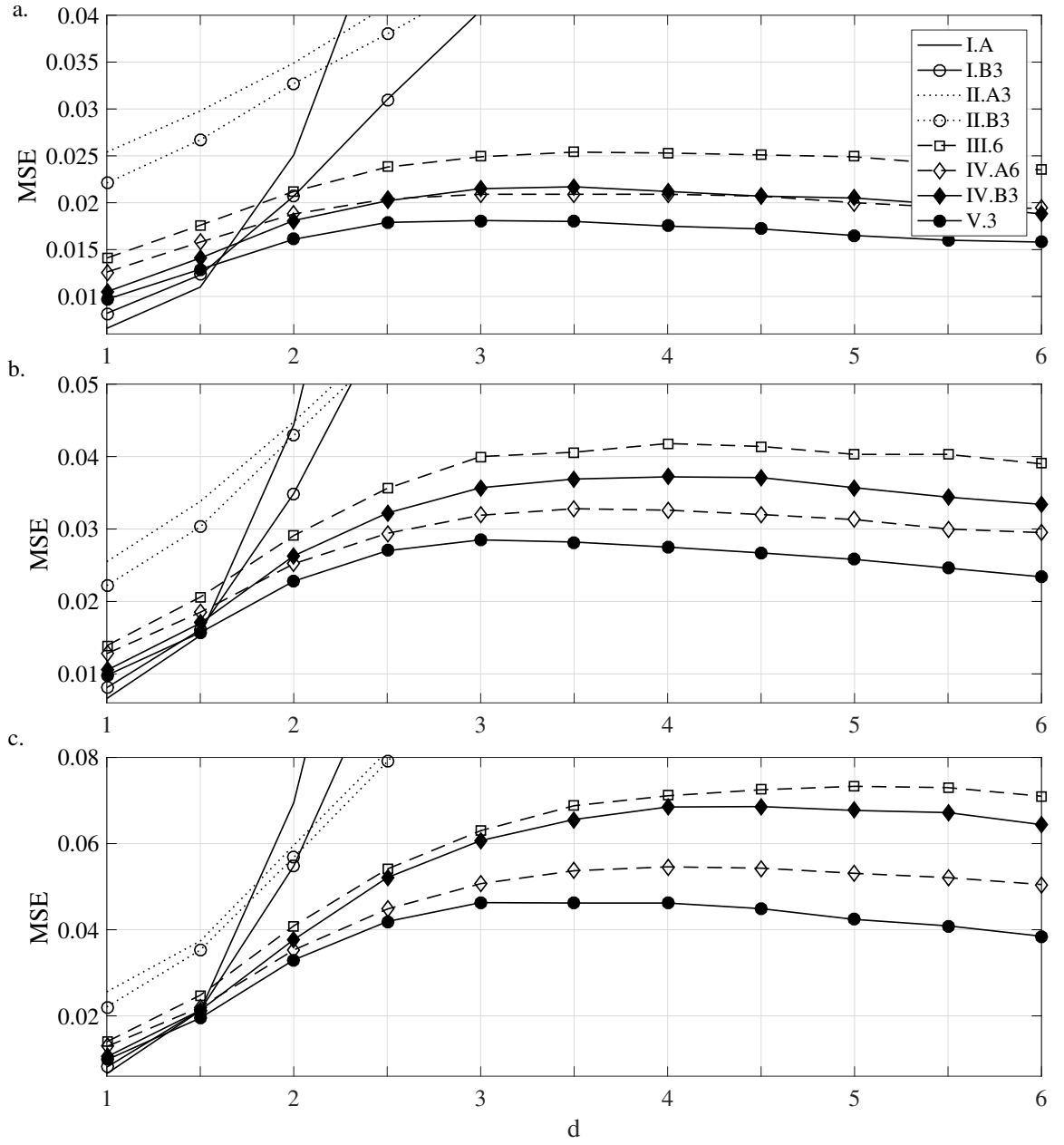


Figure B.5. Phase I efficiency of the proposed estimators for  $m = 20$  under CM 1 at  
a.  $K = 0.10$  b.  $K = 0.15$  c.  $K = 20$ .

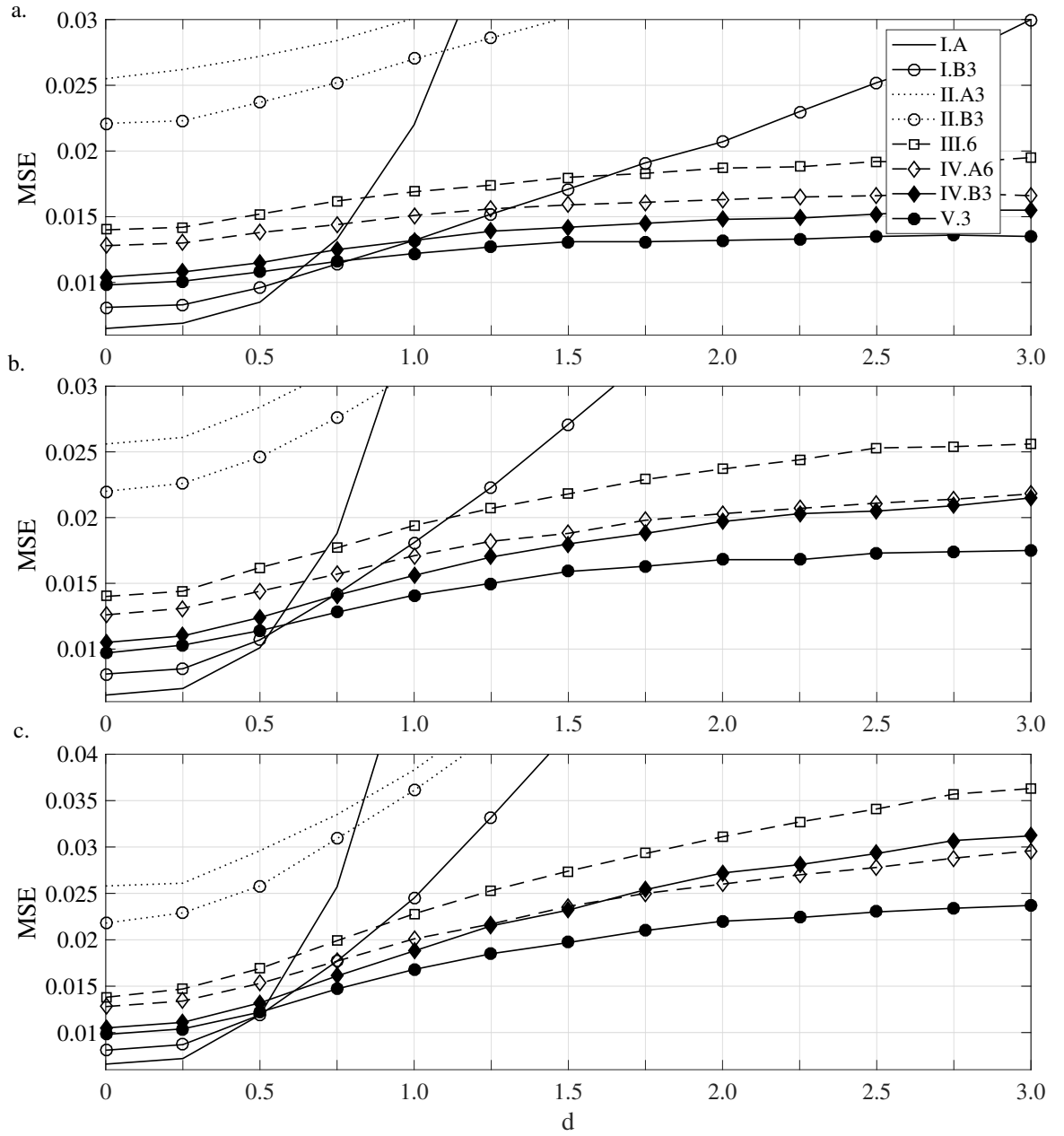


Figure B.6. Phase I efficiency of the proposed estimators for  $m = 20$  under CM 2 at  
a.  $K = 0.10$  b.  $K = 0.15$  c.  $K = 20$ .

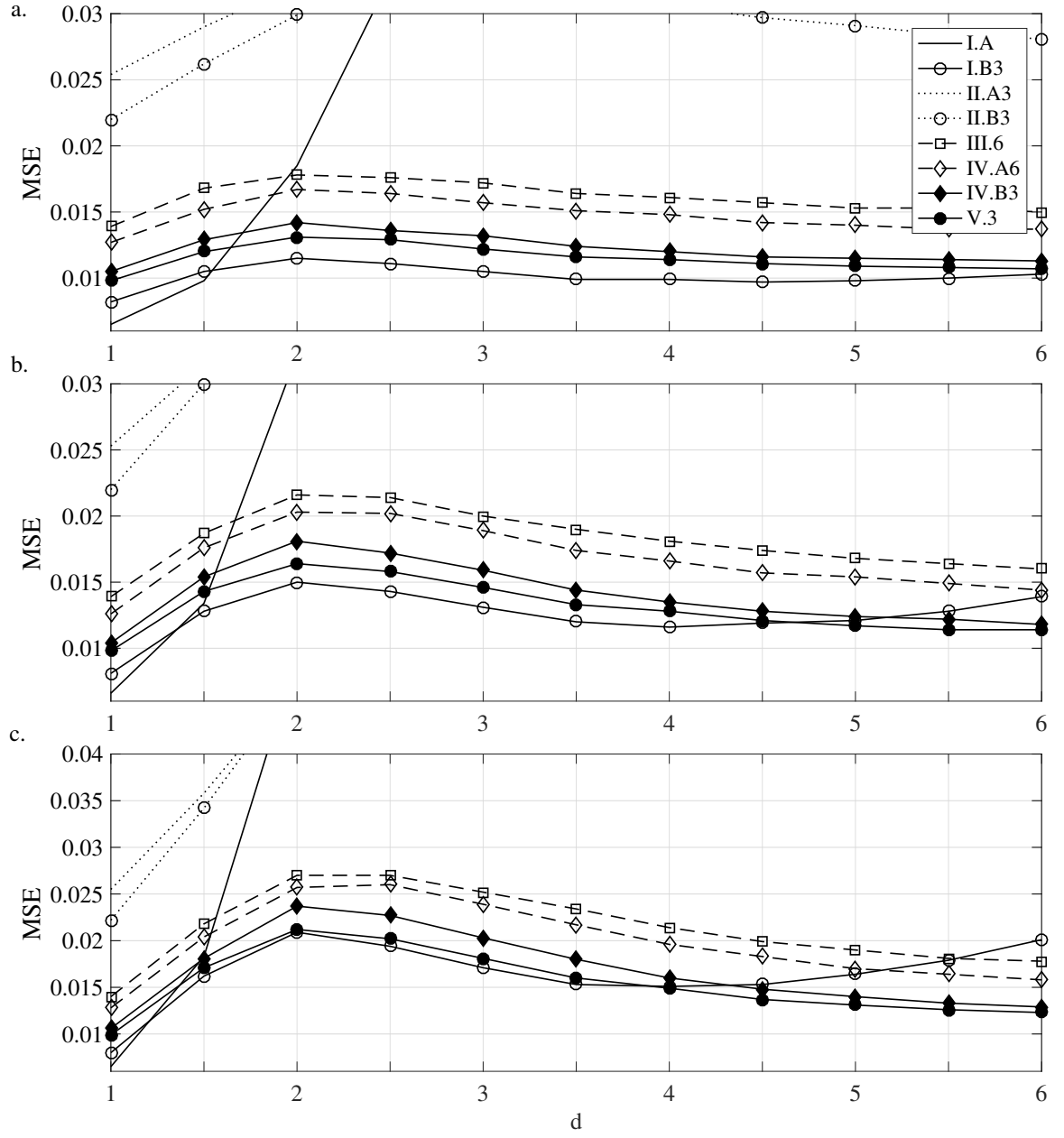


Figure B.7. Phase I efficiency of the proposed estimators for  $m = 20$  under CM 3 at  
a.  $K = 0.10$  b.  $K = 0.15$  c.  $K = 20$ .

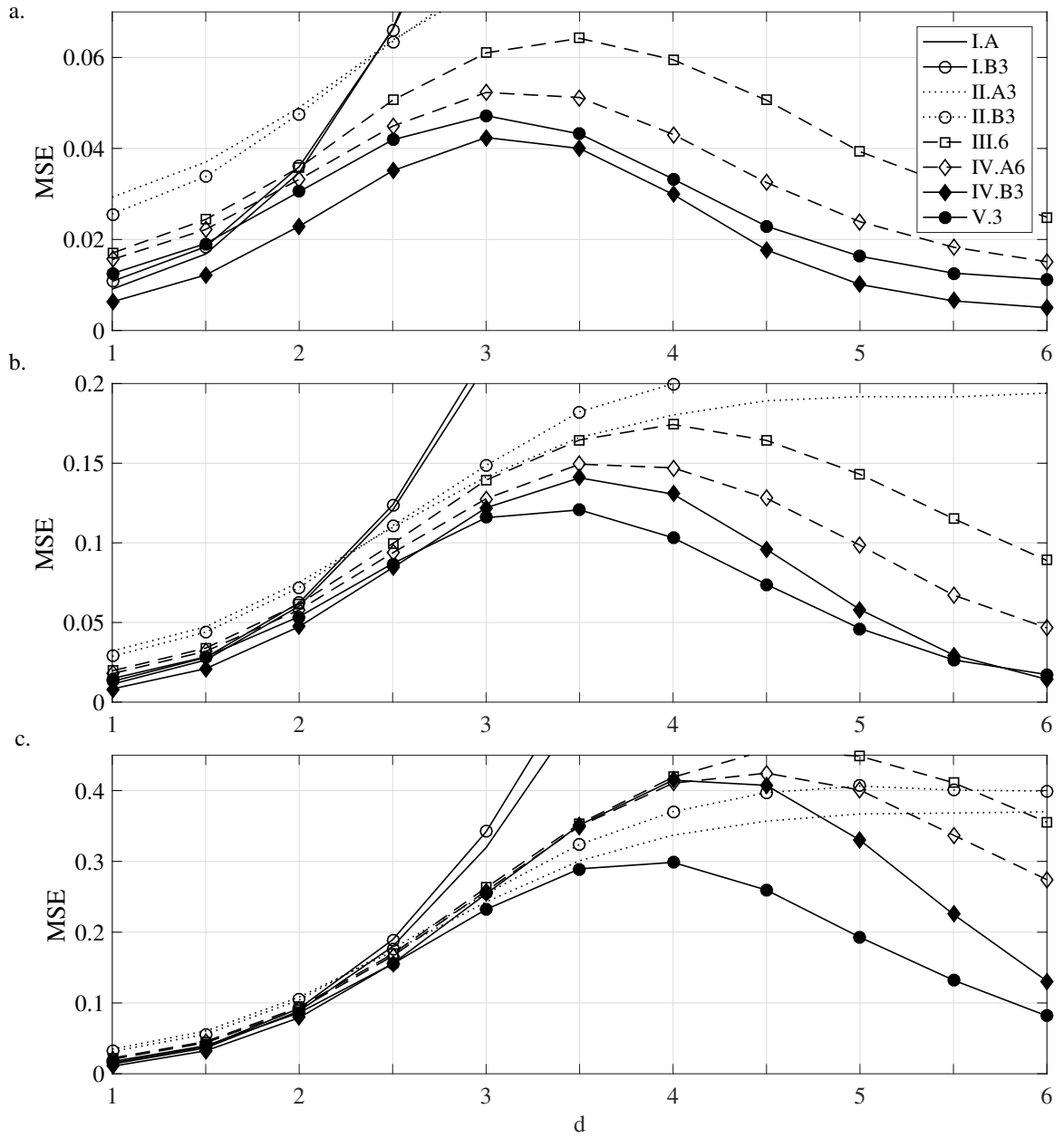


Figure B.8. Phase I efficiency of the proposed estimators for  $m = 20$  under CM 4 at  
a.  $K = 0.10$  b.  $K = 0.15$  c.  $K = 20$ .

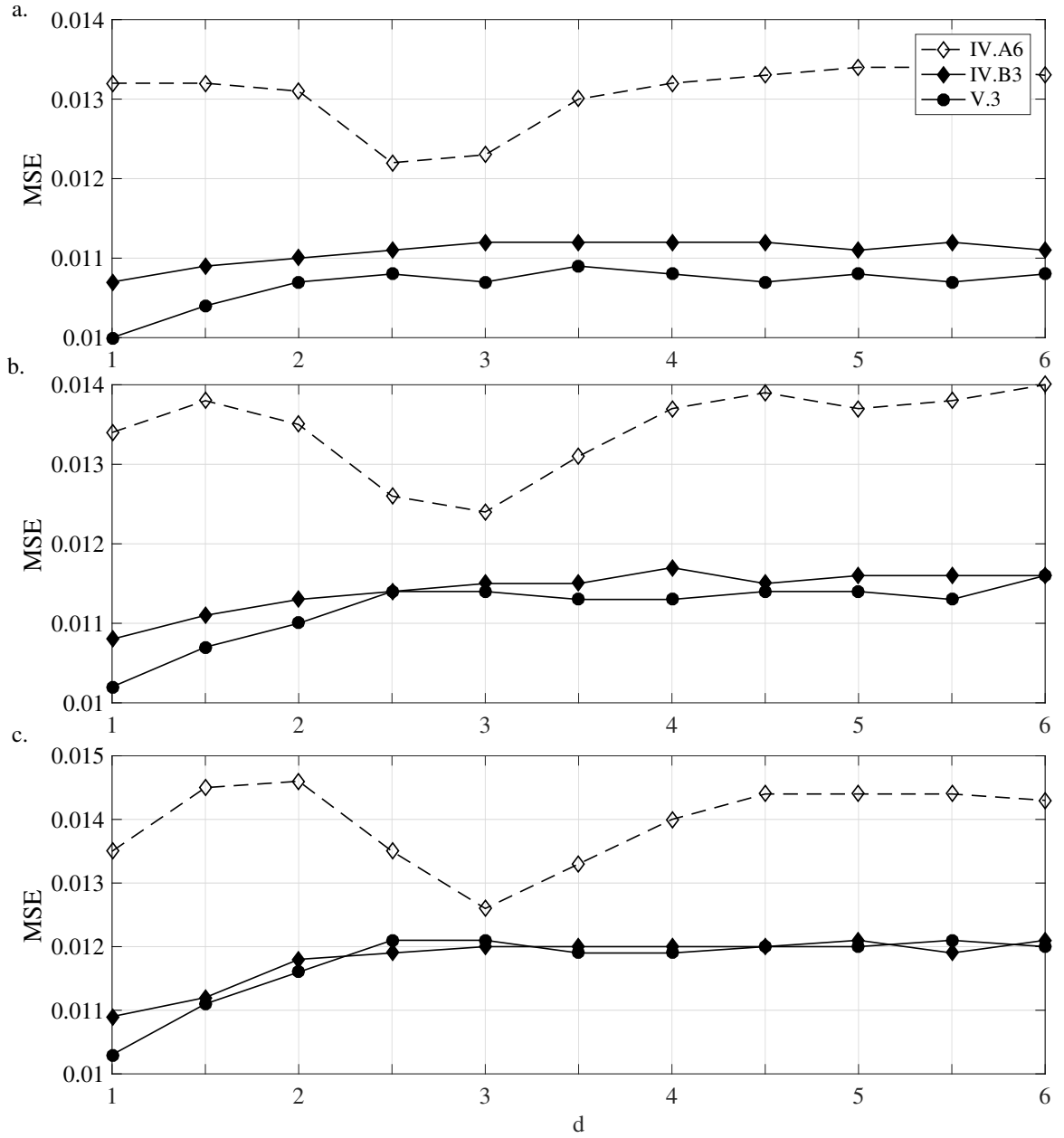


Figure B.9. Phase I efficiency of the proposed estimators for  $m = 20$  under CM 5 at  
a.  $K = 0.10$  b.  $K = 0.15$  c.  $K = 20$ .

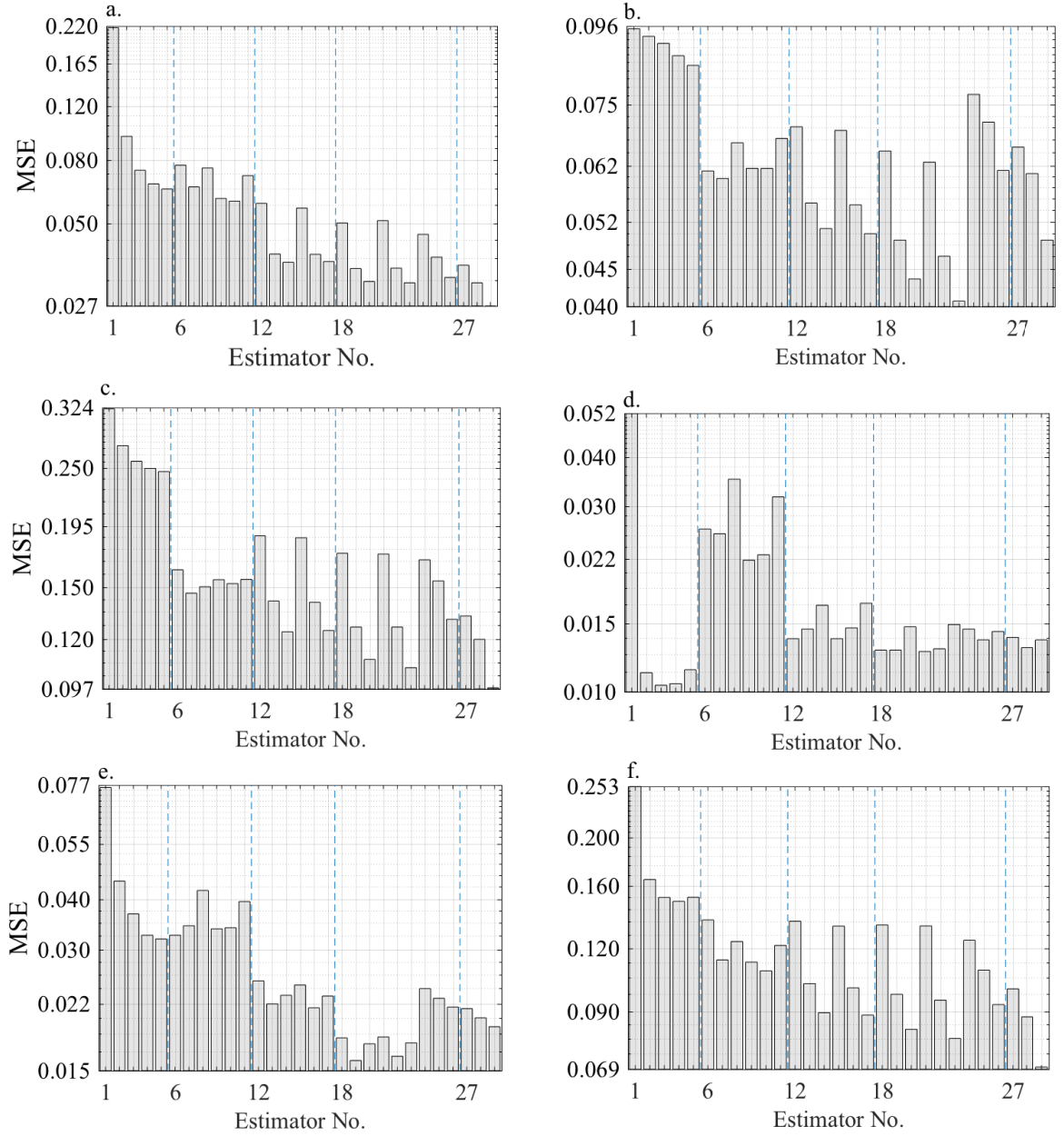


Figure B.10. Phase I efficiency of the proposed estimators for  $m = 20$  under a. SCM 1 b. SCM 2 c. SCM 3 d. SCM 4 e. SCM 5 f. SCM 6.

## APPENDIX C: PHASE II PERFORMANCES OF S-CHARTS FOR $m=50$

Phase II performances of S-Charts using proposed estimation methods (Method I-V) are shown for  $m = 50$  under normal distribution and in the presence of contaminations with various outlier densities.



Table C.1. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using normally distributed Phase I data.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0027 | 399.0    | 419.0  | 258.2       | 430.7       | 491.5       |
| <b>I.B1</b>  | 0.0027 | 400.1    | 420.7  | 257.2       | 431.0       | 495.1       |
| <b>I.B2</b>  | 0.0027 | 408.9    | 430.5  | 260.1       | 441.9       | 503.3       |
| <b>I.B3</b>  | 0.0027 | 412.4    | 435.7  | 256.4       | 452.5       | 504.5       |
| <b>I.B4</b>  | 0.0027 | 411.9    | 435.4  | 254.4       | 455.4       | 497.5       |
| <b>II.A1</b> | 0.0027 | 438.2    | 468.3  | 245.1       | 514.1       | 500.6       |
| <b>II.A2</b> | 0.0027 | 463.0    | 499.8  | 243.8       | 572.2       | 489.6       |
| <b>II.A3</b> | 0.0027 | 483.5    | 527.1  | 246.3       | 628.8       | 464.9       |
| <b>II.B1</b> | 0.0027 | 440.1    | 472.2  | 240.8       | 523.7       | 497.8       |
| <b>II.B2</b> | 0.0027 | 452.2    | 487.3  | 239.9       | 538.6       | 514.6       |
| <b>II.B3</b> | 0.0027 | 479.1    | 519.2  | 245.9       | 604.4       | 493.9       |
| <b>III.1</b> | 0.0027 | 415.1    | 437.5  | 259.4       | 453.7       | 506.3       |
| <b>III.2</b> | 0.0027 | 420.4    | 446.9  | 246.8       | 476.2       | 500.4       |
| <b>III.3</b> | 0.0027 | 441.3    | 471.9  | 246.7       | 510.6       | 515.7       |
| <b>III.4</b> | 0.0027 | 410.8    | 434.2  | 253.4       | 447.5       | 504.6       |
| <b>III.5</b> | 0.0027 | 423.7    | 450.4  | 251.0       | 478.5       | 501.5       |
| <b>III.6</b> | 0.0027 | 441.8    | 471.3  | 252.3       | 512.1       | 506.3       |
| <b>IV.A1</b> | 0.0027 | 407.1    | 429.3  | 254.3       | 445.0       | 496.3       |
| <b>IV.A2</b> | 0.0027 | 423.9    | 450.0  | 252.1       | 475.5       | 509.8       |
| <b>IV.A3</b> | 0.0027 | 435.4    | 464.2  | 249.7       | 499.0       | 509.3       |
| <b>IV.A4</b> | 0.0027 | 412.9    | 435.7  | 258.6       | 452.2       | 503.7       |
| <b>IV.A5</b> | 0.0027 | 425.3    | 451.6  | 253.0       | 477.7       | 509.5       |
| <b>IV.A6</b> | 0.0027 | 435.6    | 464.0  | 250.7       | 500.6       | 504.6       |
| <b>IV.B1</b> | 0.0027 | 409.6    | 432.6  | 256.0       | 449.4       | 502.2       |
| <b>IV.B2</b> | 0.0027 | 416.5    | 440.8  | 254.2       | 462.2       | 503.9       |
| <b>IV.B3</b> | 0.0027 | 424.7    | 450.0  | 258.6       | 479.4       | 500.1       |
| <b>V.1</b>   | 0.0027 | 414.9    | 437.1  | 261.8       | 452.4       | 506.6       |
| <b>V.2</b>   | 0.0027 | 416.4    | 440.6  | 255.8       | 458.4       | 507.9       |
| <b>V.3</b>   | 0.0027 | 420.3    | 444.7  | 255.6       | 472.5       | 502.1       |

Table C.2. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using normally distributed Phase I data.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0885 | 12.6   | 13.7   | 7.8         | 11.8        | 18.4        |
| <b>I.B1</b>  | 0.0881 | 12.8   | 14.0   | 7.8         | 11.9        | 18.8        |
| <b>I.B2</b>  | 0.0862 | 13.2   | 14.7   | 7.9         | 12.2        | 19.8        |
| <b>I.B3</b>  | 0.0864 | 13.3   | 14.9   | 7.7         | 12.3        | 20.2        |
| <b>I.B4</b>  | 0.0851 | 13.8   | 15.8   | 7.7         | 12.6        | 21.5        |
| <b>II.A1</b> | 0.0786 | 16.9   | 22.4   | 7.5         | 14.1        | 29.3        |
| <b>II.A2</b> | 0.0738 | 21.0   | 33.4   | 7.4         | 15.7        | 39.5        |
| <b>II.A3</b> | 0.0674 | 29.2   | 58.5   | 7.4         | 18.4        | 59.4        |
| <b>II.B1</b> | 0.0770 | 17.9   | 24.6   | 7.5         | 14.6        | 31.6        |
| <b>II.B2</b> | 0.0768 | 18.1   | 24.9   | 7.4         | 14.8        | 32.4        |
| <b>II.B3</b> | 0.0709 | 23.1   | 37.5   | 7.5         | 17.0        | 44.5        |
| <b>III.1</b> | 0.0850 | 13.6   | 15.3   | 7.8         | 12.5        | 20.7        |
| <b>III.2</b> | 0.0823 | 14.9   | 17.7   | 7.6         | 13.3        | 24.0        |
| <b>III.3</b> | 0.0797 | 16.2   | 20.3   | 7.5         | 14.0        | 27.5        |
| <b>III.4</b> | 0.0866 | 13.3   | 14.9   | 7.7         | 12.2        | 20.1        |
| <b>III.5</b> | 0.0821 | 15.0   | 18.0   | 7.6         | 13.3        | 24.5        |
| <b>III.6</b> | 0.0782 | 16.5   | 20.6   | 7.6         | 14.3        | 27.8        |
| <b>IV.A1</b> | 0.0864 | 13.3   | 14.8   | 7.7         | 12.2        | 20.1        |
| <b>IV.A2</b> | 0.0833 | 14.5   | 17.0   | 7.6         | 13.0        | 22.9        |
| <b>IV.A3</b> | 0.0803 | 15.7   | 19.1   | 7.6         | 13.8        | 26.0        |
| <b>IV.A4</b> | 0.0858 | 13.4   | 14.9   | 7.8         | 12.3        | 20.3        |
| <b>IV.A5</b> | 0.0829 | 14.5   | 17.0   | 7.7         | 13.0        | 23.2        |
| <b>IV.A6</b> | 0.0803 | 15.8   | 19.3   | 7.5         | 13.8        | 26.4        |
| <b>IV.B1</b> | 0.0863 | 13.3   | 14.9   | 7.7         | 12.3        | 20.2        |
| <b>IV.B2</b> | 0.0846 | 13.9   | 15.8   | 7.8         | 12.6        | 21.4        |
| <b>IV.B3</b> | 0.0821 | 14.7   | 17.3   | 7.8         | 13.1        | 23.4        |
| <b>V.1</b>   | 0.0862 | 13.3   | 14.9   | 7.8         | 12.2        | 20.2        |
| <b>V.2</b>   | 0.0853 | 13.7   | 15.5   | 7.7         | 12.5        | 21.0        |
| <b>V.3</b>   | 0.0828 | 14.4   | 16.7   | 7.8         | 13.0        | 22.7        |

Table C.3. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0023 | 439.2    | 445.2  | 485.0       | 462.7       | 364.9       |
| <b>I.B1</b>  | 0.0023 | 442.0    | 448.2  | 460.7       | 474.5       | 375.1       |
| <b>I.B2</b>  | 0.0023 | 443.7    | 450.2  | 445.4       | 484.4       | 386.8       |
| <b>I.B3</b>  | 0.0023 | 448.0    | 456.0  | 438.7       | 492.5       | 392.1       |
| <b>I.B4</b>  | 0.0023 | 457.0    | 466.1  | 436.9       | 507.4       | 401.7       |
| <b>II.A1</b> | 0.0023 | 465.9    | 478.8  | 447.7       | 525.9       | 387.5       |
| <b>II.A2</b> | 0.0023 | 479.4    | 499.7  | 441.0       | 557.2       | 381.9       |
| <b>II.A3</b> | 0.0023 | 490.5    | 521.1  | 466.3       | 574.7       | 355.2       |
| <b>II.B1</b> | 0.0023 | 458.6    | 474.1  | 433.4       | 523.4       | 375.4       |
| <b>II.B2</b> | 0.0023 | 465.7    | 482.5  | 438.1       | 536.3       | 381.4       |
| <b>II.B3</b> | 0.0023 | 485.1    | 509.4  | 459.1       | 566.1       | 375.3       |
| <b>III.1</b> | 0.0023 | 451.5    | 459.0  | 426.0       | 497.8       | 406.5       |
| <b>III.2</b> | 0.0023 | 449.4    | 461.0  | 390.5       | 509.3       | 409.6       |
| <b>III.3</b> | 0.0023 | 470.0    | 485.6  | 396.7       | 543.9       | 417.2       |
| <b>III.4</b> | 0.0023 | 447.4    | 454.7  | 422.5       | 496.0       | 401.9       |
| <b>III.5</b> | 0.0023 | 454.5    | 466.0  | 394.4       | 515.0       | 414.8       |
| <b>III.6</b> | 0.0023 | 462.2    | 476.4  | 399.9       | 529.7       | 411.0       |
| <b>IV.A1</b> | 0.0023 | 455.3    | 463.5  | 404.1       | 506.8       | 420.5       |
| <b>IV.A2</b> | 0.0023 | 450.9    | 462.1  | 379.4       | 508.8       | 423.6       |
| <b>IV.A3</b> | 0.0023 | 456.1    | 469.8  | 373.9       | 524.4       | 425.0       |
| <b>IV.A4</b> | 0.0023 | 449.3    | 457.2  | 407.8       | 500.7       | 410.1       |
| <b>IV.A5</b> | 0.0023 | 454.9    | 466.4  | 382.4       | 515.6       | 428.0       |
| <b>IV.A6</b> | 0.0023 | 468.0    | 483.6  | 379.5       | 539.1       | 431.0       |
| <b>IV.B1</b> | 0.0023 | 450.7    | 459.1  | 401.7       | 500.0       | 418.2       |
| <b>IV.B2</b> | 0.0023 | 456.7    | 466.8  | 395.8       | 514.6       | 425.7       |
| <b>IV.B3</b> | 0.0023 | 457.9    | 470.2  | 381.8       | 522.5       | 426.6       |
| <b>V.1</b>   | 0.0022 | 460.3    | 468.6  | 406.0       | 511.8       | 434.4       |
| <b>V.2</b>   | 0.0023 | 451.5    | 460.7  | 386.9       | 507.6       | 425.4       |
| <b>V.3</b>   | 0.0023 | 459.2    | 470.7  | 383.6       | 519.6       | 433.6       |

Table C.4. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0404 | 31.8   | 40.6   | 15.0        | 27.1        | 53.6        |
| <b>I.B1</b>  | 0.0453 | 28.6   | 36.8   | 13.3        | 24.3        | 48.7        |
| <b>I.B2</b>  | 0.0479 | 27.4   | 36.1   | 12.5        | 23.1        | 47.0        |
| <b>I.B3</b>  | 0.0501 | 26.6   | 35.3   | 11.8        | 22.3        | 46.2        |
| <b>I.B4</b>  | 0.0503 | 27.0   | 36.5   | 11.5        | 22.4        | 47.4        |
| <b>II.A1</b> | 0.0469 | 32.2   | 48.6   | 11.6        | 24.7        | 60.5        |
| <b>II.A2</b> | 0.0461 | 39.3   | 68.2   | 10.9        | 27.0        | 80.3        |
| <b>II.A3</b> | 0.0415 | 57.4   | 115.8  | 11.1        | 33.1        | 128.5       |
| <b>II.B1</b> | 0.0474 | 33.9   | 53.1   | 11.1        | 25.3        | 65.9        |
| <b>II.B2</b> | 0.0477 | 34.3   | 54.4   | 11.1        | 25.4        | 66.8        |
| <b>II.B3</b> | 0.0442 | 44.9   | 81.9   | 11.0        | 29.5        | 93.9        |
| <b>III.1</b> | 0.0521 | 24.8   | 31.8   | 11.5        | 21.3        | 42.1        |
| <b>III.2</b> | 0.0547 | 25.2   | 34.2   | 10.4        | 20.9        | 44.5        |
| <b>III.3</b> | 0.0538 | 27.1   | 38.3   | 10.3        | 21.8        | 49.9        |
| <b>III.4</b> | 0.0524 | 24.7   | 31.6   | 11.5        | 21.1        | 41.6        |
| <b>III.5</b> | 0.0548 | 25.1   | 33.8   | 10.4        | 20.8        | 44.4        |
| <b>III.6</b> | 0.0533 | 27.4   | 38.6   | 10.3        | 22.0        | 50.6        |
| <b>IV.A1</b> | 0.0557 | 22.9   | 28.8   | 10.9        | 19.7        | 38.5        |
| <b>IV.A2</b> | 0.0579 | 22.9   | 30.1   | 10.2        | 19.4        | 39.3        |
| <b>IV.A3</b> | 0.0572 | 24.1   | 32.4   | 10.0        | 20.0        | 42.6        |
| <b>IV.A4</b> | 0.0550 | 23.2   | 29.5   | 11.0        | 19.8        | 38.9        |
| <b>IV.A5</b> | 0.0575 | 23.1   | 30.2   | 10.2        | 19.5        | 39.8        |
| <b>IV.A6</b> | 0.0573 | 24.4   | 33.4   | 9.9         | 20.1        | 43.5        |
| <b>IV.B1</b> | 0.0560 | 22.8   | 28.9   | 10.8        | 19.6        | 38.4        |
| <b>IV.B2</b> | 0.0569 | 22.8   | 29.3   | 10.5        | 19.6        | 38.7        |
| <b>IV.B3</b> | 0.0573 | 23.6   | 31.4   | 10.2        | 19.8        | 41.1        |
| <b>V.1</b>   | 0.0572 | 21.9   | 27.3   | 10.8        | 19.1        | 36.1        |
| <b>V.2</b>   | 0.0579 | 22.0   | 27.7   | 10.4        | 19.0        | 37.1        |
| <b>V.3</b>   | 0.0586 | 22.3   | 28.8   | 10.1        | 19.1        | 38.1        |

Table C.5. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0048 | 225.2    | 244.2  | 317.6       | 215.3       | 147.6       |
| <b>I.B1</b>  | 0.0037 | 301.8    | 328.6  | 443.9       | 291.4       | 186.8       |
| <b>I.B2</b>  | 0.0033 | 331.8    | 357.6  | 478.5       | 328.6       | 206.6       |
| <b>I.B3</b>  | 0.0031 | 354.0    | 378.3  | 497.5       | 359.0       | 225.0       |
| <b>I.B4</b>  | 0.0030 | 368.5    | 393.6  | 514.4       | 374.9       | 233.4       |
| <b>II.A1</b> | 0.0028 | 382.0    | 405.4  | 533.8       | 381.8       | 253.3       |
| <b>II.A2</b> | 0.0026 | 412.8    | 441.2  | 585.5       | 418.5       | 264.6       |
| <b>II.A3</b> | 0.0027 | 421.3    | 460.0  | 632.4       | 424.2       | 249.6       |
| <b>II.B1</b> | 0.0028 | 389.6    | 414.5  | 546.0       | 393.1       | 256.8       |
| <b>II.B2</b> | 0.0027 | 398.8    | 424.0  | 556.3       | 404.3       | 261.7       |
| <b>II.B3</b> | 0.0026 | 421.5    | 454.4  | 609.8       | 431.3       | 262.7       |
| <b>III.1</b> | 0.0027 | 392.0    | 408.4  | 504.1       | 404.7       | 277.0       |
| <b>III.2</b> | 0.0025 | 418.4    | 432.9  | 493.5       | 447.8       | 312.1       |
| <b>III.3</b> | 0.0024 | 442.6    | 459.9  | 509.8       | 481.0       | 326.9       |
| <b>III.4</b> | 0.0027 | 387.9    | 404.1  | 499.0       | 399.2       | 275.3       |
| <b>III.5</b> | 0.0025 | 423.7    | 438.2  | 487.9       | 456.5       | 317.6       |
| <b>III.6</b> | 0.0024 | 434.7    | 451.1  | 498.4       | 472.7       | 323.8       |
| <b>IV.A1</b> | 0.0025 | 414.8    | 428.7  | 498.2       | 438.8       | 304.7       |
| <b>IV.A2</b> | 0.0024 | 432.4    | 444.3  | 467.9       | 474.7       | 337.4       |
| <b>IV.A3</b> | 0.0024 | 441.7    | 454.8  | 468.1       | 487.5       | 350.9       |
| <b>IV.A4</b> | 0.0026 | 406.0    | 420.0  | 494.4       | 427.5       | 299.2       |
| <b>IV.A5</b> | 0.0024 | 436.1    | 448.3  | 470.7       | 476.5       | 343.3       |
| <b>IV.A6</b> | 0.0023 | 453.2    | 468.2  | 474.8       | 507.9       | 356.7       |
| <b>IV.B1</b> | 0.0025 | 416.8    | 430.0  | 490.7       | 443.9       | 310.1       |
| <b>IV.B2</b> | 0.0025 | 419.3    | 432.2  | 478.7       | 451.2       | 319.9       |
| <b>IV.B3</b> | 0.0024 | 435.0    | 448.4  | 474.2       | 479.2       | 336.6       |
| <b>V.1</b>   | 0.0024 | 434.6    | 445.4  | 488.6       | 467.2       | 340.7       |
| <b>V.2</b>   | 0.0024 | 431.9    | 442.6  | 465.3       | 470.8       | 343.1       |
| <b>V.3</b>   | 0.0023 | 445.2    | 456.8  | 458.3       | 492.7       | 361.7       |

Table C.6. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0063 | 246.5  | 312.7  | 78.3        | 224.6       | 461.6       |
| <b>I.B1</b>  | 0.015  | 136.0  | 211.3  | 30.6        | 95.6        | 317.0       |
| <b>I.B2</b>  | 0.0193 | 109.1  | 180.5  | 24.2        | 72.2        | 254.7       |
| <b>I.B3</b>  | 0.0226 | 93.1   | 160.8  | 21.2        | 59.2        | 213.7       |
| <b>I.B4</b>  | 0.0232 | 90.4   | 157.8  | 20.3        | 57.2        | 205.8       |
| <b>II.A1</b> | 0.0209 | 90.1   | 146.0  | 23.3        | 63.0        | 194.4       |
| <b>II.A2</b> | 0.0228 | 97.6   | 170.2  | 20.2        | 62.4        | 222.7       |
| <b>II.A3</b> | 0.0213 | 131.4  | 238.6  | 20.1        | 75.7        | 337.6       |
| <b>II.B1</b> | 0.0238 | 87.2   | 147.8  | 19.9        | 58.3        | 192.0       |
| <b>II.B2</b> | 0.0245 | 87.2   | 150.2  | 19.1        | 57.0        | 195.9       |
| <b>II.B3</b> | 0.0231 | 111.7  | 201.2  | 19.2        | 67.6        | 268.4       |
| <b>III.1</b> | 0.0288 | 59.4   | 96.8   | 17.8        | 42.6        | 119.2       |
| <b>III.2</b> | 0.036  | 47.2   | 77.1   | 14.4        | 34.2        | 93.0        |
| <b>III.3</b> | 0.0368 | 47.8   | 78.7   | 13.8        | 33.9        | 95.6        |
| <b>III.4</b> | 0.0288 | 59.0   | 95.2   | 17.9        | 42.4        | 118.0       |
| <b>III.5</b> | 0.0365 | 46.6   | 75.6   | 14.1        | 34.0        | 91.9        |
| <b>III.6</b> | 0.0372 | 47.8   | 79.1   | 13.5        | 34.0        | 96.7        |
| <b>IV.A1</b> | 0.0346 | 46.5   | 73.7   | 15.3        | 34.3        | 90.9        |
| <b>IV.A2</b> | 0.0413 | 37.8   | 58.8   | 13.0        | 28.5        | 71.7        |
| <b>IV.A3</b> | 0.0421 | 37.7   | 58.9   | 12.7        | 28.4        | 71.7        |
| <b>IV.A4</b> | 0.0342 | 47.8   | 76.3   | 15.5        | 34.8        | 94.4        |
| <b>IV.A5</b> | 0.0413 | 37.9   | 58.6   | 13.0        | 28.7        | 71.8        |
| <b>IV.A6</b> | 0.0431 | 37.6   | 59.3   | 12.3        | 27.8        | 73.3        |
| <b>IV.B1</b> | 0.0358 | 45.2   | 71.7   | 14.8        | 33.1        | 87.9        |
| <b>IV.B2</b> | 0.0385 | 41.8   | 66.5   | 13.9        | 30.9        | 81.3        |
| <b>IV.B3</b> | 0.041  | 39.7   | 63.8   | 12.9        | 29.3        | 76.8        |
| <b>V.1</b>   | 0.0395 | 37.4   | 55.4   | 14.1        | 29.1        | 68.8        |
| <b>V.2</b>   | 0.0422 | 35.3   | 52.1   | 13.1        | 27.4        | 66.3        |
| <b>V.3</b>   | 0.0450 | 33.5   | 50.3   | 12.3        | 25.9        | 62.7        |

Table C.7. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0067 | 163.8    | 178.0  | 229.4       | 155.8       | 107.2       |
| <b>I.B1</b>  | 0.0052 | 220.2    | 248.2  | 334.5       | 205.2       | 129.9       |
| <b>I.B2</b>  | 0.0046 | 249.8    | 281.6  | 387.8       | 234.1       | 142.6       |
| <b>I.B3</b>  | 0.0042 | 273.6    | 306.9  | 423.0       | 259.0       | 154.2       |
| <b>I.B4</b>  | 0.004  | 286.9    | 321.9  | 446.8       | 272.5       | 160.1       |
| <b>II.A1</b> | 0.0035 | 312.5    | 340.3  | 457.3       | 300.2       | 196.3       |
| <b>II.A2</b> | 0.0032 | 350.2    | 385.3  | 531.9       | 336.5       | 208.8       |
| <b>II.A3</b> | 0.0032 | 358.2    | 402.8  | 571.2       | 340.3       | 198.1       |
| <b>II.B1</b> | 0.0034 | 327.9    | 359.0  | 494.4       | 314.0       | 201.3       |
| <b>II.B2</b> | 0.0033 | 339.9    | 371.6  | 515.7       | 328.1       | 208.4       |
| <b>II.B3</b> | 0.0032 | 362.0    | 401.6  | 566.7       | 350.6       | 208.4       |
| <b>III.1</b> | 0.0034 | 324.9    | 349.9  | 467.6       | 318.0       | 206.3       |
| <b>III.2</b> | 0.0029 | 370.0    | 392.2  | 511.4       | 375.9       | 243.0       |
| <b>III.3</b> | 0.0027 | 397.7    | 422.4  | 548.1       | 404.2       | 262.7       |
| <b>III.4</b> | 0.0034 | 321.2    | 345.6  | 463.0       | 314.3       | 205.4       |
| <b>III.5</b> | 0.0029 | 376.4    | 398.5  | 517.9       | 379.9       | 251.4       |
| <b>III.6</b> | 0.0028 | 392.6    | 416.0  | 535.3       | 402.2       | 260.6       |
| <b>IV.A1</b> | 0.0031 | 354.0    | 377.1  | 496.4       | 351.5       | 231.0       |
| <b>IV.A2</b> | 0.0027 | 392.8    | 411.2  | 511.9       | 407.0       | 272.4       |
| <b>IV.A3</b> | 0.0026 | 407.7    | 426.2  | 520.0       | 427.9       | 284.8       |
| <b>IV.A4</b> | 0.0031 | 347.8    | 370.2  | 487.8       | 347.0       | 227.6       |
| <b>IV.A5</b> | 0.0027 | 397.6    | 416.2  | 515.1       | 413.2       | 275.2       |
| <b>IV.A6</b> | 0.0025 | 420.1    | 440.2  | 532.5       | 443.9       | 291.3       |
| <b>IV.B1</b> | 0.0030 | 356.6    | 378.6  | 495.5       | 358.8       | 234.2       |
| <b>IV.B2</b> | 0.0028 | 381.3    | 402.7  | 513.2       | 388.1       | 254.7       |
| <b>IV.B3</b> | 0.0027 | 393.9    | 413.9  | 514.9       | 410.4       | 268.5       |
| <b>V.1</b>   | 0.0027 | 389.1    | 406.8  | 511.8       | 395.9       | 272.6       |
| <b>V.2</b>   | 0.0027 | 394.2    | 410.7  | 506.1       | 407.0       | 279.6       |
| <b>V.3</b>   | 0.0025 | 414.8    | 431.1  | 511.7       | 437.9       | 300.6       |

Table C.8. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0032 | 376.3  | 410.1  | 189.0       | 430.1       | 468.3       |
| <b>I.B1</b>  | 0.0069 | 266.5  | 344.3  | 67.2        | 250.5       | 502.0       |
| <b>I.B2</b>  | 0.0094 | 223.9  | 312.8  | 46.6        | 183.8       | 488.6       |
| <b>I.B3</b>  | 0.0115 | 196.3  | 290.1  | 37.8        | 147.1       | 467.4       |
| <b>I.B4</b>  | 0.0126 | 183.0  | 277.6  | 34.4        | 130.4       | 447.5       |
| <b>II.A1</b> | 0.0120 | 163.0  | 244.0  | 38.7        | 121.0       | 363.4       |
| <b>II.A2</b> | 0.0137 | 167.9  | 264.9  | 31.9        | 117.7       | 402.8       |
| <b>II.A3</b> | 0.0133 | 205.8  | 326.2  | 31.6        | 139.2       | 523.6       |
| <b>II.B1</b> | 0.0143 | 153.3  | 239.7  | 31.6        | 107.7       | 360.1       |
| <b>II.B2</b> | 0.0151 | 151.3  | 241.9  | 29.7        | 103.5       | 361.0       |
| <b>II.B3</b> | 0.0146 | 181.4  | 293.1  | 28.9        | 120.0       | 457.7       |
| <b>III.1</b> | 0.0168 | 118.2  | 188.3  | 28.0        | 81.6        | 267.6       |
| <b>III.2</b> | 0.0238 | 83.2   | 139.3  | 20.1        | 56.4        | 182.5       |
| <b>III.3</b> | 0.0253 | 81.8   | 142.1  | 18.8        | 53.4        | 180.9       |
| <b>III.4</b> | 0.0173 | 115.3  | 184.3  | 27.2        | 79.7        | 262.3       |
| <b>III.5</b> | 0.0243 | 82.4   | 139    | 19.8        | 55.5        | 180.5       |
| <b>III.6</b> | 0.0261 | 79.8   | 137.3  | 18.2        | 52.9        | 177.6       |
| <b>IV.A1</b> | 0.0212 | 90.2   | 147.5  | 22.8        | 62.1        | 195.6       |
| <b>IV.A2</b> | 0.0289 | 63.2   | 106.7  | 17.3        | 43.7        | 131.5       |
| <b>IV.A3</b> | 0.0307 | 60.7   | 103.3  | 16.1        | 41.7        | 126.7       |
| <b>IV.A4</b> | 0.0212 | 90.9   | 148.8  | 23.0        | 62.5        | 198.6       |
| <b>IV.A5</b> | 0.0290 | 63.7   | 107.7  | 17.2        | 43.9        | 132.2       |
| <b>IV.A6</b> | 0.0315 | 60.0   | 102.7  | 15.6        | 41.0        | 124.9       |
| <b>IV.B1</b> | 0.0225 | 86.0   | 142.9  | 21.6        | 58.4        | 188.2       |
| <b>IV.B2</b> | 0.0257 | 75.0   | 127.5  | 19.0        | 50.5        | 160.7       |
| <b>IV.B3</b> | 0.0291 | 67.2   | 116.2  | 16.8        | 44.7        | 144.1       |
| <b>V.1</b>   | 0.0269 | 63.5   | 101.4  | 19.0        | 45.8        | 127.8       |
| <b>V.2</b>   | 0.0301 | 57.1   | 92.7   | 17.1        | 40.7        | 115.4       |
| <b>V.3</b>   | 0.0331 | 52.2   | 85.7   | 15.5        | 37.3        | 104.4       |

Table C.9. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=0.75$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0023 | 450.2    | 457.3  | 388.9       | 496.7       | 438.3       |
| <b>I.B1</b>  | 0.0023 | 450.9    | 460.5  | 359.0       | 499.5       | 467.1       |
| <b>I.B2</b>  | 0.0024 | 442.6    | 453.3  | 342.0       | 492.5       | 463.6       |
| <b>I.B3</b>  | 0.0024 | 448.2    | 461.6  | 334.1       | 503.9       | 469.7       |
| <b>I.B4</b>  | 0.0024 | 451.2    | 466.0  | 334.1       | 510.0       | 470.0       |
| <b>II.A1</b> | 0.0023 | 463.7    | 482.2  | 336.2       | 542.2       | 453.4       |
| <b>II.A2</b> | 0.0024 | 474.3    | 499.7  | 331.2       | 578.0       | 436.1       |
| <b>II.A3</b> | 0.0024 | 489.8    | 522.8  | 342.7       | 613.5       | 409.0       |
| <b>II.B1</b> | 0.0024 | 456.9    | 477.4  | 326.3       | 542.5       | 437.6       |
| <b>II.B2</b> | 0.0024 | 466.8    | 489.6  | 324.8       | 558.3       | 446.7       |
| <b>II.B3</b> | 0.0024 | 485.6    | 514.7  | 335.2       | 598.7       | 430.7       |
| <b>III.1</b> | 0.0024 | 446.1    | 458.8  | 334.7       | 496.8       | 471.5       |
| <b>III.2</b> | 0.0025 | 444.6    | 462.6  | 309.9       | 510.1       | 473.0       |
| <b>III.3</b> | 0.0024 | 460.7    | 482.2  | 309.2       | 536.7       | 474.1       |
| <b>III.4</b> | 0.0024 | 441.0    | 454.0  | 326.9       | 492.4       | 467.5       |
| <b>III.5</b> | 0.0024 | 449.1    | 467.3  | 315.3       | 517.1       | 470.6       |
| <b>III.6</b> | 0.0025 | 456.9    | 477.7  | 311.4       | 533.8       | 468.8       |
| <b>IV.A1</b> | 0.0024 | 443.2    | 457.5  | 320.2       | 494.0       | 480.8       |
| <b>IV.A2</b> | 0.0025 | 442.6    | 459.6  | 311.6       | 504.3       | 468.3       |
| <b>IV.A3</b> | 0.0025 | 448.7    | 469.4  | 299.2       | 520.7       | 472.9       |
| <b>IV.A4</b> | 0.0024 | 441.2    | 454.9  | 324.0       | 494.0       | 473.4       |
| <b>IV.A5</b> | 0.0025 | 439.9    | 457.0  | 305.8       | 500.5       | 468.7       |
| <b>IV.A6</b> | 0.0025 | 451.0    | 472.3  | 298.5       | 524.1       | 475.4       |
| <b>IV.B1</b> | 0.0024 | 441.9    | 456.3  | 319.6       | 493.9       | 477.9       |
| <b>IV.B2</b> | 0.0024 | 440.6    | 456.4  | 312.9       | 495.5       | 475.6       |
| <b>IV.B3</b> | 0.0025 | 445.4    | 464.2  | 308.7       | 512.4       | 472.2       |
| <b>V.1</b>   | 0.0024 | 445.9    | 460.3  | 323.3       | 497.9       | 484.9       |
| <b>V.2</b>   | 0.0025 | 435.6    | 450.4  | 312.3       | 486.9       | 475.4       |
| <b>V.3</b>   | 0.0024 | 443.2    | 459.9  | 310.0       | 503.5       | 476.2       |

Table C.10. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=0.75$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0601 | 20.2   | 24.6   | 10.6        | 17.7        | 32.4        |
| <b>I.B1</b>  | 0.0672 | 17.8   | 21.1   | 9.6         | 15.9        | 28.0        |
| <b>I.B2</b>  | 0.0682 | 17.6   | 20.8   | 9.4         | 15.6        | 27.9        |
| <b>I.B3</b>  | 0.0691 | 17.5   | 21.0   | 9.2         | 15.6        | 28.3        |
| <b>I.B4</b>  | 0.0684 | 18.1   | 22.1   | 9.1         | 15.9        | 29.6        |
| <b>II.A1</b> | 0.0628 | 22.3   | 31.4   | 9.1         | 18.0        | 40.0        |
| <b>II.A2</b> | 0.0593 | 28.0   | 47.0   | 8.9         | 20.1        | 54.5        |
| <b>II.A3</b> | 0.0546 | 39.2   | 79.8   | 8.8         | 23.6        | 83.1        |
| <b>II.B1</b> | 0.0627 | 23.6   | 34.7   | 8.8         | 18.5        | 43.7        |
| <b>II.B2</b> | 0.0627 | 23.7   | 34.6   | 8.8         | 18.6        | 43.9        |
| <b>II.B3</b> | 0.0579 | 30.7   | 53.0   | 8.8         | 21.6        | 61.3        |
| <b>III.1</b> | 0.0696 | 17.3   | 20.3   | 9.2         | 15.4        | 27.6        |
| <b>III.2</b> | 0.0696 | 18.3   | 22.8   | 8.6         | 15.9        | 30.9        |
| <b>III.3</b> | 0.0679 | 19.9   | 26.3   | 8.5         | 16.8        | 34.9        |
| <b>III.4</b> | 0.0710 | 16.9   | 20.0   | 9.0         | 15.1        | 26.8        |
| <b>III.5</b> | 0.0693 | 18.5   | 23.3   | 8.6         | 16.1        | 31.2        |
| <b>III.6</b> | 0.0664 | 20.4   | 26.8   | 8.7         | 17.2        | 35.6        |
| <b>IV.A1</b> | 0.0724 | 16.4   | 19.2   | 8.9         | 14.7        | 25.9        |
| <b>IV.A2</b> | 0.0715 | 17.4   | 21.1   | 8.6         | 15.3        | 28.7        |
| <b>IV.A3</b> | 0.0698 | 18.7   | 23.9   | 8.5         | 16.0        | 31.8        |
| <b>IV.A4</b> | 0.0717 | 16.6   | 19.4   | 9.0         | 15.0        | 26.3        |
| <b>IV.A5</b> | 0.0709 | 17.6   | 21.3   | 8.7         | 15.4        | 28.9        |
| <b>IV.A6</b> | 0.0692 | 19.1   | 24.4   | 8.5         | 16.3        | 32.8        |
| <b>IV.B1</b> | 0.0723 | 16.5   | 19.3   | 8.9         | 14.8        | 25.9        |
| <b>IV.B2</b> | 0.072  | 17.0   | 20.3   | 8.7         | 15.1        | 27.4        |
| <b>IV.B3</b> | 0.0705 | 17.7   | 21.6   | 8.7         | 15.6        | 29.3        |
| <b>V.1</b>   | 0.0727 | 16.3   | 18.9   | 8.9         | 14.6        | 25.6        |
| <b>V.2</b>   | 0.073  | 16.5   | 19.5   | 8.8         | 14.6        | 26.2        |
| <b>V.3</b>   | 0.0715 | 17.3   | 20.9   | 8.7         | 15.3        | 28.1        |

Table C.11. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0025 | 418.4    | 428.4  | 490.2       | 440.6       | 323.0       |
| <b>I.B1</b>  | 0.0023 | 451.0    | 458.0  | 436.4       | 495.0       | 397.9       |
| <b>I.B2</b>  | 0.0023 | 456.7    | 464.4  | 427.2       | 503.7       | 413.2       |
| <b>I.B3</b>  | 0.0023 | 451.2    | 460.1  | 406.2       | 503.7       | 410.1       |
| <b>I.B4</b>  | 0.0023 | 452.4    | 461.9  | 405.5       | 505.5       | 413.2       |
| <b>II.A1</b> | 0.0022 | 475.1    | 488.5  | 439.6       | 538.3       | 401.7       |
| <b>II.A2</b> | 0.0023 | 470.9    | 490.6  | 424.9       | 549.2       | 379.9       |
| <b>II.A3</b> | 0.0023 | 486.2    | 516.1  | 435.8       | 581.8       | 361.7       |
| <b>II.B1</b> | 0.0023 | 461.3    | 477.8  | 417.6       | 531.5       | 388.4       |
| <b>II.B2</b> | 0.0023 | 467.0    | 483.9  | 410.8       | 540.9       | 398.4       |
| <b>II.B3</b> | 0.0023 | 489.9    | 515.4  | 442.4       | 576.4       | 386.3       |
| <b>III.1</b> | 0.0023 | 457.4    | 465.6  | 405.4       | 507.0       | 430.8       |
| <b>III.2</b> | 0.0023 | 458.8    | 472.2  | 368.4       | 525.0       | 438.9       |
| <b>III.3</b> | 0.0023 | 469.5    | 486.5  | 365.0       | 546.0       | 442.1       |
| <b>III.4</b> | 0.0023 | 449.9    | 458.5  | 391.6       | 501.0       | 426.3       |
| <b>III.5</b> | 0.0023 | 454.4    | 467.6  | 370.5       | 520.1       | 428.3       |
| <b>III.6</b> | 0.0023 | 465.9    | 482.3  | 369.0       | 542.5       | 435.5       |
| <b>IV.A1</b> | 0.0023 | 453.3    | 462.4  | 379.2       | 505.1       | 445.3       |
| <b>IV.A2</b> | 0.0023 | 458.2    | 472.0  | 359.5       | 524.7       | 445.4       |
| <b>IV.A3</b> | 0.0024 | 458.1    | 473.3  | 353.5       | 529.9       | 441.6       |
| <b>IV.A4</b> | 0.0023 | 451.9    | 460.8  | 381.3       | 502.5       | 439.5       |
| <b>IV.A5</b> | 0.0023 | 463.5    | 477.2  | 362.7       | 529.7       | 454.8       |
| <b>IV.A6</b> | 0.0024 | 462.3    | 478.6  | 356.2       | 535.0       | 443.2       |
| <b>IV.B1</b> | 0.0023 | 453.6    | 463.6  | 371.8       | 507.5       | 446.6       |
| <b>IV.B2</b> | 0.0023 | 449.2    | 460.5  | 361.3       | 506.9       | 439.3       |
| <b>IV.B3</b> | 0.0023 | 460.5    | 474.2  | 360.5       | 526.3       | 448.2       |
| <b>V.1</b>   | 0.0023 | 448.2    | 457.9  | 367.3       | 498.6       | 445.6       |
| <b>V.2</b>   | 0.0023 | 451.5    | 462.6  | 365.1       | 504.8       | 451.1       |
| <b>V.3</b>   | 0.0023 | 453.9    | 467.1  | 354.4       | 516.3       | 451.6       |

Table C.12. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0357 | 40.4   | 59.9   | 15.8        | 31.5        | 73.4        |
| <b>I.B1</b>  | 0.0508 | 25.5   | 33.5   | 11.9        | 21.5        | 43.1        |
| <b>I.B2</b>  | 0.0533 | 24.3   | 31.6   | 11.3        | 20.6        | 40.9        |
| <b>I.B3</b>  | 0.0551 | 23.7   | 31.0   | 10.8        | 20.0        | 40.7        |
| <b>I.B4</b>  | 0.0549 | 24.3   | 32.3   | 10.7        | 20.4        | 41.7        |
| <b>II.A1</b> | 0.049  | 30.7   | 46.4   | 11.2        | 23.7        | 57.4        |
| <b>II.A2</b> | 0.048  | 37.3   | 65.1   | 10.6        | 25.6        | 75.7        |
| <b>II.A3</b> | 0.044  | 52.8   | 107.4  | 10.7        | 30.5        | 116.7       |
| <b>II.B1</b> | 0.0502 | 31.6   | 49.3   | 10.6        | 23.8        | 60.3        |
| <b>II.B2</b> | 0.0504 | 32.2   | 51.0   | 10.5        | 23.9        | 62.4        |
| <b>II.B3</b> | 0.0464 | 42.3   | 76.7   | 10.5        | 28.1        | 87.7        |
| <b>III.1</b> | 0.0575 | 22.1   | 27.9   | 10.6        | 19.0        | 36.9        |
| <b>III.2</b> | 0.0598 | 22.4   | 29.3   | 9.8         | 18.8        | 39.1        |
| <b>III.3</b> | 0.0588 | 24.0   | 33.1   | 9.6         | 19.7        | 43.2        |
| <b>III.4</b> | 0.0587 | 21.5   | 27.0   | 10.4        | 18.6        | 35.9        |
| <b>III.5</b> | 0.0592 | 22.6   | 29.5   | 9.8         | 19.2        | 39.5        |
| <b>III.6</b> | 0.0576 | 24.6   | 34.0   | 9.7         | 20.2        | 44.2        |
| <b>IV.A1</b> | 0.0613 | 20.3   | 25.1   | 10.1        | 17.7        | 33.5        |
| <b>IV.A2</b> | 0.0621 | 20.9   | 26.6   | 9.7         | 17.9        | 35.6        |
| <b>IV.A3</b> | 0.0616 | 22.1   | 29.8   | 9.4         | 18.4        | 38.7        |
| <b>IV.A4</b> | 0.0608 | 20.4   | 25.3   | 10.2        | 17.8        | 33.4        |
| <b>IV.A5</b> | 0.0617 | 21.2   | 27.2   | 9.7         | 18.1        | 36.1        |
| <b>IV.A6</b> | 0.061  | 22.4   | 29.7   | 9.4         | 18.9        | 39.2        |
| <b>IV.B1</b> | 0.0619 | 20.1   | 24.9   | 10.0        | 17.6        | 33.0        |
| <b>IV.B2</b> | 0.0616 | 20.6   | 25.6   | 9.9         | 17.8        | 34.2        |
| <b>IV.B3</b> | 0.0613 | 21.3   | 27.3   | 9.7         | 18.2        | 36.1        |
| <b>V.1</b>   | 0.0627 | 19.6   | 23.9   | 10.0        | 17.2        | 31.9        |
| <b>V.2</b>   | 0.0636 | 19.6   | 24.2   | 9.8         | 17.1        | 32.3        |
| <b>V.3</b>   | 0.0637 | 20.1   | 25.4   | 9.5         | 17.3        | 33.7        |

Table C.13. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0027 | 385.3    | 400.5  | 496         | 391.8       | 275.9       |
| <b>I.B1</b>  | 0.0023 | 440.0    | 448    | 483.6       | 470.8       | 356.7       |
| <b>I.B2</b>  | 0.0023 | 439.2    | 446.5  | 460.4       | 473.0       | 368.8       |
| <b>I.B3</b>  | 0.0023 | 446.9    | 455.9  | 452.2       | 490.4       | 374.1       |
| <b>I.B4</b>  | 0.0023 | 451.4    | 460.9  | 454.7       | 498.0       | 380.0       |
| <b>II.A1</b> | 0.0023 | 460.0    | 473.2  | 487.1       | 507.8       | 363.4       |
| <b>II.A2</b> | 0.0023 | 468.5    | 488.4  | 484.9       | 527.2       | 355.7       |
| <b>II.A3</b> | 0.0023 | 479.8    | 509.8  | 509.9       | 548.2       | 335.2       |
| <b>II.B1</b> | 0.0023 | 452.0    | 467.3  | 463.1       | 506.4       | 355.2       |
| <b>II.B2</b> | 0.0023 | 461.9    | 478.9  | 469.4       | 519.5       | 364.0       |
| <b>II.B3</b> | 0.0023 | 486.7    | 512.2  | 494.4       | 557.2       | 361.5       |
| <b>III.1</b> | 0.0023 | 452.0    | 459.3  | 440.7       | 495.6       | 397.0       |
| <b>III.2</b> | 0.0023 | 454.6    | 465.5  | 409.0       | 510.5       | 408.7       |
| <b>III.3</b> | 0.0023 | 467.2    | 481.5  | 413.0       | 535.6       | 411.9       |
| <b>III.4</b> | 0.0023 | 447.8    | 455.3  | 436.0       | 493.8       | 392.7       |
| <b>III.5</b> | 0.0023 | 456.5    | 467.9  | 415.7       | 513.1       | 406.9       |
| <b>III.6</b> | 0.0023 | 462.8    | 476.9  | 410.9       | 529.2       | 405.1       |
| <b>IV.A1</b> | 0.0023 | 455.9    | 463.6  | 419.5       | 505.7       | 413.7       |
| <b>IV.A2</b> | 0.0023 | 453.3    | 463.3  | 403.5       | 507.5       | 414.4       |
| <b>IV.A3</b> | 0.0023 | 459.9    | 473.5  | 389.2       | 527.5       | 417.4       |
| <b>IV.A4</b> | 0.0023 | 452.7    | 460.2  | 423.3       | 498.8       | 409.3       |
| <b>IV.A5</b> | 0.0023 | 450.6    | 461.1  | 393.8       | 509.0       | 414.7       |
| <b>IV.A6</b> | 0.0023 | 460.1    | 474.3  | 387.2       | 528.2       | 418.4       |
| <b>IV.B1</b> | 0.0023 | 451.3    | 459.2  | 415.5       | 501.9       | 408.1       |
| <b>IV.B2</b> | 0.0023 | 459.1    | 468.4  | 413.5       | 512.9       | 420.3       |
| <b>IV.B3</b> | 0.0023 | 455.5    | 466.7  | 394.9       | 512.7       | 416.0       |
| <b>V.1</b>   | 0.0022 | 459.6    | 467.4  | 413.2       | 512.3       | 423.1       |
| <b>V.2</b>   | 0.0023 | 449.6    | 458.0  | 392.2       | 500.8       | 422.7       |
| <b>V.3</b>   | 0.0023 | 454.8    | 465.2  | 391.9       | 511.7       | 425.6       |

Table C.14. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0265 | 58.7   | 90.2   | 20.4        | 43.9        | 112.1       |
| <b>I.B1</b>  | 0.0414 | 33.4   | 47.0   | 14.1        | 27.0        | 58.8        |
| <b>I.B2</b>  | 0.0445 | 30.8   | 43.0   | 13.0        | 25.2        | 54.1        |
| <b>I.B3</b>  | 0.0466 | 29.8   | 41.7   | 12.3        | 24.3        | 53.0        |
| <b>I.B4</b>  | 0.0466 | 30.2   | 42.3   | 12.2        | 24.6        | 54.2        |
| <b>II.A1</b> | 0.0415 | 37.9   | 58.9   | 12.9        | 28.6        | 72.1        |
| <b>II.A2</b> | 0.0412 | 45.9   | 82.2   | 12.0        | 30.7        | 94.1        |
| <b>II.A3</b> | 0.0375 | 64.6   | 128.1  | 12.2        | 37.0        | 147.5       |
| <b>II.B1</b> | 0.0431 | 38.6   | 62.0   | 12.1        | 28.5        | 76.1        |
| <b>II.B2</b> | 0.0433 | 39.5   | 64.8   | 11.9        | 28.6        | 79.2        |
| <b>II.B3</b> | 0.0404 | 52.1   | 96.1   | 11.8        | 33.3        | 112.9       |
| <b>III.1</b> | 0.0496 | 26.6   | 35.0   | 11.9        | 22.4        | 45.8        |
| <b>III.2</b> | 0.0528 | 26.3   | 35.8   | 10.8        | 21.7        | 47.0        |
| <b>III.3</b> | 0.0524 | 28.0   | 39.9   | 10.5        | 22.4        | 51.4        |
| <b>III.4</b> | 0.0507 | 25.9   | 34.0   | 11.8        | 21.8        | 44.6        |
| <b>III.5</b> | 0.0524 | 26.5   | 36.1   | 10.9        | 22.0        | 47.3        |
| <b>III.6</b> | 0.0513 | 28.7   | 41.3   | 10.8        | 23.0        | 53.1        |
| <b>IV.A1</b> | 0.0536 | 24.1   | 31.0   | 11.2        | 20.5        | 41.0        |
| <b>IV.A2</b> | 0.0554 | 24.2   | 31.9   | 10.6        | 20.3        | 41.8        |
| <b>IV.A3</b> | 0.0553 | 25.4   | 35.2   | 10.2        | 20.8        | 45.3        |
| <b>IV.A4</b> | 0.0533 | 24.2   | 31.2   | 11.4        | 20.6        | 40.9        |
| <b>IV.A5</b> | 0.0552 | 24.4   | 32.6   | 10.5        | 20.5        | 42.6        |
| <b>IV.A6</b> | 0.055  | 25.6   | 35.1   | 10.2        | 21.3        | 45.7        |
| <b>IV.B1</b> | 0.0545 | 23.6   | 30.2   | 11.1        | 20.2        | 39.6        |
| <b>IV.B2</b> | 0.055  | 23.9   | 31.2   | 10.8        | 20.2        | 41.0        |
| <b>IV.B3</b> | 0.055  | 24.7   | 33.1   | 10.5        | 20.7        | 43.2        |
| <b>V.1</b>   | 0.0555 | 22.8   | 28.8   | 11.0        | 19.6        | 38.2        |
| <b>V.2</b>   | 0.0568 | 22.6   | 28.8   | 10.6        | 19.4        | 38.2        |
| <b>V.3</b>   | 0.0574 | 23.0   | 30.1   | 10.4        | 19.4        | 39.3        |

Table C.15. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 3 (d=2, K=0.10).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0022 | 453.2    | 457.3  | 481.2       | 477.8       | 393.5       |
| <b>I.B1</b>  | 0.0023 | 444.0    | 454    | 354.4       | 495.1       | 449.2       |
| <b>I.B2</b>  | 0.0024 | 437.7    | 450.6  | 326.4       | 492.7       | 461.9       |
| <b>I.B3</b>  | 0.0024 | 443.8    | 458.6  | 322.3       | 499.7       | 473.4       |
| <b>I.B4</b>  | 0.0024 | 450.5    | 466.9  | 321.0       | 511.8       | 480.4       |
| <b>II.A1</b> | 0.0022 | 471.4    | 484.5  | 426.5       | 534.6       | 407.9       |
| <b>II.A2</b> | 0.0023 | 476.7    | 497.9  | 393.2       | 567.9       | 403.0       |
| <b>II.A3</b> | 0.0023 | 492.2    | 522.3  | 414.4       | 594.5       | 376.4       |
| <b>II.B1</b> | 0.0024 | 457.3    | 476.4  | 350.7       | 540.2       | 417.8       |
| <b>II.B2</b> | 0.0024 | 466.1    | 488.6  | 340.7       | 562.2       | 425.0       |
| <b>II.B3</b> | 0.0024 | 489.6    | 518.5  | 381.0       | 595.5       | 400.5       |
| <b>III.1</b> | 0.0023 | 455.1    | 466.6  | 360.3       | 513.1       | 454.0       |
| <b>III.2</b> | 0.0024 | 448.2    | 465.3  | 322.6       | 518.9       | 452.0       |
| <b>III.3</b> | 0.0024 | 470.1    | 491.3  | 326.2       | 552.2       | 470.6       |
| <b>III.4</b> | 0.0023 | 448.1    | 459.3  | 355.0       | 505.3       | 447.5       |
| <b>III.5</b> | 0.0024 | 456.3    | 473.9  | 329.1       | 529.3       | 458.3       |
| <b>III.6</b> | 0.0025 | 455.0    | 475.2  | 319.6       | 536.8       | 448.6       |
| <b>IV.A1</b> | 0.0023 | 450.0    | 460.7  | 361.6       | 504.7       | 448.7       |
| <b>IV.A2</b> | 0.0024 | 453.7    | 469.8  | 335.2       | 520.9       | 457.9       |
| <b>IV.A3</b> | 0.0024 | 462.6    | 481.8  | 330.5       | 540.4       | 461.4       |
| <b>IV.A4</b> | 0.0024 | 439.7    | 449.8  | 356.8       | 492.1       | 438.9       |
| <b>IV.A5</b> | 0.0024 | 443.5    | 458.6  | 328.0       | 509.1       | 450.2       |
| <b>IV.A6</b> | 0.0024 | 459.2    | 479.3  | 322.1       | 540.0       | 459.8       |
| <b>IV.B1</b> | 0.0024 | 441.2    | 453.9  | 333.2       | 498.6       | 449.6       |
| <b>IV.B2</b> | 0.0024 | 445.0    | 460.2  | 324.0       | 509.0       | 457.7       |
| <b>IV.B3</b> | 0.0025 | 441.7    | 458.6  | 313.8       | 505.8       | 459.5       |
| <b>V.1</b>   | 0.0024 | 447.6    | 459.9  | 344.8       | 503.9       | 458.1       |
| <b>V.2</b>   | 0.0024 | 442.5    | 456.1  | 332.8       | 499.1       | 462.1       |
| <b>V.3</b>   | 0.0025 | 440.7    | 457.3  | 310.6       | 502.5       | 463.0       |

Table C.16. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 3 (d=2, K=0.10).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0456 | 26.4   | 31.5   | 14.0        | 23.6        | 42.1        |
| <b>I.B1</b>  | 0.0652 | 18.7   | 22.5   | 9.7         | 16.5        | 30.1        |
| <b>I.B2</b>  | 0.069  | 17.7   | 21.2   | 9.1         | 15.7        | 28.4        |
| <b>I.B3</b>  | 0.0711 | 17.2   | 20.7   | 8.8         | 15.2        | 27.9        |
| <b>I.B4</b>  | 0.0702 | 17.7   | 21.6   | 8.8         | 15.5        | 29.0        |
| <b>II.A1</b> | 0.0506 | 29.0   | 42.3   | 10.9        | 22.9        | 53.5        |
| <b>II.A2</b> | 0.0522 | 32.9   | 55.9   | 9.8         | 23.4        | 65.5        |
| <b>II.A3</b> | 0.0459 | 49.3   | 100.3  | 10.3        | 29.0        | 106.4       |
| <b>II.B1</b> | 0.058  | 26.6   | 40.7   | 9.3         | 20.2        | 50.8        |
| <b>II.B2</b> | 0.0589 | 26.7   | 41.5   | 9.1         | 20.1        | 51.0        |
| <b>II.B3</b> | 0.0523 | 37.8   | 70.9   | 9.3         | 24.7        | 78.0        |
| <b>III.1</b> | 0.0631 | 19.7   | 24.1   | 9.8         | 17.3        | 32.5        |
| <b>III.2</b> | 0.0652 | 20.3   | 26.2   | 9.0         | 17.3        | 34.9        |
| <b>III.3</b> | 0.0649 | 21.2   | 28.5   | 8.8         | 17.6        | 37.8        |
| <b>III.4</b> | 0.0641 | 19.3   | 23.5   | 9.7         | 17.0        | 31.6        |
| <b>III.5</b> | 0.0658 | 20.0   | 25.9   | 8.9         | 17.1        | 34.4        |
| <b>III.6</b> | 0.0647 | 21.6   | 29.4   | 8.7         | 17.8        | 38.6        |
| <b>IV.A1</b> | 0.0635 | 19.5   | 23.8   | 9.8         | 17.1        | 32.0        |
| <b>IV.A2</b> | 0.0656 | 19.7   | 25.0   | 9.1         | 17.0        | 33.4        |
| <b>IV.A3</b> | 0.0653 | 20.6   | 27.1   | 8.9         | 17.3        | 35.8        |
| <b>IV.A4</b> | 0.0628 | 19.7   | 24.1   | 9.8         | 17.4        | 32.3        |
| <b>IV.A5</b> | 0.0653 | 20     | 25.6   | 9.1         | 17.1        | 34.1        |
| <b>IV.A6</b> | 0.0658 | 20.7   | 27.4   | 8.7         | 17.4        | 36.5        |
| <b>IV.B1</b> | 0.0662 | 18.8   | 22.9   | 9.3         | 16.4        | 31.0        |
| <b>IV.B2</b> | 0.0681 | 18.5   | 22.9   | 9.0         | 16.1        | 30.7        |
| <b>IV.B3</b> | 0.0682 | 19.0   | 24.1   | 8.8         | 16.3        | 32.2        |
| <b>V.1</b>   | 0.0667 | 18.5   | 22.4   | 9.4         | 16.2        | 30.3        |
| <b>V.2</b>   | 0.0691 | 17.9   | 22.0   | 9.0         | 15.7        | 29.4        |
| <b>V.3</b>   | 0.0697 | 18.2   | 22.8   | 8.7         | 15.8        | 30.4        |

Table C.17. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0041 | 259.4    | 271.8  | 338.7       | 251.9       | 190.8       |
| <b>I.B1</b>  | 0.0025 | 430.5    | 443.7  | 326.8       | 494.3       | 427.2       |
| <b>I.B2</b>  | 0.0024 | 440.2    | 455.7  | 319.3       | 502.8       | 457.2       |
| <b>I.B3</b>  | 0.0024 | 444.8    | 458.9  | 328.7       | 506.8       | 456.4       |
| <b>I.B4</b>  | 0.0023 | 452.4    | 463.6  | 366.9       | 511.0       | 439.9       |
| <b>II.A1</b> | 0.0025 | 422.1    | 440.8  | 553.8       | 432.8       | 296.0       |
| <b>II.A2</b> | 0.0023 | 461.8    | 482.4  | 548.4       | 499.2       | 329.9       |
| <b>II.A3</b> | 0.0024 | 456.4    | 488.4  | 604.4       | 483.9       | 292.6       |
| <b>II.B1</b> | 0.0024 | 456.6    | 475.3  | 423.7       | 532.5       | 364.8       |
| <b>II.B2</b> | 0.0024 | 462.8    | 484.4  | 392.6       | 547.8       | 380.7       |
| <b>II.B3</b> | 0.0024 | 468.9    | 498.7  | 470.1       | 545.1       | 331.3       |
| <b>III.1</b> | 0.0023 | 450.7    | 462.5  | 390.2       | 513.6       | 405.6       |
| <b>III.2</b> | 0.0024 | 450.9    | 467.7  | 345.8       | 529.9       | 426.1       |
| <b>III.3</b> | 0.0024 | 462.8    | 481.6  | 348.7       | 545.5       | 436.2       |
| <b>III.4</b> | 0.0024 | 444.0    | 456.5  | 370.4       | 509.9       | 406.0       |
| <b>III.5</b> | 0.0024 | 452.0    | 469.2  | 338.7       | 530.8       | 432.1       |
| <b>III.6</b> | 0.0024 | 457.0    | 476.5  | 333.0       | 538.0       | 435.8       |
| <b>IV.A1</b> | 0.0023 | 446.0    | 457.6  | 390.5       | 509.2       | 400.2       |
| <b>IV.A2</b> | 0.0024 | 442.3    | 456.8  | 347.5       | 512.8       | 419.5       |
| <b>IV.A3</b> | 0.0024 | 453.2    | 470.3  | 346.4       | 531.5       | 427.7       |
| <b>IV.A4</b> | 0.0024 | 439.4    | 451.1  | 376.8       | 504.2       | 397.4       |
| <b>IV.A5</b> | 0.0025 | 441.8    | 458.1  | 339.1       | 513.4       | 419.4       |
| <b>IV.A6</b> | 0.0024 | 452.3    | 471.1  | 327.8       | 532.7       | 436.1       |
| <b>IV.B1</b> | 0.0025 | 431.1    | 444.6  | 336.0       | 495.6       | 416.3       |
| <b>IV.B2</b> | 0.0025 | 440.2    | 456.8  | 317.8       | 509.9       | 447.0       |
| <b>IV.B3</b> | 0.0025 | 443.5    | 461.5  | 313.4       | 516.2       | 449.4       |
| <b>V.1</b>   | 0.0025 | 434.4    | 448.7  | 327.9       | 500.1       | 433.9       |
| <b>V.2</b>   | 0.0025 | 438.1    | 455.4  | 309.2       | 506.3       | 458.1       |
| <b>V.3</b>   | 0.0025 | 440.6    | 459.0  | 304.8       | 510.1       | 459.3       |

Table C.18. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0089 | 159.1  | 204.9  | 61.6        | 137.0       | 291.7       |
| <b>I.B1</b>  | 0.0660 | 20.0   | 27.1   | 9.1         | 16.4        | 34.3        |
| <b>I.B2</b>  | 0.0697 | 18.2   | 23.2   | 8.9         | 15.5        | 30.1        |
| <b>I.B3</b>  | 0.0679 | 18.5   | 23.2   | 9.1         | 15.9        | 30.4        |
| <b>I.B4</b>  | 0.0624 | 20.4   | 25.8   | 9.8         | 17.5        | 34.2        |
| <b>II.A1</b> | 0.0262 | 64.9   | 101.5  | 19.6        | 47.9        | 129.3       |
| <b>II.A2</b> | 0.0341 | 56.6   | 97.8   | 14.4        | 38.3        | 120.2       |
| <b>II.A3</b> | 0.0283 | 90.6   | 172.0  | 15.8        | 52.3        | 218.9       |
| <b>II.B1</b> | 0.0477 | 36.9   | 63.2   | 10.8        | 25.6        | 74.1        |
| <b>II.B2</b> | 0.0510 | 34.9   | 60.8   | 10.0        | 24.2        | 70.5        |
| <b>II.B3</b> | 0.0411 | 59.3   | 119.9  | 11.2        | 34.1        | 133.1       |
| <b>III.1</b> | 0.0552 | 25.3   | 35.2   | 10.4        | 20.4        | 45.4        |
| <b>III.2</b> | 0.0606 | 23.3   | 32.7   | 9.4         | 18.9        | 42.0        |
| <b>III.3</b> | 0.0603 | 24.1   | 34.6   | 9.2         | 19.2        | 44.3        |
| <b>III.4</b> | 0.0578 | 24.0   | 33.5   | 10.0        | 19.4        | 42.3        |
| <b>III.5</b> | 0.0619 | 23.0   | 32.5   | 9.1         | 18.6        | 41.3        |
| <b>III.6</b> | 0.0614 | 23.8   | 34.7   | 9.1         | 18.8        | 43.7        |
| <b>IV.A1</b> | 0.056  | 24.9   | 35.0   | 10.3        | 20.0        | 44.9        |
| <b>IV.A2</b> | 0.0610 | 22.9   | 31.7   | 9.4         | 18.5        | 40.6        |
| <b>IV.A3</b> | 0.0612 | 23.1   | 32.5   | 9.2         | 18.7        | 41.7        |
| <b>IV.A4</b> | 0.0564 | 24.9   | 35.2   | 10.1        | 19.9        | 44.7        |
| <b>IV.A5</b> | 0.0628 | 22.4   | 31.4   | 9.1         | 18.0        | 39.9        |
| <b>IV.A6</b> | 0.0627 | 23.0   | 33.1   | 8.9         | 18.3        | 42.0        |
| <b>IV.B1</b> | 0.0634 | 21.4   | 29.5   | 9.3         | 17.4        | 37.4        |
| <b>IV.B2</b> | 0.0668 | 19.9   | 26.5   | 8.9         | 16.5        | 34.3        |
| <b>IV.B3</b> | 0.0666 | 20.1   | 26.9   | 8.9         | 16.7        | 34.7        |
| <b>V.1</b>   | 0.0653 | 20.4   | 27.3   | 9.1         | 16.8        | 35.5        |
| <b>V.2</b>   | 0.0693 | 18.8   | 24.7   | 8.7         | 15.9        | 31.9        |
| <b>V.3</b>   | 0.0695 | 18.9   | 24.9   | 8.6         | 15.8        | 32.2        |

Table C.19. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0051 | 207.9    | 218.3  | 271.8       | 201.7       | 152.9       |
| <b>I.B1</b>  | 0.0025 | 427.2    | 440.1  | 350.5       | 495.3       | 387.6       |
| <b>I.B2</b>  | 0.0024 | 441.9    | 455.9  | 340.7       | 510.4       | 429.8       |
| <b>I.B3</b>  | 0.0024 | 448.3    | 460.5  | 358.9       | 511.1       | 434.4       |
| <b>I.B4</b>  | 0.0023 | 453.9    | 463.3  | 402.9       | 508.2       | 412.8       |
| <b>II.A1</b> | 0.0029 | 372.3    | 396.1  | 521.6       | 366.8       | 248.0       |
| <b>II.A2</b> | 0.0025 | 434.2    | 458.7  | 581.7       | 447.7       | 290.8       |
| <b>II.A3</b> | 0.0027 | 419.2    | 455.3  | 625.4       | 423.1       | 255.1       |
| <b>II.B1</b> | 0.0025 | 439.6    | 460.5  | 483.5       | 491.1       | 318.7       |
| <b>II.B2</b> | 0.0024 | 451.7    | 473.7  | 449.6       | 518.3       | 340.5       |
| <b>II.B3</b> | 0.0025 | 446.7    | 478.7  | 533.7       | 494.0       | 290.1       |
| <b>III.1</b> | 0.0024 | 439.2    | 451.6  | 447.5       | 491.4       | 352.7       |
| <b>III.2</b> | 0.0024 | 448.2    | 463.4  | 391.7       | 519.6       | 385.3       |
| <b>III.3</b> | 0.0023 | 462.2    | 478.9  | 388.7       | 540.2       | 403.4       |
| <b>III.4</b> | 0.0024 | 436.2    | 448.7  | 416.7       | 493.1       | 360.5       |
| <b>III.5</b> | 0.0024 | 450.5    | 466.2  | 377.9       | 521.9       | 395.2       |
| <b>III.6</b> | 0.0024 | 456.5    | 474.1  | 370.8       | 534.6       | 405.8       |
| <b>IV.A1</b> | 0.0024 | 432.7    | 445.4  | 441.6       | 482.5       | 343.3       |
| <b>IV.A2</b> | 0.0024 | 437.8    | 451.3  | 386.6       | 503.3       | 376.8       |
| <b>IV.A3</b> | 0.0024 | 451.1    | 466.6  | 385.4       | 523.2       | 393.1       |
| <b>IV.A4</b> | 0.0024 | 428.9    | 441.4  | 416.3       | 484.1       | 350.8       |
| <b>IV.A5</b> | 0.0024 | 439.0    | 454.2  | 371.4       | 510.6       | 383.3       |
| <b>IV.A6</b> | 0.0024 | 450.3    | 468.3  | 355.5       | 530.7       | 407.8       |
| <b>IV.B1</b> | 0.0024 | 437.3    | 450.4  | 382.6       | 504.9       | 381.4       |
| <b>IV.B2</b> | 0.0024 | 440.1    | 454.5  | 350.5       | 510.2       | 412.8       |
| <b>IV.B3</b> | 0.0024 | 443.1    | 458.6  | 340.6       | 514.7       | 423.1       |
| <b>V.1</b>   | 0.0024 | 435.2    | 447.7  | 362.1       | 502.8       | 399.2       |
| <b>V.2</b>   | 0.0024 | 440.6    | 456.1  | 330.7       | 512.5       | 428.8       |
| <b>V.3</b>   | 0.0024 | 442.9    | 459.5  | 322.6       | 516.2       | 437.6       |

Table C.20. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.005  | 264.6  | 312.9  | 111.9       | 253.8       | 438.8       |
| <b>I.B1</b>  | 0.0595 | 24.3   | 36.6   | 9.6         | 18.6        | 44.5        |
| <b>I.B2</b>  | 0.0643 | 20.8   | 28.7   | 9.3         | 17.0        | 36.0        |
| <b>I.B3</b>  | 0.0625 | 20.9   | 27.6   | 9.6         | 17.5        | 35.5        |
| <b>I.B4</b>  | 0.0555 | 23.7   | 31.3   | 10.8        | 19.9        | 40.5        |
| <b>II.A1</b> | 0.0179 | 102.2  | 158.6  | 27.2        | 74.3        | 216.8       |
| <b>II.A2</b> | 0.0265 | 77.2   | 133.9  | 17.9        | 51.1        | 167.4       |
| <b>II.A3</b> | 0.0212 | 126.4  | 226.8  | 20.3        | 75.3        | 316         |
| <b>II.B1</b> | 0.0392 | 50.7   | 92.4   | 12.4        | 32.8        | 106.7       |
| <b>II.B2</b> | 0.0434 | 45.4   | 83.2   | 11.3        | 29.7        | 95.2        |
| <b>II.B3</b> | 0.0334 | 80.9   | 161.1  | 13.3        | 44.7        | 196.0       |
| <b>III.1</b> | 0.0454 | 33.9   | 51.3   | 11.9        | 25.8        | 64.9        |
| <b>III.2</b> | 0.0531 | 28.6   | 43.4   | 10.3        | 22.0        | 53.6        |
| <b>III.3</b> | 0.0535 | 28.8   | 43.6   | 10.1        | 22.1        | 54.9        |
| <b>III.4</b> | 0.0489 | 30.9   | 46.6   | 11.2        | 23.7        | 57.8        |
| <b>III.5</b> | 0.0546 | 28.0   | 42.6   | 10.0        | 21.4        | 53.0        |
| <b>III.6</b> | 0.0553 | 28.0   | 42.5   | 9.7         | 21.4        | 53.4        |
| <b>IV.A1</b> | 0.0452 | 34.6   | 53.6   | 11.8        | 26          | 66.5        |
| <b>IV.A2</b> | 0.0531 | 28.7   | 44.2   | 10.3        | 21.8        | 54.3        |
| <b>IV.A3</b> | 0.0539 | 28.3   | 42.6   | 10.1        | 21.7        | 53.1        |
| <b>IV.A4</b> | 0.0478 | 32.7   | 50.9   | 11.2        | 24.6        | 62.2        |
| <b>IV.A5</b> | 0.0561 | 26.8   | 40.4   | 9.8         | 20.6        | 50.1        |
| <b>IV.A6</b> | 0.0577 | 26.6   | 40.9   | 9.5         | 20.3        | 49.9        |
| <b>IV.B1</b> | 0.0549 | 27.2   | 41.5   | 10.1        | 20.7        | 50.7        |
| <b>IV.B2</b> | 0.0598 | 23.7   | 34.3   | 9.5         | 18.8        | 43.0        |
| <b>IV.B3</b> | 0.0615 | 22.9   | 32.4   | 9.3         | 18.3        | 41.0        |
| <b>V.1</b>   | 0.0581 | 24.4   | 35.1   | 9.8         | 19.3        | 44.4        |
| <b>V.2</b>   | 0.0636 | 21.6   | 30.4   | 9.1         | 17.4        | 38.0        |
| <b>V.3</b>   | 0.0647 | 21.1   | 29.1   | 9.0         | 17.2        | 37.1        |

Table C.21. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0025 | 409.7    | 418.0  | 489.4       | 416.7       | 325.2       |
| <b>I.B1</b>  | 0.0025 | 411.7    | 420.6  | 498.0       | 418.6       | 324.9       |
| <b>I.B2</b>  | 0.0025 | 415.1    | 424.4  | 504.8       | 424.0       | 325.5       |
| <b>I.B3</b>  | 0.0025 | 415.5    | 425.9  | 509.5       | 425.0       | 321.7       |
| <b>I.B4</b>  | 0.0025 | 419.5    | 430.5  | 512.1       | 431.6       | 324.4       |
| <b>II.A1</b> | 0.0023 | 449.5    | 466.0  | 558.7       | 472.4       | 326.8       |
| <b>II.A2</b> | 0.0024 | 451.9    | 474.6  | 563.7       | 479.0       | 312.3       |
| <b>II.A3</b> | 0.0024 | 457.8    | 490.9  | 601.2       | 488.7       | 291.1       |
| <b>II.B1</b> | 0.0024 | 434.4    | 452.2  | 536.9       | 459.6       | 312.4       |
| <b>II.B2</b> | 0.0024 | 444.7    | 463.6  | 546.7       | 472.8       | 318.8       |
| <b>II.B3</b> | 0.0024 | 461.1    | 488.3  | 589.5       | 494.2       | 311.1       |
| <b>III.1</b> | 0.0024 | 422.0    | 431.5  | 508.7       | 434.0       | 331.8       |
| <b>III.2</b> | 0.0024 | 430.7    | 442.9  | 513.9       | 451.5       | 332.2       |
| <b>III.3</b> | 0.0023 | 448.0    | 463.3  | 538.6       | 477.0       | 336.9       |
| <b>III.4</b> | 0.0024 | 420.4    | 429.3  | 499.2       | 433.9       | 331.1       |
| <b>III.5</b> | 0.0024 | 434.9    | 447.4  | 516.4       | 458.4       | 333.6       |
| <b>III.6</b> | 0.0024 | 437.9    | 453.4  | 518.3       | 468.5       | 326.2       |
| <b>IV.A1</b> | 0.0025 | 418.5    | 427.4  | 497.6       | 431.6       | 329.9       |
| <b>IV.A2</b> | 0.0024 | 435.7    | 446.7  | 505.9       | 463.1       | 340.7       |
| <b>IV.A3</b> | 0.0024 | 438.5    | 452.1  | 502.2       | 470.4       | 336.3       |
| <b>IV.A4</b> | 0.0024 | 427.6    | 437.0  | 509.1       | 441.6       | 337.1       |
| <b>IV.A5</b> | 0.0024 | 435.2    | 446.5  | 504.2       | 460.6       | 338.6       |
| <b>IV.A6</b> | 0.0024 | 434.8    | 448.2  | 496.6       | 469.0       | 334.6       |
| <b>IV.B1</b> | 0.0024 | 424.7    | 433.8  | 500.1       | 441.9       | 335.4       |
| <b>IV.B2</b> | 0.0024 | 424.1    | 434.4  | 503.0       | 443.3       | 331.3       |
| <b>IV.B3</b> | 0.0024 | 428.2    | 439.6  | 497.6       | 451.7       | 332.8       |
| <b>V.1</b>   | 0.0024 | 430.7    | 439.3  | 505.9       | 450.2       | 342.1       |
| <b>V.2</b>   | 0.0024 | 425.5    | 434.9  | 494.3       | 447.8       | 334.5       |
| <b>V.3</b>   | 0.0023 | 443.7    | 454.7  | 509.7       | 470.6       | 347.6       |

Table C.22. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0300 | 42.4   | 53.2   | 20.2        | 36.8        | 71.3        |
| <b>I.B1</b>  | 0.0302 | 42.9   | 54.5   | 19.6        | 37.2        | 72.9        |
| <b>I.B2</b>  | 0.0304 | 43.7   | 56.7   | 19.3        | 37.1        | 75.7        |
| <b>I.B3</b>  | 0.0311 | 44.1   | 58.5   | 18.4        | 37.1        | 78.4        |
| <b>I.B4</b>  | 0.0317 | 45.0   | 61.9   | 17.6        | 36.6        | 81.7        |
| <b>II.A1</b> | 0.0317 | 51.5   | 80.7   | 16.5        | 38.3        | 100.6       |
| <b>II.A2</b> | 0.0311 | 65.0   | 115.6  | 15.4        | 42.6        | 139.9       |
| <b>II.A3</b> | 0.0288 | 89.7   | 171.1  | 15.4        | 51.6        | 215.2       |
| <b>II.B1</b> | 0.0321 | 55.3   | 89.4   | 15.6        | 40.0        | 112.8       |
| <b>II.B2</b> | 0.0324 | 55.9   | 91.4   | 15.4        | 40.4        | 114.5       |
| <b>II.B3</b> | 0.0302 | 74.1   | 134.6  | 15.3        | 46.9        | 166.5       |
| <b>III.1</b> | 0.0325 | 42.2   | 56.3   | 17.5        | 35.3        | 74.7        |
| <b>III.2</b> | 0.0349 | 43.2   | 61.9   | 15.5        | 34.6        | 80.4        |
| <b>III.3</b> | 0.0350 | 46.8   | 70.9   | 14.8        | 35.7        | 91.7        |
| <b>III.4</b> | 0.0333 | 41.1   | 54.4   | 17.2        | 34.4        | 73.1        |
| <b>III.5</b> | 0.0348 | 43.7   | 63.7   | 15.5        | 34.5        | 82.0        |
| <b>III.6</b> | 0.0346 | 47.7   | 73.0   | 14.9        | 36.4        | 93.3        |
| <b>IV.A1</b> | 0.0337 | 40.4   | 53.7   | 17.1        | 33.9        | 71.6        |
| <b>IV.A2</b> | 0.0364 | 40.3   | 56.8   | 15.1        | 32.6        | 74.5        |
| <b>IV.A3</b> | 0.0371 | 42.6   | 63.9   | 14.3        | 32.9        | 81.4        |
| <b>IV.A4</b> | 0.0337 | 40.4   | 53.3   | 17.1        | 34.0        | 71.4        |
| <b>IV.A5</b> | 0.0365 | 40.5   | 57.6   | 15.0        | 32.5        | 74.9        |
| <b>IV.A6</b> | 0.0368 | 43.4   | 65.0   | 14.1        | 33.5        | 84.1        |
| <b>IV.B1</b> | 0.0345 | 39.7   | 52.9   | 16.6        | 33.2        | 70.5        |
| <b>IV.B2</b> | 0.0349 | 40.6   | 55.3   | 16.0        | 33.7        | 73.4        |
| <b>IV.B3</b> | 0.0359 | 41.8   | 59.6   | 15.2        | 33.4        | 78.1        |
| <b>V.1</b>   | 0.0357 | 38.0   | 50.4   | 16.1        | 31.8        | 66.9        |
| <b>V.2</b>   | 0.0366 | 37.9   | 51.0   | 15.6        | 31.4        | 67.9        |
| <b>V.3</b>   | 0.0376 | 38.8   | 55.0   | 14.7        | 31.1        | 71.1        |

Table C.23. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0056 | 187.3    | 196.4  | 244.1       | 181.3       | 137.5       |
| <b>I.B1</b>  | 0.0056 | 188.8    | 199.3  | 248.9       | 182.3       | 136.7       |
| <b>I.B2</b>  | 0.0056 | 189.4    | 202.1  | 256.8       | 180.9       | 133.7       |
| <b>I.B3</b>  | 0.0056 | 193.9    | 210.0  | 268.0       | 183.6       | 132.6       |
| <b>I.B4</b>  | 0.0056 | 195.8    | 216.1  | 279.8       | 181.8       | 129.0       |
| <b>II.A1</b> | 0.0042 | 263.7    | 291.6  | 387.2       | 248.2       | 163.7       |
| <b>II.A2</b> | 0.0039 | 293.6    | 331.1  | 456.6       | 273.4       | 169.4       |
| <b>II.A3</b> | 0.0039 | 303.9    | 349.9  | 493.3       | 281.2       | 164.1       |
| <b>II.B1</b> | 0.0042 | 267.7    | 299.7  | 409.8       | 248.9       | 160.1       |
| <b>II.B2</b> | 0.0041 | 278.1    | 313.0  | 431.6       | 258.6       | 163.1       |
| <b>II.B3</b> | 0.0039 | 301.6    | 346.8  | 491.2       | 276.8       | 165.0       |
| <b>III.1</b> | 0.0049 | 221.8    | 242.9  | 317.4       | 208.6       | 145.3       |
| <b>III.2</b> | 0.0043 | 264.8    | 295.8  | 404.4       | 247.2       | 157.7       |
| <b>III.3</b> | 0.004  | 289.4    | 325.8  | 461.6       | 270.5       | 166.2       |
| <b>III.4</b> | 0.0050 | 220.8    | 242.0  | 315.8       | 206.8       | 145.0       |
| <b>III.5</b> | 0.0043 | 266.1    | 298.3  | 408.7       | 247.1       | 157.6       |
| <b>III.6</b> | 0.0040 | 287.0    | 323.3  | 456.9       | 269.5       | 164.1       |
| <b>IV.A1</b> | 0.0049 | 223.7    | 246.4  | 325.9       | 208.9       | 145.0       |
| <b>IV.A2</b> | 0.0042 | 268.4    | 300.5  | 415.4       | 249.9       | 159.3       |
| <b>IV.A3</b> | 0.0039 | 292.8    | 328.3  | 466.3       | 275.6       | 167.6       |
| <b>IV.A4</b> | 0.0048 | 227.1    | 249.4  | 327.5       | 213.0       | 146.9       |
| <b>IV.A5</b> | 0.0041 | 276.2    | 309.9  | 435.1       | 257.9       | 161.0       |
| <b>IV.A6</b> | 0.0037 | 308.3    | 346.7  | 498.6       | 291.2       | 173.3       |
| <b>IV.B1</b> | 0.0047 | 234.3    | 259.5  | 345.5       | 217.7       | 148.4       |
| <b>IV.B2</b> | 0.0046 | 247.0    | 276.4  | 377.2       | 228.9       | 150.4       |
| <b>IV.B3</b> | 0.0042 | 271.0    | 306.5  | 435.8       | 251.5       | 155.0       |
| <b>V.1</b>   | 0.0042 | 264.3    | 291.5  | 394.5       | 248.8       | 164.7       |
| <b>V.2</b>   | 0.004  | 281.0    | 311.3  | 433.3       | 265.2       | 169.1       |
| <b>V.3</b>   | 0.0036 | 313.3    | 346.6  | 489.4       | 299.9       | 183.9       |

Table C.24. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0038 | 323.7  | 364.8  | 154.4       | 330.9       | 484.9       |
| <b>I.B1</b>  | 0.0038 | 329.7  | 372.7  | 152.4       | 339.1       | 492.6       |
| <b>I.B2</b>  | 0.0040 | 328.7  | 374.4  | 144.2       | 341.3       | 493.9       |
| <b>I.B3</b>  | 0.0041 | 337.9  | 387.8  | 137.1       | 359.0       | 509.1       |
| <b>I.B4</b>  | 0.0044 | 338.1  | 394.2  | 120.7       | 366.2       | 512.0       |
| <b>II.A1</b> | 0.0073 | 246.2  | 329.3  | 64.1        | 213.1       | 499.8       |
| <b>II.A2</b> | 0.0088 | 248.0  | 350.2  | 50.2        | 200.7       | 543.2       |
| <b>II.A3</b> | 0.0089 | 285.4  | 403.4  | 47.5        | 239.0       | 623.2       |
| <b>II.B1</b> | 0.0083 | 245.8  | 336.7  | 53.9        | 210.2       | 515.1       |
| <b>II.B2</b> | 0.0091 | 236.4  | 330.1  | 48.7        | 196.2       | 510.3       |
| <b>II.B3</b> | 0.0091 | 273.2  | 385.9  | 46.2        | 228.7       | 596.3       |
| <b>III.1</b> | 0.0060 | 272.1  | 340.0  | 83.0        | 261.5       | 488.0       |
| <b>III.2</b> | 0.0094 | 220.6  | 307.0  | 47.5        | 183.5       | 468.1       |
| <b>III.3</b> | 0.0111 | 209.1  | 305.9  | 39.0        | 161.7       | 477.8       |
| <b>III.4</b> | 0.0062 | 265.0  | 331.3  | 80.7        | 254.1       | 476.4       |
| <b>III.5</b> | 0.0098 | 218.7  | 307.6  | 45.7        | 181.3       | 474.1       |
| <b>III.6</b> | 0.0116 | 210.0  | 310.1  | 36.9        | 160.5       | 485.3       |
| <b>IV.A1</b> | 0.0064 | 262.7  | 329.9  | 77.3        | 253.4       | 473.9       |
| <b>IV.A2</b> | 0.0106 | 212.3  | 303.4  | 41.3        | 170.6       | 469.7       |
| <b>IV.A3</b> | 0.0130 | 192.0  | 290.0  | 32.8        | 141.9       | 459.1       |
| <b>IV.A4</b> | 0.0066 | 260.8  | 330.6  | 73.6        | 248.1       | 479.3       |
| <b>IV.A5</b> | 0.0115 | 200.8  | 293.0  | 38.1        | 156.0       | 457.2       |
| <b>IV.A6</b> | 0.0146 | 179.1  | 278.3  | 28.7        | 125.5       | 439.5       |
| <b>IV.B1</b> | 0.0073 | 249.2  | 322.8  | 64.3        | 230.3       | 471.9       |
| <b>IV.B2</b> | 0.0090 | 232.0  | 315.2  | 49.8        | 203.6       | 471.4       |
| <b>IV.B3</b> | 0.0113 | 210.9  | 303.5  | 37.5        | 170.6       | 468.4       |
| <b>V.1</b>   | 0.0102 | 195.2  | 274.7  | 44.9        | 159.8       | 418.6       |
| <b>V.2</b>   | 0.0123 | 175.6  | 259.4  | 36.2        | 132.9       | 399.3       |
| <b>V.3</b>   | 0.0157 | 153.5  | 242.8  | 27.8        | 105.0       | 374.9       |

Table C.25. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0074 | 142.1    | 148.1  | 182.6       | 138.4       | 106.5       |
| <b>I.B1</b>  | 0.0074 | 142.0    | 148.4  | 183.9       | 137.7       | 105.9       |
| <b>I.B2</b>  | 0.0075 | 140.8    | 147.9  | 184.0       | 135.9       | 103.6       |
| <b>I.B3</b>  | 0.0077 | 138.1    | 146.2  | 182.8       | 132.9       | 100.0       |
| <b>I.B4</b>  | 0.0078 | 137.6    | 147.9  | 186.8       | 130.7       | 96.4        |
| <b>II.A1</b> | 0.006  | 186.1    | 206.6  | 270.8       | 174.3       | 117.1       |
| <b>II.A2</b> | 0.0056 | 208.6    | 240.5  | 322.3       | 192.0       | 117.5       |
| <b>II.A3</b> | 0.0055 | 219.2    | 260.1  | 353.9       | 197.7       | 114.7       |
| <b>II.B1</b> | 0.0061 | 187.8    | 213.1  | 282.0       | 173.2       | 112.1       |
| <b>II.B2</b> | 0.0059 | 195.5    | 223.9  | 298.0       | 179.0       | 114.4       |
| <b>II.B3</b> | 0.0056 | 212.9    | 251.1  | 339.8       | 191.0       | 115.6       |
| <b>III.1</b> | 0.0070 | 154.6    | 166.8  | 211.0       | 146.3       | 107.0       |
| <b>III.2</b> | 0.0064 | 175.4    | 197.9  | 259.1       | 161.0       | 109.1       |
| <b>III.3</b> | 0.006  | 193.7    | 224.3  | 300.0       | 174.7       | 111.6       |
| <b>III.4</b> | 0.0070 | 153.3    | 165.2  | 209.3       | 144.9       | 106.7       |
| <b>III.5</b> | 0.0063 | 178.8    | 203.4  | 268.0       | 163.6       | 108.5       |
| <b>III.6</b> | 0.0061 | 192.1    | 225.3  | 302.2       | 171.7       | 110.0       |
| <b>IV.A1</b> | 0.0072 | 149.9    | 161.7  | 204.0       | 141.3       | 104.8       |
| <b>IV.A2</b> | 0.0066 | 172.0    | 195.7  | 255.8       | 156.4       | 106.0       |
| <b>IV.A3</b> | 0.0062 | 186.6    | 218.3  | 293.7       | 166.9       | 106.7       |
| <b>IV.A4</b> | 0.0071 | 152.4    | 165.3  | 209.4       | 143.5       | 105.5       |
| <b>IV.A5</b> | 0.0064 | 178.5    | 205    | 271.2       | 161.3       | 107.4       |
| <b>IV.A6</b> | 0.0060 | 196.8    | 231.6  | 314.9       | 175.3       | 109.9       |
| <b>IV.B1</b> | 0.0070 | 155.7    | 169.7  | 215.4       | 146.3       | 106.2       |
| <b>IV.B2</b> | 0.0067 | 164.6    | 184.8  | 239.0       | 150.7       | 106.2       |
| <b>IV.B3</b> | 0.0065 | 176.2    | 203.7  | 268.8       | 158.0       | 106.5       |
| <b>V.1</b>   | 0.0063 | 175.8    | 195.4  | 254.1       | 162.4       | 113.6       |
| <b>V.2</b>   | 0.0061 | 184.9    | 209.6  | 276.8       | 168.6       | 113.7       |
| <b>V.3</b>   | 0.0057 | 203.6    | 235.4  | 320.0       | 183.8       | 117.1       |

Table C.26. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0025 | 427.4  | 438.8  | 321.5       | 470.5       | 462.1       |
| <b>I.B1</b>  | 0.0024 | 440.8  | 452.5  | 333.6       | 489.4       | 469.5       |
| <b>I.B2</b>  | 0.0024 | 447.4  | 459.1  | 341.8       | 497.4       | 470.6       |
| <b>I.B3</b>  | 0.0024 | 443.4  | 456.0  | 340.7       | 498.8       | 454.8       |
| <b>I.B4</b>  | 0.0024 | 452.7  | 466.8  | 349.7       | 516.5       | 452.7       |
| <b>II.A1</b> | 0.0033 | 400.7  | 448.3  | 170.1       | 466.5       | 522.1       |
| <b>II.A2</b> | 0.0039 | 394.7  | 459.6  | 128.1       | 470.9       | 530.4       |
| <b>II.A3</b> | 0.0041 | 423.7  | 499.3  | 121.8       | 528.9       | 547.8       |
| <b>II.B1</b> | 0.0037 | 391.8  | 444.5  | 148.3       | 467.7       | 507.2       |
| <b>II.B2</b> | 0.0039 | 398.0  | 458.3  | 135.3       | 477.0       | 531.9       |
| <b>II.B3</b> | 0.0041 | 415.5  | 484.7  | 127.0       | 512.2       | 536.3       |
| <b>III.1</b> | 0.0028 | 417.1  | 442.1  | 249.6       | 479.7       | 477.6       |
| <b>III.2</b> | 0.0037 | 385.9  | 432.5  | 155.0       | 459.1       | 492.1       |
| <b>III.3</b> | 0.0043 | 381.9  | 442.7  | 120.2       | 462.8       | 507.6       |
| <b>III.4</b> | 0.0028 | 417.2  | 442.7  | 246.7       | 480.7       | 478.7       |
| <b>III.5</b> | 0.0037 | 386.9  | 435.2  | 150.1       | 463.5       | 491.4       |
| <b>III.6</b> | 0.0044 | 380.9  | 441.9  | 120.3       | 461.1       | 505.1       |
| <b>IV.A1</b> | 0.0027 | 420.7  | 443.8  | 263.0       | 483.2       | 472.5       |
| <b>IV.A2</b> | 0.0037 | 388.7  | 432.7  | 159.7       | 471.6       | 479.2       |
| <b>IV.A3</b> | 0.0043 | 376.8  | 434.6  | 120.7       | 464.9       | 486.7       |
| <b>IV.A4</b> | 0.0028 | 410.4  | 434.2  | 247.2       | 470.3       | 465.7       |
| <b>IV.A5</b> | 0.0039 | 386.1  | 437.9  | 140.2       | 467.5       | 491.2       |
| <b>IV.A6</b> | 0.0050 | 363.8  | 431.1  | 97.4        | 436.8       | 503.4       |
| <b>IV.B1</b> | 0.0029 | 410.7  | 439.9  | 225.0       | 479.2       | 480.7       |
| <b>IV.B2</b> | 0.0033 | 395.7  | 431.2  | 191.9       | 470.2       | 471.7       |
| <b>IV.B3</b> | 0.0041 | 383.1  | 433.0  | 140.4       | 468.6       | 480.5       |
| <b>V.1</b>   | 0.0037 | 375.0  | 420.0  | 156.2       | 434.7       | 494.1       |
| <b>V.2</b>   | 0.0043 | 354.4  | 408.8  | 120.7       | 409.6       | 494.4       |
| <b>V.3</b>   | 0.0058 | 325.5  | 396.1  | 83.1        | 362.8       | 503.8       |

Table C.27. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 5 (d=2, K=0.10).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0031 | 379.3    | 410.6  | 207.9       | 407.2       | 504.0       |
| <b>IV.A2</b> | 0.0032 | 387.2    | 424.9  | 192.9       | 427.2       | 519.2       |
| <b>IV.A3</b> | 0.0031 | 401.5    | 441.8  | 192.9       | 452.9       | 526.5       |
| <b>IV.A4</b> | 0.0031 | 380.3    | 410.9  | 210.5       | 412.3       | 499.6       |
| <b>IV.A5</b> | 0.0032 | 384.8    | 421.8  | 193.9       | 420.9       | 515.7       |
| <b>IV.A6</b> | 0.0031 | 401.6    | 441.7  | 191.9       | 449.2       | 528.8       |
| <b>IV.B1</b> | 0.0028 | 405.2    | 430.1  | 245.4       | 446.0       | 500.7       |
| <b>IV.B2</b> | 0.0027 | 412.9    | 438.5  | 246.3       | 459.1       | 503.2       |
| <b>IV.B3</b> | 0.0027 | 420.8    | 447.9  | 247.3       | 477.4       | 499.5       |
| <b>V.1</b>   | 0.0028 | 402.9    | 426.8  | 246.5       | 439.8       | 498.8       |
| <b>V.2</b>   | 0.0027 | 409.9    | 435.0  | 246.1       | 453.5       | 503.2       |
| <b>V.3</b>   | 0.0027 | 415.6    | 442.0  | 244.3       | 466.6       | 498.9       |

Table C.28. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 5 (d=2, K=0.10).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0964 | 12.0   | 13.4   | 6.9         | 11.0        | 18.2        |
| <b>IV.A2</b> | 0.0965 | 12.4   | 14.5   | 6.6         | 11.1        | 19.7        |
| <b>IV.A3</b> | 0.0932 | 13.3   | 16.1   | 6.7         | 11.7        | 21.7        |
| <b>IV.A4</b> | 0.0957 | 12.1   | 13.5   | 7.0         | 11.0        | 18.4        |
| <b>IV.A5</b> | 0.0950 | 12.6   | 14.7   | 6.7         | 11.3        | 20.0        |
| <b>IV.A6</b> | 0.0928 | 13.5   | 16.5   | 6.6         | 11.8        | 22.2        |
| <b>IV.B1</b> | 0.0872 | 13.3   | 15.0   | 7.6         | 12.2        | 20.5        |
| <b>IV.B2</b> | 0.0855 | 13.8   | 15.9   | 7.6         | 12.5        | 21.6        |
| <b>IV.B3</b> | 0.0831 | 14.7   | 17.4   | 7.5         | 13.2        | 23.6        |
| <b>V.1</b>   | 0.087  | 13.3   | 15.1   | 7.6         | 12.1        | 20.5        |
| <b>V.2</b>   | 0.0863 | 13.7   | 15.7   | 7.5         | 12.4        | 21.3        |
| <b>V.3</b>   | 0.0841 | 14.4   | 16.9   | 7.5         | 12.9        | 22.9        |

Table C.29. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 5 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0036 | 335.9    | 374.9  | 166.6       | 346.1       | 494.0       |
| <b>IV.A2</b> | 0.0032 | 379.8    | 418.3  | 187.9       | 412.1       | 521.8       |
| <b>IV.A3</b> | 0.0030 | 405.0    | 442.9  | 204.0       | 454.0       | 527.1       |
| <b>IV.A4</b> | 0.0035 | 340.1    | 377.7  | 172.6       | 353.1       | 490.7       |
| <b>IV.A5</b> | 0.0031 | 385.8    | 421.2  | 201.4       | 419.6       | 519.1       |
| <b>IV.A6</b> | 0.0030 | 411.8    | 447.9  | 212.4       | 460.6       | 527.6       |
| <b>IV.B1</b> | 0.0028 | 406.5    | 431.7  | 244.0       | 450.2       | 499.9       |
| <b>IV.B2</b> | 0.0028 | 413.5    | 440.2  | 242.1       | 462.5       | 503.8       |
| <b>IV.B3</b> | 0.0028 | 417.1    | 447.7  | 241.9       | 472.3       | 496.8       |
| <b>V.1</b>   | 0.0028 | 401.1    | 426.2  | 239.4       | 441.0       | 496.9       |
| <b>V.2</b>   | 0.0028 | 407.0    | 433.6  | 238.7       | 451.1       | 501.4       |
| <b>V.3</b>   | 0.0028 | 413.4    | 441.1  | 239.1       | 467.5       | 496.2       |

Table C.30. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 5 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.1110 | 10.2   | 11.1   | 6.2         | 9.4         | 15.0        |
| <b>IV.A2</b> | 0.0993 | 11.8   | 13.4   | 6.6         | 10.8        | 18.3        |
| <b>IV.A3</b> | 0.0926 | 13.2   | 15.6   | 6.8         | 11.7        | 21.3        |
| <b>IV.A4</b> | 0.1091 | 10.3   | 11.2   | 6.3         | 9.6         | 15.3        |
| <b>IV.A5</b> | 0.0955 | 12.3   | 14.1   | 6.9         | 11.2        | 19.2        |
| <b>IV.A6</b> | 0.0897 | 13.7   | 16.4   | 6.9         | 12.2        | 22.1        |
| <b>IV.B1</b> | 0.0876 | 13.4   | 15.4   | 7.5         | 12.2        | 20.8        |
| <b>IV.B2</b> | 0.0853 | 14.0   | 16.4   | 7.5         | 12.6        | 22.3        |
| <b>IV.B3</b> | 0.0831 | 14.9   | 17.9   | 7.5         | 13.1        | 24.2        |
| <b>V.1</b>   | 0.0870 | 13.5   | 15.6   | 7.5         | 12.3        | 21.1        |
| <b>V.2</b>   | 0.0865 | 13.8   | 16.1   | 7.5         | 12.3        | 21.9        |
| <b>V.3</b>   | 0.0844 | 14.5   | 17.4   | 7.4         | 12.9        | 23.4        |

Table C.31. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under CM 5 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0041 | 309.6    | 354.9  | 139.7       | 309.3       | 481.8       |
| <b>IV.A2</b> | 0.0035 | 354.4    | 397.3  | 165.4       | 375.5       | 515.1       |
| <b>IV.A3</b> | 0.0033 | 380.8    | 423.0  | 178.2       | 417.2       | 522.6       |
| <b>IV.A4</b> | 0.0038 | 323.8    | 367.8  | 153.0       | 325.6       | 495.1       |
| <b>IV.A5</b> | 0.0033 | 372.2    | 411.8  | 182.1       | 399.2       | 518.5       |
| <b>IV.A6</b> | 0.0031 | 395.3    | 434.5  | 194.2       | 436.2       | 526.5       |
| <b>IV.B1</b> | 0.0028 | 402.3    | 429.0  | 232.7       | 446.7       | 500.6       |
| <b>IV.B2</b> | 0.0028 | 409.5    | 437.0  | 235.6       | 457.0       | 503.9       |
| <b>IV.B3</b> | 0.0028 | 418.4    | 447.6  | 235.0       | 477.0       | 499.2       |
| <b>V.1</b>   | 0.0028 | 403.8    | 430.6  | 234.7       | 448.6       | 500.0       |
| <b>V.2</b>   | 0.0028 | 411.7    | 440.2  | 234.3       | 460.3       | 509.7       |
| <b>V.3</b>   | 0.0028 | 419.8    | 449.7  | 234.8       | 474.8       | 507.0       |

Table C.32. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under CM 5 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.1193 | 9.5    | 10.3   | 5.8         | 8.7         | 14.0        |
| <b>IV.A2</b> | 0.1065 | 11.0   | 12.4   | 6.2         | 10.0        | 16.9        |
| <b>IV.A3</b> | 0.1001 | 12.1   | 14.2   | 6.3         | 10.8        | 19.4        |
| <b>IV.A4</b> | 0.1168 | 9.6    | 10.4   | 5.9         | 8.9         | 14.1        |
| <b>IV.A5</b> | 0.1015 | 11.5   | 13.1   | 6.5         | 10.5        | 17.8        |
| <b>IV.A6</b> | 0.0941 | 13.0   | 15.3   | 6.7         | 11.6        | 21.0        |
| <b>IV.B1</b> | 0.0879 | 13.4   | 15.5   | 7.4         | 12.1        | 20.9        |
| <b>IV.B2</b> | 0.0857 | 14.1   | 16.6   | 7.5         | 12.6        | 22.5        |
| <b>IV.B3</b> | 0.0833 | 14.9   | 18.2   | 7.4         | 13.1        | 24.5        |
| <b>V.1</b>   | 0.0874 | 13.6   | 15.8   | 7.4         | 12.2        | 21.3        |
| <b>V.2</b>   | 0.0868 | 13.8   | 16.2   | 7.4         | 12.4        | 21.9        |
| <b>V.3</b>   | 0.084  | 14.7   | 17.8   | 7.4         | 13.1        | 24.1        |

Table C.33. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 1.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0045 | 240.8    | 257.2  | 328.0       | 232.2       | 164.6       |
| <b>I.B1</b>  | 0.0027 | 393.6    | 411.7  | 500.8       | 413.6       | 271.3       |
| <b>I.B2</b>  | 0.0026 | 408.1    | 422.3  | 469.8       | 441.3       | 300.6       |
| <b>I.B3</b>  | 0.0024 | 427.6    | 441.0  | 474.0       | 467.4       | 323.7       |
| <b>I.B4</b>  | 0.0025 | 418.4    | 431.1  | 474.5       | 455.0       | 317.8       |
| <b>II.A1</b> | 0.0026 | 402.4    | 423.2  | 542.9       | 406.8       | 275.4       |
| <b>II.A2</b> | 0.0025 | 432.0    | 456.0  | 569.7       | 450.4       | 291.5       |
| <b>II.A3</b> | 0.0025 | 438.7    | 473.9  | 619.9       | 454.0       | 269.4       |
| <b>II.B1</b> | 0.0025 | 434.9    | 455.7  | 532.7       | 465.5       | 305.2       |
| <b>II.B2</b> | 0.0025 | 434.3    | 454.5  | 511.8       | 473.1       | 309.8       |
| <b>II.B3</b> | 0.0025 | 443.4    | 472.1  | 560.9       | 480.4       | 293.0       |
| <b>III.1</b> | 0.0024 | 429.1    | 441.2  | 477.4       | 466.2       | 331.2       |
| <b>III.2</b> | 0.0024 | 443.4    | 456.1  | 436.8       | 496.2       | 365.7       |
| <b>III.3</b> | 0.0023 | 457.5    | 472.0  | 435.6       | 517.0       | 382.8       |
| <b>III.4</b> | 0.0024 | 427.3    | 439.1  | 468.1       | 465.2       | 332.4       |
| <b>III.5</b> | 0.0023 | 450.2    | 463.2  | 435.1       | 508.5       | 375.3       |
| <b>III.6</b> | 0.0023 | 463.3    | 479.3  | 437.4       | 530.1       | 381.9       |
| <b>IV.A1</b> | 0.0025 | 420.4    | 430.4  | 439.5       | 461.1       | 339.5       |
| <b>IV.A2</b> | 0.0024 | 445.4    | 457.6  | 412.4       | 505.2       | 379.6       |
| <b>IV.A3</b> | 0.0024 | 449.4    | 462.3  | 413.0       | 511.1       | 386.4       |
| <b>IV.A4</b> | 0.0024 | 435.0    | 446.2  | 461.1       | 478.6       | 347.0       |
| <b>IV.A5</b> | 0.0024 | 446.5    | 458.8  | 411.3       | 507.6       | 382.8       |
| <b>IV.A6</b> | 0.0023 | 456.6    | 471.7  | 403.8       | 525.5       | 393.1       |
| <b>IV.B1</b> | 0.0024 | 435.1    | 445.5  | 430.9       | 485.2       | 360.4       |
| <b>IV.B2</b> | 0.0024 | 439.2    | 450.2  | 409.6       | 496.0       | 379.0       |
| <b>IV.B3</b> | 0.0023 | 450.6    | 463.7  | 403.0       | 513.5       | 391.4       |
| <b>V.1</b>   | 0.0024 | 439.9    | 449.4  | 411.9       | 495.7       | 381.1       |
| <b>V.2</b>   | 0.0023 | 444.8    | 455.3  | 396.1       | 503.2       | 398.4       |
| <b>V.3</b>   | 0.0023 | 451.5    | 464.2  | 382.7       | 520.0       | 407.8       |

Table C.34. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 1.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0075 | 206.5  | 270.2  | 68.2        | 177.7       | 403.4       |
| <b>I.B1</b>  | 0.0314 | 58.8   | 102.5  | 16.0        | 39.2        | 123.7       |
| <b>I.B2</b>  | 0.0375 | 46.2   | 79.8   | 13.8        | 31.9        | 91.8        |
| <b>I.B3</b>  | 0.0396 | 41.8   | 70.9   | 13.5        | 29.5        | 81.1        |
| <b>I.B4</b>  | 0.0388 | 42.4   | 71.1   | 13.7        | 30.4        | 81.9        |
| <b>II.A1</b> | 0.0238 | 76.2   | 123.8  | 20.9        | 53.8        | 158.2       |
| <b>II.A2</b> | 0.0278 | 75.2   | 133.8  | 17.1        | 48.9        | 165.0       |
| <b>II.A3</b> | 0.0245 | 108.9  | 201.7  | 17.9        | 63.5        | 267.2       |
| <b>II.B1</b> | 0.0335 | 58.0   | 102.6  | 14.8        | 38.7        | 121.4       |
| <b>II.B2</b> | 0.035  | 56.2   | 99.4   | 13.8        | 37.3        | 117.7       |
| <b>II.B3</b> | 0.0305 | 81.2   | 153.2  | 14.8        | 48.0        | 190.5       |
| <b>III.1</b> | 0.0403 | 39.1   | 62.2   | 13.3        | 29.1        | 74.0        |
| <b>III.2</b> | 0.0471 | 32.7   | 49.9   | 11.5        | 24.8        | 61.8        |
| <b>III.3</b> | 0.0474 | 33.6   | 52.8   | 11.3        | 25.1        | 63.7        |
| <b>III.4</b> | 0.0411 | 37.6   | 58.2   | 13.2        | 28.3        | 71.5        |
| <b>III.5</b> | 0.0481 | 32.2   | 49.3   | 11.2        | 24.6        | 60.7        |
| <b>III.6</b> | 0.0486 | 33.0   | 52.0   | 10.8        | 24.9        | 62.9        |
| <b>IV.A1</b> | 0.0441 | 34.1   | 51.7   | 12.5        | 26.1        | 63.6        |
| <b>IV.A2</b> | 0.0504 | 29.1   | 42.5   | 11.0        | 22.8        | 53.7        |
| <b>IV.A3</b> | 0.0506 | 29.5   | 43.9   | 10.9        | 23.0        | 54.6        |
| <b>IV.A4</b> | 0.0437 | 34.7   | 52.8   | 12.6        | 26.6        | 64.9        |
| <b>IV.A5</b> | 0.0512 | 28.9   | 43.1   | 10.8        | 22.5        | 53.4        |
| <b>IV.A6</b> | 0.0521 | 29.3   | 44.4   | 10.4        | 22.6        | 54.9        |
| <b>IV.B1</b> | 0.0478 | 30.9   | 46.2   | 11.7        | 23.9        | 57.0        |
| <b>IV.B2</b> | 0.0512 | 28.4   | 41.8   | 11.0        | 22.2        | 51.9        |
| <b>IV.B3</b> | 0.0525 | 28.0   | 41.5   | 10.7        | 21.9        | 51.3        |
| <b>V.1</b>   | 0.051  | 27.5   | 39.2   | 11.2        | 22.1        | 49.6        |
| <b>V.2</b>   | 0.0543 | 25.6   | 36.2   | 10.6        | 20.6        | 45.1        |
| <b>V.3</b>   | 0.056  | 24.9   | 34.8   | 10.3        | 20.2        | 44.6        |

Table C.35. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 2.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0032 | 332.0    | 346.0  | 431.3       | 326.7       | 244.8       |
| <b>I.B1</b>  | 0.0031 | 341.0    | 357.1  | 454.9       | 334.2       | 245.3       |
| <b>I.B2</b>  | 0.003  | 358.9    | 377.3  | 488.5       | 354.2       | 252.5       |
| <b>I.B3</b>  | 0.0029 | 366.0    | 384.3  | 494.8       | 367.8       | 253.7       |
| <b>I.B4</b>  | 0.0028 | 380.9    | 400.6  | 509.9       | 390.2       | 260.1       |
| <b>II.A1</b> | 0.0025 | 421.2    | 440.0  | 546.0       | 435.2       | 295.7       |
| <b>II.A2</b> | 0.0024 | 443.3    | 467.2  | 568.7       | 468.6       | 302.9       |
| <b>II.A3</b> | 0.0025 | 449.3    | 482.9  | 604.7       | 475.5       | 279.6       |
| <b>II.B1</b> | 0.0025 | 419.8    | 439.9  | 539.1       | 439.1       | 292.4       |
| <b>II.B2</b> | 0.0025 | 428.7    | 450.7  | 545.1       | 454.9       | 297.7       |
| <b>II.B3</b> | 0.0025 | 449.2    | 479.1  | 592.5       | 474.7       | 293.4       |
| <b>III.1</b> | 0.0027 | 395.1    | 410.0  | 502.8       | 408.0       | 287.2       |
| <b>III.2</b> | 0.0025 | 429.5    | 445.8  | 494.1       | 466.6       | 318.6       |
| <b>III.3</b> | 0.0024 | 443.5    | 461.1  | 471.7       | 499.3       | 335.9       |
| <b>III.4</b> | 0.0027 | 396.2    | 411.5  | 504.1       | 410.1       | 284.0       |
| <b>III.5</b> | 0.0025 | 426.6    | 442.4  | 485.2       | 462.8       | 319.1       |
| <b>III.6</b> | 0.0024 | 438.2    | 455.8  | 469.9       | 488.2       | 330.7       |
| <b>IV.A1</b> | 0.0026 | 410.7    | 424.7  | 479.8       | 443.0       | 305.6       |
| <b>IV.A2</b> | 0.0024 | 440.6    | 455.3  | 442.9       | 496.3       | 349.5       |
| <b>IV.A3</b> | 0.0024 | 440.9    | 457.0  | 412.1       | 507.3       | 358.6       |
| <b>IV.A4</b> | 0.0025 | 419.2    | 434.4  | 495.9       | 449.1       | 306.7       |
| <b>IV.A5</b> | 0.0024 | 439.5    | 453.6  | 427.7       | 497.2       | 356.4       |
| <b>IV.A6</b> | 0.0024 | 446.4    | 463.7  | 400.2       | 521.7       | 370.7       |
| <b>IV.B1</b> | 0.0026 | 400.5    | 417.1  | 493.8       | 424.9       | 283.9       |
| <b>IV.B2</b> | 0.0026 | 412.2    | 428.7  | 482.1       | 446.5       | 300.2       |
| <b>IV.B3</b> | 0.0025 | 431.4    | 450.0  | 457.8       | 486.1       | 320.9       |
| <b>V.1</b>   | 0.0025 | 414.7    | 429.0  | 480.1       | 446.7       | 308.4       |
| <b>V.2</b>   | 0.0025 | 423.9    | 438.6  | 463.1       | 470.1       | 319.9       |
| <b>V.3</b>   | 0.0025 | 430.7    | 445.7  | 428.9       | 489.5       | 340.5       |

Table C.36. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 2.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0171 | 84.0   | 114.5  | 32.4        | 68.8        | 155.5       |
| <b>I.B1</b>  | 0.0188 | 82.3   | 117.2  | 28.2        | 65.5        | 158.3       |
| <b>I.B2</b>  | 0.0214 | 77.0   | 114.0  | 23.9        | 59.7        | 152.9       |
| <b>I.B3</b>  | 0.0244 | 73.3   | 114.3  | 20.2        | 53.8        | 150.8       |
| <b>I.B4</b>  | 0.0269 | 70.3   | 114.9  | 18.1        | 49.3        | 148.9       |
| <b>II.A1</b> | 0.0282 | 62.6   | 101.8  | 17.9        | 44.8        | 126.9       |
| <b>II.A2</b> | 0.0301 | 69.8   | 125.4  | 15.6        | 44.9        | 153.1       |
| <b>II.A3</b> | 0.0279 | 96.5   | 185.2  | 15.7        | 54.4        | 235.5       |
| <b>II.B1</b> | 0.0309 | 62.9   | 108.3  | 15.8        | 42.5        | 133.2       |
| <b>II.B2</b> | 0.0318 | 62.3   | 107.3  | 15.2        | 42.4        | 132.9       |
| <b>II.B3</b> | 0.0298 | 82.9   | 155.3  | 15.1        | 49.7        | 194.6       |
| <b>III.1</b> | 0.0298 | 55.9   | 86.9   | 17.3        | 41.5        | 111.4       |
| <b>III.2</b> | 0.0381 | 45.4   | 74.4   | 13.4        | 32.8        | 91.1        |
| <b>III.3</b> | 0.0405 | 45.2   | 78.0   | 12.4        | 31.2        | 92.2        |
| <b>III.4</b> | 0.0304 | 54.5   | 84.4   | 17.0        | 40.5        | 108.6       |
| <b>III.5</b> | 0.0383 | 44.6   | 72.9   | 13.4        | 32.1        | 89.2        |
| <b>III.6</b> | 0.0404 | 44.9   | 76.2   | 12.4        | 31.3        | 91.3        |
| <b>IV.A1</b> | 0.0365 | 46.3   | 74.7   | 14.1        | 33.5        | 92.8        |
| <b>IV.A2</b> | 0.0458 | 36.0   | 58.9   | 11.5        | 26.0        | 70.0        |
| <b>IV.A3</b> | 0.0489 | 34.6   | 58.1   | 10.7        | 24.6        | 68.0        |
| <b>IV.A4</b> | 0.0369 | 45.7   | 73.3   | 14.0        | 32.9        | 91.1        |
| <b>IV.A5</b> | 0.0477 | 34.4   | 56.4   | 11.1        | 24.9        | 66.5        |
| <b>IV.A6</b> | 0.0501 | 33.5   | 56.0   | 10.5        | 24.1        | 65.3        |
| <b>IV.B1</b> | 0.0334 | 53.4   | 88.0   | 15.0        | 37.4        | 110.4       |
| <b>IV.B2</b> | 0.0373 | 48.3   | 81.3   | 13.5        | 33.5        | 99.0        |
| <b>IV.B3</b> | 0.0418 | 44.0   | 76.1   | 12.0        | 29.9        | 90.2        |
| <b>V.1</b>   | 0.0380 | 43.7   | 70.7   | 13.8        | 31.4        | 86.7        |
| <b>V.2</b>   | 0.0416 | 40.1   | 65.7   | 12.6        | 28.9        | 79.7        |
| <b>V.3</b>   | 0.0462 | 36.6   | 61.3   | 11.3        | 26.0        | 72.3        |

Table C.37. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 3.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0058 | 186.5    | 199.6  | 255.3       | 179.3       | 128.1       |
| <b>I.B1</b>  | 0.0049 | 220.8    | 240.4  | 312.5       | 209.9       | 144.4       |
| <b>I.B2</b>  | 0.0047 | 235.6    | 259.0  | 338.8       | 222.3       | 150.7       |
| <b>I.B3</b>  | 0.0045 | 248.8    | 275.9  | 371.4       | 232.4       | 154.2       |
| <b>I.B4</b>  | 0.0044 | 259.0    | 289.1  | 393.3       | 242.4       | 154.5       |
| <b>II.A1</b> | 0.0035 | 317.4    | 346.1  | 462.6       | 304.5       | 199.5       |
| <b>II.A2</b> | 0.0033 | 340.1    | 375.0  | 517.0       | 326.8       | 202.8       |
| <b>II.A3</b> | 0.0033 | 351.8    | 393.9  | 558.2       | 336.4       | 197.6       |
| <b>II.B1</b> | 0.0034 | 325.6    | 356.8  | 489.0       | 313.2       | 198.4       |
| <b>II.B2</b> | 0.0033 | 334.5    | 367.5  | 509.3       | 320.4       | 202.2       |
| <b>II.B3</b> | 0.0032 | 361.7    | 404.4  | 579.6       | 347.1       | 204.5       |
| <b>III.1</b> | 0.0037 | 297.2    | 322.3  | 432.2       | 285.6       | 190.4       |
| <b>III.2</b> | 0.0032 | 342.4    | 369.0  | 499.8       | 337.2       | 214.6       |
| <b>III.3</b> | 0.0029 | 373.3    | 401.4  | 540.7       | 375.2       | 233.2       |
| <b>III.4</b> | 0.0038 | 293.1    | 318.2  | 428.0       | 281.1       | 185.9       |
| <b>III.5</b> | 0.0032 | 344.8    | 370.0  | 499.4       | 340.7       | 220.1       |
| <b>III.6</b> | 0.0030 | 364.4    | 392.4  | 533.3       | 365.8       | 228.0       |
| <b>IV.A1</b> | 0.0035 | 309.7    | 335.2  | 450.8       | 298.4       | 197.1       |
| <b>IV.A2</b> | 0.0029 | 370.4    | 395.3  | 521.4       | 375.3       | 238.0       |
| <b>IV.A3</b> | 0.0028 | 382.8    | 407.7  | 526.7       | 394.5       | 247.0       |
| <b>IV.A4</b> | 0.0035 | 311.5    | 337.5  | 456.3       | 300.3       | 196.3       |
| <b>IV.A5</b> | 0.0030 | 364.8    | 389.0  | 513.9       | 368.4       | 234.9       |
| <b>IV.A6</b> | 0.0028 | 395.0    | 419.6  | 531.4       | 411.5       | 255.8       |
| <b>IV.B1</b> | 0.0034 | 320.7    | 347.8  | 473.2       | 309.6       | 200.9       |
| <b>IV.B2</b> | 0.0032 | 341.8    | 368.4  | 503.5       | 337.2       | 214.7       |
| <b>IV.B3</b> | 0.0030 | 365.8    | 393.1  | 524.2       | 369.4       | 229.9       |
| <b>V.1</b>   | 0.0030 | 356.9    | 379.3  | 499.3       | 356.9       | 234.7       |
| <b>V.2</b>   | 0.0030 | 363.5    | 384.5  | 498.8       | 367.9       | 240.9       |
| <b>V.3</b>   | 0.0028 | 390.4    | 411.4  | 511.7       | 408.4       | 262.6       |

Table C.38. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 3.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0041 | 321.5  | 368.1  | 136.2       | 335.5       | 488.0       |
| <b>I.B1</b>  | 0.0061 | 256.3  | 322.3  | 80.9        | 239.1       | 473.0       |
| <b>I.B2</b>  | 0.0074 | 234.0  | 305.7  | 64.2        | 209.2       | 457.5       |
| <b>I.B3</b>  | 0.0087 | 222.9  | 303.8  | 52.7        | 190.3       | 463.3       |
| <b>I.B4</b>  | 0.0095 | 219.2  | 304.8  | 47.5        | 182.3       | 467.9       |
| <b>II.A1</b> | 0.0116 | 168.3  | 249.5  | 40.5        | 126.7       | 374.1       |
| <b>II.A2</b> | 0.0133 | 172.8  | 271.1  | 32.9        | 120.0       | 417.2       |
| <b>II.A3</b> | 0.0131 | 210.7  | 334.4  | 32.0        | 143.2       | 540.6       |
| <b>II.B1</b> | 0.0139 | 160.1  | 248.3  | 32.1        | 114.0       | 374.5       |
| <b>II.B2</b> | 0.0146 | 156.4  | 246.9  | 30.4        | 109.3       | 373.9       |
| <b>II.B3</b> | 0.0143 | 187.8  | 300.3  | 29.5        | 127.3       | 469.0       |
| <b>III.1</b> | 0.0128 | 153.2  | 228.4  | 36.5        | 114.9       | 342.8       |
| <b>III.2</b> | 0.0197 | 111.0  | 183.6  | 23.0        | 74.2        | 258.2       |
| <b>III.3</b> | 0.0216 | 105.2  | 179.9  | 21.0        | 67.7        | 245.5       |
| <b>III.4</b> | 0.0132 | 147.6  | 219.9  | 35.3        | 111.0       | 327.2       |
| <b>III.5</b> | 0.0199 | 108.6  | 178.7  | 23.0        | 72.9        | 249.0       |
| <b>III.6</b> | 0.022  | 105.3  | 181.1  | 20.6        | 66.5        | 249.4       |
| <b>IV.A1</b> | 0.0154 | 131.4  | 202.2  | 30.1        | 94.8        | 295.1       |
| <b>IV.A2</b> | 0.0236 | 89.2   | 152.0  | 19.8        | 58.5        | 201.9       |
| <b>IV.A3</b> | 0.0265 | 81.7   | 144.1  | 17.5        | 52.1        | 186.5       |
| <b>IV.A4</b> | 0.0154 | 131.8  | 204.0  | 29.7        | 94.4        | 295.7       |
| <b>IV.A5</b> | 0.0241 | 87.7   | 150.1  | 19.3        | 57.4        | 198.7       |
| <b>IV.A6</b> | 0.0282 | 78.6   | 140.9  | 16.6        | 49.6        | 177.3       |
| <b>IV.B1</b> | 0.0169 | 121.1  | 190.2  | 27.4        | 85.9        | 271.8       |
| <b>IV.B2</b> | 0.0201 | 107.7  | 177.1  | 22.9        | 72.3        | 250.9       |
| <b>IV.B3</b> | 0.0234 | 96.1   | 165.9  | 19.5        | 61.9        | 223.6       |
| <b>V.1</b>   | 0.0212 | 90.0   | 145.6  | 22.7        | 63.0        | 195.0       |
| <b>V.2</b>   | 0.0248 | 78.7   | 131.8  | 19.5        | 53.4        | 170.0       |
| <b>V.3</b>   | 0.0292 | 68.4   | 117.7  | 16.5        | 45.5        | 147.9       |

Table C.39. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 4.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0025 | 407.7    | 416.2  | 490.9       | 413.9       | 322.0       |
| <b>I.B1</b>  | 0.0026 | 421.4    | 439.1  | 282.7       | 471.7       | 478.3       |
| <b>I.B2</b>  | 0.0026 | 423.7    | 442.7  | 279.1       | 471.7       | 491.3       |
| <b>I.B3</b>  | 0.0025 | 429.0    | 448.1  | 286.6       | 479.7       | 490.6       |
| <b>I.B4</b>  | 0.0025 | 440.7    | 458.3  | 303.9       | 496.3       | 490.4       |
| <b>II.A1</b> | 0.0022 | 479.4    | 492.4  | 445.4       | 541.1       | 410.0       |
| <b>II.A2</b> | 0.0023 | 484.9    | 506.5  | 389.9       | 576.5       | 414.4       |
| <b>II.A3</b> | 0.0023 | 486.0    | 515.3  | 429.0       | 581.5       | 366.1       |
| <b>II.B1</b> | 0.0025 | 460.2    | 483.8  | 312.4       | 553.9       | 441.2       |
| <b>II.B2</b> | 0.0025 | 461.1    | 487.7  | 293.2       | 559.2       | 450.9       |
| <b>II.B3</b> | 0.0024 | 484.5    | 514.6  | 334.6       | 598.4       | 420.7       |
| <b>III.1</b> | 0.0024 | 445.8    | 463.5  | 309.9       | 505.7       | 484.8       |
| <b>III.2</b> | 0.0026 | 436.4    | 457.6  | 283.5       | 501.1       | 475.7       |
| <b>III.3</b> | 0.0025 | 463.9    | 489.2  | 292.1       | 549.0       | 491.4       |
| <b>III.4</b> | 0.0025 | 435.5    | 453.0  | 297.0       | 492.0       | 477.8       |
| <b>III.5</b> | 0.0025 | 441.6    | 462.7  | 291.4       | 510.5       | 472.4       |
| <b>III.6</b> | 0.0025 | 454.1    | 478.9  | 284.9       | 535.4       | 483.0       |
| <b>IV.A1</b> | 0.0028 | 405.9    | 429.6  | 245.7       | 455.1       | 485.1       |
| <b>IV.A2</b> | 0.0026 | 426.9    | 451.3  | 262.0       | 483.9       | 497.5       |
| <b>IV.A3</b> | 0.0026 | 436.5    | 461.9  | 264.7       | 507.0       | 488.9       |
| <b>IV.A4</b> | 0.0027 | 410.3    | 433.3  | 250.6       | 461.0       | 485.5       |
| <b>IV.A5</b> | 0.0027 | 428.4    | 455.0  | 252.9       | 490.1       | 499.1       |
| <b>IV.A6</b> | 0.0026 | 442.4    | 469.5  | 262.9       | 517.7       | 493.9       |
| <b>IV.B1</b> | 0.0026 | 421.5    | 441.2  | 277.5       | 481.1       | 470.3       |
| <b>IV.B2</b> | 0.0026 | 426.1    | 446.5  | 274.7       | 484.0       | 478.4       |
| <b>IV.B3</b> | 0.0026 | 433.3    | 456.2  | 271.9       | 499.0       | 480.5       |
| <b>V.1</b>   | 0.0026 | 425.2    | 445.7  | 275.6       | 483.0       | 480.4       |
| <b>V.2</b>   | 0.0026 | 420.1    | 441.2  | 270.2       | 474.9       | 481.7       |
| <b>V.3</b>   | 0.0026 | 430.2    | 454.2  | 265.5       | 493.6       | 486.7       |

Table C.40. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 4.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0298 | 43.5   | 55.7   | 20.1        | 37.2        | 74.5        |
| <b>I.B1</b>  | 0.078  | 15.2   | 17.9   | 8.3         | 13.5        | 24.1        |
| <b>I.B2</b>  | 0.079  | 15.0   | 17.4   | 8.2         | 13.5        | 23.4        |
| <b>I.B3</b>  | 0.0781 | 15.2   | 17.8   | 8.3         | 13.7        | 23.9        |
| <b>I.B4</b>  | 0.0741 | 16.4   | 19.6   | 8.6         | 14.6        | 26.3        |
| <b>II.A1</b> | 0.0491 | 29.9   | 43.8   | 11.2        | 23.6        | 55.4        |
| <b>II.A2</b> | 0.0528 | 32.3   | 53.9   | 9.9         | 23.0        | 63.9        |
| <b>II.A3</b> | 0.0457 | 49.2   | 97.9   | 10.3        | 29.3        | 106.8       |
| <b>II.B1</b> | 0.0639 | 23.6   | 36.3   | 8.6         | 18.1        | 43.7        |
| <b>II.B2</b> | 0.0654 | 23.6   | 36.7   | 8.3         | 17.8        | 44.4        |
| <b>II.B3</b> | 0.0557 | 34.5   | 64.7   | 8.9         | 22.7        | 70.3        |
| <b>III.1</b> | 0.0726 | 17.0   | 20.7   | 8.6         | 14.9        | 27.7        |
| <b>III.2</b> | 0.0734 | 17.6   | 22.2   | 8.2         | 15.1        | 29.7        |
| <b>III.3</b> | 0.0709 | 19.0   | 25.0   | 8.2         | 16.0        | 33.3        |
| <b>III.4</b> | 0.0737 | 16.7   | 20.3   | 8.5         | 14.6        | 27.1        |
| <b>III.5</b> | 0.0729 | 17.8   | 22.6   | 8.2         | 15.3        | 30.1        |
| <b>III.6</b> | 0.0708 | 19.0   | 25.2   | 8.3         | 16.0        | 33.1        |
| <b>IV.A1</b> | 0.0836 | 14.6   | 17.6   | 7.6         | 12.8        | 23.5        |
| <b>IV.A2</b> | 0.0800 | 15.6   | 19.1   | 7.7         | 13.7        | 25.8        |
| <b>IV.A3</b> | 0.0768 | 16.9   | 21.4   | 7.8         | 14.4        | 28.7        |
| <b>IV.A4</b> | 0.0829 | 14.7   | 17.9   | 7.6         | 12.9        | 23.8        |
| <b>IV.A5</b> | 0.0798 | 15.7   | 19.3   | 7.7         | 13.7        | 25.9        |
| <b>IV.A6</b> | 0.0766 | 17.1   | 22.0   | 7.7         | 14.6        | 29.1        |
| <b>IV.B1</b> | 0.0764 | 16.2   | 19.9   | 8.2         | 14.1        | 26.3        |
| <b>IV.B2</b> | 0.077  | 16.2   | 19.9   | 8.0         | 14.1        | 26.7        |
| <b>IV.B3</b> | 0.0754 | 16.9   | 21.4   | 8.1         | 14.5        | 28.3        |
| <b>V.1</b>   | 0.0771 | 16.0   | 19.6   | 8.1         | 14.0        | 26.2        |
| <b>V.2</b>   | 0.0785 | 15.7   | 19.1   | 8.0         | 13.8        | 25.5        |
| <b>V.3</b>   | 0.0772 | 16.3   | 200    | 7.9         | 14.2        | 27.0        |

Table C.41. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 5.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0028 | 379.9    | 396.1  | 493.4       | 383.8       | 270.1       |
| <b>I.B1</b>  | 0.0024 | 431.0    | 441.2  | 461.5       | 472.2       | 342.3       |
| <b>I.B2</b>  | 0.0023 | 444.0    | 453.8  | 446.4       | 490.1       | 367.5       |
| <b>I.B3</b>  | 0.0023 | 448.8    | 459.0  | 422.9       | 506.1       | 385.4       |
| <b>I.B4</b>  | 0.0024 | 441.7    | 451.5  | 415.6       | 492.5       | 385.0       |
| <b>II.A1</b> | 0.0022 | 465.7    | 478.6  | 478.1       | 519.9       | 377.5       |
| <b>II.A2</b> | 0.0023 | 471.6    | 491.6  | 462.9       | 543.0       | 365.5       |
| <b>II.A3</b> | 0.0023 | 483.4    | 514.0  | 481.0       | 560.7       | 343.1       |
| <b>II.B1</b> | 0.0023 | 455.0    | 471.1  | 441.1       | 515.0       | 366.5       |
| <b>II.B2</b> | 0.0023 | 465.0    | 483.1  | 442.5       | 532.9       | 375.7       |
| <b>II.B3</b> | 0.0023 | 492.7    | 518.8  | 472.9       | 575.2       | 373.0       |
| <b>III.1</b> | 0.0023 | 445.9    | 454.9  | 400         | 499.6       | 406.6       |
| <b>III.2</b> | 0.0024 | 450.9    | 464.3  | 368.2       | 518.1       | 422.5       |
| <b>III.3</b> | 0.0023 | 465.9    | 482.6  | 365.8       | 542.2       | 435.8       |
| <b>III.4</b> | 0.0023 | 449.7    | 458.7  | 400.0       | 503.7       | 411.7       |
| <b>III.5</b> | 0.0023 | 456.9    | 470.8  | 372.6       | 525.2       | 428.9       |
| <b>III.6</b> | 0.0024 | 461.9    | 478.3  | 370.9       | 536.2       | 423.7       |
| <b>IV.A1</b> | 0.0025 | 433.9    | 447.7  | 321.7       | 494.0       | 446.8       |
| <b>IV.A2</b> | 0.0024 | 452.2    | 469.0  | 321.7       | 517.7       | 470.5       |
| <b>IV.A3</b> | 0.0024 | 455.7    | 474.6  | 320.3       | 530.2       | 460.1       |
| <b>IV.A4</b> | 0.0024 | 440.9    | 454.7  | 330.1       | 502.8       | 450.0       |
| <b>IV.A5</b> | 0.0024 | 446.2    | 462.7  | 321.4       | 510.6       | 463.2       |
| <b>IV.A6</b> | 0.0024 | 451.2    | 470.1  | 318.0       | 526.0       | 455.2       |
| <b>IV.B1</b> | 0.0023 | 447.1    | 458.2  | 371.8       | 508.3       | 425.5       |
| <b>IV.B2</b> | 0.0024 | 443.1    | 455.3  | 360.1       | 505.2       | 424.2       |
| <b>IV.B3</b> | 0.0024 | 446.2    | 461.0  | 347.8       | 514.7       | 428.2       |
| <b>V.1</b>   | 0.0024 | 442.0    | 453.0  | 362.4       | 499.5       | 423.4       |
| <b>V.2</b>   | 0.0024 | 445.8    | 458.8  | 351.1       | 510.4       | 434.1       |
| <b>V.3</b>   | 0.0024 | 450.4    | 465.9  | 338.7       | 519.3       | 447.5       |

Table C.42. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 5.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0248 | 63.2   | 96.2   | 21.4        | 47.6        | 121.9       |
| <b>I.B1</b>  | 0.0431 | 34.4   | 52.4   | 12.9        | 26.2        | 63.8        |
| <b>I.B2</b>  | 0.0477 | 30.1   | 44.3   | 11.9        | 23.5        | 54.4        |
| <b>I.B3</b>  | 0.0508 | 27.7   | 39.8   | 11.3        | 22.1        | 49.8        |
| <b>I.B4</b>  | 0.0511 | 28.0   | 41.1   | 11.2        | 22.1        | 50.0        |
| <b>II.A1</b> | 0.0437 | 35.6   | 54.8   | 12.2        | 27.0        | 67.9        |
| <b>II.A2</b> | 0.0437 | 42.5   | 75.6   | 11.5        | 28.7        | 86.9        |
| <b>II.A3</b> | 0.0402 | 59.9   | 120.0  | 11.5        | 34.5        | 134.3       |
| <b>II.B1</b> | 0.0459 | 36.5   | 59.4   | 11.3        | 26.6        | 71.6        |
| <b>II.B2</b> | 0.0464 | 36.7   | 60.1   | 11.1        | 26.4        | 72.2        |
| <b>II.B3</b> | 0.0431 | 48.4   | 89.8   | 11.1        | 31.1        | 103.8       |
| <b>III.1</b> | 0.0545 | 24.4   | 32.8   | 10.9        | 20.4        | 42.2        |
| <b>III.2</b> | 0.0591 | 23.3   | 31.7   | 9.8         | 19.2        | 41.5        |
| <b>III.3</b> | 0.0578 | 24.9   | 35.3   | 9.7         | 20.0        | 45.2        |
| <b>III.4</b> | 0.0555 | 23.9   | 31.9   | 10.7        | 19.9        | 41.0        |
| <b>III.5</b> | 0.0586 | 23.4   | 31.8   | 9.9         | 19.3        | 41.5        |
| <b>III.6</b> | 0.0581 | 24.8   | 35.1   | 9.6         | 20.0        | 45.1        |
| <b>IV.A1</b> | 0.0685 | 18.6   | 24.0   | 9.0         | 15.7        | 30.9        |
| <b>IV.A2</b> | 0.0684 | 18.7   | 23.7   | 8.9         | 16.1        | 31.5        |
| <b>IV.A3</b> | 0.0664 | 20.2   | 26.7   | 8.8         | 17.0        | 35.0        |
| <b>IV.A4</b> | 0.0676 | 18.9   | 24.2   | 9.1         | 16.0        | 31.6        |
| <b>IV.A5</b> | 0.0678 | 19.0   | 24.2   | 8.9         | 16.3        | 32.1        |
| <b>IV.A6</b> | 0.066  | 20.5   | 27.2   | 8.8         | 17.1        | 35.8        |
| <b>IV.B1</b> | 0.0598 | 22.0   | 29.0   | 10.0        | 18.4        | 37.6        |
| <b>IV.B2</b> | 0.0608 | 21.9   | 29.1   | 9.7         | 18.4        | 37.9        |
| <b>IV.B3</b> | 0.0617 | 22.1   | 30.0   | 9.4         | 18.2        | 39.0        |
| <b>V.1</b>   | 0.0611 | 21.2   | 27.5   | 9.9         | 18.0        | 35.8        |
| <b>V.2</b>   | 0.0629 | 20.8   | 27.0   | 9.5         | 17.6        | 35.7        |
| <b>V.3</b>   | 0.0644 | 20.7   | 27.5   | 9.2         | 17.3        | 35.9        |

Table C.43. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1$  using Phase I data under SCM 6.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0049 | 213.8    | 225.5  | 282.8       | 207.4       | 154.1       |
| <b>I.B1</b>  | 0.0037 | 290.7    | 311.3  | 405.4       | 279.9       | 195.0       |
| <b>I.B2</b>  | 0.0035 | 307.4    | 330.5  | 441.0       | 296.0       | 202.9       |
| <b>I.B3</b>  | 0.0034 | 318.5    | 344.0  | 465.8       | 307.3       | 204.7       |
| <b>I.B4</b>  | 0.0033 | 330.9    | 359.0  | 491.5       | 322.9       | 205.4       |
| <b>II.A1</b> | 0.0033 | 333.6    | 360.1  | 480.6       | 323.2       | 214.1       |
| <b>II.A2</b> | 0.0029 | 377.7    | 409.3  | 559.3       | 371.0       | 233.3       |
| <b>II.A3</b> | 0.0030 | 382.2    | 426.2  | 613.8       | 367.5       | 216.6       |
| <b>II.B1</b> | 0.0030 | 362.5    | 390.1  | 531.4       | 358.3       | 229.5       |
| <b>II.B2</b> | 0.0028 | 388.5    | 418.3  | 568.4       | 390.1       | 243.8       |
| <b>II.B3</b> | 0.0029 | 393.4    | 431.5  | 602.9       | 390.4       | 231.7       |
| <b>III.1</b> | 0.0032 | 333.9    | 357.0  | 477.5       | 327.4       | 219.1       |
| <b>III.2</b> | 0.0028 | 380.3    | 402.3  | 513.3       | 389.8       | 253.6       |
| <b>III.3</b> | 0.0026 | 406.9    | 429.4  | 535.1       | 425.8       | 273.9       |
| <b>III.4</b> | 0.0032 | 333.0    | 355.1  | 468.9       | 327.1       | 219.7       |
| <b>III.5</b> | 0.0028 | 386.0    | 408.3  | 521.2       | 398.8       | 258.9       |
| <b>III.6</b> | 0.0027 | 408.0    | 431.7  | 529.5       | 431.1       | 273.3       |
| <b>IV.A1</b> | 0.0031 | 345.8    | 368.8  | 491.1       | 341.3       | 225.3       |
| <b>IV.A2</b> | 0.0027 | 397.7    | 418.2  | 509.7       | 419.9       | 270.3       |
| <b>IV.A3</b> | 0.0026 | 412.4    | 433.0  | 504.2       | 446.5       | 285.0       |
| <b>IV.A4</b> | 0.0031 | 349.0    | 373.2  | 498.3       | 346.9       | 225.4       |
| <b>IV.A5</b> | 0.0027 | 394.5    | 414.6  | 499.6       | 417.5       | 271.0       |
| <b>IV.A6</b> | 0.0026 | 420.2    | 441.1  | 495.9       | 459.0       | 294.1       |
| <b>IV.B1</b> | 0.003  | 358.5    | 380.5  | 497.6       | 362.5       | 234.9       |
| <b>IV.B2</b> | 0.0028 | 380.9    | 401.6  | 505.9       | 396.1       | 255.2       |
| <b>IV.B3</b> | 0.0027 | 403.2    | 424.2  | 502.3       | 434.8       | 275.6       |
| <b>V.1</b>   | 0.0028 | 385.2    | 403.0  | 498.5       | 398.1       | 268.9       |
| <b>V.2</b>   | 0.0026 | 404.4    | 422.1  | 498.2       | 430.0       | 285.5       |
| <b>V.3</b>   | 0.0025 | 415.3    | 430.9  | 465.3       | 457.5       | 308.5       |

Table C.44. Phase II performance of S-Charts for  $m = 50$  when  $\gamma = 1.5$  using Phase I data under SCM 6.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0052 | 257.0  | 306.4  | 105.7       | 246.1       | 431.6       |
| <b>I.B1</b>  | 0.0122 | 139.9  | 197.6  | 40.5        | 111.5       | 286.0       |
| <b>I.B2</b>  | 0.0144 | 128.3  | 189.3  | 33.2        | 98.6        | 268.2       |
| <b>I.B3</b>  | 0.0163 | 122.7  | 188.0  | 28.5        | 89.5        | 268.4       |
| <b>I.B4</b>  | 0.0172 | 124.1  | 197.6  | 26.5        | 86.1        | 287.9       |
| <b>II.A1</b> | 0.0141 | 137.9  | 211.7  | 33.6        | 100.4       | 303.7       |
| <b>II.A2</b> | 0.0179 | 128.1  | 212.7  | 25.0        | 84.6        | 301.0       |
| <b>II.A3</b> | 0.0159 | 173.2  | 289.3  | 26.5        | 108.9       | 446.9       |
| <b>II.B1</b> | 0.0203 | 108.6  | 182.3  | 22.7        | 72.0        | 248.3       |
| <b>II.B2</b> | 0.0218 | 105.7  | 181.1  | 20.9        | 68.5        | 246.4       |
| <b>II.B3</b> | 0.0197 | 140.6  | 244.1  | 21.7        | 86.0        | 360.5       |
| <b>III.1</b> | 0.0191 | 100.9  | 159.2  | 24.9        | 71.9        | 221.4       |
| <b>III.2</b> | 0.0269 | 75.8   | 130.0  | 17.6        | 50.5        | 165.9       |
| <b>III.3</b> | 0.0294 | 71.7   | 126.9  | 16.1        | 46.4        | 158.4       |
| <b>III.4</b> | 0.0195 | 98.7   | 155.5  | 24.7        | 70.3        | 215.0       |
| <b>III.5</b> | 0.0278 | 73.7   | 126.8  | 17.0        | 49.0        | 162.5       |
| <b>III.6</b> | 0.0306 | 70.2   | 125.9  | 15.4        | 44.6        | 155.4       |
| <b>IV.A1</b> | 0.0211 | 94.4   | 154.0  | 22.3        | 65.2        | 207.8       |
| <b>IV.A2</b> | 0.0306 | 65.5   | 114.6  | 15.8        | 42.6        | 142.9       |
| <b>IV.A3</b> | 0.0337 | 60.0   | 107.8  | 14.3        | 38.8        | 128.5       |
| <b>IV.A4</b> | 0.0215 | 93.4   | 151.1  | 21.7        | 64.9        | 207.6       |
| <b>IV.A5</b> | 0.0322 | 62.7   | 111.6  | 15.1        | 40.7        | 135.6       |
| <b>IV.A6</b> | 0.036  | 57.6   | 106.8  | 13.5        | 36.3        | 123.7       |
| <b>IV.B1</b> | 0.0242 | 83.3   | 138.8  | 19.4        | 56.5        | 183.9       |
| <b>IV.B2</b> | 0.0283 | 72.4   | 124.8  | 16.7        | 47.4        | 160.4       |
| <b>IV.B3</b> | 0.0322 | 64.2   | 115.4  | 14.9        | 41.2        | 139.8       |
| <b>V.1</b>   | 0.0296 | 62.7   | 105.1  | 16.6        | 42.9        | 131.8       |
| <b>V.2</b>   | 0.0339 | 53.9   | 91.7   | 14.9        | 37.0        | 111.2       |
| <b>V.3</b>   | 0.0385 | 47.8   | 83.2   | 13.0        | 32.4        | 98.6        |

## APPENDIX D: PHASE II PERFORMANCES OF S-CHARTS FOR $m=20$

Phase II performances of S-Charts using proposed estimation methods (Method I-V) are shown for  $m = 20$  under normal distribution and in the presence of contaminations with various outlier densities.



Table D.1. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using normally distributed Phase I data.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0027 | 449.9    | 484.4  | 240.4       | 538.5       | 507.9       |
| <b>I.B1</b>  | 0.0027 | 444.2    | 478.2  | 239.8       | 533.3       | 498.2       |
| <b>I.B2</b>  | 0.0027 | 453.4    | 487.7  | 243.7       | 547.9       | 494.9       |
| <b>I.B3</b>  | 0.0027 | 476.5    | 513.6  | 253.7       | 590.6       | 497.7       |
| <b>I.B4</b>  | 0.0027 | 492.8    | 532.3  | 263.5       | 619.1       | 500.6       |
| <b>II.A1</b> | 0.0027 | 526.5    | 579.7  | 268.6       | 704.6       | 455.9       |
| <b>II.A2</b> | 0.0027 | 565.3    | 631.6  | 298.3       | 762.7       | 431.7       |
| <b>II.A3</b> | 0.0027 | 607.8    | 698.2  | 336.6       | 821.6       | 394.7       |
| <b>II.B1</b> | 0.0027 | 558.4    | 617.8  | 301.0       | 743.3       | 444.9       |
| <b>II.B2</b> | 0.0027 | 572.1    | 632.6  | 326.2       | 753.6       | 448.1       |
| <b>II.B3</b> | 0.0027 | 635.6    | 722.1  | 410.3       | 820.9       | 421.8       |
| <b>III.1</b> | 0.0027 | 472.0    | 507.4  | 257.4       | 578.1       | 497.9       |
| <b>III.2</b> | 0.0027 | 516.2    | 559.4  | 280.0       | 658.4       | 483.7       |
| <b>III.3</b> | 0.0027 | 547.2    | 599.0  | 311.7       | 708.8       | 451.7       |
| <b>III.4</b> | 0.0027 | 473.4    | 510.2  | 255.4       | 584.5       | 502.5       |
| <b>III.5</b> | 0.0027 | 532.3    | 576.4  | 295.9       | 677.6       | 497.3       |
| <b>III.6</b> | 0.0027 | 558.6    | 611.1  | 324.9       | 723.1       | 460.9       |
| <b>IV.A1</b> | 0.0027 | 464.2    | 498.8  | 256.6       | 564.8       | 494.8       |
| <b>IV.A2</b> | 0.0027 | 500.6    | 542.0  | 268.4       | 635.2       | 481.8       |
| <b>IV.A3</b> | 0.0027 | 522.1    | 569.0  | 286.0       | 677.1       | 459.5       |
| <b>IV.A4</b> | 0.0027 | 468.3    | 503.3  | 255.9       | 575.4       | 489.2       |
| <b>IV.A5</b> | 0.0027 | 518.5    | 560.6  | 285.1       | 656.5       | 494         |
| <b>IV.A6</b> | 0.0027 | 546.8    | 596.2  | 304.0       | 709.3       | 479.2       |
| <b>IV.B1</b> | 0.0027 | 471.3    | 506.6  | 258.5       | 582.2       | 496.5       |
| <b>IV.B2</b> | 0.0027 | 487.1    | 525.1  | 265.1       | 608.1       | 492.7       |
| <b>IV.B3</b> | 0.0027 | 505.8    | 545.3  | 289.3       | 637.3       | 474.3       |
| <b>V.1</b>   | 0.0027 | 460.9    | 495.7  | 251.9       | 561.7       | 493.9       |
| <b>V.2</b>   | 0.0027 | 475.4    | 511.7  | 259.9       | 588.7       | 490.2       |
| <b>V.3</b>   | 0.0027 | 501.8    | 542.0  | 275.0       | 638.7       | 490.5       |

Table D.2. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using normally distributed Phase I data.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0778 | 17.9   | 25.2   | 7.4         | 14.4        | 32.0        |
| <b>I.B1</b>  | 0.0768 | 18.5   | 26.5   | 7.4         | 14.7        | 33.5        |
| <b>I.B2</b>  | 0.0753 | 19.4   | 28.2   | 7.4         | 15.3        | 35.4        |
| <b>I.B3</b>  | 0.0713 | 21.7   | 33.2   | 7.6         | 16.5        | 40.9        |
| <b>I.B4</b>  | 0.0697 | 23.4   | 37.2   | 7.5         | 17.4        | 45.0        |
| <b>II.A1</b> | 0.0601 | 42.8   | 98.1   | 7.6         | 23.1        | 92.9        |
| <b>II.A2</b> | 0.0537 | 67.8   | 166.0  | 7.9         | 29.2        | 165.2       |
| <b>II.A3</b> | 0.0457 | 127.7  | 310.6  | 8.4         | 43.3        | 380.7       |
| <b>II.B1</b> | 0.0543 | 54.5   | 123.8  | 8.0         | 27.8        | 124.7       |
| <b>II.B2</b> | 0.0527 | 58.5   | 134.0  | 8.2         | 29.6        | 135.6       |
| <b>II.B3</b> | 0.0441 | 115.8  | 277.4  | 9.0         | 44.7        | 319.9       |
| <b>III.1</b> | 0.0715 | 21.4   | 32.2   | 7.6         | 16.5        | 40.1        |
| <b>III.2</b> | 0.0634 | 30.1   | 53.2   | 7.8         | 20.7        | 61.3        |
| <b>III.3</b> | 0.0540 | 46.8   | 94.9   | 8.4         | 27.7        | 103.4       |
| <b>III.4</b> | 0.0723 | 21.2   | 32.4   | 7.5         | 16.4        | 39.6        |
| <b>III.5</b> | 0.0621 | 30.7   | 53.8   | 7.9         | 21.2        | 62.6        |
| <b>III.6</b> | 0.0543 | 46.3   | 93.3   | 8.3         | 27.6        | 102.9       |
| <b>IV.A1</b> | 0.0725 | 20.8   | 31.2   | 7.5         | 16.2        | 38.9        |
| <b>IV.A2</b> | 0.0646 | 28.1   | 48.3   | 7.8         | 19.9        | 56.1        |
| <b>IV.A3</b> | 0.0591 | 37.4   | 71.4   | 7.9         | 23.9        | 79.1        |
| <b>IV.A4</b> | 0.0718 | 21.0   | 31.5   | 7.6         | 16.4        | 39.2        |
| <b>IV.A5</b> | 0.0636 | 28.6   | 49.0   | 7.9         | 20.1        | 57.6        |
| <b>IV.A6</b> | 0.0576 | 39.0   | 74.6   | 8.1         | 24.7        | 84.0        |
| <b>IV.B1</b> | 0.0718 | 21.2   | 31.9   | 7.6         | 16.4        | 39.8        |
| <b>IV.B2</b> | 0.0685 | 23.5   | 36.8   | 7.7         | 17.9        | 44.8        |
| <b>IV.B3</b> | 0.0622 | 30.2   | 52.4   | 8.0         | 21.2        | 61.1        |
| <b>V.1</b>   | 0.0733 | 20.6   | 30.8   | 7.5         | 15.9        | 38.3        |
| <b>V.2</b>   | 0.0704 | 22.3   | 34.4   | 7.6         | 17.0        | 42.2        |
| <b>V.3</b>   | 0.0661 | 26.5   | 44.2   | 7.7         | 19.0        | 52.8        |

Table D.3. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0024 | 451.4    | 471.8  | 515.6       | 496.4       | 323.9       |
| <b>I.B1</b>  | 0.0024 | 451.1    | 473.2  | 474.8       | 509.7       | 329.4       |
| <b>I.B2</b>  | 0.0025 | 446.7    | 469.2  | 445.7       | 516.4       | 333.6       |
| <b>I.B3</b>  | 0.0024 | 463.6    | 490.5  | 462.4       | 536.7       | 336.9       |
| <b>I.B4</b>  | 0.0024 | 483.3    | 514.2  | 474.3       | 561.7       | 350.6       |
| <b>II.A1</b> | 0.0023 | 516.8    | 561.7  | 533.8       | 598.2       | 339.4       |
| <b>II.A2</b> | 0.0024 | 536.5    | 597.6  | 602.9       | 613.0       | 320.1       |
| <b>II.A3</b> | 0.0024 | 580.5    | 672.2  | 689.8       | 651.0       | 300.0       |
| <b>II.B1</b> | 0.0024 | 523.7    | 577.4  | 577.5       | 595.7       | 326.6       |
| <b>II.B2</b> | 0.0025 | 534.3    | 592.3  | 588.9       | 602.3       | 331.4       |
| <b>II.B3</b> | 0.0025 | 579.1    | 665.8  | 729.6       | 641.0       | 310.8       |
| <b>III.1</b> | 0.0024 | 467.7    | 492.7  | 444.5       | 542.5       | 353.2       |
| <b>III.2</b> | 0.0025 | 495.3    | 530.9  | 456.0       | 582.1       | 364.1       |
| <b>III.3</b> | 0.0025 | 522.8    | 569.7  | 519.3       | 610.7       | 354.5       |
| <b>III.4</b> | 0.0024 | 470.2    | 496.0  | 446.9       | 546.8       | 356.6       |
| <b>III.5</b> | 0.0024 | 501.0    | 537.9  | 473.7       | 590.0       | 360.2       |
| <b>III.6</b> | 0.0025 | 521.3    | 569.4  | 518.1       | 607.8       | 349.0       |
| <b>IV.A1</b> | 0.0024 | 474.5    | 500.8  | 431.5       | 560.4       | 362.9       |
| <b>IV.A2</b> | 0.0024 | 491.9    | 524.1  | 432.8       | 583.8       | 372.7       |
| <b>IV.A3</b> | 0.0025 | 509.2    | 550.7  | 451.7       | 604.2       | 366.1       |
| <b>IV.A4</b> | 0.0024 | 472.7    | 498.2  | 430.2       | 557.9       | 363.9       |
| <b>IV.A5</b> | 0.0024 | 502.7    | 536.7  | 441.5       | 599.6       | 380.7       |
| <b>IV.A6</b> | 0.0025 | 520.5    | 563.2  | 469.9       | 618.6       | 374.0       |
| <b>IV.B1</b> | 0.0024 | 468.9    | 494.3  | 416.7       | 553.3       | 367.5       |
| <b>IV.B2</b> | 0.0024 | 482.6    | 511.7  | 429.7       | 575.3       | 369.2       |
| <b>IV.B3</b> | 0.0024 | 502.4    | 537.1  | 466.0       | 594.2       | 371.1       |
| <b>V.1</b>   | 0.0024 | 472.5    | 497.5  | 411.8       | 560.9       | 373.4       |
| <b>V.2</b>   | 0.0024 | 478.9    | 505.6  | 408.2       | 573.3       | 380.3       |
| <b>V.3</b>   | 0.0024 | 494.6    | 527.4  | 412.2       | 600.5       | 381.4       |

Table D.4. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 1 (d=2, K=0.10).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0366 | 53.9   | 98.5   | 13.4        | 34.9        | 114.0       |
| <b>I.B1</b>  | 0.0411 | 50.1   | 94.7   | 11.8        | 31.8        | 106.1       |
| <b>I.B2</b>  | 0.0427 | 49.4   | 94.9   | 11.3        | 30.8        | 105.3       |
| <b>I.B3</b>  | 0.0419 | 53.7   | 104.5  | 11.3        | 32.6        | 118.0       |
| <b>I.B4</b>  | 0.0421 | 56.5   | 114.0  | 11.1        | 33.0        | 124.4       |
| <b>II.A1</b> | 0.0359 | 91.8   | 196.0  | 11.9        | 45.0        | 231.1       |
| <b>II.A2</b> | 0.0330 | 130.0  | 277.6  | 12.1        | 57.2        | 369.7       |
| <b>II.A3</b> | 0.0286 | 224.9  | 456.3  | 13.0        | 89.0        | 775.7       |
| <b>II.B1</b> | 0.0343 | 115.1  | 244.6  | 11.9        | 54.0        | 307.7       |
| <b>II.B2</b> | 0.0328 | 125.0  | 261.8  | 12.4        | 59.0        | 337.7       |
| <b>II.B3</b> | 0.0279 | 212.3  | 418.7  | 13.7        | 93.5        | 676.8       |
| <b>III.1</b> | 0.0439 | 48.1   | 92.0   | 11.0        | 30.2        | 102.9       |
| <b>III.2</b> | 0.0423 | 61.6   | 125.0  | 10.5        | 35.5        | 139.0       |
| <b>III.3</b> | 0.0372 | 89.7   | 188.6  | 11.5        | 46.0        | 223.1       |
| <b>III.4</b> | 0.0447 | 47.5   | 90.1   | 10.8        | 29.9        | 101.9       |
| <b>III.5</b> | 0.0415 | 62.9   | 126.9  | 10.9        | 36.3        | 143.1       |
| <b>III.6</b> | 0.0374 | 89.8   | 187.3  | 11.5        | 46.4        | 224.4       |
| <b>IV.A1</b> | 0.0465 | 43.5   | 81.5   | 10.5        | 28.0        | 91.6        |
| <b>IV.A2</b> | 0.0448 | 52.6   | 104.7  | 10.3        | 31.7        | 115.1       |
| <b>IV.A3</b> | 0.0417 | 68.3   | 141.1  | 10.6        | 37.5        | 160.0       |
| <b>IV.A4</b> | 0.0463 | 44.4   | 84.4   | 10.4        | 28.4        | 93.4        |
| <b>IV.A5</b> | 0.0445 | 53.4   | 105.4  | 10.5        | 32.0        | 116.9       |
| <b>IV.A6</b> | 0.0409 | 70.3   | 144.3  | 10.8        | 38.7        | 165.1       |
| <b>IV.B1</b> | 0.0469 | 43.5   | 82.0   | 10.3        | 28.0        | 91.7        |
| <b>IV.B2</b> | 0.0458 | 47.5   | 91.2   | 10.5        | 29.8        | 101.7       |
| <b>IV.B3</b> | 0.0434 | 58.1   | 117.1  | 10.5        | 34.0        | 129.9       |
| <b>V.1</b>   | 0.0484 | 40.3   | 74.2   | 10.2        | 26.3        | 83.8        |
| <b>V.2</b>   | 0.0480 | 42.8   | 81.1   | 10.1        | 27.3        | 89.9        |
| <b>V.3</b>   | 0.0472 | 47.8   | 93.6   | 10.0        | 29.3        | 103.7       |

Table D.5. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0050 | 244.4    | 288.4  | 407.4       | 222.6       | 123.3       |
| <b>I.B1</b>  | 0.0040 | 311.9    | 364.0  | 550.6       | 296.3       | 148.5       |
| <b>I.B2</b>  | 0.0037 | 333.7    | 385.1  | 568.6       | 329.6       | 159.6       |
| <b>I.B3</b>  | 0.0035 | 352.2    | 403.9  | 586.6       | 352.6       | 173.0       |
| <b>I.B4</b>  | 0.0034 | 371.1    | 425.8  | 614.3       | 372.9       | 180.8       |
| <b>II.A1</b> | 0.0029 | 417.9    | 477.1  | 712.1       | 405.2       | 215.5       |
| <b>II.A2</b> | 0.0028 | 442.6    | 512.6  | 767.3       | 430.8       | 218.1       |
| <b>II.A3</b> | 0.0028 | 488.6    | 594.8  | 964.1       | 465.6       | 208.0       |
| <b>II.B1</b> | 0.0029 | 435.4    | 503.8  | 761.3       | 425.9       | 218.0       |
| <b>II.B2</b> | 0.0029 | 441.1    | 515.1  | 780.2       | 434.2       | 215.3       |
| <b>II.B3</b> | 0.0029 | 471.4    | 571.2  | 926.8       | 447.1       | 209.3       |
| <b>III.1</b> | 0.0030 | 391.9    | 434.4  | 577.1       | 414.4       | 217.1       |
| <b>III.2</b> | 0.0027 | 447.1    | 495.3  | 608.3       | 481.2       | 257.0       |
| <b>III.3</b> | 0.0026 | 476.2    | 533.7  | 676.1       | 511.7       | 268.3       |
| <b>III.4</b> | 0.003  | 395.8    | 437.5  | 580.5       | 414.6       | 223.1       |
| <b>III.5</b> | 0.0028 | 442.2    | 488.0  | 604.7       | 477.7       | 259.2       |
| <b>III.6</b> | 0.0027 | 477.3    | 535.6  | 672.0       | 511.1       | 269.4       |
| <b>IV.A1</b> | 0.0028 | 411.7    | 450.2  | 549.4       | 445.6       | 241.0       |
| <b>IV.A2</b> | 0.0026 | 448.9    | 489.5  | 555.6       | 495.7       | 277.6       |
| <b>IV.A3</b> | 0.0026 | 469.1    | 515.0  | 586.4       | 520.0       | 286.0       |
| <b>IV.A4</b> | 0.0029 | 403.7    | 440.8  | 531.9       | 433.4       | 239.1       |
| <b>IV.A5</b> | 0.0026 | 454.9    | 495.4  | 554.0       | 507.3       | 283.7       |
| <b>IV.A6</b> | 0.0026 | 479.5    | 528.2  | 589.4       | 530.6       | 293.4       |
| <b>IV.B1</b> | 0.0028 | 409.5    | 447.1  | 541.4       | 443.4       | 243.5       |
| <b>IV.B2</b> | 0.0028 | 424.0    | 464.0  | 534.1       | 469.5       | 254.1       |
| <b>IV.B3</b> | 0.0027 | 451.5    | 496.4  | 562.5       | 500.0       | 270.7       |
| <b>V.1</b>   | 0.0027 | 425.7    | 459.2  | 517.3       | 471.7       | 269.8       |
| <b>V.2</b>   | 0.0026 | 436.5    | 470.8  | 501.7       | 494.4       | 282.3       |
| <b>V.3</b>   | 0.0026 | 455.0    | 492.2  | 495.8       | 518.5       | 299.5       |

Table D.6. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0073 | 300.9  | 395.4  | 58.1        | 301.5       | 534.6       |
| <b>I.B1</b>  | 0.0162 | 215.3  | 341.3  | 24.1        | 143.4       | 567.3       |
| <b>I.B2</b>  | 0.0198 | 187.5  | 315.7  | 19.8        | 109.6       | 544.4       |
| <b>I.B3</b>  | 0.0210 | 183.2  | 317.9  | 18.8        | 100.9       | 558.5       |
| <b>I.B4</b>  | 0.0207 | 182.1  | 318.0  | 19.1        | 98.6        | 563.4       |
| <b>II.A1</b> | 0.0170 | 221.5  | 377.4  | 23.1        | 129.3       | 636.2       |
| <b>II.A2</b> | 0.0171 | 262.5  | 443.3  | 22.4        | 153.7       | 751.8       |
| <b>II.A3</b> | 0.0154 | 367.9  | 592.9  | 24.5        | 241.2       | 917.6       |
| <b>II.B1</b> | 0.0184 | 242.6  | 413.0  | 21.2        | 141.9       | 700.3       |
| <b>II.B2</b> | 0.0179 | 267.0  | 454.3  | 21.9        | 153.0       | 770.5       |
| <b>II.B3</b> | 0.0155 | 372.1  | 592.5  | 25.2        | 256.7       | 923.4       |
| <b>III.1</b> | 0.0256 | 131.4  | 246.0  | 16.2        | 68.9        | 372.8       |
| <b>III.2</b> | 0.0288 | 128.7  | 253.4  | 14.3        | 63.4        | 363.1       |
| <b>III.3</b> | 0.0266 | 163.0  | 315.7  | 15.1        | 79.8        | 482.9       |
| <b>III.4</b> | 0.0261 | 129.2  | 244.5  | 15.9        | 66.9        | 368.0       |
| <b>III.5</b> | 0.0287 | 129.8  | 257.1  | 14.3        | 64.0        | 366.5       |
| <b>III.6</b> | 0.0272 | 159.2  | 308.8  | 14.8        | 78.8        | 470.1       |
| <b>IV.A1</b> | 0.0300 | 106.7  | 212.0  | 14.2        | 53.9        | 292.7       |
| <b>IV.A2</b> | 0.0329 | 100.0  | 205.2  | 12.9        | 49.7        | 261.1       |
| <b>IV.A3</b> | 0.0311 | 116.5  | 236.3  | 13.3        | 57.5        | 317.4       |
| <b>IV.A4</b> | 0.0296 | 107.3  | 210.4  | 14.2        | 54.7        | 288.4       |
| <b>IV.A5</b> | 0.0326 | 102.1  | 208.5  | 13.0        | 51.3        | 268.0       |
| <b>IV.A6</b> | 0.0314 | 120.0  | 245.3  | 13.3        | 58.3        | 324.2       |
| <b>IV.B1</b> | 0.0303 | 106.6  | 212.3  | 14.1        | 53.6        | 288.6       |
| <b>IV.B2</b> | 0.0321 | 104.6  | 212.5  | 13.2        | 51.7        | 282.5       |
| <b>IV.B3</b> | 0.0321 | 110.3  | 224.2  | 13.2        | 54.1        | 299.3       |
| <b>V.1</b>   | 0.0341 | 81.6   | 163.9  | 13.0        | 43.5        | 200.7       |
| <b>V.2</b>   | 0.0356 | 81.5   | 167.8  | 12.3        | 42.6        | 199.2       |
| <b>V.3</b>   | 0.0377 | 82.0   | 171.7  | 11.7        | 41.6        | 203.4       |

Table D.7. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0069 | 180.5    | 217.9  | 297.0       | 162.2       | 89.2        |
| <b>I.B1</b>  | 0.0056 | 238.7    | 297.7  | 447.4       | 208.5       | 102.5       |
| <b>I.B2</b>  | 0.0051 | 262.8    | 324.2  | 501.5       | 233.7       | 110.0       |
| <b>I.B3</b>  | 0.0048 | 280.5    | 346.0  | 538.9       | 252.8       | 114.5       |
| <b>I.B4</b>  | 0.0046 | 296.7    | 363.6  | 560.1       | 270.9       | 120.2       |
| <b>II.A1</b> | 0.0036 | 349.0    | 413.4  | 617.4       | 320.1       | 166.5       |
| <b>II.A2</b> | 0.0034 | 379.8    | 461.1  | 719.9       | 342.7       | 168.8       |
| <b>II.A3</b> | 0.0034 | 413.7    | 519.5  | 838.2       | 367.5       | 166.4       |
| <b>II.B1</b> | 0.0034 | 378.8    | 459.3  | 726.4       | 343.0       | 170.3       |
| <b>II.B2</b> | 0.0034 | 382.5    | 463.7  | 739.0       | 349.1       | 170.8       |
| <b>II.B3</b> | 0.0034 | 408.9    | 515.4  | 843.7       | 361.0       | 166.0       |
| <b>III.1</b> | 0.0038 | 328.9    | 382.9  | 578.0       | 312.7       | 158.6       |
| <b>III.2</b> | 0.0032 | 392.2    | 449.7  | 656.9       | 391.3       | 197.5       |
| <b>III.3</b> | 0.003  | 424.8    | 493.0  | 747.9       | 421.5       | 209.4       |
| <b>III.4</b> | 0.0037 | 331.6    | 384.6  | 575.9       | 319.1       | 161.5       |
| <b>III.5</b> | 0.0032 | 394.4    | 452.5  | 672.1       | 394.4       | 198.4       |
| <b>III.6</b> | 0.0031 | 423.0    | 489.8  | 730.8       | 420.6       | 209.7       |
| <b>IV.A1</b> | 0.0035 | 350.9    | 399.3  | 577.6       | 347.7       | 177.0       |
| <b>IV.A2</b> | 0.0030 | 403.8    | 455.1  | 625.6       | 413.9       | 216.2       |
| <b>IV.A3</b> | 0.0028 | 433.3    | 488.8  | 668.9       | 452.2       | 231.1       |
| <b>IV.A4</b> | 0.0035 | 349.5    | 398.7  | 582.3       | 343.9       | 176.6       |
| <b>IV.A5</b> | 0.0030 | 410.5    | 462.0  | 633.7       | 424.8       | 220.3       |
| <b>IV.A6</b> | 0.0029 | 437.5    | 495.5  | 675.9       | 451.8       | 235.7       |
| <b>IV.B1</b> | 0.0034 | 354.6    | 403.7  | 586.0       | 352.5       | 181.1       |
| <b>IV.B2</b> | 0.0033 | 377.6    | 430.5  | 611.2       | 382.4       | 191.5       |
| <b>IV.B3</b> | 0.0031 | 401.5    | 456.3  | 641.1       | 413.6       | 207.7       |
| <b>V.1</b>   | 0.0031 | 382.4    | 424.1  | 579.1       | 393.1       | 210.4       |
| <b>V.2</b>   | 0.0030 | 390.2    | 432.3  | 572.4       | 409.5       | 218.5       |
| <b>V.3</b>   | 0.0028 | 424.5    | 469.5  | 588.6       | 452         | 245.6       |

Table D.8. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 1 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0040 | 390.8  | 446.8  | 135.4       | 526.8       | 418.8       |
| <b>I.B1</b>  | 0.0090 | 315.5  | 415.9  | 45.8        | 362.7       | 483.9       |
| <b>I.B2</b>  | 0.0113 | 285.4  | 396.7  | 34.1        | 285.3       | 505.4       |
| <b>I.B3</b>  | 0.0124 | 282.9  | 407.2  | 31.4        | 255.8       | 540.9       |
| <b>I.B4</b>  | 0.0125 | 286.2  | 417.0  | 31.2        | 250.2       | 563.7       |
| <b>II.A1</b> | 0.0104 | 317.0  | 463.2  | 38.5        | 259.6       | 703.4       |
| <b>II.A2</b> | 0.0111 | 352.0  | 517.9  | 35.9        | 292.9       | 754.9       |
| <b>II.A3</b> | 0.0102 | 461.1  | 662.3  | 39.4        | 459.1       | 797.6       |
| <b>II.B1</b> | 0.0119 | 341.4  | 506.0  | 33.0        | 281.2       | 745.8       |
| <b>II.B2</b> | 0.0122 | 356.4  | 527.3  | 32.9        | 295.7       | 760.9       |
| <b>II.B3</b> | 0.0107 | 461.7  | 652.9  | 38.8        | 489.3       | 779.5       |
| <b>III.1</b> | 0.0162 | 217.1  | 345.2  | 24.6        | 144.6       | 575.9       |
| <b>III.2</b> | 0.0200 | 202.5  | 349.3  | 19.8        | 113.7       | 598.2       |
| <b>III.3</b> | 0.0192 | 235.0  | 401.3  | 20.4        | 135.0       | 683.3       |
| <b>III.4</b> | 0.0167 | 213.0  | 341.2  | 23.7        | 140.0       | 575.0       |
| <b>III.5</b> | 0.0202 | 204.0  | 352.5  | 19.8        | 115.9       | 603.4       |
| <b>III.6</b> | 0.0200 | 231.7  | 399.0  | 19.9        | 131.7       | 683.2       |
| <b>IV.A1</b> | 0.0198 | 180.0  | 306.9  | 20.1        | 103.3       | 521.4       |
| <b>IV.A2</b> | 0.0241 | 158.7  | 293.4  | 16.7        | 81.6        | 476.7       |
| <b>IV.A3</b> | 0.0234 | 172.0  | 315.1  | 17.2        | 90.2        | 508.9       |
| <b>IV.A4</b> | 0.0197 | 180.2  | 308.6  | 20.2        | 103.8       | 528.7       |
| <b>IV.A5</b> | 0.0239 | 161.7  | 297.6  | 16.7        | 84.0        | 483.2       |
| <b>IV.A6</b> | 0.0238 | 178.1  | 329.9  | 16.7        | 91.6        | 541.8       |
| <b>IV.B1</b> | 0.0205 | 175.6  | 302.9  | 19.3        | 100.0       | 511.8       |
| <b>IV.B2</b> | 0.0223 | 172.6  | 307.1  | 17.9        | 92.3        | 521.2       |
| <b>IV.B3</b> | 0.0237 | 173.7  | 316.7  | 16.8        | 89.4        | 523.1       |
| <b>V.1</b>   | 0.0239 | 137.2  | 251.1  | 17.3        | 74.2        | 387.0       |
| <b>V.2</b>   | 0.0257 | 134.9  | 252.5  | 16.0        | 70.9        | 383.6       |
| <b>V.3</b>   | 0.0285 | 127.0  | 247.5  | 14.5        | 64.1        | 357.0       |

Table D.9. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=0.75$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0024 | 463.3    | 484.2  | 370.6       | 554.6       | 396.4       |
| <b>I.B1</b>  | 0.0024 | 467.2    | 490.8  | 336.8       | 566.4       | 428.0       |
| <b>I.B2</b>  | 0.0025 | 469.2    | 495.6  | 325.6       | 574.8       | 426.0       |
| <b>I.B3</b>  | 0.0025 | 478.7    | 506.7  | 339.0       | 588.5       | 425.9       |
| <b>I.B4</b>  | 0.0024 | 499.6    | 531.8  | 343.6       | 619.6       | 435.2       |
| <b>II.A1</b> | 0.0024 | 529.1    | 575.0  | 368.2       | 672.1       | 402.6       |
| <b>II.A2</b> | 0.0025 | 553.6    | 614.2  | 410.7       | 699.4       | 372.6       |
| <b>II.A3</b> | 0.0025 | 598.5    | 683.3  | 496.5       | 740.2       | 348.8       |
| <b>II.B1</b> | 0.0025 | 551.3    | 604.9  | 412.6       | 690.1       | 392.0       |
| <b>II.B2</b> | 0.0024 | 565.9    | 625.2  | 444.3       | 698.0       | 388.9       |
| <b>II.B3</b> | 0.0025 | 611.3    | 696.2  | 552.1       | 732.6       | 365.0       |
| <b>III.1</b> | 0.0024 | 480.1    | 506.8  | 338.5       | 585.1       | 433.1       |
| <b>III.2</b> | 0.0025 | 520.3    | 558.3  | 356.8       | 646.4       | 435.4       |
| <b>III.3</b> | 0.0025 | 539.4    | 587.4  | 407.6       | 663.7       | 400.8       |
| <b>III.4</b> | 0.0025 | 475.0    | 501.7  | 334.9       | 580.2       | 429.0       |
| <b>III.5</b> | 0.0025 | 525.3    | 563.6  | 367.8       | 651.2       | 435.7       |
| <b>III.6</b> | 0.0026 | 547.0    | 594.8  | 402.3       | 682.1       | 415.9       |
| <b>IV.A1</b> | 0.0025 | 470.9    | 498.5  | 315.5       | 577.7       | 433.2       |
| <b>IV.A2</b> | 0.0025 | 496.4    | 529.1  | 342.6       | 613.8       | 429.0       |
| <b>IV.A3</b> | 0.0026 | 530.3    | 573.9  | 368.5       | 663.9       | 420.7       |
| <b>IV.A4</b> | 0.0025 | 473.1    | 499.7  | 326.1       | 575.6       | 432.2       |
| <b>IV.A5</b> | 0.0025 | 517.8    | 554.3  | 352.2       | 646.1       | 437.1       |
| <b>IV.A6</b> | 0.0026 | 533.1    | 576.7  | 373.6       | 669.0       | 420.6       |
| <b>IV.B1</b> | 0.0025 | 480.4    | 509.0  | 321.1       | 593.5       | 443.3       |
| <b>IV.B2</b> | 0.0025 | 484.3    | 514.8  | 330.2       | 600.1       | 429.7       |
| <b>IV.B3</b> | 0.0025 | 515.0    | 551.6  | 361.5       | 635.2       | 432.3       |
| <b>V.1</b>   | 0.0025 | 470.6    | 497.9  | 316.7       | 578.4       | 433.8       |
| <b>V.2</b>   | 0.0025 | 478.2    | 507.1  | 318.3       | 589.7       | 437.0       |
| <b>V.3</b>   | 0.0025 | 497.3    | 531.9  | 328.0       | 623.9       | 433.5       |

Table D.10. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=0.75$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0540 | 32.6   | 60.8   | 9.6         | 22.0        | 63.6        |
| <b>I.B1</b>  | 0.0594 | 28.1   | 47.6   | 8.9         | 20.0        | 54.2        |
| <b>I.B2</b>  | 0.0597 | 28.4   | 48.4   | 8.7         | 20.2        | 55.7        |
| <b>I.B3</b>  | 0.0572 | 31.6   | 57.1   | 8.9         | 21.7        | 63.0        |
| <b>I.B4</b>  | 0.0561 | 34.0   | 63.3   | 8.9         | 22.8        | 68.9        |
| <b>II.A1</b> | 0.0478 | 61.1   | 138.1  | 9.2         | 31.0        | 140.8       |
| <b>II.A2</b> | 0.0432 | 94.1   | 219.5  | 9.4         | 39.5        | 245.7       |
| <b>II.A3</b> | 0.0371 | 164.9  | 367.6  | 10.3        | 59.3        | 537.5       |
| <b>II.B1</b> | 0.0442 | 76.4   | 171.1  | 9.6         | 37.0        | 184.5       |
| <b>II.B2</b> | 0.0426 | 83.5   | 186.4  | 9.9         | 40.0        | 204.9       |
| <b>II.B3</b> | 0.0361 | 155.2  | 341.8  | 10.8        | 61.1        | 477.2       |
| <b>III.1</b> | 0.0585 | 29.6   | 50.2   | 8.8         | 21.1        | 58.4        |
| <b>III.2</b> | 0.0531 | 40.5   | 76.4   | 9.0         | 25.9        | 86.5        |
| <b>III.3</b> | 0.0462 | 61.7   | 128.5  | 9.5         | 34.3        | 142.4       |
| <b>III.4</b> | 0.0585 | 29.5   | 49.9   | 8.8         | 21.0        | 58.3        |
| <b>III.5</b> | 0.0523 | 41.6   | 79.7   | 9.1         | 26.3        | 88.9        |
| <b>III.6</b> | 0.0465 | 61.2   | 126.9  | 9.5         | 34.2        | 141.2       |
| <b>IV.A1</b> | 0.0602 | 28.2   | 47.7   | 8.7         | 20.3        | 54.7        |
| <b>IV.A2</b> | 0.0553 | 36.1   | 66.0   | 8.8         | 24.0        | 74.8        |
| <b>IV.A3</b> | 0.0503 | 48.8   | 98.1   | 9.1         | 29.0        | 107.3       |
| <b>IV.A4</b> | 0.0601 | 28.2   | 47.9   | 8.7         | 20.3        | 54.8        |
| <b>IV.A5</b> | 0.0549 | 37.0   | 67.5   | 8.8         | 24.5        | 76.4        |
| <b>IV.A6</b> | 0.0497 | 50.0   | 100.4  | 9.2         | 29.8        | 110.8       |
| <b>IV.B1</b> | 0.0599 | 28.5   | 48.1   | 8.6         | 20.5        | 55.8        |
| <b>IV.B2</b> | 0.0577 | 31.3   | 54.8   | 8.8         | 21.9        | 62.7        |
| <b>IV.B3</b> | 0.0529 | 39.7   | 74.9   | 9.1         | 25.8        | 83.5        |
| <b>V.1</b>   | 0.0618 | 26.9   | 44.1   | 8.5         | 19.6        | 52.2        |
| <b>V.2</b>   | 0.0598 | 29.0   | 49.2   | 8.6         | 20.6        | 56.9        |
| <b>V.3</b>   | 0.0572 | 34.1   | 61.1   | 8.6         | 22.8        | 70.9        |

Table D.11. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0026 | 421.0    | 448    | 514.6       | 462.4       | 273.1       |
| <b>I.B1</b>  | 0.0025 | 454.7    | 478.8  | 428.4       | 536.0       | 342.9       |
| <b>I.B2</b>  | 0.0025 | 456.3    | 480.7  | 405.3       | 545.1       | 353.7       |
| <b>I.B3</b>  | 0.0024 | 469.9    | 497.4  | 425.9       | 555.6       | 356         |
| <b>I.B4</b>  | 0.0024 | 484.0    | 513.5  | 436.3       | 574.3       | 366.5       |
| <b>II.A1</b> | 0.0023 | 512.0    | 554.9  | 504.9       | 603.0       | 343.5       |
| <b>II.A2</b> | 0.0024 | 537.9    | 599.2  | 560.2       | 626.2       | 325.5       |
| <b>II.A3</b> | 0.0024 | 578      | 665.8  | 646.3       | 664.8       | 307.3       |
| <b>II.B1</b> | 0.0024 | 527.7    | 580.9  | 536.1       | 616.6       | 334.6       |
| <b>II.B2</b> | 0.0024 | 534.2    | 590.4  | 566.8       | 614.9       | 338.6       |
| <b>II.B3</b> | 0.0024 | 584.7    | 669.7  | 708.1       | 652.1       | 322.1       |
| <b>III.1</b> | 0.0024 | 478.2    | 505.4  | 406.1       | 571.7       | 374.3       |
| <b>III.2</b> | 0.0025 | 504.9    | 541.1  | 418.5       | 611.0       | 384.4       |
| <b>III.3</b> | 0.0025 | 532.4    | 580.2  | 482.3       | 634.8       | 370.0       |
| <b>III.4</b> | 0.0024 | 473.9    | 500.1  | 404.1       | 568.1       | 376.5       |
| <b>III.5</b> | 0.0025 | 510.2    | 546.0  | 433.5       | 612.5       | 387.6       |
| <b>III.6</b> | 0.0025 | 532.9    | 581.3  | 480.4       | 629.0       | 370.8       |
| <b>IV.A1</b> | 0.0024 | 468.5    | 493.2  | 384         | 561.7       | 384.1       |
| <b>IV.A2</b> | 0.0024 | 499.9    | 533.4  | 402.7       | 607.5       | 393.2       |
| <b>IV.A3</b> | 0.0026 | 507.7    | 548.2  | 429.6       | 611.7       | 374.4       |
| <b>IV.A4</b> | 0.0024 | 481.1    | 507.1  | 392.9       | 579.1       | 393.6       |
| <b>IV.A5</b> | 0.0025 | 497.9    | 531.3  | 395.6       | 606.3       | 393.5       |
| <b>IV.A6</b> | 0.0025 | 518.2    | 560.9  | 427.8       | 625.5       | 383.4       |
| <b>IV.B1</b> | 0.0024 | 472.6    | 499.1  | 385.3       | 572.8       | 382.9       |
| <b>IV.B2</b> | 0.0025 | 474.1    | 502.2  | 378.6       | 572.5       | 385.0       |
| <b>IV.B3</b> | 0.0025 | 503.6    | 537.9  | 419.7       | 606.9       | 388.9       |
| <b>V.1</b>   | 0.0024 | 474.3    | 499.1  | 372.4       | 572.1       | 397.7       |
| <b>V.2</b>   | 0.0024 | 480.6    | 508.2  | 373.7       | 582.4       | 398.5       |
| <b>V.3</b>   | 0.0025 | 491.6    | 524.5  | 379.1       | 602.6       | 392.8       |

Table D.12. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0340 | 73.0   | 147.8  | 13.5        | 39.4        | 171.5       |
| <b>I.B1</b>  | 0.0456 | 46.3   | 93.1   | 10.7        | 28.0        | 98.2        |
| <b>I.B2</b>  | 0.0478 | 43.8   | 87.8   | 10.2        | 26.8        | 90.8        |
| <b>I.B3</b>  | 0.0461 | 47.7   | 97.4   | 10.4        | 28.5        | 101.1       |
| <b>I.B4</b>  | 0.0456 | 50.5   | 104.1  | 10.4        | 29.7        | 108.1       |
| <b>II.A1</b> | 0.0377 | 85.1   | 183.8  | 11.3        | 41.9        | 208.8       |
| <b>II.A2</b> | 0.035  | 125.5  | 272.0  | 11.5        | 53.5        | 358.4       |
| <b>II.A3</b> | 0.0302 | 207.2  | 422.2  | 12.3        | 82.1        | 707.0       |
| <b>II.B1</b> | 0.0361 | 106.7  | 229.1  | 11.3        | 49.4        | 284.4       |
| <b>II.B2</b> | 0.0349 | 115.0  | 245.2  | 11.7        | 53.3        | 308.3       |
| <b>II.B3</b> | 0.0293 | 200.2  | 404.7  | 13.2        | 85.9        | 639.6       |
| <b>III.1</b> | 0.0481 | 42.4   | 81.0   | 10.2        | 27.1        | 88.0        |
| <b>III.2</b> | 0.0454 | 54.2   | 108.7  | 10.1        | 31.9        | 120.6       |
| <b>III.3</b> | 0.0405 | 79.2   | 167.8  | 10.7        | 41.5        | 191.0       |
| <b>III.4</b> | 0.0486 | 41.3   | 78.4   | 10.1        | 26.5        | 85.1        |
| <b>III.5</b> | 0.0451 | 54.9   | 110.6  | 10.2        | 32.3        | 122.4       |
| <b>III.6</b> | 0.0405 | 78.1   | 163.0  | 10.6        | 41.7        | 187.3       |
| <b>IV.A1</b> | 0.0508 | 37.8   | 70.4   | 9.8         | 25.0        | 77.4        |
| <b>IV.A2</b> | 0.0485 | 46.0   | 89.5   | 9.7         | 28.5        | 98.9        |
| <b>IV.A3</b> | 0.0442 | 60.8   | 124.6  | 10.0        | 34.8        | 138.3       |
| <b>IV.A4</b> | 0.0508 | 37.9   | 70.8   | 9.8         | 25.0        | 77.7        |
| <b>IV.A5</b> | 0.0476 | 47.4   | 93.5   | 9.9         | 29.3        | 101.9       |
| <b>IV.A6</b> | 0.0440 | 62.3   | 127.4  | 10.0        | 35.3        | 143.4       |
| <b>IV.B1</b> | 0.0514 | 37.6   | 70.2   | 9.6         | 25.0        | 76.7        |
| <b>IV.B2</b> | 0.0504 | 41.2   | 78.7   | 9.7         | 26.3        | 86.1        |
| <b>IV.B3</b> | 0.0470 | 50.7   | 101.1  | 9.8         | 30.6        | 110.8       |
| <b>V.1</b>   | 0.0535 | 34.6   | 63.0   | 9.5         | 23.3        | 70.3        |
| <b>V.2</b>   | 0.0525 | 36.8   | 67.4   | 9.5         | 24.3        | 75.7        |
| <b>V.3</b>   | 0.0510 | 42.0   | 81.0   | 9.4         | 26.7        | 88.7        |

Table D.13. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0029 | 395.3    | 428.4  | 556.0       | 412.5       | 233.6       |
| <b>I.B1</b>  | 0.0025 | 437.5    | 463.6  | 486.7       | 493.6       | 298.1       |
| <b>I.B2</b>  | 0.0025 | 443.8    | 470.7  | 467.0       | 509.4       | 309.7       |
| <b>I.B3</b>  | 0.0025 | 453.8    | 482.4  | 473.5       | 521.9       | 316.6       |
| <b>I.B4</b>  | 0.0024 | 472.6    | 505.0  | 490.0       | 544.2       | 327.1       |
| <b>II.A1</b> | 0.0024 | 502.1    | 547.3  | 569.9       | 565.7       | 316.3       |
| <b>II.A2</b> | 0.0024 | 521.9    | 583.9  | 644.2       | 583.5       | 298.1       |
| <b>II.A3</b> | 0.0024 | 560.0    | 648.1  | 757.1       | 617.7       | 283.5       |
| <b>II.B1</b> | 0.0024 | 516.3    | 571.5  | 615.2       | 578.6       | 314.9       |
| <b>II.B2</b> | 0.0025 | 525.7    | 588.1  | 659.5       | 577.4       | 306.7       |
| <b>II.B3</b> | 0.0025 | 562.3    | 651.8  | 804.1       | 597.4       | 291.4       |
| <b>III.1</b> | 0.0024 | 462.7    | 487.6  | 463.3       | 532.2       | 344.0       |
| <b>III.2</b> | 0.0024 | 501.3    | 538.2  | 489.2       | 587.1       | 356.5       |
| <b>III.3</b> | 0.0025 | 514.5    | 563.3  | 547.6       | 587.8       | 337.0       |
| <b>III.4</b> | 0.0025 | 458.3    | 483.4  | 457.2       | 531.8       | 339.4       |
| <b>III.5</b> | 0.0024 | 504.9    | 542.4  | 489.2       | 588.6       | 358.4       |
| <b>III.6</b> | 0.0025 | 523.3    | 571.4  | 543.2       | 608.6       | 345.1       |
| <b>IV.A1</b> | 0.0024 | 459.5    | 484.4  | 430.4       | 539.8       | 348.3       |
| <b>IV.A2</b> | 0.0024 | 483.4    | 513.8  | 452.7       | 568.4       | 360.7       |
| <b>IV.A3</b> | 0.0025 | 512.6    | 555.3  | 489.2       | 606.1       | 354.4       |
| <b>IV.A4</b> | 0.0024 | 460.0    | 484.3  | 444.6       | 535.0       | 348.8       |
| <b>IV.A5</b> | 0.0024 | 502.1    | 536.8  | 464.9       | 595.7       | 368.4       |
| <b>IV.A6</b> | 0.0025 | 515.0    | 557.8  | 492.1       | 610.1       | 357.4       |
| <b>IV.B1</b> | 0.0024 | 463.5    | 488.6  | 431.9       | 542.1       | 352.3       |
| <b>IV.B2</b> | 0.0025 | 469.7    | 498.7  | 430.3       | 554.3       | 352.1       |
| <b>IV.B3</b> | 0.0025 | 478.8    | 512.3  | 459.5       | 558.2       | 348.1       |
| <b>V.1</b>   | 0.0024 | 462.1    | 486.1  | 417.7       | 543.7       | 359.3       |
| <b>V.2</b>   | 0.0024 | 469.6    | 495.6  | 414.4       | 556.8       | 366.7       |
| <b>V.3</b>   | 0.0025 | 486.7    | 519.0  | 424.4       | 584.8       | 369.7       |

Table D.14. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 2 ( $d=1$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0259 | 104.0  | 197.7  | 17.0        | 56.6        | 269.9       |
| <b>I.B1</b>  | 0.0380 | 62.9   | 127.3  | 12.3        | 35.5        | 142.6       |
| <b>I.B2</b>  | 0.0401 | 59.1   | 120.1  | 11.7        | 34.1        | 131.6       |
| <b>I.B3</b>  | 0.0394 | 62.8   | 128.3  | 11.7        | 35.3        | 141.0       |
| <b>I.B4</b>  | 0.0387 | 66.9   | 139.6  | 11.8        | 36.9        | 153.6       |
| <b>II.A1</b> | 0.0323 | 108.5  | 227.1  | 12.8        | 51.9        | 287.2       |
| <b>II.A2</b> | 0.0300 | 151.6  | 313.0  | 13.1        | 67.0        | 452.6       |
| <b>II.A3</b> | 0.0261 | 239.9  | 466.3  | 14.2        | 101.6       | 815.5       |
| <b>II.B1</b> | 0.0309 | 130.4  | 267.2  | 13.1        | 62.2        | 363.2       |
| <b>II.B2</b> | 0.0301 | 141.5  | 288.5  | 13.4        | 66.4        | 403.7       |
| <b>II.B3</b> | 0.0257 | 237.0  | 454.9  | 14.7        | 106.7       | 771.0       |
| <b>III.1</b> | 0.0420 | 53.0   | 103.9  | 11.3        | 32.3        | 115.0       |
| <b>III.2</b> | 0.0404 | 65.8   | 133.2  | 11.1        | 37.6        | 151.6       |
| <b>III.3</b> | 0.0358 | 95.3   | 197.4  | 11.8        | 49.4        | 240.8       |
| <b>III.4</b> | 0.0424 | 52.2   | 102.0  | 11.2        | 31.8        | 113.3       |
| <b>III.5</b> | 0.0398 | 67.8   | 138.1  | 11.3        | 38.0        | 155.9       |
| <b>III.6</b> | 0.0361 | 95.0   | 195.9  | 11.6        | 49.3        | 237.8       |
| <b>IV.A1</b> | 0.0449 | 47.3   | 92.6   | 10.7        | 29.3        | 100.4       |
| <b>IV.A2</b> | 0.0431 | 56.0   | 110.7  | 10.7        | 33.3        | 124.1       |
| <b>IV.A3</b> | 0.0397 | 73.2   | 151.8  | 11.0        | 39.9        | 171.5       |
| <b>IV.A4</b> | 0.0447 | 47.2   | 92.4   | 10.8        | 29.3        | 99.3        |
| <b>IV.A5</b> | 0.0428 | 57.2   | 114.5  | 10.7        | 33.8        | 126.6       |
| <b>IV.A6</b> | 0.0396 | 74.3   | 152.9  | 11.1        | 40.7        | 175.6       |
| <b>IV.B1</b> | 0.0449 | 47.4   | 91.7   | 10.7        | 29.4        | 100.1       |
| <b>IV.B2</b> | 0.0447 | 50.2   | 98.5   | 10.6        | 30.6        | 108.2       |
| <b>IV.B3</b> | 0.0419 | 61.8   | 125.9  | 10.9        | 35.4        | 139.6       |
| <b>V.1</b>   | 0.0475 | 42.1   | 79.3   | 10.3        | 27.2        | 87.0        |
| <b>V.2</b>   | 0.0469 | 44.6   | 85.6   | 10.3        | 28.2        | 93.6        |
| <b>V.3</b>   | 0.046  | 50.4   | 99.6   | 10.1        | 30.4        | 109.5       |

Table D.15. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 3 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0023 | 463.7    | 478.7  | 497.8       | 511.8       | 358.3       |
| <b>I.B1</b>  | 0.0025 | 457.9    | 483.2  | 328.0       | 558.1       | 403.7       |
| <b>I.B2</b>  | 0.0025 | 467.8    | 497.3  | 307.0       | 579.8       | 420.1       |
| <b>I.B3</b>  | 0.0025 | 472.9    | 502.8  | 319.9       | 585.3       | 420.1       |
| <b>I.B4</b>  | 0.0025 | 489.7    | 521.9  | 327.8       | 610.8       | 432.1       |
| <b>II.A1</b> | 0.0023 | 527.6    | 571.4  | 520.1       | 622.0       | 356.0       |
| <b>II.A2</b> | 0.0024 | 541.2    | 597.6  | 501.5       | 653.5       | 348.3       |
| <b>II.A3</b> | 0.0024 | 585.9    | 672.8  | 617.8       | 678.8       | 320.6       |
| <b>II.B1</b> | 0.0025 | 539.2    | 595.6  | 441.8       | 667.8       | 360.1       |
| <b>II.B2</b> | 0.0026 | 549.1    | 609.2  | 462.1       | 670.8       | 361.0       |
| <b>II.B3</b> | 0.0025 | 581.3    | 666.0  | 588.9       | 676.8       | 331.0       |
| <b>III.1</b> | 0.0024 | 481.6    | 510.1  | 369.2       | 584.8       | 398.9       |
| <b>III.2</b> | 0.0025 | 511.0    | 550.5  | 360.4       | 640.5       | 407.9       |
| <b>III.3</b> | 0.0025 | 541.0    | 590.7  | 422.2       | 666.1       | 391.9       |
| <b>III.4</b> | 0.0025 | 469.9    | 496.9  | 358.7       | 568.7       | 393.9       |
| <b>III.5</b> | 0.0025 | 508.6    | 547.0  | 371.9       | 629.2       | 403.6       |
| <b>III.6</b> | 0.0025 | 539.2    | 587.5  | 426.0       | 657.0       | 392.2       |
| <b>IV.A1</b> | 0.0024 | 476.6    | 503.4  | 365.7       | 579.3       | 401.4       |
| <b>IV.A2</b> | 0.0025 | 497.5    | 533.6  | 365.2       | 616.9       | 398.7       |
| <b>IV.A3</b> | 0.0026 | 515.4    | 560.3  | 377.8       | 643.1       | 387.4       |
| <b>IV.A4</b> | 0.0025 | 471.1    | 497.6  | 371.6       | 572.3       | 389.2       |
| <b>IV.A5</b> | 0.0025 | 502.5    | 538.5  | 368.1       | 624.6       | 405.8       |
| <b>IV.A6</b> | 0.0025 | 531.4    | 577.4  | 392.8       | 658.9       | 401.8       |
| <b>IV.B1</b> | 0.0025 | 471.6    | 500.1  | 347.0       | 577.2       | 400.2       |
| <b>IV.B2</b> | 0.0026 | 475.8    | 507.2  | 334.2       | 592.3       | 404.5       |
| <b>IV.B3</b> | 0.0025 | 500.2    | 536.0  | 363.5       | 621.0       | 405.9       |
| <b>V.1</b>   | 0.0025 | 469.0    | 496.2  | 335.8       | 572.8       | 408.0       |
| <b>V.2</b>   | 0.0025 | 474.8    | 505.5  | 328.0       | 587.1       | 410.2       |
| <b>V.3</b>   | 0.0026 | 489.6    | 524.5  | 331.1       | 613.9       | 415.1       |

Table D.16. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 3 (d=2, K=0.10).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0403 | 41.9   | 68.4   | 12.8        | 30.2        | 82.6        |
| <b>I.B1</b>  | 0.0580 | 30.2   | 51.0   | 8.8         | 21.4        | 60.5        |
| <b>I.B2</b>  | 0.0615 | 28.7   | 49.9   | 8.3         | 19.9        | 57.5        |
| <b>I.B3</b>  | 0.0593 | 30.9   | 54.9   | 8.5         | 21.2        | 61.7        |
| <b>I.B4</b>  | 0.0576 | 32.9   | 60.2   | 8.7         | 22.1        | 66.6        |
| <b>II.A1</b> | 0.0391 | 79.9   | 173.1  | 11.0        | 39.8        | 193.7       |
| <b>II.A2</b> | 0.0382 | 107.4  | 239.4  | 10.6        | 47.0        | 288.7       |
| <b>II.A3</b> | 0.0317 | 197.5  | 408.7  | 11.7        | 77.0        | 668.5       |
| <b>II.B1</b> | 0.0419 | 90.5   | 203.8  | 9.9         | 41.0        | 232.9       |
| <b>II.B2</b> | 0.0404 | 98.3   | 217.5  | 10.1        | 44.7        | 258.2       |
| <b>II.B3</b> | 0.0324 | 185.7  | 385.8  | 11.7        | 74.3        | 604.4       |
| <b>III.1</b> | 0.0525 | 36.4   | 64.0   | 9.4         | 24.6        | 75.4        |
| <b>III.2</b> | 0.0511 | 46.3   | 91.6   | 9.0         | 28.0        | 101.0       |
| <b>III.3</b> | 0.0449 | 69.0   | 146.0  | 9.6         | 36.8        | 163.2       |
| <b>III.4</b> | 0.0536 | 35.5   | 62.6   | 9.2         | 24.2        | 72.4        |
| <b>III.5</b> | 0.0496 | 48.0   | 94.8   | 9.3         | 28.9        | 106.2       |
| <b>III.6</b> | 0.0449 | 68.2   | 142.8  | 9.6         | 36.6        | 162.1       |
| <b>IV.A1</b> | 0.0532 | 35.5   | 63.0   | 9.4         | 24.1        | 72.5        |
| <b>IV.A2</b> | 0.0514 | 42.9   | 81.6   | 9.1         | 26.9        | 92.0        |
| <b>IV.A3</b> | 0.0478 | 56.7   | 118.4  | 9.4         | 31.7        | 129.3       |
| <b>IV.A4</b> | 0.0529 | 36.0   | 63.7   | 9.3         | 24.4        | 74.1        |
| <b>IV.A5</b> | 0.0506 | 44.8   | 86.7   | 9.2         | 27.9        | 97.3        |
| <b>IV.A6</b> | 0.0478 | 58.2   | 121.4  | 9.3         | 32.6        | 133.5       |
| <b>IV.B1</b> | 0.0545 | 34.8   | 61.9   | 9.1         | 23.6        | 72.0        |
| <b>IV.B2</b> | 0.0553 | 36.4   | 67.5   | 8.8         | 23.9        | 76.0        |
| <b>IV.B3</b> | 0.0516 | 44.3   | 87.0   | 9.1         | 27.2        | 95.4        |
| <b>V.1</b>   | 0.0571 | 32.0   | 56.1   | 8.8         | 22.0        | 64.8        |
| <b>V.2</b>   | 0.0575 | 33.2   | 59.5   | 8.6         | 22.3        | 68.8        |
| <b>V.3</b>   | 0.0554 | 38.0   | 72.9   | 8.8         | 24.1        | 80.0        |

Table D.17. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0038 | 297.0    | 329.3  | 446.6       | 279.8       | 179.6       |
| <b>I.B1</b>  | 0.0027 | 437.7    | 469.7  | 284.3       | 559.7       | 371.6       |
| <b>I.B2</b>  | 0.0027 | 445.0    | 476.7  | 282.0       | 564.8       | 395.2       |
| <b>I.B3</b>  | 0.0025 | 479.1    | 508.8  | 341.1       | 591.2       | 413.2       |
| <b>I.B4</b>  | 0.0024 | 492.1    | 521.4  | 383.3       | 600.0       | 401.4       |
| <b>II.A1</b> | 0.0025 | 468.1    | 518.6  | 702.9       | 481.7       | 266.8       |
| <b>II.A2</b> | 0.0024 | 513.0    | 574.6  | 717.4       | 550.1       | 283.6       |
| <b>II.A3</b> | 0.0025 | 538.7    | 635.8  | 906.3       | 544.4       | 251.2       |
| <b>II.B1</b> | 0.0025 | 523.5    | 583.6  | 505.7       | 628.1       | 317.4       |
| <b>II.B2</b> | 0.0026 | 523.9    | 587.2  | 485.3       | 625.9       | 320.9       |
| <b>II.B3</b> | 0.0027 | 555.7    | 651.5  | 700.6       | 616.3       | 275.5       |
| <b>III.1</b> | 0.0027 | 450.1    | 483.2  | 358.3       | 555.3       | 337.3       |
| <b>III.2</b> | 0.0026 | 500.8    | 543.8  | 358.9       | 632.5       | 380.1       |
| <b>III.3</b> | 0.0026 | 531.8    | 582.1  | 440.5       | 648.8       | 371.1       |
| <b>III.4</b> | 0.0027 | 453.5    | 486.7  | 355.5       | 560.9       | 345.2       |
| <b>III.5</b> | 0.0026 | 495.6    | 537.8  | 363.4       | 618.6       | 373.0       |
| <b>III.6</b> | 0.0026 | 529.0    | 579.2  | 419.1       | 646.7       | 374.8       |
| <b>IV.A1</b> | 0.0026 | 451.0    | 483.7  | 367.8       | 554.6       | 334.2       |
| <b>IV.A2</b> | 0.0027 | 476.8    | 515.1  | 358.6       | 597.0       | 361.9       |
| <b>IV.A3</b> | 0.0026 | 502.0    | 545.5  | 391.7       | 622.7       | 367.6       |
| <b>IV.A4</b> | 0.0026 | 452.1    | 486.4  | 357.8       | 561.1       | 335.6       |
| <b>IV.A5</b> | 0.0026 | 490.7    | 532.4  | 346.8       | 619.9       | 377.0       |
| <b>IV.A6</b> | 0.0027 | 511.9    | 559.2  | 382.0       | 640.8       | 376.2       |
| <b>IV.B1</b> | 0.0027 | 450.6    | 484.0  | 326.1       | 566.8       | 352.9       |
| <b>IV.B2</b> | 0.0027 | 466.2    | 502.3  | 311.9       | 597.2       | 380.5       |
| <b>IV.B3</b> | 0.0026 | 491.8    | 530.3  | 356.4       | 613.8       | 383.7       |
| <b>V.1</b>   | 0.0027 | 447.6    | 481.1  | 304.0       | 569.7       | 363.5       |
| <b>V.2</b>   | 0.0027 | 455.5    | 489.4  | 299.4       | 579.0       | 387.1       |
| <b>V.3</b>   | 0.0027 | 484.6    | 522.9  | 311.3       | 618.9       | 407.4       |

Table D.18. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0100 | 207.7  | 300.0  | 44.9        | 162.3       | 464.2       |
| <b>I.B1</b>  | 0.0606 | 35.0   | 73.0   | 8.1         | 20.6        | 73.9        |
| <b>I.B2</b>  | 0.0627 | 30.9   | 61.0   | 8.0         | 19.6        | 62.9        |
| <b>I.B3</b>  | 0.0575 | 33.9   | 65.3   | 8.6         | 21.9        | 68.6        |
| <b>I.B4</b>  | 0.0515 | 39.3   | 75.8   | 9.5         | 25.1        | 81.4        |
| <b>II.A1</b> | 0.0228 | 153.7  | 287.1  | 18.0        | 82.9        | 426.6       |
| <b>II.A2</b> | 0.0260 | 166.8  | 325.6  | 15.3        | 79.0        | 497.1       |
| <b>II.A3</b> | 0.0204 | 291.5  | 522.0  | 18.4        | 146.6       | 911.1       |
| <b>II.B1</b> | 0.0369 | 118.7  | 261.7  | 10.9        | 49.7        | 338.1       |
| <b>II.B2</b> | 0.0377 | 126.4  | 277.3  | 10.5        | 51.5        | 377.0       |
| <b>II.B3</b> | 0.0280 | 247.5  | 477.3  | 13.3        | 105.2       | 846.6       |
| <b>III.1</b> | 0.0488 | 51.7   | 108.3  | 9.4         | 28.5        | 116.0       |
| <b>III.2</b> | 0.0495 | 56.4   | 122.6  | 9.0         | 29.9        | 129.2       |
| <b>III.3</b> | 0.0431 | 78.3   | 173.0  | 9.9         | 38.6        | 190.7       |
| <b>III.4</b> | 0.0499 | 49.9   | 105.3  | 9.2         | 27.8        | 111.6       |
| <b>III.5</b> | 0.0488 | 57.2   | 123.6  | 9.2         | 30.4        | 131.0       |
| <b>III.6</b> | 0.0440 | 77.1   | 169.2  | 9.7         | 38.1        | 190.1       |
| <b>IV.A1</b> | 0.0491 | 51.4   | 108.6  | 9.3         | 28.1        | 115.7       |
| <b>IV.A2</b> | 0.0495 | 53.2   | 115.1  | 9.2         | 28.9        | 119.1       |
| <b>IV.A3</b> | 0.0465 | 65.3   | 143.0  | 9.3         | 33.5        | 153.4       |
| <b>IV.A4</b> | 0.0496 | 51.7   | 110.1  | 9.2         | 27.9        | 118.6       |
| <b>IV.A5</b> | 0.0502 | 53.5   | 116.7  | 9.0         | 28.7        | 120.8       |
| <b>IV.A6</b> | 0.0471 | 65.5   | 144.5  | 9.2         | 33.3        | 154.1       |
| <b>IV.B1</b> | 0.0538 | 45.3   | 97.9   | 8.7         | 24.8        | 99.8        |
| <b>IV.B2</b> | 0.0558 | 41.6   | 88.7   | 8.6         | 23.8        | 89.2        |
| <b>IV.B3</b> | 0.0513 | 48.5   | 104.1  | 9.0         | 27.3        | 105.5       |
| <b>V.1</b>   | 0.0577 | 38.9   | 82.4   | 8.3         | 22.4        | 84.2        |
| <b>V.2</b>   | 0.0586 | 37.5   | 78.9   | 8.2         | 22.1        | 78.8        |
| <b>V.3</b>   | 0.0569 | 40.1   | 83.2   | 8.4         | 23.5        | 86.0        |

Table D.19. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0050 | 227.2    | 256.2  | 342.3       | 211.5       | 134.7       |
| <b>I.B1</b>  | 0.0028 | 424.3    | 459.6  | 306.9       | 546.2       | 313.0       |
| <b>I.B2</b>  | 0.0027 | 440.9    | 474.0  | 302.1       | 564.7       | 355.3       |
| <b>I.B3</b>  | 0.0025 | 472.5    | 503.1  | 370.3       | 580.7       | 375.5       |
| <b>I.B4</b>  | 0.0023 | 483.5    | 511.8  | 451.3       | 569.3       | 362.3       |
| <b>II.A1</b> | 0.0029 | 408.8    | 468.0  | 700.1       | 389.0       | 212.9       |
| <b>II.A2</b> | 0.0026 | 465.6    | 530.6  | 782.2       | 463.7       | 243.5       |
| <b>II.A3</b> | 0.0027 | 494.2    | 597.0  | 954.2       | 465.8       | 216.5       |
| <b>II.B1</b> | 0.0026 | 494.9    | 562.4  | 609.1       | 550.7       | 267.4       |
| <b>II.B2</b> | 0.0027 | 493.6    | 559.5  | 569.7       | 560.0       | 273.7       |
| <b>II.B3</b> | 0.0029 | 506.2    | 609.1  | 822.9       | 521.2       | 222.5       |
| <b>III.1</b> | 0.0028 | 423.8    | 460.6  | 444.9       | 492.6       | 274.1       |
| <b>III.2</b> | 0.0027 | 477.8    | 522.7  | 428.0       | 579.7       | 321.2       |
| <b>III.3</b> | 0.0026 | 510.3    | 562.9  | 502.9       | 594.7       | 329.0       |
| <b>III.4</b> | 0.0028 | 427.4    | 463.6  | 416.0       | 505.9       | 283.7       |
| <b>III.5</b> | 0.0027 | 486.1    | 532.2  | 410.1       | 593.2       | 333.6       |
| <b>III.6</b> | 0.0027 | 509.4    | 562.4  | 484.7       | 603.6       | 331.9       |
| <b>IV.A1</b> | 0.0028 | 416.6    | 452.8  | 432.6       | 483.9       | 270.3       |
| <b>IV.A2</b> | 0.0027 | 456.1    | 496.9  | 406.7       | 551.9       | 307.5       |
| <b>IV.A3</b> | 0.0026 | 487.4    | 534.6  | 438.6       | 589.5       | 321.4       |
| <b>IV.A4</b> | 0.0028 | 423.4    | 461.6  | 421.9       | 500.4       | 273.1       |
| <b>IV.A5</b> | 0.0027 | 470.7    | 513.7  | 388.0       | 579.5       | 327.8       |
| <b>IV.A6</b> | 0.0027 | 496.8    | 545.3  | 426.8       | 609.8       | 334.9       |
| <b>IV.B1</b> | 0.0028 | 430.5    | 466.9  | 370.0       | 527.9       | 296.5       |
| <b>IV.B2</b> | 0.0027 | 451.8    | 489.3  | 346.1       | 565.5       | 330.3       |
| <b>IV.B3</b> | 0.0026 | 483.2    | 524.0  | 392.5       | 598.7       | 347.8       |
| <b>V.1</b>   | 0.0028 | 434.1    | 468.3  | 340.8       | 538.2       | 316.8       |
| <b>V.2</b>   | 0.0027 | 453.7    | 490.6  | 320.5       | 577.0       | 348.1       |
| <b>V.3</b>   | 0.0027 | 469.2    | 508.7  | 327.5       | 597.7       | 361.5       |

Table D.20. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 3 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0051 | 330.8  | 406.6  | 94.4        | 342.2       | 551.1       |
| <b>I.B1</b>  | 0.0539 | 51.5   | 117.8  | 8.5         | 24.7        | 119.9       |
| <b>I.B2</b>  | 0.0569 | 41.2   | 91.1   | 8.4         | 22.6        | 88.5        |
| <b>I.B3</b>  | 0.0511 | 43.7   | 91.0   | 9.4         | 25.5        | 93.3        |
| <b>I.B4</b>  | 0.0442 | 51.7   | 106.7  | 10.8        | 30.1        | 110.9       |
| <b>II.A1</b> | 0.0147 | 238.8  | 390.7  | 27.4        | 151.1       | 646.2       |
| <b>II.A2</b> | 0.0186 | 229    | 402.7  | 20.9        | 125.7       | 675.7       |
| <b>II.A3</b> | 0.0146 | 367.3  | 586.7  | 25.8        | 249.4       | 918.9       |
| <b>II.B1</b> | 0.0298 | 169.2  | 339.1  | 13.0        | 73.2        | 536.1       |
| <b>II.B2</b> | 0.0312 | 170.8  | 347.5  | 12.3        | 71.9        | 554.2       |
| <b>II.B3</b> | 0.0234 | 310.5  | 545.2  | 15.5        | 160.6       | 933.2       |
| <b>III.1</b> | 0.0392 | 82.4   | 172.8  | 10.8        | 40.3        | 211.8       |
| <b>III.2</b> | 0.0423 | 78.5   | 170.5  | 10.1        | 38.4        | 194.1       |
| <b>III.3</b> | 0.0376 | 103.6  | 223.3  | 10.9        | 47.8        | 275.7       |
| <b>III.4</b> | 0.0406 | 76.8   | 161.6  | 10.6        | 38.4        | 194.2       |
| <b>III.5</b> | 0.0429 | 78.2   | 171.4  | 10.0        | 37.9        | 190.8       |
| <b>III.6</b> | 0.0388 | 100.9  | 219.6  | 10.6        | 46.7        | 265.6       |
| <b>IV.A1</b> | 0.0393 | 83.1   | 175.6  | 10.8        | 40.2        | 215.6       |
| <b>IV.A2</b> | 0.0429 | 75.7   | 165.3  | 10.0        | 36.6        | 189.0       |
| <b>IV.A3</b> | 0.0406 | 86.9   | 188.6  | 10.3        | 41.4        | 223.3       |
| <b>IV.A4</b> | 0.041  | 79.3   | 167.8  | 10.3        | 38.6        | 202.7       |
| <b>IV.A5</b> | 0.0441 | 74.8   | 166.6  | 9.7         | 35.7        | 187.0       |
| <b>IV.A6</b> | 0.0422 | 85.9   | 192.7  | 10.0        | 39.6        | 217.2       |
| <b>IV.B1</b> | 0.046  | 66.3   | 146.0  | 9.5         | 32.0        | 162.9       |
| <b>IV.B2</b> | 0.0497 | 58.0   | 130.2  | 9.1         | 28.5        | 138.1       |
| <b>IV.B3</b> | 0.0471 | 63.3   | 142.7  | 9.5         | 31.3        | 148.6       |
| <b>V.1</b>   | 0.0496 | 55.7   | 122.5  | 9.0         | 28.6        | 130.1       |
| <b>V.2</b>   | 0.0529 | 49.3   | 109.9  | 8.7         | 25.8        | 110.7       |
| <b>V.3</b>   | 0.0529 | 50.5   | 112.5  | 8.7         | 26.5        | 114.3       |

Table D.21. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0026 | 417.7    | 439.3  | 558.4       | 428.4       | 284.4       |
| <b>I.B1</b>  | 0.0026 | 419.2    | 440.9  | 556.6       | 435.1       | 286.3       |
| <b>I.B2</b>  | 0.0026 | 421.9    | 446.5  | 568.2       | 437.1       | 284.7       |
| <b>I.B3</b>  | 0.0026 | 426.5    | 455.3  | 585.7       | 442.3       | 279.8       |
| <b>I.B4</b>  | 0.0026 | 437.2    | 468.6  | 602.1       | 455.3       | 284.0       |
| <b>II.A1</b> | 0.0025 | 471.1    | 517.9  | 675.8       | 494.7       | 275.3       |
| <b>II.A2</b> | 0.0025 | 491.8    | 557.4  | 743.5       | 510.3       | 264.3       |
| <b>II.A3</b> | 0.0025 | 532.2    | 629.6  | 901.6       | 538.9       | 246.6       |
| <b>II.B1</b> | 0.0025 | 483.7    | 542.4  | 726.4       | 502.4       | 271.3       |
| <b>II.B2</b> | 0.0025 | 489.4    | 551.9  | 761.2       | 503.1       | 271.3       |
| <b>II.B3</b> | 0.0026 | 523.5    | 615.4  | 913.1       | 520.1       | 254.6       |
| <b>III.1</b> | 0.0025 | 431.0    | 457.1  | 577.0       | 450.5       | 291.4       |
| <b>III.2</b> | 0.0025 | 458.9    | 499.5  | 643         | 479.9       | 287.4       |
| <b>III.3</b> | 0.0026 | 479.7    | 533.4  | 708.6       | 497.3       | 281.8       |
| <b>III.4</b> | 0.0025 | 434.9    | 463.7  | 586.2       | 455.3       | 286.3       |
| <b>III.5</b> | 0.0026 | 456.8    | 496.0  | 643.1       | 480.2       | 288.3       |
| <b>III.6</b> | 0.0026 | 468.7    | 521.1  | 689.6       | 487.6       | 277.1       |
| <b>IV.A1</b> | 0.0025 | 432.8    | 459.2  | 569.4       | 457.6       | 292         |
| <b>IV.A2</b> | 0.0026 | 446.6    | 481.7  | 597.4       | 473.2       | 289.8       |
| <b>IV.A3</b> | 0.0026 | 451.8    | 496.5  | 615.7       | 479.4       | 276.3       |
| <b>IV.A4</b> | 0.0025 | 432.5    | 459.6  | 570.7       | 456.0       | 288.8       |
| <b>IV.A5</b> | 0.0026 | 455.4    | 492.6  | 607.8       | 484.8       | 294.1       |
| <b>IV.A6</b> | 0.0026 | 472.7    | 519.2  | 650.8       | 504.5       | 289.1       |
| <b>IV.B1</b> | 0.0025 | 432.7    | 459.0  | 568.1       | 457.4       | 291.5       |
| <b>IV.B2</b> | 0.0026 | 433.7    | 464.1  | 579.0       | 456.8       | 285.6       |
| <b>IV.B3</b> | 0.0026 | 449.6    | 488.7  | 624.0       | 473.6       | 284.9       |
| <b>V.1</b>   | 0.0025 | 431.3    | 456.2  | 551.9       | 462.1       | 293.3       |
| <b>V.2</b>   | 0.0025 | 440.8    | 469.8  | 566.2       | 470.2       | 293.9       |
| <b>V.3</b>   | 0.0026 | 450.0    | 484.5  | 572.0       | 484.1       | 295.9       |

Table D.22. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0269 | 70.7   | 118.9  | 18.2        | 48.5        | 150.0       |
| <b>I.B1</b>  | 0.0269 | 73.2   | 123.7  | 17.9        | 50.2        | 155.1       |
| <b>I.B2</b>  | 0.0271 | 77.2   | 132.6  | 17.4        | 51.6        | 168.4       |
| <b>I.B3</b>  | 0.0267 | 87.1   | 153.9  | 17.1        | 55.5        | 201.1       |
| <b>I.B4</b>  | 0.027  | 95.4   | 175.1  | 16.5        | 58.5        | 222.9       |
| <b>II.A1</b> | 0.0245 | 139.7  | 266.3  | 16.9        | 73.6        | 380.5       |
| <b>II.A2</b> | 0.0232 | 196.2  | 373.2  | 16.9        | 94.6        | 603.9       |
| <b>II.A3</b> | 0.0205 | 297.5  | 524.6  | 18.2        | 153.1       | 905.0       |
| <b>II.B1</b> | 0.0233 | 178.1  | 334.6  | 17.1        | 91.9        | 515.0       |
| <b>II.B2</b> | 0.0228 | 190.8  | 355    | 17.5        | 99.3        | 559.9       |
| <b>II.B3</b> | 0.0196 | 302.7  | 529.1  | 19.4        | 165.9       | 904.0       |
| <b>III.1</b> | 0.0279 | 81.0   | 143.3  | 16.6        | 52.1        | 181.2       |
| <b>III.2</b> | 0.0275 | 109.5  | 204.7  | 15.5        | 63.5        | 271.9       |
| <b>III.3</b> | 0.0248 | 160.4  | 303.5  | 16.5        | 85.5        | 443.9       |
| <b>III.4</b> | 0.0281 | 81.0   | 143.6  | 16.5        | 52.1        | 181.7       |
| <b>III.5</b> | 0.0269 | 112.5  | 209.8  | 16.0        | 65.5        | 278.9       |
| <b>III.6</b> | 0.0249 | 154.2  | 288.4  | 16.6        | 83.2        | 422.8       |
| <b>IV.A1</b> | 0.029  | 78.1   | 139.8  | 16.0        | 50.1        | 174.5       |
| <b>IV.A2</b> | 0.0293 | 97.3   | 183.2  | 14.9        | 56.8        | 236.6       |
| <b>IV.A3</b> | 0.028  | 123.8  | 237.9  | 14.8        | 66.7        | 327.1       |
| <b>IV.A4</b> | 0.0289 | 78.2   | 139.9  | 16.2        | 49.7        | 176.5       |
| <b>IV.A5</b> | 0.0289 | 100.2  | 190.3  | 15.1        | 57.8        | 244.5       |
| <b>IV.A6</b> | 0.0276 | 127.1  | 243.7  | 15.1        | 68.6        | 334.5       |
| <b>IV.B1</b> | 0.0292 | 78.1   | 139.4  | 15.8        | 49.8        | 176.5       |
| <b>IV.B2</b> | 0.0292 | 86.0   | 156.3  | 15.4        | 53.5        | 198.9       |
| <b>IV.B3</b> | 0.0281 | 106.4  | 199.4  | 15.3        | 61.6        | 262.1       |
| <b>V.1</b>   | 0.0306 | 72.0   | 128.6  | 15.1        | 46.5        | 159.5       |
| <b>V.2</b>   | 0.0309 | 77.1   | 141    | 14.9        | 47.6        | 177.1       |
| <b>V.3</b>   | 0.0313 | 88.0   | 167.5  | 14.2        | 51.0        | 212.1       |

Table D.23. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0056 | 201.0    | 227.6  | 299.8       | 185.9       | 120.1       |
| <b>I.B1</b>  | 0.0056 | 204.7    | 234.8  | 310.6       | 187.6       | 119.7       |
| <b>I.B2</b>  | 0.0057 | 205.3    | 239.4  | 322.1       | 185.1       | 116.3       |
| <b>I.B3</b>  | 0.0057 | 210.6    | 252.9  | 344.6       | 185.4       | 113.0       |
| <b>I.B4</b>  | 0.0056 | 220.3    | 271.7  | 381.5       | 190.5       | 111.3       |
| <b>II.A1</b> | 0.0043 | 298.1    | 363.9  | 531.7       | 262.8       | 138.4       |
| <b>II.A2</b> | 0.0041 | 326.7    | 410.1  | 622.4       | 286.1       | 138.5       |
| <b>II.A3</b> | 0.0041 | 356.5    | 466.3  | 725.3       | 299.2       | 134.7       |
| <b>II.B1</b> | 0.0043 | 312.0    | 393.2  | 605.2       | 268.8       | 135.0       |
| <b>II.B2</b> | 0.0043 | 316.2    | 400.5  | 619.7       | 271.5       | 135.9       |
| <b>II.B3</b> | 0.0042 | 346.3    | 459.9  | 729.9       | 286.9       | 131.7       |
| <b>III.1</b> | 0.0051 | 240.4    | 288.3  | 409.6       | 211.8       | 125.3       |
| <b>III.2</b> | 0.0045 | 286.4    | 352.6  | 556.1       | 250.6       | 130.3       |
| <b>III.3</b> | 0.0043 | 320.1    | 403.5  | 656.1       | 278.8       | 134.3       |
| <b>III.4</b> | 0.0052 | 236.0    | 283.4  | 405.8       | 207.4       | 121.9       |
| <b>III.5</b> | 0.0045 | 287.0    | 353.4  | 553.2       | 252.2       | 131.3       |
| <b>III.6</b> | 0.0043 | 320.3    | 402.8  | 660.1       | 279.0       | 135.3       |
| <b>IV.A1</b> | 0.0051 | 239.9    | 290.0  | 425.4       | 208.9       | 122.0       |
| <b>IV.A2</b> | 0.0046 | 289.1    | 357.8  | 574.0       | 253.7       | 128.0       |
| <b>IV.A3</b> | 0.0043 | 313.4    | 388.4  | 639.7       | 281.4       | 133.1       |
| <b>IV.A4</b> | 0.0051 | 241.8    | 292.7  | 428.7       | 210.4       | 122.8       |
| <b>IV.A5</b> | 0.0044 | 297.9    | 368.2  | 597.0       | 263.2       | 132.7       |
| <b>IV.A6</b> | 0.0042 | 331.6    | 411.9  | 680.5       | 300.1       | 138.5       |
| <b>IV.B1</b> | 0.0050 | 247.1    | 299.8  | 443.6       | 215.3       | 122.8       |
| <b>IV.B2</b> | 0.0049 | 260.9    | 321.2  | 504.5       | 225.5       | 123.4       |
| <b>IV.B3</b> | 0.0046 | 290.3    | 361.7  | 597.9       | 253.4       | 127.4       |
| <b>V.1</b>   | 0.0045 | 275.1    | 329.3  | 505.1       | 246.8       | 134.7       |
| <b>V.2</b>   | 0.0044 | 287.1    | 345.4  | 544.8       | 258.5       | 135.6       |
| <b>V.3</b>   | 0.0040 | 324.4    | 390.1  | 628.8       | 303.9       | 145.7       |

Table D.24. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.15$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0040 | 381.9  | 444.9  | 130.1       | 436.3       | 544.7       |
| <b>I.B1</b>  | 0.0043 | 382.1  | 444.1  | 125.1       | 448.1       | 526.8       |
| <b>I.B2</b>  | 0.0047 | 381.4  | 445.9  | 116.0       | 454.4       | 523.5       |
| <b>I.B3</b>  | 0.0049 | 407.6  | 478.9  | 111.0       | 514.8       | 524.8       |
| <b>I.B4</b>  | 0.0055 | 415.5  | 496.4  | 96.3        | 537.8       | 523.5       |
| <b>II.A1</b> | 0.0069 | 397.0  | 520.3  | 61.7        | 441.1       | 641.8       |
| <b>II.A2</b> | 0.0076 | 426.6  | 569.5  | 55.6        | 492.8       | 658.4       |
| <b>II.A3</b> | 0.0071 | 517.8  | 678.9  | 61.6        | 718.5       | 638.1       |
| <b>II.B1</b> | 0.0077 | 432.3  | 568.3  | 56.8        | 522.3       | 634.7       |
| <b>II.B2</b> | 0.008  | 443.5  | 584.1  | 54.8        | 548.7       | 641.7       |
| <b>II.B3</b> | 0.0075 | 535.5  | 691.7  | 64.5        | 780.6       | 626.6       |
| <b>III.1</b> | 0.0069 | 355.7  | 448.1  | 67.5        | 405.2       | 554.4       |
| <b>III.2</b> | 0.0099 | 359.4  | 485.4  | 42.3        | 391.3       | 598.7       |
| <b>III.3</b> | 0.0109 | 385.4  | 530.4  | 38.2        | 427.4       | 628.5       |
| <b>III.4</b> | 0.0071 | 356.0  | 450.5  | 64.8        | 403.8       | 564.5       |
| <b>III.5</b> | 0.0099 | 362.8  | 489.2  | 42.9        | 394.0       | 599.7       |
| <b>III.6</b> | 0.0114 | 383.2  | 530.2  | 36.2        | 418.4       | 625.7       |
| <b>IV.A1</b> | 0.0076 | 345.1  | 438.7  | 60.4        | 392.8       | 546         |
| <b>IV.A2</b> | 0.0117 | 328.7  | 455.7  | 34.8        | 337.8       | 583.8       |
| <b>IV.A3</b> | 0.0134 | 332.5  | 477.3  | 29.2        | 318.2       | 611.9       |
| <b>IV.A4</b> | 0.0079 | 346.7  | 443.2  | 57.9        | 391.9       | 551.6       |
| <b>IV.A5</b> | 0.012  | 340.1  | 476.4  | 33.3        | 339.5       | 616.5       |
| <b>IV.A6</b> | 0.0142 | 337.0  | 492.6  | 27.4        | 305.0       | 649.6       |
| <b>IV.B1</b> | 0.0083 | 342.7  | 442.6  | 53.5        | 385.1       | 559.2       |
| <b>IV.B2</b> | 0.0102 | 338.6  | 449.1  | 41.7        | 378.5       | 559.8       |
| <b>IV.B3</b> | 0.0127 | 336.2  | 467.8  | 31.6        | 353.9       | 581.6       |
| <b>V.1</b>   | 0.0111 | 286.3  | 398.9  | 36.7        | 265.8       | 567.2       |
| <b>V.2</b>   | 0.0133 | 280.6  | 405.3  | 29.8        | 242.5       | 588.1       |
| <b>V.3</b>   | 0.0172 | 265.5  | 409.9  | 21.8        | 194.2       | 622.0       |

Table D.25. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0074 | 152.3    | 170.6  | 222.5       | 141.7       | 94.2        |
| <b>I.B1</b>  | 0.0074 | 151.6    | 170.9  | 222.0       | 140.8       | 92.6        |
| <b>I.B2</b>  | 0.0075 | 151.0    | 173.2  | 223.2       | 138.4       | 91.0        |
| <b>I.B3</b>  | 0.0077 | 151.4    | 179.5  | 231.9       | 135.6       | 87.1        |
| <b>I.B4</b>  | 0.0077 | 154.7    | 188.6  | 243.8       | 135.6       | 85.4        |
| <b>II.A1</b> | 0.0060 | 216.3    | 273.4  | 374.2       | 186.0       | 99.2        |
| <b>II.A2</b> | 0.0058 | 240.7    | 317.9  | 445.5       | 201.6       | 97.3        |
| <b>II.A3</b> | 0.0056 | 266.7    | 370.5  | 524.7       | 214.2       | 95.8        |
| <b>II.B1</b> | 0.0060 | 224.5    | 295.7  | 411.2       | 185.9       | 95.6        |
| <b>II.B2</b> | 0.0060 | 231.0    | 309.9  | 434.4       | 188.3       | 96.6        |
| <b>II.B3</b> | 0.0058 | 256.3    | 359.0  | 506.6       | 202.3       | 94.4        |
| <b>III.1</b> | 0.0070 | 169.5    | 205.3  | 270.0       | 148.5       | 93.6        |
| <b>III.2</b> | 0.0065 | 200.6    | 262.6  | 368.3       | 163.5       | 91.9        |
| <b>III.3</b> | 0.0063 | 226.5    | 310.1  | 454.2       | 178.8       | 91.3        |
| <b>III.4</b> | 0.0071 | 167.4    | 202.6  | 266.0       | 146.3       | 92.5        |
| <b>III.5</b> | 0.0065 | 202.1    | 263.8  | 369.5       | 165.7       | 93.5        |
| <b>III.6</b> | 0.0062 | 226.1    | 307.3  | 450.5       | 180.2       | 91.5        |
| <b>IV.A1</b> | 0.0071 | 167.8    | 204.4  | 269.3       | 146.0       | 91.5        |
| <b>IV.A2</b> | 0.0068 | 193.5    | 255.9  | 355.8       | 157.0       | 88.8        |
| <b>IV.A3</b> | 0.0065 | 217.1    | 294.6  | 434.7       | 173.0       | 88.0        |
| <b>IV.A4</b> | 0.0071 | 167.7    | 205.4  | 267.0       | 146.1       | 92.2        |
| <b>IV.A5</b> | 0.0065 | 203.6    | 268.9  | 380.8       | 165.7       | 92.2        |
| <b>IV.A6</b> | 0.0062 | 226.7    | 307.4  | 468.9       | 180.0       | 91.4        |
| <b>IV.B1</b> | 0.0071 | 169.8    | 209.4  | 279.4       | 145.7       | 91.4        |
| <b>IV.B2</b> | 0.0070 | 180.3    | 231.8  | 311.7       | 150.2       | 90.9        |
| <b>IV.B3</b> | 0.0066 | 201.4    | 267.8  | 386.4       | 161.2       | 90.3        |
| <b>V.1</b>   | 0.0066 | 188.5    | 235.3  | 323.9       | 160.5       | 96.1        |
| <b>V.2</b>   | 0.0064 | 199.9    | 255.7  | 361.5       | 166.1       | 95.6        |
| <b>V.3</b>   | 0.0060 | 227.8    | 298.1  | 462.0       | 185.1       | 97.4        |

Table D.26. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 4 ( $d=3$ ,  $K=0.20$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0025 | 457.6  | 482.4  | 302.6       | 553.5       | 441.9       |
| <b>I.B1</b>  | 0.0025 | 460.7  | 486.4  | 306.0       | 560.9       | 441.8       |
| <b>I.B2</b>  | 0.0026 | 465.7  | 492.7  | 313.9       | 566.6       | 434.2       |
| <b>I.B3</b>  | 0.0027 | 473.7  | 504.0  | 333.1       | 583.4       | 411.9       |
| <b>I.B4</b>  | 0.0028 | 483.9  | 519.3  | 336.6       | 602.0       | 402.9       |
| <b>II.A1</b> | 0.0035 | 502.7  | 577.2  | 166.7       | 718.6       | 477.3       |
| <b>II.A2</b> | 0.0038 | 520.3  | 610.6  | 150.2       | 781.9       | 459.7       |
| <b>II.A3</b> | 0.0040 | 586.3  | 701.9  | 154.8       | 918.1       | 456.0       |
| <b>II.B1</b> | 0.0040 | 511.6  | 593.2  | 157.5       | 752.3       | 453.7       |
| <b>II.B2</b> | 0.0041 | 530.3  | 615.5  | 159.3       | 777.5       | 466.9       |
| <b>II.B3</b> | 0.0042 | 595.1  | 703.9  | 174.5       | 895.4       | 449.9       |
| <b>III.1</b> | 0.0034 | 453.4  | 496.9  | 215.6       | 586.2       | 439.3       |
| <b>III.2</b> | 0.0048 | 462.6  | 532.7  | 133.9       | 660.4       | 447.5       |
| <b>III.3</b> | 0.0053 | 483.6  | 571.2  | 118.5       | 730.5       | 433.8       |
| <b>III.4</b> | 0.0033 | 453.1  | 496.2  | 216.0       | 587.8       | 438.0       |
| <b>III.5</b> | 0.0047 | 463.7  | 532.9  | 140.2       | 661.2       | 438.4       |
| <b>III.6</b> | 0.0056 | 481.2  | 571.1  | 113.4       | 736.7       | 429.7       |
| <b>IV.A1</b> | 0.0034 | 456.9  | 500.6  | 222.2       | 592.4       | 439.6       |
| <b>IV.A2</b> | 0.0050 | 439.2  | 507.1  | 125.0       | 632.0       | 420.7       |
| <b>IV.A3</b> | 0.0062 | 444.5  | 530.0  | 91.1        | 673.9       | 419.3       |
| <b>IV.A4</b> | 0.0035 | 447.8  | 491.2  | 210.1       | 582.8       | 436.3       |
| <b>IV.A5</b> | 0.0054 | 445.0  | 517.7  | 115.6       | 642.6       | 437.3       |
| <b>IV.A6</b> | 0.0067 | 452.6  | 545.9  | 84.6        | 699.3       | 441.7       |
| <b>IV.B1</b> | 0.0038 | 444.6  | 492.4  | 190.8       | 587.0       | 436.9       |
| <b>IV.B2</b> | 0.0045 | 444.9  | 502.2  | 156.3       | 608.5       | 431.7       |
| <b>IV.B3</b> | 0.0060 | 437.4  | 511.1  | 105.8       | 639.6       | 423.2       |
| <b>V.1</b>   | 0.0049 | 413.4  | 477.0  | 117.1       | 560.7       | 451.8       |
| <b>V.2</b>   | 0.0057 | 409.3  | 481.2  | 97.1        | 577.2       | 447.6       |
| <b>V.3</b>   | 0.0081 | 390.4  | 485.4  | 58.0        | 564.1       | 460.7       |

Table D.27. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 5 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0030 | 446.8    | 492.7  | 199.4       | 550.5       | 510.8       |
| <b>IV.A2</b> | 0.0031 | 482.3    | 534.4  | 205.1       | 618.7       | 513.7       |
| <b>IV.A3</b> | 0.0031 | 513.5    | 569.7  | 224.0       | 683.8       | 494.8       |
| <b>IV.A4</b> | 0.0030 | 452.4    | 497.4  | 206.4       | 554.1       | 514.5       |
| <b>IV.A5</b> | 0.0030 | 499.9    | 554.1  | 214.3       | 652.3       | 521.9       |
| <b>IV.A6</b> | 0.0030 | 531.6    | 588.2  | 239.7       | 704.4       | 505.5       |
| <b>IV.B1</b> | 0.0028 | 462.9    | 500.8  | 241.8       | 577.0       | 487.9       |
| <b>IV.B2</b> | 0.0028 | 477.2    | 517.2  | 249.2       | 603.1       | 485.0       |
| <b>IV.B3</b> | 0.0028 | 504.0    | 545.7  | 280.1       | 643.5       | 472.4       |
| <b>V.1</b>   | 0.0028 | 458.4    | 496.5  | 235.4       | 568.9       | 490.3       |
| <b>V.2</b>   | 0.0028 | 466.1    | 505.3  | 239.4       | 581.2       | 488.7       |
| <b>V.3</b>   | 0.0028 | 496.4    | 541.2  | 247.5       | 639.8       | 486.9       |

Table D.28. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 5 ( $d=2$ ,  $K=0.10$ ).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0811 | 19.0   | 29.2   | 6.7         | 14.5        | 35.5        |
| <b>IV.A2</b> | 0.0754 | 23.6   | 40.7   | 6.7         | 16.7        | 47.3        |
| <b>IV.A3</b> | 0.0676 | 31.5   | 60.8   | 7.1         | 20.0        | 66.2        |
| <b>IV.A4</b> | 0.0806 | 19.1   | 29.1   | 6.7         | 14.6        | 35.9        |
| <b>IV.A5</b> | 0.0736 | 24.3   | 42.3   | 6.8         | 17.1        | 48.4        |
| <b>IV.A6</b> | 0.0663 | 32.5   | 63.1   | 7.1         | 20.6        | 68.5        |
| <b>IV.B1</b> | 0.0729 | 21.5   | 33.4   | 7.4         | 16.3        | 40.7        |
| <b>IV.B2</b> | 0.0702 | 23.6   | 38.2   | 7.5         | 17.5        | 46.0        |
| <b>IV.B3</b> | 0.0630 | 30.8   | 55.1   | 7.8         | 21.0        | 62.8        |
| <b>V.1</b>   | 0.0748 | 20.6   | 31.5   | 7.2         | 15.8        | 38.8        |
| <b>V.2</b>   | 0.072  | 22.6   | 36.1   | 7.4         | 16.8        | 43.8        |
| <b>V.3</b>   | 0.0673 | 27.2   | 47.5   | 7.4         | 19.0        | 54.9        |

Table D.29. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 5 (d=3, K=0.15).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0033 | 425.1    | 479.7  | 167.3       | 506.4       | 542.2       |
| <b>IV.A2</b> | 0.0031 | 477.8    | 528.1  | 206.9       | 605.5       | 523.9       |
| <b>IV.A3</b> | 0.0029 | 511.1    | 561.0  | 245.3       | 668.9       | 491.0       |
| <b>IV.A4</b> | 0.0033 | 424.4    | 475.5  | 174.3       | 506.4       | 534.7       |
| <b>IV.A5</b> | 0.003  | 497.5    | 548.9  | 220.4       | 632.2       | 535.6       |
| <b>IV.A6</b> | 0.0029 | 541.6    | 594.5  | 260.1       | 705.2       | 511.2       |
| <b>IV.B1</b> | 0.0029 | 459.3    | 499.1  | 230.5       | 576.6       | 479.8       |
| <b>IV.B2</b> | 0.0028 | 471.9    | 512.9  | 239.2       | 597.8       | 479.5       |
| <b>IV.B3</b> | 0.0028 | 507.9    | 551.6  | 273.4       | 647.0       | 471.9       |
| <b>V.1</b>   | 0.0029 | 450.5    | 491.0  | 221.3       | 560.3       | 486.0       |
| <b>V.2</b>   | 0.0029 | 462.2    | 502.5  | 233.6       | 583.6       | 475.1       |
| <b>V.3</b>   | 0.0030 | 477.9    | 522.6  | 236.1       | 622.7       | 460.0       |

Table D.30. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 5 (d=3, K=0.15).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0910 | 16.1   | 24.0   | 6.1         | 12.5        | 29.7        |
| <b>IV.A2</b> | 0.0764 | 22.2   | 36.9   | 6.8         | 16.1        | 43.0        |
| <b>IV.A3</b> | 0.0663 | 31.3   | 58.4   | 7.2         | 20.5        | 65.6        |
| <b>IV.A4</b> | 0.0896 | 16.3   | 24.2   | 6.3         | 12.7        | 29.9        |
| <b>IV.A5</b> | 0.0739 | 23.1   | 37.9   | 7.0         | 16.9        | 45.4        |
| <b>IV.A6</b> | 0.0642 | 32.7   | 61.4   | 7.4         | 21.4        | 68.4        |
| <b>IV.B1</b> | 0.0730 | 22.2   | 36.1   | 7.2         | 16.5        | 42.1        |
| <b>IV.B2</b> | 0.0703 | 24.5   | 40.9   | 7.3         | 17.6        | 48.2        |
| <b>IV.B3</b> | 0.0631 | 31.3   | 57.3   | 7.8         | 21.0        | 64.5        |
| <b>V.1</b>   | 0.0754 | 21.3   | 33.7   | 7.1         | 15.9        | 40.8        |
| <b>V.2</b>   | 0.0730 | 23.3   | 38.9   | 7.1         | 16.8        | 45.4        |
| <b>V.3</b>   | 0.0684 | 27.8   | 50.7   | 7.3         | 19.0        | 56.6        |

Table D.31. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under CM 5 (d=3, K=0.20).

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.0038 | 401.2    | 465.3  | 138.0       | 465.2       | 560.7       |
| <b>IV.A2</b> | 0.0034 | 471.1    | 533.1  | 171.1       | 597.9       | 556.2       |
| <b>IV.A3</b> | 0.0032 | 510.5    | 568.6  | 208.3       | 674.6       | 521.5       |
| <b>IV.A4</b> | 0.0037 | 410.2    | 472.7  | 144.6       | 474.9       | 560.7       |
| <b>IV.A5</b> | 0.0031 | 482.0    | 537.3  | 197.0       | 609.6       | 542.9       |
| <b>IV.A6</b> | 0.0031 | 528.3    | 583.8  | 234.3       | 692.4       | 518.2       |
| <b>IV.B1</b> | 0.0030 | 448.3    | 488.9  | 218.9       | 566.9       | 472.6       |
| <b>IV.B2</b> | 0.0030 | 463.0    | 504.1  | 231.3       | 588.4       | 463.2       |
| <b>IV.B3</b> | 0.0029 | 501.2    | 547.5  | 259.4       | 654.2       | 460.5       |
| <b>V.1</b>   | 0.0030 | 454.1    | 497.6  | 212.1       | 572.3       | 486.1       |
| <b>V.2</b>   | 0.0030 | 458.0    | 502.1  | 215.7       | 591.4       | 468.6       |
| <b>V.3</b>   | 0.0031 | 477.4    | 525.5  | 221.4       | 627.0       | 458.1       |

Table D.32. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under CM 5 (d=3, K=0.20).

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>IV.A1</b> | 0.1001 | 14.7   | 22.3   | 5.6         | 11.3        | 27.0        |
| <b>IV.A2</b> | 0.0844 | 19.7   | 32.2   | 6.2         | 14.4        | 38.2        |
| <b>IV.A3</b> | 0.0719 | 27.8   | 51.2   | 6.8         | 18.6        | 57.6        |
| <b>IV.A4</b> | 0.0975 | 15.0   | 22.3   | 5.8         | 11.7        | 27.3        |
| <b>IV.A5</b> | 0.0798 | 20.9   | 34.2   | 6.5         | 15.3        | 40.3        |
| <b>IV.A6</b> | 0.0684 | 30.0   | 55.9   | 7.1         | 19.9        | 62.8        |
| <b>IV.B1</b> | 0.0739 | 22.5   | 37.2   | 7.1         | 16.4        | 43.7        |
| <b>IV.B2</b> | 0.0709 | 25.0   | 42.8   | 7.2         | 17.5        | 49.9        |
| <b>IV.B3</b> | 0.0637 | 32.1   | 60.8   | 7.6         | 21.1        | 66.3        |
| <b>V.1</b>   | 0.0759 | 21.9   | 36.4   | 7.0         | 15.9        | 42.4        |
| <b>V.2</b>   | 0.0730 | 23.9   | 41.1   | 7.0         | 17.0        | 47.5        |
| <b>V.3</b>   | 0.0696 | 28.4   | 52.7   | 7.0         | 18.8        | 58.7        |

Table D.33. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 1.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0057 | 212.2    | 248.9  | 337.6       | 193.8       | 111.7       |
| <b>I.B1</b>  | 0.0037 | 337.7    | 389.1  | 559.3       | 338.8       | 161.6       |
| <b>I.B2</b>  | 0.0034 | 367.5    | 415.2  | 542.9       | 389.8       | 187.0       |
| <b>I.B3</b>  | 0.0032 | 386.6    | 434.3  | 561.9       | 414.6       | 198.6       |
| <b>I.B4</b>  | 0.0030 | 402.3    | 452.1  | 599.5       | 424.4       | 208.9       |
| <b>II.A1</b> | 0.0031 | 394.1    | 456.3  | 685.4       | 372.3       | 197.1       |
| <b>II.A2</b> | 0.0029 | 436.1    | 509.1  | 783.2       | 416.8       | 210.0       |
| <b>II.A3</b> | 0.0029 | 465.5    | 570.3  | 928.9       | 432.4       | 194.8       |
| <b>II.B1</b> | 0.0029 | 441.9    | 512.4  | 753.7       | 439.3       | 215.3       |
| <b>II.B2</b> | 0.0029 | 448.9    | 523.4  | 784.3       | 446.6       | 218.9       |
| <b>II.B3</b> | 0.0029 | 474.5    | 576.4  | 918.3       | 454.3       | 203.4       |
| <b>III.1</b> | 0.0031 | 388.5    | 431.5  | 561.4       | 410.7       | 215.2       |
| <b>III.2</b> | 0.0027 | 451.8    | 500.7  | 568.0       | 499.8       | 263.7       |
| <b>III.3</b> | 0.0027 | 477.9    | 535.1  | 637.5       | 517.3       | 273.3       |
| <b>III.4</b> | 0.0030 | 390.1    | 432.0  | 546.1       | 415.8       | 219.6       |
| <b>III.5</b> | 0.0027 | 448.8    | 496.2  | 572.3       | 493.9       | 262.4       |
| <b>III.6</b> | 0.0027 | 482.4    | 541.3  | 636.1       | 529.4       | 272.7       |
| <b>IV.A1</b> | 0.003  | 400.6    | 441.8  | 531.1       | 434.8       | 231.0       |
| <b>IV.A2</b> | 0.0028 | 436.3    | 477.9  | 517.5       | 487.6       | 266.3       |
| <b>IV.A3</b> | 0.0026 | 466.4    | 512.9  | 560.7       | 521.6       | 285.6       |
| <b>IV.A4</b> | 0.0029 | 404.8    | 446.6  | 532.4       | 440.6       | 231.9       |
| <b>IV.A5</b> | 0.0027 | 457.3    | 502.9  | 540.6       | 514.5       | 278.6       |
| <b>IV.A6</b> | 0.0026 | 483.8    | 535.0  | 566.6       | 547.5       | 293.9       |
| <b>IV.B1</b> | 0.0029 | 403.7    | 443.3  | 510.8       | 446.5       | 237.2       |
| <b>IV.B2</b> | 0.0028 | 428.1    | 469.4  | 498.7       | 487.0       | 258.1       |
| <b>IV.B3</b> | 0.0027 | 457.9    | 501.1  | 525.5       | 521.9       | 284.9       |
| <b>V.1</b>   | 0.0028 | 416.3    | 451.1  | 464.5       | 474.6       | 263.8       |
| <b>V.2</b>   | 0.0027 | 439.6    | 476.7  | 464.7       | 510.1       | 284.5       |
| <b>V.3</b>   | 0.0026 | 461.7    | 500.7  | 455.3       | 541.5       | 310.0       |

Table D.34. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 1.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0049 | 353.9  | 426.8  | 96.0        | 412.4       | 511.0       |
| <b>I.B1</b>  | 0.0226 | 176.0  | 305.9  | 17.1        | 95.0        | 536.1       |
| <b>I.B2</b>  | 0.0269 | 146.4  | 274.1  | 14.8        | 69.7        | 465.9       |
| <b>I.B3</b>  | 0.0269 | 143.4  | 273.5  | 15.0        | 67.5        | 461.3       |
| <b>I.B4</b>  | 0.0254 | 149.6  | 285.3  | 15.9        | 71.4        | 475.6       |
| <b>II.A1</b> | 0.0138 | 258.4  | 411.3  | 28.6        | 172.9       | 680.5       |
| <b>II.A2</b> | 0.0153 | 280.8  | 463.1  | 25.4        | 172.6       | 778.5       |
| <b>II.A3</b> | 0.0133 | 396.4  | 613.6  | 28.4        | 302.3       | 906.1       |
| <b>II.B1</b> | 0.0198 | 239.5  | 417.1  | 19.3        | 130.1       | 724.0       |
| <b>II.B2</b> | 0.0203 | 245.9  | 429.4  | 18.8        | 133.2       | 742.4       |
| <b>II.B3</b> | 0.0166 | 372.0  | 595.7  | 23.1        | 254.5       | 915.8       |
| <b>III.1</b> | 0.0274 | 131.1  | 249.2  | 14.9        | 65.9        | 388.1       |
| <b>III.2</b> | 0.0311 | 124.6  | 253.9  | 13.2        | 58.0        | 359.1       |
| <b>III.3</b> | 0.0284 | 152.6  | 302.3  | 14.1        | 73.2        | 449.7       |
| <b>III.4</b> | 0.0280 | 127.6  | 245.2  | 14.7        | 63.4        | 368.2       |
| <b>III.5</b> | 0.0314 | 123.9  | 250.8  | 13.0        | 58.9        | 354.8       |
| <b>III.6</b> | 0.0295 | 150.5  | 300.6  | 13.5        | 70.8        | 448.1       |
| <b>IV.A1</b> | 0.0304 | 115.2  | 227.5  | 13.5        | 56.5        | 327.3       |
| <b>IV.A2</b> | 0.0339 | 102.8  | 214.3  | 12.5        | 49.1        | 274.3       |
| <b>IV.A3</b> | 0.0326 | 115.9  | 238.4  | 12.8        | 55.6        | 319.2       |
| <b>IV.A4</b> | 0.0307 | 115.2  | 227.1  | 13.4        | 56.1        | 328.9       |
| <b>IV.A5</b> | 0.0340 | 104.7  | 218.0  | 12.3        | 50.0        | 282.8       |
| <b>IV.A6</b> | 0.0327 | 119.9  | 247.8  | 12.5        | 56.1        | 334.2       |
| <b>IV.B1</b> | 0.0328 | 105.0  | 211.9  | 12.8        | 50.3        | 292.8       |
| <b>IV.B2</b> | 0.0354 | 97.2   | 202.3  | 11.9        | 46.0        | 261.8       |
| <b>IV.B3</b> | 0.0347 | 101.8  | 214.6  | 12.2        | 48.0        | 273.5       |
| <b>V.1</b>   | 0.0364 | 83.8   | 173.3  | 11.8        | 42.2        | 214.0       |
| <b>V.2</b>   | 0.0388 | 79.1   | 168.6  | 11.2        | 39.1        | 196.9       |
| <b>V.3</b>   | 0.0400 | 77.5   | 167.5  | 11.0        | 38.5        | 192.4       |

Table D.35. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 2.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0036 | 312.1    | 345.9  | 473.4       | 296.5       | 185.8       |
| <b>I.B1</b>  | 0.0036 | 318.0    | 355.7  | 510.5       | 299.8       | 185.1       |
| <b>I.B2</b>  | 0.0036 | 325.9    | 367.4  | 540.4       | 309.6       | 183.1       |
| <b>I.B3</b>  | 0.0036 | 333.9    | 380.2  | 572.3       | 316.4       | 180.2       |
| <b>I.B4</b>  | 0.0034 | 356.0    | 407.9  | 617.2       | 344.9       | 185.8       |
| <b>II.A1</b> | 0.0028 | 424.5    | 482.0  | 708.2       | 413.9       | 222.4       |
| <b>II.A2</b> | 0.0028 | 451.8    | 523.7  | 784.4       | 446.0       | 221.9       |
| <b>II.A3</b> | 0.0028 | 481.4    | 582.1  | 923.9       | 461.9       | 208.1       |
| <b>II.B1</b> | 0.0029 | 438.6    | 508.4  | 766.4       | 433.5       | 219.0       |
| <b>II.B2</b> | 0.0029 | 445.9    | 520.7  | 789.3       | 435.5       | 218.0       |
| <b>II.B3</b> | 0.0029 | 472.1    | 573.0  | 917.9       | 448.7       | 207.0       |
| <b>III.1</b> | 0.0032 | 366.4    | 410.1  | 596.0       | 362.3       | 205.1       |
| <b>III.2</b> | 0.0030 | 417.2    | 470.9  | 633.4       | 433.4       | 226.0       |
| <b>III.3</b> | 0.0029 | 444.3    | 508.8  | 695.6       | 460.4       | 232.2       |
| <b>III.4</b> | 0.0032 | 366.8    | 410.4  | 591.0       | 363.4       | 203.1       |
| <b>III.5</b> | 0.0030 | 414.6    | 468.4  | 632.0       | 433.0       | 221.2       |
| <b>III.6</b> | 0.0030 | 446.4    | 509.7  | 686.2       | 468.8       | 233.9       |
| <b>IV.A1</b> | 0.0032 | 370.0    | 413.8  | 567.0       | 378.9       | 205.9       |
| <b>IV.A2</b> | 0.0030 | 417.2    | 468.2  | 538.0       | 458.0       | 232.0       |
| <b>IV.A3</b> | 0.0029 | 447.0    | 505.8  | 559.2       | 499.5       | 242.1       |
| <b>IV.A4</b> | 0.0031 | 383.7    | 429.1  | 580.2       | 396.8       | 212.0       |
| <b>IV.A5</b> | 0.0030 | 426.6    | 478.4  | 539.5       | 472.8       | 240.1       |
| <b>IV.A6</b> | 0.0029 | 461.8    | 519.7  | 551.9       | 520.1       | 259.7       |
| <b>IV.B1</b> | 0.0034 | 357.3    | 406.8  | 589.2       | 355.9       | 187.7       |
| <b>IV.B2</b> | 0.0034 | 362.3    | 411.9  | 571.0       | 371.7       | 191.4       |
| <b>IV.B3</b> | 0.0032 | 398.7    | 455.7  | 575.0       | 424.9       | 205.7       |
| <b>V.1</b>   | 0.0032 | 370.3    | 415.9  | 556.0       | 382.6       | 201.5       |
| <b>V.2</b>   | 0.0032 | 381.3    | 427.9  | 535.2       | 406.8       | 209.4       |
| <b>V.3</b>   | 0.0031 | 414.4    | 465.5  | 509.3       | 463.2       | 231.9       |

Table D.36. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 2.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0116 | 189.9  | 284.9  | 38.2        | 139.2       | 437.8       |
| <b>I.B1</b>  | 0.0133 | 192.5  | 294.4  | 32.8        | 139.0       | 458.5       |
| <b>I.B2</b>  | 0.0156 | 191.2  | 300.8  | 26.9        | 133.1       | 478.4       |
| <b>I.B3</b>  | 0.0167 | 200.9  | 322.2  | 24.2        | 133.6       | 521.1       |
| <b>I.B4</b>  | 0.0186 | 206.4  | 340.7  | 21.1        | 129.6       | 558.8       |
| <b>II.A1</b> | 0.0178 | 212.1  | 365.2  | 22.5        | 123.2       | 612.8       |
| <b>II.A2</b> | 0.0178 | 256.2  | 438.8  | 21.6        | 145.8       | 752.1       |
| <b>II.A3</b> | 0.0161 | 357.1  | 582.2  | 23.4        | 228.3       | 916.9       |
| <b>II.B1</b> | 0.0186 | 250.0  | 427.2  | 20.9        | 143.6       | 730.0       |
| <b>II.B2</b> | 0.0187 | 259.6  | 440.3  | 21.0        | 151.6       | 747.8       |
| <b>II.B3</b> | 0.0161 | 368.9  | 588.6  | 23.6        | 251.5       | 910.4       |
| <b>III.1</b> | 0.0202 | 163.5  | 280.6  | 20.2        | 98.7        | 439.6       |
| <b>III.2</b> | 0.0251 | 166.4  | 306.1  | 15.7        | 86.7        | 492.3       |
| <b>III.3</b> | 0.0245 | 199.8  | 369.6  | 15.9        | 99.7        | 623.6       |
| <b>III.4</b> | 0.0209 | 157.9  | 270.6  | 19.7        | 95.0        | 426.3       |
| <b>III.5</b> | 0.0248 | 169.5  | 310.9  | 16.0        | 88.5        | 498.9       |
| <b>III.6</b> | 0.0250 | 197.5  | 364.1  | 15.7        | 99.1        | 612.9       |
| <b>IV.A1</b> | 0.0248 | 149.0  | 268.8  | 16.2        | 81.6        | 418.5       |
| <b>IV.A2</b> | 0.0313 | 136.9  | 272.1  | 12.7        | 62.5        | 416.0       |
| <b>IV.A3</b> | 0.0318 | 143.7  | 289.1  | 12.5        | 63.5        | 445.8       |
| <b>IV.A4</b> | 0.0254 | 145.8  | 265.2  | 15.7        | 78.3        | 414.7       |
| <b>IV.A5</b> | 0.0318 | 133.7  | 268.2  | 12.6        | 61.0        | 402.5       |
| <b>IV.A6</b> | 0.0321 | 143.8  | 292.6  | 12.4        | 63.0        | 437.4       |
| <b>IV.B1</b> | 0.0220 | 175.3  | 304.2  | 17.7        | 99.8        | 502.3       |
| <b>IV.B2</b> | 0.0249 | 171.7  | 305.5  | 15.6        | 93.1        | 507.2       |
| <b>IV.B3</b> | 0.0276 | 171.3  | 319.1  | 14.0        | 82.7        | 540.7       |
| <b>V.1</b>   | 0.0262 | 145.8  | 268.8  | 15.1        | 76.0        | 429.3       |
| <b>V.2</b>   | 0.0291 | 141.6  | 269.7  | 13.4        | 68.7        | 424.2       |
| <b>V.3</b>   | 0.0332 | 132.2  | 265.9  | 11.9        | 58.8        | 408.5       |

Table D.37. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 3.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| textbfI.A    | 0.0074 | 161.5    | 191.1  | 256.4       | 146.1       | 86.2        |
| <b>I.B1</b>  | 0.0066 | 187.5    | 228.4  | 311.8       | 166.0       | 93.4        |
| <b>I.B2</b>  | 0.0063 | 199.0    | 246.0  | 340.7       | 173.3       | 96.4        |
| <b>I.B3</b>  | 0.0061 | 211.7    | 266.2  | 375.3       | 182.0       | 97.8        |
| <b>I.B4</b>  | 0.006  | 219.7    | 280.8  | 406.0       | 185.3       | 97.2        |
| <b>II.A1</b> | 0.0045 | 288.0    | 354.8  | 510.3       | 254.1       | 130.8       |
| <b>II.A2</b> | 0.0042 | 321.6    | 407.2  | 611.6       | 277.4       | 134.6       |
| <b>II.A3</b> | 0.0041 | 352.6    | 463.9  | 718.3       | 295.2       | 131.1       |
| <b>II.B1</b> | 0.0044 | 304.3    | 384.3  | 581.8       | 263.5       | 129.4       |
| <b>II.B2</b> | 0.0043 | 314.5    | 399.5  | 618.0       | 271.1       | 130.6       |
| <b>II.B3</b> | 0.0043 | 344.1    | 455.4  | 714.6       | 287.3       | 128.1       |
| <b>III.1</b> | 0.0052 | 246.0    | 300.6  | 443.0       | 216.2       | 116.7       |
| <b>III.2</b> | 0.0043 | 306.5    | 376.2  | 597.6       | 274.5       | 134.7       |
| <b>III.3</b> | 0.0040 | 339.6    | 421.1  | 683.8       | 303.3       | 143.7       |
| <b>III.4</b> | 0.0051 | 248.7    | 304.5  | 448.7       | 218.0       | 117.6       |
| <b>III.5</b> | 0.0043 | 306.6    | 377.0  | 599.1       | 272.1       | 135.2       |
| <b>III.6</b> | 0.004  | 340.7    | 423.3  | 691.6       | 304.7       | 142.9       |
| <b>IV.A1</b> | 0.0049 | 257.4    | 312.4  | 469.5       | 227.7       | 122.1       |
| <b>IV.A2</b> | 0.0042 | 312.9    | 378.5  | 602.2       | 285.7       | 140.0       |
| <b>IV.A3</b> | 0.0039 | 346.0    | 419.5  | 670.2       | 322.6       | 149.3       |
| <b>IV.A4</b> | 0.0050 | 256.5    | 312.7  | 474.6       | 226.4       | 120.0       |
| <b>IV.A5</b> | 0.0041 | 319.9    | 388.4  | 621.7       | 291.7       | 141.7       |
| <b>IV.A6</b> | 0.0038 | 358.9    | 436.9  | 706.5       | 333.3       | 155.3       |
| <b>IV.B1</b> | 0.0048 | 267.1    | 325.5  | 498.8       | 236.1       | 124.0       |
| <b>IV.B2</b> | 0.0047 | 276.5    | 336.9  | 531.1       | 246.1       | 126.6       |
| <b>IV.B3</b> | 0.0043 | 314.4    | 384.9  | 626.5       | 284.8       | 136.4       |
| <b>V.1</b>   | 0.0043 | 294.0    | 348.9  | 538.1       | 269.3       | 140.2       |
| <b>V.2</b>   | 0.0041 | 313.0    | 371.1  | 578.8       | 291.0       | 147.5       |
| <b>V.3</b>   | 0.0037 | 348.1    | 410.2  | 629.3       | 337.2       | 162.0       |

Table D.38. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 3.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0031 | 416.1  | 453.7  | 212.8       | 541.3       | 398.6       |
| <b>I.B1</b>  | 0.0043 | 394.6  | 455.1  | 125.9       | 523.3       | 444.1       |
| <b>I.B2</b>  | 0.0051 | 389.6  | 461.9  | 100.4       | 521.9       | 460.9       |
| <b>I.B3</b>  | 0.0058 | 385.2  | 462.5  | 87.8        | 526.0       | 453.1       |
| <b>I.B4</b>  | 0.0061 | 401.3  | 487.1  | 82.0        | 562.7       | 461.8       |
| <b>II.A1</b> | 0.0063 | 412.9  | 533.4  | 68.1        | 484.8       | 620.0       |
| <b>II.A2</b> | 0.0070 | 438.8  | 575.2  | 61.1        | 539.2       | 627.4       |
| <b>II.A3</b> | 0.0068 | 529.7  | 690.8  | 65.1        | 759.0       | 624.0       |
| <b>II.B1</b> | 0.0078 | 422.8  | 552.8  | 55.5        | 526.9       | 610.1       |
| <b>II.B2</b> | 0.0080 | 442.9  | 580.8  | 56.0        | 559.8       | 614.8       |
| <b>II.B3</b> | 0.0072 | 537.4  | 695.0  | 65.8        | 789.0       | 616.2       |
| <b>III.1</b> | 0.0080 | 336.4  | 434.2  | 54.5        | 382.8       | 525.1       |
| <b>III.2</b> | 0.0115 | 332.3  | 469.2  | 35.2        | 317.9       | 625.1       |
| <b>III.3</b> | 0.0121 | 356.8  | 509.7  | 32.7        | 346.7       | 660.3       |
| <b>III.4</b> | 0.0082 | 334.0  | 433.2  | 52.7        | 380.1       | 532.7       |
| <b>III.5</b> | 0.0116 | 327.7  | 460.5  | 35.9        | 316.9       | 607.7       |
| <b>III.6</b> | 0.0128 | 350.6  | 505.0  | 31.3        | 328.2       | 658.5       |
| <b>IV.A1</b> | 0.0093 | 322.9  | 432.8  | 44.9        | 340.2       | 560.8       |
| <b>IV.A2</b> | 0.0137 | 285.7  | 419.2  | 28.5        | 241.9       | 603.4       |
| <b>IV.A3</b> | 0.0155 | 288.7  | 439.3  | 24.9        | 221.0       | 646.4       |
| <b>IV.A4</b> | 0.0096 | 314.5  | 420.4  | 44.3        | 331.7       | 542.7       |
| <b>IV.A5</b> | 0.0142 | 292.5  | 433.6  | 27.2        | 240.5       | 629.3       |
| <b>IV.A6</b> | 0.0157 | 300.7  | 462.0  | 24.4        | 226.2       | 685.9       |
| <b>IV.B1</b> | 0.0102 | 312.4  | 423.8  | 40.3        | 320.7       | 550.4       |
| <b>IV.B2</b> | 0.0121 | 306.6  | 430.3  | 33.3        | 294.9       | 579.0       |
| <b>IV.B3</b> | 0.0144 | 301.9  | 444.2  | 26.9        | 261.0       | 614.6       |
| <b>V.1</b>   | 0.0132 | 256.5  | 377.6  | 30.3        | 207.5       | 579.2       |
| <b>V.2</b>   | 0.0151 | 251.7  | 384.1  | 25.7        | 185.2       | 602.0       |
| <b>V.3</b>   | 0.0187 | 231.2  | 375.5  | 20.4        | 146.6       | 623.3       |

Table D.39. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 4.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0028 | 384.9    | 413.7  | 557.1       | 380.4       | 243.6       |
| <b>I.B1</b>  | 0.0027 | 450.0    | 482.4  | 266.5       | 565.1       | 427.7       |
| <b>I.B2</b>  | 0.0027 | 457.2    | 490.4  | 265.9       | 573.9       | 443.4       |
| <b>I.B3</b>  | 0.0026 | 474.6    | 506.0  | 295.8       | 591.2       | 445.8       |
| <b>I.B4</b>  | 0.0024 | 497.2    | 528.8  | 335.2       | 617.4       | 442.1       |
| <b>II.A1</b> | 0.0023 | 510.7    | 555.2  | 590.6       | 576.9       | 324.5       |
| <b>II.A2</b> | 0.0024 | 539.7    | 597.7  | 564.7       | 629.1       | 331.9       |
| <b>II.A3</b> | 0.0024 | 574.8    | 662.3  | 717.9       | 642.3       | 299.7       |
| <b>II.B1</b> | 0.0025 | 539.9    | 597.1  | 424.3       | 671.1       | 363.8       |
| <b>II.B2</b> | 0.0026 | 549.4    | 611.4  | 416.0       | 690.5       | 365.1       |
| <b>II.B3</b> | 0.0026 | 587.1    | 675.9  | 592.5       | 685.4       | 323.2       |
| <b>III.1</b> | 0.0026 | 460.2    | 491.0  | 310.6       | 575.7       | 395.9       |
| <b>III.2</b> | 0.0026 | 511.9    | 554.9  | 323.6       | 656.6       | 423.3       |
| <b>III.3</b> | 0.0026 | 544.9    | 594.8  | 395.2       | 683.6       | 408.4       |
| <b>III.4</b> | 0.0027 | 463.0    | 495.3  | 308.6       | 583.6       | 398.8       |
| <b>III.5</b> | 0.0026 | 514.9    | 558.1  | 334.7       | 652.7       | 415.9       |
| <b>III.6</b> | 0.0027 | 540.1    | 590.7  | 375.5       | 681.7       | 407.2       |
| <b>IV.A1</b> | 0.0029 | 449.9    | 489.5  | 236.8       | 583.2       | 426.3       |
| <b>IV.A2</b> | 0.0028 | 492.0    | 535.9  | 267.3       | 641.6       | 443.0       |
| <b>IV.A3</b> | 0.0028 | 513.6    | 561.8  | 302.1       | 671.9       | 421.5       |
| <b>IV.A4</b> | 0.0029 | 450.7    | 490.6  | 237.0       | 584.2       | 420.7       |
| <b>IV.A5</b> | 0.0028 | 493.5    | 536.9  | 273.0       | 641.1       | 442.0       |
| <b>IV.A6</b> | 0.0028 | 534.8    | 584.2  | 316.6       | 690.6       | 442.4       |
| <b>IV.B1</b> | 0.0027 | 462.0    | 499.1  | 277.6       | 594.7       | 406.0       |
| <b>IV.B2</b> | 0.0028 | 467.4    | 505.9  | 276.8       | 600.2       | 409.1       |
| <b>IV.B3</b> | 0.0027 | 495.4    | 535.6  | 319.8       | 631.7       | 412.6       |
| <b>V.1</b>   | 0.0028 | 449.2    | 486.0  | 255.5       | 580.0       | 405.9       |
| <b>V.2</b>   | 0.0028 | 462.8    | 501.9  | 258.5       | 600.2       | 420.7       |
| <b>V.3</b>   | 0.0028 | 481.3    | 523.2  | 273.0       | 625.4       | 422.7       |

Table D.40. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 4.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0203 | 102.5  | 170.3  | 23.1        | 68.6        | 227.8       |
| <b>I.B1</b>  | 0.0666 | 26.3   | 47.6   | 7.8         | 17.8        | 51.7        |
| <b>I.B2</b>  | 0.0673 | 25.2   | 43.4   | 7.8         | 17.8        | 49.3        |
| <b>I.B3</b>  | 0.0624 | 28.2   | 49.8   | 8.2         | 19.7        | 55.7        |
| <b>I.B4</b>  | 0.0577 | 32.4   | 59.0   | 8.7         | 21.9        | 65.6        |
| <b>II.A1</b> | 0.0328 | 98.4   | 204.9  | 13.0        | 49.8        | 246.3       |
| <b>II.A2</b> | 0.0339 | 123.5  | 265.5  | 11.8        | 54.9        | 342.2       |
| <b>II.A3</b> | 0.0276 | 226.0  | 453.3  | 13.5        | 92.3        | 771.2       |
| <b>II.B1</b> | 0.0433 | 87.7   | 201.4  | 9.6         | 38.5        | 227.0       |
| <b>II.B2</b> | 0.0423 | 95.9   | 218.9  | 9.7         | 42.0        | 253.6       |
| <b>II.B3</b> | 0.0335 | 192.2  | 405.5  | 11.4        | 72.9        | 647.1       |
| <b>III.1</b> | 0.0581 | 35.0   | 68.0   | 8.4         | 22.1        | 73.4        |
| <b>III.2</b> | 0.0551 | 42.9   | 88.7   | 8.5         | 25.4        | 93.4        |
| <b>III.3</b> | 0.0472 | 63.0   | 136.9  | 9.2         | 33.3        | 147.5       |
| <b>III.4</b> | 0.058  | 34.4   | 65.9   | 8.5         | 21.9        | 71.8        |
| <b>III.5</b> | 0.0537 | 44.4   | 91.4   | 8.6         | 26.2        | 96.1        |
| <b>III.6</b> | 0.0477 | 63.7   | 138.9  | 9.1         | 33.6        | 148.6       |
| <b>IV.A1</b> | 0.069  | 29.0   | 58.9   | 7.2         | 17.9        | 60.0        |
| <b>IV.A2</b> | 0.0626 | 34.2   | 68.1   | 7.7         | 21.1        | 71.9        |
| <b>IV.A3</b> | 0.0549 | 45.7   | 94.9   | 8.3         | 26.3        | 100.8       |
| <b>IV.A4</b> | 0.0676 | 30.4   | 62.4   | 7.3         | 18.5        | 62.6        |
| <b>IV.A5</b> | 0.0616 | 35.6   | 73.5   | 7.8         | 21.6        | 74.1        |
| <b>IV.A6</b> | 0.0545 | 47.1   | 100.3  | 8.4         | 26.7        | 104         |
| <b>IV.B1</b> | 0.0611 | 35.0   | 71.8   | 7.9         | 21.0        | 73.6        |
| <b>IV.B2</b> | 0.0615 | 34.7   | 71.2   | 7.9         | 21.0        | 72.7        |
| <b>IV.B3</b> | 0.0563 | 41.9   | 87.6   | 8.4         | 24.5        | 89.8        |
| <b>V.1</b>   | 0.0641 | 32.1   | 64.9   | 7.6         | 19.8        | 66.3        |
| <b>V.2</b>   | 0.0641 | 32.2   | 65.0   | 7.6         | 19.9        | 67.2        |
| <b>V.3</b>   | 0.0618 | 35.9   | 73.5   | 7.7         | 21.8        | 76.3        |

Table D.41. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 5.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0033 | 353.3    | 393.3  | 554.0       | 347.3       | 194.6       |
| <b>I.B1</b>  | 0.0028 | 403.7    | 438.4  | 500.9       | 447.0       | 243.4       |
| <b>I.B2</b>  | 0.0028 | 417.8    | 450.5  | 463.7       | 477.8       | 268.0       |
| <b>I.B3</b>  | 0.0026 | 440.1    | 476.0  | 490.5       | 505.1       | 280.7       |
| <b>I.B4</b>  | 0.0026 | 453.2    | 490.1  | 494.2       | 520.8       | 292.2       |
| <b>II.A1</b> | 0.0024 | 487.8    | 536.3  | 631.5       | 532.8       | 289.0       |
| <b>II.A2</b> | 0.0024 | 521.0    | 587.2  | 688.6       | 567.6       | 286.0       |
| <b>II.A3</b> | 0.0025 | 557.7    | 651.0  | 797.3       | 596.7       | 272.8       |
| <b>II.B1</b> | 0.0025 | 492.4    | 548.1  | 618.1       | 543.4       | 288.1       |
| <b>II.B2</b> | 0.0026 | 505.7    | 568.0  | 672.9       | 547.8       | 288.3       |
| <b>II.B3</b> | 0.0025 | 552.3    | 643.1  | 814.7       | 583.0       | 280.4       |
| <b>III.1</b> | 0.0025 | 455.4    | 485.7  | 459.6       | 531.9       | 318.8       |
| <b>III.2</b> | 0.0025 | 485.3    | 522.8  | 454.6       | 578.4       | 343.9       |
| <b>III.3</b> | 0.0025 | 519.2    | 568.4  | 518.5       | 603.9       | 343.8       |
| <b>III.4</b> | 0.0026 | 447.0    | 476.5  | 446.5       | 521.5       | 316.0       |
| <b>III.5</b> | 0.0025 | 497.5    | 536.9  | 469.0       | 589.5       | 352.6       |
| <b>III.6</b> | 0.0025 | 520.9    | 570.2  | 520.9       | 609.2       | 346.1       |
| <b>IV.A1</b> | 0.0026 | 458.9    | 491.5  | 343.1       | 575.7       | 358.4       |
| <b>IV.A2</b> | 0.0026 | 489.4    | 525.8  | 357.3       | 613.0       | 388.5       |
| <b>IV.A3</b> | 0.0026 | 513.6    | 558.1  | 409.7       | 632.1       | 374.4       |
| <b>IV.A4</b> | 0.0026 | 455.9    | 488.2  | 339.2       | 570.1       | 355.9       |
| <b>IV.A5</b> | 0.0025 | 499.9    | 537.3  | 375.0       | 622.2       | 391.4       |
| <b>IV.A6</b> | 0.0025 | 524.2    | 568.9  | 423.8       | 640.8       | 387.4       |
| <b>IV.B1</b> | 0.0026 | 452.3    | 483.9  | 411.1       | 540.9       | 325.1       |
| <b>IV.B2</b> | 0.0026 | 453.6    | 486.5  | 404.2       | 547.8       | 327.3       |
| <b>IV.B3</b> | 0.0026 | 484.4    | 524.5  | 428.3       | 586.2       | 342.6       |
| <b>V.1</b>   | 0.0026 | 449.3    | 478.8  | 386.8       | 542.7       | 335.6       |
| <b>V.2</b>   | 0.0026 | 457.4    | 489.8  | 379.7       | 556.8       | 343.5       |
| <b>V.3</b>   | 0.0026 | 481.3    | 520.3  | 374.1       | 599.0       | 356.1       |

Table D.42. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 5.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0189 | 149.9  | 259.7  | 22.0        | 87.6        | 408.7       |
| <b>I.B1</b>  | 0.0333 | 91.2   | 186.1  | 13.1        | 44.7        | 240.9       |
| <b>I.B2</b>  | 0.0375 | 79.5   | 168.1  | 11.8        | 39.3        | 198.5       |
| <b>I.B3</b>  | 0.0371 | 80.6   | 171.8  | 11.8        | 40.0        | 201.1       |
| <b>I.B4</b>  | 0.0367 | 82.0   | 176.3  | 12.0        | 40.7        | 202.2       |
| <b>II.A1</b> | 0.0296 | 121.8  | 248.8  | 13.9        | 59.4        | 332.4       |
| <b>II.A2</b> | 0.0282 | 163.9  | 329.3  | 14.1        | 73.1        | 498.6       |
| <b>II.A3</b> | 0.0247 | 260.1  | 492.2  | 15.0        | 115.9       | 873.4       |
| <b>II.B1</b> | 0.0293 | 145.6  | 295.0  | 13.6        | 67.4        | 422.2       |
| <b>II.B2</b> | 0.0285 | 157.7  | 317.2  | 14.0        | 73.2        | 459.8       |
| <b>II.B3</b> | 0.0239 | 256.1  | 480.2  | 15.9        | 120.6       | 829.5       |
| <b>III.1</b> | 0.0409 | 63.0   | 131.7  | 11.2        | 34.5        | 145.0       |
| <b>III.2</b> | 0.0415 | 68.7   | 145.6  | 10.7        | 36.7        | 161.0       |
| <b>III.3</b> | 0.0367 | 97.9   | 208.9  | 11.5        | 47.6        | 249.4       |
| <b>III.4</b> | 0.0421 | 60.3   | 125.7  | 11.0        | 33.2        | 135.4       |
| <b>III.5</b> | 0.0410 | 70.7   | 151.0  | 10.8        | 37.6        | 164.8       |
| <b>III.6</b> | 0.0372 | 95.5   | 202.9  | 11.3        | 47.3        | 241.5       |
| <b>IV.A1</b> | 0.053  | 44.5   | 96.6   | 9.0         | 24.6        | 96.7        |
| <b>IV.A2</b> | 0.0512 | 47.1   | 100.0  | 9.1         | 27.0        | 102.1       |
| <b>IV.A3</b> | 0.0459 | 60.9   | 129.4  | 9.8         | 33.4        | 139.2       |
| <b>IV.A4</b> | 0.0530 | 44.4   | 96.6   | 9.0         | 24.9        | 95.2        |
| <b>IV.A5</b> | 0.0502 | 48.9   | 105.1  | 9.3         | 27.9        | 106.0       |
| <b>IV.A6</b> | 0.0454 | 64.1   | 137.4  | 9.8         | 34.3        | 149.2       |
| <b>IV.B1</b> | 0.0447 | 58.1   | 124.0  | 10.2        | 31.4        | 133.3       |
| <b>IV.B2</b> | 0.0457 | 59.3   | 128.6  | 9.9         | 31.4        | 134.5       |
| <b>IV.B3</b> | 0.0436 | 67.9   | 147.9  | 10.1        | 35.3        | 159.5       |
| <b>V.1</b>   | 0.0476 | 50.7   | 106.6  | 9.8         | 28.6        | 111.6       |
| <b>V.2</b>   | 0.0482 | 52.1   | 110.8  | 9.6         | 28.7        | 116.2       |
| <b>V.3</b>   | 0.0486 | 55.2   | 119.3  | 9.3         | 30.0        | 123.7       |

Table D.43. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1$  using Phase I data under SCM 6.

| Method       | $ARFS$ | $mARL_0$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|----------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0063 | 181.9    | 207.3  | 275.9       | 167.7       | 106.4       |
| <b>I.B1</b>  | 0.0048 | 249.9    | 293.4  | 418.4       | 225.8       | 131.2       |
| <b>I.B2</b>  | 0.0046 | 264.3    | 312.9  | 464.3       | 236.7       | 136.1       |
| <b>I.B3</b>  | 0.0046 | 270.5    | 324.3  | 490.7       | 240.0       | 133.6       |
| <b>I.B4</b>  | 0.0045 | 279.5    | 337.4  | 515.5       | 248.6       | 133.2       |
| <b>II.A1</b> | 0.0041 | 307.7    | 373.1  | 545.4       | 274.5       | 144.2       |
| <b>II.A2</b> | 0.0037 | 357.1    | 438.1  | 674.2       | 318.7       | 157.9       |
| <b>II.A3</b> | 0.0037 | 386.7    | 498.0  | 790.1       | 332.6       | 148.1       |
| <b>II.B1</b> | 0.0037 | 354.2    | 432.7  | 687.6       | 318.6       | 156.2       |
| <b>II.B2</b> | 0.0037 | 367.0    | 450.2  | 722.9       | 334.1       | 158.4       |
| <b>II.B3</b> | 0.0038 | 390.4    | 504.8  | 835.0       | 338.7       | 147.2       |
| <b>III.1</b> | 0.0043 | 286.8    | 340.8  | 522.0       | 260.2       | 138.9       |
| <b>III.2</b> | 0.0037 | 351.0    | 417.5  | 658.5       | 331.5       | 162.8       |
| <b>III.3</b> | 0.0035 | 385.2    | 465.2  | 746.8       | 364.1       | 170.6       |
| <b>III.4</b> | 0.0043 | 287.5    | 341.2  | 527.3       | 261.3       | 140.0       |
| <b>III.5</b> | 0.0037 | 348.7    | 415.9  | 661.3       | 326.9       | 161.6       |
| <b>III.6</b> | 0.0035 | 386.7    | 464.4  | 737.5       | 366.6       | 174.1       |
| <b>IV.A1</b> | 0.0043 | 287.6    | 341.4  | 533.2       | 261.6       | 140.0       |
| <b>IV.A2</b> | 0.0037 | 351.3    | 415.8  | 641.0       | 337.9       | 161.3       |
| <b>IV.A3</b> | 0.0035 | 376.1    | 445.6  | 679.6       | 367.1       | 171.0       |
| <b>IV.A4</b> | 0.0043 | 293.9    | 349.6  | 546.0       | 267.5       | 141.8       |
| <b>IV.A5</b> | 0.0036 | 360.5    | 426.6  | 652.1       | 350.7       | 165.4       |
| <b>IV.A6</b> | 0.0034 | 399.7    | 472.4  | 697.4       | 400.4       | 182.4       |
| <b>IV.B1</b> | 0.0041 | 305.2    | 362.1  | 569.5       | 280.6       | 146.8       |
| <b>IV.B2</b> | 0.0039 | 323.4    | 382.0  | 599.2       | 306.0       | 152.8       |
| <b>IV.B3</b> | 0.0037 | 359.7    | 427.2  | 656.1       | 349.8       | 163.8       |
| <b>V.1</b>   | 0.0038 | 326.8    | 378.9  | 575.1       | 315.8       | 161.8       |
| <b>V.2</b>   | 0.0036 | 346.2    | 399.8  | 591.9       | 342.8       | 170.4       |
| <b>V.3</b>   | 0.0033 | 385.8    | 443.1  | 594.2       | 404.0       | 193.0       |

Table D.44. Phase II performance of S-Charts for  $m = 20$  when  $\gamma = 1.5$  using Phase I data under SCM 6.

| Method       | $ARS$  | $mARL$ | $SDRL$ | $cARL_{.1}$ | $cARL_{.5}$ | $cARL_{.9}$ |
|--------------|--------|--------|--------|-------------|-------------|-------------|
| <b>I.A</b>   | 0.0034 | 407.9  | 456.2  | 166.4       | 501.0       | 494.4       |
| <b>I.B1</b>  | 0.0081 | 302.8  | 401.6  | 53.4        | 297.6       | 560.8       |
| <b>I.B2</b>  | 0.0097 | 291.0  | 395.2  | 43.8        | 278.8       | 560.7       |
| <b>I.B3</b>  | 0.0102 | 306.7  | 421.2  | 41.4        | 296.2       | 591.3       |
| <b>I.B4</b>  | 0.0105 | 315.8  | 433.9  | 39.0        | 312.6       | 586.6       |
| <b>II.A1</b> | 0.0078 | 374.4  | 506.2  | 53.0        | 378.4       | 665.4       |
| <b>II.A2</b> | 0.0090 | 395.0  | 552.4  | 44.8        | 385.4       | 726.6       |
| <b>II.A3</b> | 0.0082 | 497.1  | 673.6  | 51.8        | 618.7       | 705.9       |
| <b>II.B1</b> | 0.0113 | 362.2  | 524.8  | 34.9        | 325.2       | 712.0       |
| <b>II.B2</b> | 0.0118 | 372.9  | 540.1  | 33.7        | 341.3       | 726.3       |
| <b>II.B3</b> | 0.0103 | 484.6  | 670.3  | 40.8        | 585.9       | 713.9       |
| <b>III.1</b> | 0.0118 | 281.5  | 402.6  | 34.3        | 245.8       | 592.4       |
| <b>III.2</b> | 0.0168 | 268.1  | 421.2  | 23.1        | 187.8       | 669.6       |
| <b>III.3</b> | 0.0167 | 296.8  | 468.1  | 23.1        | 205.2       | 725.2       |
| <b>III.4</b> | 0.0124 | 277.0  | 398.1  | 32.7        | 239.7       | 591.6       |
| <b>III.5</b> | 0.0164 | 271.3  | 425.0  | 23.3        | 190.2       | 675.2       |
| <b>III.6</b> | 0.0176 | 289.0  | 460.0  | 21.8        | 195.2       | 727.4       |
| <b>IV.A1</b> | 0.0133 | 268.1  | 389.4  | 29.5        | 229.0       | 573.3       |
| <b>IV.A2</b> | 0.0187 | 241.8  | 390.7  | 20.2        | 157.7       | 635.5       |
| <b>IV.A3</b> | 0.0202 | 240.8  | 403.8  | 18.6        | 140.4       | 682.7       |
| <b>IV.A4</b> | 0.0134 | 268.6  | 391.4  | 29.3        | 227.4       | 584.9       |
| <b>IV.A5</b> | 0.0196 | 236.6  | 388.3  | 19.4        | 146.1       | 640.6       |
| <b>IV.A6</b> | 0.0214 | 241.2  | 409.0  | 17.7        | 137.9       | 695.2       |
| <b>IV.B1</b> | 0.0150 | 253.2  | 379.5  | 26.1        | 197.8       | 581.5       |
| <b>IV.B2</b> | 0.0177 | 241.9  | 381.2  | 21.5        | 168.4       | 610.9       |
| <b>IV.B3</b> | 0.0203 | 240.5  | 397.7  | 18.6        | 147.5       | 661.5       |
| <b>V.1</b>   | 0.0188 | 207.7  | 338.8  | 20.5        | 131.5       | 565.4       |
| <b>V.2</b>   | 0.0220 | 192.5  | 328.7  | 17.2        | 109.8       | 558.4       |
| <b>V.3</b>   | 0.0259 | 176.7  | 321.4  | 15.0        | 89.4        | 554.0       |