

EXPLORING CRITICAL MATHEMATICS AWARENESS OF
UNDERGRADUATE VOLUNTARIES TEACHING MATHEMATICS

by

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ABSTRACT

EXPLORING CRITICAL MATHEMATICS AWARENESS OF UNDERGRADUATE VOLUNTARIES TEACHING MATHEMATICS

The purpose of this explorative case study was to explore the preservice mathematics teachers, who were teaching mathematics voluntarily to the students from low-income families, critical mathematics education (CME) awareness. Through this purpose, the focus group interview and case-based semi-structured individual interviews with the three participants, who were the cases, were conducted. The data were coded, and subthemes and themes emerged. As a result of the data analysis process, the participants' CME awareness, which may be the first step of the action praxis of Freire, were found as limited based on the components of CME awareness: (i) philosophical beliefs on the nature of mathematics, (ii) awareness of social justices, and (iii) views of teaching and learning mathematics. The three participants' CME awareness was discussed based on these three components. The two components, philosophical beliefs on the nature of mathematics and social justice awareness, were considered according to two different models, which categorize both in a hierarchical order from the lowest to the higher levels. Each case of all three was placed in the each of categories of those two models, and a covert relationship between the components of philosophical beliefs on the nature of mathematics and social justice awareness was found. Moreover, it was discussed that neither teaching voluntarily nor having classroom time with the socioeconomically disadvantaged students influence the participants' CME awareness. Similarly, the progress in the teacher education program and the learning pedagogy courses does not have an impact on the CME awareness. It could be stated that pre-service mathematics teachers should be supported both theoretically and practically to have CME awareness and to apply it in their classrooms.

ÖZET

GÖNÜLLÜ MATEMATİK ÖĞRETEN ÖĞRETMEN ADAYLARININ ELEŞTİREL MATEMATİK EĞİTİMİ FARKINDALIKLARININ ARAŞTIRILMASI

Bu durum çalışmasının amacı düşük gelirli ailelerin çocuklarına gönüllü matematik öğreten öğretmen adaylarının eleştirel matematik eğitimi farkındalıklarının araştırılmasıdır. Bu amaç doğrultusunda, katılımcılar ile hedef grup görüşmesi ve her üç katılımcı ile vaka-tabanlı yarı-yapılandırılmış bireysel görüşmeler yürütülmüştür. Bu veriler kodlanmış ve buna bağlı olarak alt tema ve temalar oluşturulmuştur. Data analiz sürecinin sonucunda, eleştirel matematik eğitimi farkındalığının belirlenen üç bileşenleri ((i) matematiğin doğası üzerine felsefi inançlar; (ii) sosyal adalet farkındalıkları; (iii) öğretme ve öğrenme görüşleri) üzerinden analiz edilerek, katılımcıların Freire'nin harekete geçme praksişinin ilk adımı olabilecek eleştirel matematik eğitimi farkındalıklarının yetersiz olduğu bulunmuştur. Buna ek olarak, katılımcıların eleştirel matematik eğitimi farkındalıkları bu üç bileşen üzerinden tartışılmıştır. İki bileşen olan matematiğin doğası üzerine felsefi inançlar ve sosyal adalet farkındalıkları, bu inanç ve farkındalıkları gruplayan ve her biri 3 kategoriden oluşan iki farklı modele göre değerlendirilmiştir. Bu üç durum, bu iki modelin her bir kategorisine yerleştirilmiş ve matematiğin doğası üzerine felsefi inançlar ve sosyal adalet farkındalıkları bileşenleri arasında örtük bir ilişki bulunmuştur. Ek olarak, gönüllü öğretmenlik yapmanın ya da sosyoekonomik yönden dezavantajlı öğrencilerle sınıfta zaman geçilmesinin katılımcıların eleştirel matematik eğitimi farkındalıklarını etkilemediği tartışılmıştır. Benzer şekilde, öğretmen eğitimi programındaki ve öğrenme pedagojisi derslerindeki ilerlemenin eleştirel matematik eğitimi farkındalığı üzerinde bir etkisi yoktur. Matematik öğretmen adaylarının eleştirel matematik eğitimi farkındalığına sahip olmaları ve bunu sınıflarında uygulayabilmeleri için hem teorik hem de uygulamalı olarak desteklenmesi gerektiği söylenebilir.

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LIST OF ACRONYMS/ABBREVIATIONS

CME	Critical Mathematics Education
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1. INTRODUCTION

Powerful agencies aim to tyrannize people's decisions and ideas through various channels. In today's world, advertisements and some programs on TV or in the printed publication have an effect on citizens' decisions politically, economically, and socially. People believe in some distortions they are exposed to without questioning. Individuals who are not aware of what is happening around them tend to be affected easily by others' ideologies. For example, the Mercator map locates Europe in the center and North America is shown as larger than Africa, even though Africa is three times larger in size than of North America (Frankenstein, 1994, 2013). This kind of map projection deflects a human perception to associate the significance of places according to their sizes.

Critical pedagogy was firstly proposed by Freire in 1967 to remove those negative impacts of institutions and organizations on people's decisions, especially people exposed to oppression. It is one of the most studied discourses in educational literature over the last decade. Even though the theory was proposed in the 60s for a literacy program for illiterate peasants of Brazil, it can be still adapted to today's world of neoliberalism and capitalism. This concept especially focuses on a group of people in a society who are unaware of distortions, corruption, and even artificiality. The fundamental principle of critical pedagogy is making those people aware of the contradictions by a means of literacy. As they become literate, they would ask questions about their position among oppressors, their own rights, and the structure of society. This is possible through the process of *conscientização* (conscientization), which means that "the deepening of the attitude of awareness characteristic of all emergence" (Freire, 2018, p.109).

The education system might be one of the institutions, which conveys oppressive ideals. The traditional classroom environment surpasses students' ideas and objectifies students' position during a discourse. Freire harshly criticized this system, called the banking model of education in which the teachers' responsibility is a transmission of

knowledge while students are waiting to be filled by their teachers. Teachers transmit knowledge to students through direct instruction in which students are required to take notes in silence without questioning or criticizing. Students listen to the teacher in a passive way, and so they are filled with official knowledge. In this educational system, their responsibility is to parrot the teachers' ideas.

Instead, Freire proposed a problem-posing education system, which is based on the educational praxis of reflection and action through conscientização. In this system, students construct their own knowledge collectively with the mediation of teachers. The main idea of the problem-posing education is that students have their own questions in their minds as being skeptical and curious about concepts. Critical pedagogy expects teachers to provide a classroom environment where learning takes place through dialogical interactions. In this process, teachers and students are equal partners learning from each other in the process of dialogue. Students can judge what is true based on logically and critically analyzed ideas, since teachers are not the sole authority who possess the correct knowledge.

Drawing from critical pedagogy, society and mathematics are connected to the critical mathematics education (Frankenstein, 1983; Skovsmose, 1985). Mathematics is one of the significant means that can be used to understand society and organizations. It is an indispensable part of our world to gain a control over social, economic, and political structures of society (Frankenstein, 1983). Critical mathematics education encourages students to develop the competency of critical mathematics literacy, which requires questioning existing structures of organizations and institutions using mathematics in order to take an action against inequalities and injustices. Precisely, it enhances the empowerment of students by enabling them to grasp the political and social issues through mathematics, and so, it contributes to the development of being a conscious citizen (Skovsmose, 2011). In particular, students begin to see mathematics as a tool that helps them to understand social reality while gaining mathematics competency (Skovsmose, 1998).

Through exploring the mathematics concepts together with social realities, stu-

dents would become skeptical through commonly accepted dictums in a dominant culture; thereby, the others -oppressors- cannot deceive them (Burbules & Berk, 1999). On the other hand, mathematics education can also be a tool for oppression rather than a liberation of thoughts. Philosophical conceptions of mathematics would have an impact on teachers' way of teaching and learning mathematics (Ernest, 1989, 2016a, 2016b). When teachers perceive mathematics as static and unchangeable rather than a changing and developing discipline, they would implement this belief into their classrooms by preparing classroom tasks based on clear-cut answers (Thompson, 1984). Through those classroom practices, students would perceive mathematics as an isolated discipline, nothing about real life, and they approach solutions as the only one and absolute way to solve problems (Skovsmose, 2011). Therefore, students who have difficulty solving problems are alienated by perceiving it as a natural talent, especially disadvantaged students (Frankenstein, 1983).

Even though CME aims to remove the negative impacts of oppressors on an oppressed through the use of mathematics, when it is not clearly understood, it would serve for oppressive powers perpetuating the oppression. It is important to understand the nature of mathematics, social justice issues, and teaching and learning mathematics in the context of CME. However, in Turkey, CME is narrowly focused in the education system and universities' teacher education programs. Therefore, neither teachers nor preservice teachers are expected to be knowledgeable about it. The aim of this study is to explore the CME awareness of preservice mathematics teachers who are teaching mathematics voluntarily to the students from socioeconomically disadvantaged families. Their CME awareness is important since awareness is the first step for the praxis of action through the process of conscientização. The findings of the study would give an insight to teacher educators to design teacher education courses and field experiences for preservice mathematics teachers. CME awareness is especially important for preservice mathematics teachers in order to implement CME in their classrooms as helping their students to improve their quality of life and to bring about social change.

2. LITERATURE REVIEW

2.1. Critical Pedagogy History

In 1930, the head of Frankfurt School, Max Horkheimer, launched a new research era in social sciences, which is “Critical Theory” (McKenzie, 2014). Horkheimer who was impressed by the Marxist view criticized instrumental rationality that is based on how rather than why questions. Instrumental rationality deals with any means to a specific end at any cost. It was called in Weber’s work as an “iron cage of rationality” since it takes away moral values and emotions by trapping the human decisions in order to reach an ultimate goal. Instead, in *Dialectic of Enlightenment* (1947), Adorno and Horkheimer called for a different use of the reason that focuses on the values of objectives rather than the means of process, critical theory. The best-known version of critical theory has been of Theodor Adorno and Jürgen Habermas’s work. They highlighted the radical facets of critical theory; modern society distorts human decisions, beliefs, and behaviors, and so, they become out of control of autonomy that leads to the conformity of society.

Drawing from critical theory, Brazilian educator Paulo Freire (1972) who is seen as a Godfather of “critical pedagogy”, proposed the reason behind the ignorance as coming from the domination of submerged groups leading to the “culture of silence” in which critical thinking cannot be emerged (p.30). The critical theory assumes that people’s actions and thoughts, especially ones who do not have reasoning and questioning skills, tend to be affected by others since those people have internalized their situation participating in their own domination (Frankenstein, 1983).

Freire was exiled from his own country, Brazil, since he was teaching peasants to be literate and be aware of themselves (Garcia, 2011). In his literacy programs, the peasants were learning to express their own voices rather than accepting given ideas and comments. That would mean that the peasants became to think of their exploitation and to question the backward institutions of Brazil, which would be an obstacle in

maintaining the status quo. Freire, in his leading book *Pedagogy of the Oppressed*, mentioned two terms; oppressed and oppressor, to portray a repressive society in which an oppressed are dominated by oppressors in terms of political, economic, and social aspects of society. Oppressors' main goal is to create subservience to their own desires. Oppressed would have no right to think and to criticize; they are not free and their decisions are dependent on oppressors. They do not take risks and always behave like a myrmidon, internalizing oppressors' thoughts and values. These lead them to be sensitive, and so to be manipulated easily by others' ideas. This process causes the conformity of society in which people think and act in an expected and controlled way.

To overcome the structure of a controlled system, the instrument must not be propaganda or manipulation, which are tools of hegemony (Freire, 2018); instead, critical, democratic, and active participation is necessary. Freire proposed critical pedagogy as a way of humanization and liberation (Freire, 1971). Oppressed should experience humanization by themselves as a way of "the ontological and historical vocation to be more fully human" while being the subject of the process themselves (Freire, 2018, p.66).

Although the theory of Freire, critical pedagogy, was for peasants in the Third World, it has been enhanced through the education system (Frankenstein, 1983). The education system is one of the institutions, which is a huge manifestation of available social system, giving clues about the structure of society (Freire, 2018). According to Freire, education is not neutral; it is either a tool for maintaining an available system or a tool of practice for freedom with which they discover the way towards transformation of the worlds they find themselves in (Taskin & Kucuk, 2017). Oppressors have power to regulate and determine the system of education as in the social organization and institutions; therefore, education becomes a political activity in which voices of left-outs are not taken into consideration (Akbari, 2008). The main idea of critical pedagogy is that "education must not serve as a passive reproduction of existing social relations and power relations" (Skovsmose, 1985, p.348).

In classrooms of depicted society, the only authority is a teacher (oppressor in

this situation), and students (oppressed) listen to the teacher without questioning. Therefore, students may grow without any attempt to create their own truths, and they may become accustomed to accepting others' ideas. Freire called such a system a "banking system" of education, in which students are oppressed. In other words, a student is seen as an empty vessel waiting to be filled by a teacher. It may be called a dehumanizing system, which domesticates students' minds (Monchiski, 2008).

As an alternative to the banking education, Freire proposed the ideal education system, called "problem-posing education". Critical thinking and inquiry are significant components of this system. Students are seen as independent thinkers who can criticize problems and events with their own efforts, justify their solutions and opinions, and construct their own questions to understand their surroundings clearly. Moreover, this ideal system adjusts the relationship between a student and a teacher as democratic; both of them are equal during an active learning process (Skovsmose, 1985).

On the other hand, critical pedagogy can be misused in order to protect existing unequal power relations among different social groups as a vicious cycle unless the definition of critique is specified clearly to make education the practice of emancipation. When the definition of critique is co-opted by oppressors, the education falls under domination of hegemonic powers (Skovsmose, 1994b). Pais and colleagues (2012) pointed out that the critique in education needs a more clear domestic definition; otherwise, it may lose its meaning through the social reproduction of hegemony (Brantlinger, 2014).

The concept of critique has been defined historically as a/an (1) investigation of cases, (2) verification of social problems, and (3) taking actions against doubtful social issues (Skovsmose, 1994b). Those definitions call for reflection and action that are elements of an authentic praxis for the purpose of emancipation and true humanization (Gutstein, 2016). Humanization pedagogy should be considered in which an active dialogue is constructed in between students (oppressed), as the only implementers in the process of construction of their own truths, and teachers. Only in this way, students would have individual autonomy.

2.2. Critical Pedagogy in Classrooms

Several research studies focused on obstacles teachers encounter during implementation of critical mathematics (Brantlinger, 2014; Gregson, 2013; Pais *et al.*, 2012). They mostly highlighted schooling system as an obstacle for teachers in an attempt to implement critical pedagogy education. In order to maximize resources and controlling power in hand, schools are functioning through the reproduction of existing hegemony (McLaren, 1988).

The banking concept of education, Freire called, places the knowledge outside as alienated from people where people cannot create their own knowledge (Monchinski, 2008). In such a classroom environment, the teachers' role is to transmit knowledge and follow strict curriculum aspects in a determined way. Teachers are directors of students' learning process rather than a guide and supporter, while students are just listeners as a manifestation of the "culture of silence" of a society in the classrooms (Frankenstein, 1983). Along this way, students cannot go further from memorizing factual knowledge. When they encounter different problems and situations, they would be restricted through their limited factual knowledge.

On the other hand, the main target of critical pedagogy is liberating thoughts of teachers and students. In critical school culture, under the scope of problem-posing education, teachers see students as socially constructed individuals, so instead of following top-down orders, they would try to allay the negative effects of the hegemonic systems (Kincheloe, McLaren, & Steinberg, 2011). For teachers' pedagogical practices, six learning principles in Freire's works are summarized (McLaren, 1999, p.51):

1. Students approach their learning process as grounded in the personal experiences and real life.
2. Students conceive the historical and cultural world as transformable with respect to ideological representations of reality.
3. Students make connections between the conditions of their own life and the conditions of previous meaning-making process.
4. Students create and reconstruct the way of solutions collectively by sharing with each other.

5. Through gaining literacy competencies, students realize their capability to the act of knowing as being an active subject of their learning process.
6. Students understand the myths of oppressors and take an action to transcend them.

The mission of teachers is, as a guide, to provide an environment where students express themselves confidently with regard to logically and critically analyzed ideas and classroom practices, which are relevant to students' experiences and real life. In the classroom environment, the students, therefore, approach their learning process as not isolated from their own world and surroundings. This historical and cultural world is created and reshaped by various ideological representations, and it is important for the students to realize the transformable nature of it, which can be formed collectively as an output of dialogue.

Freire's (2018, p.83) "problem-posing education regards dialogue as indispensable to the act of cognition which unveils reality". Dialogic learning and teaching consists of egalitarian, respectful aspects of communication between students and a teacher and between students; classroom communication norms rest upon mutual respect and non-dominating interactions (Burbules & Bruce, 2001). Through an active dialogue, the teacher and students learn from each other as equal learning partners who have a curiosity about a subject (Freire, 1998). The teacher initiates the discussion through why questions and promotes students to be curious through the process of inquiry.

In dialogical learning, the whole class discusses problems and multiple ways for solutions considering not only correct answers but also wrong ideas (Alrø & Skovsmose 2004). Students understand that there is not only one-way to solve a problem correctly. As a result, they begin to become aware of the fact that a teacher is not a person who knows everything, and the truth is not objective (Gutstein, 2003). Students become subjects of learning processes, constructing their own understanding and making sense of reality.

As they gain literacy skills, students become aware of their potency for being a subject of their learning process. In this way, students may pose their own questions

through the process of “conscientização”, and students may understand the organizations and institutions who hold the power (Freire, 1992). Therefore, they would become a *critical agent* who has a possession to fight against injustices and to make a difference in order to change the world in a better way (Brantlinger, 2014).

2.3. Critical Pedagogy Studies in Turkey

Research conducted in Turkey about critical pedagogy mostly focused on teachers’ views on critical pedagogy (Yilmaz & Altinkurt, 2011; Aslan & Kozikoglu, 2015; Sarigoz & Ozkara, 2015; Sahin, Demir, & Arcagok, 2016; Tasgin & Kucukoglu, 2017), the relationship between teachers’ views on critical pedagogy and their beliefs of educational philosophies (Metin & Aslan, 2014; Kozikoglu & Erden, 2018). These related studies used “Critical Pedagogy Principles Scale” developed by Yilmaz (2009) to measure teachers’ perspectives of critical pedagogy. Moreover, there are several studies (Kesik & Bayram, 2015; Buyukgoze & Findik, 2018) which evaluated Turkey’s education system under the theory of critical pedagogy.

Metin & Aslan (2014) conducted a study to find out the effects of an educational philosophy course on preschool teacher candidates’ preferences for educational philosophy and views of critical pedagogy. They found a significant difference between the teachers’ view of educational philosophy and critical pedagogy after the course. The scores of the philosophies of perennialism and essentialism decreased in the post-test, showing the change in the teachers’ perspectives through the contemporary philosophies, which focus on, in general, improving individual’s life quality by producing an environment in which students experience the world throughout an active participation process in classes. They concluded that teachers became to be aware of critical pedagogy principles after the educational philosophy course. However, the relationship between the variables of teachers’ educational philosophy preferences and their perspectives of critical pedagogy was not examined in this study.

Kozikoglu and Erden (2018) conducted a correlational survey study to determine the relationship between preservice teachers’ educational philosophy preferences and

perspectives in critical pedagogy. The results of the study showed that while there was a significant and positive relationship between the modern contemporary philosophical views (progressivism, reconstructionism, existentialism) and views of critical pedagogy principles; a significant and negative relationship was determined between traditional philosophical views (perennialism and essentialism) and critical pedagogy views. In addition, according to the results of regression analysis, the contemporary philosophical views would explain 12.8% of the variance in views of principles of critical pedagogy. Therefore, it can be concluded that preservice teachers who have a contemporary philosophical perspective are more likely to be involved in critical pedagogy principles.

Studies conducted in Turkey are limited to address teachers' classroom implications of critical pedagogy; rather they focus on teachers' perspectives on critical pedagogy measured by the Critical Pedagogy Principles Scale (Yilmaz, 2009). However, teachers' classroom applications might be different from their thoughts for some reason that might be a time restriction to cover curriculum subjects, the gap between theory and practice, or administrative sanctions at schools.

In the existing literature, critical mathematics education and as it relates to critical mathematics education, philosophy of the nature of mathematics are limitedly focused. Additionally, the questions: "What is critical pedagogy according to mathematics teachers? Are they aware of critical mathematics education? How mathematics teachers apply critical pedagogy theories as a part of their classroom practices?" are limitedly answered in the current literature of Turkey.

2.4. Critical Pedagogy and Mathematics Education

2.4.1. Critical Mathematics Education

Recently, there has been a growing amount of research in mathematics education even though the research of critical pedagogy was grounded in humanities before (Dover, 2013). Drawing upon Freire's "pedagogy of the oppressed", Frankenstein

(1983) and Skovsmose (1985) related mathematics education and critical pedagogy firstly as critical mathematics education (CME), in the USA and Europe respectively (Aslan-Tutak, Bondy, & Adams, 2011).

There are many different notions that can be framed as CME: “teaching mathematics for social justice” (Gutstein, 2003; Stinson, Bidwell, & Powell, 2012) or “social justice mathematics” (Kokka, 2017) that are based on the unifying idea of CME. In general, CME aims for students to be ready for future social and political engagements, to be conscious and participative citizens, along with the lifelong learning of mathematics as a discipline (Skovsmose, 1994b). To achieve the target of critical mathematics education, students should gain critical literacy, which “can be used for the purpose of empowerment because it can be a means to organize and reorganize interpretations of social institutions, traditions, and proposals for social reforms” (Skovsmose, 1994a, p.38).

Frankenstein (1990, 1994) defined *critical mathematical literacy* as having an ability to ask critical mathematical questions to comprehend the particular issues as well as to present relevant data to convince others on their claims. It requires understanding the politics of supposedly neutral numerical data, which leads to question taken for granted assumptions of the available structure of institutions and organizations (Frankenstein, 1994). She demanded to relate political and social issues with the instruction of mathematics to raise awareness on the issues from daily life.

Frankenstein (2011) mentioned four goals of critical mathematical literacy:

- Understanding mathematics
- Understanding mathematics of political knowledge
- Understanding politics of mathematical knowledge
- Understanding politics of knowledge.(p.53)

For instance, in a critical mathematics classroom, a question of percentages of unemployment rate requires students not only mathematics understanding but also under-

standing the politics of mathematics knowledge (Frankenstein, 2001). Mathematics is not neutral, and students should decide on the choice: which includes political judgment, who counts as unemployed, while doing mathematics at the same time. At this point, students would understand the conditions of working people that reflect what Frankenstein called the politics of mathematical knowledge. All the discussion about this problem would infer the politics of knowledge.

Similar to the critical mathematical literacy (Frankenstein, 1990), Skovsmose (1994a) introduced the notion of *mathemacy*, which means using mathematics to question distortions and corruptness of organizations. Mathemacy includes three competencies: *mathematics knowing*- using algorithm appropriately, *technological knowing*- requiring the application of mathematics while building the model, and the last *reflective knowing*- which is a critical aspect of mathemacy. Reflection is one of the significant dimensions of critical mathematical literacy or mathemacy. Barbosa (2006) claimed that a teacher tended to have the reflexive discussions more than mathematical and technological discussions, when a teacher has a socio-critical perspective owing to the fact that the competency of reflective knowing promotes to connect the role of mathematics to society and thereby developing the socio-critical perspective.

Even though applying mathematics to technological models is essential, reflective knowing is necessary to evaluate the models and so to pose further implications. If we take reflection away, technological knowing does not help them analyze and evaluate their own results and findings (Skovsmose, 1994a). Reflective knowing enables to understand social and political sanctions from the angle of mathematics, validity of mathematics applications in the corresponding context, reliability of data results, and broader context that models can be applied (Chartres, 2008).

However, solely understanding the world and reflecting on it are not sufficient to liberate the thoughts. In order to transform society, the action referring to the critical analysis is required. In Freire's praxis, reflection and action are interacting dialectically. Only reflection causes to the verbalism, while sole action yields to the pure activism (Freire, 2018). In order to reach true humanization praxis, they should

interact dialectically.

Gutstein's (2016) framework of the theory RWWM (reading and writing the world with mathematics) includes not only the reading world but also the writing world via mathematics. The framework begins with students' gain of socio-political views by the means of learning and studying mathematics to criticize reality preceding an action to change and shape society. Through this purpose, students should gain a sense of *critical agency* (Skovsmose, 2011); in other words, they should see themselves as conscious actors who have power to act and to make a difference for the world with respect to analyzed and criticized ideas.

To be a critical citizen requires students to appreciate mathematics to sense how it helps them to read the world (Gutstein, 2003). Many studies have shown that critical mathematics education provides an opportunity to change students' ideas of mathematics from consisting of dreaded subjects to a questionable and useful subject for their daily life (Andersson, 2010; Gutstein, 2016; Pais, 2012). Furthermore, most of the research have shown that students, especially the ones who have difficulty in engagement and mathematics before, have become more attentive to the discussions, and they have increased their mathematics grades at least through passing exams (Andersson, 2010; Brantlinger, 2007).

Gutstein (2003) is the pioneer, in the realm of critical mathematics education era, to conduct classroom-based studies. Gutstein's enacted two full-year mathematics course and course contents were prepared together with students. Data analysis of students' quizzes, tests, and classroom activities showed that nearly all students developed mathematical power at different levels becoming confident socially and mathematically. Students used multiple ways of finding solutions, made mathematical reasoning and justifications, and communicated their ideas in writings or by words in order to interpret social subject issues. Similarly, Tisch (2014) found that students developed the perception of ability to understand mathematics after completing social justice algebra course.

2.4.2. Mathematics Teaching and Learning in the Context of CME

Critical pedagogy is not a “one-size-fits-all” pedagogy (Stinson, Bidwell, & Powell, 2012); it changes in every different context based on the learner experiences and environment. For instance, critical pedagogy practices cannot be the same for black and white students both from low-income families that may be because of racial discriminations black students’ experience (Aslan-Tutak, Bondy, & Adams, 2011). Gutstein (2007) proposed the three forms of knowledge that teachers should support students’ critical mathematics development: community, critical and classical as a part of a theory of RWWM. It means that students need knowledge of their community, history, and culture to construct new information on it, while at the same time improving mathematics competencies and critical awareness.

Critical mathematics education tasks were contextualized to students’ daily life in several studies. Frankenstein (1990) focused on race and gender issues in her statistics class, which consists of working-class, urban adults with 30% black people and 70% women. Skovsmose (1994a) constructed a project lesson, which requires the distribution of money as a child benefit, for the elementary mathematics class. Moreover, Gutstein (2016) created the critical mathematics curriculum based on socioeconomic and racial discrimination subjects with his low-income African-American students. Through these studies, students’ ownership and attentiveness were observed towards the critical mathematics activities because of their relevancy to the context of the tasks. It is important to connect the tasks with students’ history and culture. If a given problem is relevant to their social reality, they can internalize it and be more attentive to engage in the activity and discussions.

Another important factor, which describes the quality of given education in mathematics classes, is the teacher-student relationship (Ernest, 2002). Problem-posing education is a route to reflect critically on mathematics and thereby to develop the critical mathematical literacy. In this system, learning partners construct their own knowledge and so pose their own questions reflecting on their and others’ experiences. That requires, as a method of problem-posing education, dialogic learning and teaching.

Freire (2018) suggested dialogical teaching and learning as a way of liberating thoughts in which students construct their own knowledge interacting with others through the inquiry process. In the conversation between students and teachers through the process of inquiry, both partners - the teacher and students - should take a risk for the unforeseen questions, maintaining equality between them (Freire, 2018). It would happen through active participation and involvement rather than being passive listeners.

For dialogic learning, Alrø and Skovsmose (2004) suggested Inquiry Co-operation Model (IC Model) which aims to explore knowledge collectively through the process of inquiry, interacting with other learning partners in the dialogue. The model includes the dialogic elements: “getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating” (p.47), which are stated as empirical indicators of the dialogical inquiry-based teaching and learning for the educational practices (Alrø & Skovsmose, 1996).

Through getting in contact, firstly, students would know each other having an ownership of collaborative learning process. While trying to understand a mathematics task, they ask “what if questions” by which they would discuss possible ways to solve a problem. After they locate a way to solve the problem, they would ask “why questions” to make sense of their methods. Knowledge is not fixed; a teacher and students construct it together by reflecting on and challenging it by mutual questions (Alrø & Skovsmose, 2004; Burbules & Bruce, 2001).

When the teacher and the students challenge the used methods, they would advocate their way of reasoning. Thinking aloud gives the students an opportunity to listen to different ideas and perspectives for the same problem. The teacher can also get an idea for the students learning process; therefore, the remaining part of an instruction would be based on the student current understanding. After the students hear other ideas together with provided feedback, including the teacher’s, they would reformulate their own way by comparing different methods, and in this active thinking process, they would challenge both their solutions and those of others. At the end, as a whole

class, the students would evaluate all ideas.

Moreover, drawing on Freirean dialogue, Gursel-Bilgin (2020a) stated, “love, humility, faith, mutual trust, hope, and critical thinking” as the six preconditions of the critical dialogue in the practice of peace education. According to her, those components are indispensable to build the horizontal teacher-student relations Gursel-Bilgin (2020b).

On the other hand, not all those dialogic acts have to be included, and they do not have to be presented in an order in CME implemented lessons (Alrø & Skovsmose, 1996). It is important to note that Freire suggested the dialogic teaching and learning as a departure method for problem-posing education, not as a prescriptive technique to follow (Stinson & Wager, 2012). Furthermore, classroom practices involving the indicators of dialogic teaching and learning might be grounded in “... ‘constructivist’ approaches to teaching and learning based on Vygotsky’s work. Not all such programs are necessarily Freirean, but they need to be judged in relation to the contextual specificity of their philosophy, their praxis, and their ethos of critical responsiveness with respect to bringing about a more just and humane social order.” (McLaren, 1999, p.51). McLaren suggested that critical pedagogy should not be confused with student-centered learning approaches that lack of social and political critique.

2.4.3. Mathematics as a Discipline in the Context of CME

Mathematics education is not value-neutral. Even though some people refrain from speaking of mathematics education and social justice together (Powell & Brantlinger, 2008), mathematics education has socio-political dimensions. For instance, most textbook questions require students to calculate the total cost of goods in a grocery store, which carries an ideological message that society can only be structured through the consumption of food (Frankenstein, 1983). Those kind of problems make mathematics education to serve the purpose of the capitalist and neoliberalist system, which are functioning according to the ideology of ruling class.

On the other hand, through reading mathematics, students can be aware of the asymmetric power relations, inequalities of wealth and resources, and discrimination on behalf of privileged groups and organizations (Gutsein, 2003). It is one of the ways to understand society and organizations. Therefore, mathematics might be a means of either empowerment or suppression. Skovsmose (1994a) called this a formatting power of mathematics.

Without considering the ideology of certainty of mathematics, mathematics education might serve for the oppression (Borba & Skovsmose, 1997). The ideology of certainty has its roots in the history of mathematics. According to Plato, mathematics is a static body of knowledge, and it is perfect and unquestionable inside its unshakable walls. Plato suggests the pure existence of mathematics, isolated from other disciplines. More precisely, mathematics would be completed and finished using just one set of axioms and sole logic to all the problems (Hannaford, 1998).

This perspective of mathematics confirms people that mathematics is not for everyone, just for people who are skilled from birth. That guarantees the maintenance of the existed social mechanism in which rich people will be in money, perpetuating the wealth disparity while a poor stay poor. Therefore, mathematics may become just political elite (often oppressors) aiming to protect their ascribed status (Hannaford, 1998). Borba and Skovsmose (1997) criticized the ideology of certainty in mathematics:

We see the ideology of certainty as a general and fundamental frame of interpretation for an increasing number of issues which transform mathematics in a “language of power”. This view of mathematics - as a perfect system, as a pure, as an infallible tool if well used - contributes to political control. (p.17)

Traditional mathematics classes, for instance, focus on the single correct answers. Through this true-false paradigm, communication cannot be realized as the way to develop ideas and understanding; therefore, it becomes a means to control (Skovsmose, 1994b). In other words, as focusing on a one-way solution, mathematics becomes to be seen as independent from human effect (Tisch, 2014).

There is a perception that mathematics is an objective and decontextualized discipline; therefore, individuals try to find solutions, which are taught them, rather than finding their own method by taking risk (Borba & Skovsmose, 1997). They discussed the voting theory to decide the way of the distribution of the seats in the Brazilian Congress to contestants from three different cities with different populations. Borba (1992) presented this problem in his calculus for engineering class. The result of the study showed that most of the university students used “proportional rationality”, which is to distribute the seats according to a population of cities, which is a taken-for-granted answer. However, there were more fair solutions such as the method of Banzhaf power index, which suggests the distribution of seats according to power each representative would have in the court. In this example, proportionality was seen as one way of the solution even though fairness could be accomplished through different methods.

The gatekeeper role of mathematics that keeps away some students from pursuing further education should be critiqued. CME can, therefore, be applied if the ideology of certainty in mathematics is taken into consideration. To do so, teachers’ political and ideological clarity is necessary (Bartolome, 2004), since their practices within the true-false paradigm affects students’ world-view to generate the objective views of mathematics (Borba & Skovsmose, 1997). Bartolome (2004) defined political and ideological clarity, and claimed:

“Political clarity” refers to the ongoing process by which individuals achieve ever-deepening consciousness of the sociopolitical and economic realities that shape their lives and their capacity to transform such material and symbolic conditions... “Ideological clarity” refers to the process by which individuals struggle to identify and compare their own explanations for the existing socioeconomic and political hierarchy with the dominant society’s. The juxtaposing of ideologies should help teachers to better understand if, when, and how their belief systems uncritically reflect those of the dominant society and thus maintain unequal and what should be unacceptable conditions that so many students experience on a daily basis. (p.98)

In order to challenge the ideology of certainty and apply CME, exploration of teachers’ philosophical views of the nature of mathematics is important. Ernest (1989)

suggested the three philosophies on the nature of mathematics in a hierarchical order: the instrumentalist view of mathematics, the Platonist view of mathematics, and the problem solving view of mathematics.

According to the instrumentalist view, mathematics is utilitarian, consisting of an unrelated set of rules, facts, and skills. The Platonist view of mathematics sees mathematics as static and as an invention, not a creation. Problem-solving view has the highest level in the model, and to this view, mathematics is the dynamic and expanding discipline. This view focuses on the human invention and creation, and there are no completed outputs in mathematics.

The models of beliefs on the nature of mathematics were constructed as nearly overlapping each other (Dionne, 1984; Ernest, 1989; Thompson, 1991). In the area of mathematics teacher education, Ernest's (1989) classification is to explore teachers' philosophical beliefs on the nature of mathematics is the most commonly used model. Addressing the main ideas of the other models, and also providing categorical approach to the teachers' philosophical beliefs on the nature of mathematics allows Ernest's model to be useful for analytical purposes.

2.4.4. CME Awareness

There is no consensus on the definition of awareness. Being aware means having knowledge and interpreting the case that is happening. Association of Language Awareness defines 'language awareness' as an "explicit knowledge about language, and conscious perception and sensitivity in language learning language teaching and language use" (quoted in Komorowska, 2014, p.6). Consciousness is emphasized in this definition, and being conscious is also related to the term reflection in sociology and educational sciences (Komorowska, 2014). To explore the awareness of individuals, they do not have to have information about the case, and they may reflect on their experiences. In conclusion, awareness does not strictly require being knowledgeable about an issue since individuals' knowledge might be limited.

CME awareness can be defined as knowing the aspects of CME, perceiving, and interpreting the case consciously. In order to implement CME in their classrooms, it is important for teachers to be aware of CME aspects. In particular, teachers should have the knowledge of CME, and they should interpret the CME concepts meaningfully. Ernest (2001) stated that critical mathematics education focuses on the critique stance toward mathematics teaching and mathematics itself.

Based on the literature (Alrø & Skovsmose, 2004; Ernest, 1989; Frankenstein, 1990; Skovsmose, 1994a, 1994b), CME concepts require: (i) to perceive mathematics as a changeable and corrigible discipline; (ii) to use mathematics to understand available organizations and structures and then taking an action towards them; and (iii) to see mathematics learning and teaching methods from problem-posing education perspective. Considering those domains, in order to explore CME awareness, the three components of CME awareness were determined: (i) awareness of social justices, (ii) views of teaching and learning mathematics, and (iii) philosophical beliefs on the nature of mathematics.

The first domain focuses on questioning issues of inequality and injustices through mathematics, which is also the main idea of critical mathematical literacy. CME aims for students to gain critical mathematical literacy competency, so that, they can question injustices using mathematics wisely in order to take action towards them. In this process, the teacher role is significant to provide students the learning environment in which students can develop this competency. Through this purpose, teachers themselves should be aware of those aspects to implement in classrooms. In other words, they should have awareness of social justice issues as a teacher and as a critical agent for the betterment of students (Gonzales, 2009). Marzocchi, Turner, and Druken (2019) described having the reflective conversation in social justice context as a competency that can be improved through experiencing reflective practices. Gates and Jorgenson (2009) present a three level framework to categorize understanding of social justice in an ideological field, including moderate, liberal, and radical forms. The framework begins with a capability of explaining notions of fairness and equity while not questioning the presence of a status quo, to recognizing social inequality structures, and to taking

an approach to act against structural inequalities.

The second domain is related with teachers' views of teaching and learning mathematics in the scope of CME. Freire (1987) proposed a problem-posing education system, which is contrasting with traditional methods. Through problem-posing education, students find their own solutions, and they consider multiple ways to solve the same problem. Alrø and Skovsmose (2004) defined dialogic learning and teaching as the method of problem-posing education: "a conversation with certain qualities: it is a process of inquiry, includes risk taking, and maintains equality" (p.39).

The third one is about critically thinking about mathematics itself, which is possible through questioning the mathematics related beliefs. Applying CME in the classrooms is meaningless unless the certainty of mathematics itself is questioned through philosophy: "gaining access to mathematics education without being critical of the ideology of certainty may reinforce the status quo" (Borba & Skovsmose, 1997, p.17). Individuals' philosophical beliefs on the nature of mathematics are their personal judgment about mathematics that have been shaped by their experiences as a teacher and as a student in the past and now (Raymond, 1997). Most of the studies have shown that teachers' choice of task, classroom management techniques, and so students' achievement might likely be affected by teachers' holding beliefs (Aydin & Celik, 2017; Stipek, Givven, Salmon, & Mac Gyvers, 2001). It may be implied that when teachers have traditional beliefs on nature of mathematics, it is hard to be expected from them to imply CME in their classrooms. Therefore, it is important to understand the philosophical beliefs on the nature of mathematics as a component of CME to understand the CME awareness of the participants.

3. SIGNIFICANCE OF THE STUDY

The privileged group of people bends the political, economic, and social concepts for their own benefits, oppressed ones lack any idea of what is going on, just sticking to taken-for-granted knowledge and values that are readily prepared before from somebody (Freire, 1971; Giroux, 1982). However, as Skovsmose (1994b) pointed out: “education should be public mass education and not just aimed at the elite” (p.27). This is the most significant objective of critical pedagogy, and towards this aim, Freire taught Brazil’s peasants to be literate.

In Freire’s praxis of critical pedagogy, reflection and action are interacting dialectically through the process of critical consciousness. Freire saw them as two ends of the continuum, which are inseparable to reach the liberation of thoughts. Reflection on the concrete situations is not enough to transform the world into a better place; action is required. On the other hand, an action needs to be accompanied by the process of critical reflection to reach a true praxis; otherwise, it would be pure activism (Frankenstein, 1983; Freire, 2018). In this regard, being aware of critical pedagogy can be considered as the first step of acting. Being aware requires being knowledgeable and interpreting the case that is happening. In the case of learning and teaching of mathematics, it is necessary to consider the philosophical view of a person on mathematics, mathematics learning, and mathematics teaching in order to examine the person’s awareness of critical mathematics education (CME). Perceiving mathematics as a static and infallible system reflects the status quo in society that is to understand the world in favor of privileged elite group (Gutierrez, 2007).

Participants of this study are preservice mathematics teachers who are teaching students from low-income families. They are teaching mathematics to those students voluntarily, so they are assumed to have some awareness of inequalities in education as working against it as helping those students. On the other hand, this may not be an indication of their CME awareness. Examining these preservice teachers’ CME awareness may lead to understanding the nature of CME awareness. In other words,

the preservice mathematics teachers' reflection on their experiences and the case of CME implemented instruction is important to give insights to teacher educators into the CME awareness of preservice mathematics teachers.

As a sum, the participants' CME awareness is valuable since they are teaching the students from economically disadvantaged families, and they instill in the students the same awareness through which they make sense and act on the world around them (Bartolome, 2004). The findings of the study may also be important for teacher education courses and field experience for preservice mathematics teachers. Moreover, in the CME literature, there is limited research in Turkey to focus on theory or practices of CME. Exploring philosophical beliefs on the nature of mathematics as a component of CME awareness may contribute to the literature by providing a viewpoint for CME. The study has the potential to give an insight for researchers, teacher educators, and teachers.

4. STATEMENT OF THE PROBLEM

This study aims to explore the participants-who were assumed to act towards injustices and inequalities in education- CME awareness through semi-structured focus group and individual interview questions. Being aware was defined as knowing the concepts of CME and interpreting the cases of CME. Participants are preservice mathematics teachers, and their CME awareness was investigated based on the three aspects: philosophical beliefs on the nature of mathematics, awareness of social justice, and views of teaching and learning mathematics. Therefore, this study investigated the following questions:

1. How the voluntarily teaching preservice mathematics teachers' philosophical beliefs on the nature of mathematics are related with their CME awareness?
2. How is the voluntarily teaching preservice mathematics teachers' CME awareness while reflecting on a video of mathematics instruction implementing CME?
 - 2.a. How do participants interpret the teaching and learning mathematics component of CME?
 - 2.b. How do participants interpret the awareness of the social justice component of CME?

5. METHODOLOGY

The aim of this explorative case study was to investigate CME awareness of pre-service mathematics teachers who were teaching mathematics voluntarily to students of low-income families. Their awareness was investigated based on their reflection and interpretation through interviews. Participants' awareness was significant, since it would give an insight into teacher education courses and field experiences. In this chapter, information about participants, data collection and analysis will be discussed in detail.

5.1. Design of the Study

This study called for qualitative methods to explore the research question deeply. The research questions were:

1. How the voluntarily teaching preservice mathematics teachers' philosophical beliefs on the nature of mathematics are related with their CME awareness?
2. How is the voluntarily teaching preservice mathematics teachers' CME awareness while reflecting on a video of mathematics instruction implementing CME?

The main idea of qualitative research is that meaning is created by individuals interacting with others in a social environment (Merriam, 2002). Qualitative research is used to reveal individuals' perspectives about particular phenomenon and to investigate how they interpret their experiences on it rather than understanding cause and effect relationship or predicting the hypothesis, which requires quantitative research methods (Merriam & Tisdell, 2015). Creswell (2012) differentiated the qualitative methods from the quantitative methods as which qualitative research methodology focuses on words, rather than relying on statistical analysis as in quantitative methodology. Qualitative research provides rich descriptions of meaning, process, and understanding, and it is inductive (Merriam & Tisdell, 2015).

There are different types of qualitative research methods. For this study, the

case study design was chosen to answer the research questions. Indeed, phenomenology, which is used to understand the lived experiences of people involved in a study with the phenomena that is being researched (Groenewald, 2004; Van Manen, 2016), was thought of as an alternative research design to the case study. However, the participants of this study were experiencing CME as neither a teacher nor a student; they were experiencing teaching mathematics to students from low socioeconomic families. The aim of the study was to explore their CME awareness through the focus group and individual interviews, and so interview questions and the video of instruction implementing CME were to investigate their awareness while allowing them to reflect on their experiences. Therefore, the case study was considered more appropriate through the aim of this study.

Case study is a robust research strategy, which is useful when holistic and in-depth exploration and investigation of problems are needed (Zainal, 2007). Case study is an analysis of a bounded system, a case. The case may be individuals, groups, or events (Merriam & Tisdell, 2015). In this study, the case was the school club, Education and Research Club (EREC), volunteers. Participants' individual answers were the units of analysis. However, in consequence of the focus group interview analysis, their individual answers were differentiated from each other, and thus, the participants became the cases themselves.

Case studies may be applied for different purposes: exploratory, explanatory, and descriptive (Creswell, 2007). Exploratory case study is used when investigating issues that include unclear and complex set of outcomes; explanatory case study is used to explain the causal relationship of phenomenon; and descriptive case study is applied to describe an intervention and real life context an intervention occurs (Yin, 2003). In the study, participants' interpretations and explanations regarding CME components could not be predetermined before the study. Therefore, exploratory type case study was conducted, since it aligned with the purpose of the study, which was to explore CME awareness of preservice mathematics teachers who were teaching mathematics voluntarily to socioeconomically disadvantaged students.

5.2. Sampling and Participants

The target population for this study was preservice mathematics teachers who were teaching voluntarily to the students from low-income families. Here, participants were attending the school club called Education and Research Club (EREC) to teach mathematics. The participants were teaching voluntarily to socioeconomically disadvantaged students. The participants were selected purposively to explore the way preservice mathematics teachers assumed as being aware of the issues of educational inequality that they were working against as helping disadvantaged students to make the world a more just place. They interpreted and reflected on the CME aspects during interviews in which they could reflect on open-ended questions. Furthermore, as preservice mathematics teachers, participants' CME awareness and knowledge would be informed teacher educators to design teacher preparation courses and field experiences.

EREC has many projects aiming to support education and training with the volunteers. EREC volunteers are characterized as who continue to learn and bring their differences together in a common point for a single purpose (Education and Research Club, n.d.). The purpose of these voluntary projects is to provide them with 21st century skills so that they can get up to date in a constantly changing world (Education and Research Club, n.d.). EREC contains two different projects for which they work with kids from disadvantaged backgrounds: Ilkyar and Bogazici Youth Center (Boğaziçi Gençlik Merkezi, BGM). Through Ilkyar project, volunteers work with primary school children, especially the students of village and regional boarding schools. Another project's, BGM, volunteers work to support the children of low-income families from various schools in Istanbul. The classes are held at the Faculty of Education building on weekends, Saturday and Sunday. The students of grade levels 5-8 are taught in 5 courses: Turkish, Mathematics, Science and Technology, English and Social Studies. The participants of this study were volunteers in BGM Project and teaching mathematics in those classes.

There were four preservice mathematics teachers, who were teaching mathematics at BGM, and three of them volunteered to participate in this study. One of them

did not participate due to her family's health issues. Pseudonyms of the participants were Alice, Winston, and Clarisse. The questions of general information like their motivation to teach at BGM were also asked to the participants during the focus group interview. Even though BGM's stated aim was helping students from low-income families, the participants' individual motivations in teaching voluntarily were different in some degrees. This contradicted what I have expected about the participants' motivation to teach at BGM, which was because BGM's general stated target was teaching to help the children from low-income families. Furthermore, the participants' answers related to the mathematics related beliefs differ from each other. However, I did not change the context of the research in order to investigate different cases perspective of preservice mathematics teachers on critical mathematics education awareness.

Alice: Alice's major was primary mathematics education, and she was in her second year at the university. Alice stated her motivation to teach at BGM as just having an experience of teaching in the classroom. The participant said that she had not taken a pedagogical course before when the interview was conducted.

Winston: The major of Winston was secondary mathematics education. The participant is in his third year at the university. Similar to Alice, Winston stated his motivation to teach at BGM as just having an experience of teaching in the classroom. Winston was taking a pedagogical course, Teaching Methods in Science and Mathematics, when the interview was conducted.

Clarisse: The major of Clarisse was secondary mathematics education. She is in her second year at the university. Clarisse's motivation of teaching voluntarily at BGM was having an experience of teaching in a class before becoming a teacher as well as helping other people. She also stated that she had not taken the pedagogical course before when the interview was conducted.

5.3. Data Collection

Research design was created through which participants create the knowledge discussing and criticizing. The main sources of data collection used in the study were semi-structured focus group interview and individual interviews. The unit of analysis was individual answers of the participants through the interviews. The interview as a data collection technique is explained from the perspective of constructivism in order to rationalize the methodology employed in this study.

In this explorative case study, interviews were conducted to explore preservice mathematics teachers' CME awareness, and so, the primary type of data was the responses of participants. Being aware means that having knowledge and understanding the situations that are happening. In this study, awareness was defined as knowing the CME concepts, identifying, critiquing, and interpreting the case, which is an instruction implementing CME. The participants' CME awareness was investigated under the three components: (i) philosophical beliefs on the nature of mathematics, (ii) awareness of social justices, and (iii) views of teaching and learning mathematics.

An interview would be the most appropriate method for data collection when conducting the comprehensive case study of a chosen few individuals (Merriam, 2015). Depending on the research question, the interview may be best method for the data collection process. For instance:

...if the researcher is interested, however, in what it is like for students to be in the classroom, what their experience is, and what meaning they make out that experience... then it seems to me that interviewing, in most cases, may be the best avenue of inquiry. (Seidman, 2006, p.11)

Interviews enable researchers an in-depth information to understand participants' perspectives about a phenomenon. Participants' thoughts and awareness are reflected in their words; therefore, by interviewing them, researchers would have information about their experiences and meaning they make from those experiences (Seidman, 2006). Furthermore, open-ended questions allow a researcher to ask further questions

to make clear participants' answers.

Data was collected through (i) the focus group interview and (ii) the individual semi-structured case-based interview, which would be “a unique medium for catalyzing reflection” (Abell, Bryan, & Anderson, 1998, p.493). For the focus group interview, the participants answered the open-ended interview questionnaire, consisting of general questions and mathematics related beliefs questions. For the individual interviews, the participants watched the video of instruction implementing CME, and they answered the questionnaire in three parts (introduction, case-based, and conclusion parts), including questions of social justice awareness and mathematics related beliefs.

The interview questions were prepared with the purpose of exploring participants' CME awareness. After the Coronavirus breakdown, in the second semester of the school, the university was closed, and the participants of the study could not continue to teach at BGM. Therefore, interviews were on the digital platform due to Coronavirus restrictions. Interviews were conducted in Turkish, which is participants' mother tongue, and recorded. Even though education language of participants' is English, it was thought that they would better express themselves in their mother tongue. Consents of the interviews were taken from the participants for interviews and those of voice recordings. The focus group interview was conducted firstly, and it lasted about an hour, while each case-based semi-structured individual interview lasted approximately 2 hours, compromising the time spent watching the video.

5.3.1. Focus Group Interview

Focus group interview is a data collection method with few participants who share similar experiences or supposedly similar views (Yin, 2016). The objective is to yield in depth information on participants' ideas, views, and experiences (Onwuegbuzie, Dickinson, Leech, & Zoran, 2009) allowing not only a researcher but also participants to direct conversation, similar to interviews (Bennett, 2002). Diversity in information may be provided, since group members interact with each other by discussing, agreeing, or disagreeing without interrogation (Winlow, Simm, Marvell, & Schaaf, 2013). Therefore,

in dialogue, participants may reflect on ideas and experiences that are unarticulated previously.

The school club, BGM, was the case in this study to explore CME awareness of the participants teaching voluntarily at this club to the socioeconomically underprivileged students. Therefore, the focus group interview was appropriate to explore the case, since it enables to expose “information on collective views, and the meaning that lie behind those views. They are also useful in generating a rich understanding of participants’ experiences and beliefs” (Gill, Steward, Treasure, & Chadwick, 2004, p.293). On the other hand, focus group interview may not be a convenient method due to presence of other people when sensitive and highly individual topics are discussed (Merriam & Tisdell, 2015). The topic of this study was not sensitive that the participants might be shy to talk about it; therefore, it was a slight possibility.

The questions were open-ended allowing participants to share their ideas freely. The questions addressed their motivation of participating in the program EREC-BGM, students and classroom environment, and mathematics related beliefs. Focus group interview questions are in Appendix A: Focus Group Interview Questionnaire, including focus group interview protocol in Turkish in Appendix B.

5.3.2. Case-Based Semi-Structured Individual Interview

The individual interview was prepared in the three parts: (i) introduction questions; (ii) case-based questions focusing on aspects of CME; and (iii) conclusion questions. The individual interviews were mainly conducted as case-based; most of the interview questions were prepared with respect to the case, which was the video of instruction implementing CME (The Learning Exchange, n.d.-a). The focus of the case-based semi-structured individual interview questions were mainly on the two components of CME awareness: views on teaching and learning mathematics and awareness of social justices.

The video was chosen since it shows the aspects of CME as developed in the

scope of a research of CME. The video of instruction was divided into 5 parts by the researchers: activation, problem solving, accountable talk, choosing students' works, and consolidation. The participants reflected upon at the end of each video section answering semi structured interview questions.

The instruction implementing CME in the video was developed by researchers to address social justice in mathematics classroom and presented on the website of The Learning Exchange ("About the Learning Exchange", n.d.-b), which provides research-based tools for educators in order to improve student achievements. The instruction in the video (The Learning Exchange, n.d.-a) shows one school's approach of social justice mathematics. The instruction includes mathematical and critical aspects of CME. In the supporting resource of the instruction, it is pointed out that through the mathematics tasks "students investigate fair trade, child labor and their own role as consumers and responsible citizens. Student inquiry integrates the arts, language, social studies and mathematics. They study in groups, learning how to work and communicate as a part of team. What they learn changes the way they think and the way they behave" (The Learning Exchange, n.d.-c, "Teaching Mathematics Through a Social Justice Lens", p.3). Furthermore, the context of the task is related to students' real-life issues to engage students, and making the learning process meaningful for them. This would also give the students an opportunity to reflect on their own experiences.

In the instruction, the teacher acts as a facilitator who provides students an efficient learning environment where students are collectively engaged in conceptual learning of the content. The teacher poses probing questions to facilitate the students' inquiry process while the students learn through inquiry and critical thinking. The students investigate the mathematics concept through their own methods, discussing with their peers and the teacher. The flow of the instruction of the video is described in detail in Appendix C: Individual Case-based Semi-structured Interview Protocol, and interview questions in Turkish is provided in Appendix D.

The participants of this study were needed to interpret the case based on the classroom mathematical task and its implementation process. The questions about the

mathematical task were designed under the three aspects: mathematical and critical aspects of the content and relevance of context to the students' daily life, see Table 5.1. The questions about the implementation process were developed considering dialogic learning and teaching aspects to explore the participants' CME awareness.

Case-based semi-structured individual interview questions are given for each five-section separately, in Appendix C, including Turkish translation of the questions.

Table 5.1. Summary of the Aspects of the Semi-Structured Case-Based Individual Interview.

Awareness Domain	Question Categories	Explanation of the interview question category <i>To explore participants' knowledge and interpretation of...</i>
Critical Mathematics	Critical aspect of the task	Social justice aspects of the prepared task.
	Mathematical aspect of the task	Mathematical aspects of the prepared task.
	Relevancy of the task	Context of the task in terms of its relevancy to students' real life.
Problem-Posing	Dialogical learning	The learning process of the students through inquiry, together with risk taking and collaboration while maintaining equality (Alrø & Skovsmose, 2004).
	Dialogical teaching	The teaching process of the teacher through inquiry, together with risk taking and collaboration while maintaining equality (Alrø & Skovsmose, 2004).

5.4. Data Analysis

Transcriptions from the interview were used for the data analysis. The data was coded for the thematic analysis. The themes were constructed under three components of awareness separately through aggregating similar codes together and came from the literature mostly. From the focus group data, constructed themes helped to explore the mathematics related beliefs that include two components of awareness, which are philosophical beliefs on the nature of mathematics and views of teaching and learning mathematics. The data from the case-based semi-structured individual interviews

was interpreted based on the themes for all the three components of CME awareness. Therefore, interpretation of the findings was made by analyzing both focus group and individual interview transcripts, and by reflecting on emerged themes, subthemes, and codes several times.

After data transcription of the focus group interview, the open coding was applied. The emerged codes were grouped under each question. The themes corresponding to the interview questions were “definition of mathematics, knowing mathematics, doing mathematics, teaching mathematics, and learning mathematics”. For the individual interviews, after data transcription, I applied the line-by-line coding following thematic analysis. Open codes were repeats of the participants’ own words or similar to them as a term or a concept coming from literature (Merriam & Tisdell, 2015). Then, those assigned codes were grouped according to the subjects of data, who performed the action (teacher, students) through color coding, and so, primary themes emerged; themes of teaching when the subject was teacher, themes of learning when the subject was students, and themes of social justice issues that participants comment on their experiences of social justice.

The codes were reduced by compiling the codes, which reflect similar idea after translated from Turkish to English. Those related codes were corresponding to the open-ended interview questions or what the participants talk about. The participants were not expected to answer questions correctly, and so, some codes were grouped in relation to the participants’ meaningful explanations and reasoned opinions.

The reduced codes were grouped again as subthemes as “use of tasks, classroom practices, and roles in the class” under the themes of teaching and learning. The subthemes were based on the objects of the codes. If the objects of codes were specifically about the given classroom tasks, they were given under the subtheme use of tasks. When the objects of codes were about general classroom practices, they were grouped under the subtheme of classroom practices. When the objects of codes were about subjects’ roles in the class, they were put in the subtheme of roles.

The coding process was replicated for each of three participants' data. At the end of coding, I reviewed the emerged codes and, if needed, renamed them considering the commonalities in participants' emerged codes to reflect participants' explanations on CME awareness components better. The codes were interpretive codes, which was called by Miles and Huberman (1994) as reflecting a certain depth of information to be understood clearly. For instance, the code *connecting social justice issues to mathematics* that emerged under the theme of teaching and the subtheme of use of tasks reflects the idea that a teacher connects social justice issues to mathematics through classroom tasks.

During the coding process, the transcripts were read many times comparing the codes under the emerged themes to be sure that all the relevant data were coded and the themes were coherent. The process was iterative as going back and forth between the interview transcripts and the codes. The last version of the case-based themes, subthemes and codes, which derived from the data sets of each participant, and examples of some interview excerpts are illustrated in the Tables 5.2 to 5.4.

Theme of Teaching: Theme of teaching defines the teacher's responsibilities, classroom practices, use of tasks and ways of behavior in the classroom from the perspective of dialogic teaching. It is composed of the subthemes: teacher's use of tasks, classroom practices and her role in the class.

Table 5.2. Codes of the Theme Teaching.

Subthemes of the Theme Teaching	Codes	Definition	Examples from Interview Excerpts
Use of Task	1. Connecting social justice to mathematics (Winston and Clarisse)	The teacher connects social justice issues to mathematics through the classroom tasks.	Clarisse: "In other words, [the teacher] associates it with daily life things; she also tried to make the students aware of some things. I think that is what matters. Everyone can learn mathematics in one way or another, and they can read and learn from a book. But what is important in education is not only to learn mathematics, but also to learn about life."
	2. Students being aware of injustices (Clarisse)	The teacher initiates the students' awareness of the social justice issues.	Clarisse: "In the Third World Countries, social injustice is born among children. If we are talking about education, social injustice is born among children.... That's why I think it is very nice that [the teacher] mentions this in the lesson and makes children [the students] aware of them at a young age. "
	3. Students being active in the participation (Alice, Winston, and Clarisse)	The teacher engages students in the learning process through the classroom tasks.	Clarisse: "As the teacher starts the lesson, he has the students do an activity. [The teacher says], "Take off your shoes, see where they are from," and so on. They all get involved in the event somehow... After you take off your shoes, you get involved in the event somehow, so now you do something."
Classroom Practices	1. Students directing their learning process (Alice)	The teacher initiates students to direct their learning process.	Alice: "[The teacher] was not alone in the [classroom] management; I think she also involves students in this management. It enabled them to advance this process. By making them ask questions, taking their thoughts ..."
	2. Students self-monitoring (Alice, Clarisse, and Winston)	The teacher initiates students self-monitoring rather than using traditional and direct instruction methods.	Alice: "Instead of saying it directly, she [the teacher] asked everyone... And she asked a student to explain again... She asked questions by letting the student find it himself/herself. "
	3. Probing questions (Winston and Clarisse)	The teacher asks probing questions to fix students' mistakes, to clarify their understanding, and to assess their understanding.	Winston: "...The teacher sees the student's answer but asks again to explain it, to clarify it a little more. It is a little more challenging ... She used this as a method of asking questions. 'Probing questions'... With the result from there, [the teacher] aims to correct that misunderstanding in [the student's] mind."
Roles	1. Guide (Alice, Winston, and Clarisse)	The teacher acts as a guide, rather than an authority.	Clarisse: "The teacher is also trying to guide the students, not like a teacher. She does not look down as a teacher."

Theme of Learning: Theme of learning defines the students' responsibilities and the ways of act in the classroom from the perspective of dialogic learning. It consists of

the subthemes: students' use of tasks, classroom practices, and their role in the class.

Table 5.3. Codes of the Theme Learning.

Subthemes of the Theme Teaching	Codes	Definition	Examples from Interview Excerpts
Use of Task	1. Relevance to their own life (Winston)	Students are familiar with real life issues presented in the given tasks.	Winston: "When we compare it to their real life, they say something: 'only the wearer knows where the shoe pinches'. Now, if there is such a family, acquaintance, or relative [of the students] there [in the classroom]... Especially if s/he has a hard time getting along, s/he understands the situation of an employee, who works for 40 cents an hour."
	2. Using mathematics to consume (Alice)	Students would use mathematics to consume. It is the social aspect of the task, not political.	Alice: "I think the social [aspect] is more dominant. Social aspects are like everybody getting it [things]. Here...good advertisements or cheap prices. [The teacher] is already asking these things, That is why China is preferred."
	3. Talking about SJ issues (Clarisse and Winston)	Students talk about social justice issues.	Winston: "...That is, children [the students] talking about freedom like this...talking about rights. The boy [the student] come there and says something: 'rights for children and freedoms.'"
Classroom Practices	1. Constructing their own knowledge (Alice and Winston)	Students construct their own knowledge by their efforts with the guide of the teacher.	Winston: "The students are completely themselves; the biggest role is themselves ... The student actually finds himself/herself in the teacher's guide. S/he reaches the result that needs to be achieved himself/herself."
	2. Meaningful learning (Alice, Winston, and Clarisse)	Students learn the concept in a meaningful way rather than memorizing.	Clarisse: "They are trying to understand the logic of the [content]. They have not memorized things directly. Now since it's like this [since they do not memorize], they don't think like 'I should multiply by that, divide by that.'"
	3. Learning as individuals and in groups (Alice, Winston, and Clarisse)	Students learn both in their groups and individually.	Clarisse: "Because they are all involved and trying to solve the problem. All of them try to contribute to solving the problem, whether in a group or individually. All of them are thinking, and can explain why he is doing this when the teacher asks. All of them have a logical explanation so they really got it and were really involved."
Roles	1. Expressing themselves comfortably and listening to others (Alice, Winston, and Clarisse)	Students express themselves freely and listen to others respectfully.	Alice: "A free environment. Everyone can discuss and present their own ideas...without interfering with each other, respecting each other. There is such an environment with respect and love."

Table 5.3. Codes of the Theme Learning.(cont.)

Lesson Outcomes	1. Gaining mathematics competence (Winston)	At the end of the lesson, students learn the concept structurally from low level to high.	Winston: “Okay, we’re seeing improvement. S/he [the students] first saw how the addition was done with a normal basic algorithm. After that, s/he saw that when multiplied by 10, 0 was added or [decimal place] shifted. Finally, s/he presented something fully structured that explains this. It seemed to divide the levels... But in the end, it is certain that they all gained their math skills as a result of the discussions.”
	2. Discovering not only mathematics but also ideas (Winston)	At the end of the lesson, the students discover not only mathematics but also ideas. It is the social aspect, not political.	Winston: “They [the students] discovered not only mathematics but also the idea ... I can't say much politically, but socially ...”
	3. Being aware of SJ after problem solving (Clarisse)	At the end of the lesson, students become aware of social justice issues.	Clarisse: “At the end of the lesson, [the students] realize by whom the shoes, they bought, actually made and what kind of social injustice it was. After calculating this, after they have calculated themselves [and] seen what it actually is, all the students realize what kind of social injustice it actually is.”

Theme of Social Justice: Theme of social justice defines the participant s’ ideas and examples of social justice issues, which are explained explicitly. The codes under this theme are exploitation of human rights in the Third World Countries, equality of opportunity, and equality of educational opportunity were appeared when the first participant commented on her experiences of social justice and ideas on the classroom environment of the instruction of the watched video.

Table 5.4. Codes of the Theme Social Justice.

Codes	Definition	Examples from Interview Excerpts
1. Exploitation of Human Rights (Clarisse)	Rights of people living in the 3 rd world countries are exploited, such as health and minimum wage rights...	Clarisse: "... It actually talks about how much the fashion industry exploits people in general, and especially how much it exploits people in countries we call the Third World. I watched it recently... People have to work somewhere there and the population is too high and resources are not enough."
2. Equality of opportunity (Clarisse)	The students in the classroom who are from different ethnicities and wealth conditions wear the same shoes.	Clarisse: "All of those people [the students] come from different ethnicities, but they all wear the same shoes. In other words, whether they are rich or poor, they wear the same shoes, shoes produced in the same place."
3. Equality of opportunity in education (Clarisse)	The students in the classroom who are from different ethnicities and wealth conditions get the same education.	Clarisse: "Naturally, the children [children in the documentary she watched previously] of mothers and fathers who work there are obliged to work at an early age and cannot receive education. Or even if they get education, even if they somehow go to school, they cannot get as much as [students] in other developed countries. And such an injustice is born, social injustice."

5.5. The Role of the Researcher

In this study, the interviews were semi-structured to explore the participants' CME awareness, and the researcher posed both the interview questions and follow-up questions. The role of her, in the focus group interview, was a moderator who asked the interview questions to reveal the participants' views while keeping the participants' attention on the subject of the discussion. During the interview processes, the researcher was neutral as not to influence the participants' responses, and her follow-up questions were far away from being directive for the intended and expected responses.

5.6. Validity, Reliability, and Ethical Considerations

Qualitative studies are expected to reflect the researcher's personal stances (Yin, 2016). For the quality of a study, different strategies for qualitative studies have been proposed. In order to ensure to validity, reliability, and objectivity of the study, Lincoln and Guba (1985) introduced four criteria: credibility, transferability, dependability, and confirmability as the substitutes for internal validity, external validity, reliability, and objectivity, respectively. Those criteria are not separate; the strategies to conduct the criteria are overlapping.

In this study, the data were elaborated in detail until the researcher feel saturated, and the data and emerging findings were continuously discussed with the researcher's thesis supervisor to ensure credibility. Moreover, member checking was conducted after the data analysis process, and the codes, subthemes, and themes were shared with each participant to determine whether the findings were accurate (Creswell, 2012).

Reliability is delicate in social sciences due to the unstable nature of human beings. In this study, contextual information about the study methods and participants were clearly presented. Moreover, the interview protocols, which are shown in Appendix B and Appendix C, for the focus group and case-based semi-structured interview, respectively, was created to increase the reliability of the study. Therefore, "if the findings of a study are consistent with the data presented, the study can be considered dependable" (Merriam & Tisdell, 2015, p. 252) . The interviews were audio-recorded with the consent from the participants, and they were transcribed to be analyzed. The researcher tried to be transparent while reporting the data, and her role during the study was explained in the previous section. The summary of the employed strategies are given in Table 5.5 below.

Table 5.5. Summary of the Strategy Employed for Validity and Reliability.

Criteria	Strategy Employed
Credibility	Thick description of the data Debriefing with the thesis supervisor Memberchecking
Transferability	Contextual information about study methods and participants Interview protocols
Dependability	Transparent reporting Audio-recording Transcription
Confirmability	Member checking Reflexivity

For the ethical issues, pseudonymous names for the participants were used. Moreover, before the study, the participants were sent consent form that is shown in Appendix E to inform them about the researchers, the aim of the study, and the data collection process.

6. FINDINGS

In this section, analysis of focus group and individual semi-structured interviews will be presented. The data from case-based semi-structured individual interviews were open coded and then themes were formed. As a result of thematic analysis of the individual interviews, three main themes emerged: teaching mathematics, learning mathematics, and social justice issues.

Participants' awareness of CME was explored through the framework that was identified based on the literature. This framework displays the constructs of CME awareness, and the components: awareness of social justices, views of teaching and learning mathematics, and beliefs on the nature of mathematics.

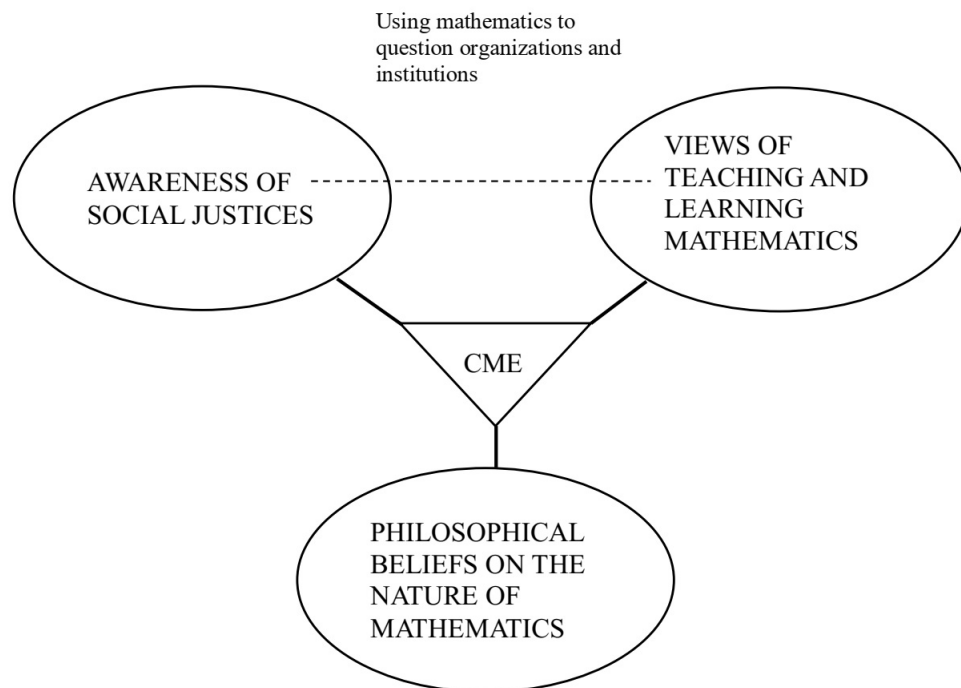


Figure 6.1. CME Awareness Components.

6.1. Mathematics Related Beliefs

All three participants were interviewed as a group, during the first phase of the data collection process. The transcript data of the focus group interview were investigated in order to explore the participants' mathematics related beliefs. I coded the transcript of participants' answers for each of the interview questions separately, which asked *the definition of mathematics*, *knowing mathematics*, *doing mathematics*, *learning mathematics*, and *teaching mathematics*.

The participants were in agreement on the idea that mathematics was applicable to real-life situations, helping us to create a solution for challenges we encounter in life; however, they defined mathematics slightly different. Clarisse defined mathematics as a language that we could create and make a change to it. In her opinion, mathematics was shaped according to human choices:

We define the addition. For example, $2+1$ is three. Thus, we cannot change ' $2+1$ is 3' within certain rules. Within the rules of mathematics, $2+1$ is always three. But if we change the rules, we can change mathematics. We can also change the results we will achieve. That is why I think that we create mathematics. I mean that we have made all of the rules, and we have actually tried to reach something according to the rules we have set.

On the other hand, the other two participants, Winston and Alice, defined mathematics as a stable discipline that we could discover. Alice stated that mathematics was a "building block" of other disciplines, like physics, chemistry, and biology, while Winston was claiming that mathematics could not be falsified, being different from science. "I mean, there is always falsification in science. Einstein comes in and says, 'that was not the case with Newton's laws'. And okay, then we throw it away. But I have never heard such a thing in mathematics before" (Winston). According to his idea, mathematics consisted of connected subjects, and its rules could not be changed easily. For instance, "... $2+1$ is not 3 in mod2. There is no number called 3 this time... We have done something so consistently from the very beginning" (Winston). Even though he addressed mathematics rules as human-made, he insisted to claim that mathematics has existed outside and we have just discovered it:

... there is a slope, there is a curve. We found a way to calculate this slope using derivative... We defined the slope of the curve. However, we do not define the slope of the mountain, that is, what is the slope of the mountain is. It already exists in nature (Winston).

He also conceptualized doing mathematics as a separate occupation for teachers and academicians: "... When I imagine myself as an academician, doing mathematics means trying to prove something crazy in such a dim light at night. But when I imagine [myself] as a teacher, doing mathematics is, in the classroom, at that moment, what you can pass on to the student".

I could conclude that Winston's beliefs on the nature of mathematics fit in Ernest's Platonist view of mathematics. Conversely, Clarisse's beliefs on nature of mathematics were closer to the Ernest's problem solving view of mathematics category.

When the participants' ideas about the meaning of *knowing mathematics* were asked, all three participants stated that it was an endless process, and no one would know it entirely. Alice and Winston described knowing mathematics by emphasizing the applicability of mathematics to real-life issues. For instance, Winston said, "If I know something, I should be able to use it in daily life when appropriate. If I am using it, [it means] I know it". In particular, "if you know that the weight [of something] in the market is 10 lira, [it means] you know mathematics" (Alice).

Clarisse explained *learning mathematics* as being aware of the learning process rather than memorizing rules. She indicated that students generally tended to memorize the rules, giving example from her class:

They just have certain rules in mind, and "I'll apply this rule on this question". For example, in the function, they programmed it in their minds like "the fixed-function was like this, then I will use this". They are just trying to use it. So they memorized which formula and where to use it, most of them have not learned it completely. That is why they cannot do it when a different question comes up, that is, when they do not understand what is happening, when they do not understand exactly what to do in the question, when they do not understand

which formula to use. Therefore, it is important not to learn the formulas in a question, but to know why that formula works in that question.

Consistently with her beliefs about learning mathematics, she stated that she tried to choose questions to eliminate students' memorization process. The questions, she chose, were "open-ended questions" which offers students a way of thinking about process: "I especially try to choose the questions that I can explain the logic [behind it] rather than [the question that the students] memorize".

The other two participants, Alice and Winston, highlighted the importance of having the ability to apply mathematics to different situations and problems. Their focus may be what and how questions when applying the correct steps to solve the problem. According to Alice, understanding the students learning requires answering the question: "Does the student know what to do there, not using the formula directly?". Similar to this claim, taking the assessment methods for the students learning into account, Winston suggested considering the students ability to apply mathematics to the real life situations rather than focusing on their test results. Even though all the three participants beliefs on learning mathematics might seem to stress applying mathematics correctly to the problems, Clarisse, unlike the other two participants, focused on knowing the reasons behind the calculations considering why questions.

Clarisse's beliefs on *teaching mathematics* were mainly learner-centered which aims constructing knowledge on students prior knowledge, as she claimed: "I think it is necessary to know what the students do not know and [then] teach accordingly". On the other hand, Alice and Winston focused on student enjoyment and attention during teaching process. Alice stated that during her teaching experiences, she aimed "to make students love [mathematics] more than teaching [them]". In addition, according to Winston, teaching was what you gave to students. It might be said that Winston saw the teaching process as a transmission of knowledge, and it had been happening through the way of "teaching problems" although he expressed not to make students "memorize the formulas".

Furthermore, the participants described their learning process when they were students as mainly traditional: “when we were at the school... I mean when I was at the school, I learned mathematics for the exam” (Winston). It, therefore, seemed that the participants’ mathematics related beliefs might be attributable to their experiences in education.

Overall, all the three participants motivation to teach mathematics voluntarily at BGM was having an experience of teaching in a classroom; however, Clarisse also added helping other people as well. Similarly, during the focus group interview, Clarisse differed from the other two participants in her answers in terms of the mathematics related beliefs. In particular, Clarisse defined mathematics as an alterable area so we could create it, while the other two participants were highlighting the static and neutral nature of mathematics. Alice focused on what questions, whereas Winston highlighted how questions for the mathematics problem solving process. Moreover, the participants reflected their classroom practices during their teaching process as parallel to their beliefs on the nature of mathematics.

6.2. Teaching and Learning Mathematics Views

Resulting from the individual interview data, three themes emerged which will be discussed in this section. The themes were teaching mathematics, learning mathematics and social justice issues. The participants emerged themes from the individual interviews were barely differentiated from each other; however, the depth of their explanations were quite different from each other. The subthemes, under the primary theme of teaching mathematics, emerged from the participants’ individual interview data were the teacher’s role, classroom practices, and use of tasks. The subthemes that were under the theme of learning were classroom practices, use of tasks, and lesson outcomes.

6.2.1. Case 1: Alice

6.2.1.1. Teaching Theme. The codes of Alice emerged under the theme of teaching and the subtheme of classroom practices were *students self-monitoring* and *students directing their learning process*; the code under the theme of teaching and the subtheme of use of tasks was *students being active in participation*. Furthermore, the participant defined the teacher's role in the classroom as a guide.

According to the participant, the teacher “was not alone in the [classroom] management. I think [the teacher] also includes the students in this management. It enabled them to advance this [learning] process. By making them ask questions, getting their thoughts...” According to her idea, the teacher was not an authority in a horizontal relationship with the students. As Alice explained, the teacher enables students a classroom environment in which the *students directing their learning process*. Commenting on teaching methods, the participant thought that the teacher was not an authority who follow direct instruction; instead, the teacher enabled the students to make their own way in the learning process by “analyzing, collecting and clearing up data”.

In spite of Alice's educational background, which was based on conventional teacher-centered learning, she stressed the importance of student-centered learning methods. When commenting on the instruction of the watched video, Alice expressed that the teacher enabled the students to realize their mistakes and answers themselves rather than saying the answers directly to them. The teacher's classroom practices through *self-monitoring* raised awareness of the students for their learning process.

Besides enabling the students to be active in their learning process, Alice also talked about the teacher's use of classroom tasks in order to make the students active in participation. “They [the students] were involved in this, and they were asked a question about what they observed. As it contributed, this may be a reason for their curiosity”. Alice mentioned taking the students' attention by using the classroom task. In the focus group interview, she had also mentioned students' attentiveness in a class

as a significant factor in her teaching process. It could be said that the participant focused on what she cared about while commenting on the instruction in the video.

The codes formed under the theme of teaching clearly showed that Alice had a student-centered teaching approach. All the subjects of the codes were students such as *students directing their learning process* and *students self-monitoring*. In the focus group interview, she had also depicted the way she taught as shaped according to the students' preferences. However, the teaching methods she had expressed for her classroom practices based on students' entertainment by rewards for the questions that were solved correctly. "...how can I connect it to the game by reducing it to the simplest while explaining the subject in general? The award-winning question came to my mind". It seemed that solving the questions correctly was important, according to the participant, rather than the way students worked on it. Furthermore, her interpretation and reflection stayed in the same frame as in the focus group interview as far away from being in-depth. This may be because Alice had not taken the pedagogical courses yet, since she was a first-year student.

6.2.1.2. Learning Theme. The codes under the learning theme and the subtheme of classroom practices were *constructing their own knowledge*, *meaningful learning*, *being active in the class*, and *learning in groups* and *individually*. Moreover, the code under the learning theme and the use of tasks subtheme was *using mathematics to consume*.

The classroom, as Alice depicted, presented students a free environment. According to her, the students interacted "without interfering with each other, with respect for each other". In this place, the students could express their ideas freely without constraints. "Whatever they think they can present it directly to the environment".

In that comfortable environment, Alice stated, the students could *construct their own knowledge* as observing and doing themselves. "Without any formula or anything the teacher was saying", the students followed their own way. The participant gave an example of students "pattern noticing" when multiplying decimal numbers with 10 and

its multipliers as to throw zero. According to her, the students explained the reason behind this pattern, working on it themselves. However, Alice did not go into details about the pattern the students found themselves and the mathematics concept of the instruction.

Alice described the students learning process as which they “put their thoughts in writing and numbers more clearly”. The students made an effort themselves by conducting “research and making comparisons”; therefore, meaningful learning happened. As far as Alice concerned, memorization did not exist in this learning process. Moreover, according to the participant, since the students involved in their learning process themselves, “there is a participation directly”. The students were active in the class “by asking questions” and “observing themselves”.

During the problem solving process, as Alice said, the students studied by communicating with each other. The advantage of *learning in groups* was “seeing something that escapes their eyes. Or in terms of helping the one [in a group] if the other does not have anything [an idea] about it”. The importance of the interaction between the students to see different ideas was emphasized by Alice. In addition, Alice remarked on the students evaluation “of their results of operations as a whole class”.

Furthermore, due to her restricted conception of social justice, the participant valued *using mathematics*, in the instruction of the watched video, *to consume*. “[The students] buy those shoes in their social life. It will be a money account”. According to her, the students would use this question of this mathematics lesson in order to buy shoes, and those preferences would depend on the marketing strategies of countries. She stated, for instance, the reasons that China was the country most of the students’ shoes were made in, were good advertisements and cheap prices. The issues like minimum wage and child labor that the students discussed were not mentioned by the participant. However, the participant vaguely touched upon the issue of the equality of opportunity:

So, if I think about the social justice thing, everyone has to have a shoe. We have to buy these. And our choices are up to us. All countries produce shoes but, for example, as China’s social advantages outweigh, I think more buy from China.

Another concern was that Alice mentioned those issues stated above as social aspects, not political. Despite the persistent reminders and the repetition of the question, the participant had difficulty answering it, and she stated that she did not know the political aspects of the instruction: “Here China’s political side... I do not know. Maybe I can say the sale, but it does not go into politics”. Moreover, despite the political and social aspects of the discussion during the consolidation part, in which the students were connecting mathematics concept they learned to socio-political constructs, interestingly, the participant indicated that there were no social and political aspects:

Social, political... Actually, it felt like they had entered mathematics a bit now. Socially... They saw how different consequences of the two different wages [of workers in China and Santorini] produced. They turned to mathematics a little. If I looked the discussion at the end, they [the students] found a method themselves. I think they got out of political and social [aspects] and went into mathematics.

Alice was the only participant who did not mention any term about social justice during her individual interview, and even, the code of using mathematics to consume, which was contrasted with the objectives of CME, was emerged. The participant also reflected a student-centered and constructive approach; for instance, she stated the meaningful learning process of the students and the way of constructing their own knowledge in the groups by discussing with others. On the other hand, in the focus group interview, Alice had mainly reflected on her classroom practices based on students’ enjoyment rather than concept learning. It could be concluded that her reflection and interpretation of the instruction in the video revealed how she experienced education in a traditionally based system even though she had a tendency to explain student-centered methods. Furthermore, the reason of her superficial explanations may be the fact that she had not taken pedagogical courses yet.

6.2.2. Case 2: Winston

6.2.2.1. Teaching Theme. The code that emerged under the subtheme of the role was a guide. The codes that emerged under the subtheme of classroom practices were

students self-monitoring and *probing questions*; while the codes emerged under the subtheme of use of task were *students being active in the participation* and *connecting social justice to mathematics*.

The participant explained the teacher's role as a *guide* as the other two participants: "The teacher's only job is to guide the students". According to him, the teacher was not an authority in the classroom, and she did not judge the students answers. Due to his experiences in traditional classrooms, he described this relationship between the teacher and the students as out of the ordinary. He might be accustomed to the asymmetrical power relations between teachers and students. During the interview, he tended to compare the teaching and learning methods and classroom norms of this class with those of Turkey.

Winston's statements about *the students self-monitoring* revealed the dilemma he faced if it was used as a teaching method in classrooms. On one hand, according to him, the teacher fixed the students misunderstandings and mistakes by asking questions. On the other hand, allowing the students a way of self-monitoring "might not always be effective" since the student had to pay utmost attention to the lesson to notice their mistakes on their own. The participant pointed out a possibility of missing a correct answer for the students as a drawback of self-monitoring. It could be seen that Winston was aware of the importance of noticing the mistakes for the students by themselves; however, he saw this noticing process as a risk to not get a correct answer.

Another code that emerged from Winston's reflection and interpretation on the teaching methods was *probing questions*. He defined the notion of probing questions as a questioning method. The participant described it as a way of clarification for the students' responses and ideas. "The teacher sees the students answer but asks again to explain it in order to clarify it a little more. It is a little more challenging". Winston emphasized the teaching method of probing questioning as an opportunity, which led the students to go deeper in their thoughts about the content.

Similar to extracted codes of the classroom practices in teaching, the codes under

the teacher's use of task mainly highlighted the significance of student involvement in the instruction. Winston claimed that collecting data about the countries, where their shoes were made, was the way for students to engage in learning activities: "We keep saying real life, everyday examples, but I guess there is no more real life than shoes. So, what she wears for 20 hours a day is a fabulous thing to grab their attention... I think it is a striking example to get attention". The task mainly constructed on the object, the shoes, which had a huge place in the students' daily life; therefore, according to the participant, it was a way for *students being active in the participation*.

As another way of enabling students participation, Winston stated, the teacher *connected social justice issue to mathematics*. The participant praised the teacher's question "what can be done to change this?" that enabled the students to think about a solution to a social problem they discussed. On the other hand, he reflected his concerns about the appropriateness of this question for that age group of students: "Is it a question to ask a child of that age? Sounds like it is not. Here people are questioning". It would be because of the fact that the participant was not familiar with the teaching environment in which social justice was integrated into mathematics. It seemed that he suggested apolitical and neutral education for that age group of students. This was accompanied by his mathematics related beliefs reflected in the focus group interview; he had, for instance, stated that there is no falsification in mathematics unlike in science. The same was seen in his answers under the theme of learning.

6.2.2.2. Learning Theme. The codes emerged under the subtheme of classroom practices were *constructing their own knowledge*, *meaningful learning*, and *being active in the participation* while the codes under the subtheme of use of task were *talking about social justice issues* and *relevance to their own life*. The codes under the subtheme of lesson outcomes were *gaining mathematical competence* and *discovering not only mathematics but also ideas*.

Winston depicted the classroom environment in which the students could share

their ideas freely. According to him, the students were “aware of their rights” in spite of their different mindsets; therefore, they would express themselves without constraints. “The child can come up there and say this easily. He says ‘I am not very good at multiplication. I am not the best. There is nothing to do. I can’t do that’...”. The participant attributed its reason to a lack of competition in the classroom since the students focused on understanding. On the other side, the participant also pointed out the classrooms in Turkey as indicating that students tended to ridicule their classmates mistakes.

In this portrayed welcoming classroom environment, constructive classroom practices were mentioned by the participant. To him, “the children have structured the ideas themselves”. Winston indicated that through the guidance of the teacher, the students *constructed their own knowledge* playing a central role in this process. Therefore, the process of meaningful learning occurred on the concept of decimals. He also mentioned that through this process, the students *learned both in groups and individually*. In this process, the students “created the problem and then collected data... analyzed it” and they explained to each other what they had found by organizing it in their groups.

Winston expressed his surprise that the students were able to *talk about social justice issues*. To talk through social justice lenses, the participant stated, required students to acquire the competence.

...children talking about freedom like this...[they] talk [told] about rights. So the kid [a student] goes [went] out there and says [said]: ‘rights for children and freedoms’. So this is [was] an important thing... In order for such a discussion to be held in our country, firstly, it is [was] necessary to bring very different things to children from the beginning.

Due to a lack of this competence, he said that unlike those students, even at the university level, he and his friends could not voice their ideas about the social justice issues in a logical manner.

Furthermore, Winston was the only participant who addressed and explained the *relevancy* of discussing tasks to the students own life. The participant stated that through the task, by which the students compared the hourly wages of workers in China and those of Canada, the students could make a connection with their life or the lives of those around them. He explained the relevancy using an idiom that had the same meaning as “only the wearer knows where the shoe pinches” when it was translated to English. On the other hand, he voiced again his concerns about the appropriateness of the task for that age group as saying “...A child of that age may not understand how little that money [the wage] is”.

Winston talked about lesson outcomes the students gained. Even so, the students understood the concept of multiplication of decimal numbers by powers of ten at different levels from high to low; the students gained *mathematical competence* at the end of the instruction. According to him, some of the students’ works reflected in-depth understanding while those of the students were reflecting less structured comprehension. It might be because “the problem provides the students a freedom”. In other words, the students would follow their own way in the problem solving process to go to the solution, and those would be different from each other as exploring different aspects of the concept. For instance, one of the groups “explored the multiplication of decimals by powers of ten as adding zero or moving the decimal point” appreciating underlying mathematics, while the other group made an addition to find a solution without realizing and applying the pattern to multiply decimals. Winston stated the importance of convenience of the problem for all the students; therefore, they could explore the concept following different paths.

As a second lesson outcome, Winston stated that the students *discovered not only mathematics but also ideas*:

The students learned mathematics from social lenses. At the end of the lesson... The critical point is I think when someone is presenting there, he says ‘40’, not ‘point 40’. Then they emphasize again, how much the difference is between them. That is why there was something saying ‘unfair’, even if they did not realize 40 cents at first, but even though it is not clear in everybody’s mind, how much 40 cents was not fair... They discovered not only mathematics but also the idea.

Not only the math operations there. It was very nice.

At the end of the instruction, the students understood mathematics considering social justice issues. At the same time, according to the participant, they realized those justice issues using mathematics. Winston stated that the students explored both mathematics and social idea; however, those ideas were related to social aspects not political: “I cannot say much politically, but socially... There is actually no p of politics [there is nothing about politics] in that class as politics we understand”.

Due to his restricted idea of politics, he thought that the existence of political aspects in a classroom could cause a clash of ideas. According to him, owing to the absence of politics, “... there isn’t much at that age anyway, but still, despite those differences, there isn’t much such a difference of opinion”. Besides seeing the difference of opinion as an unfavorable stand, another important point in his statements was the assumption of having similar ideas in that age group. In addition, Winston appreciated the students’ suggestions for the problem they solved. However, as one of the dilemmas he faced, the participant also underestimated the idea of asking someone famous to help the students inform the public about injustices based on their mathematics calculations as pointing out their age again: “[S/he] believes Michael Jordan can change things. You know, he’s still a primary school kid, we’re talking about social justice, though”.

Overall, Winston mostly conflicted with his experiences in education, which were traditionally based, while depicting the CME classroom practices and use of tasks. In particular, even though he reflected dialogic teaching and learning methods, in general, he faced the dilemmas due to his mathematics related beliefs, which might be grouped under Ernest’s (1989) Platonist view of mathematics. Even though there were codes with regard to the connection between mathematics and social justice issues, he failed to explain the social justice component by dismissing the political aspects of the classroom practices and the use of tasks.

6.2.3. Case 3: Clarisse

6.2.3.1. Teaching Theme. The code emerged under the subtheme of roles was *guide*. Under the subtheme of classroom practices, emerged codes were *students self-monitoring* and *probing questions*; while under the subtheme of use of tasks were *students being active in the participation*, *connecting mathematics to social justice issues*, and *students being aware of injustices*.

Clarisse placed the teacher's role as a guide in the classroom who approached the students like a guide instead of an authority. "...not like a teacher, she is just trying to guide them. Not looking down as a teacher". Similar to the other two participants, Clarisse mentioned the teacher's role as being far away from authoritarian, and to her, their relationship was established by mutual respect.

Parallel with her depiction of the teacher's role, Clarisse emphasized the teacher's student-centered classroom practices. According to the participant, the teacher paved the way for *students self-monitoring* rather than transmitting information directly. In particular, the teacher is "...just trying to make them [the students] understand. She is trying to make them realize their mistakes". The participant meant that the teacher made students a subject in their learning process through the method of self-monitoring:

... [The teacher] does not say the answer to the question directly. [She] does not say, 'this is the answer. It is like that for this'. Either she is trying to make the students notice himself. Or asks others, [and] they say. This way makes [the students] understand.

Another classroom practice of the teacher, Clarisse emphasized, was *probing questions* to make students realize their own mistakes. She stated that the teacher did not focus on unveiling the students mistakes; instead, the teacher tried to make students aware of their mistakes by asking elaborative questions like "why did you do this like that?, Why did you think like that?" The teacher, therefore, became knowledgeable about "why students do" the way they follow in solving the given problem. In addition,

the participant mentioned that those questions would be based on *real-life examples* if the students did not understand or if they could not explain the concept. The reason for this was to make the abstract concrete, since “the student might not imagine in an abstract way”.

Besides the teacher’s classroom practices, Clarisse commented on the way the teacher engaged students through the tasks by making them active in the participation. The teacher let them “take off your shoes [and] look at where it made in’ [Therefore] all of them become to be in the process”. Her idea was that doing something personally made the students be in the class. In Clarisse’s opinion, not only did the teacher take the students attention but initiated the students’ involvement to be in the learning process themselves as well.

Clarisse highlighted the teacher *connecting mathematics to social justice issues*. The participant mentioned that the teacher connected mathematics with its daily equivalent, and raised awareness as well; therefore, the students would “buy the shoes by thinking where they are made and how they are made with difficulty”. According to her, everyone could learn mathematics from the books; on the other hand, “what is important in education is not only to learn mathematics, but also to learn about life”. This could be interpreted as that mathematics was not a value-free discipline in her idea. In fact, in the focus group interview, Clarisse had defined mathematics as human-made that we could create, which was in accordance with those expressions.

Through given task, in this class, the teacher initiated *the students’ awareness of injustices*:

She is trying to make something noticeable; even if the people who make the shoe work for their life time, they may not buy even one shoe. Those who sell it there... For example, while the person who sells that shoe or the owner of that brand earns a 50% profit from this business, not even a dollar the workers earn in one day there. And they have worked in this business for months [and] years; however, maybe no matter how hard they work, they earn enough money to buy maybe only 2 shoes in their lifetime. Students also realize this by calculating that.

Clarisse expressed what the students discussed about social justice issues clearly, mentioning the issue of inequality of opportunity. She put emphasize on the significance of the teacher's initiation in the students' awareness about social inequality at a young age. "That is why [the teacher] ends the lesson in such a way that they [the students] can make a difference in their daily life and change things in their lives, even if small". Somewhat interestingly, even so slightly, Clarisse mentioned the action phase of critical pedagogy, which is also intended outcome of problem-posing education, by pointing out that the students would act accordingly and considering the inequalities in shoe production process.

Besides the teacher's effort to initiate the students' awareness of social justice issues, the participant emphasized the students' awareness of injustices as a lesson outcome, which was also explained under the learning theme. Clarisse was the only participant who pointed out the discussions of social inequalities that result in students' awareness.

6.2.3.2. Learning Theme. The codes emerged under the subtheme of classroom practices were *meaningful learning*, *being active in the participation*, and *learning in groups and individually*, and under the subtheme of lesson outcomes was *being aware of social justice after problem solving*.

Making connections with real-life issues leads the students to a way of *meaningful learning*. Clarisse said, "They think of its real-life counterpart. Accordingly, they are trying to calculate by multiplication, [and] addition. I think they use an appropriate method. They don't memorize, so they try to understand its logic".

In her opinion, the students did not think "to multiply with this or to divide with that" throughout memorization. Instead, they tried to grasp the idea behind the problem. Involving in the problem solving process and being aware of their mistakes and misunderstanding, she claimed, led the students to understand the real meaning of the calculations.

According to Clarisse, the students were *active in the participation* during the lesson. The students had been following the instruction, and so “they know very well what they are doing, and what the teacher is talking about, what their friends are talking about there”. since the students tried to contribute to the problem solving process in groups or individually. As being aware of what is going on around the class, the students, Clarisse stated, could share their ideas about the subject, discussed in the class.

To explain students classroom practices, Clarisse highlighted *the learning individually and in groups*. In the groups, students “try to understand each other’s ideas. They benefit from each other. And they exchange ideas”. Moreover, the students “make calculation individually, and then explain each other”. Although the participant emphasized the individual learning, the more she mentioned the learning in groups and its advantages.

As a lesson outcome, the students became to *be aware of social justice issues after problem solving process*. Clarisse stated:

At the end of the lesson, [the students] realize by whom the shoes, they bought, actually made and what kind of social injustice it was. After calculating this, after they have calculated themselves [and] seen what it actually is, all students realize what kind of social injustice it actually is. And they start thinking about what they can do about it, how they can change it. At least, the next time they buy a shoe, they will be paying attention to where it was made and by whom.

Similar to what Clarisse mentioned about the teacher’s effort in ensuring the students awareness of injustices, which was explained under the teaching theme, Clarisse pointed out the students gaining awareness of those issues by calculating and realizing themselves. Through this way, they would consider and reflect on the injustices in shoe production process that they became aware by considering “what they can do about it and how they can change it?” On the other hand, according to Clarisse’s reflection and interpretation, this awareness resulted in a special consideration and displaying sensitivity, rather than a visible action. This possible action was not on a global scale,

unlike what the students suggested in the class. For instance, the students suggested talking with Michal Jordan to announce those injustice issues to the world in order to make others be aware of those issues. In particular, Clarisse explain the action that the students might take but not pointed to the purpose of that kind of action. It seemed that even though Clarisse could explain the phase of action barely, her ideas were not mature.

6.2.3.3. Social Justice Theme. Clarisse was the only participant whose interview resulted in the theme of social justice besides the theme of teaching and the theme of learning. When commenting on the instruction in the watched video, the participant talked about injustice issues she experienced before as watching a documentary. The subthemes formed under this theme of social justice issues were *exploitation of human rights*, *equality of opportunity*, and *equality of opportunity in education*.

While watching the students discussion on the social justice issues like child rights, minimum wage, and fair employed conditions, Clarisse talked about the documentary she watched before. She addressed the issue of *the exploitation of human rights*, especially in Third World Countries. People living in those countries “do not have sufficient rights. There is no specific minimum wage there... People have to work there, no matter what they are paid”. As Clarisse stated, living conditions of those people made them vulnerable to exploitation.

Moreover, Clarisse mentioned the children of those people as being destitute of education, since they had to work at a young age. “Even if they can get education, even if they somehow go to school”, the quality of education they get was not the same with the quality of education in developed countries. This issue of *equality of opportunity in education* was touched upon while the participant was talking about the instruction of the watched video apart from the documentary. Clarisse depicted the environment of that classroom in which the students from many different cultures could get the same quality of education as students of developed countries.

Clarisse also criticized the classroom task, by which the students collected data about the countries where their shoes were made, stating her observation about the students as having the *equality of opportunity*. “Someone pays more, someone pays less but they are all wearing the same shoes, shoes from the same countries”. Unlike those who saw the shoes as a brand, Clarisse highlighted that the students wore the same shoes coming from the same countries regardless of their purchasing power.

Overall, Clarisse’s ideas on the social justice issues were consistent through the individual interview questions, focusing on the codes: *equality of opportunity* and *equality of opportunity in education*. Clarisse could clearly express what she watched in the video by certifying. Besides the constructive and student-centered teaching methods, not only did she mentioned the connection between mathematics and social justice but also she saw this connection as the students’ lesson outcome. Her mathematics related beliefs had also been far away from seeing mathematics as an isolated discipline; hence, those beliefs supported her reflection and interpretation on the case. According to her, the students would consider the difficulties shoe production workers encountered when they would buy a shoe. However, it seemed more like individual action rather than a societal action that would affect the others at the same time.

6.3. Social Justice Issues Awareness

In this part, the participants’ answers for the first and third part of the interviews will be presented, meanwhile considering the data from the second part of the interview in order to explore the participants CME awareness holistically. The findings will be demonstrated by focusing heavily on the third component of CME awareness, social justice issues.

The individual semi-structured case-based interviews consisted of three different parts; the first part includes the introduction questions; the second part includes the questions related to the instruction of the watched video; and the third part includes the conclusion questions. The introduction questions were asked to define ‘social justice’ and to consider a way of implementing social justice issues into mathematics instruction

based on the corresponding definition. The conclusion questions were required to reflect on their experiences of learning and teaching with regard to the instruction of the watched video by asking similarities and differences of this instruction with what they had gone through and advantages/disadvantages of the learning process in the instruction.

6.3.1. Case 1: Alice

Through answering the introduction questions, Alice said that she had never heard a term of social justice before, and she defined it as “equality of people in society”. She gave examples of using a gym at the school and doing grocery shopping, and those opportunities “are allowed regardless of anyone [who they are]”. At a first glance, it might be seen as an issue of equality of opportunity; however, she did not elaborate on what she answered, and the examples were barely touched on.

The participant explained the implication of social justice issues into a mathematics lesson as a conduit through which students, in an equal way, could answer questions correctly or not. It might be seen as equality of opportunity in accessing resources; however, she did not elaborate what she meant, and answering questions by having an equal time could not be interpreted as the issue of as equality of opportunity without explanations.

From her point of view, as she stated the same in the focus group interview, “mathematics should not be only mathematics.”, and she mentioned students social gain as well as mathematics gain. This social gain was explained as a way for students to “address the same question equally. This adds something to them, adds something to lesson understanding”. Moreover, as a drawback of social justice integrated mathematics instruction, the participant pointed out the time to cover curriculum concepts because of extracurricular activities as being separate from mathematics education.

After watching the video, Alice reflected that this instruction was different from what she had experienced since the system she was familiar with was teacher-centered

learning as being far away constructive methods: “The teacher comes, explains a subject, [and] asks a question. Yes, we are solving that question. But the question is usually based on [similar to] the question the teacher solved earlier”. During her teaching experience, she said that she had taught the instruction alike this in the video to teach the subject of angles. The question she asked required students to: “write me your name [using angles]. You can draw any word, letter, or kids [stickman] you want”. As a result, the students were willing to answer the question. On the other hand, “normally, if I asked to solve 10 questions, most of them did not solve it a week ago”. It seemed her focus was on the students’ enjoyment and attention rather than content learning. She had also reflected the same idea in the focus group interview as well as in the second part of the individual interview while commenting on the instruction in the video.

As an advantage of that kind of learning in the video, she claimed that the students grasped the concept by using various solving methods without memorizing. The issues she reflected were analogous to the emerging codes of *meaningful learning*, *being active in the class*, and *learning in groups* while commenting on the lesson. However, interestingly, Alice never talked about social justice issues even after watching the lesson.

As a sum, before watching the video, she could not define the term of social justice properly. She also could not interpret or reflect on the instruction in terms of social and political aspects while watching the video of the mathematics instruction implementing CME. Afterward, when conclusion questions were asked, which required a general reflection on what she watched and what she experienced, Alice still did not mention social justice issues. Because of the fact that she was not familiar with social justice issues at all, Alice failed to depict those even though she watched the video. It, therefore, could be said that one of the components of CME, awareness of social justices, was absent for this participant.

6.3.2. Case 2: Winston

Winston admitted that he was not familiar with the term of social justice in the first part of the individual interview, and he associated this term with students' efforts of seeking justice due to the emotionality and depressiveness of adolescence during their high school years. The participant stated that contemptuous attitude towards adolescents' rebellion by saying "are you going to provide justice?" evoked his definition of social justice.

Winston claimed that every subject, also social justice issues, could be implemented into mathematics, since "everything in this world has mathematics". In this process, according to him, the obstacle would be a chaotic classroom environment due to having various different ideas. On the other hand, after watching the instruction implementing CME, in the second and third part of the interview, he mentioned that this chaotic environment and discussion might be an advantage rather than a drawback. Another possible obstacle, as Winston stated, would be for teachers who worked in a private school, to speak of social justice issues once this was known by the school management. The participant thought that talking about social justice issues in the lesson might create problems for school administration and so for teachers derivatively.

Winston admitted that the instruction he watched was different from what he experienced not only as a student but also as a teacher. As he asserted in the focus group interview, in the third part of the interview, he claimed that he experienced the passive learning process, even at mathematics courses at the university, waiting for a teacher to be filled.

Nothing but a mathematics lecture, I have never taken any classes. I mean, I was not taught a different course, except for "lecture". I mean in my life, I think, have I formed the mathematics knowledge by myself? I must have done something by using my own knowledge when I was stuck somewhere, but have I constructed any concept beautifully through metacognition from the beginning? There is no way, I do not remember.

He talked about his previous schooling: "This is not an easy thing to do [the stu-

dents' discussions and presentations]. We gave our first presentations in high school". Winston stated that the presentations he made in high school were in philosophy, not in mathematics: "I made it [a presentation] in philosophy. Presentation in numerical lessons is nothing to think about". This might be, therefore, another indicator of how he experienced mathematics education in a neutral and static way.

During his teaching experience at BGM, he said that he had not been aware of the teaching strategies, and he stated that he was learning those through the pedagogical course, Teaching Methods in Science and Mathematics, which he took by the time this interview was conducted.

As advantage of this kind of learning in the video, in the conclusion part of the interview, Winston mentioned students being constructing their own knowledge with the guide of the teacher who was not authoritarian. According to him, when the source of information was a teacher, students would receive all things as what a teacher said. "Where a teacher makes a mistake, it comes to the point of 'no matter, if s/he [a teacher] says it, it is true'. Too dangerous now [It is not good]. [Students would say that] 'Let us take everything. You give it, I'll take it'" The second advantage he highlighted was classroom interaction. Those advantages were the same he had reflected while watching the instruction in the video (in the second part of the individual interview), and the codes from the second part of the interview were students' being *constructing their own knowledge*, *meaningful learning*, and *being active in the participation*.

Secondly, in his opinion, discussions in crowded classrooms would result in chaos that would provide students an opportunity to create an idea and to interact with others in a classroom. On the other hand, he had faced the dilemma while answering the introduction questions. The participant also added that during discussions, social interaction would enable students to overcome being shy, and it would open a way for students to learn together.

Overall, Winston could not define social justice in the first part of the interview, and he admitted that he was not familiar with social justice issues. The participant

claimed that he experienced neutral and static mathematics education as teacher-centered. Furthermore, during his teaching experience at BGM, he said, he had not known the teaching strategies. Moreover, in the conclusion part, he avoided talking about neither social justice issues nor the connection between mathematics and social justice even though he could interpret and reflect on what he saw while watching the video. Instead, he preferred to elaborate on constructive and student-centered teaching methods in detail because he was learning those through the pedagogical course that he took by the time this interview was conducted.

6.3.3. Case 3: Clarisse

Before watching the lesson video, Clarisse defined the notion of social justice as “having equal rights regardless of the culture they grew up in”. As an example, the participant mentioned the issue of equality of educational opportunity as a necessity of social justice. “The same type of schools...For example, not only rich children should be able to go to good private schools, or a poor child should not have to go to a bad school”. She labeled the private schools as good and public schools as bad. Obviously, Clarisse considered the equality of educational opportunity and the equality of opportunity as demanding that poor and rich could have the same qualified education.

When the integration of social justice issues into a mathematics lesson was asked, Clarisse said that she had not thought of those issues as specific to mathematics education. Then to answer this question, she addressed the issue of equity in a classroom: “Regardless of [her/his] financial situation, [s/he] should be able to receive mathematics education in the same way”. The participant thought that it would be hard to do this, and the only way was through “good teachers”. “I think that no matter what school [s/he] is in, every teacher should treat the students in that school as in the best [and] the most helpful way”. In precisely, Clarisse may mean that teachers should teach in the best way wherever or whomever they taught, pointing out the equality of educational opportunity. The notions of *equality of opportunity* and *equality of opportunity in education* had also been the codes emerged under the theme of social justice

issues in the second part of the individual interview.

Clarisse believed that teachers could make a change in providing social justice: “I think teachers can succeed in social justice in the field of education”. Bad teachers, as Clarisse stated, could prevent implementing social justice issues into a class, though. Therefore, considering her depiction of good and bad teachers, it could be said that the participant may see the implementation of social justice issues into a mathematics lesson as providing qualified education for all. She might have “faith” in believing the power of teachers to write the world even though it was not explicit, and she might have “hope” for *the equality of opportunity in education*. In addition, government support and good salary were said as contributing factors for the implementation process.

In response to the conclusion questions, Clarisse portrayed her educational experiences as a student based on a transmission of knowledge to students:

It [the lessons I took] was very different of course. Because when we were at school, I mean, in the lessons I took myself, a subject was always taught in a normal way. In no way, it was thought of in real life or anything. So, I wouldn't think either, and the teacher wouldn't make me think about it either. Only lectures were given.

Clarisse also depicted her teaching experiences as being different from the instruction she watched, in terms of the fact that she taught in a theoretical way far from being connected to real life.

When reflecting on the advantages of that kind of learning, Clarisse pointed out students' involvement in a class while on disadvantages, mentioning a possibility of wasting time to cover curriculum subjects.

I think the curriculum in Turkey. I do not know the curriculum there [the curriculum of the country in the video] but when it is thought in Turkey, the subjects might not be covered. For instance, I think high school [subjects]: limit, derivative, integral... It is impossible to do [teach] like this for each of them [subjects].

Furthermore, Clarisse never talked about social justice issues, and so, the connection of those to mathematics teaching while answering the conclusion questions, which required reflecting on the instruction they watched.

In summary, in the second part of the interview, Clarisse had been the only participant who talked about social justice issues considering her experience of watching the documentary. In response to the introduction questions, she could define this notion based on *the equality of opportunity* and *the equality of educational opportunity*, which had also been the codes from the second part of the individual interview. Even though she had explained the connection between mathematics and social justice while she was interpreting and reflecting on the instruction implementing CME, she did not mention it when the advantages and disadvantages of that kind of learning were asked in the third part of the interview. The reason for this might be the fact that she was not used to the aspects and objectives of CME both in theory and in practice although she was familiar to talk about social justice issues.

7. DISCUSSION

This study is set out to explore the participants' critical mathematics education (CME) awareness. In this sense, the participants' philosophical beliefs on the nature of mathematics, awareness of social justice issues, and views of teaching and learning mathematics were investigated as the components of CME awareness. The present findings were significant in at least two major respects. The major finding is that there may be a connection between philosophical beliefs on the nature of mathematics and social justice awareness. Secondly, experiences may influence the components of CME awareness: philosophical beliefs on the nature of mathematics, views of teaching and learning, and awareness of social justice. This might be because of the participants' ideological stances that are constructed through the structure of the complex system. In this section, the findings will be discussed in detail.

7.1. Components of CME Awareness

7.1.1. Views of Teaching and Learning Mathematics

The participants of this study had different views of teaching and learning mathematics that were derived from data from the focus group and individual interviews. In the focus group interview, Alice described mathematics teaching regarding the student enjoyment rather than content teaching, while defining mathematics learning as an ability to apply mathematics to different situations, highlighting the importance of what questions to solve problems. The second case, Winston, defined learning mathematics as the same as Alice's definition but focusing on how questions. The process and steps of the problem-solving process were important for him, and so he described teaching mathematics as a transmission of knowledge. In contrary to those views, Clarisse showed the student-centered approach because teaching mathematics, according to her idea, was to construct knowledge on students' prior knowledge. When explaining learning mathematics, she focused on why questions for the learning process, refusing the memorization.

On the other hand, when interpreting and reflecting on the case that was the CME implemented instruction in the video they watched, there were codes like *students being active in the class, constructing their own knowledge, and learning in groups and individually*, which would be correspondent to some of the dialogic elements of the Inquiry Co-operation Model (Alrø & Skovsmose, 2004). These indicators of the dialogical inquiry-based teaching and learning for the educational practices were “getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating” (Alrø & Skovsmose, 1996, p.47). Nevertheless, this could not be interpreted as the participants had a critical pedagogy teaching and learning approach, because emerged codes could be interpreted in the scope of student-centered or constructivist approaches, which critical pedagogy was rooted in. Moreover, all of the participants defined the teacher’s role as a facilitator, and according to their ideas, the teacher was not an authority who holds the sole power, contrary to their experiences in traditional classrooms (Burbules & Bruce, 2001; Freire, 1998). They might have faith in mutual trust in classrooms, seeing the relationship between the teacher and the student as equal; however, for the real critical dialogue, all of the six preconditions, which are “love, humility, faith, mutual trust, hope, and critical thinking”, should be included (Gursel-Bilgin, 2020). As McLaren (1999) claimed: “Not all such programs are necessarily Freirean...” and those programs could not be the same with critical pedagogy due to lack of political and social critique (p.51).

The codes from the individual interviews were mostly the same for the participants but the depth of their interpretations and reflections were not totally the same. Alice’s explanations were not clear or detailed, whereas Winston interpreted and reflected giving details due to the pedagogical course, he was taking. However, Winston faced the dilemmas in his explanations. For instance, when explaining *students self-monitoring*, he highlighted teaching methods of asking questions to fix students’ misunderstandings and mistakes on the one hand. On the other hand, to him, self-monitoring would be a risk to not getting correct answers. This interpretation could be explained with the frame of the ideology of certainty that was rooted in Platonist view, and “this view of mathematics - as a perfect system, as a pure, as an infallible tool if well used - contributes to political control” (Borba & Skovsmose, 1997, p.17).

In addition, Winston was in another dilemma while explaining the teacher's use of tasks to *connect social justice to mathematics*. He praised the teacher's initiation in students' thinking process to find a solution for a social problem, whereas he questioned the appropriateness of this social problem for that age group. Therefore, it could be said that his views of teaching and learning mathematics were neutral and apolitical even though he mentioned the constructive and student-centered teaching and learning methods while interpreting and reflecting on the case. Nevertheless, views of teaching and learning of the third case's, Clarisse, were consistent during focus group and individual interview, and it is not value-free. According to her idea, mathematics could be learned from the books; on the other hand, "what is important in education is not only to learn mathematics, but also to learn about life".

As a sum, even though the participants interpreted and reflected on the case from the constructive and student-centered approach in the individual interview, some part of their explanations were related to their views of teaching and learning mathematics that were identified according to the data from the focus group interview. Although the codes emerged in the individual interview, like *students being active in the class*, *construction their own knowledge*, and *learning in groups and individually*, could be interpreted as acts of dialogical teaching and learning, it couldn't be concluded that the participants had a view of problem-posing education.

7.1.2. Philosophical Beliefs on the Nature of Mathematics

According to the findings of this study, the participants' philosophical beliefs on the nature of mathematics could be categorized, to some extent, into Ernest's categorization of philosophical beliefs on the nature of mathematics. Interestingly, each case of all three could fall in the each of Ernest's (1989) categories, which are, in a hierarchical order from lower to higher, instrumental, Platonist, and problem solving views. Alice had a belief corresponding to the instrumental beliefs, while Winston's beliefs fall into the category of Platonist view, and Clarisse's belief on the nature of mathematics was as similar as the criteria of problem solving category.

They expressed their teaching practices of mathematics while teaching at BGM as in the same vein of the philosophical beliefs on the nature of mathematics according to the Ernest's model. For instance, Clarisse reflected that mathematics is changeable as being subject to human endeavor, and she expressed that she chose open-ended questions for her teaching practices. On the other hand, Winston, who hold Platonist view, mentioned solving as many as questions for his teaching practices and the importance of the transmission of knowledge to students. The first case, Alice, who had an instrumentalist belief, reflected that she, in her teaching practices, aimed toward students pleasure rather than content learning as well as using mathematics in real life situations like *using mathematics to consume* as utilitarian. Even though she did not say that mathematics rules and concepts are separate from each other, according to her opinion, "if you know that the weight [of something] in the market is 10 lira, [it means] you know mathematics".

Furthermore, all the participants, while commenting on the lesson in the video, tended to talk about what they mention about mathematics related beliefs and practices during the focus group interview; therefore, their responses in the focus group and individual interview were in consistency. The alignment between philosophical beliefs on the nature of mathematics, espoused beliefs on teaching and learning mathematics, and enacted classroom practices could be explained by Ernest's (1989) mental model for teachers.

7.1.3. Awareness of Social Justice Issues

Awareness of social justice issues is necessary but not sufficient condition for CME awareness. In all the three cases of the study, social justice and social justice issues were defined and explained limitedly even though it had been assumed that they were assumed to have some awareness of social justice issues since they were teaching mathematics voluntarily to the students of low-income families. In particular, they were expected to have awareness of social justices from the outset; however, the findings were not as expected.

Considering the emerged codes and the participants' responses, the participants' awareness of social justices could be, to some extent, grouped under the first two levels, moderate and liberate forms, of the framework that Gates and Jorgenson (2009) presented, which compromises of moderate, liberal, and radical forms of social justice. Each case had different level of social justice awareness. Unfortunately, the first case, Alice, could not be categorized according to this framework, because she failed to explain the notions of fairness and equity that the first level requires. The second case, Winston, had a moderate form of awareness, while the third case, Clarisse, had a liberal form of awareness. Those findings are in accord with recent studies indicating that preservice teachers' social justice conversations are limited.

Somewhat interestingly, one of the participants, Alice, avoided talking about social justice related issues, preferring to talk about the mathematical aspects while commenting on the instruction in the video. There were similarities between these attitudes expressed by Alice and those described by Marzocchi, Turner, and Druken (2019) whose study examined the conversations of preservice mathematics teachers around social justices. They found that the participants tended to mention mathematics firstly and social justice context secondly due to the fact that they were not used to talk about those issues in a mathematics lesson. Furthermore, several studies (Garii & Rule, 2009; Tanase & Lucey, 2017) showed that preservice teachers focus on either mathematics content or social justice issues, not both.

In particular, Alice could not identify the connection between mathematics and social justices, and even there was a code, *using mathematics to consume*, emerged during her interviews. This is what Frankenstein (1983) harshly criticized about textbooks, whose questions could have a message that life could be possible only through consumption. In the scope of CME, the context should be social justice issues such as "geopolitical issues like ecology, environmentalism, animal rights" (Ernest, 2007, p.2) or socioeconomic inequalities and injustices (Gutstein, 2006) rather than consumption issues. Through those contexts, preservice teachers and students would teach and learn mathematics in a meaningful way. Algebra, for instance, verifies that the increase in all wages at the same percentage, while the wealth gap between a poor and a rich are

increasing at that rate as well (Lesser & Blake , 2006).

Furthermore, this participant mentioned the examples of marketing strategies and advertisements as social aspects not political. Another participant who avoided talking about political aspects like Alice was Winston. Even though there were codes that emerged in his individual interview like *discovering not only mathematics but also ideas* for the students' learning outcome and teacher's way of *connecting social justices to mathematics*, the participant labeled those as social aspects, not political due to his absence of "political clarity" (Bartolome, 2004). He thought that politics causes a clash of ideas and so chaos. Preservice teachers need to appreciate the importance of grasping the mathematics concepts in the context of sociopolitical and economic realities that shape peoples' lives through political clarity (Bartolome, 2004).

The third case, Clarisse, could explain it more clearly and deeply in comparison with others. She pointed out in her individual interview that the teacher's use of tasks to *connect social justice to mathematics* and to lead *students being aware of injustices*. Especially, Clarisse evaluated *being aware of social justices after problem solving* as a lesson outcome of the students. She may have a competency of mathemacy, which requires to use mathematics to question institutions and organizations (Skovsmose, 1994a). According to her idea, mathematics is not value free and the teacher, in the video, raised the students' awareness about inequalities by using mathematics, and this made a change in their lives since they would consider those issues of inequalities between shoe workers and producers while buying a shoe, in future. Clarisse mentioned the praxis of action but it was inconsequential. Moreover, before watching the video, she couldn't clearly explain the way of implementing social justice issues into a mathematics instruction, and in the conclusion questions, she never talked about CME as the other two participants. Therefore, it can be said that Clarisse was familiar with social justice issues but not with CME.

Overall, they had a little faith in providing social justice issues based on mathematics teaching and learning, and they focused mostly on the teaching and learning techniques and methods.

7.2. Relationship between the two Components of CME Awareness: Philosophical Beliefs on the Nature of Mathematics and Social Justice Awareness

The hierarchical order of participants' level of philosophical beliefs (instrumental to problem solving) on the nature of mathematics is the same order in that of awareness of social justice issues. In particular, the first case, Alice, who hold the instrumentalist view of mathematics did not have a social justice awareness; the second case, Winston, who hold the Platonist view had the moderate level of social justice awareness; and the third case, Clarisse, who hold the problem solving view of mathematics had a liberal form of social justice awareness. Similarly, Tanase and Lucey (2017) investigated preservices' awareness of connection between mathematics, finance, and social justice issues by conducting a survey, and they concluded that most of the participants had narrow or intermediate level of mathematics conceptions, and those who tended to see mathematics as an isolated subject, were unable to articulate related social justice issues.

Table 7.1. Summary of the Findings of Philosophical Beliefs on the Nature of Mathematics and Awareness of Social Justices

Mathematics Beliefs \ Social Justice Awareness	Lack of Indication	Moderate forms	Liberal forms	Radical forms
Instrumental view	Alice			
Platonist view		Winston		
Problem solving view			Clarisse	

There may be a covert relationship between the components of philosophical beliefs on the nature of mathematics and social justice issues awareness. The relationship between the philosophical beliefs on the nature of mathematics and social justice awareness may be explained through the fact that both social justice (Gates & Jorgensen, 2009) and the philosophy of mathematics (Skovsmose, 1994b; Ernest, 1991) are ide-

ological processes. Individuals' ideological stances may be reflected throughout both the philosophical beliefs on the nature of mathematics and awareness of social justice issues as well, in the same way.

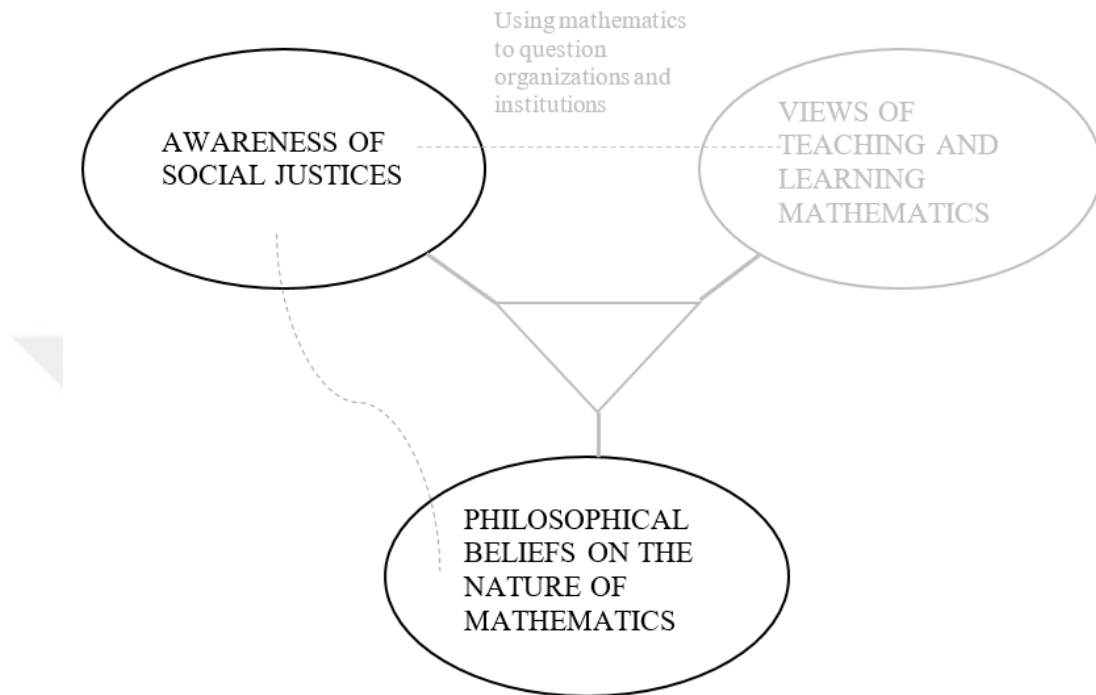


Figure 7.1. The Relationship between the Two Components of the CME Awareness: Philosophical Beliefs on the Nature of Mathematics and Awareness of Social Justices.

Critical activity is a work of philosophy (Skovsmose, 1985) and social justice issues are in the scope of ethics, which is a part of philosophy, and so, it is reasonable to consider its linkage with the philosophy of mathematics (Ernest, 2017). Furthermore, the results of Kozikoglu and Erden's (2018) study showed that there was a significant and positive relationship between the contemporary philosophical views, like progressivism and reconstructionism, and views of critical pedagogy principles, while the relationship between the traditional philosophical views and views of critical pedagogy was significant and negative. This study supports the findings of the current study on the relationship between the philosophical beliefs on the nature of mathematics and awareness of social justices.

In flat contradiction to hegemonic purposes, "an emancipation could be the result of a critique of ideology" (Skovsmose, 1994b, p.19). Through this way, attempts for

ideological and political clarity is required in teacher preparation programs, providing an environment in which preservice teachers can explore how ideologies work in a relationship with power (Bartolome, 2004).

It is necessary for preservice mathematics teachers to develop and increase those aspects of CME; in other words, they should develop critical mathematics literacy (Frankenstein, 1990) or mathemacy (Skovsmose, 1994a) to become critical agents for transformation of the world. As a result, they would implement CME in their classroom practices to promote their students to be critical agents who can use mathematics to question distortions and injustices and then taking an action towards as working for the justice world.

7.3. Influence of the Experience on the CME Awareness

Having an experience might make a significant difference in the way the participants interpret and reflect on the components of CME awareness. However, in this study, those experiences were coming from neither teaching voluntarily at BGM nor having classroom time with the socioeconomically disadvantaged students. Furthermore, the findings have also illustrated that the participants' CME awareness might not depended on the progress in the teacher education program or the learning pedagogy courses. Clarisse who had, in comparison with other two participants, the highest level of philosophical beliefs on the nature of mathematics and social justice awareness was the 2nd-year student who took limited courses on teaching and learning; whereas, Winston who hold moderate level of philosophical beliefs on the nature of mathematics and social justice awareness was the 3rd-year student. Winston was taking the course of Teaching Methods in Science and Mathematics, which may help him to examine the video from the pedagogical aspects. Due to the experience he gain from this course, he explained teaching and learning methods clearly from the constructive and student-centered approach. However, Winston admitted that even at the university level, he could not discuss those justice issues like the students in the watched video.

Somewhat interestingly, the social justice theme emerged in the individual inter-

view of the third case, Clarisse, who had relatively high level of social justice awareness, when she was talking about the documentary in which lives of Third World Countries' peoples were portrayed. Particularly, the codes under this theme, *equality of opportunity* and *equality of educational opportunity*, emerged in the first part of the individual interview, while making the definition of social justice, apart from talking about the documentary. Another previous exposure to those notions (Bartell, 2013) would make her to watch the documentary. In addition, her motivation to teach at BGM was to help others together and to have an experience of teaching in the classroom. She might have a sense of critical agency to act against injustices in education. Nevertheless, it is hard to document the life changes of individuals in the long run or to attribute any change to gaining a sense of critical agency (Gutstein, 2006).

According to Marzocchi, Turner, and Druken (2019), having a conversation in social justice context requires a skill that can be improved through experiencing reflective practices; therefore, students can feel comfortable in those discussions. On the other hand, even though studies show that even short exposure to discussion of connection between mathematics and justice issues would help participants to see their roles as agents of change (Gonzales, 2008; Gutstein, 2004, 2007), there exist also studies, which illustrate the opposite. Garii and Rule (2009), for instance, analyzed preservice teachers' lesson plans that show deficiencies in the integration of mathematics content and social justice issues even so after completing extensive training about both areas, owing to the fact that the focus of those training courses were either mathematics or social justices or that the majority of those courses were inadequate themselves.

Experiences related to mathematics and mathematics education may influence the individuals' awareness of social justice issues. The participants, in this study, reflected on how they experienced mathematics education as teacher-centered, which focused on the transmission of knowledge to students. This might explain the stand of Alice and Winston's philosophical beliefs on nature of mathematics, seeing mathematics as a certain and static area. "Teachers themselves were often educated by mathematicians who are not, in general, interested in educational or philosophical issues about uncertainty in mathematics" (Borba & Skovsmose, 1997, p.19). However, the third

case, Clarisse, had the highest-level views on the nature of mathematics despite having teacher-centered mathematics education in her previous schooling. Alternatively, experiences related to inequalities and injustices might shape their ideology and so individuals' beliefs on the nature of mathematics. The relationship between those should be analyzed carefully in order to tackle the problems in the implementation process of CME.

As a sum, according to the findings of this study, the preservice mathematics teachers' CME awareness was limited. Neither teaching at BGM nor taking teaching methods courses, but their experiences like watching a documentary might have an impact on the participants' CME awareness. Teacher educators ought to take into consideration the importance of those experiences to provide preservice and inservice teachers an environment that enables them to develop ideological clarity and awareness in the line of liberation. This environment should support them offering a chance of practice supported with theory-based learning. Therefore, through these teacher education programs, their teaching experience would be more meaningful in the way of gaining a competency of critical agency together with implementing CME in their classrooms to lead the students to become critical agents as well.

8. LIMITATIONS AND RECOMMENDATION FOR FURTHER RESEARCH

The section provides several limitations of the study and further recommendations for future research on critical mathematics education. This study explores preservice mathematics teachers' CME awareness considering the three components: philosophical beliefs of the nature of mathematics, awareness of social justice issues, and reflections on and practices of teaching and learning mathematics process. Those components were determined, corresponding to the literature; however, further research would be conducted considering different aspects of CME.

The participants of the study were chosen purposefully at the one public university to explore interpretation and reflection of preservice mathematics teachers on CME components since they were teaching students of low-income families voluntarily at school club, BGM. Moreover, more research would be needed with preservice mathematics teachers from different universities, considering their voluntarily teaching works to make generalizations about the findings of this study.

Three participants were included in the study. There were four preservice teachers teaching mathematics at BGM and one of them could not attend the interviews due to her family's health problems. To make more in-depth exploration and generalizations, further research can be done with more participants. Furthermore, the tool for data collection, in the individual interviews, was a video of the mathematics instruction implementing CME, and its language was English. It might restrict the participants' interpretations and reflections since their mother tongue is Turkish. Nevertheless, the interviews were in Turkish.

In addition, since this study is a case study and the methods of data collection were focus group and individual interviews, the literature can be expanded through experimental research and various data collection methods.

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APPENDIX A: FOCUS GROUP INTERVIEW QUESTIONNAIRE

1. Please mention about yourself.
2. What is your motivation to teach mathematics within EREC-BGM?
3. If you think a typical day of teaching, could you depict the classroom environment and your students? How could you prepare for that lesson, and how could you teach that lesson?
4. How can you describe mathematics? (What is mathematics? What does it mean to do mathematics?)
5. What does it mean to know mathematics?
6. What does it mean to learn mathematics?
7. What does it mean to teach mathematics?

APPENDIX B: FOCUS GROUP INTERVIEW PROTOCOL

ODAK GRUP MÜLAKAT PROTOKOLÜ

Merhaba,

Ben Gamze İnan, Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölüm-ü yüksek lisans öğrencisiyim. Öğretim üyesi Dr. Fatma Aslan-Tutak'ın danışmanlığında “gönüllü matematik öğretmenliği yapan üniversite öğrencilerinin eleştirel matematik eğitimi farkındalıkları” hakkında yüksek lisans tez çalışmamı yürütmekteyim. Gönüllü olduğunuz bu odak grup görüşmesine başlamadan önce, bu görüşme ile ilgili sizi tekrar bilgilendirmek istiyorum. Bu görüşme, 7 açık uçlu soru üzerinden ilerleyecek ve daha önce belirttiğim gibi yaklaşık olarak 1 saatinizi alacaktır. Soruların doğru ya da yanlış cevapları yoktur; önemli olan sizin duygu ve düşüncelerinizdir. Bu araştırmada katılımcı bilgilerinin gizliliği esas tutulmaktadır. Katılımcı isimleri yerine anonim isimler kullanılacaktır. Görüşmenin video ve ses kayıtları araştırma projemiz süresince muhafaza edilip araştırma sona erdiğinde silineceklerdir. Bu görüşmeye katılmak tamamen isteğe bağlıdır. Katıldığınız takdirde görüşmenin herhangi bir aşamasında herhangi bir sebep göstermeden onayınızı çekmek hakkına da sahipsiniz. Başlamadan önce bana sormak istediğiniz bir soru var mı? Sorunuz yok ise başlayabiliriz. Ayrıca, lütfen ara vermek istediğinizde belirtiniz. Teşekkür ederim.

Mülakat Soruları

1. Lütfen, kendinizden bahsedin.
2. EREC-BGM' de matematik öğretme motivasyonunuz nedir?
3. Sıradan, normal bir ders anlattığınız günü düşündüğünüzde, sınıf ortamını ve öğrencilerinizi tasvir eder misiniz? Siz o derse nasıl hazırlandınız ve o dersi nasıl anlatıyorsunuz?
4. Matematik nasıl tanımlarsınız? (Matematik nedir ve matematik yapmak ne demektir?)
5. Matematik bilmek ne demektir?
6. Matematik öğrenmek ne demektir?
7. Matematik öğretmek ne demektir?

APPENDIX C: INDIVIDUAL CASE-BASED SEMI-STRUCTURED INTERVIEW PROTOCOL

The link of the video of the instruction implementing CME is provided in "https://thelearningexchange.ca/projects/teaching-mathematics-through-a-social-justice-lens/?pcat=999&sess=3".

C.1. Introduction Questions

1. How would you define the notion of "social justice"? Please provide examples.
2. Based on your definition of social justice, is it possible to implement social justice issues into mathematics instruction?
 - a. If yes, how? Considering a teacher who implements social justice issues into the class, what would be obstacles and contributing factors?
 - b. If no, why? What would be the obstacles and contributing factors?

C.2. First Part: Activating Prior Knowledge

The instructor begins the lesson by activating students' prior knowledge about the cost of good, minimum wage, and fair working conditions. Students take part in the lesson by collecting data on the origin of the shoes they are wearing, noting them on the world map, and analyzing the data at the end. The context is students' real-life and they work like a researcher collecting data, analyzing, and interpreting it.

After they discussed the political knowledge of mathematics based on the collected data, the lesson problem is introduced:

Workers make 40 cents in wages for each shoe they produce in China. Assume that they make one shoe each hour. Workers who make minimum wage in Ontario earn \$10.25 for each hour they work. Compare the wage of a worker in China and in Ontario if they both work 10 hours, 100 hours and 1000 hours.

Table C.1. First Part Questions.

Mathematical Task	
Critical aspect	1. How would you explain the structure/character of the introduction and instruction tasks in terms of its political and social aspects?
Mathematical aspect	2. What would be the mathematics topic of the instruction? 3. What would be the cognitive demand of mathematics task presented at the end?
Relevancy	4. How would you explain the introduction task in terms of the real life of the students? 5. How would you explain the instruction task in terms of the students' real life?
Implementation	
Dialogical learning- students	6. How would you interpret the way students communicate with each other and with the teacher?
Dialogical teaching- teachers	7. How would you describe the way the teacher begins the instruction and directs the discussion?

C.3. Second Part: Solving The Lesson Problem

Students explore an efficient way to solve the presented lesson problem in different ways. They work collaboratively with their partners, communicating mathematics ideas verbally and showing them on the posters. The teacher circulates around the class and asks probing questions to help students clarify their thinking through conceptual understanding. At the same time, the teacher assess students' understanding to build the remaining part of the lesson on students' understanding.

Table C.2. Second Part Questions.

Task	
Critical aspect	—
Mathematical aspect	8. How would you explain the mathematical methods, students use to learn this mathematical topic? 9. How would you explain the efficiency of students' method to solve the given problem?
Relevancy	10. Do you think that students are engaging in the mathematics task? • If yes, what would be the reason for students to be attentive to the mathematics task? • If no, what would be the reason for students to not engaging to the mathematics task?
Implementation	
Dialogical learning	11. How would you explain the structure/character of small group discussions? 12. What is the role of the students in their own learning process?
Dialogical teaching	13. How does the teacher direct the small group discussions? 14. How does the teacher use questioning?

C.4. Third Part: Accountable Talk

During the part, one of the members of each group visits another two groups to see others' strategies to solve the same problem while the other member of the group stays for presentation and feedback from their peers. Students circulate to share their ideas and listen to others. Seeing multiple ways to solve the problem, hearing different ideas, and comparing the works gives students an opportunity to fill the gaps in their understanding of the concept. The teacher also walks around the class and asks questions to make students' understanding clear and push it further. At the end of the section, students return to their original groups and rearrange their poster work, reflecting on other groups' work. They will present their last work to the whole class at the consolidation part.

Table C.3. Third Part Questions.

Task	
Critical aspect	—
Mathematical aspect	—
Relevancy	—
Implementation	
Dialogical learning	15. What would be the advantages/disadvantages of seeing different methods or ideas to solve the same problem?
Dialogical teaching	16. How would you interpret the discussion, which is held by the teacher and the students?

C.5. Fourth Part: Analysis of Students Work

In previous parts, the instructor directs small group and whole class discussions to ask questions and to understand the level of students' learning. Teachers choose the sample of students to work to be presented in the class, based on those problem solving and accountable learning parts. The works of the students are chosen according to the decision of which one would help students develop their understanding of the concept.

Table C.4. Fourth Part Questions.

Task	
Critical aspect	—
Mathematical aspect	17. What would be the criteria for the teachers to choose those specific students' works that will be presented at the conclusion part of the instruction?
Relevancy	—
Implementation	
Dialogical learning	—
Dialogical teaching	—

C.6. Fifth Part: Consolidation

Students come together as a whole class to discuss their new learning. The teacher then invites the selected math partners to share their work. They connect

their mathematics learning to a larger socio political constructs. The teacher directs the discussion asking probing questions to make students' understanding clear and helps them get a conceptual understanding as she did in all the sections. The discussions between the students and between the students and the teacher focus on the big idea of the mathematics concept. At the end, they generalize a rule interpreting and criticizing the works of all.

The instructor connects the result of the problem to the social justice issue, they discussed at the beginning of the lesson, about fair working conditions for workers. Students make suggestions to improve those unfair working conditions. The class is ended with a piece of advice about making consumption after investigating into how the firms treat their workers.

Table C.5. Fifth Part Questions.

Task	
Critical aspect	18. How would you describe the structure/character of the discussion in terms of its political and social aspects?
Mathematical aspect	19. How would you describe the structure/character of the discussions in terms of its mathematical aspects?
Relevancy	20. Do you think that students are engaging in the whole-class discussion? • If yes, what would be the reason for students to be attentive to the whole-class discussion? • If no, what would be the reason for students to not engaging to the whole-class discussion?
Implementation	
Dialogical learning	21. How do the students speak when they have opinions?
Dialogical teaching	22. How does the teacher answer students' questions? 23. How would you describe the way the teacher directs the whole group discussion and concludes the instruction?

C.7. Conclusion Questions

1. Do you think that the lesson is different from other mathematics lessons you were taught or have taught?
 - If yes, how was it different from them?
 - If no, what were the similarities?
2. What would be the advantages/disadvantages of this kind of learning for students?



APPENDIX D: INDIVIDUAL INTERVIEW QUESTIONNAIRE

VAKA-TABANLI YARI-YAPILANDIRILMIŞ BİREYSEL MÜLAKAT SORULARI

Tanışma Soruları

1. “Sosyal adalet” kavramı ne demektir? Örnek vererek tanımlayınız.
2. Yaptığınız bu tanım üzerinden düşünürseniz, sosyal adalet konularını matematik derslerine uygulamak mümkün müdür?
 - Yanıtınız evet ise, nasıl? sınıfa sosyal adalet konularını uygulayan bir öğretmen düşünüldüğünde, engelleyen ve katkıda bulunan faktörler neler olurdu?
 - Yanıtınız hayır ise, neden? Engelleyen ve katkıda bulunan faktörler neler olabilir?

Birinci Bölüm Soruları

1. Politik ve sosyal yönleri dikkate alındığında derse giriş ve ders konu etkinliklerini nasıl açıklarsınız?
2. Bu derste hangi matematik konusu işlenecek olabilir?
3. Birinci bölümün sonunda verilen matematik konu etkinliğinin bilişsel talebi ne olabilir?
4. Öğrencilerin gerçek yaşamı dikkate alındığında yapılan derse giriş etkinliğini nasıl açıklarsınız?
5. Öğrencilerin gerçek yaşamı dikkate alındığında verilen matematik konu etkinliğinin nasıl açıklarsınız?
6. Öğrenci-öğrenci ve öğretmen-öğrenci etkileşimlerini nasıl yorumlarsınız?
7. Öğretmenin derse başlama ve tartışmayı yönetme metotlarını nasıl tanımlarsınız?

İkinci Bölüm Soruları

8. Öğrencilerin kullandığı öğrenme metotlarının bu matematik konusunu öğrenmede uygunluğunu nasıl açıklarsınız?
9. Öğrencilerin kullandığı öğrenme metotlarının verilen matematik problemini çözmek için uygunluğunu nasıl açıklarsınız?
10. Öğrencilerin verilen matematik problemini çözmeye katılım sağladığını düşünüyor musunuz?
 - Yanıtınız evet ise, katılım sağlamalarının sebepleri neler olabilir?
 - Yanıtınız hayır ise, katılım sağlamamalarının sebepleri neler olabilir?
11. Küçük grup tartışmalarının yapısını nasıl açıklarsınız?
12. Öğrencilerin kendi öğrenme sürecindeki rolü nedir?
13. Öğretmen soru sorma metodunu nasıl kullanmaktadır?
14. Öğretmen küçük grup tartışmalarını nasıl yönetmektedir?

Üçüncü Bölüm Soruları

15. Aynı problemi çözmek için farklı metot ve fikirleri görmenin avantajları/dezavantajları ne olabilir?
16. Öğretmen ve öğrenciler arasındaki sınıf içi tartışmayı nasıl tanımlarsınız?

Dördüncü Bölüm Soruları

17. Öğretmenlerin dersin son kısmında sunulmak üzere seçtikleri öğrenci çalışmalarında ölçüt ne olabilir?

Beşinci Bölüm Soruları

18. Politik ve sosyal boyutu dikkate alındığında sınıf içi yapılan tartışmanın yapısını nasıl açıklarsınız?
19. Matematiksel boyutu dikkate alındığında sınıf içi yapılan tartışmanın yapısını nasıl açıklarsınız?
20. Öğrencilerin bütün sınıf tartışmasına katılım sağladığını düşünüyor musunuz?
 - Yanıtınız evet ise, katılım sağlamalarının sebepleri neler olabilir?
 - Yanıtınız hayır ise, katılım sağlamamalarının sebepleri neler olabilir?
21. Öğrenciler fikirleri olduğunda nasıl ifade etmektedirler?
22. Öğretmen öğrencilerin sorularını nasıl cevaplamaktadır?
23. Öğretmenin tartışmayı yönetme ve dersi bitirme yöntemlerini nasıl tanımlarsınız?

Kapanış Soruları

1. İzlediğiniz bu ders daha önce size öğretilen veya sizin öğrettiğiniz matematik derslerinden farklı mıydı? Yanıtınız evet ise, nasıl?
2. Öğrenciler için bu tür bir öğrenmenin avantajları/dezavantajları neler olabilir?

Figure D.1. Individual Interview Questionnaire.

APPENDIX E: PARTICIPANT CONSENT FORM

KATILIMCI BİLGİ ve ONAM FORMU

Araştırmanın adı: Gönüllü Matematik Öğreten Öğretmen Adaylarının Eleştirel Matematik Eğitimi Farkındalıklarının İncelenmesi

Sayın katılımcı,

Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölümü öğretim üyesi Dr. Fatma Aslan-Tutak'ın danışmanı olduğu "Gönüllü Matematik Öğretmenliği Yapan Üniversite Öğrencilerinin Eleştirel Matematik Eğitimi Farkındalıklarının İncelenmesi" adı altında yüksek lisans tez çalışmamı yürütmekteyim. Bu çalışmanın amacı katılımcıların eleştirel matematik eğitimi farkındalıklarını incelemektir. Kararınızdan önce araştırma hakkında sizi bilgilendirmek istiyoruz. Bu bilgileri okuduktan sonra araştırmaya katılmak isterseniz lütfen bu formu imzalayıp bize ulaştırınız.

Bu araştırmaya katılmayı kabul ettiğiniz takdirde açık uçlu soruların olduğu bir hedef grup görüşmesi yapılacaktır. Bu görüşme Zoom üzerinden yapılacaktır ve yaklaşık olarak yarım saatinizi alacaktır.

İkinci olarak, sizden örnek eleştirel matematik eğitimi ders videosu üzerinden konuşacağımız 27 soruluk bir mülakat yapacağız. Bu mülakat çevrimiçi platformda, Zoom üzerinden sesli ve görüntülü görüşme ile yapılacaktır, kayda alınacaktır. Bu mülakat anket soruları tamamlandıktan sonra yapılacak ve yaklaşık olarak 1.5 saat sürecektir. Bu sürenin yaklaşık 40 dakikasını izlenilecek video bölümleri oluşturmaktadır.

Bu araştırma bilimsel bir amaçla yapılmaktadır ve katılımcı bilgilerinin gizliliği esas tutulmaktadır. Katılımcı isimleri yerine anonim isimler kullanılacaktır. Görüşmelerin video ve ses kayıtları araştırma projemiz süresince muhafaza edilip araştırma sona erdiğinde silineceklerdir.

Bu araştırmaya katılmak tamamen isteğe bağlıdır. Katıldığınız takdirde çalışmanın herhangi bir aşamasında herhangi bir sebep göstermeden onayınızı çekmek hakkına da sahipsiniz. Araştırma projesi hakkında ek bilgi almak istediğiniz takdirde lütfen araştırmacı Hatice Gamze İnan ile temasa geçiniz.

Eğer bu araştırma projesine katılmasını kabul ediyorsanız, lütfen bu formu imzalayıp bize geri yollayınız.

Ben, (katılımcının adı), yukarıdaki metni okudum ve katılmam istenen çalışmanın kapsamını ve amacını, gönüllü olarak üzerime düşen sorumlulukları tamamen anladım. Çalışma hakkında soru sorma imkanı buldum. Bu çalışmayı istediğim zaman ve herhangi bir neden belirtmek zorunda kalmadan bırakabileceğimi ve bıraktığım takdirde herhangi bir olumsuzluk ile karşılaşmayacağımı anladım.

Bu koşullarda söz konusu araştırmaya kendi isteğimle, hiçbir baskı ve zorlama olmaksızın katılmayı kabul ediyorum.

Formun bir örneğini aldım / almak istemiyorum (bu durumda araştırmacı bu kopyayı saklar).

Katılımcının Adı-Soyadı:.....
 İmzası:.....
 Adresi (varsa Telefon No, Faks No):.....
 Tarih (gün/ay/yıl):...../...../.....

Araştırmacının Adı-Soyadı:.....
 İmzası:.....
 Tarih (gün/ay/yıl):...../...../.....

Figure E.1. Participant Consent Form.