ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

ERROR ANALYSIS ON THE PARAMETERS OF THE MATERIAL BALANCE EQUATIONS FOR A SYNTHETIC GAS CAP RESERVOIR

M.Sc. THESIS

Turgay İNCEİŞÇİ

Department of Petroleum and Natural Gas Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

SENTETİK BİR GAS BAŞLIKLI REZERVUARDA MATERYAL DENGE DENKLEMİ PARAMETRELERİNDEKİ HATA ANALİZİ

YÜKSEK LİSANS TEZİ

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MAYIS 2014

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FOREWORD

In this thesis, the accuracy of the material balance calculations due to errors in PVT data, production data and the ratio of the initial gas cap volume to the oil volume was investigated on a hypothetical gas cap reservoir which was constructed and simulated using 3D simulators. The effects of different parameters on the errors in the calculated Original Oil in Place values were examined by introducing systematic error to only one parameter while taking the others at their true value.

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ABBREVIATIONS

B_g : Gas Formation Volume Factor, rb/scf
B_{gi} : Initial Gas Formation Volume Factor, rb/scf
B_{ginj} : Injected Gas Formation Volume Factor, rb/scf
B_o : Oil Formation Volume Factor at P, rb/STB
B_{oi} : Initial Oil Formation Volume Factor, rb/STB
B_w : Water Formation Volume Factor at P, rb/STB
c : Isothermal Compressibility, psi^{-1}
c_f : Formation (rock) Compressibility, psi^{-1}
c_t : Total Compressibility, psi^{-1}
c_w : Water Compressibility, psi^{-1}
E_g : Expansion of the Gas Cap Gas, rb/STB
$E_{f,w}$: Expansion of the Initial Water and the Reduction in the Pore
E_o : Expansion of Oil and Its Originally Dissolved Gas, rb/STB Volume, rb/STB
E_{ow} : Expansivity, rb/STB
E_t : Total Fluid Expansion, rb/STB
F : Underground Withdrawal, rb
GIIP : Gas Initially in Place
G_{inj} : Cumulative Gas Injected, scf
GOC : Gas Oil Contact
G_p : Cumulative Gas Produced, scf
HO : Havlena-Odeh
m : Ratio of the Initial Gas Cap Volume to the Initial Oil Volume, rb/rb
MB : Material Balance
MBE : Material Balance Equation
Mscf : Thousand Standart Cubic Foot
N : Original Oil in Place, STB
N_p : Cumulative Oil Produced, STB
OGIP : Original Gas in Place
OHIP : Original Hydrocarbon in Place
OIIP : Oil Initially in Place
OOIP : Original Oil in Place
OWC : Oil Water Contact
P : Average Reservoir Pressure at Time t, psi
P_i : Initial Reservoir Pressure, psi
ppm : Parts per Million
PVT : Pressure Volume Temperature
rb : Reservoir Barrels
RF : Recovery Factor
R_p : Producing Gas-Oil Ratio, scf/STB
R_s : Dissolved Gas-Oil Ratio at P, scf/STB
R_{si} : Initial Dissolved Gas-Oil Ratio, scf/STB

scf : Standard Cubic Feet
SPE : Society of Petroleum Engineers
STB : Stock Tank Barrel
$\mathbf{S_{wi}}$: Initial Water Saturation, fraction
$\mathbf{W_D}$: Dimensionless water influx
$\mathbf{W_e}$: Cumulative Water Influx From Aquifer, rb
$\mathbf{W_{inj}}$: Cumulative Water Injected, STB
$\mathbf{W_p}$: Cumulative Water Produced, STB
ΔP : Initial Minus Prevailing Pressure (P_i-P), psi
ΔV : Expansion of the Material due to Change in Pressure, unit of volume
Ø : Sensitivity Coefficient

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ERROR ANALYSIS ON THE PARAMETERS OF THE MATERIAL BALANCE EQUATIONS FOR A SYNTHETIC GAS CAP RESERVOIR

SUMMARY

Nowadays, for modeling reservoirs, the petroleum industry uses numerical simulators, which are basically multi-dimensional, multi-phase, dynamic material balance programs. The advancement of computer hardware and software technology and development of numerical solution methods have been also the factor for the increased use of reservoir simulation in petroleum industry. On the other hand, the application of the classical material balance approach is very simple and it provides a valuable insight into the behaviour of hydrocarbon reservoirs. Material balance methods, especially straight-line material-balance plots, are still routinely used in analyzing reservoir performance and estimate the OOIP and OGIP.

Either the classical material balance approach or numerical simulations use the same type of data to analyze the performance of the reservoirs and these data have uncertainty due to various reasons. Since data uncertainty is unavoidable, its effects should be carefully analyzed before making an important desicion. Data uncertainty in material balance techniques includes error in the measurement of laboratory pressure-volume-temperature (PVT) properties, field production data, aquifer properties and average reservoir pressures. Although the effect of pressure errors on material balance and the errors in water influx calculations have been extensively studied, the discussion of the effect of PVT errors on material balance calculations in the petroleum engineering literature is rather limited. Due to the lack of data, very often non-representative PVT correlations are selected for use or incorrect PVT data are used in material balance calculations.

In this thesis, the accuracy of the material balance calculations due to errors in PVT data were mainly investigated on a hypothetical gas cap reservoir which was constructed using 3D simulators. The possible errors in the production data and in the ratio of the initial gas cap volume to the initial oil volume (m) were also analyzed. A systematic error of +/-2% was added to each PVT parameter (R_s, R_{si}, B_o, B_{oi}, B_g and B_{gi}) to account for typical laboratory error. The amount of systematic error applied was -10% for production data (N_p and G_p) and +/-50% for the m value. Material balance calculations with different methods were performed using the erroneous data and the errors on the original oil in place (N) values were calculated according to the base case value of the output of simulator. The effects of different parameters on the errors in calculated N values were examined by introducing systematic error to only one parameter while taking the others at their true value.

Sensitivity coefficients are analyzed to provide explainations of the behavior of the errors with time.

The results of the study showed that impact of PVT errors on material balance calculations can be very significant if the recovery factor of the reservoir is quite small. However, the errors on the calculated values of N is approaching to zero for all PVT cases studied towards the end of the production period. It was observed that the effects of the errors in production data were relatively low compared to the results of the PVT data analysis. The effect of the error in N_p in N calculations was lower than the effect of G_p. It was seen that the estimation of m value 50% higher than the orginial value m causes the N values 20 % less than the base value and the estimation of m value 50 % more yields +40% error in the calculated values of N.

SENTETİK BİR GAS BAŞLIKLI REZERVUARDA MATERYAL DENGE DENKLEMİ PARAMETRELERİNDEKİ HATA ANALİZİ

ÖZET

Günümüzde çok boyutlu, çok fazlı dinamik materyal denge programları olan nümerik simulatörler petrol endüstrisinde yaygın olarak kullanılmaktadır. Petrol endüstrisindeki nümerik simulatörlerin kullanımının yaygınlaşmasında bilgisayar donanımlarındaki, yazılım teknolojilerindeki ve ayrıca nümerik çözüm yöntemlerindeki gelişmeler etkendir. Diğer taraftan, klasik materyal denge teknikleri uygulamada basit olup, hidrokarbon rezervuarlarının davranışlarının içyüzünü anlamada değerli fikirler sunmaktadır. Materyal denge teknikleri ve özellikle doğrusal materyal denge grafikleri rezervuar performanslarının analizinde, itim mekanizmalarının belirlenmesinde ve en önemlisi yerinde petrol ve gaz miktarlarının hesaplanmasında rutin olarak kullanılmaktadır.

Hem klasik materyal denge yaklaşımı hem de nümerik simülasyonlar rezervuarın performansını analiz etmek için ayni tip verileri kullanmakta ve bu veriler bazı sebeplerden ötürü belirsizlikler içermektedir. Veriler üzerindeki belirsizlikler kaçınılmaz olduğundan, önemli kararlar alınmadan önce belirsizliklerin etkileri dikkatlice araştırılmalıdır. Materyal denge tekniklerindeki veri belirsizlikleri laboratuvar basınç-sıcaklık-hacim (PVT) özelliklerinde, üretim verilerinde, akifer özelliklerinde ve ortalama rezervuar basıncında bulunabilir. Yeterli veri olmaması nedeniyle materyal denge hesaplamalarında rezervuar koşullarına uygun PVT korelasyonları kullanım için seçilmekte veya hesaplamalarda bazen yanlış PVT verileri kullanılmaktadır. Yeterli sayıda kuyudan kuyudibi basınç ölçümü yapılamaması veya yapılan kuyu testlerinin dizaynlarının iyi yapılamaması (üretim kaybı ve dolayısıyla gelir kaybını azaltmak için kuyuların test sürelerinin gerekenden az tutulması) veya kuyu sağım alanlarının doğru şekilde belirlenememesi, rezervuarı temsil eden ortalama basınç değerlerinin doğru olarak hesaplanamamasına neden olabilmektedir. Bunun yanı sıra, rezervuara komşu olan akiferin tipinin ve akifer özelliklerinin doğru tahmin edilememesi rezervuara giren su miktarının hesaplanmasında yanlış sonuçlar verebilmektedir. Materyal denge üzerindeki basınç belirsizliği ve rezervuara su girişi (W_e) hesaplamalarındaki hatalar geniş olarak çalışıldığı halde, materyal denge hesaplamalarındaki PVT hatalarının etkileri ile ilgili petrol mühendisliği literatüründeki tartışmalar daha sınırlıdır.

Bu tezdeki asıl amaç, 3B simülatörler kullanılarak oluşturulan sanal bir gaz başlıklı rezervuarda hatalı PVT verileri ile yapılan materyal denge hesaplamalarından elde edilen yerinde petrol miktarı miktarı (OIIP veya N) üzerindeki hataların büyüklüklerinin araştırılmasıdır. Üretim verilerindeki ve başlangıç gaz başlığı hacminin başlangıç petrol hacmine oranı olan m değerindeki muhtemel hataların yerinde petrol miktarı üzerindeki etkileri ayrıca analiz edilmiştir.

Tipik laboratuvar hatasını yansıtmak için her bir PVT parametresine (R_s, R_{si}, B_o, B_{oi}, B_g ve B_{gi}) % +/-2 sistematik hata eklenmiştir. R_s ile R_{si}, B_o ile B_{oi} ve B_g ile B_{gi} çiftlerinin her birinde % +/-2 hata olduğu varsayılarak yerinde petrol miktarı üzerindeki hatalar ayrıca analiz edilmiştir. Petrol üreticilerinin gelirlerini petrol ve gazın satışından elde etmeleri üretilen petrol ve gazın mümkün olan en doğru şekilde ölçülmesini zorunlu kılmıştır. Günümüzde kullanılan ölçüm ekipmanları geçmişe göre daha teknolojik olsalar da ölçtükleri değerlerde sistematik hata olma olasılığı her zaman bulunmaktadır. Üretim verilerindeki (N_p ve G_p) hataların yerinde petrol miktarı üzerindeki etkilerini araştırabilmek için, simülatörden elde edilen üretim verilerinin % 10 düşük ölçüldüğü varsayımı yapılmıştır. Başlangıç gaz başlığı hacminin başlangıç petrol hacmine oranı olan m değeri, özellikle ilk üretim safhalarındaki en belirsiz parametredir. Çalışmada kurulan model sentetik model olduğu için m değeri kesin olarak bilinmektedir. Ancak m değerindeki hataların hesaplanan yerinde petrol miktarı üzerindeki etkisini araştırabilmek için m değerinin %+/-50 hatalı hesaplandığı varsayılmıştır.

Materyal denge denkleminin doğrusal olarak ifade edilmesi ile elde edilen doğrusal materyal denge yöntemleri ile rezervuarın itim mekanizmalarına göre farklı şekilde doğru denklemleri üretilmekte ve materyal denklemindeki bilinmeyenler olan N, m ve W_e değerleri hesaplanabilmektedir. Bu çalışmada oluşturulan rezervuar modeli rezervuara su girişinin olmadığı gaz başlıklı bir rezervuardır. Doğrusal materyal denge denklemlerinin bu modele uyarlanması ile iki farklı yöntemle N hesaplanmıştır. Birinci durumda m değerinin bilindiği ancak N değerinin bilinmediği, ikinci durumda ise m ve N değerlerinin bilinmediği varsayılmıştır. Hata eklenmiş parametreler kullanılarak, materyal denge denklemi ve iki farklı doğrusal materyal denge denklemi ile yerinde petrol miktarı hesaplamaları yapılmıştır. Simülatöre hesaplattırılan yerinde petrol miktarı değeri baz alınarak N üzerindeki hatalar hesaplanmıştır. Farklı parametrelerin hesaplanan N değerleri üzerindeki etkilerini incelemek için etkisi araştırılan parametre veya parametrelerin değerine sistematik hata eklenirken diğer tüm parametreler kendi doğru değerlerinde bırakılmıştır.

Farklı parametrelerdeki hataların yerinde petrol miktarı üzerindeki etkilerini anlamak ve materyal denge hesaplamalarındaki hata analizi sonuçlarını doğrulamak için duyarlılık katsayıları ile hata analizi ayrıca tartışılmıştır. Duyarlılık katsayıları ile hangi parametrenin yerinde petrol miktarı üzerindeki etkisinin daha fazla olduğu ve grafiklerle yapılan analizlerde grafiklerin davranışları açıklanabilmiştir.

Çalışmanın sonuçları materyal denge metodları ile hesaplanan yerinde petrol miktarının, denklemlerdeki hatalı parametrelerden çok etkilendiğini göstermiştir. Özellikle rezervuardaki kurtarım faktörünün çok düşük olduğu durumlarda materyal denge hesaplamalarındaki PVT hatalarının etkilerinin çok önemli olduğu görülmüştür. Oluşturulan sentetik gaz başlıklı rezervuar modeli için PVT parametreleri içerisindeki en etkin parametrenin B_{oi} olduğu ortaya çıkmıştır. B_{oi}, B_{gi} ve R_s değerlerine pozitif hata eklenmesi durumunda N değerlerinin arttığı; B_o, B_g ve R_{si} değerlerine pozitif hata eklenmesi durumunda ise N değerlerinin azaldığı gözlenmiştir. Benzer şekilde, B_{oi}, B_{gi} ve R_s değerlerine negatif hata eklenmesinin N değerlerinin azalmasına neden olduğu; B_o, B_g ve R_{si} değerlerine negatif hata eklenmesinin ise hesaplanan N değerlerinin referans alınan değerden yüksek olmasına neden olduğu görülmüştür. Bununla beraber, hesaplanan N değerleri üzerindeki hataların çalışılan tüm PVT durumlarında üretim periyodunun sonuna doğru sıfıra yaklaştığı görülmüştür. Hatalı parametreler kullanılarak materyal denge

denklemi ile yapılan hesaplamaların doğrusal materyal denge yöntemlerine göre daha doğru sonuçlar verdiği gözlenmiştir. PVT hata miktarlarına göre daha yüksek hatalar eklenmesine rağmen, üretim verileri ve m değeri üzerindeki hataların etkileri PVT parametreleri üzerindeki hata etkileri ile karşılaştırıldığında, üretim verileri ve m değerindeki hatalarla hesaplanan N değerlerindeki hataların PVT parametrelerindeki hatalarla hesaplanan N değerlerinden çok daha düşük olduğu gözlenmiştir. Ayrıca, üretim verilerindeki hataların N üzerindeki etkilerinin, PVT parametrelerindeki hataların N üzerindeki hataya etkilerinin aksine, rezervuardan yapılan üretim miktarı arttıkça azalmadığı, gaz üretimindeki artışa bağlı olarak değişim gösterdiği gözlenmiştir. Ancak, m değerindeki hataya bağlı hesaplanan N değerlerindeki hatanın üretim periyodu boyunca çok fazla değişmediği gözlenmiştir. Petrol üretimi (N_p) üzerindeki hataların etkilerinin gaz üretimi (G_p) üzerindeki hatalara göre daha az olduğu tespit edilmiştir. Oluşturulan senteki modelde, m değerinin % 50 fazla tahmin edilmesinin, N değerlerinin baz alınan değere göre % 20 daha az olmasına neden olduğu; % 50 daha az tahmin edilmesinin ise hesaplanan N değerinin % 40 daha fazla olmasına neden olduğu görülmüştür.

PVT hataları içeren N değerindeki hataların davranışlarının duyarlılık katsayıları davranışları ile aşağı yukarı aynı parallellikte olduğu gözlenmiştir. Duyarlılık katsayısı en yüksek parametrenin B_{oi} olduğu tespit edilmiştir.

1. INTRODUCTION

The material balance equation has long been regarded as one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance. The material balance equation (MBE) can be used to estimate initial hydrocarbon volumes in place or to predict future reservoir performance or to predict ultimate hydrocarbon recovery under various types of primary driving mechanisms. Nowadays, numerical simulators are used for modeling reservoirs, which are basically multi-dimensional, multi-phase, dynamic material balance programs. The advancement of computer hardware and software technology and development of numerical solution methods have been also the factor for the increased use of reservoir simulation in petroleum industry. On the other hand, the classical material balance approach is very simple and it provides a valuable insight into the behaviour of hydrocarbon reservoirs. It is necessary to accept that material balance is complementary to simulation.

The material balance (MB) calculations require production/injection data, pressure, PVT data, aquifer properties in order to quantify the original oil in place (OOIP or N) and drive mechanisms. OOIP calculations based on material balance calculations are strongly affected by data uncertainty. Uncertainty due to data errors can be found in the above mentioned data seperately or together. Usually it is expected that production data (oil or gas) are measured with confidence since the measurement equipments are highly technologic and can be trusted as compared to the past. Another point is that the companies gain their revenues from the sale of oil or gas, accordingly, oil and gas production are in general measured quite accurately and the errors in production data are small. Reservoir pressure is more uncertain due to lack of pressure measurements from the wells in the field to find an average value of reservoir pressure. PVT data can also be uncertain since very often PVT correlations which may be non-representative or incorrect PVT data are selected for use in material balance calculations. PVT data can include errors since it may not be possible to have a representative fluid sample of the reservoir. Laboratory measurements can also have errors. Although the effect of pressure errors on material

balance has been extensively studied, there is very little discussion of the effect of PVT errors on material balance calculation in the petroleum engineering literature.

1.1 Main Objective

In this thesis, as well as the the effect of errors in PVT parameters, the effect of production data errors and the error in the ratio of the initial gas cap volume to the initial oil volume on MB calculations were investigated for a saturated synthetic gas cap reservoir created using the Petrel 2009.1 software. Systematic errors of +/-2% were introduced into the reservoir PVT properties. -10% systematic error was introduced for N_p and G_p and finally +/-50% error for m values. The systematic error was introduced in such a way that one parameter contained error while all other data were held constant at their true values. Eclipse 2009.1 black-oil simulator was used to produce all necessary data for material balance calculations. A MATLAB 2008 based program was coded for the calculation of OOIP values with Havlena and Odeh (1963, 1964) technique.

Systematic percentage errors on OOIP were calculated according to the formula,

$$\%\text{Error} = \frac{\text{Trial value} - \text{Reference value}}{\text{Reference value}} * 100 \qquad \textbf{(1.1)}$$

Eclipse 2009.1 output for OOIP was chosen as reference value in error calculations. The results of all works were illustrated graphically. The direction of change (increase or decrease) in OOIP due to the systematic errors in the data was also analysed to see how OOIP is affected.

1.2 Outline of the Thesis

This thesis is divided into six main parts including this first part.

In the first part of the thesis, a brief introduction will be given about the need for analyzing of the uncertainty of the parameters used in material balance equation and their concequences on the calculations of OOIP.

In the second chapter, the literature survey on the topic selected will be presented briefly. The methodology of analyzing uncertainty will be given at the end of the chapter.

In the third part, a general theory of material balance equation will be given. The material balance equation for gas cap reservoir without water injection and water influx case will be derived. The Havlena-Odeh straight line methods will be presented for the reservoir described above.

In the fourth part of the thesis, a synthetic gas cap reservoir will be described in detail. The construction of 3D geological model and the simulation model will be explained in detail. The grid sensitivity study and the simulation results will be presented.

In the fifth part of the thesis, error analysis of material balance parameters; such as PVT data, production data and m value will be studied to show the effects of the uncertainty of each parameter on the results of material balance calculations of N.

The last chapter will cover the conclusions derived from the study.

2. STATEMENT OF THE PROBLEM

Data quality is very important in material balance calculations. Material balance calculations may include data errors which can be found in PVT data, production data, geological data (m value) and average reservoir pressure. Production data errors are usually expected to be less than the others, since revenues of the companies are based on oil and gas sales and hence their measurements were performed with minimal errors. So, it is expected that production data errors should be minimal in material balance calculations. Average reservoir pressure is expected to be uncertain due to the lack of enough well measurements and heterogeneties throughout the reservoir. Similarly, PVT properties preferably are determined from laboratory studies conducted on the samples which are collected from the bottomhole or on the recombined samples collected from surface facility tanks. However, PVT analysis of these samples are not always available due to some reasons: a) samples collected are not reliable enough to use them in MB calculations, b) there may be no samples due to cost saving purposes. In such cases PVT properties are obtained by using emprical correlations (Ghetto et al.,1994). Although the effect of pressure errors on material balance has been widely studied, there is very little discussion of the effect of PVT errors and production data errors on material balance calculation in the petroleum engineering literature.

2.1 Literature Survey

The effects of different parameters on material balance calculations have been studied by several investigators. These studies are mainly on:

- Pressure uncertainty
- The influence of drive mechanism
- The reliability of emprical PVT correlations
- PVT uncertainty effects

- The effect of data errors on typical reservoir engineering calculations

Charles (1951) studied the effect of data errors on typical reservoir engineering calculations. He notes that important parameters such as reservoir pressure, oil in place and well damage are calculated directly from measured quantities and each measurement error has an effect on the calculated results. Material balance calculations need data which are all measured or taken from emprical correlations. Hence it is very crucial to use them as correct as possible to minimize the errors on material balance calculations.

McEwen (1961) studied pressure uncertainy for water influx reservoirs and his results showed that material balance calculations are not reliable when there is even small uncertainty in pressure. He mentioned that Havlena and Odeh's method of plotting F/E_o vs. $\sum \Delta p W_D/E_o$ to find the OOIP and aquifer constant in initially-undersaturated, water-drive oil reservoirs is not reliable if the rereservoir pressure has much uncertainty. To make the results better, he recommended plotting F vs. E_{ow}, where

$$E_{ow} = E_o + \frac{2B_{oi}c_t \sum \Delta p W_D}{1 - S_{wi}} \qquad \textbf{(2.1)}$$

for radial aquifers.

Tehrani (1976) reaarranged the material balance equation in different forms for a reservoir with water influx with small pressure uncertainty. He mentioned that material balance errors can be introduced thorough reaarranging a MBE from a 3-variable equation to a 2-variable equation. His work showed that these forms of the material balance equations were not applicable if there is uncertainty in pressure and some of them are very influenced by small pressure uncertainty.

Ghetto et al (1994) studied the reliability of emprical PVT correlations for the estimation of bubble point pressure, oil formation volume factor and solution gas oil ratio at bubble point pressure, dead oil viscosity, gas-saturated oil viscosity, undersaturated oil viscosity and isothermal compressibility. He suggested some modified emprical correlations to improve the reliability of the correlations used in the petroleum literature.

Wang and Hwan (1997) studied the influence of drive mechanisms on the OOIP estimates. According to them, the estimation of OOIP for reservoirs with water influx is more difficult than volumetric reservoirs. For a strong water drive reservoir, water influx can be estimated more accurately than OHIP. They also mentioned that if the ratio of the initial gas cap volume to the initial oil volume (m) is known, OOIP can be certainly estimated for a gas cap reservoir. However, if m is unknown, the calculation of OOIP is not reliable for high values of m (greater than 0.25) even without any water influx. On the other hand, OGIP can be calculated much more accurate than OOIP for a gas cap reservoir.

Walsh (1999) examined the effects of pressure errors on material balance plots for volumetric reservoirs and his results showed that pressure uncertainty can considerably affect the reliability of certain plots. He found that Havlena and Odeh graphical methods for reservoirs with initial gas cap have no tolerance to pressure uncertainty.

Baker et al. (2003) studied PVT uncertainty effects on material balance estimates. He found that the impact of PVT uncertainty on material balance estimates can be significant if the decline of reservoir pressure is small or if the used PVT correlation is not adjusted to field data. His results also indicated one of the reasons why a reservoir should have a significant amount of production and pressure drop before it is used for MBE calculations.

Carlos and Jose (2007) have shown the combined effects of PVT and pressure uncertainties on the material balance estimates for reservoirs wtih different drive mechanisms. Their results showed that the method F-W_e vs. E_t is the most usable method when pressure and PVT data have uncertainty. Another conclusion they presented was that the method (F-W_e)/(E_o+$E_{f,w}$) vs. (E_g+$E_{f,w}$)/(E_o+$E_{f,w}$) shows hypersensitivity to all uncertainty ranges in pressure and PVT data. However, the method F/E_t vs. W_e/E_t shows hypersensitivity to uncertainty of pressure data and moderate sensitivity to uncertainty in PVT data. According to them, pressure errors are the main source of uncertainty in material balance estimates. İn their models, R_s errors are the main source of uncertainty of PVT data in material balance estimates.

2.2 Methodology

This work focused on analyzing the errors on initial oil in place when errors are present in PVT properties of the fluid, production data or m value. A geological model for a syntethic square shaped reservoir was created with Petrel 2009.1. Grid size sensitivity was performed to find the ideal grid size that minimizes the discretization errors. Porosity, horizontal and vertical permeability values were taken as constant in the model to be able to check the results with hand calculations. The initial water saturation (S_{wi}), formation compressibility and capillary pressure were set to zero for simplicity. End points for relative permeability vs saturation data were given to Petrel 2009.1. PVT data tables were constructed according to the correlations within Petrel 2009.1.

One well of 700 STB/day production rate was completed at the bottom half of the oil zone to ensure that the effects of gas coning were eliminated. A 40-year development strategy (starting from 01.01.2013) was defined for the production well where an abondonment pressure for the well was chosen as 1000 psi and the minimum production rate of the well was set to 100 STB/day. Injection or adjacent aquifer support were not considered to improve the production. Grid properties, fluid PVT model, rock physics functions (relative permeability data and rock compressibility) and development strategy were imported to Eclipse 2009.1 simulator. The dynamic model was run for 40-year production period. The reporting time step was set to one year. The production data, pressure data and PVT data were recorded for 40 years annually.

In the first part of the thesis, material balance calculations for OOIP were performed by MB equation and Havlena and Odeh technique with the Eclipse 2009.1 yearly outputs.

The second part covers the error analysis on PVT parameters, production data and m value. All PVT parameters (R_{si}, R_s, B_{oi}, B_o, B_{gi} and B_g), production data (N_p and G_p) and the ratio of the initial gas cap volume to the initial oil volume (m) will be analyzed individually. Sensitivity coefficients were calculated for each error introduced parameter at each time step.

Using synthetic field with known dimensions and known porosity values enables us to calculate the original oil in place volume accurately by hand. Assuming that all

data produced from the simulator are true and there is no uncertainty on the data, the effect of errors introduced to the parameters will show how OOIP values are effected by the introduction of systematic errors. The results are compared with reference value of OOIP volume.

3. MATERIAL BALANCE THEORY

The material balance equation (MBE) has been used as one of the basic tools of reservoir engineers for many years for interpreting and predicting reservoir performance. The MBE can be used to estimate initial hydrocarbon volumes in place, to predict future reservoir performance and to predict ultimate hydrocarbon recovery under various types of drive mechanisms.

The equation is simply based on the principle of conservation of mass. The concept of the material balance equation was first developed by Schilthuis (1936). It is a volume balance between an underground withdrawal (cumulative production) and the expansion of the reservoir fluids resulting from a pressure drop in the reservoir and can be expressed in its simplest form as:

$$\begin{pmatrix}\text{Volume of}\\ \text{fluids}\\ \text{originally}\\ \text{in place}\end{pmatrix} = \begin{pmatrix}\text{Volume of}\\ \text{fluids}\\ \text{remained}\\ \text{in the}\\ \text{reservoir}\end{pmatrix} + \begin{pmatrix}\text{Volume of}\\ \text{produced}\\ \text{fluids}\end{pmatrix}. \qquad \textbf{(3.1)}$$

3.1 Basic Assumptions in the MBE

The basic assumptions in the material balance equation (MBE) are as follows:

Constant temperature: Changes in a reservoir generally are accepted to be at isothermal (constant temperature) conditions. Pressure-volume changes in the reservoir are assumed to be isothermal. If any temperature changes occur, they are usually small enough to be ignored without significant error.

Hydraulic connectivity: All parts of the reservoir are assumed to be hydraulically connected.

Constant reservoir volume: Reservoir volume is assumed to be constant and the formation is considered to be sufficiently competent that no significant volume change will occur through movement or reworking of the formation due to overburden pressure as the internal reservoir pressure is reduced.

Reliable production data: It is important in the application of the MB calculations to have reliable production data of oil, gas and water.

Dake (1978) points out the following characteristics regarding the material balance equation:

- it is zero dimensional which means that it is evaluated at a point in the reservoir,
- its parameters generally are not time dependent except the water influx term,
- the pressure appears explicitly in the water and pore compressibility term. However, it is implicit in the PVT parameters since B_o, R_s and B_g are functions of pressure. The water influx is also pressure dependent,
- the equation is always evaluated by comparing the current volumes at pressure p to the original volumes at p_i.

3.2 Derivation of MBE

Considering the reservoir as a tank model as illustrated in Figure 3.1., volumetric balance expressions can be derived to take into account all volumetric changes which occurs during the withdrawal of the fluids from the reservoir.

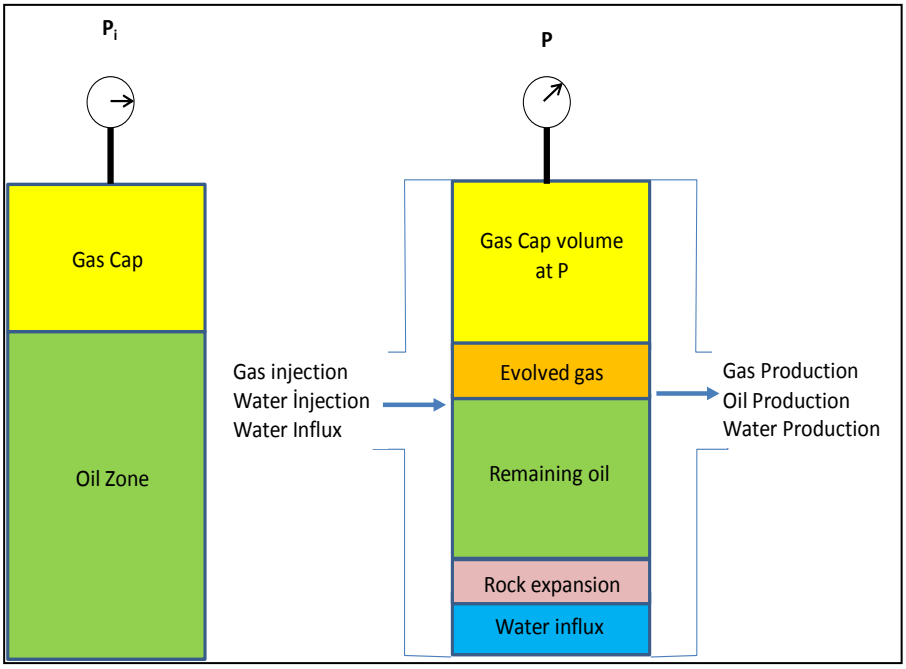

Figure 3.1 : Tank model.

The general expression of MBE is as follows:

$$\begin{pmatrix}\text{Pore volume}\\ \text{occupied}\\ \text{by the oil}\\ \text{initially in place}\\ \text{at initial p}\end{pmatrix}+\begin{pmatrix}\text{Pore volume}\\ \text{occupied}\\ \text{by the gas}\\ \text{in the gas cap}\\ \text{at initial P}\end{pmatrix}=\begin{pmatrix}\text{Pore volume}\\ \text{occupied}\\ \text{by the}\\ \text{remaining}\\ \text{oil at p}\end{pmatrix}+\begin{pmatrix}\text{Pore volume}\\ \text{occupied}\\ \text{by the gas}\\ \text{in the gas cap}\\ \text{at p}\end{pmatrix}+$$

$$\begin{pmatrix}\text{Pore volume}\\ \text{occupied by}\\ \text{the evolved}\\ \text{solution gas}\\ \text{at p}\end{pmatrix}+\begin{pmatrix}\text{Pore volume}\\ \text{occupied by}\\ \text{the net water}\\ \text{influx}\\ \text{at p}\end{pmatrix}+\begin{pmatrix}\text{Reduction}\\ \text{in pore volume}\\ \text{due to connate}\\ \text{water expansion}\\ \text{and}\\ \text{rock expansion}\end{pmatrix}+ \quad \textbf{(3.2)}$$

$$\begin{pmatrix}\text{Pore volume}\\ \text{occupied by}\\ \text{the injected gas}\\ \text{at p}\end{pmatrix}+\begin{pmatrix}\text{Pore volume}\\ \text{occupied}\\ \text{by the injected water}\\ \text{at p}\end{pmatrix}.$$

The terms in Equation 3.2 can be determined from the fluid PVT and reservoir rock properties as follows:

Pore Volume Occupied by the Oil Initially in Place at Initial Pressure

$$\text{Volume occupied by initial oil in place} = NB_{oi} \quad \textbf{(3.3)}$$

where N is the oil initially in place (STB) and B_{oi} is the oil formation volume factor at initial reservoir pressure pi (rb/STB).

Pore Volume Occupied by the Gas in the Gas Cap

$$\text{Volume of gas cap} = mNB_{oi} \quad \textbf{(3.4)}$$

where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

Pore Volume Occupied by the Remaining Oil

$$\text{Volume of the remaining oil} = (N - N_p)B_o \quad \textbf{(3.5)}$$

where N_p is the cumulative oil production (STB) and B_o is the oil formation volume factor at reservoir pressure p (rb/STB).

Pore Volume Occupied by the Gas Cap at Reservoir Pressure p

As the reservoir pressure (p_i) drops to a lower pressure (p), the gas in the gas cap expands and fills a larger volume. Assuming no gas is produced from the gas cap

during the pressure decline form p_i to p, the gas cap volume after pressure drop can be determined as:

$$\text{Volume of the gas cap at p} = \left[\frac{mNB_{oi}}{B_{gi}}\right] B_g \qquad \textbf{(3.6)}$$

where B_{gi} is the gas formation volume factor at initial reservoir pressure (rb/scf) and B_g is the current gas formation volume factor at pressure p (rb/scf).

Pore Volume Occupied by the Evolved Solution Gas

This term can be determined by applying the material balance on the solution gas as follows:

$$\begin{pmatrix}\text{volume of}\\ \text{the evolved}\\ \text{solution gas}\end{pmatrix} = \begin{pmatrix}\text{volume of gas}\\ \text{initially}\\ \text{in solution}\end{pmatrix} - \begin{pmatrix}\text{volume of}\\ \text{gas}\\ \text{produced}\end{pmatrix} - \begin{pmatrix}\text{volume of gas}\\ \text{remained}\\ \text{in solution}\end{pmatrix} \qquad \textbf{(3.7)}$$

or

$$\text{Volume of the evolved solution gas} = \left[NR_{si} - N_pR_p - (N - N_p)R_s\right]B_g \qquad \textbf{(3.8)}$$

where R_p is the net cumulative produced gas-oil ratio (scf/STB), R_s is the current dissolved gas-oil ratio at pressure p (scf/STB) and R_{si} is the initial dissolved gas to oil ratio at initial reservoir pressure (scf/STB).

Pore Volume Occupied by the Net Water Influx

$$\text{net water influx} = W_e - W_p B_w \qquad \textbf{(3.9)}$$

where W_e is the cumulative water influx (rb), W_p is the cumulative water produced (STB) and B_w is the formation volume factor of water (rb/STB).

Reduction in Pore Volume Due to Initial Water and Rock Expansion

The reduction in the hydrocarbon pore volume due to the expansion of initial water and the reservoir rock can be generally neglected for gas-cap-drive reservoir or when the reservoir pressure drops below the bubble-point pressure. The effect of these two components, however, cannot be neglected for an undersaturated oil reservoir. The water compressibility (c_w) and rock compressibility (c_f) are generally of the same order of magnitude as the compressibility of the oil.

The expansion of initial (connate) water and the reservoir rock are controlled by the compressibility of water and rock. The hydrocarbon pore volume (HCPV) compressibility as the compressibility for connate water and formation matrix can be expressed based on the general law of isothermal compressibility:

$$c = \frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T . \quad \textbf{(3.10)}$$

The reduction in the pore volume due to the expansion of the connate water in the oil zone and the gas cap is given by:

$$\text{Connate water expansion} = \left[\frac{NB_{oi}(1+m)}{1-S_{wi}}\right] S_{wi}\, c_w\, \Delta p \quad \textbf{(3.11)}$$

where Δp is the change in reservoir pressure (psi), c_w is the water compressibility coefficient (psi^{-1}) and S_{wi} is the initial (connate) water saturation (fraction).

Similarly, the reduction in the pore volume due to the expansion of the reservoir rock is given by:

$$\text{Change in pore volume} = \left[\frac{NB_{oi}(1+m)}{1-S_{wi}}\right] c_f\, \Delta p \quad \textbf{(3.12)}$$

where c_f is the formation (rock) compressibility coefficient (psi^{-1}).

Combining the expansions of the connate water and reservoir rock as represented by Equations-3.11 and 3.12 gives:

$$\text{Total changes in pore volume} = NB_{oi}(1+m)\left[\frac{S_{wi}c_w+c_f}{1-S_{wi}}\right]\Delta p. \quad \textbf{(3.13)}$$

Pore Volume Occupied by the Injection Gas and Water

Assuming that gas and water are injected for pressure maintenance purposes, the total pore volume occupied by the two injected fluids is given by:

$$\text{Total volume occupied by injection} = G_{inj}\, B_{ginj} + W_{inj} B_w \quad \textbf{(3.14)}$$

where G_{inj} is the cumulative injected gas (scf), B_{ginj} is the injected gas formation volume factor (rb/scf) and W_{inj} is the cumulative water injected (STB).

Combining Equations from 3.3 to 3.14 gives:

$$N=\frac{N_p\left[B_o+(R_p-R_s)B_g\right]-\left(W_e-W_pB_w\right)-G_{inj}B_{ginj}-W_{inj}B_w}{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]+B_{oi}(1+m)\left[\frac{S_{wi}c_w+c_f}{1-S_{wi}}\right]\Delta P}. \quad \textbf{(3.15)}$$

The above relationship is referred to as the material balance equation (MBE).

3.2.1 The MBE for gas cap reservoir

For a reservoir in which gas cap drive is the main drive mechanism and assuming that there is no adjacent aquifer to the reservoir ($W_e = 0$) and, there is no injection of water or gas for pressure support, Equation 3.15 can be written as

$$N=\frac{N_p\left[B_o+(R_p-R_s)B_g\right]+W_pB_w}{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]+B_{oi}(1+m)\left[\frac{S_{wi}c_w+c_f}{1-S_{wi}}\right]\Delta P}. \quad \textbf{(3.16)}$$

Assuming that the effect of water and pore compressibilities is also negligible as compared to the compressibility of the gas, Equation 3.16 can be reduced to

$$N=\frac{N_p\left[B_o+(R_p-R_s)B_g\right]+W_pB_w}{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]}. \quad \textbf{(3.17)}$$

3.3 The MBE as an equation of a straight line

Havlena and Odeh (1963, 1964) described the technique of interpreting the material balance as the equation of a straight line with two papers, the first paper describing the technique and the second illustrating the application to reservoir case histories (Craft and Hawkins, 1991).

The straight line starts with the MBE in the form written as:

$$N_p\left[B_o+(R_p-R_s)B_g\right]+W_pB_w = N\left[(B_o-B_{oi})+(R_{si}-R_s)B_g\right] + mNB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]+NB_{oi}(1+m)\left[\frac{S_{wi}c_w+c_f}{1-S_{wi}}\right]\Delta P+W_e+G_{inj}B_{ginj}+W_{inj}B_w. \quad \textbf{(3.18)}$$

Havlena and Odeh expressed Equation 3.18 in a form of straight line equation as:

$$F = N\,[E_o + m\,E_g + E_{f,w}] + (W_e + W_{inj}\,B_w + G_{inj}\,B_{ginj}) \quad \textbf{(3.19)}$$

in which the terms F, E_o, E_g, and $E_{f,w}$ are defined by the following relationships:

• F represents the underground withdrawal and given by:

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_p B_w \cdot \quad \textbf{(3.20)}$$

• E_o is the term which describes the expansion of oil and its originally dissolved gas and is expressed in terms of the oil formation volume factor as:

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \cdot \quad \textbf{(3.21)}$$

• E_g is the term used for the expansion of the gas-cap gas and is defined by the following expression:

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \cdot \quad \textbf{(3.22)}$$

• $E_{f,w}$ represents the expansion of the initial water and the reduction in the pore volume and is given by:

$$E_{f,w} = B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta P \cdot \quad \textbf{(3.23)}$$

3.3.1 The straight-line solution method to the MBE

The straight-line solution method is based on the plotting of a variable group versus another variable group. The selection of the variable group depends on the drive mechanism under which the reservoir is producing (Tarek, 2001). The importance of the straight-line method lies on the sequence of plotting. The deviation of data from the straight line should be carefully analyzed to find the reason for it. The future reservoir performance can be predicted by the extrapolation of a straight line. The parameters of the straight line are used to determine the unknowns which are initial oil in place (N), size of the gas cap (m), water influx (W_e) and driving mechanism of the reservoir.

Havlena and Odeh (1963, 1964) investigated several cases of varying reservoir types with Equation 3.19 and expressed that the relationship can be rearranged into the

form of a straight line. For the case studied in our thesis, Havlena-Odeh straight line material balance equation can be expressed as follows:

For a reservoir in which the gas cap expansion is the most effective driving mechanism and assuming that there is no natural water influx and the effect of water and pore compressibilities are negligible and no water injection or gas injection are applied for pressure support, the Havlena-Odeh material balance equation (Equation 3.19) can be written as:

$$F = N\,[E_o + m\,E_g]\,. \qquad \textbf{(3.24)}$$

Equation 3.24 can be used according to the number of unknowns in the equation which can be described in three ways:

1- N is unknown, m is known

Equation 3.24 indicates that a plot of F versus (E_o + mE_g) on a Cartesian scale produces a straight line through the origin with a slope of N, as shown in Figure 3.2.

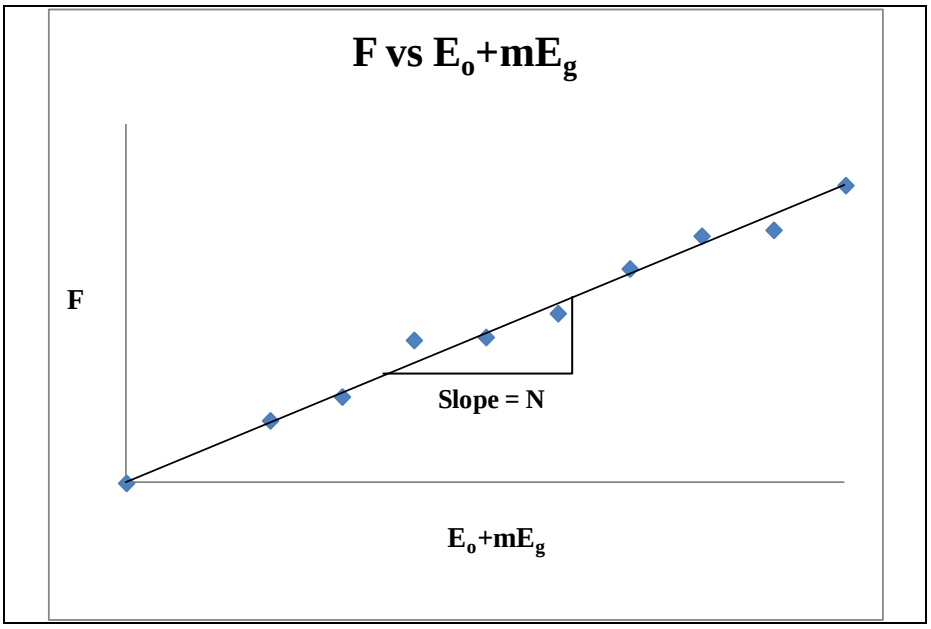

Figure 3.2 : F vs E_o+mE_g plot.

2- Both N and m are unknown

If both the values of N and m are not known, Equation 3.24 can be reexpressed as:

$$\frac{F}{E_o} = N + mN\left(\frac{E_g}{E_o}\right)\,. \qquad \textbf{(3.25)}$$

A plot of F/E_o versus E_g/E_o gives a straight line with intercept N and slope mN. m can simply be found by dividing slope to intercept. This plot is illustrated in Figure 3.3.

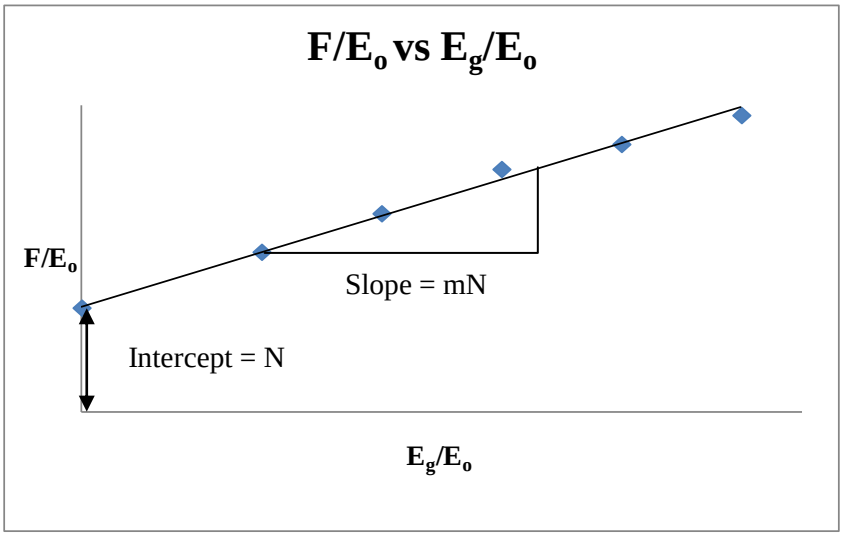

Figure 3.3 : F/ E_o vs E_g/E_o plot.

3- m is unknown, N is known

If N is known by volumetric methods but m is unknown, Equation 3.24 can be rearranged as an equation of straight line as follows:

$$\left(\frac{F}{N} - E_o\right) = m\, E_g \cdot \qquad \textbf{(3.26)}$$

Equation 3.26 states that a plot of the term (F/N - E_o) versus E_g produces a straight line passing through the origin with a slope of m. See Figure 3.4.

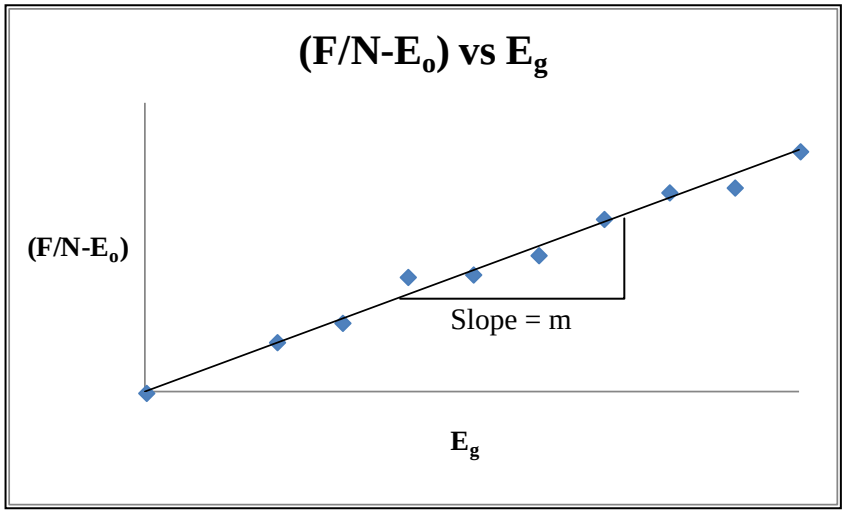

Figure 3.4 : F/N-E_o vs E_g plot.

In our case, since m is known, we will analyze mainly the case when N is unknown and m is known.

However, assuming that m is unknown, the plots of F/E_o vs E_g/E_o will also be analyzed and the results will be compared with the case where m is known.

3.4 Error Analysis

The synonymous terms uncertainty, error, or deviation are used to represent the variation in measured data. Every measurement has an uncertainty and not all uncertainties are equal. In many cases, the value of the experimental result is not measured. Actually, we measure the values of several variables and combine these data in a data reduction equation to obtain the values of the desired result. Therefore, the ability of proper combination of uncertainties from different measurements is important. Uncertainty in measurement takes place in a variety of ways: different instruments used for measurements, different observers, differences in samples, measurements taken at different times of day, etc.

Two types of errors are possible. Systematic error results from a mis-calibrated device, or a measuring technique making the measured value larger (or smaller) than the "true" value. There will remain a second type of variation in measured values of a single quantity, even when systematic errors are eliminated. These remaining variations can be classed as random errors, and can be dealt within a statistical manner.

Differential analysis is the backbone of all other sensitivity analysis techniques. Differential analysis of parameter sensitivity is based on partial differentiation of the model in aggregated form. It can be thought of as the propagation of uncertainties (Cunningham et al., 1980).

Given a functional relationship between several measured variables (x, y, z) to calculate Q what is the uncertainty in Q if the uncertainties in x, y, and z are known? The answer can get a little complicated, but it should be no surprise that the uncertainties of x, y and z "propagate" to the uncertainty of Q. It can be written that Q is a function of these variables:

$$Q = f(x, y, z) \cdot \qquad \textbf{(3.27)}$$

Equation 3.27 is a data reduction equation which is used for determining Q from the measured values of the variables x, y and z. Assuming that the variables x, y and z are uncorrelated, the uncertainty in Q is given by

$$\sigma_Q^2 = \left(\frac{\partial Q}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial Q}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial Q}{\partial z}\right)^2 \sigma_z^2 \qquad \textbf{(3.28)}$$

where $\sigma_x{}^2$, $\sigma_y{}^2$ and $\sigma_y{}^2$ are the uncertainties in the measured variables x, y and z and $\frac{\partial Q}{\partial x}$, $\frac{\partial Q}{\partial x}$ and $\frac{\partial Q}{\partial x}$ are the derivatives of Q with respect to the variables x, y and z.

A sensitivity coefficient is basically the ratio of the change in output to the change in input while all other parameters remain constant (Krieger et al., 1977).

The sensitivity coefficient, $\emptyset$, for a particular independent variable can be calculated from the partial derivative of the dependent variable with respect to the independent variable, i.e.,

$$\emptyset_x = \frac{\partial Q}{\partial x} \cdot \qquad \textbf{(3.29)}$$

If we take the ln derivative of Q with respect to the independent variable to remove the effects of units, we have

$$\emptyset_x = \frac{\partial Q}{\partial x} x \cdot \qquad \textbf{(3.30)}$$

It should be noted that from Equation 3.28, the magnitude of the uncertainty on the function Q is dependent on the uncertainty of the independent parameters x, y or z and also on their sensitivity coefficients. In other words, if a certain parameter has a larger sensitivity coefficient, then the uncertainty of this particular parameter will be more influencial on the uncertainty of Q.

3.4.1 Application of error analysis

Systematic errors were introduced into the reservoir PVT properties, m value and production data of the gas cap reservoir in such a way that one parameter contained error while all other parameters in MBE were held constant at their true value. Systematic percentage errors were calculated using Equation 1.1.

3.4.1.1 Calculation method

Assuming that the natural net water influx is negligible (W_e-W_pB_w = 0), the effect of water and pore compressibilities are also negligible and there is no water injection or gas injection, the material balance equation for a gas cap reservoir can be expressed as:

$$N=\frac{N_p\left[B_o+(R_p-R_s)B_g\right]}{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]}. \quad \textbf{(3.31)}$$

Examining Equation 3.31 reveals that there are several components of material balance that can cause errors in the calculations. The error in N in Equation 3.31 depends on drive mechanism, total pressure drop in the reservoir, pressure measurement accuracy, PVT accuracy, and production measurement accuracy (Baker et al., 2003).

In Equation 3.31 it is seen that N is a function of B_o, B_{oi}, B_g, B_{gi}, R_s, R_{si}, N_p, G_p (since $R_p=G_p/N_p$) and m. To analyze the effects of each variable in Equation 3.31 the sensitivity coefficients will be calculated and then the magnitudes of them will be compared to see which parameters have the biggest effect on N value.

The derivation of sensitivity coefficients based on the parameters of MBE is given in Appendix A.

4. 3D RESERVOIR MODELING

To conduct a simulation study, it is necessary to choose a simulator and to create a reservoir model. The first step in preparing a simulation case is to determine the representative values of the main parameters, which should reflect reservoir characteristics and operational conditions in a hypothetical gas cap field.

4.1 Introduction

For the construction of our model, the Petrel 2009.1 software was used. Petrel 2009.1 software allows the user to build reservoir models suitable for simulation, visualize simulation results, calculate volumes and design development strategies to maximize reservoir exploitation.

Eclipse 2009.1 simulator was used for dynamic simulation. Eclipse 2009.1 uses the finite difference method to solve material and energy balance equations modelling a subsurface petroleum reservoir.

4.1.1 Model description

In this study, a simple square-shaped gas cap reservoir was created with Petrel 2009.1. The dimensions of the reservoir are 3000 ft x 3000 ft x 300 ft. The gas column heigth was taken as 100 ft while the height of the oil column was 200 ft so that the ratio of the initial gas cap volume to the initial oil volume is to be 0.5. The reservoir depth is chosen in order not to have any abnormal pressure regimes. The general information for the reservoir is given in Table 4.1.

Table 4.1 : General information for the reservoir.

Surface area,ft^2	9,000,000
Net pay thickness,ft	300
Bulk volume, ft^3	2,700,000,000
Top of reservoir, ft	6550
Bottom of reservoir, ft	6850
Gas-oil contact, ft	6650

4.1.2 Well information

A single vertical well was drilled at the center of the reservoir. The total depth of the well is 6900 ft. The well was completed at the bottom half of the oil column between 6750 ft and 6850 ft in order to minimize the gas production from the expanding gas cap. Well completion data are given in Figure 4.1.

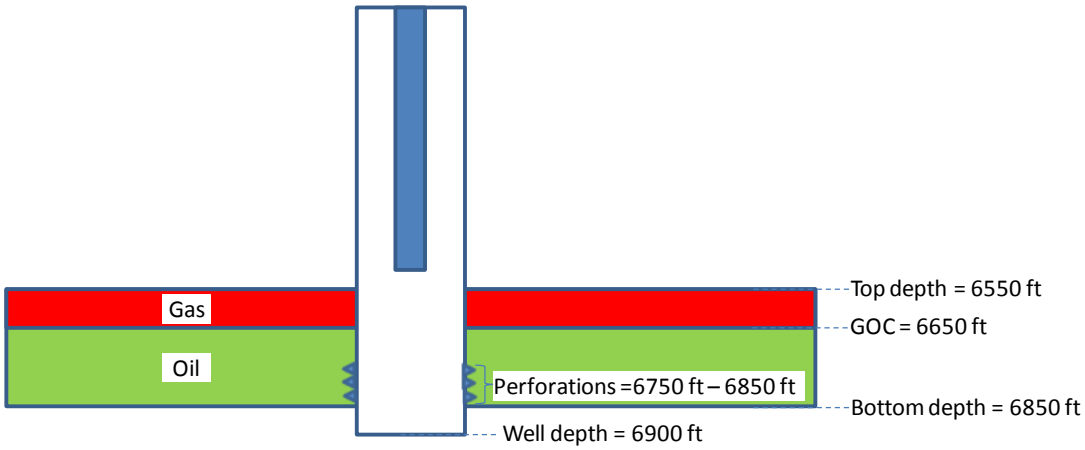

Figure 4.1 : Well completion data.

4.2 3D Geological Modeling

A simple 3D grid model was generated for the gas cap reservoir in Petrel 2009.1. Optimum grid size was decided to be 120 ft x 120 ft x 20 ft as decribed in Section 4.5. 2D framework of the generated grids can be seen in Figure 4.2.

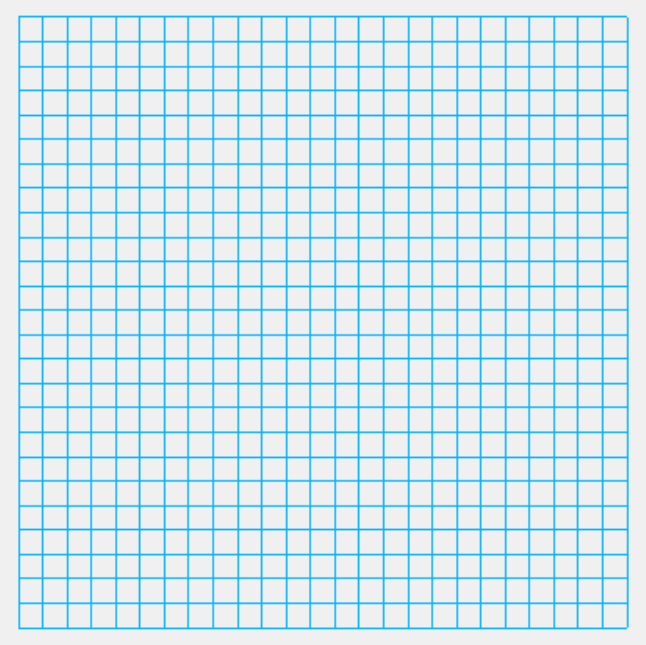

Figure 4.2 : 2D grid framework.

4.2.1 Zones and layers generation

One zone was chosen for the reservoir. “Proportional layering method” was used to identify the layers. The number of layers is chosen as 15 with equal thicknesses (20 ft). Layering of the selected model is given in Figure 4.3.

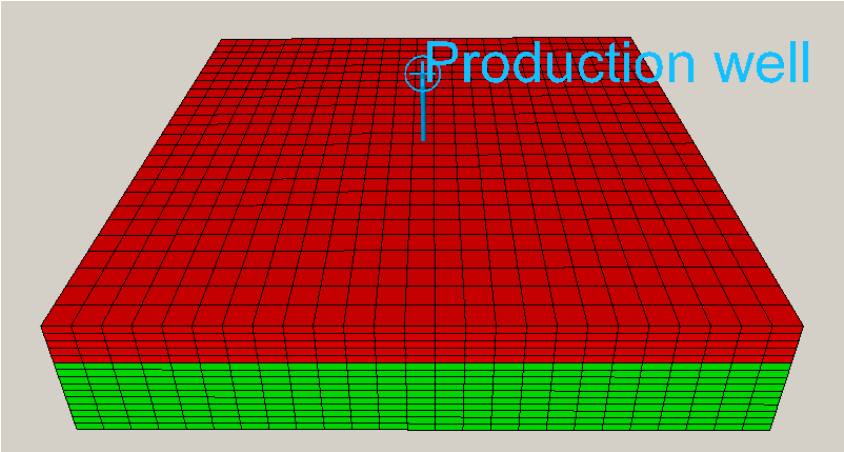

Figure 4.3 : Layering.

4.3 Petrophysical Modeling

Porosity, permeabilities in x,y and z direction, water saturation, net to gross ratio were all taken as constant and their values are given in Table 4.2.

Table 4.2 : Petrophysical data.

Porosity, fraction	0.1
Permeability in x direction, md	50
Permeability in y direction, md	50
Permeability in z direction, md	5
Water Saturation, fraction	0.0
Net to gross ratio, fraction	1.0

4.4 Volume calculation

The volume calculation results of gas cap gas (Free GIIP), dissolved gas (Dissolved GIIP) and reservoir oil (OIIP) are presented in Table 4.3.

Table 4.3 : Volume calculation results.

OIIP (STB)	22,874,560
Free GIIP (scf)	17,176,426,000
Dissolved GIIP (scf)	18,711,390,000

4.5 Grid Sensitivity

Since this study analyzes the effects of the error on the material balance equation parameters (PVT parameters, production data and m value), grid size sensitivity were performed to find the ideal grid size which minimizes the discretization errors as much as possible. The finite-difference equations are basically the well-known mass balance equations written for each phase for each grid block. The process of obtaining finite-difference equations that approximate a given differential equation is called "discretization". This discretization introduces error to the finite-difference solutions where its magnitude greatly depends on the size of grid and timestep used. Roache (1997) states that a consistent numerical method will tend to zero discretization error as the number of grid points increases and the size of the grid spacing tends to zero. The smaller the grid and timestep used, the more accurate the finite-difference solutions will be. Grid sensitivity analysis is usually performed to determine the practical values of grid and timestep size. So we will start from the biggest grid size and decrease the size of it to the acceptable size when there are no remarkable changes in the behaviour of production and pressure values between the last two grid sizes studied. The grid sizes used in grid sensitivity study are given in Table 4.4.

Table 4.4 : Grid sizes for grid sensitivity study.

Case #	2D Grid size (ft x ft)	Grid Thickness, ft	Number of 3D grids
1	3000 x 3000	20	15
2	1000 x 1000	20	135
3	600 x 600	20	375
4	200 x 200	20	3375
5	120 x 120	20	9375
6	120 x 120	10	18750

The grid sizes were chosen so that the location of the well does not coincide with grid boundaries. Starting from Case#1, the reservoir was produced at the same rate for 40 years production period and the same constraints applied for bottom hole pressure (1000 psi) and minimum production rate (100 STB/day) for all cases. Five different plots were analyzed to decide the optimum grid size:

1. Pressure vs time (Figure 4.4),
2. Oil production rate vs time (Figure 4.5),
3. Gas production rate vs time (Figure 4.6),

4. Cumulative oil production vs time (Figure 4.7),

5. Cumulative gas production vs time (Figure 4.8).

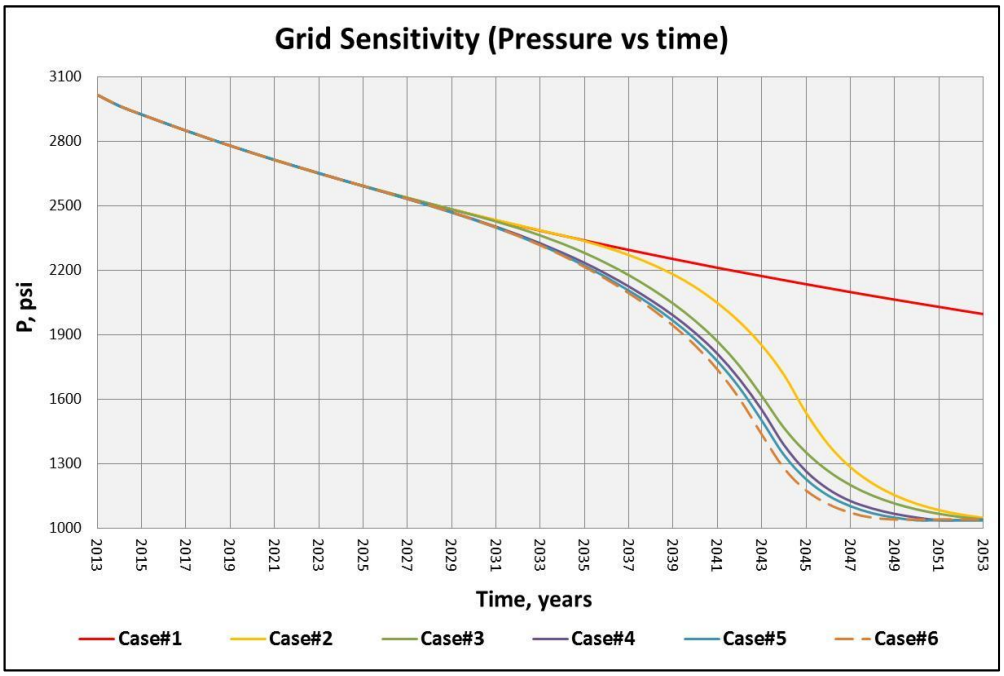

Figure 4.4 : The plot of pressure vs time.

Figure 4.4 shows that reservoir pressure does not change too much for Case#4 and Case#5. Change in layer thickness to 10 ft (Case#6) had no remarkable effect on the reservoir pressure behaviour of the field.

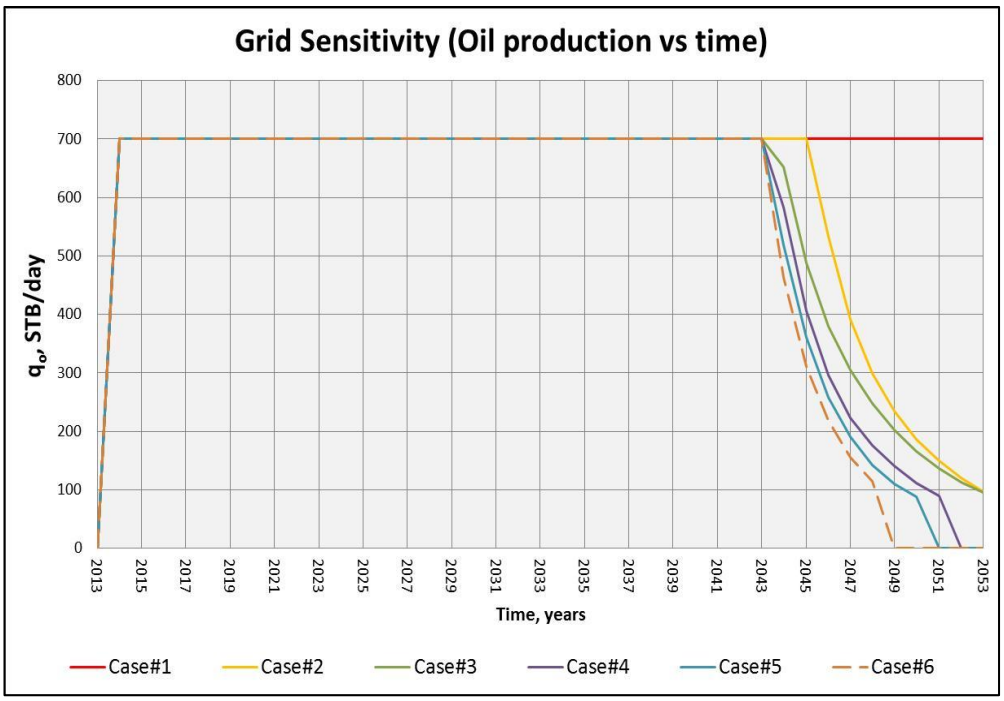

Figure 4.5 : The plot of oil production rate vs time.

Figure 4.5 and 4.6 show that the oil production and gas production rates show almost the same behaviour for Cases #4, 5 and 6. The oil production rates start to decrease at the same point in all cases (year 2043). The gas production rates for the Cases#4, 5 and 6 reaches to the peak values at year 2043 and they start to decrease as the oil production rates decrease.

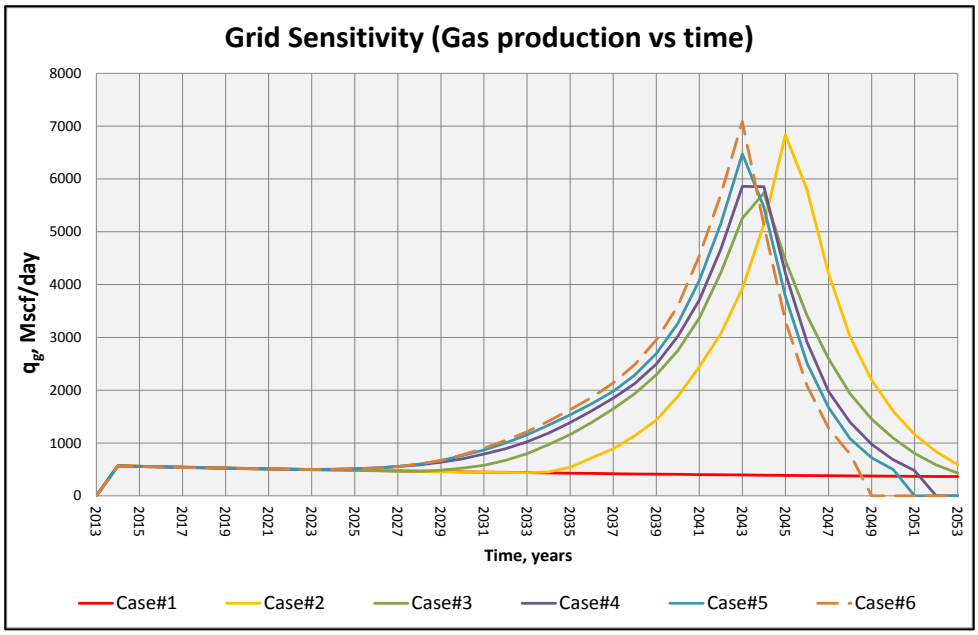

Figure 4.6 : The plot of gas production rate vs time.

The cumulative oil production and the cumulative gas production values for Cases#4, 5 and 6 are very close to each other as shown in Figures-4.7 and 4.8.

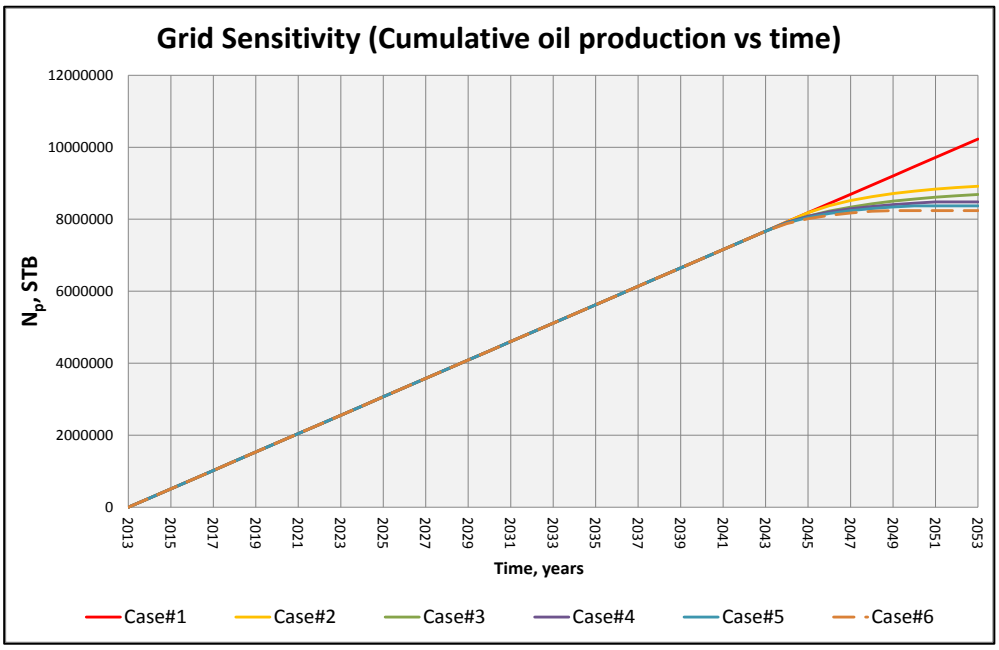

Figure 4.7 : The plot of cumulative oil production vs time.

The simulation results for Case#4 and Case#5 are quite close to each other. It means that further decrease in grid size will not contribute a remarkable decrease in discretization error, but it will significantly increase the computational time for simulation. The same procedure was applied for the vertical direction by increasing layer numbers. Layer numbers were increased for Case#5 (120ft x 120ft) grid sized model.

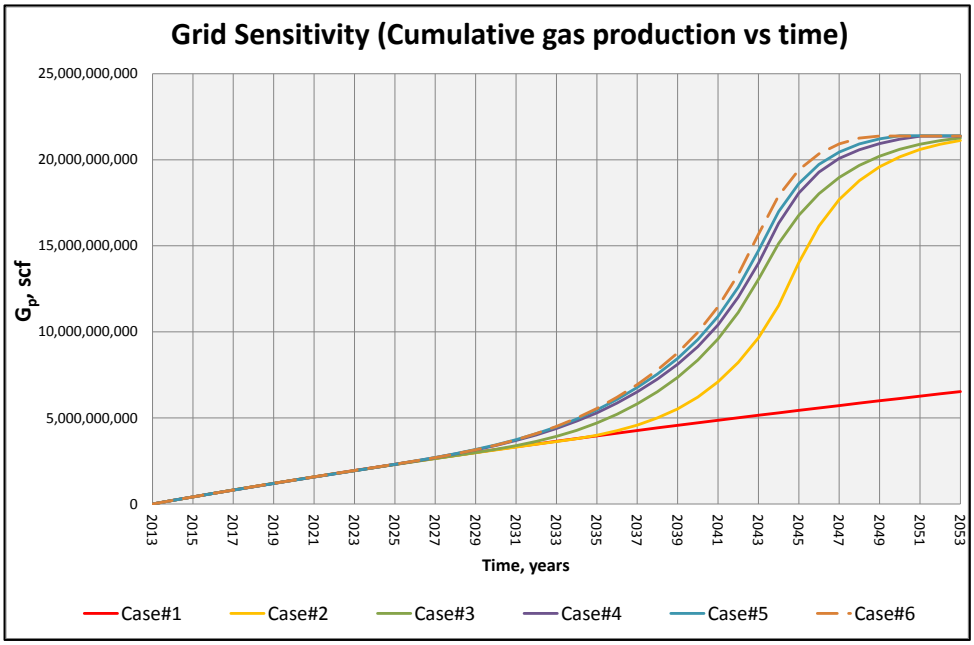

Figure 4.8 : The plot of cumulative gas production vs time.

The simulation results for Case#5 and Case#6 were very similar to each other. Therefore, Case#5 was selected for our hypothetical gas cap reservoir.

4.6 Dynamic (Simulation) Model

Simulation model was constructed by using Petrel 2009.1 simulation processes. This section includes the review of the basic reservoir engineering data used for the construction of dynamic (simulation) model, including initial reservoir conditions, PVT analysis of fluids, relative permeability data and rock properties.

4.6.1 Initial reservoir conditions

The reservoir pressure is below bubble point pressure and has an average value of 3017.54 psi. The bubble point pressure is 3500 psi. The temperature is 170 ^{o}F and it is assumed to be constant during the life of the field production.

4.6.2 PVT analysis

A fluid model for 40 API light oil and 0.6636 gravity gas was created in Petrel 2009.1 to define basic PVT parameters such as; B_o, B_g and R_s. B_o vs pressure plot, B_g vs pressure plot and R_s vs pressure plot are given in Figures-4.9, 4.10 and 4.11, respectively.

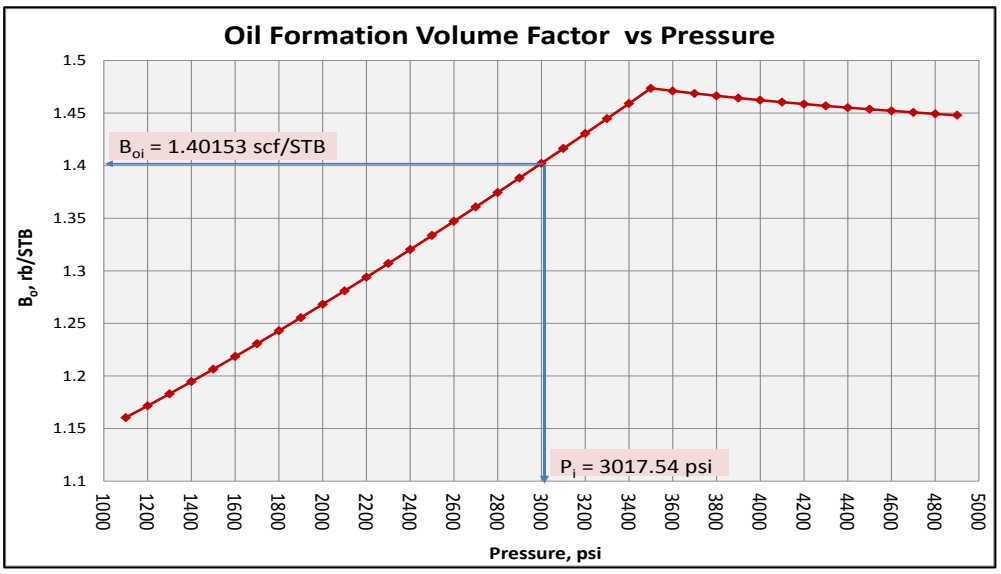

Figure 4.9 : B_o vs Pressure.

As seen in Figure 4.9, B_o values decrease as the pressure goes down from bubble point pressure (3500 psi). The initial oil formation volume factor (B_{oi}) is 1.40153 scf/STB at 3017.54 psi (average initial reservoir pressure, P_i).

The initial gas formation volume factor, B_{gi}, is 0.00093324 rb/scf at 3017.54 psi.

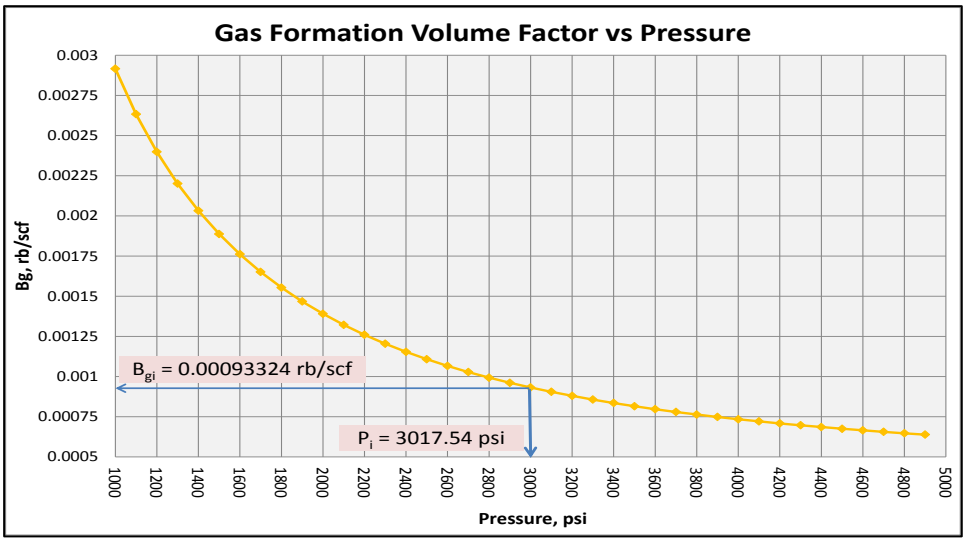

Figure 4.10 : B_g vs Pressure.

The initial gas oil ratio, Rsi, is 818 scf/STB at initial pressure of 3017.54 psi.

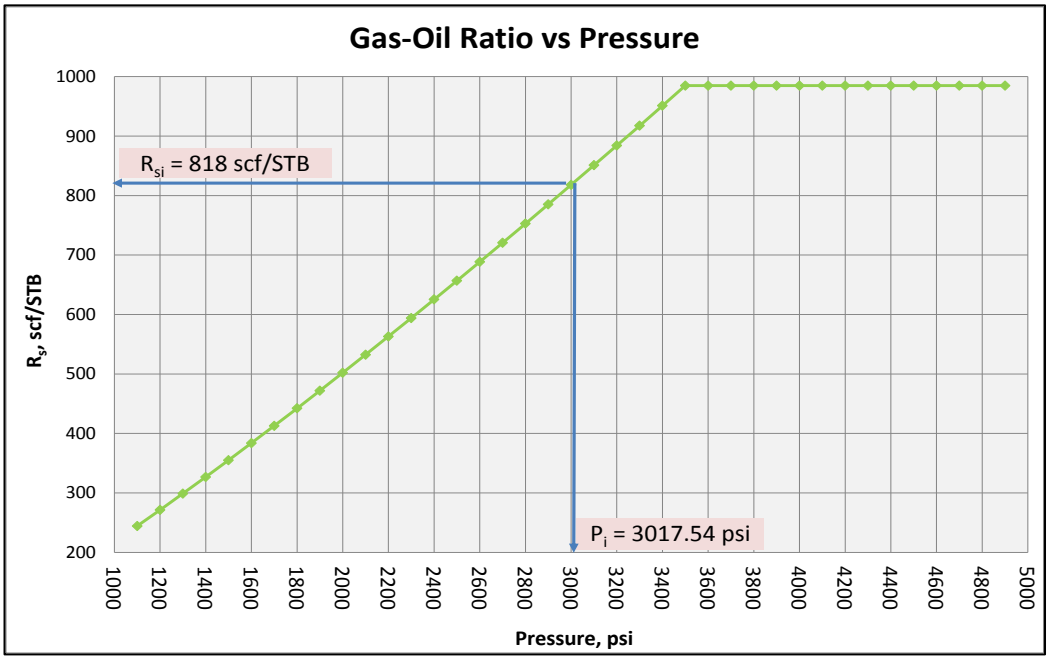

Figure 4.11 : R_s vs Pressure.

4.6.3 Relative permeability data

Relative permeability curves of reservoir fluids were created for clean sand reservoir (default) in Petrel 2009.1. Since there is no water present in our reservoir, gas and oil are two phases flowing in the reservoir. Relative permeability of oil and gas vs gas saturation curve is given in Figure 4.12. Capillary pressure was set to zero.

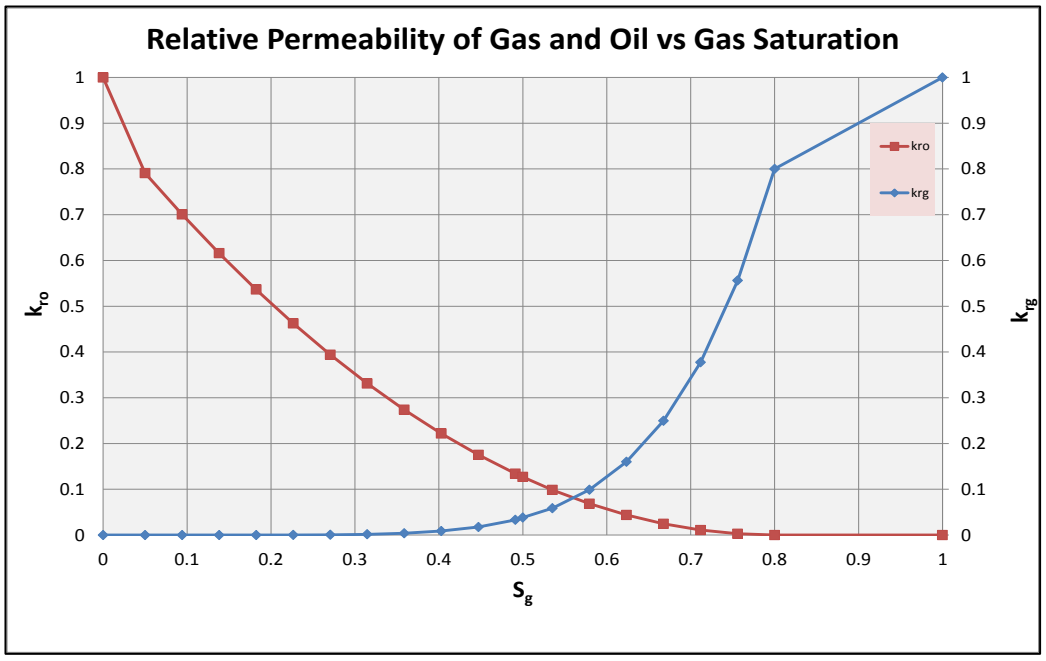

Figure 4.12 : Gas-oil relative permeability.

4.6.4 Rock properties

Rock properties were also created in Petrel 2009.1 for a consolidated sandstone (default) reservoir. The rock is considered to be incompressible for all practical purposes.

4.7 Simulation Results of the Selected Model

Table 4.5 summarizes the parameters used for the development strategy of gas cap reservoir which has grid size dimensions of 120 ft x 120 ft x 20 ft. The pressure and production data of the reservoir can be seen from Figure 4.14 to Figure 4.18.

Table 4.5 : Development strategy for simulation.

Parameters	Values
Number of production wells	1
Target production rate of a well, STB/day	700
Minimum production rate of a well, STB/day	100
Well abondonment pressure, psia	1000
Production period, years	40

The simulation starts with an oil production rate of 700 STB/day. It continues production while bottom hole pressure (BHP) of the well drops to a value of 1000 psi. At this pressure, control is changed to oil rate control. The well continues to produce at constant BHP till the oil rate of the well drops to 100 STB/day. At this time the simulation stops.

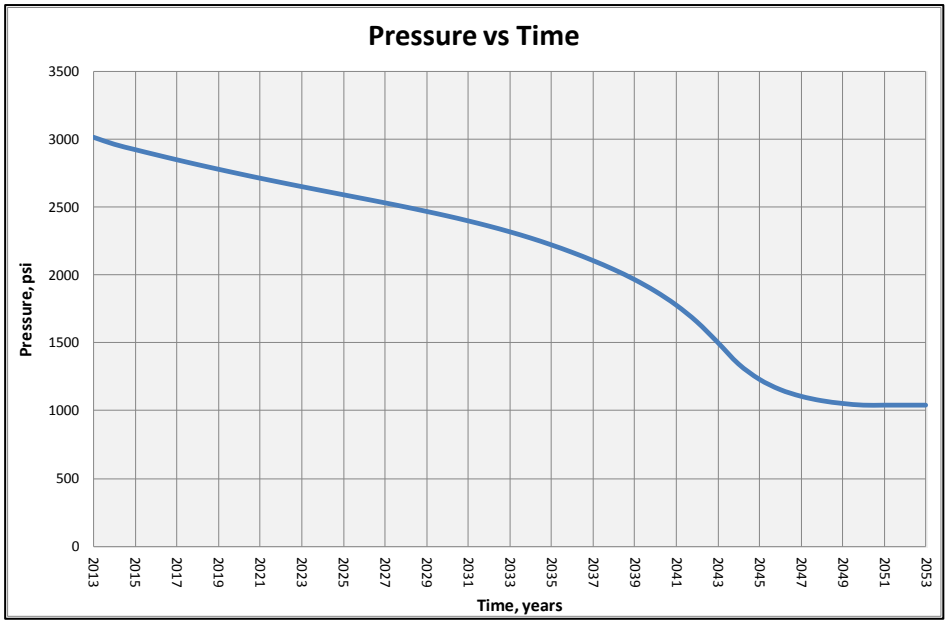

Figure 4.13 : Reservoir pressure vs time plot.

Figure 4.13 shows the change in average reservoir pressure with time. In year 2050, the reservoir pressure stabilizes since the production ceases due to the minimum production rate constraint for the well.

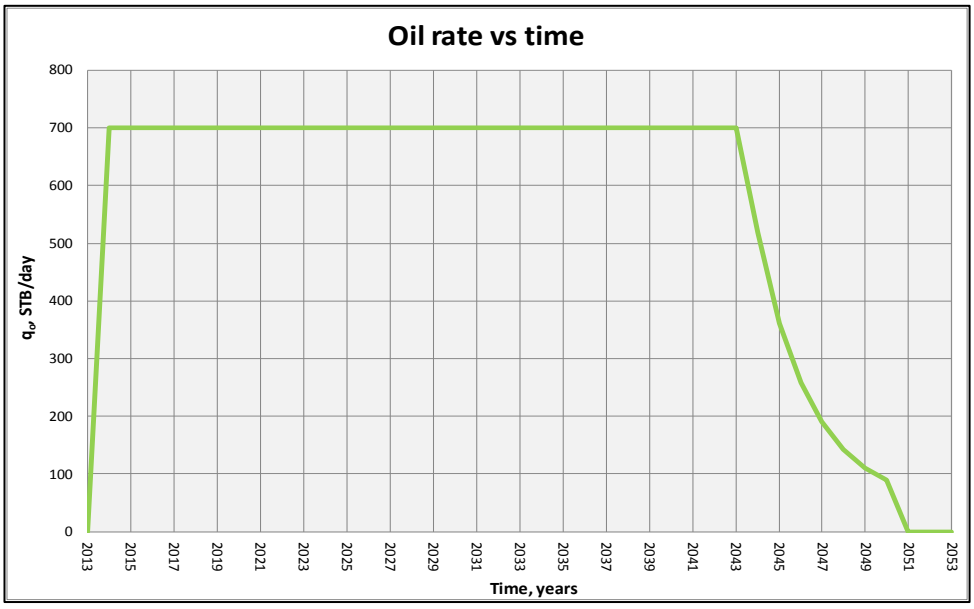

Figure 4.14 : Oil production rate vs time plot.

In Figure 4.14, oil production rate starts to decrease after year 2043 since the bottom hole pressure of the well reaches to 1000 psi which is the minimum BHP set for the well. In year 2050, the oil rate drops to zero since at this time step the minimum rate constraint of the well has been satisfied and the simulation stops.

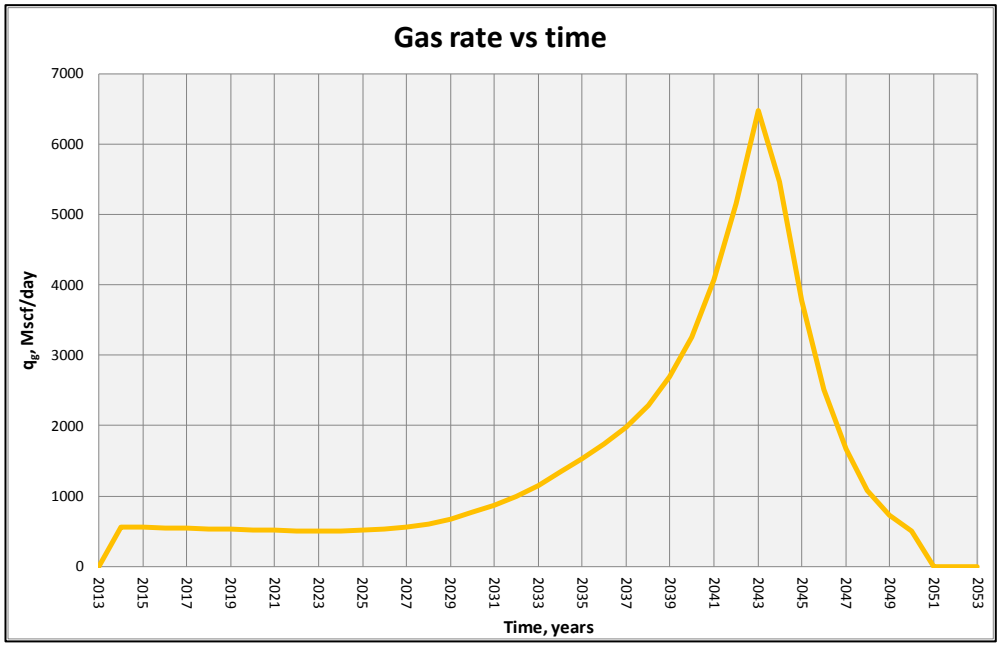

Figure 4.15 : Gas production rate vs time plot.

The cumulative oil production curve given in Figure 4.16 stabilizes towards the end of the production period since the production rate reaches to the minimum production rate assigned for the well and the simulation stops. The same behaviour is also seen in the cumulative gas production vs time plot. See Figure 4.17.

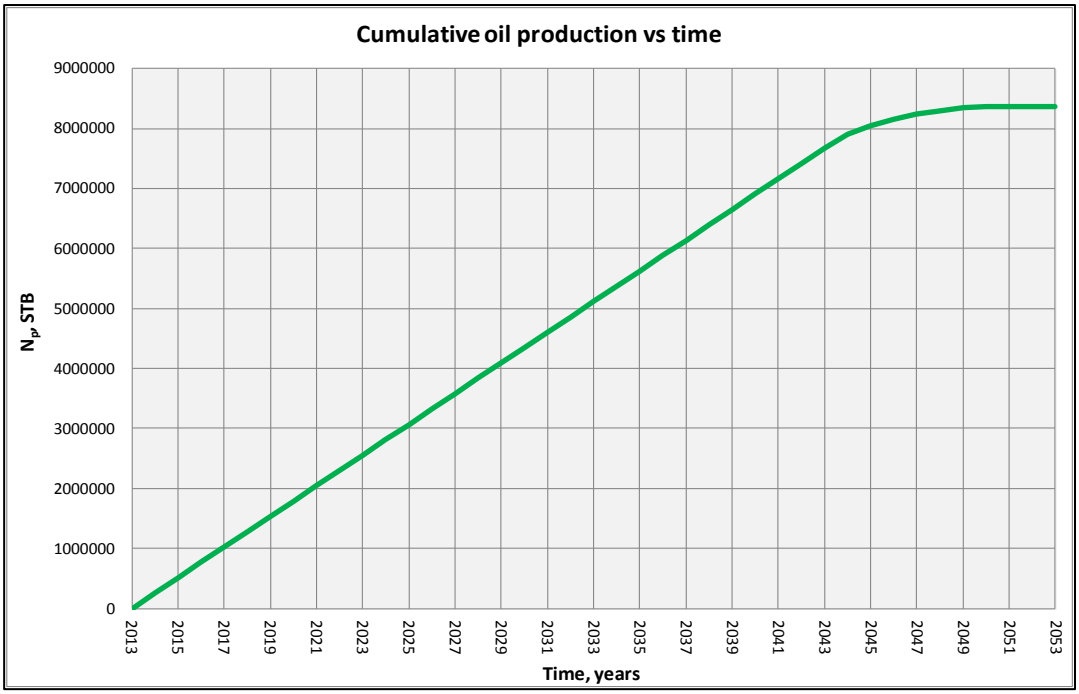

Figure 4.16 : Cumulative oil production vs time plot.

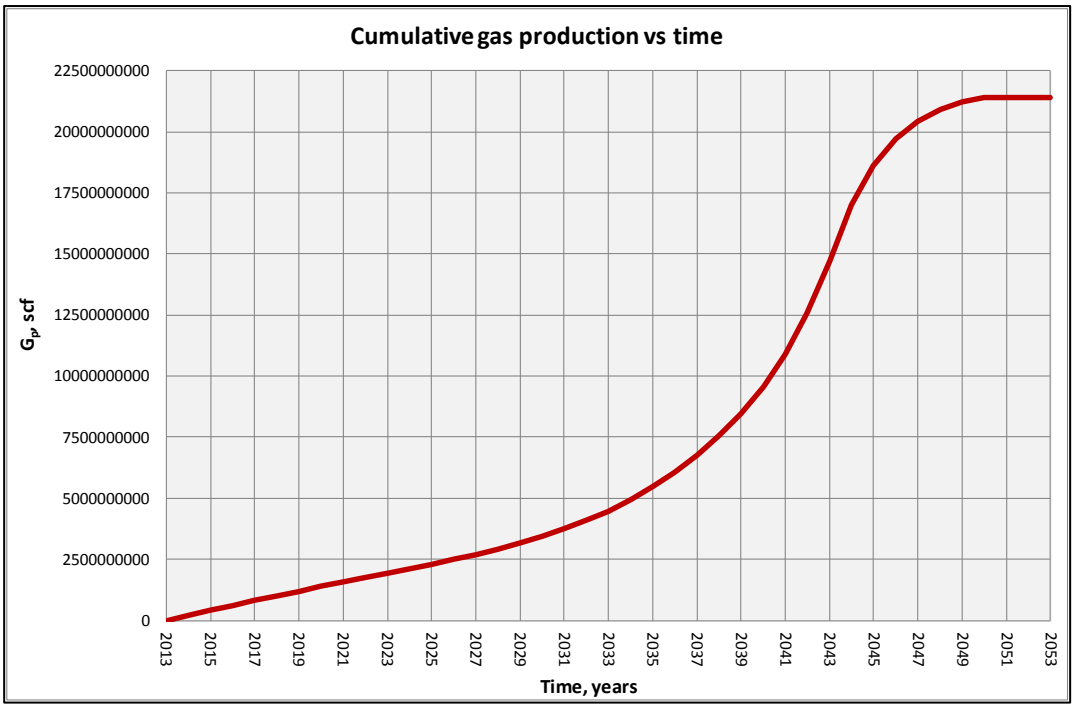

Figure 4.17 : Cumulative oil production vs time plot.

The cumulative oil production from the reservoir is 8,372,870 STB and the recovery factor (RF) is 36.6% at the end of the production period. The cumulative gas production is 21,397,670,000 scf.

5. ERROR ANALYSIS

In this part, effects of PVT data (R_s, R_{si}, B_o, B_{oi}, B_g, B_{gi}), production data (N_p and G_p) and the ratio of the initial gas cap volume to the initial oil volume (m) will be performed on original oil in place (N) calculations. The original oil in place is computed to be 22,874,560 STB. This value will serve as a reference to the N values computed from the material balance equation and the errors will be computed based on this number. MBE is used to calculate N values at each time step. Furthermore, N is also determined using the Havlena and Odeh straight line method for both cases where m is treated as known and unknown. Figure 5.1 shows the results of N calculations from Schilthuis MBE and HO.

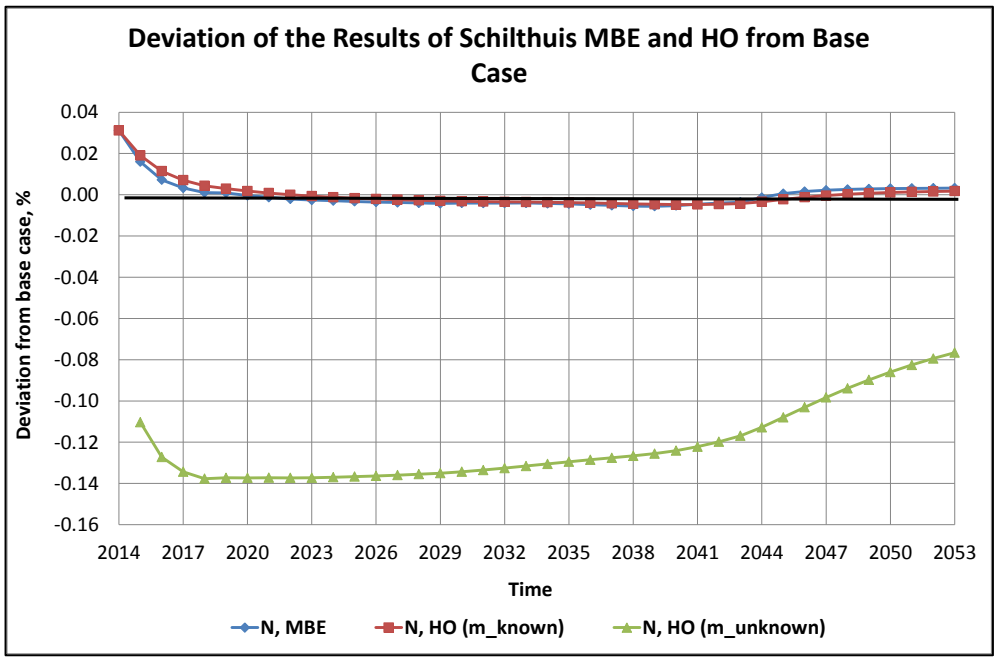

Figure 5.1 : Calculations of N from MBE and HO.

As it is seen clearly in Figure 5.1, the values of the computed N values with MBE and HO technique (m is assumed to be known) are very close to each other during the whole production period. On the other hand, the results of HO calculations for N values in the case where m is unknown are at an average of 0.12 % lower than the results of HO calculations of the case where m is known.

5.1 Effects of parameters on N (m is known)

Reference value was taken as the volumetric computation of original oil in place (N=22,874,560 STB). The effects of the parameters were presented graphically with the results of HO and MBE to compare the results. Errors were considered in the following parameters: B_o, B_{oi}, B_g, B_{gi}, R_s, R_{si}, N_p, G_p and m. All these parameters were multiplied by systematic errors given in Table 5.1 while taking the others constant.

Table 5.1 : Systematic errors applied for the error analysis of MB parameters.

Sensitivity parameter	Systematic error, %
R_s	+2 / -2
R_{si}	+2 / -2
R_s & R_{si}	+2 / -2 / +10
B_o	+2 / -2
B_{oi}	+2 / -2
B_o & B_{oi}	+2 / -2 / +10
B_g	+2 / -2
B_{gi}	+2 / -2
B_g& B_{gi}	+2 / -2 /+10
N_p	-10
G_p	-10
m	+50 / -50

5.1.1 Effects of PVT data

All PVT parameters were checked twice for +2 % and -2 % systematic errors. When analyzing two parameters together such as, B_o and B_{oi} or R_s and R_{si} or B_g and B_{gi}, +10 % error was also applied.

5.1.1.1 Effect of R_s

R_s value at each time step was multiplied with 1.02 before calculating N value. It is important to remember that the initial gas-oil ratio (R_{si}) is 818 scf/STB. After one year production, pressure decreases to 2965.8 psia and R_s value is 810.5 scf/STB at this pressure. If we multiply this value with 1.02, R_s value is getting bigger than R_{si} and the calculated N value for the first time step is not valid when systematic error is applied. So the first calculated value of N is ignored for the analysis of R_s effect on N calculations. The errors on N value under the effect of R_s are given in Figure 5.2.

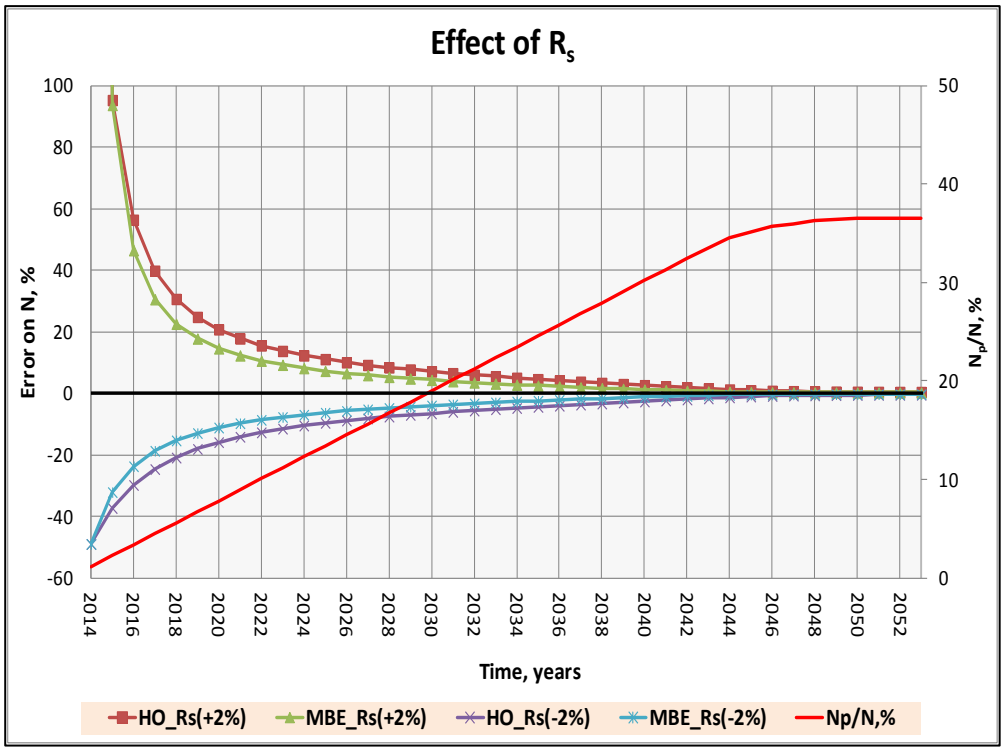

Figure 5.2 : Effect of R_s.

In Figure 5.2, it is seen that MBE calculations results are parallel with HO calculation results. +2% error of R_s value and -2 % error of R_s value have opposite effects on the N value. +2% error in R_s values increase N values of MBE and HO calculations, whereas -2% errors applied to R_s decrease the calculated N values. The increase of N value due to +2% error in R_s is 95.4 % after 2 years of production and it is getting smaller as recovery factor of the field increases. But it is important to note that the error on N value is still around 20% after a 7-year production period when recovery factor is 7.8%.

5.1.1.2 Effect of R_{si}

+/- 2% error was introduced to R_{si} value before making material balance calculations. The first value of error calculations of N (1^{st} time step) is omitted for -2% error in R_{si} value case. When we decrease R_{si} value by 2 %, this R_{si} value is getting smaller than the R_s value at this time step. Figure 5.3 shows that a 2 % decrease in R_{si} value increases N value approximately 100 % after 2 years of production and this increase is getting smaller as recovery factor of the field increases. Another point in Figure 5.3 is that +2 percent systematic error in R_{si} decreases N values about 50 % at the first time step and this decrease is getting smaller as production continues.

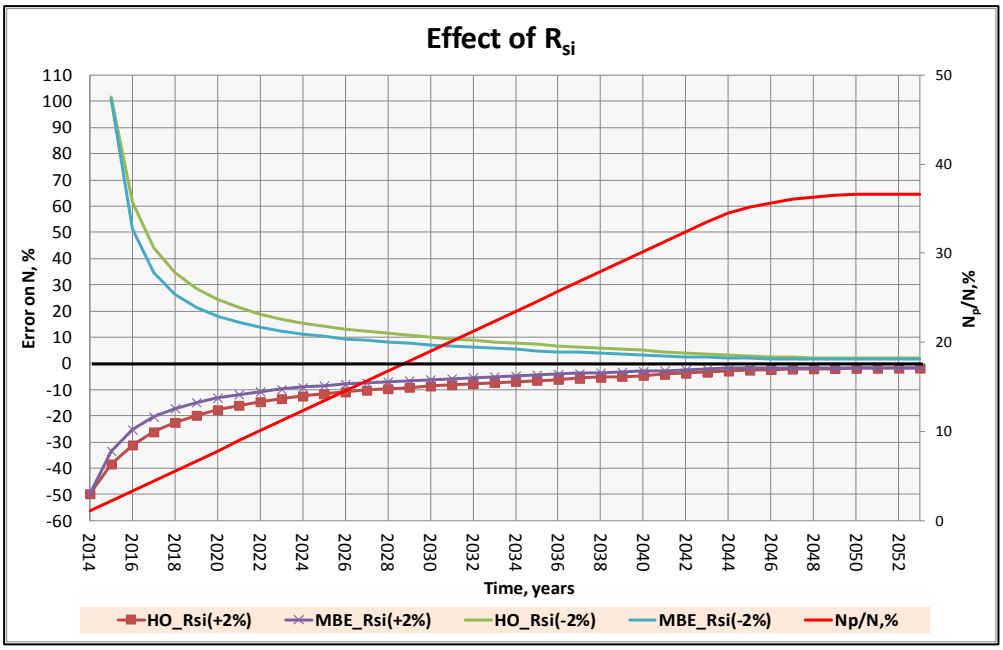

Figure 5.3 : Effect of R_{si}.

5.1.1.3 Effects of R_s&R_{si}

If we assume that +2% or -2% of systematic error is applied to R_s and R_{si} values together, the error on N values are very small compared to their single cases (Figure 5.4). The magnitude of the error on N is maximum 2.5 % during the entire production period. It means that N value is not affected too much when systematic 2 percent error on R_s and R_{si} is applied together. It can be thought that these errors of R_{si} and R_s on N value are cancelling each other.

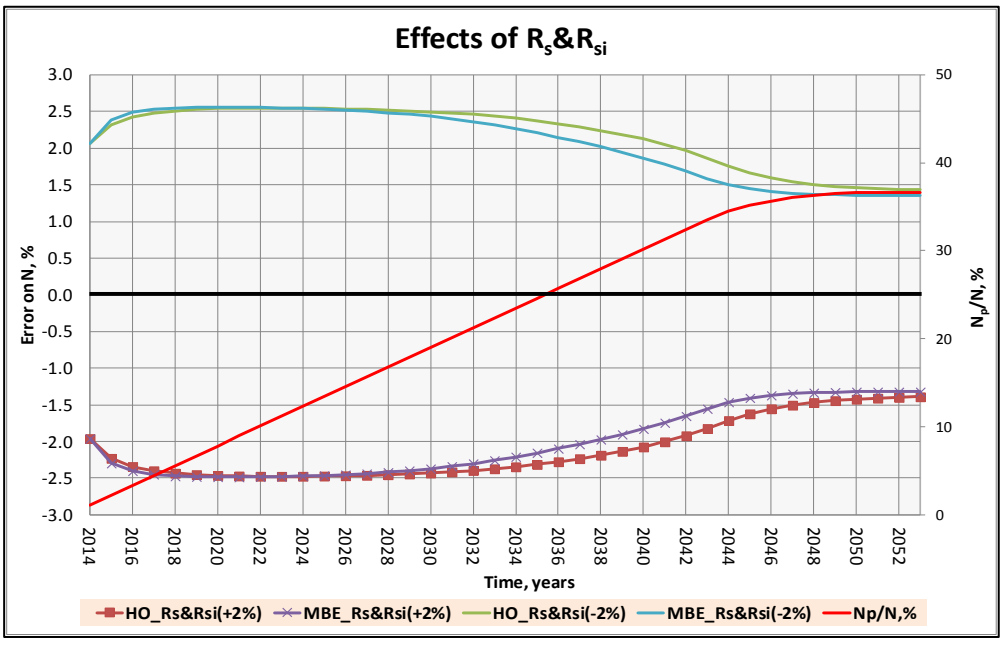

Figure 5.4 : Effects of R_s&R_{si}.

Figure 5.5 shows the effects of R_s, R_{si} and $R_s\&R_{si}$ on N on the same graph. Note that the error on N when +2 percent error is introduced to R_s and R_{si} is very close to zero as if the errors in R_s and R_{si} are cancelling each other. However when the systematic error is increased to 10%, the error on N is getting larger.

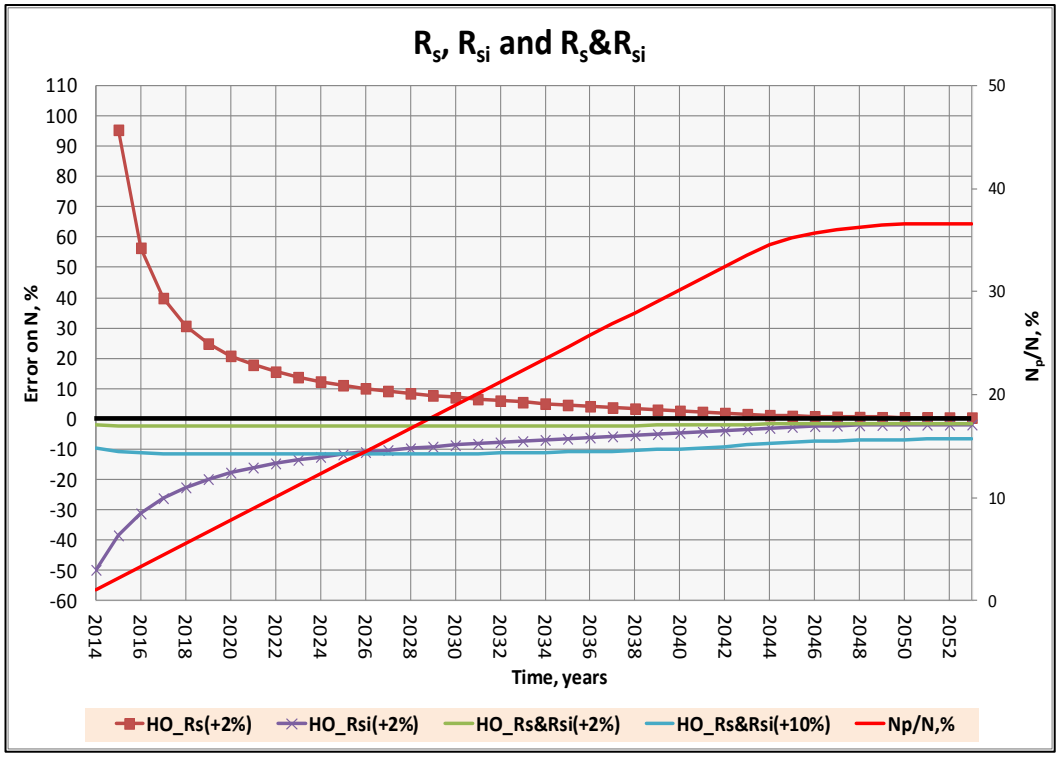

Figure 5.5 : Effects of R_s, R_{si} and $R_s\&R_{si}$.

5.1.1.4 Effect of B_o

When B_o values are multiplied by 1.02 to include 2% systematic errors, it is seen that B_o values from year 2014 to 2018 (included) are bigger than the B_{oi} value. Table 5.2 gives the calculations of B_o values at these time steps. So these calculations of N values with +2% systematic errors are excluded in the plots.

Table 5.2 : Error introduced B_o values.

Date	P,psi	B_o,rb/STB	B_o,rb/STB (+2% error introduced)
2013	P_i=3017.54	B_{oi}=1.40153	
2014	2965.80	1.3990	**1.4270**
2015	2926.21	1.3935	**1.4214**
2016	2888.35	1.3883	**1.4160**
2017	2851.44	1.3831	**1.4108**
2018	2815.70	1.3782	**1.4057**
2019	2781.34	1.3734	1.4009

Figure 5.6 illustrates the results of the effect of B_o on N calculations. +2% and -2% systematic errors were applied to B_o values. -2% systematic error on B_o gives hypersensitivity (hypersensitivity is a term used to define the errors greater than 100%) on Havlena-Odeh calculated N values and MBE calculated N values in the first 3 years production period. Table 5.3 gives these hypersensitivity values.

Table 5.3 : Hypersensitivity values of -2% of B_o errors on N.

Date	Np/N,%	HO results of -2% B_o effect	MBE results of -2% B_o effect
2014	1.1170	-224.2160023	-224.2160023
2015	2.2339	-152.0186573	803.2242493
2016	3.3509	49.91268766	140.2428151

When +2% error on B_o applied, the resulting N values are 29 % and 21.1 % smaller than the reference value at the end of a 6-year production period (year 2019) for Havlena - Odeh and MBE calculations, respectively.

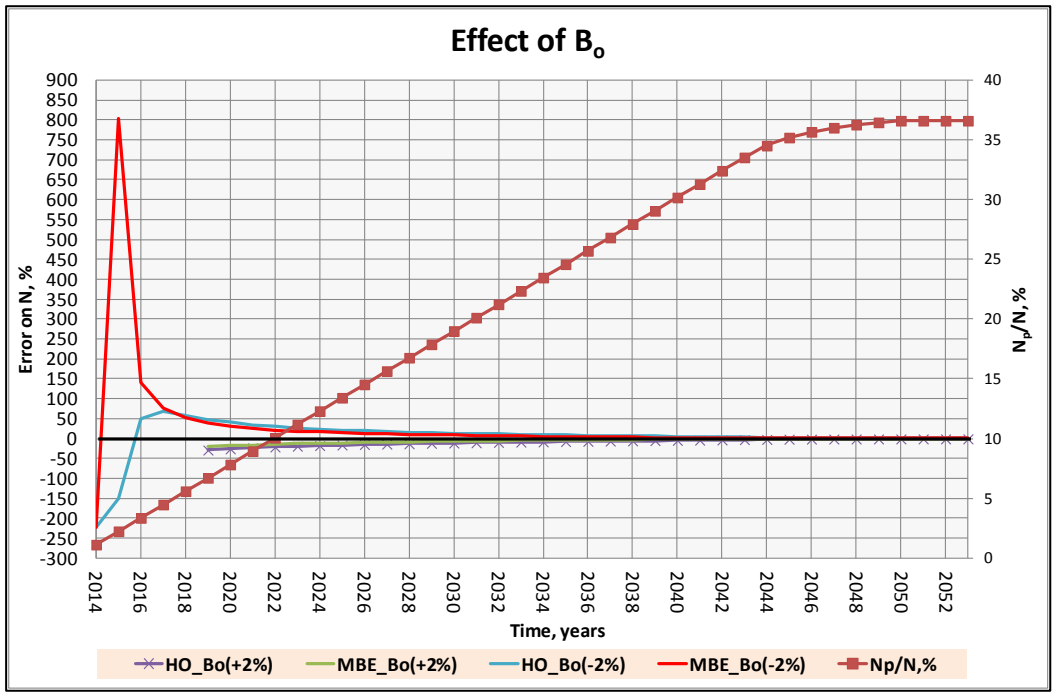

Figure 5.6 : Effect of B_o.

5.1.1.5 Effect of B_{oi}

Table 5.4 tabulates the B_o values which are greater than B_{oi} value when B_{oi} value is multiplied by 0.98 to include -2% systematic errors for B_{oi} effect on N. For that reason, MB calculations from year 2014 to 2018 (included) were not shown in the plot of B_{oi} effect (Figures-5.7 and 5.8).

Table 5.4 : Error introduced B_{oi} value.

Date	P,psi	B_o, rb/STB	B_{oi}, rb/STB (-2% error introduced)
2013	P_i=3017.54	B_{oi}=1.40153	1.37350
2014	2966	**1.39897**	
2015	2926	**1.39352**	
2016	2888	**1.38827**	
2017	2851	**1.38314**	
2018	2816	**1.37818**	
2019	2781	1.37343	
2020	2748	1.36883	

B_{oi} effect of +2% systematic error on N value shows hypersensitivity when recovery factor is low (Figure 5.7). Table 5.5 gives these hypersensitivity values.

Table 5.5 : Hypersensitivity values of +2% of B_{oi} errors on N.

Date	Np/N,%	HO results of +2% B_{oi} effect	MBE results of +2% B_{oi} effect
2014	1.1170	-228.5180691	-228.5180691
2015	2.2339	-147.7182284	761.9894295
2016	3.3509	54.8315723	141.2887103

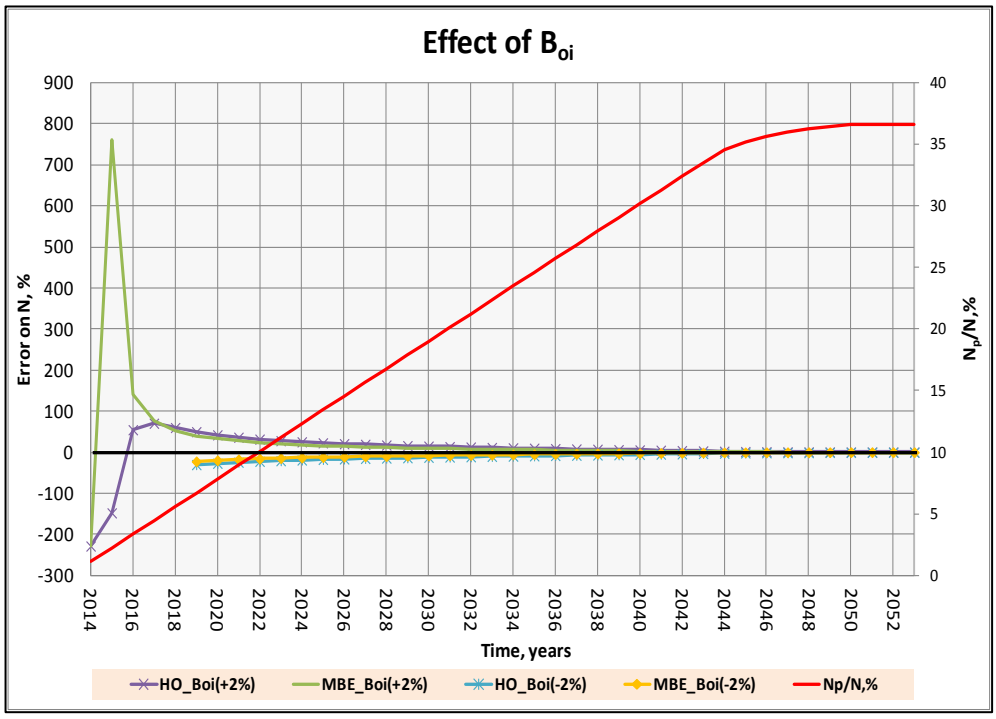

Figure 5.7 : Effect of B_{oi} (Hypersensitivity).

In Figure 5.8, the magnified plot the errors on N versus time under the effect of +2 % of B_{oi} can be seen if the hypersensitivity effects are excluded. It is important to note

that N value resulted from Havlena-odeh calculations under the influence of +2 % of B_{oi} error is 77% higher than the reference value in the year 2017 when the recovery factor is 4.47% and it is 31.8 % higher than the reference value when RF is 10%. The influence of -2% error on B_{oi} has relatively low effect than +2% error on B_{oi}. At the end of 6-year production period, the error on N is 50% when +2% error on B_{oi}, however, it is 30% when -2% error on B_{oi} is applied.

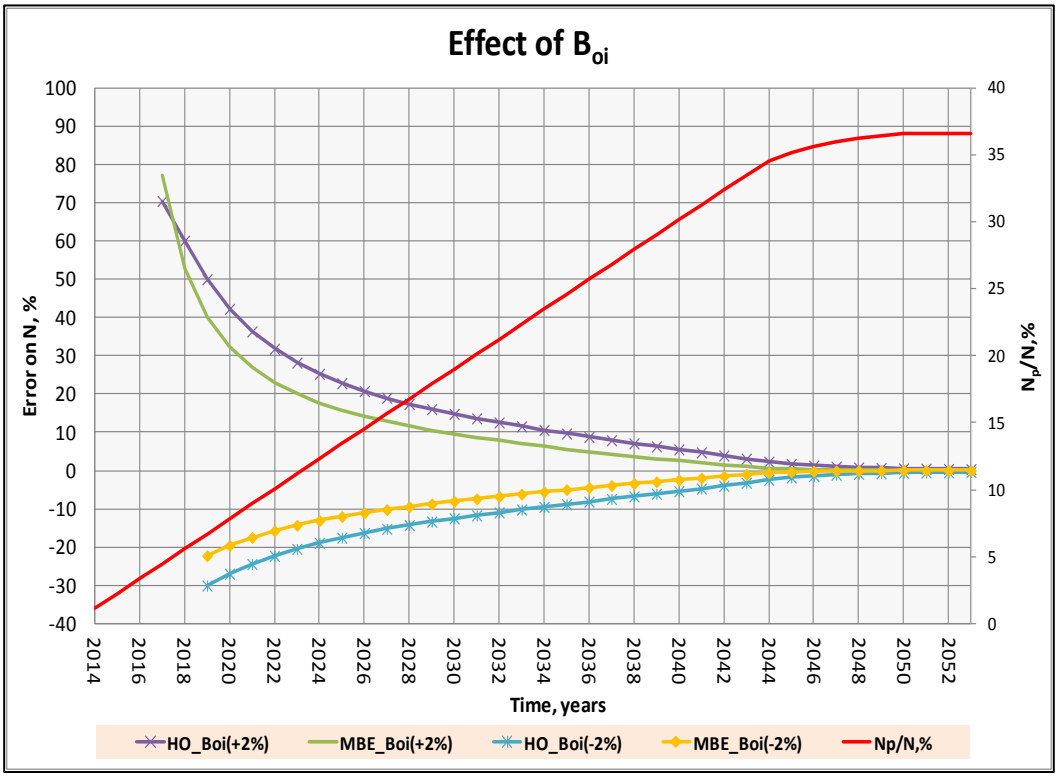

Figure 5.8 : Effect of B_{oi}.

5.1.1.6 Effects of B_o&B_{oi}

If +2% or -2% of systematic error is applied to each of B_o and B_{oi} values together, the errors on N are very small compared to their single cases (Figure 5.9). The magnitude of the error on N is maximum 1.4 % during the entire production period. It means that N value is not affected too much when systematic 2 percent error on B_o and B_{oi} is applied together.

+10% error for B_o and B_{oi} was also applied on the calculated values of N to see whether they are offsetting each other or not in the model given (Figure 5.10). This has proved that if systematic error was applied to B_o and B_{oi} values together, the magnitude of error on N does not change too much. It means that the effects of the same magnitude of errors for B_o and B_{oi} are small on the calculated values of N.

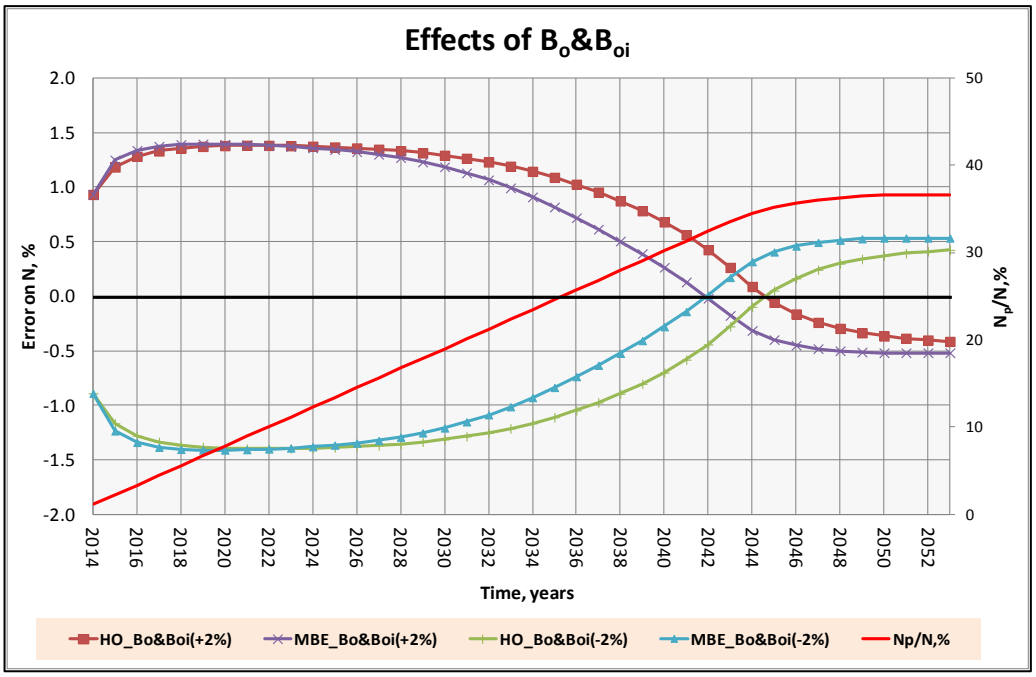

Figure 5.9 : Effects of B_o&B_{oi}.

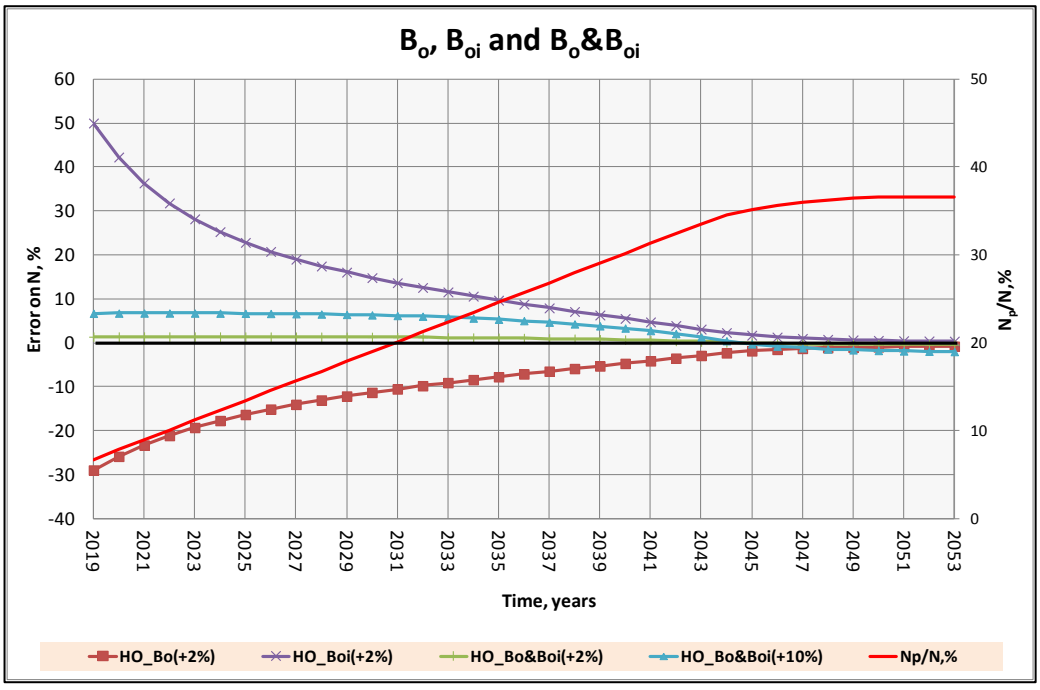

Figure 5.10 : Effects of B_o, B_{oi} and B_o&B_{oi}.

5.1.1.7 Effect of B_g

As shown in Table 5.6, when B_g value is multiplied by 0.98 to include -2% systematic errors for B_g effect on N value, B_g value at the first time step becomes lower than B_{gi} value. Since it is unphysical to have such a B_g value, MB calculations at -2% of B_g error on N at the first time step (year 2014) is excluded and not shown in the plot of B_g effect (Figure 5.11).

Table 5.6 : Error introduced B_g values.

Date	P,psi	B_g, rb/scf	B_g, rb/scf (-2% error introduced)
2013	P_i=3017.54	B_{gi}=0.00093324	
2014	2965.80	0.000947962	**0.000929002**
2015	2926.21	0.000959522	0.000940332
2016	2888.35	0.000971042	0.000951621

Figure 5.11 says that the error on N when -2% error of B_g value is applied is much more higher than the errors on N when +2% error of B_g value is applied at the end of 2-year production period. At this time step, the error on N (HO result) is nearly 95% when -2% error applied to B_g.

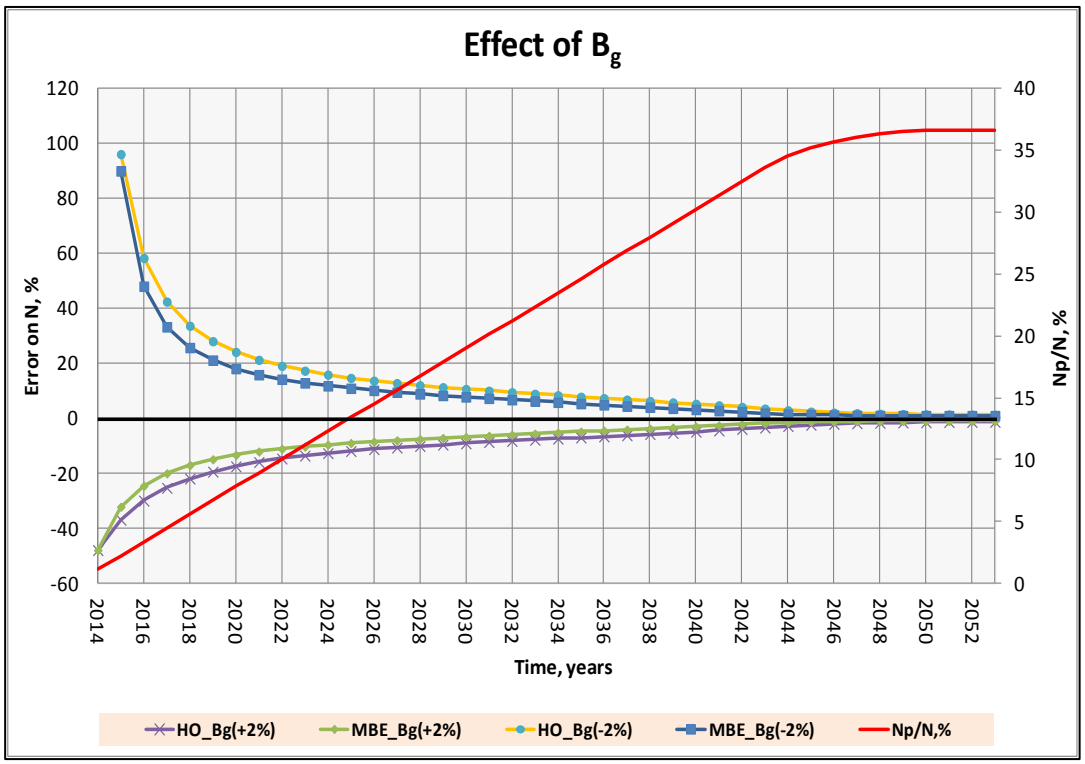

Figure 5.11 : Effect of B_g.

5.1.1.8 Effect of B_{gi}

As discussed in the case of -2% systematic errors for B_g effect on N value, when B_{gi} value is multiplied by 1.02 to include +2% systematic errors for B_{gi} effect on N value, B_g value at the first time step becomes lower than B_{gi} value (Table 5.7). Therefore, MB calculations at +2% of B_{gi} error on N at the first time step (year 2014) are excluded in the plot of B_{gi} effect (Figure 5.12).

Table 5.7 : Error introduced B_{gi} value.

Date	P,psi	B_g, rb/scf	B_g, rb/scf (-2% error introduced)
2013	P_i=3017.54	B_{gi}=0.00093324	
2014	2965.80	0.000947962	**0.000929002**
2015	2926.21	0.000959522	0.000940332
2016	2888.35	0.000971042	0.000951621

Figure 5.12 illustrates the results of B_{gi} effect on N value. +2% and -2% systematic errors were applied to B_o values. The plots of the B_{gi} effect are very similar to the plot of B_g effect (Figure 5.11). However, the error values on N are reversed for +2% error of B_{gi} and -2% error of B_{gi}.

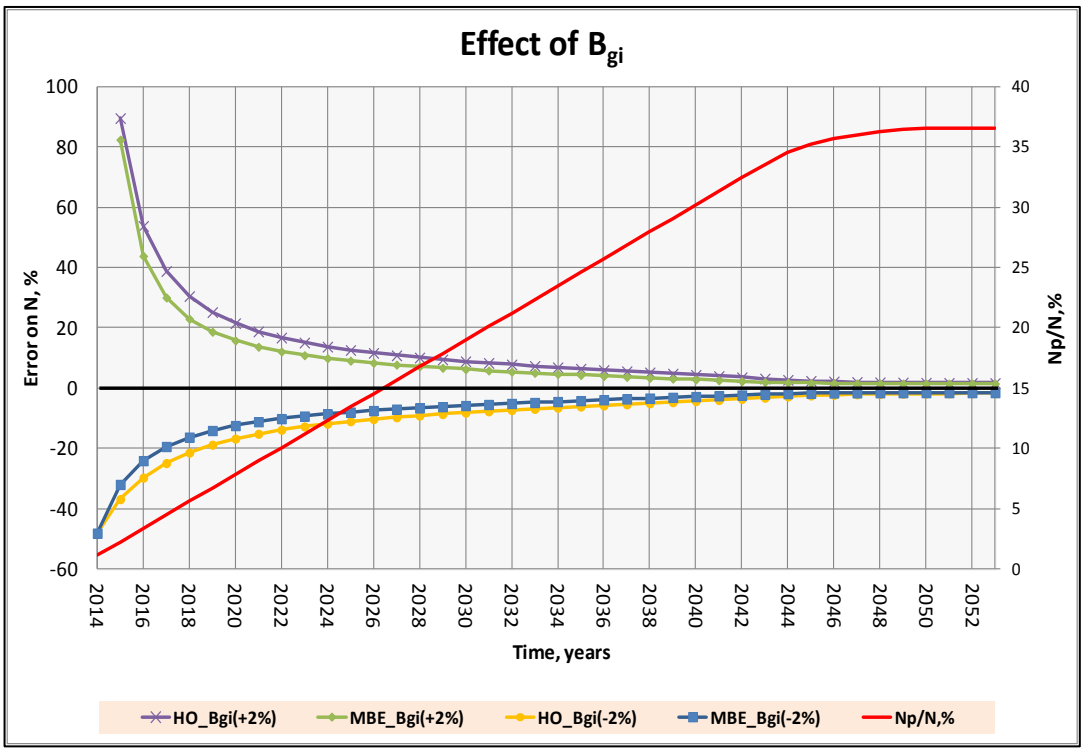

Figure 5.12 : Effect of B_{gi}.

5.1.1.9 Effects of B_g&B_{gi}

As shown in Figure 5.13, when +2% or -2% of systematic error is applied to each of B_g and B_{gi} values together, the maximum error on N values is 1.4 % during the entire production period.

One can see from Figure 5.14 that although +2% error applied to B_g and B_{gi} together has small effect on the calculated values of N, the effect increases when +10% error for B_g and B_{gi} is used in the calculations.

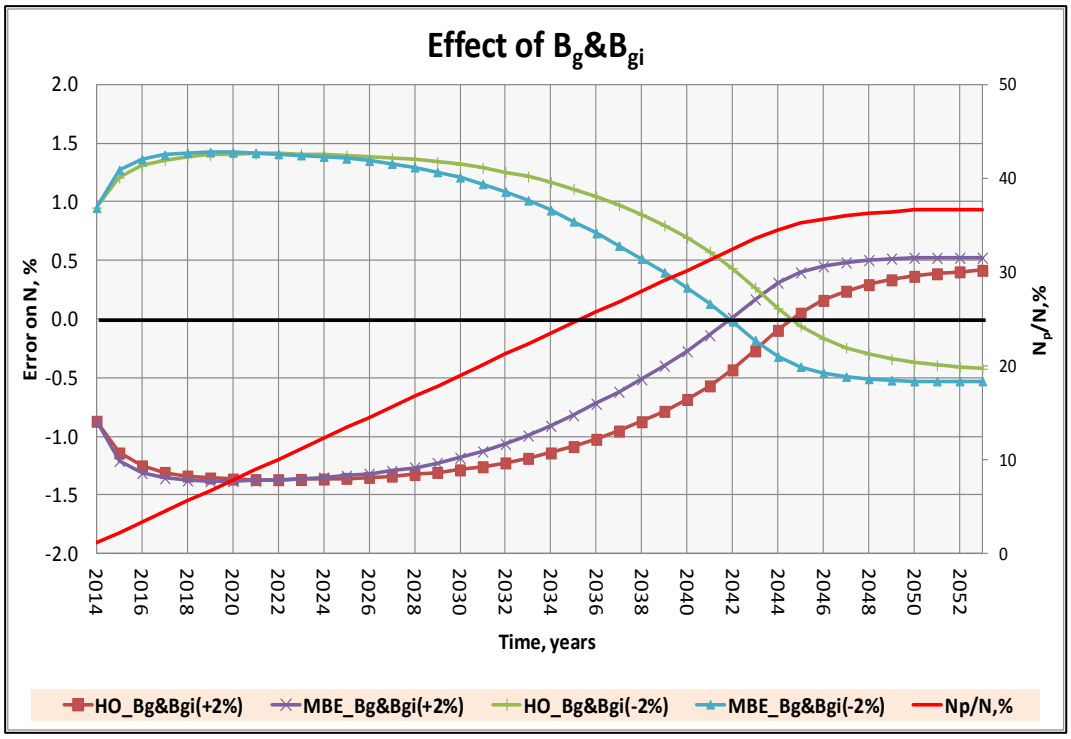

Figure 5.13 : Effects of B_g&B_{gi}.

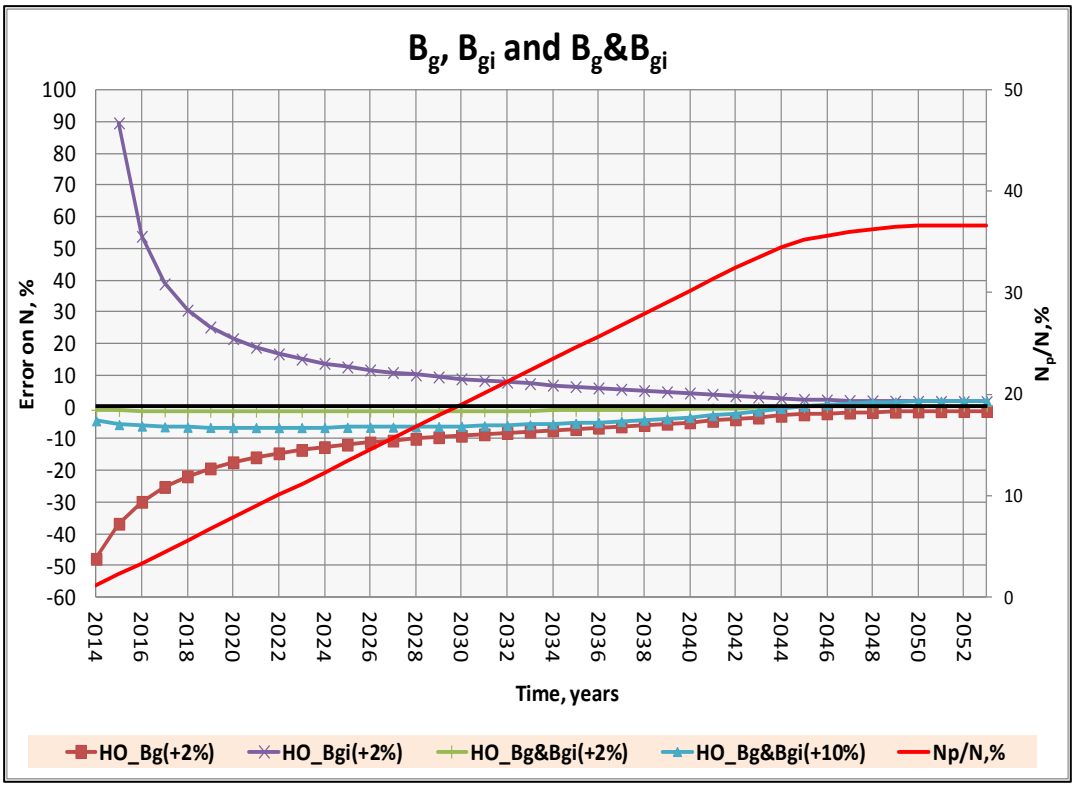

Figure 5.14 : Effects of B_g, B_{gi} and B_g&B_{gi}.

5.1.1.10 Effects of all PVT data

In order to see the effects of each PVT on the calculated values of N together, Havlena-Odeh results were illustrated in this section.

Comparison of all PVT parameters (+2 % error)

Figure 5.15 shows the comparison of the errors on N value when +2% systematic error is introduced to PVT parameters.

N value decreases when one of B_o, B_g and R_{si} is increased by +2 %. On the other hand, as we increase the values of one of B_{oi}, R_s and B_{gi} by 2%, N values increase. The most effective parameter is B_{oi} in the error on N. B_{gi} and R_s are less effective than B_{oi} on the results of error calculations on N and their magnitudes are nearly the same during the whole production period. Another point in Figure 5.15 that, all errors on N converges to zero towards the end of the production period.

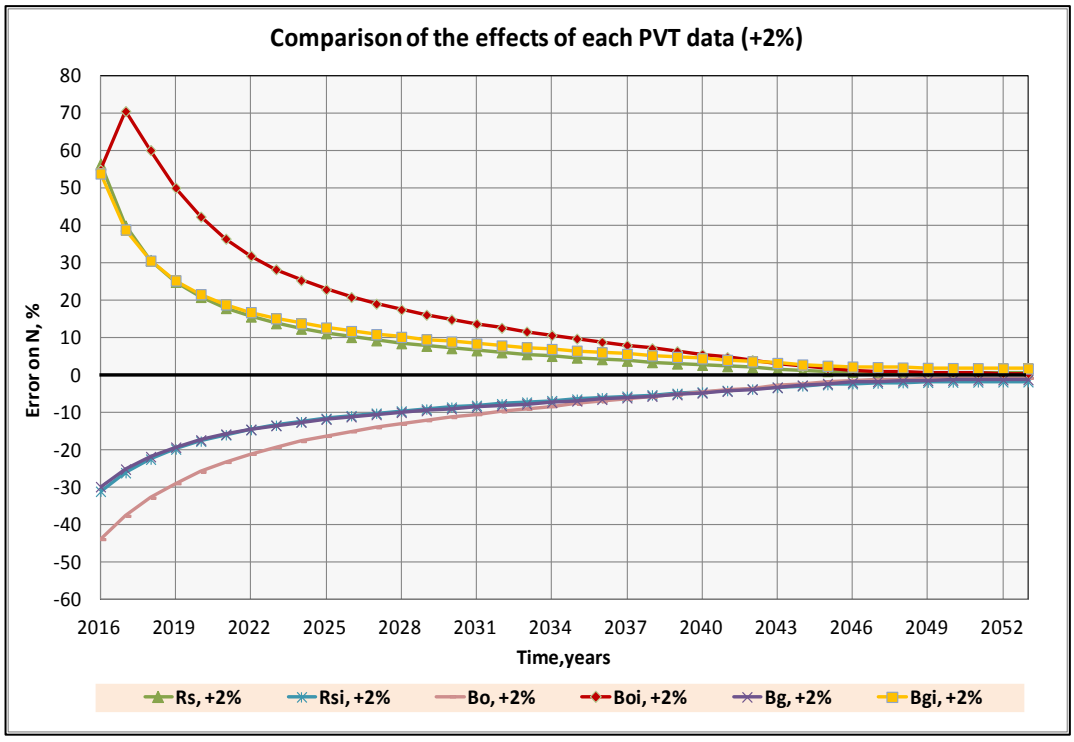

Figure 5.15 : Comparison of the effects of all PVT data (+2%).

Comparison of all PVT parameters (-2 % error)

As seen in Figure 5.16, when we decrease the value of a PVT parameter by 2 %, results are completely opposite to the results shown in Figure 5.15. N values increase when one of B_o, B_g , R_{si}, is decreased by 2 %. On the other hand, the decrease in the values of each B_{oi}, R_s or B_{gi} by 2% causes the calculated N values decrease. When -2% error for a PVT parameter is introduced to material balance calculation, we see that the most effective parameter is B_o in this case. The effects of B_g and R_{si} are very similar to each other.

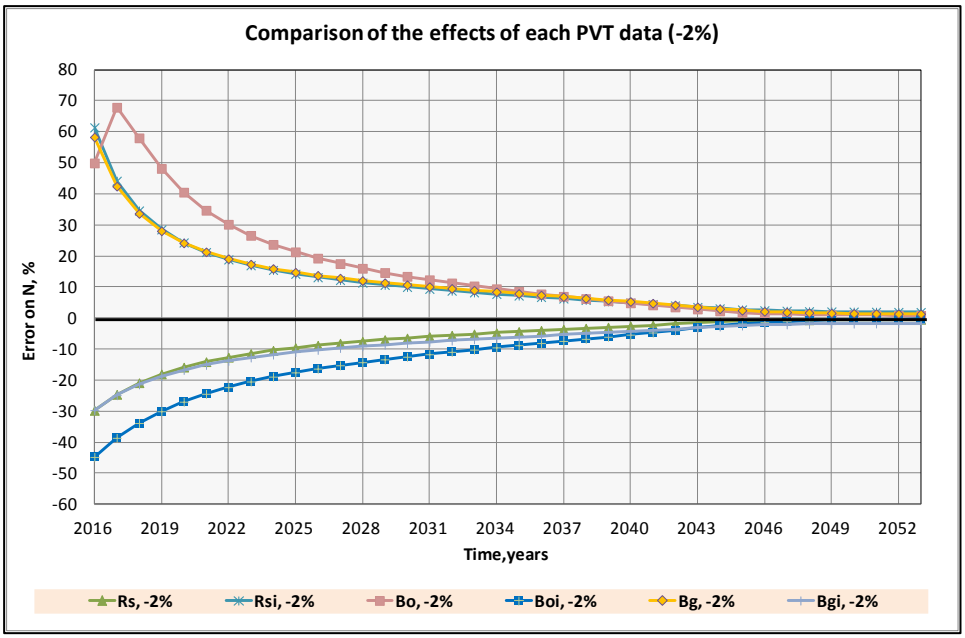

Figure 5.16 : Comparison of the sensitivities of all PVT data (-2%).

5.1.2 Effects of production data (N_p&G_p)

Production data (N_p&G_p) are generally measured correctly since the companies gain their revenues from the sales of oil and gas. In our case, assuming that the equipments used for measurement are not well calibrated and some measurement errors occur in the measurement of production of oil and gas and -10% error exist on the measured values of N_p and G_p. The errors on N values from MBE and Havlena-Odeh due to the effects of N_p and G_p are represented in Figure 5.17.

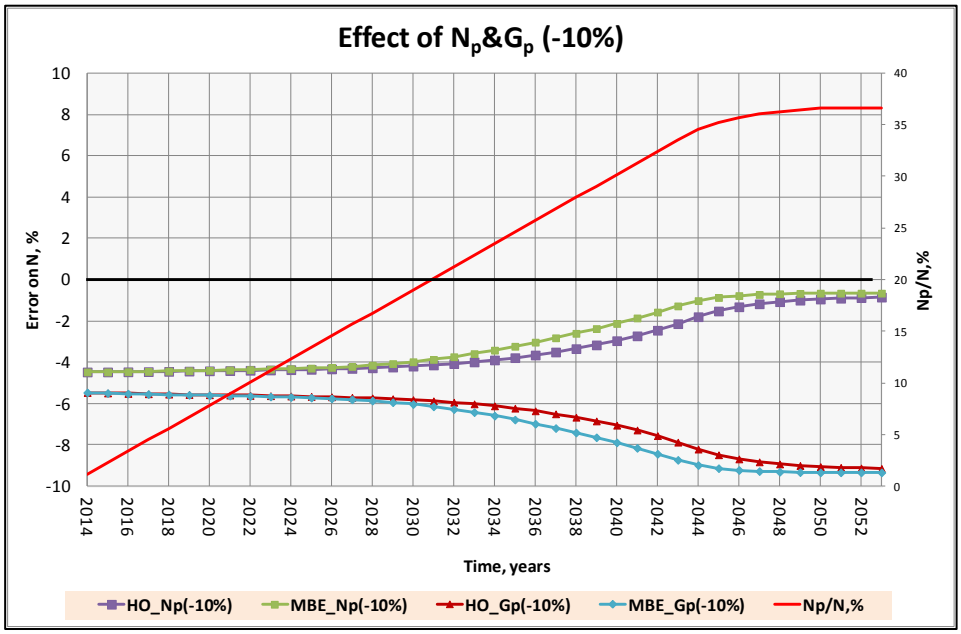

Figure 5.17 : Effects of N_p and G_p.

The errors on N due to the error on N_p is less than the error due to the error in G_p. It is also important to note that the error on N due to N_p is getting smaller after 20% recovery of oil. However, the error on N due to the error on G_p is increasing after 20% of recovery. The reason for this behaviour may be related to the increase in gas production. See Figure 4.16.

5.1.3 Effect of m value

The ratio of initial gas volume to initial oil volume (m) is higly uncertain parameter in the material balance calculations. In our case, although we exactly know the value of m, we will introduce +/- 50% error to the value of m in order to analyze the effect of m on N. The errors on N value resulting from the effect of m are represented in Figure 5.18.

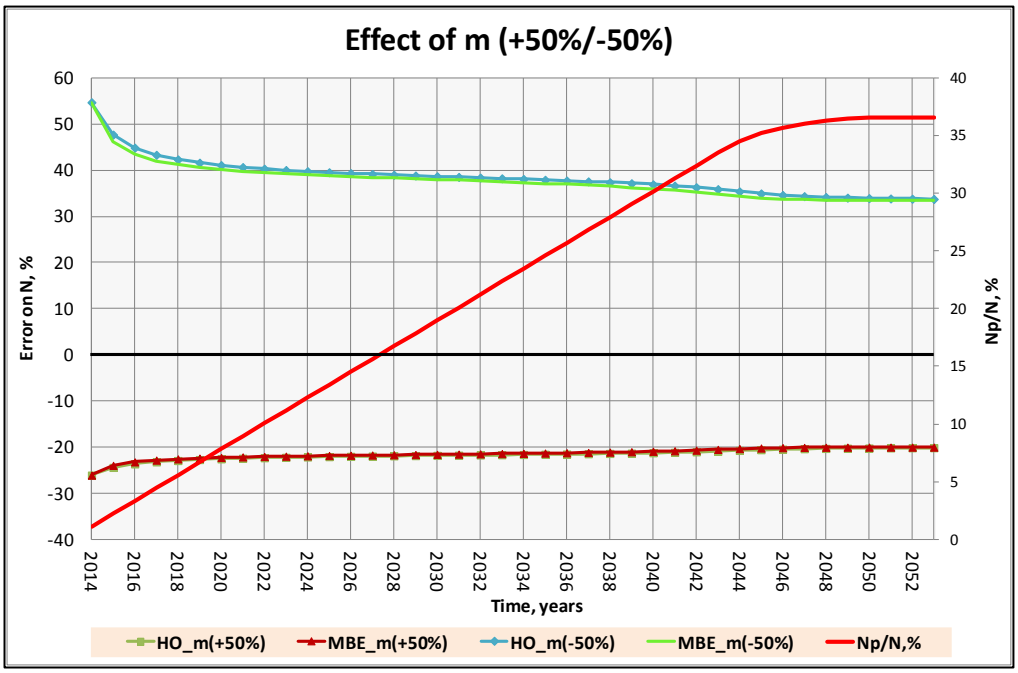

Figure 5.18 : Effect of m.

As shown in Figure 5.18, when m value is estimated 50 % higher than the original value, the calculated N values decrease by an average of 22%. However, the underestimation of m value 50% lower than the true value gives N values approximately 40% more than the reference value of N. The errors on N are stabilizing after 5-year production period when errors of m are introduced to each time step calculations. In other words, the errors on N are nearly the same during the entire production period and the errors are not converging to zero as in the case of the effects of PVT data on N.

5.2 Effects of parameters on N (m is unknown)

In this section, assuming that m value is not known, HO technique described in Section 3.4.1 will be applied to estimate the values of N and m values from the plot of F/Eo versus Eg/Eo. +2% systematic errors for PVT and -10% systematic errors for production data will be applied. Figure 5.19 shows the results of these sensitivity results.

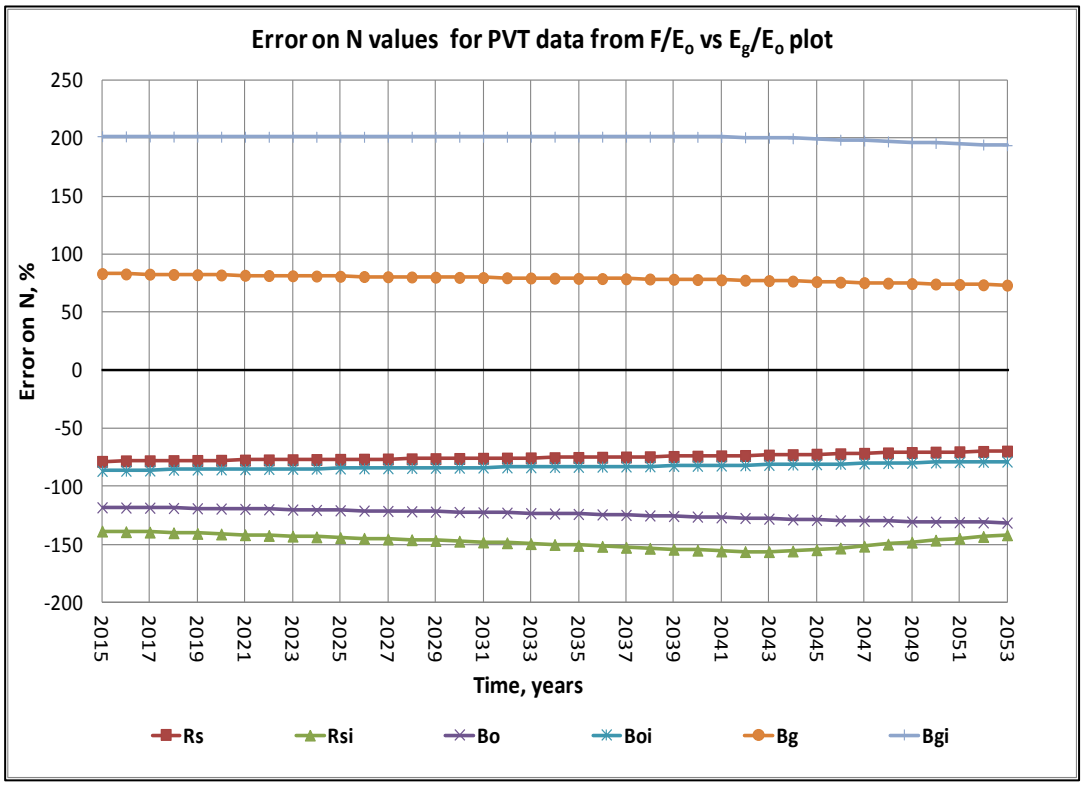

Figure 5.19 : Error on N values for PVT data from F/E_o vs E_g/E_o plot.

In Figure 5.19, it is seen that the calculated N values when applied +2 error to R_{si} or B_o are all more than 100% less than the reference value of N. The errors on N for other PVT parameters are between -80% to 200%. Another important point is that the errors on N are nearly constant during the entire production period for all PVT parameters.

The results of production data effect can be depicted in Figure 5.20. The behaviour of the plots are parallel with the method of HO when m is known. The maximum errors on N resulting from the errors in N_p and G_p are more than the case where m is known.

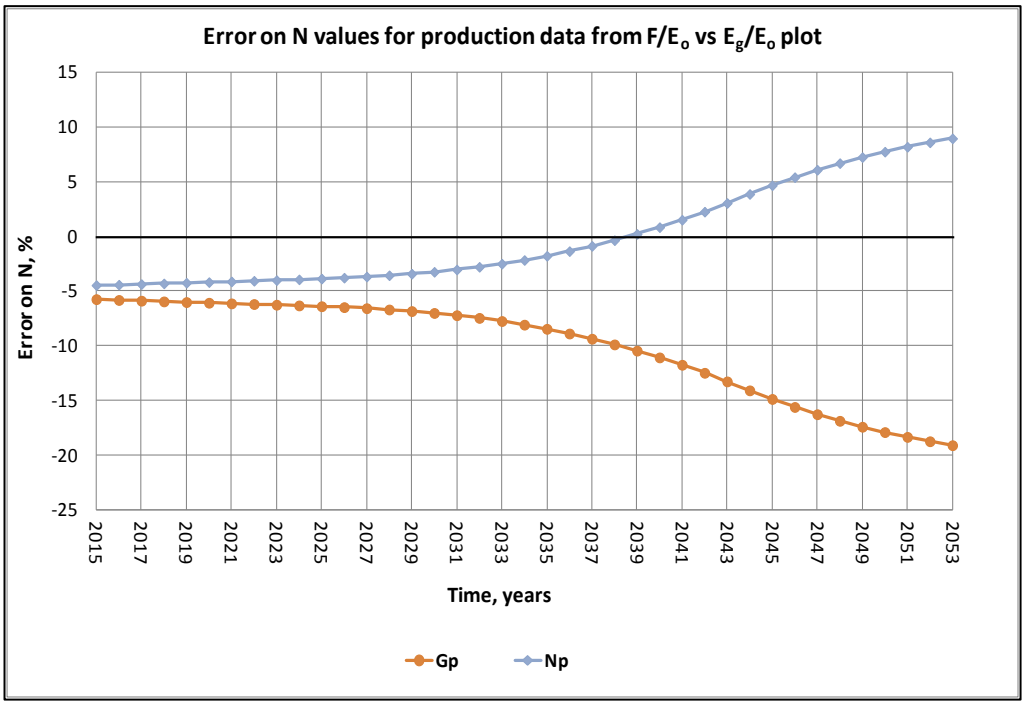

Figure 5.20 : Error on N values for production data from F/E_o vs E_g/E_o plot.

Figure 5.21 shows the results of m calculations from the plot of F/E_o vs. E_g/E_o for different parameters. All errors on m are more than 100 %.

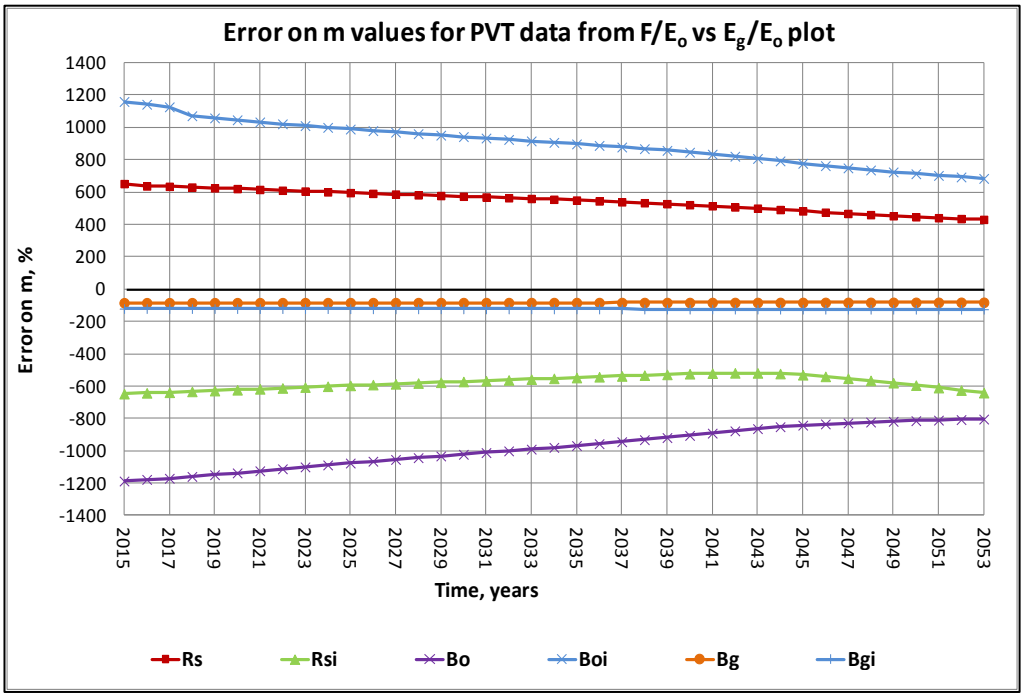

Figure 5.21 : Calculation of m from F/E_o vs E_g/E_o plot.

The comparison of the results of the methods of HO when m is known or m is unknown states that the method of plotting F/E_o vs. E_g/E_o for our gas cap reservoir with no water injection or aquifer support shows poor tolerance to PVT data

uncertainty. On the other hand, F/E_o vs. E_g/E_o plot gives more tolerable results to production data uncertainty.

5.3 Error Analysis by Sensitivity Coefficients

Sensitivity coefficients can be used to understand the behaviour of the effects of the parameters to material balance calculations. They are used to determine which parameter has more effect on N value calculations and to understand the behaviour of the plots. Appendix-A summarizes the derivation of sensitivity coefficients for each variable in MBE for a gas cap reservoir without water influx and compressibility terms. It is important to note that the derivatives are taken with respect to ln parameter to be able to compare the results with the same unit on the same graph. The procedure for analyzing the sensitivity coefficients of B_o, B_g, R_s, B_{oi}, B_{gi}, R_{si}, N_p, G_p and m value is the same as material balance calculations when error introduced to a parameter while taking the other parameters at their true values. Systematic positive errors were applied based on Table 5.1. The calculations giving unphysical values for PVT data were not shown in the plots as explained in above sections.

Figure 5.22 shows the plots of sensitivity coefficients of each PVT variable in the MBE at each time step. In Figure 5.22, the hypersensitivity of B_{oi} appears to be analogous to the hypersensitivity of the error on N plot (refer to Figure 5.7). Figure 5.23 is just a magnified view of Figure 5.22 to see the results better.

Figure 5.23 reveals that the most effective PVT parameter in MBE is B_{oi}. B_{gi} and R_s are the parameters which show also big influences on N calculations. The effects of other PVT parameters (B_o, B_g, R_{si}) to N calculations are rather less than B_{gi} and R_s sentivities. The results of sensitivity coefficients vs time plot are parallel with all PVT sensitivities (+2%) plot given in Figure 5.15. While the errors on the PVT parameters such as B_{oi}, B_{gi} and R_s are increasing the N values, the other PVT parameters such as B_o, B_g and R_{si} are decreasing the values of N. The magnitudes of the sensitivity coefficients for B_{gi} and R_s are nearly the same through the entire production life of the field. The same situation is true for the sensitivity coefficients of B_g and R_{si}.

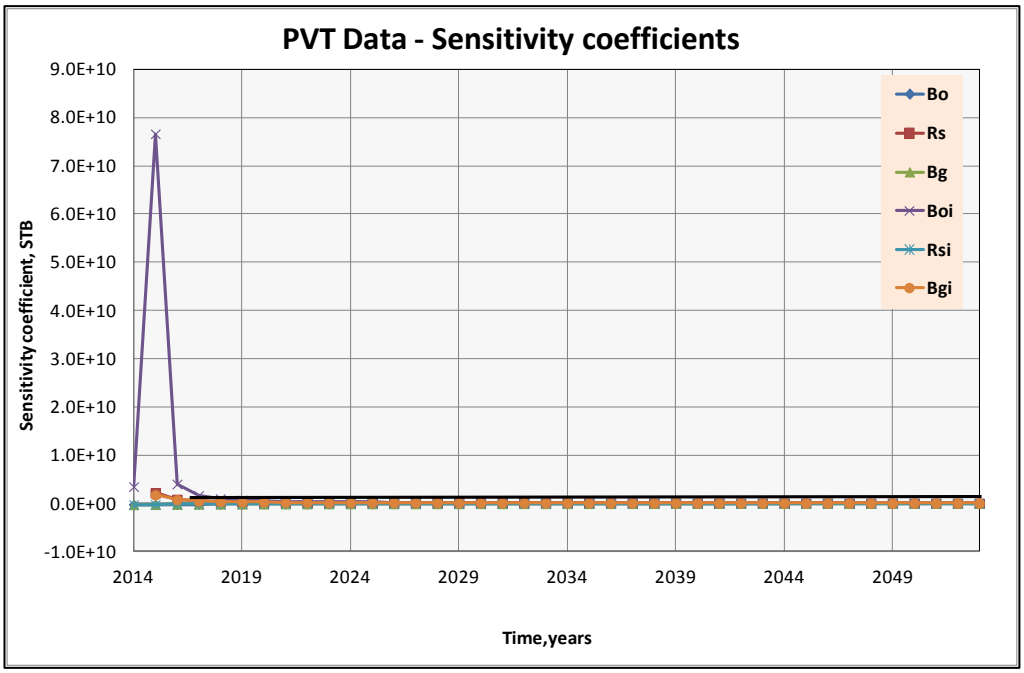

Figure 5.22 : Sensitivity coefficients for PVT data vs time plot.

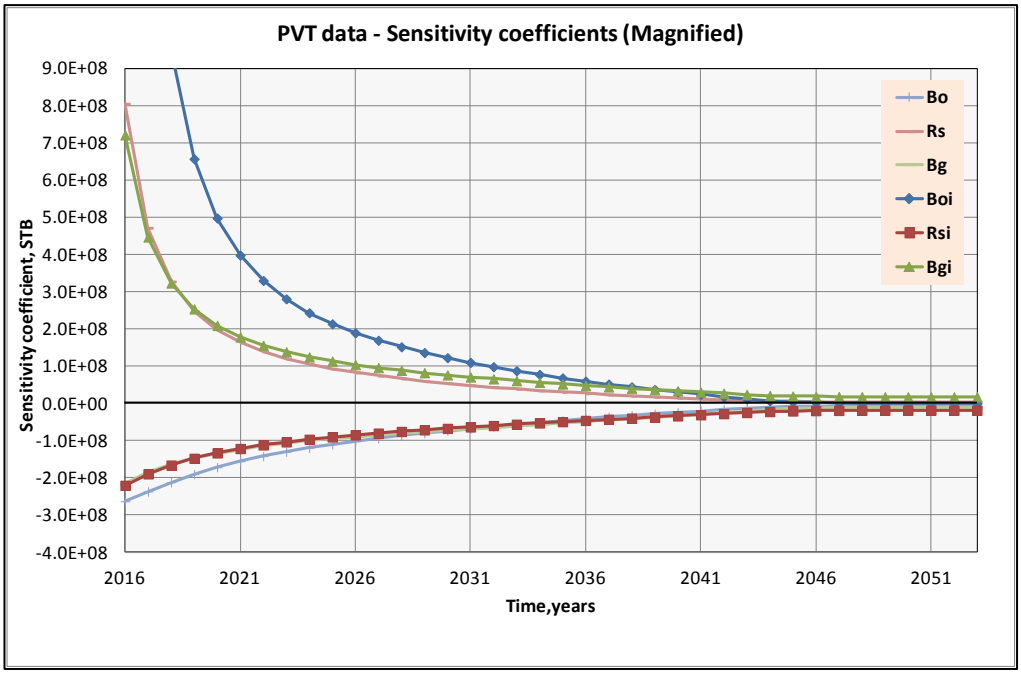

Figure 5.23 : Sensitivity coefficients for PVT data vs time plot (magnified).

As shown in Figure 5.24, the order of magnitudes of the sensitivity coefficients for production data (N_p and G_p) and m value are very low as compared to the order of magnitudes of the sensitivity coefficients of PVT parameters although the error values introduced to production data and m value were chosen bigger than errors chosen for PVT parameters. The behaviours of G_p and m curves are in line with the curves given in Figure 5.17 and Figure 5.18. In other words, the curve for m is nearly

constant during the entire production period, the values of sensitivity coefficient for G_p are increasing after 20% recovery. The behaviour of N_p curve is different than the curve given in Figure 5.17.

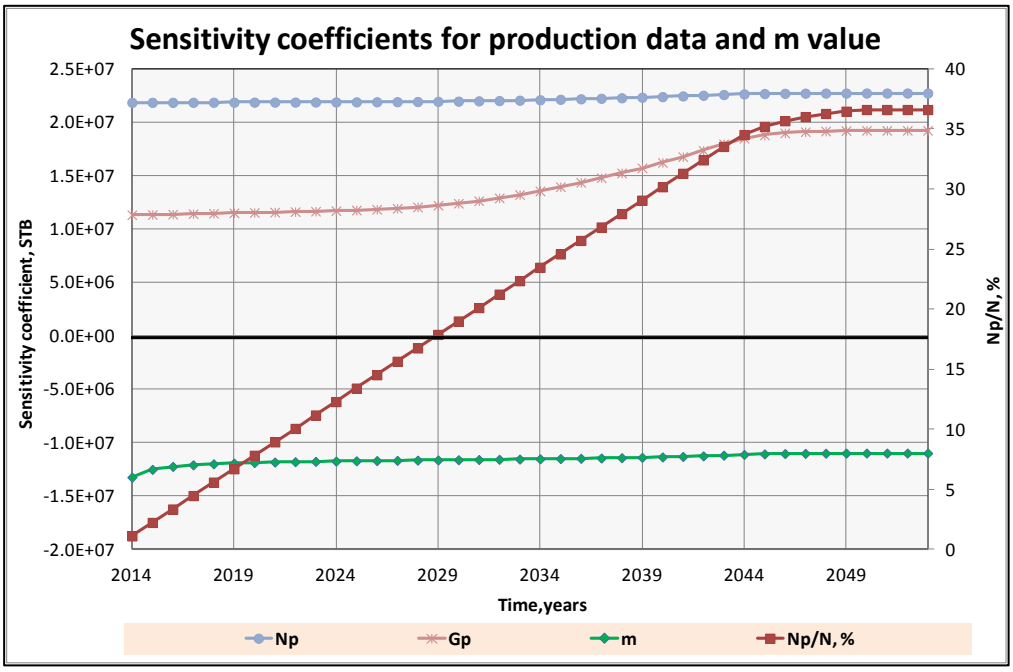

Figure 5.24 : Sensitivity coefficients for production data and m value vs time plot.

6. CONCLUSIONS

This study presents an analysis of the errors on the original oil in place calculated from material balance methos when errors are present in PVT, production data and m value. Mainly, the following conclusions can be drawn from the study:

OOIP calculations based on material balance methods are strongly affected by data uncertainty. Uncertainty due to data errors can be found in production data, PVT data and the geological parameter m.

The impacts of PVT errors on material balance calculations are observed to be significant during the early life of the reservoir for the cases considered in this study. However, the errors on the calculated values of N is approaching zero for all PVT cases studied towards the end of the production period.

MBE gives better results of OIIP than HO calculations when errors are introduced in the material balance parameters. It is an expected result since HO technique, applied for the case where m is known, uses several points to calculate OIIP value from the slope of the F vs E_o+mE_g line and all these points include errors. However, MBE uses only two sets of data points (at pressures p_i and p) and hence is only affected by the errors of the parameters at these data points.

When R_s and R_{si} values contain +/-2 percent of systematic errors together in the MB calculations, the magnitude of the error on N is small during the entire production period. It means that N value is not affected too much when +/-2 percent of systematic error on R_s and R_{si} is applied together. It can be thought that these errors in R_{si} and R_s have a cancelling effect on the calculated N values. However when the systematic error of +10 % introduced to R_s and R_{si} values together, the error on the calculated N values is also around 10% while the well produces at constant rate. The error on N decreases as the production rate of the well decreases.

The combined effect of B_o and B_{oi} or B_g and B_{gi} errors (+/-2%) on the calculations of N values from both MBE and HO methods is smaller than the combined effect of R_s and R_{si} errors.

A general conclusion can be drawn from the results of the error analysis on PVT parameters that when one of the B_o, B_g or R_{si} is increased by +2 %, the calculated N values are lower than true OIIP and N values are greater than the true OIIP if we introduce +2% error in the one of PVT parameters B_{oi}, R_s and B_{gi}. Similarly, N values increase when B_o or B_g or R_{si} is decreased by 2 %. On the other hand, as we decrease the values of one of B_{oi}, R_s, B_{gi} by 2%, N values decrease. The error on N is very huge mainly at the early life of the production period and continuously decreases towards the end of the production period.

In the case of error analysis of the production data (N_p and G_p), the error on N due to error of N_p is less than the error due to G_p. It is also important to note that the error on N due to N_p is getting smaller after 20% recovery of oil for the case studied. However, the error on N due to the error on G_p is increasing after 20% of recovery.

The estimation of m value 50 % higher than the true value of m resulted in calculated values of N that are lower than the reference value and the error on N is nearly constant during the entire production period. On the other hand, the underestimation of m value gives the calculated values of N higher and the error is 2 times more than the overestimated m values case. The errors on N are also constant during the whole production period as in the case of overestimated m case.

The method of plotting F/E_o vs. E_g/E_o for gas cap reservoir with no water injection and without aquifer support shows poor tolerance to PVT data uncertainty. On the other hand, F/E_o vs. E_g/E_o plot gives more tolerable results to production data uncertainty.

Sensitivity coefficients explain the effects of each parameter in material balance calculations. The results of the sensitivity coefficient calculations show that B_{oi}, B_{gi} and R_s are the parameters which have large effects on N calculations. The other PVT parameters have rather less sensitivity to N calculations. All results are in line with the behaviour of percentage error curves of N for the error analysis of PVT parameters.

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APPENDICES

APPENDIX A: Derivation of Sensitivity Coefficients

APPENDIX A:

Derivation of Sensitivity Coefficients

Assuming that the natural net water influx is negligible (W_e-W_pB_w = 0), the effect of water and pore compressibilities can be considered negligible and there is no water injection or gas injection, the material balance equation for a gas cap reservoir can be expressed as:

$$N=\frac{N_p\left[B_o+(R_p-R_s)B_g\right]}{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]} \quad \textbf{(A.1)}$$

or

$$N=\frac{N_pB_o+N_pR_pB_g-N_pR_sB_g}{B_o-B_{oi}+(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]}. \quad \textbf{(A.2)}$$

Rearranging Equation-A.2 for B_o gives,

$$N=\frac{N_pR_pB_g-N_pR_sB_g+N_pB_o}{(R_{si}-R_s)B_g+mB_{oi}\left[\frac{B_g}{B_{gi}}-1-\frac{1}{m}\right]+B_o}. \quad \textbf{(A.3)}$$

Taking derivative of N with respect to Bo gives,

$$\frac{\partial N}{\partial B_o}=\frac{N_p\left[(R_{si}-R_s)B_g+mB_{oi}\left(\frac{B_g}{B_{gi}}-1-\frac{1}{m}\right)\right]-N_pR_pB_g-N_pR_sB_g}{\left(\left[(R_{si}-R_s)B_g+mB_{oi}\left(\frac{B_g}{B_{gi}}-1-\frac{1}{m}\right)\right]+B_o\right)^2}. \quad \textbf{(A.4)}$$

Here, it is important to note that the unit of $\partial N/\partial B_o$ is STB^2/rb. If we take the derivative of N with respect to ln(B_o) then we have

$$\frac{\partial N}{\partial \ln(B_o)}=\frac{\partial N}{\partial B_o}B_o=\frac{N_p\left[(R_{si}-R_s)B_g+mB_{oi}\left(\frac{B_g}{B_{gi}}-1-\frac{1}{m}\right)\right]-N_pR_pB_g-N_pR_sB_g}{\left(\left[(R_{si}-R_s)B_g+mB_{oi}\left(\frac{B_g}{B_{gi}}-1-\frac{1}{m}\right)\right]+B_o\right)^2}B_o. \quad \textbf{(A.5)}$$

The unit of the derivative of N with respect to ln of any parameter will be STB. Hence, it is more consistent to compare the results with ln derivatives.

In the same way, taking the derivatives of N with respect to ln of other variables gives the following sensitivity coefficients:

Sensitivity coefficient based on B_{oi}

$$\frac{\partial N}{\partial \ln(B_{oi})} = \frac{-\left(m\frac{B_g}{B_{gi}} - m - 1\right).\left[N_p B_o + N_p B_g(R_p - R_s)\right]}{\left[B_o + B_g(R_{si} - R_s) + \left(m\frac{B_g}{B_{gi}} - m - 1\right)B_{oi}\right]^2} B_{oi} \cdot \qquad \textbf{(A.6)}$$

Sensitivity coefficient based on B_g

$$\frac{\partial N}{\partial \ln(B_g)} = \frac{N_p(R_p - R_s)\left[B_o - B_{oi}(1+m) + \left(R_{si} - R_s + \frac{mB_{oi}}{B_{gi}}\right)B_g\right]}{\left[B_o - B_{oi}(1+m) + \left(R_{si} - R_s + \frac{mB_{oi}}{B_{gi}}\right)B_g\right]^2} B_g$$

$$- \frac{\left(R_{si} - R_s + \frac{mB_{oi}}{B_{gi}}\right)\left[N_p B_o + N_p(R_p - R_s)B_g\right]}{\left[B_o - B_{oi}(1+m) + \left(R_{si} - R_s + \frac{mB_{oi}}{B_{gi}}\right)B_g\right]^2} B_g \cdot \qquad \textbf{(A.7)}$$

Sensitivity coefficient based on B_{gi}

$$\frac{\partial N}{\partial \ln(B_{gi})} = \frac{\left[N_p B_o + N_p B_g(R_p - R_s)\right]\left(mB_{oi}B_g + \left[B_o - B_{oi}(1+m) + B_g(R_{si} - R_s)\right]B_{gi}\right)}{\left[mB_{oi}B_g + \left[B_o - B_{oi}(1+m) + (R_{si} - R_s)B_g\right]B_{gi}\right]^2} B_{gi}$$

$$- \frac{\left[B_o - B_{oi}(1+m) + (R_{si} - R_s)B_g\right]\left[N_p B_o + N_p B_g(R_p - R_s)\right]B_{gi}}{\left[mB_{oi}B_g + \left[B_o - B_{oi}(1+m) + (R_{si} - R_s)B_g\right]B_{gi}\right]^2} B_{gi} \cdot \qquad \textbf{(A.8)}$$

Sensitivity coefficient based on R_s

$$\frac{\partial N}{\partial \ln(R_s)} = \frac{-N_p B_g\left[B_o - B_{oi} + B_g R_{si} + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) - B_g R_s\right]}{\left[B_o - B_{oi} + B_g R_{si} + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) - B_g R_s\right]^2} R_s$$

$$+ \frac{B_g\left(N_p B_o + N_p B_g R_p - N_p B_g R_s\right)}{\left[B_o - B_{oi} + B_g R_{si} + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) - B_g R_s\right]^2} R_s \cdot \qquad \textbf{(A.9)}$$

Sensitivity coefficient based on R_{si}

$$\frac{\partial N}{\partial \ln(R_{si})} = \frac{-B_g\left[N_p B_o + N_p B_g(R_p - R_s)\right]}{\left[B_o - B_{oi} - B_g R_s + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + B_g R_{si}\right]^2} R_{si} \cdot \qquad \textbf{(A.10)}$$

Sensitivity coefficient based on N_p

$$\frac{\partial N}{\partial \ln(N_p)} = \frac{B_o + B_g(R_p - R_s)}{\left[B_o - B_{oi} + B_g(R_{si} - R_s) + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)\right]} N_p \cdot \qquad \textbf{(A.11)}$$

Sensitivity coefficient based on m

$$\frac{\partial N}{\partial \ln(m)} = \frac{-mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)\left[N_p B_o + N_p B_g(R_p - R_s)\right]}{\left[B_o - B_{oi} + B_g(R_{si} - R_s) + B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)\right]^2} m \cdot \qquad \textbf{(A.12)}$$

Sensitivity coefficient based on G_p

If we write Equation-A.2 in terms of G_p by putting G_p/N_p instead of R_p, we get

$$N = \frac{N_p B_o + G_p B_g - N_p R_s B_g}{B_o - B_{oi} + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]} \cdot \qquad \textbf{(A.13)}$$

Taking the derivative of N with respect to lnGp gives

$$\frac{\partial N}{\partial \ln(G_p)} = \frac{B_g}{\left[B_o - B_{oi} + B_g(R_{si} - R_s) + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)\right]} G_p \cdot \qquad \textbf{(A.14)}$$

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