İSTANBUL TEKNİK ÜNİVERSİTESİ ⋆ NÜKLEER ENERJİ ENSTİTÜSÜ

COMPUTER IMPLEMENTATION OF FUZZY DECISION MAKING

(WITH AN APPLICATION TO NUCLEAR POWER PLANT SITE SELECTION)

YÜKSEK LİSANS TEZİ

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Anabilim Dalı : NÜKLEER BİLİMLER

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PREFACE

This study concerns the computer implementation of fuzzy decision–making with an application to the site selection of nuclear power plants.

Making decision about alternative nuclear power plant sites has always been a difficult problem to solve. Difficulty arises due to its complex nature since the general public's feeling of safety might be affected. It is also necessary to satisfy some conditions to overcome construction and operation difficulties. Besides that, a nuclear power plant has effect on the development of a country which is especially important for developing countries.

Some authors applied crisp rating and ranking techniques and multi-attribute utility theory because of the many interrelated factors affecting the decision of the site selection. But incorporation of objective and subjective aspects and inherent uncertainties of the future events makes the decision process complicated. In statistical decision theory, inexact knowledge about the alternatives and criteria is attributed to the uncertainty and the tools of statistics are used to arrive at an optimal final decision. But in many situations, this inexactness is due to the impreciseness, and not to the uncertainty. Fuzzy decision analysis can handle the situation in the case of the unavaliability of precise data which is most commonly the case in site selection. Also, fuzzy numbers can take into account the imprecision of the factors affecting decision–making.

The main enforcement behind this study was the encouragements of my supervisor Prof. Dr. Şarman GENÇAY willing to make the application of this analysis practical with computer codes. In computer programs the formulation given in the literature were used. Here also I wish to express my special thanks to him giving me great support in both developing and preparing this study.

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Hakan BOYACI

SUMMARY

In this study, the problem of ranking alternative sites for nuclear power plants across a set of criteria and subcriteria using fuzzy decision analysis is discussed and computer codes for ranking these alternatives both mathematically and graphically are developed. Experts' opinions are employed to rank the alternatives. Analysist (or decision maker) assigns fuzzy numbers to the experts' rankings. Then, using the fuzzy arithmetic final weights, $\overline{w}_i$, which are the fuzzy ranking assigned to alternatives A_i for $i = 1, ..., m$ are obtained. Comparing final weights, alternatives are put into order from best to worst. A list of criteria and subcriteria is given for the assesment of nuclear power plant sites. An example of site selection to illustrate the method in detail is given also.

In this method, experts do not have to be involved in the highly mathematical nature of the fuzzy decision theory. They make their ranking using ten point scale but they are aware that impreciseness in their ranking is expressed by the fuzzy numbers. Fuzzy numbers can handle the situation in the case of the unavailability of precise data which is most commonly the case in site selection. These fuzzy numbers are assigned to the experts' ranking by the analyst. Experts express their ranking by giving one or two numbers depending on his or her evaluation about the inherent impreciseness of the ranking.

It is expected that some authorities may be interested with the applications of fuzzy methods to the decision analysis related nuclear engineering, in parallel with increasing interest to the application of fuzzy thinking to the problems in many fields, because that fuzzy numbers can take into account the imprecision of the factors effecting decision making such as the site meteorology, population growth, cooling water availability, etc. In such circumstances this procedure may appear worthy for the future applications.

ÖZET

Bu çalışmada, nükleer santrallerin yer seçiminde ortaya çıkan alternatiflerin, seçilmiş kriter ve altkriterler gözönüne alınarak, bulanık karar analizi yöntemi ile sıralanması problemi üzerinde tartışılmış ve bu sıralamayı gerçekleştiren bir bilgisayar programı geliştirilmiştir. İlkönce konvansiyonel çok kriterli seçme metodları kısaca tanıtılmış, avantaj ve dezavantajları verilmiştir. Sonraki kısımda bulanık mantık kavramı ve gerekliliği açıklanmış ve uygulamadaki bazı örneklerden kısaca söz edilmiştir. Bir sonraki bölümde ise bulanık kümeler ve bulanık sayılar açıklanmış ve bazı basit özellikleriyle beraber üzerlerinde yapılan operasyonlar örnekleriyle özetlenmiştir. Bulanık mantık kullanılarak oluşturulmuş bir karar verme analizi etraflıca açıklanmış ve bu analiz yardımıyla yapılan sıralama anlatılmıştır. Yöntem, alternatiflerin sıralanmasını bir veya daha fazla sayıda uzmanların değerlendirmelerinden yararlanarak yapmaktadır.

Karar verme sürecinde, karar verici veya karar vericiler, çeşitli alternatifler arasından ve çeşitli ölçütleri (kriterleri) dikkate alarak seçim yapma veya sıralama yapma durumundadırlar. Alternatif ve ölçütlerin sayısı yüksek ise karar verme oldukça güç bir iştir. Olayların objektif ve sübjektif esasları ile gelecekteki oluşumların doğasında varolan belirsizlik sonuca gitmeyi zorlaştırır. Sayısallaştırılması mümkün olayların kolayca dikkate alınabilmesine karşılık, örneğin, nükleer güç santralı yerleşim alanındaki taşıma ve nakil imkanları ile gelecekte civarın nüfus durumu ve öneminin değerlendirilmesi kolay değildir. Böyle bir değerlendirme- nin 0–10 arasında değişen bir puanlama ile kesin belirtilmesi hem zordur, hem de uzmandan uzmana çok farklılıklar gösterebileceği gibi yalnışlıklara da sebep olabilir.

Bu gibi durumlarda söz konusu olan yalnışlık, eksiklik (inexactness), durumun istatistiksel belirsizliğinden (uncertainty) değil, sayısal değerlendirme yapılamaz (ill–defined) olmasından veya durumun hassasiyetinin olmamasından

(impreciseness) ileri gelmektedir.

İnsanoğlu bu gibi durumlarda sözel değerlendirmeler yapar. Örneğin; fazla, oldukça fazla, iyi, fena değil gibi. Ancak, karmaşık durumlar için sistematik bir yaklaşım gereklidir. Pekçok alternatifin, ölçütün, ve hatta her ölçütlere ait alt ölçütlerin bulunduğu durumlarda karar verme işlemi çok zor olabilmektedir.

Yukarıda bahsedilen ve sayısal değerlendirme yapılamıyan durumları "Bulanık Küme"lerle belirtmek, diğer bir deyişle sözel değerlendirmelere birer bulanık küme karşılık getirme olanağı vardır. 1965 yılında bilim dünyasına tanıtılan ve günümüze kadar kendini ispatlayarak geniş bir uygulama alanı yaratan "Bulanık Küme" kavramı, kullandığı bulanık mantık yardımı ile, olayları siyah, beyaz veya evet, hayır dünyasından, yani iki değerli mantığın dünyasından alıp çok değerli mantığın dünyasına götürmüştür. Böylece; yavaş, hızlı, çok hızlı gibi kavramlar bulanık kümelerle belirtilebilmektedir.

Bilgisine başvurulacak uzmanların bulanık küme kuramı bilgisi olması koşulu yoktur. Uzmanların değerlendirmeleri bu programda sözel değerlendirmeler olarak düşünülmemiş, daha farklı bir yol izlenmiştir. Klasikleşmiş kesin değerlendirmelerde 0–10 arasında puanlama yapılması bilindiği gibi yaygındır. Bu programda uzmanlardan değerlendirmelerini 0–10 arasından tam sayı seçerek yapmaları istenir, ancak her değerlendirmenin bir eksik ve bir fazla puan arasında (örneğin; değerlendirme 3 ise, 2–4 değerlerini kapsayan) üçgen üyelik fonksiyonuna ($\mu(x)$) sahip bir bulanık küme ile temsil edileceği söylenir. Bununla da kalınmayıp, uzmanlara isterlerse ardarda iki sayı ile de değerlendirmelerini yapabilmek olanağı verilmektedir. Örneğin (4, 5) gibi. Bu durumda, değerlendirme 3–6 sayılarını kapsayan ve yamuk şeklinde bir üyelik fonksiyonu ile belirtilmektedir.

Uzman değerlendirmeleri bulanık kümelerle temsil edildiğinden 0–10 arası puanlama ile yapılan değerlendirmeler klasik kesin değerlendirme olmaktan çıkmaktadır. Uzmanların değerlendirmeleri kesin sayısal değerlendirme yapılamamaktan ileri gelen belirsizliği (impreciseness) içermektedir. Bu durum, sözel değerlendirme ile eşdeğer kabul edilebilir. İzlenen yöntemin uzmanlar arasındaki değerlendirme farklarını daha iyi ortaya koyabileceği düşünülmüştür.

Uzman değerlendirmeleri bulanık sayılara dönüştürüldükten sonra, sırası ile aşağıdaki bulanık sayılar hesaplanmaktadır:

a) i 'nci alternatifin k 'ncı ölçütü ne derece tatmin ettiğini gösteren ortalama bulanık sayı hesaplanmaktadır ($\bar{\omega}_{ik}$). Bu sayının hesaplanması sırasında alt ölçütler ve alternatifler için yapılan değerlendirmeler kullanılmakta ve uzmanlar üzerinden ortalama alınmaktadır.

b) Her uzmanın değerlendirmelerinden k 'ncı ölçütün önemini belirten bulanık sayı elde edilmekte ($\bar{b}_{ik}$) ve uzmanlar üzerinden ortalama alınmaktadır ($\bar{n}_k$).

c) Elde edilen bu sayılar i 'nci alternatifin ağırlığının hesaplanmasında kullanılmaktadır ($\bar{\omega}_i$). Alternatiflerin tercihi bu ağırlıkların sıralanması ile elde edilmektedir.

Söz konusu ağırlıklar bulanık sayılardır. Bulanık sayıların nasıl karşılaştırılabileceği açıklanmış ve örnek problemin sonucu olan sıralama çalışmada verilmiştir.

Burada konumuz için temel teşkil eden örnek problemin içeriğini anlamak gereklidir. Örnek problemimizde bir nükleer guç santralinin inşa edileceği en uygun yer saptanmaya çalışılmıştır. Bunun için ref. 1'de verilen ölçüt ve altölçütler şöyle sıralanmıştır:

- Ölçüt 1 : Topoğrafya ve osenografi
  - Altölçüt 1 : Arazinin engebe durumu
  - Altölçüt 2 : Mevki incelemesi
  - Altölçüt 3 : Su derinliği
  - Altölçüt 4 : Gelgit ve gelgit akıntıları
  - Altölçüt 5 : Isıl kirlenme
- Ölçüt 2 : Jeoloji, hidroloji ve sismoloji
  - Altölçüt 1 : Jeolojik yapı
  - Altölçüt 2 : Toprak özellikleri

Altölçüt 3 : Yeraltı su haritası

Altölçüt 4 : Su kaynakları

Altölçüt 5 : Yer sarsıntıları

Altölçüt 6 : Sismik cihazlar ve mevkiileri

Altölçüt 7 : Tsunamiler

– Ölçüt 3 : Meteoroloji

Altölçüt 1 : Yağmur ve kar yağşıları

Altölçüt 2 : Fırtına, toz ve kum fırtınaları

Altölçüt 3 : Nem ve sis

Altölçüt 4 : Sıcaklık

Altölçüt 5 : Rüzgar

– Ölçüt 4 : Nakliyat

Altölçüt 1 : Anayollara yakınlık

Altölçüt 2 : Nakliyat imkanları

– Ölçüt 5 : Nüfus

Altölçüt 1 : Yoğunluk

Altölçüt 2 : Mevsimsel değişim

Altölçüt 1 : Gelecekteki nüfus

– Ölçüt 6 : Soğutma suyu

Altölçüt 1 : Kaynaklar

Altölçüt 2 : Elverişlilik

Altölçüt 3 : Uzaklık

Altölçüt 4 : Özellikleri

– Ölçüt 7 : Konstrüksiyon, destek servisler ve şebeke suyu

Altölçüt 1 : Arazi planlama zorlukları

Altölçüt 2 : Konstrüksiyon için arazi özellikleri

Altölçüt 3 : Konstrüksiyon işgücü

Altölçüt 4 : Konstrüksiyon zorlukları

Altölçüt	5	:	Elektrik
Altölçüt	6	:	Gaz
Altölçüt	7	:	Drenaj ve kanalizasyon
Altölçüt	8	:	Dağıtım hatları
Altölçüt	9	:	Şebeke suyu elverişliliği
Altölçüt	10	:	Şebeke suyu özellikleri
Altölçüt	11	:	Şebeke suyu dağıtım sistemi

Uzmanların yaptığı değerlendirmeler sayfa 39'daki tablo 5.3 ve sayfa 40'daki tablo 5.4'de verilmiştir. Bu değerlendirmelerin yardımıyla yapılan klasik ağırlıklı ortalama ve bulanık sıralama yönteminin sonuçları sayfa 40 ve 41'de verilmiştir. Sonuçlardan da gö rüleceği gibi her iki yöntem aynı sıralamayı vermektedir. Bulanık sıralamanın sonucu aynı zamanda sayfa 40'da şekil 5.3'de grafik olarak da gösterilmiştir. Bu grafikten de görüleceği gibi en iyi alternatif olarak gözüken 3. alternatifin kendine en yakın alternatif olan 2. alternatifle kesiştiği noktanın ordinat değeri 0.94'tür. Bu değer bizim üstünlük için baz kabul ettiğimiz sigma'nın değerinden (0.95) küçüktür. Bu da bize kullandığımız bulanık sıralama yönteminin sağlıklı bir sonuç verdiğini göstermektedir.

CONTENTS

I. INTRODUCTION [1], [2], [3]

In multi-attribute decision making, an analyst or a group of analysts are faced with the problem of choosing among alternatives, considering a number of criteria; this becomes a difficult task if the number of alternatives and criteria are large. Incorporation of objective and subjective aspects and inherent uncertainities of the future events also makes the decision process more complicated. Quantifible objectives like cost can easily be considered. But, the importance of transportation possibilities at the site of a nuclear power plant or the importance of population and its future growth around the site can not be expressed precisely on a ten point scale. In statistical decision theory, inexact knowledge about the alternatives and criteria is attributed to the uncertainity and the tools of statics are used to arrive at an optimal final decision. But in many situations, this inexactness is due to the impreciseness, not to the uncertainity.

In such an inexact situation due to the imprecision, human beings usually use linguistic variables like "important", "not important", "big", "small", "good", "poor". Inexactness in these terms are not due to uncertainity but due to ill-defined terms. Normal human beings can easily judge the situation and take proper action. But if the situation is complicated, as in the case of site selection with many alternatives and criteria to be considered, the final decision is achieved by a systematic approach.

Fuzzy sets are a concept that can bring the reasoning used by computers closer to that used by people. With fuzzy sets, human concepts like "small", "big", "young", "old", "high" or "low" can be translated into a form usable by computers. Indeed, fuzzy sets and fuzzy logic are now finding wider and wider application in a broad range of problem solving, from industrial process control and pattern recognation to weather prediction, medical diagnosis and agricultural planning.

The main advantage of using fuzzy subsets is to overcome the difficulties of decision making in a fuzzy situation represented by ill-defined terms. Inherent imprecision of such terms make crisp ranking very difficult and application of statistical decision theory doubtful. The situation can be handled by the analyst, ranking these quantities verbally which is the normal behaviour of human beings to account for inherent imprecision. The verbal ranking are then represented by fuzzy subsets. The final ranking for alternatives from best to worst can be obtained using fuzzy operations.

In this study, a computer implementation of fuzzy decision analysis is given. This computer program can be applied to any decision problem. Here, it is applied to the site selection problem for a nuclear power plant which is a difficult decision having many fuzzy situations related to the present and to the future.

II. CONVENTIONAL MULTICRITERIA SELECTION METHODS[4]

In order to make an election among alternatives we usually consider a group of criteria. So we need some methods to rank the alternatives according to these criteria after we gathered all the data about them.

These methods can be classified in three groups:

i) Methods that reduce many criteria into one criterion which defines the perfect selection (Single Criterion Methods).
ii) Methods that define a final ranking (ordering) considering the ranking of alternatives with respect to each criterion (Single Ranking Methods).
iii) Methods which test the validity of some preassumptions.

Now let us discuss these methods, briefly.

II.1 Definitions

Since our problem is choosing among alternatives (or putting them in order), considering a number of criteria, we have two sets **A** and **C**.

$$\mathbf{A} = \{A_i/i \in (1,\ldots,n)\}, \text{the set of alternatives that will be ranked.}$$

$$\mathbf{C} = \{C_j/j \in (1,\ldots,m)\}, \text{the set of criteria}$$

It is also necessary to define a set of weights **P**, because in many situations the weights (or the degree of importance) of each criterion are not equal.

$$\mathbf{P} = \{p_j/j \in (1,\ldots,m)\}, \text{the set of weights.}$$

It will be easier to show these sets **A** and **C** on a single table. This table is shown in fig. 2.1 and named as "$[A \times C]$ Matrix" or "Situation Table" where X_{ij} represents, how well A_i satisfies criterion C_j.

A \ C	C_1	C_2	C_j	C_{m-1}	C_m
A_1					
A_2					
A_i			X_{ij}		
A_{n-1}					
A_n					
P	P_1	P_2	P_j	P_{m-1}	P_m

Figure 2.1. $[A \times C]$ Matrix or Situation Table

After this table has been set we need to find a *single* order of preference of the elements $A_i \in \mathbf{A}$. The methods which can realize this situation will be given in the following sections.

II.2. Single Criterion Methods

II.2.1. *Weighted Average:*

It is the most widely used method in many fields when multidimensional examination is needed.

The characteristics of this method is to reduce many criteria into a single one. This reduction can be shown mathematically as follows:

$$(A_i) = \frac{\sum_{j=1}^{m} P_j \times X_{ij}}{\sum_{j=1}^{m} P_j} \tag{2.1}$$

By this equation, ranking of A_i's considering m criteria is reduced to the consideration of one criterion. Then, the order of preference of alternatives A_i is equal to the order of (A_i). Clearly, $A_h > A_i$ if

$$(A_h) > (A_i) \tag{2.2}$$

II.2.2. *Probabilistic Average:*

This method has been developed by using "utility theory" in statistics.

First of all, acceptibility function $U_i(C_j)$ is defined for every alternative $(A_i \in \mathbf{A})$. The aim of this definition is to determine the desirability levels of the criterion C_j for alternative A_i.

After determination of these functions for every A_i, a single value $\mu(A_i)$ is found as

$$\mu(A_i) = \sum_{j \in C} \bar{p}_j \times U_i(C_j) \tag{2.3}$$

$$\sum_j \bar{p}_j = 1 \tag{2.4}$$

$\mu(A_i)$ gives an average value for the alternative A_i showing how well A_i satisfies the criteria. Then, ranking between A_i's can be made putting $\mu(A_i)$ in an order.

The calculation of the weight probabilities $\bar{p}_j$'s is an important task. It will be convenient to make inquiry among people who are qualified in this subject.

II.3. Single Ranking Methods

II.3.1. *Weighted Ranking:*

In this method the alternatives are put in an order according to criteria. If these rankings are denoted by $s_j(A_i)$, it is obvious that $s_j(A_i)=1$ when A_i satisfies criterion C_j best. Then we will have to calculate the values

$$\nu(A_i) = \sum_{j=1}^{m} s_j(A_i) \times p_j \tag{2.5}$$

by the help of weights p_j's explained in the previous methods. Thus, j criteria are all considered and a single value for each alternative is obtained. Then the last ranking of alternatives is achieved by ordering $\nu(A_i)$'s. This time, in order to say $A_h > A_i$, it will be sufficient to seek for the condition

$$\nu(A_h) < \nu(A_i) \tag{2.6}$$

This method is easy, but it is not used frequently because of a disadvantage. If two elements have the same rank for the same criterion, that is if they are equal to eachother, and especially if the weight of this criterion is big, an unhealthy ranking will be obtained.

II.3.2. *Democratic Method:*

This is a very practical method when the weights for criteria cannot be considered.

The first step of this method is to make a ranking as in the previous method. That is, it is necessary to make the rankings

$$A_h > A_i > A_k > A_\ell \tag{2.7}$$

for every criterion C_j.

At the next step, rankings for each pair of the alternatives are taken into consideration. If at least the half of the criteria of the set **C** approve the relation (>) for a pair "A_i, A_k", the group formed by this condition is accepted otherwise it is rejected.

Arrow diagram is defined to make the process practical which can be found in details in the references given above.

II.4. Preassumption Methods

II.4.1. *Consecutive Permutations:*

The major characteristics of this method and the following methods are to examine the acceptibility degrees of some preassumptions or rankings instead of combining the criteria or rankings and reducing them into a single one.

The situation table $(A \times C)$ that we will use here can include either numerical values or some evaluations like "good, bad" or "advantageous, disadvantageous" etc.

The first thing to do is to set up an ordinary rankings between elements A_i of the set **A** which is called $\{S_i\}$. The second step will be the testing of the acceptibility degrees of these hypothetical rankings by the help of a coefficient which will be calculated.

In order to perform this test, the partial acceptance coefficient "a_{ih}" and the partial rejection coefficient "a_{hi}" are calculated for every ranking $\{S_i\}=\{S_1, S_2, \ldots, S_{n!}\}$.

As a definition, "a_{ih}" is the addition of all the weights that proves the hypothesis $A_i > A_h$ and vice versa for "a_{hi}".

In order to find the certain and acceptable ranking it is required to calculate the "general acceptance degree". The ranking S_i which has the biggest acceptance degree among the others is the one which is most prefered. This "general acceptance degree" $A(S_i)$ is defined as;

$$A(S_i) = \sum a_{ih} - \sum a_{hi} \tag{2.8}$$

II.4.2. *General Acceptance Degree:*

Calculations in previous method may be very long and need computer programming. Method is simplified and a practical solution is obtained which is called "General Acceptance Degree". The details of the method are given in ref.4.

II.4.3. *Average Deviations Method:*

The method which will be discussed here deals with pre–rankings as the previous two methods do, but instead of maximizing the coefficients obtained from these pre–rankings it calculates the differences of these pre–rankings from the real ranking tables which are made separately for each criterion.

The theory used in this method is based on the Set Theory largely. As it is known, in this theory the validity of an assumption is indicated as the number 1 and the invalidity as the number 0. So the theoretical pre–ranking tables can be formed by the help of these two numbers. For example the pre–ranking $a > b > c$ that can be made for a set of three elements, can be indicated as the matrix $(A \times A)$ of n^2 elements as;

A \ A	a	b	c
a		1	1
b	0		1
c	0	0	

Table 2.1. $(A \times A)$

It will be possible to form the real ranking tables that will be obtained for any criterion C_j, by the help of a similar table.

When these two tables are formed, to see the number 1 for (A_i, A_h) on both tables will mean that the criterion C_j approves the assumption $A_i > A_h$ on theoretical ranking. Conversely to see the number 0 at the same places on both tables will mean that the criterion C_j approves a negativeness.

By this way, it will be possible to test the real ranking of all C_j's against every theoretical ranking. Two acceptances can be made for this test.

The first acceptance is to search for the minimum of the differences "d_2" at both tables and the second is to search for the maximum of the cooperations "d_1" again at both tables.

The theoretical ranking confirming the condition

$$\max \pi_1 = \max \sum_j P_j \times d_1 \tag{2.9}$$

or the condition

$$\min \pi_2 = \min \sum_j P_j \times d_2 \tag{2.10}$$

appears as the ranking seeked for, by considering the weights P_j of every criterion.

II.4.4. *The ELECTRE Method:*

Utility function theory which is applied by addition or comparision operations is criticized because of the unavailibility of comparision of some elements according to some criteria.

Departing from this consideration, a "superiority" relation and method which is more realistic and considers every situation is developed and used successfully. According to this method, "$A_i \, S \, A_h$" means that decision center (person or persons) sees the risk of prefering A_i to A_h at the level of acceptance.

Again according to this superiority relation "S", it can be said that writing "$A_i \, S \, A_h$" means that there are valid reasons not to accept the superiority of A_h.

Firstly the superiority relation "S" mentioned above is formed. Two conditions needed to form this relation can be pointed out as follows:

To be able to say that an element $A_i \in \mathbf{A}$ is superior to another element $A_h \in \mathbf{A}$ and to write "$A_i \, S \, A_h$", two conditions must be taken into consideration at the same time. They are;

1) The total of the weights of the criteria saying that A_i is as good as A_h must be high,
2) The difference $|X_j(A_h) - X_j(A_i)|$ obtained as a result of consideration of the
   criterion j which tells A_i is not better than A_h should be small

The first condition acts towards forming the majority for acception of A_i, so it is named as "harmony condition" and requires the calculation of a harmony coefficient "$U_m(A_i, A_h)$".

The second condition deals with the attitudes against the superiority of A_i, so it is known as "disharmony condition". It is necessary to calculate a disharmony coefficient "$U_z(A_i, A_h)$" in order to consider this condition.

The second step of the method is to draw arrow diagrams after calculation of all coefficients which lead us to the conclusion.

III. FUZZY–LOGIC CONCEPT

III.1 Why Fuzziness ? [3], [5]

With the increase of computer usage, quantitative techniques are being utilized for the analysis of humanistic systems such as economic, urban, social, biological systems where animate behavior of the system constituents plays a dominant role. These techniques are adaptations of the methods that have been developed for the analysis of mechanistic systems which employ the laws of mechanics, electromagnetism and thermodynamics. Inspiration of the great success of these methods in mechanistic systems brought the idea that the same techniques can also be applied to the humanistic systems. But in reality, these traditional quantitative techniques of mechanistic system analysis are not well–suited for dealing with humanistic systems. The reason is the *principle of incompatibility* which says that when the complexity of a system increases the possibility of making precise and significant statements about its behavior decreases. Thus, precise quantitative analyses of humanistic system behaviors have less fitness to the real world problems which involve humans.

The essential elements in human thinking are not numbers but labels of fuzzy sets where the transition of the objects from membership to nonmembership is gradual rather than abrupt. In human reasoning neither traditional two–valued nor multivalued logic is used. It is a logic with fuzzy truths, fuzzy connectiveness and fuzzy rules of inference that is used. This fuzzy–logic plays a basic role in human ability to summarize information.

Most of the tasks executed by humans do not demand a high level of precision during their execution. Becuse of this imprecision human brain takes a big advantage by encoding the "task–relevant" information into labels of fuzzy sets which link to the basic data. By this way, the bunch of information reaching the brain in various ways is reduced to the simple information that is needed to perform the specific task with the minimal level of precision. The ability of human brain to use fuzzy sets and its summarizing capability is one of the most important advantage of it. It is also the basic characteristic of the human brain which differs its intelligence from that of machine intelligence used in todays digital computers.

Although fuzziness is undesirable, the flexibility of fuzzy sets makes them favorable against conventional sets becuse they are not limited by the rigidity of mathematical reasoning and computer programs. For example, feedback oscillations and "hunting" in process controllers are common problems because of their extremely precise rules. That is overpreciseness of the process control in judging during decision making often causes worse behaviors than human controllers who use fuzzy rules of thumb.

Humans tend to use words instead of numbers to interpret the system behavior. With fuzzy sets linguistic variables such as "young", "tall", "low", and modifiers such as "not", "very" can be translated directly into a computer program by using membership functions outlining them. This direct translation is very important when processes are too complicated to be understood fully in terms of accurate mathematical relations. In this situation with fuzzy sets processes can be controlled successfully with rules of thumb.

III.2 Fuzzy–Logic Applications [3]

Fuzzy–logic has found applications in process controllers, expert systems, pattern recognition, computer vision, robotics and natural language understanding. We will discuss examples of some of these applications briefly.

One of the first commercial applications of fuzzy–logic to process control was the development of a microprocessor controller for cement kilns. A cement kiln is a rotating chamber in which ground limestone and clay react with each other at temperatures between 1000 and 1400 degrees celcius. Because of the complexity of the chemical reactions and few possible measurements of the internal state of the kiln, it is hard to control the process. But a skilled operator can manage to control the outputs within the desired limits with fairly high efficiency. The operator overcomes the difficulty by applying 40 or 50 rules of thumb to control the inputs.

The main problem in automating the process is that at any given time many of the rules apply to some extent and they cause confliction. A human controller tries to balance the force of the rules and how well they apply by various rules of thumb untill they hold true. These rules themselves are expressed in fuzzy terms such as "rather high", "slightly reduce" instead of quantitatively.

The controller uses fuzzy–logic to deal with this situation. Firstly, the program defines the fuzzy terms "low", "high", "OK" and "normal" for the measured quantities by relating the exact value of the quantities to fuzzy sets with membership functions. Similarly the program defines the terms such as "medium", "negative", "high negative", "small positive" for the adjustments in the control parameters. Using these relations, rules of thumb can be set up according to any condition.

In operation, the extension of the application of each rule is determined by the controller according to the actual condition of the kiln that satisfies the condition stated in the rule. The final action is determined as the weighted average of the actions held by each rule depending on the degree to which the conditions of the rule are satisfied.

Both savings in fuel and a more stable product quality have been reported by the users of this fuzzy–logic controller.

Another self learning fuzzy–logic controller was developed in Japan. Here the motion of a car is controlled by a fuzzy–logic controller with 20 control rules. According to the distance of the car from side and back walls of the turn and its current heading, these rules recommend a specific change in direction. Just as in the kiln controller, this change of the car's direction is a weighted average of the results of each rule. Here the input variables are described in terms of linguistic variables as in the kiln controller instead of precise quantities to simplify matters.

This controller mounted to a model car drives the car through angled corridors by automatically deducing the fuzzy control rules through a computer analysis of the operators' control actions. Their control actions point to fuzzy–logic controller the way of the construction of the rules where the fuzzy–logic can derive it's own rules automatically.

Another application of fuzzy–logic controller in Japan is the automatic operation of a train (ATO) by using predictive fuzzy–logic controller. The criteria that this controller uses during operation are safety, riding comfort, accurate stopping, minimal running time and minimal energy consumption. Field tests have shown that "Fuzzy ATO" has the skill of an experienced human operator running a train.

The predictive fuzzy–logic controller used in the Fuzzy ATO differs from conventional fuzzy–logic controllers by the form of its rules. In conventional fuzzy–logic controller the form of the rules operating is like "if the system is in state S then do C". But in predictive fuzzy–logic controller the rules are expressed in terms of desired states such as "if the desired state is S then do C". The weights of the control rules in predictive fuzzy control are determined by predicting the operation results of the actions described in the rules.

During the last years, expert systems have become the most important applications of artificial intelligence. A typical expert system has imprecise, incomplete or not totally reliable information in the knowledge base because of reflecting human expertise. For this reason, an expert system gives the answer to a question qualified with a "certainty factor" that is giving an idea about the degree of confidence of the answer.

To achieve this certainty factor expert sytems use probability–based methods. But much of this uncertainty comes from the fuzziness and incompleteness of data instead of its randomness. So, the computed values of the certainty factor mostly lack in reliability.

Fuzzy–logic provides a systematic basis for the computation of certainty factors using fuzzy numbers dealing with the fuzziness, incompleteness and randomness of the management of uncertainty in expert systems.

Fuzzy analysis can be used to model natural activity as well as human activity. For example, chinese meteorologists used a fuzzy model for determining the best rubber tree growing areas. They used fuzzy boundaries for the most desirable range of decision influencing factors. They reached good harmony with actual results after testing the reliability of the method.

The other major areas of application are in pattern recognition, computer vision and robotics. Overprecision is the most frequent problem in computer vision and pattern recognition. In recognizing an object various features such as size, width, breadth, shape are used for determining the compatibility to a certain object. Fuzzy characteristic definitions instead of sharp edges often improve the efficiency.

As examples of application, computer vision for dynamic scenes developed at the University of Michigan using fuzzy categories during analysis, and fuzzy computer vision to a simple system developed in Bulgaria allowing a robot arc welder to follow a seam accurately can be considered.

Another important area of application is in the understanding of natural language. Fuzzy sets can overcome the difficulties of natural–language communication with computers using the linguistic variables which the fuzzy–logic based on.

IV. FUZZY SETS and FUZZY NUMBERS

IV.1. FUZZY SETS [3], [6]

A fuzzy set, as its name implies, is a class with fuzzy boundaries: the class of cheap bicycles, big numbers, low mountains, or blond men or women for instance. Such a class may be characterized by associating a grade of membership with every object that could be in the class. Suppose we are considering the members of the IEEE and the fuzzy set of young members. If Albert is a member of the IEEE and his age is 45, his grade of membership in the set may be, say 0.3 . The fuzzy set "young", then, is defined by its membership function; it associates with every member of the IEEE his or her grade of membership in the set "young".

Basically the grade of membership is subjective in nature; it is a matter of definition rather than measurement. Thus the number 0.3 in the case of Albert may be viewed as a response to the question: "Albert is 45 years old and is a member of the IEEE. To what degree (on a scale of 0 to 1) does the label 'young member of the IEEE' apply to him ?".

In a way not well understood at present, humans have a remarkable ability to assign a grade of membership to a given object without a conscious understanding of how the grade is arrived at. For example, a student in a course would have no difficulty in assigning the professor a grade of membership in the fuzzy set of good teachers. Indeed, the assignment would usually be arrived at almost instantaneously without any conscious analysis of the factors that enter into the assessment of the grade of membership in the class.

In the theory of fuzzy sets, a number of attempts have been made to assign the grade of membership on a more rational basis. In one approach, a method based on betting is employed, somewhat similiar to the approach used in defining subjective probabilities. In another approach, the grade of membership is interpreted as a measure of consensus.

Since the grade of membership is both subjective and dependent on context, there is not much point in treating it as a precise number. Thus, in many applications it is sufficient to represent the grade of membership in a fuzzy set as a fuzzy number. The degree of membership might be expressed as "close to 0.8" or "approximately 0.6", or as "quite high" or "very low" and so on.

The grade of membership is not a probability. Basically it is a measure of the compatibility of an object with the concept represented by a fuzzy set. (For example, 0.3 is the compatibility of Albert with the concept "*young*" defined by the fuzzy set of young members of the IEEE; 0.3 is not the probability that Albert is a young member of the IEEE).

IV.1.1. Concept, Notation, Terminology and Basic Properties

Let X be a universal set with A a classical subset of X. Then, $A = \{x\}$ denoting that x is a member of A. We may also use the membership function concept to indicate which members of the universal set are the members of A. The membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases} \tag{4.1}$$

Thus, $A = \{x|\mu_A(x)\}$ which includes every x in X with $\mu_A(x) = 1$. Now, let us assume that $\mu_A(x)$ may take any value in the interval [0,1]. This case may be interpreted as: x is a member of subset $\tilde{A}$ strongly, more or less, a little; if $\mu_A(x)$ is near to *one*, neither too near to *zero* nor to *one*, or near *zero* respectively. $\tilde{A}$ is a subset of X which has no sharp boundary. $\mu_A(x)$ shows that to what extent x is a member of $\tilde{A}$. Set $\tilde{A}$ is called fuzzy subset of X.

A finite *fuzzy* subset of X is expressed as

$$\tilde{A} = \mu_1 x_1 + \ldots + \mu_n x_n \tag{4.2}$$

or, equivalently, as

$$\tilde{A} = \mu_1/x_1 + \ldots + \mu_n/x_n \tag{4.3}$$

where the μ_i, $(i = 1, ..., n)$, represent the *grades of membership* of the x_i in X. Unless stated to the contrary, the μ_i are assumed to lie in the interval [0,1], with 0 and 1 denoting *no* membership and *full* membership, respectively.

In case of having continuous membership function, following notation may be used;

$$\tilde{A} = \int \mu_A(x)/x \tag{4.4}$$

The points in X at which $\mu_A(x) > 0$ constitute the *support* of $\tilde{A}$. The points at which $\mu_A(x) = 0.5$ are the *crossover* points of $\tilde{A}$.

Example 4.1. Assume

$$X = \{a, b, c, d\}$$

Then, we may have

$$A = \{a, b, d\}$$

and

$$\tilde{A} = 0.3a + 0.9b + d .$$

as nonfuzzy and fuzzy subsets of X, respectively. If

$$X = \{0, 0.1, 0.2, 1\}$$

then the fuzzy subset of X would be expressed as, say,

$$\tilde{A} = 0.3/0. + 0.9/0.1 + +1/1 .$$

If $X = [0, 1]$, then $\tilde{A}$ might be expressed as

$$\tilde{A} = \int_0^1 \frac{1}{1 + x^2}/x$$

which means that $\tilde{A}$ is a fuzzy subset of the unit interval [0,1] whose membership function is defined by

$$\mu_A(x) = \frac{1}{1 + x^2}$$

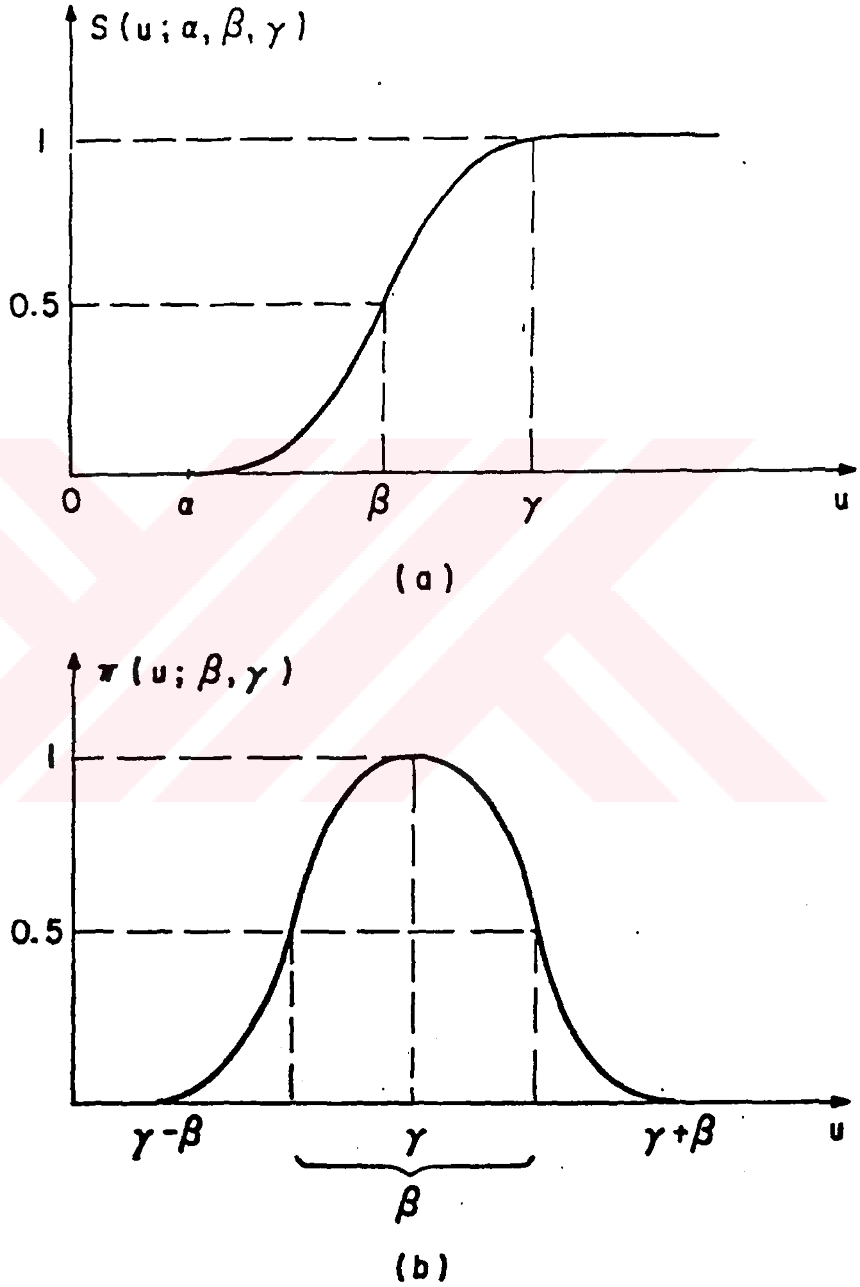

Figure 4.1. Plots of S and π Functions

In many cases, it is convenient to express the membership function of a fuzzy subset of the real line in terms of a standart function whose parameters may be adjusted to fit a specified membership function in an approximate fashion. Two such functions, of the form shown in Fig. 4.1, are defined below (For the sake of simplicity, fuzzy subsets will be represented without ˜ in the rest of the study):

$$\begin{aligned} S(u;\alpha,\beta,\gamma) &= 0 && for\ u \le \alpha \\ &= 2\Big(\frac{u-\alpha}{\gamma-\alpha}\Big)^2 && for\ \alpha \le u \le \beta \\ &= 1 - 2\Big(\frac{u-\gamma}{\gamma-\alpha}\Big)^2 && for\ \beta \le u \le \gamma \\ &= 1 && for\ u \ge \gamma \end{aligned}$$

and

$$\begin{aligned} \pi(u;\beta,\gamma) &= S(u;\gamma-\beta,\gamma-\frac{\beta}{2},\gamma) && for\ u \le \gamma \\ &= 1 - S(u;\gamma,\gamma+\frac{\beta}{2},\gamma+\beta) && for\ u \ge \gamma \end{aligned}$$

In $S(u;\alpha,\beta,\gamma)$, the parameter β, $\beta = \frac{\alpha+\gamma}{2}$, is the crossover point. In $\pi(u;\beta,\gamma)$, β is the bandwidth, that is, the separation between the crossover points of π, while γ is the point at which π is unity.

IV.1.2. Operations On Fuzzy Sets

If F and G are fuzzy subsets of X, their *union*, $F \cup G$, *intersection*, $F \cap G$, *bounded–sum*, $F \oplus G$, and *bounded–difference*, $F \ominus G$, are fuzzy subsets of U defined by

$$F \cup G \triangleq \int_U \mu_F(u) \vee \mu_G(u)/u \tag{4.5}$$

$$F \cap G = \int_U \mu_F(u) \wedge \mu_G(u)/u \tag{4.6}$$

$$F \oplus G = \int_U 1 \wedge (\mu_F(u) + \mu_G(u))/u \tag{4.7}$$

$$F \ominus G = \int_U 0 \vee (\mu_F(u) - \mu_G(u))/u \tag{4.8}$$

where $\vee$ and $\wedge$ denote max and min, respectively. The *complement* of F is defined by

$$F' = \int_U (1 - \mu_F(u))/u \tag{4.9}$$

or, equivalently,

$$F' = X \ominus F \tag{4.10}$$

It can readily be shown that F and G satisfy the identities

$$(F \cap G)' = F' \cup G' \tag{4.11}$$

$$(F \cup G)' = F' \cap G' \tag{4.12}$$

$$(F \oplus G)' = F' \ominus G \tag{4.13}$$

$$(F \ominus G)' = F' \oplus G \tag{4.14}$$

Although it is traditional to use the symbol $\cup$ to denote the union of nonfuzzy sets, in the case of fuzzy sets it is adventageous to use the symbol $+$ in place of $\cup$ where no confusion with the arithmetic sum can result. This convention is employed in the following example, which is intended to illustrate (4.5), (4.6), (4.7), (4.8) and (4.9).

Example 4.2. In universal set $X = \{a, b, c, d\}$ fuzzy subsets F and G are expressed by

$$F = 0.4a + 0.9b + d$$

$$G = 0.6a + 0.5b$$

we have

$$F + G = 0.6a + 0.9b + d$$

$$F \cap G = 0.4a + 0.5b$$

$$F \oplus G = a + b + d$$

$$F \ominus G = 0.4b + d$$

$$F' = 0.6a + 0.1b + c$$

IV.1.3. Linguistic Variables

Informally, a linguistic variable, χ, is a variable whose values are words or sentences in a natural or artificial language. For example, if age is interpreted as a linguistic variable, then its term–set, $T(\chi)$, that is, the set of its linguistic values, might be

$$\begin{aligned} T(\text{age}) =& \textbf{young} + \textbf{old} + \textbf{very young} + \textbf{not young} + \textbf{very old} \\ &+ \textbf{very very young} + \textbf{rather young} \\ &+ \textbf{more or less young} + \dots \end{aligned} \qquad (4.15)$$

where each of the terms in T(age) is a label of a fuzzy subset of a universe of discourse, say $X = [0, 100]$.

A linguistic variable is associated with two rules: (a) a *syntatic rule*, which defines the well–formed sentences in $T(\chi)$; and (b) a *semantic rule*, by which the meaning of the terms in $T(\chi)$ may be determined. If Y is a term in $T(\chi)$, then its *meaning* (in a denotational sense) is a subset of X. A *primary term* in $T(\chi)$

is a term whose meaning is a *primary fuzzy set*, that is, a term whose meaning must be defined a priori, and which serves as a basis for the computation of the meaning of the nonprimary terms in $T(\chi)$. For example, the primary terms in (4.15) are **young** and **old** whose meaning might be defined by their respective compatibility functions μ_{young} and μ_{old}. From these, then, the meaning — or, equivalently, the compatibility functions — of the non–primary terms in (4.15) may be computed by the application of a semantic rule.

Rule for **very**

$$\mu_{\text{very young}} = (\mu_{\text{young}})^2 , \tag{4.16}$$

Rule for **more or less**

$$\mu_{\text{more or less old}} = (\mu_{\text{old}})^{1/2} , \tag{4.17}$$

and rule for **not very**

$$\mu_{\text{not very young}} = 1 - (\mu_{\text{young}})^2 . \tag{4.18}$$

For illustration, plots of the compatibility functions of these terms are shown in Fig. 4.2.

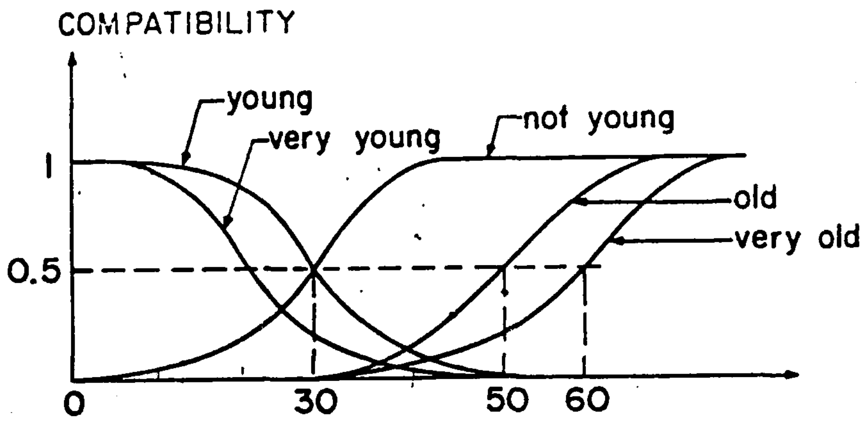

Figure 4.2 Compatibility Functions of **young** and its Modifications

IV.2. FUZZY NUMBERS [1]

IV.2.1. Definition

The fuzzy subsets are described by

$$(\alpha/\beta \;\;,\;\; \gamma/\delta) \tag{4.19}$$

or

$$(\alpha[s_1,\; s_2]/\beta \;\;,\;\; \gamma/\delta[t_1,\; t_2]) \tag{4.20}$$

where α, β, γ, $\delta \in \{0,\; 1, \ldots\; L\}$, with L a positive integer, and $\alpha \leq \beta \leq \gamma \leq \delta$. Fig. 4.3 shows the construction of the membership function of the fuzzy set represented by (4.19) which is zero at the left of α and at the right of δ. Line segments connect the points $(\alpha, 0)$, $(\beta, 1)$ and $(\beta, 1)$, $(\gamma, 1)$ and $(\gamma, 1)$, $(\delta, 0)$. If $\alpha = \beta$ and $\gamma = \delta$; then, the shape of the function becomes rectangular. If β, α are equal, then, the membership function has a triangular shape. The fuzzy set given by (4.20) is similar to the first one but the points $(\alpha, 0)$, $(\beta, 1)$ and $(\gamma, 1)$, $(\delta, 0)$ are connected by the segments of parabolas with their equations given respectively by:

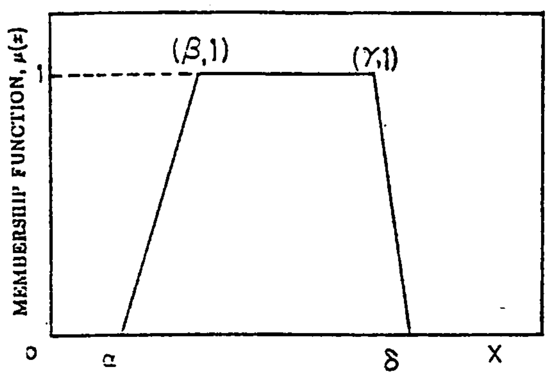

Figure 4.3. Membership function of a fuzzy number given in the form of $(\alpha/\beta\,, \gamma/\delta)$

$$x = s_1\mu^2 + s_2\mu + \alpha \quad , \quad x = t_1\mu^2 + t_2\mu + \delta \tag{4.21}$$

where μ is the value of the membership function, x represents the members of the subset on the horizantal axis, s_1, s_2 and t_1, t_2 are the coefficients. The fuzzy subsets (4.19) and (4.20) are called fuzzy numbers. Formal definition of fuzzy numbers can be found in ref. 8. We also need a fuzzy number with the sides of its membership function defined by cubic functions. In this case, we will have three coefficients in the square brakets of (4.20). It is easy to construct various fuzzy numbers having different interpretations. For example, (6/7, 7/8) may represent "approximately 7", "between 5 and 6" may be represented by (5/5, 6/6), "at least 4" by (4/4, 4/6). Non–fuzzy numbers may also be given in the form of $(\alpha/\alpha, \alpha/\alpha) = \alpha$. Necessary operations on fuzzy numbers will be given here briefly.

IV.2.2. Addition and Multiplication of Fuzzy Numbers

Let us consider two fuzzy numbers represented by (see fig. 4.4)

$$(\alpha_1/\beta_1\,,\,\gamma_1/\delta_1) \quad , \quad (\alpha_2/\beta_2\,,\,\gamma_2/\delta_2) \tag{4.22}$$

Addition and multiplication of these two fuzzy numbers can be carried out as follows:

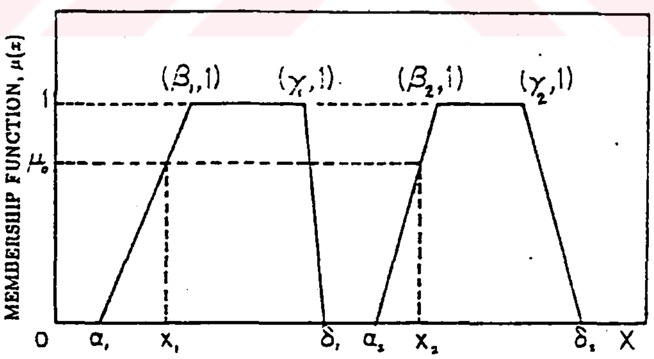

Figure 4.4. Two fuzzy numbers for the explanation of fuzzy operations

a) *Addition:* equations of the left hand sides of given membership functions can be written respectively as

$$\mu(x) = \frac{x}{\beta_1 - \alpha_1} - \frac{\alpha_1}{\beta_1 - \alpha_1} \quad , \quad \mu(x) = \frac{x}{\beta_2 - \alpha_2} - \frac{\alpha_2}{\beta_2 - \alpha_2} \tag{4.23}$$

Substituting the coordinates of the points (x_1, μ_o), (x_2, μ_o) respectively, and adding them,

$$x_3 = x_1 + x_2 = \mu_o(\beta_1 - \alpha_1 + \beta_2 - \alpha_2) + \alpha_1 + \alpha_2 \,. \tag{4.24}$$

This result allow us to obtain the left hand side equation of the membership function of the resulting fuzzy number as

$$\mu(x) = \frac{x - (\alpha_1 + \alpha_2)}{(\beta_1 - \alpha_1) + (\beta_2 - \alpha_2)} \,. \tag{4.25}$$

It is easy to show that $\mu(x) = 0$ at $x = \alpha_1 + \alpha_2$ and $\mu(x) = 1$ at $x = \beta_1 + \beta_2$. A similar result is obtained for the right hand side of the membership function. Thus, addition of two fuzzy numbers can be written as

$$\begin{aligned}(\alpha_1/\beta_1\,, \gamma_1/\delta_1) &\oplus (\alpha_2/\beta_2\,, \gamma_2/\delta_2) \\ &= ((\alpha_1 + \alpha_2)/(\beta_1 + \beta_2)\,, (\gamma_1 + \gamma_2)/(\delta_1 + \delta_2))\end{aligned} \tag{4.26}$$

where the notation $\oplus$ is used to indicate fuzzy addition (this addition is different then the one given in (4.7)).

b) *Multiplication*: The definition of multiplication implies that (following an approach similar to the one used in addition)

$$x_4 = x_1 \cdot x_2 = [\mu_o(\beta_1 - \alpha_1) + \alpha_1][\mu_o(\beta_2 - \alpha_2) + \alpha_2] \,. \tag{4.27}$$

Replacing x_4 by x and μ_o by $\mu(x)$ the equation of the left hand side of the membership function of the resulting fuzzy number can be written as

$$x = (\beta_1 - \alpha_1)(\beta_2 - \alpha_2)\mu^2(x) + [\alpha_2(\beta_1 - \alpha_1) + \alpha_1(\beta_2 - \alpha_2)]\mu(x) + \alpha_1\alpha_2 \,. \tag{4.28}$$

Then, one can easily write the multiplication of two fuzzy numbers as

$$(\alpha_1/\beta_1\,, \gamma_1/\delta_1) \odot (\alpha_2/\beta_2\,, \gamma_2/\delta_2) = (\alpha_1\alpha_2[s_1, s_2]/\beta_1\beta_2\,, \gamma_1\gamma_2/\delta_1\delta_2[t_1, t_2]) \tag{4.29}$$

where the notation $\odot$ is used to indicate fuzzy multiplication. The coefficients in the square brakets are used to construct the equations of the left and right sides of the membership function as given in eqs. (4.21). Here (see eq. (4.27) for s_1 and s_2. t_1, t_2 are obtained in a similar way),

$$\begin{aligned} s_1 &= (\beta_1 - \alpha_1)(\beta_2 - \alpha_2)\,, \\ s_2 &= \alpha_2(\beta_1 - \alpha_1) + \alpha_1(\beta_2 - \alpha_2)\,, \\ t_1 &= (\delta_1 - \gamma_1)(\delta_2 - \gamma_2)\,, \\ t_2 &= -[\delta_2(\delta_1 - \gamma_1) + \delta_1(\delta_2 - \gamma_2)]\,. \end{aligned} \tag{4.30}$$

For our calculations, we also need the addition of the fuzzy numbers having parabolic sided membership funtions. Repeating the same procedure given above for addition, one can easily show that

$$\begin{aligned} &(\alpha_1[s_1', s_2']/\beta_1\,, \gamma_1/\delta_1[t_1', t_2']) \oplus (\alpha_2[s_1'', s_2'']/\beta_2\,, \gamma_2/\delta_2[t_1'', t_2'']) \\ &\qquad = ((\alpha_1 + \alpha_2)[s_1, s_2]/(\beta_1 + \beta_2)\,, (\gamma_1 + \gamma_2)/(\delta_1 + \delta_2)[t_1, t_2])) \end{aligned} \tag{4.31}$$

where

$$\begin{aligned} s_1' + s_1'' &= s_1 \quad , \quad s_2' + s_2'' = s_2\,, \\ t_1' + t_1'' &= t_1 \quad , \quad t_2' + t_2'' = t_2\,. \end{aligned} \tag{4.32}$$

The equations of the left and right sides of the membership function of the fuzzy number obtained at the right side of eq. (4.31) can easily be written (see eqs. (4.21))

The last two necessary relations are given below: Using the multiplication procedure one can show that

$$\begin{aligned} &(\alpha_1/\beta_1\,, \gamma_1/\delta_1) \odot (\alpha_2[s_1, s_2]/\beta_2\,, \gamma_2/\delta_2[t_1, t_2]) \\ &\qquad = (\alpha_1\alpha_2[u_1, u_2, u_3]/\beta_1\beta_2\,, \gamma_1\gamma_2/\delta_1\delta_2[v_1, v_2, v_3])\,. \end{aligned} \tag{4.33}$$

The equations of the left and right sides of the membership function are given respectively as

$$
\begin{aligned}
x &= u_1\mu^3 + u_2\mu^2 + u_3\mu + \alpha_1\alpha_2 \ , \\
x &= v_1\mu^3 + v_2\mu^2 + v_3\mu + \delta_1\delta_2
\end{aligned} \tag{4.34}
$$

where (see eqs. (4.30))

$$
\begin{aligned}
u_1 &= s_1(\beta_1 - \alpha_1) \ , \\
u_2 &= s_1\alpha_1 + s_2(\beta_1 - \alpha_1) \ , \\
u_3 &= s_2\alpha_1 + \alpha_2(\beta_1 - \alpha_1) \ , \\
v_1 &= -t_1(\delta_1 - \gamma_1) \ , \\
v_2 &= t_1\delta_1 - t_2(\delta_1 - \gamma_1) \ , \\
v_3 &= t_2\delta_1 - \delta_2(\delta_1 - \gamma_1) \ .
\end{aligned} \tag{4.35}
$$

Finally, repeating the addition procedure

$$
\begin{aligned}
&(\alpha_1[u_1', u_2', u_3']/\beta_1 \, , \gamma_1/\delta_1[v_1', v_2', v_3']) \oplus (\alpha_2[u_1'', u_2'', u_3'']/\beta_2 \, , \gamma_2/\delta_2[v_1'', v_2'', v_3'']) \\
&\quad = \left((\alpha_1+\alpha_2)[u_1, u_2, u_3]/(\beta_1+\beta_2) \, , (\gamma_1+\gamma_2)/(\delta_1+\delta_2)[v_1, v_2, v_3]\right)
\end{aligned} \tag{4.36}
$$

where

$$
\begin{aligned}
u_1' + u_1'' &= u_1 \ , \ u_2' + u_2'' = u_2 \ , \ u_3' + u_3'' = u_3 \ , \\
v_1' + v_1'' &= v_1 \ , \ v_2' + v_2'' = v_2 \ , \ v_3' + u_3'' = v_3 \ .
\end{aligned} \tag{4.37}
$$

IV.2.3. Comparision of fuzzy numbers

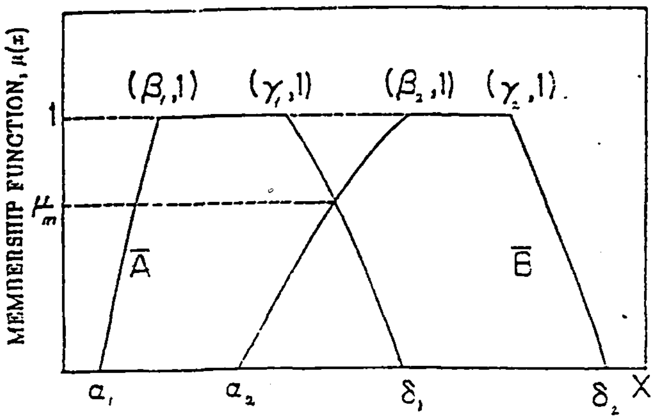

Figure 4.5. Comparison of Fuzzy Numbers

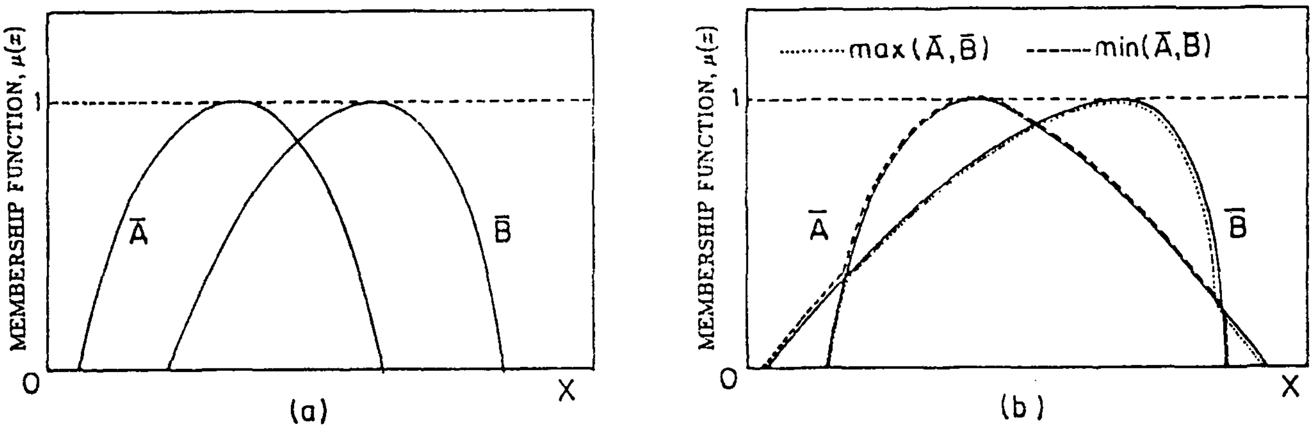

Figure 4.6. Fuzzy number $\bar{B}$ dominates fuzzy number $\bar{A}$

The analyst has to compare fuzzy numbers to rank fuzzily–rated alternatives. Comparison of two fuzzy numbers seems quite different than the comparison of two ordinary numbers. Here, a simple and practical rule is given. For example, if we compare the fuzzy numbers which are represented by the membership functions given in fig. 4.5, the conclusion is that $\bar{B} \geq \bar{A}$ (fuzzy numbers are denoted by an overbar). The membership function $\mu(x)$ of max$[\bar{A}, \bar{B}]$ is obtained as follows: a horizantal line is drawn ($\mu(x) = y_o$ = constant) and y_o is associated to the x which corresponds to the abscissa of the intersection point with the larger abscissa. Repeating the same procedure for $0 \leq y_o \leq 1$ membership function $\mu(x)$ of max$[\bar{A}, \bar{B}]$ is obtained. Evaluation should be done separately for increasing and decreasing parts of the membership functions. If the membership functions have more than one intersection, as is shown in fig. 4.6.(b), the membership function of the max or min fuzzy number differs from the membership functions of given fuzzy numbers. This means that in such a case the max or min of given fuzzy numbers is different than the given numbers.

A more appropriate method is to find the possibility of $\bar{B} > \bar{A}$ and $\bar{A} > \bar{B}$. Possibility is defined as

$$e(\overline{B} > \overline{A}) = \max_{y \geq x} \min[\mu_A(x), \mu_B(y)] \tag{4.38}$$

$e(\bar{B} > \bar{A})$ is in the sense that if $y > x$ in all existing pairs (x, y) which satisfies

$\mu_A(x) = \mu_B(y) = 1$ then $e(\bar{B} > \bar{A}) = 1$. If we apply this definition to the fuzzy numbers whose membership functions are given in fig. 4.5 the conclusion is $e(\bar{B} \geq \bar{A}) = 1$ and $e(\bar{A} > \bar{B}) = \mu_m$. If $\mu_m < \sigma$ where σ is a positive number less than one then we say that $\bar{B}$ dominates $\bar{A}$ and can be written as $\bar{B} > \bar{A}$. In this case it is clear that $\beta_2 > \gamma_1$. If $\mu_m > \sigma$ then we say that $\bar{B}$ does not dominate $\bar{A}$ and $\bar{A}$ does not dominate $\bar{B}$, and write $\bar{B} \approx \bar{A}$. The constant σ is chosen by the analyst depending on the needs of the problem.

V. FUZZY RANKING OF ALTERNATIVES [1], [7]

In most decision making situations, the state of the system is seldom known exactly. If the state of the system is known exactly, then one may select the alternative which has maximum utility. With inexact knowledge about the state of the system, the choice of the best alternative is not so simple. For handling this type of situation, statistical decision theory assumes that this inexact knowledge about the state of the system may be attributed to uncertainty, and then the optimal decision may be taken using the tools of statistics. In many cases this inexact knowledge about the state of the system is correctly attributed to uncertainty, but there are systems where it is incorrect to consider this inexactness due to uncertainty. This inexactness may be due to impreciseness. Such situations are quite familiar to human beings who are accustomed to using linguistic variables such as small, very small, medium, very large, etc. These terms give rise to inexactness which is due to ill–defined terms, rather than due to uncertainty. The application of statistical decision theory is not proper in such situations.

However, it is well known that every normal human being is capable of understanding the meaning of such ill–defined terms and of taking proper action if the situation is not too complex. However, if there are many criteria and many alternatives available, then some mechanized or systematic approach to decision making is needed.

V.1. Experts' Ranking

Assume that we have m alternatives to be ranked from best to worst which will be denoted by $A_1, A_2, ..., A_m$. An analyst or a team of analysts will study the problem and achieve the decision selecting from a set of alternatives, those which best satisfy the previously decided criteria, $C_1, C_2, ..., C_K$. Each criterion may have R^k subcriteria $(k = 1, 2, \dots, K)$ and the $r-th$ subcriterion of $k-th$

criterion will be denoted by c_{kr} $(r = 1, 2, ..., R^k)$. The decision is made with the help of the testimony of n experts, $J_1, J_2, ..., J_n$.

In our method, the experts are asked: (a) to rank the criteria assigning them one or two positive integer numbers from 0 to 10 which indicates the increasing importance of each criterion from "very unimportant" to "very important" with respect to an overall objective. For example, assignment of 3 to a criterion shows that its importance is around 3 and the analyst assigns a fuzzy number to this ranking which is described by the membership function given in fig. 5.1. It has triangular shape with the top at $x = 3$. If an expert assigns two numbers, say 4 and 5, this has the meaning that importance of the criterion is in the range of 4 and 5. The corresponding fuzzy number has a trapezoidal membership function which is shown on the same figure. The base of the membership functions, depending on having one or two numbers on the experts ranking list are taken as two or three units respectively. (b) to rank each subcriterion in a similar way according their relative importance among others in the same criterion. (c) to rank alternatives indicating, how well alternatives satisfy each subcriterion.

Rankings are done assigning the numbers as explained above. In the last case (c), the numbers from 0 to 10 cover the range of "very poor" to "very good" in the expert's opinion.

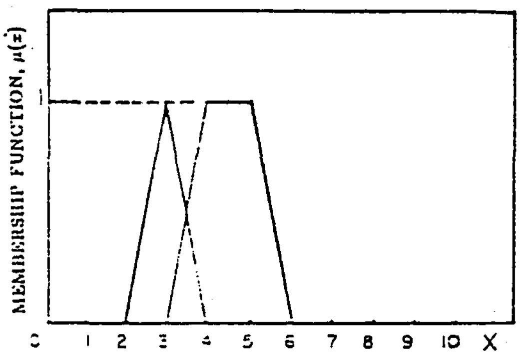

Figure 5.1. Fuzzy description of expert ranking "3" and "4,5"

Since experts' rankings are represented by fuzzy numbers, their assignments of integer numbers to criteria and alternatives do not mean that rankings are done precisely in a ten point scale as in crisp ranking. It should be pointed out to the experts by the analyst that the imprecisenesses in their rankings are expressed by the fuzzy numbers. This is a rather similar process to verbal ranking. In this method the differences between experts' ranking may be better expressed because of a wide variety of fuzzy number sets that can be constructed according to their ranking. However, there is no restriction on the experts not to use crisp ranking. Experts' crisp ranking for some of the criteria and alternatives do not create a problem since non–fuzzy numbers can be described in the form of $(\alpha/\alpha, \alpha/\alpha)$ as mentioned above.

V.2. Fuzzy Ranking

Fuzzy ranking of alternatives is accomplished following the steps given below :

1) The analyst assignes fuzzy numbers to each expert ranking in the form of (4.19).

2) The weighted average rating $(\bar{\omega}_{ik})$, indicating how well A_i satisfies C_k is obtained using the rating made for the alternatives, criteria and subcriteria, and then averaging accross the experts.

3) Fuzzy number $(\bar{b}_{kj})$ which is a measure of the importance of criterion C_k with respect to an overall objective for expert J_j is obtained. And averaging across the experts, fuzzy number $\bar{n}_k$ is found.

4) The fuzzy weight $(\bar{\omega}_i)$ that alternative A_i receives is obtained as

$$\overline{w}_i = (1/KL) \odot [(\overline{w}_{i1} \odot \overline{n}_1) \oplus (\overline{w}_{i2} \odot \overline{n}_2) \oplus \ldots \oplus (\overline{w}_{iK} \odot \overline{n}_K)] \ . \tag{5.1}$$

where K is the number of criteria and L is the highest grade that an expert can give (L=10 in our case). The calculation of above mentioned fuzzy numbers are given in details in ref. 1.

Fuzzy weights $\overline{w}_i$ are found using the given definitions and formulas in chapter IV. Fuzzy weights are then used by the analyst to chose the most preferred alternative or to put alternatives, A_i, in an order of preference. Let e_{ij} $(i, j = 1, 2, ..., m)$ be the possibility that fuzzy weights $\overline{w}_i \geq \overline{w}_j$ as given in eq. (4.38). It is proposed [9] to define sets H_i and to put all undominated alternatives to the set H_1. The set H_2 will include all undominated alternatives after removing those that are in H_1. This process terminates in a set, say H_d. Then, highest ranking alternatives are in H_1 while lowest ranking alternatives are included in H_d. The set H_i may contain more than one alternative which have the same level in ranking. If H_1 has more than one alternative the process may be repeated for the ranking among these alternatives. The constant σ should also be taken to the consideration (see section IV. 2. 3). A computer program is prepared and applied to some examples which are given in the following section.

V.3. Examples

As an application, a hypothetical example (see table 5.1) is ranked by the Fuzzy–Logic method. For comparing the results with conventional methods, this example is also ranked by the Weighted Average method (see section II.2.1.). The similarity between two methods can be considered good. But, Probabilistic Average method is better (see section II.2.2.) for comparison. First one is chosen because of its easy to apply nature.

The application of the equation (2.1) given in the section II.2.1 to our example of multi expert and criteria having subcriteria case is as follows:

i) The eq. (2.1) is rewritten for our case as

$$(A_i^k) = \frac{\sum_{j=1}^{m} (c_{kr}) \times A_i^{kr}}{\sum_{j=1}^{m} (c_{kr})} \tag{5.2}$$

where (A_i^k) is for (A_i) in equ. (2.1) for the criterion C_k. (A_i^{kr}) corresponds X_{ij} in equ. (2.1) which represents how well A_i satisfies subcriterion r of criterion k (which is denoted by c_{kr}). (c_{kr}) is used instead of P_j in equ. (2.1) which represents the subcriteria weights. $r(r = 1, \ldots, R^k)$ is the number of subcriteria in each criterion.

ii) For the last step, equ. (2.1) may be written as

$$(A_i) = \frac{\sum_{j=1}^{m} (C_k) \times (A_i^k)}{\sum_{j=1}^{m} (C_k)} \tag{5.3}$$

where (C_k) is the criterion weight. Final ranking of the alternatives are made according to the values (A_i)'s calculated for each of the alternatives.

In this example three criteria, C_k, each having three subcriteria, c_{kr}, are used for ranking three alternatives, A_i, by three experts, J_j. While ranking the alternatives by the weighted average method, two valued rankings of the fuzzy–logic method such as "7,8" or "3,4" are averaged and become "7.5" and "3.5" respectively.

Table 5.1 Experts' ranking of the hypothetical example

				A_1			A_2			A_3		
	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3
C_1	5	4, 5	4									
c_{11}	9, 10	10	9	7, 8	7	6, 7	8	8	8	8	8	10
c_{12}	7	5, 6	6, 7	8	9	7	7	8	6	9, 10	10	9
c_{13}	7	4	5, 6	7	7	7	7	7	7	8	7, 8	8, 9
C_2	7, 8	7, 8	7, 8									
c_{21}	8	6	7	3	4	5	6	5	4	6	9	7, 8
c_{22}	8	8, 9	9	5	5	5	6	6	6	8	7, 8	7
c_{23}	7, 8	9	8, 9	3, 4	4	4, 5	5	5, 6	6	7, 8	6	6, 7
C_3	8	8	8									
c_{31}	8	8	8	7	6, 7	6	6	6	6	8, 9	9	8
c_{32}	3	3	3	10	10	10	10	10	10	9	10	9, 10
c_{33}	5	7	6	4	4	4	8	8	8	6, 7	7, 8	8, 9

The result of the comparison is given below.

a) Results of fuzzy ranking of the hypothetical example (see fig. 5.2):

$$\overline{w}_1 = (2.08[0.015, 0.242, 1.270]/3.61\,,\ 4.10/6.24[-0.012, 0.300, -2.435])\,,$$
$$\overline{w}_2 = (2.57[0.015, 0.255, 1.432]/4.27\,,\ 4.69/7.01[-0.012, 0.310, -2.614])\,,$$
$$\overline{w}_3 = (3.07[0.015, 0.270, 1.594]/4.94\,,\ 5.74/8.31[-0.011, 0.316, -2.872])$$

The points of intersections on fig. 5.2 are found as A(4.18, 0.96), B(4.50, 0.79), C(4.81, 0.94). Refering to the sections IV. 2. 3 and V.2 and choosing σ =0.95, the obvious conclusion is that the ranking of the alternatives is in the order of A_3, A_2, A_1.

b) Results of the hypothetical example's ranking by the weighted average method:

$$(A_1) = 5.77\,,$$
$$(A_2) = 6.71\,,$$
$$(A_3) = 8.00\,,$$

According to these values, the alternatives are ranked in the order of A_3, A_2, A_1.

As it can be seen, both of the methods give the same ranking. The computer program for the weighted average method is also given at the appendix.

The results of the example for a nuclear power plant site selection given in Ref.1 are obtained by applying each method. The criteria and subcriteria list of this example is given in table 5.2. Table 5.3 shows ranking of subcriteria and alternatives by three experts. Table 5.4 gives the experts ranking of criteria.

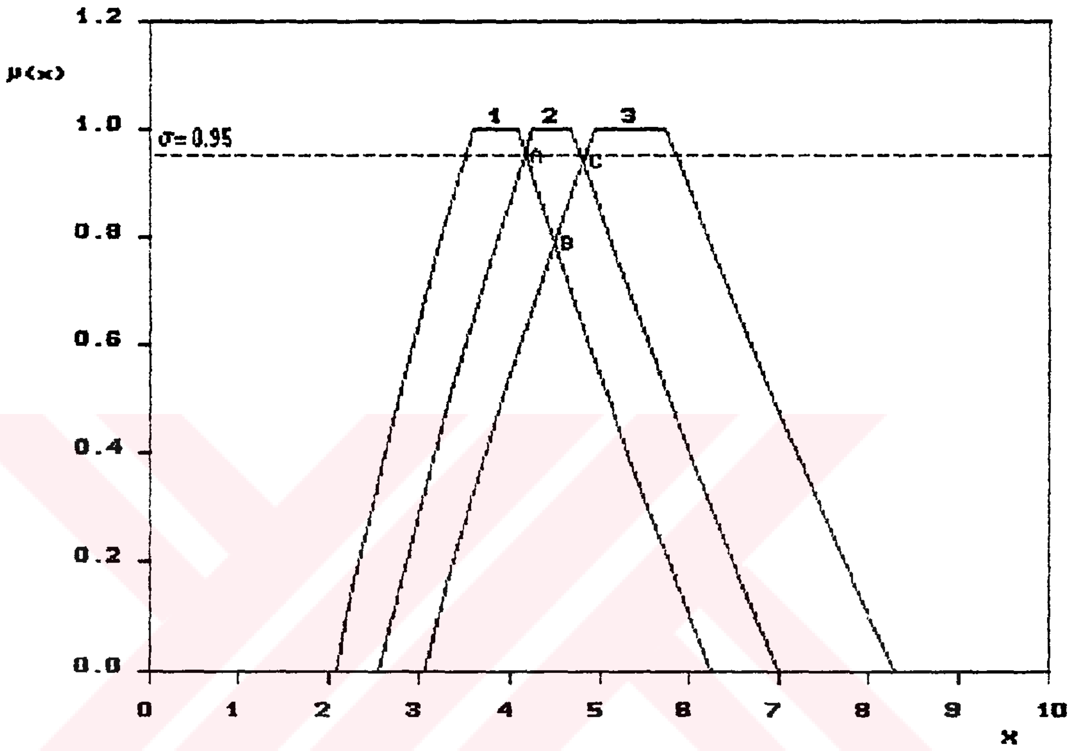

Figure 5.2 Graphical representation of the results of hypothetical example given in table 5.1

Table 5.2 List of the criteria for the assesment of the sites

Symbol	Criteria	Subcriteria
C_1	Topography and oceanography	
c_{11}		Ground level and easy access
c_{12}		Site survey
c_{13}		Water debth
c_{14}		Tides and tidal currents
c_{15}		Thermal pollution
C_2	Geology, hydrology and seismology	
c_{21}		Geological structure
c_{22}		Soil characteristics
c_{23}		Underground water tables
c_{24}		Water sources
c_{25}		Earthquakes
c_{26}		Seismic equipments and locations
c_{27}		Tsunamis
C_3	Meteorology	
c_{31}		Rainfall and snowfall
c_{32}		Thunderstorm, dust and sandstorm
c_{33}		Humidity, fogs and mists
c_{34}		Temperature
c_{35}		Wind
C_4	Transportation	
c_{41}		Proximity to main arteries
c_{42}		Transportation capability
C_5	Population	
c_{51}		Density
c_{52}		Seasonal variation
c_{53}		Future population
C_6	Cooling water	
c_{61}		Sources
c_{62}		Availability
c_{63}		Distance
c_{64}		Characteristics
C_7	Construction, services and domestic water	
c_{71}		Land planning difficulties
c_{72}		Land properties for construction
c_{73}		Labor for construction
c_{74}		Construction difficulties
c_{75}		Electricity
c_{76}		Gas
c_{77}		Drainage and sewerage
c_{78}		Transmission, distribution lines
c_{79}		Availability of domestic water
c_{710}		Characteristics of domestic water
c_{711}		Distr. syst. for domestic water

Table 5.3 Experts' ranking of subcriteria and alternatives

	Weights			A_1			A_2			A_3		
c_{kr}	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3
C_1												
c_{11}	9,10	10	9	7,8	7	6,7	8	8	7	8	8	7,8
c_{12}	7	5,6	4,5	8	9	9	7	8	8	9,10	10	8
c_{13}	7	4	5,6	7	7	6	7	7	9	6	7,8	6
c_{14}	4	3,4	5	10	10	10	10	10	10	10	10	10
c_{15}	9	8	7,8	7	7	6,7	6	6	5	9,10	8,9	7
C_2												
c_{21}	8	6	6,7	3	4	4	6	5	6,7	6	9	8,9
c_{22}	8	8,9	7	5	5	6	6	6	6	8	7,8	7
c_{23}	7	7	5,6	4	4,5	3	6	7	7	7	6	7
c_{24}	8	6	8	3	3	4	6	7	7	9	10	8
c_{25}	10	10	9	5	5	6,7	8	6,7	7,8	10	10	10
c_{26}	3,4	7	4	10	10	9	8,9	9	8	10	9,10	9
c_{27}	6	5	5	10	10	10	9	10	9	10	10	10
C_3												
c_{31}	8	8	9,10	7	6,7	7	6	6	7	8,9	9	9,10
c_{32}	3	3	2,3	10	10	10	10	10	9	9	10	10
c_{33}	5	7	8	4	4	3,4	8	8	6	6	5	4,5
c_{34}	5	5,6	3	8	7,8	9	9	8,9	10	10	10	9
c_{35}	10	9,10	10	3	4,5	5	8	6	8	10	8	10
C_4												
c_{41}	6	6	8	5	5	6	8	8	7	10	10	8
c_{42}	10	10	7,8	7	5	7	8	8,9	9	10	9	9,10
C_5												
c_{51}	8	7,8	7	3	3	4,5	9,10	8,9	7	10	7,8	9,10
c_{52}	8	9	7	2	3	4	5	4	4	6	6	7
c_{53}	8	8	9	3	4	4	3,4	2,3	3	7	8,9	7
C_6												
c_{61}	7	7,8	9	6	6	5,6	7,8	7,8	9	10	8,9	10
c_{62}	10	10	10	6	7	7	8	8	9	10	10	9
c_{63}	5	6	6	7	6	6	5	6	5	10	9	9
c_{64}	7	7,8	6	7	6	6	8	9	8	10	8	9
C_7												
c_{71}	6	7,8	6	5	4,5	6	8	6	6	10	10	9
c_{72}	8	9	7	4	5	5,6	8	8	7	10	10	10
c_{73}	6	7	7	6	6	6,7	8	8,9	8	9,10	8	8
c_{74}	7	7	7	6	7	7	7	6	6	8	10	8
c_{75}	9	9	10	7	6,7	5,6	7	8	10	9	9	10
c_{76}	5	5	4	7	6	8	10	10	9	8	8,9	8
c_{77}	5	7	7	8	8	8	10	10	10	9	9	10
c_{78}	8	7	10	7	7	7	7, 8	9	7,8	10	10	10
c_{79}	10	10	10	6	6,7	6	8,9	10	10	8,9	9,10	9
c_{710}	6	8	5,6	6	5,6	7	8	7,8	6,7	10	9	9
c_{711}	6	7,8	8	4	5	5	5	6	5	8	7,8	7

Table 5.4 Experts' ranking of criteria

	Experts Ranking			Fuzzy Description ($\bar{b}_{kj}$)			Averaging Across the Experts ($\bar{n}_k$)
	J_1	J_2	J_3	J_1	J_2	J_3	
C_1	5	4,5	5,6	(4/5, 5/6)	(3/4, 5/6)	(4/5, 6/7)	(3.67/4.67, 5.33/6.33)
C_2	7,8	7,8	6	(6/7, 8/9)	(6/7, 8/9)	(5/6, 6/7)	(5.67/6.67, 7.33/8.33)
C_3	8	8	6,7	(7/8, 8/9)	(7/8, 8/9)	(5/6, 7/8)	(6.33/7.33, 7.67/8.67)
C_4	6,7	7,8	5,6	(5/6, 7/8)	(6/7, 8/9)	(4/5, 5/6)	(5/6, 6.67/7.67)
C_5	9,10	9,10	8,9	(8/9, 10/10)	(8/9, 10/10)	(7/8, 9/10)	(7.67/8.67 , 9.67/10)
C_6	9,10	10	9,10	(8/9, 10/10)	(9/10, 10/10)	(8/9, 10/10)	(8.33/9.33, 10/10)
C_7	5,6	5	6	(4/5, 6/7)	(4/5, 5/6)	(5/6, 6/7)	(4.33/5.33, 5.67/6.67)

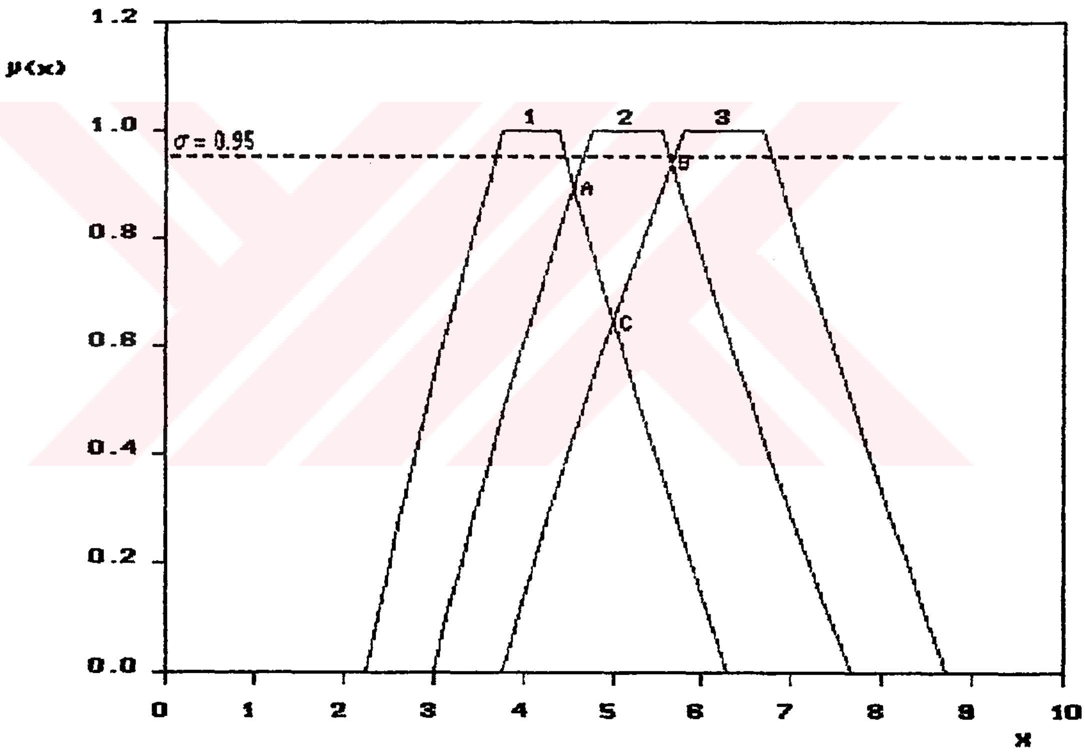

Figure 5.3 Graphical representation of the results of the sites ranking given in table 5.3

a) Results of fuzzy ranking of the sites:

$$\bar{w}_1 = (2.25[0.014, 0.234, 1.283]/3.78\,,\ 4.40/6.30[-0.007, 0.228, -2.139])\,,$$

$$\bar{w}_2 = (3.00[0.014, 0.253, 1.517]/4.78\,,\ 5.55/7.67[-0.007, 0.234, -2.351])\,,$$

$$\bar{w}_3 = (3.76[0.014, 0.271, 1.752]/5.80\,,\ 6.70/8.73[-0.005, 0.185, -2.211])$$

The points of intersections on fig. 5.3 are A(4.57, 0.90), B(5.66, 0.94), C(5.01, 0.65). Here also the ranking of the sites is in the order of A_3, A_2, A_1 by choosing σ=0.95.

b) Results of the sites ranking by the weighted average method:

$$(A_1) = 5.67\,,$$
$$(A_2) = 7.17\,,$$
$$(A_3) = 8.68\,,$$

The order of the sites is A_3, A_2, A_1 by these results.

As a conclusion it can be said that in both examples fuzzy ranking method gives results in harmony with that of the weighted average method which is the most widely used conventional ranking method.

VI. CONCLUSION

The main advantage of using fuzzy subsets is to overcome the difficulties of decision making in a fuzzy situation. The method enables us to make decision with inexact knowledge about state of the system. Expert may have many difficulties in crisp ranking, especially in the consideration of the events developing in time, like population and transportation in the vicinity of the site of a nuclear power plant. Fuzzy decision method may solve such difficulties.

A computer program which is an implementation of fuzzy decision analysis is given in Appendix A which can be applied to the problems in any field. The main feature of the program is its capability of making fuzzy decision analysis with the testimony of more than one experts to rank the alternatives. Consideration of subcriteria included in each criterion is possible.

In the presented method the expert need not to be involved in the highly matematical nature of fuzzy decision theory. They make their ranking using a ten–point scale, but they are aware that impreciseness in their ranking is expressed by fuzzy numbers which are assigned to the experts' ranking by the analyst.

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APPENDIX A. COMPUTER PROGRAMS FOR FUZZY RANKING OF ALTERNATIVES

A1. Fortran Program for Fuzzy Ranking of the Alternatives Mathematically

```
      PROGRAM FUZZY

      DIMENSION Rk(20),B(20,10,4),Nk(20,4)
      DIMENSION D(20,20,10,4),P(20,20,4),AvN(20),PN(20,20,4)
      DIMENSION A(10,20,20,10,4),M(10,20,20,4)
      DIMENSION Wik(10,20,4),S1(10,20),S2(10,20),T1(10,20),T2(10,20)
      DIMENSION Wi(10,4),U1(10),U2(10),U3(10),V1(10),V2(10),V3(10)

      REAL Nk,M
      INTEGER Ck,Rk,Ai,r

      OPEN (1,FILE='VARS_WS.IN',STATUS='OLD')
      OPEN (2,FILE='VARS.OUT',STATUS='unknown')
      OPEN (3,FILE='B(kj).OUT',STATUS='unknown')
      OPEN (4,FILE='N(k).OUT',STATUS='unknown')
      OPEN (5,FILE='D(krj).OUT',STATUS='unknown')
      OPEN (6,FILE='P(kr).OUT',STATUS='unknown')
      OPEN (7,FILE='PN(kr).OUT',STATUS='unknown')
      OPEN (8,FILE='A(ikrj).OUT',STATUS='unknown')
      OPEN (9,FILE='M(ikr).OUT',STATUS='unknown')
      OPEN (10,FILE='W(ik).OUT',STATUS='unknown')
      OPEN (11,FILE='W(i).OUT',STATUS='unknown')
      OPEN (12,FILE='COEFFS',STATUS='unknown')

C     ******************************************************************************
C     **************** INPUT OF THE VARIABLE NUMBERS *****************
C     ******************************************************************************

      READ(1,*)
      READ(1,*)
      READ(1,*)  Ai
      WRITE(2,*) ' ALTERNATIVE NUMBER = ' , Ai
      READ(1,1)
    1 FORMAT (//)
      READ(1,*)  Jj
      WRITE(2,*)
      WRITE(2,*) ' EXPERT NUMBER = ' , Jj
      READ(1,1)
      READ(1,*)  Ck
      WRITE(2,*)
      WRITE(2,*) ' CRITERION NUMBER = ' , Ck
      READ(1,1)
      READ(1,*)  (Rk(k) , k = 1,Ck)

      DO 5 k = 1,Ck
    5 WRITE(2,*)  Rk(k)
```

```
C     ********************************************************************************
C     *************** INPUT OF THE CRITERION WEIGHTS ******************
C     ********************************************************************************

      WRITE(3,*)
      WRITE(3,*)  ' *************** CRITERION WEIGHTS *************** '
      WRITE(3,*)
      READ(1,2)
    2 FORMAT (//////)

      DO 10 k = 1,Ck
          READ(1,*)
          READ(1,*)
          WRITE(3,*)
      DO 10 j = 1,Jj
          READ(1,*)  (B(k,j,n) , n = 1,4)
          WRITE(3,*)  'C' , k , ',' , 'J' , j , ':'
          WRITE(3,*)
          WRITE(3,*)  'B(k,j) =' , (B(k,j,n) , n = 1,4)
   10     WRITE(3,*)

C     ********************************************************************************
C     *************** AVERAGING THE CRITERION WEIGHTS *****************
C     ********************************************************************************

      WRITE(4,*)
      WRITE(4,*)  ' *********** AVERAGED CRITERION WEIGHTS *********** '
      WRITE(4,*)

      DO 15 k = 1,Ck

          DO 20 n = 1,4
              Nk(k,n) = 0.0

              DO 25 j = 1,Jj
   25             Nk(k,n) = Nk(k,n) + B(k,j,n)

   20          Nk(k,n) = Nk(k,n) / Jj

          WRITE(4,*)  'C' , k , ':'
          WRITE(4,*)
          WRITE(4,*)  'N(k) =' , (Nk(k,n) , n = 1,4)
   15     WRITE(4,*)

C     ********************************************************************************
C     *************** INPUT OF THE SUBCRITERION WEIGHTS ***************
C     ********************************************************************************

      WRITE(5,*)
      WRITE(5,*)  ' ************* SUBCRITERION WEIGHTS ************** '
      WRITE(5,*)
      READ(1,3)
    3 FORMAT (////)
```

```
      DO 30 k = 1,Ck
         READ(1,4)
 4       FORMAT (///)
         WRITE(5,*)
      DO 30 r = 1,Rk(k)
         READ(1,*)
         READ(1,*)
         WRITE(5,*)
      DO 30 j = 1,Jj
         READ(1,*)  (D(k,r,j,n) , n = 1,4)
         WRITE(5,*)  'C' , k , r , ',' , 'J' , j , ':'
         WRITE(5,*)
         WRITE(5,*)  ' D(k,r,j) = ' , (D(k,r,j,n) , n = 1,4)
30       WRITE(5,*)

C    ********************************************************************************
C    *************** AVERAGING THE SUBCRITERION WEIGHTS ***************
C    ********************************************************************************

      WRITE(6,*)
      WRITE(6,*)  ' ********* AVERAGED SUBCRITERION WEIGHTS ********** '
      WRITE(6,*)

      DO 35 k = 1,Ck
         WRITE(6,*)
      DO 35 r = 1,Rk(k)

         DO 40 n = 1,4
            P(k,r,n) = 0.0

            DO 45 j = 1,Jj
45             P(k,r,n) = P(k,r,n) + D(k,r,j,n)

40          P(k,r,n) = P(k,r,n) / Jj

         WRITE(6,*)  'C' , k , r , ':'
         WRITE(6,*)
         WRITE(6,*)  'P(k,r) =' , (P(k,r,n) , n = 1,4)
35       WRITE(6,*)

C    ********************************************************************************
C    ************** NORMALIZING THE SUBCRITERION WEIGHTS **************
C    ********************************************************************************

      WRITE(7,*)
      WRITE(7,*)  ' ********* NORMALIZED SUBCRITERION WEIGHTS ******** '
      WRITE(7,*)

      DO 50 k = 1,Ck
         WRITE(7,*)
         AvN(k) = 0.0

         DO 55 r = 1,Rk(k)
55          AvN(k) = AvN(k) + (P(k,r,2) + P(k,r,3)) / 2.
```

```
      DO 50 r = 1,Rk(k)

         DO 60 n = 1,4
60          PN(k,r,n) = P(k,r,n) * 10.0 / AvN(k)

         WRITE(7,*)  'C' , k , r , ':'
         WRITE(7,*)
         WRITE(7,*)  'PN(k,r) =' , (PN(k,r,n) , n = 1,4)
50       WRITE(7,*)

C    ******************************************************************************
C    *************** INPUT OF THE ALTERNATIVE WEIGHTS *****************
C    ******************************************************************************

      WRITE(8,*)
      WRITE(8,*) ' *************** ALTERNATIVE WEIGHTS *************** '
      WRITE(8,*)
      READ(1,3)

      DO 65 k = 1,Ck
         READ(1,3)
         WRITE(8,*)
      DO 65 r = 1,Rk(k)
         READ(1,*)
         READ(1,*)
         WRITE(8,*)
      DO 65 j = 1,Jj
         READ(1,*)  ((A(i,k,r,j,n) , n = 1,4) , i = 1,Ai)
         WRITE(8,*)
      DO 65 i = 1,Ai
         WRITE(8,*)  'C' , k , r , ',' , 'J' , j , ',' , 'A' , i , ':'
         WRITE(8,*)
         WRITE(8,*)  'A(i,k,r,j) =' , (A(i,k,r,j,n) , n = 1,4)
65       WRITE(8,*)

C    ******************************************************************************
C    *************** AVERAGING THE ALTERNATIVE WEIGHTS ****************
C    ******************************************************************************

      WRITE(9,*)
      WRITE(9,*)  ' ********** AVERAGED ALTERNATIVE WEIGHTS ********** '
      WRITE(9,*)

      DO 70 i = 1,Ai
         WRITE(9,*)
      DO 70 k = 1,Ck
         WRITE(9,*)
      DO 70 r = 1,Rk(k)

         DO 75 n = 1,4
            M(i,k,r,n) = 0.0

            DO 80 j = 1,Jj
80             M(i,k,r,n) = M(i,k,r,n) + A(i,k,r,j,n)

75          M(i,k,r,n) = M(i,k,r,n) / Jj
```

```
      WRITE(9,*)  'A' , i , ',' , 'C' , k , r , ':'
      WRITE(9,*)
      WRITE(9,*)  'M(i,k,r) =' , (M(i,k,r,n) , n = 1,4)
70    WRITE(9,*)

C     ****************************************************************************************
C     ***** CALCULATING THE CRITERION RANKING OF THE ALTERNATIVES *****
C     ****************************************************************************************

    WRITE(10,*)
    WRITE(10,*)  ' ***** CRITERION RANKING OF THE ALTERNATIVES ***** '
    WRITE(10,*)

    DO 85 i = 1,Ai
      WRITE(10,*)
    DO 85 k = 1,Ck
      S1(i,k) = 0.0
      S2(i,k) = 0.0
      T1(i,k) = 0.0
      T2(i,k) = 0.0

      DO 90 n = 1,4
        Wik(i,k,n) = 0.0
      DO 90 r = 1,Rk(k)
90      Wik(i,k,n) = Wik(i,k,n) + PN(k,r,n) * M(i,k,r,n) / 10.0

      WRITE(10,*)  'A' , i , ',' , 'C' , k , ':'
      WRITE(10,*)
      WRITE(10,*)  'W(i,k) =' , (Wik(i,k,n) , n = 1,4)
      WRITE(10,*)

      DO 95 r = 1,Rk(k)
        S1(i,k) = S1(i,k) + (M(i,k,r,2) - M(i,k,r,1)) * (PN(k,r,2) - PN(k,r,1) ) / 10.0
        S2(i,k) = S2(i,k) + (PN(k,r,1) * (M(i,k,r,2) - M(i,k,r,1)) + M(i,k,r,1) * (PN(k,r,2)
     *             - PN(k,r,1))) / 10.0
        T1(i,k) = T1(i,k) + (M(i,k,r,4) - M(i,k,r,3)) * (PN(k,r,4) - PN(k,r,3)) / 10.0
95      T2(i,k) = T2(i,k) - (PN(k,r,4) * (M(i,k,r,4) - M(i,k,r,3)) + M(i,k,r,4) * (PN(k,r,4)
     *             - PN(k,r,3)))  / 10.0

      WRITE(10,*)  ' COEFFICIENTS : '
      WRITE(10,*)
      WRITE(10,*)  ' S1(i,k) = ' , S1(i,k) , ' S2(i,k) = ' , S2(i,k)
      WRITE(10,*)  ' T1(i,k) = ' , T1(i,k) , ' T2(i,k) = ' , T2(i,k)
85    WRITE(10,*)

C     ****************************************************************************************
C     ***** CALCULATING THE OVERALL RANKING OF THE ALTERNATIVES *****
C     ****************************************************************************************

    WRITE(11,*)
    WRITE(11,*)  ' ****** OVERALL RANKING OF THE ALTERNATIVES ****** '
    WRITE(11,*)
```

```
      DO 100 i = 1,Ai
         U1(i) = 0.0
         U2(i) = 0.0
         U3(i) = 0.0
         V1(i) = 0.0
         V2(i) = 0.0
         V3(i) = 0.0

         DO 105 n = 1,4
            Wi(i,n) = 0.0
         DO 105 k = 1,Ck
105         Wi(i,n) = Wi(i,n) + Nk(k,n) * Wik(i,k,n) / (Ck * 10.0)

         WRITE(11,*)  'A' , i , ':'
         WRITE(11,*)
         WRITE(11,*)  'W(i) =' , (Wi(i,n) , n = 1,4)
         WRITE(11,*)

         DO 110 k = 1,Ck
            U1(i) = U1(i) + S1(i,k) * (Nk(k,2) - Nk(k,1)) / (Ck * 10.0)
            U2(i) = U2(i) + (S1(i,k) * Nk(k,1) + S2(i,k) * (Nk(k,2) - Nk(k,1))) / (Ck * 10.0)
            U3(i) = U3(i) + (S2(i,k) * Nk(k,1) + Wik(i,k,1) * (Nk(k,2) - Nk(k,1))) / (Ck * 10.0)
            V1(i) = V1(i) - T1(i,k) * (Nk(k,4) - Nk(k,3) ) / (Ck * 10.0)
            V2(i) = V2(i) + (T1(i,k) * Nk(k,4) - T2(i,k) * ( Nk(k,4) - Nk(k,3))) / (Ck * 10.0)
110         V3(i) = V3(i) + (T2(i,k) * Nk(k,4) - Wik(i,k,4) * (Nk(k,4) - Nk(k,3))) / (Ck * 10.0)

         WRITE(11,*)  ' COEFFICIENTS : '
         WRITE(11,*)
         WRITE(11,*) 'U1(i)=',U1(i), ' U2(i) =' ,U2(i),' U3(i) =',U3(i)
         WRITE(11,*) 'V1(i)=',V1(i), ' V2(i) =' ,V2(i),' V3(i) =',V3(i)
         WRITE(11,*)
100      WRITE(12,*) U1(i),U2(i),U3(i),Wi(i,1),V1(i),V2(i),V3(i),Wi(i,4)


      STOP
      END
```

A2. C++ Program for Displaying the Ranked Alternatives Graphically

```
#include <graphics.h>
#include <stdlib.h>
#include <stdio.h>
#include <math.h>
#include <conio.h>
#include <string.h>

char *A = "COEFFS", *B = "POINTS";
char laf[30], laf1[2];
float garip;

void main(void)
{
FILE *dat, *cik;
register int i;
int gdriver = DETECT, gmode, errorcode;
int basx, basy, sonx, sony, xm, ym;
int n, m, ii, ar=0, art=0;
float xxa, xxb, yya, yyb;
float xxa1, xxb1, xxa2, xxb2;
float uzun;
float y[]= {0.,0.,0.,0.,0.};
float x[5];
int j;
int xx, yy, col, s;
int xtx, xty, ytx, yty;
float x1, y1, stepc;
float step, kay, kax, st, sigma;
float d[9][8];
char textx[40], texty[40];
char oku[100], ch='A';
clrscr();

/////////////////////////////////
/// READING THE FILE ///
/////////////////////////////////

i=0;
if ((dat = fopen(A, "r"))== NULL) {
                                    printf("%s can not be opened ! \n",A);
                                       exit(1);
                                     }
do {
   for(j=0; j<8; j++) {
                     fscanf(dat,"%s",oku);
                     d[i][j] = atof(oku);
                     }
   i++;
   }
```

```
/////////////////////////////////////////////////
/// DRAWING OF THE X--AXIS ///
////////////////////////////////////////////////

step=1;
stepc=(sonx-basx)/10.0;
for(i=0; i<11; i++) {
                        x1=basx+i*stepc;
                        garip=i*step;
                        sprintf(laf,"%1.0f",garip);
                        outtextxy(x1-6,sony+16,laf);
                        line(x1,sony+6,x1,sony);
                        }
rectangle(basx,basy,sonx,sony);

////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////
/// DRAWING OF THE GRAPHICS AND WRITING OF THE CURVE INTERCEPTS ///
////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////////

col=16;
st=.1;
for(j=0; j<s; j++) {
                        xx= basx+kax*(d[j][3]);yy= sony;
                        col--;
                        setcolor(col);
                        moveto(xx,yy);
                        for(y1=0; y1<=1.01; y1+=st) {
                                    xxa= d[j][0]*y1*y1*y1+d[j][1]*y1*y1+d[j][2]*y1+d[j][3];
                                    xxb= kax*xxa; xx= basx+xxb;
                                    yy= sony-kay*y1;
                                    lineto(xx,yy);
                                    }
                        uzun=basx+kax*(d[j][4]+d[j][5]+d[j][6]+d[j][7]);
                        sprintf(laf,"%d",j+1);
                        outtextxy(xx+(uzun-xx)/2-4,sony-kay-10,laf);
                        line(xx,sony-kay,uzun,sony-kay);
                        moveto(xx,yy);
                        for(y1=1; y1>=-.01; y1-=st) {
                                xx= basx+kax*(d[j][4]*y1*y1*y1+d[j][5]*y1*y1+d[j][6]*y1+d[j][7]);
                                yy= sony-kay*y1;
                                lineto(xx,yy);
                                }
                        }
setcolor(YELLOW);
outtextxy(basx+425,sony+30,textx);
outtextxy(basx-80,30,texty);
if ((cik = fopen(B, "w"))== NULL) {
                                    printf("%s can not be opened ! \n",B);
                                exit(1);
                                }
```

```
while(!feof(dat));
fclose(dat);
s=i-1;
printf("Sigma= ");
scanf("%f",&sigma);

/////////////////////////////////////////////////////////////////////
/// INITIALIZATION OF THE GRAPHICS CARD ///
/////////////////////////////////////////////////////////////////////

 registerbgidriver((void(*)(void))EGAVGA_driver);
 registerbgidriver((void(*)(void))Herc_driver);
 registerbgifont((void(*)(void))small_font);
 initgraph(&gdriver, &gmode, "");
 errorcode = graphresult();
 if (errorcode != grOk) {
                    printf("Graphics Error ! : %s\n", grapherrormsg(errorcode));
                    printf("Press any key to halt:");
                    getch();
                    exit(1);
                    }
xm = getmaxx();
ym = getmaxy();
basx = 0.2*xm;
basy = 10;
sonx = 0.9*xm;
sony = 0.7*ym;
kax = 0.7*xm/10;
kay = (sony-basy)/1.2;

//////////////////////////////////////////////////////////
/// LABELS OF THE X AND Y AXISES ///
//////////////////////////////////////////////////////////

strcpy(textx,"X");
strcpy(texty,"µ(x)");

//////////////////////////////////////////////
/// DRAWING OF THE Y--AXIS ///
//////////////////////////////////////////////

step = 0.2;
stepc = (sony-basy)/6.0;
settextstyle( 0, HORIZ_DIR, 1 );
setcolor(9);
for(i=0; i<7; i++) {
                x1=basy+i*stepc;
                garip=1.21-i*step;
                sprintf(laf,"%1.1f",garip);
                outtextxy(basx-38,x1-6,laf);
                line(basx-6,x1,basx,x1);
                 }
```

```
setcolor(BLUE);
sprintf(laf,"%s","Point");
outtextxy(5,sony+50,laf);
sprintf(laf,"%s","µ(x)");
outtextxy(5,sony+62,laf);
sprintf(laf,"%s","x");
outtextxy(5,sony+74,laf);
st=.002;
n=0;
for(i=0; i<s-1; i++) {
                fprintf(cik,"%d %s\n",i+1,". curve intercepts");
                for(j=i+1; j<s; j++) {
                            y[1]=0.;y[2]=0.;y[3]=0.;y[4]=0.;
                            for(y1=1; y1>=0; y1-=st) {
                              xxa1= d[i][0]*y1*y1*y1+d[i][1]*y1*y1+d[i][2]*y1+d[i][3];
                              xxa2= d[i][4]*y1*y1*y1+d[i][5]*y1*y1+d[i][6]*y1+d[i][7];
                              xxb1= d[j][0]*y1*y1*y1+d[j][1]*y1*y1+d[j][2]*y1+d[j][3];
                              xxb2= d[j][4]*y1*y1*y1+d[j][5]*y1*y1+d[j][6]*y1+d[j][7];
                              garip=fabs(xxa1-xxb1);
                              if (garip<.01 && y[1] ==0.) {
                                              y[1]=y1;
                                              x[1]=xxa1;
                                              }
                              garip=fabs(xxa1-xxb2);
                              if (garip<.01 && y[2] ==0.) {
                                              y[2]=y1;
                                              x[2]=xxa1;
                                              }
                              garip=fabs(xxa2-xxb1);
                              if (garip<.01 && y[3] ==0.) {
                                              y[3]=y1;
                                              x[3]=xxa2;
                                              }
                              garip=fabs(xxa2-xxb2);
                              if (garip<.01 && y[4] ==0.) {
                                              y[4]=y1;
                                              x[4]=xxa2;
                                              }
                              }
                            settextstyle( 2, HORIZ_DIR, 4 );
                            for(ii=1; ii<5; ii++) {
                                    if(y[ii]!=0.) {
                                    sprintf(laf1,"%c",ch);
                                    setcolor(15-i);
                                    outtextxy(basx+kax*x[ii]+4,sony-kay*y[ii]-4,laf1);
                                  settextstyle( 0, HORIZ_DIR, 1 );
                                    if(ar==10) {
                                            art=44;
                                          ar=0;
                                            setcolor(BLUE);
                                            sprintf(laf,"%s","Point");
                                            outtextxy(5,sony+95,laf);
                                            sprintf(laf,"%s","µ(x)");
                                            outtextxy(5,sony+107,laf);
                                            sprintf(laf,"%s","x");
                                            outtextxy(5,sony+119,laf);
                                            setcolor(15-i);
```

```
                                    }
                        outtextxy(53*ar+60,sony+50+art,laf1);
                        sprintf(laf,"%1.3f",y[ii]);
                        outtextxy(53*ar+60,sony+62+art,laf);
                        sprintf(laf,"%1.3f",x[ii]);
                        outtextxy(53*ar+60,sony+74+art,laf);
                        ch++;
                      ar++;
                       }
                      }
        printf(cik,"%c      µ(x)= %1.3f %1.3f %1.3f %1.3f \n",ch,y[1],y[2],y[3],y[4]);
                  }
          n++;
          }
setcolor(GREEN);
setlinestyle(3,1,1);
line(basx,sony-sigma*kay,sonx,sony-sigma*kay);
fclose(cik);
getch();
closegraph();
}
```

APPENDIX B. COMPUTER PROGRAM FOR COMPARISON

```fortran
      PROGRAM NORM

      DIMENSION Rk(20),B(20,10),Nk(20)
      DIMENSION D(20,20,10),P(20,20),Pt(20)
      DIMENSION A(10,20,20,10),M(10,20,20)
      DIMENSION Wik(10,20),Wi(10)

      REAL Nk,M,Nkt
      INTEGER Ck,Rk,Ai,r

      OPEN (1,FILE='nVARS_WS.IN',STATUS='OLD')
      OPEN (2,FILE='nVARS.OUT',STATUS='unknown')
      OPEN (3,FILE='nB(kj).OUT',STATUS='unknown')
      OPEN (4,FILE='nN(k).OUT',STATUS='unknown')
      OPEN (5,FILE='nD(krj).OUT',STATUS='unknown')
      OPEN (6,FILE='nP(kr).OUT',STATUS='unknown')
      OPEN (8,FILE='nA(ikrj).OUT',STATUS='unknown')
      OPEN (9,FILE='nM(ikr).OUT',STATUS='unknown')
      OPEN (10,FILE='nW(ik).OUT',STATUS='unknown')
      OPEN (11,FILE='nW(i).OUT',STATUS='unknown')

C     **************** INPUT OF THE VARIABLE NUMBERS ****************

      READ(1,*)
      READ(1,*)
      READ(1,*)  Ai

      WRITE(2,*) ' ALTERNATIVE NUMBER = ' , Ai

      READ(1,1)
    1 FORMAT (//)
      READ(1,*)  Jj

      WRITE(2,*)
      WRITE(2,*) ' EXPERT NUMBER = ' . Jj

      READ(1,1)
      READ(1,*)  Ck

      WRITE(2,*)
      WRITE(2,*) ' CRITERION NUMBER = ' . Ck

      READ(1,1)
      READ(1,*)  ( Rk(k) , k = 1,Ck )

      DO 5 k = 1,Ck
    5    WRITE(2,*)  Rk(k)

C     **************** INPUT OF THE CRITERION WEIGHTS ****************

      WRITE(3,*)
      WRITE(3,*)  ' *************** CRITERION WEIGHTS *************** '
      WRITE(3,*)
```

```
      READ(1,2)
    2 FORMAT (//////)

      DO 10 k = 1,Ck

         READ(1,*)
         READ(1,*)

         WRITE(3,*)

      DO 10 j = 1,Jj

         READ(1,*)   B(k,j)

         WRITE(3,*)  'C' , k , ';' , 'J' , j , ':'
         WRITE(3,*)
         WRITE(3,*)  'B(k,j) =' , B(k,j)
   10    WRITE(3,*)

C     *************** AVERAGING THE CRITERION WEIGHTS ****************

      WRITE(4,*)
      WRITE(4,*)  ' *********** AVERAGED CRITERION WEIGHTS *********** '
      WRITE(4,*)

      DO 15 k = 1,Ck

         Nk(k) = 0.0

         DO 25 j = 1,Jj

   25       Nk(k) = Nk(k) + B(k,j)

         Nk(k) = Nk(k) / Jj

         WRITE(4,*)  'C' , k , ':'
         WRITE(4,*)
         WRITE(4,*)  'N(k) =' , Nk(k)
   15    WRITE(4,*)

C     *************** INPUT OF THE SUBCRITERION WEIGHTS ****************

      WRITE(5,*)
      WRITE(5,*)  ' ************* SUBCRITERION WEIGHTS ************** '
      WRITE(5,*)

      READ(1,3)
    3 FORMAT (////)

      DO 30 k = 1,Ck

         READ(1,4)
    4    FORMAT (///)

         WRITE(5,*)
```

```
      DO 30 r = 1,Rk(k)

         READ(1,*)
         READ(1,*)

         WRITE(5,*)

      DO 30 j = 1,Jj

         READ(1,*)  D(k,r,j)

         WRITE(5,*)  'C' , k , r , ',' , 'J' , j , ':'
         WRITE(5,*)
         WRITE(5,*)  ' D(k,r,j) = ' , D(k,r,j)
 30      WRITE(5,*)

C     *************** AVERAGING THE SUBCRITERION WEIGHTS ***************

      WRITE(6,*)
      WRITE(6,*)  ' ********* AVERAGED SUBCRITERION WEIGHTS ********** '
      WRITE(6,*)

      DO 35 k = 1,Ck

         WRITE(6,*)

      DO 35 r = 1,Rk(k)

         P(k,r) = 0.0

         DO 45 j = 1,Jj

 45         P(k,r) = P(k,r) + D(k,r,j)

         P(k,r) = P(k,r) / Jj

         WRITE(6,*)  'C' , k , r , ':'
         WRITE(6,*)
         WRITE(6,*)  'P(k,r) =' , P(k,r)
 35      WRITE(6,*)

C     *************** INPUT OF THE ALTERNATIVE WEIGHTS ****************

      WRITE(8,*)
      WRITE(8,*) ' *************** ALTERNATIVE WEIGHTS *************** '
      WRITE(8,*)

      READ(1,3)

      DO 65 k = 1,Ck

        READ(1,3)

        WRITE(8,*)
```

```
      DO 65 r = 1,Rk(k)

         READ(1,*)
         READ(1,*)

         WRITE(8,*)

      DO 65 j = 1,Jj

          READ(1,*)  ( A(i,k,r,j) , i = 1,Ai )

          WRITE(8,*)

      DO 65 i = 1,Ai

          WRITE(8,*)  'C' , k , r , '.' , 'J' , j , '.' , 'A' , i , ':'
          WRITE(8,*)
          WRITE(8,*)  'A(i,k,r,j) =' , A(i,k,r,j)
  65      WRITE(8,*)

C     ************** AVERAGING THE ALTERNATIVE WEIGHTS ****************

      WRITE(9,*)
      WRITE(9,*)  ' ********** AVERAGED ALTERNATIVE WEIGHTS ********** '
      WRITE(9,*)

      DO 70 i = 1,Ai

          WRITE(9,*)

      DO 70 k = 1,Ck

          WRITE(9,*)

      DO 70 r = 1,Rk(k)

          M(i,k,r) = 0.0

          DO 80 j = 1,Jj

  80            M(i,k,r) = M(i,k,r) + A(i,k,r,j)

          M(i,k,r) = M(i,k,r) / Jj

          WRITE(9,*)  'A' , i , '.' , 'C' , k , r , ':'
          WRITE(9,*)
          WRITE(9,*)  'M(i,k,r) =' , M(i,k,r)
  70      WRITE(9,*)

C     ***** CALCULATING THE CRITERION RANKING OF THE ALTERNATIVES ******

      WRITE(10,*)
      WRITE(10,*)  ' ***** CRITERION RANKING OF THE ALTERNATIVES ***** '
      WRITE(10,*)
```

```
      DO 85 i = 1,Ai

         WRITE(10,*)

      DO 85 k = 1,Ck

         Wik(i,k) = 0.0
         Pt(k) = 0.0

         DO 90 r = 1,Rk(k)

            Wik(i,k) = Wik(i,k) + P(k,r) * M(i,k,r)
 90         Pt(k) =  Pt(k) + P(k,r)

         Wik(i,k) = Wik(i,k) / Pt(k)

         WRITE(10,*)  'A' , i , ',' , 'C' , k , ':'
         WRITE(10,*)
         WRITE(10,*)  'W(i,k) =' , Wik(i,k)
 85      WRITE(10,*)

C     ****** CALCULATING THE OVERALL RANKING OF THE ALTERNATIVES *******

      WRITE(11,*)
      WRITE(11,*)  ' ****** OVERALL RANKING OF THE ALTERNATIVES ****** '
      WRITE(11,*)

      DO 100 i = 1,Ai

         Wi(i) = 0.0
         Nkt = 0.0

         DO 105 k = 1,Ck

            Wi(i) = Wi(i) + Nk(k) * Wik(i,k)
 105        Nkt = Nkt + Nk(k)

         Wi(i) = Wi(i) / Nkt

         WRITE(11,*)  'A' , i , ':'
         WRITE(11,*)
         WRITE(11,*)  'W(i) =' , Wi(i)
 100     WRITE(11,*)


      STOP
      END
```

AUTOBIOGRAPHY

I was born in Ankara on 02. Mayıs. 1967. I finished primary school at Konya in 1978. Then I entered Bursa Anadolu High School (including secondary school) and was graduated from there in 1985. The same year I entered Mechanical Engineering Department of the Boğaziçi University. On August, 1990 I received my BSc degree in mechanical engineering. Then I entered Institute for Nuclear Energy of the Istanbul Technical University. In 1991 I was appointed research assistant at the same institute.