**İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY**

# APPLICATION OF ELECTRIC CIRCUIT SYSTEMS TO A GEOTHERMAL SYSTEM

**Msc. Thesis By**

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# ELEKTRİK DEVRE SİSTEMLERİNİN BİR JEOTERMAL SİSTEME UYGULANMASI

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## NOMENCLATURE

| | |
|---|---|
| A | : Area |
| C | : Capacitor |
| c | : Compressibility |
| d | : Diameter |
| E | : Voltage Source |
| G | : Conductance |
| h | : Entalpy |
| i | : Circuit |
| k | : Permeability |
| L | : Inductor |
| l | : length |
| P | : Pressure |
| Q | : Heat |
| R | : Resistance |
| t | : Time |
| V | : Voltage |
| W | : Mass |

### Greek Symbols

| | |
|---|---|
| $\Phi$ | : Porosity |
| $\rho$ | : Density |
| $\alpha$ | : Recharge Constant |
| $\kappa$ | : Storage Capacity |
| $\Delta$ | : Difference |

### Subscripts

| | |
|---|---|
| a | : Aquifer |
| e | : Water Influx |
| k | : Cross Sectional |
| L | : Loss |
| p | : Production |
| r | : Reservoir |
| T | : Total |

**APPLICATION OF ELECTRIC CIRCUIT SYSTEMS TO A GEOTHERMAL SYSTEM**

## SUMMARY

Geothermal reservoirs like oil and/or gas reservoirs and aquifers are basically hydraulic systems. Modeling pressure and temperature change due to production in geothermal reservoir, material balance and/or energy balance approaches are used. However, there exist no analytical solutions because geothermal reservoir system properties changes with time. For this reason, some complex simulations are used to examine the behavior of a geothermal system. Softwares used to analyze a geothermal system are mainly based on the petroleum industry. But, these softwares are generally expensive and requires expertice.

Electrical circuit solutions, with the appropriate analogies, are used to solve and to examine many different engineering systems. In the literature; however, the electrical analogies are used to solve mostly for mechanical system related subjects. Other then one exception there is no example in the literature that a hydraulic system is solved using circuit system theory. The exception example, found in the literature, is simply a flow between two tanks that both of them are open to atmosphere, and fluid levels in one of the tanks are kept constant. The tanks are connected to each other with a thin pipe. Influx is allowed for one of the tank and discharge at constant rate is allowed for other tank.

Methods used to analyze electric circuits systems, can be applicable to analyze hydraulic systems by taking into account some principles. It is possible to make a liaison between the hydraulic and electrical systems when a hydraulic system is appropriately defined and transformed into a circuit system. To bring up this liaison, first, the equivalency of the elements in the both systems should be determined physically. Main interest elements in hydraulic system can be chosen as production rate and respective pressure changes that are measurable. Electric circuit system equivalencies of these hydraulic elements are given in the literature as current and voltage (both are measurable), respectively. The main objective of this study is to

physically describe and solve a geothermal system by using theorems of electric circuit systems.

Modeling a geothermal system with an equivalent electric circuit system makes it possible to simulate for different production rates-pressure variation scenarios not only theoretical but also experimental in laboratory.

**ELEKTRİK DEVRE SİSTEMLERİNİN**
**BİR JEOTERMAL SİSTEME UYGULANMASI**

## ÖZET

Jeotermal sistemlerin üretime bağlı basınç ve sıcaklık değişimlerini bulabilmek için materyal balans ve/veya enerji balans yaklaşımı kullanılmaktadır. Ancak, jeotermal rezervuar sistem özelliklerinin zamana bağlı olması nedeniyle, analitik çözümü yoktur. Bu nedenle jeotermal sistemlerin davranışlarının incelenmesinde karmaşık yazılımlar kullanılmaktadır. Bu konuda genellikle petrol endüstrisinin sağladığı ticari yazılımlar mevcuttur. Ancak, bu yazılımlar hem pahalı hem de kullanımı uzmanlık gerektirmektedir.

Elektrik devre çözümleri uygun elektriksel analojiler kullanılarak birçok mühendislik sistemin çözümünde ve incelenmesinde kullanılmaktadır. Literatürde elektriksel analoji kullanılarak mühendislik problemlerinin çözümünde daha çok mekanik sistemler göz önüne alınmış ve uygulamaları verilmiştir. Akışkan sistemi elektriksel analoji uygulamasında bir örnek dışında yapılmış çalışma yoktur ve bu çalışma da iki sabit akışkan seviyeli tank arasındaki akışı modellemek üzerinedir. Tanklar birbirine ince bir boru ile bağlanmıştır. Bir tanka beslenmeye izin verilirken diğer tankta sabit debide bir dışa akış vardır.

Elektrik devrelerinin analizi için kullanılan yöntemler, bazı değişkenler göz önünde tutularak hidrolik sistemlerin analizinde de kullanılabilir. Sistem uygun olarak kullanıldığında, bu sistemler ile elektrik devre sistemlerinin arasında bir paralellik kurmak söz konusudur. Bu paralelliği ortaya koymak üzere öncelikle fiziksel olarak, elektrik devrelerini oluşturan elemanlara karşılık hidrolik sistemlerde hangi elemanların bulunduğunu belirlemek gerekir. Bu elemanlar belirlendikten sonra, elektrik elemanları için kullanılan akım ve gerilim büyüklüklerine karşı düşecek iki hidrolik büyüklüğün tanımlanması ve bunların ölçülmesi söz konusu olacaktır. Bu çalışmanın temel amacı bir jeotermal sistemi elektrik devre sistemleriyle tanımlamak ve gene elektrik devre sistemleri kullanılarak çözmektir.

jeotermal rezervuar sistemini elektrik devre sistemleri ile modellemek farklı üretim debilerine karşılık oluşacak basınç düşüm senaryolarını hem teorik olarak hem de deneysel olarak simule edilmesini sağlayacaktır.

# 1. INTRODUCTION

The geothermal energy is simply the heat of the earth. It can be described as the under pressured accumulated liquids, like hot water, steam or gas, which are found at different parts of the underground earth layers or as the thermal energy that hot dry rocks contain[1,2].

This energy coming from subsurface layers is used to produce electricity, to heat the buildings and greenhouses, to heat at industrial processes, to maintain some chemicals and to serve for touristic purposes [1,2]. As it is mentioned that the geothermal energy is used in many different industries. An alteration of production in the geothermal system can affect these industries. There are many different methods to examine the results of alterations in a geothermal system. The material balance and/or energy balance methods are two frequently used approaches for a reservoir.

The material balance and the energy balance approaches are applied to reveal the pressure and temperature changes related to production from a geothermal system. However, there exist no analytical solutions since the properties of a geothermal system are time independent. Consequently, some complex simulations using numerical solutions methods are applied to observe the behavior of a geothermal reservoir system. Generally these simulations, which are based on petroleum industry, are very expensive and require an expert to be used.

The natural geothermal systems are defined as an engineering system so called as fluid or hydraulic systems. Electrical circuit systems are successfully used to simulate some engineering systems, mostly mechanical systems. Different analogies are created by defining bilateral equivalency of elements of systems that are listed in table 1.1.

**Table 1.1.** Elements Equivalency In Different Systems [3].

| System | Voltage Element | Circuit Element |
|---|---|---|
| **Mechanic** | Velocity Difference | Force |
| **Mechanic (Rotationary)** | Angular Velocity Difference | Moment |
| **Electric** | Voltage Difference | Current |
| **Fluid** | Pressure Difference | Volumetric Flow Rate |
| **Thermal** | Temperature Difference | Heat Flow Rate |

It is possible to make a liaison between the hydraulic and electrical systems when the system is appropriately designed. To bring up this liaison, first,

1. It is possible to describe any hydraulic system in terms of an electrical system by using basic elements of a circuit system.
2. Before describing such an equivalency, it is vital to clearly understand the physics of an examined system that is subject to be transformed into an electrical system. Once the transformation is accomplished, solution is sought by honoring the use of the electric circuit system theorems.
3. Element wise equivalency of the both system must be determined.
4. In general, production rate and pressure response are the elements that are chosen for a hydraulic system, and current and voltage are the elements that are chosen for an electrical system.
5. Once the transformation is accomplished, solution is sought by honoring the usage of electric circuit system theorems.

This study broadens our view to describe the behavior of a geothermal system while using electrical circuit solutions. The improved solutions provide to set up an equivalent circuit system for the examined geothermal system, consequently, electric circuits systems ensure to simulate the system for different conditions in a simple manner.

## 1.1. The Resource of The Geothermal Energy

The sub layers of the earth differentiate in physical and chemical properties from the surface to the centre. The upper part of the subsurface is called lithosphere containing the crust and the upper mantle. Beneath them is found the core. The heat is mainly coming from the temperature differences between these layers. A simple view of these layers can be seen in Figure 1.1. [4,5]

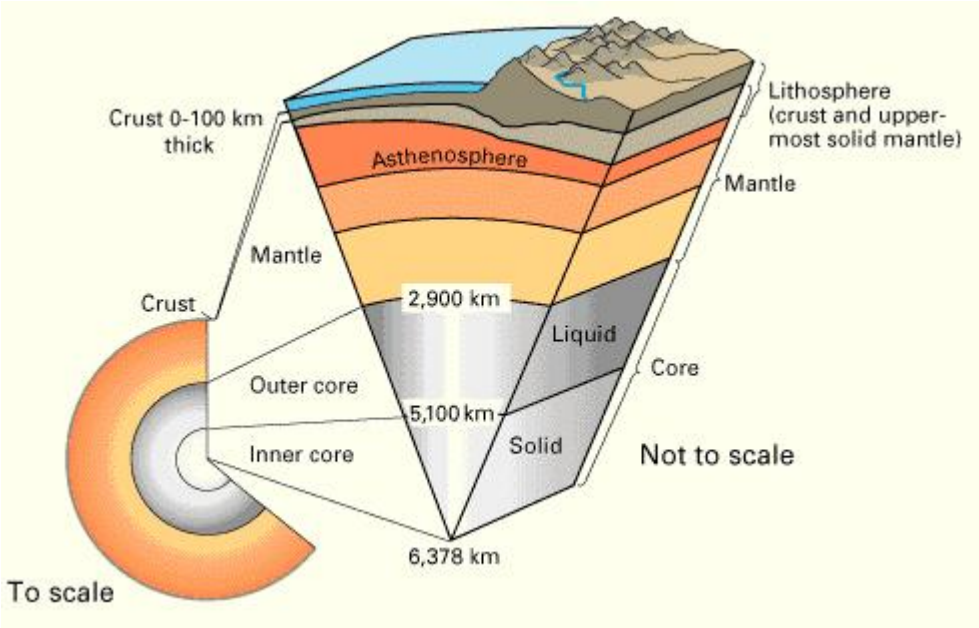


**Figure 1.1.** The Sub Layers Of The Earth [2]

It can be said that geothermal heat originates from Earth's fiery consolidation of dust and gas over 4 billion years ago. At earth's core - 4,000 miles (6400 kilometers) deep - temperatures may reach over 9,000 degrees $^{0}$F (5000 $^{0}$C). [4]

The heat from the earth's core continuously flows outward. It conducts to the surrounding layers of rock, the mantle. When temperatures and pressures become high enough, some mantle rock melts and forms magma. Then, because it is less dense than the surrounding rock layers , the magma rises, moving slowly up toward the earth's crust, while carrying the heat from the below layers as depicted in Figure 1.2. [2,4]

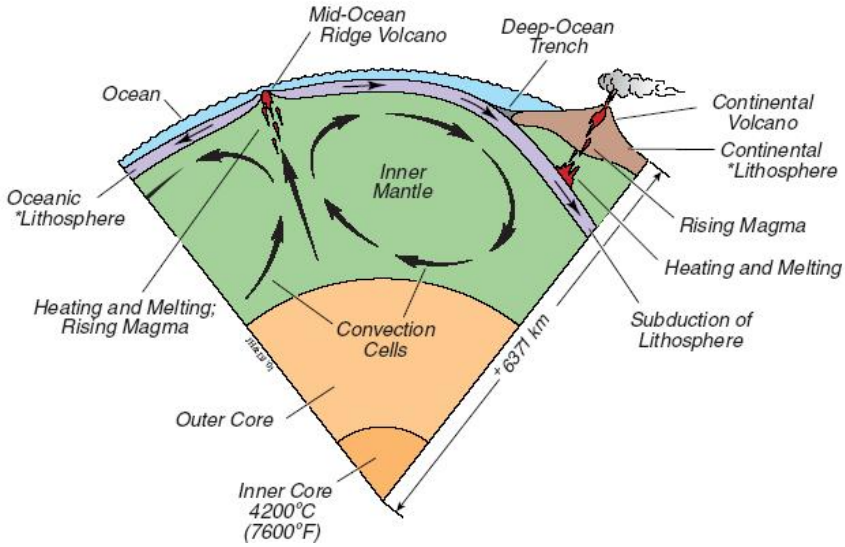


**Figure 1.2.** The mechanism Of The Earth Dynamics [2]

Sometimes hot magma reaches to the surface, and then it is called lava. But mostly, the magma remains below earth's crust, heating nearby rock and water, which comes by rainwater that has seeped deep into the earth, sometimes as hot as 700 $^{0}$F as shown in Figure 1.3. Some of this hot geothermal water travels back up through faults and cracks and reaches the earth's surface as hot springs or geysers. Although some of the geothermal water comes to surface, most of them remain at the deep underground, and it is trapped in cracks and porous rocks. This natural collection of hot water is called a geothermal reservoir. [2,4]

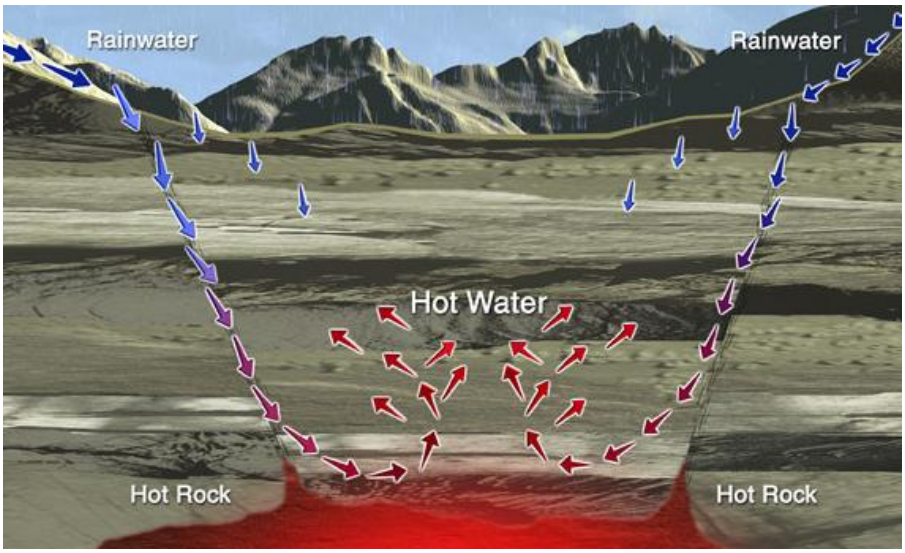


**Figure 1.3.** The mechanism Of The Geothermal Systems [2]

## 1.2. Geothermal System

A geothermal system consists of three main components: heat source, reservoir and the fluid carrying the heat. A heat source can be high temperature magma which reach to the surface or near surface or it can be –for low temperature system- the increased temperature with increased depth (This is called geothermal gradient.) as drawn in Figure 1.4. [6]. The first line indicates the boiling point of the pure water. The second line point out the temperature profile of the circulated water which is recharged at the point A and flow out from the point E. The line passing from the line F and G and going through the increasing depth imply the temperature profile of the formations on the geothermal field until the magma.

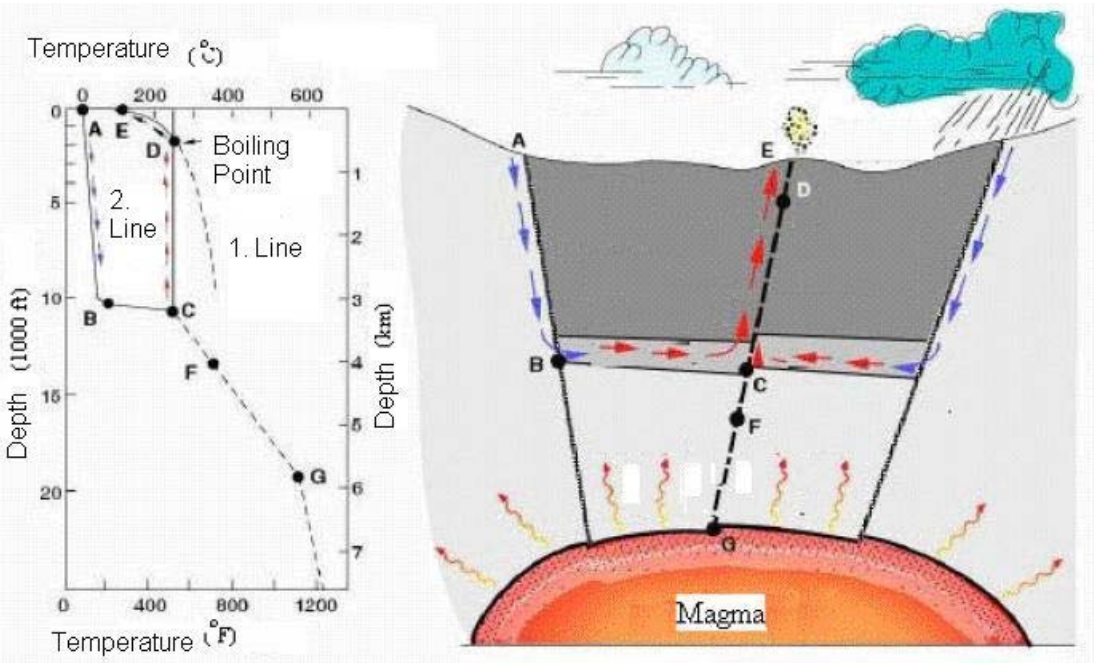


**Figure 1.4.** The Geothermal Gradient and A Model Of The Fluid Flow At The System [2]

High gradients (up to 11°F/100 ft, or 200°C/km) are observed along the oceanic spreading centers (for example, the Mid-Atlantic Rift) and along island arcs (for example, the Aleutian chain). The high rates are due to molten volcanic rock (magma) rising to the surface. Low gradients are observed in tectonic subduction zones because of thrusting of cold water-filled sediments beneath an existing crust. The tectonically stable shield areas and sedimentary basins have average gradients that typically vary from 0.82–1.65°F/100 ft (15–30°C/km). [6]

Measurements of thermal gradient data in Japan range widely and over short horizontal distances between to 0.6–4.4°F/100 ft (10–80°C/km). The Japanese Islands are a volcanic island arc that is bordered on the Pacific side by a trench and subduction complex. The distribution of geothermal gradients is consistent with the tectonic settings. In the northeastern part of Japan, the thermal gradient is low on the Pacific side of the arc and high on the back-arc side. The boundary between the outer low thermal gradient and the high thermal gradient regions roughly coincides with the boundary of the volcanic front [6].

The second component of the geothermal system is a reservoir. A reservoir is a permeable, cracked rock where the fluid is found and circulated. Generally, there exist some impermeable rocks layers are found at the upper part of the reservoir.

The geothermal fluid is meteoric water at most of the case. It can be at liquid or gas phase at the reservoir depending on the temperature and pressure. This fluid mostly contains some chemicals and gas like $CO_2$ and $H_2S$ [6].

The mechanism of the geothermal system is based on the convection of the heat by fluids. Because of the warming up is formed occurs the heat convection current. Consequently, this warming up causes the fluid to expand thermally. The heated low density fluid has a tendency to go up in the system and it change place with the cold high density fluid coming from the edge of the system. In natural heat convection current systems, the lower part has the tendency of rising the temperature and oppositely, the upper parts need to decrease the temperature as depicted in Figure 1.5 [6].

The model shown in Figure1.5 is quiet simple. In fact, it is very tough to create a model for a real geothermal reservoir system. The simulations of the systems need an enormous quantity of data depending on the high temperature, and are quiet hard works. In a geothermal system present many different geological layers which have many different physical and chemical properties.

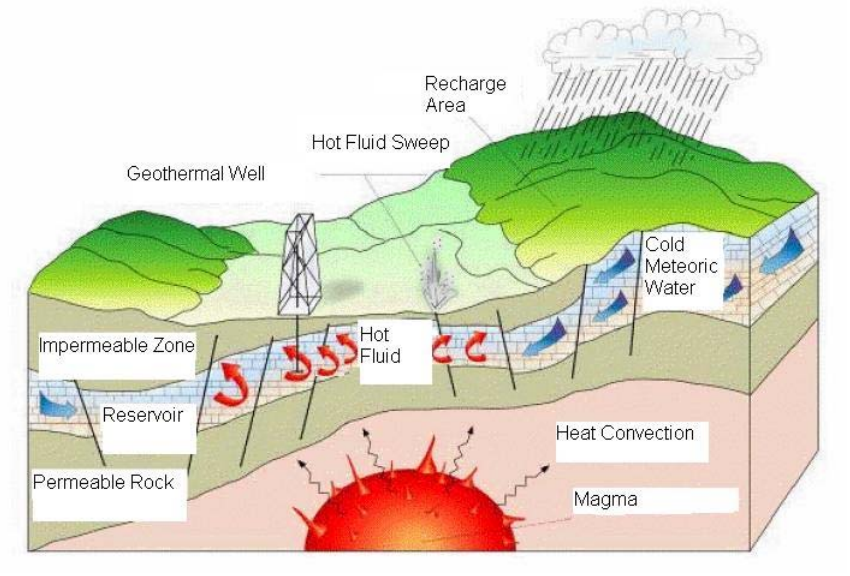


**Figure 1.5.** An Ideal Geothermal System [2]

### 1.3. Reservoir Modeling

The behaviors of the geothermal reservoirs can be designed in many different ways. In numerical models, the reservoirs are taken in 3 dimensions and every property of the fluids, of the rocks and of the well are taken into account. The straightness of these models is related not only to the uprightness of the input data, but also the majority of them [7,8].

At the early time of the production, volumetric modeling method, which needs geological, geophysical data and the properties of the rocks and fluids, can be applicable. By taking reservoir area, thickness, porosity and fluid properties, the volume of the reservoir and the amount of the fluid containing by the reservoir can be predicted with this method. Geological model is also needed and very important. Most of the time there is no accurate geological model [7,8].

Volumetric method considers the reservoir pore volume (PV) occupied by fluids at initial conditions and at later conditions after some fluid production and associated pressure reduction. The later conditions often are defined as the reservoir pressure at which production is no longer economical. Volumetric method is used early in the life of a reservoir before significant development and production. This method, however, can also be applied later in a reservoir's life and often is used to confirm estimates from material-balance calculations. The accuracy of volumetric estimates depends on the availability of sufficient data to characterize the reservoir's aerial

extent and variations in net thickness. Obviously, early in the productive life of a reservoir when few data are available to establish subsurface geologic control, volumetric estimate is least accurate. As more wells are drilled and more data become available, the accuracy of these estimates improves.

Another modeling procedure is decline curve analysis, where the data of the reservoir production and the variation of these data with respect to time are utilized. The future reservoir performance prediction is the purpose of this method. But, there have to be enough production data for this method to be applicable [7,8].

The basis of decline curve analysis is to match past production performance histories or trends with a model. Assuming that future production continues to follow the past trend, these models can be used to estimate original fluid in place and to predict ultimate reserves at some future reservoir abandonment pressure or economic production rate. Or, the remaining productive life of a well or the entire field can be determined. In addition, the estimation of the individual well flowing characteristics, such as formation permeability, can be done with decline-type-curve analysis.

Decline-curve methods, however, are applicable to individual wells or entire field. Decline-curve analysis techniques offer an alternative to volumetric methods and history matching with reservoir simulations for estimating original fluid in place and reserves. Application of decline-curve analysis techniques is most appropriate when more conventional volumetric methods are not accurate or when sufficient data are not available to justify complex reservoir simulation.

Unlike volumetric methods that can be used early in the productive life of a reservoir, decline curve can not be applied until some development has occurred and a production trend is established.

Again, it must be emphasized that the basis of decline-curve analysis for estimating fluid in place and reserves at some future abandonment condition is the assumption that future production performance can be modeled with past history. Any changes in field development strategies or production operation practices could change the future performance of a well and significantly affect reserve estimates from decline-curve techniques.

Lumped Parameter Modeling is rather frequently used model for the simulation of the geothermal reservoirs. This method is similar to the volumetric modeling method.

But, in this approach the reservoir is taken as dimensionless. The reservoir is entirely described and by taking the data as input and output from reservoir, the reservoir pressure, temperature and production data can be predicted with respect to time. It is frequently used as follows; a quite simple model by taking the reservoir volume, recharge, total heat, the production and the fluid loss as input data. In Figure 1.6, the parameters taking into account at dimensionless reservoir modeling is shown [7,8].

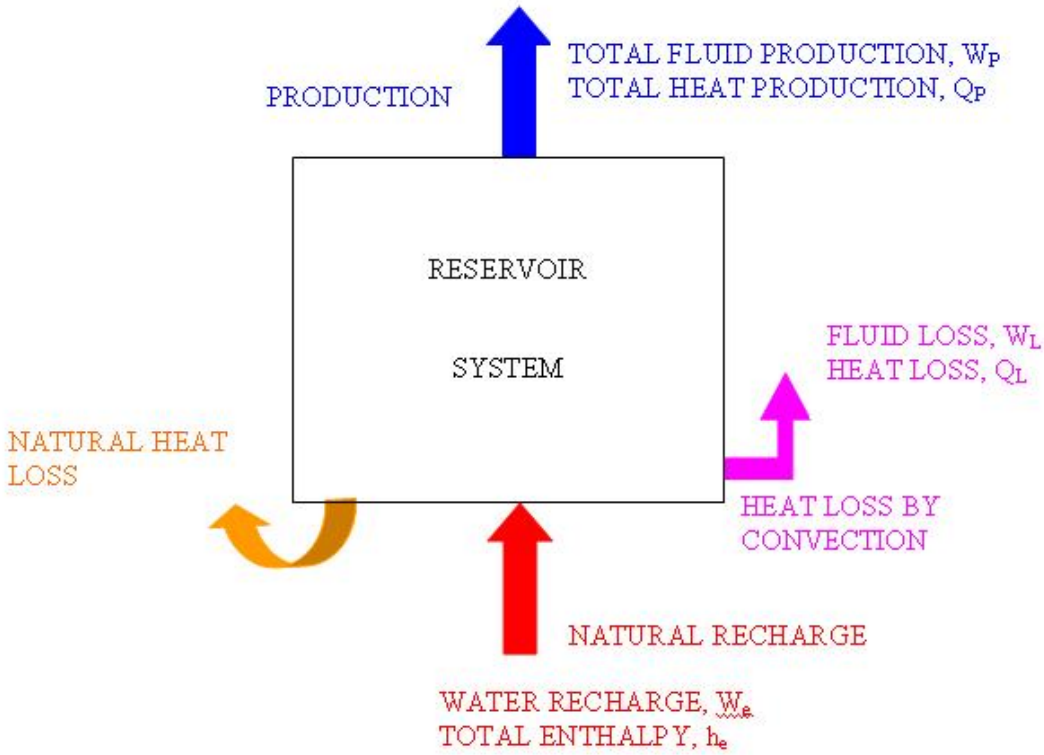


**Figure 1.6.** A Dimensionless Reservoir Model [8]

There exist several used lumped parameter models. The reason for this model to be used more frequently is that it needs little history of production and minimum information about the reservoir parameters [9].

### 1.3.1. Dimensionless Reservoir Modeling

In this modeling method, the geothermal system is presented by three components: the reservoir, the aquifer and the recharge source.

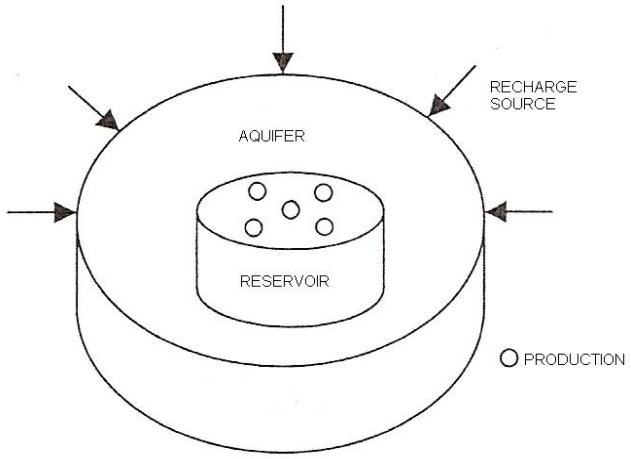


**Figure 1.7.** The Components of Geothermal System [8]

In Figure 1.7, the reservoir and the aquifer is considered as tanks, and at everypoint in the tanks the average properties are used. [10]

The dimensionless reservoir modeling to be used at some conditions:

- The production flow rate should be stable during the production operations.
- At warm episodes, the flow rate should be at minimum levels.
- At cold episodes, the flow rate should be at maximum levels.

The most frequently used tank models to designate the geothermal systems are:

a. 1 reservoir with recharge ( 1 Tank Model)

b. 1 reservoir – 1 aquifer with recharge ( 2 Tank Model )

c. 1 reservoir – 2 aquifer with recharge ( 3 tank Model )

d. 2 reservoir – 1 aquifer ( or no aquifer) with recharge

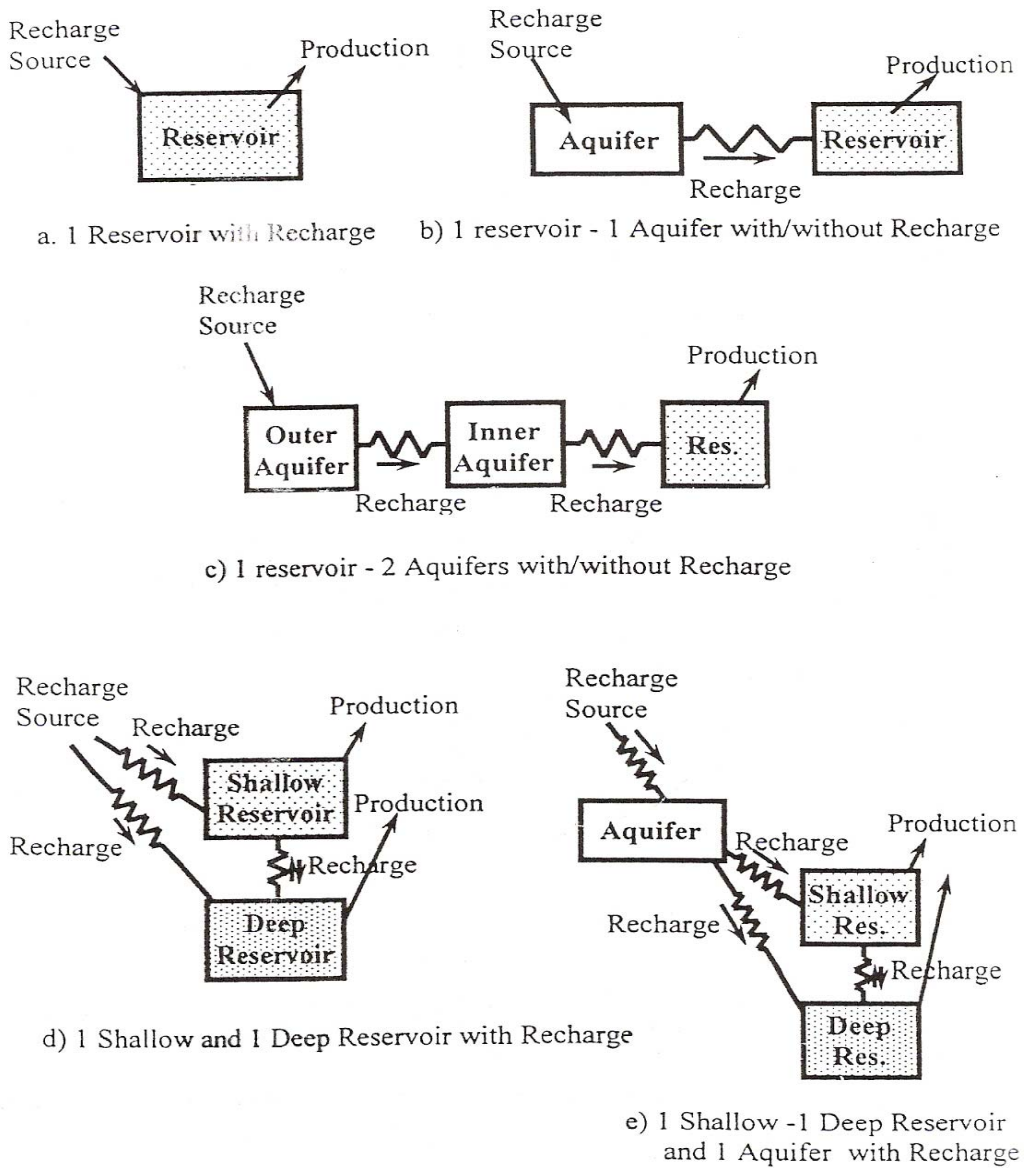


**Figure 1.8.** Tank Systems Used In Modeling [9]

In this study, one tank and two tank systems, as shown in Figure 1.8.a and 1.8.b are defined. As it can be seen from Figure 1.8.a, the system is mainly constituted by two main parts. A reservoir tank, where the production is allowed and a recharge source tank which is assumed to be constant pressure. The whole system is affected by the production from the reservoir tank. It can be seen from Figure 1.8.b, the two tank system is mainly constituted by three main parts. A reservoir tank, where the production and/or injection operation is running, an aquifer tank and a recharge source tank which is assumed to be constant pressure. Due to the net production from reservoir tank, water enters to the reservoir and decreases the pressure in the aquifer.

In this way, the whole system is affected by the production from the reservoir tank. The material balance studies are made on the effect of production on pressure drop.

Figure 1.9 illustrates the early- and late-time reservoir pressure drawdown for the open and closed two-tank lumped models for the case of constant production. As long as the production rate is constant, all models exhibit three distinct pressure decline regions. The first one, the early-time region, reflects the linear decline behavior of the reservoir tank alone ; a Cartesian plot of P versus t yields a straight line with slope of $w_{pnet}/\kappa_r$. This early-time linear decline behavior is shown by all models (one- and two-tank models, open and closed) [9,11].

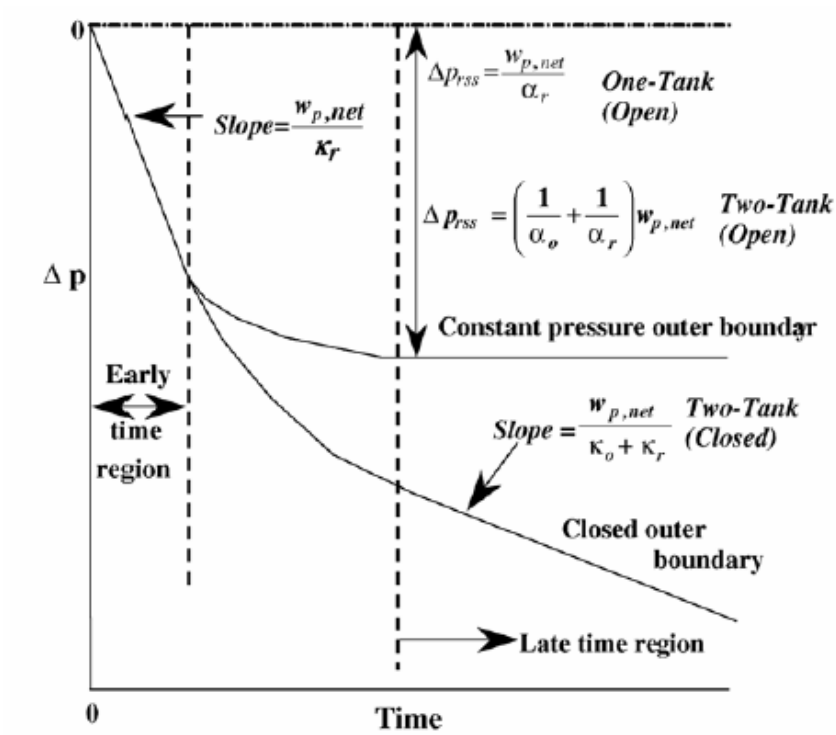


**Figure 1.9.** Pressure Drawdown in Open and Closed Two-tank Lumped Models for Constant Production Case. [11]

In this study, the two-tank system is analyzed as it is designed like an electrical circuit system. The main purpose for this study is;

- The pressure drop behavior in a geothermal system due to production by using the theorems of electric circuit systems.
- Determining reservoir parameters (constants).
- Predicting the future production performance of the reservoir with determined reservoir parameters.

# 2. BASIC ELECTRICAL CIRCUIT ANALYSIS

In the previous chapter, the geothermal energy and some modeling methods for its representation are described. Before using electrical analogy, some basic electrical properties, rules and elements will be described in the following parts of this section. The purpose of this study is to model one tank and two tank geothermal reservoir system using electric circuit analogy.

## 2.1. Electric Current and Voltage

The undefined quantities of circuit theory are mostly taken as electric current and voltage, in some specific applications charge and flux. As current and voltage are undefined quantities, only how they are determined by means of measurement is important. So first what is meant by current and voltage will be explained, and then how they are related to charge and flux will be given.

An electric circuit is essentially a pipeline that facilitates the transfer of charge from one point to another. The unit of electric charge is the coulomb. Ordinary matter is made up of atoms which have positively charged nuclei and negatively charged electrons surrounding them. The influence of charges is characterized in terms of the forces between them (Coulomb's law) and the electric field and voltage produced by them. The rate of flow of electric charge is called electric current and is measured in amperes. The time rate of change of charge constitutes an electric current. Mathematically, the relationship is expressed as;

$$i(t) = \frac{dq(t)}{dt} \qquad \text{or} \qquad q(t) = \int_{-\infty}^{t} i(x)dx \tag{2.1}$$

where i and q represent current and charge, respectively. The basic unit of current is the ampere (A) and 1 ampere is 1 coulomb per second.

Voltage is electric potential energy per unit charge, measured in joules per coulomb. It is often referred to as "electric potential", which then must be distinguished from

electric potential energy by noting that the "potential" is a "per-unit-charge" quantity. Like mechanical potential energy, the zero of potential can be chosen at any point, so the difference in voltage is the quantity which is physically meaningful. The difference in voltage measured when moving from point A to point B is equal to the work which would have to be done, per unit charge, against the electric field to move the charge from A to B. Mathematically, the relationship is expressed as;

$$v(t) = \frac{d\phi(t)}{dt} \qquad \text{or} \qquad \phi(t) = \int_{-\infty}^{t} v(x)dx \tag{2.2}$$

In order to show the relation between current and voltage we have to state how we measure these quantities, so we have to first define current reference in relation to voltage reference. These are shown in Figure 2.1. The direction of the current flow arrow is directed through the negative (–) to positive (+) sides of the voltage flow. This shows that the current and voltage is a well-adjusted pair [12,13].

**Figure 2.1.** The Reference Direction of Current and Voltage

## 2.2. Kirchhoff Current and Voltage Laws

Gustav Robert Kirchhoff, a German scientist, indicated some rules by taking into consideration the undefined quantities, current and voltage, given at the previous section. The first law is Kirchhoff's current law (KCL), which express that the algebraic sum of the currents entering any node is zero. The mathematical representation of the law is;

$$\sum_{j=1}^{N} i_j(t) = 0 \tag{2.3}$$

Where $i_J(t)$ is the jth current entering to the node through branches j and N is the number of branches connected to the node.

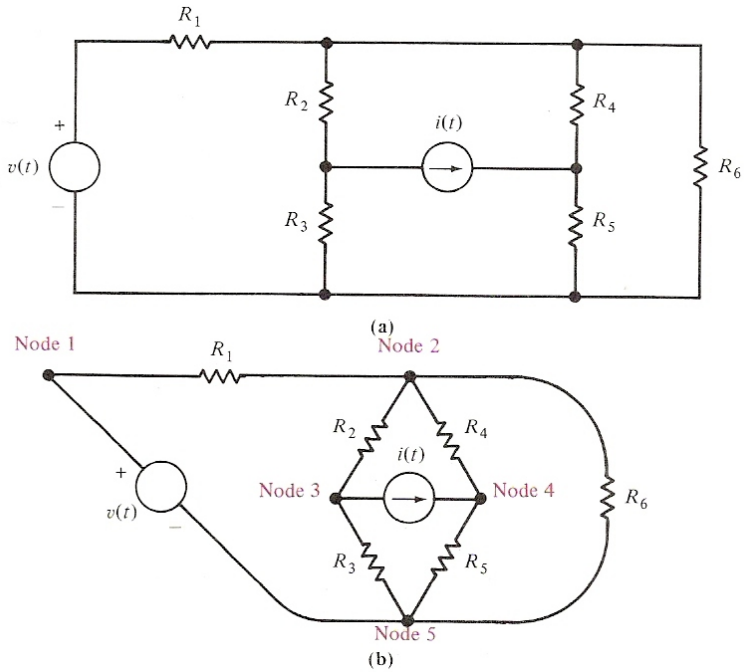


**Figure 2.2.** (a) Example circuit (b) circuit in (a) with nodes illustrated [13]

Applying Kirchhoff's current law to the node shown in the Figure 2.2;

$$i_1(t) + [-i_2(t)] + i_3(t) + i_4(t) + [-i_5(t)] = 0 \tag{2.4}$$

The currents entering to the node are taken positive and the currents leaving the node are negative. Alternatively, the equation can be written as;

$$i_1(t) + i_3(t) + i_4(t) = i_2(t) + i_5(t) \tag{2.5}$$

So Kirchhoff's first law indicates, the sum of the currents entering to a node is equal to the sum of the currents leaving the node.

Kirchhoff's second law, Kirchhoff's voltage law (KVL), indicates that the algebraic sum of the voltages around any loop is zero. In the application of KVL, a loop is generally referenced clockwise, and KVL is enforced in the direction of the current.

$$\sum V_{acrossbranches} = 0 \tag{2.6}$$

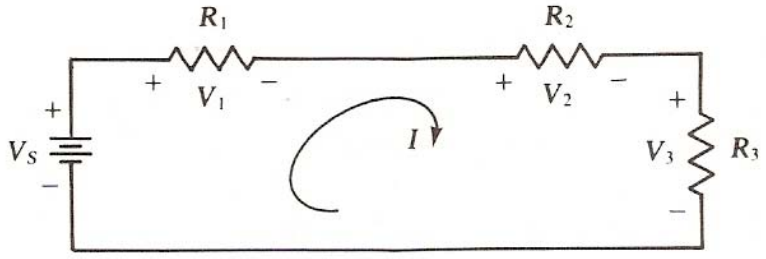


**Figure 2.3.** A Loop with Series of Circuits [12]

In Figure 2.3, the sum of the voltage drops through the resistors, $V_1 + V_2 + V_3$ is equal to the voltage rise $V_S$ across the voltage source.

$$V_S = V_1 + V_2 + V_3 \tag{2.7}$$

## 2.3. Certain Circuit Elements

The elements which will be defined are two-terminal devices that are completely characterized by the current and the voltage at the terminals. These elements are broadly classified as being either active or passive. The distinction between these two classifications depends essentially upon one thing – whether they supply or absorb energy. An active element is capable of generating energy and a passive element cannot generate energy [14].

### 2.3.1. Independent Sources

An independent voltage source is a two-terminal element that maintains a specified voltage between its terminals regardless of the current through it. Mathematical representation of independent voltage source is $v(t) = f(t)$ , $t \geq 0$ where *f(t)* is any function of time. The general symbol for an independent source, a circle, is shown in Figure 2.4.a. As the figure indicates, terminal A is v(t) volts positive with respect to terminal B. If the voltage is time invariant, the symbol shown in Figure 2.4.b is sometimes used. This symbol, which is used to represent a battery, illustrates that terminal A is V volts positive with respect to terminal B, where the long line on the top and the short line on the bottom indicate the positive and negative terminals, respectively, and thus the polarity of the element.

In contrast to the independent voltage source, the independent current source is a two-terminal element that maintains a specified current regardless of the voltage across its terminals. The general symbol for an independent current source is shown in Figure 2.4.c, where i(t) is the specified current and the arrow indicates the positive direction of current flow.

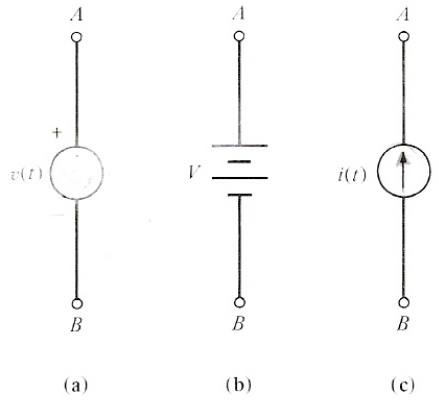


**Figure 2.4.** Symbols for Independent Voltage Source; Constant Voltage Source and Independent Current Source [15].

### 2.3.2. Resistance

In the following, the definition of ideal, linear, two terminal resistance will be given. Resistance is a property of materials that resists the movement of electrons. This action of resistance makes it essential to implement a voltage to cause current to flow. In the SI unit, the resistance (R) is described by ohm with symbol Ω, the Greek letter omega. The symbol for resistance is R.

The current (I) is proportional with the voltage (V) that is applied to the conductor. The relation between V and I is,

$$V(t) = Ri(t) \tag{2.8}$$

where current I is in amperes, voltage V is in volts and resistance R is in ohms. This relation is known as Ohm's Law. As it can be seen easily from the ohm's law, it is easy to conclude that the greater the resistance is, the less the current occurs for constant voltage. The inverse of resistance can be used sometimes and is called conductance (defined with an inverse omega). It is symbolized with G and the unit is siemens or mho. The above defined resistance is a passive circuit element, since it consumes the energy [14].

### 2.3.3. Capacitance and Inductance

#### 2.3.3.1. Capacitance

Capacitance is the measure of the ability of the component to store charge. A capacitor is a device specially designed to have a certain amount of capacitance. The concept of capacitance is simply indicated on the study of a specific kind of capacitor: the parallel-plate capacitor.

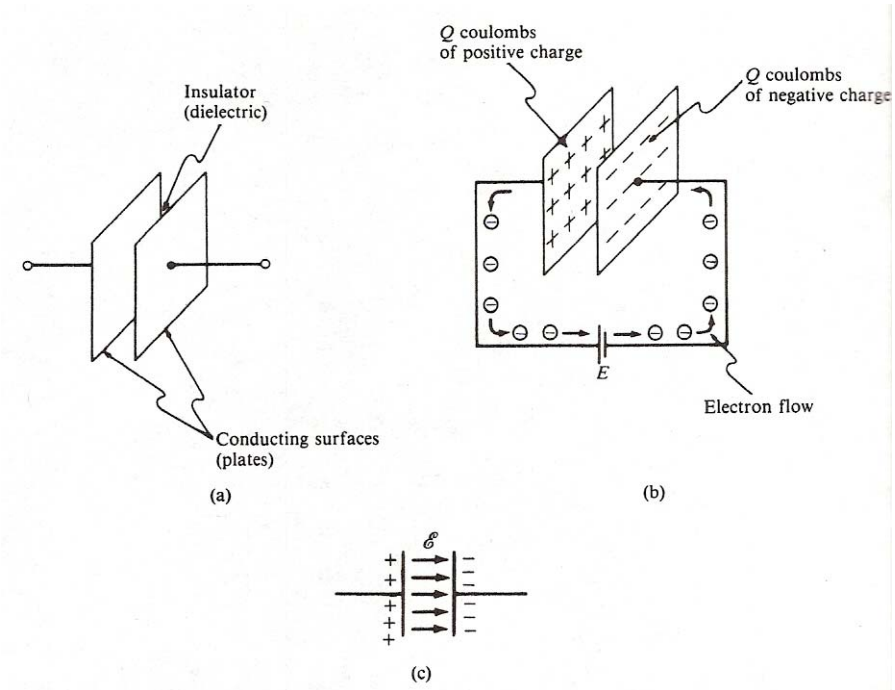


**Figure 2.5.** Parallel – Plate Capacitor [15].

In Figure 2.5 (a), two parallel plates are separated by an insulator. This is called the basic structure of a parallel-plate capacitor. The insulator found between two plates is called the dielectric.

In Figure 2.5 (b), a voltage source is connected across the two plates. Electrons will flow through the negative terminal and develop a negative charge on that plate. Electrons on that plat force back an equal number of electrons from the other plate, while leaving it with a positive charge. That means an electric field is set up in the dielectric between the plates, and the direction of the field is such that it drives electrons off the positively charged plate shown in Figure 2.5. (c).

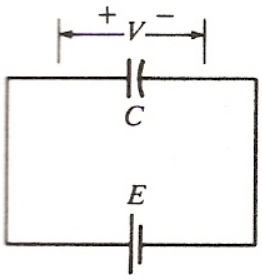


**Figure 2.6.** Schematic Diagram of The Charged Capacitor [14]

The curved line in the symbol for capacitor C usually represents the negative plate. The **Figure 2.6** shows that voltage V is developed through the capacitor as a result of the charge stored on its plates. The polarity of V sets against the voltage source E. When the charge becomes so large that V = E, there will be no electron movement on the battery. The capacitor is said to be fully charged at that time.

Capacitance is described as the ratio of the charge stored by a capacitor to the voltage V across it.

$$C = \frac{Q}{V} \tag{2.9}$$

The units of capacitance is farads (F), named after the English physicist Michael Faraday.

As a circuit element the definition of the capacitor is;

$$i = C\frac{dv}{dt} \tag{2.10}$$

where C is a constant called the capacitance.

If the q-v characteristic is time varying, the capacitance C becomes some prescribed function of time C(t).

$$q(t) = C(t)v(t) \tag{2.11}$$

It follows from the Equation 2.11 that,

$$i(t) = C(t)\frac{dv(t)}{d(t)} + \frac{dC(t)}{dt}v(t) \tag{2.12}$$

The current is a time-varying linear capacitor differs from equation 2.11 not only in the replacement of C by C(t), but also in the presence of an extra term.

Suppose a linear capacitor in **F**igure 2.6. is driven by a current source i(t). The corresponding voltage at any time t is obtained by integrating both sides of equation 2.11 from $\tau = -\infty$ to $\tau = t$. Assuming $v(-\infty)=0$, the equation 2.13 is obtained;

$$v(t) = \frac{1}{C}\int_{-\infty}^{\tau} i(\tau)d\tau \tag{2.13}$$

Note that unlike the resistor voltage which depends on the resistor current only at one instant of time t, the above capacitor voltage depends on the entire past history (i.e. $-\infty < \tau < t$ ) of $i(\tau)$. Hence, capacitor has memory.

Now suppose the voltage $v(t_0)$ at some time $t_0 < t$ is given, the equation 2.12 integrated from $t = -\infty$ to t becomes;

$$v(t) = v(t_o) + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \qquad t \geq t_o \tag{2.14}$$

In other words, instead of specifying the entire past history, only specifying v(t) at some conveniently chosen initial time $t_0$ is needed. In effect the initial condition $v(t_0)$ summarizes the effect of $i(\tau)$ from $\tau = -\infty$ to $\tau = t_0$ on the present value of v(t).

### 2.3.3.2. Inductance

In Figure 2.7 (a), it is shown a loop of wire, the lower portion of which is located in a magnetic field. As the loop moves upward, the lower portion moves vertically through the magnetic field. There exist no voltage source connected to the loop and a current flow around the loop. Therefore, a voltage must exist to flow the current. The motion of a conductor through a magnetic field creates a voltage across the ends of the conductor. In Figure 2.7 (b), a straight section of conductor moving through a magnetic field can be seen. There exist no external loop across which current can flow, however a voltage is nevertheless induced across the ends of the conductor. The moving conductor is simply an open-circuited voltage source [14].

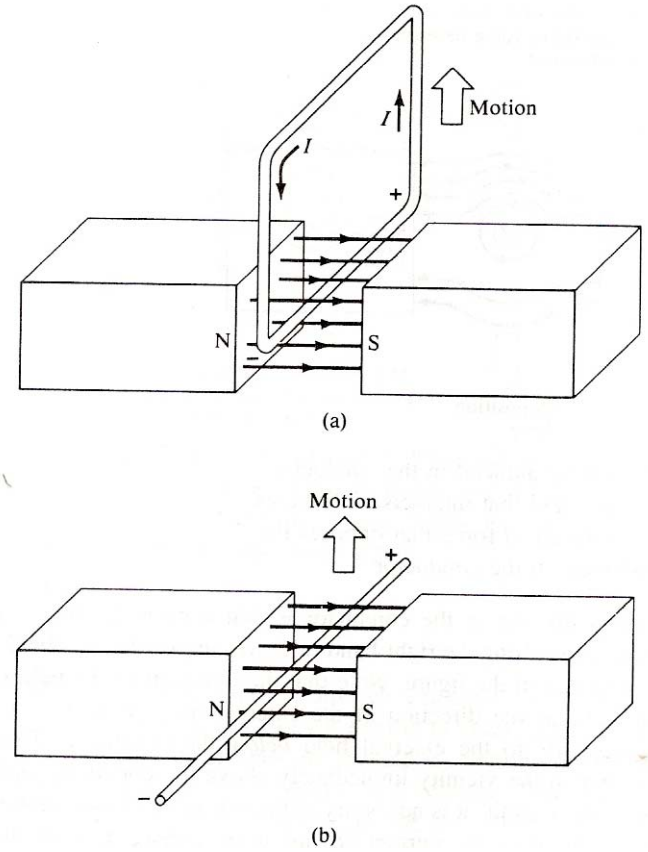


**Figure 2.7**. Electromagnetic Induction [14]

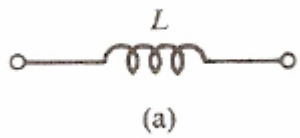


**Figure 2.8.** Schematic Symbol and Equivalent Circuit of an Inductor [15]

Figure 2.8 (a) shows an ideally pure inductor.

As a circuit element the definition of the inductor is;

$$v = L\frac{di}{dt} \tag{2.15}$$

where L is a constant called the inductance.

If the φ-i characteristic is time varying, the inductance L becomes some prescribed function of time L(t).

$$\phi(t) = L(t)i(t) \tag{2.16}$$

It follows from the equation 2.16 that,

$$v(t) = L(t)\frac{di(t)}{d(t)} + \frac{dL(t)}{dt}i(t) \tag{2.17}$$

The voltage is a time-varying linear inductor differs from equation 2.16 not only in the replacement of L by L(t), but also in the presence of an extra term.

By duality of conductor and inductor, inductor has also memory and that the inductor current is given by;

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau \qquad t \geq t_o \tag{2.18}$$

## 2.4. Graph Theory

### 2.4.1. Graph

In order to represent any circuit element without ambiguity, and in order to write linearly independent equations in a systematic way, graph theory from mathematics is used in circuit theory. A graph is defined by two sets, one set contains the nodes the other set contains the branches, so G={S(n), S(b)}[16].

### 2.4.2. Definitions on Graph Theory

#### 2.4.2.1. Digraph

Let a given graph G has a sub-graph $G_y$ whose characteristics are;

1. $G_y$ has n elements and n+1 nodes,
2. The elements in $G_y$ $e_1$,$e_2$, $e_3$,$e_4$ …and nodes $d_1$,$d_2$,$d_3$,$d_4$…can be ordered as nodes of the element $e_k$ are $d_k$ and $d_{k+1}$.
3. The grades of the nodes $d_1$ and $d_{n+1}$ are 1, and all the other nodes' grades are 2.

The graph which has these properties is said to be a digraph. In the Figure 2.9 , a digraph is depicted. In a digraph, nodes, which grades are 1, are called end nodes, and the others are called inner nodes.

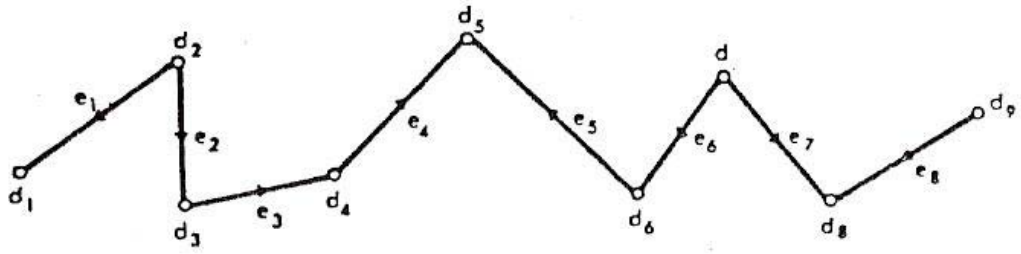


**Figure 2.9.** A Digraph $G_y$ [16].

### 2.4.2.2. Connected Graph

In a given graph G, if there exists at east one way between any two nodes, this graph is called connected graph. In the case of non-existence of this way between nodes, the graph is called separated graph.

In the Figure 2.10, the graph is separated graph. There exists no way between the nodes $d_1$ and $d_6$.

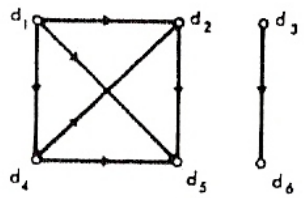


**Figure 2.10.** A separated Graph [16].

### 2.4.2.3. Loop

A sub-graph $G_c$ of a graph G, which has the properties described below, is called a loop.

1. The sub-graph $G_c$ is a connected graph.
2. The grades on the $G_c$ are 2.

Depends on the description above, the graph named loop are constituted of a digraph whose end nodes are connected. This explanation emphasizes that there are two different ways between any two nodes of a loop [16].

### 2.4.2.4. Tree

A sub-graph $G_T$ of a graph G, which has the properties described below, is called a tree.

1. The sub-graph $G_T$ is a connected graph.
2. $G_T$ has every nodes of the graph G.
3. $G_T$ has no loop inside.

The elements of the tree are called branches. Depending on the description above, there are two main characteristics on a tree. First, there is only and only one way between any two nodes of a tree. If there is no way between two nodes, the tree should be separated and if there are more than one way, there is a loop. These two conditions oppose with the properties 1 and 3 of a tree. So, the first characteristic is proven.

Secondly, number of branches on a tree is one less than number of nodes on a tree[17].

### 2.4.2.5. Cut-set

A sub-graph $G_k$ of a graph G, which has the properties described below, is called a cut-set.

1. If the elements related to $G_k$ are removed from the graph G, the left graph has two pieces.
2. None of the sub-graphs of $G_k$ has the property described at the first condition.

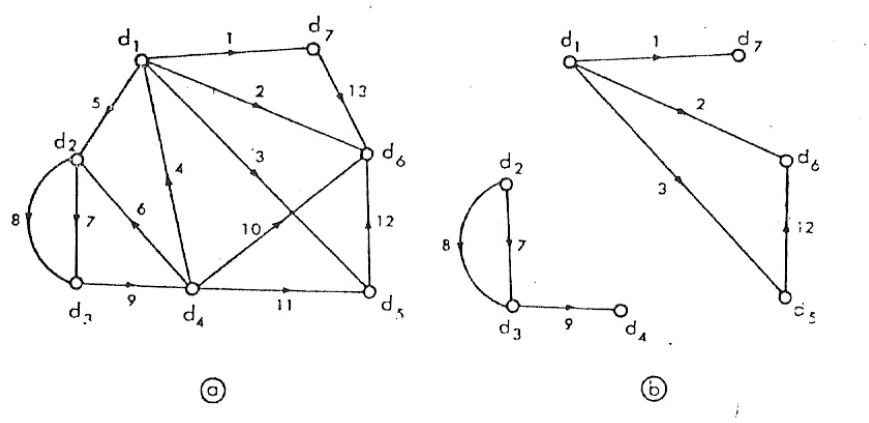


**Figure 2.11.** (a) A Connected Graph, (b) A sub-graph [17]

In Figure 2.11, when the elements 4,5,6,10,11 and 13 are removed, the sub-graph showed at b is maintained. $G_k$ = {4,5,6,10,11,13}is not a cut-set. $G_{k1}$ ={4,5,6,10,11} and $G_{k2}$ = {4,5,10,11,13} provide the first condition. On the other hand, nor $G_{k1}$ or $G_{k2}$ can not be a cut-set, because $G_{k3}$ = {4,5,10,11}, a common sub-graph of both, provide the first condition. $G_{k3}$ is a cut-set.

#### 2.4.2.6. Fundamental Cut-set

Every $n_d$-1 branches of a tree $G_T$, selected from a connected graph formed of $n_e$ elements and $n_d$ nodes, constitute a cut-set with the situation that other elements exist only in $G_T$. In this manner, defined $n_d$-1 cut-set is called fundamental cut-set. Depending on the description above, there are four main characteristics on a fundamental cut-set.

1. If there exists no loop on a sub-graph of G, this graph can be taken as a sub-graph of a selected tree $G_T$ in G.
2. If $G_c$ is a loop of connected graph G, a $G_T$ in G can be chosen that this $G_c$ is described as a fundamental loop.
3. If $G_k$ is a cut-set in G, a tree in G can be chosen that $G_k$ is a cut-set described by this tree.
4. $G_1$ and $G_2$ are two sub-graph of G and provide the conditions below;
    a. $G_1$ and $G_2$ has no common elements
    b. There exists no loop in $G_1$.
    c. $G_2$ has no cut-set of G.

In this manner, a tree GT in G can be chosen that G1 is left inside, and G2 is left outside this tree [17].

#### 2.4.2.7. Fundamental Loop

Every $n_e$ - $n_d$+1 cut-set of a tree $G_T$, selected from a connected graph formed of $n_e$ elements and $n_d$ nodes, constitute a tree with the situation that other elements exist only in $G_T$. This described loop is called fundamental loop.

In a connected graph, more than one different tree can be chosen generally. So, the number of fundamental loop can be that much [17].

## 2.5. Differential Equations for Circuits

The circuits containing capacitors, inductors and resistors give rise to dynamical systems. The methods for dealing with networks of capacitors and inductors are the same as for resistors, except that the concept of resistance has to be extended slightly to accommodate the fact that the two-port models for inductors and capacitors are differential equations and not algebraic equations. For linear systems, the differential equations are usually solved by transforming them (Laplace or Fourier), so the equations end up algebraic in the end [18].

## 2.6. Generation of State Equation

The standard form of the state equations is given in matrix form by,

$$\dot{x} = Ax + Bu \tag{2.19}$$

Where x is a column vector of state variables, and u is a column vector of input variables. Since the required solution variables may not be state variables an output equation is also required and is given in matrix form by,

$$y = Cx + Du \tag{2.20}$$

where y is a column vector of output variables.

Note that the matrices A, B, C and D are not the topological matrices. However, they are matrices which describe circuit components and their connections. The first requirement of state variable analysis is for an algorithm which determines the A, B, C and D matrices for any circuit. Secondly the state equations must be solved to yield steady-state frequency and transient response to any input.

The essential features of solution by combined cut-set and tie-set analysis which must be incorporated in the generalized procedure may be summarized as follows:

1. A tree must be drawn to include all capacitors, some resistors, and no inductors. Such a tree is called a proper tree.
2. Cut-set equations for the capacitive tree branches and tie-set equations for the inductive link branches from the basis of the set of state equations.
3. Tree resistor currents and link resistor voltages which appear as variables must be eliminated from the set of equations using further cut-set and tie-set equations.

4. Similar procedures are required to express to output variables in terms of state variables.

There will be one first order differential equation for each variable. The variables should be taken one by one. If the variable is a capacitor voltage, identify the basic cutset containing that capacitor. The differential equation is given by the KCL equation for that basic cutset. If the variable is an inductor current, identify the basic loop containing that inductor. The KVL equation for that basic loop will yield the desired differential equation. In both cases, the final stage involves some substitution and algebraic manipulation to express the right hand side in terms of the other variables and the sources [18].

## 3. ELECTRICAL ANALOGY

The methods which are used to analyze electric circuits systems can be applicable to analyze hydraulic systems by taking into account some variables. It is possible to make a similarity between the hydraulic and electrical systems when the system is physically defined. To bring up this liaison, first, it must be determined the equivalency of the electric systems' elements to those of hydraulic systems. Then, two elements from hydraulic systems, pressure and flow rate, have to be described to be equivalent to the voltage and the current of the electrical system. It can be possible to describe hydraulic systems by using electrical systems at the following part of this chapter. The similarity created between hydraulic and electrical systems is called electrical analogy.

Geothermal systems are natural hydraulic systems. To use this similarity in geothermal reservoir systems is a different approach. In the following subsections, examination and application of electric circuit systems on a geothermal reservoir system using material balance approach is going to be developed, and the physical behavior of the both systems is going to be compared.

### 3.1. Definition of One Tank System

System at which applied the electrical circuit elements is the one tank system described. The system mainly consists of a recharge source which feeds the reservoir. The system is schematically shown in Figure 3.1. [8,9]

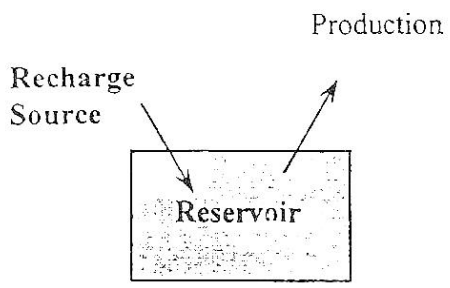


**Figure 3.1.** The One Tank Model [10]

The geothermal system shown in Figure 3.1 can be depicted as shown in Figure 3.2 in order to construct the electrical analogy easily.

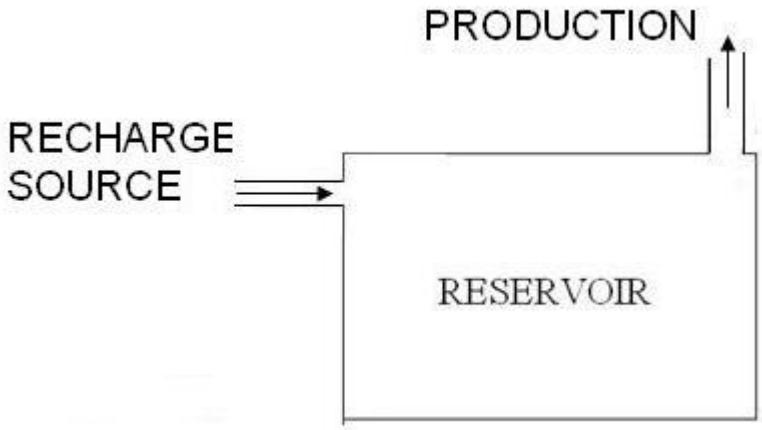


**Figure 3.2.** A Schematical View of the One Tank System

At this section, the hydraulic system is given in Figure 3.2 will be modeled by electric circuits.

To describe the system in that way, a voltage resource is put to the surface and is called $P_o$. The geothermal fluid is stored in the tank. It is appropriate to put a capacitor in electrical system instead this storage capacity. While the fluid flow goes on through well, it changes the media. While the fluid flows to well, it passes from the formation to the well. There is constriction of fluid flow area inside the well. This causes a resistance through flow so that a resistance is placed instead. Finally, an electric circuit system shown in Figure 3.3 is formed. [19,20]

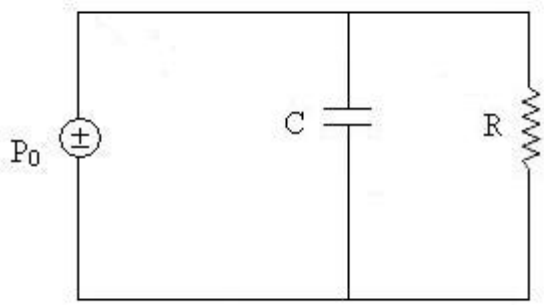


**Figure 3.3.** Electric Circuit System Formed From One Tank Model

### 3.2. Application of The Mass Balance Equilibrium To One Tank System

To describe this system, a current and a voltage value is needed. At the frontier part of this section, the equations to find the value of voltage $V_C$ will be derived. The value I, for the geothermal system flow rate (q), is taken constant in this study.

The graph of the electric circuit system is drawn in Figure 3.4.

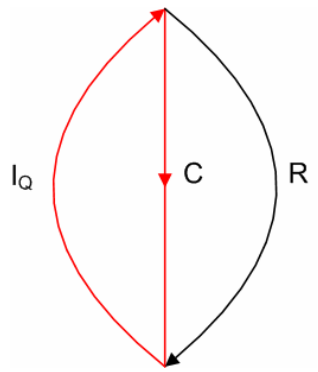


**Figure 3.4.** Electric Circuit System Graph

The loop is chosen depending on the properties which are described in previous section, and it is shown in red.
Using the definition for $i_C$ gives;

$$i_C = c\frac{dv_C}{dt} = -i_{R_C} + i_Q = -\frac{v_C}{R} + i_Q \tag{3.1}$$

$$\frac{dv_C}{dt} = -\frac{1}{RC}v_C + \frac{i_Q}{C} \tag{3.2}$$

Writing Equation 3.2 in s-domain gives,

$$sv_C(s) - v_C(0) = -\frac{1}{RC}v_c(s) + \frac{i_Q}{s} \tag{3.3}$$

$$\left(s + \frac{1}{RC}\right)v_C(s) = v_C(0) + \frac{i_Q}{s} \tag{3.4}$$

$$v_C(s) = \frac{v_C(0)}{(s + \frac{1}{RC})} + \frac{i_Q}{s^2 + \frac{s}{RC}} \tag{3.5}$$

Writing equation 3.5 in real time domain;

$$v_c(t) = e^{-\frac{1}{RC}t}(v_C(0) - \frac{i_Q}{RC}) + \frac{i_Q}{RC} \tag{3.6}$$

### 3.3. Definition of the Two Tank System

The system at which applied the electrical circuit elements is the two tank system described in Chapter 1. The system mainly consists of a recharge source which feeds the aquifer, a reservoir which is fed by the aquifer. The system is schematically shown in Figure 3.5. [8,9]

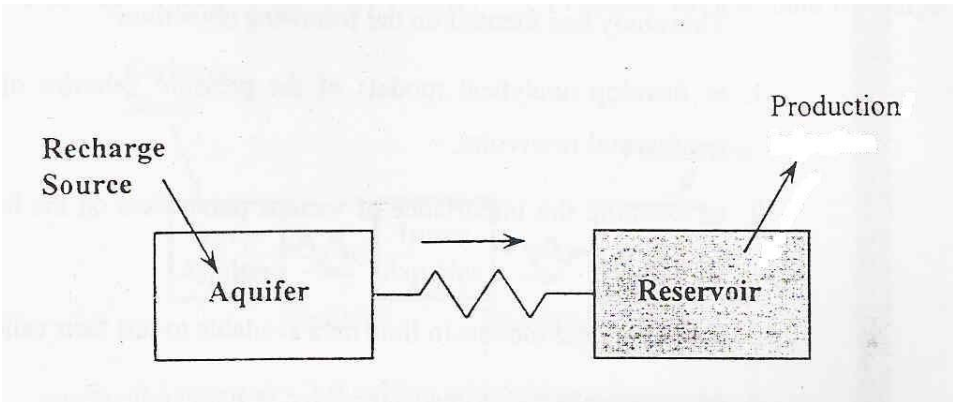


**Figure 3.5.** The 2 Tank Model of the System [10]

This two tank model is reformed as a two closed tanks which is connected with a tube. The geothermal system shown in Figure 3.5 can be depicted as shown in Figure 3.6 in order to construct the electrical analogy easily.

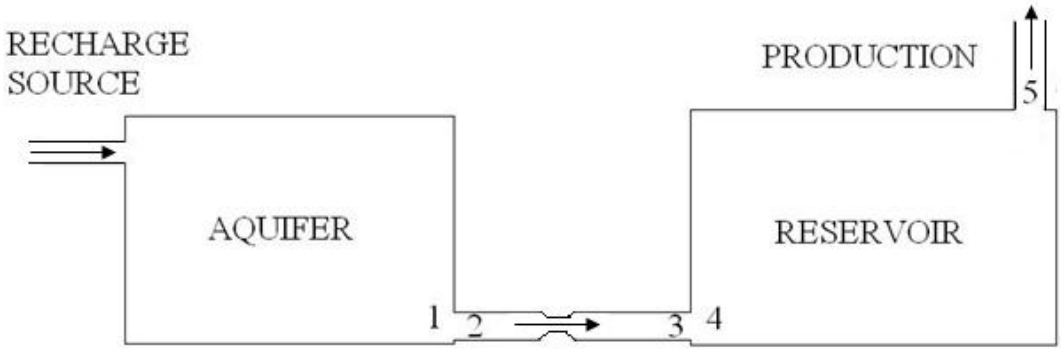


**Figure 3.6.** A Schematical View of the Two Connected and Closed Tank System

### 3.4. Application of The Mass Balance Equilibrium To Two Tank System

At this section, the hydraulic system is given in **Figure 3.6** will be modeled by electric circuits. To use electrical analogy, the equivalency of the system elements shown in **Table 3.1** is used.

**Table 3.1**. Electric Equivalency of Hydraulic System

| **Hydraulic Elements** | | **Electric Elements** | |
|---|---|---|---|
| **Symbol** | **Quantity** | **Symbol** | **Quantity** |
| $q$ | Flow Rate | $i$ | Current |
| $P$ | Pressure | $v$ | Voltage |
| $V$ | Tank Volume | $C$ | Capacitance |
| $R$ | Resistance to Flow | $R$ | Resistance |

To describe the system in that way, a voltage resource is put to the surface and is called $P_o$. In the first tank, there is storage of the geothermal fluid. It is appropriate to put a capacitor in electrical system instead this storage capacity. While the flow goes on from Point 1 to Point 2, there occurs a resistance due to the restriction. The raison for this resistance is that the media of the aquifer and the media between aquifer and reservoir have different properties. This difference can create a resistance through fluid flow. So between the Point 1 and 2, a resistance is placed. From the Point 2 to 3, there exists an electromotive force due to the change of the surroundings (from the aquifer to the reservoir). It is appropriate to put an inductance between the Point 2 and 3.But; also there must be a resistance parallel to inductance. Since the geothermal system permeability can simply be assumed extremely high due to

fractured system, this resistance can be taken as zero, i.e., pure inductance. Again, while the flow goes on from the Point 3 to the Point 4, there occurs a resistance due to the constriction. The raison for this resistance to occur is the same with the resistance between the points 1 and 2. At the second tank area, it can be said that there is storage of the fluid at the reservoir so that a capacitance can be placed between the Point 4 and 5 . While the fluid flow goes on through 5, it changes the media. While the fluid flows to Point 5, it passes from the formation to the well. There is constriction of fluid flow area inside the well. This can occur a resistance through flow so that a resistance is placed instead. Finally, an electric circuit system shown in Figure 3.7 is formed. [19,20]

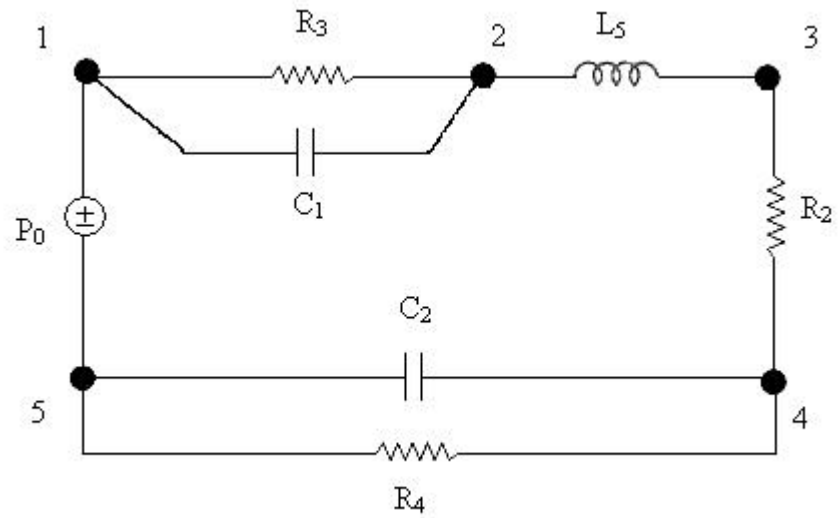


**Figure 3.7.** Electric Circuit System Formed From the Two Tank Model

To describe this system, a current and a voltage value is needed. At the frontier part of this section, the equations to find the value of current as $i_{L5}$ and the value of the voltage as $V_{C1}$ and $V_{C2}$ will be derived. The derivation of the value $i_L$ will be derived but in fact the value I, for the geothermal system flow rate (q), is taking constant for this study. And Also $V_{C1}$ is assumed to be constant for this sudy.

The solution for $V_{C1}$ are given below and detailed formulations are described in Appendix-A.

The graph of the electric circuit system is drawn at the Figure 3.8.

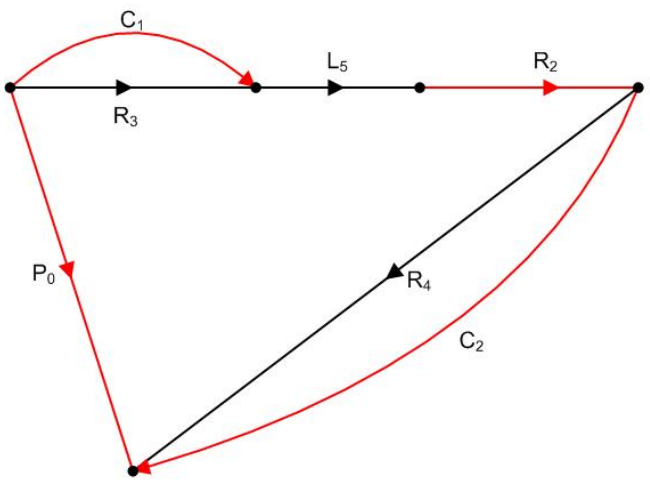


**Figure 3.8.** Electric Circuit System Graph

The loop is chosen depending on the properties which are described in previous section, and it is shown in red.

Using the definition for $i_{C1}$ gives;

$$i_{C_1} = c_1 \frac{dv_{C_1}}{dt} = i_{L_5} - i_{R_3} = i_{L_5} - \frac{v_{C_1}}{R_3} \tag{3.7}$$

For $i_{C2}$;

$$i_{C_2} = c_2 \frac{dv_{C_2}}{dt} = i_{L_5} - i_{R_4} = i_{L_5} - \frac{v_{C_2}}{R_4} \tag{3.8}$$

Using the definition of inductance for $V_{L5}$ gives,

$$v_{L_5} = L_5 \frac{di_{L_5}}{dt} = -v_{c_1} + Po - v_{c_2} - v_{R_2} \tag{3.9}$$

where $V_{R2}=R_2 i_{L5}$.

Writing equation in the matrix form of equation 3.9,

$$\frac{d}{dt}\begin{bmatrix} v_{C_1} \\ v_{C_2} \\ i_{L_5} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_3 c_1} & 0 & \frac{1}{c_1} \\ 0 & -\frac{1}{R_4 c_5} & \frac{1}{c_2} \\ -\frac{1}{L_5} & -\frac{1}{L_5} & \frac{R_2}{L_5} \end{bmatrix} \begin{bmatrix} v_{C_1} \\ v_{C_2} \\ i_{L_5} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_5} \end{bmatrix} P_0 \tag{3.10}$$

Solution of the Equation 3.10 is detailed in the Appendix-A and gives in Laplace space;

$$v_{C_2}(s) = \frac{as^2 + bs + c}{ks^3 + ms^2 + ns + p} \tag{3.11}$$

### 3.4.1. Interpretation of Equation 3.11

To explain the equation 3.11, the situation of the roots of the denominator should be clarify to observe the behavior of the equation. There are three probabilities for the solution of the equation 3.11 depending on the roots:

1. s can be in the form of

   $$s = \gamma + iw \tag{3.12}$$

   In this case, in the general solution, the behavior of the equation should be as;

   $$F(t) = e^{\gamma t} \qquad \gamma > 0 \tag{3.13}$$

   This means while t goes to infinity, the solution goes also to infinity. This shows that the system is unstable, which is not possible for our system.

2. $\gamma = 0$ and jw is not equal to zero.

   That means s= jw. For the possible condition, jw can be double root or not.

   If jw is not double root, the solution will bring with the cosine and sinus terms like;

   $$F(t) = k_1 \cos wt + k_2 \sin wt \tag{3.14}$$

   The explanation for this equation is that while t goes to infinity, the equation is restricted and will make oscillations between limited values due to cosine and sinus terms.

   In the case of jw having double root, the solution will be in the form of,

   $$F(t) = k_3\, t \cos t + k_4\, t \sin t \tag{3.15}$$

While t goes to infinity, the equation goes to infinity due to the t terms beside the cosine and sinus terms. This is not possible for our system because it has an unstable state [18].

3. $\gamma < 0$

   For this case, while t goes to infinity, the solution of the equation converges to zero with an exponential drop. In the situation of having a double root, there will be terms with $t^2e^{\gamma t}$ and $t\ e^{\gamma t}$. Even in this case, the equation will converge to zero with an exponential drop because the term $e^{\gamma t}$ is the dominant parameter in the equation.

### 3.4.2. Evaluation of The Constants in Equation 3.11

In this section, the constants in the general solution of the equation 3.11 will be determined. The equation of $V_{C2}$ is representing the change of pressure and/or the pressure drop related to time in the geothermal system. As it can be seen from the general solution, equation 3.11 contains some constants. These constants are lettered as a,b,c,k,m,n and p. Actually, these constants includes some parameters from the electric circuit elements which are resistance, inductance and capacitance. This section describe the constants of the equation can be appropriately similar to that of the properties of the geothermal system.

#### 3.4.2.1. Evaluation of Constant "a"

$$a = V_{C_1}(0)$$

- $V_{C1}(0)$ : The pressure in aquifer at the initial condition.

#### 3.4.2.2. Evaluation of Constant "b"

$$b = \frac{i_{L_5}(0)}{c_1} - \frac{R_2 v_{C_1}(0)}{L_5 R_4 C_2}$$

• $i_{L5}$ (0) is clearly the flow rate through the reservoir from the aquifer. It can also be said that $I_L$ (0) is the recharge flow rate.

• $C_1$: This term as meaning contains capacity. It can be referred as the capacity of the aquifer. In other words, it can simply be the volume of the aquifer. In the field application, its unit is $km^3$.

• $R_2$ is a property that includes the physical properties of the aquifer. These properties create a resistance through the flow. These properties are the porosity of the aquifer media ( $\Phi_a$), the permeability at the aquifer ( $k_a$ ) and the compressibility of the aquifer formation ( $C_{ta}$ ).

• $L_5$: It is not quiet simple to determine the equivalent of the inductance at the geothermal system. First of all the definition of the inductance should be taking into account, which is [3];

$$v = L\frac{\partial i}{\partial t} \tag{3.16}$$

To describe inductance in a hydraulic system, the voltage term v turns to pressure P and the current term i turn to flow rate Q. So the Equation is transformed to;

$$P = n\frac{\partial Q}{\partial t} \tag{3.17}$$

The value of n in the equation 3.17 can give us what inductance means in the geothermal system. The deeper we go to the philosophy of the equation, the more clear definition we get.

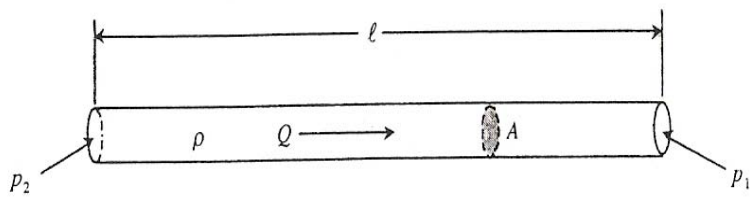


**Figure 3.9.** Flow in A Pipe [3]

In Figure 3.9, a simple view is shown. Depending on the figure, the equation which describes the flow through the pipe is;

$$P_2 - P_1 = P_{21} = \frac{\rho l}{A}\frac{\partial Q}{\partial t} \tag{3.18}$$

The term ρl/A is determining the term of inductance in our system but with some differences. The raison for this difference is that the equation is generated for a flow in a pipe but actually we are in a porous, permeable media. The term of the density is equal to 1 because it is water (assumption). So that the Equation 3.19 is;

$$P_{21} = \frac{l}{AS_P}\frac{\partial Q}{\partial t} \tag{3.19}$$

The new term "$S_P$" determines the effect of pore size on the area, which is related to permeability or it can be said that the term $S_P$ is a function of the permeability k.

- $R_4$ is also a resistance property that occurs while the fluid enters to the well. It can be said that it is a resistance because of the constriction of the area inside the well. In Figure 3.10, it is clearly described.

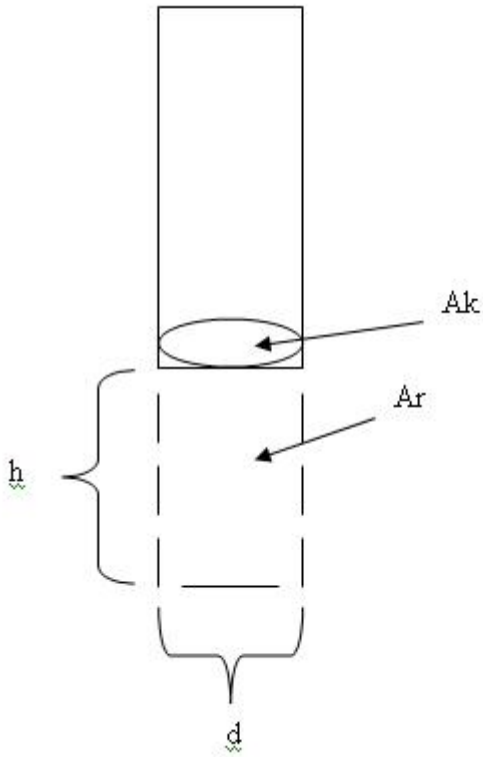


**Figure 3.10.** The Vertical Cross Section of the Well

$$A_k = \frac{\pi d^2}{4} \tag{3.20}$$

$$A_r = \pi d h \tag{3.21}$$

$$\frac{A_r}{A_k} = \frac{\pi d h}{\pi d^2} = \frac{4h}{d} \tag{3.22}$$

- $C_2$: This term as meaning contains capacity. It can be referred as the capacity of the reservoir. In other words, it can simply be the volume of the reservoir. In the field application, its unit is $km^3$.

### 3.4.2.3. Evaluation of Constant "c"

$$c = \frac{v_{C_1}(0)}{L_5 C_2} - \frac{R_2 v_{C_1}(0)}{R_4 c_2 L_5} - \frac{v_{C_2}(0)}{c_1 L_5} + \frac{i_{L_5}(0)}{R_4 c_2 c_1}$$

- $V_{C2}(0)$: This term is the bottom hole pressure at the time t = 0.
- $I_{L5}$ (0) is clearly the flow rate through the reservoir from the aquifer. It can also be said that $I_L$ (0) is the recharge flow rate.

### 3.4.2.4. Evaluation of Constants "k,m,n and p"

$$k = \frac{1}{L_5}$$

$$m = \frac{1}{R_4 c_2 L_5} + \frac{1}{R_3 c_1 L_5} - \frac{R_2}{(L_5)^2}$$

$$n = \frac{1}{R_4 c_2 L_5} - \frac{R_2}{R_4 c_2 (L_5)^2} - \frac{R_2}{R_3 c_1 (L_5)^2} + \frac{1}{L_5 c_1} + \frac{1}{c_2 L_5}$$

$$p = \frac{1}{R_4 c_2 c_1 L_5} + \frac{1}{R_3 c_1 c_2 L_5} - \frac{R_2}{(L_5)^2 R_4 c_2}$$

The parameters inside the constants k,m,n and p are defined in the previous sections.

## 4. MODEL VERIFICATION

In Figure 1.9; there exist three distinct pressure decline regions when the fluid in the reservoir is produced. The first one so-called early-time region reflects the linear decline behavior of the reservoir tank alone. Cartesian plot of pressure (P) versus time (t) relationship yields a straight line behavior. The second one, also named transient region, the line exponentially drops until the third region where the pseudo steady state is observed. In the third region, a straight line behavior so-called as pseudo steady state or late time is observed. As long as the production rate is constant, the early time pressure response of a closed system is same as of an open system. The pressure drop changes linearly with the production both in early time and in late time regions, but with different slopes. Pressure drop rate in late time is very small with time.

Laugernes Field, found in Iceland, is chosen to test the model and compare the results with the observed real field data as shown in figure 4.1. It is clear to understand from figure 4.1 that production rate is not constant and varies seasonally that is the general characteristic for a geothermal reservoir. During the winter times production rate is increased due to increasing demand for heating, and production rate is decreased during the summer period. In addition, the production rates changes when only the winter or summer periods are examined alone. However, it is also clear that the production rates are fairly constant for the first 9 pick data points in the winter times where the maximum pressure drop is recorded.

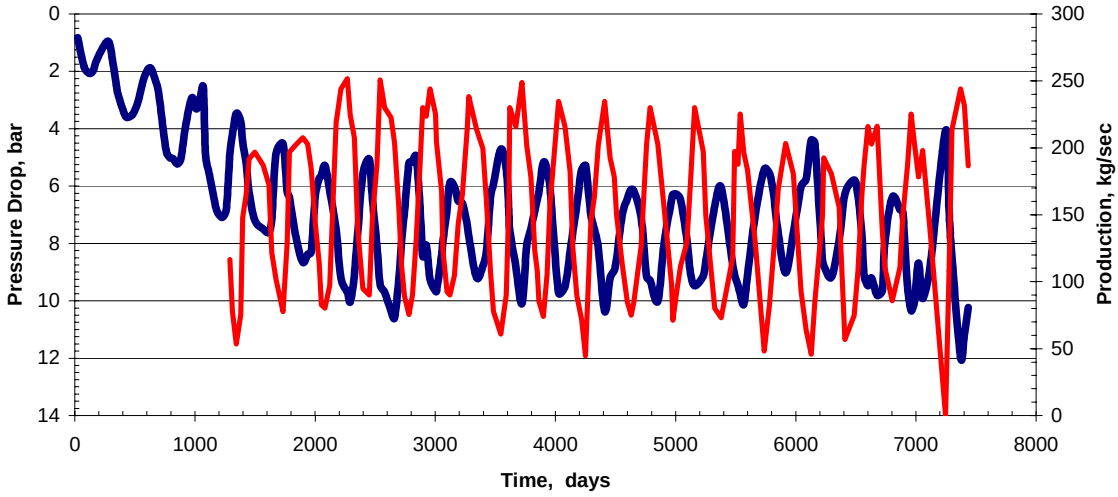


**Figure 4.1.** Observed Pressure, Production and Time Relation for Laugernes Field [10].

Depending on the real field data, the constants of Equation 3.6 are generated by using least square curve fitting method. Before curve fitting, pressure values at the maximum production rates periods were selected, and representing pressure versus time behavior is regenerated, as shown in figure 4.2.

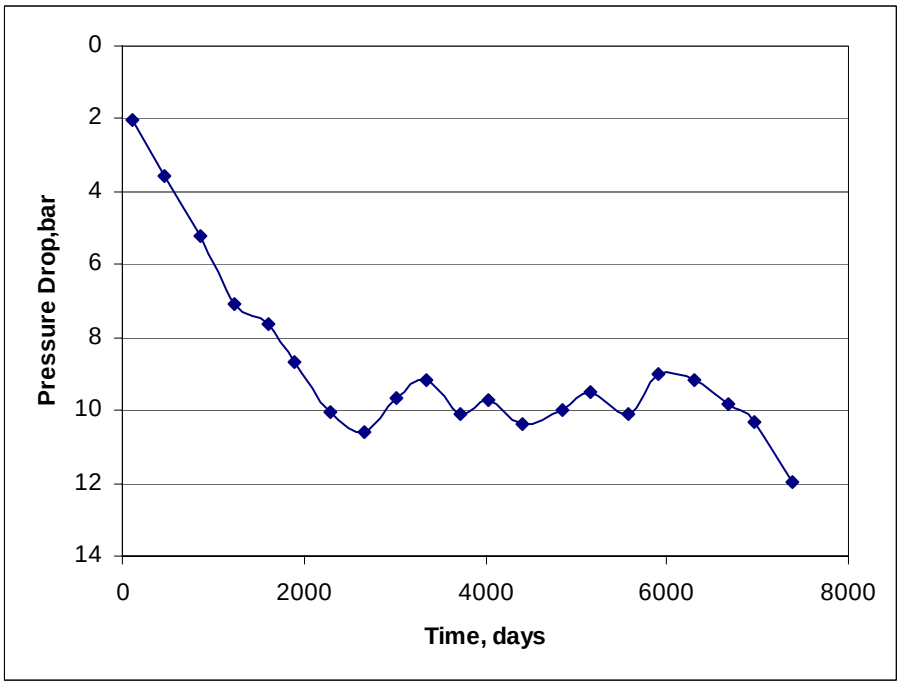


**Figure 4.2.** The Pressure Drop and Time Relation Data of The Laugernes Field.

Then curve fitting is applied on three different selected data sets. The first curve fitting is done using the first 6 data points representing 1616 days of production , the

second curve fitting is done using the first 9 data points representing 2661 days of production, and the third curve fitting is done using the all data points representing 7373 days of production.

Curve fitting using the first 6 data points is explained in details as follows:

Taking the natural logarithm of both sides of the equation 3.6 gives;

$$\ln(V) = \ln\left[v_c(0) - \frac{i_Q}{RC}\right]e^{-\frac{t}{RC}} \tag{4.1}$$

where $V = v - \frac{i_Q}{RC}$.

Setting as $\ln(V) = V$, $\ln\left[v_c(0) - \frac{i_Q}{RC}\right] = K$ and $-\frac{1}{RC} = n$, the equation becomes;

$$V = K + nt \tag{4.2}$$

The 6 first data points used are given at table 4.1.

**Table 4.1.** Six First Bottom Pick Data Points of The Laugernes Field.

| t,days | 78,192 | 117,043 | 462,649 | 856,884 | 1226,954 | 1616,024 |
|---|---|---|---|---|---|---|
| P,bar | 1,8366 | 2,057 | 3,563 | 5,229 | 7,086 | 7,615 |
| W | 3,3733 | 4,2312 | 12,6945 | 27,3424 | 50,2114 | 57,9882 |

The weighted normal equations are calculated;

155,8415 K + 1082,935 n = 282,5062 (4.3)

1082,935 K + 7590,039 n = 1996,905 (4.4)

Solving equations 4.3 and 4.4 for K and n yields,

K = -1,80071 and n = 0,52

Then, equation 3.6 becomes,;

$$v_c(t) = e^{-1,80071+0,52\ln(t)} - 0,16518 \tag{4.5}$$

Value for iq/RC is generated from initial condition for $v_c$ at time t=0.

The same calculations are made for the first 9 data points and gives;

$$v_c(t) = e^{-2,1156+0,5675\ln(t)} - 0,12055 \tag{4.6}$$

For the all data points, the generated curve fitting equation becomes,

$$v_c(t) = e^{-1,6087+0,4673\ln(t)} - 20015 \tag{4.7}$$

To compare the results obtained from the curve fitting equations with respect to real field data, pressure versus time relationship is plotted as depicted in figure 4.2. Since the production rates are fairly constant for the first 9 data points, the results obtained from both the 6 data points and the 9 data points are in very good agreement with the observed field data. It should be noted that the 9 points production rate variations compare to that those of 6 points are larger; therefore, slightly less match for the 9 data points is expected. On the other hand, when the all data points are used, the curve fit match is not good as expected due to production rate varies enormously. It should be noted that the proposed model given this study is derived for constant production rate condition. Extrapolation is made from the curve fitting equations for 7500 days of production to predict future production performance when constant production rate is granted and plotted in figure 4.3. As shown in figure 4.3, future production performances obtained from the 6 data points and the 9 data points are in good agreement to each other. The main difference causing approximately 2 bars of pressure drop at 7500 days is due to slightly production rate differences between the observed 6 data points and 9 data points that are used for curve fitting. It is also clear from figure 4.3 that three flow regimes namely early time linear behavior, mid time non-linear behavior, and late time linear behavior are also observed. Modeling variable rate production from a geothermal reservoir is possible using electric circuit system if the model is developed concerning switch theory that is beyond the objective of this study.

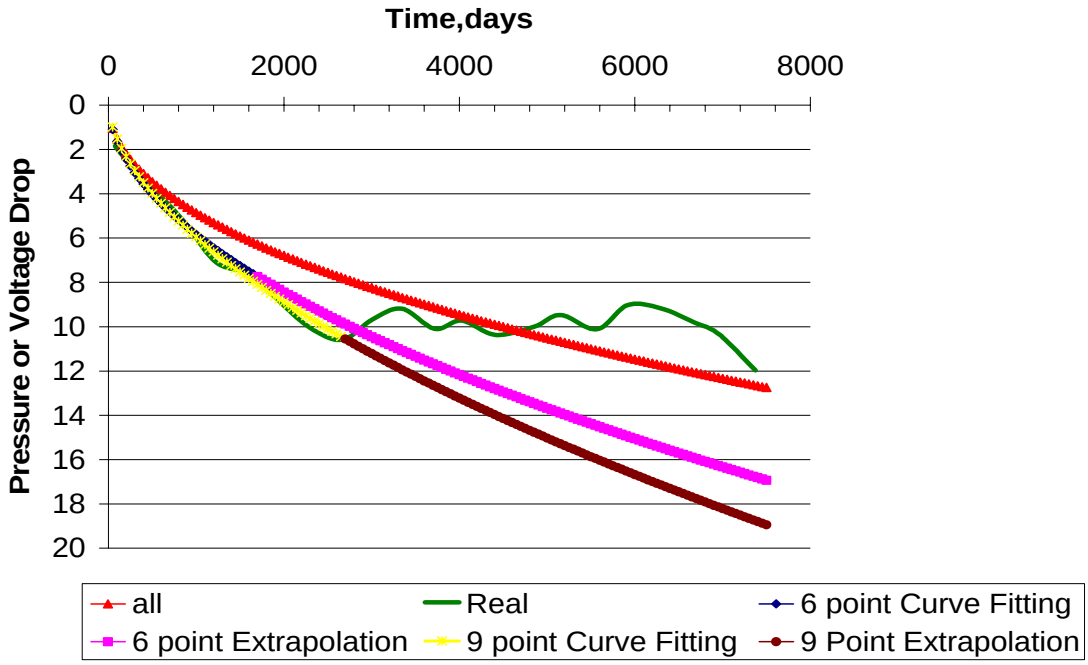


**Figure 4.3.** Curve Fitting and Extrapolation Obtained from the Proposed Model Using Real Field Data for 3 Cases.

## 5. CONCLUSIONS

There is a tight relationship between an engineering system such as a reservoir and the other systems such as electric circuit system if the physical phenomena of the related systems are well defined, and the main parameters of the both systems are related to each other one to one. In other words, any parameter or element in an engineering system should be approximately linked to representative element in the other system. Examined system or engineering system in this study is a geothermal reservoir, and electric circuit system is the system that a representative solution is sought. Once the equivalency of the related parameters in both systems is determined, general solution principles in circuit system are applied. To realize such solution, first step is to set up an analogy between the systems while preserving the physics of the phenomenon.

Applicability of electric circuits on a geothermal reservoir using material balance approach is the basis of this study. Following findings are concluded from this study.

This is the first time in the literature that a geothermal reservoir is modeled with an electric circuit system.

One tank and two tank model is considered for the examined geothermal reservoir system.

Material balance concept is used when developing analogy for circuit system. However, after setting up equivalent circuit, solution of the system was obtained by applying only the circuit system theory.

Equivalency of the parameters in the both system is determined after the analogy is realized. Equivalency of the parameters of pressure drop and production rate in a geothermal system is voltage drop and current variation in a circuit system, respectively. Basic elements representing a geothermal system as circuit system are capacitance, inductance, resistance and voltage source. Kirchhoff laws are applied to solve the equivalent circuit system. Constants in the developed model are defined in terms of physically.

Developed model for one tank system is verified with a real field data obtained from Laugernes Geothermal Field, Iceland. Constants of the model are determined from least square curve fitting method. Excellent curve fitting is obtained when the production rate is fairly constant, indicating the first 1600 days. However, the match, as expected, is not good when the production rate varies enormously. Extrapolation for the future production performance is obtained from the data that represent negligible variations on the production rate. The model yields approximately 17 bars (6 point extrapolation) and 19 bars (9 point extrapolation) of pressure drop instead of 12 bars observed in the field at 7500 days of production if the production rate of the field during 7500 days were the same that of the production rate observed in the first 1600 days. Since the production rate variations on the 9 data points are larger; consequently, pressure drop obtained from the 6 data point extrapolation is more representative for the examined Laugernes Geothermal field.

Three distinct behaviors obtained from the proposed model are observed and defined below;

- a sharp linear drop so-called steady state flow regime shown in early time
- a steady linear drop so-called pseudo steady state flow regime shown in late time
- a non-linear drop so-called transient flow regime shown in between early and late time.

Geothermal system represented by two tanks system is modeled with electric circuit system, and the solutions are given in Laplace domain. Solution of the model involves in more parameters. The model yields three roots that the behavior of only one root gives the expected behavior of a reservoir opened to production.

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## APPENDIX - A

In this section, the derivation of the equation 3.11 is given in details.

Using the definition for $i_{C1}$ gives;

$$i_{C_1} = c_1 \frac{dv_{C_1}}{dt} = i_{L_5} - i_{R_3} = i_{L_5} - \frac{v_{C_1}}{R_3} \tag{A-1}$$

For $i_{C2}$;

$$i_{C_2} = c_2 \frac{dv_{C_2}}{dt} = i_{L_5} - i_{R_4} = i_{L_5} - \frac{v_{C_2}}{R_4} \tag{A-2}$$

Using the definition of inductance for $V_{L5}$ gives,

$$v_{L_5} = L_5 \frac{di_{L_5}}{dt} = -v_{c_1} + Po - v_{c_2} - v_{R_2} \tag{A-3}$$

where $V_{R2}=R_2 i_{L5}$.

Writing in the matrix form of equations A-1,A-2 and A-3,

$$\frac{d}{dt}\begin{bmatrix} v_{C_1} \\ v_{C_2} \\ i_{L_5} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_3 c_1} & 0 & \frac{1}{c_1} \\ 0 & -\frac{1}{R_4 c_5} & \frac{1}{c_2} \\ -\frac{1}{L_5} & -\frac{1}{L_5} & \frac{R_2}{L_5} \end{bmatrix} \begin{bmatrix} v_{C_1} \\ v_{C_2} \\ i_{L_5} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_5} \end{bmatrix} P_0 \tag{A-4}$$

The solution should be in the form of the equation A-4,

$$X(s) = (sU - A)^{-1} = X(0) + (sU - A)^{-1}.BW(s) \tag{A-5}$$

$$(sU-A)^{-1}=\begin{bmatrix} s+\frac{1}{R_3c_1} & 0 & -\frac{1}{c_1} \\ 0 & s+\frac{1}{R_4c_2} & -\frac{1}{c_2} \\ \frac{1}{L_5} & \frac{1}{L_5} & s-\frac{R_2}{L_5} \end{bmatrix} \tag{A-6}$$

$$(sU-A)^{-1}=\frac{Ek^T(sU-A)}{\det(sU-A)} \tag{A-7}$$

$$Ek(sU-A)=\begin{bmatrix} Ek_{11} & Ek_{12} & Ek_{13} \\ Ek_{21} & Ek_{22} & Ek_{23} \\ Ek_{31} & Ek_{32} & Ek_{33} \end{bmatrix} \tag{A-8}$$

Where;

$$Ek_{11}=(s+\frac{1}{R_4c_2})(s-\frac{R_2}{L_5})+\frac{1}{L_5c_2} \tag{A-9}$$

$$Ek_{12}=(-1)\frac{1}{L_5c_2} \tag{A-10}$$

$$Ek_{13}=-(s+\frac{1}{R_4c_2})\frac{1}{L_5} \tag{A-11}$$

$$Ek_{21}=(-1)\frac{1}{L_5c_1} \tag{A-12}$$

$$Ek_{22}=(s+\frac{1}{R_3c_1})(s-\frac{R_2}{L_5})+\frac{1}{L_5c_1} \tag{A-13}$$

$$Ek_{23}=(-1)(s+\frac{1}{R_3c_1})\frac{1}{L_5} \tag{A-14}$$

$$Ek_{31}=(s+\frac{1}{R_4c_2})(\frac{1}{c_1}) \tag{A-15}$$

$$Ek_{32}=(-1)(s+\frac{1}{R_3c_1})(-\frac{1}{c_2}) \tag{A-16}$$

$$Ek_{33}=(s+\frac{1}{R_3c_1})(s+\frac{1}{R_4c_2}) \tag{A-17}$$

Solving the equation A-4 for $V_{C2}(s)$ gives,

$$v_{C_2}(s) = \frac{1}{\det(sU - A)}\begin{bmatrix} Ek_{11} & Ek_{21} & Ek_{31} \end{bmatrix}.\begin{bmatrix} v_{C_1}(0) \\ v_{C_2}(0) \\ i_{L_5}(0) \end{bmatrix} \tag{A-18}$$

Solving the determinant part of the equation A-18 gives,

$$\det(sU - A) = \left[ (s + \frac{1}{R_3 c_1})(s + \frac{1}{R_4 c_2})(\frac{1}{L_5})(s - \frac{R_2}{L_5}) + \frac{1}{c_1}(s + \frac{1}{R_4 c_2})\frac{1}{L_5} + (s + \frac{1}{R_3 c_1})\frac{1}{c_2}\frac{1}{L_5} \right] \tag{A-19}$$

The rest of the A-18 is named as A;

$$A = Ek_{11} v_{C_1}(0) + Ek_{21} v_{C_2}(0) + Ek_{31} i_{L_5}(0) \tag{A-20}$$

$$A = \left( (s + \frac{1}{R_4 c_2})(s - \frac{R_2}{L_5}) + \frac{1}{L_5 c_2} \right).v_{C_1}(0) - \frac{v_{C_2}(0)}{c_1 L_5} + (s + \frac{1}{R_4 c_2})\frac{1}{c_1} i_{L_5}(0) \tag{A-21}$$

$$A = \left( s^2 - s\frac{R_2}{L_5} + s\frac{1}{R_4 c_2} - \frac{R_2}{R_4 c_2 L_5} + \frac{1}{L_5 c_2} \right) v_{C_1}(0) - \frac{v_{C_2}(0)}{c_1 L_5} + s\frac{i_{L_5}(0)}{c_1} + \frac{i_{L_5}(0)}{R_4 c_2 c_1} \tag{A-22}$$

$$A = s^2 \left[ v_{C_1}(0) \right] + s\left[ \left( \frac{i_{L_5}(0)}{c_1} - \frac{R_2 v_{C_1}(0)}{L_5 R_4 c_2} \right) \right] + \left[ \frac{v_{C_1}(0)}{L_5 c_2} - \frac{R_2 v_{C_1}(0)}{R_4 c_2 L_5} - \frac{v_{C_2}(0)}{c_1 L_5} + \frac{i_{L_5}(0)}{R_4 c_2 c_1} \right] \tag{A-23}$$

$$A = as^2 + bs + c \tag{A-24}$$

$$v_{C_2}(s) = A.\frac{1}{\det(sU - A)} \tag{A-25}$$

$$\det(sU - A) = \left( s^2 + \frac{s}{R_4 c_2} + \frac{s}{R_3 c_1} + \frac{1}{R_4 c_2} \right)(s - \frac{R_2}{L_5})\frac{1}{L_5} \tag{A-26}$$

$$\det(sU - A) = s^3\left[\frac{1}{L_5}\right] + s^2\left[\frac{1}{R_4 c_2 L_5} + \frac{1}{R_3 c_1 L_5} - \frac{R_2}{(L_5)^2}\right]$$
$$+ s\left[\frac{1}{R_4 c_2 L_5} - \frac{R_2}{R_4 c_2 (L_5)^2} - \frac{R_2}{R_3 c_1 (L_5)^2} + \frac{1}{L_5 c_1} + \frac{1}{c_2 L_5}\right] \tag{A-27}$$
$$+ \left[\frac{1}{R_4 c_1 c_2 L_5} + \frac{1}{R_3 c_1 c_2 L_5} - \frac{R_2}{(L_5)^2 R_4 c_2}\right]$$

$$\det(sU - A) = ks^3 + ms^2 + ns + p \tag{A-28}$$

$$v_{C_2}(s) = \frac{as^2 + bs + c}{ks^3 + ms^2 + ns + p} \tag{A-29}$$

## AUTOBIOGRAPHY

Author was born in İstanbul on August 21, 1980. He started to his education life in Ataköy 1986. He graduated from Saint-Michel French High School in 1998. Having joined to Petroleum and Natural Gas Engineering Department of Mining Faculty at ITU in 1998, he graduated from this department in June 2003 as a Petroleum and Natural Gas Engineer. He decided continuing to study master program of the same department of Science and Technology Institute at ITU. He was also charged as a research assistant in Department of Petroleum and Natural Gas Engineering at ITU and he has continued studying presently as research assistant since 2005.