AN OPTIMAL SOLUTION FOR THE MULTI-AGENT RENDEZVOUS PROBLEM APPEARING IN COOPERATIVE CONTROL

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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The multi-agent rendezvous problem appearing in cooperative control is considered in this thesis. There are various approaches to this topic as the objectives and problem set-ups vary in real-life rendezvous problems. Some of the applications can be given as the coordination of autonomous mobile robots or unmanned air vehicles (UAVs) for joint tasks, and motion planning for vehicle convoys. The problem is basically on providing a rendezvous for mobile agents at a specified or unspecified destination. What makes the topic interesting is maintaining a coordination between the mobile agents so that the agents reach the rendezvous point simultaneously. Early or late arrivals are not desired.

An energy optimal solution is obtained for the problem. Imperfect road conditions, obstacles, internal problems of the agents or similar disturbances are also tried to be handled. As these factors are included in the problem, it is assumed that the agents communicate between each other at specified time instants exchanging information about their expected arrival times in order to maintain a common rendezvous time among the team.

The solution is initially derived for rendezvous in one-dimensioned space. Then, the problem configuration is altered for two-dimensioned motions, and the target point is assumed to be moving in order to extend the solution to possible practical applications. The effect of increasing disturbance on the control input and time delays in the communication are also discussed.

Keywords: Multi-Agent Rendezvous Problem, Optimal Control, Cooperative Control, Minimum Energy Control, Calculus of Variations.

ÖZET

İŞBİRLİKLİ KONTROLDE GÖRÜLEN ÇOK ARAÇLI BULUŞMA PROBLEMİ İÇİN OPTİMAL BİR ÇÖZÜM

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Bu tezde, işbirlikli kontrolde görülen çok araçlı buluşma problemi ele alınmaktadır. Bu konuya yapılan yaklaşımlar, gerçek hayattaki buluşma problemlerinin amaçlarının ve problem düzeneklerinin birbirlerinden farklı olması nedeniyle çeşitlilik gösterir. Problemin bazı uygulamalarına örnek olarak, hareketli özerk robotların veya insansız hava araçlarının (İHA) birleşik görevler için koordine edilmesi ve araç konvoyları için hareket planlaması verilebilir. Problem temel olarak, hareketli araçların belirli veya belirsiz bir varış noktasında buluşmalarının sağlanmasına dayalıdır. Konuyu ilginç kılan şey, erken veya geç varışlar istenen durumlar olmadığından, buluşmanın aynı anda gerçekleşmesini sağlayacak şekilde hareketli olan araçlar arasında bir koordinasyonun sağlanmasıdır.

Problem için enerji açısından optimal bir çözüm elde edilirken, mükemmel olmayan yol durumlarının, engellerin, araçların içsel problemlerinin veya benzer bozucu etkilerin de ele alınmasına çalışılmaktadır. Bu etkenler probleme dahil edildiğinden, araçların takım içerisinde ortak bir buluşma zamanı belirleyebilmek için, birbirleri arasında muhtemel varış zamanları hakkında bilgi alış verişinde bulunmak üzere haberleştikleri kabul edilmektedir.

Çözüm başlangıçta bir boyutlu uzaydaki buluşma için geliştirilmektedir. Sonrasında ise, problemin konfigürasyonu iki boyutlu hareketler için değiştirilmekte ve çözümün muhtemel pratik uygulamaları kapsayacak şekilde genişletilebilmesi işin hedef noktasının hareket ettiği varsayılmaktadır. Ayrıca, kontrol girdisi üzerindeki bozucu etkilerin ve haberleşmedeki zaman gecikmelerinin artması durumları da tartışılmaktadır.

Anahtar Kelimeler: Çok Araçlı Buluşma Problemi, Optimal Kontrol, İşbirlikli Kontrol, Minimum Enerji Kontrolü, Değişimler Hesabı.

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to mom and dad...

Anneme ve babama...

Chapter 1

INTRODUCTION

In this thesis, we investigate optimal solutions for a rendezvous problem appearing in cooperative control. The problem in question is getting $N > 1$ vehicles in a task force to reach a specified point at the same time instant. It is assumed that these vehicles communicate with each other exchanging information about their expected final time to reach the rendezvous point in order to avoid early or late arrivals, and this communication occurs at discrete time instants.

The interesting part of the problem is that the final times of the vehicles may change during the mission due to unforeseen events, like obstacles on the road or internal problems of the vehicles, and there might be time delays in the communication between the vehicles. Therefore, a satisfactory solution for the stated rendezvous problem should take these conditions into account and provide reliable results in order to be applicable.

Before attacking the problem, let us give some information about the concepts related with the subject and present a brief information about the similar studies in the literature.

1.1 Cooperative Control

Research on control of multi-vehicle systems performing cooperative tasks gained importance in the late 1980s [1] when several researchers began investigations in multiple mobile robot systems [2]. Since then, the interest in this topic has increased significantly thanks to the development of inexpensive and reliable wireless communications systems and the application fields in military operations [1].

The most popular problems of cooperative control of mobile robots involve groups or teams of autonomous vehicles cooperating to complete a mission [3]. Basically, the success of the mission can only be attained when none of the vehicles or groups that are performing separate tasks fail, i.e. each individual performing the corresponding task must succeed. The interesting part of the problem is that the vehicles or the groups have to perform coordinated actions [3] in order to complete their individual tasks.

At this point, it is helpful to give a concise explanation about what is meant by *cooperative control* before going into more detail in the subject. A comprehensive study about the recent researches on the topic can be found in [1].

Consider a group of vehicles aiming to complete a task and the corresponding overall performance function below.

$$J = \int_0^T L(x, \alpha, u)dt + V(x(T), \alpha(T)) \tag{1.1}$$

where x is the states, α is the roles, T is the final time that the task should be completed, L is the incremental cost of the task and V represents the terminal cost of the task. Notice that (1.1) is a typical cost function.

A task is called *decoupled*, if the cost function J can be written as

$$J = \sum_{i=0}^{N} \left(\int_0^T L^i(x^i(t), \alpha^i(t), u^i(t))dt + V^i(x^i(T), \alpha^i(T)) \right) \tag{1.2}$$

where

i : the index corresponding to vehicle i

$x^i(t)$: the state of vehicle i at time t

$\alpha^i(t)$: the role of vehicle i at time t that is subject to change during the task

$u^i(t)$: the input controlling the state of vehicle i at time t

$x^i(T)$: the final value of the state of vehicle i

$\alpha^i(T)$: the final value of the role of vehicle i

otherwise, the task is called *coupled* or namely *cooperative* which means that the task performance depends on the joint locations, roles and inputs of the vehicles [1].

Then, *cooperative control* can simply be defined as determining the control law (i.e. the control input $u^i(t)$) that solves the coupled performance function which is the dual of (1.2) corresponding to a cooperative task defined above.

1.2 Optimal Control

Optimal control can basically be described as the problem of determining a control law for a given system while satisfying the specified optimality conditions. A precise mathematical description can be given as finding an *admissible control* $u^*(t)$ which forces the following system with $x(t)$ as the state, $u(t)$ as the control input and t is time

$$\dot{x}(t) = f\left(x(t), u(t), t\right) \tag{1.3}$$

to follow an *admissible trajectory* $x^*(t)$ that *minimizes* or *maximizes* the performance measure in the form

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t)dt \tag{1.4}$$

with $u^*(t)$ being *the optimal control input* and $x^*(t)$ being *the optimal trajectory* [4]. Here, f, g, h are specified functions that satisfy certain assumptions, see e.g. [4].

In this study, the solutions for the multi-agent rendezvous problem are to be optimal with constraints like fixed initial and final states, and the cost to be minimized is the control energy. Therefore, we will form a performance measure as in (1.4) that involves the constraints to be considered, and solve for the optimal input resulting in a successful rendezvous.

1.3 Rendezvous Problem

The multi-agent rendezvous problem in this thesis is an optimal control problem appearing in cooperative control. The idea is having a number of mobile agents arrive at a meeting point, namely *the rendezvous point*, at the same time. The crucial point is assuring that the agents perform cooperative actions by arranging themselves according to the information gathered from the other agents in the team [5].

Actually, the title "rendezvous problem" is a broad one, and it is a general name for various problems in cooperative control some of which are;

- The problem of two aircrafts aiming to meet at a non-specified point at a predefined final time [6],
- The problem of trajectory planning for the vehicles in a team aiming to reach the neighborhood of a point not before the other vehicles in the team [7],
- The problem of path planning for a robot aiming to reach a target [8, 9],
- The coordination of unmanned aerospace vehicles (UAVs) to reach a target point [10],
- The problem of enabling mobile users in a location, tracking and rendezvous with a variety of mobile entities [11],
- The problem of conflict management in a multi-user computer network [12],
- The problem of motion planning for vehicle convoys [13],
- The problem of multi-agent rendezvous [14–17],

Probably, the most popular one of the problems above is the multi-agent rendezvous problem which is the subject discussed in this thesis. The popularity is basically due to variety of applications in military operations ranging from cooperative attack in land operations to cooperative control of unmanned air vehicles (UAVs) for rendezvous in air operations. In the literature there are many versions of this problem involving constraints such as rendezvous with fixed final time, time optimal rendezvous with unspecified final states, and energy efficient rendezvous. Next, some of the solutions to similar rendezvous problems are presented.

In one of the studies related with the multi-agent rendezvous, the problem of determining a meeting point while minimum energy consumption is taken into account is discussed [18]. In that paper, a multi-robot team, which consists of autonomous mobile robots trying to meet at a single point for a mission, is considered, and two solutions about the minimum energy consumption during the travels towards the rendezvous point are proposed. The interesting part of the problem is that the cost of travel for each robot is different and the goal is to find an energy efficient solution considering the robots in the team as a whole. Although the proposed solutions are successful in finding a rendezvous point, the proposed algorithms do not consider a timing constraint. In addition, as indicated in the paper, the solutions assume a reliable communication between the robots, and communication delays or loss of information about the current positions of the robots are not handled.

A similar problem on multi-agent rendezvous is considered in [15,16]. In that setting, there are $N > 1$ vehicles aiming to meet at an unspecified point which is regarded as rendezvous by sensing the current positions of the neighboring mobile agents that are within their *sensing region*. The presented solution is basically determining decentralized control algorithms for the agents. In other words, the solution just guarantees the rendezvous for the agents but does not include the constraints on the control energy, the final states (velocity and acceleration) of the agents, and imperfect communication.

Another related study about the multi-agent rendezvous problem is on the stability of mobile robot rendezvous [8]. In this study, a mobile robot aiming to reach a target point is considered, and an optimal control is derived. In that problem, the destination point and the final time is specified for an autonomous robot, and the robot aims to arrive the rendezvous point on time while evaluating its current states with respect to the rendezvous point and arranging itself accordingly via applying a step control acceleration. Both 1-D and 2-D solutions are provided assuming that the states are known and there is no noise or disturbance in the system. Besides, the solutions are derived for a single autonomous robot and thus exclude a cooperative control scheme and communication with a central point or any other agent. Moreover, the resulting control inputs have large magnitudes and there is a possibility of instability as the robots gets closer to the rendezvous point, for that reason a limit is placed on the applied control input.

A very similar work is presented in [19] in which a multi-robot rendezvous problem is discussed. The goal of the problem is to determine control laws for N robots moving in the horizontal plane in order to reach a moving robot at the same time. The motion of the moving robot, namely *the reference-robot* is not a priori known by other robots aiming to catch it. It is assumed that all robots move faster than the reference robot, the motion of the reference robot is continuous and there is a reliable communication between the leader robot and the others, and a sensory system in order to determine the position of the reference robot. In that paper two approaches for the solution are presented.

- First one is the leader-follower approach in which the leader of the team tracks the position of the reference robot and the team members try to follow the leader. Since the leader perfectly tracks the reference-robot and the others catch the leader eventually, all the robots catch the reference robot consequently. In that approach, it is also assumed that the followers move faster than their leader.
- The second one is the reference-robot approach in which all the robots sense the position of the reference robot continuously and try to catch it. Since all the robots move faster than the reference robots, all the robots in the team catches the reference-robot successfully.

The solutions are obtained via relative kinematic equations and they are successful as all the robots catch the reference-robot at the same time. However, the solutions include neither time nor energy optimality constraints, and it is assumed that there is no communication deficiency like lost or delayed information signals between the robots.

1.4 Thesis Contribution and Organization

In the following chapters, the multi-agent rendezvous problem in question is described, the mathematical solution is derived and the application results are presented. The major advantage of the solutions given here is being optimal in terms of the control energy used by the mobile agents while providing a successful rendezvous with pre-specified initial and final states. Moreover, delayed communication for simultaneous cooperative rendezvous is handled.

Remaining parts of the thesis are organized as follows. In Chapter 2, the configuration of the multi-agent rendezvous problem is described and the basics of the mathematical solution are introduced together with some simple illustrative examples. The application of the solution to the problem is given in Chapter 3. The solutions for rendezvous in 1-D in the presence/absence of communication problems modeling delayed or lost information about the estimated times of arrivals are presented. The solution is extended to 2-D and the case of moving target point is addressed for practical applications. The effect of increasing disturbance on the control input and time delays in the communication are also discussed. Chapter 4 includes the conclusions and some notes about possible complementary studies to be made.

Chapter 2

PROBLEM STATEMENT AND PRELIMINARIES FROM OPTIMAL CONTROL THEORY

In this chapter, the description of the multi-agent rendezvous problem is given. Basic concepts and tools from optimal control theory are described in order to establish a background for the problem solution.

2.1 Description of the Problem

As stated in the introduction briefly, we will discuss a rendezvous problem appearing in cooperative control. Remember that there are $N > 1$ vehicles in a task force and their goal is to reach a target point(rendezvous point) at the same time instant spending as low energy as possible. It is assumed that they communicate with each other at specified time instants during their mission in order to arrange their states so that they all arrive the target point at the same time instant, neither before nor after.

Consider the rendezvous problem for N vehicles illustrated in Figure 2.1. The vehicles aim to reach the target point p_t at the same time instant. Basically, one can assign a final time t_f for the mission and inform the vehicles to be at the target point at time t_f and the vehicles arrange their acceleration or velocity accordingly. However, if one of the vehicles fails to be at p_t at time t_f due to an internal problem or bad road conditions, the mission may not be completed successfully. In order to overcome that problem, it is better to utilize an information exchange between the vehicles about their position, velocity and acceleration or just the estimated time of arrival at discrete time instants. Thus, at each information exchange instant the vehicles can use the information that they received from the other vehicles in the team as a feedback to arrange their states and try to catch the fastest or the slowest of the team in order to arrive the rendez-vous point p_t at the same instant.

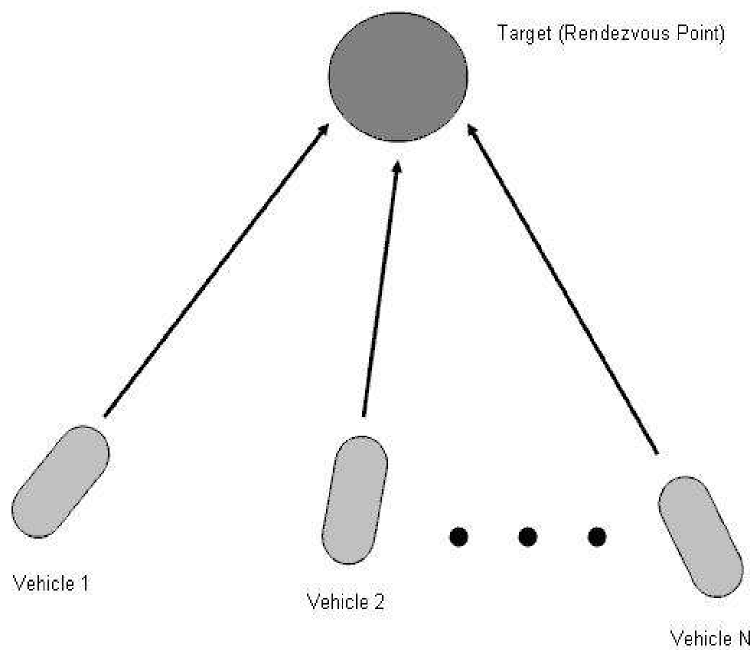


Figure 2.1: Rendezvous problem for N vehicles

Consider the following dynamical model for the vehicles

$$\dot{x}_i(t) = f(x_i(t), u_i(t), w_i(t)) \tag{2.1}$$

where f is a known function of the state x_i, input u_i and the disturbance w_i. The state x_i consists of the position, velocity and the acceleration of the ith vehicle in space. In order to simplify the problem we may consider the following linear case assuming that $w_i(t) = 0$:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) \tag{2.2}$$

where $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The target point is assumed be fixed at the origin. In order to determine the solution guaranteeing the success of the mission, the following quadratic cost function should be solved for the minimizing optimal control input $u_i(t)$:

$$J_i(t) = \int_t^{t_i^f} (\|x_i(\tau)\|^2_{Q_i} + \|u_i(\tau)\|^2_{R_i}) d\tau + \|x_i(t_i^f)\|_{Q_i^f} \tag{2.3}$$

where Q_i, R_i and Q_i^f are the weighting matrices of appropriate dimensions. What makes the problem interesting is that in the equation above, the final time t_i^f is time varying, and is updated at discrete time instants t_k depending on the feedback received from the other vehicles. There are two choices:

$$t_i^f(t) = \min\{t_{1f}(t), t_{2f}(t), t_{3f}(t), \ldots, t_{Nf}(t)\} \tag{2.4}$$

or

$$t_i^f(t) = \max\{t_{1f}(t), t_{2f}(t), t_{3f}(t), \ldots, t_{Nf}(t)\} \tag{2.5}$$

where $t_{jf}(t)$ is the expected time of arrival for the jth vehicle at time t, and is a function of position, velocity, acceleration and control input of the jth vehicle.

We can denote $t_{jf}(t)$ as $t_{jf}(t) = p(x_{jp}(t), v_{jp}(t), a_{jp}(t), u_j(t))$, where $x_{jp}(t)$, $x_{jv}(t)$, $x_{ja}(t)$ and $u_j(t)$ are the position, velocity, acceleration and control input of the jth vehicle at time t, respectively. Here, p is a function to determine the final time of a vehicle by assuming that current optimal control input $u_i^*(t)$ will not be subject to any change during the rest of the travel. Note that, $t_i^f(t)$ is generated from a data set of N received by the ith vehicle up to time t. In order to have a more realistic problem, it is convenient to assume that there are time delays in the communications between each vehicle, i.e.:

$$t_i^f(t) = \min\{t_{1f}(t - h_{1i}(t)), t_2^f(t - h_{2i}(t)), t_3^f(t - h_{3i}(t)), \ldots, t_N^f(t - h_{Ni}(t))\} \quad (2.6)$$

or

$$t_i^f(t) = \max\{t_1^f(t - h_{1i}(t)), t_2^f(t - h_{2i}(t)), t_3^f(t - h_{3i}(t)), \ldots, t_N^f(t - h_{Ni}(t))\} \quad (2.7)$$

where $h_{ji}(t)$ represents the time delay in the one way communication from vehicle j to vehicle i at time instant t.

2.2 Preliminaries from Optimal Control Theory

In this section the solution of the rendezvous problem described in Section 2.1 is presented using basic principles from optimal control theory. For the time being, suppose that the information exchange between the vehicles is perfect and not effected by the delay. Then, we can proceed with the solution of the quadratic cost function minimization problem.

2.2.1 Calculus of Variations

Suppose that the state $x_i(t)$ is composed of position and velocity of the ith vehicle only, i.e.

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2.8)$$

where $Q_i = I_{2\times2}$, $R_i = I_{2\times1}$, $Q_i^f = I_{2\times2}$ and the initial and the final conditions x_{i0} and x_{if} are known.

Optimal solution of the quadratic cost function minimization problem defined by (2.3) can be obtained by "Calculus of Variations" which is a well-known method for solving optimal control problems. As explained in [4], the necessary conditions for optimality in order to solve the problem are:

$$\begin{aligned} \dot{x}_i^* &= \frac{\partial \mathcal{H}}{\partial p}(\dot{x}_i^*(t), \dot{u}_i^*(t), \dot{p}_i^*(t), t) \\ \dot{p}_i^* &= \frac{\partial \mathcal{H}}{\partial x}(\dot{x}_i^*(t), \dot{u}_i^*(t), \dot{p}_i^*(t), t) \\ 0 &= \frac{\partial \mathcal{H}}{\partial u_i}(\dot{x}_i^*(t), \dot{u}_i^*(t), \dot{p}_i^*(t), t) \end{aligned} \tag{2.9}$$

for all t $\in [t_0, t_f]$, where $\mathcal{H}$ is the Hamiltonian defined as

$$\mathcal{H}(x(t), u(t), p(t), t) \triangleq g(x(t), u(t), t) + p^T(t)\,[f(x(t), u(t), t)] \tag{2.10}$$

and

$$\begin{aligned} &\left[\frac{\partial h}{\partial x}(x^*(t_f) - p^*(t_f))\right]^T \partial x_f + \\ &\left[\mathcal{H}(x^*(t_f), p^*(t_f), t_f) + \frac{\partial h}{\partial t}(x^*(t_f), t_f)\right] \partial t_f = 0 \end{aligned} \tag{2.11}$$

for the system and the cost function below

$$\dot{x}_i(t) = f(x_i(t), u_i(t), t) \tag{2.12}$$

$$J(u) = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t)dt \tag{2.13}$$

At this point, it is not difficult to construct an analogy between (2.3) and (2.13) as follows

$$\begin{aligned} h(x(t_f), t) &= \|x_i(T_i^f(t))\|_{Q_i^f} \\ g(x(t_f), t) &= \|x_i(\tau)\|_{Q_i}^2\| + \|u_i(\tau)\|_{R_i}^2 \end{aligned} \tag{2.14}$$

In (2.11), $p^*(t)$ represents the Lagrange multipliers $p_1^*(t), p_1^*(t), \ldots, p_n^*(t)$ which are selected as follows.

$$\dot{p}^*(t) = -\left[\frac{\partial f}{\partial x}(x^*(t), u^*(t), t)\right]^T p^*(t) - \left[\frac{\partial g}{\partial x}(x^*(t), u^*(t), t)\right] \tag{2.15}$$

Note that, $p(t)$ is also called *costate* and the equation above is called *costate equations*. By solving the equations (2.9) and (2.11), the costates, the optimal control input and the output trajectory can be obtained easily.

If we apply (2.9)-(2.15) to our simplified problem, we can have the following formulation [4].

The Hamiltonian for the problem is

$$\begin{aligned}\mathcal{H}(x(t), u_i(t), p_i(t), t) = \quad & \frac{1}{2}x^T Q_i x_i(t) + \frac{1}{2}u^T R_i u_i(t) + \\ & p^T A_i x(t) + p_i^T(t) B_i u_i(t) \end{aligned} \tag{2.16}$$

Then, the necessary conditions for the solution in order to exist are

$$\dot{x}_i^* = A_i x_i^*(t) + B_i u_i^*(t) \tag{2.17}$$

$$\dot{p}_i^* = \frac{\partial \mathcal{H}}{\partial x_i} = -Q_i x_i^*(t) - A_i^T p_i^*(t) \tag{2.18}$$

$$0 = \frac{\partial \mathcal{H}}{\partial u} = R_i u_i^*(t) + B_i^T p_i^*(t) \tag{2.19}$$

Notice that the optimal control input $u_i(t)$ can be obtained from (2.19) as

$$u_i^*(t) = -R_i^{-1} B_i^T p_i^*(t) \tag{2.20}$$

Substituting (2.20) into (2.17) yields

$$x_i^*(t) = A_i x^*(t) - B_i R_i^{-1} B_i^T p_i^*(t) \tag{2.21}$$

Combining (2.21) and (2.18) we have $2n$ linear differential equations which are formulated below.

$$\left[\begin{array}{c}\dot{x}_i^*(t)\\ \hline \dot{p}_i^*(t)\end{array}\right] = \left[\begin{array}{c|c} A_i & -B_iR_i^{-1}B_i^T \\ \hline -Q_i & -A_i^T\end{array}\right]\left[\begin{array}{c}x_i^*(t)\\ \hline p_i^*(t)\end{array}\right]$$

$$\left[\begin{array}{c}\dot{x}_i^*(t)\\ \hline \dot{p}_i^*(t)\end{array}\right] = \Phi\left[\begin{array}{c}x_i^*(t)\\ \hline p_i^*(t)\end{array}\right] \tag{2.22}$$

where Φ is the transition matrix. In order to solve (2.22) we need $2n$ boundary conditions, and we already have them as $x_i(t_0) = x_i^0$ and $x_i(t_f) = x_i^f$. The rest of the solution in order to determine the Lagrange multipliers $p_i^*(t)$, the control input $u_i^*(t)$ and the states $x_i^*(t)$ is trivial and can be obtained easily after solving (2.22) for $x_i^*(t)$ and $p_i^*(t)$ by following the steps explained in [20]. Remember that the solution of a set of the $2n$ linear differential equations will be in the form below

$$\left[\begin{array}{c}x_i^*(t)\\ \hline p_i^*(t)\end{array}\right] = c_1e^{\lambda_1 t}v_1 + c_2e^{\lambda_n t}v_2t + \ldots + c_ne^{\lambda_n t}v_n \tag{2.23}$$

where c_is are the coefficients, and λ_is and v_is are the eigenvalues and the eigenvectors of Φ, respectively. Now, let us look at some sample results obtained by the explained method.

The solutions for $x_i^0 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, $x_i^f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $t_i^0 = 0$, $t_i^f = 10$ and $t_i^f = 5$ are depicted in Figures 2.2 and 2.3, respectively. In that figures, it can be observed that the optimal control input $u_i^*(t)$ for the cost function in (2.3) brings the vehicle to the rendezvous point at nearly $t = 5$ whereas the rendezvous time was specified as $t_i^f = 10$ initially, and no change were made during the travel.

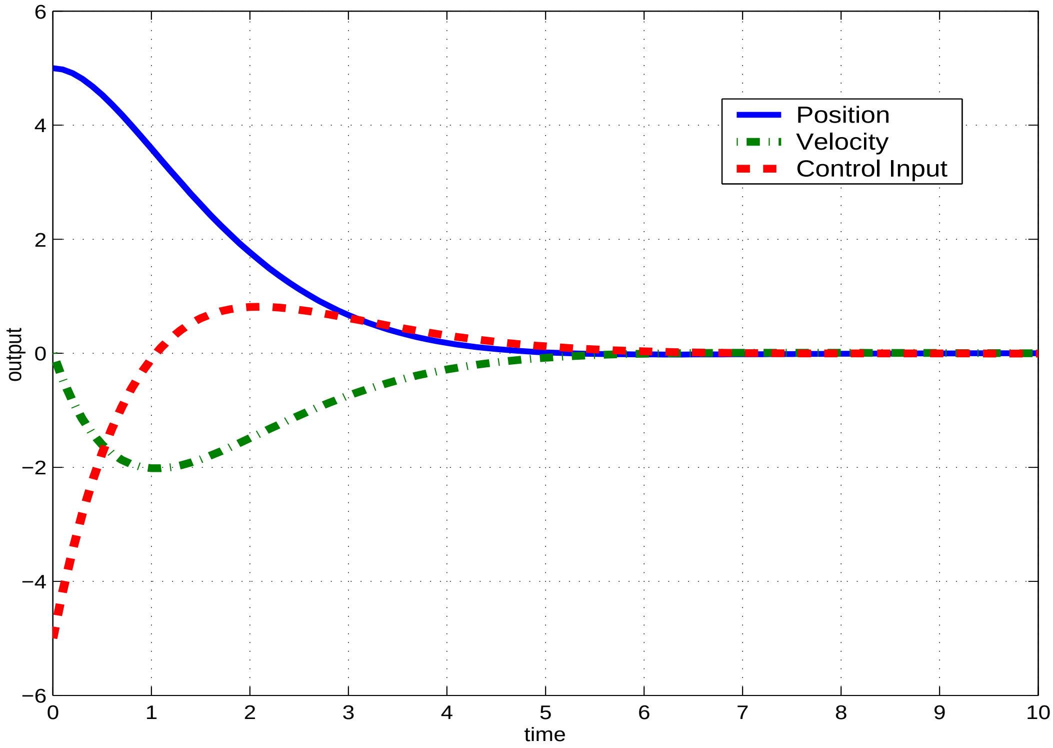


Figure 2.2: Position, velocity and control input for $t_i^0 = 0$, $t_i^f = 10$

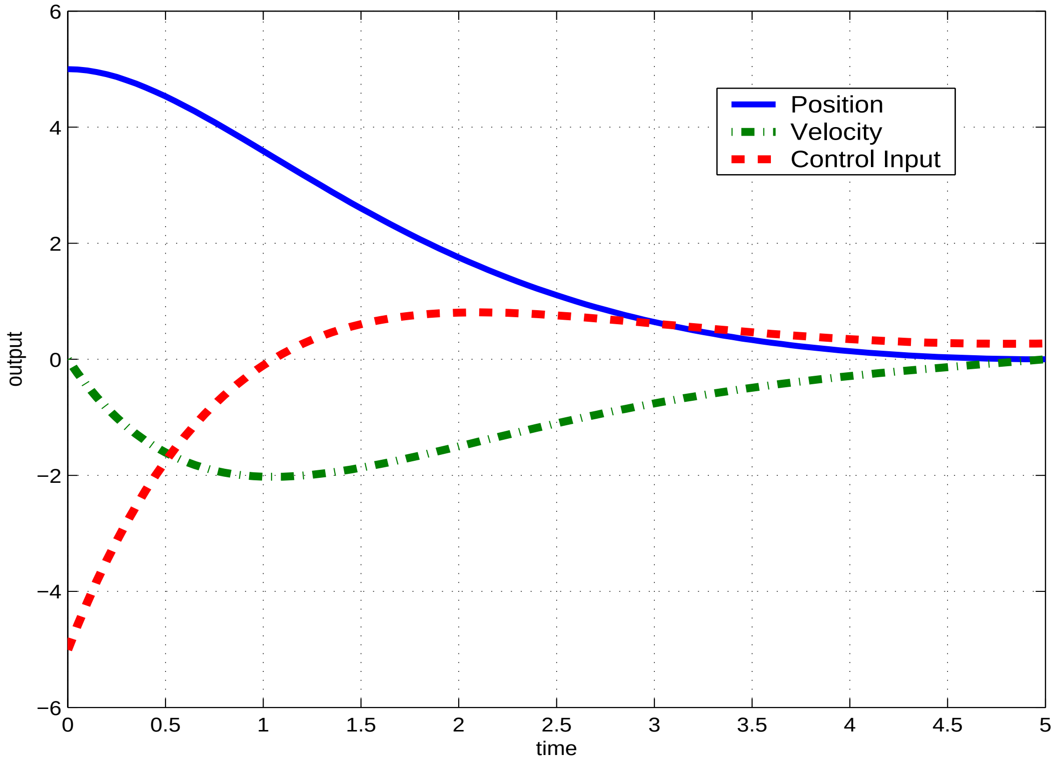


Figure 2.3: Position,velocity and control input for $t_i^0 = 0$, $t_i^f = 5$

This result is basically due to the configuration of the cost function, because the optimal control input is determined by taking the constraints for the control energy and the difference between the current and final states to be reached into account directly. In other words, the control input tries to bring the vehicle to the rendezvous point just on time using minimum control energy but it also forces the states to converge to zero as soon as possible. Thus, we observe that the vehicle reaches the target point much before the specified time.

Moreover, as seen below for $x_i^0 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, $x_i^f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $t_i^0 = 0$ and $t_i^f = 3$, if the specified final time is close to the departure time (i.e. the travel time is short) the control input grows too much, and such a case is not acceptable due to the practical limits of the controller. Therefore, a realizable solution is needed and the next section addresses this problem.

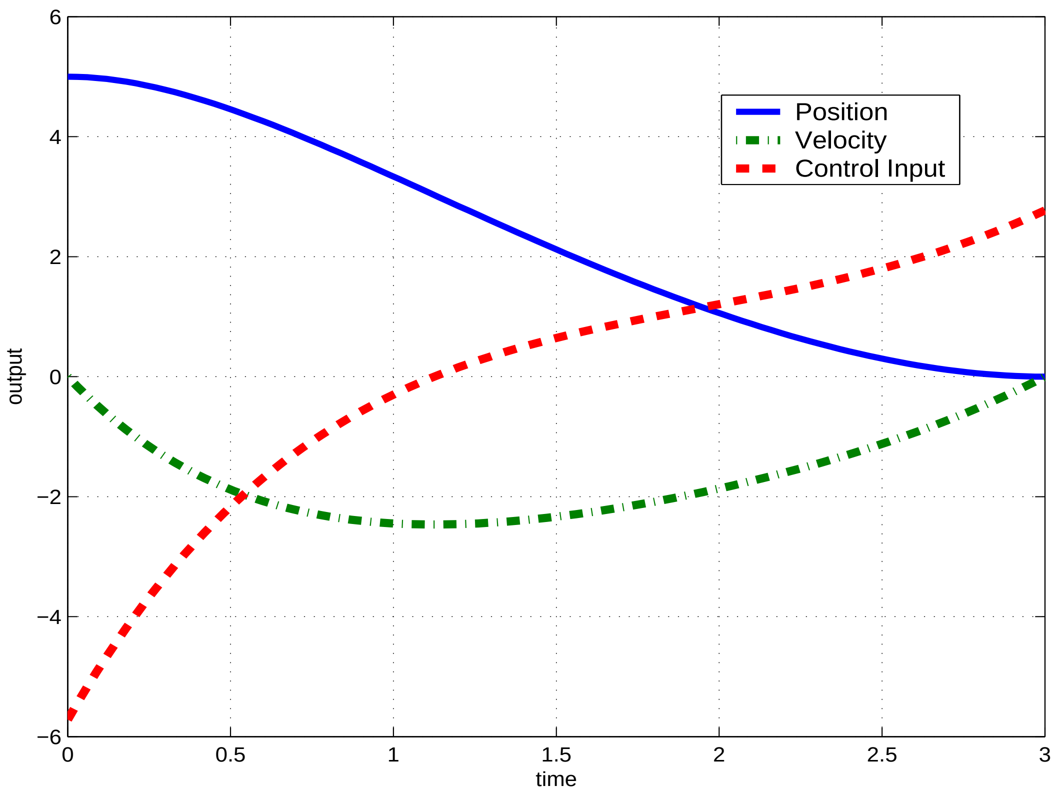


Figure 2.4: Position, velocity and control input for $t_i^0 = 0$, $t_i^f = 3$

2.2.2 Minimum Energy Control

Regarding the results obtained by solving the cost function in (2.3), it can be concluded that the obtained control input does not solve the problem appropriately due to the constraints included in cost function. Having known the reasons, it might be possible to improve the solution.

For instance, the term $\|x_i(t_i^f)\|_{Q_i^f}$ has no relevance with the minimization of the control input to be used since $x_i^f = 0$, and it can be excluded. Moreover, including the term $\|x_i(t)\|_{Q_i}^2$ in the cost function to be minimized results in a struggle for an early arrival to the rendezvous point (i.e. t_i^f) and consequently the usage of more energy in the control input. So, we may also exclude that term in the cost function and have the new cost function to be minimized as follows.

$$J_i(t) = \frac{1}{2}\int_{t_i^0}^{t_i^f} (\|u_i(t)\|_{R_i}^2)dt \tag{2.24}$$

Now, the question is "How can we guarantee that the vehicle reaches the rendezvous point having excluded the states in the cost function?" and the answer is simple. The control input and the states are related by the system equation in (2.2) with A_i and B_i as in (2.8). Therefore, although the term related with the states is not involved in (2.24), the optimal solution of (2.24) gives not only the minimum energy signal (i.e. the control input $u_i(t)$) but also the desired state trajectory since the solution is based on the time constraint of t_i^0 and t_i^f, and the boundary conditions of $x_i(t_i^0) = x_i^0$ and $x_i(t_i^f) = x_i^f$. Thus, we will solve another cost function minimization problem known as "Minimum Energy Control Problem".

Solution of the minimum energy control problem is based on "Calculus of Variations Method" and basically aims to minimize the control energy in the cost function of (2.24). Thus, it is expected that the optimal control input will not grow much enabling the vehicle to reach the target point just on time.

As minimum energy control control is a very well known issue, the lengthy derivation of the optimal control input and the minimum cost can be found in many books like [21], [6] and [4]. Here, the derivation is skipped and only the solution is presented, however the reader is referred to [21] for a detailed derivation with a comprehensive explanation on the topic.

Notice that the controllability Gramian for the system in (2.2) with A_i and B_i as in (2.8)

$$W_c(t_i^0, t_i^f) = \int_{t_i^0}^{t_i^f} e^{A_i(t_i^0-\tau)} B_i B_i^T e^{A_i^T(t_i^0-\tau)} d\tau \tag{2.25}$$

is nonsingular for any $t_i^f > t_i^0$ since (A_i, B_i) is a controllable pair. Then, the minimum energy control signal $u_i(t)$ can be obtained as

$$u_i^*(t) = -B_i^T e^{A_i^T(t_i^0-t)} W_c^{-1}(t_i^0, t_i^f)(x_i^0 - e^{A_i(t_i^0-t_i^f)} x_i^f) \tag{2.26}$$

Alternatively, the minimum energy control signal can be written in terms of the reachability Gramian W_r as

$$W_r(t_i^0, t_i^f) = \int_{t_i^0}^{t_i^f} e^{A_i(t_i^f-\tau)} B_i B_i^T e^{A_i^T(t_i^f-\tau)} d\tau \tag{2.27}$$

and

$$u_i^*(t) = -B_i^T e^{A_i^T(t_i^f-t)} W_r^{-1}(t_i^0, t_i^f)(x_i^f - e^{A_i(t_i^f-t_i^0)} x_i^0) \tag{2.28}$$

Then, the minimum energy value J_i^* can be determined as

$$\begin{aligned} J_i^* &= \frac{1}{2}\int_{t_i^0}^{t_i^f} \|u_i^*(t)\|^2 dt \\ &= \frac{1}{2}(x_i^0 - e^{A_i(t_i^0-t_i^f)}x_i^f)^T W_c^{-1}(t_i^0,t_i^f)(x_i^0 - e^{A_i(t_i^0-t_i^f)}x_i^f) \end{aligned} \tag{2.29}$$

For the controllable pair (A_i, B_i) in (2.8) and the boundary conditions $x_i^0 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and $x_i^f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ with $t_i^0 = 0$ and $t_i^f = 5$, W_c and $u_i^*(t)$ are calculated as

$$\begin{aligned} W_c(t_i^0, t_i^f) &= \begin{bmatrix} \frac{1}{3}(t_i^f - t_i^0)^3 & -\frac{1}{2}(t_i^f - t_i^0)^2 \\ -\frac{1}{2}(t_i^f - t_i^0)^2 & (t_i^f - t_i^0) \end{bmatrix} \\ &= \begin{bmatrix} \frac{125}{3} & -\frac{25}{2} \\ -\frac{25}{2} & \frac{5}{3} \end{bmatrix} \\ u_i^*(t) &= \frac{12}{25}t - \frac{6}{5} \end{aligned} \tag{2.30}$$

Then, the trajectory of $x_i(t)$ can easily be calculated as

$$\begin{aligned} x_i^*(t) &= e^{A_i(t-t_i^0)} + \begin{bmatrix} \frac{2}{25}t^3 - \frac{3}{5}t^2 \\ \frac{6}{25}t^2 - \frac{6}{5}t \end{bmatrix} \\ &= \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{2}{25}t^3 - \frac{3}{5}t^2 \\ \frac{6}{25}t^2 - \frac{6}{5}t \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{2}{25}t^3 - \frac{3}{5}t^2 \\ \frac{6}{25}t^2 - \frac{6}{5}t \end{bmatrix} \end{aligned} \tag{2.31}$$

which is plotted in Figure 2.5.

The optimal solution for $t_i^f = 10$ is also plotted in Figure 2.6. Comparing Figures 2.2 and 2.3 with Figures 2.5 and 2.6, one can observe that the minimum energy control solution provides the desired rendezvous results unlike the former one.

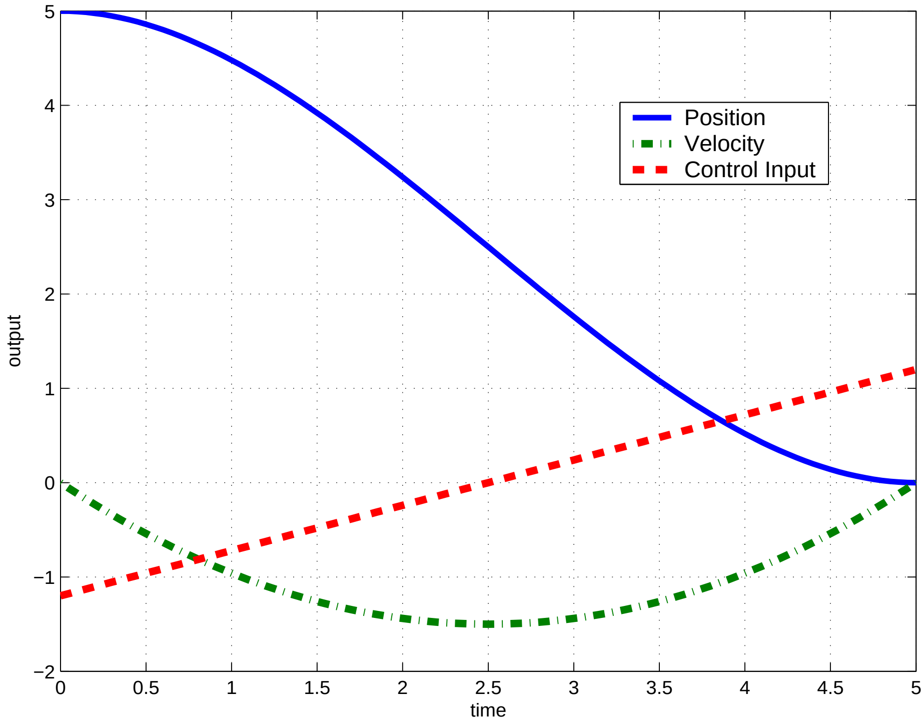


Figure 2.5: Position, velocity and control input for $t_i^0 = 0$, $t_i^f = 5$ in minimum energy control

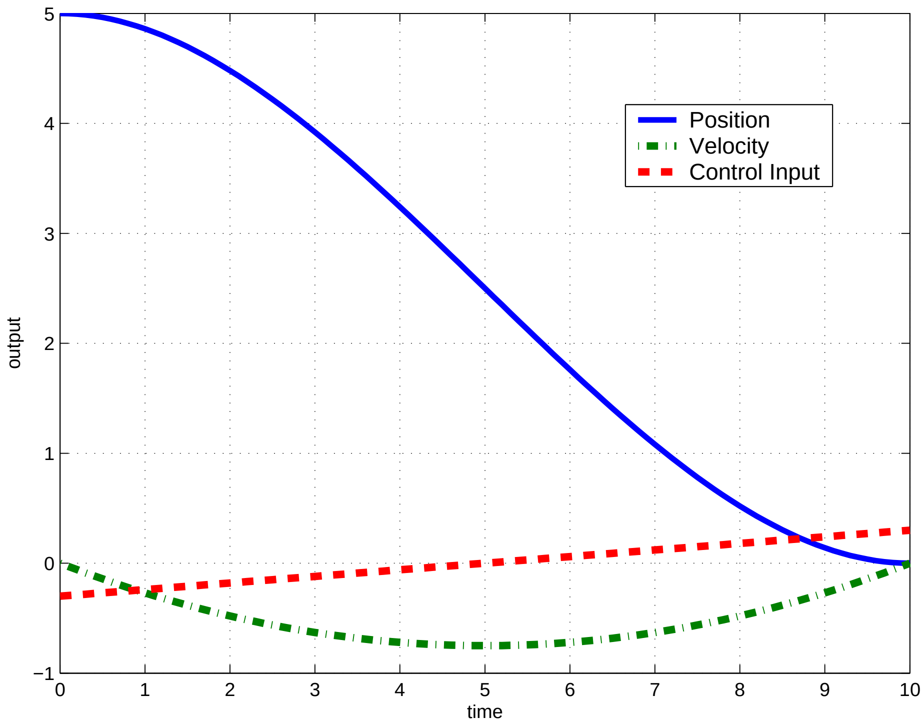


Figure 2.6: Position, velocity and control input for $t_i^0 = 0$, $t_i^f = 10$ in minimum energy control

2.3 Adaptation of the Solution to the Multi-Agent Rendezvous Problem

Up to this point, the solution was for the optimal control of one vehicle, and now the question is: "How can we include other vehicles in the team and the communication between them in order to update the rendezvous instant and get all of the vehicles to be at the target point just at the rendezvous time?" The answer is discussed next.

Remember that the optimal solution of the rendezvous problem is $u_i^*(t)$ which minimizes the cost function in (2.24) with respect to the boundary conditions (i.e. the initial and the final states). Now, suppose that the vehicles communicate at each discrete time instant t_k and the rendezvous time is updated or remain unchanged with respect to the received information. Then, we can solve for $u_i^*(t)$ by accepting x_i^0 as the value of the states at the communication instant and x_i^f as the value of the states at the rendezvous instant.

Knowing the exact values of the states at the communication instant and the required distance to be traveled, it is not so difficult to solve for $t_i^{f_{new}}$ which is the new rendezvous time for vehicle i. In order to determine $t_i^{f_{new}}$ at time t_k, it is assumed that the current optimal control input $u_i^*(t)$ will not change during the rest of the travel. That means the trajectory of the ith vehicle's position will remain the same during the rest of the travel. As seen in (2.31), the trajectory of the position of vehicle i (i.e. $x_{ip}(t)$) is a third order polynomial of t. Similarly, $x_{ip}(t)$ will be a fifth order polynomial of t when we include the acceleration in our model. So, we can express $x_{ip}(t)$ as

$$x_{ip}(t) = a_n t^n + a_{n-1} t^{n-1} + a_{n-2} t^{n-2} + \ldots + a_1 t + a_0 \qquad (2.32)$$

Then, we can express the distance to be traveled by vehicle i as

$$\begin{aligned} x_{ip}^{f} - x_{ip}^{0} &= \Delta x_{ip} \\ &= a_n (t_i^{f_{new}})^n + a_{n-1}(t_i^{f_{new}})^{n-1} + \quad \ldots \quad + a_1 t_i^{f_{new}} - \\ &\quad \big(a_n (t_k)^n + a_{n-1}(t_k)^{n-1} + \quad \ldots \quad + a_1(t_k)\big) \end{aligned} \tag{2.33}$$

in order to determine $t_i^{f_{new}}$. Here, x_{ip}^{0} and x_{ip}^{f} are the positions of vehicle i at the communication instant t_k and the rendezvous time, respectively. We can determine the coefficients $a_n, a_{n-1}, \ldots, a_1$ at the communication instant by using the previous values of x_{ip}. Then, we can solve (2.33) for $t_i^{f_{new}}$ and determine the new rendezvous time for vehicle i.

By repeating this procedure at the communication instants, each vehicle can determine its own expected arrival time, and the rendezvous time can be found accordingly by (2.4) or (2.5). Notice that choosing (2.4) instead of (2.5) is better for the success of the mission since (2.5) means that the vehicles in the team follows the slowest vehicle and this may result in increasing t_i^f values tending to infinity.

A sample solution for the 2-vehicle rendezvous problem below is presented next:

$$\begin{aligned} &\dot{x}_i = A_i x_i(t) + B_{1i} u_i(t) + B_{2i} w_i(t), \\ &t_0 = 0, \quad t_f = 10, \\ &x_1(t_0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, x_1(t_f) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\ &x_2(t_0) = \begin{bmatrix} 15 \\ 0 \end{bmatrix}, x_2(t_f) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \tag{2.34}$$

where $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_{1i} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_{2i} w_i(t)$ is an appropriate random signal accounting for imperfect road conditions or any other reason disturbing the current position of the vehicle i, and vehicles communicate every 2 seconds.

Looking at Figure 2.8 in detail, we can see that the change in the rendezvous times (i.e. t_i^f) starts at $t = 2$ and the control inputs are updated accordingly as obviously seen in Figure 2.7. Notice that same updating process is repeated at instants $t = 4, 6, 8$ until the rendezvous time $t = 8.9062$.

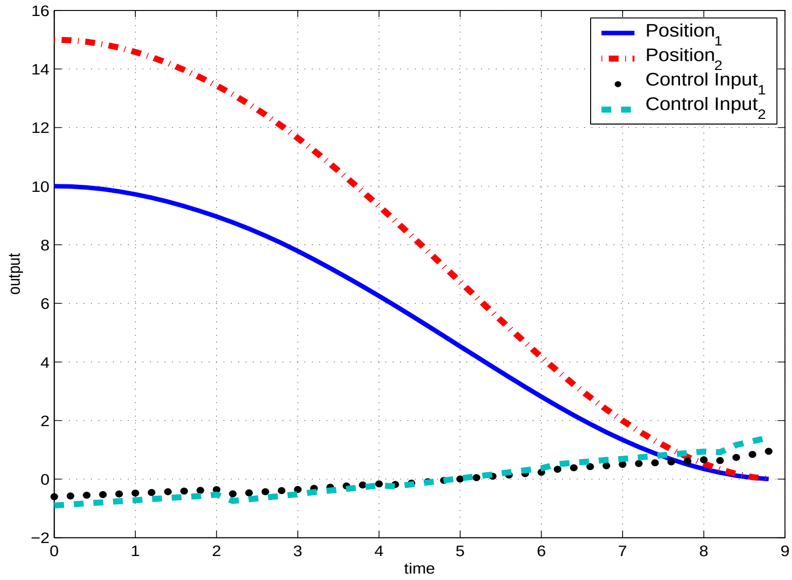


Figure 2.7: Position & control inputs vs. time for $t_i^0 = 0$, $t_i^f = 10$ in minimum energy control

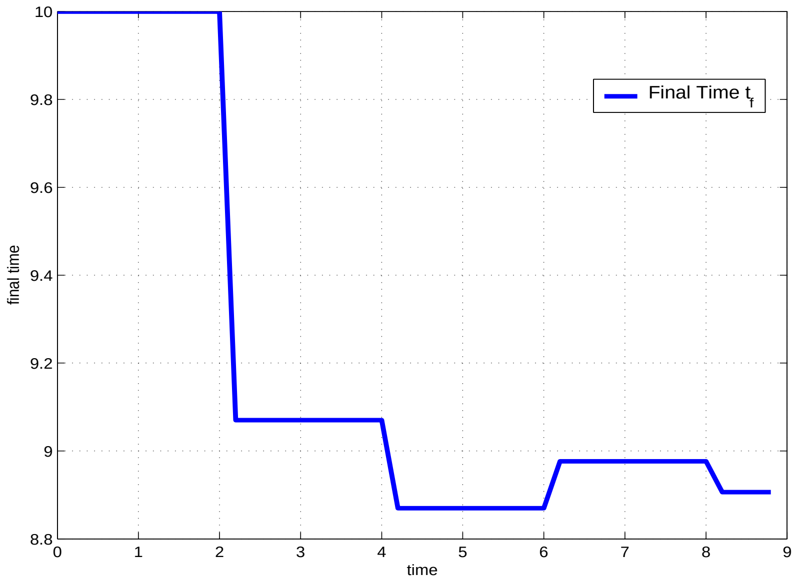


Figure 2.8: Change in t_f at communication instants for $t_i^0 = 0$, $t_i^f = 10$ in minimum energy control

Chapter 3

APPLICATION TO RENDEZVOUS PROBLEMS

In this chapter, we will present application results for the solution derived in Section 2.2.2 for more realistic problems like the multi-vehicle rendezvous problems in 1-D with fixed target (i.e. rendezvous point) and the multi-vehicle rendezvous problems with moving target in 2-D. In addition, the effect of imperfect communication in the form of time delays will be discussed.

3.1 Multi-Agent Rendezvous Problem in 1-D

In this section, the application results for rendezvous problems in 1-D will be given. Moreover, time delays modeling late or lost rendezvous time information will be considered.

3.1.1 Three-Vehicle Rendezvous Problem

Let us consider the following dynamical model for a three-vehicle rendezvous problem:

$$\begin{aligned}
&\dot{x}_i = A_i x_i(t) + B_i(1 + w_i(t))u_i(t), \\
&t_0 = 0, \quad t_f = 20, \\
&x_1(t_0) = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, x_1(t_f) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\
&x_2(t_0) = \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}, x_2(t_f) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\
&x_3(t_0) = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}, x_3(t_f) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
\end{aligned} \tag{3.1}$$

where $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and w_i is an appropriate random signal accounting for imperfect road conditions or any other reason disturbing the current control input of the vehicle i. Vehicles are assumed to exchange information every 4 second and t_i^f is calculated as explained in Section 2.3.

In this model, the vehicles start at rest with zero acceleration and the input $u_i(t)$ is used to control the rate of change in the acceleration. Furthermore, the disturbances caused by the imperfect conditions (obstacles on the road, internal problems of the agents or other uncertainties) are included in the model as negative or positive effects in the control input. These disturbances are handled by $w_i(t)$ as random changes of $\pm$ 2 % in the optimal control input $u_i^*(t)$ during the travel.

For this problem, W_c and $u_i^*(t)$ can be calculated at each communication instant as indicated below. Note that, the formula for W_c and $u_i^*(t)$ are valid until the next communication instant and are updated upon the next final time information.

$$W_c = \begin{bmatrix} (\frac{1}{2}t_0^2 - t_0 t + \frac{1}{2}t^2)^2 & (\frac{1}{2}t_0^2 - t_0 t + \frac{1}{2}t^2)(t_0 - t) & (\frac{1}{2}t_0^2 - t_0 t + \frac{1}{2}t^2 \\ (\frac{1}{2}t_0^2 - t_0 t + \frac{1}{2}t^2(t_0 - t)) & (t_0 - t)^2 & (t_0 - t) \\ (\frac{1}{2}t_0^2 - t_0 t + \frac{1}{2}t^2) & (t_0 - t) & 1 \end{bmatrix}$$

$$u_i(t) = -B_1^T e^{A_i^T (t_0 - t)} W_c^{-1} (x_i^0 - e^{A_i (t_i^0 - t)} x_i^f) \tag{3.2}$$

The simulation results for that problem are shown in Figures 3.1, 3.2 and 3.3. Recall that t_f is determined as explained in Section 2.3. As seen in the figures, the control inputs are updated when final time is updated, and the vehicles reach the rendezvous point simultaneously.

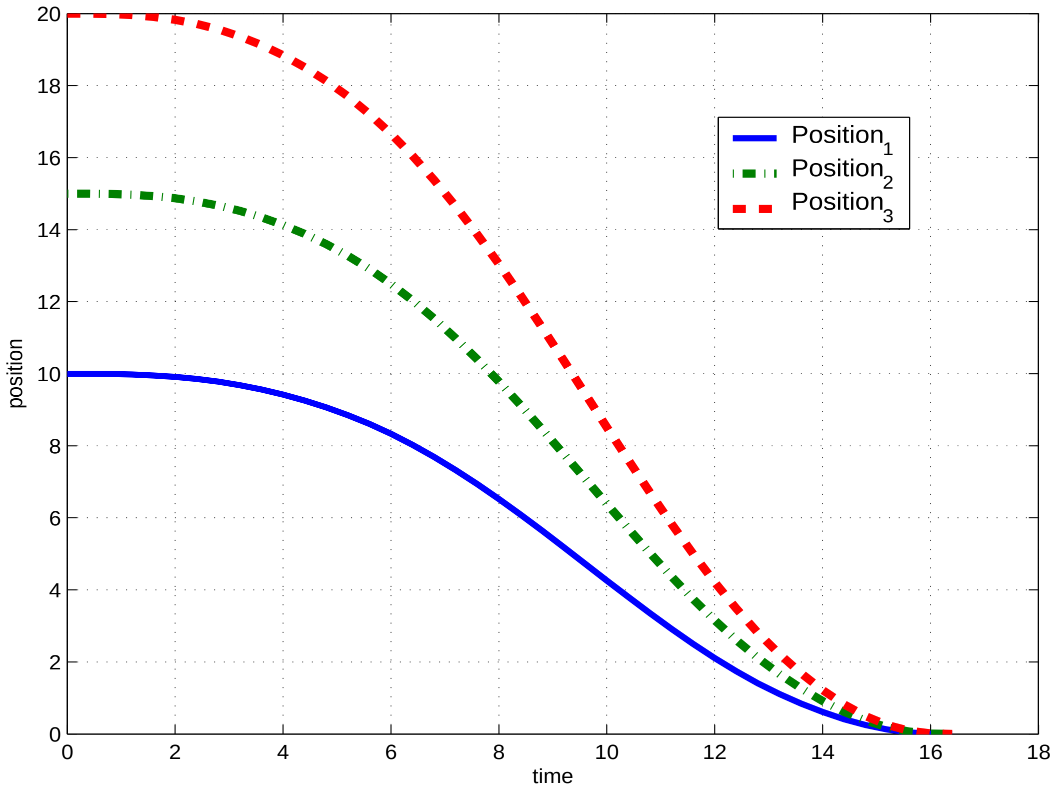


Figure 3.1: Positions vs. time for rendezvous in 1-D without communication problems

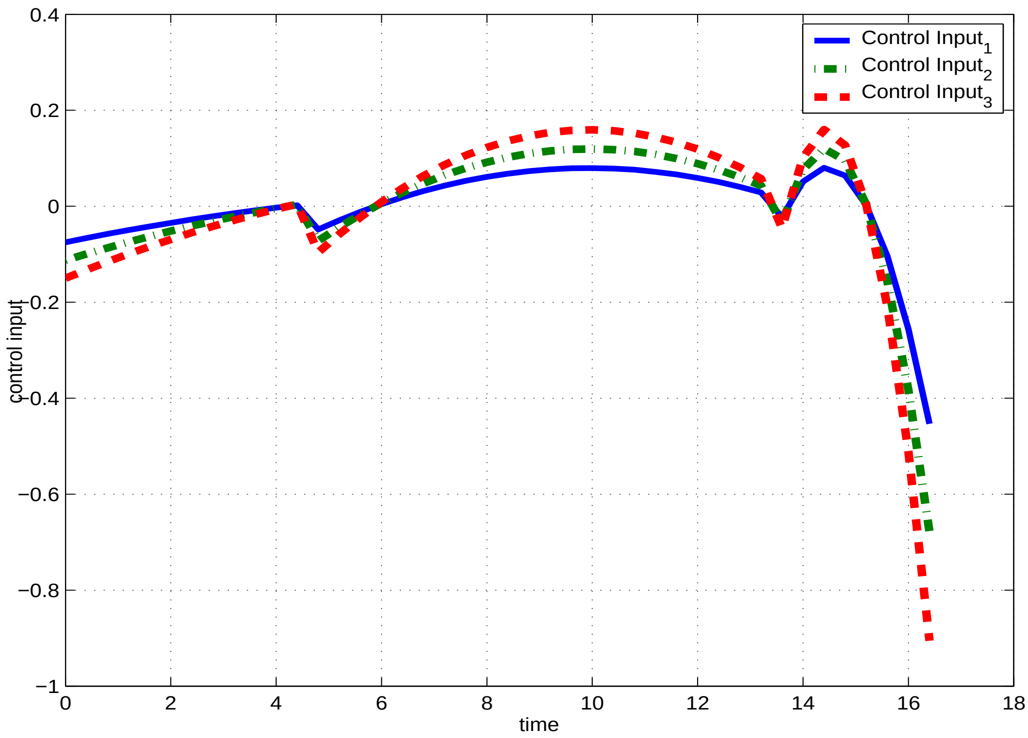


Figure 3.2: Control inputs vs. time for rendezvous in 1-D without communication problems

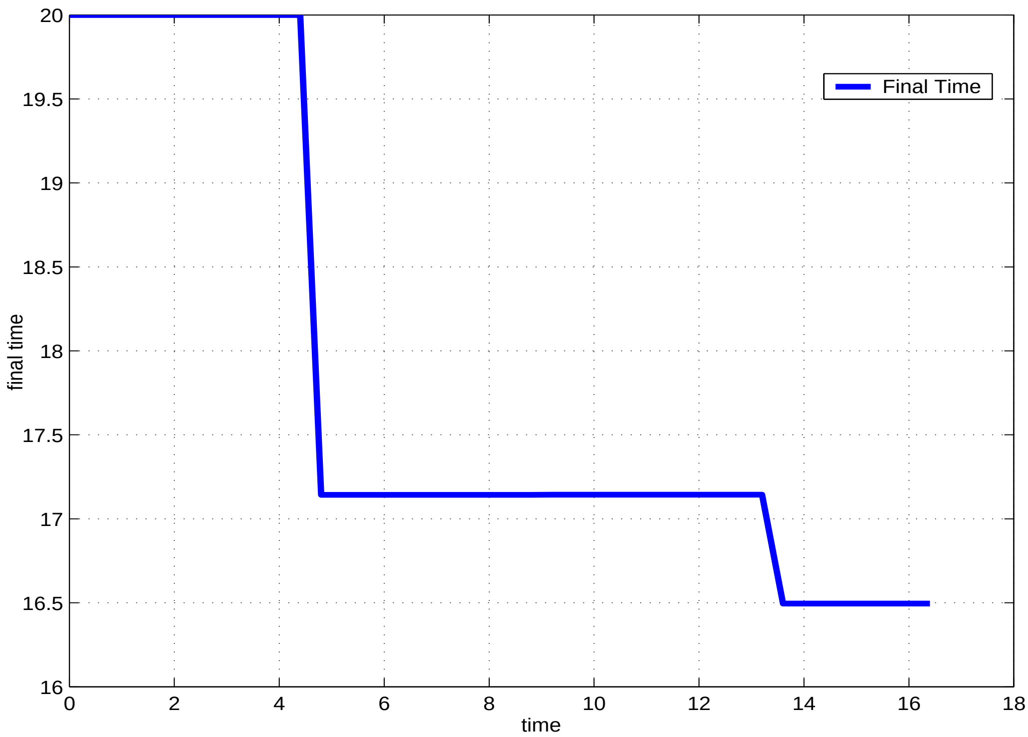


Figure 3.3: Change in t_f at communication instants for rendezvous in 1-D without communication problems

3.1.2 Three-Vehicle Rendezvous Problem in the Presence of Communication Problems

Remember that the deficiencies in the information exchange were not considered in Section 3.1.1. Next, the communication problems are included in the problem statement in order to discuss the effects of late or missed information about the rendezvous times occurring during the task.

Let us begin with a simple but illustrative case assuming that the vehicles in the task force inform a central unit about their own final time, i.e. rendezvous instant, during the operation, the central unit determines the overall final time by finding the minimum of the final times, and informs all the vehicles in the task force back about the new rendezvous instant to reach the target point. In this case, it is assumed that the delays occur only in the information signals from the vehicles to the central unit, and not the other way around for simplicity.

In order to model the lost or late signals, it is reasonable for the central unit to specify a maximum delay h_{max} and wait for all the information to be gathered. If all the information comes before the maximum waiting time, then the central unit calculates the rendezvous time and informs all the vehicles. However, if there are still some missing information even though the maximum waiting time limit is reached, then the central unit calculates the rendezvous time assuming that the previous successful information is still valid for the vehicles that could not send their final time information. The reason for such an evaluation is simply the concern about the success of the mission, that even one of the vehicles cannot be at the rendezvous point on time, let the others be.

Thus, we have the following formulation for the final time t_f

$$
\begin{aligned}
t^a_{if}(k) &= \begin{cases} t_{if}(k) & \text{if } h_{ik} \le h_{max} \\ t_{if}(k-1) & \text{if } h_{ik} > h_{max} \end{cases} \\
t_f(k) &= \min\{t^a_{1f}(k), t^a_{2f}(k), \ldots, t^a_{Nf}(k)\}
\end{aligned}
\tag{3.3}
$$

where

$t_f(k)$: calculated final time at communication instant t_k

$t^a_{if}(k)$: assumed final time for vehicle i in the presence of time delay

$t_{if}(k)$: final time for vehicle i to be transmitted to the central unit in order to determine $t^a_{if}(k)$

h_{ik} : communication delay from vehicle i to the central unit

h_{max} : maximum delay limit applied by the central unit after which final time is calculated and transmitted to the all vehicles in the task force

Note that h_{max} is chosen as 1 second and h_{ik}s are modeled as

$$h_{ik} \sim |N(0, 0.9h_{max})| \tag{3.4}$$

i.e. we have normally distributed positive time delay values. Below are the simulation results of the above configuration.

Instant(k)	1	2	3	4
Vehicle 1	19.9982	16.0275	16.0283	167.3708
Vehicle 2	14.0863	19.9977	16.0280	16.0277
Vehicle 3	20.0015	37.8605	16.0277	16.0277

Table 3.1: Calculated Final Time t_f w.r.t. the communication instant

Instant(k)	1	2	3	4
Final time (sec)	14.0863	16.0275	16.0277	16.0277

Table 3.2: Final Times in the absence of time delays h_{ik}s

By looking at Table 3.1 and Table 3.2, we can see that the t_{if} information is processed at each communication instant and the t_f is determined by taking the minimum of these t_{if}s. Since the effect of delay is not considered, there is no deviation from the calculated values for t_f. However, this is not the case when the communication delay is taken into account as seen next.

Instant(k)	1	2	3	4
Vehicle 1	1.7243	0.0321	0.2711	0.8425
Vehicle 2	2.3702	1.0971	0.4884	0.0759
Vehicle 3	1.4989	0.6383	1.4527	0.7019

Table 3.3: Communication delays h_{ik} in sec. for corresponding instants k

In Table 3.3, we can see that some of the time delays h_{ik} are greater than the threshold value h_{max}. That means the central unit will not be able to evaluate the t_{if} information coming from the corresponding vehicle and that will result in a deviation from the calculated t_f values in the absence of delay.

Instant(k)	1	2	3	4
Final time (without delay)	14.0863	16.0275	16.0277	16.0277
Final time (with delay)	20.0000	16.0275	16.0275	16.0275

Table 3.4: Final Times in the presence of time delays h_{ik}s

Looking at Table 3.1 and Table 3.4, we can see that at the first communication instant (i.e. $k = 1$) the final time t_f should be 14.0863 since it is the minimum of the t_{if}s at that instant. However, as seen in Table 3.3, the time delays from the vehicles to the central unit are greater than h_{max} and for that reason the central unit regards t_{if}s as they did not change w.r.t. their previous value, and accepts them as 20.0000 ($t^a_{if}(k)$). Therefore, t_f is determined as $\min\{20.0000, 20.0000, 20.0000\} = 20$ rather than $\min\{19.9982, 14.0863, 20.0015\} = 14.0863$.

Similarly, the time delay h_{2k} for vehicle 2 is greater than h_{max} in the second communication instant. Even though this alters the flow of the algorithm, the final time result $t_f(k = 2)$ is not affected since $\min\{16.0275, 19.9977, 37.8605\}$ and $\min\{16.0275, 20.0000, 37.8605\}$ give the same result as 16.0275.

In the last communication instant, time delay from vehicle 3 to the central unit is greater than h_{max} as seen in Table 3.3. Therefore, $t_i^f(k = 3)$ is regarded as 16.0275, which is the previous value of it, and $t_f(k = 3)$ is determined as $\min\{16.0283, 16.0280, 16.0275\} = 16.0275$ rather than $\min\{16.0283, 16.0280, 16.0277\} = 16.0277$.

Now, let us take a look at the simulation results which are plotted in Figures 3.4 - 3.8

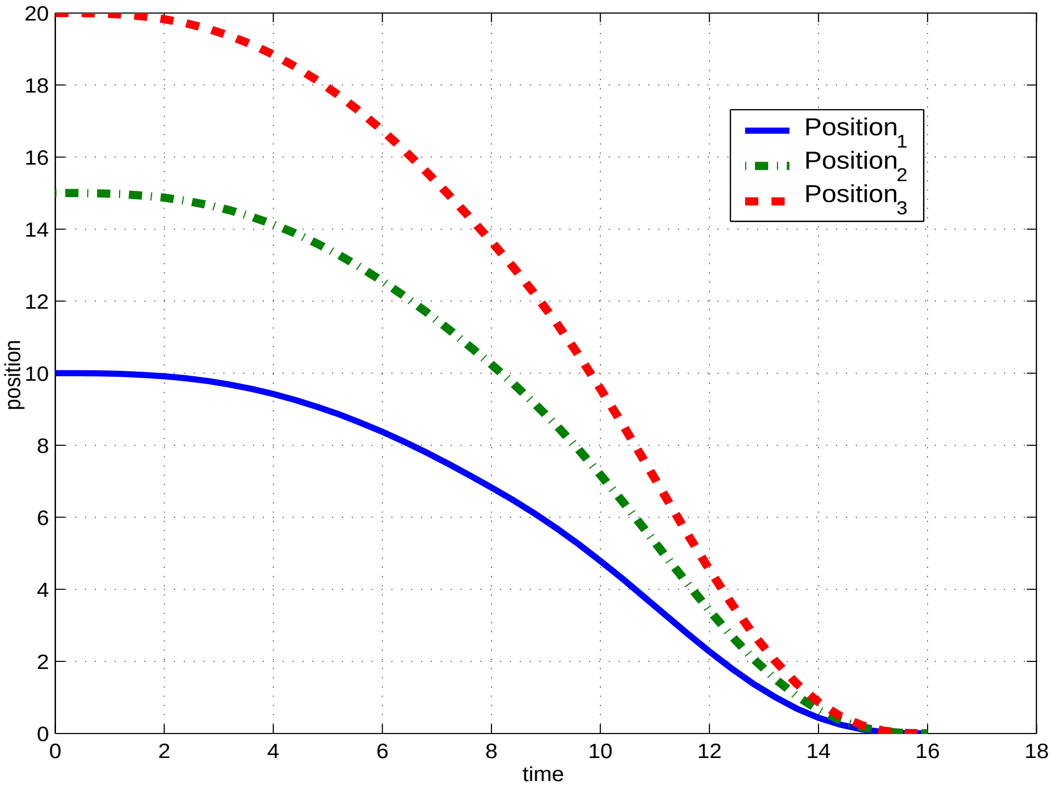


Figure 3.4: Positions vs. time for rendezvous in 1-D with one-way time delays in communication

As seen in Figure 3.4, all of the 3 vehicles in the task force reach the rendezvous point at the same time, which means that the mission is completed successfully. Now, let us look at the trajectories of the other states (i.e. velocity and acceleration), the control input and the rendezvous time.

Looking at Figures 3.5-3.8, it can be observed that the only change in the final time, consequently the control input, the acceleration and the velocity, occurs just after $t = 8$ (to be precise $t = 8.0321$). That time instant is evidently $t(k = 2) + h_{12}$, the instant when the final time information of vehicle 1 is obtained.

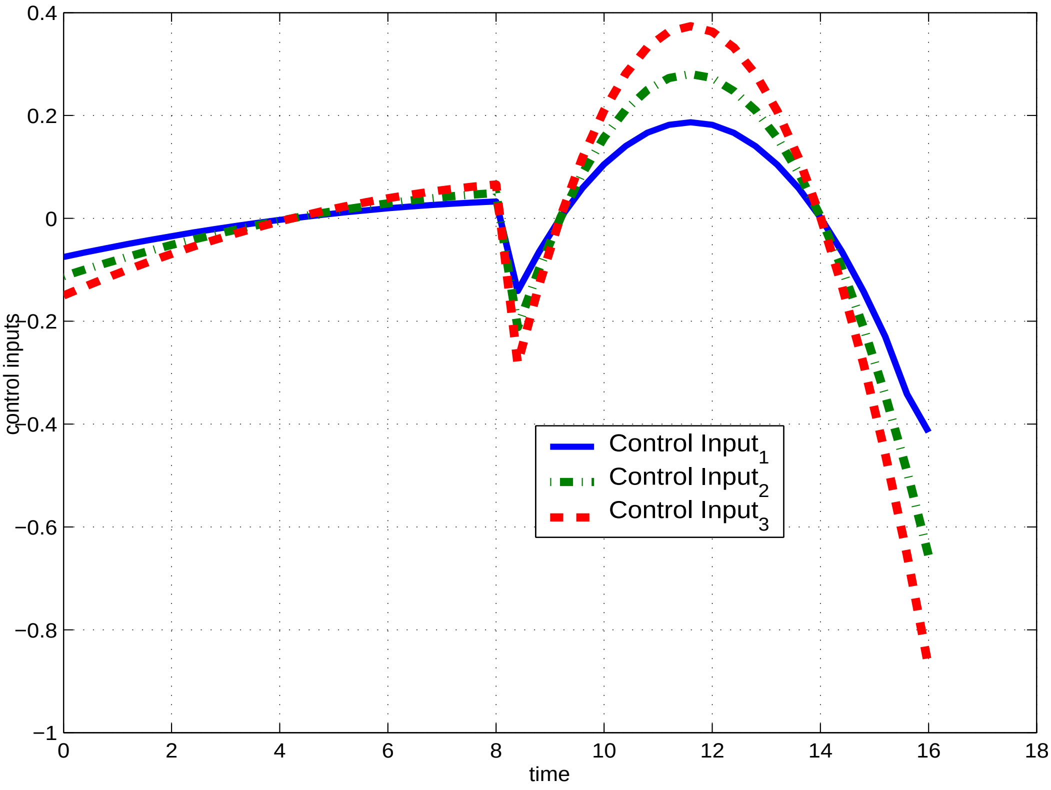


Figure 3.5: Control inputs vs. time for rendezvous in 1-D with one-way time delays in communication

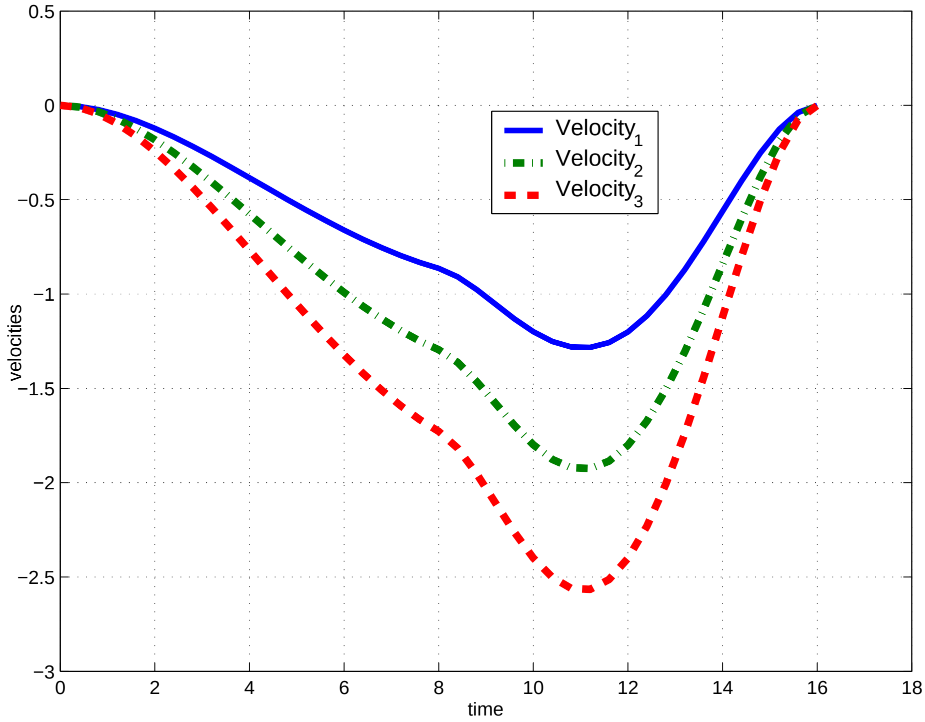


Figure 3.6: Velocities vs. time for rendezvous in 1-D with one-way time delays in communication

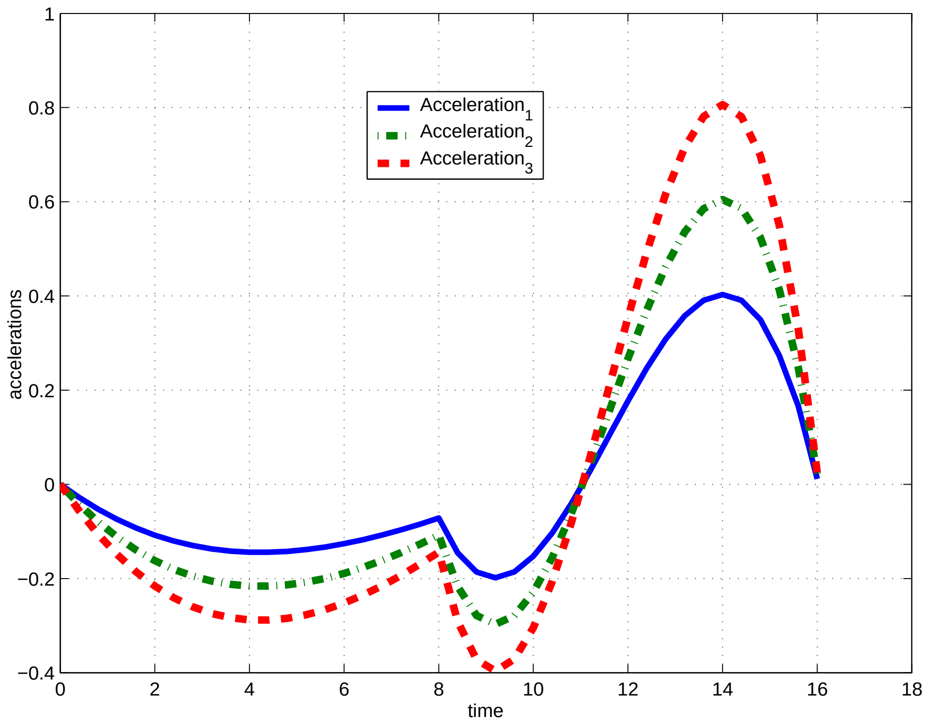


Figure 3.7: Accelerations vs. time for rendezvous in 1-D with one-way time delays in communication

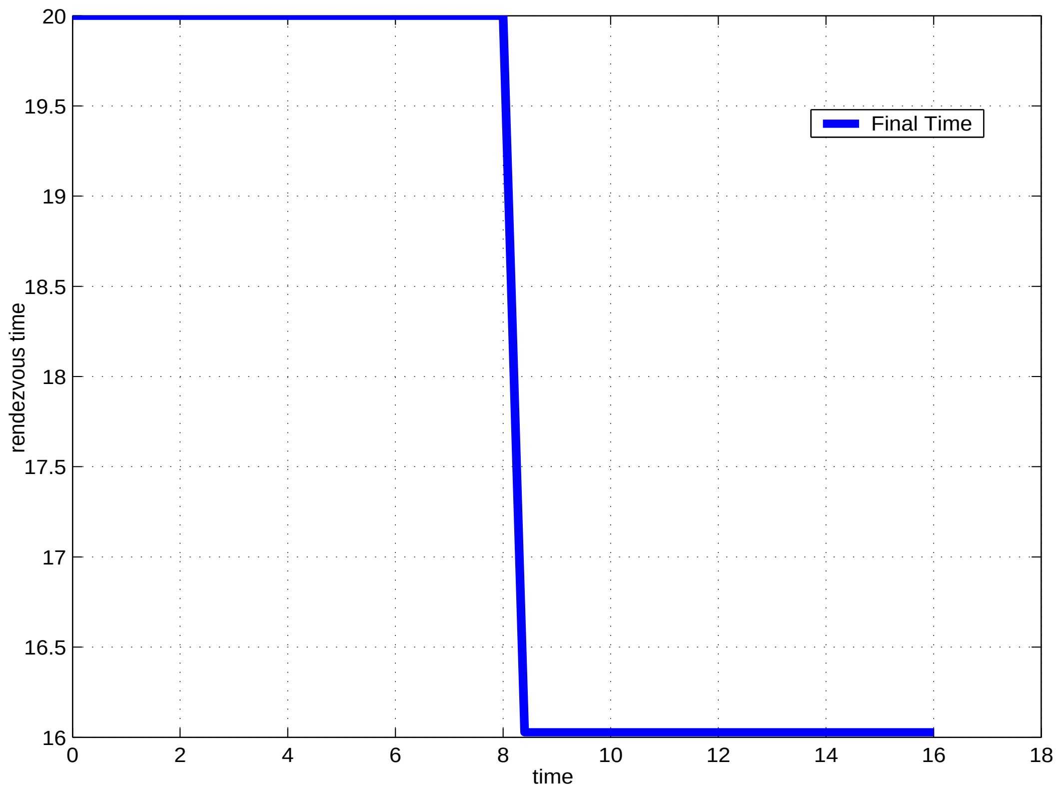


Figure 3.8: Change in t_i^fs at communication instants for rendezvous in 1-D with one-way time delays in communication

Notice that, previously solved problem configuration is an illustrative example in order to see the flow of the algorithm in the case of time delays in the communication, and it should be improved in order to be much more realistic and applicable. Therefore, it is more reasonable to assume that the time delays are not just between the vehicles and the central unit, but rather between the vehicles. In other words, the vehicles in the task force send their final time information not to the central unit but to each other, and each vehicle determines its individual final time accordingly. So, if the time delay h_{ij} from vehicle i to vehicle j is greater than the acceptable limit whereas the time delay h_{ji} from vehicle j to vehicle i is less than or equal to the limit, vehicle j cannot utilize the information coming from vehicle i while vehicle i can use the information sent from vehicle j. For that reason, the final time for each vehicle can be different from the final times of the other vehicles and that can cause some deviations from the expected results.

Then, we have the following formulation for the final time $t_i^f(k)$

$$
\begin{aligned}
t_{if}^a(k) &= \begin{cases} t_{if}(k) & \text{if } h_{ij}(k) \leq h_{max} \\ t_{if}(k-1) & \text{if } h_{ij}(k) > h_{max} \end{cases} \\
t_i^f(k) &= \min\{t_{1f}^a(k), t_{2f}^a(k), \ldots, t_{Nf}^a(k)
\end{aligned} \tag{3.5}
$$

where

$t_i^f(k)$: final time to be used by vehicle i at t_k
(determined upon receiving the information from the other vehicles)

$t_{if}(k)$: calculated final time of vehicle i at t_k

$t_{if}^a(k)$: assumed final time for vehicle i due to the time delay

$h_{ij}(k)$: communication delay from vehicle i to vehicle j at t_k

h_{max} : maximum delay limit applied by each vehicle after which corresponding final time is calculated and used for completing the task

Note that h_{max} is chosen as 1 second and $h_{ij}(k)$s are modeled as in 3.4. The simulation results of the explained configuration are presented next.

Instant(k)	1	2	3
Vehicle 1	20.0015	19.9960	22.9341
Vehicle 2	19.9985	19.9973	16.8726
Vehicle 3	20.0015	16.8714	15.3113

Table 3.5: Calculated Final Times $t_i^f(k)$ w.r.t. the communication instant

Instant(k)	1	2	3
Final time (sec)	19.9985	16.8714	15.3113

Table 3.6: Final Times in the absence of time delays $h_{ij}(k)$s

By looking at Table 3.5 and Table 3.6, we can see that the $t_{if}(k)$ information would be processed at each communication instant and the $t_i^f(k)$ would be determined by taking the minimum these $t_i^f(k)$s in the absence of time delays. However, as seen next, time delays between the vehicles will change the flow of the algorithm and the result.

$$h(1) = \begin{bmatrix} 0 & 0.2423 & 0.5864 \\ 0.5430 & 0 & 0.0178 \\ 0.6499 & 0.7575 & 0 \end{bmatrix}$$

$$h(2) = \begin{bmatrix} 0 & 0.7216 & 0.2115 \\ 0.6973 & 0 & 0.4059 \\ 0.7724 & 0.5591 & 0 \end{bmatrix}$$

$$h(3) = \begin{bmatrix} 0 & 0.3521 & 1.0979 \\ 0.3825 & 0 & 0.3672 \\ 1.8155 & 0.3548 & 0 \end{bmatrix}$$

Communication delays $h_{ij}(k)$ for corresponding instants k are seen above $(ij)^{th}$ entry of the corresponding 3x3 matrix represents the time delay from vehicle i to vehicle j). We can observe that some of the time delays h_{ij} are greater than the threshold value $h_{max} = 1$ sec. For instance, the time delay from vehicle 3 to vehicle 1 (i.e. $h_{13}(3)$) at third communication is 1.0979 and the time delay from vehicle 1 to vehicle 3 (i.e. $h_{31}(3)$) at the same instant is 1.8155. That means vehicle 1 will not be able to evaluate the final time information coming from vehicle 3 and vice a versa. This will result in a deviation from the final time values calculated in the absence of delay.

Instant(k)	1	2	3
Vehicle 1	19.9985	16.8714	16.8714
Vehicle 2	19.9985	16.8714	15.3113
Vehicle 3	19.9985	16.8714	15.3113

Table 3.7: Final Times in the presence of time delays $h_{ij}(k)$s

Looking at Table 3.5 and Table 3.7, we can see that at third communication instant (i.e. $k = 3$) the final time $t_f(k = 3)$ should be 15.3113 since it is the minimum of the t_{if}s $\{22.9341, 16.8726, 15.3113\}$. However, the time delays $h_{13}(3)$ and $h_{31}(3)$ are are greater than h_{max} and for that reason vehicles 1 and 3 determine their final times as $t_1^f = \min\{22.9341, 16.8726, 16.8714\} = 16.8714$ and $t_3^f = \min\{19.9960, 16.8726, 15.3113\} = 15.3113$ by taking the previous final time informations of each other into account rather than the current ones. Therefore, vehicle 1 cannot be at the rendezvous point on time, whereas vehicles 2 and 3 can.

The simulation results are depicted in Figures 3.9-3.13. As seen in Figure 3.9, vehicles 2 and 3 meet at the rendezvous point on time whereas vehicle 1 is away from that point. Although that means the mission is not completed successfully, it can be said that the result is still satisfactory since vehicle 1 is very close to the rendezvous point.

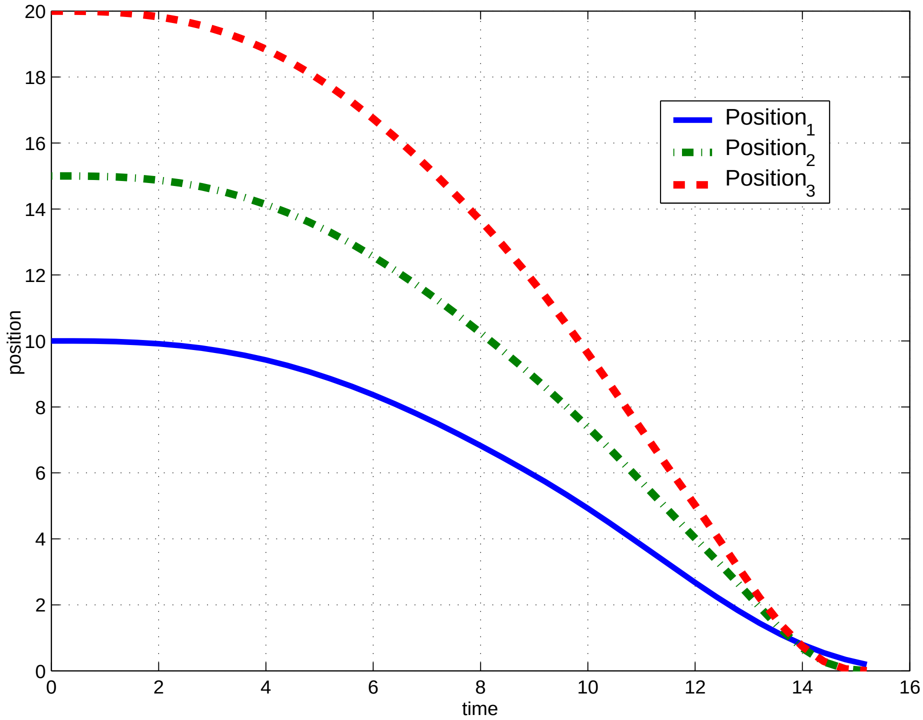


Figure 3.9: Positions vs. time for rendezvous in 1-D including communication problems

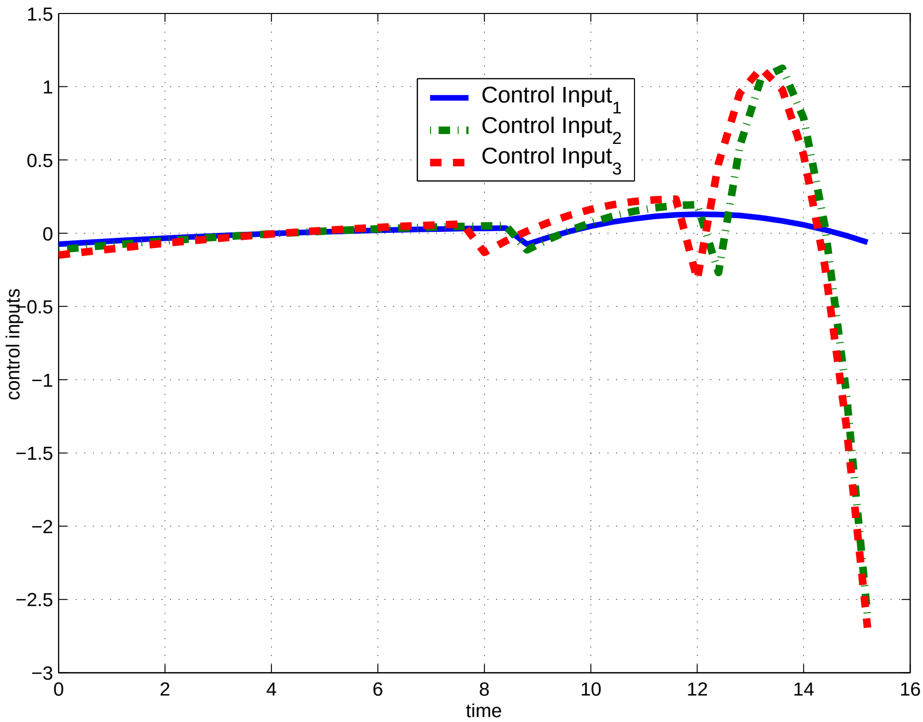


Figure 3.10: Control inputs vs. time for rendezvous in 1-D including communication problems

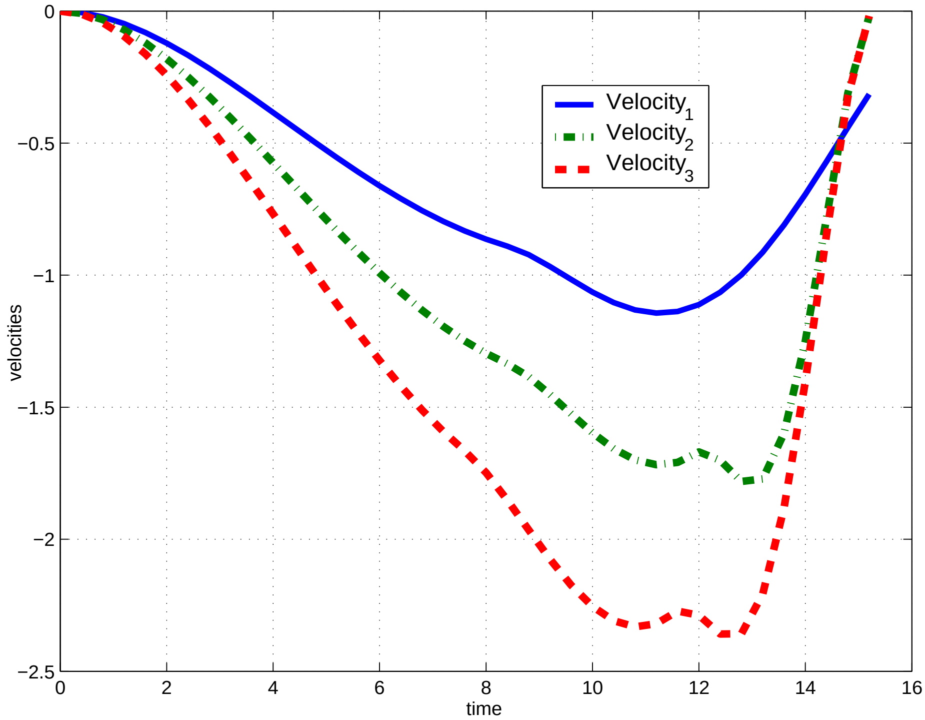


Figure 3.11: Velocities vs. time for rendezvous in 1-D including communication problems

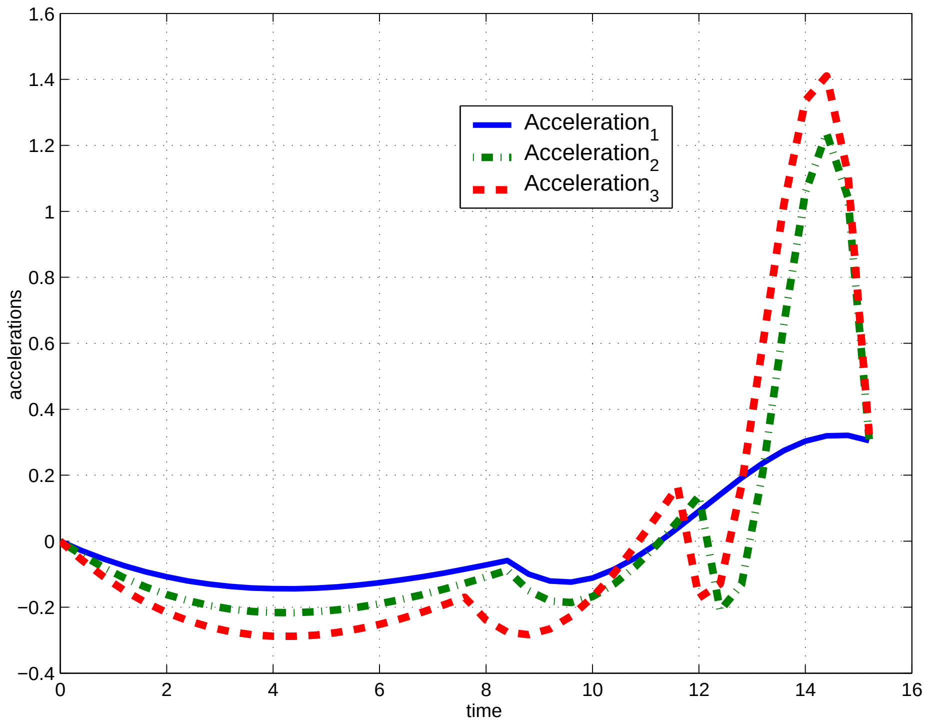


Figure 3.12: Accelerations vs. time for rendezvous in 1-D including communication problems

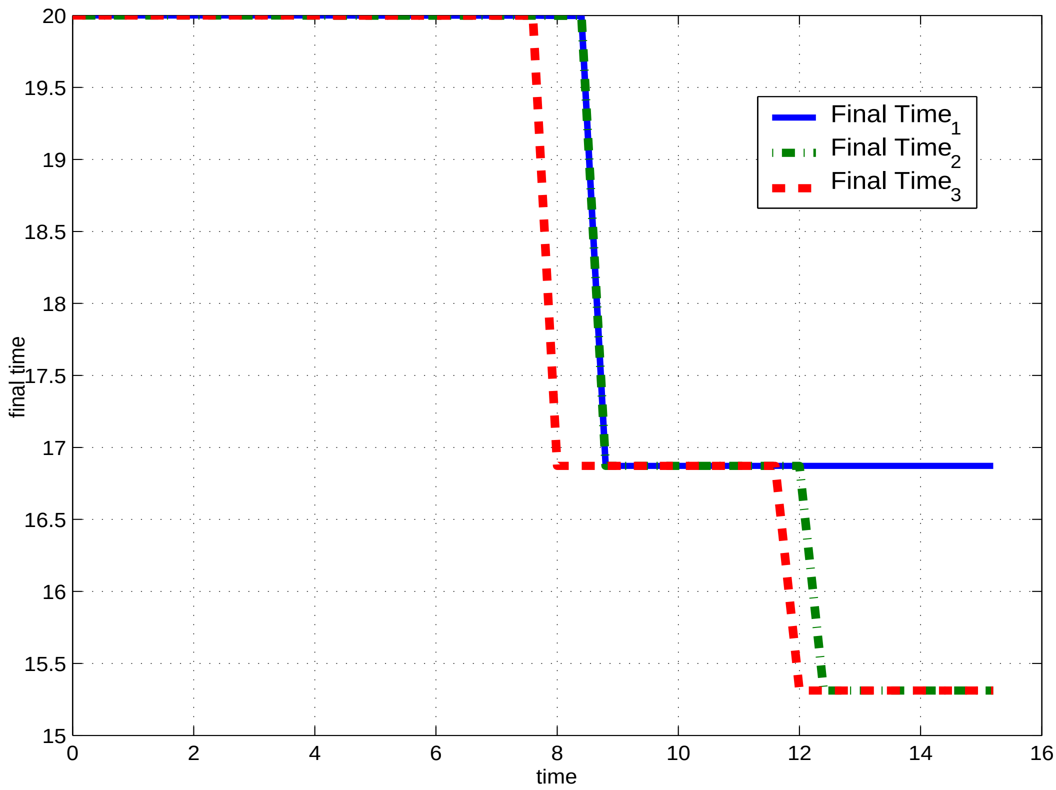


Figure 3.13: Change in t_i^fs at communication instants for rendezvous in 1-D including communication problems

In Figures 3.14-3.18, results of another simulation is presented. Although the problem configuration is same as before, the effect of random disturbance $w_i(t)$ is different. As seen in the plots, the vehicles reach the rendezvous point just at the same time, indicating that the mission is accomplished.

Instant(k)	1	2	3
Vehicle 1	20.0047	14.1826	14.1847
Vehicle 2	20.0046	19.9969	14.1848
Vehicle 3	20.0042	19.9969	14.1851

Table 3.8: Calculated Final Times $t_i^f(k)$ w.r.t. the communication instant

Instant(k)	1	2	3
Final time (sec)	20.0042	14.1826	14.1847

Table 3.9: Final Times in the absence of time delays $h_{ij}(k)$s

By looking at Table 3.5 and Table 3.6, we can see that the $t_{if}(k)$ information would be processed at each communication instant and the $t_i^f(k)$ would be determined by taking the minimum these $t_{if}(k)$s in the absence of time delays. However, as seen next, time delays between the vehicles will change the flow of the algorithm but not the results.

$$h(1) = \begin{bmatrix} 0 & 0.1265 & 0.7810 \\ 0.5896 & 0 & 0.8066 \\ 0.2932 & 0.5562 & 0 \end{bmatrix}$$

$$h(2) = \begin{bmatrix} 0 & 0.2583 & 0.7373 \\ 0.5576 & 0 & 1.0757 \\ 1.8981 & 0.2728 & 0 \end{bmatrix}$$

$$h(3) = \begin{bmatrix} 0 & 0.9126 & 0.2327 \\ 0.0312 & 0 & 0.5483 \\ 0.4585 & 0.9214 & 0 \end{bmatrix}$$

Communication delays $h_{ij}(k)$ for corresponding instants k are seen above $(ij)^{th}$ entry of the corresponding 3x3 matrix represents the time delay from vehicle i to vehicle j). We can observe that some of the time delays h_{ij} are greater than the threshold value h_{max}. However, this does not change the calculation results for $t_i^f(k)$s as seen in Table 3.10.

Instant(k)	1	2	3
Vehicle 1	20.0042	14.1826	14.1847
Vehicle 2	20.0042	14.1826	14.1847
Vehicle 3	20.0042	14.1826	14.1847

Table 3.10: Final Times in the presence of time delays $h_{ij}(k)$s

The plots of the simulation are presented next.

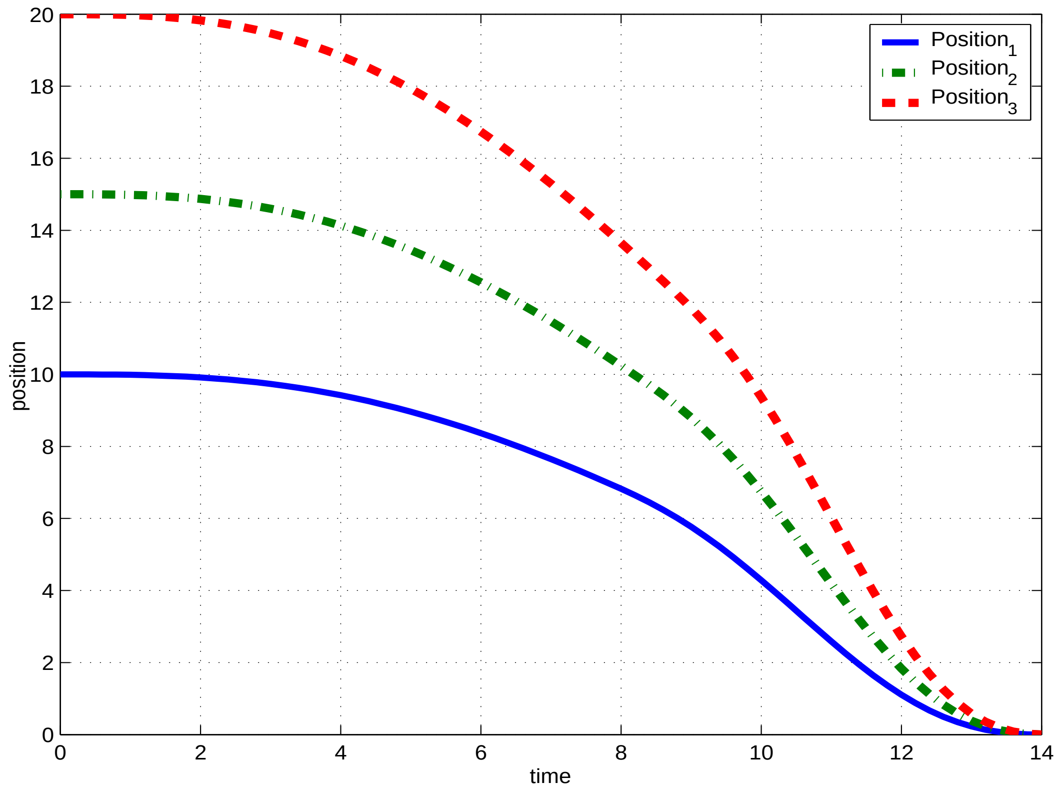


Figure 3.14: Positions vs. time for rendezvous in 1-D including communication problems

As seen above, all of the vehicles meet at the rendezvous point on time which means that the mission is completed successfully.

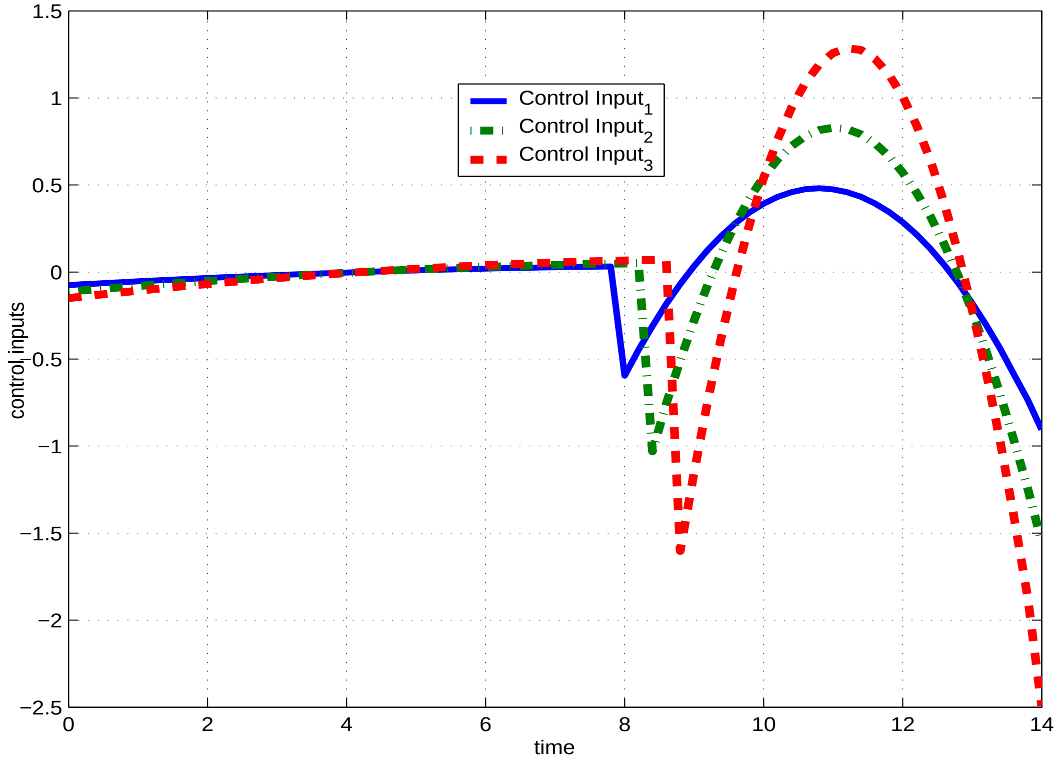


Figure 3.15: Control inputs vs. time for rendezvous in 1-D including communication problems

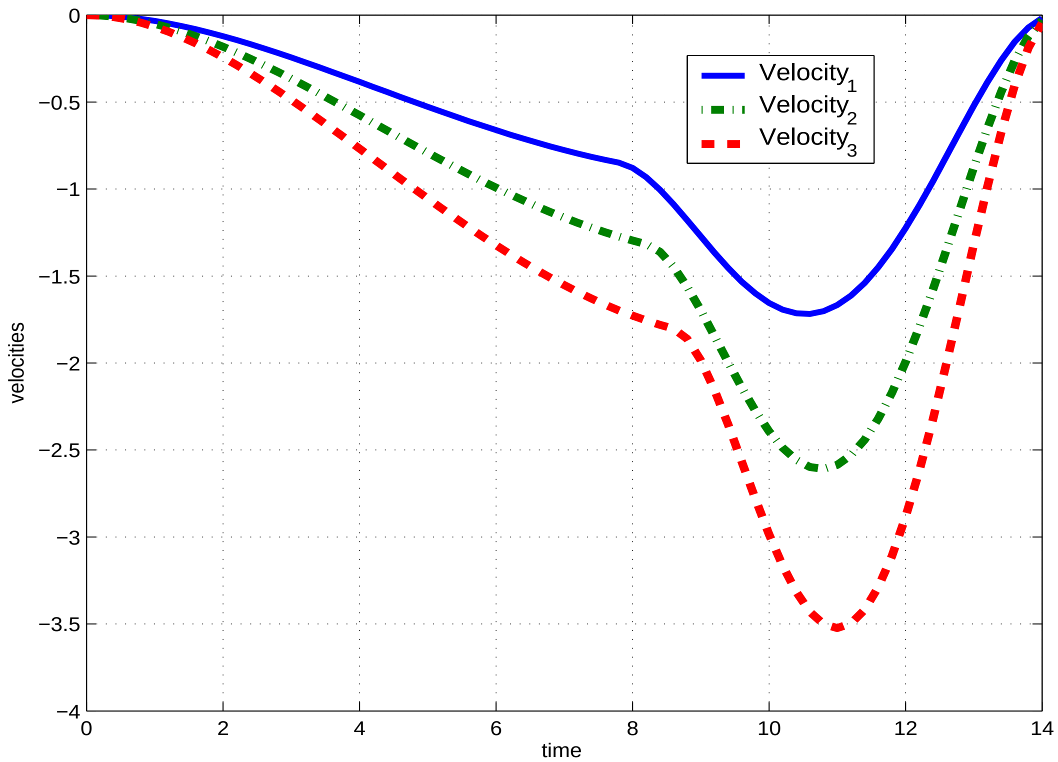


Figure 3.16: Velocities vs. time for rendezvous in 1-D including communication problems

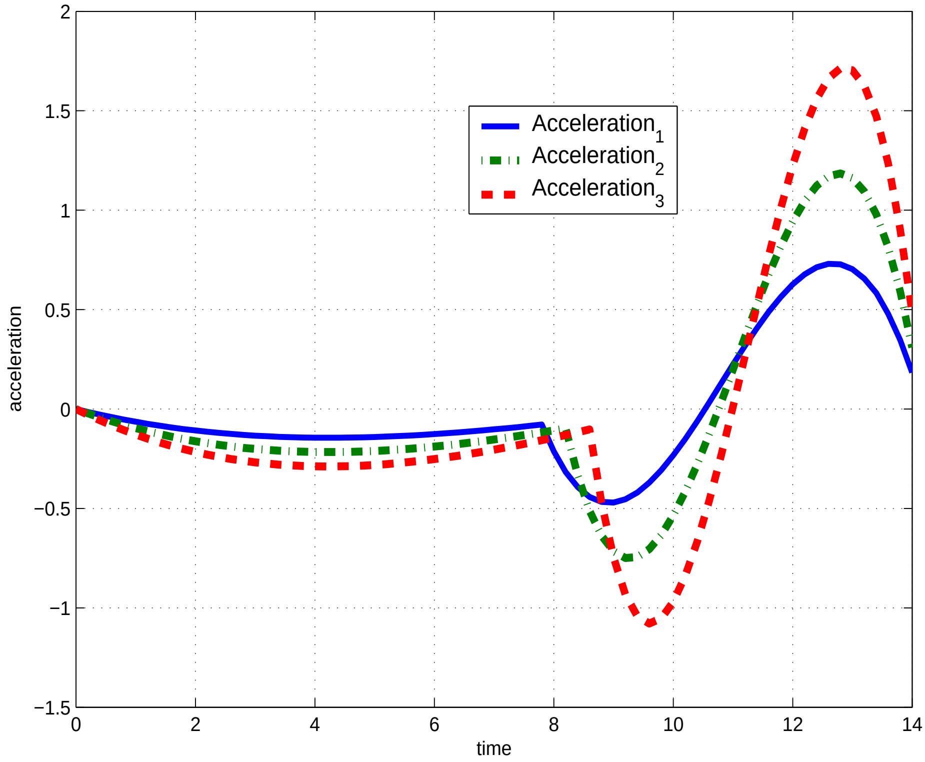


Figure 3.17: Accelerations vs. time for rendezvous in 1-D including communication problems

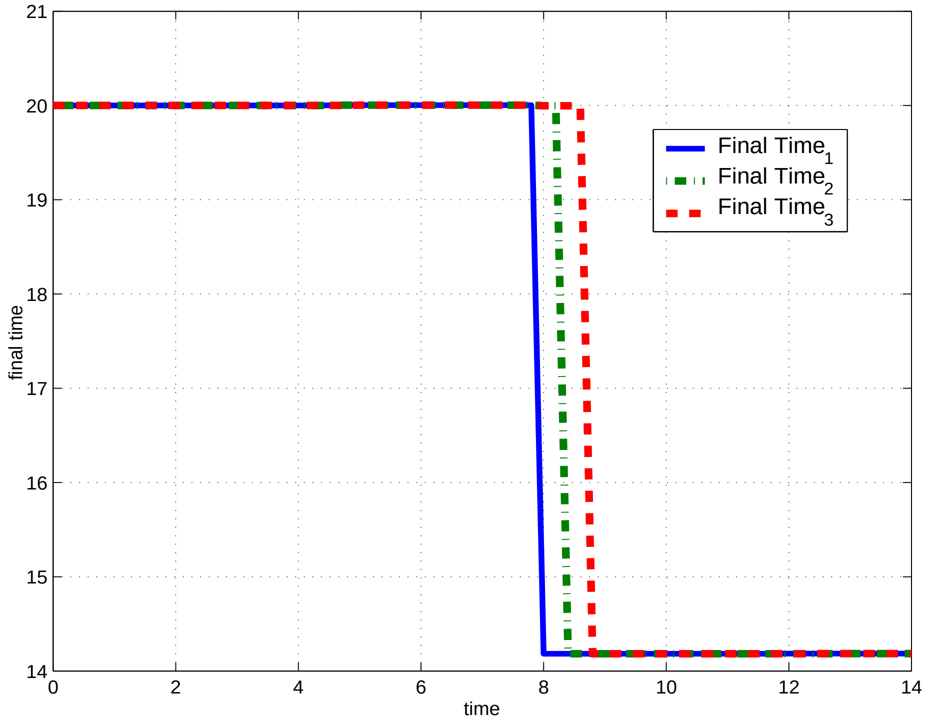


Figure 3.18: Change in t_i^fs at communication instants for rendezvous in 1-D including communication problems

3.2 Multi-Agent Rendezvous Problem in 2-D

Naturally, a 2-D solution for the multi-agent rendezvous problem would be much more realistic than a 1-D solution for practical applications. Having obtained reliable solutions for the problem in 1-D, it is not so difficult to extend them to 2-D. The rendezvous problem in 2-D is divided into subproblems in 1-D on x and y axes, and they are solved as separate rendezvous problems in 1-D. Thus, at every communication instant we have two different expected final times for each vehicle as final times for x and y components of the states. Then, the minimum of the two is chosen as the expected arrival time for the corresponding vehicle and we proceed as explained in Section 2.3 to obtain the solution.

Next, the solution results of the 2-D multi-agent rendezvous problem for fixed and moving target points are presented.

3.2.1 2-D Rendezvous Problem with Fixed Target

Let us consider the dynamic model below for the rendezvous problem in 2-D.

$$\begin{aligned} \dot{x}_i &= A_i x_i(t) + B_i \left((\begin{bmatrix} 1 & 1 \end{bmatrix} + w_i(t)) \cdot u_i(t) \right), \\ t_0 &= 0, \quad t_f = 10 \end{aligned} \tag{3.6}$$

where $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and w_i is an appropriate random signal accounting for imperfect road conditions or any other reason disturbing the current control input of vehicle i. Here, $x_i(t)$ and $u_i(t)$ are 3×2 and 1×2 matrices, respectively and "$\cdot$" denotes the element wise multiplication. It is assumed that w_i is in the form of $\begin{bmatrix} w_{ix} & w_{iy} \end{bmatrix}$ and causes random changes of $\pm$ 2 % in x and y components of the optimal control input $u_i^*(t)$. Vehicles are assumed to exchange information every 2 second and t_i^f is calculated as explained in Section 2.3 without considering time delays in the communication. Initial and final states are

$$x_1(t_0) = \begin{bmatrix} 10 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, x_1(t_f) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$x_2(t_0) = \begin{bmatrix} 15 & 10 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, x_2(t_f) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$x_3(t_0) = \begin{bmatrix} 20 & 15 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, x_3(t_f) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{3.7}$$

Recall that we found an *optimal* solution for the multi-agent rendezvous problem. Therefore, the solution of the problem in 2-D indicates an expected result that the trajectories of the vehicles aiming to reach the target point are basically straight lines between them and the target point as seen in Figure 3.19. Actually, this is a natural consequence of the optimality constraint in the problem that the *minimum energy path* from a vehicle to the target point should be a straight line avoiding any extra usage of control energy. On the other hand, since the vehicles communicate among themselves and inform each other about their individual final times, the velocities of the vehicles should be changing their trajectories at the communication instants when the final time of the rendezvous is updated as seen in Figure 3.20.

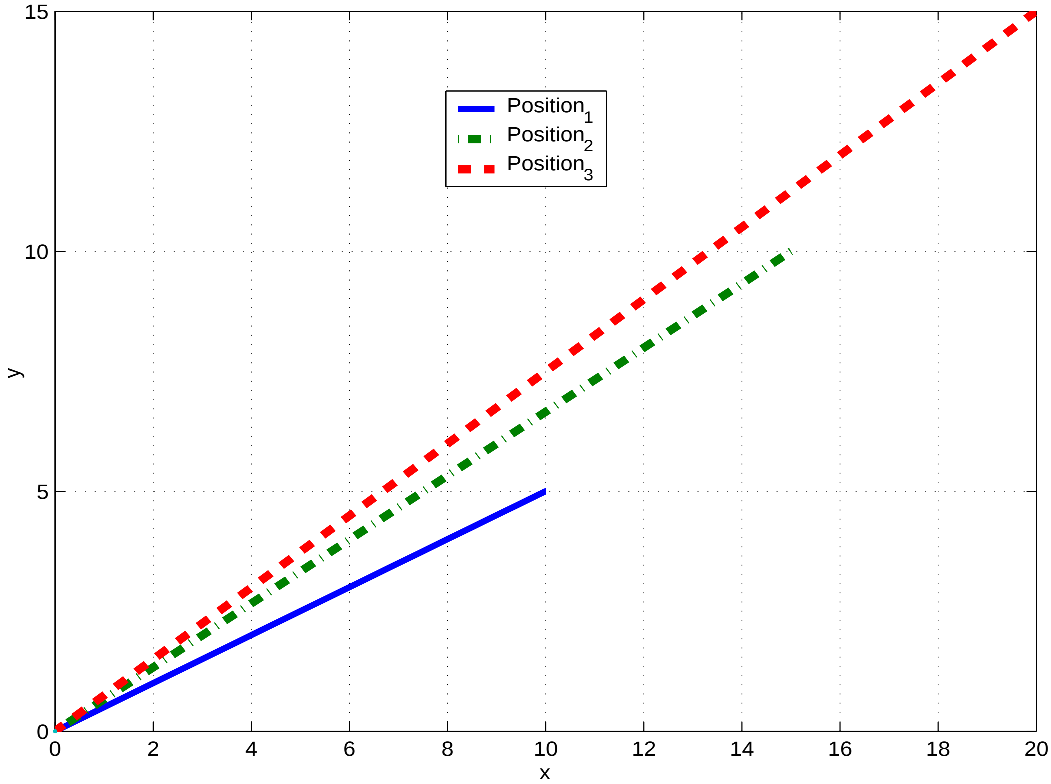


Figure 3.19: Position trajectories for rendezvous in 2-D with fixed target

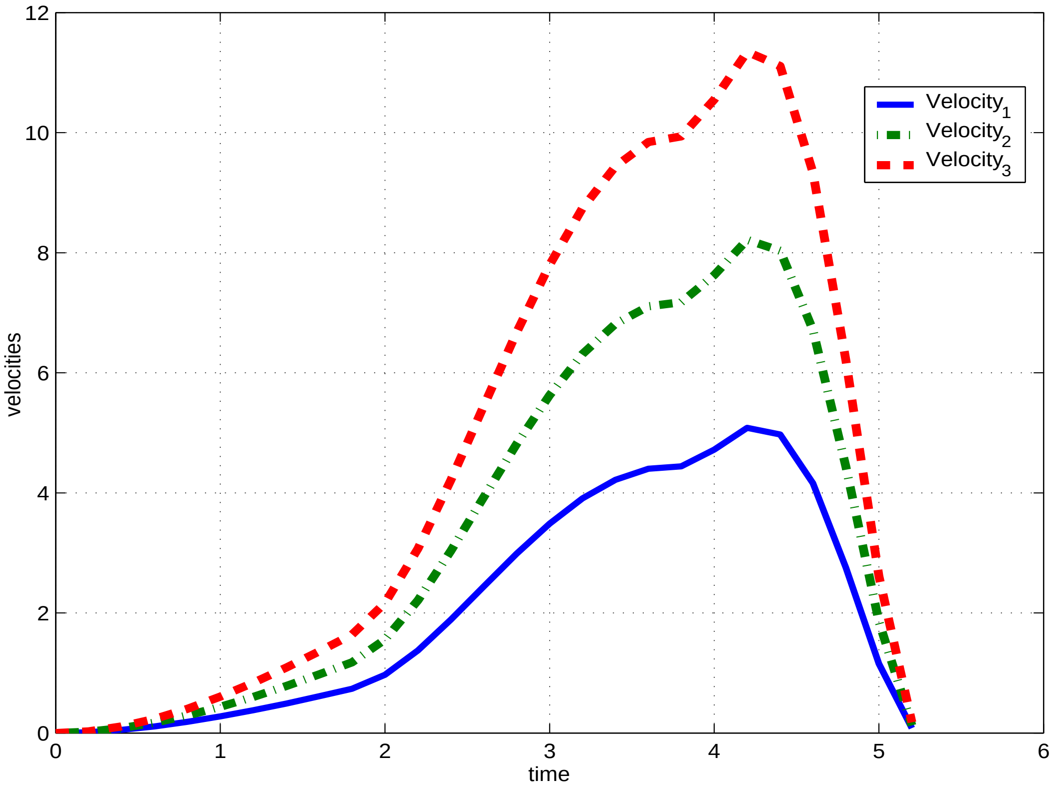


Figure 3.20: Velocities vs. time for rendezvous in 2-D with fixed target

3.2.2 2-D Rendezvous Problem with Moving Target

The multi-agent rendezvous problem becomes more interesting when the rendezvous point, namely the target, is not stationary but instead mobile. Remember that such a configuration is discussed in [14] and [19] for mobile autonomous robots. However, in these studies the proposed solutions were not optimal and a perfect communication scheme was assumed. Now, let us look at the application of our optimal solution to the 3-vehicle rendezvous problem with a moving target in 2-D, and observe the results. We have the following assumptions.

- The motion of the target is not a priori known by the vehicles,
- The position and the velocity of the target is determined by the vehicles via sensors at every time instant,
- The target moves with constant velocity,
- The vehicles can move faster than the target.

The constant velocity motion of the target is modeled as

$$\begin{bmatrix} p_{tx} & p_{ty} \\ v_{tx} & v_{ty} \\ a_{tx} & a_{ty} \end{bmatrix} = \begin{bmatrix} \frac{t}{4} & \frac{t}{4} \\ \frac{1}{4} & \frac{1}{4} \\ 0 & 0 \end{bmatrix}$$

where

p_{tx}, p_{ty} : current x and y components of the position of the target

v_{tx}, v_{ty} : current x and y components of the velocity of the target

a_{tx}, a_{ty} : current x and y components of the acceleration of the target.

The problem configuration is same as in 3.6 except the initial and final states. Also, $w_i(t)$ has an effect of $\pm 5\%$ random changes in x and y components of $u_i^*(t)$. The initial states are given as

$$x_1(t_0) = \begin{bmatrix} 10 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_2(t_0) = \begin{bmatrix} 0 & 15 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_3(t_0) = \begin{bmatrix} 20 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (3.8)$$

Since the target moves with constant velocity, the final states can be defined as

$$x_i(t_f) = \begin{bmatrix} p_{ix}(t_f) & p_{iy}(t_f) \\ v_{ix}(t_f) & v_{iy}(t_f) \\ a_{ix}(t_f) & a_{iy}(t_f) \end{bmatrix} = \begin{bmatrix} p_{tx} & p_{ty} \\ v_{tx} & v_{ty} \\ 0 & 0 \end{bmatrix}$$

where

$p_{ix}(t_f), p_{iy}(t_f)$: x and y components of the position of vehicle i at t_f

$v_{ix}(t_f), v_{iy}(t_f)$: x and y components of the velocity of vehicle i at t_f

$a_{ix}(t_f), a_{iy}(t_f)$: x and y components of the acceleration of vehicle i at t_f

In addition, it is assumed that there is no time delay in the communication. The results are shown in Figures 3.21 and 3.22

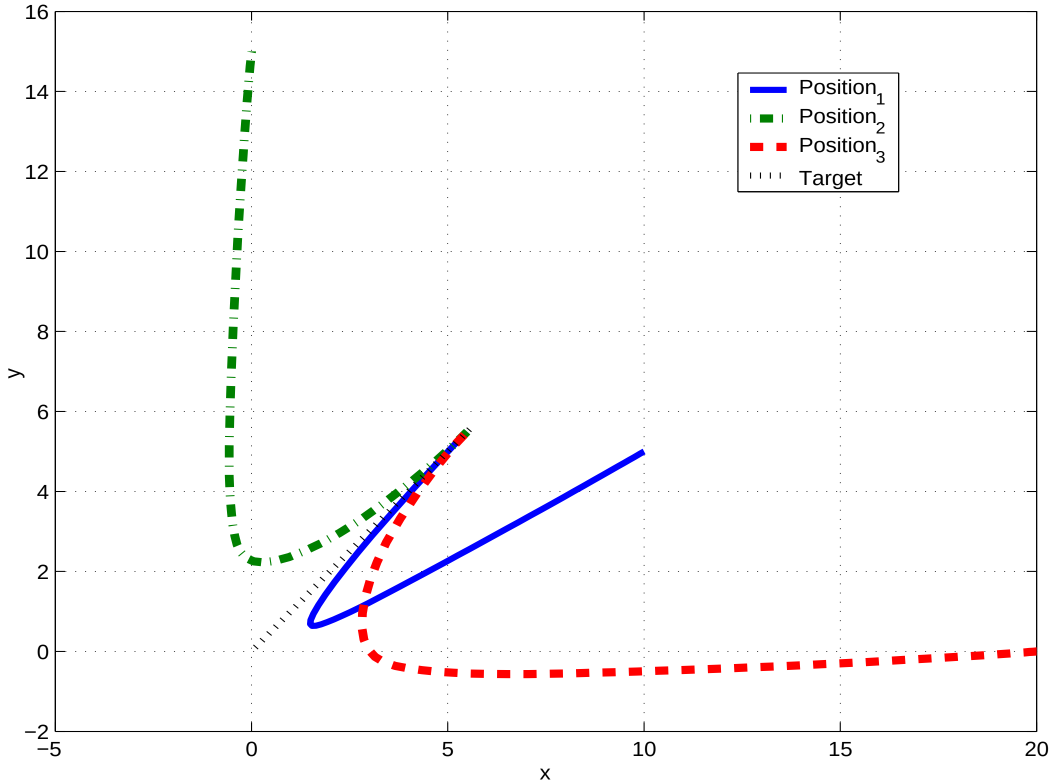


Figure 3.21: Position trajectories for rendezvous in 2-D with moving target

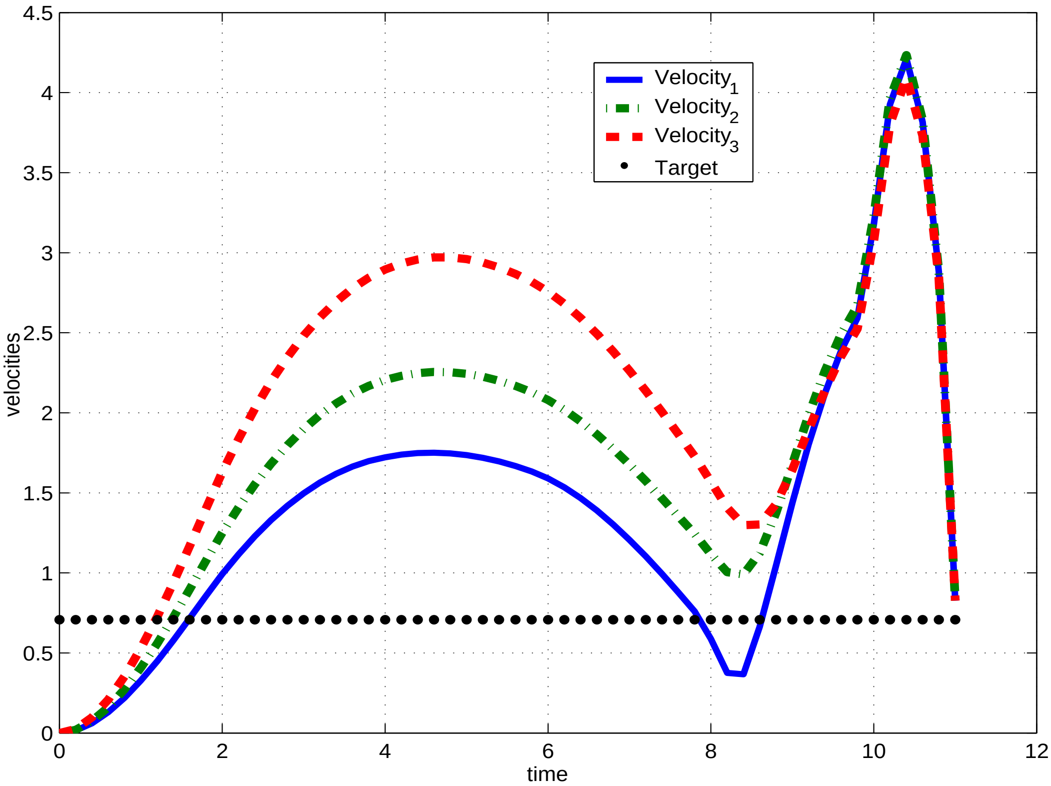


Figure 3.22: Velocities vs. time for rendezvous in 2-D with moving target

As seen in Figures 3.21 and 3.22 the vehicles catch the moving target successfully. Now, let us go one step further and consider the time delays in the communication.

3.2.3 2-D Rendezvous Problem with Moving Target in the Presence of Communication Problems

As seen in Figures 3.21 and 3.22, it can be concluded that the solution works for the moving target problem. On the other hand, the solution should be discussed in the presence of time delays or lost information signals which are very natural events to be faced with. Next, the result of the application to the problem is presented with and without considering the uncertainties which were involved in the model as $w_i(t)$s.

The problem configuration is given as below without including $w_i(t)$s in the model.

$$\begin{aligned} \dot{x}_i &= A_i x_i(t) + B_i u_i(t), \\ t_0 &= 0, \quad t_f = 10, \end{aligned}$$

$$x_1(t_0) = \begin{bmatrix} 10 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_2(t_0) = \begin{bmatrix} 0 & 15 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_3(t_0) = \begin{bmatrix} 20 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (3.9)$$

where A_i and B_i are as in (3.6) and the motion of the target is modeled same as before. Due to time delay, final times of each vehicle are calculated as explained in (3.5). $h_{ij}(k)$ is modeled as in in (3.4) and h_{max} is chosen as 0.5 sec.

As seen in Figures 3.23 and 3.24, which are the simulation results without the effect of $w_i(t)$s, the vehicles catch the moving target and the mission is completed successfully.

However, if we look at Figure 3.25 we can see that the final times are not equalized successfully. Consequently, the final states of the vehicles are not completely same as their target. Evidently, this is due to the communication delays between the vehicles since these delays prevent the vehicles to accurately determine the actual rendezvous time of the team. Nonetheless, the result is quite satisfactory.

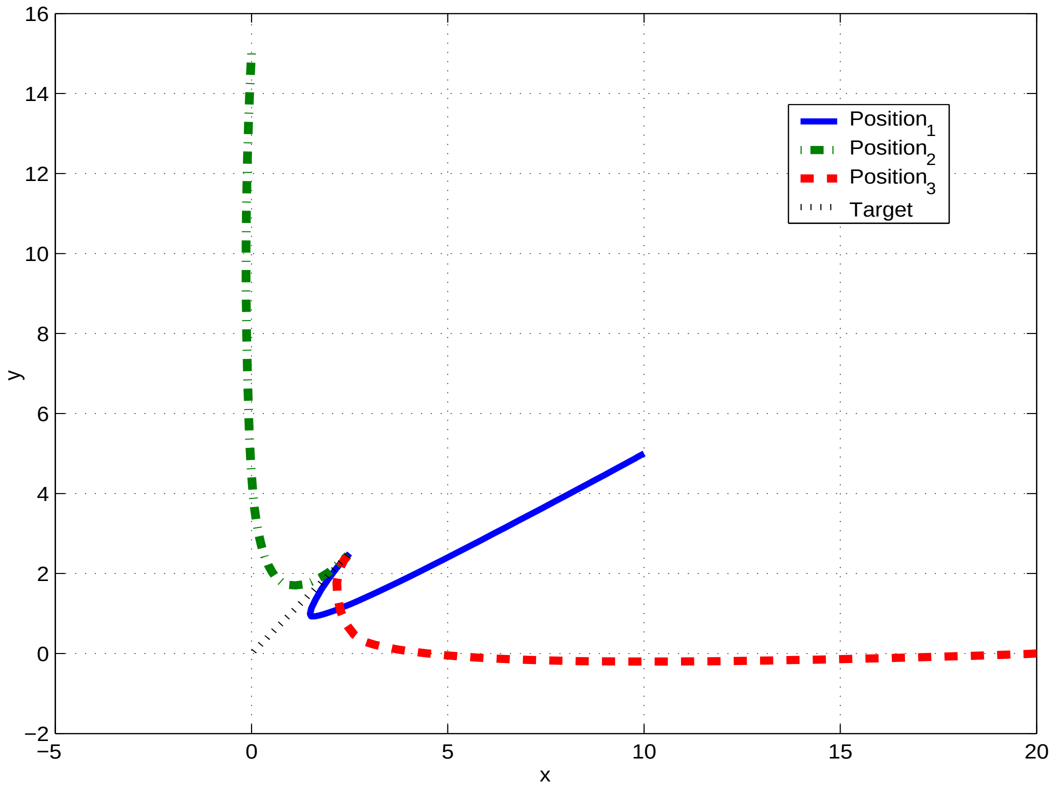


Figure 3.23: Position trajectories for rendezvous in 2-D with moving target

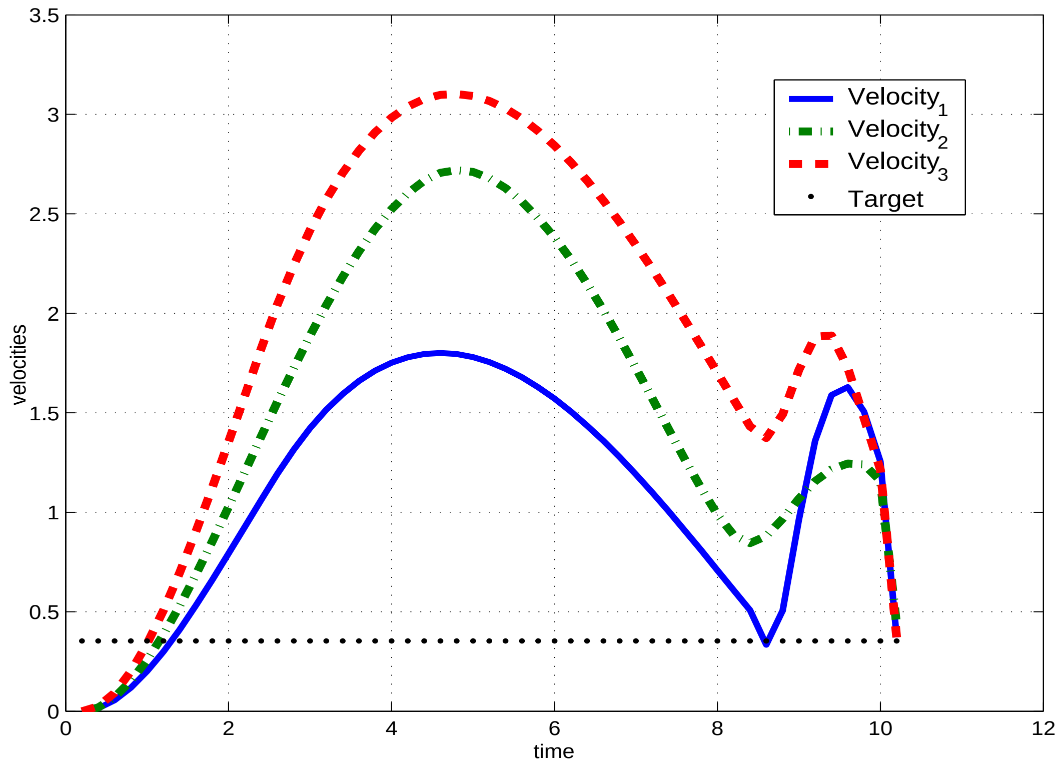


Figure 3.24: Velocities vs. time for rendezvous in 2-D with moving target

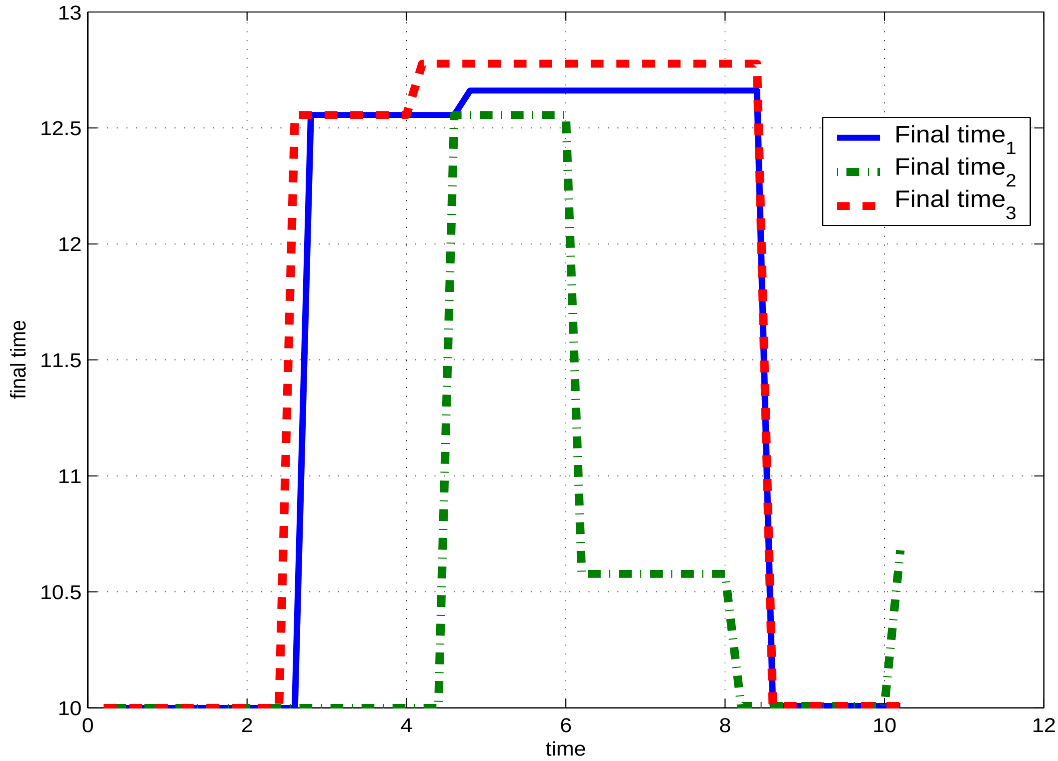


Figure 3.25: Change in t_i^fs at communication instants for rendezvous in 2-D with moving target

Now, let us look at the solution if the uncertainties are taken into account by including $w_i(t)$ in the model as a disturbance in the control input. Note that, missed or late information signals (i.e. the unavailable information) were also taken into account (as in the previous example) while obtaining the following solution by utilizing time delays in the communication between the vehicles. The problem configuration is given as

$$
\begin{aligned}
\dot{x}_i &= A_i x_i(t) + B_i \left((\begin{bmatrix} 1 & 1 \end{bmatrix} + w_i(t)) \cdot u_i(t) \right), \\
t_0 &= 0, \quad t_f = 10, \\
x_1(t_0) &= \begin{bmatrix} 10 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_2(t_0) = \begin{bmatrix} 0 & 15 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad x_3(t_0) = \begin{bmatrix} 20 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}
\tag{3.10}
$$

The motion of the target is modeled as

$$
\begin{bmatrix} p_{tx} & p_{ty} \\ v_{tx} & v_{ty} \\ a_{tx} & a_{ty} \end{bmatrix} = \begin{bmatrix} \frac{t}{2} & \frac{t}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix}
$$

and $w_i(t)$ has an effect of $\pm 5\%$ random changes in x and y components of $u_i^*(t)$. The time delays are modeled as before.

Results of the solution are depicted in Figures 3.26-3.28. Looking at Figure 3.28, we observe that the time delays in the communication cause the vehicles to determine the rendezvous time differently during the mission, however thanks to successful information exchange towards the end of the travel, the final rendezvous times are very close. Consequently, we can see that the vehicles achieve a successful rendezvous as plotted in Figure 3.26. However, the differences in the rendezvous times caused by the time delays in the communication, result in deviations from the aimed velocity value of the moving target at the instant of rendezvous as seen in Figure 3.27.

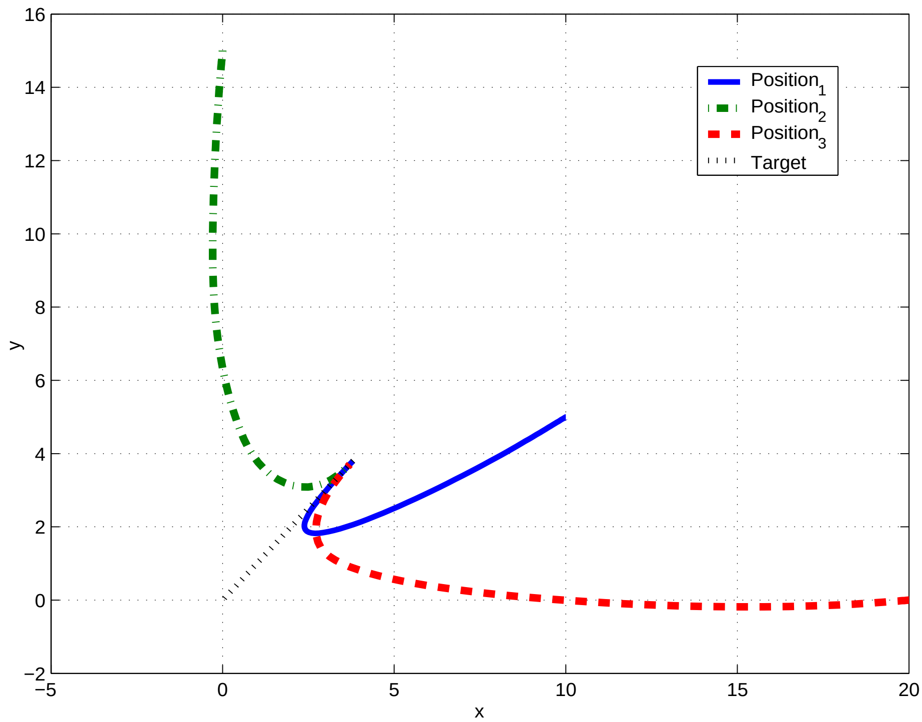


Figure 3.26: Position trajectories for rendezvous in 2-D with moving target including communication problems

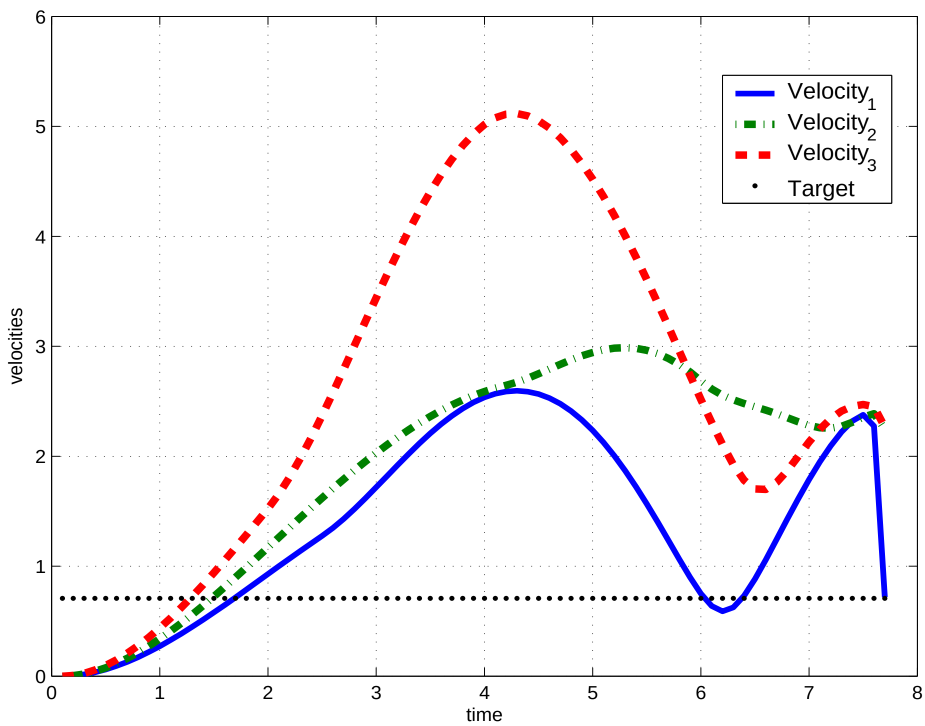


Figure 3.27: Velocities vs. time for rendezvous in 2-D with moving target including communication problems

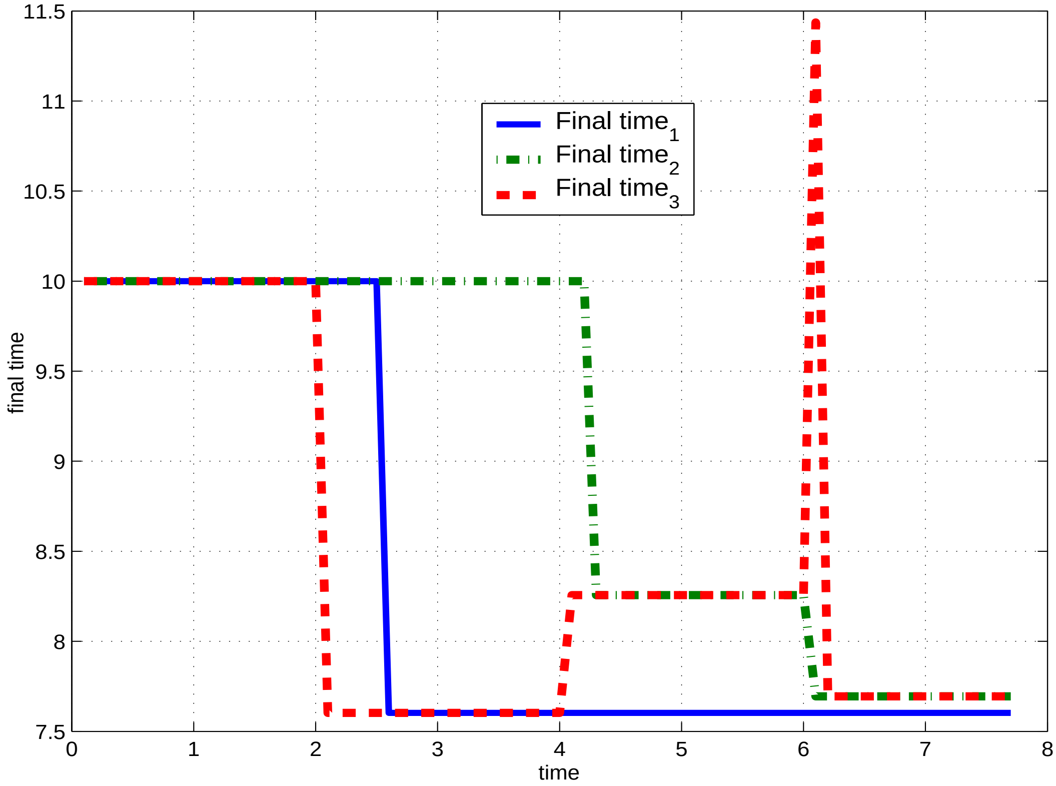


Figure 3.28: Change in t_i^fs at communication instants for rendezvous in 2-D with moving target including communication problems

3.3 Effects of Increasing Disturbance and Time Delay on the Solution

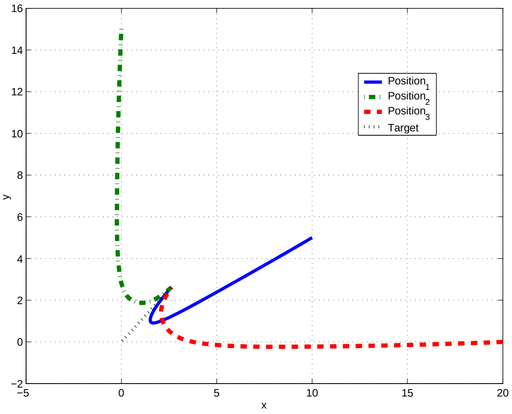
In this part, we present the solution results for increasing disturbance $w_i(t)$ and time delays $h_{ij}(k)$. Various simulations are shown for each case and the results are discussed. The problem configuration is chosen as in (3.10) and the constant velocity motion of the target is modeled as

$$\begin{bmatrix} p_{tx} & p_{ty} \\ v_{tx} & v_{ty} \\ a_{tx} & a_{ty} \end{bmatrix} = \begin{bmatrix} \frac{t}{4} & \frac{t}{4} \\ \frac{1}{4} & \frac{1}{4} \\ 0 & 0 \end{bmatrix}.$$

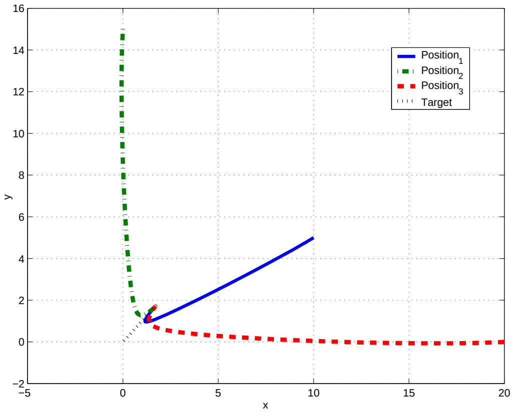
3.3.1 Increasing Disturbance

In order to observe the effect of disturbance clearly, it is assumed that there is no time delay in the communication between the vehicles. The disturbance $w_i(t)$ is chosen to be in the form of $\pm$ 0, 2, 2.5, 3.33, 5 and 10 % random changes in the optimal control input $u_i^*(t)$. Results are shown in Figures 3.29-3.31. The observations for increasing disturbance (i.e. the uncertainties) can be summarized as below.

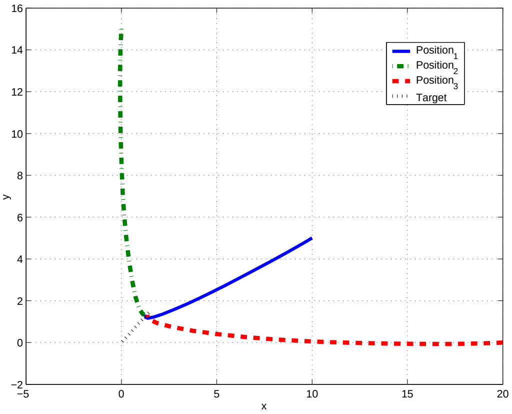
- Since the disturbances model the uncertainties and they are random, the rendezvous time and the magnitudes of the disturbances are not related directly.
- The vehicles catch the moving target successfully in all of the cases, because the rendezvous time is common for the vehicles thanks to the perfect communication.
- In general, the magnitudes of the control inputs (except the maximum values appearing due to the updates in the final time at close instants to the rendezvous) get larger as the effects of disturbances increase.



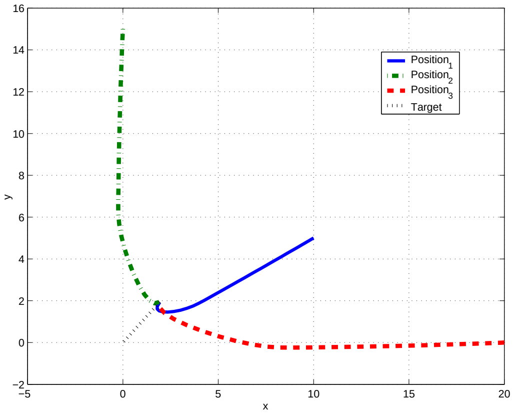
(a) $\pm$ 0 % random changes in $u_i^*(t)$



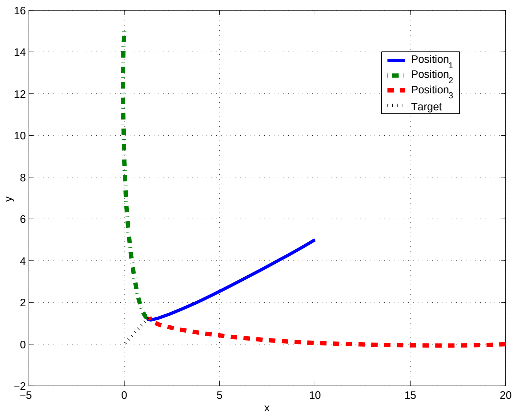
(b) $\pm$ 2 % random changes in $u_i^*(t)$



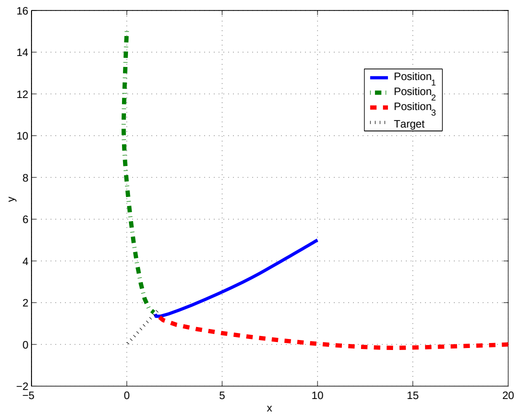
(c) $\pm$ 2.5 % random changes in $u_i^*(t)$



(d) $\pm$ 3.33 % random changes in $u_i^*(t)$

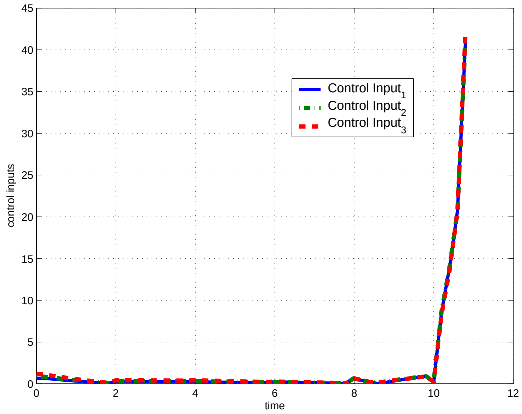


(e) $\pm$ 5 % random changes in $u_i^*(t)$

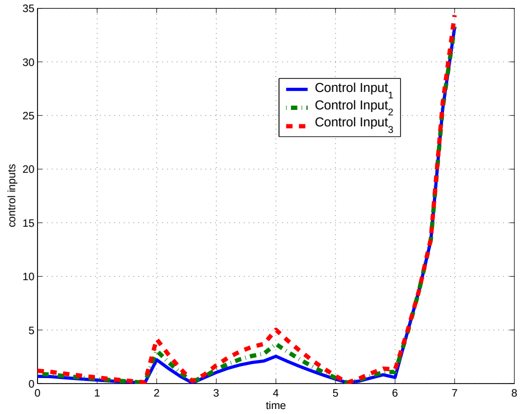


(f) $\pm$ 10 % random changes in $u_i^*(t)$

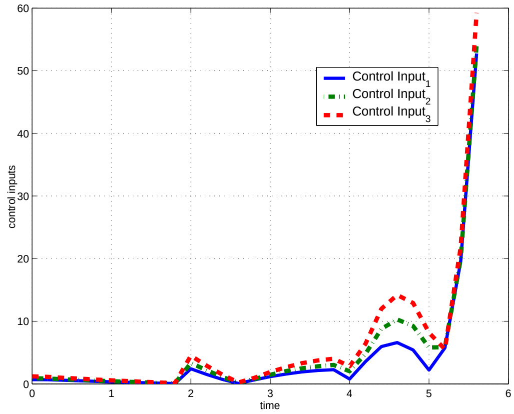
Figure 3.29: Position trajectories for increasing disturbances



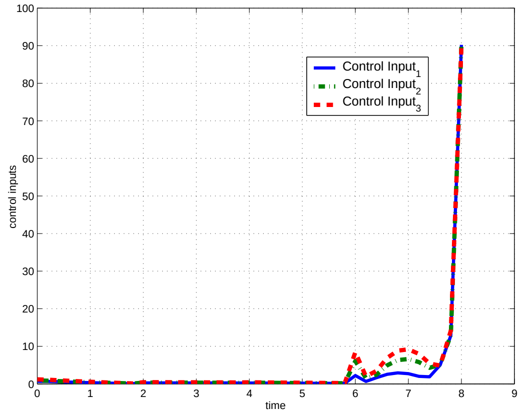
(a) $\pm$ 0 % random changes in $u_i^*(t)$



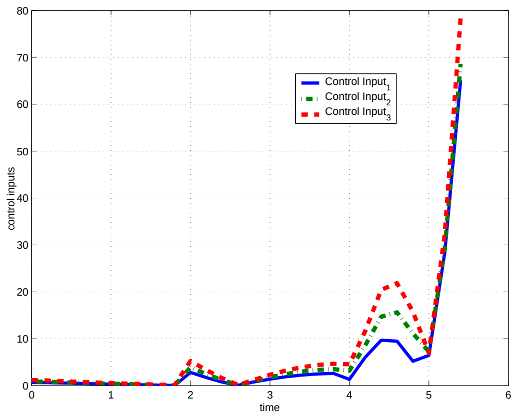
(b) $\pm$ 2 % random changes in $u_i^*(t)$



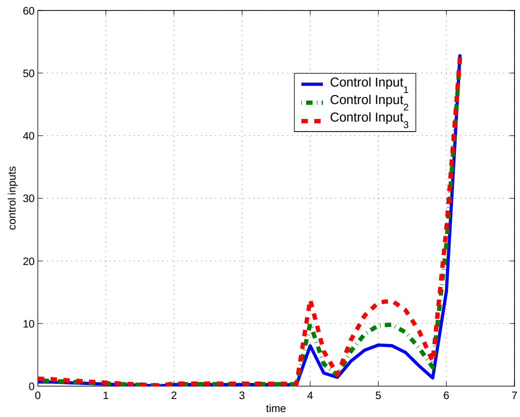
(c) $\pm$ 2.5 % random changes in $u_i^*(t)$



(d) $\pm$ 3.33 % random changes in $u_i^*(t)$

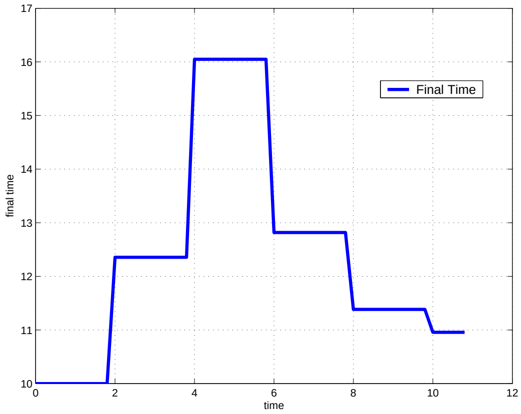


(e) $\pm$ 5 % random changes in $u_i^*(t)$

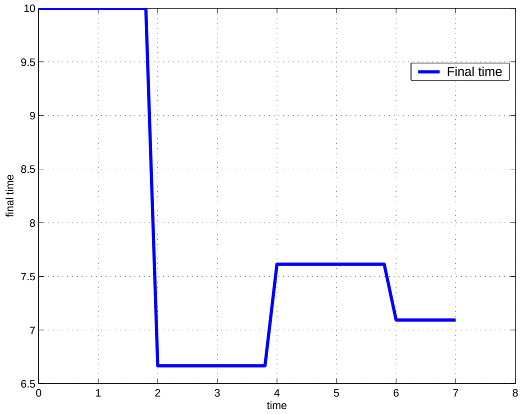


(f) $\pm$ 10 % random changes in $u_i^*(t)$

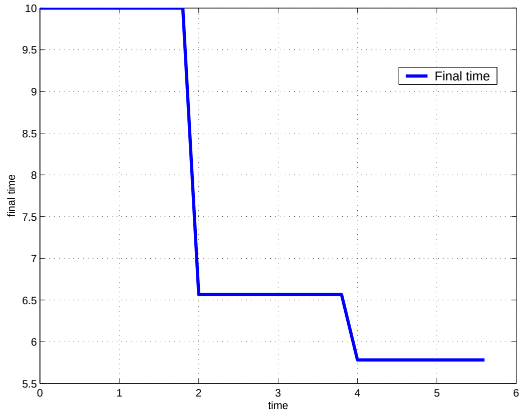
Figure 3.30: Magnitudes of the control inputs vs. time for increasing disturbances



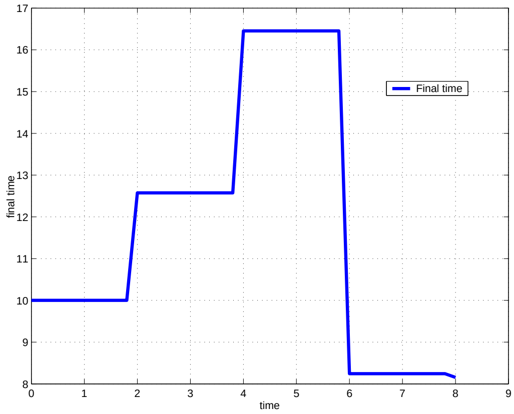
(a) ± 0 % random changes in $u_i^*(t)$



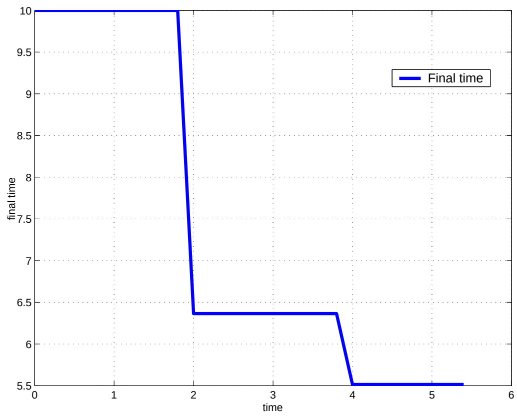
(b) ± 2 % random changes in $u_i^*(t)$



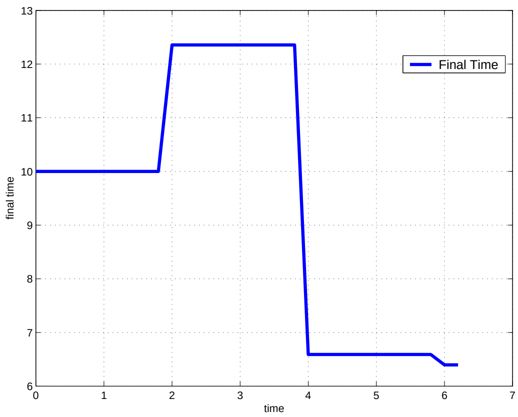
(c) ± 2.5 % random changes in $u_i^*(t)$



(d) ± 3.33 % random changes in $u_i^*(t)$



(e) ± 5 % random changes in $u_i^*(t)$



(f) ± 10 % random changes in $u_i^*(t)$

Figure 3.31: Final times vs. time for increasing disturbances

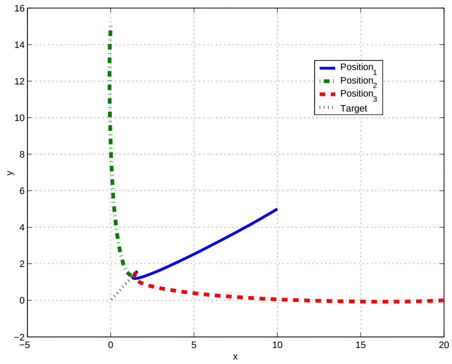
3.3.2 Increasing Time Delay

In order to observe the effect of increasing time delay in the communication, solutions corresponding to various time delays are obtained. The waiting time h_{max} is 0.5 sec as before. The delay model is similar to (3.4)

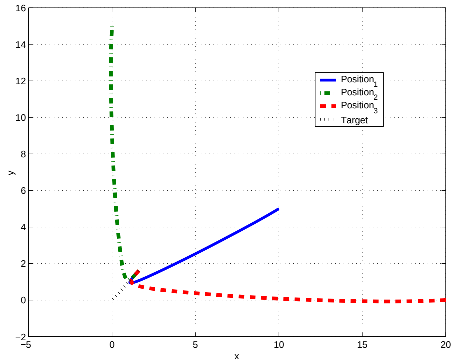
$$h_{ik} \sim |N(h_{mean}, 0.05)| \tag{3.11}$$

i.e. we have normally distributed positive time delay values with mean h_{mean} and variance 0.05. Thus, we can observe the effect of time delay in the communication by increasing its mean while keeping the variance small. We obtain the solutions for different values of h_{mean} as 0.1, 0.3, 0.4, 0.45, 0.49, 0.5, 0.8 and 1.3 sec. It is assumed that there is no disturbance (i.e. $w_i(t) = 0$). Results are shown in Figures 3.32-3.34. The observations for increasing waiting time h_{max} can be summarized as below.

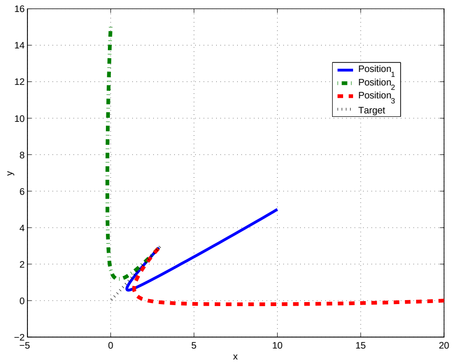
- Increasing the mean of the random time delay results in deviations in rendezvous time information. If time delays exceed the maximum waiting time at a communication instant, the vehicles cannot have a common rendezvous time until the next successful communication.
- If most of the time delays are greater than the waiting time during the travel, the success of the mission depends on reliable information exchanges as the rendezvous time approaches. If successful communication is not achieved towards the end of the travel, the rendezvous does not occur.
- As expected, the rendezvous is not achieved if time delays in the communication are greater than the maximum waiting time. Therefore, the waiting time should be chosen carefully according to the expected time delays in the communication.



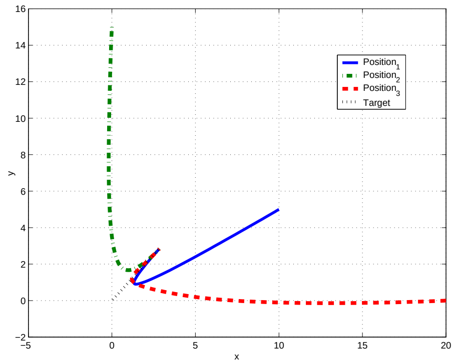
(a) $h_{mean} = 0.1$ sec



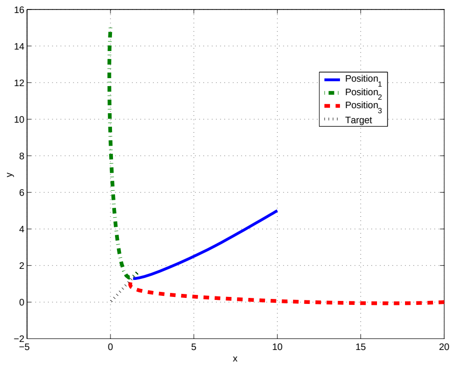
(b) $h_{mean} = 0.3$ sec



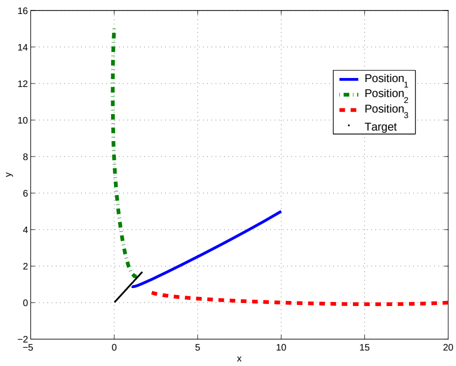
(c) $h_{mean} = 0.4$ sec



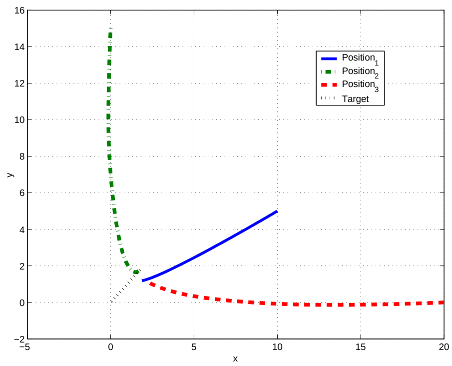
(d) $h_{mean} = 0.45$ sec



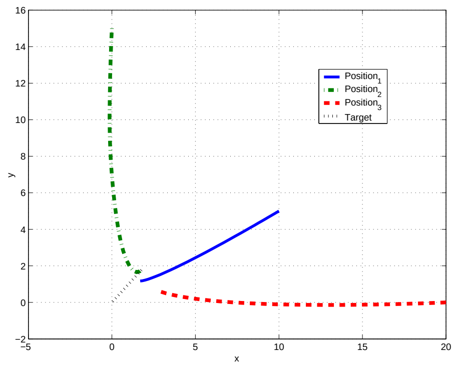
(e) $h_{mean} = 0.49$



(f) $h_{mean} = 0.5$ sec

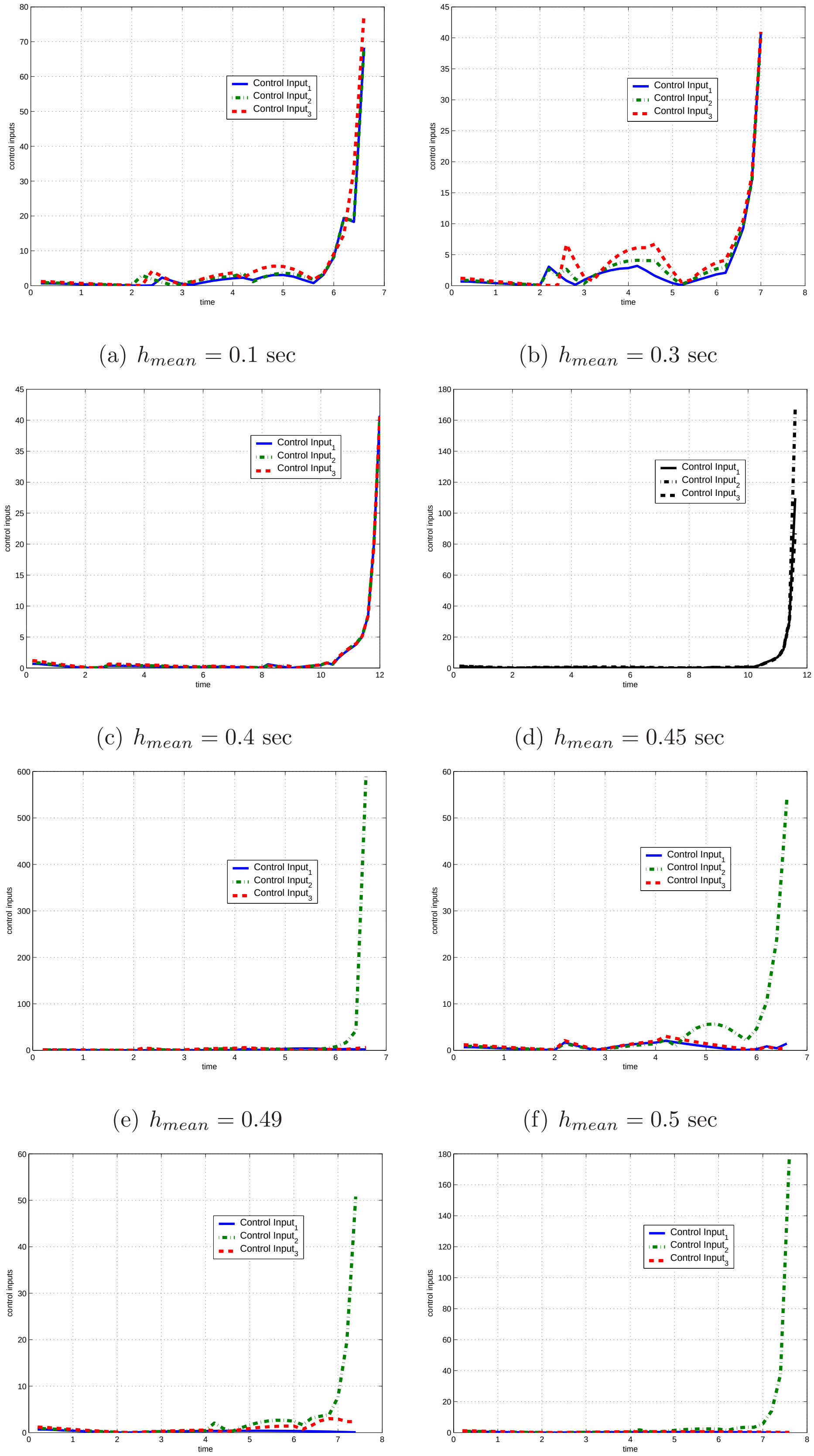


(g) $h_{mean} = 0.8$ sec



(h) $h_{mean} = 1.1$ sec

Figure 3.32: Position trajectories for increasing time delay in the communication



(a) $h_{mean} = 0.1$ sec

(b) $h_{mean} = 0.3$ sec

(c) $h_{mean} = 0.4$ sec

(d) $h_{mean} = 0.45$ sec

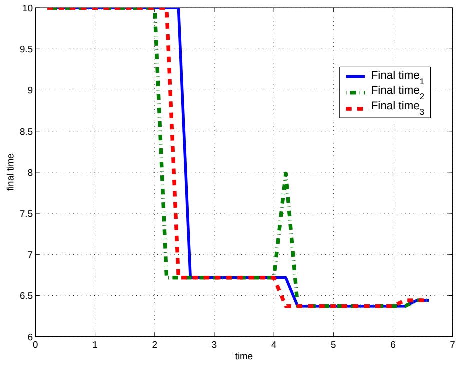
(e) $h_{mean} = 0.49$

(f) $h_{mean} = 0.5$ sec

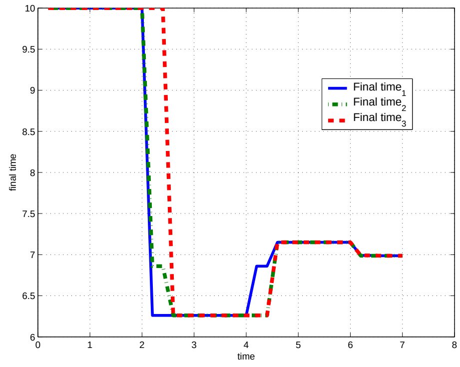
(g) $h_{mean} = 0.8$ sec

(h) $h_{mean} = 1.1$ sec

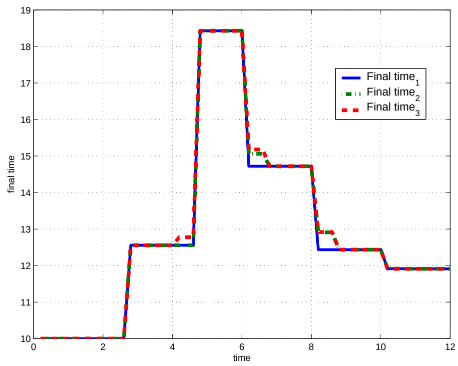
Figure 3.33: Magnitudes of the control inputs vs. time for increasing time delay in the communication



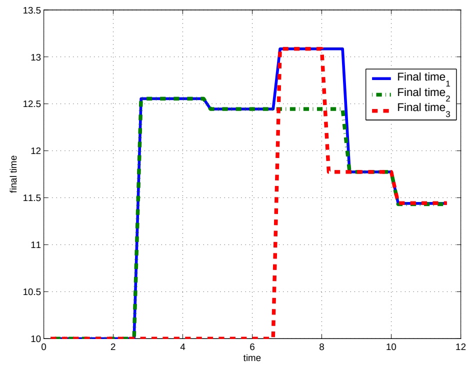
(a) $h_{mean} = 0.1$ sec



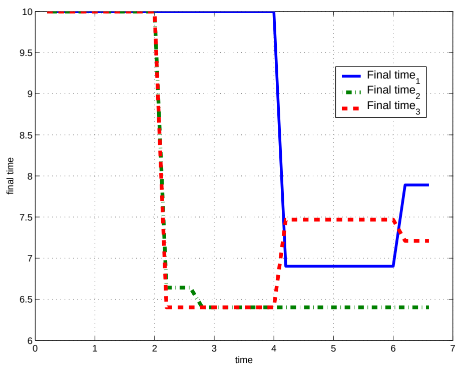
(b) $h_{mean} = 0.3$ sec



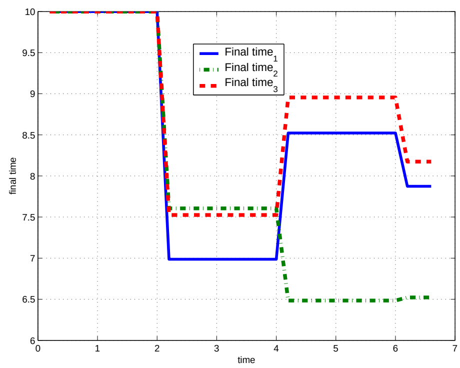
(c) $h_{mean} = 0.4$ sec



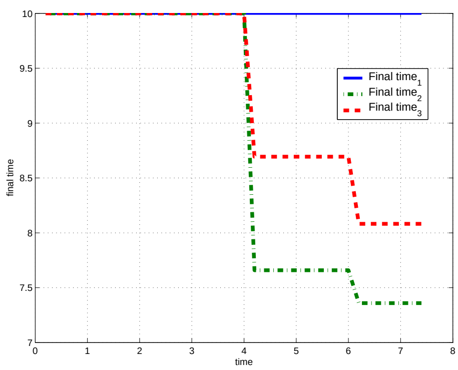
(d) $h_{mean} = 0.45$ sec



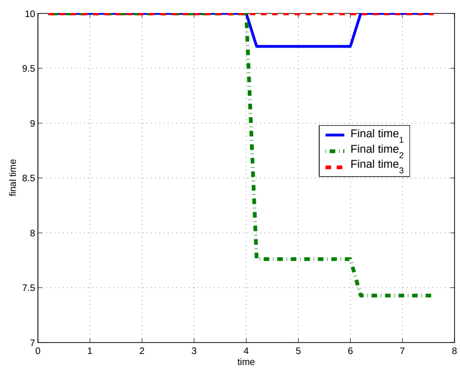
(e) $h_{mean} = 0.49$



(f) $h_{mean} = 0.5$ sec



(g) $h_{mean} = 0.8$ sec



(h) $h_{mean} = 1.1$ sec

Figure 3.34: Final times vs. time for increasing time delay in the communication

Chapter 4

CONCLUSIONS

In this study, the multi-agent rendezvous problem is discussed. The problem consists of $N > 1$ mobile agents, which are trying to reach a destination (i.e. the rendezvous point) simultaneously. It is assumed that the initial and the final states of the agents as well as a target value for the rendezvous time are specified. Even it seems reasonable to expect the agents to reach the destination at the pre-specified rendezvous time, desired rendezvous may not be possible due to some unforeseen events like obstacles on the road, internal problems of the agents or any other disturbance. Therefore, a communication scheme is utilized between the agents in order to enable an exchange of information about the estimated times of arrivals of the agents with regular time intervals. The reason for having the information exchanges at discrete time instants is simply the cost of reliable continuous communication.

In order to solve this multi-agent rendezvous problem, an appropriate cost function is defined and solved for the minimizing control input. The solution for the problem is based on minimum energy control which is a very useful extension of calculus of variations, and the results are quite satisfactory that the optimal solutions obtained at each communication instant successively realize a simultaneous rendezvous despite unexpected disturbances. Initially, the solution is given in 1-D and does not take into account the problems in information exchange, like delayed or lost signals. Therefore, an algorithm is proposed for handling delayed or lost information, and the solution is extended to 2-D including moving target point case. In this case the agents try to catch a mobile target simultaneously. Such a problem appears in several applications, e.g. optimal motion or trajectory planning for UAVs, robot teams and vehicle convoys. The effect of increasing disturbance on the control input and time delays in the communication are also discussed.

In addition, it is possible to make some improvements in order to have a more comprehensive solution. For instance, it may be a good complementary study to develop an estimator for the signals carrying information about the rendezvous times of the agents since these signals may be delayed and noisy or completely lost, rather than using the proposed algorithm in the solution for handling the same problem. Also, another complementary study may be adapting collision/obstacle avoidance to the solution which is very important for vehicles or robots working autonomously.

APPENDIX A

Matlab code for the solution of rendezvous in 1-D

```
clear;
clc;

A = [0 1 0;0 0 1;0 0 0];
B = [0;0;1];
x1_0 = [10;0;0];
x2_0 = [15;0;0];
x3_0 = [20;0;0];
x0_1 = x1_0;
x0_2 = x2_0;
x0_3 = x3_0;
xf = [0;0;0];ts = 0;
tf = 20;
tb= ts;
te= tf;
inc = (tf-ts)/50;
hmax = (tf-ts)/20;
hmean = 0;
hst = hmax*0.90;
syms t p;
x_1 = [];
u_1 = [];
x_2 = [];
u_2 = [];
x_3 = [];
u_3 = [];
tem = [];
tmr = [];
tm = tb;
tm_c = tm;
t_t1 = [];
t_t2 = [];
t_t3 = [];
te_t = [te;te;te];
te_calc = [te;te;te];
te_r = [te];
h =[0;0;0];

while tm < te
    % Expected arrival times are determined here
    if length(tmr) > 0 && mod(length(tmr),10) == 0
        [i1 i2] = size(te_calc);
        in = length(tmr);
        c1 = ([(tmr(in-4)^4) (tmr(in-4)^3) (tmr(in-4)^2) ...
        tmr(in-4) 1; (tmr(in-3)^4) (tmr(in-3)^3) ...
        (tmr(in-3)^2) tmr(in-3) 1;(tmr(in-2)^4) ...
        (tmr(in-2)^3) (tmr(in-2)^2) tmr(in-2) 1;...
        (tmr(in-1)^4) (tmr(in-1)^3) (tmr(in-1)^2)...
        tmr(in-1) 1;(tmr(in)^4) (tmr(in)^3) (tmr(in)^2)...
        tmr(in) 1] \ ...
        [x_1(2,in-4);x_1(2,in-3);x_1(2,in-2);...
        x_1(2,in-1);x_1(2,in)]);

        t_t_1 = roots([(c1(1)/5) (c1(2)/4) (c1(3)/3)...
        (c1(4)/2) c1(5)-((c1(1)/5)*(tmr(in)^5)+(c1(2)/4)*...
        (tmr(in)^4)+(c1(3)/3)*(tmr(in)^3)+(c1(4)/2)*...
        (tmr(in)^2)+c1(5)*tmr(in)+(xf(1)-x_1(1,in)))]);
        t_t1 = [t_t1 t_t_1];
        temp1 = [];
        for i=1:1:5
            if abs(imag(t_t_1(i)))≤1e-3
                t_t_1(i) = real(t_t_1(i));
            end
        end
```

```
        for i=1:1:5
         temp1(i) = isreal(t_t_1(i)) && 0<t_t_1(i);
        end
        te_1 = max(temp1'.*t_t_1);

        c2 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
        (tmr(in-4)^2) tmr(in-4) 1; (tmr(in-3)^4)...
        (tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
        (tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
        tmr(in-2) 1; (tmr(in-1)^4) (tmr(in-1)^3)...
        (tmr(in-1)^2) tmr(in-1) 1; (tmr(in)^4)...
        (tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
        [x_2(2,in-4);x_2(2,in-3);x_2(2,in-2);...
        x_2(2,in-1);x_2(2,in)]);

        t_t_2 = roots([(c2(1)/5) (c2(2)/4)...
        (c2(3)/3) (c2(4)/2) c2(5)-((c2(1)/5)*...
        (tmr(in)^5)+(c2(2)/4)*(tmr(in)^4)+...
        (c2(3)/3)*(tmr(in)^3)+(c2(4)/2)*...
        (tmr(in)^2)+c2(5)*tmr(in)+(xf(1)-...
        x_2(1,in)))]);

        t_t2 = [t_t2 t_t_2];
        temp2 = [];
        for i=1:1:5
            if abs(imag(t_t_2(i)))≤1e-3
                t_t_2(i) = real(t_t_2(i));
            end
        end
        for i=1:1:5
         temp2(i) = isreal(t_t_2(i)) && 0<t_t_2(i);
        end
        te_2 = max(temp2'.*t_t_2);

        c3 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
        (tmr(in-4)^2)  tmr(in-4) 1;(tmr(in-3)^4)...
        (tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
        (tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
        tmr(in-2) 1;(tmr(in-1)^4) (tmr(in-1)^3)...
        (tmr(in-1)^2) tmr(in-1) 1;(tmr(in)^4)...
        (tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
        [x_3(2,in-4);x_3(2,in-3);x_3(2,in-2);...
        x_3(2,in-1);x_3(2,in)]);

        t_t_3 = roots([(c3(1)/5) (c3(2)/4)...
        (c3(3)/3) (c3(4)/2) c3(5)-((c3(1)/5)*...
        (tmr(in)^5)+(c3(2)/4)*(tmr(in)^4)+...
        (c3(3)/3)*(tmr(in)^3)+(c3(4)/2)*...
        (tmr(in)^2)+c3(5)*tmr(in)+(xf(1)-...
        x_3(1,in)))]);

        t_t3 = [t_t3 t_t_3];
        temp3 = [];
        for i=1:1:5
            if abs(imag(t_t_3(i)))≤1e-3
                t_t_3(i) = real(t_t_3(i));
            end
        end
        for i=1:1:5
         temp3(i) = isreal(t_t_3(i)) && 0<t_t_3(i);
        end
        te_3= max(temp3'.*t_t_3);

        te_calc = [te_calc [te_1;te_2;te_3]];
        te_r =[te_r min([te_1;te_2;te_3])];
        tm_c = tm;
        % Time delay matrix is formed here
        h1 = (hmean) + hst*abs(randn);
        h2 = (hmean) + hst*abs(randn);
        h3 = (hmean) + hst*abs(randn);
        h = [h [h1;h2;h3]];
        te_t = [te_t [te;te;te]];
    end

%------------------------------------------------------------
% Rendezvous time information is updated below
    [in1 in2] = size(h);
    for i=1:3
        if (h(i,in2) ≤ hmax) && ((tm_c + h(i,in2))≤ tm)
            te_t(i,in2) = te_calc (i,in2);
        end
    end
    te = min(te_t(:,(in2)));

%-----------------------------------------------------------------
% Terms related with the Controllability Matrix W_c are below
    Wc  = int(expm(A*(tb-t))*B*B'*expm(A'*(tb-t)),t,tb,te);
    eat = expm(A'*(tb-t));
    ea  = expm(A*(tb-te));

%------------------------------------------------------------------
% Here, control inputs are calculated and disturbances are added
    up_1 = - B'*eat*inv(Wc)*(x0_1-ea*xf);
    up_1 = up_1;
    up_1 = up_1 + (up_1/50)*((rand>0.9)*randn);
    up_2 = - B'*eat*inv(Wc)*(x0_2-ea*xf);
    up_2 = up_2;
    up_2 = up_2 + (up_2/50)*((rand>0.9)*randn);
    up_3 = - B'*eat*inv(Wc)*(x0_3-ea*xf);
    up_3 = up_3;
    up_3 = up_3 + (up_3/50)*((rand>0.9)*randn);


%-------------------------------------------------
% Solutions are obtained below
    d_1 = int(expm(A*(p-t))*B*up_1,t,tb,p);
    xp1_1 = subs(d_1,p,tm);
    up1_1 = subs(- B'*eat*inv(Wc)*(x0_1-ea*xf),t,tm);
    xp_1 = ((expm(A*(tm-tb))*x0_1) + (xp1_1  ));
    d_2 = int(expm(A*(p-t))*B*up_2,t,tb,p);
    xp2_1 = subs(d_2,p,tm);
    up2_1 = subs(- B'*eat*inv(Wc)*(x0_2-ea*xf),t,tm);
    xp_2 = ((expm(A*(tm-tb))*x0_2) + (xp2_1  ));
    d_3 = int(expm(A*(p-t))*B*up_3,t,tb,p);
    xp3_1 = subs(d_3,p,tm);
    up3_1 = subs(- B'*eat*inv(Wc)*(x0_3-ea*xf),t,tm);
    xp_3 = ((expm(A*(tm-tb))*x0_3) + (xp3_1  ));
```

```
%------------------------------------------------------------
% Here, solutions are added into corresponding
% matrices
    x_1 = [x_1 xp_1];
    u_1 = [u_1 up1_1];
    x_2 = [x_2 xp_2];
    u_2 = [u_2 up2_1];
    x_3 = [x_3 xp_3];
    u_3 = [u_3 up3_1];

%------------------------------------------------------------
% Required simulation updates are made below
    tem = [tem te];
    tmr = [tmr tm];
    tb = tm;
    tm = tm + inc;
    x0_1 = xp_1;
    x0_2 = xp_2;
    x0_3 = xp_3;
end
```

APPENDIX B

Matlab code for the solution of rendezvous in 2-D

```
clear;
clc;

A = [0 1 0;0 0 1;0 0 0];
B = [0;0;1];
x1_0 = [10 5;0 0;0 0];
x2_0 = [0 15;0 0;0 0];
x3_0 = [20 0;0 0;0 0];
x0_1 = x1_0;
x0_2 = x2_0;
x0_3 = x3_0;
xf = [0 0;0 0;0 0];
xf_1 = [];
ts = 0;
tf = 10;
tb= ts;
te= tf;
inc = (tf-ts)/50;
hmax = 0.5;
hmean = 0;
hst =  hmax*0.90;
syms t p;
x_1 = [];
u_1 = [];
x_2 = [];
u_2 = [];
x_3 = [];
u_3 = [];
tem = [];
tmr = [];
tm = tb;
tm_c = tm;
t_t1 = [];
t_t2 = [];
t_t3 = [];
te_t = [];
te_calc = [te;te;te];
te_r = [te];
te_h = ones(3,1)*te;
te_rec = [];
h = zeros(3);
st_1 = [];
st_2 = [];
st_3 = [];

while (tm) < min(te)
    % Expected arrival times are determined here
    if length(tmr) > 1 && mod(length(tmr),10) == 0
        in = length(tmr);
        c1 = ([(tmr(in-4)^4) (tmr(in-4)^3) (tmr(in-4)^2)...
        tmr(in-4) 1; (tmr(in-3)^4) (tmr(in-3)^3)...
        (tmr(in-3)^2) tmr(in-3) 1; (tmr(in-2)^4)...
        (tmr(in-2)^3) (tmr(in-2)^2) tmr(in-2) 1;...
        (tmr(in-1)^4) (tmr(in-1)^3) (tmr(in-1)^2)...
        tmr(in-1) 1; (tmr(in)^4) (tmr(in)^3) (tmr(in)^2)...
        tmr(in) 1] \...
        [x_1(2,(2*in-9));x_1(2,(2*in-7));x_1(2,(2*in-5));...
        x_1(2,(2*in-3));x_1(2,(2*in-1))]);

        t_t_1 = roots([(c1(1)/5) (c1(2)/4) (c1(3)/3)...
        (c1(4)/2) c1(5)-((c1(1)/5)*(tmr(in)^5)+(c1(2)/4)...
        *(tmr(in)^4)+(c1(3)/3)*(tmr(in)^3)+(c1(4)/2)*...
        (tmr(in)^2)+c1(5)*tmr(in) + (xf(1,1)-...
        x_1(1,(2*in-1))))]);
```

```
t_t1 = [t_t1 t_t_1];
temp1 = [];
for i=1:1:5
    if abs(imag(t_t_1(i)))≤1e-3
        t_t_1(i) = real(t_t_1(i));
    end
end
for i=1:1:5
 temp1(i) = isreal(t_t_1(i)) && 0<t_t_1(i);
end
te_1_x = max(temp1'.*t_t_1);

in = length(tmr);
c1 = ([(tmr(in-4)^4) (tmr(in-4)^3...
(tmr(in-4)^2) tmr(in-4) 1;(tmr(in-3)^4)...
(tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
(tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
tmr(in-2) 1;(tmr(in-1)^4) (tmr(in-1)^3)...
(tmr(in-1)^2) tmr(in-1) 1;(tmr(in)^4)...
(tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
[x_1(2,(2*in-8));x_1(2,(2*in-6));...
x_1(2,(2*in-4)); x_1(2,(2*in-2));...
x_1(2,(2*in))]);

t_t_1 = roots([(c1(1)/5) (c1(2)/4)...
(c1(3)/3) (c1(4)/2) c1(5)-((c1(1)/5)*
(tmr(in)^5)+(c1(2)/4*(tmr(in)^4)+...
(c1(3)/3)*(tmr(in)^3)+(c1(4)/2)*...
(tmr(in)^2)+c1(5)*tmr(in) + (xf(1,1)-...
x_1(1,(2*in)))))]);

t_t1 = [t_t1 t_t_1];
temp1 = [];
for i=1:1:5
    if abs(imag(t_t_1(i)))≤1e-3
        t_t_1(i) = real(t_t_1(i));
    end
end
for i=1:1:5
 temp1(i) = isreal(t_t_1(i)) && 0<t_t_1(i);
end
te_1_y = max(temp1'.*t_t_1);
te_1 = min(te_1_x,te_1_y);

c2 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
(tmr(in-4)^2) tmr(in-4) 1;(tmr(in-3)^4)...
(tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
(tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
tmr(in-2) 1; (tmr(in-1)^4) (tmr(in-1)^3)...
(tmr(in-1)^2) tmr(in-1) 1;(tmr(in)^4)...
(tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
[x_2(2,(2*in-9));x_2(2,(2*in-7));...
x_2(2,(2*in-5)); x_2(2,(2*in-3));...
x_2(2,(2*in-1))]);

t_t_2 = roots([(c2(1)/5) (c2(2)/4)...
(c2(3)/3) (c2(4)/2) c2(5)-((c2(1)/5)*...
(tmr(in)^5)+(c2(2)/4*(tmr(in)^4)+...
(c2(3)/3)*(tmr(in)^3)+(c2(4)/2)*...
(tmr(in)^2)+c2(5)*tmr(in) + (xf(1,1)-...
x_2(1,(2*in-1)))))]);

t_t2 = [t_t2 t_t_2];
temp2 = [];
for i=1:1:5
    if abs(imag(t_t_2(i)))≤1e-3
        t_t_2(i) = real(t_t_2(i));
    end
end
for i=1:1:5
 temp2(i) = isreal(t_t_2(i)) && 0<t_t_2(i);
end
te_2_x = max(temp2'.*t_t_2);

c2 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
(tmr(in-4)^2)  tmr(in-4) 1; (tmr(in-3)^4)...
(tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
(tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
tmr(in-2) 1; (tmr(in-1)^4) (tmr(in-1)^3)...
(tmr(in-1)^2) tmr(in-1) 1; (tmr(in)^4)...
(tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
[x_2(2,(2*in-8));x_2(2,(2*in-6));...
x_2(2,(2*in-4)); x_2(2,(2*in-2));...
x_2(2,(2*in))]);

t_t_2 = roots([(c2(1)/5) (c2(2)/4)...
(c2(3)/3) (c2(4)/2) c2(5)-...
((c2(1)/5)*(tmr(in)^5)+(c2(2)/4)*...
(tmr(in)^4)+(c2(3)/3)*(tmr(in)^3)+...
(c2(4)/2)*(tmr(in)^2)+...
c2(5)*tmr(in) + (xf(1,1)-...
x_2(1,(2*in))))]);

t_t2 = [t_t2 t_t_2];
temp2 = [];
for i=1:1:5
    if abs(imag(t_t_2(i)))≤1e-3
        t_t_2(i) = real(t_t_2(i));
    end
end
for i=1:1:5
    temp2(i) = isreal(t_t_2(i)) && 0<t_t_2(i);
end
te_2_y = max(temp2'.*t_t_2);
te_2 = min(te_2_x,te_2_y);

c3 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
(tmr(in-4)^2)  tmr(in-4) 1;...
(tmr(in-3)^4) (tmr(in-3)^3) (tmr(in-3)^2)...
tmr(in-3) 1; tmr(in-2)^4) (tmr(in-2)^3)...
(tmr(in-2)^2) tmr(in-2) 1;(tmr(in-1)^4)...
(tmr(in-1)^3) (tmr(in-1)^2) tmr(in-1) 1;...
(tmr(in)^4) (tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
[x_3(2,(2*in-9));x_3(2,(2*in-7));...
x_3(2,(2*in-5)); x_3(2,(2*in-3));...
x_3(2,(2*in-1))]);

t_t_3 = roots([(c3(1)/5) (c3(2)/4) (c3(3)/3)...
(c3(4)/2) c3(5)-((c3(1)/5)*(tmr(in)^5)+...
```

```
        (c3(2)/4)*(tmr(in)^4)+(c3(3)/3)*...
        (tmr(in)^3)+(c3(4)/2)*(tmr(in)^2)+c3(5)*...
        tmr(in)+(xf(1,1)-x_3(1,(2*in-1))))]);

        t_t3 = [t_t3 t_t_3];
        temp3 = [];
        for i=1:1:5
            if abs(imag(t_t_3(i)))≤1e-3
                t_t_3(i) = real(t_t_3(i));
            end
        end
        for i=1:1:5
         temp3(i) = isreal(t_t_3(i)) && 0<t_t_3(i);
        end
        te_3_x = max(temp3'.*t_t_3);

        c3 = ([(tmr(in-4)^4) (tmr(in-4)^3)...
        (tmr(in-4)^2) tmr(in-4) 1; (tmr(in-3)^4)...
        (tmr(in-3)^3) (tmr(in-3)^2) tmr(in-3) 1;...
        (tmr(in-2)^4) (tmr(in-2)^3) (tmr(in-2)^2)...
        tmr(in-2) 1; (tmr(in-1)^4) (tmr(in-1)^3)...
        (tmr(in-1)^2) tmr(in-1) 1; (tmr(in)^4)...
        (tmr(in)^3) (tmr(in)^2) tmr(in) 1] \...
        [x_3(2,(2*in-8));x_3(2,(2*in-6));...
        x_3(2,(2*in-4)); x_3(2,(2*in-2));...
        x_3(2,(2*in))]);

        t_t_3 = roots([(c3(1)/5) (c3(2)/4)...
        (c3(3)/3) c3(4)/2) c3(5)-((c3(1)/5)*...
        (tmr(in)^5)+(c3(2)/4)*(tmr(in)^4)+...
        (c3(3)/3)*(tmr(in)^3)+(c3(4)/2)*...
        (tmr(in)^2)+c3(5)*tmr(in)+ (xf(1,1)-...
        x_3(1,(2*in))))]);

        t_t3 = [t_t3 t_t_3];
        temp3 = [];
        for i=1:1:5
            if abs(imag(t_t_3(i)))≤1e-3
                t_t_3(i) = real(t_t_3(i));
            end
        end
        for i=1:1:5
         temp3(i) = isreal(t_t_3(i)) && 0<t_t_3(i);
        end
        te_3_y = max(temp3'.*t_t_3);
        te_3 = min(te_3_x,te_3_y);

        te_calc = [te_calc [te_1;te_2;te_3]];
        te_r =[te_r min([te_1;te_2;te_3])];
        tm_c = tm;
        % Time delay matrix is formed here
        h_t = hmean + hst*abs(randn(3));
        h = [h h_t];
        te_t = [te_t te_h];
        te_rec = [te_rec te];
    end
%-------------------------------------------------------
% Rendezvous time information is updated below
    [in1 in2] = size(h);
    in21 =in2/3;
    for i=1:1:3
        for j=0:1:2
            if (h(i,in2-j) ≤ hmax) && ((tm_c + h(i,in2-j))≤ tm)
                te_h(i,3-j) = te_calc (3-j,in21);
            end
            if (i==(3-j))
                te_h(i,3-j) = te_calc (3-j,in21);
            end
        end
    end
    te = min(te_h');
%-------------------------------------------------------------------
% Terms related with the Controllability Matrix W_c are below
    Wc1  = int(expm(A*(tb-t))*B*B'*expm(A'*(tb-t)),t,tb,te(1));
    eat1 = expm(A'*(tb-t));
    ea1  = expm(A*(tb-te(1)));
    Wc2  = int(expm(A*(tb-t))*B*B'*expm(A'*(tb-t)),t,tb,te(2));
    eat2 = expm(A'*(tb-t));
    ea2  = expm(A*(tb-te(2)));
    Wc3  = int(expm(A*(tb-t))*B*B'*expm(A'*(tb-t)),t,tb,te(3));
    eat3 = expm(A'*(tb-t));
    ea3  = expm(A*(tb-te(3)));
%---------------------------------------------------------------------
% Here, control inputs are calculated and disturbances are added
    up_1= - B'*eat1*inv(Wc1)*(x0_1-ea1*xf);
    up_1 = up_1;
    up_1 = up_1  + (up_1/50).*((rand>0.9)*randn(1,2));
    up_2 = - B'*eat2*inv(Wc2)*(x0_2-ea2*xf);
    up_2 = up_2;
    up_2 = up_2  + (up_2/50).*((rand>0.9)*randn(1,2));
    up_3 = - B'*eat3*inv(Wc3)*(x0_3-ea3*xf);
    up_3 = up_3;
    up_3 = up_3  + (up_3/50).*((rand>0.9)*randn(1,2));
%----------------------------------------------------------------
% Solutions are obtained below
    d_1 = int(expm(A*(p-t))*B*up_1,t,tb,p);
    xp1_1 = subs(d_1,p,tm);
    up1_1 = subs(- B'*eat1*inv(Wc1)*(x0_1-ea1*xf),t,tm);
    xp_1 = ((expm(A*(tm-tb))*x0_1) + (xp1_1  ));
    d_2 = int(expm(A*(p-t))*B*up_2,t,tb,p);
    xp2_1 = subs(d_2,p,tm);
    up2_1 = subs(- B'*eat2*inv(Wc2)*(x0_2-ea2*xf),t,tm);
    xp_2 = ((expm(A*(tm-tb))*x0_2) + (xp2_1  ));
    d_3 = int(expm(A*(p-t))*B*up_3,t,tb,p);
    xp3_1 = subs(d_3,p,tm);
    up3_1 = subs(- B'*eat3*inv(Wc3)*(x0_3-ea3*xf),t,tm);
    xp_3 = ((expm(A*(tm-tb))*x0_3) + (xp3_1  ));
%----------------------------------------------------------------
% Here, solutions are added into corresponding matrices
    x_1 = [x_1 xp_1];
    u_1 = [u_1 up1_1];
    x_2 = [x_2 xp_2];
    u_2 = [u_2 up2_1];
    x_3 = [x_3 xp_3];
    u_3 = [u_3 up3_1];
%----------------------------------------------------------------
% States are collected in single matrices for each vehicle
    st_1 = [st_1; xp_1(1,1) xp_1(1,2) xp_1(2,1)...
            xp_1(2,2) xp_1(3,1) xp_1(3,2)...
            up1_1(1,1)up1_1(1,2)];
```

```
    st_2 = [st_2; xp_2(1,1) xp_2(1,2) xp_2(2,1)...
            xp_2(2,2) xp_2(3,1) xp_2(3,2)...
            up2_1(1,1) up2_1(1,2)];
    st_3 = [st_3; xp_3(1,1) xp_3(1,2) xp_3(2,1)...
            xp_3(2,2) xp_3(3,1) xp_3(3,2)...
            up3_1(1,1) up3_1(1,2)];
%---------------------------------------------------------------
% Required simulation updates are made below
    tem = [tem te'];
    tb = tm;
    tm = tm + inc;
    tmr = [tmr tm];
    x0_1 = xp_1;
    x0_2 = xp_2;
    x0_3 = xp_3;
    xf = [(tm/4) (tm/4);(1/4) (1/4);0 0];
    xf_1 = [xf_1; xf(1,1) xf(1,2) xf(2,1)...
            xf(2,2) xf(3,1) xf(3,2)];
end
%---------------------------------------------------------------
% Magnitudes of velocities, accelerations, control inputs,
% target vel. and acce. are calculated here
v1 = sqrt((st_1(:,3)).^2 + (st_1(:,4)).^2);
v2 = sqrt((st_2(:,3)).^2 + (st_2(:,4)).^2);
v3 = sqrt((st_3(:,3)).^2 + (st_3(:,4)).^2);
a1 = sqrt((st_1(:,5)).^2 + (st_1(:,6)).^2);
a2 = sqrt((st_2(:,5)).^2 + (st_2(:,6)).^2);
a3 = sqrt((st_3(:,5)).^2 + (st_3(:,6)).^2);
u1 = sqrt((st_1(:,7)).^2 + (st_1(:,8)).^2);
u2 = sqrt((st_2(:,7)).^2 + (st_2(:,8)).^2);
u3 = sqrt((st_3(:,7)).^2 + (st_3(:,8)).^2);
vf = sqrt((xf_1(:,3)).^2 + (xf_1(:,4)).^2);
af = sqrt((xf_1(:,5)).^2 + (xf_1(:,6)).^2);
```

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