

CHOCOLATE PRODUCTION LINE SCHEDULING: A CASE STUDY

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

SEPTEMBER 2006

Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

CHOCOLATE PRODUCTION LINE SCHEDULING: A CASE STUDY

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September 2006, 73 pages

This study deals with chocolate production line scheduling. The particular production line allows producing multiple items at the same time. Another distinguishing property affecting the planning methodology is that an item can have different production capacities when produced in different product combinations which are called production patterns in this study. Planning is done on a 12 weeks rolling horizon. There are 21 products and 103 production patterns covering all the production possibilities. The subject of the study is to construct an algorithm that gives 12 weeks' production values of each product and to construct the shift based scheduling of the first week of the planning horizon. The first part is Master Production Scheduling (MPS) and the objective is minimizing the shortage and overage costs. A mathematical modeling approach is used to solve the MPS problem. The second part is the scheduling part which aims to arrange the production patterns obtained from the MPS module within the shifts for the first week of the planning horizon considering the setup times.

The MPS module is a large integer programming model. The challenge is finding a reasonable lower bound whenever possible. If it is not possible, finding a reasonable upper bound and seeking solutions better than that is the main approach. The scheduling part, after solving MPS, becomes a TSP and the setup times are sequence independent. In this part, the challenge is solving TSP with an appropriate objective function.

Keywords: Chocolate Production Line Scheduling, Master Production Schedule, Traveling Salesman Problem, Integer Programming

ÖZ

ÇİKOLATA ÜRETİM HATTI ÇİZELGELEMESİ: BİR VAKA ÇALIŞMASI

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Tez Yöneticisi: Prof. Dr. Ömer KIRCA

Eylül 2006, 73 sayfa

Bu çalışmanın konusu olan hat aynı anda birden fazla ürünün üretilmesine imkan veren özellikte bir çikolata üretim hattıdır. Hattın planlama yöntemleri üzerinde etkisi olan bir başka özelliği ise bir ürünün birlikte üretildiği diğer ürünlere bağlı olarak üretim kapasitesinin değişmesidir. Aynı anda üretilen ürün gruplarına bu çalışmada üretim şablonları denecektir. Planlama 12 haftalık döngüsel bir ufukta yapılmaktadır. 21 ürün ve bu ürünlerin tüm üretim seçeneklerini kapsayan 103 üretim şablonu mevcuttur. Çalışmanın konusu, her ürünün 12 haftada ne kadar üretileceği bilgisini veren bir algoritmanın tasarımı ve ilk hafta için, ilk bölümün sonuçlarını kullanarak, vardiya bazında çizelgeleme yapacak bir metodun geliştirilmesidir. İlk bölüm, Ana Üretim Programının (AÜP) oluşturulmasıdır. Amacı yok satma ve fazla üretme maliyetlerinin toplamını minimize etmektir. Bu bağlamda, matematiksel modelleme temelli bir algoritma geliştirilmiştir. İkinci bölümün amacı ise, AÜP’den alınan sonuçlar kullanılarak, üretim şablonlarının, planlama ufkunun ilk haftası için, vardiya bazında, setup zamanları düşünülerek sıralanmasıdır.

AÜP büyük boyutlu bir tamsayı programlama modelidir. Uğraş konusu mümkün olduğu durumlarda kabul edilebilir bir alt sınır bulup ona yaklaşmaktır. Eğer bu mümkün değilse, kabul edilebilir bir üst sınır bulup ondan daha iyi sonuçlar araştırmaktır.

Çizelgeleme kısmı ise, AÜP'den sonuçları aldıktan sonra sıra bağımsız kurulum zamanları ile bir Gezgin Satıcı Problemine (GSP) dönüşmektedir. Bu kısımda ise uğraş uygun bir amaç fonksiyonu ile GSP'ni çözmektir.

Anahtar Kelimeler: Çikolata Üretim Hattı Çizelgelemesi, Ana Üretim Programı, Gezgin Satıcı Problemi, Tamsayı Programlama

To my parents and my brother

ACKNOWLEDGEMENTS

I would like to thank to Prof. Dr. Ömer Kırca, the supervisor of this study, for his valuable guidance throughout my study. I would also thank to Assoc. Prof. Sedef Meral for the motivating attitude throughout my master study.

This study is the result of the support and understanding of Mr. Etkin Binbaşıoğlu and Mr. Hakan Polatoğlu. I wish to express my appreciation to Adem Doğar for his valuable suggestions.

I would also like to thank to my parents, Nizamettin Çölova and Gülseren Çölova, as they never give up on me throughout my life. The last but not the least, I am very thankful to my brother Melih Çölova for his delighting attitude and for his helps in editing the document.

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CHAPTER 1

INTRODUCTION

The firm that is considered in this study operates in the Fast Moving Consumer Goods (FMCG) sector since 1961. It produces biscuits, crackers, cakes, chocolates and bars. Number of different products that the firm produces is approximately 300. The Firm's products are sold mainly in domestic market. It produces its products to be sold in about 200000 sales points around Turkey.

The firm, because of low customer loyalty in the FMCG sector, pays great attention to the availability of its products on the shelves. There are so many levels that a firm must be successful to satisfy the availability at salespoints, but the first is to work with production plans. For that purpose, at the end of the year 2002, the Systems Design team of the firm started to implement a brand new approach for the Firm's production and inventory management activities, called Production and Inventory Management System (PIMS). PIMS consists of a number of sub-modules which are production orders management, MPS, scheduling, labor planning, and MRP relations.

The sales department develops the product based weekly demand forecasts for 12 weeks which means the total expected shipment amount from the plants to the customer for a week and a product. These data are called the "production orders". The planning department evaluates these production orders using the MPS module which is a linear programming based mathematical model. Planning department transfers the information of the amounts met from the orders and amounts that cannot be met from the orders because of the capacity constraints or some other reasons to the sales department. Then a commitment is done so that the production

of agreed upon amounts are guaranteed by the production firm. Due to the capacity constraints, the model should be designed in such a way that it enables the production of some products earlier than the time that is going to be sold. So, an inventory management model is needed to manage this situation. According to the inventory management system, all the production goes into the reserved inventory, the amount that is agreed on to be prepared is released at the beginning of each week, the remaining part of the inventory is kept as a reserved inventory. This procedure continues on a rolling horizon basis every week. Figure 1.1 describes the flow of the procedure.

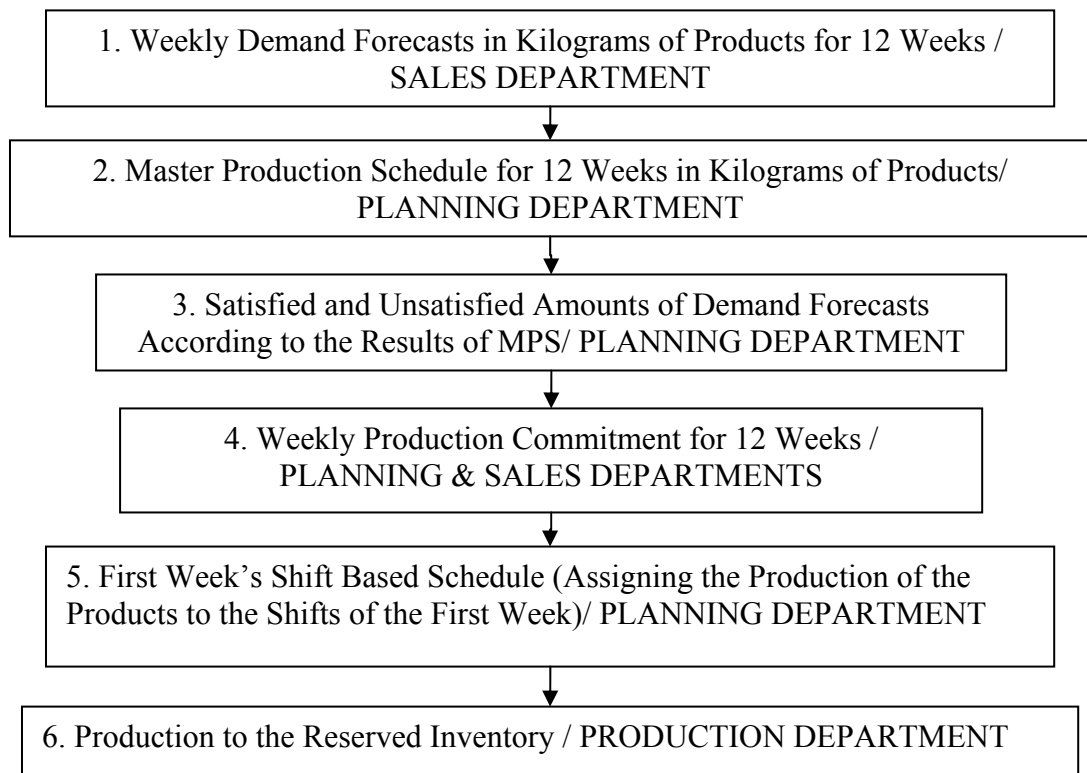


Figure.1.1 Production Planning Hierarchy

The detailed scheduling of the MPS's first week plan is done by the Scheduling model. This model assigns the products to the shifts considering the setup times. Then, the production is done according to this shift based schedule.

Besides the Firm's relatively long history in the snacking industry, it has not been long ever since the production of bars and chocolate started. By the end of the year 2003, bar production started and by the end of the year 2004 so did the chocolate production. Chocolate production line has some characteristics that make it special from the view point of planning and production relative to the biscuit, cracker and cake (classical) production lines. First is that while only one product can be produced on the classical lines simultaneously, multiple products can be produced on the chocolate production line at the same time. Another diversity is the production rate that differs for the same product as it is produced together with different products on the line at the same time. These two differences of the chocolate production line make it impossible to use the existing MPS module, since it assumes fixed capacities for the products. Also, the decision becomes "how many of which product combinations should be produced each week" from "how much of which product to be produced each week". This situation makes impossible to implement the second step of the Planning Procedure with the current design (Figure 1.1). Moreover, it becomes "assigning the product combinations to shifts" from "assigning products to shifts" which causes the step 5 of the Planning Procedure to be useless unless modified (Figure 1.1).

The discussed features of the chocolate production line, incorporates a new concept which is the production patterns. The chocolate production line allows producing multiple products at the same time, but the production rate of one product differs in different production patterns. The production patterns shelter the information that, of what products it is consisting of and the hourly production rate of the products in it.

Production patterns replaces the product base modeling approach to the production pattern base modeling approach, meaning that, in the MPS model the decision variable is no longer the production amounts of the products for each week directly, rather the decision variable is the number of shifts that a production pattern is going to be produced in each week and next, the production amount of the products for each week is calculated using this data.

The basic assumption of the production planning process is shifts are not preemptive. That is to say, the time bucket for production is a shift. Only one setup is allowed in a shift. This makes the decision variable, which is the number of shifts that a production pattern is going to be produced in each week, an integer decision variable. Also, if the decision variable is not integer then the solution of MPS will be away from reality since the capacity is very flexible with the introduction of patterns.

Once the MPS is solved, its first week's plan is frozen to be produced. Other eleven weeks master plan can be changed with the new demand information at the following week. Obtaining the first week's plan, the production sequence of the patterns for the first week should be decided. This part is called the "Scheduling Module". In the Scheduling Module multiple setups should be considered during a production change since the changes are not between the products any longer, it is so between the production patterns which consist of multiple products.

These motivations led to this study which aims to develop mathematical models by incorporating multiple products for MPS and Scheduling modules and to develop solution methodologies for these models.

The MPS model with its new formulation becomes an integer model. Thus, obtaining an optimum solution becomes much more difficult compared to the original linear MPS model. This challenge led to making some approximations to find acceptable solutions. Linear relaxations and some methodologies to reduce the number of the decision variables are applied to find acceptable solutions. Then, an algorithm is developed to find an acceptable solution to the problem incorporating the integer MPS model and its approximation based derivatives. Eventually, the obtained solution is compared with the outputs of the existing MPS system.

After obtaining solutions to the MPS model, the second phase is scheduling the production patterns obtained from the new MPS model. The production patterns of the first week are transferred to the Scheduling model. The Scheduling model

assigns the patterns to the shifts of the first week using a mathematical model which is constructed as a Traveling Salesman Problem (TSP). The objective of the scheduling model is arranging the production patterns so as to maximize the production time for each product in a shift considering the setup times according to a weight given, using the shortage and overage values for products. In the scheduling model, optimum value is obtained by solving the TSP based model.

In this thesis, there exist 6 chapters. In Chapter 2, the characteristics of the Chocolate Production Line, the inner customers and the suppliers of the planning process, the information flow between them, the aim and the boundaries of the study are discussed in detail. In Chapter 3, related literature on MPS and Scheduling are introduced. In Chapter 4, the test data that is used in the design stage is introduced, the design methodology of the MPS is explained in detail and the developed model is tested with the current model and a heuristic model. In Chapter 5, the design methodology of the Scheduling Module is explained. Finally, in Chapter 6, conclusions and further research areas are briefed.

CHAPTER 2

PROBLEM DEFINITION AND SCOPE OF THE STUDY

In this chapter, the specific chocolate production line is described; consecutively, the structure of the production and the characteristics affecting the planning procedure is explained. Second, the planning environment and its relationships with the other systems like Sales Forecasting System (SFS) and Material Requirements Planning (MRP) is discussed for the case. Finally, the scope of the study is stated.

2.1. The Chocolate Production Line

The Chocolate Production Line (CPL) consists of a number of machines to inject liquid chocolate into the molds, to cool the chocolate, to construct some figures and to pack. Production process of a product consists of a combination of these machines in a given order. Figure 2.1 illustrates the CPL's layout. Trays of products enter the line and are processed in a given order. In Figure 2.1, Product A flows through 4 process points and Product B flows through 2 process points. Trays are fed into the line in a given order. For example, according to the production rate of the products 2 trays of B and 1 tray of A is fed in a loop. Also, the products A and B form a production pattern. A list of the production patterns is given in Appendix A.

Because of the characteristics and technological constraints, at most three products are produced at the same time. Other than that, the products are classified into three groups and any single product of an arbitrary group cannot be produced at the same time with another product from the same group. For example, in the Figure 2.1

products A and B are of different groups. This constraint is guaranteed while the production patterns are formed. That is, two products from the same product group cannot be involved in a production pattern, because products in the same group use the same resources.

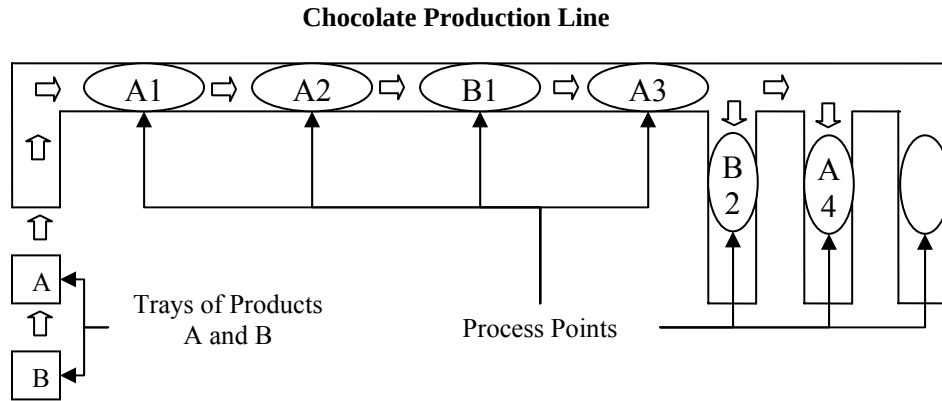


Figure. 2.1 The Sketch of the Chocolate Production Line

The time bucket for the production is a shift. In a shift, a given production pattern is produced.

Setup time for a product in a pattern depends on the preceding products in the pattern produced in the preceding shift. The setup time is present for a product if a product from the same product group was present in the pattern produced in the preceding shift. Otherwise, setup time for that product is “0” in that shift. Setup times are not sequence dependent.

Production of a product can start independent of other products in the pattern. If the setup takes shorter for a product relative to the other products in a pattern, the production can start for that product even if the setups are in progress for the others.

There is a predefined loss ratio during the production of each product. This ratio includes the setup, scrap and the break-downs.

2.2. The Planning Environment

The production is planned by the planning department of the Firm which is marketing department's client for the demand forecasts and supplier of the purchasing department for the material requirements also, supplier of the production department for the shift based production plan. (Figure 2.2)

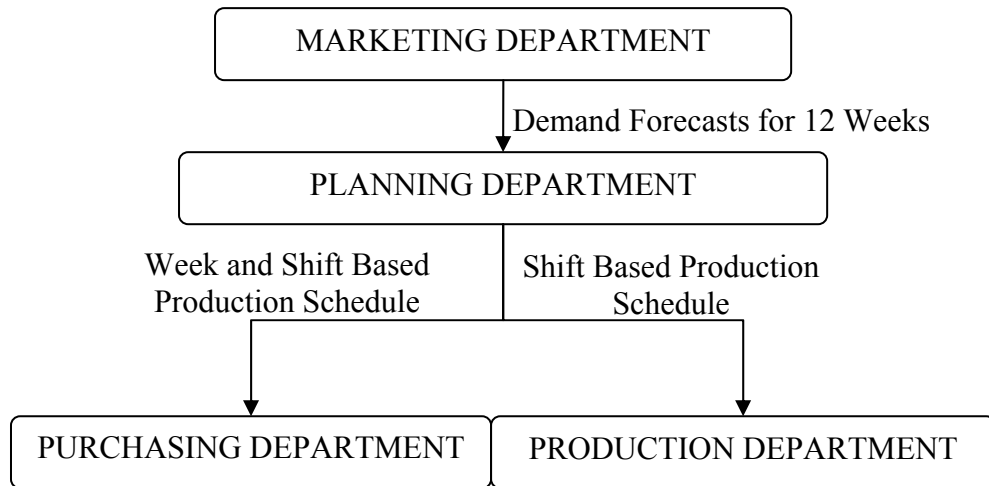


Figure 2.2 Clients and Suppliers of the Planning Department

Analyzing marketing department's role in the planning process, marketing department makes weekly demand forecasts for each product that will be produced on CPL in kilograms using several forecasting techniques. These forecast figures imply that, in any week, the total shipment from the chocolate plant to the customers will be approximately that much. According to the commitments between production firm and marketing firm, the production firm guarantees the availability of the products in the ordered quantities at the beginning of each week. Weekly forecasts cover a period of 12 weeks, beginning from two weeks ahead of the planning week. Namely, planning week is the week in which the planning process is run. (Figure 2.3) According to Figure 2.3, the first demand forecast figure is for the week 1.

Marketing department renews demand forecasts every week on a rolling horizon basis. Planning department, makes the plan for the week 0 (Figure 2.3) on a shift basis –which product is going to be produced in which shift- and it is frozen. Other weeks' plans are made in kilograms of each product for a week. These figures are used by the purchasing department to run the MRP model. MRP model runs for the first 18 shifts of the week 0 and remaining weeks 1 through 11 in Figure 2.3.

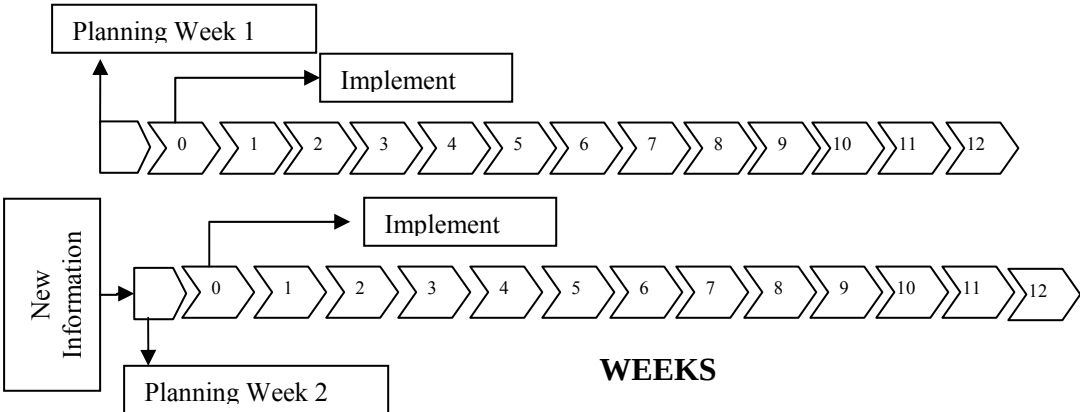


Figure 2.3 Planning Horizon

Since the product is highly replaceable in the market, any order that cannot be satisfied at the beginning of the week is considered as a lost sale meaning that no backorders are allowed in the planning process.

2.3. The Scope of the Study

Scheduling of CPL consists of determining the quantities to be produced of each product for 12 weeks and determining which pattern is produced in which shift for the first week.

The costs associated with CPL scheduling are:

- (1) Shortage cost: Since backordering is not allowed; any demand that is not satisfied is a lost sale. The cost of a lost sale is the loss of the profit from

that much of the demand that is not satisfied. Moreover, any demand that is not satisfied will cause inconvenience thus, this inconvenience should be reflected to the shortage cost. So, the real shortage cost is the multiplication of the loss of profit with a coefficient greater than 1. This coefficient is a compromise of the marketing and the production firms.

- (2) Overage cost: Overage cost is the inventory holding cost. The inventory holding cost is calculated on a weekly basis. Also, it includes only the financial cost of the inventory holding. The cost of production of a product is assumed to be the fund that is the basis for financial loss calculation. The interest rate is the firm's own calculated weekly ROI.

The first part of the study aims to minimize the total cost associated with shortage and the overage costs over the 12 weeks period. This is, namely, the Master Production Scheduling (MPS) part.

The second part of the study aims to schedule the MPS results for the shifts of the first week, meaning that every shift of week 1 is combined with a production pattern. The main objective in the second part is to arrange the patterns so that the production quantities are nearest to the MPS results for the products without a shortage problem and the production quantities are nearest to the demand values for the products with a shortage problem.

CHAPTER 3

LITERATURE SURVEY

First part of the study is the design of the MPS model and solution methodology for the model. The studies on MPS mostly focus on the effect of freezing the master plans in different ways.

Lin, Kraje, Leongb, Bentonb (1993) examine the effects of environmental factors such as cost structure, bill of material (BOM) structure, cumulative lead time, magnitude of MPS change costs, and the magnitude of forecast error on the choice of frozen interval for a single end item in an uncertain environment where a rolling schedule is used. Xie, Zhao and Lee (2002) investigate the impact of the freezing multi item MPS on a rolling horizon basis where the demand is uncertain and also simulate an MPS system and observe the effects of the environmental factors on the selection of the MPS freezing factors. For generating an MPS they use a single level capacitated lot sizing methodology, but for the cases that the capacity is not sufficient they have implemented a net requirement reduction algorithm to make the capacity sufficient.

Das, Rickard, Shah and Macchietto (2000) propose an information system to link the aggregated production plan, master production plan, and the short term scheduling models. While doing this, they have used a linear model for aggregated production planning and also for the MPS part they have used a model very similar to the Karajewski and Ritzman (1993)'s model which requires the inputs beginning inventory levels, lot sizes, and demand forecasts.

Han, Duplaga and Kimb (1994) discuss the master production scheduling procedure at Hyundai Motor Company. The firm's approach to the MPS problem is arranging monthly meetings and providing an agreed upon MPS at these meetings. The particular MPS which is the subject of this study is so specific that no directly related study has been encountered during the search. Its uniqueness comes from the production with predefined patterns and instability of the production rate for the products. Moreover, chocolate production and planning studies are searched and no related study has been found, neither.

For the second part which is sequencing the first week's production patterns according to an objective function is the TSP problem. The problem is to find the shortest path connecting the predefined nodes which are acquired in the MPS part.

TSP problem is a well-known problem with numerous solution algorithms. The general formulation is as follows and first formulated by Duntzig, Fulkerson and Johnson (1954):

$$\min \sum_{\forall i} \sum_{\forall j} c_{ij} x_{ij}$$

st

$$\sum_{\forall j} x_{ij} = 1 \quad \forall i$$

$$\sum_{\forall i} x_{ij} = 1 \quad \forall j$$

$$\sum_{i,j \in M} x_{ij} \leq |M| - 1, \quad \forall M \subset N \text{ such that, } |M| \geq 2 \text{ and } \{1\} \notin M$$

x_{ij} is a binary variable denoting if node j is visited after node i . N is the set of cities. The last set of constraints is for eliminating subtours.

Miller, Tucker and Zemlin (1960) proposed new constraints to eliminate the sub tours. The model is called MTZ. The constraints are in the form as below:

$$u_i - u_j + A_{ij}x_{ij} \leq B_{ij} \quad \forall i, j$$

U_i 's are real variables which are called “sequencing variables”. U_i represents the position of the i^{th} node if the fixed constants A_{ij} and B_{ij} is taken as $(n-1)$ and $(n-2)$ respectively, where n represents the number of nodes.

Desrochers, Laporte (1991) proposed new constraints for the subtour elimination constraints of the TSP and they called the new model as the Lifted MTZ. They have shown that Lifted MTZ gives better solutions especially on the symmetric TSP.

For solving TSP, heuristic algorithms are highly employed to obtain a solution for large scale problems in a reasonable time. Aart (1988) used simulated annealing for the solution of the TSP, Fietcher (1990) employed a tabu search algorithm for TSP, genetic algorithms are used by Potvin (1996).

Although MTZ formulation is not a very effective way of solving the TSP, in this study with the specific objective function, and the size of the problem not being too large CPLEX is able to solve it in a very small amount of time. So, this formulation is used while modeling.

CHAPTER 4

MPS MODULE DESIGN

The planning module consists of two sub-modules. These are the MPS module and the scheduling module. In this chapter, the work carried out for the MPS module is discussed.

Since the production rate differs for the same product depending on the pattern it is produced in, it is not possible to simply construct a model deciding how many kilograms to produce for each week of which product. The question to answer first is how many of each pattern should be produced each week so that minimum cost is acquired, leading to the production amounts of each product for each week of the planning horizon. Thus, a model is constructed to give the answer.

In this part, the notation of the MPS model is introduced. After that the test data used in the models are examined. Then, the methodology for finding a solution to the MPS module is defined in detail. Finally, the results are tested.

4.1 Notation

Three indices are used while modeling. First one is for the products (j). Index (j) can take values between 1 and 21, since there are 21 products. The index (t) refers to the weeks of the planning horizon, and it is between 0 and 12. Week 0 is the first week in which production can be done. Week 12 is the week for which the last demand forecast exists. The index (i) refers to the production patterns. They are between 1 and 103. There are 103 patterns covering all production possibilities. Zero production is not considered as a pattern.

Parameters are listed below: (all the monetary values are given in YTL.)

P_j : Markup of product j ;

Q_j : Production cost of product j ;

D_{jt} : Demand forecast of product j in week t ; demand forecast for week 0 does not exist and the last demand forecast is for week 12. Demand is given in kilograms.

r : Weekly interest rate; this is the weekly ROI for the firm.

l : Loss ratio; loss ratio contains setup times, scrap and breakdowns for the production line. This number is taken as a constant for all patterns.

C_{ij} : Production rate of product j while it is produced in pattern i ; for the products that are included in pattern i , this number is greater than 0, otherwise it is equal to 0. Production rate is given in kilograms per hour.

a_j : Beginning inventory level of product j ; for all products there is a given inventory level at the beginning of the planning horizon, this number refers to that value in kilograms.

α : The weight of the shortage cost in the objective function; this weight is greater than 1.

Decision variables are listed below:

Y_{it} : The number of the pattern i which is produced in week t .

TC : Total cost;

SC : Shortage cost;

OC : Overage cost;

I_{jt}^+ : Inventory on hand of product j at the beginning of week t ;

I_{jt} : Shortage of product j at the beginning of the week t ;

Z_{jt} : Total production amount of product j in the week t ;

4.2. Test Data and Parameter Values

For some of the parameters used in the model there are standard values that are obtained by discussions with the firm managers and by observations. These values

are not considered as a test data and the models are not run for different values of these parameters. These parameters and their values are given below; but, since the profit and cost parameters are confidential, they are not listed.

r: The in house used ROI figure is 0.004 for May of 2006.

l: Depending on the planning departments works, loss ratio is used as 0.15.

C_{ij}: Production rate values are given in kilograms for the products. (Appendix A)

a_j: For every run the beginning inventory was considered to be 0 for each product.

α: The inconvenience factor is taken as 0.15, so α is equal to 1.15.

Unlike the other parameters, demand forecast values are used as a test data with multiple demand sets. Demand for the firm's products is highly seasonal with a lag of a year. So, as to test data, demand forecasts done in the first week of each month of the second half of 2005 and the first half of 2006 are used. Thus, twelve demand sets are used as the test data.

The test data contains situations that the average capacity of the chocolate production line is smaller and greater than the demand forecasts. So, the shortage and overage situations can be observed with the test data. The total amount of demand forecasts among the twelve weeks planning horizon is compared with the total 12 weeks capacity of CPL in Figure 4.1. For the test data 1, 10, 11, and 12 the total capacity is above the total demand forecasts, whereas the test data 3, 5, and 6 are more than the total capacity in various amounts. Also, for the test data 2, 4, 7, 8 and 9 the total demand forecast is near the total capacity.

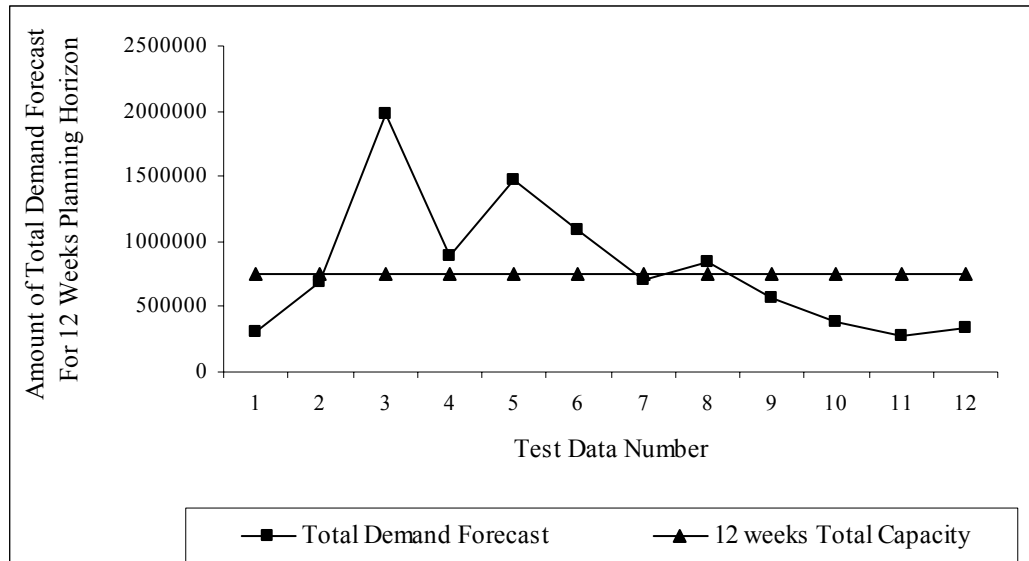


Figure 4.1 Total of the demand forecasts in the test data and the total 12 weeks capacity

Moreover, when the test data is analyzed individually, among the twelve weeks planning horizon, the scenarios of (a) increasing demand, (b) decreasing demand, (c) steady demand forecasts with respect to amounts are covered. In Figure 4.2, four samples of the test data are represented.

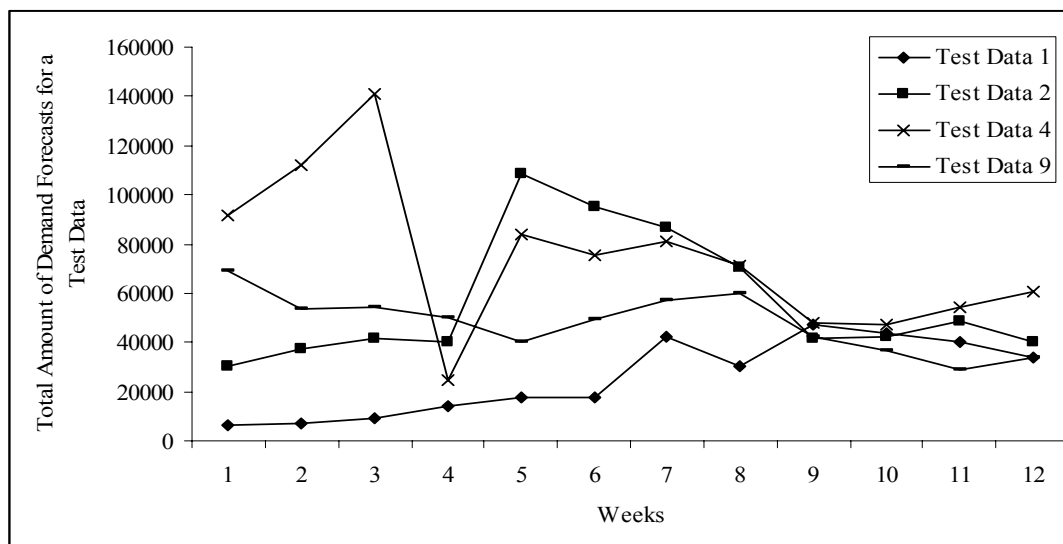


Figure 4.2 Samples of the test data

4.3. Methodology

Since the production rate differs for the same product within different patterns; it is not possible to build a linear model. Instead, a model to determine the number of the patterns to be produced in each week should be constructed. Since the time unit for production is a shift, the decision variable becomes the number of shifts to be produced of each pattern in each week. Since the shifts are assumed to be non-preemptive, the decision variable becomes an integer decision variable.

Initially, the number of the shifts in a week is assumed to be fixed and equal to 18, meaning that every day contains 3 shifts and there are 6 regular working days. (In the Section 4.4, the overtime shifts will be considered.)

Below, the preliminary MPS model is introduced. In this model, there are 12 weeks and 103 patterns meaning that there exist 1236 integer decision variables and each decision variable can take values between 0 and 18.

MPS-1

$$\min (TC = \alpha SC + OC) \quad (4.3.1)$$

Subject to

$$SC = \sum_{\forall j} \sum_{\forall t} P_j \times Q_j \times I_{jt}^- \quad (4.3.2)$$

$$OC = \sum_{\forall j} \sum_{\forall t} [(Q_j \times I_{jt}^+) / 12] \times [(1 + r)^{12} - 1] \quad (4.3.3)$$

$$I_{j(t+1)}^+ - I_{j(t+1)}^- = I_{jt}^+ + Z_{jt} - D_{j(t+1)} \quad ; \quad For \ \forall t < 12, \ For \ \forall j \quad (4.3.4)$$

$$Z_{jt} = \sum_{\forall i} Y_{ti} \times 8 \times (1 - l) \times C_{ij} \quad ; \quad For \ \forall t < 12, \ For \ \forall j \quad (4.3.5)$$

$$\sum_{\forall i} Y_{ti} \leq 18 \quad ; \quad For \ \forall t < 12 \quad (4.3.6)$$

$$I_{j0}^+ = a_j \quad ; \quad For \ \forall j \quad (4.3.7)$$

$$Y_{ti} \text{ integer variable} \quad (4.3.8)$$

The definitions of the equations of the model are as follows:

(4.3.1) Objective function to be minimized: TC value is the sum of the shortage and overage costs with the given weights. This value refers to the total cost for the entire planning horizon.

(4.3.2) Total shortage cost equation: Shortage cost for the entire planning horizon is equal to the summation of loss of profits for each week's shortages for each product. Loss of profit from a product for a week is calculated as the multiplication of the markup of the particular product with the cost of producing it and the shortage quantity for that week.

(4.3.3) Total overage cost: Overage cost for the entire planning horizon is equal to multiplication of the YTL value of the average inventory level with the 12 weeks' interest rate.

(4.3.4) Inventory balance: Inventory position (inventory position can be a negative value since shortages can occur.) at the beginning of a week is equal to the demand forecast of the current week subtracted from the summation of the inventory on hand at the beginning of the previous week and the previous week's production of that product.

(4.3.5) Weekly production quantity: Total weekly production of a product in a given week is equal to the summation of the production of that product in any pattern in any shift in that week. The production quantity of a product in a pattern for a week is found as the multiplication of the number of the patterns to be produced that week with the working hours in a shift and with the loss ratio and with the hourly production rate of the product in that particular pattern.

(4.3.6) Maximum number of shifts in a week: The maximum number of the shifts in which a pattern is decided to be produced can be 18.

(4.3.7) Beginning inventory: The beginning inventory of each product is introduced.

(4.3.8) Non preemptive shifts: The number of the patterns to be produced in a week must be integers since the unit production time is a shift.

The MPS-1 model is coded in GAMS, version 20.1 and solved using CPLEX solver, and run on a 2.8 Ghz. Pentium(R) 4 CPU and 256 RAM. The test data defined in section 4.2 was used. The model is run for 30 minutes for each data set. The results are shown in Table 4.1.

Table 4.1 The results of MPS-1 running for 30 minutes for each data set

Demand Sets	Objective Function Value	Gap (%)	Overage Cost	Shortage Cost
1	9898	59.75	6465	2985
2	37629	78.29	7829	25913
3	744726	1.00	4417	643750
4	104612	5.28	6722	85121
5	389946	1.27	5162	334590
6	154306	2.35	6449	128570
7	16862	49.72	7358	8264
8	29412	33.06	20685	5624
9	16707	61.55	5733	9542
10	9582	78.59	7590	4000
11	10243	69.29	8255	1728
12	10154	70.47	7253	2522

CPLEX solver uses branch and bound method with cuts. The results show that, for some demand sets (demand set 3, 4, 5, and 6) the gap between the best obtained feasible solution and the last updated lower bound in branch and bound tree is reasonably low. For these demand sets, the common property is one of the cost elements (overage or shortage cost) is remarkably higher than the other. Unlike the sets 3, 4, 5, and 6; the remaining demand sets end up with high gaps and the cost elements are closer to each other as to their values. The reason for lower gaps can be explained as follows:

(a) Higher overage cost, lower shortage cost: Since the inventory cost is very high, the model mostly focuses on reducing inventory cost and avoids additional inventory for further periods than the processed week. The model, approximately, runs 12 separate periods. In other words, the 12 periods do not have the connection by means of inventory. So, the model behaves like 12 distinct, small models trying to minimize the difference between the production quantity and the demand for a product in a week. As a result, it converges to the optimum faster.

(b) Higher shortage cost, lower inventory cost: Since, the capacity is insufficient; the model tries to maximize each week's production without considering the inventory to be kept for further weeks. That is to say, it is already impossible to keep inventory for further weeks' demand from the processed week because of the capacity problem. So, the model behaves like 12 distinct, small production maximization models. As a result it converges to the optimum faster.

Analyzing the results of the model from the point of numbers of shifts to produce a pattern for each week, namely the decision variable Y_{ti} ; it is observed that too few figures are greater than 6. By this motivation, the value of Y_{ti} is considered to be bounded by 6. This has a narrowing affect on the branching process. So, a new constraint is added to MPS.

The new model is given below:

MPS-2

min (4.3.1)

Subject to

(4.3.2), (4.3.3), (4.3.4), (4.3.5), (4.3.6), (4.3.7), (4.3.8)

$Y_{ti} \leq 6$ (4.3.9)

Using the same test data and the same hardware, the model is run. The results of the runs are given in Table 4.2.

Table 4.2. The results of bounded MPS-2 running for 30 minutes for each data set

Demand Sets	Objective Function Value	Gap (%)	Overage Cost	Shortage Cost
1	8820	55.24	8569	219
2	36039	12.12	7345	24952
3	744720	0.94	4394	643760
4	105277	5.86	6494	85897
5	389845	1.24	5216	334460
6	154400	2.38	6659	128470
7	17232	50.75	7016	8883
8	27821	29.25	6163	18832
9	15749	59.21	4875	10141
10	11011	76.11	6945	3535
11	10186	70.15	6589	3127
12	10439	71.59	7352	2683

The gaps are still greater than the acceptable values. However, in some demand sets; an improvement is obtained by bounding the Y_{ti} variable. The improvements in the objective function values are compared in Table 4.3.

Table 4.3. Comparison of the MPS-1 with MPS-2

Demand Sets	Objective Function Value of MPS1	Objective Function Value of MPS2	Improvement
1	9898	8820	1078
2	37629	36039	1590
3	744726	744720	6
4	104612	105277	-665
5	389946	389845	101
6	154306	154400	-94
7	16862	17232	-370
8	29412	27821	1591
9	16707	15749	958
10	9582	11011	-1429
11	10243	10186	57
12	10154	10439	-285

For some demand sets, however, no improvement can be obtained. As a matter of fact, bounding the Y_{ti} variable does not provide a significant advantage for reaching the optimum.

When the solution process is observed in GAMS, after finding a feasible solution and obtaining dramatic improvements at the beginning, CPLEX starts to improve

the lower bound. Yet, this process continues so slowly resulting in high gaps. So, the model is run longer to see if in fact the solutions are not so far from the optimum. The model is run for 3 hours for the samples 1, 7, 9, 10, 11, 12 since they have high gaps. The results are given in Table 4.4.

Table 4.4 Three hours long runs for MPS-2

Demand Sets	Objective Function Value	Gap	Overage Cost	Shortage Cost
1	8820	53.23	8568	219
7	17232	50.72	7016	8883
9	1539	57.8	6541	7624
10	11011	76.01	6945	3535
11	9036	65.96	7077	1702
12	9845	69.74	8997	737

These runs also do not improve the solution considerably for all samples. A longer run which is 24 hour long is also tried for the sample 1, but no significant improvement can be obtained.

4.3.1. Linear Relaxation

After not being able to reduce the gap in the integer model, as a second step, finding an approximate solution is considered. To make an approximation, first, the linear relaxation of the integer model MPS-2 is constructed and the solutions are observed. It is realized that for the data sets that are close to the optimum solution in MPS-2 (test data 3, 4, 5, and 6), solutions of the linear relaxation and the integer model are very similar in terms of the objective function and the patterns. Spinning off from this observation, two algorithms are developed to find an acceptable solution. The first algorithm, fixes the values of the number of the patterns to be produced in each week to 0 in MPS-2, if the same decision variable is “0” in the linear relaxation, then solves MPS-2. The second algorithm rounds the number of the patterns to be produced in each week found in the linear relaxation of MPS-2 up or down. The details are given below for linear relaxation and two algorithms and their results are examined.

In the linear relaxation, the variable Y_{ii} is taken as a continuous variable instead of an integer one. The model is given below.

MPS-3

Min (4.3.1)

Subject to

(4.3.2); (4.3.3); (4.3.4); (4.3.5); (4.3.6); (4.3.7); (4.3.9)

MPS-2 is selected to apply the relaxation, because in the trial runs it is seen that if the linear model is not bounded, it produces very little amounts of each product and produces a very large amount of a few products. To overcome this situation, Y_{ii} variable is bounded. The results of the linear model and the integer model are given in Table 4.5. It is observed that, for the samples 3, 4, 5, 6 the total cost of the integer model is very close to the total cost of the linear model. Subsequently, the production numbers of the patterns for each week is investigated; it is observed that similar patterns are decided to be produced by the linear and the integer models.

These facts led the study to investigate some connections between the linear model and the integer model so that some improvement can be obtained in the objective function. For this purpose, two different methodologies are considered.

Table.4.5 Comparison of the linear and integer models

Demand Sets	Objective Function Value of MPS-2	Objective Function Value of MPS-3	Gap	Gap(%)	Solution Time of MPS-3 (seconds)
1	8820	0		8820	100.0
2	36039	28393		7646	21.2
3	744720	730753		13967	1.9
4	105277	94537		10740	10.2
5	389845	377608		12237	3.1
6	154400	144846		9554	6.2
7	17232	4747		12485	72.5
8	27821	16467		11354	40.8
9	15749	2555		13194	83.8
10	11011	33		10978	99.7
11	10186	0		10186	100.0
12	10439	0		10439	100.0

In the first method, call it MPS-3.1, the linear model is solved and the patterns that are greater than 0 are taken as a parameter to the integer model and the integer model has the additional constraint “production number of a pattern in a week is 0 if the production number of the same pattern is 0 in the linear model”.

The algorithm is as follows:

In the linear relaxation model, for the sake of simple explanation, the name of the decision variable defining the number of shifts that pattern i is produced in the week t , is referred as $Y_{linear_{ti}}$ and the name of the same variable in the integer model is referred as Y_{ti} .

Step.1 Solve MPS-3.

Step.2 Assign $Y_{ti} = 0$ if $Y_{linear_{ti}} = 0$ for $\forall t, i$

Step.3 Solve MPS-2

The models are solved using the same test data. The results are given in Table 4.6.

Table 4.6 Results of MPS-3.1 running for 30 minutes

Demand Sets	Objective Function Value	Gap	Overage Cost	Shortage Cost
1	12044	39.6	8867	2762
2	39837	15.8	9222	26621
3	748450	1	5328	646190
4	106142	5.53	7228	86012
5	392710	1.15	5656	336570
6	158621	2.65	6744	132070
7	18780	41.94	8587	8863
8	30331	25.5	7609	13757
9	14834	34.2	8023	5922
10	14494	37.65	9627	4231
11	16112	30.6	10796	4623
12	14307	38.45	10648	3181

It can be seen that, the high gap values still exist. Moreover, in Table 4.7, the comparison of this method solutions are given and the objective function values are not improved by this method.

Table 4.7 Comparison of the results of the MPS-2 with MPS-3.1

Demand Sets	Objective Function Value of MPS-2	Objective Function Value of MPS-3.1	Improvement	Improvement(%)
1	8820	12044	-3224	-36.6
2	36039	39837	-3798	-10.5
3	744720	748450	-3730	-0.5
4	105277	106142	-865	-0.8
5	389845	392710	-2865	-0.7
6	154400	158621	-4221	-2.7
7	17232	18780	-1548	-9.0
8	27821	30331	-2510	-9.0
9	15749	14834	915	5.8
10	11011	14494	-3483	-31.6
11	10186	16112	-5926	-58.2
12	10439	14307	-3868	-37.1

In the second method, call MPS-3.2, the linear model solutions are rounded up or down in the integer model. Additional constraints are added to the MPS to satisfy that the Y_{ti} variables are between the rounded up value of the $Ylinear_{ti}$ and the rounded down value of the $Ylinear_{ti}$. For the cases $Ylinear_{ti}$ is equal to 0, Y_{ti} is not bounded.

The algorithm is as follows:

Step.1 Solve MPS-3

Step.2 Solve MPS-2 with the additional constrains for every t and i where $Ylinear_{ti}$ is greater than 0.

$$\lfloor Ylinear_{ti} \rfloor \leq Y_{ti} \leq \lceil Ylinear_{ti} \rceil$$

$\lfloor x \rfloor$ means the nearest integer smaller than x and $\lceil x \rceil$ means the nearest integer greater than x.

The models are solved in succession using the same test data. The results are given in Table 4.8.

Table.4.8. Results of the MPS-3.2 running for 30 minutes

Demand Sets	Objective Function Value	Gap	Overage Cost	Shortage Cost
1	12255	68.3	11679	500
2	40010	18.7	7935	27891
3	748215	1	4088	647070
4	107340	4.76	5802	88296
5	391413	1.16	5162	335870
6	156850	2.84	6319	130890
7	18743	43.5	7091	10132
8	30622	29.34	5725	21649
9	16951	59.51	8900	7000
10	11797	74.9	9494	2002
11	10177	66.6	7275	2522
12	11810	75.12	7621	3642

It can be seen that, the high gap values still exist. In Table 4.9, the comparison of the MPS-3.2 solutions is listed. No improvement can be obtained by this method.

Table 4.9 Comparison of the results of the MPS-3.2 with MPS-2

Demand Sets	Objective Function Value of MPS-2	Objective Function Value of MPS-3.2	Improvement	Improvement(%)
1	8820	12255	-3435	-38.9
2	36039	40010	-3971	-11.0
3	744720	748215	-3495	-0.5
4	105277	107340	-2063	-2.0
5	389845	391413	-1568	-0.4
6	154400	156850	-2450	-1.6
7	17232	18743	-1511	-8.8
8	27821	30622	-2801	-10.1
9	15749	16951	-1202	-7.6
10	11011	11797	-786	-7.1
11	10186	10177	9	0.1
12	10439	11810	-1371	-13.1

In fact, MPS-3.1 covers MPS-3.2. In MPS-3.2 the integer variable becomes “0” if the continuous variable is “0” like in MPS-3.1 and in MPS-3.1 nonzero variables can take any value whereas in MPS-3.2 nonzero variables can only take rounded up or down value of the corresponding variable in the linear relaxation model.

Both algorithms did not work since the variable numbers are still very high. So, another approach based on the reduction of the number of variables is considered.

4.3.2. Aggregation of Weeks

Because of not being able to solve the models in the linear relaxation within acceptable gaps, another approach is designed. The main motivation is that only the first week's plan is frozen and implemented, other eleven weeks are changing with the new demand information input every week on the rolling horizon. So, it is considered to aggregate last eleven weeks' demand according to a rule, to solve the model with the aggregated weeks and then decompose the aggregated weeks to individual weeks. The details of the model are explained in this section.

Since no considerable amount of improvement is obtained in most cases with the linear relaxation based algorithms, a new approach is constructed to have better solutions. The idea comes from the fact that, for the 12 weeks planning horizon only the first week is concrete in terms of demand forecasts, other eleven weeks' demand forecasts can be changed for the following planning week. So, it is logical to aggregate last eleven weeks in a fashion that the total number of integer decision variables is reduced. For that purpose, the twelve week planning period is divided into four periods where a period is a combination of weeks. The aggregation is done like below:

Period 1: First week of the planning horizon, week 1 in Figure 4.3.

Period 2: The combination of weeks 2, 3, and 4 in Figure 4.

Period 3: The combination of weeks 5, 6, 7, and 8 in Figure 4.3.

Period 4: The combination of weeks 9, 10, 11, and 12 in Figure 4.3.

The demand forecasts of the weeks are summed for the periods and new demand forecasts for the periods are constructed.

The main idea is to reduce the number of the decision variables to reach a better solution and then to disaggregate the periods into their original form.

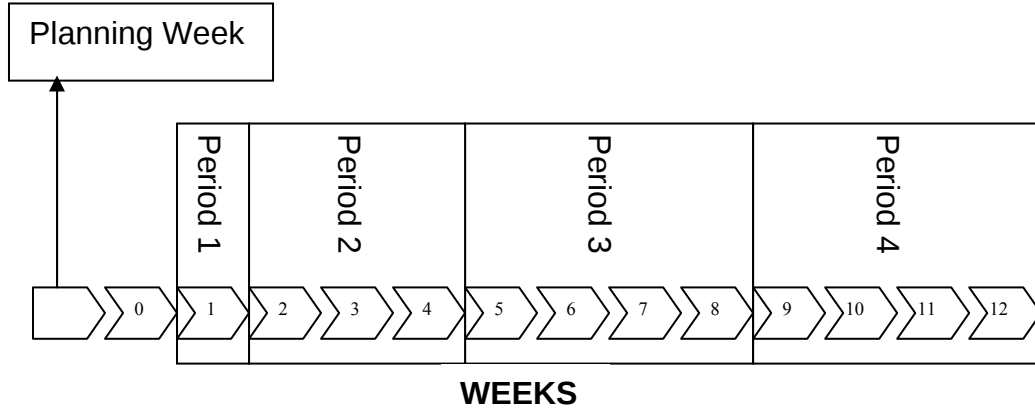


Figure 4.3 Week aggregation

Two consecutive models are run in this methodology. The highlights of these two phases are as follows:

Phase 1: The demand forecasts of the weeks are summed for the periods. The number of available shifts is rearranged according to the total number of weeks in a period. Next, the MPS-2 module is run with the same constraints.

The model of phase 1 which is named as MPS-4.1 is constructed as follows:

MPS-4.1

$$\min (TC = \alpha SC + OC) \quad (4.3.2.1)$$

Subject to

$$SC = \sum_{\forall j} \sum_{\forall p} P_j \times Q_j \times I_{jp}^- \quad (4.3.2.2)$$

$$OC = \sum_{\forall j} \sum_{p=1} [(Q_j \times I_{jp}^+)] \times [(1+r)^1 - 1] + \sum_{\forall j} \sum_{p=2} [(Q_j \times I_{jp}^+)] \times [(1+r)^3 - 1] + \sum_{\forall j} \sum_{p=3,4} [(Q_j \times I_{jp}^+) / 2] \times [(1+r)^8 - 1] \quad (4.3.2.3)$$

$$I_{j(p+1)}^+ - I_{j(p+1)}^- = I_{jp}^+ + Z_{jp} - DP_{j(p+1)}; \quad For \forall p < 4; For \forall j \quad (4.3.2.4)$$

$$Z_{jp} = \sum_{\forall i} YP_{pti} \times 8 \times (1-l) \times C_{ij}; \quad For \forall p < 4, For \forall j \quad (4.3.2.5)$$

$$\sum_{\forall i} YP_{pi} \leq 18; \quad For p = 1 \quad (4.3.2.6)$$

$$\sum_{\forall i} YP_{pi} \leq 54; \quad For p = 2 \quad (4.3.2.7)$$

$$\sum_{\forall i} YP_{pi} \leq 72; \quad For p = 3,4 \quad (4.3.2.8)$$

$$I_{j0}^+ = a_j; \quad For \forall j \quad (4.3.2.9)$$

$$YP_{pi} \text{ integer variable} \quad (4.3.2.10)$$

In this model, the index t, is replaced by the index p, which indicates the period number which consists of a number of weeks. Also, instead of the demand parameter D, DP is used indicating total demand forecast for a period. Besides, decision variable Y is changed with YP to differentiate from the next model's notation.

The overage cost constraint (4.3.2.3), is the sum of the costs of the inventory for each period. And the inventory cost for a period is calculated as the average inventory level in YTL multiplied with the period's interest rate.

Constraints (4.2.3.6), (4.3.2.7), (4.3.2.8) bound the number of available shifts in a period; the number of available shifts during a period differs according to the number of weeks in a period.

The other constraints are similar to the MPS model.

After solving Phase 1, the YP variables are transferred to Phase 2 as parameters. Phase 2 is explained below:

Phase 2: This model is the MPS-2 model with additional constraints implying that the total of the number of each week's production of a pattern for a period should be equal to the period's total production of the same pattern.

The model of Phase 2 is given below with the name MPS-4.2:

MPS-4.2

Min (4.3.1)

Subject to

(4.3.2); (4.3.3); (4.3.4); (4.3.5); (4.3.6); (4.3.7); (4.3.8);

$$\sum_{t=1} Y_{ti} \leq YP_{1i} \quad ; \quad For \quad \forall i \quad (4.3.2.11)$$

$$\sum_{t=2}^4 Y_{ti} \leq YP_{2i} \quad ; \quad For \quad \forall i \quad (4.3.2.12)$$

$$\sum_{t=5}^8 Y_{ti} \leq YP_{3i} \quad ; \quad For \quad \forall i \quad (4.3.2.12)$$

$$\sum_{t=9}^{12} Y_{ti} \leq YP_{4i} \quad ; \quad For \quad \forall i \quad (4.3.2.14)$$

The constraints (4.3.2.11), (4.3.2.12), (4.3.2.13), (4.3.2.14) assure that the total number of the production of each pattern is equal to the Phase 1's results for the periods' production number of patterns.

The two consecutive models, which are named as MPS-4, run and solutions are listed in Table 4.10. Aggregation of weeks methodology is named as MPS-4.

Table 4.10 Results of MPS-4 running for 30 minutes

Demand Sets	Objective Function Value	Gap of the MPS-4.1	Gap of the MPS-4.2	Overage Cost of MPS-4	Shortage Cost of MPS-4
1	7437	58.00	1	6297	990
2	44836	6.13	1	7016	32886
3	768340	0.10	1	4824	66393
4	111904	1.50	1	7823	9055
5	416680	0.90	1	9042	354460
6	163739	1.50	1	7023	136280
7	14360	42.79	1	6808	6567
8	33336	14.97	9.8	6755	23113
9	15149	67.70	10	7900	6300
10	10495	75.3	3.94	6373	3583
11	8460	59.7	0.9	6490	17143
12	8764	82.5	5.75	7289	1282

MPS-4.1 has high gaps where MPS-4.2 has small gaps. However, comparing the results of the models with the MPS model in Table 4.11 in some demand patterns considerable amount of progress has been realized by this methodology. Especially, the improvements are drastic in samples 1, 7, 11, and 12 and a smaller improvement is realized in sample 9 and 10.

Table 4.11 Comparison of MPS-4 and MPS-2

Demand Sets	Objective Function Value of MPS-2	Objective Function Value of MPS-4	Improvement	Improvement (%)
1	8820	7437	1383	15.7
2	36039	44836	-8797	-24.4
3	744720	768340	-23620	-3.2
4	105277	111904	-6627	-6.3
5	389845	416680	-26835	-6.9
6	154400	163739	-9339	-6.0
7	17232	14360	2872	16.7
8	27821	33336	-5515	-19.8
9	15749	15149	600	3.8
10	11011	10495	516	4.7
11	10186	8460	1726	16.9
12	10439	8764	1675	16.0

4.3.3. A Hybrid Method

With aggregation of weeks, an improvement is made from the point of the objective function but, in MPS-4.1 there exists high gaps. To overcome this problem, the method used in MPS-3.1 that is defined in Linear Relaxation section is applied to MPS-4.1 of the week aggregation method. Namely, the following algorithm is applied on MPS-4.1.

Step.1 Solve MPS-4.1 with the linear relaxation. Call the decision variable YP_{linear} in the linear model.

Step.2 Assign $YP_{pi} = 0$ if $YP_{linear}_{pi} = 0$

Step.3 Solve MPS-4.1 with integer variables.

Step.4 Solve MPS-4.2

The results of the defined model, call MPS-5, are given in the Table 4.12.

Table 4.12 Results of the MPS-5 running for 5 minutes.

Demand Sets	Objective Function Value	Gap of the MPS-4.1	Gap of the MPS-4.2	Overage Cost	Shortage Cost
1	11676	4.69	2.89	8353	2890
2	40319	1.06	1	8183	27945
3	764436	0.43	0.91	5743	659730
4	113259	1.56	0.93	8609	90999
5	418864	1.72	0.75	8700	356660
6	167597	1.58	0.66	8362	138470
7	15688	2.95	1	8437	6304
8	29500	7.48	3.92	7544	19092
9	17586	11.48	5.21	9518	7016
10	15229	1.56	4.63	8165	6142
11	14237	2.10	1.68	8913	4629
12	12800	12.28	4.88	8694	3569

The computational time is shorter for the Hybrid Method. The run time is set to five minutes and reasonable gaps are obtained with that runtime. When compared to MPS-4, this model has better objective function values for some instances. (Table 4.13)

Table 4.13 Comparison of MPS-5 with MPS-4

Demand Sets	Objective Function Value of MPS-4	Objective Function Value of the Hybrid MPS-5	Improvement	Improvement(%)
1	7437	11676	-4239	-57.0
2	44836	40319	4517	10.1
3	768340	764436	3904	0.5
4	111904	113259	-1355	-1.2
5	416680	418864	-2184	-0.5
6	163739	167597	-3858	-2.4
7	14360	15688	-1328	-9.2
8	33336	29500	3836	11.5
9	15149	17586	-2437	-16.1
10	10495	15229	-4734	-45.1
11	8460	14237	-5777	-68.3
12	8764	12800	-4036	-46.1

Nevertheless, when compared to the MPS-2 solutions, the Hybrid Method has better objective function value in only one sample, sample 7. The comparison is made in Table 4.14.

Table 4.14 Comparison of MPS-5 with MPS-2.

Demand Sets	Objective Function Value of MPS-2	Objective Function Value of MPS-5	Improvement	Improvement (%)
1	8820	11676	-2856	-32.4
2	36039	40319	-4280	-11.9
3	744720	764436	-19716	-2.6
4	105277	113259	-7982	-7.6
5	389845	418864	-29019	-7.4
6	154400	167597	-13197	-8.5
7	17232	15688	1544	9.0
8	27821	29500	-1679	-6.0
9	15749	17586	-1837	-11.7
10	11011	15229	-4218	-38.3
11	10186	14237	-4051	-39.8
12	10439	12800	-2361	-22.6

4.3.4. Summary of the MPS Models

Since the original MPS model is a very large integer program it is a challenging work to obtain an acceptable near optimum solution. Thus, a number of models are

constructed and evaluated in Chapter 4. Here, in Table.4.15, the summary of the models that are used are listed to build a clear understanding of the models.

Table 4.15 The summary of the MPS models

Model Name	Description	Section
MPS-1	Integer MPS Model.	4.3
MPS-2	MPS1 with the addition of constraints bounding the number of shifts that can be produced in a week for a product.	4.3
MPS-3	Linear relaxation of the MPS2.	4.3.1
MPS-3.1	MPS2 with the additional constraints assigning the number of shifts that can be produced to "0" for the products whose number of shifts to be produced is equal to "0" in the MPS-3.	4.3.1
MPS-3.2	MPS2 with the additional constraints bounding the number of shifts to be produced for a product between the nearest smaller integer and the nearest bigger integer number that is obtained by solving MPS-3.	4.3.1
MPS-4	Model obtained with the aggregation of weeks.	4.3.2
MPS-4.1	Phase 1 of MPS-4, new formation of MPS-2 with the aggregated weeks.	4.3.2
MPS-4.2	Phase 2 of MPS-4, decomposes the solution of the aggregated weeks to individual weeks.	4.3.2
MPS-5	Hybrid model, applies the methodology of MPS-3.1 to MPS-4.1 and then solves MPS-4.2	4.3.3

Moreover, to clearly state the flow of the work done, the models that are constructed and the linkages between them are given in the Figure 4.4.

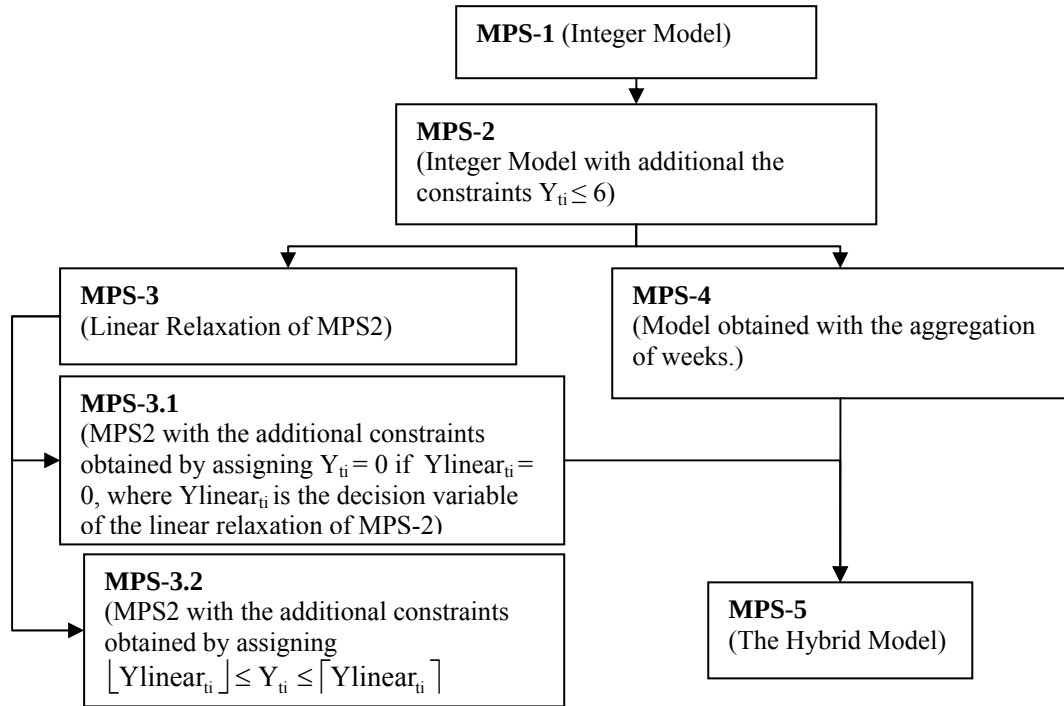


Figure 4.4 The Linkages between the MPS models

4.3.5. Obtaining the Final Solution for MPS

Evaluating the results, three facts are realized:

1. MPS-4, sometimes, has much better solutions than MPS-2, and sometimes slightly worse than MPS-2.
2. MPS-5 sometimes has better solutions than MPS-4.
3. The MPS-5, sometimes, has better solutions than MPS-2 in the test data.
4. MPS-3.1 and MPS-3.2 do not provide any better result than either MPS-2 or MPS-4.

Depending on these facts, finding an upper bound with the hybrid model (MPS-5) for the problem is decided to be the first step of the solution. Next, week aggregation model is run. The upper bound is updated with the solution of the week aggregation model (MPS-4) if it results in a better solution. Finally, original MPS model (MPS-2) is run. The result is compared with the upper bound. If the upper

bound is higher than the MPS-2 solution, then the final solution is taken as the result of MPS-2, else the solution is taken as the model which generates the upper bound.

With the motivation defined above, an algorithm is designed. The algorithm is defined as follows:

Step.1 Implement Hybrid Method (MPS-5).

Final Solution = Solution of MPS-5,

Upper Bound = Objective function value of MPS-5,

Go to step 2.

Step.2 Implement Week Aggregation model (MPS-4).

If Objective function value of MPS-4 is less than the Upper Bound

Then, Final Solution = Solution of MPS-4,

Upper Bound = Objective function value of MPS-4, Go to step 3

Else, go to step 3

Step.3 Implement MPS-2.

If Objective function value of MPS-2 is less than the Upper Bound

Then, Final Solution = Solution of MPS-2, Stop,

Else, stop.

The algorithm runs for 65 minutes, as the sum of the solution times of the three models. The results of the algorithm are shown in Table 4.16.

Table 4.16 The Results of the Algorithm

Test data	Objective Function Value of MPS-5	Objective Function Value of MPS-4	Objective Function Value of MPS-2	Best Solution
1	11676	7437	8820	7437
2	40319	44836	36039	36039
3	764436	768340	744720	744720
4	113259	111904	105277	105277
5	418864	416680	389845	389845
6	167597	163739	154400	154400
7	15688	14360	17232	14360
8	29500	33336	27821	27821
9	17586	15149	15749	15149
10	15229	10495	11011	10495
11	14237	8460	10186	8460
12	12800	8764	10439	8764

4.3.6. Comparison with the Current System

The results obtained from the newly developed MPS model are obtained by comparing 2 heuristics and the integer MPS model results and selecting the best solution among them. The integer MPS model is for some of the test data, gives solutions with high gaps, meaning that it is not for sure that the obtained solution is near the optimum value. Although integer model's solutions are compared with the discussed heuristic models, the comparison between the current system MPS results and the newly developed MPS model results is carried out to find out how much benefit is gained, or the solutions of the new model is not any better than the current system.

Current model is a linear model with the objective function of minimizing the total cost of shortages and overages. This model only decides the production quantities of the products. The model is as follows:

$$\text{Min } TC \quad (4.3.6.1)$$

Subject to

$$TC = SC + OC \quad (4.3.6.2)$$

$$SC = \sum_{\forall t} \sum_{\forall j} mpsslack_{jt} \times Q_j \times P_j \quad (4.3.6.3)$$

$$OC = \sum_{\forall j} \sum_{\forall t} mpss_{jt} \times Q_j \times r \quad (4.3.6.4)$$

$$mpss_{j0} = a_j \quad \text{For } \forall j \quad (4.3.6.5)$$

$$mpss_{jt} + mpsslack_{jt} \geq D_{jt} \quad \text{For } \forall t < 12, \text{ For } \forall j \quad (4.3.6.6)$$

$$mpss_{jt} + mpsy_{jt} + mpsslack_{jt} = D_{jt} + mpss_{j(t+1)} \quad \text{For } \forall t < 12, \text{ For } \forall j \quad (4.3.6.7)$$

$$\sum_{\forall j} mpsy_{jt} / C_j \leq 18 \times 8 \times (1 - l) \quad \text{For } \forall t < 12 \quad (4.3.6.8)$$

Indices j and t are for the products and weeks, respectively. Decision variables are as follows:

$mpss_{jt}$: beginning inventory level for product j and week t ,

$mpsy_{jt}$: amount of production for product j in week t ,

$mpsslack_{jt}$: amount of shortage for product j in week t ,

TC, SC, OC : total cost, shortage cost and overage cost; respectively.

The parameters are same with the previous models except C_j , which is the production rate of product j . In the previous models, production rate has another index for the production pattern. But, in the current model, the patterns are not included. In this model, all the products are assumed to be produced single and production rate of a product is taken as the production rate of that product when it is produced alone. The production rates of current model are included in Patterns 83 through 103 in Appendix A.)

The objective is minimizing the total cost (4.3.6.1). The constraints 4.3.6.2 to 4.3.6.4 are equations for the total cost, shortage cost and overage cost, respectively. Constraint 4.3.6.5 is for stating the beginning inventory levels. Constraint 4.3.6.6 states that demand for a product is less than the beginning inventory level for a week plus shortage amount. Constraint 4.3.6.7 is the inventory balance constraint. Finally, 4.3.6.8 is the capacity constraint.

To be able to make a comparison between the current model and the newly developed model, the test data used in this study is employed in the current model. The other parameters are made to be equal for all the runs for both models. The current model is run and inventory levels and shortage values; $mpss_{jt}$ and $mpsslack_{jt}$, are obtained. These two values are inserted in the corresponding overage and shortage calculations of the MPS-2 model, namely equations 4.3.2 and 4.3.3. Then, the objective function value of the MPS-2 is calculated using equation 4.3.1. The results of the comparison runs are given in the Table 4.17

Table 4.17. Comparison of the current system results and the results obtained from the developed MPS

Test data	Developed Model Solutions	Current Model Solutions	Cost Advantage (%)
1	7437	8740	14.91
2	36039	138127	73.91
3	744720	916611	18.75
4	105277	225486	53.31
5	389845	550603	29.20
6	154400	305654	49.49
7	14360	100546	85.72
8	27821	138955	79.98
9	15149	60573	74.99
10	10495	25645	59.08
11	8460	11685	27.60
12	8764	9834	10.88

The solutions are the overall cost of the planning horizon including the shortage and overage costs. The developed system's total costs are better than the current model's solutions for all the instances. The cost advantage of the developed system differs between the range 14% and 86%.

4.3.7. Comparison with the Item-by-Item Heuristic

Showing that the developed heuristic provides a significant improvement in terms of overall cost compared with the current model, in this section, the developed heuristic is compared with another heuristic methodology which is named as Item-

by-item heuristic. Kırca and Kökten(1994) developed Item-by-item heuristic to be able to find a solution to the Capacitated Lot Sizing Problem (CLSP) with realistic dimensions. The heuristic uses an Item-by-item approach for the problem. The heuristic selects a set of items among all items according to a selection criterion and schedules these items along the entire planning horizon and resumes until no item is left to schedule. Kırca and Kökten showed that the Item-by-item heuristic, on the average, outperforms some other well known heuristics.

Item-by-item heuristic starts with selecting N items among all items where N is found as the number of items to be scheduled in each iteration such that the solution could be obtained in a reasonable computational time.

Item-by item algorithm is aligned for the chocolate production line MPS model. The number of items to be scheduled in each iteration is selected to be “1” after constructing trial runs and observing that even for 2 products sometimes the problem becomes unsolvable in a reasonable computational time since, a product is associated with at least five production patterns which are the integer variables of MPS. The product selection criterion is decided to be the product with the most profit meaning that the focus is on the shortage minimization. The reason for that preference is that the shortage cost is significantly higher than the overage cost in the trial runs. Eventually, the item-by-item algorithm is constructed and the algorithm is outlined below.

Item-by-item Algorithm for MPS

Step 0. Set the iteration counter $k = 1$, $\forall_{it} = 0$ for all i and t , where $\forall_{it}$ is the number of pattern i to be produced in week t . Call the number of shifts available for week t as S_t . Set $S_t = 18$,

Step 1. Select an item among the items which are not scheduled yet by using a selection criterion which is selecting the most profitable one. Call the product V_k .

Step 2. Solve MPS-2 for V_k replacing all Y_{it} 's with $(\bar{Y}_{it} + Y_{it})$ where Y_{it} is the decision variable explained in the previous sections. Call the total number of shifts scheduled at the iteration k as P_t

Step 3. Update $\bar{Y}_{it}$ as $\bar{Y}_{it} + Y_{it}$, $S_t = S_t - P_t$,

Step 4. If no item remains to be scheduled then stop, else $k = k + 1$ and go to step 1.

The algorithm is coded and run for all the test data. Results are given in Table 4.18

Table 4.18 Comparison of the Item-by-item Heuristic's Solutions with The Developed Heuristic's solutions.

Demand Sets	Objective function value of the Developed Heuristic	Objective Function Value of Item-by-item Heuristic	Overage Cost	Shortage Cost	Solution Time (seconds)
1	7437	40027	39442	508	255
2	36039	133900	22140	97180	310
3	744720	929210	9538	799720	305
4	105277	213590	24419	164500	190
5	389845	564310	20945	472490	240
6	154400	290060	19141	235580	345
7	14360	112850	51767	53115	240
8	27821	125320	36171	77516	180
9	15149	84409	54254	26222	130
10	10495	69892	58351	10036	205
11	8460	56695	52618	6153	200
12	8764	56227	55453	673	125

Although, Item-by-item heuristic has significantly low computational times with respect to the developed algorithm, the results of the item-by-item approach are worse significantly in terms of overage, shortage and total cost.

4.3.7. Evaluation of the Results

After reaching the final solution with the algorithm defined in Section 4.3.5, the model is tested with more test demand data. The original MPS model (MPS-2), the

week aggregation model (MPS-4) and the hybrid model (MPS-5) are run and the best solution is obtained. The results of the three models are given in Appendix C. The final solution is compared with the current model and Item-by-item model. The models are run with other twelve demand data. The results are consolidated with the first twelve test data which is used in the design stage and listed in Table 4.19.

Table 4.19 Results of the models in terms of costs for 24 test data

Test Data	Developed Model			Item-by-item Model			Current Model		
	Shortage Cost	Overage Cost	Overall Cost	Shortage Cost	Overage Cost	Overall Cost	Shortage Cost	Overage Cost	Overall Cost
1	990	6297	7437	508	39442	40027	0	8740	8740
2	24952	7345	36039	97180	22140	133900	106505	15646	138127
3	643760	4394	744720	799720	9538	929210	777889	22038	916611
4	85897	6494	105277	164500	24419	213590	181775	16444	225486
5	334460	5216	389845	472490	20945	564310	460858	20616	550603
6	128470	6659	154400	235580	19141	290060	247384	21162	305654
7	6567	6808	14360	53115	51767	112850	72563	17098	100546
8	18832	6163	27821	77516	36171	125320	104454	18832	138955
9	6300	7900	15149	26222	54254	84409	39237	15450	60573
10	3583	6373	10495	10036	58351	69892	12701	10808	25645
11	17143	6490	8460	6153	52618	56695	3401	7772	11685
12	1282	7289	8764	673	55453	56227	0	9834	9834
13	11912	6030	19729	160397	14230	198687	51155	18621	77450
14	2488	6715	9577	60974	9962	80083	8035	13435	22675
15	1165	5166	6506	37481	6799	49903	0	6612	6612
16	1217	6084	7484	27146	7064	38283	0	8107	8107
17	3124	7895	11488	23266	18188	44945	1554	19899	21688
18	1446	5717	7380	15163	8406	25844	0	3418	3418
19	815	5728	6667	26559	9103	39646	0	3139	3139
20	1072	6827	8060	26469	9191	39630	0	6795	6795
21	1543	6582	8357	25275	9218	38285	0	7277	7277
22	1175	7637	8988	27805	7495	39471	0	8274	8274
23	2871	6892	10109	65116	9135	84020	16803	12267	31591
24	10722	5817	18183	91264	6500	111455	18292	9132	30168
Average	54658	6438	68554	105442	23314	144448	87609	12559	113319

Results show that the results of the developed model are better than the current system and also, than Item-by-item heuristic solutions in terms of overall cost. Moreover, item-by-item heuristic and the current model's solutions are worse than the developed model in terms of overage costs for individual test data and at the

average. For some instances, Item-by-item heuristic and the current model gives better solutions than the developed model in terms of shortage costs. But, the developed model, for most of the test data, results in better shortage costs, also on the average the developed model outperforms other two models in terms of shortage cost.

The performance of the developed model is also measured by the total production quantities over the planning horizon. The developed model is compared with the other two models in Table 4.20.

Current model is referred to as “reference model” and Item-by-item heuristic’s and developed model’s production quantities are given as percentages of the reference model. “Percent production” column refers to the percentage values. It is observed that developed model results in larger production values than other two models. The developed model also has better results in terms of cost. So, larger production quantities are achieved with less cost compared to the other two methods.

Table 4.20 Results of the models in terms of production quantities for 24 test data

Test Data	Current Model (Reference Model)		Item-by-item Heuristic		Developed Model	
	Total Production		Total Production		Total Production	
	Quantity (kilograms)	Percent Production	Quantity (kilograms)	Percent Production	Quantity (kilograms)	Percent Production
1	265671	100	291890	109.9	306010	115.2
2	452118	100	408421	90.3	592096	131.0
3	669489	100	514884	76.9	801917	119.8
4	471300	100	382137	81.1	614142	130.3
5	604266	100	421028	69.7	717223	118.7
6	579048	100	454491	78.5	711484	122.9
7	496275	100	493700	99.5	676890	136.4
8	558332	100	524239	93.9	752664	134.8
9	447015	100	430467	96.3	553100	123.7
10	327829	100	299252	91.3	371580	113.3
11	255237	100	243678	95.5	283040	110.9
12	309465	100	315322	101.9	332410	107.4
13	565886	100	483799	85.5	653350	115.5
14	405784	100	345637	85.2	431010	106.2
15	211035	100	175936	83.4	217690	103.2
16	252360	100	231091	91.6	253770	100.6
17	428569	100	444210	103.6	471680	110.1
18	98833	100	110942	112.3	103930	105.2
19	90338	100	87135	96.5	93812	103.8
20	204338	100	246126	120.5	254120	124.4
21	223971	100	231621	103.4	251740	112.4
22	261468	100	276855	105.9	286670	109.6
23	331933	100	315635	95.1	382770	115.3
24	278932	100	231138	82.9	303250	108.7

4.4. Overtime Formulation

The MPS models discussed up to this section are structured with the assumption that a week is six days long and a week consists of 18 shifts. But, except the normal working days, there is an opportunity that on Sundays, three shifts are available to be used at the expense of overtime wages.

The cost of working on Sundays is assumed to be the total overtime wages paid to the employees. The overtime expense is calculated to be 1750 YTL per shift. The three shifts are not necessarily be opened together, rather, it is possible to open 1,2 or 3 shifts in a day as for the overtime.

The models are revised with the additional overtime variables and constraints.

4.4.1. The MPS Model Modification

The additional variable v_t is added to the integer decision variables indicating how many overtime shifts should be opened in the expense of the additional overtime costs.

The model is as follows:

$$\min (TC = \alpha SC + OC + OV) \quad (4.4.1.1)$$

Subject to

$$SC = \sum_{\forall j} \sum_{\forall t} P_j \times Q_j \times I_{jt}^- \quad (4.4.1.2)$$

$$OC = \sum_{\forall j} \sum_{\forall t} [(Q_j \times I_{jt}^+) / 12] \times [(1 + r)^{12} - 1] \quad (4.4.1.3)$$

$$OV = \sum_{\forall t} 1750 \times v_t \quad (4.4.1.4)$$

$$I_{j(t+1)}^+ - I_{j(t+1)}^- = I_{jt}^+ + Z_{jt} - D_{j(t+1)} \quad ; \quad For \ \forall t < 12 \ , \ For \ \forall j \quad (4.4.1.5)$$

$$Z_{jt} = \sum_{\forall i} Y_{ti} \times 8 \times (1 - l) \times C_{ij} \quad ; \quad For \ \forall t < 12 \ , \ For \ \forall j \quad (4.4.1.6)$$

$$\sum_{\forall i} Y_{ti} \leq 18 + v_t \quad ; \quad For \quad \forall t < 12 \quad (4.4.1.7)$$

$$v_t \leq 3 \quad ; \quad For \quad \forall t < 12 \quad (4.4.1.8)$$

$$I_{j0}^+ = a_j \quad ; \quad For \quad \forall j \quad (4.4.1.9)$$

$$Y_{ti}, v_b \text{ integer variable} \quad (4.4.1.10)$$

In the objective function (4.4.1.1), the OV variable indicating the overtime cost is added. The value of the OV is calculated in (4.4.1.4) as one overtime shift costs 1750 YTL. In (4.4.1.7), the number of overtime shifts is added to the maximum number of shifts available in a week. Finally, in (4.4.1.8) the maximum number of overtime shifts in a week is stated.

4.4.2. Week Aggregation Model Overtime Formulation

The week aggregation model is a two phase model. In the first phase, the model is formulated for the four periods described in the Section 4.3.1.2. The four periods' overtime variables are $v1_p$, $v2_p$, $v3_p$ and $v4_p$.

The first phase of the model is as follows:

$$\min (TC = \alpha SC + OC + OV) \quad (4.4.2.1)$$

Subject to

$$SC = \sum_{\forall j} \sum_{\forall p} P_j \times Q_j \times I_{jp}^- \quad (4.4.2.2)$$

$$OC = \sum_{\forall j} \sum_{p=1} [(Q_j \times I_{jp}^+) \times [(1+r)^1 - 1]] + \sum_{\forall j} \sum_{p=2} [(Q_j \times I_{jp}^+) \times [(1+r)^3 - 1]] + \sum_{\forall j} \sum_{p=3,4} [(Q_j \times I_{jp}^+) / 2] \times [(1+r)^8 - 1] \quad (4.4.2.3)$$

$$OV = \sum_{\forall p} 1750 \times (v1_p + v2_p + v3_p + v4_p) \quad (4.4.2.4)$$

$$I_{j(p+1)}^+ - I_{j(p+1)}^- = I_{jp}^+ + Z_{jp} - DP_{j(p+1)} ; \quad For \forall p < 4 ; \quad For \quad \forall j \quad (4.4.2.5)$$

$$Z_{jp} = \sum_{\forall i} YP_{pi} \times 8 \times (1-l) \times C_{ij} \quad ; \quad For \forall p < 4, For \quad \forall j$$

(4.4.2.6)

$$\sum_{\forall i} YP_{pi} \leq 18 + v1_p \quad ; \quad For \quad p = 1 \quad (4.4.2.7)$$

$$\sum_{\forall i} YP_{pi} \leq 54 + v2_p \quad ; \quad For \quad p = 2 \quad (4.4.2.8)$$

$$\sum_{\forall i} YP_{pi} \leq 72 + v3_p \quad ; \quad For \quad p = 3 \quad (4.4.2.9)$$

$$\sum_{\forall i} YP_{pi} \leq 72 + v4_p \quad ; \quad For \quad p = 4 \quad (4.4.2.10)$$

$$v1_p \leq 3 \quad ; \quad For \quad p = 1 \quad (4.4.2.11)$$

$$v2_p \leq 9 \quad ; \quad For \quad p = 2 \quad (4.4.2.12)$$

$$v3_p \leq 12 \quad ; \quad For \quad p = 3$$

(4.4.2.13)

$$v4_p \leq 12 \quad ; \quad For \quad p = 4 \quad (4.4.2.14)$$

$$I_{j0}^+ = a_j \quad ; \quad For \quad \forall j \quad (4.4.2.15)$$

$$YP_{pi}, v1_p, v2_p, v3_p, v4_p \quad \text{integer variable} \quad (4.4.2.16)$$

The overtime cost is added to the objective function. In (4.4.2.7), (4.4.2.8), (4.4.2.9), (4.4.2.10) the maximum number of the available shifts with the addition of the overtime shifts is stated. And in (4.4.2.11), (4.4.2.12), (4.4.2.13), (4.4.2.14) the maximum number of available overtimes are stated for each period.

After solving the first phase, the YP variables are transferred to the second phase as parameters. Model of the second phase is below:

Min (4.4.1.1)

Subject to

(4.4.1.2), (4.4.1.3), (4.4.1.4), (4.4.1.5), (4.4.1.6), (4.4.1.7), (4.4.1.8), (4.4.1.9), (4.4.1.10),

$$\sum_{t=1} Y_{ti} \leq YP_{li} \quad ; \quad For \quad \forall i \quad (4.4.2.17)$$

$$\sum_{t=2}^4 Y_{ti} \leq YP_{2i} \quad ; \text{ For } \forall i \quad (4.4.2.18)$$

$$\sum_{t=5}^8 Y_{ti} \leq YP_{3i} \quad ; \text{ For } \forall i \quad (4.4.2.19)$$

$$\sum_{t=9}^{12} Y_{ti} \leq YP_{4i} \quad ; \text{ For } \forall i \quad (4.4.2.20)$$

The constraints (4.4.2.17), (4.4.2.18), (4.4.2.19), (4.4.2.20) assure that the total number of the production of each pattern is equal to the first phase's results for the periods' production number of patterns.

4.4.3. The Hybrid Model Overtime Formulation

As described in Section 4.3.3., the week aggregation model is solved in three phases with the additional phase that solves the first phase of the week aggregation model with continuous decision variables. This algorithm is modified for the overtime extension.

The algorithm is described below:

1. Solve first phase of the week aggregation model with continuous decision variables. Call the decision variable $YPlinear$. $v1_p$, $v2_p$, $v3_p$ and $v4_p$ variables remain integer.
2. Assign $YP_{pi} = 0$ if $YP\ linear_{pi} = 0$ before solving the first phase of the week aggregation model
3. Solve first phase of the week aggregation model with integer variables YP_{pi} 's.
4. Solve the second phase of the week aggregation model.

4.4.4. Evaluation of the Results

The three models are run and the solutions are evaluated. The one with the best solution is selected. The solution time is set to be 30 minutes for these runs also.

The results of the overtime extended model are listed in Table 4.21. In Table.4.21, also, the objective function result of the model without overtime shifts is listed.

Table 4.21 Results of the model with the overtime shifts running for 30 minutes

Test Data	Overage Cost	Shortage Cost	Overtime Cost	Total Cost	Total Cost of the Model Without the Overtime Shifts
1	1242	6180	0	7609	7437
2	7307	25609	0	36759	36039
3	3936	604480	19250	718340	744720
4	6494	85897	0	105277	105277
5	4373	308780	15750	375225	389845
6	6785	112370	14000	150008	154400
7	4399	9690	0	15543	14360
8	6266	21418	0	30898	27821
9	7569	6757	0	15340	15149
10	6571	2486	0	9431	10495
11	6428	1922	0	8639	8460
12	6514	2842	0	10182	8764

Overtime shifts improve the solution for the four of the samples and do not change the solution significantly for the others, but it is obvious that with the introduction of the new overtime integer decision variables, the number of the integer variables increases. So; for some instances the objective value is worse than the other one, however the differences are not large.

4.5. Summary of MPS Module Design

To find the optimum production sketch of the 12 weeks planning horizon, an MPS module is designed deciding on the weekly production numbers of patterns to minimize the total cost which consists of shortage and overage costs. The model is constructed and the solution is searched. The optimum solution, however, could be difficult to reach by the basic mathematical model. The model does easily find a feasible solution; however, it cannot converge to an optimum solution. The gap between the best feasible solution and the lower bound in the branch and bound tree is, for some instances, cannot be reduced to acceptable values. Afterwards, a new methodology is applied to solve the problem.

The motivation is that the difference of the linear solution and the integer solution of the model is very small for the instances that the integer MPS model could find a near optimum value meaning that it could reach a solution with low gaps. This comes with the idea of the connection of the linear model and the integer model. First of all, two different connections types are built between the linear model and the integer model which are:

1. First, the MPS is solved linearly. Then the production patterns for weeks that are equal to 0 and the ones that the linear model chooses not to produce for a week are set to 0 in the integer model too. This methodology reduces the number of the decision variables considerably. But, unfortunately, no better solutions are obtained, moreover the high gaps still stands.
2. Second method is that the linear MPS is solved and the production number of patterns is obtained as continuous variables, then these variables are passed to the integer MPS model and the integer model rounds the continuous variables up or down. Nevertheless, this method did not end up with better solutions; neither, moreover this method gives worse solutions compared to the previous one.

Reducing the number of the decision variables is the main approach to the problem so; another option is implemented which is combining weeks in a manner that they form new periods. The combination is done in such a way that, the first week is Period 1, weeks 2,3 and 4 are Period 2; weeks 5,6,7 and 8 are Period 3 and weeks 9,10,11 and 12 are Period 4.

The MPS is divided into two parts which are, in turn, finding the production numbers for periods and dividing the aggregated production numbers into weeks. This model is run and the solutions are evaluated. Although the gaps are still high especially for the first part, the objective function value is improved for some test data. For this method, the gaps are still high. To reduce the gaps and to find an upper bound for the solution, the methodology described above which is linearly solving the problem and setting the 0 variables to 0 in the integer model, is

implemented for the first part of the method. Run time is set to 5 minutes and small gaps are obtained in the run time, but it slightly improves some objective function values.

This work leads to the methodology that solving three models, the original MPS model, week aggregation model, and hybrid model and taking the best solution among them.

The results are compared with the current model and Item-by-item heuristic and it is shown that the developed heuristic method generates better solutions than both of them. Also, during the comparison, the test data is increased to twenty four.

For the last part of Chapter 4, the overtime extension is considered in the model. A decision variable is added to the model to indicate the overtime shifts that are opened. The three methods are revised according to this extension.

CHAPTER 5

THE SCHEDULING MODEL DESIGN

In this chapter, the work carried out for the Scheduling module is discussed.

In Chapter 4, the MPS results are obtained by the designed methodology. Referring to the results of the MPS module, the obtained decision variables are:

- Production amounts of each pattern for each of 12 weeks.
- Production quantities of each product for 12 weeks.
- Weekly shortage and overage quantities of each product.

As described in Chapter 2, after obtaining the 12 weeks production quantities, first week of 12 weeks planning horizon is frozen and the allocation of patterns to the shifts should be done.

In this chapter, the allocation of patterns to the shifts is done according to a specific objective. The setup structure, the structure of the model, the formation of the objective function, and the evaluation of results are discussed.

5.1. The Setup Structure

The characteristics of the production line results in some unique properties for the setup structure. The production is done with production patterns but, no single setup exists between patterns. Patterns include products and setups are realized between products, not patterns.

There are three product types which are produced using different process points of the production line. As a matter of fact, the products which are in the same group realize setups during a change between patterns. No setup occurs between the products of separate product groups. Also, the setup times are not sequence dependent.

The start of the production of products in a pattern is not dependent to other products production start in a shift, meaning that the production of a product can start even though the production of another product is not started in the same pattern because of a continuing setup.

5.2. The Scheduling Model

5.2.1. The TSP-like Structure of the Scheduling Model

The scheduling model is to arrange the patterns with minimum objective function. There are a number of patterns to be arranged which can be referred to as nodes. The basic characteristics are as follows:

- There are a number of patterns to be visited.
- There is a cost of visiting a node after another one related with the setups between the patterns.
- The shortest path connecting all the nodes should be found.

The basic characteristics state that the scheduling problem is much like so called Traveling Salesman Problem. The TSP is simply modeled as follows:

$$\begin{aligned} & \min \sum_{\forall i} \sum_{\forall j} c_{ij} x_{ij} \\ & \text{st} \\ & \sum_{\forall j} x_{ij} = 1 \quad \forall i \end{aligned}$$

$$\sum_{\forall i} x_{ij} = 1 \quad \forall j$$

x_{ij} binary

In the model, c_{ij} refers to the cost of going from one node after another with the indices i and j .

x_{ij} is the decision variable stating that node i is visited after node j . This decision variable is a binary variable taking the value 1 if i and j are visited respectively and 0 otherwise.

This formulation does not consider the subtours. Subtours are not allowed in the scheduling model, indicating that the starting node is visited last after visiting all other nodes.

Miller, Tucker and Zemlin (1960) proposed new constraints to eliminate the sub tours.

$$u_i - u_j + A_{ij}x_{ij} \leq B_{ij} \quad \forall i, j$$

U_i 's are real variables which are called the sequencing variables. U_i represents the position of the i^{th} node if the fixed constants A_{ij} and B_{ij} are taken as $(n-1)$ and $(n-2)$ respectively, where n represents the number of nodes.

This formulation is not a very efficient way of solving the TSP, but for the case, the trials which will be shown later in this chapter proved that the solution time is very short. So, Miller, Tucker and Zemlin (1960)'s additional constraints are used to eliminate the sub tours.

5.2.2. The Linkage between the MPS and the Scheduling Modules

In the MPS module, the number of each pattern to be produced and the shortage, overage values are found. These values are transferred to the Scheduling module. The reason for transferring the shortage and overage values is for giving weights to the products while allocating patterns to shifts. The detailed explanation is discussed in the next section.

Values of the first week's MPS results are considered only. Also, patterns decided to be produced for the first week are decided to be transferred to the scheduling module. As a rule of thumb, arranging the blocks of patterns is the same with arranging each pattern for shifts since the setup time is zero if the production goes with the same pattern in the next shift. So, scheduling each shift is identical to scheduling blocks of patterns. So, the numbers of patterns are not transferred to the scheduling model, but only the information of the patterns to be produced is transferred to the scheduling model. In Figure 5.1, a sample of patterns and the network structure of the scheduling model is represented.

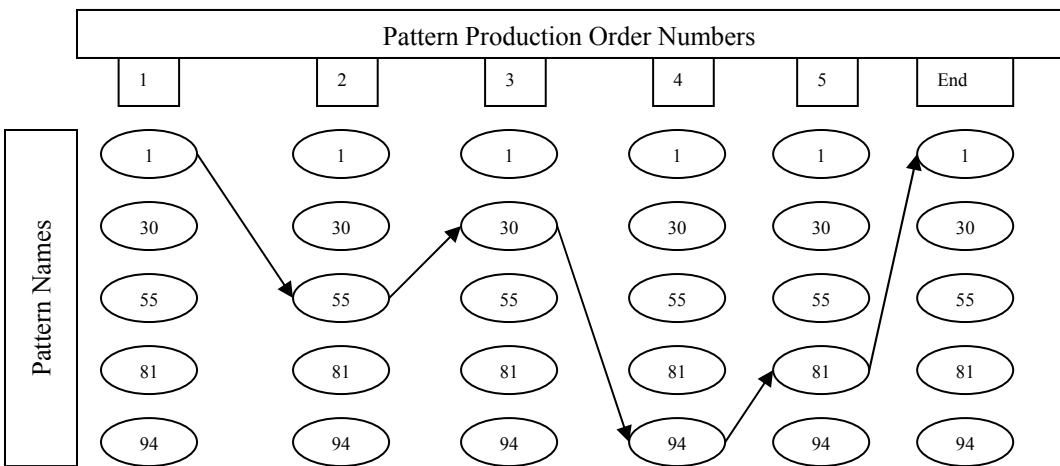


Figure 5.1 Structure of the Scheduling model

In Figure 5.1, the patterns with the names 1, 30, 55, 81, and 94 are transferred from the MPS model to the Scheduling model. The number of shifts that these patterns

are going to be produced is not transferred. For example, pattern 1 will be produced in 5 shifts, pattern 30 will be produced in 2 shifts, etc. But this information is not used in the Scheduling model. The scheduling model decides which pattern is produced after another after selecting a pattern as the starting node. In Figure 5.1, pattern 1 is the starting node, and then respectively, 55, 30, 94, and 81 are decided to be produced. As obvious, the trip between the nodes ends at the starting point. Each connector between the nodes is associated with a cost, which is the length between two nodes. The cost structure is explained in the next section.

5.2.3. The Objective Function Formation of the Scheduling Model

The scheduling module is modeled as a TSP problem. In the TSP problem, the objective function is formulated as follows: $\min \sum_{\forall i} \sum_{\forall j} c_{ij} x_{ij}$

The variable c_{ij} is the distance matrix between the cities in the TSP, in the scheduling model it is the cost of changing pattern i to pattern j. So, its value is very important due to its effect on the results.

Three alternative methods are considered to be employed in the objective function as the c_{ij} variable.

- Minimizing total setup times: When a pattern is changed with another one, setups occur between the products of the same group. Depending on the number of products in two consecutive patterns at most 3 setups will be realized. An example for the number of products in a pattern and setups is in Figure 5.2.

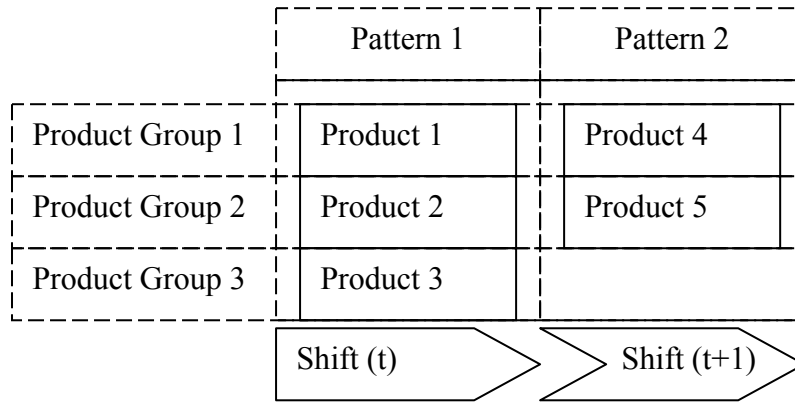


Figure 5.2 An example for the number of products in a pattern and the setups

In the Figure 5.2, pattern 1 consists of the products 1,2, and 3; pattern 2 consists of products 4 and 5. Products 1 and 4 are in Product Group 1, products 2 and 5 are in Product Group 2 and product 3 is in the Product Group 3. Setups occur between the products of the same product group. So, setups between the product 1 and 4, and between the products 2 and 5 are realized. Since after product 3, no other product exists in the pattern 2 of the product group 2, no setup exists in product group 3, neither. There are two setups changing the production from pattern 1 in shift (t) to pattern 2 in shift (t+1).

Total setup means the sum of the all setup times of the products during a change in the patterns. So, the objective function is “the sum of all total setup times in the week 1.”

This formation is, however, has a deficiency; since it does not consider the overages and shortages of the products obtained in MPS in that week. This formulation will try to minimize the total setup times giving more importance to a product with high setup times but not having a shortage problem.

- **Maximizing total production amount:** Another alternative objective function is maximizing the total production amount of the products in the week. When all products are considered, there are some products which have higher capacities than the others. So maximizing the total production amount will force the model to

arrange shorter setup times for the products with high production rates regardless of considering whether they have shortages for that week.

- **Minimizing the total weighted setup time:** The third alternative is to minimize the total weighted setup time. The weight is given according to the product's needed hours to be produced. Time needed for production of a product in hours is calculated as follows:

1. The products shortage and overage value is obtained from the MPS module.
2. If a shortage exists for a product then the first week's demand value is assumed to be the needed amount, else, namely, an overage exists for a product, then the first weeks production value obtained from the MPS module is assumed to be the needed amount.
3. The needed production time for a product is found by simply dividing the needed production amount by the average of its production rates among the patterns that are decided to be produced in that week. This weight parameter is called w_j where j is the product index.

This formulation has the advantage of considering the shortage and overage situations over other two formulations.

5.2.4. The Scheduling Model

The model is constructed as a TSP model with MTZ subtour elimination constraints. Simple notation is as follows:

Indices:

i, l : production pattern index

j, k : product indices

Parameters:

$Setup_{jk}$: Setup time of changing to product k from product j in minutes,

w_j : weight of product j in the objective function which is described in the previous section.

$$w_j = \begin{cases} D_{j1} / (\text{average production rate of } j) & \text{;if } \Gamma_{jt} \geq 0 \text{ in the MPS} \\ Z_{j0} / (\text{average production rate of } j) & \text{;if } \Gamma_{jt}^+ > 0 \text{ in the MPS} \end{cases}$$

b_{ij} : parameter indicating that product j is in the production pattern i .

$$b_{ij} = \begin{cases} 1 & \text{if product } j \text{ is in the production pattern } i \\ 0 & \text{if product } j \text{ is not in the production pattern } i \end{cases}$$

$ctrl_i$: Control parameter indicating that pattern i is decided to be produced in the MPS module.

$$ctrl_i = \begin{cases} 1 & \text{if pattern } i \text{ is decided to be produced in MPS} \\ 0 & \text{if pattern } i \text{ is not decided to be produced in MPS} \end{cases}$$

n : total number of pattern types to be produced in the MPS module.

Decision variables:

x_{il} : binary variable indicating that pattern i is produced after pattern l .

u_i : continuous variable representing the position of the pattern i .

The model is given below:

$$\text{Min } \sum_{\forall i,l} x_{il} \sum_{\forall j,k} w_k b_{ij} b_{lk} (\text{setup}_{jk}) \quad (5.2.4.1)$$

st

$$\sum_{\forall i} x_{il} = 1 \quad \text{for } ctrl_l = 1 \quad (5.2.4.2)$$

$$\sum_{\forall l} x_{il} = 1 \quad \text{for } ctrl_i = 1 \quad (5.2.4.3)$$

$$u_i - u_l + (n-1)x_{il} \leq (n-2) \quad \text{for } ctrl_j = 1, ctrl_l = 1 \quad (5.2.4.4)$$

$$X_{il} \text{ are binary variables} \quad (5.2.4.5)$$

$$U_i \geq 0$$

(5.2.4.6)

The objective function (5.2.4.1) represents the total weighted setup time. Equation (5.2.4.2) and (5.2.4.3) represents that only one arrival and departure can be realized on a node. The inequality (5.2.4.4) eliminates the subtours.

5.3. Evaluation of the Model and the Results

The model is coded in GAMS and solved in CPLEX solver. The computational time is fairly short, that is, the result is obtained in at most two seconds in all trials.

The performance of the model is compared for three models with the objective functions defined in Section 5.2.3. The model is run for the 12 test data which are obtained by running MPS module with the 12 demand data and transferring the output of MPS to the Scheduling module. The results are listed in Table 5.1.

Table 5.1. Results of the TSP model for three separate objective functions

Test data	Minimize total weighted setup time		Minimize total setup time		Maximize total production amount	
	Total setup time for the products with shortage problem		Total setup time for the products with shortage problem		Total setup time for the products with shortage problem	
	(hours)	(hours)	(hours)	(hours)	(hours)	(hours)
1	0.00	8.75	0.00	8.75	0.00	13.00
2	0.25	6.50	1.00	6.50	2.25	14.25
3	0.50	2.00	0.50	2.00	2.00	6.00
4	3.00	8.00	2.25	7.00	6.00	16.00
5	0.50	2.25	0.50	2.00	4.00	7.25
6	2.00	7.00	2.75	6.75	4.00	14.50
7	1.25	5.50	2.25	5.50	7.00	15.50
8	1.25	3.50	1.50	2.75	6.00	11.25
9	2.75	5.75	2.75	5.25	5.50	14.25
10	0.25	1.50	0.25	1.50	1.50	1.50
11	0.50	6.00	1.25	5.25	2.00	14.00
12	0.25	7.00	0.25	6.75	0.50	18.00
Average	1.04	5.31	1.27	5.00	3.40	12.13

In Table 5.1, there are three kinds of models in comparison which are (1) minimizing the weighted total setup time, (2) minimizing the total setup time, and (3) maximizing the total production amount. Each model is compared by means of two criteria. First one is the “total setup time for the products with shortage problem”. This criterion aims to measure the efficiency of the model at focusing on setup times of the products with shortages. Second criterion is the total setup time which measures the overall performance of the model.

Model (1) performs better than two other models in terms of setup times for the products with shortages. For all instances except Test Data 4, Model (1) gives better or equal solutions than Model (2), and on the average outperforms both other models. While Model (1) outperforms other two in terms of setup times for products with shortages, Model (2) has better solutions than Model (1) in terms of total setup times. Results are slightly better for some test data for Model (2) than Model (1), whereas for some test data results are equal in terms of total setup times. But, this is not an unexpected result because the weighted setup minimization focuses on minimizing setup times for products with shortages. This is a result of assigning weights to products with shortages by considering the demand value, not the production value like products with overages. Since the demand value is greater than the production value for products with shortages Model (1) pays more attention to these products than it does so in Model (2). The total setup minimization performance for Model (1) is, also, not considered to be significantly worse than Model (2). Moreover, Model (1) outperforms Model (3) in terms of all performance measures for all the test data.

5.4. The Summary of the Scheduling Module Design

The MPS results are obtained in the Chapter 4. Having the information which of the patterns should be produced for the first week, the second step is to arrange the patterns so that a certain objective function which is mainly a function of the setup times is optimized.

The scheduling algorithm is modeled as a TSP model, and the sub tour elimination is accomplished by the MTZ formulation.

The objective function is formulated as the weighted sum of the setup times. The weight is the number of hours needed to produce the needed amount for that product. The needed amount is obtained in such a way that if a product has shortage problem the needed amount is assumed to be the demand value otherwise the needed amount is assumed to be the production amount obtained from the MPS module. Another alternative is to formulate the objective function as the sum of all the setups incurred during the week. Also, maximizing the total production is considered as a third objective function formulation.

The three models with alternative objective function formulations were compared by the total of the setup times of the products which have shortage problem and by the total setup time for all products. The objective function obtained by the weighted production time methodology performs better than the other methods.

CHAPTER 6

CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

The thesis study is carried out at a firm that produces products for the snacking market. The product range is biscuits, crackers, cakes, chocolate and bars. The particular product type whose production is scheduled in this study is chocolate.

Chocolate production is fairly new to the company that the firm has only two years of experience. The methods that are employed to schedule the production of other products are not useful for chocolate production line scheduling because of the difference between the particular production line and the other ones.

As a matter of fact, the firm's production methodology remains the same roughly, but the algorithms used to solve the two major module of the planning process are useless for the chocolate production line. These modules are MPS module and the scheduling module.

The uselessness of the current algorithms for the two modules is because of the difference of the chocolate production line from the view point of production technology and equipment. The chocolate production line can produce multiple items at the same time and the production capacities of the products differ when produced with different products.

In the study, new methodologies are designed to be able to make the scheduling. The first part is the MPS module. For the MPS module, it is a challenging job to have the optimum values when the number of the decision variables is considered. The problem is modeled and solved; but the solutions are very far from the lower

bounds to be acceptable. The major problem is to find a reasonable lower bound for the problem. However, it was impossible even though the model is run for 3 hours.

The second way is finding a reasonable upper bound and to test the solution with respect to the upper bound. Two methodologies are designed for this purpose. First one is to aggregate the weeks of the planning horizon to four periods. Thanks to this approach, the number of decision variables is reduced by four times. After solving the MPS model with the four periods, the second phase is to disaggregate the periods to weeks sticking to the results of the first part. This methodology results in better objective function values than the original MPS model for some instances. But for some instances the original MPS model is fairly better. Moreover, especially at the first phase of the week aggregation model the high gaps between the best feasible solution and the lower bound found in the branch and bound tree remains. So, another methodology is designed to be able to solve faster and with low gaps.

The second method is to solve the linear relaxation of the first phase of the week aggregated model and then solving this phase again with integer variables but for this time the decision variables which are not decided to be produced in the linear model are assumed to be zero in the integer model. This method ends up with very low gaps and short computational times. Then the results are taken as an input for the second phase of the week aggregation model.

The three models' results are evaluated and an algorithm is designed to take the best of three solutions as the final solution.

Overtimes are not considered up to that point in the models. A new variable is added to consider the overtime. The models are revised according to overtime extension and the results are improved from the point of the objective function value and the solution time did not change.

As to the scheduling part, the output of the MPS module is taken as an input and the scheduling problem is modeled as a TSP. The objective function is taken as the total weighted setup time for the week. The weights are the production time to produce the needed amount of a product. The needed production amount is assumed to be the demand value if a shortage is observed for that product in the MPS module, else the production amount decided in the MPS is assumed to be the needed amount for that product. The weight is calculated by dividing the needed production amount to the average production rate of the product. The scheduling results are quite reasonable. Moreover, compared to the simple setup time minimization and total production time maximization approaches better results are obtained for the products that have a shortage problem.

In MPS model, the solution is obtained by heuristic methods and compared with current model and another heuristic that was originally designed for the capacitated lot sizing problem (CLSP). More methods designed for the CLSP can be searched and modified to obtain solutions to MPS model. For scheduling part, TSP solution can be done with other solution algorithms in case the size of the problem gets larger. Moreover, as a general view point to the problem, the freezing period of the rolling horizon can be considered again taking into account the uncertainty of the demand data and the environmental factors like lead times, magnitude of MPS change costs, and the magnitude of forecast errors. (Lin, Kraje, Leongb, Bentonb, 1993) The performance of the Chocolate Production Line Scheduling can be tested on the rolling horizon as a further research. The performance can be measured on the rolling horizon with different demand sets which have various characteristics as increasing, decreasing, randomly fluctuating patterns.

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APPENDIX A: List of the Production Patterns

Pattern Number	Product Codes			Capacity (Kg.s per hour)			
	Product1	Product2	Product3	Product1	Product2	Product3	Total
1	28050	28064		448	192	0	640
2	28050	28068		439	241	0	680
3	28050	28065		513	161	0	674
4	28050	28071		560	134	0	694
5	28050	28070		494	132	0	625
6	28050	28063		433	89	0	522
7	28050	26063		470	75	0	545
8	28051	28064		279	177	0	456
9	28051	28068		274	223	0	497
10	28051	28065		323	136	0	459
11	28051	28071		346	115	0	461
12	28051	28070		230	140	0	370
13	28051	28063		270	82	0	353
14	28051	26063		230	76	0	307
15	28052	28064		363	230	0	593
16	28052	28068		336	309	0	645
17	28052	28065		425	202	0	627
18	28052	28071		479	168	0	647
19	28052	28070		346	140	0	485
20	28052	28063		329	114	0	443
21	28052	26063		346	76	0	422
22	28053	28064		380	250	0	630
23	28053	28068		369	313	0	683
24	28053	28063		361	115	0	477
25	28053	28070		512	128	0	640
26	28054	28064		258	281	0	538
27	28054	28068		250	351	0	601
28	28054	28063		243	129	0	372
29	28054	28070		302	140	0	442
30	28055	28064		302	287	0	590
31	28055	28068		293	359	0	652
32	28055	28070		346	140	0	485
33	28055	28063		285	132	0	417
34	28056	28064		384	246	0	630
35	28056	28068		360	323	0	683
36	28056	28063		352	119	0	471
37	28056	28070		494	132	0	625
38	28057	28064		202	287	0	489
39	28057	28068		195	359	0	554
40	28057	28063		190	132	0	322
41	28057	28070		230	140	0	370
42	28058	28064		273	308	0	581
43	28058	28068		252	405	0	658
44	28058	28063		245	148	0	394
45	28058	28070		288	140	0	428

Pattern Number	Product Codes			Capacity (Kg.s per hour)			
	Product1	Product2	Product3	Product1	Product2	Product3	Total
46	28059	28065		513	161	0	674
47	28059	28063		433	89	0	522
48	28059	28071		560	134	0	694
49	28060	28065		323	136	0	459
50	28060	28063		270	82	0	353
51	28060	28071		346	115	0	461
52	28061	28065		346	224	0	570
53	28061	28063		285	132	0	417
54	28067	28064		384	246	0	630
55	28067	28068		360	323	0	683
56	28067	28063		352	119	0	471
57	28067	28070		494	132	0	625
58	28069	28063		240	161	0	400
59	28070	28063		132	165	0	297
60	28070	26063		149	82	0	231
61	28050	28063	28065	367	63	139	568
62	28050	28063	28070	401	60	87	548
63	28050	28063	28071	420	56	120	596
64	28051	28063	28065	242	54	126	421
65	28051	28063	28070	260	52	80	391
66	28051	28063	28071	263	52	111	426
67	28052	28063	28065	301	69	153	523
68	28052	28063	28070	316	73	100	489
69	28052	28063	28071	321	74	139	534
70	28053	28063	28070	307	89	100	496
71	28054	28063	28070	216	90	114	420
72	28055	28063	28070	253	91	116	460
73	28056	28063	28070	337	76	104	517
74	28057	28063	28070	169	91	116	376
75	28058	28063	28070	228	98	124	450
76	28059	28063	28070	401	60	87	548
77	28059	28063	28071	420	56	120	596
78	28060	28063	28070	260	52	80	391
79	28060	28063	28071	263	52	111	426
80	28061	28063	28070	253	91	116	460
81	28061	28063	28071	258	93	162	513
82	28067	28063	28070	337	76	104	517
83	28059			672			672
84	28050			672			672
85	28053			619			619
86	28056			576			576
87	28067			576			576
88	28069			303			303
89	28051			403			403
90	28060			403			403

Pattern Number	Product Codes			Capacity (Kg.s per hour)			
	Product1	Product2	Product3	Product1	Product2	Product3	Total
91	28054			351			351
92	28057			266			266
93	28061			399			399
94	28052			576			576
95	28055			399			399
96	28058			324			324
97	28064			379			379
98	28065			278			278
99	28068			497			497
100	28071			223			223
101	28070			165			165
102	28063			189			189
103	26063			93			93

APPENDIX B: An Example of the 12 weeks Master Plan

		Weeks											
		0	1	2	3	4	5	6	7	8	9	10	11
Production Pattern Number	2	2		1									
	5				2	2	3	2	2	2	3	1	1
	12				2	2	1	2	5			2	1
	16	5	5		2	2							
	19							2	2	2	3	1	2
	25				2	2	1	2		3	2	2	1
	28						2				1	1	
	30									1			1
	32	3	4	5	1	1	2	2	1	3	3	2	1
	43												1
	47	1						1					1
	56		3	2									
	61				1	1		1		1		1	
	66							1	3				1
	78				1		3					1	
	80						2		1				
	81				2	2		1	1		1	1	
	82				2	2	1	2		1		1	1
	85										2		
	86					2	1	1		3		2	
	87										2		1
	88			1	1	1	1		3			1	
	90				1	1	1	1		2	1	2	1
	94	2	2	2	1								
	95	2	3	2									
	100			1									
	101	1	1	1									
	104	2		1									
	105			2									
	108												5
Total		18	18	18	18	18	18	18	18	18	18	18	18

APPENDIX C: Results of MPS-2, MPS-4, MPS-5 for 24 Test Data

Test Data	Hybrid Model(MPS-5)			Aggregation of Weeks (MPS-4)			Original Model (MPS-2)		
	Shortage Cost	Overage Cost	Overall Cost	Shortage Cost	Overage Cost	Overall Cost	Shortage Cost	Overage Cost	Overall Cost
1	2890	8353	11676	990	6297	7437	219	8569	8820
2	27945	8183	40319	32886	7016	44836	24952	7345	36039
3	659730	5743	764436	66393	4824	768340	643760	4394	744720
4	90999	8609	113259	9055	7823	111904	85897	6494	105277
5	356660	8700	418864	354460	9042	416680	334460	5216	389845
6	138470	8362	167597	136280	7023	163739	128470	6659	154400
7	6304	8437	15688	6567	6808	14360	8883	7016	17232
8	19092	7544	29500	23113	6755	33336	18832	6163	27821
9	7016	9518	17586	6300	7900	15149	10141	4875	15749
10	6142	8165	15229	3583	6373	10495	3535	6945	11011
11	4629	8913	14237	17143	6490	8460	3127	6589	10186
12	3569	8694	12800	1282	7289	8764	2683	7352	10439
13	15856	8302	26538	11912	6030	19729	13419	6718	22150
14	3305	8677	12478	2488	6715	9577	4534	5290	10505
15	3600	8099	12240	1165	5166	6506	4009	4850	9461
16	4032	9365	14003	1217	6084	7484	3806	6635	11013
17	4226	10514	15374	3124	7895	11488	1465	9926	11611
18	1185	7188	8551	1446	5717	7380	1167	7456	8799
19	1134	7076	8381	815	5728	6667	1970	5545	7810
20	1395	7859	9463	1072	6827	8060	3323	6641	10463
21	1060	8547	9767	1543	6582	8357	3843	6672	11091
22	1881	8533	10697	1175	7637	8988	3198	6995	10674
23	3792	8740	13102	2871	6892	10109	4152	7858	12634
24	13548	8532	24113	10722	5817	18183	11789	5041	18599