

A DECISION SUPPORT SYSTEM FOR
WORKFORCE MANAGEMENT IN CALL CENTERS

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by

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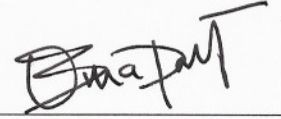
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ABSTRACT

A Decision Support System for Workforce Management in Call Centers

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Shift design, workforce allocation and call allocation are the major problems in call center management. The aim of the call center manager is to allocate and dynamically update the workforce so that the incoming calls are answered in the shortest possible time, above certain service level measures. The software tools developed to aid decision making in these areas use models that are based on Erlang-C calculations. However, the strict assumptions of Erlang-C often lead to invalid decisions. For this reason, especially at peak times during the day, dynamic updates in the applied design are inevitable. In this study, a framework for a decision support system (DSS) is developed for designing the shifts and allocating the agent workforce to the shifts in a call center, so that target service levels are met. In the proposed system, shifts are designed by solving a linear optimization model. Using this solution as the input, a simulation model is developed to dynamically update the workforce so that the minimum required service level is met at all times. The proposed DSS is applied to an existing call center system, alternative designs are generated and compared.

KISA ÖZET

Çağrı Merkezlerinde İşgücü Yönetimi İçin

Bir Karar Destek Sistemi

Burak Gedikoğlu

Vardiya tasarımı, personel tahsisi ve çağrı tahsisi çağrı merkezi yönetiminin en önemli problemleridir. Çağrı merkezi yöneticisinin amacı, gelen çağrıların belirlenen performans ölçütleri üzerinde en kısa zamanda cevaplanmasını sağlayacak işgücünü tahsis etmek ve dinamik olarak güncellemektir. Bu alanlarda karar vermeye yardımcı olmayı amaçlayan yazılımların temeli Erlang-C hesaplamalarına dayanmaktadır; fakat Erlang-C'nin katı varsayımları bazen geçersiz kararlara yol açabilmektedir. Bu yüzden, özellikle günün yoğun zamanlarında, uygulanan tasarımda dinamik güncellemeler kaçınılmaz olmaktadır. Bu çalışmada, hedef servis seviyelerini sağlayacak şekilde, çağrı merkezinde vardiya tasarımı ve iş gücünün vardiyalara tahsisi için bir Karar Destek Sistemi çerçevesi geliştirilmektedir. Önerilen sistemde, vardiyalar bir doğrusal eniyileme modelini çözerek tasarlanmakta; doğrusal eniyileme modelinin sonuçlarını girdi olarak kullanarak ihtiyaç duyulan en düşük servis seviyesini sağlamak amacıyla dinamik güncellemelerin yapılabilmesi için bir benzetim modeli geliştirilmektedir. Önerilen Karar Destek Sistemi, gerçek bir çağrı merkezi sistemi üzerinde uygulanmakta, alternatif tasarımlar oluşturulmakta ve karşılaştırılmaktadır.

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1. INTRODUCTION

In the last fifteen years, with the increasing trend of the economy from manufacturing towards the service sector, call centers have become a well-known phenomenon throughout the world. The total number of call centers has increased substantially and the size of call centers has grown larger. Some of these call centers employ 500-2,000 agents and it is estimated that about 2,000,000 agents work in nearly 10,000 call centers across the globe (Koole and Pot, 2005).

Several developments, change in the economy, modification of key factors of being successful and productive in the market can be mentioned among the reasons for the growth of industry. However, in today's competitive world, the most important and critical factor in business success is being able to respond rapidly to customer requests. Therefore, together with the gross development of call centers; quality of customer service and customer relationship management has become vital for the companies.

From a different perspective, with the growing number of agents working in the sector, labor costs constitute a substantial part of the business expenses and reducing these costs while increasing the productivity has become a considerable aim. Eventually it becomes essential for call center managers to organize job routing, scheduling and staffing operations effectively by the use of special workforce management (WFM) systems.

Workforce management systems are analysis tools and computer programs that forecast call volumes, determine staffing levels, work schedules, and monitor performance indicators. Although WFM tools introduce many benefits, they have

also several drawbacks due to the use of Erlang-C approximations. In today's complex and multi-skilled call centers, the strict assumptions of Erlang-C are often invalid and in several cases the resulting solutions are misleading. Actually it is very difficult to use the Erlang-C model for "what-if" scenarios generated to evaluate call center design, call handling strategies and call routing options. Thus the use of an alternative tool becomes inevitable, and with its dynamic and developing nature, system simulation appears to be an appropriate method to be used in what-if analysis in workforce management.

In this study, a Decision Support System (DSS) is proposed for designing shifts and allocating the agent workforce to the shifts in a call center so that the target service levels are met in all periods. In the proposed system, shifts are designed by solving a linear optimization model. Using this solution as the input, a simulation model is developed to dynamically update the workforce allocation so that the minimum required service levels are met at all times.

One of the leading call center companies in Turkey was chosen for this study. Currently, the call center provides services in three different locations with different agent and seat capacities. A major problem is to allocate the calls between these locations, while attaining the target service and utilization levels. At present, shift scheduling and staff allocation decisions are made by using a workforce management tool. However, they still have problems in reaching the target service levels at peak hours, mostly due to the reasons explained above. Increasing the number of agents allocated to the shifts results in severely low utilizations in the periods when the call rates are low. Generating shifts with flexible durations appears to be a reasonable option for this problem. The aim of this study is to develop an easy to use

environment by ARENA and MS-Excel that will work together with the existing tool and enhance its solution quality by generating valid results and by allowing the real time assessment of numerous alternatives on flexible shift durations in multiple locations. As an illustration, an alternative workforce schedule and staff allocation is designed by using the proposed system and it is shown that the proposed design outperforms the solution of the current system.

The organization of the thesis is as follows: In Section 2, the related background on call center processes and performance measures is introduced and the uses of simulation and optimization techniques in the current workforce management systems are discussed. In Section 3, a literature review on the basic models developed for workforce scheduling and staffing is provided. In Section 4, a DSS is designed for workforce management in the form of an integrated optimization model and a simulation model. In Section 5, alternative workforce designs are generated and evaluated as an illustration of the proposed DSS. In Section 6, the framework of the DSS environment is provided. Section 7 includes the conclusion and the summary.

2. CALL CENTER OVERVIEW

Dawson (2004) defines a call center as “a physical location where calls are placed, or received, in high volume for the purpose of sales, marketing, customer service, telemarketing, technical support or other specialized business activity.” In a similar manner, Mehrotra (1997) defines a call center as “any group whose principal business is talking on the telephone to customers or prospects” (Mehrotra, 1997, as cited in Koole and Pot, 2005).

The early definitions described a call center as a place of doing business only by a phone which uses a database with an automatic call distribution system. However, in today’s business environment, the definition can be expanded in two ways.

First, it is expanded to include call-making and call-taking organizations that were originally overlooked, like collections and fundraising organizations, and both internal and external help desks. And more controversially, it is expanded to include centers that handle not only phone calls but other types of contacts like e-mail, fax, chat sessions or video interactions. The term “contact center” rather than “call center” is used in the industry when different media types for customer contact are used. In addition, there are mainly two types of calls handled by the call center. Inbound calls are calls that are initiated by a caller and answered by a call center and outbound calls are initiated by a call center and answered by the dialed party (Mehrotra and Fama, 2003; Dawson, 2004). For instance, in the call center of a bank, calls that are made for the purpose of marketing or sales of loans or credit cards are outbound calls and calls that are received for the purpose of customer service or money transactions are inbound calls.

Around 30 years ago, call centers first were recognized as organized corporate departments. In general, with some limited exceptions, only airline reservation centers, catalog ordering companies and problem solvers like GE Answer Center widely operated call centers. In the early 1990s, with the development of Automatic Call Distributor, which was a customized, proprietary phone system designed to funnel as many calls as possible, large call centers that could afford the investment in technology become able to handle huge volumes (Dawson, 2004).

More recently, with the development of client/server software systems, open phone systems and browser, PC LANs based desktops for data access, any call center can have an advanced call handling and customer management system, even those with ten agents or less (Dawson, 2004).

As companies have learned that service quality and speed are the key factors in attracting potential customers and maintaining current customers, the common perception of the call center has changed. It is no longer conceived of as a high living facility. In fact, it is now considered as a tool used to gain competitive advantage. In some industries, like financial services, hospitals, and catalog retailing, a call center can mean the difference between being in business and not being in business. In other industries like cable television and utilities, call centers have become the centerpieces of corporate attempts to make service faster and easier and to improve their company images in customers' minds (Dawson, 2004).

Nowadays, it can be stated as a business rule that any company that sells any product or service should have a call center because it is the most effective and efficient way to reach and also be reached by customers.

2.1. Processes in a Call Center

In a call center, calls are processed briefly as follows. First, a customer dials a toll-free telephone number to contact a call center. If all of the phone lines are busy at that time, the customer can be blocked and receive a busy signal. At first, calls may be connected to an Interactive Voice Response (IVR) unit. The latest generation of voice-recognition technology allows IVR to interpret complex user commands, so the system receives the call and prompts some announcements and serves a series of interactive menus. It is simply a computer system that allows customers to enter information in response to some predefined questions. The customer then gets the information needed back from the system through a recorded voice or a synthesized voice. In other words, customers may self-serve and they are able to complete the service interaction at the IVR. The duration of an IVR experience may last several minutes or less depending on the type of the call, but in each condition, the resulting inbound telecommunication expenses generally are covered by the call center (Avramidis and L'Ecuyer, 2005; Duder and Rosenwein, 2001; Dawson, 2004).

In some cases, the IVR system can handle all of the needs of the customer so an agent's intervention is not required. However, if the call cannot be handled totally by the IVR system, it is passed from the IVR to an Automatic Call Distributor (ACD). An ACD is a specialized switch designed to route the call to a skilled agent; if no agent is available, then the call is placed in a queue. If one or more agents are idle, then a customer call is routed to the agent that has been idle for the longest duration. Otherwise, if all agents are busy, the call enters a queue of the selected option on a FIFO basis or according to the predefined priority rules of the call center. At this point, the customer either waits until a suitable agent is free to handle a new

call or abandons the system if the waiting time is too long. This type of abandonment is referred as a “reneged” call. In other words, reneging occurs when a customer joins a queue on arrival but later decides to leave the system. Reneging is often entity-dependent. Each customer entering a queue has a different tolerance threshold for the waiting time in the queue. This threshold is also called as the “tolerance time” of a customer. A customer abandons the system when the waiting time reaches his or her tolerance threshold.

Sometimes an arriving call leaves the system before joining the queue due to the existing queue capacity of the call center. In most service systems, the amount of waiting space is limited, so when all the space is occupied, an arriving call will not be able to enter the system and will be “balked.” In short, upon arrival, a customer may be balked, may abandon or may take the service depending on the queue capacity of the call center and the tolerance threshold of the customer (Duder and Rosenwein, 2001; Kelton, 2004; Avramidis and L’Ecuyer, 2005).

2.2. Performance Measures in a Call Center

A call center can measure its performance according to many different criteria. It is important to establish performance measures that are appropriate to the company's business and the industry within which that company operates. The most commonly used terms are listed below (Duder and Rosenwein, 2001; Nucomm International, 2005).

Average Speed of Answer (ASA):

ASA is the average delay in the queue experienced by a customer waiting for an agent. Depending on the call center's policy and performance requirements, there are two possible definitions:

- i) The time a caller had to hold in the agent queue before being answered. Callers who are answered immediately are factored into the calculation with a zero ASA.
- ii) The time a caller who was not answered immediately had to hold in the agent queue. Callers who are answered immediately are excluded from the calculation.

Service Level:

Service level can be defined as the percent of the time in which a service goal is reached. In other words, it is the percentage of the calls in the queue that are answered within certain duration. The 80/20 rule is generally accepted as the service goal, which means at least 80% of the incoming calls are to be handled within 20 seconds. There are three important service levels that the scheduler has to be concerned about:

- i) Target: The company's stated goal for consistent service to customers.
- ii) Forecast: The company's forecasted service level based on the call forecasts for a given period.
- iii) Actual: The realized service levels on a given period.

Service level is the crucial measure for the determination of accurate staffing levels.

Abandonment Rate:

Abandonment rate is the percentage of callers who hang up before their call is answered by an agent or before they make a selection in an IVR unit. Andrews and Parsons (1993) developed a linear regression model that showed a strong correlation between ASA and percent abandonment (Andrews and Parsons, 1993, as cited in Duder and Rosenwein, 2001), i.e., the abandonment rate increases with the ASA.

Average Talk Time (ATT):

ATT is the average duration that an agent actually spends in talking with the customer per call, excluding the hold time and after call work time that are explained below.

Average Hold Time:

It measures the average amount of time a customer spends on hold. The measurement includes the time in which a customer is placed on hold after initially speaking with a customer service representative. During this period, the customer waits after he initiates his problem to the agent and the agent is busy with the related process. Customers may be displeased when they are held in the line and customer satisfaction significantly decreases when holding time gets longer.

Average After Call Work Time (AWT):

Sometimes an agent has to do some extra work for the customer process, like doing post calls or contact works such as notations to the customer database recording the interaction, etc. The agent is unavailable for other calls during this period although the customer has already left the system. The average of this measurement is also called “Average Wrap Time.”

Average Handle Time (AHT):

AHT is the total time to handle a call or a contact, on average. It is the summation of Average Talk Time, Average Hold Time and Average After Call Work Time.

Agent Utilization:

Utilization can be explained as the percent of the available time of agents that is spent actually for handling incoming calls. Utilization can be calculated as:

$$\text{Utilization} = \frac{\text{Number of Calls Handled} \times \text{Average Handle Time}}{\text{Time Length of Period} \times \text{Number of Agents}}$$

where “Time Length of Period” is the length of the period during which the mean arrival rate is assumed to be stationary. “Number of Agents” is the number of available agents in this period. “Number of Calls Handled” is the number of calls answered in this period.

A call center is “living dangerously” if it drives towards 100% utilization. It means that it is operating at maximum available capacity and if actual call volume exceeds the forecasted value even slightly, queue sizes and abandonment rates are likely to rise.

While each of these measurements may be used to evaluate call center performance, it is critical to focus on those items that are business-critical for the organization.

2.3. Challenges for Call Center Management

In today's competitive industry, managing a call center has many challenges. Basically, call center managers must strike a balance between three important competing interests, which are costs, service quality and employee satisfaction (Mehrotra and Fama, 2003).

On a daily or long-term strategic basis, executives and managers must answer various questions to determine which decision support models are valuable for them. These questions are listed below (Mehrotra and Fama, 2003):

- How many agents should we have on staff with which particular skills?
- How should we schedule these agents' shifts, breaks, lunches, training, meetings and other activities?
- How many calls of which type do we expect at which times?
- How quickly do we want to respond to each type of inbound call?
- How should we cross-train our agents? How should we route our calls to make the best use of these resources?
- Given a forecast, a routing design, and an agent schedule, how well will the system perform?
- What is our overall capacity? How will a spike in call volumes impact our overall performance?
- How is our center doing right now? What has changed since we did our last forecast and published our schedules? If the changes are significant, what can I do to respond to minimize the impact on the rest of the day or week?

Although answers of all of these questions are important for managers, staff allocation and shift scheduling play important roles in the overall success of call centers by directly effecting the three key factors (cost, service quality and employee satisfaction), as mentioned above. Therefore, in the remainder of this study, the decisions and models on staffing and scheduling will be emphasized and a DSS model will be introduced.

2.4. Workforce Management

Workforce Management (WFM) is mainly interested in the allocation of labor resources over time. Labor allocation is typically a tactical problem with a time horizon of a few weeks. The function of WFM is to determine the requirements and the way of meeting these. As mentioned in previous sections, it is crucial to determine the correct way of staffing and scheduling since it directly affects the productivity, profitability and efficiency of call centers. Strategies must be determined carefully, because if overstaffing occurs, the utilization of the agents will drop and expenses will increase. On the other hand, understaffing causes a decrease in performance measures like service level and average speed of answer.

The typical sequence of steps in WFM is as follows (Buffa et al., 1976, as cited in Ingolfsson et al., 2002; Bhulai et al., 2005):

1. Demand forecasting
2. Labor staffing
3. Shift scheduling
4. Staff allocation to the shifts

Demand forecasting is concerned mainly with the prediction of the future amount of calls that will be handled by the call center. Labor staffing translates these calls into the number of required agents where the target service level is met. Shift scheduling is the generation of shifts where the staffing levels are met. Staff allocation refers to the assignment of the employees to the designed shifts so that the minimum staff levels determined in Step 2 are attained. The steps that constitute a basis for WFM are further explained below.

2.4.1. Demand Forecasting

Among the four steps, forecasting the demand is the most critical one because the following steps use the output of forecasting as an input for their analysis to schedule the workforce to meet target service level and other performance criteria. In order to run a call center effectively, forecasts should be made for different time periods such as yearly, monthly, weekly, daily, hourly, half hourly or, for peak periods on a quarter hourly basis. Long-term forecasts, like yearly and monthly predictions, are used for planning operational changes, budgeting and staff planning, training, and scheduling vacations. Mid-term forecasts, such as weekly and daily forecasts, are used for workforce staffing and scheduling. Short-term forecasts, such as hourly forecasts, show how well a call center is staffed for the current day (Klungle, 1997).

2.4.2. Labor Staffing

Labor staffing is a medium duration attempt to identify a workforce's ideal size and composition. In a staffing problem, depending on the requirements, the day is divided into periods (e.g., 15 minutes, 30 minutes, 1 hour) and the number of

agents for each period is determined according to constraints and target service levels. These constraints should be determined carefully and their impact on staffing should be considered. Constraints may be considered separately as per call, per period or per shift depending on the situation (Thompson, 1997; Avramidis and L'Ecuyer, 2005).

2.4.3. Shift Scheduling

Shift scheduling (developing a single day's work schedule) and tour scheduling (developing a weekly work schedule) are expected outcomes of labor staffing problems. Management often wishes to deliver a specified level of service as less costly as possible by maximizing the level of service delivered by a fixed number of agents (Thompson, 1997).

The need to schedule shifts is an ordinary process in the service sector. Restaurants, bank branches, retail stores, and airline check-in areas are some examples of organizations that need to schedule shifts to match the supply of employees providing services with the demand of customers that typically varies over time. Call centers are perhaps the largest sector that needs shift scheduling. Labor is the primary costs of call centers so shift scheduling has a gross impact on cost saving and productivity improvements (Ingolfsson et al., 2002).

Unlike labor staffing, shift scheduling has a much shorter planning horizon. The time period may be a day, a week or even a month, varying according to size, the number of agents and expectations (Thompson, 1997).

2.4.4. Staff Allocation

After a set of reasonable work schedules has been specified in the previous step, agents are allocated to the shifts according to the defined allocation criteria of the company. The decision variables are the number of agents in each work shift. It does not effect staffing directly, but assures that agents are allocated to a feasible set of work schedules. More constrained version of the problem may emerge when there is a fixed set of available agents to be scheduled for the day or the week, where each agent has a specific set of skills. In this condition, call center managers come across more complex scheduling and allocation problems (Avramidis and L'Ecuyer, 2005).

Staff allocation has two common forms (Thompson, 1997):

- i) A higher-flexibility form in which employees are not guaranteed a set number of hours over the planning horizon and are generally part-time employees.
- ii) A lower-flexibility form in which employees are guaranteed a set number of hours over the planning horizon and are usually full-time employees.

2.5. Workforce Management Systems

Workforce management systems are computer programs, software and analysis tools that forecast incoming call volumes, determine staffing levels and schedules, and monitor the appropriateness of the outcomes to these schedules. A workforce management system is like a large spreadsheet that uses historical data, analyzes scenarios by changing a series of inputs to predict performance indicators and produces desired output like work schedules (Klungle, 1997). Their forecasting capabilities are quite good in cases of low variability. In addition, with the

development of the technology and integration of computer-based monitoring systems, workforce management systems provide the simultaneous observation of the performance indicators of call centers and enable what-if analysis.

Klungle (1997) summarizes the key strengths and weaknesses of Workforce Management Systems and states that the weaknesses are due mostly to the strict assumptions of Erlang-C, absence of skill-based routing capability, the limited capability in forecasting and handling randomness.

2.5.1. Erlang-C and Its Shortcomings

Most of the WFM systems use Erlang-C calculations in Step 2 of WFM, to find out the minimum number of staff required to answer the expected number of incoming calls in a given time period. Erlang-C is a formula suggested by A. K. Erlang and it is an updated version of Erlang-B, where it is assumed that the blocked customers who find all servers busy are cleared so the blocked customers are lost. Erlang-C model is similar to Erlang-B model, but it assumes that the blocked customers will wait in a queue as long as necessary for a server to become available (Cooper, 2000). In this model, the probability of customers who find all servers busy, $C(s,a)$ is given by the Erlang-C (or Erlang delay) formula as a function of the number of identical servers, s , and the expected work load, a , as,

$$C(s,a) = \frac{\frac{a^s}{s!(1-\rho)}}{\sum_{k=0}^{s-1} \frac{a^k}{k!} + \frac{a^s}{s!(1-\rho)}}, \quad (2.1)$$

where

$$\rho = \begin{cases} \frac{a}{s} & \text{if } a < s \\ 1 & \text{if } a \geq s \end{cases} . \quad (2.2)$$

Here, a is the offered load in erlangs, computed as $a = \lambda\tau$, where λ is the arrival rate and τ is the average service time. The quantity ρ is the server utilization (the fraction of time, on average, that a server is busy), and hence $C(s, a) = 1$ when $\rho = 1$.

The probability that a customer will wait in the queue more than t units of time before beginning service, $P(t)$ is

$$P(t) = C(s, a) e^{-(1-\rho)s\mu t} \quad (2.3)$$

where $\mu = 1/\tau$ is the service rate. For instance, in a 10-server system operating at 80% utilization, the fraction of customers who will wait longer than one average service time is given by the right-hand side of formula with $s=10$, $\rho=80\%$ (and therefore $a=8$ and $C(s, a) = C(10, 8) = 0.4092$) and $t = \tau = 1/\mu$: $P(\tau) = 0.05538$ (Cooper, 2000).

It can be seen in the Equation (2.3) that given the number of agents, the call arrival rate and service rate, one can approximate the probability that the average waiting time in the queue is at least t time units. Considering the definition of the service level introduced in Section 2.2; $1-P(t)$ is the probability that a customer waits at most t time units in the queue. For example, for a given service rate, μ and arrival rate, λ , the Equation (2.3) can be used to estimate the required number of agents, s , that leads a service level of 80/20, i.e., 80% of all calls should be answered in at most 20 seconds, such that $t=20$ seconds., $P(t)=1-0.8=0.2$.

Although most workforce management systems use the Erlang-C formula, there are major difficulties and problems due to the static nature of the Erlang model. Erlang-C uses the basic M/M/s queuing model and this model is based on the assumptions listed below (Klungle, 1997; Duder and Rosenwein, 2001):

- The number of arrivals of inbound calls are assumed to have a Poisson distribution.
- Agent service times are assumed to have an exponential distribution.
- A fixed number of agents (s) are assumed to be available.
- All servers are identical, having the same skill and service time distributions.
- The queuing rule is First Come First Served (FCFS) and the service process consists of a single phase.
- Queue length is unlimited and thus no blocking, balking or reneging occurs.

Apart from these assumptions, the performance of the predictions based on Erlang approximations differs with the size of the call center. While in larger, high volume call centers, Erlang predictions are very close to reality, in smaller, lower volume call centers they can vary significantly (Dawson, 2004).

In today's complex and multi-skilled call centers, many of the assumptions listed above can be accepted as invalid and the use of Erlang calculations may give inaccurate results. This leads to over or under staffing and unproductive scheduling. In addition, Erlang-C is not flexible enough to be used for "what if" scenarios that determine call center design, call handling strategies and call routing options (Klungle, 1997).

Because of these shortcomings of the Erlang-C model, the use of an alternative tool becomes inevitable. Among other tools, with its dynamic and

developing nature, simulation emerges to play an important role in call center design and in workforce management systems.

2.5.2. Use of Simulation in WFM Systems

Simulation can be defined as “a broad collection of methods and applications to mimic the behavior of real systems, usually on a computer with appropriate software” (Kelton, 2004). Computer simulation deals with models of systems. Call centers are typical examples of one of these systems with its structure and processes. Solutions obtained via simulation can move beyond the limitations of Erlang calculations and overcome many of the difficulties of analytical models and assumptions.

With the movement towards the skill-based routing of calls and due to developments in the technology, Erlang-C is basically outdated with its strict assumption of identical labors and calls. In addition, rather than a FCFS rule, most of the call centers prefer to give priorities to some customers in order not to make them wait longer. In this manner, simulation provides priority options that can be modified according to changing user preferences. Furthermore, Erlang can not handle peak values accurately and runs with average values for key performance measures leading to static results. However, simulation provides a dynamic and flexible environment in which these options can be settled. Moreover, while Erlang-C is incapable of representing interactions in the system, simulation can give detailed information about interaction between components (Klungle, 1997; Kelton, 2004).

The use of Erlang-C in the second step of WFM (labor staffing) leads to validity problems for other mathematical models that use outputs of Erlang-C as

inputs in the third and fourth steps of WFM, namely shift scheduling and staff allocation. In order to overcome this problem, simulation is proposed to check the validity of the results dynamically and make what-if analysis easier and more reliable. Simulation is not a substitution for Erlang-C and related mathematical models, but it is a “supportive integrated model” that enables the decision maker to monitor the validity of the results.

3. LITERATURE REVIEW

The problems of labor staffing and scheduling have received substantial attention in the literature and many researchers have offered different models and algorithms for shift scheduling and agent allocation in call centers. There have also been different studies on single-skill and multi-skill call centers. The most relevant papers on scheduling in call centers are stated in this section.

Models for employee scheduling have a long history in the literature. The first study on this subject was Edie's (1954) classic study of traffic delays at tollbooths. Edie proposed a combination of empirical analysis and formulas for stationary queuing systems to find out the staffing requirements required to ensure a specified level of service. After Edie's study, the first integer programming formulation was the following set covering type model proposed by Dantzig (1954):

$$\text{Minimize } \sum_{j \in J} c_j x_j \quad (3.1)$$

$$\text{Subject to } \sum_{j \in J} a_{tj} x_j \geq b_t, t \in T$$

$$x_j \geq 0, \quad \text{and integer, } j \in J$$

where J represents the set of all shifts, T represents the planning periods that the shift schedule covers, b_t is the estimated demand for period t ; c_j is the cost of assigning an employee to shift j ; a_{tj} is equal to one if period t is a work period for shift j and zero otherwise, and x_j is an integer variable defined as the number of employees assigned to shift $j, j \in J$ (Mehrotra et al., 2000).

With this formulation, Dantzig (1954) showed the use of linear integer programming to determine the shift schedules and staff allocation that will meet

specified requirements in each planning period at minimum cost. Most models dealing with shift scheduling in single-skill environment with multi-period are based on the standard set-covering model presented by Dantzig (Ingolfsson et al., 2002; Bhulai et al., 2005).

As previously stated in Section 2.4, WFM consists of four steps: demand forecasting, labor staffing, shift scheduling and staff allocation to the shifts. Most researchers follow this general approach in different WFM analysis tools.

In recent years, Green et al. (2001) has suggested a new approach called the SIPP (Stationary Independent Period by Period) approach to determine the staffing requirements in each period based on a stationary M/M/s queuing system which assumes stationary arrivals and service processes. The aim is to determine the smallest number of employees or servers required to provide a specified level of service. However, this approach does not give adequate results and is unreliable in many situations. Therefore, different models are generated to extend the range of situations where SIPP provides reliable employee requirements. The lag max approach is the most promising way that replaces the average arrival rate over a planning period with the maximum over a planning period of a shifted version of the arrival rate function.

Ingolfsson et al. (2002), Thompson (1997), and Atlason et al. (2004) propose some approaches to integrate steps two and three of WFM, where minimum agent requirements and shift schedules are determined simultaneously.

Ingolfsson et al. (2002) propose a method to find low cost employee shift schedules that guarantee that target service level is met or exceeded. As previously discussed, most approaches use a two-step procedure. First, they determine the

minimum employee requirements and then finding a minimum cost schedule that provides the required number of employees in every period. According to Ingolfsson et al. (2002), due to the approximations used in the first step, the two-step approach usually results in infeasible or suboptimal solutions. Therefore, their method iterates between two components: a schedule evaluator and a schedule generator. The iteration begins with the schedule evaluator that uses the randomization method to calculate transient service levels and then it identifies infeasible intervals, where the service level is below the target level. The schedule generator solves a series of integer programs to produce schedules. Then, each schedule is transferred to the schedule evaluator for evaluation of the cost, level of service and other possible results of using the schedule. The procedure terminates when a feasible solution is found. Although the method does not guarantee optimality, it provides a lower bound on the minimum cost.

One of the motivating examples of our study is by Thompson (1997), who introduces two new models of the labor staffing and scheduling problems that overcome the limitations of the existing models. He distinguishes between the aggregate threshold service level (the overall level of service that management wishes to provide to customers) and the minimum acceptable service level (the lowest level of customer service that management considers acceptable in any planning period). For instance, management may wish to serve at least 80% of all customers arriving during a week within 30 seconds of their arrival (the threshold level of service), but wants every planning period to serve at least 50% of customers within one minute of their arrival (the minimum acceptable service level).

Mainly, these distinctive models have important attributes. First, both models guarantee the delivery of a minimally acceptable level of service in all periods. Second, while one model can identify the least expensive way of delivering a specified aggregate level of customer service, the other model can identify the highest level of service attainable with a fixed amount of labor. Third, managers can identify the pareto relationship between customer service and labor costs with the use of these models. Last, a degree of control over service levels that is not possible with existing models is provided by these models (Thompson, 1997).

Atlason, et al. (2004) and Henderson and Mason (1998) propose a general methodology based on the cutting plane method of Kelley Jr. (1960) to optimize the scheduling of agents in a single-call type and single-skill call center, under service-level constraints. The cutting plane method, which is another motivating example of our study, solves a linear (integer) program to find the staffing levels, and this solution is used as an input for a simulation model to calculate the service level. If the service level is not satisfactory, new constraints are added to the linear program and reiterated. Briefly, the method combines simulation with integer programming and cut generation to solve resource allocation problems where the objective function or constraints are evaluated by using simulation (Atlason et al., 2004).

The traditional cutting plane algorithm of Kelley works as follows: First, the call center staffing problem is converted into a linear integer problem with the service level constraints. Then, the linear integer problem is solved and a simulation is run with the staffing levels obtained from the solution. If the service levels meet the target service level constraints as approximated by the sample average then the algorithm stops with an optimal solution. On the other hand, if a service level

constraint is violated then a new linear constraint is added to the problem that eliminates the current solution but does not eliminate any feasible solutions to the sample average approximation (Atlason et al., 2004; Henderson and Mason, 1998).

A recent study by Atlason et al. (2004) generally fits the framework of Kelley's cutting plane method, but it differs from the traditional method only in that they use a simulation to generate the cuts and evaluate the function values instead of having an algebraic form for the function and using analytically determined gradients to generate the cuts.

Recently, with the increasing volume of multi-skill call centers, some researchers have been studying the modification of the existing models for multi-skill call centers. Cezik and L'Ecuyer (2005) describe a generalization of the method from Atlason et al. (2004) in the context of an equivalent problem in multi-skill call centers. Due to the complexity of multi-skill call centers, the computation time of the algorithm is relatively longer than single-skill call centers. This is a serious drawback of the algorithm because some phases like rostering require an iterative procedure with small changes so the simulation has to be run for a very long time which is not practical. Furthermore, Koole and Pot (2005) also propose a two-step method to generate shifts in multi-skill call centers. They split the day into consecutive time intervals. In the first step, the optimal staffing levels for each skill group and each interval are determined. In the second step, shifts are composed such that the staffing level in each interval is met.

With the inspiration given by all of these previous studies, a DSS is prepared to perform the third and fourth steps of the WFM in this study. The proposed model differs from the previous models by integrating a mathematical model with a system

simulation model. Most of the researchers use a stepwise approach by using various mathematical models but the use of computer system simulation as a supportive model is not a common method in the literature. Therefore, we propose a DSS for the WFM as an alternative approach to solve shift scheduling and staff allocation problems.

4. DSS DESIGN FOR WORKFORCE MANAGEMENT

In this section, a Decision Support System (DSS) will be proposed for shift design and workforce allocation. Development of the DSS will be explained in application with a case in one of the leading call center companies of Turkey (which will be called “Company A”).

4.1. Company Overview

Company A is one of the biggest call center companies in Turkey. It operates in mainly three locations. These locations will be referred as Location 1, Location 2 and Location 3, respectively, from smaller to larger seat capacities. The seat and agent capacities of each location are shown below in Table 4.1.

Table 4.1. Agent and Seat Capacities of the Locations

LOCATIONS	Number of Available Agents	Number of Available Seats
Location 1	222	80
Location 2	313	170
Location 3	375	240
Total	910	490

Agent capacity identifies the maximum number of agents that can work in each location in a day since an agent works a single shift each day. In Company A, there is no agent transfer between locations. Agents usually work in the same location every time and do not change their location unless they request a permanent move to another location. In addition to agent capacity, locations also have seat capacities. Seat capacity shows the maximum number of agents that can work at any time in a location.

Locations put some constraints on working hours, i.e., Location 1 works on a 24 hour basis, whereas Location 2 and 3 do not have night shifts between 02:00 and 08:00. Thus, Location 1 is the only location that can handle calls between 02:00 and 08:00.

Company A segments its customers into four groups: I1, I2, I3 and I4, respectively, from higher priority to lower priority. Calls are answered in accordance to their priorities rather than a FCFS rule.

Furthermore, Company A provides approximately 100 different services, grouping these services under three main headings: Routine Calls, Regular Campaign Calls and Temporary Campaign Calls. Routine Calls are for services that exist permanently in the IVR system; Regular Campaign Calls involve campaigns provided in regular periods during a year, and Temporary Campaign Calls are for services in temporary campaigns that can be set anytime.

4.2. Problem Definition and Objective

Company A has the problem of creating a weekly work schedule (tour schedule) for the call center staff at a low cost while attaining a satisfactory service level for incoming calls. Minimum service level is based on an 80/20 rule, which means that at least 80% of the incoming calls should be handled within 20 seconds. Therefore, they need a weekly schedule which enables them to meet the target service level and satisfy constraints while keeping their utilization on a rational level.

Second, Company A wants to make their workforce management system more flexible by allowing flexible shift durations. Currently the duration of a shift is fixed at 8 hrs/day. However, they try to allocate flexible shift durations to prevent

excess agent allocation in some periods, increase productivity and utilization. Flexibility provides a more dynamic environment for the call center so that it is able to cope with peak values in certain periods. In order to design a flexible shift schedule, the following questions should be answered:

- What will be the duration, start and finish time of each shift? (Shift scheduling problem)
- How many agents will work in each shift? (Staff allocation problem)

As discussed in Section 2.4, the shift scheduling problem requires the estimated figures for the minimum staff levels in each period. Based on Erlang-C estimation techniques, these values are estimated in Step 2. However, noting the drawbacks of Erlang-C calculations, as discussed in Section 2.5.1, a DSS is developed that enables the decision maker to try different scenarios on scheduling and agent allocation. The proposed DSS consists of two steps: First, with the given inputs and constraints, we formulate a linear optimization problem and solve it to determine the shift schedules and staff allocation in each shift. Then this solution is used as an input for the simulation model generated to observe the service levels within a day. Microsoft Excel Solver Add-In is used to solve the linear programming problem and Arena software (version 10.00) is used for simulation modeling and analysis.

4.3. Linear Programming Model

The aim of the LP model is to prepare weekly schedule of shifts with flexible durations and to allocate the agents to these shifts given the estimated agent requirements in each period required to attain the target service levels. In this section, development and the solution of the LP model will be described.

4.3.1 Development of the LP Model

Depending on the historical data, Company A makes call forecasts for 15 minute intervals in a week. Accordingly, the minimum number of required agents for each 15-minute period is calculated by using the Erlang formula.

In the staffing problem, the day is divided into hourly slots. A shift starts at the beginning of an hour and lasts for four to eight hours. Thus, there can be shifts lasting four, five, six, seven or eight hours in the same day and throughout the week.

The decision variables of the problem are defined as follows:

G_i : The number of employees that start to work at the beginning of slot i , $i= 8, 9, \dots, 22$ in a day.

Q_j : The number of employees that end work at the end of slot j , $j=11, \dots, 24, 25$ in a day ($j=25$ represents 01:00 A.M).

X_{ij} : The number of agents allocated to the shift which starts at the beginning of slot i and ends at the end of slot j , $i= 8, 9, \dots, 22$ and $j= 11, \dots, 24, 25$.

Location 1 is the only location that is active between 02:00 and 08:00 A.M. Company A determines a fixed number of agents for the 02:00 – 08:00 shift, enough to satisfy the minimum requirements during this period. For this reason, the LP Model considers the shift scheduling problem between 08:00 A.M in a day and 02:00 A.M the following day, i.e., there are 18 hours to be considered for allocation. Agents arrive at the call center between 08:00 – 22:00 and depart between 12:00 – 02:00 allowing a minimum of four hours and a maximum of eight hours of shift duration.

Labor costs constitute the major part of the operational costs in Company A. If the wages are paid on an hourly basis that can differ between slots in a day, the total payments to the agents is simply written as:

$$\text{Minimize Total Payments to the Agents} = \sum_{i=8}^{22} \sum_{j=11}^{25} X_{ij} a_{ij}$$

where a_{ij} is the agent payment for the shifts between slots i and j .

In our system, wages are paid on a monthly basis and constant for all agents. Thus, $a_{ij}=a$ can be omitted in the objective function which is simplified as:

$$\text{Minimize Total Number of Allocated Agents} = \sum_{i=8}^{22} \sum_{j=11}^{25} X_{ij}$$

There are 92 constraints which can be organized into eight groups, as shown below.

A. Minimum workforce requirements should be met in each slot.

$$\sum_{i=8}^k G_i - \sum_{j=12}^k Q_{j-1} \geq \text{Min. staff requirement in slot } k, k=8, \dots, 24, 25.$$

where $G_i=0$ for $i=23, 24, 25$.

The daily allocation of agents to the shifts should satisfy the minimum hourly workforce requirements. In Table 4.2, the values shown in the right side columns are the minimum number of required agents calculated by the Erlang-C formula for each slot and each day of the week.

Table 4.2. Constraints for Minimum Workforce Requirements

Intervals	Number of Agents Working in a Slot	Sign	Min. Number of Required Agents in a Slot						
			M	T	W	TH	F	ST	SN
08.00-09.00	G_8	$\geq$	52	58	63	73	58	52	42
09.00-10.00	G_8+G_9	$\geq$	118	129	124	131	133	119	83
10.00-11.00	$G_8+G_9+G_{10}$	$\geq$	174	178	158	174	191	164	120
11.00-12.00	$G_8+G_9+G_{10}+G_{11}$	$\geq$	208	212	212	190	241	196	162
12.00-13.00	$G_8+G_9+G_{10}+G_{11}+G_{12}-Q_{11}$	$\geq$	250	257	274	247	260	229	209
13.00-14.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}-Q_{11}-Q_{12}$	$\geq$	274	268	285	256	232	246	211
14.00-15.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}-Q_{11}-Q_{12}-Q_{13}$	$\geq$	241	215	229	217	228	212	191
15.00-16.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}-Q_{11}-Q_{12}-Q_{13}-Q_{14}$	$\geq$	215	220	233	209	228	219	179
16.00-17.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}$	$\geq$	234	241	246	235	233	224	179
17.00-18.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}$	$\geq$	230	241	228	213	238	217	183
18.00-19.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}$	$\geq$	234	240	228	219	255	256	189
19.00-20.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}$	$\geq$	245	258	230	232	254	263	195
20.00-21.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}$	$\geq$	249	271	235	240	240	265	205
21.00-22.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}$	$\geq$	234	271	226	242	224	247	212
22.00-23.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}$	$\geq$	208	240	204	198	193	201	173
23.00-00.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}$	$\geq$	144	152	136	143	139	164	120
00.00-01.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}-Q_{23}$	$\geq$	77	83	79	79	81	87	104
01.00-02.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}-Q_{23}-Q_{24}$	$\geq$	34	38	38	37	38	38	50

In this group of constraints, it is considered that the number of agents working in each one hour slot must be equal to or greater than the estimated minimum requirements. For instance, the minimum number of agents required from 08:00-09:00 on Monday is 52. Until the 12:00-13:00 slot, there are only entering agents to the system due to the minimum four working-hour limitation. The earliest departures can occur at 12:00. After this slot, there are entering agents as well as departing ones from the system based on four to eight hour shift durations. Therefore, the number of agents working in a slot is found by adding the new agents that start working at the beginning of the current slot to the number of agents in the previous slot and subtracting the agents that depart at the end of previous slot.

B. An agent should work for at least four hours.

$$\sum_{i=8}^k G_i \geq \sum_{j=11}^{k+3} Q_j, \quad k=8, 9, \dots, 21.$$

Table 4.3. Constraints for at Least Four Working Hours

G	Sign	Q
G_8	$\geq$	Q_{11}
G_8+G_9	$\geq$	$Q_{11}+Q_{12}$
$G_8+G_9+G_{10}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}$
$G_8+G_9+G_{10}+G_{11}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}$
$G_8+G_9+G_{10}+G_{11}+G_{12}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}+Q_{23}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}$	$\geq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}+Q_{23}+Q_{24}$

This constraint shows that if an agent starts to work, he has to work for at least four hours. For instance, as is seen in the first row of Table 4.3, the number of agents that start work at 8:00 (G_8) should be equal to or greater than the number of agents that end work at 12:00 (Q_{11}). If this is not the case, it means that some of the agents depart before working four hours. In case of equality, all of the agents that start working at 8:00 work for four hours and leave at 12:00. If it is greater, we understand that some of the workers will work for more than four hours. In addition, as is demonstrated in the subsequent rows of the same table, new agents that start work and end work at the current period are added to the previous equation to ensure that the total number of agents who start working before a certain period must be greater than or equal to the total number of agents that depart until four hours later than this period.

C. An agent should work no more than eight hours.

$$\sum_{i=8}^k G_i \leq \sum_{j=11}^{k+7} Q_j, \quad k=8, 9, \dots, 17.$$

Table 4.4. Constraints for at Most Eight Working Hours

G	Sign	Q
G_8	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}$
G_8+G_9	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}$
$G_8+G_9+G_{10}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}$
$G_8+G_9+G_{10}+G_{11}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}$
$G_8+G_9+G_{10}+G_{11}+G_{12}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}+Q_{23}$
$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}$	$\leq$	$Q_{11}+Q_{12}+Q_{13}+Q_{14}+Q_{15}+Q_{16}+Q_{17}+Q_{18}+Q_{19}+Q_{20}+Q_{21}+Q_{22}+Q_{23}+Q_{24}$

It is stated that agents cannot work more than eight hours, so the duration of a shift can be at most eight hours. For instance, as is given in the first row of Table 4.4., the number of agents that start working at 08:00 (G_8) must be less than the total number of agents that depart at 12:00 (Q_{11}), 13:00 (Q_{12}), 14:00 (Q_{13}), 15:00 (Q_{14}) and 16:00 (Q_{15}). Consequently, all agents that start working at 08:00 must leave work before 16.00.

D. The total number of agents allocated in a day should not exceed agent capacity

$$\sum_{i=8}^{22} G_i \leq \text{Agent capacity}$$

The total agent capacity of the three locations of Company A is 910, which is also given in Table 4.1. During a day, the total number of agents allocated to all shifts should not exceed the total number of available agents.

E. The total number of agents in a period should not exceed the total seat capacity.

$$\sum_{i=8}^k G_i - \sum_{j=12}^k Q_{k-1} \leq \text{Total seat capacity}, \quad k=8, \dots, 24, 25.$$

Table 4.5. Constraints for Seat Capacity

Intervals	# Agents working in a slot	Sign	Seat Capacity
08.00-09.00	G_8	$\leq$	490
09.00-10.00	G_8+G_9	$\leq$	490
10.00-11.00	$G_8+G_9+G_{10}$	$\leq$	490
11.00-12.00	$G_8+G_9+G_{10}+G_{11}$	$\leq$	490
12.00-13.00	$G_8+G_9+G_{10}+G_{11}+G_{12}-Q_{11}$	$\leq$	490
13.00-14.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}-Q_{11}-Q_{12}$	$\leq$	490
14.00-15.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}-Q_{11}-Q_{12}-Q_{13}$	$\leq$	490
15.00-16.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}-Q_{11}-Q_{12}-Q_{13}-Q_{14}$	$\leq$	490
16.00-17.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}$	$\leq$	490
17.00-18.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}$	$\leq$	490
18.00-19.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}$	$\leq$	490
19.00-20.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}$	$\leq$	490
20.00-21.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}$	$\leq$	490
21.00-22.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}$	$\leq$	490
22.00-23.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}$	$\leq$	490
23.00-00.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}$	$\leq$	490
00.00-01.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}-Q_{23}$	$\leq$	490
01.00-02.00	$G_8+G_9+G_{10}+G_{11}+G_{12}+G_{13}+G_{14}+G_{15}+G_{16}+G_{17}+G_{18}+G_{19}+G_{20}+G_{21}+G_{22}-Q_{11}-Q_{12}-Q_{13}-Q_{14}-Q_{15}-Q_{16}-Q_{17}-Q_{18}-Q_{19}-Q_{20}-Q_{21}-Q_{22}-Q_{23}-Q_{24}$	$\leq$	490

F. At the end of the day, all agents should leave work.

$$\sum_{i=8}^{22} G_i = \sum_{j=11}^{25} Q_j$$

This equality shows that all agents that start working in any hour of a day must leave work the same day. Thus, the sum of all incoming agents is equal to the sum of all leaving agents between 08:00 – 02:00.

G. The total number of agents who start working at the beginning of slot i should be equal to the sum of the agents allocated to the shifts that start at the beginning of slot i.

$$G_k = \sum_{j=k+3}^{k+7} X_{kj} \quad k= 8, 9, \dots, 22.$$

where $X_{kj}=0$ for $j>25$.

Table 4.6. Constraints for Shift Start Times

G	Sign	X
G ₈	=	X ₈₁₁ +X ₈₁₂ +X ₈₁₃ +X ₈₁₄ +X ₈₁₅
G ₉	=	X ₉₁₂ +X ₉₁₃ +X ₉₁₄ +X ₉₁₅ +X ₉₁₆
G ₁₀	=	X ₁₀₁₃ +X ₁₀₁₄ +X ₁₀₁₅ +X ₁₀₁₆ +X ₁₀₁₇
G ₁₁	=	X ₁₁₁₄ +X ₁₁₁₅ +X ₁₁₁₆ +X ₁₁₁₇ +X ₁₁₁₈
G ₁₂	=	X ₁₂₁₅ +X ₁₂₁₆ +X ₁₂₁₇ +X ₁₂₁₈ +X ₁₂₁₉
G ₁₃	=	X ₁₃₁₆ +X ₁₃₁₇ +X ₁₃₁₈ +X ₁₃₁₉ +X ₁₃₂₀
G ₁₄	=	X ₁₄₁₇ +X ₁₄₁₈ +X ₁₄₁₉ +X ₁₄₂₀ +X ₁₄₂₁
G ₁₅	=	X ₁₅₁₈ +X ₁₅₁₉ +X ₁₅₂₀ +X ₁₅₂₁ +X ₁₅₂₂
G ₁₆	=	X ₁₆₁₉ +X ₁₆₂₀ +X ₁₆₂₁ +X ₁₆₂₂ +X ₁₆₂₃
G ₁₇	=	X ₁₇₂₀ +X ₁₇₂₁ +X ₁₇₂₂ +X ₁₇₂₃ +X ₁₇₂₄
G ₁₈	=	X ₁₈₂₁ +X ₁₈₂₂ +X ₁₈₂₃ +X ₁₈₂₄ +X ₁₈₂₅
G ₁₉	=	X ₁₉₂₂ +X ₁₉₂₃ +X ₁₉₂₄ +X ₁₉₂₅
G ₂₀	=	X ₂₀₂₃ +X ₂₀₂₄ +X ₂₀₂₅
G ₂₁	=	X ₂₁₂₄ +X ₂₁₂₅
G ₂₂	=	X ₂₂₂₅

Constraints for shift start times are given in Table 4.6. There can be five different shifts that start at the same time referring to the shifts having the durations of four, five, six, seven or eight hours. The total number of agents who start working at the beginning of slot i (G_i) must be equal to the sum of the total number of agents who are assigned to the shifts that start at hour i and end at the end of slot j where j is (i+3), (i+4), (i+5), (i+6) or (i+7), respectively, until i=19:00. As shifts must end at 02:00 and the minimum four hours and maximum eight hours working restriction is

indispensable, there are four shifts that start at 19:00, three that start at 20:00, two that starts at 21:00 and only one that starts at 22:00.

H. The total number of agents that quit at the end of slot j must be equal to the sum of the agents allocated to the shifts that end at the end of slot j .

$$Q_j = \sum_{i=j-7}^{j-3} X_{ij}, \quad j=11, \dots, 24, 25.$$

where $X_{kj}=0$ for $i < 8$.

Table 4.7. Constraints for Shift End Times

Q	Sign	X
Q_{11}	=	X_{811}
Q_{12}	=	$X_{812}+X_{912}$
Q_{13}	=	$X_{813}+X_{913}+X_{1013}$
Q_{14}	=	$X_{814}+X_{914}+X_{1014}+X_{1114}$
Q_{15}	=	$X_{815}+X_{915}+X_{1015}+X_{1115}+X_{1215}$
Q_{16}	=	$X_{916}+X_{1016}+X_{1116}+X_{1216}+X_{1316}$
Q_{17}	=	$X_{1017}+X_{1117}+X_{1217}+X_{1317}+X_{1417}$
Q_{18}	=	$X_{1118}+X_{1218}+X_{1318}+X_{1418}+X_{1518}$
Q_{19}	=	$X_{1219}+X_{1319}+X_{1419}+X_{1519}+X_{1619}$
Q_{20}	=	$X_{1320}+X_{1420}+X_{1520}+X_{1620}+X_{1720}$
Q_{21}	=	$X_{1421}+X_{1521}+X_{1621}+X_{1721}+X_{1821}$
Q_{22}	=	$X_{1522}+X_{1622}+X_{1722}+X_{1822}+X_{1922}$
Q_{23}	=	$X_{1623}+X_{1723}+X_{1823}+X_{1923}+X_{2023}$
Q_{24}	=	$X_{1724}+X_{1824}+X_{1924}+X_{2024}+X_{2124}$
Q_{25}	=	$X_{1825}+X_{1925}+X_{2025}+X_{2125}+X_{2225}$

Constraints for shift end times are given in Table 4.7. It is required that the total number of agents that leave at the end of slot j (Q_j) should be equal to the sum of agents allocated to the shifts that end at the end of slot j .

According to these constraints and the objective function, a formulation table is prepared for seven days in MS-Excel. Excel Solver Add-In is used for the solution of this linear programming problem and the required “shifts in a day” as well as the

“number of agents” allocated to each of these shifts are determined. The results are shown in the next section.

4.3.2. Results of the LP Model

As a result of the linear programming solution we obtain the number of agents that start work and end work in each slot as well as the number of agents allocated to each shift.

Table 4.8. Number of Entering and Leaving Agents per Slot

		HOURS																	
		8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
MONDAY	G	69	79	36	34	35	22	22	19	48	41	43	34	25	1	0			
	Q				0	0	36	29	12	70	51	19	35	22	22	54	74	50	34
TUESDAY	G	61	73	49	36	39	10	31	17	34	49	54	51	26	7	0			
	Q				0	0	67	18	16	48	52	32	25	10	31	80	63	56	41
WEDNESDAY	G	105	53	0	93	25	9	19	49	79	40	10	17	11	0	0			
	Q				0	0	75	45	11	26	0	106	11	9	19	65	63	40	38
THURSDAY	G	100	58	21	34	41	2	16	37	59	38	56	19	14	4	0			
	Q				0	0	55	30	15	58	29	26	41	2	16	37	59	54	77
FRIDAY	G	58	75	71	49	41	52	55	56	91	43	19	19	0	0	0			
	Q				0	21	0	0	36	75	95	78	51	42	30	61	58	43	38
SATURDAY	G	119	41	33	14	21	18	46	38	66	31	22	21	24	0	0			
	Q				0	1	80	31	35	13	33	14	21	18	46	38	66	31	67
SUNDAY	G	42	60	51	46	12	0	26	6	16	50	51	43	16	5	0			
	Q				0	0	36	11	0	55	51	55	4	0	28	25	8	101	50

Table 4.8 demonstrates the weekly allocation of the agents that start work at the beginning of each slot (G_i) and end work at the end of each slot (Q_i). As can be seen from the table, there is usually no need for agents to begin working after 21:00 and 22:00 with small exceptions. Again, except for Friday, agents do not leave work until 14:00. That means, agents that start work at the beginning of the day have to work for a minimum of six hours.

Based on the results in Table 4.8, the LP Model provides the weekly shift schedule and the agent allocation to each shift, as shown in Table 4.9.

Table 4.9. Allocation of the Agents to Shifts

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-12.00	0	0	0	0	0	0	0
08.00-13.00	0	0	0	0	0	0	0
08.00-14.00	36	44	59	55	0	72	31
08.00-15.00	27	10	37	30	0	21	11
08.00-16.00	6	7	9	15	58	26	0
09.00-13.00	0	0	0	0	27	1	0
09.00-14.00	0	23	16	0	0	8	6
09.00-15.00	2	8	8	0	14	10	0
09.00-16.00	6	6	2	0	26	10	0
09.00-17.00	70	36	26	58	33	13	55
10.00-14.00	0	0	0	0	0	0	0
10.00-15.00	0	0	0	0	0	0	0
10.00-16.00	0	3	0	0	0	0	0
10.00-17.00	0	9	0	0	0	0	0
10.00-18.00	36	37	0	21	33	33	51
11.00-15.00	0	0	0	0	0	0	0
11.00-16.00	0	0	0	0	0	0	0
11.00-17.00	0	3	0	0	0	0	0
11.00-18.00	15	8	0	8	23	0	0
11.00-19.00	19	24	93	26	27	14	46
12.00-16.00	0	0	0	0	0	0	0
12.00-17.00	0	0	0	0	0	0	0
12.00-18.00	0	7	0	0	0	0	0
12.00-19.00	0	8	14	0	0	0	9
12.00-20.00	35	25	11	41	19	21	4
13.00-17.00	0	0	0	0	0	0	0
13.00-18.00	0	0	0	0	0	0	0
13.00-19.00	0	0	0	0	0	0	0
13.00-20.00	0	0	0	0	0	0	0
13.00-21.00	22	10	9	2	0	18	0
14.00-18.00	0	0	0	0	0	0	0
14.00-19.00	0	0	0	0	0	0	0
14.00-20.00	0	0	0	0	0	0	0
14.00-21.00	0	0	0	0	8	0	0
14.00-22.00	22	31	19	16	31	46	26
15.00-19.00	0	0	0	0	0	0	0
15.00-20.00	0	0	0	0	0	0	0
15.00-21.00	0	0	0	0	8	0	0
15.00-22.00	0	0	0	0	0	0	1
15.00-23.00	19	17	49	37	54	38	4
16.00-20.00	0	0	0	0	0	0	0
16.00-21.00	0	0	0	0	0	0	0
16.00-22.00	0	0	0	0	0	0	1
16.00-23.00	19	17	16	0	0	0	9
16.00-00.00	29	17	63	59	41	66	6
17.00-21.00	0	0	0	0	0	0	0
17.00-22.00	0	0	0	0	0	0	0
17.00-23.00	1	18	0	0	0	0	6
17.00-00.00	22	16	0	0	0	0	2
17.00-01.00	18	16	40	38	34	31	42
18.00-22.00	0	0	0	0	0	0	0
18.00-23.00	8	15	0	0	0	0	0
18.00-00.00	10	13	0	0	0	0	0
18.00-01.00	12	13	0	16	0	0	14
18.00-02.00	13	13	10	40	34	22	37
19.00-23.00	6	13	0	0	0	0	6
19.00-00.00	8	13	0	0	0	0	0
19.00-01.00	10	13	0	0	0	0	26
19.00-02.00	11	13	17	19	25	21	11
20.00-00.00	5	4	0	0	0	0	0
20.00-01.00	10	11	0	0	0	0	14
20.00-02.00	10	11	11	14	5	24	2
21.00-01.00	0	3	0	0	0	0	5
21.00-02.00	1	4	0	4	0	0	0
22.00-02.00	0	0	0	0	0	0	0
02.00-08.00	20	20	25	25	25	30	40
TOTAL	528	559	536	523	525	525	463

As seen in Table 4.9, the total number of agents required per day varies between 463 and 559, which is significantly less than the number of available agents given in Table 4.1 as 910. The solution generates a flexible schedule in which the duration of a shift is between four to eight hours per day. It follows that the duration of shifts is usually between seven to eight hours per day. However, shorter shifts provide flexibility to the system, especially during peak hours. Using this table as a base for shift design, executives can find out which shifts are needless, which shifts have to be fixed and which shifts may be merged with previous or following ones.

Table 4.10. Hourly Allocation of the Agents in the Proposed System

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-09.00	69	61	105	100	58	119	42
09.00-10.00	148	134	158	158	158	160	102
10.00-11.00	184	183	158	179	191	194	153
11.00-12.00	218	219	251	213	241	208	199
12.00-13.00	252	258	276	254	260	229	211
13.00-14.00	274	268	285	256	232	246	211
14.00-15.00	261	232	229	217	271	212	201
15.00-16.00	251	231	233	224	320	219	195
16.00-17.00	287	249	300	267	277	249	212
17.00-18.00	257	251	314	247	278	267	207
18.00-19.00	249	254	324	273	255	256	207
19.00-20.00	265	273	235	267	254	263	195
20.00-21.00	255	274	235	240	240	265	207
21.00-22.00	234	271	226	242	224	247	212
22.00-23.00	212	240	207	226	193	201	184
23.00-00.00	158	160	141	189	139	164	159
00.00-01.00	84	97	79	130	98	98	150
01.00-02.00	34	41	38	77	64	67	50

Table 4.10 shows the number of agents that should work in each slot throughout the week in order to satisfy the service level constraints. Obviously these values are greater than or equal to the minimum requirements given in Table 4.2.

The optimal allocation of agents in Table 4.10 shows the total number of agents allocated to the three locations of Company A. In order to provide input for the simulation model in the next step, it should be allocated in each location

separately. Noting that the agents have identical skills, it is decided to allocate the agents to different locations in accordance to the seat capacities. Therefore, as can be calculated from given seat capacities in Table 4.1, Location 1 has 16% of the total available seats respectively. Similarly, Location 2 and 3 have 35% and 49% of total available seats. Accordingly, the tables for the weekly allocation of agents for each hour in each location are given in Table A.3, A.4 and A.5 in the Appendix. These tables will be used as inputs for the weekly resource schedule of each location in the simulation model which will be developed in the next section.

4.4. Simulation Model

As previously stated, in complex call center systems, it is better to use LP with other analysis tools. In this section, a simulation model is constructed in order to monitor performance measures and update the current workforce design.

Arena software is used for developing and simulating the call center system. Figure 4.1 shows a general IVR tree based simulation model of Company A.

First of all, it should be stated that the calls that are handled only by the IVR system are beyond the scope of this model, because those operations do not require agent interaction and there is no relation with agent scheduling. In the simulation model, calls are created by the CREATE arrivals module. According to the weekly call forecast which is given in the Table A.1, the numbers of arrivals for quarter-hour periods are entered in the model. The graphical representation of the arrivals on Monday is shown in Figure 4.2 as an example where the arrival rates are hourly rates. The quarter-hourly call rates in Table A.1 are multiplied by four in order to obtain hourly rates.

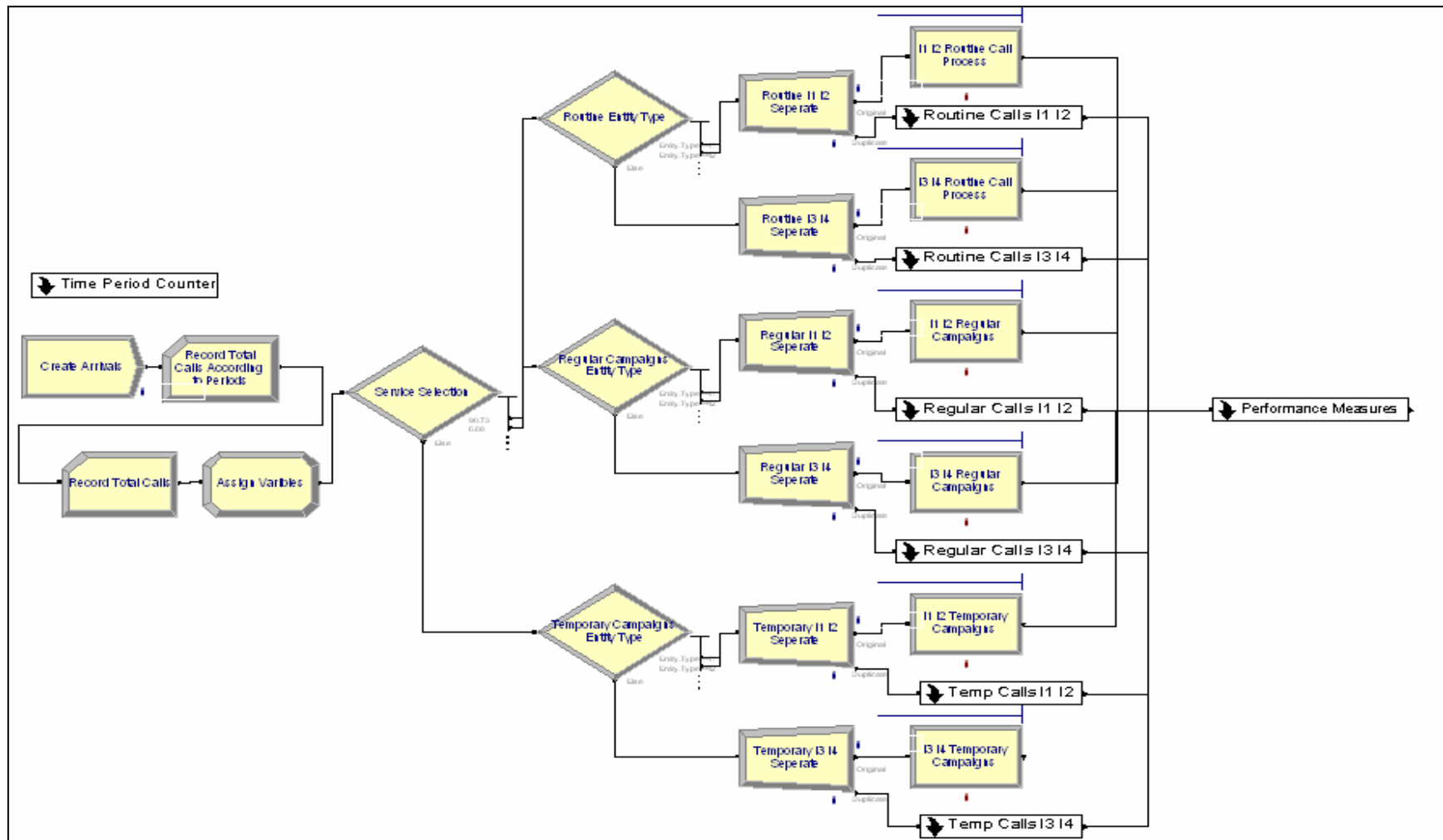


Figure 4.1. Overview of the simulation model

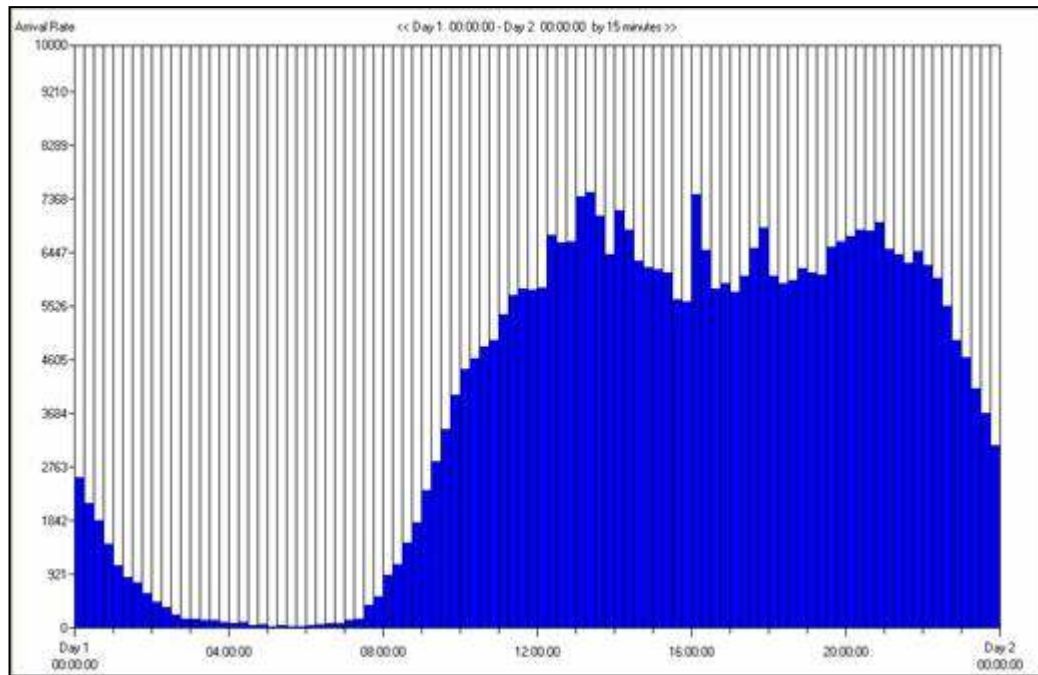


Figure 4.2. Arrival rates on Monday

Upon call arrival, the customer dials his unique customer number (ID). As stated previously, Company A classifies customers into four groups (I1, I2, I3 and I4) according to their priorities. I1 and I2 are considered as high priority customers/calls, while I3 and I4 are low priority customers/calls. Therefore, when a customer enters his customer ID immediately upon arrival, the system labels his entity type and gives him priority in agent service. According to historical data, the percentages of each entity type are 16.21% for I1, 31.05% for I2, 35.74% for I3 and 17.00% for I4. In the simulation model, the entity type is assigned as an attribute in the ASSIGN module. Moreover, the system assigns a unique customer number (Customer #), which is used later, to each entity.

Upon entrance, customers choose one of the three groups of services: Routine Calls, Regular Campaign Calls and Temporary Campaign Calls. According to historical data, 90.73% of the calls are Routine Calls, 6.68% are Regular Campaign Calls and 2.59% are Temporary Campaign Calls. In Figure 4.1, the service selection

module enables this allocation. It is a module that allocates the calls to the service types according to these percentages.

After choosing the service type, the call is allocated to one of the three locations by an automatic call distributor (ACD). An ACD is a computer tool that can be modified according to the current situation or expectations for the future. Currently, Company A sends the calls with I1 and I2 entity types in a preferred order to Location 1 and Location 2. Agents in Location 3 do not handle I1 and I2 type calls. Therefore, these types of calls are sent first to Location 1. If all agents are busy in Location 1, they are transferred to Location 2 and if all agents are also busy in this location, the customer waits in the joint queue of Locations 1 and 2. The allocation rule for I3 and I4 type calls is a preferred order of Location 3, Location 2 and Location 1, respectively.

As previously mentioned, Location 1 is the only location that operates for six hours between 02:00 and 08:00, so all incoming calls during this period are handled by agents in Location 1 regardless of their entity types. In the simulation model as shown in Figure 4.1, three DECIDE modules (Routine Entity Type, Regular Campaign Entity Type and Temporary Campaign Entity Type) allocate the calls to the locations according to their entity types.

While the customers wait in the queue, two key concepts should be considered: Entity balking and entity abandonment (reneging), which were discussed above in Section 2.1. In this study, we assume that the queue capacity of Company A is infinite because currently the queue capacity is so high that there is enough room for all calls. Thus, all calls can enter the system without balking. On the other hand, the entity abandonment option must be added to the system, because arriving

customers wait only a limited period of time before departing from the queue. If the tolerance time of the customer is less than the total waiting time in the queue, the customer abandons the system before reaching an agent. This tolerance time is generated from a normal distribution which has a mean of 120 seconds and a standard deviation of 30 seconds, and is assigned to a call entity as an attribute in the first ASSIGN module.

Modeling Abandonment

Modeling the abandonment requires additional modules and a submodel where abandonment check will take place. The idea is to remove a call from the queue when its renege time has elapsed. In order to create a mechanism to detect this condition, we add a SEPARATE module which makes a duplicate of each entity. While the original entity, which represents the actual customer, enters the service queue, the duplicate entity enters a submodel where the abandonment check will take place. As an illustration, overview of the submodel for Routine Calls with I1 I2 entity types is given below in Figure 4.3.

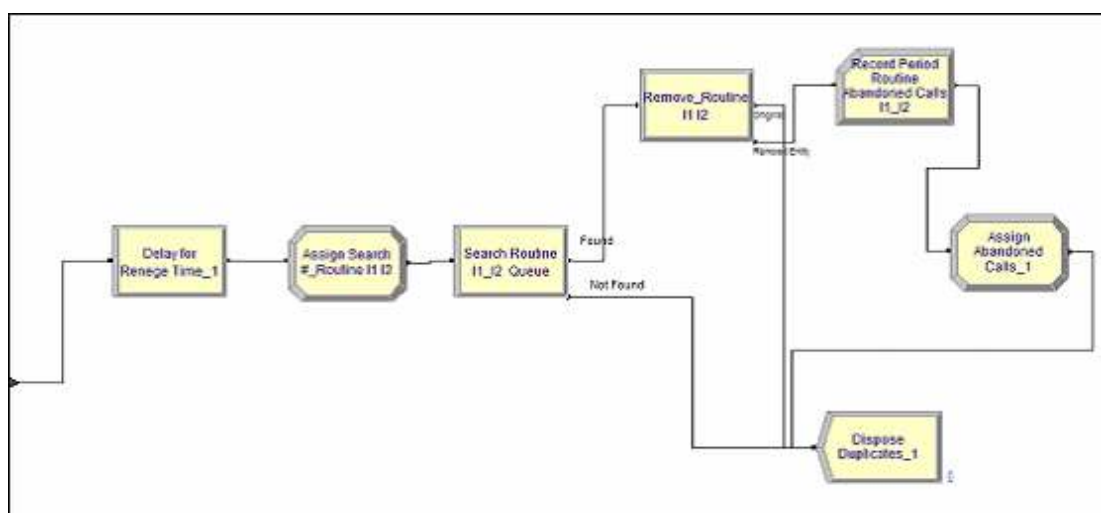


Figure 4.3. Overview of abandonment check submodel

When the duplicate entity enters the submodel, it is delayed by renege time, which is assigned as an attribute “renege time” upon arrival. After the delay, the entity is sent to an Assign module where the value of the attribute “Customer #” is assigned to a new variable called “Search #.” Then, the duplicate entity enters a SEARCH module to find the queue position of the original entity for which the attribute “Customer # = Search #.” The SEARCH module searches over the entire queue range, from 1 to NQ (the number of entities in queue); if the queue is empty or the original entity has already left the queue, the duplicate entity is sent to the DISPOSE module. Otherwise, if the original entity is found, its rank in the queue is saved in the variable J and the entity is sent to a REMOVE module. The REMOVE module removes an entity from the queue and sends it to another place in the model. In the model, the customer with rank J is removed from the related queue. While the original entity is sent to the DISPOSE module the duplicate entity is sent to the RECORD and ASSIGN modules in order to increment the variables for the number of abandoned calls. Finally, it is disposed from the system.

Modeling Processes

The customers that are not reneged from the system enter the PROCESS modules in accordance to their service and entity types. As seen in the model, in each service type, there are two types of PROCESS modules: I1-I2 PROCESS and I3-I4 PROCESS modules. After allocating the calls to one of the locations according to entity types, an idle agent is checked and service is performed by the PROCESS modules. The service times of the agents are assumed to follow a triangular distribution with the values minimum 125, most likely 141 and maximum 152 seconds. The hourly allocations of agents to the locations which are given in Table

A.3, A.4 and A.5 are entered into the Agent Schedule for the related locations in the simulation model as an input. Without loss of generality we assume that all agents have identical service time distributions.

Modeling Statistical Performance Measures

When the service is accomplished, the call is disposed from the system. However, in the simulation model, some extra modules are needed in order to calculate the required performance indicators per period.

The system must reset some indicators at the end of every 15 minutes, so one sub-model named “time period counter” is added to the model. The overview of this module is shown in Figure 4.4

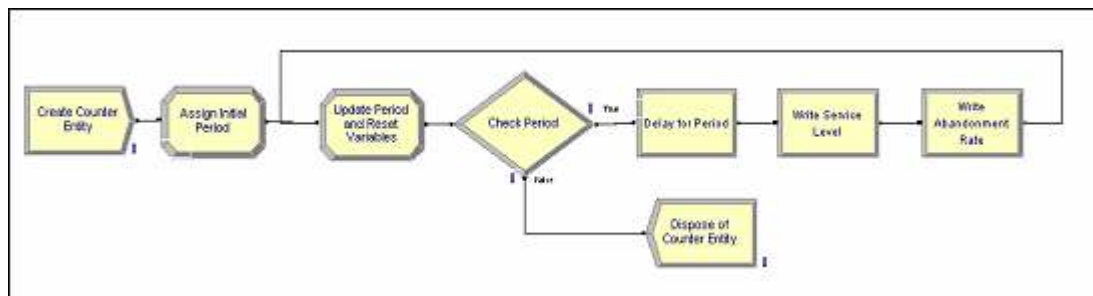


Figure 4.4. Overview of time period counter submodel

In this sub-model, a period counter entity increments the period by one at the end of every 15 minutes and resets the variables tracking the number of processed calls and number of abandoned calls. The general idea is to create an initial entity at the beginning of the simulation, delay the entity for 15 minutes, calculate the service level and the abandonment rate, write to an output file, increment the time period by one, reset the variables and delay for another 15 minutes. This process iterates in every 15 minutes until the end of the week. In the “Check Period” DECIDE module,

the model checks whether we still have more periods in the week. If we do, it sends the entity to the next DELAY module. Otherwise, the entity is disposed from the system at the end of the last period.

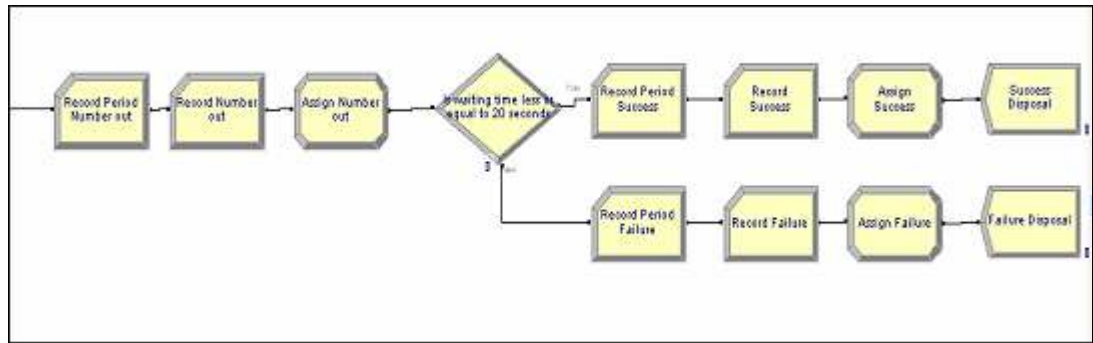


Figure 4.5. Overview of performance measures submodel

After receiving their service by the agent, calls enter another submodel called the “performance measures” submodel which is shown in Figure 4.5. In this submodel, a RECORD module called the “Record number out” counts the number of answered calls, i.e., the number of calls that leave the system after receiving service. Then, another DECIDE module checks whether the total waiting time of the customer in the queue was less than 20 seconds or not. If it was less than 20 seconds, it is recorded as a “success” and the number of successes is incremented by 1, otherwise it is a “failure” and number of failures is incremented by 1.

In the time period counter submodel, as period counter entity determines and increments the period, the required statistics can be obtained per period. At the end of each period, the service levels per period are calculated as,

$$\text{Service Level for Period } i = (\text{Period } i \text{ Success}) / (\text{Period } i \text{ Number Answered})$$

where the “Period i Success” is the number of calls answered in 20 seconds in period i and the “Period i Number Answered” is the number of calls that are answered in period i.

Similarly, the abandonment rate per period is found by,

$$\text{Abandonment Rate for Period } i = (\text{Period } i \text{ Abandoned}) / (\text{Period } i \text{ Total Calls})$$

where “Period i Abandoned” is the number of abandoned calls in period i and “Period i Total Calls” is the total number of calls that have entered the system in period i.

Service levels and abandonment rates for each period are the key performance indicators of Company A. As the staffing varies throughout the day, system performance can vary significantly from one period to another. However, in order to manipulate the staffing schedule to improve performance of the call center, we need to know what periods are under or over-staffed. Thus, in order to dynamically observe these indicators, Write modules, called the Write Period Service Level and Write Abandonment Rate, shown in Figure 4.4, record service levels for each period in an MS Excel file automatically in order to plot these results for each period in a graph. As we do not need other indicators (ASA, utilization, etc..) on a period basis, the weekly evaluation of these default measures are obtained from category overview report provided by Arena at the end of the simulation run.

5. COMPARISON OF ALTERNATIVE SCENARIOS

In this section, the simulation results of three scenarios will be compared. The first scenario is the current workforce schedule of Company A; the second one is an alternative system developed by solving the LP model only, as suggested in Section 4; and the third scenario is obtained by further improving the second scenario in the simulation environment.

We simulate the developed call center model using Arena 10.00 for seven days and ten replications. The system and collected statistics are initialized between replications in order to obtain ten statistically independent and identical replications each starting at time zero and running for seven days. In all scenarios, the same weekly call forecasts given in Table A.1 are used as an input.

5.1. First Scenario: Current System

In order to detect the performance of the current schedule of Company A and to compare it with the proposed models in this study the simulation model is run with the current agent allocation of Company A as shown in Table A.2. Accordingly, the total number of agents working in all locations per day varies between 700 and 850.

The model for the current system is run for seven days and the performance measures on service levels, abandonment rates, average speed of answer (ASA) and utilization of the system are obtained.

Service levels for each period are shown in Table A.6. As seen in this table, with the current schedule, Company A can handle approximately every call within 20 seconds; so service levels in most of the periods is 100%. There are few exceptions in some periods that have service level slightly below 100%. As an example, the

service levels for Saturday are plotted in Figure 5.1. The dotted lines represent the 95% interval estimations for the expected service levels in each 15 minute period.

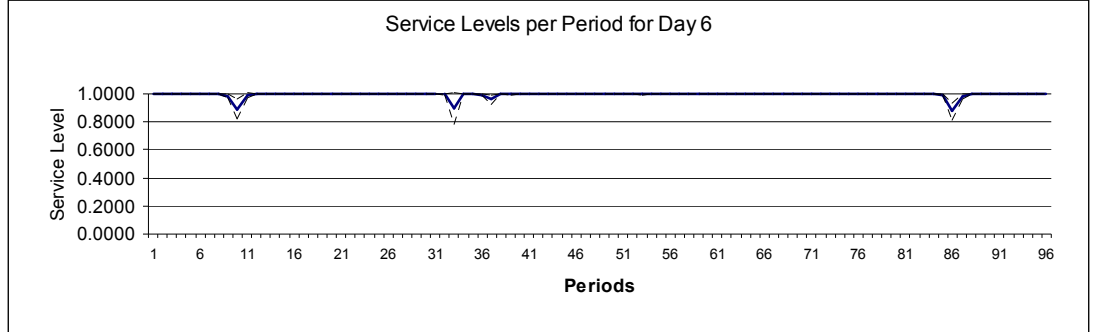


Figure 5.1. Service levels on Day 6 in the current system

At first look, the call center system may seem to be a very productive one since the minimum requirement for the service level is 80%. However, service levels are generally 100% or close to 100% in all periods. It means that Company A allocates more agents to the shifts than the required to achieve its 80/20 target service level. In any case, a call center may achieve 100% service levels with excess workforce, but the goal is to achieve the target with the minimum number of agents and the least cost. Since the service levels are very high, the abandonment rate is zero in almost all periods and 95% interval estimations for the expected ASA is very low for all entity types, as seen in Table 5.1.

Table 5.1. Average Speed of Answer in the Current System

Entity Type	ASA (Seconds)	Half-Width for 5% Significance Level
I1	0.0340	0.0134
I2	0.1138	0.0206
I3	0.0312	0.0023
I4	0.0690	0.0137

Considering the high service levels, we should evaluate the utilization of the system in order to show the real productivity of the system. Agent utilization was briefly explained in Section 2.2, but it should be mentioned here that there are two types of default utilizations measures in Arena: “instantaneous utilization” and “scheduled utilization.”

“Instantaneous utilization is calculated by computing the utilization, $u(t)$, at a particular instant in time (i.e., $u(t) = [\text{number of resource units busy at time } t] / [\text{number of resource units scheduled at time } t]$), and then calculating a time-weighted average of $u(t)$ over the whole run to produce the value shown in the reports” (Kelton, 2004).

On the other hand, “scheduled utilization is the time-average number of units of the resource that are busy (taken over the whole run), divided by the time-average number of units of the resource that are scheduled (over the whole run)” (Kelton, 2004).

Under different conditions, both of the reported utilizations can provide us useful information. Instantaneous utilization shows how busy the resource was over the entire simulation run and counts zero capacity periods as zero utilization periods. On the other hand, scheduled utilization tells us how busy the resource was over time that it was available at all and capacity was non-zero.

95% interval estimations for instantaneous and scheduled utilizations of the current system are given below.

Table 5.2. Instantaneous and Scheduled Utilizations in the Current System

Locations	Instantaneous Utilization	Scheduled Utilization	Half-Width for 5% Significance Level
Location 1	0.8050	0.8743	0.0013
Location 2	0.3311	0.4568	0.0042
Location 3	0.5025	0.6856	0.0037

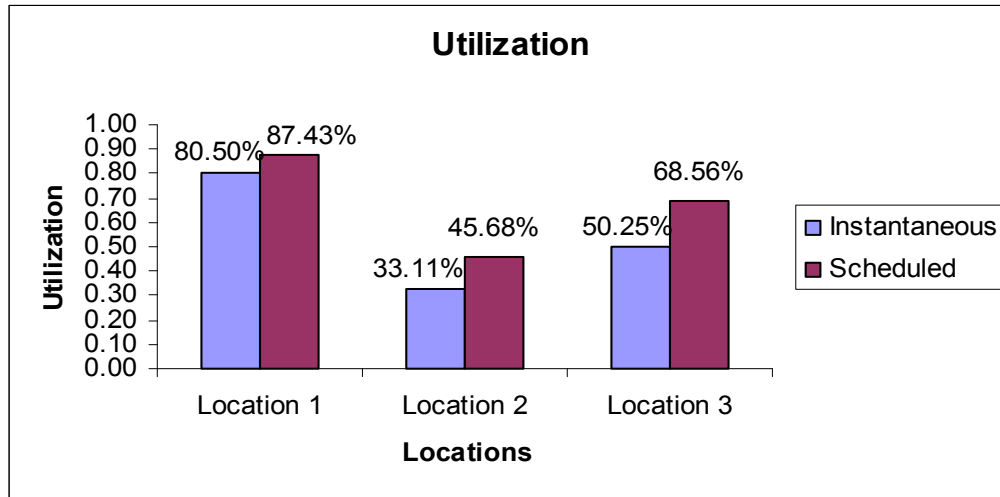


Figure 5.2. Instantaneous and scheduled utilizations in the current system

As is seen in Figure 5.2, while instantaneous and scheduled utilizations of Location 1 are rather close to each other, the utilizations of the other locations have significant variance. The reason is that Location 1 works during the entire run, but Location 2 and Location 3 do not work between 02:00 and 08:00. Instantaneous utilization considers this period as a zero-utilization period so it gives a lesser value than scheduled utilization does. In addition, it is recommended that it is more reasonable to use scheduled utilization if the scheduled capacities vary among different positive values over time rather than varying between zero and a single positive constant (Kelton, 2004). In our case, as some of the locations' capacities are zero during a quarter of the day and capacities vary over time, we will consider scheduled utilization values rather than instantaneous utilization.

As seen in Figure 5.2, while the utilization of Location 1 is at the normal rate, utilizations of other locations are very low. The reason for low utilization is excess workforce. The current schedule used by Company A guarantees high service levels, but with low efficiency and effectiveness. The reason for this result is the incapability of the current methods used to prepare better schedules and existence of fixed shift structure in Company A. In the next section, an alternative design which uses the LP model results as an input is generated and analyzed.

5.2. Second Scenario: LP Solution

In the second scenario, the agent schedules suggested by the LP model are used in the simulation model as appears in Table A.3, A.4 and A.5 for each location, respectively. As the LP model uses given call forecasts to determine these schedules, it is assumed that forecasts are exactly correct and forecasting errors are ignored. As a result of the simulation run, service levels for each period are tabulated in Table A.7. Those values are the average values obtained through ten replications. As an example, daily graph for the service levels in each period is given in Figure 5.3 for Day 1. Service level graphs for the other six days can be referred in Appendix from Figures A.1 to A.6.

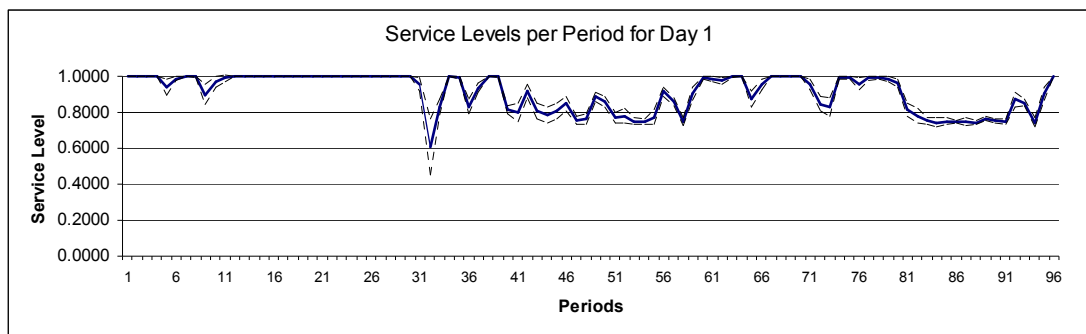


Figure 5.3. Service levels on Day 1 in Scenario 2

It follows from service level figures that although the call center has difficulty in meeting its target service level of 80% during peak hours, it is above the target during most of the periods.

In all days, there is a significant drop in service levels between period 29 and 36, which represents the time period between 07:00 – 09:00. The reason for this decrease is the existence of a single shift between 02:00 and 08:00 in Location 1. The number of agents in the night shift is kept at a minimum which is usually insufficient to handle the significantly increased call rate after 07:00. The first shift of a day starts at 08:00 and the system is initially quite loaded due to the remaining calls from the previous hour. In this case, there may be two solutions to prevent this decrease in the service level. The first one is to increase the number of agents working in the night shift in order to meet target service level between 07:00 – 08:00. This solution could increase the service levels, but on the other hand may decrease the utilization during the rest of the night shift. Therefore, the second option which is to start the first shift of the day at 07:00 rather than 08:00 in one of the locations may be more feasible. This option is considered as a third scenario in the next section.

It can be followed from Table A.7 that there are other periods when service levels are below the target. The reason for low service levels in these hours is mostly due to the assumption of independent periods in Erlang-C which the following LP model is based on. Actually, the performance in one period directly affects the next period by the remaining calls from the previous period. Therefore, the service levels are sometimes less than the expectations.

Another reason for the deviation from the target service levels is the existence of priorities in the queuing discipline. Shift designs and staff allocations that are based on FCFS assumptions of Erlang-C may lead to decreased service levels in this case. Moreover, the abandonment option is not valid in Erlang-C approximations although it exists in the real system.

The second performance indicator is the abandonment rate. The weekly abandonment rates in the second scenario are shown in Table A.8 and plotted below for Day 1 in Figure 5.4. Abandonment rate figures for the other six days can be referred in Appendix from Figure A.7 to A.12.

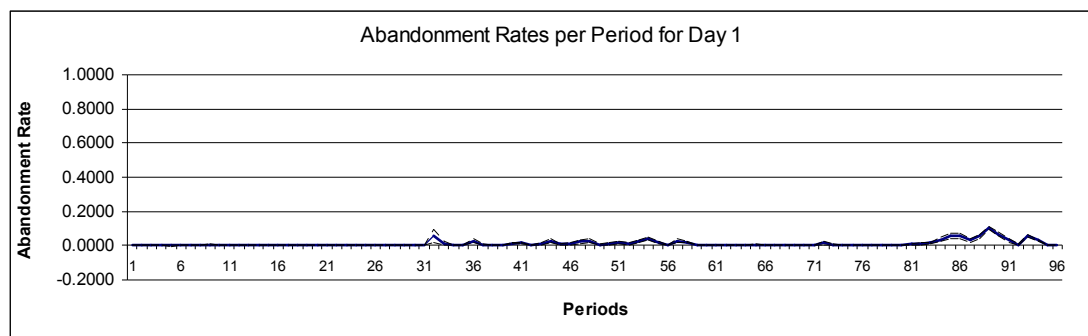


Figure 5.4. Abandonment rates on Day 1 in Scenario 2

As service levels are generally over 80%, abandonment rates are quite low. The target is to allow the minimum abandoned calls in all periods. As seen in the graphs, with small exceptions where abandonment rates are between 5% and 20%, in most of the periods, they are zero or close to zero. One may think of increasing the workforce size in some shifts to decrease the abandonment rate.

The third performance output we need is the utilization of the system. 95% interval estimations for the utilizations of each location are given in Table 5.3.

Table 5.3. Instantaneous and Scheduled Utilizations in Scenario 2

Locations	Instantaneous Utilization	Scheduled Utilization	Half-Width for 5% Significance Level
Location 1	0.8230	0.8527	0.0006
Location 2	0.6276	0.8320	0.0022
Location 3	0.6630	0.8868	0.0011

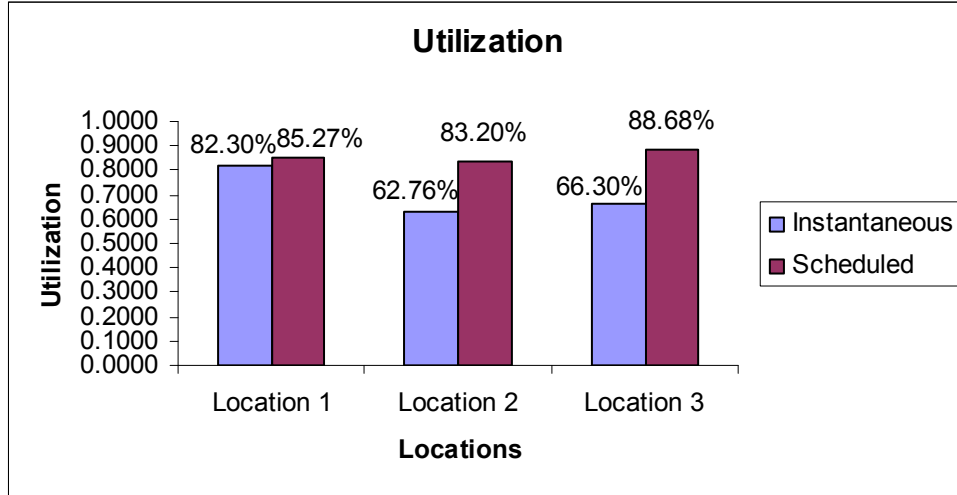


Figure 5.5. Instantaneous and scheduled utilizations in Scenario 2

The average utilization values show that the proposed schedule, obtained by the proposed LP model, makes the system more productive than the current schedule. If we compare the utilization values of current and alternative schedule, we see that utilizations of Location 2 and Location 3 are significantly greater in Scenario 2, while utilization of Location 1 is approximately the same. Let us recall that the total number of agents used is significantly smaller in Scenario 2. Thus, we should expect higher utilization.

In Figure 5.5, all of the locations have reasonable scheduled utilization values between 80% and 90%. Although low utilizations should be avoided, very high utilizations that are close to 100% are not desired as well, considering the risk of a

sudden increase in the call rate resulting in large queues and high abandonments. Thus, the resulting utilizations seem to be satisfactory for each location.

The last performance output is average speed of answer, which is given below in Table 5.4.

Table 5.4. Average Speed of Answer in Scenario 2

Entity Type	ASA (Seconds)	Half-Width for 5% Significance Level
I1	1.2963	0.0313
I2	16.1968	0.2301
I3	0.5122	0.0119
I4	1.9784	0.0646

Accordingly, ASA for I1, I3 and I4 type entities are very low, whereas I2 entities wait approximately 16 seconds. However, it is desired to service the higher priority customers in a shorter amount of time. In this case, the average waiting time of I2 calls is more than the average waiting time of lower priority calls, I3 and I4, since I1 and I2 calls are handled by only two locations, i.e., Location 1 and Location 2, whereas I3 and I4 calls are handled by all of the three locations. Since Location 3 has the maximum capacity among three locations, it provides more room for the calls and I3 and I4 calls wait less than I2 calls do.

In order to overcome this problem and to increase the service levels in predefined periods, a third scenario is generated in the next section.

5.3. Third Scenario: Improvement by Simulation

Simulation provides the opportunity to try different alternative scenarios. Therefore, based on the previous results obtained by simulation, a third scenario is developed in order to decrease average speed of answer for I2 customers, while

satisfying the minimum service level requirement in all periods. There are three basic modifications in the third scenario:

First, it is assumed that an arriving call can be allocated to any of the three locations in accordance to the number of available agent capacities. Thus, there is only one location set which includes three locations, and furthermore all locations handle all type of calls. An arriving call is directed to the location which has the maximum number of idle agents.

Second, the first shift of the day in Location 3 starts in 07:00 instead of 08:00 to avoid the significant decrease in the service levels between 07:00 and 08:00.

Third, we allocate additional agents in some shifts in order to increase service levels. Since agents in each location have identical skills, additional agents can be allocated to any location. In this case, we allocate them to Location 3 because of its higher seat capacity. Service levels of the Scenario 2 in Table A.7. are evaluated to decide which periods need additional agents. The number of additional agents in each shift is shown in Table 5.5.

Table 5.5. Allocation of Additional Agents to the Shifts

Day	Shift	Number of Additional Agents
Monday	20:00 - 02:00	15
Tuesday	08:00 - 14:00	15
Tuesday	20:00 - 02:00	15
Wednesday	10:00 - 16:00	10
Wednesday	20:00 - 02:00	15
Thursday	09:00 - 13:00	5
Thursday	20:00 - 02:00	15
Friday	08:00 - 12:00	30
Friday	22:00 - 02:00	5
Saturday	12:00 - 16:00	5
Saturday	20:00 - 02:00	10
Sunday	08:00 - 12:00	20
Sunday	20:00 - 00:00	15

The simulation model is run with these additions for seven days and ten replications and the related performance outputs are obtained.

The resulting service levels for each period are given in Table A.9. The daily service level graph for Day 1 is shown in Figure 5.6 as an illustration. Service level graphs for the other six days can be referred in Appendix from Figure A.13 to A.18.

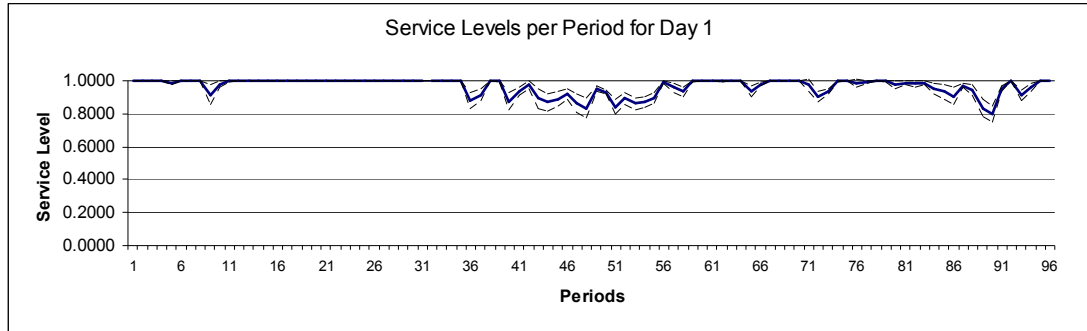


Figure 5.6. Service levels on Day 1 in Scenario 3

Service levels show that allocating additional agents to some shifts and starting the first shift of the day at 07:00 in Location 3 increase the service levels above the targets in almost all periods. There are still some exceptional periods in some other days where the service levels are slightly below 80%. Additional agents can be allocated to the shifts including these hours; however, because of the minimum four working-hours requirement, this may result in significant decreases in the overall utilization, and thus should be handled carefully.

As service levels are on the rational level, abandonment rates are usually zero or slightly above zero in each period. Due to the increase in the overall performance of the call center in Scenario 3, the number of abandoned calls also decreases. Abandonment rates for each period are given in Table A.10 and plotted in Figure 5.7 for Day 1. The graphs for the other six days can be referred in Appendix from Figures A.16 to A.24.

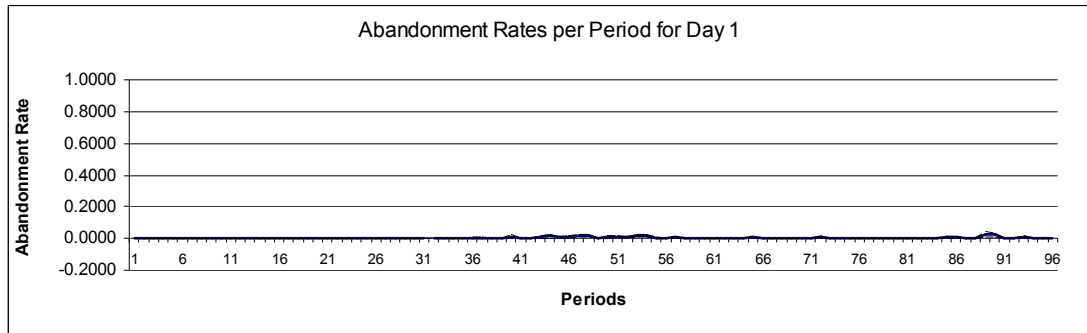


Figure 5.7. Abandonment rates on Day 1 in Scenario 3

95% interval estimations for instantaneous and scheduled utilizations of Scenario 3 are given below.

Table 5.6. Instantaneous and Scheduled Utilizations in Scenario 3

Locations	Instantaneous Utilization	Scheduled Utilization	Half-Width for 5% Significance Level
Location 1	0.6154	0.6687	0.0004
Location 2	0.6378	0.8836	0.0007
Location 3	0.6907	0.9032	0.0010

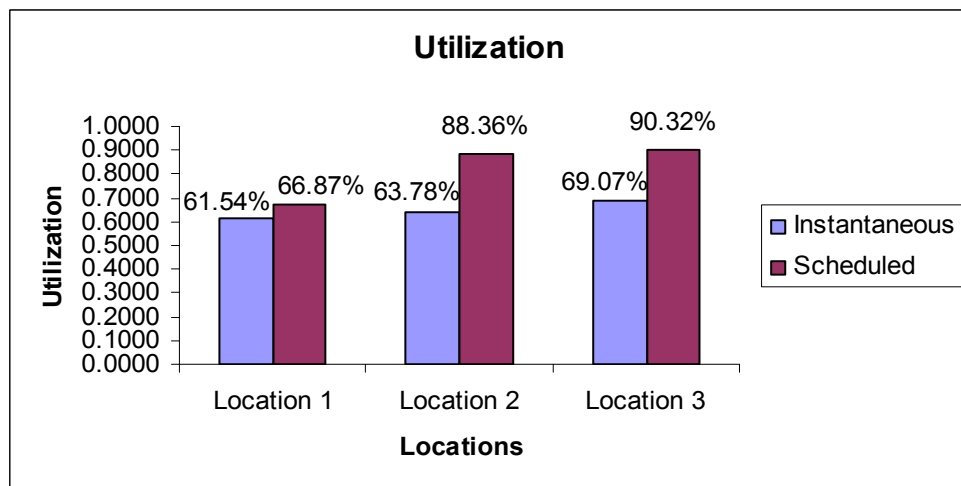


Figure 5.8. Instantaneous and scheduled utilizations in Scenario 3

The order of the utilizations of the locations change due to the new allocation rule of Scenario 3. Location 3, which has the highest seat capacity, has the highest

utilization value and Location 1, which has the smallest seat capacity, has the smallest utilization. Since the calls are allocated to the locations in accordance to the remaining capacity for idle agents, Location 3 handles more calls than other locations. Utilization values by location can change by applying different allocation alternatives but the important measure is the total utilization of the three locations. In this case, the overall utilization of the Company A is quite reasonable.

Finally, 95% interval estimations for the average speed of answer for different entity types is obtained in Table 5.7.

Table 5.7. Average Speed of Answer in Scenario 3

Entity Type	ASA (Seconds)	Half-Width for 5% Significance Level
I1	0.1762	0.0023
I2	0.2955	0.0143
I3	1.1693	0.0336
I4	8.5078	0.3213

Accordingly, the ASA for I2 customers is significantly reduced after allocating higher priority calls to Location 3 as well together with Location 1 and 2. It follows that higher priority customers wait less than the lower priority customers in the new scenario. Thus, the order of the average speed of answer by the entity type is very satisfactory due to its appropriateness to the priority rule of Company A.

This is an alternative model the aim of which is to improve some of the performance measures that do not give satisfactory results in the current and proposed models in the previous sections. Depending on the situation, several alternatives may be applied and the results can be monitored with the help of the proposed DSS model.

6. A FRAMEWORK FOR THE DSS ENVIRONMENT

The primary aim of this DSS is to support the decision makers of a call center in the decision-making process on shift scheduling and agent allocation. The decision maker is the main “user” of DSS. In the framework, the user is the person who interacts with the system by entering additional inputs into the system and by taking and evaluating the outputs from the DSS in order to use them in the decision-making process. The decision maker communicates with the DSS via a user interface. In the current DSS, almost all interactions are in an Excel environment. However, it can be enhanced by developing a much more user-friendly environment in Visual Basic. It is known that Visual Basic is compatible with Excel and Arena environments. The framework of the DSS generated in this study is demonstrated in Figure 6.1.

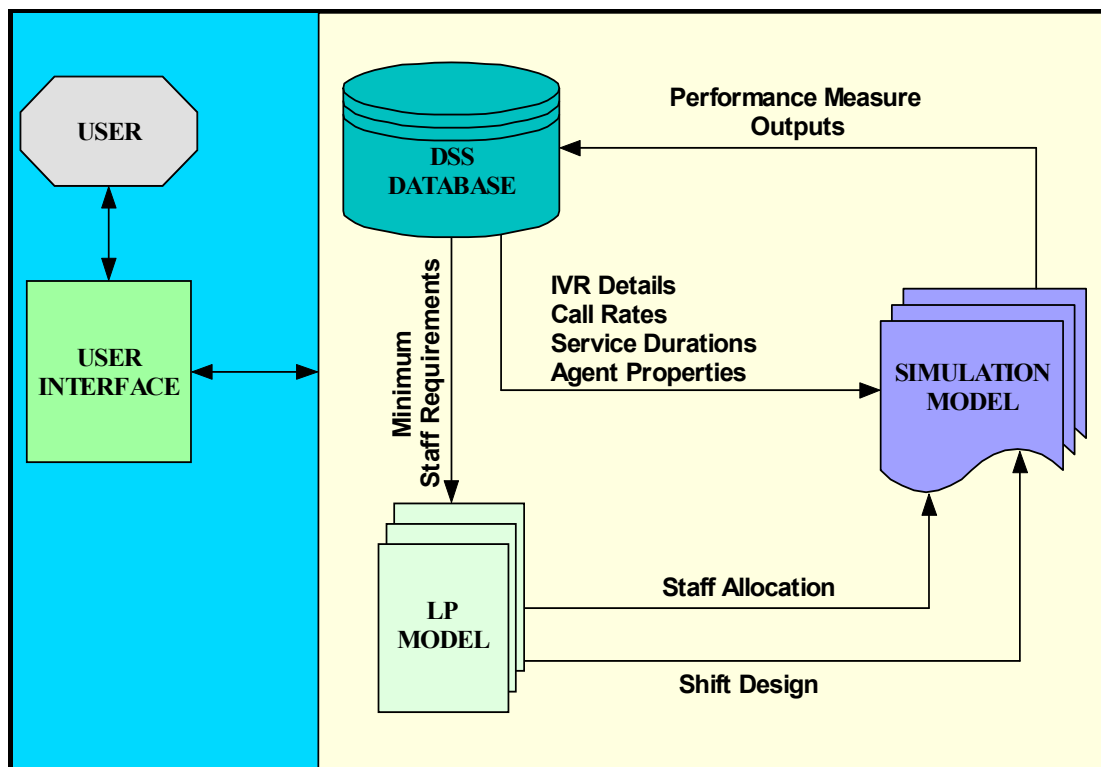


Figure 6.1. Structure of the Decision Support System (DSS)

The proposed DSS has three main components which are the DSS database, the LP model and the simulation model.

The database keeps all of the records of the call center system. The historical call data, service durations, agent properties, shift schedules and agent allocations are kept in the database of the current workforce system. The DSS system makes the necessary updates when new schedules are generated by the DSS models.

The LP Model receives the input data for the minimum staff requirements of the call center, and processes it to provide shift schedules and staff allocations for the simulation model. All input and output data are stored in the Microsoft Excel environment.

The last component of the DSS is the simulation model. The Arena simulation system uses the input (shift design and agent allocation) provided by the LP model and extracts the necessary info on call rates, IVR details, service durations and agent properties from the DSS database. The simulation model is run by the inputs obtained in the Excel environment, the resulting performance measures are recorded and the output reports are prepared again in Excel in the required formats. The user can generate several workforce designs in this environment, and evaluate and compare their performances. The finalized workforce design is used to update the current schedule in the database of the WFM system.

7. CONCLUSION

In this thesis, the aim is to generate a flexible DSS environment that works with adequate models and provides valid and precise solutions in the real time for WFM. Although Erlang-C approximations commonly are used and often give adequate results, they may be misleading in some conditions because of its shortcomings for complex systems. Noting that the what-if analysis options of the existing WFM tools mostly are based on Erlang approximations, we propose that both linear programming and real time system simulation modeling should be used in WFM tools to generate more valid results.

In the proposed DSS, assuming that the call forecasts are exactly correct, linear programming uses the minimum staff requirements provided by Erlang-C calculations to schedule the shifts and allocate staff to the shifts. Then, using the LP solution as an input, the simulation monitors the performance outputs and provides the required information to dynamically update the workforce allocation so that the minimum required service level is met at all times. As an illustration, we generate an alternative WFM design using LP modeling and further enhance its performance by system simulation in Arena. It is shown that integrating system simulation and linear programming models is a promising approach to solving shift design and staff allocation problems in call center management.

As future research, proposed DSS environment can be enhanced by user-friendly interfaces for data input, model access and output analysis. Excel Solver is incapable of solving large scale problems so new solver tools which are compatible with MS-Excel environments should be used. Furthermore, both models can be

improved to allow more constraints, like personnel transportation cost, absenteeism, skill-based routing and part-time workers.

APPENDIX

Table A.1. Weekly Call Forecast of Company A

WEEKLY CALL FORECAST									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	649	755	645	695	684	748	888
2	00:15	00:30	540	574	546	560	556	629	705
3	00:30	00:45	464	468	465	456	486	522	662
4	00:45	01:00	363	408	415	356	415	405	523
5	01:00	01:15	271	313	317	279	352	325	412
6	01:15	01:30	222	222	258	250	234	263	325
7	01:30	01:45	198	216	227	224	219	196	267
8	01:45	02:00	151	178	156	162	148	166	254
9	02:00	02:15	113	100	124	124	114	135	209
10	02:15	02:30	94	84	98	116	95	122	171
11	02:30	02:45	59	72	77	104	85	100	151
12	02:45	03:00	41	64	64	75	65	83	114
13	03:00	03:15	43	51	50	72	44	62	82
14	03:15	03:30	36	55	74	67	41	49	76
15	03:30	03:45	36	59	72	64	42	42	56
16	03:45	04:00	29	47	46	43	24	38	47
17	04:00	04:15	23	36	37	47	23	33	47
18	04:15	04:30	28	35	43	32	23	33	36
19	04:30	04:45	16	31	43	24	21	29	42
20	04:45	05:00	10	26	32	29	22	24	30
21	05:00	05:15	10	25	35	24	18	25	31
22	05:15	05:30	12	17	27	20	17	20	25
23	05:30	05:45	11	17	20	22	19	21	19
24	05:45	06:00	8	28	22	18	20	32	31
25	06:00	06:15	15	19	21	15	18	25	28
26	06:15	06:30	17	23	26	28	28	27	33
27	06:30	06:45	23	42	30	48	33	39	41
28	06:45	07:00	23	48	49	67	37	36	41
29	07:00	07:15	37	49	61	96	49	47	49
30	07:15	07:30	40	58	83	123	84	73	67
31	07:30	07:45	103	114	126	206	112	107	124
32	07:45	08:00	136	160	173	265	167	156	160
33	08:00	08:15	228	267	262	345	271	239	191
34	08:15	08:30	277	325	351	378	310	290	247
35	08:30	08:45	366	428	462	506	403	361	281
36	08:45	09:00	458	469	551	689	511	435	319
37	09:00	09:15	595	675	650	802	693	624	425
38	09:15	09:30	719	801	738	833	749	749	482
39	09:30	09:45	855	949	910	889	1008	927	616
40	09:45	10:00	1003	1038	1037	988	1134	897	648
41	10:00	10:15	1113	1067	1175	1131	1261	972	706
42	10:15	10:30	1160	1157	1044	1158	1298	1059	761
43	10:30	10:45	1213	1251	984	1246	1251	1196	836
44	10:45	11:00	1237	1362	1090	1211	1396	1251	912
45	11:00	11:15	1350	1415	1100	1287	1503	1275	1037
46	11:15	11:30	1430	1345	1358	1276	1605	1315	1016
47	11:30	11:45	1462	1504	1608	1299	1725	1310	1136
48	11:45	12:00	1454	1547	1736	1334	1780	1443	1213

Table A.1. Weekly Call Forecast of Company A (Continued)

WEEKLY CALL FORECAST									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	1465	1546	1641	1481	1737	1388	1280
50	12:15	12:30	1689	1695	1727	1596	1690	1486	1332
51	12:30	12:45	1657	1691	1731	1602	1689	1500	1360
52	12:45	13:00	1663	1721	1992	1704	1613	1534	1419
53	13:00	13:15	1855	1785	1969	1699	1470	1604	1346
54	13:15	13:30	1875	1797	1784	1761	1445	1635	1387
55	13:30	13:45	1773	1679	1903	1609	1547	1600	1389
56	13:45	14:00	1607	1690	1736	1560	1546	1525	1310
57	14:00	14:15	1797	1501	1576	1585	1521	1515	1294
58	14:15	14:30	1711	1467	1437	1527	1516	1450	1322
59	14:30	14:45	1578	1485	1580	1453	1663	1414	1337
60	14:45	15:00	1551	1428	1686	1387	1556	1424	1253
61	15:00	15:15	1544	1550	1735	1379	1514	1513	1232
62	15:15	15:30	1530	1532	1570	1425	1570	1534	1241
63	15:30	15:45	1412	1517	1470	1442	1569	1522	1214
64	15:45	16:00	1405	1449	1609	1468	1596	1419	1204
65	16:00	16:15	1865	1647	1831	1743	1556	1496	1204
66	16:15	16:30	1624	1858	1866	1703	1595	1611	1234
67	16:30	16:45	1459	1547	1596	1541	1685	1539	1217
68	16:45	17:00	1484	1579	1475	1471	1562	1508	1233
69	17:00	17:15	1445	1632	1564	1380	1613	1464	1176
70	17:15	17:30	1517	1858	1566	1527	1593	1532	1261
71	17:30	17:45	1633	1565	1596	1474	1686	1457	1270
72	17:45	18:00	1724	1574	1540	1450	1659	1510	1272
73	18:00	18:15	1516	1505	1521	1452	1673	1612	1203
74	18:15	18:30	1484	1529	1424	1363	1651	1610	1216
75	18:30	18:45	1496	1574	1482	1342	1633	1698	1212
76	18:45	19:00	1546	1581	1462	1487	1647	1714	1238
77	19:00	19:15	1529	1643	1535	1513	1637	1592	1248
78	19:15	19:30	1517	1669	1457	1544	1694	1737	1260
79	19:30	19:45	1639	1668	1431	1487	1616	1708	1247
80	19:45	20:00	1664	1694	1522	1453	1626	1772	1245
81	20:00	20:15	1687	1793	1560	1539	1683	1819	1384
82	20:15	20:30	1710	1892	1681	1697	1701	1873	1407
83	20:30	20:45	1710	1830	1699	1670	1614	1803	1371
84	20:45	21:00	1745	1941	1508	1680	1593	1820	1451
85	21:00	21:15	1629	1922	1515	1645	1590	1918	1556
86	21:15	21:30	1609	1845	1454	1737	1631	1747	1519
87	21:30	21:45	1570	1805	1602	1668	1501	1646	1445
88	21:45	22:00	1619	1894	1615	1599	1425	1484	1286
89	22:00	22:15	1559	1742	1595	1520	1362	1472	1305
90	22:15	22:30	1507	1716	1485	1396	1421	1448	1261
91	22:30	22:45	1386	1645	1252	1336	1291	1363	1113
92	22:45	23:00	1239	1483	1253	1172	1193	1214	1021
93	23:00	23:15	1165	1239	1083	1110	1075	1278	939
94	23:15	23:30	1034	1090	975	1000	998	1227	832
95	23:30	23:45	924	948	858	936	919	1047	748
96	23:45	00:00	786	835	760	822	765	889	699

Table A.2. Hourly Allocation of the Agents in the Current System

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-09.00	111	122	135	135	132	87	80
09.00-10.00	181	209	210	223	226	175	121
10.00-11.00	267	296	304	280	282	255	164
11.00-12.00	292	296	311	287	304	270	228
12.00-13.00	333	308	335	305	310	270	236
13.00-14.00	405	380	395	387	375	335	306
14.00-15.00	405	380	395	387	375	335	306
15.00-16.00	397	389	387	379	367	351	300
16.00-17.00	411	431	394	405	378	377	315
17.00-18.00	385	395	369	369	329	358	323
18.00-19.00	400	375	342	349	362	341	313
19.00-20.00	417	403	362	369	341	336	316
20.00-21.00	385	402	349	375	334	346	342
21.00-22.00	313	330	289	293	269	281	272
22.00-23.00	313	330	289	293	269	281	272
23.00-00.00	313	306	279	270	259	239	272
00.00-01.00	207	249	190	186	170	163	133
01.00-02.00	90	138	96	94	89	104	73

Table A.3. Hourly Allocation of the Agents in Location 1 Obtained by LP Solution

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-09.00	11	10	17	16	9	19	7
09.00-10.00	24	21	25	25	25	26	16
10.00-11.00	29	29	25	29	30	31	24
11.00-12.00	35	35	40	34	39	33	32
12.00-13.00	40	41	44	41	42	37	34
13.00-14.00	44	43	46	41	37	39	34
14.00-15.00	42	37	37	35	43	34	32
15.00-16.00	40	37	37	36	51	35	31
16.00-17.00	46	40	48	43	44	40	34
17.00-18.00	41	40	50	40	44	43	33
18.00-19.00	40	41	52	44	41	41	33
19.00-20.00	42	44	38	43	41	42	31
20.00-21.00	41	44	38	38	38	42	33
21.00-22.00	37	43	36	39	36	40	34
22.00-23.00	34	38	33	36	31	32	29
23.00-00.00	25	26	23	30	22	26	25
00.00-01.00	13	15	13	21	16	16	24
01.00-02.00	5	7	6	12	10	11	8

Table A.4. Hourly Allocation of the Agents in Location 2 Obtained by LP Solution

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-09.00	24	21	37	35	20	42	15
09.00-10.00	52	47	55	55	55	56	36
10.00-11.00	64	64	55	63	67	68	53
11.00-12.00	76	77	88	75	84	73	70
12.00-13.00	88	90	96	89	91	80	74
13.00-14.00	96	94	100	90	81	86	74
14.00-15.00	91	81	80	76	95	74	70
15.00-16.00	88	81	81	78	112	77	68
16.00-17.00	100	87	105	94	97	87	74
17.00-18.00	90	88	110	87	97	94	72
18.00-19.00	87	89	113	96	89	90	72
19.00-20.00	93	95	82	93	89	92	68
20.00-21.00	89	96	82	84	84	93	72
21.00-22.00	82	95	79	85	78	87	74
22.00-23.00	74	84	72	79	68	70	64
23.00-00.00	55	56	49	66	49	57	56
00.00-01.00	29	34	27	46	34	34	53
01.00-02.00	12	14	13	27	23	23	17

Table A.5. Hourly Allocation of the Agents in Location 3 Obtained by LP Solution

Slots	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
08.00-09.00	35	31	54	51	30	61	21
09.00-10.00	75	69	81	81	80	82	52
10.00-11.00	94	93	81	91	97	99	78
11.00-12.00	111	112	128	109	123	106	101
12.00-13.00	129	132	141	130	132	117	108
13.00-14.00	140	137	145	131	119	126	108
14.00-15.00	133	118	117	111	138	108	102
15.00-16.00	128	118	119	114	163	111	100
16.00-17.00	146	127	153	136	141	127	108
17.00-18.00	131	128	160	126	142	136	105
18.00-19.00	127	130	165	139	130	131	106
19.00-20.00	135	139	120	136	129	134	99
20.00-21.00	130	140	120	122	122	135	106
21.00-22.00	119	138	115	123	114	126	108
22.00-23.00	108	122	105	115	98	103	94
23.00-00.00	81	81	72	97	71	83	81
00.00-01.00	43	49	40	67	50	50	77
01.00-02.00	17	21	19	39	33	34	25

Table A.6. Service Levels in the Current System

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	00:15	00:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	00:30	00:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	00:45	01:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	01:00	01:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	01:15	01:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	01:30	01:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	01:45	02:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	02:00	02:15	1.0000	1.0000	0.9914	0.9856	0.9958	0.9833	0.9426
10	02:15	02:30	1.0000	1.0000	0.9885	0.9898	1.0000	0.8904	0.1496
11	02:30	02:45	1.0000	1.0000	1.0000	0.9991	1.0000	0.9898	0.0548
12	02:45	03:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.4073
13	03:00	03:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9023
14	03:15	03:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	03:30	03:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	03:45	04:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	04:00	04:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	04:15	04:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	04:30	04:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	04:45	05:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	05:00	05:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	05:15	05:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	05:30	05:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	05:45	06:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	06:00	06:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	06:15	06:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	06:30	06:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	06:45	07:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	07:00	07:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	07:15	07:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	07:30	07:45	1.0000	0.9969	1.0000	0.9220	1.0000	1.0000	0.9920
32	07:45	08:00	1.0000	0.9964	0.9933	0.4915	1.0000	0.9969	0.7302
33	08:00	08:15	0.9234	0.9635	0.9736	0.7331	0.8859	0.8942	0.8192
34	08:15	08:30	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9878
35	08:30	08:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
36	08:45	09:00	1.0000	1.0000	1.0000	0.9997	1.0000	0.9943	1.0000
37	09:00	09:15	1.0000	1.0000	1.0000	0.9917	1.0000	0.9576	0.9982
38	09:15	09:30	1.0000	1.0000	1.0000	1.0000	1.0000	0.9989	1.0000
39	09:30	09:45	1.0000	1.0000	1.0000	1.0000	1.0000	0.9978	0.9982
40	09:45	10:00	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
41	10:00	10:15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9989	0.9946
42	10:15	10:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
43	10:30	10:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44	10:45	11:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9957
45	11:00	11:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9732
46	11:15	11:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
47	11:30	11:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
48	11:45	12:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.6. Service Levels in the Current System (Continued)

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000
50	12:15	12:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
51	12:30	12:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
52	12:45	13:00	1.0000	0.9999	0.9993	1.0000	1.0000	1.0000	0.9952
53	13:00	13:15	1.0000	1.0000	0.9995	1.0000	1.0000	0.9971	0.9922
54	13:15	13:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
55	13:30	13:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
56	13:45	14:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
57	14:00	14:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
58	14:15	14:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
59	14:30	14:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
60	14:45	15:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
61	15:00	15:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
62	15:15	15:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
63	15:30	15:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
64	15:45	16:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
65	16:00	16:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
66	16:15	16:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
67	16:30	16:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
68	16:45	17:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
69	17:00	17:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
70	17:15	17:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
71	17:30	17:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
72	17:45	18:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
73	18:00	18:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
74	18:15	18:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
75	18:30	18:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
76	18:45	19:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
77	19:00	19:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
78	19:15	19:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
79	19:30	19:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
80	19:45	20:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
81	20:00	20:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
82	20:15	20:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
83	20:30	20:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
84	20:45	21:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
85	21:00	21:15	1.0000	0.9998	1.0000	1.0000	1.0000	0.9920	1.0000
86	21:15	21:30	1.0000	1.0000	1.0000	0.9998	0.9933	0.8733	1.0000
87	21:30	21:45	1.0000	1.0000	1.0000	0.9999	0.9904	0.9815	1.0000
88	21:45	22:00	1.0000	1.0000	0.9960	1.0000	1.0000	0.9994	1.0000
89	22:00	22:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
90	22:15	22:30	1.0000	1.0000	0.9987	1.0000	1.0000	1.0000	1.0000
91	22:30	22:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
92	22:45	23:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
93	23:00	23:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
94	23:15	23:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
95	23:30	23:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
96	23:45	00:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.7. Service Levels in Scenario 2

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	1.0000	0.7186	0.9609	0.9486	1.0000	0.9739	0.9976
2	00:15	00:30	1.0000	0.6059	0.8945	0.6772	1.0000	0.7532	0.8793
3	00:30	00:45	1.0000	0.8259	0.9954	0.8097	1.0000	0.6944	0.7179
4	00:45	01:00	1.0000	0.9969	1.0000	0.9874	1.0000	0.9059	0.8793
5	01:00	01:15	0.9394	0.7162	0.9061	0.9906	1.0000	0.9930	1.0000
6	01:15	01:30	0.9887	0.4783	0.6449	0.6352	1.0000	0.8336	0.9969
7	01:30	01:45	0.9995	0.6409	0.8469	0.6991	1.0000	0.7769	0.9996
8	01:45	02:00	0.9986	0.7993	0.9664	0.9198	1.0000	0.9822	1.0000
9	02:00	02:15	0.8993	0.8811	0.8547	0.9300	0.9922	0.9747	1.0000
10	02:15	02:30	0.9681	0.9980	0.9789	0.8270	0.9364	0.6237	0.6567
11	02:30	02:45	0.9892	0.9947	1.0000	0.9906	1.0000	0.9888	0.9456
12	02:45	03:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	03:00	03:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	03:15	03:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	03:30	03:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	03:45	04:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	04:00	04:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	04:15	04:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	04:30	04:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	04:45	05:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	05:00	05:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	05:15	05:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	05:30	05:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	05:45	06:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	06:00	06:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	06:15	06:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	06:30	06:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	06:45	07:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	07:00	07:15	1.0000	1.0000	1.0000	0.9987	1.0000	1.0000	1.0000
30	07:15	07:30	1.0000	1.0000	1.0000	0.9528	0.9894	1.0000	1.0000
31	07:30	07:45	0.9537	0.8595	0.9838	0.6080	0.9964	1.0000	1.0000
32	07:45	08:00	0.6061	0.3297	0.5462	0.5851	0.6857	0.9728	1.0000
33	08:00	08:15	0.8322	0.5006	0.5566	0.5694	0.2566	0.4455	0.9977
34	08:15	08:30	1.0000	0.9794	0.9976	0.9428	0.8349	0.9354	0.9975
35	08:30	08:45	0.9935	0.7750	1.0000	1.0000	0.7515	0.9965	0.7362
36	08:45	09:00	0.8314	0.6471	0.9958	0.8413	0.5864	1.0000	0.5595
37	09:00	09:15	0.9363	0.6418	0.9750	0.6794	0.3316	1.0000	0.3122
38	09:15	09:30	1.0000	0.9611	1.0000	0.9245	0.7617	0.9893	0.5253
39	09:30	09:45	0.9967	0.8065	0.9974	0.9991	0.8024	0.9989	0.9470
40	09:45	10:00	0.8153	0.7341	0.8940	0.9208	0.7460	0.9951	0.8898
41	10:00	10:15	0.8000	0.7514	0.7352	0.7776	0.7068	0.9852	0.7849
42	10:15	10:30	0.9174	0.9292	0.7232	0.7971	0.7353	0.9829	0.8278
43	10:30	10:45	0.8093	0.8118	0.8419	0.7662	0.9055	0.9690	0.9861
44	10:45	11:00	0.7842	0.7539	0.8068	0.7354	0.8357	0.9123	0.9853
45	11:00	11:15	0.8073	0.7354	0.8056	0.7344	0.7515	0.8231	0.8218
46	11:15	11:30	0.8507	0.8744	0.9984	0.8860	0.7762	0.7922	0.8673
47	11:30	11:45	0.7584	0.8342	0.9364	0.9636	0.7736	0.8392	0.9883
48	11:45	12:00	0.7653	0.7523	0.8139	0.9665	0.7548	0.7992	0.9654

Table A.7. Service Levels in Scenario 2 (Continued)

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	0.8864	0.7836	0.7428	0.7863	0.7417	0.7395	0.9259
50	12:15	12:30	0.8600	0.8692	0.9157	0.8816	0.9183	0.7662	0.8654
51	12:30	12:45	0.7721	0.7947	0.8875	0.9306	0.9997	0.7955	0.8432
52	12:45	13:00	0.7809	0.7664	0.8254	0.8318	1.0000	0.7434	0.8131
53	13:00	13:15	0.7514	0.7338	0.7667	0.7545	1.0000	0.7426	0.8169
54	13:15	13:30	0.7475	0.7566	0.7842	0.7763	1.0000	0.7352	0.8638
55	13:30	13:45	0.7733	0.8117	0.8756	0.7772	1.0000	0.7771	0.7293
56	13:45	14:00	0.9177	0.9181	0.8643	0.9451	1.0000	0.8856	0.8724
57	14:00	14:15	0.8580	0.9606	0.9801	0.9368	1.0000	0.9745	0.9527
58	14:15	14:30	0.7488	0.9054	0.8429	0.7681	1.0000	0.8383	0.9200
59	14:30	14:45	0.9037	0.9253	0.8118	0.7434	1.0000	0.7384	0.7986
60	14:45	15:00	0.9927	0.9375	0.7654	0.7785	1.0000	0.7506	0.8323
61	15:00	15:15	0.9861	0.8721	0.7756	0.8549	1.0000	0.7533	0.9486
62	15:15	15:30	0.9760	0.7458	0.7446	0.8937	1.0000	0.7457	0.9159
63	15:30	15:45	0.9972	0.7881	0.7855	0.8750	1.0000	0.7496	0.9030
64	15:45	16:00	1.0000	0.8296	0.7780	0.7996	1.0000	0.7386	0.9084
65	16:00	16:15	0.8715	0.8252	0.7361	0.7625	1.0000	0.7580	0.9439
66	16:15	16:30	0.9541	0.7883	0.8673	0.7734	1.0000	0.7475	0.9578
67	16:30	16:45	0.9998	0.7753	0.9913	0.9841	1.0000	0.8733	0.9933
68	16:45	17:00	1.0000	0.8931	1.0000	1.0000	1.0000	0.9875	0.9980
69	17:00	17:15	1.0000	0.8782	1.0000	1.0000	1.0000	0.9981	1.0000
70	17:15	17:30	0.9995	0.7956	1.0000	0.9864	1.0000	0.9991	0.9834
71	17:30	17:45	0.9558	0.7898	1.0000	0.9585	1.0000	1.0000	0.9270
72	17:45	18:00	0.8473	0.9539	1.0000	1.0000	1.0000	1.0000	0.9842
73	18:00	18:15	0.8285	0.9907	1.0000	0.9974	1.0000	0.9935	0.9725
74	18:15	18:30	0.9909	0.9969	1.0000	1.0000	1.0000	0.9784	0.9904
75	18:30	18:45	0.9925	0.9631	1.0000	1.0000	1.0000	0.8595	0.9972
76	18:45	19:00	0.9570	0.9341	1.0000	1.0000	1.0000	0.7537	0.9931
77	19:00	19:15	0.9895	0.9583	0.9998	1.0000	1.0000	0.8272	0.9731
78	19:15	19:30	0.9956	0.9837	0.9326	0.9992	1.0000	0.8536	0.9516
79	19:30	19:45	0.9828	0.9849	0.9602	1.0000	1.0000	0.7812	0.8678
80	19:45	20:00	0.9604	0.9539	0.9097	1.0000	1.0000	0.7852	0.8732
81	20:00	20:15	0.8153	0.8535	0.7999	1.0000	1.0000	0.7552	0.7820
82	20:15	20:30	0.7807	0.7576	0.7602	0.8327	0.9900	0.7480	0.7579
83	20:30	20:45	0.7523	0.7439	0.7611	0.7520	0.9939	0.7440	0.7291
84	20:45	21:00	0.7442	0.7550	0.7476	0.7539	0.9966	0.7476	0.7612
85	21:00	21:15	0.7501	0.7615	0.8281	0.7404	0.9827	0.7589	0.7702
86	21:15	21:30	0.7474	0.7494	0.8091	0.7491	0.8327	0.7494	0.7624
87	21:30	21:45	0.7471	0.7468	0.7799	0.7459	0.7462	0.7233	0.7484
88	21:45	22:00	0.7433	0.7549	0.7600	0.7443	0.8870	0.8693	0.8388
89	22:00	22:15	0.7652	0.7534	0.7642	0.8349	0.9759	0.9895	0.9493
90	22:15	22:30	0.7520	0.7571	0.7530	0.8873	0.8301	0.8499	0.8936
91	22:30	22:45	0.7479	0.7516	0.8290	0.9696	0.7381	0.7363	0.8751
92	22:45	23:00	0.8730	0.7986	0.9814	0.9982	0.9178	0.8413	0.9836
93	23:00	23:15	0.8524	0.8992	0.9733	0.9985	0.9824	0.9233	1.0000
94	23:15	23:30	0.7437	0.7227	0.7208	1.0000	0.8256	0.7721	1.0000
95	23:30	23:45	0.9057	0.8564	0.8510	1.0000	0.7094	0.7219	1.0000
96	23:45	00:00	0.9992	0.9908	0.9786	1.0000	0.8984	0.9163	1.0000

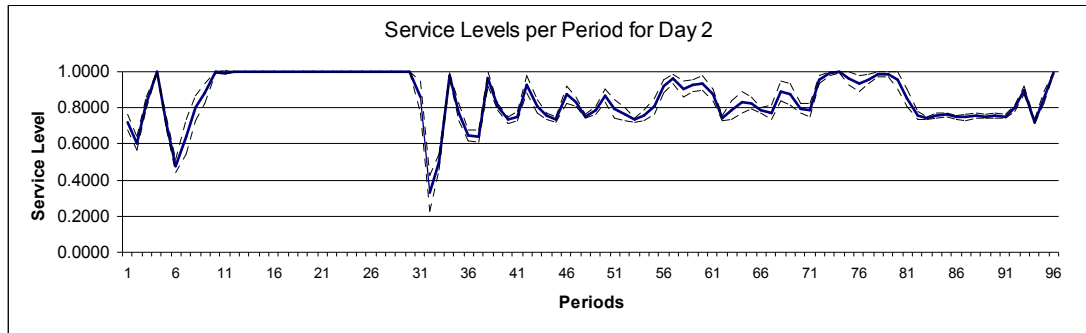


Figure A.1. Service levels on Day 2 in Scenario 2

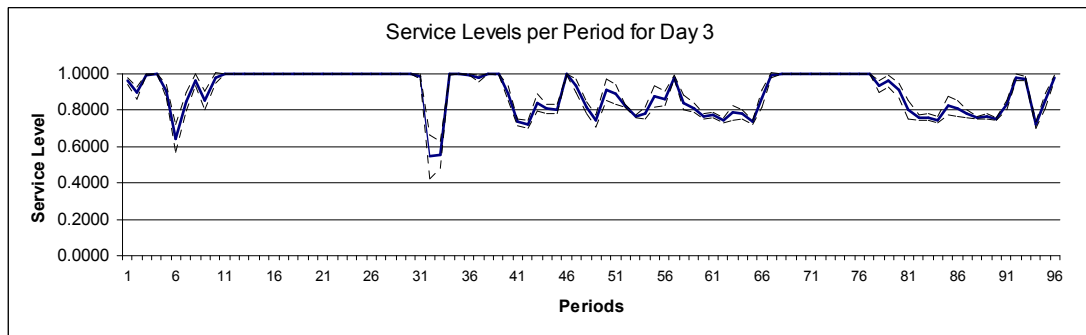


Figure A.2. Service levels on Day 3 in Scenario 2

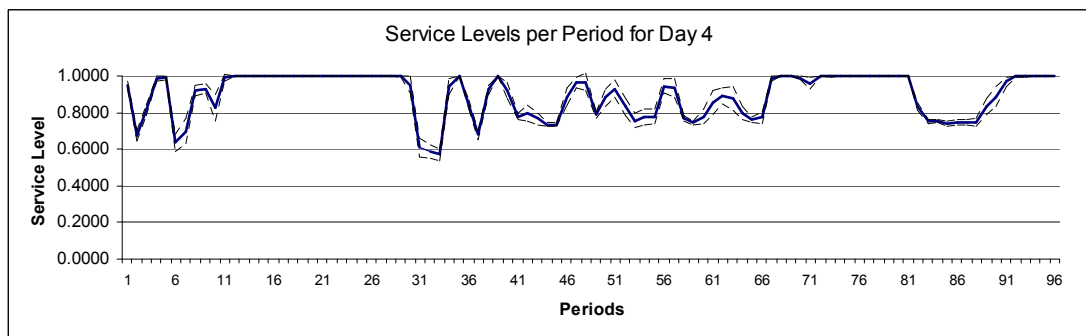


Figure A.3. Service levels on Day 4 in Scenario 2

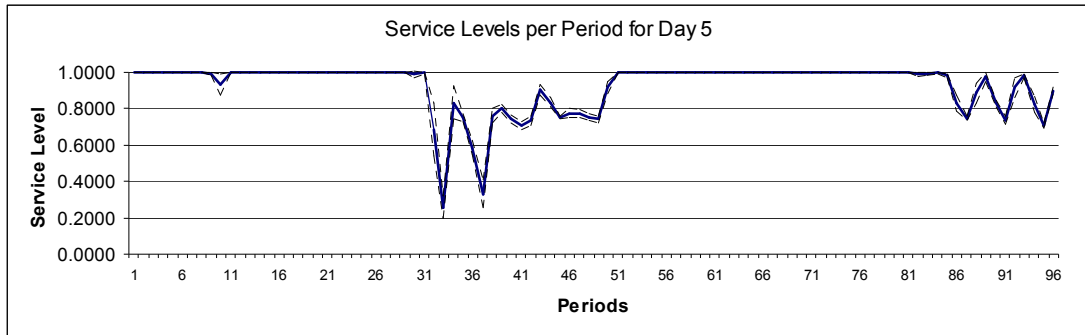


Figure A.4. Service levels on Day 5 in Scenario 2

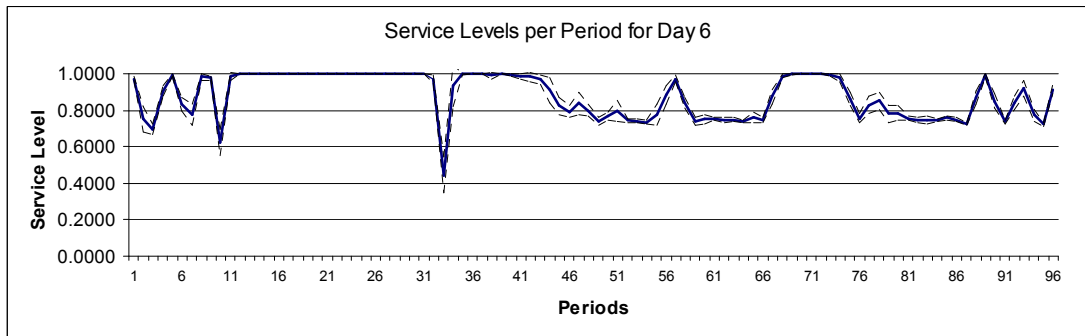


Figure A.5. Service levels on Day 6 in Scenario 2

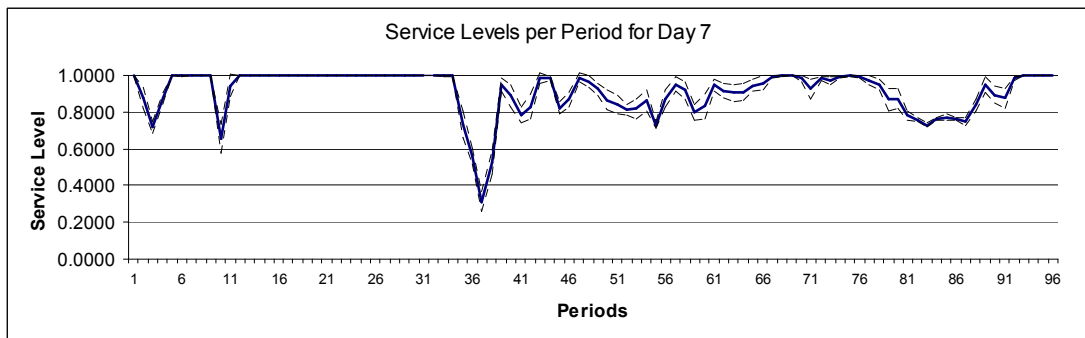


Figure A.6. Service levels on Day 7 in Scenario 2

Table A.8. Abandonment Rates in Scenario 2

ABANDONMENT RATES									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	0.0000	0.1766	0.0042	0.0250	0.0000	0.0023	0.0008
2	00:15	00:30	0.0000	0.1102	0.0028	0.1130	0.0000	0.0747	0.0201
3	00:30	00:45	0.0000	0.0149	0.0000	0.0128	0.0000	0.0560	0.0445
4	00:45	01:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0028	0.0033
5	01:00	01:15	0.0014	0.1506	0.0304	0.0016	0.0000	0.0000	0.0000
6	01:15	01:30	0.0000	0.1095	0.0391	0.0639	0.0000	0.0335	0.0000
7	01:30	01:45	0.0000	0.0500	0.0025	0.0251	0.0000	0.0209	0.0000
8	01:45	02:00	0.0000	0.0120	0.0000	0.0025	0.0000	0.0000	0.0000
9	02:00	02:15	0.0034	0.0154	0.0276	0.0628	0.0019	0.0253	0.0000
10	02:15	02:30	0.0000	0.0000	0.0000	0.0090	0.0027	0.1563	0.1092
11	02:30	02:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	02:45	03:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	03:00	03:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	03:15	03:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	03:30	03:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	03:45	04:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17	04:00	04:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	04:15	04:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	04:30	04:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	04:45	05:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	05:00	05:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	05:15	05:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	05:30	05:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	05:45	06:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	06:00	06:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	06:15	06:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	06:30	06:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	06:45	07:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	07:00	07:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	07:15	07:30	0.0000	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000
31	07:30	07:45	0.0000	0.0048	0.0007	0.1210	0.0000	0.0000	0.0000
32	07:45	08:00	0.0562	0.1063	0.0471	0.3496	0.0312	0.0000	0.0000
33	08:00	08:15	0.0131	0.1205	0.1045	0.2993	0.2045	0.1031	0.0000
34	08:15	08:30	0.0000	0.0009	0.0000	0.0068	0.0187	0.0099	0.0004
35	08:30	08:45	0.0000	0.0428	0.0000	0.0000	0.0551	0.0000	0.0473
36	08:45	09:00	0.0286	0.1372	0.0003	0.0252	0.2155	0.0000	0.1288
37	09:00	09:15	0.0042	0.1143	0.0002	0.1064	0.3225	0.0000	0.2935
38	09:15	09:30	0.0000	0.0005	0.0000	0.0024	0.0503	0.0000	0.1012
39	09:30	09:45	0.0000	0.0565	0.0000	0.0000	0.0965	0.0000	0.0005
40	09:45	10:00	0.0133	0.1436	0.0128	0.0023	0.2177	0.0000	0.0100
41	10:00	10:15	0.0143	0.0728	0.1055	0.0412	0.2534	0.0000	0.0337
42	10:15	10:30	0.0011	0.0017	0.0418	0.0091	0.0468	0.0001	0.0097
43	10:30	10:45	0.0112	0.0204	0.0056	0.0473	0.0024	0.0006	0.0003
44	10:45	11:00	0.0283	0.1062	0.0336	0.0464	0.0272	0.0035	0.0000
45	11:00	11:15	0.0094	0.0645	0.0272	0.0664	0.0802	0.0071	0.0194
46	11:15	11:30	0.0107	0.0028	0.0000	0.0015	0.0170	0.0093	0.0065
47	11:30	11:45	0.0257	0.0211	0.0003	0.0002	0.0283	0.0036	0.0000
48	11:45	12:00	0.0284	0.0630	0.0326	0.0011	0.0574	0.0436	0.0002

Table A.8. Abandonment Rates in Scenario 2 (Continued)

ABANDONMENT RATES									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	0.0042	0.0213	0.0162	0.0248	0.0349	0.0310	0.0005
50	12:15	12:30	0.0131	0.0090	0.0020	0.0029	0.0035	0.0291	0.0041
51	12:30	12:45	0.0172	0.0151	0.0021	0.0012	0.0000	0.0086	0.0037
52	12:45	13:00	0.0116	0.0299	0.0632	0.0208	0.0000	0.0329	0.0267
53	13:00	13:15	0.0289	0.0385	0.0734	0.0198	0.0000	0.0578	0.0088
54	13:15	13:30	0.0410	0.0244	0.0116	0.0425	0.0000	0.0361	0.0109
55	13:30	13:45	0.0166	0.0057	0.0099	0.0150	0.0000	0.0142	0.0135
56	13:45	14:00	0.0008	0.0009	0.0038	0.0004	0.0000	0.0024	0.0039
57	14:00	14:15	0.0270	0.0008	0.0021	0.0029	0.0000	0.0008	0.0005
58	14:15	14:30	0.0216	0.0020	0.0050	0.0572	0.0000	0.0173	0.0025
59	14:30	14:45	0.0004	0.0006	0.0235	0.0443	0.0000	0.0237	0.0191
60	14:45	15:00	0.0001	0.0004	0.0929	0.0157	0.0000	0.0237	0.0074
61	15:00	15:15	0.0001	0.0143	0.1251	0.0097	0.0000	0.0737	0.0002
62	15:15	15:30	0.0000	0.0252	0.0422	0.0013	0.0000	0.0805	0.0027
63	15:30	15:45	0.0000	0.0146	0.0094	0.0068	0.0000	0.0605	0.0027
64	15:45	16:00	0.0000	0.0093	0.0414	0.0182	0.0000	0.0248	0.0012
65	16:00	16:15	0.0054	0.0162	0.0871	0.1161	0.0000	0.0274	0.0001
66	16:15	16:30	0.0004	0.1003	0.0020	0.0184	0.0000	0.0346	0.0002
67	16:30	16:45	0.0000	0.0170	0.0000	0.0000	0.0000	0.0010	0.0000
68	16:45	17:00	0.0000	0.0023	0.0000	0.0000	0.0000	0.0001	0.0001
69	17:00	17:15	0.0000	0.0046	0.0000	0.0000	0.0000	0.0001	0.0000
70	17:15	17:30	0.0000	0.0770	0.0000	0.0001	0.0000	0.0000	0.0002
71	17:30	17:45	0.0004	0.0199	0.0000	0.0003	0.0000	0.0000	0.0009
72	17:45	18:00	0.0150	0.0004	0.0000	0.0000	0.0000	0.0000	0.0002
73	18:00	18:15	0.0055	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
74	18:15	18:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
75	18:30	18:45	0.0000	0.0011	0.0000	0.0000	0.0000	0.0116	0.0000
76	18:45	19:00	0.0003	0.0009	0.0000	0.0000	0.0000	0.0239	0.0000
77	19:00	19:15	0.0000	0.0005	0.0000	0.0000	0.0000	0.0063	0.0000
78	19:15	19:30	0.0000	0.0001	0.0003	0.0000	0.0000	0.0102	0.0014
79	19:30	19:45	0.0001	0.0000	0.0002	0.0000	0.0000	0.0120	0.0050
80	19:45	20:00	0.0006	0.0006	0.0017	0.0000	0.0000	0.0314	0.0029
81	20:00	20:15	0.0066	0.0072	0.0150	0.0000	0.0000	0.0568	0.0529
82	20:15	20:30	0.0134	0.0331	0.0680	0.0553	0.0000	0.0657	0.0776
83	20:30	20:45	0.0167	0.0368	0.0929	0.0629	0.0000	0.0369	0.0363
84	20:45	21:00	0.0362	0.0571	0.0189	0.0696	0.0000	0.0397	0.0531
85	21:00	21:15	0.0576	0.0752	0.0133	0.0553	0.0001	0.0892	0.1095
86	21:15	21:30	0.0573	0.0524	0.0097	0.0793	0.0328	0.0613	0.0995
87	21:30	21:45	0.0316	0.0376	0.0549	0.0505	0.0243	0.0256	0.0566
88	21:45	22:00	0.0553	0.0524	0.0784	0.0293	0.0008	0.0038	0.0067
89	22:00	22:15	0.1048	0.0686	0.1026	0.0074	0.0001	0.0005	0.0003
90	22:15	22:30	0.0687	0.0827	0.1000	0.0029	0.0531	0.0298	0.0060
91	22:30	22:45	0.0302	0.0636	0.0105	0.0001	0.0257	0.0352	0.0013
92	22:45	23:00	0.0049	0.0085	0.0002	0.0000	0.0020	0.0073	0.0000
93	23:00	23:15	0.0569	0.0505	0.0127	0.0000	0.0000	0.0037	0.0000
94	23:15	23:30	0.0300	0.0733	0.0771	0.0000	0.0414	0.0642	0.0000
95	23:30	23:45	0.0011	0.0037	0.0078	0.0000	0.0516	0.0318	0.0000
96	23:45	00:00	0.0000	0.0000	0.0000	0.0000	0.0018	0.0038	0.0000

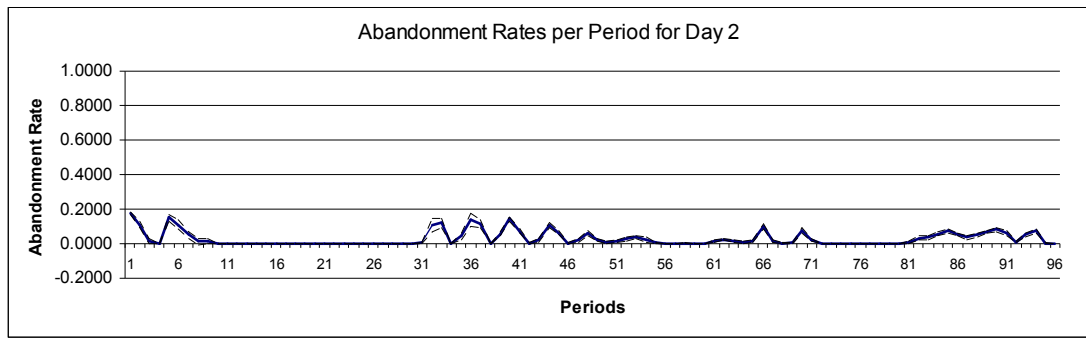


Figure A.7. Abandonment rates on Day 2 in Scenario 2

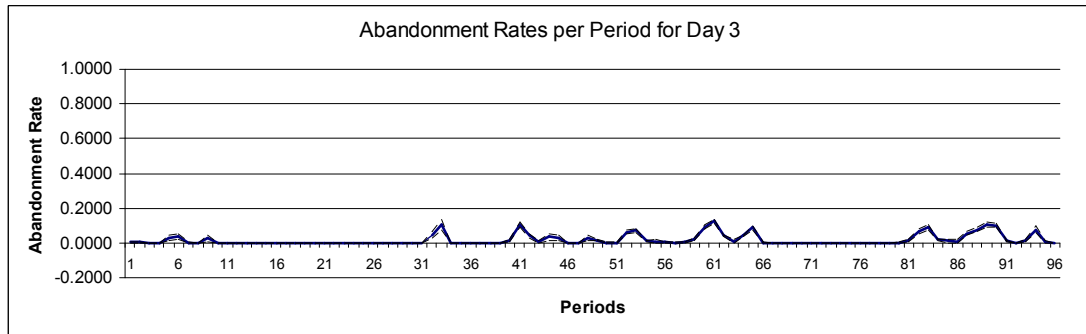


Figure A.8. Abandonment rates on Day 3 in Scenario 2

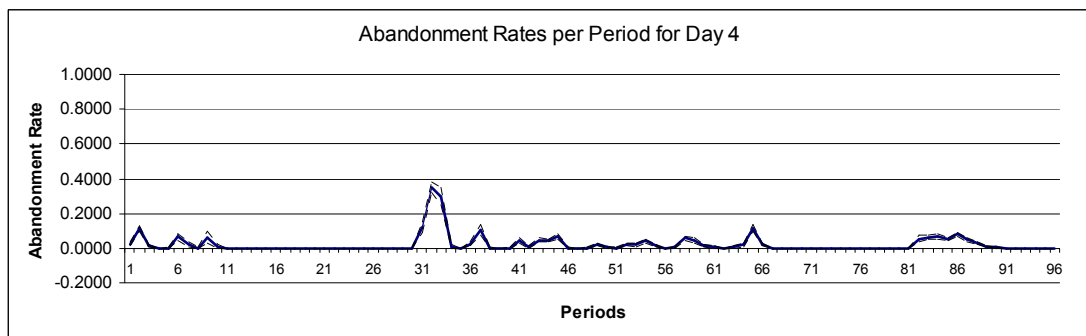


Figure A.9. Abandonment rates on Day 4 in Scenario 2

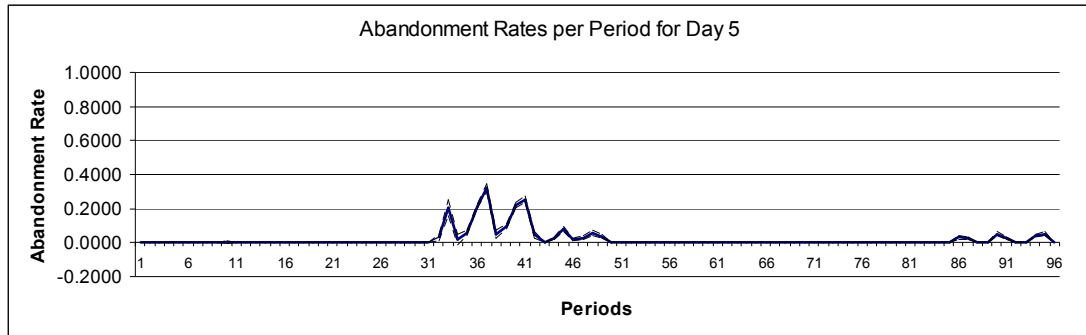


Figure A.10. Abandonment rates on Day 5 in Scenario 2

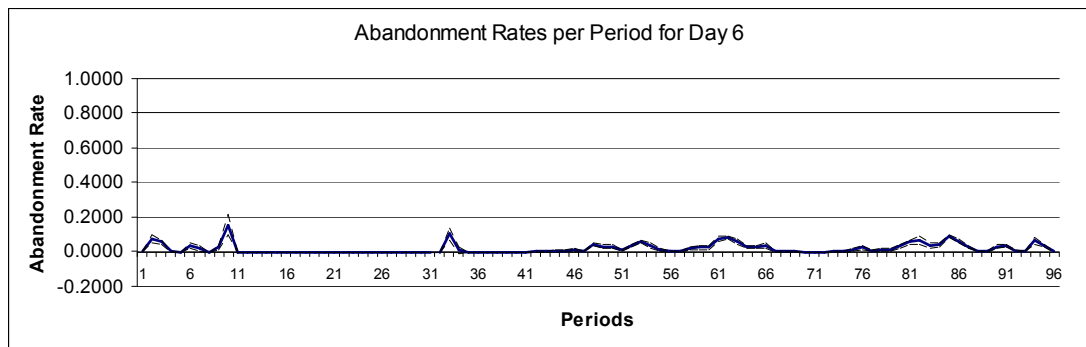


Figure A.11. Abandonment rates on Day 6 in Scenario 2

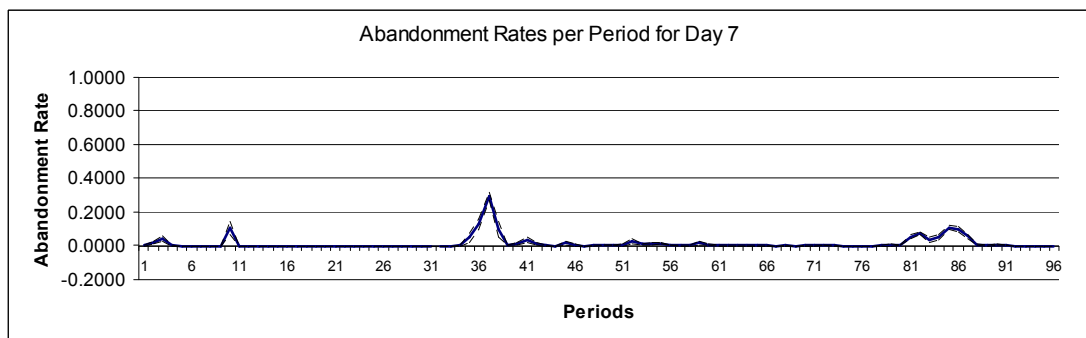


Figure A.12. Abandonment rates on Day 7 in Scenario 2

Table A.9. Service Levels in Scenario 3

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	1.0000	0.8593	0.9958	0.9828	1.0000	0.9991	1.0000
2	00:15	00:30	1.0000	0.8732	0.9998	0.8388	1.0000	0.8894	0.9762
3	00:30	00:45	1.0000	1.0000	1.0000	0.9749	1.0000	0.9678	0.9481
4	00:45	01:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9966
5	01:00	01:15	0.9869	0.9968	1.0000	1.0000	1.0000	1.0000	1.0000
6	01:15	01:30	1.0000	0.9968	0.9966	0.9992	1.0000	1.0000	1.0000
7	01:30	01:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	01:45	02:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	02:00	02:15	0.9133	0.8695	0.8429	0.9261	0.9942	1.0000	1.0000
10	02:15	02:30	0.9791	1.0000	0.9643	0.8476	0.8131	0.7043	0.7343
11	02:30	02:45	0.9962	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987
12	02:45	03:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	03:00	03:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	03:15	03:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	03:30	03:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	03:45	04:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	04:00	04:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	04:15	04:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	04:30	04:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	04:45	05:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	05:00	05:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	05:15	05:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	05:30	05:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	05:45	06:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	06:00	06:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	06:15	06:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	06:30	06:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	06:45	07:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	07:00	07:15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	07:15	07:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	07:30	07:45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	07:45	08:00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	08:00	08:15	1.0000	1.0000	1.0000	0.9997	1.0000	1.0000	1.0000
34	08:15	08:30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	08:30	08:45	0.9986	0.9844	1.0000	1.0000	1.0000	1.0000	0.9985
36	08:45	09:00	0.8798	0.9677	1.0000	0.8410	0.9917	1.0000	0.8858
37	09:00	09:15	0.9145	0.8456	0.9998	0.6678	0.6760	1.0000	0.7263
38	09:15	09:30	1.0000	1.0000	1.0000	0.9969	0.9789	1.0000	0.7793
39	09:30	09:45	0.9976	0.9931	0.9978	1.0000	0.9999	0.9980	1.0000
40	09:45	10:00	0.8736	0.8709	0.9291	0.9885	0.9536	1.0000	1.0000
41	10:00	10:15	0.9363	0.8934	0.7234	0.8394	0.8009	0.9977	0.9960
42	10:15	10:30	0.9761	1.0000	0.9289	0.9643	0.9700	1.0000	0.9997
43	10:30	10:45	0.8915	0.9858	0.9984	0.8877	0.9996	0.9960	1.0000
44	10:45	11:00	0.8679	0.8960	0.9990	0.8272	0.9943	0.9691	0.9982
45	11:00	11:15	0.8893	0.8923	0.9875	0.7891	0.9504	0.9095	0.9014
46	11:15	11:30	0.9217	0.9990	1.0000	0.9885	0.9456	0.9421	0.9329
47	11:30	11:45	0.8625	0.9583	0.9868	0.9982	0.9166	0.9854	1.0000
48	11:45	12:00	0.8335	0.9387	0.8613	0.9979	0.8300	0.8184	0.9963

Table A.9. Service Levels in Scenario 3 (Continued)

SERVICE LEVELS									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	0.9489	0.9570	0.8497	0.8930	0.8113	0.7640	0.9490
50	12:15	12:30	0.9301	0.9805	0.9884	0.9728	0.9539	0.9224	0.9076
51	12:30	12:45	0.8425	0.9898	0.9802	0.9908	0.9917	0.9656	0.9586
52	12:45	13:00	0.8930	0.9887	0.8141	0.9182	0.9999	0.9171	0.8811
53	13:00	13:15	0.8592	0.9415	0.6946	0.8491	1.0000	0.8443	0.9100
54	13:15	13:30	0.8677	0.9336	0.8648	0.8049	1.0000	0.8296	0.9214
55	13:30	13:45	0.8921	0.9773	0.9214	0.8660	1.0000	0.9805	0.9037
56	13:45	14:00	0.9917	0.9941	0.9268	0.9833	1.0000	0.9955	0.9581
57	14:00	14:15	0.9564	0.9815	0.9893	0.9802	1.0000	0.9959	0.9923
58	14:15	14:30	0.9317	0.9571	0.9796	0.7466	1.0000	0.9328	0.9631
59	14:30	14:45	0.9982	0.9519	0.9382	0.7680	1.0000	0.8836	0.8598
60	14:45	15:00	1.0000	0.9637	0.8082	0.8900	1.0000	0.8323	0.9439
61	15:00	15:15	0.9988	0.8844	0.6947	0.9688	1.0000	0.7932	0.9960
62	15:15	15:30	0.9972	0.8751	0.8134	0.9681	1.0000	0.7412	0.9773
63	15:30	15:45	0.9999	0.8808	0.9824	0.9612	1.0000	0.7860	0.9799
64	15:45	16:00	1.0000	0.9329	0.9281	0.8991	1.0000	0.8737	0.9851
65	16:00	16:15	0.9361	0.8630	0.7932	0.7035	1.0000	0.8930	0.9716
66	16:15	16:30	0.9770	0.7839	0.9839	0.9193	1.0000	0.8553	0.9959
67	16:30	16:45	1.0000	0.8419	0.9999	1.0000	1.0000	0.9712	0.9960
68	16:45	17:00	1.0000	0.9949	1.0000	1.0000	1.0000	0.9953	0.9980
69	17:00	17:15	1.0000	0.9865	1.0000	1.0000	1.0000	1.0000	1.0000
70	17:15	17:30	0.9995	0.7868	1.0000	0.9994	1.0000	0.9999	0.9951
71	17:30	17:45	0.9745	0.8649	1.0000	0.9943	1.0000	1.0000	0.9958
72	17:45	18:00	0.9062	0.9818	1.0000	0.9994	1.0000	1.0000	0.9901
73	18:00	18:15	0.9363	0.9943	1.0000	0.9996	1.0000	0.9974	0.9932
74	18:15	18:30	0.9977	0.9978	1.0000	1.0000	1.0000	0.9937	0.9990
75	18:30	18:45	0.9999	0.9949	1.0000	1.0000	1.0000	0.9324	0.9969
76	18:45	19:00	0.9839	0.9870	1.0000	1.0000	1.0000	0.8917	0.9997
77	19:00	19:15	0.9954	0.9812	0.9916	1.0000	1.0000	0.9350	0.9971
78	19:15	19:30	1.0000	0.9984	0.9651	0.9994	1.0000	0.9393	0.9791
79	19:30	19:45	0.9960	0.9971	0.9745	0.9985	1.0000	0.9061	0.9921
80	19:45	20:00	0.9722	0.9924	0.9582	1.0000	1.0000	0.8893	0.9829
81	20:00	20:15	0.9862	0.9583	0.8572	1.0000	0.9996	0.7766	0.8635
82	20:15	20:30	0.9826	0.9577	0.8940	0.9142	0.9356	0.7928	0.8407
83	20:30	20:45	0.9855	0.9484	0.8501	0.8582	0.9234	0.9034	0.9539
84	20:45	21:00	0.9488	0.9068	0.9223	0.8547	0.9918	0.9588	0.9246
85	21:00	21:15	0.9321	0.8753	0.9942	0.9098	0.9844	0.8449	0.7757
86	21:15	21:30	0.9061	0.8813	0.9859	0.9136	0.7734	0.7798	0.7886
87	21:30	21:45	0.9711	0.9524	0.8851	0.9228	0.7700	0.9015	0.9486
88	21:45	22:00	0.9431	0.8859	0.8398	0.9926	0.9269	0.9913	0.9973
89	22:00	22:15	0.8334	0.8972	0.7446	0.9960	0.9898	0.9991	0.9979
90	22:15	22:30	0.7953	0.7854	0.7294	0.9999	0.8277	0.9391	0.9792
91	22:30	22:45	0.9515	0.8559	0.9282	1.0000	0.8286	0.8465	0.9820
92	22:45	23:00	0.9991	0.9872	0.9996	0.9995	0.9716	0.9800	1.0000
93	23:00	23:15	0.9089	0.9122	0.9922	1.0000	0.9999	0.9908	1.0000
94	23:15	23:30	0.9586	0.8251	0.9040	1.0000	0.8422	0.8644	1.0000
95	23:30	23:45	1.0000	0.9961	0.9778	1.0000	0.8000	0.8554	1.0000
96	23:45	00:00	1.0000	1.0000	1.0000	1.0000	0.9880	0.9997	1.0000

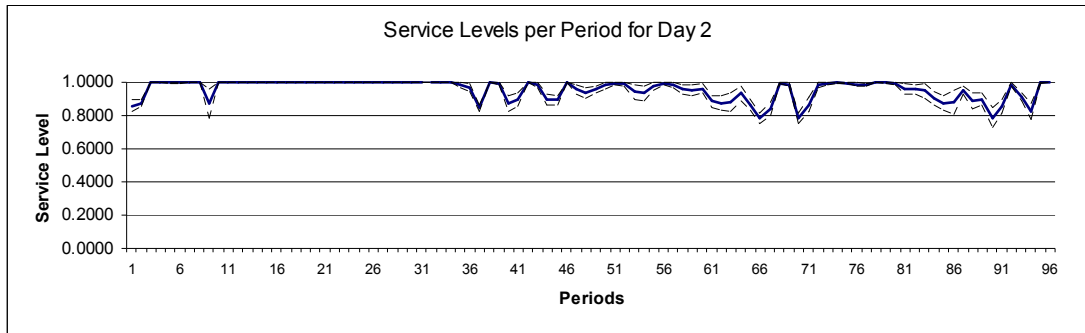


Figure A.13. Service levels on Day 2 in Scenario 3

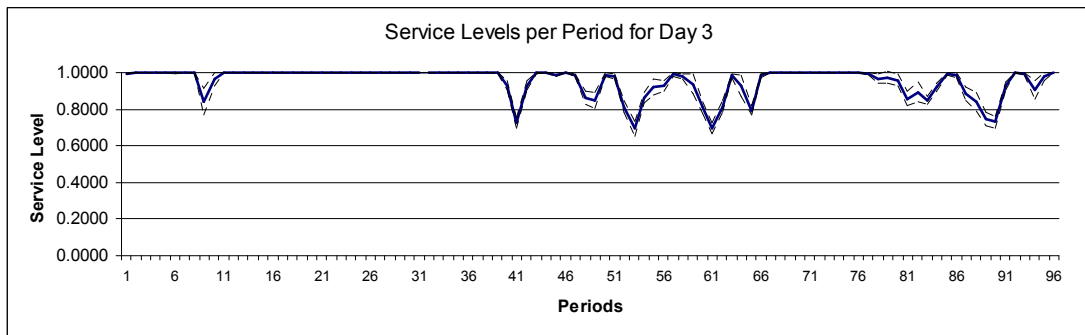


Figure A.14. Service levels on Day 3 in Scenario 3

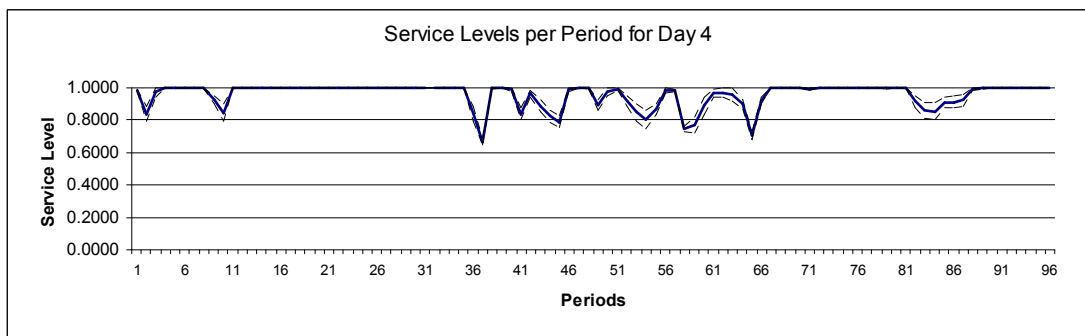


Figure A.15. Service levels on Day 4 in Scenario 3

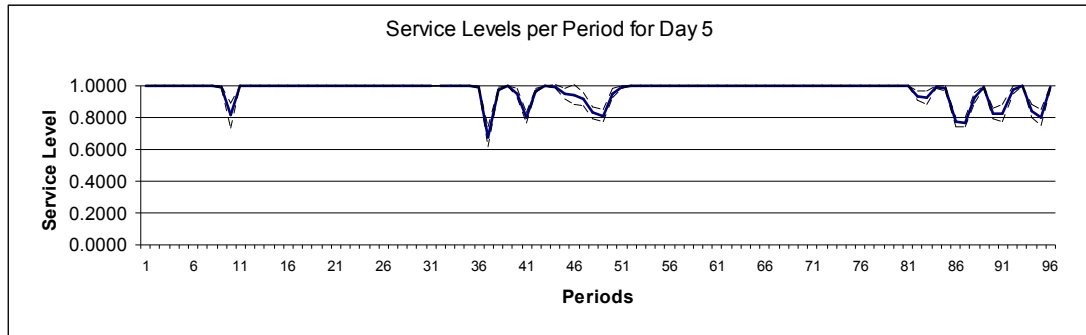


Figure A.16. Service levels on Day 5 in Scenario 3

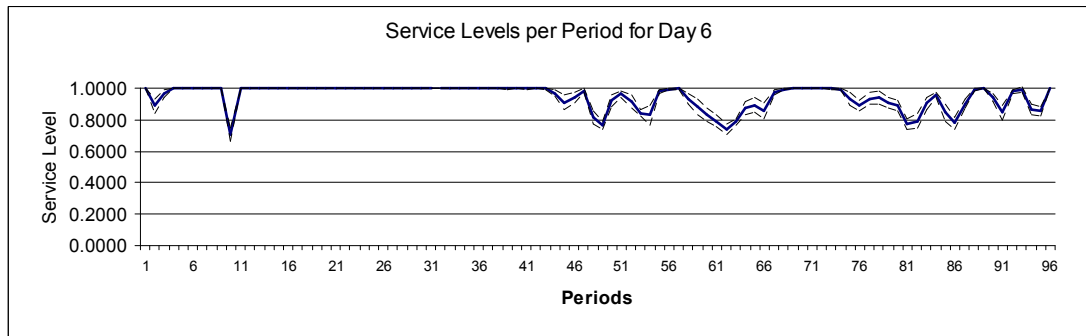


Figure A.17. Service levels on Day 6 in Scenario 3

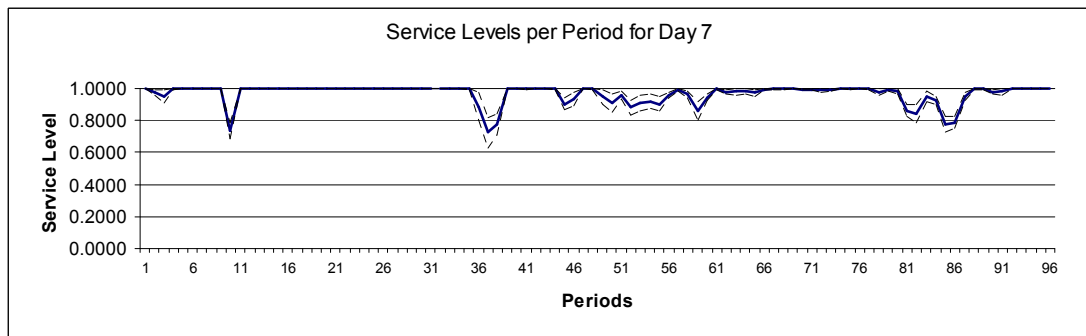


Figure A.18. Service levels on Day 7 in Scenario 3

Table A.10. Abandonment Rates in Scenario 3

ABANDONMENT RATES									
Period	From	To	M	T	W	TH	F	ST	SN
1	00:00	00:15	0.0000	0.0265	0.0000	0.0076	0.0000	0.0003	0.0000
2	00:15	00:30	0.0000	0.0098	0.0000	0.0129	0.0000	0.0069	0.0010
3	00:30	00:45	0.0000	0.0000	0.0000	0.0002	0.0000	0.0004	0.0016
4	00:45	01:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	01:00	01:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	01:15	01:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	01:30	01:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	01:45	02:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	02:00	02:15	0.0018	0.0105	0.0472	0.0633	0.0156	0.0000	0.0000
10	02:15	02:30	0.0000	0.0000	0.0009	0.0069	0.0150	0.1873	0.0936
11	02:30	02:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	02:45	03:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	03:00	03:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	03:15	03:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	03:30	03:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	03:45	04:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17	04:00	04:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	04:15	04:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	04:30	04:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	04:45	05:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	05:00	05:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	05:15	05:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	05:30	05:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	05:45	06:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	06:00	06:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	06:15	06:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	06:30	06:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	06:45	07:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	07:00	07:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	07:15	07:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
31	07:30	07:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32	07:45	08:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
33	08:00	08:15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
34	08:15	08:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
35	08:30	08:45	0.0000	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000
36	08:45	09:00	0.0037	0.0006	0.0000	0.0224	0.0000	0.0000	0.0073
37	09:00	09:15	0.0042	0.0258	0.0000	0.0760	0.0749	0.0000	0.1106
38	09:15	09:30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0254
39	09:30	09:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
40	09:45	10:00	0.0137	0.0205	0.0062	0.0003	0.0026	0.0000	0.0000
41	10:00	10:15	0.0015	0.0092	0.0569	0.0128	0.0334	0.0000	0.0000
42	10:15	10:30	0.0015	0.0000	0.0004	0.0007	0.0004	0.0000	0.0000
43	10:30	10:45	0.0083	0.0003	0.0001	0.0159	0.0000	0.0000	0.0000
44	10:45	11:00	0.0165	0.0105	0.0000	0.0181	0.0001	0.0022	0.0001
45	11:00	11:15	0.0075	0.0110	0.0000	0.0204	0.0040	0.0098	0.0116
46	11:15	11:30	0.0080	0.0000	0.0000	0.0001	0.0020	0.0035	0.0086
47	11:30	11:45	0.0156	0.0012	0.0002	0.0001	0.0121	0.0004	0.0000
48	11:45	12:00	0.0180	0.0023	0.0248	0.0001	0.0273	0.0292	0.0000

Table A.10. Abandonment Rates in Scenario 3 (Continued)

ABANDONMENT RATES									
Period	From	To	M	T	W	TH	F	ST	SN
49	12:00	12:15	0.0009	0.0003	0.0127	0.0057	0.0209	0.0222	0.0044
50	12:15	12:30	0.0076	0.0002	0.0006	0.0005	0.0008	0.0014	0.0070
51	12:30	12:45	0.0120	0.0000	0.0003	0.0000	0.0000	0.0015	0.0049
52	12:45	13:00	0.0061	0.0002	0.0574	0.0067	0.0000	0.0053	0.0163
53	13:00	13:15	0.0164	0.0026	0.0690	0.0142	0.0000	0.0144	0.0076
54	13:15	13:30	0.0181	0.0014	0.0117	0.0374	0.0000	0.0090	0.0045
55	13:30	13:45	0.0054	0.0000	0.0084	0.0063	0.0000	0.0001	0.0041
56	13:45	14:00	0.0000	0.0001	0.0028	0.0001	0.0000	0.0000	0.0005
57	14:00	14:15	0.0062	0.0005	0.0004	0.0043	0.0000	0.0003	0.0001
58	14:15	14:30	0.0009	0.0004	0.0001	0.0552	0.0000	0.0037	0.0024
59	14:30	14:45	0.0000	0.0022	0.0061	0.0318	0.0000	0.0084	0.0086
60	14:45	15:00	0.0000	0.0012	0.0418	0.0090	0.0000	0.0146	0.0003
61	15:00	15:15	0.0000	0.0131	0.0905	0.0007	0.0000	0.0407	0.0000
62	15:15	15:30	0.0001	0.0084	0.0238	0.0005	0.0000	0.0537	0.0004
63	15:30	15:45	0.0000	0.0112	0.0001	0.0009	0.0000	0.0296	0.0008
64	15:45	16:00	0.0000	0.0007	0.0079	0.0075	0.0000	0.0106	0.0003
65	16:00	16:15	0.0062	0.0106	0.0408	0.0798	0.0000	0.0095	0.0009
66	16:15	16:30	0.0002	0.0726	0.0001	0.0005	0.0000	0.0093	0.0000
67	16:30	16:45	0.0000	0.0174	0.0000	0.0000	0.0000	0.0001	0.0000
68	16:45	17:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
69	17:00	17:15	0.0000	0.0014	0.0000	0.0000	0.0000	0.0000	0.0000
70	17:15	17:30	0.0000	0.0708	0.0000	0.0000	0.0000	0.0000	0.0000
71	17:30	17:45	0.0013	0.0117	0.0000	0.0000	0.0000	0.0000	0.0000
72	17:45	18:00	0.0125	0.0007	0.0000	0.0000	0.0000	0.0000	0.0010
73	18:00	18:15	0.0006	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
74	18:15	18:30	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000
75	18:30	18:45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0068	0.0000
76	18:45	19:00	0.0018	0.0001	0.0000	0.0000	0.0000	0.0094	0.0000
77	19:00	19:15	0.0000	0.0002	0.0001	0.0000	0.0000	0.0039	0.0000
78	19:15	19:30	0.0000	0.0000	0.0011	0.0000	0.0000	0.0050	0.0002
79	19:30	19:45	0.0002	0.0000	0.0001	0.0000	0.0000	0.0084	0.0001
80	19:45	20:00	0.0001	0.0001	0.0035	0.0000	0.0000	0.0086	0.0002
81	20:00	20:15	0.0001	0.0017	0.0117	0.0000	0.0000	0.0453	0.0181
82	20:15	20:30	0.0003	0.0020	0.0154	0.0135	0.0040	0.0227	0.0137
83	20:30	20:45	0.0002	0.0010	0.0288	0.0071	0.0039	0.0024	0.0013
84	20:45	21:00	0.0007	0.0095	0.0060	0.0156	0.0001	0.0011	0.0064
85	21:00	21:15	0.0059	0.0180	0.0001	0.0039	0.0003	0.0287	0.0532
86	21:15	21:30	0.0067	0.0120	0.0000	0.0071	0.0588	0.0410	0.0227
87	21:30	21:45	0.0002	0.0002	0.0105	0.0041	0.0328	0.0058	0.0006
88	21:45	22:00	0.0026	0.0105	0.0184	0.0000	0.0026	0.0001	0.0000
89	22:00	22:15	0.0274	0.0054	0.0474	0.0000	0.0000	0.0000	0.0000
90	22:15	22:30	0.0252	0.0369	0.0430	0.0000	0.0478	0.0070	0.0030
91	22:30	22:45	0.0021	0.0126	0.0024	0.0000	0.0129	0.0140	0.0003
92	22:45	23:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
93	23:00	23:15	0.0129	0.0255	0.0023	0.0000	0.0002	0.0007	0.0000
94	23:15	23:30	0.0005	0.0193	0.0068	0.0000	0.0281	0.0326	0.0000
95	23:30	23:45	0.0000	0.0000	0.0006	0.0000	0.0125	0.0129	0.0000
96	23:45	00:00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

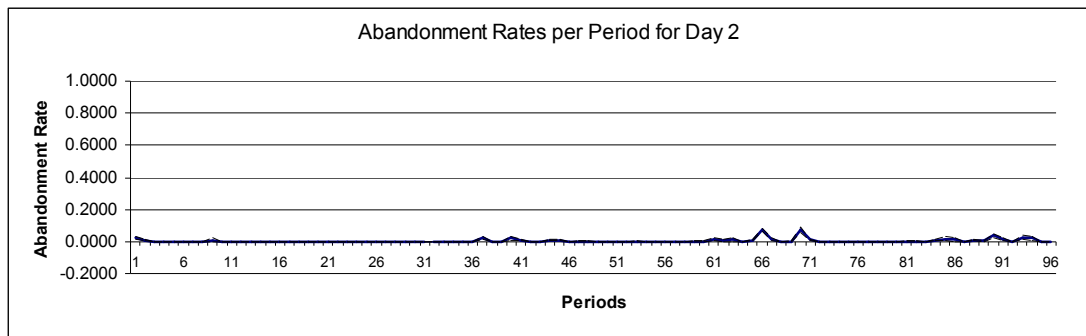


Figure A.19. Abandonment rates on Day 2 in Scenario 3

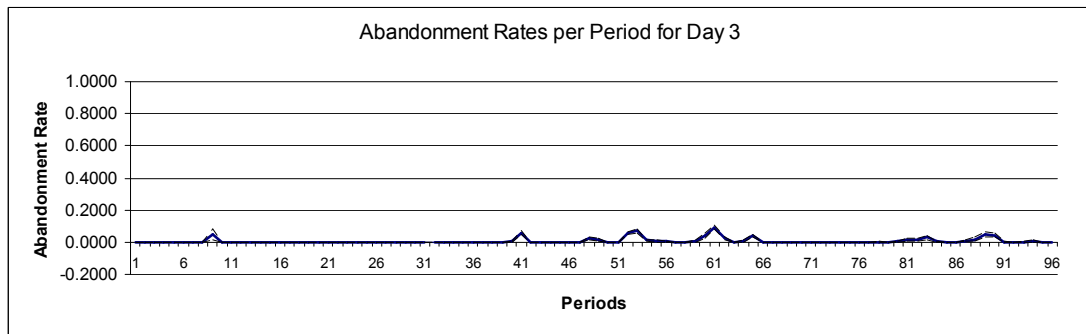


Figure A.20. Abandonment rates on Day 3 in Scenario 3

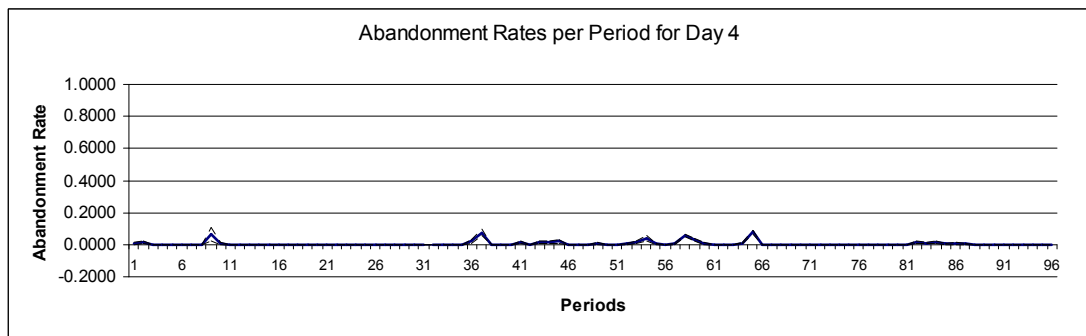


Figure A.21. Abandonment rates on Day 4 in Scenario 3

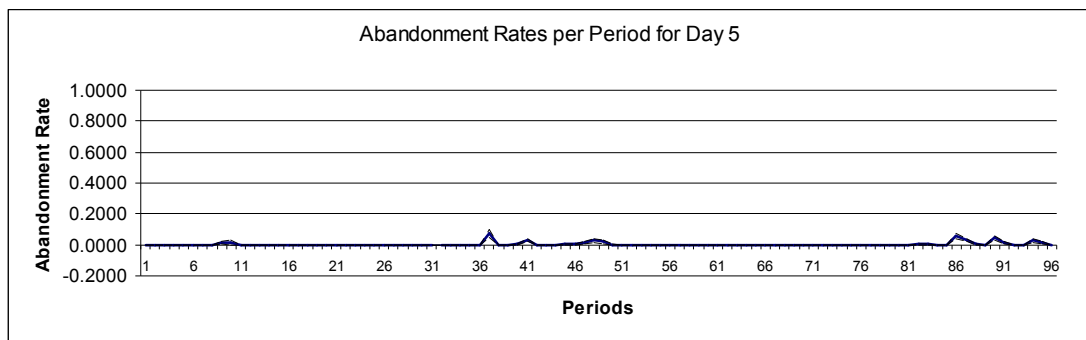


Figure A.22. Abandonment rates on Day 5 in Scenario 3

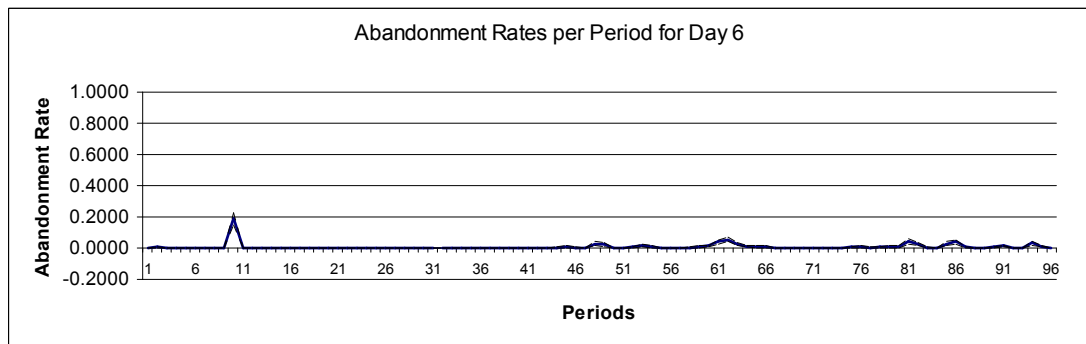


Figure A.23. Abandonment rates on Day 6 in Scenario 3

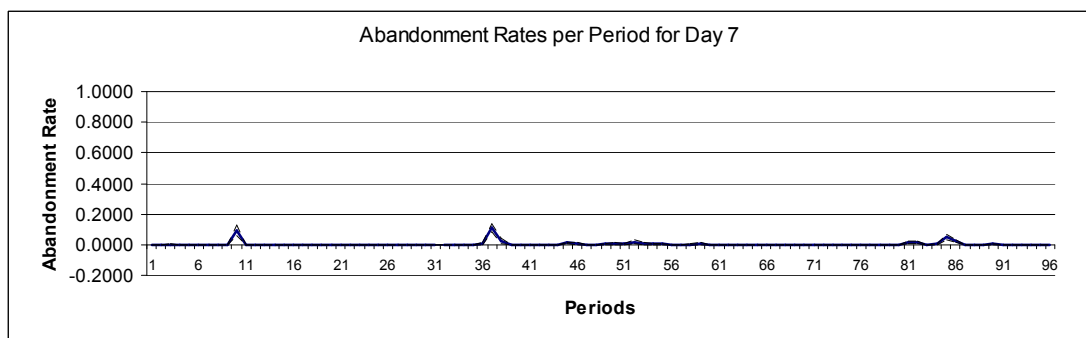


Figure A.24. Abandonment rates on Day 7 in Scenario 3

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