Merve ŞAHİN

M.S. Thesis in Industrial Engineering

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FORECASTING AVIATION SPARE PARTS DEMAND

by

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APPROVAL PAGE

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

Accurate demand forecasts provide high levels of customer service while inaccurate forecasts may result in out of stock costs and thereby becoming loss of sales. To construct the appropriate inventory policy, using high accuracy demand forecasting is crucial especially in the repair and maintenance industry such as aircraft service parts because of the high cost of aircraft and the expense of such repairable spares. These parts generally show an intermittent demand structure that is characterized by demand data that has many time periods with zero demands. Accurate forecasting of these kind of parts can result in substantial cost savings because items with intermittent demand can have a value of up to 60% of the total stock value for all items. However, variability and occurrence uncertainty of these parts raise challenges to forecast with high accuracy when traditional forecasting methods are used. In this study, several intermittent demand forecasting techniques are applied on Turkish Airlines service parts data subsequent to be categorized. These techniques are obtained from the literature and some results of them employed to compare with Artificial Neural Networks application. Performance of these forecasting methods is evaluated using the appropriate accuracy measures. Consequently, it is aimed to provide guidance to the airline practitioners to determine the best forecasting method for each demand item based on its characteristic properties considering different accuracy measures for intermittent demand data. In this way, selection of the most appropriate forecasting method can provide an input to determine inventory parameter to decrease inventory costs.

Keywords: Categorization of demand data, Intermittent Demand Forecasting, Croston Method and its variants, Artificial Neural Networks, Spare Parts Management

HAVACILIKTA YEDEK PARÇALARIN TALEP TAHMİNİ

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ÖZ

Doğru talep tahminleri yüksek müşteri hizmeti sağlarken yanlış tahminler stokta bulunmama ve bunun sonucunda satışların kaybı ile sonuçlanabilir. Özellikle onarım ve bakım sektöründe uçak ve yedek parçalarının yüksek maliyetli olması sebebiyle yüksek doğrulukta talep tahmini kullanmak uygun stok politikası kurmak için önemlidir. Bu parçalar çoğu zaman sıfır talepli zaman dilimi bulunduran kesikli bir talep yapısı gösterirler. Kesikli talep yapısına sahip bu parçaların stok değerinin toplam stok değerinin %60 ına kadar ulaşabilmesi nedeniyle bu tür parçaların doğru tahmini önemli maliyet kazançlarıyla sonuçlanabilir. Fakat klasik tahmin metotları kullanıldığında talebin değişkenliği ve belirsizliği yüksek doğrulukta tahmin edilmesini zorlaştırır. Bu çalışmada, Türk Hava Yolları yedek parçaları verisi categorize edildikten sonra çeşitli kesikli talep tahminleme teknikleri uygulandı. Bu teknikler; Croston, Croston metodunun varyantları, diğer kesikli talep tahminleme metotları ve Yapay Sinir Ağlarıdır. Bu tekniklerin performansı uygun değerlendirme ölçütleri kullanılarak değerlendirildi. Sonuç olarak her bir parça için kendi karakteristik özelliklerine göre en iyi tahminleme metodunu farklı değerlendirme ölçütleri dikkate alındığında belirlemede havayolu uygulayıcılarına rehber olması sağlandı. Böylece en uygun tahminleme metodunun seçimi, envanter yönetiminde stok maliyetlerini azaltmak için envanter parametresini belirleyen bir girdi sağlayabilir.

Anahtar Kelimeler: Talep verisinin kategorizasyonu, Kesikli talep tahmini, Croston yöntemi ve türevleri, Yapay sinir ağları, Yedek parça yönetimi

To my family

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SYMBOL/ABBREVIATION

ADI : Average Inter Demand Interval
ANN : Artificial Neural Networks
ARTA : Autoregressive to Any Algorithm
BP : Back-Propagation
CV : Coefficient of Variation
EWMA : Exponentially Weighted Moving Average
GMAMAE : Geometric Mean of the Arithmetic Mean of the Absolute Errors
GRMSE : Geometric Root Mean Square Error
GRNN : Generalized Regression Neural Network
IFM : Integrated Forecasting Method
LTD : Lead Time Demand
MAD : The Mean Absolute Deviation
MAD/A : Mean Absolute Deviation to Average
MAPE : Mean Absolute Percentage Error
MASE : Mean Absolute Scaled Error
MLP : Multi-Layered Perceptron
MSE : The Mean Squared Error
PB : Percentage Better
RBF : Radial Basis Function
RMSE : The Root Mean Squared Error
RNN : Recurrent Neural Network
SKU : Stock Keeping Unit
sMAPE : Symmetric MAPE
UAPE : Mean Unbiased Absolute Percentage Error

CHAPTER 1

INTRODUCTION

1.1 PROBLEM DEFINITION

In most companies, customers are demanding faster service in competitive environment. As such, planning activities based on forecasts are becoming more important to enable the availability of spare parts in manufacturing and service systems. Demand for spare parts show irregular pattern with a high degree of uncertainty as to when and at what level demand will occur. This type of demand is handled as non smooth in this study and it is characterized as erratic, intermittent or lumpy according to its statistical properties.

Non smooth items may represent as much as 50 percent of the products a firm handles. In this regard, it is vital and complex to estimate these items cost of modern aircraft and their spares repair contributes to the total investment of airline operators. Excessive downtime costs may result from underestimating spare parts demand, it is therefore crucial to employ special techniques in comparison with the smooth and continuous case in order to forecast accurately since they show irregular demand pattern. If inter arrival time between nonzero demands is very high, keeping the continuous inventories of these items may cause holding costs because of the lack of an appropriate forecasting method. Thus, managing spare parts is a critical operational issue in manufacturing and service industry. This can be due to small improvements that may be converted to considerable cost savings.

The issue of decreasing stock out and holding costs has taken interest in academic literature in decades and people focus on forecasting methods which increase forecast accuracy for non smooth demand types in the areas of management, economics and engineering. Besides, computerized forecasting solutions which consider different performance measures are necessary for the revenue and the profit of a company. Typical high performance companies tend to improve robust demand forecasting techniques and process which leads to fewer inventories, and better customer satisfaction.

1.2 THEORETICAL BACKGROUND

Forecasting and stock control difficulties arise from the intermittent nature of the underlying demand patterns. If spare parts are managed efficiently, cost savings may be achieved due to their availability. Management of spare parts entails the categorisation of the relevant stock keeping units (SKU) and forecasting accurately to facilitate decision making. In this regard, an appropriate forecasting and stock control method will most probably increase spare parts availability, and reduce inventory costs.

Since the seminal work of Croston in 1972 in the area of forecasting for intermittent demand, a limited number of researches have been conducted on its implication to inventory management although these items comprise a substantial portion of the inventory population in parts (Mak and Huang, 1999; Hernandez, 1999; Botter and Fortuin 2000; Boylan et al., 2008; Porras and Dekker, 2008). Let alone the fact that effective management of uncertain demand may be a source of competitive advantage. Therefore, this research aspires to take forward the current state of knowledge on forecasting intermittent demand. In this context, some researchers have proposed methods for forecasting an uncertain non smooth demand, where others have focused on the management of the manufacturing resources while facing an uncertain irregular demand to obtain competitive advantage (Bartezzaghi et al., 1999). It can be said that most papers in this area focus on the control of inventories of intermittent demand SKUs, assuming that an appropriate estimator to forecast future demand requirements (Silver, 1970; Ward, 1978; Schultz, 1987; Watson, 1987; Dunsmuir and Snyder, 1989; Segerstedt, 1994).

1.3 OBJECTIVE OF THE STUDY

Irregular demand patterns make demand forecasting challenging and forecast errors can be costly in terms of unmet demand or obsolescent stock. In this context, forecasting methods which are developed for non smooth data type have been addressed in this study to compare their performances. There are various established forecasting methods in literature, like exponential smoothing, Croston's method with its variants and bootstrapping. It can be argued that the Croston's method is the norm for intermittent demand problems, and dynamic demand rate forecasts. In recent years, artificial neural network approach has been used to forecast intermittent time series. A key neural network limitation that is addressed in this study is the small time series sample size in forecasting.

A common problem for irregular demand pattern is the need to forecast demand with the highest possible degree of accuracy. The accuracy of forecasting methods is closely related to characteristic of demand data (Boylan et al., 2008). The need of producing more accurate time series forecasts remains an issue in both conventional and soft computing techniques; therefore innovative methods have been developed in literature. Moreover, traditional time-series methods may sometimes not capture the nonlinear pattern in data. In this context, artificial neural network modelling is a logical choice to overcome the limitations cited and improve forecast accuracy. The overall aim of this study is to clarify which forecasting methods are more reasonable for which type of demand data considering different accuracy measures. To achieve this, the objectives of this study are:

- to explore the problem of spare parts management in modern industries, explaining spare parts and spare parts demand properties for the purpose of categorization;
- to explain and compare the intermittent demand forecasting methods by minimization of used error measures that have been given more attention in scientific literature;
- to investigate the neural network models, one of which have gained the more attention in the recent scientific literature;
- to compare intermittent demand forecasting methods and neural networks on real industrial data considering different accuracy measures.

1.4 THESIS STRUCTURE

Chapter 1 of the thesis introduces the problem, provides justification for solving the problem, and states the scope and objective of the research. Chapter 2 reviews the literature related to the problem. In Chapter 3, a real data set that contains irregular demand series of spare parts demand from the airline industry is categorized. Chapter 4 describes different intermittent demand forecasting techniques used to forecast the non smooth demand and analyses the results according to different performance measures. An analysis of the forecast accuracy results is presented to determine the best suitable method for each data type. A general concept of spare parts demand management is also given. Chapter 5 represents the neural networks application compared with the other forecasting methods which are selected as outstanding in Chapter 4. Finally, Chapter 6 contains the summary of the forecasting performance of the techniques used in this thesis and the research results as well as recommendations for future work. The conclusions derived from this study provide guidance for the industry users.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Forecasting has a great importance and widely used for every stage of supply chains from any supplier to customer. Good forecasts lead to good decisions such a determining how many items can be produced and how can we decide available inventory level to minimize stock out and holding inventory costs. Most firms cannot wait for demand if it is needed to maintain production uninterruptedly. Planning future demand to meet customer orders as they occur comes into prominence in terms of sustaining continuous production. The only way that balancing demand fluctuations' unfavourable effect is keeping inventory. However, there is a trade off in decision making whether make to stock in case of the customer orders or make to order when it is need. Make to stock decision contains inventory costs such as carrying, holding costs. However, make to order decision caused to the customer waits so long as lead time. Some customers are reluctant to wait as far as the lead time because of the structure of supply chain process that is the next step wholly dependent to previous step so waiting any time affects all process directly. As a result stock out costs can be seen in the make to order decision while holding costs can be seen in the make to stock decision. It is needed to keep a certain level of inventory in case of customer orders by providing balance between holding and stock out costs. The inventory costs of a product depend on using forecasting methods how accurately. Firms use these methods to determine how many products should be produced or ordered for the next period. Inventory planners aim to reduce costs of inventory with high customer service levels using appropriate forecasting methods.

2.1.1 Demand Forecasting

Firms need to plan their activities for the future using historical demand data. At this point, forecasts play a key role in decision making to operate activities. Accurate demand forecasting decreases uncertainties and risks in business management while inaccurate demand forecasting increases uncertainties and risks. For example when actual demand mismatches the demand forecast, it can be seen stock out or holding costs in other words demand is coming but it cannot be turning into sales due to a lack of availability or there is not any demand but some amount of product is held in inventory in case of customer order.

There are two approaches to make a demand forecasting. First one is qualitative method that can be used in short term forecasting and rely on subjective opinions. Second one is quantitative method that is generally used for long-term forecasting. It considers historical data to find out any statistical correlations such seasonality, trend and cycles. It is expected that past demand data statistical relationships can be carried forward to the future to estimate of future demand. Qualitative and quantitative forecasting approaches can be used in collaboration with or singly to make pricing decisions and determining future capacity requirements in organization.

Demand forecasting is essential in supply chain management (Kilger et al., 2000). In academic literature authors have studied on accurate demand forecasting to provide efficient supply chain management. Gupta and Maranas (2003) indicate that demand uncertainty can be dealt with using demand forecasting and supply network planning model to minimize costs in the chemical industry. If an organization can not apply any accurate demand forecasting method for future demand, every department in the firm such as production planning, inventory management, workforce scheduling, financial planning can be affected. Inaccurate forecasts prevent efficient control on the system and cause high costs and low customer service level. However, by applying accurate demand forecasting, the cash flow in the firm increases because of unnecessary costs are eliminated so it is possible to allocate this cash to any other areas in the firm that may be needed to. Demand forecasting with minimum error ensures costs related to inventory as possible as low while customer satisfaction is high. Therefore, achieving demand forecasting accuracy is the first step to minimize inventory costs in supply chain management.

2.1.2 Forecasting for Spare Parts

Forecasts that are used for the planning of inventory levels are practically the biggest challenge in big organisations such as aerospace or automotive industries because of the infrequent demand structure. Infrequent demand occurrences and irregular demand sizes, when demand occurs, can be defined as non smooth. Non smooth type of demand typically can be seen for a large number of spare parts, is very difficult to forecast with high accuracy which is caused to the inventory system costs. Especially spare parts in some of the organizations show usually highly varied pattern and it leads to holding or stock out costs.

The selection of the forecasting method is generally based on the values of parameters that specify the characteristics of the demand data (Boylan et al., 2008) for non smooth demand data. We considered forecasting methods and accuracy measures for smooth demand and non smooth demand individually as the following.

2.2 FORECASTING METHODS FOR SMOOTH DEMAND

Conventional forecasting methods can be divided into two categories in general as the following:

- Causal methods which is related to explanatory variables
- Time-series methods which is related to the history of demand

2.2.1 Causal Methods

Causal methods are used when the historical data cannot be available in the system. Causal forecasting methods are based on a known relationship between the factors to be forecasted and other current factors. It aims to find out which factors have the greatest impact. For example, the demand for the spare part can be expected to increase dependent on past usage patterns such as flying hours. It is possible to determine spare part necessity by the help of explanatory variables such as average demand interval and variance related to using machines (Ghobbar A., 2003).

2.2.2 Time Series Methods

Time series methods are forecasting techniques that based on analysis of historical data. They make the assumption that past patterns in data can be used to forecast future because of there may be similarity between current and future data in terms of any pattern. Time series methods are appropriate when you can find out a reasonable statistical relationship in data at particular points. They are generally used for short term forecasting (say as 12 months or less). Firms generally use traditional forecasting methods which are given in the following to forecast the demand efficiently.

2.2.2.1 Moving Averages

A simple moving average is the average of a data series over a selected time period. As time goes on the oldest data is removed from the moving average calculation and replaced by the most recent data. This allows the moving average to "move" thereby keeping pace. It gives equal weight to each data period. Different time horizons analyzing all period lengths change from 2 to 12.

2.2.2.2 Weighted and Exponential Moving Averages

A weighted moving average provides to reduce the lag by giving more weight to recent data, thereby allowing the moving average to respond more quickly to current market conditions. The most popular version is the linearly weighted moving average. An exponential moving average gives more weight to recent data similar to weighted moving average.

2.2.2.3 Exponential Smoothing

Exponential smoothing methods are used when well direct forecasts are needed over the short term. It refers to a particular type of method applied to time series data, either to produces smoothed data or to forecast. In its simple computational form, forecasting for the next period by forming a weighted combination of the last observation and the last forecast: where α is a parameter called the smoothing coefficient, smoothing factor, or smoothing constant. Values of α are restricted such that $0 < \alpha < 1$. This method gives greater weight to demand in more recent periods, and less weight to demand in earlier periods as exponentially As such, it can be said that α is the relative weight given to the most recent data in the series, and its choice is up to the analyst and the field of study. When forecasts are done on a monthly basis, smoothing constant value that is between 0.1 and 0.3 is mostly appropriate for exponential smoothing (Silver et al., 1998).

The general form of exponential smoothing is given by, where *Ft* represents the smoothed estimate, *Xt* the actual value at time *t* and *α* the smoothing constant, which has a value between 0 and 1.

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t \tag{2.1}$$

2.2.2.4 Double Exponential Smoothing

If the data contains any pattern such as a trend, the forecasting technique must consider the trend because of disregarding the trend will cause the forecast to underestimate or to overestimate actual demand. Double exponential smoothing has two smoothing constants, α and β, and uses two smoothing equations to deal with this problem: one for the value of the series (the intercept) and one for the trend (the slope).

The smoothing constants may have the same value but for most applications more stability is given to the slope estimate $\beta \leq \alpha$. S_t is the value of intercept (series) at time t and G_t is the value of slope (trend) at time t. They are formulated as the following:

$$S_t = \alpha X_t + (1-\alpha)(S_{t-1} + G_{t-1}) \tag{2.2}$$

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta) G_{t-1} \tag{2.3}$$

The τ-step forecast made in period t is:

$$F_{t,t+\tau} = S_t + \tau G_t \tag{2.4}$$

2.2.2.5 Winter’s Method

If the data show trend as well as seasonality double exponential smoothing does not work well so it is needed third parameter that considers seasonality. Winter’s method catches level, trend and seasonality with the following parameters respectively (α, β, γ) all these parameters have values ranging between 0 and 1. Winter's method prediction function is;

$$D_t = (\mu + G_t)c_t + \varepsilon_t \tag{2.5}$$

In this formula μ is the intercept at time t = 0 excluding seasonality, G as the trend or slope component, c_t is the multiplicative seasonal component in period t and ε_t is the error.

$$S_t = \alpha\left(\frac{D_t}{C_{t-N}}\right) + (1-\alpha)(S_{t-1} + G_{t-1}) \tag{2.6}$$

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1} \tag{2.7}$$

$$c_t = \gamma\left(\frac{D_t}{S_t}\right) + (1-\gamma)c_{t-N} \tag{2.8}$$

The τ-step forecast made in period t is:

$$F_{t,t+\tau} = (S_t + \tau G_t)c_{t+\tau-N} \tag{2.9}$$

In the above equations S_t is the current level of deseasonalized series, G_t is trend updated like Holt's method, c_t is the seasonal factor and N is the length of seasonal period. The function tries to find the optimal values of α, β and γ by minimizing the one-step prediction error. Initial values for S_t, G_t and c_t are computed by applying a simple decomposition in the trend and seasonal components using moving averages on the first period (a simple linear regression on the trend component can be used to initialize values of level and trend).

2.2.2.6 Box-Jenkins Methods

The Box and Jenkins methodology was defined by statisticians, George Box and Gwilym Jenkins in 1970. In this method, ARIMA (autoregression and moving average integrated) models are used to find the best fit of a time series to past values of these time series. An ARIMA model is used for forecasting rely on an appropriate ARIMA process fitting it to the data. Using ARIMA processes is the best way to define the data with regard to provide flexible possible models. Box-Jenkins modelling procedure involved an iterative three-stage process of model selection, parameter estimation and model checking. Recent explanations of the process (Makridakis and Hyndman, 1998) often add a preliminary stage of data preparation and a final stage of model application. The ARIMA model is expressed as ARIMA (p,q) where p is a number of autoregressive parameters and q is a number of moving average parameters. It can be defined as,

$$X_t = \emptyset_1 \, X_{t-1} + \emptyset_2 X_{t-2} + \; \dots \dots + \; \emptyset_p X_{t-p} - \emptyset_1 \varepsilon_{t-1} - \emptyset_2 \varepsilon_{t-2} - \; \dots \dots - \emptyset_q \varepsilon_{t-q} + \varepsilon_t \tag{2.10}$$

2.3 PERFORMANCE MEASURES FOR SMOOTH DEMAND

Forecasting methods' accuracy can be evaluated considering the forecast errors which results from the forecasts. Given a set of n observations of the series D_t, and the corresponding forecasts F_t, it can be defined statistics based on the error term (which $e_t = D_t - F_t$) that is used to define accuracy of the forecasts. Some of traditional performance measures that are used to measure forecasting techniques' accuracy as the following. These measures can be applied to any set of forecasts irrespective of which technique had been used.

2.3.1 The Mean Deviation (MD)

The mean deviation is a simple error statistic that is computed as the arithmetic average of forecast errors. It is used to characterize the bias of a forecasting method. Mean deviation takes negative value if the forecast underpredicts the actual or it takes positive value if forecast overpredicts the actual. Small mean deviation does not imply that the errors are small it only shows biasness. Mean deviation can be defined as;

$$MD = (1/n)\sum_{i=1}^{n} e_t \tag{2.11}$$

$e_1, e_2 \dots e_n$ is the forecast errors observed over n periods.

2.3.2 The Mean Absolute Deviation (MAD)

The mean absolute deviation deals with the cancelling out problem by averaging the absolute value of the errors. Thus the MAD represents the average size of the errors irrespective of under forecasts or over forecasts. The MAD is a traditional and popular error measure in logistics and inventory control systems because it is easy to calculate and convenient to interpret. MAD can be defined as the following;

$$MAD = (1/n)\sum_{i=1}^{n}|e_i| \tag{2.12}$$

2.3.3 The Mean Squared Error (MSE)

The Mean Squared Error is obtained by averaging the squares of the forecast errors. This procedure eliminates the cancelling out problem similar to MAD. The MSE is the equivalent of the variance of the forecast errors and it is statistically appropriate to measure of errors. For a given item, it is generally used to compare the accuracy of various forecasting methods to find the best method that minimizes the MSE. It can be defined as,

$$MSE = (1/n)\sum_{i=1}^{n}e_i^2 \tag{2.13}$$

2.3.4 The Root Mean Squared Error (RMSE)

The Root Mean Squared Error is simply the square root of the MSE. It represents the standard deviation of the forecast errors for unbiased forecasts. The RMSE is more intuitive to interpret according to MSE. Formula of RMSE is defined as the following;

$$RMSE = \sqrt{(1/n)\sum_{i=1}^{n}e_i^2} \tag{2.14}$$

2.3.5 Mean Absolute Percentage Error (MAPE)

The absolute size of each forecast error can be expressed as a percentage of the actual demand and then these percentages are averaged in MAPE calculation. If a time series contain any pattern, forecasts are affected by the noise in the long run. So it is necessary considering the noise to be accurate in the long run. MAPE can be used especially for time series which draws any pattern. It is represented as the following;

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t-\hat{Y}_t}{Y_t}\right| \tag{2.15}$$

2.4 FORECASTING METHODS FOR NON-SMOOTH DEMAND

Accurate forecasted data can drive financial planning and production, distribution and logistics management, parts and service management (Lawrence et al, 2006). In operational planning, Bartezzaghi et al. (1999) argued that 'uncertainty reduction methods' are used for demand uncertainty by decision makers in forecasting demand. However, when demand has number of periods with no demand and the high variability of demand size many of these uncertainty reduction methods may perform weakly.

Time series methods are commonly used for regular demand data type in forecasting. In practice, exponential smoothing methods are often used in forecasting, with appropriate variants for the demand data which presents regular shape in its history (Boylan and Syntetos, 2008). Though, these forecasting methods are insufficient for non smooth demands and therefore forecasting this type of demand requires special techniques regarding the characteristics of the demand pattern in comparison with the smooth and continuous case. Forecasting techniques which are used for intermittent demand can be explained in details as the following.

2.4.1 Croston's method

The exponential smoothing and Croston methods are most frequently used for low and intermittent demand forecasting; in particular Croston's method is constructive and also significantly superior to exponential smoothing in case of the infrequent demand occurrence. Croston's method forecasts the non-zero demand size and the inter-arrival time between successive demands using exponential smoothing individually, with forecasts being updated only after demand occurrences. The inter arrival time is identified as the period between two consecutive non-zero demands (Croston, 1972).

Let:

Y (t) be the estimate of the mean size of a nonzero demand, P(t) be the estimate of the mean interval between nonzero demands, and Q be the time interval since the last nonzero demand. Croston forecasting method updates values of Y(t) and P(t) according to the following procedure;

```
        If $X(t) = 0$ then
$Y(t) = Y(t-1)$
$P(t) = P(t-1)$
$Q = Q + 1$
        Else
$Y(t) = \alpha X(t) + (1-\alpha)Y(t-1)$
$P(t) = \alpha Q + (1-\alpha)P(t-1)$
$Q = 1$
```

Figure 2.1 Croston's Algorithm

Croston's method assumes the estimate of mean demand per period as the following;

$$M(t) = \frac{Y(t)}{P(t)} \tag{2.16}$$

2.4.1.1 Syntetos and Boylan Variation of Croston's Method

Syntetos and Boylan pointed out that Croston's original method is biased (Syntetos and Boylan, 2001). They proved it as the following:

$$E(C_t) = E\left[\frac{Y_t}{P_t}\right] \approx \mu/p(1 + \frac{\alpha}{2-\alpha}.(\frac{p-1}{p})) \tag{2.17}$$

and, in particular, for α=1 that

$$E(C_t) = E\left[\frac{Y_t}{P_t}\right] = E\left[\frac{X_t}{Q_t}\right] = \mu\left[-\frac{1}{p-1}\ln\left(\frac{1}{p}\right)\right] \tag{2.18}$$

Based on (2.18) and ignoring the term p/p −1, Syntetos and Boylan proposed a new estimator given as below:

$$SB_t = \left(1 - \frac{\alpha}{2}\right)\frac{Y_t}{P_t} \tag{2.19}$$

2.4.1.2 Leven-Segerstedt Variation of Croston's Method

Levén and Segerstedt (2004) suggested a modification of Croston's method to obtain a method that works for both slow and fast moving items. They aim to reduce the bias indicated by Syntetos and Boylan, 2001. Their estimator is updated as follows:

$$LS_t = \alpha \frac{X_t}{Q_t} + (1-\alpha) LS_{t-1} \qquad (2.20)$$

They referred to the Syntetos and Boylan unbiased approach on the Croston's original method, but indicated that their estimator is not affected from such a bias.

2.4.1.3 Vinh's Variation of Croston's Method

The variation of Croston's method estimates the mean demand per period by applying exponentially weighted average forecasting method (Vinh, 2005). This approach similar to Croston's method on account of considering the non-zero demand size and the inter-arrival time between successive demands using exponential smoothing individually, but there is a significant difference due to the last two estimations are handled to update values of Y(t) and P(t) in forecasting. Figure (2.2) represents the procedure of this method. If the current demand is zero Y(t) and P(t) values are updated as the Croston's method otherwise if demand is nonzero these parameters are updated according to following equations different from the Croston's method.

If $X(t) \neq 0$ then

$$Y(t) = \alpha X(t) + \alpha(1-\alpha) Y(t-1) + (1-\alpha)^2 Y(t-2)$$

$$P(t) = \alpha Q + \alpha(1-\alpha) P(t-1) + (1-\alpha)^2 P(t-2)$$

$Q = 1$

Figure 2.2 Vinh's variation of Croston's method

$$\alpha + \alpha(1-\alpha) + (1-\alpha)^2 = 1 \qquad (2.21)$$

The estimate of mean demand per period;

$$M(t) = \frac{Y(t)}{P(t)} \tag{2.22}$$

2.4.2 Size Interval Method

Johnston and Boylan presented the Size-Interval method (SI) to meet intermittent demand requirements. The method was based on Croston's concept of building demand estimates from constituent elements. The demand arrival process though was assumed to be Poisson rather than Bernoulli and consequently the inter-demand intervals were taken as exponentially rather than geometrically distributed. SI was compared with EWMA on theoretically generated demand data over a wide range of different average inter-demand intervals, smoothing constant values and sizes of demand (Johnston and Boylan, 1996). According to these comparisons SI method is superior to EWMA for inter-demand intervals greater than 1.25 forecast revision periods.

2.4.3 Chua's Method

Chua's proposed method to forecast intermittent demand which exhibits non smooth behaviour is also a modification of Croston's algorithm. In Croston's algorithm, the historical demand is separated into two series, one representing the non-zero demand and other representing the inter arrival time. Croston's method cannot work well on lumpy data, which shows highly varied demand size and inter arrival time between two successive demands. The forecast will be biased and will tend to zero because there will be a lot of zeroes in the lumpy data. This method provides a solution to this kind of irregular demand type.

Chua et al. (2008) argued that some problems may arise because there are too many zeroes in the interval series to forecast the next inter arrival time. Instead of just knowing the next inter arrival period, he recommended that the forecaster should know when the next lump of demand will occur and how the length of the lump spans. Chua's proposed algorithm attempts to solve the problem of forecasting lumpy demand by separating the historical demand into three series instead of two. As in Croston's algorithm, one of the series contains the nonzero demands. The other two series contain the inter arrival time between lumps of demand and length of the lump spans. These series are used in the following equations:

The mathematical expression in the algorithm for lumpy demand is as the following:

$$\hat{p}_t = \alpha q_t + (1-\alpha)\hat{p}_{t-1} \tag{2.23}$$

$$\hat{s}_t = \alpha r_t + (1-\alpha)\hat{s}_{t-1} \tag{2.24}$$

$$L_t = \alpha y_t + (1-\alpha)L_{t-1} \tag{2.25}$$

$$T_t = \alpha L_t + (1-\alpha)T_{t-1} \tag{2.26}$$

$$\hat{z}_{t+k} = (2L_t - T_t) + (k+1)\frac{\alpha}{1-\alpha}(L_t - T_t) \tag{2.27}$$

Where

y_t refers to the t^{th} non-zero demand for period t;

$\hat{z}_t$ refers to the forecast for the $(t+1)^{th}$ non-zero demand;

r_t refers to the number of periods of the t^{th} lump of demand spans;

$\hat{s}_t$ refers to the forecast of the number of periods the $(t+1)^{th}$ lump of demand spans;

q_t refers to the interval between the $(t-1)^{th}$and t^{th} lump of demand;

$\hat{p}_t$ refers to the forecast for the interval between the the t^{th} and $(t+1)^{th}$ lump of demand;

Using Brown's exponential smoothing L_t and T_t refers to the current level estimate and trend estimate consequtively. Simple exponential smoothing was used by Chua on the interarrival time between lumps of demand and on the number of periods lumps of demand spans to get $\hat{p}_t$ and $\hat{s}_t$. Brown's exponential was then used for smoothing to get $\hat{z}_t$, $\hat{z}_{t+1}$,........, $\hat{z}_{t+\hat{s}_t}$ which represents $\hat{s}_t$ forecasts of non-zero demand. Consequently, the next lump of data will occur $\hat{p}_t$ periods later and lump of spans will take $\hat{s}_t$ periods with demand values of $\hat{z}_t$, $\hat{z}_{t+1}$,........, $\hat{z}_{t+\hat{s}_t}$.

2.4.4 MCARTA

In modelling non smooth type of demand, traditional methods are insufficient because of the many time periods with zero demands. In this regard, the two algorithm stages proposed by Rossetti and Varghese (2008) to generate demand include demand occurrence and determining nonzero demand sizes. The demand at time t, Y_t is modelled as a function of the squared coefficient of variation CV^2 ($Y_{t,NZ}$), lag 1 correlation coefficient of nonzero demand $\emptyset_{1,NZ}$ and probability of zero demand π_z. In order to combine the probability of zero demand, $\mathrm{X_t} \in \{0,1\}$ and $\{\mathrm{X_t}: t \in I^t\}$ is a stochastic process leading to the occurrence of demand in any period t and I^t is the set of times at which the demand is observed. The demand occurrence process was modelled as the following two state Markov chain process, where, $p_{ij} = P\{X_t = j|X_{t-1} = i\}$ represents the transition probabilities.

The Markov chain model decides whether the next state is zero demand or nonzero demand. If the state value randomly generated is 0, this causes a zero demand. In case of the state value randomly generated is 1, a nonzero demand is generated according to an assumed underlying nonzero demand process Y_{NZ}. The geometric distribution is selected because it is appropriate to generate non zero demand. Lag 1 correlation is initiated the demand process by the autoregressive to any algorithm (ARTA). The ARTA generates correlated demands $\mathrm{Y_{t,NZ}} \sim \mathrm{ARTA}\,(\mathrm{G(Y_{NZ})}, \emptyset_{1,\mathrm{NZ}})$ with correlation coefficient $\emptyset_{1,NZ}$ and geometric distribution. Thus demand characteristics such a correlation coefficient and coefficient of variation are protected. AR(1) has an underlying demand process with a normal noise $\sim Normal\,(0, (1 - \emptyset_{1,NZ}^2))$ in the autoregressive to any algorithm (Varghese, 2009).

The probability distribution of total demand is similar to normal distribution for smooth demand type in traditional statistical forecasting methods suppose that. It can, however, be said that it cannot fit any basic distribution exactly. Therefore, it is crucial to determine which distribution model is the best fit to this type of demand. Any parametric forecasting approach considers distributional fitting assumptions; however, the need for modelling free approach arises for the indefiniteness of the demand distribution. Non parametric method related to intermittent demand forecasting is given in details in the following section.

2.4.5 Non Parametric Forecasting by Bootstrapping

Having said that if the data become more irregular, the true demand size distribution may not fit with any theoretical distribution where the efficiency of any parametric approach is challengeable. It can be argued that further improvements can be achieved using non-parametric bootstrapping approaches when lumpy demand is met. Non-parametric bootstrapping approaches rely upon sampling randomly individual observations from the demand history to build a histogram of the lead time demand distribution. The seminal work of Efron (1979), was followed by considerable literature that dealt with bootstrapping approaches for forecasting irregular demand items (Snyder, 2002; Willemain et al., 2004; Porras and Dekker, 2008). It is worth noting that Willemain et al. (2004) claimed improvements in forecasting accuracy achieved over parametric approaches.

2.4.5.1 Willemain's Method

In order to simulate a whole distribution for lead-time demand rather a point forecast, a combination of the Markov process, bootstrapping and jittering is used by Willemain et al. (2004) as a modelling free approach in intermittent demand forecasting. After obtaining historical demand data, markov model predicts transition probabilities for two states. Then sequence of zero and nonzero values are generated according to Markov equations depend on the last observed demand. From the historical data non zero demand values are selected randomly to use for non zero states. Neighbourhood values of the non zero demand provide more variability in the demand data. Non zero demand values that have not been observed in the past obtained by the way jittering. To get lead time demand (LTD) forecast it is necessary to sum of the predicted values over the forecast horizon. LTD values are sorted and have found the related distribution.

2.4.5.2 Hua's Method

An integrated forecasting method (IFM) is developed by Hua et al. (2006) to predict auto correlated nonzero demands and presented two methods of assessing forecast methods. A first order Markov process is used in their forecasting method, first estimating the nonzero demand, and then the lead time distribution. A variable to adjust for nonzero demand is specified by the user, then a bootstrap to estimate the lead time distribution is used. Exponential smoothing, Croston method, and a modified bootstrap, which is the slightly same as the bootstrap method described by Willemain et al. (2004), were compared by their new method and performance measure.

2.4.6 Neural Networks

Neural networks link inputs and outputs adaptively in quantitative models during a learning process. The data is processed along different layers, while learning occur through adjustment of the weights connecting the units appearing in layers. The input and output layers' association is established at the final iteration. At this point in time, work in the area remains dynamic, led by cognitive statisticians and engineers.

It is interesting to note that ANNs have been recently recognised to solve many forecasting and decision making problems. As a matter of fact, time and effort has been spent for time series forecasting. Therefore, numerous ANN applications can be found in this area.

The relation between the independent and dependent variables can be underestimated by the traditional statistical time-series methods as pointed out by Hill et al. (1996). During the model building process, the underestimated relations are inflexible to modification. Let alone the fact that necessary data transformations can fail due to traditional methods. Biased estimates could be led to by the presence of outliers in data (Iman and Conovar, 1983). In this regard, Gutierrez et al. (2007) suggests recalibration on all previous data by the use of these models to capture the nonlinear pattern in data. To overcome these limitations, it can be argued that ANN modelling is a reasonable choice. It is worth noting that good approximations to any relationship can be provided by ANN models (White, 1992).

A Radial Basis Function (RBF) network was used by Carmo and Rodrigues (2004) to apply ANN modelling in their application of Gaussian radial or elliptical basis function networks aiming to generate better predictive performance than alternative models using irregular type of demand. The most widely used method, a multi-layered perceptron (MLP) trained by a back-propagation (BP) algorithm was adopted by Gutierrez et al. (2007) coming up with an evaluation of whether the ANN based approach is a superior alternative to traditional approaches for modelling and forecasting irregular demand.

Amin-Naseri and Rostami Tabar (2007) compared traditional methods with neural networks by applying generalized regression neural network (GRNN) and Elman recurrent neural network (RNN) for forecasting lumpy demand of spare parts. Their study shows that neural networks result is more accurate forecasting.

The latest study in this area of Nasiri Pour et al. (2008) based on the characteristic of lumpy demand patterns of spare parts that is a hybrid forecasting approach consisting of a multi-layered perceptron neural network and a traditional recursive method for forecasting future demands. In their study, the multi-layered perceptron neural network is used to forecast the existence of non-zero demands, and then a traditional recursive method is used to estimate the volume of non-zero demands. Their hybrid approach was compared to Syntetos and Boylan approximation and recently developed neural network approaches, namely: multi-layered perceptron neural network, generalized regression neural network, and recurrent neural network. Their approach resulted in more accurate forecasts than traditional methods.

2.5 PERFORMANCE MEASURES FOR NON-SMOOTH DEMAND

In evaluating the performance of forecasting methods in a supply chain, smaller forecasting errors for smooth demand could occur as opposed to non smooth demand data. The literature review shows two types of performance measures which are used for non smooth demand. The first type relies on the comparison per period forecast errors. In the second type, forecasts are used to determine inventory control parameters and compare the average inventory or service levels (Syntetos and Boylan, 2008).

If the demand shows more irregular pattern, the need for interpretable forecast errors may arise as well as measuring with high accuracy. Due to the difficulty of interpretation, relative performance measures that imply the forecast errors on a comparative basis are needed. As such, each error can be identified as a percentage of actual demand in this comparative process.

As far as this research is concerned, no studies found consistent outstanding performance using either Croston-type or traditional methods. It can be argued that Croston-type methods perform better on average (Willemain et al, 1994; Johnston and Boylan, 1996; Ghobbar and Friend, 2003; Regattieri et al, 2005; Boylan and Syntetos, 2006), however, some findings show that better results can be obtained using the traditional methods (Eaves, 2002; Eaves and Kingsman, 2004). The following section explains the performance measures which are developed for non smooth demand.

2.5.1 Modification of MAPE

MAPE is convenient to deduce for smooth demand as it defines accuracy as a percentage of the errors. In the following formula; n is the number of historical forecasts included in the error measure, $\boldsymbol{Y_t}$ is the observation at time t and $\boldsymbol{\hat{Y}_t}$ is the forecast of demand at time t. The difference between $\boldsymbol{Y_t}$ and $\boldsymbol{\hat{Y}_t}$ is divided by the actual value of $\boldsymbol{Y_t}$. Computing comes in handy in regards to the percentage error where it compares error of predicted time series that differ in time.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t-\hat{Y}_t}{Y_t}\right| \qquad (2.28)$$

As a matter of fact, intermittent demand has many times zero demand which results in a division by zero arising as a problem, therefore, the aforementioned measure is not suitable. An alternative measure is suggested by Makridakis and used by Makridakis and Hibon (2000) as a solution, namely; Symmetric MAPE (sMAPE):

$$sMAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|Y_t-\hat{Y}_t|}{|Y_t+\hat{Y}_t|/2} \qquad (2.29)$$

Syntetos (2001) and Boylan and Syntetos (2006) considered the sMAPE to be 200 for any period when the actual demand is zero, regardless of the error size. It can be argued that the sMAPE does not provide a satisfactory comparison of forecasting methods.

2.5.2 Mean Absolute Deviation to Average (MAD/A)

Due to the computing simplicity of the MAD measure, it is commonly used as defined below:

$$MAD = \frac{1}{n}\sum_{t=1}^{n}\left|Y_t - \hat{Y}_t\right| \tag{2.30}$$

Having said that the division by zero problem occurs in this measure as well, four alternatives have been suggested to surmount this problem, namely: the MAD/Average ratio, the geometric mean absolute error, percentage better method, and the mean absolute scaled error. MAD/A can be applied to non smooth demand as the following:

$$\frac{\mathrm{MAD}}{\mathrm{Average}} = \frac{\frac{1}{n}\sum_{t=1}^{n}\left|Y_t - \hat{Y}_t\right|}{\frac{\sum_{t=1}^{n} Y_t}{n}} \tag{2.31}$$

Hyndman observed that the MAD/A assumes that the data is constant over time. Since it does not consider seasonality, this measure may become undependable for seasonal non smooth data (Hyndman, 2006).

2.5.3 Geometric Root Mean Square Error (GRMSE)

Newbold and Granger (1974) used firstly the GRMSE in forecasting comparisons. The difficulty of sensitiveness to outliers of MSE is reduced when GRMSE is used as a relative measure (Fildes, 1992), however it is sensitive to zero error of any one estimation. Computation of this measure is straightforward as below;

$$GRMSE = \left(\prod_{i=1}^{n}\left|Y_t - \hat{Y}_t\right|\right)^{1/2n} \tag{2.32}$$

Where Y_t represents the demand at period t and $\hat{Y}_t$represents the forecast made for the period t, n is the number of the demand periods in the performance block.

2.5.4 Geometric Mean of the Arithmetic Mean of the Absolute Errors

Regardless of the problem size, the geometric mean (based on series) of the arithmetic mean (based on time) can be used considering mean absolute errors (GMAMAE) as the following:

$$GMAMAE = \left(\prod_{i=1}^{N}\left(\frac{1}{n_i}\sum_{t=1}^{n_i}\left|Y_{it} - \hat{Y}_{it}\right|\right)\right)^{1/N} \tag{2.33}$$

This measure is robust for large forecast errors provided that the remaining errors are stable. It is also not severely affected by trend or seasonality (Boylan and Syntetos, 2006).

2.5.5 Percentage Better (PB)

Percentage better method is an alternative that is straightforward to use as a measure for the non smooth demand data. Based on any performance criteria, one forecast method can be compared to another to reveal which method has the least error in each one of the series (Syntetos and Boylan, 2005).

2.5.6 Mean Absolute Scaled Error (MASE)

Hyndman (2006) suggested a new error measure for non smooth demand, known as the mean absolute scaled error (MASE), is formulated as follows:

$$\text{MASE}=\text{mean}(|q_t|) \tag{2.34}$$

$$q_t = \frac{Y_t-\hat{Y}_t}{\frac{1}{n-1}\sum_{i=2}^{n}|Y_t-Y_{t-1}|} \tag{2.35}$$

The errors are scaled relying on the MAD from the naive forecasting method. Hyndman and Koehler (2006) claimed that the measure is robust to outliers and is valid for all type of series.

2.5.7 Mean Unbiased Absolute Percentage Error (UAPE)

The modification of MAPE as an unbiased absolute percentage error (UAPE) is proposed by Makridakis (1993). Y_t refers to the actual demand at time index t, F_t refers to the forecast at time index t, and error at time index t, $\varepsilon_t=Y_t-F_t$. The mean of UAPE is given by the following equation:

$$MeanUAPE = mean\left(\frac{200|\varepsilon_t|}{(Y_t+F_t)}\right) \tag{2.36}$$

UAPE is unbiased in terms of the scale of the error. However, Collopy et al. (2000) suggest that positive and negative errors are dealt with in the same way in UAPE which makes it appropriate for non smooth demand type.

It can be deduced as such, in view of the foregoing, that conventional performance measures are insufficient for non smooth demand type which has irregular structure and frequent zero values. Hence, to obtain more reliable results, modification of conventional performance measures or new developed accuracy measures for the non smooth demand that is explained above can be used. Willemain (2004) suggest that the entire demand distribution should be attended by accuracy metrics as opposed to the quality of the point forecast only. However, comparing on item basis is impractical. In this regard, Willemain suggested a uniform distribution solution to deal with this problem. One way to look at the demand distribution is that it is needed to estimate errors in each demand level. In other words, it is necessary to know how often there was one unit of demand compared to how often a method forecasted one unit of demand.

On the other hand, relative error metrics which average the ratios of the errors from a current method to the errors of a naive method that can describe each error as a ratio to an average error from a baseline method (Syntetos and Boylan, 2008) of non smooth demand type.

2.6 CATEGORIZATION OF DEMAND DATA

An important issue in the operational management of the spare parts is the selection of the forecasting method that provides the highest degree of accuracy using historical demand data (Varghese and Rossetti, 2008). At this point, historical data can be divided into several categories to select the appropriate forecasting method for each category to facilitate decision making. Different categories of products are considered in regards to stock control policies and appropriate demand forecasting methods by Williams (1984). Then this categorizing demand notion is further explored by Eaves and Kingsman in the forecasting and inventory control (Kostenko and Hyndman, 2006).

2.6.1 Demand Data Categorization

Forecasting demand with the highest possible degree of accuracy remains a problem especially when demand data shows irregular form. Thus, there can be connection between the accuracy of forecasting method and the characteristic of demand data (Boylan and et al, 2008). It is essential to identify which forecasting method is superior for each demand type especially in cases where non smooth demand data are met. At this point, an enhanced overview of the number of SKUs to overcome is provided by the categorization schemes (Altay, 2011).

The characteristics of demand data are derived from two parameters: the average inter demand interval (ADI) and the coefficient of variation (CV). ADI measures the average number of time periods between two successive demands which indicates the intermittence of demand, while the CV indicates the erraticness of demand size variability. The formula of the coefficient of variation is:

$$CV = \frac{\sqrt{\frac{\sum_{i=1}^{n}(\varepsilon_{ri} - \varepsilon_{a})^{2}}{n}}}{\varepsilon_{a}} \tag{2.37}$$

Where n is the number of periods, and ε_{ri} and ε_{a} are the actual and average demand for spare parts in period i, respectively. Demand patterns are classified into four categories as considered from the occurrence of both the pattern and the size of demand. In all cases, x indicates the cut off value (ADI = 1.32) for ADI, which measures the average number of time periods between two consecutive demands, and y indicates the corresponding cut off value ($CV^2 = 0.49$) for CV^2, which is equal to the square of the standard deviation of the demands divided by the average demand.

2.6.2 The Specification of Demand Patterns

In order to determine the most appropriate forecasting method, issues of the specification and categorisation of non smooth demand patterns are necessary. To compose areas of superior performance, forecasting methods are compared, then the demand patterns are categorised based on the results (Johnston and Boylan, 1996). The following sections define demand patterns as from the literature.

2.6.2.1 Smooth Demand

When the average inter demand interval is less than 1.32 and the coefficient of variation is less than 0.49, this demand data is categorised as slow demand according to the scheme of Syntetos et al. (2005). This irregular type of demand has slow movement due to low average demand per period. Therefore, it can be modelled as a Bernoulli process if time is treated as discrete and a Poisson process (Syntetos, 2001). Although it is not irregular as intermittent and erratic that caused difficulties of forecasting and inventory control, normality assumption is not available. Williams (1984) identified that slow moving items which are associated with low sporadicity and average demand during the lead time are less than ten units.

2.6.2.2 Intermittent Demand

When the average inter demand interval is bigger than 1.32 and the coefficient of variation is less than 0.49, this type of demand is defined as intermittent demand. This type of demand appears to be random, with many time periods having no demand but not highly variable. Silver (1998) defined intermittent demand as ‘infrequent in the sense that the average time between consecutive transactions is considerably larger than the unit time period, the latter being the interval of forecast updating’. Johnston and Boylan (1996) defined intermittent demand as the inter-demand interval greater than 1.25 inventory review periods.

2.6.2.3 Lumpy Demand

Lumpy demand has many time periods having no demand. However, when the demand occurs, high variable demand sizes occur. Vereecke and Verstraeten (1994) defined lumpy demand items as “items whose demand frequency is less than 4 times a year”.

Inter-demand interval and the coefficient of variation of demand sizes are needed to specify the demand item as lumpy. The condition of $ADI > 1.32$ and $CV^2 > 0.49$ imply the existence of lumpy demand. According to Altay (2011), it can be said that due to excessive stocks and low customer service levels, lumpy demand reveals the biggest challenge for spare parts in forecasting and inventory management.

2.6.2.4 Erratic Demand

Erratic demand has highly variable demand sizes when demand occurs and low number of time periods between two consecutive demands. The conditions of $ADI \leq 1.32$ and $CV^2 > 0.49$ examines whether items are being erratic or not. Silver (1970) defined an erratic item as “one having primarily small demand transactions with occasional very large transactions”.

2.6.3 Demand Categorization Schemes

In 1984, Williams initiated the idea of categorizing demand patterns and studied the classification of products according to demand type, and appropriate methods of forecasting demand for different categories of products. Similarly, a number of authors explored the demand categorization approach using schemes that are generated with rules in order to select convenient forecasting method and inventory control parameters. The following section presents the studies on categorizing demand data as from the literature.

2.6.3.1 Williams Categorization Scheme

Williams (1984) defined two parameters to classify demand that rely on variance partition. Component parts of the lead time variance are: variance of the order sizes, transaction variability, and variance of the lead-times with assumptions that random demand arrivals and constant lead times (Altay, 2011). According to his equation;

$\frac{CV^2(x)}{\lambda \bar{L}}$ represents how the squared coefficient of variation of demand during lead time. It is related to lumpiness of the demand data.

$\frac{1}{\lambda \bar{L}}$ represents the number of lead times between consecutive demands meaning that how often demand occurs or show intermittency degree.

This categorization approach aims to find the most appropriate forecasting method and inventory control for each category. The property of lumpiness depends on the variability of the demand size when demand occurs. As such, the degree of lumpiness that the data has is shown by the value of the cut-off values are chosen based on the underlying empirical data set. For this reason this approach’s validity for all data set might not be continued.

2.6.3.2 Johnston and Boylan Categorization Scheme

Johnston and Boylan (1996) suggested new notion to categorize demand items using comparisons of some methods such as Croston and EWMA (Exponentially Weighted Moving Average). When the average demand interval is greater than 1.25 forecasting periods, Croston method that is developed for intermittent demand give more accurate results than EWMA method. Therefore, one of the property of intermittent demand is determined with the term of average demand interval.

2.6.3.3 Eaves Categorization Scheme

Eaves (2002) defined a new categorization of intermittent demand scheme using three measures: transaction variability, demand size variability, and lead time variability which are similar to Williams' categorization. The cut-off values are determined from the data set.

2.6.3.4 Syntetos and Boylan Categorization Scheme

Syntetos and Boylan (2005) categories of demand depend on mean square errors of each forecasting methods. They developed a method to reduce biasness effect of Croston's method, and defined four categories of demand, namely: erratic, lumpy, smooth, and intermittent. Their scheme is based on the mean inter-demand interval (p) and the squared coefficient of variation of demand sizes when demand occurs (v). Their comparison between the mean standard error performances of Croston's method, EWMA, and the SBA suggested ($p = 1{,}32$ and $v = 0{,}49$) as cut off values (Syntetos and Boylan, 2001). Validity of this approach was tested using 3000 SKU which belong to the automotive industry. SB revealed that smooth demand is best forecasted using Croston method and SB is appropriate for the other areas. This approach identified CV^2 that is a new parameter in categorizing the demand concept. This scheme is represented in Figure (2.3).

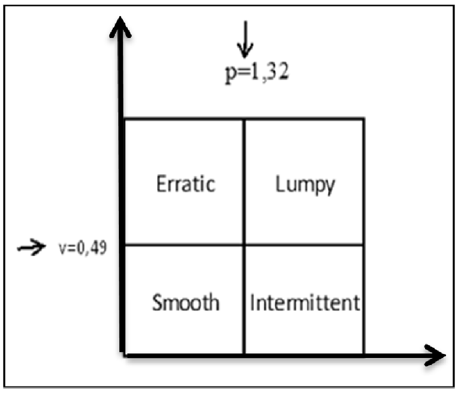


Figure 2.3 Syntetos and Boylan scheme (2005)

According to their approach, forecasting methods for each category are determined in the following table.

Table 2.1 Appropriate forecasting methods for each demand categories

Demand Pattern Condition	Demand Type	Appropriate forecasting method
ADI $\leq$ p; $CV^2 > v$	Erratic	Syntetos and Boylan
ADI> p; $CV^2 > v$	Lumpy	Syntetos and Boylan
ADI $\leq$ p; $CV^2 \leq v$	Smooth	Croston
ADI > p; $CV^2 \leq v$	Intermittent	Syntetos and Boylan

It is both the number of non zero demand occurring periods and the size of demand that are taken into consideration in order to classify demand as lumpy or slow. If demand has not been classified as lumpy or slow then it falls into the “normal” category. In this last case, if the variability of the demand sizes is greater than the specified cut-off value, the SKU is defined as erratic. Therefore, it is mainly the variability of the demand sizes that is considered in order to define an SKU as erratic.

2.6.3.5 Boylan, Syntetos and Karakostas Categorization Scheme

Boylan et al. (2008) suggested an expansion of the categorization in Syntetos et al. (2005) using different theoretical statistical distributions to implement stock control policies. It can be argued that the categorization is a best-practice approach rather than an analytical rule where the stock control policies are assigned to each sub-group based on professional experience. Boylan et al. (2008) determined to rearrange the cut-off value of CV^2 from 0.49 to 0.32 as almost 50 percent of the first data set had zero variance of demand size. They are called slow instead of intermittent as in Syntetos et al. (2005), and the remnants are named lumpy.

2.6.3.6 Rossetti and Varghese Categorization Scheme

Demand type is characterized by intermittence which is how often a transaction (or a non-zero demand) occurs and lumpiness which the variability of the non-zero demand. The mean interval between transactions μ_I or probability of zero demand π_Z is intermittence. The squared coefficient of variance CV^2_{NZ} of non-zero demand is lumpiness (Williams, 1984; Syntetos, 2001; Syntetos et al., 2005a; Boylan et al., 2008; Eaves, 2002). These authors developed a demand classification scheme that is based on the intermittence and lumpiness of the demand. Intermittence and lumpiness both introduce variability into the demand process. Willemain et al. (1994) pioneered the consideration of correlation between consecutive demand sizes and consecutive demand intervals. Varghese and Rossetti (2008) also considered lag 1 correlation between consecutive non-zero demands.

Varghese (2009) described bursty demand in their categorization scheme that consecutive periods of non zero demand is following when there is no demand or a demand occurs.

It can be deduced from the literature that several demand scenarios often overlap with intermittent demand, namely: bursty demand, lumpy demand, slow demand, sporadic demand, erratic demand, irregular demand which are illustrated as in Figure (2.4).

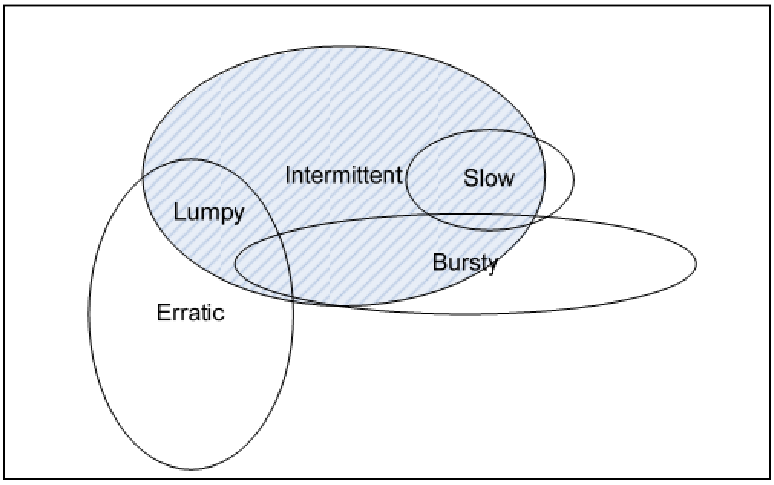


Figure 2.4 Rossetti and Varghese categorization scheme (2009)

2.7 INVENTORY CONTROL FOR NON SMOOTH DEMAND

Since the seminal work by Croston in 1972 demand forecasting and stock control have been examined independently. In spite of the weaknesses, limited theoretical and empirical research has been conducted on their interaction. As far as inventory management as concerned, the light is shed on the control of inventories, assuming that an appropriate forecasting method is in place to estimate future demand requirements (Silver, 1970; Ward, 1978; Schultz, 1987; Watson, 1987; Segerstedt, 1994). On the other hand, as far as intermittent demand forecasting is concerned, stock control does not determine the consequences of employing specific estimators. A limited number of researchers shed light on forecast accuracy that it is to be distinguished from the stock control performance of the estimators utilised (Sani and Kingsman, 1997; Eaves and Kingsman, 2004; Syntetos and Boylan, 2006 and Strijbosch et al., 2000). As far as performance measurement is concerned, Boylan and Syntetos (2006) considered the importance from a practitioner perspective in regards of capturing the combined forecasting stock control operation through service level and inventory costs related metrics.

Recent empirical investigations on the performance of various intermittent demand forecasting approaches were conducted by Willemain et al. (2004) and Syntetos and Boylan, (2005). Boylan and Syntetos (2006) argued that “no matter what inventory system is in use, the accuracy-implication metrics of stock-holding costs and service-level should always be used, since this is of prime importance to the organisation. These measures should be used not only when it is difficult to assess forecast error directly. By keeping an inventory method fixed, accuracy-implication metrics offer a direct comparison of the effects of using different forecasting methods.” Thus, stock-holding cost and service level measures are of utmost importance in assessing the performance of an inventory management system.

Teunter and Duncan (2009) used six methods of forecasting demand by evaluating a new performance measure that compares target to achieved service level, they showed that the original Croston’s method as well as the Syntetos & Boylan and the Levén & Segerstedt variants outperform moving average and exponential smoothing.

CHAPTER 3

DEMAND DATA CATEGORIZATION

Forecasting and stock control requires the categorization of demand data for decision making. Categorizing the relevant items is of utmost importance to the management of spare parts in order to apply the most appropriate forecasting method. Using a suitable forecasting approach determined by demand data categorization may increase spare parts availability and also reduce inventory costs.

In this study, the data set is described based on its historical demand patterns to classify them in consideration of Syntetos scheme. The application of the classification scheme considers the mapping of the demand items. Using categorized demand data, appropriate forecasting methods which are developed for non smooth data can be identified for each demand pattern.

3.1 SPARE PART DEMAND DATA

In this study, a demand categorization approach is applied to a real data set. The data set contains 90 SKU items, have irregular pattern, chosen from the Turkish Airlines inventory spare part items and each item's demand values covers a period of 62 months from 2005 to 2010. These spare parts were selected among others for the diversity of their demand variability and various numbers of periods with no demand. According to Syntetos categorization scheme, cut off values such as average inter demand interval and coefficient of variation is used in this study to categorise demand data set. The cut off value for average demand is 1, 32 and the coefficient of variation is 0, 49. In order for the data to start with non-zero values, eliminating the beginning zero values of the data set is necessary as the first step. Otherwise, the leading zeroes may provide inaccurate information about the first inter arrival interval because there is no information about it. In this context, inter arrival time is the period between two consecutive non-zero. Because the next data value is not known whether it is a non-zero or not, the next inter arrival time has to be forecasted from the last non-zero value based on the previous values of inter arrival time.

3.2 DATA ANALYSIS

Descriptive statistics of the data set is represented in the following table.

Table 3.1 Descriptive statistics of the data set

	Demand					Demand Size					Interdemand Interval				
90 Series	Mean	SE Mean	TrMean	Variance	CoefVar	Mean	SE Mean	TrMean	Variance	CoefVar	Mean	SE Mean	TrMean	Variance	CoefVar
Minimum	0,2742	0,0616	0,2321	0,2351	63,53	1,05	0,05	1	0,05	21,3	1,0167	0,0167	1	0,0167	12,7
Lower Quartile	0,605	0,124	0,518	0,957	130,11	1,61725	0,1795	1,51575	0,79875	46,6	1,6205	0,20725	1,38325	1,3515	57,0825
Median	1,46	0,3035	1,25	5,713	160,19	2,782	0,402	2,3005	5,39	65,485	2,107	0,322	1,8775	2,594	72,77
Upper Quartile	3,81425	0,798	3,419	39,475	187,385	5,78725	0,898	5,4925	33,22225	92,5125	2,95825	0,52975	2,51125	4,97575	82,485
Maximum	1734	337	1473	7026982	361,34	1955	369	1684	7498492	185,02	3,8	1,35	3,615	28,56	178,12

3.3 CATEGORIZATION OF SPARE PART DEMAND

After trimming the data, inter demand interval values as a number of time periods between two successive non zero values are computed. The average of these computed values gives the ADI value of data which is compared with the cut off value (ADI = 1.32). Coefficient of variation value which measures nonzero values deviation in the historical data is computed for each data series and compared with the corresponding cut off value ($CV^2 = 0.49$). To categorize each series the following Syntetos' procedure is used as the following;

- The condition ADI ≤ 1,32; $CV^2 \leq 0,49$ tests for stock keeping units (SKUs), which are not very intermittent and erratic (i.e. faster moving parts, or parts whose demand pattern does not raise any significant forecasting or inventory control difficulties);

- The condition ADI > 1,32; $CV^2 \leq 0,49$ tests for low demand patterns with constant, or more generally not highly variable, demand sizes (i. e. not very erratic);

- The condition ADI> 1,32; $CV^2 > 0,49$ tests for items with lumpy demand. Lumpy demand may be defined as a demand with large differences between each period's requirements and with a large number of periods having zero requests;

- The condition ADI ≤ 1,32; $CV^2 > 0,49$ tests for items with erratic (irregular) demand with rather frequent demand occurrences (i. e. not very intermittent).

After the categorization the number of series for each demand types are computed. The results are given in the following table.

Table 3.2 Number of data series according to demand categories

Demand Pattern Condition	Demand Type	Number of data series
ADI ≤ 1,32; CV2 > 0,49	Erratic	11
ADI> 1,32; CV2 > 0,49	Lumpy	30
ADI ≤ 1,32; CV2 ≤ 0,49	Smooth	4
ADI > 1,32; CV2 ≤ 0,49	Intermittent	45

The following Figure represents the data set plotted in Minitab software;

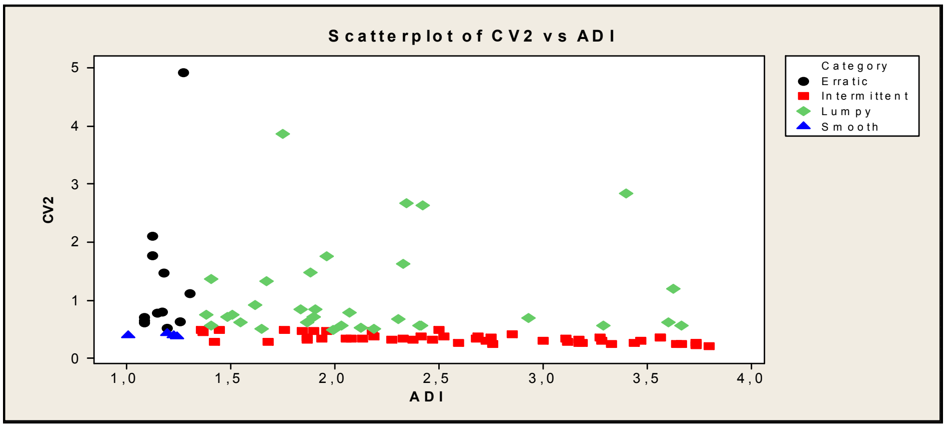


Figure 3.1 Demand Data Pattern in Minitab Output

Most of data series shows intermittent pattern as can be seen in the aforementioned figure. The following figure represents a sample of each demand category (intermittent, lumpy, erratic, smooth) based on biennial demand history for the Turkish Airlines service parts.

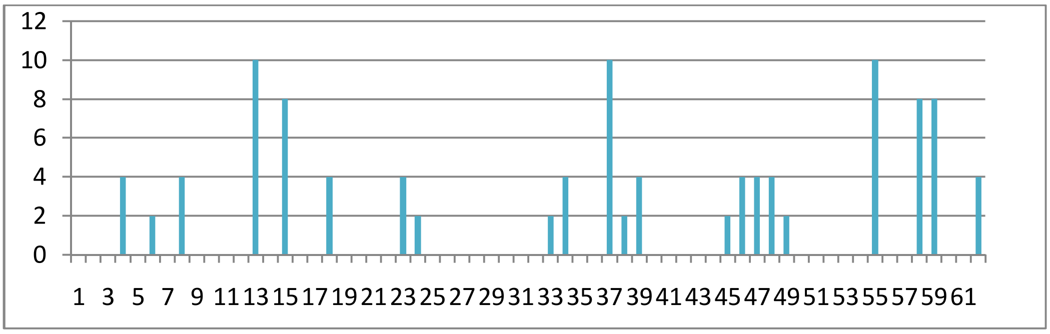


Figure 3.2 An example of Intermittent data

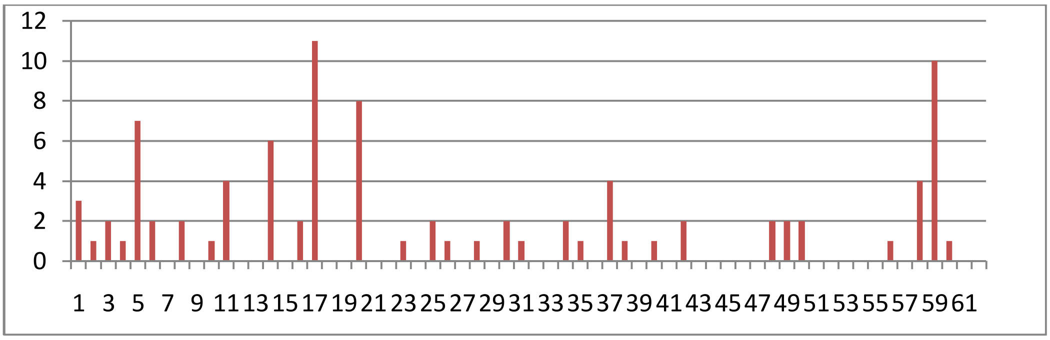


Figure 3.3 An example of Lumpy data

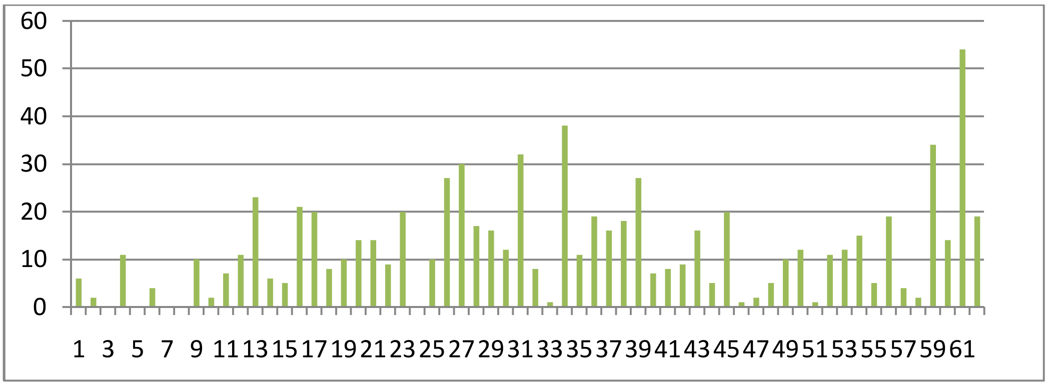


Figure 3.4 An example of Erratic data

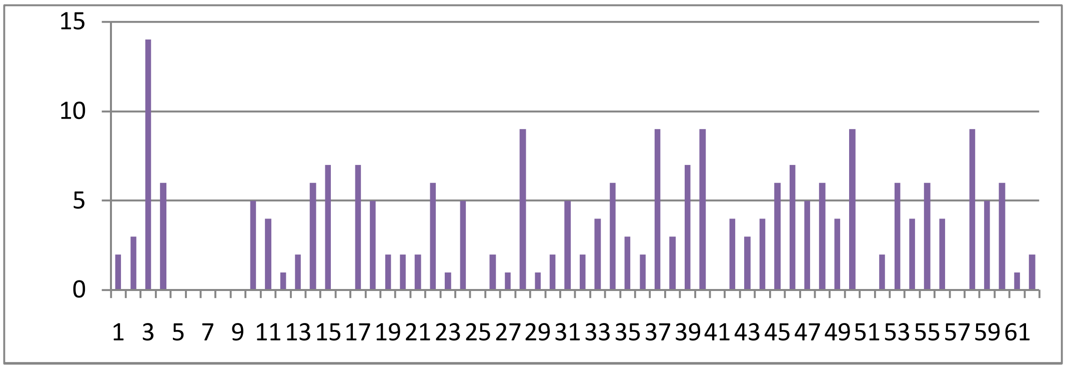


Figure 3.5 An example of Smooth data

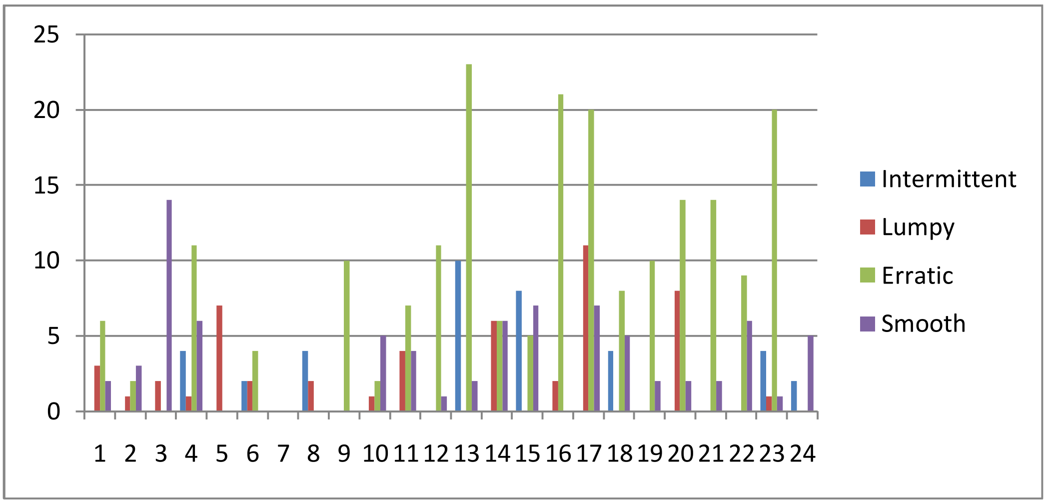


Figure 3.6 Each demand pattern examples based on the data set

In view of the foregoing, categorization rules comprise a crucial element for forecasting and inventory management. Limited work has been conducted in this area. The question remains, how demand patterns should be classified for operational management. More attributes should be extracted from the data set to exhibit properties of demand data. For this reason, more detailed attributes are determined in Appendix A.1 to guide practitioners about which attributes of the demand combination could fully describe the characteristic of demand. After that, specific forecasting method should be employed depending on these characteristics of the demand pattern.

CHAPTER 4

FORECASTING SPARE PARTS DEMAND

Exponential smoothing methods and variations are often used for smooth demand pattern, as well as to forecast spare parts requirements (Snyder, 2002). Exponential smoothing methods, however, place more weight on the most recent data. As such, it underestimates the size of the demand when it occurs and it overestimates the long term average demand. Besides, biased forecasting methods cause unreasonably high stocks. Different methods are needed for non-smooth demand pattern which show no demand for some periods. Therefore, forecasting methods which are especially developed for non-smooth demand data were applied in this study for selected non smooth demand data from airline industry.

The data set that includes 90 items and covers 62 months from 2005 to 2010 represents demand data of stock keeping items in monthly inventories. This data set is selected from 321 data series to analyze, because it met the requirement that it contains greater than or equal to 15 non-zero values. Traditional forecasting methods which are developed for non smooth demand patterns are applied to this data set. In this chapter, selected forecasting methods were performed using constant and optimum alpha parameters as explained below.

4.1 FORECASTING USING CONSTANT ALPHA PARAMETER

The data set had been classified as intermittent, lumpy, erratic and smooth in the previous chapter. According to this data categorization, eight forecasting methods which are convenient for non smooth demand patterns were applied to this data set. Firstly, the length of initialization block is determined as the first 12 periods of data to initialize values required for methods based on recursive formula (such as the mean inter-demand interval for Croston's method). Subsequently, alpha value is obtained as 0.2 to update forecasts in each forecasting technique. Finally, the last 12 data are used in the 'performance block', in which performance statistics are calculated with constant alpha value (0.2) considering different accuracy measures.

4.2 FORECASTING USING OPTIMUM ALPHA PARAMETER

In this model the demand data series have been divided into four blocks. The first 12 data is used for the initialization stage (i=12). Then, periods of data from 13 to 50 were selected for calibration (c=38) purpose. Finally, after the updating and optimizing procedures, the last 12 data (p=12) were used for measuring performance of the related forecasting method. This procedure is explained in the following Figure 4.1.

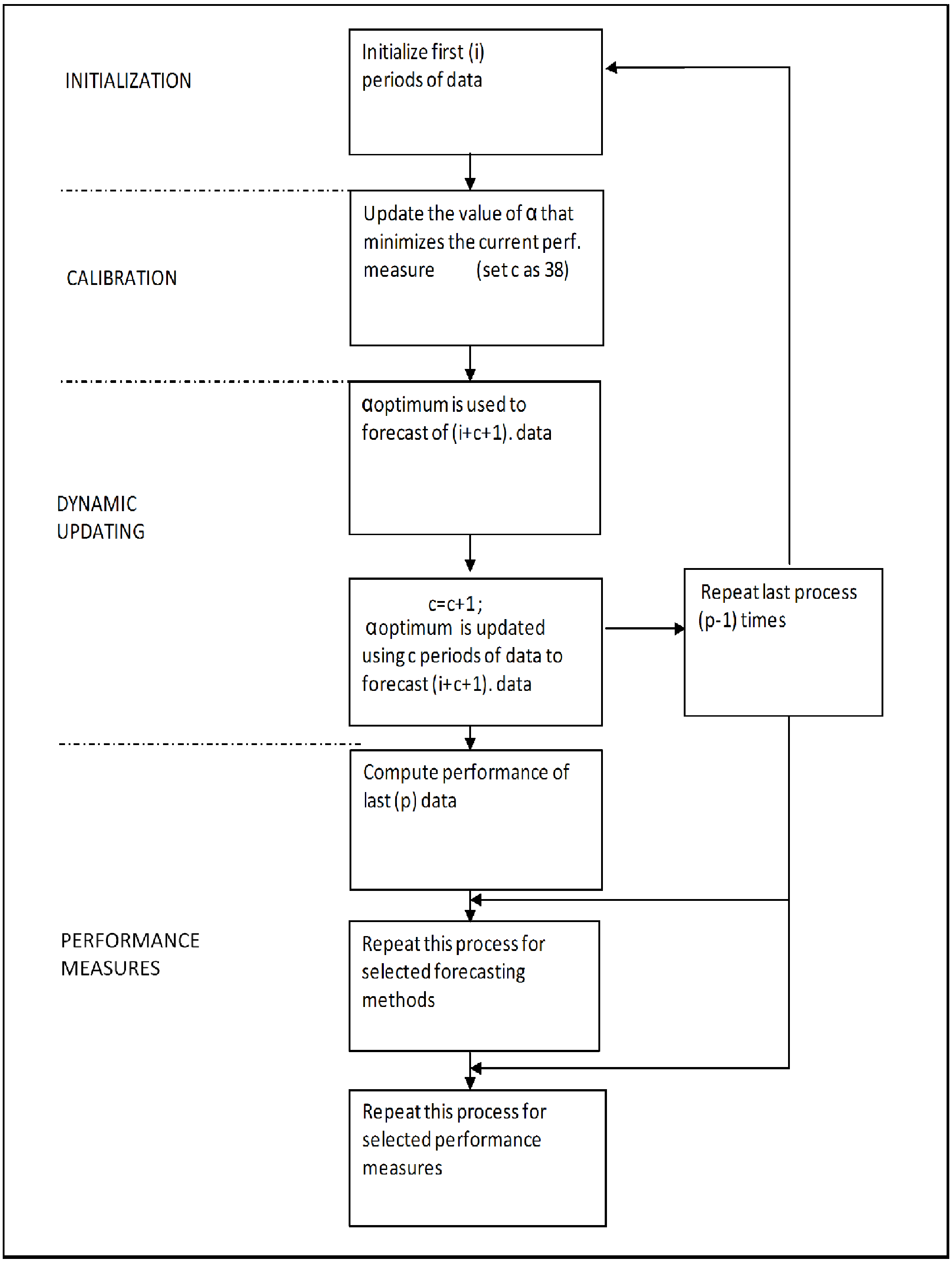


Figure 4.1 Forecasting Using Optimum Alpha Parameter

(i) initialization: First i periods of data is used for the initialization purpose with α value is 0.2 (i=12). Average of non zero demand sizes and intervals are obtained from the first “i” period individually to initialize.

(ii) calibration: The value of α that minimizes the current performance measure can be updated by SOLVER in Excel using the c period of data ($c_{initial}$=38)

(iii) dynamic updating of performance block: $\alpha_{optimum}$ which is found in calibration is used to forecast the (i+c+1) data in particular. Then set c=c+1 and $\alpha_{optimum}$ is updated using c periods of data to forecast the (i+c+1) data in particular. The $\alpha_{optimum}$ value is updated to minimize based on the current performance measure by SOLVER considering c period which is increased by one in each step of updating data to forecast. Overall, it can be said that SOLVER optimizes (p-1) times of the α value.

(iiii) performance measure block: In this step, performance of last (p) data is computed according to current performance measure until this process is repeated for selected intermittent demand forecasting methods are completed. Finally, all the process is repeated for each performance measure.

In this model, optimum α is used instead of using fixed values for α as in earlier studies (i.e 0.2). The new approach is evaluated by considering eight established intermittent demand forecasting methods. The comparison is performed on 90 real demand data series from the airline industry. Various performance measures have been explained in Chapter (2), It is worth noting that RMSE, MADA, MASE, GRMSE and GMAMAE are used for this study and are employed to compare forecasting methods.

4.3 EMPLOYED DEMAND FORECASTING METHODS

This research come up with two modification of Croston method and they are compared to current intermittent demand forecasting methods utilizing aforementioned two model approach. Modification of Croston methods which are included in the comparative study, as follows:

4.3.1 Proposed Syntetos' adjustment to Vinh's variation of Croston's Method

Syntetos' adjustment coefficient is applied to Vinh's variation of Croston's method in order to eliminate biasness affect on this variation. This adjustment of Vinh's variation works as in Figure (4.2).

```
If $X(t) = 0$ then
    $Y(t) = Y(t-1)$
    $P(t) = P(t-1)$
    $Q = Q + 1$
Else
    $Y(t) = \alpha X(t) + \alpha(1-\alpha)Y(t-1) + (1-\alpha)^2 Y(t-2)$
    $P(t) = \alpha Q + \alpha(1-\alpha)P(t-1) + (1-\alpha)^2 P(t-2)$
    $Q = 1$
With $\alpha + \alpha(1-\alpha) + (1-\alpha)^2 = 1$
```

Figure 4.2 Vinh's variation of Croston method

Let:

Y (t) be the estimate of the mean size of a nonzero demand, P(t) be the estimate of the mean interval between nonzero demands, and Q be the time interval since the last nonzero demand as mentioned in Section 2.4.1.

The estimate of mean demand per period;

$$M(t) = \left(1 - \frac{\alpha}{2}\right)\frac{Y(t)}{P(t)} \tag{4.1}$$

4.3.2 Two Parameter Croston

According to the seminal work of Croston (1972) the sequence of intermittent demands converts into two constituent sequences (of inter-order interval durations, and the quantum of non-zero demands) and applies SES to each series separately, with an update to forecast parameters being done only when a non-zero demand occurs.

It is, however, worth noting that the algorithm developed by Croston does not clearly specify how the estimated demand is to be generated once all the necessary parameters are available. Therefore, Croston's method with some modification is implemented. First, some notation upon which the algorithm based on Croston's method is introduced as the following.

α: Smoothing constant for non-zero demand

β: Smoothing constant for inter-order interval

Y_t: Forecast of non-zero demand in period t

M_t: Forecast of demand in period t

P_t: Forecast of the inter-order interval in period t

Q: Time elapse since the last non-zero demand was observed in periods. This is used to update the estimated inter-order interval each time the non-zero demand occurs.

α : smoothing constant for demand size

β: smoothing constant for demand interval

```
        If $X(t) = 0$ then
$Y(t) = Y(t-1)$
$P(t) = P(t-1)$
$Q = Q + 1$
        Else
$Y(t) = \alpha X(t) + (1-\alpha)Y(t-1)$
$P(t) = \beta Q + (1-\beta)P(t-1)$
$Q = 1$
```

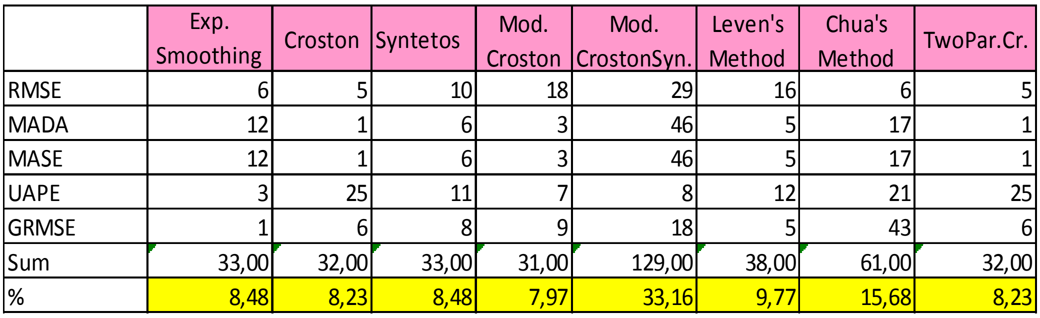
Figure 4.3 Two Parameter Croston

4.4 RESULTS

Intermittent demand forecasting methods (Exponential smoothing, Croston, Syntetos, Modified Croston, Leven's and Chua's method) and the modification of Croston methods (ModCrostonSyn.,Two parameter Croston) are compared regarding different performance measures.

The following two tables represent the output of eight forecasting methods according to four accuracy measures when considering constant alpha model. In Table 3, we can see that the modified Croston Syntetos variation is the best based on being approximately 33% which is the highest overall percentage of the sum of accuracy measures. In Table 4, modified Croston Syntetos variation has the biggest score which emphasises that it is the first ranked method that gives best result based on average of accuracy measures. Table (4.1) represents the overall winning percentage of each forecasting method based on accuracy measures for the constant alpha model.

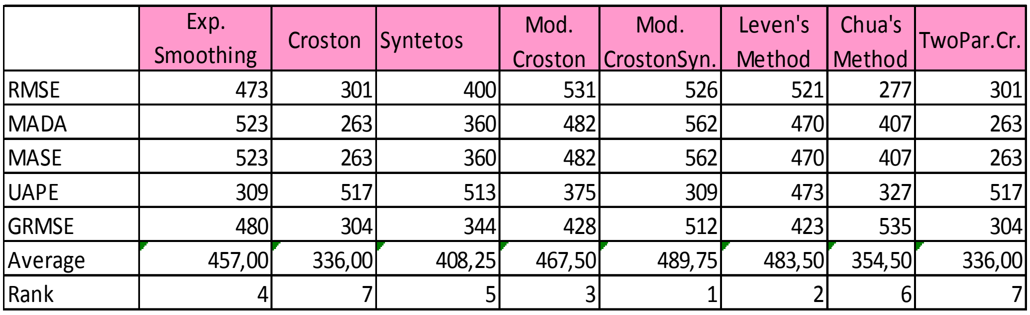
Table 4.1 Winning Percentages of Forecasting Methods based on different Accuracy Measures



	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
RMSE	6	5	10	18	29	16	6	5
MADA	12	1	6	3	46	5	17	1
MASE	12	1	6	3	46	5	17	1
UAPE	3	25	11	7	8	12	21	25
GRMSE	1	6	8	9	18	5	43	6
Sum	33,00	32,00	33,00	31,00	129,00	38,00	61,00	32,00
%	8,48	8,23	8,48	7,97	33,16	9,77	15,68	8,23

Scores of each forecasting method based on accuracy measures for constant alpha model are represented in Table (4.2).

Table 4.2 Scores of Forecasting Methods based on different Accuracy Measures for Constant Alpha



	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
RMSE	473	301	400	531	526	521	277	301
MADA	523	263	360	482	562	470	407	263
MASE	523	263	360	482	562	470	407	263
UAPE	309	517	513	375	309	473	327	517
GRMSE	480	304	344	428	512	423	535	304
Average	457,00	336,00	408,25	467,50	489,75	483,50	354,50	336,00
Rank	4	7	5	3	1	2	6	7

The following two tables represent the output of eight forecasting methods according to four accuracy measures when considering optimum alpha model. In Table (4.3), we can see that Leven and Segerstedt method is the best one where it is approximately 34% higher in regards to overall highest percentage based on sum of accuracy measures. In Table (4.4), Leven and Segerstedt method has the highest score which emphasises that it is the first ranked method which gives best result based on average of different accuracy measures.

Table 4.3 Highest Percentages of Forecasting Methods based on Accuracy Measures for Optimum Alpha

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
RMSE	15	2	10	7	9	26	10	11
MADA	13	2	7	3	10	34	16	5
MASE	7	0	4	0	2	74	3	0
GRMSE	10	7	13	3	8	35	9	5
Sum	35,00	4,00	21,00	10,00	21,00	134,00	29,00	16,00
%	9,00	1,03	5,40	2,57	5,40	34,45	7,46	4,11

Scores of each forecasting method based on accuracy measures for optimum alpha model is represented as the following Table (4.4)

Table 4.4 Scores of Forecasting Methods based on Accuracy Measures for Optimum Alpha Model

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
RMSE	471	384	400	423	401	439	431	291
MADA	465	366	371	361	409	506	447	243
MASE	516	279	413	284	427	651	480	189
GRMSE	501	335	402	349	329	565	476	283
Average	484,00	343,00	394,67	356,00	412,33	532,00	452,67	241,00
Rank	2	7	5	6	4	1	3	8

The overall highest percentage of each forecasting method based on accuracy measures for constant alpha and optimum alpha model is represented in Table (4.5).

Table 4.5 Overall Winning Percentage of Forecasting Methods

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
0,20	8,48	8,23	8,48	7,97	33,16	9,77	15,68	8,23
Optimum alpha	9,00	1,03	5,40	2,57	5,40	34,45	7,46	4,11

Scores of forecasting methods when α value is 0.2 and optimum are represented in Table (4.6).

Table 4.6 Scores of Forecasting Methods

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven's Method	Chua's Method	TwoPar.Cr.
0,20	457	336	408	468	490	484	355	336
Optimum alpha	484	343	395	356	412	532	453	241

In Table (4.7), geometric mean (across series) of the arithmetic mean (across time) of the absolute errors (GMAMAE) of forecasting methods are represented.

Table 4.7 Overall comparisons of forecasting methods based on GMAMAE considering different α values

Forecast Metd α	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. Croston Syn.	Leven's Method	Chua's Method	TwoPar.Cr.
0,2	2,2322	2,9599	2,7966	2,3675	2,3063	2,3604	2,2755	2,9599
Optimum alpha	2,1846	2,5166	2,4572	2,5011	2,5289	2,1426	2,1542	2,93

RMSE results of optimum alpha and constant alpha are converted to normalized values which is ranged between 0 and 1. This procedure is applied by using the following normalization formula;

$$N = \frac{X - X^{min}}{X^{max} - X^{min}} \qquad (4.2)$$

In this formula X^{min} and X^{max} are the minimum and maximum values of row respectively. X is transformed into range of [0, 1] using this equation. If $X = X^{min}$, then N=0. If $X = X^{max}$, then N=1. In the following table, the results of a comparison between optimum alpha and constant alpha according to normalized values are presented in Table (4.8).

Table 4.8 Average of Normalized RMSE values using optimum and constant alpha values

Methods	Exp.Smoothing	Croston	Syntetos	Mod.Cr.	Mod.Cr.Syn.	Leven	Chua's Metho	TwoPar.Cr.
Normalized Optimum α Avg	0,231435984	0,341243	0,328569	0,306024	0,35439142	0,336074	0,339946071	0,602464
Normalized Constant α Avg	0,215665211	0,689486	0,487895	0,217252	0,22100189	0,170846	0,632381006	0,6894862

The data set have been categorized in Chapter (3) as 45 numbers of intermittent data, 30 numbers of lumpy data, 11 numbers of erratic data and 4 numbers of smooth data. Allocation of each data types can be seen in Figure (4.4).

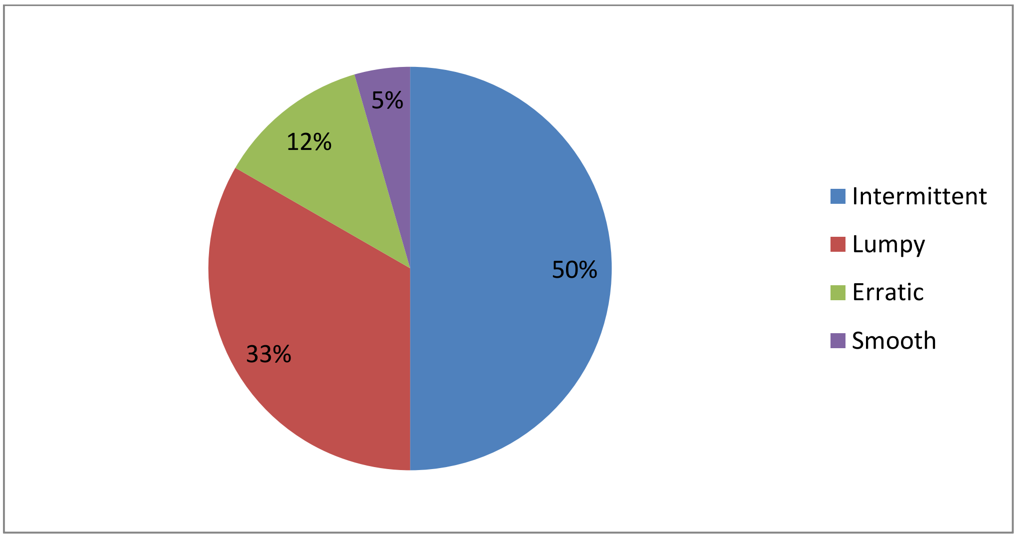


Figure 4.4 Allocation of data types that are employed

In Table (4.9), average scores of each data type according to forecasting methods based on RMSE performance measure are represented.

Table 4.9 Comparisons of forecasting methods based on RMSE for each demand data type

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven	Lumpy	TwoPar.Cr.
Intermittent	5,333	4,356	4,444	4,6444	4,667	4,8667	5,08889	2,6
Lumpy	5,233	4,167	4,333	4,6	3,867	5,1	4,6	4,1
Erratic	4,545	4,364	5,091	5	5,182	3,9091	4,36364	3,545
Smooth	6	3,75	3,5	5,25	4,5	6	4	3

In Table (4.10), average scores of each data type according to forecasting methods based on MAD/A performance measure are represented.

Table 4.10 Comparisons of forecasting methods based on MAD/A for each demand data type

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven	Lumpy	TwoPar.Cr.
Intermittent	4,93	4,57	4,45	3,86	4,90	5,60	4,98	2,714
Lumpy	5,71	3,81	4,10	3,97	4,16	5,97	5,58	2,710
Erratic	5,73	3,64	3,45	5,27	5	5,36	4,55	3
Smooth	4,50	4	4,75	4,50	4,75	6,75	3,75	3

In Table (4.11), average scores of each data type according to forecasting methods based on MASE performance measure are represented.

Table 4.11 Comparisons of forecasting methods based on MASE for each demand data type

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn	Leven	Lumpy	TwoPar.Cr.
Intermittent	5,467	2,956	4,511	3,267	4,978	7,3556	5,48889	1,956
Lumpy	6,267	2,767	4,433	2,833	4,5	7,5667	5,86667	1,767
Erratic	5,182	4	4,82	3,45	4,545	7,4545	3,54545	3
Smooth	6,25	4,75	6	3,5	4,5	2,75	4,5	3,75

In Table (4.12), average scores of each data type according to forecasting methods based on GRMSE performance measure are represented.

Table 4.12 Comparisons of forecasting methods based on GRMSE for each demand data type

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn.	Leven	Lumpy	TwoPar.Cr.
Intermittent	5,60	4,07	4,29	4,16	3,44	6,36	5,09	3
Lumpy	5,97	3,07	4,43	3,53	3,80	6,50	5,70	3
Erratic	4,36	3,64	4,82	3,91	4,27	5,36	5,64	4
Smooth	5,5	5	5,75	3,25	3,25	6,25	3,5	3,5

In Table (4.13), overall average scores of each data type according to forecasting methods based on average of performance measures (RMSE, MAD/A, MASE, GRMSE) are represented.

Table 4.13 Overall scores of forecasting methods versus different data types

	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. CrostonSyn	Leven	Lumpy	TwoPar.Cr.
Intermittent	5,33	3,99	4,42	3,98	4,50	6,04	5,16	2,57
Lumpy	5,79	3,45	4,32	3,73	4,08	6,28	5,44	2,89
Erratic	4,95	3,91	4,55	4,41	4,75	5,52	4,52	3,39
Smooth	5,56	4,38	5,00	4,13	4,25	5,44	3,94	3,31

Considering different performance measures, the accuracy comparison results can indicate which forecasting method performs better for each of the four demand categories, namely; Intermittent, Lumpy, Erratic, and Smooth or per each demand item. Rankings of all used forecasting methods for each demand item is represented in Appendix A.2.

The purpose of conducting any categorisation exercise is to choose the most appropriate forecasting (and may be inventory control) methods. It seems more reasonable to compare alternative estimation procedures initially for the purpose of indicating the superior performance regions. Based on the results, the demand patterns can be categorised. The decision about when each method performs best will be based on characteristics of demand. In other words, superior method will be identified based on the inter-demand interval and the squared coefficient of variation of the demand sizes. In this way, cut-off values are used for the inter-demand interval and the squared coefficient of variation enables specifying areas of superior performance for each of the methods considered. The decision about when each method performs best is based on the inter-demand interval and the squared coefficient of variation of the demand sizes can be seen in Figure (4.5).

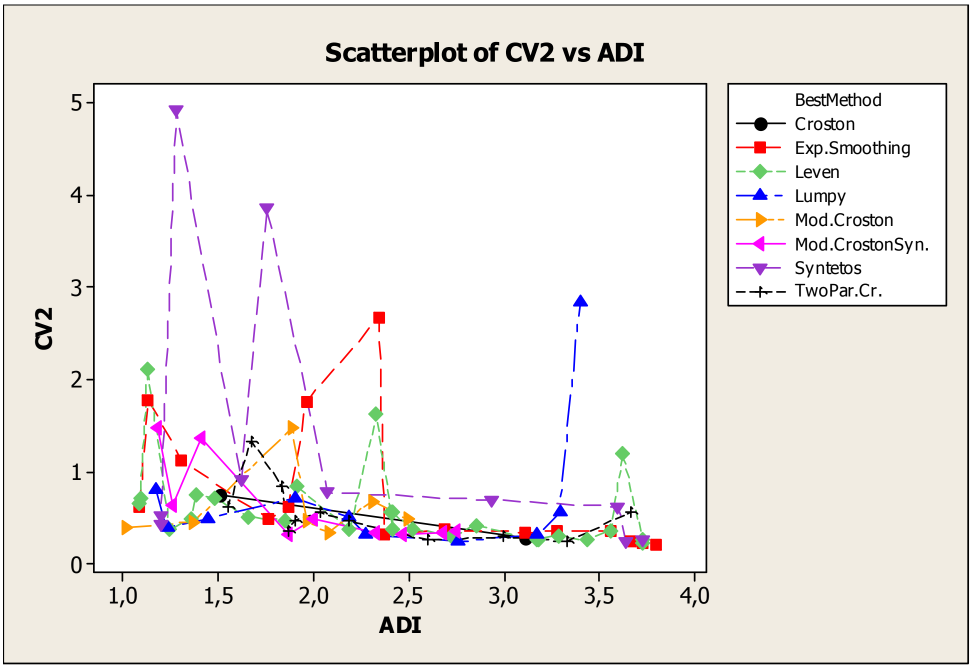


Figure 4.5 Selection best forecasting method for mapped demand data using its properties

In this study, the following steps are recommended for efficient spare parts management;

1. Obtain historical data set
2. Categorize data set based on historical demand attributes to identify and map appropriate area.
3. Forecasting methods that is suggested for this area are compared using related data series considering different feasible accuracy measures (it is assumed that each of them has equal importance),
4. Results of accuracy measures of each forecasting methods are averaged to find out overall winner forecasting method for used data set,
5. Overall winner forecasting method is used throughout lead time to determine inventory rules considering customer service level and inventory policy limitations that is identified by the company.

CHAPTER 5

NEURAL NETWORKS APPLICATIONS IN SPARE PARTS FORECASTING

The purpose of this chapter is to apply neural network models and compare these results with traditional methods which are highlighted in Chapter (4) on selected real industrial data respect to different performance measures.

It is worth noting that Artificial Neural Networks (ANN) do not differ essentially from standard statistical models in that they are a network of many simple computing units called neurons or cells, which are highly interconnected and organized in layers. Information processing is performs by neurons that convert received inputs into processed outputs. The linking arcs of these neurons generate and store knowledge regarding the relationship between different nodes. The ANN models are able to perform a variety of tasks and achieve significant results. There is considerably growing interest in the application of neural networks in scientific studies related to non smooth demand.

As a matter of fact ANN is an adaptive, nonlinear system based on the operation of biological neural networks that contain a collection of connected neurons to produce complex results by performing a function from data by changing system parameters during operation instead of using time series forecasting. After the training phase, the ANN parameters are fixed and the system is deployed to solve the problem at hand (the testing phase) to optimize a performance criterion. Mathematical model of ANN is explained in the following section.

5.1 MATHEMATICAL MODEL

It is essential to consider the following issues when creating a functional model of the biological neuron: (i) model the strength of the connection between an input and a neuron as weights for which negative and positive values replicate values inhibitory and stimulator connections respectively, (ii) model the neuron cell where the modified inputs are summed according to weights to construct linear combination, (iii) control the output of the neuron to range between 0 and 1, or -1 and 1 (Haykin, 1998). Figure (5.1) represents a basic view of the neural network.

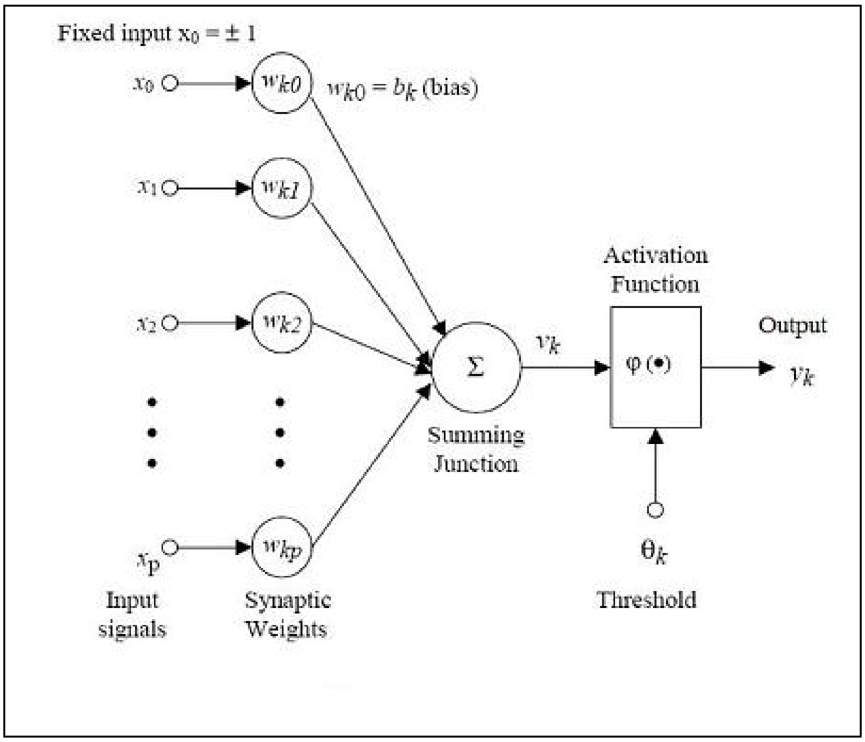


Figure 5.1 Basic view of neural network

From this model the interval activity of the neuron is shown to be:

$$v_k = \sum_{j=1}^{p} w_{kj} x_j \tag{5.1}$$

The output of the neuron, y_k, would therefore be the outcome of some activation function on the value of v_k.

5.2 ACTIVATION FUNCTION

The activation function alters the output of a neuron in a neural network between 0 and 1, or -1 and 1. In general there are three types of activation functions, denoted by Φ. First, values of 0 and 1 are taken respectively where the summed input is less than or greater than/ equal to the threshold value given by the Threshold Function, as follows:

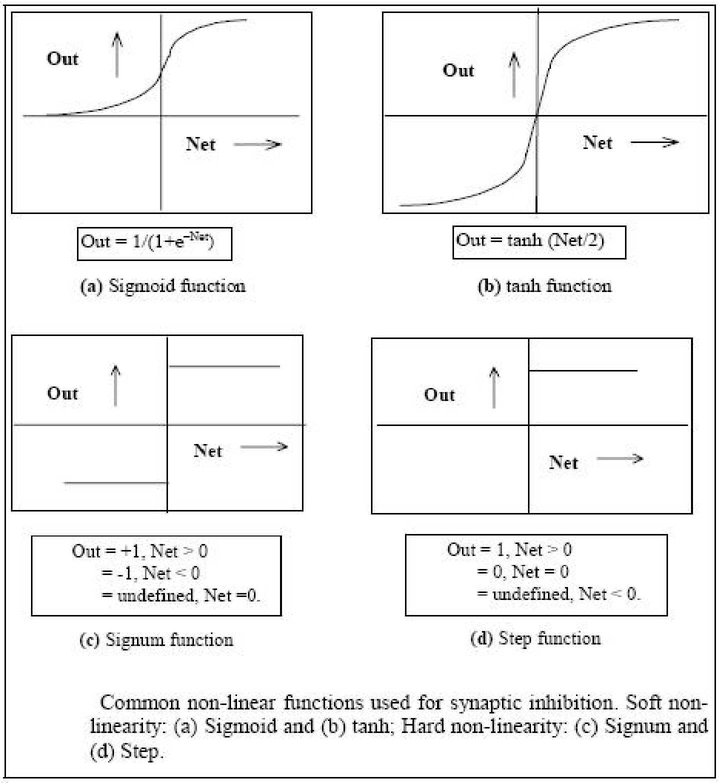
$$\varphi(v)=\begin{cases}1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0\end{cases} \tag{5.2}$$

Secondly, similarly to the threshold function values of 0 or 1 are taken. Values between 0 and 1 are also taken by the Piecewise-Linear function depending on the amplification factor in a certain region of linear operation, as follows:

$$\varphi(v)=\begin{cases}1 & v \geq \frac{1}{2} \\ v & -\frac{1}{2} > v > \frac{1}{2} \\ 0 & v \leq -\frac{1}{2}\end{cases} \tag{5.3}$$

Third, the Sigmoid function can range between -1 to 1 as the following hyperbolic tangent function:

$$\varphi(v)=\tanh\left(\frac{v}{2}\right)=\frac{1-\exp(-v)}{1+\exp(-v)} \tag{5.4}$$



Out = $1/(1+e^{-Net})$

(a) Sigmoid function

Out = tanh (Net/2)

(b) tanh function

Out = +1, Net > 0
= -1, Net < 0
= undefined, Net =0.

(c) Signum function

Out = 1, Net > 0
= 0, Net = 0
= undefined, Net < 0.

(d) Step function

Common non-linear functions used for synaptic inhibition. Soft non-linearity: (a) Sigmoid and (b) tanh; Hard non-linearity: (c) Signum and (d) Step.

Figure 5.2 Activation function of neural network

5.3 NEURON MODEL

A model of a neuron has three basic parts: input weights scale values used as inputs to the neuron, a summer adds all the scaled values together, and an output function produces the final output of the neuron. A bias is added to the system which can be represented by a weight with a constant input of one as represents in Figure (5.3).

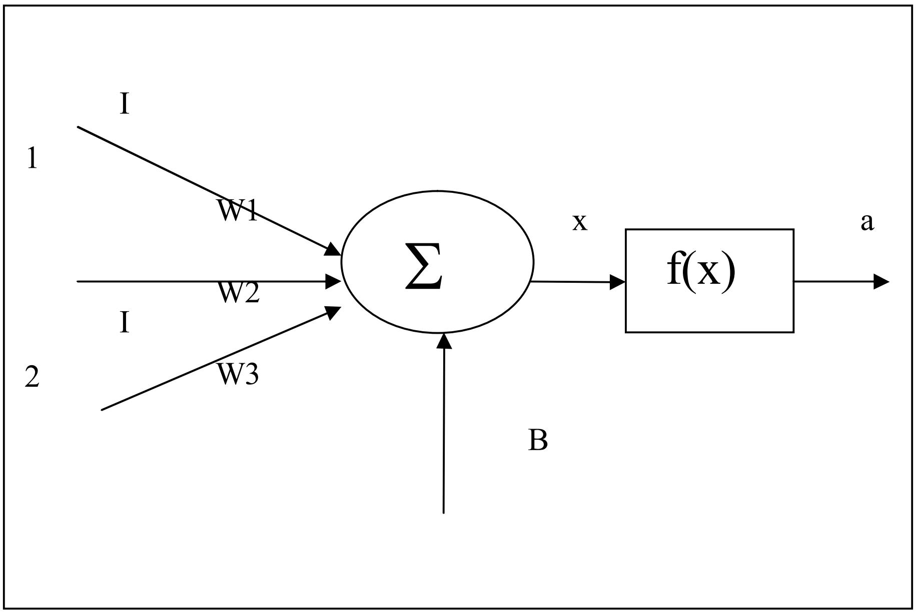


Figure 5.3 Neuron model

Where **I1**, **I2**, and **I3** are the inputs, **W1**, **W2**, and **W3** are the weights, **B** is the bias, **x** is an intermediate output, and **a** is final output. The equation for **a** is given by $a = f(W_1 I_1 + W_2 I_2 + W_3 I_3 + B)$ where f could be any function. Most often, f is the sign of the argument (i.e. 1 if the argument is positive and -1 if the argument is negative), linear (i.e. the output is simply the input times some constant factor), or some complex curve used in function matching. It is used as a first case where f is the sign of the argument for two reasons: it closely matches the 'all or nothing' property seen in biological neurons.

5.4 NEURON LAYER

Inputs are tied to neurons and each neuron produces its own output as represented in the equations below:

$$a_1 = f_1(\vec{W}_1 \bullet \vec{I} + B_1) \tag{5.5}$$

$$a_2 = f_2(\vec{W}_2 \bullet \vec{I} + B_2) \tag{5.6}$$

$$a_3 = f_3(\vec{W}_3 \bullet \vec{I} + B_3) \tag{5.7}$$

The weights are added to represent the weights of one neuron as a row of a matrix. For simplification, the input vector and the biases can be represented as the final form of one layer of artificial neurons, as follows:

$$\vec{a} = f(W \cdot I + B) \tag{5.8}$$

The finished equation for the three layer network in this equation is given by:

$$a = f(W_3 \cdot f(W_2 \cdot f(W_1 \cdot \vec{I} + \vec{B}_1) + \vec{B}_2) + \vec{B}_3) \tag{5.9}$$

5.5 TRAINING OF NEURAL NETWORKS

A set of inputs has to be configured to produce the desired set of outputs where the bond of these connections can be set either by: (i) aforementioned knowledge to set the weights explicitly, or (ii) feeding teaching patterns to train the neural network based on learning rule to change its weights. Haykin (1998) defines learning in the context of neural networks as: “Learning is a process by which the free parameters of a neural network are adapted through a process of stimulation by the environment in which the network is embedded.” Learning styles are categorised as: (i) supervised learning in which input-output pairs containing the neural network are provided by the system itself or an external teacher, (ii) unsupervised learning in which an output is trained to respond to clusters of pattern within input to develop its own representation of the input stimuli, (iii) reinforcement learning in which good (rewarding) or bad (punishable) grades for actions are based on the environmental response to adjust its parameters until an equilibrium state occurs.

5.6 NEURAL NETWORKS IN FORECASTING

Neural network models are broadly used in forecasting applications. The ability of ANN to deal with high non linear functions synthesis, which are very hard to model as a mathematical expression facilitates higher evaluation speed, and adaptability to changing. Neural networks learn from sample demand data which are used for training. The training helps the neural networks to understand the relationships of the data presented to them and try to capture delicate functional relationships among the data even if the underlying relationships are unknown.

5.6.1 Number of Input Nodes

The most critical decision variable for a time series forecasting problem is the number of input nodes which corresponds to the number of variables in the input vector used to forecast future values. The important input variable in forecasting demand form the dimension of the input vector.

5.6.2 Data Normalization

When variables are loaded into a neural network, be re-scaling the numeric range of the loaded variable to neural network scaling format is essential. There are two main numeric ranges the networks commonly operate in which are zero to denote (0, 1), and minus one to one denote (-1, 1).

5.6.3 Activation Function

The output of hidden layers is an activation function applied to the sum of the weighted values, which is then fired on to the next layer. The sigmoid (logistic) function is used in the networks using back propagation as the training algorithm since it has a simple derivative, a fact used in implementing the back propagation algorithm.

5.6.4 Training Sample and Test Sample

A training sample and a test sample are required for building an ANN forecaster used for ANN model development and evaluating the forecasting ability of the model respectively. The division of the data into the training and test sets is of major importance. The performance of the network can be measured using the test patterns for verification to examine how well it generalizes on patterns it has never seen before.

5.6.5 Back propagation Feed forward Networks

The generalization of out-of-sample is the major concern in the performance of a system that is trained by examples. Memorizing the training set will most probably result in well within sample training and badly out-of-sample. In designing a neural network, several variables must be determined: (i) number of input nodes (neurons), (ii) number of hidden layers and hidden nodes, (iii) number of output nodes.

The number of layers for this type of ANN will be determined via experimenting or by trial and error as it is problem dependent. The input, hidden and output layer architecture were built in order to test and compare their effectiveness when forecasting non smooth demand.

5.6.6 Number of Hidden Neurons

To construct well designed neural network, the need to decide the number of hidden neurons is determined by trial and error. The accuracy of the determined hidden layers will impact the network’s temptation to generalise and learn, resulting in either memorising the problem and generalising later when too many are used or generalising without learning the pattern when too few are used.

5.6.7 Learning Rate

The value of the learning rate affects the performance of the neural networks. It is not practical to determine the optimal setting for the learning rate before training, and, the optimal learning rate changes during the training process, as the algorithm moves across the performance surface.

The benefit of learning lays in reducing error which is achieved by modifying the weights leading to an output node for the next time the same pattern is used determined by the learning rate times the error. For the back propagation models the learning rate was set at 0.1 in this study.

5.6.8 Momentum

The training time is improved on the back propagation algorithm, which improves the stability. This momentum factor is commonly set around 0.9 in this study that determines the proportion of the last weight change that is added into the new weight change.

5.6.9 Stopping Criteria

The ANN model in this study trains the data set and computes an average error factor during training. After the learning events are completed with no improvement to the test set, the MSE for our model is reached. The network interrupts training and the weights for every pattern are saved at the point where the MSE for our model on the test set is at its minimum value.

5.7 NEURAL NETWORKS IN SPARE PARTS FORECASTING

When demand is non smooth, traditional forecasting methods may perform inadequately in capturing the nonlinear pattern in data. In this situation it can be reasonable to use artificial neural network modelling because it provides the capacity of learning from the environment, the adaptability and non-linearity. In the field of spare parts forecasting, data sets of inputs associated to output (the forecast) are, generally, the starting point; therefore an algorithm that compares the output of a NN with the actual values of demand and then corrects the weights. The forecasted value is the output of a neural network that forecasts spare parts demand which is generally only one, while the inputs might be numerous.

In this study real data sets of 20 types of spare parts demand in Turkish Airlines inventory items have been used. These spare parts were chosen among others for the diversity of their ADI which is greater than 1.32 and CV^2 which is lower than 0.49 and is classified into intermittent demand category. The data were handled for 64 monthly periods from 2005 to 2010. The data series were divided into three sets, namely: training, validation, and test sets. Data series have been divided into three blocks to apply neural network models, namely: initialization, validation, and performance measurement.

From 62 monthly observations, 12 observations have been used in initialization, 38 observations are used in validation while the last 12 observations are used as the test set. In order to evaluate the performance of the neural networks, their forecasts are compared to the three traditional best forecasting methods from explained in Chapter (4), which are exponential smoothing, modified Croston Syntetos' method, and Leven's method. For these forecasting methods the data series have been divided into three blocks, namely: initialization, validation, and performance measurement. The 'initialization block' is used to initialize values required for methods based on recursive formulas. In the 'calibration block', the optimal smoothing constants are identified based on the selected performance measure. Finally, the optimal smoothing constants are used to update forecasts in the 'performance block' in which performance statistics are calculated.

The following sections presents an application of neural network methods for forecasting twenty different items that exhibit non smooth demand structure and compared with other forecasting methods.

5.7.1 Generalized Regression Neural Network

As opposed to the back propagation method, an iterative training procedure is not essential according to the Generalized Regression Neural Network (GRNN) proposed by Spetch (1991) in which any arbitrary function between input and output vectors are estimated from the training data and used for the estimation of continuous variables based on kernel regression.

The GRNN consists of four layers which are input layer, pattern layer, summation layer, and output layer. A schematic diagram of GRNN architecture is presented in Figure (5.4).

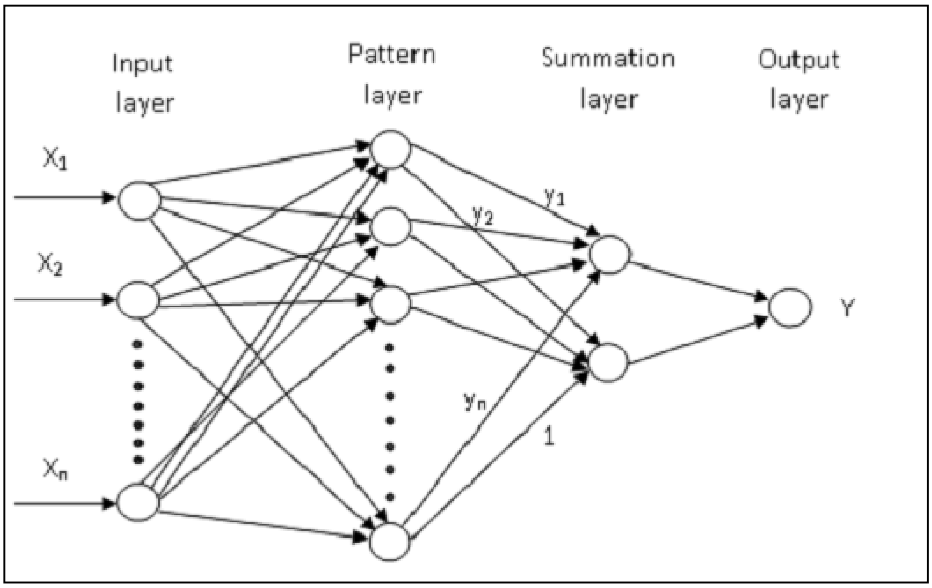


Figure 5.4 Schematic diagram of GRNN architecture, adapted from Spetch

Inspection of Figure (5.4) reveals that the degree of non-linearity in the relationships between inputs, and inputs and outputs is reduced by applying smoothing parameters. The input variables for the GRNN are as follows:

1. The demand at the end of the immediately preceding target period (lag 1).

2. The number of consecutive period with no demand transaction immediately preceding target period.

3. The number of periods separating the last two nonzero demand transactions as of the end of the immediately preceding target period.

4. The mean of demand for four period immediately preceding target periods.

As opposed to back propagation algorithm, in GRNN there are no training parameters, such as learning rate and momentum. However, there is a smoothing factor that is used when the network is applied to new data.

5.7.2 Recurrent Neural Network

The difference between a recurrent neural network and a feed forward neural network is containing at least one feedback loop which may have hidden layers or self-feedback in which the output of a neuron is fed back into its own input.

Naseri and Tabar (2008) modified RNN to forecast intermittent demand of spare parts using the basic Elman-type RNN (Elman and Zipser, 1987). Their network consists of four layers, namely: input layer, hidden layer, a context layer and an output layer as in the following Figure (5.5)

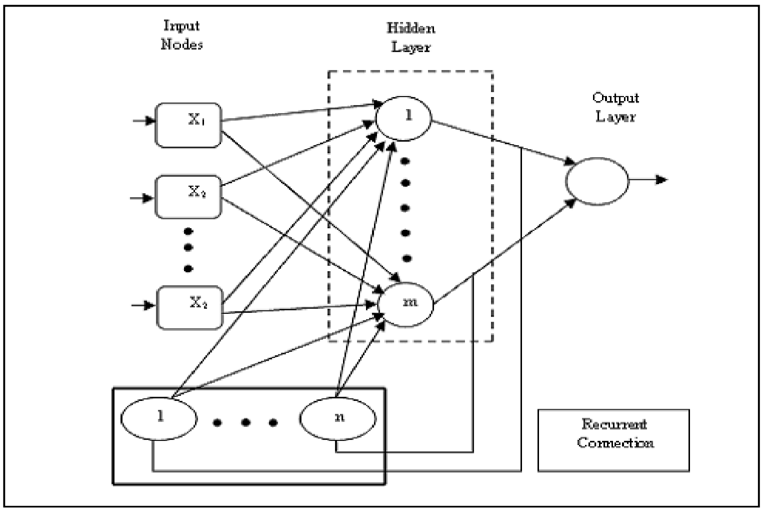


Figure 5.5 Schematic diagram of Elman type RNN architecture

Input parameters that are defined in RNN as the following;

1. The demand at the end of the immediately preceding target period (lag 1).

2. The number of consecutive periods with demand transaction, immediately preceding target period.

3. The number of consecutive period with no demand transaction, immediately preceding target period.

4. The number of periods separating the last two nonzero demand transactions as of the end of the immediately preceding target period.

5. The number of period(s) between target period and first nonzero demand immediately preceding target period.

6. The number of period(s) between target period and first zero demand immediately preceding target period.

7. The mean of demand for six periods immediately preceding target period.

8. The maximum demand among six periods immediately preceding target period.

5.7.3 Multi Layer Perceptron (MLP) with Input Set A

BPA is widely spread as a learning rule for multi-layer perceptron for a training set of data. Gutierrez et al. (2007) adopted a MLP trained by a BP algorithm using three layers of MLP, namely: one input layer for input variables, one hidden unit layer, and one output layer. The hidden layer consists of three nodes which is one output unit used in the output layer. All the input nodes were fully connected to all the hidden nodes which were connected to the output node. The latter represents the estimation of the demand transaction for the current period. In this study, MLP application learning rate value of 0.1 and a momentum factor value of 0.9 is used. The architectural graph in the following Figure (5.6) illustrates the layout of a multilayer perceptron neural network.

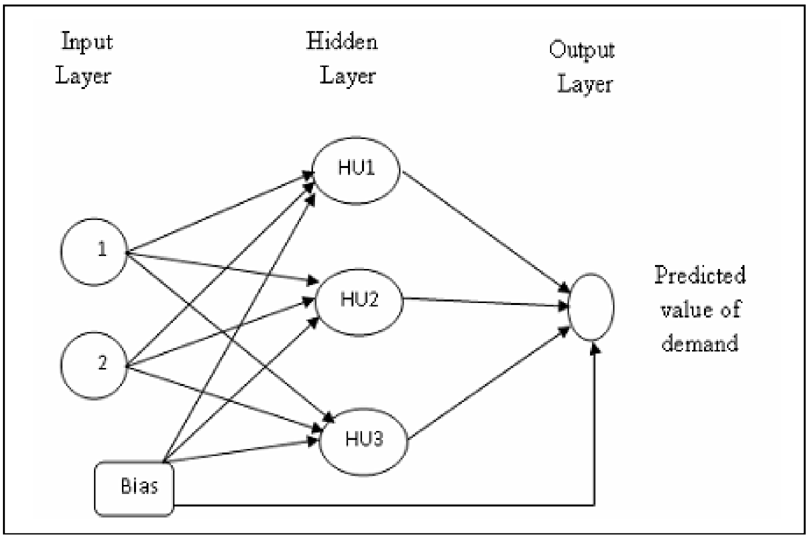


Figure 5.6 Schematic diagram of MLP (Gutierrez et al, 2007)

Tan-sigmoid and saturated linear transfer functions have been used in this study for hidden and output layers. In this regard, Naseri and Tabar (2008) adapted RNN to forecasting non smooth demand of spare parts using eight variables for input nodes. In this study, these defined variables used for input nodes in input layer adapted multi layer perceptron network structure

The software Alyuda Neurointelligence which is an ANN based forecasting program is used in this research for neural networks modelling. First, input and target values are normalized as in the following formula.

$$S_i(scaled) = \left[\frac{S_i - \min(S)}{\max(S) - \min(S)} \right] x2 - 1 \tag{5.10}$$

Where S is the time series of variable under consideration, S_i is the value of observations and S_i (scaled) is the normalized value. Range 1 to 50 neurons have been examined in the hidden layer to find the best number of neurons where one with minimum error was selected. When an input is presented to the network, the training algorithm attempts to adjust the weights so that the desired output is produced. In this research BPA have been used as training algorithm with learning rate of 0.1. Adaptive calibration algorithm was used for the specification of adjusting parameters of network.

Figure (5.7) represents an example of the network model that is used in Alyuda Neurointelligence software. The best network model (8-9-1) that indicates the number of inputs used, optimum number of hidden layer according to used learning algorithm and output respectively.

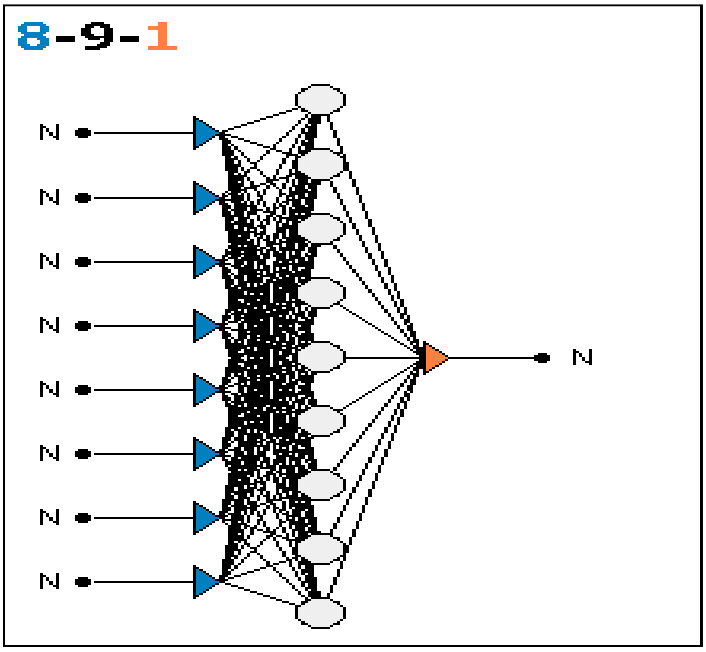


Figure 5.7 Network architecture of MLP using input set A (Alyuda screenshot)

Error distribution of this network is represented in Figure (5.8) and actual versus output graph is represented in Figure (5.9), as follows:

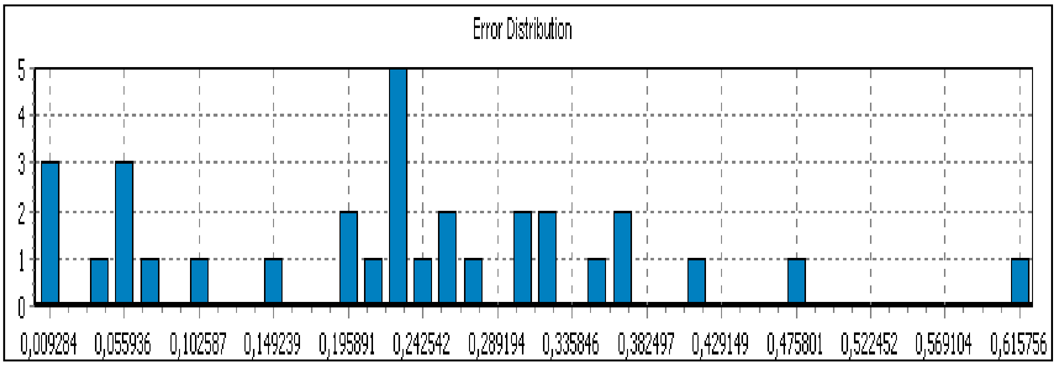


Figure 5.8 Error distribution of MLP with input set A

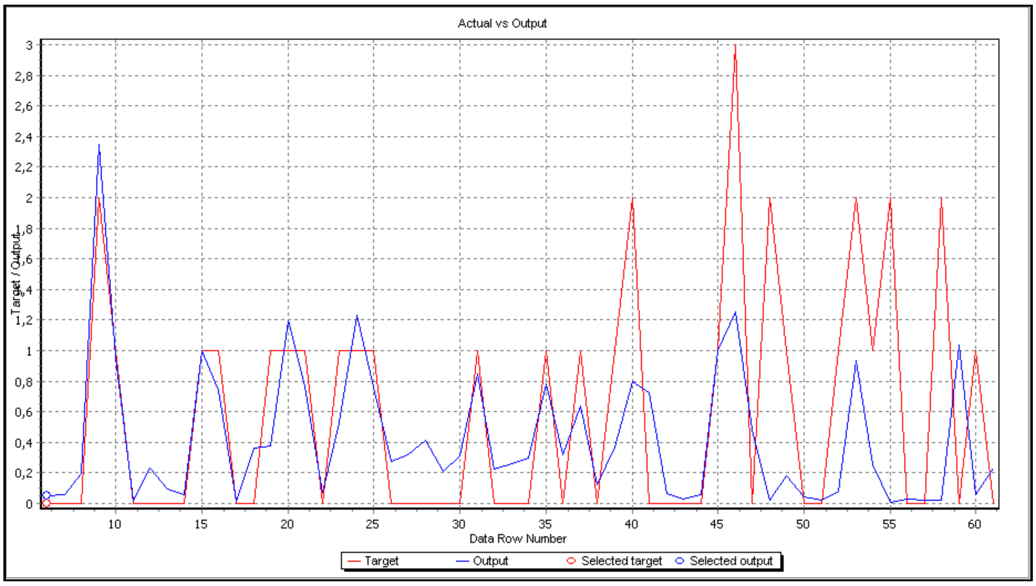


Figure 5.9 Actual versus output of MLP with input set A

5.7.4 Multi Layer Perceptron (MLP) with Input Set B

In the network model for this research, eleven input variables are used for input layer according to the same network model in Section (5.7.3) of this Chapter, as follows:

1. The number of consecutive period with no demand transaction, immediately preceding target period.
2. The number of periods separating the last two nonzero demand transactions as of the end of the immediately preceding target period.
3. The number of demand intervals from the last nonzero demand observation until the current period.
4. The maximum demand among six periods immediately preceding target period.
5. The mean of demand for six periods immediately preceding target period.
6. The number of demand intervals between two non zero demands immediately preceding target period.
7. $\hat{p}_t$; refers to the forecast for the interval between the the t^{th} and $(t+1)^{th}$ lump of demand;

$$\hat{\mathrm{p}}_{\mathrm{t}} = \alpha \mathrm{q}_{\mathrm{t}} + (1-\alpha)\hat{\mathrm{p}}_{\mathrm{t}-1} \tag{5.11}$$

8. $\hat{\mathrm{s}}_{\mathrm{t}}$; refers to the forecast of the number of periods the $(t+1)^{th}$ lump of demand spans;

$$\hat{\mathrm{s}}_{\mathrm{t}} = \alpha \mathrm{r}_{\mathrm{t}} + (1-\alpha)\hat{\mathrm{s}}_{\mathrm{t}-1} \tag{5.12}$$

9. L_{t} ; refers to the current level estimate using Brown's exponential smoothing.

$$\mathrm{L}_{\mathrm{t}} = \alpha \mathrm{y}_{\mathrm{t}} + (1-\alpha)\mathrm{L}_{\mathrm{t}-1} \tag{5.13}$$

10. T_{t} refers to the current trend estimate using Brown's exponential smoothing.

$$\mathrm{T}_{\mathrm{t}} = \alpha \mathrm{L}_{\mathrm{t}} + (1-\alpha)\mathrm{T}_{\mathrm{t}-1} \tag{5.14}$$

Where

y_{t} refers to the t^{th} non-zero demand for period t;

$\hat{z}_t$ refers to the forecast for the $(t+1)^{th}$ non-zero demand;

r_{t} refers to the number of periods of the t^{th} lump of demand spans;

q_{t} refers to the interval between the $(t-1)^{th}$and t^{th} lump of demand;

Figure (5.10) represents an example of the network model that is related to application of the multi layer perceptron with input set B. The best network model is found as (10-12-1), number of input, hidden layer and output respectively.

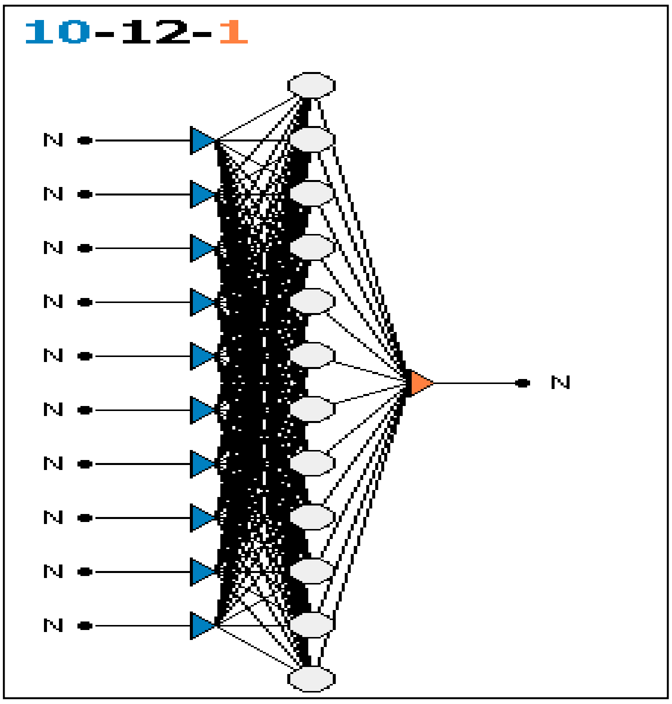


Figure 5.10 Network architecture of MLP with input set B (Alyuda screenshot)

Figure (5.11) shows error distribution of the network model where Figure (5.12) shows the actual versus output graph.

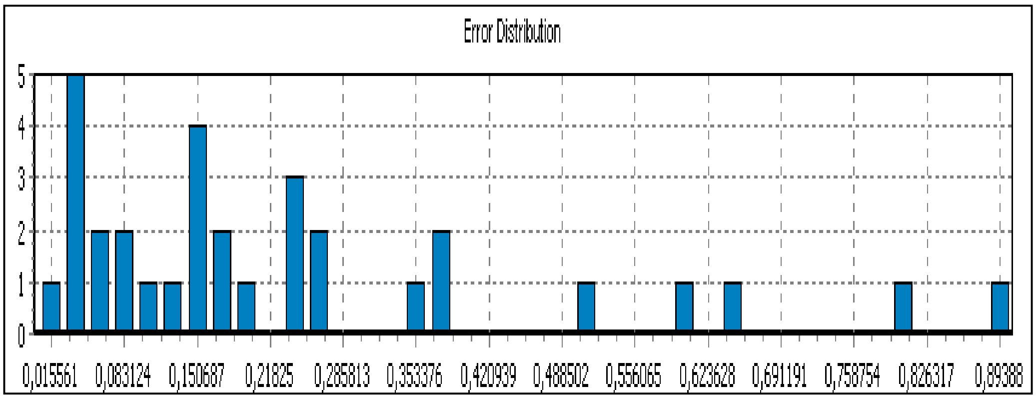


Figure 5.11 Error distribution of MLP with input set B

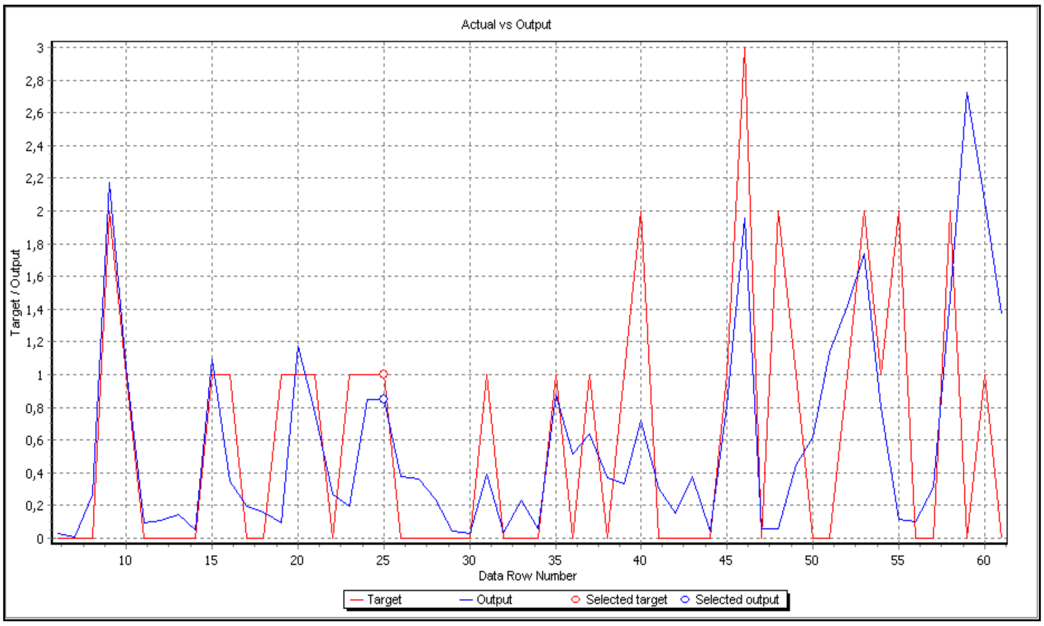
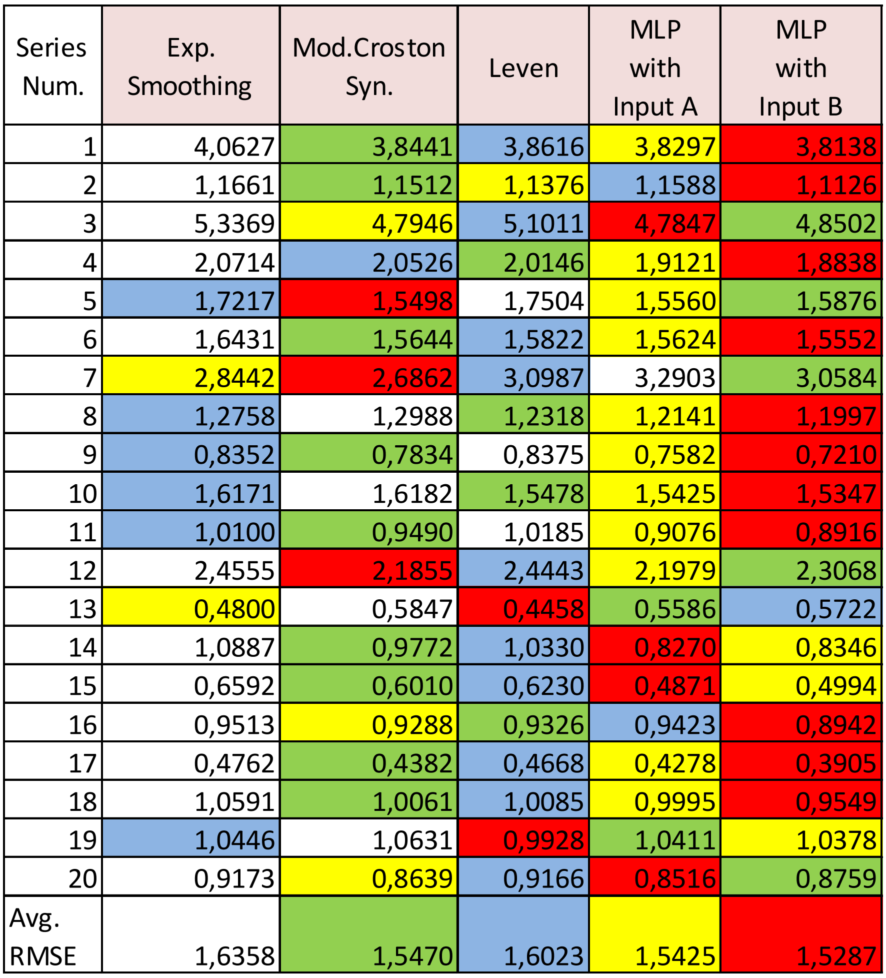


Figure 5.12 Actual versus output of MLP with input set B

5.8 RESULTS

Modified Croston Syntetos' method, Exponential Smoothing, and Leven's method, are the three forecasting methods selected from Chapter (4) which outstand compared to other demand forecasting models to evaluate performance of neural networks on a dataset of 20 real time series. In comparisons α is used as 0.2 for other three forecasting methods. The performance of methods were evaluated using four error measures due to their straightforwardness to interpret for non smooth demand data namely: RMSE, MAD/A, MASE, GRMSE that are explained in Chapter (2), on the final 12 observations. Results are represented in the following tables (Table 5.1, Table 5.2, Table 5.3, Table 5.4). Comparisons indicate that NNs forecasting have superior performance in comparison to other forecasting methods considering two types of MLP input model (A and B).

Table 5.1 Comparisons based on RMSE

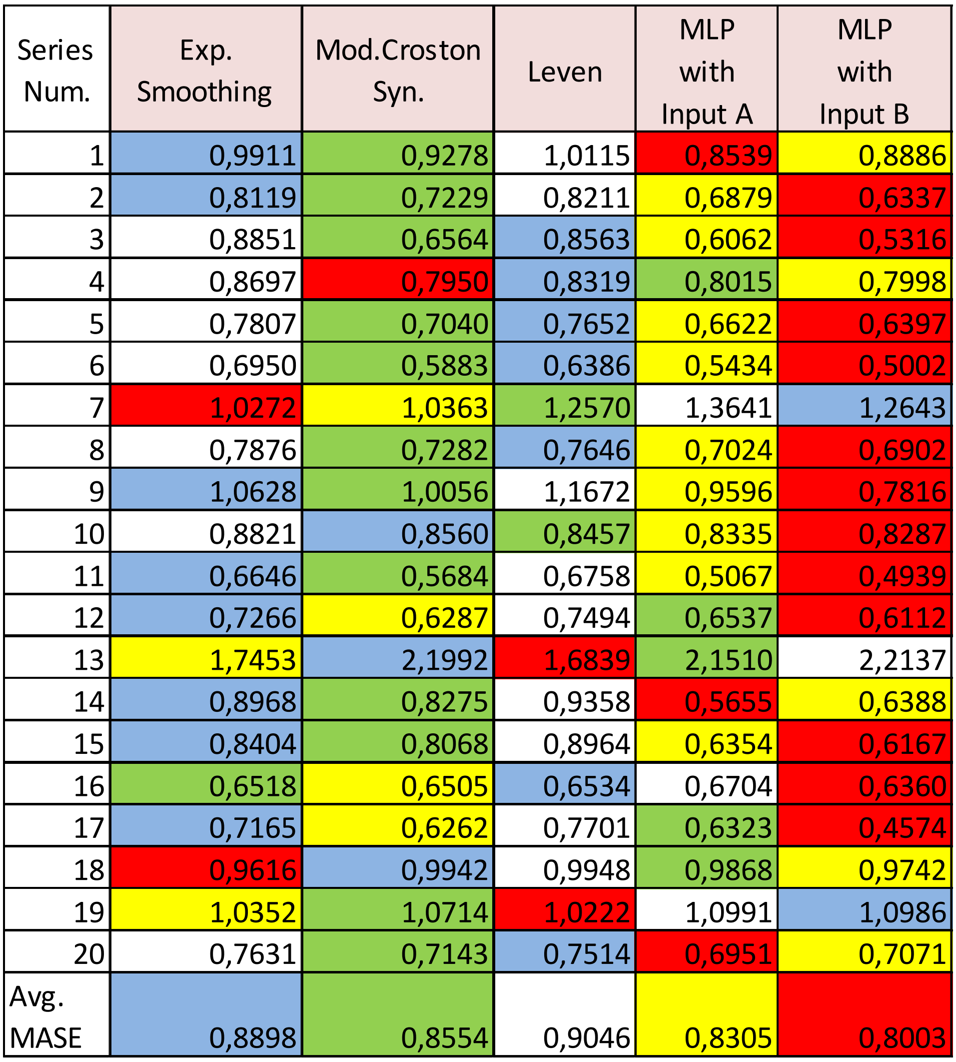


Series Num.	Exp. Smoothing	Mod.Croston Syn.	Leven	MLP with Input A	MLP with Input B
1	4,0627	3,8441	3,8616	3,8297	3,8138
2	1,1661	1,1512	1,1376	1,1588	1,1126
3	5,3369	4,7946	5,1011	4,7847	4,8502
4	2,0714	2,0526	2,0146	1,9121	1,8838
5	1,7217	1,5498	1,7504	1,5560	1,5876
6	1,6431	1,5644	1,5822	1,5624	1,5552
7	2,8442	2,6862	3,0987	3,2903	3,0584
8	1,2758	1,2988	1,2318	1,2141	1,1997
9	0,8352	0,7834	0,8375	0,7582	0,7210
10	1,6171	1,6182	1,5478	1,5425	1,5347
11	1,0100	0,9490	1,0185	0,9076	0,8916
12	2,4555	2,1855	2,4443	2,1979	2,3068
13	0,4800	0,5847	0,4458	0,5586	0,5722
14	1,0887	0,9772	1,0330	0,8270	0,8346
15	0,6592	0,6010	0,6230	0,4871	0,4994
16	0,9513	0,9288	0,9326	0,9423	0,8942
17	0,4762	0,4382	0,4668	0,4278	0,3905
18	1,0591	1,0061	1,0085	0,9995	0,9549
19	1,0446	1,0631	0,9928	1,0411	1,0378
20	0,9173	0,8639	0,9166	0,8516	0,8759
Avg. RMSE	1,6358	1,5470	1,6023	1,5425	1,5287

Table 5.2 Comparisons based on MAD/A

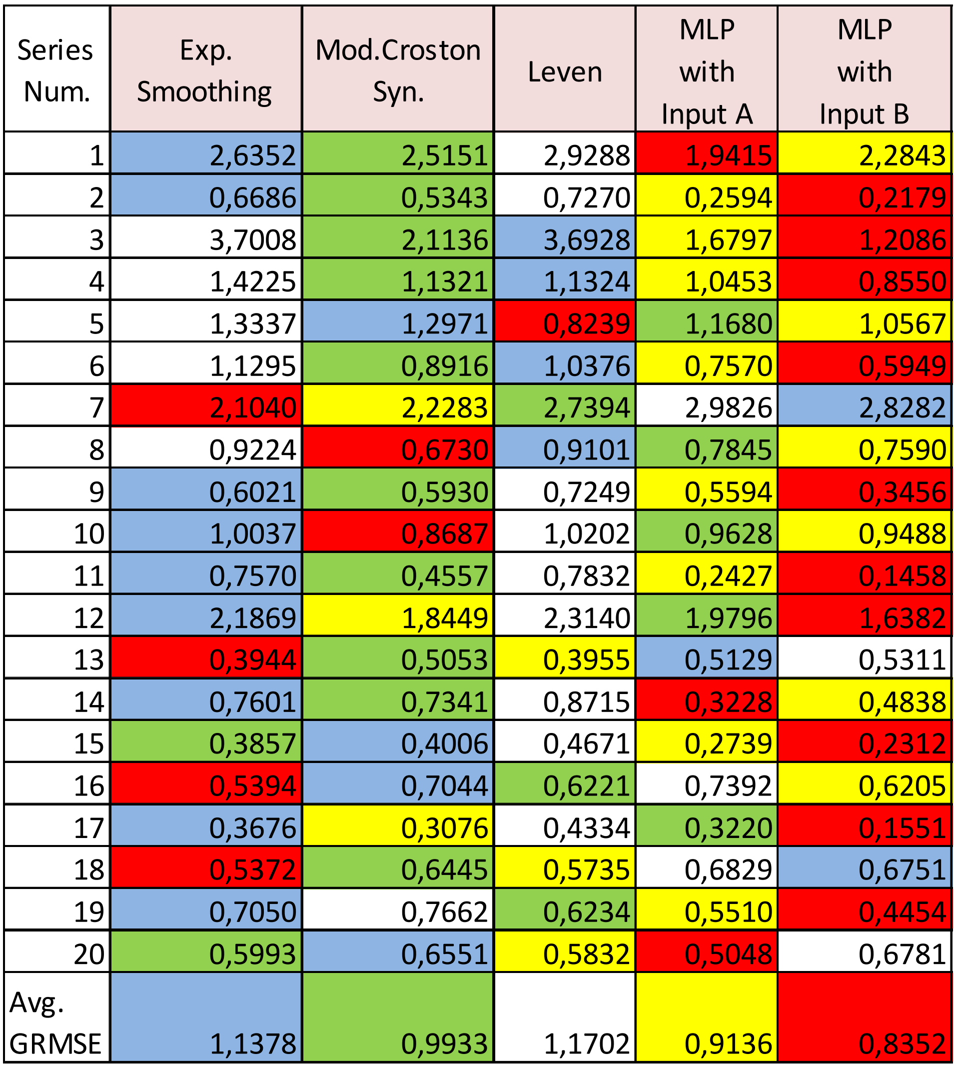
Series Num.	Exp. Smoothing	Mod.Croston Syn.	Leven	MLP with Input A	MLP with Input B
1	1,3214	1,2370	1,3487	1,1386	1,1848
2	1,3194	1,1747	1,3343	1,1179	1,0298
3	1,9342	1,4344	1,8711	1,3247	1,1616
4	0,8349	0,7632	0,7987	0,7694	0,7678
5	1,3383	1,2069	1,3117	1,1352	1,0966
6	1,5164	1,2835	1,3934	1,1856	1,0914
7	2,3969	2,4180	2,9330	3,1829	2,9501
8	0,6695	0,6189	0,6499	0,5971	0,5867
9	1,7005	1,6090	1,8676	1,5353	1,2505
10	0,7560	0,7337	0,7249	0,7144	0,7103
11	0,7595	0,6496	0,7724	0,5791	0,5645
12	1,5340	1,3273	1,5821	1,3800	1,2903
13	0,6545	0,8247	0,6314	0,8066	0,8301
14	2,1524	1,9860	2,2459	1,3571	1,5332
15	1,9610	1,8825	2,0917	1,4826	1,4390
16	0,8889	0,8871	0,8910	0,9142	0,8672
17	1,6718	1,4612	1,7969	1,4753	1,0674
18	0,9616	0,9942	0,9948	0,9868	0,9742
19	0,9411	0,9740	0,9293	0,9991	0,9987
20	1,1023	1,0317	1,0853	1,0040	1,0213
Avg. MADA	1,3322	1,2350	1,3773	1,1938	1,1260

Table 5.3 Comparisons based on MASE



Series Num.	Exp. Smoothing	Mod.Croston Syn.	Leven	MLP with Input A	MLP with Input B
1	0,9911	0,9278	1,0115	0,8539	0,8886
2	0,8119	0,7229	0,8211	0,6879	0,6337
3	0,8851	0,6564	0,8563	0,6062	0,5316
4	0,8697	0,7950	0,8319	0,8015	0,7998
5	0,7807	0,7040	0,7652	0,6622	0,6397
6	0,6950	0,5883	0,6386	0,5434	0,5002
7	1,0272	1,0363	1,2570	1,3641	1,2643
8	0,7876	0,7282	0,7646	0,7024	0,6902
9	1,0628	1,0056	1,1672	0,9596	0,7816
10	0,8821	0,8560	0,8457	0,8335	0,8287
11	0,6646	0,5684	0,6758	0,5067	0,4939
12	0,7266	0,6287	0,7494	0,6537	0,6112
13	1,7453	2,1992	1,6839	2,1510	2,2137
14	0,8968	0,8275	0,9358	0,5655	0,6388
15	0,8404	0,8068	0,8964	0,6354	0,6167
16	0,6518	0,6505	0,6534	0,6704	0,6360
17	0,7165	0,6262	0,7701	0,6323	0,4574
18	0,9616	0,9942	0,9948	0,9868	0,9742
19	1,0352	1,0714	1,0222	1,0991	1,0986
20	0,7631	0,7143	0,7514	0,6951	0,7071
Avg. MASE	0,8898	0,8554	0,9046	0,8305	0,8003

Table 5.4 Comparisons based on GRMSE



Series Num.	Exp. Smoothing	Mod.Croston Syn.	Leven	MLP with Input A	MLP with Input B
1	2,6352	2,5151	2,9288	1,9415	2,2843
2	0,6686	0,5343	0,7270	0,2594	0,2179
3	3,7008	2,1136	3,6928	1,6797	1,2086
4	1,4225	1,1321	1,1324	1,0453	0,8550
5	1,3337	1,2971	0,8239	1,1680	1,0567
6	1,1295	0,8916	1,0376	0,7570	0,5949
7	2,1040	2,2283	2,7394	2,9826	2,8282
8	0,9224	0,6730	0,9101	0,7845	0,7590
9	0,6021	0,5930	0,7249	0,5594	0,3456
10	1,0037	0,8687	1,0202	0,9628	0,9488
11	0,7570	0,4557	0,7832	0,2427	0,1458
12	2,1869	1,8449	2,3140	1,9796	1,6382
13	0,3944	0,5053	0,3955	0,5129	0,5311
14	0,7601	0,7341	0,8715	0,3228	0,4838
15	0,3857	0,4006	0,4671	0,2739	0,2312
16	0,5394	0,7044	0,6221	0,7392	0,6205
17	0,3676	0,3076	0,4334	0,3220	0,1551
18	0,5372	0,6445	0,5735	0,6829	0,6751
19	0,7050	0,7662	0,6234	0,5510	0,4454
20	0,5993	0,6551	0,5832	0,5048	0,6781
Avg. GRMSE	1,1378	0,9933	1,1702	0,9136	0,8352

Figure (5.13) represents the relative performance of forecasting methods with respect to average of accuracy measures that considered in this chapter (RMSE, MAD/A, MASE, GRMSE) on data set of 20 series. MLP with input set B outperforms the other methods in that it has the minimum average value in most of series.

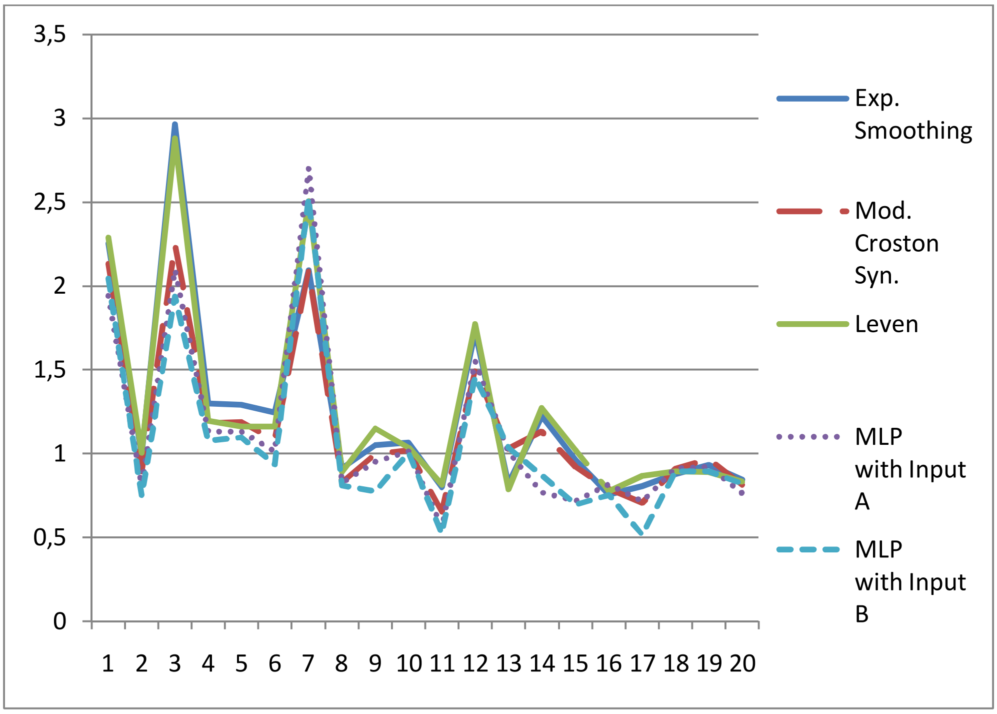


Figure 5.13 Comparison of forecasting methods based on average accuracy measure results

The findings point to accuracy and effectiveness of neural networks in decision making due to based on data-driven self-adaptive methods and find out invisible relationships in data to model complex systems.

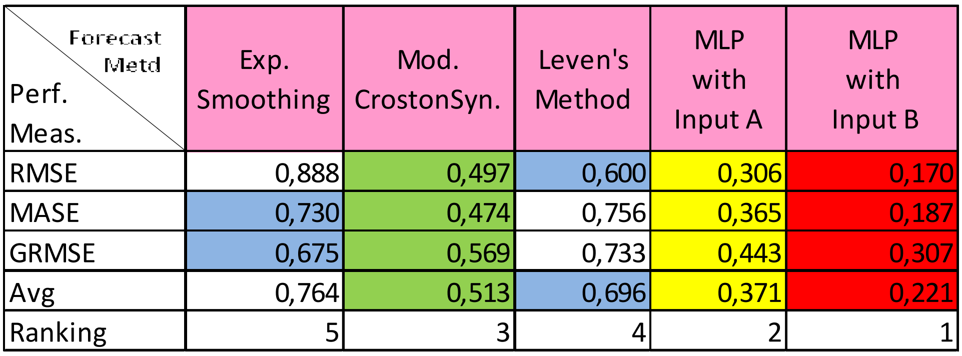
Table 5.5 represents geometric mean of the arithmetic mean is of the absolute errors (GMAMAE) for five methods. As the table demonstrates MLP with Input Set B has a better performance than those other methods.

Table 5.5 GMAMAE results of NN models vs. Other Forecasting Methods

Exp. Smoothing	Mod. CrostonSyn.	Leven's Method	MLP with Input A	MLP with Input B
1,117	1,045	1,132	1,001	0,955

Overall comparison results of NN models and other forecasting methods based on selected performance measures and ranking of all methods are given in Table 5.6 as the following:

Table 5.6 Overall Comparison Results of NN models vs. Forecasting Methods



Forecast Metd Perf. Meas.	Exp. Smoothing	Mod. CrostonSyn.	Leven's Method	MLP with Input A	MLP with Input B
RMSE	0,888	0,497	0,600	0,306	0,170
MASE	0,730	0,474	0,756	0,365	0,187
GRMSE	0,675	0,569	0,733	0,443	0,307
Avg	0,764	0,513	0,696	0,371	0,221
Ranking	5	3	4	2	1

CHAPTER 6

CONCLUSION AND FURTHER RESEARCH

Managing a service parts inventory is a complicated business in modern industry. One wants to simultaneously keep stocks low to contain inventory costs and keep stocks high to insure availability. This classic inventory management dilemma can be eased by accurate demand forecasts. Unfortunately, while demand forecasting is difficult in general, it can be especially difficult to forecast demand for service parts. This is because service parts demand is intermittent. Intermittent demand data contain a large percentage of zero values, with random non-zero demands mixed at random times. Classical time series forecasting methods usually perform poorly when applied to intermittent demand.

One of the objectives of this research is to explore the problem of spare parts management in modern industries, explaining spare parts and spare parts demand properties for the purpose of categorization. Therefore, to estimate the best forecasting technique used to handle spare parts demand problems, historical demand data of spare parts were classified depending on its properties such as average inter demand interval and coefficient of variation based on Syntetos categorization scheme. Four distinct groups of spare parts were determined based on that classification, namely: intermittent, lumpy, smooth, and erratic. Thus, this categorization scheme gives an idea in regards to outstanding performance for each region in consideration of the selected parameters that show the properties of demand such as demand size variability and demand arrival variability. Further attributes of demand series are defined to classify demand series considering different and reasonable candidate parameters in Chapter (3).

The second objective of this research is to o explain and compare the spare parts demand forecasting methods that have been given more attention in the recent scientific literature. Spare parts demand forecasting models are used considering the optimum alpha model presented in Chapter (4). The smoothing parameter of the forecasting methods is optimized dynamically via Excel SOLVER to give minimum error instead of using a fixed smoothing factor values as in earlier studies. This approach is used in eight forecasting methods, namely: Exponential Smoothing, Croston's method, SBA, Modified Croston, Modified Croston Syntetos, Leven's method, Two parameter Croston and Chua's method. The generated forecasts are compared against the actual data for evaluating the best fit of forecasted data. The comparison is performed on 90 real intermittent demand data series from the airline industry. The forecasting techniques are evaluated on the basis of RMSE, GRMSE, MASE, and MAD/A to reveal the efficiency of the forecasting techniques. Categorization by demand frequency and demand size variability allows the system to choose the best forecasting method. There may be additional benefit in determining best forecasting methods for each individual spare parts item according to its properties such as average inter-demand interval and coefficient of variation. In this study, the data is mapped according to its properties which are represented by the best forecasting method as plotted in the related graph in Chapter (4). In this way optimal forecasting method can be selected based on historical demand patterns. To explore this graph different forecasting accuracy metrics are considered.

The third objective of this research is to investigate the neural network models, one of which have gained the more attention in the recent scientific literature. Although there are various forecasting methods developed for intermittent demand series in literature, using neural networks in spare parts forecasting can be reasonable in terms of capturing non linear relations in data set. For this reason we handled neural networks for spare parts forecasting as explained in Chapter (5).

The fourth objective of this research is to apply traditional forecasting methods and neural networks on real industrial data and compare the results. The comparison between the performance of NN forecasts and the three traditional time-series methods which demonstrated outstanding performance, has proven that NN models are generally superior to three traditional forecasting models even under a relatively simple network structure, using four different forecast performance measures. Overall results related to the comparison indicate that MLP with Input Set B has a better performance than those other methods.

Results of this study show that traditional performance measures for forecast errors cannot be taken as the only indicators for the choice among different demand forecasting methods. Thus, companies need to have proper forecast accuracy measures.

There is a scope in order to increase the performance of inventory planning systems, and modifications are required for the interaction between forecasting and stock control in terms of their effects on system performance.

In view of the foregoing, it can be said in general terms that demand sporadicity and irregularity result in demand patterns that are difficult to forecast and manage. It is interesting to note that some exceptions to the rule may occur. It is necessary to understand the sources of non-smooth pattern because the management may try to influence the sources in order to reduce the level of non-smoothness by use of the efficient methods. In this regard, a theoretical methodology from the start to finish should be developed to deal with these sort of problems.

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APPENDIX

Table A.1 Candidate classification parameters

Data	CV^2	ADI	CV^2ADI	Lag1CorrIntrvl	Lag1CorrNZ	P(01)	P(11)
1	0,3494	2,7500	0,6542	0,3418	0,1736	0,3250	0,4286
2	0,2647	3,7333	0,7131	0,3216	0,3516	0,2889	0,2500
3	0,7084	1,9032	0,4666	0,1029	0,1282	0,5517	0,4688
4	2,1119	1,1321	0,1197	0,1031	0,0860	0,8571	0,8889
5	0,3617	3,5625	0,7561	0,1731	0,1363	0,2273	0,4118
6	0,4692	1,9667	1,0666	0,2687	0,0440	0,4333	0,6129
7	0,2438	3,6667	1,5266	0,2201	0,0000	0,2444	0,3125
8	0,3156	3,1765	0,4737	0,1867	0,5904	0,3256	0,2222
9	0,3720	2,1923	0,4830	0,2045	0,1187	0,4242	0,5000
10	0,5176	1,2000	0,1701	0,0361	0,2470	0,7778	0,8654
11	0,4681	1,9032	0,5403	0,2717	0,0237	0,4483	0,6250
12	0,3445	3,5625	0,3254	0,1994	0,1981	0,3256	0,2222
13	0,4505	1,3721	0,2524	0,0647	0,0164	0,7647	0,7273
14	0,4794	1,7647	0,5461	0,1127	0,0105	0,4800	0,6389
15	0,2969	2,7273	1,3212	0,0144	0,1114	0,3158	0,4348
16	0,3680	1,2500	0,1770	0,0822	0,1121	0,8333	0,7755

17	0,3918	1,2340	0,2059	0,1135	0,2108	0,6923	0,8125
18	0,2362	3,3333	0,8039	0,2047	0,0556	0,2619	0,4211
19	0,2522	3,7333	0,3738	0,1341	0,4781	0,3409	0,1176
20	0,2261	3,7333	0,3738	0,2230	0,4523	0,3111	0,1250
21	0,3272	2,6818	1,2744	0,3433	0,3651	0,4054	0,3750
22	0,2082	3,8000	0,5857	0,2312	0,0714	0,2889	0,1875
23	0,4338	1,2000	0,3920	0,0714	0,0432	0,6000	0,8824
24	0,3212	1,8750	0,5271	0,1483	0,2260	0,3333	0,5200
25	0,3262	2,0800	0,6638	0,1785	0,0954	0,3714	0,5385
26	0,3907	1,0169	0,0161	0,0172	0,3350	1,0000	0,9833
27	0,3198	2,2800	0,5373	0,2337	0,0591	0,4000	0,4615
28	0,3784	2,4167	0,3710	0,1940	0,1748	0,3611	0,4400
29	0,4883	1,4500	0,3291	0,0404	0,0879	0,7368	0,6905
30	0,2552	3,4375	0,6767	0,0740	0,0919	0,3182	0,1765
31	0,6779	2,3077	0,6117	0,0068	0,0198	0,3529	0,5556
32	0,3121	2,4706	1,3429	0,4786	0,3546	0,1860	0,6111
33	0,2550	2,6000	0,6187	0,2083	0,2174	0,2222	0,4375
34	0,4581	2,1852	1,0799	0,2337	0,2410	0,4242	0,5000
35	0,7363	1,5128	0,6392	0,0213	0,0335	0,5714	0,7000
36	0,6211	1,8710	0,6473	0,0368	0,3884	0,4138	0,6563
37	0,4927	2,0000	0,6753	0,1752	0,2249	0,4839	0,5000
38	0,6540	1,0909	0,0698	0,1000	0,0746	1,0000	0,9107
39	0,2549	3,1765	0,4687	0,0521	0,2500	0,3721	0,1667
40	0,5500	2,4091	0,2201	0,2663	0,1457	0,4722	0,3600
41	0,2464	3,6429	0,1688	0,0607	0,3249	0,3478	0,0000
42	1,3302	1,6765	0,9268	0,1035	0,0224	0,5000	0,6571

43	0,4954	2,1923	0,3067	0,0536	0,0221	0,5588	0,3333
44	0,4790	2,5000	2,6886	0,3399	0,1859	0,2857	0,6154
45	0,8405	1,9130	2,6558	0,0771	0,0913	0,1944	0,7200
46	3,8543	1,7576	0,6737	0,0382	0,0650	0,5652	0,6579
47	4,9221	1,2826	0,3075	0,1882	0,4946	0,6154	0,8333
48	0,4694	1,8519	1,1870	0,4340	0,0609	0,3125	0,6552
49	0,3212	2,3750	0,2900	0,4305	0,2000	0,5000	0,2800
50	0,3311	2,3333	0,6816	0,0872	0,0945	0,3611	0,5200
51	2,6660	2,3478	0,5873	0,2633	0,1635	0,3611	0,4800
52	0,6085	1,0909	0,1004	0,3889	0,0364	0,8000	0,9286
53	0,2891	3,0000	1,1433	0,4820	0,0105	0,2000	0,5000
54	0,3677	2,6875	0,9732	0,1081	0,0895	0,2273	0,4118
55	0,3547	1,8710	0,5145	0,1966	0,1454	0,4828	0,5938
56	1,1187	1,3111	0,2056	0,0049	0,2016	0,8462	0,7917
57	2,8274	3,4000	2,2343	0,0472	0,1603	0,2381	0,4737
58	1,4710	1,8889	0,4234	0,2013	0,0371	0,6154	0,5429
59	1,7597	1,9667	0,4438	0,0256	0,0696	0,6154	0,5714
60	1,6154	2,3333	0,3451	0,0384	0,1130	0,5185	0,5588
61	1,1908	3,6250	2,0182	0,0523	0,0268	0,2683	0,5000
62	0,6393	1,2609	0,2559	0,0191	0,1822	0,7143	0,7872
63	0,5623	3,6667	0,2940	0,1539	0,2612	0,3409	0,1176
64	0,2319	3,7333	0,2200	0,0316	0,2885	0,3111	0,1250
65	1,4741	1,1875	0,1687	0,2044	0,1178	0,6667	0,8571
66	1,7764	1,1346	0,0986	0,0365	0,7610	1,0000	0,8704
67	0,9114	1,6216	0,4728	0,1490	0,0863	0,5652	0,6579
68	1,3602	1,4146	0,2981	0,0211	0,1323	0,6875	0,7778

69	0,4145	2,8571	0,8503	0,2317	0,5490	0,2093	0,5556
70	0,6903	2,9333	0,5751	0,1870	0,1564	0,3182	0,2353
71	0,5575	3,2941	2,9712	0,0357	0,0826	0,2143	0,5789
72	0,6146	3,6000	0,6203	0,1390	0,2515	0,2667	0,3125
73	0,3740	2,5238	0,6691	0,3768	0,2923	0,3077	0,4091
74	0,8324	1,8387	0,6959	0,2699	0,4911	0,4138	0,6563
75	0,6078	1,5526	0,4192	0,1371	0,0023	0,7000	0,6585
76	0,7114	1,4872	0,3539	0,1837	0,2252	0,6667	0,7209
77	0,2647	3,1875	0,5540	0,5328	0,2917	0,2791	0,3333
78	0,3387	3,1111	0,5712	0,0660	0,0959	0,3659	0,3000
79	0,3490	3,2778	0,5235	0,3530	0,2184	0,3333	0,2632
80	0,5007	1,6571	0,4731	0,2138	0,1577	0,6400	0,5833
81	0,5520	2,4167	0,6241	0,3789	0,0333	0,3714	0,4615
82	0,4898	1,3636	0,4491	0,1055	0,0179	0,5625	0,8000
83	0,2364	2,7647	0,7609	0,0069	0,2879	0,2558	0,3889
84	0,7945	1,1778	0,1078	0,0150	0,0855	0,5714	0,8298
85	0,7171	1,0943	0,0815	0,0784	0,0777	1,0000	0,9286
86	0,5622	2,0357	0,6163	0,1767	0,0067	0,4063	0,5517
87	0,3029	3,2857	5,6874	0,0851	0,2553	0,0652	0,8667
88	0,7545	1,3889	0,2452	0,0758	0,1331	0,5217	0,7105
89	0,2731	3,1176	0,4935	0,3315	0,1234	0,3256	0,2778
90	0,7739	2,0741	0,8130	0,2131	0,0239	0,4242	0,5357

Table A.2 Ranking of non smooth demand forecasting methods for each demand item

Series	Exp. Smoothing	Croston	Syntetos	Mod. Croston	Mod. Croston Syn.	Leven	Lumpy	TwoPar.Cr.
1	5	4	3	2	1	7	6	8
2	6	3	1	4	2	7	5	8
3	3	6	7	5	4	2	1	8
4	6	2	3	7	8	1	5	4
5	3	5	7	4	6	1	2	8
6	7	5	4	1	2	8	6	3
7	1	4	5	6	7	2	3	8
8	2	4	5	6	7	8	1	3
10	2	7	6	5	8	1	3	4
11	7	3	1	5	6	8	4	2
12	6	3	4	2	5	7	8	1
13	1	5	6	7	4	2	3	8
14	7	2	5	1	3	8	4	6
15	1	5	4	3	2	6	8	7
16	3	7	6	4	5	1	2	8
17	2	8	7	5	6	1	4	3
18	2	6	7	4	5	3	1	8
19	8	4	6	5	7	3	2	1
20	3	6	7	4	5	1	2	8

21	2	5	6	7	4	1	3	8
22	6	4	3	2	1	8	7	5
23	1	6	7	5	4	2	3	8
25	2	3	1	5	4	6	7	8
26	6	4	3	2	1	5	7	8
27	6	2	3	1	4	7	8	5
28	6	4	7	1	3	2	8	5
29	2	4	3	5	6	7	1	8
30	3	6	7	4	5	1	2	8
31	2	4	3	6	7	8	1	5
32	2	6	7	5	4	1	3	8
33	4	7	6	1	8	2	3	5
34	7	2	4	3	1	6	5	8
35	6	4	5	3	2	8	7	1
36	5	3	2	6	7	8	4	1
37	8	1	2	3	4	6	7	5
38	1	8	7	5	6	3	2	4
39	3	4	5	2	1	7	6	8
40	8	6	2	4	3	1	7	5
41	2	6	7	5	4	1	3	8
42	3	7	6	4	5	1	2	8
43	5	3	1	4	2	7	6	8
44	5	4	2	6	7	3	8	1

45	3	7	6	4	5	8	1	2
47	4	5	2	1	3	7	6	8
48	2	3	4	6	7	1	8	5
49	7	2	1	5	6	8	3	4
50	7	4	1	5	2	8	6	3
51	2	7	5	6	4	1	3	8
52	1	6	7	5	4	2	3	8
53	6	3	4	2	1	7	5	8
54	1	6	7	5	4	2	3	8
55	1	2	7	4	6	5	3	8
57	2	6	5	7	8	4	3	1
59	1	5	4	6	7	2	3	8
62	6	2	3	4	5	8	7	1
63	1	5	6	3	4	8	2	7
64	2	5	6	7	4	3	1	8
65	6	4	5	1	2	8	3	7
66	1	6	7	8	5	3	2	4
67	3	8	7	5	6	1	4	2
68	2	5	4	6	8	1	7	3
69	4	8	5	2	1	6	3	7
70	7	3	2	5	6	8	4	1
71	1	5	6	7	4	2	3	8
72	7	4	5	2	1	3	8	6

74	1	3	2	5	6	7	8	4
75	7	4	1	2	3	6	8	5
77	7	3	2	6	1	5	8	4
78	2	4	5	6	8	1	3	7
79	6	3	1	2	4	5	7	8
80	3	6	7	4	5	2	1	8
81	4	2	1	3	5	7	6	8
82	2	6	4	5	7	1	3	8
83	3	4	7	8	6	2	5	1
84	2	6	5	3	4	7	8	1
85	2	6	4	7	8	1	3	5
87	3	6	7	5	4	1	2	8
88	1	7	3	6	5	4	2	8
89	1	7	6	5	4	3	2	8
90	3	5	6	2	8	1	7	4