

İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

**FORMATION CONTROL OF ROBOT CLUSTERS WITH KINEMATIC
CONSTRAINTS**

**M.Sc. Thesis by
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**KİNEMATİK KISITLAR İLE ROBOT SÜRÜLERİNİN FORMASYON
KONTROLÜ**

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FOREWORD

I would like to express my deep appreciation and thanks for my advisor Prof. Dr. Hakan Temeltas for everything he thought me, for the new perspectives he brought me, for his great tolerance, and his excellent guidance.

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FORMATION CONTROL OF ROBOT CLUSTERS WITH KINEMATIC CONSTRAINTS

SUMMARY

Control of multiple vehicles raises new challenges that do not exist in single vehicle systems. Some examples can be given in areas of communication, coordinated path planning, sensor fusion and formation control. Among these areas, the formation control is possibly the most attractive one for researchers in recent years, since flocking and formation navigation are essential operations for multivehicle systems.

In this work a system for formation control of arbitrary number of autonomous vehicles is presented. The presented system uses a kinematic model with nonholonomic constraints and double integrator model for vehicle dynamics. This study also improves the method for obstacle avoidance of the vehicle group by means of an artificial vector rotation field defined on obstacle surfaces. Interactions of vehicles are modeled via artificial potentials similar to the gravitational and magnetic potentials and formation graphs. The presented system can flock and navigate in any formation. In potential field based approach, a virtual leader is used to navigate the vehicles and safe group navigation in formation is ensured in an environment with obstacles while non of the vehicles have the a-priori knowledge about the place and positioning of obstacles. The thesis is organized as follows: Kinematic and dynamic model of mobil robots is presented in second chapter, potential field based formation control of robot clusters is presented in third chapter, formation graphs based formation control of robot clusters is presented in fourth chapter, simulation studies is presented in fifth chapter, conclusion is presented in last chapter.

KİNEMATİK KISITLAR İLE ROBOT SÜRÜLERİNİN FORMASYON KONTROLÜ

ÖZET

Çok araçlı sistemlerin kontrolü tek araçlı sistemelerin kontrolüne ek olarak iletişim, yörünge planlaması, formasyon kontrolü gibi ek problemler getirmektedir. Çok araçlı sistemlerin belli görevleri yerine getirirken belli dizilim şekillerini koruyarak hareket etmeleri beklenmektedir. Araçların belli dizilim şekillerini koruyarak hareketleri bir problem olarak araştırılmaktadır ve formasyon kontrolü problemi olarak tanımlanmıştır. Formasyon kontrolü son yıllarda en çok çalışma yapılan alanlardan biridir. Çok araçlı sistemlerin formasyon kontrolü merkezi yöntemler ve merkezi olmayan yöntemler olmak üzere iki ana başlık altında incelenmektedir. Merkezi yöntemlerde tüm araçlar için bir noktadan kontrol sistemi önerilirken, merkezi olmayan yöntemlerde araçlar arası etkileşimler tanımlanarak her aracın verilen amaç doğrultusunda kendi kontrolünü sağlaması önerilmektedir.

Bu çalışma, çok araçlı sistemlerin formasyon kontrolü üzerinedir. Çok araçlı sistemlerin formasyon kontrolü için iki yöntem kullanılmıştır. Birinci yöntemde araçların belli bir dizilim şekliyle ilerlemesi için sanal lider ve yapay potansiyeller kullanılmıştır. Yapay potansiyeller araçlar-aracalar, araçlar engeller ve araçlar sanal lider arası etkileşimlerin modellenmesinde kullanılmıştır. İkinci yöntemde ise araçlar arası etkileşim yapısal graflar ile modellenmiştir. İncelenen sistemde araçların kinematik modelinde non-holonamik kısıtlar kullanılmıştır. Araçların dinamik modeli için double-integrator modeli kullanılmıştır.

Her iki yöntem için kinematik kısıtlı araç kinematik modeli ile simülasyon çalışmaları yapılmıştır. Bu çalışmada ayrıca engelden sakınım için sanal vektör rotasyon alanı kullanılmıştır. Araçların engellerin yeri hakkında ön bilgisi bulunmamaktadır. Farklı dizilim şekilleri ile farklı engelli alanlar üzerinde çeşitli senaryolar denenmiştir.

1. INTRODUCTION

1.1 Robot Clusters

Robot clusters are thought to have several advantages over single robot systems. The most common motivations for developing robot cluster system solutions are that building several simple robots is much easier than having a single powerful robot, the task complexity is too high for a single robot to execute, robot clusters can solve problems faster by work-sharing, the task is distributed, use of robot clusters increases robustness as a result of redundancy of the robots.

Military discovery missions, mine cleaning, search and rescue tasks are some examples in which robot clusters have advantages compared to single robots.

The issues that are considered in developing robot cluster solutions are the task requirements and the sensory and effector capabilities of the available robots.

The types of the robots considered in the study of robot cluster systems are those robots that move in the environment, such as ground vehicles, aerial vehicles, or underwater vehicles [1].

There are some more problems in control of robot clusters compared to control of single robot systems. Examples are communication between robots [2], path planning [3, 4], usage of many sensors in the same environment [5]. Many of the studies are on formation control. Formation control is essential because robot cluster is expected to do its tasks in a certain formation [6].

1.2 History Of The Studies In The Field

The beginning of the works on robot cluster systems is in the 1980s, the field grew very fast, and includes large amount of research. At the most general level, approaches to robot cluster systems fall into one of two broad categories: collective swarm systems and intentionally cooperative systems [1].

Collective swarm systems are those in which robots execute their own tasks with only minimal need for knowledge about other robot cluster members. These systems includes a large number of homogeneous mobile robots, these robots use local control laws to generate global team behaviors. There is little communication between robots. On the other hand, robots in intentionally cooperative systems have knowledge of the presence of other robots in the environment and act together based on the state, actions, or capabilities of other robots in the cluster in order to execute the same task.

In intentionanally cooperative systems, robots act according to the actions or state of other robots, and realize either strongly or weakly cooperative solutions. Strongly cooperative solutions require robots to act in harmony to achieve the goal and execute the task.

Classification of the approaches can also be defined as centralized approach and decentralized approach. In centralized approach every indivual robot in the cluster acts according to the orders from one point. In decentralized approach, the behaviour of the system is the sum of the interactions of all individuals with their neigbours [7]. One of the examples in nature is the bird swarms. The studies on bird swarms show that there is no leader in the swarm and the organization of the swarm is the result of interactions of every single individual with neighbours [8, 9].

The following methods are investigated for formation control: behaviour based methods [10], artificial potential field based methods [11, 12], formation graph based methods [13]. Behaviour based methods model the interaction between robots by natural behaviours. In potential field based methods, an artificial potential field is assumed between robots and environment. Artificial potential field based approach is proposed by Khatib in [14]. Leonard and Fiorelli proposed use of a virtual leader in [12]. Murray and Saber suggested use of graphs and artificial potentials.

1.3 Architectures For Robot Clusters

Several different philosophies for robot cluster architectures are possible, the most common are centralized, hierarchical, decentralized, and hybrid. Many robot cluster control architectures developed over the years. Three examples are presented to illustrate the spectrum of control architectures. The first one is the Nerd Herd, it is

representative of a pure swarm robotics approach using large numbers of homogeneous robots. The second is the alliance, it is representative of a behavior based approach that enables coordination and control of heterogeneous robots without coordination. The third is distributed robot architecture (DIRA), it is a hybrid approach that enables both robot autonomy and coordination in heterogeneous robot clusters [1].

1.3.1 The nerd herd

One of the first studies of social behaviors in robot clusters was realized by Mataric with results being demonstrated on the Nerd Herd team of twenty identical robots as shown in Fig. 1.1. This work is an example of swarm robotic systems. The decentralized control approach was based in this architecture, and it is assumed that all robots are homogeneous with relatively simple individual capabilities, such as detecting obstacles and other robot cluster members. A set of basic social behaviors is defined and demonstrated, including obstacle avoidance, homing, aggregation, dispersion, following, and safe moving.



Figure 1.1 : The nerd herd team

These basic behaviors are combined in various ways to form more complex social behaviors, including flocking (composed of safe moving, aggregation, and dispersion), surrounding (composed of safe moving, following, and aggregation), herding (composed of safe moving, surrounding, and flocking), and searching for

resources (composed of safe moving, dispersion, following, homing, and flocking) [1].

1.3.2 Alliance architecture

Another example of robot cluster architectures is the alliance architecture, developed by Parker for fault-tolerant task sharing in heterogeneous robot clusters. This approach offers an architecture of behavior sets and motivations for achieving action selection without agreement between robots. Behavior sets group low-level behaviours together for the execution of a task. The motivations consist of levels of impatience and acceptance that can raise and lower a robot's interest in activating a behavior set corresponding to a task that must be realized [1].

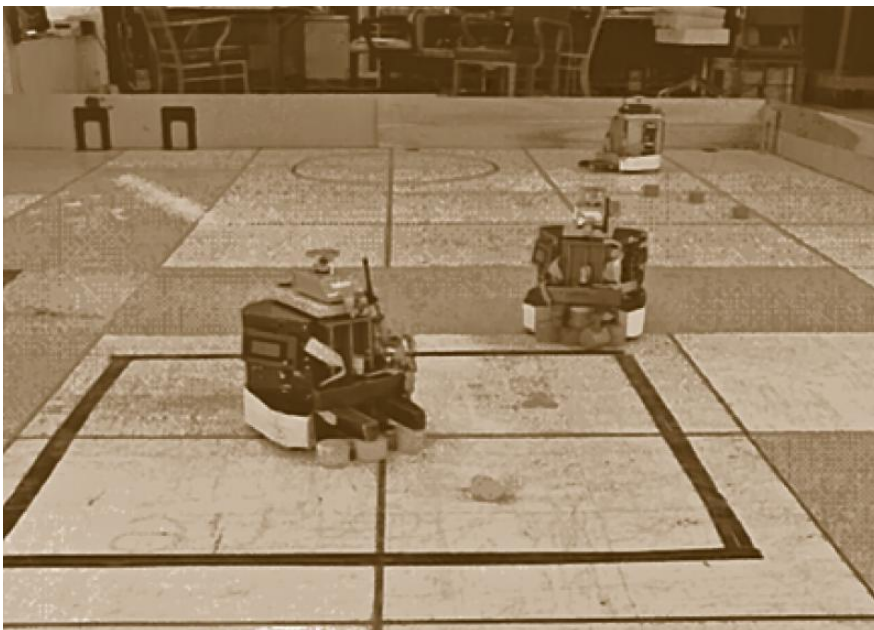


Figure 1.2 : Alliance architecture

1.3.3 Distributed robot architecture

Simmons developed a hybrid architecture called the distributed robot architecture (DIRA). Similar to the Nerd Herd and alliance architectures, the DIRA approach allows autonomy in individual robots. However, unlike the previous approaches, DIRA also facilitates coordination between robots. This approach is based on layered architectures that are popular for single-robot systems. In this approach, each robot's control architecture consists of a planning layer that decides how to achieve high-level goals; an executive layer that synchronizes agents, sequences tasks, and

monitors task execution; and a behavioral layer that interfaces to the robot's sensors and effectors. Each of these layers interacts with those above and below it. Additionally, robots can interact with each other by direct connections at each of the layers [1].

This architecture is demonstrated in a team of three robots; a crane, a moving eye, and a mobile manipulator; performing a construction assembly task as shown in Fig. 1.3. This task requires the robots to work together to connect a beam at a given location. In these demonstrations, a foreman agent decides which robot should move the beam at which times. Initially, the crane moves the beam near the platform based on encoder feedback. The foreman then sets up a behavioral loop between the roving eye and the crane robot to servo the beam closer to the platform. Once the beam is close enough, the foreman orders the moving eye and the mobile manipulator to make the arm to catch the beam. After contact is made, the foreman tasks the moving eye and the mobile manipulator to coordinate to move the beam to the desired point, and complete the task [1].

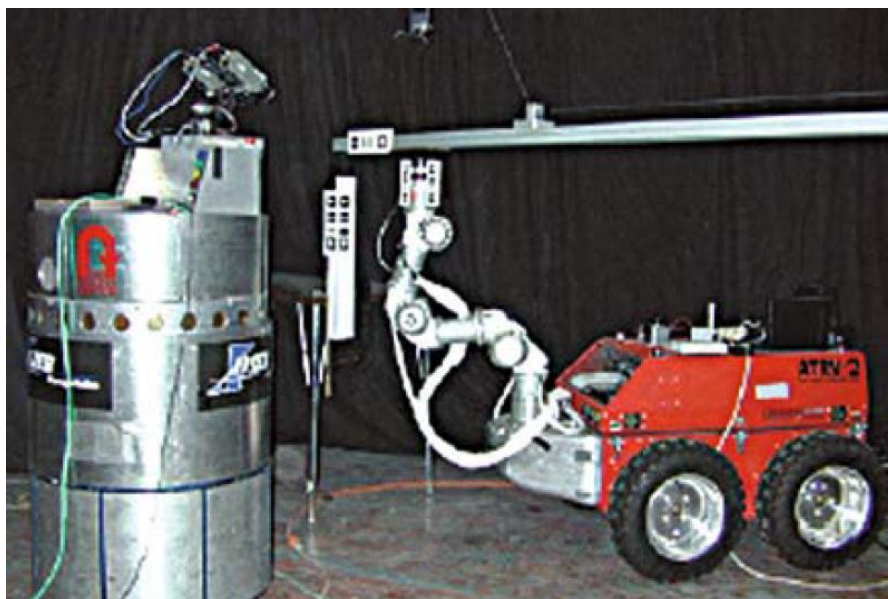


Figure 1.3 : Distributed robot architecture

1.3.4 Swarm robots

Swarm robotics systems are often called collective robotics, indicating that individual robots are often unaware of the actions of other robots in the system except the distances to the neighboring robots. These approaches aim to achieve a

desired team-level global behavior from the interaction dynamics of individual robots following relatively simple local control laws. These systems assume very large numbers of robots (at least dozens, and often hundreds or thousands). Swarm robotic approaches achieve high levels of redundancy because robots are assumed to be identical, and thus interchangeable with each other [1].

Many types of swarm behaviors are studied, such as searching for resources, flocking, chaining, herding, aggregating, and protecting. The majority of these swarm behaviors deal with distributed robot cluster motions, requiring robots to coordinate motions either relative to other robots, relative to the environment, relative to external agents, relative to robots and the environment, relative to all (other robots, external agents, and the environment)

Much of the current research in swarm robotics is aimed at developing specific solutions to one or more of the swarm behaviors. Some of these swarm behaviors received more attention like flocking, searching for resources. In general, most current work in the development of swarm behaviors is aimed not just at demonstrating group motions that are similar to biological systems, but also at understanding the formal control theoretic principles that can predictably converge to the desired group behaviors, and remain in stable states. Demonstration of physical robot swarms is both a hardware and a software challenge. The first demonstrations were by Mataric, involving about twenty physical robots performing aggregation, dispersion, and flocking. This work defined suitable basis behaviors as primitives for structuring more complex systems. More recently, McLurkin developed an extensive catalog of swarm behavior software, and demonstrated these behaviors on about a hundred physical robots called the SwarmBot robots, developed by iRobot [1]. They are shown in Fig. 1.4.

He created several group behaviors, such as avoid many robots, disperse from source, disperse from leaves, disperse uniformly, compute average bearing, avoid many robots, follow the leader, orbit group, navigate gradient, cluster on source, and cluster into groups. A swarm of a hundred and eight robots used the developed dispersion algorithms in an empty schoolhouse of area of about three hundred square meters, and were able to locate an object and lead a human to its location [1].

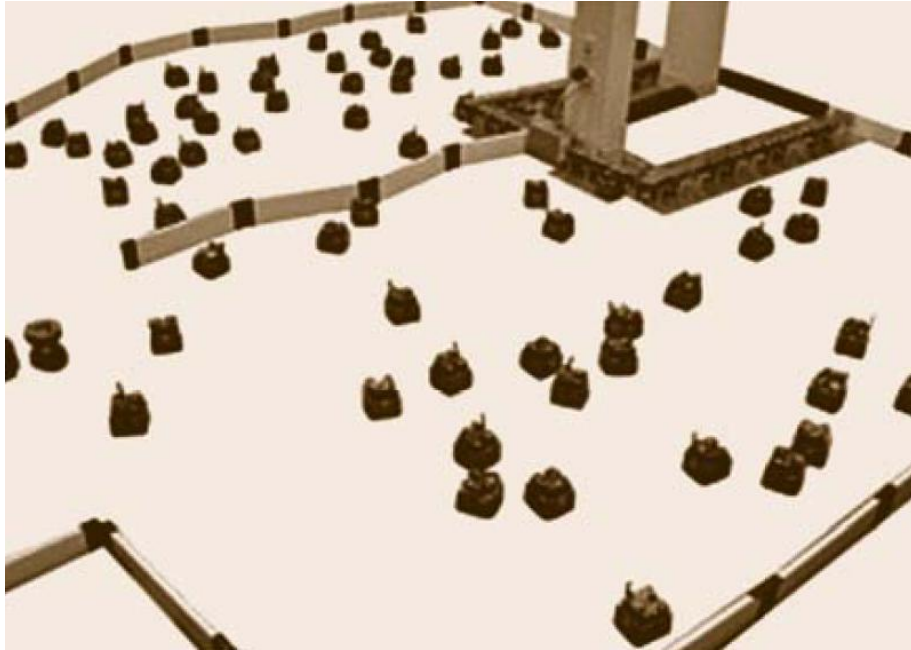


Figure 1.4 : SwarmBot robots

The European Union sponsored several swarm robot projects, smaller sized individual robots are more frequently used in newer projects. As an example, the i-swarm project is aimed at developing millimeter-sized robots with talent of sensing, computing, and having power for performing natural swarming behaviors and collective perception tasks. This project is both a hardware and a software challenge, developing very small robots that are fully autonomous and can perform meaningful cooperative behaviors will require significant advances in the current state of the art.

One of the important examples of the studies in this field is the US multi university swarms led by the University of Pennsylvania. Research in this project is aimed at developing a new system theoretic framework for swarming, developing models of swarms and swarming behavior, analyzing swarm formation, stability, and robustness, producing emergent behaviors for active perception, and developing algorithms for distributed localization [1].

Besides the hardware challenges of dealing with large numbers of small robots, there are many important software challenges that remain to be solved. From a practical perspective, the usual approach to creating homogeneous multirobot swarms is to hypothesize a possible local control law or laws, and then study the resulting group behavior, iterating until the desired global behavior is obtained. However, the longer-term objective is to be able to both predict group performance based on known local

control laws, and to generate local control laws based upon a desired global group behavior. Active research by many investigators is keeping on to develop solutions to these key research challenges.

1.4 Problem

In this study, the work in [15] is closely investigated. In [15] formation control of multi vehicles is investigated, two approaches are examined in this study. The first one is artificial potentials based approach, the other is formation graphs based approach. In [15], the work in [11] is based for potential field based approach and contributions to this work is accomplished and [13] is investigated for formation graph approach and contributions to this work is performed.

In this thesis, contributions to the work in [15] is realized. In [15] vehicles are modeled as point masses in two dimensional space, this model is enhanced to two dimensional vehicles with orientation angles. This vehicle model has orientation in two dimensional space, it has angular position and velocity. These vehicles have kinematic constraints in two dimensional space.

1.5 Organization Of The Thesis

In the first chapter; history of the studies on multirobot systems, basic concepts in this area and examples of some architectures are presented. In the second chapter, kinematic and dynamic model of robots are defined. Two control approaches are studied in the thesis. Potential Field Based formation control of robot clusters is examined in third chapter. Formation graphs based formation control of robot clusters is examined in fourth chapter. Simulation studies are presented in fifth chapter. Conclusion is mentioned in sixth chapter.

2. KINEMATIC AND DYNAMIC MODEL OF MOBIL ROBOTS

Each vehicle in the vehicle groups has the state vector $x_i = (q_{xi} \ q_{yi} \ \mu_i \ p_i \ w_i)^T$, $x \in R^5$, where q_{xi} and q_{yi} represent position vector, μ_i represents orientation angle and p_i , w_i represents linear and angular velocities respectively for the vehicle i and this can be shown in Figure 2.1. Dynamical equations can be derived by the following nonlinear equation set:

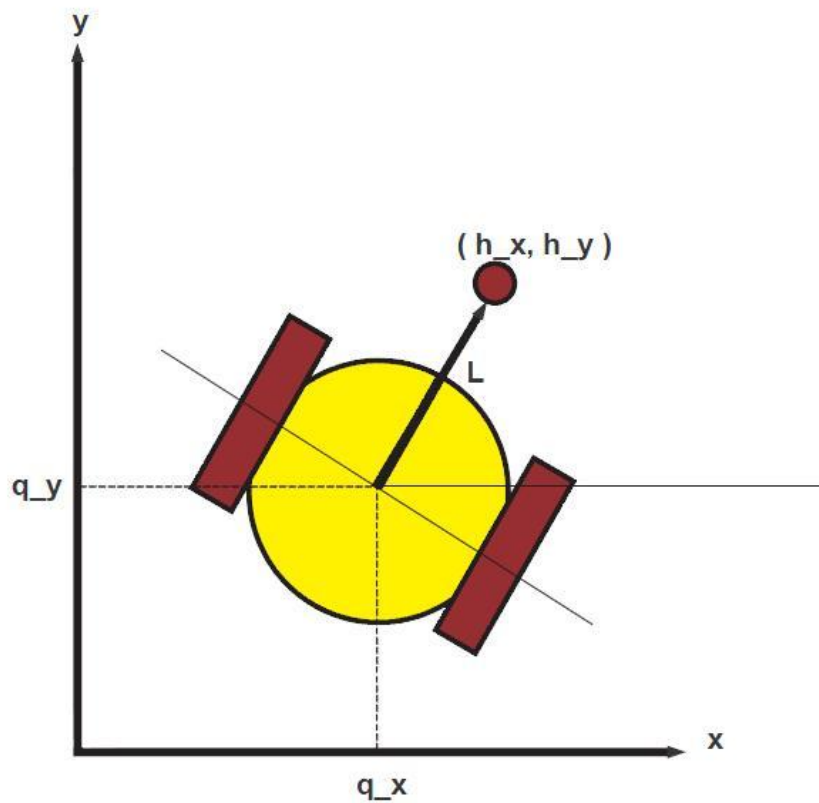


Figure 2.1 : Handle point

where F_i and T_i are force and torque inputs affecting center of the vehicle i . As it is seen in the state equation, state transition terms are nonlinear while the input terms are in linear relationship. Hence the state equation, in general, can be given as $\dot{x}_i = f(x_i) + g_i u_i$. It is obvious that the input vector u for vehicle i is formed as $u_i = (F_i \ T_i)$. Due to constraints in the wheels of the vehicles, the input variable F may not cause a motion along some directions.

In our general approach those forces are derived from gradient operator of the potential fields. Hence, lets assume that the external forces are applied a specific point instead of the center of the vehicle, namely, handle point, as it is shown in Figure 2.1. The distance of the point from the center is given by L parameter. Then point h_i is defined by:

$$h_i = q_i + L_i \begin{pmatrix} \cos \theta_i \\ \sin \theta_i \end{pmatrix} \quad (2.1)$$

$$h_i = \begin{pmatrix} \cos \theta_i - L_i \sin \theta_i \\ \sin \theta_i \quad L_i \cos \theta_i \end{pmatrix} \begin{pmatrix} p_i \\ w_i \end{pmatrix} \quad (2.2)$$

It is possible to define a closed map such that $\mathcal{f}(x_i) : R^5 \rightarrow R^5$ which maps the state vector x_i into a state vector assigned at handle point h_i :

$$\beta_i = \Sigma(x_i) = \begin{pmatrix} q_{xi} + L_i \cos \theta_i \\ q_{yi} + L_i \sin \theta_i \\ p_i \cos \theta_i - L_i w_i \sin \theta_i \\ p_i \sin \theta_i - L_i w_i \cos \theta_i \\ \theta_i \end{pmatrix} \quad (2.3)$$

$$\beta_i = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5) \quad (2.4)$$

The mapping \mathcal{f} between x_i and β_i is diffeomorphism [16]. The inverse mapping supply position and velocity vectors for the handling point. The orientation of a vehicle μ_i is uncontrollable as a result of the inverse mapping, however the orientation will always be aligned with the velocity vector in translational motion. For the sake of simplicity, the vehicle dynamic model will then be assumed as a double integrator in order to get rid of inertial parameters such as m_i and J_i . Thus, the dynamic of vehicle i is represented by:

$$\ddot{h} = \dot{p} \quad (2.5)$$

Formation control of these vehicles is explained in the following chapter.

3. POTENTIAL FIELD BASED FORMATION CONTROL

Vehicle dynamic is modeled as double integrator. In a flocking motion three types of potential fields exist: One between a vehicle and the virtual leader, the other between a vehicle and its neighbors and the last one is between a vehicle and the obstacles around. These potentials are explained in the following sections.

3.1 Interactions Between Vehicle And Leader

The field defined between a vehicle and a leader is an attractive field that attracts the vehicle towards the leader. For some or all vehicles a field having a global minimum at a desired distance from the virtual leader is defined. This field create a force at the direction of its gradient which attracts the vehicle at that desired location as illustrated in Figure 3.1.

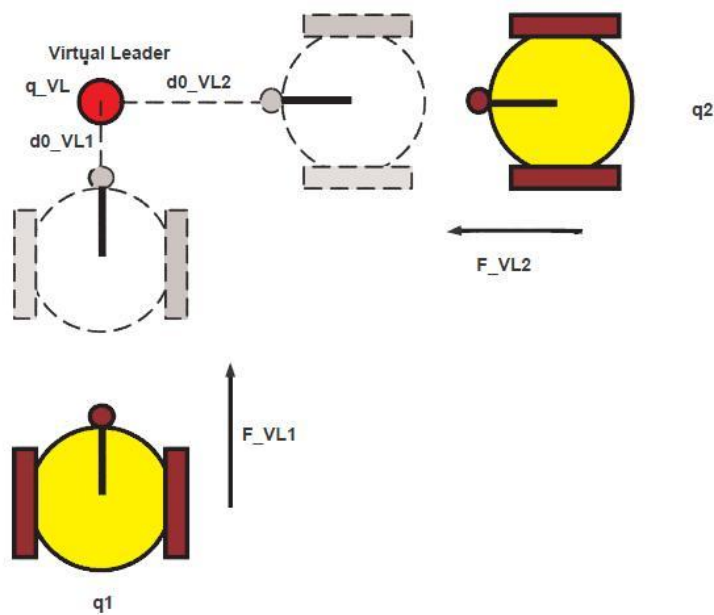


Figure 3.1 : Vehicle leader interaction

Let q_i be the position of the vehicle, q_{VL} be the position of the virtual leader, d_{i_VL0} be the desired distance of i th vehicle to the virtual leader and k_{VL} is a scalar coefficient. The potential field is defined in equation 3.1:

$$U_{VL,i} = \frac{1}{2} k_{VL} \|q_{VL} - q_i - d_{i,VL_0}\|^2 \quad (3.1)$$

The attractive force that acts on the vehicle is thus:

$$F_{VL} = k_{VL}(d_{i,VL} - d_{i,VL_0}) \quad (3.2)$$

$$d_{i,VL} = q_{VL} - q_i \quad (3.3)$$

3.2 Communication Between Neighboring Vehicles

In a desired formation, a neighboring relation between the vehicles is defined. In such a case, vehicle i is required to align its position according to a limited set of vehicles $j \in J_i$. Here J_i is the set of neighboring vehicles for i th vehicle. For example in the triangular shape in Figure 3.2, vehicles 2 and 3 are neighbors for vehicle 1, thus vehicle 1 is required to align its position according to vehicles 2 and 3.

The neighboring relationship is a unidirectional relationship.

This alignment is maintained by an artificial potential field defined in between neighboring vehicles. This field is formulated as below:

$$U_{ij} = \frac{1}{2} k_{ij} \|q_j - q_i - d_{ij0}\|^2 \quad (3.4)$$

Say q_i and q_j are positions of i th and j th vehicles respectively and k_{ij} is a scalar coefficient for intervehicle force between i th and j th vehicles. The attractive force that acts on the vehicle is as expressed in equation 3.5 and 3.6.

$$F_{ij} = k_{ij}(d_{ij} - d_{ij0}) \quad (3.5)$$

$$d_{ij} = q_j - q_i \quad (3.6)$$

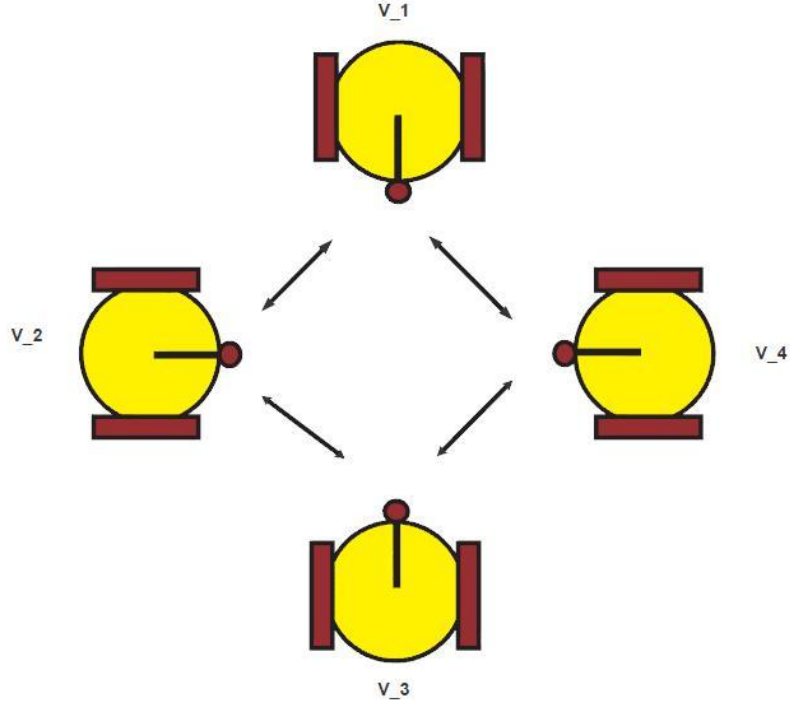


Figure 3.2 : Neighboring relationship

To simplify the expression we may take all k_{ij} coefficients equal $k_{ij} = k_{IV}$ for all vehicle connections, and express the total inter-vehicle force for a vehicle as in equation 3.7.

$$F_{IV} = k_{IV} \sum_{j \in J_i} F_{ij} \quad (3.7)$$

It is important to note that neighboring relation is defined by the final formation shape. For a running system, from the beginning to the end neighbors of a vehicle are the same regardless of their position in the space. This means a constant and stable communication link is assumed to exist in between the neighboring vehicles.

3.3 Interactions Between Vehicles And Obstacles

Say a vehicle is moving within a formation and aligning itself to the virtual leader and neighboring vehicles according to the formation constraints specified by di_{VL0} and $dij0$. In order to avoid obstacles a vehicle detects the obstacles around itself by sensing the smallest distance to that obstacle which is expressed as dOB . The idea is illustrated in Figure 3.3.

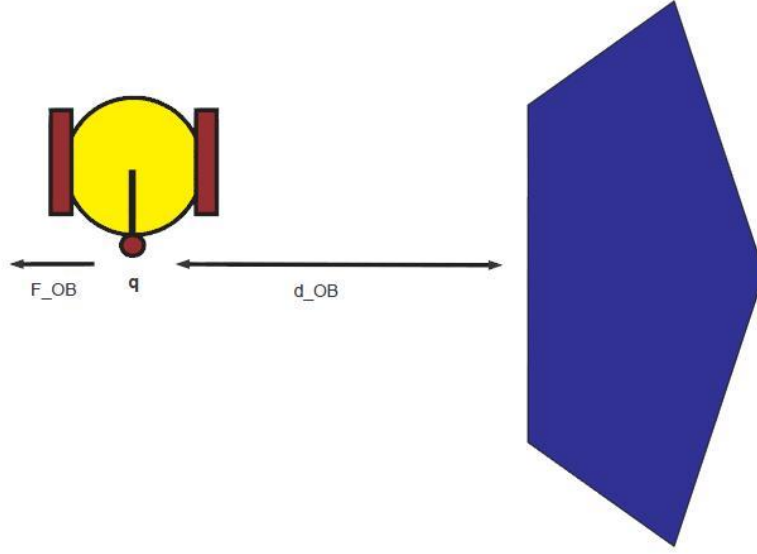


Figure 3.3 : Vehicle obstacle interaction

Say the closest point of an obstacle to the vehicle is q_{OB} . So the obstacle force on a vehicle is as expressed in equation 3.8 and 3.9. Here n is the number of obstacles the vehicle is sensing, d_{OBm} is the distance to the m th obstacle and k_{OB} is a scalar coefficient for obstacle force.

$$F_{OB} = \sum_{m=1}^n \frac{k_{OB}}{d_{OBm}} \quad (3.8)$$

$$d_{OBm} = q_i - q_{OBm} \quad (3.9)$$

In order to increase obstacle avoidance we increase the leader following behavior by rotating the obstacle force in the direction of virtual leader. To do this, a vector field I is defined around obstacles. This field takes a vector F which is directed at a direction a as its argument and turns it halfway towards a desired direction Q . The resulting direction of vector F is then β as expressed in equations 3.10 and 3.11.

$$F_{new} = \Phi(F) = |F| (i \cos \theta_{\beta} + j \sin \beta) \quad (3.10)$$

$$\beta = \begin{cases} \frac{a+\theta}{2} & \text{if } a - \theta < \pi \\ \frac{a+\theta}{2} + \pi & \text{if } a - \theta > \pi \end{cases} \quad (3.11)$$

When the vehicle is moving under close to the obstacles we want to improve the leader following behavior. This is achieved by moving to be closer to the leader when an obstacle is detected. In other words, by turning the effect of obstacle forces towards the leader's direction. Say QOB is the direction of FOB and QVL is the direction of the virtual leader. We apply the vector field I to the obstacle forces. The resulting force is and its direction is given in equation 3.12 and 3.13.

$$F_{OB_{n\epsilon w}} = \Phi (F_{OB}) \quad (3.12)$$

$$\Phi (F_{OB}) = |F_{OB}| (i \cos \theta_{OB_{n\epsilon w}} + j \sin \theta_{OB_{n\epsilon w}}) \quad (3.13)$$

$$\theta_{OB_{n\epsilon w}} = \begin{cases} \frac{\theta_{OB} + \theta_{VL}}{2} & \text{if } \theta_{OB} - \theta_{VL} < \pi \\ \frac{\theta_{OB} + \theta_{VL}}{2} + \pi & \text{if } \theta_{OB} - \theta_{VL} > \pi \end{cases} \quad (3.14)$$

The rotation of obstacle force is illustrated in Figure 3.4. This modification shows greater performance for cornerings. In corners when virtual leader turns sharp corners FVL and FOB acts on opposite directions for the vehicles in the back of the group. Since FVL is proportional to distance to leader and FOB is inversely proportional to distance to obstacle, vehicle in this Figure ends up moving back and forth towards the obstacle surface. The rotation prevents this and makes the vehicle follow the leader more when an obstacle exists.

3.4 Shape Of The Formation

Formation shape is given by desired intervehicle distances. Say a vehicle i has neighbors in the set J_i and desired distances d_{ij0} for each element in J_i where intervehicle potential U_{ij} is 0. The set J_i determines the positioning of vehicle i according to its neighbors in the specified formation. The set $J = \{J_i; i = 1 \dots n\}$ where n is the number of vehicles gives the shape of the formation.

Since every vehicle is connected to a non-empty set of neighboring vehicles, circle, line, triangle, or any arbitrary shape can be given by distance constraints.

As a geometrical example, in the circular formation in Figure 3, vehicle 2 desired to position itself at d_{210} , d_{240} and d_{250} where for example $d_{21} = (-a, -b)$ and $d_{24} = (a, b)$. The vehicle set q and the neighboring set J for this shape is given in equation 3.15 and 3.16 respectively.

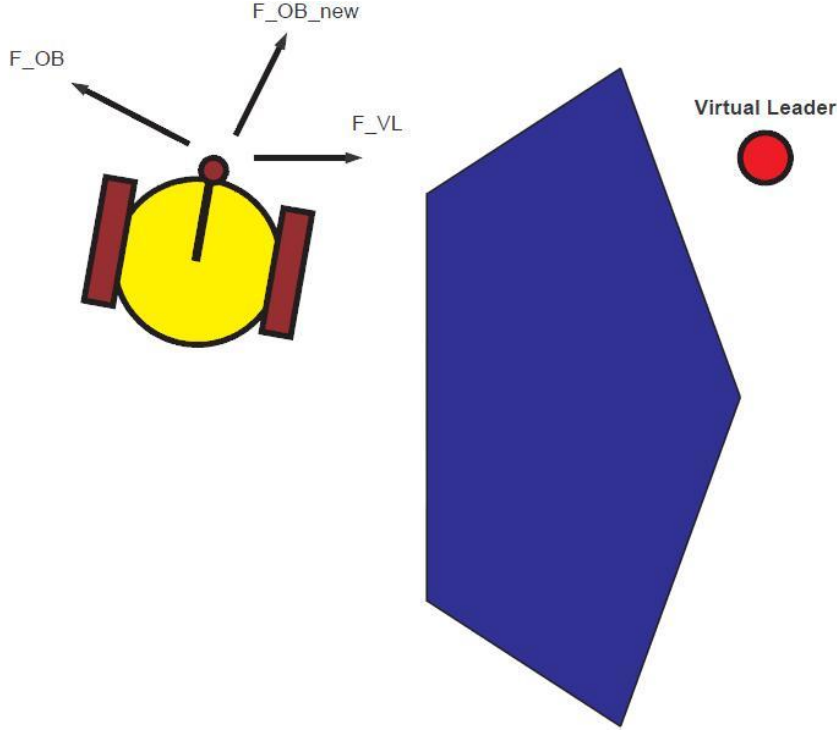


Figure 3.4 : Rotation of obstacle force

$$q = \{q_1, q_2, q_3, q_4, q_5, q_6\} \quad (3.15)$$

$$J = \begin{matrix} \{q_2, q_3\}, \\ \{q_1, q_4\}, \\ \{q_1, q_6\}, \\ \{q_2, q_5\}, \\ \{q_2, q_3, q_4, q_6\}, \\ \{q_3, q_5\} \end{matrix} \quad (3.16)$$

Distances between neighbors can be expressed as a distance set $D = \{D_i; i = 1 \dots n\}$ where each element D_i corresponds to J_i and contains distances between i th vehicle and vehicles in J_i . The D set for triangular shape is given in equation 3.17.

$$D = \begin{matrix} \{(b,a), (-b,a)\}, \\ \{(-b,-a), (b,a)\}, \\ \{(b,-a), (-b,a)\}, \\ \{(-b,-a), (-2b,0)\}, \\ \{(b,-a), (-b,-a), (-2b,0), (-2b,0)\}, \\ \{(b,-a), (2b,0)\} \end{matrix} \quad (3.17)$$

3.5 Movement

The vehicles move by the double integrator dynamics given in equation (7). Each vehicle calculates its own separate control signal u using the net force acting on itself. According to the theorem in Olfati-Saber and Murray [13] a damping component is added to ensure stability. The damping component expressed in

equation 3.16 is similar to the frictional force, acting in the opposite direction of speed. In this equation k_f is the damping coefficient and p is the speed.

$$F_D = -k_f \frac{p}{\sqrt{1 + \|p\|}} \quad (3.18)$$

Net force F_{NET} is the sum of virtual leader force F_{VL} , intervehicle forces F_{IV} and obstacle forces F_{OB} and expressed in equation 3.19. The control signal for a vehicle is expressed in equation 3.20.

$$F_{NET} = F_{VL} + F_{IV} + F_{OB} \quad (3.19)$$

$$u = F_{NET} - F_D \quad (3.20)$$

The control signal for a vehicle is limited as expressed in equation 3.21.

$$\bar{u} = \begin{cases} +\lambda & \text{if } u > \lambda \\ u & \text{if } -\lambda < u < \lambda \\ -\lambda & \text{if } u < -\lambda \end{cases} \quad (3.21)$$

4. FORMATION GRAPHS BASED FORMATION CONTROL

In this section, several basic concepts and the control system presented in Olfati-Saber and Murray [13] are summarized. It is important to note that this study provides the base and the proofs for the ideas and formulations presented here and interested reader is encouraged to read it also.

A directed graph $G = (V; E)$ consists of a vertex set $V = \{v_1, v_2, \dots, v_n\}$ and an edge set E . Here $E \subset V^2$ and each e_{ij} element of E is defined as $e_{ij} = (v_i; v_j)$ for vertices v_i and v_j that have a connection. First element, v_i is called "head" and the second element v_j is called "tail" of the edge. If the head and the tail of an edge are the same graph is said to contain a self-loop at that vertex. Throughout this article we will assume graphs that contain no self-loops. The connectivity number of vertex v_i is defined as the edges leaving v_i and denoted by $|v_i|$. Connectivity number of a vertex is the number of neighbors of that vertex. The degree of a graph is defined as $\max(|v_i|)$ where $i \in [1; n]$ and denoted as $\deg(G)$. Note that $\deg(G)$ is the maximum number of neighbors that any vertex has in G .

Similarly a formation graph is a triplet $G = (V_e, C, D)$ that consists of an extended vertex set V_e , a connectivity set C and a distance set D . V_e is defined as $\{V_\infty; v\}$ where v_∞ is a virtual vertex at infinity and used only as a notational element in order to make connectivity numbers for all vertices equal. This is required in order to represent C and D sets as matrices. C and D are $[\deg(G) \times n]$ matrices.

C matrix shows the neighboring relations and is defined as in equation 4.1.

$$C_{ij} = \begin{cases} j & ; \text{if } V_i \text{ aligns itself according to } V_j \\ \infty & ; \text{otherwise} \end{cases} \quad (4.1)$$

D matrix shows the distance constraints for each edge and is defined as in equation 4.2

$$d_{ij} = \begin{cases} \|q_j - q_i\| & ; \text{if } V_i \text{ alligns itself according to } V_j \\ b & ; \text{otherwise} \end{cases} \quad (4.2)$$

Note that neighboring relations are one-way, meaning that if v_j is a neighbor to v_i , v_i may not be a neighbor to v_j , thus C and D matrices are not symmetrical. Here q_j and q_i show the positions for i th and j th vehicles respectively, and d_{ij} is the desired euclid distance between the two. An example formation graph is shown in Figure 4.1. Corresponding V_e and C matrix are given in equation 4.3 and 4.4.

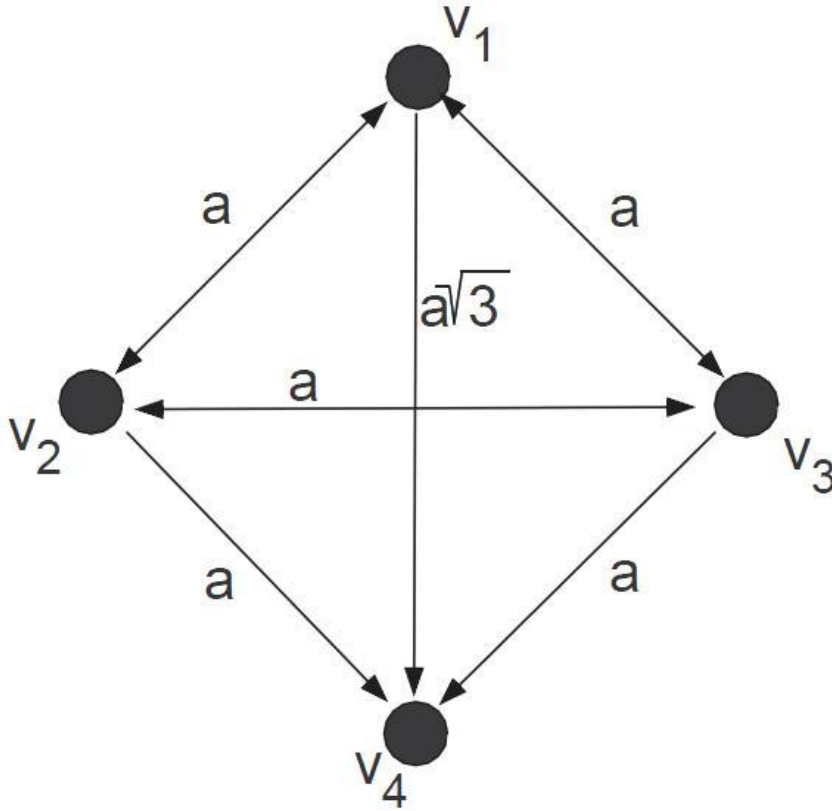


Figure 4.1 : Formation graph

$$V_e = \{ V_1, V_2, V_3, V_4, V_\infty \} \quad (4.3)$$

$$C = \begin{matrix} & V_2 & V_3 & V_\infty \\ V_1 & & & \\ V_2 & & & \\ V_3 & & & \\ V_4 & & & \\ V_\infty & & & \end{matrix} \quad (4.4)$$

Consider n identical vehicles with point mass dynamics and a formation graph G . Here q is the state information of vehicles. In this work state information is 2D

position information which implies that p and u are similar to velocity and acceleration respectively.

$$\dot{q}_i = p_i \quad (4.5)$$

$$\dot{p}_i = u_i \quad (4.6)$$

In order to obtain the control signal for such a system, artificial structural potential $V(q)$ is used. The definition of $V(q)$ is given in equation 4.7.

$$V(q) = \langle \Phi(q), \Phi(q) \rangle \quad (4.7)$$

Here $I(q)$ is the structural constraint vector and $I(q) = \{Q_1, \dots, Q_s\}$ where s is the edge count of graph G . Q_i is defined in equation 4.8 and may be interpreted as the distance to the desired d_{ij} value for the edge between i th and j th vehicles.

$$\phi_1(q_i, q_j) = \|q_i - q_j\| - d_{ij} \quad (4.8)$$

With the help of Lagrangian and Hamiltonian equations for the system, Murray et al. [3] states that the control signal in equation 4.9 achieves local stabilization of vehicles to the formation specified by graph G .

$$u_i = \frac{\bar{u}}{|J_i|} \sum_{j \in J_i} \lambda_1 \sigma(\|q_i - q_j\| - d_{ij}) u_{ij} - \bar{u} \lambda_2 \sigma(p_i) \quad (4.9)$$

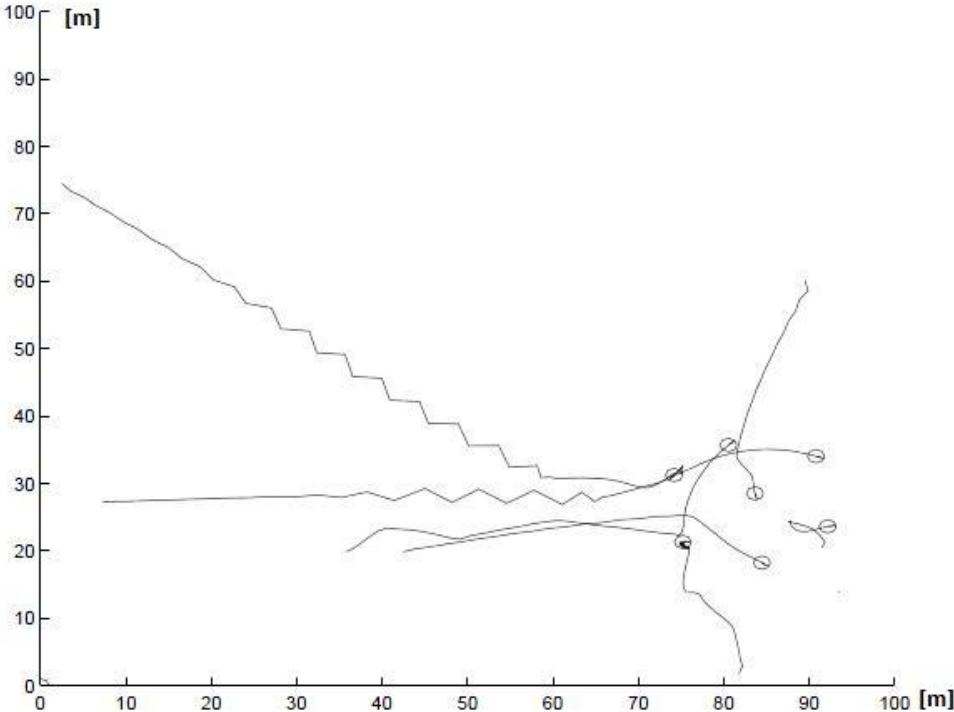
$$u_{ij} = \frac{q_j - q_i}{\|q_j - q_i\|} \quad (4.10)$$

Here \bar{u} is an upper bound for u and $\sigma(y)$ is defined as in equation 4.11. λ_1 and λ_2 are two real numbers such that $\lambda_1 + \lambda_2 = 1$.

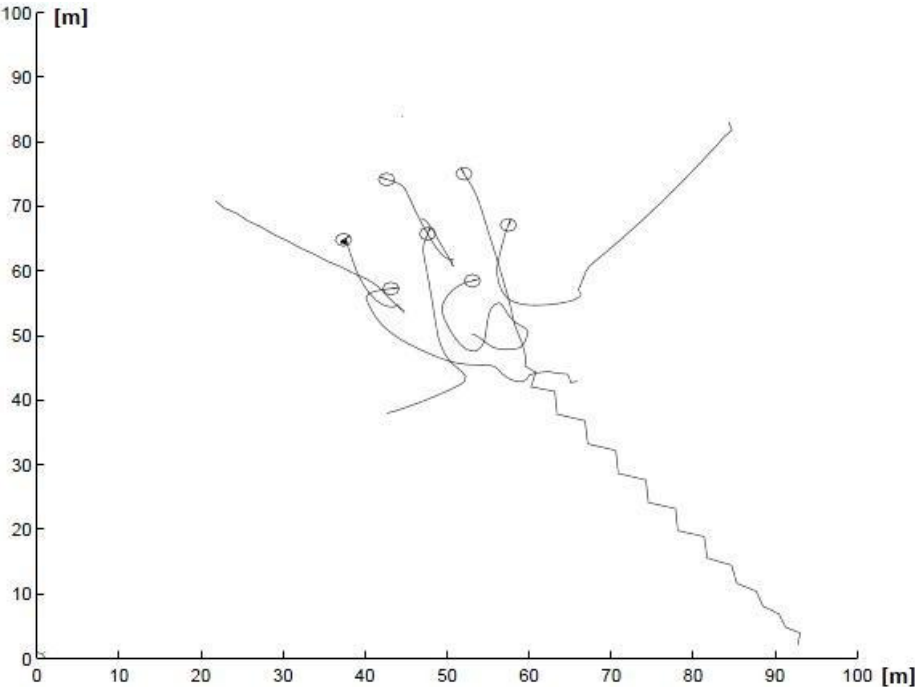
$$\sigma(y) = \frac{y}{\sqrt{1 + \|y\|^2}} \quad (4.11)$$

As stated and proved in [13] the control law stated above guarantees local formation stabilization with bounded feedback, which means vehicles starting from any initial conditions in the space forms and keeps the desired formation. However the direction of formation is not specified. A graph needs to be rigid and non-foldable in order to be able to represent a unique formation. Simulations of this system shows that different initial conditions may yield to different flocking points and formation orientations.

Figure 4.2 shows two sample runs of a system for a group of 7 vehicles starting from different initial conditions. In (a) and (b) vehicles construct the formation in different locations and formations are rotated arbitrarily.



(a)



(b)

Figure 4.2 : Sample runs

5. SIMULATION STUDIES

5.1 Potential Field Based Formation Control Simulation Studies

To demonstrate the flocking and navigation properties of our approach we present two simulations. In the first simulation six vehicles starting from arbitrary initial positions in 2D space forms a diamond around a virtual leader as shown in Figure 5.1.

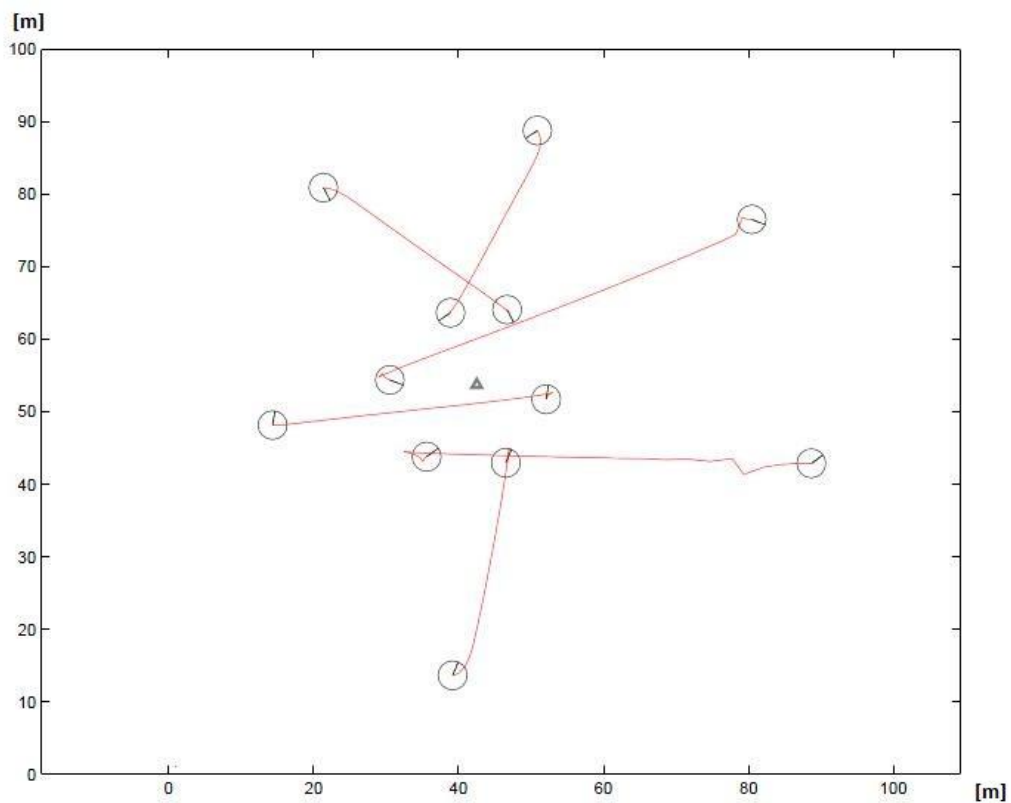


Figure 5.1 : Flocking

In Figure 5.1 circles around show the starting positions of vehicles. k parameters are selected as: $k_{VL} = 0.01$, $k_{IV} = 0.005$ and $k_f = 0.2$. Trajectories of vehicles are shown as lines in this Figure. For all vehicles horizontal and vertical distances to virtual leader are also given in Figure 5.2. Figures a to f show the horizontal differences of the six vehicles to the virtual leader. The final distance of first vehicle to virtual leader is 5 m. The final distance of second vehicle to virtual leader is 0 m.

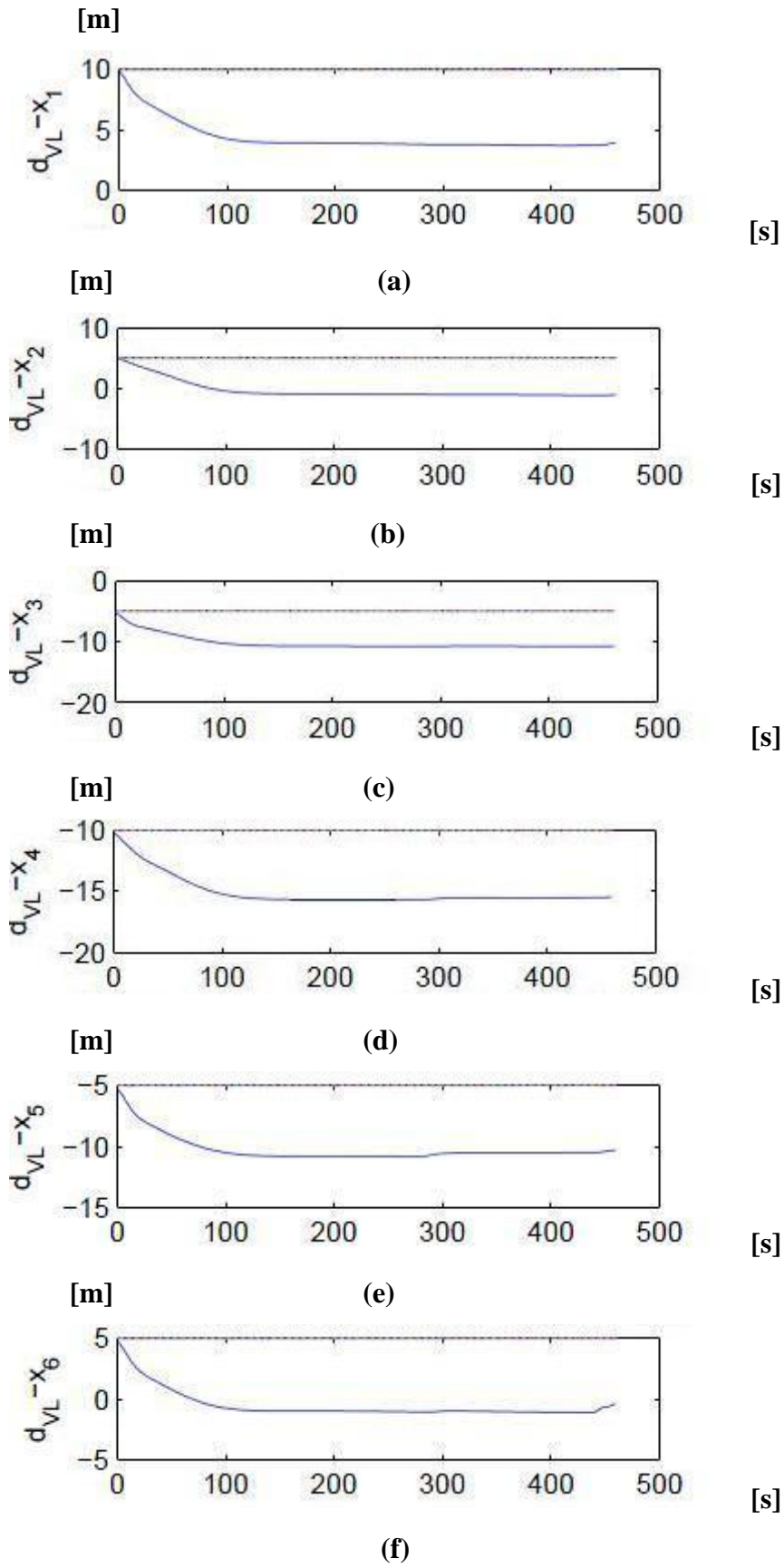


Figure 5.2 : Horizontal distances to virtual leader

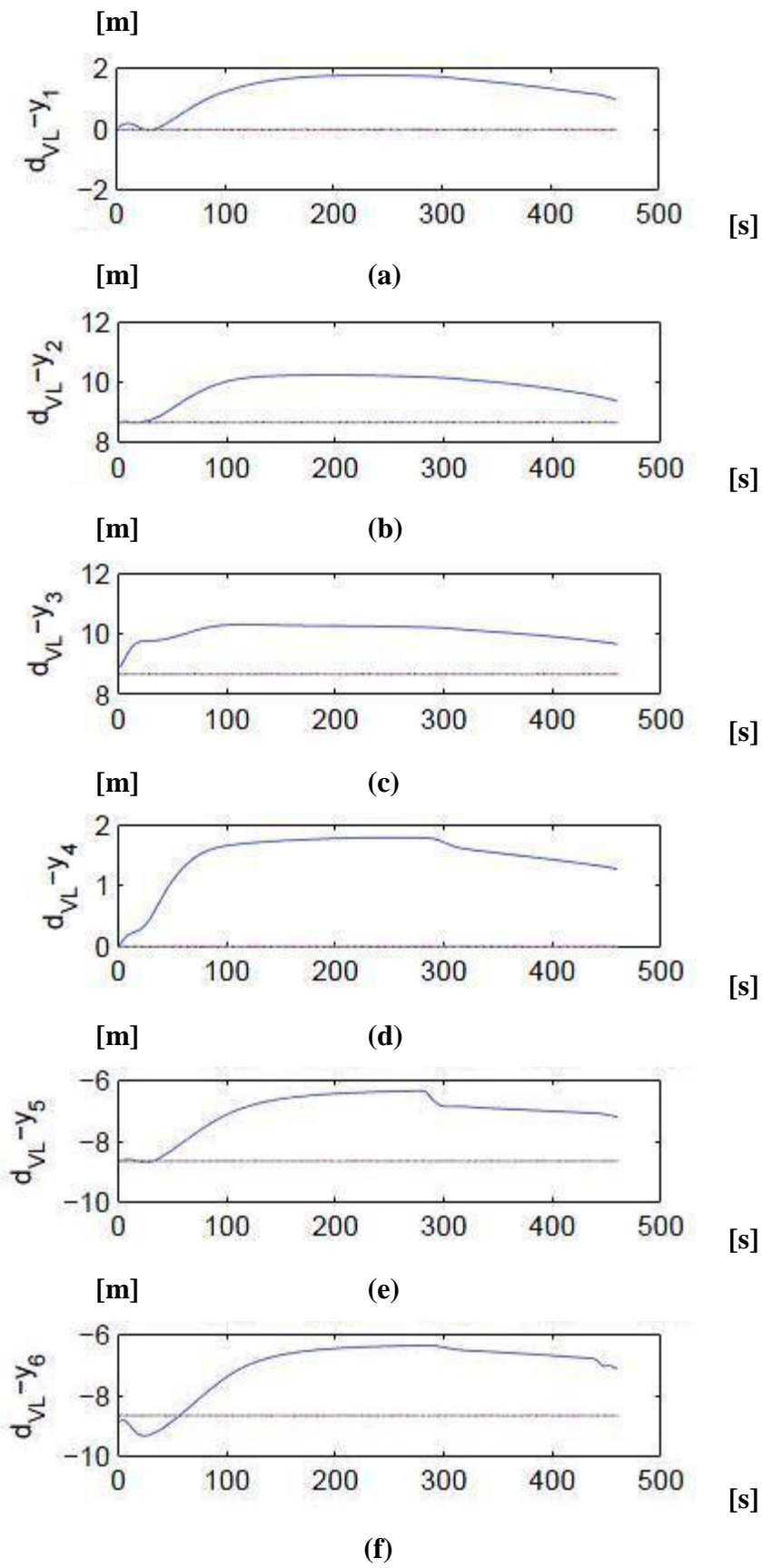
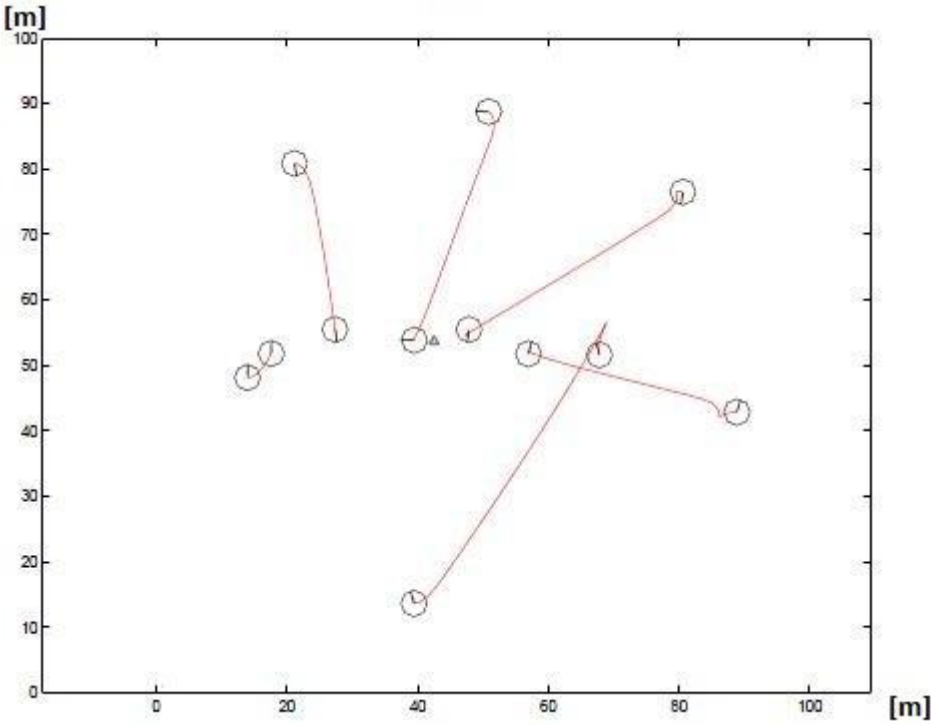
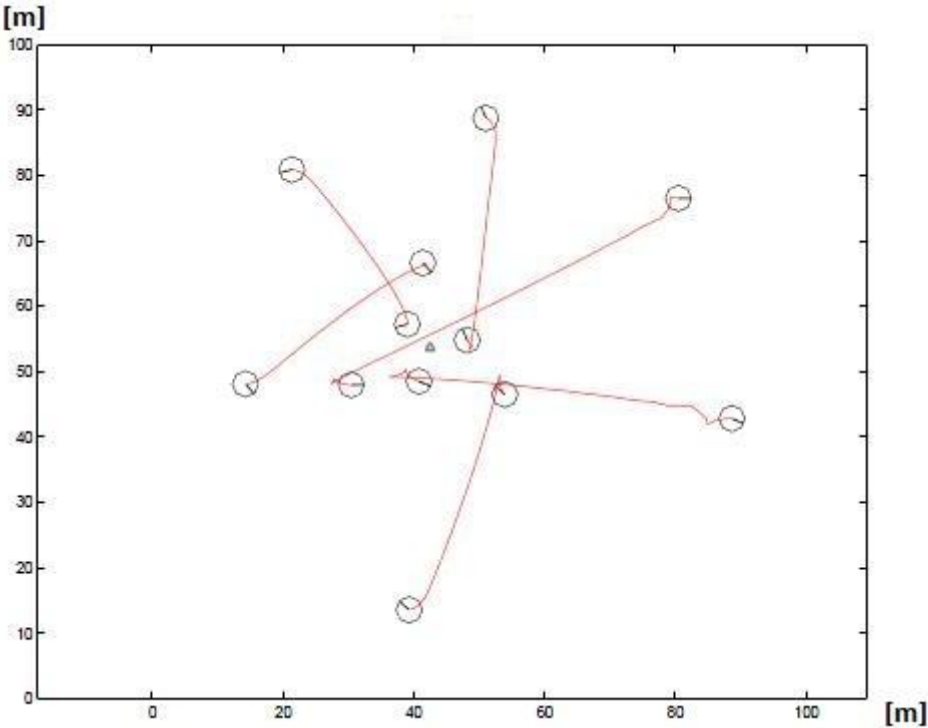


Figure 5.3 : Vertical distances to virtual leader

The final distance of third vehicle to virtual ledar is -10 m. The final distance of fourth vehicle to virtual leadar is -15m.



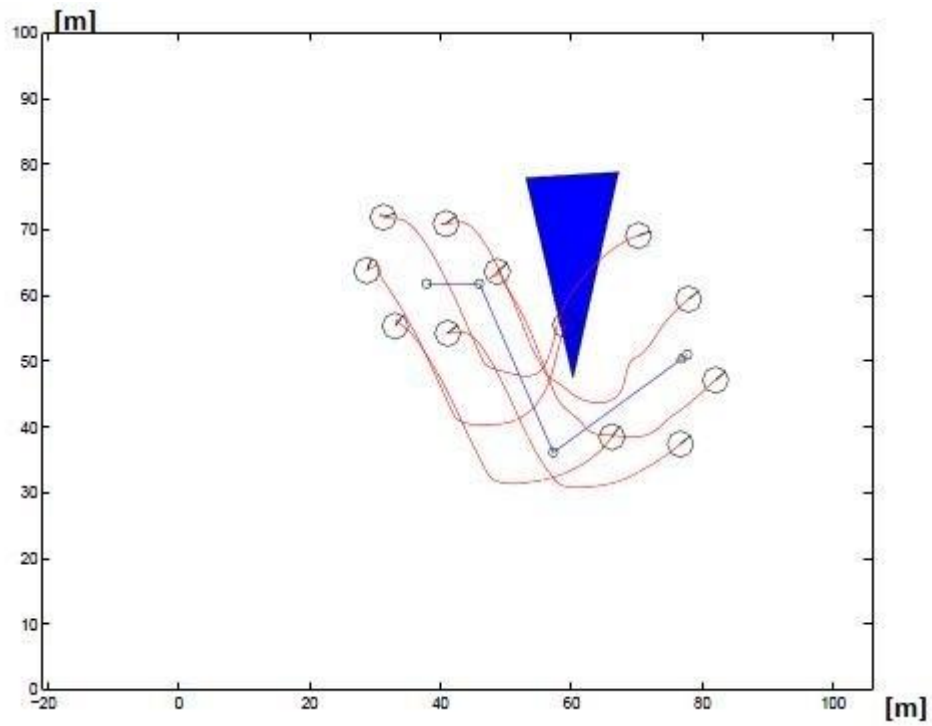
(a)



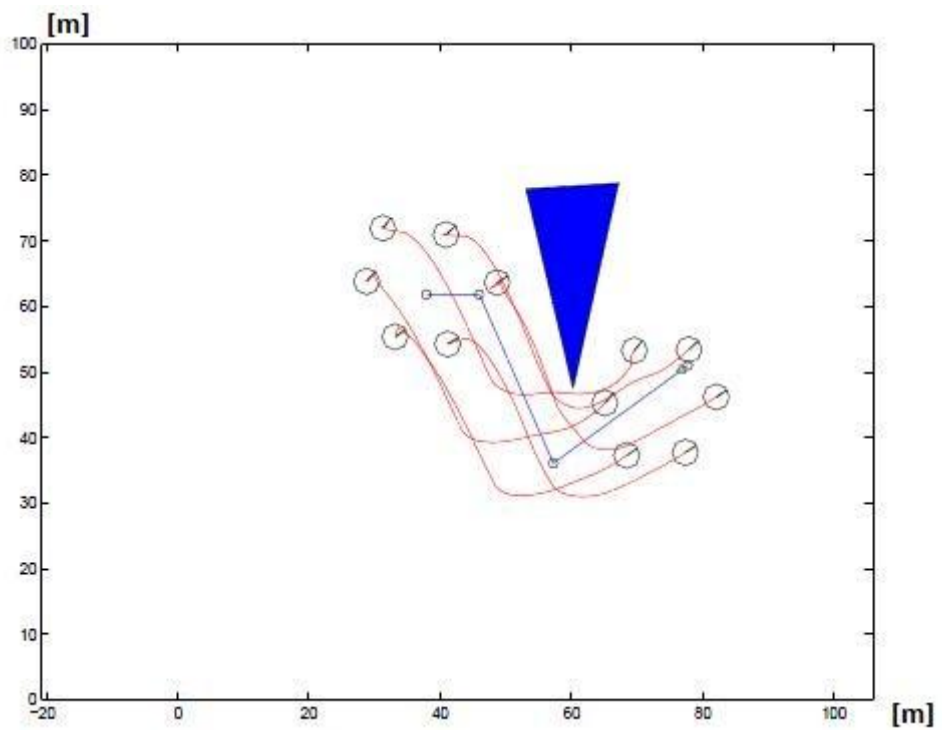
(b)

Figure 5.4 : Line and triangular formations

The final distance of fifth vehicle to virtual leader is -10 m. The final distance of sixth vehicle to virtual leader is -10 m.

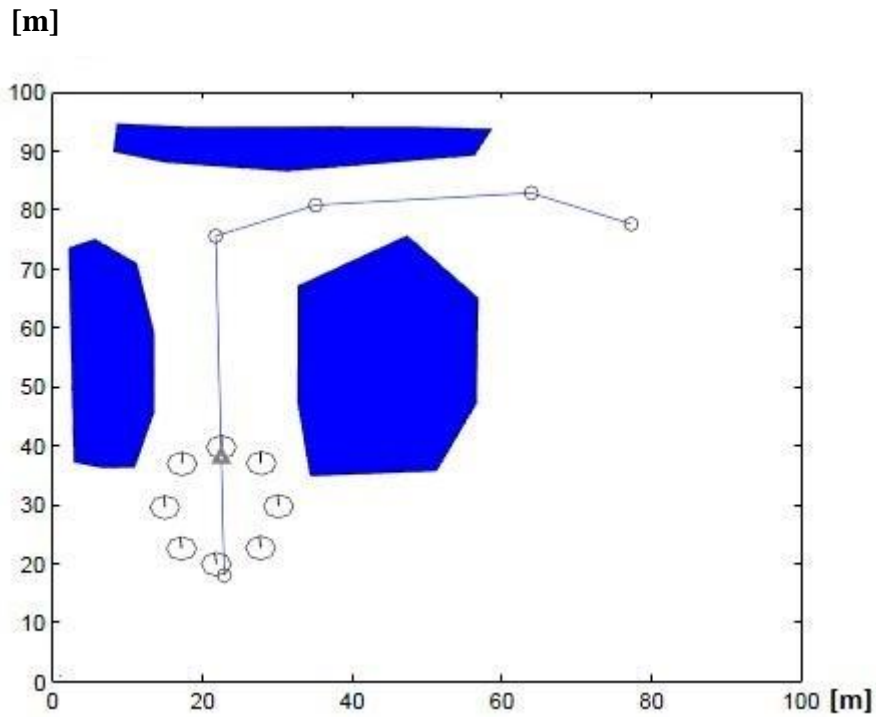


(a)

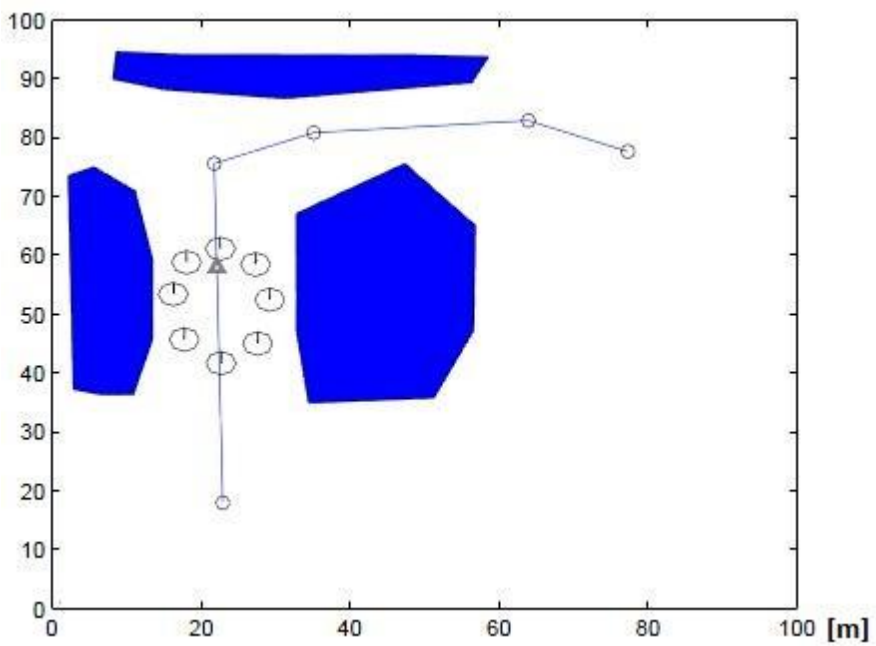


(b)

Figure 5.5 : Improved obstacle avoidance



(a)



(b)

Figure 5.6 : Navigation through obstacles

Different shapes are also possible. In Figure 5.4 a line and a triangle formations are shown.

In these Figures, vehicles starting from arbitrary locations form desired shapes around the virtual leader. In (a) vehicles form a line around the virtual leader. In (b) vehicles form a triangle around the virtual leader.

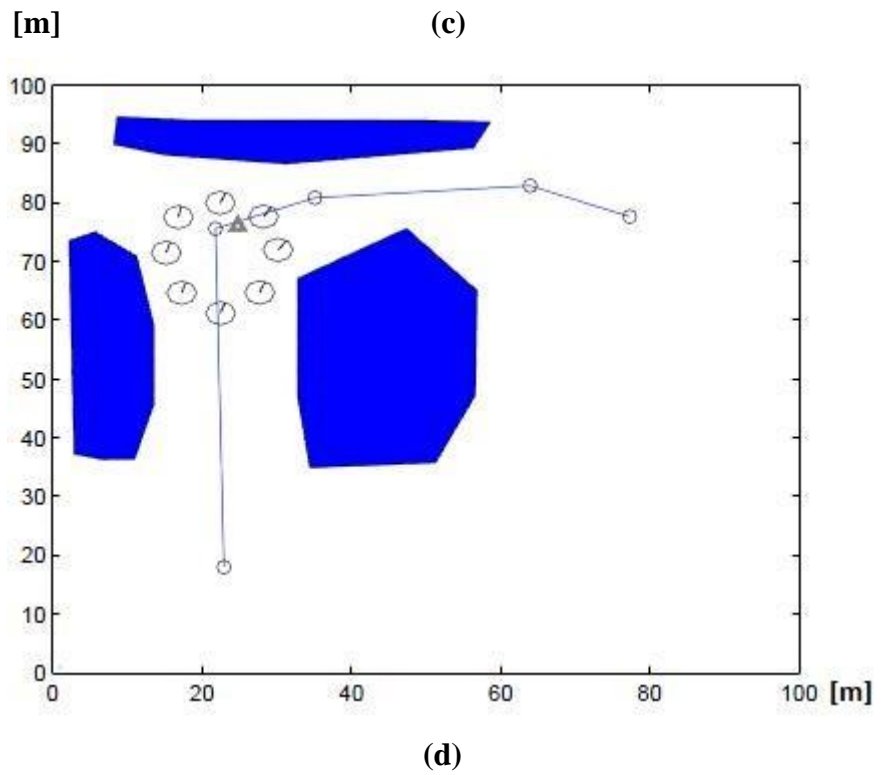
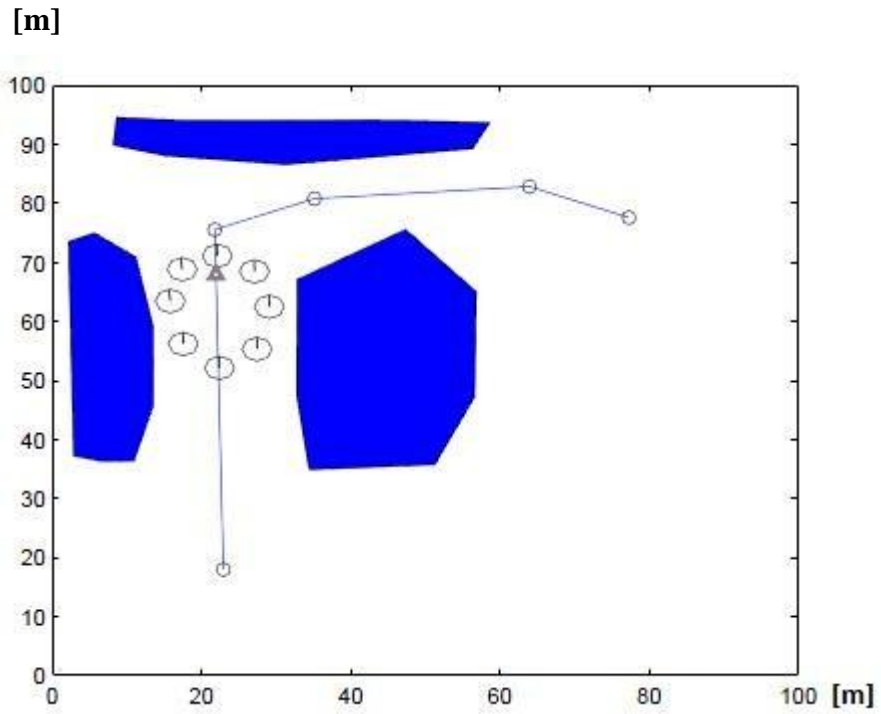


Figure 5.6 : (continued) Navigation through obstacles

To demonstrate the improved obstacle avoidance performance of the system due to artificial rotational vector fields, two simulations are presented in Figure 5.5. In (a) obstacle forces are not rotated as in Elkaim and Siegel [10].

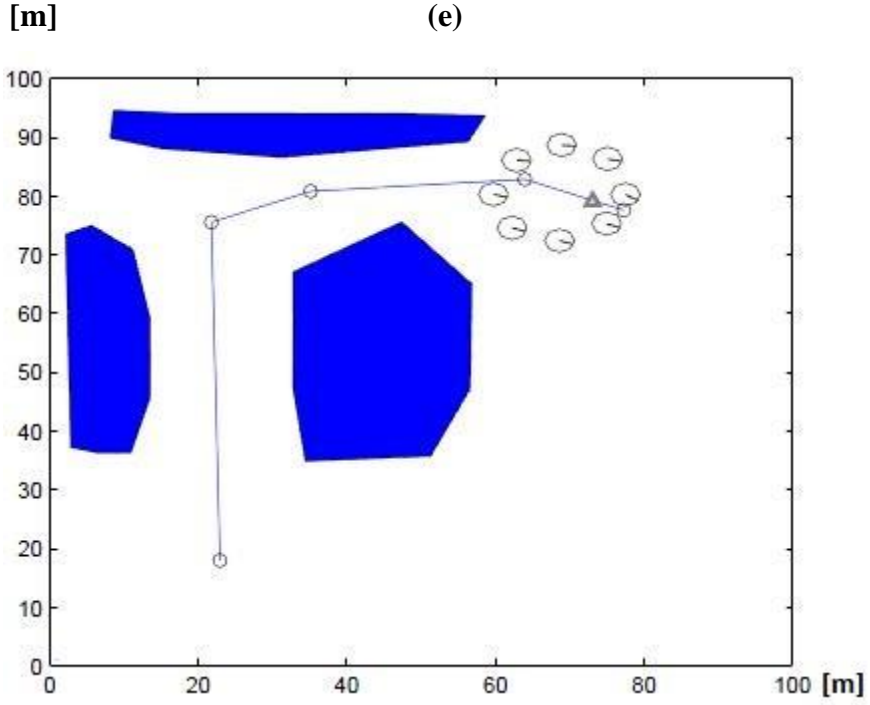
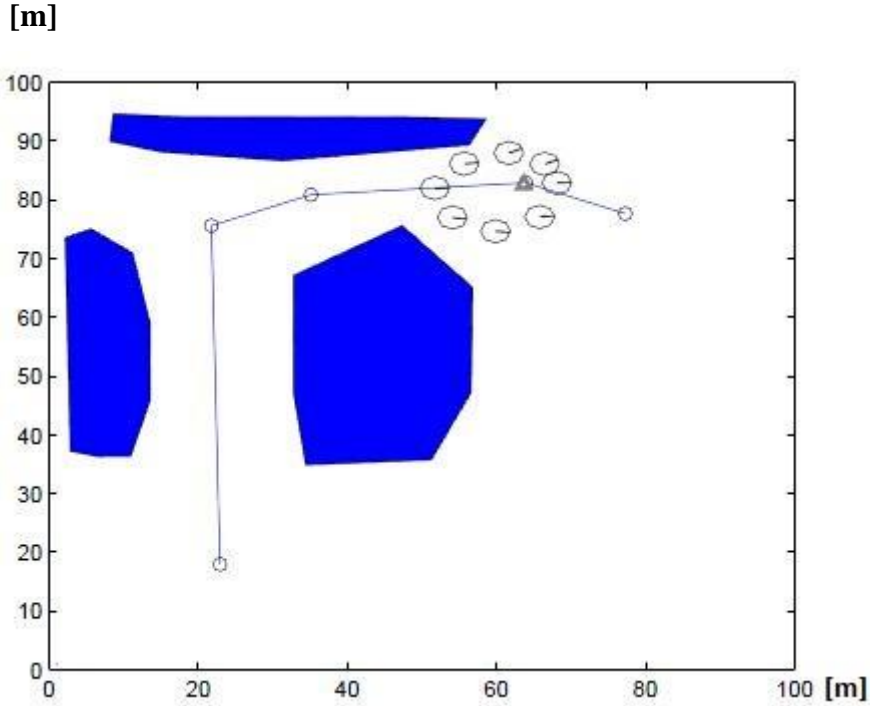


Figure 5.6 : (continued) Navigation through obstacles

Here the vehicles at the back of the group cannot follow the leader while it turns a sharp corner around the obstacle and eventually they hit the obstacle. In (b) obstacle force is rotated and as seen in Figure, all of the vehicles in the group can successfully follow the leader in the same configuration.

As we said before this method can handle any number of vehicles and ensures collision free navigation in an environment with obstacles. In the second simulation given in Figure 5.6 we show 12 vehicles in a circle formation navigating successfully between obstacles.

In Figure 5.6 the blue straight lines show the virtual leader's path and vehicles are navigating in circle formation around the leader.

In (a) vehicles narrow the circle as they enter the passage, and in (b) vehicles can be seen as they change the shape of the circle according to the shape of the environment.

Note that vehicles have no a-priori knowledge about the shape and positioning of obstacles. In (c-e) it is observed that the group can handle sharp corners also.

In (f) as the vehicles get out of the passage the original circle is formed again. Note that all the vehicles kept the formation and followed the virtual leader successfully.

Vertical distances to virtual leader are given in Figure 5.3. Figures a to f show the vertical differences of the six vehicles to the virtual leader. The final distance of the first vehicle to virtual leader is 1 m. The final distance of the second and third vehicles to virtual leader is 9 m. The final distance for the fourth vehicle to virtual leader is -1 m. The final distances of the fifth and sixth vehicles to virtual leader is -7 m.

5.2 Formation Graphs Based Formation Control Simulation Studies

In this section, some numerical simulations are presented in order to illustrate the behavior of various systems. The first simulation shows seven vehicles, whose initial positions are set randomly. The vehicles form a V-shape formation.

Figures 5.7 illustrate seven vehicles forming a V-shape formation, whose initial

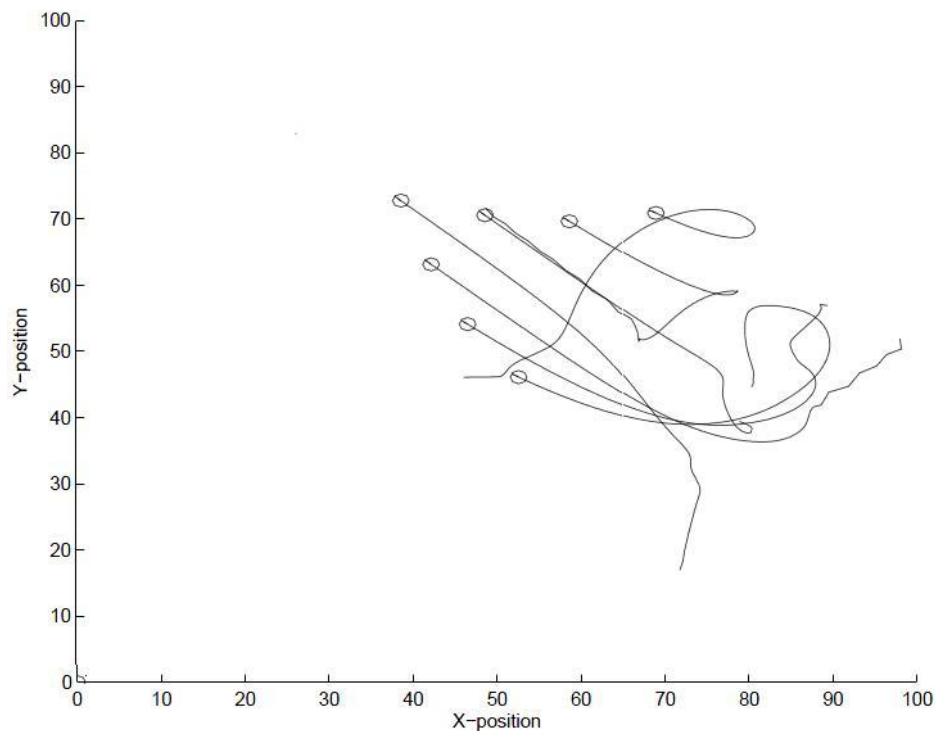
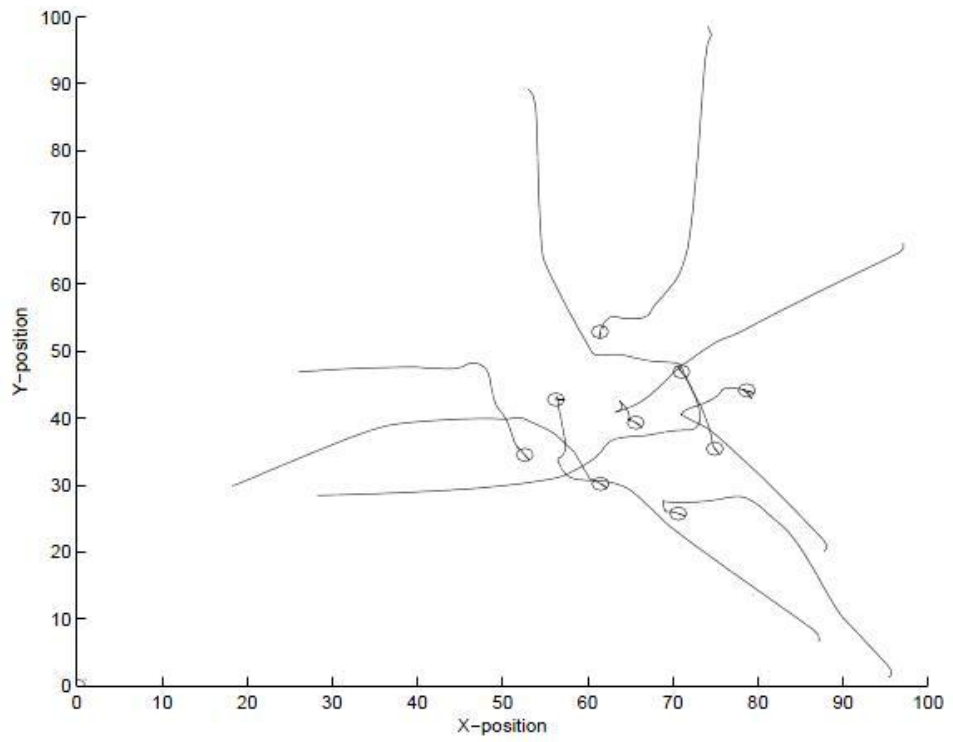


Figure 5.7 : V - shape formation

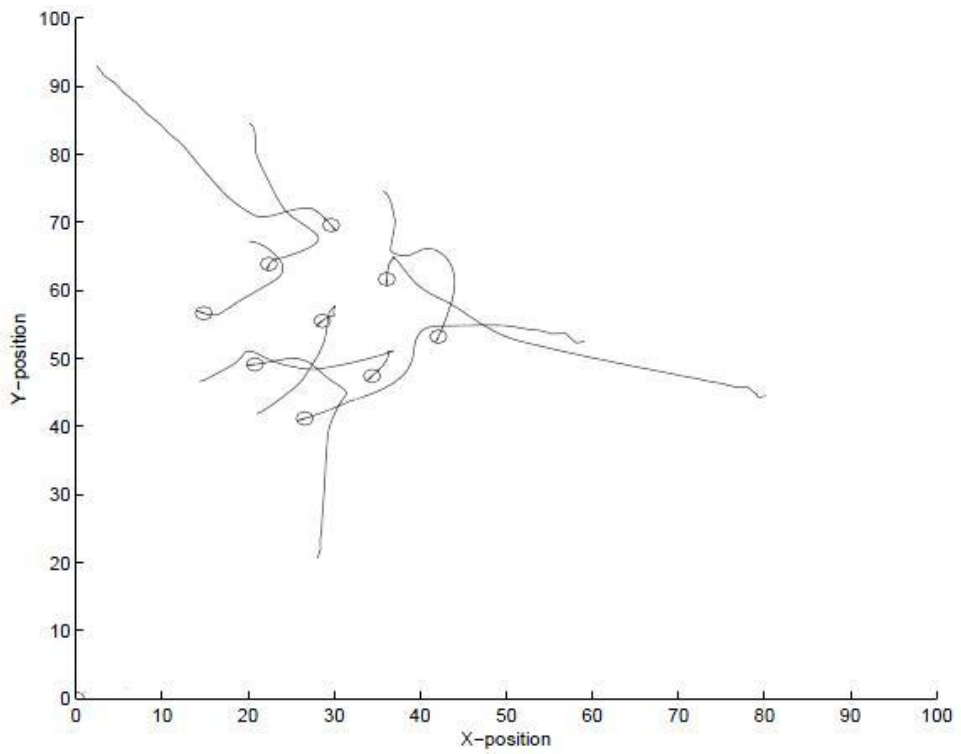
positions are set randomly.

On the contrary, Figure 5.8 shows square formations under randomly chosen initial conditions. Both in (a) and (b) the vehicles form a square shape formation, but they begin from different initial positions.

Figure 5.9 shows linear formations under randomly chosen initial conditions. In (a) and (b) five vehicles form a line formation but they begin from different initial positions.

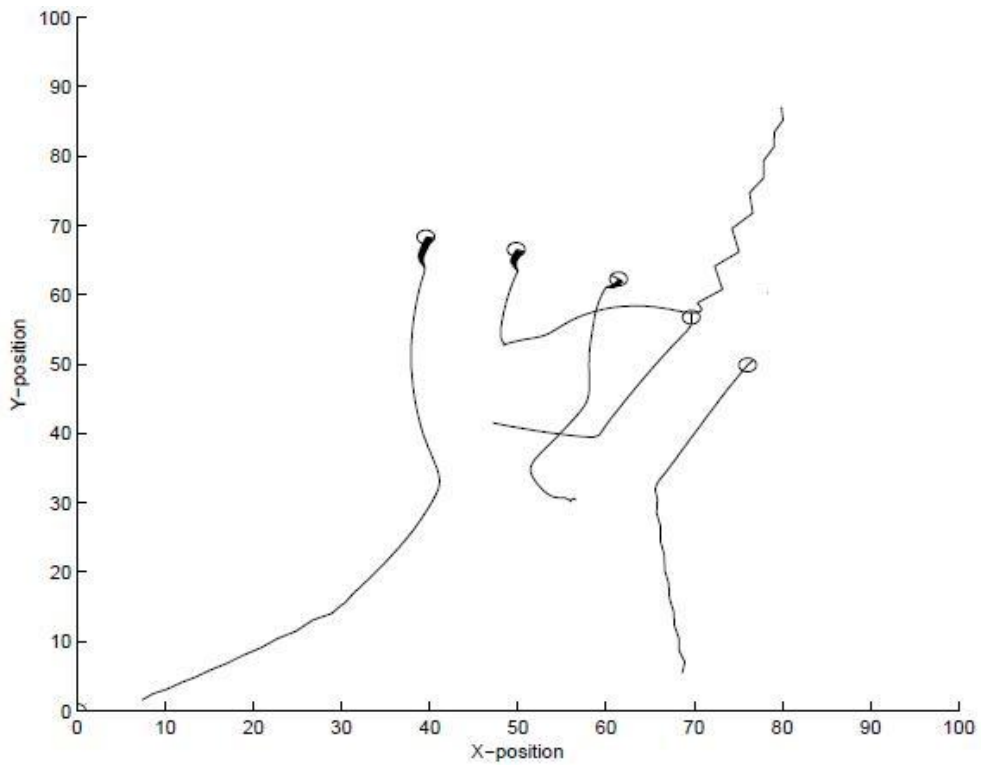


(a)

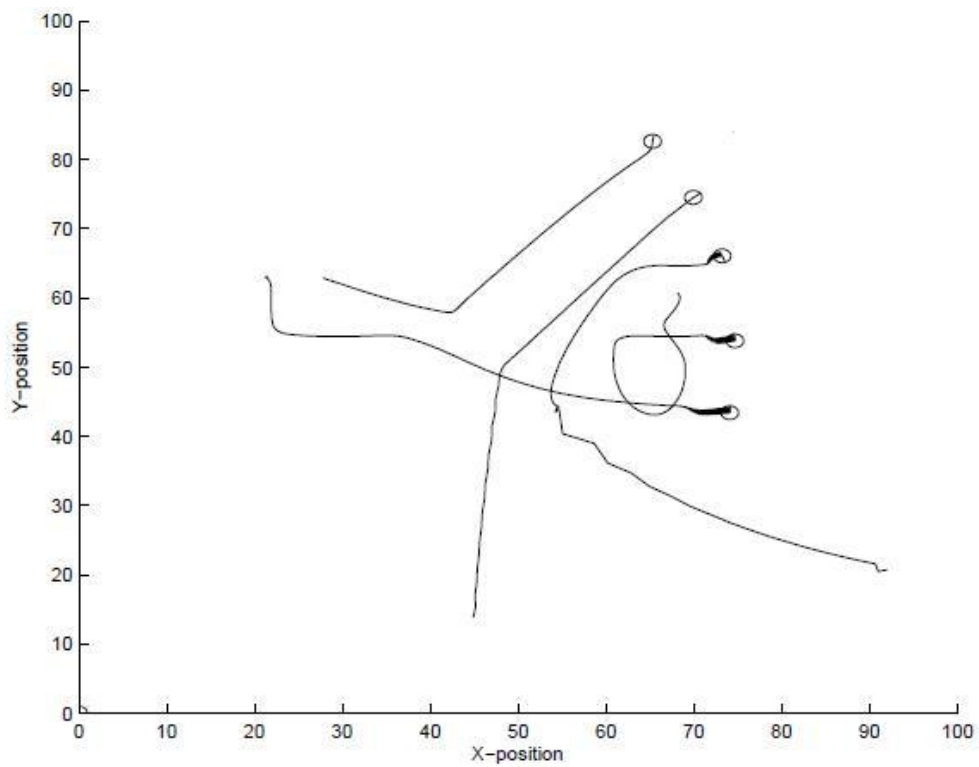


(b)

Figure 5.8 : Square shape formation



(a)



(b)

Figure 5.9 : Linear formation under randomly chosen initial conditions

6. CONCLUSION

We have investigated the flocking and navigation of multiple vehicle systems using a decentralized control approach. Each vehicle uses kinematic model with nonholonomic constraints and double integrator dynamical models for differential drives types of mobile robots. Two approaches are used for control laws; potential fields and formation graphs. In artificial potential fields approach a damping factor is used to ensure asymptotic stability. For group navigation a virtual leader is used. The presented system is realized in simulation level and it is seen that a multi vehicle system can flock and navigate in any formation successfully independent of obstacles in the environment. It is important to note that for safe navigation the virtual leader is required to follow a collision free path.

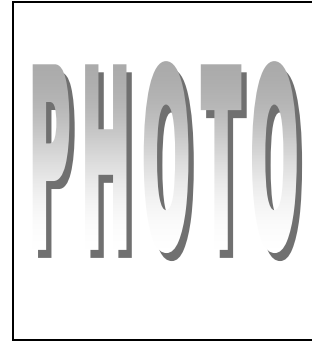
Further study includes leaderless navigation models and safe group navigation when virtual leader is passing through obstacles. In formation graph approach, various examples of formations are given as simulation results. Further study includes definition of a dynamical neighborhood in order to prevent collisions between non-neighboring vehicles in the final formation.

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- **Sisli Ufuk Y.**, Temeltas Hakan, 2008, Decentralized Formation Control of Multi Vehicles Systems with Non-Holonomic Constraints Using Artificial Potential Field, *IFAC World Congress*, Seoul, South Korea.