LEASING AND REMANUFACTURING STRATEGIES WITH PRICING AND INVENTORY CONTROL

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ABSTRACT

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We consider infinite horizon inventory policies for a company which satisfies customer demand by either selling or leasing a product. If the product is leased, it can be leased for a different number of periods. The customer demand follows a Poisson process whose mean is a function of the price. The company has manufacturing, refurbishing and remanufacturing options for building-up inventory. The new products and the products returned by the customers that are subject to remanufacturing or refurbishing process are collected into the same finished goods inventory. It is assumed that significantly worn-out ones among the returned products are disposed of from the system. In the model, we consider an $(S-1, S)$ type policy for controlling the inventory and determine the optimal inventory level as well as the optimal leasing and selling price of the product. Our analytical model assumes that a used product can return to the system infinitely many times.

We use the optimal price and inventory levels obtained from the analytical model in a simulation model which considers the more realistic case of a finite number of returns. We calculate the average profit per unit time including revenue from leasing and sales, inventory holding costs in the system, backorder and lost sales costs. We discuss the results of the analytical model and compare them with the simulation model results, and provide insights for the decision makers.

ÖZET

KİRALAMA VE YENİDEN ÜRETİM STRATEJİLERİNDE FİYATLANDIRMA VE ENVANTER KONTROLÜ

Bu çalışmada, ürünlerini kiralayarak veya satarak müşteri taleplerini karşılayan bir firmanın fiyatlandırma ve envanter politikaları incelenmektedir. İncelediğimiz modelde sisteme gelen kiralama ya da satın alma müşteri talepleri verilen fiyatın bir fonksiyonu olacak şekilde Poisson sürecine göre gerçekleşmektedir. Kiralanan ya da satın alınan ürünlerin müşteride geçirdikleri süreler birbirinden farklı rassal değişkenler olabilmektedir. Müşteride kalma süresi sona eren ürünler yeniden üretim için sisteme geri dönebilmekte, dönmeyen ürünler için ise üretim siparişi açılmaktadır. Yeni ürünler ile müşteriden dönüp, yeniden üretim işleminden geçen ürünler aynıbitmiş ürün envanterine gönderilmektedir. Müşteriden döndüğünde yeniden üretilemeyecek kadar yıpranmış olan ürünler ise sistemin dışına atılmaktadır. Modelde $(S-1, S)$ envanter politikasıaltında, en iyi envanter seviyesi ile en iyi kiralama ve satış fiyatları belirlenmektedir.

Geliştirilen modelde ürünlerin sonsuz defa yeniden üretilip müşteriye dönebileceği varsayılmıştır. Bu modelin çözümünden elde edilen en iyi fiyat ve envanter seviyeleri kullanılarak, benzetim yardımı ile müşteriden dönen ürünlerin ancak sınırlı sayıda yeniden üretim işleminden geçip müşteriye dönebileceği durumda sistemin birim zamandaki beklenen karı hesaplanmıştır. Bu kar hesaplanırken, satış ve kiralama gelirleri ile ürünü envanterde tutma maliyeti ve müşteriyi bekletme veya kaybetme maliyeti göz önünde bulundurulmuştur. Modelin analitik sonuçları, kurulan benzetim modeli ile karşılaştırmalar ve karar vericiler için çıkarımlar tartışılmaktadır.

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$a_{l,i}$	Market size of the product that will be leased with option i
a_s	Market size of the product that will be sold
b	Backorder cost
c_f	Cost of finished-good
$c_{l,i}$	Unit refurbishing cost of a product
c_m	Unit manufacturing cost of a product
c_r	Unit remanufacturing cost of a product
d_i	Depreciation rate for a product leased
f_i	A factor in leasing industry
h_f	Holding cost of finished-good
i	The number of lease options
I	Inventory carrying charge
$k_{l,i}$	The change in demand of leasing customer per unit change in leasing price of a product
k_s	The change in demand of buying customer per unit change in selling price of a product
K	The number of subintervals in a decision making period
m_i	the duration of a lease agreement with respect to the number of subintervals
N	The number of outstanding products that would be in the inventory after necessary lead times
$N_{l,i}$	The number of products that are leased and are at customer
N_r	The number of products that are sold and are at customer
p_i	Proportion of products that will be refurbished after being leased
$P_{l,i}$	Price of a leased product
P_s	Price of a sold product
q_i	Proportion of products that will not be refurbished after being leased
$Q_{l,i}$	The number of products that will be refurbished per unit time

Q_m	The number of products that will be manufactured per unit time
Q_r	The number of products that will be remanufactured per unit time
$\widetilde{Q}_{l,i}$	The number of products at refurbishing and will be sent to the inventory
$\widetilde{Q}_m$	The number of products at manufacturing and will be sent to the inventory
$\widetilde{Q}_r$	The number of products at remanufacturing and will be sent to the inventory
r	The return probability of sold products
rn	The number of times that a returned product can be reprocessed
$T_{l,i}$	The time that a product spends at customer who chooses lease option i
T_m	The time that a product spends in manufacturing
T_r	The time that a product spends in remanufacturing
T_{rc}	The time that a sold product spends at customer
$T_{r,i}$	The time that a leased product spends in refurbishing
$\alpha_{l,i}$	The proportion between leasing price and selling price of a product
β	Interest rate for the subinterval
$\widetilde{\beta}$	Annual interest rate
$\lambda_{l,i}$	Arrival rate of leased products
λ_s	Arrival rate of sold products

1. INTRODUCTION

In recent years, companies started to implement new strategies in order to improve their profitability. One of the concepts that may have a significant role on the profitability is reprocessing the returned products. In this study, we consider two types of reprocessing structures: refurbishing and remanufacturing. Refurbishing is "servicing and/or renovation of older or damaged equipment to bring it to a workable or better looking condition" (http://www.businessdictionary.com/). Remanufacturing, on the other hand, is "a series of manufacturing steps acting on an end-of-life part or product in order to return it to like-new or better performance, with warranty to match." (http://www.remanufacturing.org.uk). However, it differs from the other recovery processes since it is very detailed. For instance, let's consider a bicycle returned by a customer. If we only dye the frame of the bicycle, this process is refurbishing. However, if we disassemble the returned product and repair the wheels and the pedals and reassemble the bicycle, this process is called a remanufacturing.

In this study, we consider leasing and selling strategies. Leasing is obtaining the products without investing capital on that product and formally it can be defined as "An instrument conveying the possession of real property for a fixed period of time in consideration of the payment of rent." (http://www.lectlaw.com/). The leasing customer uses the product until the end of the period which is specified on the leasing contract. In our problem, customers arrive to the system either to lease or to buy the product. A leasing customer may lease the product for different periods depending on the lease option chosen by the customer. However, customers know that at the end of the lease period they have to return the product. The returned products are evaluated and if they are not highly depreciated they are refurbished. On the other hand, when the customer arrives if there is no on-hand finished goods inventory the demand is either backordered or lost.

In leasing industry, refurbishing is an important strategy that affects the profit of the system. Since the leasing customers have to return the product, in order to lease again or sell these products, they need to be renewed. Refurbishing process increases the income obtained from the leased products, because a renewed product can be leased at a higher price. Another factor that affects the revenue obtained in leasing a product is setting the right price for the products, since the pricing strategy affects the product demand.

A customer who does not prefer to lease the product has the option of buying it, if there is a product available in the finished goods inventory the customer demand is satisfied. Otherwise, it is backordered. The customer who buys the product may return the product back to the system with some probability. The returned products are remanufactured and sent to the finished goods inventory. We use an $(S-1, S)$ base stock policy in order to replenish the inventory. We assume that there is only one type of product and all the products that are refurbished, remanufactured or manufactured are put into the same finished goods inventory. Thus, if the buying customer does not return the product, or if the returned product from the leasing customer is not refurbishable a manufacturing order is given. In our model, manufacturing is used for the following purposes: For obtaining a brand-new product, for replacing a highly depreciated product while it is at the leasing customer and for compensating a product that will not return from the buying customer.

We model the system as an infinite horizon problem, and we calculate the expected refurbished, remanufactured and manufactured quantities. We also calculate the expected costs that are incurred in different stations, and at the inventory control stage of the system. Our objective is to maximize the expected profit per unit time in the long run by optimizing the order-up-to level and the selling price of the product. The base stock level affects the system profit because if there is no product in the inventory the demand is backordered or lost. Thus, either a backordering cost or a loss sale cost is incurred. Also, in case there is no available product on hand, manufacturing option has to be used in order to satisfy the customer demand and a manufacturing cost is incurred. On the other hand, if there are excessive number of products in the finished goods inventory, then holding cost will increase and the profit of the system will decrease.

In this study, we consider leasing and remanufacturing strategies with pricing and inventory control. To the best of our knowledge, this study is the first one that combines remanufacturing, leasing, pricing and inventory control issues. We, first model the system, solve it analytically to determine optimal price and order-up-to level and find the corresponding expected profit per unit time. However our analytical results are based on two main assumptions: (1) If not fulfilled, a leasing customer demand is backorder. (2) Returned products are reprocessed infinitely many times. We relax some assumptions of the analytical model and build a simulation model in order to get a more realistic setting. The analytical model provides an initial search point for the price and order-up-to level values tried out in the simulation model. The simulation model considers price and inventory pairs in various scenarios. The simulation model results are fitted to an equation in order to obtain the optimal price for different pairs of the selling price and order-up-to level with a given set of system parameters.

This thesis is organized as follows: Chapter 2 presents the literature review about remanufacturing, leasing, pricing, inventory control and positions our research in the literature. Chapter 3 gives the problem description, model formulations in detail with corresponding assumptions and the analytical results of the model. Chapter 4 explains the simulation model, points out the differences between the analytical and simulation models and discusses the results of these models. Also, explains the function fitted according to the simulation model results. Finally Chapter 5 concludes the thesis and makes suggestions for the future research.

2. LITERATURE REVIEW

In this chapter, we review the literature related to various aspects of the problem that we present in this thesis. We investigate leasing and selling strategies with pricing and inventory control. Remanufacturing is another important part of this research because we deal with both new and remanufactured products. In section 2.1, we discuss literature on remanufacturing and in section 2.2, we discuss leasing literature that contains concurrent leasing and selling strategies and different aspects of leasing concept. In section 2.3, we discuss pricing literature and especially revenue management strategies by deciding the optimal price. Also at the end of this section we briefly discuss the literature on inventory control and queuing networks. In conclusion we explain the contribution of our study in the literature.

2.1. Remanufacturing

Remanufacturing is an important strategy that helps to recover the value of used products. Remanufacturing is a growing interest area and its benefits to the total profit of the system are investigated by many researchers. In remanufacturing process there are important concepts that affect profitability of the system such as: inventory control, optimal price and quantity, minimizing life cycle cost of remanufactured items, rate of returns, the design and structure of reverse supply chain and the competition between the new and the remanufactured products. One of the most important issues in remanufacturing is inventory control of remanufacturable products. Van Der Laan et al. (1999) contributes to the literature by introducing a model that has a hybrid system in which the customer demand is satisfied by both output of the manufacturing process and the output of remanufacturing process. They first sort the returned products according to their quality and if the quality is low, these products are recycled. On the other hand, if the quality is too low then the products can be disposed. After the remanufacturing and manufacturing processes the products are put in a serviceable inventory and they analyze the effects of remanufacturing in a pull and push production/inventory system. They state that an efficient planning and control system in a hybrid system is more complex than a traditional system without remanufacturing. They conclude that once the management decides to remanufacture, the chosen control policy must be in combination with other efficiency improving actions such as: reducing uncertainty in the timing and quality of returned products, collection of data about understanding the correlation between demand and returns.

Bayindir et al. (2003) investigate the benefits of remanufacturing in inventory-related costs. They define inventory-related costs as, holding cost of work in process (WIP) inventory, holding and backorder costs of the end item. They analyze the effects of different system parameters such as lifetime of a product, supplier lead time, capacity restrictions of production facilities, and investigate the conditions in which remanufacturing option is the most beneficial. In order to minimize remanufacturing and inventory-related costs, they determine optimal order-up-to-level and return ratio of the product. This study is the first one that considers the effects of finite capacity on inventory related costs. They achieve this by incorporating the effect of finite capacity on lead times and WIP inventory level in the objective function. They observe that when there is a capacity restriction, utilizing both options (remanufacturing and manufacturing) reduces inventory-related costs.

Many researchers discuss how remanufacturing affects the quantity of products in open/closed queuing networks. Toktay et al. (2000) decide an ordering policy that minimizes the total expected procurement, inventory holding, and lost sales cost. As a product they consider Kodak's single-use cameras. They assume that return flows of used cameras are uncertain and they model the system as a closed queuing network. In order to adaptively estimate and control the return flows, they develop a heuristic. Also Toktay et al. (2000) investigate the effects of demand rate, length of the product's life cycle and procurement delay on the total expected cost. They conclude that the two most important operational factors are dynamic estimation number of cameras that will be returned to the system and the return probabilities of product.

The price of the remanufactured products is another important criterion that affects the long run average profit. Ray et al. (2005) study three pricing strategies:

uniform price for all customers, age-independent price differentiation between new and replacement customers and age-dependent price differentiation between new and replacement customers. They compare the profit potential of different pricing schemes. In this study, two different customer types are investigated that are first-time buyers and the customers who already own the product but are willing to replace it with a new one. They discuss a price discrimination policy by offering trade-in rebate to the replacement customers. They provide important results that may helps managers in deciding the price for the new customers and optimal trade-in rebate for the replacement customers. Ketzenberg and Zuidwijk (2009) also discuss the effect of price on the profit of remanufactured products. In this study, customers are assumed to be sensitive to both the price and the return policy. They study the selling price, return policy and the quantity of new products to purchase. They analyze a deterministic model and then extend it to a stochastic model in which the demand and the returns are uncertain. They also focus on the effect of investments to increase speed of recovery, and to reduce market and return uncertainty. They observe that the product type (functional or innovative) is important in understanding how should the firms invest.

Optimal price and quantity of remanufactured products is also discussed by Vorasayan and Ryan (2006a) because both the price and the quantity of remanufactured products have significant impacts on manufacturer's total profit, they investigate the optimal price for a product and proportion to remanufacture. They introduce a model which includes sale, return, refurbishment and resale processes in an open queuing network. They design a numerical study to show the cost characteristics of the new and the remanufactured products. They also analyze the impact of pricing strategy on the demand of the remanufactured items. They conclude that price and the proportion of the returns are significant in the profitability of system. The decision of how much to remanufacture depends on the certainty conditions and the demand of the customers. In another study, Vorasayan and Ryan (2006b) investigate the best policy to manage product returns, and analyze a generalized queuing model which includes manufacturing, customers, return (collecting), refurbishing and resale stages. They model the system as a Jackson queuing network in which each stages can be analyzed independently. They find the total profit of the system with respect to four different attributes, namely variability of manufacturing and refurbishing, the number of times a product can be refurbished, the return probability of products and quality grades of returned products. They show that prioritizing the returns according to their quality grades increases the total profit due to significant inventory and cost reductions.

Quality grades have great importance in remanufacturing because they are used to model depreciation level of returned product. In such models the cost of the process chances according to quality grades. Souza et al. (2002) examine the case of production planning and control for a remanufacturer who has the option to sell the returned products or to remanufacture them. They consider the percentage of each quality grade on each station, remanufacturing costs, and processing time parameters to decide the optimal remanufacturing policy. They model the remanufacturing facility as a multiclass open queuing network in which customer demand and used products' arrivals are modeled as renewal processes. After defining and evaluating the model analytically they simulate the system in order to test well-performing dispatching rules and to evaluate the impact of those rules on service level of inaccuracies in grading and sorting the used products. They provide insights on how to make significant production decisions to maximize the long run average profits of the remanufacturer. Another important study that sheds light into the importance of quality levels in remanufacturing is introduced by Denizel et al. (2010). They consider production planning of remanufactured products in a deterministic demand and return stream setting. The remanufactured products have different and uncertain quality levels and the production capacity is finite. They conclude that remanufacturing cost increases as quality of returned product decreases and decide that any unused product returns may be salvaged according to their quality level. They investigate for the optimal quantity that should be remanufactured and the inventory that should be carried over for the next periods.

The structure of the reverse supply chain influences the price and the profitability of the whole system. Guide and Pentico (2003) observe the reverse logistics models assume that product returns are exogenous so they propose a model in which they can control product returns. Their model consists of three stages which are defined as product acquisition, production and operational control and demand management and product pricing. They introduce a framework in which a planner can decide which methods of product returns are the most profitable. Atasu and Çetinkaya (2006) also discuss how the timing and the quantity of shipments of used products influences the decisions on production, inventory, remanufacturing quantity and price of a product. They investigate the effects of timing of returns on the profitability of manufacturer-collector pair and they develop a cost optimization model. Also they analyze the properties of optimal shipment frequency to understand how the speed of reverse supply chain affects the profitability. They conclude that when the time and the quantity of returns are considered, the collection method of used products becomes important factor in efficient operation of reverse supply chain. Therefore, they observe that the fastest reverse supply chain is not always the most efficient one. Savaskan and Wassenhove (2006) model a direct and indirect used product collection system. In the direct used product collection system the manufacturer collects the used products directly from the consumers and in the indirect product collection system the retailers accept the product returns. They examine how the structure of reverse channel affects the profits and decide that if a direct collection system is used channel profits are proportional to the amount of returns on collection effort. On the other hand, when an indirect collection system is used, the profit of the channel is affected by the competition between the retailers. When they compare direct collection and indirect collection, they observe that an indirect collection model can benefit from economies of scale in transportation of used products. Therefore, they conclude that an indirect collection model is a more preferred alternative for the manufacturer.

An important issue in remanufacturing is time delays experienced in the collection of used products since they may limit the options available for the reuse process. (Guide et al, 2006). In this study, the analysis shows that the monetary benefits of decreasing delays and improving network responsiveness influences transportation responsiveness. Toktay et al. (2003) discuss the methods for affecting product return rates and they review data-driven methods that help in forecasting product return flows. They assume that future returns are a function of past sales. This study contributes on clarifying the flow characteristics of used products, customer segment, ease of return and rebate policies. Spengler and Stölting (2008) not only consider the return flows and but also deal with the life cycle of a product. They assume that the life time of product is the time that the company has to deal with the product. The model of product project life cycle has three main periods as preparation, production and after care. They investigate remanufacturing concept in production cost and they present a strategic framework for evaluating remanufacturing strategies. Also they carry out an analysis of life cycle cost calculation approach by the help of a case study. They use a case study a life cycle cost calculation approach and calculate the remanufacturing costs.

The technology which is used during remanufacturing of the used products is important, since it influences the value that can be recovered from the remanufactured product. (Debo et al, 2005). In this study the authors consider pricing and technology selection problem for the remanufacturable products. They assume that the customer profile is heterogeneous in their willingness to pay for a remanufactured product and that the customers value the remanufactured products less than the new ones. They claim that producing a remanufacturable product can be profitable in some markets because it becomes possible to reach low-end-consumers. In addition, remanufacturing a returned product costs less than producing a new product. However, another important point is to choose the best pricing and production technology which will fit the target market. Zikopoulos and Tagaras (2008) examine the attractiveness of sorting before disassembly in remanufacturing of products. They study the conditions to invest in the required technology which result in higher profits. They consider single period, two-level reverse supply chain. They first examine the system and without sorting that will maximize the expected profits under demand and quality uncertainty. Then, they analyzed the system with sorting the in order to test the quality of returns and compare these two systems in terms of profitability. They conclude that the economic comparison between the system with and without sorting before disassembly shows that sorting will be beneficial in order not to remanufacture products more than the quantity which is required.

In most of the studies, it is accepted that remanufacturing is desirable and profitable for the firms. However, there are situations in which firms may not prefer remanufacturing the used product since it is possible that the remanufactured products may cannibalize the sales of new products. Thus the competition between the new and the remanufactured products becomes an important topic in remanufacturing. Ferguson and Toktay (2006) provides insights on questions such as, is it possible for the remanufactured products to cannibalize sales of new product, should and OEM remanufacture its own product, and how should it price the remanufactured and the new product, how should the OEM decide the recovery strategy in the case of the existance of third-party remanufacturer, and the last but not the least one is what is the impact of fixed and variable costs on these recovery strategies. In this study, they consider a two period, single firm and single product model. They state assumption on customer demand, lifetime of the product, and variable and fixed cost of the model. They observe that two concerns affect the OEM's decision about the remanufacturing: cost and cannibalization. They conclude that the OEM should choose a remanufacturing strategy over a collection strategy, because a relative cost of collecting versus processing shows the profitability of strategy. They also conclude that if the unit cost of manufacturing increases the relative advantage of remanufacturing increases. In that case the OEM should set up remanufacturing operation to preempt entry by the third-party remanufacturer. A related study is done by Ferrer and Swaminathan (2006). They analyze a model in which customers cannot distinguish the remanufactured and the original products. They consider two period model in where the manufacturer only produces in the first period and has right to make new and remanufactured products in the next period. They investigate how the price affects profit in the dynamic across periods over time and they decide the optimal quantities, prices and the optimality conditions for the firms either sell both products types (new and remanufactured) or sell only new products. They assume the market can be monopolistic or duopolistic and so an independent operator has access to the market and can set a price for the remanufactured product. They observe that as the marginal cost of remanufacturing decreases the value of making new products in the future periods decreases. They conclude that it is a game theoretical approach such as the OEM set a price for the remanufactured product and the independent operator hopes to attract the customer who concerns the price set by OEM is high. Then, the independent operator may reach to some of the customers in the market. Thus competition between the OEM and independent operator becomes important while dealing with the potential profit of the remanufacturing.

2.2. Leasing

Leasing is simply a contract between a seller and a customer that provides an asset to the customer for a specified period of time and payments. In literature the efficiency of leasing vs. selling is widely discussed. Desai and Purohit (1998) examine the effects of concurrent strategies of leasing and selling for durable goods. They explain the strategy of leasing and selling in marketing context and then state the conditions in which coexistence of both options is optimal. They also investigate what are the long-term implications of leasing to the customers and selling to another set of customers. They discuss whether this mixed strategy of leasing and selling affects the competition of leased and sold products. They evaluate the benefits and the costs of leasing and selling by considering two important criteria: (1) the future competition of leases and sales, (2) the customers targeted either through leases or through sales. They also analyze pure leasing and pure selling strategies and then compare them. They conclude that leasing dominates selling only under certain conditions but under uncertain conditions a concurrent strategy is optimal and the combination of selling and leasing better for manufacturer than either pure selling or pure leasing.

Bhaskaran and Gilbert (2005) investigate the effects of complementary products on leasing and selling decisions. They define complementary products as the products that are usually produced by an independent firm and have a significant effect on demand of durable goods. They deal with the interaction between durable goods and the complementary products and the effects of this interaction on the strategy of manufacturer for leasing and/or selling its product. They observe that if there is a low level of interaction (availability of complementary product does not highly affect the demand of durable goods) then the manufacturer should lease the products. Conversely, if availability of complementary products highly affects the demand of durable goods then the manufacturer should prefer selling strategy. To this end, with respect to the degree of complementarity the authors state the optimal region for the hybrid leasing and selling strategy and for pure leasing or pure selling strategy.

Leasing and selling strategies are also discussed by Goering and Pippenger (2009). They deal with exchange rates and concurrently leasing and selling policies for the durable goods. They assume that rental price is linearly related to the available stock in each period and selling price is discounted stream of lease prices. They explain that increases in exchange rates also cause increase in the sales price of durable products. According to chosen rental fraction of products they try to maximize total profit of the firm. They observe that leasing is preferred to selling if the exchange rates do not vary over time and concurrent strategy is optimal in case of expected future exchange rate is lower than the current rate.

Leasing affects the profitability of the firms but also it has environmental effects since it has a significant role in product recovery and remarketing. Agrawal et al. (2010) discuss whether leasing is green or not and how leasing is extending effective life of products. In addition to that, they try to answer the questions about: (1) the optimal integrated strategy of remarketing and disposal, (2) the optimal conditions for selling firms in a trade-in program and (3) the environmental implications of leasing and selling. They find that for the firms which choose selling strategy to employ trade-ins is optimal unless the production cost is high. They also state when durable goods are considered leasing dominates selling directly to the customers. They conclude that life-cycle environmental impact is strongly related to the production and disposal cost. For intermediate production cost leasing dominates selling but in case of high production cost both of the options have the same total impact.

In leasing literature, adverse selection and the market of lemons, and game theoretical approaches in leasing are also important concepts. Hendel and Lizzeri (2002) discuss the role of leasing under adverse selection in used car industry. According to this paper, leasing contracts are strengthened when the products are more durable, by this reason there is an increase in individual car leases. Gilligan (2004) also investigates leasing strategies for durables under adverse selection. He concerns aircraft industry and the market of lemons. He compares the market under perfect and asymmetric information. He mentions the negative and significant relationship between trading volume and the depreciation level of product and also states that leasing mitigates adverse selection.

2.3. Pricing

Pricing is significant for the overall profitability of the organization and it is a process of determining the value of the products. It is also important in product positioning. The pricing decision sets demand of the products and it either keeps the customers satisfied or sends them to the competitor. Certainly the needs and the wants of the customer are converted into demand only if the customer has the willingness and capacity to buy the product. Thus pricing remains a very complex issue in almost all industries and both in the past and nowadays it is a widely discussed issue. Hinterhuber (2004) introduce a framework for pricing decisions that guides for new product pricing and price re-positioning strategies for the existing products. The related study shows that price highly affects profitability of the firm; however costs and the volumes of the products are important from managerial point of view. The author states that the assumption of high prices cause low market share is wrong; he comes up with the idea of if the prices reflect high customer value, and high market share can be obtained. The proposed framework consists of the following steps: (1) defining the pricing objective, and (2) analyzing key elements of pricing decisions. To sum up, he explains the ranges of profitable prices, and economic value (customer value) analysis which is a tool that can be used to justify price increases.

In pricing strategies one of the mainly used is dynamic pricing (time-based) which is a kind of price discrimination in which the company sets the price according to different factors and depending on time (year, month, day, etc.). Dynamic pricing is common among industries and helps to maximize profits because it assigns the prices according to levels of demand and willingness to pay. Maglaras and Meissner (2006) consider a revenue maximization problem in which the decision maker either chooses a dynamic pricing policy or, by a fixed price; selects a dynamic role that controls the allocation of capacity for different products. They assume that if the firm is monopolist then a dynamic pricing strategy optimizes expected revenues, but if there is a competitive environment, choosing a dynamic capacity allocation rule is optimal. Elmaghraby and Keskinocak (2003) contribute to the dynamic pricing literature with a survey of applications. They state that charging a customer a price is not easy and to set the right price the company should have information about the demand. Nowadays, obtaining information about the customer is easier because of the new technology in communication channels. New decision support tools are developed for a better pricing strategy that contains optimization methods. They investigate dynamic pricing strategies under various assumptions or short and long life cycle products and they combine inventory and pricing decisions.

Adida and Perakis (2006) study the combination of the dynamic pricing and inventory control. They assume that demand is uncertain, no backorders are allowed and it is a make-to-stock manufacturing system with multi-product. The contribution of the study is introducing robust optimization ideas into the dynamic pricing model. Dynamic pricing policy is important to set the best price but also from another point of view the customer's acceptance for the unpredictable changing prices is more important. Morris (2001) study a simulation based dynamic pricing model in which purchasing behavior of the customers affect the price decisions of the company over time. He presents a learning curve simulator that is designed for analyzing agent based pricing strategies in a finite time horizon and this tool is used to determine effective dynamic pricing policies. Fleischmann, et al. (2005), discuss a dynamic pricing model for coordinated sales and operations. They develop a deterministic model that shows a price and demand effect as well as stockpiling and consumption effect. They assume that price and market stockpile influence demand, demand influences consumption and consumption influences the market stockpile. According to market stockpile they decide the optimal price in each period.

Revenue management is widely discussed in literature because it is a process that can significantly increase revenues of firms by an efficient inventory management and pricing. Revenue management concept helps allocating the right inventory to the right customer at the right price in capacity-constrained industries. A related study is introduced by Bitran and Caldentey (2003). They investigate the relation between dynamic pricing and revenue management. They set a generic framework to explain the revenue management concept. In this framework capacity is fixed and the objective is to find the optimal pricing strategy which will maximize the revenue collected over the selling horizon.

Significant literature exists on inventory management and queuing models because oof the importance of these concepts in various industries. Rubio and Wein (1996) study in setting base stock levels in a product-form queuing network. They explain the relationship between make-to-stock system and open queuing systems. They model the system as open Jackson network. They analyze a production control policy in which unsatisfied demand is backordered and if the amount of product is below a base stock level another unit of that product is released. They assume that facility produces multiple products. The work in process inventory has a steady state distribution and cost minimizing base stock level for each product is significant in steady state distribution of the product's total work-in-process inventory. They identify important formulas for the optimal base stock levels.

Another combination of inventory models and queuing networks is studied by Wu and Dong (2008). They combine these two systems to deal with performance modeling and analysis in multi-product manufacturing logistics networks. They introduce a multi-class inventory queue model and a job queue decomposition strategy to analyze major performance measures. To conclude, this study contributes to the literature by investigating multiple types of products produced at each step of manufacturing network with finite capacity restriction. Bai et al. (2004) also study inventory-queue models by considering inter-departure times. They concern probability distribution and coefficient of variations of inter-departure times for base stock inventory queues with birth and death production processes. They analyze $M/M/1$, $M/M/c$ and $M/M/\infty$ inventory queues and their results show that inventory queues are more complicated than other queue models.

Aras et al. (2010), discuss the optimal inventory and pricing policies for remanufacturable leased products. They consider a company which leases new products and sells remanufactured products that are returned at the end of lease period. They use dynamic programming formulation for obtaining the optimal price of remanufactured products and the optimal payment policy for the leased products. They mention the importance of remanufacturing in economy and discuss the uncertainty in the quantity, quality and the timing of returned products. To this end, leasing becomes an important strategy since it helps to have information about the return process. Leasing is useful to obtain consistent flow of used products for remanufacturing because they assume that the leased products return at the end of agreement thus they know how many products will be returned. However if the number of products in the stock is not enough to satisfy customer demand they order new products from a supplier. They assume that the returned products may only be remanufactured once. In this study, they present a numerical study that gives insights on the factors that affect the quantity and the price of products.

2.4. Positioning Our Research in the Literature

In this study, we consider an infinite horizon model where a product can be sold and leased for a number of different periods. We assume that selling and leasing options are available for both new and remanufactured products. Besides, in our analytical model, we assume that the returned products may be remanufactured for infinitely many times and in our simulation model they can be remanufactured for several times related to their depreciation level. It is assumed that significantly worn-out ones among the returned products are disposed of from the system.

In the model that we consider the customer demand for leasing and selling occurs according to a Poisson process whose mean is a function of the price. The company has manufacturing, refurbishing and remanufacturing options for building-up inventory. The new products and the products returned by the customers that are subject to a remanufacturing or refurbishing process are collected into the same finished goods inventory. In the model, we consider an $(S-1, S)$ type of policy. Specifically, when an item goes out of the inventory and is not bound to return (for remanufacturing

Table 2.1. Summary of Literature Survey

	Remanufacturing	Leasing	Pricing	Inventory Control
Aras et al. (2010)	X	X	X	
Bayindir et al. (2003)	X			X
Toktay et al. (2000)	X			X
Desai and Purohit (1998)		X		
Bhaskaran and Gilbert (2005)		X		
Vorasayan Ryan (2006)	X		X	X
Hinterhuber (2003)			X	
Adida and Perakis (2006)			X	
Van der Laan et al. (1999)	X		X	X
Ray et al. (2005)	X		X	
Guide and Pentice (2003)				X
Atasu and Çetinkaya (2006)	X		X	X
Souza et al. (2002)	X			
This thesis (2011)	X	X	X	X

or refurbishing) then a manufacturing order is given immediately. We determine the optimal the inventory level, and the optimal leasing and selling price of the product. We compute average profit per unit time including, revenue from leasing and selling, costs from holding inventory and costs due to not being able to satisfy customer demand.

To the best of our knowledge this thesis is the first study that combines remanufacturing, leasing, pricing and inventory control issues in a queuing theoretic framework. Table 2.1 is formed according to the four important topics that are covered in this thesis. Our difference from Aras et al. (2010) is that our study considers optimal inventory policies. Differently from their study we assume that leasing and selling options can be considered for both new and remanufactured products. Moreover, our model model is an infinite horizon model as opposed to Aras et al. (2010). Leasing is an important strategy to increase the profit. Desai and Purohit (1998) and Bhaskaran and Gilbert (2005) deal with leasing and selling but we also provide insights about managing the products that are returned from leasing and selling.

Remanufacturing is another important strategy which is widely discussed in the literature. Bayindir et al. (2003) consider remanufacturing and inventory related costs, and our difference from their study is dealing with leasing and pricing strategies that they do not incorporate in their model. Toktay et al. (2000) consider the effects of remanufacturing on inventory control. They deal with the return flows of Kodak's single-use cameras. Similarly, we consider the return flows of products that are sold, but we also consider the return flows of leased products and the potential of them to be remanufactured. Pricing strategies for new and remanufactured products are discussed by Ray et al. (2005) but we also consider pricing of leased products. Vorasayan and Ryan (2006b) study a generalized queuing network which includes manufacturing, customers return, refurbishing and resale stages. Our study includes these stages too, besides we optimize inventory level and leasing and selling prices of the product.

3. PROBLEM DESCRIPTION, MODEL FORMULATION AND ANALYTICAL RESULTS

In this chapter, we describe the problem and define the model that we develop in this thesis. In Section 3.1, we explain the demand structure and the pricing scheme. Also, we mention the leasing and selling mechanisms of the product. In Section 3.2, we describe refurbishing, remanufacturing and manufacturing processes of the system. The last section of this chapter, Section 3.3, includes the description of inventory control related aspects of the model.

3.1. Demand Structure and Pricing

We consider a single product system in which customers arrive either to buy or lease a product. We assume that the customers arrive according to a Poisson process. A leasing customer may opt for one of M different leasing options and each leasing option i, (i=1,2,...M) consists of different time durations. The arrival rate for customers who choose the lease option i is denoted by $\lambda_{l,i}$. We assume that all the products leased by each customer are returned to the system at the end of the lease period, which is dictated by the lease contract.

On the other hand, the customers who opt for buying the product arrive with rate λ_s. We assume that a buying customer will return the product with probability (w.p) r and upon arrival, the manufacturer knows whether a buying customer will return the product or not. We assume that a base stock policy is used to give a manufacturing order when a buying customer will not return the product, which happens w.p. $1 - r$.

As in Ray et.al, 2005, we assume a linear demand function in which the demand rate and the product price are inversely proportional. Equation 3.1 gives the demand function used in our model.

$$\lambda_s = a_s - k_s P_s. \tag{3.1}$$

The selling price of the product is denoted by P_s where the market size of the product is denoted by a_s. The slope k_s in Equation (3.1) represents the unit decrease in arrival rate λ_s when product price is increased by one unit, thus it is a measure of consumers' price sensitivity. The demand function gives the average number of products that customers are willing to buy at a given price and a higher price results in a lower demand. Since the demand rate of buying customers, λ_s must be a nonnegative number, the selling price, P_s must satisfy the following condition:

$$0 \leq P_s \leq \frac{a_s}{k_s}. \tag{3.2}$$

We use a demand function which has a similar structure as Equation 3.1 for the customers who lease the product. We assume that market size $a_{l,i}$ depends on leasing option i and $k_{l,i}$ represents the change in demand of leasing customer per unit change in leasing price of a product. The leasing price in leasing option i is denoted by $P_{l,i}$ and the corresponding demand rate is

$$\lambda_{l,i} = a_{l,i} - k_{l,i} P_{l,i}. \tag{3.3}$$

As long as $0 \leq P_{l,i} \leq (a_{l,i}/k_{l,i})$, the demand rate of the leasing customers will be nonnegative, and when the leasing price of the product $P_{l,i}$ less than $(a_{l,i}/k_{l,i})$ the demand rate of the leasing customers will be positive. It means there will be a positive fraction who would be willing to lease a product. If the leasing price of the product $P_{l,i}$ equals to 0 the demand rate of leasing customers would be equal to the market size of the products which is the maximum amount of demand possible.

There are M different options each of which dictates different time durations, the arrival rate of customers who choose for leasing would be $\sum_{i=1}^{M} \lambda_{l,i}$. Therefore, the total demand rate of customers is

$$\lambda_T = \lambda_s + \sum_{i=1}^{M} \lambda_{l,i}. \tag{3.4}$$

We assume that the leasing price, $P_{l,i}$ depends on the selling price P_s of the product and is proportional to it. We denote this proportion by $\alpha_{l,i}$. We assume that if the lease duration is long $\alpha_{l,i}$ will be higher, thus the leasing price will be closer to the selling price of the product.

$$P_{l,i} = \alpha_{l,i} P_s. \tag{3.5}$$

We decide $\alpha_{l,i}$ values based on the model built in Aras et.al (2010). In their study, a decision making period is divided into subintervals and the number of these subintervals is denoted by K. For instance, if the decision making period is a year and the subintervals are the months, K will be equal to 12. The duration of a lease agreement in terms of the number of subintervals is denoted by m_i and $\alpha_{l,i}$ is given by

$$\alpha_{l,i} = f_i \frac{\beta(1 - \beta^{m_i K})}{1 - \beta}, \tag{3.6}$$

where β is the interest rate for the subinterval, and $\widetilde{\beta}$ is the annual interest rate, where

$$\beta = \frac{1}{1 + \frac{\widetilde{\beta}}{K}}. \tag{3.7}$$

The factor f_i is used in leasing industry to take depreciation and interest rate into account:

$$f_i = \frac{d_i}{m_i K} + (1 - \frac{d_i}{2})\frac{\widetilde{\beta}}{K}, \tag{3.8}$$

where d_i is the depreciation rate for a product leased and d_i is between 0 and 1. The factor f_i depends on the number of subintervals that a product is leased and the interest rate of these subintervals. The length of the lease period affects depreciation rate d_i of the leased product. If the leasing period is longer, d_i will be higher. Calculation of f_i is due to financing (www.leaseguide.com/lease08.htm). In our study we use this factor just to obtain the proportion between leasing price and selling price.

We assume that all the leased products will return to the system; however a decision whether they are refurbishable or not is made according to their level of depreciation. Depending on the length of the leasing period, the time required to refurbish a product will vary. For instance, if a product lease period is long, the refurbishing time will be longer than refurbishing time of a product with a shorter leasing period. We model this by assuming that a certain percentage q_i of the leased products will be highly depreciated and their condition will not permit refurbishing and, hence they are dismantled. In order to compensate for these products, a manufacturing order is given. Remanufacturing option is used to process the sold products that are returned to the system and the manufacturing option is used to produce brand-new products.

In the following section, we explain the how we manage the inventory and how to calculate total expected profit of the system and the related steady state quantities in the different stations of the system.

3.2. Inventory Control

In our model, we assume that when a customer (either leasing or buying) arrives to the system, her demand is satisfied if there is any product in the inventory. Otherwise, customer demand is backordered. We use an $(S-1, S)$ base stock policy to replenish the inventory when a customer demand arrives and then the finished goods inventory level decreases by one. In our model, the finished goods inventory can be filled up by using refurbishing, remanufacturing and manufacturing operations. In refurbishing, remanufacturing and manufacturing stations, we assume that the product service times follow a general distribution G and there is no capacity restriction in the stations. Product arrivals to each station follow Poisson processes, thus each station can be modelled as an $M/G/\infty$ queue. Regardless of the renewing process used, all the products are put to the same finished goods inventory and hence, we assume that a customer is not able to differentiate a remanufactured/refurbished or a new product.

Therefore, in order to calculate profit generated by the system, we need to calculate N, the number of outstanding products that are not in the inventory but will be at the finished goods after necessary lead times. Note that, from now on we assume that the system is in steady state, and therefore all the random variables used are the steady state. The produts are either at a customer or being processed. N can be calculated as

$$N = \sum_{i=1}^{M} N_{l,i} + N_r + \sum_{i=1}^{M} \widetilde{Q}_{l,i} + \widetilde{Q}_r + \widetilde{Q}_m, \tag{3.9}$$

where $N_{l,i}$ is the number of products that are at leasing customers and when they are returned they will be refurbished and then put in to the inventory. It is a random variable which follows a Poisson distribution. N_r represents the number of products that are sold and are at customer and N_r is a random variable with Poisson distribution. $\widetilde{Q}_{l,i}$ is the number of products that are returned from the leasing customers and are at the refurbishing process whereas $\widetilde{Q}_r$ is the the number of products that are returned from the buying customers and are being remanufactured. $\widetilde{Q}_m$ is the number of products that are in manufacturing process. All the variables in Equation (3.9) are Poisson random variables with the mean given in Equation (3.10). The expected value of N is,

$$E[N] = \sum_{i=1}^{M} p_i\lambda_{l,i}T_{l,i} + r\lambda_s T_{rc} + \sum_{i=1}^{M} p_i\lambda_{l,i}T_{r,i} + r\lambda_s T_r + (1-r)\lambda_s T_m + \sum_{i=1}^{M} q_i\lambda_{l,i}T_m. \tag{3.10}$$

The first term in Equation (3.10) is the expected value of $N_{l,i}$, since $\lambda_{l,i}$ is the demand rate of leased product. Let p_i be the percentage of leased products that upon return will be refurbished and put into the finished goods inventory. $T_{l,i}$ is the time period that a customer who chooses leasing option i uses the product. At the end of that time period the leasing customer returns the product and a decision is made whether a product is refurbishable. The second term in Equation (3.10) is the expected value of N_r. We assume that a customer who buys a product will return it after a constant time T_{rc}. Since λ_s is the demand rate of buying customers, and r is the probability that a sold product will be returned to the system, the expected number of products at buying customers and are returned to the system after T_{rc} is $r\lambda_s T_{rc}$. The expected value of $\widetilde{Q}_{l,i}$ is

$$E[\widetilde{Q}_{l,i}] = p_i \lambda_{l,i} T_{r,i}, \tag{3.11}$$

which is the average number of products in refurbishing station. The time that the returned products spend at refurbishing station is denoted by $T_{r,i}$, in Equation (3.11). The longer time a product stays in the lease, it's depreciation level will be higher. We assume that for each leasing option i, the time that a returned product spends in refurbishing is different.

The expected value of $\widetilde{Q}_r$, i.e., the number of products on average that will be processed in remanufacturing station is

$$E[\widetilde{Q}_r] = r\lambda_s T_r, \tag{3.12}$$

where T_r is the time that a product spends in remanufacturing process.

In manufacturing station while calculating the expected number of products that are processed in manufacturing station $E[\widetilde{Q}_m]$, has two components. The first component is the expected number of products that are returned from the lessee at the end of a lease period but are not refurbishable is equal to $\sum_{i=1}^{M} \lambda_{l,i} q_i T_m$, where q_i is the proportion of returns that is not permitted to join in refurbishing process and T_m is the time that a product spends at manufacturing station. Note that we allow p_i to change with leasing option i and we have $1 - p_i$ equal to q_i. On the other hand, the second component is related to the products that will not return from the buying customers, since the proportion of these products is $(1-r)$ and the number of products on average that will be manufactured is $(1 - r)\lambda_s T_m$, the expected value of $\widetilde{Q}_m$ is

$$E[\widetilde{Q}_m] = (1 - r)\lambda_s T_m + \sum_{i=1}^{M} q_i \lambda_{l,i} T_m. \tag{3.13}$$

In this study, our objective is to maximize expected profit per unit time with respect to S and P_s jointly. The profit function is defined as

$$Profit(S, P_s) = \sum_{i=1}^{M} P_{l,i}\lambda_{l,i} + P_s\lambda_s - TC, \tag{3.14}$$

where TC represents for the total expected cost.

$$TC = \sum_{i=1}^{M} E[Q_{l,i}]c_{l,i} + E[Q_r]c_r + E[Q_m]c_m + E[h_f(S - N)^+ + b(N - S)^+]. \tag{3.15}$$

The sum of the first and the second terms in Equation (3.14) is the revenue obtained from lease and sales. Since for each lease option, the leasing price of the product is different to calculate the total demand of the product we sum up the revenue of the product over M options. The third term in Equation (3.14) represents the total expected cost. In Equation (3.15) the second term represents the expected cost of refurbishing, where $c_{l,i}$ is the cost of refurbishing per product and $Q_{l,i}$ is the random variable that denotes steady state number of leased products that can be refurbished per unit time. $Q_{l,i}$ is distributed as Poisson with mean $E[Q_{l,i}]$:

$$E[Q_{l,i}] = p_i \lambda_{l,i}. \tag{3.16}$$

We assume that upon arrival of a customer, we know whether this customer will return the product to the system or will not return. Let Q_r denote the number of sold products that will be remanufactured per unit time, yet they are still used by buying customers. Q_r is a Poisson distributed random variable with mean:

$$E[Q_r] = r\lambda_s, \tag{3.17}$$

thus $E[Q_r]c_r$ is the expected cost of remanufacturing, where c_r represents the unit cost of remanufacturing a product.

The manufacturing process used to compensate for the products that will not return from the buying customer or for the products that are returned from the lessee at the end of lease period, but are not in a refurbishable condition. Therefore, in order to increase customer service level manufacturing becomes an important part of the system.

Let Q_m denote the steady state quantity that represents the number of products that needs to be manufactured per unit time and Q_m is a Poisson distributed random variable with mean:

$$E[Q_m] = (1-r)\lambda_s + \sum_{i=1}^{M} q_i \lambda_{l,i}, \tag{3.18}$$

where $(1-r)\lambda_s$, represents the average number of sold products that will not be returned and the second term is the average number of products that will not be in refurbishable condition.

As we stated, other costs that occur in the system are the holding and backorder costs. We consider that if inventory level S is greater than N, the number of products that would be in the inventory after a while but they are either at a customer or being processed, there will be a holding cost denoted by h_f, otherwise, there will occur a backorder cost denoted by b.

Let c_f denote the average unit cost of a product per unit time, which placed into the finished goods inventory. The holding cost per product per unit time is

$$h_f = Ic_f, \tag{3.19}$$

where I is the inventory carrying charge, usually taken as the interest rate in the market.

Since a product placed into the finished goods inventory may be one of the three operations, namely manufacturing, remanufacturing and refurbishing, in order to calculate c_f, we need

$$\frac{\sum_{i=1}^{M} E[Q_{l,i}]}{\lambda_T} c_{l,i} = \frac{\sum_{i=1}^{M} p_i \lambda_{l,i}}{\lambda_T} c_{l,i}, \tag{3.20}$$

where λ_T and $c_{l,i}$ denote the total customer arrival rate to the system and refurbishing cost per product respectively. First term in Equation (3.20) is the long-term probability that a product in the finished goods inventory is from refurbishing process. Hence, Equation (3.20) represents part of the expected cost of a finished product due to refurbishing. Similarly, Equation (3.21) represents the expected cost portion of a finished product due to remanufacturing:

$$\frac{E[Q_r]}{\lambda_T} c_r = \frac{r\lambda_s}{\lambda_T} c_r, \tag{3.21}$$

Equation (3.22) is the portion of the expected cost of a finished product due to manufacturing:

$$\frac{E[Q_m]}{\lambda_T} c_m = \frac{(1-r)\lambda_s}{\lambda_T} c_m + \frac{\sum_{i=1}^{M} q_i \lambda_{l,i}}{\lambda_T} c_m. \tag{3.22}$$

The first term in Equation (3.22) is the expected cost portion due to unreturned sold products and the second one is the one due to leased products that are returned is not in a refurbishable condition.

Therefore, the expected cost of a finished product c_f can be approximated by

$$c_f = \frac{\sum_{i=1}^{M} E[Q_{l,i}]c_{l,i} + E[Q_r]c_r + E[Q_m]c_m}{\lambda_T}, \tag{3.23}$$

Since we obtain holding cost h_f, with a given backorder cost b, the expected cost of holding and backorder is

$$E[h_f(S-N)^+ + b(N-S)^+] = h_f\sum_{n=0}^{S}(S-n)P_n + b\sum_{n=S+1}^{\infty}(n-S)P_n, \tag{3.24}$$

where P_n is the steady state probability that there are n outstanding products in the system; i.e $P(N=n)$=P_n. The expected number of shortages

$$E[(N-S)^+] = \sum_{n=S+1}^{\infty}(n-S)P_n, \tag{3.25}$$

however, since $(N-S)^+$ can be written as

$$(N-S)^+ = (S-N)^+ + (N-S), \tag{3.26}$$

therefore, the expected value of this quantity is

$$E[(N-S)^+] = E[(S-N)^+] + E[N] - S, \tag{3.27}$$

thus the right-hand-side of the Equation (3.24) can be written as

$$h_f\sum_{n=0}^{S}(S-n)P_n + b\sum_{n=S+1}^{\infty}(n-S)P_n = (h_f+b)\sum_{n=0}^{S}(S-n)P_n + bE[N] - bS. \tag{3.28}$$

We maximize the profit of the system per unit time and the profit function is concave in S for a given P_s since the revenue function is concave in S and cost function is convex in S the profit given by Equation (3.14) is sum of two concave functions so it is concave. Thus for a given price P_s by the help of difference equations we find the optimal inventory level S^*. To this end, the optimal solution S^* occurs at a point where the first difference of the cost function exceeds 0 the first time.

$$S^* = \min\{S : P(N \leq S)\} \geq \frac{b}{h_f + b}. \tag{3.29}$$

3.3. Numerical Study and Computational Results

We model the system as customers arrive either to buy or lease a product. Arrivals occur according to a Poisson Process. Leasing customers' arrival rate is denoted by $\lambda_{l,i}$ and in our numerical analysis, we assume that we have three leasing options (M=3). According to the first leasing option the customer will use the product for 180 days (6 months), according the second leasing option this time period will be equal to 360 days and the last option consists of 540 days of time period. In the model, the time periods are denoted by $T_{l,i}$ where i=1,2,3.

On the other hand, a customer may be willing to buy the product, in this case we assume that the customer will use the product for a constant period of time and we denote this period by T_{rc}. In our numerical study T_{rc} equals to 720 days. The demand rate of sold products is denoted by λ_s. The demand of the products depends on selling of the price which is a decision variable. The demand rate of sold products equals to $a_s - k_s P_s$ where P_s denotes the selling price of the product and a_s is the market size of the product and k_s is the slope and a measure of buying customers' price sensitivity. For the leasing customers a similar model is considered and $\lambda_{l,i}$ equals to $a_{l,i} - k_{l,i} P_{l,i}$. In our numerical study, we assume that $a_{l,i}$ for all i is the same and it equals to 2, also we assumed that the market size products sold, a_s equals to $a_{l,i}$. The selling price of the product is defined in the region [0,a_s/k_s] so if a_s=2 and k_s=0.25 then $P_s \in [0, 8]$. There is a relation between selling price and leasing price of the product and this relation is explained as $P_{l,i}$ equals to $\alpha_{l,i} P_s$.

In order to calculate $\alpha_{l,i}$ values first of all we need to set; m_i, $\widetilde{\beta}$, K, and d_i.

We define m_i as the duration of lease agreement in terms of subintervals and the first leasing period is 180 days, however we choose subintervals as months and K=12, thus m_1=0.5, m_2=1, and m_3=1.5. The annual interest rate $\widetilde{\beta}$ is set to 0.1. The depreciation rate of the product is also considered and there are two different sets for d_i. These sets are given in Table 3.1.

Table 3.1. Coefficients of Depreciation

	d_1	d_2	d_3
data set1	0.1	0.2	0.3
data set2	0.25	0.50	0.75

First of all, β must be calculated as it is defined in Equation (3.7), and is found to be 0.99. Then to calculate $\alpha_{l,i}$ values f_i must be found. By using Equation (3.8) f_i paramaters are obtained as f_1=0.025, f_2=0.024 and f_3=0.024. To this end, for the first data set $\alpha_{l,i}$ values are calculated as $\alpha_{l,1}$=0.15, $\alpha_{l,i}$=0.27 and $\alpha_{l,3}$=0.39.

To summarize, we have two data sets and in Table 3.2, we show the summary of parameters based on the second data set and with respect to these sets we obtained two sets of $\alpha_{l,i}$ values.

Table 3.2. Parameters of data sets

d_1	d_2	d_3	f_1	f_2	f_3
0.1	0.2	0.3	0.025	0.024	0.024
0.25	0.5	0.75	0.049	0.048	0.047

Since the region is defined for the possible selling and leasing prices of the product, the region, for the demand rate of buying and leasing customers can be obtained. Leasing price depends on the selling price of the product and the corresponding coefficients, $\alpha_{l,i}$ are given in Table 3.3 for two different data sets. Then, hence $P_s \in [0, 8]$,for the first data set $P_{l,1} \in [0, 1.2]$, $P_{l,2} \in [0, 2.16]$ and $P_{l,3} \in [0, 3.12]$ and for the second data set $P_{l,1} \in [0, 2.24]$, $P_{l,2} \in [0, 4.32]$ and $P_{l,3} \in [0, 6.16]$.

According to the demand function defined in the model, demand rate depends

Table 3.3. The proportion of leasing price to selling price

	$\alpha_{l,1}$	$\alpha_{l,2}$	$\alpha_{l,3}$
data set 1	0.15	0.27	0.39
data set 2	0.28	0.54	0.77

on the price of the product since we obtain the price we can calculate λ_s and $\lambda_{l,i}$ for each leasing option i. Thus, we obtain that $\lambda_s \in [0, 2]$, $\lambda_{l,1} \in [1.7, 2]$, $\lambda_{l,2} \in [1.46, 2]$ and $\lambda_{l,1} \in [1.22, 2]$.

Then to calculate profit generated by the system, first we calculate N, the number of outstanding products that are in the inventory but they will be at the finished goods after necessary lead times. These produts are either at a customer or being processed. In order to calculate N we need to know return probability of a sold product r, which is set to two different values and the first one is 0.7 and the second one is 0.4. Note that all of the leased products will be returned but with probability p_i they will be in a refurbishable condition. This probabilities are given in Table 3.4 for each leasing option i.

Table 3.4. Probability of being in a refurbishable condition

p_1	p_2	p_3
0.9	0.8	0.7

In order to find number of outstanding products we need to know the time spent at refurbishing, remanufacturing and manufacturing stations. The time spent in refurbishing station changes according to time spent at lease and is denoted by $T_{r,i}$ and for each i the number of days are given in Table 3.5

Table 3.5. The time spent in refurbishing station

$T_{r,1}$	$T_{r,2}$	$T_{r,3}$
1	2	3

The time spent at remanufacturing station is denoted by T_r and it equals to 3 days where manufacturing time is denoted by T_m and equals to 6 days.

Our objective is to maximize the profit of the system and since we know expected quantities that will be refurbished, remanufactured and manufactured we need the costs of these processes. Unit cost of refurbishing per unit time for each leasing option i is different with respect to length of lease period and the costs are given in Table 3.6.

Table 3.6. Unit costs of refurbishing process

$c_{l,1}$	$c_{l,2}$	$c_{l,3}$
0.1	0.2	0.3

Remanufacturing cost per unit, c_r, equals to 0.5 and manufacturing cost per unit, c_m equals to 1. Since all the unit costs and expected quantities are known, the expected cost of refurbishing, remanufacturing and manufacturing stations can be calculated. In Figure 3.1, for the first data set where $\alpha_{l,1}$=0.15, $\alpha_{l,i}$=0.27 and $\alpha_{l,3}$=0.39 the cost of refurbishing with respect to the selling price is given as:

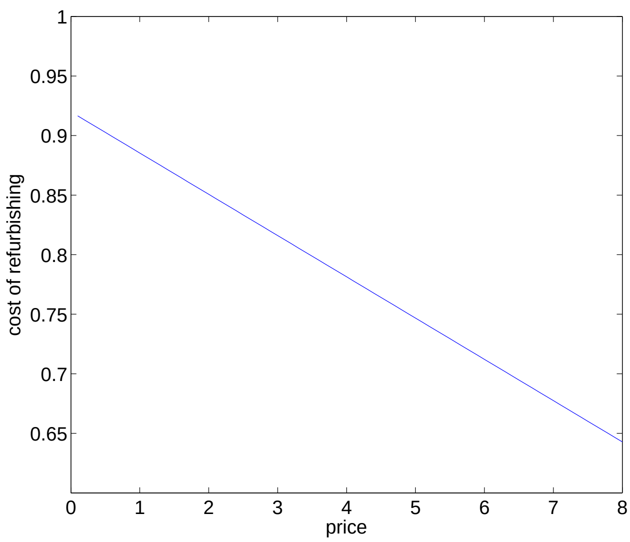

Figure 3.1. The expected cost of refurbishing vs. price

As it is shown in Figure 3.1, the cost of refurbishing process decreases linearly when the price increases. the reason for that is the decrease in the demand. Note that, there is a linear relationship between the price and the demand of the product, thus if the price increases the demand will decrease therefore, at the end of lease period the number of returned products will decrease too. The refurbishing process will be applied to less number of product thus the total cost of refurbishing will decrease.

When the price increases the number of returned products from the buying customers decreases because the demand of the products decreases. Therefore, the expected remanufacturing cost per unit time decreases linearly when the price increases.

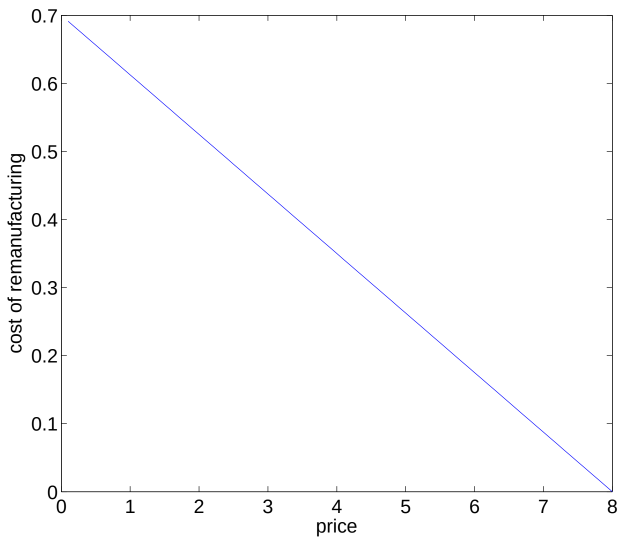

Figure 3.2. The expected cost of remanufacturing vs. price

The expected manufacturing cost per unit time decreases by the same reason of decrease in refurbishing and remanufacturing costs. However it is observed that the expected manufacturing cost is higher than the expected refurbishing and remanufacturing costs because manufacturing order is given for both the products which are not refurbishable and not returned from the buying customers. In Figure 3.3 the inverse linear relationship between the price and the manufacturing cost is shown.

On the other hand there are inventory holding and backorder costs that affect the total cost of the system. In our numerical study, backorder cost, b equals to 2 and to obtain holding cost per unit per unit time, h_f firstly we need to find cost of finished good, c_f. The calculation of c_f is given in Equation (3.23) and h_f equals to inventory

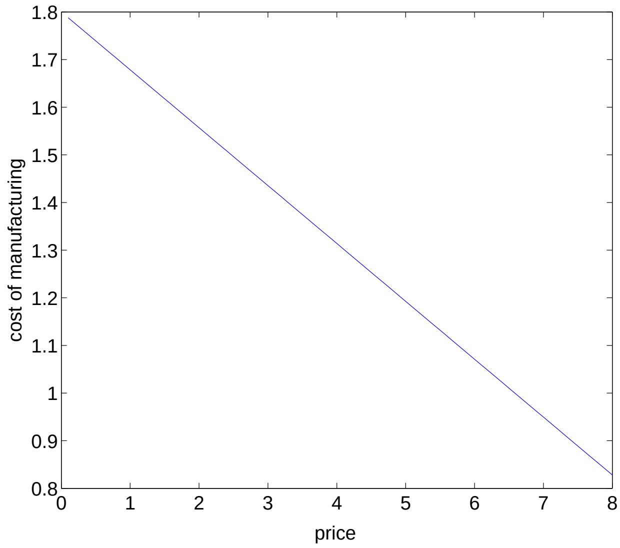

Figure 3.3. The expected cost of manufacturing vs. price

carrying charge times the unit cost of finished goods per unit time and the inventory carrying charge is set to 0.003. For the selling price equal to 8 first data set is used in which $\alpha_{l,i}$ is found as 0.15, 0.27 and 0.39 and r is set to 0.7 and c_f is found equal to 0.3358.

Therefore, the expected number of backorders and the number of products that will be hold in the finished goods inventory and the related cost is found by solving the Equation (3.28). Thus for a given price P_s by the help of difference equations we find the optimal inventory level S^*. The optimal S^* is calculated by obtaining the fractile that equals to $b/(h_f + b)$ and in the numerical study it is equal to 0.9995.

The cost of the system includes processing cost as refurbishing, remanufacturing and manufacturing, and it also includes holding and backorder costs. The expected cost of the system with respect to the product price linearly decreases. (see Figure 3.4). The reason for that is the decrease in the demand of the product for both leasing and buying. Since all the processing costs decrease when the price increases the total of these costs will decrease, too. Also, it is known that the total number of products in the system decrease.

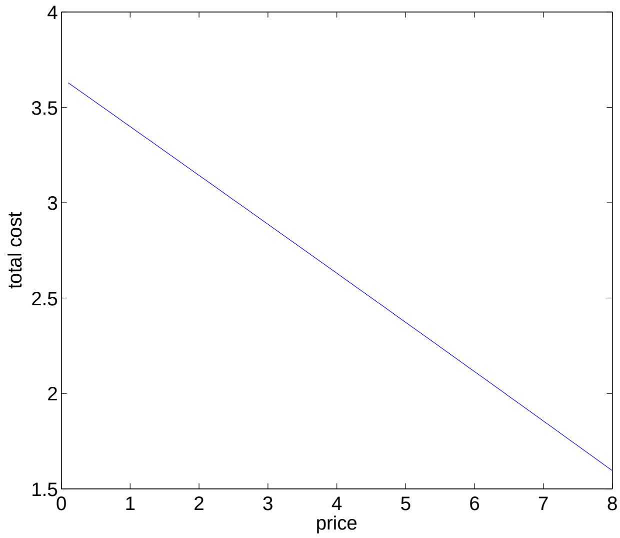

Figure 3.4. The total cost vs. price

The revenue of the system is obtained from the sales and leasing options. The number of products that is sold or leased depends on the price of the product. When the price increases the revenue will increase too but if the price continues to increase the decrease of the demand will decrease the number of sold or leased products therefore the revenue will decrease. Figure 3.5 shows the revenue function per unit time with respect to the selling price of the product.

Our objective is to maximize the expected profit of the system with respect to the selling price and inventory level S. The obtained profit function is concave, so we can obtain a global optimum price value. According to this function, the best price which maximizes the profit and satisfies the customer demand is obtained when it equals to 6.2. The maximum profit for this data set equals to 8.3928 where the revenue equals to 10.4555 and the overall cost equals to 2.0627. Figure 3.6 show the concavity of the profit function with respect to the selling price.

The shapes of the revenue function and the profit function are similar, to this end, we also investigate the revenue maximization results. We ignore the costs that are incurred in the system and obtain the optimal price that will maximize the revenue. The price obtained from revenue maximization is close to the price obtained from profit maximization.

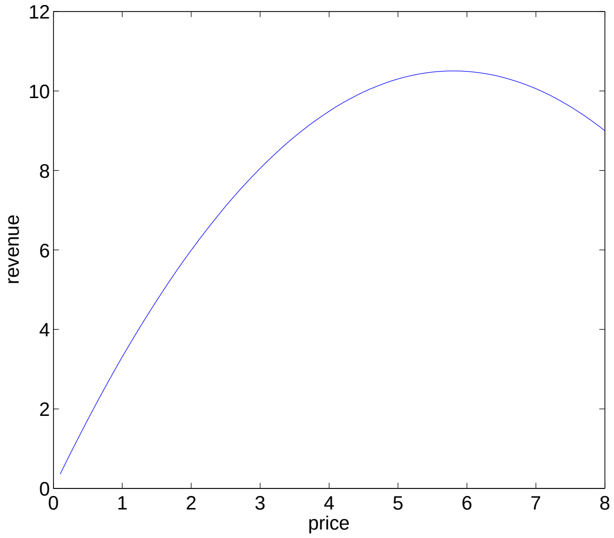

Figure 3.5. The revenue of the system vs. price

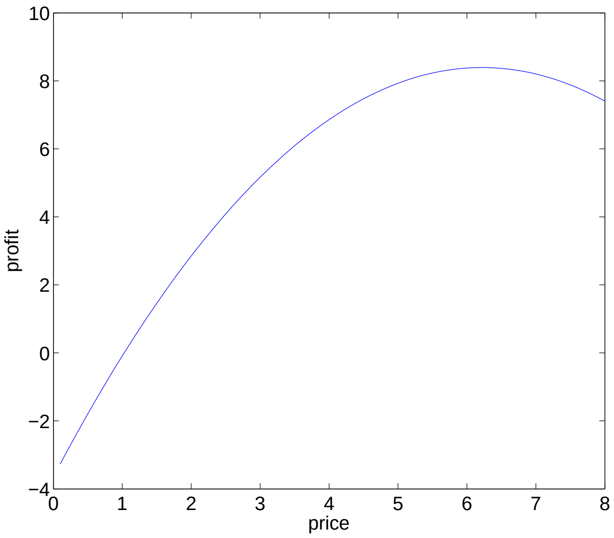

Figure 3.6. The profit of the system vs. price

For the same data set when the return probability of a sold product r is changed from 0.7 to 0.4 the profit decreases from 8.3928 to 8.3272, however the shape of the function does not change. The reason of the decrease in the profit is the increase in the backorder cost and manufacturing cost. Because for each of product not returned we give a manufacturing order. We also investigate how the optimal inventory level

S changes with respect to the price. In Figure 3.7 we see that the optimal inventory level decreases as the selling price increases because if the price increases the demand for the product decreases and therefore, the inventory order-up-to level decreases.

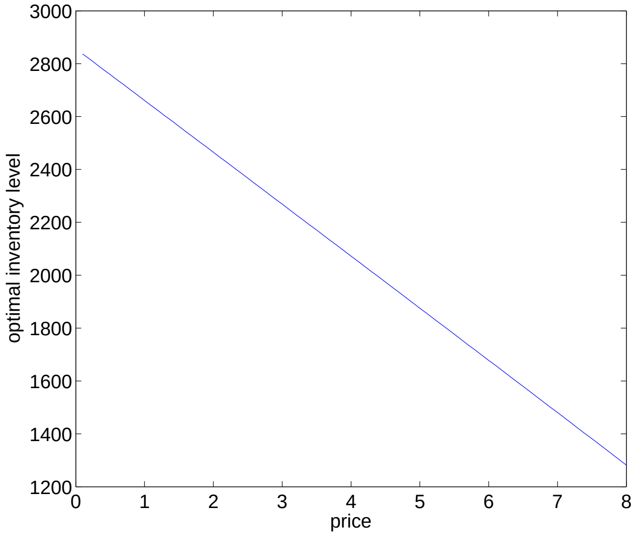

Figure 3.7. The optimal inventory level vs. price

All the observations that are done for the first data set are also done for the second data set where $\alpha_{l,1}$=0.28, $\alpha_{l,i}$=0.54 and $\alpha_{l,3}$=0.77. The total cost of the system is shown in Figure 3.8 where the total cost linearly decreases as the price increases.

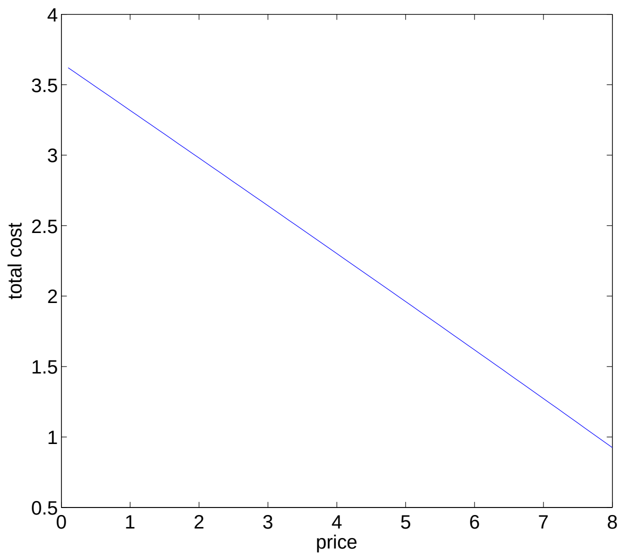

Figure 3.8. The total cost vs. price for the second data set

The revenue obtained by the second data set is higher than the one obtained in the first data set, because the $\alpha_{l,i}$ values are higher than the values in the first set, therefore the leasing prices of this set are higher. The revenue generated by the system is also concave and as the price exceeds the level equals to 5.3 the revenue decreases.

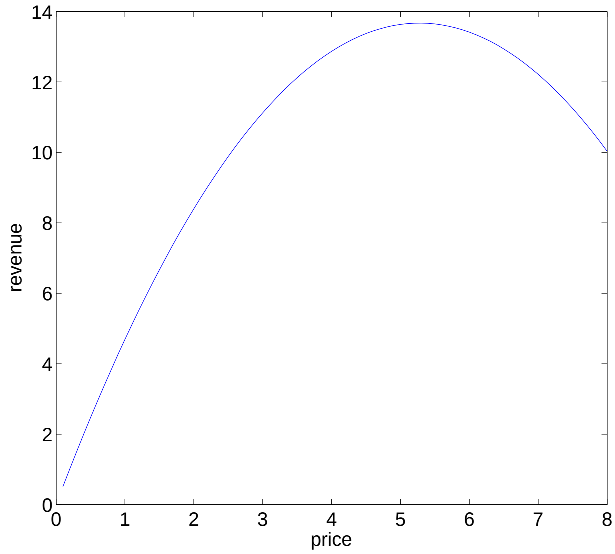

Figure 3.9. The revenue of the system vs. price for the second data set

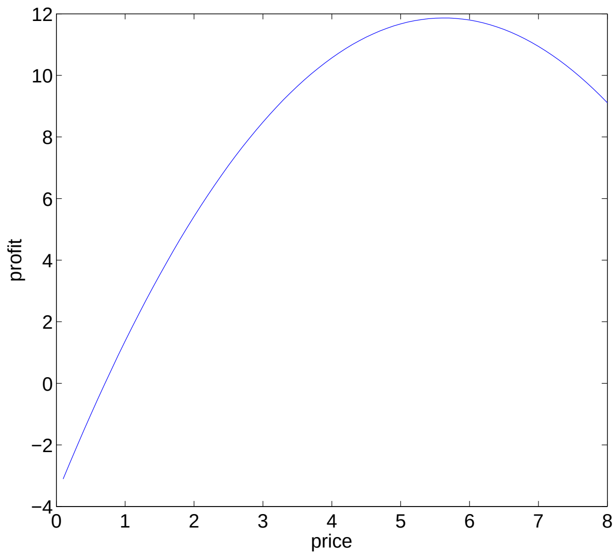

Figure 3.10. The profit of the system vs. price for the second data set

The profit function for the second data set is also concave. The maximum profit is reached when the price equals to 5.6 which is less than the optimal price obtained for the first data set. The shape of the profit function is also similar to the shape of the revenue function. For the same data set if the return probability of a sold product r is changed from 0.7 to 0.4 the profit decreases from 11.8634 to 11.7752.

As a result of the analytical model, we can proof that the average profit is concave in S and according to numerical example results, we observe that the average revenue is concave with respect to the price where the average cost is convex. Therefore, we obtain that the average profit is concave with respect to the price. These observations depend on the numerical studies presented this section. For the holding and backorder costs we proof that the inventory level is optimal thus these costs are minimized with respect to that optimal inventory level S. However the proof of the average cost and the average profit that are not shown analytically. Therefore in the following chapter, by using the simulation model we present more realistic calculations.

Another important result of the analytical model is the similarity of the revenue and profit functions for both data sets. The results obtained from the revenue maximization is close to the results of the profit maximization. Therefore, Chapter 4 simulation model and the computations are build based on these results.

4. SIMULATION MODEL AND COMPUTATIONAL RESULTS

In our study, firstly we model the system and then solve it analytically to find optimal selling price and order-up-to levels in order to maximize the profit of the system per unit time. We make two important assumptions such as all the unsatisfied customer demands are backordered and the returned products can be reprocessed infinitely many times. Then, in order to obtain more realistic cases, we build a simulation model with using ARENA software. We relax two main assumptions of the analytical model. We limit the number of times that a product can be reprocessed. Also, we assume that when a leasing customer arrives if there is no on-hand finished goods inventory the customer is lost, thus in the model a loss sale cost will be incurred. The objectives of the simulation model are: (1) Relaxing the assumptions of the analytical model, (2) Providing insights on how profit, optimal price and inventory level changes with various system parameters, and (3) Fitting an equation from the simulation results to find the optimal price and order-up-to level pairs for a given set of system parameters.

4.1. Description of The Simulation Environment

We simulate the system as an arriving customer may either be a leasing customer or a purchasing customer. A customer who wants to lease the product may choose one of the different lease options i where $(i = 1, 2, ..., M)$. Arrival rates change with respect to the product price since the demand is a function of the price. We assume that arrivals occur according to Poisson processes and since the demand rates of the customers are different, we design the arrivals as different streams. We assume that the customers who want to lease the product have 3 options with different time durations, thus we create 3 arrival mechanisms with different arrival rates.

Upon a leasing customer arrives to the system, the type of the customer is defined. A type shows that which lease option is chosen by the customer. For instance, if the customer chooses lease option 1, she is named as type 1 customer. Also, at the end of lease period, the customer will return the product and if the product is not highly depreciated it will refurbished. Classifying the customers helps in order to classify the returned products. Because the probability of being highly depreciated changes with respect to the lease option. If the customer leases the product for a long time, the product will be highly depreciated with a higher probability and highly depreciated products are dismantled from the system. Also, returned products assumed to be in a refurbishable condition will not spend the same time in the refurbishing station. The time spent in the refurbishing station changes according to the length of lease period. Figure 4.1 shows the arrivals and classification of the customers. It also shows that in order to satisfy the demand, we check the finished goods inventory, and if there is no product the customer is lost.

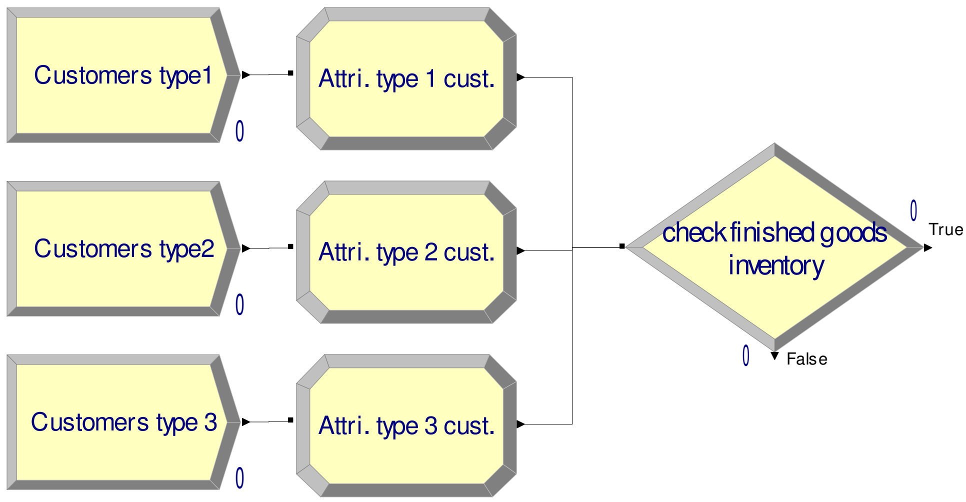

Figure 4.1. Arrival mechanism of leasing customer

We also consider that a customer may prefer to buy the product (see Figure 4.2). Arrival rate of buying customers' demand is different from the leasing customers' demand and it depends on the selling price of the product. In order to satisfy the customer demand, we check the finished goods inventory. We assume that with a certain probability, we know whether the customer will return the product at the end-of lifetime of the product. The information about the return potential of a sold product is important, because if the product will not be returned, we give a manufacturing order. In Figure 4.2, "Signal" module shows that a manufacturing order is given upon

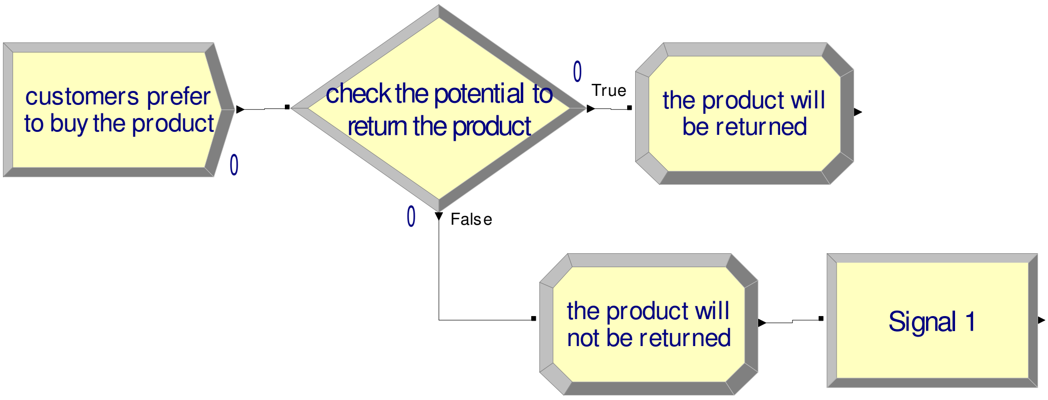

Figure 4.2. Arrival mechanism of buying customers

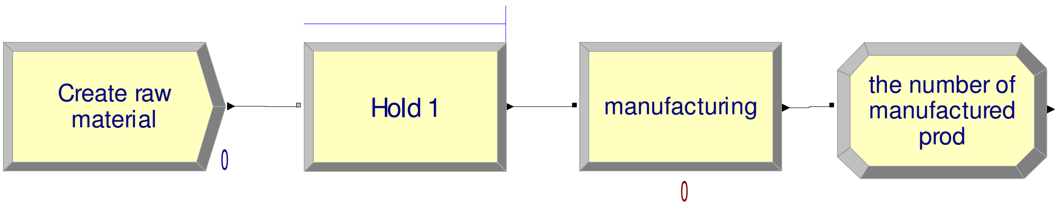

Figure 4.3. The manufacturing system in the simulation model

an arriving customer is the one that will not return the product.

We assume that, there is no capacity restriction for the manufacturing station, thus a new product may be produced when it is needed. A manufacturing order is given in two cases: (1) the returned product from the lessee is highly depreciated and has to be dismantled, (2) a sold product will not be returned by the customer.

4.2. Computational Setting

We simulate the system as a customer arrives either to buy or to lease a product. We assume that the buying and the leasing customers arrive according to Poisson processes with rates λ_s and $\lambda_{l,i}$, respectively. As in the analytical model, λ_s equals to $a_s - k_s P_s$ and $\lambda_{l,i}$ equals to $a_{l,i} - k_{l,i} P_{l,i}$. In our numerical study, a_s equals to $a_{l,i}$ and they are assumed to be equal to 2. Also, k_s equals to $k_{l,i}$ and the value of them is equal to 0.25. $P_{l,i}$ is assumed to be a fraction of P_s. This fraction is denoted by

$\alpha_{l,i}$ which changes with respect to the leasing option i. We assume that there are 3 leasing options. We run the model for two different values of $\alpha_{l,i}$ that are the same as in the analytical model. In the first case, $\alpha_{l,i}$ values are equal to 0.15, 0.27, 0.39 and in the second case they are equal 0.28, 0.54 and 0.77. The selling price and the leasing price are chosen according to the analytical model results since these results provide an initial search point for the price and order-up-to level S. Then, we investigate the closer values of the price and the inventory level to find where the maximum profit is obtained. Since we are not able to try every price and inventory level, we have to limit our search and then decide in these limits which profit is the highest one.

In this model we are dealing with steady state quantities thus we choose a long run length which is equal to 5000 days in ARENA software. We test longer and shorter time periods in order to reach steady state quantities and we determine that a period of 5000 days is enough. Also, in order to decrease the effect of variation we replicate the runs for 5 times and take the average of these 5 replications. In total, 921 scenarios are applied for different price and inventory levels. For 12 different cases, first for a given price, the effects of inventory level on the profit is investigated. Then, for a given inventory level different price levels are put into the system and the effect of price on the profit is investigated. To this end, the compound effect of price and inventory level on the profit of the system is discussed.

4.2.1. Purchasing and Remanufacturing

In our model, we assume that by the time customer arrives if there is any product in the inventory, she is satisfied and upon arrival, we know whether this customer will return the product or not. If there is no product in the finished goods inventory the customer is backordered until the time there is an available product in the inventory. We assume that the buying customer will use the product for 720 days and with probability r will return the product. This probability is set equal to 0.7 and 0.4 in different scenarios. Also, we assume that if the customer will not return the product we give a manufacturing order for a product. The returned products (from the buying customers) are remanufactured and this process takes 3 days per product. After remanufacturing process the products are put into the same finished goods inventory. Manufacturing time is longer than refurbishing or remanufacturing times and it is set to 6 days.

In this model, we are able to show that when a buying customer arrives we know the type of customer (will she return the product or not) so if the customer will not return the product we give a manufacturing order upon the customer is satisfied. Thus, while the product is at customer another product is manufactured in order to satisfy a possible incoming demand. We assume that for a manufacturing order we do not need to wait until the end of the time which is defined as the buying customer uses the product.

4.2.2. Leasing and Refurbishing

In our system, the leasing customers are divided into three groups such as who want to lease the product for 180 days, 360 days and 540 days. However, leasing a product is possible only when there is a product available in the inventory, thus we check the finished goods inventory. For the leasing customers, we assume that upon a customer arrives we know the type of the customer and the type of customer shows how long the customer will lease the product. We also know that at the end of lease period the customer will return the product, however if the returned product is not in a refurbishable condition we give a manufacturing order and then the product that is not refurbishable will be dismantled. Otherwise, if the product is not highly depreciated it can be refurbished and the time that the product will spend in refurbishing station is known. If the product is leased for 180 days it will spend 1 day in refurbishing; if the product is leased for 360 days, it will spend 2 days in refurbishing and if the product is leased for 540 days it will spend 3 days while being refurbished. The probability of being in a refurbishable condition for a returned product changes with respect to the time spent at lessee. The probability of being refurbishable for a product leased for 180 days is 0.9 and for the product leased for 360 days is 0.8 while for the product leased for 540 days it equals to 0.7. All the refurbished products are sent to the same finished goods inventory. The parameters of the leasing and refurbishing is summarized in Table 4.1.

Table 4.1. Parameters of leasing and refurbishing

Leasing Duration	180	360	540
Refurbishing Duration	1	2	3
Refurbishing Probability	0.9	0.8	0.7

4.2.3. Parameters of The System

In the simulation model, we determine 12 different sets and these sets change according to three different factors which are $\alpha_{l,i}$, return probability of sold product r, and number of times that the product is reprocessed which is denoted by rn. Note that $\alpha_{l,i}$ is the proportion of leasing price to the the selling price of the product. We investigate two levels of $\alpha_{l,i}$, the first level is 0.15, 0.27, 0.39 and the second level is 0.28, 0.54, 0.77. These values are the same as the ones in the analytical model and they are obtained from the formula given in Equation (3.6). The return probability r is set to two different values that are 0.7 and 0.4. The reprocessing number after the return rn is set to 1, 2 and 3 in different scenarios.

Other parameters such as refurbishing, remanufacturing and manufacturing costs and the times spent in these processes are the same as in the analytical model. While doing the computations, we fix some the parameters and they are summarized in Table 4.2

Table 4.2. Costs in the simulation model

$c_{l,1}$	$c_{l,2}$	$c_{l,3}$	c_r	c_m	h_f	b
0.1	0.2	0.3	0.5	1	0.001	2

In the system, all the costs are assumed to be per unit. In the analytical model, there was no loss sale cost, however in simulation model, we assume that the unsatisfied leasing customer will not wait for an available product and this will cause a loss sale. The loss sale cost equals to \$0.15 per unit. The other parameters that change in different cases are shown in Table 4.3.

Table 4.3. Parameters of the 12 cases

Case number	$\alpha_{l,1}$	$\alpha_{l,2}$	$\alpha_{l,3}$	r	rn
1	0.28	0.54	0.77	0.4	1
2	0.28	0.54	0.77	0.4	2
3	0.28	0.54	0.77	0.4	3
4	0.15	0.27	0.39	0.4	1
5	0.15	0.27	0.39	0.4	2
6	0.15	0.27	0.39	0.4	3
7	0.15	0.27	0.39	0.7	3
8	0.15	0.27	0.39	0.7	2
9	0.15	0.27	0.39	0.7	1
10	0.28	0.54	0.77	0.7	3
11	0.28	0.54	0.77	0.7	2
12	0.28	0.54	0.77	0.7	1

4.3. Analysis of The Simulation Results

In simulation model, as an initial state we use the optimal price and inventory levels obtained from the analytical model. While calculating the profit of the system the revenue obtained from leasing and sales and the costs which are incurred in various stations of the system are considered. These costs are refurbishing, remanufacturing, manufacturing costs and holding, backorder and loss sale costs. All the costs are calculated per day. The revenue is also calculated per day, thus the expected profit is obtained per day. The results of the runs are summarized in Table 4.4.

In the first case, the proportion of the leasing price to the selling price of the product is 0.28 and 0.54 and 0.77, leased for 180, 360 and 540 days, respectively. The return probability of a sold product equals to 0.4 and a returned product can be reprocessed only once. For the prices: 5, 5.4, 5.6, 5.8, 6 and 6.2 different inventory levels such as 1250, 1300, 1350, 1400, 1450, 1500, 1550, 1600, 1650, 1700 are tried out and according to the generated profits near to the maximum profit new runs are

Table 4.4. Results of the 12 cases

Case number	Order-up-to level	price	profit
1	1475	5.7	10.9034
2	1450	5.7	11.3577
3	1450	5.8	11.5883
4	1900	6.3	7.2838
5	1825	6.4	7.8309
6	1850	6.2	8.0844
7	1925	6.2	8.1170
8	1925	6.2	7.8410
9	1950	6.2	7.2951
10	1575	5.8	11.6493
11	1575	5.8	11.396
12	1600	5.8	10.9168

applied. Then, the prices such as 5.7, 5.75, 6.4 and 6.6, the new inventory levels such as 1375, 1425, 1475, 1525 are tried out. According to the run results for the case 1 the optimal price is 5.8 and the optimal inventory level is 1450. In Figure 4.4, for a given price (5.8), the graph of the profit with respect to the inventory level is shown. On the

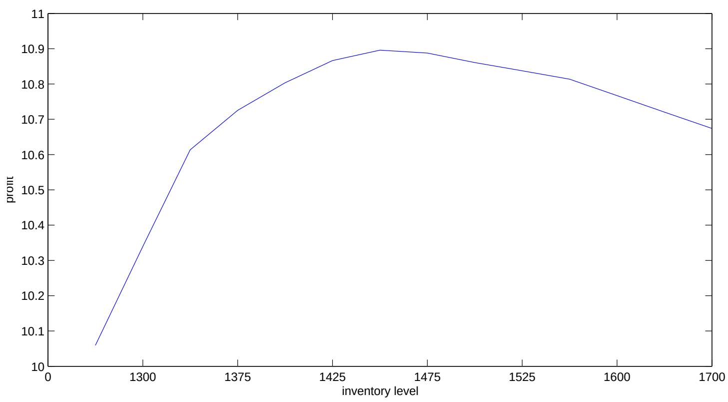

Figure 4.4. The profit vs. inventory level for the price 5.8

other hand, for the given best inventory level equal to 1450 the pattern of the profit with respect to the price can be shown as in Figure 4.5.

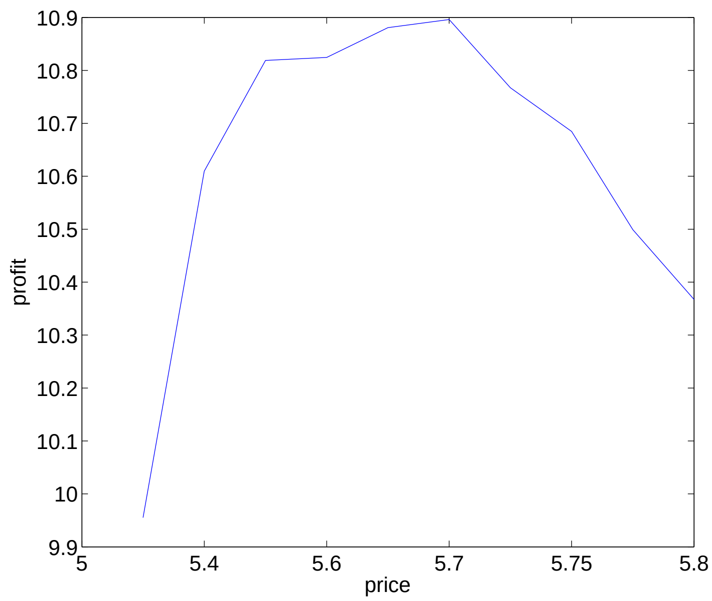

Figure 4.5. The profit vs. the price for the inventory level 1450

Although while simulating the system we make assumptions differently from the analytical model, the unit costs and the time spent in various stations of the system are the same as in the analytical model. Thus in order to compare the results of two models for the first case, the summary of the analytical model results are given in Table 4.5.

Table 4.5. Summary of analytical model results of the first case

Inventory Level (S^*)	Cost	Revenue	Profit	Price (P_s^*)
1244	1.8071	13.582	11.775	5.7

The profit obtained from the analytical model is higher which is reasonable, because in the simulation model we assume that the returned products cannot be reprocessed infinitely many times. Also, since we are not able to sell or lease the same product more than twice we have to keep more products in the finished goods inventory in order to satisfy the customer demand. Thus, in the simulation model the optimal inventory level S^* is higher than the one in the analytical model. According to simulation model the calculated profits of the first case are given in Table 4.6 for different combinations of the selling price and inventory levels.

Table 4.6. The profit with respect to price and inventory level

	S													
price	1250	1300	1350	1375	1400	1425	1450	1475	1500	1525	1550	1600	1650	1700
5	8.74863	9.041327	9.338547	9.504804	9.656913	9.79977	9.955178	10.08705	10.19612	10.31996	10.4159	10.60483	10.58085	10.6244
5.4	9.40851	9.740816	10,02466	10.2075	10.35042	10.48526	10.60979	10.71004	10.78877	10.82375	10.79551	10.73113	10.68459	10.63817
5.6	9.70899	10.05288	10.34989	10.50798	10.66071	10.70114	10.81896	10.80318	10.80569	10.81088	10.78479	10.73475	10.68843	10.64218
5.7	9.89169	10.18781	10.47339	10.64331	10.74763	10.78361	10.82461	10.90349	10.84195	10.83549	10.81231	10.76587	10.7193	10.67278
5.75	9.93345	10.26354	10.57158	10.6493	10.77243	10.82626	10.88081	10.88179	10.85729	10.8359	10.81251	10.76618	10.71922	10.67295
5.8	10.0591	10.33969	10.61344	10.72551	10.80331	10.86648	10.8961	10.8876	10.86085	10.83739	10.81379	10.76707	10.72036	10.67389
6	10.2255	10.50768	10.73322	10.822	10.82747	10.7905	10.76723	10.74391	10.72058	10.69708	10.674	10.6277	10.58126	10.53523
6.2	10.4521	10.71227	10.76826	10.77734	10.73108	10.70793	10.68467	10.66117	10.63783	10.61448	10.59134	10.54482	10.49805	10.45141
6.4	10.5796	10.6466	10.5917	10.56583	10.54569	10.5219	10.49904	10.47556	10.45233	10.42914	10.40628	10.35976	10.31329	10.26711
6.6	10.5553	10.50704	10.46039	10.4373	10.41396	10.39069	10.36725	10.34391	10.3206	10.29735	10.27403	10.2276	10.18161	10.13513

We design simulation scenarios, in order to understand the effect of the number of times the same product is reprocessed. According to the run results, when the number of times a returned product is reprocessed increases the profit increases too, which is meaningful because the same product may be leased or sold for so many times. The expected profit of the simulation model also approximates to the expected profit obtained from the analytical model. It can be observed that the number of times that a product is reprocessed affects the profit of the system and while this number goes to infinity the profit continues to increase hence the profit obtained from the analytical model is the highest one. However, when we discuss the optimal price and the inventory level for the case 2, the optimal price equals to 5.7 and the optimal inventory level is 1450. In addition to that, for the case 3 the optimal price equals to 5.8 and the optimal inventory level equals 1450. When we compare first three case the profit generated by the system is the highest in the third case where rn equals to 3. The results of the scenarios based on the second and third cases are given in Appendix A. Also, the figures that show the profit of the system with respect to the price and the inventory level are given in Appendix A.

In the simulation model, we also investigate how the proportion of the leasing price to the selling price, $\alpha_{l,i}$ affects the profit generated by the system, the optimal price and inventory levels. The first case and the fourth cases are compared to understand how the profit, optimal price and inventory levels are affected by the level of $\alpha_{l,i}$. For the fourth case, the simulation model results show that the maximum profit is reached at a point where the price equals to 6.3 and the order-up-to level equals to 1900. Figure 4.6 shows, for the given optimal order-up-to level, how the expected profit changes with respect to the selling price. On the other hand, Figure 4.7 shows, for the given optimal selling price how the expected profit changes with respect to the order-up-to-level.

For the fourth case, the profit is calculated for different combinations of the selling price and and inventory level. The results (see Table A.3 in Appendix A) show that the optimal profit decreases when $\alpha_{l,i}$ values decrease from 0.28, 0.54 and 0.77 to 0.15, 0.27 and 0.39 for $i = 1, 2, 3$. The reason of the decrease in the profit can be explained by the decrease in the revenue obtained from the leasing customers. Moreover, the

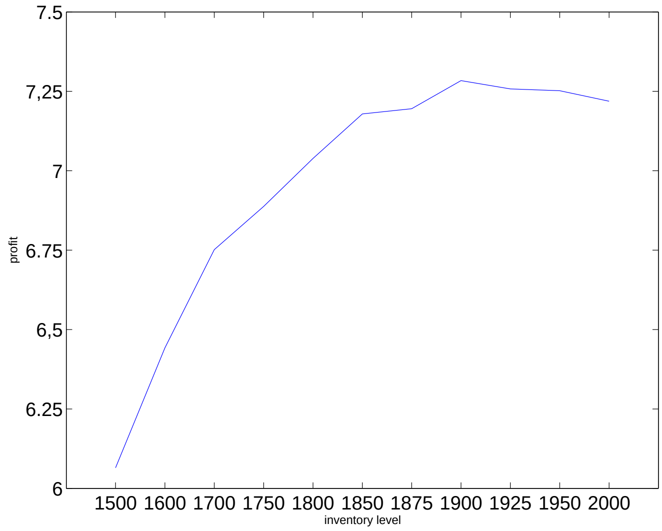

Figure 4.6. The profit vs. the inventory level for the case 4

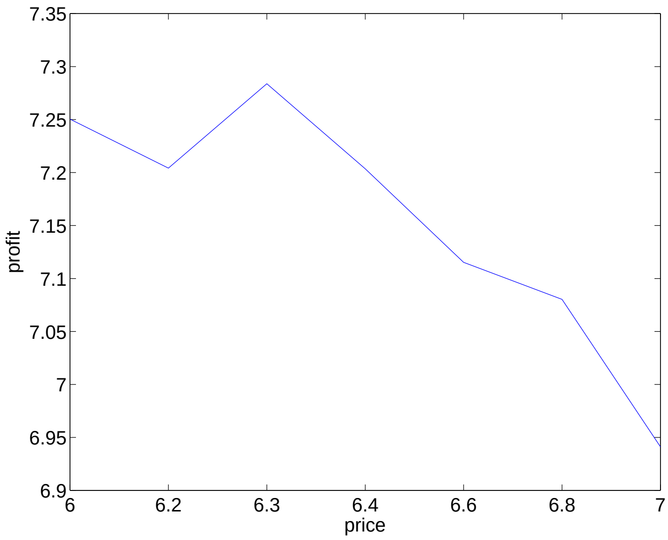

Figure 4.7. The profit vs. the price for the case 4

simulation results show that the calculated profit is less than the expected profit of the analytical model for the same case. The analytical model results of the fourth, fifth and sixth cases are the same since in the analytical model we do not consider a limit on the number of times that a returned product is reprocessed. Thus the corresponding results are given in Table 4.7.

Table 4.7. Summary of the analytical model results of the fourth case

Inventory Level (S^*)	Cost	Revenue	Profit	Price (P_s^*)
1523	2.1005	10.428	8.3272	6.3

Another important criteria that affects the profit of the system is the return probability of sold products. Two levels of this probability, 0.7 and 0.4 are considered in different scenarios. When we compare the sixth and seventh cases the return probability of a sold product is equal to 0.4 in the case 6 and it is equal to 0.7 in the case 7. When we analyze the results of the case 6 scenarios, the profit equals to 8.0844 where the selling price is equal 6.2 and the order-up-to level is equal to 1850.

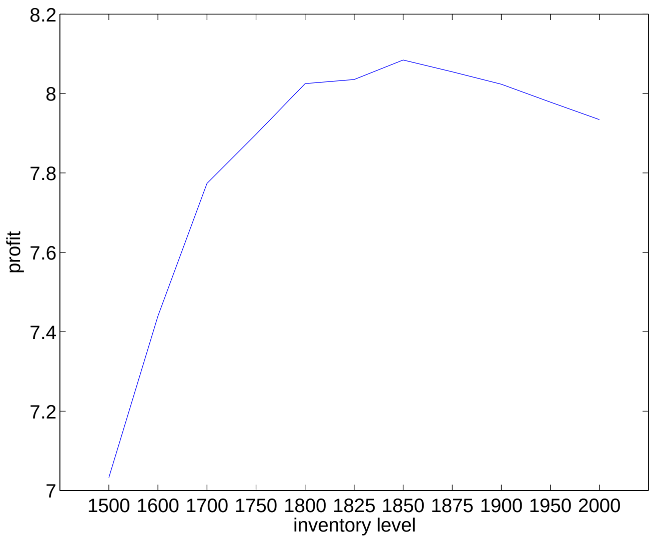

Figure 4.8. The profit vs. the inventory level for the case 6

On the other hand, the results of the senarios designed based on the case 7 parameter levels, show that the highest profit equals to 8.1170 where the selling price equals to 6.2 and the order-up-to level equals to 1925. As a result, when the return probability of a sold product is low, the profit will decrease. Also, the optimal inventory

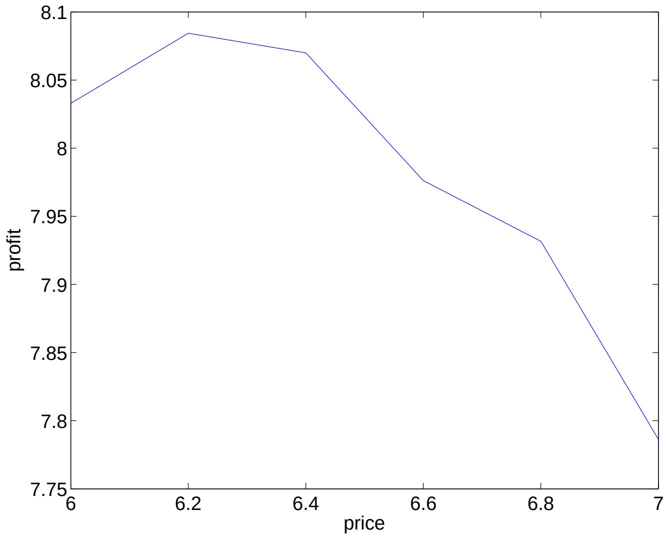

Figure 4.9. The profit vs. the price for the case 6

level increases as the return probability increases. Because, for the products that will not return from the buying customers, we give a manufacturing order; however for the products that will return we wait until the end of period that the customer uses the product. Therefore, in order to decrease backorder and loss sale costs the optimal order-up-to level is increased. The graph of the profit with respect to the inventory level and the selling price is given in Figure 4.10 and Figure 4.11, respectively. The summary of results for the seventh, eighth and ninth cases are given in Appendix A.

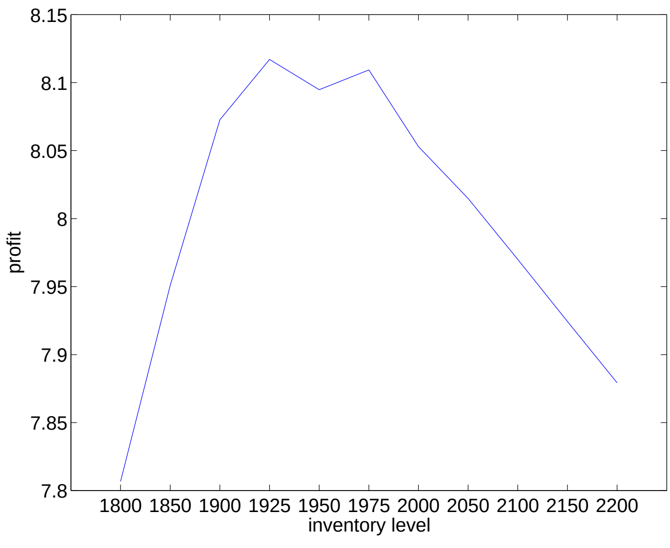

Figure 4.10. The profit vs. the inventory level for the case 7

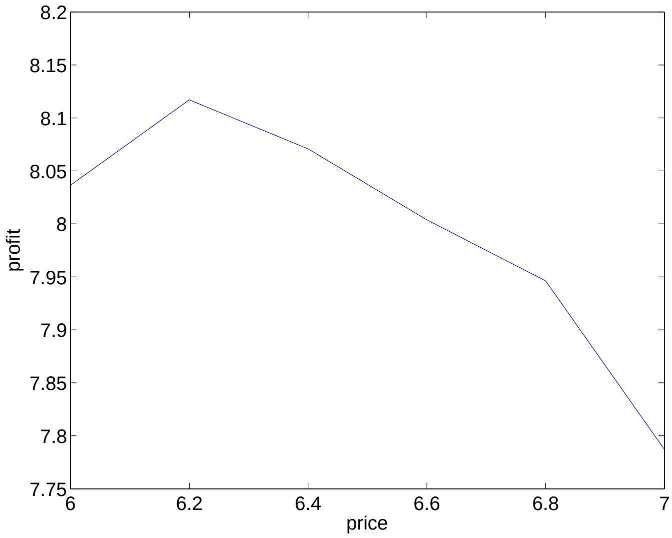

Figure 4.11. The profit vs. the price for the case 7

We also design scenarios for the tenth, eleventh and twelfth cases where $\alpha_{l,i}$ equals to 0.28, 0.54, 0.77 for $i = 1, 2, 3$, r equals 0.7 and rn equals 3, 2, 1 respectively. When we compare the case 7 and case 10 where the only difference is the value of $\alpha_{l,i}$. We observe that, when $\alpha_{l,i}$ is higher the profit of the system will be higher, since the revenue obtained from the leasing customers is higher.

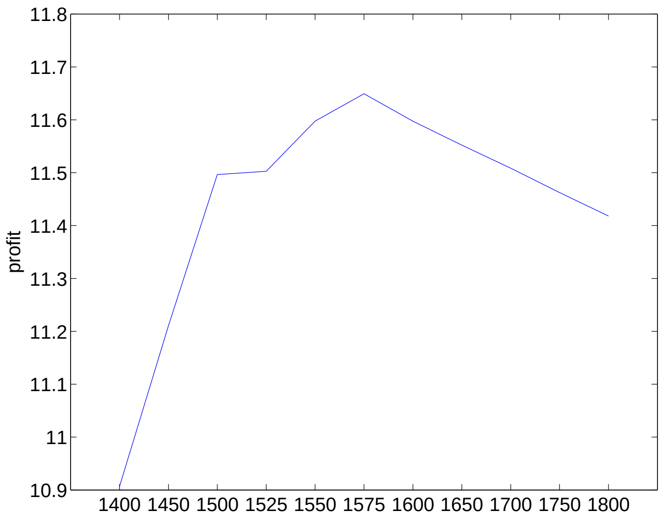

Figure 4.12. The profit vs. the inventory level for the case 10

On the other hand, when we compare scenario results of the case 11 and the case 12, we observe that if the number of times that a returned product is reprocessed, is

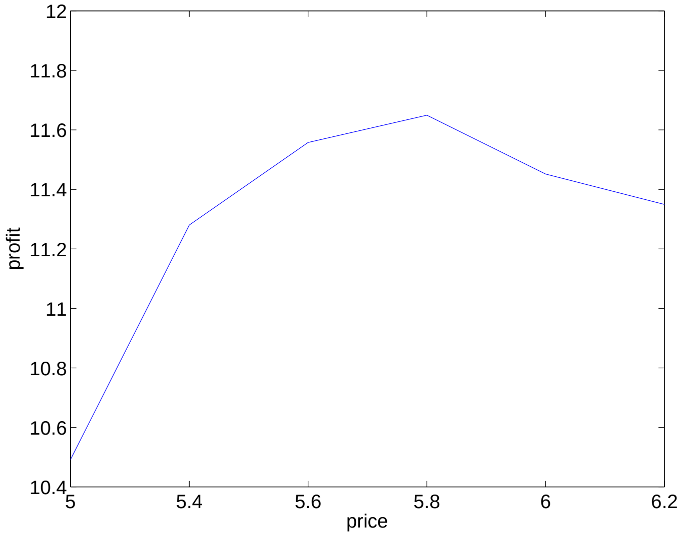

Figure 4.13. The profit vs. the price for the case 10

high the profit generated by the system will be higher.

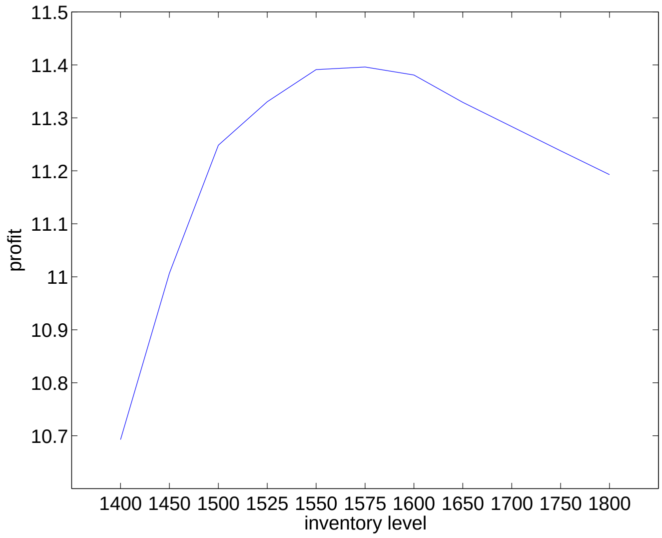

Figure 4.14. The profit vs. the inventory level for the case 11

According to these results, for a given price the profit first increases, then decrease with respect to the inventory level. The reason of the increase in profit is the decrease of the backorder and the loss sale costs. This result also means the customer demand is more satisfied. However, after a certain level of the inventory, the profit decreases because the increase in the holding cost cannot be compensated by the decrease in the backorder and loss sale costs. For a higher price level when the inventory level

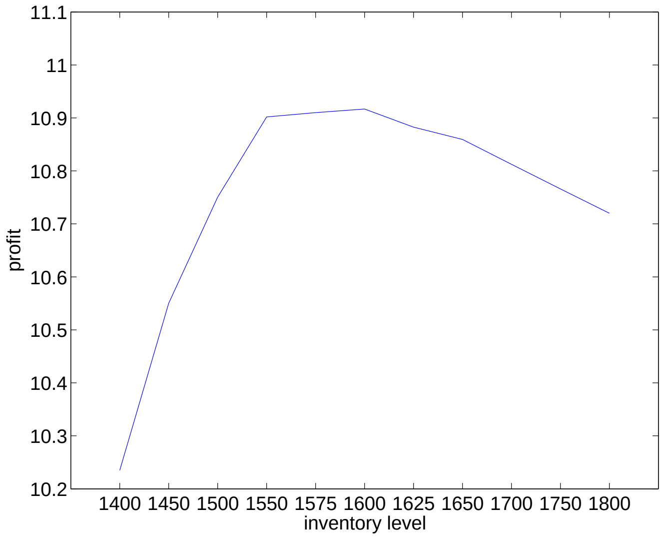

Figure 4.15. The profit vs. the inventory level for the case 12

is increased the profit also increases, however again after a certain inventory level the profit decreases. When a higher price is set it is observed that usually the maximum profit is reached at a lower order-up-to level S, since at a higher price the demand of the product will be lower. On the other hand, for a given order-up-to-level S, when the price increases, the profit increases up to a level. However after a certain level, increasing the price will not increase the profit, because the demand decreases.

The simulation model give insights about how should the price and the base stock level S^* set, in order to maximize the profit while satisfying the leasing and selling customer demand best. In the simulation model, we use the results of the analytical model as an initial price and inventory level. Then, for different price and inventory levels we run the simulation model and analyze results. However, although we have done 921 scenarios, certainly there are more cases that can be tried to see how the profit changes id different scenarios. Thus, we determine that by fitting the results that are obtained from the simulation model to a function in order to make possible to try out more combinations of the price and inventory level.

4.4. An Approximation for The Average Profit

In simulation model, we tried out different price and inventory levels in order to find optimal pairs that would maximize the profit. Therefore, we aim to fit an equation from the simulation model results. In order to find the function which fits the data best, we use a tool (http://zunzun.com) which is a function finder. This tools also searches whether the fitted function is linear, exponential, cubic, quadratic, logarithmic, trigonometric, etc. At the end of the search, the tool gives a rank for type of the function and the quadratic function has one of the highest rank. Thus, we choose the equation that is a quadratic type of function, also from the analytical and simulation model results we know that the shape of the profit function is suitable to be a quadratic function.

The data which is used to find the best function is the one obtained by using the results of case 1 with parameters defined in Table 4.3, because the results generated for the first case is very detailed. There are 140 data points that can be fed into the tool in order to obtain a function which gives the best results. After putting the data points into the tool it gives the following function:

$$P(S, P_s) = a + bP_s + cS + dP_s^2 + eS^2 + fP_sS. \tag{4.1}$$

The selling price is denoted by P_s, the inventory level is denoted by S and the obtained profit is denoted by $P(S, P_s)$. The coefficients of the equation are given in the Table 4.8.

Therefore, since a, b, c, d, e and f are known by putting new price and inventory levels into the function one can obtain new profit values. The function is quadratic and the shape of the function is concave thus we are able to find the global optimum where the profit of the system is the maximum. As a result the obtained points are P_s^*=5.754, S^*=1562 and $P(S, P_s)^*$=10.8569. When we compare the results of the simulation model and results obtained from the function defined in Equation (4.1) we

Table 4.8. Coefficients of the function

a = -68.4989
b = 15.3301
c = 0.04513
d = -0.8592
e = -0.000008
f = -0.00348

can conclude that the best price and the best inventory level for this data set is 5.8 and 1450, respectively. The profit obtained from the model is 10.8961, thus actually when we search near the optimal solution, we find another function where the coefficients are given in Table 4.9.

Table 4.9. Coefficients of the second function

a = -173.2847
b = 42.4647
c = 0.08207
d = -2.8007
f = -0.000014
g = -0.006676

According to this new function the optimal price equals to P_s^*= 5.8225, the optimal inventory level is found as S^*=1475 and the optimal profit is $P(S, P_s)^*$= 10.8853. The obtained values are closer the ones found as a result of simulation model. To sum up, fitting the results obtained from the simulation model help the decision maker getting new profit values with respect to the different combinations of the price and inventory level. The results generated by the fitted function are consistant with the results of the analytical and simulation model thus one can use the function in order to decide the optimal price and order-up-to level S that will maximize the profit of the system.

5. CONCLUSIONS

In this study, we consider a single product system in which customers arrive according to Poisson processes and request either to lease or to buy a product. If a customer prefers to lease the product, she chooses one the lease options given by the manufacturer. At the end of the lease period, the customer return the product to the system. The returned product may either be refurbished or dismantled. The decision to refurbish or to dismantle the product depends on whether the product is depreciated. On the other hand, if a customer prefers to buy the product, she may return the product to the system with a certain probability. Returned products from the buying customers are remanufactured. If a product returned from lease is not refurbishable or a sold product is not returned from the customer, a manufacturing order is given. We assume that a manufacturing order is given according to an $(S-1, S)$ policy.

We model the system as an infinite horizon problem with the objective of maximizing the expected profit per unit time with respect to the selling price and the base stock level S. After building the analytical model, to find the optimal selling price and order-up-to level a search algorithm is used which is coded in MATLAB. In the analytical model, we assume that the returned products can be reprocessed infinitely many times. However, in the real world, this is not possible because each time a product is reprocessed, its level of depreciation increases. Hence, after a certain number of reprocessing operations, the product has to be dismantled. Another assumption we make in the analytical model is that if a leasing customer's demand is not fulfilled from the finished goods inventory it is backordered. In order to relax these two assumptions we built a simulation model. In the simulation model, 921 scenarios are designed for 12 different cases. Each case consists of different combinations of the following system parameters: $\alpha_{l,i}$ the proportion of leasing price to selling price; r, the return probability of the sold product; rn the number of times that the returned product is reprocessed.

Both the analytical and simulation model results show that for a given price, the profit first increases then decreases with respect to the inventory level. The reason for the decrease in profit after a certain level of inventory level is that the increase in holding cost cannot be compensated any more by the revenue obtained from the lease and the sales. On the other hand, for a given inventory level the profit first increases then decreases with respect to the price. The reason of this result can be explained as follows: An increase in price up to a point, increases the revenue generated by the system. However, after that point decrease in demand cannot be compensated by the price increase.

The simulation model results also provide insights on how the profit, optimal price and order-up-to level S changes with various system parameters. The results show that, if the number of times that the returned product is reprocessed decreases, the average profit decreases. This is very reasonable because for each dismantled product a manufacturing order has to be given and the manufacturing cost incurred will increase. Also, limiting the number of times of reprocessing causes an increase in backorder and loss sale costs. Moreover, if the return probability of sold products increases the profit generated by the system increases.

We used the simulation model results to obtain a function that finds the approximate profit for a given price and inventory level. Since the profit is a function of two decision variables the function is 3 dimensional thus a 3D function finder is used provided in (http://zunzun.com). This function is used to find the profit without running the simulation model for different price and inventory level combinations. By using the same function, the profit is optimized by taking the partial derivatives of the function with respect to the decision variables. By using the fitted function, different combinations of price and inventory level pairs can be tried to get the profit of the system. The results generated by the function are consistant with the analytical and simulation model results.

As a future study, the loss sale case can be incorporated into the analytical model because the current model is valid under the backorder assumptions. Another extension may be on the assumption that the customers are not able to differentiate the reprocessed and brand-new products. Choice behaviour of the customers who are able to differentiate between the reprocessed and brand-new product may be incorporated in a new model. Moreover, we assume that the market is monopolistic. Thus in a future study the remanufacturers may compete on the remanufactured product market. In another extension, the competition of the leased and sold products can be added to the model then the payment structure of the leased products may be changed since the payment structure affects the leased product demand.

APPENDIX A: FIGURES AND TABLES OF THE SIMULATION MODEL RESULTS

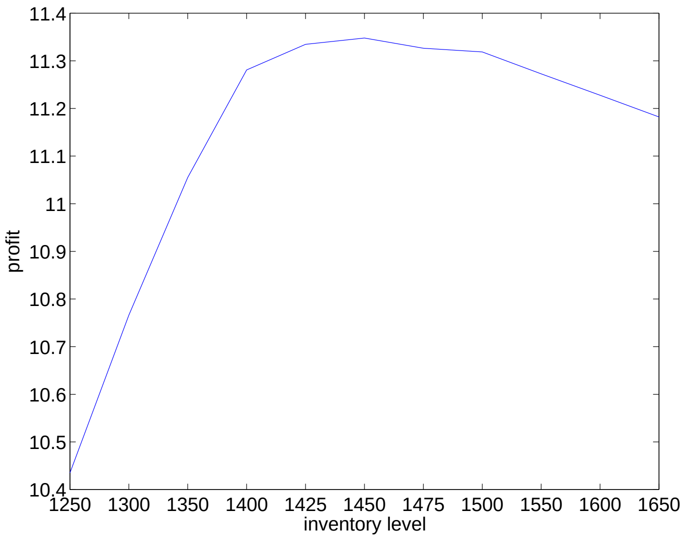

Figure A.1. The profit vs. the inventory level for the case 2

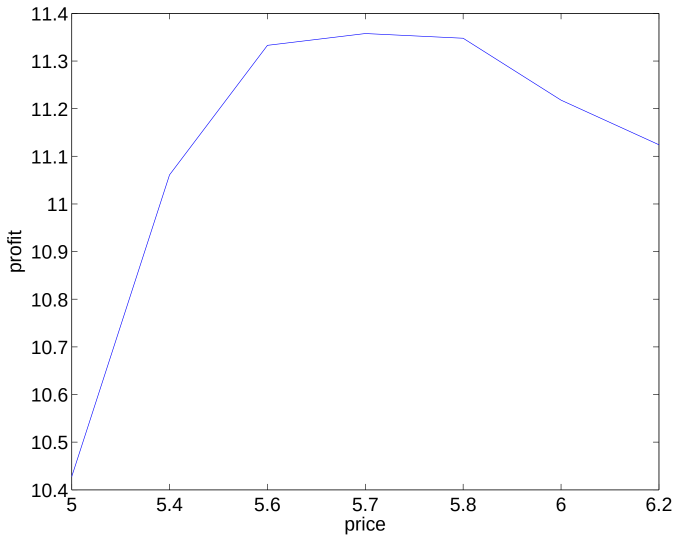

Figure A.2. The profit vs. the price for the case 2

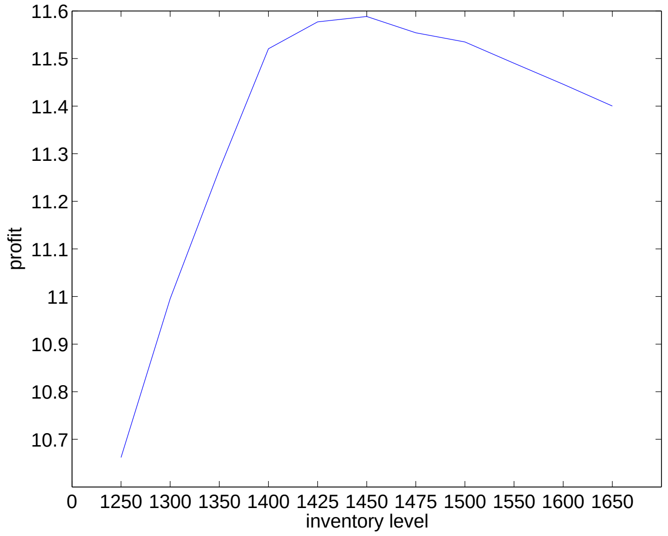

Figure A.3. The profit vs. the inventory level for the case 3

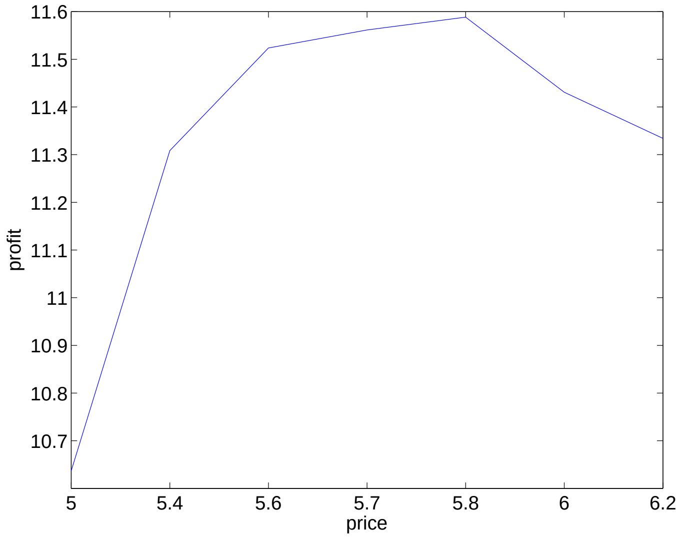

Figure A.4. The profit vs. the price for the case 3

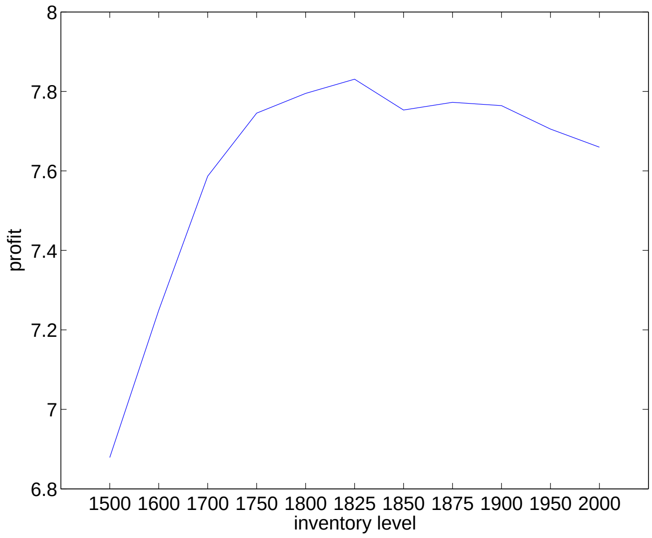

Figure A.5. The profit vs. the inventory level for the case 5

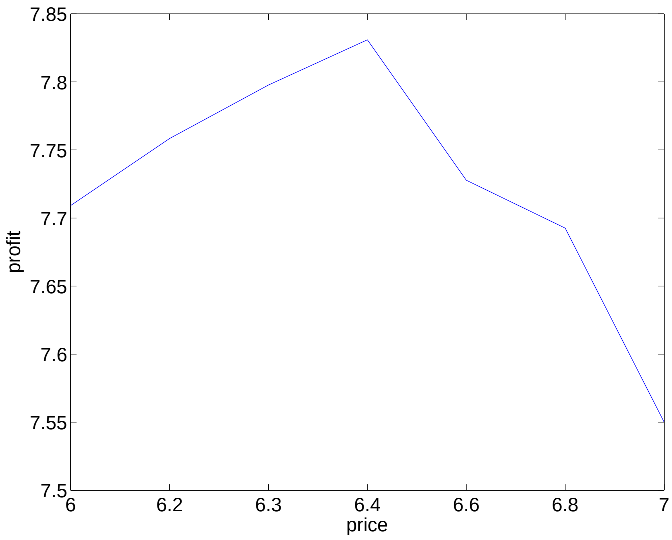

Figure A.6. The profit vs. the price for the case 5

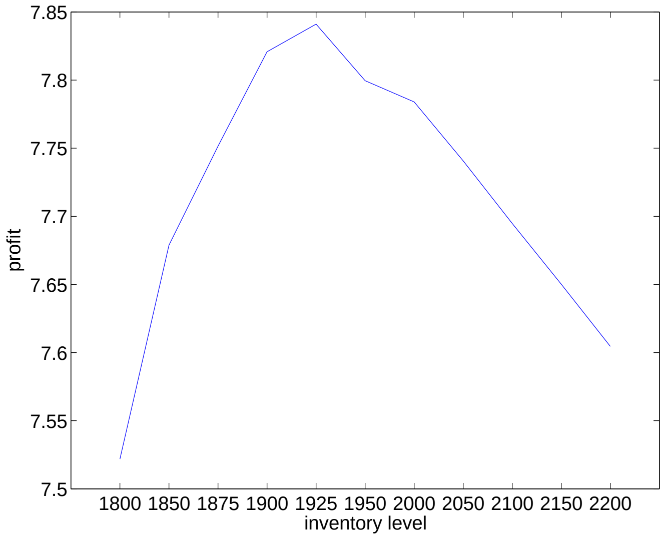

Figure A.7. The profit vs. the inventory level for the case 8

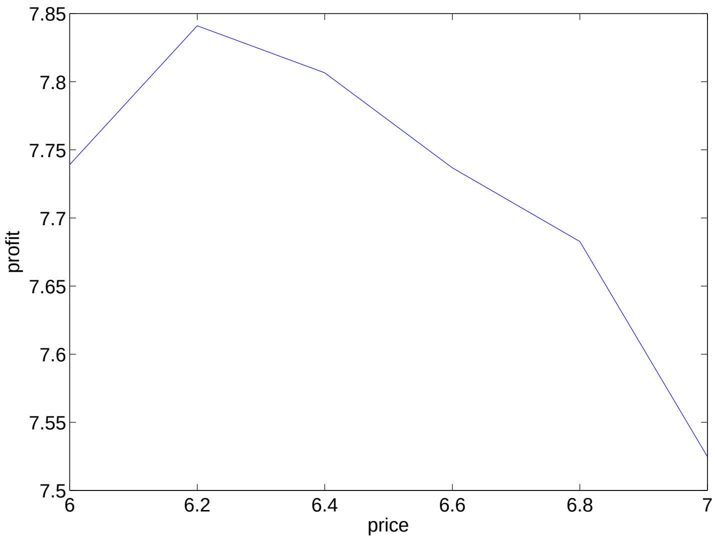

Figure A.8. The profit vs. the price for the case 8

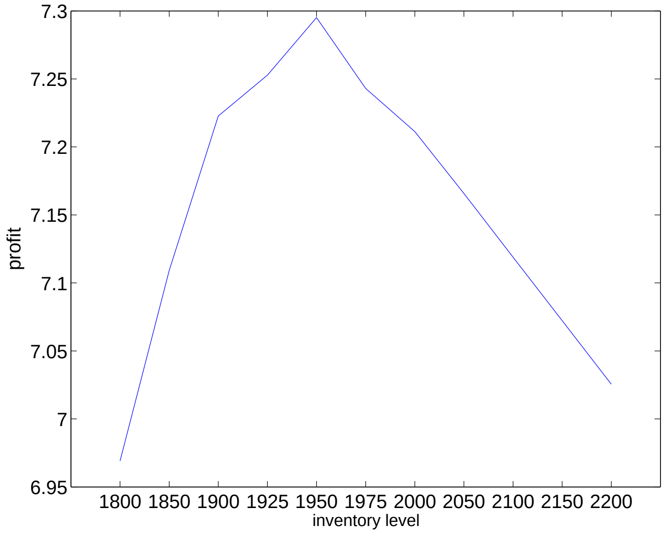

Figure A.9. The profit vs. the inventory level for the case 9

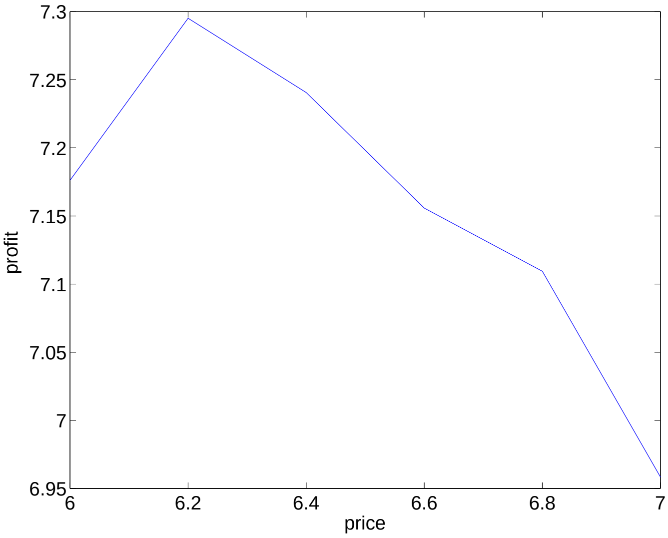

Figure A.10. The profit vs. the price for the case 9

Table A.1. Case 2 Results

	S										
price	1250	1300	1350	1400	1450	1500	1525	1550	1575	1600	1650
5	9.1593	9.4956	9.8229	10.1355	10.2540	10.4276	10.5631	10.6821	10.9499	11.0884	11.1638
5.4	9.8555	10.2029	10.4973	10.8043	10.9601	11.0609	11.1875	11.2495	11.2694	11.2094	11.1992
5.6	10.1415	10.4854	10.7931	11.0971	11.1864	11.333	11.3269	11.2825	11.2480	11.2074	11.1615
5.7	10.3051	10.6242	10.9579	11.1995	11.2669	11.3577	11.3052	11.3182	11.2797	11.2345	11.1893
5.8	10.4348	10.0552	11.2807	11.3345	11.3477	11.3263	11.3186	11.2725	11.2276	11.1821	11.1928
6	10.6677	10.9827	11.2504	11.2581	11.2407	11.2179	11.1949	11.1725	11.1273	11.0821	11.0368
6.2	10.9071	11.0910	11.2069	11.1692	11.1468	11.1242	11.1091	11.0793	11.0342	10.9886	10.9427

Table A.2. Case 3 Results

	S										
price	1250	1300	1350	1400	1425	1450	1475	1500	1550	1600	1650
5	9.3882	9.6908	10.0089	10.3188	10.4830	10.6370	10.7860	10.9264	11.1919	11.3337	11.3778
5.4	10.0271	10.3772	10.7071	11.0358	11.1197	11.3084	11.4519	11.4409	11.4728	11.4381	11.3931
5.6	10.3312	10.6888	10.9978	11.3211	11.4529	11.5237	11.5285	11.4971	11.4788	11.4290	11.3843
5.7	10.4717	10.8510	11.1445	11.4245	11.5122	11.5614	11.5664	11.5565	11.5028	11.4584	11.4135
5.8	10.6617	10.9955	11.2664	11.5204	11.5769	11.5883	11.5541	11.5348	11.4901	11.4458	11.4003
6	10.8603	11.2260	11.3878	11.4877	11.4534	11.4308	11.4085	11.3861	11.3407	11.2961	11.2516
6.2	11.1250	11.2921	11.4095	11.3798	11.3563	11.3340	11.3115	11.2895	11.2443	11.1992	11.1545

Table A.3. Case 4 Results

	S										
price	1500	1600	1700	1750	1800	1850	1875	1900	1925	1950	2000
6	5.9612	6.5506	6.8720	6.9886	7.0762	7.1900	7.1953	7.2504	7.2100	7.1942	7.1475
6.2	6.2999	6.6599	6.9634	7.0792	7.1677	7.2122	7.2158	7.2041	7.1677	7.1444	7.0976
6.3	6.0646	6.4424	6.7515	6.8877	7.0387	7.1788	7.1954	7.2838	7.2577	7.2521	7.2190
6.4	6.4255	6.7568	7.0695	7.1540	7.1961	7.2025	7.2031	7.2035	7.1863	7.1474	7.1008
6.6	6.4575	6.7935	7.0831	7.2153	7.1491	7.1724	7.1384	7.1153	7.0920	7.0686	7.0223
6.8	6.4906	6,8456	7.0961	7.2087	7.1579	7.1268	7.1035	7.0803	7.0567	7.0336	6,9872
7	6.4908	6.8625	7.0705	7.0905	7.0381	6.9871	6.9641	6.9411	6.9182	6.8952	6.8485

Table A.4. Case 5 Results

	S										
price	1500	1600	1700	1750	1800	1825	1850	1875	1900	1950	2000
6	6.7111	7.0798	7.3777	7.5227	7.6506	7.7091	7.8110	7.8016	7.7987	7.7640	7.7188
6.2	6.7851	7.1795	7.4954	7.6392	7.7695	7.7584	7.7944	7.7866	7.7545	7.7111	7.6658
6.3	6.8469	7.1697	7.5450	7.6830	7.7750	7.7977	7.7840	7.7863	7.7433	7.6908	7.6560
6.4	6.8786	7.2491	7.5866	7.7454	7.7951	7.8309	7.7533	7.7726	7.7642	7.7052	7.6597
6.6	6.1967	7.2654	7.6071	7.7303	7.7508	7.7277	7.7132	7.6905	7.6673	7.6222	7.5768
6.8	7.0010	7.3687	7.6525	7.7257	7.7080	7.6925	7.6699	7.6476	7.6246	7.5796	7.5342
7	6.9794	7.3660	7.6295	7.6267	7.5959	7.5499	7.5275	7.5051	7.4823	7.4365	7.3913

Table A.5. Case 6 Result

	S										
price	1500	1600	1700	1750	1800	1825	1850	1875	1900	1950	2000
6	6.9380	7.3014	7.6664	7.7943	7.9193	8.0182	8.0329	8.0789	8.0621	8.0338	7.9899
6.2	7.0327	7.4393	7.7734	7.8973	8.0250	8.0352	8.0844	8.0546	8.0235	7.9784	7.9342
6.4	7.1372	7.4759	7.8596	7.9955	8.0553	8.0308	8.0699	8.0230	8.0352	7.9726	7.9274
6.6	7.1608	7.5694	7.9034	8.0272	8.0046	7.9916	7.9762	7.9537	7.9303	7.8857	7.8408
6.8	7.2251	7.5897	7.9075	7.9746	7.9750	7.9539	7.9316	7.9084	7.8863	7.8419	7.7966
7	7.2100	7.6044	7.8743	7.8706	7.8119	7.8091	7.7851	7.7645	7.7426	7.6967	7.6527

Table A.6. Case 7 Results

	S										
price	1800	1850	1900	1925	1950	1975	2000	2050	2100	2150	2200
6	7.6808	7.8180	7.9776	8.0365	8.0548	8.0823	8.0763	8.0866	8.0429	7.9974	7.9527
6.2	7.8067	7.9508	8.0726	8.1170	8.0947	8.1093	8.0529	8.0148	7.9700	7.9243	7.8792
6.4	7.9335	8.0587	8.0947	8.0707	8.0666	8.0395	8.0411	7.9940	7.9497	7.9052	7.8600
6.6	7.9454	8.0200	8.0313	8.0038	7.9812	7.9586	7.9372	7.8919	7.8462	7.8022	7.7579
6.8	7.9825	7.9905	7.9678	7.9461	7.9235	7.9013	7.8787	7.8337	7.7898	7.7442	7.6997
7	7.8974	7.8661	7.8093	7.7870	7.7650	7.7427	7.7201	7.6759	7.6308	7.5859	7.5416

Table A.7. Case 8 Results

	S										
price	1800	1850	1875	1900	1925	1950	2000	2050	2100	2150	2200
6	7.3921	7.5722	7.5856	7.6680	7.7391	7.7934	7.8128	7.8103	7.7642	7.7119	7.6740
6.2	7.5218	7.6788	7.7515	7.8208	7.8410	7.7995	7.7839	7.7407	7.6947	7.6501	7.6045
6.4	7.6739	7.7415	7.7727	7.8347	7.8065	7.8173	7.7660	7.7212	7.6755	7.6304	7.5850
6.6	7.7123	7.7621	7.7789	7.7500	7.7367	7.7134	7.6677	7.6224	7.5766	7.5321	7.4863
6.8	7.7179	7.7392	7.7274	7.7052	7.6827	7.6599	7.6150	7.5607	7.5243	7.4786	7.4330
7	7.6237	7.6077	7.5706	7.5472	7.5247	7.5023	7.4566	7.4120	7.3664	7.3219	7.2762

Table A.8. Case 9 Results

	S										
price	1800	1850	1900	1925	1950	1975	2000	2050	2100	2150	2200
6	6.8762	7.0265	7.1077	7.1711	7.1761	7.2417	7.2409	7.2098	7.1854	7.1383	7.0918
6.2	6.9690	7.1091	7.2227	7.2527	7.2951	7.2430	7.2112	7.1658	7.1189	7.0723	7.0255
6.4	7.0937	7.1968	7.2235	7.2253	7.2405	7.2064	7.2031	7.1548	7.1082	7.0620	7.0157
6.6	7.1790	7.2349	7.1973	7.1811	7.1557	7.1323	7.1090	7.0626	7.0161	6.9697	6.9232
6.8	7.1682	7.1639	7.1558	7.1327	7.1094	7.0858	7.0624	7.0165	6.9699	6.9235	6.8773
7	7.0923	7.0543	7.0046	6.9815	6.9583	6.9354	6.9123	6.8656	6.8189	6.7723	6.7261

Table A.9. Case 10 Results

	S										
price	1400	1450	1500	1525	1550	1575	1600	1650	1700	1750	1800
5	9.3243	9.8043	10.0036	10.1548	10.2989	10.4918	10.6314	10.9198	11.1754	11.3429	11.4265
5.4	10.1842	10.5167	10.8368	10.9959	11.1639	11.2803	11.3965	11.5673	11.5291	11.4814	11.4369
5.6	10.5432	10.8677	11.2058	11.3456	11.4770	11.5577	11.5885	11.5493	11.5047	11.4608	11.4155
5.8	10.9066	11.2107	11.4964	11.5027	11.5974	11.6493	11.5973	11.552	11.5085	11.4625	11.4179
6	11.2089	11,4700	11.5293	11.4957	11.4729	11.4514	11.4288	11.3836	11.3387	11.2941	11.2489
6.2	11.3880	11.4310	11.4158	11.3934	11.3714	11.3494	11.3269	11.2822	11.2367	11.1910	11.1472

Table A.10. Case 11 Results

	S										
price	1400	1450	1500	1525	1550	1575	1600	1650	1700	1750	1800
5	9.1671	9.4803	9.7797	9.9548	10.0804	10.2495	10.3923	10.6847	10.9203	10.1068	11.2057
5.4	9.9607	10.2980	10.6206	10.783	10.9342	11.0634	11.1866	11.3133	11.3047	11.2454	11.1992
5.6	10.3320	10.6800	10.9532	11.1122	11.2322	11.3213	11.3361	11.3890	11.2756	11.2296	11.1845
5.8	10.6926	11.0067	11.2483	11.3303	11.3910	11.396	11.3808	11.3292	11.2836	11.2378	11.1928
6	10.9819	11.2422	11.3237	11.2854	11.2542	11.2308	11.2079	11.1627	11.1178	11.0731	11.0271
6.2	11.1651	11.2391	11.1991	11.1764	11.1541	11.1313	11.1087	11.0633	11.0170	10.9718	10.9261

Table A.11. Case 12 Results

	S										
price	1400	1450	1500	1550	1575	1600	1625	1650	1700	1750	1800
5	8.7315	9.0487	9.3449	9.6356	9.7625	9.9299	10.0534	10.1802	10.4182	10.5861	10.6501
5.4	9.5010	9.8465	10.1498	10.4279	10.5590	10.6992	10.7817	10.7695	10.7789	10.7564	10.7101
5.6	9.8921	10.2387	10.5090	10.7405	10.8376	10.8775	10.8456	10.8711	10.7943	10.7478	10.7019
5.8	10.2348	10.5502	10.7504	10.9019	10.9099	10.9168	10.8825	10.8593	10.8125	10.7659	10.7201
6	10.5276	10.7193	10.8420	10.7966	10.7733	10.7500	10.7268	10.7039	10.6576	10.6116	10.5652
6.2	10.7179	10.7933	10.7522	10.7055	10.6820	10.6587	10.6355	10.6123	10.5661	10.5195	10.4737

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