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SÜLEYMAN DEMİREL UNIVERSITY
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INVESTIGATING OF EARLY UNIVERSE THROUGH EQUATION OF STATE OF QUARK GLUON PLASMA

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THE DEGREE OF DOCTOR OF PHILOSOPHY
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COMMITMENT

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Hamid ALZAKI

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ABSTRACT

Ph.D. Thesis

INVESTIGATING OF EARLY UNIVERSE THROUGH EQUATION OF STATE OF QUARK GLUON PLASMA

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In this thesis we study the evolution of thermodynamics parameters in an epoch of the early universe. The universe in that epoch was filled by a fluid of quark gluon plasma.The thermodynamics parameters we study are the energy density, pressure density and entropy density. The time evolution of these parameters can be obtained upon solving Friedmann differential equations of the energy density, of the general relativity. In this thesis we provide analytic solutions for the Friedmann differential equation in the framework of the bag models. In addition, we present the results of the analytic solutions for pressure density and entropy density. Based on the analytic solutions, we present the plot of the time evolutions of these thermodynamics parameters and a discussion about their behavior will be provided. This will shed light on that epoch of the early universe.

Keywords: Quark-gluon plasma, Friedmann differential equations, early universe.

2020, 58 pages

ÖZET

Doktora Tezi

İŞBİRLİKÇİ GRİ YÖNEYLEM ARAŞTIRMASI OYUNLARI

Hamid Farhan Ashour ALZAKI

Süleyman Demirel Üniversitesi
Fen Bilimleri Enstitüsü
Fizk Anabilim Dalı

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Bu tezde bazı termodinamik parametrelerin evrenin erken bir döneminde değişimleri incelenmiştir. Bahsi edilen dönemde evren kuvark-gluon plazması adı verilen sıvı formda bir maddeyle dolu idi. Çalışılan termodinamik parametreler enerji yoğunluğu, basınç yoğunluğu ve entropi yoğunluğudur. Bu parametrelerin zamanla değişimleri, Genel Görelilik'in bir denklemi olan ve enerji yoğunluğuna bağlı olarak yazılabilen Friedmann diferansiyel denklemi analitik olarak çözülerek elde edilebilmektdir. Bu tez çalışmasında Friedmann denklemine bag model yaklaşımı çerçevesinde analitik çözümler elde dilmiştir. Ayrıca, basınç yoğunluğu ve entropi yoğunluğu için analitik çözümler sunulmuştur. Analitik çözümler temel alınarak ilgili termodinamik parametrelerin zamana bağlı değişimlerini gösteren grafikler ve davranışları üzerine tartışmalar sunulmuştur. Bu tartışmaların evrenin o erken evresine ışık tutacağını düşünmekteyiz.

Anahtar Kelimeler: kuvark-gluon plazması, Friedmann diferansiyel denklemi, erken evresi.

2020, 58 sayfa

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Hamid Farhan Ashour ALZAKI
ISPARTA, 2020

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LIST OF SYMBOLS AND ABBREVIATION

QGP	**:** Quark Gluon Plasma
EOS	**:** Equation Of State
QCD	**:** Quantum Chromo-dynamic
MIT	**:** Massachusetts Institute of Technology
SM	**:** Standard Model
RHIC	**:** Relativistic Heavy Ion Collider
CERN	**:** The European Organization for Nuclear Research
SPS	**:** Super Proton Synchrotron
BNL	**:** Brookhaven National Laboratory
ε	**:** Total Energy density
T	**:** Temperature
P	**:** Pressure
t	**:** Time

1. INTRODUCTION

Hadrons are composed of quarks and can interact through strong nuclear force. Hadrons can be classified into baryons and mesons. Baryons are made of three quarks or three anti-quarks while mesons consist of quark and anti-quark. Protons and neutrons are examples of baryons. Gluons are massless particles that mediate the strong nuclear interaction (force) between quarks or in other words gluons are the mediator of the strong nuclear interaction between quarks (Markum et al., 1985). Protons and neutrons combine together to form the nucleus of an atom. Thomsen, Rutherford and Chadwick found, experimentally, that atoms contain nucleus and electrons orbiting. Later, many other particles such as muons, neutrinos and hadrons were observed. A variety of precision measurements were made since the 1960s to investigate the internal structures of protons and neutrons (Lipkin, 1973). In 1964, the quark model was first presented by Murray Gilman and Zweig, independently. According to the quark model, hadrons are combinations of quarks or anti-quarks or both of them together (Griffiths, 2008). The quark model is a successful model as it is verified experimentally up to date.

The main forces (interactions) governing our universe are gravity, electromagnetic, the weak and strong nuclear forces. The strong nuclear force is responsible for the confinement of quarks and gluons forming the hadrons and also the confinement of protons and neutrons in the nucleus of the atom and thus, is the dominant force at the nucleus scale. For instances, the main forces can be scaled as: gravitational force is $\sim 10^{-39}$, the weak interaction is $\sim 1/137$ and the strong interaction is ~ 1 (Busza et al., 2018). At low energies, the strong nuclear interaction is really strong. This can be the reason that the observation of free quarks is difficult. However, at sufficient large values of energy, the strong nuclear force becomes loose enough to create a state consists of free quarks and gluons. This state is known as Quark Gluon Plasma (QGP).

Quantum chromodynamics (QCD) is widely accepted theory describing the strong nuclear interaction. Collins, QCD have asymptotic freedom i.e. the QCD coupling constant decreases with the increasing of the momentum transfer q^2 (Collins and Perry, 1975; Yang, 2006). The QCD coupling constant controls the strength of the interactions between quarks/gluons and also between quarks and gluons. Thus, at large momentum transfer q^2, which means high energies, the QCD coupling constant becomes small enough to allow quarks and gluons to be free. This property of QCD leaded Perry and others to firstly proposed the existence of QGP as a new form of matter at very high temperatures and densities in the 1970s. QGP is also believed to exist as a phase of the early universe after parts of a thousandth of a second following the Big Bang. QGP played an important role in the achievement of hadronization process, a process in which quarks and gluons are confined together to form hadrons, that leaded to the formations of protons and neutrons essential for having atoms and thus matter forming stars, planets and all material structures in our universe. Fig 1. shows the expansion of the universe following the big bang.

The Big Bang is widely accepted theory to describe how our universe emerges and its evolution with time. According to practical observations, it appears that the universe began to form 13.7 billion years ago. Since then, the universe is believed to have gone through three stages in its formation. One of these stages of the universe is the stage happened after very short time estimated to be a 10^{-6} of a second following the Big Bang. In this stage, the universe was so hot and was filled with QGP fluid. In the next stage of universe, the protons and neutrons were formed due to the hadronization process, and these formed nuclei that captured electrons to form hydrogen atoms. With the formation of the electrically neutral hydrogen, the cosmic microwave background radiation appeared, which we can measure today with our modern devices. After the formation of hydrogen, it began to cluster into stars, galaxies and quasars, clusters of galaxies and clusters of enormous galaxies.

1.1. Research Objective

The goal of the study in this thesis is to find analytic solutions for the Friedmann differential equation that describes the time evolution of the energy density in the beginning of the universe. In this study, we consider bag models as a framework to derive the equations of state of QGP, which are required for searching for solutions for the Friedmann differential equation. For some bag models, we will show that possible analytic solutions can exist. Based on these solutions, we will show the time evolution of the thermodynamic parameters in the early universe when it was field by a fluid of QGP. These thermodynamic parameters include, energy density, pressure density and entropy density.

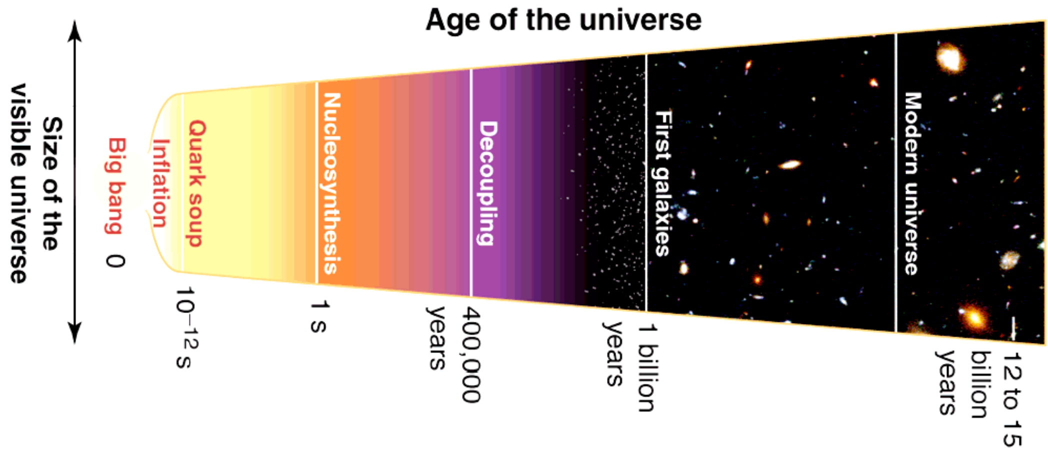

Figure 1.1. The Expansion of the universe(Perlmutter et al., 1998).

1.2. Thesis Organization

In chapter one we give a brief introduction related to quark gluon plasma.

In chapter two we briefly review the standard model, its ingredient relevant to the quark gluon plasma.

Chapter three: contains a introduction to the equations of state of the Quark Gluon Plasma in different models based on MIT bag model.

In chapter four we discuss cosmological model of the universe based on general relativity and Einstein field equations and mathematical concepts related to it.

Chapter five: contains the analytic solutions for the Fried-mann differential equations governing the variation of the energy density, the temperature and the pressure with time in some models and the related analysis regarding these thermodynamic parameters.

Finally in chapter six we give the conclusion.

2. The Standard Model

The Standard Model (SM) is a model that describes in a unified formalism both the weak and the electromagnetic forces. The particle content of the standard model is shown in Figure 2.1.

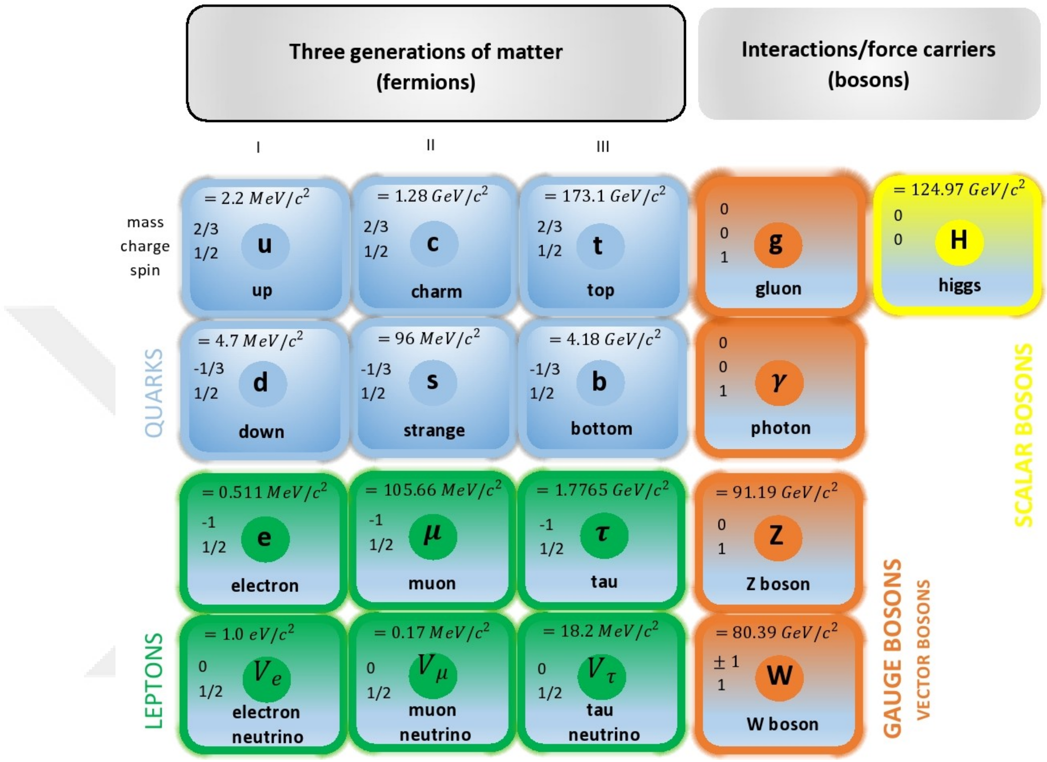

Figure 2.1. The Standard model (Rolnick and Huston, 1994).

The particles are characterized by their spin, their mass, and their quantum numbers (charges) that determine their interactions. The fermionic content of the SM (spin = 1/2 particles) is organized in three family with identical quantum numbers and different masses. Table 2.1 introduces a detail of the quarks and leptons.

Table 2.1. Quarks and Leptons

Quarks			Leptons		
Flavour	Mass (GeV/c^2)	Charge	Flavour	Mass (GeV/c^2)	Charge
u	0.002	2e/3	ν_e	$< 8 \times 10^{-9}$	0
d	0.004	-e/3	e	5.110×10^{-4}	-e
c	1.2	2e/3	ν_μ	$< 2.7 \times 10^{-4}$	0
s	0.120	-e/3	μ	0.1057	-e
t	173	2e/3	ν_τ	< 0.035	0
b	4.2	-e/3	τ	1.777	-e

Table 2.2. Quark quantum numbers

Quark quantum numbers	**up**	**down**	**charm**	**strange**	**top**	**bottom**
I_3−3-component of isospin	$1/2$	$-1/2$	0	0	0	0
S - strangeness	0	0	0	-1	0	0
C - charm	0	0	1	0	0	0
B−bottomness	0	0	0	0	0	-1
T−topness	0	0	0	0	1	0

2.1. Quarks

Quarks are charged particles. They were proposed to explain the structure of hadrons. Quarks are classified into six quarks (Collins, 1975); up (u), down (d), charm (c),strange (s), top (t) and bottom (b). Their masses ranges from few MeV to 173 GeV as shown in Table 2.1. All quarks have electric charges, the quarks u, c and t have charge -$\frac{2}{3}e$ and the quarks d, s and b have charge -$\frac{1}{3}e$. Each quark is said to represent a separate flavour (Markum et al., 1985). Particles made up of quarks are called Hadrons. Hadrons with spin integer are called mesons and those with spin half-integer are called baryons. The proton and the neutron are example of baryons. Each of them made up of three quark. $|p>=|uud>$, $|n>=|udd>$. Fig.2.2 provides the structure of proton.

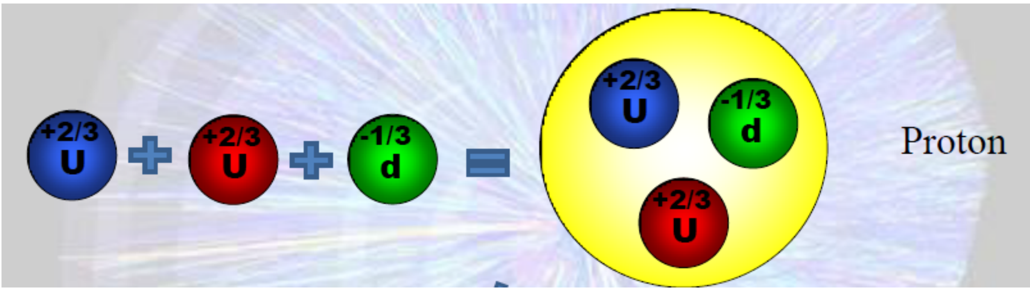

Figure 2.2. proton charge (Enstrom, 1998).

Generally, the quark family consists of six types, with different flavors, all of which carry a colored charge and an electric charge. Despite extensive searches in many experiments, no free quark was detected. It is therefore believed that quarks are confined within the hadrons, and that the potential of the strong force between quarks increases as the distance increases. It needs a high amount of energy to separate two quarks from each other. Thus, the hadrons are neutral-colored particles. Therefore, the quarks that make up the hadrons must contain a mixture of color that makes the particle "colorless." For example, three possible colour configurations to form colorless spin zero pion.

$$| \pi^+ >= \begin{cases} u_r \bar{d}_{\bar{r}} > \\ u_b \bar{d}_{\bar{b}} > \\ u_g \bar{d}_{\bar{g}} > \end{cases}$$

where $r, b.g$ refer to red, blue and green colors respectively.

2.2. Gluons

Gluons transmit the strong nuclear force between quarks or between themselves. Gluon is one of the gauge bosons such as photon, graviton and $W^{\mp}$, Z bosons (Dicus et al., 1987). The gluon is a spin-1 particle with no mass. The effective range of the force is of the order of 10^{-15} m (Lehnert, 2013; Gilani, 2004). Gluons carry both colour and anticolour charges. Below are possible colour combinations of colorless gluons

- Red anti-red $r\bar{r}$, red anti-blue $r\bar{b}$, red anti-green $r\bar{g}$
- Green anti-red $g\bar{r}$,green anti-green $g\bar{g}$,green anti-blue $g\bar{b}$
- Blue anti-red $b\bar{r}$,blue anti-green $b\bar{g}$,blue anti-blue $b\bar{b}$

Since the gluons themselves carry colors they can interact with each other (Pickering and Cushing, 1986). This is the main difference between strong interaction and electromagnetic interaction, gluons can be released and absorbed, created and annihilated, when a strong interaction is involved in a process. Since quarks make up hadrons such as the proton and the neutron, gluons mediates the force that keep the proton and the neutron together. The strong nuclear force that makes the nucleons in a nucleus bind together. One result of the features of the strong interaction is that when the quarks are near each other, the strength of the strong force decreases. This feature is the asymptotic freedom at small distances between quarks (Bethke, 2002).

The strong running coupling constant α_s gives a measure of the strength of the strong nuclear interaction. The interactions of quarks and gluons through the strong interaction are described by QCD theory (Shirkov and Solovtsov, 1997). QCD has complex field equations which make it difficult to perform accurate calculations. Instead of that, it may be necessary to use perturbative ways. Perturbative ways rely on expressing the required calculations as a power chain for the expansion parameter. This parameter must be selected in such a way that the higher order terms in the expansion are too small that they can be neglected (Jain and Munczek, 1991; Wang et al., 2002). Expansion parameter is given as

$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(33-2N_f\right)\ln\left(\frac{Q^2}{\Lambda^2}\right)} \tag{2.1}$$

Where, N_f denotes number of quark flavours, Λ is a scaling parameter while Q stands for the momentum transfer. From this, it can be easily seen that how α_s decreases when the momentum transfer is increased. This corresponds to a decrease in alpha when the distance from a quark decreases which, implies asymptotic freedom and color confinement.

2.3. Quantum Chromo-dynamics

QCD is a type of quantum field theory, it is called also quantum-mechanical gauge theory of the strong nuclear interaction. It was formulated in the 1960s and 1970s, in similarity with quantum electrodynamics (Pascual and Tarrach, 1984). QCD is a kind of quantum field theory based on a non-abelian gauge theory, with symmetry group SU(3) where the QCD analog of electric charge, in the familiar electromagnetic interaction, is a property called color. There are two strange properties in QCD which are

- Asymptotic freedom: if the energy of the reaction between quarks and gluons increases, a steady decrease in the strength of the interaction occurs (Bethke, 2007). This prediction of QCD was first discovered in the early 1970s by Hugh David Poltizer, Frank Wilkesk and David Gross, for which they received the Nobel Prize in Physics in 2004.
- Confinement, which says that the strong nuclear force between quarks increases as the quarks move away from each other. Because of this, one needs infinite energy to separate two quarks away from each other. As a consequence the quarks remain together in the form of hadrons, protons or neutrons, for example.

Although QCD has not been proven analytically, many scientists believe that this theory is correct, because it has been able to determine the cause of the constant failure to search for free quarks (Johnson, 1979).

The underlying principle of the Lagrangian of QCD is the requirement of invariance under the local rotation in the colors space. As it is known, the complex rotations of an arbitrary vector in three-dimensional space are described by unitary 3×3 matrices of unit determinant which formes symmetry group of the gauge transformation $SU(3)$. The generators can be taken to be eight linearly zero trace independent hermitian 3×3 matrices. The general Hermitian matrix λ_i, that can be parameterized with eight real numbers $a, b, ...h$ is:

$$\begin{pmatrix} a & c-id & e-if \\ c+id & b & g-ih \\ e+if & g+h & -a-b \end{pmatrix} \tag{2.2}$$

They correspond to the eight Gell-Mann matrices (Carozzi et al., 2000), and are symbolized by $\lambda_i, i = 1, 2, ...8$. These matrices achieve the commutative relations $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$ with f_{abc}, the $SU(3)_C$ structure constants, that are real and completely antisymmetric under interchange of any pair of indices. The derivation of the Lagrangian of QCD method is based on modifying the Lagrangian of the quantum electrodynamics (QED) to suit the strong interaction's gauge invariance (Müller, 1992). The Lagrangian of QED is given by

$$L_{QED} = i\bar{\psi}\gamma^{\mu}\left(\partial_{\mu} + ieA_{\mu}\right)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{2.3}$$

where A_{μ} is the field of the electromagnetic vector,$and\ F^{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is a tensor of the electromagnetic field force.

The color property implies that a quark's wave function has three color space components $\psi = (\psi_r, \psi_g, \psi_b)$. A transformation of a color gauge $\psi \rightarrow U\psi$ is defined by a unitary matrix U (3×3) with $detU = 1$, can indeed be given as an imaginary exponential of a hermitian matrix L. The linear combinations of all traceless Hermitian of the eight matrixes (Gell-Mann, 2010) are as described above

$$L = \frac{1}{2}\sum_{a=1}^{8}\theta_a\lambda_a \tag{2.4}$$

$$[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c \quad \text{and} \quad [\lambda_a, \lambda_b]_+ = \frac{4}{3}\delta_{ab} + 2d_{abc}\lambda_c \tag{2.5}$$

The eight generators in the Lie Group $SU(3)$ are called Gell-Mann matrixes, or $\frac{1}{2}\lambda_a$ to be more precise. The real parameter θ_a must differ in space to make the rotation U is dependent on space, $\theta_a = \theta_a(x)$. It is leading to

$$U(x) = \exp\left[\frac{1}{2}\sum_{a=1}^{8}\theta_a(x)\lambda_a\right] \tag{2.6}$$

Therefore, $\psi \to U(x)\psi$ and

$$\partial_\mu[U(x)\psi] = U\partial_\mu\psi + \left(\partial_\mu U\right)\psi = U\left[\partial_\mu\psi + U^*\left(\partial_\mu U\right)\psi\right] \tag{2.7}$$

Because $U\partial_\mu\psi$ is on the right side, a color potential needs to be added, similarity of the electromagnetic potential A_μ in QED. This potential can be denoted as $\widehat{A}_\mu$ and can be expressed as

$$\widehat{A}_\mu(x) = \frac{1}{2}\sum_{a=1}^{8}A_\mu^a(x)\lambda_a \tag{2.8}$$

The changes $\widehat{A}_\mu(x)$ in the color rotation are given by

$$\widehat{A}_\mu \to U^*\widehat{A}_\mu U - i\frac{1}{g}U^*\left(\partial_\mu U\right) \tag{2.9}$$

In the Dirac equation, the potential can be then chosen so that the total derivative is invariant under this gauge transformation. This can be done through replacing the derivative with the covariant derivative $D_u = \partial_\mu \to \left(\partial_\mu - ig\widehat{A}_\mu\right)$.

The next step consists of adding a kinetic term to Lagrangian, similar to the corresponding one in QED, $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. Here the field strength tensor $F_{\mu\nu}$ has to be improved in order to maintain the theory gauge invariant. Because there are no interactions between two photons, QED is an abelian gauge theory. However, QCD is a local theory of non-abelian $SU(3)$ where there are interactions between gluons, the generators are non-commutative. This leads to a change in the field strength tensor

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{abc}A_\mu^b A_\nu^c \tag{2.10}$$

And this description makes it invariant under transformation of the local color gage $F_{\mu\nu} \to U^*F_{\mu\nu}U$. Thus, the QCD Lagrangian can be expressed as

$$L_{QCD} = i\bar{\psi}\gamma^\mu\left(\partial_\mu - ig\widehat{A}_\mu\right)\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} \tag{2.11}$$

Finally, the general Hermitian matrix Gell-Mann matrixes λ_i introduced above, can be parameterized as

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.12}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.13}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.14}$$

As mentioned before, they satisfy the commutative relations $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$. The $SU(3)_C$ structure constant f_{abc} has real values and it is antisymmetric under interchange of any pair of indices. The non-vanishing values of f_{abc} are

$$f_{123} = 1,\ f_{458} = f_{678} = \sqrt{3}/2,\ f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2} \tag{2.15}$$

3. EQUATION OF STATE OF QUARK GLUON PLASMA

In general, equations of state are thermodynamic equations that relate mathematically two or more state functions describing a characteristic of the matter. These equations, can describe characteristics of solids, fluids and mixtures of fluids. For example, the state functions can be the pressure, temperature, volume or internal energy (Borsányi et al., 2014). One of the earliest state equations is that of the ideal gas, that is essentially accurate for low-pressure and moderate temperatures of gases. Yet at higher pressures and lower temperatures this equation is increasingly unreliable, and fails to expect condensation from a gas to liquid. Hence a number of much more reliable state equations for gases and liquids were already developed. Currently, there is no single equation of state which provides optimality the properties of all materials within all conditions. Besides predicting the behavior of gases and liquids, there are also equation of state for predicting solid volumes, including the transformation of solids from one crystalline state to another. There are equations of state, which model the interior of stars. A similar term is the ideal liquid-state equation used in cosmology. The laws that expressed the equations of state include

- Boyle's Law in 1662, Boyle observed that the volume of gas is inversely proportional to pressure. This can be described in mathematical form as $PV = constant$
- Charles's law in 1787, the French physicist Jacques Charles observed that over the same 80 kelvin interval oxygen, nitrogen, hydrogen, carbon dioxide and air were expanding to the same degree. Joseph Louis Gay-Lussac later published results of similar experiments in 1802 which suggested a linear relationship between volume and temperature $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
- Dalton's law of partial pressure in 1801, the pressure P of a gas mixture is equal to the total of the pressure of all the component gases. This can be expressed mathematically for n species as $P_{total} = P_1 + P_2 + P_3 + ... + P_n$.
- The ideal gas law in 1834, Emile Clapeyron put Boyle's law together with Charles' law in the ideal gas law that formulated as $PV = R(T + 273.15)$ where P,T and V are pressure, temperature and volume, respectively, R is gas constant.

Our knowledge of the equation of state of quark gluon plasma has been constantly increased because of the experimental results from heavy ion collision, modern astrophysical measurements and also because of the progress in lattice quantum chromo-dynamics calculations (a quantum field theory in which the strong interaction is described in terms of interaction between quarks mediated by gluon exchange, both quarks and gluons). New results on the equations of state of quark gluon plasma encourage researchers to investigate the early universe when it was in quark gluon plasma state.

3.1. Fermi-Dirac Distribution

Fermions follow the Paulis exclusion principle. Thus no more than one particle can occupy the same energy state. It means that the number of particles n_i is not allowed to exceed the number of states g_i in the ith group. The group of g_i states can be split into two individual subgroups. The first subgroup is n_i of the states that contain one particle. The other subgroup then becomes $g_i - n_i$ and must not contain particles. As a consequence, the number of the different ways that the states can be split can is given as

$$\frac{g_i!}{n_i!\,(g_i - n_i)!} \tag{3.1}$$

which represents the contribution to the number of microstates obtained from the ith group. Hence, one can find the total number of microstates which corresponds to an allowed distribution,n_i, as

$$t_{FD}\,(n_i) = \prod_i \frac{g_i}{n_i!\,(g_i - n_i)!} \tag{3.2}$$

It should be noted that, the macrostate conditions imply that

$$\sum_i n_i = N \tag{3.3}$$

and

$$\sum_i n_i \varepsilon_i = U \tag{3.4}$$

where U is the energy. Using the Stirlings approximation to simplify $\ln t_{DF}$, one finds that

$$\ln t_{FD} = \sum_i \{g_i \ln g_i - n_i \ln n_i - (g_i - n_i) \ln (g_i - n_i)\} \tag{3.5}$$

In order to maximize $\ln t_{DF}$, one can use the Lagrange method. Thus, for the maximum condition

$$d(\ln t) + \alpha d(N) + \beta d(U) = 0 \tag{3.6}$$

where α and β are a multiplier for the number condition and for the energy condition respectively. The parameter α can be determined from the number of particles N. It can be regarded as a potential for particle number and can be given as $\alpha = \frac{\mu}{k_b T}$ where μ is the chemical potential and T is the temperature. On the other hand, the parameter β can be determined by ensuring that the distribution describes an assembly with the correct energy U. It can be thought as potential for energy and is given by $\beta = -\frac{1}{k_b T}$. Upon substitution of $\ln t = \ln t_{FD}$ and the expressions of N and U given in Eq.3.3 and Eq.3.4 and performing differentiation, we get the following equation

$$\sum_i \{\ln [(g_i - n_i)/n_i] + \alpha + \beta \varepsilon_i\} dn_i = 0 \tag{3.7}$$

The above equation indicates that for each i we have

$$\ln [(g_i - n_i)/n_i] + \alpha + \beta \varepsilon_i = 0 \tag{3.8}$$

the preceding equation can be rewritten as

$$n_i = g_i / [\exp(-\alpha - \beta \varepsilon_i) + 1] \tag{3.9}$$

This can be expressed as a distribution function

$$f_{FD}(\varepsilon) = n_i / g_i = 1/[\exp(-\alpha - \beta \varepsilon) + 1] \tag{3.10}$$

which represent the Fermi Dirac distribution function.

3.2. Bose Einstein Distribution

Regarding bosons, any number of them is allowed to occupy any one particle state. Thus, in the ith group we have g_i states occupied by n_i identical particles without any restrictions on the occupation number. A common microstate can be expressed in terms of $g_i - 1$ lines and n_i crosses. The lines stands for the divisions between the states, g_i. On the other hand, the crosses denote an occupying particle. Then, the number of ways in which $(n_i + g_i - 1)$ symbols can be arranged into n_i cross and $(g_i - 1)$ lines is given by

$$\frac{(g_i + n_i - 1)!}{n_i!\,(g_i - 1)!} \tag{3.11}$$

Since we deal with large numbers of g_i states, one can drop the term -1 in both the numerator and denominator and thus, the number of microscopic states for a given allowed distribution of bosons is approximated as

$$t_{BE}\,(n_i) = \prod_i \frac{(g_i + n_i)!}{n_i!g_i!} \tag{3.12}$$

Proceeding as in the case of fermion above, we get

$$\ln t_{BE} = \sum_i \{(g_i + n_i)\ln(g_i + n_i) - n_i \ln n_i - g_i \ln g_i\} \tag{3.13}$$

and hence, after using the Lagrange method to maximize $\ln t_{BE}$ and after differentiation we obtain

$$\sum_i \{\ln[(g_i + n_i)/n_i] + \alpha + \beta\varepsilon_i\}\, dn_i = 0 \tag{3.14}$$

which indicates that

$$\ln[(g_i + n_i)/n_i] + \alpha + \beta\varepsilon_i = 0 \tag{3.15}$$

The above equation can be rewritten as

$$n_i = g_i / [\exp(-\alpha - \beta\varepsilon_i) - 1] \tag{3.16}$$

and finally

$$f_{BE}(\varepsilon) = n_i/g_i = 1/[\exp(-\alpha - \beta\varepsilon) - 1] \tag{3.17}$$

The previous equation represents the Bose-Einstein distribution function.

3.3. MIT Bag Model

The MIT Bag Model is the most widely used model that express the equations of state of the QGP in the framework of bag models. It is simple model that was developed in 1974 at the Massachusetts Institute of Technology in Cambridge and it attracted the attention of researchers in the field of hadron physics (Sladek et al., 1974). In the bag models, quarks are allowed to move freely inside a bag that is subjected to a fixed external pressure to equal the internal pressure and thus keeps the bag stable (Johnson et al., 1975).

The MIT bag model rely on several assumptions that can be summarized in the following. Firstly, quarks can not leave the bag. Secondly, outside the bag there is no quarks can be found. Thirdly, quarks masses are neglected. The internal pressure acting on the surface of the bag is balanced by the effect of an external pressure denoted by $\mathcal{B}$. It should be noted that the internal pressure is due to the kinetic energy of the quarks inside the bag while the external pressure is exerted by the vacuum on the surface of the bag. As a consequence of the balance of these pressures, the energy momentum conservation at the bag surface is guaranteed and the confinement results from this pressure balance.

Below, we derive the equations of state of the QGP in the MIT bag model based on the assumptions discussed above for QGP containing only up and down quarks with neglected interactions inside the plasma (Garcia et al., 2010). The degrees of freedom of the gluons and quarks can be denoted as N_g ,N_q respectively. They can be calculated as follows:

$$N_g = 2 \times 8 = 16 \tag{3.18}$$

where 2 accounts for the number of polarizations of each gluon and 8 to account for its allowed colors.

$$N_q = 2 \times 3 \times 2 = 12 \tag{3.19}$$

where 2 accounts for the number of quarks that we have and 3 to account for allowed colors for each quark and finally 2 for quark spin. We proceed now to calculate the energy densities of the quarks and anti-quarks in the quark-gluon plasma. there will be little more quarks than anti-quarks generally in the QGP created from normal atomic nuclei. The energy densities for a quark and an anti-quark can be calculated in terms of the temperature via

$$\begin{aligned} E_q &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{[e^{(k-\frac{1}{3}\mu)/T}+1]} \\ E_{\bar{q}} &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{[e^{(k+\frac{1}{3}\mu)/T}+1]} \end{aligned} \tag{3.20}$$

where the chemical potential μ accounts for the amount of energy needed to add other quark to the plasma at zero temperature. Evaluating the integrations,one gets

$$E_q + E_{\bar{q}} = \frac{7\pi^2}{120}T^4 + \frac{\mu^2}{36}T^2 + \frac{\mu^4}{648\pi^2} \tag{3.21}$$

Turning now to the gluons, the energy density is given by

$$E_g = \int \frac{d^3k}{(2\pi)^3} \frac{k}{(e^{k/T} - 1)} = \frac{\pi^2 T^4}{30} \tag{3.22}$$

Denoting the total energy density of the QGP by ε and the total pressure of the QGP as p, on finds that

$$\begin{aligned} \varepsilon &= \mathcal{B} + \frac{E_k}{V} \\ p &= -\mathcal{B} + \frac{1}{3}\frac{E_k}{V} \end{aligned} \tag{3.23}$$

In the above equation E_k is the total internal energy that originate from the kinetic energies of both quarks and gluons inside the bag. Hence, one finds that

$$E_k = \big(16E_g + 12(Eq + E_{\bar{q}})\big)V \tag{3.24}$$

Consequently, we find that

$$\begin{aligned} \varepsilon &= \mathcal{B} + 16E_g + 12(Eq + E_{\bar{q}}) \\ p &= -\mathcal{B} + \frac{1}{3}\bigg(16E_g + 12(Eq + E_{\bar{q}})\bigg). \end{aligned} \tag{3.25}$$

For the case of a baryon-number symmetric quark-gluon plasma the chemical potential vanishes and hence we finally obtain

$$\begin{aligned} \varepsilon &= \frac{37\pi^2}{30}T^4 + \mathcal{B} \\ p &= \frac{37\pi^2}{90}T^4 - \mathcal{B}. \end{aligned} \tag{3.26}$$

One can estimate the order of the transition temperature from QGP phase to hadron phase. This temperature is denoted by T_c and is referred as critical temperature. The hadronic phase is composed of massless charged and neutral pions that has equations of state given as

$$\varepsilon = 3N_f \frac{\pi^2}{90} T^4 \tag{3.27}$$

$$p = N_f \frac{\pi^2}{90} T^4 \tag{3.28}$$

where as before the energy density is denoted by ε and the pressure is p and $N_f = 3$ is the internal degrees of freedom of the pions. From equating the equations of pressure in the two thermodynamics systems, the Hadron phase and the quark gluon plasma phase we can write

$$3\frac{\pi^2}{90} T_c^4 = 16\frac{\pi^2}{90} T_c^4 + 24\frac{7\pi^2}{720} T_c^4 - B \tag{3.29}$$

which leads to

$$T_c = (\frac{45B}{17\pi^2})^{\frac{1}{4}} \tag{3.30}$$

Finally, the entropy density S can be found using the following relation

$$S = \frac{\varepsilon + p}{T} \tag{3.31}$$

3.3.1. Energy density and pressure of QGP case made of only quarks

Based on quantum mechanics, the average number of particles per cell, in case of fermions such as quarks, can be expressed as

$$n_{av} = \frac{g}{e^{(E-\mu)/kT} + 1} \tag{3.32}$$

here E is the particle energy, k is the usual Boltzmann constant, T is the temperature, μ is the chemical potential. In the above equation, g denotes the number of different quantum states that the particle may have within the cell. For particle with momenta lie between p and $p + dp$ in one cubic centimeter, the number density can be expressed as

$$n(p)dp = \frac{g}{e^{(E-\mu)/kT} + 1} \frac{4\pi p^2}{h^3} dp \tag{3.33}$$

where h is the Planck constant and the volume of elementary cell in a phase space is given by $h^3 = \Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z$. The particle energy E is related to its momentum p through the relation

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \tag{3.34}$$

Where m is the mass of the particle. In the limit of massless fermions, $E = pc$ and hence Eq.3.33 turns to be

$$n(p)dp = \frac{g}{e^{(pc-\mu)/kT}+1}\frac{4\pi p^2}{h^3}dp \tag{3.35}$$

In natural units we have $\hbar = c = k = 1$, and thus Eq.3.35 simplifies to

$$n(p)dp = \frac{g}{e^{(p-\mu)/T}+1}\frac{4\pi p^2}{(2\pi)^3}dp \tag{3.36}$$

we can calculate the energy density of all particles in the unit volume as follows

$$\varepsilon = \int_0^\infty E(p)n(p)dp \tag{3.37}$$

since $E = p$,the above equation becomes

$$\varepsilon = \int_0^\infty pn(p)dp \tag{3.38}$$

from Eq.3.36 and by substituting in the above equation we get

$$\varepsilon = \frac{g_q}{2\pi^2}\int_0^\infty \frac{p^3 dp}{e^{(p-\mu)/T}+1} \tag{3.39}$$

where we set $g = g_q$. In the fermion case we take $\mu = 0$ and by substituting in the above equation we get

$$\begin{gathered}
\varepsilon_q = \tfrac{g_q}{2\pi^2}\int_0^\infty \tfrac{p^3 dp}{e^{p/T}+1} \\
\Longrightarrow \varepsilon_q = \tfrac{g_q T^4}{2\pi^2}\int_0^\infty \tfrac{z^3 dz}{e^z+1} \quad [p/T = z] \\
\Longrightarrow \varepsilon_q = \tfrac{g_q T^4}{2\pi^2}\int_0^\infty \tfrac{z^3 dz}{e^z(1+e^{-z})} \\
\Longrightarrow \varepsilon_q = \tfrac{g_q T^4}{2\pi^2}\int_0^\infty e^{-z}z^3\left(1+e^{-z}\right)^{-1}dz
\end{gathered} \tag{3.40}$$

From Refs.(Csernai, 1994; Boggs, 2019), the following equations were obtained

$$\varepsilon_q = \frac{7}{8}\frac{\pi^2}{30}g_q T^4 \tag{3.41}$$

$$p_q = \frac{7}{8}\frac{\pi^2}{90}g_q T^4 \tag{3.42}$$

Thus, the entropy density S_q in this case become

$$S_q = \frac{\varepsilon_q + p_q}{T} \tag{3.43}$$

which finally is given by

$$S_q = \frac{7g_q}{180}\pi^2 T^3 \tag{3.44}$$

3.3.2. Energy density and pressure of QGP case made of only Gluons

In the boson case we take $\mu = 0$ and make the replacements $g_q \leftrightarrow g_g$ and $+1 \leftrightarrow -1$ in in Eq.3.39 to get

$$\begin{aligned} \varepsilon_g &= \tfrac{g_g}{2\pi^2}\int_0^\infty \tfrac{p^3 dp}{e^{p/T}-1} \\ \Longrightarrow \varepsilon_g &= \tfrac{g_g T^4}{2\pi^2}\int_0^\infty \tfrac{z^3 dz}{e^z-1} \quad [p/T = z] \\ \Longrightarrow \varepsilon_g &= \tfrac{g_g T^4}{2\pi^2}\int_0^\infty \tfrac{z^3 dz}{e^z(1-e^{-z})} \\ \Longrightarrow \varepsilon_g &= \tfrac{g_g T^4}{2\pi^2}\int_0^\infty e^{-z}z^3\left(1-e^{-z}\right)^{-1} dz \end{aligned} \tag{3.45}$$

From the (Csernai, 1994; Boggs, 2019)the following equations were obtained

$$\varepsilon_g = \frac{\pi^2}{30} g_g T^4 \tag{3.46}$$

$$P_g = \frac{\pi^2}{90} g_g T^4 \tag{3.47}$$

Thus, the entropy density in this case, S_g, can be written as

$$S_g = \frac{\varepsilon_g + p_g}{T} \tag{3.48}$$

which finally is given by

$$S_g = \frac{2g_g}{45}\pi^2 T^3 \tag{3.49}$$

3.4. Model 1 and model 2

In model 1 and model 2, changes in the equation of state of QGP of the MIT bag model has been introduced to describe the lattice QCD data. These modifications include a reduction in the Stephan-Boltzmann constant and an introduction of another temperature dependent term (linear or quadratic) in the pressure and also in the energy density and finally a bag constant term with negative sign. The equations of states are given as (Begun et al., 2012)

$$p_1 = \frac{\delta_1}{3}T^4 - AT - B_1, and \quad \varepsilon_1 = \delta_1 T^4 + B_1 \tag{3.50}$$

from which we get the following relation:

$$p_1[\varepsilon_1(t)] = \frac{1}{3}[\varepsilon_1(t) - 4B_1] - A[\frac{\varepsilon_1(t) - B_1}{\delta_1}]^{\frac{1}{4}} \tag{3.51}$$

where $\delta_1 = 4.73$, $A = 3.94T_c^3$ and $B_1 = -2.37T_c^4$ (Begun et al., 2012). For model 2 we have (Pisarski, 2006)

$$p_2 = \frac{\delta_2}{3}T^4 - CT^2 - B_2, and \quad \varepsilon_2 = \delta_2 T^4 - CT^2 + B_2 \tag{3.52}$$

that gives the following relation

$$p_2[\varepsilon_2(t)] = \frac{1}{3\delta_2}\left[\delta_2\left(\varepsilon_2(t) - 4B_2\right) - C[C + \sqrt{C^2 + 4\delta_2(\varepsilon_2(t) - B_2)}\right] \tag{3.53}$$

where $\delta_2 = 4.73$, $A = 3.94T_c^3$ and $B_2 = -2.37T_c^4$ (Begun et al., 2012).

4. COSMOLOGICAL MODEL OF UNIVERSE

Einstein's general theory of relativity widely improved our ideas and thoughts about the space and time concepts (Alpher and Herman, 1948). Einstein field equations relate the distribution of matter and energy within Universe to the geometry of the spacetime. Cosmological models attempt to explain the reasons of the presence of the Universe and to predict its evolution during time. Einstein introduced the equations that describe a universe filled with matter based on his theory of general relativity. Einstein assumed that the universe had to be static and introduced the cosmological constant, into the field equations to fulfill this assumption. However, Willem de Sitter, Friedmann and Lemaitre shortly after that obtained cosmological solutions to the field equations for the case of non-static universes.

4.1. General Relativity And Gravitation

Albert Einstein worked on his theory of gravitation between 1907 and 1915. This famous theory is known as General relativity and implies that the observed gravitational effect between masses results from their warping of space time. Due to the remark that the Einstein field equations are nonlinear, solutions for these equations are hard to be obtained. To overcome this difficulty, Einstein used approximation methods to get predictions of the General relativity. However, in mid 1916, Karl Schwarzschild successfully obtained the first non-trivial exact solution to the Einstein field equations (Batista et al., 2012; Ashtekar et al., 2015; Liddle and Lyth, 2000).
Space-time in the light of general relativity is not flat Minkowski space of special relativity but rather is a curved space-time and can be modelled using a mathematical structure named as the apseudo-Riemannian manifold. Clearly, the general relativity is an extension of the special relativity to include curved space-time. Mass and momentum flux bend space-time in a way portrayed by the tensor field conditions of Einstein (Pavšič, 2006; Foster and Nightingale, 2010). The Newtonian picture considers gravity as one of the main forces in the universe that helps to accelerate particles in Euclidean space while time is absolute. On the other hand and in the perspective of the general relativity, there is no gravitational force.

4.2. The Tensor

Tensors are the natural and logical generalization of vectors, as vectors are important and necessary for a large number of mathematical studies of physical phenomena. There are many applications of tensors in different branches of mathematics, for instances in mechanics, and fluids (Heinbockel, 2001; Carroll, 1997; Sergeev, 1984). Let a space V_N in N of the dimensions, take a set of axes in this space as follows
$X^1, X^2, X^3, ...X^N$, shortly $X^i, 1 \leq i \leq N$. Take another set of axes in the same space V_N, let it be $\bar{X}^\alpha, 1 \leq \alpha \leq N$. It is clear that

$$X^i \equiv X^i(\bar{X}^1, \bar{X}^2, \bar{X}^3, ..., \bar{X}^N) \quad 1 \leq i \leq N \tag{4.1}$$

$$\bar{X}^\alpha \equiv \bar{X}^\alpha(X^1, X^2, X^3, ...X^N) \quad 1 \leq \alpha \leq N \tag{4.2}$$

We can now express the cartesian axes in terms of spherical axes and vice versa in a three dimensional space. Let (X, Y, Z) be the cartesian coordinates and (r, θ, ϕ) are the spherical coordinates. Then, we find that

$$\begin{aligned} X^1 &= X = r \sin\theta \cos\phi \\ X^2 &= Y = r \sin\theta \sin\phi \\ X^3 &= Z = r \cos\theta \end{aligned} \tag{4.3}$$

And

$$\begin{aligned} r &= \sqrt{X^2 + Y^2 + Z^2} \\ \theta &= \tan^{-1} \frac{\sqrt{X^2 + Y^2}}{Z} \\ \phi &= \tan^{-1} \frac{Y}{X} \end{aligned} \tag{4.4}$$

When we differentiate the equations, we get

$$dX^i = \frac{\partial X^i}{\partial \bar{X}^1} \bar{X}^1 + \frac{\partial X^i}{\partial \bar{X}^2} \bar{X}^2 + ... + \frac{\partial X^i}{\partial \bar{X}^N} \bar{X}^N \equiv \sum_{\alpha=1}^{N} \frac{\partial X^i}{\partial \bar{X}^\alpha} \bar{X}^\alpha \tag{4.5}$$

and

$$d\bar{X}^{\alpha} = \sum_{i=1}^{N} \frac{\partial \bar{X}^{\alpha}}{\partial X^{i}} dX^{i} \tag{4.6}$$

Density, pressure, temperature are scalar quantities whose specification in any coordinate system requires only knowing their magnitudes. On the other hand, vectors have directions and hence to specify them we need to describe their magnitudes and their directions. Both scalers and vectors are particular cases of a more general object called a tensor. The tensor B^{ij} is a second rank tensor. It transforms as

$$\bar{B}^{ij} = \frac{\partial \bar{x}^{j}}{\partial x^{k}} \frac{\partial \bar{x}^{i}}{\partial x^{l}} B^{kl} \tag{4.7}$$

Where, B^{ij} are called components of covariant tensor. According to the rule of transformation, covariant tensor

$$\bar{B}_{ij} = \frac{\partial x^{k}}{\partial \bar{x}^{i}} \frac{\partial x^{l}}{\partial \bar{x}^{j}} B_{kl} \tag{4.8}$$

The components of mixed tensor B^{i}_{j}, according to the rule of transformation are represented as follows

$$B^{i}_{j} = \frac{\partial \bar{x}^{i}}{\partial x^{k}} \frac{\partial x^{l}}{\partial \bar{x}^{j}} B^{k}_{l} \tag{4.9}$$

4.3. The Einstein Field Equations

Einstein field equations are considered as the basic field equations in general relativity and are given as (Einstein and Davis, 2013).

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} \tag{4.10}$$

The left side of the equation, $G_{\mu\nu}$, represent space-time geometry, $T_{\mu\nu}$ express the matter and energy distribution and k: is the Einstein constant. Since, $\mu, \nu = 0, 1, 2, 3$: so the above equation represent a set of 16 equations. In the above equation we have

R: curvature scalar

$R_{\mu\nu}$: curvature tensor.

$g_{\mu\nu}$: metric tensor that we use it to build the space time fabric and can be consider the gravitational potential in the space time fabric.

$T_{\mu\nu}$: energy momentum tensor, it expresses the energy matter at each of the points of space time fabric that cause the curvature in this fabric.

4.4. The Energy Momentum Tensor

The energy momentum tensor is a second rank tensor that characterizes the flow and distribution of momentum and energy in an area of space time. It is specified by sixteen components. Only 10 of them are independents because the energy momentum tensor is symmetric. As mentioned earlier, it is usually symbolized as $T_{\mu\nu}$. The components of the energy momentum are measured in units of energy density (Jm^{-3}) in the International System of Units while, in natural units they are measured in GeV^4 units. In matrix form we have

$$T_{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{01} & T^{11} & T^{12} & T^{13} \\ T^{02} & T^{12} & T^{22} & T^{23} \\ T^{03} & T^{13} & T^{23} & T^{33} \end{pmatrix} \tag{4.11}$$

Below we describe each component of the energy momentum tensor (Diaz, 2010)

- T^{00} is the local energy density.
- $T^{i0} = T^{0i}$ is the average of stream of the energy per unit area divided by the speed of light, C, at right angles to the i-direction where $[i = 1,2,3]$.
- $T^{ij} = T^{ji}$is the average of stream of the i-component of momentum per unit area at right angles to the j-direction where $[i\&j = 1,2,3]$.

In our case study, we consider Quark Gluon plasma as an ideal fluid and therefore the treatment of its energy momentum tensor is the same as that of an ideal fluid. The QGP is characterized by mass density ρ, a four-velocity u_μ and pressure p that works equally in all directions at a point. In an event with the metric tensor $g_{\mu\nu}$, the components of the energy momentum tensor are given by

$$T_{\mu\nu} = (\rho + \frac{p}{c^2})\, u_\mu u_\nu - p\, g_{\mu\nu} \tag{4.12}$$

Or we can write it as

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - p\, g_{\mu\nu} \tag{4.13}$$

where the speed light $C = 1$ in natural unit and ε is the energy density. At any space time event, and if we are limited to using only locally inertial frames with cartesian coordinates, the metric tensor can be taken as the Minkowski metric (Sundermeyer, 1982; Lambourne, 2010). Thus, the components of the energy momentum tensor will be given as

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - p\,\eta_{\mu\nu} \tag{4.14}$$

in a frame where $u_\mu = \big(1,0,0,0\big)$ we finally get

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{4.15}$$

4.5. Ricci Tensor

The Riemann curvature tensor denoted as $R^{\gamma}_{\alpha\beta\gamma}$ can determine whether the space is flat or curved, If Riemann tensor vanishes at all points in a space then the space is flat. The Ricci tensor $R_{\alpha\beta}$ can be obtained from contracting the first and last indices in Riemann tensor that it is to say

$$R_{\alpha\beta} \equiv \sum_{\gamma} R^{\gamma}_{\alpha\beta\gamma} \tag{4.16}$$

The Ricci scalar R, that appears in the Einstein field equation, is obtained through further contracting of the indices on the Ricci tensor

$$R \equiv \sum_{\alpha,\beta} g^{\alpha\beta} R_{\alpha\beta} \tag{4.17}$$

Ricci tensor describes the curvature of the space, it is symmetric i.e. $R_{\alpha\beta} = R_{\beta\alpha}$. It has four indices, each of them takes four values. So it has $4^4 = 256$ components. However, there are only 20 independent components in four-dimensional space-time due to the symmetry. In general, in an n-dimensional Riemannian space, the Riemann tensor $R^{\ell}ijk$ is defined as

$$R^{\ell}ijk = \frac{\partial \Gamma^{\ell}ik}{\partial x^{j}} - \frac{\Gamma^{\ell}ij}{\partial x^{k}} + \sum_{m} \Gamma^{m}ik\Gamma^{\ell}mj - \sum_{m} \Gamma^{m}ij\Gamma^{\ell}mk \tag{4.18}$$

Where the quantities $\Gamma^{i}jk$, are connection coefficients. These connection coefficients are important to help to perform the differentiation in curved space. They can be defined from the differentiation of the basis vectors $\hat{e}$ of the space (Alsing et al., 2011) as

$$\frac{\partial \hat{e}_j}{\partial x^k} = \sum_i \Gamma^i jk \hat{e}_i \tag{4.19}$$

The above equation shows that $\Gamma^i jk$ is merely indicating the component of the rate of change of the basis vector $\hat{e}_j$ regarding changes in the coordinate x^k in the direction of basis vector $\hat{e}_i$. The connection coefficient has a direct relation to the metric tensor $g_{\mu\nu}$ through the relation below

$$\Gamma^i jk = \frac{1}{2} \sum_\ell g^{i\ell} \left(\frac{\partial g_{\ell k}}{\partial x^j} + \frac{\partial g_{j\ell}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^\ell} \right) \tag{4.20}$$

where the dual metric tensor g^{ij} is the matrix inverse of g_{ij} satisfying

$$\sum_k g^{ik} g_{kj} = \delta^i_j \tag{4.21}$$

with $\delta^i_j = 1$ for $i = j$ and $\delta^i_j = 0$ for $i \neq j$.
The metric tensor components ($g_{\mu\nu}$) are pure numbers quantities, showing that the connection coefficients $\Gamma^i jk$ can be expressed in units of m^{-1}, while the Ricci tensor ($R_{\mu\nu}$) and the curvature scalar R can both be expressed in units of m^{-2}. Upon recalling the appropriate units of $T_{\mu\nu}$, it is easy to show that $\frac{8\pi G}{C^4}$ has the right units to be the Einstein constant k and the components of the energy momentum tensor can be expressed in units of Jm^{-3}.

4.6. The Space-Time Of General Relativity

In Newtonian mechanics and in special relativity, the forces result in an accelerated motion. Newtonian gravity is an example of such force but, general relativity varies greatly in dealing with gravity. In this theory, gravity is identified with curves of space-time itself. The accelerated motion under the action of gravity is replaced by free motion in warped space-time. The world-lines of free particles are geodesics of this space-time and for a small spot of space-time, its geometry is still Minkowskian. The line element ds^2, the infinitesimal generalization of the space-time separation of two points, in Minkowski space-time can be expressed as

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{4.22}$$

When referring to this points (t,x,y,z) in Minkowski space-time, we can write it in the form $x^\mu = (x^0,x^1,x^2,x^3)$. The former equations can be written in terms of the Minkowski metric tensor with signature $(-,+,+,+)$

$$\eta_{\mu\nu} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.23}$$

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \tag{4.24}$$

and in Minkowski metric with signature $(+,-,-,-)$, we find that

$$\eta_{\mu\nu} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{4.25}$$

$\eta_{\mu\nu} = diag.(1,-1,-1,-1)$ and so we get

$$ds^2 = \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu}\, dx^\mu\, dx^\nu \tag{4.26}$$

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2d\tau^2 \tag{4.27}$$

where

$$d\tau^2 = \frac{ds^2}{c^2} \quad \Rightarrow \quad d\tau^2 = dt^2\left(1 - \frac{dx^2+dy^2+dz^2}{c^2dt^2}\right) \tag{4.28}$$

$$v^2 = \frac{dx^2+dy^2+dz^2}{dt^2} \tag{4.29}$$

$$d\tau = dt\sqrt{1-\beta^2} \tag{4.30}$$

In the spherical coordinate system we have

$$ds^2 = dt^2 - dr^2 - r^2\, d\theta^2 - r^2\, sin^2\theta\, d\phi^2 \tag{4.31}$$

In Minkowski space-time the point

$$x^\mu = (x^0,x^1,x^2,x^3) = (t,r,\theta,\phi) \tag{4.32}$$

The Minkowski Space-time metric tensor's coefficients $\eta_{\mu\nu}$ are constant, which shows that the Minkowski space-time of the special relativity is flat.
The coefficient of the metric tensor are functions of the coordinates in curved time of space in Riemannian space. Therefore in curved space, in order to compute a line element ds^2, the metric tensor $\eta_{\mu\nu}$ must be replaced by the general metric tensor $g_{\mu\nu}$ which has coefficients as coordinate function. Thus we have

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^{\mu}\, dx^{\nu} \tag{4.33}$$

4.7. Robertson-Walker geometry of space.

In spite of there are lots of various structures in the universe, all astronomical observations tell us that, the Universe is homogeneous and hence appears isotropic in all locations. This is frequently referred as the cosmological principle.
In 1935, Arthur Walker of the University of Liverpool and Howard Robertson of the California Institute of Technology showed independently that there is a one space time metric describing all relativistic models that are homogeneous and isotropic which is named as Robertson Walker metric. The cosmic space-time element is given as (Cai and Kim, 2005; Frieman et al., 1990)

$$ds^2 = c^2(dt)^2 - \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{4.34}$$

where t symbolize cosmic time, $x^1, x^2 and\ x^3$ are position coordinates, and the metric coefficients $g_{\mu\nu}$ are functions of $t, x^1, x^2\ and\ x^3$. We can combine this requirement into the metric by pressing that the cosmic time enters the metric coefficients $g_{\mu\nu}$ just through a scaling function $a^2(t)$. We can write

$$ds^2 = c^2(dt)^2 - a^2(t) \sum_{\mu,\nu=0}^{3} h_{\mu\nu} dx^{\mu} dx^{\nu};\ h_{\mu\nu} = g_{\mu\nu} \tag{4.35}$$

In terms of co-moving coordinates $r, \theta\ and\ \phi$ we find that

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2) \right] \tag{4.36}$$

where K is the cosmology constant that describes the geometry of the spatial part of space time. According to its values, we have closed universe if k= 1, flat universe if k= 0 and open universes if K = -1. It should be noted that a(t) is scale factor that gives the information about distance ratios at different times. When a(t) increases with time, the fundamental observers become more widely separated with time. As a consequence, the galaxies containing those fundamental observers get further apart indicating that the universe is expanding. The Robertson-Walker metric tensor $g_{\mu\nu}$ is given as

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-Kr^2} & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix} \tag{4.37}$$

4.8. The Hubble Law And The Expansion Of The Universe

In 1922, astronomer Edwin Hubble analyzed the light of galaxies using the observations recorded in the massive Mount Wilson Observatory. The observatory was the largest optical astronomical one at that time, By analyzing the spectrum of the observed galaxies, he concluded that their spectrum deviates towards red. It means that these galaxies move away from each other. This is because, if these galaxies are approaching to us or to each other, their spectrum will deviate towards the blue or violet. Hubble deduced that the speed of the galaxy relative to its distance from us in the universe is 55 km / s per million parsec. Parsec is equal to 3.26 light years, or about 17 km/s per million light years. There is another unit, Mega Parsec, which is equal about 3 million light years. This relationship is called the Hubble constant. This constant is one of the most important numbers in cosmology because it is necessary to estimate the size and age of the universe. This long awaited confirmation indicates the rate at which the universe is expanding from the primitive big bang, which is quite opposite to Einstein's thought of a stable universe. The rate of the change of the proper distance d_p, distance between two fundamental observers or their galaxies, with respect to cosmic time define the so-called the proper radial velocity v_p. In terms of v_p and d_p the Hubble parameter $H(t)$ is given as

$$H(t) = \frac{v_p}{d_p} \tag{4.38}$$

$H(t)$ can be defined from the scale factor $a(t)$ as follows

$$H \equiv H(t) = \frac{1}{a(t)}\frac{da(t)}{dt} \equiv \frac{\dot{a}}{a} \tag{4.39}$$

Positive values of v_p indicate that fundamental observers or their galaxies are moving away from each others while negative values of v_p indicating that they are coming toward each others.

4.9. The Energy Density Of Universe

Universe consists of vacuum, matter and photons. Consequently, the total energy density of the universe is the sum of energy densities of these three ingredients. It can be expressed as

$$\varepsilon(t) = \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{rad} \tag{4.40}$$

The physicist Alexander Friedman in 1922 was able to prove that the universe is in state of expansion through his equations which are named Friedman's equations which later was supported by the observations of Hubble in 1928. These equations are a set of equations in cosmology governing the expansion of the universe within the framework of general theory of relativity. Friedman's famous equations are as follows

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_R) - \frac{K}{a^2} + \Lambda, \tag{4.41}$$

When the universe expands the density of mater ρ_m and density of radiation ρ_R decreases $\rho_m \propto \frac{1}{a^3}$ $and \rho_R \propto \frac{1}{a^4}$. De-sitter solved the Friedman's equations with only dark energy which showed that the universe exponentially accelerated

$$\left(\frac{\dot{a}}{a}\right)^2 = \Lambda \tag{4.42}$$

Thus the solution was as follows

$$a = ce^{\sqrt{\Lambda}t} \tag{4.43}$$

It is called also De-sitter space, in the language of general relativity which is the maximally symmetric vacuum solution of Einstein's field equations with a positive cosmological constant (corresponding to a positive vacuum energy density and negative pressure). From Eq.4.41 we can find the following equation

$$\dot{H} = -4\pi G(p + \varepsilon) + \frac{K}{a^2} \tag{4.44}$$

The energy momentum tensor is conserved and thus (Kirkby, 1971)

$$\dot{\varepsilon} + 3H(\varepsilon + p) = 0 \tag{4.45}$$

The modern measurements indicated that the universe is flat and consequently K = 0. So, from Eq.4.44 and Eq.4.45 we get

$$-\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + p)} = \sqrt{\frac{8\pi G}{3}} dt \tag{4.46}$$

With which we can find the temporal evolution of the energy density ε.

5. RESULTS AND DISCUSSION

This chapter summarizes our results for the variation of the energy density (ε), the temperature (T), the entropy density and the pressure density in the era of an early universe filed with QGP fluid. The results here are for several cases. A case where QGP is made only of quarks, a case where QGP is made only of gluons and a third case where QGP is made of both quarks and gluons together. In all these cases we consider the MIT bag model as our framework for the equations of states of the QGP. We start first by solving the corresponding differential equation to obtain analytic solutions. We then graphically represent the analytic solutions showing the time variations of the energy density (ε), the temperature (T), the entropy density and the pressure density. In our analysis, we use the following initial conditions (Kalaydzhyan and Shuryak, 2015; Sanches Jr et al., 2015):

$$\varepsilon_i(t_i) = 10^7\,\mathrm{MeV/fm}^3 \qquad \text{at} \qquad t_i = 10^{-9}\ s \tag{5.1}$$

The time started at the electro-weak phase transition, $t_i = 10^{-9}\ s$, and ended at $t_f = 10^{-4}\ s$ at which QCD phase transition was occurred.

5.1. Natural Units

In our analysis we use natural units. This unit system is preferable for easy calculations in which the speed of sound in vacuum and the reduced planck constants are taken to be unity.

The standard unit of length in particle physics is the femtometre, where $1fm = 10^{-15}\ m$. For instance, the radius of the proton is $\sim 1.0\ fm$. Usually the cross sections in "barns are determined. Barn in Nuclear Physics (barn, symbol b) is a very small unit area that matches the nuclear level of an atom. It is used in nuclear physics and particle physics to define the nuclear section of a nucleus and its probability of entering into a reaction with a primary particle such as a proton, neutron, gamma ray, or others. Barn is also used to define a primary particle collision segment together in dispersion experiments (Antchev et al., 2013). $1b = 10^{-}28$. Energy is measured in GeV, where 1 $GeV = 1.6 \times 10^{-10} J$. We were used to the use of units measuring times in seconds and distances in metres. The speed of light in such units takes the value close to $3 \times 10^8\ ms^{-1}$.

Instead, we could have chosen to use time unit in seconds and distance in light seconds. In such units, light speed takes the value c=1 light-second per second so that c can be left out. Natural units adopting in particles, nuclear physics and astrophysics, where $\hbar = 1, c = 1$ and the unit of energy is the GeV. Thus, the length, area, time, rate, momentum and mass of all fundamental quantities can be expressed in terms of GeV power (McWeeny, 1973; Bartlett, 1974). For any system the total energy ε, momentum p and mass m are related as follows by the relativistic formula

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \quad (5.2)$$

Thus,$\varepsilon, P \ andm$ have the same units in GeV. Length, time in GeV^{-1} and area in GeV^{-2}

Table 5.1 illustrates how different conventional units are related in the unit system of nature

Table 5.1. Basic physical quantities in both SI and Natural Units systems

Quantity	Dimensions		Conversions
	SI Units	Natural Units	
mass	kg	E	$1\ GeV = 1.8 \times 10^{-27}\ kg$
length	m	1/E	$1\ GeV^{-1} = 0.197 \times 10^{-15}\ m$
time	s	1/E	$1\ GeV^{-1} = 6.58 \times 10^{-25}\ s$
energy	$kg.m^2/s^2$	E	$1\ GeV = 1.6 \times 10^{-10}\ Joules$
momentum	$kg.m/s$	E	$1\ GeV = 5.39 \times 10^{-19}\ kg.m/s$
velocity	m/s	-	$1 = 2.998 \times 10^{8}\ m/s\ (= c)$
angular momentum	$kg.m^2/s$	-	$1 = 1.06 \times 10^{-34}\ J.s\ (= h)$
cross-section	m^2	1/E^{2}	$1\ GeV^{-2} = 0.389\ mb = 0.389 \times 10^{-31}\ m^2$
force	$kg.m/s^2$	E^{2}	$1\ GeV^2 = 8.19 \times 10^{5}\ Newton$
charge	$C = A.s$	-	$1 = 5.28 \times 10^{-19}\ Coulomb;\ e = 0.303\ or\ = 1.6 \times 10^{-19}\ C$

5.2. Analytical solutions for the differential equations of the thermodynamic parameters of QGP in MIT Bag Model

In this subsection we present our results for obtaining analytical solutions for the differential equations of the thermodynamic parameters of QGP in some versions of the MIT Bag Model.

5.2.1. QGP case made of only Gluons

We start with the equations of the state of the energy density and pressure of a QGP formed only of gluons which are given as

$$\begin{aligned} \varepsilon_g &= \frac{8}{15}\pi^2 T^4 + B, \\ p_g &= \frac{8}{45}\pi^2 T^4 - B, \end{aligned} \tag{5.3}$$

By differentiating both sides of the energy density equation we obtain

$$d\varepsilon_g = \frac{4 \times 8\pi^2 T^3}{15} dT \tag{5.4}$$

Adding the two equations in Eq.5.3, we get

$$\varepsilon_g + p_g = \frac{32}{45}\pi^2 T^4 \tag{5.5}$$

By substituting the above equation in the differential equation

$$\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + P)} = -\sqrt{\frac{8\pi G}{3}} dt \tag{5.6}$$

and performing integration we get

$$\int_{T_0}^{T} \frac{1}{T\sqrt{\frac{8\pi^2}{15}T^4 + B}} dT = -\int_{t_0}^{t} \sqrt{\frac{8\pi G}{3}} dt \tag{5.7}$$

Let $x = T^2 \Rightarrow dx = 2T dT$. By substituting in the above equation, we get

$$\frac{1}{2}\int_{\sqrt{x_0}}^{\sqrt{x}} \frac{1}{x\sqrt{\frac{8\pi^2}{15}x^2 + B}} dx = -\int_{t_0}^{t} \sqrt{\frac{8\pi G}{3}} dt \tag{5.8}$$

$$\frac{\sqrt{15}}{4\sqrt{2}\pi}\int_{\sqrt{x_0}}^{\sqrt{x}}\frac{1}{x\sqrt{x^2+\frac{15B}{8\pi^2}}}dx=-\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.9}$$

$$\frac{\sqrt{15}}{4\sqrt{2}\pi}\int_{\sqrt{x_0}}^{\sqrt{x}}\frac{1}{x\sqrt{x^2+\alpha^2}}dx=-\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \;\; where \;\; \alpha^2=\frac{15B}{8\pi^2} \tag{5.10}$$

$$-\frac{\sqrt{15}}{4\alpha\sqrt{2}\pi}\left[\ln|\frac{\alpha+\sqrt{x^2+\alpha^2}}{x}|\right]_{\sqrt{x_0}}^{\sqrt{x}}=-\left[\sqrt{\frac{8\pi G}{3}}t\right]_{t_0}^{t} \tag{5.11}$$

$$\ln\frac{T_0^2[\alpha+\sqrt{T^4+\alpha^2}]}{T^2[\alpha+\sqrt{T_0^4+\alpha^2}]}=\frac{K}{a}(t-t_0) \;\; ;a=\frac{\sqrt{15}}{4\alpha\sqrt{2}\pi},k=\sqrt{\frac{8\pi G}{3}} \tag{5.12}$$

$$\frac{T_0^2[\alpha+\sqrt{T^4+\alpha^2}]}{T^2[\alpha+\sqrt{T_0^4+\alpha^2}]}=e^{\frac{K}{a}(t-t_0)} \tag{5.13}$$

$$\frac{T^2[\alpha+\sqrt{T_0^4+\alpha^2}]}{T_0^2[\alpha+\sqrt{T^4+\alpha^2}]}=e^{-\frac{K}{a}(t-t_0)} \tag{5.14}$$

$$\frac{T^2}{[\alpha+\sqrt{T^4+\alpha^2}]}=C \;\; ;C=\frac{T_0^2}{[\alpha+\sqrt{T_0^4+\alpha^2}]}e^{-\frac{K}{a}(t-t_0)} \tag{5.15}$$

$$T^2=\frac{2\alpha C}{1-C^2} \tag{5.16}$$

$$T=\sqrt{\frac{2\alpha C}{1-C^2}} \tag{5.17}$$

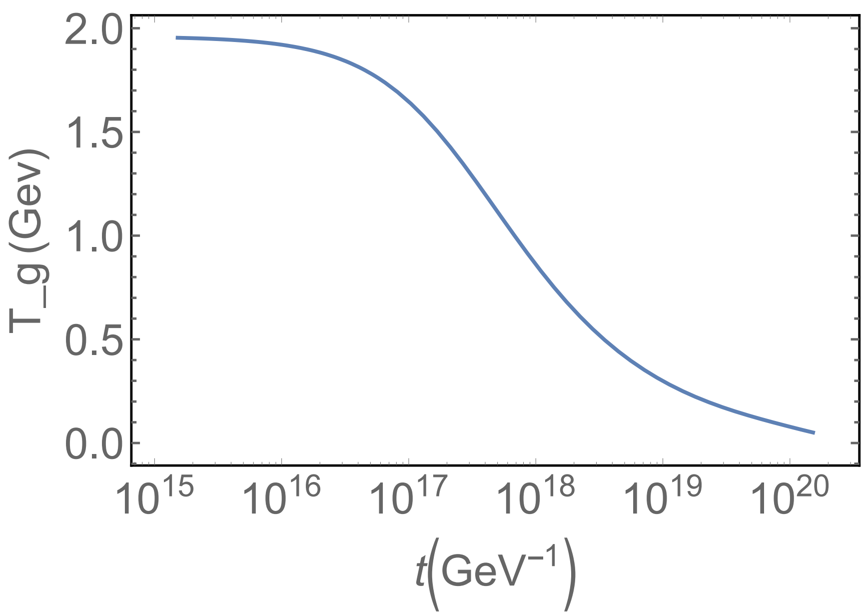

Figure 5.1. Temperature versus time of gluon in MIT bag model for QGP case made of only Gluons.

This solution gives the evolution of the temperature in the time interval mentioned above. Working in natural units, we find that

$$\varepsilon_i(t_i) = 10^7\,\mathrm{MeV/fm^3} = 7.674\times 10 (GeV)^4 \; at \; t_i = 10^{-9}\; s = 1.52\times 10^{15}(GeV)^{-1} \tag{5.18}$$

In Fig. 5.1 we show the plot of the evolution of the temperature in the time interval where universe was filled with QGP. From the figure, one can notice that, the temperature was very high at the beginning of the time interval and then began to drop dramatically during this period.

Using the expression of the temperature given in Eq.5.17, we can find the energy density, pressure and entropy, respectively, as

$$\varepsilon_g = \frac{8\pi^2}{15}\left(\frac{2\alpha C}{1-C^2}\right)^2 + B \tag{5.19}$$

$$p_g = \frac{8\pi^2}{45}\left(\frac{2\alpha C}{1-C^2}\right)^2 - B \tag{5.20}$$

$$S_g = \frac{37}{45}\pi^2\left[\sqrt{\frac{2\alpha C}{1-C^2}}\right]^3 \tag{5.21}$$

In Fig. 5.2 we show the graphical representation of the variation of energy density with time for the same case. By looking at the figure, one can notice that the energy density was very high and then began to drop dramatically during this period

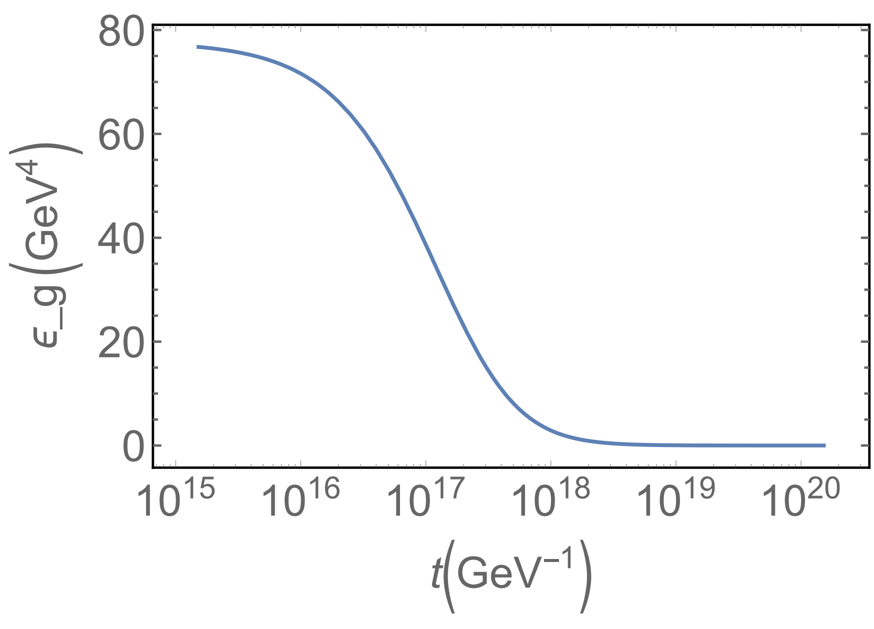

Figure 5.2. Energy density versus time of gluon in MIT bag model for QGP case made of only Gluons.

Fig. 5.3 presents our result for the graphical the time variation of the pressure. We also note that the pressure was very high and then began to decrease during this period

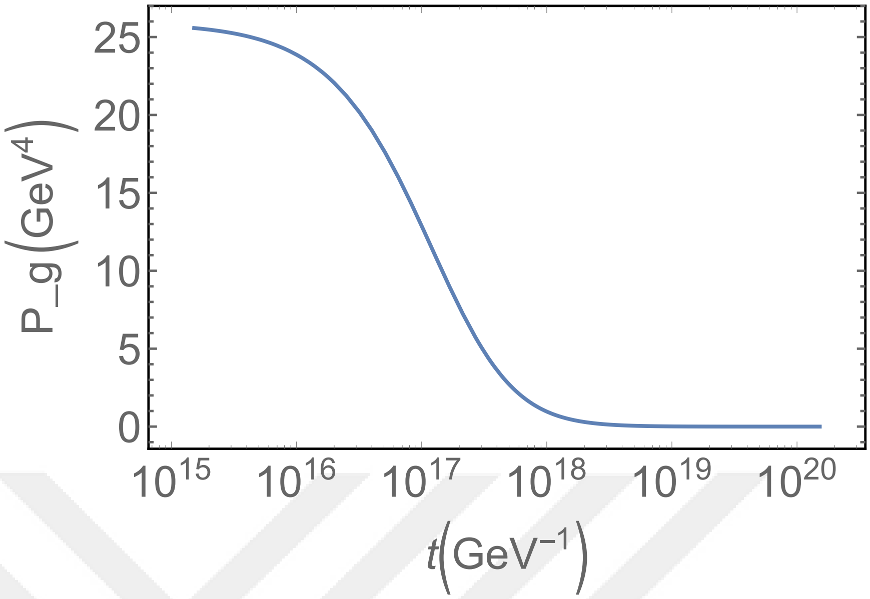

Figure 5.3. Pressure versus time of gluon in MIT bag model for QGP case made of only Gluons.

Before we comment on the entropy density graph, let us give an idea of entropy. Entropy is an important and essential term for understanding many of the phenomena around us in the universe. Entropy is viewed through several sciences. Through statistical mechanics, it is a measure of the amount of energy within any physical system in which this energy cannot be used to produce work, which is the second law of the thermodynamics. As for entropy from the perspective of physics, it is the automatic change that occurs in the system, and it is known that the automatic changes tend to generally be directed towards achieving a balance in the differences inherent within any system, whether this system is physical or chemical, so that the differences are reduced to the least possible amount; to bring the system to a state of equilibrium.

In Fig. 5.4 we show the graphical representation of the time evolution of entropy density, We note that the entropy density was very high and then began to decrease during this period of QGP.

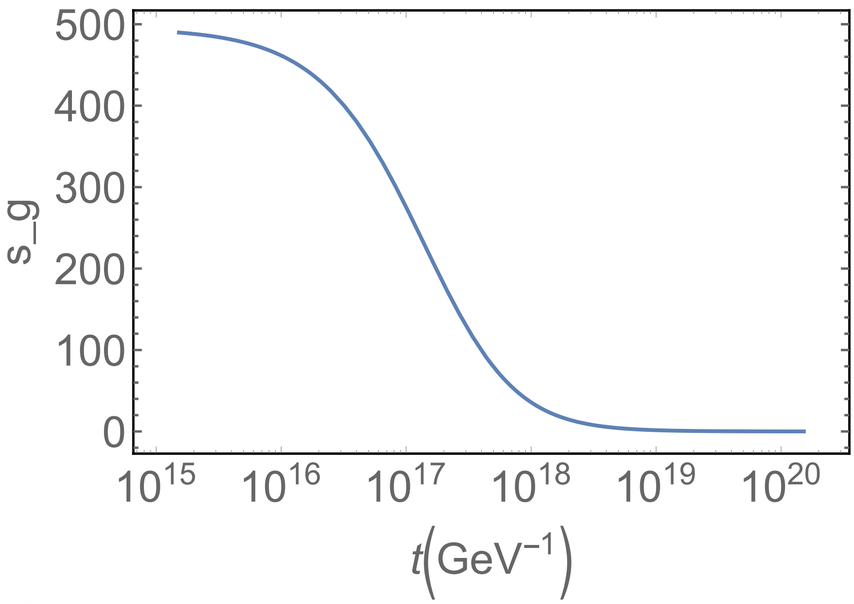

Figure 5.4. Entropy density versus time in MIT bag model for QGP case made of only Gluons.

5.2.2. QGP made of only quarks case

The equations of the state of QGP made of only quarks are given as

$$\begin{aligned} \varepsilon_q &= \frac{7}{10}\pi^2 T^4 + B, \\ p_q &= \frac{7}{30}\pi^2 T^4 - B, \end{aligned} \tag{5.22}$$

By differentiating both sides of the energy density equation we obtain

$$d\varepsilon_q = \frac{14}{5}\pi^2 T^3 dT \tag{5.23}$$

From Eq.5.22, by adding the two equations, we get

$$\varepsilon_q + p_q = \frac{28}{30}\pi^2 T^4 \tag{5.24}$$

By substituting in the differential equation

$$\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + P)} = -\sqrt{\frac{8\pi G}{3}}dt \tag{5.25}$$

and performing integration, we get

$$\int_{T_0}^{T} \frac{1}{T\sqrt{\frac{7\pi^2}{10}T^4+B}}dT = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.26}$$

Let $x = T^2 \Rightarrow dx = 2TdT$. By substituting in Eq.5.26, we get

$$\frac{1}{2}\int_{\sqrt{x_0}}^{\sqrt{x}} \frac{1}{x\sqrt{\frac{7\pi^2}{10}x^2+B}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.27}$$

Thus we get

$$\frac{1}{2}\int_{\sqrt{x_0}}^{\sqrt{x}} \frac{1}{x\sqrt{\frac{7}{10}\pi^2(x^2+\frac{10B}{7\pi^2})}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.28}$$

$$\frac{\sqrt{10}}{2\sqrt{7}}\int_{\sqrt{x_0}}^{\sqrt{x}} \frac{1}{x\sqrt{(x^2+\beta^2)}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \ \ where \ \ \beta^2 = \frac{10B}{7\pi^2} \tag{5.29}$$

$$-\frac{\sqrt{10}}{2\beta\sqrt{7}\pi}\left[\ln|\frac{\beta+\sqrt{x^2+\beta^2}}{x}|\right]_{\sqrt{x_0}}^{\sqrt{x}} = -\left[\sqrt{\frac{8\pi G}{3}}t\right]_{t_0}^{t} \tag{5.30}$$

$$\frac{1}{2\sqrt{B}}\ln\frac{T_0^2[\beta+\sqrt{T^4+\beta^2}]}{T^2[\beta+\sqrt{T_0^4+\beta^2}]} = K(t-t_0) \ \ ;k = \sqrt{\frac{8\pi G}{3}} \tag{5.31}$$

$$\frac{T_0^2[\beta+\sqrt{T^4+\beta^2}]}{T^2[\beta+\sqrt{T_0^4+\beta^2}]} = e^{2K\sqrt{B}(t-t_0)} \tag{5.32}$$

$$\frac{T^2[\beta+\sqrt{T_0^4+\beta^2}]}{T_0^2[\beta+\sqrt{T^4+\beta^2}]} = e^{-2K\sqrt{B}(t-t_0)} \tag{5.33}$$

$$\frac{T^2}{[\beta+\sqrt{T^4+\beta^2}]} = C \ \ ;C = \frac{T_0^2}{[\beta+\sqrt{T_0^4+\beta^2}]}e^{-2K\sqrt{B}(t-t_0)} \tag{5.34}$$

$$T^2 = C\beta + C\sqrt{T^4+\beta^2} \tag{5.35}$$

$$T^2 = \frac{2\beta C}{1-C^2} \tag{5.36}$$

$$T = \sqrt{\frac{2\beta C}{1-C^2}} \tag{5.37}$$

Below in Fig.(5.5) we show the plot of the time variation of temperature in the case of QGP made of only quarks

From Eq.5.37, we can find the expressions of the energy, pressure and entropy densities for the same state of QGP, respectively

$$\varepsilon_q = \frac{7\pi^2}{10}(\frac{2\beta C}{1-C^2})^2 + B \tag{5.38}$$

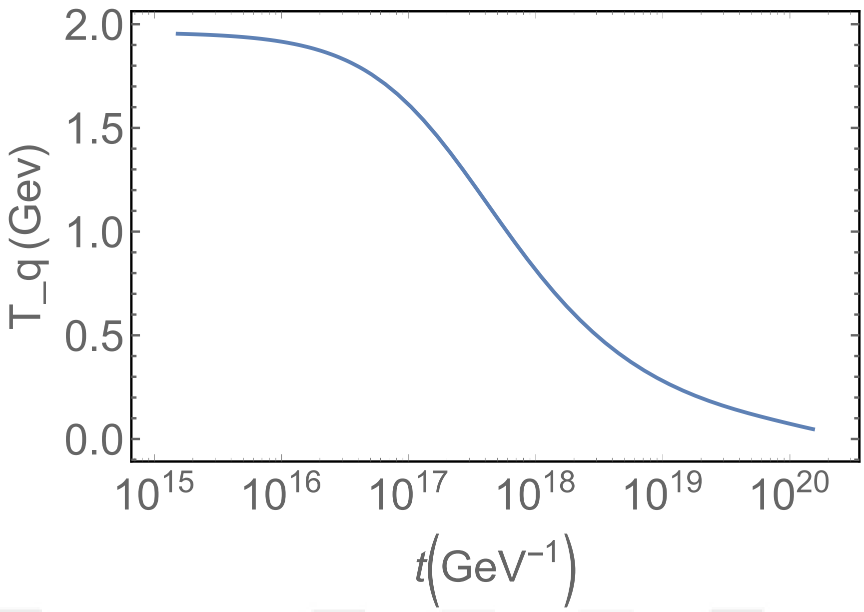

Figure 5.5. Temperature variation with time in case of MIT bag model where QGP made of only quarks.

$$p_q = \frac{7\pi^2}{30}(\frac{2\beta C}{1-C^2})^2 - B \tag{5.39}$$

$$S_q = \frac{37}{45}\pi^2[\sqrt{\frac{2\beta C}{1-C^2}}]^3 \tag{5.40}$$

In Fig. 5.6 we show the graphical representation of the time variation of the energy density. By looking at the figure, one can notice that the energy density was very high and then began to drop dramatically during this period

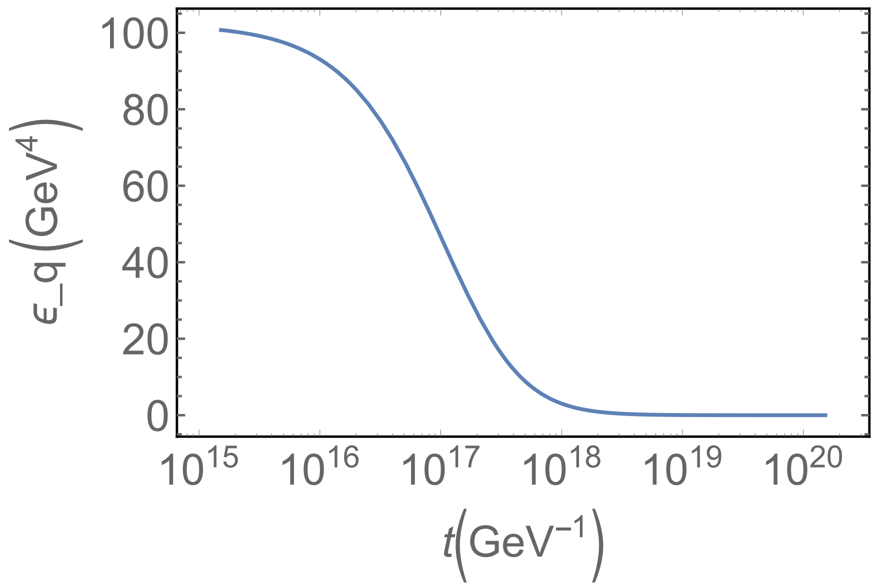

Figure 5.6. Energy density versus time in case of MIT bag model where QGP made of only quarks.

In Fig. 5.7 we show the graphical representation of the variation of the pressure with time. We also note that the pressure was very high and then began to decrease during this period

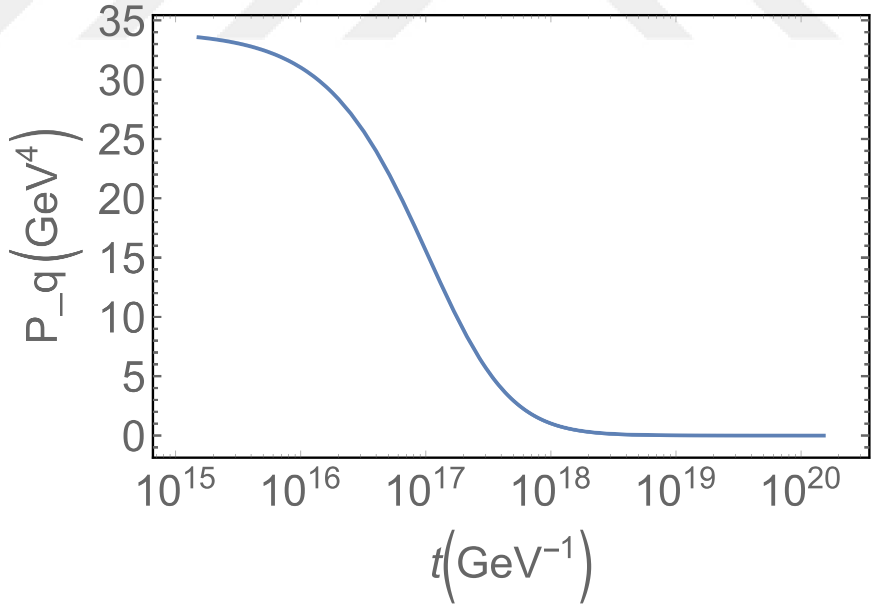

Figure 5.7. Pressure versus time in MIT bag model for QGP case made of only quarks.

In Fig. 5.8 we show the graphical representation of the of the entropy density versus time. We note that the entropy was high and then began to decrease during this period

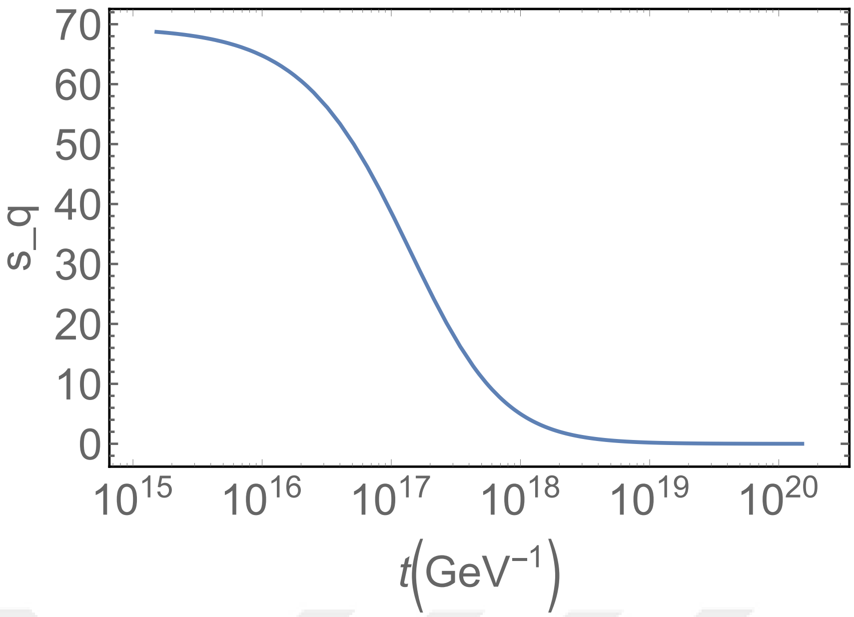

Figure 5.8. Entropy versus time in MIT bag model for QGP case made of only quarks.

5.2.3. A case of QGP made of both quarks and gluons

The energy and pressure densities of QGP state in this case are given by (Schäfer et al., 1984; Fogaça et al., 2010):

$$\begin{aligned} \varepsilon &= B + \frac{37\pi^2}{30} T^4, \\ p &= -B + \frac{37\pi^2}{90} T^4 \end{aligned} \tag{5.41}$$

By differentiating both sides of the energy density equation we obtain

$$d\varepsilon = \frac{4 \times 37\pi^2 T^3}{30} dT \tag{5.42}$$

By adding the equations of the energy and pressure densities we get

$$\varepsilon + p = \frac{74}{45} \pi^2 T^4 \tag{5.43}$$

Upon substituting in the differential equation

$$\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + P)} = -\sqrt{\frac{8\pi G}{3}} dt \tag{5.44}$$

and performing the integration we get

$$\int_{T_0}^{T} \frac{1}{T\sqrt{\frac{37\pi^2}{30}T^4+B}}dT = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.45}$$

$$\int_{T_0}^{T} \frac{1}{T\sqrt{\frac{37\pi^2}{30}T^4+B}}dT = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.46}$$

Let $x = T^2 \Rightarrow dx = 2TdT$. By substitute in above Eq. we get

$$\frac{1}{2}\int_{x_0}^{x} \frac{1}{x\sqrt{\frac{37\pi^2}{30}x^2+B}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.47}$$

$$\frac{\sqrt{30}}{2\sqrt{37\pi^2}}\int_{x_0}^{x} \frac{1}{x\sqrt{x^2+\frac{30B}{37\pi^2}}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \tag{5.48}$$

$$\frac{\sqrt{30}}{2\sqrt{37\pi^2}}\int_{x_0}^{x} \frac{1}{x\sqrt{x^2+\gamma^2}}dx = -\int_{t_0}^{t}\sqrt{\frac{8\pi G}{3}}dt \;\; where \;\; \gamma^2 = \frac{30B}{37\pi^2} \tag{5.49}$$

$$-\frac{\sqrt{30}}{2\gamma\sqrt{37\pi^2}}\left[\ln|\frac{\gamma+\sqrt{x^2+\gamma^2}}{x}|\right]_{x_0}^{x} = -\left[\sqrt{\frac{8\pi G}{3}}t\right]_{t_0}^{t} \tag{5.50}$$

$$\ln\frac{T_0^2[\gamma+\sqrt{T^4+\gamma^2}]}{T^2[\gamma+\sqrt{T_0^4+\gamma^2}]} = \frac{K}{a}(t-t_0) \;\; ;a = \frac{\sqrt{30}}{2\gamma\sqrt{37\pi^2}}, k = \sqrt{\frac{8\pi G}{3}} \tag{5.51}$$

$$\frac{T_0^2[\gamma+\sqrt{T^4+\gamma^2}]}{T^2[\gamma+\sqrt{T_0^4+\gamma^2}]} = e^{\frac{K}{a}(t-t_0)} \tag{5.52}$$

$$\frac{T^2[\gamma+\sqrt{T_0^4+\gamma^2}]}{T_0^2[\gamma+\sqrt{T^4+\gamma^2}]} = e^{-\frac{K}{a}(t-t_0)} \tag{5.53}$$

$$\frac{T^2}{[\gamma+\sqrt{T^4+\gamma^2}]} = C \;\; ;C = \frac{T_0^2}{[\gamma+\sqrt{T_0^4+\gamma^2}]}e^{-\frac{K}{a}(t-t_0)} \tag{5.54}$$

$$T^2 = \frac{2\gamma C}{1-C^2} \tag{5.55}$$

$$T = \sqrt{\frac{2\gamma C}{1-C^2}} \tag{5.56}$$

Below we show the plot of the temperature versus time. We note that the temperature was high and began to decrease in the period under study.

From the Eq.5.56, we can find the energy, pressure and entropy densities respectively as

$$\varepsilon = \frac{37\pi^2}{30}(\frac{2\gamma C}{1-C^2})^2 + B \tag{5.57}$$

$$p = \frac{37\pi^2}{90}(\frac{2\gamma C}{1-C^2})^2 - B \tag{5.58}$$

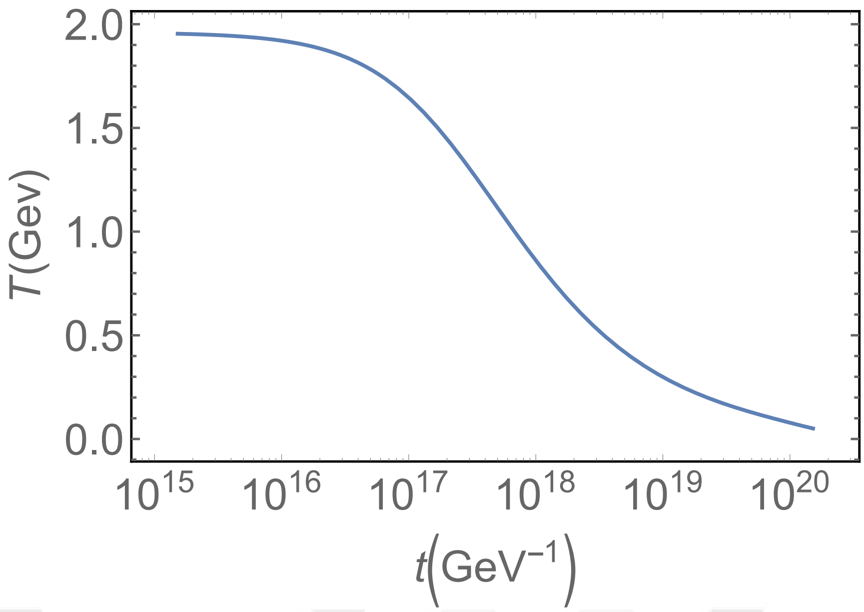

Figure 5.9. Temperature as a function of time in MIT bag model for QGP case made of both quarks and gluons.

$$S = \frac{74}{45}\pi^2[\sqrt{\frac{2\gamma C}{1-C^2}}]^3 \tag{5.59}$$

In Fig. 5.10 we show the graphical representation of the variation of the energy density with time. By looking at the figure, one can notice that the energy density was very high and then began to drop dramatically during this period

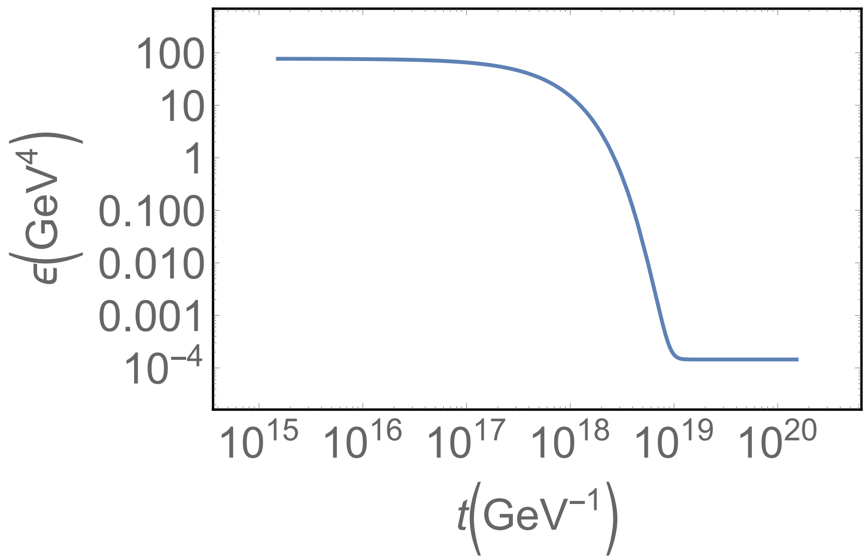

Figure 5.10. Energy density as a function of time for QGP case made of both quarks and gluons.

In Fig. 5.11 we show the graphical representation of the time evolution of the pressure density in this case of QGP. By looking at the figure, one can notice that the pressure density was very high and then began to drop dramatically during this period of time.

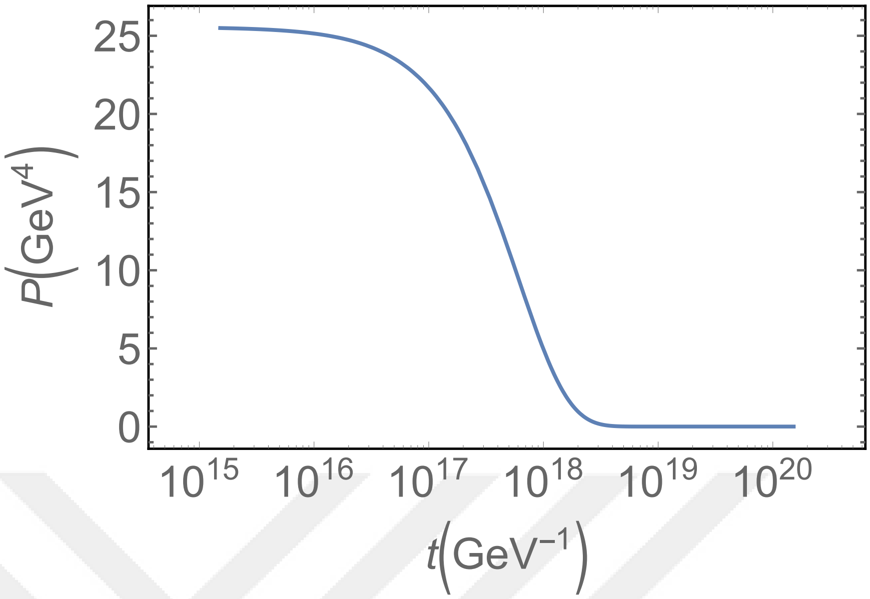

Figure 5.11. Pressure versus time in MIT bag model for QGP case made of both quarks and gluons.

In Fig. 5.12 we show that the variation of the entropy density with time.

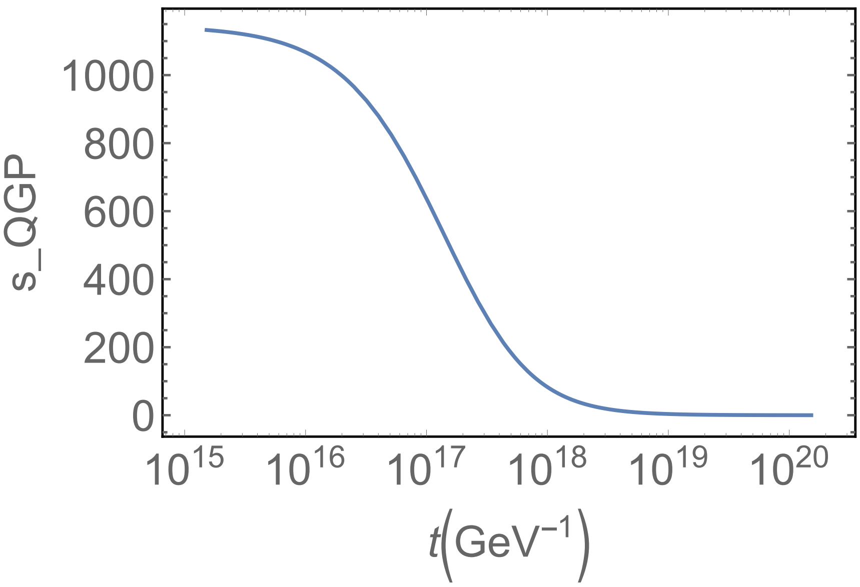

Figure 5.12. Entropy density versus time in MIT bag model for QGP case made of both quarks and gluons.

5.3. Numerical solutions for the differential equations of the thermodynamic parameters of QGP in Model1 and Model2

In this subsection we present our results for obtaining numerical solutions for the differential equations of the thermodynamic parameters of QGP in some modified versions of the MIT Bag Model, namely Model1 and Model2. These models have complicated forms of the differential equations and thus analytic solutions are difficult to be obtained. Rather, numerical solutions can be easily obtained and the results will be shown graphically in the following.

5.3.1. Model1

In this model, the energy density ε_1 and the pressure density P_1 as functions of temperature T are given by

$$\varepsilon_1(T) = \sigma T^4 + B, \quad p_1(T) = \frac{\sigma}{3} T^4 - AT^2 - B \tag{5.60}$$

Eliminating the temperature, we obtain

$$p_1[\varepsilon_1(t)] = \frac{1}{3}[\varepsilon_1(t) - 4B_1] - A[\frac{\varepsilon_1(t) - B_1}{\delta_1}]^{\frac{1}{4}} \tag{5.61}$$

where the factors by (Begun et al., 2012) $\delta_1 = 4.73$, $A = 3.94T_c^3$ and $B_1 = -2.37T_c^4$. Upon substituting the expression of $p1$ in the preceding equation in the differential equation

$$\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon+P)} = -\sqrt{\frac{8\pi G}{3}}dt \tag{5.62}$$

and solving numerically we get the plot shown in Fig. 5.13 which gives the graphical representation of the time evolution of energy density of the QGP in model1. By looking at the figure, one can notice that the energy density was very high and then it decreases during this period. The numerical solutions obtained in this step can be used to obtain corresponding solutions of the temperature and pressure using the equations of the state listed above.

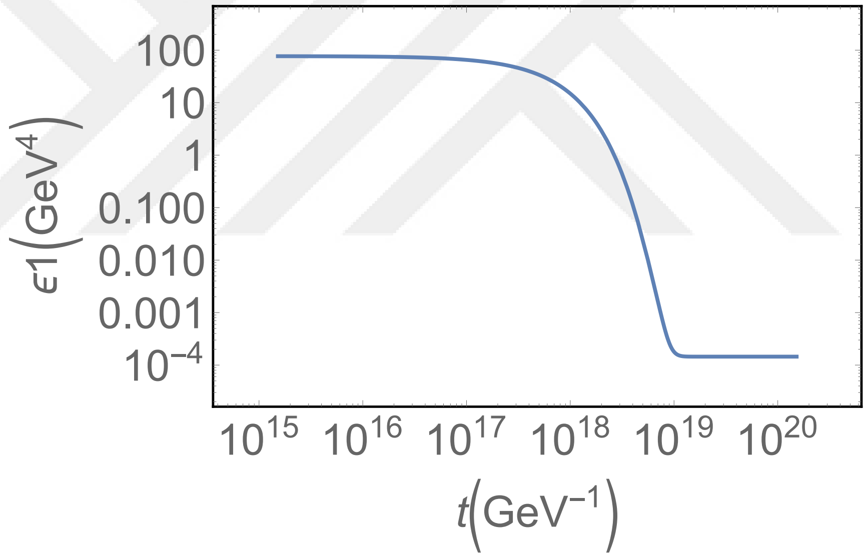

Figure 5.13. Time evolution of energy density of QGP in model1.

In Fig. 5.14 we show the graphical representation of the time evolution of temperature of the QGP in model1. As can be seen from the figure, the temperature was high and then began to drop as one expect due to the cooling of the universe.
In Fig. 5.15 we show the graphical representation of the time evolution of pressure of the QGP in model1 , By looking at the figure, we notice that the pressure was very high and then it decreases.

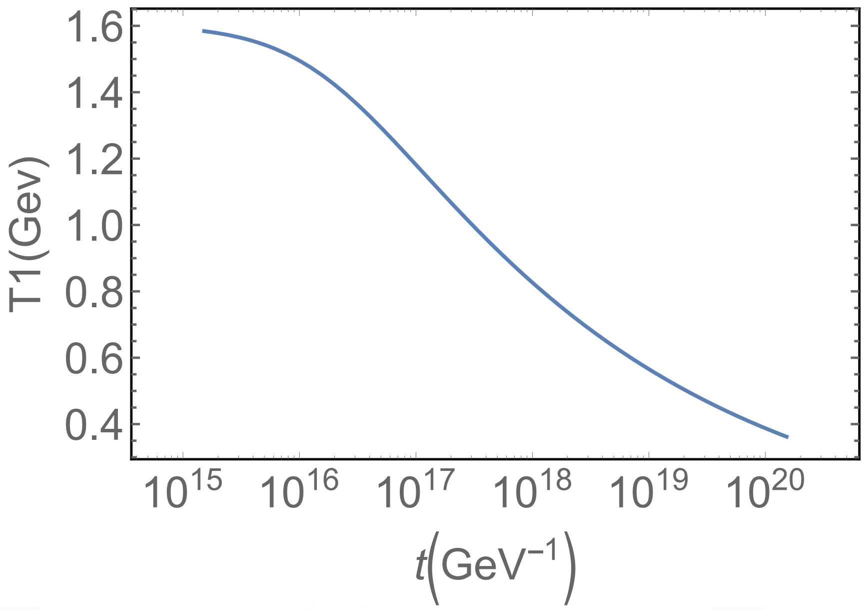

Figure 5.14. Time evolution of temperature of QGP in model1.

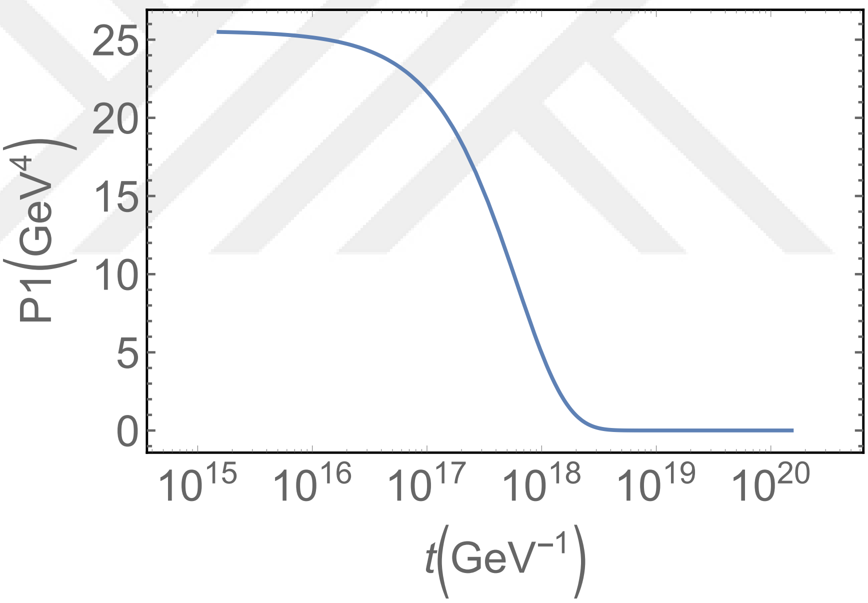

Figure 5.15. Time evolution of pressure of QGP in model1.

5.3.2. Model2

In this model, the energy density ε_2 and the pressure density p_2 as function of temperature T, are given by

$$\varepsilon_2(T) = \sigma T^4 - CT^2 + B_2, \quad p_2(T) = \frac{\sigma}{3}T^4 - CT^2 - B_2 \tag{5.63}$$

Eliminating the temperature, we obtain

$$p_2[\varepsilon_2(t)] = \frac{1}{3\delta_2}[\varepsilon_2(t) - 4B_2 - C[C + \sqrt{C^2 + 4\delta_2(\varepsilon_2(t) - B_2)}] \tag{5.64}$$

where the factors from (Begun et al., 2012). $\delta_2 = 13.01$, $c = 6.06T_c^2$ and $B_2 = -2.34T_c^4$. Upon substituting the expression of p_2 in the preceding equation in the differential equation

$$\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + P)} = -\sqrt{\frac{8\pi G}{3}}dt \tag{5.65}$$

and solving numerically s we get the plot shown in Fig. 5.16. The plot shows the graphical representation of the time evolution of energy density of the QGP in model2. Clearly from the figure that, the energy density was very high at the beginning of the time interval and then began to drop during this time period. The numerical solutions obtained in this step can be used to obtain corresponding solutions of the temperature and pressure using the equations of the state listed above.

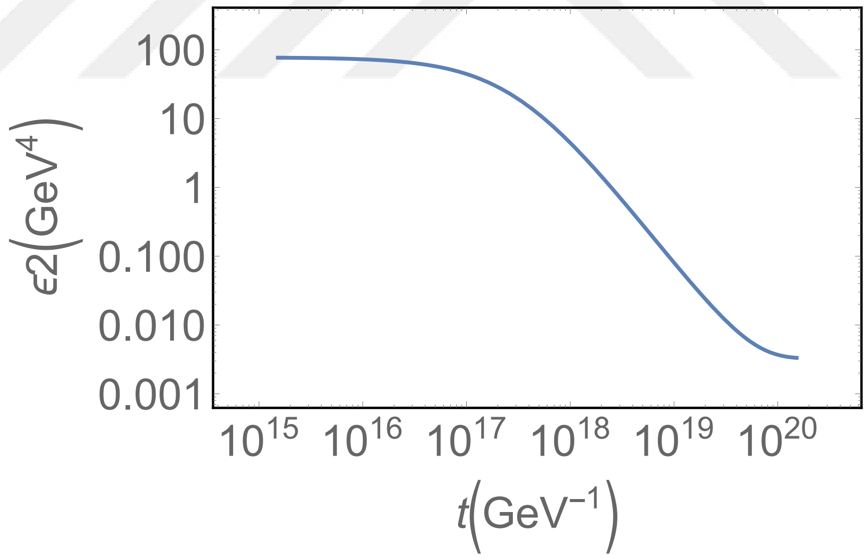

Figure 5.16. Time evolution of energy density of QGP in model2.

In Fig. 5.17 we show the graphical representation of the time evolution of temperature of the QGP in model.2. As expected, the figure shows that the temperature started to decrease after it was high in the period studied.

In Fig. 5.18 we show the graphical representation of the time evolution of pressure of the QGP in model.2. One can notice that the pressure was very high and then it

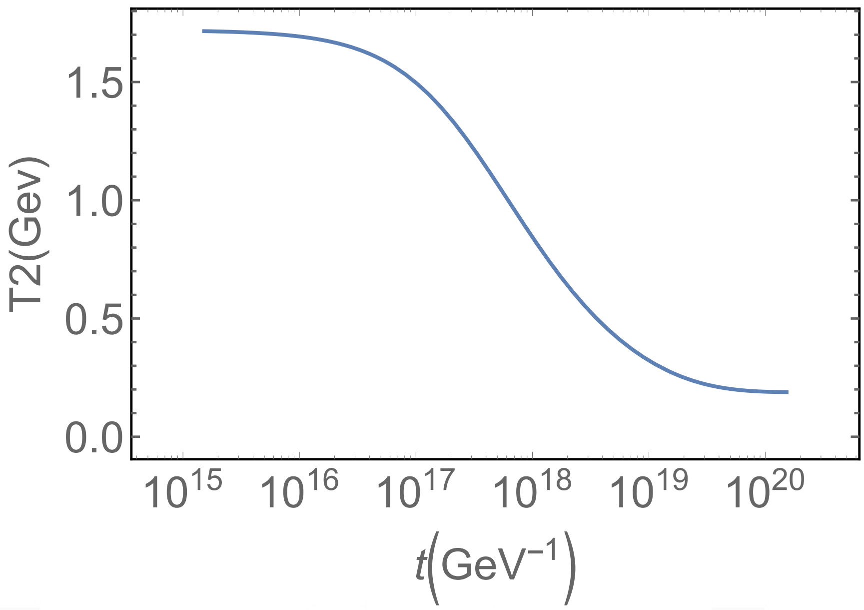

Figure 5.17. Time evolution of temperature of QGP in model2.

decreases.

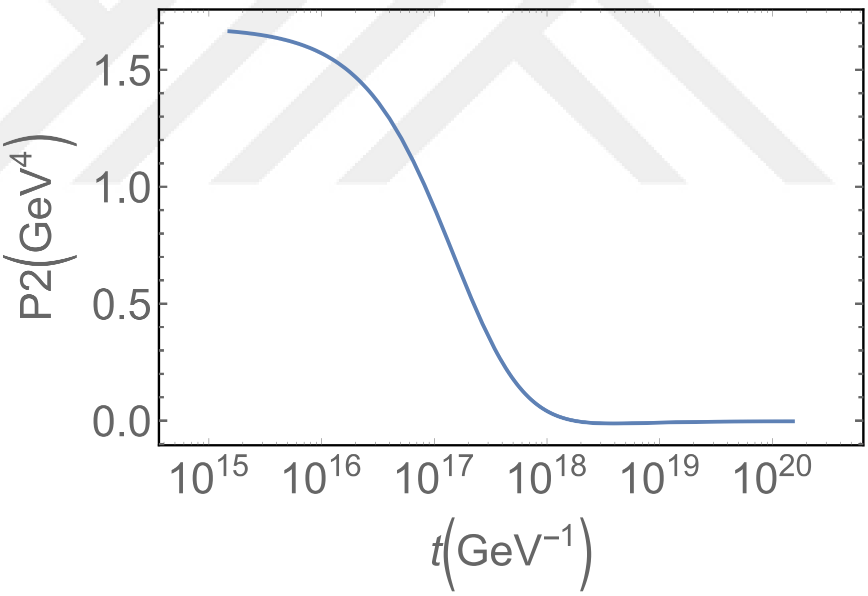

Figure 5.18. Time evolution of pressure of QGP in model2.

6. CONCLUSIONS

In the light of the progress in both experimental and theoretical sides, our understanding of the properties of QGP has been improved significantly, comparing to that one several decades before. The progress, regarding the equations of state of QGP, was made due to measurements obtained from heavy ion collision experiments and development in lattice QCD calculations. The impact on the equations of state of QGP can affect our understanding of an early era of the universe thought to be filled with dense QGP plasma for some short time interval after the big bang. During the era of QGP, the relevant parameters consist of the total energy density, pressure density, temperature and entropy density. With the help of the equations of state of QGP, one can study the time evolution of thermodynamic parameters like energy, pressure, temperature and entropy densities. In this thesis, we have used the equations of state of QGP in the MIT bag model and two updated versions, to take into account the recent progress, to solve the Friedman's differential equations that govern the time variation of the energy density, temperature, and pressure. We have showed that for some models, analytic solutions can be obtained and for other models numerical solutions are easily more obtained. The time variation of the thermodynamic parameters are then shown to give an idea of what happen in that era of universe. For different models, different results are obtained, showing that the equations of state of QGP can affect our understanding for what happened in that era of the early universe.

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