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NEUTROSOPHIC SUPERHYPER TOPOLOGICAL SPACES

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By Muslim Abdulazeez Noah ZAINEL

June 2023

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Mathematics.

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ABSTRACT

NEUTROSOPHIC SUPERHYPER TOPOLOGICAL SPACES

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June 2023

This thesis has been partitioned into four chapters, the first chapter dedicates to preliminaries and basic concepts, while chapter two comes as first attempt in building a new type of neutrosophic topological spaces, the aim is to shed the light on a new structure known as the n^{th}-power set $P^n(X)$ of a set X, this new kind of sets enables us to create and built new topology spaces called Neutrosophic SuperHyper Topological Spaces, the n^{th}-power sets are the optimal representation for the applications in our real world. In this chapter, new concepts and theorems related to this new topologies have been discussed, which are neutrosophic open n^{th}-power set, neutrosophic closed n^{th}-power set, as well as, the closures and the interiors are defined with their properties. Many of relations for these concepts have been introduced.

Chapter three contains new concepts presents for the first time linking the concept of the neutrosophic n^{th}- power sets with the traditional neutrosophic bi-topological spaces. The types of the topological spaces in neutrosophic theory are always changed depending upon the structure of the sets, in this chapter, the Neutrosophic SuperHyper Bi-Topological Spaces has been fathomed. As well as, we presented pairwise neutrosophic n^{th}- power open set, pairwise neutrosophic n^{th}- power closed set, thirteen new theorems, some new proposition, corollaries, and applied numerical examples to support the new notions. The definitions of the neutrosophic interior and the neutrosophic closure of $P^n(X)$ defined and adapted on Bi-Topological spaces have been presented.

Finally, Conclusions and some inspiration future search lines have been presented in chapter four to attract other researchers to keep on going to enrich this studying fields.

2023, 44 pages

Keywords: Neutrosophic superhyper bi-topological spaces, Neutrosophic topological spaces, Neutrosophic hyper-operation and neutrosophic hyper-structures

ÖZET

NÖTROSOFİK SÜPERHİPER TOPOLOJİK UZAYLAR

Muslim Abdulazeez Noah ZAINEL
Matematik, Yüksek Lisans
Tez Danışmanı: Doç. Dr. Gonca DURMAZ GÜNGÖR
Eş Danışman: Prof. Dr. Huda E. Khalid
Haziran 2023

Bu tez dört bölüme ayrılmıştır. İlk bölüm, ön bilgiler ve temel kavramlara ayrılmıştır. İkinci bölüm, yeni bir tür nötrösofik topolojik uzay oluşturma çabası olarak ortaya çıkar ve bir kümenin n^{th} inci gücü $P^n(X)$ olarak adlandırılan yeni bir yapıya ışık tutmayı amaçlar. Bu yeni tür küme, nötrösofik SüperHiper Topolojik Uzaylar adı verilen yeni topoloji uzayları oluşturmamıza olanak sağlar. n^{th} inci güç kümeleri, gerçek dünyadaki uygulamalar için en uygun temsili sağlar. Bu bölümde, bu yeni topolojilerle ilgili yeni kavramlar ve teoremler tartışılmıştır. Bunlar arasında nötrösofik açık n^{th} inci güç kümesi, nötrösofik kapalı n^{th} inci güç kümesi, kapanışlar ve içler ile ilgili tanımlar ve özellikler yer alır. Bu kavramlar için birçok ilişki tanıtılmıştır.

Üçüncü bölüm, nötrösofik n^{th} inci güç kümeleri kavramını geleneksel nötrösofik bi-topolojik uzaylarla ilk defa bağlayan yeni kavramları içermektedir. Nötrösofik teorideki topolojik uzayların türleri her zaman kümelerin yapısına bağlı olarak değişir. Bu bölümde, Nötrösofik SüperHiper Bi-Topolojik Uzaylar açıklanmıştır. Ayrıca, çiftli nötrösofik n^{th} inci güç açık küme, çiftli nötrösofik n^{th} inci güç kapalı küme, on üç yeni teorem, bazı yeni önermeler, sonuçlar ve yeni kavramları desteklemek için uygulamalı sayısal örnekler sunulmuştur. $P^n(X)$' in nötrösofik iç ve nötrösofik kapanışının Bi-Topolojik uzaylara uygun olarak tanımlanması sunulmuştur.

Son olarak, dördüncü bölümde sonuçlar ve gelecekteki araştırma yönlendirmeleri sunulmuş, diğer araştırmacıların bu çalışma alanını zenginleştirmeye devam etmeleri için ilham verilmeye çalışılmıştır.

2023, 44 sayfa

Anahtar Kelimeler: Nötrosofik süperhiper bi-topolojik uzaylar, Nötrosofik topolojik uzaylar, Nötrosofik hiper-işlem ve nötrosofik hiper-yapılar

PREFACE AND ACKNOWLEDGEMENTS

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Muslim Abdulazeez Noah ZAINEL
Çankırı 2023

CONTENTS

LIST OF SYMBOLS

$cl_P^n(P^n(x))$	Closure of the n^{th}-power set $P^n(x)$
$F_{P^n(x)}^{(\{x\})}$	Falsity membership function of the element $\{x\}$ in the n^{th}-power set $P^n(x)$
$\tau^{\iota stpair}$	First topological space of the bi-topological spaces
$I_{P^n(x)}^{(\{x\})}$	Indeterminacy membership function of the element $\{x\}$ in the n^{th}-power set $P^n(x)$
$int_p^n(P^n(x))$	Interior of the n^{th}-power set $P^n(x)$
V	Maximum operator
Λ	Minimum operator
$\#_2$	Neutrosophic binary hyper operator
$\#_m$	Neutrosophic m-ary hyper operator
$P^n(x_{\alpha,\beta,\gamma})$	Neutrosophic n^{th}-power point
$(\tau_1^{istpair}, \tau_2^{2ndpair}, P^n(x))$	Neutrosophic superhyper bi-topological spaces (NSHBI-TS)
$NSHSO$	Neatrosophic superhyper supra operator
$\tau^{neutrotopo}$	Neutrosophic super hyper topological
τ_n^*	Neutrosophic super hyper supra topological space
$P^x(x)\ n^{th}$	Power set of a set *X*
$\tau^{2nd\ pair}$	Second topological space of the bi-topological spacces
$T_{P^n(x)}^{(\{x\})}$	Truthmembership function of the element $\{x\}$ in the n^{th}-power set $P^n(x)$

LIST OF FIGURES

1 INTRODUCTION

There is no doubt that the neutrosophic theory has been originated by the Romanian-American scientist Florentin (Smarandache 1995).When he suggested a new kind of philosophy that carry the insight of taking any idea or any issue from three corners or three frames, he claimed existing truth part of the issue, the opposite part is the falsity part of the issue, while the middle part of the issue is the indeterminate part which positioned between the truth and the falsity. F. Smarandache be able to redefined all traditional sets and fields, mathematical operations, mathematical analysis, calculus, geometry, optimization theory, operation research, all theoretical parts of probability and statistics, topological spaces, and other fields of knowledge. As of 1995 till now, F. Smarandache collected dozens of scientists around the globe such as but not limited to A.A. Salama from Egypt, Huda E. Khalid from Iraq, Said Broumi from Morroco, Ishaani Priyadarshinie from USA, Maikel Yelandi Leyva Vázquez from South of America, M. Ganster from Australia, Xiaohong Zhang from China, Fernando A. F. Ferreira from Portugal, Vasantha Kandasamy from India, Giorgio Nordo from Italy, Sergey Gorbachev from Russia, (Smarandache 2016-2018-2019, Smarandache and Vazquez 2021), they working with him under two reputed international journals (Neutrosophic Sets and Systems journal briefly NSS journal which issued by the University of New Mexico and International Journal of Neutrosophic Science briefly IJNS journal) those two journals are indexed by dozens of repositories, such as Scopus, Amazon Kindle (USA), General Science Index, ProQuest (USA), Cengage Thompson Gale (USA), Google Books (USA), Cengage Learning (USA), Google Scholar (USA), DOAJ (Sweden), Index Copernicus (Poland), Engineering Village ,Elsevier, Ei_Compendex source list (Netherland),… etc.

Neutrosophic logic and theory is more general than fuzzy logic and theory that originated by Azerbaijani -American scientist Lotfi A. Zadeh on 1965 (Atanassov 1983). Neutrosophic logic focuses on redefined the parts of knowledge according membership functions. Also, neutrosophic theory is more general than the intuitionistic fuzzy logic and theory that setup by Bulgarian scientist Krassimir on (Atanassov 1983), he significantly extended fuzzy sets theory by launching the concept of

"Intuitionistic Fuzzy Sets" and investigated their basis properties depending upon membership functions and non- membership functions. However, this chapter sheds the light on the basic concepts of the neutrosophic theory especially the neutrosophic topological spaces with most impacting works in this field of science.

1.1 Neutrosophic Knowledge

Set theory is the fundamental concept in mathematics developed by a Russian Mathematician George Cantor in 1877. He showed that the points on two dimensional square has a one to one correspondent with points on different line segment leading to the development of dimensional theory. Frechet and Hausdorff along with others studied general topology. Hausdorff, the German mathematician, following the footsteps of Cantor developed set theory. Set theory enabled us to study various precise concepts in mathematics. But in real life situation we do come across many imprecise concepts or uncertain situation. If a class has fifty students say, to distinguish the taller/stronger students we are left with some short of uncertainty or vagueness. We can overcome the vagueness by fixing the percentage of membership namely the percentage of membership enables us to find out the level of inexactness. This theory is known as fuzzy theory (Thivagar *et al.* 2018).

The neutrosophic theory (i.e. Neutrosophic Set / Logic / Probability / Statistics / Measure / Pre-calculus / Calculus / Algebraic Structures ...etc. (Smarandache 1998-1999, Smarandache and Khalid 2018) as a new mathematical tool is dealing with problems involving indeterminacy, inconsistent data, or incompleteness information. The most general definition, the classification, and many real examples of Indeterminacies from our everyday life, utilized in the neutrosophic theories and their applications, are presented in the upcoming subsections in an understandable manner. “Indeterminacy” should not be taken into the narrow sense of a lexical dictionary, but as something that is in between the opposites. Because of dealing with various types of indeterminacies (vague, unclear, uncertain, conflicting, incomplete, hesitancy, neutrality, unknown, etc.) related to the data or to the procedures employed in our real world, we may extend by neutrosophication any classical scientific or cultural crisp concept from any field of knowledge to a corresponding neutrosophic (un-crisp) concept, since in our world more things are indeterminate or partially indeterminate than completely determinate (Smarandache 2021).

1.2 Neutrosophic Triplets

Let $\langle B \rangle$ be an item (concept, notion, idea, sentence, theory etc.) and $\langle antiB \rangle$ its opposite. In between the opposites $\langle B \rangle$ and $\langle antiB \rangle$, there is a neutral (or indeterminacy) part, denoted by $\langle neutB \rangle$.

The $\langle neutB \rangle$ is neither $\langle B \rangle$ nor $\langle antiB \rangle$, or sometimes the $\langle neutB \rangle$ is a mixture of partial $\langle B \rangle$ and partial $\langle antiB \rangle$. Of course, we consider the neutrosophic triplets $(\langle B \rangle, \langle neutB \rangle, \langle antiB \rangle)$ that make sense in the world, and there are plenty of such triplets in our everyday life(Smarandache 1998).

Example 1.2.1 (Smarandache 2021) [(Friend, Neutral, Enemy), (Positive, Zero, Negative), (Win, Tie – game Lose), (Short, Medium, Tall), (True, Partially-true & Partially-false, False), (True, Indeterminacy, False), (Membership, Partially-membership & Partially-non-membership, Non-membership), (White, gray, Black), etc.]

Definition 1.2.1 (The Neutrosophic Set Triplet) (Smarandache 2021)

The neutrosophic set membership-value of an element x with respect to a give set M is: $NS(x) = (T, I, F)$, where

T = The degree of membership of the element x with respect to the set M;

I = The indeterminate-degree of membership or non-membership of the element x with respect to the set M;

F = The degree of nonmembership of the element x with respect to the set M; or T = membership, I = indeterminacy, F = non-membership.

This section is mainly dedicated to the basic concepts of the neutrosophic theory that presented by F. Smarandache (Smarandache 1999), and A.A. Salama (Salama and AL-Blowi 2012), given that the components T, I, F refer to the membership, indeterminacy, and non-membership functions respectively, where $]0^{-}, 1^{+}[$ is non-standard unit interval.

Definition 1.2.2 (Smarandache 1999)

Let T, I, F be real standard or non-standard subsets of $]0^{-}, 1^{+}[$, with

$$\sup\ \_T = t\_sup,\ \ \inf\ \_T = t\_inf$$

$$\sup\ \_I = i\_sup,\ \ \inf\ \_I = i\_inf$$

$$\sup\ \_F = f\_sup,\ \ \inf\ \_F = f\_inf$$

$$\mathrm{n} - sup = t - sup + i - sup + f - sup$$

$$\mathrm{n} - inf = t - inf + i - inf + f - inf$$

where, T, I, F are called neutrosophic components.

Definition 1.2.3 (Salama and AL-Blowi 2012)

Let X be a non-empty fixed set. A neutrosophic se A is an object having the form

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\},$$

where $\mu_A(x), \sigma_A(x), and\ \gamma_A(x)$ are the degrees of membership function, indeterminacy function, and non-membership function respectively.

Definition 1.2.4 (Salama and AL-Blowi 2012)

The universal neutrosophic set 1_N, and the empty neutrosophic set 0_N, each one of them has four types or patterns as follow:

The empty set 0_N can be defined as one of the following four formulas, adhering to take the zero value in at least in its truth component:

- $0_{N1} = \{\langle x, 0,0,1\rangle : x \in X\}$
- $0_{N2} = \{\langle x, 0,1,1\rangle : x \in X\}$
- $0_{N3} = \{\langle x, 0,1,0\rangle : x \in X\}$
- $0_{N4} = \{\langle x, 0,0,0\rangle : x \in X\}$

while, the universal set 1_N can be defined as one of the following four formulas, adhering to take the one value in at least in its truth component:

- $1_{N1} = \{\langle x, 1,0,0\rangle : x \in X\}$
- $1_{N2} = \{\langle x, 1,0,1\rangle : x \in X\}$
- $1_{N3} = \{\langle x, 1,1,0\rangle : x \in X\}$
- $1_{N4} = \{\langle x, 1,1,1\rangle : x \in X\}$

Definition 1.2.5 (Salama and AL-Blowi 2012)

Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a neutrosophic set on X, then the complement of the set A (represented as A^c) can be defined in three kinds of complements:

- $A^{c1} = \{\langle x, 1 - \mu_A(x), \sigma_A(x), 1 - \gamma_A(x)\rangle : x \in X\}$,

- $A^{c2} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$,
- $A^{c3} = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 1.2.6 (Salama and AL-Blowi 2012)

Let X be a non-empty set, and let A & B are two neutrosophic sets defined as:

$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$,

then there are two definitions for inclusion $(A \subseteq B)$ as follow:

1- $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), and\ \gamma_A(x) \geq \gamma_B(x), \forall\ x \in X$

2- $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), and\ \gamma_A(x) \geq \gamma_B(x), \forall\ x \in X$

Definition 1.2.7 (Salama and AL-Blowi 2012)

Let X be a non-empty set, and let A & B are two neutrosophic sets defined as:

$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$.

The intersection operation $(A \cap B)$, and the union operation $(A \cup B)$ can be defined as follow:

1- The intersection operation $(A \cap B)$ is defined in three patterns:
- $A \cap B = \{\langle x, \mu_A(x).\mu_B(x), \sigma_A(x).\sigma_B(x), \gamma_A(x).\gamma_B(x)\rangle : x \in X\}$, or
- $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$, or
- $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$.

2- The union operation $(A \cup B)$ is defined in two patterns:
- $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \sigma_B(x)\rangle : x \in X\}$, or

- $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \sigma_B(x)\rangle : x \in X\}$.

So, we can generalize the above mentioned operations:

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic sets in X, then $\cap A_j$ may be defined as:

- $\cap A_j = \{\langle x, \wedge_{j\in J} \mu_{Aj}(x), \wedge_{j\in J} \sigma_{Aj}(x), \vee_{j\in J} \gamma_{Aj}(x)\rangle : x \in X\}$, or
- $\cap A_j = \{\langle x, \wedge_{j\in J} \mu_{Aj}(x), \vee_{j\in J} \sigma_{Aj}(x), \vee_{j\in J} \gamma_{Aj}(x)\rangle : x \in X\}$,

while, $\cup A_j$ may be defined as:

- $\cup A_j = \{\langle x, \vee_{j\in J} \mu_{Aj}(x), \wedge_{j\in J} \sigma_{Aj}(x), \wedge_{j\in J} \gamma_{Aj}(x)\rangle : x \in X\}$,
- $\cup A_j = \{\langle x, \vee_{j\in J} \mu_{Aj}(x), \vee_{j\in J} \sigma_{Aj}(x), \wedge_{j\in J} \gamma_{Aj}(x)\rangle : x \in X\}$.

1.3 Neutrosophic Topological Spaces

A.A. Salama was the first scientist who generalized both fuzzy topological spaces (Chang 1968), and intuitionistic fuzzy topological spaces (Coker 1997), to the neutrosophic topological spaces. He introduced new articles concerning to this subject on 2012. Later, there were dozens of manuscripts new concepts improving neutrosophic topological spaces (Ahmed *et al*. 2021, Gündüz and Bayramov 2020, Al-Nafee *et al.* 2019-2020, Salama *et al*. 2022. For more extension, the upcoming fundamentals have been introduced:

Proposition 1.3.1 (Salama and AL-Blowi 2012)

For any neutrosophic set A, the following inclusions are true:

- $0_x \subseteq A$

- $A \subseteq 1_x$

Definition 1.3.1 (Salama and AL-Blowi 2012)

A neutrosophic topology on a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms

- $0_N, 1_N \in \tau$.
- $\cap G_i \in \tau$, all $\{G_i : i \in J\} \subseteq \tau$.
- $\cup G_i \in \tau$, all $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic topological space, and any neutrosophic set in τ is known as neutrosophic open set in X. The elements of τ are called open neutrosophic sets. A neutrosophic set F is closed if and only if its complement (i.e. F^c) is neutrosophic open set.

Example 1.3.1 (Salama and AL-Blowi.2012)

Let $X = \{x\}$, and $A = \{\langle x, 0.5, 0.5, 0.4\rangle : x \in X\}$, $B = \{\langle x, 0.4, 0.6, 0.8\rangle : x \in X\}$, $D = \{\langle x, 0.5, 0.6, 0.4\rangle : x \in X\}$, $C = \{\langle x, 0.4, 0.5, 0.8\rangle : x \in X\}$, then the family $\tau = \{0_x, 1_x, A, B, C, D\}$ of neutrosophic sets in X is neutrosophic topology on X.

1.4 Layout of This Thesis

Chapter two comes as first attempt in building a new type of neutrosophic topological spaces, the aim is to shed the light on a new structure known as the n^{th}-power set $P^n(X)$ of a set X, this new kind of sets enables us to create and built new topology spaces called Neutrosophic SuperHyper Topological Spaces, the n^{th}-power sets are the optimal representation for the applications in our real world. In this chapter, new concepts and theorems related to this new topologies have been discussed, which are neutrosophic open n^{th}-power set, neutrosophic closed n^{th}-power set, as well as, the closures and the interiors are defined with their properties. Many of relations for these concepts have been introduced.

Chapter three contains new concepts presents for the first time linking the concept of the neutrosophic n^{th}- power sets with the traditional neutrosophic bi-topological spaces. The types of the topological spaces in neutrosophic theory are always changed depending upon the structure of the sets, in this chapter, the Neutrosophic SuperHyper Bi-Topological Spaces has been fathomed. As well as, we presented pairwise neutrosophic n^{th}- power open set, pairwise neutrosophic n^{th}- power closed set, thirteen new theorems, some new proposition, corollaries, and applied numerical examples to support the new notions. The definitions of the neutrosophic interior and the neutrosophic closure of $P^n(X)$ defined and adapted on Bi-Topological spaces have been presented.

Finally, Conclusions and some inspiration future search lines have been presented in chapter four to attract other researchers to keep on going to enrich this studying fields.

2 MATERIALS AND METHODS

2.1 Neutrosophic SuperHyper Topological Spaces: Original Notions and New Insights

This chapter comes as first attempt in building a new type of neutrosophic topological spaces, the aim is to shed the light on a new structure known as the n^{th}-power set $P^n(X)$ of a set X, this new kind of sets enables us to create and built new topology spaces called Neutrosophic SuperHyper Topological Spaces, the n^{th}-power sets are the optimal representation for the applications in our real world. In this chapter, new concepts and theorems related to this new topologies have been discussed, which are neutrosophic open n^{th}-power set, neutrosophic closed n^{th}-power set, as well as, the closures and the interiors are defined with their properties. Many of relations for these concepts have been introduced.

The concepts of the neutrosophic n^{th}-power set of a set, SuperHyperGraph and Pliothogenic n-SuperHyperGraph, SuperHyperAlgebra, n-ary (Classical-/Neutro-/Anti-) HyperAlgebra have been firstly introduced by the father of neutrosophic theory (Smarandache 2016). As the introduction for Neutrosophic SuperHyper Topological Spaces which is until yet is fathomless branch of science, in this section we recalling the fundamental definitions of the neutrosophic logic with preliminaries of related n^{th}-power set of a set. The essential theory of neutrosophic was introduced and built (Smarandache 1995-1998-1999). Any mathematician who tracking the trace of this knowledge will easily deduce that the neutrosophic theory was rapidly and broadly radiated through Neutrosophic Sets and Systems journal, and International Journal of Neutrosophic Science, these two journals are very active and reputed journals indexed by dozens of repositories, encyclopedias, and identifications' websites especially Scopus database.

This chapter has been organized as follow:

We present some basic preliminaries in section 2, while section 3 has been dedicated to submit a new structure of neutrosophic topology called Neutrosophic SuperHyper Topological Spaces, in this section and for the first time, this type of topology was discussed in details. The main core of this article is in section 3 which is contain definitions, theorems, and corollaries covered the new subject that introduced firstly in this chapter which is named Neutrosophic SuperHyper Bi-Topological Spaces. The last section is the conclusion section.

2.2 System of Sub-System of Sub-Sub-System

A system may be a set, space, organization, association, team, city, region, country, etc. One consider both: the static and dynamic systems. With respect to various criteria, such as: political, religious, economic, military, educational, sportive, touristic, industrial, agricultural, etc. A system S is made up of several sub-systems $S_1, S_2, \dots, S_p$, for integer $p \geq 1$; then each su-system S_i, for $i \in \{1,2, \dots, p\}$ is composed of many sub-sub-systems $S_{i1}, S_{i2}, \dots, S_{ip_i}$, for integer $p_i \geq 1$; then each sub-sub-systems S_{ij}, for $j \in \{1,2, \dots, p_i\}$ is composed sub-sub-sub-systems $S_{ij1}, S_{ij2}, \dots, S_{ijp_j}$, for integers p_j; and so on the following example of systems made of Sub-Sub-Sub-Systems (four levels).

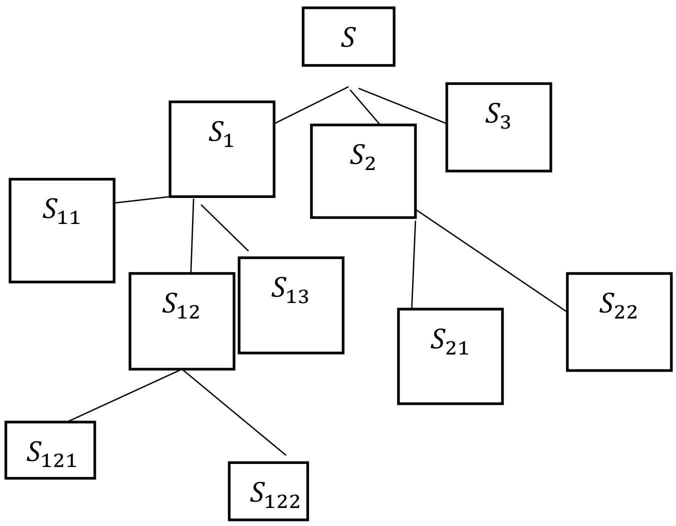


Figure 2.1 Using a tree-graph representation

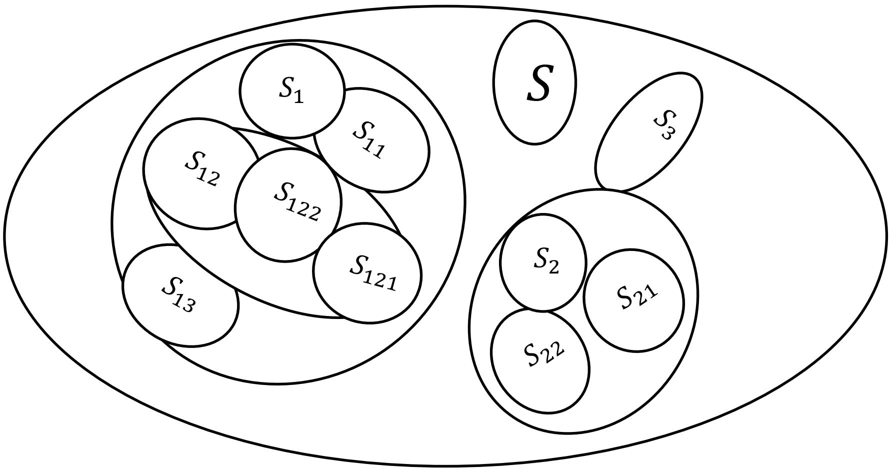
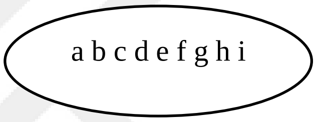
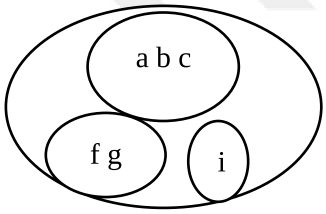
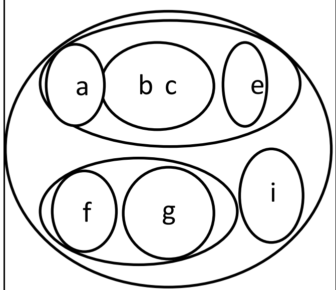


Figure 2.2 Using a geometric representation

$P^0(S) = S$ $= \{a, b, c, d, e, f, g, h, l\}$ 1 level of pairs of braces		Level 1
$P^1(S) = \{\{a, b, c, d, e\}, \{f, g, h\}, \{l\}\}$ 2 level of pairs of braces i.e. a pair of braces {} inside, another pair of braces {}, or {… {…} ...}	 2 level of closed curves	Level 2
$P^2(S) = P(P(S))$ $= \{\{\{a\}, \{b, c, d\}, \{e\}\}, \{\{f\}, \{g, h\}\}, \{l\}\}$ 3 levels of pairs of braces	 3 level of closed curves	Level 3

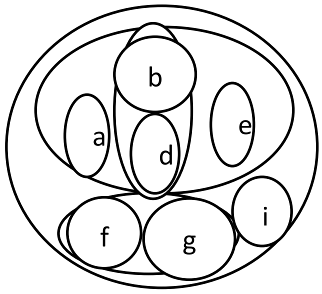
$P^3(S) = P(P^2(S))$ $= \{\{\{a\},\{b,c\},\{d\},\{e\}\},\{\{f\},\{g,h\}\},\{l\}\}$ 4 levels of pairs of braces	 4 level of closed curves	Level4

Figure 2.3 Using an algebraic representation through pairs of braces {}

2.3 Definition of n^{th} -Power of a set

The n^{th} -Power of a set was firstly introduced by F. Smarandache at al.2016 where $P^n(S)$ represents the n^{th} -PowerSet of the set S, for integer $n \geq 1$, is recursively defined as:

$P^2(S) = P(P(S))$, $P^3(\mathrm{S}) = P\left(P\big(P(S)\big)\right)$,..., $P^n(S) = P(P^{n-1}(S))$, where $P^0(S) = S$, $and\ P^1(S) = P(S)$, i.e. $P^0(S) \subset P^1(S) \subset P^2(S) \subset \cdots \subset P^{n-1}(S) \subset P^n(S)$.

The n^{th}-PowerSet of a Set is better reflect for our complex reality, since a set S (that may represent a group, a society, a country, a continent, etc.) of elements (such as: people, objects, and in general any items) is organized onto subsets $P(S)$, which on their turns are also organized onto subsets of subsets, and so on, that is our world (F. Smarandache. 2022).

Example 2.3.1 Suppose that the set of the grandparents represents the power set $P^2(S) = P(P(S))$, then the first offspring is the parents themselves which can be regarded as the power set $P(S)$, and the second offspring is the non-empty set $P^0(S) = S$, i.e. $S = P^0(S) \subset P^1(S) \subset P^2(S)$.

The following medical case study would be appropriate to demonstrate the importance of the power set concept:

There are many diseases and conditions that can be passed on through genes. Some of these diseases include Down syndrome, hemophilia, hypertension, sickle cell anemia, and cystic fibrosis. Most genetic diseases are a combination of mutations in multiple genes, often in combination with environmental factors. There are three groups of genetic diseases, each with their own causes: Monogenetic diseases, multifactorial inherited diseases, and chromosomal abnormalities.

The couple of husband can be represented as PowerSet $P(S)$, it is important to know what $P(S)$ have inherited a genetic disease from their parents (i.e. represented the non-empty set $P^0(S) = S$ as grandparents) and to remember that the above mentioned genetic diseases can be passed on to their descendants (i.e. the offspring which is mathematically denoted by the power set $P^2(S) = P(P(S))$. If S & $P(S)$ are aware of possible diseases that can be inherited to $P(S)$& $P^2(S)$ respectively, contact a specialist and see what S & $P(S)$ can do to help $P(S)$& $P^2(S)$ and avoid serious problems later. By working together with the help of family and doctor, the health risks can be avoided instead of taking their toll later.

2.4 Neutrosophic HyperOperation and Neutrosophic HyperStructures

In the classical HyperOperation and classical HyperStructures, the empty-set $\emptyset$ does not belong to the power set, (i.e. $P_*(H) = P(H)/\{\emptyset\}$). Nonetheless, in the real world we encounter many situations when HyperOperation # is indeterminate, for example $a \# b = \emptyset$ (unknown or undefined), or partially indeterminate, for example: $a \# b = \{ [0.2, 0.3], \emptyset\}$. In our everyday life, there are many more operations and lows that have some degrees of indeterminacy (vagueness, unclearness, unknowingness, contradiction, etc.), than those that are totally determined. That’s why in 2016 the scientists F. Smarandache have extended the classical HyperOperation to the Neutrosophic HyperOperation, by taking the whole power $P(H)$ (that includes the empty-set $\emptyset$ as well), instead of. $P_*(H)$(That does not include the empty-set $\emptyset$), as follow.

2.4.1 Definition of Neutrosophic HyperOperation:

Let U be a universe of discourse and H be a non-empty set, $H \subset U$. A Neutrosophic Binary HyperOperation $\#_2$ is defined as follows:

$\#_2: H^2 \rightarrow P(H)$, where H is a discrete or continuous set, and $P(H)$ is the powerset of H that includes the empty-set $\emptyset$.

Neutrosophic m-ary HyperOperation $\#_m$ is defined as:

$\#_m: H^m \rightarrow P(H)$, for integer $m \geq 1$. Similarly, for $m = 1$ one gets a Neutrosophic Unary HyperOperation.

2.4.2 Neutrosophic SuperHyper Topological Spaces

This section gives an original creativity neutrosophic mathematical structure for new notion named as Neutrosophic SuperHyper Topological Spaces (NSHTS) defined under a new kind of sets called neutrosophic n^{th}-power set $P^n(X)$.

Definition 2.4.2.1 Let X be a non-empty set, $P^n(X)$ is the neutrosophic n^{th}-power set of a set X, for integer $n \geq 1$. A Neutrosophic SuperHyper Topological space on $P^n(X)$ is a subfamily $\tau^{neutrotopo}$ of $N(P^n(X))$, and satisfying the following axioms:

1- The neutrosophic universal n^{th}-power set $1_{P^n(X)}$,and the neutrosophic empty n^{th}-power set $0_{P^n(X)}$ both are belonging to $\tau^{neutrotopo}$.

2- Any arbitrary (finite on infinite) union of members of $\tau^{neutrotopo}$ belong to $\tau^{neutrotopo}$.

3- $\tau^{neutrotopo}$ is closed under finite intersection of members of $\tau^{neutrotopo}$ (i.e., the intersection of any finite number of members of $\tau^{neutrotopo}$ belongs to $\tau^{neutrotopo}$).

Then $(\tau^{neutrotopo}, P^n(X))$ is called Neutrosophic SuperHyper Topological Spaces (NSHTS). Because of the definition of (NSHTS) via neutrosophic n^{th}-power open sets that commonly used in this manuscript, the family of neutrosophic sets $\tau^{neutrotopo}$ of the n^{th}-power sets are commonly called a (NSHTS) on the neutrosophic n^{th}-power sets $P^n(X)$.

A subpowerset $P^{m1}(C) \subseteq P^{m2}(X)$ for integers $m1 \leq m2$ is to be closed in $(\tau^{neutrotopo}, P^n(X))$ if its complement $P^{m2}(X)/P^{m1}(C)$ is an open set.

Example 2.4.2.1 What is the difference between $P^1(x)$ & $P^2(x)$ in the structured of the Neutrosophic SuperHyper topological spaces $(\tau^{neutrotopo}, P^n(X))$, and how it effects on the distribution of the internal elements? Take a look on the following example.

Suppose $X = \{a, b, c\}$ with the following

$$P^1(x) = \left\{ \{a, T = 0.3, I = 0.1, F = 0.6\}, \{b, c\} \begin{array}{r} T = \{0.7,0.4\} \\ I = \{0,0.3\} \\ F = \{0.4,0.3\} \end{array} \right\},$$

$$P^2(x) = \left\{ \{a, T = 0.3, I = 0.1, F = 0.6\}, \{\{b\},\{c\}\} \begin{array}{r} T = \{\{0.7\},\{0.4\}\} \\ I = \{\{0\},\{0.3\}\} \\ F = \{\{0.4\},\{0.3\}\} \end{array} \right\}.$$

For more details, we can see that in $P^1(x)$ the element a affected by its membership functions $\{0.3, 0.1, 0.6\}$ directly, while the element(s) $\{b, c\}$ has (have) two kinds of affected (directed affect) and (indirect affect) as follow:

- The element b has a separate direct affect by its membership functions {0.7,0,0.4}, and the element c has a separate direct affect by its membership functions {0.4,0.3,0.3}.
- The structured element $\{b, c\}$ have common indirect affected by their membership functions $\{0.7,0.4\}, \{0,0.3\}, \{0.4,0.3\}$.

This is a very harmonic with the previous example (1.3) stated in section one, by expressing the elements a, b as the parents (husband and wife), each one of them can affected separately by the inherited genes from their parents, also, they will crossing their parents' gene to their offspring mutually and their descendants will be affected directly by their parents and indirectly by their grandparents.

Then $(\tau^{neutrotopo}, P^n(X))$ is the Neutrosophic SuperHyper Topological space, where:

$$\tau^{neutrotopo} = \{0_{P^n(X)}, 1_{P^n(X)}, P^1(x), P^2(x)\}.$$

Definition 2.4.2.2 Let $P^n(X)$ be a neutrosophic n^{th}-power set over a non-empty set X, the neutrosophic interior and the neutrosophic closure of $P^n(X)$ are respectively defined as:

$$int^n\big(P^n(X)\big) = \cup \{P^m(X): P^m(X) \subseteq P^n(X), P^m(X) \in \tau^{neutrotopo}\},$$

this means that for the same collection of the neutrosophic n^{th}-power set $P^n(X)$, all $P^m(X)$ given that $m \leq n$ regarded as interior for $P^n(X)$.

$$cl^n\big(P^n(X)\big) = \cap \{P^h(X): P^n(X) \subseteq P^h(X), (P^h(X))^c \in \tau^{neutrotopo}\}.$$

Definition 2.4.2.3 The following mathematical phrases are true for any two neutrosophic ${n_1}^{th}$-power set $P^{n1}(Y1)$ and ${n_2}^{th}$-power set $P^{n2}(Y2)$ on the neutrosophic

n^{th}-power set $P^n(X)$, given that $n_1, n_2 \leq n$, and that there is no restrictions on the relation between n_1 and n_2:

1. $T_{P^{n1}(Y1)}(\{x\}) \leq T_{P^{n2}(Y2)}(\{x\}), I_{P^{n1}(Y1)}(\{x\}) \leq I_{P^{n2}(Y2)}(\{x\})$ and $F_{P^{n1}(Y1)}(\{x\}) \geq F_{P^{n2}(Y2)}(\{x\})$, for integers $n_1, n_2 \geq 1$, and for all $\{x\} \subseteq P^n(X)$ iff $P^{n1}(Y1) \subseteq P^{n2}(Y2)$.
2. $P^{n1}(Y1) \subseteq P^{n2}(Y2)$ and $P^{n2}(Y2) \subseteq P^{n1}(Y1)$ iff $P^{n1}(Y1) = P^{n2}(Y2)$, given that $n_1 = n_2$.
3. $P^{n1}(Y1) \cap P^{n2}(Y2) = \{\langle \{x\}, \min\{T_{P^{n1}(Y1)}(\{x\}), T_{P^{n2}(Y2)}(\{x\})\}, \min\{I_{P^{n1}(Y1)}(\{x\}), I_{P^{n2}(Y2)}(\{x\})\},$

$$\max\{F_{P^{n1}(Y1)}(\{x\}), F_{P^{n2}(Y2)}(\{x\})\}\rangle : \{x\} \subseteq P^n(X)\}$$

4. $P^{n1}(Y1) \cup P^{n2}(Y2) = \{\langle \{x\}, \max\{T_{P^{n1}(Y1)}(\{x\}), T_{P^{n2}(Y2)}(\{x\})\}, \max\{I_{P^{n1}(Y1)}(\{x\}), I_{P^{n2}(Y2)}(\{x\})\},$

$$\min\{F_{P^{n1}(Y1)}(\{x\}), F_{P^{n2}(Y2)}(\{x\})\}\rangle : \{x\} \subseteq P^n(X)\}$$

In general, the union or the intersection of any arbitrary members of neutrosophic n^{th}-power set $P^{ni}(X)_{i \in I}$ are defined by:

$$\bigcap_{i \in I} P^{ni}(X) = \{\langle \{x\}, \inf\left\{T_{P^{ni}(\{x\})}\right\}, \inf\left\{I_{P^{ni}(\{x\})}\right\}, \sup\left\{F_{P^{ni}(\{x\})}\right\}\rangle : \{x\} \subseteq P^n(X)\},$$

$$\bigcup_{i \in I} P^{ni}(X) = \{\langle \{x\}, \sup\left\{T_{P^{ni}(\{x\})}\right\}, \sup\left\{I_{P^{ni}(\{x\})}\right\}, \inf\left\{F_{P^{ni}(\{x\})}\right\}\rangle : \{x\} \subseteq P^n(X)\}.$$

5. The neutrosophic n^{th}-power universal set $P^n(X)$ is denoted by $1_{P^n(X)}$,and it is exist if and only if the following conditions are holding together:

$$T_{P^n(\{x\})} = 1_{P^n(X)}, I_{P^n(\{x\})} = 1_{P^n(X)}, and\ F_{P^n(\{x\})} = 0_{P^n(X)}.$$

6. The neutrosophic n^{th}-power empty set $P^n(X)$ is denoted by $0_{P^n(X)}$, and it is exist if and only if the following conditions are holding together:

$$T_{P^n(\{x\})} = 0_{P^n(X)}, I_{P^n(\{x\})} = 0_{P^n(X)}, and\ F_{P^n(\{x\})} = 1_{P^n(X)}.$$

7. Let $P^{n1}(Y1) \subseteq P^{n2}(Y2)$, given that $n_1 \leq n_2$, then the complementary of $P^{n1}(Y1)$ concerning to $P^{n2}(Y2)$ is defined as follow:

$$P^{n1}(Y1) \setminus P^{n2}(Y2) = \{\langle \left|T_{P^{n1}(Y1)}(\{x\}) - T_{P^{n2}(Y2)}(\{x\})\right|, \left|I_{P^{n1}(Y1)}(\{x\}) - I_{P^{n2}(Y2)}(\{x\})\right|, 1_{P^n(X)} - \left|F_{P^{n1}(Y1)}(\{x\}) - F_{P^{n2}(Y2)}(\{x\})\right|\rangle\}.$$

8. Clearly, the neutrosophic complement of $1_{P^n(X)}$ and $0_{P^n(X)}$ are defined as:

$$(1_{P^n(X)})^c = \langle T_{P^n(\{x\})} = 0_{P^n(X)}, I_{P^n(\{x\})} = 0_{P^n(X)}, F_{P^n(\{x\})} = 1_{P^n(X)}\rangle = 0_{P^n(X)},$$

$$(0_{P^n(X)})^c = \langle T_{P^n(\{x\})} = 1_{P^n(X)}, I_{P^n(\{x\})} = 1_{P^n(X)}, F_{P^n(\{x\})} = 0_{P^n(X)}\rangle = 1_{P^n(X)}.$$

Proposition 2.4.2.1 Let $P^{n1}(X), P^{n2}(X), P^{n3}(X), and\ P^{n4}(X) \subseteq N(P^n(X))$ without any restrictions on the relations between $n_1, n_2, n_3, n_4, and\ n$, then the following mathematical statements are true:

Let $P^{n1}(X) \subseteq P^{n2}(X), and\ P^{n3}(X) \subseteq P^{n4}(X)$, given that $n_1 \leq n_2$, & $n_3 \leq n_4$, this implies that

$$P^{n1}(X) \cap P^{n3}(X) \subseteq P^{n2}(X) \cap P^{n4}(X),$$

$(P^{n1}(X)^c)^c = P^{n1}(X)$, also if $P^{n2}(X)^c \subseteq P^{n1}(X)^c \Longrightarrow P^{n1}(X) \subseteq P^{n2}(X)$,

$$(P^{n1}(X) \cap P^{n2}(X))^c = P^{n1}(X)^c \cup P^{n2}(X)^c,$$

$$(P^{n1}(X) \cup P^{n2}(X))^c = P^{n1}(X)^c \cap P^{n2}(X)^c.$$

Definition 2.4.2.4 Let X be a non-empty set, $P^n(X)$ is the n^{th}-power neutrosophic set of a set X, for integer $n \geq 1$. If α, β, γ be real standard or non-standard subsets of

$]^-0,1^+[$, then the neutrosophic n^{th}-power set $P^n(x_{\alpha,\beta,\gamma})$ is called a neutrosophic n^{th}-power point, and it is defined by:

$$P^n\left(x_{\alpha,\beta,\gamma}(y)\right) = \begin{cases} \langle \alpha_{P^n(x)}, \beta_{P^n(x)}, \gamma_{P^n(x)} \rangle, & if\ P^n(x) = P^n(y) \\ \langle 0_{P^n(x)}, 0_{P^n(x)}, 1_{P^n(x)} \rangle, & if\ P^n(x) \neq P^n(y) \end{cases}$$

for $x, y \in X, and\ P^n\left(x_{\alpha,\beta,\gamma}\right), P^n(y) \subseteq P^n(X)$, here $P^n(y)$ is called the support of $P^n(x_{\alpha,\beta,\gamma})$.

Definition 2.4.2.5 Let $P^{n1}(X) \in N(P^n(X))$, the belonging operation of the neutrosophic n^{th}-power point $P^n(x_{\alpha,\beta,\gamma})$ to $P^{n1}(X)$ (i.e. $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$) is satisfied if and only if $T_{P^{n1}(\{x\})} \geq \alpha, I_{P^{n1}(\{x\})} \geq \beta, F_{P^{n1}(\{x\})} \leq \gamma$.

Definition 2.4.2.6 A sub-collection τ_n^* of neutrosophic n^{th}-power set $P^n(X)$ on a non-empty set X is said to be Neutrosophic SuperHyper Supra Topological Space on X if the n^{th}-power sets $0_{P^n(X)}, 1_{P^n(X)} \in \tau_n^*$ and $\underset{i \in I}{\overset{\infty}{\cup}} P^{ni}(X) \in \tau_n^*$ for $\{P^{ni}(X)\}_{i=1}^{\infty} \in \tau_n^*$. Then $(\tau_n^*, P^n(X)$) is called Neutrosophic SuperHyper Supra Topological Space on X.

3 RESULTS AND DISCUSSION

3.1 Neutrosophic SuperHyper Bi-Topological Spaces

We have published this chapter in the journal of Neutrosophic Sets and Systems (NSS), Vol. (51), Pp: 33 -45, (2022), NSS journal is indexed by Scopus repository of Quarter three, and it is issuing by University of New Mexico, USA (Khalid 2023).

This chapter contains new concepts presents for the first time linking the concept of the neutrosophic n^{th}- power sets with the traditional neutrosophic bi-topological spaces. The types of the topological spaces in neutrosophic theory are always changed depending upon the structure of the sets, in this chapter, the Neutrosophic SuperHyper Bi-Topological Spaces has been fathomed. As well as, we presented pairwise neutrosophic n^{th}- power open set, pairwise neutrosophic n^{th}- power closed set, thirteen new theorems, some new proposition, corollaries, and applied numerical examples to support the new notions. The definitions of the neutrosophic interior and the neutrosophic closure of $P^n(X)$ defined and adapted on Bi-Topological spaces have been presented.

Definition 3.1.1 Let $({\tau_1}^{1stpair}, P^n(X)), and\ ({\tau_2}^{2ndpair}, P^n(X))$ be two different Neutrosophic SuperHyper topological spaces on X. Then $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ is called Neutrosophic SuperHyper Bi-Topological space (NSHBI-TS).

Definition 3.1.2 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be a (NSHBI-TS). A collection of a neutrosophic n^{th}-power set $N = \{\langle\{x\}: T_{P^n(\{x\})}, I_{P^n(\{x\})}, F_{P^n(\{x\})}\rangle: \{x\} \subseteq P^n(X)\}$ over $P^n(X)$ is said to be a pairwise neutrosophic n^{th}-power open set in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ if there exist a neutrosophic n^{th}-power open set $N_1 = \{\langle\{x\}: T_{P^{n1}(\{x\})}, I_{P^{n1}(\{x\})}, F_{P^{n1}(\{x\})}\rangle: \{x\} \subseteq P^n(X)\}$ in ${\tau_1}^{1stpair}$ and a neutrosophic n^{th}-power open set $N_2 = \{\langle\{x\}: T_{P^{n2}(\{x\})}, I_{P^{n2}(\{x\})}, F_{P^{n2}(\{x\})}\rangle: \{x\} \subseteq P^n(X)\}$ in ${\tau_2}^{2ndpair}$ such that

$N = N_1 \cup N_2 = \{\langle\{x\},\ T_{P^n(\{x\})} = \max\{T_{P^{n1}(\{x\})}, T_{P^{n2}(\{x\})}\},$
$I_{P^n(\{x\})} = \max\{I_{P^{n1}(\{x\})}, I_{P^{n2}(\{x\})}\}, F_{P^n(\{x\})} = \min\{F_{P^{n1}(\{x\})}, F_{P^{n2}(\{x\})}\}\rangle : \{x\} \subseteq P^n(X)\}.$

Definition 3.1.3 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be a (SHNBI-TS). A collection of a neutrosophic n^{th}-power set $C = \{\langle\{x\}: T_{P^c(\{x\})}, I_{P^c(\{x\})}, F_{P^c(\{x\})}\rangle : \{x\} \subseteq P^n(X)\}$ over $P^n(X)$ is said to be a pairwise neutrosophic n^{th}-power closed set in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ if its neutrosophic complement is a pairwise neutrosophic n^{th}-power open set in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$. Clearly, a neutrosophic n^{th}-power set C over $P^n(X)$ is a pairwise neutrosophic n^{th}-power closed set in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ if there exist a neutrosophic n^{th}-power closed set $C_1 = \{\langle\{x\}: T_{P^{c1}(\{x\})}, I_{P^{c1}(\{x\})}, F_{P^{c1}(\{x\})}\rangle : \{x\} \subseteq P^n(X)\}$ in $({\tau_1}^{1stpair})^c$, and a neutrosophic n^{th}-power closed set $C_2 = \{\langle\{x\}: T_{P^{c2}(\{x\})}, I_{P^{c2}(\{x\})}, F_{P^{c2}(\{x\})}\rangle : \{x\} \subseteq P^n(X)\}$ in $({\tau_2}^{2ndpair})^c$ such that $C = C_1 \cap C_2 = \{\langle\{x\},\ T_{P^c(\{x\})} = \min\{T_{P^{c1}(\{x\})}, T_{P^{c2}(\{x\})}\},$ $I_{P^c(\{x\})} = \min\{I_{P^{c1}(\{x\})}, I_{P^{c2}(\{x\})}\},\ F_{P^c(\{x\})} = \max\{F_{P^{c1}(\{x\})}, F_{P^{c2}(\{x\})}\}\rangle : \{x\} \subseteq P^n(X)\}$, where $({\tau_i}^{ipair})^c = \{(N)^c \subseteq N(P^n(X)) : N \subseteq {\tau_i}^{ipair}, i = 1st, 2nd$. The family of all pairwise neutrosophic n^{th}-power open/closed sets in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ is denoted by PNn^{th}POS in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ / PNn^{th}PCS in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$, respectively.

Theorem 3.1.1 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space. Then,

1. $0_{P^n(x)}, and\ 1_{P^n(x)}$ are pairwise neutrosophic n^{th}-power open/closed sets.
2. An arbitrary neutrosophic union of pairwise neutrosophic n^{th}-power open sets is a pairwise neutrosophic n^{th}-power open set.
3. An arbitrary neutrosophic intersection of pairwise neutrosophic n^{th}-power closed sets is a pairwise neutrosophic n^{th}-power closed set.

Proof: 1. Let $0_{P^{n1}(x)},\ 0_{P^{n2}(x)} \subseteq {\tau_1}^{1stpair}, {\tau_2}^{2ndpair}$ respectively and $n_1 + n_2 = n$. Since

$$0_{P^{n1}(x)} \cup 0_{P^{n2}(x)} = 0_{P^{n}(x)},$$

$0_{P^n(x)}$ is a PNn^{th}POS in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$. Similarly, $1_{P^n(x)}$ is a PNn^{th}PCS in $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$.

2. Suppose

$$\{N_i = \langle \{x\}: T_{P^{ni}(\{x\})}, I_{P^{ni}(\{x\})}, F_{P^{ni}(\{x\})} \rangle : i \in I\} \subseteq \text{PN}n^{th}\text{POS in } ({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X)).$$

Then each N_i is a pairwise neutrosophic n^{th} power open set for all $\in I$, this implies that there exist $N_i^1 \in {\tau_1}^{1stpair}$and $N_i^2 \in {\tau_2}^{2ndtpair}$ such that $N_i = N_i^1 \cup N_i^2$ for all $i \in I$ which implies that

$$\bigcup_{i \in I} N_i = \bigcup_{i \in I} [N_i^1 \cup N_i^2] = \left[\bigcup_{i \in I} N_i^1\right] \cup \left[\bigcup_{i \in I} N_i^2\right].$$

Now, since ${\tau_1}^{1stpair}$ and ${\tau_2}^{2ndpair}$ are both Neutrosophic SuperHyper Topological Spaces on the neutrosophic n^{th}-power set $P^n(X)$, then $\left[\bigcup_{i \in I} N_i^1\right] \subseteq {\tau_1}^{1stpair}$, and $\left[\bigcup_{i \in I} N_i^2\right] \subseteq {\tau_2}^{2ndpair}$. Therefore, $\bigcup_{i \in I} N_i$ is a pairwise neutrosophic n^{th}-power open set.

It is immediate from the definition.

Corollary 3.1.1 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space. Then, the family of all pairwise neutrosophic n^{th}-power open sets is a Neutrosophic SuperHyper Supra Topological Space (NSHSTS) on X. This (NSHSTS) is denoted by τ_{12}^{supra}.

Theorem 3.1.2 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space. Then,

1. Every ${\tau_1}^{1stpair}, {\tau_2}^{2ndpair}$- neutrosophic n^{th}-power open set is a pairwise neutrosophic n^{th}- power open set, i.e. ${\tau_1}^{1stpair} \cup {\tau_2}^{2ndpair} \subseteq \tau_{12}^{supra}$.

2. Every ${\tau_1}^{1stpair}, {\tau_2}^{2ndpair}$- neutrosophic n^{th}- power closed set is a pairwise neutrosophic n^{th}- power closed set, i.e. $({\tau_1}^{1stpair})^c \cup \left({\tau_2}^{2ndpair}\right)^c \subseteq (\tau_{12}^{supra})^c$.

3. If ${\tau_1}^{1stpair} \subseteq {\tau_2}^{2ndpair}$, then $\tau_{12}^{supra} = {\tau_2}^{2ndpair}$ and $(\tau_{12}^{supra})^c = ({\tau_2}^{2ndpair})^c$.

Proof. Straightforward.

Definition 3.1.4 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space, and $P^n(X) \in N(P^n(X))$.The pairwise neutrosophic closure of $P^n(X)$, denoted by $cl_p^n(P^n(X))$, is the neutrosophic intersection of all pairwise neutrosophic closed supra n^{th}-power sets of $P^n(X)$, i.e., $cl_p^n\left(P^n(X)\right) = \cap \{P^m(X) \in \left(\tau_{12}^{supra}\right)^c : P^n(X) \subseteq P^m(X)\}$. It is clear that $cl_p^n\left(P^n(X)\right)$ is the smallest pairwise neutrosophic n^{th}-power closed set containing $P^n(X)$.

Theorem 3.1.3 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space, and $P^{n1}(X), P^{n2}(X) \in N(P^n(X))$, without restrictions on the relations between $n1, n2, n$. Then, the following mathematical statements are true:

1. $cl_p^n\left(0_{P^{ni}(x)}\right) = 0_{P^{ni}(x)}$ and $cl_p^n\left(1_{P^{ni}(x)}\right) = 1_{P^{ni}(x)}$, $i = 1,2$.
2. $P^n(X) \subseteq cl_p^n(P^n(X))$.
3. $P^n(X)$ is a pairwise neutrosophic n^{th}-power closed set if and only if $cl_p^n(P^n(X)) = P^n(X)$.
4. $P^{n1}(X) \subseteq P^{n2}(X) \Longrightarrow cl_p^n\left(P^{n1}(X)\right) \subseteq cl_p^n(P^{n2}(X))$.
5. $cl_p^n\left(P^{n1}(X)\right) \cup cl_p^n\left(P^{n2}(X)\right) \subseteq cl_p^n(P^{n1}(X) \cup P^{n2}(X))$.
6. $cl_p^n\ [\ cl_p^n\left(P^{ni}(X)\right] = cl_p^n(P^{ni}(X)$, i.e., $cl_p^n(P^{ni}(X)$ is a pairwise neutrosophic n^{th}-power closed set.

Proof. Straightforward.

Theorem 3.1.4 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space, and $P^{n1}(X) \in N(P^n(X))$. Then,

$$P^{n1}(x_{\alpha,\beta,\gamma}) \in cl_p^n(P^{n1}(X)) \Leftrightarrow U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \cap P^{n1}(X) \neq 0_{P^{n1}(x)},$$
$$\forall\, U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \in \tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma})),$$

where $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right)$ is any pairwise neutrosophic n^{th}-power open set contains $P^{n1}(x_{\alpha,\beta,\gamma})$, and $\tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma}))$ is the family of all pairwise neutrosophic supra n^{th}-power open set contains $P^{n1}(x_{\alpha,\beta,\gamma})$.

Proof: Let $P^{n1}(x_{\alpha,\beta,\gamma}) \in cl_p^n(P^{n1}(X))$, and suppose that there exist

$$U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \in \tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma})),$$

such that $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \cap P^{n1}(X) = 0_{P^{n1}(x)}$. Then $P^{n1}(X) \subseteq (\,U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right))^c$, thus

$$cl_p^n(P^{n1}(X)) \subseteq cl_p^n\left(U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right)\right)^c = (U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right))^c$$

which implies that $cl_p^n(P^{n1}(X)) \cap\ U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) = 0_{P^{n1}(x)}$, this is a contradiction. hence $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \cap P^{n1}(X) \neq 0_{P^{n1}(x)}$.

Conversely, assume that $P^{n1}(x_{\alpha,\beta,\gamma}) \notin cl_p^n(P^{n1}(X))$, then $P^{n1}(x_{\alpha,\beta,\gamma}) \in (cl_p^n(P^{n1}(X)))^c$. Thus, $(cl_p^n(P^{n1}(X)))^c \in \tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma}))$, therefore, by hypothesis, $(cl_p^n(P^{n1}(X)))^c \cap P^{n1}(X) \neq 0_{P^{n1}(x)}$, this is a contradiction. Hence we get $P^{n1}(x_{\alpha,\beta,\gamma}) \in cl_p^n(P^{n1}(X))$.

Theorem 3.1.5 Let $({\tau_1}^{1stpair}, {\tau_2}^{2ndpair}, P^n(X))$ be Neutrosophic SuperHyper Bi-Topological space. A neutrosophic n^{th}-power set $P^{n1}(X)$ over $P^n(X)$ is a pairwise neutrosophic n^{th}-power closed set if and only if $P^{n1}(X) = cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$.

Proof: Suppose that $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power closed set and $P^{n1}(x_{\alpha,\beta,\gamma}) \notin P^{n1}(X)$. Then $P^{n1}(x_{\alpha,\beta,\gamma}) \notin cl^{n}_{p}(P^{n1}(X))$. Thus, there exists $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \in \tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma}))$ such that $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \cap P^{n1}(X) = 0_{P^{n1}(x)}$. Again, since $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) \in \tau_{12}^{supra}(P^{n1}(x_{\alpha,\beta,\gamma}))$, then there exists $P^{m1}(X) \in \tau_1{}^{1stpair}$ and $P^{m2}(X) \in \tau_2{}^{2ndpair}$ such that $U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right) = P^{m1}(X) \cup P^{m2}(X)$. Consequently, $\left(P^{m1}(X) \cup P^{m2}(X)\right) \cap P^{n1}(X) = 0_{P^{n1}(x)}$, this implies that $P^{m1}(X) \cap P^{n1}(X) = 0_{P^{n1}(x)}$, and $P^{m2}(X) \cap P^{n1}(X) = 0_{P^{n1}(x)}$. Since $P^{n1}(x_{\alpha,\beta,\gamma}) \in U\left(P^{n1}(x_{\alpha,\beta,\gamma})\right)$, then either $P^{n1}(x_{\alpha,\beta,\gamma}) \in P^{m1}(X)$ or $P^{n1}(x_{\alpha,\beta,\gamma}) \in P^{m2}(X)$, this implies that either $P^{n1}(x_{\alpha,\beta,\gamma}) \notin cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X))$ or $P^{n1}(x_{\alpha,\beta,\gamma}) \notin cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$. Therefore, $P^{n1}(x_{\alpha,\beta,\gamma}) \notin cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$. Thus, $cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X)) \subseteq P^{n1}(X)$. On the other hand, we have $P^{n1}(X) \subseteq cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$. Hence, $P^{n1}(X) = cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$.

Conversely, suppose that $P^{n1}(X) = cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$. Since, $cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X))$ is a neutrosophic n^{th}-power closed set in $(\tau_1{}^{1stpair}, P^{n}(X))$, and $cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$ is a neutrosophic n^{th}-power closed set in $(\tau_2{}^{2ndpair}, P^{n}(X))$, so, by definition (3.4), $cl^{n}_{\tau_1{}^{1stpair}}(P^{n1}(X)) \cap cl^{n}_{\tau_2{}^{2ndpair}}(P^{n1}(X))$ is a pairwise neutrosophic n^{th}-power closed set in $(\tau_1{}^{1stpair}, \tau_2{}^{2ndpair}, P^{n}(X))$, consequently, $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power closed set.

Corollary 3.1.2 Let $(\tau_1{}^{1stpair}, \tau_2{}^{2ndpair}, P^{n}(X))$ be Neutrosophic SuperHyper Bi-Topological space. Then, $cl^{n}_{p}\left(P^{n}(X)\right) = cl^{n}_{\tau_1{}^{1stpair}}\left(P^{n1}(X)\right) \cap cl^{n}_{\tau_2{}^{2ndpair}}\left(P^{n1}(X)\right), \forall\, P^{n1}(X) \in N(P^{n}(X))$.

Definition 3.1.5 A SuperHyper operator $\psi : N(P^n(X)) \rightarrow N(P^n(X))$ is called a neutrosophic SuperHyper Supra closure operator if it satisfies the following conditions for all $p^{n1}(X), p^{n2}(X) \subseteq N(P^n(X))$.

1. $\psi(0_{P^n}(X)) = 0_{P^n}(X)$.

2. $P^{n1}(X) \subseteq \psi(P^{n1}(X)$.

3. $\psi(P^{n1}(X) \cup \psi(P^{n2}(X)) \subseteq \psi(P^{n1}(X) \cup (P^{n2}(X))$.

4. $\psi(\psi(P^{n1}(X)) = \psi(P^{n1}(X))$.

Theorem 3.1.6 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). Then, the operator $cl_p^n : N(P^n(X)) \rightarrow N(P^n(X))$ which defined by:

$$cl_p^n(P^{n1}(X)) = cl_{\tau^{1stpair}}^n(P^{n1}(X)) \cap cl_{\tau^{2ndpair}}^n(P^{n1}(X)).$$

It is the Neutrosophic SuperHyper Supra closure operator and it is induced, a unique Neutrosophic SuperHyper Supra topology is given by:

$$\{P^{n1}(X) \in N(P^n(X)) : cl_p^n((P^{n1}(X))^c) = (P^{n1}(X))^c\}.$$

Proof: Straight forward

Definition 3.1.6 Let $(\tau_1^{1st\ pair}, \tau_2^{2nd\ pair}, P^n(X))$ be a Neutrosophic SuperHyper Bi-Topological Space and $P^{n1}(X) \in N(P^{n1}(X))$. The pairwise neutrosophic interior of $P^{n1}(X)$, denoted by $int_p^n(P^{n1}(X))$ is the neutrosophic union of all pairwise neutrosophic n^{th} -power open subsets of $P^{n1}(X)$, $i.e.$,

$$int_p^n(P^{n1}(X) = U\,\{P^m(X) \in \mathcal{T}^{neutrotopo} : P^m(X) \subseteq P^{n1}(X)\}.$$

Obviously; $int_p^n(P^n(X))$ is the biggest pairwise neutrosophic n^{th}-power open set contained in $P^{n1}(X)$.

Example 3.1.1 Suppose $X = \{a, b, c\}$ with the following neutrosophic n^{th}-power sets:

$$P^{n1}(X) = \left\{ \{\mathrm{a}, \mathrm{T} = 0.3, \mathrm{I} = 0.1, F = 0.6\}, \{b, c\} \begin{matrix} T = \{0.7, 0.4\} \\ \mathrm{I} = \{0, 0.3\} \\ F = \{0.4, 0.3\} \end{matrix} \right\},$$

$$P^{n2}(X) = \left\{ \{\mathrm{a}, \mathrm{T} = 0.5, \mathrm{I} = 0.2, F = 0.7\}, \{\{b\}, \{c\}\} \begin{matrix} T = \{\{0.6\}, \{0.1\}\} \\ \mathrm{I} = \{\{0\}, \{0.5\}\} \\ F = \{\{0.4\}, \{0.4\}\} \end{matrix} \right\},$$

$$P^{n3}(X) = \left\{ \{\mathrm{a}, \mathrm{b}\} \begin{matrix} T = \{0.3, 0.2\} \\ \mathrm{I} = \{0.4, 0.9\} \\ F = \{0.6, 0.9\} \end{matrix}, \{c, 0.5, 0.1, 0.8\} \right\},$$

$$P^{n4}(X) = \left\{ \{\{\mathrm{a}\}, \{\mathrm{b}\}\} \begin{matrix} T = \{\{0.1\}, \{0.7\}\} \\ \mathrm{I} = \{\{0.2\}, \{0.8\}\} \\ F = \{\{0.6\}, \{0.9\}\} \end{matrix}, \{c, 0.2, 0.5, 0.3\} \right\},$$

$$P^{n5}(X) = \{\{\mathrm{a}, 0.1, 0.1, 0.7\}, \{b, 0.7, 0.6, 0.1\}, \{c, 0.2, 0.4, 0.3\}\},$$

$$P^{n6}(X) = \left\{ \{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \begin{matrix} T = \{0.3, 0.7, 0.4\} \\ \mathrm{I} = \{0.1, 0, 0\} \\ F = \{0.6, 0.5, 0.3\} \end{matrix} \right\}.$$

Then $(\tau_1^{1stpair}, \tau_2^{2stpair}, P^n(X))$ is a Neutrosophic SuperHyper Bi-Topological Space (NSHBI-TS), where

$$\tau_1^{1stpair} = \{0_{P^n}(X), 1_{P^n}(x), P^{n1}(X), P^{n2}(X), P^{n3}(x), P^{n4}(X)\}$$

while,

$$\tau_2^{2ndpair} = \{0_{P^n}(X), 1_{P^n}(X), P^{n5}(X), P^{n6}(X)\}.$$

Obviously,

$$\tau^{neutrotopo} = \tau_1^{1stpair} \cup \tau_2^{2ndpair} \cup \{P^{n1}(X) \cup P^{n5}(X), P^{n2}(X) \cup P^{n5}(X), P^{n3}(X) \cup P^{n5}(X)\}.$$

Because all the neutrosophic n^{th}-power sets $P^{n1}(X) \cup P^{n5}(X), P^{n2}(X) \cup P^{n5}(X), P^{n3}(X) \cup P^{n5}(X)$ are not belonging to either $\tau_1^{1stpair}$ nor to $\tau_2^{2ndpair}$.

Example 3.1.2 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be the same (NSHBI-TS) as in Example 3.1.1, and let:

$$G = \left\{ \begin{matrix} & T = \{0.6, 0.4\} & \\ \{a, b\} & I = \{0.5, 0\} & , \{c, 0.4, 0.7, 0.3\} \\ & F = \{0.7, 0.3\} & \end{matrix} \right\}$$

be a neutrosophic n^{th}-power set over $P^n(X)$. Now to find $cl_p^n(G)$, we need to determine pairwise neutrosophic n^{th}-power closed sets in $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$. We can conclude that:

$$(P^{n1}(X))^c = \left\{ \{a, T = 0.7, I = 0.9, F = 0.4\}, \{b, c\} \begin{matrix} T = \{0.3, 0.6\} \\ I = \{1, 0.7\} \\ F = \{0.6, 0.7\} \end{matrix} \right\},$$

$$(P^{n2}\,(X))^c = \left\{ \{a\,, T = 0.5\,, I = 0.8\,, F = 0.3\}, \{\{b\},\{c\}\} \begin{matrix} T = \{\{0.4\}\,,\{0.9\}\} \\ I = \{\{1\}\,,\{0.5\}\,\} \\ F = \{\{0.6\},\{0.6\}\} \end{matrix} \right\},$$

$$(P^{n3}\,(X))^c = \left\{ \{a\,,b\} \begin{matrix} T = \{0.7\,,0.8\} \\ I = \{0.6\,,0.1\,\} \\ F = \{0.4,0.1\} \end{matrix}, \{c, 0.5,0.9,0.2\} \right\},$$

$$(P^{n4}\,(X))^c = \left\{ \{\{a\},\{b\}\} \begin{matrix} T = \{\{0.9\},\{0.3\}\} \\ I = \{\{0.8\},\{0.2\}\,\} \\ F = \{\{0.4\},\{0.1\}\} \end{matrix}, \{c, 0.8,0.5,0.7\} \right\},$$

$$(P^{n5}\,(X))^c = \{\{a\,,0.9,0.9,0.3\},\{b,0.3,0.4,0.9\},\{c,0.8,0.6,0.7\}\ \},$$

$$(P^{n6}\,(X))^c = \left\{ \{a\,,b,c\} \quad \begin{matrix} T = \{0.7\,,0.3,0.6\} \\ I = \{0.9\,,1,1\,\} \\ F = \{0.4,0.5,0.3\} \end{matrix} \right\}.$$

and we can write each of the following unions $(P^{n1}\,(X) \cup P^{n5}\,(X)\,)^c$, $(P^{n2}\,(X) \cup P^{n5}\,(X)\,)^c$, $(P^{n3}\,(X) \cup P^{n5}\,(X)\,)^c$ in two different ways as follow:

1. The first solution will go to $(P^{n1}\,(X) \cup P^{n5}\,(X)\,)^c$ in two different ways: either by taking the pattern of $P^{n1}\,(X)$:

$$P^{n1}\,(X) \cup P^{n5}\,(X) = \left\{ \{a\,,0.3\,,0.1\,,0.6\},\{b,c\} \begin{matrix} T = \{0.7\,,0.4\} \\ I = \{\ 0.6\,,0.4\,\} \\ F = \{0.1,0.3\} \end{matrix} \right\},$$

$$(P^{n1}\,(X) \cup P^{n5}\,(X))^c = \left\{ \{a\,,0.7\,,0.9\,,0.4\},\ \{b,c\} \begin{matrix} T = \{0.3\,,0.6\} \\ I = \{\ 0.4\,,0.6\,\} \\ F = \{0.9,0.7\} \end{matrix} \right\},$$

or by taking the pattern of $P^{n5}\,(X)$,

$$P^{n1}\,(X) \cup\, P^{n5}\,(X) = \big\{\{\mathrm{a}\,,0.3{,}0.1{,}0.6\}, \{b, 0.7{,}0.6{,}0.1\}, \{c, 0.4{,}0.4{,}0.3\}\big\},$$

$$(P^{n1}\,(X) \cup\, P^{n5}\,(X))^{c} = \big\{\{\mathrm{a}\,,0.7{,}0.9{,}0.4\}, \{b, 0.3{,}0.4{,}0.9\}, \{c, 0.6{,}0.6{,}0.7\}\big\}.$$

Here and by the ability and the flexibility of neutrosophic n^{th} -power sets, we saw that the term $(P^{n1}\,(X) \cup P^{n5}\,(X))^{c}$ had written in to different ways.

2. The following work goes for finding the term $(P^{n2}\,(X) \cup P^{n5}\,(X))^{c}$ in two different ways: either by taking the pattern of $P^{n2}\,(X)$:

$$P^{n2}(X) \cup\, P^{n5}\,(X) = \left\{ \{\mathrm{a}\,, 0.5, 0.2\,, 0.7\}, \{\{b\},\{c\}\} \begin{array}{r} T = \{\{0.7\}\,,\{0.2\}\} \\ \mathrm{I} = \{\{0.6\}\,,\{0.5\}\,\} \\ F = \{\{0.1\},\{0.3\}\} \end{array} \right\},$$

$$(P^{n2}\,(X) \cup\, P^{n5}\,(X))^{c} = \left\{ \{\mathrm{a}\,, 0.5\,, 0.8\,, 0.3\}, \{\{b\},\{c\}\} \begin{array}{r} T = \{\{0.3\}\,,\{0.8\}\} \\ \mathrm{I} = \{\{0.4\}\,,\{0.5\}\,\} \\ F = \{\{0.9\},\{0.7\}\} \end{array} \right\},$$

or by taking the pattern of $P^{n5}\,(X)$:

$$P^{n2}(X) \cup\, P^{n5}\,(X) = \big\{\{\mathrm{a}\,,0.5{,}0.2{,}0.7\}, \{b, 0.7{,}0.6{,}0.1\}, \{c, 0.2{,}0.5{,}0.3\}\ \big\},$$

$$(P^{n2}\,(X) \cup\, P^{n5}\,(X))^{c} = \big\{\{\mathrm{a}\,,0.5{,}0.8{,}0.3\}, \{b, 0.3{,}0.4{,}0.9\}, \{c, 0.8{,}0.5{,}0.7\}\ \big\}.$$

3. The last work goes for finding the $(P^{n3}\,(X) \cup P^{n5}\,(X))^{c}$ in two different ways: either by taking the pattern of $P^{n3}\,(X)$:

$$P^{n3}(X) \cup P^{n5}(X) = \left\{ \begin{matrix} T = \{0.3\,, 0.7\} \\ \{\mathrm{a}\,,\mathrm{b}\}\ \mathrm{I} = \{0.4\,, 0.9\,\}, \{c, 0.5{,}0.4{,}0.3\} \\ F = \{0.6{,}0.1\} \end{matrix} \right\},$$

$$(P^{n3}(X) \cup P^{n5}(X))^c = \left\{ \begin{matrix} T = \{0.7\,, 0.3\} \\ \{\mathrm{a}\,,\mathrm{b}\}\ \mathrm{I} = \{0.6\,, 0.1\,\}, \{c, 0.5{,}0.6{,}0.7\} \\ F = \{0.4{,}0.9\} \end{matrix} \right\}.$$

or by taking the pattern of $P^{n5}(X)$:

$$P^{n3}(X) \cup P^{n5}(X) = \left\{ \{\mathrm{a}\,, 0.3{,}0.4{,}0.6\}, \{b, 0.7{,}0.9{,}0.1\}, \{c, 0.5{,}0.4{,}0.3\} \right\},$$

$$(P^{n3}(X) \cup P^{n5}(X))^c = \left\{ \{\mathrm{a}\,, 0.7{,}0.6{,}0.4\}, \{b, 0.3{,}0.1{,}0.9\}, \{c, 0.5{,}0.6{,}0.7\} \right\}.$$

By the definition of the inclusion that stated in Chapter 2, the first point of it, we see the pairwise neutrosophic n^{th}-power sets which contain G are:

$$(P^{n3}(X))^c \text{ and } 1_{p^n(X)} = \left\{ \begin{matrix} T = \{1, 1\} \\ \{\mathrm{a}\,,\mathrm{b}\}\ \mathrm{I} = \{1, 1\,\}, \{c, 1{,}1{,}0\} \\ F = \{0{,}0\} \end{matrix} \right\}.$$

It follows that: $cl_p^n(G) = (P^{n3}(X))^c \cap 1_{p^n(X)} = (P^{n3}(X))^c$. Therefore $cl_p^n(G) = (P^{n3}(X))^c$. It is worthy to mention that all neutrosophic n^{th}-power closed sets

$$\begin{bmatrix} \left(P^{n1}(X)\right)^c, \quad \left(P^{n2}(X)\right)^c, \quad \left(P^{n4}(X)\right)^c, \quad \left(P^{n5}(X)\right)^c, \left(P^{n3}(X) \cup P^{n5}(X)\right)^c, \\ (P^{n2}(X) \cup P^{n5}(X))^c, (P^{n1}(X) \cup P^{n5}(X))^c \end{bmatrix},$$

all are do not contain G.

Example 3.1.3 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be the same NSHBI-TS as in Example 3.1.1, and let:

$$M = \left\{ \{a\,, 0.4\,, 0.5\,, 0.2\}, \{b, c\} \begin{array}{r} T = \{0.8\,, 0.8\} \\ \mathrm{I} = \{0.7\,, 0.5\,\} \\ F = \{0{,}0.2\} \end{array} \right\}.$$

Again, from the definition of inclusion (i.e. Def. 2.3.4, the first point of it), we see the pairwise neutrosophic n^{th}-power sets which containing in M are:

$$0_{P^n(X)}, P^{n\,1}(X), and\ P^{n\,1}(X) \cup P^{n\,5}(X)$$

of the first pattern tracking the trace of the pattern of $P^{n\,1}(X)$, i.e.

$$P^{n\,1}(X) \cup P^{n\,5}(X) = \left\{ \{a\,, 0.3\,, 0.1\,, 0.6\}, \{b, c\}, \begin{array}{r} T = \{0.7\,, 0.4\} \\ \mathrm{I} = \{0.6\,, 0.4\,\} \\ F = \{0.1{,}0.3\} \end{array} \right\}.$$

It is clear that $P^{n\,1}(X) \subseteq M, \left(P^{n\,1}(X) \cup P^{n\,5}(X)\right) \subseteq M, 0_{P^n(X)} \subseteq M$. Therefore,

$$int_P^n(M) = \left(P^{n\,1}(X) \cup P^{n\,5}(X)\right) \cup P^{n\,1}(X) \cup 0_{P^n(X)} = P^{n\,1}(X) \cup P^{n\,5}(X).$$

Theorem 3.1.6 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS), and $P^{n\,1}(X), P^{n\,2}(X) \in N(P^n(X))$. Then the following mathematical statements are true:

1. $int_P^n\left(0_{P^n(X)}\right) = 0_{P^n(X)}$ and $int_P^n\left(1_{P^n(X)}\right) = 1_{P^n(X)}$,
2. $int_P^n\left(P^{n1}(X)\right) \subseteq P^{n1}(X)$,
3. $P^{n\,1}(X)$ is a pairwise neutrosophic n^{th}-power open set if and only if $int_P^n\left(P^{n1}(X)\right) = P^{n1}(X)$,
4. $P^{n1}(X) \subseteq P^{n2}(X) \Longrightarrow int_P^n\left(P^{n1}(X)\right) \subseteq int_P^n\left(P^{n2}(X)\right)$,

5. $int_P^n\left(P^{n1}(X) \cap P^{n2}(X)\right) \subseteq int_P^n\left(P^{n1}(X)\right) \cap int_P^n\left(P^{n2}(X)\right)$,

6. $int_P^n[int_P^n(P^{n1}(X))] = int_P^n(P^{n1}(X))$.

Proof: The poofs of the six facts mentioned in this theorem are straight forwards.

Theorem 3.1.7 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS), and $P^{n\,1}(X) \in N(P^n(X))$. Then

$$P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_P^n\left(P^{n1}(X)\right) \Leftrightarrow \exists P^n(z_{\alpha,\beta,\gamma}) \in \tau^{neutrotopo}(P^n\left(x_{\alpha,\beta,\gamma}\right))$$

such that $P^n(z_{\alpha,\beta,\gamma}) \subseteq P^{n1}(X)$.

Proof: Straight forward.

Theorem 3.1.7 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). A neutrosophic n^{th}-power set $P^{n1}(X)$ over $P^n(X)$ is a pairwise neutrosophic n^{th}-power open set if and only if $P^{n1}(X) = int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$.

Proof: Let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power open. Since $int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$ and $int_{\tau_2^{2ndpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$ (by th. 3.19/ second point of it), then $int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$.

Now let $P^n\left(x_{\alpha,\beta,\gamma}\right) \in P^{n1}(X)$. Then, either there exists $P^n\left(z_{\alpha,\beta,\gamma}^1\right) \in \tau_1^{1stpair}$ such that $P^n\left(z_{\alpha,\beta,\gamma}^1\right) \subseteq P^{n1}(X)$ or there exists $P^n\left(z_{\alpha,\beta,\gamma}^2\right) \in \tau_2^{2ndpair}$ such that $P^n\left(z_{\alpha,\beta,\gamma}^2\right) \subseteq P^{n1}(X)$, thus $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_1^{1stpair}}^n P^{n1}(X)$ or $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_2^{2ndpair}}^n P^{n1}(X)$, Hence, $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X)) \Rightarrow P^{n1}(X) \subseteq int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$, and so

$$P^{n1}(X) = int^{n}_{\tau_1^{1stpair}}(P^{n1}(X)) \cup int^{n}_{\tau_2^{2ndpair}}(P^{n1}(X)).$$

Conversely, since $int^{n}_{\tau_1^{1stpair}}(P^{n1}(X))$ is a neutrosophic n^{th}-power open set in $(\tau_1^{1stpair}, P^{n}\,(X))$ and $int^{n}_{\tau_2^{2ndpair}}(P^{n1}(X))$ is a neutrosophic n^{th}-power open set in $(\tau_2^{2ndpair}, P^{n}\,(X))$, then by definition (3.15) we can conclude that $int^{n}_{\tau_1^{1stpair}}(P^{n1}(X)) \cup int^{n}_{\tau_2^{2ndpair}}(P^{n1}(X))$ is a pairwise neutrosophic n^{th}-power open set in $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^{n}\,(X))$. Thus, $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power open set.

Corollary 3.1.3 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^{n}\,(X))$ be (NSHBI-TS). Then,

$$int^{n}_{p}P^{n1}(X) = int^{n}_{\tau_1^{1stpair}}(P^{n1}(X)) \cup int^{n}_{\tau_2^{2ndpair}}(P^{n2}(X)).$$

Definition 3.1.7 An operator $\phi\colon N(P^{n}(X)) \to N(P^{n}(X))$ is called a Neutrosophic SuperHyper Supra Operator (NSHSO) if it satisfies the following conditions: all $P^{n1}(X), P^{n2}(X) \in N(P^{n}(X)$.

1. $\phi(0_X) = 0_X$.
2. $\phi(P^{n1}(X)) = P^{n1}(X)$.
3. $\phi(P^{n1}(X)) \cap (P^{n2}(X)) \subseteq \phi(P^{n1}(X)) \cap \phi(P^{n2}(X))$.
4. $\phi\left(\phi(P^{n1}(X))\right) = \phi(P^{n1}(X))$.

Theorem 3.1.8 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^{n}\,(X))$ be (NSHBI-TS). Then, the operator

$$int^{n}_{P} : N(P^{n}(X)) \to N(P^{n}(X))$$

which is defined by:

$$int_P^n\left(P^{n1}(X)\right) = int_{\tau_1^{1stpair}}^n\left(P^{n1}(X)\right) \cup int_{\tau_2^{2ndpair}}^n\left(P^{n1}(X)\right),$$

is Neutrosophic SuperHyper Supra Interior Operator (NSHSIO), and it is induced, a unique neutrosophic SuperHyper supra topology given by

$$\{P^{n1}(X) \in N\left(P^n(X)\right): int_P^n\left(P^{n1}(X)\right) = P^{n1}(X)\},$$

which is precisely $\tau^{neutrotopo}$.

Proof: Straight forward.

Theorem 3.1.9 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS) and $P^{n1}(X) \in N\left(P^n(X)\right)$. Then, $int_P^n\left(P^{n1}(X)\right) = \left(cl_P^n\left(P^n(X)\right)^c\right)^c$ and $cl_P^n\left(P^{n1}(X)\right) = (int_P^n(P^{n1}(X))^c)^c$.

Proof: Straight forward.

Definition 3.1.8 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS), $P^{n1}(X) \in N\left(P^n(X)\right)$, and $P^n(X_{(\alpha,\beta,\gamma)}) \in N\left(P^n(X)\right)$. Then $P^{n1}(X)$ is said to be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$, if there exists a pairwise neutrosophic n^{th}-power open set $P^{n2}(X)$ such that $P^n\left(x_{\alpha,\beta,\gamma}\right) \in P^{n2}(X) \subseteq P^{n1}(X)$. The family of pairwise neutrosophic n^{th}-power neighborhood of neutrosophic n^{th}-power point $P^n\left(x_{\alpha,\beta,\gamma}\right)$ denoted by $P_{\tau^{neutrotopo}}^{n1}(x_{\alpha,\beta,\gamma})$.

Theorem 3.1.10 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS), and let $P^{n1}(X) \in N\left(P^n(X)\right)$. Then $P^{n1}(X)$is a pairwise neutrosophic n^{th}-power open set if and only if $P^{n1}(X)$is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic n^{th}-power points.

Proof: Let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power open set, and let $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$. Then $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X) \subseteq P^{n1}(X)$, therefore $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$ for each $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$.

Conversely, suppose that $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic n^{th}-power points, and $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$. Then there exist a pairwise neutrosophic n^{th}-power open set $P^{n2}(X)$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n2}(X) \subseteq P^{n1}(X)$. Since

$$P^{n1}(X) = \bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} \{P^n(x_{\alpha,\beta,\gamma})\} \subseteq$$
$$\bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} P^{n2}(X) \bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} P^{n1}(X) = P^{n1}(X).$$

It follows that $P^{n1}(X)$is a union of all pairwise neutrosophic n^{th}-power open sets. Hence, $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power open set.

Proposition 3.1.1 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS), and

$$\{P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma}) : P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))\}$$

be a system of pairwise neutrosophic n^{th}-power neighborhoods. Then,

1. For every $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma}), P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$,
2. $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$ and $P^{n1}(X) \subseteq P^{n3}(X) \Longrightarrow P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$,
3. $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma}) \Longrightarrow \exists P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$ such that $P^{n3}(X) \subseteq P^{n1}(X)$ and $P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(y_{\alpha',\beta',\gamma'})$, for every $P^n(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

The proofs of the first two mathematical statements are straight forwards. For the third statement, let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$, then there exists a pairwise neutrosophic n^{th}-power open set $P^{n3}(X) \in$

$\tau^{neutrotopo}$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n1}(X)$. Since $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n3}(X)$ can be written, then $P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$. From theorem 3.27, if $P^{n3}(X)$ is a pairwise neutrosophic n^{th}-power open set, then $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic points, i.e., $P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(y_{\alpha',\beta',\gamma'})$, for every $P^n(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

Corollary 3.1.4 Let $P^{n1}(X), P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma}) \Rightarrow P^{n1}(X) \cap P^{n3}(X) \notin P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$.

Explanation for the above corollary. Actually, if $P^{n1}(X), P^{n3}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$, there exist $P^{n4}(X), P^{n5}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$ such that

$$P^n(x_{\alpha,\beta,\gamma}) \in P^{n4}(X) \subseteq P^{n1}(X) \text{ and } P^n(x_{\alpha,\beta,\gamma}) \in P^{n5}(X) \subseteq P^{n3}(X).$$

But $P^{n4}(X) \cap P^{n5}(X)$ does not need to be a pairwise neutrosophic n^{th}-power open set. Therefore, $P^{n1}(X) \cap P^{n3}(X)$ does not need to be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$.

Theorem 3.1.11 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). Then $P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma}) = P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$ for each $P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))$.

Proof: Let $P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))$ be any neutrosophic n^{th}-power point and $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$. Then there exists a pairwise neutrosophic n^{th}-power open set , $P^{n3}(X) \in \tau^{neutrotopo}$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n1}(X)$. If $P^{n3}(X) \in \tau^{neutrotopo}$, there exist $P^{n31}(X) \in \tau^{1stpair}$ and $P^{n32}(X) \in \tau^{2ndpair}$ such that , $P^{n3}(X) = P^{n31}(X) \cup P^{n32}(X)$. Since $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) = P^{n31}(X) \cup P^{n32}(X)$, then $P^n(x_{\alpha,\beta,\gamma}) \in P^{n31}(X)$ or $P^n(x_{\alpha,\beta,\gamma}) \in P^{n32}(X)$. So, $P^n(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \subseteq P^{n3}(X) \subseteq P^{n1}(X)$ or $P^n(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \subseteq P^{n3}(X) \subseteq P^{n1}(X)$. In this case,

$P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma})$ or $P^{n1}(X) \in P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$, that is, $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$.

Conversely, suppose that $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$. Then $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma})$ or $P^{n1}(X) \in P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$. Hence, there exists $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \in \tau^{1stpair}$ or $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \in \tau^{2ndpair}$ such that $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \subseteq P^{n1}(X)$ and $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \subseteq P^{n1}(X)$. As a result, $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \cup P^{n32}(X) = P^{n3}(X) \subseteq P^{n1}(X)$. Such that $P^{n3}(X) \in \tau^{neutrotopo}$, i.e., $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$.

Definition 3.1.9 An operator $\#: N(P^{n}(X)) \rightarrow N(P^{n}(X))$ is called a neutrosophic Supra n^{th}-power neighborhood operator if it satisfies the following conditions:

$$\forall\, P^{n1}(X), P^{n3}(X) \in N(P^{n}(X))$$

1. $\forall P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma})), P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$,
2. $P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma}))$ and $P^{n1}(X) \subseteq P^{n3}(X) \Longrightarrow P^{n3}(X) \in \#P^{n}(x_{\alpha,\beta,\gamma})$
3. $P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma})) \Longrightarrow \exists\, P^{n3}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma}))$, such that $P^{n1}(X) \subseteq P^{n3}(X)$ and $P^{n1}(X) \in \#\left(P^{n}(y_{\alpha',\beta',\gamma'})\right), P^{n}(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

Theorem 3.1.12 Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^{n}(X))$ be neutrosophic SuperHyper Bi-Topological Space. Then, the operator $P^{n1}_{\tau^{neutrotopo}}: N(P^{n}(X)) \rightarrow N(P^{n}(X))$ which is defined by $P^{n1}_{\tau^{neutrotopo}}(P^{n}(x_{\alpha,\beta,\gamma})) = P^{n1}_{\tau^{1stpair}}(P^{n}(x_{\alpha,\beta,\gamma})) \cup P^{n1}_{\tau^{2ndpair}}\left(P^{n}(x_{\alpha,\beta,\gamma})\right)$, is neutrosophic Supra n^{th}-power neighborhood operator and it is induced, a unique neutrosophic Supra topology given by $\{P^{n1}(X) \in N(P^{n}(X)): \forall P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)\ for\ P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})\}$ which is precisely $\tau^{neutrotopo}$.

4 CONCLUSIONS AND RECOMMENDATION

It is obvious from the previous three chapters that new types and original topological spaces (i.e. Neutrosophic SuperHyper Topological Spaces $\tau^{neutrotopo}$, and Neutrosophic SuperHyper Bi-Topological Spaces ($\tau_1^{1stpair}, \tau_2^{2ndpair}$)) have been defined depending upon n^{th}-power set $P^n(X)$, indeed these new types of sets took its importance from its existence in the neutrosophic environment. The n^{th}-power sets $P^n(X)$ that defined by the neutrosophic notion adheres to define each element in the set by three membership functions (truth, indeterminate, and falsity) membership functions gave us new insights to define new kinds of topological spaces depending on modern structure these sets.

The original type of mathematical structure that represented by the neutrosophic superhyper topological spaces had the ability to define the neutrosophic universal n^{th}-power set $1_{P^n(X)}$, the neutrosophic empty n^{th}-power set $0_{P^n(X)}$, unmatched mathematical relations (i.e. less than or equal relation, greater than or equal relation, the inclusion relation, the union relation, the intersection relation, and the belonging relation,). However, new propositions have been defined, and we redefined the De Morgan's theorem, setting up unprecedented of neutrosophic n^{th}-power point. Also, we introduced Neutrosophic SuperHyper Supra Topological Spaces, the definition of the neutrosophic n^{th}-power open sets, the definition of the neutrosophic n^{th}-power closed sets and dozens of theorems and corollaries. This study include a novel definitions for the neutrosophic closure and the neutrosophic interior of $P^n(X)$. Furthermore, the definitions of the pairwise neutrosophic closure and the definition of the pairwise neutrosophic interior of $P^n(X)$ have been introduced, the flexibility of the neutrosophic superhyper topological spaces depending upon the neutrosophic n^{th}-power sets enables us to build some new and important numerical examples which had two techniques in representing the same mathematical operation and two different results gained (i.e. example 3.17 and example 3.18). Finally, the pairwise neutrosophic n^{th}-power neighborhood had been defined.

There are unlimited and broad notions and mathematical aspects to work in the neutrosophic SuperHyper Topological Spaces such as but not limited to:

1. Irreducible sets in view of the neutrosophic n^{th}-power sets $P^n(X)$.
2. Convergent sequences in neutrosophic superhyper topological Spaces.
3. Continuity and compactness notions in the neutrosophic superhyper Topological Spaces.
4. Hausdorff distance in the neutrosophic superhyper topological Spaces.
5. Kuratowski convergence in the neutrosophic superhyper topological Spaces.
6. Wijsman convergence in the neutrosophic superhyper topological Spaces.

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