



**MODELLING OF RAILWAY VEHICLE
SUSPENSION SYSTEM AND VIBRATION
CONTROL**

DISSERTATION

Aslı SOYİÇ LEBLEBİCİ

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**MODELLING OF RAILWAY VEHICLE SUSPENSION SYSTEM AND
VIBRATION CONTROL**

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Dissertation

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ABSTRACT

Modelling of Railway Vehicle Suspension System and Vibration Control

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The main objective of this thesis is to model and control the suspension systems of railway vehicles. Since track irregularities are the main vibration source for railway vehicles, the railway track structure and track irregularities were examined in detail in the thesis. The Hunt linear track model was adapted to the vehicle model and simulation studies were carried out. In addition, for the measurement data taken from the Konya-Polatlı high-speed main line, an analytical model with the characteristics of these data was created using statistical methods. To investigate the railway vehicle performances, half-car and full-car models of the vehicle were obtained. Semi-active and active controllers were designed that can increase passenger comfort and safety. The *skyhook* and *groundhook* variable value dampers are used for semi-active controllers. Gradient-based method and linear matrix inequalities were used to solve the problem that finding optimal values of these dampers. As a solution method to active controller design problem, $\mathcal{H}_2$, $\mathcal{H}_\infty$ with $\mathcal{H}_2/\mathcal{H}_\infty$ modern control methods and the LQG method have been used. Polytopic vehicle suspension models were obtained for the uncertainty of the vehicle body mass and the Hertzian spring stiffness, whose value could not be determined clearly. Active controllers were synthesized for these polytopic models. We synthesized low-order controllers by using *MATLAB HIFOO* toolbox. All theoretical results were compared regarding bode diagrams and the RMS values of vehicle response. The vehicle model has been updated to include flexible and rigid modes. With the active controller design, rigid and flexible modes have been suppressed.

Keywords: Active suspension; Wheel-rail interaction; Multi-objective control; Railway vehicle linear modelling; Railway track irregularities.

ÖZET

Demiryolu Taşıt Süspansiyon Sisteminin Modellenmesi ve Titreşim Kontrolü

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Bu tezde, demiryolu taşıtlarının süspansiyon sistemlerinin modellenmesi ve kontrolü amaçlanmıştır. Trenlerde titreşimin ana kaynağının yoldaki bozulmalar olduğu bilinmekte olup, tezde öncelikli olarak demiryolu yol yapısı ve yol bozuklukları incelenmiştir. Hunt doğrusal yol modeli araç modeline adapte edilmiş ve benzetim çalışmaları yapılmıştır. Ayrıca Konya-Polatlı hızlı tren hattından alınan ölçüm verileri için istatistiksel yöntemler kullanılarak bu verilerin karakteristiklerini taşıyan analitik bir model oluşturulmuştur. Yol bozukluklarının araç üzerindeki etkilerini araştırmak için aracın yarım-arac ve tam-arac modelleri elde edilmiştir. Aracın pasif araç yanıtlarında gözlemlenen zıt ilişkileri çözebilecek ve yolcu konforunu ve güvenliğini artırabilecek yarı-aktif ve aktif kontrolcüler tasarlanmıştır. Yarı-aktif kontrolcü olarak *skyhook* ve *groundhook* değişken değerli sönümleyiciler kullanılmıştır. Bu sönümleyicilerin en iyi değerlerini bulabilmek için lokal minimum bulma metodu ile doğrusal matris eşitsizlikleri kullanılmıştır. Araç yanıtlarının optimizasyonu için aktif kontrolcü tasarım problemine çözüm metodu olarak LQG metodu, $\mathcal{H}_2$, $\mathcal{H}_\infty$ ile $\mathcal{H}_2/\mathcal{H}_\infty$ modern kontrol yöntemleri uygulanmıştır. Yol-arac modelinde değeri net belirlenemeyen tekerlek-ray arası Hertz yay sabiti ile araç gövdesi kütlelerinin belirsizliği için aracın politopik modelleri elde edilmiş ve bu modeller için aktif kontrolcüler sentezlenmiştir. Aktif kontrolcü boyutlarının büyük olması maliyetleri artıracığı için düşük boyutlarda kontrolcü tasarımları *MATLAB HIFOO* araç kutusu kullanılarak gerçekleştirilmiştir. Tüm teorik sonuçlar bode diyagramları ve araç yanıtları RMS değerleri açısından karşılaştırılmıştır. Gelecekte yapısal olarak hafifletilmiş araçların tasarımları planlanmakta olup, hafifletilmiş araçlarda esnek modların etkileneceği bilindiğinden araç modeli esnek ve rijit modları kapsayacak şekilde güncellenmiştir. Aktif kontrolcü tasarımı ile hem rijit modları hem de esnek modlar baskılanmaya çalışılmıştır.

Anahtar Kelimeler: Aktif süspansiyon; Tekerlek-yol etkileşimi; Çok amaçlı kontrol; Demiryolu aracının doğrusal modellemesi; Demiryolu yol düzensizlikleri.

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I would like to express my eternal gratitude to my family, especially to my parents for their everlasting love and support in me.

This thesis is dedicated to my sister Arzu SOYİ, who taught me to read when we were children.

Aslı SOYİ LEBLEBİCİ

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with “scientific plagiarism detection program” used by Eskişehir Technical University, and that “it does not have any plagiarism” whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.

Aslı SOYİÇ LEBLEBİCİ

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GLOSSARY

z_s	: Vertical displacement of the car body at the center of gravity
θ_s	: Car body pitch angle
φ_s	: Car body roll angle
z_{b1}	: Vertical displacement of the front bogie
z_{b2}	: Vertical displacement of the rear bogie
θ_{b1}	: Front bogie pitch angle
θ_{b2}	: Rear bogie pitch angle
φ_{b1}	: Front bogie roll angle
φ_{b2}	: Rear bogie roll angle
z_{wij}	: ij^{th} wheel vertical displacement, for $i, j = 1, 2$.
φ_{wij}	: ij^{th} wheelset roll angle, for $i, j = 1, 2$.
v	: Vehicle velocity
A_v, A_r	: Track roughness coefficient
A^T	: Transpose of A
j	: $\sqrt{-1}$
$X \otimes Y$	: Kronocker product of X and Y
I_n	: $n \times n$ identity matrix
$\mathbf{0}$	: $m \times n$ null matrix
$Q^\dagger$	: pseudo inverse of Q
$\mathcal{H}\{M\}$	: $M + M^T$
$\text{sign}\{.\}$	: signum function
$H\{.\}$	: Heaviside step function

1. INTRODUCTION

The history of rail transportation began in the 1830s with the introduction of freight transportation on the Liverpool-Manchester line, and it has continued to operate ever since [5]. Trains for passenger and freight transport are known as safer, cheaper, more comfortable, and more environmentally friendly vehicles than air transport and road transport. Thus the demand for high-speed transportation increases and competition with other transportation systems arises, it becomes imperative to maintain these advantages [6, 7].

A typical rail vehicle comprises body and suspension components, as defined by Iwnicki [8, 9]. The main body components include the car body, bogie frames, and wheelsets, while the suspension components consist of springs and dampers. The suspension elements can be classified into two distinct systems: the secondary and primary suspensions. The secondary suspensions between the car body and bogie frames primarily contribute to passenger comfort [10, 11]. On the other hand, the primary suspension systems connect the wheelsets to the bogie frames and are responsible for ensuring safety. Research suggests that railway suspension systems should fulfill the following requirements:

- (i) Support the weight (change) of the vehicle.
- (ii) Guide the vehicle on the track.
- (iii) Isolate the car body and bogies from vibrations.

In the era of mechatronic design, computer simulations can help identify and prevent many issues before the production of a vehicle, leading to cost reductions [12, 8]. However, developing an accurate vehicle model is crucial to these simulations. References [8, 9, 13, 14] have highlighted that railway vehicles possess complex structures with six degrees of freedom: longitudinal, lateral, and vertical motions, as well as rotational motions (roll, pitch, and yaw). Although higher-order systems provide the most accurate representation of vehicle dynamics, lateral and vertical motions exhibit weak coupling. Therefore, utilizing selected degrees of freedom for specific modeling is possible. For instance, in ride comfort analysis, according to the international standard “*ISO:2631-*

2:Mechanical vibration and shock evaluation of human exposure to whole-body vibration Part-2” humans are most sensitive to vertical vibrations in the low-frequency range ($\leq 50 \text{ Hz}$) [15, 16]. Consequently, side and full-view models can assess the rail vehicle bodies’ bounce, heave, and roll acceleration. The human body’s susceptibility to acceleration depends on the frequency and magnitude influenced by track irregularities resulting from manufacturing defects and wear on the track layers. For safety, the global standard “*EN 13848-1:2019 Railway applications. Track. Track geometry quality Characterization of track geometry*” in [17], suggests measuring track irregularities at specific intervals using track-recording-car (TRC). However, these measurement results cannot be directly used in vehicle-track, bogie-track, or wheelset-track models in the frequency domain. A detailed mathematical model of the track must be generated to achieve accurate results in computer simulations [9, 18, 19, 20].

In the literature, random track irregularity is typically assumed to follow a stationary, ergodic Gaussian process characterized by a single power spectral density (PSD) function [21, 22]. The PSD function describes the frequency content of the stochastic track irregularity data [9, 2]. In a report by the U.S.A. Federal Railroad Administration in [2], track geometry measurements from 30 zones in the United States were collected to provide an analytical characterization of track geometry. This report employed a statistical approach to characterize track geometry and explain the relationships between track geometry parameters and the impact of curvature, gage, alignment, and surface variations. Researchers can utilize the state space approach to incorporate these track models as disturbance inputs in vehicle dynamics, as summarized in [23, 24]. In addition to statistical modeling of track irregularities for measured data, the finite element method (FEM) is commonly used to model the track [4, 25, 26]. The FEM model enables strain/stress and fatigue analysis of the track. However, this model is unsuitable for linear control theory due to its higher degrees of freedom. Therefore, researchers require a control-oriented equivalent model.

During operation, the load transferred between the track and wheels varies spatially and with speed. The resulting dynamic forces between the wheel and rail couple the track and car body, generating significant vibrations felt on the car body and under the rail

surface. In offline dynamic analysis, the vehicle and track dynamics are typically assumed to be decoupled and considered separately [9, 27]. However, due to the dynamic forces between the wheel and rail, it is unrealistic to completely separate these two subsystems from each other [27, 28, 29]. A lumped-parameter model, employing a mass-spring-damper system, was developed in [29, 30] to summarize the behavior of track degradation. In [26], this lumped model was utilized for railway vehicle inputs, and an active controller design was implemented using a Linear-Quadratic-Gaussian (LQG) regulator. While the results are satisfactory, this model was derived specifically for track degradations caused by harmonic loads. Therefore, the model needs to be further developed depending on the simulation objectives.

These track irregularities cause body vibrations on the vehicle body which can negatively impact passenger comfort. Therefore, improving ride comfort has become a significant focus in the design and construction of vehicles to enhance passenger acceptance of rail transportation [31]. This comfort problem can be achieved by attenuating vehicle body vibrations through passive, semi-active, and active suspension systems. Passive suspension systems typically consist of springs and pneumatic or oil dampers, offering simplicity and cost-effectiveness. Optimization of passive elements can provide a solution to enhance ride comfort [14, 32]. However, passive suspensions have limitations, such as the inability to utilize external power sources, thus limiting their functionality to energy storage or dissipation. Furthermore, due to conflicting performance objectives of the vehicle, it is challenging to simultaneously improve all system outputs even with finely optimized passive suspensions [6, 32, 33, 34].

Semi-active suspension systems, positioned between passive and active systems in terms of performance, offer cost-effectiveness and consume significantly less power [13, 35, 36]. These systems employ dampers with adjustable values and suitable control algorithms. Studies have shown that these dampers can improve vehicle suspension specifications [37]. Among the semi-active control methods, skyhook dampers are commonly preferred [38]. Semi-actively or actively controlled skyhook damping (also referred to as groundhook damping) methods have been extensively studied in railways over the past decades [31, 39, 40, 41, 42, 43, 44, 45, 46] and the references therein. In practical appli-

cations, a different configuration of skyhook-modal control involves connecting an electromagnetic actuator between the center of the car body and an auxiliary mass, along with hydraulic actuators in parallel with secondary suspension elements. This approach has encouraged the study of skyhook damping control due to its ability to achieve approximately 60% improvements in car body vertical accelerations with less power consumption. However, studies have shown that while skyhook damping can yield approximately 50% better comfort performance on straight tracks, it may increase secondary suspension deflections when the vehicle encounters deterministic track inputs [40]. In [40], linear filters were designed to optimize the trade-off between random and deterministic track inputs. Tuning the damper value in this context is a crucial challenge [33]. Therefore, this thesis was employ two approaches to determine optimal semi-active damper values. In [36, 47], the gradient method was utilized to identify optimum skyhook and groundhook damper values, supported by vehicle performance analysis in the time domain. Additionally, in [48], the semi-active problem was addressed through LMIs. It should be noted that a disadvantage of skyhook/groundhook damping is that the force generated by a semi-active damper depends on the speed of the damper's movement. Consequently, at low speeds, the production of large forces is restricted [8, 35]. Due to the inherent contradictions that are challenging to overcome with passive or semi-active systems, active control offers an alternative solution.

In the study [49], it was claimed that active systems perform well, especially in more complex problems. The first study on active system approaches in ground transportation was presented by [50], stating that active suspension design can address design problems that were previously unsolvable. An actively controlled system comprises subsystems such as actuators, transducers or sensors, and a controller to implement the chosen control algorithm [10]. Consequently, active systems require complex instrumentation and external power sources. Comprehensive literature surveys on active suspension design in railway vehicles are presented in [6, 51], containing a wide literature survey about active suspension design in railway. The studies concluded that fully active systems have not yet been extensively used in operational railways due to their high costs relative to the benefits gained. Vehicle manufacturers have also identified disadvantages such as reliability issues, high maintenance costs, and complexity as hindrances to implementing

active systems [10]. However, these challenges continue to fuel interest in the optimal design of active suspensions. Numerous studies have been conducted in the railway domain [11, 52, 53, 54, 55, 56, 57, 58, 59, 60, 13, 61, 62, 7, 33, 63, 64] and can be classified into three main categories:

- (i) Active secondary suspension (for ride comfort)
- (ii) Active primary suspension (for improvement on running safety and curving performance)
- (iii) Tilting train (ride comfort on curves)

Designing active secondary suspensions aims to improve passenger comfort and keep suspension deflections within safety limits. Over the past few decades, studies in the field of railway systems have focused on utilizing classical and modern control techniques to design active suspension systems. In [56], an adaptive PID controller is designed for Manchester benchmark vehicles to enhance passenger comfort under different track conditions. Simulation results demonstrate the high performance achieved by the passive system responses. The "track-vehicle-human" model formulation was presented in [7, 26] where an active suspension based on a Linear-Quadratic-Gaussian (LQG) control method was designed. Simulations were conducted using the side-view model of a typical high-speed passenger wagon. However, due to the absence of roll motion in the side view, the railway vehicle is modeled as a full-car seventeen-degree-of-freedom (17 DOF) system [63]. An LQG controller design was discussed to observe roll motion effects, and simulations were performed using Manchester benchmark vehicle parameters [65]. It is worth noting that there are constraints on the achievable performances of railway vehicles, and it may be more realistic to define the problem as a multi-objective control problem. Linear matrix inequalities have emerged as a powerful design technique for multi-objective control problems [66]. In [59], an H_∞ active and skyhook semi-active controller is designed to reduce lateral vibration effects on comfort. Simulation results show active and semi-active controlled vehicles achieve the desired performance on straight tracks and large-radius curves. There are no significant internal differences between the control approaches regarding ride comfort evaluation. Reducing vibration effects on vehicles can

also be achieved by reducing sprung or unsprung masses. Lighter experimental analysis of the influence of the passengers on flexurals and reduced-weighted railway vehicle bodies have been proposed [24, 67, 68]. It is well known that smaller masses vibrate more. In [67], the effect of car body mass on vehicle performance is thoroughly studied. Results with active systems for mitigating mass indicate that a lightweight body significantly minimizes the force effects on the track but may compromise passenger comfort. Moreover, lighter body masses in railway vehicles increase the vehicle's flexibility, which becomes evident in ride comfort since bending vibrations due to the car body's structural flexibility occur at lower natural frequencies, where humans are more sensitive. As a result, studies on railway vehicle comfort have shifted towards flexible-bodied vehicle modeling and control [69, 70]. Reference [71] investigates the influence of flexible modes on passenger ride comfort in detail.

Since the future railway vehicle design is planned with lightweight structural bodies, flexible body modeling has gained prominence in vehicle dynamic analysis and control. In [72] incorporates active control methodology for both the flexible and rigid modes of the vehicle. [33] presented an active primary suspension design that was examined using $\mathcal{H}_\infty$ control, and the results demonstrated the effectiveness of active primary suspensions in minimizing wheel forces. In these studies, since ride comfort analysis is important, modeling all the modes is unnecessary, then only lower modes are considered to be controlled. However, the neglected modes are still present in the system. In practical implementation, there may be challenges as the controller may excite unmodeled flexible body modes, known as the “spillover effect” [10, 73]. In other words, actuator forces for reducing the vibration of the lower modes will also influence the residual modes of the structure and produce undesirable vibration due to the unmodeled dynamics [74]. While the change of sensor position is a solution for reducing the spillover effect [75], another solution to exceed the spillover effect may be altering the position of actuators [76].

1.1. The Scope of the Work

The contents of this thesis and our contributions are as follows:

In Chapter 2, the focus is on track modeling techniques. The chapter begins by

exploring the influence of track parameters on vehicle dynamic responses. In this chapter, various track parameters are discussed concerning their impact on the behavior of railway vehicles. Next, the rail irregularity is modeled using two approaches:

(i) A lumped-track model simplifies the rail irregularity as a concentrated disturbance acting on the vehicle at specific locations along the track. This model provides a simplified representation of rail irregularity.

(ii) System identification methods are used to model the rail irregularity in the frequency domain. The rail profile is modeled by obtaining the power spectral density (PSD) function from track displacement data collected from the Konya-Polatlı high-speed main line. Two specific methods, namely the Welch method and subspace identification, are employed to estimate the PSD function. Furthermore, a stochastic rail model problem is studied to highlight the practical relevance of the track modeling techniques. This analysis demonstrates the applicability of the developed models in capturing the stochastic behavior of rail irregularities. The findings and results presented in this chapter were published in [26, 77].

Chapter 3 focuses on utilizing a six-degree-of-freedom (6 DOF) model to investigate the impact of secondary suspension parameters on the output responses of the AM-96 Bombardier high-speed train. This suspension model is widely used due to its simplicity and can be employed to analyze comfort, suspension deflection, and safety responses regarding root-mean-square (RMS) quantities. By varying the passive suspension parameters of the vehicle, this study provides valuable insights into the suspension design problem, as an optimized passive suspension can offer satisfactory ride quality and potentially reduce the cost of active control forces. Furthermore, the effects of track quality and speed on vehicle responses are also explored. Understanding how these factors influence vehicle behavior is crucial for optimizing suspension design and ensuring safe and comfortable operation. To broaden the scope of the study, the analysis is extended to include freight wagons. At the same speed, the effects of freight trains on track dynamics are more pronounced compared to passenger trains. Therefore, investigating the impact of speed on freight wagons provides valuable insights into these vehicles' dynamic behavior and operational considerations. The results of the chapter were published in [32, 33, 78].

In Chapter 4, the research continues from Chapter 3 by focusing on applying semi-active suspension systems. Specifically, the skyhook control (for comfort), groundhook control (for safety), and hybrid on/off control systems are applied to a nine-degree-of-freedom (9 DOF) model utilizing Siemens ICE parameters. This extended model incorporates the vehicle's roll motion, providing a more realistic representation of the dynamics. The designs of the semi-active control systems are conducted in the discrete-time domain. The track measurements obtained from the Konya-Polatlı main line, collected by the Turkish State Railways-Railway Research and Technology Center (TCDD-DATEM), are directly applied as excitation inputs to the vehicle model. A control-oriented formulation is adopted to address the semi-active control problem, and Linear Matrix Inequalities are employed to solve the problem. The results presented both in the time and frequency domain are extensively discussed. The chapter results were presented in [79] and published in [36, 47, 48].

Chapter 5 focuses on investigating the influence of the rail profile from the study conducted in [2] on the design of active suspensions for different vehicle models. Initially, assuming rigid contact between the wheel and rail, an optimization problem is defined within the framework of active suspension design for a half-car model. The solution to this optimization problem is discussed, considering the goal of minimizing car body accelerations and secondary suspension deflections using the LQG methodology. Later, an elastic contact between the rail and the wheel is assumed, and the six-degree-of-freedom vehicle model is augmented to ten-degree-of-freedom. Next, an elastic contact assumption is introduced between the rail and wheel, leading to the augmentation of the vehicle model from a six-degree-of-freedom (6 DOF) to a ten-degree-of-freedom (10 DOF) configuration. The impact of elastic contact on the dynamics of the system is studied, and the effect of roll motion is incorporated into the augmented model. This leads to formulating a seventeen-degree-of-freedom (17 DOF) full-car model. The Linear- Quadratic-Regulator with the Kalman estimation method is applied to achieve active control. Additionally, an $\mathcal{H}_\infty$ output-feedback control method is utilized for both the secondary and primary suspensions to minimize wheel-rail forces. Particularly, impressive results are observed for active solutions in the primary suspension, especially in the middle-frequency range. The results of the chapter were published in [61, 7, 63, 62, 67].

Chapter 6 focuses on conducting a parametric study to assess the dependence of closed-loop responses on the Hertzian spring stiffness used to model the elastic contact. The spring stiffness is constrained to a predefined interval, and a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ state feedback controller is designed to ensure satisfactory performance within the uncertainty set. The study utilizes seventeen-degrees-of-freedom vehicle models with Siemens ICE passenger train parameters. Through the parametric study, the influence of different values of the Hertzian spring stiffness on the closed-loop system responses is evaluated. This analysis provides insights into the system's sensitivity to variations in elastic contact modeling. The results of this chapter were published in [28, 67, 80].

In Chapter 7, the focus is on investigating the flexible modes of the vehicle body and their impact on active control strategies. The future of railway vehicles aims to be lightweight to achieve energy efficiency, improved comfort, and enhanced safety. It has been observed that a lighter body can significantly affect flexible modes, particularly at low frequencies. Motivated by this, the vehicle's mathematical model is updated by incorporating the first two flexible body modes and rigid body motions. This updated model allows for a more comprehensive representation of the vehicle's dynamics, considering both rigid and flexible modes. An active controller for the secondary suspension is studied using $\mathcal{H}_2$ output feedback control. The goal of the active control approach is to suppress both the rigid and flexible modes of the vehicle, leading to improved ride performance. By effectively managing the dynamics of the vehicle's body, the active control system aims to enhance passenger comfort. The results of this chapter were published in [72, 81, 82].

2. HIGH SPEED RAILWAY TRACK

Chapter 2 provides an in-depth analysis of railway track characteristics and modeling techniques. It begins by highlighting that while the ideal track should be straight, in practice, it exhibits deviations and irregularities due to factors like fatigue or manufacturing defects. These deviations include curves, transitions, s-shaped features, switches, level crossings, joints, weld points, and track irregularities. It is emphasized that design geometry (track layout) and deviations from it (irregularities, roughness, and track geometry quality) should be treated separately as they have distinct effects on vehicle dynamics. It is noted in [83] that the train encounters great variability in track conditions throughout its lifetime. The importance of modeling track irregularities representing the entire railway network was emphasized. Therefore, detailed knowledge of the track geometry is essential for accurate modeling [20].

The main focus of this chapter is on track irregularities, which are the main source of vibrations experienced by vehicles. The effects of irregular tracks impact both comfort and safety during train travel [26, 84]. Therefore, extensive research has been conducted in both industry and academia to model track irregularities [77, 22, 83, 85]. These irregularities can be classified into two main categories: deterministic and random. Deterministic irregularities refer to consistent records from repeated experiments under the same conditions. On the other hand, random irregularities exhibit varying records, leading to a stochastic process [9, 86]. Random track irregularities are commonly assumed to be stationary and ergodic Gaussian processes, allowing for analytical characterization using statistical methods [7, 77, 2, 24, 23].

This chapter summarizes the high-speed railway track structure, providing an overview of its components. Existing modeling techniques and analytical characteristics are examined, focusing on the lumped-parameter track modeling approach, which describes track behavior using a one-degree-of-freedom (1DOF) model. The chapter then delves into the detailed discussion of random track irregularities, emphasizing their significance as the main disturbance input for railway vehicles. Finally, an analytical model is developed based on measurements obtained from the Konya-Polatli high-speed main line, allowing for a practical understanding of track behavior in real-world scenarios.

2.1. High Speed Railway Track

The railway track can be classified into two main classes: slab tracks, generally for trams, metros, and bridges, and ballasted tracks with wooden or concrete sleepers for conventional and high-speed trains. Figure 2.1 shows a ballasted track structure that can be grouped into two categories: superstructure and substructure. The structure is shown in Figure 2.2.

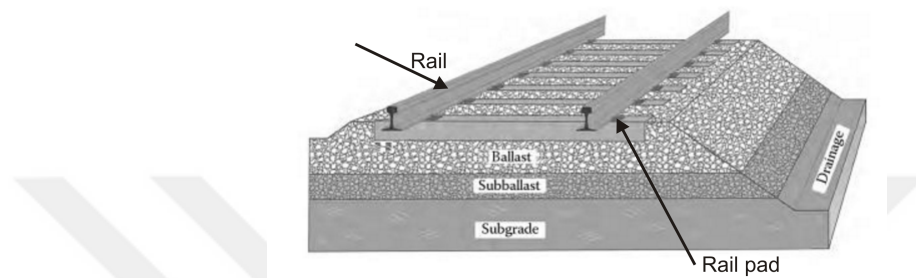


Figure 2.1. Railway track layers [3].

The ballasted track has many advantages, such as relatively low construction costs, simple replacement of track components, good drainage properties, good elasticity, good damping of noise, etc. [87, 88]. The tasks of ballasted track layers are defined in [89] and summarized below.

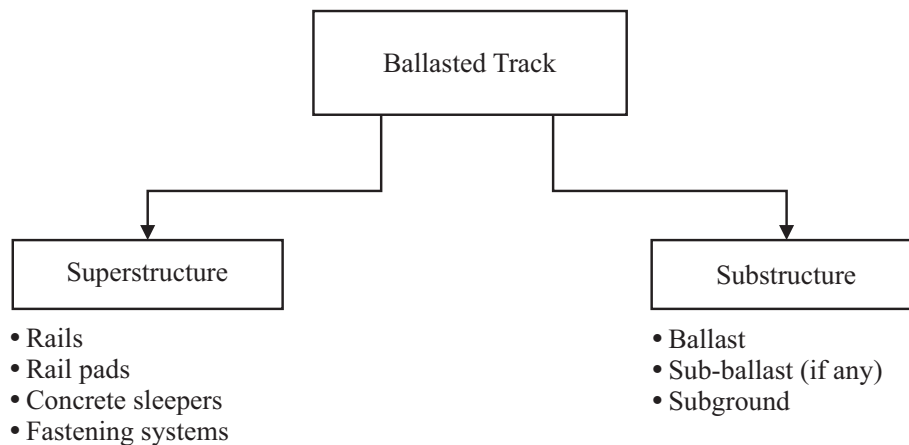


Figure 2.2. Ballasted-track structure.

- **The rail** is the most important layer at the top of the track. The primary task is guiding the railway vehicle. The rail also distributes the loads from the vehicle

to the sleepers and from the track to the vehicle. The rail conducts the signals for signalization systems, and for the catenary system, it acts as an electrical conductor.

- **The rail pads** are placed between rails and sleepers. The main task of the rail pads is to decrease the dynamic wheel-rail forces to protect the whole system.
- **The sleepers** primary task is load distribution from rail to the ballast and from ballast to the rail. In the past, wooden sleepers were used because timber was readily available. In modern railway tracks generally, concrete sleepers are preferred because of their durability and long service life. The sleepers also should provide an anchorage for fastening.
- **The ballast** is the substructure's first layer and provides fast fluid drainage. The ballast layer also reduces the stress during the load transfer and should provide stability and elasticity to the track.
- **The sub-ballast** main task is to separate ballast and sub-ground layers.
- **The subground** is the last layer bearing and distributing the dynamic loading resultant downward along its infinite depth. This layer includes existing soil and rock.

2.1.1. The railway track analytical characterization

Railway track is a source of excitation that can induce oscillations, vibrations, and noise in trains, tracks, and their surrounding environment. In other words, the railway track is the excitation source for the vehicle. The importance of obtaining accurate models for the vehicle, track, and wheel/rail contact, as the railway track serves as the input to the vehicle model in various studies, should not be ignored. The chapter highlights the significance of understanding and characterizing these excitation sources to accurately simulate and analyze the dynamic behavior of railway systems. The following sources of excitation are summarized as follows,[89]:

1. *Ground vibrations and noise*: These vibrations cause discomfort for people and buildings.

- (a) The trains are the main source of ground vibrations transferred from the track to the buildings.
- (b) The vehicle speed can be handled as another source for ground vibrations.
- (c) In addition to the human discomfort in the buildings, the ground vibrations may damage especially the historical buildings located nearby the track [90].

2. *Either short or long wavelength irregularities:*

- (a) The short/long wavelength irregularities of the track, either vertically or laterally, may induce vibrations and noise, which cause traveling discomfort for the passengers or goods.

3. *Deficiencies or excesses in the cant:*

- (a) They cause lateral forces in the curves, which affect passengers traveling comfort badly.

4. *Rails:* The ideal rail should be straight, but the rail is neither straight nor perfect because of manufacturing errors or the damages of train operations.

- (a) The corrugations that appear can be explained as short wavelength periodic irregularities. The corrugations cause high-frequency vibrations of the rail and wheel and influence both comfort and safety in railways.

5. *Wheel-out-of roundness:*

- (a) Short wavelength irregularities may be formed on block-braked wheels that deteriorate the comfort of passengers or goods.

Since the track geometry greatly impacts the comfort of passengers and goods, the track dynamics should be discussed in detail. In the last century, because of the high maintenance costs, track modeling has a large space in research. The track modeling can be categorized as follows:

- (i) Lumped-parameter model

(ii) Rail beam on continuous support

(iii) Rail beam on discrete support

The continuous track model, which has a large number of degrees of freedom, can be simulated with Finite Element Analysis (FEM), which is unsuitable for linear control theory. Instead, the lumped-parameter track model is introduced as a control-oriented alternative. The lumped-parameter track model simplifies the continuous track model by representing it with a single rigid mass supported by linear spring and damper elements. This simplified model allows for easier analysis and control design. Figure 2.4 visually represents the lumped-parameter track model. By utilizing the lumped-parameter track model, researchers can develop control strategies and optimize the dynamic behavior of the track system.

2.2. Lumped-parameter model

In this thesis, we studied the lumped parameter track model with one degree of freedom to describe the track behaviors. Figure 2.3 shows a two-layer discrete track model and equivalent single-layer track, which can also be explained by a beam on a Winkler foundation [30]. All track parameters are given in Table 2.1 [1].

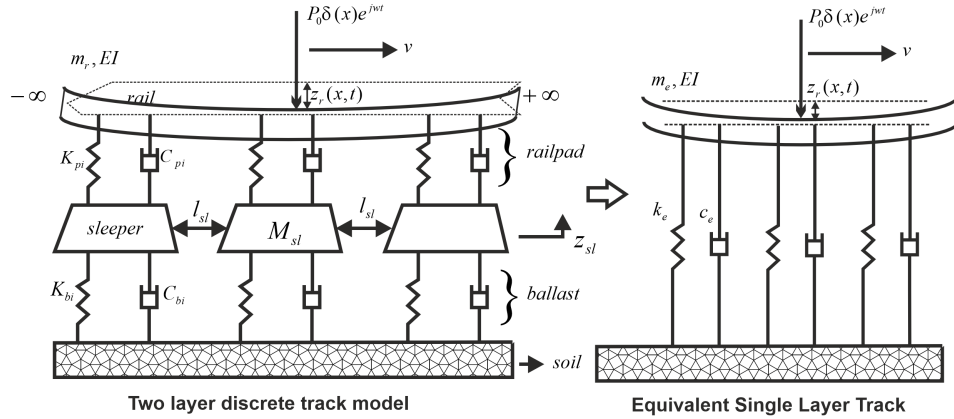


Figure 2.3. Two layer and equivalent single layer track.

Under a harmonic load with parameters $k_p = K_{pi}/l_s$, $k_f = K_{bi}/l_s$, and $m_{sl} = M_{sl}/2l_{sl}$, to obtain the equivalent lumped track model the following conversion principles should be used [26].

- (i) The strain energy on the springs over width $\delta(x)$ should be equal for both the two-layer and single-layer models. Then,

$$\frac{1}{k_e} = \frac{1}{k_f} + \frac{1}{k_p}, \quad (2.1)$$

- (ii) For viscous damping, energy dissipated per unit time should be equal, and, equivalent damping c_e will be,

$$c_e = \frac{c_p k_f^2 + c_f k_p^2}{(k_f + k_p)^2}, \quad (2.2)$$

- (iii) Conversion of masses needs constant kinetic energy and equivalent mass m_e ,

$$m_e = \frac{m_e k_f^2 + m_{sl} k_p^2}{(k_f + k_p)^2}. \quad (2.3)$$

From Figure 2.3, the differential equation of the single layer track model for width $\delta(x)$ can be written as:

$$EI \frac{\partial^4 z_r(x, t)}{\partial x^4} + m_e \frac{d^2 z_r(x, t)}{dt^2} + c_e \frac{dz_r(x, t)}{dt} + k_e z_r(x, t) = P_0 \delta(x) e^{j\omega t}. \quad (2.4)$$

The $z_r(x, t)$ is the particular solution of Equation (2.4) and refers to the rail vertical displacement.

$$z_r(x, t) = \tilde{z}(x) e^{j\omega t} \quad (2.5)$$

If the Equation (2.5) is substituted into the Equation (2.4), the differential equations of a static beam on Winkler foundation would be obtained as in Equation (2.6) with $\tilde{k} = (-m_e \omega^2 + c_e j\omega + k_e)$.

$$EI \frac{\partial^4 \tilde{z}(x)}{\partial x^4} + \tilde{k} \tilde{z}(x) = P_0 \delta(x) \quad (2.6)$$

The $\tilde{z}(x)$ can be written as Equation (2.7) by solving the Equation (2.6) with $\tilde{\beta} = \left(\frac{\tilde{k}}{4EI}\right)^{0.25}$ and has a maximum when $x = 0$.

$$\tilde{z}(x) = \frac{P_0 \tilde{\beta}}{2\tilde{k}} e^{-\tilde{\beta}x} (\cos \tilde{\beta}x + \sin \tilde{\beta}x) \quad (2.7)$$

The receptance of the track at the maximum is defined with $H_1(\omega)$ in Equation (2.8)

which can be also written as in Equation (2.9).

$$H_1(\omega) = \frac{z(0, t)}{P_0 e^{j\omega t}} = \frac{\tilde{\beta}}{2k} \quad (2.8)$$

$$H_1(\omega) = \frac{\beta}{2k_e} \left(1 - \frac{m_e \omega^2}{k_e} - \frac{j\omega c_e}{k_e}\right)^{-3/4} \quad (2.9)$$

Here β is static equivalent of $\tilde{\beta}$ and corresponding to $\beta = (\frac{k_e}{4EI})^{0.25}$. After using the Maclaurin series by neglecting higher terms than ω^2 , the receptance will be:

$$H_1(\omega) = \frac{\frac{\beta}{2k_e}}{1 - \frac{3m_e \omega^2}{4k_e} + \frac{3c_e j\omega}{4k_e}} \quad (2.10)$$

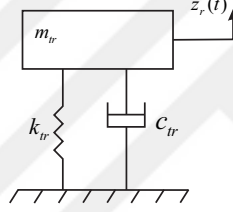


Figure 2.4. Lumped track model.

To obtain the equivalent lumped parameter track model with a single-layer track, the receptances should be equal under the same harmonic load. Figure 2.4 shows the equivalent lumped track model with the equation of motion:

$$m_{tr} \ddot{z} + c_{tr} \dot{z} + k_{tr} z = P_0 e^{j\omega t}, \quad (2.11)$$

Then, the receptance of the lumped track model can be obtained from Equation (2.11) as follows:

$$H_2(\omega) = \frac{1}{k_{tr} + c_{tr} j\omega - m_{tr} \omega^2} \quad (2.12)$$

The equivalent mass, spring, and damper parameters of the lumped-track model can be determined by equalizing the receptances $H_1(\omega)$ and $H_2(\omega)$ and listed below [29, 30, 91].

$$k_{tr} = \frac{2k_e}{\beta}, \quad (2.13)$$

$$c_{tr} = \frac{3c_e}{2\beta}, \quad (2.14)$$

$$m_{tr} = \frac{3m_e}{2\beta} \quad (2.15)$$

Table 2.1. *Ballasted Track Parameters [1]*

Track Component	Description	Values
UIC60 rail (60E1)	Bending stiffness, (EI)	6.4 MNm^2
	Shear stiffness, (G)	250 MN
	Mass per unit length, (m_r)	60 kg/m
	rotational inertia per unit length	0.24 kgm
	Elastic modulus of the rail, (E)	$2.1 \times 10^{11} \text{ N/m}^2$
	Poisson's ratio, ν	0.3
	Railhead radius, r_r	0.30 m
Rail pad	Linear stiffness, K_{ri}	80 MN/m
	Linear viscous damping, C_{ri}	15 kNs/m
	Side length	0.15 m
Sleeper	Mass, M_{sl}	250 kg
	sleeper distance, l_{sl}	0.65 m
Ballast	Linear stiffness, K_{bi}	30 MN/m
	Linear viscous damping, C_{bi}	31 kNs/m

The lumped-parameter track model provides a simplified representation of the track system. But, it is derived under the assumption of harmonic loads, which may not capture the exact behavior of track irregularities in real-world scenarios.

2.3. Random Track Profile

In railways, high-track irregularities may cause unsafe operations for rail vehicles. Therefore, checks and maintenance of the track geometry are necessary and should be operated periodically. Long-term maintenance planning is a cost-optimal management problem to keep track of irregularities under safety limits. For modern railway tracks, data are collected for longitudinal evenness. If the track irregularities exceed the limits, the vehicle speed should be decreased on this part until the track is maintained to the required quality [87, 20, 17, 89, 92].

The random track irregularities can be classified into four main categories according to the [17, 2, 9] and shown in Figure 2.5a and Figure 2.5b.

1. *Vertical profile*, an average elevation between right and left rail,

2. *Cross level*, difference elevation between right and left rail,
3. *Alignment*, the average lateral positions between right and left rail,
4. *Gauge*, the horizontal distance between right and left rail.

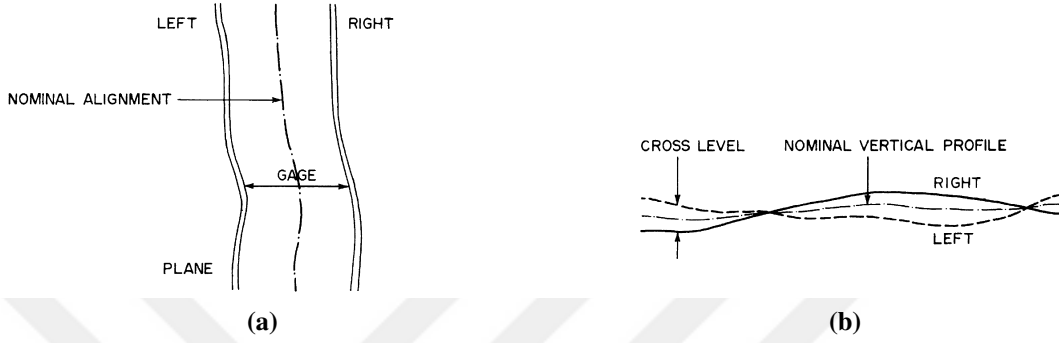


Figure 2.5. (a) *Gage and alignment* (b) *Cross-level and vertical profile* [9]

It is necessary for the acceptance of track construction or maintenance works to measure these track geometry. The measurement can be applied via, [20, 93, 94];

- (i) Discrete working systems, such as hand measurements (Corrugation Analysis Trolley)
- (ii) Continuously working systems, such as track recording car.

In the aforementioned report [2], the Federal Railroad Administration (FRA) conducted an extensive data collection effort in the United States. The objective was to gather track geometry data from 30 different zones, covering a total distance of 150 miles. This data was acquired using the Track-Recording-Car AMTRACK under the track safety standards. The primary aim of this data collection initiative was to obtain a comprehensive understanding of the typical variations in track geometry. The FRA could derive the power spectral density (PSD) functions for the longitudinal levels and track alignment by analyzing the collected data. These PSD functions provide valuable insights into the frequency content and characteristics of the track irregularities within the wavelength range from 3 m to 150 m. Equation (2.16) shows the cross-level track irregularity PSD function in the temporal domain with roughness constant A_v and cut-off frequencies Ω_r , Ω_c .

This analytical model is commonly used for offline simulations of railway vehicle vertical vibrations studies.

$$S_r(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_c^2)(\Omega^2 + \Omega_r^2)} \quad (2.16)$$

The cross-level track irregularities were classified into eight classes by FRA (the last two are only hypothetical), corresponding to the track roughness value and maximum allowable speed limits. Table 2.2 lists the track classes where class-1 means the worst quality track while class-8 is substituted for the best. Note that the cut-off frequencies are independent of the classes and are fixed as $\Omega_r = 0.146 \text{ rad/m}$, $\Omega_c = 0.825 \text{ rad/m}$, while the roughness parameter A_v is frequently changing during traveling [80].

Table 2.2. *Vehicle Speed Limits [2]*

Track Quality Class	A_v in (10^{-6} m^{-1})	Vehicle Speed Limits (km/h)	
		Passenger	Freight
Class-1	0.4868	24	16
Class-2	0.3387	48	40
Class-3	0.2328	97	64
Class-4	0.1566	129	96
Class-5	0.1058	145	129
Class-6	0.07197	177	—
Class-7	0.04234	201	—
Class-8	0.0254	257	—

In Turkey, the Turkish State Railways Research Center (TCDD-DATEM) has a track recording car named Piri Reis which has the same specifications as high-speed mainline trains. The measurements have been done with Piri Reis periodically. The chord measuring technique is used to collect data. Chord measuring systems rely on the principle that a baseline is formed between two points on the rails, and at a third position, the distance between this baseline and the rail at this position is measured. The ‘sensors’ may be of ‘mechanical’ type such as wheels or without contact using optical methods as on Piri Reis. When railways started to measure the irregularities on their tracks, there was a necessity to find indicators to characterize the quality of the measured track geometry. In Turkey (also in Europe), EN 13848-1 defines the measurement principles for track gauge, longitudinal level, alignment, cross-level, and twist. In this standard, for longitudinal level and

alignment, three different wavelength filters $D1$, $D2$, and $D3$, are used [20, 95, 17].

- (i) Waves between 3 and 25 m (corresponding filter is $D1$) originate from ballast and subsoil.
 - (ii) The wave range $D2 : 25\ m < \lambda \leq 70\ m$ corresponds to the track layout irregularities.
 - (iii) $D3 : 70\ m < \lambda \leq 150\ m$ filter originate from problems in the track layout. This wave band is only used when the vehicle speed exceeds 250 km/h .
- ★ In addition to these track irregularities, the faults in the waves below 3 m are caused by corrugation, production tolerances, joints, or weld points, which refer to high-frequency vibrations.

Equation (2.17) clearly explains the relationship between wavelength and frequency. It states that longer wavelengths correspond to lower-frequency excitations, while shorter wavelengths lead to higher-frequency excitations. This relationship is essential in the context of track irregularities.

$$f = \frac{v}{\lambda} \quad (2.17)$$

Regarding track irregularities, the wavelengths typically range from $\lambda = 0.03$ to 200 meters. Depending on the vehicle speed v , these irregularities can induce excitation frequencies ranging from 0 to 3000 Hz. The vehicle and the track interaction influences the specific range of excitation frequencies. Table 2.3 presents a summary of the most common causes of excitations and their associated wavelength ranges. This information is crucial for understanding the irregularities affecting the track and the corresponding frequency content that impacts the vehicle dynamics.

If the measurements can be modeled mathematically in the frequency domain, it may be possible to estimate where the degradation occurs. For longitudinal level measurement data collected from Polatlı-Konya high-speed mainline section between 75.5 - 78.00 km in Turkey, a mathematical model was obtained using system identification techniques in [77]. These data were collected due to the standard EN13848 in [17] using Piri

⁰In most cases only the irregularity in the vertical direction is measured and analysis

Table 2.3. *Wavelength and caused element of excitation*

Wavelength	Caused by
0-2 m	• corrugation • production tolerances • wear of rail shape
2-3 m ⁰	joints or weld points
3-25 m	• rail roughness or rail irregularities
25-70 m	• mechanisms in ballast and subsoil
≥ 70 m	• track layout irregularities
	• problems in track layout

Reis TRC on different dates. We developed an analytical presentation with three different approaches. The results have been compared with the Federal Railroad Administration (FRA) standards [2] because of common use in literature.

2.3.1. The Welch method

Figures 2.6a-2.7a show the rail profile collected data from Konya-Polatli main line. From the figures, it can be concluded that the wavelength range and the line speed affect the track's irregularity magnitudes and densities. When the higher magnitudes occur in the long wave intervals, the shorter magnitudes are created by the short wave band [77]. If the vehicle has a traveling speed v , the track irregularity generates a dynamic excitation frequency $f = 2\pi nv$ in rad/s with wavenumber n . The vehicle speed and the wavenumber also influence the excitation frequency.

The measured track profile is generally considered a realization of a stationary ergodic Gaussian random process that can be described via PSD functions. The Welch method is the best procedure for PSD estimation with minimum bias and variance [96, 77].

Let the collected data from the left and right rail be shown with $z_L[k]$ and $z_R[k]$, defined as jointly stationary zero-mean discrete processes. The cross-correlation $R_{z_R z_L}(\tau)$

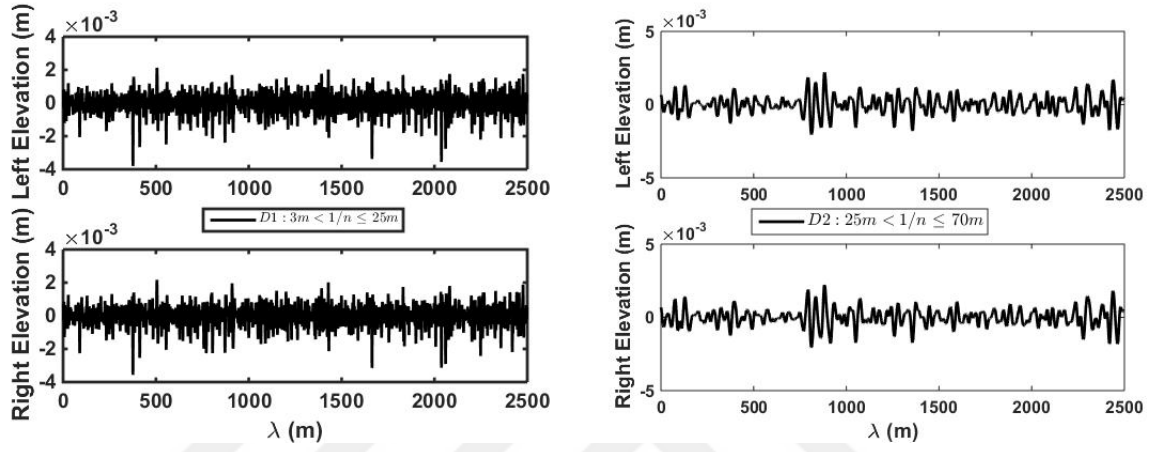


Figure 2.6. The left and right rail profile collected data (a) in the wave range D_1 at $v = 200$ km/h (b) in the wave range D_2 at $v = 200$ km/h

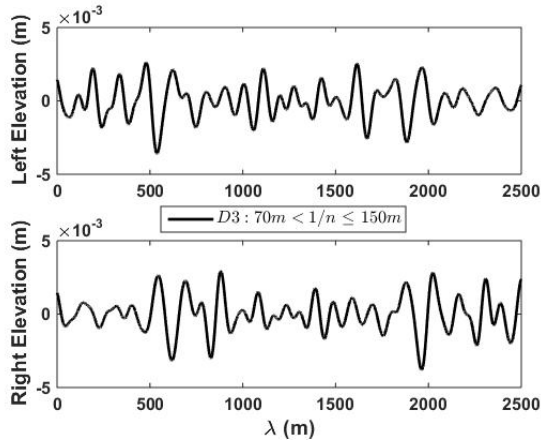


Figure 2.7. (a) The left and right rail profile collected data in the wave range D_3 at $v = 200$ km/h

and cross(auto)-PSD $G_{z_R z_L}(z)$ matrices of them are calculated from the Equation (2.18).

$$\begin{aligned} R_{z_R z_L}(\tau) &= E\{z_R[k + \tau]z_L^T[k]\} \\ G_{z_R z_L}(z) &= \sum_{\tau=-\infty}^{\infty} R_{z_R z_L}(\tau)z^{-\tau} \end{aligned} \quad (2.18)$$

where X^T denotes transpose of X and $E\{X\}$ shows the expected value of random matrix X for all $\tau \in R$. The Welch method can be applied by dividing the data signal into R successive blocks of K samples to form the periodogram for each block and averaging. Then the Fast Fourier transform for $i = 1, \dots, R$ will be

$$G_{Z_R z_L}^i = \frac{1}{K} Z_{R(i)}(\theta, K) Z_{L(i)}^H(\theta, K)$$

where

$$\begin{aligned} Z_{R(i)}(\theta, K) &= \sum_{k=0}^{K-1} z_R[k + iK]e^{-j\theta k} \\ Z_{L(i)}(\theta, K) &= \sum_{k=0}^{K-1} z_L[k + iK]e^{-j\theta k} \end{aligned}$$

The power spectral density is obtained by the Welch estimation method as follows:

$$G_{z_R z_L}^W(e^{j\theta}) = \frac{1}{R} \sum_{i=0}^{R-1} G_{z_R z_L}^{(i)}(\theta, K) \quad (2.19)$$

at the equidistantly spaced frequencies $\theta_k = \frac{2\pi k}{K}$ for $k = 0, \dots, K/2$. Figure 2.8a shows the estimated auto-PSDs of three filtered versions of the collected data from the Konya-Polatlı mainline left rail for $K = 2500$, and $R = 1$. Figure 2.8b gives the real rail track profile for both the left and right parts of the track. It is obvious from the figure that there are only minor differences between the left and right parts of the track on the high-frequency band. Also, the PSD functions are compared with the FRA cross-level track irregularity defined in Equation (2.16). Although the FRA model is commonly used in the literature, Figure 2.8b shows that the FRA model could not accurately describe the high-speed mainline track irregularities.

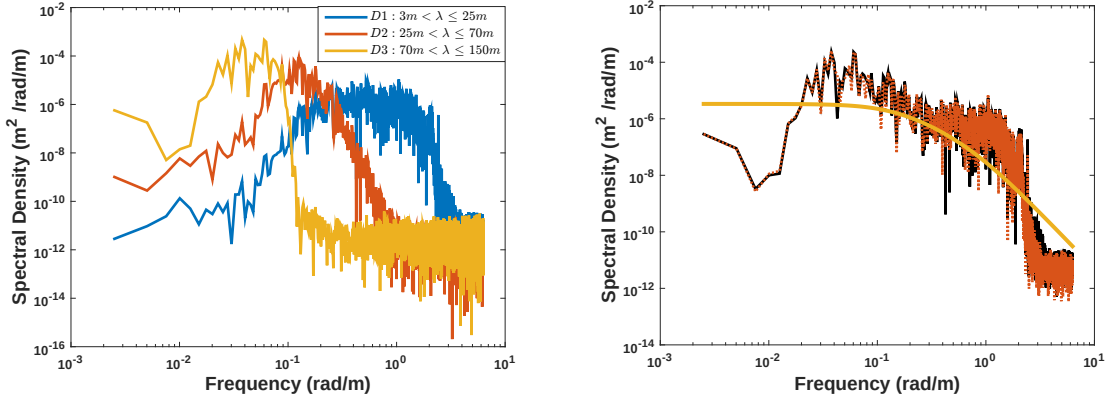


Figure 2.8. (a) The Welch auto-PSD estimates of the collected data for the left track (b) The Welch auto-PSD estimates of the collected data for the left track (—), for the right track (...) and the FRA class-6 cross level PSD (—)

2.3.2. Curve fitting method

In railways, the curve fitting method that provides a rational representation of the track irregularity auto-PSD function is commonly used. The two-slope approximation was applied to fit the track profile auto-PSD in Figure 2.8b. The G_{CF2} in Equation (2.20) shows the two-line spectrum of the track profile obtained by two single slope lines at the frequency $2\pi n_0 = 1 \text{ rad/m}$.

$$G_{CF2}(2\pi n) = \begin{cases} \rho \left(\frac{n}{n_0}\right)^{-\delta_1} & 2\pi n \leq 2\pi n_0, \\ \rho \left(\frac{n}{n_0}\right)^{-\delta_2} & 2\pi n \geq 2\pi n_0. \end{cases} \quad (2.20)$$

where $\rho = 0.05 \times 10^{-6} \text{ m}^2/(\text{rad/m})$, $\delta_1 = 2.2$, $\delta_2 = 6.1$. The three-slope model also was developed by dividing the spectrum into three wavebands with the intersecting frequencies $2\pi n_1 = 0.2 \text{ rad/m}$ and $2\pi n_2 = 2 \text{ rad/m}$. The three-slope spectrum is defined with $G_{CF3}(2\pi n)$ and given by Equation (2.21).

$$G_{CF3}(2\pi n) = \begin{cases} \rho_l \left(\frac{n}{n_0}\right)^{-\delta_l} & 2\pi n \leq 2\pi n_1, \\ \rho_m \left(\frac{n}{n_0}\right)^{-\delta_m} & 2\pi n_1 \leq 2\pi n \leq 2\pi n_2, \\ \rho_h \left(\frac{n}{n_0}\right)^{-\delta_h} & 2\pi n \geq 2\pi n_2. \end{cases} \quad (2.21)$$

with the parameters $\rho_l = 0.0427 \times 10^{-6}$, $\rho_m = 0.232 \times 10^{-6}$, $\rho_h = 14.7 \times$

$10^{-4} \text{ m}^2/(\text{rad}/\text{m})$ and $\delta_l = 2.2$, $\delta_m = 1$, $\delta_h = 13.8$. Figures 2.9a-2.9b shows the curve fitting method results. The three-slope model approximation fits much better to the Welch estimated PSD than the FRA model. If a detailed analysis is needed, then the curve fitting method with more than three-slope lines is suggested [77], but in vehicle dynamic analysis, discontinuity is a problem. Then a better identification method should be studied.

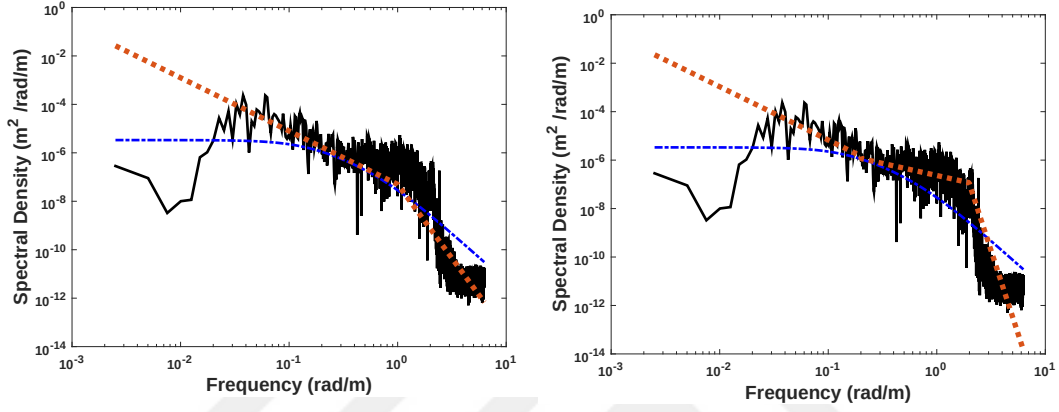


Figure 2.9. The track irregularity spectrum the Welch estimated PSD:(—); The FRA track class-6:(—); the curve fitting method:(...) (a) with the two-slope two-slope approximation (b) with the three-slope approximation

2.3.3. Subspace identification

The right and left track roughness excite the wheels with displacements $w_{11}(\lambda)$ and $w_{12}(\lambda)$ in the distance λ . Figure 2.10 shows the eight inputs track model with central roughness $\xi_M(\lambda)$ and rotational roughness $\beta(\lambda)$. There is no relationship between the stochastic processes $\xi_M(\lambda)$ and $\beta(\lambda)$. The wheel displacements can be defined using the central and rotational roughness in Equation (2.22).

$$\begin{bmatrix} w_{11}(\lambda) \\ w_{12}(\lambda) \end{bmatrix} = \begin{bmatrix} 1 & l_p \\ 1 & -l_p \end{bmatrix} \begin{bmatrix} \xi_M(\lambda) \\ \beta(\lambda) \end{bmatrix} \quad (2.22)$$

Then, the auto and cross-PSD functions of the wheel displacements can be summa-

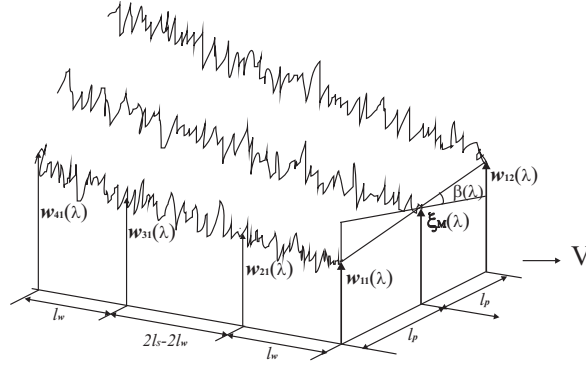


Figure 2.10. Eight inputs track model.

rized like:

$$\begin{bmatrix} G_{w_{11}} \\ G_{w_{12}} \\ G_{w_{11}w_{12}} \end{bmatrix} = \begin{bmatrix} 1 & l_p^2 \\ 1 & l_p^2 \\ 1 & -l_p^2 \end{bmatrix} \begin{bmatrix} G_M \\ G_\theta \end{bmatrix} \quad (2.23)$$

where $G_\theta(n)$, $G_M(n)$ denotes the auto-PSD with the spatial frequency n . The stochastic processes $\xi_M(\lambda)$ and $\beta(\lambda)$ are modeled using the subspace identification method. Figure 2.11 shows the results that fit much better than other methods, especially in the low and mid-frequency bands.

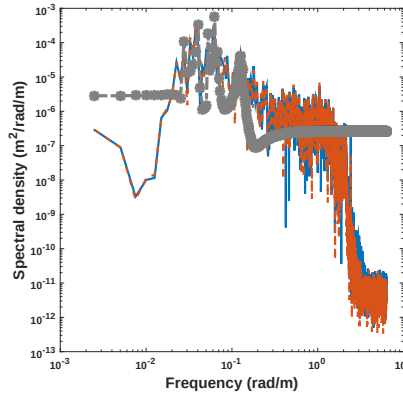


Figure 2.11. Results for Subspace algorithm

2.4. Summary

The railway track structure has multiple layers, each with specific tasks and responsibilities. These layers include the subgrade, ballast, sub-ballast, and rail. The form, materials, fatigue, and deterioration of these layers can significantly influence the vibra-

tions experienced by the vehicles traveling on the track. Therefore, modeling the track is of utmost importance in understanding and predicting the system's behavior. In this chapter, we utilized the Hunt lumped-track model to discuss the behaviors of the track layers. This model can be adapted to the linear mathematical model of the vehicle, allowing for a comprehensive analysis of the vehicle-track interaction. However, it should be noted that the lumped-track model was derived under harmonic load assumption and may not capture all the distortions that can occur on the track layers.

Regular track measurements were taken at specific intervals by TRC to monitor and prevent degradations on the track layers. These measurements provide valuable information that can be used to estimate the state of the track layers. We developed a mathematical representation of track irregularities in the frequency domain. The Welch estimation and subspace identification methods were employed to analyze the track measurements collected from the Konya-Polatlı high-speed main line. The resulting stochastic representation was compared to the widely used Federal Railroad Administration (FRA) track irregularity models' power spectral density (PSD) functions. The presented mathematical model of track irregularities was expected to be utilized in railway vehicle dynamics research. It provides a summary of the behaviors exhibited by a typical high-speed railway track, enabling researchers to gain insights into the dynamic interactions between the track and the vehicles traveling on it.

3. THE EFFECT OF PASSIVE SUSPENSION PARAMETERS

Railway vehicles have environmental friendliness, safety, and comfort advantages in transportation systems. However, to enhance their acceptance and competitiveness, trains need to achieve higher speeds [97]. After the 1990s, several countries, such as Japan, Germany, and France, have made significant advancements in high-speed railway transportation, setting new speed records. For instance, Japan achieved a 602 km/h speed with Maglev trains in 2015. Despite these remarkable achievements, it is crucial to focus on the long-term behavior of the vehicle and the track to ensure safe and reliable operations [4]. Figure 3.1 provides an overview of the short-term and long-term behaviors of the vehicle and track. Track degradations have emerged as a significant concern among the two main fatigue problems affecting long-term behaviors. Track maintenance, particularly on rails and sub-tracks, can be challenging and costly.

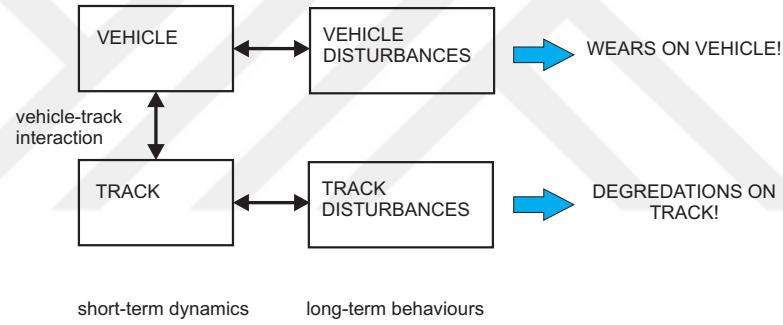


Figure 3.1. *Track and vehicle short-long term features [4]*

Simulation tools play a crucial role in reducing costs by allowing for extensive simulation studies before the production of railway vehicles. To ensure realistic simulations, developing detailed models of both the vehicles and the rail track is essential. Passenger trains typically exhibit rigid modes in the car body that occur at frequencies around 1 Hz, e.g., Table 3.1 provides examples of rigid modes for a specific wagon model (AM96) [19]. However, human bodies are more sensitive to vibrations in the low-frequency range, particularly frequencies below 15 Hz [15, 98]. Consequently, achieving better ride comfort involves suppressing vibrations within this frequency range.

Traveling at higher speeds leads to forced vibrations in a wide frequency range which requires an analysis of the mid and high-frequency vibrations and the correspond-

Table 3.1. *Rigid body modes*

Rigid modes	Heave	Pitch
Car body	1.17 Hz	1.53 Hz
Bogie	1.83 Hz	8.40 Hz
Car body damping ratio	0.1212	0.1577
Bogie damping ratio	0.0464	0.0752

ing noise [99]. Table 3.2 lists the vehicle modes where vibration in the frequency domain occurs[4]. The table shows that the rigid-bodied models can be sufficient for comfort research since the elastic modes dominate the mid and high-frequency ranges. In most studies, the track and vehicle dynamics have been handled separately. However, the bases of the vehicle-track coupling dynamics theory show that vehicle and track are two inseparable subsystems since the track is the vehicle's main source of vibration and the vehicle is the main vibration source for the track [27]. The investigation of the railway vehicle's dynamics, its long and short-term behaviors both in frequency and time domain, and the suppression of body modes with active and semi-active control form the basis of this thesis.

Table 3.2. *Modes of the vehicle [4]*

Frequency Range	Carbody	Bogie	Wheelset
0 – 50 Hz			
Low Freq. Range Rigid Model	Ride comfort	Running Stability	Running stability
50 – 500 Hz	-structural vibra- tions	-fatigue -wheelset bearings	-grumbling noise -hammering
Mid-Freq.Range Elastic model	-internal noise		
0.5 – 20 kHz			-fatigue
High Freq. Range Elastic Model		fatigue	-rolling noise -acoustic noise

3.1. Railway Vehicle Structure

Figure 3.2 shows a typical modern railway passenger vehicle wagon structure. The structure consists of two main bodies: *car body* and *bogies*. The first designed trains were defined as a simple box on wheels. But, in time, the bogies become the inevitable part of the vehicle that does not exist on-road vehicles. The bogies are connected to the car

body via the secondary suspension systems and fastened to the wheelsets using primary suspension systems.

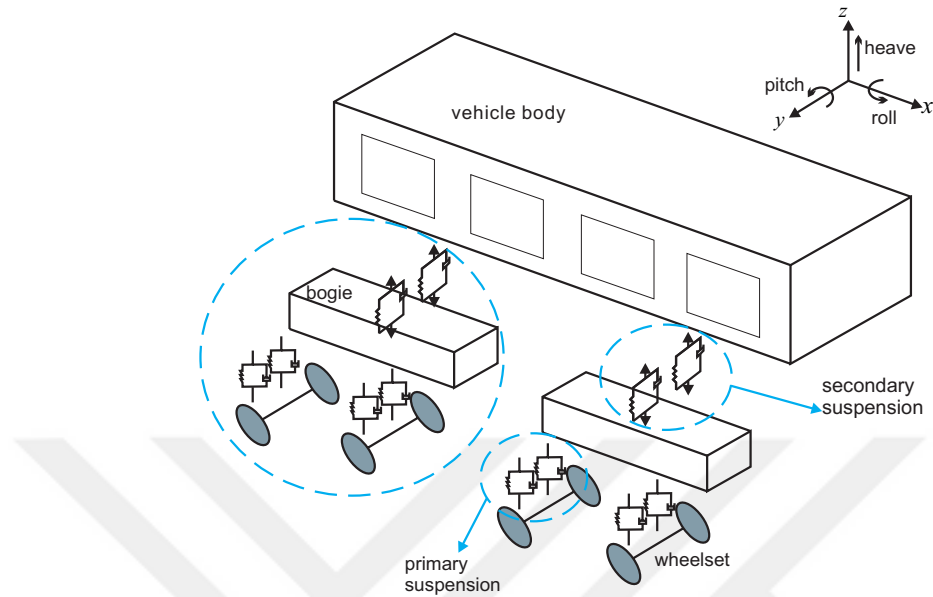


Figure 3.2. A typical high speed passenger vehicle.

The bogies can be classified into three groups according to their number of axles as shown in Figure 3.3, [100, 101, 8]:

- (i) *One-axle bogie*: Not commonly used since direct transmission of the impacts to the car.
 - (i) *Two-axle bogie*: Commonly used for passenger cars, decrease the impact of track irregularities at the suspension points.
 - (i) *Three-axle bogie*: Often used in locomotives, they are equipped with more complex structures and affect the running performance with hard maintenance specifications.
- The three-axle bogies are often used in locomotives.

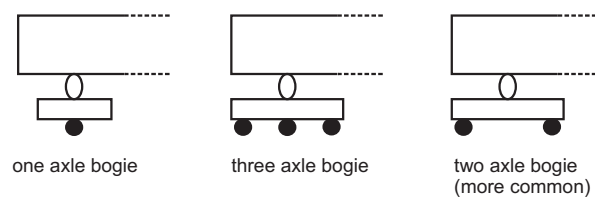


Figure 3.3. Bogies classes according to the number of axles.

The other classification criterion is a connection point to the vehicle, as shown in Figure 3.4. The bogies can be fastened to the vehicle as either articulated (jacobs) or non-articulated bogies. The articulated bogies are positioned between the car bodies. Although they are used in TGV with two-axles and Spanish Tolgo with single-axle, because of the increased axle load and difficult maintenance the manufacturers do not prefer [100]. In Germany, the discussion has been realized about the bogie configurations as cars with two-bogies or Jacobs solution. Two preliminary conclusions of this consideration show that [12],

- (i) The Jacobs solution, since it uses the reduced number of bogies, does not diminish the train set's length or mass.
- (ii) The Jacobs solution has the following advantages;
 - ★ The reduced number of bogies provides an aerodynamic resistance
 - ★ Improved ride comfort and reduced lineside noise
 - ★ The passenger transition simplicity between coaches

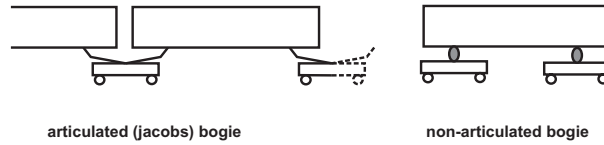


Figure 3.4. *Bogie types according to the connection point.*

However, for passenger vehicles, a pair of two-axle non-articulated bogies are commonly used where the wheelsets are connected in an H-shaped frame [8]. In this thesis, we assumed that the rigid car body m_s is supported by two-axle non-articulated welded H-form bogies. Figure 3.5 represents the structure of this bogie.

3.2. Passenger Vehicle Dynamics

Higher speeds during traveling on railways increase vibrational effects on vehicle bodies, and this causes deteriorations in passenger comfort. Filtering the vibrations of the car body's vertical accelerations in a specific frequency range could provide more comfortable driving [102]. In railway vehicles, the secondary suspensions are directly related

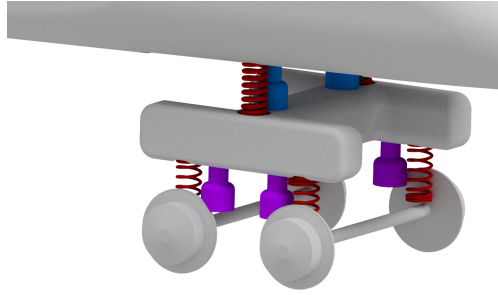


Figure 3.5. *H-welded non-articulated bogie*

to comfort. The structure of a typical freight wagon uses only springs as a passive suspension system, and a damper parallel to the suspension spring is connected in passenger vehicles [9, 30, 32]. Higher speed comfort demands nowadays can be met via suspension systems such that:

1. Optimization of passive suspension systems
2. Semi-active suspension system designs
3. Active suspension design

Optimization of passive suspension systems can be handled as a first step to improve vehicle performance and decrease semi-active/active suspension costs [11, 52]. This optimization can be achieved with numerical solutions before testing railway vehicles in the laboratory. As an example, in [14], a high-speed railway vehicle is modeled with three degrees-of-freedom (3DOF) and ten degrees-of-freedom (10 DOF) mathematical models in the Japanese Railways Laboratory. The best spring stiffness and vehicle mass values are recognized with test setups that take advantage of natural frequency adjustments. Since the vehicle's natural frequency depends on the secondary suspension spring stiffness and the body mass, obtaining the optimal values may be possible by fixing the natural frequency. The theoretical discussions also support the practical results. In [34], the researchers analyze different configurations for passive suspensions to determine the best form.

In this chapter, we analyze the dynamic behavior of the railway passenger vehicle and determine the effect of the passive suspension elements on the overall dynamics. Figure 3.6 gives the side view of a railway passenger vehicle (called half car since it cut

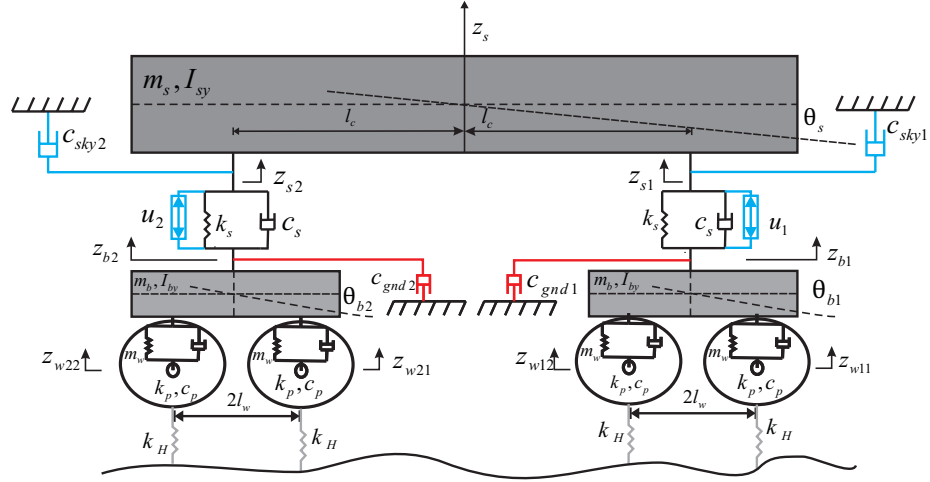


Figure 3.6. Side view of the high speed railway vehicle.

to half longitudinally from the center). The car body mass is denoted by m_s with I_{sy} pitch moment of inertia. It is connected to the bogie frame with the secondary suspension system consisting of spring k_s and damper c_s . Two bogie masses m_b with I_{by} pitch moment of inertia are fastened to the wheelsets using primary suspension pairs of linear spring k_p and damper element c_p . In this chapter, we used the Bombardier AM-96 railway vehicle passenger coach parameters for the simulations. This passenger train belongs to Bombardier and is used in Belgium, especially for long distances. The vehicle half-car mathematical model is obtained under the assumptions listed below.

1. All masses are assumed rigid and have freedoms of heave and pitch motions.
2. The contact between the wheel and rail is also assumed as rigid, and no wheel jump happens when the vehicle runs on the track.
3. All suspension elements are linear.
4. The railway vehicle guides a rough track with a constant speed v .

Under these assumptions, the primary and secondary suspension forces for $i, j = 1, 2$ can be summarized as:

$$\begin{aligned} F_i &= -k_s(z_{si} - z_{bi}) - c_s(\dot{z}_{si} - \dot{z}_{bi}), \\ F_{ij} &= -k_p(z_{bij} - z_{wij}) - c_p(\dot{z}_{bij} - \dot{z}_{wij}) \end{aligned} \quad (3.1)$$

Here $i = 1$ represents the front bogie, and $i = 2$ is the rear bogie, while the same goes for j describing the wheelsets. By using the Equation (3.1), the dynamical equation can be derived as follows:

- For carbody;

$$\begin{aligned} m_s \ddot{z}_s &= \sum_{i=1}^2 (F_i - u_i) \\ I_{sy} \ddot{\theta}_s &= l_c \sum_{i=1}^2 (-1)^{i+1} (F_i - u_i) \end{aligned} \quad (3.2)$$

- For front bogie ($i = 1$) and rear bogie ($i = 2$);

$$\begin{aligned} m_b \ddot{z}_{bi} &= -F_i + u_i + \sum_{j=1}^2 F_{ij} \\ I_{by} \ddot{\theta}_{bi} &= l_w \sum_{j=1}^2 (-1)^{j+1} F_{ij} \end{aligned} \quad (3.3)$$

Let's the state vector x , disturbance input z_w and the regulated output z can be concentrated as,

$$\begin{aligned} x &= [s_1, s_2, s_{11}, s_{12}, s_{21}, s_{22}, \dot{z}_{s1}, \dot{z}_{s2}, \dot{z}_{b11}, \dot{z}_{b12}, \dot{z}_{b21}, \dot{z}_{b22}]^T, \\ z_w &= [z_{w11}, z_{w12}, z_{w21}, z_{w22}]^T, \\ u &= [u_1, u_2]^T, \\ z &= [s_1, s_2, \ddot{z}_s, \ddot{\theta}_s]^T, \\ y &= [s_1, s_2]^T. \end{aligned}$$

s_1, s_2 denote the front and rear relative displacement on the secondary suspension, and $s_{11}, \dots, s_{22}$ is used for front-front, front-rear, rear-front, and rear-rear primary suspension travels. The relationships between secondary suspension and primary suspension node displacements for $i, j = 1, 2$ can be explained via Equation (3.4).

$$\begin{aligned} z_{si} &= z_s + (-1)^{(i+1)} l_c \theta_s, \\ z_{bij} &= z_{bi} + (-1)^{(j+1)} l_w \theta_{bi} \end{aligned} \quad (3.4)$$

The state-space representation is obtained below,

$$\mathcal{P} : \left\{ \begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_2 & D_{21} & D_{22} \\ C_y & D_{y1} & D_{y2} \end{array} \right] \begin{bmatrix} x \\ \dot{z}_w \\ u \end{bmatrix} \right. \quad (3.5)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0_{6 \times 6} & T_M \\ -K S_M T_M^T & -C S_M T_M^T T_M \end{bmatrix}, \quad B_1 = \begin{bmatrix} N \\ -C S_M T_M^T N \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0_{6 \times 2} \\ -S_M T_M^T \Gamma \end{bmatrix}, \quad C_2 = \begin{bmatrix} \Gamma^T & 0_{2 \times 6} \\ -\Gamma^T K S^{-1} S_M & -\Gamma^T C S^{-1} S_M T_M \end{bmatrix}, \\ D_{21} &= 0_{4 \times 4}, \quad D_{22} = \begin{bmatrix} 0_{2 \times 2} \\ -M_s^{-1} S_s^T \end{bmatrix}, \quad C_y = \begin{bmatrix} \Gamma^T & \mathbf{0} \end{bmatrix}, \\ D_{y1} &= \mathbf{0}, \quad D_{y22} = \mathbf{0} \\ S_s &= \begin{bmatrix} 1 & l_c \\ 1 & -l_c \end{bmatrix}, \quad S_b = \begin{bmatrix} 1 & l_w \\ 1 & -l_w \end{bmatrix}, \quad T_s = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad T_b = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \\ T_M &= \begin{bmatrix} I_2 & -T_s S_b^{-1} & -T_b S_b^{-1} \\ 0_{2 \times 2} & I_2 & 0_{2 \times 2} \\ 0_{2 \times 2} & 0_{2 \times 2} & I_2 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} I_2 \\ 0_{4 \times 2} \end{bmatrix}, \quad N = \begin{bmatrix} 0_{2 \times 4} \\ -I_4 \end{bmatrix}, \end{aligned}$$

$$M_s = \text{diag}(m_s, I_{sy}),$$

$$M_b = \text{diag}(m_b, I_{by}),$$

$$M = \text{diag}(M_s, M_b, M_b),$$

$$S = \text{diag}(S_s, S_b, S_b),$$

$$K = \text{diag}(k_s I_2, k_p I_4),$$

$$C = \text{diag}(c_s I_2, c_p I_4),$$

$$S_M = S M^{-1} S^T$$

and when $u = 0$, the configuration is valid for a passive suspension systems.

3.2.1. Vehicle-track model

In this thesis, only vertical vibrations will be considered. The vehicle is assumed to travel on a track, represented by a cross-level track irregularity. This track irregularity model can be found in the Federal Railroad Administration report and will be exploited during the thesis. We need a function that explains track input behaviors in the frequency domain to discuss vehicle performance. Since the power spectral densities (PSDs) describe the frequency content of the data, in simulations, the PSD of track irregularity will be used.

The relationship between track irregularity PSD $S_r(w)$, vehicle transfer function $H(w)$, and the vehicle system response $S_z(w)$ will be established as follows:

$$S_z(w) = H(jw)H(-jw)S_r(w) = |H(w)|^2 S_r(w) \quad (3.6)$$

In this thesis, we will use the vertical profile and cross-level track irregularities as excitation inputs. The FRA cross-level shaping filter PSD is defined in the spatial domain as,

$$S_r(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (3.7)$$

where A_v track roughness parameter and Ω denotes the spatial frequency in rad/m . In the dynamic analysis, the frequency needs to be converted into the temporal domain by using,

$$\Omega(rad/m) = \frac{w(rad/s)}{v(m/s)} \quad (3.8)$$

Substituting Equation (3.8) into the Equation (3.7), then

$$S_r\left(\frac{w}{v}\right) = \frac{A_v v^2 \Omega_c^2}{(w^2 + v^2 \Omega_r^2)(w^2 + v^2 \Omega_c^2)} \quad (3.9)$$

To obtain $S_r(w)$, the Equation (3.9) should be divided by speed v [7, 71],

$$S_r(w) = \frac{A_v v^3 \Omega_c^2}{(w^2 + v^2 \Omega_r^2)(w^2 + v^2 \Omega_c^2)} \quad (3.10)$$

The PSD function $S_r(w)$ can be expressed as an output spectrum of a linear-time-

invariant filter with transfer function $G_r(jw)$ and a unit intensity white noise input as [78],

$$S_r(w) = G_r(jw)G_r(-jw)^T \quad (3.11)$$

Then z_{w11} can be expressed with a transfer function

$$G_r(s) = \frac{\Omega_c \sqrt{A_v} v^3}{s^2 + (\Omega_r + \Omega_c)vs + \Omega_r \Omega_c v^2}. \quad (3.12)$$

Let's the corner frequencies are $w_c = v\Omega_c$ and $w_r = v\Omega_r$. Then the state-space representation of $G_r(s)$ is,

$$\begin{aligned} \dot{x}_{tr} &= A_{tr}x_{tr} + B_{tr}\eta \\ z_{w11} &= C_{tr}x_{tr} \end{aligned} \quad (3.13)$$

with

$$\begin{aligned} A_{tr} &= \text{diag}(-w_r, -w_c), \\ B_{tr} &= \frac{w_c \sqrt{A_v} v}{w_r - w_c} \begin{bmatrix} -1 & 1 \end{bmatrix}^T, \\ C_{tr} &= \begin{bmatrix} 1 & 1 \end{bmatrix} \end{aligned}$$

Since the disturbance input of the plant $\mathcal{P}$ is velocity, then the integrated vehicle-track model can be expressed with

$$S_z(\omega) = s^2 |H_{z\dot{z}_w}(\omega)|^2 S_r(\omega) \quad (3.14)$$

Generally, the root-mean-square (RMS) value is used in vehicle performance tests or feature supply analysis. The calculation is easy and provides driving quality estimation on track models where the probability distributions are assumed to be stationary. The auto-correlation and PSD functions are related to a Fourier transform for a stationary random process.

$$R_z(t) = E[z^2(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_z(w) e^{jw\tau} dw \quad (3.15)$$

where $E[\cdot]$ denotes the expected value. Then the root-mean-square (RMS) values of

vehicle outputs by substituting Equation (3.14) into the Equation (3.15) for $t = 0$ and written as,

$$R_z(0) = E[z^2(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H_{zz_w}(\omega)|^2 S_r(\omega) d\omega \quad (3.16)$$

The output RMS values of the half-car model in Figure 3.6 in terms of vehicle and track parameters can be represented as follows:

$$R_z(0) = \frac{A_v v \omega_c^2}{2\pi} \int_{-\infty}^{\infty} \frac{|H_{zz_w}(\omega)|^2}{(\omega^2 + \omega_r^2)(\omega^2 + \omega_c^2)} d\omega \quad (3.17)$$

3.3. The Influences of the Secondary Suspension Passive Elements on Ride Performance

In this section, the secondary suspension elements are going to be examined. The simulations are carried out with *MATLAB*. It is assumed that the AM-96 train goes on a straight line with FRA track class-6 irregularity characteristics and a constant speed of 160 km/h.

3.3.1. The effect of spring stiffness

The natural frequency of a body can be explained by the frequency at which the body wants to vibrate [87]. The characteristic equation of the vibration is represented with ω_n denoting the natural frequency and ζ the damping ratio as a damped system [9, 103].

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad (3.18)$$

The damped natural frequency ω_d can be calculated from Equation (3.18),

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (3.19)$$

We follow the procedure below to find the 6DOF vehicle natural responses. The body equations of motions in matrix form will be,

$$\begin{bmatrix} m_s & 0 \\ 0 & I_{sy} \end{bmatrix} \begin{bmatrix} \ddot{z}_s \\ \ddot{\theta}_s \end{bmatrix} = -c_s \begin{bmatrix} 1 & 1 \\ l_c & -l_c \end{bmatrix} \begin{bmatrix} \dot{z}_s + l_c \dot{\theta}_s - \dot{z}_{b1} \\ \dot{z}_s - l_c \dot{\theta}_s - \dot{z}_{b2} \end{bmatrix} - k_s \begin{bmatrix} z_s + l_c \theta_s - z_{b1} \\ z_s - l_c \theta_s - z_{b2} \end{bmatrix}$$

By taking Laplace transform,

$$\begin{bmatrix} m_s s^2 + 2c_s s + 2k_s & 0 \\ 0 & I_{cy} s^2 + 2l_c^2 c_s s + 2l_c^2 k_s \end{bmatrix} \begin{bmatrix} Z_s(s) \\ \theta_s(s) \end{bmatrix} = k_s \begin{bmatrix} Z_{b1}(s) \\ Z_{b2}(s) \end{bmatrix} + \dots \\ + c_s s \begin{bmatrix} 1 & 1 \\ l_c & -l_c \end{bmatrix} \begin{bmatrix} Z_{b1}(s) \\ Z_{b2}(s) \end{bmatrix} \quad (3.20)$$

Since the natural frequency and the damping ratio of the car body are calculated for free vibrations of the mechanical system, then Equation (3.20) will be written as;

$$\begin{bmatrix} (m_s s^2 + 2c_s s + 2k_s) Z_s(s) \\ (I_{cy} s^2 + 2l_c^2 c_s s + 2l_c^2 k_s) \theta_s(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3.21)$$

Then from Equation (3.21) and Equation (3.18), the natural frequencies and damping ratios for heave and pitch motions can be obtained as,

$$\omega_{n1} = \sqrt{\frac{2k_s}{m_s}}, \quad \zeta_{n1} = \frac{c_s}{\sqrt{2k_s m_s}} \quad \text{for car body heave motion} \\ \omega_{n2} = l_c \sqrt{\frac{2k_s}{I_{sy}}}, \quad \zeta_{n2} = \frac{l_c c_s}{\sqrt{2k_s I_{sy}}} \quad \text{for car body pitch motion} \quad (3.22)$$

The equations above show that the secondary suspension spring stiffness directly affects the car body's natural frequencies while the mass is inversely proportional. The damping ratio ζ_{n1} and ζ_{n2} are directly influenced by the damping coefficient c_s . The overshoot produced on the accelerations will be inversely proportional to them. Figures 3.7a-3.7b show the variations of the car body mode when the spring stiffness k_s changes in the range of $[0.1k_s, 100k_s]$, consistent with the higher spring values shift the natural frequen-

cies to the higher frequencies which may cause some overlapping with the bogie modes.

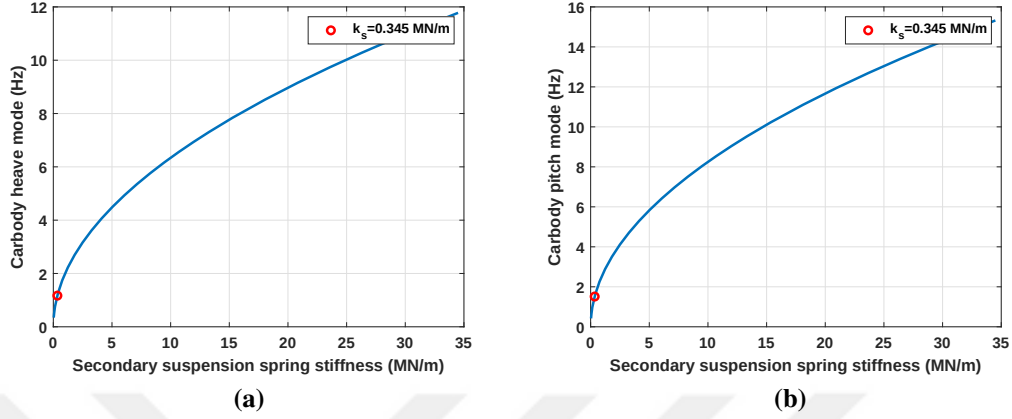


Figure 3.7. Car body modes (a) Heave motion (b) Pitch motion

Figures 3.8a-3.8b show how changes in the spring stiffness value influence car body vertical and pitch RMS accelerations. At first, the vertical acceleration rises almost linearly, but after a while, it becomes fixed. This is because a high spring stiffness leads to stiffening the spring, which causes elasticity loss. The other effect of spring stiffness

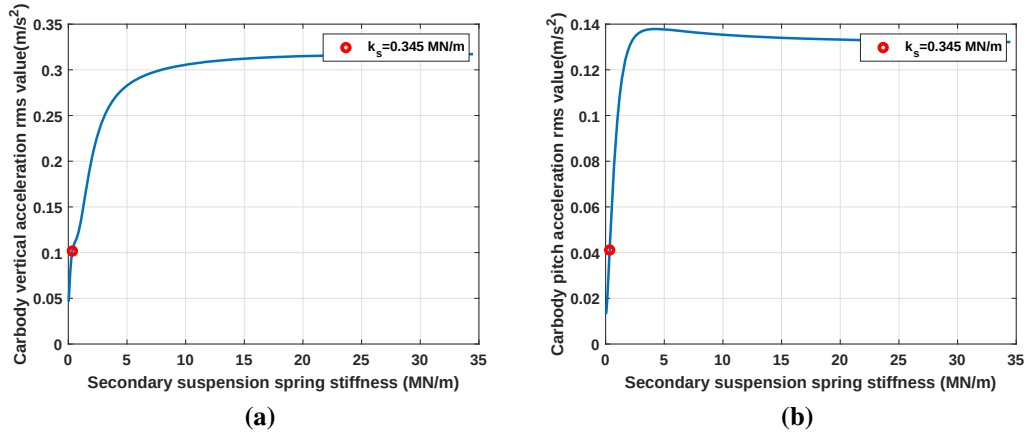


Figure 3.8. (a) The car body vertical acceleration rms values vs secondary suspension spring stiffness (b) The car body pitch acceleration rms values vs secondary suspension spring stiffness

value on the vehicle body can be handled on roll motion. The railway vehicles experienced centrifugal forces during the curve. These forces cause roll motions on the vehicle. To keep roll motions at an acceptable level, spring stiffness k_s should be quite large. Then,

in optimization studies, the only tunable parameters will be the damping coefficient and the body mass [32].

3.3.2. The effect of the damping coefficient

In railways, dampers are usually chosen as viscous or friction-damping devices which can control the oscillations in the primary/secondary suspension of the vehicle. The damper characteristics are symmetric in secondary suspension and asymmetric in primary suspension. The asymmetric nature may provide greater force in extension than in compression. Thus, asymmetric dampers are used for the motion of the wheel over a convex irregularity causing larger forces than negotiating a concave one, then leading to a decrease in the wheel load but an increase in the risk of derailment [8, 9]. Table 3.1 gives the railway vehicle natural frequencies, which are calculated by using Equation (3.22). Since the maximum peak occurs at nearly 1.2 Hz for car body vertical acceleration, below this frequency, the increases in the secondary damping may decrease the car body's vertical acceleration while raising the secondary suspension travel. However, above 1.2 Hz, the wheel-hop mode will start to occur, which causes increases in car body vertical acceleration responses as seen in Figure 3.9. Therefore, it can be concluded as the damping characteristics of the vehicle's suspension system play a significant role in determining the comfort conditions experienced by passengers. We observed that increased damping can lead to a rise in the root mean square value of car body vertical acceleration, indicating poorer ride comfort [32, 104].

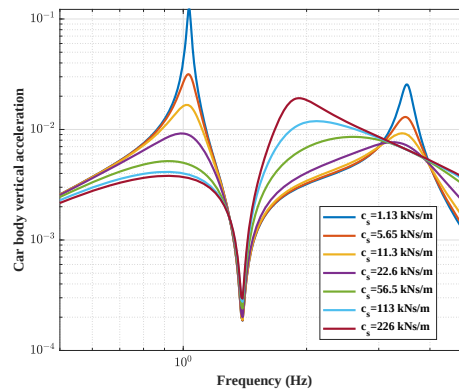


Figure 3.9. Secondary suspension damper value effect on wagon body vertical acceleration in frequency domain

In this section, to analyze the damping coefficient effect on vehicle responses, let's change the secondary suspension damping coefficient c_s from $0.1c_s$ to $100c_s$. Figures 3.10a-3.10b will provide the simulation performed in this range. From the figures, it is easy to observe the opposite relationship between vertical/pitch acceleration and suspension travel, similar to the automobile trade-offs which is discussed in reference [105]. The optimal value for AM-96 damper can be chosen as 0.55, marked with a red circle.

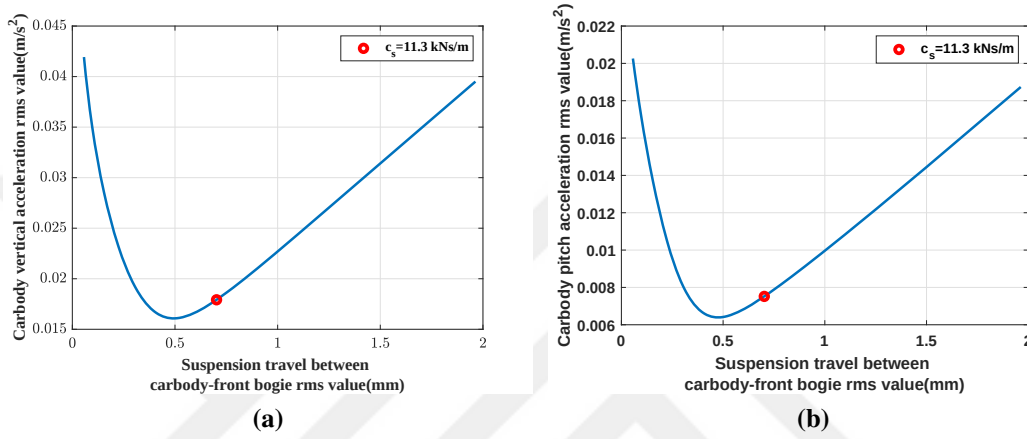


Figure 3.10. (a) Car body vertical acceleration rms values vs front secondary suspension travel rms values
(b) Car body pitch acceleration rms values vs front secondary suspension travel rms values in terms of secondary suspension damping coefficients

The car body mass is another parameter that directly influences the ride performance. It may vary depending on the number of passengers and luggage occupancy or structural elements. Changing mass influences the body's natural frequency, as expected and shown in Equation (3.22). The heavier wagon causes less vibrations on the body but creates higher forces. Then, the effect of the mass will be analyzed in Chapter 6.

3.3.3. Track quality effects

It is evident from Equation (3.17) that the track roughness parameter affects the RMS responses directly. From this point of view, the RMS values are calculated for each track class in compliance with defined speed limits. The worst results for suspension deformation were obtained for the track Class-4 as shown in Figure 3.11a and according to the track class RMS responses of the vehicle are given in Figures-3.12 - 3.12.

The bode diagrams in Figures 3.13a-3.13b can be expressed as rising track quality

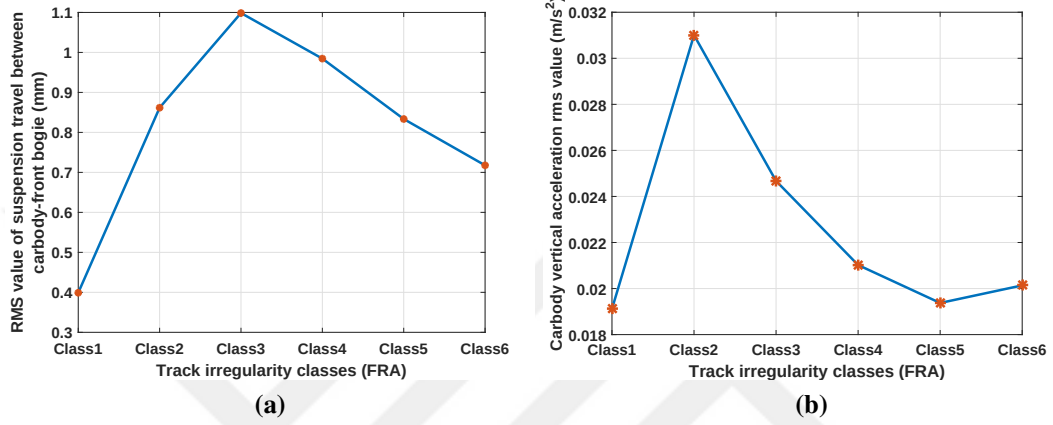


Figure 3.11. Track quality effect on (a) The secondary suspension travel rms value between car body and front bogie (b) on carbody vertical acceleration rms value

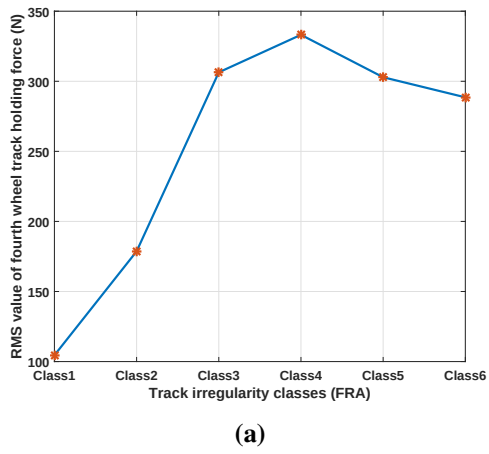


Figure 3.12. Track quality effect on the wheel force

shifts the second mode to the high frequencies. On the other hand, it is observed that for lower frequencies, the RMS value of the car body heave acceleration decreases. According to the Figure 3.13a, though ensuring the speed limit, the track Class-2 is the worst class because of its worst effect on the car body's vertical acceleration.

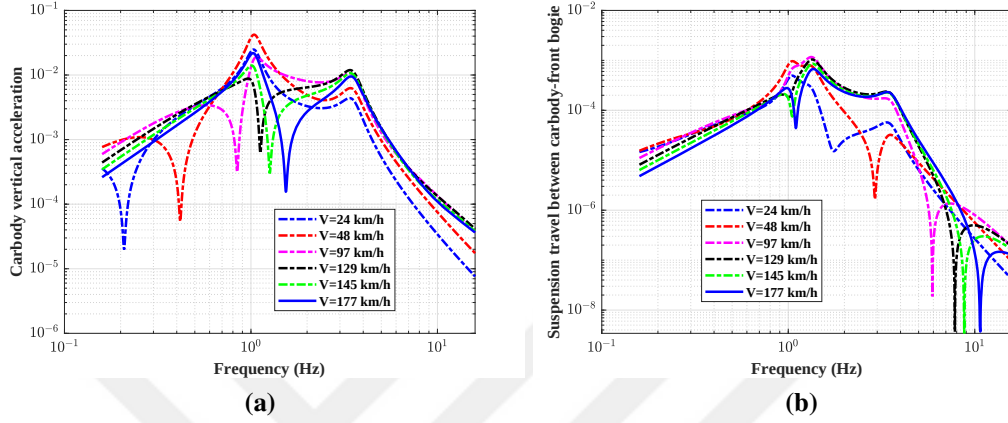


Figure 3.13. Track quality effect on (a) Carbody vertical acceleration in frequency domain (b) on the secondary suspension travel between car body and front bogie in frequency domain

3.3.4. Velocity effect

Vehicle speed is another important parameter that affects the vehicle RMS responses defined in Equation (3.17). For track Class-6, the vehicle is simulated by changing the traveling speed from 40 km/h to 220 km/h . As shown in Figures 3.14-3.15, raising the speed increases wheel-track holding forces while dramatically deteriorating passenger comfort.

According to Equation (3.17), higher speeds are expected to increase the dynamic response on the passenger/freight train. The studies [106, 107] show that, at the same speed, the freight train dynamics are dramatically influenced than the passenger trains. A solution to separate the lines for freight and passenger vehicles is suggested. However, the freight trains still cause ground vibrations due to their heavier loads, especially in the 4-30 Hz frequency range. In this frequency range, when the amplitude of the vibration is large, the vibration can be felt. In addition, it may cause disturbances for people living near the railways and damage to lineside structures [108, 109]. Therefore the freight train should be specially treated. Designing lightweight bodies may seem like an easy solution.

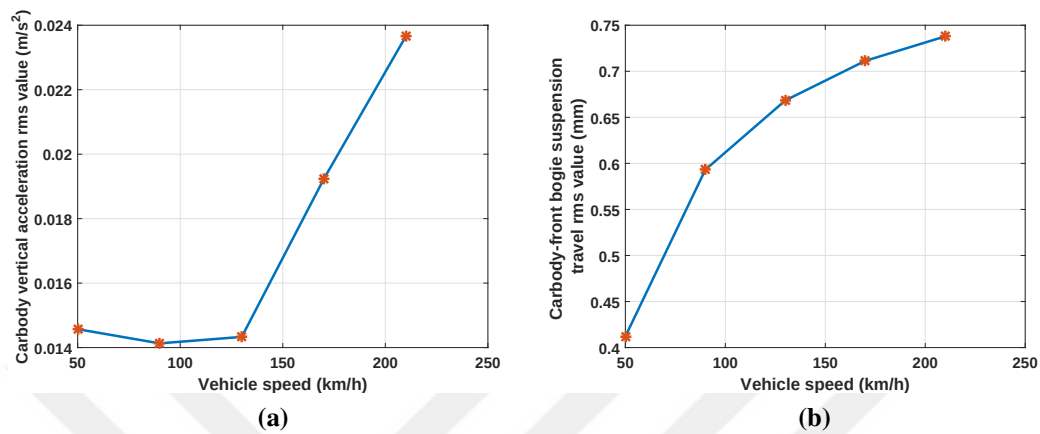


Figure 3.14. Vehicle speed effect on (a) Carbody vertical acceleration rms value (b) on the secondary suspension travel rms value between car body and front bogie

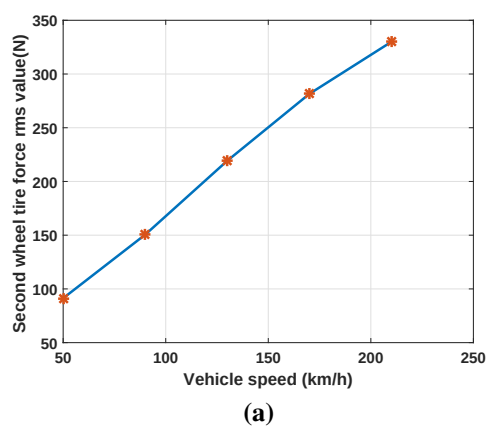


Figure 3.15. Vehicle speed effect on second wheel force

Although the lightweight freight wagon design is one of the significant applications of using aluminum in railways, it has many drawbacks, such as energy efficiency, lower wheel forces, retaining a high salvage value when recycled at the end of their life cycle and additionally large body vibrations [110].

3.4. Freight Wagon Dynamics

Unlike passenger wagons, freight wagons have no primary suspensions between wheels and bogie frames. For the secondary suspension, dry friction is generally used for freight trains [9]. The friction elements are load dependent, meaning the friction damping level varies with the axle load. It needs less maintenance, so these elements have low costs [111]. However, in some European countries, freight vehicles have primary suspensions. The French National Railways (SNCF) developed a new bogie type for freight wagons with lighter and shorter structures. The bogie Y21 was designed in 1960 and upgraded to Y25 bogies in 1966. The Y25 bogie has been a UIC standard freight bogie since 1967. The Y25 bogies have a suspension structure with friction dampers and intertwined coil springs instead of leaf springs [107, 112, 109]. The outer coil of intertwined coil springs is connected to the bogie frames using Lenoir-Link and carries the vertical load to the bogie center. In this section, it is assumed that the Y25 freight vehicle goes on a straight line with a constant speed, and during the riding, no wheel jumps occur. The following section will discuss the vehicle speed and the track quality effects on vehicle response.

3.4.1. The influence of speed on riding performances

In this part of the thesis, to examine speed effects on freight train riding, for track class-5, the vehicle performance is simulated by changing speed from 16 km/h to 130 km/h . As shown in Figures 3.16-3.17, rising speed increases wheel forces and the car body vibrations. Also, it may be concluded that the freight wagon wheels have larger dynamic forces at the same track quality than the passenger wagons'. At higher speeds, wheel forces are increased, which may cause rises in ground vibrations.

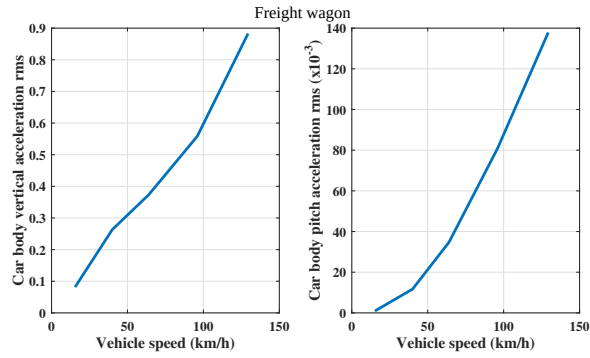


Figure 3.16. Effects of vehicle speed onto the RMS values of freight wagon body vertical accelerations

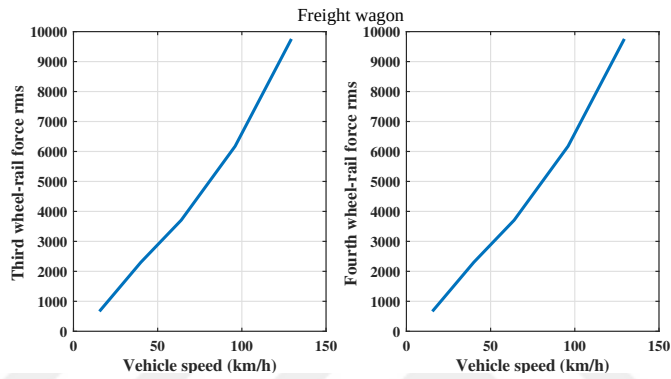


Figure 3.17. Effects of vehicle speed onto the RMS values of the freight wagon wheel forces

3.4.2. The effects of track quality

The track roughness parameter stands for the track quality, and its effects can be examined using Table 2.3. Let's calculate the RMS values of the Y25 freight wagon for each track class in compliance with defined speed limits. The worst results for suspension deformation were obtained for the track Class-5 as shown in Figures 3.18-3.19. Also, it can be concluded that even if ensuring the speed limits, a higher quality track and high speeds cause the transmitted wheel forces are getting higher.

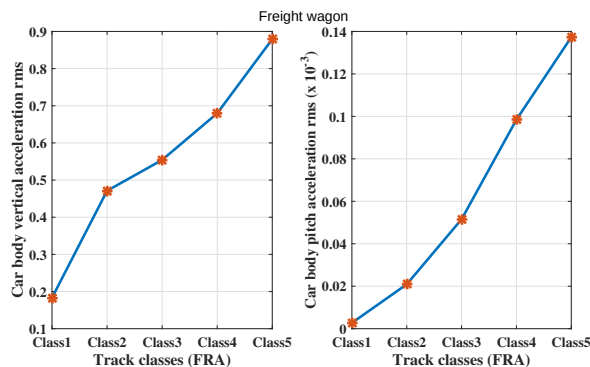


Figure 3.18. Effects of track quality onto the RMS values of the freight wagon body vertical accelerations

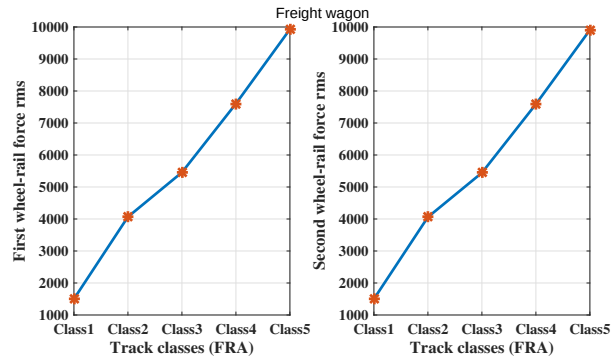


Figure 3.19. Effects of track quality onto the RMS values of the freight wagon wheel forces

3.5. Summary

In this chapter, a high-speed railway passenger vehicle was modeled as a 6DOF half-car model. The parametric studies of mass, spring, damping, track quality, and vehicle speed were presented to define the performance limitations of the vehicle. The RMS results and the frequency responses of the vehicle outputs were discussed by tuning these parameters. It was shown that the track quality affected the driving performance. As expected, the results showed that higher speeds cause uncomfortable travels with larger force transmission to the track. Traveling with better comfort requires the trains to move on a smoother track. We can conclude that all passive elements should be tuned separately to achieve better performance. However, since there is a strong conflict between the performance indicators, the benefit of passive suspension optimization will be limited.

4. SEMI-ACTIVE CONTROL

In the previous section, we have seen that in passive systems, the suspension parameters are optimized according to the safety and comfort requirements. But there is a contradiction, and they can not be improved with a passive suspension system. As a result, the necessity of developing semi-active or active suspension systems has been revealed. Regarding performance achievement, the semi-active systems keep their place between the passive and active systems [113, 97, 114, 35]. The main role of the semi-active systems is to ensure passenger comfort and safety by isolating the body from the ground vibrations. They are based on replacing the passive damper with an adjustable damper. Control algorithms can change the damping values of these dampers in some intervals. Since the damper force depends on the velocity of the sprung and unsprung masses, these semi-active systems have upper and lower bounds which restrict the performance. However, semi-active systems consume less power than active systems, which makes them attractive to designers. Unlike the active systems, the semi-active systems can not cause the suspension system to become unstable [45]. The most commonly used semi-active control algorithm is implemented in vehicles using skyhook dampers, groundhook dampers, or a combination of them (hybrid control) [36, 115].

In this section, as a semi-active controller skyhook, groundhook, and hybrid control algorithms are applied to the railway vehicle with rigid wheel-rail contact. In order to get better performances the optimal values of the damper should be determined. The gradient method and linear matrix inequalities were used to obtain the ultimate values that provide more comfortable and safer riding conditions.

4.1. The Skyhook Control

Skyhook dampers are the most common semi-active control elements for vehicles. Because of the active systems' high costs, in [38], the skyhook dampers were developed and suggested as semi-active force generators to control the vibrations. The skyhook damper can be defined as the damper connected between the body and the sky as in Figure 4.1. Here the abbreviation s denotes sprung mass (car body) while u refers to unsprung mass (bogie). The main idea is to apply high damping to the sprung mass while

the sprung and unsprung mass comes close to or move away from each other.

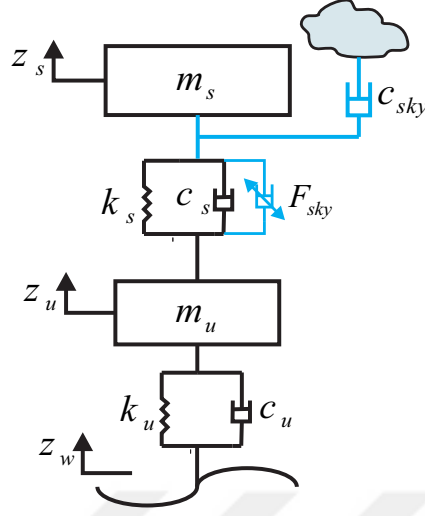


Figure 4.1. The skyhook control algorithm

Realizing these connections physically is impossible, so this control algorithm is explained mathematically in Equation (4.1). Since the force is directly related to the absolute velocity of the body, the skyhook damper is named as absolute damper.

$$F_{sky} = c_{sky} \dot{z}_s \operatorname{sign}(z_{rel}) H(z_{rel}), \quad \text{with} \quad z_{rel} = \dot{z}_s(\dot{z}_s - \dot{z}_u) \quad (4.1)$$

Here sign is the signum function, and $H(z_{rel})$ is the Heaviside step function (or unit-step function).

4.2. The Groundhook Control

Similar to a skyhook damper, the groundhook damper is a semi-active control element. It is achieved by connecting a damper with a variable value between the unsprung mass and the ground, as in Figure 4.2. The dynamic behavior is developed by applying lower damping to the unsprung mass while the sprung and unsprung mass come close to or move away from each other and can be formulated with Equation (4.2). The expected performance is to suppress the unsprung mass body accelerations on the high frequencies.

$$F_{gnd} = -c_{gnd} \dot{z}_u \operatorname{sign}(\tilde{z}_{rel}) H(-\tilde{z}_{rel}), \quad \text{with} \quad \tilde{z}_{rel} = \dot{z}_u(\dot{z}_s - \dot{z}_u), \quad (4.2)$$

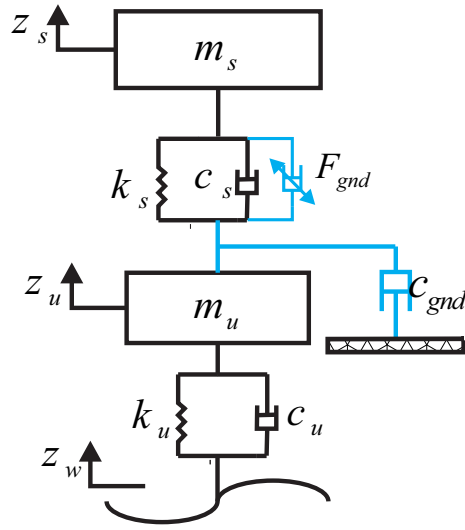


Figure 4.2. The groundhook control algorithm

4.3. The Hybrid Control

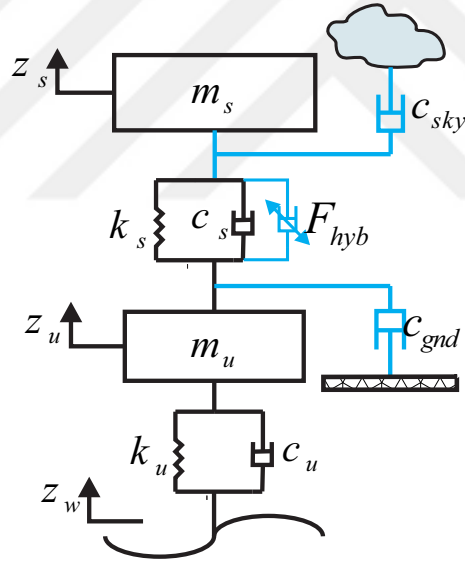


Figure 4.3. The hybrid control algorithm

Hybrid control is a semi-active control algorithm that is a combination of skyhook and groundhook methods and configuration is shown in Figure 4.3. According to the hybrid control method, the velocities of the sprung and unsprung masses should be controlled at every time instant. In hybrid control, high damping is applied to the sprung mass if skyhook conditions occur. If groundhook conditions occur, low damping will be applied to the unsprung mass. If both conditions are met, the damping ratios proportional to the purpose of the controller design are applied to the masses [116, 117]. The hybrid

control algorithm may also be explained as below.

Algorithm 1 Hybrid control algorithm

Require: $G = [0, ..., G_{max}]$, $\mu \in [0, 1]$.

- 1: Define the hybrid control force $F_{hyb} = \mathcal{G}z_{vel}$ with $\mathcal{G} > 0$.
- 2: Choose μ such that,
- 3: **if** *main purpose is getting better comfort* **then**,

$$\mu \in [0.5, 1]$$

- 4: **else if** *main purpose is getting safer travel* **then**,

$$\mu \in [0, 0.5]$$

- 5: **end if**
 - 6: **for** $p=1:\text{length}\{G\}$ **do**,
 - 7: $\mathcal{G} = G(p)$,
 - 8: **for** $t = 0 : t_{max}$ **do** ,
 - 9: Measure $\dot{z}_s(t)$ and $\dot{z}_u(t)$,
 - 10: **if** $\dot{z}_s(t) (\dot{z}_s(t) - \dot{z}_u(t)) > 0$ & $-\dot{z}_u(t) (\dot{z}_s(t) - \dot{z}_u(t)) \neq 0$ **then**,
 - $z_{vel}(t) = \mu \dot{z}_s(t)$
 - 11: **else if** $-\dot{z}_u(t) (\dot{z}_s(t) - \dot{z}_u(t)) > 0$ & $\dot{z}_s(t) (\dot{z}_s(t) - \dot{z}_u(t)) \neq 0$ **then**,
 - $z_{vel}(t) = (1 - \mu) \dot{z}_u(t)$
 - 12: **else if** $-\dot{z}_u(t) \dot{z}_s(t) > 0$ **then**,
 - $z_{vel}(t) = \mu \dot{z}_s(t) + (1 - \mu) \dot{z}_u(t)$,
 - 13: **else**
 - $z_{vel}(t) = 0$
 - 14: **end if**
 - 15: **end for**
 - 16: **end for**
 - 17: Detect optimum value for $\mathcal{G}$.
-

Detecting skyhook and groundhook dampers' and hybrid design parameters' optimal values requires a detailed study. The next subsection will deal with the lot.

4.4. Gradient Method

In this section, a gradient-based algorithm will be used to find the optimal value of the damper. The RMS outputs of the vehicle are calculated for a range of skyhook dampers, and the optimal values will be detected by finding the global minimum of these quantities. It is assumed that the half-car model moves on Konya-Polatlı high-speed main line at a constant speed. In other words, the Konya-Polatlı high-speed mainline track measurement data will excite the high-speed railway vehicle. The data used here is obtained from Turkish State Railways, collected by Piri Reis track recording car using optical sensors placed on axles. Namely, the data is equivalent to the wheel's displacement measurements from a reference point. Therefore the plant for the 6DOF vehicle model that is defined in the previous section with $\mathcal{P}$ should be updated from wheel displacement vector z_w to the output vector z by redefining a new state vector as,

$$\tilde{x} := x - B_1 z_w$$

To realize change of states, the following conditions to be satisfied are,

1. The A matrix should be full rank (invertible)
2. The plant should be strictly proper

After change of variables the new state space equation with $\tilde{B}_1 = AB_1$ and $\tilde{D}_{11} = C_2B_1$ will have the following form.

$$\tilde{\mathcal{P}} : \begin{cases} \dot{\tilde{x}} = A\tilde{x} + \tilde{B}_1 z_w + B_2 u \\ z = C_2 \tilde{x} + \tilde{D}_{11} z_w + D_{22} u \end{cases} \quad (4.3)$$

Implementing the skyhook (groundhook) algorithm requires absolute and relative velocities for every instant. For this, the plant $\tilde{\mathcal{P}}$ in Equation (4.3) should be transformed into the discrete-time domain. Since the samples of Konya-Polatlı high-speed mainline track are collected from each 0.5 m, then sampling time will be $t_s = 0.5/v$, and the plant $\tilde{\mathcal{P}}$ is transformed into discrete time by using the zero-order-hold (ZOH) method [103].

The discrete-time state space equation is obtained in Equation (4.4).

$$\mathcal{P}_d : \begin{cases} \tilde{x}[k+1] = A_d \tilde{x}[k] + B_1^d z_w[k] + B_2^d u[k] \\ z[k] = C_2^d \tilde{x}[k] + D_{21}^d z_w[k] + D_{22}^d u[k] \end{cases} \quad (4.4)$$

The skyhook dampers are implemented both on the front and rear suspension. So, the front and rear skyhook dampers were chosen as $c_{sky1} = c_{sky2} = c_{sky}$ to decouple the modes from each other. Then the control force $u[k]$ can be written as,

$$u[k] = C_y^d x[k] = \begin{bmatrix} c_{sky} \dot{z}_{s1}[k] \operatorname{sign}(\dot{z}_{s1}[k](\dot{z}_{s1}[k] - \dot{z}_{b1}[k])) H(\dot{z}_{s1}[k](\dot{z}_{s1}[k] - \dot{z}_{b1}[k])) \\ c_{sky} \dot{z}_{s2}[k] \operatorname{sign}(\dot{z}_{s2}[k](\dot{z}_{s2}[k] - \dot{z}_{b2}[k])) H(\dot{z}_{s2}[k](\dot{z}_{s2}[k] - \dot{z}_{b2}[k])) \end{bmatrix} \quad (4.5)$$

The closed loop output vector $z_{cl}[k]$ can be obtained by substituting the control forces in Equation (4.5) into the plant $\mathcal{P}_d$. Also, Konya-Polatli mainline measurements are directly applied for $z_w[k]$. Then the closed-loop state space will be;

$$\mathcal{P}_{dcl} : \begin{cases} \tilde{x}_{cl}[k+1] = (A_d + B_2^d C_y^d) \tilde{x}_{cl}[k] + B_1^d z_w[k] \\ z_{cl}[k] = (C_2^d + D_{22}^d C_y^d) \tilde{x}_{cl}[k] + D_{21}^d z_w[k] \end{cases} \quad (4.6)$$

To analyze performances of the skyhook dampers, the RMS values are calculated from Equation (4.7) where z_{cl}^j is the j^{th} element of the output vector, L is the length of z_{cl}^j . Since we have 5000 data from Konya-Polatli mainline measurements, L equals 5000 [96]. Here, the vehicle parameters are used for French TGV. Then, Figures 4.4a-4.5b show the changes in RMS car body vertical acceleration in terms of skyhook damper value in the range of $c_{sky} \in [0, 100] \text{ kNs/m}$. We can see from the figures that the best skyhook damping value was determined as $c_{sky} = 61 \text{ kNs/m}$, which is the global minimum of car body vertical acceleration.

$$RMS(z_{cl}^j) = \frac{1}{L} \sum_{n=1}^{L+1} z_{cl}^j[n] z_{cl}^{j*}[n], \quad j = 1, \dots, 4. \quad (4.7)$$

Let's observe the time domain responses by fixing the skyhook damping value at $c_{sky} = 61 \text{ kNs/m}$. The results are presented in Figures 4.5a - 4.5b. The real-time skyhook

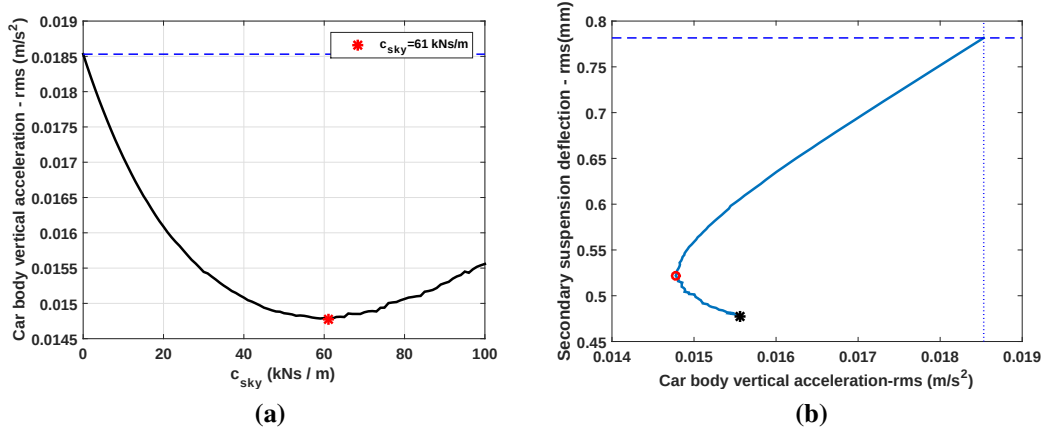


Figure 4.4. Skyhook damping effects on (a) the carbody vertical acceleration rms values (b) the secondary suspension travel between car body and front bogie rms value

damper design greatly reduced vertical acceleration and suspension deflections of the car body. In the previous section, it was seen that high damping may decrease the car body's vertical acceleration, and the same observation may be seen from the results. To examine the roll response, the nine degrees-of-freedom (9 DOF) vehicle mathematical model shown in Figure (4.6) will be explained.

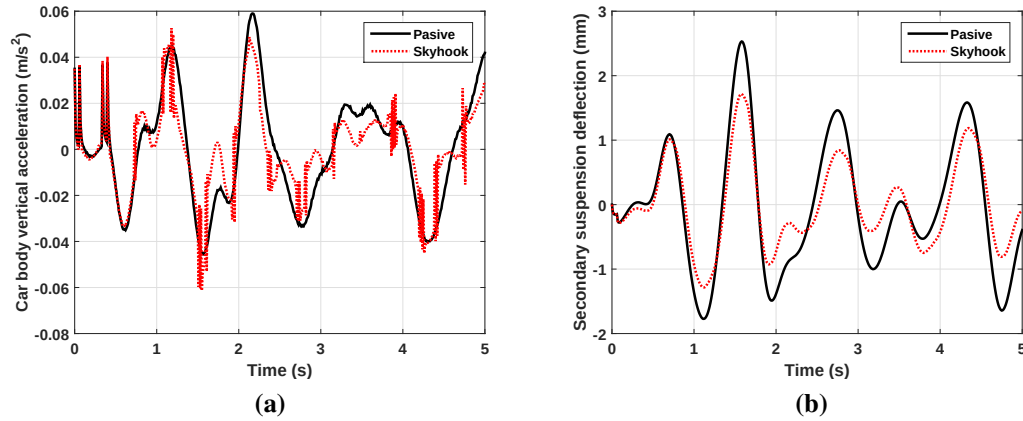


Figure 4.5. Skyhook on/off control time domain results (a) the carbody vertical acceleration (b) the secondary suspension travel between car body and front bogie

4.4.1. 9DOF vehicle model

A similar notation used in Figure 3.6 will be used in this section. Since the roll motion is added to the dynamics, the state vector in the plant $\mathcal{P}$ in Equation (3.5) is reformed as $x = [x_s, x_{bf}, x_{br}]$ with vectors $x_s = [z_s, \theta_s, \varphi_s]^T$, $x_{bf} = [z_{b1}, \theta_{b1}, \varphi_{b1}]^T$,

$x_{br} = [z_{b2}, \theta_{b2}, \varphi_{b2}]^T$ and the plant will be adopted accordingly. The roll angles of the car body and bogies are denoted with $(\varphi_s, \varphi_{b1}, \varphi_{b2})$ respectively.

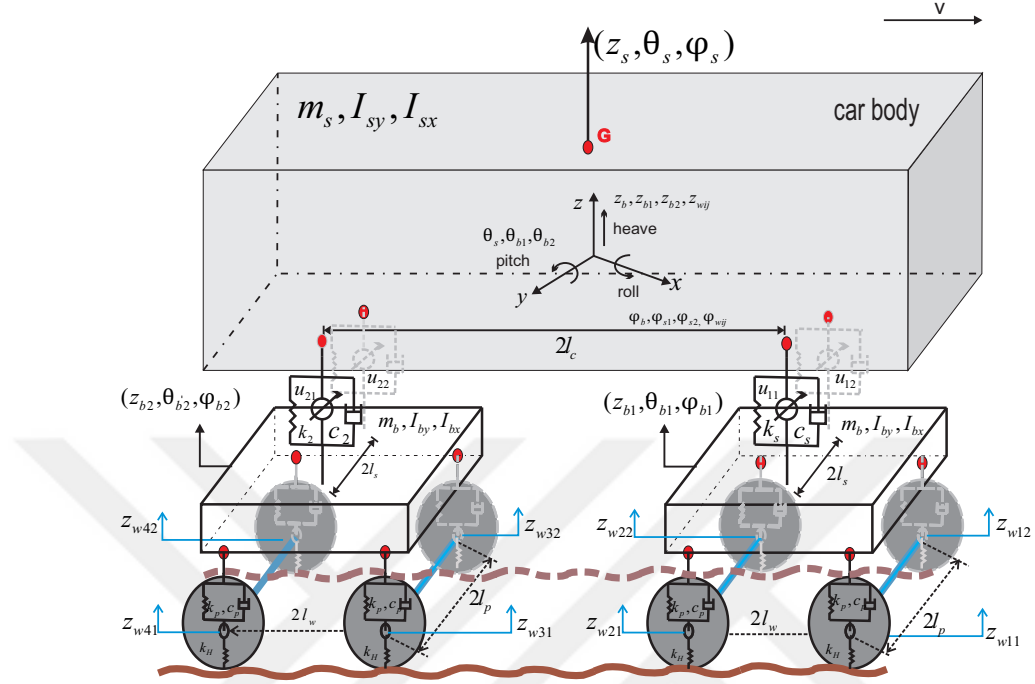


Figure 4.6. 9 DOF mathematical model of a wagon

The relative displacements can be defined as follows:

$$\begin{aligned} \begin{bmatrix} z_{11} & z_{12} & z_{21} & z_{22} \end{bmatrix}^T &= S_c x_s, \\ \begin{bmatrix} z'_{11} & z'_{12} & z'_{21} & z'_{22} \end{bmatrix}^T &= T_c S_c x_{bf} + T_t S_c x_{br}, \\ \begin{bmatrix} z_{b11} & z_{b12} & z_{b21} & z_{b22} \end{bmatrix}^T &= S_t x_{bf}, \\ \begin{bmatrix} z_{b31} & z_{b32} & z_{b41} & z_{b42} \end{bmatrix}^T &= S_t x_{br}, \end{aligned}$$

$$S_c = \begin{bmatrix} 1 & l_c & -l_s \\ 1 & l_c & l_s \\ 1 & -l_c & -l_s \\ 1 & -l_c & l_s \end{bmatrix}, \quad S_t = \begin{bmatrix} 1 & l_w & -l_p \\ 1 & l_w & l_p \\ 1 & -l_w & -l_p \\ 1 & -l_w & l_p \end{bmatrix},$$

$$T_c = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad T_t = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$$

Then, force equations for the secondary suspensions (F_{ij}) and primary suspensions (F_{bmn}) can be re-written as in Equation (4.8) for $i, j = 1, 2$, $m = 1, \dots, 4$, and $n = 1, 2$, as,

$$\begin{aligned} F_{ij} &= -k_s(z_{ij} - z'_{ij}) - c_s(\dot{z}_{ij} - \dot{z}'_{ij}), \\ F_{bmn} &= -k_p(z_{bmn} - z_{wmn}) - c_p(\dot{z}_{bmn} - \dot{z}_{wmn}) \end{aligned} \quad (4.8)$$

The equations of motions are updated with new control vector $u = [u_{11}, u_{12}, u_{21}, u_{22}]$ as below:

- For car body;

$$\begin{aligned} m_s \ddot{z}_s &= \sum_{i=1}^2 \sum_{j=1}^2 F_{ij} - u_{ij}, \\ I_{sy} \ddot{\theta}_s &= \sum_{j=1}^2 -l_c \{F_{2j} - u_{2j}\} + l_c \{F_{1j} - u_{1j}\}, \\ I_{sx} \ddot{\varphi}_s &= \sum_{i=1}^2 -l_s \{F_{i1} - u_{i1}\} + l_s \{F_{i2} - u_{i2}\}, \end{aligned} \quad (4.9)$$

- For front ($i = 1$) bogie and rear ($i = 2$) bogie

$$\begin{aligned} m_b \ddot{z}_{bi} &= \sum_{j=1}^2 \{-F_{ij} + u_{ij}\} + \sum_{m=2i-1}^{2i} \sum_{n=1}^2 F_{bmn}, \\ I_{by} \ddot{\theta}_{bi} &= \sum_{n=1}^2 l_w \{-F_{b(2i)n} + F_{b(2i-1)n}\}, \\ I_{bx} \ddot{\varphi}_{bi} &= \sum_{m=2i-1}^{2i} l_p \{-F_{bm1} + F_{bm2}\} + \dots \\ &\quad + l_s \{F_{i1} - u_{i1}\} - l_s \{F_{i2} - u_{i2}\} \end{aligned} \quad (4.10)$$

The state-space matrices in the plant $\tilde{\mathcal{P}}$ are translated into:

$$A = \begin{bmatrix} 0_{9 \times 9} & I_9 \\ -M^{-1}\tilde{K} & -M^{-1}\tilde{C} \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0_{9 \times 8} \\ k_1 M^{-1}\tilde{S}_t^T \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0_{9 \times 8} \\ c_1 M^{-1}\tilde{S}_t^T \end{bmatrix},$$

$$\tilde{B}_1 = B_{11} + AB_{12}, \quad B_2 = \begin{bmatrix} 0_{9 \times 4} \\ -M^{-1}(T\tilde{S}_c)^T \end{bmatrix},$$

with,

$$\tilde{M}_s = \text{diag}(m_s, I_{sy}, I_{sx}),$$

$$\tilde{M}_b = \text{diag}(m_b, I_{by}, I_{bx}),$$

$$M = \text{diag}(\tilde{M}_s, \tilde{M}_b, \tilde{M}_b),$$

$$K_s = \text{diag}(k_s I_{12}, k_p I_8),$$

$$C_s = \text{diag}(c_s I_{12}, c_p I_8),$$

$$T = \begin{bmatrix} I_4 & -T_c & -T_t \end{bmatrix},$$

$$T_s = \text{diag}(-T^T T, -I_8),$$

$$\tilde{S}_c = I_3 \otimes S_c,$$

$$\tilde{S}_t = \begin{bmatrix} 0_{8 \times 3} & I_2 \otimes S_t \end{bmatrix},$$

$$\tilde{S}_s = \begin{bmatrix} \tilde{S}_c^T & \tilde{S}_t^T \end{bmatrix}^T,$$

$$\tilde{K} = \tilde{S}_s^T K_s T_s \tilde{S}_s$$

$$\tilde{C} = \tilde{S}_s^T C_s T_s \tilde{S}_s$$

Determining the skyhook optimal value requires the RMS values of the car body vertical accelerations to be calculated and plotted for $0 \leq c_{sky} \leq 100 \text{ kNs/m}$. It is seen that the value 68 kNs/m provides the global minimum for the vertical acceleration, but it increases the RMS car body roll acceleration. Since the car body roll acceleration also has a significant effect on passenger comfort, the value of the skyhook damping may be chosen as $c_{sky} = 26 \text{ kNs/m}$, which minimizes the RMS car body roll acceleration while decreasing the RMS car body heave acceleration. Figures 4.7-4.8 show the skyhook damping effects on vehicle outputs for Siemens ICE train parameters [84].

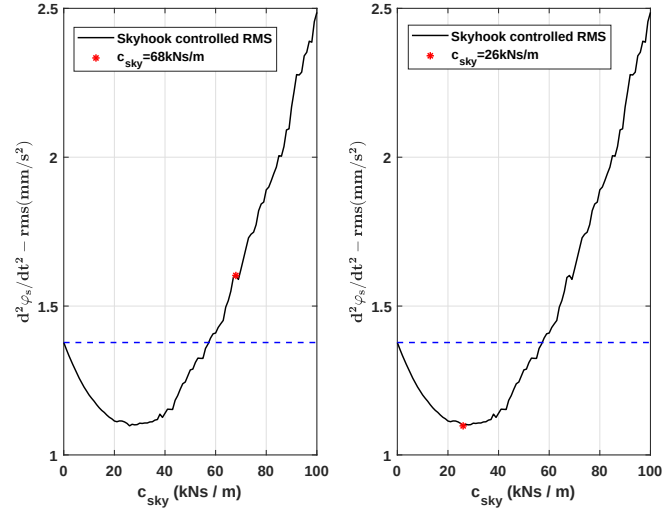


Figure 4.7. On/off skyhook damping effects on car body roll acceleration

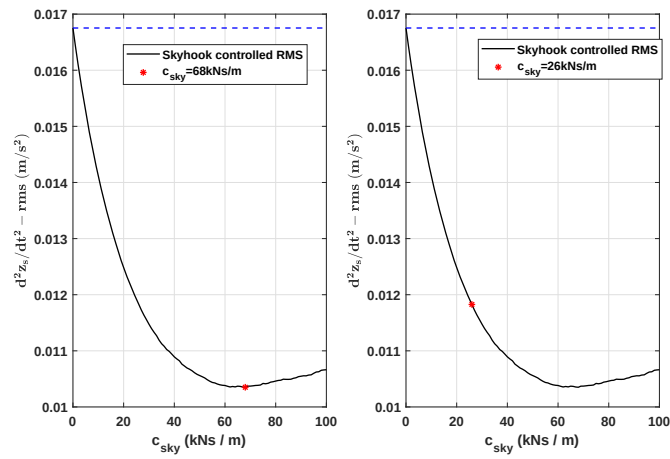


Figure 4.8. On/off skyhook damping effects on car body vertical acceleration

The time domain results are provided in Figures 4.9- 4.10. We can see that the skyhook damper design provides better responses than the uncontrolled vehicle. It should be noted that with a skyhook design, the overall damping of the secondary suspension increases. Since the damping increases, the car body vibration is suppressed better and more effectively. Let's discuss the groundhook damper design for 9 DOF full-car vehicle analytical models.

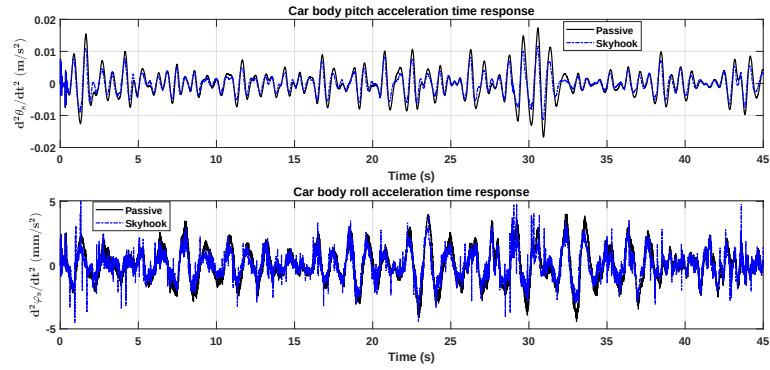


Figure 4.9. On/off skyhook damping time domain results for $c_{sky} = 26 \text{ kN s/m}$

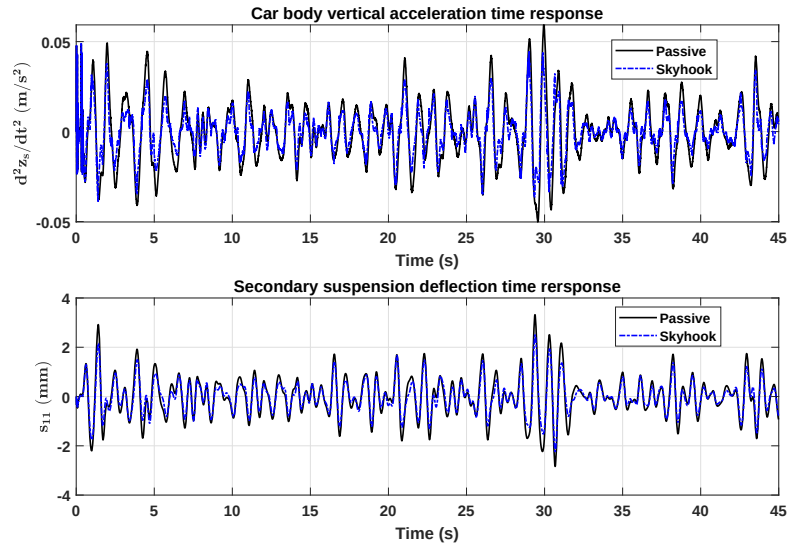


Figure 4.10. On/off skyhook damping time domain results for $c_{sky} = 26 \text{ kN s/m}$

Here, the front and rear groundhook dampers were selected as equal. The effects of groundhook control on car body roll motions Figures 4.11 is studied. Here, $c_{gnd} = 24 \text{ kN s/m}$ provides the global minimum for car body heave acceleration and gives better results than passive values in roll acceleration. The time domain results by

fixing the groundhook damper on 24 kNs/m are shown in Figures 4.12-4.13. Although the groundhook provides better ride comfort than the passive system, the results are less impressive than the skyhook damping. Also, getting smaller suspension deflections by using a groundhook damping connection is impossible. Therefore, it is suggested to consider the hybrid control as a semi-active controller.

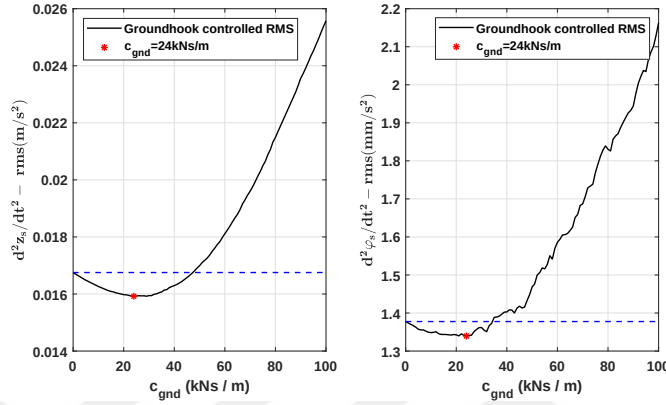


Figure 4.11. On/off groundhook damping effects on car body roll acceleration

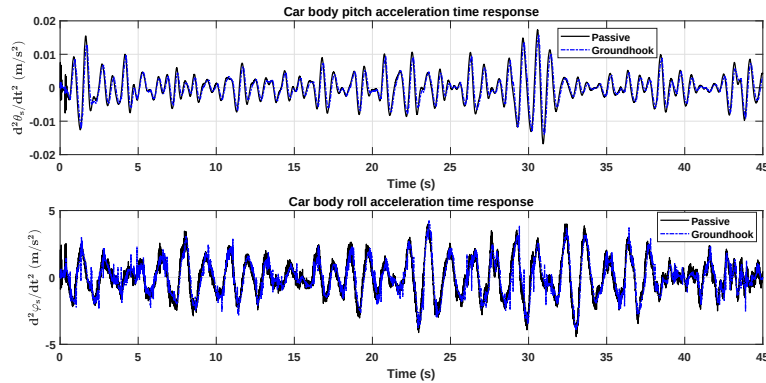


Figure 4.12. On/off groundhook damping time domain results for Siemens ICE 9DOF mathematical model

Figures 4.14a-4.14b for the 6DOF half car model and Figures 4.15-4.16 for the 9DOF full car model show the hybrid time domain results. The performances of the designed skyhook, groundhook, and hybrid control algorithm in terms of RMS quantities are listed in Table 4.1. Although the best suspension deflection results were obtained from the skyhook control, for comfort, the hybrid design performs much better. The designed semi-active controllers are in real-time, in order to investigate the frequency domain effects, the problem should be re-defined in the control-oriented framework and solved by using LMIs.

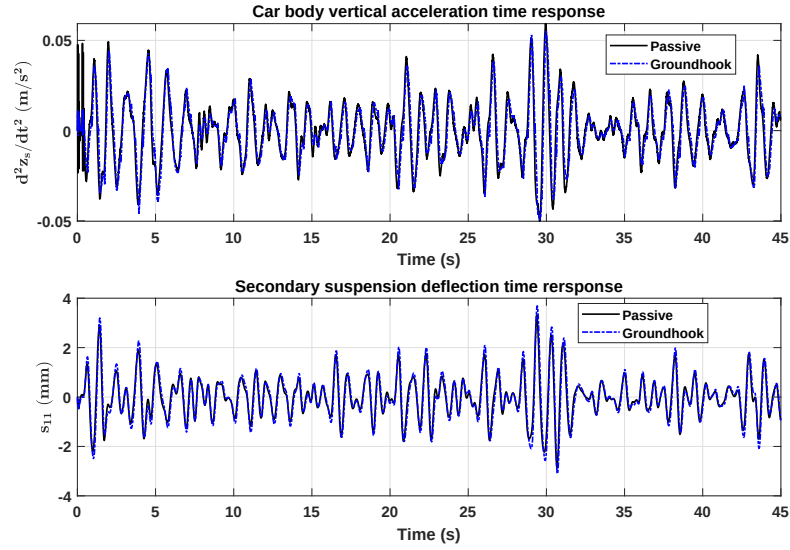


Figure 4.13. On/off groundhook damping time domain results for Siemens ICE 9DOF mathematical model

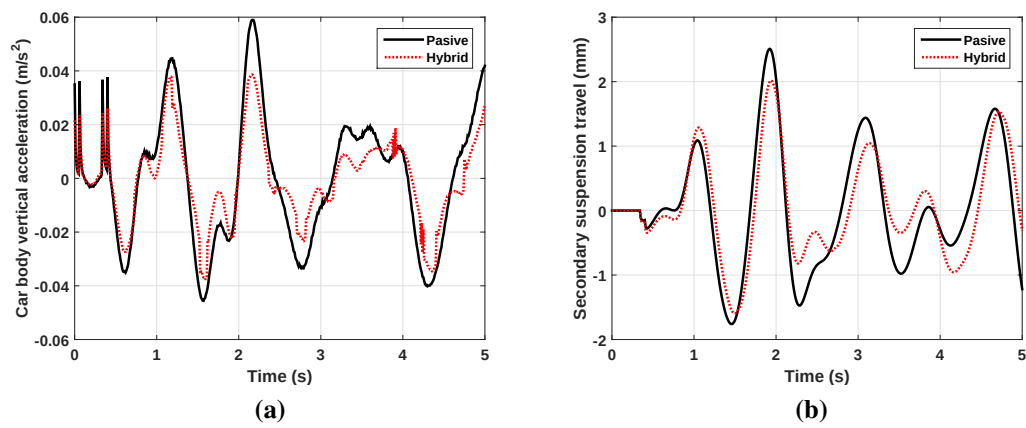


Figure 4.14. Hybrid control time domain results (a) the carbody vertical acceleration (b) the secondary suspension travel between car body and front bogie

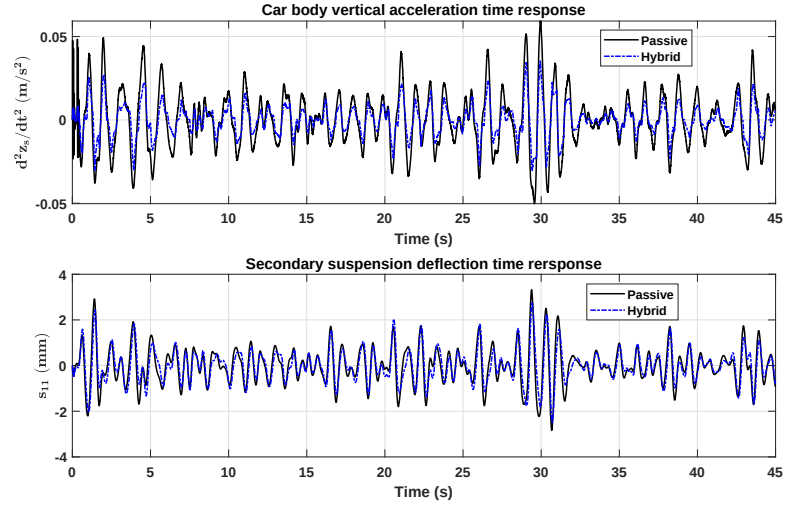


Figure 4.15. Hybrid control time domain results for Siemens ICE 9DOF mathematical model

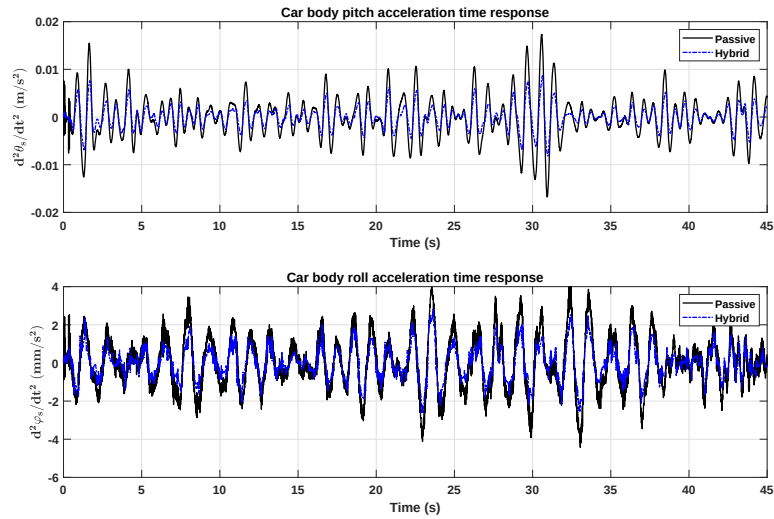


Figure 4.16. Hybrid control time domain results for Siemens ICE 9DOF mathematical model

Table 4.1. The % rms performances of the output vector

6 DOF Model Output	Skyhook	Groundhook	Hybrid
$\ddot{z}_s$	20.25	21	34.07
$\ddot{\theta}_s$	13.48	26	29.7
s_1	33.2	-17	20.7
s_2	32.6	-17.6	20.11
9DOF Model Output	Skyhook	Groundhook	Hybrid
$\ddot{z}_s$	29.4	4.96	44.4
$\ddot{\theta}_s$	29.08	12.10	48.33
$\ddot{\varphi}_s$	20.32	2.76	37.8
s_{11}	23.037	-11.44	12

4.5. LMI Based Optimization

Let's define the multi-objective problem into the following problem.

Problem 4.5.1 For a given positive number γ , design an output feedback controller $u = K(s)y$ that minimize the objective function J under defined constraints in Equation (4.11) and stabilize the closed loop system.

$$\begin{aligned}
 J : \quad & \min_{K \in RH_\infty} \|T_{z_2\eta}\|_2^2 \\
 \text{s.t.} \quad & \|T_{z_\infty\eta}\|_\infty^2 < \gamma^2
 \end{aligned} \tag{4.11}$$

Here $T_{z_2\eta}$ and $T_{z_\infty\eta}$ refer to the closed loop transfer function from η to output vectors $z_2 = [\ddot{z}_s, \ddot{\theta}_s]$ and $z_\infty = [s_1, s_2]$ respectively. The controller gain matrix K can be found by using *state-feedback control method* or *static-output-feedback control method*.

In order to design a *state-feedback controller* or *static-output-feedback controller*, let's reconstruct the state space equation for the vehicle-track augmented system. The vehicle 6DOF model state space and cross-level track irregularity state-space forms were given in Chapter 3. Since it is assumed that the vehicle is traveling on a straight track with a constant speed, there is a time delay τ_{ij} between front and rear wheel inputs such as,

$$z_{wij} = z_{w11}(t - \tau_{ij}), \quad i, j = 1, 2. \tag{4.12}$$

where

$$\tau_{11} = 0, \quad \tau_{12} = \frac{2l_w}{v}, \quad \tau_{21} = \frac{2l_c}{v}, \quad \tau_{22} = \frac{2(l_c + l_w)}{v}$$

This time delay can be explained via 2nd order Padè approximation which has the following state-space form with state vector x_{pade} .

$$\begin{bmatrix} \dot{x}_{pade} \\ z_w \end{bmatrix} = \begin{bmatrix} A_{pade} & B_{pade} \\ C_{pade} & 0 \end{bmatrix} \begin{bmatrix} x_{pade} \\ z_{w11} \end{bmatrix} \quad (4.13)$$

Then the augmented state-space equation for vehicle-track system will be,

$$\begin{aligned} \dot{\hat{x}} &= \hat{A}\hat{x} + \hat{B}_1\eta + \hat{B}_2u \\ z &= \hat{C}_2\hat{x} + \hat{D}_{22}u \end{aligned} \quad (4.14)$$

Here $\hat{x} = [x, x_{pade}, x_{tr}]^T$ where the state vectors x and x_{tr} were defined in Chapter 3, and the matrices with appropriately sized null matrix $\mathbf{0}$ are given below.

$$\begin{aligned} \hat{A} &= \begin{bmatrix} A & B_1C_{pade}A_{pade} & B_1C_{pade}B_{pade}C_{tr} \\ \mathbf{0} & A_{pade} & B_{pade}C_{tr} \\ \mathbf{0} & \mathbf{0} & A_{tr} \end{bmatrix}, \quad \hat{B}_1 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ B_{tr} \end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix} B_2 \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \\ \hat{C}_2 &= \begin{bmatrix} C_2 & D_{21}C_{pade}A_{pade} & D_{21}C_{pade}B_{pade}C_{tr} \end{bmatrix}, \quad \hat{D}_{22} = D_{22} \end{aligned}$$

4.5.1. State-feedback control

The full-state feedback control assumes that all the states are to be measurable. In other words, the measured vector is given in $y = C_y\hat{x}$ with $C_y = I_{n \times n}$, where n is the degree of the system. The state feedback law can be written as,

$$u(t) = K_{SF}\hat{x}(t) \quad (4.15)$$

where K_{SF} is the $n \times n$ gain matrix. The closed loop system in Equation (4.16) can be found by inserting Equation (4.15) into the plant state space equation in Equation (4.14) .

$$\begin{aligned}\dot{\hat{x}} &= (\hat{A} + \hat{B}_2 K_{SF})\hat{x} + \hat{B}_1 \eta \\ z &= (\hat{C}_2 + \hat{D}_{22} K_{SF})\hat{x} \\ z_\infty &= \hat{C}_\infty \hat{x}\end{aligned}\tag{4.16}$$

The listed linear matrix inequality (LMI) formulations are used to find K_{SF} which solves the optimization problem J [66].

1. **H_∞ Performance:** The closed loop system satisfies the condition $\|T_{z_\infty \eta}\|_\infty < \gamma$ (for $\gamma > 0$) if and only if there exists a symmetric matrix $X_\infty > 0$ such that:

$$\begin{pmatrix} \mathcal{H}^1 \left\{ \left((\hat{A} + \hat{B}_2 K_{SF}) X_\infty \right) \right\} & * & * \\ \hat{B}_1 & -I & * \\ \hat{C}_\infty X_\infty & \hat{D}_{\infty 1} & -\gamma^2 I \end{pmatrix} < 0, \tag{4.17}$$

2. **H_2 Performance:** The H_2 norm of the closed loop transfer function from η to z does not exceed $\nu > 0$ if there exist two symmetric matrices $X_2 > 0$ and $W > 0$ such that:

$$\begin{pmatrix} \mathcal{H} \left\{ \left((\hat{A} + \hat{B}_2 K_{SF}) X_2 \right) \right\} & \hat{B}_1 \\ * & -I \end{pmatrix} < 0, \tag{4.18}$$

$$\begin{pmatrix} X_2 & (\hat{C}_2 + \hat{D}_{22} K_{SF}) X_2 \\ * & W \end{pmatrix} > 0, \tag{4.19}$$

$$\text{Trace}(W) < \nu^2 \tag{4.20}$$

¹ $\mathcal{H}\{M\} = M + M^T$

The inequalities are solved for a single Lyapunov matrix X that enforces both H_∞ and H_2 performances.

$$X := X_\infty = X_2 \quad (4.21)$$

With the change of variables as $Y := K_{SF}X$, LMI formulation of the multi-objective state-feedback synthesis problem will be transferred into the following,

- Minimize $\rho\gamma^2 + \sigma\text{Trace}(W)$ over Y, X, W, γ^2 such that

– H_∞ synthesis:

$$\begin{pmatrix} \mathcal{H}\left\{\begin{pmatrix} \hat{A}X + \hat{B}_2Y \end{pmatrix}\right\} & * & * \\ \hat{B}_1 & -I & * \\ \hat{C}_\infty X & \hat{D}_{\infty 1} & -\gamma^2 I \end{pmatrix} < 0, \quad (4.22)$$

$$\gamma^2 < \gamma_0^2 \quad (4.23)$$

– H_2 synthesis

$$\begin{pmatrix} \mathcal{H}\left\{\begin{pmatrix} \hat{A}X + B_2Y \end{pmatrix}\right\} & \hat{B}_1 \\ \hat{B}_1^T & -I \end{pmatrix} < 0, \quad (4.24)$$

$$\begin{pmatrix} X & \hat{C}_2X + \hat{D}_{22}Y \\ * & W \end{pmatrix} > 0 \quad (4.25)$$

$$\text{Trace}(W) < \nu_0^2 \quad (4.26)$$

In practice, it is impossible to measure every state because of the technical reasons and the high costs. So it is beneficial to study the static output feedback technic because of its simple design with fewer measurements.

4.6. Static Output Feedback Design

The static-output-feedback (SOF) gain is summarized in Equation (4.27) where $\hat{K} := K_{SOF}\hat{C}_y$.

$$u(t) = \hat{K}\hat{x}(t) \quad (4.27)$$

$K_{SOF} : K_{SOF} \in R^{m \times q}$ output control gain matrix,

$\hat{C}_y : \hat{C}_y \in R^{q \times n}$ observed output matrix with $q < n$

When an output gain matrix $\hat{K} = K_{SOF}\hat{C}_y$ is required then the following problem arises:

Problem 4.6.1 Find a solution of the LMI given in Equations (4.22) - (4.26) such that the product YX^{-1} factors as below.

$$YX^{-1} = K_{SOF}\hat{C}_y \quad (4.28)$$

Since it is difficult to directly calculate the gain matrix K_{SOF} , following change of variables are suggested in [118, 119, 120] as,

$$\begin{aligned} X &:= QX_QQ^T + RX_RR^T \\ Y &:= Y_RR^T \end{aligned} \quad (4.29)$$

Here $X_Q > 0$, $X_R > 0$ are symmetric matrices and Y_R is a general matrix. The basis of $\ker(\hat{C}_y)$ forms the Q matrix and the matrix R can be calculated from $R = \hat{C}_y^\dagger + QL$. $\hat{C}_y^\dagger$ denotes the pseudo inverse of $\hat{C}_y$ and L is an arbitrary matrix chosen by the designer. But the LMI optimization problem may be infeasible if the matrix L chosen is inappropriate, [120]. To overcome this problem [119] suggests a two step procedure to define L .

- **Step 1:** Solve the state feedback control problem and find the Lyapunov matrix X which is defined in equation (4.21).
- **Step 2:** If the state feedback optimization problem is feasible, then L can be chosen as

$$L = Q^\dagger X \hat{C}_y^T (\hat{C}_y X \hat{C}_y^T)^{-1} \quad (4.30)$$

Then the static output feedback controller $K_{SOF}\hat{C}_y = YX^{-1}$ can be obtained by Lemma 1.

Lemma 1: Consider the equality $K_{SOF}\hat{C}_y = YX^{-1}$ with the variables in Equation (4.29), if the matrix $X > 0$, then the matrices X_R , X_Q are nonsingular and the static output feedback gain matrix $K_{SOF} = Y_R X_R^{-1}$.

Then if equation (4.29) is placed into the equations (4.22 - 4.26) the static output feedback LMI constraints will be obtained as below.

- Minimize $\rho\gamma^2 + \sigma\text{Trace}(W)$ over Y_R , X_R , X_Q , W , γ^2 such that

1. H_∞ Synthesis

$$\begin{pmatrix} \mathcal{H} \left\{ \left(\hat{A}QX_QQ^T \right) + \left(\hat{A}RX_RR^T \right) + \left(\hat{B}_2Y_RR^T \right) \right\} & * & * \\ \hat{B}_1^T & -I & * \\ \hat{C}_\infty QX_QQ^T + C_\infty RX_RR^T + \hat{D}_{\infty 2}Y_RR^T & \hat{D}_{\infty 1} & -\gamma^2 I \end{pmatrix} < 0, \quad (4.31)$$

$$\gamma^2 < \gamma_0^2 \quad (4.32)$$

2. H_2 Synthesis

$$\begin{pmatrix} \mathcal{H} \left\{ \left(\hat{A}QX_QQ^T \right) + \left(\hat{A}RX_RR^T \right) + \left(\hat{B}_2Y_RR^T \right) \right\} & * \\ \hat{B}_1^T & -I \end{pmatrix} < 0, \quad (4.33)$$

$$\begin{pmatrix} QX_QX^T + RX_RR^T & \hat{C}_2QX_QQ^T + \hat{C}_2RX_RR^T + \hat{D}_{22}Y_RR^T \\ * & W \end{pmatrix} > 0, \quad (4.34)$$

$$\text{Tr}(W) < \nu_0^2 \quad (4.35)$$

Figures 4.17a-4.18a show the static-output feedback controlled and state feedback controlled vehicle responses. It can be concluded from these figures it is possible to

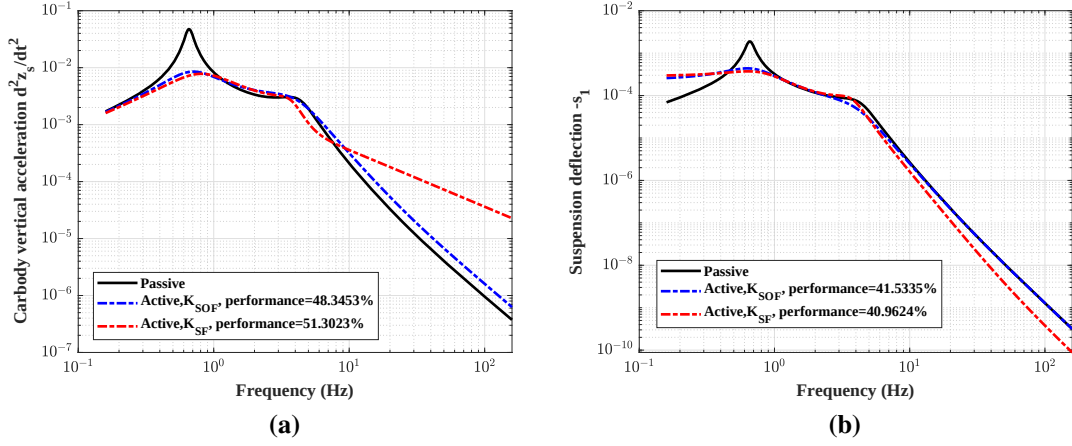


Figure 4.17. State-feedback and static output-feedback controllers design (a) carbody vertical acceleration bode plot (b) secondary suspension travel bode plot

achieve performances of the state-feedback controller with fewer measurements by designing a static-output feedback controller. From this point of view, in the next section, to get optimum skyhook (groundhook) design parameters, we will configure the static output feedback gain matrix K_{SOF} .

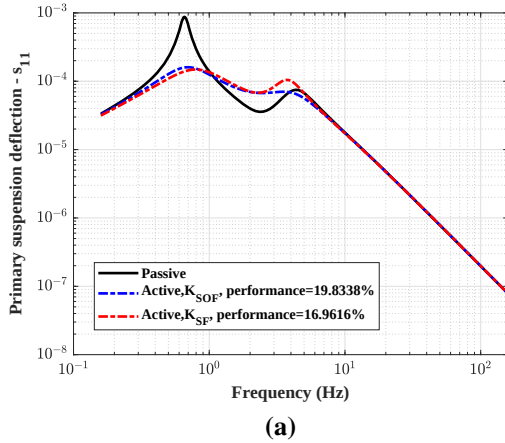


Figure 4.18. (a) State-feedback and static output-feedback controllers design primary suspension travel bode plot

4.7. Semi-Active Controller Design via LMI Optimization

4.7.1. Skyhook damping

The main objective of this section is to design an active suspension control that consists of the “skyhook” damping method. In order to reduce vertical oscillations of the

vehicle body the secondary suspension active control forces in terms of skyhook dampers can be given by the control-oriented formula in Equation (4.36) [121].

$$u_i = -c_{skyi}(\dot{z}_{si} - \dot{z}_{bi}) - (1 - \alpha_i)c_{skyi}\dot{z}_{bi} \quad i = 1, 2. \quad (4.36)$$

Here $c_{sky1} > 0$, $c_{sky2} > 0$ are the damper values and $\alpha_1, \alpha_2 \in [0, 1]$ are the design parameters. $\alpha_1 = \alpha_2 = 0$ means that the control forces u are independent of the vertical velocities of the bogies which implies the oscillations of bogies. The performances of the active control depend on choosing the values of the c_{skyi} , α_i for $i = 1, 2$. In this section, the LMI-based optimization method was used to detect these parameters.

$$K_{SOF} = \begin{bmatrix} c_{sky1} & 0 & -\alpha_1 c_{sky1} & 0 \\ 0 & c_{sky2} & 0 & -\alpha_2 c_{sky2} \end{bmatrix} \quad (4.37)$$

The static output feedback gain K_{SOF} will be synthesized for the measurements $y = [\dot{z}_{s1}, \dot{z}_{s2}, \dot{z}_{b1}, \dot{z}_{b2}]^T$. The solution may be possible by determining the LMI variables Y_R and X_R as in Equation(4.38).

$$Y_R = \begin{bmatrix} Y_{R11} & 0 & Y_{R13} & 0 \\ 0 & Y_{R22} & 0 & Y_{R24} \end{bmatrix}, \quad X_R = \begin{bmatrix} X_{R11} & 0 & 0 & 0 \\ 0 & X_{R22} & 0 & 0 \\ 0 & 0 & X_{R33} & 0 \\ 0 & 0 & 0 & X_{R44} \end{bmatrix} \quad (4.38)$$

By using Lemma-1, Equation (4.37) and Equation (4.38), c_{skyi} and α_i can be found from the static-output feedback solution as,

$$c_{sky1} = \frac{Y_{R11}}{X_{R11}}, \quad \alpha_1 = \frac{X_{R11}Y_{R13}}{Y_{R11}X_{R33}}, \quad c_{sky2} = \frac{Y_{R22}}{X_{R22}}, \quad \alpha_2 = \frac{X_{R22}Y_{R24}}{Y_{R22}X_{R44}}$$

Table 4.2 lists the obtained parameters c_{skyi} and α_i via LMI solutions. However, these results are obtained without constraint on the vehicle outputs. This problem is resolved by adding constraints on secondary suspension deflections. The main goal is getting better comfort conditions while keeping the suspension travels below the passive

Table 4.2. Skyhook dampers and design parameters values

Variable	Values for $z_\infty = [\]$	Values for $z_\infty = [s_1 \ s_2]^T$
c_{sky1}	53.409 kNs/m	40.613 kNs/m
c_{sky2}	53.568 kNs/m	38.012 kNs/m
α_1	0.1872	0.0014
α_2	0.1875	0.0014

ones. Figures 4.19a-4.20a present the optimal solutions. The results are dramatic, especially in the low-frequency range.

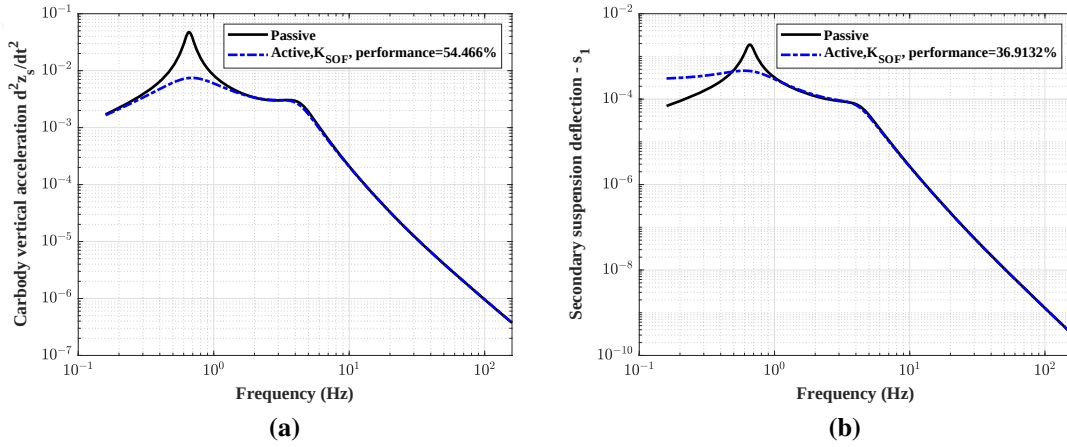


Figure 4.19. Skyhook damping static output feedback design for $z_\infty = [s_1 \ s_2]^T$ (a) carbody vertical acceleration bode plot (b) secondary suspension travel bode plot

Before concluding, we should remark that the skyhook damping method is directly related to ride comfort. But this is not enough. In addition to better comfort conditions, the controller should also satisfy the safety criteria. Then the groundhook dampers should be considered.

4.7.2. Groundhook damping

The main objective is designing an active suspension control consisting of “ground-hook” damping, directly related to unsprung mass performance. Similar to skyhook damping, groundhook damping can be formalized as a control-oriented model like as:

$$u_i = c_{gndi}(\dot{z}_{si} - \dot{z}_{bi}) - (1 - \beta_i)c_{gndi}\dot{z}_{si}, \quad i = 1, 2 \quad (4.39)$$

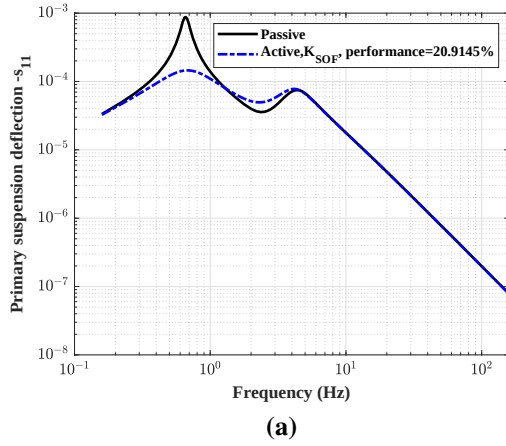


Figure 4.20. (a) Skyhook damping static output feedback design for $z_\infty = [s_1 \ s_2]^T$ primary suspension travel bode plot

The main task of the groundhook dampers is suppressing the oscillations on the unsprung mass. Then the output vectors are selected as $z_2 = [\ddot{z}_{b1} \ \ddot{z}_{b2}]^T$ and $z_\infty = [s_1 \ s_2]^T$ with measurements $y = [\dot{z}_{s1} \ \dot{z}_{s2} \ \dot{z}_{b1}, \ \dot{z}_{b2}]^T$. Under this modification, Problem 4.5.1 can be solved with the static output feedback design method for Equation 4.40 by using the LMI variables Y_R and X_R as in Equation (4.38). Let's define a new control matrix in the following form.

$$K_{SOF} = \begin{bmatrix} \beta_1 c_{gnd1} & 0 & -c_{gnd1} & 0 \\ 0 & \beta_2 c_{gnd2} & 0 & -c_{gnd2} \end{bmatrix} \quad (4.40)$$

The matrix consists of groundhook design parameters organized in the Equation (4.40). The results are provided in Figures 4.21a - 4.22a. The bode plots and the RMS performances show impressive results now on bogie vibrations and safety, especially for the second mode. However, another trade-off happens since it could not prevent the increases in the car body's vertical acceleration. Therefore, the hybrid controller design will be incorporated to take advantage of both skyhook damping on the ride comfort and groundhook damping on the safety criteria.

4.7.3. Hybrid control

The hybrid control combines the skyhook and groundhook damping methods to take advantage of both. The gain matrix in Equation(4.41) summarizes the hybrid method

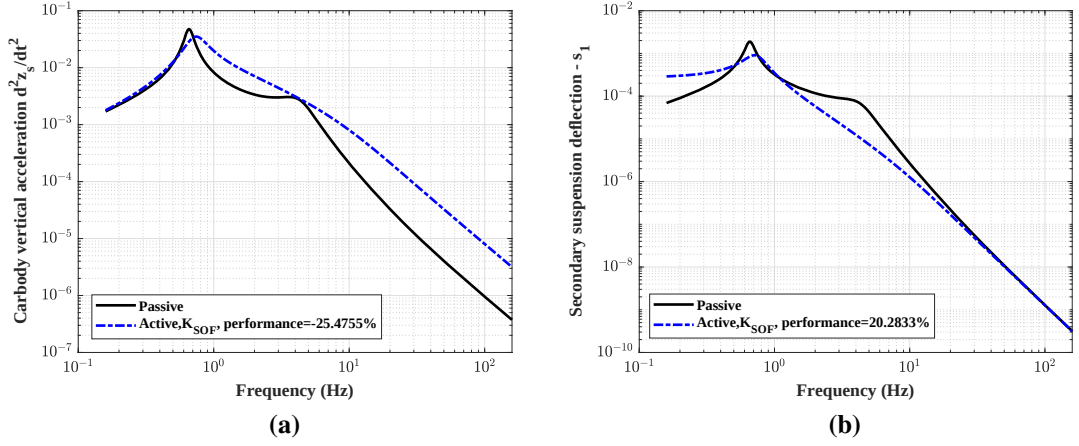


Figure 4.21. Groundhook damping (a) carbody vertical acceleration bode plot (b) secondary suspension deflection bode plot

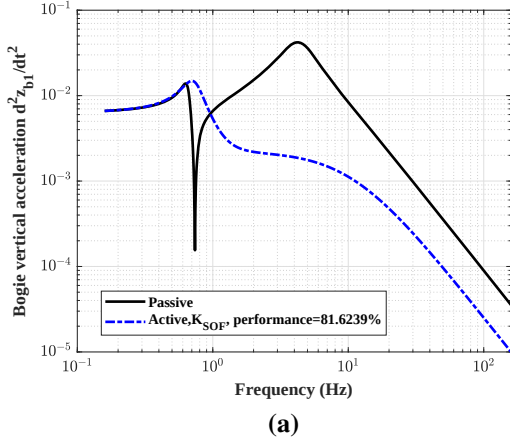


Figure 4.22. (a) Groundhook damping method front bogie vertical acceleration bode plot

with $\mu_1, \mu_2 \in [0, 1]$ with measurement vector $y = [\dot{z}_{s1}, \dot{z}_{s2}, \dot{z}_{b1}, \dot{z}_{b2}]^T$. With $\mu_i = 0$, the hybrid control turns into purely skyhook damping, and if $\mu_i = 1$, the hybrid control becomes groundhook control. Let's organize the gain matrix K_{SOF} as follows,

$$K_{SOF} = (\Lambda\Pi)\Lambda_{SKY} + (\Pi - \Lambda\Pi)\Lambda_{GND} \quad (4.41)$$

where

$$\Lambda_{SKY} = \text{diag}(c_{sky1}, c_{sky2}, \alpha_1 c_{sky1}, \alpha_2 c_{sky2}),$$

$$\Lambda_{GND} = \text{diag}(\beta_1 c_{gnd1}, \beta_2 c_{gnd2}, c_{gnd1}, c_{gnd2}),$$

$$\Lambda = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}, \quad \Pi = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

LMI solution provides the best values of the skyhook and groundhook parameters. If the main purpose is getting better ride comfort without any degradation on the safety criteria, μ_1, μ_2 are chosen in the $[0.5, 1]$ range. On the other hand, if the main purpose is to provide safer travels without any decreases in comfort, the μ_1, μ_2 should be chosen in the $[0, 0.5]$ range.

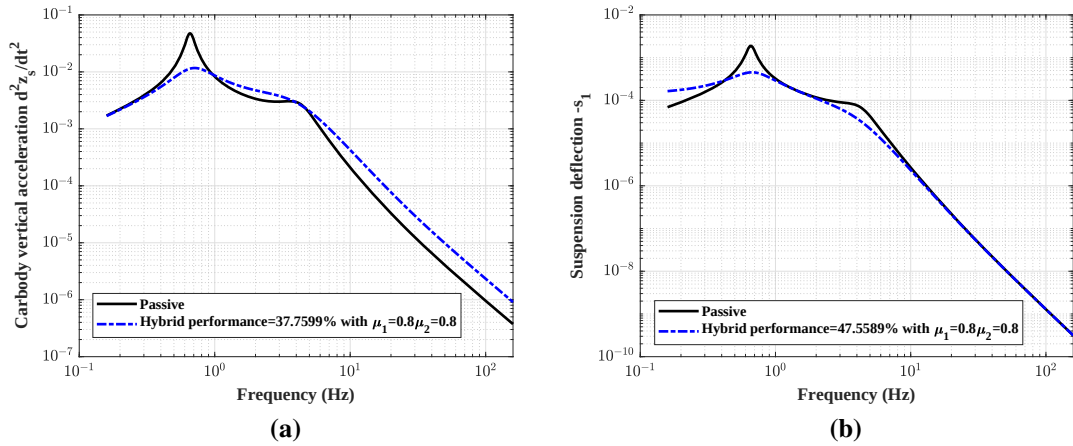


Figure 4.23. Hybrid solution (a) carbody vertical acceleration bode plot (b) secondary suspension bode plot

The results were obtained for $\mu_1 = \mu_2 = 0.8$ and given in Figures 4.23a-4.24a. These impressive results show that it is possible to increase comfort conditions with safer travel by using semi-active controllers in railways.

4.8. Summary

In this chapter, the semi-active control methods were applied to both the 6DOF and 9DOF vehicle models. The RMS value of the car body's vertical acceleration was formulated in terms of skyhook / groundhook damper values. The optimal damping values were

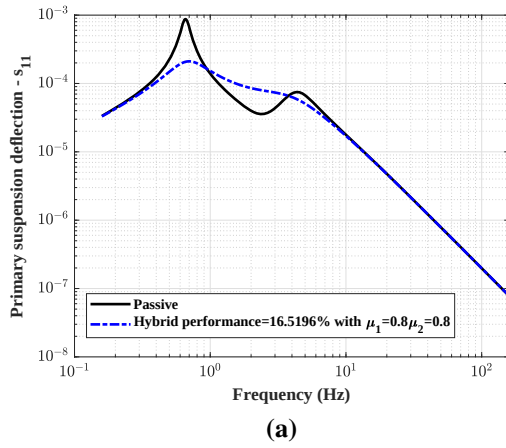


Figure 4.24. (a) *Hybrid solution primary suspension deflection bode plot*

determined both from the gradient and LMI methods. It was seen that the pure skyhook or groundhook dampers could not perform well on both safety and comfort criteria. Then the hybrid control was added to bring the advantages of both dampers. We observed that conflict still exists between the performance indicators. A multi-objective active control problem was formulated to overcome these difficulties, and solutions were obtained using LMIs such as Chapter 6.

5. ACTIVE SUSPENSION DESIGN

Ride comfort plays a major role in designing and constructing vehicles to improve the acceptance of rail transportation by passengers. It can be possible via secondary suspension that can minimize the car body's vertical accelerations on the straight track and lateral accelerations on the curves caused by the track irregularities. The studies have shown that some functionalities that could not be achieved by using passive or semi-active suspensions could be succeeded by active suspensions [122, 7]. The earliest successful mainline testing with lateral and vertical secondary suspensions was done in British Rail in 1983, and as a result, 35% - 50% improvements were obtained in ride comfort [123]. These results encouraged the researchers to work on the active control of railway vehicles. Various semi-active/active suspension system designs have been widely studied theoretically and experimentally [9], though some early impressive results, the only train with active suspension in running service in Europe is Fiat Pendolino and Shinkansen in Japan [124]. In Austria, the Siemens SGP experimental vehicle with semi-active secondary vertical dampers and fully-active pneumatic actuators is used [125].

For the last decades, actively controlled suspension design problems come into prominence in railways since mechatronic developments have been incorporated with vehicles. In this part of the thesis, the actively controlled suspension design for railway vehicle secondary and primary suspension systems will be studied in detail. The main goal is to analyze the vehicle body vibrations and find analytical methods to reduce their effects while getting better ride quality with smaller primary and secondary suspension deflections. It is impossible to overcome this problem via passive/semi-active systems since there is a trade-off between railway vehicle outputs.

5.1. LQG Control

This section will use the LQG method as an active controller. Figure 5.1 shows the representative block diagram of the LQG method, which consists of two parts: *Kalman filtering* and *Linear Quadratic Regulator (LQR)*.

In LQR design, measuring all states in the system is necessary, which is impossible because of the unavailability of capable sensors to measure all states. Therefore, the

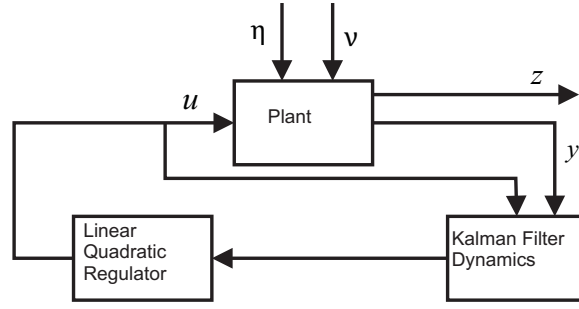


Figure 5.1. The LQG control separation principle [13]

Kalman filter can be used as an estimator [7]. Let's define the measurement vector as $y = [s_1, s_2]$ and used state-vector $\hat{x}$ as defined in the plant $\mathcal{P}$ in Equation (4.14) previous Chapter and measurement noise ν as,

$$y = \hat{C}_y \hat{x} + \nu \quad (5.1)$$

with $\hat{C}_y = [I_2 \ 0]$ and satisfies,

$$E\{\eta\eta^T\} = 1, \quad E\{\eta\nu\} = 0, \quad E\{\nu\nu^T\} = \epsilon I_2$$

Here ϵ is a small positive number. Then the model based estimator have a form:

$$\dot{x}_e = \hat{A}x_e + \hat{B}_2u + \tilde{L}(y - y_e) \quad (5.2)$$

where x_e is the estimated state vector and $y_e = \hat{C}_y x_e$, $\tilde{L}$ represent the Kalman filter gain which minimizes the estimation error $e := \hat{x} - x_e$. Then the feedback control law $u = -K_{lqr}x_e$ minimizes the cost function $\mathcal{J}$ in Problem (5.1.1) with user-defined weighting matrices Q and $R = \rho I_n$ where $\rho \in (0, 1)$.

Problem 5.1.1 Find an output feedback controller that minimizes the objective function $\mathcal{J}$ while satisfying closed loop system stability.

$$\mathcal{J} = \int (z^T Q z + u^T R u) dt \quad (5.3)$$

The closed loop system can be obtained for $x_{cl} = [\hat{x} \ x_e]^T$ as in Equation (5.4), and the

problem is solved via the *MATLAB lqr* command.

$$\begin{aligned}\dot{x}_{cl} &= \begin{bmatrix} \hat{A} & -\hat{B}_2 K_{lqr} \\ \tilde{L}\hat{C}_y & \hat{A} - \hat{B}_2 K_{lqr} - \tilde{L}\hat{C}_y \end{bmatrix} x_{cl} + \begin{bmatrix} \hat{B}_1 \\ \mathbf{0} \end{bmatrix} \eta \\ z &= \begin{bmatrix} \hat{C}_2 & -\hat{D}_{22} K_{lqr} \end{bmatrix} x_{cl}\end{aligned}\quad (5.4)$$

Table 5.1 lists the RMS performances of LQG controlled railway vehicle half-car model with Bombardier AM96 wagon parameters. As can be seen from the table, the LQG-controlled vehicle has a 27.6% improvement in the car body heave acceleration and a 23.5 % improvement in car body pitch acceleration. Although these are impressive results on accelerations, the front bogie secondary and primary relative displacements remain almost the same. However, the rear bogie secondary suspension deflection was reduced by 34.6 %. This is expected since the LQG controller is placed on the secondary suspension and directly influences the outputs related to it. The front suspension deflection remained unchanged since all the control effort was concentrated on the rear one. We can conclude that the LQG-controlled vehicle provides better comfort than the passive system while suppressing the suspension deflection.

Table 5.1. *The LQG controlled vehicle performances*

Output	Passive RMS	Active RMS	Performance Improvement(%)
s_1	0.0021	0.0021	-
s_2	0.0026	0.0017	34.6
s_{11}	0.0005	0.0005	-
s_{12}	0.0006	0.0006	-
s_{21}	0.0007	0.0006	14.3
s_{22}	0.0007	0.0007	-
$\ddot{z}_s$	0.0865	0.0626	27.6
$\ddot{\theta}_s$	0.0243	0.0186	23.5

Let's illustrate the performance of the LQG-controlled vehicle performances in the frequency range with Figures 5.2a-5.3b. The controller works well, especially in the low-frequency band, which improves the first body mode of the system.

Let's plot the actuator forces u_1 , u_2 in Figure 5.4. We can see that two actuators have RMS values of 972.2 and 1074 at most that means the active control problem defined above does not require large actuator forces [61, 7]. Before continuing, pay attention that

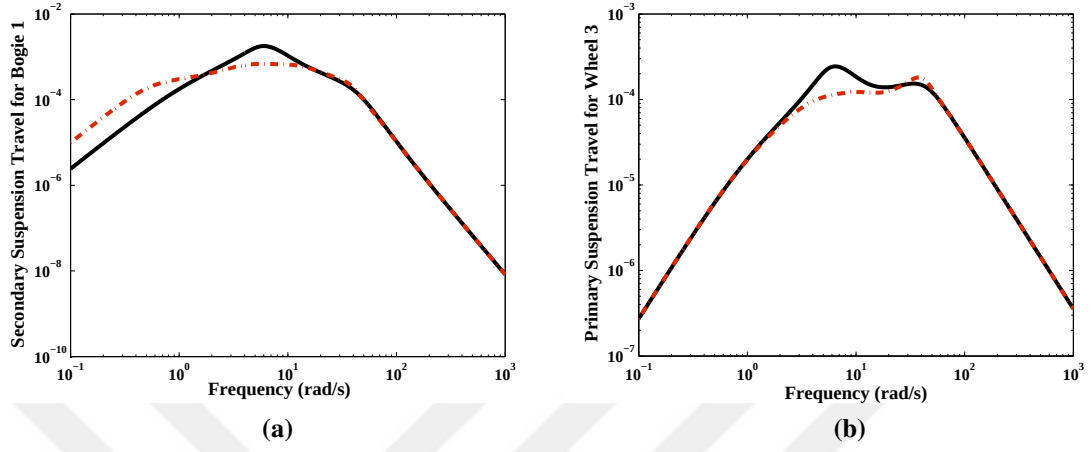


Figure 5.2. (—) denotes passive suspension and (---) refers to actively controlled vehicle response (a) Secondary suspension travel of front bogie bode plot (b) Primary suspension travel between rear bogie and third wheel bode plot

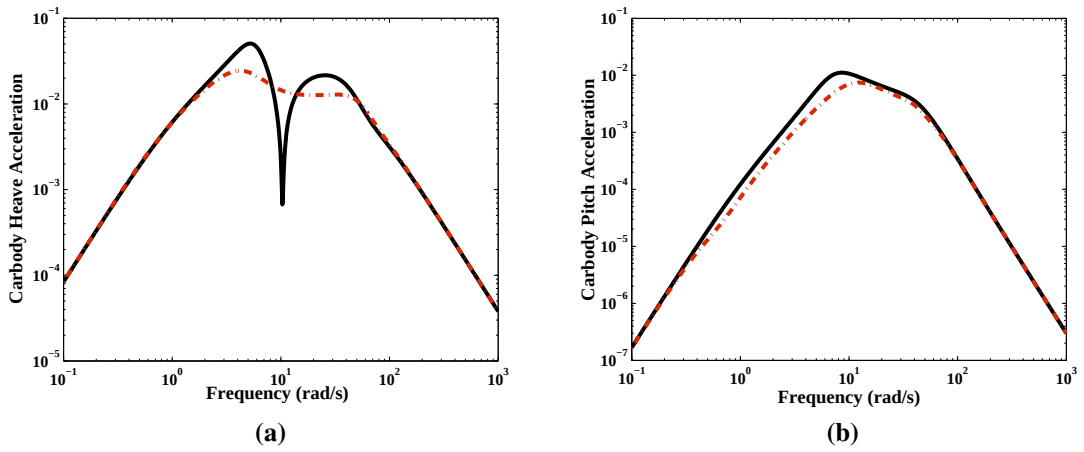


Figure 5.3. (—) denotes passive suspension and (---) refers to actively controlled vehicle response (a) Car body heave acceleration bode plot (b) Car body pitch acceleration bode plot

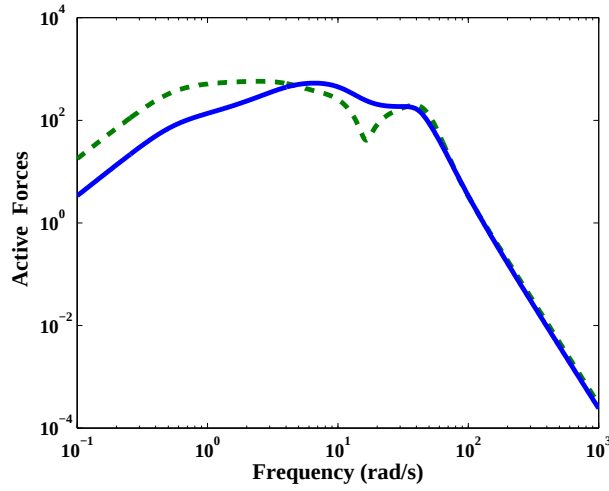


Figure 5.4. *Active control forces*

in LQG design, the controller has the same size as the plant and the results are highly dependent on the weighting matrices Q and R . It may be possible that observe better suspension deflections for different values of weighting matrices.

In railway vehicle dynamic analysis for simulation purposes, various models representing the vehicle dynamics may be used. So far we have considered the 6 DOF and 9 DOF models which are obtained by assuming a rigid contact between wheels and rail. If it would be said in a nutshell, in rigid contact the wheel and the rail act as welded. In other words, the wheel displacements are equal to rail displacement.

To see active controller behaviors, the elastic contact (more realistic) also can be studied [8, 126, 28]. The elastic contact between the wheel and rail has highly nonlinear behavior. The Hertzian contact theory is widely used in multibody dynamic analysis since it can explain the behavior. In the Hertzian contact theory, the elastic contact of two metal round surfaces can be modeled by a nonlinear or linearized spring [127, 93, 87]. This spring value is parameter-dependent and changes over time. Figure 5.5 shows the wheel-rail contact and the equivalent mathematical model in terms of elastic contact. In the figure, k_H is the Hertzian spring, m_w is the wheel mass with z_w vertical displacement, and w denotes the rail vertical displacement.

Then the force between wheel and rail with δ elastic compression can be derived as:

$$F_H = -c_H \delta^{3/2} \quad (5.5)$$

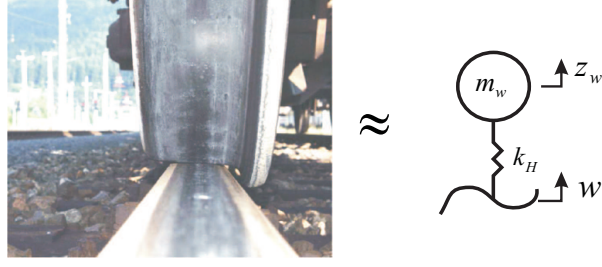


Figure 5.5. *The wheel-rail elastic contact equivalent mathematical model.*

where c_H is the Hertzian contact constant in $Nm^{-3/2}$ and defined by Equation (5.6).

$$c_H = \frac{2E}{3(1-\nu^2)}(r_r r_w)^{1/4} \quad (5.6)$$

Here E is the elastic modulus of the rail in N/m^2 , ν is the Poisson's ratio of the rail, r_r is the rail head radius in m , and r_w represents the rolling radius of the wheel in m . The linearized Hertzian contact stiffness k_H with static wheel load F_s is given in:

$$k_H = \frac{3}{2}c_H^{2/3}F_s^{1/3} \quad (5.7)$$

Equation (5.7) means that the Hertzian spring stiffness has an approximate value and depends on the rail's material and size, wheel size, and static load. Since the wheel and rail wear in time, the radii become smaller. According to the fullness of the vehicle, the static wheel load also may vary. Therefore the Hertzian spring and the wheel-rail force would not stay the same.

In this part of the thesis, at first, a new half-car model is obtained with ten degrees of freedom (10DOF). Since the model will require the elastic wheel-rail contact, let's write the equations of motions for the wheels,

$$m_w \ddot{z}_{wij} = F_{wij} - F_{ij}, \quad \text{for } i, j = 1, 2. \quad (5.8)$$

where $F_{wij} = -k_H(z_{wij} - r_{ij})$. Then the state vector should be updated as $x = [x_1, \dots, x_{20}]^T$

with states:

$$\begin{aligned}
x_1 &= s_1, & x_2 &= s_2, & x_3 &= s_{11}, & x_4 &= s_{12}, & x_5 &= s_{21}, \\
x_6 &= s_{22}, & x_7 &= z_{w11}, & x_8 &= z_{w12}, & x_9 &= z_{w21}, & x_{10} &= z_{w22}, \\
x_{11} &= \dot{z}_{s1}, & x_{12} &= \dot{z}_{s2}, & x_{13} &= \dot{z}_{b11}, & x_{14} &= \dot{z}_{b12}, & x_{15} &= \dot{z}_{b21}, \\
x_{16} &= \dot{z}_{b22}, & x_{17} &= \dot{z}_{w11}, & x_{18} &= \dot{z}_{w12}, & x_{19} &= \dot{z}_{w21}, & x_{20} &= \dot{z}_{w22}.
\end{aligned}$$

and the disturbance input $r = [r_{11}, r_{12}, r_{21}, r_{22}]^T$. The state space equation from input r to outputs z , and y have the form of

$$\mathcal{P}_{10DOF} : \begin{cases} \dot{x} = Ax + B_1 r + B_2 u \\ z = C_2 x + D_{21} r + D_{22} u \\ y = C_y x \end{cases} \quad (5.9)$$

where the state space matrices will be augmented as,

$$\begin{aligned}
A &= \left[\begin{array}{cc|cc} \mathbf{0} & \mathbf{0} & T_M & N \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I_4 \\ \hline -K S_M T_M^T & \mathbf{0} & -C S_M T_M^T T_M & C S_M N \\ -k_p M_w^{-1} N^T & -k_H M_w^{-1} & -c_p M_w^{-1} N^T & c_p M_w^{-1} \end{array} \right], \\
B_1 &= \begin{bmatrix} \mathbf{0} \\ -k_H (M_w^{-1} N^T)^T \end{bmatrix}, \quad B_2 = \begin{bmatrix} \mathbf{0} \\ -S_M T_M^T \Gamma \\ \mathbf{0} \end{bmatrix}, \\
C_2 &= \begin{bmatrix} I_6 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\Gamma^T K S_M T_M^T & \mathbf{0} & -\Gamma^T C S_M T_M^T T_M & \Gamma^T C S_M N \end{bmatrix}, \\
D_{21} &= 0_{8 \times 4}, \quad D_{22} = \begin{bmatrix} 0_{6 \times 2} \\ -M_s^{-1} S_s^T \end{bmatrix}, \quad C_y = \begin{bmatrix} \Gamma^T & \mathbf{0} \end{bmatrix}
\end{aligned}$$

where S_s , M_s , K , C , S_M , T_M , Γ were defined for 6DOF vehicle model in Chapter 3 with Equation (3.5) and $M_w = m_w I_4$.

Here we are going to use the notation of vertical track profile (from the FRA report) shown in Figure 5.6 since it points out the difference between the rail head. In [2], the



Figure 5.6. Vertical rail profile

vertical track profile is represented with the PSD function,

$$S_r(\Omega) = A_v \frac{\Omega_2^2(\Omega^2 + \Omega_1^2)}{\Omega^4(\Omega^2 + \Omega_2^2)} \quad (5.10)$$

The track classes with roughness constant and upper-speed limits are given in Table 5.2.

Table 5.2. Vertical Profile Track Irregularity PSD Function Parameters [62]

Track Class	A_v in 10^{-6} m	Passenger Train Speed limit
1 (poorest)	1.6748	24
2	0.9540	48
3	0.5300	97
4	0.2968	129
5	0.1675	145
6 (best quality)	0.0954	177

$S_r(\Omega)$ is the PSD function between the white noise input η and the rail vertical profile r_{11} in a relation as given in Equation (5.11).

$$S_r(\Omega) = |G_r(\Omega)|^2 S_\eta(\Omega) \quad (5.11)$$

with the transfer function $G_r(\Omega)$ of the track profile. In Equation (5.11), there is a singular point at $\Omega = 0$. So, $G_r(\Omega)$ will be modified with $\hat{G}_r(\Omega)$ as,

$$|G_r(\Omega) - \hat{G}_r(\Omega)| \rightarrow 0$$

for all $\Omega > 0$ and $\gamma = 0.01$.

$$\hat{G}_r(\Omega) = \sqrt{A_v \Omega_2^2} \frac{(\Omega + \Omega_1)}{(\Omega^2 + \gamma^2)(\Omega + \Omega_2)} \quad (5.12)$$

This is a fair representation that can be seen from Figure 5.7 [32], and can be used safely during the simulations. When we consider the half-car modes, the rail inputs at rear

wheels are time-delayed versions of the front wheels and to represent delays a 2nd order Padè approximation can be used. For obtaining the 10DOF half-car analytical model, the track-vehicle state-space matrices in Equation (4.14) should be updated with vertical profile track irregularity dynamic by using the procedure in Section 3.2.1.

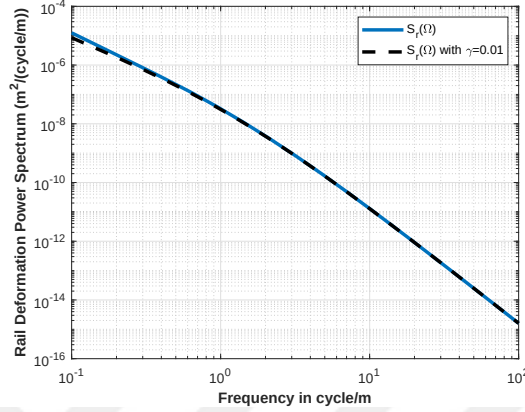


Figure 5.7. *The PSD of track irregularity*

Now, let's design an LQG controller that solves Problem 5.1.1 for the vehicle that has an elastic contact between wheels and rail and runs with a constant speed on the irregular track defined in Equation (5.11).

The simulation results are in Figures 5.8a - 5.9b. There is 3rd peak on the figures, which comes from wheel dynamics with elastic contact and happens at $\approx 1000 \text{ rad/s}$. Our LQG controller is responsible for suppressing low-body modes, so there is no significant improvement in the high-frequency range. However, we can see an improvement in all performance indicators in the frequency range where the human body is more sensitive.

The LQG control methods were also tested under changing road conditions. The RMS results for both passive and LQG-controlled vehicles are shown in Figures 5.10a-5.11b as bar plots and . This performance convenience to the design such that for changing quality conditions of the track, the active-controlled vehicle may perform well.

Although the LQG results in ride comfort, the half-car models don't hint at roll motion. For a rigid model of a railway vehicle 17DOF full-car model consists of all vertical car body and bogie modes that come from heave, pitch, and roll motions, and heave and roll modes of wheelsets. To study the effect of roll motion, the model is extended to a 17DOF full-car model, and the effect of the LQG controller will be examined.

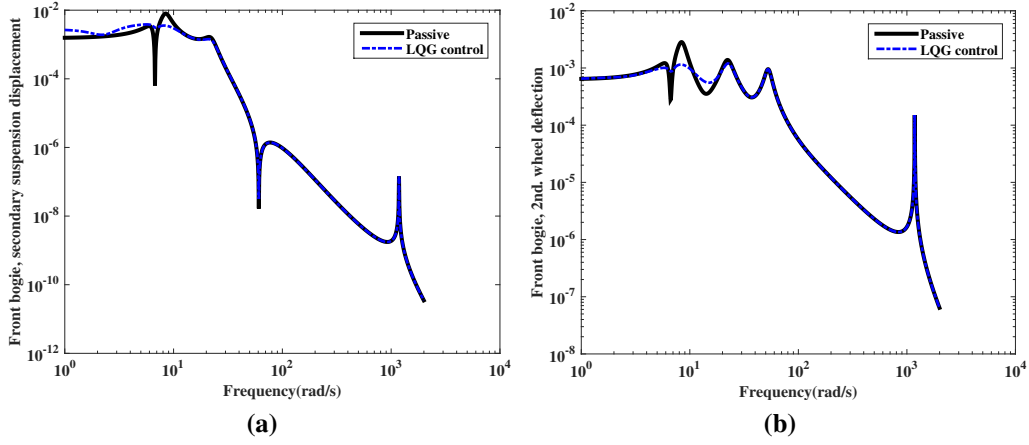


Figure 5.8. The frequency response of (a) front bogie secondary suspension deflection (b) primary suspension deflection between front bogie - 2nd wheel

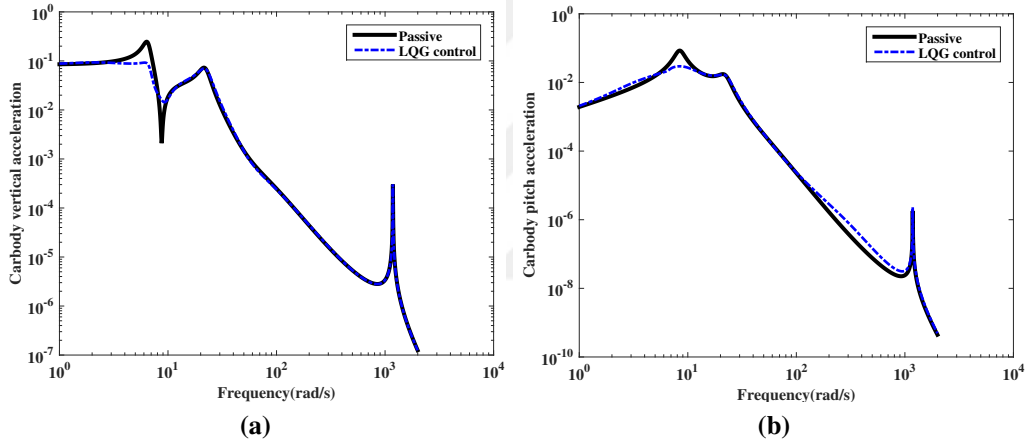


Figure 5.9. The frequency response of (a) car body vertical acceleration (b) car body pitch acceleration

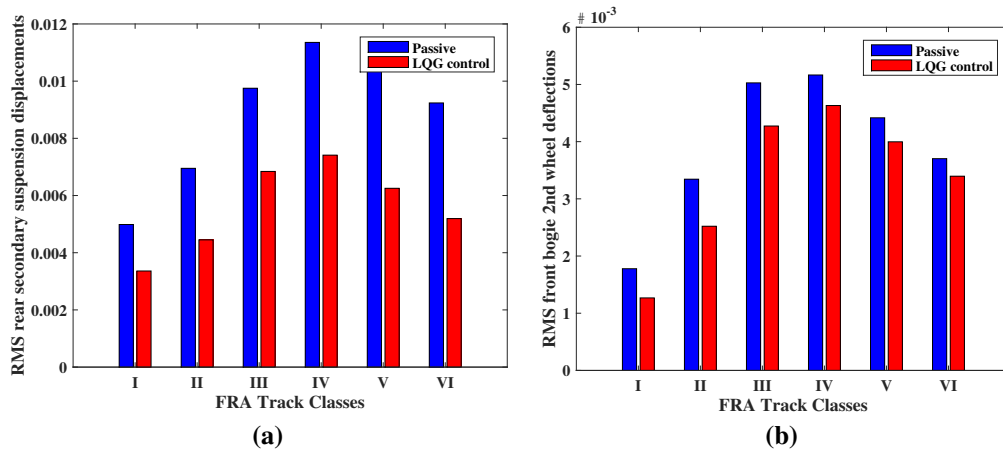


Figure 5.10. The passive and LQG controlled vehicle RMS results of (a) the rear bogie secondary suspension deflections for all track classes (b) the primary suspension deflection for all track classes

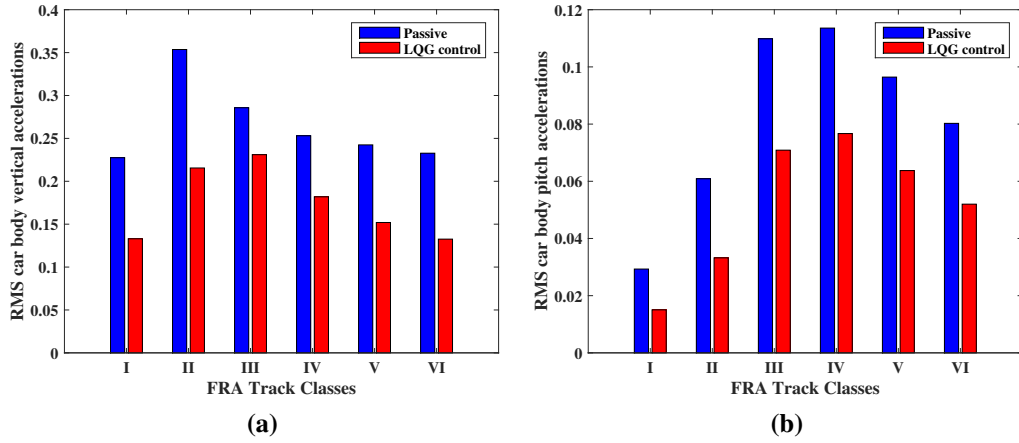


Figure 5.11. The passive and LQG controlled vehicle RMS results of (a) car body vertical acceleration for all track classes (b) the car body pitch acceleration for all track classes

Let's add the wheelsets heave (z_{wm}) and roll (φ_{wm}) motions for $m = 1, \dots, 4$ to the 9 DOF vehicle model discussed in the previous chapter. Now, the wheel forces with an elastic contact between the wheel and rail can be written as:

$$F_{wmn} = -k_H (z_{wmn} - w_{mn}), \quad n = 1, 2 \quad \text{and} \quad m = 1, \dots, 4 \quad (5.13)$$

where k_H denotes the Hertzian contact spring around 10^9 N/m . And the equations of motions for wheelsets with primary suspension forces F_{bmn} defined in Equation (3.1) are,

- For wheelsets ($m=1, \dots, 4$)

$$m_w \ddot{z}_{wm} = -F_{bmn} + \sum_{n=1}^2 F_{wmn}, \quad (5.14)$$

$$I_{wx} \ddot{\varphi}_{wm} = l_p \{F_{bm1} - F_{bm2}\} + l_p \{F_{wm2} - F_{wm1}\}$$

Then by modifying the state and output vectors as $x = [x_s \ x_{bf} \ x_{br} \ x_w]^T$ with $x_w = [z_{wm} \ \varphi_{wm}]^T$, $z = [\ddot{x}_s \ (z_{ij} - z'_{ij})]$ and $y = [z_{ij} - z'_{ij}]^T$, the state-space equation will be changed from input w to output vector such that:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_2 x + D_{21} w + D_{22} u, \end{aligned} \quad (5.15)$$

where

$$\begin{aligned}
A &= \begin{bmatrix} 0_{17 \times 17} & I_{17} \\ \hat{M}^{-1}(\hat{K} + K_H) & \hat{M}^{-1}\hat{C} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0_{17 \times 8} \\ -\hat{M}^{-1}K_H W \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0_{17 \times 4} \\ -\hat{M}^{-1}P \end{bmatrix}, \\
C_2 &= \begin{bmatrix} \tilde{\Gamma}A & P^T & 0_{4 \times 17} \end{bmatrix}^T, \quad D_{21} = 0_{7 \times 8}, \quad D_{22} = \begin{bmatrix} \tilde{\Gamma}B_2 \\ 0_{4 \times 4} \end{bmatrix},
\end{aligned}$$

with matrices defined in subsection 4.4.1 and below,

$$\begin{aligned}
\hat{M} &= \text{diag}(\tilde{M}_s, \tilde{M}_b, \tilde{M}_b, S_w^{-T} \tilde{M}_w S_w^{-1}, S_w^{-T} \tilde{M}_w S_w^{-1}), \\
\hat{K} &= \begin{bmatrix} \tilde{S}_s^T K_s \tilde{T} \tilde{S}_s & \tilde{S}_s^T K_s \tilde{Y}_s \\ \tilde{Y}_s^T K_s \tilde{S}_s & -\tilde{Y}_s^T K_s \tilde{Y}_s \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} \tilde{S}_s^T C_s \tilde{T} \tilde{S}_s & \tilde{S}_s^T C_s \tilde{Y}_s \\ \tilde{Y}_s^T C_s \tilde{S}_s & -\tilde{Y}_s^T C_s \tilde{Y}_s \end{bmatrix}, \\
W &= [0_{9 \times 8} \quad I_8]^T, \quad P = \begin{bmatrix} T_l S_c & 0_{4 \times 8} \end{bmatrix}^T, \\
\tilde{T} &= \text{diag}(T_s, Z_s), \\
\tilde{M}_w &= \text{diag}(m_w, I_{wx}, m_w, I_{wx}) \\
K_H &= \text{diag}(0_{9 \times 9}, -k_H I_8), \\
Z_s &= \text{diag}(0_{4 \times 4}, -I_4), \\
Y &= [0_{4 \times 8} \quad I_8]^T, \\
\tilde{Y}_s &= \begin{bmatrix} 0_{8 \times 8} \\ Y \end{bmatrix}, \\
T_s &= -T_l^T T_l, \\
\tilde{\Gamma} &= \begin{bmatrix} 0_{3 \times 14} & I_3 & 0_{3 \times 17} \end{bmatrix}
\end{aligned}$$

Here, I_n is the n by n identity matrices, $0_{m \times n}$ denotes the m by n null matrix, $\otimes$ is the Kronecker product, and $\text{diag}(\cdot)$ refers to (block) diagonal. For the design of a feedback law, we consider the suspension travels $y = [s_{ij}]^T$, which can be written as:

$$y = C_y x. \quad (5.16)$$

The main goal is to minimize body accelerations by measuring the secondary suspension's relative displacements. It is assumed that the vehicle moves with a constant speed v on an irregular track with a PSD function $S_w(\Omega) = \frac{A_v}{(\Omega+\alpha)^2}$ [28, 39, 63].

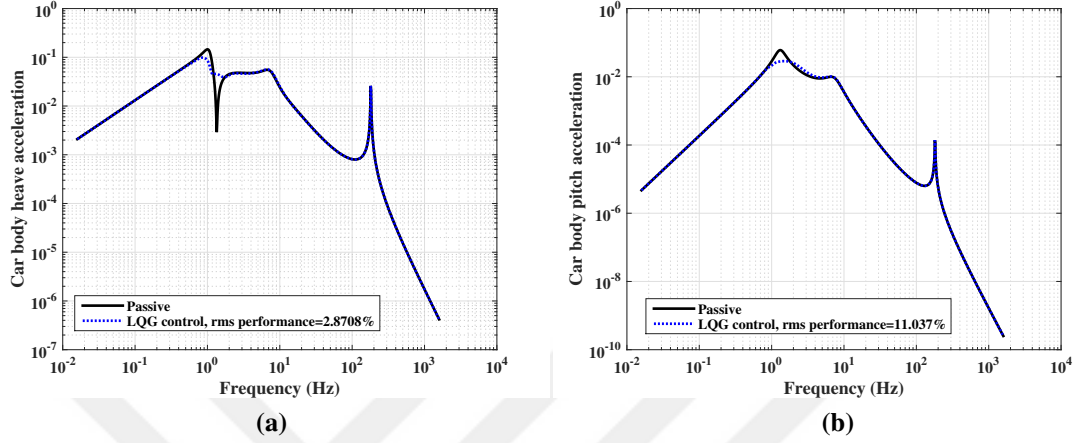


Figure 5.12. The passive and LQG controlled vehicle bode plot for (a) carbody vertical acceleration (b) carbody pitch acceleration

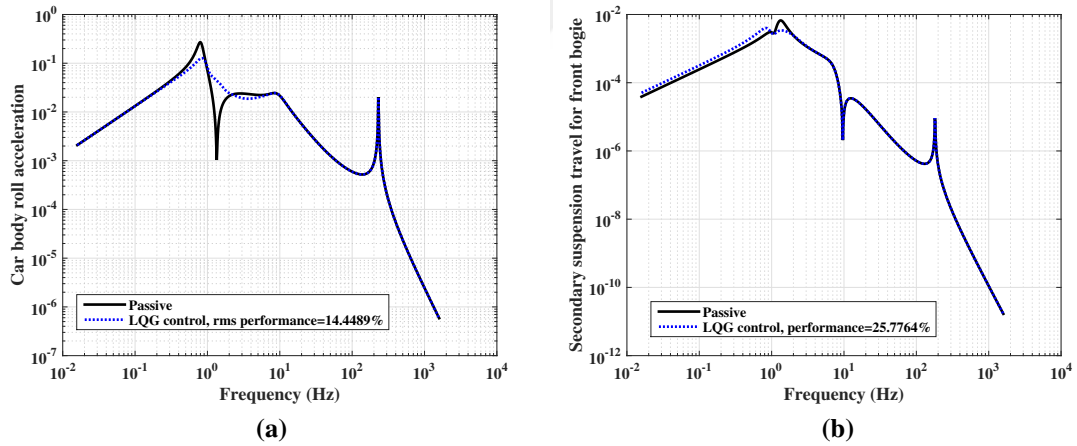


Figure 5.13. The passive and LQG controlled vehicle bode plot for (a) carbody roll acceleration (b) the secondary suspension deflection

Then benchmark parameters in [65] are used and MATLAB simulations results for LQG control are given in Figures 5.12a-5.13b. The results show that it is possible to get better comfort with less suspension deflections by active controllers. Although, the LQG control method is highly parameter dependent which means that these results are not the best. Different results may be obtained by adjusting the parameters. Also, note that the LQG controller has the same size as the plant which may be a problem for the

microcontroller if implemented on the real system since it may increase the cost. Now, by using MATLAB *HIFOO* control toolbox, we will synthesize low-order controllers.

5.2. $\mathcal{H}_\infty$ Fixed-Order Controller Design

The problem will be redefined for *H-Infinity-Fixed-Order-Optimization* (HIFOO) controller as,

Problem 5.2.1 Find an output-feedback controller $u = \mathcal{K}(s)y$ that internally stabilizes the closed loop system while satisfying

$$\begin{aligned} \min \quad & J_\infty = \|\Lambda_{\infty 3} T_{z_a \eta}\|_\infty \\ \text{s.t.} \quad & \|\Lambda_{\infty 1} T_{y \eta}\|_\infty < \sigma_1 \\ & \|\Lambda_{\infty 2} T_{z_t \eta}\|_\infty < \sigma_2 \end{aligned} \quad (5.17)$$

for chosen positive weighting matrices $\Lambda_{\infty 1}$, $\Lambda_{\infty 2}$, $\Lambda_{\infty 3}$ and positive numbers σ_1 , σ_2 .

Simulations are performed in MATLAB for a 10DOF half-car vehicle model for $z_a = [\ddot{z}_s, \ddot{\theta}_s]$ and $z_t = [s_{11}, \dots, s_{22}]$ and various controllers with order q_c were synthesized. The RMS performances of the outputs of interest were listed in Table 5.3. From the table and RMS performances, it can be concluded that the passive values can also be improved by using smaller-size controllers. Despite the high order and weight dependence, the LQG controller still performs best.

Table 5.3. The fixed-order $\mathcal{H}_\infty$ Controllers Performances

Output	$(q_c = 4)$	$(q_c = 6)$	$(q_c = 8)$	$(q_c = 10)$	(LQG)
s_1	13.03	-7.09	13.03	13.63	14.0317
s_2	0.89	-	0.89	0.90	0.8953
s_{11}	13.04	-	-	13.14	15.0452
s_{12}	13.04	-	-	13.14	13.2682
s_{21}	22.08	-	-	21.01	22.8078
s_{22}	22.08	-	22.08	21.01	22.3459
$\ddot{z}_s$	39.27	40.12	31.17	38.47	49.2727
$\ddot{\theta}_s$	39.55	52.29	44.55	39.55	39.6905

Until now, only active secondary suspension design has been studied. Unlike the automotive industry, active/semi-active controllers in railways may also be used in vehicle

primary suspension. The main task of the primary suspension is to provide safer riding. On the other hand, it may be possible to get better comfort conditions via well-designed primary suspensions since the vibrations will be filtered before reaching the car body. Then as a comparison study, active primary suspension system performances will be examined. It is expected to provide a complete approach to the reader in terms of active suspension design.

5.3. Active Primary Suspension Control

The primary suspension system's main task is to improve the wheel-rail interaction, while the secondary suspension should provide better comfort. Since there is a contradiction between the ride comfort and the wheel-rail forces in the passive setup, an active/semi-active controller may provide a solution [128, 11, 129]. In the previous chapters, the semi-active and active control methods have been applied to the secondary suspension systems to increase the ride quality. In this section of the thesis, the active primary suspension system is designed by using $\mathcal{H}_\infty$ output-feedback control to reduce the wheel-rail forces.

5.3.1. $\mathcal{H}_\infty$ output-feedback control

Figure 5.14 shows the 6 DOF model of the vehicle with active primary suspension. The mathematical representation of the model can be obtained by updating the model in Figure 3.6 with regulated output vector as $z_\infty = [F_{ij} \ddot{z}_s \ddot{\theta}_s]^T$ while F_{ij} defined in Equation (3.1) and the new control force $\tilde{u} = [u_{11} u_{12} u_{21} u_{22}]^T$. After doing the necessary modifications in Equations (3.2-3.3) the following equations of motions are obtained as,

▷ For car body;

$$\begin{aligned} m_s \ddot{z}_s &= \sum_{i=1}^2 F_i \\ I_{sy} \ddot{\theta}_s &= l_c \sum_{i=1}^2 (-1)^{i+1} F_i \end{aligned} \tag{5.18}$$

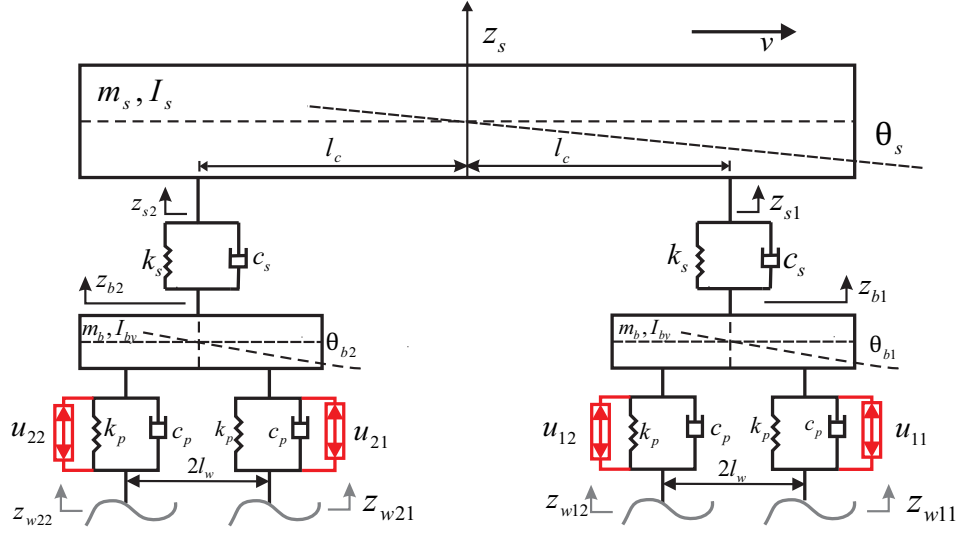


Figure 5.14. 6 DOF railway vehicle model with active primary suspension control.

▷ For front bogie (with $i = 1$) and for rear bogie (with $i = 2$)

$$\begin{aligned}
 m_b \ddot{z}_{bi} &= -F_i + \sum_{j=1}^2 (F_{ij} - u_{ij}) \\
 I_{by} \ddot{\theta}_{bi} &= l_w \sum_{j=1}^2 (-1)^{j+1} (F_{ij} - u_{ij})
 \end{aligned} \tag{5.19}$$

Here, it was assumed that the Siemens ICE train with parameters in Table A4 is moving on a tangential track. The $\mathcal{H}_\infty$ output feedback controller is designed to ensure the vehicle stability and comfort conditions on a tangential track. It is assumed that the accelerometers were used for feedback from the controller. The cost function $\mathcal{J}$ was re-defined as in Equation (5.20) with positive weights $\Delta_f, \Delta_v, \Delta_{pf}$.

$$\mathcal{J} = \Delta_f \|T_{F_{ij} \dot{z}_w}\|_\infty^2 + \Delta_v \|T_{\ddot{z}_s \dot{z}_w}\|_\infty^2 + \Delta_{pf} \|T_{\ddot{\theta}_s \dot{z}_w}\|_\infty^2 \tag{5.20}$$

First, the measurement vector $y = [\ddot{z}_{s1} \ \ddot{z}_{s2}]^T$ is considered. Then for active primary suspension design, the accelerometers are placed on the primary suspension connection nodes of the bogies, and the new measurement vector is obtained as $\tilde{y} = [\ddot{z}_{b11} \ \ddot{z}_{b12} \ \ddot{z}_{b21} \ \ddot{z}_{b22}]^T$. Here, the number of sensors and actuators doubles which in turn increases the cost. For increasing the applicability and decreasing the cost let's modify $\tilde{y}$ as $\hat{y} = [\ddot{z}_{b1} \ \ddot{z}_{b2}]^T$ by mounting the accelerometers at the centre of the bogies.

In the simulations, the weighting matrices are selected as $\Delta_f = \frac{\beta_f}{\|G_{F_{ij}r}\|_\infty}$, $\Delta_v = \frac{\beta_v}{\|G_{\ddot{z}_s r}\|_\infty}$ and $\Delta_p = \frac{\beta_p}{\|G_{\ddot{\theta}_s r}\|_\infty}$ with selected positive numbers in Table 5.4. Here, G denotes the open loop transfer function.

Table 5.4. *The positive weighting numbers*

Controller	β_f	β_v	β_p
$\mathcal{K}_s$	1.75	2.5	1.9
$\mathcal{K}_p$	2	1.2	1.2
$\tilde{\mathcal{K}}_p$	2	1.2	1.2

The $\mathcal{K}_s$ denotes the active controller for secondary suspension with the measurement vector y . The $\mathcal{K}_p$ and $\tilde{\mathcal{K}}_p$ represent the active controllers for primary suspension with the measurement vectors $\tilde{y}$ and $\hat{y}$ respectively. The problem was solved by using MATLAB *hinfmix* command.

Since there is a rigid contact between the wheels and the rail, the simulation results are obtained by assuming that the controlled vehicle driving on a straight track has track irregularity defined in Equation (3.7). From the RMS performances in Table 5.5 we can see that designing an active controller for primary suspension gives impressive improvements on the wheel forces. The active primary suspension not only decreases them but also reduces the car body acceleration. This is expected since vibrations are filtered at first hand on the bogies. Another important conclusion we can derive is that it is possible to obtain better results with fewer measurements on the primary suspension. Figures 5.15a-5.16b give the frequency responses of the vehicle outputs with passive, active secondary, and active primary suspensions. Remember, the secondary suspension control provided improvement only in the low-frequency range while the primary suspension control works better both at the low and middle-frequency range.

Table 5.5. *The RMS % performances of the controllers*

Output	$\mathcal{K}_s$	$\mathcal{K}_p$	$\tilde{\mathcal{K}}_p$
F_{ij}	—	61	62
$\ddot{z}_s$	20.6	14.9	14.8
$\ddot{\theta}_s$	9.3	15.9	16

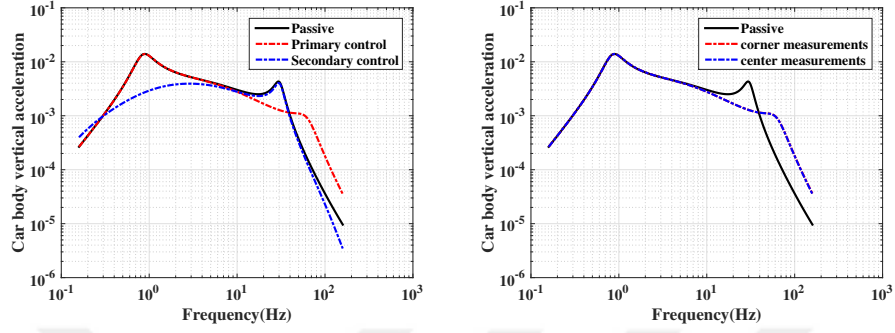


Figure 5.15. (a) Car body vertical acceleration bode plot with $\mathcal{K}_s$ and $\mathcal{K}_p$ (b) Car body vertical acceleration bode plot with $\mathcal{K}_p$ and $\tilde{\mathcal{K}}_p$

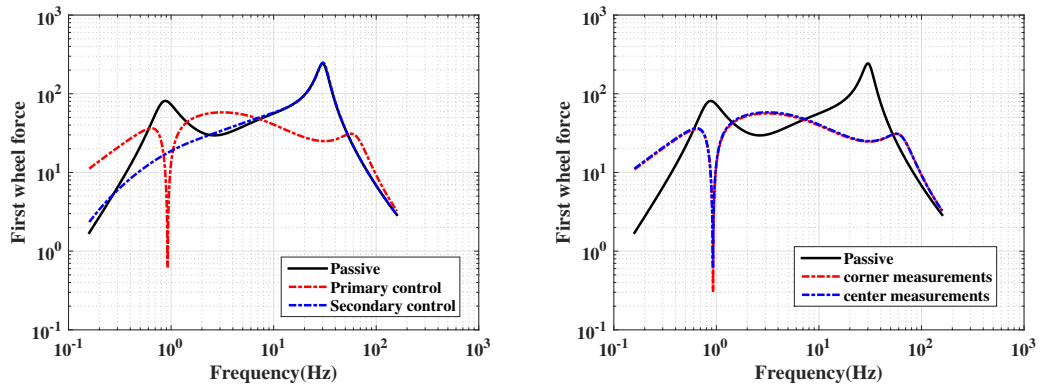


Figure 5.16. (a) Primary suspension forces between rst wheel-front bogie bode plot with $\mathcal{K}_s$ and $\mathcal{K}_p$ (b) Primary suspension forces between rst wheel-front bogie bode plot with $\mathcal{K}_p$ and $\tilde{\mathcal{K}}_p$

5.4. Summary

In this chapter, the LQG control methodology was initially used to design an active secondary suspension that minimizes the car body's vertical acceleration and secondary and primary suspension deflections simultaneously. Although promising improvements were obtained, it does not mean that these results were best. There is a margin of improvement by tuning the weighting matrices considering the elastic contact between the wheel and the rail, the model was extended to 10DOF, but now low-order controllers were synthesized by using the open-access tool HIFOO to improve the vehicle's comfort and safety. These controllers' performances were seen to be as good as LQG-controlled vehicles and may encourage the user to test and realize the vehicle. With the experience gained here, the method was also applied to the full vehicle model, including the roll motion. Then all actively controlled secondary suspension system was seen to provide decreased acceleration on the car body without increasing wheel forces. Finally, the active control problem was re-defined for primary suspension to obtain more impressive results on wheel-rail forces. It was seen that wheel-rail forces are decreased by approximately half-and-half supporting less vertical acceleration on the car body demand.

6. POLYTOPIC VEHICLE SUSPENSION MODELS

In the previous chapters, while solving the active/ semi-active problems, it was assumed that all vehicle and track parameters are linear and exact. This assumption is unrealistic since some design parameters may vary during driving. For instance, the vehicle travels on a straight rough surface line where the quality of the line can differ during driving. The wheel-rail contact is another concern that should be addressed where the force transfers of vehicle-track take place. The elastic contact between the wheel and rail is parameter-dependent and changes over time. Therefore, the designed multi-objective controller should guarantee vehicle performances for all possible values of the design parameters.

In this chapter, the parameter-dependent system will be formalized for the changing quality of the tracks. Later, a multi-objective controller will be synthesized for an acceptable range of Hertzian contact stiffness. The study will conclude with an analysis of where the body mass changes.

6.1. A Parameter Dependent System

A linear-time-invariant system P can be modeled as a parameter-dependent system with the parameter vector $\rho = [\rho_1, \dots, \rho_n]$. Suppose that an LTI system can be decomposed into the form of $P(\rho) = P^0 + \sum_{i=1}^n \rho_i P^i$ [66], where,

$$P(\rho) = \left(\begin{array}{c|c} \frac{A(\rho) + jE(\rho)}{C(\rho)} & \frac{B(\rho)}{D(\rho)} \end{array} \right), \quad P^i = \left(\begin{array}{c|c} \frac{A^i + jE^i}{C^i} & \frac{B^i}{D^i} \end{array} \right) \quad (6.1)$$

with

$$\begin{aligned} A(\rho) &= A^0 + \sum_{i=1}^n \rho_i A^i, & B(\rho) &= B^0 + \sum_{i=1}^n \rho_i B^i \\ C(\rho) &= C^0 + \sum_{i=1}^n \rho_i C^i, & D(\rho) &= D^0 + \sum_{i=1}^n \rho_i D^i \\ E(\rho) &= E^0 + \sum_{i=1}^n \rho_i E^i, \end{aligned}$$

The half-car model will be employed first, in the polytopic formulation. It was assumed that the train goes on a straight track which has cross-level track irregularity defined in Equation (3.7). The time delays between wheels were ignored. Then state-space presen-

tation from white noise velocity input $\tilde{\eta}$ to $\dot{z}_w$ is given below.

$$\begin{aligned}\dot{\tilde{x}}_{tr} &= \tilde{A}_{tr} + \sqrt{A_v} \tilde{B}_{tr} \tilde{\eta} \\ \dot{z}_w &= \tilde{C}_{tr} \tilde{x}_{tr} + \sqrt{A_v} \tilde{D}_{tr} \tilde{\eta}\end{aligned}\tag{6.2}$$

where $\tilde{A}_{tr} = I_4 \otimes A_{tr}$, $\tilde{B}_{tr} = I_4 \otimes B_{tr}$, $\tilde{C}_{tr} = I_4 \otimes A_{tr} C_{tr}$, $\tilde{D}_{tr} = I_4 \otimes C_{tr} B_{tr}$ with the matrices A_{tr} , B_{tr} , C_{tr} are defined in Chapter 3 in Equation (3.13). The track-vehicle augmented state-space was obtained by substituting Equation (6.2) into the Equation (3.5).

$$\begin{aligned}\dot{\tilde{x}} &= \tilde{A} \tilde{x} + \sqrt{A_v} \tilde{B}_1 \tilde{\eta} + \tilde{B}_2 u \\ z_2 &= \tilde{C}_2 \tilde{x} + \sqrt{A_v} \tilde{D}_{21} \tilde{\eta} + \tilde{D}_{22} u \\ z_\infty &= \tilde{C}_\infty \tilde{x} + \sqrt{A_v} \tilde{D}_{\infty 1} \tilde{\eta} + \tilde{D}_{\infty 2} u\end{aligned}\tag{6.3}$$

Here exogenous output is $z_2 = [\ddot{z}_s, \ddot{\theta}_s]^T$ and the regulated output is $z_\infty = [F_{11}, \dots, F_{22}]^T$ with the matrices below.

$$\begin{aligned}\tilde{A} &= \begin{bmatrix} A & B_1 \tilde{C}_{tr} \\ \mathbf{0} & \tilde{A}_{tr} \end{bmatrix}, \quad \tilde{B}_1 = \begin{bmatrix} B_1 \tilde{D}_{tr} \\ \tilde{B}_{tr} \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} B_2 \\ \mathbf{0} \end{bmatrix}, \\ \tilde{C}_2 &= \begin{bmatrix} C_2 & D_{21} \tilde{C}_{tr} \end{bmatrix}, \quad \tilde{D}_{21} = D_{21} \tilde{D}_{tr}, \quad \tilde{D}_{22} = D_{22}, \\ \tilde{C}_\infty &= \begin{bmatrix} C_\infty & D_{\infty 1} \tilde{C}_{tr} \end{bmatrix}, \quad \tilde{D}_{\infty 1} = D_{\infty 1} \tilde{D}_{tr}, \quad \tilde{D}_{\infty 2} = D_{\infty 2}.\end{aligned}$$

Let's formulate the problem for synthesizing the controller which can meet the comfort and handling properties for different track qualities. The parameter-dependent representation of the track-vehicle system with track roughness A_v is as follows:

$$P(A_v) = P^0 + \sqrt{A_v} P^1\tag{6.4}$$

where the quadruplets

$$P^0 = \left(\tilde{A}, \begin{bmatrix} \mathbf{0} & \tilde{B}_2 \end{bmatrix}, \begin{bmatrix} \tilde{C}_\infty & \tilde{C}_2 \end{bmatrix}^T, \begin{bmatrix} \mathbf{0} & \tilde{D}_{\infty 2} \\ \mathbf{0} & \tilde{D}_{22} \end{bmatrix} \right),$$

$$P^1 = \left(\mathbf{0}, \begin{bmatrix} \tilde{B}_1 & \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix}^T, \begin{bmatrix} \tilde{D}_{\infty 1} & \mathbf{0} \\ \tilde{D}_{21} & \mathbf{0} \end{bmatrix} \right)$$

Then, the robust multiobjective control design problem is:

Problem 6.1.1 *Design a state feedback controllers $u = \mathcal{K}\tilde{x}$ that provides stability of the closed loop system and minimizes the cost function J such that:*

$$\begin{aligned} \min \quad & J = \Phi \|T_{z_2\eta}\|_2^2 \\ \text{s.t.} \quad & \|T_{z_\infty\eta}\|_\infty < \gamma \end{aligned} \tag{6.5}$$

for $\Phi > 0$ and $\gamma > 0$.

All the closed loop poles are restricted to lie on the defined range $\mathcal{D} = \{z = x+iy, -40 < x < -0.001, |y| < 70\}$ shown with Figure 6.1. This feasible region allows the controller to be bounded and stabilize the closed-loop system. It is obtained by trial and error. The active control problem will be synthesized for two-different conditions:

- (i) For fixed track quality $A_v = 0.0719 \times 10^{-6}$ (class-6) the controller “ $\mathcal{K}_f$ ” is designed
- (ii) For different track qualities where $\sqrt{A_v} \in [\sqrt{0.0254 \times 10^{-6}}, \sqrt{0.1566 \times 10^{-6}}]$ (track classes 4-8) the controller “ $\mathcal{K}_p$ ” is designed.

This optimization problem can be solved by using LMIs. Figures 6.2a-6.2b show the simulation results that are obtained by MATLAB *msfsyn* command. We observe that the first mode, directly related to passenger comfort, is well suppressed while the second mode keeps its passive state for $\mathcal{K}_f$ and $\mathcal{K}_p$. From the graphs on Figures 6.2a-6.2b, we can observe that the controller performances on different track qualities are quite similar to the fixed roughness controller, and this motivates the designer to synthesize polytopic controllers when it is hard to know about the track roughness [21].

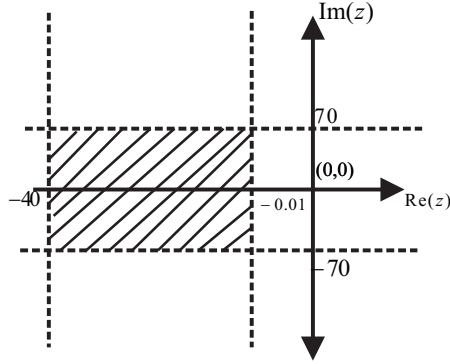


Figure 6.1. The closed-loop poles region $\mathcal{D}$

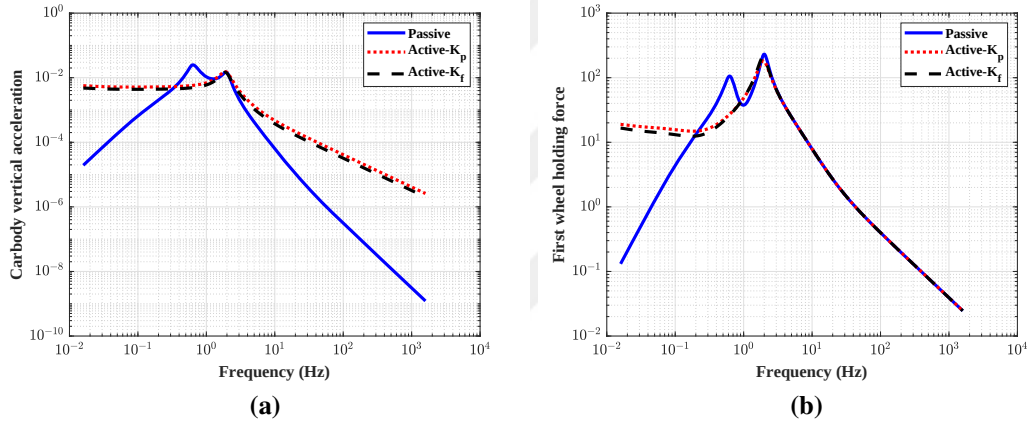


Figure 6.2. Frequency response of (a) the carbody vertical acceleration (b) wheel-rail force

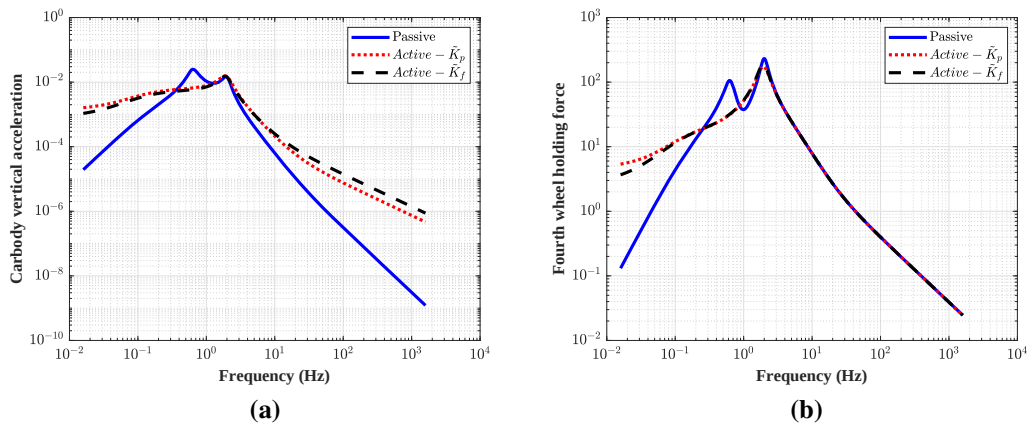


Figure 6.3. Frequency response of (a) the carbody vertical acceleration (b) wheel-rail force

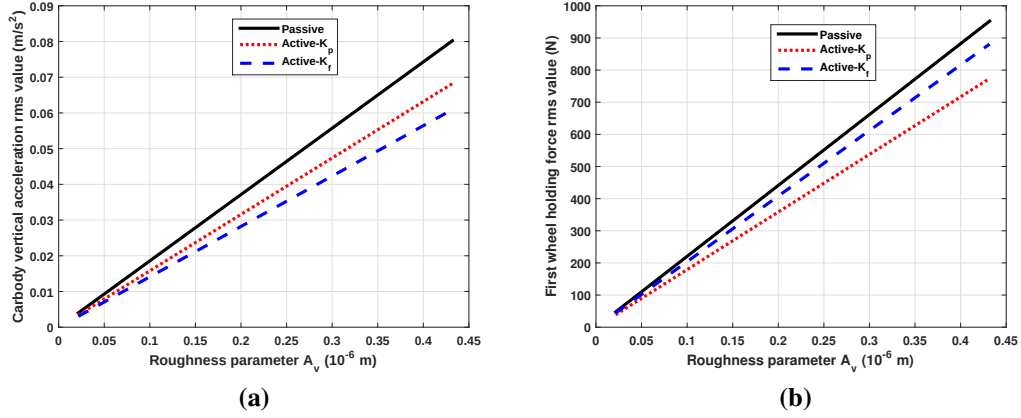


Figure 6.4. RMS response for increasing A_v (a) the carbody vertical acceleration (b) wheel-rail force

Let's integrate the handling requirement into the optimization as $z_2 = [F_{ij}]$ for $i, j = 1, 2$ to be minimized while the ride quality $z_\infty = [\ddot{z}_s \ddot{\theta}_s]^T$ is a constraint. Now the polytopic controller $\mathcal{K}_f$ will be replaced with $\tilde{\mathcal{K}}_f$ and $\tilde{\mathcal{K}}_p$ instead of $\mathcal{K}_p$. The RMS results of the solution of Problem 6.1.1 are given in Table 6.1. We can see that the designed controller will perform well if the track roughness parameter is precisely known. Also, when the track roughness is confined in a range, the designed polytopic controller would suppress the wheel-rail forces better. Figures 6.3a-6.3b show the frequency responses of the passive and active suspension. The polytopic control effectively solves impressive where discomfort problems. Figures 6.4b-6.4a shows the passive and actively controlled vehicle RMS results in terms of roughness parameter A_v . It may be concluded from the figures that the active controller gives better results in ride comfort for all track conditions, especially if the track roughness is known exactly. Additionally, the polytopic controller is more successful in improvement of the wheel-rail forces that are transferred between the wheel and rail at the contact point.

The simulation above was performed for the rigid contact between the wheels and rail. Since the 6DOF model was used, the car body roll motion and the wheel-rail contact behavior were ignored. However, as well as the impact of the car roll motion on passenger comfort, the wheel-rail interaction affects the wheel forces as a wheel. Then modeling wheel-rail contact and influences on active controller design for a full-car model will be analyzed.

Table 6.1. *Passive and actively controlled vehicle rms responses*

Output	Passive	Performances			
		$\mathcal{K}_f$	$\mathcal{K}_p$	$\tilde{\mathcal{K}}_f$	$\tilde{\mathcal{K}}_p$
$\ddot{z}_s \text{ (m/s}^2\text{)}$	0.0134	0.0102 (24.04%)	0.0114 (14.94%)	0.0110 (17.64%)	0.0117 (12.35%)
$\ddot{\theta}_s \text{ (m/s}^2\text{)}$	0.0023	0.0019 (17.82%)	0.0021 (7.98%)	0.0016 (27.60%)	0.0017 (23.86%)
$z_\infty \text{ (N)}$	231.32	224.82 (2.81%)	213.71 (7.62%)	229.80 (0.6%)	223.17 (3.47%)

6.1.1. Wheel-rail contact

The trains are known as vehicles that move on a track at higher speeds under big axle loads. The higher axle loads cause higher wheel-rail dynamic forces transmitted to the track during driving. The results of higher dynamic forces can be summarized as follows [28].

- (i) Ground vibrations occur, which are transferred to the buildings located nearby the track. It may cause human discomfort in these buildings or damage to the structure.
- (ii) These dynamic forces may create irregularities or worn wheel/rail profiles, which can increase the vibrations on the vehicle and decrease the passenger comfort indirectly.

Since these forces are transferred at the wheel-rail contact point, another important problem is arised for railways as modeling contact points to the closest reality. In [130], it was emphasized that many parameters affect the value of tire damping coefficient in road vehicles and the difficulties of calculating its exact value. Similarly, in railway vehicles, the approximate value of the wheel-rail elastic contact spring coefficient may be calculated. In Chapter 5, the elastic contact is explained in detail. Equation (5.7) shows that the Hertzian spring stiffness has an approximate value and depends on the rail's material and size, wheel size, and static wheel load. Then, in this part of the thesis, to consider all details about elastic contact effects on vehicle performance, and 17 DOF vehicle models will be used. An affine parameter-dependent system with the parameter k_H will be obtained and determined. The possible range in the safety limits for the k_H . All simulations

are realized by using the commonly used high-speed passenger train French TGV parameters listed in Table A5, and it is assumed that the vehicle moves on a rough track which is modeled as $S_w(\phi) = \frac{A_v}{(\Omega + \alpha)^2}$. The state space form in Equation (5.15) should be updated with output vectors as $z_\infty = [\ddot{z}_s \ \ddot{\theta}_s \ \ddot{\varphi}_s]^T$ and $z_2 = [-k_H(z_{wij} - w_{ij})]^T$ for $i = 1, \dots, 4$ and $j = 1, 2$. The range for the Hertzian spring stiffness k_H is defined as $[k_H - \rho, k_H + \rho]$ with an uncertainty $\rho \in [\rho_{min}, \rho_{max}]$.

The parameter limits are listed below.

1. The wheel radius variations are permitted in the range $\mp 10\%$ for an unloaded and loaded vehicle [131]. The rail head radius changes are assumed as $\mp 10\%$ and the range $\mathcal{R}_1$ is calculated as:

$$\mathcal{R}_1 : \{c_H | (0.81)^{1/4} c_H \leq c_H \leq (1.27)^{1/4} c_H\} \quad (6.6)$$

where c_H is defined in Equation (5.6).

2. The static wheel load for unloaded and loaded vehicles will be in the range $\mathcal{R}_2$. The loaded vehicle is assumed that 60 people each have 70 kg are traveling in the vehicle.

$$\mathcal{R}_2 : \{F_s | F_s \leq 1.05 F_s\} \quad (6.7)$$

The Hertzian spring stiffness range can be obtained as $\mathcal{R}_{k_H}$. In other words, $\mathcal{R}_{k_H}$ corresponds to the permitted -3.45% decreases and 5.8% increases on k_H .

$$\mathcal{R}_{k_H} = \mathcal{R}_1^{2/3} \cup \mathcal{R}_2 : \{k_H | 0.97 k_H \leq k_H \leq 1.06 k_H\} \quad (6.8)$$

When the stiffness changes, it was seen in Equation (6.9) that the natural frequency will be affected. The wheelset heave (ω_{n1}) and roll (ω_{n2}) modes are directly proportional to the square root of the Hertzian spring stiffness as,

$$\omega_{n1} = \frac{1}{2\pi} \sqrt{\frac{2k_H}{m_w}}, \quad \omega_{n2} = \frac{1}{2\pi} \sqrt{\frac{2k_H l_p^2}{I_{wx}}} \quad (6.9)$$

In Figure 6.5a, these mode changes in terms of k_H . The higher values of k_H shift the

wheel hop mode to the higher frequencies but do not affect the magnitude of the car body's vertical acceleration. On the other hand, higher values of k_H decrease the damping factor will increase the overshoot of the wheel-rail forces as shown in Figure 6.5b. Therefore, optimization of the wheel-rail forces without deformations on the car body accelerations in the vertical direction for varying k_H should be investigated. For this, define the following problem.

Problem 6.1.2 *For the augmented track-vehicle system the generalized eigenvalue minimization problem is defined such that :*

$$\begin{aligned}
 \min \quad & \alpha \\
 \text{s.t.} \quad & \alpha > 0 \\
 & \hat{A}_0^T Q + Q^T \hat{A}_0 < 0 \\
 & A_i Q^T + Q A_i^T < \alpha (\hat{A}_0 Q^T + Q \hat{A}_0^T), \quad \text{for } i = 1, 2
 \end{aligned} \tag{6.10}$$

with the invertible matrix $Q = Q^T$ and $A_1 = \rho_{\min} \hat{A}_1$ and $A_2 = \rho_{\max} \hat{A}_1$.

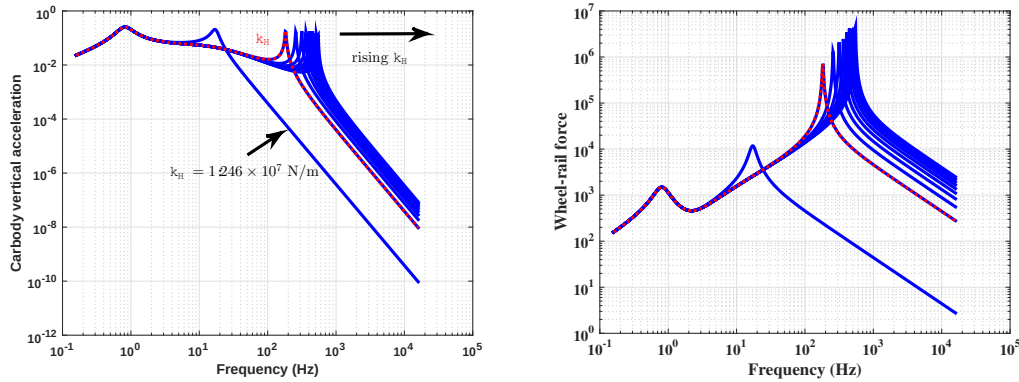


Figure 6.5. The linearized Hertzian spring k_H effects on (a) car body vertical acceleration frequency response (b) on the wheel-rail force frequency response, (...) represents the calculated k_H

Before designing a controller, at first, the quadratic stable range for k_H should be determined since the system is stable if and only if it is quadratically stable [132, 66]. The Lyapunov matrix $P = Q^{-1}$ guarantees the quadratic stability range $[-4.2 \times 10^7, 4.2 \times 10^7]$, which means the $\pm 3.37\%$ variations on k_H .

Again MATLAB *msfsyn* command is used to solve the problem with the constraints on the car body heave, roll, and pitch accelerations. The results are obtained for $\Phi_2 = \text{diag}(0.3\|G_{\ddot{z}_s\eta}\|_\infty^{-1}, 0.21\|G_{\ddot{z}\theta_s\eta}\|_\infty^{-1}, 1.5\|G_{\ddot{\phi}_s\eta}\|_\infty^{-1})$ with $\gamma = 1$ and the weight matrix Φ_2 . Here $G_{\ddot{z}_s\eta}$, $G_{\ddot{z}\theta_s\eta}$, $G_{\ddot{\phi}_s\eta}$ show the open-loop transfer functions from the outputs $\ddot{z}_s$, $\ddot{z}\theta_s$, $\ddot{\phi}_s$ to the input η .

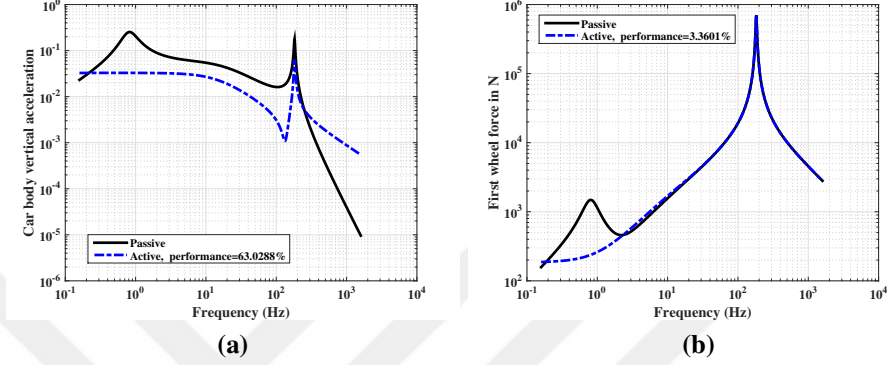


Figure 6.6. (a) Car body vertical acceleration bode plot (b) The wheel-rail force bode plot

The simulation results are demonstrated in Figures 6.6a-6.6b for the passive and active frequency responses of the car body heave acceleration and the wheel-rail dynamic forces for French TGV. Although the designed controller is placed between the car body and bogies, meaningful results are obtained for the wheel-rail dynamic forces at the low-frequency band, $\approx 3.36\%$. Improvements are achieved, especially on the heave acceleration with 63% and the roll acceleration with 57.33%.

6.2. The Car Body Mass Effects on Active Control of Railway Vehicle

In railways, the train should be designed to meet comfort requirements while transferring less dynamic loads from trains to track. Since the passive systems are not sufficient for changing weights, speeds, and track conditions, the active controller provides an expensive solution. Besides, the signals for feedback may not be measurable which requires the integration of the estimation methods into the control systems [67]. Therefore the future plan is to design lighter-bodied vehicles to achieve comfort and safety requirements with an energy-efficient structure. The lightweight design may reduce the transferred dynamic forces to the track but it causes higher accelerations on the car body which disturb passengers more [68]. From this point of view, in this part of the thesis,

the car body's mass effect on vehicle responses should be studied. The 17DOF vehicle model in Figure 4.6 is used with Siemens ICE high-speed train parameters in Table A4. The state space representation was updated with $z_{\infty 1} = [z_{wij} - w_{ij}]^T$, $z_{\infty 2} = [\ddot{z}_s \ddot{\theta}_s \ddot{\varphi}_s]^T$ and $z_{\infty} = [z_{\infty 1}^T z_{\infty 2}^T]^T$.

It is assumed that the vehicle is traveling on an irregular track with a constant speed v . A stochastic model in [133] for the collected data, from the right (r) and left (l) parts of the Qinhuangdao-Shenyang mainline in China is developed. The PSD function in Equation (6.11) represents the track model with the parameters $A^r = 0.8609$, $B^r = 0.2213$, $A^l = 0.8857$, $B^l = 0.3789$ and the spatial frequency Ω in *cycle/m*.

$$S_v(\Omega) = \frac{A^{(r,l)} B^{(r,l)2}}{\Omega^2(\Omega^2 + B^{(r,l)2})} \quad (6.11)$$

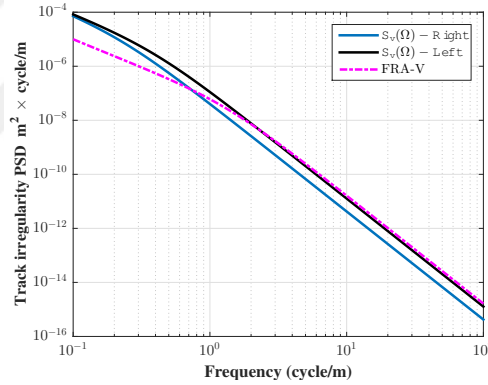


Figure 6.7. Track irregularity PSD of Qinhuangdao-Shenyang main line

The PSD function $S_v(\Omega)$ is plotted in Figure 6.7 for both the right and left tracks. From the figure, it can be seen that the specifications of this line converge to the FRA vertical profile track irregularity in class-5 quality. Here, we're going to develop a polytopic model to study the effect of mass on vehicle behaviors. At first, the maximum stable range for the car body mass is determined by,

$$\Delta_m : [m_s - m_0, m_s + m_0], \quad m_0 \in [m_0^{\min}, m_0^{\max}]$$

where the range is found as $m_0 \in [0.33m_s, 0.66m_s]$. The car body mass changes in this

range will affect the static wheel load directly and can be calculated from the gravitational acceleration $g = 9.81 \text{ m/s}^2$ as,

$$F_s = \frac{(m_s + 2 \times m_b + 4 \times m_w) \times g}{8} \quad (6.12)$$

The constant wheel-rail interaction during whole driving depends on the relationship between the dynamic wheel-rail forces (F_{dyn}^{ji}) and the static wheel forces (F_s) such that,

$$F_{dyn}^{ji} \leq F_s$$

where F_{dyn}^{ji} denotes the dynamic force between ji^{th} wheel and the rail as,

$$F_{dyn}^{ji} = -k_H(z_{wji} - w_{ji}), \quad \text{for } j = 1, \dots, 4 \quad \text{and } i = 1, 2. \quad (6.13)$$

Figure 6.8a shows the dynamic force variations according to the car body mass and the Hertzian spring k_H . Figure 6.8b represents the dynamic force RMS values versus static force RMS values according to the variations of the car body mass m_s in the range Δ_m .

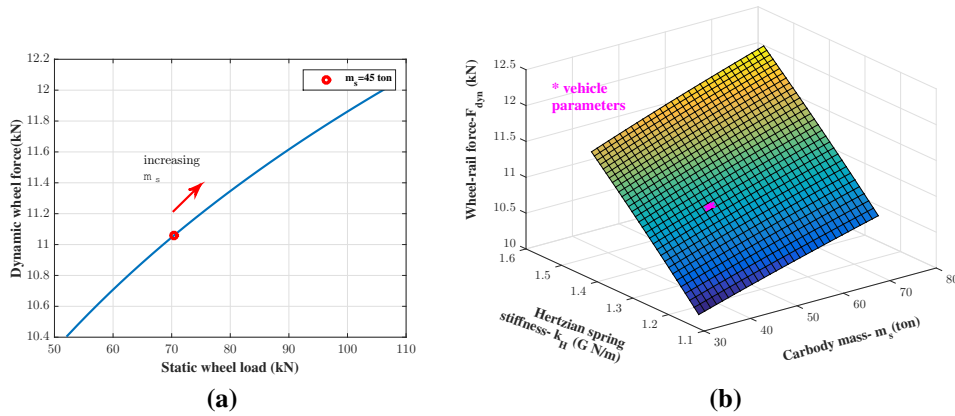


Figure 6.8. (a) The dynamic wheel-rail forces changes in terms of k_H and m_s (b) The car body mass m_s effects on dynamic and static forces

Figure 6.9a represents the car body heave acceleration RMS values versus the dynamic wheel-rail forces for different car body masses. As expected from the vibration theory, the heavier bodies vibrate less and hence provide better comfort. On the other

hand, lighter bodies have smaller dynamic forces which means safer and more energy-efficient design. The figures also support these remarks that can be defined as a trade-off for the train dynamics.

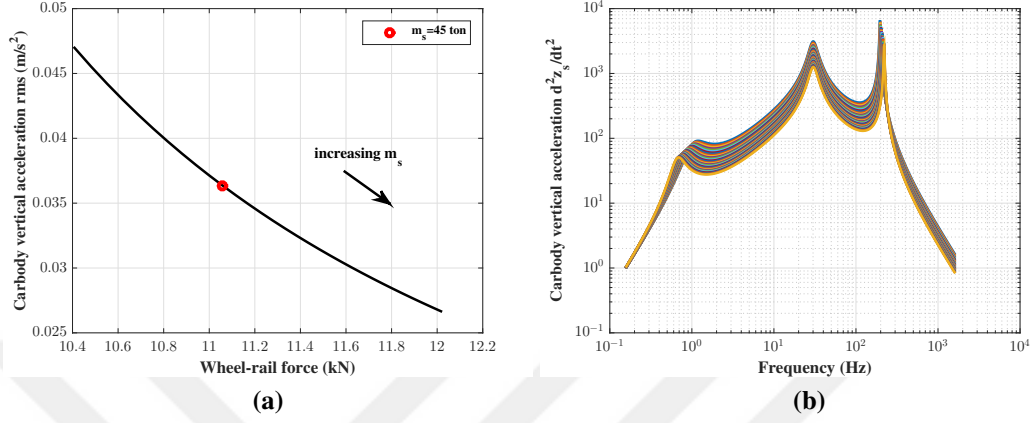


Figure 6.9. (a) The car body heave acceleration and dynamic wheel-rail force changes in terms of car body mass m_s (b) m_s influences on passenger comfort in the frequency domain

If the effect of body mass on the heave acceleration is investigated, it would be seen that the body mass does not only affect the magnitude of the acceleration but also influences the natural frequencies (see Equation (3.22)). However, the car body gets lighter, and the body and the wheel hop modes when become closer to each other, as in Figure 6.9b. Figure 6.10 illustrates this effect in detail.

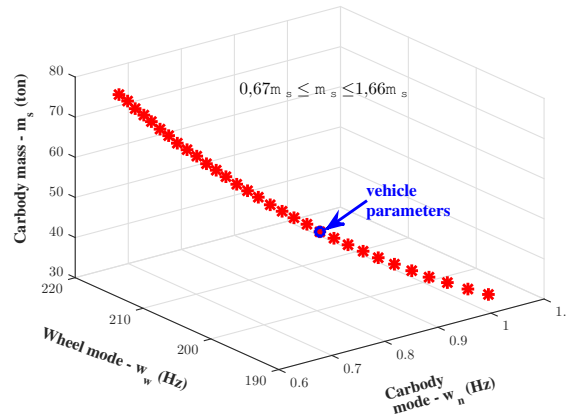


Figure 6.10. The car body mass m_s effects on the vehicle modes.

Since the mass of the wagon may vary due to changing number of passengers or loads a polytopic controller should be designed to guarantee the vehicle's safety and comfort conditions. We are going to employ the $\mathcal{H}_\infty$ controller to perform this requirement.

Consider the following problem,

Problem 6.2.1 For given $\psi_1, \psi_2, \psi_3, \psi_4 > 0$ find a state feedback controller $u = K(s)x$ which stabilizes the closed loop system and minimizes the objective function J_∞ .

$$J_\infty = \psi_1 \|T_{z_{\infty 1} w}\|_\infty^2 + \psi_2 \|T_{\ddot{z}_s w}\|_\infty^2 + \psi_3 \|T_{\ddot{\theta}_s w}\|_\infty^2 + \psi_4 \|T_{\ddot{\varphi}_s w}\|_\infty^2 \quad (6.14)$$

Here, T denotes the closed loop system transfer function from input w to the related outputs $z_{\infty 1}$, $\ddot{z}_s$, $\ddot{\theta}_s$ and $\ddot{\varphi}_s$ respectively. The weight functions $\psi_1, \psi_2, \psi_3, \psi_4$ are selected as $\lambda_1 \|G_{z_{\infty 1} w}\|_\infty^{-1}$, $\lambda_2 \|G_{\ddot{z}_s w}\|_\infty^{-1}$, $\lambda_3 \|G_{\ddot{\theta}_s w}\|_\infty^{-1}$ and $\lambda_4 \|G_{\ddot{\varphi}_s w}\|_\infty^{-1}$ with the open loop transfer functions G from w to the outputs $z_{\infty 1}$, $\ddot{z}_s$, $\ddot{\theta}_s$ and $\ddot{\varphi}_s$. The four different controllers are designed for different car body mass conditions and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ listed in Table 6.2.

Table 6.2. The selected parameters

Parameter	K_∞	K_∞^l	K_∞^h	K_∞^p
λ_1	100	100	100	100
λ_2	55	55	50	50
λ_3	32	30	20	20
λ_4	55	50	80	80

In the Table 6.2, K_∞ denotes the $\mathcal{H}_\infty$ design with vehicle real m_s and the K_∞^l is designed with the same objective but a 20 % lighter. The K_∞^h includes the 20 % heavier body mass while K_∞^p is designed for polytopic systems with $m_s \in \Delta_m$.

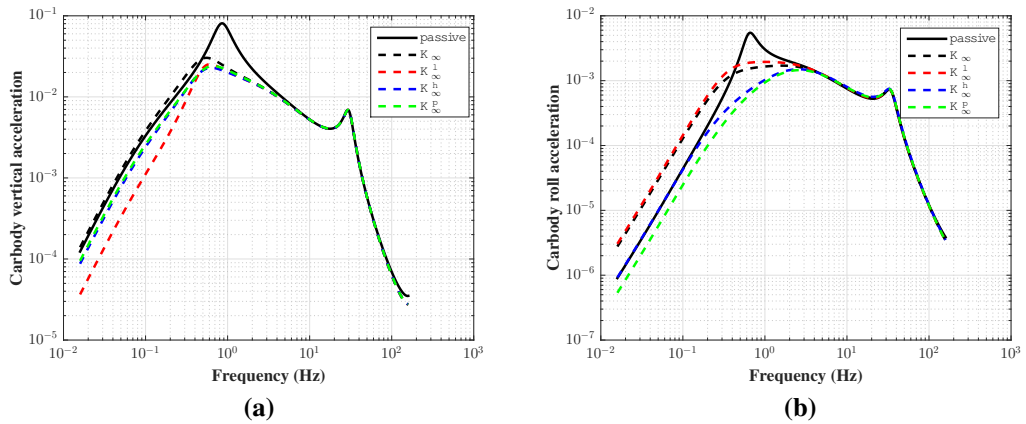


Figure 6.11. Frequency response of (a) car body vertical acceleration (b) car body roll acceleration

The controlled system is tested on Qinhuangdao-Shenyang main line track profile by using numerical simulations in MATLAB. The RMS values are displayed in Table 6.3 and the Figures 6.11a-6.11b show the frequency response plots.

Table 6.3. *The controllers rms % performances.*

Output	K_{∞}	K_{∞}^l	K_{∞}^h	K_{∞}^p
$\ddot{z}_s$	35.18	38.61	39.07	38
$\ddot{\theta}_s$	26.12	27.30	29.05	21
$\ddot{\varphi}_s$	32.7	28	52.2	54.5
F_{dyn}^{ji}	< 1	< 1	< 1	< 1

It is concluded from the figures that the controllers succeed to suppress the first mode of the vehicle which is directly related to passenger comfort. The controllers designed for the polytopic mass and the heavier mass give the best results, especially on the roll acceleration of the car body. According to the rms performances, the improvements in the heave acceleration of the body are between 35 % and 40 %. This is an achievement since there are no increases in the wheel-rail dynamic forces.

6.3. Summary

In this chapter, the parameter-dependent system was presented, for varying track quality. Then, a polytopic controller was designed for a half-car vehicle model on different track conditions. The Hertzian contact theory was exploited to describe the elastic contact between the wheel and the rail. The feasible range for the Hertzian spring stiffness was defined and the maximum stability range for the controller was found. For this stable range, an $\mathcal{H}_{\infty}$ state-feedback control method was used to decrease the wheel-rail forces while keeping the body's vertical accelerations under the passive values. The results especially on comfort were striking. On the other hand, the applicability was still a discussion since the state-feedback design needs all states to be measured which may not be a feasible assumption.

The effect of the car body mass on vehicle response was investigated. The results show that passenger comfort could not be achieved at higher speed by only decreasing the rigid body mass. However, lighter bodies may shift the flexible body modes to the low

frequencies. Reducing the mass of the vehicle body, decreased the force that was applied to the track and increased the vibrations on the body.



7. MULTI-OBJECTIVE CONTROL FOR ELASTIC BODIED RAILWAY VEHICLE

Railway transportation is very cost-intensive compared to other transportation systems. Therefore, railway vehicles must be designed as energy efficient and cost-effective which may be possible by making use of new lightweight designs and straightforward mechanical configurations. On the other hand, in the lightweight vehicle design, the flexible body modes of the vehicle which refer to *elastic modes* in the mechanical engineering literature will shift towards the frequency values that the human body is sensitive to, and hence modeling of the vehicle's flexible body modes also will become necessary. To achieve this design, it has become inevitable to apply mechatronic systems that include sensors, actuators, and control systems to the vehicle from the earliest stages of development.

The Finite-Element (FE) method is widely used to model whole vehicle modes. The FE models have a large number of degrees of freedom which is inevitable for classical control design. Then, the reduced order model of the original system is used for controller design. When the reduced order controller is applied to the original system, the actuator forces not only reduce the vibration of lower modes but also produce undesirable vibrations due to the unmodeled dynamics. This situation is named as control spill-over. In addition, the sensors may also sense the deflection from unmodeled modes (observation spill-over). These spill-over effects may cause performance degradation and even system instability [10, 73, 74].

Consequently, the significance of flexible body modeling has garnered attention in the analysis and control of vehicle dynamics. Despite ongoing debates within the academic and industrial realms regarding its necessity and feasibility, the active control of a vehicle can enhance comfort and ensure safer transportation for both passengers and cargo. This chapter provides an extensive discussion on not only the rigid body modes but also the flexible modes.

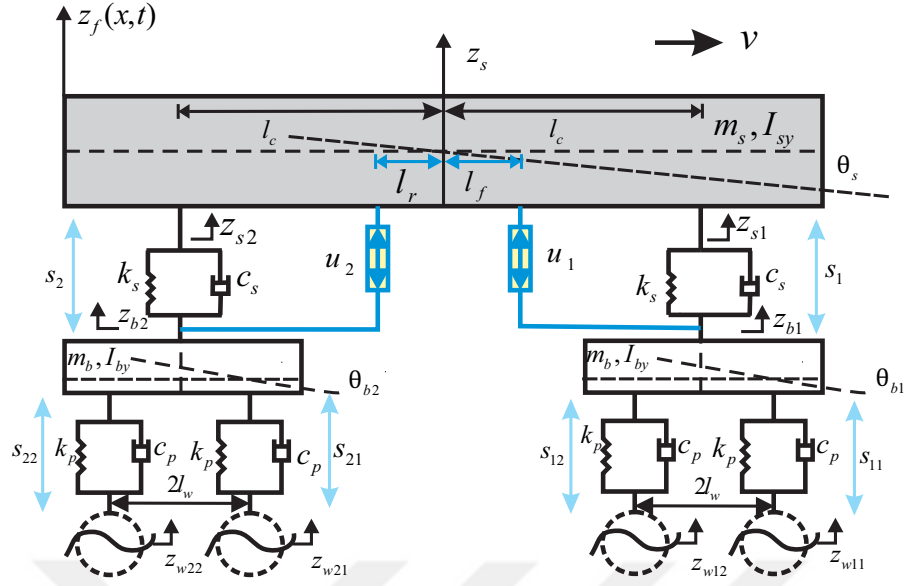


Figure 7.1. Flexible bodied railway vehicle 6DOF mathematical model.

7.1. Flexible Bodied Vehicle

Figure 7.1 presents a mathematical depiction of a flexible-bodied railway vehicle, illustrating the vertical displacement $z_f(x, t)$. The car body is approximated as a thin beam, considering its length significantly surpasses its depth by approximately tenfold. By assuming negligible shear deformations, the Euler-Bernoulli beam theory is employed to represent the flexible-bodied vehicle [134, 72]. The equation of motion for the forced vibration of uniform cross-section beams is as follows.

$$EI \frac{\partial^4 z_f(x, t)}{\partial x^4} + \rho A_f \frac{\partial^2 z_f(x, t)}{\partial t^2} + \mu_f I \frac{\partial z_f(x, t)}{\partial t} = Q(x, t) \quad (7.1)$$

The beam's Young's modulus of elasticity is denoted as E , the second moment of inertia as I , the flexural rigidity as ρ , internal damping ratio μ_f , and the cross-sectional area of the beam as A_f . Assuming $Q(x, t) = 0$, the free vibration solution can be derived, enabling the calculation of natural frequencies. By applying the separation principle, the vertical displacement $z_f(x, t)$ can be expressed in Equation (7.2), in terms of the mode $q(t)$ and the corresponding mode shape function $W(x)$ as,

$$z_f(x, t) = W(x)q(t) \quad (7.2)$$

Substiute Equation (7.2) into the Equation (7.1).

$$\frac{EI}{\rho A_f} \frac{\partial^4 W(x)q(t)}{\partial x^4} + \frac{\partial^2 W(x)q(t)}{\partial t^2} + \frac{\mu_f I}{\rho A_f} \frac{\partial W(x)q(t)}{\partial t} = 0 \quad (7.3)$$

The Equation (7.3) can be written as,

$$\frac{1}{W(x)} \frac{EI}{\rho A_f} \frac{\partial^4 W(x)}{\partial x^4} = -\frac{1}{q(t)} \left\{ \frac{\partial^2 q(t)}{\partial t^2} + \frac{\mu_f I}{\rho A_f} \frac{\partial q(t)}{\partial t} \right\} = \omega^2 \quad (7.4)$$

such that,

$$\frac{d^4 W(x)}{dx^4} = \beta_f^4 W(x), \quad \text{with} \quad \beta_f^4 = \frac{\omega^2 \rho A_f}{EI} \quad (7.5)$$

$$\frac{d^2 q(t)}{dt^2} + \frac{\mu_f I}{\rho A_f} \frac{dq(t)}{dt} + \omega^2 q(t) = 0 \quad (7.6)$$

The solution of Equation (7.5) can be expressed as,

$$W(x) = C_1 e^{\beta_f x} + C_2 e^{-\beta_f x} + C_3 e^{j\beta_f x} + C_4 e^{-j\beta_f x} \quad (7.7)$$

Here C_1, C_2, C_3, C_4 are constants and can be determined from the known boundary conditions. Equation (7.7) can be expressed more conveniently as,

$$W(x) = C_1 \cos \beta_f x + C_2 \sin \beta_f x + C_3 \cosh \beta_f x + C_4 \sinh \beta_f x \quad (7.8)$$

or

$$\begin{aligned} W(x) = & C_1 (\cos \beta_f x + \cosh \beta_f x) + C_2 (\cos \beta_f x - \cosh \beta_f x) + \dots \\ & + C_3 (\sin \beta_f x + \sinh \beta_f x) + C_4 (\sin \beta_f x - \sinh \beta_f x) \end{aligned} \quad (7.9)$$

The function $W(x)$ is known as the normal mode (characteristic function) of the beam and ω is called the natural frequency. For any beam, there will be an infinite number of normal modes with one natural frequency associated with each normal mode [135]. If

the i^{th} natural frequency is denoted as ω_i and the corresponding mode $q_i(t)$ with normal mode $W_i(x)$, the total free vibration response of the beam can be written as,

$$z_f(x, t) = \sum_{i=1}^{\infty} W_i(x) q_i(t) \quad (7.10)$$

Let's put Equation (7.10) into the Equation (7.1).

$$\sum_{i=1}^{\infty} \left\{ EI \frac{\partial^4 W_i(x)}{\partial x^4} \right\} q_i(t) + \sum_{i=1}^{\infty} \left\{ \rho A_f \frac{\partial^2 q_i(t)}{\partial t^2} \right\} W_i(x) + \sum_{i=1}^{\infty} \left\{ \mu_f I \frac{\partial q_i(t)}{\partial t} \right\} W_i(x) = Q(x, t) \quad (7.11)$$

By using Equation (7.5), Equation (7.11) becomes

$$\sum_{i=1}^{\infty} \left\{ \omega_i^2 \rho A_f W_i(x) \right\} q_i(t) + \sum_{i=1}^{\infty} \left\{ \rho A_f \frac{\partial^2 q_i(t)}{\partial t^2} \right\} W_i(x) + \sum_{i=1}^{\infty} \left\{ \mu_f I \frac{\partial q_i(t)}{\partial t} \right\} W_i(x) = Q(x, t) \quad (7.12)$$

Multiply Equation (7.12) by $W_j(x)$ and integrate over the length of the beam to obtain

$$\begin{aligned} \sum_{i=1}^{\infty} \left\{ \omega_i^2 \int_0^L \rho A_f W_i(x) W_j(x) dx \right\} q_i(t) + \sum_{i=1}^{\infty} \left\{ \int_0^L \rho A_f W_i(x) W_j(x) dx \right\} \frac{\partial^2 q_i(t)}{\partial t^2} + \dots \\ \sum_{i=1}^{\infty} \left\{ \frac{\mu_f I}{\rho A_f} \int_0^L \rho A_f W_i(x) W_j(x) dx \right\} \frac{\partial q_i(t)}{\partial t} = \int_0^L Q(x, t) W_j(x) dx \end{aligned} \quad (7.13)$$

According to the orthogonality condition for the normal modes [134], orthogonality relation can be expressed in the following form:

$$\int_0^L \rho A_f W_i(x) W_j(x) dx = \delta_{ij}, \quad (7.14)$$

where

$$\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \quad (7.15)$$

In the view of orthogonality conditions, the following modal equation can be written from Equation (7.13).

$$\frac{d^2 q_i(t)}{dt^2} + 2\zeta_i \omega_i \frac{dq_i(t)}{dt} + \omega_i^2 q_i(t) = Q_i(x, t) \quad (7.16)$$

where $Q_i(x, t)$ is the generalized force corresponding to the i^{th} mode with the natural frequency $\omega_i = \sqrt{\frac{EI}{\rho L^4}} \lambda_i^2$ with $\beta_f^i = \frac{\lambda_i}{L}$ and ζ_i is the damping ratio of the corresponding mode [136].

In this part of the thesis, the equations of motion for the flexible modes of the railway vehicle can be expressed by using the following equation.

$$Q_i(x, t) = \frac{1}{m_s} \left\{ \tilde{F}_1 W_i(l_1) + \tilde{F}_2 W_i(l_2) - u_1 W_i(l_3) - u_2 W_i(l_4) \right\} \quad (7.17)$$

where

$$\begin{aligned} \tilde{F}_i &= -k_s [\tilde{z}_{si} - z_{bi}] - c_s [\dot{\tilde{z}}_{si} - \dot{z}_{bi}] , \\ \tilde{z}_{si} &= z_{si} + \Gamma_q^T q, \quad \text{for } i = 1, 2, \\ q &= [q_1 \dots q_n]^T \end{aligned}$$

Equation (7.18) provides the representation of the characteristic function (mode shape function), which is obtained by applying boundary conditions to Equation (7.9) and depends on the roots λ_i that are the solutions of the equation $\cos(\lambda_i L) \cosh(\lambda_i L) - 1 = 0$ [136].

$$W_i(l) = \cosh(\lambda_i l) + \cos(\lambda_i l) - \frac{\cosh(\lambda_i L) - \cos(\lambda_i L)}{\sinh(\lambda_i L) - \sin(\lambda_i L)} \times (\sinh(\lambda_i l) + \sin(\lambda_i l)) \quad (7.18)$$

Considering that the human body is more sensitive to vibrations in the low-frequency range ($\leq 15 \text{ Hz}$), in addition to rigid body modes in Figure 7.2, only the first two flexible bending modes are taken into consideration which have the shapes shown in Figure 7.3.

Then the state-space equations can be defined as in Equation (7.19) in terms of open-loop-transfer functions $\tilde{G}$ and the new state vector $\tilde{x} = [x^T \ q^T \ \dot{q}^T]^T$ with the regulated output $\tilde{z} = [\ddot{z}_s \ \ddot{\theta}_s \ s_1 \ s_2 \ \ddot{q}^T]^T$ for $n = 2$ [72]. Here x is defined in Chapter 3 with

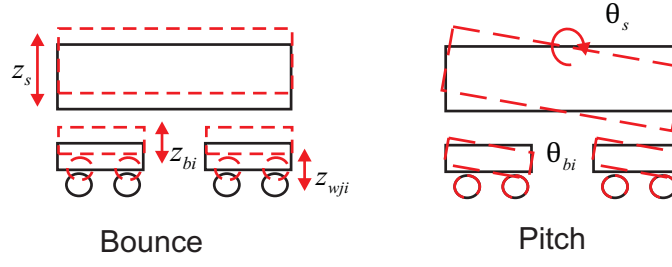


Figure 7.2. Rigid body vertical modes in railway vehicle half car model.

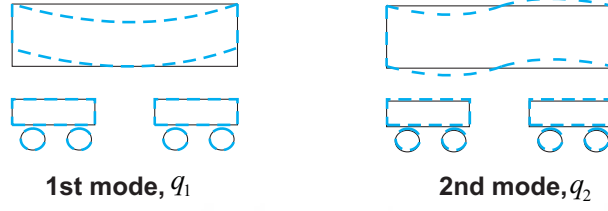


Figure 7.3. First two flexible car body mode.

Equation (3.5). Then,

$$\begin{bmatrix} \tilde{z} \\ y \end{bmatrix} = \tilde{G} \begin{bmatrix} \dot{z}_w \\ u \end{bmatrix} \quad (7.19)$$

where

$$\tilde{G} = \left[\begin{array}{c|c} C_2^f(sI - A^f)^{-1}B_1^f + D_{21}^f & C_2^f(sI - A^f)^{-1}B_2^f + D_{22}^f \\ \hline C_y^f(sI - A^f)^{-1}B_1^f + D_{y1}^f & C_y^f(sI - A^f)^{-1}B_2^f + D_{y2}^f \end{array} \right],$$

$$A^f = \begin{bmatrix} A & A_{rf} \\ A_{fr} & A_{ff} \end{bmatrix}, \quad B_1^f = \begin{bmatrix} B_1 \\ 0_{2n \times 4} \end{bmatrix}, \quad B_2^f = \begin{bmatrix} B_2 \\ 0_{n \times 2} \\ -\frac{\Gamma_u}{m_s} \end{bmatrix},$$

$$A_{rf} = \begin{bmatrix} 0_{6 \times n} & (\Gamma_q \Gamma^T)^T \\ -k_s M^{-1}(\Gamma_q \Gamma_1)^T & -c_s M^{-1}(\Gamma_q \Gamma_1)^T \end{bmatrix}, \quad A_{fr} = \begin{bmatrix} 0_{n \times 6} & 0_{n \times 6} \\ -\frac{k_s}{m_s} \Gamma_q \Gamma^T & -\frac{c_s}{m_s} \Gamma_q \Gamma_1 \end{bmatrix},$$

$$A_{ff} = \begin{bmatrix} 0_{n \times n} & I_n \\ -(\Omega + \frac{k_s}{m_s} \Gamma_q \Gamma_q^T) & -(\zeta + \frac{c_s}{m_s} \Gamma_q \Gamma_q^T) \end{bmatrix}, \quad C_2^f = \begin{bmatrix} C_2 & C_{2rf} \\ C_{2fr} & C_{2ff} \end{bmatrix},$$

$$\begin{aligned}
C_{2rf} &= \begin{bmatrix} \Gamma_q^T & 0_{2 \times n} \\ -k_s M_s^{-1} S_s^T \Gamma_q^T & -c_s M_s^{-1} S_s^T \Gamma_q^T \end{bmatrix}, \quad C_{2fr} = \begin{bmatrix} -\frac{k_s}{m_s} \Gamma_q \phi_a & -\frac{c_s}{m_s} \Gamma_q \Gamma_1 \end{bmatrix}, \\
C_{2ff} &= \begin{bmatrix} -(\Omega + \frac{k_s}{m_s} \Gamma_q \Gamma_q^T) & -(\zeta + \frac{c_s}{m_s} \Gamma_q \Gamma_q^T) \end{bmatrix}, \quad D_{21}^f = \begin{bmatrix} D_{21}^T & 0_{4 \times n} \end{bmatrix}^T, \\
D_{22}^f &= \begin{bmatrix} D_{22}^T & -(\frac{\Gamma_u}{m_s})^T \end{bmatrix}^T, \quad C_y^f = \begin{bmatrix} C_y & \Gamma^T & 0_{2 \times n} \end{bmatrix}, \quad D_{y1}^f = D_{y1}, \quad D_{y2}^f = D_{y2}, \\
\Gamma_q &= \begin{bmatrix} W_1(l_1) & W_2(l_2) & \dots & W_n(l_1) \\ W_1(l_2) & W_2(l_2) & \dots & W_n(l_2) \end{bmatrix}, \quad \Gamma_u = \begin{bmatrix} W_1(l_3) & W_2(l_3) & \dots & W_n(l_3) \\ W_1(l_4) & W_2(l_4) & \dots & W_n(l_4) \end{bmatrix}, \\
\Omega &= \text{diag}(\omega_1^2, \dots, \omega_n^2), \\
\zeta &= \text{diag}(2\zeta_1 \omega_1, \dots, 2\zeta_n \omega_n), \\
\Gamma_1 &= \begin{bmatrix} S_s & -T_s & -T_b \end{bmatrix}, \\
\Gamma_u &= \begin{bmatrix} S_u & -T_s & -T_b \end{bmatrix}, \\
S_u &= \begin{bmatrix} 1 & l_f \\ 1 & -l_r \end{bmatrix}
\end{aligned}$$

The matrices A , B_1 , B_2 , C_2 , D_{21} , D_{22} , C_y , D_{y1} , D_{y2} , Γ , S_s , T_s , T_b are given for plant $\mathcal{P}$ in Chapter 3 and Chapter 5. In the next section, the $\mathcal{H}_2$ output-feedback control scheme is applied to the vehicle suspension with a measurement vector $y = [s_1 \ s_2]^T$, aiming to attenuate both the rigid and flexible modes. The $\mathcal{H}_2$ control structure is depicted in Figure 7.4. Let's define the following problem

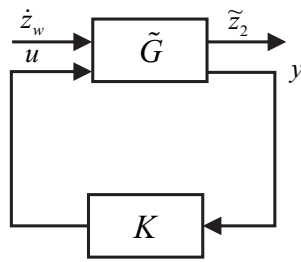


Figure 7.4. $\mathcal{H}_2$ output-feedback control diagram.

Problem 7.1.1 Find an output feedback controller $u = \mathcal{K}y$ that minimizes the objective function while ensuring the stability of the closed-loop system, with F_{li} and $\alpha_i > 0$.

$$\left(F_{li} = \left[\tilde{G}_{11} + \tilde{G}_{12} \mathcal{K} (I - \tilde{G}_{22} \mathcal{K})^{-1} \tilde{G}_{22} \right] \right)$$

$$\min_{\mathcal{K} \in \mathcal{RH}_2} J_f = \sum_{i=1}^6 \alpha_i \|F_{li}(\tilde{G}, \mathcal{K})\|_2^2 \quad (7.20)$$

The controller is specifically designed for Siemens ICE train, considering the parameters listed in Table A4, and it involves replacing the controllers at $l_f = l_r = l_c$. The performance of the controller is then simulated on FRA random track cross-level irregularities defined in Equation (3.7). The obtained results for both rigid and flexible

Table 7.1. RMS performances of controller $\mathcal{K}$

Output $\times 10^{-3}$	Rigid				Flexible	
	$\ddot{z}_s$	$\ddot{\theta}_s$	s_1	s_2	$\ddot{q}_1$	$\ddot{q}_2$
Passive rms	0.46	0.46	8.9	1.4	0.369	0.028
Case 1						
$l_f = l_c$	0.33	0.312	5.1	0.78	0.346	0.026
$l_r = l_c$	+27.8%	+31.8%	+42%	+45%	+6.2%	+6.6%
Case 2						
$l_f = \frac{l_c}{2}$	0.33	0.32	5.2	0.78	0.49	0.041
$l_r = \frac{l_c}{2}$	+27.84%	+31.77%	+42%	+45%	-33%	-45%
Case 3						
$l_f = \frac{L}{4} + \frac{l_c}{2}$	0.329	0.311	5.1	0.776	0.81	0.037
$l_r = \frac{L}{4} + \frac{l_c}{2}$	+27.7%	+31.8%	+42%	+45%	-22%	-59.5%
Case 4						
$l_f = \frac{L}{4} + \frac{l_c}{2}$	0.33	0.31	5.1	0.78	0.45	0.045
$l_r = \frac{l_c}{2}$	+27.75%	+31.77%	41.98%	+45%	-120%	-31.7%
Case 5						
$l_f = \frac{l_b}{2}$	0.329	0.311	5.1	0.775	0.48	0.037
$l_r = \frac{L}{4} + \frac{l_c}{2}$	+27.85%	+31.75%	+41.98%	+45%	-31.5%	-33.2%

body modes are summarized in Table 7.1. It is important to note that the performance of flexible (Equation (7.16)) and rigid body pitch modes (Equation (3.2)) is dependent on their respective locations. Consequently, the designed controller $\mathcal{K}$ is connected to different points on the car body. The flexible body modes exhibit varying performances based on the controller's location. Asymmetrically placed controllers negatively affect the first two flexible modes, while symmetric placements yield better performances. Notably, the best results are achieved by positioning the controllers on the secondary suspension. To support these findings, output frequency response plots are provided in Figures 7.5a-7.6b.

Table 7.2 presents two additional controllers that address the same problem but are implemented in different locations. The design of the first controller, $\tilde{\mathcal{K}}$, involves placing

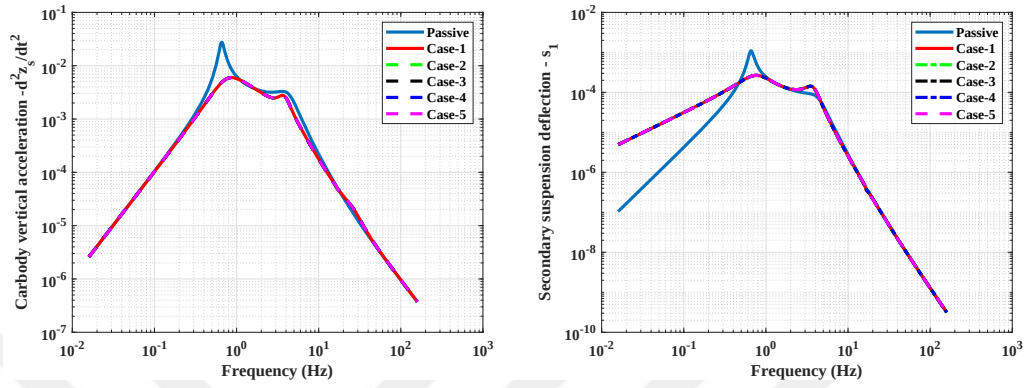


Figure 7.5. (a) Car body vertical acceleration frequency response (b) Secondary suspension deflection frequency response

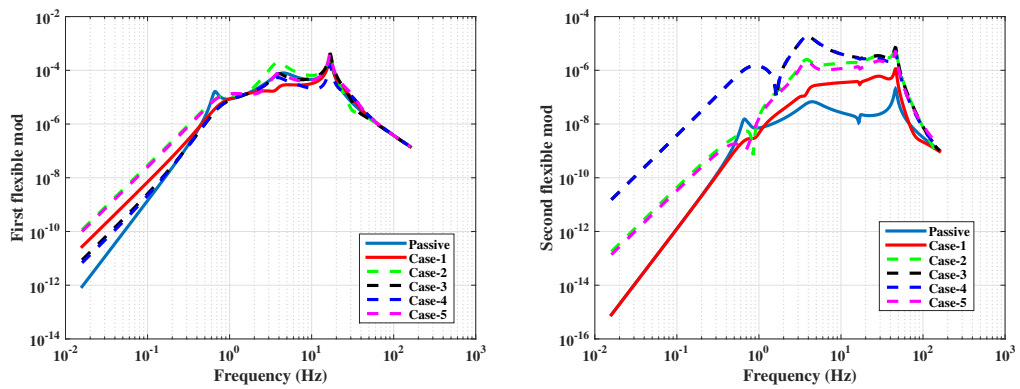


Figure 7.6. (a) First flexible mode frequency response (b) Second flexible mode frequency response

Table 7.2. RMS performances of controllers $\tilde{\mathcal{K}}$ and $\mathcal{K}_{in}$

Output ($\times 10^{-3}$)	Active rms (with $\tilde{\mathcal{K}}$)	Active rms (with $\mathcal{K}_{in}$)
s_1	0.33 (+28%)	0.27 (+40%)
s_2	0.31 (+32%)	0.27 (+41%)
$\ddot{z}_s$	5.1 (+42%)	3 (+66%)
$\ddot{\theta}_s$	0.78 (+45%)	0.64 (+55%)
$\ddot{q}_1$	0.35 (+6%)	0.35 (4%)
$\ddot{q}_2$	0.026 (+7%)	0.025 (9%)

the controller outside the secondary suspension systems by selecting $l_f = l_r = \frac{L}{4} + \frac{l_c}{2}$. This positioning is based on the observation that the car body vertical accelerations increase towards the endpoints of the vehicle. Despite this change in location, the performances of the rigid modes, except for pitch acceleration, remain unchanged compared to the previously designed controller. Another controller, $\mathcal{K}_{in}$, is designed for the active control forces in the wagon. Simulation results indicate that $\mathcal{K}_{in}$ provides the best performances for both rigid and flexible body vibrations. However, it is important to note that while it may be proposed as a solution, the practical applicability of $\mathcal{K}_{in}$ should be thoroughly examined and discussed. It is crucial to consider the practical implications and feasibility of these controllers in real-life scenarios [72].

The results demonstrate the feasibility of controlling flexible body modes using an appropriate active controller. However, when considering a full wagon, the effectiveness of the controller may diminish. This is because the roof and floor of the vehicle body move out of phase with each other, and the car body mode shapes in 3D become more complex, which cannot be accurately represented by the simplified beam model. Therefore, it is suggested to employ a finite element model (FEM) for dynamic analysis, which can capture the complex behavior of the entire structure across a wide range of frequen-

cies. However, FEM models have a large number of degrees of freedom, often exceeding 500,000 which makes them impractical for control theory [8]. To address this challenge, an alternative analytical approach is proposed, namely the elastic box-type model supported by bogies. In the box-type modeling approach, the wagon is assumed to consist of six panels (front, rear, side, roof, and floor panels) connected by springs, as depicted in Figures 7.7-7.8. However, a significant challenge in box-type modeling is determining the appropriate spring values. Although some solutions have been proposed in the literature to address this issue, determining the accurate spring values remains a problem [137].

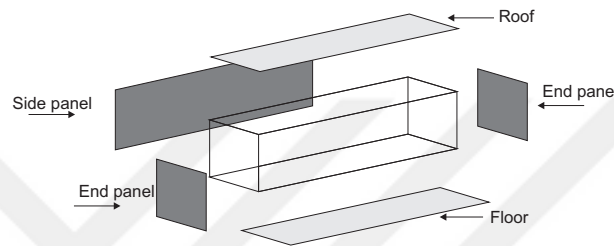


Figure 7.7. 3D analytical model.

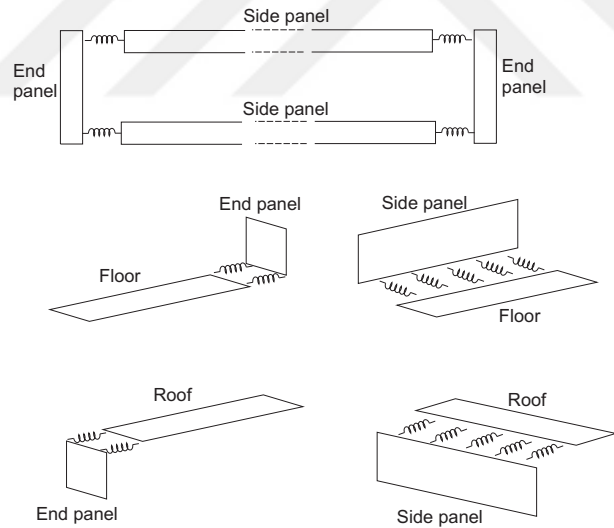


Figure 7.8. Elastic box type modelling connections.

However, according to [138], in the low-frequency range, the free-free beam model for a Shinkansen train with an aluminum alloy body aligns well with field measurements. The first bending mode is observed at approximately 9.48 Hz, while the first torsional mode occurs at around 13.4 Hz. Given that the primary objective is to enhance passenger comfort by utilizing active controllers to reduce both rigid and flexible body modes in

the low-frequency range, the full body of the passenger wagon is modeled as a free-free Euler-Bernoulli beam. In this simplified model, only the first bending and torsional modes are considered. To achieve the desired control objectives, the $\mathcal{H}_\infty$ control method is applied using LMIs to the vehicle's secondary suspension. The results obtained from this approach are then discussed in detail.

7.1.1. Active control for flexible bodied full-vehicle model

In this section of the thesis, we focused on the analysis and control of low-frequency rigid and flexible body modes. To facilitate this analysis, the car body is modeled as an Euler-Bernoulli beam. The equations of motion for the flexible modes can be expressed as follows:

For bending modes:

$$\begin{aligned} \ddot{q}_{bk} + 2\xi_{bk}\omega_{bk}\dot{q}_{bk} + \omega_{bk}^2 q_{bk} = \frac{2}{\rho AL} \Big\{ & \tilde{F}_{11}W_{bk}(l_1) + \tilde{F}_{12}W_{bk}(l_1) + \dots \\ & + \tilde{F}_{21}W_{bk}(l_2) + \tilde{F}_{22}W_{bk}(l_2) - u_{11}W_{bk}(l_1) + \dots \\ & - u_{12}W_{bk}(l_1) - u_{21}W_{bk}(l_2) - u_{22}W_{bk}(l_2) \Big\} \end{aligned} \quad (7.21)$$

For torsional modes:

$$\begin{aligned} \ddot{q}_{\phi p} + 2\xi_{\phi p}\omega_{\phi p}\dot{q}_{\phi p} + \omega_{\phi p}^2 q_{\phi p} = \frac{2l_s}{\rho JL} \Big\{ & -\tilde{F}_{11}W_{\phi p}(l_1) + \tilde{F}_{12}W_{\phi p} + \dots \\ & -\tilde{F}_{21}W_{\phi p}(l_2) + \tilde{F}_{22}W_{\phi p}(l_2) + \dots \\ & + u_{11}W_{\phi p}(l_1) - u_{12}W_{\phi p}(l_1) + \dots \\ & + u_{21}W_{\phi p}(l_2) - u_{22}W_{\phi p}(l_2) \Big\} \end{aligned} \quad (7.22)$$

where the $W_{bk}(l_{ij})$ is the k^{th} mode shape function of bending and $W_{\phi p}(l_{ij})$ represents the p^{th} mode shape function of torsional modes. These mode shape functions are defined in Equation (7.23) and Equation (7.24), respectively.

$$\begin{aligned} W_{bk}(l_{ij}) = \cosh(\lambda_k l_{ij}) + \cos(\lambda_k l_{ij}) - \frac{\cosh(\lambda_k L) - \cos(\lambda_k L)}{\sinh(\lambda_k L) - \sin(\lambda_k L)} \times \\ \{ \sinh(\lambda_k l_{ij}) + \sin(\lambda_k l_{ij}) \}, \end{aligned} \quad (7.23)$$

$$W_{\phi p}(l_{ij}) = \cos \frac{p\pi}{L} l_{ij} \quad (7.24)$$

Then the vertical relative displacements between the car body and bogies can be updated as:

$$\begin{aligned} \tilde{s}_{ij} = & \{z_s - l_c(-1)^i \theta_s + l_s(-1)^j \varphi_s\} - z_{bij} + \dots \\ & + \sum_{k=1}^r W_{bk}(l_{ij}) q_{bk} + l_s(-1)^j \sum_{p=1}^t W_{\phi p}(l_{ij}) q_{\phi p} \end{aligned} \quad (7.25)$$

The updated vertical displacements $\tilde{s}_{ij}$ will modify the forces acting on the secondary suspensions, resulting in the transformed forces $\tilde{F}_{ij}$.

$$\tilde{F}_{ij} = -k_s s_{ij} - c_s \dot{s}_{ij} - k_s W_b q_b - k_s W_{\phi} q_{\phi} - c_s W_b \dot{q}_b - c_s W_{\phi} \dot{q}_{\phi} \quad (7.26)$$

For modeling, only the first bending mode ($k=1$) at approximately 9.48 Hz and the first torsional mode ($p=1$) around 13.4 Hz were considered [138]. To represent the system in state-space form, the Equation (5.15) should be rewritten with the new state vector $\tilde{x} = [x \ q_{b1} \ \dot{q}_{b1} \ q_{\phi 1} \ \dot{q}_{\phi 1}]^T$.

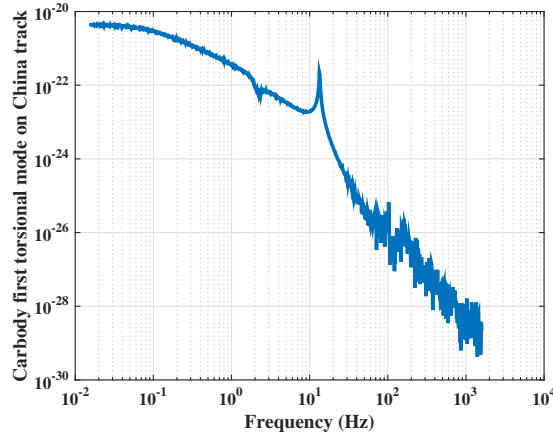


Figure 7.9. The carbody first torsional mode frequency response

In this section, it is assumed that the train is operating on the test track of the Qinhuangdao-Shenyang high-speed railway line at a constant speed of 200 km/h. According to the findings of the study by Bokaeian et al. [138], when completely correlated irregularities are considered, the torsional flexural mode is not excited. As a result, on this

particular track, the torsional mode becomes negligible and is effectively removed from the design problem. This is depicted in Figure 7.9, which illustrates the disappearance of the torsional mode and its associated velocity. Then, the multiobjective control problem is:

Problem 7.1.2 *Design a state feedback controller $u = \mathcal{K}x$ that minimizes the cost function $\psi_1 \|T_{z_{\infty 1} w}\|_{\infty}^2 + \psi_2 \|T_{z_{\infty 2} w}\|_{\infty}^2$ for $\psi_1, \psi_2 > 0$.*

In the given context, $T_{z_{\infty 1} w}$ and $T_{z_{\infty 2} w}$ represent the closed-loop transfer functions from the disturbance w to the outputs $z_{\infty 1} = [s_{wimn}]^T$ and $z_{\infty 2}$, respectively. Two different active controllers, $\mathcal{K}_R$ and $\mathcal{K}_F$, are designed for $z_{\infty 2} = [\ddot{z}_s \ddot{\theta}_s \ddot{\varphi}_s]^T$ and $z_{\infty 2} = [\ddot{z}_s \ddot{\theta}_s \ddot{\varphi}_s \ddot{q}_{b1}]^T$, respectively. Figures 7.10a-7.11b display the frequency responses of a passive system and the actively controlled flexible-bodied vehicle. The results indicate that when the objective function includes the first bending mode acceleration, it leads to improved performance in terms of bending mode acceleration and car body vertical acceleration. Furthermore, the controller $\mathcal{K}_R$ demonstrates better performance in roll and pitch accelerations. These findings suggest that the energy requirements of $\mathcal{K}_F$ are met through both roll and pitch accelerations.

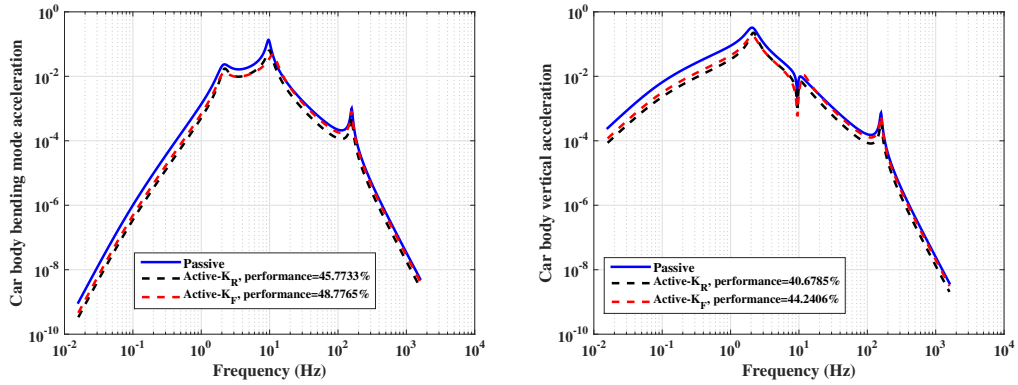


Figure 7.10. (a) First flexible mode frequency response (b) Car body vertical acceleration frequency response

In the preceding discussions, the significance of actuator placement, which greatly influences the flexible modes and the performance of active controllers for flexible-bodied vehicles, has been emphasized. However, these results do not provide any insights into the safety aspects of the vehicle. Therefore, in this section of the thesis, an extensive study on

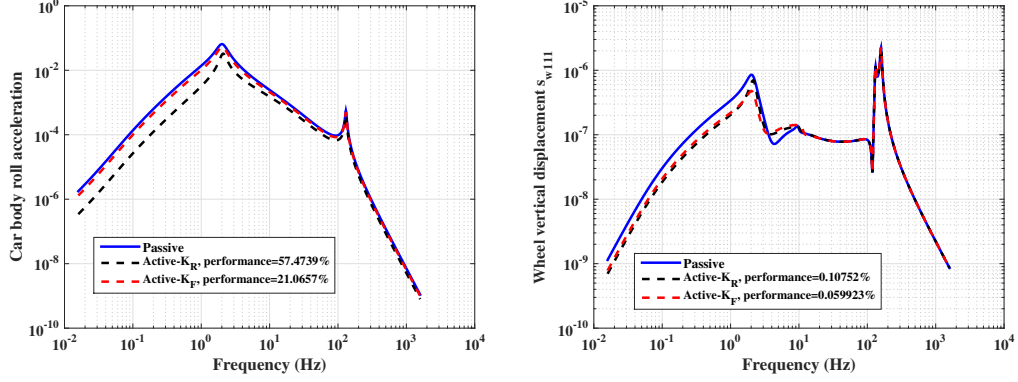


Figure 7.11. (a) The carbody roll acceleration frequency response (b) The wheel-rail vertical displacement frequency response

active control for bogies is conducted, specifically focusing on safety considerations in the middle-frequency range [139]. It is assumed that the rigid car body is supported by two-axle non-articulated welded H-form bogies. The elastic contact between the wheels and the rail is taken into account to capture realistic dynamics. For simulations and analysis, the parameters of the Siemens ICE train, as provided in Table A4 are used.

7.1.2. Active control for railway vehicle with flexible bodied bogies

In addition to the rigid body modes of a 10-degree-of-freedom vehicle model, the front and rear bogie flexible modes (q_{1i} , q_{2i}) are incorporated into the corresponding vertical displacements. This augmentation allows for a more comprehensive representation of the vehicle's dynamics, accounting for both rigid and flexible body modes. As a result, the dynamic forces acting on the secondary and primary suspensions can be updated as follows:

$$\begin{aligned}\tilde{F}_i &= -k_s [z_{si} - \tilde{z}_{bi}(l_1, t)] - c_s [\dot{z}_{si} - \dot{\tilde{z}}_{bi}] , \\ \tilde{F}_{1i} &= -k_p [\tilde{z}_{b1i}(l_2, t) - z_{w1i}] - c_p [\dot{\tilde{z}}_{b1i}(l_2, t) - \dot{z}_{w1i}] , \\ \tilde{F}_{2i} &= -k_p [\tilde{z}_{b2i}(l_3, t) - z_{w2i}] - c_p [\dot{\tilde{z}}_{b2i}(l_3, t) - \dot{z}_{w2i}] ,\end{aligned}\tag{7.27}$$

for $i = 1, 2$. The state-space equations in $\mathcal{P}_{10DOF}$ are updated with the state vector $x_f = [z_{si} - z_{bi} \ z_{bij} - z_{wij} \ \dot{z}_{si} \ \dot{z}_{bij} \ \dot{z}_{wij} \ q_{1i} \ q_{2i} \ \dot{q}_{1i} \ \dot{q}_{2i}]^T$, the regulated output $z = [\ddot{z}_s \ \ddot{\theta}_s \ q_{11} \ q_{21}]^T$ and $z_\infty = [F_{wij}]^T$, the disturbance input $r = [r_{11} \ \dots \ r_{22}]^T$ and the control input $u = [u_1 \ u_2]^T$ for $i, j = 1, 2$.

The modes of the vehicle body, bogie, and wheels are presented in Table 7.3. The natural frequencies ω_{1i} and ω_{2i} represent the i^{th} natural frequencies of the front and rear bogies, respectively. These frequencies are influenced by factors such as the elastic modulus of the bogie material, the cross-section area, and the moment of inertia of the beam. The associated damping ratios are denoted by ζ_{1i} and ζ_{2i} . It is important to note that variations in the damping ratios can result in a reduction of the peak amplitudes at the corresponding frequencies, as depicted in Figure 7.12. The simulation results highlight that as the damping ratio of the first bending mode of the bogies approaches unity, the system's behavior closely approximates that of a rigid body in terms of ride comfort. These findings emphasize the significance of damping ratios in controlling and optimizing the performance of the system. By adjusting the damping ratios, it is possible to mitigate vibrations and enhance ride comfort by reducing the amplitudes at the specific frequencies associated with the bogie modes.

Table 7.3. *Vehicle modes.*

Car body	Damped freq. (Hz)
Heave	0.7839
Pitch	0.9366
Bogie	Damped freq. (Hz)
Heave	3.5840
Pitch	4.0298
Wheel	
Heave	167.2614
Flexible mode of the bogies	
1st bending, $\omega_{11}/2\pi$	38.2086
2nd bending $\omega_{12}/2\pi$	105.2393

In this section, the disturbance input is represented by the track vertical profile, as given in Equation (5.10). The influence of the track roughness parameter on the vehicle response can be explained through a scaling effect. Figure 7.13 illustrates the impact of changes in track quality on the car body's vertical acceleration in the frequency domain. According to the figure, as the track roughness increases, the magnitude of the car body's vertical acceleration also increases. In other words, poor track conditions result in higher car body acceleration. This demonstrates the direct relationship between track quality and ride comfort, with rougher tracks leading to more pronounced vibrations and discomfort for passengers.

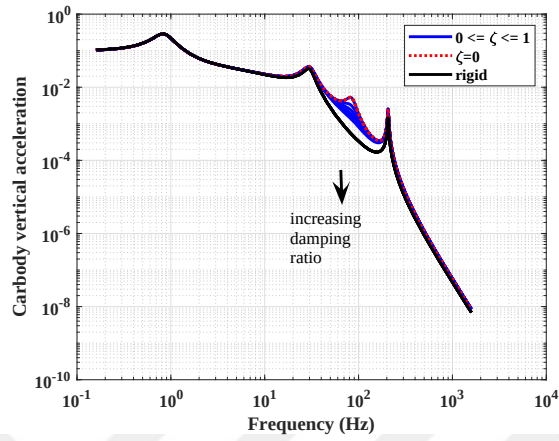


Figure 7.12. Damping ratio of the bogie first flexible mod effects on carbody vertical acceleration.

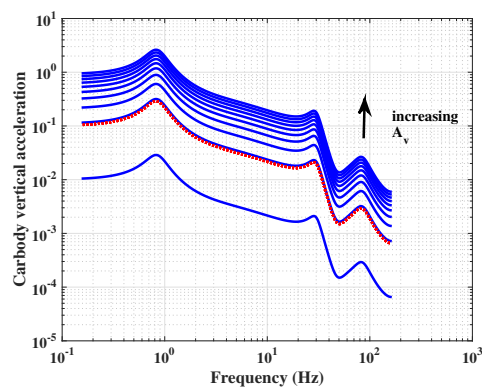


Figure 7.13. Track quality effect on the carbody vertical acceleration.

To address these challenges and improve both ride comfort and safety under varying track qualities, mixed $\mathcal{H}_2/\mathcal{H}_\infty$ active controllers should be designed. These controllers aim to optimize both the ride comfort and safety aspects of the vehicle's performance throughout the entire driving process. By integrating both $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control techniques, the designed controllers provide a balanced approach that minimizes the effects of disturbances and ensures robust stability and performance. This enables the vehicle to deliver better ride comfort while maintaining safe operating conditions, even when encountering different track qualities. The utilization of mixed $\mathcal{H}_2/\mathcal{H}_\infty$ active controllers offers a comprehensive solution that addresses the trade-off between ride comfort and safety in diverse track conditions. By considering the specific requirements and constraints of each track quality in Problem 7.1.3, the state-feedback controller will aim to optimize the overall performance of the vehicle and provide passengers with a comfortable and safe travel experience. Let's define the objective function as

$$J_b = \sum_{k=1}^4 [\lambda_k \|T_{z_k \eta}\|_\infty^2 + \beta_k \|T_{z_{\infty k} \eta}\|_2^2] \quad (7.28)$$

Here $T_{z_k \eta}$ and $T_{z_{\infty k} \eta}$ represent the closed-loop transfer function from zero-mean white noise input η to the k^{th} element in the output vector z and z_∞ . The track-vehicle model from zero-mean white noise input to vehicle output is given in the previous chapter. By choosing suitable values for the positive gains λ_k , β_k it is possible to prioritize the reduction of either of the transfer function. Then, the optimization procedure can be cast into the following problem:

Problem 7.1.3 Find state feedback controllers $u = \mathcal{K}_f x$ for fixed track quality (FRA-class6) and $u = \mathcal{K}_p x$ for different track qualities that minimize the objective function J_b in Equation (7.28) and stabilize the closed loop system.

The design problem was solved using Linear Matrix Inequalities (LMIs), and the results were obtained from MATLAB *hinfmix* command. The simulations are presented in Figures 7.14a-7.15b. These figures illustrate the performance of the designed active control system compared to the passive system. From the figures, it can be observed that in the mid-frequency range, the car body vertical accelerations show improved results

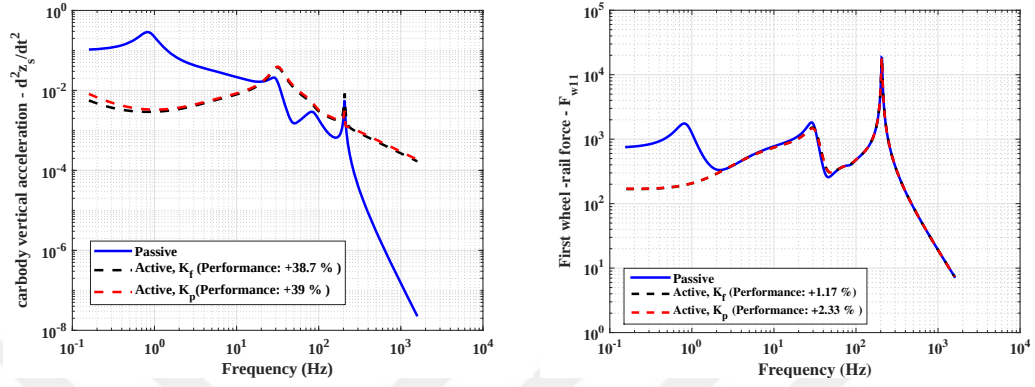


Figure 7.14. (a) Car body vertical acceleration frequency response (b) The force frequency response between first wheel-rail

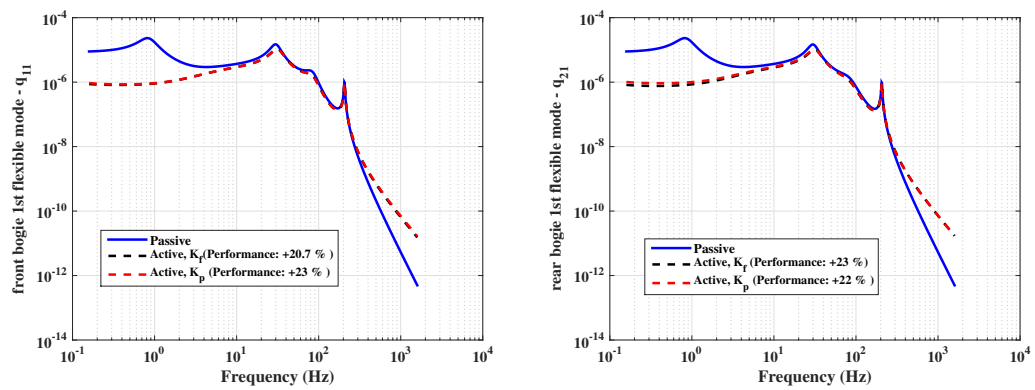


Figure 7.15. (a) Front bogie first flexible mode q_{11} frequency response (b) Rear bogie first flexible mode q_{21} frequency response

compared to the passive system. This improvement is achieved by reducing the influence of the bogie flexible modes on the car body vibrations. As a result, the active control system provides better ride comfort and stability in this frequency range. In the lower frequencies, all output variables exhibit improved performance compared to the passive system. This indicates that the active control system effectively enhances the comfort and safety of the transportation by reducing vibrations and oscillations in this frequency range. The active control system proves beneficial for providing a more comfortable and safer transport experience. At high frequencies, the active control system maintains performance levels similar to the passive system slightly higher. This is attributed to the placement of the designed controller on the secondary suspension, which focuses on addressing the mid and low-frequency range responses. By protecting the passive values in the high-frequency range, the active control system ensures stability and maintains the desired performance characteristics.

7.2. Summary

In this chapter, the consideration of both rigid and flexible modes of the vehicle body was introduced. An approximate linear model was derived, incorporating both rigid and flexible modes, specifically for a half car, utilizing the Euler-Bernoulli beam theory. To address the suppression of both rigid and flexible body modes under 20 Hz, an active controller was designed using the $\mathcal{H}_2$ output feedback control method. Simulation results demonstrated the feasibility of effectively suppressing both flexible and rigid body modes through the designed active control system. This finding has potential implications for lightweight train designs and control strategies, as it highlights the possibility of achieving improved performance by addressing both rigid and flexible modes. Another solution for the vehicle is filtering the vibrations on the unsprung mass bogies. Although the bogies are known as the most important component of the vehicle there are fewer studies about the bogie dynamics. In this chapter, the historical development of the bogie and the planned design of the bogies in the future were investigated in detail. For the low and mid frequencies, the active secondary suspension control was developed for variant track qualities that were concluded with better vehicle performances.

In particular, the active secondary suspension control was developed for various

track qualities, aiming to enhance overall vehicle performance. The results concluded that implementing active control techniques in the secondary suspension system led to improved vehicle performance, especially for low and mid frequencies. By investigating both the body modes and the dynamics of bogies, this chapter provides valuable insights into the control and design aspects of railway vehicles. The findings contribute to the understanding of lightweight train designs, the suppression of vibrations, and the development of active control strategies for improved vehicle performance.



8. CONCLUSIONS

The contributions of this thesis are as follows:

- 1). The thesis begins by analyzing the body and suspension components of a typical high-speed passenger railway vehicle. A linear half-car mathematical model was derived, and the stochastic track irregularity was adopted from Federal Railroad Administration report as a disturbance input using a state-space approach. The optimization of the passive system is discussed to achieve better comfort and safety performances under design constraints.
- 2). The influence of vehicle speed on comfort and safety was investigated, with the suggestion of a new straight-line building for passenger vehicles only being considered as an expensive solution. Instead, designing vehicles with lighter bodies was proposed. The effects of lightweight design on both rigid and flexible structures are examined, noting the shifts in natural frequencies and changes in body accelerations. It was shown that lighter vehicles can provide improved comfort and safety with smaller active forces, promoting energy efficiency and cost reduction.
- 3). The impact of track irregularities on railway vehicles was addressed. Track modeling techniques involving system identification methods were used to characterize the track in the frequency domain, comparing the results obtained from field measurements on the Konya-Polatlı high-speed main line in Turkey with the FRA power spectral density model.
- 4). The ride comfort problem was formulated as an active control problem. Initially, a half-car vehicle model was considered, and an objective function was defined to minimize the RMS car body vertical accelerations and secondary suspension travel values using LQG control. The problem was later reformulated as a multi-objective control problem with safety constraints, solved using LMIs. Active controller designs for the primary suspension were shown to improve ride comfort, with HIFOO controllers offering less complex designs and better performance enhancements.
- 5). Semi-active controllers were explored as a cost-effective alternative, as they do not require external energy sources. The design of a skyhook damper for better comfort

conditions was discussed, with two different methods employed to synthesize the best damper value. Gradient-based and LMI-based approaches were utilized, and a hybrid controller combining skyhook and groundhook dampers was proposed to address safety concerns. Simulation results demonstrate good comfort conditions, but large suspension deflections are observed with deterministic track inputs.

- 6). A seventeen-degree-of-freedom full ride model for a high-speed railway passenger vehicle was developed, and the LQG method was used to design the active suspension system. HIFOO controllers were employed to handle complexity and allow the user a flexible controller selection.
- 7). The importance of the wheel-rail contact in determining the vehicle's performance was highlighted in a case study for both half and full-car models. The contact can be modeled as either rigid or elastic, with the latter represented by a high-stiffness spring. Robust H_∞ controllers were designed to accommodate variations in the spring stiffness. Numerical simulations demonstrate the possibility of achieving a good compromise between ride comfort and safety requirements with well-designed active controllers, even with variations in the spring stiffness.

8.1. Future Works

The following topics need further attention:

- We have not studied the lateral dynamics of the railway vehicle. Since lateral vibrations have bad effects on ride comfort, especially in the curves, this subject should also be discussed.
- In railways, the derailment criteria is used for safety analysis. The derailment criteria is based on the wheel's total lateral and total vertical forces ratio. Therefore the vertical and lateral coupled models have to be obtained.
- In this thesis, mostly rigid-bodied vehicle models were discussed. To analyze flexible body modes, the vehicle was assumed as an elastic beam. Since the roof and floor of the railway vehicle body move out of phase with each other, the car body mode shapes in 3D become more complex and could not be

explained via simplified beam theory. Then, an analytical model should be obtained for the elastic-bodied wagon.



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APPENDIX

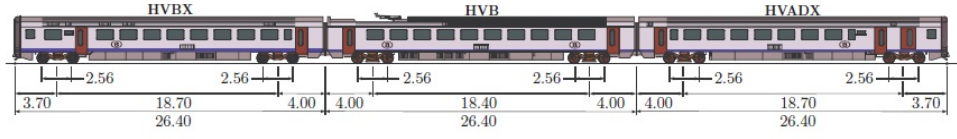


Figure A1. Bombardier AM96 Electrical Unit[19]

Table A1. Bombardier-AM96 railway vehicle parameters [19]

Variable	Description	Value
c_s	Secondary suspension damping coefficient per bogie	22.6 kNs/m
c_p	Primary suspension damping coefficient per wheelset	3.7 kNs/m
I_{sy}	Car body mass moment of inertia	0.63 Mkgm^2
I_{by}	Bogie mass moment of inertia	760 kgm^2
k_s	Secondary suspension spring stiffness per bogie	0.69 MN/m
k_p	Primary suspension spring stiffness per wheelsets	1.30 MN/m
l_c	Half of the bogie base	9.2 m
l_w	Half of the wheelbase	1.28 m
L	Length of the coach	26 m
m_s	Car body mass (1/2)	12600 kg
m_b	Bogie mass (1/2)	3450 kg
m_w	Wheels mass	850 kg

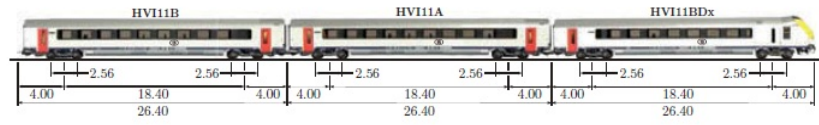


Figure A2. Bombardier I11 Electrical Unit[19]

Table A2. Bombardier-I11 railway vehicle parameters [19]

Variable	Description	Value
c_s	Secondary suspension damping coefficient per bogie	11.3 kNs/m
c_p	Primary suspension damping coefficient per wheelset	3.750 kNs/m
I_{sy}	Car body mass moment of inertia	0.67 Mkgm^2
I_{by}	Bogie mass moment of inertia	1475 kgm^2
k_s	Secondary suspension spring stiffness per bogie	0.12 MN/m
k_p	Primary suspension spring stiffness per wheelsets	0.475 MN/m
l_c	Half of the bogie base	9.2 m
l_w	Half of the wheelbase	1.28 m
m_s	Car body mass (1/2)	13420 kg
m_b	Bogie mass (1/2)	6800 kg

Table A3. *The Benchmark vehicle parameters [65].*

Variable	Description	Value
c_s	Secondary suspension damping coefficient	$20 \times 10^3 \text{ N s/m}$
c_p	Primary suspension damping coefficient	$4 \times 10^3 \text{ N s/m}$
I_{sx}	Car body roll moment of inertia	56.8 ton m^2
I_{sy}	Car body pitch moment of inertia	1970 ton m^2
I_{bx}	Bogie roll moment of inertia	1.722 ton m^2
I_{by}	Bogie pitch moment of inertia	1.476 ton m^2
I_{wx}	Wheelset roll moment of inertia	1.120 ton m^2
k_s	Secondary suspension spring stiffness	$0.43 \times 10^6 \text{ N/m}$
k_p	Primary suspension spring stiffness	$1.22 \times 10^6 \text{ N/m}$
k_H	Hertzian spring stiffness	$1.174 \times 10^9 \text{ N/m}$
l_c	Half of the distance between bogie centers	9.5 m
l_w	Half of the distance between wheel centers	1.28 m
l_s	Half of transverse distance of secondary suspension	1.025 m
l_p	Half of transverse distance of primary suspension	1.025 m
m_s	Car body mass	32 ton
m_b	Bogie mass	2.615 ton
m_w	Wheelset mass	1.813 ton
r_w	Wheel radius	0.46 m



Figure A3. Siemens ICE train[19]

Table A4. Siemens ICE high speed train parameters [84].

Variable	Description	Value
c_s	Secondary suspension damping coefficient	$29.2 \times 10^3 \text{ Ns/m}$
c_p	Primary suspension damping coefficient	$21.9 \times 10^3 \text{ Ns/m}$
I_{sx}	Car body roll moment of inertia	$10.35 \times 10^4 \text{ kgm}^2$
I_{sy}	Car body pitch moment of inertia	$2.391 \times 10^6 \text{ kgm}^2$
I_{bx}	Bogie roll moment of inertia	$2.366 \times 10^3 \text{ kgm}^2$
I_{by}	Bogie pitch moment of inertia	$4.989 \times 10^2 \text{ kgm}^2$
I_{wx}	Wheelset roll moment of inertia	678 kgm^2
k_s	Secondary suspension spring stiffness	$0.324 \times 10^6 \text{ N/m}$
k_p	Primary suspension spring stiffness	$28.2 \times 10^6 \text{ N/m}$
k_H	Hertzian spring stiffness	$1... \times 10^9 \text{ N/m}$
l_c	Half of the distance between bogie centers	8.5 m
l_w	Half of the distance between wheel centers	1.4 m
l_s	Half of transverse distance of secondary suspension	1.142 m
l_p	Half of transverse distance of primary suspension	1 m
m_s	Car body mass	45 ton
m_b	Bogie mass	3.09 ton
m_w	Wheelset mass	1.56 ton
r_w	Wheel radius	0.46 m
F_s	Static wheel load	81.4 kN

Table A5. French TGV high speed train parameters [84].

Variable	Description	Value
c_s	Secondary suspension damping coefficient	$40 \times 10^3 \text{ N s/m}$
c_p	Primary suspension damping coefficient	$38 \times 10^3 \text{ N s/m}$
I_{sx}	Car body roll moment of inertia	$7.42 \times 10^4 \text{ kgm}^2$
I_{sy}	Car body pitch moment of inertia	$2.74 \times 10^6 \text{ kgm}^2$
I_{bx}	Bogie roll moment of inertia	$1.6 \times 10^3 \text{ kgm}^2$
I_{by}	Bogie pitch moment of inertia	$1.7 \times 10^2 \text{ kgm}^2$
I_{wx}	Wheelset roll moment of inertia	1067 kgm^2
k_s	Secondary suspension spring stiffness	$0.35 \times 10^6 \text{ N/m}$
k_p	Primary suspension spring stiffness	$0.7 \times 10^6 \text{ N/m}$
k_H	Hertzian spring stiffness	$1... \times 10^9 \text{ N/m}$
l_c	Half of the distance between bogie centers	7.85 m
l_w	Half of the distance between wheel centers	1.5 m
l_s	Half of transverse distance of secondary suspension	1.025 m
l_p	Half of transverse distance of primary suspension	1.025 m
m_s	Car body mass	44 ton
m_b	Bogie mass	1.7 ton
m_w	Wheelset mass	1.9 ton
r_w	Wheel radius	0.43 m
r_r	Rail head radius	0.30 m
F_s	Static wheel load	67.4 kN
E	Elastic modulus of the rail	$2.1 \times 10^{11} \text{ N/m}^2$
ν	Poissons' ratio	0.3

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