

DESCENT CONTROL ALGORITHM DESIGN FOR SHIPBORNE
SEA-SKIMMING MISSILES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TUNA GÜN AKSU

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
AEROSPACE ENGINEERING

JANUARY 2024

Approval of the thesis:

**DESCENT CONTROL ALGORITHM DESIGN FOR SHIPBORNE
SEA-SKIMMING MISSILES**

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ABSTRACT

DESCENT CONTROL ALGORITHM DESIGN FOR SHIPBORNE SEA-SKIMMING MISSILES

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January 2024, 96 pages

Sea-skimming missiles are missiles which fly very close to sea surface. This is to avoid detection by radar or make it difficult for defense systems to intercept the missile. Sea-skimming missiles are expected to descend to the surface of the water to start avoiding detection as early as possible right after the launch. However, this steep descent is a challenging task because of the platform movements, ballistic phase with the booster, and different environmental disturbances. It is also expected that the missile's descent should be fast without any overshoot because of the sea below. Thus, a robust and rapid descent control system design is one of the main challenges for sea-skimming missiles. This thesis proposes modified proportional-derivative controller with predictive model for calculating the system input for the powered descent to the sea surface, with no or minimum overshoot. The effectiveness of the proposed method is evaluated through extensive simulations and compared against traditional control approaches and guidance methods.

Keywords: Sea-skimming Guidance, Shipborne, Robust Altitude Control, Steep De-

scent, Trajectory Shaping



ÖZ

GEMİYLE TAŞINAN SU SATHINA YAKIN UÇAN FÜZELER İÇİN İNİŞ KONTROL ALGORİTMASI TASARIMI

Aksu, Tuna Gün

Yüksek Lisans, Havacılık ve Uzay Mühendisliği Bölümü

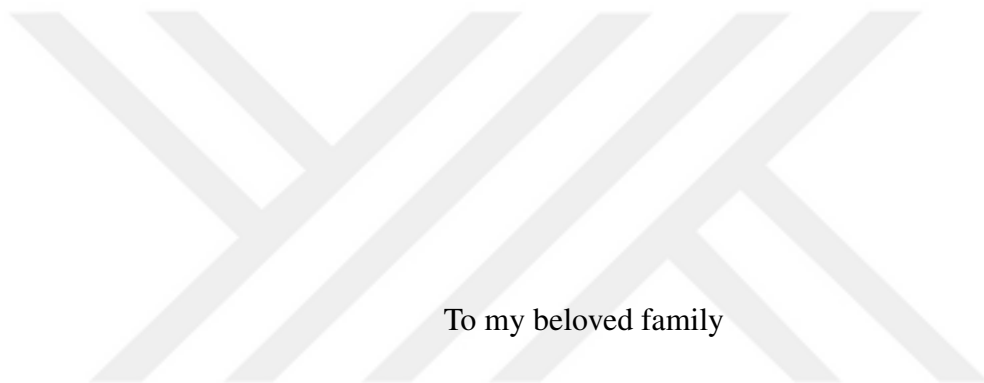
Tez Yöneticisi: Doç. Dr. Halil Ersin Söken

Ocak 2024 , 96 sayfa

Denizi sıyrarak uçan füzeler su sathına yakın uçan füzelerdir. Bu, radar tarafından tespit edilmeyi önlemek veya savunma sistemlerinin füzeyi engellemesini zorlaştırmak amacıyla yapılmaktadır. Su sathına yakın uçan füzelerin, tespit edilmesini engellemek için fırlatıldıktan hemen sonra su yüzeyine inmesi beklenir. Ancak, bu dik iniş, platform hareketleri, katı yakıtlı roket ile balistik uçuş fazı ve farklı çevresel bozucular nedeniyle zorlu bir görevdir. Ayrıca, aşağıda deniz olduğu için füzenin inişinin hızlı ve üst aşım yapmadan gerçekleşmesi beklenmektedir. Bu nedenle, gürbüz ve hızlı bir iniş kontrol sistemi tasarımı, su sathına yakın uçan füzeler için ana zorluklardan biridir. Bu tez, füzenin deniz yüzeyine hızlı ve gürbüz iniş yapabileceği şekilde sistem girdisini hesaplamak için kestirim modeli ile değiştirilmiş oransal-türevli bir kontrolcü önermektedir. Kontrolcünün amacı üst aşım olmaksızın veya minimum seviyede aşım ile deniz yüzeyine motorla iniş yapabilmeyi hedeflemektedir. Önerilen yöntemin etkinliği, kapsamlı simülasyonlar yoluyla ve geleneksel kontrol yaklaşımları ve güdüm metotlarıyla karşılaştırılarak değerlendirilmiştir.

Anahtar Kelimeler: Su Sathına Gdm, Gemiyle Taşınan, Grbz Ykseklik Kontrol, Dik İniş, Yrnge Şekillendirme





To my beloved family

ACKNOWLEDGMENTS

I would like to express my deepest gratitude and appreciation to my supervisor, Assoc. Prof. Halil Ersin Söken, for his unwavering guidance, invaluable insights, patience, and constant encouragement. His mentorship and enthusiasm played a pivotal role in shaping the trajectory of this work.

Acknowledgments are extended to the thesis committee members for their insightful comments.

I am thankful to ROKETSAN for providing the academic opportunity and pleased to express my gratitude to my colleagues Özgün Dülger and Görkem Eşsiz for their time, energy, and knowledge. I learned so much from them. I would also like to thank my colleagues in my department and office; their friendship and support were invaluable.

Sincere thanks are expressed to my close friends for their support, understanding, and love.

I am forever grateful to my parents, Hürriyet Aksu and Ali İrfan Aksu, for their unwavering support, unending love, and patience. Their encouragement and self-sacrifices have made me who I am today. I would also like to thank my brother, Avni Arda Aksu, for the person he is. I am in awe of the man you have become and will be forever grateful that you are my brother. Thank you all for your faith in me; I know that I am very lucky to have you by my side.

Finally, I want to express my deepest appreciation to my dearest, Ayşe İpek Talu. Your love, support, understanding, and encouragement were a constant source of strength. Your belief in my capabilities motivated me to persevere and excel in my endeavors.

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LIST OF ABBREVIATIONS AND SYMBOLS

DOF	Degree of Freedom
PD	Proportional-Derivative
MPD	Modified Proportional-Derivative
BPNG	Biased Proportional Navigation Guidance
IVG	Impact Vector Guidance
GENEX	Generalized Vector Explicit Guidance
LOS	Line-of-Sight
ECI	Earth-Centered Inertial
ECEF	Earth-Centered Earth-Fixed
NED	North-East-Down
m	mass
I_{yy}	moment of inertia about body y-axis
l_{ref}	reference length
S_{ref}	reference area
T, T_0	temperature, sea-level temperature
h, h_0	height, initial height
v, v_0	velocity, initial velocity
a, a_0	acceleration, initial acceleration
a_z	acceleration in z-axis of the missile body frame
δ	deflection of control surface
θ, θ_0	pitch angle, initial pitch angle
θ_b	booster misalignment angle
α	angle of attack
γ	flight path angle

$\dot{\lambda}$	line-of-sight rate
λ	line-of-sight angle
ϵ_{lead}	lead angle
ϵ_{look}	look angle
$\vec{v}$	velocity vector
$\bar{V}$	velocity column matrix
$\vec{v}_{closing}$	closing velocity vector
$\vec{r}$	relative vector from missile to target
R	magnitude of relative vector from missile to target
Q	dynamic pressure
C_x	body x-axis aerodynamic coefficient
C_z	body z-axis aerodynamic coefficient
C_m	pitching moment coefficient
F_{aero}^b	aerodynamic force in the missile body frame
M_{aero}^b	aerodynamic moment in the missile body frame
t_p	predicted time at pullout
ω_n	natural frequency
ζ	damping ratio
h_{com}	altitude command
d_t	time step
F_i	ECI reference frame
F_e	ECEF reference frame
F_b	body-fixed reference frame of the missile
F_t	body-fixed reference frame of the target
F_n	vehicle-carried reference frame
F_r	release(launch) reference frame

CHAPTER 1

INTRODUCTION

1.1 Motivation and Problem Definition

The utilization of missiles stands as a cornerstone in modern defense systems, pivotal for both offensive and defensive operations across diverse global landscapes. These precision-guided weapons embody cutting-edge technology, integrating sophisticated guidance, propulsion, and targeting systems. From rudimentary designs to highly advanced iterations, missiles have undergone a transformative journey, revolutionizing warfare paradigms. Their adaptability across land, air, and sea domains, coupled with technological innovations, has not only reshaped military strategies but also posed complex challenges in the realms of engineering, aerodynamics, and control systems.

The trajectory of missile technology has consistently pursued expanded ranges and improved capabilities. The focus on long-range missiles has brought attention to their multifaceted advantages, offering logistical efficiencies, operational cost-effectiveness, and, notably, strengthening security measures, such as the pivotal role of long-range missiles in scenarios involving confrontations between warships with comprehensive defense systems. Their presence provides a strategic advantage by enabling engagement with threats from safe distances, significantly reducing risks by allowing operations far beyond the hostile reach of defense systems. This critical capability is important across all military operations, emphasizing the significance of extended-range solutions in fortifying security paradigms.

The evolution of cruise missiles has transformed modern warfare, offering precision strike capabilities across diverse operational scenarios. Tomahawk, BrahMos, AGM-158 JASSM and Harpoon missile systems are such examples. The Tomahawk cruise

missile, recognized for its range spanning from 1,000 to 2,500 kilometers, excels in precision and stealth, adeptly engaging land-based targets while evading enemy defenses. Likewise, the BrahMos missile, boasting a supersonic speed and a range of approximately 300 to 500 kilometers, remains a versatile choice for anti-ship and land-attack missions, ensuring swift and accurate strikes against surface targets. The AGM-158 JASSM, covering a range of 370 to over 1,000 kilometers, showcases exceptional penetration capabilities in heavily defended airspace, delivering precision strikes against fortified targets. Additionally, the AGM/UGM/RGM-84 Harpoon, with a range of about 67 to 280 kilometers, is specifically designed for sea-skimming anti-ship missions, leveraging its low-altitude flight to precisely target naval vessels. These diverse capabilities among cruise missiles underscore their pivotal role in modern military strategies, offering distinct advantages across varying operational landscapes.

Sea-skimming missiles also play a crucial role in modern naval warfare. These missiles are specifically designed to fly at extremely low altitudes, skimming the surface of the water, with the primary objective of avoiding radar detection and thwarting interception by defense systems. By flying at extremely low altitudes just above the surface of the water, these missiles significantly reduce their radar cross-section, making them inherently stealthier and more challenging to detect for enemy warships equipped with radar systems. Their proximity to the sea surface provides a natural cover from radar detection and interception, allowing them to exploit the clutter and curvature of the Earth to evade early detection. Additionally, the low-altitude flight profile of sea-skimming missiles makes it harder for ship-based defense systems to effectively engage and intercept them. The swift descent to sea level just before impact maximizes their accuracy and destructive potential, posing a formidable challenge to a warship's defensive measures, including missile interception systems and electronic countermeasures [1]. By assuming simple geometry for the earth surface, radar horizon can be calculated. The results are as in Figure 1.1.

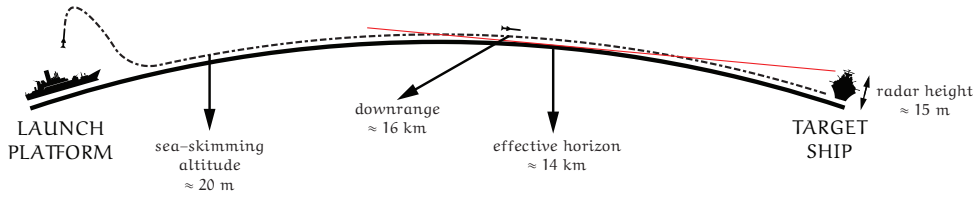


Figure 1.1: Radar Horizon Illustration.

As illustrated in Figure 1.1, based on a radar antenna height of 15 meters above sea level on a warship ([2], [3]), the radar's effective horizon can be estimated at approximately 14 kilometers from the warship. Under ideal conditions—assuming perfect weather without clutter—the radar can detect a sea-skimming missile at an altitude of 20 meters above the sea from a distance of up to 16 kilometers away from the warship. Overall, the sea-skimming technique offers a potent advantage by enhancing the probability of successfully penetrating a warship's defenses and striking the intended target with devastating effect. However, the ability to descend rapidly and smoothly to the sea surface poses a significant challenge for sea-skimming missiles due to various factors, including platform movements, ballistic phases with boosters, and varying environmental conditions. The steep descent of sea-skimming missiles is a critical maneuver that must be executed precisely and efficiently. A swift descent to the sea surface allows the missile to establish a low profile early on, minimizing the time it is susceptible to detection. However, achieving a fast descent without overshooting poses a complex control problem.

This study aims to address the challenges associated with the steep descent phase of sea-skimming missiles and proposes a modified PD (proportional-derivative) altitude controller with predictive model for calculating the system input required to achieve a powered descent to the sea-skimming altitude level while minimizing or eliminating the overshoot. The effectiveness of the proposed method will be thoroughly evaluated through various scenarios such as encompassing different initial conditions, booster thrust vector misalignment, and environmental conditions. Moreover, its performance will be compared against traditional control and guidance approaches that are commonly employed in sea-skimming missile systems. Through the rigorous analysis and comparison of results, this study aims to demonstrate the promising potential of

the newly proposed control system in overcoming the hardships of steep descent for sea-skimming missiles.

1.2 Literature Review

In literature, different approaches have been made about control algorithms for sea-skimming missiles. In [4], an optimal guidance method using LQR (Linear Quadratic Regulator) is implemented, however initial launch and booster phase of the missile is not considered in this study. [5], proposes full-state feedback controller using Kalman filter (KF) to estimate noisy radar altimeter measurements and uses a similar open-loop model for estimation but same considerations as in [4] are left out. Also, for inner loop autopilot with altitude control loops, such as attitude and acceleration autopilots, are analyzed [6]. As a result, acceleration autopilot is chosen for the inner loop. Some studies concentrate on flight at sea-skimming level under disturbances such as sea state and sensor noise [7, 8]. There are also studies covering determination of the optimum flight altitude under sea wave disturbances [9]. Although, these studies use similar inner autopilot and estimations from acceleration to altitude, concerns about overshoot and booster phase remains. Model predictive control approach is also discussed in recent years for missiles and unmanned aerial vehicle (UAV) as processors are getting powerful. In [10], similar predictive model is used to produce inputs to the system to control altitude of the vehicle by using model predictive control.

Since this study involves some form of trajectory shaping, guidance methods are also considered as solution. Impact angle and path following are two topics that surge in attention and popularity. Especially, path following algorithms utilized by cruise missiles, anti-ship missiles and unmanned aerial vehicles to follow complex and long routes, avoid obstacles. Also, impact angle control can be utilized by this study since it can be considered that if impact vector angle selected as zero degrees, missile will approach target parallel to the ground. Impact angle control problem can be investigated using optimal control and proportional navigation approaches. An optimal control based approach which proposes generalized form of the energy minimization can be derived from [11].

In [12], it starts from optimal control law and its application to proportional navigation. Afterward, generalized vector explicit guidance (GENEX) is derived and applied to two different scenarios. Firstly, the terminal phase of an air-to-air missile and, secondly, from launch to the terminal phase of an air-to-surface missile are investigated. Through a detailed analysis, the authors demonstrate the efficacy and potential applications of GENEX, emphasizing its suitability for diverse mission profiles and its contribution to advancing guidance system capabilities. For moving or stationary targets, the GENEX method shows the ability to achieve selected performance criteria such as miss distance and final relative orientation of the missile and target. In [13], methods for aircraft trajectory generation and guidance proposed. It aims to efficiently guide an aircraft along a specified path by generating intermediate waypoints synthetically. Implementation of the virtual target is used to generate these waypoints. Although it can generate the desired paths, overshoot criteria is not necessarily met.

In paper [14], trajectory shaping algorithms using virtual target following are investigated. Limitations of pure pursuit-based planar path following is shown. Also, virtual target following algorithm uses a curved path generated from a radius of curvature, which is similar to the shapes occur in pure pursuit-based planar path algorithm, analyzed and compared to the planar path following algorithm. For curved-path following algorithm initial motion that vehicle does in examples given in the study may not be appropriate for the problem this thesis trying to solve. However, study shows promising results both in the aspects of trajectory and overshoot.

In [15], biased proportional navigation with exponentially decaying error algorithm is proposed, analyzed and compared to the other studies and methods. The proposed method utilizes neither range-to-go nor time-to-go but can control impact angle and follow paths. Study starts with derivation of the proposed guidance method which includes selection of gains, utilization of damping ratio while selecting the gains and biased proportional navigation guidance law introduced in [16]. Also, derivation of the guidance law which will be called biased proportional navigation guidance (BPNG) in this thesis is the alternative derivation of the guidance law in [11]. It is seen from the study that both results are promising and does not demand high acceleration magnitudes given that gains are selected properly.

In [17], two different impact vector guidance (IVG) laws, which are line-of-sight (LOS) angle referenced and velocity referenced, are introduced. They are based on effective pure proportional navigation. Acceleration is obtained using the closing speed and applied perpendicular to the velocity vector. Thus it does not require speed change. Although this study proposes IVG to control final impact direction in 3D vector form, 2D guidance law derivation using BPN guidance law is also presented and analyzed. The proposed guidance law is effective for controlling the impact angle and trajectory shaping against stationary and moving targets.

1.3 Contributions and Novelties

The contribution of this study may be stated as such:

- Proposing a novel method for powered descent to the sea-skimming altitude as soon as possible without overshoot by designing and using a proportional-derivative controller, and without requiring high levels of computing power.
- Implementation and design of biased proportional navigation guidance, impact vector guidance and general explicit vector guidance to the descent problem using the moving virtual target method.
- Implementation of real life problems which makes launch phase of the missile more complex and nonlinear. Analyzing these problems with different methods and discussion of the applied methods for the RGM-84 Harpoon missile.

1.4 The Outline of the Thesis

The remainder of this paper is organized as follows.

In Chapter 2, an overview of the base missile RGM-84 is provided, encompassing general knowledge, selection criteria, physical parameters, aerodynamic characteristics, and the propulsion model. Additionally, this chapter includes the equations of motion for 3-degree-of-freedom (3-DOF) flight simulation, along with the dynamics of the virtual target and the environmental model.

In Chapter 3, the design of the acceleration autopilot and an evaluation of its performance are presented. This chapter encompasses the acquisition of a linear missile model for the pitch plane, achieved through the utilization of aerodynamic coefficients. Moreover, the full state-feedback method is applied to the model.

In Chapter 4, various altitude and trajectory control/guidance methods are introduced. These methods encompass the implementation of classic approaches, trajectory shaping techniques. Also, a novel approach to descent problem is proposed. The chapter details their respective design procedures, including derivations.

In Chapter 5, various analysis methods are explored, including the motion of the platform, the impact of booster thrust vector misalignment, and the effects of different environmental conditions. This chapter presents simulation results for each of these methods.

In Chapter 6, a comprehensive summary, conclusions drawn from the research findings, and recommendations for future work in this subject area are presented.



CHAPTER 2

FLIGHT SIMULATION AND MISSILE MODEL

This study models the atmospheric flight of a sea-skimming cruise missile in 3 degrees of freedom (3-DOF). Focusing on the missile's descent to sea level, only the motion in the pitch plane is considered. As depicted in Figure 2.1, the system model integrates aerodynamics, propulsion (with booster and turbojet), control actuator system, target and environment models. An acceleration autopilot is introduced to implement commands from guidance algorithms. The red guidance block represents the implementation of algorithms to solve the steep and safe descent problem to the sea-skimming altitude. Consequently, a nonlinear simulation environment is created using MATLAB and Simulink.

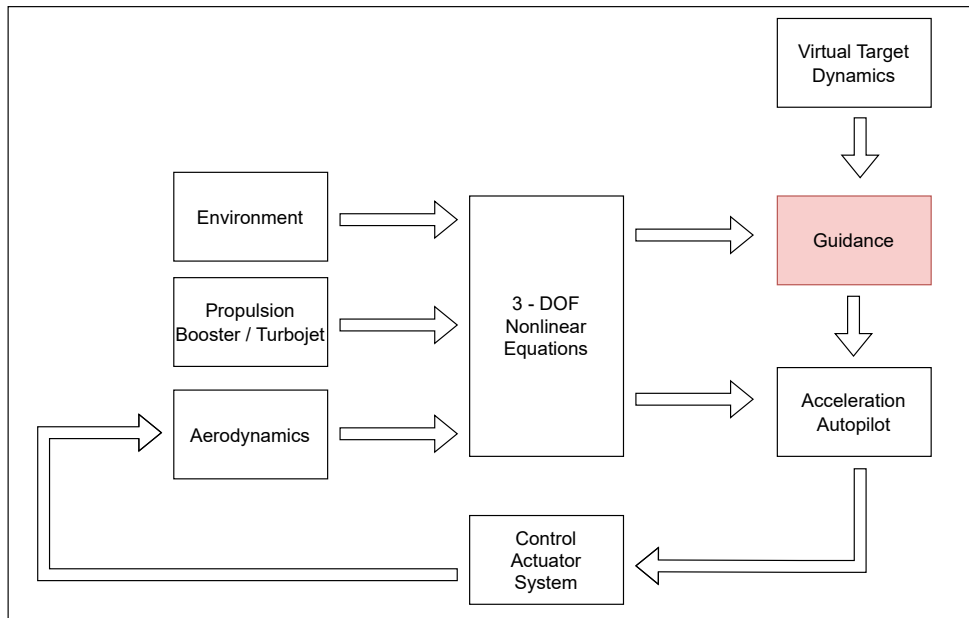


Figure 2.1: Block Diagram of the Simulation Model.

2.1 Harpoon Missile

To construct a simulation model a well-known missile is selected. As a baseline missile, RGM-84 Harpoon is selected. The RGM-84 Harpoon is a well-known anti-ship missile developed and produced by McDonnell Douglas (later acquired by Boeing) primarily for use by the United States Navy [18]. Since their inception, the Harpoon series has transcended national boundaries, becoming an indispensable asset in the naval inventories of numerous countries worldwide. It's a shipborne, long-range, all-weather, subsonic missile designed to strike and destroy enemy surface ships and capable of sea-skimming.

Although this thesis will concern about the shipborne variant (surface-to-surface missile), an inherent advantage of the Harpoon missile family lies in its adaptability across diverse platforms such as surface ships, submarines, and aircraft, thus offering strategic flexibility to naval forces across the globe.

The Harpoon missile has a range typically varying between approximately 67 to 150 nautical miles (124 to 278 kilometers), depending on the specific variant. It travels at subsonic speeds, typically around 0.8 Mach, allowing for precise targeting and engagement of enemy ships.

The RGM-84 Harpoon uses a combination of inertial guidance, radar altimeter and an active radar seeker to navigate towards its target [19]. This guidance system allows the missile to follow a predetermined flight path, with sea-skimming, while actively searching for and locking onto the intended enemy vessel. For propulsion systems, a solid-fuel rocket motor and a turbojet engine is employed. These solid-fuel engines provide the initial thrust required to launch the missile from its platform (the ship) and propel it towards its target. Turbojet engines are incorporated, allowing for sustained propulsion during the missile's flight. This switch to turbojet engines provided increased range and allowed the missile to cruise at subsonic speeds while skimming at low altitudes over the water surface, thereby reducing the risk of detection by enemy radar systems. All avionics divided by sections can be seen from Figure 2.2.

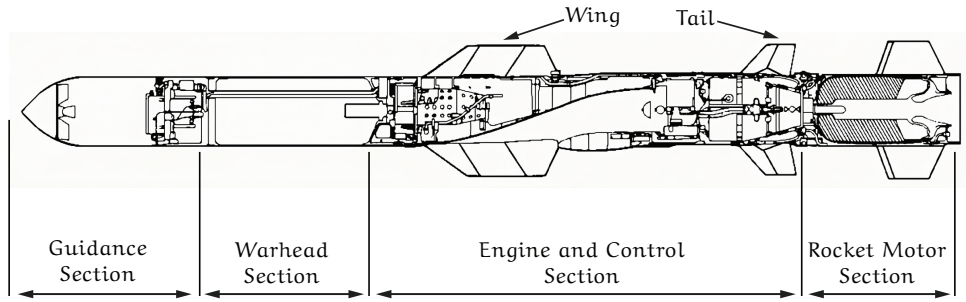


Figure 2.2: RGM-84 Harpoon Missile Configuration. [20]

Figure 2.3 illustrates a generic mission plan for a surface-to-surface anti-ship sea-skimming cruise missile. The sequence commences with the creation of the mission plan, followed by the missile's launch from a platform ship utilizing a solid-fuel rocket booster. Subsequently, the missile swiftly descends to sea-skimming altitude, minimizing the possibility of detection by enemy ship radars. During the midcourse guidance phase, which may involve evasive maneuvers and maneuvers dictated by geographical requirements, the mission plan progresses. The culmination of the mission plan occurs during the terminal phase of the missile's trajectory.

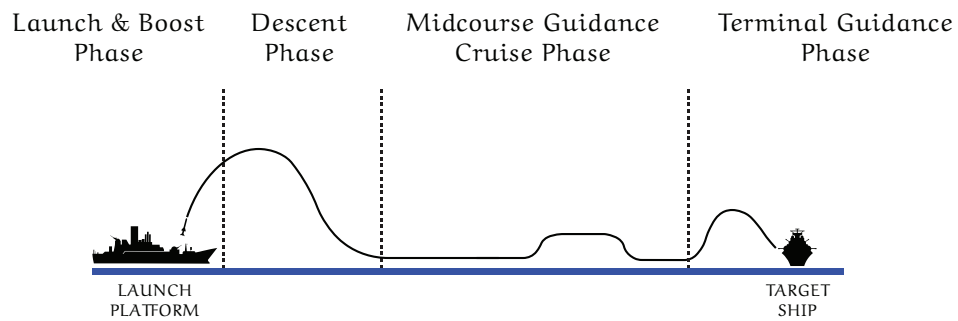


Figure 2.3: Sample Mission Plan.

Given that the missile under study is a shipborne cruise missile designed for sea-skimming capabilities, its swift descent to sea level is one of the utmost importance. As a result, using Harpoon as a baseline missile model to construct a Harpoon-like system for a simulation to study algorithms developed and implemented makes real-life sense.

2.2 Reference Frames and Coordinate Systems

Reference frames, also known as coordinate systems, provide a consistent and standardized way to describe the position, velocity, and attitude of a missile at any given moment in time. They are an essential tool for modeling, analyzing, and controlling missile behavior.

2.2.1 ECI and ECEF Reference Frame

An inertial reference frame is defined with respect to a fixed point or to a point traveling at a constant speed and it is non-rotating. The center of the inertial reference frame used in this document, which is the Earth-Centered Inertial (ECI) reference frame, is the center of mass of the earth. Within this reference frame, the orientation is defined as follows:

- The x-axis points to the vernal equinox.
- The z-axis points to the North Pole.
- The y-axis completes the right-hand coordinate system.

Earth-Centered Earth-Fixed (ECEF) reference frame, on the other hand, has the same origin and z-axis with the ECI coordinate frame but is not an inertial reference frame. ECEF reference frame rotates with the Earth. Within this reference frame, the orientation is defined as follows:

- The x-axis points to the Prime (Greenwich) Meridian.
- The z-axis points to the North Pole.
- The y-axis completes the right-hand coordinate system.

ECI reference frame is denoted as F_i , and ECEF reference frame is denoted as F_e . In this study, the rotation of the earth will be neglected. As a result, the ECI and the ECEF reference frames coincide.

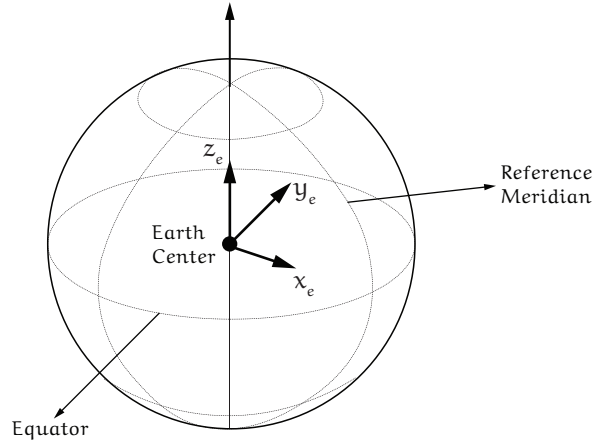


Figure 2.4: Earth-Centered Earth-Fixed (ECEF) Reference Frame.

2.2.2 Body-Fixed Reference Frame

The body-fixed reference frame is a right-handed coordinate system positioned at the missile's center of gravity. It is affixed to the missile in a way that maintains its alignment with the missile's movements and rotations. Within this reference frame, the orientation is defined as follows:

- The x-axis extends toward the missile's nose.
- The z-axis forms a 90-degree angle with the x-axis, residing in the xz-plane of symmetry.
- The y-axis is positioned to satisfy the right-handed coordinate system criteria.

The missile's body-fixed reference frame is denoted as F_b , and the target's body-fixed reference frame is denoted as F_t .

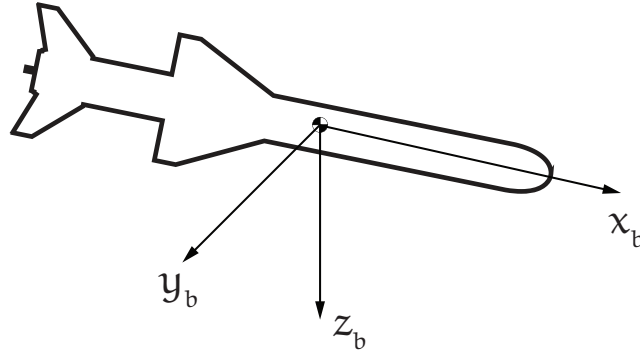


Figure 2.5: Body-Fixed Reference Frame.

2.2.3 Vehicle-Carried (NED) Reference Frame

The vehicle-carried reference frame, referred to as the NED frame, is another right-handed coordinate system positioned at the missile's center of gravity. Vehicle-carried reference frame is denoted F_n . This frame moves with the missile but does not rotate with it. Instead, the directions of its axes are constant. Within this reference frame, the orientation is defined as follows:

- The x-axis points to the north.
- The y-axis points to the east.
- The z-axis points downward, perpendicular to the reference ellipsoid.

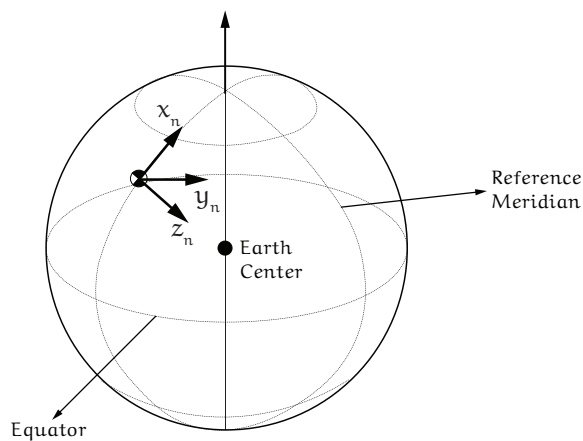


Figure 2.6: Vehicle-Carried (NED) Reference Frame.

A reference frame is selected to visualize and make calculations between the missile and the target concerning each other. This reference frame is stationary at the selected origin; in this study, it is the launch point of the missile. Unlike the vehicle-carried NED reference frame, this local frame does not move with the missile. Since its origin is the release (launch) point from the ship, this frame will be donated as F_r . Although the origin is at the launch points, its z-axis coincides with the sea level. The calculations will be done as below.

Initial coordinates of the missile in latitude (lat_0) and longitude (lon_0) are expressed in radians. By using equation A.25, $\hat{C}^{(r,e)}$ is calculated. Also, by using the initial coordinates on Geographic coordinate system, the ECEF position of the missile, $\vec{r}_{er_0}^{(e)}$ is obtained. After the launch of the missile, position of the missile with respect to the reference frame F_r is calculated by using the below equation.

$$\vec{r}_{rb}^{(r)} = \hat{C}^{(r,e)}(\vec{r}_{eb}^{(e)} - \vec{r}_{er_0}^{(e)}) \quad (2.1)$$

2.3 Equations of Motion

The equations of motion in 6 degrees of freedom (6-DOF) can be organized into translational and rotational dynamics, comprising three force and three moment equations each. However, nature of the problem lies at the pitch plane motion for safe descent to the sea level. Thus, modelling the pitch plane motion is sufficient for the analysis of the algorithms and dynamics. Although, 3 degrees of motion will be modelled all equations will be obtained and linearized from the 6-DOF equations of motion which is defined in Appendix A. Also, all the assumptions will be defined below.

Firstly, since motion will be defined in pitch (X-Z) plane and the missile is assumed to be symmetric below assumptions will be made.

$$I_{xy} = I_{yz} = I_{xz} = 0 \quad (2.2)$$

$$I_{yy} = 0 \quad (2.3)$$

Also roll and yaw motion related parameters will be assumed as below.

$$\dot{p} = p = \phi = 0 \quad (2.4)$$

$$\dot{r} = r = \psi = v = 0 \quad (2.5)$$

As a result of these assumption for force in Y -axis and moment in X and Z axes below assumptions will be made.

$$\overline{F}_{ib}^{(b)}(2) = Y = 0 \quad (2.6)$$

$$\overline{G}_{ib}^{(b)}(1) = L = 0 \quad (2.7)$$

$$\overline{G}_{ib}^{(b)}(3) = N = 0 \quad (2.8)$$

Since, the subject of the thesis is the launch and immediate control actions after the launch, simulations will stopped after either failure, ditching into the sea, or missile's convergence to the commanded sea-skimming altitude. Thus, the missile will not be covering much distance and rotation of the earth can be assumed zero.

$$\overline{\omega}_{e/i}^{(e)} = 0 \quad (2.9)$$

As a result of all these assumptions below equations are used. Coordinate transformation matrix from vehicle-carried reference frame to body-fixed reference frame can be used as below equation 2.10.

$$\widehat{C}^{(b,n)} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \quad (2.10)$$

After rearranging force and acceleration equations they are defined as below.

$$X - mg \sin \theta = m(\dot{u} + wq) \quad (2.11)$$

$$Z + mg \cos \theta = m(\dot{w} - uq) \quad (2.12)$$

2.11 and 2.12 are translational equations of motion. For the rotational equations of motion. Rotational equation 2.13 can be defined as below after assumptions.

$$M = I_{yy} \dot{q} \quad (2.13)$$

To calculate Euler angle, θ , euler integration method, as in Appendix A.1, will be used. Other angles such as the angle of attack, α , the flight path angle, γ will be calculated as below.

$$\alpha = \tan^{-1} \frac{w}{u} = \tan^{-1} \frac{\vec{v}_{eb}^{(b)}(3)}{\vec{v}_{eb}^{(b)}(1)} \quad (2.14)$$

$$\gamma = \tan^{-1} \frac{\vec{v}_{eb}^{(n)}(3)}{\vec{v}_{eb}^{(n)}(1)} \quad (2.15)$$

Defined angles can be seen from Figure 2.7.

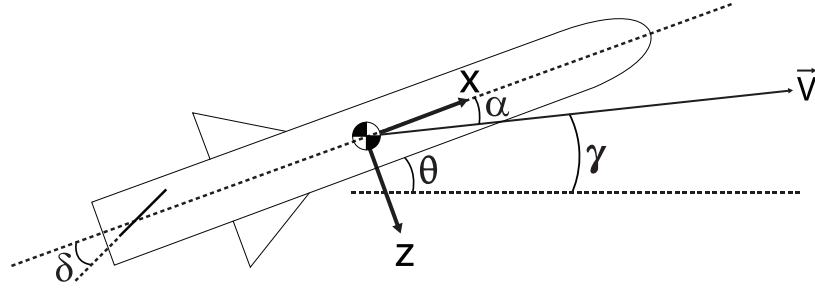


Figure 2.7: Missile Model Definition.

2.4 Physical Parameters and Aerodynamics

To construct a 3-DOF simulation, geometrical, inertial, and aerodynamic data of the selected base missile are needed. Missile parameters are borrowed from existing works [21], [22], [20].

Firstly, missile mass (m), without solid-fueled rocket booster, is determined as 500 kg. Missile mass variation due to turbojet's fuel consumption is neglected. This is due to the fact that this study, is interested in initial phase of the missile after its launch. Thus distance travelled for the inspected time interval of the simulation is very short with respect to missile's capability.

Moment of inertia about body y -axis at reference center of gravity, I_{yy} is also required for modelling and autopilot design purposes. Moment of inertia of a perfect cylinder with the exact dimensions is approximately 621 kg.m². Same assumption about the low mass concentration at the radome and guidance section of the missile 2.2, as in [21] is done. As a result, I_{yy} is scaled with a factor of 0.8 thus determined as 500 kg.m² in this study. Also, center of gravity is assumed to be at the 2 m from the nose of the missile as in Figure 2.8. Change in center of gravity is also neglected since change in mass because of the fuel is neglected. Summary of the selected parameters can be seen from Table 2.1.

Table 2.1: Inertial Parameters of the Missile

Parameter	Value
m , kg	500
I_{yy} , kg.m ²	500
l_{ref} , m	0.343
S_{ref} , m ²	0.092

Secondly, missile geometry for the base missile, which is based on the Harpoon missile (RGM-840), are determined. A software tool called MISSILE DATCOM [23], commonly referred to in the aerospace industry, is used to predict how missiles and projectiles behave in flight. It calculates aerodynamic characteristics like lift, drag, and stability to help engineers design and optimize missile systems for better performance. In this study, since 3-DOF simulation will be constructed, only the required aerodynamic coefficients are calculated by using the missile geometry and dimensions given in Figure 2.8.

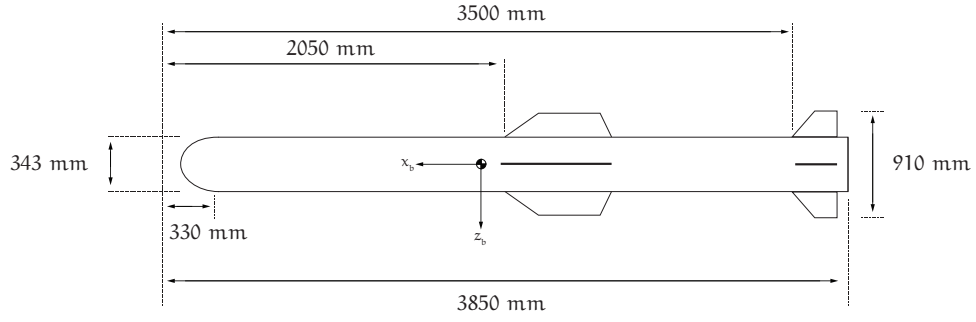


Figure 2.8: Missile Geometry and Dimensions.

Aerodynamic coefficients are evaluated for a range of Mach numbers between $[0.2, 0.9]$, angle of attack between $[-20^\circ, 20^\circ]$ and control surface deflection angles between $[-15^\circ, 5^\circ]$. All these values are summarized in Table 2.2.

Table 2.2: Aerodynamic Analysis Parameters and Values

Parameter	Range of Values
Mach Number, M	$[0.2, 0.9]$
Angle Of Attack, α	$[-20^\circ, 20^\circ]$
Control Surface Deflection, δ	$[-15^\circ, 15^\circ]$

Figure 2.9 shows negative of the axial force coefficient, which is the missile body x -axis force coefficient. It can be said that after 0.8 Mach, drag rises. This shows that 0.8 Mach can be selected as the cruise speed of the missile.

Aerodynamic stability derivatives required for 3-DOF simulation and autopilot design, which are the missile body z -axis force coefficient C_z and pitching moment coefficient C_m , are obtained for the analysis parameters given in Table 2.2 as in Figures 2.10, 2.11, 2.12 and 2.13. Linear regions for the missile can be seen from the figures and derivation for the state-space modelling required for the autopilot will be done using obtained aerodynamic database.

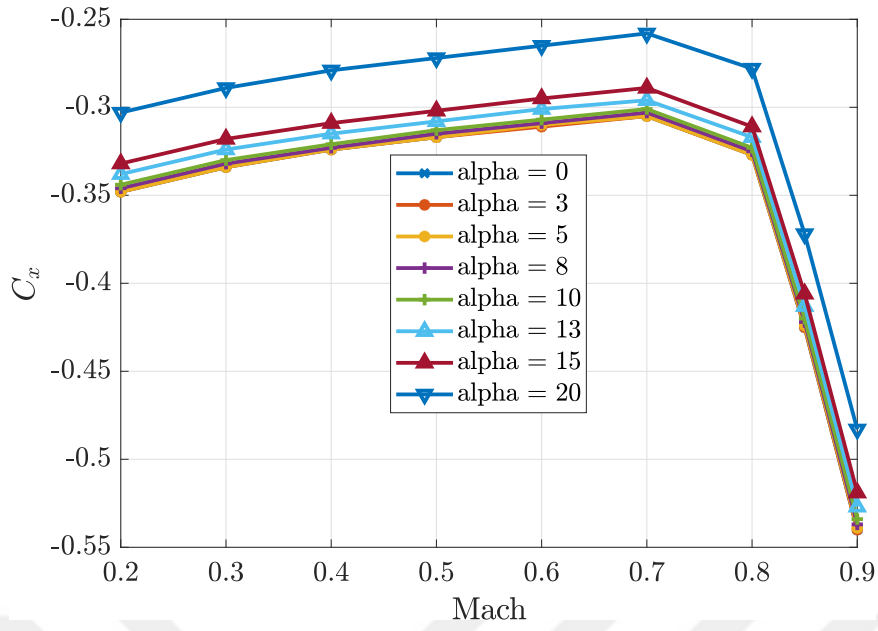


Figure 2.9: Missile Body x -Axis Force Coefficient versus Mach Number.

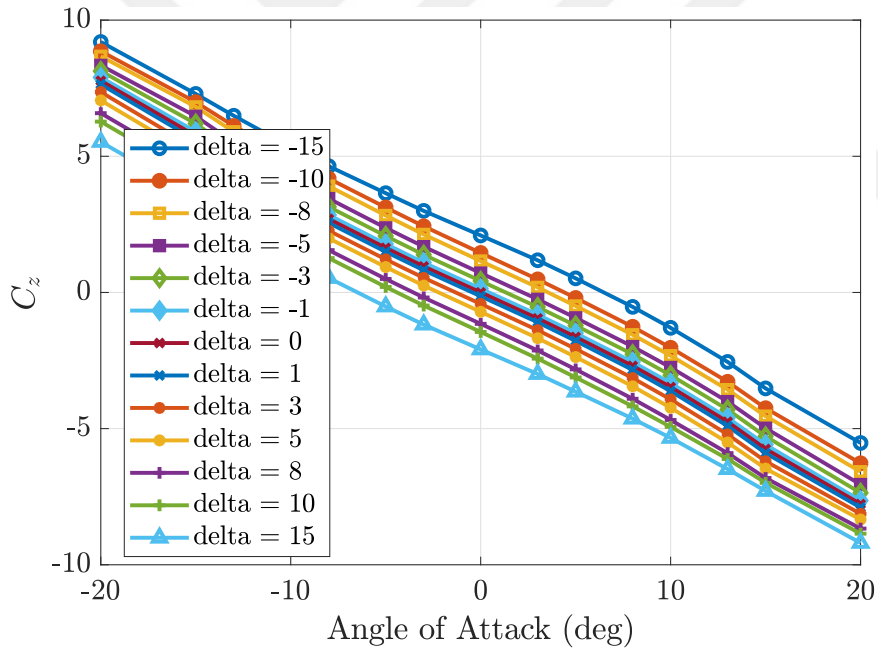


Figure 2.10: Missile Body z -Axis Force Coefficient versus Angle of Attack.

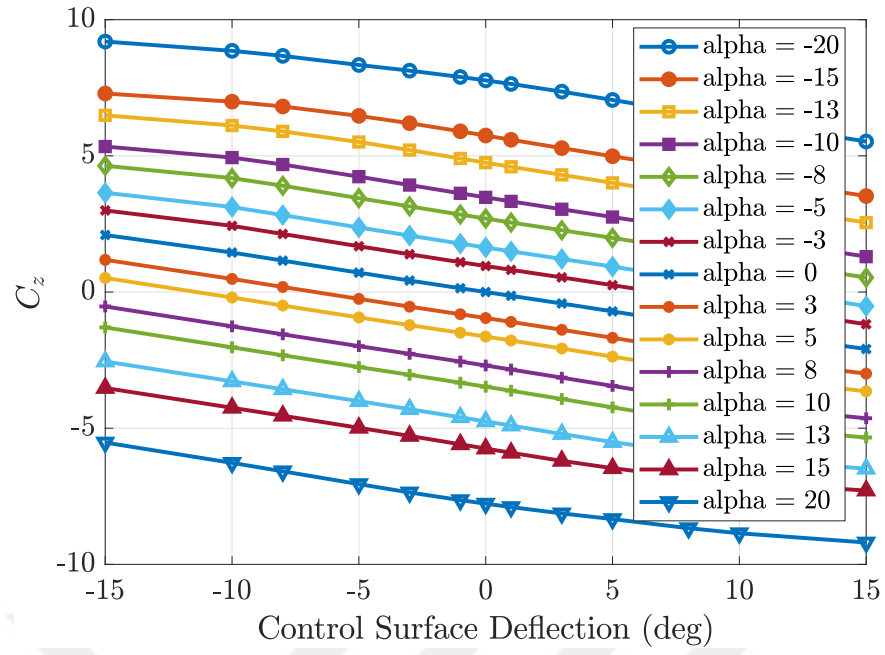


Figure 2.11: Missile Body z -Axis Force Coefficient versus Control Surface Deflection.

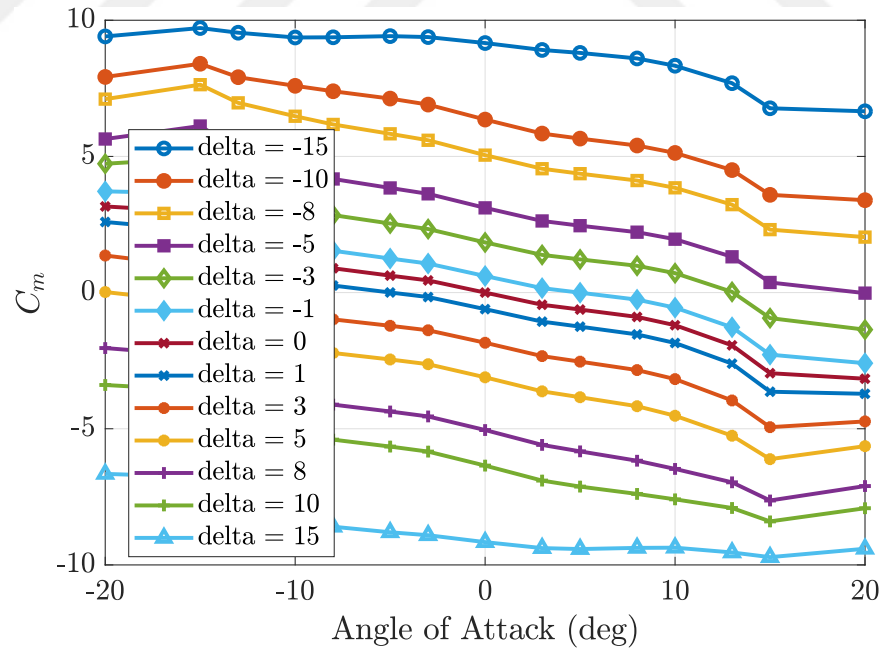


Figure 2.12: Missile Pitching Moment Coefficient versus Angle of Attack.

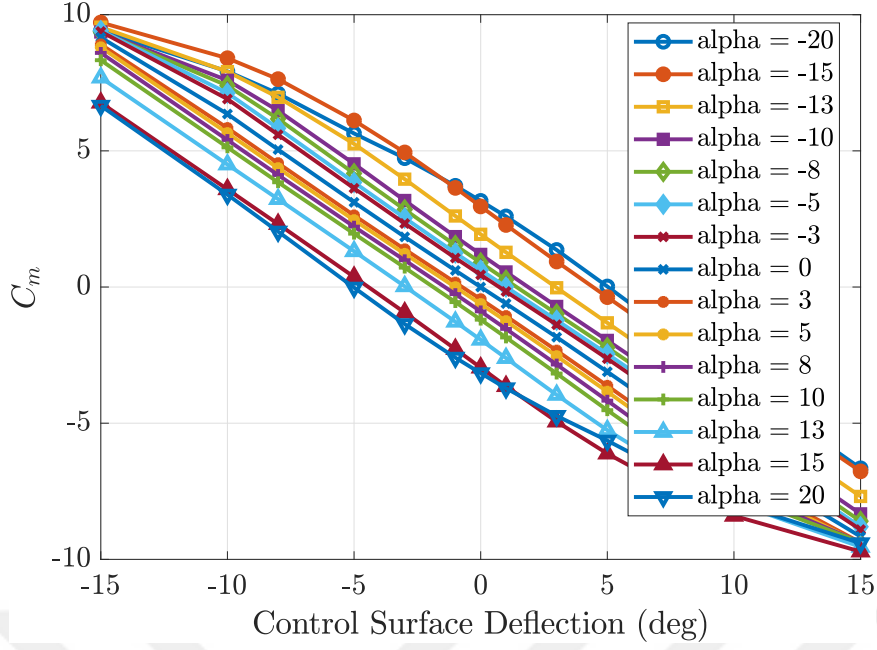


Figure 2.13: Missile Pitching Moment Coefficient versus Control Surface Deflection.

2.5 Actuator System

In this study, the autopilot gives commands to the control actuation system (CAS) as control surface deflection, δ . Since this study only focuses on the dynamics within the pitch plane, the autopilot generates commands only for the elevator. Actuator systems are driven by electrical motors which themselves have angle and angular rate limits. As a result, elevator commands are realized as a second order transfer function as below.

$$\frac{\delta}{\delta_{com}} = \frac{\omega_{cas}^2}{s^2 + 2\zeta_{cas}\omega_{cas}s + \omega_{cas}^2} \quad (2.16)$$

With the limits defined, CAS becomes nonlinear and all parameters are defined in below Table 2.3.

Table 2.3: CAS Parameters

Parameters	Value	Unit
Natural Frequency (ω_{cas})	15	Hz
Damping Ratio (ζ_{cas})	0.8	—
Angle Limit (δ_{lim})	15	$^{\circ}$
Angular Rate Limit ($\dot{\delta}_{lim}$)	120	$^{\circ}/s$

2.6 Propulsion Systems

Shipborne Harpoon, RGM-84, comes with a solid-propellant booster and a turbojet engine. Booster is for the missile to gain elevation and sufficient velocity for the turbojet to start. Also, for the descent, cruise, and terminal phase of the missile, RGM-84 uses its turbojet engine. In this study, both of the propulsion systems are modeled.

2.6.1 Booster

Shipborne cruise missiles usually use solid-fueled rocket booster for the ballistic launch phase. It is required for the missile to gain elevation and speed so that turbojet engine can be started successfully. RGM-84 uses A/B44G-2 or -3 solid-fueled rocket booster, which is a single phase rocket [24], and information can be found on [25] and [26]. When impulse of the booster comes to an end, booster detaches from the missile, and ignition process of the turbojet engine starts. Solid-fuel rocket boosters burn out through a certain time with a certain propulsion. For this study, booster thrust with 45000 N of force with 3 seconds of burn out time is considered, which is valid for $T_0 = 15^{\circ}C$ sea-level temperature.

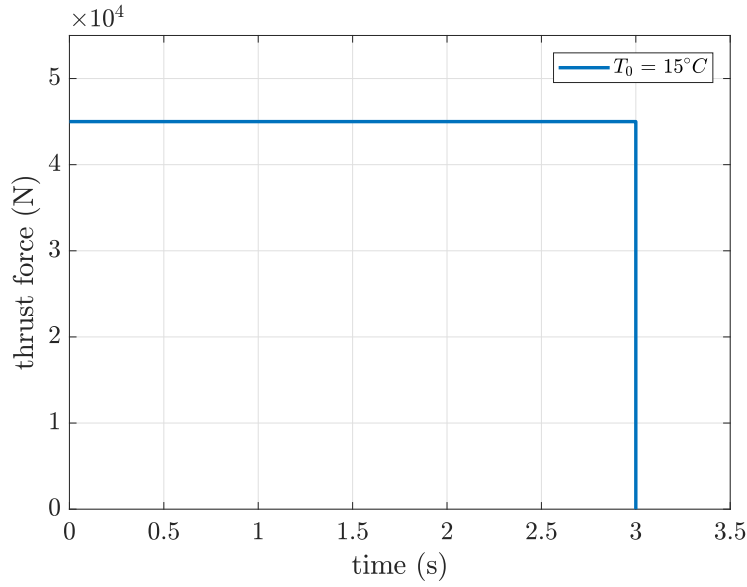


Figure 2.14: Booster Thrust Profile. [26]

Assuming that the booster's total impulse is constant, Figure 2.15 shows the booster specific impulse profile for different ambient temperatures. This is not a far off assumption since studies such as [27] and [28] show similar change of specific impulse with respect to the changing temperature. These studies show that as the temperature rises, fuel in the booster burns out faster, which means burn rate increases.

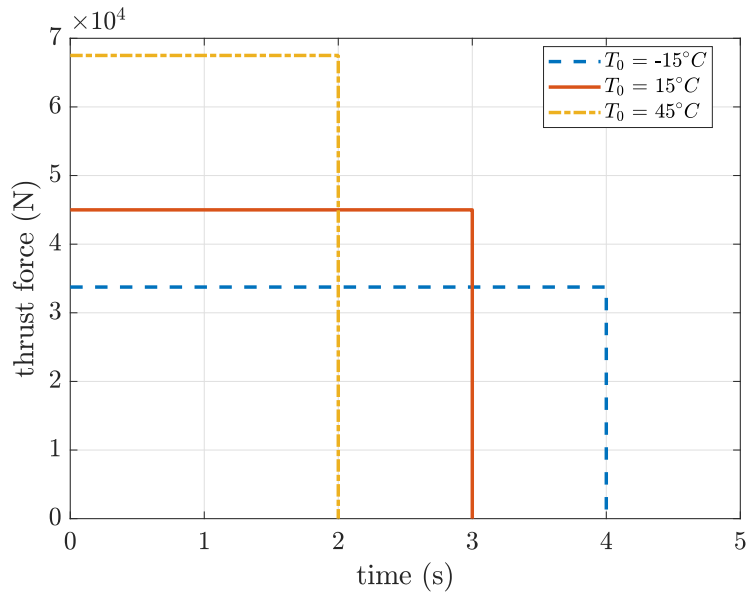


Figure 2.15: Booster Thrust Profile at Different Sea-Level Ambient Temperature Conditions.

2.6.2 Turbojet

After booster separates from the main body, required thrust is supplied by the turbojet. For this study main purpose of the turbojet is to provide enough thrust, in the limits of the selected turbojet engine, for missile to hold at the desired Mach number.

Harpoon uses Teledyne CAE J402-CA-400 turbojet engine to provide sustainer propulsion for the weapon system [29]. Teledyne J402 is 300 daN class turbojet engine [30]. It weighs 46 kg and it has one single-stage axial and single-stage centrifugal compressor and a single-stage axial turbine.

For the implementation of the turbojet to the simulations, windmilling phase is also considered. Windmilling refers to a turbojet engine when the engine's parts spin due to airflow, even when the engine isn't running or producing power [31]. In literature, for a similar sized turbojet engine, which is 350 daN class Microturbo TR60 turbojet engine, after booster separation, windmilling process duration is assumed to be 8 seconds as it is presented in a previous study [32]. Last two seconds of the windmilling, turbojet will start producing the thrust for the idle condition. Turbojet engine will generate thrust according to the performance model in Table 2.4.

A basic speed control loop, with speed error input and RPM (rotation per minute) command output, is also integrated into the system. As a result of this, the missile can increase its speed to the desired cruise Mach number after the boost phase.

To implement dynamics of the engine, more mature version of the J402-CA-400 is taken into consideration from the literature [33]. It is assumed that it takes 6 seconds to reach full thrust from the windmilling. Also from the [33] and [31], it is appropriate to apply a rate limiter for the thrust which is selected as 500 N/s. In this way it takes 4 seconds to reach full thrust from idle.

Table 2.4: Turbojet RPM vs. Thrust.

RPM (%)	0	80	100
Thrust (N)	0	1000	3000

Since this study concerned the very start of the cruise missile's life cycle, the change in the missile's mass because of the consumed fuel is assumed to be zero. Additionally, even though it will not undergo scrutiny, the review of results will involve referencing the flight envelope depicted in Figure 2.16 for commentary.

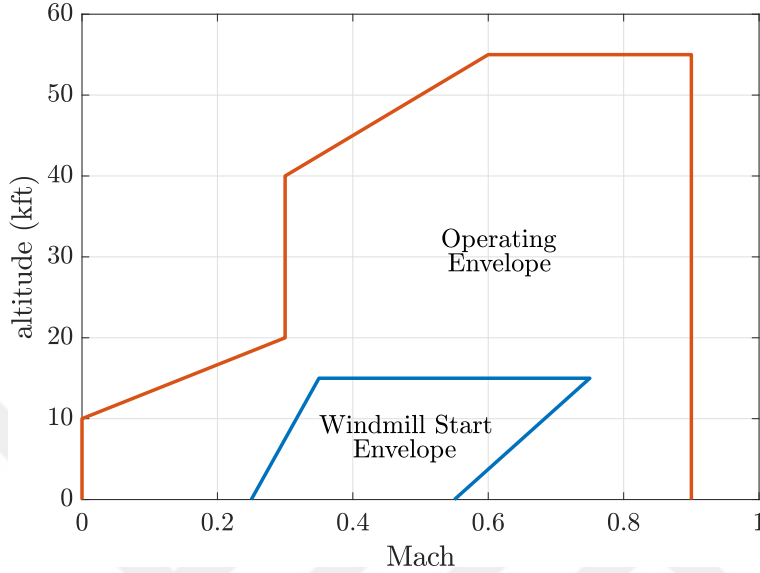


Figure 2.16: Turbojet Flight Envelope. [33]

2.7 Virtual Target Dynamics and Trajectory

Firstly, by altering equation 2.1, initial position of target in F_r can be calculated as below.

$$\bar{r}_{rt_0}^{(r)} = \hat{C}^{(r,e)}(\bar{r}_{et_0}^{(e)} - \bar{r}_{er_0}^{(e)}) \quad (2.17)$$

After that by integrating the target velocity, $\vec{v}_{et}^{(n)}$, with the initial position calculated in 2.17, $\vec{r}_{rt}^{(r)}$ is calculated. Different approaches for virtual target modelling options are investigated. Since, problem considers trajectory tracking, tail chase is considered. While constant speed target can be used, in this study target motion simulated such that, the missile, the pursuer, maintains its distance from the target in a constant, selected distance. To do that a strategy from [13] is adopted.

$$\vec{v}_{et}^{(n)}(1) = r^* \frac{\vec{v}_{eb}^{(n)}(1)}{R} \quad (2.18)$$

r^* is the constant pursuit distance, $\vec{r}_{bt}^{(r)}$ is the relative vector from the missile body to the target body resolved in F_r . Range R is the magnitude of the $\vec{r}_{bt}^{(r)}$. Relative vector can be calculated as below.

$$\vec{r}_{bt}^{(r)} = \vec{r}_{rt}^{(r)} - \vec{r}_{rb}^{(r)} \quad (2.19)$$

To use in guidance algorithms, some angle definitions should be done. Below Figure 2.17 shows angles used in guidance algorithms for trajectory shaping and tracking.

$\dot{\lambda}$ is the line of sight (LOS) rate, λ is the line of sight (LOS) angle, ϵ_{look} is the look angle, ϵ_{lead} is the lead angle. Calculation of these angles in pitch plane can be done as follows.

$$\dot{\lambda} = \frac{\vec{r}_{bt}^{(r)} \times \vec{v}_{closing}}{R} \quad (2.20)$$

$\vec{v}_{closing}$ is the relative velocity between missile and target velocity as in equation 2.21.

$$\vec{v}_{closing} = \vec{v}_{et}^{(n)} - \vec{v}_{eb}^{(n)} = \vec{V}_T - \vec{V}_M \quad (2.21)$$

Magnitude of different velocity vectors are expressed as in Equations 2.22, 2.23 and 2.24.

$$V_M = |\vec{v}_{eb}^{(n)}| \quad (2.22)$$

$$V_T = |\vec{v}_{et}^{(n)}| \quad (2.23)$$

$$V_c = |\vec{v}_{closing}| \quad (2.24)$$

LOS angle is calculated as below.

$$\vec{u}_{bt}^{(r)} = \frac{\vec{r}_{bt}^{(r)}}{R} \quad (2.25)$$

$$\lambda = \text{atan}\left(\frac{\vec{u}_{bt}^{(r)}(1)}{-\vec{u}_{bt}^{(r)}(3)}\right) \quad (2.26)$$

Then lead and look angles are calculated from below relations.

$$\epsilon_{lead} = \lambda - \gamma \quad (2.27)$$

$$\epsilon_{look} = \lambda - \theta \quad (2.28)$$

All angles expressed in this thesis can be summed up as in Figure 2.17.

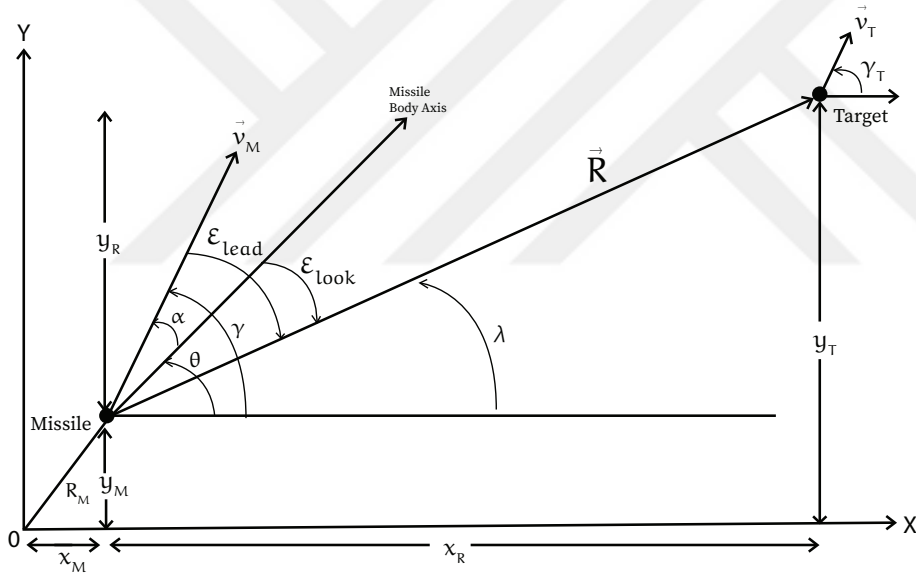


Figure 2.17: Guidance Angles Definitions. [34]

2.8 Environment Model

In this study, an environment model is adopted to better model the earth. Air temperature, air density, speed of sound, gravitational force and acceleration are calculated. Also, wind is modeled in this block and converted to the body-fixed reference frame.

Air temperature, density and speed of sound are calculated as in Equations 2.29, 2.30 and 2.31. The standard atmosphere model explained in [35], which utilizes lapse rate is used.

$$T = T_0 - Lh \quad (2.29)$$

Air density is calculated as below.

$$\rho = \rho_0 \left(\frac{T}{T_0} \right)^{\frac{g}{LR_{gc}} - 1} \quad (2.30)$$

Speed of sound is calculated as below.

$$a = \sqrt{\gamma_{sh} R_{gc} T} \quad (2.31)$$

To model the gravity using MATLAB, "WGS84 Gravity Model", is utilized. Besides the wind is modelled as below.

$$\bar{v}_{ea}^{(n)} = V_w \begin{bmatrix} -\cos(\theta_w) \\ 0 \\ 0 \end{bmatrix} \quad (2.32)$$

$$\bar{v}_{ea}^{(b)} = \hat{C}^{(b,n)} \bar{v}_{ea}^{(n)} \quad (2.33)$$

By calculating all above and using the below relations, Mach number and dynamic pressure are also calculated. Firstly, the wind velocity is introduced to calculate the airspeed, V_∞ .

$$V_{\infty} = |\vec{v}_{eb}^{(b)} - \vec{v}_{ea}^{(b)}| \quad (2.34)$$

Then Mach number is obtained as in 2.35.

$$M = \frac{V_{\infty}}{a} \quad (2.35)$$

Dynamic pressure is calculated as in 2.36.

$$Q = \frac{1}{2} \rho V_{\infty}^2 \quad (2.36)$$

Environmental model parameters are defined as Table 2.5 below.

Table 2.5: Environmental Model Parameters.

Parameter	Name	Unit	Value
T	temperature at given altitude	K	-
T_0	ambient temperature at sea level	K	-
L	lapse rate	K/m	0.0065
h	altitude	m	-
ρ	density	kg/m ³	-
ρ_0	density at sea level	kg/m ³	-
g	acceleration due to gravity	m/s ²	-
R_{gc}	characteristic gas constant	J/(kg.K)	287
a	speed of sound	m/s	-
γ_{sh}	specific heat ratio	-	1.4
V_w	constant wind speed	m/s	-
θ_w	direction of wind	rad	-
V_{∞}	airspeed	m/s	-
M	Mach number	-	-
Q	dynamic pressure	kg/(m.s ²)	-

CHAPTER 3

ACCELERATION AUTOPILOT DESIGN

In this study, the guidance and control commands for the steep descent phase are realized with the pitch autopilot. For this purpose, a full-state feedback pitch acceleration autopilot is selected. Acceleration autopilot realizes acceleration command, a_z (acceleration in the z-axis of missile body frame), by changing control surface deflection. In the literature, the full state-feedback controller method for different applications is discussed in detail in [36]. Also, in [37], how to constitute a state-feedback closed loop controller for aerospace vehicles with aerodynamic and flight stability analysis is studied. In [38], the application of the selected method is implemented for a cruise missile.

The pitch acceleration autopilot considered in this study is full state feedback control structure upon the LTI (linear time invariant) system in equations (3.1) and (3.2).

$$\dot{x} = Ax + Bu \quad (3.1)$$

$$y = Cx + Du \quad (3.2)$$

To construct these matrices Equations 2.12 and 2.13 are used. While linearizing these equations some approximations are made. Compared to the aerodynamic forces gravitational force is small. Also, small angle assumption with the constant speed is applied to the equation 2.14, thus it can be expressed as below.

$$\alpha = \tan^{-1} \frac{w}{u} \approx \frac{w}{u} \quad (3.3)$$

$$\dot{\alpha} \approx \frac{\dot{w}}{u} \approx \frac{\dot{w}}{V} \quad (3.4)$$

Thus, rearranging the equation 2.12 with these assumptions, below equation is obtained.

$$\dot{w} = \frac{Z}{m} + uq = a_z + qV \quad (3.5)$$

Then, by using equation 3.4 with the equation 3.5, also rearranging and adding equation 2.13, below equations can be obtained.

$$\dot{\alpha} = \frac{Z}{mu} + q \cong \frac{Z}{mV} + q \cong \frac{a_z}{V} + q \quad (3.6)$$

$$\dot{q} = \frac{M}{I_{yy}} \quad (3.7)$$

Force and moment equations can be expressed in a linearized way as below.

$$a_z = Z_\alpha \alpha + Z_q q + Z_\delta \delta \quad (3.8)$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_\delta \delta \quad (3.9)$$

Where δ is for the elevator deflection angle. Since, the selected autopilot has acceleration state, to obtain state-space system with a_z below steps are followed. By using equation 3.8 below relation is obtained.

$$\alpha = \frac{a_z - Z_q q - Z_\delta \delta}{Z_\alpha} \quad (3.10)$$

Then by substituting equation 3.10 into the equation 3.9 gives the relation in equation 3.11.

$$\dot{q} = M_\alpha \left(\frac{a_z - Z_q q - Z_\delta \delta}{Z_\alpha} \right) + M_q q + M_\delta \delta \quad (3.11)$$

Let's take the derivative of equation 3.8 and substituting Equations 3.6 and 3.11 into the appropriate places, new equation 3.12 is obtained.

$$\begin{aligned} \dot{a}_z = & \left(\frac{Z_\alpha}{V} + \frac{Z_q M_\alpha}{Z_\alpha} \right) a_z + \left(Z_\alpha + Z_q M_q - \frac{Z_q^2 M_\alpha}{Z_\alpha} \right) q \dots \\ & \dots + \left(Z_q M_\delta - \frac{Z_q M_\alpha Z_\delta}{Z_\alpha} \right) \delta + (Z_\delta) \dot{\delta} \end{aligned} \quad (3.12)$$

Rearranging 3.11 gives equation 3.13.

$$\dot{q} = \left(\frac{M_\alpha}{Z_\alpha} \right) a_z + \left(M_q - \frac{Z_q M_\alpha}{Z_\alpha} \right) q + \left(M_\delta - \frac{Z_\delta M_\alpha}{Z_\alpha} \right) \delta \quad (3.13)$$

Dimensional aerodynamic derivatives can be obtained as below.

$$Z_\alpha = Q S_{ref} C_{z_\alpha} \frac{1}{m} \quad (3.14)$$

$$Z_q = Q S_{ref} C_{z_q} \frac{l_{ref}}{2mV} \quad (3.15)$$

$$Z_\delta = Q S_{ref} C_{z_\delta} \frac{1}{m} \quad (3.16)$$

$$M_\alpha = Q S_{ref} l_{ref} C_{m_\alpha} \frac{1}{I_{yy}} \quad (3.17)$$

$$M_q = Q S_{ref} l_{ref} C_{m_q} \frac{l_{ref}}{2I_{yy}V} \quad (3.18)$$

$$M_\delta = Q S_{ref} l_{ref} C_{m_\delta} \frac{1}{I_{yy}} \quad (3.19)$$

Aerodynamic stability derivative values were derived by analyzing the linear regions of C_z and C_m . These values are tabulated in Table 3.1 for Mach numbers ranging from 0.5 to 0.8. It is noteworthy that the selected cruise speed is 0.8 Mach, and under most conditions, the booster speeds up the missile above 0.5 Mach. Therefore, only data from Mach 0.5 to 0.8 are included in the table. The calculations for C_{z_α} and C_{m_α} are based on angles of attack within the range of $[-15^\circ, 15^\circ]$.

Similarly, the calculations for C_{z_δ} and C_{m_δ} also employ angles of attack from $[-15^\circ, 15^\circ]$.

As for C_{z_q} and C_{m_q} , these values are derived from DATCOM analysis.

Table 3.1: Aerodynamic Stability Derivatives

Derivatives (1/rad)	0.5 Mach	0.6 Mach	0.7 Mach	0.8 Mach
C_{z_α}	-21.4	-21.52	-21.41	-21.42
C_{z_q}	-92.30	-93.56	-96.94	-99.12
C_{z_δ}	-7.36	-7.49	-7.87	-8.12
C_{m_α}	-6.98	-7.23	-8.36	-9.03
C_{m_q}	-352.08	-360.96	-380.33	-395.63
C_{m_δ}	-32.24	-32.79	-34.47	-35.55

Using the transfer function in 2.16 and combining with Equations 3.12 and 3.13, state-space system in Equations 3.1 and 3.2 is obtained as below.

$$A = \begin{bmatrix} \left(\frac{Z_\alpha}{V} + \frac{Z_q M_\alpha}{Z_\alpha}\right) & \left(Z_\alpha + Z_q M_q - \frac{Z_q^2 M_\alpha}{Z_\alpha}\right) & \left(Z_q M_\delta - \frac{Z_q M_\alpha Z_\delta}{Z_\alpha}\right) & (Z_\delta) \\ \left(\frac{M_\alpha}{Z_\alpha}\right) & \left(M_q - \frac{Z_q M_\alpha}{Z_\alpha}\right) & \left(M_\delta - \frac{Z_\delta M_\alpha}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \quad (3.20)$$

$$x = \begin{bmatrix} a_z \\ q \\ \delta \\ \dot{\delta} \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_n^2 \end{bmatrix} ; \quad u = \delta_{com}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} ; \quad D = 0$$

Also, to eliminate steady-state error, integral of the acceleration error as in equation 3.21.

$$x_i = \int (a_{z_{com}} - a_z) dt = \int a_{z_{err}} dt \quad (3.21)$$

Then, closed loop full state feedback structure can be expressed with the control gains in equation 3.22.

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} A - BK_s & BK_i \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_{zcom} \quad (3.22)$$

Where,

$$K = \begin{bmatrix} K_s & K_i \end{bmatrix} \quad (3.23)$$

$$K_s = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \quad (3.24)$$

Closed-loop block diagram structure can also be shown in the Figure 3.1.

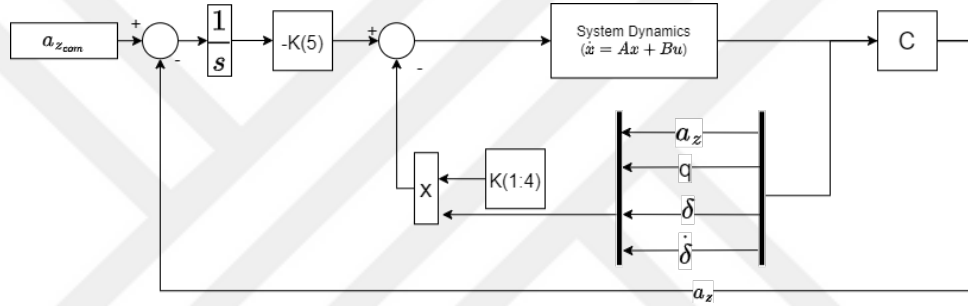


Figure 3.1: Pitch Acceleration Autopilot Block Diagram

Closed loop transfer function of the block diagram will be referred as $G(s)$, where $G(s) = \frac{a}{a_{com}}(s)$. To calculate control gains (K) pole placement method is used. Placing the poles to the desired places is done with the MATLAB function "place" as explained in [39].

Since Harpoon is a subsonic shipborne sea-skimming missile, design altitudes and Mach is selected accordingly as in Table 3.2. This study will be focusing the start of the flight so center of gravity will be assumed constant.

Table 3.2: Design Points

Parameters	Value
Mach Number Array	[0.5 0.6 0.7 0.8 0.9]
Altitude Array (m)	[0 1000]

It is important note that, booster speeds up the missile above 0.5 Mach thus design points starts from there. 0.8 Mach and sea-level is selected as the critical design condition. So, the autopilot design and methods will be shown for this point but will be applied for the other design points as well.

To decide how to select desired location of the poles, first open loop poles of the critical design point are examined. They are as in below equation 3.25.

$$p_{1,2} = -0.6758 \pm 5.0937i \quad (3.25)$$

$$p_{3,4} = -75.3982 \pm 56.5487i \quad (3.26)$$

$p_{3,4}$ belong to the CAS dynamics, which will not be changed when placing the poles. Dominant poles, poles that have real parts closer to zero, is shifted 8 times to the left. Damping ratio of the pole is selected as 0.92. Pole of the integral state $a_{z_{err}}$ is placed in a way that it has the same natural frequency with the dominant poles. Shifted poles can be seen in equation 3.27.

$$p_1 = -5.8767 \quad (3.27)$$

$$p_{2,3} = -5.4066 \pm 2.3032i \quad (3.28)$$

$$p_{4,5} = -75.3982 \pm 56.5487i \quad (3.29)$$

To conduct frequency domain analysis of the system Bode diagram of the closed loop system given in Figure 3.2 can be examined. Bandwidth of the system is defined as the magnitude of the Bode diagram falls below 0.7079 ($\cong -3$ dB). It can be seen from the Figure 3.2 bandwidth of the system is 0.56 Hz. Gain margin of the system is 4.22 ($\cong 12.51$ dB), phase margin of the system is -180° . These margins of the system explained in [36] and shows that closed system is stable and performing as desired.

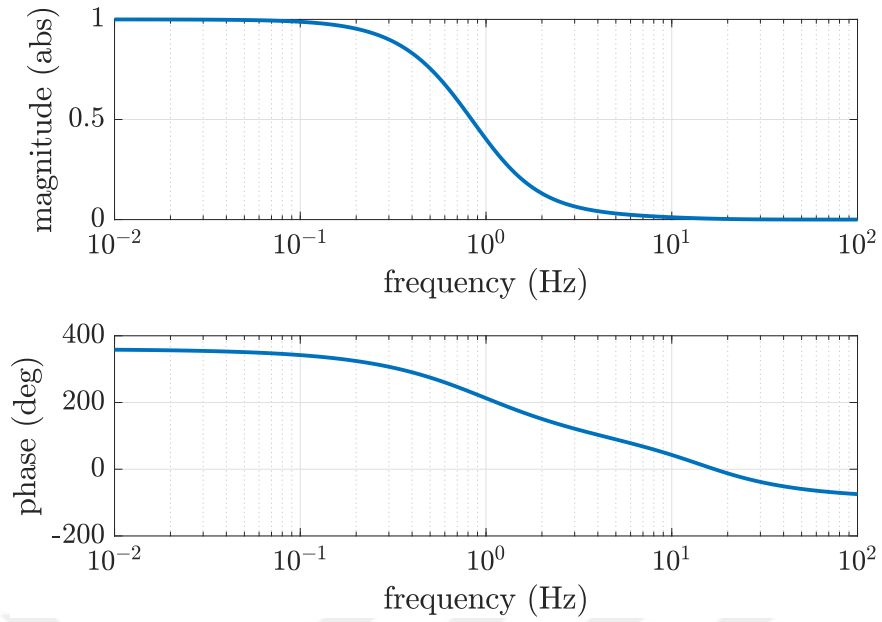


Figure 3.2: Pitch Acceleration Autopilot Closed Loop Bode Diagram.

Step response of the critical design point is shown below Figure 3.3. It can be seen that no overshoot is observed and settling time of the pitch acceleration autopilot is 1.1 seconds as desired.

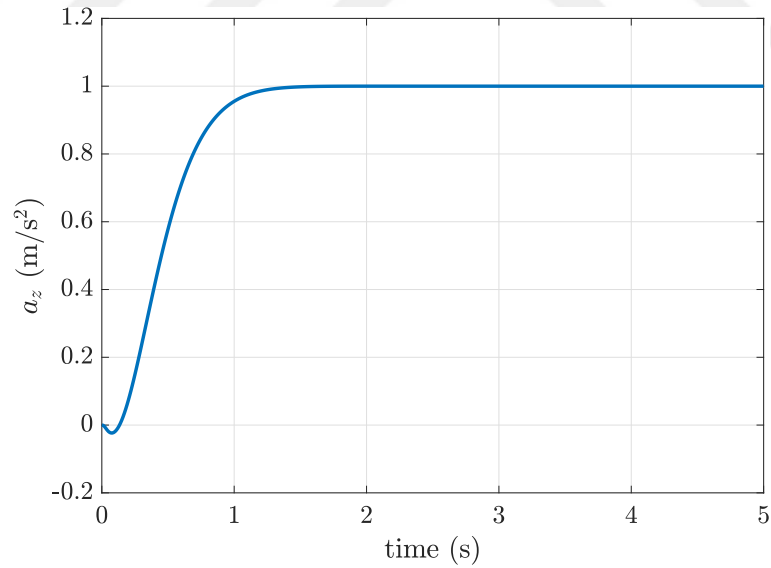


Figure 3.3: Pitch Acceleration Autopilot Closed Loop Step Response.

At last, while designing the autopilot, acceleration limits for the system are also determined. To do that, the maximum angle of attack limit and stability region of the missile from the aerodynamics are considered. Angle of attack, α , limit range is determined to be between $[-15, 15]$ degrees. Corresponding acceleration values in linear design are used as the acceleration limits in the model and can be seen in Figure 3.4.

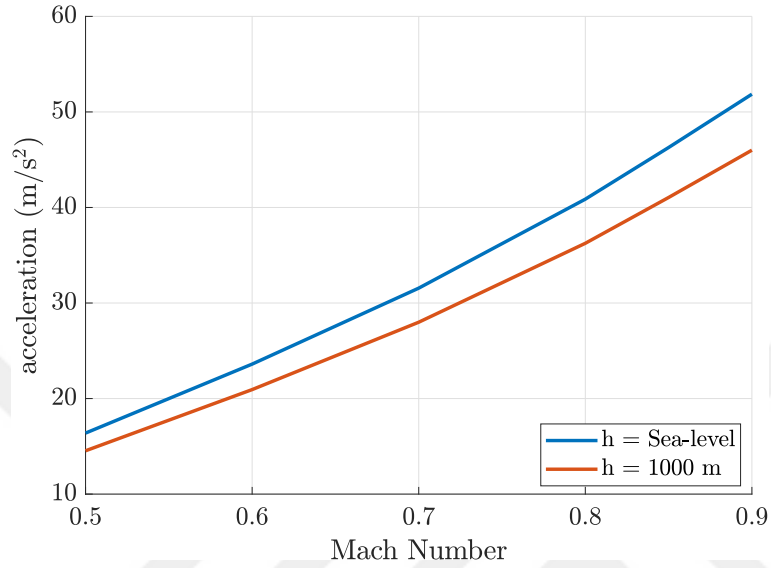


Figure 3.4: Pitch Acceleration Autopilot Acceleration Limits.

CHAPTER 4

DESIGN OF DESCENT ALGORITHMS

In this chapter, implemented control and guidance methods implemented will be explained. In this thesis, classical altitude control, proportional-derivative (PD), biased proportional navigation guidance (BPNG), impact vector guidance (IVG), generalized vector explicit guidance (GENEX) and proposed algorithm modified PD (MPD) altitude controller is used. Although these methods are selected, different methods are also can be adapted. While these methods have been chosen, it's worth noting that alternative approaches, such as model predictive control (MPC), could be considered. However, the decision was made against it due to the current challenges in real-time applicability.

4.1 Classical Altitude Control (PD)

An altitude controller is needed to generate the acceleration command for the pitch acceleration autopilot to track the altitude command. Altitude controller will be designed over the acceleration autopilot closed loop controller. First, relation between acceleration and altitude will be defined as in Figure 4.1.

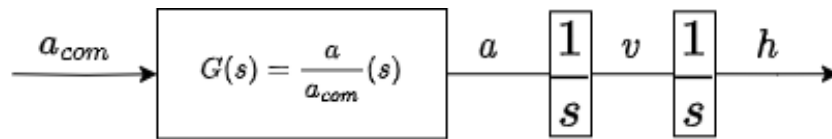


Figure 4.1: Block Diagram of the Altitude Dynamics.

Where $G(s)$ is transfer function of the pitch acceleration autopilot. It can be seen in Figure 4.1 integrating negative of the body z -axis acceleration, which autopilot uses, to obtain normal acceleration and integrating twice will result the altitude profile of the missile given that initial conditions are known. So, in block diagrams 4.1 and 4.2 a is normal acceleration, v is vertical speed or altitude rate and h is defining altitude. Thus, the transfer function from acceleration command to altitude can be expressed as in equation 4.1.

$$H(s) = \frac{h}{a_{com}} = G(s) \frac{1}{s^2} \quad (4.1)$$

It is concluded that acceleration autopilot is a type-0 system which means with the double integrator added to the system to obtain altitude profile, transfer function, from acceleration command to altitude as in equation 4.1, has now become a type-2 system. It is known that for step and ramp inputs, type-2 system is not expected to have steady-state error thus PD controller is adapted as in previous studies [8] and [22]. Figure 4.2 demonstrates block diagram of the PD altitude controller.

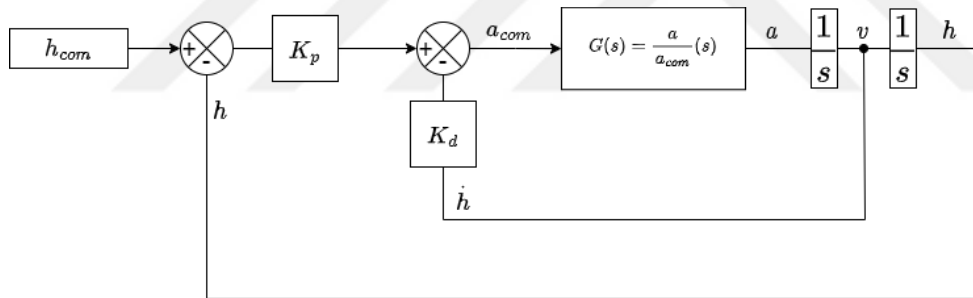


Figure 4.2: Block Diagram of PD Altitude Control System.

Closed loop transfer function is expressed as in equation 4.2.

$$H_{cl}(s) = \frac{h}{h_{com}}(s) = \frac{K_p H(s)}{1 + K_p H(s) + s K_d H(s)} \quad (4.2)$$

Figure 4.3 shows the comparison between Bode diagrams of the two transfer functions $G(s)$ and $H_{cl}(s)$.

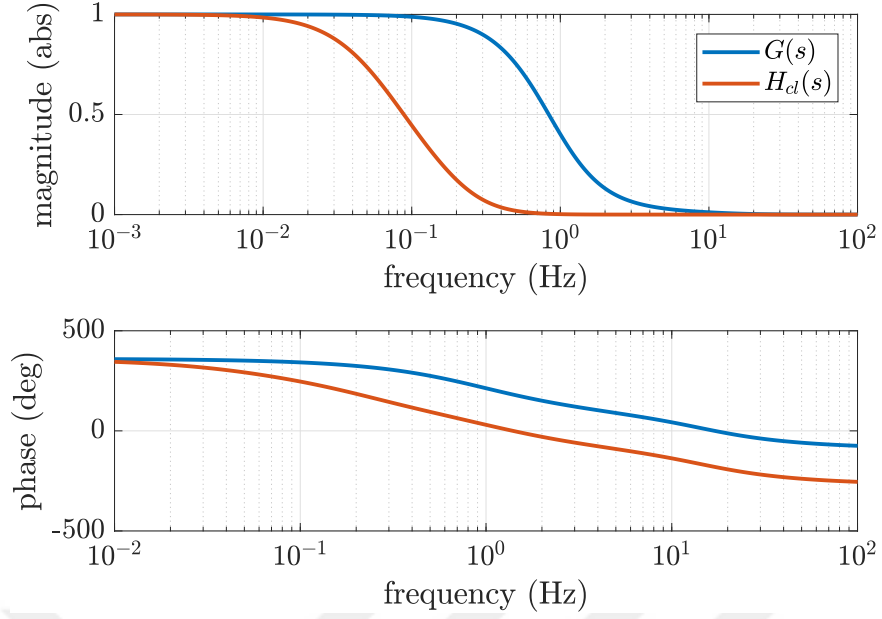


Figure 4.3: Comparison Between Bode Diagrams of $G(s)$ and $H_{cl}(s)$.

Since safe descent to the sea-level is crucial for the missile, most important criteria is zero overshoot. While setting proportional and derivative gains, respectively K_p and K_d , trial and error method will be used. Other than the overshoot criteria settling time is another performance parameter. Although having agile dynamics is not a requirement for the cruise missiles settling time still taken into consideration.

While selecting controller gains, analysis shows that the faster the system becomes less successful it is for descending to the sea-level. This issue and a solution will be proposed in section 4.5. Although making controller very slow shows some level of success, it is not desirable for a PD altitude controller which will remain in use after descending to the sea level to hold the altitude at the desired level. It is important to note that this algorithm will be utilized for all guidance methods once the descent to the commanded altitude occurs while cruising.

Since this algorithm could be used for cruising at sea-level, with this in mind proportional gain K_p is set to 0.2 and derivative gain K_d is set to 0.85. Step response with the chosen controller gains can be seen in Figure 4.4.

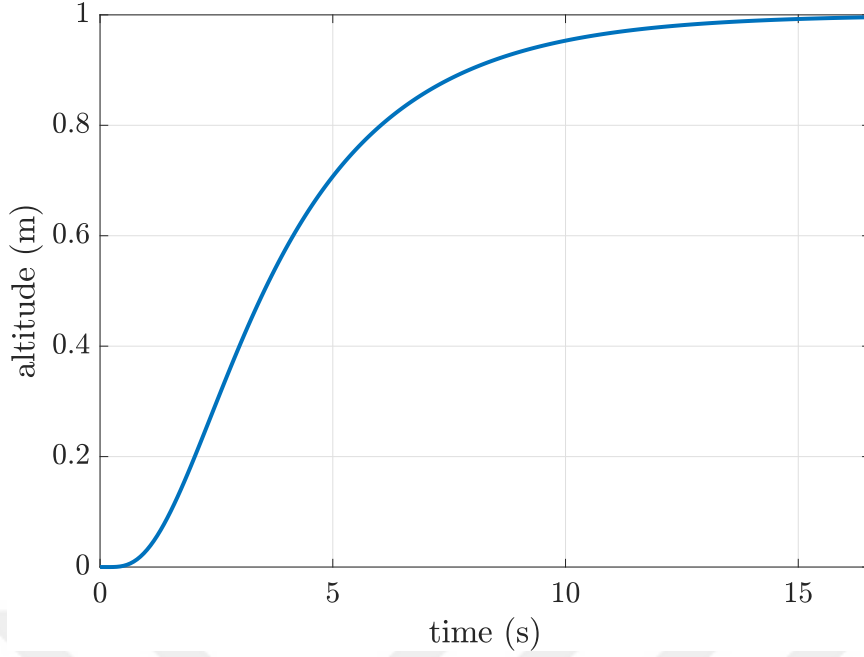


Figure 4.4: Altitude Tracking PD Controller Step Response

For the ideal case scenario, initial conditions of the missile and environmental conditions are shown in Table 4.1. Throughout this study, altitude command will be set as 20 m.

Table 4.1: Nominal Simulation Conditions

h_0 (m)	θ_0 (deg)	q_0 (deg/s)	θ_b (deg)	z_b (m)	$T(^{\circ}\text{C})$	V_w (m/s)
10	30	0	0	0	15	0

In Figure 4.5, ideal case scenario simulation results are displayed. In Figure 4.5b, $a_{z_{com}}$ is acceleration command in body z-axis from benchmark PD altitude controller, $a_{z_{upperlim}}$ and $a_{z_{lowerlim}}$ are acceleration limits from autopilot design section as it shown in Figure 3.4. The simulation starts with booster ignition and launching of the missile from a ship with an initial pitch angle. During the boost phase, which takes 3 seconds for the booster to run out, the missile gains altitude and speed. At the end of this phase, booster is separated, and pitch control starts. First, altitude controller produces positive acceleration command, in the missile body axis, to turn the missile velocity vector downwards. In the meantime, the missile starts to slow down due to the aerodynamic drag. After 6 seconds of windmilling and startup, the turbojet

engine starts to generate thrust. The missile speeds up again while it completes its descent and performs negative acceleration pullout maneuver for transition to cruise altitude.

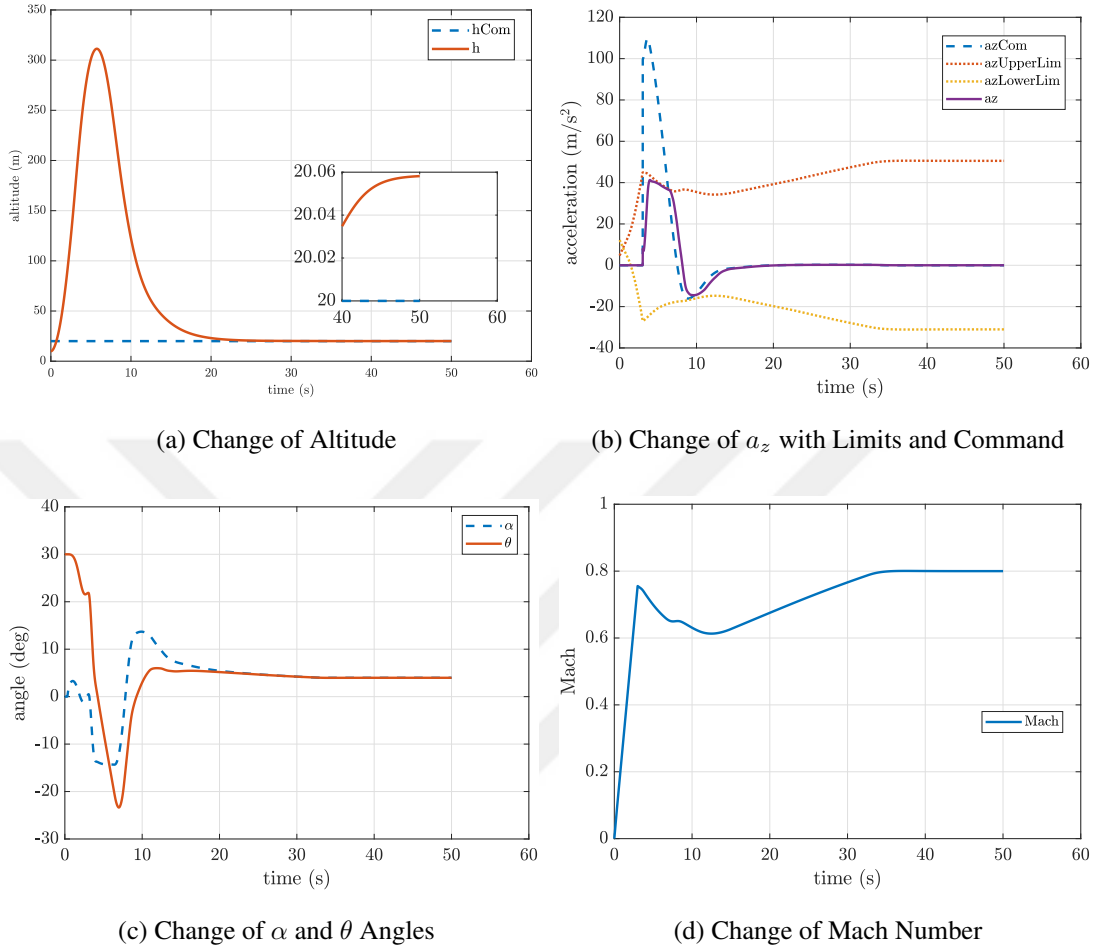


Figure 4.5: Simulation Results with PD Altitude Controller with Nominal Initial Conditions.

4.2 Biased Proportional Navigation Guidance (BPNG)

In this study, biased proportional navigation (biased PN) guidance method, which is defined in [15] as an alternative derivation of the guidance law proposed in [11], is used as one for the benchmark methods.

The biased PN guidance law is as in equation 4.3 from [16].

$$\dot{\gamma} = N\dot{\lambda} + b \quad (4.3)$$

Where N is navigation constant (gain) and b is bias. Based on the linear engagement geometry shown in Figure 4.6, in planar motion LOS rate can be calculated as in equation 4.4.

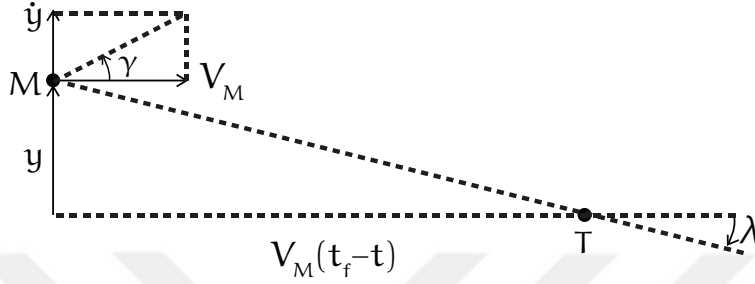


Figure 4.6: Linear Engagement Geometry for BPNG.

$$\dot{\lambda} = -\frac{\dot{y}}{V_M(t_f - t)} - \frac{y}{V_M(t_f - t)^2} = \frac{\lambda - \gamma}{t_f - t} \quad (4.4)$$

Then, by integrating 4.3 from current to final time as in [15], following equation is obtained.

$$\gamma_f - \gamma = N(\lambda_f - \lambda) + b(t_f - t) \quad (4.5)$$

Then solving equation 4.5 with 4.3 for b , equation 4.6 is calculated as below.

$$\dot{\gamma} = N\dot{\lambda} + \frac{N\lambda - \gamma - (N - 1)\gamma_f}{t_f - t} \quad (4.6)$$

Where γ_f is final thus the desired impact angle for the problem. Denominator of the second term of the equation 4.6, represent time to go, t_{go} . It is commonly calculated as in equation 4.7 used in literature such as [40].

$$t_f - t = t_{go} \approx \frac{R}{V_C} \quad (4.7)$$

Although t_{go} can be calculated as in equation 4.7, because of the dynamic nature of the problem, which is the descent to the sea-level without overshooting, calculation will be modified as in equation 4.8.

$$t_{go} \approx \frac{\Delta h}{V_d} \quad (4.8)$$

Where

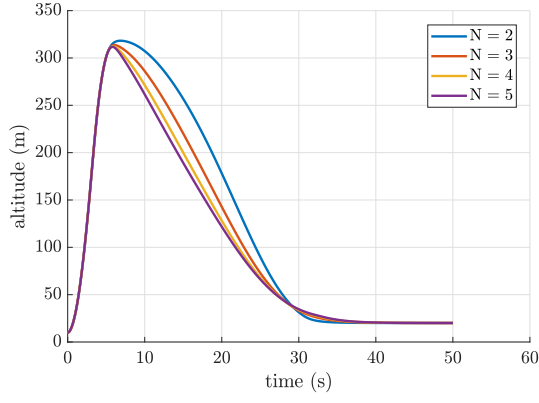
$$V_d = \bar{v}_{eb}^{(n)}(3) \quad (4.9)$$

$$\Delta h = h - h_{com} \quad (4.10)$$

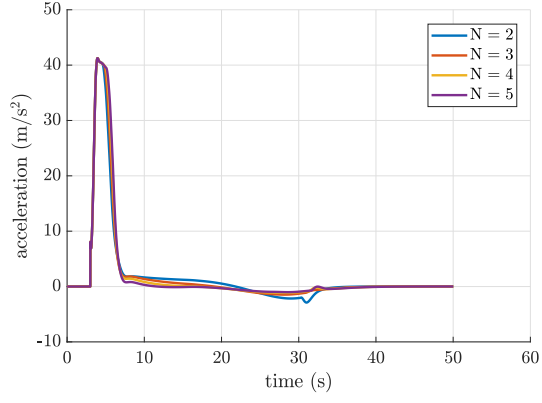
In this way, more accurate time-to-go calculation which is descending to the target altitude is calculated. Then acceleration command for the autopilot can be obtained by using equation

$$a_{com} = N\dot{\lambda}V_C + \frac{N\lambda - \gamma - (N-1)\gamma_f}{t_{go}}V_d \quad (4.11)$$

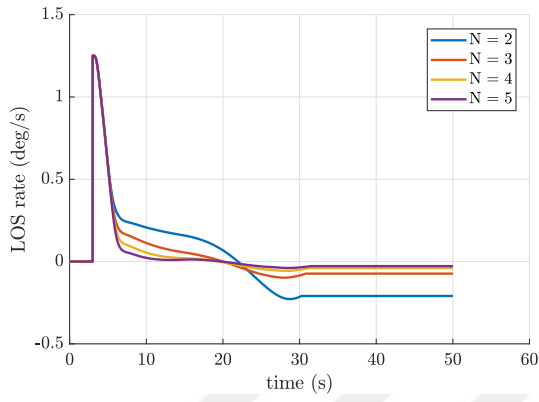
Now that guidance method is set, optimum navigation constant N for the descent problem is investigated. Simulation results with different navigation constants with the nominal initial conditions defined in 4.1 are shown in Figure 4.7.



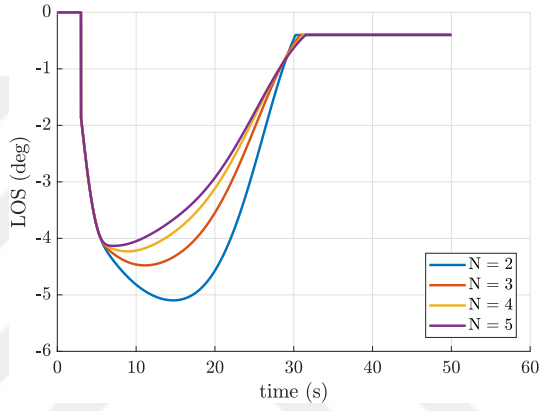
(a) Time versus Altitude for Different Navigation Constants



(b) Time versus Acceleration for Different Navigation Constants



(c) Time versus LOS Rate for Different Navigation Constants



(d) Time versus LOS for Different Navigation Constants

Figure 4.7: Simulation Results for Different Navigation Constants.

By using Figure 4.7 and the study [11], where it says guidance strategy becomes optimal with respect to total control effort, N is selected as 3. Results with the nominal initial conditions are shown in Figure 4.8. It can be seen that after the booster separation, BPNG commanded the maximum acceleration for missile to the descent as intended. After that it descends without using abilities of the missiles to the fullest but does not overshoot. Thus, the angle of attack profile of the missile through the flight is around its trim condition. Also, uses potential energy of the missile fairly well.

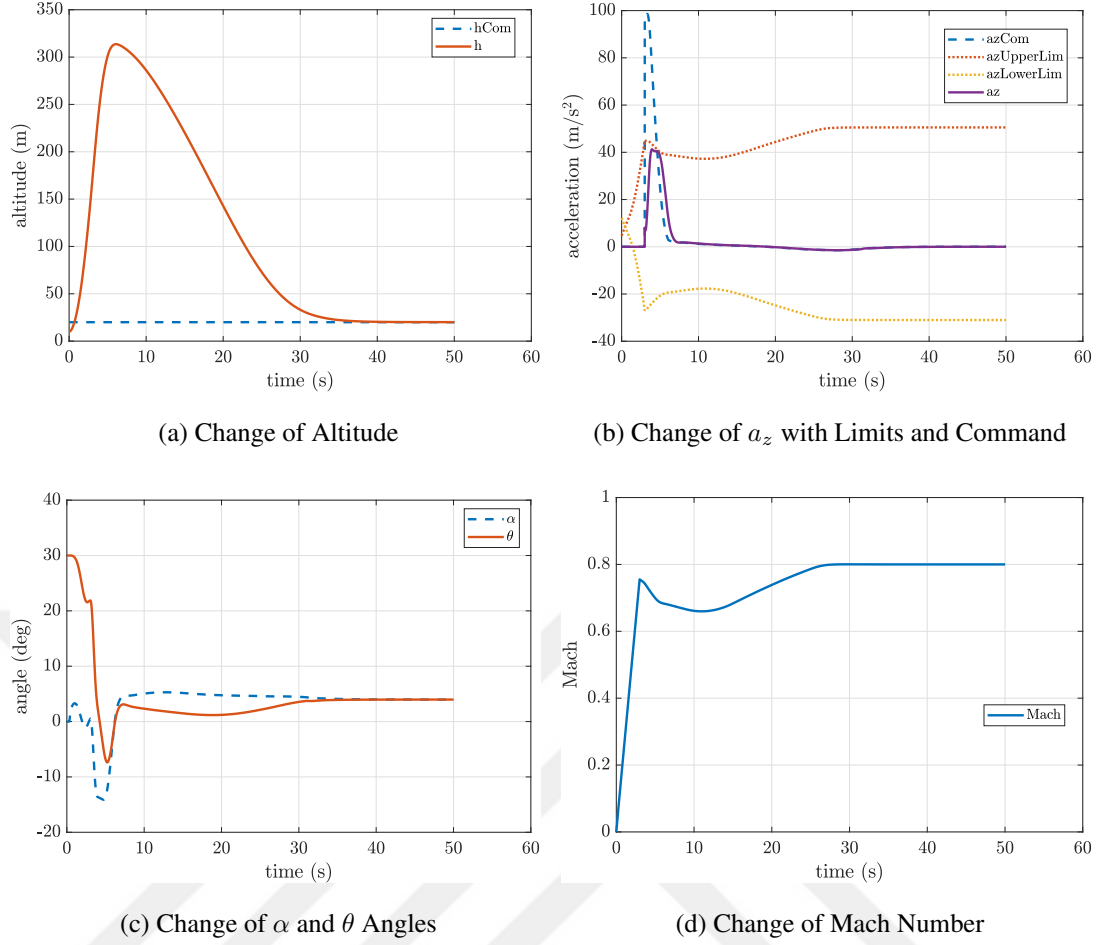


Figure 4.8: Simulation Results with BPNG with Nominal Initial Conditions.

4.3 Impact Vector Guidance (IVG)

The impact vector guidance (IVG), which is the method introduced in [17], is based on effective pure proportional navigation (EPPN). Since, this thesis investigates fast and robust descent to the sea-level, methods with impact angle control and trajectory optimization are needed. In linear domain, impact angle error is expressed with respect to line-of-sight(LOS) and velocity are the two alternative representations. Then combining these alternative gives control over either final LOS angle or the flight path angle. Thus, to apply trajectory shaping in two dimensions with IVG, guidance law is derived in linear domain. By using the Figure 4.9 below relations are defined.

$$\lambda = -\frac{y}{V_c(t_f - t)} \quad (4.12)$$

$$\dot{\lambda} = -\left(\frac{\dot{y}}{V_c(t_f - t)} + \frac{y}{V_c(t_f - t)^2}\right) \quad (4.13)$$

$$\hat{\gamma} = \frac{\dot{y}}{V_c} \quad (4.14)$$

Where y is the vertical distance between the target and the missile, V_c is the closing speed and $\hat{\gamma}$ is the relative flight path angle as expressed in the Figure 4.9.

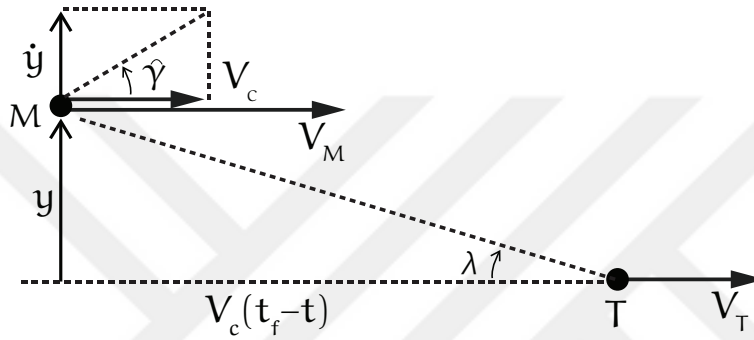


Figure 4.9: Linear Engagement Geometry.

As in equation 4.3, biased PN law is expressed as in below equation 4.15.

$$\dot{\hat{\gamma}} = N\dot{\lambda} + b \quad (4.15)$$

Thus to convert $\dot{\hat{\gamma}}$ in equation 4.15 to acceleration, equation 4.14 is used. As a result

$$\ddot{y} = a = V_c(N\dot{\lambda} + b) \quad (4.16)$$

$$b = \frac{\hat{\gamma}_f - \hat{\gamma} - N(\lambda_f - \lambda)}{t_f - t} \quad (4.17)$$

Integrating both sides of the equation 4.15 from t to t_f gives two alternative results in equations 4.18 and 4.19.

$$b_r = \dot{\lambda} + (N - 1) \frac{\lambda - \lambda_f}{t_f - t} \quad (4.18)$$

$$b_v = N\dot{\lambda} + (N - 1) \frac{\hat{\gamma} - \hat{\gamma}_f}{t_f - t} \quad (4.19)$$

Where b_r is the angle error expressed with respect to LOS and b_v is the angle error expressed with respect to the velocity. In equation 4.19, relative flight path angle is used but to have a selected final flight path angle against a moving target, below modification is adopted as in equation 4.20.

$$b_v = N\dot{\lambda} + (N - 1) \frac{\gamma - \gamma_f}{t_f - t} \quad (4.20)$$

Time-to-go is estimated in similar way as in 4.2, equation 4.7.

$$t_f - t = t_{go} \approx -\frac{R}{\dot{R}} = \frac{R}{V_c} \quad (4.21)$$

Then, using equations 4.18, 4.20 and 4.21, and by substituting them into equation 4.16

$$\dot{\gamma}_r = \frac{V_c}{V_M} \{ (N + 1)\dot{\lambda} + \frac{V_c}{R} (N - 1)(\lambda - \lambda_f) \} \quad (4.22)$$

$$\dot{\gamma}_v = \frac{V_c}{V_M} \{ 2N\dot{\lambda} + \frac{V_c}{R} (N - 1)(\gamma - \gamma_f) \} \quad (4.23)$$

These equations are modified as it done in 4.2, for equation 4.8. As a result below equations 4.24 and 4.25 are obtained as a function of t_{go} .

$$\dot{\gamma}_r = \frac{V_c}{V_M} \{ (N + 1)\dot{\lambda} + (N - 1)(\lambda - \lambda_f) \frac{1}{t_{go}} \} \quad (4.24)$$

$$\dot{\gamma}_v = \frac{V_c}{V_M} \{ 2N\dot{\lambda} + (N - 1)(\gamma - \gamma_f) \frac{1}{t_{go}} \} \quad (4.25)$$

In Figure 4.10, results of the different approaches, which are LOS angle referenced approach (LOS-ref.), $\dot{\gamma}_r$, and velocity referenced approach (Vel-ref.), $\dot{\gamma}_v$, are shown.

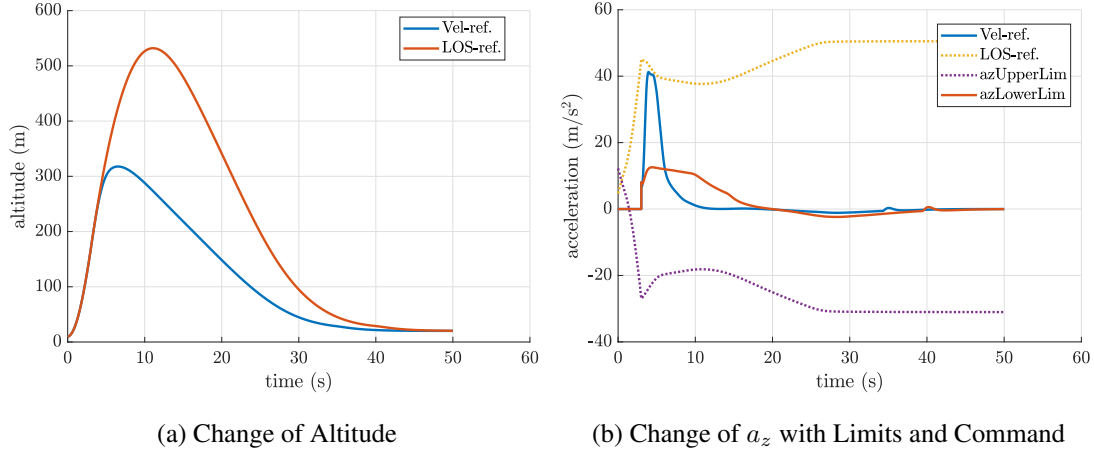


Figure 4.10: Simulation Results with IVG for Different Approaches with Nominal Initial Conditions.

By using Figures 4.10 and 4.11, it can be said that velocity referenced approach is more suitable for the steep descent problem. From Figure 4.10a, it can be seen descent starts quicker and the Figure 4.10b shows that velocity referenced approach uses more of the abilities of the missile thus velocity referenced approach is selected. Also, from the Figure 4.11, it is shown why the difference in acceleration commands are exist.

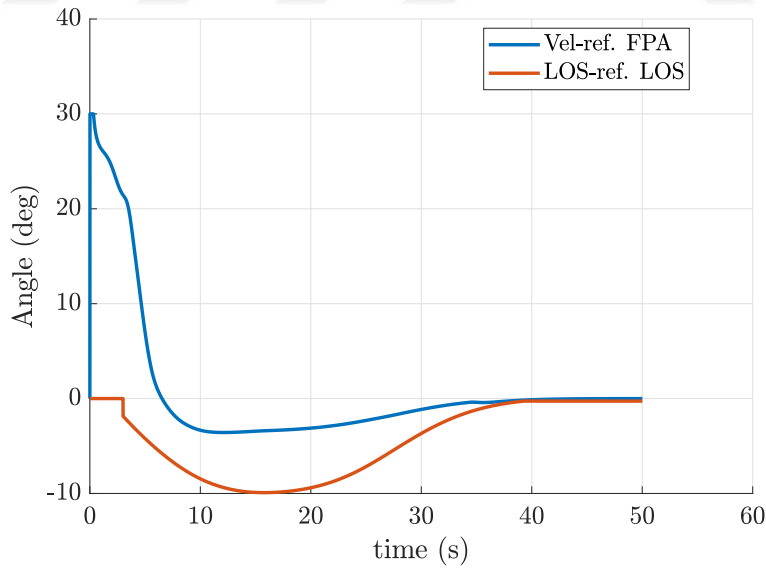


Figure 4.11: Change of λ and γ Angles

Figure 4.12 shows that after $N = 2$ change in performance is minimal and in the light of a previous study [17], it is known that control effort becomes optimum for most of the cases when navigation constant is selected as 4. As a result, in scope of this study, N is selected as 4.

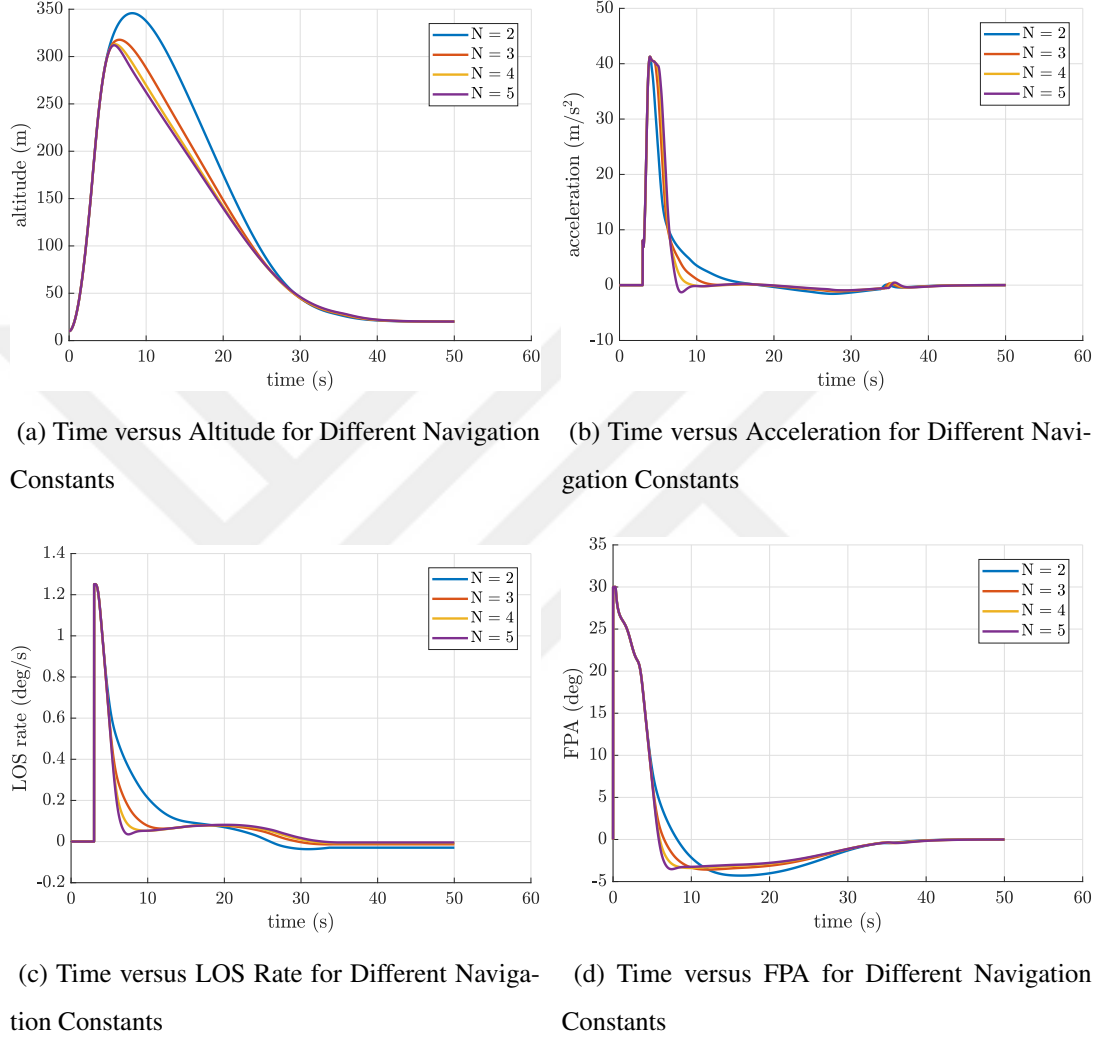


Figure 4.12: Simulation Results for Different Navigation Constants.

Results with the nominal initial conditions are shown in Figure 4.13. It is seen that maximum acceleration command is given by the guidance method, however before available angle of attack limit is reached, acceleration command is damped by the algorithm. Although, it slows down the missile, safe descent is obtained.

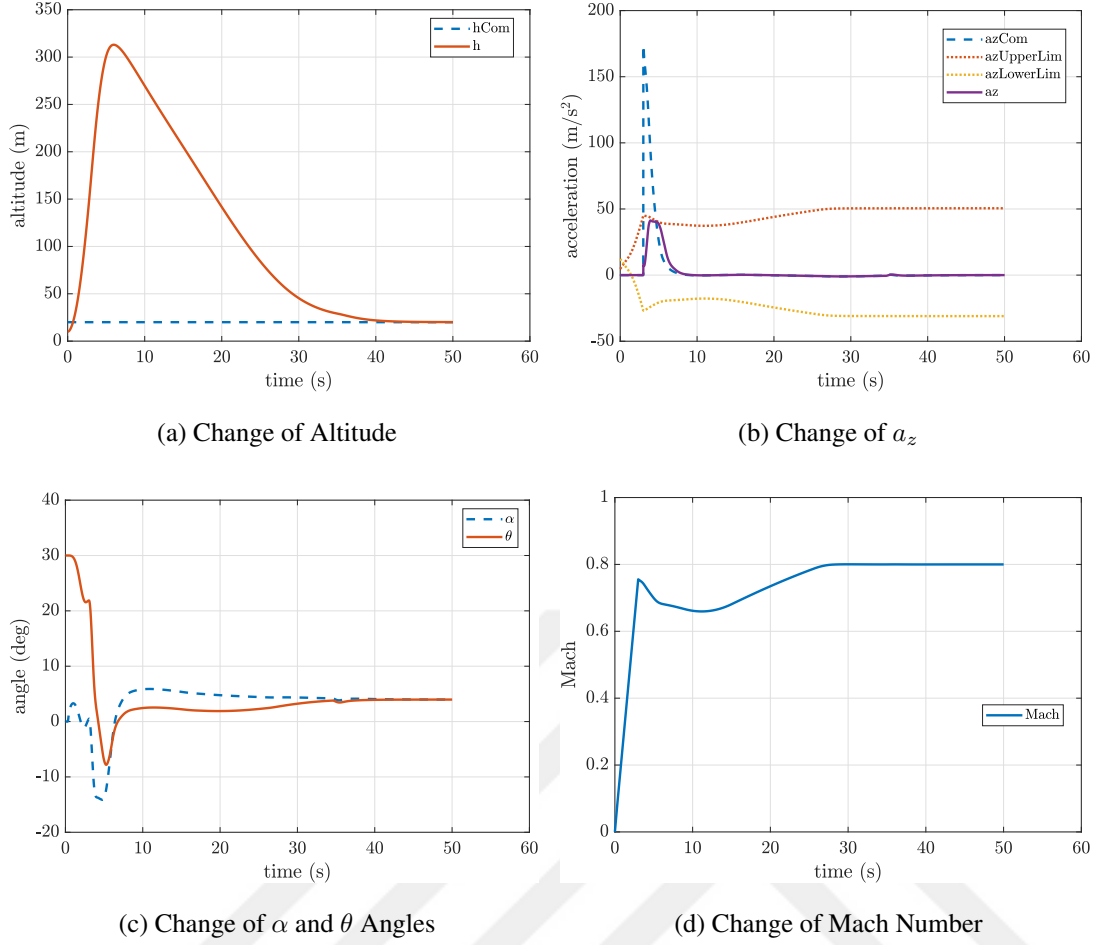


Figure 4.13: Simulation Results with IVG with Nominal Initial Conditions.

4.4 Generalized Vector Explicit Guidance (GENEX)

Study done in [12] summarized below for better understanding of the guidance approach. After a linear state equation and boundary conditions are set in equation 4.26, cost function is selected as in equation 4.27, where x is the state, b may be time-varying, u is control and unbounded, $T = t_f - t$ is time to go and $n \geq 0$ is an integer.

$$\dot{x} = Ax + bu \quad ; \quad x(t_0) = x_0 \quad ; \quad x(t_f) = x_f = 0 \quad (4.26)$$

$$J = \int_0^{T_0} \frac{u^2}{2T^n} dT = \int_0^{T_0} L(u, T) dT \quad (4.27)$$

It is known that from the minimum principle of Pontryagin, when Hamiltonian is minimized, control u is optimal. Also, a fundamental matrix $M(T)$ is defined as in equation 4.28.

$$\frac{dM}{dT} = MA \quad M(T=0) = 1 \quad (4.28)$$

Then $Q(t)$ function is defined as below.

$$Q(T) = \int_0^T (Mb)(Mb)^T T^n dT \quad (4.29)$$

As a result optimal control u^* can be calculated as below.

$$u^* = -(Mb)^T Q^{-1} MxT^n \quad (4.30)$$

Now that optimal control approach is defined, its application for the proportional navigation guidance will be handled as below. First, lateral separation of a missile and its final location and zero effort miss (ZEM) can be defined as in equations 4.31 and 4.32.

$$y = y_f - y_M \quad (4.31)$$

$$z = y_f - y_M - \dot{y}_M T \quad (4.32)$$

Then by differentiating equation 4.32, $\dot{z}$ is obtained as in equation 4.33 since $\dot{T} = -1$.

$$\dot{z} = -\dot{y}_M - \ddot{y}_M T - \dot{y}_M \dot{T} = -\ddot{y}_M T \quad (4.33)$$

Select missile acceleration as the control and the state as the ZEM. As a result, state equation is obtained as below.

$$\dot{x} = -uT \quad ; \quad x(0) = x_0 \quad ; \quad x_f = 0 \quad (4.34)$$

This is form as in equation 4.26 where $A = 0$ and $b = -T$. Also from the definition of the fundamental matrix in equation 4.28, $M = I$.

$$Q = \int_0^T T^{n+2} dT = \frac{T^{n+3}}{n+3} \quad (4.35)$$

Then, substitute equation 4.35 into the equation 4.30 gives

$$u^* = T \left[(n+3)/T^{n+3} \right] x T^n = \left[(n+3)/T^2 \right] x \quad (4.36)$$

Where u^* is the proportional navigation acceleration command consists of x is the state ZEM, guidance gain and time-to-go. Now, previous method can be extended to add also desired final velocity as the second ZEM state. So, the states can be defined as below equations 4.37 and 4.38.

$$x_1 = z = y_f - y_M - \dot{y}_M T \quad (4.37)$$

$$x_2 = v = \dot{y}_f - \dot{y}_M \quad (4.38)$$

Where v is the difference between current and desired final velocity. Then control input matrix b can be defined as in equation 4.39.

$$\dot{z} = -uT \quad (4.39)$$

$$\dot{v} = -u \quad (4.40)$$

And terminal conditions are applied as $z = 0$ and $v = 0$ at $T = 0$. Then state equations can be rewritten as in equation 4.41.

$$\begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} -T \\ -1 \end{bmatrix} u \quad (4.41)$$

Then, $Q(T)$ is reconstructed and after applying method defined before, optimal input is obtained as below.

$$u^* = 1/T^2 \left[K_1(y_f - y_M - \dot{y}_M T) + K_2(\dot{y}_f - \dot{y}_M)T \right] \quad (4.42)$$

Where K_1 and K_2 defined as in equation 4.43.

$$K_1 = (N + 2)(N + 3) \quad (4.43)$$

$$K_2 = -(N + 1)(N + 2) \quad (4.44)$$

Selection of N will be done by optimization in coming parts. N is used to modify the curvature of the trajectory which is crucial in implementation to the solution of this problem.

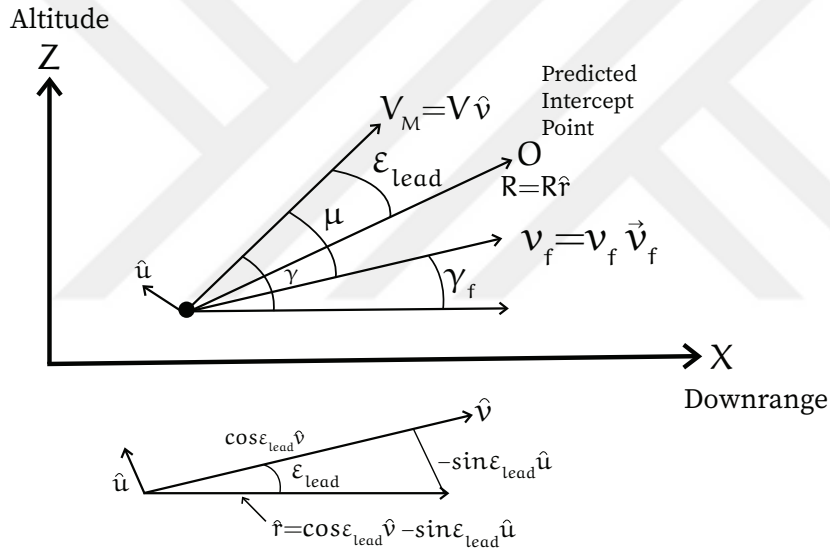


Figure 4.14: Engagement Geometry. [12]

Assuming that solution equation 4.42 is applicable to the x and z coordinates in similar form, a vector formulation of the guidance law can be written as in equation 4.45.

$$u = 1/T^2 \left[K_1(R_f - R_M - V_M T) + K_2(V_f - V_M)T \right] \quad (4.45)$$

Where V_M is missile velocity, V_f is the missile final velocity.

$$R_f - R_M = R\hat{r} \quad ; \quad V_M = V\hat{v} \quad ; \quad V_f = V\hat{v}_f \quad ; \quad T = R/V \quad (4.46)$$

Where R is range, V is average closing speed, and time-to-go can be estimated as range over closing speed. Then by substituting equation 4.46 into equation 4.45 then taking the component of the u normal to the velocity vector equation 4.47 is obtained.

$$u = \frac{V^2}{R} \left[K_1(\hat{r} - \hat{v}\cos(\epsilon_{lead})) + K_2(\hat{v}_f - \hat{v}\cos(\mu)) \right] \quad (4.47)$$

Where lead angle ϵ_{lead} as defined in Figure 2.17, and velocity error μ are as in equation 4.48.

$$\cos(\epsilon_{lead}) = \hat{r}\hat{v} \quad ; \quad \cos(\mu) = \hat{v}_f\hat{v} \quad (4.48)$$

Since solution in equation 4.47 is 3 dimensional but this thesis requires solution in single plane below solution will be implemented in thesis.

$$\hat{r} = \hat{v}\cos(\epsilon_{lead}) - \hat{u}\sin(\epsilon_{lead}) \quad (4.49)$$

$$\hat{v}_f = \hat{v}\cos(\mu) - \hat{u}\sin(\mu) \quad (4.50)$$

Where $\mu = \gamma - \gamma_f$. After equations 4.49 and 4.50 substituted into equation 4.47, 4.51 is obtained.

$$a_{com} = V^2/R[-K_1 * \sin(\epsilon_{lead}) - K_2\sin(\gamma - \gamma_f)]\hat{u} \quad (4.51)$$

What is defined as heading error in equation 4.51 is explained as lead angle in Figure 2.17.

To apply defined acceleration in descent to the sea-skimming problem below modification and strategy are adapted. Firstly, equation 4.51 is rewritten as below.

$$a_{com} = \frac{V_c}{t_{go}}[-K_1 * \sin(\epsilon_{lead}) - K_2\sin(\gamma - \gamma_f)]\hat{u} \quad (4.52)$$

Where time-to-go calculated as in equation 4.8, $t_{go} \approx \Delta h/V_d$, with the similar goal in mind. Also V_c taken as V_M as it was in the derivation. Furthermore, it should be noted that the choice of N , thus K_1 and K_2 gains, may be used to weigh on the curvature of the trajectory or the final position and orientation. In Figure 4.15, different navigation constants are investigated. It can be seen that for $N = -1$ best results for the steep descent problem is obtained. In this study setup with the moving virtual target, while increasing N , stability is decreasing thus the oscillations. This is expected since first term of the guidance law is about the ZEM and tries to trajectory to having zero heading error by forcing missile to gravitate to the target, while second term enforces on the final velocity and orientation.

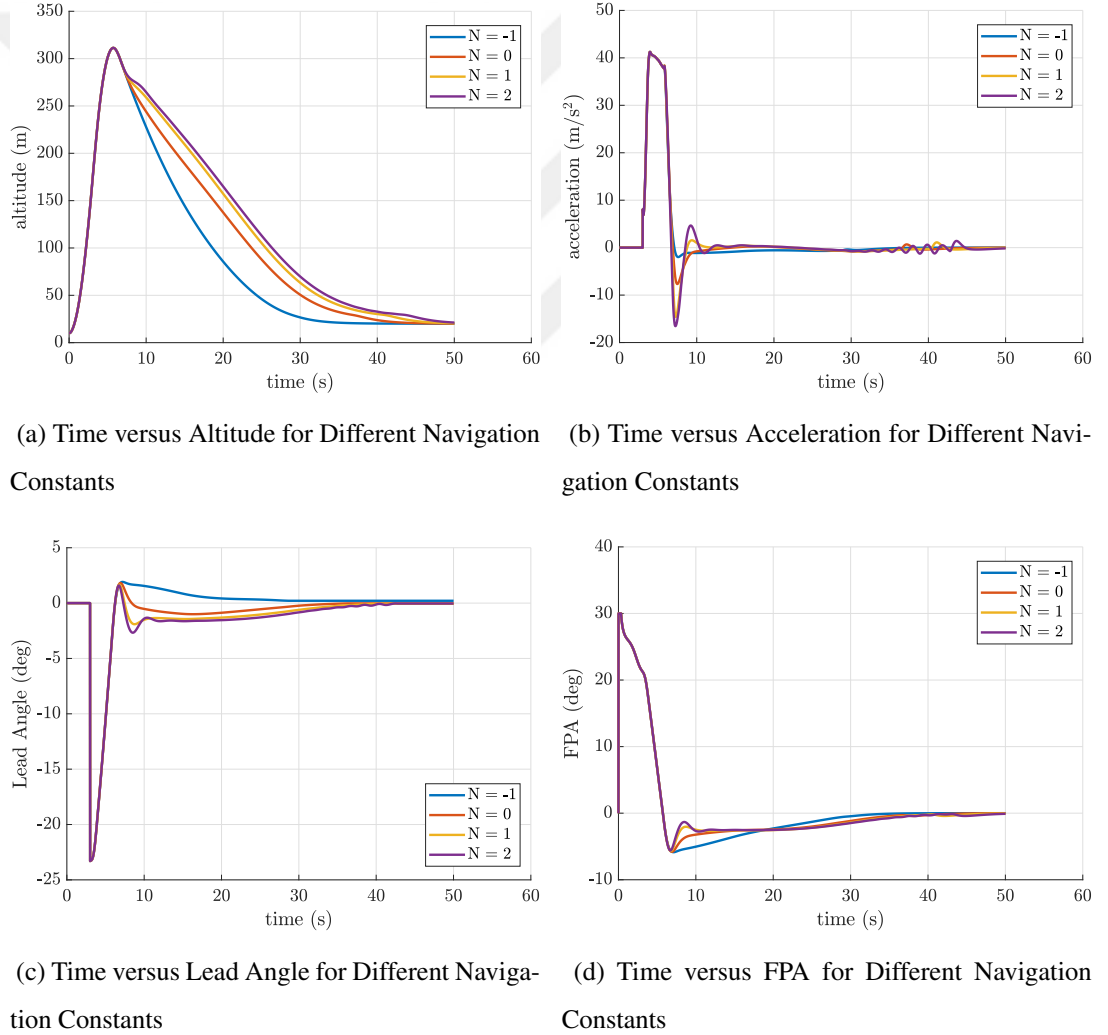


Figure 4.15: Simulation Results for Different Navigation Constants.

After $N = -1$ is selected, result with the nominal initial conditions are presented in Figure 4.16.

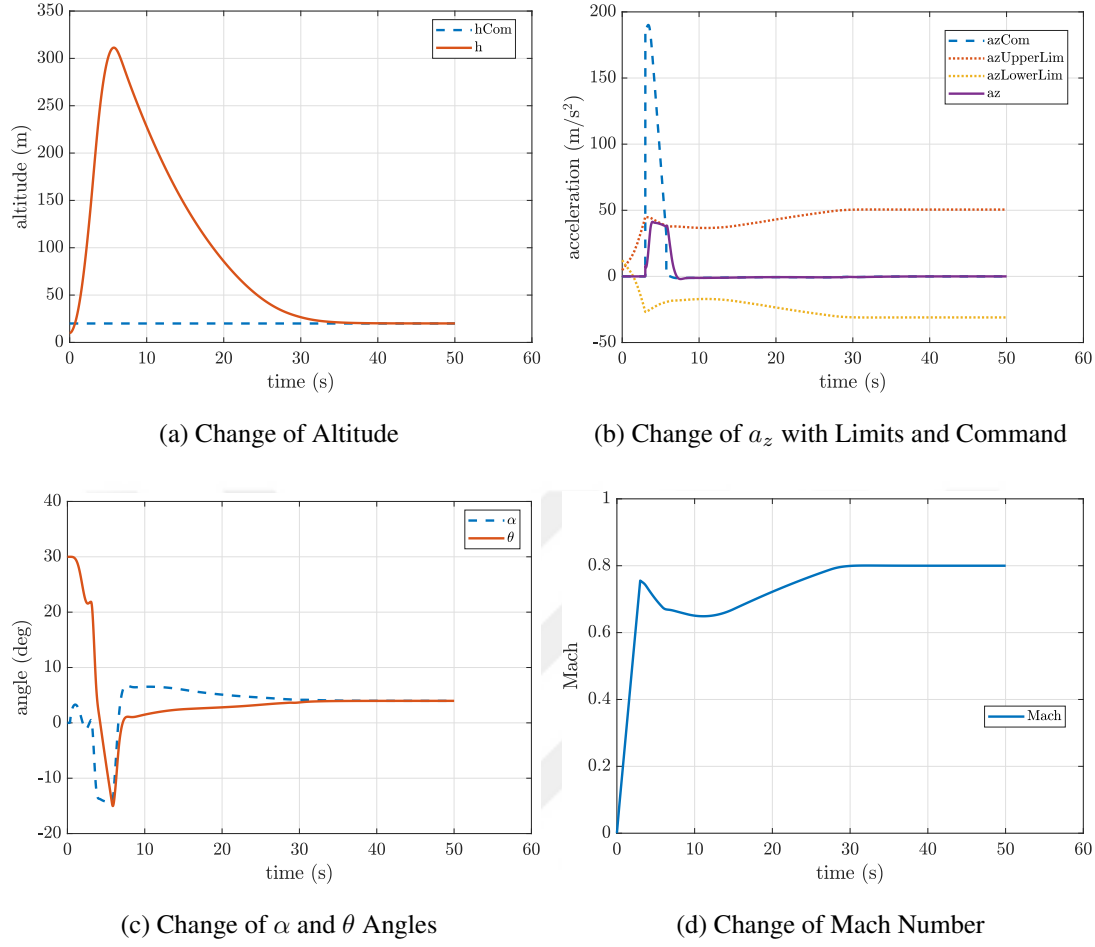


Figure 4.16: Simulation Results with GENEX with Nominal Initial Conditions.

4.5 Modified PD Altitude Control (MPD)

PD altitude controller in section 4.1 can easily fail under non-ideal conditions. When the whole launch envelope is considered, a more generic and robust approach is needed. This study proposes to control, limit, and predict the dive speed using a model to predict missile states and then produce acceleration command to autopilot. If a missile can limit its dive speed at each instant according to the instantaneous conditions, it can achieve a quick but safe descent. Details of this proposed steep descent algorithm, a modified PD (MPD) altitude controller, are as follows.

A basic model is used to predict missile dynamics by taking consideration of autopilot dynamics. As in previous study [10], to construct this model, acceleration autopilot is modeled as a first-order linear model with a time constant. Transfer function of this expression from acceleration command to acceleration is as in equation 4.53.

$$\frac{a(s)}{a_{com}(s)} = \frac{1}{\tau_{st}s + 1} \quad (4.53)$$

Where $\tau = 1/\tau_{st}$. τ_{st} is the settling time of the acceleration autopilot in this case 1.1 seconds. This means $\tau \cong 0.9$. Basic model to be predicted is constructed as in equation 4.54, where h is the missile height, v is the vertical speed and a is the upward acceleration. This model reflects that at any time when altitude controller commands constant acceleration a_{com} , acceleration autopilot will realize that command with a time constant τ . Thus, starting from commanding acceleration, the missile's states can be predicted at a future time with enough precision using this model.

$$\begin{bmatrix} \dot{h} \\ \dot{v} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau \end{bmatrix} \begin{bmatrix} h \\ v \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tau \end{bmatrix} a_{com} \quad (4.54)$$

The missile should dive as fast as possible for an optimum descent phase. However, it should not perform any overshoot when transitioning to commanded altitude. This is only possible with a pullout maneuver performed at the last minute with acceleration just within missile limits. With initial conditions at hand, equation 4.54 can be expressed in frequency domain as a set of equations 4.55.

$$\dot{h} = v \quad (4.55)$$

$$sH - h_0 = V \quad (4.56)$$

$$\dot{v} = a \quad (4.57)$$

$$sV - v_0 = A \quad (4.58)$$

$$s(sH - h_0) - v_0 = A \quad (4.59)$$

$$s^2H - sh_0 - v_0 = A \quad (4.60)$$

$$\dot{a} = -\tau A + \tau A_{com} \quad (4.61)$$

$$sA - a_0 = -\tau A + \tau A_{com} \quad (4.62)$$

$$s(sV - V_0) - A_0 = -\tau A + \tau A_{com} \quad (4.63)$$

$$s^2V - sv_0 - a_0 = -\tau A + \tau A_{com} \quad (4.64)$$

$$s(s^2H - sh_0 - v_0) - a_0 = -\tau A + \tau A_{com} \quad (4.65)$$

$$s^3H - s^2h_0 - sv_0 - a_0 = -\tau A + \tau A_{com} \quad (4.66)$$

Also, by replacing a_{com} in this model with a_{uplim} and considering it as a step input, equation 4.67 is generated.

$$A_{com}(s) = \frac{a_{uplim}}{s} \quad (4.67)$$

For $t \geq 0$, solution of equation 4.54 as a set of equations in frequency domain can be found using inverse Laplace transform. In accordance with equation 4.67, using equation 4.68, time solution of acceleration equation is defined as in equation 4.71.

$$sA(s) - a_0 + \tau A(s) = \tau A_{com}(s) \quad (4.68)$$

$$A(s) = \frac{a_0}{s + \tau} + \frac{\tau}{s + \tau} \frac{a_{uplim}}{s} \quad (4.69)$$

$$A(s) = (a_{uplim}) \frac{1}{s} + (a_0 - a_{uplim}) \frac{1}{s + \tau} \quad (4.70)$$

$$a(t) = a_{uplim} + (a_0 - a_{uplim})e^{-\tau t} \quad (4.71)$$

In the same way, using equation 4.72, velocity solution is defined as in equation 4.75.

$$s^2V(s) - sv_0 - a_0 + \tau A(s) = \tau A_{com}(s) \quad (4.72)$$

$$V(s) = \frac{v_0}{s} + \frac{a_0}{s^2 + \tau s} + \frac{\tau}{s^2 + \tau s} \frac{a_{uplim}}{s} \quad (4.73)$$

$$V(s) = (v_0 + \frac{a_0 - a_{uplim}}{\tau}) \frac{1}{s} + (a_{uplim}) \frac{1}{s^2} + (\frac{-a_0 + a_{uplim}}{\tau}) \frac{1}{s + \tau} \quad (4.74)$$

$$v(t) = (v_0 + \frac{a_0 - a_{uplim}}{\tau}) + (a_{uplim})t + (-a_0 + a_{uplim})e^{-\tau t} \quad (4.75)$$

Lastly, altitude of the missile with respect to time is solved as in equation 4.79.

$$s^3H(s) - s^2h_0 - sv_0 - a_0 + \tau A(s) = \tau A_{com}(s) \quad (4.76)$$

$$H(s) = \frac{h_0}{s} + \frac{v_0}{s^2} + \frac{a_0}{s^3 + \tau s^2} + \frac{\tau}{s^3 + \tau s^2} \frac{a_{uplim}}{s} \quad (4.77)$$

$$H(s) = (h_0 + \frac{-a_0 + a_{uplim}}{\tau^2}) \frac{1}{s} + (v_0 + \frac{a_0 - a_{uplim}}{\tau}) \frac{1}{s^2} \dots \dots + (a_{uplim}) \frac{1}{s^3} + (\frac{a_0 - a_{uplim}}{\tau^2}) \frac{1}{s + \tau} \quad (4.78)$$

$$h(t) = (h_0 + \frac{-a_0 + a_{uplim}}{\tau^2}) + (v_0 + \frac{a_0 - a_{uplim}}{\tau})t \dots \dots + (a_{uplim}) \frac{t^2}{2} + (\frac{a_0 - a_{uplim}}{\tau^2})e^{-\tau t} \quad (4.79)$$

Initial conditions for t_0 , which are a_0 , v_0 , h_0 , are known together with the acceleration limit a_{uplim} and acceleration autopilot dynamics parameter τ is known from autopilot design. Thus, with the equations 4.71, 4.75 and 4.79, for $t \geq 0$, states of the system can be predicted with the assumption of missile will pull acceleration in upper limit. This model prediction is accurate enough for controller to work for the descent phase of the sea-skimming missile. Proposed MPD with prediction model is as in Figure 4.17.

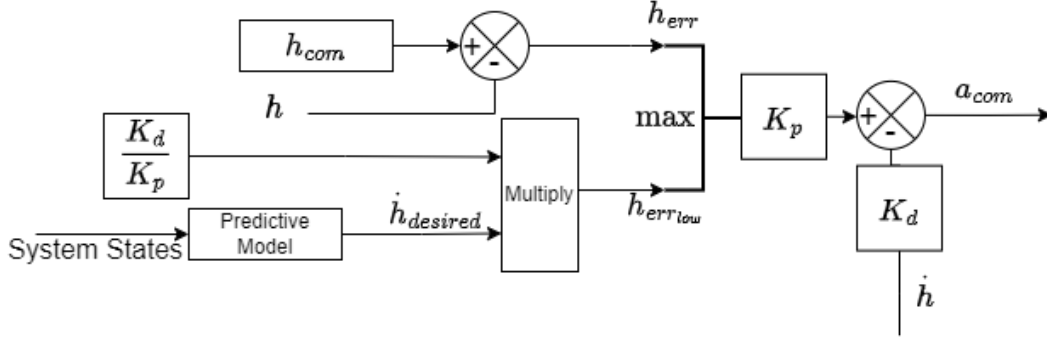


Figure 4.17: Block Diagram of MPD Altitude Control System.

In Figure 4.18, flowchart of the proposed steep descent algorithm is shown. This flowchart explains how the proposed method works. h_{com} is already decided before missile launched from ship. Pitch control starts with separation of booster from the missile. For every time step t_i , missile states h_0 , v_0 , a_0 and a_{lim} are fed to the algorithm. Then, to predict the pullout time, t_p , equation 4.75 is solved by using Newton-Raphson method for $v(t_p) = 0$, knowing that at the end of the pullout maneuver vertical speed will be zero. If solution cannot be found in limited iteration then it can be said that missile has positive upward velocity and it will continue ascending further with this model. Hence, there is no need to limit dive speed of the missile. As a result of this, predetermined $\dot{h}_{desired}$ which is -100 m/s is set. In the case of an existence of a solution, $h(t_p)$ is calculated by using equation 4.79 and compared with the h_{com} . If $h(t_p)$ is not higher than the h_{com} , it means that the system has a tendency to overshoot. Thus, precautions are needed and $\dot{h}_{desired}$ is set to zero. In the case of, $h(t_p)$ is higher than the h_{com} , then optimum dive speed is calculated by using equation 4.80. Determining this dive speed limit is the one main outcomes of this algorithm.

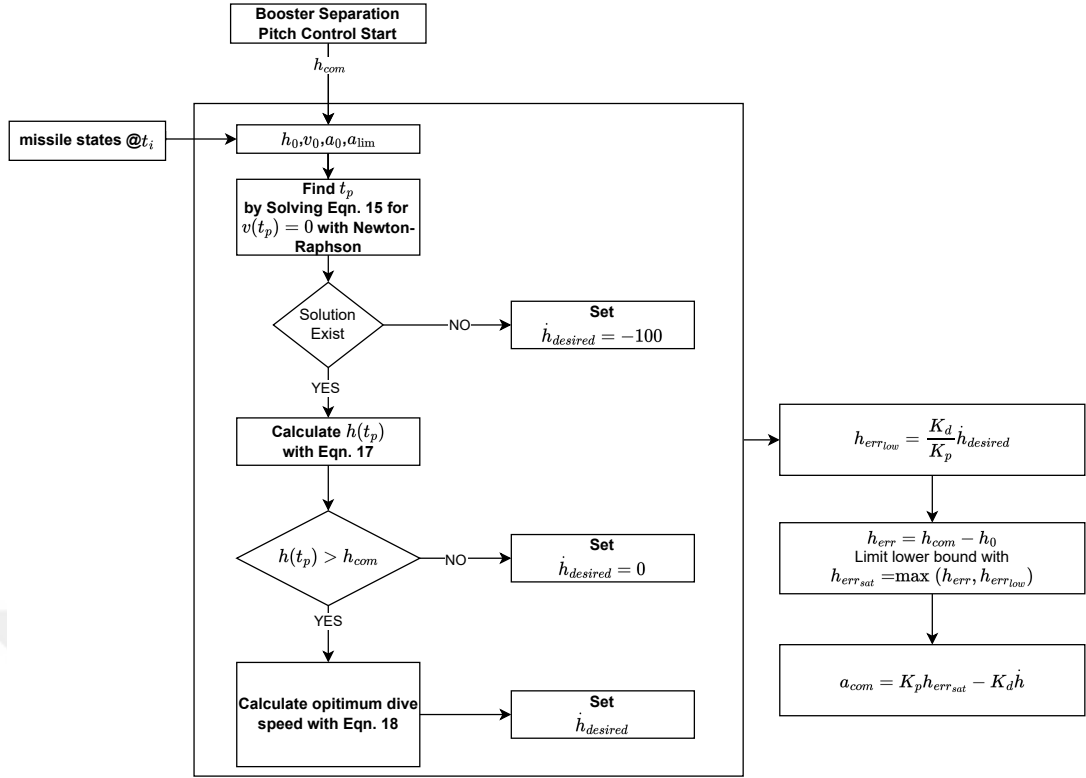


Figure 4.18: Proposed Steep Descent Algorithm's Flowchart.

Equation 4.79 can be defined in reverse for a certain time with the assumption of pullout maneuver ends right at the desired altitude h_{com} . Desired dive speed $\dot{h}_{desired}$ is calculated using equation 4.80.

$$\begin{aligned} \dot{h}_{desired} = v_0 = & \left(h_{com} - \left(h_0 + \frac{-a_0 + a_{uplim}}{\tau^2} \right) - (a_{uplim}) \frac{t_p^2}{2} \dots \right. \\ & \left. \dots - \left(\frac{a_0 - a_{uplim}}{\tau^2} \right) e^{-\tau t} \right) / t_p - \left(\frac{a_0 - a_{uplim}}{\tau} \right) \end{aligned} \quad (4.80)$$

Since system only has an altitude controller and not an altitude rate controller, one way to control altitude rate is; saturating the altitude error such that proportional and derivative parts of the PD altitude controller cancels each other and set acceleration command to zero. This is achieved as follows.

First of all, guidance method is defined as in equation 4.81.

$$a_{com} = K_p(h_{err}) - K_d\dot{h} \quad ; \quad h_{err} = h_{com} - h_0 \quad (4.81)$$

Then lower bound of the altitude error $h_{err_{low}}$ is calculated by using guidance gains and desired altitude rate as in equation 4.82.

$$h_{err_{low}} = \frac{K_d}{K_p}\dot{h}_{desired} \quad (4.82)$$

After that, altitude error is saturated as in equation 4.83.

$$h_{err_{sat}} = \max(h_{err}, h_{err_{low}}) \quad (4.83)$$

Finally, with this conscious introduction of the non-linearity, controller method is rearranged as in equation 4.84.

$$a_{com} = K_p h_{err_{sat}} - K_d\dot{h} \quad (4.84)$$

As a result, missile dive speed cannot exceed $\dot{h}_{desired}$, so the altitude rate is controlled indirectly.

It should be noted that, controller gains K_p and K_d are taken as in 4.1 but it is seen from the analysis that even with faster controller dynamics MPD algorithm works. This means that faster altitude control can be taken advantage of when MPD is not limiting the $\dot{h}$.

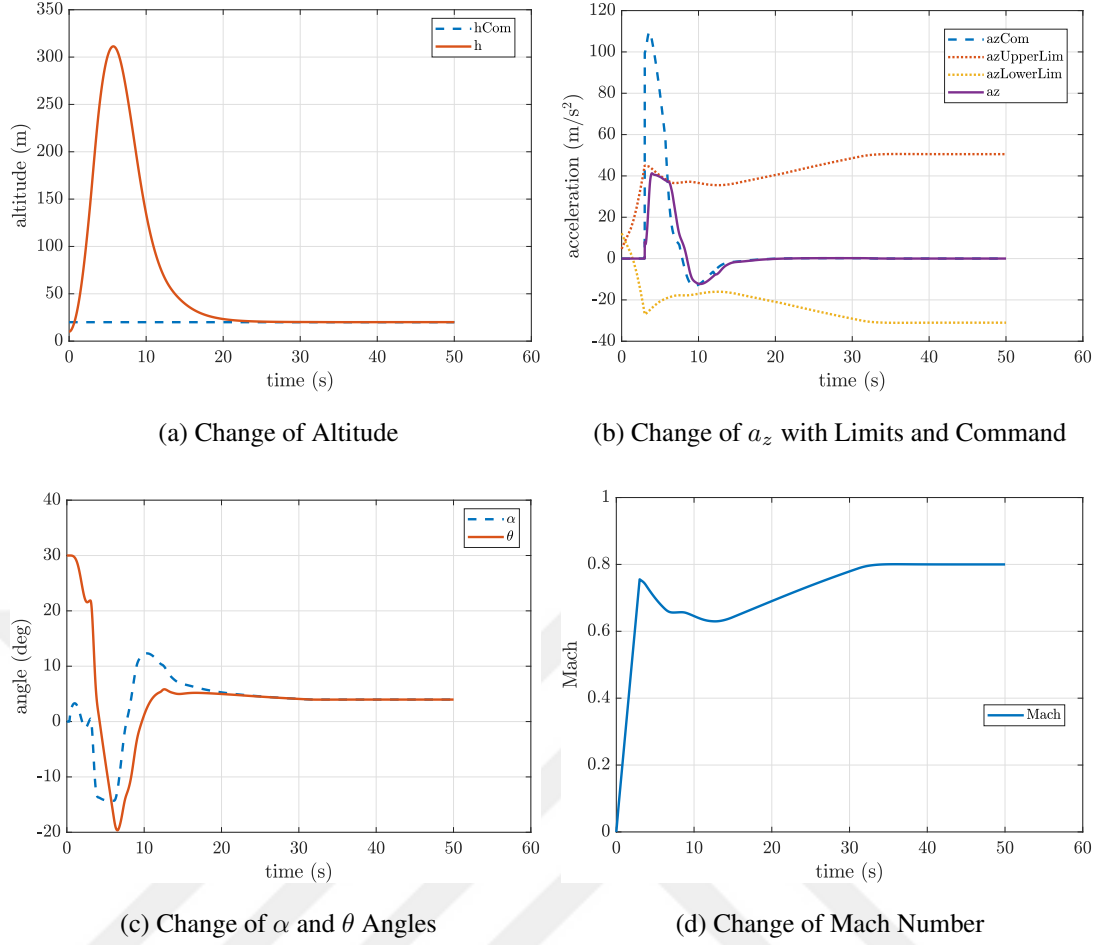


Figure 4.19: Simulation Results with MPD with Nominal Initial Conditions.

In Figure 4.19, missile climbs above 300 m. At the peak altitude, guidance methods based on MPD produce maximum positive acceleration commands along the missile's body axis, resulting in a pronounced downward acceleration. To prevent the missile's vertical speed from dangerously escalating, the MPD controller intentionally moderates the vertical velocity during the dive phase, successfully executing the pullout maneuver.

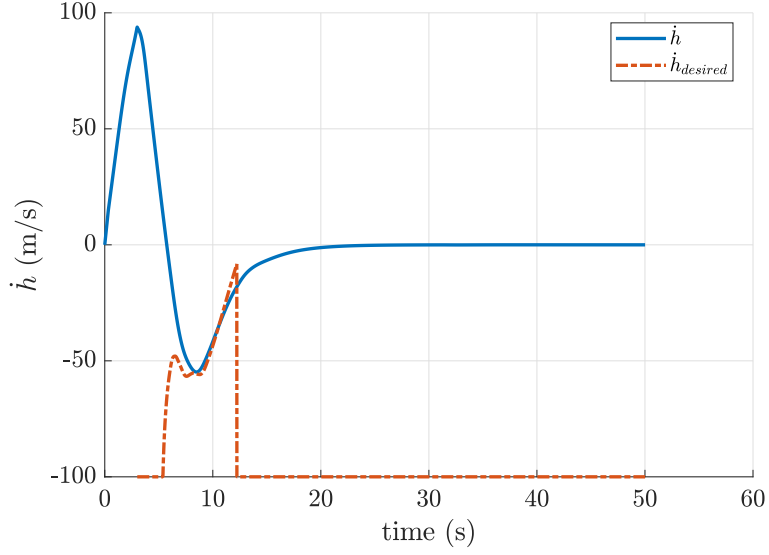


Figure 4.20: $\dot{h}$ and $\dot{h}_{desired}$ Results with MPD with Nominal Initial Conditions.

Figure 4.20 demonstrates the missile's $\dot{h}$ profile under the MPD controller, providing a clearer interpretation of the algorithm's effectiveness. Throughout the dive phase, the MPD controller continually determines the optimal dive speed, $\dot{h}_{desired}$, computes the error saturation limit, and subsequently applies the guidance method. The command-pursuit profile seen between $6 < t < 13$ seconds is the precious outcome of this algorithm which constitutes the proposed steep descent approach with MPD altitude control.

CHAPTER 5

ANALYSIS OF DESCENT ALGORITHMS

5.1 Problem Definitions for Analysis

This section aims to introduce several significant real-world problems encountered in the domains of ship motion, the misalignment of booster thrust vectors, and the effects of different environmental conditions. Results will be diversified into three main failure modes: failure, overshoot and not converged. The failure mode is the one where the missile hits the surface of the water; overshoot mode is the one where the missile gets down below the commanded altitude, and not the converged mode is the one where the missile is not at the desired altitude in the desired simulation time, which is 50 seconds. Also, one more time parameter will be presented which is the average convergence time. It is calculated as time when missile altitude is within the 5% of the altitude command and stays in that threshold without overshooting. Since the analysis is conducted using the full factorial method, wherein all possible combinations for each variable are tested, certain initial conditions may fall outside the missile launch envelope. As the focus of this thesis does not encompass the launch envelope, instances where the missile hits the water before booster separation in all five methods will not be classified as failed runs. Instead, they will be denoted as instances outside the launch envelope.

5.1.1 Platform Motion

Initial conditions of the missile largely depend on the platform that the missile is launched from. Since the platform in the present study is a ship, its motion on the surface directly affects missile launch conditions. Especially, rough sea conditions

result in large ship movements. Figure 5.1 displays both translational and rotational ship motions. In this study, only motion about x-axis of the ship is considered. It causes missile's launch angle θ_0 and pitch angular rate q_0 to deviate from the desired nominal condition.

Table 5.1: Analysis Parameters for Platform Motion.

Parameter	Value
$h_0(\text{m})$	10
θ_0 (deg)	10, 20, 30, 40, 50
q_0 (deg/s)	-20, -10, 0, 10, 20
θ_b (deg)	0
z_b (m)	0
$T(^{\circ}\text{C})$	15
V_w (m/s)	0

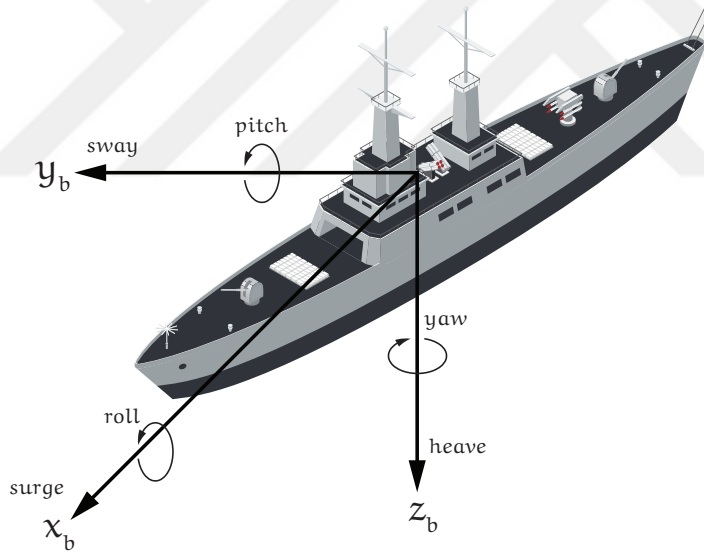


Figure 5.1: Ship Principal Axes.

Standard motions of typical ships are defined in [41] for different sea states. As a challenging launch condition, sea state 6 is assumed and roll motion of the platform is presumed to be ± 20 deg roll angle and ± 20 deg/s roll rate. This motion will directly affect the missile pitch motion initialization. Thus, analysis parameters happens to be as of in Table 5.1.

5.1.2 Booster Thrust Vector Misalignment

The booster thrust axis was nominally aligned to pass through the missile center of gravity. However, because of assembly or production errors, thrust vector may be misaligned. Thus, the effect of thrust vector misalignment on the missile throughout the boost phase is a major concern that must be examined. θ_b is the misalignment angle of the thrust vector with the body x-axis of the missile. z_b is the misalignment of the thrust vector from the center of the body x-axis of the missile along the body z-axis of the missile. Both misalignment types are expressed in Figure 5.2.

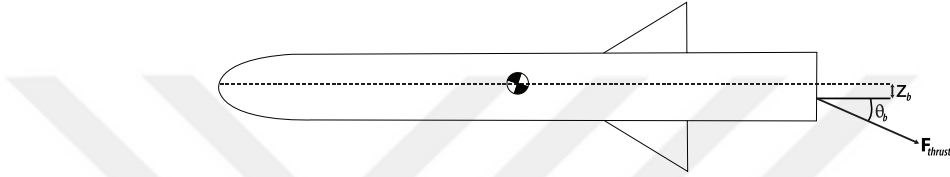


Figure 5.2: Thrust Vector Misalignment Diagram.

Thrust vector misalignment for two different alignment issues calculated as below.

$$\vec{F}_b = F'_b \begin{bmatrix} \cos\theta_b \\ 0 \\ -\sin\theta_b \end{bmatrix} \quad (5.1)$$

Where F'_b is amount of thrust booster can provide, $\vec{F}_b$ is the thrust vector.

$$\vec{M}_b = (-\vec{l}_{wb} - \vec{c}g + z_b(0, 0, 1)) \times \vec{F}_b \quad (5.2)$$

Where $\vec{M}_b$ is pitch moment vector, $\vec{l}_{wb}$ is missile length vector with booster, $\vec{c}g$ is the center of gravity vector. By using these calculations thrust vector misalignment is added to the simulation. Maximum misalignment values for θ_b and z_b are assumed as 0.3 deg and 0.001 m. Table 5.2 shows the analysis parameters.

Table 5.2: Analysis Parameters for Booster Thrust Vector Misalignment.

Parameter	Value
$h_0(\text{m})$	10
θ_0 (deg)	30
q_0 (deg/s)	0
θ_b (deg)	-0.3, -0.25,...0 ... 0.25, 0.3
z_b (m)	-0.001, -0.0005, 0, 0.0005, 0.001
$T(^{\circ}\text{C})$	15
V_w (m/s)	0

5.1.3 Effects of Different Environmental Conditions

Different environmental conditions, such as ambient temperature and wind, will be introduced, and the effects of these parameters on the propulsion system and aerodynamic parameters will be investigated. For the sake of simplicity of the analysis, the effect of wind on the sea, defined as sea states, is not considered. Thus, because of the wind or temperature, platform motion is not added to the environmental condition analyses.

In this study, wind from two different directions and magnitudes is modeled. Knowing the air speed erroneously results in lower acceleration limits in autopilot and higher or lower angles of attack. Also, different ambient temperature changes performance of the booster as it is explained in 2.6.1 and shown in Figure 2.15. Table 5.3 shows the analysis parameters.

Table 5.3: Analysis Parameters for Effects of Different Environmental Conditions Analysis.

Parameter	Value
$h_0(\text{m})$	10
$\theta_0 (\text{deg})$	30
$q_0 (\text{deg/s})$	0
$\theta_b (\text{deg})$	0
$z_b (\text{m})$	0
$T(^{\circ}\text{C})$	-15, 15, 45
$V_w (\text{m/s})$	-20, 0, 20

5.2 Results and Comparison

Firstly, for all methods, simulations are done with the nominal initial conditions defined in Table 4.1 with the altitude command, h_{com} , of 20 m, and plotted below Figure 5.3.

It can be seen from Figure 5.3a, PD and MPD methods are the fastest to descend, followed by GENEX, BPNG and IVG. Figure 5.3b, also supports the results since more of the available acceleration limit from the angle of attack utilized by PD and MPD. The main difference is that MPD tries to check it states thus determines the $\dot{h}_{lim}$. The amount of acceleration missile utilized at the initial stage of the descent after booster separation is one of the significant determinants of the descent performance. Figure 5.3c shows that PD and MPD controllers are almost at the limit set by the autopilot while other methods, after the initial unpowered flight, easier on the angle of attack limits. Although it is not modelled, high angle of attacks also results in low Mach numbers which causes problems for the turbojet engine to start or operate.

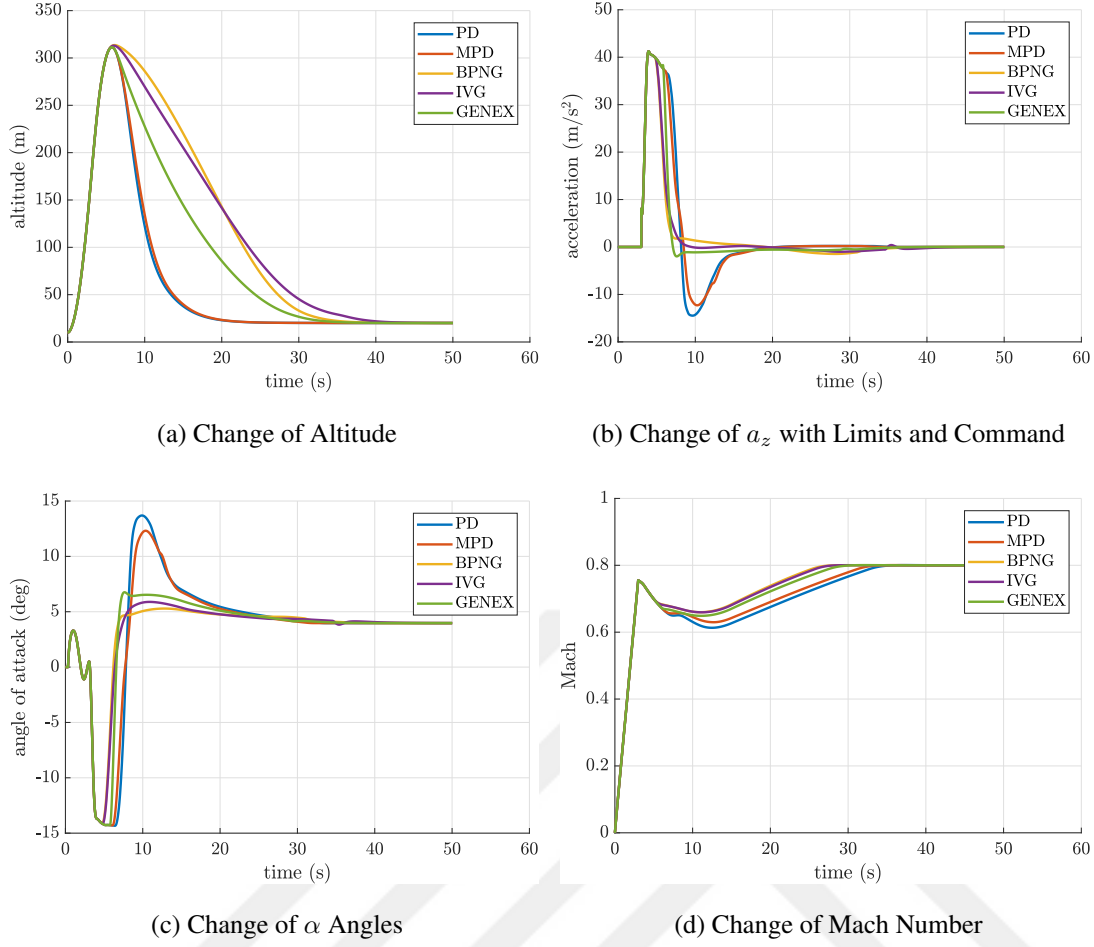


Figure 5.3: Simulation Results with All Methods Together with Nominal Conditions.

5.2.1 Platform Motion Results

Total of 25 conditions are simulated for the analysis of platform motion under different initial θ_0 and q_0 , and results are tabulated in Table 5.4. From the results it is concluded that 3 analysis conditions are outside of the launch envelope.

Table 5.4: Batch Results for Platform Motion Analysis.

Method	Failure	Overshoot	Not Converged	Avg. Convergence Time
PD	41%	4.5%	0	19.8 sec
MPD	0	0	0	25.2 sec
BPNG	0	0	0	38.1 sec
IVG	0	0	18%	39.9 sec
GENEX	0	0	0	35.2 sec

Table 5.4 illustrates that only the PD controller failed. For PD method, almost all of the failed runs are the ones that where missile comes from high altitudes with low Mach number. PD altitude controller cannot achieve a successful pullout maneuver and missile ditches into sea. The average convergence time indicates the PD method as the fastest, closely followed by the MPD method. Subsequently, GENEX, BPNG, and IVG follow in that order. Since IVG method is slowest, it is also reflected on 4 not converged runs. For a more comprehensive understanding, runs 96-100, involving all five methods, have been selected for review. The initial conditions of these simulations are detailed in Table 5.5.

Table 5.5: Batch Simulation Run 96-100 Initial Conditions

h_0 (m)	θ_0 (deg)	q_0 (deg/s)	θ_b (deg)	z_b (m)	$T(^{\circ}\text{C})$	V_w (m/s)
10	40	20	0	0	15	0

Figure 5.4a displays the simulation outcomes corresponding to the initial conditions specified in Table 5.5, wherein the PD method failed, and the IVG method did not reach the commanded altitude within 50 seconds. The PD altitude controller was unable to execute a successful pullout maneuver as explained above, leading the missile to impact the sea. Conversely, the MPD algorithm managed successful maneuvers without overshooting. However, due to its rapid response, it resulted in higher angles of attack, delaying the convergence to the commanded Mach number due to the stringent pullout maneuver. The BPNG, IVG, and GENEX methods steadily descend to the designated altitude while experiencing a gradual increase in speed, although they do not fully utilize the missile's capabilities.

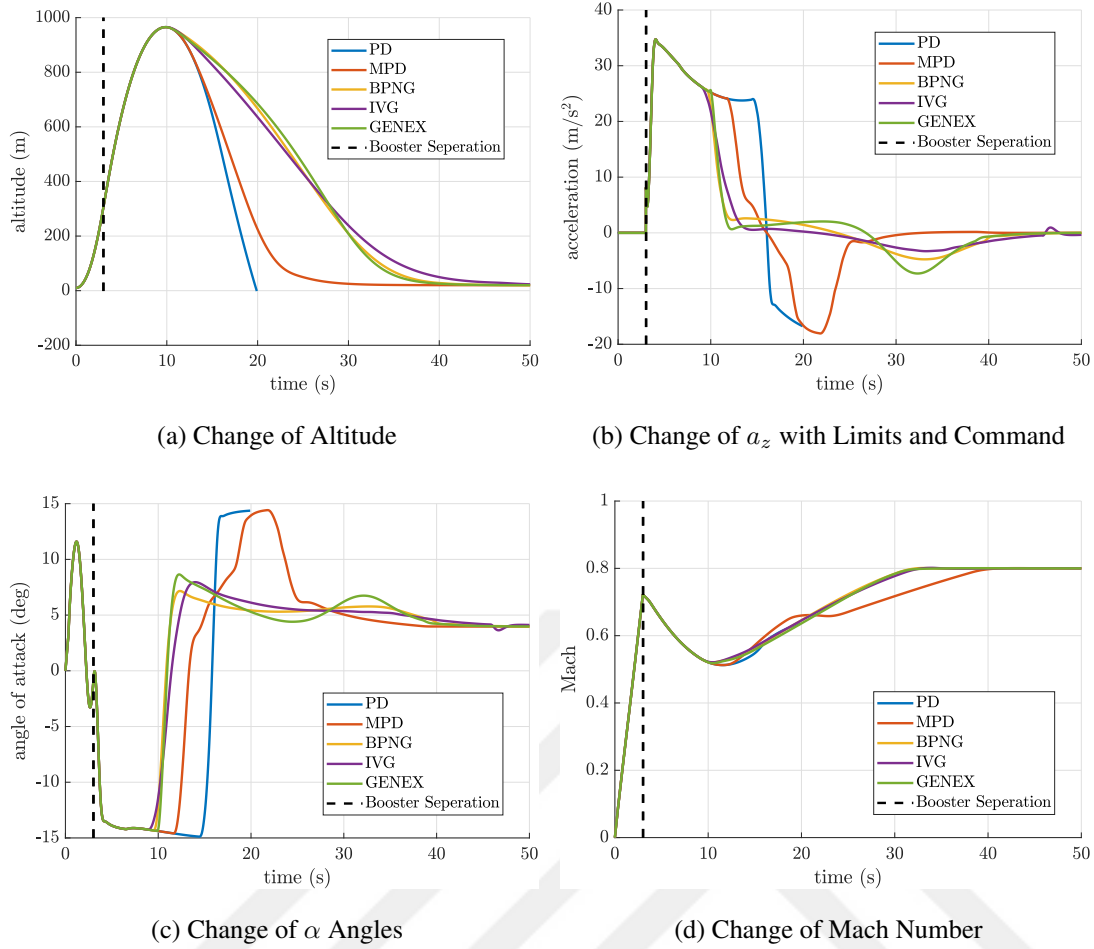


Figure 5.4: Simulation Results for the Initial Conditions Set in Table 5.5.

5.2.2 Booster Thrust Vector Misalignment Results

All 65 conditions, total of 325 simulation runs, are simulated for the analysis of the booster thrust vector misalignment under different initial θ_b and z_b . From the results it is concluded that 17 analysis conditions are outside of the launch envelope.

Table 5.6: Batch Results for Booster Thrust Vector Misalignment Analysis.

Method	Failure	Overshoot	Not Converged	Avg. Convergence Time
PD	62.5%	4%	0	18.7 sec
MPD	20.8%	2%	2%	27.3 sec
BPNG	16.7%	4%	16.7%	39.2 sec
IVG	12.5%	2%	33%	39.5 sec
GENEX	14.6%	2%	14.6%	36.1 sec

Table 5.6 shows that, booster thrust vector misalignment affects every method and all methods are failed, did overshoot and not converged in time. Although IVG method has the least failures, it is also the slowest. Detailed analysis shows that guidance methods are more robust to the changes with respect to MPD since they do not rely on the missile model to be correct and thrust vector misalignment may result in performance of the missile to behave out of ordinary. However, total of successful simulation run number still the highest for MPD method and it is still the second fastest.

Table 5.6 demonstrates that misalignment of the booster thrust vector affects all methods, leading to failures, overshooting, and failure to converge within the specified time. Despite IVG method exhibiting the fewest failures, it is also the slowest. A detailed analysis indicates that guidance methods exhibit greater robustness to thrust vector misalignment compared to MPD, as they do not depend on the accuracy of the missile model. Since, thrust vector misalignment can cause the missile's performance to deviate from the expected behavior. However, the total count of successful simulation runs remains highest for the MPD method, which also ranks as the second fastest.

Table 5.7: Batch Simulation Run 71-75 Initial Conditions

h_0 (m)	θ_0 (deg)	q_0 (deg/s)	θ_b (deg)	z_b (m)	$T(^{\circ}\text{C})$	V_w (m/s)
10	30	0	-0.2	0.001	15	0

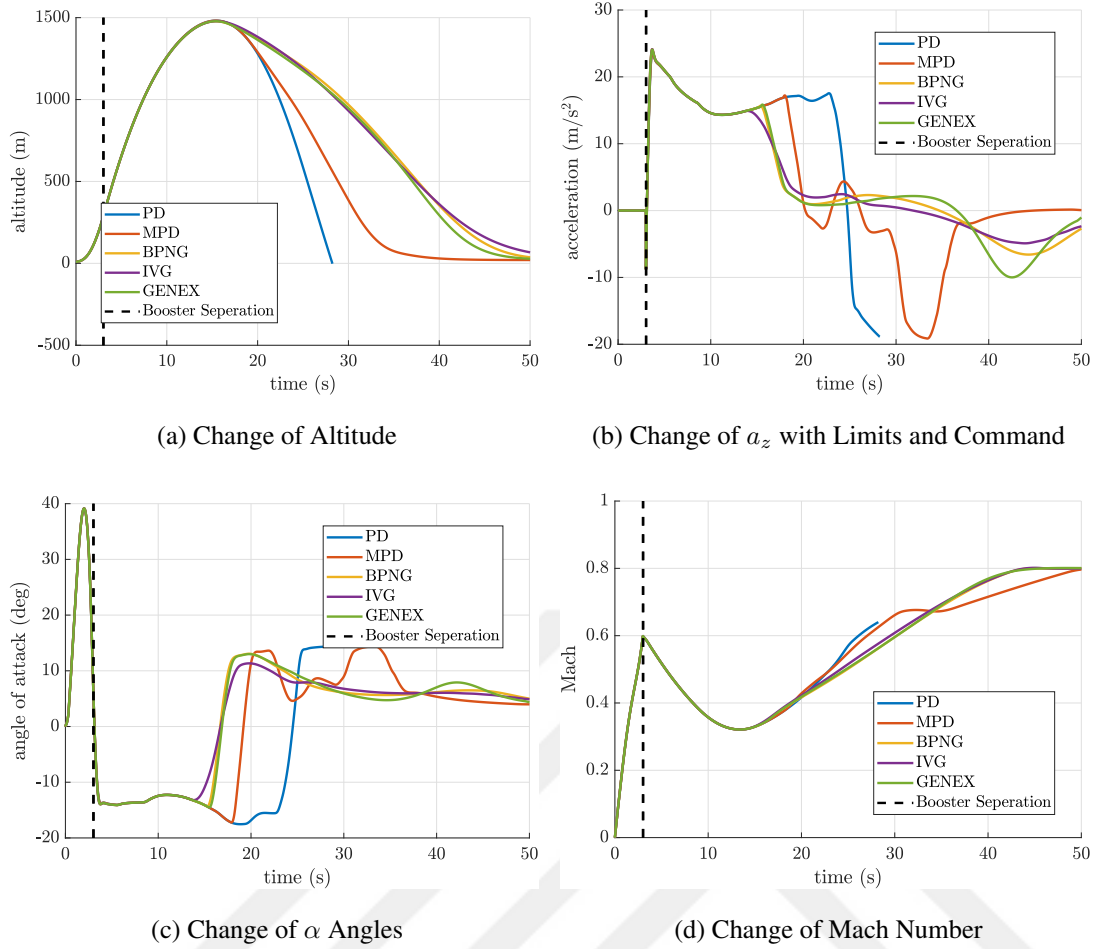
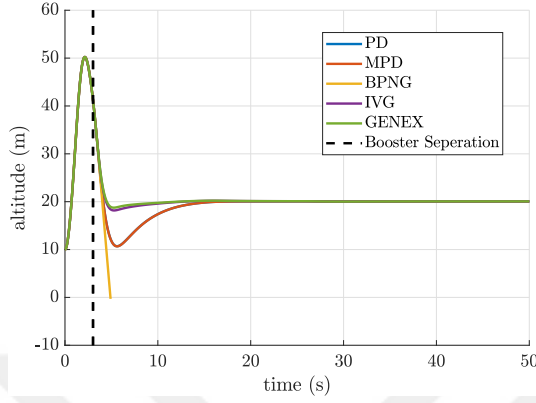


Figure 5.5: Simulation Results for the Initial Conditions Set in Table 5.7.

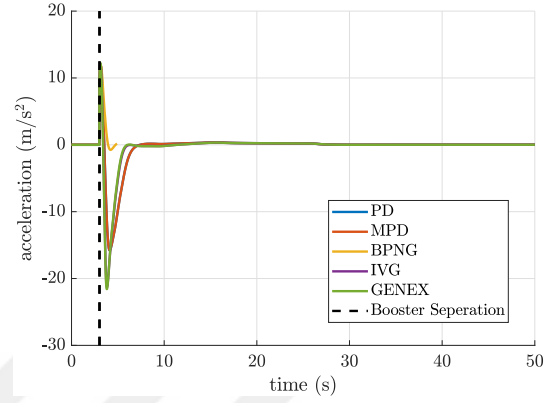
Figure 5.5 shows batch results which PD hit the water and IVG is not converged. From Figures 5.5b and 5.5c, it can be seen that flight profile is somewhat chaotic with both misalignment types exist. Angle of attach and acceleration profile of the guidance methods are linear than that of PD and MPD. MPD also shows over the limit angles of attack but successfully concludes the maneuver.

Table 5.8: Batch Simulation Run 241-245 Initial Conditions

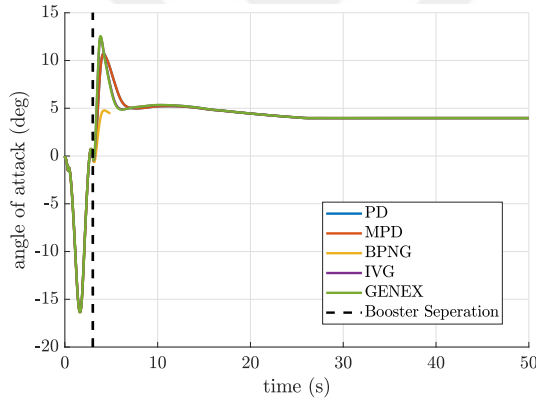
h_0 (m)	θ_0 (deg)	q_0 (deg/s)	θ_b (deg)	z_b (m)	T (°C)	V_w (m/s)
10	30	0	0.15	0.0005	15	0



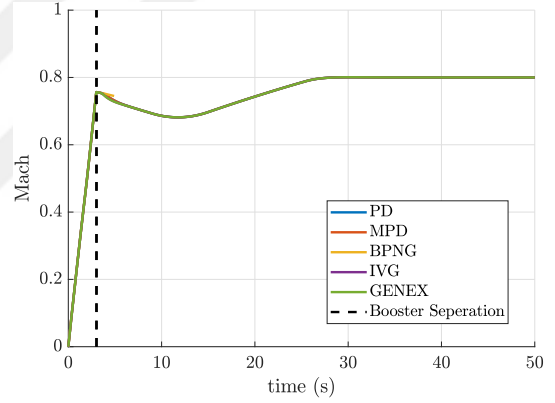
(a) Change of Altitude



(b) Change of a_z with Limits and Command



(c) Change of α Angles



(d) Change of Mach Number

Figure 5.6: Simulation Results for the Initial Conditions Set in Table 5.8.

In Figure 5.6, the initial altitude gained from the booster appears lower compared to Figure 5.5. In all methods, there is an overshoot, while the BPNG method fails. Notably, the PD and MPD exhibit greater overshoot than the IVG and GENEX. The substantial overshoot of the MPD suggests that the algorithm miscalculated the necessary acceleration for the pullout maneuver due to the altitude error and downward speed after booster separation. On the contrary, the GENEX and IVG demonstrate a more robust response to the low initial altitude.

5.2.3 Effects of Different Environmental Conditions Results

Total of 9 conditions are simulated for the analysis of environmental conditions with the conditions shown in Table 5.3.

Table 5.9: Batch Results for Effects of Different Environmental Conditions Analysis.

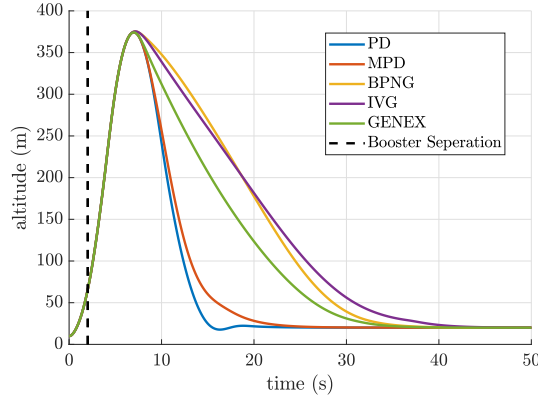
Method	Failure	Overshoot	Not Converged	Avg. Convergence Time
PD	22%	11%	0	22.2 sec
MPD	0	0	0	23.6 sec
BPNG	0	0	0	37.5 sec
IVG	0	0	0	42.0 sec
GENEX	0	0	0	35.2 sec

From Table 5.9 it can be seen that only PD controller failed. Detailed examination of the runs shows that 2 runs that are failed and 1 run with overshoot when wind is from behind the missile, which is $V_w = -20$ (m/s). Also, the average convergence time again shows that the PD method is the fastest, with the MPD method a close second. They are followed by GENEX, BPNG and IVG respectively. To get better understanding runs from 1-5 selected for the review (total of 5 methods). Initial conditions of these simulations are as in Table 5.10.

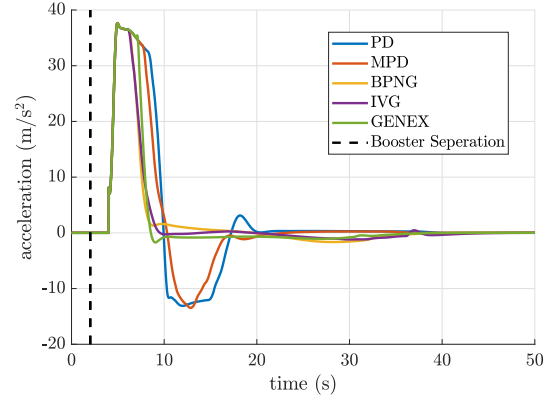
Table 5.10: Batch Simulation Run 1-5 Initial Conditions

h_0 (m)	θ_0 (deg)	q_0 (deg/s)	θ_b (deg)	z_b (m)	$T(^{\circ}\text{C})$	V_w (m/s)
10	30	0	0	0	-15	-20

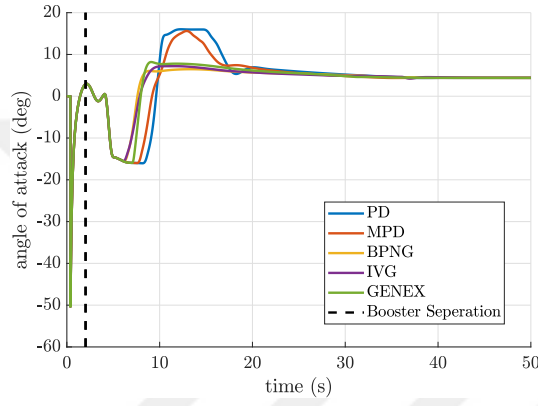
Figure 5.7 demonstrates that a colder ambient temperature extends the booster phase. High angles of attack during the boost phase are evident. The PD method exhibits overshoot, unlike the other methods. Despite the PD method attempting to prevent failure by sacrificing speed, it still experiences overshoot. This occurs due to the increased downward speed caused by the acceleration command. Consequently, an overshoot still occurs despite applying maximum acceleration to raise the nose within the autopilot's constraints.



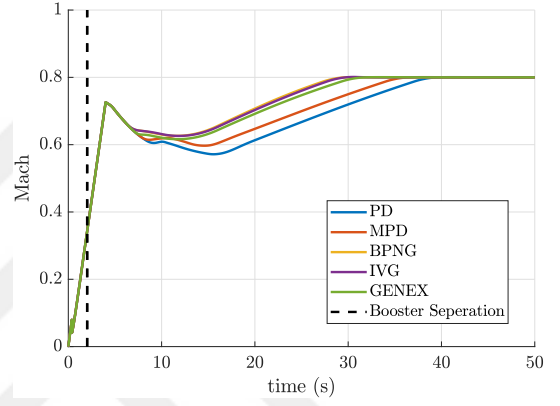
(a) Change of Altitude



(b) Change of a_z with Limits and Command



(c) Change of α Angles



(d) Change of Mach Number

Figure 5.7: Simulation Results for the Initial Conditions Set in Table 5.10.

Regarding the failed simulation runs with the PD method, the initial ascent of the missile with the booster results in higher altitudes. Therefore, the missile experiences prolonged dives with higher acceleration commands, which results in higher downward speed, leading to impact with the water.

5.2.4 Final Results

To comprehensively analyze the performance of the methods, all problem definitions were utilized to generate a more comprehensive analysis. Table 5.3 displays the initial conditions for the variables. Similar to all analyses conducted in this thesis, these simulation runs also employ the full factorial method. A total of 5625 conditions are simulated for analyzing each method, resulting in 28125 simulation runs. The results indicate that 1739 analysis conditions lie outside the launch envelope of the missile.

Table 5.11: Analysis Parameters for All Effects Together.

Parameter	Value
$h_0(\text{m})$	10
$\theta_0 (\text{deg})$	10, 20, 30, 40, 50
$q_0 (\text{deg/s})$	-20, -10, 0, 10, 20
$\theta_b (\text{deg})$	-0.3, -0.15, 0, 0.15, 0.3
$z_b (\text{m})$	-0.001, -0.0005, 0, 0.0005, 0.001
$T(^{\circ}\text{C})$	-15, 15, 45
$V_w (\text{m/s})$	-20, 0, 20

The table depicted in Table 5.12 demonstrates results akin to those obtained from the separate analyses. The majority of failures are still attributed to the PD method. The average convergence time, computed from successful simulation runs, indicates a notably faster performance for the PD method due to its lower number of such runs compared to other methods. The MPD method consistently ranks second across all failure categories, albeit with slight differences. Particularly in the 'not converged' section, the MPD method exhibits promising results. While the IVG method has the fewest failure runs, it also records the highest count of instances where convergence was not achieved. Following the MPD method, GENEX displays the most promising performance metrics. Additionally, the average distance traveled is provided in Table 5.12. It is evident that time and position are directly proportional due to a constant Mach speed command applied throughout simulation runs. Notably, a faster descent results in less distance traveled before reaching the sea-skimming level. As a result, missile remains less visible to the enemy radar.

Table 5.12: Analysis Results for All Effects Together.

Method	Failure	Overshoot	Not Converged	Avg. Conv. Time	Avg. Dist
PD	64.5%	4.4%	0	19.7 sec	4042.6 m
MPD	28.5%	3%	1.4%	28.2 sec	5661.2 m
BPNG	24.3%	2.8%	18.6%	39.4 sec	9039.6 m
IVG	20.7%	1.9%	35.3%	39.7 sec	9341.4 m
GENEX	22.2%	1.9%	17.1%	36.3 sec	8138.4 m

To provide a clearer visual representation of the results, Figure 5.8 was obtained. Similar observations mentioned in the preceding paragraph can also be inferred from the figure. In comparison to the PD method, the success rates of other methods show an increase.

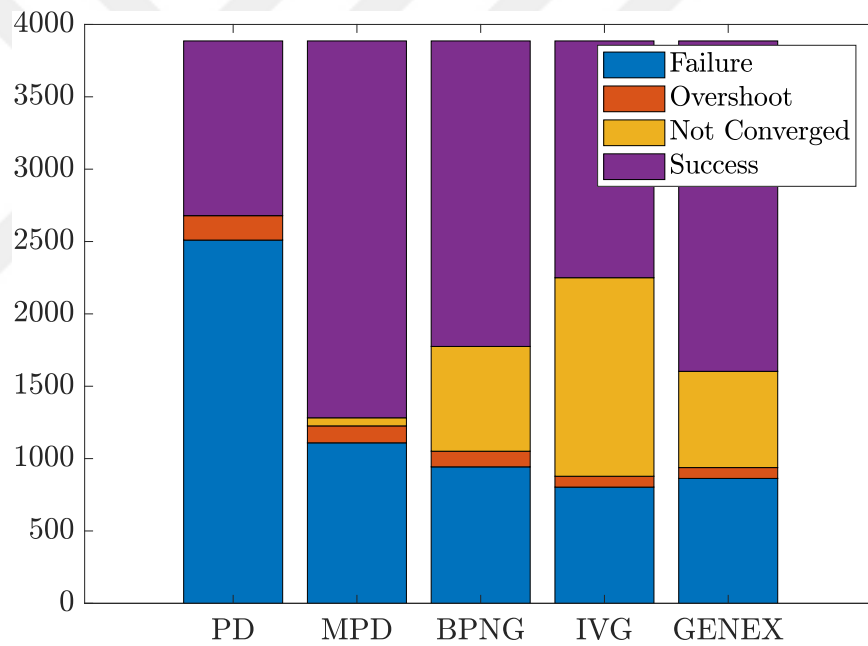


Figure 5.8: Analysis Results for All Effects Together for All Methods.



CHAPTER 6

CONCLUSIONS

In this thesis, safe and quick descent to the sea-skimming altitude of an anti-ship cruise missile considered. Initially, a 3-degree-of-freedom (3-DOF) simulation environment for the missile was established, encompassing aerodynamics, CAS dynamics, turbojet, booster, and environmental modeling. Utilizing the outcomes of the aerodynamic analysis, an acceleration autopilot design was developed. Furthermore, different types of descent algorithms were devised. The classical PD altitude control method was employed, alongside a novel approach that involves modifying the PD altitude controller through the limitation of altitude rate using model prediction. Recognizing that addressing the issue encompasses not only control perspectives but also trajectory shaping and the control of the final impact angle, specific guidance methods were implemented within the scope of this study. The analysis and discussion of these diverse descent algorithms were conducted, drawing insights from real-world problems.

Upon comparing the methods under nominal conditions, it becomes evident that utilizing PD and MPD methods for descent proves to be the swiftest approaches. In terms of guidance methods, the rankings consistently depict GENEX, BPNG, and IVG in order of descending speed, with GENEX being the fastest and IVG the slowest. Notably, when focusing solely on the safe descent aspect of the problem, the guidance algorithms also demonstrate a high level of success. Furthermore, it is notable that guidance algorithms typically necessitate lower acceleration utilization, leading to a slower response but mitigating drastic changes in the angle of attack.

Among the three distinct real-world problems examined in this thesis, the investigation reveals that booster thrust vector misalignment stands out as the scenario where

the majority of failures occur. This occurrence primarily stems from the booster phase, recognized as one of the more nonlinear stages within the flight profile explored in this study. Subsequent to booster separation, the missile's states hold critical significance for ensuring a safe and expedited descent. Moreover, the impact of wind dynamics, particularly in altering the average descent time, was notably observed. Additionally, the platform's motion highlighted that even when the booster fails to provide the missile with adequate elevation due to variations in the initial launch angle and angular velocity, for the runs stays in the assumptions accepted in this study, responses remained robust, although the occurrence of PD failures increased.

The results obtained from the analysis indicate that the proposed MPD method demonstrates promising outcomes compared to classical or other guidance methods, showcasing a steeper descent rate and robustness in addressing real-world problems. Nevertheless, it is important to note that while the MPD method does not noticeably slow down simulation analysis, the sheer volume of operations required in each simulation step demands greater computational power.

For this topic, following considerations can be investigated for future work:

- Missile model can be extended to 6-DOF simulation, since all real world presented also affects other channels such as roll and yaw. Coupling effects among various flight dynamics also can be investigated.
- Obtained aerodynamic data assumed to be correct thus autopilot always behaves as intended. Performance of these descent algorithms can be investigated with aerodynamic uncertainty in these parameters.
- Feedback signals from the avionics such as IMU, radar altimeter and control actuator system assumed perfect. These avionics can be modelled and performance of the descent algorithms can be investigated.
- Since this study strictly investigates launch/boost and descent phases of the missile, sea-state is not modelled. For more detail analysis of the initialization of the missile or to extent scope of the study to all phases of a sea-skimming missiles sea-state also can be investigated.

- Booster and turbojet models can be modelled more comprehensively. The performance of the turbojet for powered descent may be investigated with different ambient temperatures.

In conclusion, the proposed method exhibits promising outcomes across all the real-world problems presented and addressed in this thesis. It demonstrates an increase in overall success rates for each individual case as well as when considered collectively, showcasing the robustness of this method.





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APPENDIX A

DERIVATION OF EQUATIONS OF MOTION

First, the sum of the forces acting on a body is equal to the product of that body's mass and its acceleration relative to the ECI frame.

$$\sum \overline{F}_{ib}^{(b)} = m \overline{a}_{ib}^{(b)} \quad (\text{A.1})$$

To find the acceleration, linear and angular velocity can be defined as follows,

$$\overline{v}_{ib}^{(b)} = \overline{v}_{eb}^{(b)} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \& \quad \overline{\omega}_{ib}^{(b)} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \& \quad \tilde{\omega}_{ib}^{(b)} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (\text{A.2})$$

$$\overline{a}_{ib}^{(b)} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (\text{A.3})$$

And sum of all the acting forces,

$$\overline{F}_{ib}^{(b)} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \hat{C}^{(b,n)} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad (\text{A.4})$$

Transformation matrix,

$$\hat{C}^{(b,n)} = \begin{bmatrix} \cos \psi \cos \theta & \sin \psi \cos \theta & -\sin \theta \\ \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \phi \\ \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \theta \cos \phi \end{bmatrix} \quad (\text{A.5})$$

And we place force and acceleration equations together,

$$X - mg \sin \theta = m(\dot{u} + wq - vr) \quad (\text{A.6})$$

$$Y + mg \cos \theta \sin \phi = m(\dot{v} - wp + ur) \quad (\text{A.7})$$

$$Z + mg \cos \theta \cos \phi = m(\dot{w} + vp - uq) \quad (\text{A.8})$$

After we rearrange the above equations,

$$\dot{u} = \frac{X}{m} - g \sin \theta + vr - wq \quad (\text{A.9})$$

$$\dot{v} = \frac{Y}{m} + g \cos \theta \sin \phi + wp - ur \quad (\text{A.10})$$

$$\dot{w} = \frac{Z}{m} + g \cos \theta \cos \phi + uq - vp \quad (\text{A.11})$$

These three equations are translational equations of motion. For the angular equations of motion, the total moment on the body must be equal to the change in the angular momentum of the body. This can be shown as below,

$$\sum \vec{G}_{ib}^{(b)} = \frac{d\vec{H}_{ib}^{(b)}}{dt} \quad (\text{A.12})$$

Angular momentum,

$$\vec{H}_{ib}^{(b)} = I \cdot \vec{\omega}_{ib}^{(b)} \quad (\text{A.13})$$

Inertia matrix,

$$\hat{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \quad (\text{A.14})$$

$$\tilde{\omega}_{ib}^{(b)} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (\text{A.15})$$

$$\frac{d\vec{H}_{ib}}{dt}^{(b)} = \dot{\vec{H}}_{ib}^{(b)} + \tilde{\omega}_{ib}^{(b)} \hat{I} \tilde{\omega}_{ib}^{(b)} \quad (\text{A.16})$$

$$\vec{G}_{ib}^{(b)} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \quad (\text{A.17})$$

After we placed the equations together,

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix} \quad (\text{A.18})$$

$$L = I_{xx}\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} + q(-I_{xz}p - I_{yz}q + I_{zz}r) - r(-I_{xy}p + I_{yy}q - I_{yz}r) \quad (\text{A.19})$$

$$M = -I_{xy}\dot{p} + I_{yy}\dot{q} - I_{yz}\dot{r} - p(-I_{xz}p - I_{yz}q + I_{zz}r) + r(I_{xx}p - I_{xy}q - I_{xz}r) \quad (\text{A.20})$$

$$N = -I_{xz}\dot{p} - I_{yz}\dot{q} + I_{zz}\dot{r} + p(-I_{xy}p + I_{yy}q - I_{yz}r) - q(I_{xx}p - I_{xy}q - I_{xz}r) \quad (\text{A.21})$$

Rearranging above equations as below, translational and angular dynamics equations can be shown as in their open forms as below.

$$L = I_{xx}\dot{p} + I_{xy}(-\dot{q} + rp) + I_{xz}(-\dot{r} - qp) + I_{yz}(r^2 - q^2) + rq(I_{zz} - I_{yy}) \quad (\text{A.22})$$

$$M = I_{yy}\dot{q} - I_{xy}(\dot{p} + rq) - I_{xz}(r^2 - p^2) + I_{yz}(-\dot{r} + pq) + rp(I_{xx} - I_{zz}) \quad (\text{A.23})$$

$$N = I_{zz}\dot{r} - I_{yz}(\dot{q} + rp) - I_{xy}(p^2 - q^2) - I_{xz}(\dot{p} - qr) + pq(I_{yy} - I_{xx}) \quad (\text{A.24})$$

Below equation is the transformation matrix from ECEF to vehicle-carried NED coordinate frame.

$$\hat{C}^{(n,e)} = \begin{bmatrix} -\cos \lambda \sin \mu & -\sin \lambda \sin \mu & \cos \mu \\ -\sin \lambda & \cos \lambda & 0 \\ -\cos \lambda \cos \mu & -\sin \lambda \cos \mu & -\sin \mu \end{bmatrix} \quad (\text{A.25})$$

Below equation defines rotation of the earth wrt. Body-fixed coordinate frame.

$$\bar{\omega}_{e/i}^{(b)} = \hat{C}^{(b,n)} \hat{C}^{(n,e)} \bar{\omega}_{e/i}^{(e)} \quad (\text{A.26})$$

Below is the angular velocity of the vehicle-carried NED coordinate frame relative to the ECEF coordinate frame in the body-axis coordinate frame.

$$\bar{\omega}_{n/e}^{(b)} = \begin{bmatrix} \frac{\bar{v}_{ib}^{(b)}(2)}{N_e + h} \\ -\frac{\bar{v}_{ib}^{(b)}(1)}{M_e + h} \\ -\frac{\bar{v}_{ib}^{(b)}(2)}{N_e + h} \tan \mu \end{bmatrix} \quad (\text{A.27})$$

By using constants at defined below equations, angular velocity in equation A.27 can be found.

$$R_{Ea} = 6,378,137.0 \text{ m} \quad (\text{A.28})$$

$$f = \frac{1}{298.257223563} \quad (\text{A.29})$$

$$R_{Eb} = R_{Ea}(1 - f) = 6,356,752.0 \text{ m} \quad (\text{A.30})$$

$$e = \frac{\sqrt{R_{Ea}^2 - R_{Eb}^2}}{R_{Ea}} = 0.08181919 \quad (\text{A.31})$$

$$M_e = \frac{R_{Ea}(1 - e^2)}{1 - e^2 \sin^2 \mu} \quad (\text{A.32})$$

$$N_e = \frac{R_{Ea}}{\sqrt{1 - e^2 \sin^2 \mu}} \quad (\text{A.33})$$

Thus, the angular velocity of the body relative to the vehicle-carried NED frame in the body frame is obtained as follows.

$$\bar{\omega}_{b/n}^{(b)} = \bar{\omega}_{ib}^{(b)} - \bar{\omega}_{e/i}^{(b)} - \bar{\omega}_{n/e}^{(b)} \quad (\text{A.34})$$

$$\bar{\omega}_{b/n}^{(b)} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad (\text{A.35})$$

A.1 Euler Integration Method

For determination of the attitude of the aircraft, we can use derivatives of Euler angles as in equation A.36.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix} \bar{\omega}_{b/n}^{(b)} \quad (\text{A.36})$$

A.2 DCM Integration Method

Determination of angular velocity and derivative of DCM can be found as in equation A.37.

$$\tilde{\omega}_{b/n}^{(b)} = \hat{C}^{(b,n)} \dot{\hat{C}}^{(n,b)} \quad (\text{A.37})$$

$$\dot{\hat{C}}^{(n,b)} = \hat{C}^{(n,b)} \tilde{\omega}_{b/n}^{(b)} \quad (\text{A.38})$$

After we integrate derivative of DCM, to find Euler angles we use values in DCM as below.

$$\phi = \text{atan}_2 \hat{C}^{(n,b)}(3, 2), \hat{C}^{(n,b)}(3, 3) \quad (\text{A.39})$$

$$\theta = \text{atan}_2 -\hat{C}^{(n,b)}(3, 1), \sqrt{\hat{C}^{(n,b)}(3, 2)^2 + \hat{C}^{(n,b)}(3, 3)^2} \quad (\text{A.40})$$

$$\psi = \text{atan}_2 \hat{C}^{(n,b)}(2, 1), \hat{C}^{(n,b)}(1, 1) \quad (\text{A.41})$$

A.3 Quaternion Integration Method

We can obtain derivatives of quaternions, by using quaternions and angular velocity as in equation A.42.

$$\dot{\check{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -P & -Q & -R \\ P & 0 & R & -Q \\ Q & -R & 0 & P \\ R & Q & -P & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad (\text{A.42})$$

$$\check{q} = \int \dot{\check{q}} dt \quad (\text{A.43})$$