

**OPTIMAL INVENTORY POLICIES IN
SERIAL SUPPLY CHAINS
MULTI-ECHELON INVENTORY MANAGEMENT:
A CASE STUDY IN THE PORCELAIN INDUSTRY**

Nuri Mert ONUR

ABSTRACT

In the first chapter, we have summarized the basics of supply chain management. Integration along the Supply Chain and Natures of Supply Chain Management problems observed.

In the second chapter, models and potential problems in inventory management have been studied. Chapter II describes a review of the most important contributions in lot sizing problems for single and multi-stage of reorder cycle time models, including some approaches with the power of two restrictions, and the application of the algorithms.

In the last chapter, the results of the empirical study carried out in Yıldız Porcelain Factory have been presented. To carry out all the necessary calculations to assess the required values, we have prepared a visual basic macro through using the properties of the excel microsoft office program.

In the empirical study the principal objective is to find a solution to the problem of determining the total costs in multi-stage serial systems in the production process of porcelain substances using the Szendrovits, Andrew Z. algorithm approach, satisfying the power of two restrictions. Other secondary objective is to determine the effectiveness of the power of two approach, comparing the results obtained in the dissertation prepared by Faik Başaran in 1993. The algorithm developed is based on the assumption of a multi-stage serial system.

ÖZ

Birinci bölümde, tedarik zincir yönetimi konusunun tanımı ve özellikleri anlatılmıştır.Tedarik zinciri ve tedarik zincir yönetimindeki karşılaşılan problemler incelenmiştir.

İkinci bölümde, envanter yönetimindeki modeller ve potansiyel problemler gösterilmiştir.Ayrıca ikinci bölümde yeniden sipariş verme süresi modellerinde ikinin kuvveti algoritmasını içerecek şekilde tek ve çok kademe için kısımlara ayırma problemleri incelenmiştir.

Son Bölümde, Porselen Sanayii alanında uygulanan çalışmanın sonuçları gösterilmiştir.Sonuçlara ulaşmak için Yıldız Porselen Fabrikasından Faik Başaran tarafından alınan veriler mikrosoft ofis programı olan excel’de visual basic bilgisayar dilinde makro yazılarak uygulanmış ve sonuçlar ortaya konulmuştur.

Yapılan uygulama çalışmasının birincil amacı ikinin kuvveti algoritmasını Andrew Z. Szendrovits’in algoritmasını kapsayacak şekilde çok kademeli seri sistem olan porselen sanayii örneğinde uygulayıp toplam maliyetleri bulmaktır.İkincil amaç ikinin kuvveti algoritmasına göre bulunan değerleri 1993 yılında İstanbul’da Faik Başaran tarafından hazırlanmış doktora tezindeki değerlerle karşılaştırıp algoritmanın efektifliğini ortaya çıkarmaktır.Uygulanan algoritma çok kademeli seri sistem için tasarlanmıştır.

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TABLE OF CONTENTS

ABSTRACT	iii
ÖZ.....	iv
LIST OF TABLES.....	x
LIST OF FIGURES.....	xi
ABBREVIATIONS.....	xii
INTRODUCTION	1
1. CONCEPTUAL FRAMEWORK.....	2
1.1. Supply Chain Management.....	2
1.1.1. Basics of Supply Chain Management.....	2
1.1.1.1. Definition of Supply Chain Management	4
1.1.1.2. Integration along the Supply Chain	5
1.1.1.3. Natures of Supply Chain Management Problems.....	6
1.1.1.4. Important Issues in Efficient Supply Chain Planning.....	9
1.1.1.5. Push-based versus Pull-based Supply Chain.....	10
1.1.1.5.1. Push-based Supply Chain System.....	10
1.1.1.5.2. Pull-based Supply Chain System	11
2. MODELS AND POTENTIAL PROBLEMS IN INVENTORY	
MANAGEMENT	13
2.1. Lot Sizing Problems and Models as a Remedy to Lot Sizing Problems	13
2.1.1. Single Stage Models	13
2.1.2. Multi-Stage Models	15
2.2. Types of Inventory Models.....	18
2.2.1. The Basic EOQ Model.....	21
2.2.1.1. Multiple Items EOQ Models	22
2.2.1.2. Resource Constrained Multiple Items EOQ Models	22

2.2.1.3.	EOQ For Multiple Items With One Constraint	22
2.2.1.4.	EOQ For Multiple Items With Two Constraint.....	26
2.2.2.	Dynamic Lot Sizing Models	30
2.2.2.1.	Example For Dynamic Lot Sizing Models.....	30
2.2.2.1.1.	Period Order Quantity	31
2.2.2.1.2.	Fixed Period Demand.....	32
2.2.2.1.3.	Lot For Lot Rule (L4L)	34
2.2.2.1.4.	Silver-Meal Method	35
2.2.2.1.5.	Wagner-Whitin Algorithm	37
2.2.3.	The Model by Crowston, Wagner, and Williams.....	43
2.2.3.1.	Simple Extensions of the Model.....	43
2.2.4.	Reorder Cycle Time Problems	44
2.2.5.	Power – of – two Policy.....	47
3.	EMPIRICAL STUDY	49
3.1.	Purpose & Scope Of The Study	49
3.1.1.	Purpose.....	49
3.1.2.	Scope.....	49
3.2.	History of Porcelain.....	50
3.3.	Kinds of Porcelain	54
3.4.	History of Yıldız Porcelain Factory	55
3.5.	Production Structure Studied	56
3.6.	Information of Production Structure	64
3.7.	Methodology	98
3.8.	Problem Definition.....	98
3.9.	Maxwell and Muckstadt Approach	98
3.10.	Implementation of the Empirical Study	101
	Large Brimmed Soup Bowl	110
	LBSB Lid.....	111

Brimmed Regular Plate Ø29 cm.....	112
Brimmed Regular Plate Ø25 cm.....	113
Brimmed Regular Plate Ø19 cm.....	114
Brimmed Hollow Plate Ø21 cm	115
Brimmed Oval Plate Ø35 cm	116
Brimmed Oval Plate Ø21 cm	117
Large Brimmed Compote Bowl.....	118
Brimmed Compote Bowl.....	119
Brimmed Creamer	120
Brimmed Creamer Saucer	121
Brimmed Salt Shaker.....	122
Brimmed Lemon Plate.....	123
CONCLUSION	125
REFERENCES	127

LIST OF TABLES

Table 1 : Energy Used Per Unit According To Various Departments In The Porcelain Facility.....	67
Table 2 : Operation Report On The Raw Materials Consumed And Stock Status.....	68
Table 3: Total Cost At Various Production Stages (Clay, Kaoline, Feldspar Constitute The Initial Matter; Zinc-Ocside, Tin-Ocside, Barrium Carbonade And Talk Constitute The Anxiliary Initial Matter).....	71
Table 4 : Productive Motions At Stages	72
Table 5 : Sales Motions At Stages	73
Table 6 : Molding Clay Preparation Costs	74
Table 7 : The Lathe Clay Preparation Costs	76
Table 8 : Molding Costs	78
Table 9 : Lathe Costs	80
Table 10: Glaze Preparation Costs	82
Table 11: Glazing Kilns Cost	84
Table 12: Cost Of Technical Decoration	86
Table 13: Cost Of Technical Decoration Kilns	89
Table 14: The Summary Of Information to Define Unit Costs.....	91
Table 15: Informations of Set Products	93
Table 16: Set Product Unit Costs	94
Table 17: Stage Preparation Cost Ratio	95
Table 18: Standart Occupation Time of Set Products (Minutes)	96
Table 19: The Sum of Factory Order and Holding Costs of Set Products.....	97
Table 20: The Total Cost Values Summerized In the Study	126

LIST OF FIGURES

Figure 1: A push-based supply chain system-----	11
Figure 2: A pull-based supply chain system -----	12

ABBREVIATIONS

The following assumptions are used in the different parts of the study :

- A (G) = The arc set corresponding to G,
- A_i = Fixed setup costs for $i \in N(G)$,
- C = An arbitrary node,
- c_n = The value subtracted from every n. stage after the production,
- D = Demand
- G = Represents the acyclic directed graph corresponding to the production and distribution system.
- g_i = $h_i \lambda_i / 2$, the average echelon holding cost per unit time for operation i when $T_i = 1$ (the same unit time used to determine the demand),
- h_i = The echelon holding costs, h_i , for $i \in N(G)$
- H_i = Total production cost in n.stage,
- k = The optimal value of M_i for $i \in N(G)$,
- K_i = The setup cost of the production in every n. stage,
- K_n = Is an integer,
- M_i = represent the multiple of the base planning period per reorder interval for operation i for $i \in N(G)$,
- $N(G)$ = Represents the node set,
- P_n = Production rate of the n.stage
- S_n = The value subtracted from every n. stage of the setup cost
- T = (T_n : $n \in N$) of positive numbers,
- $T^*(k)$ = The corresponding optimal solution,
- T_i = represent the reorder interval at operation i for $i \in N(G)$,
- TL = Be the base planning period, measured in unit time (minutes, days, weeks, months, year, etc.),

INTRODUCTION

The principal objective of this research is to find a solution to the problem of determining the total cost in multi-stage serial systems in the production process of porcelain substances using the Szendrovits, Andrew Z. algorithm approach, satisfying the power of two restrictions. Other secondary objective is to determine the effectiveness of the power of two approach, comparing the results obtained in the dissertation prepared by Faik Başaran in 1993.

The algorithm developed is based on the assumption of a multi-stage serial system. A stage might consist of an operation such as procurement of raw materials or fabrication of parts. The serial structure is the simplest type of multi-stage structures in which materials enter the first (1) stage and progressively pass through a sequence of stages until final product exits at the last (6) stage. These stages of our case are Lathe or Molding Clay Preparation (changes up to the product produced), Lathe or molding (changes up to the product produced), Glaze Preparation, Glazing Kilns, Technical Decoration and finally Technical Decoration Kilns.

Demand for each end item is assumed to occur at a constant and continuous rate, and is given for a planning horizon of n periods. Production is instantaneous and no backorders are allowed and unconstrained capacity at each node is assumed. The cost function is composed by the fixed setup cost and the holding cost. Fixed setup costs and echelon holding costs are changed at each stage.

It is assumed that the cycle length should satisfy the power of two restrictions, which applies zero inventory ordering and stationary-nested policies. A stationary policy is one in which each facility uses a fixed order quantity and a fixed interval time between successive orders. In a nested policy each facility orders every time any of its suppliers orders.

The organization of the document is as follows. Chapter II describes a review of the most important contributions in lot sizing problems for single and multi-stage models, for reorder cycle time models, including some approaches with the power of two restrictions, and the application of the algorithms. In the last chapter, the results of an empirical study carried out in Yıldız Porcelain Factory have been presented.

1. CONCEPTUAL FRAMEWORK

1.1. Supply Chain Management

In the past few years, interest in supply chain management has grown dramatically. This interest has forced many firms to adjust and analyze their supply chains. In most cases, however, this has been done based on experience and intuition; very few analytical models or design tools have been used in this process. In this chapter, we summarize the basics of supply chain management.

1.1.1. Basics of Supply Chain Management

An accelerating trend toward globalization marked the latter half of the twentieth century and the beginning of the present one. It is common to see a company design, produce and distribute products through a global network to provide the best customer service at the lowest price. Coordination throughout the entire logistical system must be planned and managed, because of the impact in costs that it represents to the companies and their opportunity to compete in today's global market.

The central issue in the supply chain performance is the inventory management. Inventories are present at every stage of the supply chain as raw materials to finished goods. The inventory acts as a buffer against any uncertainty, but holding inventory is costly and runs the risk of product deterioration and obsolescence. The focus of inventory problems traditionally has been on lot size determination. Supply occurs in discrete batches or lots and items proceeds through a

sequence of stages. The issue of the lot sizing is to determine how large these lots should be trying to find the best balance between fixed costs and inventory holding costs. Ford Harris in 1915 introduced the classic Economic Lot Size Model which serves as reference for many other research studies.

Therefore, the lot sizing problem can be formulated as the problem of determining the reorder interval time, because of a functional relationship between the lot size and the manufacturing cycle time. Due to the fact that this problem is continuous and that the reorder optimal interval can take any positive real value, is often impractical to implement it. This is referred to a discrete problem imposing the restriction that the reorder interval can take only positive integer values. *Maxwell and Muckstadt (1985)* explain the advantages of formulating the problem in terms of reorder intervals rather than in terms of lot sizes. They establish three principal reasons for this: (1) the experience that production planning is more naturally centered around the frequency of production because it dictates the numbers of set-ups, the requests for tooling and fixtures, and the demands on the material handling system, (2) the mathematical representation of the model is simplified, and (3) from a scheduling point of view it is often practical to keep reorder intervals constant in the face of minor changes to demand forecasts and to adjust lot sizes accordingly.

A special case is given by considering the discrete problem with the power of two restrictions in which the reorder interval is constraint to be not only integer, but also a power of two. The power-of-two policy was developed by *Roundy (1985)*. It considers the problem of determining the reorder interval instead of the reorder quantity and has the advantage of an easy implementation, even if the system is very complex and it is known that the cost of the optimal solution for the discrete problem using the power-of-two solution of a continuous problem is within about 6% of the cost of the optimal solution of the continuous problem without those restrictions.

Implementing power of two policies makes production scheduling easier, and ensures that production cycles regenerate as frequently as possible, so that inventory imbalances that in practice can be easily corrected. Although considerable research has been devoted to traditional methods of search, optimization using such methods

is not that efficient, particularly in finding a solution for very complex search space. Furthermore, significant less attention has been paid to stochastic search and optimization techniques like genetic algorithms. *Khouja, Michalewicz and Wilmot (1998)* presented a genetic algorithm for solving the Economic Lot Size Scheduling Problem finding better solutions than the iterative dynamic programming approach. Genetic algorithms have been employed to solve optimization problems across all disciplines and interests, obtaining global optimal or near optimal solutions in complex search spaces. Their simplicity permits their use to solve difficult problems, showing an important reduction in the computational time.

1.1.1.1. Definition of Supply Chain Management

Supply chain management or logistics management refers to the management of the flow of goods from points-of-origin to points-of-consumption. In the past, a variety of names have been used according to Lambert and Stock (1993):

Physical distribution	Materials Management
Distribution	Materials logistics management
Distribution engineering	Logistics
Business logistics	Quick-response systems
Marketing logistics	Industrial logistics
Distribution logistics	

Nowadays, supply chain management and logistics management seem to be the most widely accepted term. The Council of Logistics Management, one of the largest and most prestigious groups of logistics professionals, provides the excellent definition of logistics management as following:

"Logistics management is the process of planning, implementing and controlling the efficient, cost effective flow and storage of raw material, in-process inventory, finished goods, and related information from point-of-origin to point-of-consumption for the purpose of conforming to customer requirements."

Supply chain management or logistics management is a vital part of a firm's operation. Logistics is the third-largest source of cost of doing business for a typical firm after manufacturing and marketing. Efficient and effective management of the logistics function can have a substantial impact. Logistics cost is reduced, profitability is improved, and the level of customer service is increased. There are a number of key factors in supply chains, Arnold and Chapman (2000):

- *A supply chain includes all activities and processes to supply a product or service to an end customer.*
- *Any number of companies can be linked in the supply chain.*
- *A customer can be a supplier to another customer so the total chain can have a number of supplier/customer relationships.*
- *While the distribution system can be direct from supplier to customer, it can contain a number of intermediaries (distributors) such as wholesalers, warehouses, and retailers.*
- *Product or services usually flow from supplier to customer and design and demand information usually flows from customer to supplier.*

1.1.1.2. Integration along the Supply Chain

Basically, the integrated supply chain management concept refers to administering all supply chain activities as an integrated system. Integrating all distribution-related activities in the supply chain as mentioned in the previous section can reduce total operating costs of a company. Without this integrated approach, the costs to satisfy customer demand and expectations will be higher. A company must make a decision that coordinates all set of activities within the supply chain or business interfaces. The following are the list of critical business interfaces within the supply chain.

- Supplier-purchasing
- Purchasing-production
- Production-marketing

- Marketing-distribution
- Distribution-intermediary (wholesaler and/or retailer)
- Intermediary-customer/end-user

These business interfaces must be considered as a whole since uncoordinated decisions involving these activities could cause a build up of inventory along the supply chain. Now, the decisions of purchasing are not only concerning about the low per unit costs for raw material, but also need to consider the production to achieve the lowest per-unit production costs. All decisions within the business interfaces must be made under the same goal, which is minimize the inventory holding costs and logistics costs or total operating costs of the firm. Management should strive to minimize the total operating costs rather than the cost of each activity. Attempts to reduce the cost of individual activities may lead to increased total costs. For example, consolidating finished goods inventory in a small number of distribution centers will reduce inventory carrying costs and warehousing costs but may lead to an increase in freight expense or a lower sales volume. On the other hand, savings associated with large volume purchases may increase the inventory carrying costs. So, reductions in one cost may lead to increase in the costs of other activities. Effective supply chain management can be accomplished only by viewing logistics as an integrated system, and also minimizing its total operating cost subject to the company's customer service objectives.

1.1.1.3. Natures of Supply Chain Management Problems

Generally supply chain management problems involve the decision on how products are to move through the supply and distribution channels, and at the operational level, this includes decision on how to fill a recently received customer order, how to respond to a temporary transportation rate reduction, and also how to route the current customer orders. Each day the supply chain system operates to move the products smoothly and efficiently through the channel. Basically the planning in supply chain management can be divided into four major decision areas: customer

service standards, distribution network configuration, inventory policy or deployment, and transportation system selection and routing.

Customer service standards: The design of supply chain system greatly affects the level of customer service. Conversely, the level of customer service to be provided definitely impacts the design of supply chain systems. High levels of service normally use decentralized inventories at several locations and the use of, sometime, more expensive forms of transportations. Low levels of service generally require the use of less expensive forms of transportations and allow centralized inventories at few locations. It is known that high levels of service equates to high logistics costs. So, the first priority in supply chain planning must be the proper setting of customer service levels. Ballou (1999) suggests that effective supply chain planning should start with a survey of customer service needs and desires.

Distribution network configuration: Distribution network decision involves how to place the stocking points and the sourcing points in the supply chain system. This also includes the number, location, and size of the facilities and assigning market demands to each facility. Generally distribution network problem includes all product movements and associated costs starting from plants/suppliers all the way to end customers. Finding the minimum assignment cost is the ultimate goal of distribution network planning. The following are the key questions in distribution network problem:

- What are the best number, location, and size of stocking points?
- Which plants/suppliers should serve which stocking points/facilities?
- Which products should be shipped directly from plants/suppliers to customers and which should be transshipped through the warehousing system?

Inventory policy: In general two strategies, push inventory and pull inventory are involved in managing inventory throughout a supply chain. The push inventory

strategy refers to a make-to-stock policy while a pull inventory policy refers to a demand-drive policy. More details on the push and pull inventory policies will be presented again in a later section. An effective inventory policy tries to reduce the number of stocking points throughout the supply chain system. This will reduce the amount of inventory carried in the system including the safety stocks. However, the cost reduction associated with inventory consolidation is in trade-off with higher transportation costs. With fewer stocking points, smaller outbound shipment sizes with higher shipping charges must be weighed against larger shipment sizes of inbound goods that travel through longer distances to the marketplace. Therefore, the distribution network decision must be sensitive to the inventory deployment and control policies used. This indicates that inventory policy directly affects the distribution network decision and the whole supply chain planning. The following are common questions related to inventory policy:

- What turnover ratio should be maintained?
- Which products should be maintained at which stocking points?
- What level of product availability should be maintained in inventory?
- Which method of inventory control is best?
- Should push or pull inventory strategies be used?

Transport selection and routing: Transportation selection and routing decisions directly affect the supply chain decisions. The number, size and location of stocking points depend on the transportation policies of the company as much as inventory policies. As the number of stocking points increases, fewer customers will be assigned to any one point, the mode of transportation may change and this will affect the transportation cost. The following are questions related to the transportation system selection and routing:

- Which customers should be served out of which stocking points?
- Which transportation types, truckload (TL) or less than truckload (LTL), should be assigned to which customers?
- Which modes of transportation, Rail, Truck, Air, Water, or Pipeline, should be used?

1.1.1.4. Important Issues in Efficient Supply Chain Planning

Cost trade-offs: Supply chain planning needs to balance all conflicting costs such as transportation costs versus inventory costs, production costs versus distribution costs, and ultimately customer service costs versus all supply chain costs. All issues in the supply chain must be considered as a whole to avoid any suboptimal plans. Both facility location and distribution issues must be addressed at the same time, since output of facilities location decision is the input to the distribution system and are economically related to one another.

Consolidation: Consolidation happens when small shipments are consolidated to form a large shipment to gain the economies of scale. For example, two or more customer orders might be combined with other customer orders received at other time periods to form a large shipment if possible. Consolidation strategy will lower average per-unit shipping costs. This also avoids shipping small quantities of items over long distances at high per-unit transport rate. In general, the concept of consolidation will be useful when the quantities shipped are small.

Postponement: The key idea of postponement is "to ship as much as you can as far as you can before committing to the end product." The final product processing and distribution are delayed until a customer order is received. This is done to avoid increasing total inventory level throughout the company logistics network and the possibility of obsolete stocks. Postponement can be classified into five types; Labeling, Packaging, Assembly, Manufacturing, and Time.

Mixed strategy: A mixed strategy allows an optimal strategy to be established for separate product groups. Usually mixed strategy leads to lower costs than a single or global strategy. In general, single strategies can benefit from economies of scales and administrative simplicity however they ineffectively perform when the product groups vary in terms of cube, weight, order size, sales volume, and customer service requirements. Examples of a mixed strategy include using of some public warehousing along with privately owned space, shipping product directly from the plants along with

from the warehouses, and filling customer order from a single warehouse along with instances of shipping from multiple warehouses for some products.

1.1.1.5. Push-based versus Pull-based Supply Chain

Supply chain or logistics systems are normally categorized as push-based or pull-based systems. In a push-based supply chain system, long-term forecasts are used to determine a firm's production. On the other hand, in a pull-based supply chain system, production is demand driven, and therefore is directly related to actual customer demands instead of a forecast. With actual demands, a firm can decrease inventory both at the retail and the manufacturing levels and also decrease the variability in the system due to lead-time reduction. A significant reduction in system inventory level and costs make a pull-based system more superior to a push-based system. The trend today is toward pull-based system even though it is more difficult to implement than a push-based system. The succeeding sections summarize key concepts of these two supply chain systems.

1.1.1.5.1. Push-based Supply Chain System

In a push-based supply chain system, production decisions are based on long-term forecasts. Orders from the retailer's warehouses are used to forecast customer demand. This system is appropriate where production or purchase quantities exceed the short-term requirements of the inventories. However, a firm may have the problem of overstocking or excess inventory. The excess inventory could become obsolete, damaged, or nonfunctional because of age. High inventory leads to high inventory cost. A push-based system also produces larger and more variable production batches and this can impact the customer service levels, since the system has the inability to meet changing demand patterns. Moreover, a push-based supply chain increases transportation costs, heightens inventory levels and heightens manufacturing costs, due to inability to meet or react to changing market conditions. Figure 1.1 shows a push-based system.

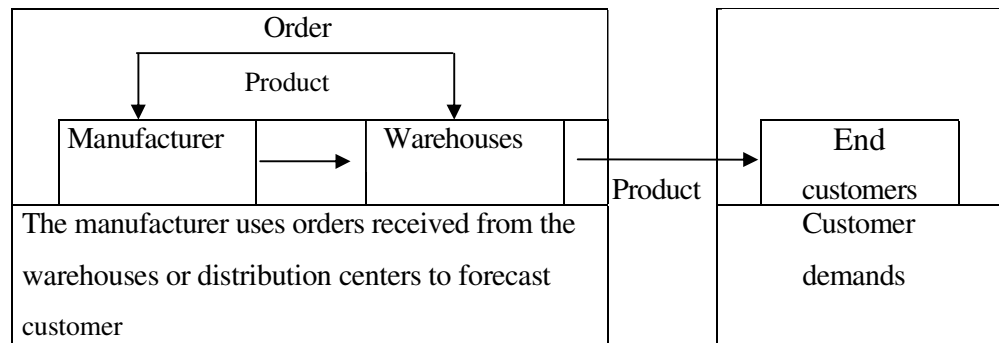


Figure 1: A push-based supply chain system

1.1.1.5.2. Pull-based Supply Chain System

In pull-based supply chain system, actual customer demands rather than forecast are used in driving production or orders. In a pull-based system, the supply chain uses fast information flow to transfer information about customer demand to all stocking points and manufacturing facilities. This leads to a decrease in lead times, a decrease in inventories throughout the supply chain, and a decreasing in variability in the system. Pull-based system gives a significant reduction in system inventory and system costs. However, it is often difficult to implement when lead times are long. Furthermore, it is more difficult to take advantage of economies of scale in manufacturing and transportation since systems are not planned far ahead in time. To successfully apply a pull-based system, it is important to determine the procurement costs and lead time effects against inventory carrying costs. Since demand and lead time sometimes cannot be known with certainty, a firm must plan for the situation where not enough stock may be on hand to fill customer requests. In addition to the regular stock that is maintained for the purpose of meeting average demand and average lead time, an increment of inventory, safety stock, is added. Currently, there are two methods for controlling inventory in a pull-based system; 1) the reorder point method and 2) the

period review method. Some firms also use a combination of these two. In this study, the reorder point method is used in the models developed. For more information about the reorder point method and inventory control, consult Ballou (1999). Figure 1.2 shows a pull-based supply chain system.

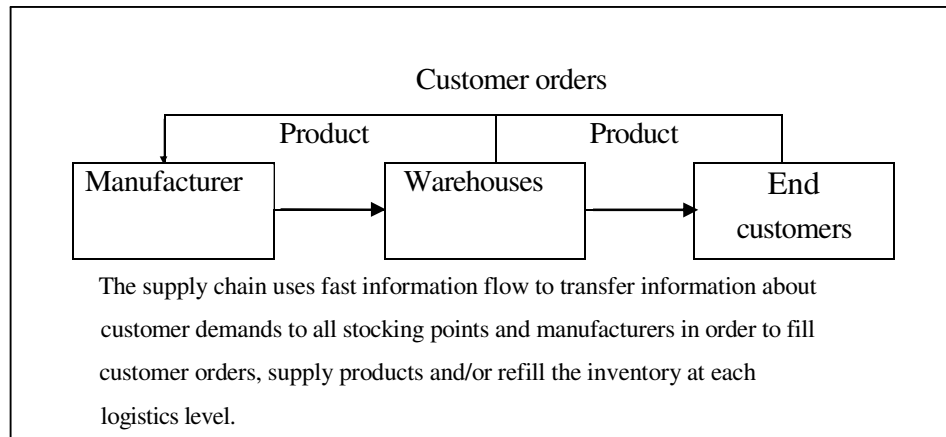


Figure 2: A pull-based supply chain system

2. MODELS AND POTENTIAL PROBLEMS IN INVENTORY MANAGEMENT

Inventory problems have been studied for many years. This review describes some of the most important contributions in this field. It includes methods used to solve single and multi-stage lot sizing problems. For multi-stage systems, some models are shown that deal with special cases like capacity constraint and joined setup costs. Finally, it is presented some algorithms applications that can be considered as previous work in lot sizing problems.

2.1. Lot Sizing Problems and Models as a Remedy to Lot Sizing Problems

2.1.1. Single Stage Models

For many years the main focus of the inventory theory has been in the lot size determination. Many authors try to solve the single stage problem. The classic Economic Lot Size Model, introduced by Ford Harris in 1915, is a very basic model that considered a warehouse facing constant demand for a single item. It assumes constant fixed cost, instantaneous batch delivery following a deterministic lead time, all replenishment orders are for the same quantity and no shortages are allowed. The total cost per time $TC(Q)$, is composed by ordering cost, product purchase cost and inventory holding cost.

$$TC(Q) = \text{ordering cost} + \text{purchased cost} + \text{inventory holding cost}$$

$$TC(Q) = AD/Q + CD + hQ/2 \text{ (Equation 1.1)}$$

Based on the cycle inventory level over time, the inventory level decreased constantly from the order quantity size (Q) to zero each cycle, and averages $Q/2$. The process repeats each time Q units are sold (every $T=Q/D$), integrating over this cycle length it can be found the average inventory, I .

$$I = \frac{1}{Q/D} \int_0^{Q/D} (Q - tD) dt = D/Q \left(Qt - Dt^2/2 \right) \Big|_0^{Q/D} = D/Q \left(Q^2/D - Q^2/2D \right) = Q/2$$

This yields :

$$dTC(Q)/dQ = -AD/Q^2 + h/2 = 0$$

$$Q^* = \sqrt{2AD/h}$$

Another important issue in the EOQ model is the definition of the total cost for the optimal quantity (Q^*). In this case, ordering and holding costs are equal.

$$AD/Q^* = AD/\sqrt{2AD/h} = \sqrt{ADh}/2$$

Inventory holding cost per period is

$$hQ^*/2 = (h\sqrt{2AD/h})/2 = \sqrt{ADh}/2$$

The total cost using the optimum lot size quantity is determined by :

$$TC(Q^*) = \sqrt{ADh}/2 + CD + \sqrt{ADh}/2 = 2 \sqrt{ADh}/2 + CD = \sqrt{2Adh} + CD$$

$$TC(Q^*) = \sqrt{2Adh} + CD$$

All this description has been provided to describe the relationship between the order quantity and the reorder cycle time both assumes to be constant. The Economic Order Quantity is used as reference point in a lot of methods proposed later.

Veinott (1967) showed that a broad class of problems (including deterministic single and multi-facility economic lot size) can be formulated as minimizing a concave function over the solution set of Leontief substitution system. To understand what does this mean it is necessary to introduce some concepts. A matrix A is called Leontief if it has exactly one positive element in each column and there is a nonnegative (column) vector x for which Ax is positive. The linear program for finding a (column) vector $x = (x_j)$, called optimal, is given by:

Objective function:

Minimizes cx

Subject to: $Ax = b, x \geq 0$,

If A is Leontief, and $b \geq 0$, it is a Leontief substitution system and has $X(b) \in S$ as its solution set. S is the set of programs x for which $x_i x_j = 0$ for all pairs (i,j) in a

specified set. In applications it is often appropriate to impose additional restrictions of the form $x_i x_j = 0$, for example in production problems if it is possible to produce only one product in each period.

Leontief substitution systems seem to provide a natural setting for studying inventory models with concave costs. Their applications are on single and multi-facility lot size problem, lot-size-smoothing and warehousing models. Their algorithms required a computational effort that increases algebraically with the size of the problem instead of exponentially.

2.1.2. Multi-Stage Models

Multi-echelon inventory systems can be used to optimize the deployment of inventory in a supply chain. Multi-stage manufacturing situations (raw materials, components, subassemblies, assemblies) are conceptually very similar to multi-echelon inventory systems. Multi-echelon models examine the entire system, searching better solutions for the entire chain, not each stage independently. This coordination has the advantage of given better global solutions. In the multi-stage systems there have been a lot of contributions in serial, assembly, distribution, general and some special structures.

Clark and Scarf (1960) introduced the echelon stock concept which permits some very convenient mathematical simplifications. They define the echelon stock of echelon j (in general multi-echelon system) as the number of units in the system that are at, or have passed through, echelon j but have as yet not been specifically committed to outside customers. They considered the problem of determining optimal purchasing quantities in a multi-stage serial and distribution models. Echelon j stock may often be considered to be the facility j value-added inventory. The Clark-Scarf model allows stochastic demand and convex holding costs, but setup costs are assumed to be associated with no more than two facilities.

Crowston, Wagner and Henshaw (1972) made a comparison of exact and heuristics routines for lot size determination in multi-stage assembly systems. They concluded that economic lot sizes in multi-stage assembly systems can be determined by dynamic programming for problems of moderate size, while heuristic search routines appear to be promising for large problems. Using these results *Crowston, Wagner and Williams (1973)* present a model for multi-stage assembly systems to compute a set of optimal lot sizes so that the lot size at each facility is a positive integer multiple of the lot size at its successor facility. It is important to mention that they considered the serial system as a special case of the assembly system. Their model assumes constant continuous final demand, instantaneous production at each stage and infinite planning horizon.

A few years later, *Williams (1982)* proved that the well known theorem by Crowston, Wagner and Williams (1973) shows to be defective. The theorem establishes that an optimal solution to the batch size determination problem for multi-echelon production/inventory assembly structures is characterized by a set of lot sizes, such that the lot size at each stage must be an integer multiple of the lot size at its successor stage. The theorem proved to be defective at the point that results were extended from two level systems to more general assembly systems.

Schwarz (1973) deals with a one-warehouse n-retailer deterministic inventory system with known demands. As a conclusion, he shows that the form of the optimal policy can be very complex for more than four retailers and he argues for restricting attention to a simpler class of strategies (where each location's order quantity does not change with time) and develops an effective heuristic for finding good solutions.

Schwarz and Schrage (1975) make use of the myopic strategy. Myopic policies optimize a given objective function with respect to any two stages and ignore multi-stage interaction effects. Optimal and near optimal policies were proposed for multi-echelon production/inventory assembly systems under continuous review with constant demand over and infinite planning horizon. Schwarz and

Schrage model was widely used as a standard among the multi-stage production/inventory models.

Szendrovits (1981) presented a comment on the optimality in Schwarz and Schrage model, considering that their restrictions could be helpful to facilitate analytical tractability, but do not necessarily lead to optimal inventory policies as claimed by the authors. Szendrovits showed that a lower cost solution could be obtained in sample problems when the integrality constraint was violated. The example provided a lower cost solution by permitting two lots at a given stage to provide the total input for the three lots at its successor stages.

Later, *Blackburn and Millen (1985)* proposed simple cost modifications to improve the global optimality of the Schwarz and Schrage procedure. The effectiveness of these alternative modifications was tested through a series of simulation experiments. A new formulation of the lot sizing problem in multi-stage assembly systems which leads to an effective optimization algorithm was proposed by *Afentakis, Gavish and Karmarkar (1984)*. The problem was reformulated in terms of echelon stock which simplifies its decomposition by a Lagrangean relaxation method. A Branch and Bound algorithm which uses the bounds obtained by the relaxation was developed and tested.

A significant amount of work in this area has focused on evaluating the performance of the proposed techniques. *Blackburn and Millen (1985)* examined seven different heuristic algorithms, six combinations of methods and four cost modification procedures. A series of simulation experiments was conducted and it was concluded that the combination methods when used with some of the cost modifications result in enhanced performance in comparison to other sequential approaches. *Axsäter (1986)* analyzed the applicability in practice of some standard lot sizing problems and the way in which some adjustments can be considered. Assumptions in lot sizing models and the extent to which these assumptions are valid in practical situations are discussed.

A branch-and-bound based algorithm for optimal lot sizing of products with a complex product structure was proposed by *Afentakis and Gavish (1986)*. It assumed unconstrained production facilities and suggested that the formulation of the lot sizing problem in terms of its echelon stock, and the use of Lagrangean relaxation, seems to yield efficient algorithms. *Afentakis (1987)* developed an improved heuristic method for the dynamic lot-sizing problem in multi-stage production systems. This is a generalization of the single stage Wagner-Within algorithm, and attempts to optimize over all stages simultaneously, while building the production plans in a forward manner.

Billington, Blackburn, Maes, Millen, and Wassenhove (1994) examined the performance of heuristics found effective for the capacitated multiple-product, single stage problem in multi-stage settings. This study is one of the most comprehensive in terms of the number of methods examined and the conditions under which they were examined. The single-stage heuristics are: *Dixon/Silver (1981)*, *Lambrecht and Vanderveken (1979)*, the *Dogramaci, Panayiotopoulos and Adam (1981)*, and different versions of the ABC heuristics of *Maes and Van Wassenhove (1986)*. These heuristics are altered in two ways: (1) they allow the inclusion of the cost modification procedures developed by Blackburn and Millen, and (2) the feasibility routines have been modified to work in multi-stage environments. Both modifications attempt to coordinate decisions made across stages concerning lot sizes.

2.2. Types of Inventory Models

Inventory models come in all shapes, sizes, colors and varieties. In general, the assumptions that one makes about three key variables determines the essential structure of the model. These variables are demand, costs, and physical aspects of the system.

A. *Demand*: The assumptions that one makes about demand are usually the most important in determining the complexity of the model.

a. *Deterministic and stationary*: The simplest assumption is that the demand is constant and known. These are really two different assumptions: one, that the demand is not anticipated to change, and the other is that the demand can be predicted in advance. The simple EOQ model is based on constant and known demand.

b. *Deterministic and time varying*: Changes in demand may be systematic or unsystematic. Systematic changes are those that can be forecasted in advance. Lot sizing under time varying demand patterns is a problem that arises in the context of manufacturing final products from components and raw materials.

c. *Uncertain*: We use the term uncertainty to mean that the distribution of demand is known, but the exact values of the demand cannot be predicted in advance. In most contexts, this means that there is a history of past observations from which to estimate the form of the demand distribution and the values of the parameters. In some situations, such as with new products, the demand uncertainty could be assumed but some estimate of the probability distribution would be required.

d. *Unknown*: In this case even the distribution of the demand is unknown. The traditional approach in this case has been to assume some form of a distribution for the demand and update the parameter estimates using Bayes rule each time a new observation becomes available.

B. *Costs*: Since the objective is to minimize costs, the assumptions one makes about the cost structure are also important in determining the complexity of the model

a. *Averaging versus discounting:* When the time value of Money is considered, costs must be discounted rather than averaged.

b. *Structure of the order cost:* The assumptions that one makes about the order cost function can make a substantial difference in the complexity of the resulting model.

c. *Time varying costs:* Most inventory models assume that costs are time invariant. Time varying costs can often be included without increasing the complexity of the analysis.

d. *Penalty costs:* Most stochastic, and many deterministic models, include a specific penalty, p , for not being able to satisfy a demand when it occurs. In many circumstances p can be difficult to estimate. For that reason, in many systems one substitutes a service level for p . The service level is the acceptable proportion of demands filled from stock, or the acceptable proportion of order cycles in which all demand is satisfied.

C. *Other distinguishing physical aspects:* Inventory models are also distinguished by the assumptions made about various aspects of the timing and logistics of the model. Some of these include:

a. *Lead time assumptions:* The lead time is defined as the amount of time that elapses from the point that a replenishment order is placed until it arrives. The lead time is a very important quantity in inventory analysis; it is a measure of the system response time. The simplest assumption is that the lead time is zero. This is, of course, analytically expedient but not very realistic in practice. It makes sense only if the time required for replenishment is short compared with the time between reorder decisions.

The most common assumption is that the lead time is a fixed constant. The analysis is much more complicated if the lead time is assumed to be a random variable. Issues such as order crossing (that is, orders not arriving in the same sequence that they were placed), and independence must be considered.

b. *Backordering assumptions*: Assumptions are required about the way that the system reacts when demand exceeds supply. The simplest and most common assumption is that all exceeds demand is backordered. Backordered demand is represented by a negative inventory level. The other extreme is that all excess demand is lost. This latter case, known as lost sales, is most common in retailing environments.

Mixtures of backordering and lost sales have also been explored. Various alternatives exist for mixture models. One is that a fixed fraction of demands is backordered and a fixed lost. Another is that customers are willing to wait a fixed time for their orders to be filled.

c. *The review process*: Continuous review means that the level of inventory is known at all times. This has also been referred to as transactions reporting because it means that each demand transaction is recorded as it occurs. Modern supermarkets with scanning devices at the checkout counter are an example of this sort of system (assuming, of course, that the devices are connected to the computer used for stock replenishment decisions).

d. *Changes which occur in the inventory during storage*: Traditional inventory theory assumes that the inventory items do not change character while they are in stock.

2.2.1. The Basic EOQ Model

The assumptions of the basic EOQ model are :

1. Demand is known with certainty and fixed at λ units time.

2. Shortages are not permitted.
3. Lead time for delivery is instantaneous.
4. There is no time discounting of Money. The objective is to minimize average costs per unit time over an infinite time horizon.
5. Costs include K per order, and h per unit held per unit time.

2.2.1.1. Multiple Items EOQ Models

The classical EOQ model is for a single item.

* What happens when we have more than one item?

* Answer: Simply calculate the EOQ of each item, if there is no interaction among items.

i : 1,2, ...,m (number of items)

$$EOQ_i = \sqrt{\frac{2A_i D_i}{H_i}}$$

2.2.1.2. Resource Constrained Multiple Items EOQ Models

What if multiple items share common resources such as; Budget, Storage capacity, or both.

Then the $EOQ_i = \sqrt{\frac{2A_i D_i}{H_i}}$ procedure is no longer adequate because common resources are limited, and results may violate the resource constraints.

2.2.1.3. EOQ For Multiple Items With One Constraint

Suppose that we have a budget with the investment capacity of C than total investment in inventory shouldn't exceed C dollars.

Resource constraint:

$$\sum_{i=1}^m c_i Q_i \leq C$$

Where;

i = 1, 2, ..., m (number of items)
 Q_i = lot size of item i
 c_i = unit cost of item i
 C = maximum amount that will be invested

Objective:

To minimize the total annual inventory cost,

$$TC_i(Q) = c_i D + H_i \frac{Q_i}{2} + \frac{A_i D_i}{Q_i} \quad \text{total annual cost of item i}$$

$$TC = \sum_{i=1}^m TC_i(Q) \quad \text{total annual cost of all items}$$

Subject to:

$$\sum_{i=1}^m c_i Q_i \leq C$$

Lagrange Multiplier (λ) can be used to consider constraint in objective function (TC):

Lagrangian Function:

$$TC(Q, \lambda) = \sum_{i=1}^m TC_i(Q) + \lambda \left[\sum_{i=1}^m c_i Q_i - C \right]$$

The Lagrange multiplier acts as a penalty to reduce each Q*_i to minimize cost while enforcing the constraint.

Solution Procedure:

1. Solve the unconstrained problem:

Find: $EOQ_i = \sqrt{\frac{2A_i D_i}{H_i}}$ for $i = 1, 2, \dots, m$

If the constraint is satisfied this solution is the optimal one.

2. If this is not the case, set $TC(Q, \lambda)$.

$$TC(Q, \lambda) = \sum_{i=1}^m TC_i(Q) + \lambda \left[\sum_{i=1}^m c_i Q_i - C \right]$$

3. Obtain Q^*_i and λ^* by solving $(m+1)$ equations given by:

$$\frac{dTC(Q, \lambda)}{dQ_i} = 0 \quad \Rightarrow \quad \frac{-A_i D_i}{Q_i^2} + \frac{H_i}{2} + \lambda c_i = 0 \quad (1)$$

$$\frac{dTC(Q, \lambda)}{d\lambda} = 0 \quad \Rightarrow \quad \sum_{i=1}^m c_i Q_i = C \quad (2)$$

Solve (1) for Q^*_i

$$Q^*_i = \sqrt{\frac{2A_i D_i}{H_i + 2\lambda c_i}} \quad \text{for } i = 1, 2, \dots, m$$

Substitute Q^*_i into (2):

$$\sum_{i=1}^m c_i Q_i = C \quad (2)$$

$$Q^*_i = \sqrt{\frac{2A_i D_i}{H_i + 2\lambda c_i}} \quad \text{for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m c_i \sqrt{\frac{2A_i D_i}{H_i + 2\lambda c_i}} = C$$

Solve for λ^* and then determine Q^*_i values

EXAMPLE A

	Floppy Drive 1	Floppy Drive 2
A =	\$50	\$50
i =	20%	20%
c =	\$50	\$80
H =	\$10	\$16
D =	250	484

Investment budget is limited to \$5000.

$$50(Q_1) + 80(Q_2) \leq 5000$$

Step 1: Solve the unconstrained problem

$$EOQ_i = \sqrt{\frac{2A_i D_i}{H_i}}$$

$$EOQ_1 = 50 \text{ units}$$

$$EOQ_2 = 55 \text{ units}$$

$$50(50) + 80(55) = \$6900 > 5000$$

Resource constraint is violated, not optimal

Step 2: Obtain Q^*_i and λ^* by applying method of lagrange multiplier

$$Q^*_i = \sqrt{\frac{2A_i D_i}{H_i + 2\lambda c_i}} \quad \text{for } i = 1, 2$$

$$Q^*_1 = \sqrt{\frac{2(50)(250)}{10 + 2\lambda(50)}} = \sqrt{\frac{25000}{10 + 100\lambda}} = \frac{50}{\sqrt{1 + 10\lambda}}$$

$$Q^*_2 = \sqrt{\frac{2(50)(484)}{16 + 2\lambda(80)}} = \sqrt{\frac{48400}{16 + 160\lambda}} = \frac{55}{\sqrt{1 + 10\lambda}}$$

Constraint:

$$c_1 Q^*_1 + c_2 Q^*_2 = 5000$$

Step 3: Substitute Q^*_i into constraint to solve for λ^* :

$$Q^*_{_1} = \frac{50}{\sqrt{1+10\lambda}} \quad Q^*_{_2} = \frac{55}{\sqrt{1+10\lambda}}$$

Constraint:

$$c_1 Q^*_{_1} + c_2 Q^*_{_2} = 5000$$

$$50 \frac{50}{\sqrt{1+10\lambda}} + 80 \frac{55}{\sqrt{1+10\lambda}} = 5000$$

$$\frac{6900}{\sqrt{1+10\lambda}} = 5000 \quad \Rightarrow \quad \sqrt{1+10\lambda} = 1.38$$

Step 4: Determine $Q^*_{_1}$ values using λ^* :

$$Q^*_{_1} = \frac{50}{\sqrt{1+10\lambda}} \quad Q^*_{_2} = \frac{55}{\sqrt{1+10\lambda}}$$

$$\sqrt{1+10\lambda} = 1.38$$

$$Q^*_{_1} = \frac{50}{1.38} = 36 \text{ units}, \quad Q^*_{_2} = \frac{55}{1.38} = 40 \text{ units}$$

2.2.1.4. EOQ For Multiple Items With Two Constraint

Two common constraints in inventory systems are space and budget. When both are involved in the same system, we will extend the procedure to a two-constraint case:

The problem formulation:

$$\text{minimize } TC(Q) = \sum_{i=1}^m TC_i(Q) = \sum_{i=1}^m \left(c_i D + H_i \frac{Q_i}{2} + \frac{A_i D_i}{Q_i} \right)$$

subject to :
$$\sum_{i=1}^m c_i Q_i \leq C \quad (\text{Budget constraint})$$

$$\sum_{i=1}^m f_i Q_i \leq F \quad (\text{Space constraint})$$

$$Q_i \geq 0 \quad i = 1, 2, \dots, m$$

Solution Procedure:

1. Solve the unconstrained problem. If both constraints are satisfied, this solution is the optimal one.
2. Otherwise include one of the constraints, say budget, and solve a one-constraint problem to find Q_i . If the space constraint is satisfied, this solution is the optimal one.
3. Otherwise, repeat the process for only the space constraint.
4. If both single-constraint solutions do not yield the optimal solution, then both constraints are active, and the Lagrangian equation with both constraints must be solved:

$$TC(Q, \lambda_1, \lambda_2) = \sum_{i=1}^m TC_i(Q) + \lambda_1 \left[\sum_{i=1}^m c_i Q_i - C \right] + \lambda_2 \left[\sum_{i=1}^m f_i Q_i - F \right]$$

EXAMPLE B

Consider example A. Company has a total of 2000 units of space to store disk drives.

Disk drive 1 requires 25 units of space,
Disk drive 2 requires 40 units of space.

Problem Formulation:

$$\text{minimize } TC(Q) = \sum_{i=1}^m TC_i(Q) = 50(250) + 10 \frac{Q_1}{2} + \frac{50(250)}{Q_1} + 80(484) + 16 \frac{Q_2}{2} + \frac{50(484)}{Q_2}$$

Subject to: $50(Q_1) + 80(Q_2) \leq 5000 \quad (\text{budget constraint})$

$$25(Q_1) + 40(Q_2) \leq 2000 \quad (\text{space constraint})$$

$$Q_1, Q_2 \geq 0$$

Step 1: Solve the unconstrained problem

$$EOQ_i = \sqrt{\frac{2A_i D_i}{H_i}}$$

$$EOQ_1 = \sqrt{\frac{2(50)(250)}{10}} = 50 \text{ units /order}$$

$$EOQ_2 = \sqrt{\frac{2(50)(484)}{16}} = 55 \text{ units /order}$$

$$(1) \quad \text{Budget constraint:} \quad 50(50) + 80(55) = 6900 > 5000 \quad \text{not satisfied!}$$

$$(2) \quad \text{Space constraint:} \quad 25(50) + 40(55) = 3450 > 2000 \quad \text{not satisfied!}$$

EOQ values obtained here cannot be used!

Step 2: solve the problem only with budget constraint:

$$Q^*_i = \sqrt{\frac{2A_i D_i}{H_i + 2\lambda c_i}} \quad \text{for } i = 1, 2$$

$$Q^*_1 = \sqrt{\frac{2(50)(250)}{10 + 2\lambda(50)}} = \frac{50}{\sqrt{1 + 10\lambda}}$$

$$Q^*_2 = \sqrt{\frac{2(50)(484)}{16 + 2\lambda(80)}} = \frac{55}{\sqrt{1 + 10\lambda}}$$

Budget Constraint:

$$c_1 Q^*_1 + c_2 Q^*_2 = 5000$$

$$50 \frac{50}{\sqrt{1 + 10\lambda}} + 80 \frac{55}{\sqrt{1 + 10\lambda}} = 5000, \quad \sqrt{1 + 10\lambda} = 1.38$$

$$Q^*_1 = \frac{50}{\sqrt{1+10\lambda}} = \frac{50}{1.38} = 36 \text{ units /order}$$

$$Q^*_2 = \frac{55}{\sqrt{1+10\lambda}} = \frac{55}{1.38} = 40 \text{ units /order}$$

Check the budget constraint solution to see if it satisfies the space constraint:

Space constraint:

$$25(36) + 40(40) = 2500 > 2000 \text{ not satisfied!}$$

Step 3: Solve the problem only with space constraint:

$$Q^*_i = \sqrt{\frac{2A_i D_i}{H_i + 2\lambda f_i}} \quad \text{for } i = 1, 2$$

$$Q^*_1 = \sqrt{\frac{2(50)(250)}{10 + 2\lambda(25)}} = \frac{50}{\sqrt{1+5\lambda}}$$

$$Q^*_2 = \sqrt{\frac{2(50)(484)}{16 + 2\lambda(40)}} = \frac{55}{\sqrt{1+5\lambda}}$$

Space Constraint:

$$f_1 Q^*_1 + f_2 Q^*_2 = 2000$$

$$25 \frac{50}{\sqrt{1+5\lambda}} + 40 \frac{55}{\sqrt{1+5\lambda}} = 2000 \quad \sqrt{1+5\lambda} = 1.73$$

$$Q^*_1 = \frac{50}{\sqrt{1+5\lambda}} = \frac{50}{1.73} = 28 \text{ units /order}$$

$$Q^*_2 = \frac{55}{\sqrt{1+5\lambda}} = \frac{55}{1.73} = 32 \text{ units /order}$$

$$(1) \quad \text{Budget constraint:} \quad 50(28) + 80(32) = 3960 < 5000 \text{ satisfied!}$$

$$(2) \quad \text{Space constraint:} \quad 25(28) + 40(32) = 1980 < 2000 \text{ satisfied!}$$

Optimal solution:

$$Q^*_1 = 28 \text{ units /order} \quad Q^*_2 = 32 \text{ units /order}$$

Compare three different set of Q_i values in terms of total annual inventory cost.

Q_1	Q_2	TC(Q)	Budget Constraint	Space Constraint	Increase (%)
50	55	52600	infeasible	infeasible	0
36	40	52672	feasible	infeasible	0.14
28	32	52819	feasible	feasible	0.42

2.2.2. Dynamic Lot Sizing Models

Dynamic lot sizing will be preferred when demand is lumpy, that is, demand is not uniform during the planning horizon.

Methods:

1. **Period order quantity**
The average lot size desired is divided by the average period demand
2. **Fixed period demand**
Ordering m periods of demand, m = selected fixed period
3. **Lot for lot (L4L)**
The order quantity is always the demand for one period
4. **Silver-Meal method (SM)**
Heuristic approach to aim at a low-cost solution that is not necessarily optimal
5. **Wagner-Whitin**
An optimization approach to lumpy demand

2.2.2.1. Example For Dynamic Lot Sizing Models

Suppose for a certain product type you need to produce weekly demand below:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

A = \$50 per order

H = \$0.5 per unit per week

Assumption:

Lead time is known with certainty (fixed lead time)

2.2.2.1.1. Period Order Quantity

The average lot size desired is divided by the average period demand

Example:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

For weekly demand given above evaluate POQ for Q = 125 units, 140 units, and 275 units.

Average weekly demand = $975 / 8 = 122$ units per week

For Q = 125

T = fixed period between orders = $125 / 122 = 1.02 = 1$ week

For Q = 140

T = fixed period between orders = $140 / 122 = 1.14 = 1$ week

For Q = 275

T = fixed period between orders = $250 / 122 = 2.25 = 2$ weeks

Lot size (Q) = 125 units per order, fixed period between orders (T) = 1 week

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	125	25
2	25	75	125	75
3	75	175	125	25
4	25	200	125	-50
5	-50	150	125	-75
6	-75	100	125	-50
7	-50	75	125	0
8	0	100	125	25

Shortage occurs in weeks 4,5 and 6. If shortage is not allowed, this ordering policy is not acceptable.

Lot size (Q) = 140 units per order, fixed period between orders (T) =1 week

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	140	40
2	40	75	140	105
3	105	175	140	70
4	70	200	140	10
5	10	150	140	0
6	0	100	140	40
7	40	75	140	105
8	105	100	140	145

Total inventory cost = 8 orders (\$50 /order) + 515 units (\$0.5 /unit)
= \$657.5

Lot size (Q) = 275 units per order,

Fixed period between orders (T) = 2 weeks

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	275	175
2	175	75		100
3	100	175	275	200
4	200	200		0
5	0	150	275	125
6	125	100		25
7	25	75	275	225
8	225	100		125

Total inventory cost = 4 orders (\$50 /order) + 975 units (\$0.5 /unit)
= \$687.5

2.2.2.1.2. Fixed Period Demand

Ordering m periods of demand, m = selected fixed period

Example:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

For weekly demand given above evaluate FPD for T = 2 weeks, 4 weeks, and 8 weeks.

For T = 2 weeks

$Q_1 = 175$ units, $Q_2 = 375$ units, $Q_3 = 250$ units, $Q_4 = 175$ units

For T = 4 weeks

$Q_1 = 550$ units, $Q_2 = 425$ units

For T = 8 weeks

$Q_1 = 975$ units

Fixed period between orders: (T) = 2 weeks

Lot size: $Q_1 = 175$ units, $Q_2 = 375$ units, $Q_3 = 250$ units, $Q_4 = 175$ units

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	175	75
2	75	75		0
3	0	175	375	200
4	200	200		0
5	0	150	250	100
6	100	100		0
7	0	75	175	100
8	100	100		0

Total inventory cost = 4 orders (\$50 /order) + 475 units (\$0.5 /unit)
= \$437.5

Fixed period between orders: (T)= 4 weeks

Lot size: $Q_1 = 550$ units, $Q_2 = 425$ units

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	550	450
2	450	75		375
3	375	175		200
4	200	200		0
5	0	150	425	275
6	275	100		175
7	175	75		100
8	100	100		0

Total inventory cost = 2 orders (\$50 /order) + 1575 units (\$0.5 /unit) = \$887.5

Fixed period between orders: (T)= 8 weeks

Lot size: $Q_1 = 975$ units

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	975	875
2	875	75		800
3	800	175		625
4	625	200		425
5	425	150		275
6	275	100		175
7	175	75		100
8	100	100		0

Total inventory cost = 1 order (\$50 /order) + 3275 units (\$0.5 /unit)
= \$1687.5

2.2.2.1.3. Lot For Lot Rule (L4L)

The order quantity is always the demand for one period

Example:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

For weekly demand given above evaluate L4L rule.

Lot size per order:

$Q_1 = 100$ units, $Q_2 = 75$ units, $Q_3 = 175$ units, $Q_4 = 200$ units

$Q_5 = 150$ units, $Q_6 = 100$ units, $Q_7 = 75$ units, $Q_8 = 100$ units

Week	Beginning Inventory	Demand	Order	End Inventory	
1	0	100	100	0	
2	0	75	75	0	
3	0	175	175	0	
4	0	200	200	0	
5	0	150	150	0	
6	0	100	100	0	
7	0	75	75	0	
8	0	100	100	0	
					Total
inventory cost = 8 orders (\$50 /order) + 0 (\$0.5 /unit)					
= \$400					

2.2.2.1.4. Silver-Meal Method

Aim to achieve the minimum average cost per period for the m-period span.

The average cost per period includes ordering and inventory holding costs and expressed as given below:

$$K(m) = \frac{1}{m} (A + HD_1 + 2HD_2 + \dots + (m-1)HD_m)$$

Where;

- m = number of demand periods to be ordered in the present time.
- A = fixed ordering cost per order
- H = inventory holding cost per unit per period
- K(m) = average cost per period during m periods

Solution procedure:

Compute K(m) for m = 1,2,...,m

Stop when, $K(m+1) > K(m)$, i.e. the period in which the average cost per period start to increase.

Order the quantity equals to m periods demand.

$$Q_i = D_1 + D_2 + \dots + D_m$$

Q_i is the quantity ordered in period i, and it covers m periods into the future. The process repeats at period (m+1) and continues through the planning horizon.

Example:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

A = \$50 per order H = \$0.5 per unit per week

Apply the Silver-Meal method to place orders.

Solution:

For Q_1 :

1. $m=1$ $K(1) = 50$
2. $m=2$ $K(2) = 1/2 (50 + 0.5(75)) = 43.75 < K(1)$
3. $m=3$ $K(3) = 1/3 (50 + 0.5(75) + (2)(0.5)(175)) = 87.6 > K(2)$
 STOP

$$Q_1 = 100 + 75 = 175 \text{ units}$$

Next order should arrive in week 3, so continue for Q_3 .

For Q_3 :

1. $m=1$ $K(1) = 50$
2. $m=2$ $K(2) = 1/2 (50 + 0.5(200)) = 75 > K(1)$
 STOP

$Q_3 = 175$ units next order should arrive in week 4, so continue for Q_4 .

For Q_4 :

1. $m=1$ $K(1) = 50$
2. $m=2$ $K(2) = 1/2 (50 + 0.5(150)) = 62.5 > K(1)$
 STOP

$Q_4 = 200$ units next order should arrive in week 5, so continue for Q_5 .

For Q_5 :

1. $m=1$ $K(1) = 50$
2. $m=2$ $K(2) = 1/2 (50 + 0.5(100)) = 50 \leq K(1)$
3. $m=3$ $K(3) = 1/3 (50 + 0.5(100) + (2)(0.5)(75)) = 58.3 > K(2)$
 STOP

$$Q_5 = 150 + 100 = 250 \text{ units}$$

Next order should arrive in week 7, so continue for Q_7 .

For Q_7 :

1. $m=1$ $K(1) = 50$
2. $m=2$ $K(2) = 1/2 (50 + 0.5(100)) = 50 \leq K(1)$
All demand periods are considered! STOP.

$$Q_7 = 75 + 100 = 175 \text{ units}$$

Order :	1	2	3	4	5
Week :	1	3	4	5	7
Quantity (lot size) :	175	175	200	250	175

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	175	75
2	75	75		0
3	0	175	175	0
4	0	200	200	0
5	0	150	250	100
6	100	100		0
7	0	75	175	100
8	100	100		0

$$\begin{aligned} \text{Total inventory cost} &= 5 \text{ orders } (\$50 / \text{order}) + 275 (\$0.5 / \text{unit}) \\ &= \$387.5 \end{aligned}$$

~~Wagner-Whitin~~

2.2.2.1.5. Wagner-Whitin Algorithm

The original formulation of this problem is due to Wagner & Whitin [1958] (henceforth referred to as WW). They assume :

- a. Shortages are not permitted.
- b. Starting inventory is zero. This assumption may be relaxed by netting out starting inventory from the first period's (or additional period's, if necessary) demand.

c. Only linear costs of holding and fixed order costs are present. Note that means that the total cost function is concave. WW's results hold under the more general assumption:

d. The holding cost is a concave function of the ending inventory in each period and the ordering cost is a concave function of the order quantity in each period.

WW is an optimization procedure based on dynamic programming to find optimum order quantity policy Q_i with a minimum cost solution.

WW evaluates all possible ways of ordering to cover demand in each period of the planning horizon.

Wagner-Whitin replaces EOQ for the case of lumpy demand.

Cost of placing order:

$$K(t, m) = A + H \left(\sum_{j=t+1}^m (j - t) D_j \right)$$

Where;

$K(t, m)$ = total cost of quantity ordered for m periods ahead,

A = ordering cost,

H = inventory holding cost per unit per period,

D_j = demand at period j

$t = 1, 2, \dots, N$ and $m = t+1, t+2, \dots, N$

For each period minimum cost is defined as:

$$K^*(m) = \min_{t=1,2,\dots,m} \{K^*(t-1) + K(t, m)\}$$

$K^*(0) = 0$ and $K^*(N)$ is defined as the least cost solution.

Example:

Weeks	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100

$A = \$50$ per order $H = \$0.5$ per unit per week

Apply the Wagner-Whitin algorithm to determine optimal order quantities.

Solution:

For $m=1, t=1$

$$K(1,1) = 50 + 0.5 \left(\sum_{j=1+1}^1 (j-1)D_j \right) = 50 + 0 = 50$$

$$K^*(1) = K^*(0) + K(1,1) = 0 + 50 = 50$$

For $m=2, t=1,2$

$$K(1,2) = 50 + 0.5 \left(\sum_{j=1+1}^2 (j-1)D_j \right) = 50 + 0.5(75) = 87.5$$

$$K(2,2) = 50 + 0.5 \left(\sum_{j=2+1}^2 (j-2)D_j \right) = 50 + 0 = 50$$

$$K^*(2) = \min \begin{cases} K^*(0) + K(1,2) = 0 + 87.5 = 87.5 \\ K^*(1) + K(2,2) = 50 + 50 = 100 \end{cases}$$

$$K^*(2) = 87.5$$

For $m=3, t=1,2,3$

$$K(1,3) = 50 + 0.5(75 + (2)175) = 262.5$$

$$K(2,3) = 50 + 0.5(175) = 137.5$$

$$K(3,3) = 50$$

$$K^*(3) = \min \begin{cases} K^*(0) + K(1,3) = 0 + 262.5 = 262.5 \\ K^*(1) + K(2,3) = 50 + 137.5 = 187.5 \\ K^*(2) + K(3,3) = 87.5 + 50 = 137.5 \end{cases}$$

$$K^*(3) = 137.5$$

For $m=4$, $t=1,2,3,4$

$$K(1,4) = 50 + 0.5(75 + (2)175 + (3)200) = 562.5$$

$$K(2,4) = 50 + 0.5(175 + (2)200) = 337.5$$

$$K(3,4) = 50 + 0.5(200) = 150$$

$$K(4,4) = 50$$

$$K^*(4) = \min \begin{cases} K^*(0) + K(1,4) = 0 + 562.5 = 562.5 \\ K^*(1) + K(2,4) = 50 + 337.5 = 387.5 \\ K^*(2) + K(3,4) = 87.5 + 150 = 237.5 \\ K^*(3) + K(4,4) = 137.5 + 50 = 187.5 \end{cases}$$

$$K^*(4) = 187.5$$

For $m=5$, $t=1,2,3,4,5$

$$K(1,5) = 50 + 0.5(75 + (2)175 + (3)200 + (4)150) = 862.5$$

$$K(2,5) = 50 + 0.5(175 + (2)200 + (3)150) = 562.5$$

$$K(3,5) = 50 + 0.5(200 + (2)150) = 300$$

$$K(4,5) = 50 + 0.5(150) = 125$$

$$K(5,5) = 50$$

$$K^*(5) = \min \begin{cases} K^*(0) + K(1,5) = 0 + 862.5 = 862.5 \\ K^*(1) + K(2,5) = 50 + 562.5 = 612.5 \\ K^*(2) + K(3,5) = 87.5 + 300 = 387.5 \\ K^*(3) + K(4,5) = 137.5 + 125 = 262.5 \\ K^*(4) + K(5,5) = 187.5 + 50 = 237.5 \end{cases}$$

$$K^*(5) = 237.5$$

For $m=6$, $t=1,2,3,4,5,6$

$$K(1,6) = 50 + 0.5(75 + (2)175 + (3)200 + (4)150 + (5)100) = 1112.5$$

$$K(2,6) = 50 + 0.5(175 + (2)200 + (3)150 + (4)100) = 762.5$$

$$K(3,6) = 50 + 0.5(200 + (2)150 + (3)100) = 450$$

$$K(4,6) = 50 + 0.5(150 + (2)100) = 225$$

$$K(5,6) = 50 + 0.5(100) = 100$$

$$K(6,6) = 50$$

$$K^*(6) = \min \begin{cases} K^*(0) + K(1,6) = 0 + 1112.5 = 1112.5 \\ K^*(1) + K(2,6) = 50 + 762.5 = 812.5 \\ K^*(2) + K(3,6) = 87.5 + 450 = 537.5 \\ K^*(3) + K(4,6) = 137.5 + 225 = 362.5 \\ K^*(4) + K(5,6) = 187.5 + 100 = 287.5 \\ K^*(5) + K(6,6) = 237.5 + 50 = 287.5 \end{cases}$$

$$K^*(6) = 287.5$$

For $m=7, t=1,2,3,4,5,6,7$

$$K(1,7) = 50 + 0.5(75 + (2)175 + (3)200 + (4)150 + (5)100 + (6)75) = 1337.5$$

$$K(2,7) = 50 + 0.5(175 + (2)200 + (3)150 + (4)100 + (5)75) = 950$$

$$K(3,7) = 50 + 0.5(200 + (2)150 + (3)100 + (4)75) = 600$$

$$K(4,7) = 50 + 0.5(150 + (2)100 + (3)75) = 337.5$$

$$K(5,7) = 50 + 0.5(100 + (2)75) = 250$$

$$K(6,7) = 50 + 0.5(75) = 87.5$$

$$K(7,7) = 50$$

$$K^*(7) = \min \begin{cases} K^*(0) + K(1,7) = 0 + 1337.5 = 1337.5 \\ K^*(1) + K(2,7) = 50 + 950 = 1000 \\ K^*(2) + K(3,7) = 87.5 + 600 = 687.5 \\ K^*(3) + K(4,7) = 137.5 + 337.5 = 475 \\ K^*(4) + K(5,7) = 187.5 + 250 = 437.5 \\ K^*(5) + K(6,7) = 237.5 + 87.5 = 325 \\ K^*(6) + K(7,7) = 287.5 + 50 = 337.5 \end{cases}$$

$$K^*(7) = 325$$

For $m=8, t=1,2,3,4,5,6,7,8$

$$K(1,8) = 50 + 0.5(75 + (2)175 + (3)200 + (4)150 + (5)100 + (6)75 + (7)100) = 1687.5$$

$$K(2,8) = 50 + 0.5(175 + (2)200 + (3)150 + (4)100 + (5)75 + (6)100) = 1250$$

$$K(3,8) = 50 + 0.5(200 + (2)150 + (3)100 + (4)75 + (5)100) = 850$$

$$K(4,8) = 50 + 0.5(150 + (2)100 + (3)75 + (4)100) = 537.5$$

$$K(5,8) = 50 + 0.5(100 + (2)75 + (3)100) = 325$$

$$K(6,8) = 50 + 0.5(75 + (2)100) = 187.5$$

$$K(7,8) = 50 + 0.5(100) = 100$$

$$K(8,8) = 50$$

$$K^*(8) = \min \begin{cases} K^*(0) + K(1,8) = 0 + 1687.5 = 1687.5 \\ K^*(1) + K(2,8) = 50 + 950 = 1300 \\ K^*(2) + K(3,8) = 87.5 + 850 = 937.5 \\ K^*(3) + K(4,8) = 137.5 + 537.5 = 675 \\ K^*(4) + K(5,8) = 187.5 + 325 = 512.5 \\ K^*(5) + K(6,8) = 237.5 + 187.5 = 425 \\ K^*(6) + K(7,8) = 287.5 + 100 = 387.5 \\ K^*(7) + K(8,8) = 325 + 50 = 375 \end{cases}$$

$K^*(8) = 375$

Period t:	1	2	3	4	5	6	7	8
Demand	100	75	175	200	150	100	75	100
m:	1	2	3	4	5	6	7	8
1	50	87.5	262.5	562.5	862.5	1112.5	1337.5	1687.5
2		100	187.5	387.5	612.5	812.5	1000	1300
3			137.5	237.5	387.5	537.5	687.5	937.5
4				187.5	262.5	362.5	475	675
5					237.5	287.5	362.5	512.5
6						287.5	325	425
7							337.5	387.5
8								375
min	50	87.5	137.5	187.5	237.5	287.5	325	375
t*,m*	---	1,2	3,3	4,4	5,5	---	6,7	8,8
i:	1	2	3	4	5	6	7	8
Q _i :	175	0	175	200	150	175	0	100

We placed an order in period 6 for demand in weeks 6 and 7, so next we will examine period 5.

Order :	1	2	3	4	5	6
Week :	1	3	4	5	6	8
Quantity (lot size) :	175	175	200	150	175	100

Week	Beginning Inventory	Demand	Order	End Inventory
1	0	100	175	75
2	75	75		0
3	0	175	175	0
4	0	200	200	0

5	0	150	150	0
6	0	100	175	75
7	75	75		0
8	0	100	100	0

$$\begin{aligned}\text{Total inventory cost} &= 6 \text{ orders } (\$50 / \text{order}) + 150 (\$0.5 / \text{unit}) \\ &= \$375\end{aligned}$$

2.2.3. The Model by Crowston, Wagner, and Williams

QaM then installation stock is created at F_n and the average level of such inventory is a complicated function of Q^{TM} and $Q_{a(s)}$. We show the installation stock at each stage of a 3-stage serial production process with $Q_1 = 6Q_3$ and $Q_2 = 2Q_3$. The echelon stock for each stage of the system is shown to form the familiar sawtooth pattern of the ordinary Wilson lot size formula. Given the assumption of constant demand, the average echelon inventory at stage F_n is $(Q_n - 1)/2$. Thus the total holding and setup cost for the echelon stock will be

$$(1) f_n(Q_n) = DS_n / (Q_n) + (Q_n - 1) h_n / 2$$

and the total cost for the system, s , will be

$$(2) T = \sum f_n(Q_n)$$

This may be rewritten as

$$(3) T = \sum \{ DS_n / K_n Q_N + (K_n Q_{N-1}) h_n / 2 \}$$

For a particular vector K^j the optimal value Q_N would be

$$(4) Q_N^j = \{ (2D \sum S_n / K_n^j) / \sum h_n K_n^j \}^{1/2}$$

2.2.3.1. Simple Extensions of the Model

In this section we briefly consider a special case of noninstantaneous production and the case of transfer delay between stages. If we

assume production rate p_n at F_n and given $p_n \geq p_{0n}$ then the result of theorem 1 applies.

$$(5) TC_n = DS_n / K_n Q_n + [(K_n Q_n - 1) / 2] [1 - D / p_n] h_n$$

$(2DS^*/H_n)^m$. This implies the carrying cost of a unit of in-process inventory of F_n is a function of the total value of its components. Our model indicates that this results in double-counting. Finally, we would suggest that if heuristic decision rules are constructed for the more complicated case of multiple successors, incremental holding costs are again appropriate.

2.2.4. Reorder Cycle Time Problems

Based on the traditional Economic Order Quantity model showed before, the time between two consecutive orders, called reorder interval, is constant and proportional to the order quantity. The lot sizing problem can be formulated as the determination of the reorder cycle interval. $T=Q/D$, and ignoring the production cost (because it won't affect for the comparison), the optimum reorder interval can be derived. As mentioned before, this problem is continuous and the reorder optimal interval can take any positive real value. However, their solution presents some difficulties. This is the reason to solve it as a discrete problem, imposing the restriction that the reorder interval can take only positive integer values.

There are several reasons to formulate the lot sizing model in terms of reorder intervals as described before. A lot of authors have been developing new techniques to solve the problem in terms of this point of view; some of the most important are mentioned here.

Elmaghraby (1978) analyzed the economic lot scheduling problem (ELSP), which arises from the desire to accommodate the cyclical production pattern when several products are made on a single facility. This work reviews the contributions to the problem, and extends the analysis in four directions: (1) offers an improved

analytical approach based on dynamic programming. It tries to guarantee feasibility at the outset, by imposing some constraints on the cycle times, then to optimize the individual cycle duration subject to the imposed constraint. The solution obtained in this manner is feasible and optimal over its set of solutions; (2) a test of feasibility of a given set of parameters, through an integer linear programming formulation; (3) a systematic procedure for escape from infeasibility, when the set of parameters were judge infeasible; and (4) a procedure for the determination of a basic period for a given set of multipliers to achieve a feasible schedule.

Szendrovits (1975) presented the functional relationship between the production lot size, the manufacturing cycle time and the average process inventory in a production system, and illustrated the resulting effect on the conventional Economic Lot Quantity model. He treats the manufacturing cycle time as a function of the lot size in a multistage production system. This model was called the economic production quantity (EPQ). This study challenges the widely accepted doctrine of the efficiency of long production runs.

Roundy (1985) introduced two simple policies called q-optimal integer-ratio and optimal power-of-two, which are proved to be 94% and 98% effective. The effectiveness of a policy is 100% times the ratio of the minimum of the average cost over all policies to the average cost of the policy in question. Both policies are very efficient and their most important advantage is the flexibility it allows in choosing the order intervals to correspond to easily-implemented time periods.

The power-of-two policy is a special case of the discrete problem for determining the reorder cycle time, in which the reorder interval is constraint to be not only integer, but also a multiple of two. It allows us to obtain an extremely efficient algorithm which produces a policy having an average cost within 2% of the minimum possible. *Mitchell (1987)* extended Roundy's results for the backlogging problem, obtaining a 98% effective policy for the backlogging problem in $O(N \log N)$ time.

Maxwell and Muckstadt (1985) presented an algorithm that can be used to find consistent and realistic reorder intervals for each item in large-scale production distribution systems. Attention was restricted to policies that are nested, stationary, and a power-of-two multiple of a base planning period. The model that results from the assumptions is an integer nonlinear programming problem. It was showed that the solution to this problem is similar to that of the economic lot size problem with a modified echelon holding cost for an operation, to reflect the precedence constraints of the production-distribution system.

Roundy (1986) studied a multi-product multi-stage production inventory system in continuous time. In process and finished goods were referred to as products and inventories of a single item held at different locations were treated as different products. External demand can occur for any or all of the products at a constant, product-dependent rate. In the new policy defined by Roundy each product uses a stationary interval of time between successive orders, and the ratio of the order intervals of any two products is an integer power of two. The effectiveness of an optimal power-of-two policy is at least 98%. The algorithm is efficient for very large systems.

Jackson, Maxwell and Muckstadt (1988) had reviewed the Maxwell and Muckstadt (1985) model, proving a useful invariance property of the optimal partition of such systems, and used these results as the basis for algorithms to solve a capacitated version of the Maxwell-Muckstadt model. They suggest that the algorithm perform well in cases characterized by many operations per work center, however this reasoning was based on limited argument and experience with practical examples. This approach can be effectively used to establish reorder intervals in many industrial environments.

The power-of-two policy has been extended to solve more complex problems, showing that it maintains its effectiveness. One of the major complications in managing multi-item inventory systems stems from the fact that various

components, in particular, setup costs, are often jointly incurred between several distinct items. It is presented two cases with joint setup costs where power-of-two policy were applied successfully.

Jackson, Maxwell and Muckstadt (1985) presented an efficient procedure for the joint replenishment problem under the restriction that the reorder intervals must be power of two times a base period length. To solve the joint replenishment problem requires answering two questions: (1) what is the optimal time between major setups? , and (2) what is the optimal reorder interval for each item. They demonstrate by analytic means rather than experimentation that the worst case performance is within 6% of optimality. The performance bound is more than adequate given the typical errors in estimates of the setup costs, the holding costs, and the demand rate.

Federgruen and Zheng (1992) extended the results obtained by Roundy (1985) to a general joint setup cost structure. The joint cost structure often reflects economies of scale which invoke the need for careful coordination of the items replenishment strategies, and the joint replenishment problem is the most multi-item inventory model with joint setup costs. They derived two efficient algorithms to compute an optimal power-of-two policy. The problem of determining the optimal power-of-two policy can be formulated as a nonlinear mixed integer program.

Federgruen, Zheng and Queyranne (1992) generalized Roundy's results. They considered a production-distribution network represented by a general directed acyclic network showing that the power-of-two policies are close to optimal in a general class of production-distribution networks with general joint setup costs.

2.2.5. Power – of – two Policy

The power-of-two policy has been used in industry for many years, and extensive research studies on the efficiency of this restriction have been done. Based

on that, the present approach includes the power of two restrictions. The new approach is compared with the methodology for the implementation of the power-of-two policy, presented *Maxwell and Muckstadt (1985)* as a nonlinear integer problem. One of the main contributions of this research is a methodology that could be easily implemented particularly in industrial applications, and that could be used to develop future studies including additional restrictions as capacity constraints.

It is shown that several research studies have been done for many years focusing in lot size determination for single stage systems like the classic Economic Lot Size Model by Harry Ford, and multi-stage inventory systems as *Clark and Scarf (1960)*, *Afentakis and Gavish (1986)* and *Schwarz (1973)*. Some applications for multi-stage models make use of a myopic strategy were the objective function is optimized based on any two stages, as done by Schwarz and Schrage (1975).

After the formulation of the lot sizing problem as the problem of determining the reorder cycle time, a lot of authors have been developed new techniques, like Elmaghraby (1978) who proposed an analytical approach based on dynamic programming. Moreover, Roundy (1985) introduced two policies called q-optimal integer-ratio and optimal power-of-two, which are proved to be 94% and 98% effective. The power-of-two policy is a special case of a discrete problem for determining the reorder cycle time, in which the reorder interval is constraint to be not only integer, but also a power of two. Consequently, *Maxwell and Muckstadt (1985)*, *Roundy (1986)* and Federgruen and Zheng (1992), proved the advantages of this policy applying it to problems with additional restrictions.

The next chapter describes algorithm developed to solve a problem of determining the reorder cycle time determination in multi-stage serial system, considering the power-of-two restrictions.

3. EMPIRICAL STUDY

3.1. Purpose & Scope Of The Study

3.1.1. Purpose

The principal objective of this research is to find a solution to the problem of determining the total cost in multi-stage serial systems in the production process of porcelain substances using the Szendrovits, Andrew Z. algorithm approach, satisfying the power of two restrictions. Other secondary objective is:

To determine the effectiveness of the power of two approach, comparing the results obtained in the dissertation prepared by Faik Başaran in 1993.

3.1.2. Scope

The algorithm developed is based on the assumption of a multi-stage serial system. A stage might consist of an operation such as procurement of raw materials or fabrication of parts. The serial structure is the simplest type of multi-stage structures in which materials enter the first (1) stage and progressively pass through a sequence of stages until final product exits at the last (6) stage. These stages of our case are Lathe or Molding Clay Preparation (changes up to the product produced), Lathe or molding (changes up to the product produced), Glaze Preparation, Glazing Kilns, Technical Decoration and finally Technical Decoration Kilns.

Demand for each end item is assumed to occur at a constant and continuous rate, and is given for a planning horizon of n periods. Production is instantaneous and no backorders are allowed and unconstrained capacity at each node is assumed. The cost function is composed by the fixed setup cost and the holding cost. Fixed setup costs and echelon holding costs are changed at each stage.

It is assumed that the cycle length should satisfy the power of two restrictions, which applies zero inventory ordering and stationary-nested policies. A stationary policy is one in which each facility uses a fixed order quantity and a fixed interval time between successive orders. In a nested policy each facility orders every time any of its suppliers orders.

The organization of the document is as follows. Chapter II describes a review of the most important contributions in lot sizing problems for single and multi-stage models, for reorder cycle time models, including some approaches with the power of two restrictions, and the application of the algorithms. In the last chapter, the results of an empirical study carried out in Yıldız Porcelain Factory have been presented.

3.2. History of Porcelain

Porcelain is a type of hard semi-translucent ceramic generally fired at a higher temperature than glazed earthenware, or stoneware pottery. It is white, but mildly translucent and can be decorated to provide colour. Porcelain, pronounced POUR suhlihn, is a type of ceramics highly valued for its beauty and strength. It is often called china, or chinaware, because it was first made in China. Porcelain is characterized by whiteness, a delicate appearance, and translucence (ağabeylity to let light through). Because it is the hardest ceramic product, porcelain is used for electrical insulators and laboratory equipment. However, porcelain is known primarily as a material for high-quality vases and tableware, as well as for figurines and other decorative objects. The type of porcelain that is used for such purposes produces a bell-like ring when struck.

Porcelain differs from other types of ceramics in its ingredients and in the process by which it is produced. Two common types of ceramics--earthenware and stoneware—are made from a single natural clay, which is then fired (baked). In many cases, the object is coated with a glassy substance called glaze. Firing at a low temperature produces earthenware, a porous material. Earthenware can be made

waterproof by glazing. Firing at a high temperature produces stoneware, a hard, heavy material. Stoneware is nonporous without glazing.

Unlike earthenware and stoneware, porcelain is basically made from a mixture of two ingredients-kaolin and petuntse. Kaolin is a pure white clay that forms when the mineral feldspar breaks down. Petuntse is a type of feldspar found only in China. It is ground to a fine powder and mixed with kaolin. This mixture is fired at temperatures from about 2280_F (1250_C) to 2640_F (1450_C). At these extreme temperatures, the petuntse vitrifies that is, it melts together and forms a nonporous, natural glass. The kaolin, which is highly resistant to heat, does not melt and therefore allows the item to hold its shape. The process is complete when the petuntse fuses itself to the kaolin. The Chinese probably made the first true porcelain during the Tang dynasty (618-907). The techniques for combining the proper ingredients and firing the mixture at extremely high temperatures gradually developed out of the manufacture of stoneware. During the Song dynasty (960-1279), Chinese emperors started royal factories to produce porcelain for their palaces. Since the 1300's, most Chinese porcelain has been made in the city of Jingdezhen.

For centuries, the Chinese made the world's finest porcelain. Collectors regard many porcelain bowls and vases produced during the Ming dynasty (1368-1644) and Qing dynasty (1644-1912) as artistic treasures. Porcelain makers perfected a famous blue and white underglazed porcelain during the Ming period. Painting over the glaze with enamel colors also became a common decorating technique at this time. During the Qing period, the Chinese developed a great variety of patterns and colors and exported porcelain objects to Europe in increasing numbers. By the 1100's, the secret of making porcelain had spread to Korea and to Japan in the 1500's. Workers in these countries also created beautiful porcelain objects. A Japanese porcelain called Kakiemon was first produced during the 1600's. It features simple designs on a white background. Another well-known Japanese porcelain called Imari ware, or Arita, is famous for its dense decorations in deep blue and red.

European porcelain; as early as the 1100's, traders brought Chinese porcelain to Europe, where it became greatly admired. However, it was so rare and expensive that only wealthy people could afford it. As trade with the Orient grew during the 1600's, porcelain became popular with the general public. The custom of drinking tea, coffee, and chocolate became widespread and created a huge demand for porcelain cups and saucers. European manufacturers responded by trying to make hard-paste porcelain themselves, but for a long time they failed to discover the secret. Nevertheless, some of their experiments resulted in beautiful soft-paste porcelain. The first European soft-paste porcelain was produced in Florence, Italy, about 1575. By the 1700's, porcelain manufactured in many parts of Europe was starting to compete with Chinese porcelain. France, Germany, Italy, and England became the major centers for European porcelain production.

French porcelain; France became famous during the 1700's as the leading producer of soft-paste porcelain. The first factories were established at Rouen, St. Cloud, Lille, and Chantilly. The most celebrated type of soft-paste porcelain was first produced at Vincennes in 1738. In 1756, the factory was moved to the town of Sevres. Its soft-paste porcelain became known as Sevres. The earliest Sevres had graceful shapes and soft colors. Sevres pieces produced from 1750 to 1770 were decorated with brilliant colors and heavy gilding. Many of these pieces had richly colored backgrounds and white panels painted with birds, flowers, landscapes, or people. Sevres is also noted for its fine figurines of biscuit (unglazed porcelain). Beginning in 1771, a hard-paste porcelain industry developed near Limoges, where kaolin deposits had been discovered. By the 1800's, Limoges had become one of the largest porcelain centers in Europe. An American named David Haviland opened a porcelain factory at Limoges in 1842 to make tableware for the American market. Haviland porcelain features soft colors that blend together and small floral patterns.

German porcelain; A German chemist named Johann Friedrich Bottger discovered the secret of making hard-paste porcelain in 1708 or 1709. This discovery led to the establishment of a porcelain factory in Meissen in 1710. Meissen porcelain is sometimes called Dresden because Bottger first worked near the city. For nearly a

century, it surpassed in quality all other hard-paste porcelain made in Europe. The great success of Meissen porcelain can be partly attributed to the fine artists who decorated it. They painted the wares with an amazing variety of colors and designs. Johann Horoldt (or Heroldt), who became chief painter in 1720, produced beautiful Chinese and Japanese as well as European designs. Johann Kandler, who worked from about 1730 to 1770, is famous for his exquisite figures of animals and people. Political disorder in Germany and competition from Sevres porcelain drove the Meissen factory into decline during the late 1700's. It continued to operate but did not make wares of the same artistic quality.

English porcelain; England is well known as the center for the production of bone china. Before the invention of bone china, the English manufactured fine soft-paste porcelain at Chelsea, Bow, and Derby. Most of this English porcelain was styled after Oriental and Continental designs. Worcester porcelain, first produced in 1751, is one of the oldest and best English porcelains. During its early years, the Worcester factory produced soft-paste porcelain, much of it decorated with Chinese designs in blue underglaze. Since the 1760's, it has manufactured bone china in a wide variety of colors and patterns. Josiah Spode developed a bone china paste that became the standard English paste in 1800. Spode china featured a large number of designs but was especially noted for its exotic birds. Most of the famous English Wedgwood ware is not porcelain at all, but earthenware or stoneware. Nevertheless, its classical Greek figures and reliefs became enormously popular and had a great influence on porcelain designs throughout Europe.

Modern porcelain; Technical advances enabled the porcelain industry to produce porcelain in large quantities. Today, extensive porcelain making is carried out in the United States, Europe, and Japan. Some notable examples of fine contemporary porcelain are American Lenox, German Rosenthal, and Japanese Noritake.

3.3. Kinds of Porcelain

There are three main kinds of porcelain: (1) hard-paste porcelain, (2) soft-paste porcelain, and (3) bone china. The differences between these types of porcelain are based on the material from which they are made. This material is called the body or paste.

Hard-paste porcelain, which is sometimes called true porcelain or natural porcelain, has always been the model and ideal of porcelain makers. It is the type of porcelain first developed by the Chinese from kaolin and petuntse. Hard-paste porcelain resists melting far better than other kinds of porcelain. For this reason, it can be fired at higher temperatures. These hot temperatures cause the body and the glaze to become one. When hard-paste porcelain is broken, it is approximately impossible to distinguish the body from the glaze.

The proportions of kaolin and petuntse in hard-paste porcelain may vary. The porcelain is said to be severe if the percentage of kaolin is high, and mild if the percentage of kaolin is low. Most collectors of porcelain prefer mild porcelain because of its mellow, satiny appearance. In comparison, severe porcelain may seem harsh and cold.

Soft-paste porcelain, sometimes called artificial porcelain, was developed in Europe in an attempt to imitate Chinese hard-paste porcelain. Experimenters used a wide variety of materials in their efforts to produce a substance that was hard, white, and translucent. They eventually developed soft-paste porcelain by using mixtures of fine clay and glasslike substances. These materials melt at the high temperatures used in making hard-paste porcelain. For this reason, soft-paste porcelain is fired at lower temperatures and does not completely vitrify--that is, it remains somewhat porous. Breaking a piece of soft-paste porcelain reveals a grainy body covered with a glassy layer of glaze. Although soft-paste porcelain was invented in imitation of true porcelain, it has merits of its own. Most of it is creamy in tone, and some people

prefer this color to pure white. In addition, the colors used to decorate it merge with the glaze to produce a soft, silky effect that appeals to many collectors.

Bone china is basically made by adding bone ash (burned animal bones) to kaolin and petuntse. English porcelain makers discovered this combination of ingredients about 1750, and England still produces nearly all the world's bone china. Though not as hard as true porcelain, bone china is more durable than soft-paste porcelain. The bone ash greatly increases the translucence of the porcelain. (http://www.artistictile.net/pages/Info/Info_tile1.html)

3.4. History of Yıldız Porcelain Factory

Sultan Abdulhamid II established Yıldız Porcelain Factory in 1890 at the suggestion of the French ambassador M. Paul Cambon. Known at that time as the Imperial Porcelain Factory, it was established to meet the interior decoration needs of the Ottoman Palace. At this time there was a high demand for porcelain from both the court and the wealthy classes, as a result of which large quantities of porcelain were imported from European countries at high prices. This economic consideration must have been the crucial factor in the decision to open a local factory. The Imperial Porcelain Factory was built on a flat area in Yıldız Palace Park at the personal instigation of the sultan. Experts from the Sevres and Limoges factories in France assisted in setting up the factory, and the latest European technology, including porcelain moulds, were imported. Trial production at the factory began in 1892 but two years later the great earthquake of 1894 caused serious damage to the building. The same year it was repaired by chief palace architect Raimondo d'Aronco, and production recommenced. From 1894 onwards in addition to vases, wall plates and other primarily decorative objects, wash basins, writing sets, dinner, tea and coffee services, plates for visiting cards, lidded bowls, dishes, jugs for asure, bonbon dishes in the form of water melons, and other items for daily use also began to be produced. The main subjects of the designs were portraits of the sultans, panoramas of Istanbul, figures of women and children, mythological and allegorical scenes, arabesque scrollwork, floriate patterns and rococo style country scenes. The decorators included

well known painters such as Hazret-i ,Sehriyari Ali Ragip, Enderuni Abdurrahman, Omer Adil, A. Nicot, E. Narcice, L Avergne, and Tharet. Consequently the Imperial Porcelain Factory, whose primary purpose was to produce decorative porcelain for the palace and court circles, also played a significant role in the development of Turkish art. After sultan Abdulhamid II was deposed in 1909 production at the factory was stopped until 1911, when its former administrators persuaded the government to reopen it. During the War of Independence this factory produced ceramic insulators for linking telegraph wires. It was closed down again in 1920, and in 1936 was liquidated. In 1957 the state textile and ceramics conglomerate Sumerbank reopened the factory. Since 1995 Yıldız Porcelain Factory has been a museum-factory operating under the auspices of the Department of National Palaces. As well as producing ware in traditional designs with the object of keeping the art of Turkish porcelain alive, the factory produces limited edition reproductions of originals in "the National Palaces Porcelain Collection".

This factory made a key contribution to the synthesis between European and Turkish art. Production here has continued uninterrupted, and its high quality products have helped to preserve the art of Turkish porcelain and acquaint people all over the world with its traditional designs.

Today the Yıldız Porcelain Factory produces both items in modern designs and reproductions of its exquisite early ware, so that the public can enjoy the art of a bygone age in their homes as well as in museums. (www.basbakanlik.gov.tr)

3.5. Production Structure Studied

Quartz, feldspar and kaoline, which are the raw materials used in making porcelain, are brought to the facility in large quantities as 15-20 tonne blocks and go through various tests so as to be determined whether they are homogeneous and whether they meet the required physical and chemical qualities. The raw materials found to have feasible qualities are mixed in required proportions and ground to very fine particles and cleaned from iron with magnets. The vacuumed pugs are measured

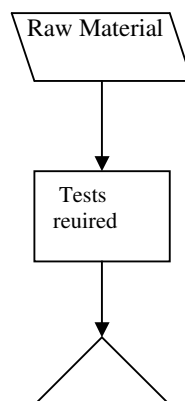
to be 20 mms in diameter. While some part of the the clay which has rested for two months has gained formable quality is sliced in required thickness to be used over the lathe, water is added to the rest to obtain casting material. after the formation through casting or the lathe the handles are fixed to the product. Until this process, the extra residue can be recycled. Biscuit product taken from the lathe are taken into drying compartments where they are dried with air ventiation of 45 °C. Casting material is dried in room temperature so as to achieve stable solid thickness. Although during the first drying a certain hardness is achieved, a biscuit firing in a kiln of about 900-1200 °C is also necessary so that the china would not crack during the glazing stage. Although glazing is applied through dipping the product into a large tank, spraying is also used with larger products. Opaque, glossy or translucent glazed products go through a second firing for 16 hours in a 1500 °C kiln.

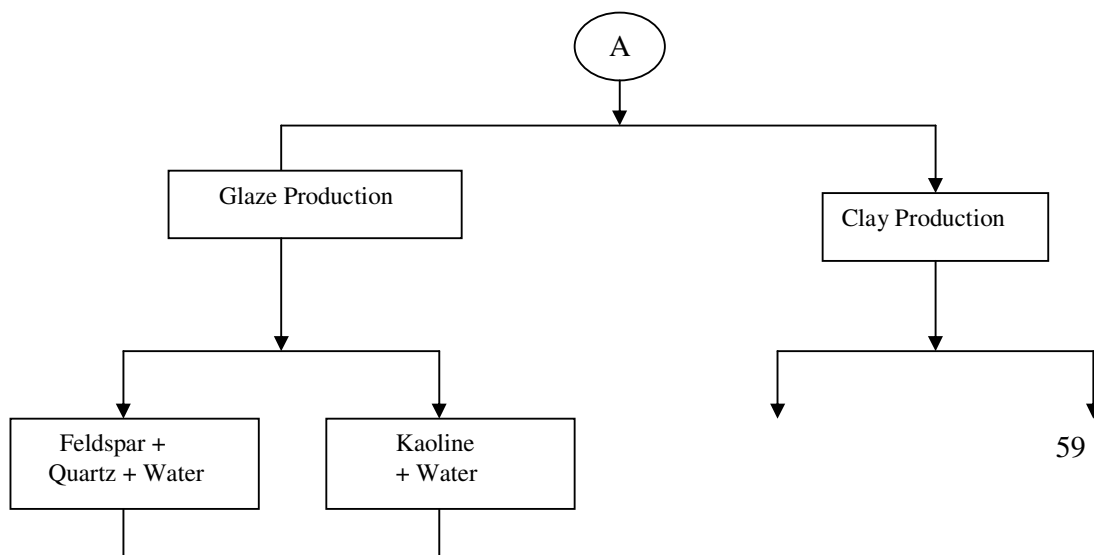
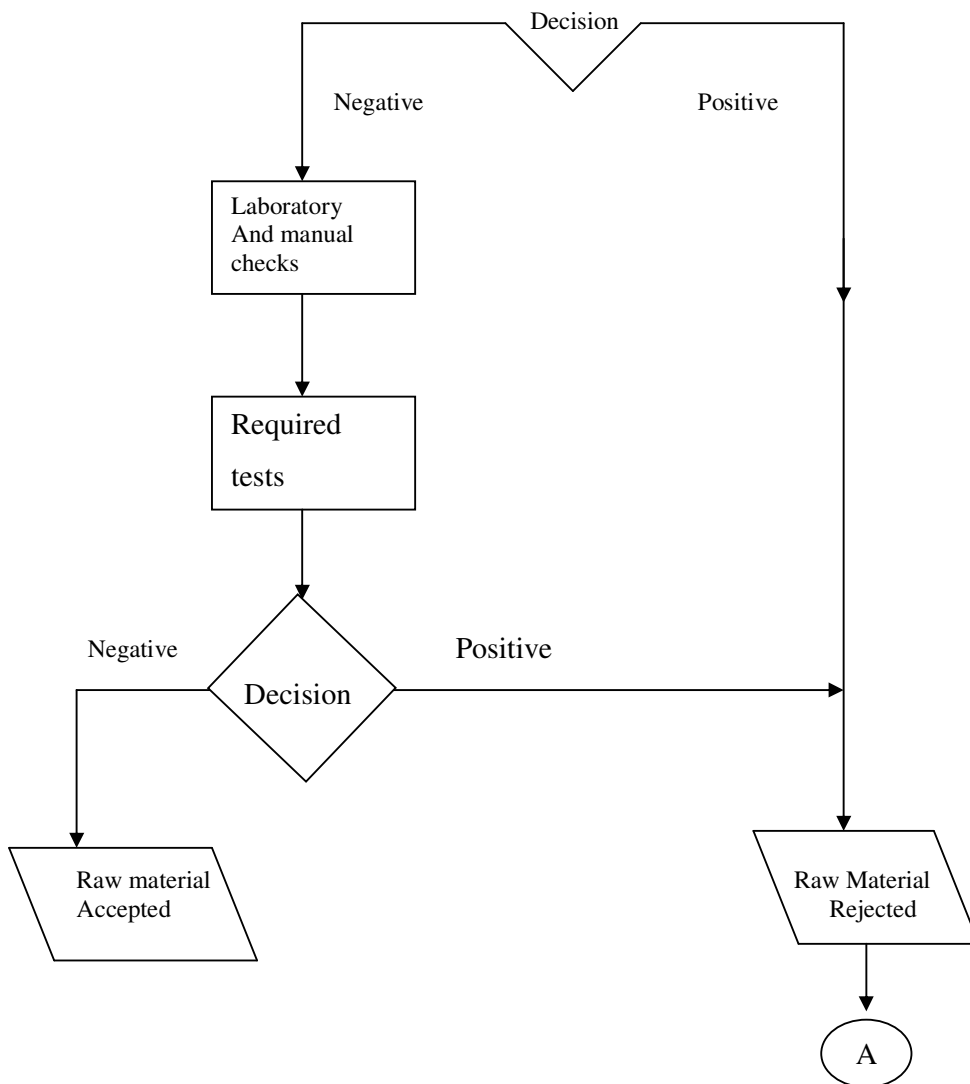
After the glazing stage the product has achieved a water proof quality. China, gastronomically and hygienically very practical, is used in hotels as well as in homes, and the products used in hotels are washed in industrial dishwashers so more durable products against scratches, cracks and hits have been developed. Most importantly these products are thicker than home chinaware.

Of the products with no decorations, the ones with no faults such as deformation, cracks and fingerprints are graded as first quality while the ones with acceptable faults are graded as second quality and the ones with surface cracks, chamot and deformations are considered as loss. These products are destroyed as they are impossible to recycle. Enamels which are applied over the glazing, are open to outside effects and are transferred onto the plate from paper on which the decoration is printed through screen painting technique. The decoration is expected to resist at least 350 washings. The decorated product is fired in the tunnel kiln for 4 hours which makes the enamel unite with the glazing. The elegant enamel has become the chinaware's most visible quality. Then, the product fresh from the kiln is inspected for the final time, wrapped and marketed.

Generally speaking, the executions of the marketing department starts with the determination of an demand in coordination with the factory which manufactures various products. The capacity of the facility is considered to be the number one element at the planning stage of the production programme whereas the demand is considered to be number two. While the chain stores executives i.e. experienced staff who have worked in positions like area managers and the representatives of manufacturers decide on the quantity of the products with certain decorations to be sold, the marketing department operates the coordination between various sales areas and the facility. Because it is the area representatives who operates face to face with the customers and who are experienced on the consumers' tendencies. This group consisting of 30 people are so experienced on the taste- design, purchase power- price and sales capacity in different seasons in different regions of Turkey that the sales price of enameled goods can be determined by area managers.

The production and distribution stages summerized above is shown in the following diagram.





Feldspar +
Quartz + Water



Kaoline
+
Water



Grinding

B



To Glazing Tanks



C



Pomping to the filter
press



60

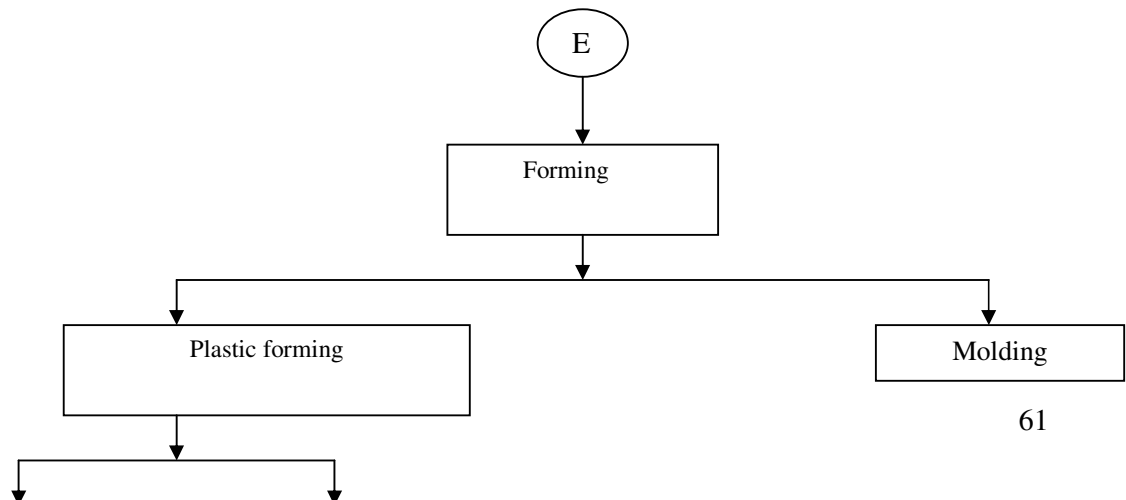
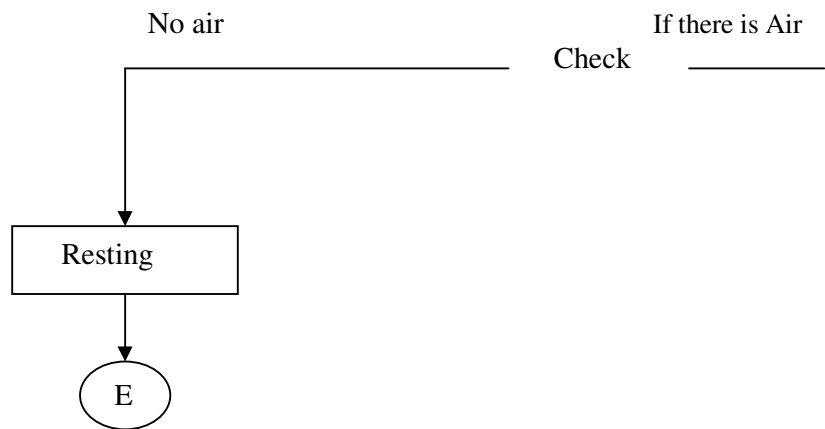


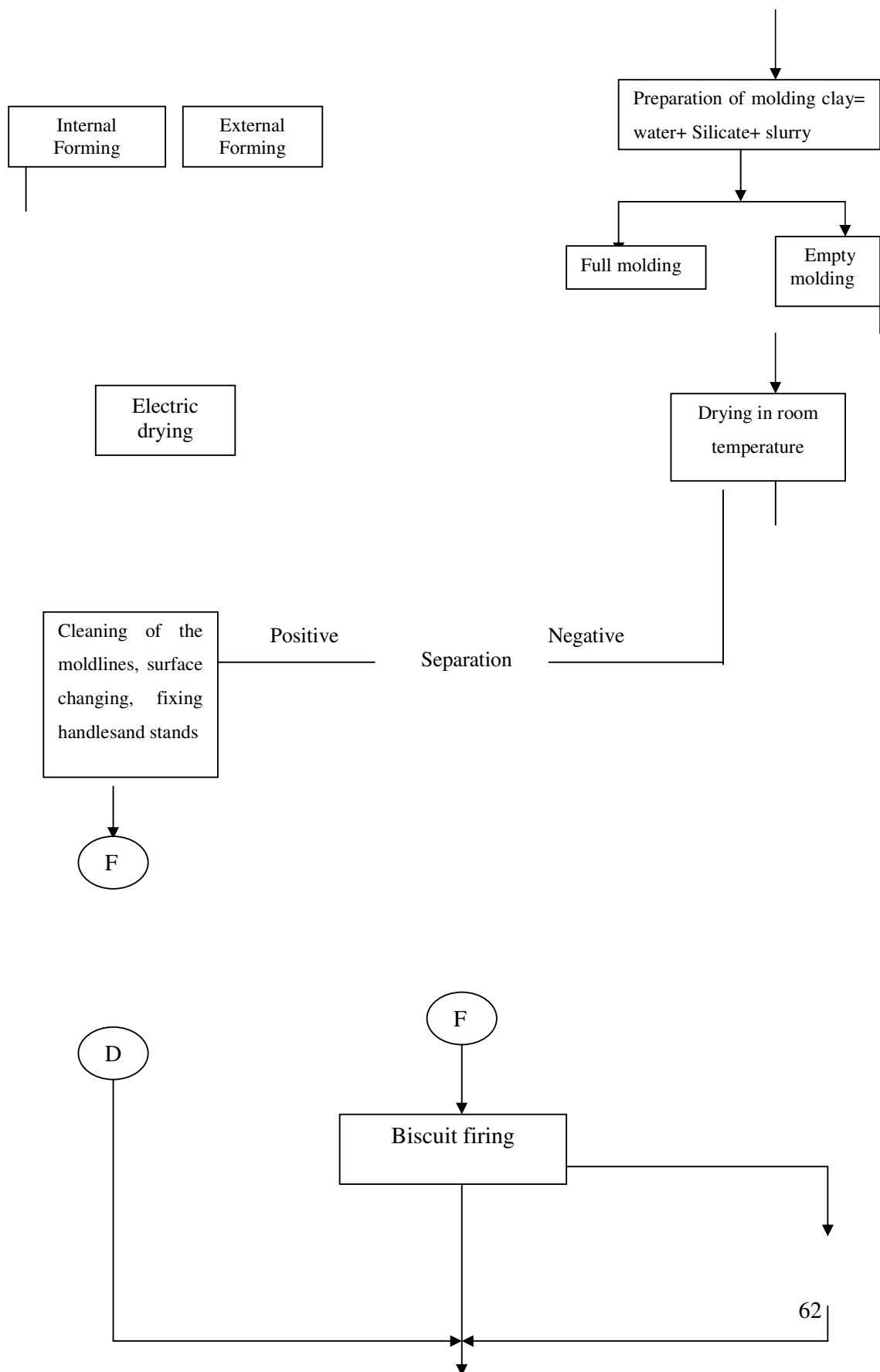
D

Draining excess water in
the filter press

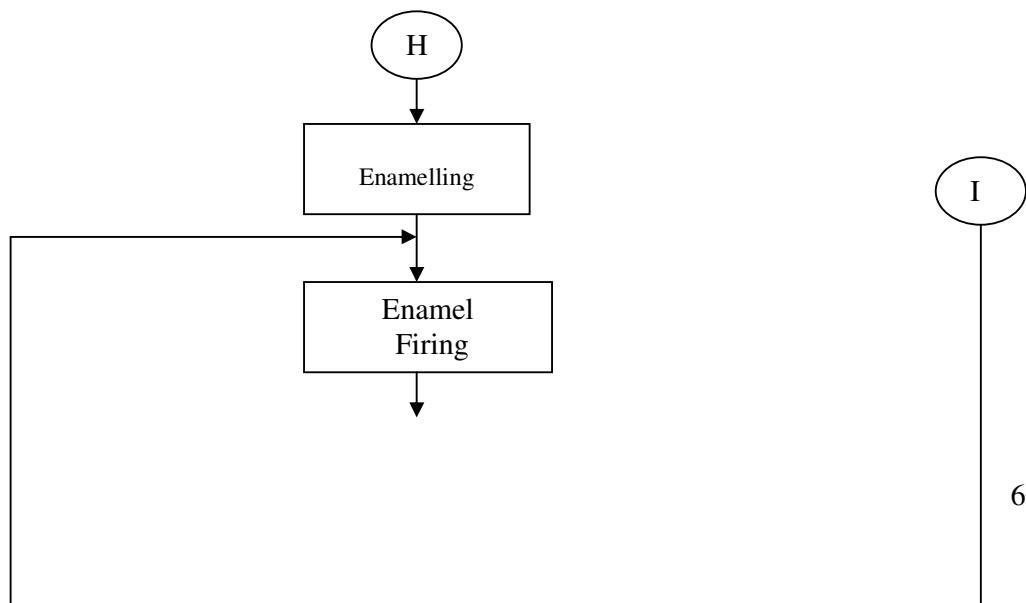
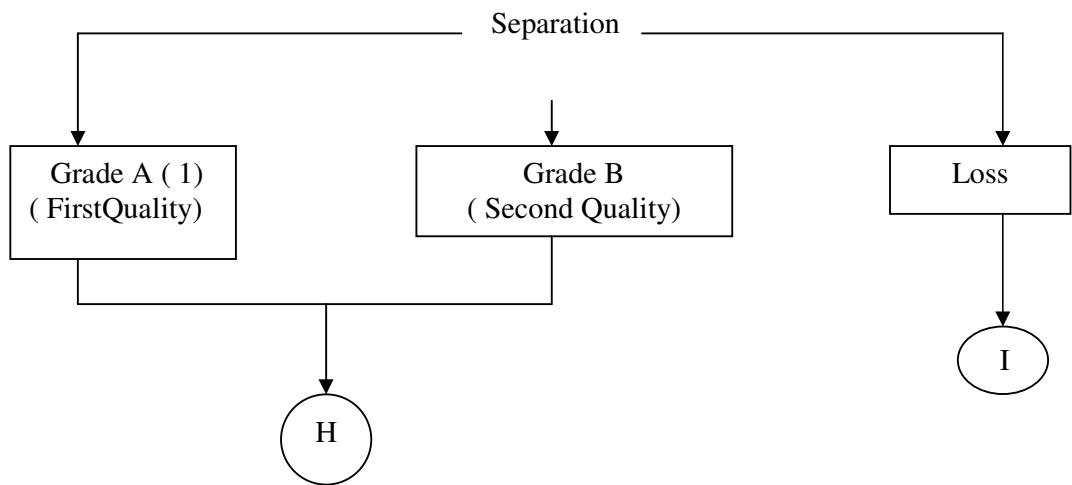
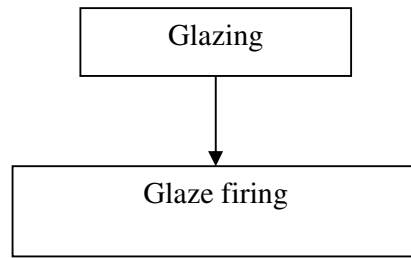
Transferring to vacuum
Press

Vacuuming





Enamelling



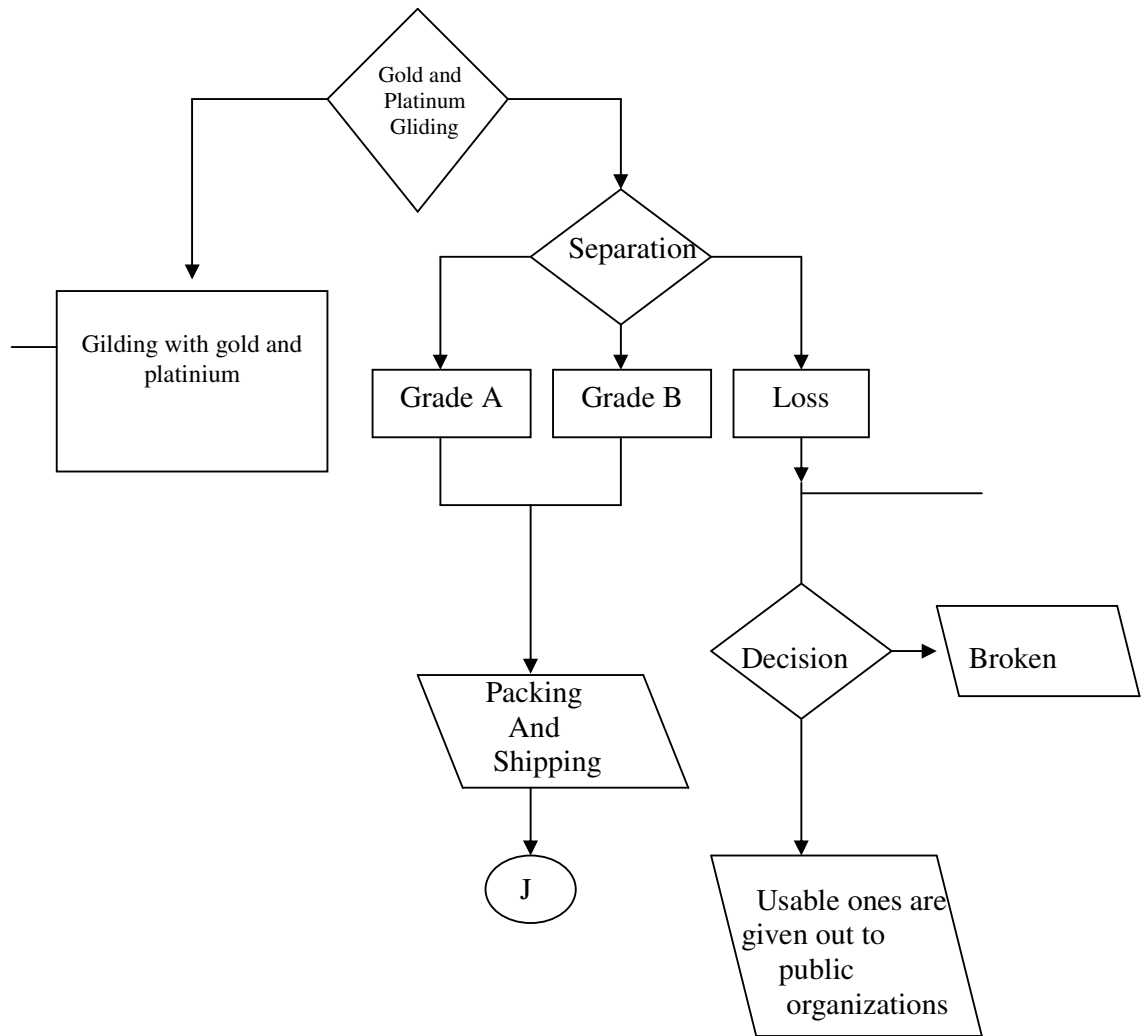


Figure 3 : The production and distribution stage diagram of the porcelin factory

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.156-161.

3.6. Information of Production Structure

Various calculations on the cost of Chinaware products form factors and firing factors(coefficient figures) of each product unit are used. Form factors are values representing difficulties faced during the production and decoration stages of whiteware, whereas firing factor stands for degree of use of the kilns per unit product.

For the form factor, each product is given the value of 1 then for the products exceeding 30 cm in diameter another value of 0.5 is added for each extra 5 cm. Weight is the key element on the determination of the difficulty factor and after 5 kgs, 0.1 per 5 kg unit, and after 25 kgs 0.2 is added to the calculation. For the products which takes more than 60 minutes to mold a figure of 0.1 is added to the forming factor for each 30 minute period. With non fixing formings should the product weight less than 1.5 kgs, the forming factor is directly calculated as 1. If handles etc are to be fixed to the body or the lids another 0.25 is added for each piece fixed.

With the plates, forming factor after 15 cm at the biscuit stage a figure of 0.5 for each 5 cm, and after 25 cms 1 per 5 cm is added to the initial factor 1. When calculating the forming factors of bowls, should the product at the biscuit stage is less than 5 cm in diameter the forming factor is considered 1, as the diameter increases another 0.2 is added for each 5 cm.

While calculating the volumes of the products going into the kilns used in the determination of their firing factors, the products are to roughly resemble a cylinder or a rectangular prism. Before the volumes are calculated, the size of the product at the biscuit stage is increased 1.5 cm. At this stage if the height of the product is less than 6 cm, the height is accepted as 6 cm. Products reaching 6 to 10 cm in height with the additions are taken as 10 cm. However, with the products exceeding 10 cm without the 1.5 cm addition, the value found with the addition is the base. Additionally, with stacked neatly firings number of shelves and the shelf heights should be taken into consideration. The structure of the product and glazing show differences when they are loaded on top of each other. For instance larger products such as kettles and vases should be piled in the kiln with cake plates taking extra care so as to protect the decoration and cake plates should be put in the kiln with cups so that different combinations help to make best use of the kiln capacity. For this reason although certain rules are tried to follow to calculate the firing factor, the result is gained with a certain approximation.

$$\text{Firing Factor} = \frac{\text{Volume of the Prism}}{1000 \cdot k}$$

k in this formula stands for the number of layers. 1000 stands for the dividend as 1 firing factor is given 1000 cm³ volume.

In the following section cost records of the porcelain facility where the application study is performed. The records used here are the records for compote and cake sets out of 300 different items. In the facility where the production lasts 24 hours, the kilns are never turned off so glazing and decoration kilns are operated in three shifts. In the other departments a workday consists of 8 hours and when necessary the employees work overtime even at the weekends. When calculating the unit costs, total costs of a department within a period and total weights of firing-forming factors determining the property of the production activity are considered.

Items like power, machinery, construction maintenance and spare parts costs have been used as setup costs of various departments. Management of the facilities, and items such as main labour costs, indirect materials, superstructure maintenance, and amortization constitute the fixed costs. Seasonal cost ratio of holding cost is determined %10 of the total production cost at various stages. Chinaware (porcelain) does not lose its quality in time so once the product is produced and placed in the warehouse it only requires the lighting and protection services enabling us to use the factors such as the crackability of the product and the change in the trends.

Data from both the marketing department of the production facility and marketing company of the holding have been used to calculate unit cost figures. To calculate the costs of the sets amount and cost ratio of different decorations, the records of the household appliances department of the corporation have been

executed. Since common tendency of the corporate management is to keep more stock than the demand, costs related to the orders by standing have been ignored.

Table 1: Energy Used Per Unit According To Various Departments In The Porcelain Facility

	Amount produced (kg)	Electricity Used (kw hr)	Unit (kg) ÷ kw hr	Steam (tone)	Unit (tone) ÷ amount produced
Casting	6 277	1 205	5.21		

Lathe	24 989	2 048	12.20	20 000	1.25
Glazing Kilns	31 266	291 690	0.11		
Tecnical Decoration	15 730	10 000	1.27		
Hand Made Decoration	1 250	8 967	0.14		
Tiles	657	1 904	0.35		
Boilers	25 240t	273	92.45		
Work shops		200			
Offices		200			
Heating				5 240	
Total		316 487		25 240	

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.166

Table 2: Operation Report On The Raw Materials Consumed And Stock Status

Mud (kg)	Monthly Comsumption	Assembly Comsumption	Stock
Sındırgı Kaoline	27 300	195 736	69 956
Dağardı Feldspat	11 100	79 232	23 607

Brown Clay	2 500	15 244	6 972
Bentonite	1 700	11 260	612
Esiri Clay	1 100	11 034	22 292
Uşak Kaoline	1 250	9 139	23 000
Eskişehir Kaoline	560	3 839	5 962
Total	45 510	325 484	
Glaze (kg)			
Dağardı Feldspar	2 800	16 495	
Silisium	700	5 238	3 706
Marble Powder	600	4 620	2 868
Eskişehir Kaoline	194	1 884	
Tin deocide	212	1 599	419
Dolomite	200	1 484	1 768
Zinc-oxide	150	1 080	128
Talk	130	965	803
Fe ₂ O ₃	15	371	1 995
Barium carbonate	22	109	79
Borax	8	18	48
Total	5 031	33 863	

**Table 2: Operation Report On The Raw Materials Consumed And Stock Status
(Continued)**

Sodium silicate	75	525	650
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Sodium carbonate	25	175	43
Total	100	700	
Clay produced	49 151	378 923	
Glaze produced and consumed	5 031	33 863	
Clay used in production	51 629	349 393	
Clay loss	522	3 530	
Transfer	49 151-51 629-522	378 923-349 393-3 530	
Clay stock	= -3000	= 26000	

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.167-168.

Table 3: Total Cost At Various Production Stages (Clay, Kaoline, Feldspar Constitute The Initial Matter; Zinc-Ocside, Tin-Ocside, Barrium Carbonade And Talk Constitute The Anxiliary Initial Matter)

	Initial Material	Anxiliary Initial Material	Labour	General Production Costs	Total
Clay production	66 513 679	41 721 628	53 671 047	320 823 366	488 729 720
White ware(Porcelain)	426 733 181	281 952 748	700 577 793	2 508 360 921	3 917 624 643
Technically decorated Porcelain	2 674 643 000	124 866 823	444 256 424	878 503 167	4 122 269 414
Hand decorated Porcelain	352 588 131	63 764 384	496 731 175	1 321 248 494	2 234 332 184
Shield	904 045	22 313 499	47 698 369	191 412 136	262 328 049
Molding-Modeling	2 899 910	19 001 774	77 964 543	208 908 567	308 774 794
Vaste Of Production		9 295 000			9 295 000
General Total	3 524 281 946	532 915 856	1 826 899 351	5 429 256 651	11 343 353 800

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.169.

Table 4: Productive Motions At Stages

	(Kg)	Number	Total (TL)
Clay production	269 950		488 729 720
White ware (Porcelain)			
Molding	46 414	157 779	1 311 767 382
Lathe	190 952	713 923	2 605 857 261
Total	237 366	871 702	3 917 624 643
Technically decorated Porcelain	162 356	655 744	4 122 269 414
Hand decorated Porcelain	16 830	36 399	2 234 332 184
Shield			262 328 049
Molding-Modeling			308 774 794
Vaste Of Production			9 295 000
General Total			11 343 353 800

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.170.

Table 5: Sales Motions At Stages

	(Kg)	Number	Total (TL)
White ware(Porcelain)			
Molding	719	6 395	54 709 905
Lathe			
Total	719	6 395	54 709 905
Technically decorated Porcelain	162 356	655 744	4 122 269 414
Hand decorated Porcelain	16 830	36 399	2 234 332 184
Grand Total			6 411 311 503

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.171.

Table 6: Molding Clay Preparation Costs

Clay	283 390
Kaoline	1 172 798
Feldspar	531 131
Other	131 440
Main Production Initial Matter Total	2 11 759
Other Direct Chemicals	1 476 443
Direct Initial Matter Ingredients	3 595 202
Main Labour	960 862
Over Time	265 283
Bonus	351 785
Direct Labour	1 577 930
Total	5 173 132
Management Of Production Facilities	292 596
Power	70 35
Water	434 984
Steam	110 326
Electric Maintenance	1 153 160
Machinery Maintenance	1 524 247
Construction Maintenance	1 128 271
Transportation	267 915
Workers Cafeteria	425 355
Total Costs	5 407 212
Direct General Production Costs	4 497 080
Total general production costs	9 904 292
Monthly Total	15 077 424

Table 6: Molding Clay Preparation Costs (Continued)

Weekends And Holidays	632 500
Additional Payments	850 956
Vacational Wages	277 970
Food in Cash	27 296
Labor housing aid	104 209
Communications (Bus Pass)	98 417
Employee saving cuts	154 745
Other Frinelve Benefits	206 609
Accidents and Illnesses	44 112
Sick Leaveness	162 724
Maternity	27 121
Old age / disability	298 326
Labour costs	2 884 985
Other Benefit and Services	721 238
Super Structure Maintenance	368 633
Facilities Amortization	635 995
Total	1 625 866
Spare parts	38 682
Equipment	311 750
Repair and Construction material	2 615
Stationary	2 579
Clothes	376 111
Electirical Appliances	8 885
Other Materials	13 106
Indirect Material Total	753 728
Grand Total	20 342 003

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.172-173.

Table 7: The Lathe Clay Preparation Costs

Clay	867 975
Kaoline	3 592 072
Feldspar	1 626 761
Other	402 576
Main Production Initial Matter Total	6 489 384
Other Direct Chemicals	4 522 086
Direct Initial matter	11 011 470
Main Labour	2 942 950
Overtime	812 514
Bonus	1 077 455
Direct Labour Costs	4 832 919
Total	15 844 389
Managment Production Facilities	896 169
Power	207 855
Water	1 332 281
Steam	337 911
Electric Maintenance	3 531 927
Machinery Maintenance	4 668 499
Construction Maintenance	3 455 693
Transportation	820 578
Workers Cafeteria	1 302 788
Total Costs	16 553 701
Direct General Production Costs	13 773 764
Total General Production Costs	30 327 465
Monthly Total	46 171 854

Table 7: The Lathe Clay Preparation Costs (Continued)

Weekends and Holidays	1 937 234
Additional Payments	2 606 327
Vacational Wages	851 374
Food in Cash	83 601
Housing Aid	319 175
Communications (Bus pass)	301 433
Employee Saving Cuts	473 956
Other Fringe Benefits	632 806
Accidents and Illnesses	135 105
Sick Leave	498 395
Maternity	83 066
Old age / Disability	913 720
Labour Costs	8 836 192
Other Benefit and Services	2 209 023
Super Structure Maintenance	1 129 056
Facilities Amortization	1 641 658
Total	4 979 737
Spare Parts	118 466
Equipment	954 772
Repair and Construction Material	8 005
Stationary	7 901
Clothes	1 151 884
Electrical Appliances	27 213
Other Material	40 140
Indirect Material Total	2 308 381
Grand Total	62 296 164 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.174-175.

Table 8: Molding Costs

Main Labour	22 416 603
Overtime	1 912 831
Bonus	6 715 790
Direct Labour Costs	31 045 224
Facilities Management	762 649
Power	65 612
Water	1 017 745
Steam	516 268
Electric Maintenance	4 047 123
Machinery Maintenance	4 279 589
Transportation	3 761 096
Workers Cafeteria	5 971 296
Total Costs	20 421 378
Direct general production costs	49 357 007
Total General Production Costs	69 778 385
Monthly Total	100 823 609

Table 8: Molding Costs (Continued)

Weekends and Holidays	9 558 130
Additional Payment	11 627 977
Vacational Wages	3 751 037
	3 409 930
Marital Aid	213 847
Mortal Aid	17 090
Food in Cash	265 956
Employee Housing Aid	1 684 92
Bus-pass	1 348 119
Employe saving cust	2 777 731
Other Social Aids	3 567 712
Accidents and Illnesses	776 664
Sick Leave	2 779 852
Maternity	463 309
Old age / Disability	5 124 878
Labour Costs	47 366 624
Facitities Amortization	69 750
Spare Parts	30 984
Hand Tools	599 974
Equipment	1 526 855
Repair and Construction Material	266 664
Stationary	7 887
Clothes	4 253 714
Electirical Appliances	36 691
Other Material	724 486
Indirect Material Total	7 446 955
Grand Total	155 436 938 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.176-177.

Table 9: Lathe Costs

Main labour	13 527 359
Overtime	2 122 074
Bonus	4 612 084
	20 261 517
Direct Labour Costs	
Facilities Management	11 028 626
Power	608 114
Water	1 356 994
Steam	21 166 993
Electric Maintenance	4 047 123
Machinery Maintenance	5 706 118
Transportation	2 820 822
Workers Cafeteria	4 478 472
Total costs	51 213 262
Direct General Production Costs	37 090 941
Total General Production Costs	88 304 203
Monthly Total	108 565 720

Table 9: Lathe Costs (Continued)

Weekends and Holidays	7 322 462
Additional Payments	9 461 170
Vacational Wages	1 533 978
	355 931
Food In Cash	253 694
Labor Housing Aid	912 446
Communications (Bus-pass)	1 094 225
Employee Saving Cuts	2 012 900
Other Frinelve Benefits	2 516 728
Accidents and Illnesses	620 053
Sick Leave	2 198 910
Maternity	366 485
Old age / Disability	4 031 336
Labour Costs	32 680 318
Facilities Amortization	3 240 166
Spare Parts	1 197 211
Had Tools	1 874
Equipment	272 569
Repair and Construction Material	57 842
Clothes	2 077 809
Electirical Appliances	356 268
Other Material	120 017
Indirect Material Total	4 083 590
Grand Total	148 569 794 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.178-179.

Table 10: Glaze Preparation Costs

Clay	174 748
Kaoline	723 187
Feldspar	327 513
Other	81 050
Main Production Initial Matter Total	1 306 498
Other Direct Chemicals	910 425
Direct Initial Matter	2 216 923
Main Labour	592 500
Overtime	163 582
Bonus	216 922
Direct Labour Costs	973 004
Total	3 189 927
Managment of Production Facilities	180 424
Power	41 847
Water	268 226
Steem	68 031
Electric Maintenance	711 078
Machinery Maintenance	939 902
Construction Maintenance	695 730
Transportation	165 206
Workers Cafeteria	262 289
Total costs	3 332 733
Direct General Production Costs	2 773 053
Total General Production Costs	6 105 786
Monthly Total	9 295 713

Table 10: Glaze Preparation Costs (Continued)

Weekends and Holidays	390 021
Additional Payments	524 728
Vacational Wages	171 406
Food in Cash	16 832
Labor Housing Aid	64 259
Communications (Bus-pass)	60 687
Employee Saving Cuts	95 421
Other Fringe Benefits	127 402
Accidents and Illnesses	27 201
Sick Leave	100 341
Maternity	16 723
Old age / Disability	133 958
Labour Costs	1 778 979
Other Benefit and Services	444 739
Structure Maintenance	227 311
Facilities Amortization	330 513
Total	1 002 563
Spare Parts	23 851
Equipment	192 223
Repair and Construction Material	1 612
Stationary	1 591
Clothes	231 907
Electrical Appliances	5 480
Other Material	8 080
Indirect Material Total	464 744
Grand Total	12 541 999 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.180-181.

Table 11: Glazing Kilns Cost

Main Labour	18 119 340
Overtime	1 673 519
Bonus	6 012 166
Direct Labour	25 805 025
Management Production Facilities	28 768 152
Power	58 815 691
Water	1 049 285
Steam	481 369
Electric Maintenance	1 699 792
Machinery Maintenance	1 797 427
Transportation	3 356 779
Workers Cafeteria	5 329 381
Total Costs	101 197 876
Direct General Production Costs	55 523 146
Total General Production Costs	156 821 022
Monthly Total	182 626 047

Table 11: Glazing Kilns Cost (Continued)

Weekends and Holidays	13 265 499
Additional Payments	13 700 766
Vacational Wages	4 465 381
Seniority	1 656 722
Marial Aid	420 150
Mortal Aid	14 199
Food in Cash	277 407
Employee Housing	1 678 176
Bus-pass	1 586 738
Employee saving Cost	2 895 007
Other Frinege Benefits	3 508 083
Accidents and Illnesses	844 577
Sick Leave	3 062 672
Maternity	510 445
Old age / Disability	5 614 898
Labour Costs	53 500 720
Structure Amortization	40
Facilities Amortization	188 696
Total	188 736
Hand Tools	241 141
Equipment	3 198 458
Repair and Construction Material	1 054 220
Stationary	19 226
Clothes	6 569 687
Electirical Appliances	2 362 057
Other Material	1 487 107
Indirect Material Total	14 931 896
Grand Total	251 247 399 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.182-183.

Table 12: Cost Of Technical Decoration

Main Labour	34 573 840
Overtime	4 530 546
Bonus	11 623 746
Direct Labour	50 728 132
Managment Production Facilities	2 095 565
Power	3 200 598
Water	678 497
Steam	516 268
Electric Maintenance	1 349 041
Transportation	4 858 083
Workers Cafeteria	7 712 923
Total Costs	20 410 975
Direct General Production Costs	84 701 176
Total General Production Costs	105 112 151
Paintings	140 484
Gold Gilding	3 773 929
Dyes	19 928 413
Other Chemical Substances	176 172
Other Substances	708 782
Direct Materials	24 727 780
Monthly Total	180 568 063

Tablo 12: Cost Of Technical Decoration (Continued)

Weekends and Holidays	18 949 377
Additional Payments	17 410 400
Vacational Wages	7 049 362
Seniority	2 017 547
Marial Aid	222 441
Food in Cash	602 123
Employee Housing Aid	2 427 660
Bus-pass	2 013 587
Employe Saving Cost	4 505 360
Other Social Benefits	6 857 182
Accidents and Illnesses	1 265 874
Sick Leave	4 599 145
Maternity	766 525
Old age / Disability	8 431 771
Labour Costs	77 118 354
Official Payment	2 462 226
Bonus	242 458
The Hardness of Business	120 110
Difficulty of Finding Personnel	280 256
Private Service Reparations	1 252 552
Official's Child Addition	51 230
Official Cure Payment	109 795
Official Food Expences	9 778
Employee Housing	104 078
Clipping from the Official Sum	234 989
Retired Official Savings Bank's Lot	465 639
The Payment of Savings Bank	135 544
Official Salary and Outcomes	5 468 655

Tablo 12: Cost Of Technical Decoration (Continued)

Facilities Amortization	137 333
Hand Tools	471 360
Repair and Construction Material	11 977
Stationary	24 249
Clothes	2 326 757
Electirical Appliances	193 151
Other Material	275 686
Indirect Material Total	3 303 180
Grand Total	266 595 585 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.184-185-186.

Table 13: Cost Of Technical Decoration Kilns

Main labour	10 641 517
Over time	982 861
Bonus	3 530 954
Direct Labour	15 155 332
Managament Production Facilities	16 895 582
Power	34 542 549
Water	616 247
Steam	282 708
Electric Maintenance	998 290
Machinery maintenance	1 055 632
Transportation	1 971 441
Workers cafeteria	3 129 954
Total costs	59 492 403
Direct general production costs	32 608 832
Total general production costs	92 101 235
Monthy total	107 256 567

Table 13: Cost Of Technical Decoration Kilns (Continued)

Weekends and holidays	7 790 849
Additional payments	8 046 484
Vacational vagen	2 622 525
Seniority	972 995
Marial aid	246 754
Mortal aid	8 339
Food in cash	162 921
Employee hosing	985 595
Bus-pass	931 893
Employe saving cust	1 700 242
Other Social Benefits	2 060 302
Accidents and Illnesses	496 022
Sick leave	1 798 712
Maternity	299 785
Old age / disability	3 297 639
Labour costs	31 421 057
Bulding and facilities amortication	110 846
Had tools	28 686
Equipment	380 479
Repair and Constrution material	125 408
Stationary	2 286
Clothers	781 512
Electirical Appliances	280 984
Other Material	176 901
Indirect Material Total	1 776 256
Grand Total	140 564 726 TL

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, pp.187-188.

Table 14: The Summary Of Information to Define Unit Costs

Production	Σ Forming Factor	Σ Firing Factor	Number	Σ kg	Σ Cost
Molding Clay				6 277	20 342 003
Preparation					
Lathe Clay				37 843	62 296 164
Preparation					
Molding	54 786		21 762	6 277	155 436 938
Lathe	140 908		71 663	34 989	148 569 794
Glaze				31 266	12 541 999
Preparation					
Glazing Kiln		304 523			251 247 399
Tecnical					
Decoration	147 617		93 596	15 730	266 595 585
Tecnical					
Decoration Kilns		194 755	225 058	35 690	140 564 726

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.189.

Table 15: Informations of Set Products

	Set 12 persons	Set 12 Persons	26 Piece dinner set	Set for 6 Persons	Cake Set	Season Demand	Loss Ratio	Forming Factor	Kiln Factor	Weight per Unit (kg)
Large Brimmed Soup Bowl	1	1				3200	% 44.0	3.03	5.72	1.810
LBSB Lid	1	1				3200	% 35.0	1.25	4.04	0.369
Brimmed Regular Plate Ø29 cm	2	1			1	10400	% 6.5	3.00	6.3	0.790
Brimmed Regular Plate Ø25 cm	12	6	6			79400	% 3.5	2.50	7.45	0.530
Brimmed Regular Plate Ø19 cm	12	6	6		6	97400	% 6.3	1.58	2.81	0.222
Brimmed Hollow Plate Ø21 cm	12	6	6			94400	% 7.3	1.81	3.42	0.353
Brimmed Oval Plate Ø35 cm	2	1				7400	% 10.8	2.85	5.87	0.760
Brimmed Oval Plate Ø21 cm	12	6				42400	% 10.4	2.02	2.44	0.280
Large Brimmed Compote Bowl	2	1		1		6400	% 5.4	1.77	5.18	0.503
Brimmed Compote Bowl	12	6	6	6		70400	% 6.6	1.00	1.28	0.166
Brimmed Creamer	1					2200	% 7.7	2.25	1.69	0.156
Brimmed Creamer Saucer	1					2200	% 2.5	1.00	1.06	0.160
Brimmed Salt Shaker	4	2	2			19800	% 4.0	1.00	0.44	0.076
Brimmed Lemon Plate	1	1				3200	% 6.0	2.25	1.45	0.109
Total	75	38	26	7	7					

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.190.

Table 16: Set Product Unit Costs

	Molding Clay Preparation	Molding	Lathe Clay Preparation	Lathe	Glaze Preparation	Glazing Kilns	Whiteware Cost per Unit	Technical Decoration	Technical Decoration Kilns	Decoration Costs Per Unit	Total Cost
Large Brimmed Soup Bowl	6 905	12 989			5 311	8 403	33 608	4 364	3 879	8 243	41 851
LBSB Lid	3 795	7 138			2 919	4 618	18 470	2 396	2 132	4 528	22 998
Brimmed Regular Plate Ø29 cm			1 256	3 056	1 901	5 021	11 234	2 272	1 906	4 178	15 412
Brimmed Regular Plate Ø25 cm			657	1 985	995	4 631	8 628	1 752	2 086	3 838	12 106
Brimmed Regular Plate Ø19 cm			359	1 633	541	2 269	4 802	2 107	1 496	3 603	8 405
Brimmed Hollow Plate Ø21 cm			551	1 813	836	2 681	5 881	2 039	1 540	3 579	9 460
Brimmed Oval Plate Ø35 cm	2 911	9 556			2 240	5 723	20 430	3 033	2 496	5 529	25 959
Brimmed Oval Plate Ø21 cm	887	5 602			682	1 969	9 140	3 129	1 511	4 640	13 780
Large Brimmed Compote Bowl			958	1 220	1 451	4 947	8 576	2 010	4 162	6 172	14 748
Brimmed Compote Bowl			186	1 272	282	721	2 461	2 600	752	3 352	5 813
Brimmed Creamer	386	4 875			297	1 065	6 623	2 729	819	3 548	10 171
Brimmed Creamer Saucer			157	713	236	990	2 096	2 675	1 899	4 574	6 670
Brimmed Salt Shaker	139	1 608			108	206	2 061	3 004	530	3 534	5 595
Brimmed Lemon Plate	250	4 523			193	847	5 813	2 899	747	3 646	9 459

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.191.

Table 17: Stage Preparation Cost Ratio

<u>(Power+ Machinery+ Construction) Maintenance+ Parts</u>	x 100
Total Department Cost	
Molding Clay Preparation	% 18.80
Lathe Clay Preparation	% 18.80
Molding	% 5.36
Lathe	% 6.80
Glaze Preparation	% 18.80
Glazing Kiln	%1.39
Tecnical Decoration	% 0.51
Tecnical Decoration Kilns	% 1.46

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.192

Table 18: Standart Occupation Time of Set Products (Minutes)

	Molding clay preparation	Molding	Lathe clay prep	Lathe	Blare prep	Glazing kiln	white ware cost per unit	Tecnical decoration	Tecnical decoration kilns	Decoration cost per	Total cost
Large Brimmed Soup Bowl	1.07	17.12			0.87	31.72	50.78	15.38	20.28	35.66	86.44
LBSB Lid	0.22	6.49			0.18	12.02	18.91	5.83	7.68	13.51	32.42
Brimmed Regular Plate Ø29 cm			2.83	6.68	0.38	28.84	38.73	6.99	18.44	25.43	64.14
Brimmed Regular Plate Ø25 cm			2.38	5.57	0.25	24.03	32.23	5.82	15.37	21.19	53.42
Brimmed Regular Plate Ø19 cm			1.81	5.34	0.19	18.26	25.60	4.42	11.68	16.10	41.70
Brimmed Hollow Plate Ø21 cm			1.26	8.05	0.17	17.40	26.88	8.43	11.13	19.56	46.44
Brimmed Oval Plate Ø35 cm	0.45	14.79			0.36	27.40	43.00	13.28	17.52	30.80	73.80
Brimmed Oval Plate Ø21 cm	0.17	10.48			0.13	19.42	30.20	9.41	12.42	21.83	52.03
Large Brimmed Compote Bowl			1.80	4.45	0.24	17.02	23.51	4.66	10.88	15.54	39.05
Brimmed Compote Bowl			0.59	7.87	0.08	9.61	18.15	8.25	6.15	14.40	32.55
Brimmed Creamer	0.09	11.68			0.07	16.25	28.09	10.49	10.39	20.88	48.97
Brimmed Creamer Saucer			0.09	5.19	0.08	9.61	14.97	4.66	6.52	11.18	26.15
Brimmed Salt Shaker	0.05	5.19			0.04	4.23	9.51	4.66	2.70	7.36	16.87
Brimmed Lemon Plate	0.06	11.68			0.05	13.94	25.73	10.49	8.91	19.40	45.13

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.193.

Table 19: The Sum of Factory Order and Holding Costs of Set Products

Large Brimmed Soup Bowl	1 750	1 750	1 750			
LBSB Lid	1 236	1 236	1 236			
Brimmed Regular Plate Ø29 cm	1 928	3 856	1 928			1 928
Brimmed Regular Plate Ø25 cm	2 280	27 360	13 680	13 680		
Brimmed Regular Plate Ø19 cm	860	10 320	5 160	5 160		5 160
Brimmed Hollow Plate Ø21 cm	1 047	12 564	6 282	6 282		
Brimmed Oval Plate Ø35 cm	1 796	3 592	1 796			
Brimmed Oval Plate Ø21 cm	747	8 964	4 482			
Large Brimmed Compote Bowl	1 585	3 170	1 585		1 585	
Brimmed Compote Bowl	392	4 704	2 352	2 352	2 352	
Brimmed Creamer	517	517				
Brimmed Creamer Saucer	324	324				
Brimmed Salt Shaker	135	540	270	270		
Brimmed Lemon Plate	444	444	444			
Total		79 341	40 965	27 744	3937	7 088

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.201.

3.7. Methodology

The methodology used in the present work includes three major tasks: (1) development of the algorithm to solve the proposed problem, (2) measurement of the effectiveness of the algorithm, and (3) models are programmed through using the computer program of Excel to build a macro using visual basic codes.

To satisfy the objectives of this research, it is necessary to follow some steps described in detail next. The first part of this section is the problem definition, trying to clearly establish the restrictions considered. Next, two ways to solve the proposed problem are presented: (1) the optimal power-of-two policy formulated as a nonlinear integer-programming problem, proposed by *Maxwell and Muckstadt (1985)*; and (2) the algorithm approach using the power-of-two restrictions. These models are programmed through using the computer program of Excel to build a macro using visual basic codes . The codes built are available in Appendix A.

The algorithm approach and the optimal power-of-two methodology, *Maxwell and Muckstadt (1985)*, other methods cased in the dissertation of Faik Bařaran are compared to define the effectiveness of the algorithms.

3.8. Problem Definition

An algorithm to determine the reorder cycle time in multi-stage serial system is developed. Demand for each end item is assumed to occur at a constant and continuous rate. Production is instantaneous and no backorders are allowed. Fixed setup costs and echelon holding costs are changed at each stage. The capacity at each node is unconstraint.

3.9. Maxwell and Muckstadt Approach

A power-of-two policy, as described by *Roundy (1986)*, is a sequence $T = (T_n: n \in \mathbb{N})$ of positive numbers with the following three properties. First, orders

for product n are placed once every $T_n > 0$ units of time beginning at time zero. Second, $T_n = 2^{K_n} \beta$ for all products n and for some $1 \leq \beta < 2$, where K_n is an integer. Finally, the Zero-Inventory Property holds that an order is placed for a product only when the inventory of that product is zero.

Maxwell and Muckstadt (1985) presented a method for computing power-of-two policy, based on the assumptions presented previously in the problem definition. Let G represents the acyclic directed graph corresponding to the production and distribution system. Let $N(G)$ represents the node set and $A(G)$ the arc set corresponding to G . The costs considered in the model are fixed setup costs A_i , for $i \in N(G)$, and the echelon holding costs, h_i , for $i \in N(G)$.

Let T_i for $i \in N(G)$, represent the reorder interval at operation i and let T_L be the base planning period, measured in unit time (minutes, days, weeks, months, year, etc.). The reorder interval for each operation is expressed as a multiple of T_L . Let M_i for $i \in N(G)$ represent the multiple of the base planning period per reorder interval for operation i . Also, for all $i \in N(G)$, let $g_i = h_i T_L / 2$, the average echelon holding cost per unit time for operation i when $T_i = 1$ (the same unit time used to determine the demand).

The model can be stated as:

$$\text{Minimize } \sum_{i \in N(G)} [A_i / T_i + g_i T_i]$$

Subject to:

$$T_i = M_i T_L, i \in N(G),$$

$$M_i \leq M_j, (i, j) \in A(G),$$

$$M_i = 2^{k_i}, k_i = 0, 1, 2, 3, \dots$$

This formulation is called Problem P. Problem P is a large-scale, nonlinear integer programming problem. In practical situations, the sets $N(G)$ and $A(G)$ could contain many thousands of elements. To solve Problem P they used a two step

procedure. In the 30 first step they solved the relaxed version of this problem to establish what group of operations must have identical reorder intervals. The mathematical formulation of the relaxed problem, which is called Problem RP, replaces for each $i \in N(G)$ the integrality constraint on M_i with the constraint $M_i \geq 1$, and replaces the requirement that $T_i \geq T_L$ with $T_i \geq 0$.

Problem RP (relaxed problem) is

$$\text{Minimize } \sum_{i \in N(G)} [A_i / T_i + g_i T_i]$$

$$\text{Subject to: } T_i \geq T_j \geq 0 \quad (i,j) \in A(G),$$

Jackson, Maxwell and Muckstadt (1988) showed the characterization of the optimal solution. They established the correspondence between the solutions of problem RP and ordered partitions of the graph G . Define a sub graph G' of the graph G to consist of a subset $N(G')$ of the node set $N(G)$ together with the associated arc set $A(G')$ where $(i,j) \in A(G')$ if and only if $i \in N(G')$, $j \in N(G')$, and $(i,j) \in A(G)$. An ordered collection of sub graphs $(G_1, G_2, \dots, G_n)$ of G is said to be ordered by precedence if for any $1 \leq p < q \leq n$ there does not exist a node $j \in N(G_p)$ and a node $j \in N(G_q)$ such that $(i,j) \in A(G)$. That is, no node in $N(G_p)$ precedes any node in $N(G_q)$ if $q > p$. The collection of subgraph $(G_1, G_2, \dots, G_n)$ forms an ordered partition of the graph G if (a) the node subsets $N(G_1), N(G_2), \dots, N(G_n)$ form a partition of the node set $N(G)$, and (b) the collection is ordered by precedence. A directed cut of a sub graph G' is simply an ordered (binary) partition (G', G'^+) of the sub graph G' . Suppose that the reorder intervals share a common value: $T_i = T$ for all $i \in N(G')$. Then the optimal value of T is given by:

$$T = [\sum_{i \in N(G)} A_i / \sum_{i \in N(G)} g_i]^{1/2}$$

Letting

$$A(G') = \sum_{i \in N(G)} A_i \text{ and}$$

$$g(G') = \sum_{i \in N(G)} g_i$$

Then $T = (A(G') / g(G'))^{1/2}$.

The optimal solution of problem P can be found if the solution to problem RP is known. The optimal value of M_i for $i \in N(G)$ can be found by calculating

$$k = \left\lceil -\log_2 T_L - \log_2 \sqrt{2} + \log_2 \{K(G') / g(G')\} \right\rceil$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to x . Using this k_i the optimum M_i 's are obtained substituting k on $M_i = 2^{k_i}$. More details are provided in *Maxwell and Muckstadt (1985)*.

3.10. Implementation of the Empirical Study

As a matter of the scope of this empirical study, the basic models which were also cased in the dissertation by Faik Basaran have been examined and studied in detail. However, a different approach, namely "Power of Two Policy" has been selected and used to calculate the reorder time intervals and then compute order quantities. The algorithm of Muckstadt and Roundy for finding the optimal ordered partition of a serial system has been also undertaken in this study. This algorithm can be used to find the optimal ordered partition for a serial system. It consists of three main steps. In the first, we find the clusters, or equivalently, an optimal partition of G ; in the second, we solve problem(3.2), in the third, we solve problem (3.1). And in the last stage we find the optimal solution. The below are the details of this algorithm:

Using the definitions, assumptions, form of the policies considered, and the echelon inventory method for calculating holding costs, we see that the reorder interval model is

$$z_9 = \underset{i \in N(G)}{\text{minimize}} \sum [K_i / T_i + g_i T_i], \quad (3.1)$$

subject to $T_i = 2^l T_L, \quad l \in \{0, 1, \dots\},$

$$T_i \geq T_{i-1} \geq 0.$$

This problem is a nonlinear, integer programming problem. The integer decision variable is l . However, due to its special structure we can easily solve it even when G is an arbitrary acyclic graph. As was the case for the single stage system, problem turns out to have very close relationship to its following relaxation :

$$z_{10} = \text{minimize } \sum_{i \in N(G)} [K_i / T_i + g_i T_i], \quad (3.2)$$

subject to $T_i \geq T_{i-1} \geq 0.$

Now suppose we are given an optimal partition $G_1, \dots, G_N$ of the serial system graph G . Let $T^*(k)$ be the corresponding optimal solution. Now let us see how we can find the optimal powers-of-two solution to problem (3.1). Let $T_i = T(k)$, $i \in N(G)$, $k = 1, \dots, N$, where we find $T(k)$ by solving

$$\text{minimize } \sum_{i \in N(G_k)} [K_i / T(k) + g_i T(k)] \quad (3.4)$$

subject to $T(k) = 2^l T_L, \quad l \in \{0, 1, \dots\}$

Given an arbitrary node set C ,

We define $T^*(C) = [(\sum_{i \in C} K_i) / (\sum_{i \in C} g_i)]^{1/2}$

The Algorithm for Serial systems:

Step 1. Find an optimal partition of G.

(a) Set $C^i \leftarrow \{i\}$ and (i) $i = 1$ for all $1 \leq i \leq n$, and $S = (1, 2, \dots, n)$.

Set $j = 2$. Note: (i) is the node that precedes i in the sequence S.

(b) If $T^*(C^j) \geq T^*(C^{\sigma(j)})$, go to Step 1d; otherwise, collapse $C^{\sigma(j)}$ into C^j by setting $C^j \leftarrow C^{\sigma(j)} \cup C^j$, $\sigma(j) \leftarrow \sigma(\sigma(j))$, and $S \leftarrow S \setminus \{\sigma(j)\}$.

(c) If $\sigma(j) > 0$, go to Step 1b

(d) Set $j \leftarrow j + 1$. if $j \leq n$, go to Step 1b

(e) Re-index the clusters $(C_i : i \in S)$ so that $S = (1, 2, \dots, N)$ and if

$j \in C^i$, $k \in C^l$, and $j < k$ then $i < l$

Comment: $(C^k : k \in S)$ are the clusters. The optimal partition is $(G_k : k \in S)$ where G_k is the subgraph of G induced by C^k . Thus $C^k = N(G_k)$.

Step 2. Find the Solution to problem (3.2).

For each cluster C^k , $k \in S$, set

$$T^*(k) = T^*(C^k) = \left[\left(\sum_{i \in C^k} K_i \right) / \left(\sum_{i \in C^k} g_i \right) \right]^{1/2}$$

For each $i \in C^k$ set $T_i^* = T^*(k)$

Step 3. Find the Solution to problem (3.4).

$$Z = \text{minimize } \sum_{i \in N(G_k)} [K_i/T(k) + g_i^* T(k)]$$

subject to $T_i = 2^l T_L$, $l \in \{0, 1, \dots\}$.

For each $i \in C^k$ set $T_i = 2^l T_L$ where $2^l \geq T^*(k)/\sqrt{2} T_L > 2^{l-1}$

It is sometimes desirable to impose a uniform lower and / or upper bound which applies to all reorder intervals. Suppose we add to problem (3.1) the constraint

$$2^{-1} T_L \leq T_i \leq 2^1 T_L \quad \forall i \in N(G).$$

If $i \in C^k$, an optimal solution to this version of (3.1) is obtained by selecting $T_i = 2^1 T_L$ if $T^*(k) \leq 2^1 T_L$, $T_i = 2^{-1} T_L$ if $T^*(k) \geq 2^1 T_L$, and selecting T_i

as in Step 3 above if $2^l T_L \leq T^*(k) \leq 2^l T_L$. If this done all claims of optimality and near-optimality made in the following still apply.

The relaxation (3.2) of problem (3.1) was first formulated and solved in a more general setting by Schwarz & Schrage [1975].

After the calculation of order quantities using the given data of demand for each part and deriving reorder time intervals via the algorithm mentioned before, the method proposed by Szendrovits has been employed in finding average stage inventory costs (I_n), average fixed costs (F_n), and finally total costs (TC). All these cost components have been computed using the following formulas (Szendrovits, 1981, p. 1083):

The average stage inventory costs can be obtained by multiplying the time-weighted stage inventory costs per cycle by the number of cycles per unit time, D/Qx . For stage 1 the average stage inventory cost per period is:

$$I_1 = c_1 (Q_1^2/2P_1 + Q_1^2/2D) (D/Q_1) = Q_1 (c_1/2) (D/P_1 + 1)$$

For stage 2, where $Q_2 = Q_1/k$, we obtain:

$$\begin{aligned} I_2 &= c_2 [k Q_2^2 / 2P_2 + k Q_2^2 / 2P_1 + k (k-1) / 2 Q_2^2 (1/P_2) - (1/P_1)] D/Q_1 \\ &= Q_1 (c_2/2) (1/k) [(D/P_2 + D/P_1) + (k-1) (D/P_2 - D/P_1)] \end{aligned}$$

The average fixed costs per period are:

$$F = D (K_1 + kK_2) / Q_1$$

The cost function for the two stage system, where $P_2 < P_1$

$$Z(Q_1, k) = I_1 + I_2 + F$$

subject to $Q_2 = Q_1/k$, $k > 1$, $k = \text{integer}$.

Note that $Z(Q_1, k)$ is convex on Q_1 ; thus, for a given A it is easy to determine the optimal Q_1 .

To carry out all the necessary calculations to assess the required values, I have prepared a visual basic macro through using the properties of the excel program. This replication of the algorithm on the excel has given the below results:

```
Sub test()
```

```
Dim Demand As Double
```

```
Dim MaxQ As Double
```

```
Dim CI(1 To 6) As Double
```

```
Dim F As Double
```

```
Dim TotalCost As Double
```

```
Dim p(1 To 6) As Double
```

```
Dim M(1 To 6) As Double
```

```
Dim K(1 To 6) As Double
```

```
Dim S(1 To 6) As Double
```

```
Dim Q(1 To 6) As Double
```

```
Dim H(1 To 6) As Double
```

```
Dim T(1 To 6) As Double
```

```
Dim TPrime(1 To 6) As Double
```

```
Dim C(1 To 6) As Double
```

```
Dim sigma(1 To 6) As Double
```

```
Dim C1, C2, C3, C4, C5, C6 As Double
```

```
Dim Constant As Double
```

```
Dim SumK, SumH As Double
```

```
Dim I, j, n As Integer
```

```
Demand = Cells(3, 3)
```

```
For I = 1 To 6
```

```

    C(I) = I
Next

For I = 1 To 6
    sigma(I) = I - 1
Next

Constant = Cells(3, 5)

For I = 1 To 6
    p(I) = Cells(5 + I, 7)
Next

For I = 1 To 6
    S(I) = Cells(5 + I, 4)
Next

For I = 1 To 6
    K(I) = Cells(5 + I, 2)
Next

For I = 1 To 6
    H(I) = Cells(5 + I, 3)
Next

For I = 1 To 6
    T(I) = Constant * Application.Power(K(I) / H(I), 0.5)
Next

For j = 2 To 6
    If (T(C(j)) < T(C(sigma(j)))) Then
        'do nothing
    
```

Else

$\text{sigma}(j) = \text{sigma}(j - 1)$

$C(j) = C(j - 1)$

$\text{SumK} = 0$

$\text{SumH} = 0$

 For $n = C(j)$ To j

$\text{SumK} = \text{SumK} + K(n)$

$\text{SumH} = \text{SumH} + H(n)$

 Next

 For $n = C(j)$ To j

$T(n) = \text{Constant} * \text{Application.Power}(\text{SumK} / \text{SumH}, 0.5)$

 Next

End If

Next

For $I = 1$ To 6

$\text{Cells}(15 + I, 6) = C(I)$

$\text{Cells}(15 + I, 7) = T(I)$

Next

For $I = 1$ To 6

 For $n = 0$ To 100

 If $(T(I) < \text{Application.Power}(2, n))$ Then

$TPrime(I) = \text{Application.Power}(2, n)$

$\text{Cells}(15 + I, 8) = \text{Application.Power}(2, n)$

```

        n = 100
    End If

Next

Next

For I = 1 To 6
    Q(I) = Demand * TPrime(I)
    Cells(23 + I, 7) = Q(I)
Next

MaxQ = 0
For I = 1 To 6
    If (Q(I) > MaxQ) Then
        MaxQ = Q(I)
    End If
Next

For I = 1 To 6
    M(I) = Application.Round(Q(I) / Q(6), 0)
    Cells(23 + I, 8) = M(I)
Next

For I = 1 To 5
    If (p(I + 1) >= p(I)) Then
        CI(I) = Q(6) * H(I) / 2 * 1 / M(I) * (Demand * 250 / p(I) + Demand * 250 / p(I
+ 1) + (M(I) - 1) * Demand * 250 * (1 / p(I) - 1 / p(I + 1)))
    Else
        CI(I) = Q(6) * H(I) / 2 * 1 / M(I) * (Demand * 250 / p(I) + Demand * 250 / p(I
+ 1))
    End If

```

Next

For I = 6 To 6

$CI(I) = Q(6) * H(I) / 2 * 1 / M(I) * (Demand * 250 / p(I) + (M(I) - 1) * Demand * 250 * (1 / p(I)))$

Next

For I = 1 To 6

$Cells(23 + I, 9) = CI(I)$

Next

F = 0

For I = 1 To 6

$F = F + Demand * 250 * S(I) * M(I) / Q(6)$

Next

$Cells(24, 10) = F$

TotalCost = F

For I = 1 To 6

$TotalCost = TotalCost + CI(I)$

Next

$Cells(24, 11) = TotalCost$

End Sub

The study has finally produced the following numerical findings obtained through applying the suggested algorithm for twelve different items:

Large Brimmed Soup Bowl

	DEMAND	12,8	Const.	24,46001		
	I	0,000261				
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	3188	38663	1298	5607	7,0237341	111701
2 Molding	1890	33056	696	12293	5,8487416	6981
3 Glaze Preparation	1194	20763	998	4313	5,8656182	137379
4 Glazing Kilns	196	16450	117	8286	2,6699416	11304
5 Technical Decoration	79	8164	22	4342	2,406128	7771
6 Technical Decoration Kilns	57	3822	57	3822	2,9870932	17679

Results:

	Groups	T*	T	Total Holding Cost
1	1	7,023734144	8	915,4781289
2	2	5,855258265	8	678,254734
3	2	5,855258265	8	426,8802121
4	4	2,669941634	4	158,9796992
5	5	2,60548754	4	74,33202214
6	5	2,60548754	4	39,80273011

Per day total holding cost	2293,728
Per year Total Hold. Cost	573431,9

Qi	Ki	I	ΣF	Total Cost
102,4	2	241026,9774	386250	1530832,639
102,4	2	387902,524		
102,4	2	81425,20632		
51,2	1	292624,938		
51,2	1	123892,8152		
51,2	1	17710,17818		

LBSB Lid

	DEMAND 12,8 Const. 24,46001 I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	1752	21246	713	3082	7,0240104	543273
2 Molding	1039	18164	383	6755	5,8500383	18416
3 Glaze Preparation	656	11409	549	2370	5,8652268	664000
4 Glazing Kilns	107	9039	64	4554	2,6612666	29830
5 Technical Decoration	43	4485	12	2384	2,395023	20501
6 Technical Decoration Kilns	31	2101	31	2101	2,9711495	46688

Results:

	Groups	T*	T	Total Holding Cost
1	1	7,024010391	8	503,0885944
2	2	5,855902574	8	372,7529643
3	2	5,855902574	8	234,5542113
4	4	2,661266571	4	87,18200614
5	5	2,59275619	4	40,73534656
6	5	2,59275619	4	21,7966473

Per day total holding cost	1260,11
Per year Total Hold. Cost	315027,4

Qi	Ki	I	ΣF	Total Cost
102,4	2	48856,19225	212312,5	448757,0822
102,4	2	80799,02693		
102,4	2	16369,64531		
51,2	1	60942,12453		
51,2	1	25791,12299		
51,2	1	3686,470185		

Brimmed Regular Plate Ø29 cm

	DEMAND 41,6 Const. 13,56797 I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Lathe Clay Preparation	911	14501	236	1020	3,400754	42233
2 Lathe	675	13481	208	2848	3,0360284	17892
3 Glaze Preparation	467	10633	357	1544	2,8434496	314526
4 Glazing Kilns	110	9089	70	4951	1,4926339	12433
5 Technical Decoration	40	4138	12	2260	1,3339813	17099
6 Technical Decoration Kilns	28	1878	28	1878	1,656709	19445

Results:

	Groups	T*	T	Total Holding Cost
1	1	3,400753978	4	542,8351525
2	2	3,036028434	4	461,6720703
3	3	2,843449607	4	347,7892681
4	4	1,492633874	2	153,7452228
5	5	1,442499081	2	64,95629133
6	5	1,442499081	2	34,40307277

Per day total holding cost	1605,401
Per year Total Hold. Cost	401350,3

Qi	Ki	I	ΣF	Total Cost
166,4	2	249596,7317	214000	1766690,358
166,4	2	325979,1996		
166,4	2	192315,0619		
83,2	1	546246,9023		
83,2	1	196768,0486		
83,2	1	41784,41347		

Brimmed Regular Plate Ø25 cm

	DEMAND317,6Const. 4,910451 I0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Lathe Clay Preparation	549	11557	124	533	1,0702495	50218
2 Lathe	425	11024	135	1850	0,9641537	21458
3 Glaze Preparation	290	9174	187	808	0,8730539	478080
4 Glazing Kilns	103	8366	64	4567	0,5448552	14922
5 Technical Decoration	39	3799	9	1743	0,4975298	20536
6 Technical Decoration Kilns	30	2056	30	2056	0,5931581	23329

Results:

	Groups	T*	T	Total Holding Cost
1	1	1,070249507	2	1233,088616
2	2	0,964153726	1	882,189621
3	3	0,873053891	1	670,4660362
4	4	0,544855204	1	449,9564921
5	5	0,533067882	1	196,5529182
6	5	0,533067882	1	115,2668596

Per day total holding cost	3547,521
Per year Total Hold. Cost	886880,1

Qi	Ki	I	ΣF	Total Cost
635,2	2	4846310,388	168250	3747938,99
317,6	1	6768445,231		
317,6	1	7993754,494		
317,6	1	12205630,13		
317,6	1	4385776,023		
317,6	1	1111214,725		

Brimmed Regular Plate Ø19 cm

DEMAND 389,6 Const. 4,433556						
I 0,000261						
	Ki	Hi	s	C	Ti	Pn
1 Lathe Clay Preparation	346	8059	68	291	0,9186491	66033
2 Lathe	278	7768	111	1522	0,8387258	22382
3 Glaze Preparation	167	6246	102	439	0,7249523	629053
4 Glazing Kilns	65	5807	32	2237	0,4690649	19636
5 Technical Decoration	33	3570	11	2096	0,4262605	27041
6 Technical Decoration Kilns	22	1474	22	1474	0,5416451	30699

Results:

	Groups	T*	T	Total Holding Cost
1	1	0,918649124	1	755,9933081
2	2	0,838725763	1	673,1889834
3	3	0,724952312	1	484,7588041
4	4	0,469064925	1	360,4251322
5	5	0,462962697	1	214,6200658
6	5	0,462962697	1	96,98822883

Per day total holding cost	2585,975
Per year Total Hold. Cost	646493,6

Qi	Ki	I	ΣF	Total Cost
389,6	1	9147345,787	86500	3758481,23
389,6	1	6819335,939		
389,6	1	6223664,335		
389,6	1	9685608,251		
389,6	1	4711356,069		
389,6	1	911005,8464		

Brimmed Hollow Plate Ø21 cm

	DEMAND 377,6 Const. 4,503453 I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Lathe Clay Preparation	454	9006	104	447	1,0111315	94857
2 Lathe	350	8559	123	1690	0,910685	14847
3 Glaze Preparation	227	6869	157	679	0,8186756	703059
4 Glazing Kilns	70	6190	37	2644	0,478905	20607
5 Technical Decoration	33	3546	10	2029	0,4344435	14178
6 Technical Decoration Kilns	23	1517	23	1517	0,5545195	32216

Results:

	Groups	T*	T	Total Holding Cost
1	1	1,011131476	2	1115,117828
2	2	0,910685002	1	772,0186815
3	3	0,818675598	1	565,689838
4	4	0,478904999	1	375,2103795
5	5	0,473626207	1	207,8426504
6	5	0,473626207	1	97,79873114

Per day total holding cost	3133,678
Per year Total Hold. Cost	783419,5

Qi	Ki	I	ΣF	Total Cost
755,2	2	6251587,302	139500	4339099,25
377,6	1	10491416,13		
377,6	1	6115037,52		
377,6	1	13134903,85		
377,6	1	6419304,029		
377,6	1	839243,4269		

Brimmed Oval Plate Ø35 cm

DEMAND 29,6 Const. 16,08481 I 0,000261						
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	1612	24347	547	2364	4,1388111	265600
2 Molding	1065	21983	512	9044	3,5403576	8081
3 Glaze Preparation	553	12939	421	1819	3,3252785	332000
4 Glazing Kilns	132	11120	80	5643	1,7524692	13086
5 Technical Decoration	52	5477	16	3017	1,5672783	9000
6 Technical Decoration Kilns	36	2460	36	2460	1,9458059	6822

Results:

	Groups	T*	T	Total Holding Cost
1	1	4,138811081	8	954,3419624
2	2	3,540357644	4	606,1219526
3	3	3,325278479	4	338,295635
4	4	1,752469161	2	151,9613363
5	5	1,693670637	2	68,33905027
6	5	1,693670637	2	37,01662656

Per day total holding cost	2156,077
Per year Total Holding Cost	539019,1

Qi	Ki	I	ΣF	Total Cost
236,8	4	170004,4799	523250	2246580,456
118,4	2	595861,4429		
118,4	2	112558,0314		
59,2	1	456768,1995		
59,2	1	309152,9019		
59,2	1	78985,40018		

Brimmed Oval Plate Ø21 cm

	DEMAND 169,6 Const. 6,719682 I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	660	13120	167	720	1,5071407	703059
2 Molding	493	12400	300	5302	1,339866	11405
3 Glaze Preparation	193	7098	128	554	1,1080501	919385
4 Glazing Kilns	65	6544	27	1942	0,6697054	18463
5 Technical Decoration	38	4602	16	3113	0,6106147	12701
6 Technical Decoration Kilns	22	1489	22	1489	0,8167944	28870

Results:

	Groups	T*	T	Total Holding Cost
1	1	1,507140687	2	911,1206963
2	2	1,339865982	2	795,7299264
3	3	1,108050109	2	410,8898401
4	4	0,669705363	1	209,9258322
5	5	0,666929717	1	139,9175855
6	5	0,666929717	1	54,97594195

Per day total holding cost	2522,56
Per year Total Hold. Cost	630640

Qi	Ki	I	ΣF	Total Cost
339,2	2	2101642,475	313750	1.221.790
339,2	2	3909201,929		
339,2	2	705018,4481		
169,6	1	3126928,922		
169,6	1	1875919,055		
169,6	1	185442,6491		

Large Brimmed Compote Bowl

	DEMAND 25,6 Const. 17,29584 I 0,000261				Ti	Pn
	Ki	Hi	s	C		
1 Lathe Clay Preparation	676	14072	180	778	3,7908545	66400
2 Lathe	496	13294	83	1137	3,3408324	26858
3 Glaze Preparation	413	12157	273	1178	3,1878913	498000
4 Glazing Kilns	140	10979	69	4878	1,9530992	21067
5 Technical Decoration	71	6101	10	2000	1,8658226	25648
6 Technical Decoration Kilns	61	4101	61	4101	2,1094135	32956

Results:

	Groups	T*	T	Total Holding Cost
1	1	3,790854537	4	357,1622282
2	2	3,34083238	4	301,7592852
3	3	3,187891287	4	265,8060125
4	4	1,953099238	2	143,4022564
5	5	1,967369758	2	76,2894313
6	5	1,967369758	2	57,9180393

Per day total holding cost	1202,337
Per year Total Hold. Cost	300584,3

Qi	Ki	I	ΣF	Total Cost
102,4	2	60282,34229	151500	587362,7955
102,4	2	81096,46884		
102,4	2	49272,85663		
51,2	1	155518,7931		
51,2	1	69304,30693		
51,2	1	20388,02767		

Brimmed Compote Bowl

	DEMAND 281,6 Const. 5,214892					
	I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Lathe Clay Preparation	209	5604	35	151	1,0070926	202576
2 Lathe	174	5453	87	1185	0,9315415	15187
3 Glaze Preparation	87	4268	53	229	0,7445481	1494000
4 Glazing Kilns	34	4039	10	711	0,4784625	37311
5 Technical Decoration	24	3328	13	2587	0,4428527	14487
6 Technical Decoration Kilns	11	741	11	741	0,6353786	58302

Results:

	Groups	T*	T	Total Holding Cost
1	1	1,007092641	2	516,6330442
2	2	0,931541523	1	374,5140516
3	3	0,744548107	1	243,9400279
4	4	0,478462466	1	182,5193938
5	5	0,48365482	1	146,3749796
6	5	0,48365482	1	38,24755405

Per day total holding cost	1502,229
Per year Total Hold. Cost	375557,3

Qi	Ki	I	ΣF	Total Cost
563,2	2	1965927,743	61000	13589872,61
281,6	1	3595268,062		
281,6	1	1162186,002		
281,6	1	3836602,533		
281,6	1	2842905,811		
281,6	1	125982,4555		

Brimmed Creamer

	DEMAND 8,8 Const. 29,49989					
	I 0,000261					
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	431	9740	73	313	6,2055387	1328000
2 Molding	358	9427	261	4614	5,7487721	10233
3 Glaze Preparation	97	4813	56	241	4,187919	1707429
4 Glazing Kilns	41	4572	15	1050	2,7935674	22065
5 Technical Decoration	26	3522	14	2715	2,5346169	11394
6 Technical Decoration Kilns	12	807	12	807	3,5972795	34510

Results:

	Groups	T*	T	Total Holding Cost
1	1	6,205538701	8	143,4131837
2	2	5,748772092	8	131,4108273
3	3	4,18791895	8	56,37010042
4	4	2,79356737	4	31,26481395
5	5	2,763874616	4	22,68857715
6	5	2,763874616	4	6,709307712

Per day total holding cost	391,8568
Per year Total Hold. Cost	97964,2

Qi	Ki	I	ΣF	Total Cost
70,4	2	18569,27705	51313	150215,3436
70,4	2	35670,22769		
70,4	2	4277,536156		
35,2	1	23559,95086		
35,2	1	15920,40299		
35,2	1	905,4488554		

Brimmed Creamer Saucer

	DEMAND 8,8 Const. 29,49989 I 0,000261				Ti	Pn
	Ki	Hi	s	C		
1 Lathe Clay Preparation	179	6501	30	127	4,8950459	1328000
2 Lathe	149	6374	49	664	4,5103208	23029
3 Glaze Preparation	100	5710	44	192	3,9039323	1494000
4 Glazing Kilns	56	5518	14	976	2,9718268	37311
5 Technical Decoration	42	4542	14	2661	2,8367523	25648
6 Technical Decoration Kilns	28	1881	28	1881	3,5991915	54994

Results:

	Groups	T*	T	Total Holding Cost
1	1	4,895045853	8	82,13760083
2	2	4,510320841	8	77,22011117
3	3	3,903932299	4	51,24553536
4	4	2,971826816	4	39,36302349
5	5	3,079643113	4	31,37692147
6	5	3,079643113	4	15,6458585

Per day total holding cost	296,9891
Per year Total Hold. Cost	74247,26

Qi	Ki	I	ΣF	Total Cost
70,4	2	5560,028728	16125	63911,56213
70,4	2	10716,97772		
35,2	1	6073,616589		
35,2	1	14056,73547		
35,2	1	10054,83514		
35,2	1	1324,368477		

Brimmed Salt Shaker

DEMAND 54,24658 Const. 11,88162 I 0,000261						
	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	158	5437	26	113	2,02546 33	2390400
2 Molding	132	5324	86	1522	1,87086 93	23029
3 Glaze Preparation	46	3802	20	88	1,30691 81	2988000
4 Glazing Kilns	26	3714	3	203	0,99412 55	84766
5 Technical Decoration	23	3511	15	2989	0,96166 54	25648
6 Technical Decoration Kilns	8	522	8	522	1,47090 71	132800

Results:

	Groups	T*	T	
1	1	2,025463316	4	193,5523453
2	2	1,870869294	2	141,4252976
3	3	1,306918075	2	76,86306941
4	4	1,019168882	2	65,61637028
5	5	1,019168882	2	61,24046205
6	5	1,019168882	2	11,39519259

Per day total holding cost	550,0927
Per year Total Hold. Cost	200783,8

Qi	Ki	I	ΣF	Total Cost
216,9863014	2	87680,4691	23000	577814,9708
108,4931507	1	171388,6163		
108,4931507	1	33933,1345		
54,24657534	1	138763,584		
108,4931507	1	120157,4412		
108,4931507	1	2891,725657		

Brimmed Lemon Plate

DEMAND	12,8	Const.	24,46001
I	0,000261		

	Ki	Hi	s	C	Ti	Pn
1 Molding Clay Preparation	363	9096	47	203	4,8863522	1992000
2 Molding	316	8893	242	4281	4,6107978	10233
3 Glaze Preparation	74	4612	36	157	3,0983316	2390400
4 Glazing Kilns	38	4455	12	835	2,2590437	25722
5 Technical Decoration	26	3620	15	2884	2,0729512	11394
6 Technical Decoration Kilns	11	736	11	736	2,9902956	40242

Results:

	Groups	T*	T	Total Holding Cost
1	1	4,886352231	8	167,0011816
2	2	4,61079781	8	158,4117891
3	3	3,09833158	4	49,33442995
4	4	2,259043674	4	39,28477568
5	5	2,254309855	4	30,70221952
6	5	2,254309855	4	7,670672256

Per day total holding cost	452,4051
Per year Total Hold. Cost	113101,3

Qi	Ki	I	ΣF	Total Cost
102,4	2	36595,92311	40750	244498,0923
102,4	2	71192,66686		
51,2	1	14846,45574		
51,2	1	46218,71524		
51,2	1	33396,06783		
51,2	1	1498,263506		

CONCLUSION

The main contribution of this research is the development of the algorithm approach to determine the reorder cycle time in multi-stage serial system, considering the power of two restrictions.

The algorithm developed shows to be effective, because it provides solutions in costs Optimal solutions are obtained 75 percent of the time for small problems. The algorithm programmed in excel (macro in visual basic) and using a Pentium 4, provides solutions in less than one second for small problems.

The algorithm is conducted to measure the ability to provide solutions of the same quality in different problems instances. The experiment shows that algorithm gives very accurate solutions, comparing the results obtained in the other algorithms.

A comparison between power of two algorithm and another heuristics developed in this area. The total cost values obtained in the study are summarized in the accompanying table.

As it can be seen from the table, the existing method (method 5) which has been used in the factory is not effective. It is the worst method since the total cost has the highest value. The method 3 (Peterson and Silver's) is also not effective. It is the second worst method in all. It has the second highest total cost value. On the other hand the other four methods, method 1, method 2, method 4 and our main method (method 6) have total cost values approximately near to each other.

$17.699.188 \leq 17.933.825 \leq 18.238.911 \leq 22.042.954 \leq 80.751.763 \leq 270.166.199$.
The method 4 is the most effective method with having the least total cost value while the others having a bigger total cost value. Only finding the total cost of the Brimmed Oval Plate Ø21 cm, our method power of two policy is the best effective one since it has the smallest total cost value.

Table 20: The Total Cost Values Summerized In the Study

	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Large Brimmed Soup Bowl	1.126.341	1.074.427	18.612.381	1.033.294	1.598.951	1.530.832
LBSB Lid	545.860	506.667	10.221.347	440.867	572.030	448.757
Brimmed Regular Plate Ø29 cm	736.982	717.858	5.004.862	744.624	2.632.592	1.766.690
Brimmed Regular Plate Ø25 cm	3.747.080	3.730.631	7.952.717	2.982.999	73.071.238	3.747.938
Brimmed Regular Plate Ø19 cm	2.514.883	2.507.217	4.251.750	2.558.164	56.049.899	3.758.481
Brimmed Hollow Plate Ø21 cm	3.698.482	3.692.103	7.143.803	3.569.385	78.304.415	4.339.099
Brimmed Oval Plate Ø35 cm	1.310.901	1.275.535	13.092.904	1.795.896	4.394.043	2.246.580
Brimmed Oval Plate Ø21 cm	2.497.986	2.477.857	7.775.161	2.416.083	31.320.130	1.221.790
Large Brimmed Compote Bowl	419.578	342.605	1.659.070	320.254	763.674	587.362
Brimmed Compote Bowl	1.060.694	1.054.242	2.109.826	1.225.824	19.743.470	1.358.987
Brimmed Creamer	141.365	137.789	1.195.428	120.186	171.019	150.215
Brimmed Creamer Saucer	71.972	57.523	168.709	60.208	79.444	63.911
Brimmed Salt Shaker	212.811	208.263	738.661	294.589	1.216.383	577.814
Brimmed Lemon Plate	153.976	151.108	825.144	136.815	248.911	244.498
TOTAL	18.238.911	17.933.825	80.751.763	17.699.188	270.166.199	22.042.954

Method 1: Crowston – Wagner – Williams **Method 2:** Economic Order Quantity **Method 3:** Peterson – Silver **Method 4:** Schwarz – Schrage

Method 5: The Existing Method in the Factory **Method 6:** Power-of-Two Policy

Source: Faik Başaran, Çok Kademeli Üretim Dağıtım Yapıları Stok Sistemleri ve Porselen Sanayii Uygulaması, PhD Thesis, Istanbul University, Istanbul, 1993, p.259.

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