

AUGUST 2019

Ph.D in Mechanical Engineering

BURAK ŞAHİN

**REPUBLIC OF TURKEY
GAZİANTEP UNIVERSITY
GRADUATE SCHOOL OF NATURAL & APPLIED SCIENCES**

**A THEORETICAL AND EXPERIMENTAL STUDY ON
ASYMMETRIC SPUR GEARS**

**Ph.D THESIS
IN
MECHANICAL ENGINEERING**

**BY
BURAK ŞAHİN
AUGUST 2019**

**A THEORETICAL AND EXPERIMENTAL STUDY ON
ASYMMETRIC SPUR GEARS**



**Ph.D. Thesis
in
Mechanical Engineering
Gaziantep University**

**Supervisor
Prof. Dr. Nihat YILDIRIM**

**by
Burak ŞAHİN
August 2019**



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REPUBLIC OF TURKEY
GAZİANTEP UNIVERSITY
GRADUATE SCHOOL OF NATURAL & APPLIED SCIENCES
DEPARTMENT OF MECHANICAL ENGINEERING

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Gears

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Exam Date : 28.08.2019

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Burak ŞAHİN

ABSTRACT

A THEORETICAL AND EXPERIMENTAL STUDY ON ASYMMETRIC SPUR GEARS

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Ph.D. in Mechanical Engineering

Supervisor: Prof. Dr. Nihat YILDIRIM

August 2019

266 pages

The aim of this thesis is to investigate asymmetric spur gears for helicopter transmission units in terms of static and dynamic performance by conducting theoretical and experimental works.

This thesis is a part of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) [1] which was launched at METU Technopark Facility within the scope of contract signed between Undersecretariat for Defence Industries (SSM) and Turkish Aerospace Industries, Inc. (TAI) on June 26, 2013.

A user friendly software has been developed to generate macro geometry of asymmetric gears for checking geometrical restrictions and finite element analyses. Software allows analysis and design of asymmetric spur gears for root stress including micro profile design for minimum peak to peak transmission error (PPTE) and obtain smooth transmission error (TE) curves under design or any desired off-design load.

Back to back test rig has been designed, manufactured and assembled in the scope of this thesis and asymmetric gear design project for helicopter power transmission units [1]. By using this test rig, experiments have been conducted to measure static and dynamic transmission error and construct TE curves for symmetrical and asymmetrical tooth spur gears. Dual asymmetry (asymmetry for tooth and root fillet profiles) have been investigated for further decrease in tooth root bending stress of spur gears.

Key Words: Asymmetric spur gears, asymmetry of tooth profile, asymmetry in root profile, dual asymmetry, gear design, static transmission error, dynamic transmission error, back to back gear test rig, finite element analysis, tooth root bending stress.

ÖZET

ASİMETRİK DÜZ DİŞLİLER ÜZERİNE TEORİK VE DENEYSEL BİR ÇALIŞMA

ŞAHİN, Burak

Doktora Tezi, Makina Mühendisliği Bölümü

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Ağustos 2019

266 sayfa

Bu tezin amacı, teorik ve deneysel çalışmalar yürüterek helikopter güç aktarım ünitelerinde kullanılabilecek asimetrik düz dişlilerin statik ve dinamik performanslarının incelenmesidir. Bu tez, TAI (Türk Havacılık ve Uzay Sanayi A.Ş) ile Savunma Sanayi Müsteşarlığı (SSM) arasında 26.06.2013 tarihinde imzalanan anlaşma ile ODTÜ Teknoparkta kurulmuş olan DKTM (Döner Kanat Teknoloji Merkezi) tarafından desteklenen Özgün Helikopter Programı [1] kapsamında yürütülmüştür.

Asimetrik düz dişlilerin geometrik limitlerinin kontrolü ve sonlu eleman analizleri için makro geometri oluşumunu yapabilen kullanıcı dostu bir yazılım geliştirilmiştir. Yazılım ayrıca asimetrik dişlilerin diş dibi gerilmeleri açısından analiz ve tasarımına izin verdiği gibi dişli çifti için minimum iletim hatası elde edilmesi ve farklı yükler altında iletim hatası grafiklerinin türetilmesine de olanak sağlar.

Bu tez ve helikopter dişli tasarımı projesi [1] kapsamında, sırt sırta bir dişli test düzeneği tasarım, imalat ve montajı yapılmıştır. Bu düzenek kullanılarak, simetrik ve asimetrik düz dişliler için statik ve dinamik iletim hatalarının ölçümleri yapılmıştır. Simetrik düz dişlilere kıyasla diş dibi gerilmesinde azalma sağlayan asimetrik profil uygulanmıştır. Buna ek olarak hem evolvent diş profiline hem de diş dibi radyusuna çift asimetri uygulanmış ve daha az gerilme değeri elde edilmiştir.

Anahtar Kelimeler: Asimetrik düz dişliler, diş profilinde asimetri, diş dibi radyusunda asimetri, çift asimetri, dişli tasarımı, statik iletim hatası, dinamik iletim hatası, sırt sırta dişli test düzeneği, sonlu eleman analizi, sonlu eleman analizi, diş dibi gerilmesi.



***‘Dedicated to my beloved parents and family,
my glorious country and
my ideals’***

ACKNOWLEDGEMENTS

First of all, I would like to express my sincere appreciation to my supervisor, Prof. Dr. Nihat YILDIRIM for his guidance, insightful conversations and support throughout this thesis. I am thankful for his encouragement and motivation. It was a privilege to study with him, share time in workshop and conduct experiments in laboratory together.

I would like to express my endless thanks to Assist. Prof. Dr. Abdullah AKPOLAT for his invaluable encouragement and moral support during my thesis studies. It was an honor to study and discuss with him.

I would like to thank my thesis committee, Prof. Dr. Ömer EYERCİOĞLU and Assoc. Prof. Dr. Zihni Burçay SARIBAY for their support and constructive criticism during my study.

I am grateful to Assoc. Prof. Dr. H. İbrahim ÇELİK for his supports. He always provided heartfelt support and encouragement. I would like to thank Mr. Ömer YILDIRIM for his technical support for software, sharing time for studying together up to late hours and brotherhood.

I would like to express my deepest appreciation and love to my wife, Bircan GENÇ ŞAHİN and my daughter, Erva for their support, patience, encouragement. Without their devotion, neither my life nor my studies would be complete.

This thesis is a part of the project of “Improvement of Design and Analysis Methods for High Performance Asymmetric Spur Gears”. It has been supported by Rotary Wing Technology Center (DKTM) under the cooperation of Turkish Aerospace Industries (TAI), Undersecretariat for Defence Industries (SSM) and Gaziantep University in the scope of Indigenous Helicopter Program.

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CHAPTER 1

INTRODUCTION

1.1 General Introduction

A theoretical and experimental study on asymmetric tooth profile spur gears has been conducted in this thesis. This thesis is a part of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) which was launched at METU Technopark Facility within the scope of contract signed between Undersecretariat for Defence Industries (SSM) and Turkish Aerospace Industries, Inc. (TAI) on June 26, 2013.

The study mainly focuses on asymmetric tooth profile gears which can be an alternative to symmetric one especially used in helicopter transmission units because of their high load carrying capacity and low weight.

User friendly software for symmetric and asymmetric tooth spur gears has been developed in scope of this thesis and project of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) [1]. The software has capabilities such as macro geometry generation, tooth root stress calculations based on international gear standard of ISO [2], stiffness calculations based on thin slice theory, micro profile design for minimum transmission error based on Harris map and, mathematical modeling and simulation to obtain dynamic transmission error for symmetric and asymmetric tooth spur gears.

1.2 An Overview of Asymmetric Spur Gears

Power transmission is a very crucial duty for many machines and systems which do work for industry. Because power generated by source is not directly usable by the machines and systems generally. It is required to be converted and transmitted from power source to power consuming equipment.

When two smooth cylinders are mounted on shafts with parallel axes and are pressed together lengthwise, it is possible to transmit power from one shaft to the other by friction drive. If there is no slippage during the contact, such rotating cylinders will

ensure a smooth drive and cylinders' speeds are inversely proportional to the diameters of the cylinders. This relation applies if the driving and the driven cylinders are perfectly accurate and the cylinders are said to produce uniform velocity transmission. However, this is impossible to achieve in practice because it does not produce a positive drive owing to the slippage which may be caused by various factors. To overcome the problem of slippage, toothed wheels or gears are used which produce accurate transmission of angular velocity [3].

There are three distinct types of gears in terms of shaft positions (Figure 1.1):

- power and motion transfer between parallel shafts including spur and helical gears,
- between intersecting shafts such as bevel gears,
- between neither parallel nor intersecting shafts such as worm, worm wheel, crossed helical gears and hypoid gears.

Gear tooth geometry is also an important concern to provide uniform motion transfer between shafts. Mating teeth of gears acting against each other to produce rotary motion are similar to cams. When the tooth profiles are designed so as to produce a constant angular velocity ratio during meshing, these are said to have conjugate action. Many tooth profile forms such as involute, cycloidal and circular can theoretically fulfill the basic requirements such as uniform motion transfer and constant angular velocity during gear meshing (conjugate action). Involute profile, which with few exceptions, is in universal use for gear teeth. It is insensitive to center distance change and allows different operating center distance without changing the velocity ratio [3].

Gear design includes two main criteria namely the static and dynamic performances. Static performance usually deals with global kinematic and geometric requirements and strength analysis of gear materials under load while dynamic performance usually deals with vibration and noise of gears. Gears experience two kinds of stress mainly; contact stress at tooth surface and bending stress at tooth root. Gear tooth strength relates to the ability to resist tooth breakage under the effect of tooth root bending stress; gear tooth surface durability relates to the ability to resist pitting under the effect of surface pressure between meshing teeth.

Design criteria of gears determine some parameters like safety, reliability, service life, performance and all these parameters are affected by many geometrical and working

condition parameters. Due to a vast amount of parameters and complexity of gears and gearbox design, designers need to have a comprehensive knowledge of gearing supported by well accepted equations, design procedures such as Lewis bending equation, AGMA stress equation and international standards regarding gears.

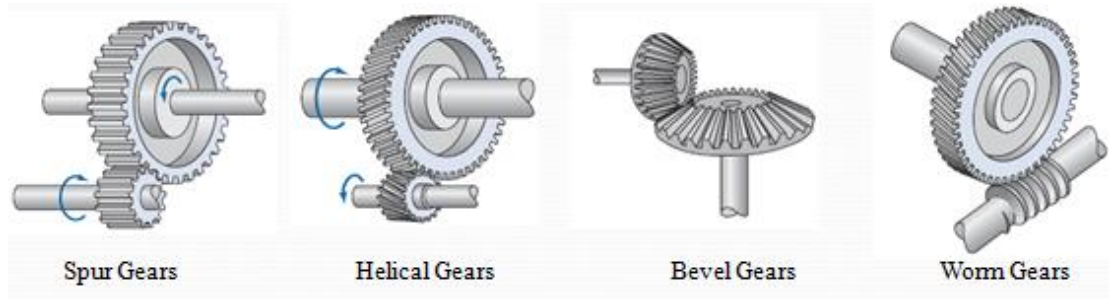


Figure 1.1 Gear types in terms of shaft arrangement

In the design of heavily loaded transmission gears as used in aerospace applications, it is often necessary to increase bending strength (in other words decrease bending stress) while maintaining load carrying capacity or increase the load carrying capacity while maintaining bending stress/strength. [4].

In a standard gear which has symmetrical teeth, both left and right sides of a gear tooth profile have the same geometry/shape and hence similar bending and contact strength capacities. However, in most practical cases, both the forward and backward rotations are not always used for power transmission [5-6].

A method of achieving the goals of improved gear performance is the design of gears with asymmetrical teeth. That is, the pressure angle on the drive side is different from the pressure angle on the coast side. It is possible to design teeth with the greater pressure angle on either the drive side or the coast side, and each method can have its advantages. Asymmetric teeth are well suited for cases where the torque is transmitted only, or mainly, in one direction. Because of the asymmetric teeth, designers are able to create gear drives capable of handling greater torque in the same amount of space, or they are able to reduce the amount of space required to handle the same amount of torque [4].

In many gear transmissions, a tooth load on one flank is significantly higher and is applied for longer periods of time than for the opposite one; an asymmetric tooth shape reflects this functional difference. With asymmetric gears, the standard symmetric

tooling gear rack is modified by altering the pressure angle of one of its flanks [7]. Two sides of the gear tooth are functionally different for most gears. Even if one side (drive side) is significantly loaded for longer periods, the opposite side (coast side) is unloaded or slightly loaded for short duration only. Thus asymmetric tooth (Figure 1.2) is well suited for cases where the torque is transmitted mainly, in one direction, [8] as in the case of most aerospace applications.

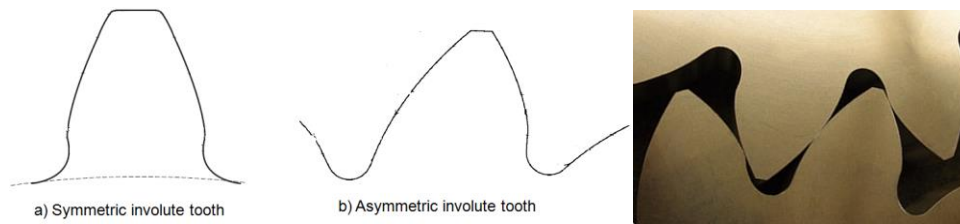


Figure 1.2 Symmetric-asymmetric involute gear tooth profile and asymmetric spur gear pair

1.3 Motivation of Thesis

Gear mass, noise and vibration level can be significantly reduced by designing and producing gear tooth in asymmetric form (in macro level and by applying proper profile modifications in micro level). Similarly by providing standard pressure angle at coast side and higher pressure angle at drive side, both bending and contact strength can be significantly improved with less mass [8]. Symmetric gears are being designed with standard procedures, whereas in the asymmetric gears, decision of suitable pressure angles at coast side and drive side are more crucial. Though the increase in pressure angle increases bending strength, there is an upper limit of the pressure angle to avoid tooth peaking and to have safe contact ratio.

In addition, decision of gear tooth fillet radius also influences the upper limit of pressure angle. Hence in the design of asymmetric gears, these factors are to be considered in addition to the symmetric gear design procedures. Decision on pressure angle alters many crucial factors such as critical tooth width, tooth form factor, stress concentration factor, load sharing factor, and magnitude of moment acting on a single tooth [3-4]. Hence there is a need for an expert numerical software/system to predict the above said crucial factors and design an asymmetric gear possessing required bending (and contact) stresses for a particular specific application.

Even by designing gears (symmetric or asymmetric) in a safe way statically in terms of bending and contact stresses, designer needs to have knowledge about gear dynamic behavior and keep it under control to reduce transmission error of gear pair and as a consequence decrease in noise and vibration.

Gear dynamics has been a subject of intense interest to the gearing area during the last few decades and dynamic loads and vibration are a major concern for gear drives at high speeds. Simulation of meshing of gear drives performed by application of tooth contact analysis and experimental tests of gear drives have confirmed that transmission errors (TE) are the prime cause of noise and vibrations of the gear drives [9].

The definition of transmission error is given as “The difference between the actual position of the output gear and the position it would occupy if the gear drive were perfectly conjugate”. This may be expressed as angular displacement or as linear displacement at the pitch point [10]. The causes of transmission error are elastic deflections under load, geometrical errors and geometrical modifications. Transmission errors causing dynamic loads in gears affect not only the gear vibrations and noise but also tooth bending fatigue and surface failure. Therefore, the most important objective in gear design is the minimization of dynamic loads and transmission errors.

The gears’ load-carrying calculation procedure is based on the assumption that gear’s teeth are rigid and produced with perfect geometry. When loaded, these teeth experience deflection, both at the base of the teeth and on the tooth flank. These deflections cause modification in the relative motion of teeth flanks in opposition to the ideal one and consequently generate transmission error [11].

During the meshing of gear pair, there appears so-called contact shock due to the contact of two new teeth coming into mesh. This impact produces noise and amplifies inaccuracies in the pitch and cause deformation of the teeth under load. In order to reduce the impact influence, the involute in the tip region is modified through a relief curve. This process is called profile modification at the tip and depends on elastic gear tooth deflection that needs to be compensated [12]. Tip relief profile modification (Figure 1.3) is defined as thickness $\Delta s(d)$ of the material removed along the tooth flank with reference to the nominal involute profile. To define changes in tooth thickness

$\Delta s(d)$, tooth tip diameter d_a , profile relief at tooth tip (C_a) and diameter at the beginning of correction (d_k) [12] have to be calculated by:

$$\Delta s(d) = C_a \frac{d-d_k}{d_a-d_k} \quad (1.1)$$

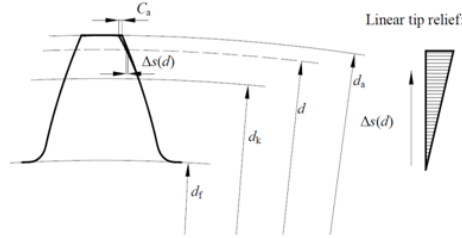


Figure 1.3 Linear tip relief modification

The requirements set to modern gear boxes are really tough to meet: They need to be even more efficient, stronger, smaller, quieter, and easier to produce and finally they need to cost less. Despite the impressive progress made in the recent years regarding the analysis and simulation, experiments are still essential. There are many modes of failure that may appear when the load carrying capacity of a gear pair is exceeded. Tooth breakage, pitting and micro-pitting as well as excessive wear or even scuffing are the most common failure types to be taken into consideration in design process. Besides the load carrying capacity, there are also other important parameters like efficiency and dynamic behavior that need to be experimentally investigated. Test rigs that allow gears to operate under predefined speed and torque conditions are therefore needed in such research and development works [13].

All these requirements expected to be satisfied by gears demand more research on alternative designs of gears and following tasks are steps planned to be accomplished:

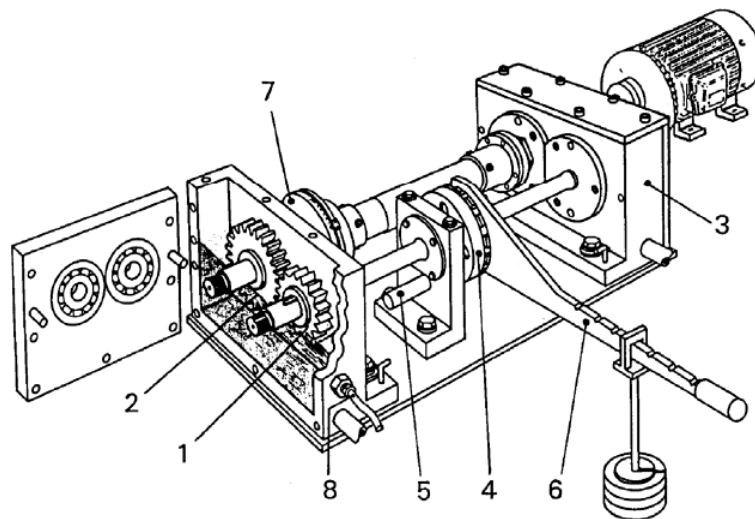
- ✓ Macro geometry construction of asymmetric spur gears,
- ✓ Design and analysis of asymmetric spur gears based on tooth root bending stress theoretically,
- ✓ Micro geometry design of gears for minimized transmission error including design and analysis of profile relief of gear teeth geometry in micro scale,
- ✓ Experimental studies to evaluate gear performance including quasi static transmission error comparisons.

Any theoretical improvements of the gear sets and gearboxes must be verified by through mechanical testing in a controlled environment before it is really applied in a

commercial design. Therefore, theoretical design and analyses are necessary before any manufacture of gears, and also experimental studies are necessary to validate the theoretical studies in terms of gear design procedure/parameter effectiveness or performances. Actual working conditions can be set or determined with relatively high accuracies during experimental studies. However, experimental studies on gear performance require much more time and effort than what theoretical studies need.

There are some test rigs which are conventionally used to evaluate gear and gearbox performance in terms of static and dynamic behavior, service life, reliability, noise and vibration; single tooth bending test rig, single tooth flank test rig and back to back test rig.

A conventional back-to-back test rig configuration consists of a test gearbox and a slave gearbox of identical gear ratio, which are coupled together (Figure 1.4). The test and slave gearboxes are similar in the design. The test torque is generated by applying equal and opposite torques to the gears in the slave and test box for example by “winding-up” a torsion bar between slave and test box. The external power drive compensates for the losses in the system (mesh and bearing friction, etc.), overcoming the inertial resistance and controls the rotational speed.



- | | |
|---------------|---------------------------|
| 1 Test Pinion | 5 Locking Pin |
| 2 Test Wheel | 6 Load Lever and Weights |
| 3 Slave Gear | 7 Torque Measuring Clutch |
| 4 Load Clutch | 8 Temperature Sensor |

Figure 1.4 Back to back gear test rig

1.4 Purpose of the Thesis

This study comprises both theoretical and experimental efforts. The purpose of this study is to:

- conduct a theoretical study on asymmetric spur gears to construct gear macro geometry,
- conduct a theoretical study on asymmetric spur gears to determine bending stress-strength of gear tooth,
- conduct a theoretical study on asymmetric spur gears to construct gear micro geometry including profile modification for quasi static transmission error calculations and
- conduct an experimental study on asymmetric spur gears to verify the results of theoretical calculations for micro profile and rating the gear pair performance.

Theoretical part of study includes following purposes:

- Geometrical construction,
- Design and analysis of macro geometry,
- Analysis of tooth root stress under load,
- Design and analysis of micro geometry,
- Micro profile modifications,
- Calculation of peak to peak transmission error (PPTE) values,
- Development of user friendly software for design and analysis of asymmetric spur gears for purpose aforementioned.

Experimental part of the study aims to verify the results of theoretical predictions/calculations for a reasonable rating of asymmetric gear pair performance. Experiments have been conducted on back to back gear test rig.

1.5 Structure of the Thesis

Structure of thesis is explained in this section briefly.

Chapter 2 includes literature survey on asymmetric tooth profile gears. The works of researchers from 1990s to nowadays are presented for macro profile design, micro profile design, analytical and numerical efforts and performance tests.

Chapter 3 is based on macro geometry generation of asymmetric tooth profile spur gears. Case studies were started with symmetric tooth profile spur gear pairs and then were maintained for asymmetric ones. Some comparisons with macro geometries from KissSoft, commercial gear software were performed.

Chapter 4 consists of tooth root bending stress calculations for symmetric and asymmetric tooth spur gears. Some modifications of root bending stress equations were modified for asymmetric gears.

Finite element analyses were carried out for evaluation of symmetric and asymmetric tooth profile spur gears in Chapter 5.

Symmetrical and asymmetrical tooth spur gears were designed for micro geometrical parameters to obtain minimum values of peak to peak transmission error and smooth transmission error curves for design and off-design loads based on Harris Map. In-house software was developed to perform these tasks by depending on principles of transmission error theory in Chapter 6.

Chapter 7 presents construction of a back to back test rig developed in the scope of a distinctive gear design project was presented with measurements of static transmission error (STE) for symmetric and asymmetric tooth spur gear pairs.

Dynamic transmission error measurements of both symmetric and asymmetric spur gears were performed in Chapter 8.

Mathematical model studies of spur gears and software developed for this purpose were given in Chapter 9.

All efforts were made for asymmetric tooth profile spur gears during thesis were discussed in Chapter 10.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

Gears are the one of the main components of power transmission elements. They are used to transmit power, torque and speed between parallel and non-parallel shafts. Gear pairs are required to be designed for a safe meshing and desired service life under intended design torque. Gear performance needs to be evaluated in terms of root bending stress, contact stress, gear dynamics including micro profile modification and transmission error by obtaining gear geometry accurately. A number of studies are available in literature for different design criteria; macro geometry generation, bending stress calculations based on theoretical and finite element studies, experimental works, mathematical modeling and dynamic analysis, and micro profile design for minimized transmission error.

2.2 Macro Geometry of Asymmetric Spur Gears

Geometry of asymmetric profile gears differs from symmetric one due to difference in pressure angle of drive and coast side flanks of tooth. There is some commercial software namely KissSoft [14], Romax [15], Dontyne [16] which are used to generate gear geometry, calculate geometrical parameters, tooth root bending stress and gear surface contact stress for spur gears. But no commercial software which is capable of generating asymmetric gear tooth profile and all gear geometry has been encountered. Generation of macro geometry of asymmetric profile gears is very significant and necessary for stress analysis but there is no commercial software to generate these required profiles of asymmetric gears. Researchers intend to perform finite element analyses of asymmetrical tooth profile gear should prepare his/her own software to generate asymmetric gear geometry. Some studies about asymmetric gear generation presented in literature are reviewed in this section.

Agrawal et al [17] compared bending stress carrying capacity of an asymmetric tooth gear to symmetric one which transmits same torque. The bending stress carrying capacity was calculated in Ansys [18] by structural analysis of the gear tooth under a load. Ansys was used as finite element software and 3-Dimensional model of the gear geometry has been considered. It was concluded that a smaller asymmetric gear transmits same torque transmitted by a larger symmetric gear and thus it is useful for aviation vehicle where weight of the vehicle comes into prominence.

Alipiev [19] proposed a new method called as realized potential to design symmetric and asymmetric gears. This method is principally based on the determination of the realized potential and existence areas of involute gear drives by determining the minimum number of teeth of gear for different number of teeth of pinion in symmetric and asymmetric gear pairs.

Alipiev [20] intended to apply a theory of involute gear meshing conditions based on a new generalized basic rack of an asymmetric profile. The basics of the generalized theory were stated to a considerable extend by proposing a model for gear cutting using two different rack-type cutters and presenting geometrical relationships.

Alipiev et al [21] presented a method of realized potential to determine lowest possible minimum number of teeth for symmetrical and asymmetrical tooth profile spur gears based on standard and non-standard basic racks. Pressure angle of gear rack cutter, tooth addendum, tip radius, tooth tip sharpening, fillet radii limitations were taken into account for four and five teeth asymmetric spur gears. Areas of existence for undercut and undercut-free gears were determined by keeping transverse contact ratio and tooth tip land thickness at required levels.

Chira et al [22] developed software by using Matlab [23] and Autoslip [24] to generate symmetrical and asymmetrical gear tooth profile including involute and root regions. Normal module, teeth number and pressure angles for both side of gear tooth profile are inputs parameters for software. 2D model of gear is obtained by using developed software. It was stated that this software can calculate transmission error, efficiency, bending and contact stress but there is no evident actually; case studies or calculations were not presented.

Chira et al [25] prepared software which is capable of designing, modelling and analyzing the geometrical and functional parameters of asymmetric tooth spur gears. It was stated that asymmetric gears are more advantageous than symmetric one in terms of performance. Gear tooth profiles were generated in this study but no calculation of root bending and surface contact stress is presented.

In reference Joshi [26], gear models were generated by using a prepared macro based on Pro/Engineer Wildfire [27] by entering gear pair basic parameters into gear geometry equations. The structural analysis of the spur gear tooth model was carried out using the finite element analysis in Pro/Mechanica (tool of Pro Engineer) [28] which is an application of Pro/Engineer. Moreover, the effect of change of fillet radius on the strength of spur gear involute teeth was investigated in Pro/Engineer by taking the load of 100 kN at the highest point of single tooth contact (HPSTC).

Kargin [29] investigated possible applications of involute gear transmissions with asymmetric gear profiles; three levels of transmissions were identified. Each level was characterized by a particular set of engagement angles, operational parameters, types of gear, and manufacturing methods. This classification was made to permit easy identification of these levels' properties and facilitates the design of such transmissions.

Kargin [30] determined transition curves at the base of asymmetric gear teeth analytically. In the first stage, by roulette constructions associated with initial gear circumferences that roll over one another, the coordinates of points of such transition curves were found as the tooth tips slip over the transition curves at the base. In the second stage, a radial gap was constructed so that its outline facilitates simple tooth manufacture and minimum stress concentration at the tooth root.

Kargin [31] carried out a review of asymmetric tooth profile gear transmission including generation of gear profile for different asymmetry factor which is defined as ratio of pressure angle value of drive side to that of coast side. Transmissions were classified and case studies were presented for three different groups of asymmetry factor value which is greater but very close to unity; between 1.25 and 1.40; between 2.0 and 6.0.

In ref Phalake [32], it was aimed to generate the profile of asymmetric spur gear teeth and determine bending stresses using ANSYS [18] and compare the results between symmetric spur gear and the asymmetric spur gear. Gear tooth coordinates were obtained in prepared software in Excel [33]. Gear profile was generated and finite element analyses were carried out using CREO [34] and ANSYS [18] respectively for symmetric and asymmetric spur gears.

Singh and Senthilvelan [8] modified existing symmetric gear design procedures including stress modifying factors namely tooth form factor, stress concentration factor and load sharing factor to calculate the bending stress of asymmetric tooth gears. It is stated that tooth thickness at critical section of asymmetric tooth root plays an important role for reducing the bending stress due to increase in tooth thickness. Similarly, the distance between the critical section and point of intersection between tooth axis and direction of force of contact (in way the moment arm) also reduces the bending stress in case of asymmetric tooth profile (usually with larger pressure angle on drive side).

Sondur and Dharashivkar [35] modeled asymmetric involute gears with different pressure angles by using Catia V5 R15 [36]. Finite element analysis was carried out for comparison of root bending stress of asymmetric gears by Ansys [18]. Authors claims that there is a 20% stress reduction as a result of change of drive side pressure angle from 20 degree to 35 degree by keeping coast side pressure angle as 20 degree and location of maximum bending stress remains constant.

Vojtkova [37] prepared software to obtain geometric profile of asymmetric gear by checking top land thickness and occurrence of interference. Asymmetric tooth gears were illustrated by taking limits of pressure angle, top land thickness and contact ratio into consideration. Calculation of reduced radii of curvature was conducted and specific sliding curves of both pinion and gear were drawn separately.

Fetvacı and Imrak [38] developed a mathematical model to design asymmetric spur gears. Cutting tool and generated gear surface were obtained by using developed computer program. The influence of coast side pressure angle on the stress at the root of the tooth was also investigated by performing finite element analyses of asymmetric gears with different coast side pressure angles. The results demonstrated that using greater coast side pressure angle decreases stresses at the root of the tooth.

Fetvacı [39] presented a study of mathematical model to generate geometry of asymmetric profile spur gears for involute and trochoid sections based on mathematical model. Trajectory of gear cutting tool during generation process for pinion type shaper and hobbing was investigated. Involute and root sections of asymmetric spur gears were modelled for different tip corner radii of cutting tool.

2.3 Tooth Stress-Strength of Asymmetric Spur Gears

Gear tooth root bending stress and surface contact stress are two important design concerns for gears. For design and analysis studies, it is important to check these two stress values against regarding strengths of gears materials. There are some international standards providing relations and procedures for stress calculations of symmetric tooth profile gears. However, there is not any validated and reliable bending stress calculation procedure for the asymmetric spur gears yet. Despite the fact that literature below promises a benefit from asymmetry, a need still arises for a reliable and experimentally validated calculation procedure for the tooth root bending stress of asymmetric tooth shape spur gears. Some literature works for calculations of tooth root bending stress and contact stress, and finite element analyses are summarized under this section.

Abdullah and Jweeg [40] developed an analytical method based on a trial graphical method has been introduced previously to find the bending stress for symmetrical and asymmetrical involute gear teeth. Gears were selected with and without profile correction for this study. Geometrical and operating parameters for gears such as tooth loading angle, tooth critical section thickness, the load height from this section and Lewis form factor were determined to calculate tooth root bending stress. More accurate results were obtained in shorter time than trial graphical method.

In the study of Abdulwahhab and Abdullah [41], a method was presented for enhancing bending stress performance of helical gears by achieving asymmetry on involute gear pairs. Modifications were performed on Lewis equation to estimate root stress of asymmetric tooth helical gears and finite element analyses were conducted. Optimum helix angle was determined as 22.5 degree by loading gear at tip of tooth and near middle of tooth; stress reductions were obtained by 1.73% and 16.87% respectively.

Sekar and Muhtuveerappan [42] adapted methods presented in ISO standards for calculation of tooth form and stress concentration factors, critical root section thickness, bending moment arm and root stress for asymmetric spur gears. Several finite element analyses were conducted by using 2D, Plane 82, 8 noded quadratic elements. Root bending stress and load sharing were determined for asymmetric tooth gear pairs for tip loading and loading at HPSTC. Greater pressure angle was suggested by authors for drive side of flanks to reduce bending stress.

Çavdar et al [43] conducted finite element analyses for symmetric and asymmetric spur gears to determine tooth bending stress. They compared results and stated that maximum stress is changing but location remains unchanged (Vice versa was stated in Reference [42]; maximum stress location changes). It was claimed that stress reduction of 35-39% was obtained by using asymmetric profile compared to symmetric gear with 20 degree pressure angle while this reduction decreased to 20% for comparison of gear with 32 degree.

In reference study of Çavdar et al [44], asymmetric tooth spur gear pairs were modelled for different gear parameters such as pressure angles of drive and coast side flanks by keeping teeth number, pitch diameter, tip diameter, profile shift, face width and root fillet radius constant to evaluate root bending stress, Hertzian contact pressure, sliding velocity and contact ratio without any numerical studies.

Chira [45] used optimizing software on the base of genetic algorithms for designed 2D model of the asymmetric gears. The main aim was to minimize the Hertzian contact stress, bending stress, transmission error and sliding velocity. Asymmetric gears with greater drive side pressure angle were preferred in this study.

Chira and Bănică [46] presented a study on the relative slide between the flanks of the asymmetric teeth. A method of determining the variation of the relative speed and the specific sliding during meshing was brought into attention. Software has been developed to achieve these purposes for an asymmetric gear with an asymmetry coefficient higher than unity, an asymmetry coefficient lower than one and a symmetric gear designed for the same distance between the axes and the same number of teeth.

Chira and Bănică [47] presented a study for relative sliding between the surface flanks of symmetric and asymmetric tooth gears. Lower relative sliding velocity was obtained for asymmetric gears compared to symmetric one.

Costopoulos and Spitas [48] proposed a new asymmetric gear teeth geometry called asymmetric half-involute gear teeth to increase bending load capacity by combining good meshing properties of driving involute geometry and the increased strength of non-involute curves. Bending load carrying capacity of these gears were investigated using finite element analyses. Researchers founded that reduction in bending stress up to 28% is possible by the proposed asymmetric gear teeth geometry.

In the paper of Dadhaniya et al. [49], the effect of greater pressure angle of drive side on bending stress and tooth deformation under load were investigated by finite element analyses for asymmetric gears.

Deng et al. [6] carried out finite element analyses to investigate the influence of pressure angles in the drive and coast side profiles on bending stress at the root of the tooth. Bending stiffness of gear teeth was also calculated by the same method. Only a single gear tooth was modelled and a concentrated normal load was applied at HPSTC. As a result, it was demonstrated that using a larger pressure angle in the coast side profile reduces bending stress at the root of the tooth and increases bending stiffness of the tooth whereas load sharing ratio was not affected by coast side pressure angle.

Deng et al [50] investigated the effect of asymmetry of tooth profile on root bending stress for gears with larger pressure angle of coast side by keeping that of drive side constant (20 degree) and all remaining parameters namely; module, teeth number, addendum and profile shift coefficient. Several finite element analyses were conducted to evaluate root bending stress and tooth bending stiffness by loading gear tooth at gear tip depending on increased pressure angle for coast side of tooth profile. Authors emphasized that greater coast side pressure angle makes positive contribution for both cases.

Dharashivkar et al [51] explained a method to evaluate the bending stress at the tooth root of symmetric and asymmetric involute spur gears using 3D photo elasticity method. Authors used the method ISO/TC-60 [52] to calculate the root stresses theoretically. CATIA V5 R15 [36] and ANSYS [18] were used to generate gear models

and carry out finite element analyses respectively. Results obtained by theoretical, experimental and FEA studies were compared.

Dragar and Kamondi [53] dealt with tooth root stress calculation for non-symmetric tooth shape based on tooth form factor which is related to gear geometry. The calculation procedure and gear models were given without any case study and comparison.

Francesco and Marini [4] made an adaptation of ISO 6336-3 [2] methodology for bending stress calculations of asymmetric gears. They aimed to provide a procedure for calculation of asymmetric gears' bending stress without need of finite element analyses. Factors used in bending stress calculations in ISO 6336-3 [2] and mainly affected by changing tooth profile as a result of asymmetry namely form factor, notch factor etc. were calculated based on tooth root critical thickness of asymmetric profile gears were estimated. Thereafter bending stress values were determined for ISO 6336-3 [2] method and finite element analysis (FEA); the maximum difference (between them) was 13%.

Francesco and Marini [5] stated that greater pressure angle for drive or coast side (necessity for asymmetry in gears) of tooth has higher load carrying capacity or higher bending strength respectively. They made finite element analyses of symmetric and asymmetric tooth gears by making only pressure angle different for both side by keeping all other parameters constant. Authors emphasized that bending stress reduction of 18.5% was obtained when there was a difference of 12.5 degree between drive (17.5degree) and coast (30degree) side pressure angles.

Ingole et al [54] aimed stress relieving by making circular holes on the purpose of reduction of stress concentration for asymmetric spur gears. Finite element analyses were carried out for asymmetric tooth spur gears with and without stress relief holes. Strain gauge measurements were conducted for bending stress at root and results were compared. Authors stated that the best solution was obtained for stress relieving holes at center of tooth of asymmetric profile gear.

Kapelevich [7] made a study on symmetric and asymmetric tooth gears for loading of single flank (unidirectional) or both flanks (bidirectional). Symmetric tooth gears with 25 degree of pressure angle was taken a baseline (reference) for comparison of bending

and contact stresses and sliding velocity. Greater value of drive side pressure angle was preferred in analyses; decrease in surface contact stress (6-12%) and specific sliding velocity (6-32%) were obtained while there is increase in bending stress values (7-8%).

Kapelevich and Shekhtman [55] presented a rating method for asymmetric tooth gears by their bending and contact stress levels in comparison with symmetric tooth gears whose rating was defined by standards. This method application is based on finite element analysis (FEA) for bending stress definition and the Hertzian equation for contact stress. It defines equivalency factors for practical asymmetric tooth gear design and rating by conversion of the asymmetric tooth geometry into the comparable symmetric tooth geometry and definition of its generating rack. Maximum bending stresses using the 2D or 3D FEA and maximum contact stresses for both asymmetric and comparable symmetric gear teeth using the Hertz equation were calculated. Bending and contact stress conversion coefficients were derived.

Kapelevich and Shekhtman [56] stated that contact stress reduction, load capacity increase and size-weight reduction are the main advantages of asymmetric tooth gears. Due to lack of published standard for asymmetric gears, asymmetric gear pair was modelled as comparable symmetric ones to calculate root bending and surface contact stress analytically without need for FEA analysis studies.

In reference [57], Kapelevich analyzed asymmetric gears in terms of root stress based on three methods such random search, trigonometric functions approximation and FEA. Direct gear design (non-traditional) method was used for asymmetric external and internal gears for two stage planetary gear box of TV7 – 117S turboprop jet. Tooth root stress values were calculated for low and high contact ratio symmetric and asymmetric gears (equal contact ratios for each group) and stress reduction was obtained.

Kapelevich [58] underlined that the main advantage of asymmetric gears is contact stress reduction on drive flanks. Direct gear design method of the asymmetric tooth gears were presented by not depending on conventional cutting tool design for gear manufacturing. In this paper, tooth fillet profile optimization was carried out by three methods such as random search method locating fillet points, trigonometric functions

for fillet approximation and FEA for stress calculation. Asymmetric tooth profile gear was implemented to two stage planetary gearbox of TV7-117S turboprop engine of Russian airplane IL-114. It was stated that low weight to output torque ratio was obtained for gearbox with asymmetric gears; about 50% of lower ratio compared to that with symmetric gears.

Kapelevich [59] developed the basic geometric theory of the asymmetric gear teeth and carried out experimental analyses on a two-stage planetary gear reducer with asymmetric teeth of a turbo-prop engine. Experimental analyses were supplemented by finite element analysis results. Author concluded that asymmetric gear with larger drive side pressure angle results an increase in load carrying capacity and a significant reduction in vibration level in addition to reduction of weight and dimensions of gearbox.

Karpat [60] carried out an analytical and numerical investigation concerning tooth bending stress analysis and dynamic analysis of symmetric and asymmetric spur gear drives. Asymmetric gears which have larger pressure angle on drive flank with respect to coast side were investigated. Dynamic loads, dynamic factors, static and dynamic transmission errors were computed by a developed computer program. This program uses a numerical method based on torsional stiffness. As a result, it was determined that increasing of drive side pressure angle reduces tooth bending stresses. Besides, increasing both drive side pressure angle and tooth height provides advantages concerning dynamic loads and static transmission errors.

Karpat and Ekwaro-Osire [61] carried out a numerical study to investigate the efficiency of asymmetric spur gears with high contact ratio (HCR). The numerical results were compared with analytical and experimental results available in the literature. The influence of asymmetric gear specifications such as drive side pressure angle and tooth height also was investigated from the point of view of sliding velocity and power loss. Based on the numerical results, it was demonstrated that using HCR asymmetric spur gears instead of the HCR symmetric spur gears decrease predicted power losses under lower speed or higher torque.

In this study [62], asymmetric profile involute spur gears in terms of root bending stress, deflection, mesh stiffness, dynamic loads and dynamic factor were investigated by Karpat et al. Profile shifted symmetric and asymmetric (with no relief) spur gears

were compared for form factor, stress concentration factor and root bending stress based on DIN 3990-3 [63] and finite element analyses (for quadrilateral 8 node Plane 82 element).

Karpat et al [64] presented a research study of application areas of asymmetric tooth spur gears. Several finite element analyses were conducted to determine bending stress for tensile and compressive side of tooth flanks. Root section thickness at a distance from two times module from gear tooth tip was calculated for different asymmetric gear pairs. In addition, Hertzian pressure and stiffness were determined based on pressure angle of both flanks.

Kumar et al [65] performed finite element analyses to optimize the dedendum (based on gear root circle diameter optimization) for the chosen module of 3 mm, 18 teeth, and pressure angle of 20-34 ° asymmetric spur gears.

Kumar et al [66] examined bending stress of asymmetric tooth spur gear pairs. The influence of ratio of top land thickness to base diameter and asymmetry factor on cutter tip radius and contact ratio was investigated for different working center distance. Several case studies were carried out to determine maximum bending stress at root fillet region for loading gears at HPTC and HPSTC. Obtaining of maximum stress reduction of 43.7% and 15.9 were implied respectively for loading conditions abovementioned.

Litvin et al. [9] developed an asymmetric pinion profile which slightly differs from conventional involute profile for face gear drive applications in helicopter transmission. Simulation of meshing and stress analysis of new geometry were carried out by finite element method and reduction of more than 10% in contact and bending stress magnitudes has been proven.

Mallesh et al [67] focused on the effect of profile shift on tooth root bending stress for asymmetric spur gears with greater drive side pressure angle. A great many finite element analyses for different sets of symmetric and asymmetric spur gears subjected to a load at HPSTC were carried out. Comparison has been made for symmetric and asymmetric spur gear tooth using Lewis equation and finite element analysis software. With no information about cutter tip radius, authors remarked on obtaining 20-25% reduction in stress by increasing the drive side pressure angle up to 30 degrees.

Mallesh et al [68] generated asymmetric spur gear tooth profile for different pressure angles on drive and coast sides and estimated the critical section. Bending stress analysis has been performed and bending stress distribution for symmetric and asymmetric spur gear drives using ANSYS [18]. Comparison of bending stress analysis has been performed for symmetric and asymmetric spur gear tooth at critical section.

Mallesh et al [69] presented a study of geometrical model generation of symmetric and asymmetric tooth profile spur gears and finite element analyses of those gears. Effect of different values of module and teeth number for gear pairs with different pressure angle values were evaluated for bending stress.

Marimuthu and Muhtuveerappan [70] intended to determine optimum profile shift for normal contact ratio (NCR) and high contact ratio (HCR) asymmetric gears based on method of direct gear design. Two isograms for asymmetric tooth gears (for NCR and HCR) were obtained to state their area of existence. Multi pair contact analysis (MPCA) was done to estimate load sharing and finite element analyses were conducted for full rim gear models.

Marimuthu and Muhtuveerappan [71] presented a study for investigation of the effects of addendum height, teeth number and module on load sharing of normal contact ratio asymmetric tooth profile spur gears (with greater pressure angle of drive side) by using MPCA models. Three teeth gear models were used in finite element analyses. Increase in addendum and normal module increases bending stress whereas stress reduction of bending and contact stress is obtained by increase of teeth number.

In reference study of Muni et al [72], asymmetric spur gear pairs were optimized for maximum bending stress based on method of direct gear design. Some isograms were developed for conditions of minimum contact ratio, avoiding interference and undercutting, pressure angle at addendum circle and working pressure angle to show existence of possible designs.

Novikov et al [73] presented a research and development study of production of gearbox with asymmetric planet gears for turboprop engine of TV7-117S. Photo elastic and finite element models were studied to investigate effect of asymmetry on root bending stress for this application. Gears were grouped based on transmission error

and then assembled depending on transmission error based to provide an effective engagement and load distribution. Low weight to output torque ratio was obtained by using asymmetric gears in planet gearbox of turboprop engine.

Patel and Vaghela [74] carried out a study based on fixing coast side pressure angle while increasing pressure angle of drive side flank to decrease root stress. Several finite element analyses were conducted for asymmetric gears with greater pressure angle of drive side. Load carrying capacity has been increased from 2150 N to 3820 N without compromising bending stress.

Pedersen [75] performed a numerical investigation on asymmetric gears to reduce bending stress by changing the geometry of the cutting tool. It was mentioned that a significant reduction in bending stress can be achieved by designing the tooth root thicker and by changing the root shape where the stress concentration occurs. Two new asymmetric elliptic rack cutter geometries were suggested and maximum bending stress reduction was found with larger drive side pressure angle. Based on the results of analyses, reduction in bending stress up to 44.3% was achieved.

Phalake et al [76] performed bending stress analysis of the symmetric and asymmetric spur gear to design spur gears to improve the bending load capacity of the teeth. Tooth fillet at root of tooth and pressure angle were modified, and bending stress was determined using ANSYS [18]. Comparisons between symmetric and asymmetric spur gears were done on the basis of results by ANSYS in terms of maximum bending stress.

Prajapati and Vaghela [77] aimed to show the effects of different gear parameters on tooth bending stress at the critical section of asymmetric spur gear by reviewing current papers. They mentioned about minimizing bending stress by optimizing pressure angle for drive and coast side of gear tooth profile.

Prajapati and Vaghela [78] estimated tooth thicknesses of critical section and tip for asymmetric spur gears by increasing drive side pressure angle and keeping that of coast side constant based on involute tooth thickness equations.

Prajapati et al [79] presented limiting parameters for optimizing bending stress of asymmetric spur gear pairs. Reduction of root bending stress by depending on pressure angle increase and contact ratio decrease as a result of this increase in pressure angle

were explained. Tip thickness decreases with increase of drive side pressure angle were also introduced in their work.

Prajapati and Vaghela [80] introduced an optimization study for bending stress at root section of asymmetric spur gears. By depending on drive side pressure angle (by keeping that of coast side constant) tooth tip thickness land, transverse contact ratio, tooth form factor, critical section thickness and root bending stress were used as objective functions in analyses.

Sekar and Muhtuveerappan [81] made a study for estimation of load sharing, bending stiffness and bending stress of asymmetric tooth helical gears (conventionally and direct designed). Area of existence diagram was used to select gear pair based on requirements. Finite element analyses were made to evaluate the influence of gear ratio, transverse contact ratio, top land thickness and number of teeth on performance parameters abovementioned.

Shankin et al [82] presented a study of asymmetric gear application for planetary gearbox of aerospace propulsion of a light multipurpose helicopter. Two stages of gearbox were modernized by changing asymmetric gear with symmetric ones. Symmetric gears were replaced by asymmetric ones with pressure angles of 32 and 20 for drive and coast flanks respectively. Stress reductions of 9% were obtained for tooth root and contact stresses.

Spitas et al [83] performed a parametric investigation of the total effect of the cutter tip radius and the dedendum on the clearance and the resulting tooth bending strength. It is aimed to obtain a stronger tooth form; analytical calculations, computerized generation and finite element studies were done. Non-dimensional modelling was used to obtain results applicable to entire gear families.

Vaghela and Patel [84] conducted a series of finite element analyses for asymmetric spur gears to investigate the effect of increasing drive side pressure angle on root bending stress by taking minimum allowable limit of tooth tip thickness into account. Authors stated that increase in pressure angle limits maximum value of root fillet radius but no practical example or case study was presented in their paper.

Vaghela and Patel [85] investigated the bending stress reduction in spur gears in asymmetric tooth form by finite element analyses. Greater value of pressure angle for

drive side was preferred and root bending stress reduction was obtained. In this study, authors suggested using lower face width for calculated stress value and making lighter gears by this means.

Vojtková [86] searched benefits of use of asymmetric profile gears (instead of symmetric one) and developed software to control the shape of the gear teeth for calculation of the reduced radii of curvature of the asymmetrical teeth, Hertzian pressures, the pressure angles limits and specific sliding velocity.

Vojtková [87] discussed the effect of tooth asymmetry on radii of curvature for asymmetric teeth spur gears. Reduced radii of curvature in extreme point along engagement including pitch point, specific sliding and surface contact stress were investigated. It was stated that greater pressure angle for drive side improves reduced radii of curvature and reduces contact stress.

Vojtková [88] presented a study on investigation of the change of radii of curvature and Hertz contact stress by depending on change of pressure angle values for drive and coast side of gear tooth profile. The contact position of engaging gear tooth is different for different pressure angle of drive side. This affects radii of curvature so Hertzian contact stress. It was stated that the significant reduction in contact stresses at pitch point (23%) and start of active profile (43%) were obtained by using 39.5 and 18 degree of pressure angle for drive side and coast side respectively.

In study of Wang et al [89], symmetric and asymmetric gear sets (with greater pressure angle of drive side) were analyzed based on the method of edge based smoothed point interpolation (ES-PIM) for different meshing and different loading conditions. Bending stress distribution on root fillet region was also investigated in this paper.

Wen et al [90] made an investigation of load carrying capacity of asymmetric tooth profile internal and external gears in planetary gear transmission. Optimization for gear contact and bending stresses based on gear cutting tool parameters such as pressure angle for drive side and dedendum was carried out. Gear tooth strengths were increased by 12.69% and 10.14% respectively for contact and bending performance.

Yang [91] developed an assembly model of pinion and gear for profile shifted asymmetric helical gear drives. Tooth contact analyses were carried out using three dimensional finite element analyses and kinematic errors were investigated for spur

and helical gears. Helical and spur gears were compared from the point of view of stress analysis and it was demonstrated that asymmetric helical gears is superior to the symmetric one.

Brown et al [92] carried out an experimental investigation on helicopter main drive system to analyze contact and bending stresses of symmetric and asymmetric gears. Designed and manufactured gear specimens with circular root fillet profile were tested experimentally to determine bending fatigue strength and wearing resistance. Experimental tests were supplemented by finite element analyses and results of asymmetric gear analyses were compared with ones of symmetric gear analyses. Based on the test results, they concluded that bending fatigue load capacity of asymmetric gear was 16% greater compared to symmetric gear. Also, the mean endurance limit of gear with optimized fillet profile was estimated to be 12% above the standard fillet profile gear.

2.4 Experimental Studies for Tooth Root Bending Stress of Symmetric and Asymmetric Spur Gears

All theoretical predictions require validation by some means and it is generally the experimental work required for such validations. Here, too, not enough literature work is available on the asymmetric gears. Only few bending stress measurements were accomplished but no transmission error measurements were mentioned. Therefore, measurements of both bending stress and transmission error in quasi-static conditions will be a good help to validate the likely theoretical achievements.

Gasparini et al [93] investigated different gears (ground-unground, shot peened) of Agusta Westland helicopter transmissions by loading five teeth. Relationship between root stress and applied load was researched in detail based on the methods of AGMA 2101 D04 [94], FEA and strain gauge measurements. Fatigue limits obtained in tests were higher than those of AGMA [94] and ISO [2] standards. Authors considered that these comparisons are not fully meaningful because standards are not specific aerospace gears.

Eyercioğlu et al [95] designed a test fixture for a high frequency push pull fatigue test device to investigate fatigue life of through hardened and induction hardened spur gears by conducting single tooth bending fatigue tests. The loading was applied at

HPSTC in form of sinusoidal and fluctuated between preload and maximum design load. 10^7 cycles was selected to determine endurance strength for tests as specified in AGMA standards. Effect of increase in surface quality at root area on endurance limit of forged gears was investigated; improvement of 5% and 12.5% was obtained respectively for forged gears and forged polished gears at root.

Rajesh et al [96] intended to design standard and increased tooth number gears, design gear tooth bending tester fixture, measure strain at gear roots, determine bending stress based on AGMA [94] and compare results. One tooth models of gears with no profile shift, positive and negative profile shift were manufactured and tested for tooth root bending strain. Gear root stresses were calculated by AGMA equations for same models. Loading during tests was carried out at gear tooth tip for different load values. Reduction in bending stress of 35% was obtained for negative profile shifted gears compared to standard gears with no relief.

Zhang et al [97] evaluated bending fatigue characteristics of spur gear pair made of 20CrMoH by conducting single tooth bending methods. During tests, pulsating cyclic loading was applied and S-N curves were obtained; fatigue limits were tabulated for different reliabilities. Zwick/Amsler 250 HFP 5100 high resonance fatigue test machine equipped with a test fixture was used for all tests by providing loading at HPSTC.

Akata et al [98] dealt with performance of gears used in pumps by using INSTRON 8501 (50 kN capacity, 0.5-100 Hz). Two groups of gears (machined and closed die hot upset) were evaluated for fatigue performance based on three point bending test. Same gear pair was studied by finite element analysis. Results from experimental works and FEA differ by 8%.

Ravai Nagy and Lobontiu [99] presented an approach for measurement of root bending stress (strain) of asymmetric profile gears by using test rig. The procedure was explained but no result of any test was given in the study.

Ravai Nagy and Lobontiu [100] made a research study for design and manufacturing of asymmetric gears including bending fatigue test of those. Researches developed a single tooth test rig based on tests of only one tooth of symmetric and asymmetric spur gears. Loading was carried out by providing mesh of this tooth with a rack. Maximum

load carrying capacity of asymmetric tooth spur gears for bending was evaluated and an increase of 10-16% was obtained when compared to symmetric one.

Karpat and Çavdar [101] investigated the design of a test rig (as a plan of future planned study) for evaluation of asymmetric tooth profile gears for wind turbine gearboxes in terms of efficiency, fatigue life, acoustic, and dynamic loads and wear of gears.

2.5 Micro Profile Design and Transmission Error Prediction of Asymmetric Spur Gears

Transmission error is one of main excitation source of noise of gears and it has possible negative effects on gear operation namely high vibration levels transmitted via gear body, shaft, bearing, housing and casing, high noise levels of both air borne and structure borne types, kinematic inaccuracy causing problems in accurate positioning applications. Transmission error under design load is the main cause of all these negative effects and has to be minimized in amplitude and optimized in shape to get a smooth TE curve.

There are some efforts to minimize transmission error values for gear pairs by aiming of improving their dynamic performance, vibration and noise. Some literature works are summarized here.

Yıldırım and Munro [102] developed a systematic approach to design of profile relief for low and high contact ratio spur gears (by introducing double relief as a corner stone for high contact ratio micro profile), and evaluate TE and tooth loads by involving manufacturing tolerances. Results of calculations were verified by experimental works for TE values under different loads. The importance and limitations of different relief types namely short, intermediate and long relief were stated. Minimum value of peak to peak value of transmission error (PPTE) was obtained for design load and off design loads by applying intermediate relief type to optimize TE for any required load value.

Yıldırım and Munro [103] introduced a new relief type; double relief by using short and long relief and advantages of both were brought together. It was stated that this new type of relief is superior to other relief types in terms of values of PPTE, tooth loading and number of smooth TE curves. Theoretical calculations were supported by experimental works in this study.

Yıldırım et al [104] designed HCR spur gears with double relief for helicopter transmission units and desired performance has been achieved whereas performance of low and high contact ratio spur gears with different profile modifications was limited below intended level. Double relieved gear pair was tested; low PPTE and noise reduction of 7-11 dBA were obtained for full load range of gears.

Munro et al [105] presented a method allows measurement of tooth pair stiffness of spur gear pairs throughout path of contact or presence of corner contact. Transmission error curves of HCR spur gears under different loads for constant (12 GPa) and variable (12-8 GPa) mesh stiffness values were derived based on Harris map by avoiding test rig deflections during tests.

Munro et al [106] dealt with study of transmission error outside of path of contact for spur gears and compared results to experimental works. Theoretical calculations presented on paper differed from actual measurements of TE values; the difference changes from 9.5 to 12.0 micron. Theoretical TE values calculated are smaller than values obtained by experiments.

Tavakoli and Houser [107] developed a method to calculate static TE and load sharing for LCR and HCR spur gears. This work is also an optimization study by applying linear or parabolic tip and root relief. Mesh compliance, TE and tooth load curves were derived for tip and root relief.

Palmer and Fish [108] designed micro profile of loaded spur gears with and without adjacent pitch error values. Gears with different amount and extent of relief values were examined for no relief, very short, short, intermediate, long and very long tip relief. In addition to 2D models, 3D models of FEA with parallel strips were used for variable stiffness in this study. Double relief was applied to high contact ratio (HCR) gears in this study. It was stated that a correctly designed relief for HCR gear produce lower TE.

Houser et al [109] made a comparison between theoretical and experimental results of transmission error values for spur and helical gears. Experimental measurements of spur and helical gear sets were made on a loaded test rig (driven by DC dynamometer and eddy current brake) whereas load distribution program (LDP) [110] was used for theoretical predictions.

Tesfahunegn et al [111] studied the effects of shape (linear and circular profile) and amount of profile modification on transmission error, bending and contact stress numerically and experimentally. They analyzed five teeth model instead of modelling of full gear in order to reduce analysis time based on plane strain finite element and results were compared with experimental studies. The difference in PPTE of experimental and numerical studies changed between %12 and %27 based on FEA values. The minimum values of PPTE for linear and circular profile modifications were obtained by applying different amount of modification for same design load. It was stated that the shape of profile modification has not significant effect on gear strength while their amount affects it. When profile modified gears are supposed to higher torque than design torque, circular modification is better choice.

Not much work has been found on the micro geometry design of asymmetric gears. The bending stress advantage of asymmetry, if combined with the correct and optimum design of micro geometry, will make spur gear preferring engineer's hand more powerful against helical gear preferring engineer without carrying the burden of axial tooth/shaft forces. Therefore, correct design of micro geometry of asymmetric gears is very important to minimize transmission error as low as possible for a better dynamic performance and less vibration.

Karpat and Ekwaro-Osire [112] presented a study on the wear of asymmetrical profile spur gears. The effect of tip relief modification on the wear was investigated and dynamic analysis was aimed to define static transmission error (STE). It was shown that the wear depth and the dynamic load decrease with the increase in tip relief.

Karpat and his co-authors [113] made 2D finite element analyses for evaluation of asymmetric tooth gears' stiffness values by changing pressure angle of drive flank of tooth (coast side was hold constant as 20degree) and teeth numbers by keeping all other gear parameters same. The effects of these parameters on tooth stiffness were evaluated and their curves (versus contact positions) were presented.

Litvin et al. [114] designed asymmetric modified spur gears by combining involute gear and double crowned pinion to stabilize the bearing contact and to reduce TE. Finite element analyses of proposed spur gear geometry were carried out. It was demonstrated that the proposed modification on gear geometry enables to reduce TE, contact and bending stresses for the drive side profiles.

Abdullah et al [115] studied on experimental measurement of TE of symmetric (no relief), asymmetric profile of 14,5-25 degree pressure angle (no relief) and asymmetric profile of 14,5-25 degree pressure angle (with relief) spur gears. Test rig was designed and constructed to measure torsional vibration. Dynamic transmission error, dynamic factor and noise level were compared to symmetric one.

Kapelevich and Shekhtman [116] aimed an optimum solution for gear drives with high performance by combining increased load carrying capacity and efficiency for LCR and HCR asymmetric profile spur gears in addition to low TE. Authors stated that an effective value of contact ratio and TE are affected by manufacturing tolerances and deflections in real working conditions, but only bending and contact deflections of gear tooth were taken into consideration.

2.6 Mathematical Modeling and Dynamic Analysis of Asymmetric Spur Gears

Even by designing gears (symmetric or asymmetric profiles) in a safe way statically in terms of bending and contact stresses, designer needs to have knowledge about gear dynamic behavior and keep it under control to reduce TE of gear pair and as a consequence decrease in noise and vibration.

Gear dynamics has been a subject of intense interest to the gearing area during the last few decades and dynamic loads and vibration are a major concern for gear drives at high speeds. Simulation of meshing of gear drives performed by application of tooth contact analysis and experimental tests of gear drives have confirmed that transmission error (TE) are the prime cause of noise and vibrations of the gear drives [9].

Dynamic analysis and mathematical modeling are very significant to evaluate dynamic performance of gear pairs and whole gearbox including shafts and bearing in addition to gears. Some research studies intended for these achievements for symmetric and asymmetric gears for different degree of freedom (DOF) models are explained in this section.

Kahraman and Blankenship [117] concentrated on the influence of relieving tip of gear tooth linearly on torsional vibration of spur gears. A power recirculation type gear test rig was used to measure dynamic transmission error (DTE) of spur gear pairs with tip relief of 10 micron starts at different roll angle (24.8, 23.6, 22.2 and 20.9 degree) and extends to tip of gear tooth. Dynamic transmission error of spur gear pair was

measured under different design torques and at speed range of 600-4100 rpm by a power recirculating test rig. An optimum tip relief was intended to minimize DTE.

Kahraman and Blankenship [118] investigated the effect of contact ratio on dynamic characteristics of spur gear pair by measuring DTE experimentally and proposed a single degree of freedom (SDOF) analytical model to estimate gear pair dynamics. The details of test rig were presented in paper. Root mean square (RMS) values of DTE were given for different contact ratio gear pairs. Integer contact ratio such as 2.0 and 3.0 was claimed to give better gear dynamics with lower TE in this work.

Hotait and Kahraman [119] conducted several speed up and speed down experiments to measure DTE (by linear accelerometers) and dynamic root strain (by strain gauges) for modified and unmodified spur gear pairs under different torque values through a back to back test rig. The change of natural frequency of gear mesh with increased torque values was emphasized because of relationships of load-mesh stiffness and actual increased load-contact ratio in this paper. A linear relation of dynamic factor and DTE was focused.

Tamminana et al [120] developed deformable body based on finite elements and simplified discrete model for prediction of dynamic behavior of tip relieved and non-relieved spur gear pairs and estimated dynamic factor (DF) and DTE for modified and unmodified gears. Both of models were validated by experimental results which presented in previous studies [118-119]. Discrete model presented here depends on deformable body model to calculate stiffness and STE and this is main disadvantage.

Kahraman et al [121] developed a finite element model of gear-rotor system by including gears-shafts-bearings and taking coupling between torsional and transverse vibration into consideration. In this model, tooth separation was neglected and constant mesh stiffness was used. Natural frequencies and mode shape of system and the significant effect of bearing compliances on gear rotor dynamics were shown.

Al-Shhyab and Kahraman [122] proposed a torsional dynamic model by taking gear backlash, mesh stiffness fluctuations, torsional flexibilities of shafts, tooth modifications and manufacturing errors into account. Equations of motions were solved by using Harmonic Balance Method (HBM) to evaluate dynamic performance of gear trains.

Kahraman and Singh [123] examined nonlinear dynamic response of spur gear pair with backlash under both internal and external excitations. Harmonic balance method and digital simulation technique were used to develop steady state solutions and results were compared to experimental ones presented in literature.

Wang et al [124] established a 6-DOF model to estimate dynamic behaviors of spur gears pairs. Runge-Kutta method was used to solve nonlinear dynamic equations. In addition, numerical simulations were carried out to investigation of sidebands of modulation. TE and time varying mesh stiffness (TVMS) were predicted under modulation and non - modulation conditions. To verify prediction of modulation sidebands, an experiment was performed.

In reference paper of Jia et al [125], a two stage gearbox including gears, bearings and shafts were modeled mathematically by coupling transverse and torsional motions of these power transmission elements. Profile and pitch errors, normal contact force and friction force between gear teeth were included in dynamic models. The curves of torsional mesh stiffness, load sharing ratio, dynamic friction coefficient between gears, amplitude and phase modulations were obtained at the end of dynamic analysis.

Howard et al [126] presented a multi degree of freedom (16-DOF) dynamic model for spur gears to investigate the effect of friction on gear case vibration. Results were compared for model including and excluding friction between gear pair. A crack of 4.7 mm in a single tooth of gear was involved in dynamic model, a reduction of tooth stiffness was observed as a result. Equations of motions were solved by using ordinary differential equation (ODE) solvers of Matlab [23] to simulate whole gear system.

Karpat et al. [127] presented a paper which gave the results of dynamic analyses of involute spur gears with asymmetric teeth. The instantaneous dynamic loads of symmetric and asymmetric gears were determined by developing a dynamic model in MATLAB [23]. Two dimensional finite element analyses were also performed to calculate the deflections of both the asymmetric and the symmetric gear teeth. It was shown that increasing the drive side pressure angle increases dynamic factor (DF) and reduces static transmission error (STE). DF decreases with increase of addendum coefficient. Besides, amplitudes of the STE harmonics reduce significantly with the increase of contact ratio.

Karpat et al [128] introduced a computer program to analyze symmetric and asymmetric tooth spur gears' dynamic behaviors and to calculate transmitted torque, dynamic load and STE. A mathematical model of gear pairs including friction but excluding profile errors was developed to determine dynamic behavior of asymmetric gear pairs. Developed software also generates gear models for finite element works. By applying load at five different points along length of contact, single tooth stiffness was determined.

In the study of Karpat and Ekwaro-Osire [129], asymmetric profile instead of symmetric one was suggested for increasing load carrying capacity and minimizing dynamic loads for HCR gears. Equations of motion for gear pairs were generated to determine dynamic load as a function of contact position during meshing. The influence of contact ratio, addendum, stiffness, pressure angle and rotational speed on dynamic loads were investigated. Lower dynamic load was obtained for asymmetric tooth gears while compared to symmetric profile gears.

Hasan et al [130] prepared a nonlinear one degree of freedom dynamic model to investigate effect of tooth profile errors on dynamic load factor for symmetric and asymmetric profile gears by considering shaft and bearings as rigid and neglecting inertias of motor and load. Authors studied also the influence of transmitted load and tooth asymmetry on TE and DF. Fourth order Runge-Kutta method was used to solve derived equations to obtain dynamic load factor and transmission error for both symmetric and asymmetric tooth gears.

Liou et al [131] simulated gear pairs by NASA gear dynamics code (DANST) to investigate the effect of contact ratio on dynamic load for LCR and HCR spur gears. DANST is based on 4-DOF by including motor, gear pair and load effects. DF was estimated for LCR and HCR spur gears for different contact ratio values versus rotational speeds. Authors obtained some important findings as lower dynamic load for HCR gears, increasing contact ratio yields in reduced dynamic load and combined effect of contact ratio and operating speed on DF.

Yoon and Rao [132] applied cubic splines as new gear tooth profile and analyzed this type tooth profile gear pairs in addition to involute spur gears (with linear and parabolic relief) for gear dynamics including TE and dynamic tooth loads. To evaluate dynamic behaviors of gear pairs (obtaining dynamic tooth load and dynamic factor), a 4-DOF

dynamic model was prepared consisting the effects of gear mesh, input and output loads by not taking lateral movements of bearing and shafts into consideration for involute and cubic spline profile gears. A better dynamic performance was obtained for cubic spline tooth gears for dynamic load and natural frequencies.

Arıkan and Kaftanoğlu [133] analyzed spur gear pairs for dynamic load and root stress. A 2-DOF model including friction between gear teeth, tooth profile modifications, pitch and profile errors was used to obtain dynamic load and dynamic factors. Finite element analyses were carried out to determine variable mesh stiffness.

Özgüven [134] developed a 6-DOF nonlinear dynamic model with spur gears with two shafts, bearings, load and prime mover. TVMS, STE, backlash, tooth separation and double side impacts were taken into consideration in dynamic analyses. Mesh stiffness was calculated by using a software, STEP developed in reference study [107] by including shear deformations, bending, base rotation and Hertzian deflection. Torsional compliances of the shafts and transverse compliances of bearings with damping of both were included in dynamic model.

Özgüven and Houser [135] proposed a SDOF nonlinear dynamic model for calculation of dynamic mesh load, DF and DTE by consisting of gear errors, profile modification and time varying effect of mesh stiffness and damping by including STE of high speed spur gear pairs. Results of dynamic parameters estimated by dynamic model were compared to previous experimental works and compatible values were obtained.

Özgüven and Houser [136] reviewed mathematical models prepared for gear dynamics comprehensively and grouped them as simple dynamic factor models, models with tooth compliance, models for gear dynamics, models for geared rotor dynamics and models for torsional vibrations. Authors analyzed presented papers in terms of analytical, empirical and semi empirical works in terms of parameters included in mathematical models, and constant and time varying parameters.

Öztürk et al [137] made an optimization study for reduction of DTE for predefined rotational speed range and torque by developing a SDOF and MDOF lumped models. Identical gear pair mounted on relatively short shaft and longer shaft was simulated by using SDOF and MDOF models respectively. Approximate values of PPDTE are obtained for short shaft by using both dynamic models because of rigidity of the shaft

whereas shaft flexibility plays significant role in dynamic response of gear system for long shaft, and both models yields different results of PPDTE.

Öztürk et al [138] proposed two different dynamic models (SDOF and MDOF) to reduce DTE under a range of torque and speed values and made an optimization study for minimization of STE and DTE for spur gear pairs. Objective function was selected the variation of TE, multiple optimization was applied for amount of relief and start of relief for ideal tooth profiles of pinion and gear separately.

Maliha et al [139] presented a nonlinear dynamic model coupled with linear finite element models of shafts for a gear system and dynamic characteristics namely dynamic gear load, dynamic bearing load, DTE and bearing displacements were stated based on this model. Nonlinearity in the system was assumed to be coming from backlash between mating gears. External excitation was due to external static torque while variation of mesh stiffness with time excited the gear system internally in this model.

Doğruer and Özgüven [140] investigated the influence of transverse compliances of shaft and bearings on dynamic load of gear pair using nonlinear mathematical model by including nonlinearity because of backlash. The significance of interaction between effect of transverse shaft flexibility and backlash nonlinearity was focused in terms of gear system dynamics and mesh mode frequency.

Gürkan and Özgüven [141] included elasticity and clearance of bearings and backlash in their MDOF dynamic model of gear system. Whereas constant average mesh stiffness was used in model, the excitation effect of time varying mesh stiffness on gear system was not ignored by taking STE into account as a displacement. Results of mathematical model were compared to some experimental works presented in literature and close values were obtained in terms of DF and DTE.

Özgüven and Özkan [142] developed a software which calculates whirl speeds and dynamic response of multi bearing rotor systems in their study. Including rotary inertia, gyroscopic moment, axial load, damping of internal and viscous, and shear deformation, multi bearing (flexible bearings) rotor system is simulated by finite element analysis.

Liu and Parker [143] studied the effects of load sharing, varying mesh stiffness, profile modification and loss of contact on multi mesh gear system vibration by using three different developed models namely TVMS model, STE model and no load transmission error (NLTE) model. The results obtained from these three models were compared based on finite element benchmark and the best agreement was obtained by TVMS method.

Cai [144] developed an equivalent SDOF dynamic model for helical gear pairs by including nonlinear tooth separation without spacing and run out errors. Effects of shaft deviation and pressure angle errors on gear pair vibration were also investigated. A modified stiffness function was also proposed for helical gears used in dynamic analyses.

A 6-DOF nonlinear dynamic model was developed by Divandari et al [145] to investigate the influence of localized tooth defect and modifications of tooth on gear dynamics by involving the effects of gear errors. The model used in this paper is the same as the model developed by Özgüven [134] and it includes nonlinearity due to backlash, tooth separation and backside collision.

Faggioni et al [146] proposed a nonlinear dynamic model to optimize gear pair vibration to reduce vibration level by taking STE and maximum amplitude of DTE as objective functions. TVMS value including relief effects was obtained based on finite element analyses.

Moradi and Salarieh [147] described dynamic system consists of gears through SDOF model in terms of DTE, super and sub harmonics in addition to primary resonance of gear system. An investigation of the effect of stiffness, damping and excitation torque of engine was accomplished. Backlash was involved in the model as displacement type nonlinear effect and third order smooth polynomial equations were solved and, approximate values with difference of 5% were obtained in terms of DTE.

Iida et al [148] proposed a vibration (coupled torsional and flexural vibration) model of shaft in gearing system containing gear eccentricity and unbalance of mass. Natural frequencies of the system were calculated without taking coupling effect into consideration. The geometrical eccentricity of gears was taken as an excitation effect in vibration of gearing system.

Kim et al [149] took translational movement of gear pairs because of bearing deformation into account in their dynamic model regarding pressure angle and contact ratio as time varying parameters. Equations of motion based on six coordinates (two of rotational and four translational) were derived by substituting kinetic energy, potential energy and Rayleigh's dissipation function into Lagrange's equation. They were solved by applying Newmark time integration method for different damping and stiffness values. It was stated that mesh stiffness has a significant influence on vibration of gear mesh deformation, whereas it is not too much effective on radial vibration of gear pair.

Shen et al [150] used method of incremental harmonic balance (IHB) to analyze nonlinear dynamics of gear pair including STE, backlash, time varying stiffness, and damping ratio. Effect of multi order harmonics was also compared based on IHB and numerical method. It was found that with the increase of the damping ratio, single side and double side impact behaviors of gear can be analyzed and gear dynamics can be controlled.

Hu et al [151] developed a finite element dynamic model of high speed gear rotor bearing system by involving the effects of backlash, profile modification (short and long relief), time varying mesh stiffness on gearing system's dynamics. Time varying mesh stiffness was calculated by using two methods; finite element and analytical method based on curve fitting of tooth stiffness. For short and long relief with different amount of relief values, effect of profile modification on gear dynamic behavior in terms of DTE was investigated. A reasonable short tooth modification was recommended for improvement of gearing system whose details are presented in paper.

Siyu et al [152] investigated effects of dynamic backlash (because of center distance error), time varying mesh stiffness and friction on gear system by introducing a MDOF nonlinear mathematical model. The difference between the effects of constant and dynamic backlash on dynamics of gears was studied and nonlinear equations of motions were solved by using Runge-Kutta method. It was obtained that gearing system can go into chaotic motion earlier because of dynamic backlash effect.

Eritenel and Parker [153] proposed a nonlinear mathematical model including nonlinearity of mesh stiffness and mesh force for gears in the cases of partial and total

contact loss between gear pair by applying a perturbation method. Conducted solution of equation of motion includes the decrease in contact ratio of gear pairs because of misalignment and lead modifications.

Eritenel et al [154] presented a study of investigation of effects of tooth contact deflection under load on backlash and tip modification using finite element software, CALYX [155] for plastic gears. Authors researched possible backside contact with corner contact, distributions of load and pressure for gears with no relief and linear relief at tooth tip. No backside contact was obtained because of tooth deflection and increased backside gap occurred with increasing load.

Lin et al [156] proposed a parallel shaft spur gear model for simulating dynamic loads by investigating effective parameters such as inertia values of power source, load, shafts and, shaft stiffness, gear tooth stiffness, friction and load sharing on gear pair dynamics. 4-DOF mathematical model was prepared and governing equations were derived by considering the tooth separation and backside collision, but results were presented in Ref [157].

Lin et al [157] presented solutions to governing equations generated in paper [156] to obtain dynamic load. An increase in dynamic load was obtained with increased speed up to the natural frequency of the system while there was a dramatic decrease at speed beyond natural frequency. It was found that tooth stiffness effect on system dynamic response is superior to that of shaft.

Jia et al [158] presented a dynamic model for a gear system consists of three shafts and two gear pairs of a two stage gearbox with 26-DOF. Pitch error, profile error, friction, localized tooth crack and time varying mesh stiffness were included in dynamic model to investigate their effects on gearing system's dynamic performance. It was found that different values of pitch and profile errors have district influences on gearbox vibration.

Zhou et al [159] proposed a 16-DOF dynamic model by considering bearing clearance, transmission error, couple effect of lateral-torsional vibration. The influence of rotational speed, eccentricity and bearing clearance on dynamic behavior of gear system was investigated by using Runge-Kutta method. It was found that any increase in eccentricity gear's vibration amplitude increases, so eccentricity should be avoided

and bearing clearance selection is significant because of its important influence on gear system dynamics.

Saxena et al [160] made a modal analysis of gearing system including ball bearings by using ANSYS Workbench [18] and ADAMS [161] to predict natural frequencies in different modes, to investigate the effect of variation of bearing stiffness on natural frequencies and to study Campbell diagram. It was found that natural frequencies is greatly affected by variation of bearing stiffness values in between 10^7 and 10^{10} N/m. Effect of rotational speed on natural frequencies was also investigated in Campbell diagram.

Tang et al [162] investigated the influence of static transmission error, time varying mesh stiffness and backlash on dynamic behavior of spiral bevel gear pairs by expressing static transmission error as a combination of functions of parabolic and sine curves. Effect of sine and parabolic transmission error on dynamic characteristics of spiral bevel gears was investigated. It was found that sine format excitation gives higher performance and weak tooth surface impact at low velocity.

Abbes et al [163] presented an approach which is based on finite element analyses supported with elastic foundation theory for modeling spur gear pair. The effect of rim thickness of gear, eccentricity and tooth crack on dynamic response was investigated by taking time varying characteristics of mesh stiffness into consideration.

Lin et al [164] presented analytical software which computes static transmission error and its frequency spectrum for different tooth profile modifications. Dynamic loading was compared for unmodified and linearly modified tooth spur gear pairs. Optimum profile modification was intended to obtain minimum dynamic load. A 4-DOF model was generated and equations of motion were solved to obtain dynamic factor for different rotational speeds by giving specifications of gear pairs whereas no information about shafts and bearing were presented.

Lee et al [165] presented a dynamic response simulation for HCR spur gear pairs to obtain dynamic load, dynamic root stress (based on Heywood formula) and contact stress under the effect of various linear profile modifications. A dynamic model which includes gear pair, motor, load and shafts was proposed by only focusing on contact along length of contact and eliminating off line contact situations. Differential

equations of motions were linearized and solved by an iterative procedure. It was found that optimum profile modification amount is different for minimized dynamic load and stress, and excess modification has a detrimental effect than lower modification.

Saxena et al [166] investigated the dynamic behavior of multi mesh gearing transmission and estimate natural frequencies by using modal analyses techniques. By changing bearing stiffness values, natural frequencies were evaluated for different mode shapes. It has been shown that bearing stiffness has a critical effect on natural frequencies of the system.

Terauchi et al [167] investigated the effect of profile modification on tooth of spur gears in terms of dynamic load, gear noise and scoring by conducting experiments on gas carburized case hardened chrome-molybdenum steel gears using power-circulating test rig. Strain at the root of gear was measured by placing strain gauges at root fillet section of gear tooth. Noise was also measured by condenser microphone and sound level meter. Gears with and without profile modifications were tested to investigate dynamic characteristics. Less sound level was obtained because of profile modification.

2.7 Manufacturing and Measurement of Asymmetric Spur Gears

Similar to design and analysis of asymmetric gears, manufacturing and measurement of gear profile differs from symmetric tooth profile gears. In literature, there are too many works intended for gear design and analysis contrast to manufacturing and measurement (after manufacturing of asymmetric gear pairs). Less published papers compared to previous sections are presented here.

Ravai Nagy and Lobontiu [168] studied on different grinding methods of gears' flanks and some other issues relating to the manufacturing of the gearbox including asymmetric teeth gears and also testing of those. Study includes design of flexural strength test rig without any test result.

Ravai Nagy and Lobontiu [169] suggested grinding methods for nonstandard (asymmetric profile) gears by modifying grinding wheels. They presented required rotational speed and feed of the grinding wheel based on gear pair material and applied heat treatment. Surface roughness values of both flanks of asymmetric gear pairs after grinding process were measured.

Kapelevich [170] presented definitions for measurement of symmetric and asymmetric tooth spur and helical gears which are directly designed and generated. Measurement over balls or pins was defined based on involute geometry equations by taking different pressure angles for drive and coast side into account. In addition to all these, CMM inspection issues were defined for directly designed gears.



CHAPTER 3

MACRO GEOMETRY GENERATION OF ASYMMETRIC SPUR GEARS

3.1 Introduction

In this chapter, formulae for macro geometry generation of symmetric tooth profile gears have been explained. Modifications of these formulae for asymmetric tooth gear have been carried out for tooth profile generation including involute and trochoid portions. Some case studies have been done to obtain asymmetric tooth gear profiles and compared directly with KissSoft [14] for symmetric tooth profile gears whereas a different procedure has been implemented for asymmetric ones (explained during this chapter).

3.2 Involute Symmetric Tooth Profile Spur Gears

Power is not always usable directly in which produced so power and motion transmission is essential for many machines and systems. Gears are designed to fulfill functions to transmit power or angular motion from one shaft to another by means of progressive engagement of teeth. Power can be transmitted in the application of speed decreasing (torque increasing) or speed increasing (torque decreasing).

Gear tooth profile is expected to provide a uniform motion / power transmission between shafts with a nonslip and smooth drive. Due to these reasons, the shape and size of tooth profile is very essential. Many tooth profile forms such as involute, cycloidal and circular can theoretically fulfill the basic requirements such as uniform motion transfer and constant angular velocity during gear meshing (conjugate action).

Involute profile is in universal use for gear teeth with few exceptions. Involute gear tooth profile provides constant angular velocity and the meshing tooth forms specific geometrical characteristics. It is insensitive to center distance change and allows different operating center distance without changing the velocity ratio.

Involute tooth form is obtained by unwinding a string hold tight from circumference of a base circle (Figure 3.1). The involute curve is also expressed as mathematical form in terms of vectorial angle in radians:

$$\theta = \tan \phi - \phi = \text{inv}(\phi) \quad (3.1)$$

Where θ and ϕ are vectorial and pressure angles respectively in radians and inv is involute function.

R_B , R_C and R are base radius, radius of curvature related to Radius of R and any radius on involute curve respectively (shown in Figure 3.2). R_C and R are dependent on base radius. These all parameters are related to each other (Eq. 3.2-3.3).

$$\cos \phi = \frac{R_B}{R} \quad (3.2)$$

$$R_C = \sqrt{R^2 - R_B^2} \quad (3.3)$$

Base radius is radius of the base circle which involute curve generates from (Figure 3.2 and 3.4). Pressure angle is the angle between line of action and the common tangent to the two circles at pitch points of meshing gears (Figure 3.3). Pressure angle for any radius varies from one radius to another; it is specific to each radius on involute curve. Gear teeth which are identical in terms of module and teeth number but differ for values of pressure angle are presented in Figure 3.5. It is clear to say that increase in pressure angle yields thicker root section but cause decrease in tooth tip thickness.

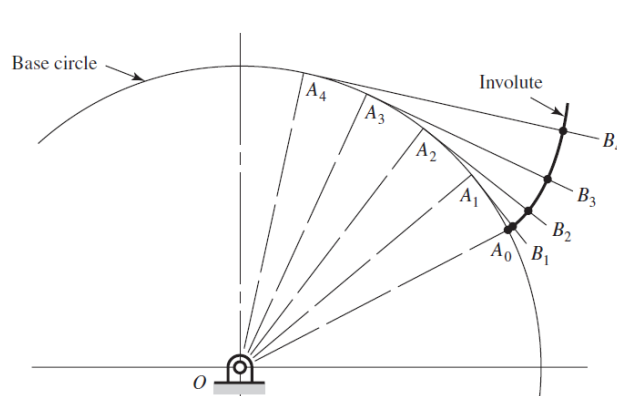


Figure 3.1 Involute generation [171]

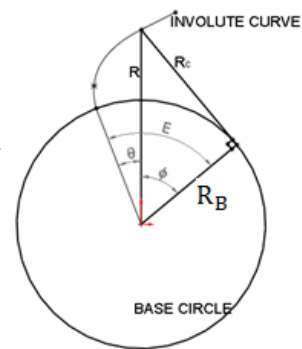


Figure 3.2 Involute curve

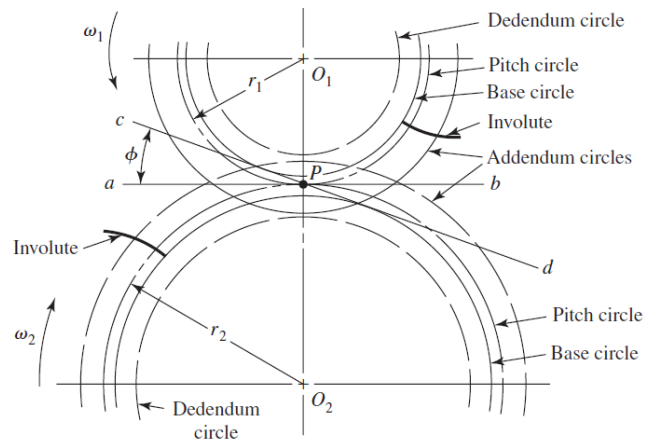


Figure 3.3 Pressure angle [171]

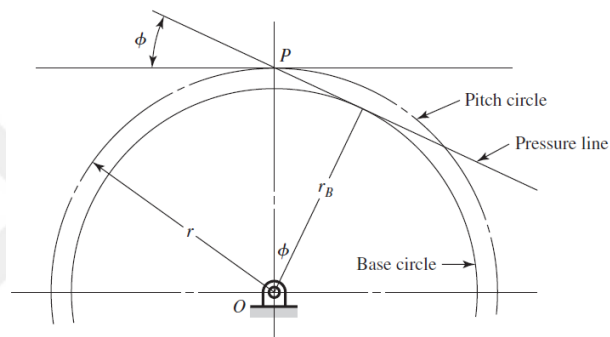


Figure 3.4 Base circle and pressure angle [164]

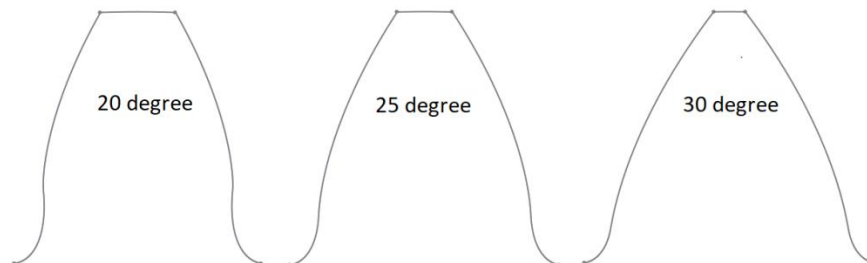


Figure 3.5 Symmetric gear teeth with pressure angle values of 20, 25 and 30degree

Spur gears of involute profile are simplest and the most common type of gears. The teeth of a spur gear are parallel to the axis of rotation as shown in Figure 3.6a. Spur gears are used to transmit motion between parallel shafts which encompasses the majority of applications. A pair of mating spur gears is illustrated in Figure 3.6b.

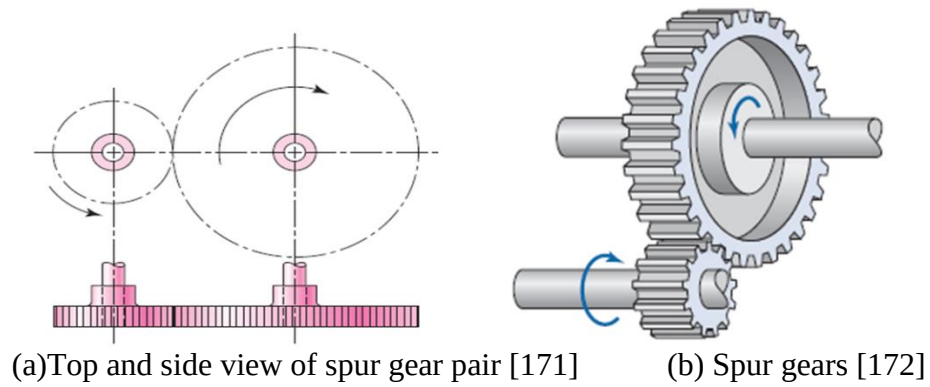


Figure 3.6 Spur gear pair

3.3 Involute Asymmetric Tooth Profile Spur Gears

Symmetric tooth profile spur gears have identical tooth flanks at both side of tooth in terms of pressure angle. On the other side, in many applications in which gear pairs are utilized, rotation is usually for same direction without need of any reverse movement. Therefore, it is not required to manufacture gear pairs of identical tooth forms (in terms of pressure angle) at both side of gear tooth. As stated up to now, when no reversal movement is needed, asymmetric tooth gears can be an alternative to symmetric one for transmission.

Asymmetric tooth profile gears differ from symmetric owing to different pressure angles and so base radii for drive and coast side flanks (Figure 3.7). Different pressure angles are intended for both flanks to take advantage of asymmetry in terms of superior bending strength as compared to symmetric gears (deeply discussed in Chapter 4).

Regardless of tooth profile of symmetrical or asymmetrical, design of a gear necessitates checking against likely gear failures due to bending, contact and scuffing mainly. On the other hand, there is a requirement for some tools to estimate whether gear pairs are to be in safe or not during meshing. These can be some standards including all equations, tables and figures, finite element works or experimental facilities.

Unfortunately, no standard for asymmetric gears has been encountered in literature. So anyone who intends to study on asymmetric profile gears can make finite element analysis, conduct some experiments or modify equations used for symmetric gears by depending on gear tooth geometry. These all requires macro geometry generation of

asymmetric tooth gears for evaluation of gear pair performance. Because researcher needs to generate gear geometric models for finite element works. In addition, gear pairs should be designed and manufactured for conducting experiments. Ultimately, investigation of gear macro geometry is substantial to improve any formulae already presented and well accepted for symmetric gears. For these requirements stated above, macro geometry generation of asymmetric tooth spur gears is essential and explained in this study.

Asymmetric spur gear has non identical pressure angles for drive and coast flank. Different pressure angle values yield different base circles for each flank of tooth (Figure 3.7 and 3.8).

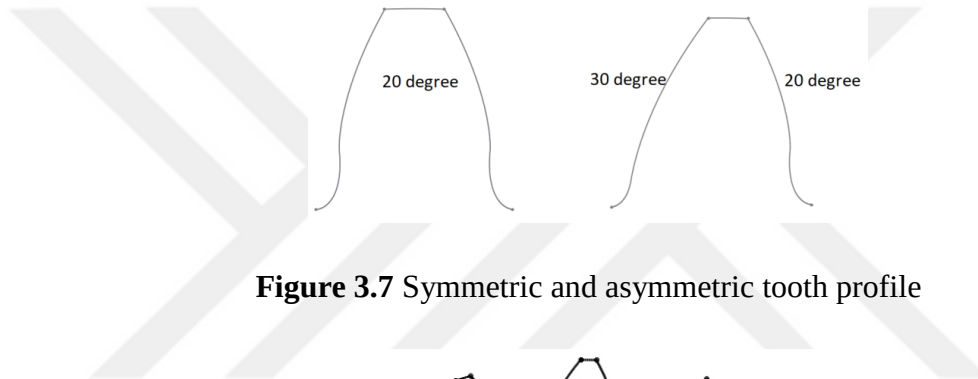


Figure 3.7 Symmetric and asymmetric tooth profile

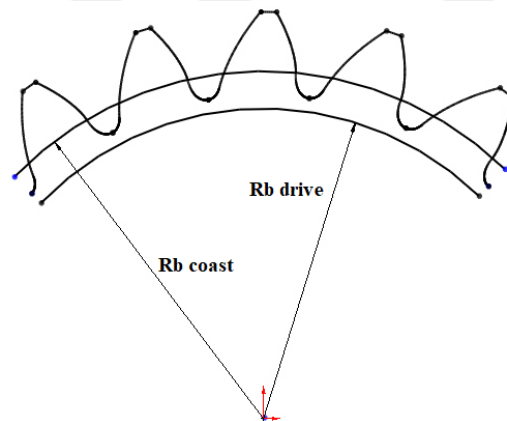


Figure 3.8 Asymmetric teeth with two different base circles

3.4 Gear Geometry Generation

Gears can be produced by different gear manufacturing and generation methods. The generation of involute spur gears (as shown in Figure 3.9.a, b and c) by a rack-cutter (MAAG), hob cutter, or pinion cutter (Fellows shaping) is a widespread practice in industry [173].

Gear generating process by using gear cutting tool called as hob is hobbing. The cutting tool, hob is a straight sided cutter and it is also named as rack (in the form of a spur gear of infinite teeth numbers). Cutting tool conjugates with gear to be produced during generation process. Gear blank rotates whereas cutter moves longitudinally as long as gear generation (Figure 3.10). When generation is completed, gear is obtained including two different curves; involute and trochoid on tooth flank and at the root region respectively (Figure 3.11).

A hob (Figure 3.12) is a gear cutting tool used to produce gears with identical pressure angle and module (or diametral pitch) without taking teeth numbers of gears into consideration. Hobs are gear cutting tools which are obtainable in industry for standard pressure angle, module and tip radius.

Gears generated by same hob are able to mesh together because of having identical specific parameters of normal module and pressure angle. Sum of the length of β and R equals to dedendum of gear where R is tip radius of cutting tool. TP and TH are hob tooth space and hob tooth thickness respectively.

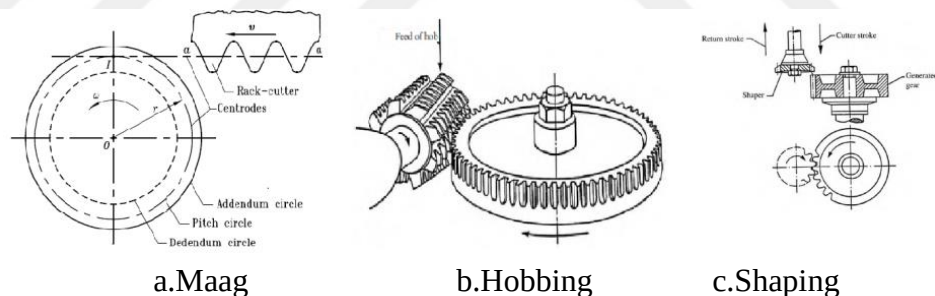


Figure 3.9 Gear generating methods [173]

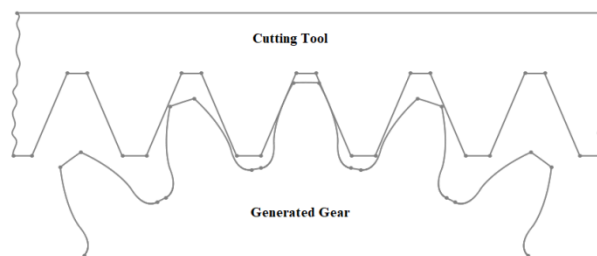


Figure 3.10 Gear hobbing process

Tooth thickness of gear (on pitch circle) is equal to tooth space of hob. During generation of gears, hob moves a distance of $(TP+TH)$ while gear rotates via an angle of $(TP+TH)/R_G$, where R_G is pitch radius.

Gear tooth geometry including involute and trochoid portion can be plotted on Cartesian coordinate. Equations (3.4 – 3.27) required to generate symmetric gear tooth profile in this study are obtained from Ref [171] and explained briefly below.

Involute portion of tooth with respect to gear tooth center is generated by starting at a point where tooth thickness and pressure angle are known. For a start, this point is generally selected pitch point with a pressure angle of ϕ_1 , pitch radius of R_1 and tooth thickness of CTT_1 .

$$\theta_1 = \tan \phi_1 - \phi_1 \quad (3.4)$$

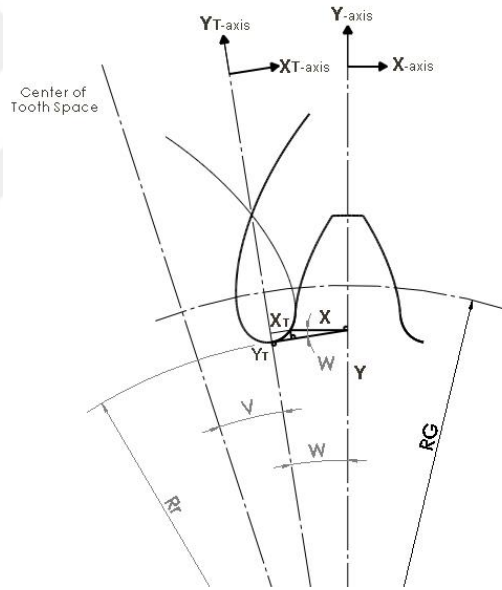


Figure 3.11 Involute and trochoid profiles

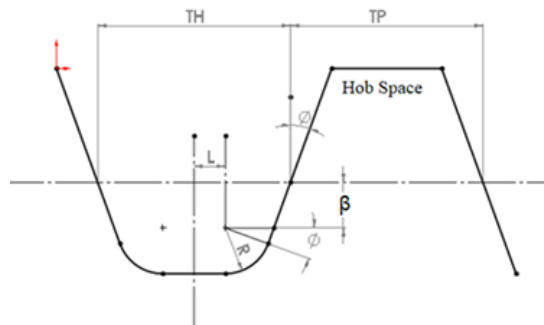


Figure 3.12 Hob geometry for symmetrical tooth

$$A = \theta_1 + \frac{0.5 * CTT_1}{R_1} \quad (3.5)$$

$$B = A - \theta_2 \quad (3.6)$$

$$\phi_2 = \cos^{-1} \frac{R_B}{R_2} \quad (3.7)$$

$$\theta_2 = \tan \phi_2 - \phi_2 \quad (3.8)$$

$$CTT_2 = 2 * R_2 * \left(\frac{0.5 * CTT_1}{R_1} + \theta_1 - \theta_2 \right) \quad (3.9)$$

The coordinates of X and Y are calculated for any point of involute curve:

$$X = R_2 \sin B \quad (3.10)$$

$$Y = R_2 \cos B \quad (3.11)$$

Whereas

ϕ_2 : Pressure angle at any radius,

θ_2 : Involute angle at any radius,

R_2 : Any radius on involute curve,

CTT_2 : Circular tooth thickness at any radius.

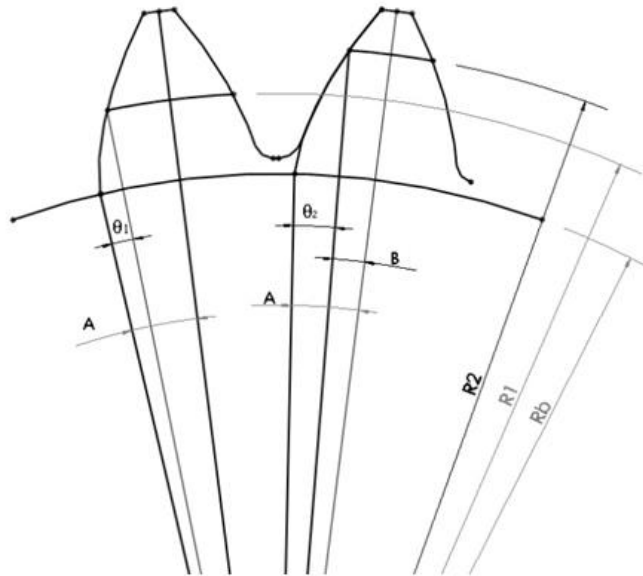


Figure 3.13 Calculating tooth thickness at any point on involute curve

A software routine is used based on all these relationships to obtain involute curve. Routine is started by using pressure angle of zero (on base circle) to obtain the nearest point to tooth tip with a difference of 0.125mm.

Hob tip radius generates the shape and radius of curvatures of gear root section which is restricted by root diameter and involute curve. Gear root includes several radius of curvature because at any point on root has different center and radius. To calculate trochoid coordinates, following formulae are used:

$$W + V = \frac{0.5*(TH+TP)}{R_G} \quad (3.12)$$

$$V = \frac{L}{R_G} \quad (3.13)$$

Where L is distance between the center of the hob tooth and point Z.

$$L = \frac{TH}{2} - \beta \tan \phi - \frac{R}{\cos \phi} \quad (3.14)$$

$$V = \frac{0.5*(TH+TP)-L}{R_G} \quad (3.15)$$

It can be seen that trochoid curve is generated by Point Z, when hob moves a distance of $R_G * E$ and gear blank rotates via an angle of E (Figure 3.14).

$$X_Z = R_Z \sin(T - E) \quad (3.16)$$

$$Y_Z = R_Z \cos(T - E) \quad (3.17)$$

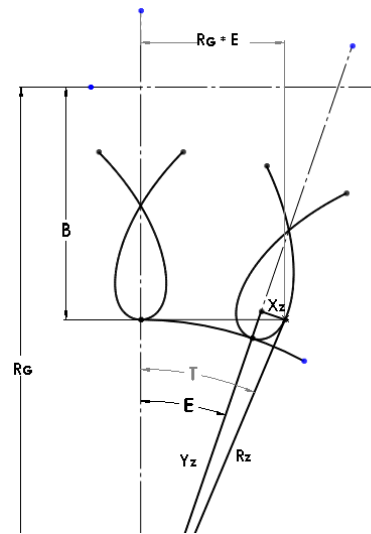


Figure 3.14 Trochoid generated by Point Z

where

$$\cos T = \frac{R_G - \beta}{R_Z} \quad (3.18)$$

$$\sin T = \frac{R_G * E}{R_Z} \quad (3.19)$$

$$X_Z = (R_G * E) * \cos E - (R_G - \beta) * \sin E \quad (3.20)$$

$$Y_Z = (R_G - \beta) * \cos E + (R_G * E) * \sin E \quad (3.21)$$

Actual value of coordinates of trochoid is found by adding hob tip radius to generated coordinates (Figure 3.11):

$$X_T = X_Z + R * \cos A \quad (3.22)$$

$$Y_T = Y_Z - R * \sin E \quad (3.23)$$

Trochoid coordinates with respect to the system via tooth center is finally evaluated:

$$\sin W = \frac{X_T + X \cos W}{Y} \quad (3.24)$$

$$\cos W = \frac{Y_T - X \sin W}{Y} \quad (3.25)$$

$$X = Y_T \sin W - X_T \cos W \quad (3.26)$$

$$Y = Y_T \cos W + X_T \sin W \quad (3.27)$$

Up to here, tooth profile generation consisting of involute and trochoid has been explained based on geometry and movement of hob (gear cutting tool) by using Figures of 3.10 - 3.14 and Equations of 3.4 - 3.27.

In this study, hob geometry (Figure 3.15) and equations are going to be modified and given hereupon for generation of asymmetric tooth profile involute gear. An asymmetric tooth spur gear is modeled as two comparable symmetric tooth spur gears to generate gear tooth and gear complete models (in this chapter) and evaluate tooth root bending stress and related parameters (in next chapter) as shown in Figure 3.16. These two symmetric gears refer to drive and coast side of asymmetric gear in terms of pressure angle and base circle radius.

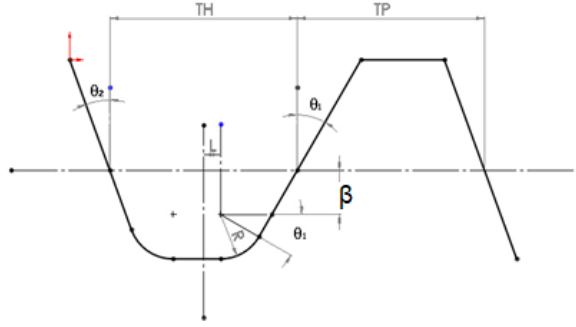


Figure 3.15 Hob geometry for asymmetrical tooth

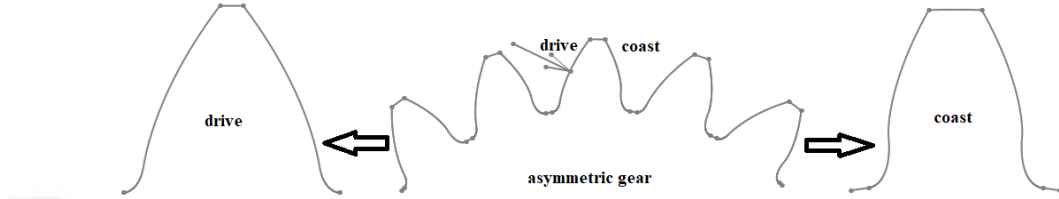


Figure 3.16 Modelling asymmetric gear as two symmetric gears

All equations presented for symmetric tooth gears up to now, have been modified to obtain asymmetric tooth gear involute profile and given below for drive and coast side flanks (Eq. 3.28-3.37).

$$\theta_{1d,c} = \tan \phi_{1d,c} - \phi_{1d,c} \quad (3.28)$$

$$A_{d,c} = \theta_{1d,c} + \frac{0.5 * CTT_1}{R_1} \quad (3.29)$$

$$B_{d,c} = A_{d,c} - \theta_{2d,c} \quad (3.30)$$

$$\phi_{2d,c} = \cos^{-1} \frac{R_{Bd,c}}{R_2} \quad (3.31)$$

$$\theta_{2d,c} = \tan \phi_{2d,c} - \phi_{2d,c} \quad (3.32)$$

$$CTT_{2d} = 2 * R_2 * \left(\frac{0.5 * CTT_1}{R_1} + \theta_{1d} - \theta_{2d} \right) \quad (3.33)$$

$$CTT_{2c} = 2 * R_2 * \left(\frac{0.5 * CTT_1}{R_1} + \theta_{1c} - \theta_{2c} \right) \quad (3.34)$$

$$CTT_2 = 0.5 * (CTT_{2d} + CTT_{2c}) \quad (3.35)$$

$$X_{d,c} = R_2 \sin B_{d,c} \quad (3.36)$$

$$Y_{d,c} = R_2 \cos B_{d,c} \quad (3.37)$$

Whereas

$\phi_{2d,c}$: Pressure angle at any radius for drive and coast side flank,

$\theta_{2d,c}$: Involute angle at any radius for drive and coast side flank,

R_2 : Any radius on involute curve (it is same for both flanks on involute curve),

CTT_2 : Circular tooth thickness at any radius (it is sum of half thickness of drive and coast side tooth flanks)

3.5 Software

In scope of this thesis, in-house software has been prepared to generate symmetric and asymmetric tooth spur gear profiles based on explained gear geometry generation principles above. Software has a geometrical construction module which consists of Main Window, Input Parameters Window, Tooth Profile Drawing Window, Results Window and Error Warnings Window as shown in Figure 3.17-3.22.

Main window includes sub windows such as input parameters, tooth profile drawing, results and error message windows.

Input window is used to enter gear pair parameters; geometrical parameters such as module, teeth numbers of pinion and gear, pressure angle of drive and coast side, addendum, dedendum, hob tip radii of drive and coast side, face width; working parameters such as power and angular velocity.

Tooth profile drawing module of software provides driver and driven gear tooth profile including involute and trochoid section for symmetric and asymmetric spur gear tooth profile.

Results window shows geometrical and stress calculation results for driver and driven gears separately (Figure 3.21).

Error warnings - message module gives warning if there is exceed the limit for gear pair parameters such as tooth tip thickness and interference (Figure 3.22).

It is possible to save coordinates of gear tooth profile as a “text file” editable by most text editing programs like editor, excel, word etc. (Figure 3.23). The saved text file can be used to draw 2D and 3D model of gears by Solidworks [174] (Figure 3.24). The

models generated by in-house software are going to be used for finite element analyses in Chapter 5.

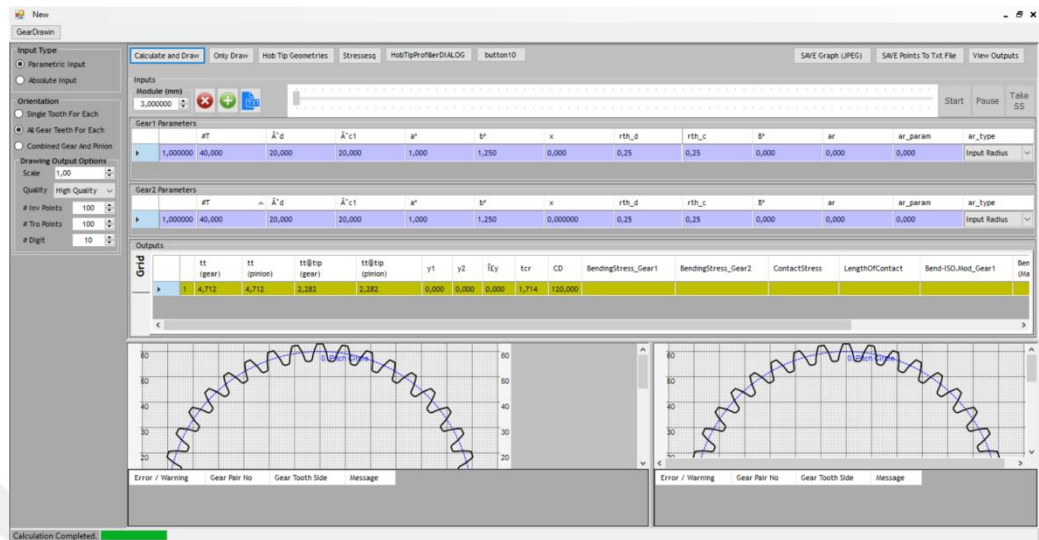


Figure 3.17 Main window (in-house software)

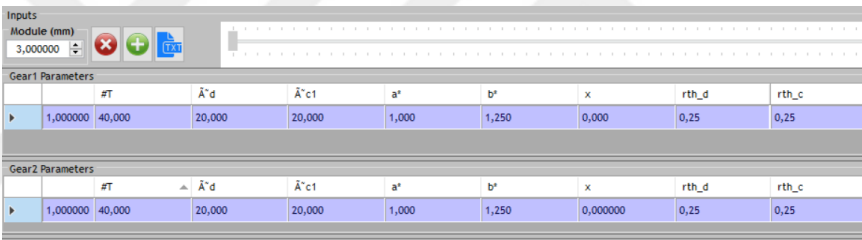


Figure 3.18 Input parameters window (in-house software)

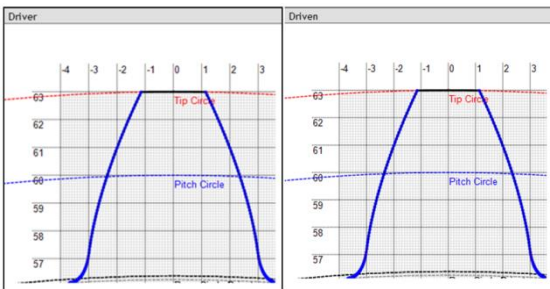


Figure 3.19 Symmetric tooth profile generating windows (in-house software)

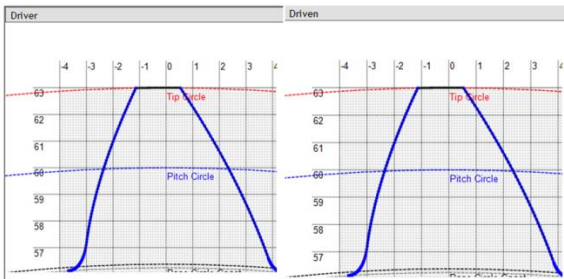


Figure 3.20 Asymmetric tooth profile generating windows (in-house software)

Calculation Results			
RESULTS			
PROPERTY	PINION	GEAR	UNIT
Normal Module	3,000000		mm
Teeth Number	40,000000	40,000000	-
Gear Ratio	1,000000		-
Center Distance (Working)	120,000000		mm
Center Distance (Working)	120,000000		mm
Center Distance (Reference)	120,000000		mm
Involute Function Of Normal Pressure Angle Drive Side	0,014904		?
Involute Function Of Working Normal Pressure Angle Drive Side	0,014904		?
Involute Function Of Working Transvers Pressure Angle Drive Side	0,014904		?
Transverse Pressure Angle (Radian) Drive Side	0,3491		Radian
Transverse Pressure Angle (Degree) Drive Side	20,0000		Degree
Working Normal Pressure Angle (Radian) Drive Side	0,3491		Radian
Working Normal Pressure Angle (Degree) Drive Side	20,0000		Degree
Working Transverse Pressure Angle (Radian) Drive Side	0,3491		Radian
Working Transverse Pressure Angle (Degree) Drive Side	20,0000		Degree
Pressure Angles			
PROPERTY	PINION	GEAR	UNIT
Pressure Angle (Drive Side)	20,0000		Degree
Pressure Angle (Coast Side)	20,0000		Degree
Working Normal Pressure Angle (Drive Side)	20,0000		Degree
Working Transverse Pressure Angle (Drive Side)	20,0000		Degree
Pressure Angle At Tip (Drive Side)	26,4986	26,4986	Degree
Pressure Angle At Tip (Coast Side)	26,4986	26,4986	Degree
Pressure Angle At SAP (Drive Side)	12,9195	12,9195	Degree
Pressure Angle At LPSTC (Drive Side)	18,8536	18,8536	Degree
Pressure Angle At HPSTC (Drive Side)	21,1300	21,1300	Degree
Pressure Angle At EAP (Drive Side)	26,4986	26,4986	Degree

Figure 3.21 Results window (in-house software)

Error Warnings		
Gear	Tooth Side	Message
Driver	Both	Tip Tooth Thickness is not in allowable limits. ($tt@tip > 0.25 \times \text{Module}$)
Driven	Both	Tip Tooth Thickness is not in allowable limits. ($tt@tip > 0.25 \times \text{Module}$)

Figure 3.22 Tip thickness limit exceed - error message window (in-house software)

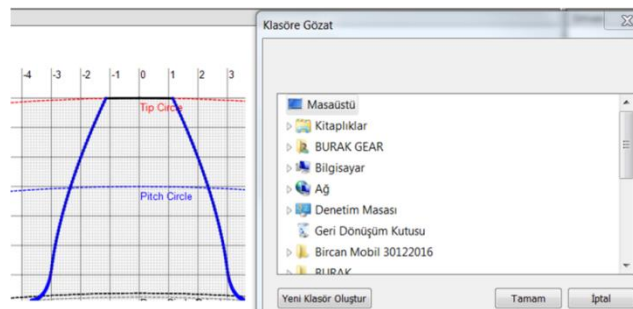


Figure 3.23 Saving file of gear tooth profile coordinates (in-house software)

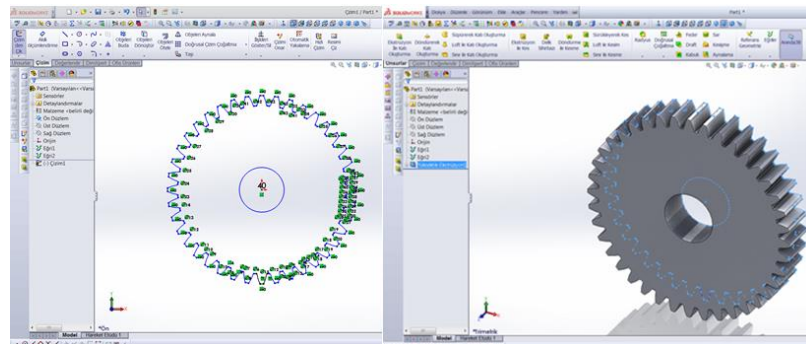


Figure 3.24 3D gear model generation by using Solidworks [174]

3.6 Case Studies of Macro Geometry Generation

A graphical comparison of macro geometry construction of symmetric spur gears constructed by using KissSoft [14] and in house software is presented in Figure 3.25-3.31. Gear parameters used for macro geometry generation of symmetric gear profiles (SP) are given in Table 3.1.

Red lines refer to geometry obtained from in-house software whereas blue stars are belonging to KissSoft outcomes. These macro geometries obtained from both tools coincide with each other and satisfy. Gear pairs with same gear parameters except root fillet radii (SP1-SP7) were studied for macro geometry generation. When they are compared, it is easy to say that gear profiles from both tools are identical.

Table 3.1 Symmetric tooth spur gear pair parameters of Case SP1-SP7

Parameter	Value	Unit
Module	3	mm
Teeth number	40	-
Pressure angle	20	degree
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
	Cutter tip radius coefficient	
Case SP1	0.10	-
Case SP2	0.20	-
Case SP3	0.25	-
Case SP4	0.30	-
Case SP5	0.35	-
Case SP6	0.40	-
Case SP7	0.45	-

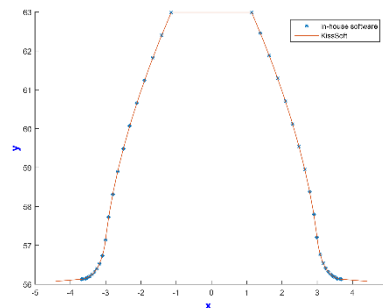


Figure 3.25 Macro geometry generation of Case SP1 (KissSoft and software)

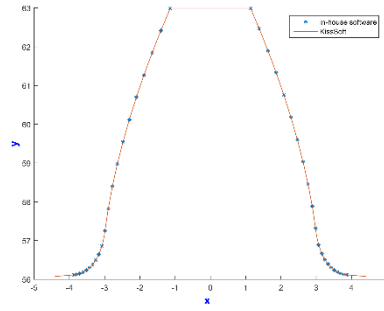


Figure 3.26 Macro geometry generation of Case SP2 (KissSoft and software)

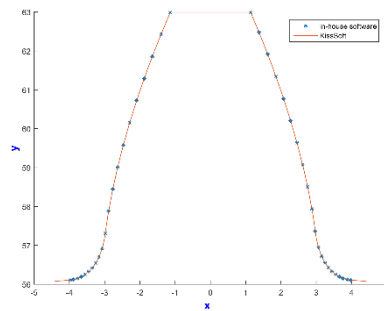


Figure 3.27 Macro geometry generation of Case SP3 (KissSoft and software)

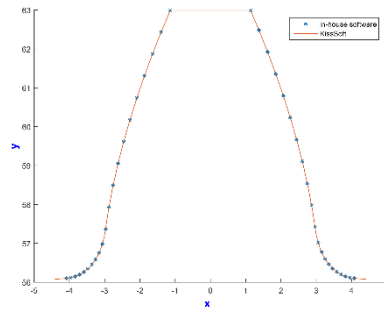


Figure 3.28 Macro geometry generation of Case SP4 (KissSoft and software)

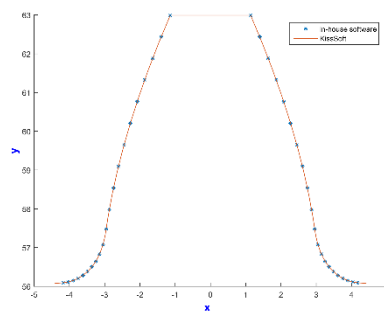


Figure 3.29 Macro geometry generation of Case SP5 (KissSoft and software)

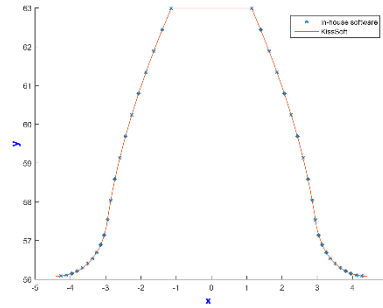


Figure 3.30 Macro geometry generation of Case SP6 (KissSoft and software)

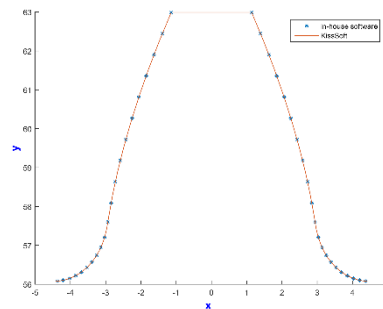


Figure 3.31 Macro geometry generation of Case SP7 (KissSoft and software)

Three more cases for symmetric tooth spur gear pairs with same individual gear parameters and different teeth number values were generated by using both tools. Macro geometries are presented for SP8, SP9 and SP10 in Figure 3.32, 3.33 and 3.34 respectively. Geometries satisfy for both tools without any deviation.

Table 3.2 Symmetric tooth spur gear pair parameters of Case SP8-SP10

Parameter	Value	Unit
Module	1	mm
Pressure angle	20	degree
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Cutter tip radius coefficient	0.45	-
Teeth number		
Case SP8	20	-
Case SP9	40	-
Case SP10	120	-

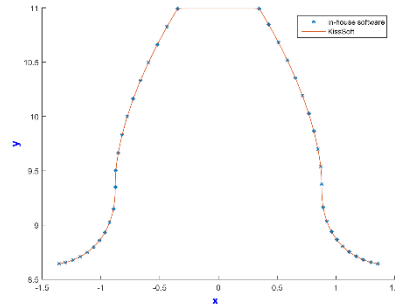


Figure 3.32 Macro geometry generation of Case SP8 (KissSoft and software)

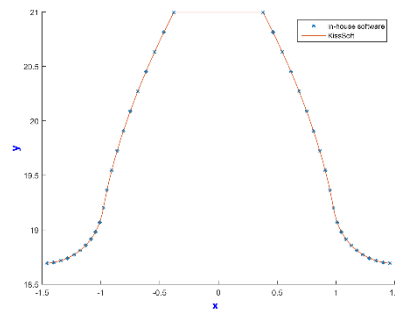


Figure 3.33 Macro geometry generation of Case SP9 (KissSoft and software)

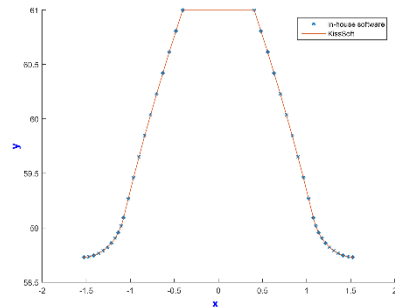


Figure 3.34 Macro geometry generation of Case SP10 (KissSoft and software)

Case studies of SP11 and SP12 are gear pairs with module of 3mm and teeth number of 40. They differ from each other in terms of pressure angle and root fillet radius coefficients. Case SP11 have 25degree of pressure angle and root fillet radius coefficient of 0.3 whereas corresponding values are 30degree and 0.1 respectively for Case SP12. Gear geometries coincide with each other for software and KissSoft's outcomes of Case SP11 and SP12 as presented in Figure 3.35 and 3.36. Solid line in red color is the result of KissSoft while blue stars refer to geometry obtained from software.

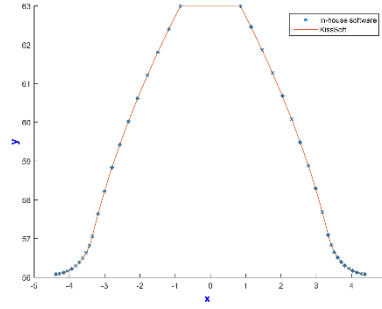


Figure 3.35 Macro geometry generation of Case SP11 (KissSoft and software)

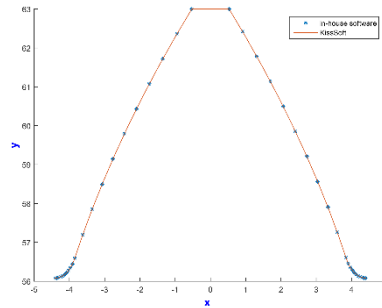


Figure 3.36 Macro geometry generation of Case SP12 (KissSoft and software)

Asymmetric tooth profile is required to be generated and compared but no commercial software which constructs macro geometry of asymmetric spur gears has been encountered during survey. So an asymmetric tooth is considered as two representative symmetric tooth profiles. For instance, an asymmetric tooth with pressure angles of 30 degree and 20 degree for drive and coast sides respectively are generated. These two representative symmetric teeth profiles are constructed by KissSoft for comparison. Comparison of drive and coast side tooth profile of asymmetric gear is carried out by superimposing profiles obtained by in-house software and KissSoft as shown in Figure 3.37-3.42.

Several case studies for asymmetric profile (AP) have been presented in this thesis. Asymmetric spur gears with greater pressure angle of drive side (AP1, AP2 and AP3) and greater value of coast side tooth flank cases (AP4, AP5 and AP6) were analyzed for macro geometry generation by KissSoft (generating two comparable symmetric) and software. Gear parameters for Case AP1, AP2, AP3, AP4, AP5 and AP6 are presented in Table 3.3-3.8. Generated macro geometries for these cases are illustrated in Figure 3.37-3.42. Gear profiles are in accordance with each other for both tools.

Table 3.3 Asymmetric tooth spur gear pair parameters of Case AP1

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	30/20	30/30	20/20	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.11	0.11		-

Table 3.4 Asymmetric tooth spur gear pair parameters of Case AP2

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	30/25	30/30	25/25	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.11	0.11		-

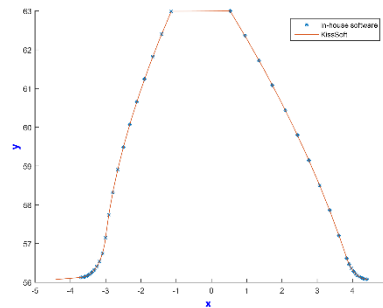
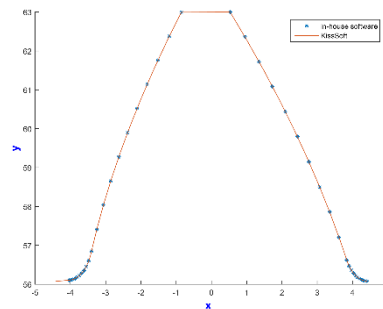
**Figure 3.37** Macro geometry generation of Case AP1(KissSoft and software)**Figure 3.38** Macro geometry generation of Case AP2 (KissSoft and software)

Table 3.5 Asymmetric tooth spur gear pair parameters of Case AP3

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	25/20	25/25	20/20	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.31	0.31		-

Table 3.6 Asymmetric tooth spur gear pair parameters of Case AP4

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	25/30	25/25	30/30	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.11	0.11		-

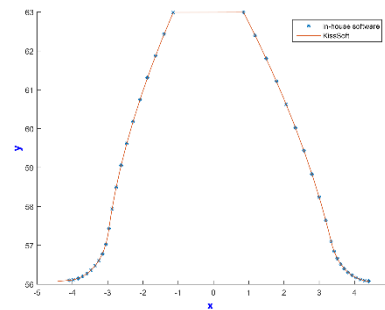
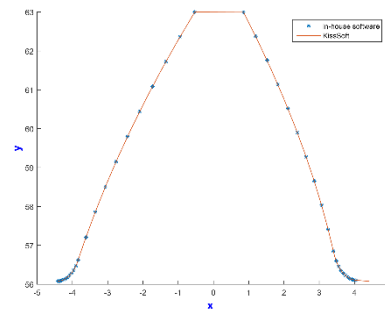
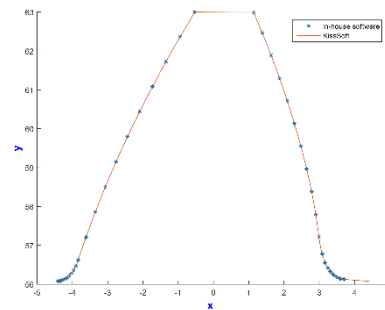
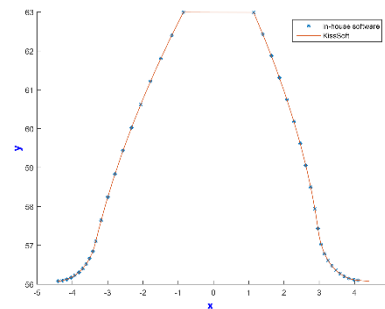
**Figure 3.39** Macro geometry generation of Case AP3 (KissSoft and software)**Figure 3.40** Macro geometry generation of Case AP4 (KissSoft and software)

Table 3.7 Asymmetric tooth spur gear pair parameters of Case AP5

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	20/30	20/20	30/30	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.11	0.11		-

Table 3.8 Asymmetric tooth spur gear pair parameters of Case AP6

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	20/25	20/20	25/25	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient	0.31	0.31		-

**Figure 3.41** Macro geometry generation of Case AP5 (KissSoft and software)**Figure 3.42** Macro geometry generation of Case AP6 (KissSoft and software)

In addition to macro geometry generation of spur gears with symmetric and asymmetric tooth and symmetric trochoidal root profiles, geometry of spur gears with asymmetric root profile (AR) [175] and *dual asymmetry* (DA) [176-178] for both tooth and root profiles were generated. For this aim, Case AR1 and DA1 were studied for macro geometry generation. General gear parameters for these cases are presented in Table 3.9 and 3.10 respectively. Generated macro geometries for Case AR1 and DA1 are presented in Figure 3.43 and 3.44 respectively.

Table 3.9 Asymmetric tooth spur gear pair parameters of Case AR1

Parameter	Symmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	20/20	20/20		degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient(drive/coast)	0.45/0.20	0.45	0.20	-

Table 3.10 Asymmetric tooth spur gear pair parameters of Case DA1

Parameter	Asymmetric gear	Representative symmetric gears		Unit
		Gear 1	Gear 2	
Module	3	3		mm
Teeth number	40	40		-
Pressure angle (drive/coast)	30/20	30/30	20/20	degree
Addendum coefficient	1.00	1.00		-
Dedendum coefficient	1.25	1.25		-
Cutter tip radius coefficient(drive/coast)	0.10/0.40	0.10	0.40	-

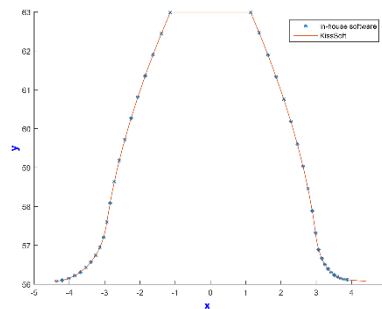


Figure 3.43 Macro geometry generation of Case AR1 (KissSoft and software)

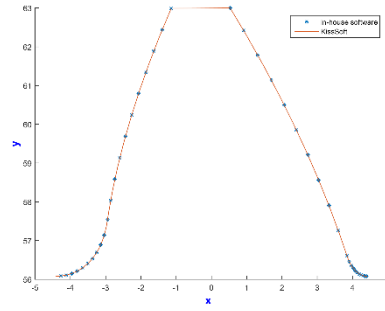


Figure 3.44 Macro geometry generation of Case DA1 (KissSoft and software)

CHAPTER 4

TOOTH ROOT BENDING STRESS CALCULATIONS FOR SPUR GEARS

4.1 Introduction

Gears are one of the main elements of mechanical power transmission units, their invent and history dates back to old times. Whereas there are substantial amount of literature, published works, well accepted formulae-equations and international standards such as ISO [2], AGMA [94] and DIN [63] for symmetric profile gears, studies have kept going to improve their performances in terms of load carrying capacity. ISO 6336:2006 is one of the well-known gear standards and consists of five parts given in Table 4.1. These parts involves calculations of basic principles, introduction and general influence factors, tooth bending strength, surface durability (pitting), strength and quality of gear materials and service life under variable load.

Gears generally operate in unidirectional for many applications; reverse motion is rarely encountered. So during gear operation, one side of gear tooth (drive side) is loaded heavily for long time duration whereas the other side (coast side) is unloaded or lightly loaded for short time. Asymmetric gears (Figure 4.1) can be an alternative to symmetric gears for power transmission without reversal movement.

Asymmetric tooth profile spur gear is formed by the two involutes of different base circles; coast side and the drive side profiles as shown in the Figure 4.2. Pressure angle of drive side flank is modified for the design of the asymmetric spur gear so that one can get more contacting area.

Regardless of tooth profile of symmetrical or asymmetrical, design of a gear necessitates checking against likely gear failures due to bending, contact and scuffing mainly. On the other hand, there is a requirement for some tools to estimate whether asymmetric tooth gear pairs are to be in safe or not during meshing. These can be some standards including all equations, tables and figures, finite element works or experimental facilities. Unfortunately, no standard for asymmetric gears is encountered in literature. So anyone who intends to study on asymmetric profile gears

can carry out finite element analysis, conduct some experiments or modify existing equations of symmetric tooth gears for asymmetric tooth gears by depending on gear tooth geometry. The third of these, modifying formulae already presented for symmetric gears is investigated to have a tool for bending stress calculation of asymmetric gears in this chapter.

Asymmetric gears have different gear tooth profile than symmetric gears so it is not possible to calculate tooth root bending stress of asymmetric gears by using standard formulae. Design and analysis of asymmetric spur gears require some modifications on formulae presented in ISO 6336:2006 Part 3 [2].

Table 4.1 ISO 6336:2006 parts

Standard	Part	Scope
ISO 6336:2006	1	Basic principles, introduction and general influence factors
ISO 6336:2006	2	Calculation of surface durability (pitting)
ISO 6336:2006	3	Calculation of tooth bending strength
ISO 6336:2006	5	Strength and quality of materials
ISO 6336:2006	6	Calculation of service life under variable load
ISO 53:1998	-	Cylindrical gear for general and heavy engineering - Standard basic rack tooth profile
ISO 54:1996	-	Cylindrical gear for general and heavy engineering - Module
ISO 1328:1997	1	Cylindrical gears-ISO system of accuracy- Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth
ISO 1328:1997	2	Cylindrical gears-ISO system of accuracy- Definitions and allowable values of deviations relevant to radial composite deviations and run out information

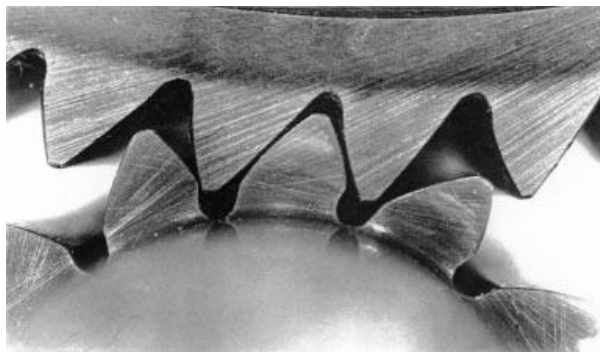


Figure 4.1 Asymmetric spur gear pair

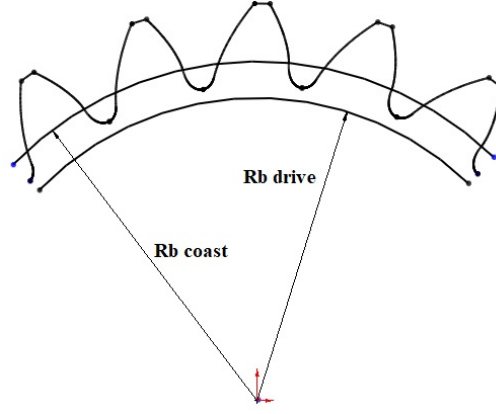


Figure 4.2 Asymmetric tooth gear with two different base circles

4.2 Modification of Formulae of ISO 6336:2006 Part 3 [2]

ISO 6336 Part 3, Calculation of load capacity of spur and helical gears [2] specifies the fundamental formulae for use in tooth bending stress calculations of involute external or internal spur and helical gears. ISO 6336 Part 3 [2] suggests following formulae to calculate tooth root bending stress of symmetric spur gears.

$$\sigma_{F0} = \frac{F_t}{b m_n} Y_F Y_S Y_\beta Y_B Y_{DT} \quad (4.1)$$

Form factor (Y_F) and stress concentration factor (Y_S) are calculated by following formulae developed by ISO [2]:

$$Y_F = \frac{\frac{6h_{Fe}}{m_n} \cos \alpha_{Fen}}{\left(\frac{S_{Fn}}{m_n}\right)^2 \cos \alpha_n} \quad (4.2)$$

$$Y_s = (1.2 + 0.13L) q_s^{\left[\frac{1}{1.21 + \frac{2.3}{L}}\right]} \quad (4.3)$$

$$L = \frac{S_{Fn}}{h_{Fe}} \quad (4.4)$$

$$q_s = \frac{S_{Fn}}{2\rho_F} \quad (4.5)$$

Notations for these equations are given in Table 4.2.

In this study, some modifications on formulae of stress calculation factors such as form factor and stress concentration factor are required for asymmetric gears. Helix angle

factor, rim thickness factor and deep tooth factor values are assumed as unity because of;

- ✓ Helix angle is zero for spur gears.
- ✓ Rim thickness effect is omitted as yet.
- ✓ Contact ratio is smaller than 2.5 for gears which this study is focused on.

Table 4.2 Parameters required for equations of 4.1-4.5

Symbol	Unit	Explanation
F_t	Newton	Nominal tangential load
b	mm	Face width
m_n	mm	Normal module
Y_F	-	Form factor
Y_S	-	Stress concentration factor
Y_β	-	Helix angle factor
Y_B	-	Rim thickness factor
Y_{DT}	-	Deep tooth factor
α_{Fen}	Radian	Load direction angle, relevant to direction of application of load at the outer point of single pair tooth contact of virtual spur gears
α_n	Degree	Normal pressure angle
h_{Fe}	mm	Bending moment arm for tooth root stress relevant to load application at the outer point of single pair tooth contact
s_{Fn}	mm	Tooth root chord at the critical section
ρ_F	mm	Radius of root fillet
q_s	-	Notch parameter; $q_s = s_{Fn} / 2\rho_F$

Tooth root chord at the critical section (tooth root thickness) and bending moment arm (shown in Figure 4.3) are very important parameters for calculation of form factor and stress concentration factor so tooth root bending stress. ISO 6336 - Part 3 [2] suggests an assumption; tooth root chord at the critical section is at a position where root fillet curve is tangent to a line which makes a 30 degree angle with symmetry axis of gear tooth.

In this study, tooth root chord at the critical section (tooth root critical thickness) for asymmetric gear are calculated by modelling asymmetric spur gear as two symmetric gears as shown in Figure 4.4. Two different thicknesses for gear models of drive and coast sides of asymmetric tooth (Figure 4.4) are summed up and divided into two. This

new value is taken as tooth root thickness of asymmetric spur gear. Then, bending moment arm is calculated based on this new thickness. Two substantial stress modifying factors; form factor (Y_F) and stress concentration factor (Y_S) are calculated depending on this calculated tooth root thickness and bending moment arm of asymmetric gear. Tooth root bending stress is evaluated by using all these parameters.

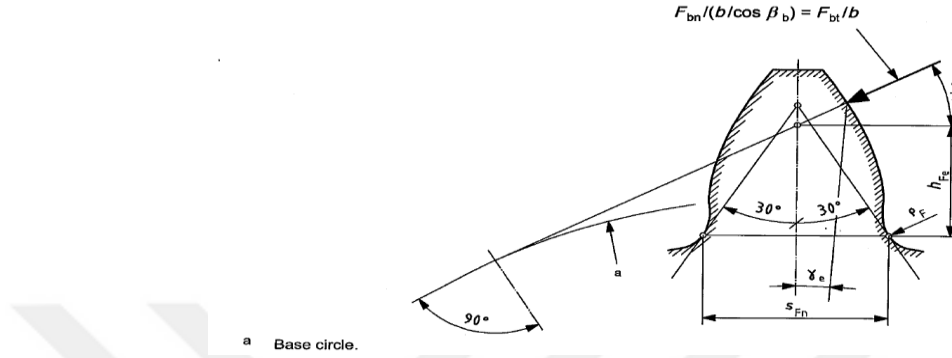


Figure 4.3 Determinations of normal chord of tooth root critical section [2]

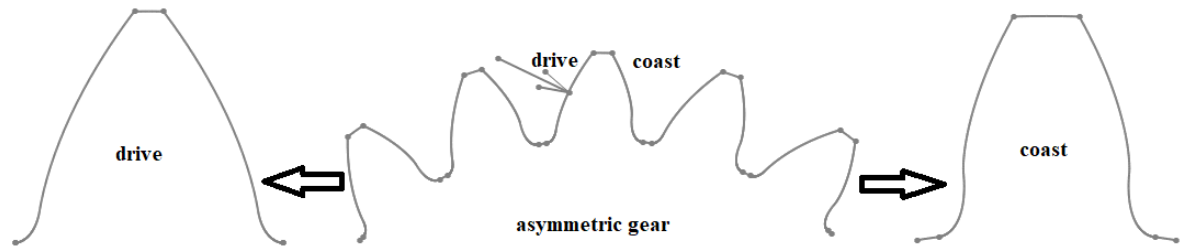


Figure 4.4 Modelling asymmetric gear as two symmetric gears (presented previously in Chapter 3)

4.3 Software for Tooth Root Stress Calculation of Asymmetric Tooth External Spur Gears

There is a growing trend in usage of asymmetric gears so it is needed for focusing on studies for asymmetric gear design and analysis. In-house software has been prepared within the scope of this thesis to calculate tooth root bending stress and all of the geometrical and stress related parameters.

Geometrical construction module of in-house software for symmetric and asymmetric spur gear was presented in previous chapter. In this section of thesis, a module of in-house software for calculation of tooth root bending stress of asymmetric spur gear is presented (Figure 4.5).

Module, teeth number of pinion and gear, pressure angles of drive and coast sides of gear, coefficients of addendum, dedendum, profile shift and root fillet radius are inputs and can be entered by user to software as shown in Figure 4.6-4.9. Tooth root critical thickness, bending moment arm, form factor (Y_F), stress concentration factor (Y_S), other related parameters and tooth root bending stress are calculated and presented in Figure 4.10.

Figure 4.5 shows the GearDraw software interface. The 'Inputs' section has 'Module (mm)' set to 3.000000. The 'Gear1 Parameters' and 'Gear2 Parameters' tables are as follows:

	#T	A^d	A^{c1}	a^p	b^p	x	rth_d	rth_c
Gear1 Parameters	1,000000	40,000000	30,000000	20,000000	1,000	1,250	0,000	0,200000
Gear2 Parameters	1,000000	40,000000	30,000000	20,000000	1,000	1,250	0,000	0,200000

Figure 4.5 Tooth root bending stress calculation of asymmetric spur gear (software)

Figure 4.6 shows the same GearDraw software interface, but with a red circle highlighting the 'Module (mm)' input field and the A^d and A^{c1} columns in the 'Gear1 Parameters' and 'Gear2 Parameters' tables.

Figure 4.6 Selections of module and teeth numbers (software)

Figure 4.7 shows the same GearDraw software interface, but with a red circle highlighting the A^d and A^{c1} columns in the 'Gear1 Parameters' and 'Gear2 Parameters' tables.

Figure 4.7 Selections of pressure angles of drive and coast sides (software)

Figure 4.8 shows the same GearDraw software interface, but with red circles highlighting the a^p , b^p , and x columns in the 'Gear1 Parameters' and 'Gear2 Parameters' tables.

Figure 4.8 Selections of coefficients of addendum, dedendum and profile shift

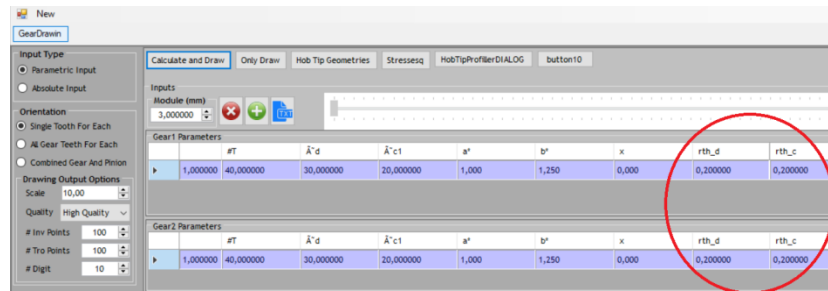


Figure 4.9 Selection of coefficients of root fillet radii (software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	1,206023	1,206023	mm
Radius of Root Fillet (Coast Side)	1,229115	1,229115	mm
AB1 (Drive Side)	0,866025	0,866025	-
AAA1 (Drive Side)	35,623027	35,623027	-
AAA2 (Drive Side)	326,483883	326,483883	-
AAA3 (Drive Side)	0,009446	0,009446	-
AAA4 (Drive Side)	3,083955	3,083955	-
AAA5 (Drive Side)	2699,999955	2699,999955	-
den (Drive Side)	122,617962	122,617962	mm
E (Drive Side)	-0,155279	0,571182	mm
G(Drive Side)	-1,050000	-1,050000	mm
H (Drive Side)	-0,966070	-0,978177	mm
Ang (Drive Side)	0,147263	0,136618	mm
Theta (Drive Side)	0,899935	0,910579	mm
Gama E (Drive Side)	0,026279	0,026279	Radian
Form Factor Pressure Angle in Radian (Drive Side)	0,559472	0,559472	Radian
Form Factor Pressure Angle in Degree (Drive Side)	32,055405	32,055405	Degree
Involute Function of Normal Pressure Angle (Drive Side)	0,053751	0,053751	Radian
Involute Function of Form Factor Pressure Angle (Drive Side)	0,066742	0,066742	Radian
Bending Moment Arm for Tooth Root Stress (Drive Side)	3,819953	3,819953	mm
Bending Moment Arm for Tooth Root Stress (Coast Side)	3,071520	3,071520	mm
Tooth Root Normal Chord 1	7,792081	7,792081	mm
Tooth Root Normal Chord 2	6,407793	6,407793	mm
Tooth Root Normal Chord	7,099937	7,099937	mm
Tooth Root Normal Chord (Hypotenuse)	7,139276	7,139276	mm
AAB4(Drive Side)	4,850618	4,850618	-
AAB5(Drive Side)	6,579398	6,579398	-
Form Factor (Drive Side)	1,356404	1,356404	-
L (Drive Side)	1,858645	1,858645	-
AAB6(Drive Side)	0,408587	0,408587	-
Notch Parameter (Drive Side)	2,943534	2,943534	-
Stress Correction Factor (Drive Side)	2,240916	2,240916	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,794331	0,794331	-
YPYS*YEps (Drive Side)	2,414438	2,414438	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	135,454330	135,454330	MPa

Figure 4.10 Tooth root bending stress parameters' results (software)

4.4 Case Studies of Tooth Root Bending Stress

Case studies of calculation of tooth root bending stress for symmetric and asymmetric tooth spur gears are presented here. Comparison of bending stress at root of symmetric gears is made by using an in-house software, KissSoft [14] and literature works. Whereas results are verified with literature works and commercial gear software for symmetric gears, there is no tool to compare asymmetric gear root stress. Therefore, asymmetric gear root stress results are compared with literature works in this chapter.

4.4.1 Case Studies of Symmetric Spur Gears

Case studies for tooth root bending stress calculations have been initially presented for symmetric tooth external spur gears. Results of some literature works have been firstly

compared to in-house software's results by analyzing same gear pairs in terms of root bending stress and related parameters.

4.4.1.1 Case S1

Ristic and Kramberger [179] calculated the tooth root critical section thickness at root, bending moment arm and tooth root bending stress for symmetric spur gears. Same case studies are performed in scope of this thesis for spur gears whose parameters are given in Table 4.3. Gear parameters such as tooth root critical section thickness and bending moment arm are calculated and compared in Table 4.4 and 4.5 respectively.

The biggest difference is 1.2 % and 4.6% between Ref [179] and current study for tooth root critical thickness and bending moment arm respectively. Software has always given higher value for root critical thickness and bending moment arm when results are compared to Ref [179].

Table 4.3 Gear pair parameters of Case S1

Parameter	Value		Unit
Module	24		mm
Pressure angle	20		Degree
Face width	350		mm
Torque	2528.8		kNm
	Pinion	Gear	
Teeth number	20	96	-
Profile shift coefficients	0.000	0.326	-

Table 4.4 Comparison of tooth root thickness of Case S1

Cutter tip radius (mm)	Coefficient of cutter tip radius	Root thickness(mm)		Difference (%)
		Ref [179]	in-house software	
4.56	0.190	55.64	56.11	-0.8
5.47	0.227	55.37	55.94	-1.0
6.38	0.266	55.14	55.76	-1.1
7.29	0.304	55.00	55.57	-1.0
8.22	0.342	54.86	55.37	-0.9
9.12	0.380	54.54	55.20	-1.2
10.03	0.418	54.42	55.01	-1.1
10.94	0.456	54.31	54.81	-0.9

Table 4.5 Comparison of bending moment arm of Case S1

Cutter tip radius (mm)	Coefficients of cutter tip radius	Bending moment arm (mm) Ref [179]	Bending moment arm (mm) in-house software	Difference (%)
4.56	0.190	26.17	26.90	-2.7
5.47	0.227	25.67	26.50	-3.1
6.38	0.266	25.26	26.10	-3.2
7.29	0.304	24.90	25.69	-3.1
8.22	0.342	24.54	25.27	-2.9
9.12	0.380	23.99	24.87	-3.5
10.03	0.418	23.54	24.46	-3.8
10.94	0.456	22.95	24.05	-4.6

4.4.1.2 Case S2

Conventional gear manufacturing processes are recommended in Spitas et al [83]. Gear pair parameters are presented in Table 4.6. Firstly, some of case studies presented for different clearance and dedendum alternatives were performed by using in-house software. Results for both studies are compared in Table 4.7. There is difference between stress values calculated by Spitas et al [83] and current study whereas there is no difference between results of in-house software and KissSoft [14] for same gear pair (Table 4.8).

Table 4.6 Gear pair parameters of Case S2

Parameter	Value	Unit
Module	1	mm
Pressure angle	20	Degree
Teeth number	20	-
Addendum coefficient	1.0	-

Table 4.7 Comparison of tooth root stress of Case S2 [83]

Standard	Profile	Cutter tip radius coefficient	Dedendum coefficient	Root stress (MPa)	
				Ref [83]	in-house software
ISO 53	A	0.38	1.25	2.51	2.86
ISO 53	B	0.30	1.25	2.65	3.02
ISO 53	C	0.25	1.25	2.73	3.13
Optimum	-	0.47	1.12	2.24	2.55

Table 4.8 Comparison of tooth root stress with KissSoft [14]

Standard	Profile	Cutter tip radius coefficient	Dedendum coefficient	Root stress (MPa)	
				in-house software	KissSoft
ISO 53	A	0.38	1.25	2.86	2.86
ISO 53	B	0.30	1.25	3.02	3.02
ISO 53	C	0.25	1.25	3.13	3.14
Optimum	-	0.47	1.12	2.55	2.55

4.4.1.3 Case S3

Sekar and Muthuveerappan [42] studied on symmetric and asymmetric spur gears for tooth root bending stress. In this section, symmetric case studies are compared to software results. Gear pair parameters are presented in Table 4.9. Results are given for tooth root thickness, bending moment arm and tooth root stress in Table 4.10, 4.11 and 4.12 respectively. Tooth root bending stress versus contact position along length of contact is presented in Figure 4.11 in reference study. Maximum root bending stress can be read as 26.3 MPa from that curve.

Gear parameters are entered to in-house software as shown in Figure 4.12 and 4.14 whereas outputs are presented in Figure 4.13 and 4.15 for Gear Pair 1 and 2 of Case S3 respectively. The biggest difference is % 3.0 between stress values calculated by Ref [42] and current study.

Table 4.9 General gear parameters of Case S3

Parameter	Gear Pair 1	Gear Pair 2	Unit
Module	1.0		mm
Teeth number (driver gear)	20		-
Teeth number (driven gear)	20		-
Pressure angle	20	30	degree
Addendum coefficient	1.00		-
Dedendum coefficient	1.25		-
Cutter tip radius coefficient (fully rounded)	0.47	0.11	-

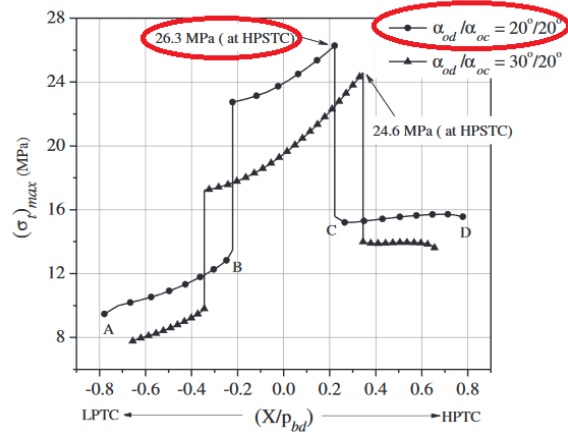


Figure 4.11 Tooth root stress of Case S3 - Gear Pair 1 [42]

Table 4.10 Comparison of tooth root thickness of Case S3 [42]

Gear Pair	Tooth root critical thickness (mm)	
	Ref [42]	In-house software
1	1.96	1.96
2	2.36	2.36

Table 4.11 Comparison of bending moment arm of Case S3 [42]

Gear Pair	Bending moment arm (mm)	
	Ref [42]	In-house software
1	1.05	1.05
2	1.39	1.39

Table 4.12 Comparison of tooth root stress of Case S3 [42]

Gear Pair	Tooth root stress (MPa)	
	Ref [42]	In-house software
1	26.30	26.83
2	26.70	27.56

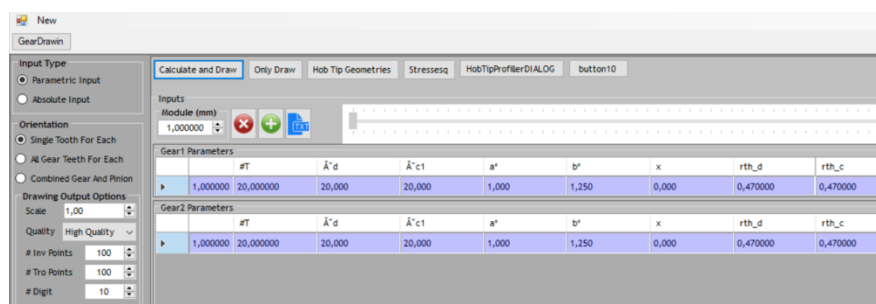


Figure 4.12 Gear parameters of Case S3 - Gear Pair 1 (software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	0,628801	0,628801	mm
Radius of Root Fillet (Coast Side)	0,628801	0,628801	mm
Tooth Root Normal Chord	1,956641	1,956641	mm
Tooth Root Normal Chord (Hypotenuse)	1,956641	1,956641	mm
AAB4(Drive Side)	3,597562	3,597562	-
AAB5(Drive Side)	5,945487	5,945487	-
Form Factor (Drive Side)	1,652643	1,652643	-
L (Drive Side)	1,861455	1,861455	-
AAB6(Drive Side)	0,408899	0,408899	-
Notch Parameter (Drive Side)	1,555852	1,555852	-
Stress Correction Factor (Drive Side)	1,727657	1,727657	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,731746	0,731746	-
YF*YS*YEps (Drive Side)	2,089281	2,089281	-
Pitchline Velocity	2,617994		m/sec
TangentialLoad	9,396926		Newton
Face Width	1,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	26,830118	26,830118	MPa

Figure 4.13 Tooth root stress parameters of Case S3 - Gear Pair 1 (software)

Figure 4.14 Gear parameters of Case S3 - Gear Pair 2 (software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	0,403469	0,403469	mm
Radius of Root Fillet (Coast Side)	0,403469	0,403469	mm
Tooth Root Normal Chord	2,360625	2,360625	mm
Tooth Root Normal Chord (Hypotenuse)	2,360625	2,360625	mm
AAB4(Drive Side)	4,825968	4,825968	-
AAB5(Drive Side)	7,122786	7,122786	-
Form Factor (Drive Side)	1,475929	1,475929	-
L (Drive Side)	1,691699	1,691699	-
AAB6(Drive Side)	0,389169	0,389169	-
Notch Parameter (Drive Side)	2,925411	2,925411	-
Stress Correction Factor (Drive Side)	2,156202	2,156202	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,822432	0,822432	-
YF*YS*YEps (Drive Side)	2,617307	2,617307	-
Pitchline Velocity	2,617994		m/sec
TangentialLoad	8,660254		Newton
Face Width	1,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	27,560401	27,560401	MPa

Figure 4.15 Tooth root stress parameters of Case S3 - Gear Pair 2 (Software)

4.4.1.4 Case S4

In this case, tooth root stress is calculated based on international gear standards [2, 52] (by using in-house software) and KissSoft [14]. Case studies of identical gear pair of

same module, teeth number, and root fillet radii with different pressure angle are carried out. Gear pair parameters are given in Table 4.13

Tooth root stress calculated based on ISO 6336-3 [2] for symmetric spur gears of different cutter tip radius coefficient values are calculated by in-house software and KissSoft. The results are presented in Table 4.14. The biggest difference is 0.15% and 0.04% for tooth root critical thickness and tooth root stress respectively. There is almost no difference between bending moment arm values. Screen images of KissSoft related to these calculations and results are presented in Figures of 4.16-4.23. Entering input parameters and taking results from in-house software are presented in Figure 4.24 - 4.27.

Table 4.13 General gear parameters of Case S4

Parameter	Pair 1	Pair 2	Pair 3	Unit
Module	3.0			mm
Teeth number (driver/driven)	40 /40			-
Pressure angle	20	25	30	deg
Root fillet radius coefficient	0.10			-
Face width	20			mm
Torque	160.43			Nm

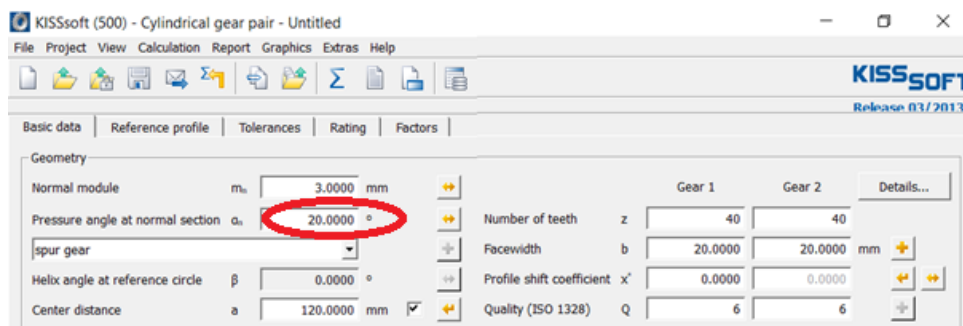


Figure 4.16 KissSoft - Gear pair parameters input (Case S4 - Gear Pair 1)

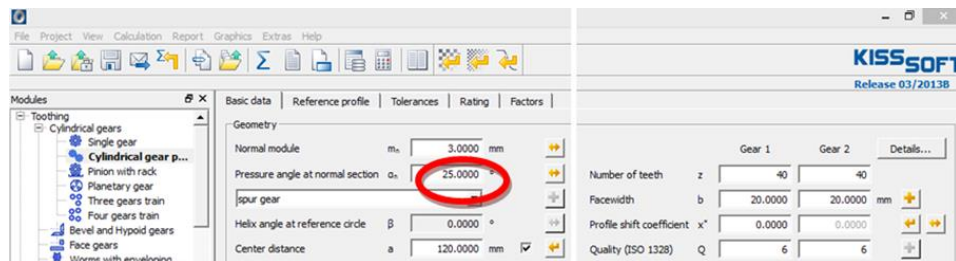


Figure 4.17 KissSoft - Gear pair parameters input (Case S4 - Gear Pair 2)

The screenshot shows the KissSoft interface for gear pair parameters. The 'Basic data' tab is active. The 'Geometry' section has the following values: Normal module $m_n = 3.0000$ mm, Pressure angle at normal section $\alpha_n = 30.0000^\circ$ (circled in red), Helix angle at reference circle $\beta = 0.0000^\circ$, and Center distance $a = 120.0000$ mm. The 'Details' section shows: Number of teeth $z = 40$ for both Gear 1 and Gear 2, Facewidth $b = 20.0000$ mm, Profile shift coefficient $x^* = 0.0000$, and Quality (ISO 1328) $Q = 6$.

Figure 4.18 KissSoft - Gear pair parameters input (Case S4 - Gear Pair 3)

The screenshot shows the KissSoft interface for gear reference parameters. The 'Gear 1' and 'Gear 2' tabs are active. The 'Input of coefficient' section has the following values: Dedendum coefficient $h_{fp}^* = 1.2500$, Root radius coefficient $\rho_{fp}^* = 0.1000$, Addendum coefficient $h_{ap}^* = 1.0000$, Protuberance height coefficient $h_{pfp}^* = 0.0000$, Protuberance angle $\alpha_{pfp} = 0.0000^\circ$, Tip form height coefficient $h_{fsp}^* = 0.0000$, Ramp angle $\alpha_{rp} = 0.0000^\circ$, and Tip alteration $k'm_n = 0.0000$ mm.

Figure 4.19 KissSoft - Gear reference parameters input (Case S4 - Gear Pair 1-2-3)

The screenshot shows the KissSoft interface for gear rating parameters. The 'Strength' tab is active. The 'Calculation method' is set to 'ISO 6336:2006 Method B'. The 'Driving gear' is 'Gear 1' and the 'Working flank' is 'right flank'. The 'Application factor' K_A is 1.2500. The 'Reference gear' is 'Gear 1'. The 'Power' P is 42.0005 kW, 'Torque' T_1 is 160.4300 Nm, 'Speed' n_1 is 2500.0000 1/min, and 'Required service life' H is 20000.0000 h.

Figure 4.20 KissSoft - Gear rating parameters input (Case S4 - Gear Pair 1-2-3)

3. TOOTH ROOT STRENGTH

Calculation of Tooth form coefficients according method: B
(Calculate tooth shape coefficient YF with addendum mod. x)

		----- GEAR 1 -----	----- GEAR 2 -----
Tooth form factor	[YF]	1.40	1.40
Stress correction factor	[YS]	2.36	2.36
Working angle (°)	[alfFn]	19.04	19.04
Bending lever arm (mm)	[hF]	3.18	3.18
Tooth thickness at root (mm)	[sFn]	6.42	6.42
Tooth root radius (mm)	[roF]	1.04	1.04
$(hF^* = 1.059/1.059 \quad sFn^* = 2.139/2.139 \quad roF^* = 0.346/0.346 \quad dsFn = 113.11/113.11 \quad alfsFn = 30.00/30.00)$			
Contact ratio factor	[Yeps]	1.000	1.000
Helical load factor	[Ybet]	1.000	1.000
Deep tooth factor	[YDT]	1.000	1.000
Gear rim factor	[YB]	1.000	1.000
Effective facewidth (mm)	[hfft]	20.00	20.00
Nominal stress at tooth root (N/mm²)	[sigF0]	147.18	147.18
Tooth root stress (N/mm²)	[sigF]	250.21	250.21

Figure 4.21 KissSoft - Root stress parameters' calculations (Case S4 - Gear Pair 1)

3. TOOTH ROOT STRENGTH

Calculation of Tooth form coefficients according method: B
(Calculate tooth shape coefficient YF with addendum mod. x)

	----- GEAR 1 -----	----- GEAR 2 -----
Tooth form factor	[YF] 1.26	1.26
Stress correction factor	[YS] 2.48	2.48
Working angle (°)	[alfFen] 24.95	24.95
Bending lever arm (mm)	[hF] 3.53	3.53
Tooth thickness at root (mm)	[sFn] 7.11	7.11
Tooth root radius (mm)	[roF] 1.03	1.03
(hF* = 1.178/1.178 sFn* = 2.370/2.370 roF* = 0.342/0.342 dsFn = 113.10/113.10 alfsFn = 30.00/30.00)		
Contact ratio factor	[Yeps] 1.000	1.000
Helical load factor	[Ybet] 1.000	1.000
Deep tooth factor	[YDT] 1.000	1.000
Gear rim factor	[YB] 1.000	1.000
Effective facewidth (mm)	[beff] 20.00	20.00
Nominal stress at tooth root (N/mm²)	[sigF0] 139.07	139.07
Tooth root stress (N/mm²)	[sigF] 237.22	237.22

Figure 4.22 KissSoft - Root stress parameters' calculations (Case S4 - Gear Pair 2)

3. TOOTH ROOT STRENGTH

Calculation of Tooth form coefficients according method: B
(Calculate tooth shape coefficient YF with addendum mod. x)

	----- GEAR 1 -----	----- GEAR 2 -----
Tooth form factor	[YF] 1.14	1.14
Stress correction factor	[YS] 2.60	2.60
Working angle (°)	[alfFen] 30.55	30.55
Bending lever arm (mm)	[hF] 3.93	3.93
Tooth thickness at root (mm)	[sFn] 7.87	7.87
Tooth root radius (mm)	[roF] 1.01	1.01
(hF* = 1.309/1.309 sFn* = 2.622/2.622 roF* = 0.337/0.337 dsFn = 113.09/113.09 alfsFn = 30.00/30.00)		
Contact ratio factor	[Yeps] 1.000	1.000
Helical load factor	[Ybet] 1.000	1.000
Deep tooth factor	[YDT] 1.000	1.000
Gear rim factor	[YB] 1.000	1.000
Effective facewidth (mm)	[beff] 20.00	20.00
Nominal stress at tooth root (N/mm²)	[sigF0] 131.51	131.51
Tooth root stress (N/mm²)	[sigF] 228.23	228.23

Figure 4.23 KissSoft - Root stress parameters' calculations (Case S4 - Gear Pair 3)

Table 4.14 Comparison of root stress for Case S4 by in-house software and KissSoft

Gear Pair	Tooth root thickness (mm)		Bending moment arm (mm)		Tooth root stress (MPa)	
	in-house software	KissSoft	in-house software	KissSoft	in-house software	KissSoft
1	6.41	6.42	3.18	3.18	147.25	147.18
2	7.11	7.11	3.53	3.53	139.12	139.07
3	7.86	7.87	3.93	3.93	131.55	131.51

Calculate and Draw

Inputs

Module (mm)

3,000

Teeth Number [Driver, Driven]

40 40

Pressure Angle (deg) [Drive, Coast]

20,00 20,00

Addendum Coef. (mn)

1,000

Dedendum Coef. (mn)

1,250

Hob Tip Radius Coef. (mn) [Drive, Coast]

0,1000 0,1000

Profile Shift (mn) [Driver, Driven]

0,0000 0,0000

Face Width (mm)

20,00

Power (Watt)

42000,000

Angular Velocity Of Driver (rpm)

2500,0

Calculate and Draw

Inputs

Module (mm)

3,000

Teeth Number [Driver, Driven]

40 40

Pressure Angle (deg) [Drive, Coast]

25,00 25,00

Addendum Coef. (mn)

1,000

Dedendum Coef. (mn)

1,250

Hob Tip Radius Coef. (mn) [Drive, Coast]

0,1000 0,1000

Profile Shift (mn) [Driver, Driven]

0,0000 0,0000

Face Width (mm)

20,00

Power (Watt)

42000,000

Angular Velocity Of Driver (rpm)

2500,0

Calculate and Draw

Inputs

Module (mm)

3,000

Teeth Number [Driver, Driven]

40 40

Pressure Angle (deg) [Drive, Coast]

30,00 30,00

Addendum Coef. (mn)

1,000

Dedendum Coef. (mn)

1,250

Hob Tip Radius Coef. (mn) [Drive, Coast]

0,1000 0,1000

Profile Shift (mn) [Driver, Driven]

0,0000 0,0000

Face Width (mm)

20,00

Power (Watt)

42000,000

Angular Velocity Of Driver (rpm)

2500,0

Figure 4.24 Parameter input (in-house software) (Case S4 - Gear Pair 1-2-3)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Bending Moment Arm for Tooth Root Stress (Drive Side)	3,177898	3,177898	mm
Bending Moment Arm for Tooth Root Stress (Coast Side)	2,324782	2,324782	mm
Tooth Root Normal Chord 1	6,414059	6,414059	mm
Tooth Root Normal Chord 2	6,414059	6,414059	mm
Tooth Root Normal Chord	6,414059	6,414059	mm
Tooth Root Normal Chord (Hypotenuse)	6,470545	6,470545	mm
AAB4(Drive Side)	4,295455	4,295455	-
AAB5(Drive Side)	6,008110	6,008110	-
Form Factor (Drive Side)	1,398713	1,398713	-
L (Drive Side)	2,018334	2,018334	-
AAB6(Drive Side)	0,425613	0,425613	-
Notch Parameter (Drive Side)	3,085708	3,085708	-
Stress Correction Factor (Drive Side)	2,362310	2,362310	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,687692	0,687692	-
YF*YS*YEps (Drive Side)	2,272268	2,272268	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	147,246073	147,246073	MPa

Figure 4.25 Root stress parameters (in-house software) (Case S4 - Gear Pair 1)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Bending Moment Arm for Tooth Root Stress (Drive Side)	3,534770	3,534770	mm
Bending Moment Arm for Tooth Root Stress (Coast Side)	1,633822	1,633822	mm
Tooth Root Normal Chord 1	7,107077	7,107077	mm
Tooth Root Normal Chord 2	7,107077	7,107077	mm
Tooth Root Normal Chord	7,107077	7,107077	mm
Tooth Root Normal Chord (Hypotenuse)	7,356912	7,356912	mm
AAB4(Drive Side)	5,086456	5,086456	-
AAB5(Drive Side)	6,409829	6,409829	-
Form Factor (Drive Side)	1,260176	1,260176	-
L (Drive Side)	2,010619	2,010619	-
AAB6(Drive Side)	0,424822	0,424822	-
Notch Parameter (Drive Side)	3,464126	3,464126	-
Stress Correction Factor (Drive Side)	2,477392	2,477392	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,746261	0,746261	-
YF*YS*YEps (Drive Side)	2,329790	2,329790	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	139,124610	139,124610	MPa

Figure 4.26 Root stress parameters (in-house software) (Case S4 - Gear Pair 2)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Bending Moment Arm for Tooth Root Stress (Drive Side)	3,927296	3,927296	mm
Bending Moment Arm for Tooth Root Stress (Coast Side)	0,900319	0,900319	mm
Tooth Root Normal Chord 1	7,863566	7,863566	mm
Tooth Root Normal Chord 2	7,863566	7,863566	mm
Tooth Root Normal Chord	7,863566	7,863566	mm
Tooth Root Normal Chord (Hypotenuse)	8,426047	8,426047	mm
AAB4(Drive Side)	5,950140	5,950140	-
AAB5(Drive Side)	6,764282	6,764282	-
Form Factor (Drive Side)	1,136827	1,136827	-
L (Drive Side)	2,002285	2,002285	-
AAB6(Drive Side)	0,423965	0,423965	-
Notch Parameter (Drive Side)	3,887234	3,887234	-
Stress Correction Factor (Drive Side)	2,596737	2,596737	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,794331	0,794331	-
YF*YS*YEps (Drive Side)	2,344898	2,344898	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	131,552982	131,552982	MPa

Figure 4.27 Tooth root stress parameters (in-house software) (Case S4 - Gear Pair 3)

4.4.1.5 Case S5

Case studies are performed for a gear pair with same module, teeth number, pressure angle and different root fillet radii. General parameters are given in Table 4.15. Results of tooth root stress of in-house software and KissSoft are presented in Table 4.16.

The biggest difference is 0.2 % between results for tooth root bending stress values calculated by KissSoft and in-house software. This difference is so small and it is due to round up of digits when calculating of stress by using related factors.

Table 4.15 General gear parameters of Case S5

Gear Parameters		
Parameter	Value	Unit
Module	3	mm
Pressure angle	20	Degree
Teeth number	40/40	-
Torque	160.43	Nm

Table 4.16 Comparison of root bending stress for Case S5 with KissSoft [14]

Gear Pair	Coefficients of cutter tip radius	Tooth Root Stress (MPa)	
		in-house software	KissSoft [14]
1	0.10	147.50	147.18
2	0.15	140.55	140.50
3	0.20	134.39	134.35
4	0.25	128.68	128.65
5	0.30	123.13	123.33
6	0.35	118.19	118.35
7	0.38	115.50	115.50
8	0.40	113.53	113.65
9	0.45	109.12	109.21
10	0.47	107.40	107.50

4.4.2 Case Studies of Asymmetric Spur Gears

In this part, case studies for tooth root bending stress calculations of asymmetric spur gears are presented based on literature works.

4.4.2.1 Case A1, A2 and A3

Sekar and Muthuveerappan [42] studied on asymmetric spur gears for bending tooth root stress in addition to symmetric ones. Their study includes calculations of tooth

root bending stress based on ISO standard [52] and finite element analyses. Gear pair parameters are presented in Table 4.17 and 4.18.

Root stress value for Case A1 is illustrated in Figure 4.28. Some screen shots from in-house software are presented in Figure 4.29, 4.30, 4.31 for Gear pair 1 and Figure 4.32, 4.33, 4.34 for Gear pair 2. Results are given in Table 4.19, 4.20 and 4.21.

Table 4.17 General gear parameters of Case A1, A2 and A3 [42]

Parameter	Value	Unit
Module	1	mm
Teeth number	20/20	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Cutter tip radius coefficient	Fully rounded	-
Center distance	20	mm

Table 4.18 Pressure angle values of Case A1, A2 and A3 [42]

Gear Pair	Pressure Angle		Unit
	Drive Side	Coast Side	
A1	30	20	Degree
A2	25	20	Degree
A3	30	25	Degree

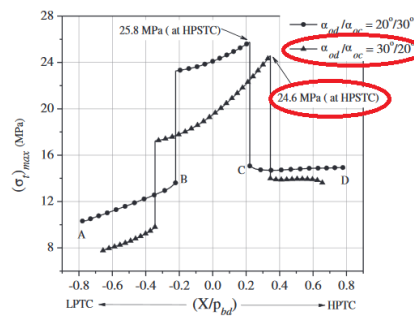


Figure 4.28 Tooth root stress of Case A1 [42]

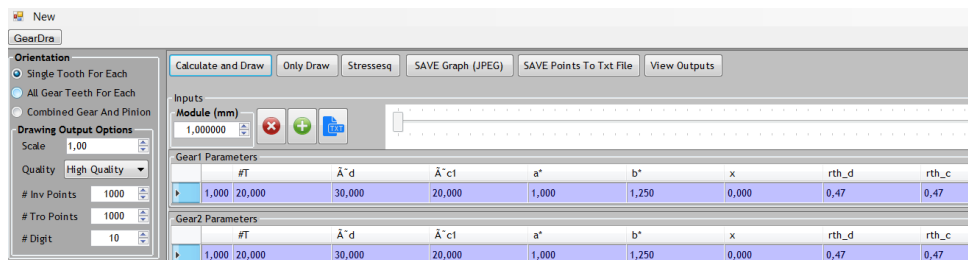


Figure 4.29 Input parameters of Case A1 (in-house software)

FrmStressCalculation

Input Parameters

Power (Watt) Angular Velocity (rpm) Facewidth (mm)

Figure 4.30 Loading parameters of Case A1 (in-house software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	0,620985	0,620985	mm
Radius of Root Fillet (Coast Side)	0,628801	0,628801	mm
den (Drive Side)	21,001355	21,001355	mm
E (Drive Side)	-0,207644	0,001338	mm
G (Drive Side)	-0,780000	-0,780000	mm
H (Drive Side)	-0,869353	-0,890252	mm
Ang (Drive Side)	0,256630	0,238617	mm
Theta (Drive Side)	0,790568	0,808580	mm
Gama E (Drive Side)	0,047654	0,047654	Radian
Form Factor Pressure Angle in Radian (Drive Side)	0,601067	0,601067	Radian
Form Factor Pressure Angle in Degree (Drive Side)	34,435581	34,435581	Degree
Involute Function of Normal Pressure Angle (Drive Side)	0,053751	0,053751	Radian
Involute Function of Form Factor Pressure Angle (Drive Side)	0,084637	0,084637	Radian
Load Direction Angle in Radian (Drive Side)	0,553413	0,553413	Radian
Load Direction Angle in Degree (Drive Side)	31,708201	31,708201	Degree
AAB1(Drive Side)	20,359442	20,359442	-
AAB2(Drive Side)	19,345018	19,345018	-
AAB3(Drive Side)	-1,578834	-1,578834	-
Bending Moment Arm for Tooth Root Stress (Drive Side)	1,296429	1,296629	mm
Tooth Root Normal Chord	2,149233	2,149233	mm
Nominal Tooth Root Stress (Drive Side)	25,048913	25,048913	MPa

Figure 4.31 Output parameters of Case A1 (in-house software)

New

GearDra

Orientation

☒ Single Tooth For Each

☐ All Gear Teeth For Each

☐ Combined Gear And Pinion

Drawing Output Options

Scale

Quality

Inv Points

Tro Points

Digit

Calculate and Draw Only Draw Stressesq SAVE Graph (JPEG) SAVE Points To Txt File View Outputs

Inputs

Module (mm)

Gear1 Parameters

#T	$\tilde{A}^*d$	$\tilde{A}^*c1$	a*	b*	x	rth_d	rth_c
1,000 20,000	25,000	20,000	1,000	1,250	0,000	0,47	0,47

Gear2 Parameters

#T	$\tilde{A}^*d$	$\tilde{A}^*c1$	a*	b*	x	rth_d	rth_c
1,000 20,000	25,000	20,000	1,000	1,250	0,000	0,47	0,47

Figure 4.32 Input parameters of Case A2 (in-house software)

FrmStressCalculation

Input Parameters

Power (Watt) Angular Velocity (rpm) Facewidth (mm)

Figure 4.33 Loading parameters of Case A2 (in-house software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	0,625050	0,625050	mm
Radius of Root Fillet (Coast Side)	0,628801	0,628801	mm
den (Drive Side)	20,765507	20,765507	mm
E (Drive Side)	-0,096909	0,001338	mm
G (Drive Side)	-0,780000	-0,780000	mm
H (Drive Side)	-0,880427	-0,890252	mm
Ang (Drive Side)	0,247074	0,238617	mm
Theta (Drive Side)	0,800124	0,808580	mm
Gama E (Drive Side)	0,059254	0,059254	Radian
Form Factor Pressure Angle in Radian (Drive Side)	0,509687	0,509687	Radian
Form Factor Pressure Angle in Degree (Drive Side)	29,202907	29,202907	Degree
Involute Function of Normal Pressure Angle (Drive Side)	0,029975	0,029975	Radian
Involute Function of Form Factor Pressure Angle (Drive Side)	0,049261	0,049261	Radian
Load Direction Angle in Radian (Drive Side)	0,450433	0,450433	Radian
Load Direction Angle in Degree (Drive Side)	25,807883	25,807883	Degree
AAB1(Drive Side)	20,134381	20,134381	-
AAB2(Drive Side)	19,392644	19,392644	-
AAB3(Drive Side)	-1,589695	-1,589695	-
Bending Moment Arm for Tooth Root Stress (Drive Side)	1,165717	1,165717	mm
Tooth Root Normal Chord	2,047283	2,047283	mm
Nominal Tooth Root Stress (Drive Side)	26,097868	26,097868	MPa

Figure 4.34 Output parameters of Case A2 (in-house software)

Table 4.19 Comparison of tooth root thickness of Case A1, A2 and A3

Gear Pair	Tooth root critical thickness (mm)		
	Ref [42] based on ISO	Ref [42] (FEA)	software
A1	2.192	2.307	2.149
A2	2.067	2.149	2.047
A3	2.263	2.374	2.241

Table 4.20 Comparison of bending moment arm of Case A1, A2 and A3

Gear Pair	Bending moment arm (mm)		
	Ref [42] based on ISO	Ref [42] (FEA)	software
A1	1.290	1.356	1.297
A2	1.168	1.228	1.166
A3	1.338	1.406	1.341

Table 4.21 Comparison of tooth root stress of Case A1, A2 and A3

Gear Pair	Tooth Root Stress (MPa)		
	Ref [42] based on ISO	Ref [42] (FEA)	software
A1	23.7	24.6	25.1
A2	24.5	25.3	26.1

4.4.2.2 Case A4

Kapelevich [55] modeled an asymmetric spur gear tooth as comparable symmetric gear teeth in his study. In that method, pressure angle of drive and coast sides of asymmetric tooth are summed up and divided into two. Then, this value is taken as pressure angle of comparable symmetric gear teeth. Author calculated tooth root stress of comparable symmetric gear teeth based on ISO standard [2] and analyzed gear by using finite element method. For asymmetric gear, only finite element analyses were done and a conversion coefficient was searched to calculate root stress of an asymmetric gear by modelling it as comparable symmetric gears. Gear pair parameters are given in Table 4.22 and 4.23 but root radius coefficient value is missing. Author states that a root optimisation is carried out to minimize root stress so no root radius value is given in Ref [55]. In our study, no optimisation for gear root has been performed. Instead, maximum usable cutting tool tip radius is preferred for thicker root section and lesser root bending stress by obtaining trochoid root form. Screen views from software for Case Study A4 are presented in Figure 4.35-4.37. Tooth root stress

values obtained from reference paper, current study and KissSoft for comparable symmetric gear are presented in Table 4.24 respectively. But root stresses are given based on only reference paper and current study for asymmetric gear owing to lack of asymmetric gear module of KissSoft.

Table 4.22 General gear parameters of Case A4

Parameter	Value	Unit
Module	5	mm
Teeth number (pinion/gear)	20/49	-
Torque (pinion)	900	Nm
Rotational speed (pinion)	1000	rpm
Center distance	172.5	mm

Table 4.23 Gear pair parameters of Case A4

	Asymmetric gear		Comparable symmetric gear	Unit
Pressure Angle (drive/coast)	35/20		27.5	Degree
Addendum coefficient	0.921	1.080	0.951	-
Dedendum coefficient	1.151	1.081	1.127	-
Profile shift coefficient	0.06	-0.06	0.06 -0.06	-
Root radius coefficient	-		0.327	-

Table 4.24 Comparison of tooth root stress of Case A4

Case A4 – Gear pairs	Tooth Root Stress (MPa)		
	Ref [55]	in-house software	KissSoft
Asymmetric	295	353	-
Comparable symmetric	309	336	336

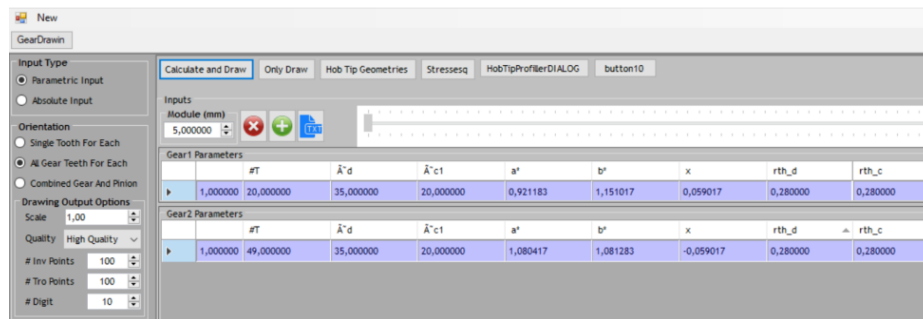


Figure 4.35 Gear pair parameters of Case A4 (in-house software)

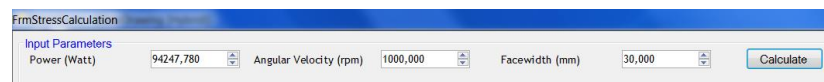


Figure 4.36 Loading parameters of Case A4 (in-house software)

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	2,215337	2,046280	mm
Radius of Root Fillet (Coast Side)	2,284423	2,075717	mm
Bending Moment Arm for Tooth Root Stress (Drive Side)	6,022198	6,831404	mm
Bending Moment Arm for Tooth Root Stress (Coast Side)	4,173818	4,979159	mm
Tooth Root Normal Chord 1	12,877452	13,482254	mm
Tooth Root Normal Chord 2	9,775553	10,454103	mm
Tooth Root Normal Chord	11,326502	11,968179	mm
Tooth Root Normal Chord (Hypotenuse)	11,476330	12,110661	mm
AAB4(Drive Side)	4,203549	4,693325	-
AAB5(Drive Side)	5,825743	6,623045	-
Form Factor (Drive Side)	1,385911	1,411163	-
L (Drive Side)	1,880792	1,751935	-
AAB6(Drive Side)	0,411034	0,396380	-
Notch Parameter (Drive Side)	2,556383	2,924374	-
Stress Correction Factor (Drive Side)	2,124548	2,184634	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,836732	0,836732	-
YF*YS*YEps (Drive Side)	2,463701	2,579537	-
Pitchline Velocity	5,235988		m/sec
Tangential Load	18000,000000		Newton
Face Width	30,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	353,332037	369,944783	MPa

Figure 4.37 Output parameters of Case A4 (in-house software)

4.5 Asymmetric Root Profile

Gears are critical power transmission elements and are expected to carry high loads for long service life. Their load carrying capacity and service lives are affected significantly owing to stresses occur at root section (root bending stress) and tooth flanks (contact stress). By focusing on service life based on root bending stress, there are some methods to decrease maximum stress level at gear tooth root section or improve strength of root section (stress concentration region of root) in terms of bending. Some of these methods are increasing pressure angles of tooth profiles symmetrically for both flanks, using asymmetric tooth profile form for gears and asymmetric root profile applications [176-178]. These all have advantages in terms of gear performance with some disadvantages.

Increasing pressure angles symmetrically for both flanks of gear tooth profile provides larger tooth thickness at root section but smaller radius of curvature at root. Maximum allowable limit of root fillet radius also decreases by increasing pressure angle. Smaller fillet radius yields to higher stress concentration at gear tooth root section. Radius of curvature at root section refers to transition from the involute tooth profile curve to the trochoid root curve. Small radius of curvature at gear root section is highly unwanted because of its additive effect on gear root stress concentration.

Increase in pressure angle without changing any other gear parameter yields stronger root region (Table 4.10) because of increase in root critical section thickness but small

radius of curvature at transition region from involute to trochoid or vice versa. When pressure angle increases, bending moment arm also increases (Table 4.11).

As presented in this chapter for Case S3, increasing pressure angle from 20 degree to 30 degree (by keeping all others of gear parameters constant) yields decrease of ultimate allowable limit of root fillet radius coefficient from 0.47 to 0.11 (Table 4.9), increase of tooth root thickness from 1.96mm to 2.36mm (Table 4.10), increase of bending moment arm from 1.05mm to 1.39mm (Table 4.11).

Tooth flanks of a gear tooth are functionally different for many gear drives; tooth load on one flank is significantly higher and is applied for longer periods of time than for the opposite one. So the geometry of the drive side does not have to be symmetric to the coast side. This allows for the design of gears with asymmetric teeth. Asymmetric gears are good solutions for unidirectional applications of non-reverse motions. Different pressure angles are used to make gear tooth asymmetric. Larger pressure angle yields thicker root critical section thickness but lesser thickness at tooth tip. Therefore it is not possible to increase pressure angle of one flank of tooth beyond a limit for avoiding pointed tip. Contact ratio is also badly affected by increasing drive side pressure angle. These negative aspects and restrictions make asymmetry limited in tooth.

As in the case of tooth profile, symmetry in gear root section is not a must; it is a result of gear generation processes. Gear root section (root fillet radii) can be in asymmetric form as an alternative to the symmetric trochoid root fillet radii. Symmetric tooth profile gear with asymmetric root is preferred by some researchers [19, 83, 175-178] to reduce bending stress at tooth root.

Akpolat [175] applied asymmetric root to symmetric tooth profile spur gears to improve bending performance. It was stated that using symmetric tooth gear with asymmetric root instead of symmetric root, 10% less bending stress at gear root was obtained.

Akpolat et al [176] studied the effect of asymmetric fillet on tooth root bending stress by keeping all other gear parameters constant. Symmetric and asymmetric tooth profile spur gears with symmetric and asymmetric root fillet were investigated in terms of root stress. When results were compared to reference condition (symmetric tooth with

symmetric root fillet coefficient of 0.38), almost 10–11% reduction in bending stress was achieved.

Şahin et al [177] stated that dual asymmetry for both tooth profile and root fillet of spur gears to decrease root bending stress further. Stress reductions of 9-15% were obtained for spur gear in terms of root bending stress by employing dual asymmetry to both tooth and fillet profiles.

Şahin et al [178] investigated performance of asymmetric tooth gear under bending loading of gear tooth. Symmetric, circular and asymmetric root fillet was applied to symmetric tooth spur gears to decrease root bending stress under same loading conditions. For further decrease of bending stress of spur gear, asymmetric root fillet was applied to the asymmetric tooth gear. Up to 15% stress reduction was taken by applying different level of asymmetry in root and tooth.

In this thesis, symmetric tooth spur gears with symmetric and asymmetric root fillets were firstly analyzed to decrease root bending stress. Then asymmetric tooth spur gears with symmetric root fillet were evaluated for bending stress at tooth root. In addition, a combined method of applying asymmetry to both of tooth profile (involute) and root fillet profile (trochoid) is preferred in this study. This method is called as dual asymmetry.

Many case studies were carried out to evaluate the effect of asymmetry on tooth profile and root region together and separately. Abbreviations for these studies are given in Table 4.25.

Table 4.25 Case study alternatives for gear tooth and root profiles

Case Study Alternatives	Abbreviations
Symmetric Tooth - Circular Root	ST-CR
Symmetric Tooth - Symmetric Root	ST-SR
Symmetric Tooth - Asymmetric Root	ST-AR
Asymmetric Tooth - Symmetric Root	AT-SR
Asymmetric Tooth - Asymmetric Root	AT-AR

4.5.1 Symmetric Tooth – Asymmetric Root Profile Spur Gears

Though gear tooth root region is theoretically an inactive part of tooth; maximum bending stress occurs in the same region during meshing. Form of fillet and radius of

curvature are important parameters to obtain a smooth transition from active profile to inactive gear root. Fillet radius (Figure 4.38) and fillet region of gear tooth are results of gear generating methods. They are highly dependent on gear cutting tool geometry and tool tip radius. Considering only the rack cutter (which represents also hob cutter) for manufacturing gears with involute toothing, the tooth profile for such a rack becomes a straight line as seen in Figure 4.39.

ISO [180] recommends maximum value as 0.38 of root fillet coefficient for standard basic rack (Figure 4.39) with standard addendum and dedendum coefficients (Table 4.26).

Symmetric tooth profile gear with different fillet profile types such as symmetrical and asymmetrical root fillets were studied to evaluate the effects of root profile and cutter tip radius coefficients on bending stress (cases ST-CR, ST-SR and ST-AR).

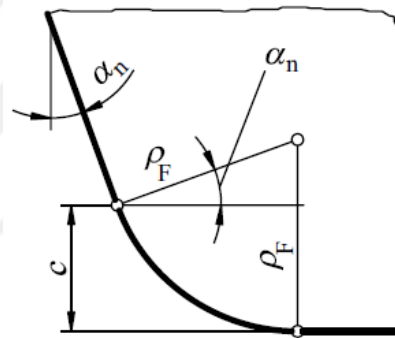


Figure 4.38 Fillet (cutter tip) radius of basic rack tooth profile [180]

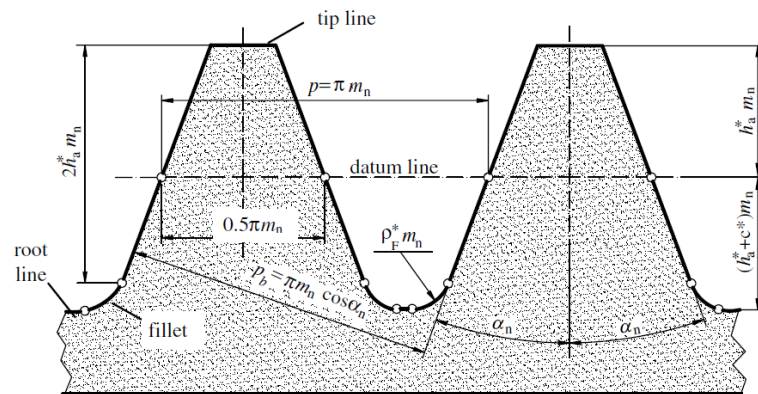


Figure 4.39 Basic tooth rack profiles [180]

Table 4.26 Basic rack tooth profiles [180]

Symbol	Types of basic rack tooth profile (m is the module)				
	Explanation	A	B	C	D
α_n	Normal pressure angle	20°	20°	20°	20°
h_{aP}	Addendum coefficient for basic rack	1 m	1 m	1 m	1 m
c_P	Clearance coefficient for basic rack	0.25 m	0.25 m	0.25 m	0.40 m
h_{fP}	Dedendum coefficient for basic rack	1.25 m	1.25 m	1.25 m	1.40 m
ρ_{fP}	Root fillet radius	0.38 m	0.30 m	0.25 m	0.39 m

Table 4.27 General gear parameters

Parameter	Value	Unit
Module	3	mm
Teeth Number (driver/driven)	40/40	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Torque	160.43	Nm
Face width	20	mm
Center distance	120	mm
Pitch diameter	120	mm
Rim thickness ((=Root Dia.-Bore Dia.)/2)	31.25 ((=112.5-50.0) / 2)	Mm
Rim backup ratio (Rim thickness/tooth height)	4.63	-

Bending stress values evaluated by in-house software for symmetric gears of D20C20, D25C25 and D30C30 with different fillet geometries of symmetrical and asymmetrical ones are presented in Table 4.28. It is easier to compare effect of root fillet type and coefficient of tool tip radius on bending stress when case studies are grouped based on pressure angle values for D20C20, D25C25 and D30C30 in itself.

With symmetric tooth profile of D20C20, root bending stress value decreases from 115.36MPa to 107.42MPa for the root shapes of symmetric with cutting tool tip radius coefficient (ρ_f^*) of 0.38, (suggested by ISO 53 [180]), and symmetric with cutting tool tip radius coefficient of 0.47 respectively. Here it is seen that symmetric root with cutting tool tip radius coefficient of 0.47 has resulted in a better performing fillet with possibly larger curvature and less stress concentration. It is clear from results in Table 4.29 that increasing cutting tool tip radius coefficient from 0.38 to 0.47 provides a stronger tooth root section (with 6.9% less stress) for the current gear data. This stress reduction will certainly improve the gear service life in bending.

Larger cutting tool tip radius coefficient of 0.47 is suggested by Spitas et al. [83], Akpolat [175], Akpolat et al. [176], Sahin et al [177] and Sahin et al [178] as a means to reduce bending stress. No interference problem has been experienced with the cutting tool tip radius coefficients up to 0.47 as stated in reference studies [83, 175-178] too.

Table 4.28 Root bending stress of different root shapes with symmetric tooth profile

Case	α		Tooth Profile (DXXCYY)	Load (N)	ρ_f^*		σ_b in-house software(MPa)
	α_d	α_c			ρ_{fd}^*	ρ_{fc}^*	
ST-SR0	20		D20C20	2845.402	0.30		123.13
ST-SR1	20		D20C20	2845.402	0.38		115.36
ST-SR2	20		D20C20	2845.402	0.47		107.42
ST-SR3	25		D25C25	2950.215	0.31		117.17
ST-SR4	30		D30C30	3087.442	0.11		130.45
ST-AR1	20		D20C20	2845.402	0.58	0.36	98.26
ST-AR2	25		D25C25	2950.215	0.43	0.20	106.57
ST-AR3	25		D25C25	2950.215	0.53	0.10	98.68
ST-AR4	30		D30C30	3087.442	0.12	0.10	129.23

Table 4.29 Root bending stress reduction of symmetric tooth profile (D20C20) in cases of ST-SR1, ST-CR1, ST-SR2 and ST-AR1

Case	α		Tooth Profile (DXXCYY)	ρ_f^*		σ_b in-house software (MPa)	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-SR1	20		D20C20	0.38		115.36	Ref. value
ST-SR2	20		D20C20	0.47		107.42	6.90
ST-AR1	20		D20C20	0.58	0.36	98.26	14.82

Such an improvement obtained by cutter tip radius change naturally causes an inclination to further increase the tip radii but increasing radius beyond 0.47 is not limitless. There is a limit for the radius beyond which cutting tool tip start to sharpen at the midpoint and a tool form dis-order occurs as shown in Figure 4.40. Such problems regarding cutting tool form dis-order were also explained by [175-178].

Due to the limitation of tool tip sharpening and form dis-order, it is not possible to make tool tip radius larger beyond the limit of 0.47 for symmetric fillets. However any further increase in tool tip radius without harm to tool itself and the tooth would be likely to help reduce bending stress further. An alternative means of increasing the tip

radius (without harming the tool) is to change the form of the cutter and fillet by shifting from symmetric tool tip to asymmetric tool tip shape.

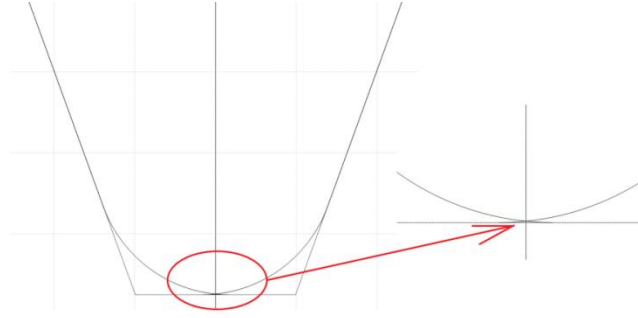


Figure 4.40 Cutting tool tip with coefficient of 0.55 for both sides [175-178]

Asymmetrical cutting tool tip (without any dis-order as seen in Figure 4.41) will help increase critical tooth thickness and decrease stress concentration more to decrease bending stress at tooth critical section. An asymmetric cutter tip radius of basic rack is given in Figure 4.41. From this figure, for a standard thickness tooth with zero backlash, (tooth thickness coefficient, $C_s = 0.5$) following equations can be written:

$$\pi m_n (1 - C_s) = \frac{P_c}{2} = \frac{\pi m_n}{2} \quad (4.6)$$

$$t' = \left(\frac{\pi m_n}{2} - 2h_{aP} \tan \alpha \right) \quad (4.7)$$

$$A = \frac{\rho_{fd}}{\cos \alpha} \quad \text{and} \quad B = \frac{\rho_{fc}}{\cos \alpha} \quad (4.8)$$

$$A + B = t' + \rho_{fd} \tan \alpha + \rho_{fc} \tan \alpha \quad (4.9)$$

$$\frac{\rho_{fd} + \rho_{fc}}{\cos \alpha} = t' + (\rho_{fd} + \rho_{fc}) \tan \alpha \quad (4.10)$$

$$(\rho_{fd} + \rho_{fc})_{max} = \frac{t' \cos \alpha}{1 - \sin \alpha} \quad (4.11)$$

$$(\rho_{fd} + \rho_{fc})_{max} = \left(\frac{\pi m_n}{2} - 2h_{aP} \tan \alpha \right) \frac{\cos \alpha}{1 - \sin \alpha} \quad (4.12)$$

$$(\rho_{fd}^* + \rho_{fc}^*)_{max} = \left(\frac{\pi}{2} - 2h_{aP}^* \tan \alpha \right) \frac{\cos \alpha}{1 - \sin \alpha} \quad (4.13)$$

Equations of 4.12 and 4.13 provide the maximum value for summation of cutter tip radii and radii coefficients respectively for drive and coast side profiles. Eq. 4.13 is a

general one for both symmetric and asymmetric fillets (cutter tip radii) and can be compared with the symmetric ($\rho_{fd}^* = \rho_{fc}^* = \rho_f$) case studies.

Coefficients of asymmetric cutting tool tip of 0.58 and 0.36 for drive (ρ_{fd}^*) and coast side (ρ_{fc}^*) (Case ST-AR1) are the maximum limitations for D20C20 of symmetric gears (obtained by equations given above). There is a significant reduction of 10.2 % when Case ST-AR1 is preferred instead of Case ST-SR1 (in Table 4.29). Beyond these limits of root fillet coefficients (stated for Case ST-AR1), any value will cause interference as shown in Figure 4.42. These limit values, however, change with pressure angle of the tooth [175]. Increase in pressure angle causes a thicker tooth at root section while a reduced tooth space near root land. This then will decrease maximum limit of the fillet radius hence the tool tip radius. The upper limit for usable root fillet radius (or cutting tool tip radius) depends upon pressure angles of both flanks of involute tooth profile as shown in Figure 4.43.

For D25C25, having a pressure angle of 25degrees, 0.31 (Case ST-SR3) is the maximum limit for coefficient of symmetric cutter tip radius with a root stress of 117.17MPa. A stress reduction of almost 15.78% (98,68MPa), with reference to Case ST-SR3 can be obtained if an asymmetric trochoidal fillet (coefficients of 0.53 and 0.10 for drive and coast sides) is used as in Case ST-AR3, (Table 4.30).

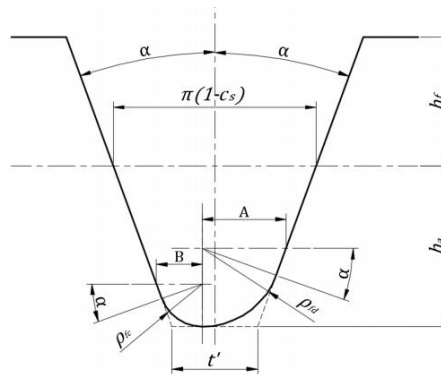


Figure 4.41 Asymmetric cutting tool tip [175-178]

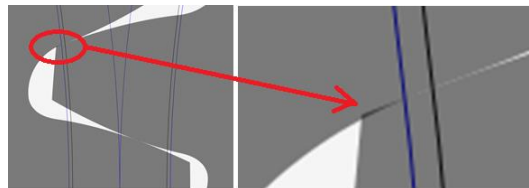


Figure 4.42 Interference simulations of symmetric tooth gear of D20C20 with asymmetric root fillet ($\rho_{fd}^* = 0.70$, $\rho_{fc}^* = 0.24$) (Interference occurs) [175-178]



Figure 4.43 Root fillets and root regions for cases of ST-SR2, ST-SR3 and ST-SR4 (Having different symmetric pressure angles with limiting symmetric cutter tip radii coefficients)

Table 4.30 Root bending stress values of symmetric tooth profile (D25C25) in cases of ST-CR2, ST-SR3, ST-AR2 and ST-AR3

Case	α		Tooth Profile (DXXCYY)	ρ_f^*		σ_b in-house software (MPa)	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-SR3	25		D25C25	0.31		117.17	Ref. value
ST-AR2	25		D25C25	0.43	0.20	106.57	9.05
ST-AR3	25		D25C25	0.53	0.10	98.68	15.78

Because of decreasing upper limit of root fillet radius with pressure angle's increase, only small asymmetry in root is allowed for 30 degree of pressure angle for symmetric tooth profile spur gears. Whereas 0.11 of root fillet coefficient is maximum limit for symmetric root, 0.12 and 0.10 can be used for asymmetric root. Therefore, there is not too much stress reduction from ST-SR4 to ST-AR4 as given in Table 4.31.

Table 4.31 Root bending stress values of symmetric tooth profile (D30C30) cases of ST-CR3, ST-SR4 and ST-AR4

Case	α		Tooth Profile (DXXCYY)	ρ_f^*		σ_b in-house software (MPa)	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-SR4	30		D30C30	0.11		130.45	Ref. value
ST-AR4	30		D30C30	0.12	0.10	129.23	0.94

With symmetric profile spur gears (having different pressure angles of 20, 25 and 30degrees), an analysis of the effect of different root fillet forms have been performed above. Trochoidal roots (symmetrical and asymmetrical) generated by different cutter tip designs were compared in terms of maximum tooth root bending stress under a constant transmitted torque. In both trochoidal roots maximum allowable root radii were employed. Followings are the summary of bending stress reductions obtained for symmetric tooth profiles by different root fillet designs:

- 14.82 % for D20C20, (for design change from ST-SR1 to ST-AR1)
- 15.78 % for D25C25, (for design change from ST-SR3 to ST-AR3)
- 0.94 % for D30C30, (for design change from ST- SR4 to ST-AR4).

4.5.2 Asymmetric Tooth – Asymmetric Root Profile Spur Gears

In previous section, some case studies for symmetric tooth profile with different fillet design alternatives were presented. In this section, asymmetric tooth with symmetric fillet was first analyzed to reduce root stress. Next, asymmetric tooth gears with asymmetrical fillets were analyzed. General gear pair parameters for asymmetric tooth cases are same as the symmetric tooth profile cases, and given in Table 4.27. Case studies of asymmetric tooth profile gears are performed for constant torque and reference/nominal center distance value for interference free meshing. Bending stress calculations for asymmetric tooth gears with symmetric and asymmetric fillet at root using in-house software were performed and results are presented in Table 4.32.

When case studies are grouped based on pressure angles, it is easier to compare effects of root fillet type and coefficient of tool tip radius on bending stress. Therefore it is better to present stress values in different groups based on pressure angles of both sides of asymmetric tooth profile gear. There are significant stress reductions like:

- 13.27 % for D25C20, (for design change from AT-SR1 to AT-AR1),
- 16.17 % for D30C20, (for design change from AT-SR2 to AT-AR2),
- 10.32 % for D30C25, (for design change from AT-SR3 to AT-AR3),
- 9.26 % for D25C30, (for design change from AT-SR4 to AT-AR6),
- 14.49 % for D20C25, (for design change from AT-SR5 to AT-AR4) and
- 14.87 % for D20C30 (for design change from AT-SR6 to AT-AR5).

These results show that there is significant stress reduction (up to of 16.17 % for greater pressure angle of drive side and almost 14.87 % for greater pressure angle of coast side) for asymmetric tooth profile spur gears with asymmetric root fillets.

All calculations and analysis up to here are based on constant torque loading. Under constant torque, tooth load is calculated based on base radius of drive side flank (hence pressure angle of drive side). Each gear pair having different pressure angles on drive

side will carry a different tooth load. Increase in pressure angle decreases base radius hence increases load on tooth (under constant torque).

Table 4.32 Root bending stress values of asymmetric tooth (AT) with symmetric root (SR) and asymmetric root (AR) profiles calculated by in-house software

Case	α		Tooth Profile DXXCYY	Tooth Force (N)	ρ_f^*		σ_b in-house software (MPa)
	α_d	α_c			ρ_{fd}^*	ρ_{fc}^*	
AT-SR1	25	20	D25C20	2950.21	0.395		116.15
AT-SR2	30	20	D30C20	3087.44	0.30		125.55
AT-SR3	30	25	D30C25	3087.44	0.21		127.37
AT-SR4	25	30	D25C30	2950.21	0.21		119.70
AT-SR5	20	25	D20C25	2845.40	0.395		107.93
AT-SR6	20	30	D20C30	2845.40	0.30		109.74
AT-AR1	25	20	D25C20	2950.21	0.59	0.22	100.74
AT-AR2	30	20	D30C20	3087.44	0.56	0.10	105.25
AT-AR3	30	25	D30C25	3087.44	0.35	0.10	114.22
AT-AR4	20	25	D20C25	2845.40	0.58	0.19	92.29
AT-AR5	20	30	D20C30	2845.40	0.48	0.10	93.42
AT-AR6	25	30	D25C30	2950.21	0.32	0.10	108.62

Compared with constant torque loading, under constant tooth load/force condition, comparison of bending stress for pairs having different pressure angles of drive flanks will not be a reliable method. Because tooth load value will be different for each case of pressure angles of drive flanks (Table 4.32) and it is not suggested to analyze the bending stress under constant tooth load.

Therefore, it is important that enough attention is given to the conditions of tooth form and the fillet geometry of the drive flanks of meshing pairs to clearly understand what causes change in bending stress results. For example, gear pairs of D20C20 with different types of fillet (symmetric and asymmetric) geometries will usually show the effect of fillet form on the bending stress. Whereas gear pairs of D20C20 and D25C20 with a fixed type of fillet (symmetric or asymmetric) geometry will usually show the effect of tooth form on the bending stress despite a slight difference in the coefficients of cutter tip radius due to tooth root space differences for different pressure angles.

4.6 Conclusion

In this chapter, in-house software has been prepared for root stress calculation of symmetric and asymmetric tooth profile spur gears. Root stress for spur gears with both profiles can be evaluated through software by depending on ISO standard [2] for symmetric tooth spur gears and some modified formulae of ISO [2] in the scope of this thesis for asymmetric tooth spur gears

Case studies have been carried out to verify results by comparing with those of KissSoft commercial gear software (for symmetric tooth gears) and literature works (for both symmetric and asymmetric tooth spur gears).

Gear parameters such as module, teeth number, pressure angle, face width; coefficients of addendum, dedendum, profile shift and cutting tool tip radius and loading parameters namely power, torque, load and speed have been preferred for identical values presented in published papers without changing of even digits after dot.

Works presented in Ref [55, 83 and 179] depends on constant torque (instead of constant load) whereas Sekar and Muhtuveerappan [42] prefers constant load for their analyses of symmetric and asymmetric tooth gears.

In practical applications, checking of loads on gears during meshing is extremely difficult so shaft torques is generally kept under control for analyses in our study regardless of symmetric or asymmetric tooth. Therefore case studies were conducted under constant torque.

Load and highest point of single tooth contact (HPSTC) vary by depending on pressure angle. These changes have been taken into consideration in this thesis. But many studies in literature (almost all of them), constant/identical load is applied and evaluation is carried out based on this idea (constant load transmission). Actually, increasing pressure angle yields decreased base circle radius and increased load on gear tooth (under constant torque). Therefore, constant/identical loading in calculations and analyses for symmetric and asymmetric tooth spur gears does not give accurate results owing to difference from real conditions.

In this study, all calculations are based on an already manufactured gearbox whose operating conditions and geometrical limitations predetermined and not prone to be

changed. Therefore, any gear pair, symmetric or asymmetric, for a constant center distance and input-output values of power and speed can be used in these analyses.

Comparison of gear pairs of symmetric or asymmetric tooth in terms of bending stress gives completely different idea about advantages and disadvantages for usage. This gives an answer about whichever is more suitable for intended application. Because increase in pressure angle of drive side flank results in decrease of base circle. As a result, load on gear tooth and bending moment arm is going to increase in addition to increase of critical section thickness at gear root. Thickness increase in root provides less stress while increase of load and moment arm yields higher value of bending stress.

Several case studies were performed by using literature data and results were compared for symmetric and asymmetric tooth profile external spur gears. When tooth root critical thickness, bending moment arm and tooth root bending stress were computed by using in-house software and compared to those of reference studies. Percentage differences are presented in Table 4.33. Results obtained via commercial gear software, KissSoft [14] are very close to in-house software's based on ISO standards.

Table 4.33 Maximum difference of tooth critical thickness, bending moment arm and tooth root bending stress for symmetric gears

Difference (%) of results of in house software with	Parameters		
	Tooth Critical Thickness (mm)	Bending Moment Arm (mm)	Root Stress (MPa)
Ristic [179]	1.2	4.5	-
Spitas [83]	-	-	13.0
Sekar and Muthuveerappan [42]	0	0	3.0
KissSoft [14]	0.15	0	0.23

Asymmetric tooth profile spur gears were designed and evaluated to reduce root bending stress in addition to symmetric ones. Many effective parameters on root stress of asymmetric gears such as pressure angle, tooth thickness, bending moment arm, form factor, stress concentration factor and other influential factors on stress were taken into consideration.

More efforts to reduce root bending stress further were made by applying dual asymmetry to tooth and root profiles for symmetrical and asymmetrical tooth spur gears.

By interpreting results, it can be concluded that asymmetrical root application to symmetric tooth profile spur gears yields reduction of tooth root bending stress under constant torque. Stress reductions of 14.82 and 15.78 for symmetrical tooth spur gears were obtained for D20C20 and D25C25 respectively when bending stress of ST-AR1 compared to ST-SR1 and bending stress of ST-AR3 compared to ST-SR3. But less stress reduction (almost zero) was achieved for symmetric spur gear with pressure angle of 30degree (D30C30) owing to decreased allowable ultimate limit of root fillet radius with increased pressure angle.

Considering asymmetric spur gear with symmetric and asymmetric root fillets, the lowest and highest stress reductions of 10.32% and 16.17% were obtained for asymmetric gear with greater pressure angle of drive side. The situation is not too much different for asymmetric tooth gears with greater pressure angle of coast side. The lowest and highest reduction of root stresses for asymmetric gears with greater pressure angle of coast side is 9.26% and 14.87 % respectively.

These all shows that asymmetric root fillet application to symmetric and asymmetric tooth spur gears provides significant reductions in gear tooth root bending stress. Applying asymmetry to the tooth and root profiles is called as dual asymmetry in this thesis and reference studies [177-178]. Dual asymmetry is also a very good solution to reduce stress at tooth root for spur gears. Keeping all mentioned above in mind and taking limitations into consideration, asymmetric root application to spur gears regardless of symmetric or asymmetric tooth can yield stress reduction. Dual asymmetry also provides reduction in gear root stress by depending on relationship between pressure angle and root fillet usable limit.

CHAPTER 5

FINITE ELEMENT ANALYSIS OF SYMMETRIC AND ASYMMETRIC TOOTH PROFILE SPUR GEARS FOR TOOTH ROOT BENDING STRESS

5.1 Introduction

In previous chapter, tooth root bending stresses were calculated for symmetric and asymmetric tooth spur gears based on ISO standards by using in-house software prepared in scope of this thesis. In this chapter, finite element analyses (FEA) are conducted to evaluate root bending stresses of spur gear pairs for different case studies.

5.2 Finite Element Methods

Finite element method is one of the numerical techniques which are commonly used in engineering applications by computers. There are many applications of finite element analyses such as linear and nonlinear, static and dynamic, stress and deflection analysis, heat transfer and forced vibrations. For analyzing a mechanical component, FEA divides the structure into small and simple elastic parts (elements). Each element is defined by material and geometrical properties. By using polynomial functions with matrix operations, partial differential equations are developed, which are assembled to obtain a system of algebraic equations determining values of the unknowns for global elastic structure. Some advantages of the discretization to finite elements are; precise representation of complex geometry, inclusion of dissimilar materials and tracing the local effects.

Computer software programs are useful tools to make simulations of models and evaluate mechanical behaviors under working conditions. Finite Element Method (FEM) is a numerical technique solving engineering problems by simulating the models of studied parts by also modelling the loading and boundary conditions.

The idea of FEM has been got up by the need to complex analysis of elasticity and structure problems, because its principal is dividing large problem into small and simple parts facilitate access to assembled solution.

Over years a lot of FEA studies of gears have been achieved and published with widely different modeling settings, with two dimensional (2D) and three dimensional (3D) analyses, and most of these studies have produced the promised results.

Finite Element Method which has capability to give accurate information of tooth root bending stress. This method is used in this study to find maximum bending stress at critical section of gear root.

In this thesis, Finite Element Method, through MSC programs [181-182] are used to simulate spur gear models and present gear root stress values. Number of 2D models is analyzed to calculate tooth root bending stress for symmetric and asymmetric spur gears.

The 2D element shapes which are commonly used in general-purpose commercial FEM software (Figure 5.1). Many geometric shapes and different number of nodes and elements are used for specific applications.

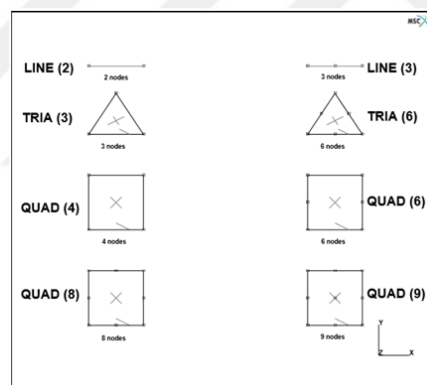


Figure 5.1 2D elements in Marc [181]

5.3 Material and Method

In this study, four different software have been used to conduct case studies.

- The first one is in-house software which have been already developed and explained in Chapter 3.
- The second one is Solidworks [174].
- The third one is SimXpert [182].
- Marc [181] is the last one.

The first one (in-house software) is used to generate complete geometry of gear including involute and root regions for finite element analysis (Chapter 3) and to calculate gear root bending stress values based on gear standards (Chapter 4).

Generating the X and Y coordinates of gear tooth geometry including involute and root (trochoid) portions are performed by using in-house software. Gear pair parameters namely module, teeth numbers, drive and coast side pressure angles, coefficients of addendum, dedendum, profile shift and root fillet radii can be entered as inputs to in-house software by user as shown in Figure 4.5 – 4.9 (presented in previous chapter).

Gear models are firstly generated and checked graphically for driver and driven gears via in-house software as shown in Figure 5.2.

In-house software gives chance to save the coordinates of gear tooth profile in a text file for drive and coast side of gear tooth profile of driver and driven gears separately (Figure 5.3 and 5.4). Points obtained by using generated coordinates are converted to 2D gear model in Solidworks [174] to use them in meshing and FEA analyses (Figure 5.5-5.7).

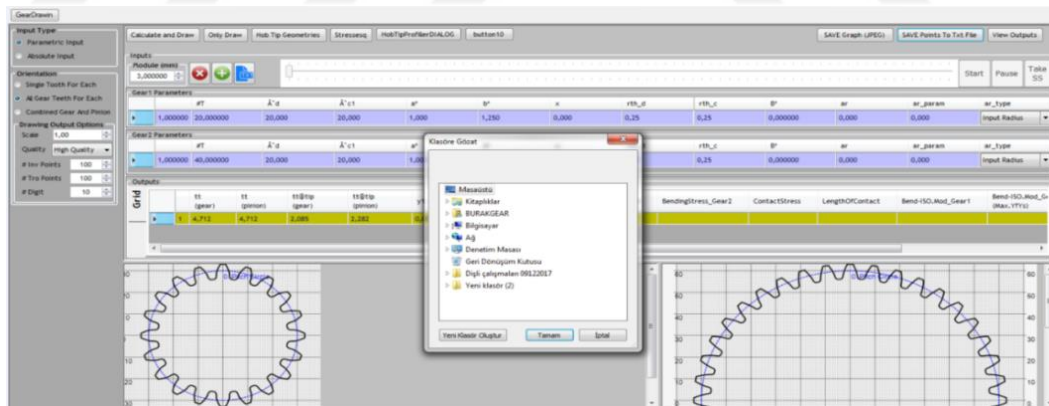


Figure 5.2 Gear model generation and saving file (in-house software)

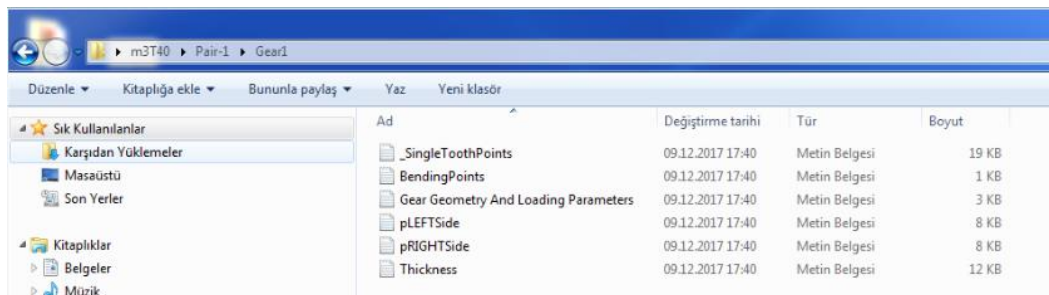


Figure 5.3 Coordinates of left and right sides of gear tooth profile saved in text files

The screenshot displays a software interface with two main data tables for gear tooth profiles. The left table, titled 'pLEFTSide - Not Deften', and the right table, titled 'pRIGHTSide - Not Deften', both have columns: Dosya, Düzen, Başım, Görünüm, and Yarım. Each table contains 16 rows of numerical data representing the coordinates of the gear tooth profile.

Dosya	Düzen	Başım	Görünüm	Yarım
4.1240169407	-56.0986183811	0.000000	-1.1409343781	-62.9896679523
4.1075288595	-56.099403229	0.000000	-1.1670988935	-62.9345622290
4.0910588382	-56.1014806211	0.000000	-1.1931207003	-62.8794474738
4.0746088730	-56.1032391294	0.000000	-1.2189994518	-62.8243238468
4.0581809580	-56.1052157020	0.000000	-1.2447348042	-62.7691913084
4.0417770850	-56.1074101941	0.000000	-1.2703264065	-62.7140506193
4.0253992421	-56.1098224619	0.000000	-1.2957739027	-62.6589013405
4.0090494129	-56.1124523634	0.000000	-1.3210769311	-62.6037438331
3.9927295758	-56.1132997587	0.000000	-1.3462351249	-62.5485782586
3.9764417029	-56.1143645110	0.000000	-1.3712481115	-62.4934047787
3.9601877595	-56.1216464868	0.000000	-1.3961155124	-62.4382235554
3.9439697025	-56.1251455570	0.000000	-1.4208369435	-62.3830347511
3.9277894802	-56.1288015981	0.000000	-1.4454120145	-62.3278385283
3.9116490313	-56.1327944928	0.000000	-1.4698403290	-62.2726350500
3.8955502835	-56.1369441312	0.000000	-1.4941214843	-62.2174244793
3.8794951531	-56.1413104125	0.000000	-1.5182550711	-62.1622069798
3.8634855439	-56.14584932457	0.000000	-1.5422406735	-62.1069827154
3.8475233465	-56.1506925517	0.000000	-1.5660778688	-62.0517518503
3.8316104367	-56.1557082647	0.000000	-1.5897662273	-61.9965143491
3.8157486754	-56.1609403339	0.000000	-1.6133051121	-61.941279766
3.7999399071	-56.1663887256	0.000000	-1.6366946789	-61.8860212982
3.7841859591	-56.1720534250	0.000000	-1.6599338761	-61.8307656795
3.7684886404	-56.1779344389	0.000000	-1.6830224439	-61.7755042867
3.7528497408	-56.1840317979	0.000000	-1.7059599149	-61.7202372861

Figure 5.4 Coordinates of gear tooth profile saved in a text file (in-house software)

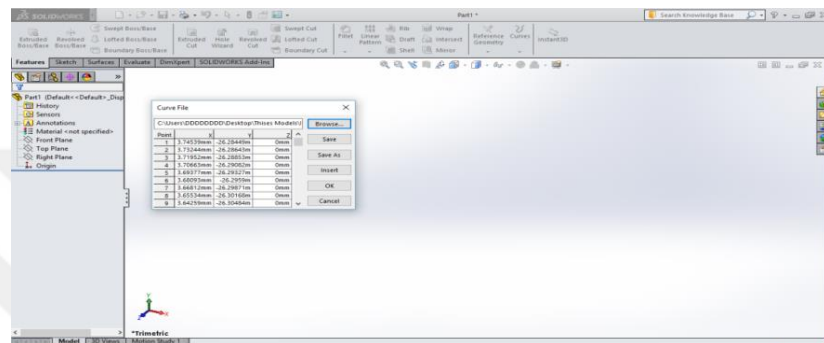


Figure 5.5 Importing coordinates of a single tooth profile into Solidworks [174]

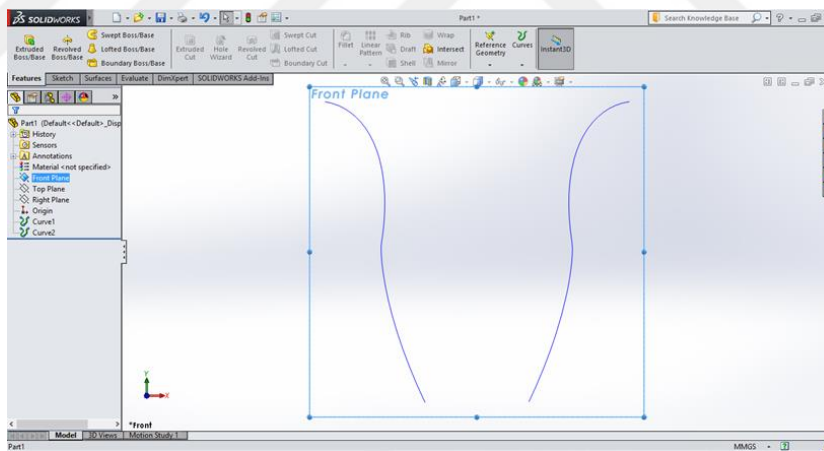


Figure 5.6 Macro geometric profile of a single tooth created in Solidworks [174]

Gear model meshing is done by SimXpert [182] for normal and mapped mesh models as shown in Figure 5.8-5.14.

Meshing process is executed by using SimXpert [182] which has superior and flexible meshing tools providing high quality meshes with automatic and manual steps. The used structural elements are mostly linear rectangular and triangular type.

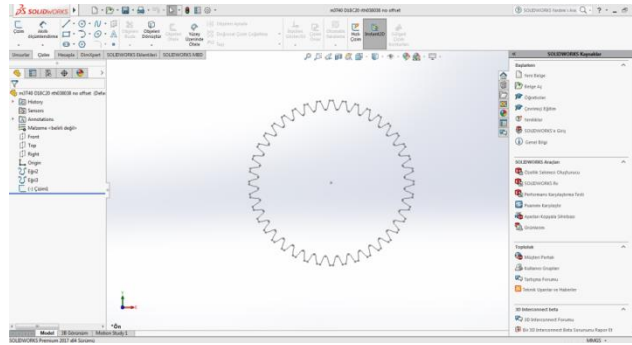


Figure 5.7 Whole gear tooth model created in Solidworks

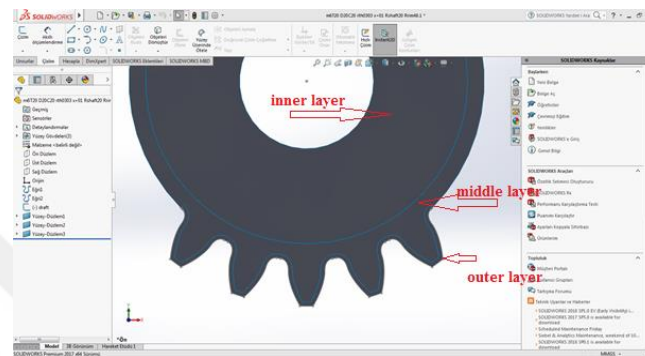


Figure 5.8 Generating three different surfaces for mapped meshing of a 5Teeth model

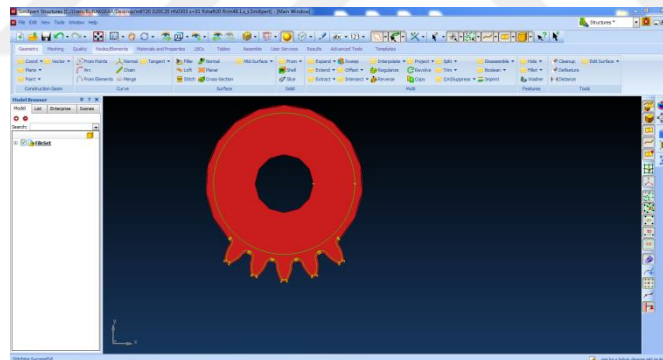


Figure 5.9 Obtaining 3Layer 5Teeth gear model to SimXpert [182]

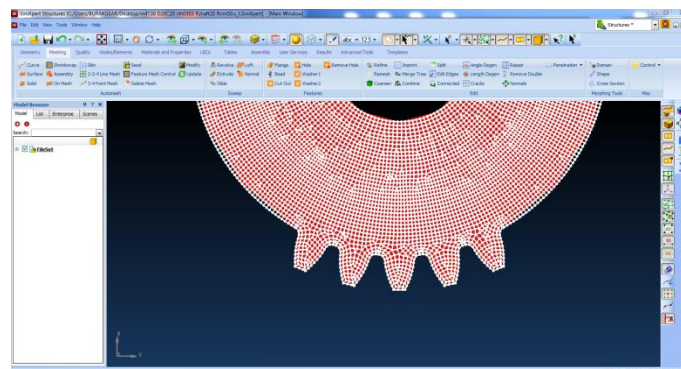


Figure 5.10 Conventional meshing (SimXpert) [182]

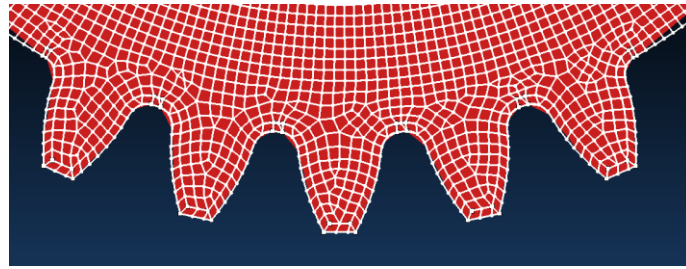


Figure 5.11 Conventional meshing (close view) (SimXpert) [182]

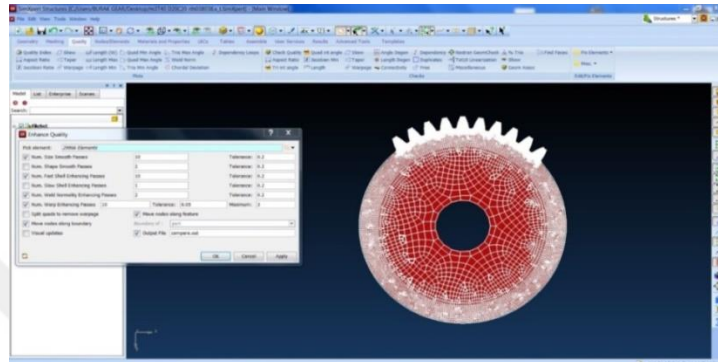


Figure 5.12 Mapped meshing (SimXpert) [182]

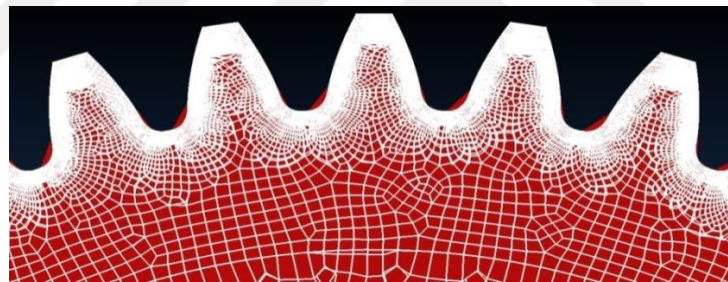


Figure 5.13 Mapped meshing (close view) (SimXpert) [182]

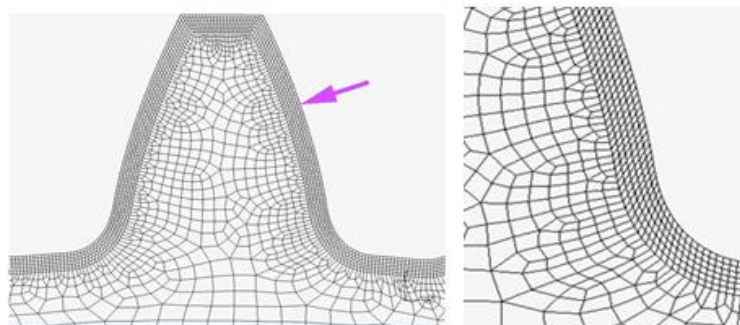


Figure 5.14 Mapped meshing of One-Tooth (close view) (Marc) [181]

Mapped meshing (Figure 5.14-5.16) is more suitable for gear root bending stress to shorten required analysis time. The most critical meshing surface is the most outer one because maximum root stress occurs at surface of the tooth root region. Model surfaces

are divided into surfaces which have different mesh density according to their criticality for precision of finite element analysis. This procedure is usually applied to reduce time consumption without losing the accuracy of the important required results. FEA are carried out by using Marc [181] which is one of the products of MSC Software Company.

MSC Marc [181] is a finite element solver technology proceeds advanced simulations. It has capability to simulate all types of nonlinearities including contact.

Two-dimensional gear models which have been already meshed by SimXpert [182] are imported to Marc workspace as Nastran bulk data in order to complete analysis and post processing.

Boundary and loading conditions are presented in Figure 5.15-5.17. Gear center is fixed and gear tooth is loaded at HPSTC. Element types are selected in Marc (Plane strain with full integration), type 11 with 8 nodes for rectangular element, and type 6 with 6 nodes for triangular element. Each node has six degrees of freedom. Thickness of body is given as face width of the gear. The material is specified for all models as standard steel of Elastic-Plastic Isotropic property with Young's modulus of 210 GPa and a Poisson's ratio of 0.3. By using Links (RBE2), nodes in the inner circle are fixed where gears are assumed as fitted to the shaft, the remaining gear body nodes are free for rotations and displacements. The analyses are conducted using a plain strain analysis.

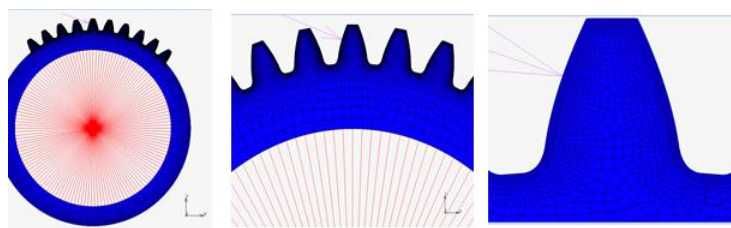


Figure 5.15 Applying boundary conditions to gear model (fixing center node, Marc)

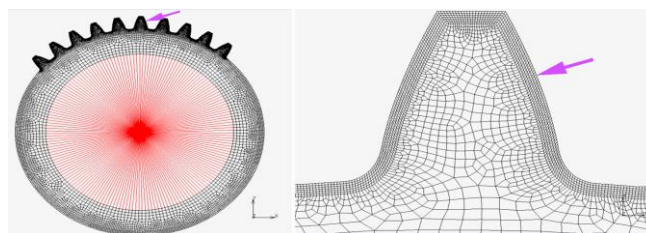


Figure 5.16 Loading at HPSTC (Marc)

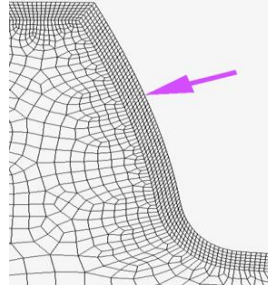


Figure 5.17 Loading at HPSTC (close view)

5.4 Case Studies

In this section, series of finite element analysis have been firstly performed for symmetric and asymmetric tooth profile spur gears based on published works. Then, some additional case studies were carried out for gear pairs (to be used for experiments in back to back test rig). Abbreviation of case studies is presented in Table 5.1. Cases presented for finite element works are entitled as sequences of cases given in Chapter 4 to prevent confusion and provide continuity.

Gear models of different teeth number (3Teeth, 5Teeth, 10Teeth, etc.) can be used for tooth root bending stress analysis instead of whole teeth wheel. Three teeth, five teeth or any other model can be selected for finite element analyses. But same number of teeth model is preferred as in reference articles during thesis' studies. 5teeth model prepared by using SimXpert is presented in Figure 5.18.

One gear can be analyzed by applying a point load (or pressure) perpendicular to tooth surface (Figure 5.19) or two gears can be brought together for simulation of load transmission (Figure 5.20).

Table 5.1 Abbreviation for case studies of symmetric and asymmetric spur gears

Symmetric tooth	Asymmetric tooth
S6	A5
S7	A6
S8	A7
S9	A8
S10	A9
S11	A10
S12	A11

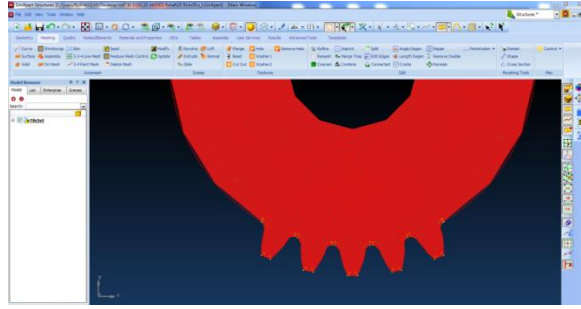


Figure 5.18 Five teeth model for FEA

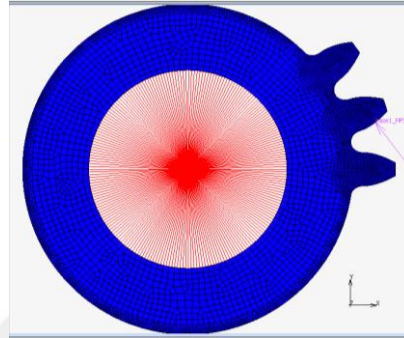


Figure 5.19 Tooth loading at HPSTC

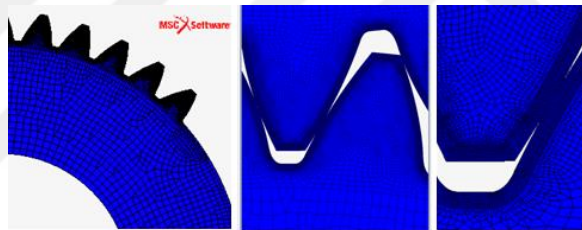


Figure 5.20 Gear pair meshing simulation in FEA

5.4.1 Case Studies of Symmetric Tooth Profile Spur Gear

Many researchers have published papers for finite element analyses of symmetric tooth profile spur gears up to now. We intended to select reference papers which focus on asymmetric spur gears besides symmetric ones to verify results of our own analysis for both tooth profiles of gears.

5.4.1.1 Case S6-S7-S8

In this section of the thesis, finite element analyses were performed for the reference article of Sekar and Muthuveerappan [42]. Gear parameters are given in Tables of 5.2 and 5.3. A gear model with coding D20C20 refers to a symmetric tooth with drive side pressure angle of 20degree and coast side pressure angle of 20degree, whereas coding D25C20 refers to an asymmetric tooth with drive side pressure angle of 25degree and

coast side pressure angle of 20 degree. Three teeth model studied in reference article is presented in Figure 5.21.

Firstly, Case S6 was performed for symmetric gear pairs studied in reference work [42]. Tooth root bending stress was determined analytically and numerically in related reference paper. Two additional cases were carried out to be sure about that our own finite element works' results in this section. They (Case S7 and S8) were compared with results of in-house software and KissSoft. All analyses and calculations were done for constant load (as same as in reference).

Tooth root bending stress value (for Case S6) obtained from FEA in reference paper is shown in Figure 5.22. Three teeth model was analyzed for finite element works (Figure 5.23). It is similar to model studied in reference article except loading type as already shown in Figure 5.21. Single point load was applied in this thesis whereas meshing of gear pair was preferred in reference paper.

Quadrilateral (Quadra-8) element type was preferred while element size was selected as 0.1 mm. During analysis, 2D gear model was fixed at center and load was applied at radius of 20.484mm as stated in reference work.

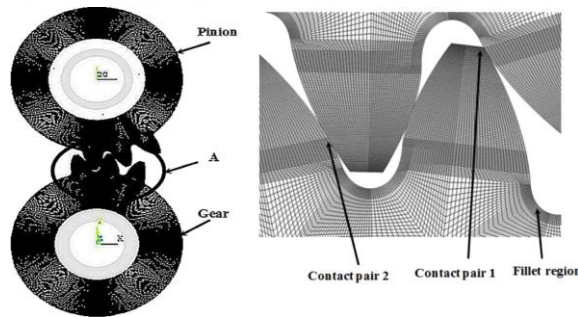


Figure 5.21 Gear body and three-teeth model [42]

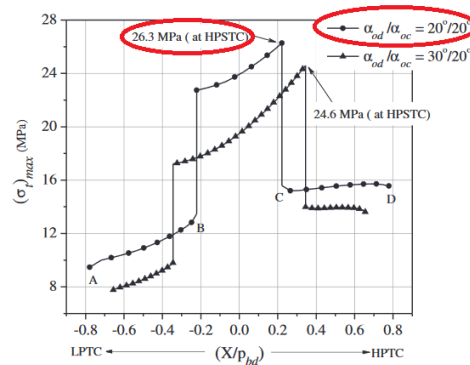


Figure 5.22 Root stress versus contact position curve [42]

Table 5.2 Gear pair parameters for Case S6, S7 and S8

Gear pair parameter	Value	Unit
Module	1	mm
Teeth number	20	-
Gear ratio	1	-
Addendum factor	1.00	-
Dedendum factor	1.25	-
Addendum modification factor	0	-
Face width	Unit width	mm
Normal load	10	N
Rack cutter type	Full round rack cutter	-
Contact ratio	1.5568	-

Table 5.3 Pressure angle and cutter tip radius coefficients for Case S6, S7 and S8

Parameters	Cases		
	Case S6	Case S7	Case S8
Pressure angle (degree)	D20C20	D25C25	D30C30
Cutter tip radius coefficient	0.472	0.318	0.110

Tooth root bending stress results for Case S6, S7 and S8 are presented in Table 5.4. According to the results obtained from finite element analyses in this thesis, the maximum tooth root stress is 26.06 MPa whereas it is 26.30 MPa for same gear model (Case S6) in reference article [42].

Results of software were already compared to KissSoft's ones in previous chapter (Chapter 4) for cases of this reference work [42]. But if necessary to compare for once, results obtained by in-house software (based on ISO) and KissSoft commercial software for each case were very close to each other; the largest difference between them was %0.73.

In reference paper [37], there is only one FEA analysis for symmetric spur gears (Case S6). We intended to improve our FEA knowledge and background by making more studies (Case S7 and S8). Results obtained from in-house software, finite element analyses conducted in thesis studies and reference paper are presented in Table 5.4. In-house software and KissSoft estimate very close values while lower stress values were obtained by FEA.

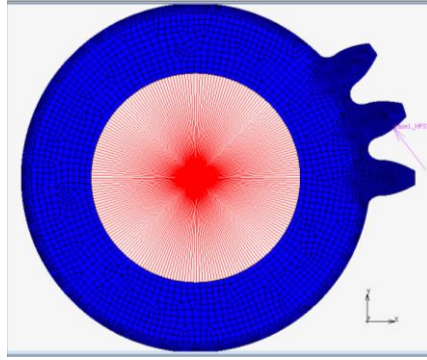


Figure 5.23 3-teeth model and boundary conditions for Case S6 (Marc)

Table 5.4 Gear tooth root bending stress for Case S6, S7 and S8

Calculation methods	Root bending stress (MPa)		
	Case S6	Case S7	Case S8
In-house software (based on ISO)	26.79	27.09	27.50
KissSoft (based on ISO)	26.81	27.07	27.70
FEA (current study)	26.06	25.19	25.89
FEA (reference article)	26.30	-	-

5.4.1.2 Case S9

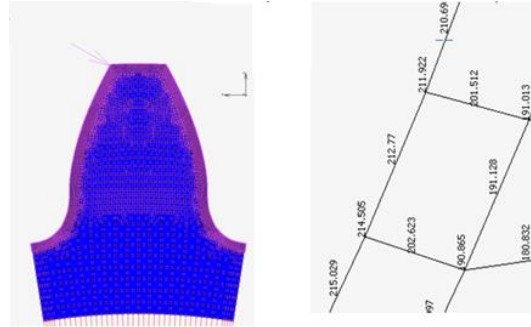
Another reference paper [43] was studied for symmetric gear pair whose parameters are presented in Table 5.5. Gear loading model is given in Figure 5.24. Element type and mesh size were selected Q8 and 0.05 mm respectively for finite element analysis. Tooth root thickness and root bending stress values obtained by reference study works of finite element analyses and calculations based on DIN [63], and current study are presented in Table 5.6.

Table 5.5 Gear pair parameters for Case S9

Gear Pair Parameter	Value	Unit
Module	2	mm
Teeth number	20	-
Pressure angle	D20C20	degree
Gear ratio	1	-
Addendum factor	1.00	-
Dedendum factor	1.25	-
Normal load	101.6	N/mm

Table 5.6 Gear tooth root bending stress for Case S9

Calculation methods	Root stress (MPa)	Root thickness (mm)
Current study (based on FEA)	215	3.789
Ref [43] (based on DIN [63])	204	-
Ref [43] (based on FEA)	210	3.857

**Figure 5.24** Stress concentrations on gear tooth [43]

5.4.1.3 Case S10-S11-S12

The current PhD study consists of theoretical and experimental studies. A back to back test rig is intended to be designed and used; therefore three different gear pairs (Case S10-S11-S12) were selected for finite element analyses and experiments. These gear pairs' parameters are presented in Table 5.7.

Three different gear pairs in terms of pressure angles were generated for FEA based on given gear parameters in Table 5.7. Gear tooth models, boundary conditions and loading conditions are presented in Figure 5.25 and 5.26.

Table 5.7 Gear pair parameters for Case S10, S11 and S12

Gear Pair Parameter	Value			Unit
Module	3			mm
Teeth number	40			-
Gear ratio	1			-
Addendum factor	1.00			-
Dedendum factor	1.25			-
Face width	20			mm
Rack cutter tip radius coefficient	0.1			-
Torque	160.428			Nm
Pressure angle	Cases			
	S10	S11	S12	
	D20C20	D25C25	D30C30	degree

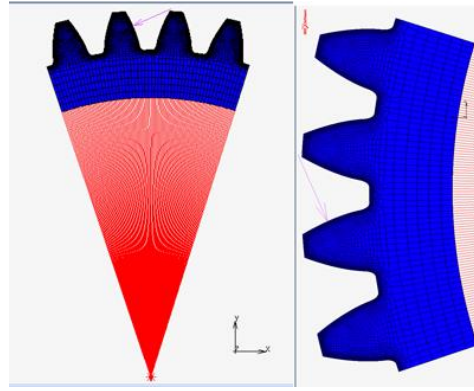


Figure 5.25 Gear tooth model, loading condition and mesh for Case S10

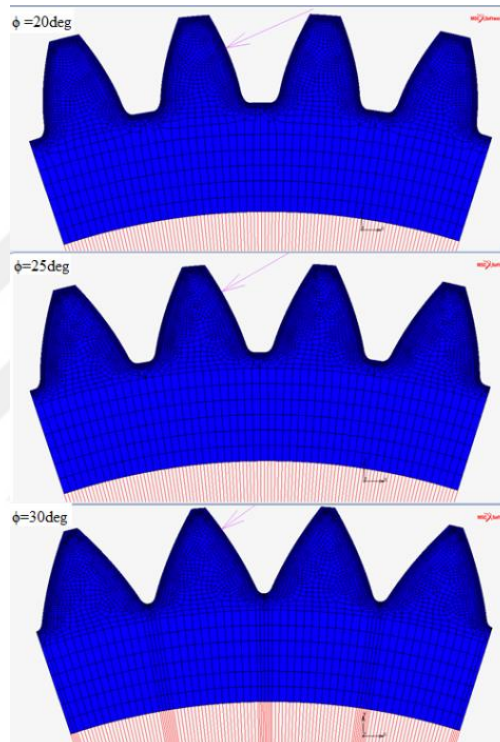


Figure 5.26 Gear tooth model, loading condition and mesh (Case S10-S11-S12)

For these gear pairs, different finite element analyses were carried out. Q8 (quadrilateral 8), 0.1 mm (sometimes 0.05mm) and mapped meshing were preferred for element type, element size and meshing respectively.

Tooth root bending stresses calculated for Case S10, S11 and S12 are presented in Table 5.8. Finite element analyses achieve lower stress level when compared other tools, whereas KissSoft and in-house software produce same level of tooth root bending stress.

Table 5.8 Gear pair root bending stress for Case S10, S11 and S12

Calculation Methods	Root Bending Stress (MPa)		
	Case S10	Case S11	Case S12
in-house software (based on ISO)	147.24	139.12	131.55
KissSoft	147.18	139.07	131.51
FEA	136.90	122.60	112.50

5.4.2 Case Studies of Asymmetric Tooth Profile Spur Gear

After completing case studies for symmetric spur gears, finite element analyses of asymmetric ones were conducted for root bending stress.

5.4.2.1. Case A5 and A6

Reference study [42] focused on symmetric and asymmetric spur gears for bending tooth root stress. The study includes calculations of tooth root bending stress based on ISO [2] and finite element analysis. In previous chapter, case studies for asymmetric spur gears were performed (by in-house software) and results were compared to reference paper results.

In this section, FEA studies were conducted for asymmetric gear pair analyzed previously in Ref [42]. General gear parameters presented in Table 5.9 are same for both cases whereas Case A5 and A6 are distinguished in terms of pressure angle in Table 5.10.

Similar to reference paper [42] (as shown in Figure 5.21), gear tooth model was prepared for three teeth model (Figure 5.27). 2D gear model was prepared and fixed at center. Quadrilateral (Quadra-8) element type was preferred and element size was taken as 0.1 mm. Load was applied at radius of 21.00mm as stated in reference paper.

Root bending stress results are presented in Table 5.11. FEA result of reference paper [42] is 24.60 MPa (as shown in Figure 5.28). When results of finite element analyses conducted in reference and current study are compared, it is obvious to say that the difference is almost zero.

Table 5.9 Gear pair parameters for Cases of A5 and A6

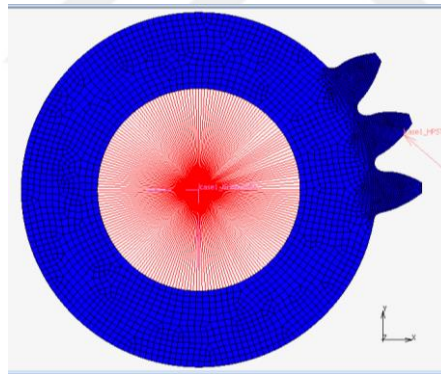
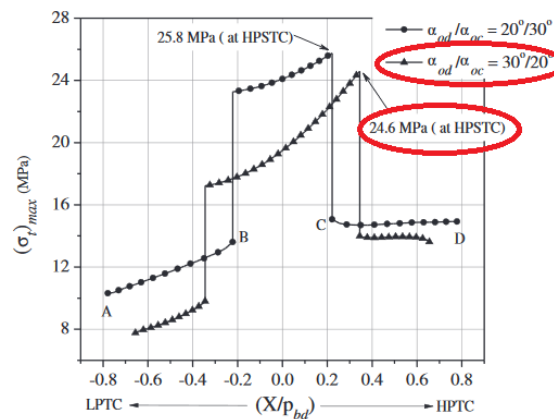
Gear pair parameter	Value	Unit
Module	1	mm
Teeth number (driver/driven)	20/20	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Center distance	20	mm
Rack cutter tip radius coefficient	Fully rounded	-

Table 5.10 Pressure angles of drive and coast side for Cases of A5 and A6

Case no	Pressure angle		Unit
	Drive side	Coast side	
A5	30	20	Degree
A6	25	20	Degree

Table 5.11 Comparison of tooth root stress with Ref [42]

Case no	Pressure angle	
	Ref [42] based on FEA	Current study based on FEA
A5	24.60	24.59
A6	25.30	25.22

**Figure 5.27** Three teeth model and boundary conditions for Case A5**Figure 5.28** Root stress values for Case A5 [42]

5.4.2.2. Case A7

Asymmetric gear with normal module of 2mm and teeth number of 20 of Reference [43] were analyzed in this case. Gear pair parameters and root bending stress values are given in Table 5.12 and 5.13 respectively. Stress analysis of reference paper [43] is illustrated in Figure 5.29. Element type and mesh size were selected as Q8 and 0.05mm respectively for finite element analysis of Case A7. Tooth root thickness and root bending stress values obtained by reference and current studies are presented in Table 5.13.

Table 5.12 Gear pair parameters for Case A7 [43]

Gear pair parameter	Value	Unit
Module	2	mm
Teeth number	20	-
Pressure angle	D20C26	degree
Gear ratio	1	-
Addendum factor	1.00	-
Dedendum factor	1.25	-
Normal load	101.6	N/mm

Table 5.13 Gear pair root bending stress for Case A7

Calculation methods	Root stress (MPa)	Root thickness (mm)
Current study (FEA)	175	4.113
Ref [43] (DIN [63])	184	-
Ref [43] (FEA)	178	4.096

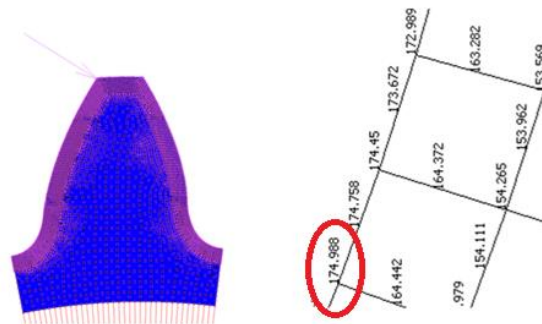


Figure 5.29 Maximum stress on gear tooth root [43] (Case A7)

5.4.2.3. Case A8-A9-A10-A11

Cases studies of A8-A9-A10-A11 were studied by conducting finite element analyses and ISO [2] based calculations of asymmetric tooth spur gears. Gear pair parameters

are presented in Table 5.14. Asymmetric spur gear pairs differing in terms of pressure angles were generated for finite element analyses. For given gear parameters, different pressure angles for drive and coast sides of tooth profile were selected based on different combinations of 20, 25 and 30degree such as D20C25, D20C30, D25C20 and D30C20. Software results are presented in Figure 5.30-5.33. FEA results are presented with in-house software results (in Table 5.15).

Table 5.14 Gear pair parameters for Case A8-A9-A10-A11

Gear pair parameter	Value	Unit
Module	3	mm
Teeth number	40	-
Gear ratio	1	-
Addendum factor	1.00	-
Dedendum factor	1.25	-
Face width	20	mm
Rack cutter tip radius coefficient	0.1	-
Torque	160.428	Nm

Table 5.15 Comparison of tooth root stress for Case A8-A9-A10-A11

Case No	Gear Pair	Tooth root stress (MPa)	
		in-house software	FEA
A8	D20C25	138.61	128.5
A9	D20C30	130.43	121.6
A10	D25C20	147.38	129.7
A11	D30C20	146.94	121.3

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	1,039317	1,039317	mm
Radius of Root Fillet (Coast Side)	1,025811	1,025811	mm
Tooth Root Normal Chord 1	6,414059	6,414059	mm
Tooth Root Normal Chord 2	7,107077	7,107077	mm
Tooth Root Normal Chord	6,760568	6,760568	mm
Tooth Root Normal Chord (Hypotenuse)	6,769981	6,769981	mm
AAB4(Drive Side)	4,772102	4,772102	-
AAB5(Drive Side)	6,008110	6,008110	-
Form Factor (Drive Side)	1,259007	1,259007	-
L (Drive Side)	2,127371	2,127371	-
AAB6(Drive Side)	0,436463	0,436463	-
Notch Parameter (Drive Side)	3,252408	3,252408	-
Stress Correction Factor (Drive Side)	2,470637	2,470637	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,687692	0,687692	-
YF*YS*YEps (Drive Side)	2,139100	2,139100	-
Pitchline Velocity	15,707963		m/sec
Tangential Load	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	138,616621	138,616621	MPa

Figure 5.30 Tooth root stress calculated by software for Case A8

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	1,039317	1,039317	mm
Radius of Root Fillet (Coast Side)	1,011460	1,011460	mm
Tooth Root Normal Chord 1	6,414059	6,414059	mm
Tooth Root Normal Chord 2	7,863566	7,863566	mm
Tooth Root Normal Chord	7,138812	7,138812	mm
Tooth Root Normal Chord (Hypotenuse)	7,178039	7,178039	mm
AAB4(Drive Side)	5,321024	5,321024	-
AAB5(Drive Side)	6,008110	6,008110	-
Form Factor (Drive Side)	1,129127	1,129127	-
L (Drive Side)	2,246394	2,246394	-
AAB6(Drive Side)	0,447655	0,447655	-
Notch Parameter (Drive Side)	3,434376	3,434376	-
Stress Correction Factor (Drive Side)	2,592107	2,592107	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,687692	0,687692	-
YF*YS*YEps (Drive Side)	2,012748	2,012748	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	130,428856	130,428856	MPa

Figure 5.31 Tooth root stress calculated by software for Case A9

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	1,025811	1,025811	mm
Radius of Root Fillet (Coast Side)	1,039317	1,039317	mm
Tooth Root Normal Chord 1	7,107077	7,107077	mm
Tooth Root Normal Chord 2	6,414059	6,414059	mm
Tooth Root Normal Chord	6,760568	6,760568	mm
Tooth Root Normal Chord (Hypotenuse)	6,769981	6,769981	mm
AAB4(Drive Side)	4,602561	4,602561	-
AAB5(Drive Side)	6,409829	6,409829	-
Form Factor (Drive Side)	1,392666	1,392666	-
L (Drive Side)	1,912590	1,912590	-
AAB6(Drive Side)	0,414498	0,414498	-
Notch Parameter (Drive Side)	3,295230	3,295230	-
Stress Correction Factor (Drive Side)	2,374773	2,374773	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,746261	0,746261	-
YF*YS*YEps (Drive Side)	2,468084	2,468084	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	147,382898	147,382898	MPa

Figure 5.32 Tooth root stress calculated by software for Case A10

Tooth Root Bending Stress Factors			
PROPERTY	PINION	GEAR	UNIT
Radius of Root Fillet (Drive Side)	1,011460	1,011460	mm
Radius of Root Fillet (Coast Side)	1,039317	1,039317	mm
Tooth Root Normal Chord 1	7,863566	7,863566	mm
Tooth Root Normal Chord 2	6,414059	6,414059	mm
Tooth Root Normal Chord	7,138812	7,138812	mm
Tooth Root Normal Chord (Hypotenuse)	7,178039	7,178039	mm
AAB4(Drive Side)	4,903882	4,903882	-
AAB5(Drive Side)	6,764282	6,764282	-
Form Factor (Drive Side)	1,379373	1,379373	-
L (Drive Side)	1,817743	1,817743	-
AAB6(Drive Side)	0,403991	0,403991	-
Notch Parameter (Drive Side)	3,528963	3,528963	-
Stress Correction Factor (Drive Side)	2,390514	2,390514	-
Contact Ratio Factor for Tooth Root Stress(Drive Side)	0,794331	0,794331	-
YF*YS*YEps (Drive Side)	2,619235	2,619235	-
Pitchline Velocity	15,707963		m/sec
TangentialLoad	2673,803044		Newton
Face Width	20,000000		mm
Helix Angle Factor	1,000000		-
Rim Thickness Factor	1,000000		-
Deep Tooth Factor	1,000000		-
Nominal Tooth Root Stress (Drive Side)	146,943783	146,943783	MPa

Figure 5.33 Tooth root stress calculated by software for Case A11

5.4.3 Case Studies of Asymmetric Root Fillet

For 2-D Finite Element analysis of the spur gears, MSC Software MARC [181] is used to simulate the working conditions (meshing under design load) of different gears with consistent boundary and loading conditions. Tooth load is applied at the highest point of single tooth contact (HPSTC) as pressure from element edge and gears are fixed inside the bore diameter as shown in Figure 5.34. Element types are quadratic eight node (Q8) and triangular six node (T6) elements. Element size on the tooth surface contour is about 0.10 mm. Total numbers of elements and nodes are around 36485 and 111770 respectively. FEA model and bending stress results for Case ST-SR3 (Table 4.28) are presented in Figure 5.34 and 5.35 respectively.

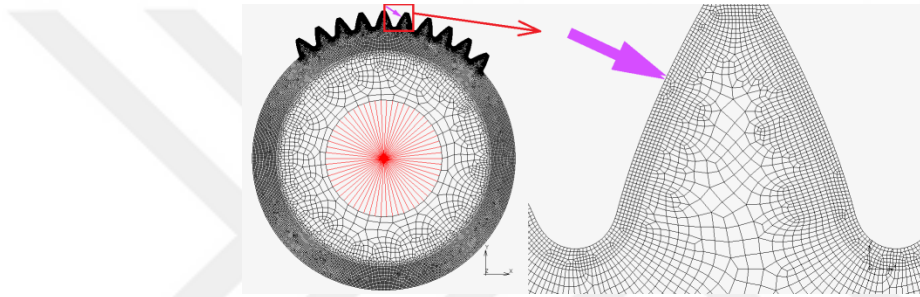


Figure 5.34 Finite element model of symmetric tooth profile gear with symmetric root fillet (Case ST-SR3)

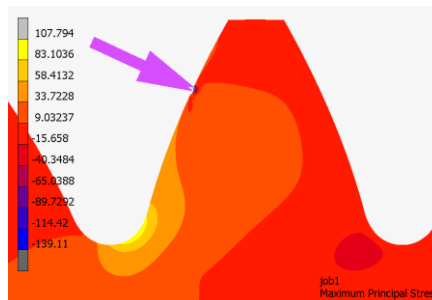


Figure 5.35 Finite element result of root bending stress for symmetric tooth profile gear with symmetric root fillet (Case ST-SR3)

Bending stress values evaluated by the finite element analyses for symmetric gears of D20C20, D25C25 and D30C30 with different fillet geometries such as circular, symmetrical and asymmetrical ones are presented in Table 5.16. It is easier to compare effect of root fillet type and coefficient of tool tip radius on bending stress when case

studies are grouped based on pressure angle values for D20C20, D25C25 and D30C30 in itself.

With symmetric tooth profile of D20C20, root bending stress value decreases from 113.34MPa to 108.69MPa by using circular root fillet instead of trochoidal fillet with radius coefficient of 0.38 (suggested by ISO [180] standard). When tool tip radius coefficient of 0.47 is preferred, root stress decreases to 105.98MPa.

Here it is seen that on the contrary to expectations circular root fillet does not produce the least stress, because symmetric root with cutting tool tip radius coefficient of 0.47 has resulted in a better performing fillet with possibly larger curvature and less stress concentration.

However, circular root fillet does still produce less bending stress compared to the case of symmetric root with cutting tool tip radius coefficient of 0.38. It is clear from results in Table 5.17 that increasing cutting tool tip radius coefficient from 0.38 to 0.47 provides a stronger tooth root section (with 6.5% less stress) than what a full circular root form provides (with 4.1% less stress) for the current gear data. Compared with the circular root form, almost 2.5% more reduction in stress (due to increased cutter tip radii) will certainly improve the gear service life in bending.

Table 5.16 Root bending stress of different root shapes with symmetric tooth profile

Case	α		Tooth profile (DXXCYY)	Load (N)	ρ_f^*		σ_b FEA(MPa)
	α_d	α_c			ρ_{fd}^*	ρ_{fc}^*	
ST-CR1	20		D20C20	2845.402	Circular		108.69
ST-CR2	25		D25C25	2950.215	Circular		111.37
ST-CR3	30		D30C30	3087.442	Circular		125.19
ST-SR0	20		D20C20	2845.402	0.30		120.82
ST-SR1	20		D20C20	2845.402	0.38		113.34
ST-SR2	20		D20C20	2845.402	0.47		105.98
ST-SR3	25		D25C25	2950.215	0.31		107.79
ST-SR4	30		D30C30	3087.442	0.11		123.54
ST-AR1	20		D20C20	2845.402	0.58	0.36	101.72
ST-AR2	25		D25C25	2950.215	0.43	0.20	100.67
ST-AR3	25		D25C25	2950.215	0.53	0.10	95.98
ST-AR4	30		D30C30	3087.442	0.12	0.10	121.49

For D25C25, having a pressure angle of 25degree, 0.31 (Case ST-SR3) is the maximum limit for coefficient of symmetric cutter tip radius with a root stress of 107.70MPa. A circular fillet with radius of 1.166mm (Case ST-CR2) can be used but with a resulting root stress slightly more than the trochoidal fillet stress (111.37MPa). A stress reduction of almost 14% (95.98MPa), with reference to circular fillet, can be obtained if an asymmetric trochoidal fillet (coefficients of 0.53 and 0.10 for drive and coast sides) is used as in Case ST-AR3, (Table 5.18).

Results for D30C30 are presented in Table 5.19. Increase in pressure angle decreases allowable limit of root fillet radius coefficients for D30C30. This yields low level of stress reduction by changing root fillet from symmetric to asymmetric.

Table 5.17 Root bending stress reduction of symmetric tooth profile (D20C20) in Case ST-SR1, ST-CR1, ST-SR2 and ST-AR1 under the load of 2845N

Case	α		Tooth profile (DXXCYY)	ρ_f^*		σ_b FEA (MPa)	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-SR1	20		D20C20	0.38		113.34	Ref. value
ST-CR1	20		D20C20	circular		108.69	4.10
ST-SR2	20		D20C20	0.47		105.98	6.50
ST-AR1	20		D20C20	0.58	0.36	101.72	10.25

Table 5.18 Root bending stress values of symmetric tooth profile (D25C25) in Case ST-CR2, ST-SR3, ST-AR2 and ST-AR3 under the load of 2950N

Case	α		Tooth profile (DXXCYY)	ρ_f^*		σ_b FEA (MPa)	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-CR2	25		D25C25	circular		111.37	Ref. value
ST-SR3	25		D25C25	0.31		107.79	3.21
ST-AR2	25		D25C25	0.43	0.20	100.67	9.61
ST-AR3	25		D25C25	0.53	0.10	95.98	13.82

Table 5.19 Root bending stress values of symmetric tooth profile (D30C30) Case ST-CR3, ST-SR4 and ST-AR4 under the load of 3087N

Case	α		Tooth profile (DXXCYY)	ρ_f^*		σ_b FEA	Reduction %
	α_d	α_c		ρ_{fd}^*	ρ_{fc}^*		
ST-CR3	30		D30C30	circular		125.19	Ref. value
ST-SR4	30		D30C30	0.11		123.54	1.32
ST-AR4	30		D30C30	0.12	0.10	121.49	2.96

With symmetric profile spur gears (having different pressure angles of 20, 25 and 30degree), an analysis of the effect of different root fillet forms have been performed above.

Circular roots and trochoidal roots generated by different cutter tip designs were compared in terms of maximum tooth root bending stress under a constant transmitted torque. In both circular and trochoidal roots maximum allowable root radii were employed. Followings are the summary of bending stress reductions obtained for symmetric tooth profiles by different root fillet designs:

- 10.25 % for D20C20, (root stress decreases from 113.34MPa to 101.72MPa owing to design change from ST-SR1 to ST-AR1)
- 13.82 % for D25C25, (root stress decreases from 111.37MPa to 95.98MPa owing to design change from ST-CR2 to ST-AR3)
- 2.96 % for D30C30, (root stress decreases from 125.19MPa to 121.49MPa owing to design change from ST-CR3 to ST-AR4).

In addition to case studies for symmetric tooth profile with different fillet design alternatives, asymmetric tooth with symmetric fillet is first used to reduce root stress. Next, asymmetric tooth with asymmetrical fillet design (dual asymmetry) is investigated to improve gear bending life performance.

General gear pair parameters for asymmetric tooth cases are same as the symmetric tooth profile cases, and given in Table 4.28. Case studies of asymmetric tooth profile gears are performed for constant torque and reference/nominal center distance value for interference free meshing. FEA model and bending stress result of finite element analysis for case of AT-AR2 are presented in Figure 5.36 and 5.37 respectively. Tooth root bending stress values are presented in Table 5.20 for all of asymmetric tooth cases.

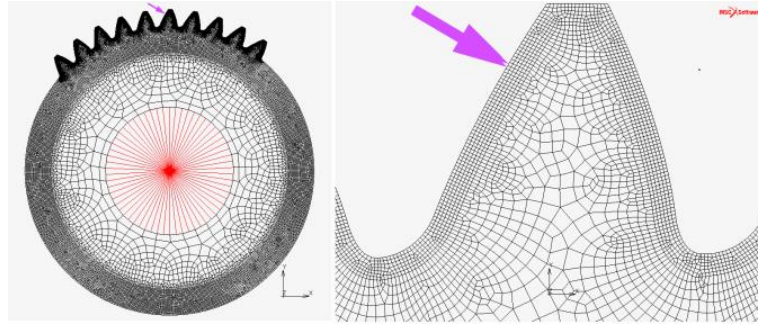


Figure 5.36 Finite element model of asymmetric tooth profile gear with asymmetric root fillet (Case AT-AR2)

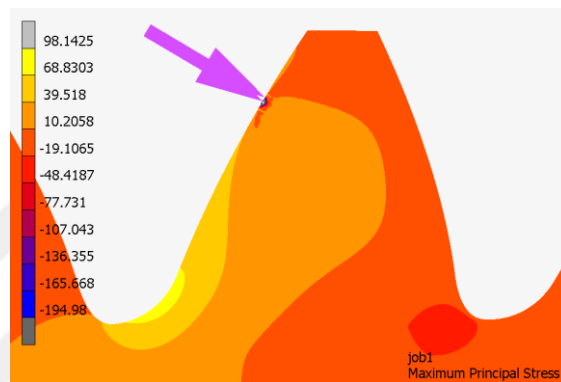


Figure 5.37 Finite element result of root bending stress for asymmetric tooth profile gear with asymmetric root fillet (Case AT-AR2)

When case studies are grouped based on pressure angles it is easier to compare effects of root fillet type and coefficient of tool tip radius on bending stress. Therefore it is better to present stress values in different groups based on pressure angles of both sides of asymmetric tooth profile gear. There are significant stress reductions like:

- 7.27 % for D25C20, (root stress decreases from 106.60MPa to 98.85MPa owing to design change from AT-SR1to AT-AR1),
- 11.27 % for D30C20, (root stress decreases from 110.61MPa to 98.14MPa owing to design change from AT-SR2 to AT-AR2),
- 10.17 % for D30C25, (root stress decreases from 114.00MPa to 102.41MPa owing to design change from AT-SR3 to AT-AR3),
- 8.84 % for D25C30, (root stress decreases from 112.21MPa to 102.29MPa owing to design change from AT-SR4 to AT-AR6),
- 8.47 % for D20C25, (root stress decreases from 105.53MPa to 96.59MPa owing to design change from AT-SR5 to AT-AR4) and

- 10.70 % for D20C30 (root stress decreases from 107.80MPa to 96.26MPa owing to design change from AT-SR6 to AT-AR5).

Table 5.20 Root bending stress values of asymmetric tooth (AT) with symmetric root (SR) and asymmetric root (AR) profiles calculated by finite element analysis

Case	α		Tooth profile (DXXCYY)	Load (N)	ρ_f^*		σ_b FEA (MPa)
	α_d	α_c			ρ_{fd}^*	ρ_{fc}^*	
AT-SR1	25	20	D25C20	2950.215	0.395		106.60
AT-SR2	30	20	D30C20	3087.442	0.30		110.61
AT-SR3	30	25	D30C25	3087.442	0.21		114.00
AT-SR4	25	30	D25C30	2950.215	0.21		112.21
AT-SR5	20	25	D20C25	2845.402	0.395		105.53
AT-SR6	20	30	D20C30	2845.402	0.30		107.80
AT-AR1	25	20	D25C20	2950.215	0.59	0.22	98.85
AT-AR2	30	20	D30C20	3087.442	0.56	0.10	98.14
AT-AR3	30	25	D30C25	3087.442	0.35	0.10	102.41
AT-AR4	20	25	D20C25	2845.402	0.58	0.19	96.59
AT-AR5	20	30	D20C30	2845.402	0.48	0.10	96.26
AT-AR6	25	30	D25C30	2950.215	0.32	0.10	102.29

5.5 Conclusion

To sum up, the aim of this chapter is to determine tooth root stress by finite element analyses for symmetric and asymmetric gears. Up to now, several finite element analyses have been carried out for both types of gear profiles through Marc [181]. For preparation finite element analyses of gear, in-house software and Solidworks [174] have been used to generate gear tooth profile coordinates for involute and root curves, to create 2D whole gear model, and to generate three different surfaces for a better meshing.

SimXpert [182] was preferred for meshing processes. Once the gear models were created and mesh generation was completed, Marc [181] was used to complete a finite element analysis of the gear tooth under load and bending stress distribution was obtained for the tooth root.

To obtain further stress reduction for symmetric and asymmetric tooth spur gears, different root fillet profiles were applied. Circular and trochoidal (symmetric and asymmetric) roots generated by different cutter tip designs symmetric profile spur

gears (having different pressure angles of 20, 25 and 30degree) were compared in terms of maximum tooth root bending stress under a constant transmitted torque.

By applying different root fillet types to spur gears with symmetric tooth profile, root stress reductions were obtained. For symmetric tooth spur gear pair (D20C20), when asymmetric root fillet radius coefficients (0.58 for drive side and coast side for 0.36) in Case ST-SR1 instead of symmetric root fillet radius (0.38 for both sides of tooth) for Case ST-AR1 were applied, 10.25% of root bending stress reduction was obtained.

Once asymmetric root fillet with the coefficient of 0.53 for drive and 0.10 for coast side flanks (ST-AR3) of symmetric tooth with pressure angle of 25degree (D25C25) were preferred to circular root fillet (ST-CR2), root stress reduction of 13.82% was achieved.

As regards to symmetric tooth spur gear pair with pressure angle of 30degree (D30C30), a stress reduction of 2.96 was obtained in the case of using asymmetric root fillet radius coefficients of 0.12 and 0.10 respectively for drive and coast sides of tooth profile (ST-AR4) rather than symmetric root with circular shape (ST-CR3).

To investigate combined effect of asymmetric tooth and root profiles on tooth root bending stress, dual asymmetry were applied to both profiles of tooth and root and different percentages of stress reductions were obtained.

Regarding to asymmetric spur gears (D25C20), when asymmetric root fillet with coefficient of 0.59 for drive and 0.22 for coast sides were applied in place of symmetric root fillet (0.395 of fillet coefficient) a reduction of 7.27% in root stress was gained. Changing root fillet design of asymmetric tooth (D30C20) from symmetric (AT-SR2) to asymmetric one (AT-AR2) yields 11.27% stress reduction. When it comes to asymmetric gear (D30C25), asymmetric root fillet coefficients of 0.35 and 0.10 for drive and coast sides (AT-AR3) respectively were chosen instead of symmetric root with coefficient of 0.21 (AT-SR3) and stress reduction of 10.17% was obtained as consequence. Design variation of root fillet from symmetric coefficient of 0.21 (AT-SR4) to asymmetric one of 0.32 and 0.10 for drive and coast flanks (AT-AR6) produces less stress with reduction of 8.84%.

In respect to asymmetric spur gears, there are two different case studies for greater pressure angle of coast side than drive side. For first case (D20C25), 8.47% stress

reduction in root was obtained by means of applying asymmetric root fillet coefficients of 0.58 and 0.19 for drive and coast side (AT-AR4) instead of symmetric coefficient of 0.395 (AT-SR5). Considering second case (D20C30), 10.70% reduction in stress is possible by applying asymmetric root fillet (0.48 and 0.10 for drive and coast, AT-AR5) rather than symmetric root fillet with coefficient of 0.30, AT-SR6).



CHAPTER 6

TRANSMISSION ERROR THEORY AND MICRO GEOMETRY DESIGN

6.1 Transmission Error Theory

Gear design criteria include two main points of static and dynamic performances. Static performance usually deals with global kinematic requirements, geometrical requirements and strength analysis of gear materials under load. Dynamic performance of gears is just as important as performance under static conditions during meshing and it is dependent on vibration level during gear meshing due to loading conditions, manufacturing and mounting errors. Vibration cause to noise during operation and it is a proof of non-uniform load distribution of tooth contact.

Theoretically a pair of gear with the perfect involute tooth profiles and tooth spacing transmits a uniform motion between the shafts. But in reality, motion is not transmitted uniformly. Due to deflection of the teeth under load, interference, in other words corner contact (Figure 6.1) occurs between the teeth of the incoming pair and this causes impact loads on the teeth and instantaneous non uniform motion transfer between the gears.

Transmission error (TE) is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. The ideal position represents a condition of perfect gear box, without geometrical errors and deflections. TE can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along a line of action at base circle [102].

In other words, (TE) is defined as the difference between the actual position of the output gear and the position it would occupy if the gears were perfectly conjugate. It can be defined in angular units, or can be expressed in linear units along the line of action. TE is generally converted to linear motion along the line of action which is tangential to the base circles of the mating gears. The mathematical formulation of TE in linear units is given in Eq. 6.1:

$$TE = r_{b2} * (\theta_2 - \frac{T_1}{T_2} * \theta_1) \quad (6.1.)$$

Where

θ_1 : Angular rotation of the driving gear θ_2 : Angular rotation of the driven gear

T_1 : Tooth number of the driving gear T_2 : Tooth number of the driven gear

r_{b2} : Base radius of the driven gear

The static transmission error (STE) is strongly related to the kinematic accuracy of gear trains, dynamic load of gear tooth and gear noise. It is defined as the difference between the theoretical position of the output gear, with perfect geometric accuracy and rigid drive, and the actual output position at a low enough speed [102].

TE has long been established as main source of vibration and unpleasant noise of gear pairs during operation. It is highly unwelcome due to its negative effect on gear performance. In this study, the main purpose is to minimize quasi static transmission error which is strongly related to gear dynamic performance and influence noise, vibration and harshness (NVH) characteristics adversely by relieving gear teeth tip.

TE is one of main excitation source of gears and it has possible negative effects on gear operation namely high vibration levels transmitted via gear body, shaft, bearing, housing and casing, high noise levels of both air borne and structure borne types, kinematic inaccuracy causing problems in accurate positioning applications. TE under design load is the main cause of all the negative effects and has to be minimized in amplitude and optimized in shape to get a smooth TE curve.

For a smooth meshing during gear operation, the production of gears with minimum geometrical errors is first step and very critical to reduce TE value. But loading effect or gear tooth flexibility under load cause to TE for an even error free gear pair. Adjacent pitch error and tooth deflection both cause corner contact (CC) (Figure 6.1). It is effective at the start and end of mesh. In addition, both adjacent pitch error (PE) and the teeth deflection will cause a position variation of the output gear hence transmission error of gear pairs.

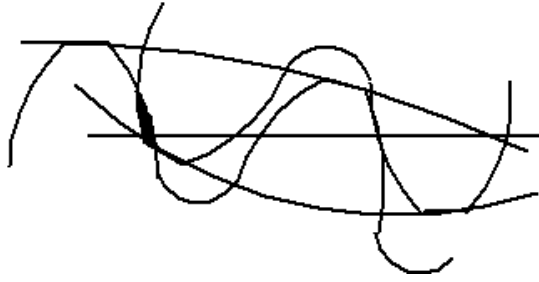


Figure 6.1 Corner contact [102]

Corner contact causes an instantaneous variation in transmission error curve as well as the physical damage of tooth surface under heavy loads. An intentional profile modification is very vital to avoid interferences. For this purpose, the tip of the driven flank or the root of the driving flank or both has to be modified. A definite amount of material is removed to provide a smooth meshing of gear pair. Intentional profile modifications of gears at tooth tips of both driving and driven gears are illustrated in Figure 6.2a and b respectively.

Profile modification for linear type can be characterized by two parameters namely; amount of relief and extent of relief (Figure 6.3). Amount of relief is the thickness of material removed from tip of the tooth in a plane normal to the involute curve. Extent of relief shows how far the relief extends down the tooth from the tip (Figure 6.3).

Extent of relief, although can be measured in roll angle, is usually given as linear dimension between start and end points of the relief along the path of contact. Extent of relief in linear units is calculated by multiplying the required roll angle difference between start and end of relief by the base radius (r_b) in Equation 6.2.

$$\text{Extent of relief} = r_b * (\text{roll angle}_{\text{end}} - \text{roll angle}_{\text{start}}) \quad (6.2)$$

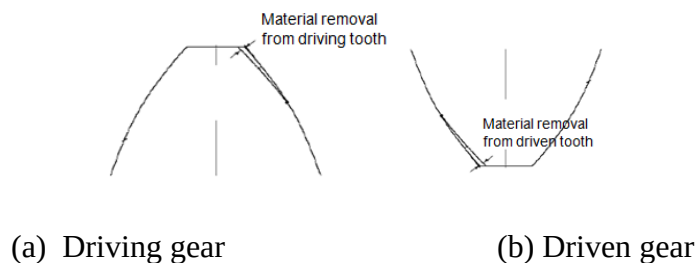


Figure 6.2 Application of tip relief for teeth to prevent corner contact

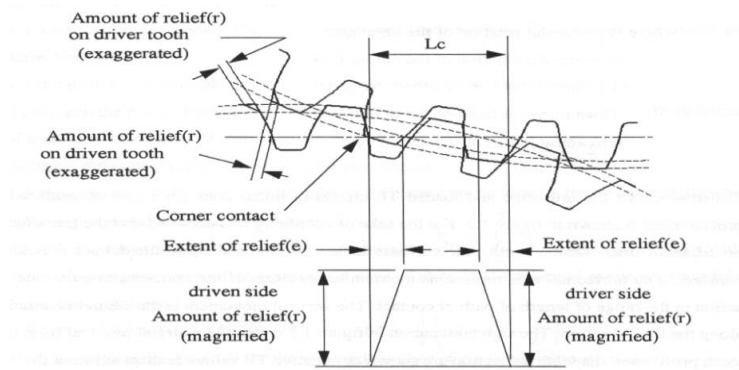


Figure 6.3 Geometry of the tooth contact and profile relief along path of contact

Operating gears at speeds away from the system's resonant speeds is important. However, systems will generally be multiple degree of freedom systems with multiples of coupled and uncoupled resonant frequencies. In addition, the resonance is not hit by only the main excitation frequency but also by its harmonics and sub harmonics. There is a direct relationship between STE values and, vibration and noise levels. Smoothing the STE curve under operating load, either by profile relief or some other means, comes front to avoid harmonics hitting problem and high vibration and noise levels.

Construction of loaded and unloaded TE curves in linear units for a pair of gears with profile relief is shown in Figure 6.3. For ease understand of topic, it is assumed that the gears in mesh have no adjacent pitch errors and, tooth pair stiffness and transmitted load are constant. The vertical axis refers to TE in linear units while horizontal axis represents the nominal positions of contact along the line of action. When the contact point is in single tooth pair contact region (SC), total load is carried by that one pair whereas the contact point is in double tooth pair contact region (DC) the all of the load is shared by two pairs (regardless of whether equally or not). Applying a proper relief on the gear teeth and also constructing the expected STE curve under load to check the suitability of that relief is very essential. Loaded static transmission error (LSTE) and no load static transmission error (NLSTE) curves are constructed for a constant tooth load and constant tooth pair stiffness with a linear type of profile relief (Figure 6.4). Relief shape is generally linear which the simplest form of relief is. There are other relief shapes mathematically, but manufacturing cost must be taken into account. It can be linear, concave or convex (Figure 6.5).

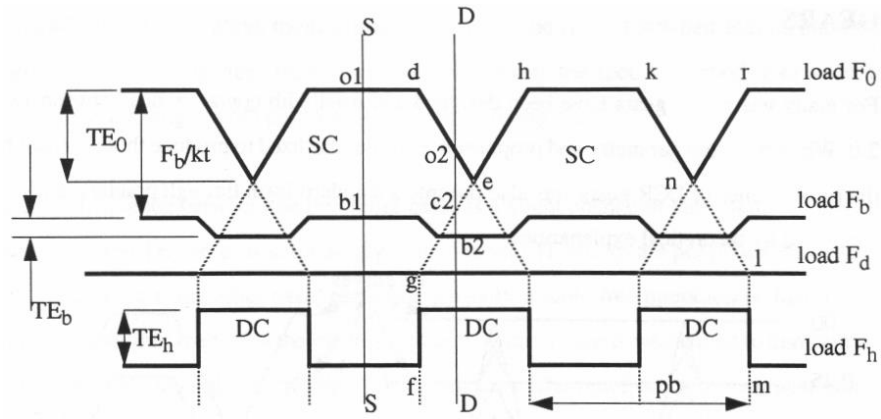


Figure 6.4 Constructions of the TE curves for low contact ratio (LCR) spur gears

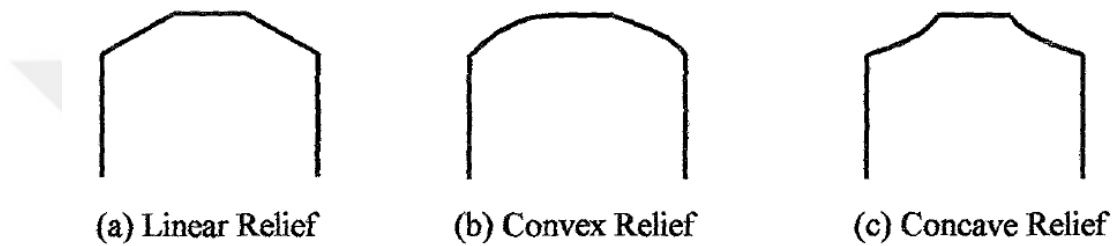


Figure 6.5 Relief types [102]

The procedure for constructing NLSTE and LSTE curves are presented as follows (Figure 6.4):

- a) Construct the boundary profiles of the mating tooth pairs such as curves f-g-h-k-l-m.
- b) Displace them by the base pitch of the gears along the horizontal axis.
- c) Construct the no load TE curve by following the top boundary borders of the each tooth boundary profile, curve d-e-h-k-n-r.
- d) To construct the loaded TE curve under any load, such as design load F_b , calculate the tooth pair deflection, $\frac{F_b}{k_t}$.
- e) Take a slice, for example S-S, downwards representing the deflection of teeth) by the amount of teeth deflection, $\frac{F_b}{k_t}$.
- f) Check if this new point is in the double pair region.
- g) If answer is NO as in the case of slice S-S, mark the position of the new point b_1 which is the actual position of the output gear.

- h) If answer is YES then use the load constraint $F_b = \sum_{i=1}^2 F_i$ to find the actual position in the double pair region. For instance, in case of slice D-D the actual position (point b_2) is founded by equations:

$$F_b = k_t(o_2 - b_2) + k_t(c_2 - b_2)$$

- i) Repeat the procedure e-i for the required number of contact positions (slices).
j) Connect the new marked points ($b_1, b_2, etc.$) to construct the loaded TE curve.

There are two cases of profile relief such as long and short relief (Figure 6.6). They define the useful limits beyond which is not profitable to go in terms of corner contact, tooth loading and TE. The amount of relief at teeth tip is same for short and long relief cases and it is equal to sum of the tooth pair deflection whereas there is difference for extent of relief values for both cases.

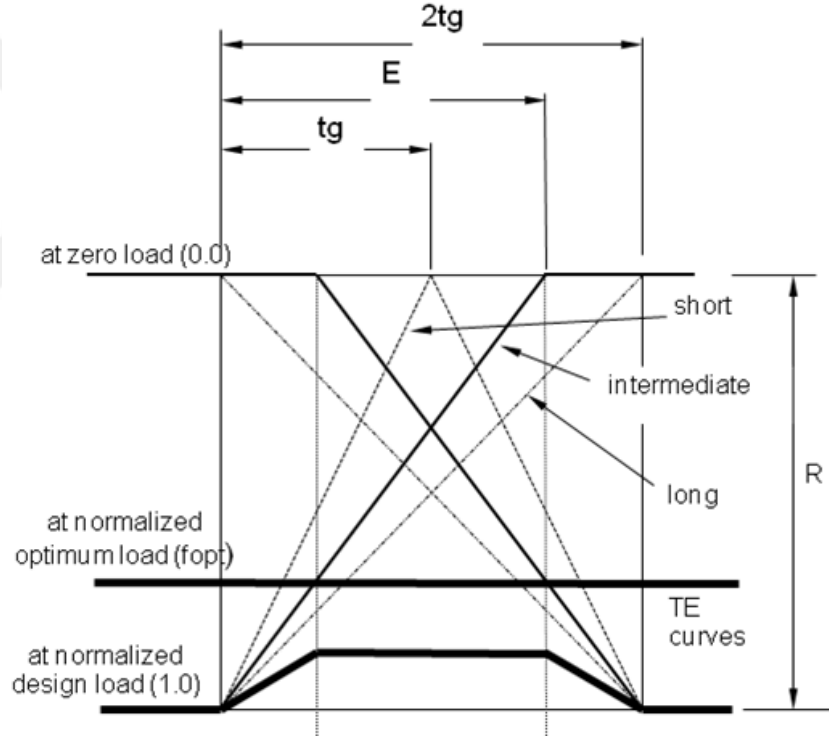


Figure 6.6 Profile relief geometry

6.2 Software for Calculation of Transmission Error

In the scope of this study, user friendly software has been prepared to design and analysis of micro profile of symmetric and asymmetric external spur gears. General view of software is illustrated in Figure 6.7.

Gear parameters such as module, teeth numbers of driver and driven gears, pressure angle for drive and coast side, coefficients of addendum-dedendum-hob tip radius-profile shift and, face width, modulus of elasticity, Poisson's ratio, angular velocity and design load values are entered to software (Figure 6.8).

Based on these geometrical and loading parameters, length of contact, base pitch and transverse contact ratio are calculated. Tooth pair stiffness along length of contact is evaluated based on thin slice theory and presented. User can use these calculated values or enter any other value into software for stiffness (Figure 6.9). Tooth pair stiffness and mesh stiffness curves derived by software are presented in Figure 6.10 and 6.11 respectively.

Relief parameters (for short, intermediate and long relief) namely; amount and extent of relief values are calculated based on stiffness and load transmitted by gear teeth pairs in mesh for minimized transmission error. Then user is allowed to select parameters suggested by software or can prefer entering any other value by selection of one of radio buttons for optimum defined and user defined (Figure 6.12).

Different design load percentages can be selected by user for TE calculations and curves. For each load, different colors are recommended by software to prevent confusion about which curve belongs to which load.

Peak to peak transmission error (PPTE) values for different percentages of design load are evaluated by depending on all these calculated and selected parameters (Figure 6.13).

TE curves under different design load values are presented in Figure 6.14. Each curve is in different color as selected from software given in Figure 6.13. The smoothest curve (the black curve, presented at lowermost in Figure 6.14) is obtained for design load (%100) for long relief.

Load diagrams, contact stress, sliding velocity and $P \times V$ (contact stress*sliding velocity) curves are illustrated in Figure 6.15-6.18.

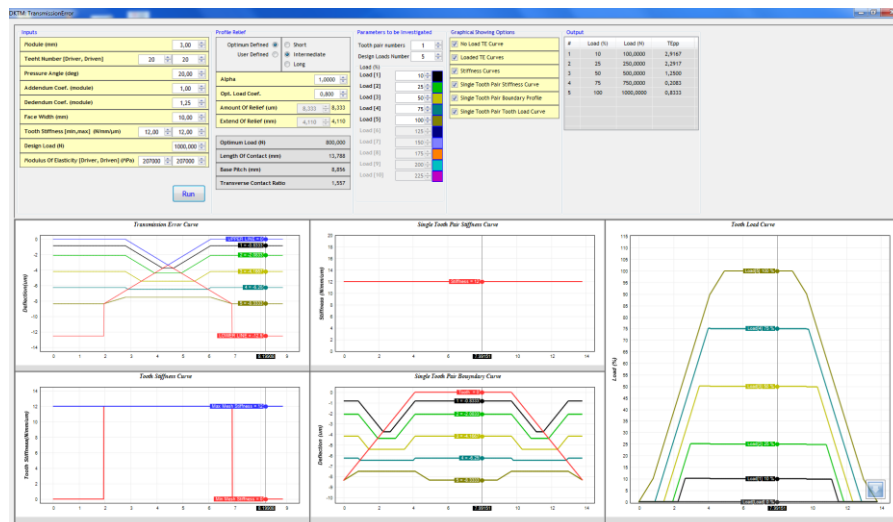


Figure 6.7 General view of in-house software

The 'Run' dialog box contains the following input parameters:

- Module (mm): 3,000
- Teeth Number [Driver, Driven]: 40, 40
- Pressure Angle (deg) [Drive, Coast]: 20,000, 20,000
- Addendum Coef. (mm): 1,000
- Dedendum Coef. (mm): 1,250
- Hob Tip Radius Coef. (mm) [Drive, Coast]: 0,300, 0,300
- Profile Shift Coef. (mm) [Driver, Driven]: 0,000, 0,000
- Face Width (mm): 20,000
- Modulus Of Elasticity (MPa) [Driver, Driven]: 207000, 207000
- Posion's Ratio (MPa) [Driver, Driven]: 0,300, 0,300
- Design Load (N): 7484,790
- Angular Velocity Of Driver (rpm): 1400

Figure 6.8 Software interface for input parameters for gear micro profile design

The 'Tooth Pair Stiffness' section has two radio buttons: 'From Geometry' (selected) and 'From MinMax Value'. Below it, 'Stiffness Values (GPa) [Kmin, Kmax]' are set to 10,00 and 10,00.

The 'Relief Inputs' section contains the following parameters:

- Optimum Load (N): 7484,790
- Length Of Contact (mm): 15,176
- Base Pitch (mm): 8,856
- Transverse Contact Ratio: 1,714

Figure 6.9 Software interface for stiffness and relief parameters

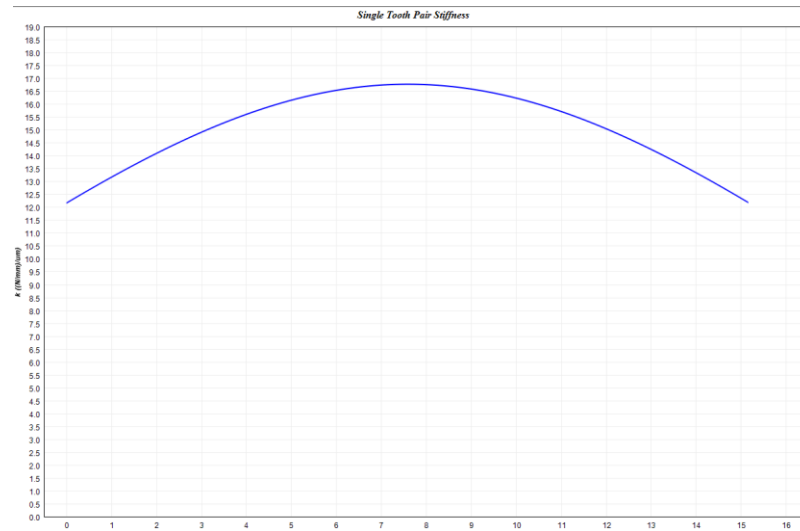


Figure 6.10 Tooth pair stiffness curve (in-house software)

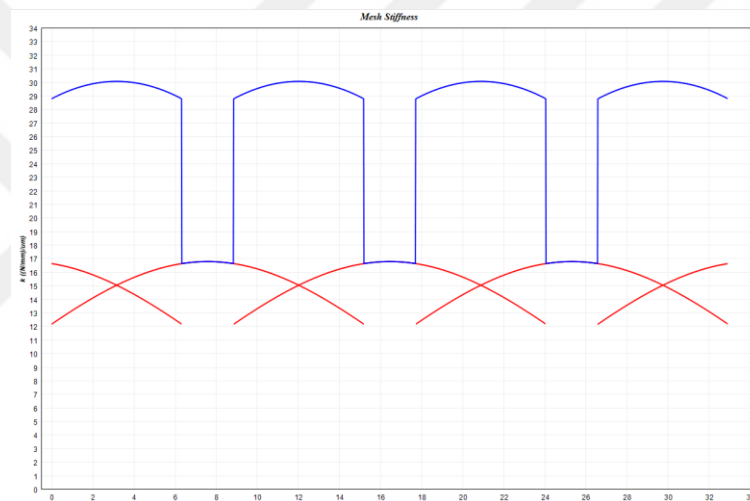


Figure 6.11 Mesh stiffness curve (in-house software)

Relief Inputs		Relief Inputs	
Optimum Load (N)	7484,790	Optimum Load (N)	7484,790
Length Of Contact (mm)	15,176	Length Of Contact (mm)	15,176
Base Pitch (mm)	8,856	Base Pitch (mm)	8,856
Transverse Contact Ratio	1,714	Transverse Contact Ratio	1,714
☒ Optimum Defined ☐ User Defined	☐ Short ☐ Intermediate ☒ Long	☐ Optimum Defined ☒ User Defined	☐ Short ☐ Intermediate ☒ Long
Alpha	1,000	Alpha	1,000
Opt. Load Coef.	1,000	Opt. Load Coef.	1,000
Amount Of Relief (um)	22,527	Amount Of Relief (um)	22,527
Extend Of Relief (mm)	6,319	Extend Of Relief (mm)	6,319

Figure 6.12 Software interface for relief design alternatives

Parameters to be Investigated

Tooth pair: 3

Design Loads: 5

Load (%)

Load [1]	100,00	Load [6]	0,00
Load [2]	75,00	Load [7]	0,00
Load [3]	50,00	Load [8]	0,00
Load [4]	25,00	Load [9]	0,00
Load [5]	0,00	Load [10]	0,00

Graphical Showing Options

Outputs

#	Load (%)	Load (N)	TEpp
1	100	7484,7...	1,4337
2	75	5613,5...	3,9005
3	50	3742,3...	6,3672
4	25	1871,1...	8,8340
5	0	0,0000	11,2743

Figure 6.13 Software interface for design load percentage and PPTE values for these loads

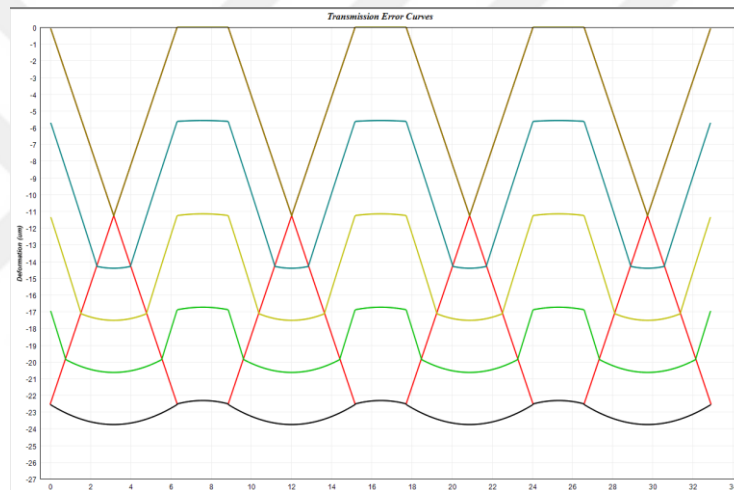


Figure 6.14 Transmission error curves for different load values (in-house software)

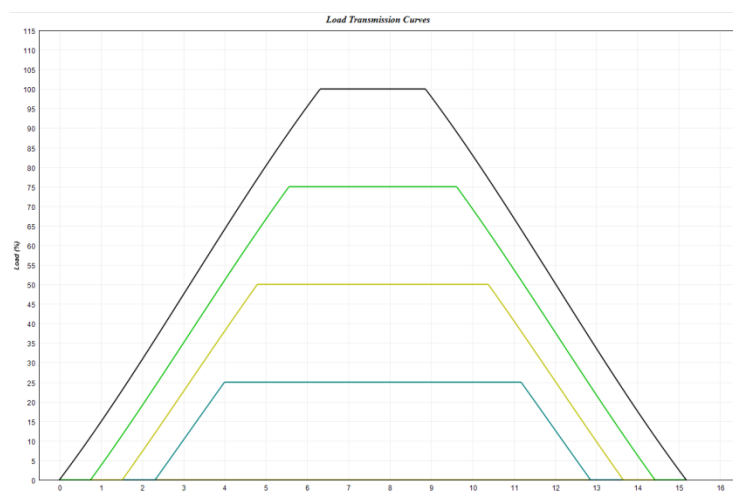


Figure 6.15 Load diagrams (in-house software)

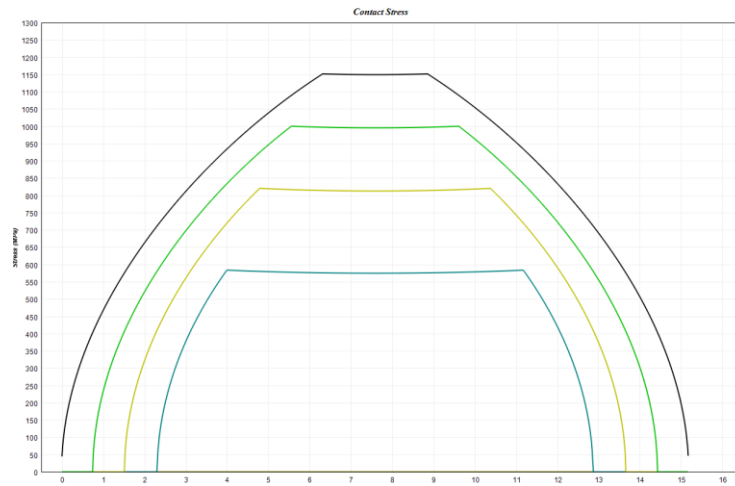


Figure 6.16 Contact stress (in-house software)

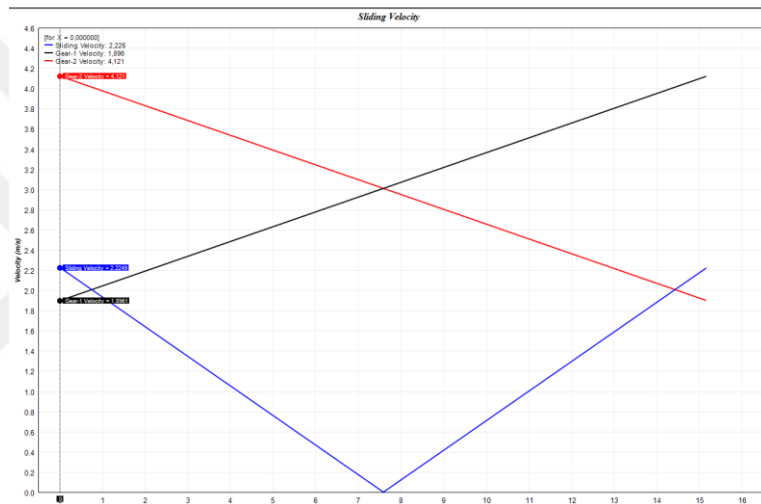


Figure 6.17 Sliding velocity curves (in-house software)

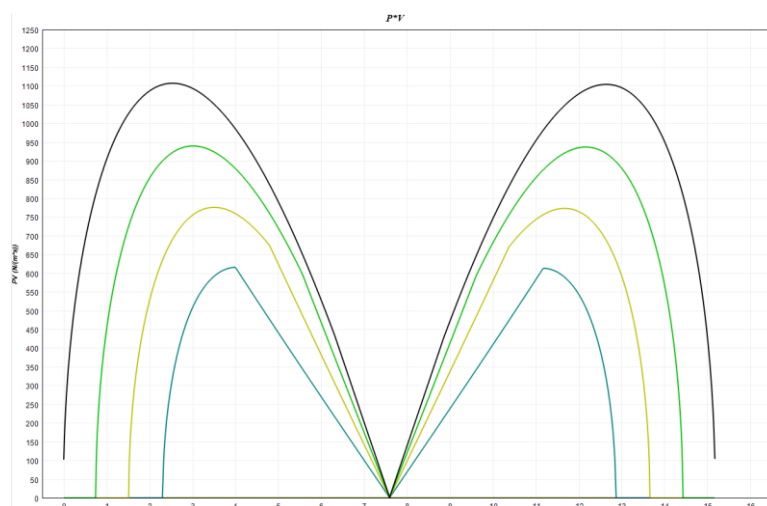


Figure 6.18 PxV curves (in-house software)

Up to now, TE theory, a well-accepted method of Harris Map to estimate TE values and construction of TE curves under different loads are explained. Then developed in-house software based on this significant and valuable knowledge is introduced by supporting some illustrative figures given above.

In addition to these all, FEA analyses to obtain TE curves are carried out in part of the study. Procedure followed for analyses of mapped meshed gear pairs are explained by supporting figures. One-tooth gear model (Figure 6.19) was used to evaluate stiffness along length of contact whereas five-teeth-model (Figure 6.20) was preferred for quasi static transmission error analysis.

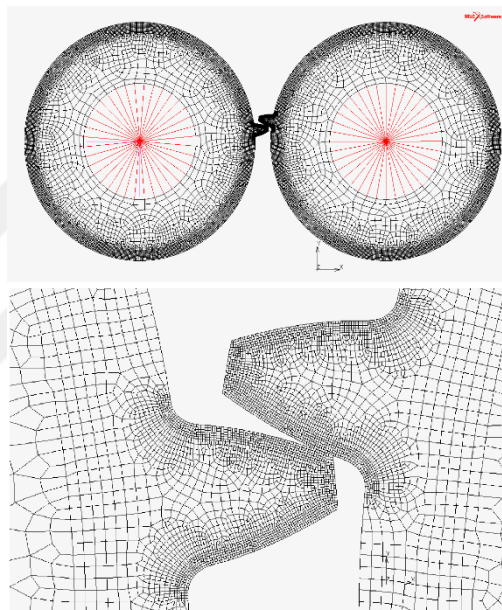


Figure 6.19 One-tooth gear finite element model

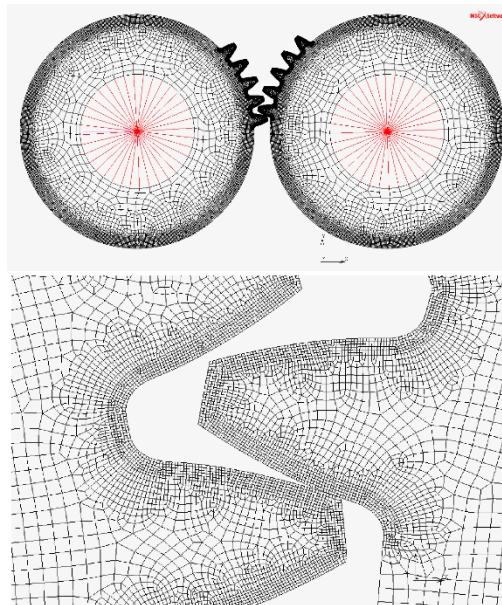


Figure 6.20 Five-tooth gear finite element model

6.3 Case Studies

In this study, static transmission error (STE) is analyzed for intended micro profile design of symmetrical and asymmetrical tooth external spur gear pairs. TE curves are drawn and PPTE values are evaluated based on Harris map and FEA works.

6.3.1 Case Studies of Symmetric Tooth Profile Spur Gear Pairs

Transmission error analyses for symmetrical tooth profile spur gear pairs have been carried out and presented in this chapter. Some of case studies were performed for analytical prediction based on Harris Map (in-house software) and numerical analyses (FEA). Some comparisons between these different evaluation methods and sometimes with literature works have been presented under this section. Abbreviations for case studies of symmetric profile external spur gears are given in Table 6.1.

Table 6.1 Abbreviation for case studies of symmetric spur gears

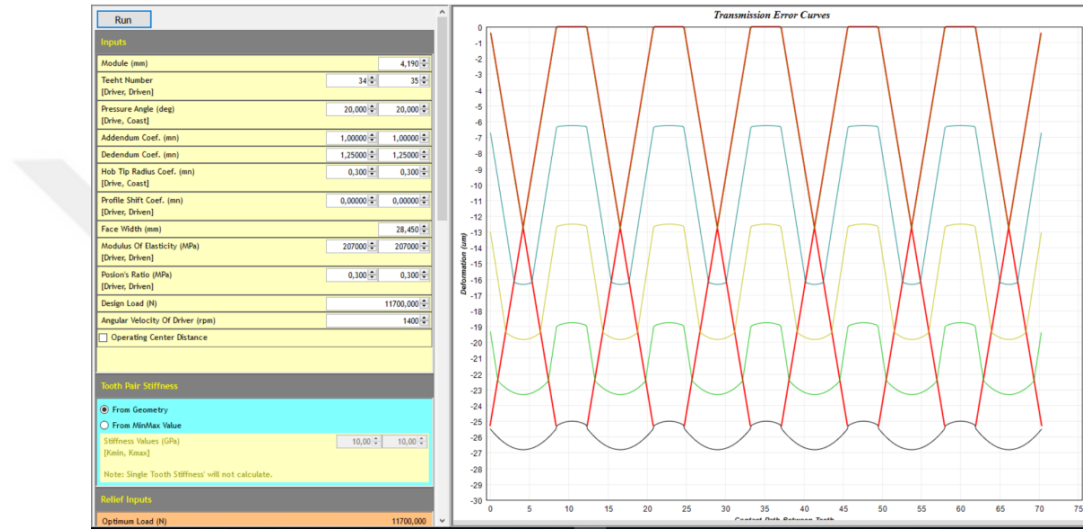
Cases	DXXCXX	Relief Type
S13	D20C20	Long relief
S14	D20C20	Long relief
S15	D20C20	No relief
S16	D25C25	No relief
S17	D30C30	No relief
S18	D20C20	Long relief
S19	D25C25	Long relief
S20	D20C20	Long relief
S21	D20C20	Short relief
S22	D20C20	Long relief

6.3.1.1 Case S13 (D20C20 – Long Relief)

Tavakoli et al [107] gave peak to peak transmission error (PPTE) value as 4.8 μm in their study. In-house software estimated it as 4.82 μm for 100 % of design load (11,7kN) based on required parameters given in Table 6.2. Loads were selected as percentage of 25%, 50%, 75%, and % 100 of design load in reference study. TE curves are given in Figure 6.21 for each load values. PPTE values calculated by in-house software are tabulated in Table 6.3.

Table 6.2 General gear parameters of Case S13 (D20C20 – Long Relief)

Parameter	Value	Unit
Pressure angle (drive/coast)	20/20	Degree
Normal module	4.19	mm
Pinion tooth number	34	-
Gear tooth number	35	-
Design load	11700	Newton
Face width	28.45	mm
Amount of relief	26	μm
Extent of relief	10	mm

**Figure 6.21** TE curve for Case S13 (D20C20 – Long Relief) (in-house software)**Table 6.3** PPTE values for Case S13 (D20C20 – Long Relief) (in-house software)

Output			
#	Load (%)	Load (N)	TEpp
1	25	2925,0241	12,4622
2	50	5850,0481	9,9159
3	75	8775,0722	7,3697
4	100	11700,0962	4,8235

6.3.1.2 Case S14 (D20C20 – Long Relief)

In this case, results of reference paper [102] and analytical prediction based on in-house software were compared. Gear parameters for transmission error analysis carried out in the study of Yıldırım et al [102] are presented in Table 6.4. PPTE value was given as 1.00 μm in reference study while 1.05 μm has been calculated in current study. TE curves for design and off-design loads are presented in Figure 6.22 and 6.23

for current and reference study respectively. PPTE values in reference paper [102] (measured and predicted) and predicted by current study (based on Harris map) are very close to each other (with a difference of 0.05 μm).

Table 6.4 General gear parameters of Case S14 (D20C20) [102]

Parameter	Value	Unit
Pressure Angle (drive/coast)	20/20	Degree
Normal module	6.35	mm
Tooth number	32	-
Gear tooth number	32	-
Design load	300	N/mm
Face width	20.0	mm
Relief type	Long Relief	-

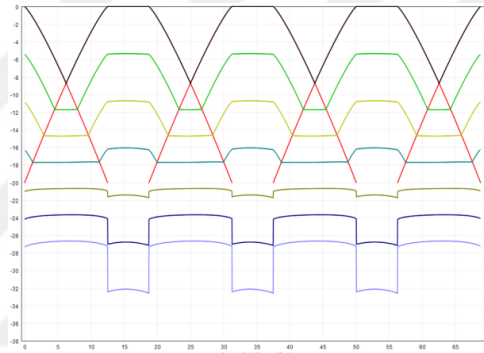


Figure 6.22 TE curves for Case S14 (D20C20) (in-house software)

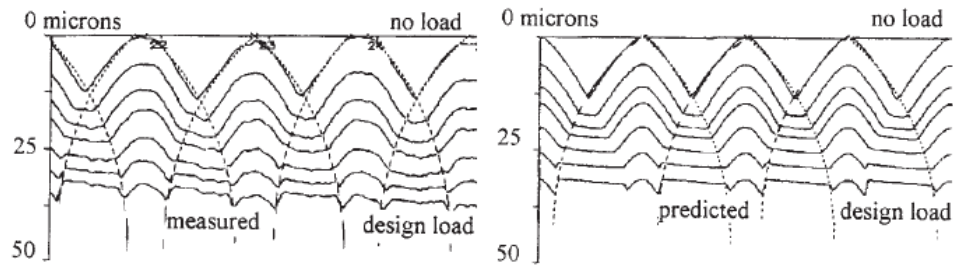


Figure 6.23 Measured and predicted TE curves of Case S14 (D20C20) [102]

6.3.1.3 Case S15-S16-S17 (Without Relief)

Several case studies were performed to evaluate tooth pair stiffness and by depending on those values, transmission error for different symmetric profile spur gears with no relief were determined by using in house software and finite element analysis tools. General gear parameters for gear pairs of Cases of S15, S16 and S17 are given in Table 6.5. Pressure angles, design loads and length of contact for each case are given in Table 6.6. Minimum and maximum tooth pair stiffness values are presented in Table 6.7

based on finite element analysis and software for cases. Meshing position in curves obtained by FEA was illustrated as contact position numbers (1, 2 and 3 refers to start of contact, pitch point and end of contact respectively) instead of values in Figure 6.24. Because length of contact values vary for different gear profiles by depending on pressure angle change.

The maximum differences are 7.97% and -2.92% for maximum and minimum stiffness values calculated by finite element analysis and software (Cases of S15, S16 and S17). Transmission error curves derived by both tools are very similar in terms of regime given in Figure 6.25-6.26 (Case S15), Figure 6.27-6.28 (Case S16) and Figure 6.29-6.30 (Case S17). PPTE values are presented in Table 6.8, 6.9 and 6.10 for same cases. PPTE is minimum for no relief gear profile when no load is transmitted whereas it increases with increasing load. The maximum difference is at level of 1.0 micron between software and FEA calculations for stiffness calculations.

Table 6.5 General gear parameters of Case S15, S16 and S17

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Design torque	160.43	Nm
Face width	20	mm

Table 6.6 Gear parameters of Case S15, S16 and S17

Parameter	Value			Unit
	S15	S16	S17	
Pressure angle (drive/coast)	20	25	30	degree
Design load	2845.46	2950.06	3087.28	N
Length of contact	15.18	12.91	11.25	mm

Table 6.7 Minimum and maximum tooth pair stiffness values of Case S15-S16-S17

Cases	Stiffness (N/mm/ μ m)			
	FEA		In-house software	
	min	max	min	max
S15	11.71	16.82	11.41	15.69
S16	13.33	19.14	13.12	19.39
S17	14.70	21.70	14.27	23.43

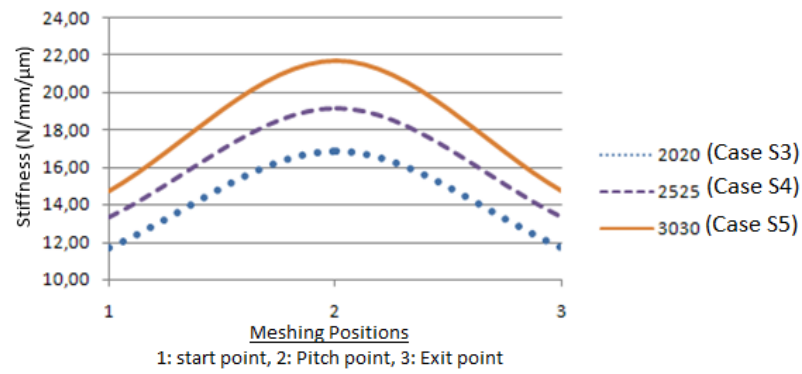


Figure 6.24 Stiffness curves of Case S15, S16 and S17 (FEA)

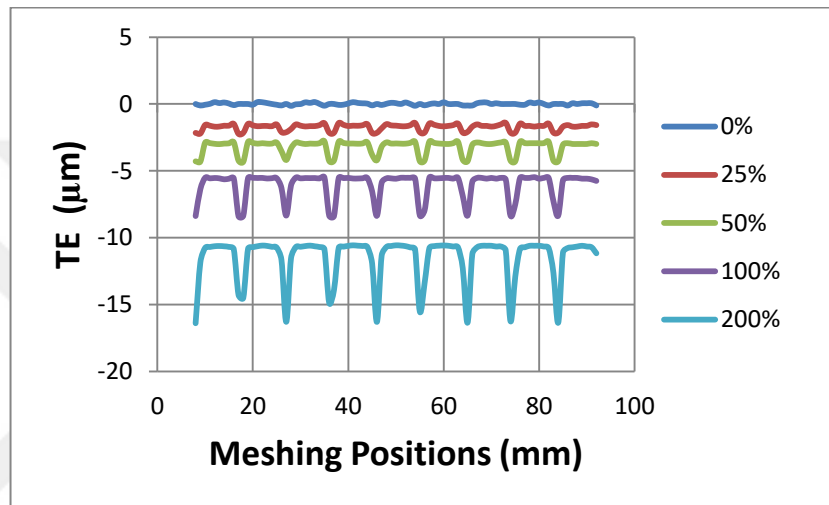


Figure 6.25 TE curves of Case S15 (D20C20) (FEA)

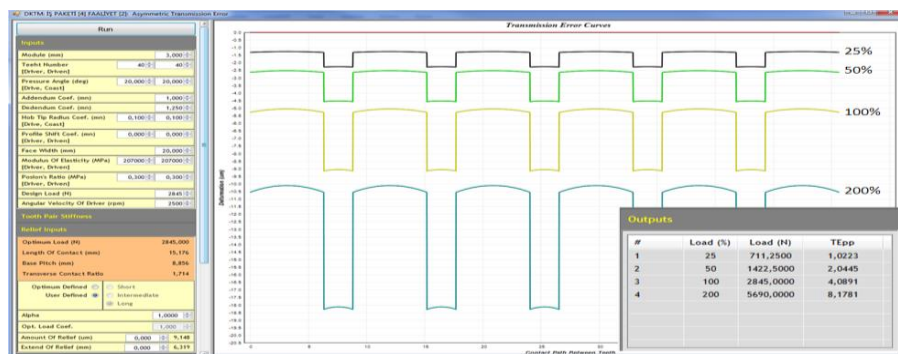


Figure 6.26 TE curves of Case S15 (D20C20) (in-house software)

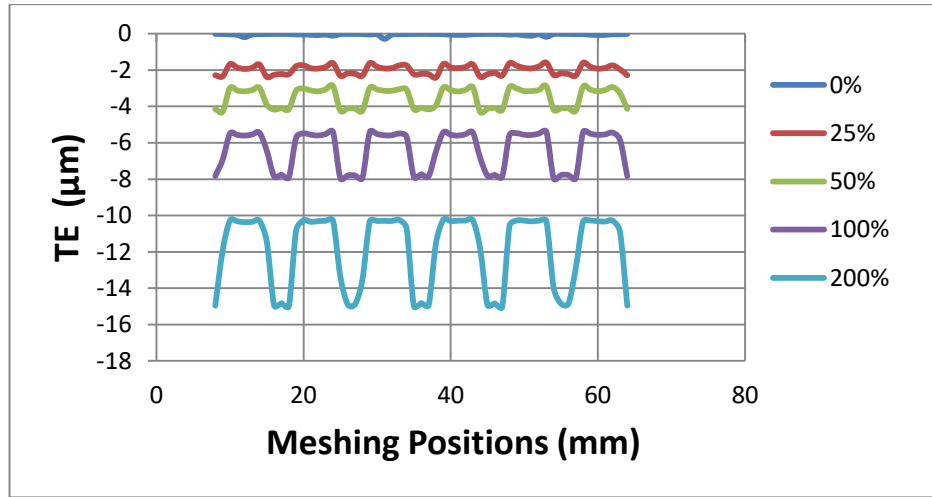


Figure 6.27 TE curves of Case S16 (D25C25) (FEA)

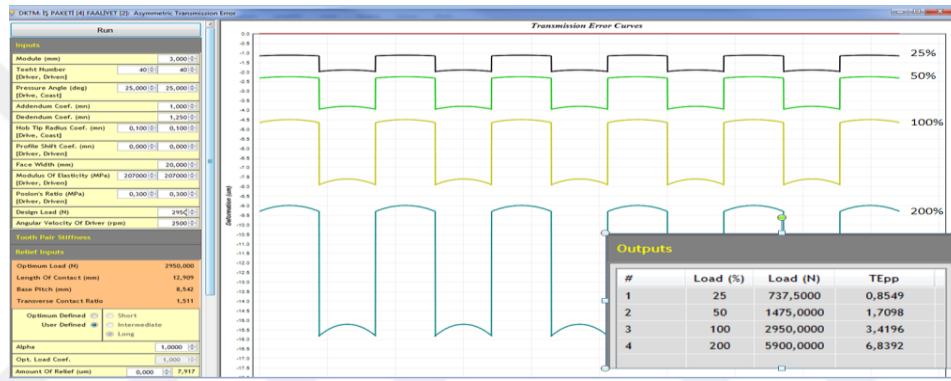


Figure 6.28 TE curves of Case S16 (D25C25) (in-house software)

Table 6.8 PPTE values of Case S15 (D20C20)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0.0000	0.2875	-0.2875
25	1.0223	0.9529	0.0694
50	2.0445	1.6632	0.3813
100	4.0891	3.0389	1.0502

Table 6.9 PPTE values of Case S16 (D25C25)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0.0000	0.2694	-0.2694
25	0.8549	0.9682	-0.1133
50	1.7098	1.5711	0.1387
100	3.4196	2.7183	0.7013

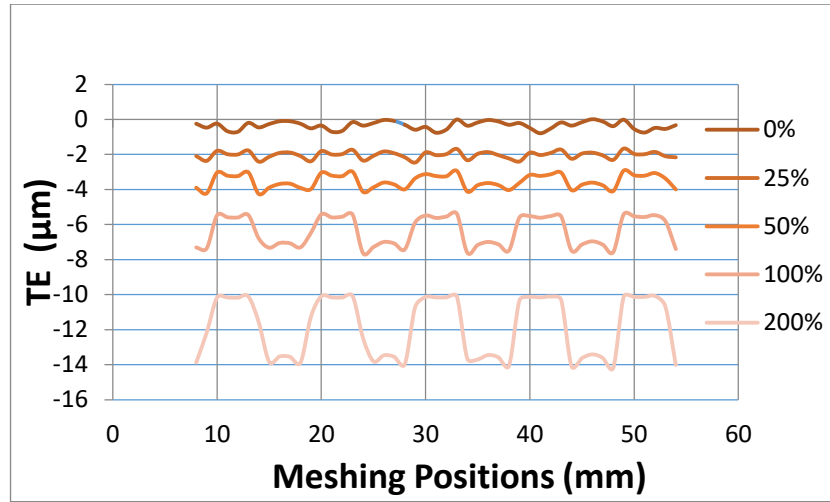


Figure 6.29 TE curves of Case S17 (D30C30) (FEA)

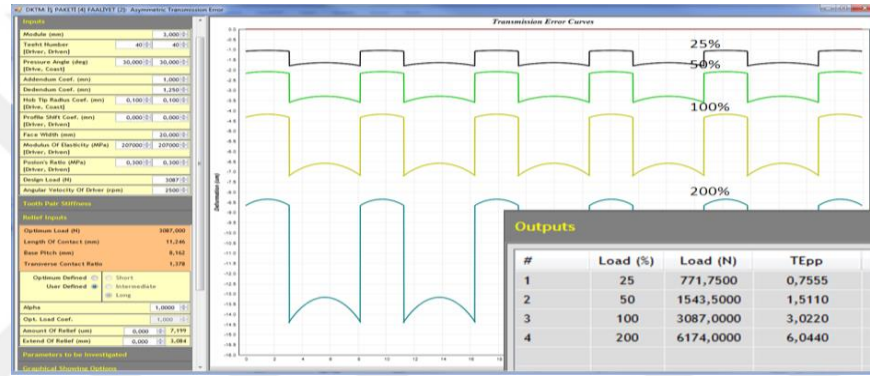


Figure 6.30 TE curves of Case S17 (D30C30) (in-house software)

Table 6.10 PPTE values of Case S17 (D30C30)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0.0000	0.8002	-0.8002
25	0.7555	0.9561	-0.2006
50	1.511	1.3614	0.1496
100	3.022	2.4578	0.5642

6.3.1.4 Case S18 (D20C20 - Long Relief) - S19 (D25C25 - Long Relief)

Similar gear pairs as given in Case S15 and S16 but with long relief in this time were analyzed for quasi static transmission error. General gear parameters for macro and micro geometry are presented in Table 6.11 and 6.12. Quasi static transmission error curves under different design loads are given in Figures of 6.31-6.34 whereas PPTE values are presented in Tables of 6.13 and 6.14. It can be deduced from tables presented below; the maximum difference for calculated PPTE values by software and FEA are 0.58 micron and 0.30 micron for Case S18 and S19 respectively. Software

predicts generally bigger values for Case S18 whereas FEA gives higher values for Case S19.

Table 6.11 General gear parameters of Case S18 and S19

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Design torque	160.43	Nm
Face width	20	mm

Table 6.12 Pressure angle and relief parameters of Case S18 and S19

Parameter	Value		Unit
	S18	S19	
Pressure angle (drive/coast)	20/20	25/25	degree
Amount of relief	9.128	7.917	μm

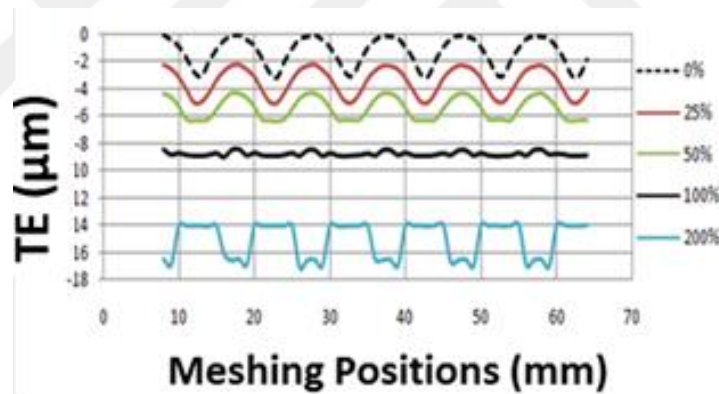


Figure 6.31 TE curves of Case S18 (D20C20) (FEA)

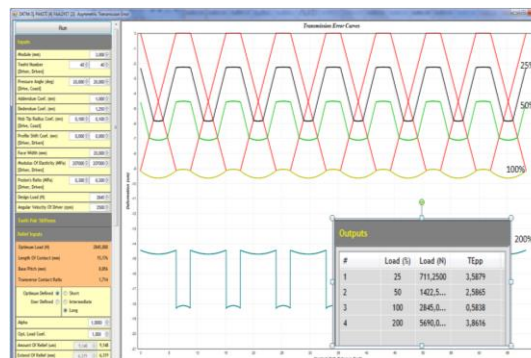


Figure 6.32 TE curves of Case S18 (D20C20) (in-house software)

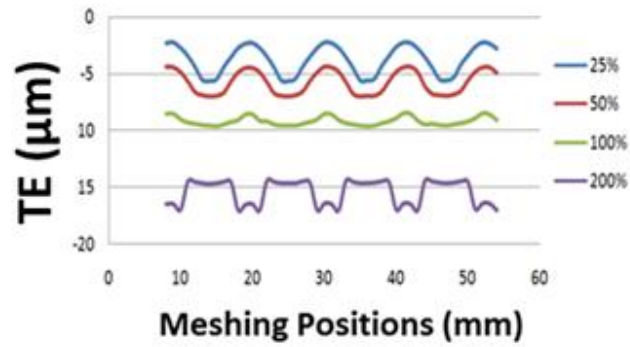


Figure 6.33 TE curves of Case S19 (D25C25) (FEA)

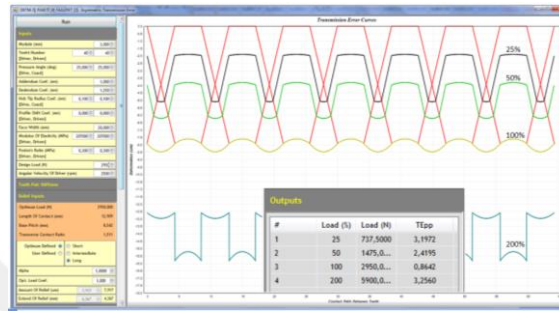


Figure 6.34 TE curves of Case S19 (D25C25) (in-house software)

Table 6.13 PPTE values of Case S18 (D20C20)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	3.588	3.200	0.388
50	2.586	2.000	0.586
100	0.584	0.700	-0.116

Table 6.14 PPTE values of Case S19 (D25C25)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	3.197	3.500	-0.303
50	2.419	2.500	-0.081
100	0.864	0.900	-0.036

6.3.1.5 Case S20 (D20C20 – Long Relief) – S21 (D20C20– Short Relief)

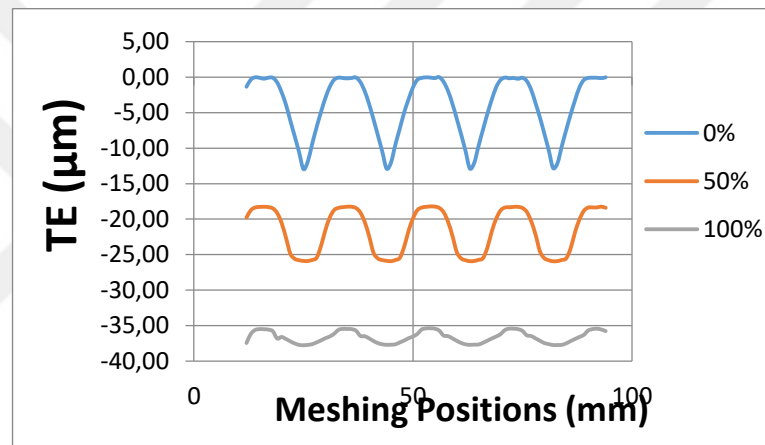
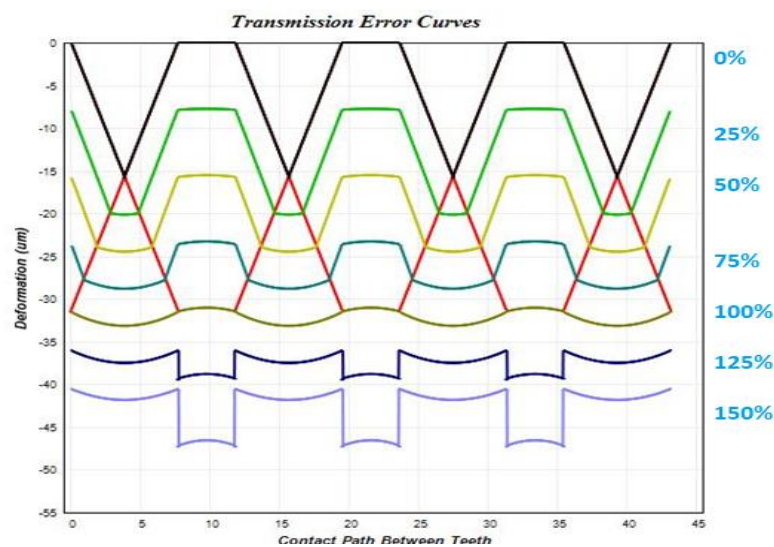
For cases of S20 and S21, same gear pair with different relief types is analyzed for quasi static transmission error. General gear parameters for macro and micro geometries are presented in Table 6.15 and 6.16. Quasi static transmission error curves under different design loads are given in Figures of 6.35-6.38. Peak to peak transmission error are presented in Table 6.17 and 6.18.

Table 6.15 General gear parameters of Case S20-S21

Parameter	Value	Unit
Normal module	4	mm
Teeth number of pinion	30	-
Pressure angle	20	Degree
Gear ratio	1	-
Face width	15	mm

Table 6.16 Relief parameters of Case S20-S21

Parameter	Value		Unit
	S20	S21	
Relief type	Long	Short	-
Amount of relief	31.46	31.46	μm
Extent of relief	7.17	3.89	mm

**Figure 6.35** TE curves of Case S20 (D20C20 - Long relief) (FEA)**Figure 6.36** TE curves of Case S20 (D20C20-Long relief) (in-house software)

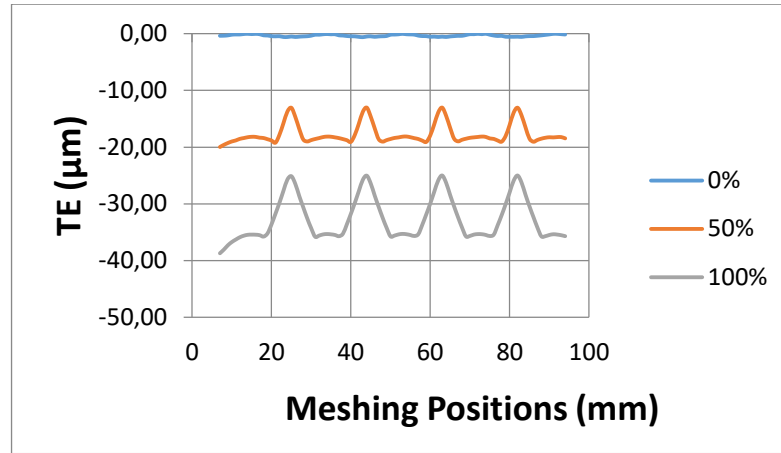


Figure 6.37 TE curves of Case S21 (D20C20 - Short relief) (FEA)

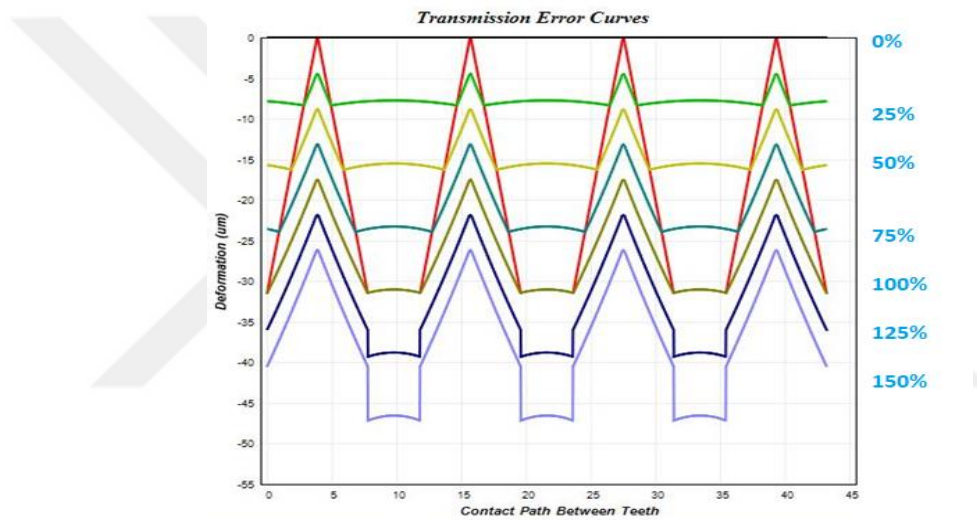


Figure 6.38 TE curves of Case S21 (D20C20-Short relief) (in-house software)

Table 6.17 PPTE values of Case S20 (D20C20 - Long relief)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	15.760	12.950	2.81
50	8.956	7.730	1.23
100	2.114	2.390	-0.27

Table 6.18 PPTE values of Case S21 (D20C20 - Short relief)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0	0.6	-0.60
50	7.500	6.190	1.31
100	13.980	10.900	3.08

6.3.1.6 Case S22 (D20C20 - Long Relief)

In this section, a symmetric spur gear has been designed for minimum transmission error under design load (100% of load). General gear parameters and calculated relief parameters for minimum value of PPTE (by in-house software) are given in Table 6.19. Transmission error curves are presented in Figure 6.39 and 6.40.

Table 6.19 Gear parameters of Case S22 (D20C20)

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm
Relief type	Long	-
Amount of relief	30	μm
Extent of relief	6.319	mm

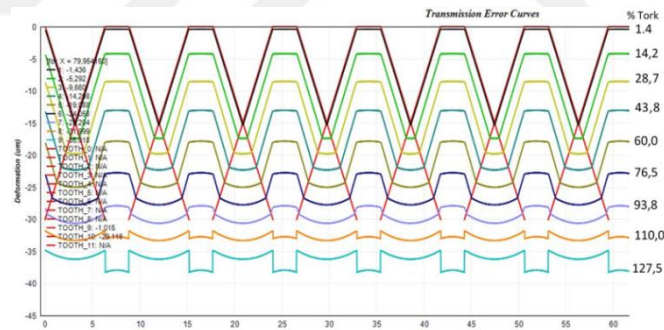


Figure 6.39 TE curves of Case S22 (D20C20-Long relief) (in-house software)

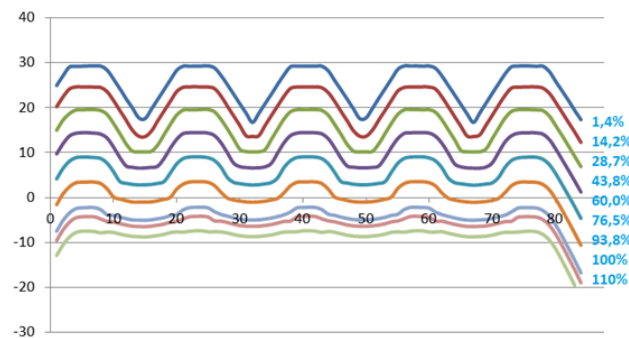


Figure 6.40 TE curves of Case S22 (D20C20 – Long relief) (FEA)

6.3.2 Case Studies of Asymmetric Tooth Profile Spur Gear Pairs

Asymmetric profile spur gears have been analyzed for transmission error; transmission error curves have been constructed and peak to peak transmission error (PPTE) values have been evaluated. Abbreviation for case studies of asymmetric profile external spur gears is given in Table 6.20. In this section, several case studies were performed to evaluate transmission error values for asymmetric profile spur gears with no relief and with relief by using in-house software and finite element analysis tools.

Table 6.20 Abbreviation for case studies of asymmetric spur gears

Cases	DXXCXX	Relief Type
A12	D20C25	No Relief
A13	D20C30	No Relief
A14	D25C20	No Relief
A15	D30C20	No Relief
A16	D20C25	Long Relief
A17	D20C30	Long Relief
A18	D25C20	Long Relief
A19	D30C20	No Relief
A20	D30C20	Long Relief
A21	D30C20	Short Relief
A22	D30C20	Long Relief
A23	D20C30	Long Relief

6.3.2.1 Case A12, A13, A14 and A15 (Without Relief)

Asymmetric profile spur gears with no relief were firstly analyzed for micro profile in this section. Gear parameters are given in Table 6.21 and 6.22 for gear pairs of Case A12, A13, A14 and A15. Minimum and maximum tooth pair stiffness values are presented in Table 6.23 based on finite element analysis and in-house software. Stiffness curves are presented for FEA in Figure 6.41.

Transmission error curves of asymmetric profile spur gears without relief derived by in-house software and finite element analysis are very similar in terms of regime given in Figure 6.42-6.43, 6.44-6.45, 6.46-6.47 and 6.48-6.49. PPTE values are presented in Table 6.24, 6.25, 6.26 and 6.27 for Cases of A12, A13, A14 and A15 respectively.

Table 6.21 General gear parameters of Case A12, A13, A14 and A15

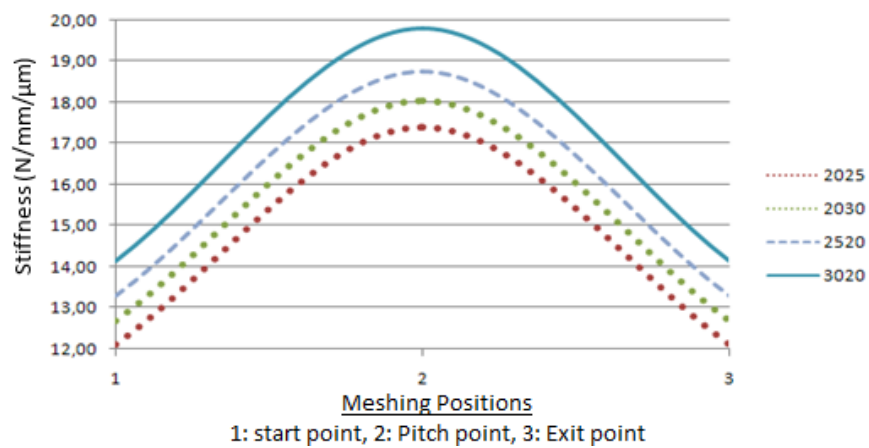
Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Design torque	160.43	Nm
Face width	20	mm

Table 6.22 Gear parameters of Case A12, A13, A14 and A15

Parameter	Cases				Unit
	A12	A13	A14	A15	
Pressure angle (drive/coast)	20/25	20/30	25/20	30/20	Degree
Relief type	No	No	No	No	-
Design load	2845.26	2845.26	2950.06	3087.28	N
Length of contact	15.18	15.18	12.91	11.25	mm

Table 6.23 Tooth pair stiffness values for Case A12, A13, A14 and A15

Stiffness (N/mm/ μ m)				
Cases	FEA		In-house software	
	min	max	min	max
A1	12.10	17.40	11.65	16.00
A2	12.67	18.02	11.88	16.30
A3	13.31	18.76	12.88	19.08
A4	14.13	19.80	13.77	22.84

**Figure 6.41** Stiffness curves of Case A12, A13, A14 and A15 (FEA)

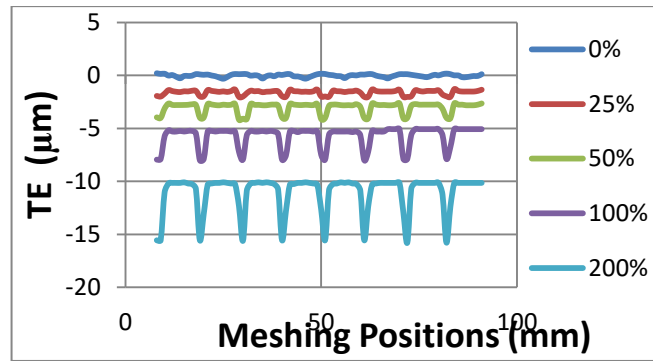


Figure 6.42 TE curves of Case A12 (D20C25-No relief) (FEA)

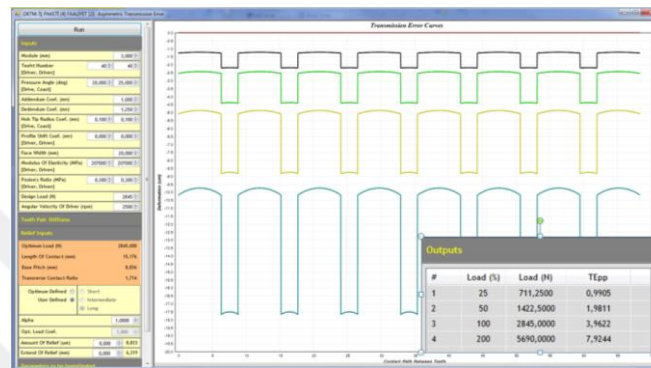


Figure 6.43 TE curves of Case A12 (D20C25-No relief) (in-house software)

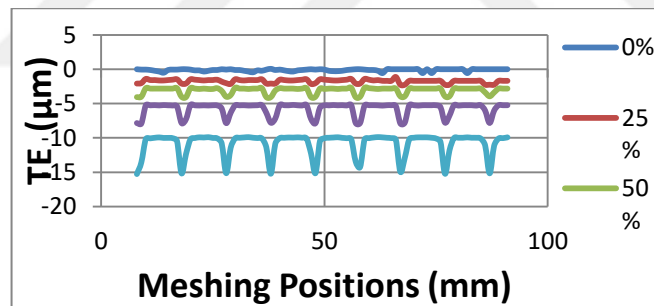


Figure 6.44 TE curves of Case A13 (D20C30-No relief) (FEA)

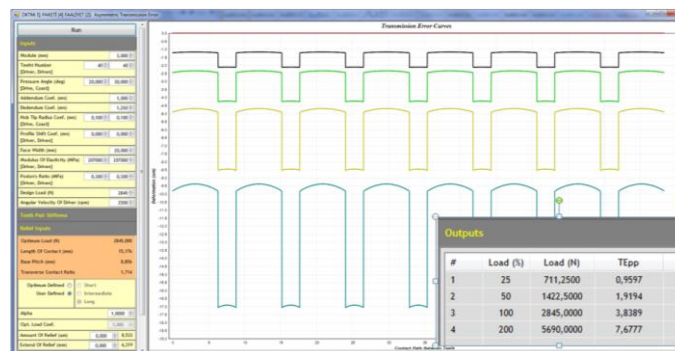


Figure 6.45 TE curves of Case A13 (D20C30-No relief) (in-house software)

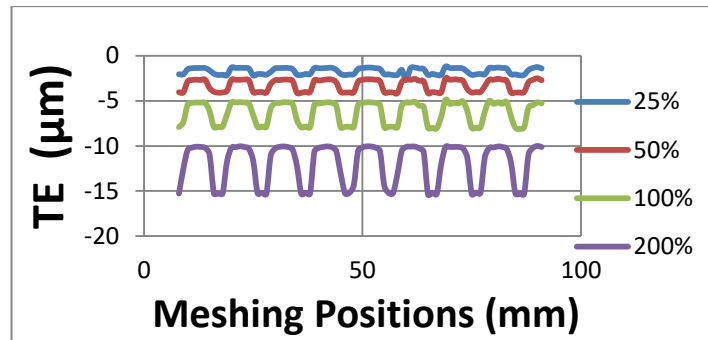


Figure 6.46 TE curves of Case A14 (D25C20-No relief) (FEA)

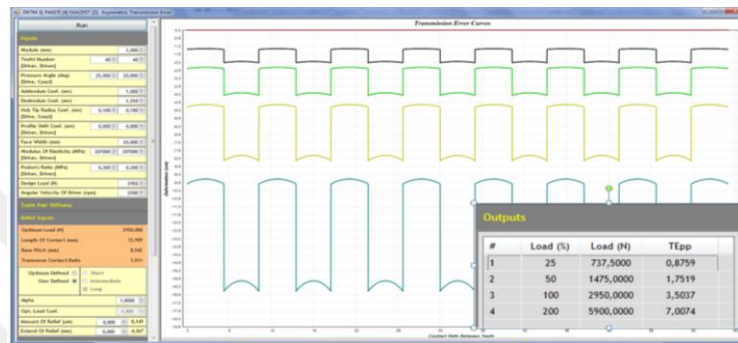


Figure 6.47 TE curves of Case A14 (D25C20-No relief) (in-house software)

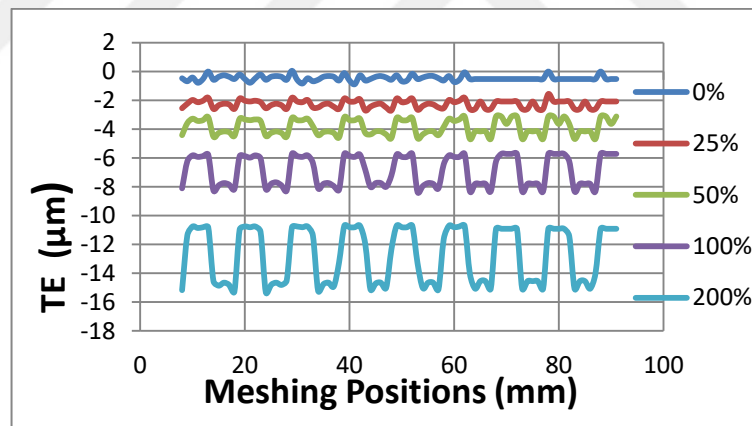


Figure 6.48 TE curves of Case A15 (D30C20-No relief) (FEA)

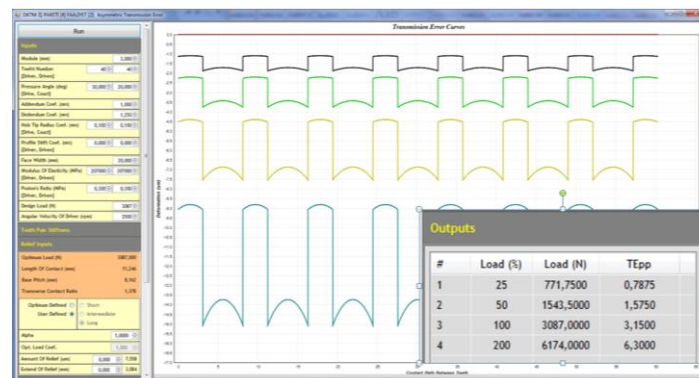


Figure 6.49 TE curves of Case A15 (D30C20-No relief) (in-house software)

The difference between peak to peak transmission error values by software and FEA is not too much bigger than 1.0 micron here.

Table 6.24 PPTE values of Case A12 (D20C25)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	0.9905	0.7386	0.2519
50	1.9811	1.5336	0.4475
100	3.9622	2.9318	1.0304

Table 6.25 PPTE values of Case A13 (D20C30)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	0.9597	1.1276	-0.1679
50	1.9194	1.3531	0.5663
100	3.8389	2.7176	1.1213

Table 6.26 PPTE values of Case A14 (D25C20)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	0.8759	1.4900	-0.6141
50	1.7519	1.6422	0.1097
100	3.5037	3.2083	0.2954

Table 6.27 PPTE values of Case A15 (D30C20)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	0.7875	1.1432	-0.3557
50	1.5750	1.5589	0.0161
100	3.1500	2.6500	0.4999

6.3.2.2 Case A16, A17 and A18 (With Long Relief)

Asymmetric profile spur gears with long relief were designed for minimum transmission error in this section. General gear parameters and relief parameters for each case are presented in Table 6.28 and 6.29. Transmission error curves of asymmetric profile spur gears with long relief derived by in-house software and finite element analysis are very similar in terms of regime given in Figure 6.50-6.51, 6.52-6.53 and 6.54-6.55. Peak to peak transmission error values are presented in Table 6.30, 6.31 and 6.32 for Cases of A16, A17 and A18 respectively.

Table 6.28 General gear parameters of Case A16-A17-A18

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Design torque	160.43	Nm
Face width	20	mm

Table 6.29 Gear parameters of Case A16-A17-A18

Parameter	Cases			Unit
	A16	A17	A18	
Pressure angle (drive/coast)	20/25	20/30	25/20	Degree
Relief type	Long	Long	Long	-
Design load	2845.26	2845.26	2950.06	N
Length of contact	15.18	15.18	12.91	mm
Amount of relief	8.833	8.533	8.149	μm

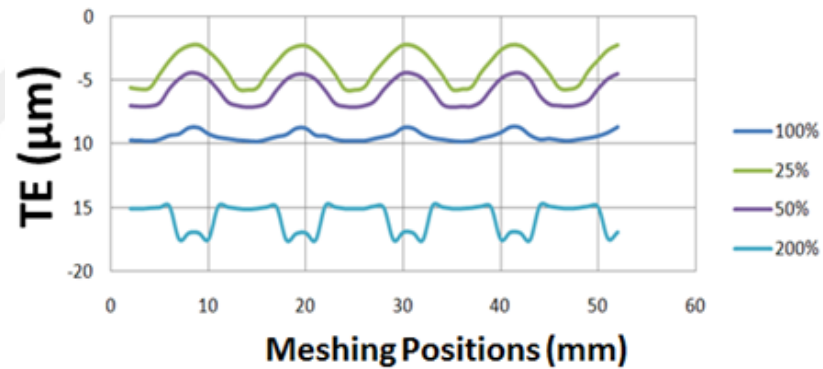


Figure 6.50 TE curves of Case A16 (D20C25-Long relief) (FEA)

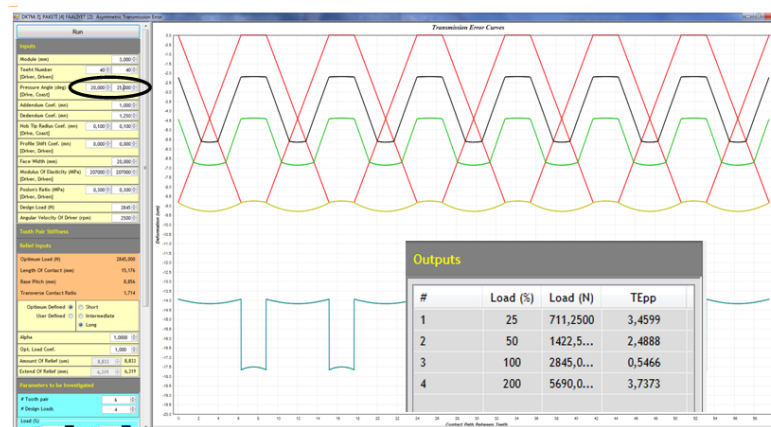


Figure 6.51 TE curves of Case A16 (D20C25-Long relief) (in-house software)

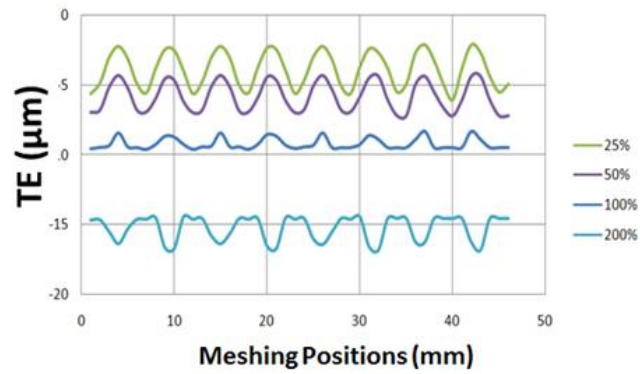


Figure 6.52 TE curves of Case A17 (D20C30-Long relief) (FEA)

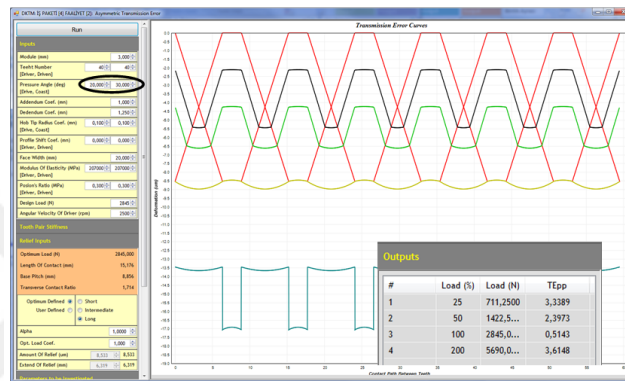


Figure 6.53 TE curves of Case A17 (D20C30-Long relief) (in-house software)

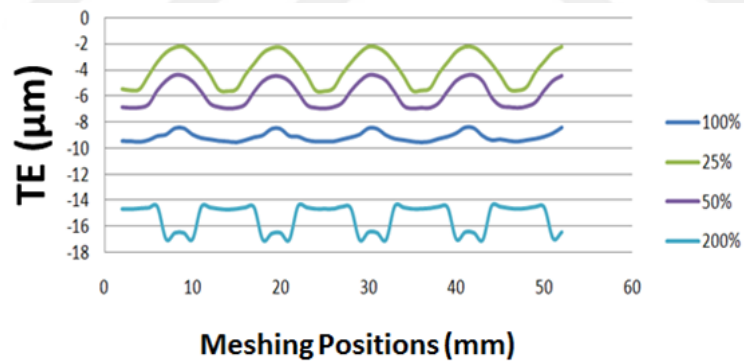


Figure 6.54 TE curves of Case A18 (D25C20-Long relief) (FEA)

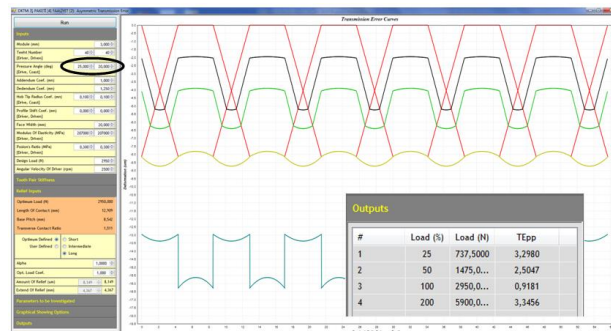


Figure 6.55 TE curves of Case A18 (D25C20-Long relief) (in-house software)

Table 6.30 PPTE values of Case A16 (D20C25)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	3.460	3.500	-0.04
50	2.490	2.500	-0.01
100	0.547	0.600	-0.05

Table 6.31 PPTE values of Case A17 (D20C30)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	3.390	3.900	-0.561
50	2.397	2.500	-0.103
100	0.514	0.500	0.014

Table 6.32 PPTE values of Case A18 (D25C20)

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
25	3.298	3.800	-0.502
50	2.505	2.800	-0.295
100	0.918	1.000	-0.082

6.3.2.3 Case A19 (No Relief), A20 (Long Relief) and A21 (Short Relief)

Asymmetric profile spur gears with long relief, short relief and without relief were designed. General gear parameters and relief parameters for Case A19, A20 and A21 are presented in Table 6.33 and 6.34.

Table 6.33 General gear parameters of Case A19, A20 and A21

Parameter	Value	Unit
Normal module	4	mm
Teeth number of pinion	30	-
Pressure Angle (drive/coast)	30/20	Degree
Gear ratio	1	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm

TE curves of in-house software and FEA are very similar in terms of regime but PPTE values differ from each other. Maximum difference values between both works are

1.24, 3.38 and 1.97 for Case A19 (no relief), A20 (Long relief) and A21 (short relief) respectively. Obtained difference is greater for full design load of Case A19 and A20 whereas bigger difference is gotten for no load condition of Case A21.

Table 6.34 Relief parameters of Case A19, A20 and A21

Parameter	Cases			Unit
	A19	A20	A21	
Pressure angle (drive/coast)	30/20	30/20	30/20	Degree
Relief type	No	Long	Short	-
Amount of relief	0	25.514	25.514	μm
Extent of relief	0	3.843	1.922	mm

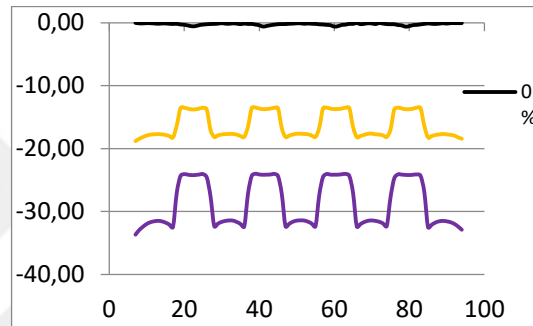


Figure 6.56 TE curves of Case A19 (D30C20-No relief) (FEA)

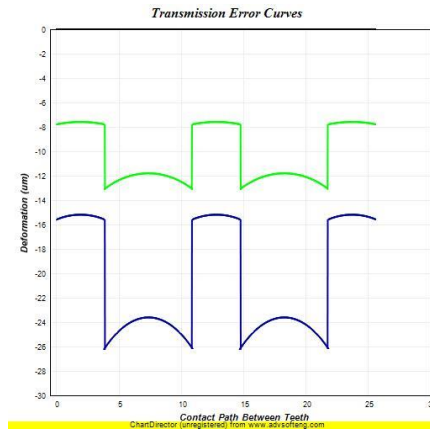


Figure 6.57 TE curves of Case A19 (D30C20-No relief) (in-house software)

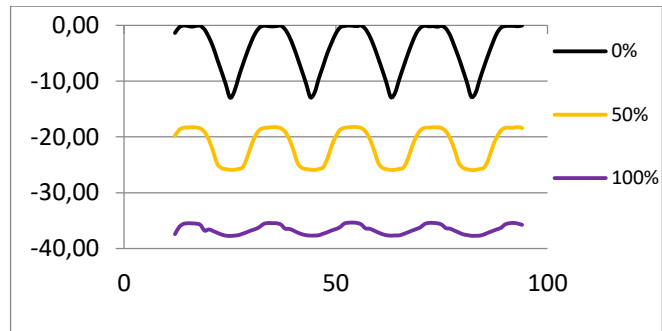


Figure 6.58 TE curves of Case A20 (D30C20-Long relief) (FEA)

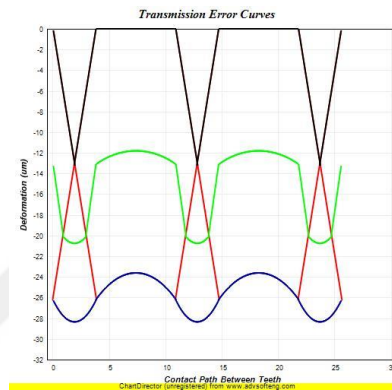


Figure 6.59 TE curves of Case A20 (D30C20-Long relief) (in-house software)

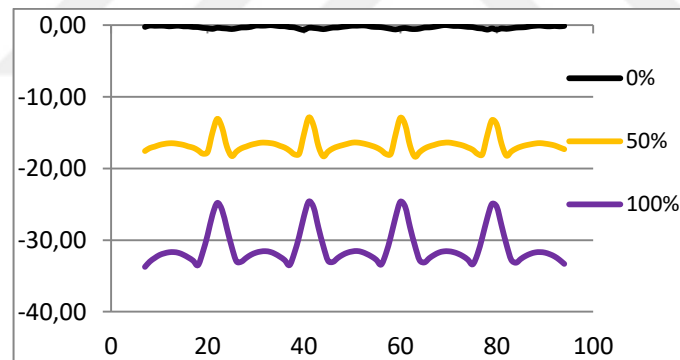


Figure 6.60 TE curves of Case A21 (D30C20-Short relief) (FEA)

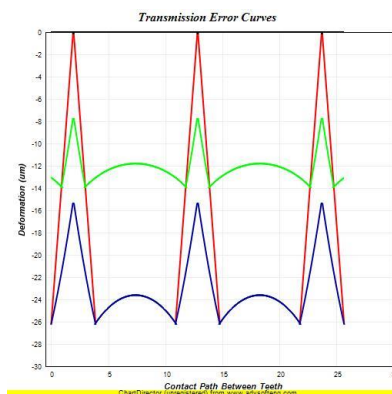


Figure 6.61 TE curves of Case A21 (D30C20-Short relief) (in-house software)

Table 6.35 PPTE values of Case A19

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0	0.57	-0.57
50	5.07	4.97	0.10
100	10.14	8.90	1.24

Table 6.36 PPTE values of Case A20

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	13.15	9.77	3.38
50	8.94	6.51	2.43
100	4.73	2.25	2.48

Table 6.37 PPTE values of Case A21

PPTE (μm)			
Load (%)	In-house software	FEA	Difference (μm)
0	0.12	0.67	-0.55
50	6.2	5.47	0.73
100	10.87	8.90	1.97

6.3.2.4 Case A22

An asymmetrical tooth spur gear pair which has been designed, manufactured and tested in back to back test rig (presented in Chapter 7) has been analyzed for micro profile design. Macro and micro geometry parameters of this gear pair are given in Table 6.38. PPTE values obtained based on Harris Map are presented in Table 6.39.

Table 6.38 General gear parameters of Case A22

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Pressure Angle (drive/coast)	30/20	Degree
Gear ratio	1	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm
Relief type	Long	-
Amount of relief	24.6	μm
Extent of relief	3.084	mm

Table 6.39 PPTE values of Case A22

Torque (Nm)	PPTE (μm) (In-house software)
0	12.33
18	12.07
74	11.05
133	9.99
202	8.76
270	7.53
340	6.32
410	5.04
480	3.75

This gear pair based on given macro and micro geometrical parameters given in Table 6.38 were manufactured and tested. TE values and PPTE curves are presented in later chapter.

6.3.2.5 Case A23

An asymmetrical tooth spur gear pair which is similar to study given in previous case in terms of many of gear pair parameters but differs from it for pressure angle and relief parameters were analyzed in this case study. Macro and micro geometry parameters of this gear pair are given in Table 6.40. TE curves under different percentages of design torque in Figure 6.62 respectively.

Table 6.40 General gear parameters of Case A23

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Pressure Angle (drive/coast)	20/30	Degree
Gear ratio	1	-
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm
Relief type	Long	-
Amount of relief	29.2	μm
Extent of relief	6.319	mm

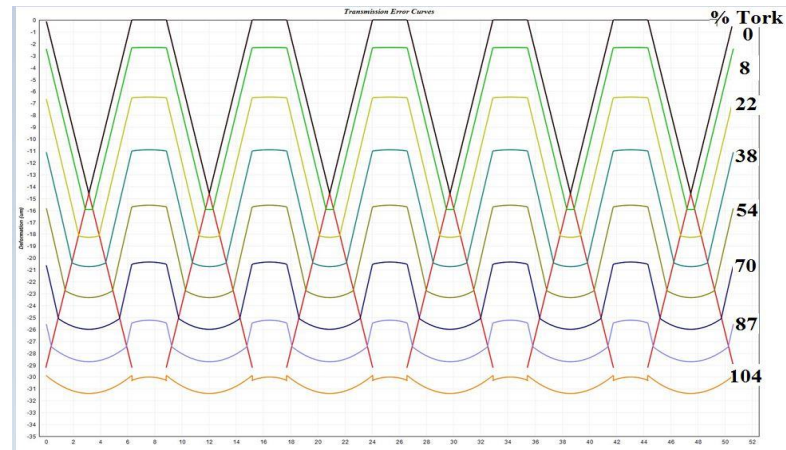


Figure 6.62 TE curves of Case A23 (D20C30-Long relief) (in-house software)

This gear pair was also manufactured and tested by using back to back gear test rig after in chapter 7. PPTE values and TE curves obtained from analytical and experimental works are presented together, compared and discussed in later chapter.

CHAPTER 7

BACK TO BACK GEAR TEST RIG AND TRANSMISSION ERROR MEASUREMENT

7.1 Introduction

In this chapter, a back to back gear test rig (Figure 7.1-7.2) which is already designed, manufactured and assembled in the scope of a distinctive project of gear design for helicopters as a part of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) with the support of Undersecretariat for Defence Industries (SSM) and Turkish Aerospace Industries, Inc. (TAI) [1]. Test rig is explained in detail and some results of measurements conducted by using this test rig in this thesis' works are presented.

7.2 Back to Back Gear Test Rig

Back to back gear test rig consists of two gearboxes (Figure 7.3); first one is test gear box (symmetric and asymmetric profile spur gears can be mounted and tested) and other one is slave gear box with herringbone gear pair. These gearboxes are connected to each other via coupling and torsional rods. In slave gearbox, herringbone gears are preferred to provide more reliable, quieter and less vibration and producing less bearing loads during working.

Test rig allows user to change and mount gears to be tested easily. To provide easy assembly and disassembly, the positions and mounting conditions of gears, shafts, bearings, housings, couplings and measurement devices were taken into consideration during design of gear test rig. Both gear boxes is closed with removable cover to prevent oil spillage during tests. Bearings and housings which are ready to remove are illustrated for test and slave gearboxes in Figure 7.4.

Shafts of test and slave gearboxes are connected to each other through torsional rods (Figure 7.5) and couplings (Figure 7.5-7.6). Torsional rods have low torsional stiffness

so they behave as torsional couplings between shafts of gearboxes and prevent transfer of vibration between gearboxes.

Couplings provide torque transmitting between gearboxes by connecting torsional rods which are in between test and slave gearboxes are presented in Figure 7.5-7.6. These multi-hole couplings have many holes placed in an angular offset order and can provide locking of a specific torque in the system by this means.

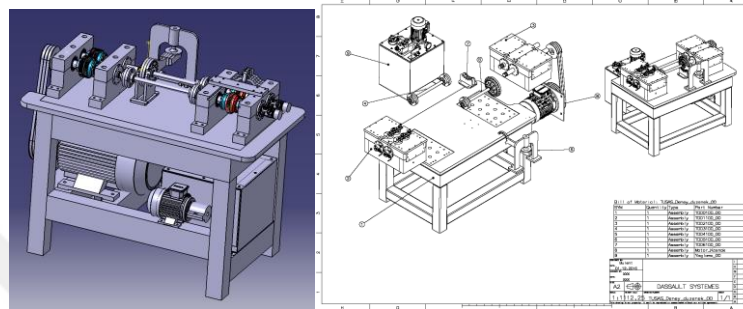


Figure 7.1 General drawings of test rig



Figure 7.2 General view of test rig



Figure 7.3 Test and slave gearboxes of test rig



Figure 7.4 Bearing and housings of test and slave gearboxes

Applying a torque to the system and measuring it is very essential to assure checking performance of the designed gears. Strain gauge measurement ensures applying intended torque for tests. Thus, strain gauges of Micro Measurements CEA-06-062UV-350 Model (Figure 7.7) were used by attaching them on torsional rods (Figure 7.8-7.9).

To calibrate the holes on the couplings in terms of load, strain gauges were firstly calibrated through applying torque by hanging measured weights to a measured distance on torque wrench (Figure 7.10). Calibration results of these gauges are listed in Table 7.1 and calibration graph is presented in Figure 7.11. Conversion factor can be read as 0.596 Nm / micro strain for calibration.



Figure 7.5 Torsional rods and multi-hole couplings

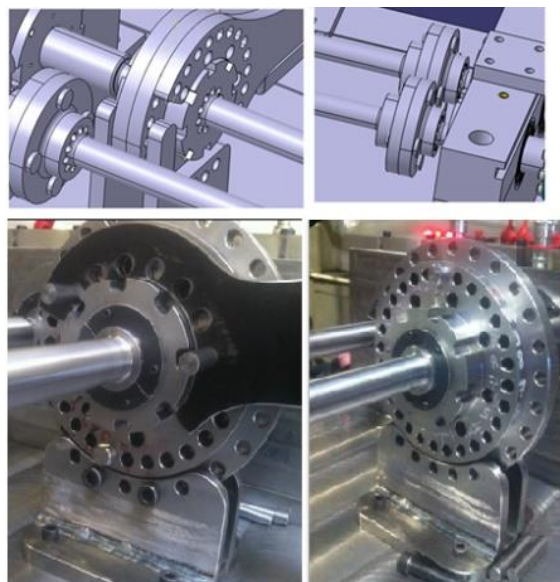


Figure 7.6 Multi-hole couplings

General Purpose Strain Gages—Shear/ Torque Pattern

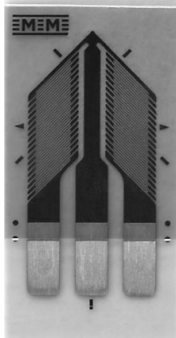
GAGE PATTERN DATA			
	GAGE DESIGNATION See Note 1	RESISTANCE (OHMS) See Note 2	OPTIONS AVAILABLE See Note 3
	CEA-XX-062UV-350 CEA-XX-062UV-500	350 $\pm$ 0.4% 500 $\pm$ 0.4%	P2, SP35 SP35
DESCRIPTION Two-element 90° rosette for torque and shear-strain measurement. Sections have a common electrical connection. Exposed solder tab area is 0.04 x 0.07 in [1.0 x 1.8 mm].			

Figure 7.7 Strain gauge of Micro Measurements CEA-06-062UV-350 for torque application



Figure 7.8 Strain gauges mounting on torsional rod



Figure 7.9 Strain gauges on torsional rod

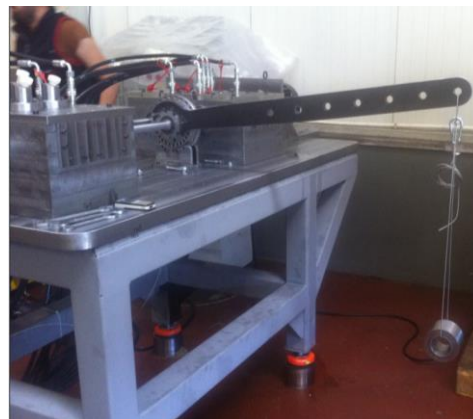


Figure 7.10 Torque calibrations by using locking device strain gauge measurement

After completing the calibration of strain gauges on torsional rods, holes placed in angular offset on couplings were calibrated in terms of torque by applying load with a torque wrench (Figure 7.12) and inserting pins to the calibrated holes (Figure 7.13-7.14), and measuring strain owing to applied torque (Figure 7.15).

After disassembly of tested gear pair and mounting of each new gear pair to be tested, holes of couplings are required to be calibrated. Torques obtained by inserting pins to the holes of couplings was measured by using calibrated strain gauges (Figure 7.15).

Holes of the couplings which pins were inserted into (Figure 7.16) were calibrated in terms of torques (Table 7.2) for symmetric profile spur gear pair under design torque of 422Nm.

Table 7.1 Strain gauges calibration data

Applied Load (kg)	Moment Arm Length (mm)	Torque Calculated (Nm)	Torque (Nm)	Applied Torque (Nm)	Measured Strain ($\mu\epsilon$)
0	-	0		0	0
Weight of Torque Wrench (w)	-	Torque Applied by Wrench (T_o)		-	59
w + 3.40	981	$T_o + 3.4 * 9.81 * 0.981$	$T_o + 32.72$	66	112
w + 6.80	982	$T_o + 6.8 * 9.81 * 0.981$	$T_o + 65.51$	99	168
w + 10.2	984	$T_o + 10.2 * 9.81 * 0.984$	$T_o + 98.46$	131.6	223
w + 13.6	986	$T_o + 13.6 * 9.81 * 0.986$	$T_o + 131.55$	165.8	281
w + 17.0	988	$T_o + 17.0 * 9.81 * 0.988$	$T_o + 164.76$	199.4	338
w + 20.4	989	$T_o + 20.4 * 9.81 * 0.989$	$T_o + 197.92$	235.4	399
w + 23.8	990	$T_o + 23.8 * 9.81 * 0.990$	$T_o + 231.14$	262.5	445
w + 27.4	991	$T_o + 27.4 * 9.81 * 0.991$	$T_o + 266.37$	298	505
w + 30.8	993	$T_o + 30.8 * 9.81 * 0.993$	$T_o + 300.03$	331.5	562
w + 34.2	994	$T_o + 34.2 * 9.81 * 0.994$	$T_o + 333.49$	364.6	618
w + 37.6	996	$T_o + 37.6 * 9.81 * 0.996$	$T_o + 367.38$	399	676
w + 40.6	996	$T_o + 40.6 * 9.81 * 0.996$	$T_o + 396.69$	426	722

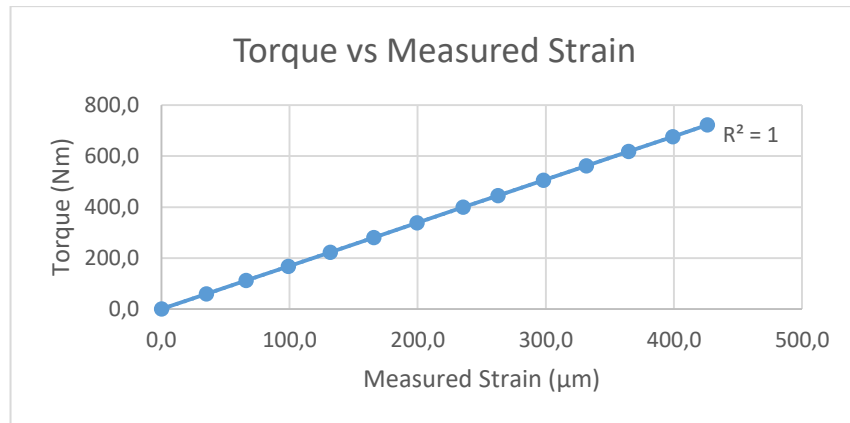


Figure 7.11 Obtained calibration graphs of strain gauges on torsional rod



Figure 7.12 Designed and manufactured torque wrench for test rig



Figure 7.13 Loading by torque wrench on couplings and inserting pins-1

Lubrication system (Figure 7.17) provides access of oil to each point required to be lubricated during rotation. To provide a good lubrication, pressurized oil is transferred through grooves (Figure 7.18).



Figure 7.14 Loading by torque wrench on couplings and inserting pins-2

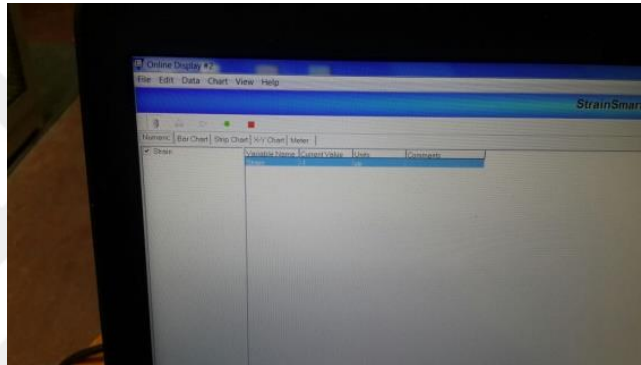


Figure 7.15 Strain gauge measurements by using Strain Smart [183]



Figure 7.16 Pin inserted couplings



Figure 7.17 Lubrication System

As presented in Table 7.3, similar calibrations are required in the case of change of gear pair with new pair for tests and torque application direction variation. For same gear pair presented in Table 7.2, new calibration data is presented in Table 7.3 because of rotational direction change from CCW to CW.

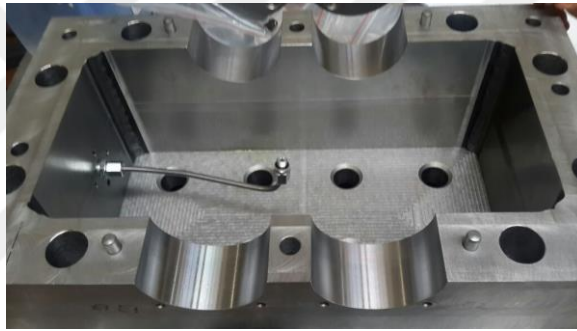


Figure 7.18 Lubrication pipe mounted on gearbox

Table 7.2 Calibration for holes of coupling (symmetric profile spur gear pair, CCW direction under design load of 422Nm)

Hole no (CCW)	Measured Strain ($\mu\epsilon$)	Calculated Torque (Nm)	% Torque (for Design Load of 422Nm)
0	13	7.7	1.8
1	102	60.0	14.2
2	205	121.0	28.7
3	313	184.7	43.7
4	429	253.0	60.0
5	546	322.0	76.3
6	671	396.0	93.8
7	786	464.0	110.0
8	906	535.0	126.7

Table 7.3 Calibration for holes of coupling (symmetric profile spur gear pair, CW direction under design load of 422Nm)

Hole no (CW)	Measured Strain ($\mu\epsilon$)	Calculated Torque (Nm)	% Torque (for Design Load of 422Nm)
0	-1	0	0
1	-58	34.2	8
2	-156	92	21.8
3	-169	99.7	23.6
4	-384	226	53.7
5	-503	297	70
6	-619	365	86.5
7	-744	439	104
8	-863	509	120

The most important parameters to be measured by this test rig is no load static transmission error (NLSTE) and loaded static-dynamic transmission error (LSTE) for symmetric and asymmetric tooth profile spur gears through optical encoders (Figure 7.19).

Before measurements of TE through encoders, they were calibrated by using previously calibrated dial indicators with a precision of 0.01 mm/line. In this application, the magnetic base of the dial indicator was mounted on the gearbox housing. Calibration was carried out by contacting dial indicator to the tip of the gear tooth. Under no-load condition, one of the shafts was held constant and the other one was rotated with the amount of backlash between mating teeth of gears. At the same time, when the signals coming from two encoders and angular difference of rotations of both shafts were read, the linear movement of tooth tip was measured by using dial indicator (Figure 7.20). During this calibration, the outputs of encoders were saved as an angular difference and the difference between start and end of rotation was evaluated as 0.2134 degrees (0.7238-0.5104).

The linear movement of the tip of the tooth (at the radius of 62.5mm) was read 0.24mm from dial indicator. Angular difference from encoders was converted into linear movement by using contact radius (62.5mm) and was compared with readings from dial indicator.

The angular difference value coming from encoders was 0.2134 degrees (equals to 0.0037245radian). The linear movement (arc) was calculated by multiplying the angular difference with radius of the tooth tip ($0.0037245 \times 62.5\text{mm} = 0.233\text{mm}$).

Reading from dial indicator was 0.24mm. The difference of value measured by encoders and the reading from dial indicator was 7micron.



Figure 7.19 Encoder mounted on test rig shafts



Figure 7.20 Calibrations of encoders by using dial indicator

This check which was carried out by using dial indicator showed the correctness of encoders based on optical measurement. The deviation (0.007mm) could not be related to encoders and could occur because of reading error of value from dial indicator and non-positioning at exact radius (62.5mm).

During operation of gear test rig (test and slave gearboxes), shafts and bearings are exposed to vibration and shock in addition to gears at high speeds. While angular vibration measurements of gears can be carried out through encoders, linear vibrations of bearing and shafts under high speeds and frequencies can be obtained by using sensors. It was decided to use non-contact Eddy current sensors (Micro-Epsilon, EPS08, eddy NCDT series 3100) (Figure 7.21) to determine vibrations which are under or above micro level. Positioning of these sensors on back to back test rig for intended (7.22-7.23) and manufactured one (Figure 7.24-25).

Eddy current sensors were positioned with a distance of 0.3mm-0.5mm to the disk of inertia which allows measurement of vibrations of bearings and shafts, and mounted on shafts of test gearbox (Figure 7.24). Two different sensors were used to take measurements from two different locations on each shaft of test gearbox through four sensors. By this means, vibrations in lateral and rocking can be determined.



Figure 7.21 Micro-Epsilon Eddy current sensor (EPS08, eddyNCDT)

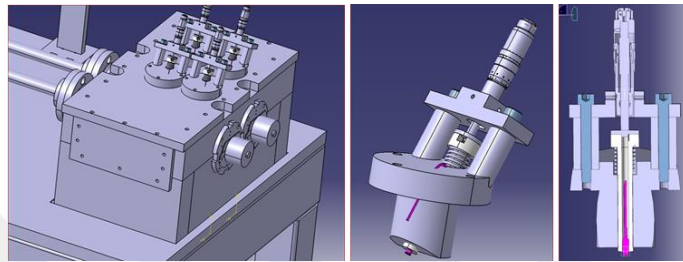


Figure 7.22 Eddy current sensors positioning on intended design of test rig-1

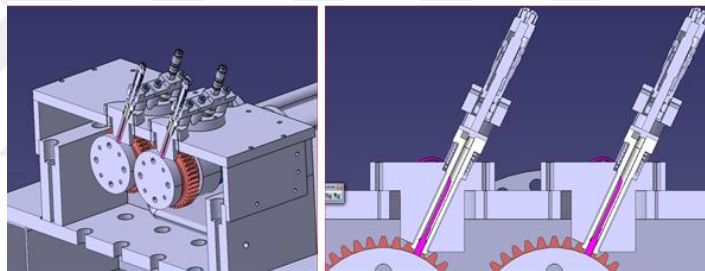


Figure 7.23 Eddy current sensors positioning on intended design of test rig-2

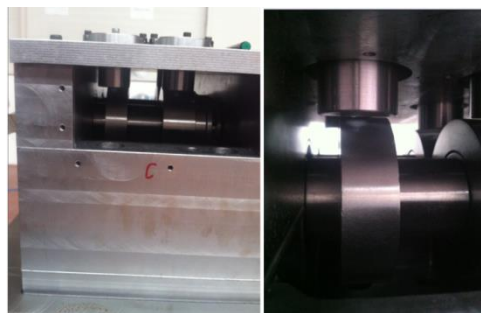


Figure 7.24 Non-contact measurement locations of eddy current sensors on test rig

Each of Eddy current sensors was calibrated by using sensitive micrometer before each test as shown in Figure 7.26. This provides accurate positioning of sensors and achievement of correct measurements (Figure 7.27). Insize branch of micrometer was

calibrated by Johnson's Blocks before using it for mounting Eddy current sensors to test rig. This micrometer was used to calibrate sensors from time to time.

Eddy current sensors were obtained with its own software of calibration for each case of each mounting. Sensors were calibrated by using software of calibration (Micro Epsilon, Figure 7.28) and calibrated micrometer. Calibration and pretests of sensors are shown in Figure 7.28-7.29. Eddy current sensors were positioned at a distance of 480 micron from target surface. These sensors are capable of measuring vibration and movement from a distance of ± 400 micron for maximum frequency of 25 kHz. By assuming that a 40-teeth test gear is rotating at a maximum speed of 6000rpm, tooth passing frequency is calculated as 4000Hertz $((6000/60)*40)$. If it is assumed that third harmonics of vibrations occur, frequency will be 12kHz. This value is in the limit of sensor's measuring capability and can be measured easily by using these sensors.



Figure 7.25 Positioning of Eddy current sensors on test rig



Figure 7.26 Insize micrometer



Figure 7.27 Positioning and calibration of Eddy current sensors by using micrometer

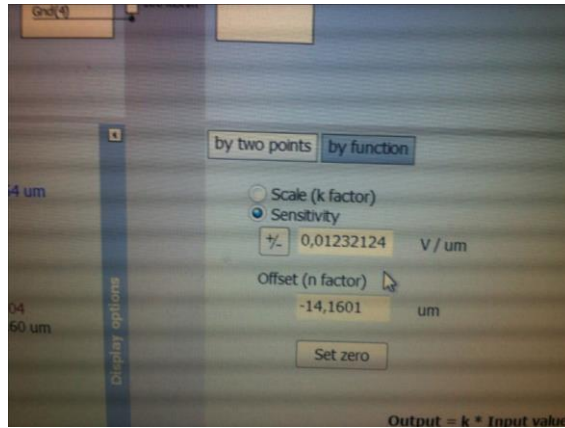


Figure 7.28 Calibration software of Eddy current sensors (Micro Epsilon)

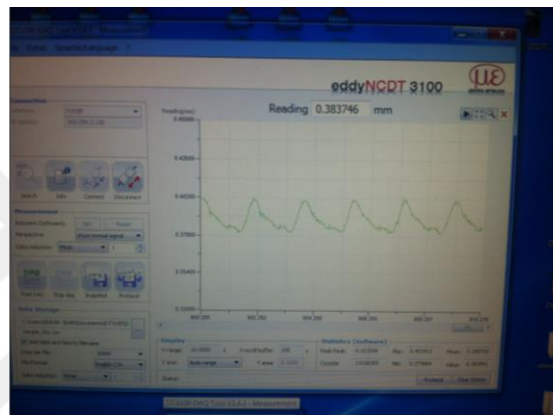


Figure 7.29 Calibrations of Eddy current sensors and pretests

7.3 Static Transmission Error (STE) and Micro Profile Measurements of Symmetric and Asymmetric Tooth Profile Spur Gear Pairs

Symmetric and asymmetric tooth profile spur gear pairs were designed and manufactured with a relieved tip in this study. Gear pairs were designed to provide the predefined center distance of gearbox with gear ratio of unity regardless of symmetric or asymmetric profile.

These gears were evaluated in terms of static transmission error values and smoothness of TE curves by using in-house software based on Harris map and finite element works based on deflection analyses of gear tooth under load. In this section of thesis, symmetric and asymmetric tooth profile spur gear pairs (designed, manufactured and assembled to the test rig) were tested for static transmission error (STE) by using back to back test rig [1].

7.3.1 Measurement of Tooth Micro Profile of Symmetric Spur Gear Pair (Case S22-Long Relief)

A symmetric spur gear pair (Case S22) which was analyzed for transmission error in Chapter 6 was evaluated by testing it through back to back gear test rig. Gear pair parameters (already presented in Table 6.19) were given here again (Table 7.4) to provide continuity of reading flow. Micro profile of gear pair of Case S22 was firstly measured to know how far obtained geometry is from the desired one (or how close to) as shown in Figure 7.30. Gear profile, lead and index measurements were presented in Figure 7.31-7.33. Intended relief parameters and obtained micro profile of gear pair by measurements for Case S22 is presented in Table 7.5. As it is understood from values given in the table, mean values of measurements are very close to designed value of relief parameters; the difference is 0.3 micron for reliving of left flank whereas 1.4 micron for right one.

Table 7.4 Gear parameters of Case S22

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Pressure Angle	20	Degree
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm
Relief Type	Long	-
Amount of relief	30	μm
Extent of relief	6.319	mm

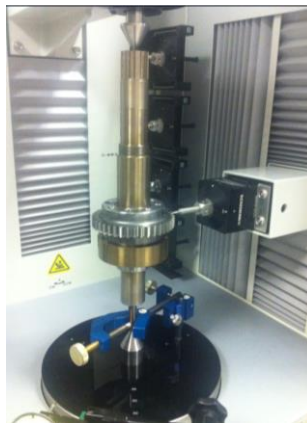


Figure 7.30 Measurement of tooth surface topology of gear pair (Case S22)

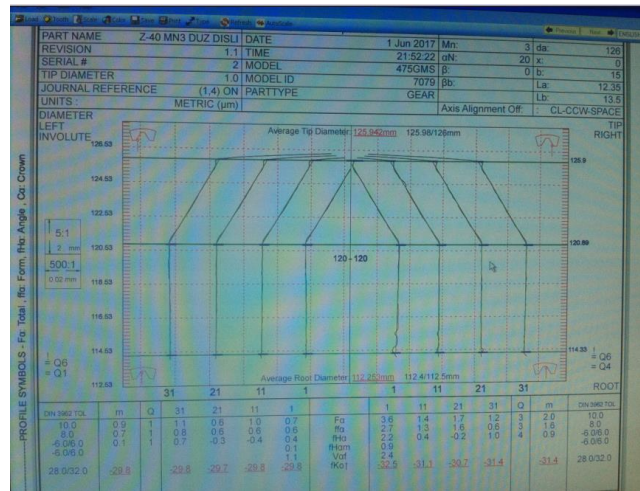


Figure 7.31 Measurements of profile and lead of gear pair (Case S22)-1

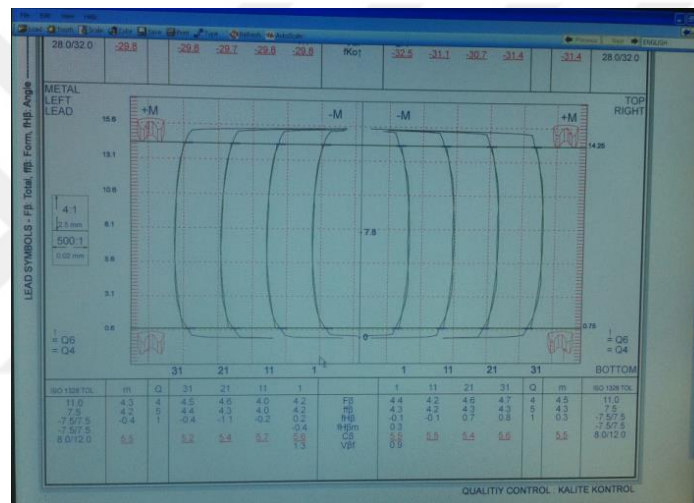


Figure 7.32 Measurements of profile and lead of gear pair (Case S22)-2

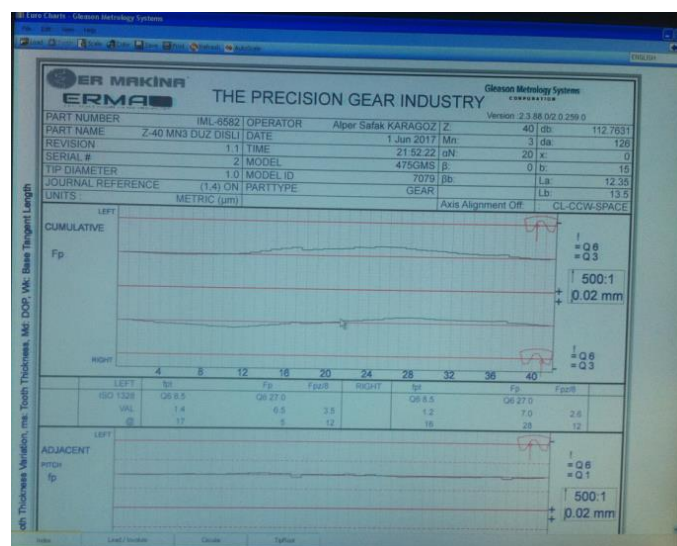


Figure 7.33 Measurements of index of gear pair (Case S22)

Table 7.5 Micro geometry parameters of symmetric profile spur gear pair of Case S22 (intended and measured)

Relief	Amount of relief (μm)			Extend of relief (mm)	Radius of relief start (mm)
Designed	30.0			6.319	120.891
Measured	Min	Mean	Max		
Left flank	29.7	29.7	29.8	6.319	120.89
Right flank	30.7	31.4	32.5	6.319	120.89

7.3.2 Static Transmission Error (STE) Measurement of Symmetric Tooth Profile Spur Gear Pair (Case S22-Long Relief)

Designed gear pair of Case S22 was manufactured for intended macro and micro profile (Figure 7.34). To support analytical and numerical studies (carried out in previous chapter), experimental measurements were intended in this thesis. To that end, symmetric profile gear pair of Case S22 was firstly mounted on back to back test rig, then pretests were conducted to check working of all elements of gearboxes and test rigs without any problem, and a healthy lubrication of both gearboxes (Figure 7.35). Some quasi-static transmission error curves for pretests were also obtained and this gave chance to check running of measurement devices (Figure 7.36).

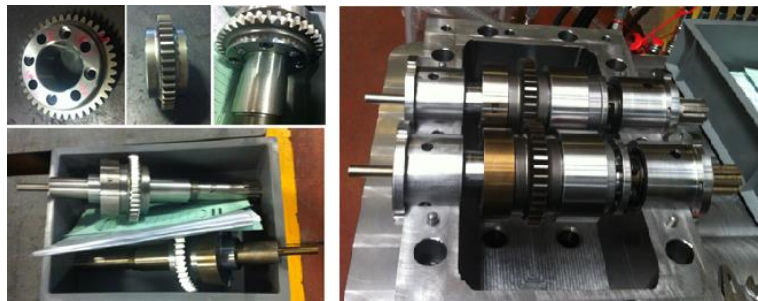


Figure 7.34 Symmetric tooth profile spur gear pair of Case S10



Figure 7.35 Conducting pretests of static transmission error measurements

After performing pretests, some repeatability tests were conducted to verify repeatability of static transmission error measurements by back to back test rig for symmetric profile spur gear pair (Case S22) under different percentages of design torque values (7Nm and 396Nm) at operating speed of 40rpm (in CCW) by means of couplings and pins (Figure 7.37-7.39).

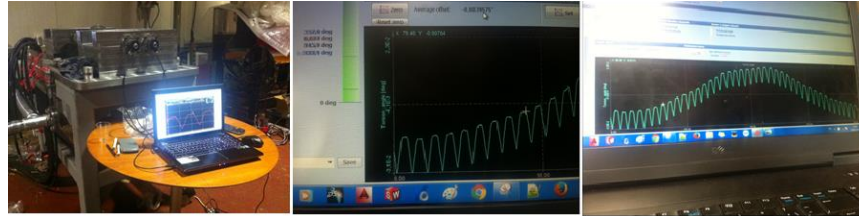


Figure 7.36 Static transmission error curves of pretests

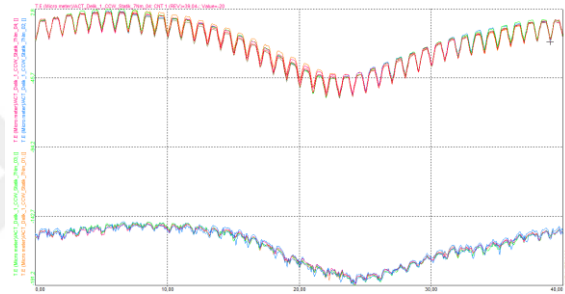


Figure 7.37 Repeatability tests for STE of Case S22 under load of 7Nm and 396Nm (CCW)

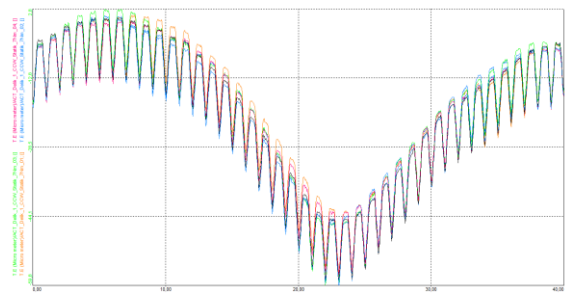


Figure 7.38 Repeatability tests for STE of Case S22 under load of 7Nm (CCW)

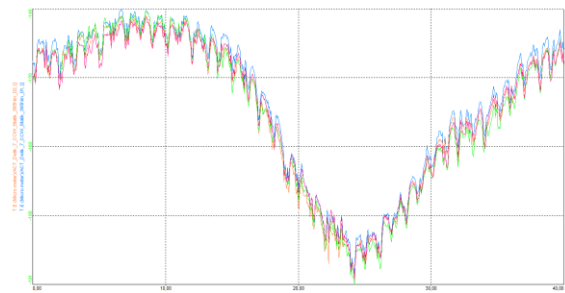


Figure 7.39 Repeatability tests for STE of Case S22 under load of 396Nm (CCW)

For same gear pair (Case S22) in the case of CW rotation, repeatability tests of STE were obtained as in Figure 7.40-7.49. By starting with lowest torque value of 7.7Nm (1.8%), torque value was increased from one to another through inserting pins into couplings holes. STE curves for each torque value were obtained. Blue curves refer to test at the start of measurements whereas orange ones belong to later ones. It can be deduced from figures, TE curves can be repeatable completely in terms of PPTE values and regime of curves based on teeth. But there is deviation in terms of DC positions of curves under low load values at start and end of tests. DC position is related to elastic deformations of all elements of gearbox which are exposed to high torques during transmission. As this deformable element can be gear tooth itself; bearings, shafts, couplings, couplings pin holes and pins may be effected during high load transmitting. It is possible to say that geometries of coupling pins holes and pins are effective for DC position variations under low loads.

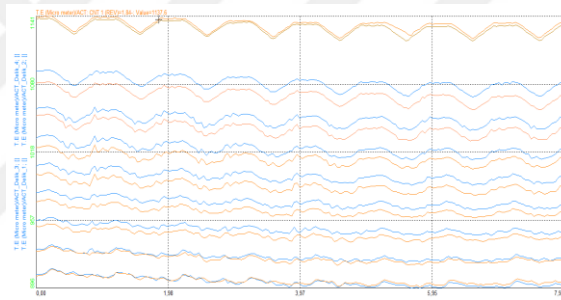


Figure 7.40 Repeatability tests for STE of Case S22 for all percentages of design load (CW)

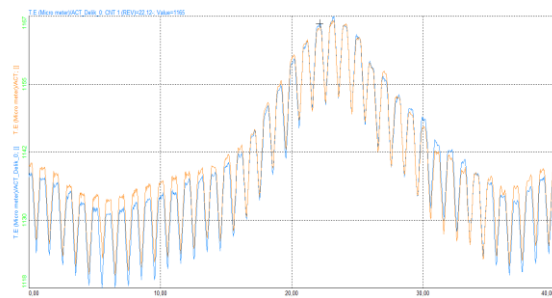


Figure 7.41 Repeatability tests for STE of Case S22 under no load (CW)

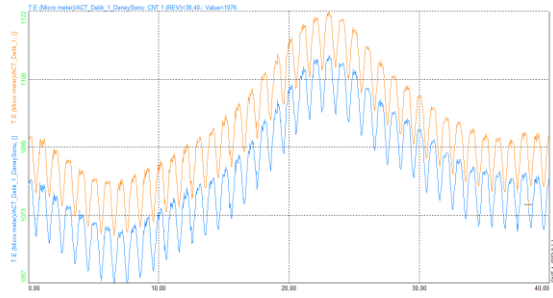


Figure 7.42 Repeatability tests for STE of Case S22 under load of 34Nm (8%) (CW)

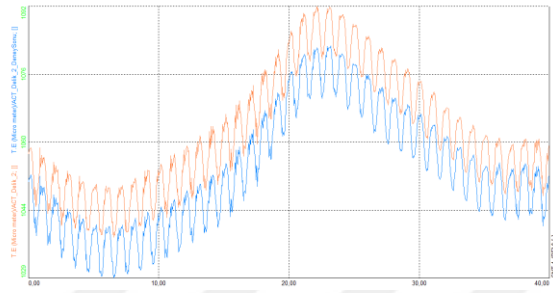


Figure 7.43 Repeatability tests for STE of Case S22 under load of 92Nm (21.8%) (CW)

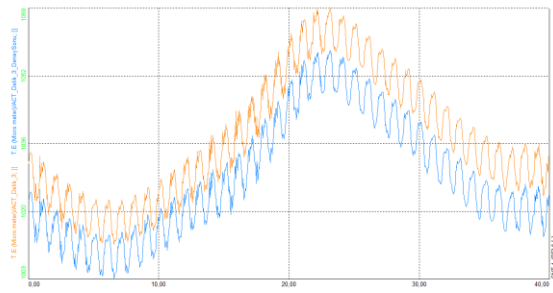


Figure 7.44 Repeatability tests for STE of Case S22 under load of 99.7Nm (23.6%) (CW)

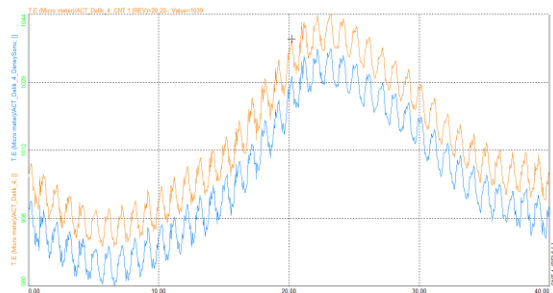


Figure 7.45 Repeatability tests for STE of Case S22 under load of 226Nm (53.7%) (CW)

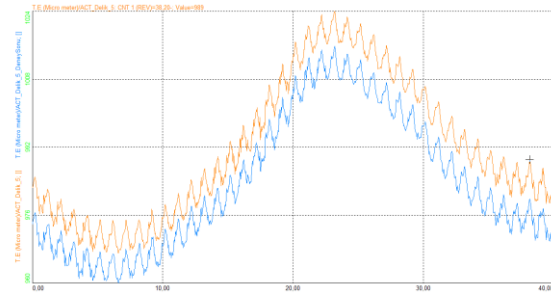


Figure 7.46 Repeatability tests for STE of Case S22 under load of 297Nm (70%) (CW)

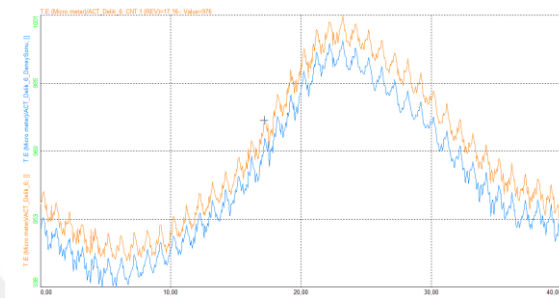


Figure 7.47 Repeatability tests for STE of Case S22 under load of 365Nm (86.5%) (CW)

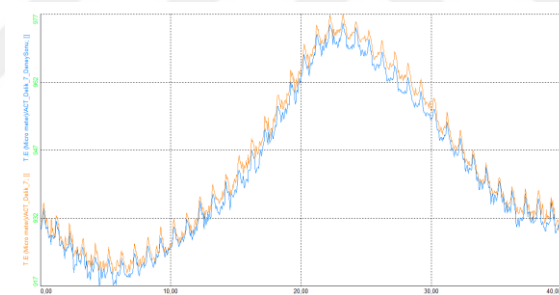


Figure 7.48 Repeatability tests for STE of Case S22 under load of 439Nm (104%) (CW)

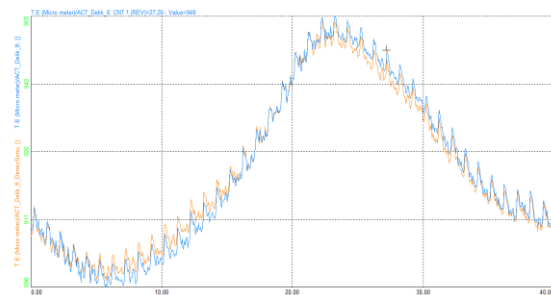


Figure 7.49 Repeatability tests for STE of Case S22 under load of 509Nm (120%) (CW)

TE curves for all of the percentages of design loads were presented in Figure 7.50 (for all teeth) and Figure 7.51 (for 10-teeth). TE curves obtained by in-house software and

FEA are given in Figure 7.51 and 7.52 respectively. PPTE values of analytically predicted by in-house software (Chapter 6), numerically analyzed based on finite element works (Chapter 6) and experimentally measured by back to back test rig were given in Table 7.6. When PPTE values were taken as basis and the comparison of values obtained by in-house software and FEA were performed, the biggest difference is read as 1.9 micron from Table 7.7. Difference is getting bigger from lowest load (6Nm) to 121Nm then goes down sharply for 185Nm (0.1 micron). Then difference increases in negative direction; this means that FEA works gives higher PPTE values from now on.

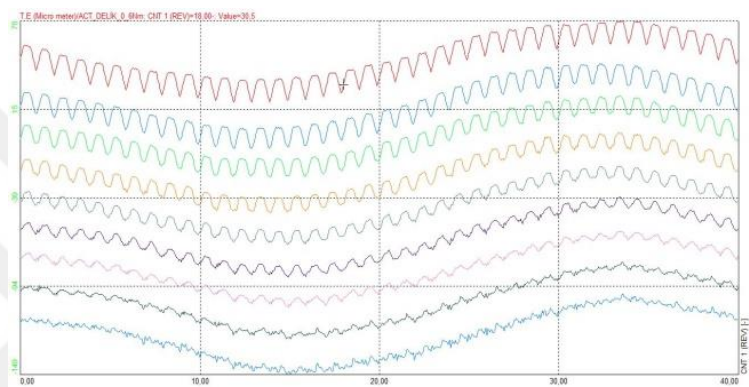


Figure 7.50 TE curves of Case S22 obtained by test rig (40-Teeth)

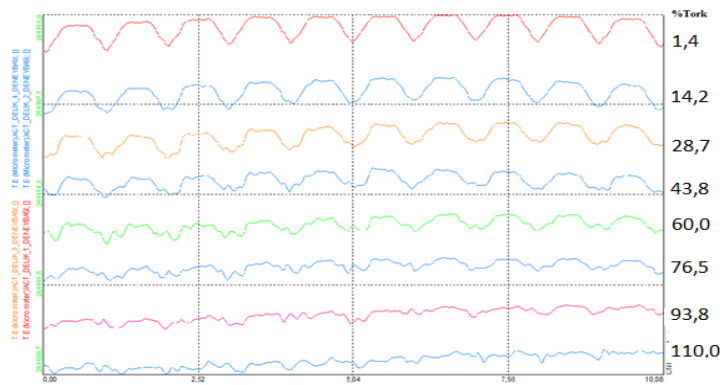


Figure 7.51 TE curves of Case S22 obtained by test rig (10-Teeth)

Table 7.6 PPTE values of Case S22 (D20C20)

Torque (Nm)	Torque Percentages of Design Torque (%)	PPTE (μm)		
		Analytically calculated (in-house software)	Numerically Analyzed (FEA)	Experimentally measured
6	1.4	14.9	14.7	14.2
60	14.2	13.2	12.0	12.4
121	28.7	11.3	11.0	9.4
185	43.8	9.3	10.0	9.2
253	60.0	7.2	7.0	8.3
323	76.5	5.0	5.0	5.8
396	93.8	2.7	3.0	3.2
464	110	1.5	2.0	1.7

Table 7.7 Difference between values of PPTE obtained by in-house software and FEA for Case S22

Torque (Nm)	Torque Percentage of Design Torque	PPTE (μm)		
		Software (based)	FEA	Difference
6	1.4	14.9	14.2	0.7
60	14.2	13.2	12.4	0.8
121	28.7	11.3	9.4	1.9
185	43.8	9.3	9.2	0.1
253	60.0	7.2	8.3	-1.1
323	76.5	5.0	5.8	-0.8
396	93.8	2.7	3.2	-0.5
464	110.0	1.5	1.7	-0.2

For comparison of results of in-house software and experimental measurements, the situation is little bit different; difference for PPTE values is almost zero for the minimum load (6Nm), it increases by 1.0 micron for 60Nm and decreases suddenly to 0.3 micron for 121Nm. Experimental works yields higher PPTE values for 185Nm, 396Nm and 464 Nm whereas there is no difference for 323Nm (Table 7.8).

Table 7.8 Difference between values of PPTE obtained by in-house software and measurements for Case S22

Torque (Nm)	Torque Percentage of Design Torque	PPTE (μm)		
		Software (based)	Measurement	Difference
6	1.4	14.9	14.7	0.2
60	14.2	13.2	12.0	1.2
121	28.7	11.3	11.0	0.3
185	43.8	9.3	10.0	-0.7
253	60.0	7.2	7.0	0.2
323	76.5	5.0	5.0	0.0
396	93.8	2.7	3.0	-0.3
464	110.0	1.5	2.0	-0.5

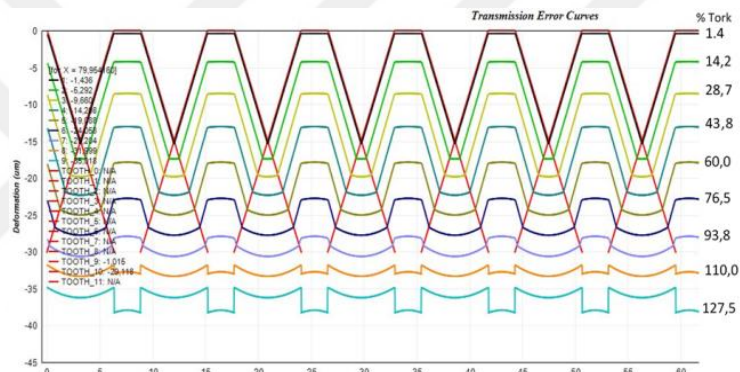


Figure 7.52 TE curves of Case S22 obtained by in-house software

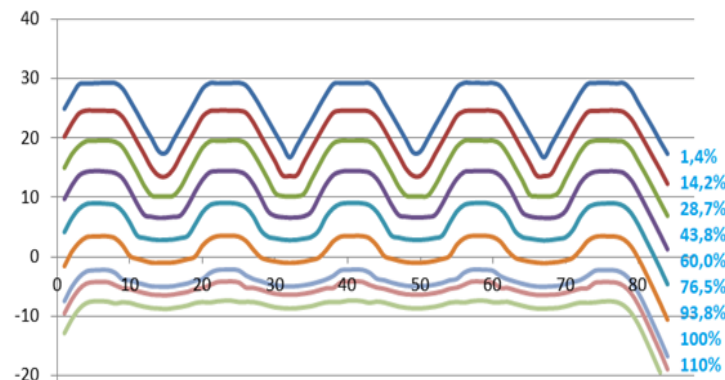


Figure 7.53 TE curves of Case S22 obtained by FEA

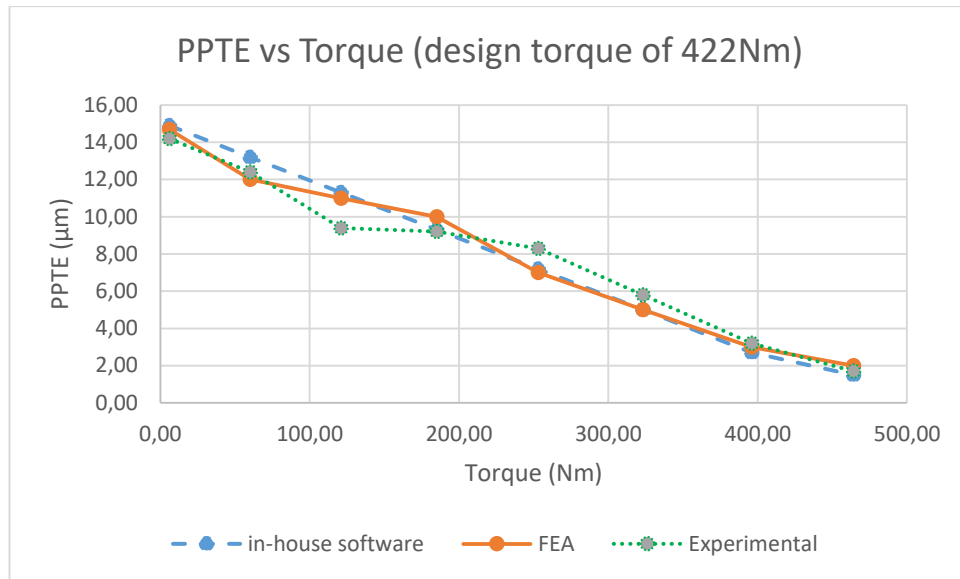


Figure 7.54 Experimental and theoretical PPTE values vs Torque (Nm) for Case S22

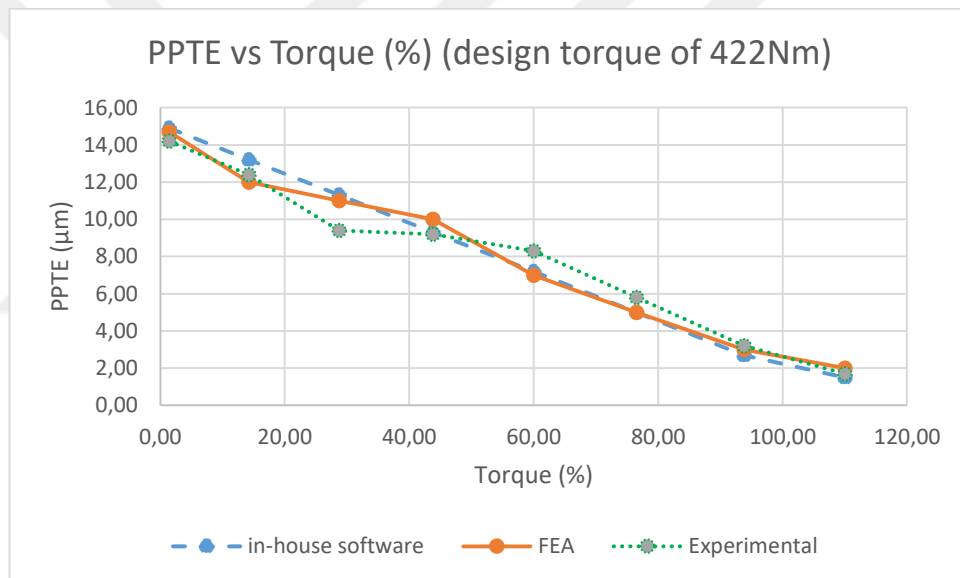


Figure 7.55 Experimental and theoretical PPTE values vs Torque (%) for Case S22

As seen in the comparison of theoretical and experimental TE curves, and PPTE values for symmetrical tooth spur gear pair (Figure 7.54-7.55), values and curves regimes are very close to each other. These all show that micro geometry design and developed in-house software satisfy with each other substantially.

7.3.3 Measurement of Tooth Micro Profile of Asymmetric Spur Gear Pair (Case A22-D30C20)

An asymmetric spur gear pair of Case A22 (given in Figure 7.56) which was analyzed for transmission error in Chapter 6 was evaluated by measurement via gear test rig here. Gear pair parameters (already presented in Chapter 6) were given here again

(Table 7.9). Before starting for tests of manufactured asymmetric tooth gears, micro profile of gear pair of Case A22 was measured in terms of gear profile and lead (Figure 7.57-7.59).

Intended relief parameters and measured micro profile of gear pair for Case A22 are presented in Table 7.10. It can be concluded from values presented in Table 7.10, obtained relief parameters of left flank are very close to designed values with 0.1 micron of difference while relief parameters of right side flank are less than intended ones; the difference is 3.4 micron.

Table 7.9 Gear parameters of Case A22

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	40	-
Gear ratio	1	-
Pressure Angle	30/20	Degree
Addendum coefficient	1.00	-
Dedendum coefficient	1.25	-
Hob tip radius coefficient	0.3	-
Design torque	422	Nm
Face width	15	mm
Relief Type	Long	-
Amount of relief	24.6	μm
Extent of relief	3.084	mm

Table 7.10 Micro geometry parameters of asymmetric profile spur gear pair of Case A22 (intended and measured)

Relief	Amount of relief (μm)			Extend of relief (mm)	Radius of relief start (mm)
Designed	24.6			3.084	122.618
Measured	Min	Mean	Max		
Left flank (C20)	24.1	24.5	24.8	3.084	122.62
Right flank (D30)	20.9	21.2	21.7	3.084	122.62



Figure 7.56 Asymmetric tooth profile spur gear pair of Case A22 (D30C20)



Figure 7.57 Measurement of tooth surface topology of gear pair (Case A22)

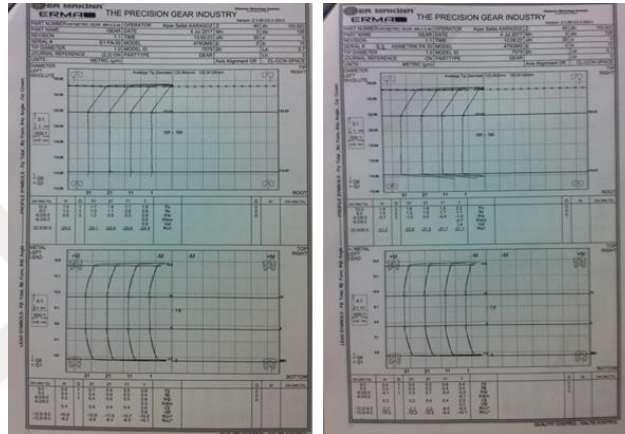


Figure 7.58 Measurements of profile and lead of gear pair of Case A22 (left flank)

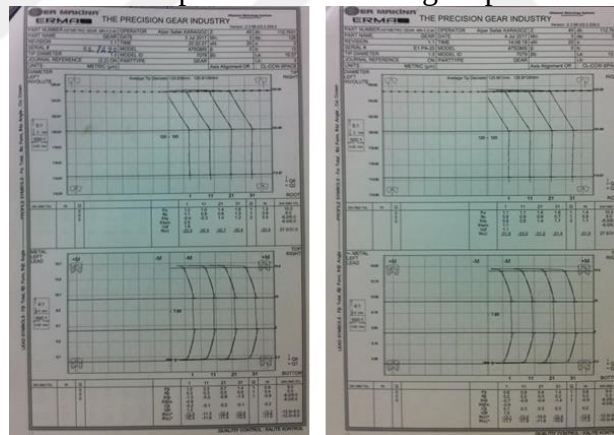


Figure 7.59 Measurements of profile and lead of gear pair of Case A22 (right flank)

7.3.4 Static Transmission Error (STE) Measurement of Asymmetric Tooth Profile Spur Gear Pair (Case A22-D30C20)

Asymmetric tooth profile spur gear pair of Case A22 designed previously was manufactured for intended macro and micro profile as shown in Figure 7.60. Experimental studies were carried out in addition to analytical and numerical works to evaluate micro geometry design of asymmetric gears.

Static transmission error curves for asymmetric tooth spur gear pair of Case A22 (D30C20-Long Relief) under all percentages of design load are presented in Figure 7.61-7.72 as separate curves for each curve and all together in one curve. TE curves under low load values are more similar to no load transmission error (NLTE) curve whereas it deviates from this format with increasing load. TE curves obtained by in-house software were presented in Figure 7.73.



Figure 7.60 Asymmetric spur gear pair of Case A22

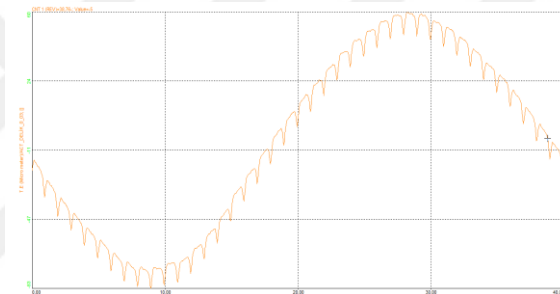


Figure 7.61 STE curve of asymmetric spur gear pair (Case A22) under no load

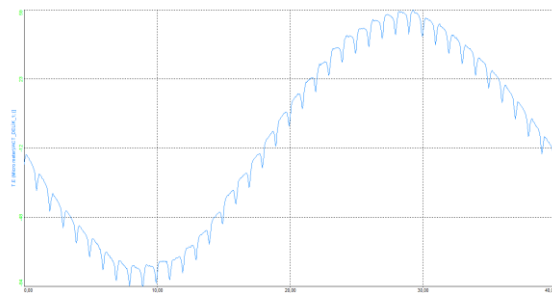


Figure 7.62 STE curve of asymmetric spur gear pair (Case A22) under load of 18Nm

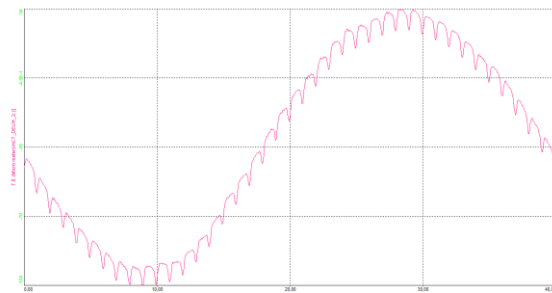


Figure 7.63 STE curve of asymmetric spur gear pair (Case A22) under load of 74Nm

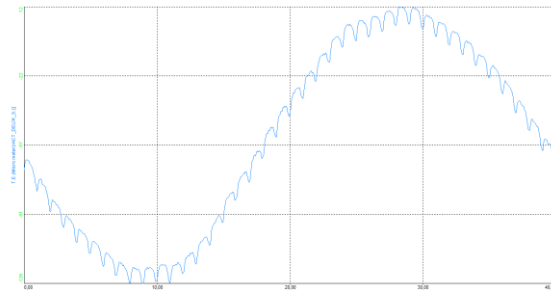


Figure 7.64 STE curve of asymmetric spur gear pair (Case A22) under load of 133Nm

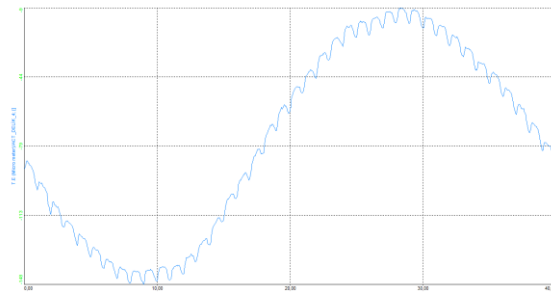


Figure 7.65 STE curve of asymmetric spur gear pair (Case A22) under load of 202Nm

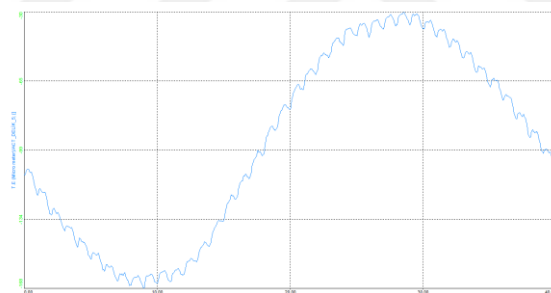


Figure 7.66 STE curve of asymmetric spur gear pair (Case A22) under load of 270Nm

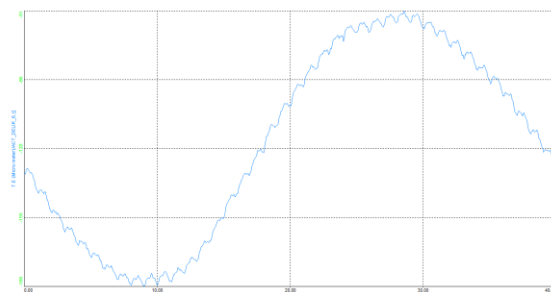


Figure 7.67 STE curve of asymmetric spur gear pair (Case A22) under load of 340Nm

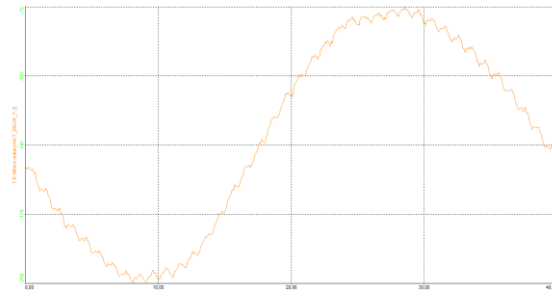


Figure 7.68 STE curve of asymmetric spur gear pair (Case A22) under load of 410Nm

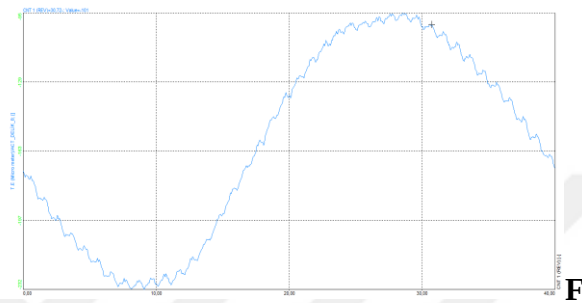


Figure 7.69 STE curve of asymmetric spur gear pair (Case A22) under load of 480Nm

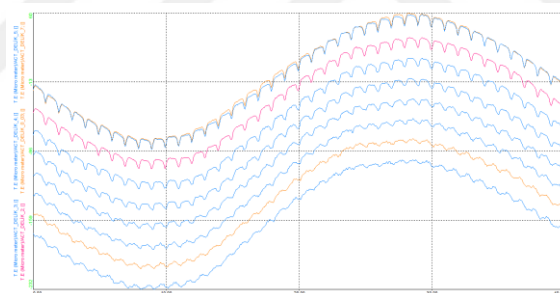


Figure 7.70 STE curve of asymmetric spur gear pair (Case A22) for all load values (0-18-74-133-202-270-340-410-480 Nm)

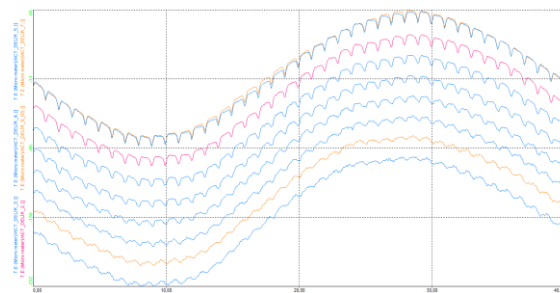


Figure 7.71 Transmission Error Curves of Case A22 for Whole Teeth (Experiment)

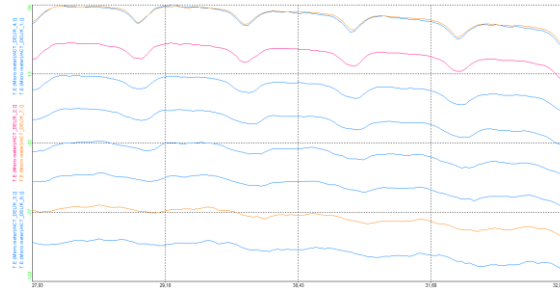


Figure 7.72 Transmission Error Curves of Case A22 (Experiment)

When PPTE values obtained by in-house software and experiments are compared, the biggest difference is read as 1.53micron (Table 7.11). Difference is very low for no load condition and under the loads of 18Nm and 74Nm. It reaches 0.99 micron for 133Nm whereas decreases to 0.76 for 202Nm. It takes highest value as 1.53 for 270Nm and starts to decrease from there as follows; 1.32 micron, 1.04 micron and 0.75 micron for 340Nm, 410Nm and 480Nm respectively.

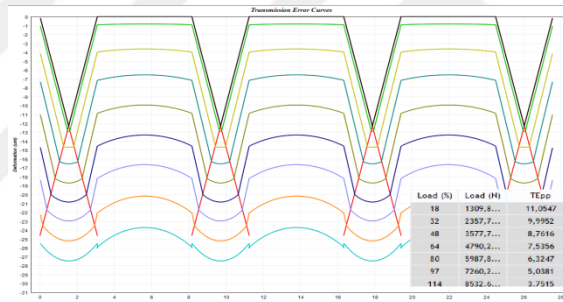


Figure 7.73 Transmission Error Curves of Case A22 (In-house Software)

Table 7.11 PPTE values of Case A22

Torque (Nm)	Torque Percentages of Design Torque (%)	PPTE (μm)		
		Analytically calculated (in-house software)	Experimentally measured	Difference
0	0	13.00	12.33	-0.67
18	4.0	12.00	12.07	0.07
74	17.5	11.00	11.05	0.05
133	31.5	9.00	9.99	0.99
202	47.8	8.00	8.76	0.76
270	64.0	6.00	7.53	1.53
340	80.0	5.00	6.32	1.32
410	97.0	4.00	5.04	1.04
480	114.0	3.00	3.75	0.75

When theoretical and experimental PPTE curves versus torque for asymmetrical tooth spur gear pair are generated in same figure (Figure 7.74-7.75), it is obvious to say that values and curves regimes are very close to each other. These all show that micro geometry design and prepared in-house software satisfy with each other substantially.

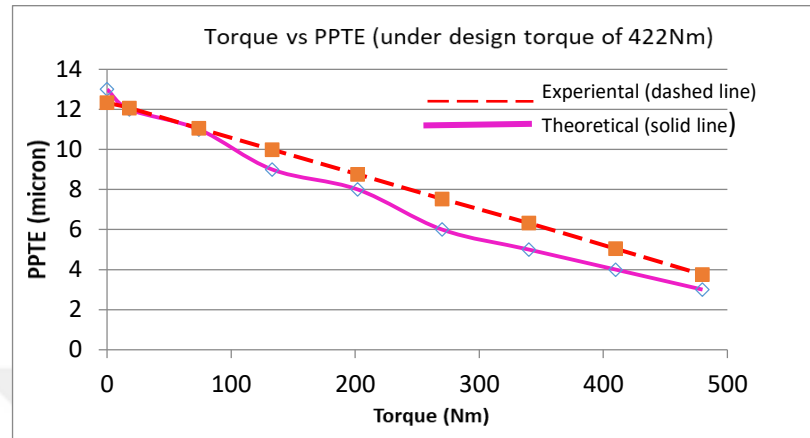


Figure 7.74 Experimental and theoretical PPTE values vs Torque (Nm) for Case A22

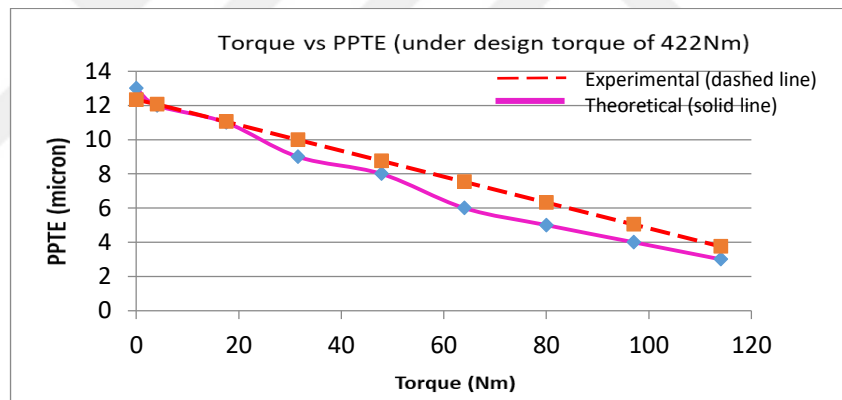


Figure 7.75 Experimental and theoretical PPTE values vs Torque (%) for Case A22

CHAPTER 8

MEASUREMENTS OF DYNAMIC TRANSMISSION ERROR FOR SYMMETRICAL AND ASYMMETRICAL TOOTH SPUR GEAR PAIRS

8.1 Introduction

Spur gear pairs with symmetrical and asymmetrical tooth profile were designed with a relieved tip to minimize peak to peak transmission error (PPTE) values in Chapter 6. These gear pairs were manufactured and tested in terms of static transmission error (STE) in Chapter 7 by using gear test rig developed in the scope of this thesis and a distinctive project of gear design for helicopters [1].

Operational speed and transmitted load have significant effects on dynamics of gears and gearboxes regardless of symmetric or asymmetric tooth profile. Therefore, symmetrical tooth spur gear pair (Case S22) and asymmetrical tooth spur gear pair (A22 and A23) were tested for evaluation of dynamic transmission error (DTE) values under different loads and at different operating speeds. Dynamic transmission error (DTE) curves and dynamic frequency response curves are presented in this chapter.

8.2 Measurements of Dynamic Transmission Error for Symmetrical Tooth Spur Gear Pairs

Symmetric tooth spur gear pair (Case S22) was firstly tested to check for repeatability of dynamic transmission error measurements in this section. Repeatability curves for dynamic transmission error of S22 under same torque but at different times (start and end of tests) are presented in Figure 8.1-8.4.

Blue and orange curves were constructed at the start and end of repeatability tests respectively. These curves show that measurement devices attached on gear test rig and measurement methods are reliable and can be used for measurements under different loads, at different speeds and different times.

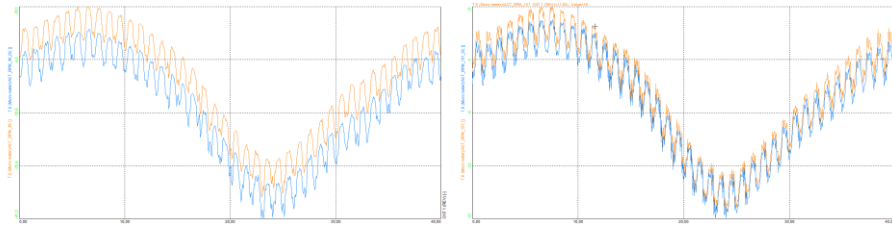


Figure 8.1 Repeatability tests for DTE of Case S22 for 36rpm and 151rpm

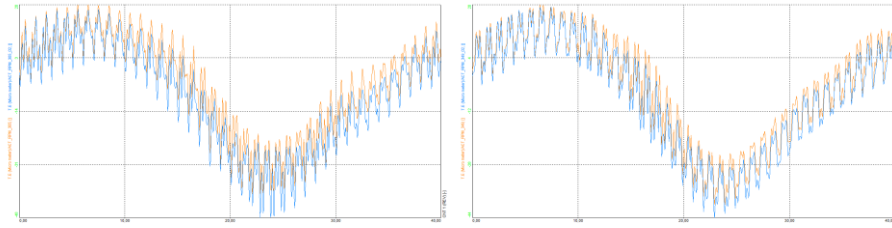


Figure 8.2 Repeatability tests for DTE of Case S22 for 305rpm and 349rpm

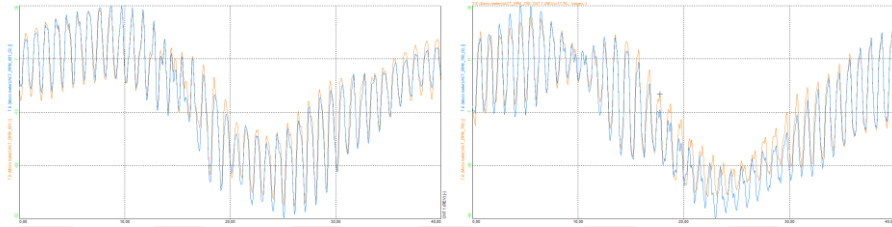


Figure 8.3 Repeatability tests for DTE of Case S22 for 601rpm and 700rpm

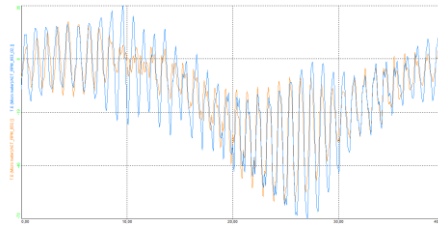


Figure 8.4 Repeatability tests for DTE of Case S22 for 853rpm

DTE curves for symmetric tooth spur gear pair (Case S22) under the load of 185Nm in the rotation of CCW direction were obtained by measurement using back to back gear test rig (Figure 8.5-8.11).

4th harmonics of DTE are seen for 181rpm and 233rpm in Figure 8.5 and 8.6 respectively. 3rd harmonic are obvious and transition from 3rd to 2nd harmonics for 386rpm can be seen in Figure 8.7. When it comes to 564rpm, 2nd harmonics appears partially whereas transition from 2nd to 1st harmonics can be seen apparently (Figure 8.8).

DTE curves of 721rpm and 755rpm are presented in Figure 8.9 together. First harmonics is apparent in Figure 8.9 and it is clearly seen in Figure 8.10 for 1030rpm. Tooth is about to lose the contact and load is very close to zero. At 1054rpm, load on the tooth is almost zero and contact loss is about to occur whereas tooth load is significant for same position in the bottom of curve (Figure 8.11).

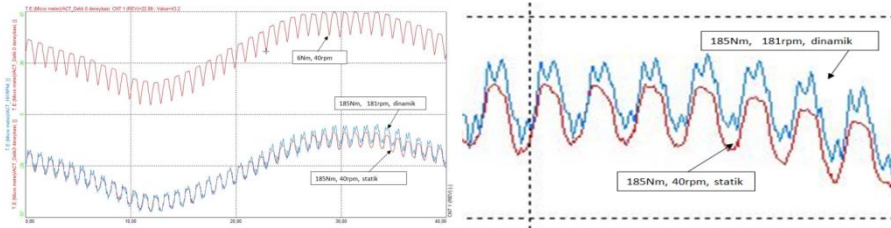


Figure 8.5 DTE curves of Case S22 at 40rpm and 181rpm

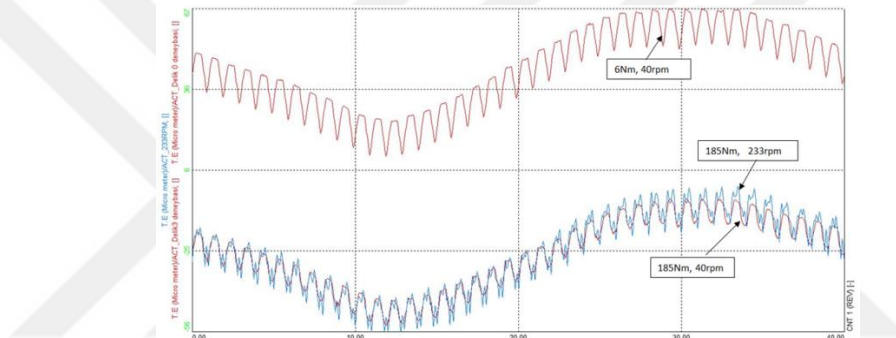


Figure 8.6 DTE curves of Case S22 at 40rpm and 233rpm

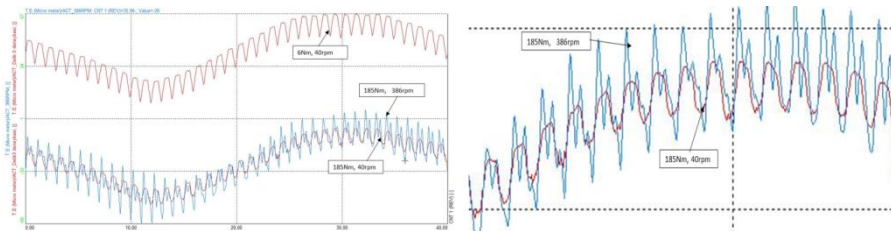


Figure 8.7 DTE curves of Case S22 at 40rpm and 386rpm

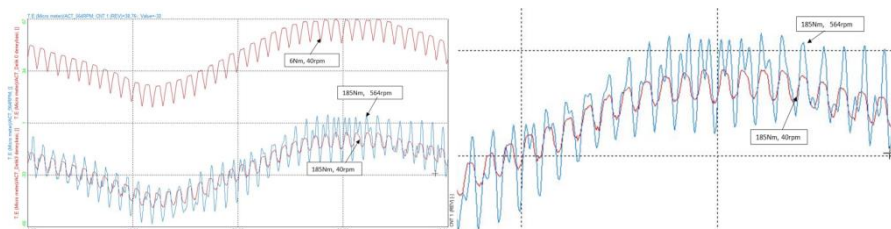


Figure 8.8 DTE curves of Case S22 at 40rpm and 564rpm

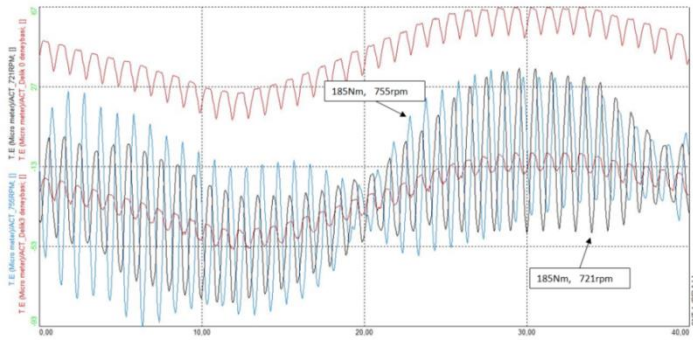


Figure 8.9 DTE curves of Case S22 at 721rpm and 755rpm

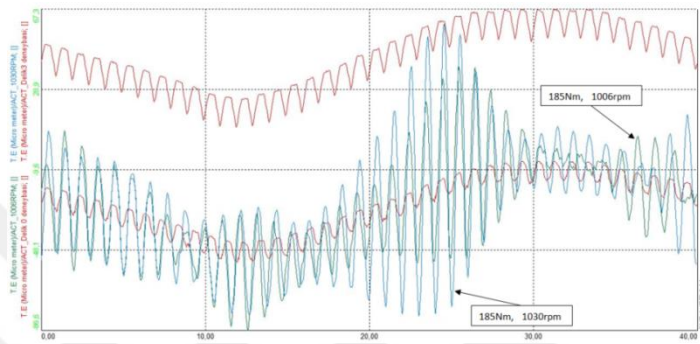


Figure 8.10 DTE curves of Case S22 at 1006rpm and 1030rpm

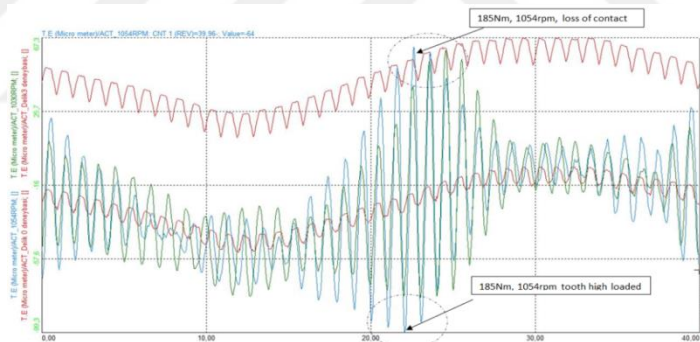


Figure 8.11 DTE curves of Case S22 at 1030rpm and 1054rpm

Dynamic frequency-response curve under 185Nm for different rotational speeds is presented in Figure 8.12 by obtaining it from dynamic transmission error curves under same torque presented above. As it can be concluded from Figure 8.12, three different resonance speeds are obvious; first one is about 750rpm, second one is in between 1000rpm and 1200 rpm, third one is about 2200-2600rpm. In addition, harmonic resonances are apparent at 250rpm and 386rpm.

When same gear pair (Case S22) were run under 396Nm (94%) instead of 185Nm, dynamic response curve was gotten as shown in Figure 8.13. It is possible to say that

there are more resonance frequencies than one. Dynamic frequency response curve of 396Nm differs from that of 185Nm for following circumstances:

Amplitude of frequency at 1065rpm increases whereas frequency amplitude at 2200rpm-2600rpm decreases. Frequency response at 750rpm shifts to 810rpm and amplitude falls off under increased load. This shows that resonance is directly related to the tooth stiffness.

Dynamic frequency response curve of Case S22 under 323Nm (76%) is presented in Figure 8.14. When load is decreased to 323Nm (from 396Nm) frequency response at 810rpm decreased to 791rpm but amplitude is almost same. This is also evident of the relationship between resonance and tooth pair stiffness.

In addition to DTE frequency response curves, linear shaft deflection frequency response curves can be constructed by using Eddy current sensors. Dynamic shaft deflection response curve for the load of 323Nm is given in Figure 8.15. There is an apparent resonance value at about 800rpm (791rpm) in this curve and this resonance value is visible in dynamic transmission error frequency response curve. Resonances can be seen at 1200rpm and 2500rpm for this frequency response curve similar to response curves presented above. When these amplitude values are taken into consideration, occurrence of these resonances is predicted based on angular deflection more than linear shaft deflection.

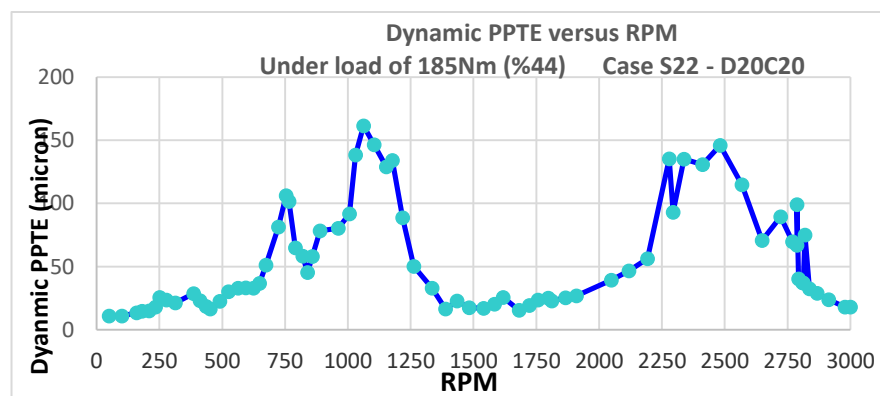


Figure 8.12 DTE response curve (experimentally obtained) for Case S22(185Nm)

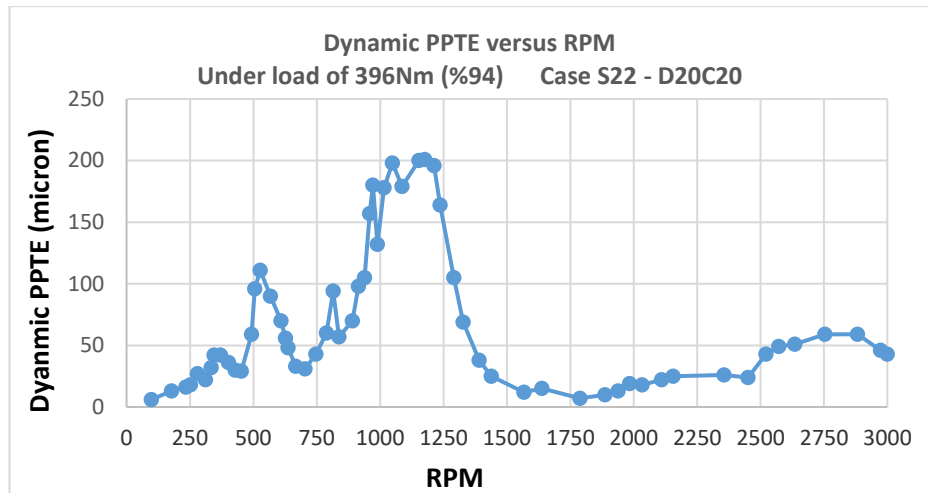


Figure 8.13 DTE response curve (experimentally obtained) for Case S22(396Nm)

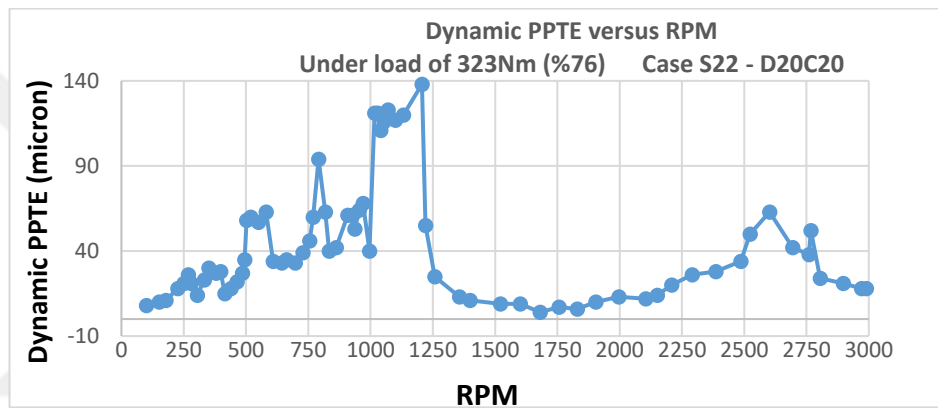


Figure 8.14 DTE response curve (experimentally obtained) for Case S22(323Nm)

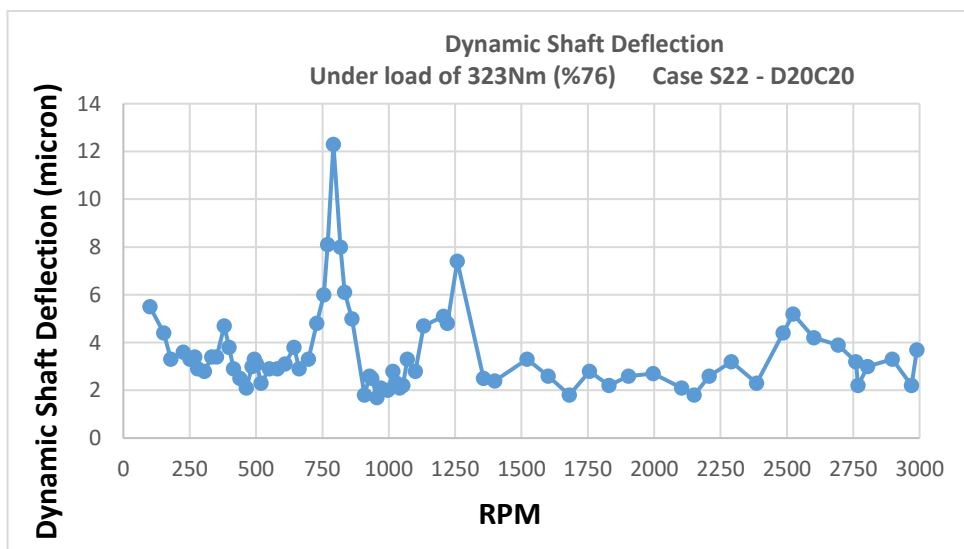


Figure 8.15 Dynamic shaft deflection response curve for Case S22 (323Nm)

8.3 Measurements of Dynamic Transmission Error for Asymmetrical Tooth Spur Gear Pairs

Asymmetric tooth spur gears of Cases A22 and A23 were analyzed to evaluate their dynamic performances using gear test rig [1]. Dynamic transmission error curves for asymmetric tooth spur gear pairs of Case A22 under the load of 202Nm in the rotation of CCW direction are presented in Figure 8.16-8.43. These curves show the variation of peak to peak dynamic transmission error values, DTE curves' regime and DC positions under the effect of operational speeds. Dynamic frequency response curve obtained from DTE curves for the load of 202Nm is presented in Figure 8.44. For another off-design load (410Nm) for same gear pair (Case A22), dynamic response curve is given in Figure 8.45. Frequency response curves for both of the load values (202Nm and 410Nm) are presented together in Figure 8.46.

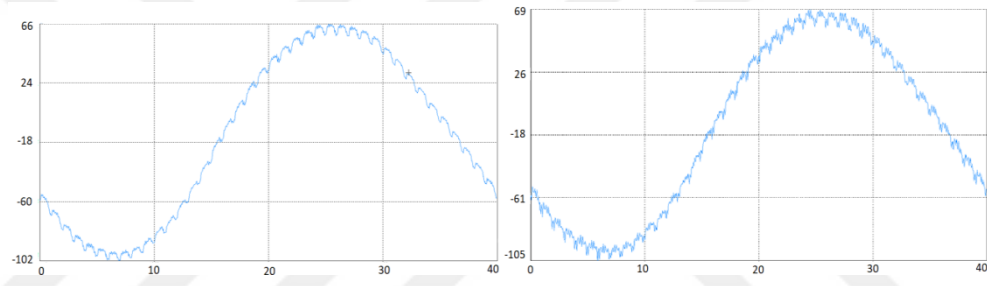


Figure 8.16 DTE curves of Case A22 at 108rpm and 151rpm

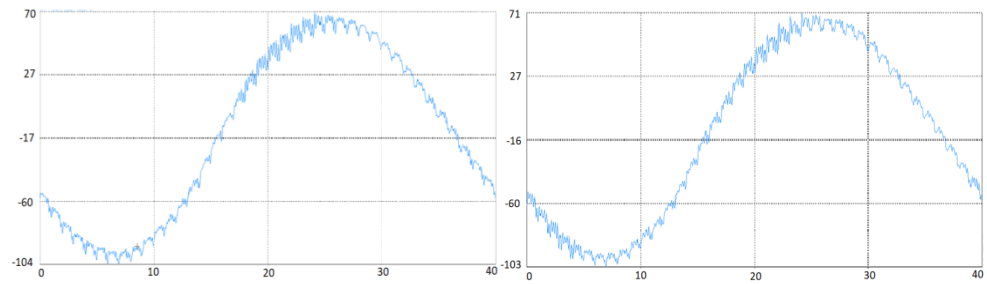


Figure 8.17 DTE curves of Case A22 at 187rpm and 199rpm

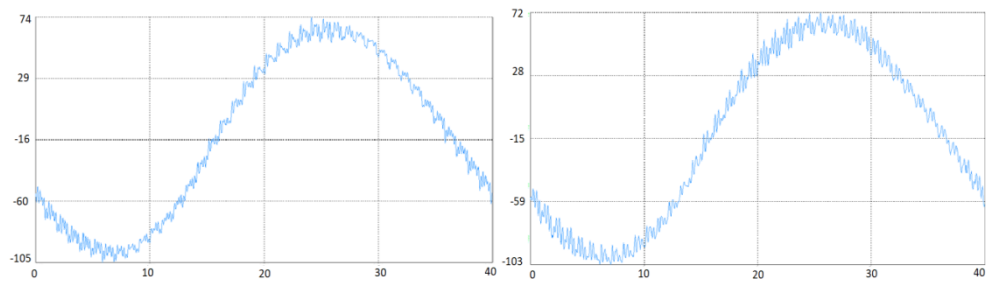


Figure 8.18 DTE curves of Case A22 at 222rpm and 247rpm

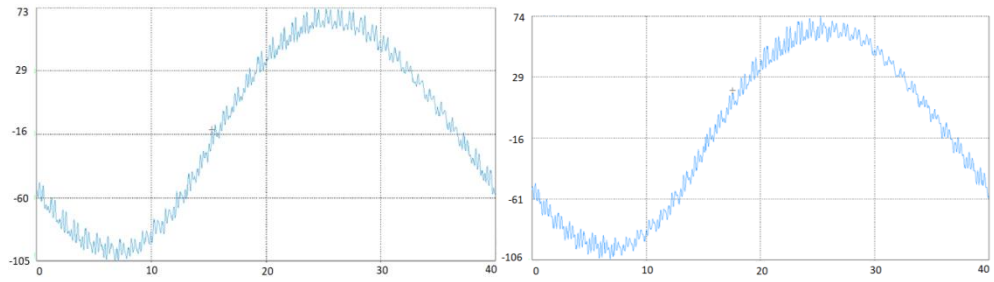


Figure 8.19 DTE curves of Case A22 at 259rpm and 265rpm

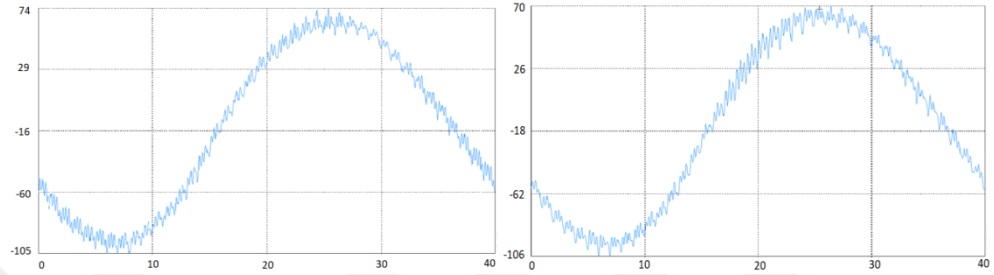


Figure 8.20 DTE curves of Case A22 at 281rpm and 295rpm

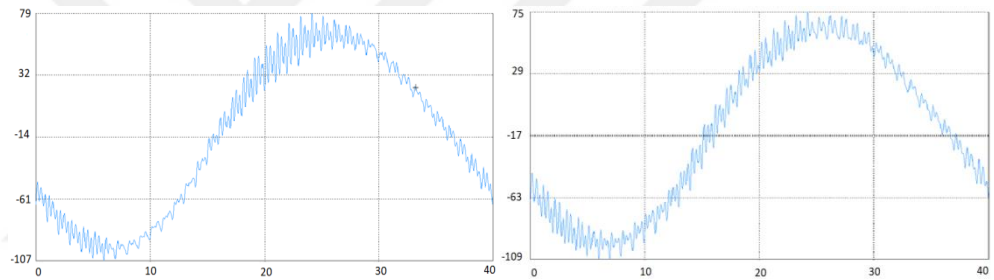


Figure 8.21 DTE curves of Case A22 at 312rpm and 339rpm

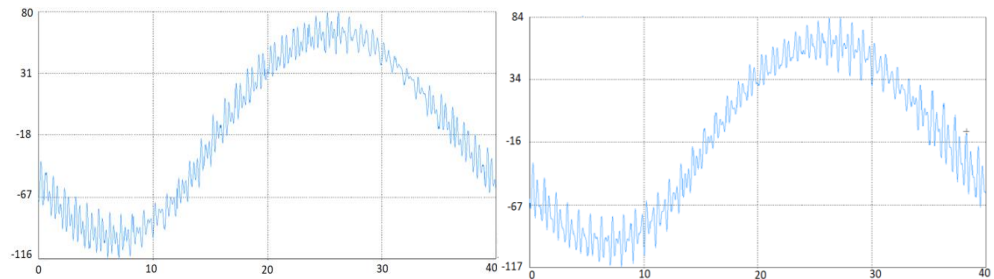


Figure 8.22 DTE curves of Case A22 at 361rpm and 377rpm

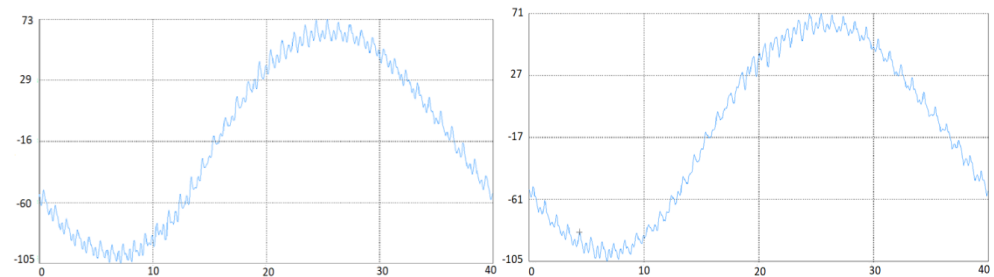


Figure 8.23 DTE curves of Case A22 at 410rpm and 435rpm

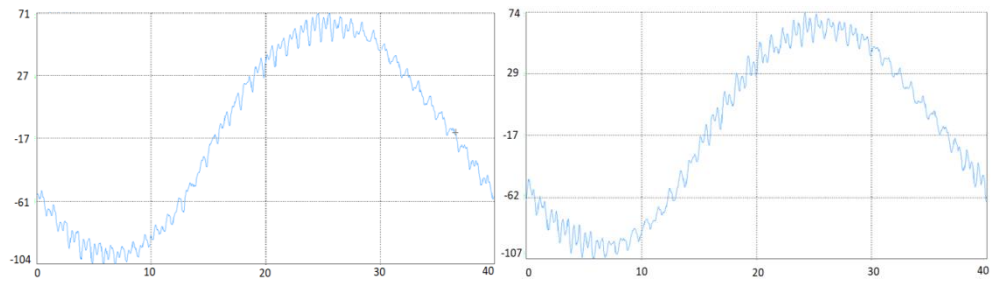


Figure 8.24 DTE curves of Case A22 at 457rpm and 495rpm

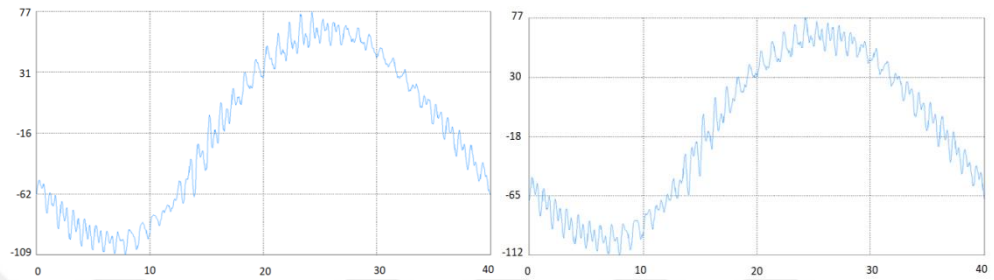


Figure 8.25 DTE curves of Case A22 at 535rpm and 552rpm

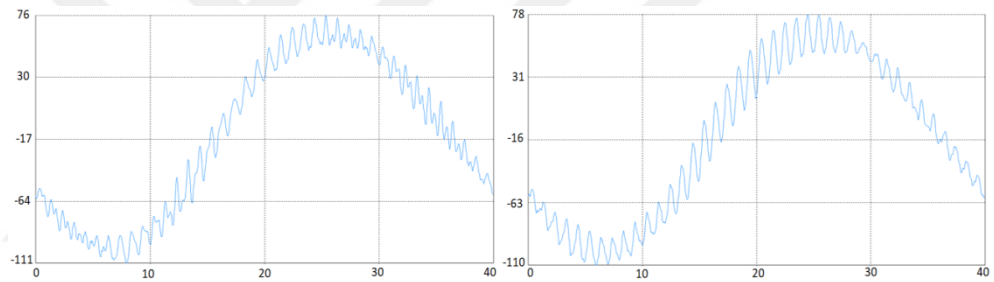


Figure 8.26 DTE curves of Case A22 at 593rpm and 629rpm

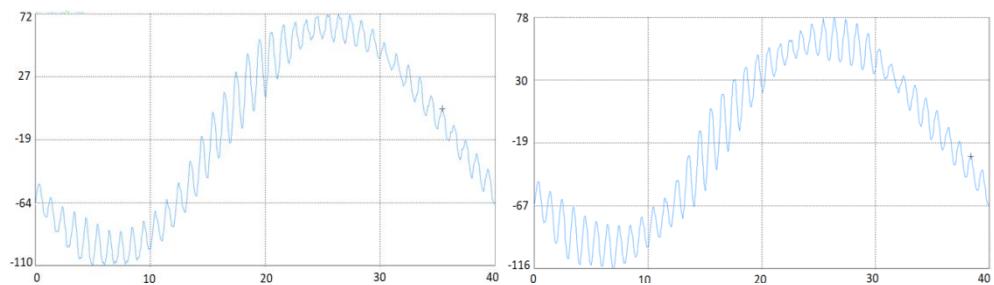


Figure 8.27 DTE curves of Case A22 at 661rpm and 682rpm

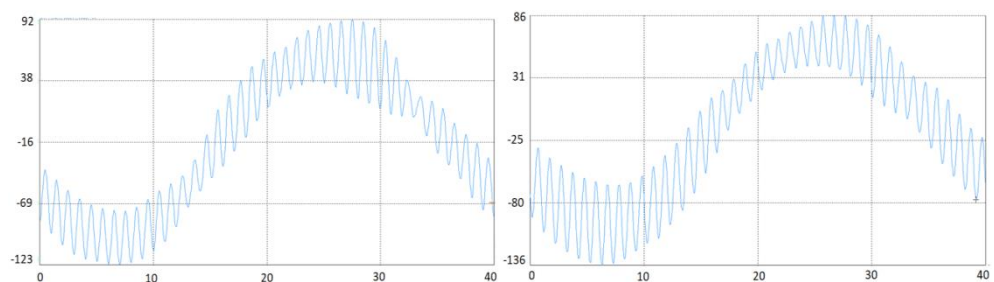


Figure 8.28 DTE curves of Case A22 at 722rpm and 747rpm

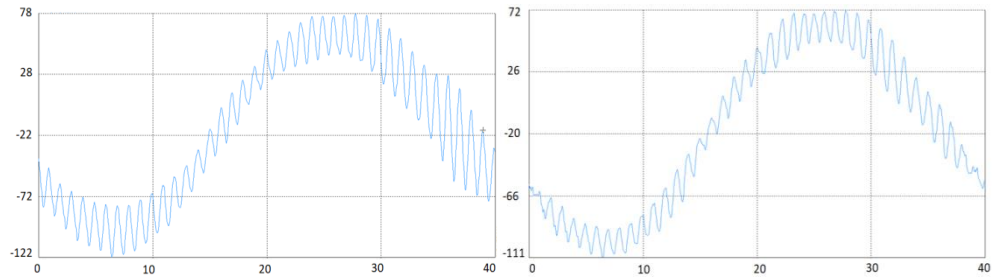


Figure 8.29 DTE curves of Case A22 at 771rpm and 798rpm

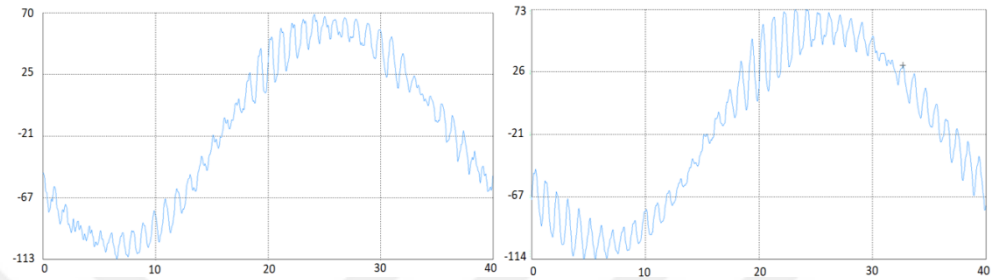


Figure 8.30 DTE curves of Case A22 at 851rpm and 876rpm

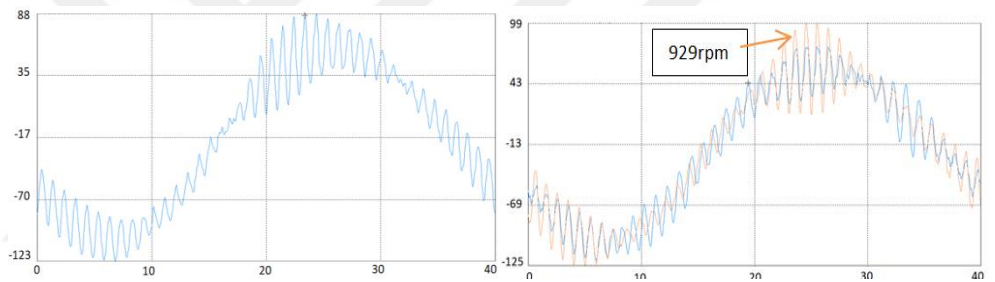


Figure 8.31 DTE curves of Case A22 at 896rpm and 929-957rpm

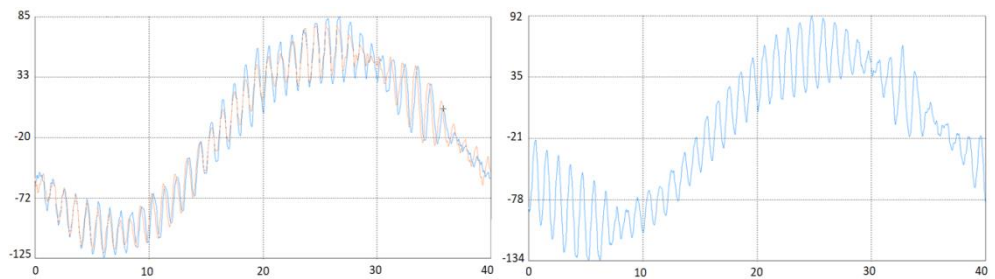


Figure 8.32 DTE curves of Case A22 at 957-977 and 1007rpm

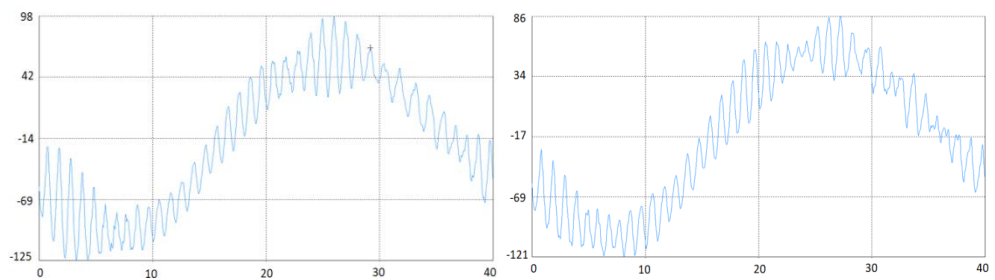


Figure 8.33 DTE curves of Case A22 at 1042rpm and 1069rpm

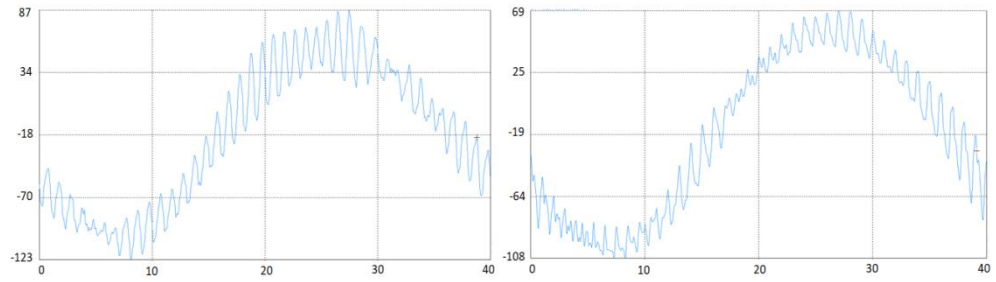


Figure 8.34 DTE curves of Case A22 at 1100rpm and 1218rpm

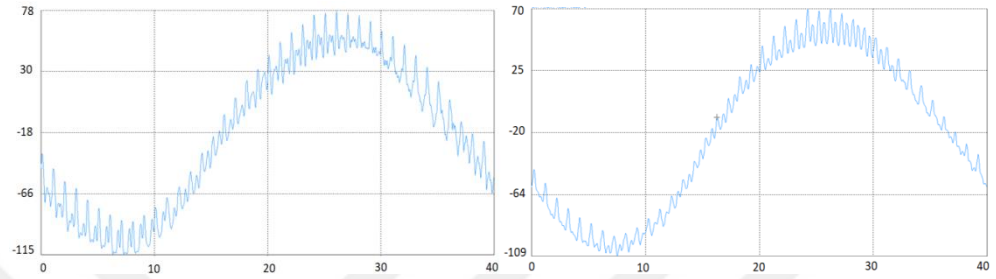


Figure 8.35 DTE curves of Case A22 at 1290rpm and 1402rpm

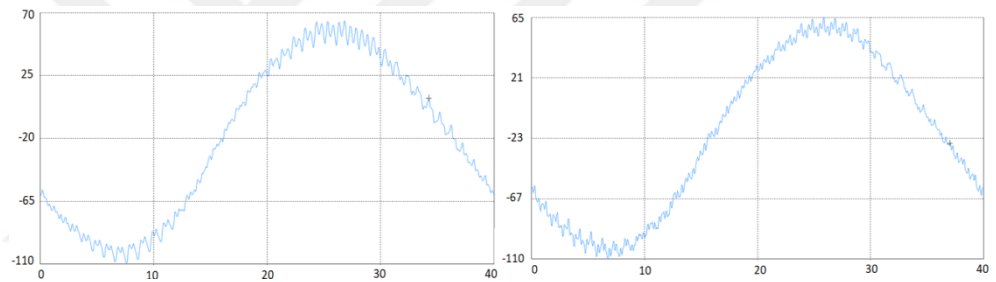


Figure 8.36 DTE curves of Case A22 at 1510rpm-1643rpm

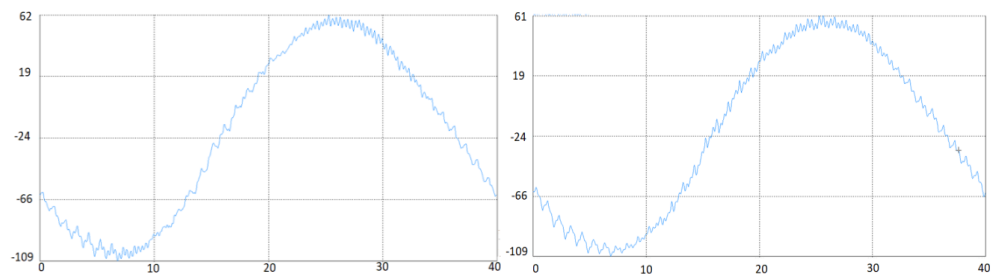


Figure 8.37 DTE curves of Case A22 at 1735rpm-1816rpm

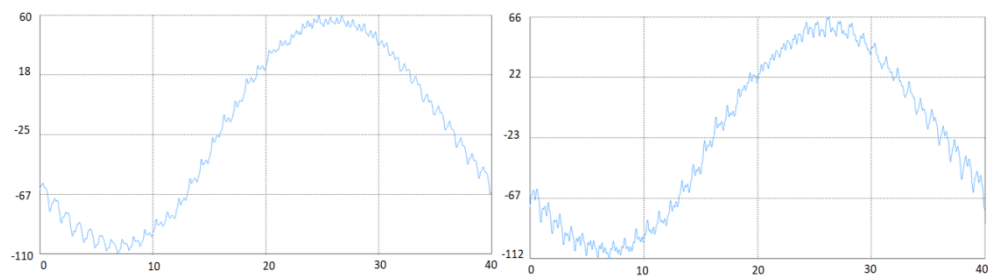


Figure 8.38 DTE curves of Case A22 at 1917rpm-2022rpm

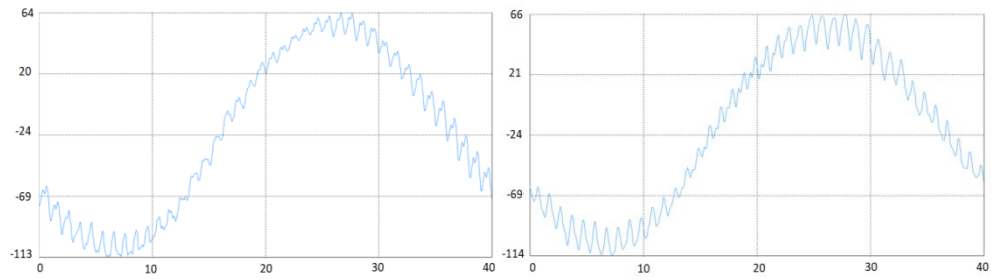


Figure 8.39 DTE curves of Case A22 at 2119rpm-2225rpm

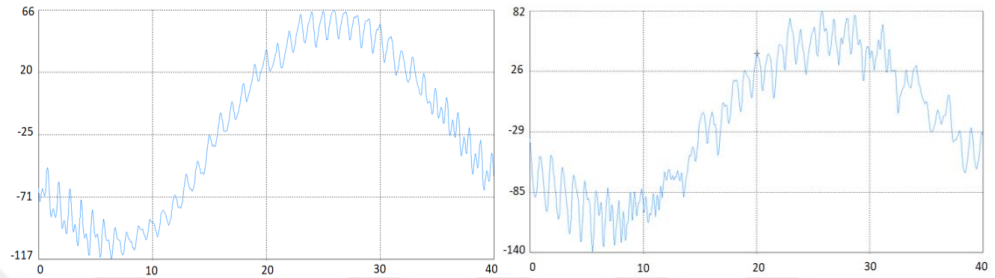


Figure 8.40 DTE curves of Case A22 at 2339rpm-2430rpm

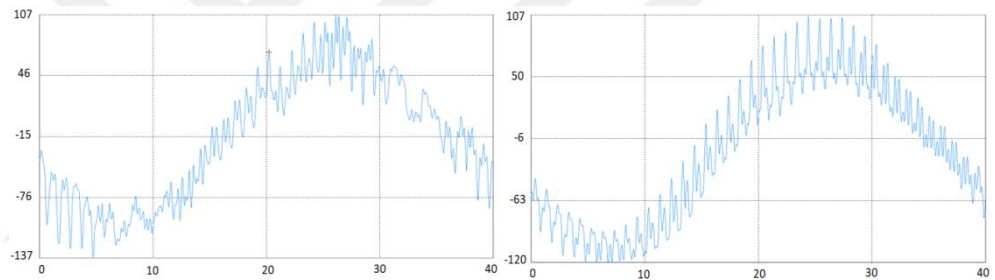


Figure 8.41 DTE curves of Case A22 at 2532rpm-2625rpm

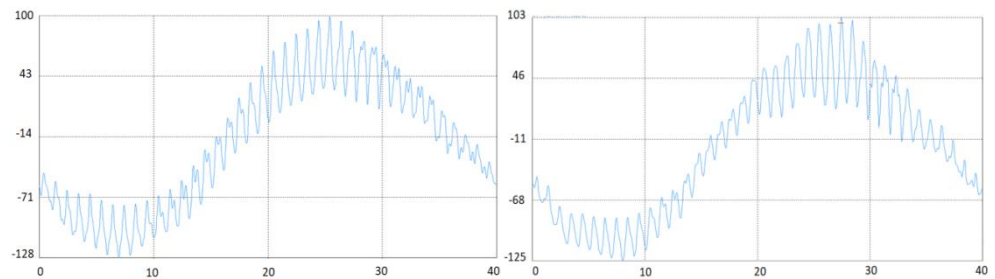


Figure 8.42 DTE curves of Case A22 at 2721rpm-2804rpm

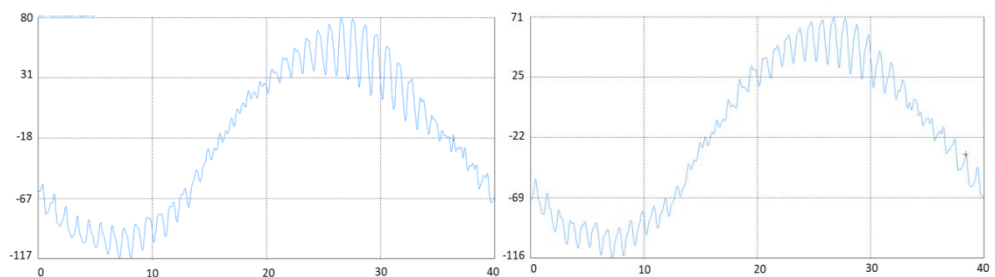


Figure 8.43 DTE curves of Case A22 at 2917rpm-3000rpm

As it can be deduced from Figure 8.44, three different resonance speeds are obvious; first one is about 750rpm, second one is in between 1000rpm and 1200 rpm, third one is about 2500rpm. In addition, harmonic resonances are apparent at 400rpm, 900rpm and 2800rpm. When same gear pair (Case A22) are run under 410Nm (97%) instead of 202Nm (48%), dynamic response curve is obtained as shown in Figure 8.45. Dynamic frequency response curve of 410Nm differs from that of 202Nm for following circumstances (Figure 8.46):

Amplitude of frequency at 1050rpm increases and frequency amplitude at 2500rpm shifts to 2700rpm with decreased amount. Frequency response at 750 rpm shifts to 800rpm and amplitude increases.

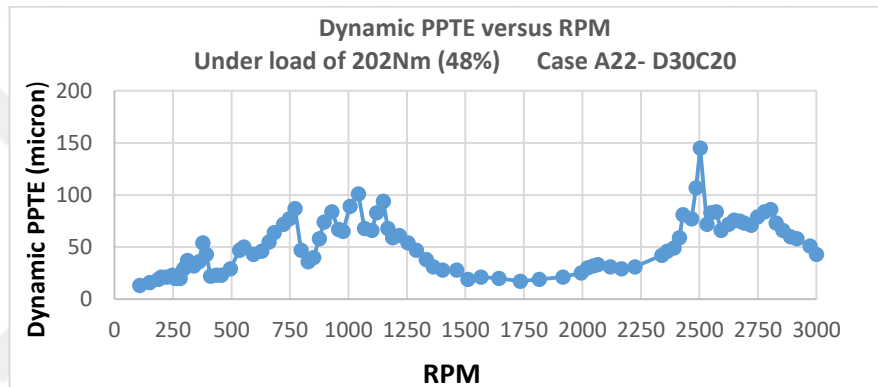


Figure 8.44 DTE response curve (experimentally obtained) for Case A22 (202Nm)

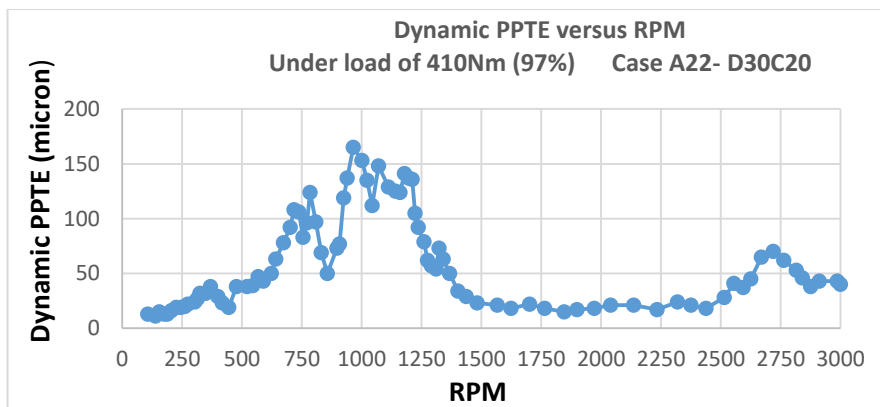


Figure 8.45 DTE response curve (experimentally obtained) for Case A22 (410Nm)

Dynamic frequency response curves for asymmetric tooth spur gear pairs of Case A23 (D20C30) under the load of 227Nm (54%) and 438Nm (104%) are presented in 8.47 and 8.48 respectively. Frequency response curves for the loads of 227Nm (54%) and 438Nm (104%) are presented together in Figure 8.49.

Resonance values at 775rpm, 1000-1200rpm and 2500 rpm were obtained under the load of 227Nm whereas 900 rpm, 1200rpm and 2700 rpm can be seen as resonances for 438Nm in Figure 8.47 and 8.48 respectively.

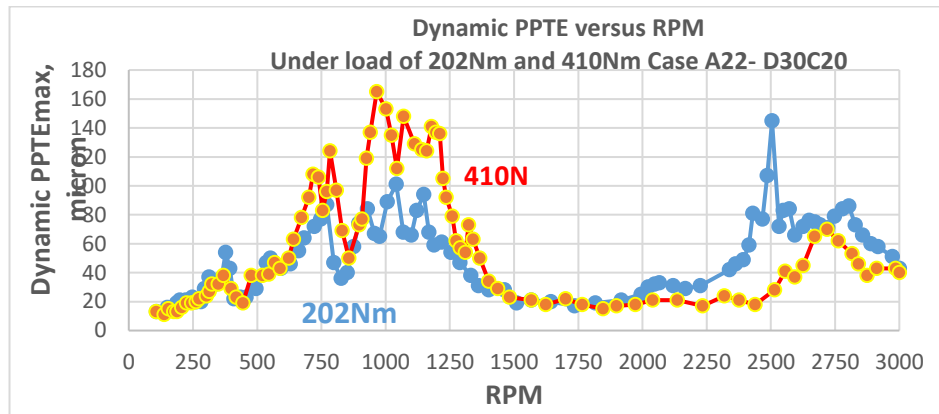


Figure 8.46 DTE response curve (experimentally obtained) for Case A22 (202Nm and 410Nm)

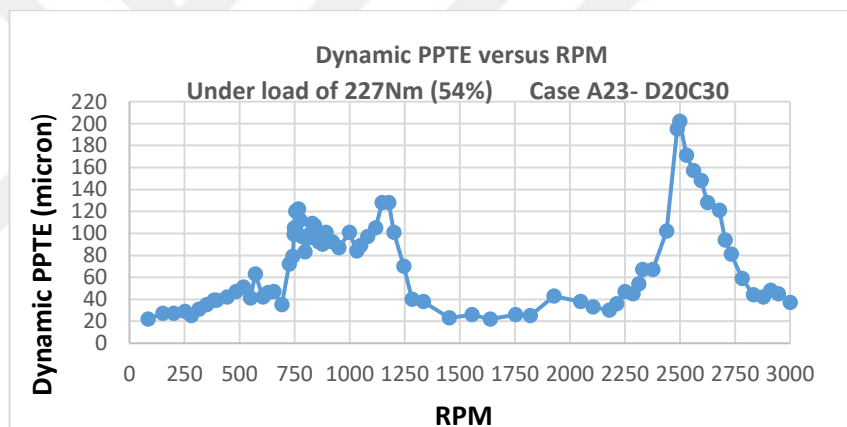


Figure 8.47 DTE response curve (experimentally obtained) for Case A23 (227Nm)

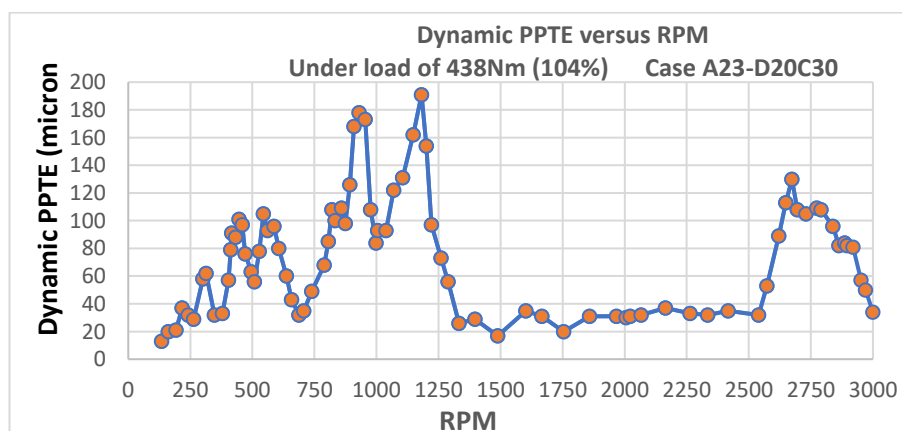


Figure 8.48 DTE response curve (experimentally obtained) for Case A23 (438Nm)

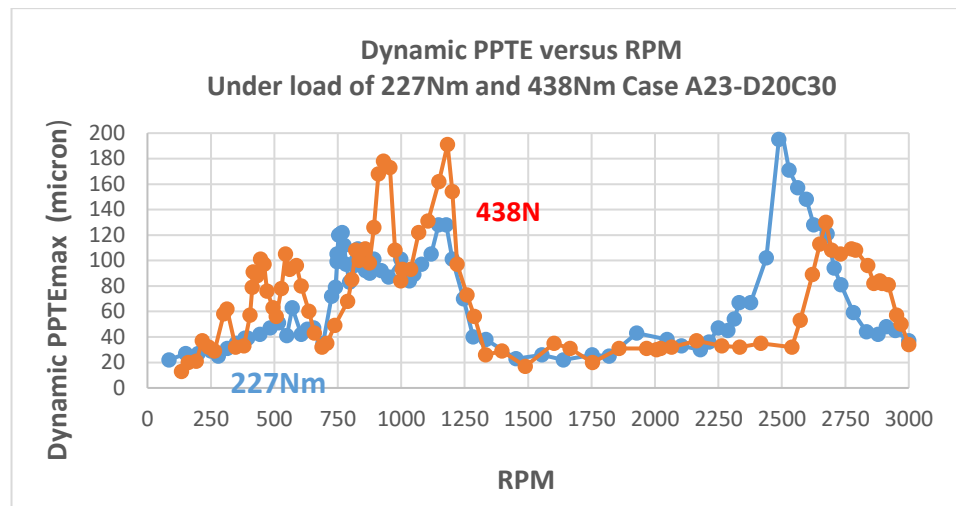


Figure 8.49 DTE response curve (experimentally obtained) for Case A23 (227Nm and 438Nm)

CHAPTER 9

MATHEMATICAL MODELING OF SYMMETRIC AND ASYMMETRIC TOOTH SPUR GEAR PAIRS

9.1 Introduction

Even by designing gears (symmetric or asymmetric) in a safe way statically in terms of bending and contact stresses, designer needs to have knowledge about gear dynamic behavior and keep it under control to reduce transmission error of gear pair and as a consequence decrease in noise and vibration.

Gear dynamics has been a subject of intense interest to the gearing area during the last few decades and dynamic loads and vibration are a major concern for gear drives at high speeds. Simulation of meshing of gear drives performed by application of tooth contact analysis and experimental tests of gear drives have confirmed that transmission errors (TE) are the prime cause of noise and vibrations of the gear drives [9]. Dynamic analysis and mathematical modeling are very significant to evaluate dynamic performance of gear pairs and whole gearbox including shafts, bearing in addition to gears. There is vast amount of literature on gear dynamics and mathematical modeling of gear systems for different degree of freedom models. Some research studies intended for these achievements for symmetric and asymmetric gears for different degree of freedom models are explained in Chapter 2.

In this thesis, two degree of freedom (2-DOF) model was firstly developed to evaluate dynamic factor for spur gear pairs under different load and at different operating speeds. Then multi degree of freedom model for spur gear pair including shafts and bearings effects on gear pair dynamics was constructed. Case studies for both models are presented in this chapter.

9.2 Two degree of freedom (2-DOF) Mathematical Modeling of Gear Pairs

A 2 DOF torsional model for spur gear pair has been proposed based on reference study [133] including friction between gear pair (Figure 9.1). The equations of motion governing the angular displacements of the pinion and the gear, including forces due to friction between the contacting teeth are derived by using the free body diagrams of

gear pair and presented in Eq. 9.1 and 9.2. Equation of motions are converted and presented in Eq. 9.9 and 9.10.

The equation of motion in terms of transmission error is obtained by subtracting the above equations side by side and by including the viscous damping force and excluding gear profile errors.

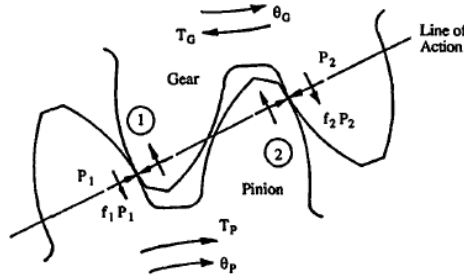


Figure 9.1 Free body diagram of gears [133]

$$J_G \ddot{\theta}_G = R_b(P_1 + P_2) - T_G \pm \rho_{G1} f_1 P_1 \pm \rho_{G2} f_2 P_2 \quad (9.1)$$

$$J_P \ddot{\theta}_P = -r_b(P_1 + P_2) - T_P \pm \rho_{P1} f_1 P_1 \pm \rho_{P2} f_2 P_2 \quad (9.2)$$

$$x_G = R_b \theta_G \quad (9.3)$$

$$x_P = r_b \theta_P \quad (9.4)$$

$$J_G = m_G R_b^2 \quad (9.5)$$

$$J_P = m_P r_b^2 \quad (9.6)$$

$$M = \frac{m_P m_G}{m_P + m_G} \quad (9.7)$$

$$P_S = \frac{T_G}{R_b} = \frac{T_P}{r_b} \quad (9.8)$$

$$m_G \ddot{x}_G = (P_1 + P_2) - P_S \pm \frac{\rho_{G1} f_1 P_1 \pm \rho_{G2} f_2 P_2}{R_b} \quad (9.9)$$

$$m_P \ddot{x}_P = -(P_1 + P_2) + P_S \pm \frac{\rho_{P1} f_1 P_1 \pm \rho_{P2} f_2 P_2}{r_b} \quad (9.10)$$

$$\ddot{x}_r = 2 \vartheta \ddot{x}_r \sqrt{\frac{k_1(s_{P1} m_G + s_{G1} m_P) + k_2(s_{P2} m_G + s_{G2} m_P)}{m_P m_G}} + \frac{k_1(s_{P1} m_G + s_{G1} m_P) + k_2(s_{P2} m_G + s_{G2} m_P)}{m_P m_G} x_R = \frac{(m_G + m_P)}{m_P m_G} P_S \quad (9.11)$$

$$x_r = x_p - x_g \quad (9.12)$$

$$\dot{x}_r = \dot{x}_p - \dot{x}_g \quad (9.13)$$

$$\ddot{x}_r = \ddot{x}_p - \ddot{x}_g \quad (9.14)$$

$$k_1 = \frac{k_{G1}k_{P1}}{k_{G1} + k_{P1}} \quad (9.15)$$

$$k_2 = \frac{k_{G2}k_{P2}}{k_{G2} + k_{P2}} \quad (9.16)$$

$$s_{G1} = 1 \pm \frac{\rho_{G1}f_1}{R_b} \quad (9.17)$$

$$s_{G2} = 1 \pm \frac{\rho_{G2}f_2}{R_b} \quad (9.18)$$

$$s_{P1} = 1 \pm \frac{\rho_{P1}f_1}{r_b} \quad (9.19)$$

$$s_{P2} = 1 \pm \frac{\rho_{P2}f_2}{r_b} \quad (9.20)$$

Table 9.1 Nomenclature for equations of 9.1-9.20

Symbol	Explanation
$J_{P,G}$	Polar moment of inertias of pinion and gear
P_s	Steady state force resulting from the driving torque
$P_{1,2}$	Dynamic load at first contact and second contact points
$T_{P,G}$	Torque values of pinion and gear
$\rho_{G1,2}$	Radius of curvature of gear at first and second contact points
$\rho_{P1,2}$	Radius of curvature of pinion at first and second contact points
$f_{1,2}$	Coefficient of friction for contact points of first, second
r_b, R_b	Base radius of pinion, gear
$\theta_{P,G}$	Angular displacement of pinion, gear
$m_{P,G}$	Reduced mass of pinion, gear
M	Equivalent mass
$k_{1,2}$	Combined stiffness at first and second contact points
$x_{P,G}$	Displacement of tooth profile of pinion and gear
x_r	Transmission error

If the speed of the pinion is greater than gear's one, the sign is positive otherwise it is negative for the relations given above. Equivalent stiffness values of k_1 and k_2 at the contact points are calculated by equations of 9.15 and 9.16 by combining the stiffness of contacting teeth as springs connected in series.

MyDynamicModel

CALCULATE

Parameters Of Study

Design Load: 7484,790 N

Input Speed: 1400,000 rpm

Parameters Of Gears

	Pinion	Gear
Gears' Damping Coef	0,10	-
Density	7850,0 kg/m^3	7850,0 kg/m^3
Gear Mass	auto 1,332 kg	1,332 kg

X-Axis Type

Lc: 15,1758149892709

pb: 8,85639430228065

CR: 1,71354328537099

DynFac vs RPM

X-Axis Type

☒ Time Based

☐ Teeth Number

Figure 9.3 Software interface for input parameters for 2DOF mathematical model

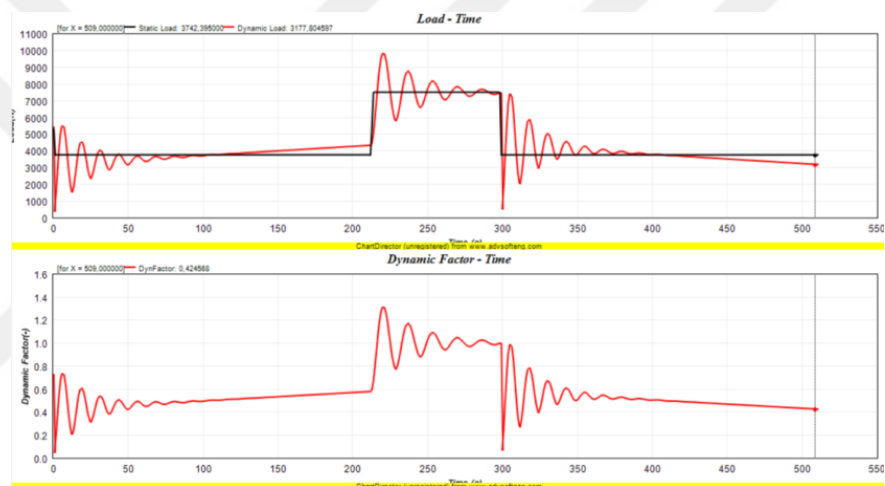


Figure 9.4 Software interface for gear mesh loads (dynamic load and static load sharing) and dynamic factor

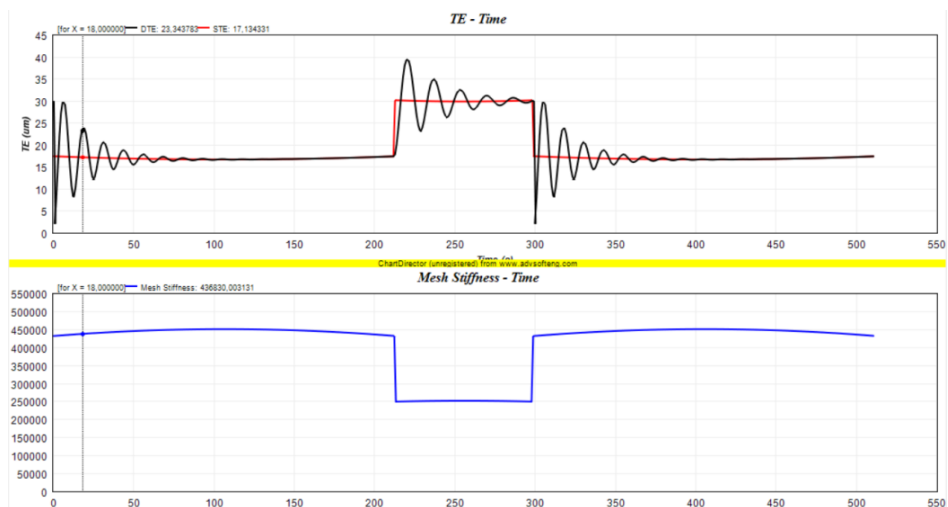


Figure 9.5 Software interface for dynamic transmission error (DTE) and mesh stiffness

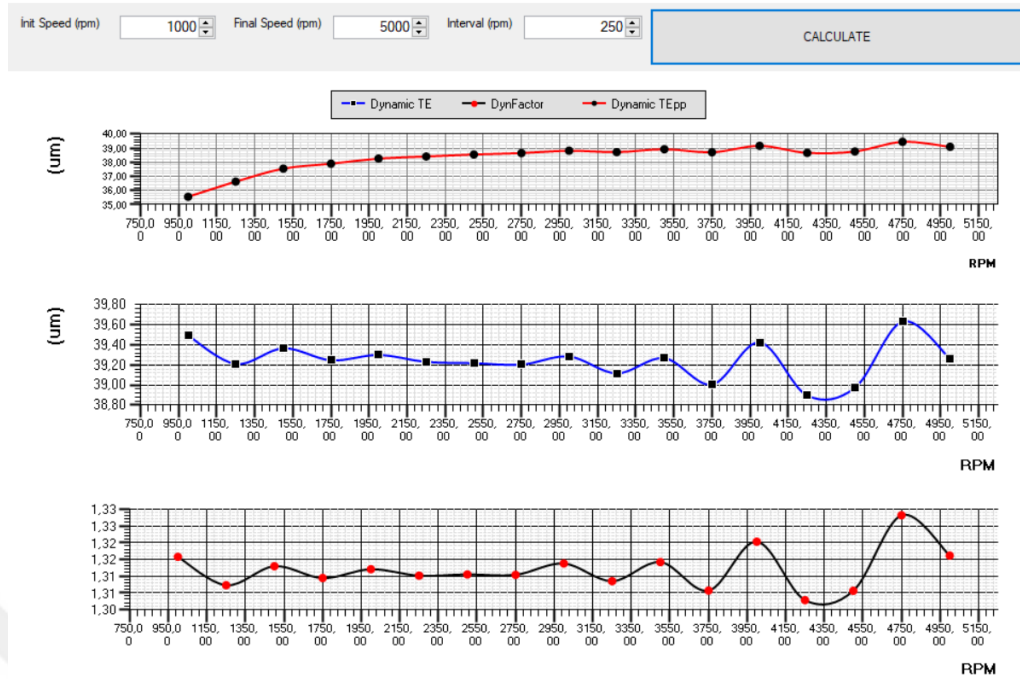


Figure 9.6 Software interface for PPDTE, maximum DTE and DF versus rpm

9.4 Case Studies of Symmetric Spur Gears (2-DOF)

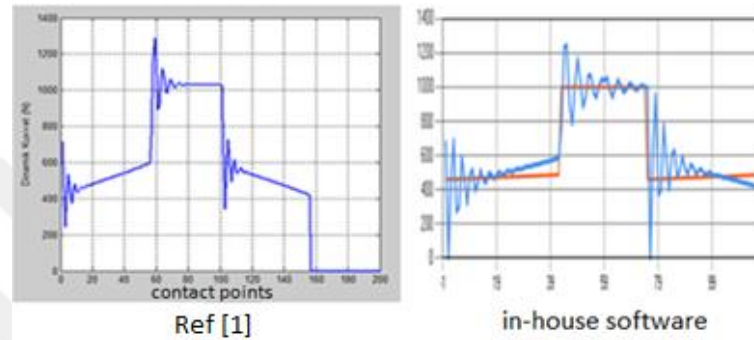
Some case studies were performed to verify our 2-DOF mathematical model's results with literature works. Some case studies studied in the scope of a distinctive project of gear design for helicopters [1] have been analyzed and presented here. In addition, some analyses given in literature were carried out to compare dynamic load and factor curves.

9.4.1 Case S23

A symmetrical tooth profile spur gear pair was analyzed for dynamic performance based on in-house software developed to solve governing equation of motions. Gear pair parameters such as normal module, teeth number, pressure angle and face width, operating conditions such as load and speed with mesh damping coefficient are presented in Table 9.2. Dynamic load curves obtained in the Ref [1] and by in-house software were presented in Figure 9.7. These curves have similar oscillations and maximum load values (almost 1300N) whereas there is static load of 1000 Newton.

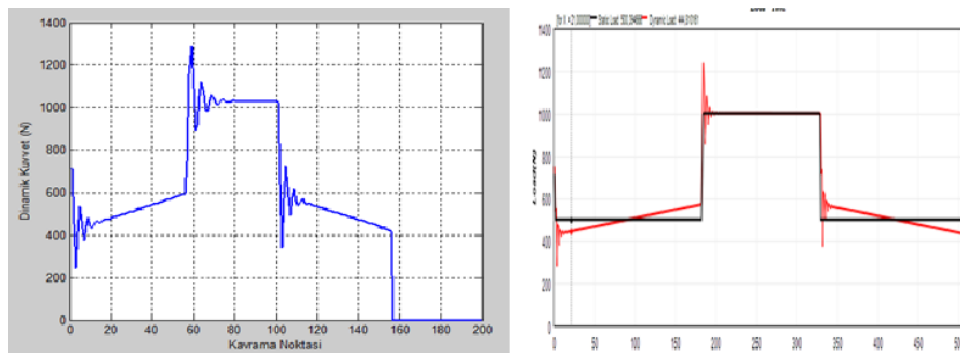
Table 9.2 Gear pair parameters – Case S23

Parameter	Value	Unit
Normal module	3	mm
Teeth number of pinion	20	-
Gear ratio	1	-
Pressure Angle	20	Degree
Load	1000	N
Face width	25.4	mm
Rotational speed	1500	rpm
Mesh damping	0.10	-

**Figure 9.7** Dynamic load curves– Case S23

9.4.2 Case S24

A symmetrical spur gear pair whose parameters are same with those of Case S23 except mesh damping coefficient was evaluated in term of dynamic load. Mesh damping coefficient is 0.17 for this case. Dynamic load curves for reference and current study are presented in Figure 9.8.

**Figure 9.8** Dynamic load curves– Case S24

9.4.3 Comparison of Case S23 and S24

Case S23 differs from S24 in terms of mesh damping coefficient whereas all other parameters of gear pair and loading conditions are same. These two cases were

compared to check our model correctness in terms of reflecting effective parameters on gear dynamics. Mesh damping increase yields in decrease of oscillation of dynamic load curve for Case S24 (with mesh damping ratio of 0.17) when compared to Case S23 (with lower value of mesh damping ratio) as shown in Figure 9.9.

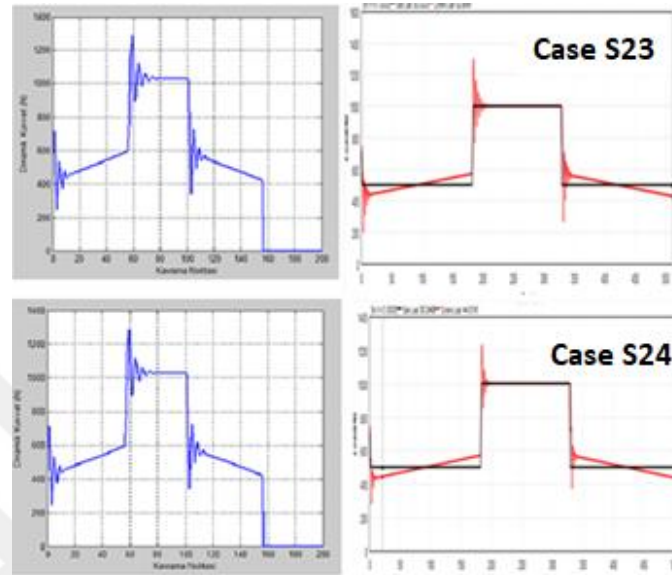


Figure 9.9 Dynamic load curves for Case S23 and S24

9.4.4 Case S25

A symmetric spur gear pair with gear parameters presented in Table 9.3 was analyzed and dynamic load curves are given in Figure 9.10. The load given in pound force refers to 3300N whereas our software estimates this dynamic force as 3500N.

Table 9.3 Gear pair parameters of Case S25

Parameter	Value	Unit
Normal module	3.18	mm
Teeth number	36	-
Pressure Angle	20	Degree
Load	2668.93	N
Face width	25.4	mm
Rotational speed	1600	rpm
Mesh damping	0.10	-

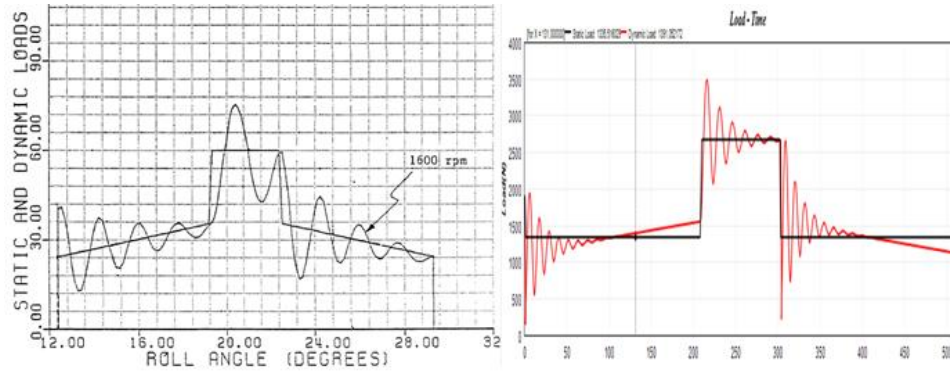


Figure 9.10 Dynamic load curves - Case S25 (Ref [157] and in-house software)

9.4.5 Case S26

A symmetrical tooth profile spur gear pair whose gear parameters presented in Table 9.4 was analyzed and dynamic load curves were generated (Figure 9.11). Dynamic factor versus rotational speed are presented in Figure 9.12 and 9.13 for Ref [1] and in-house software respectively.

Peak values are lower and speeds corresponding to these peak values are shifted slightly for curve constructed by in-house software. This can stem from the difference between models developed in reference and current study.

Table 9.4 Gear pair parameters of Case S26

Parameter	Value	Unit
Normal module	3.18	mm
Teeth number	36	-
Pressure Angle	20	Degree
Load	1000	N
Face width	25.4	mm
Rotational speed	1000	rpm
Mesh damping	0.10	-
Gear mass	1.7496	kg

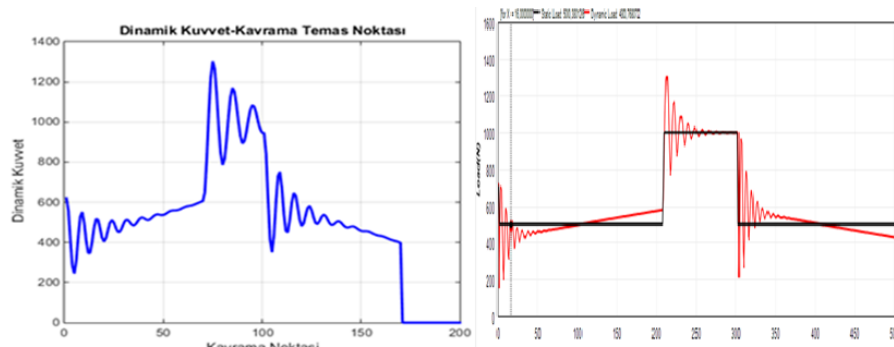


Figure 9.11 Dynamic load curves of Case S26

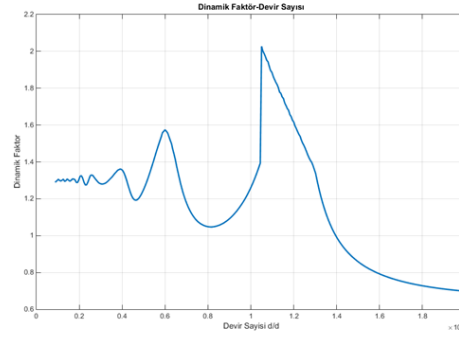


Figure 9.12 Dynamic factor versus rotational speed - Case S26 [1]

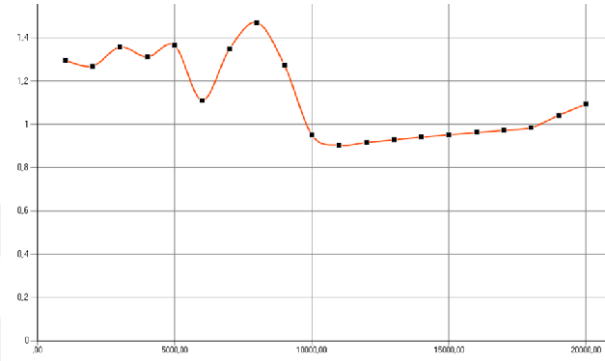


Figure 9.13 Dynamic factor versus rotational speed - Case S26 (in-house software)

9.4.6 Case S27

Symmetric tooth spur gear pair studied in Ref [131] was analyzed in terms of dynamic load factor and curves of load factor versus rotational speed were illustrated in Figure 9.14 and 9.15. Dynamic load factor were calculated lower than reference study. This can be because of eliminating shaft and bearing flexibilities in our 2-DOF model.

Table 9.5 Gear pair parameters – Case S27

Parameter	Value	Unit
Normal module	3.18	mm
Teeth number	32	-
Pressure Angle	20	Degree
Load	8890	N
Face width	25.4	mm
Rotational speed	1000	rpm
Mesh damping	0.10	-

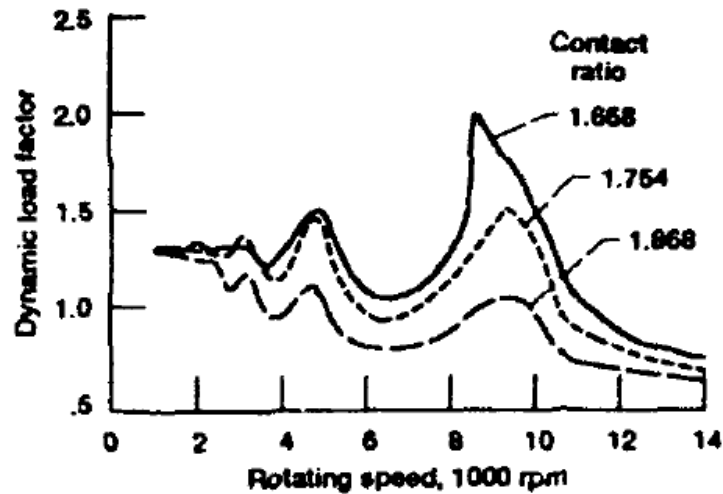


Figure 9.14 Dynamic factor versus rotational speed - Case S27 [131]

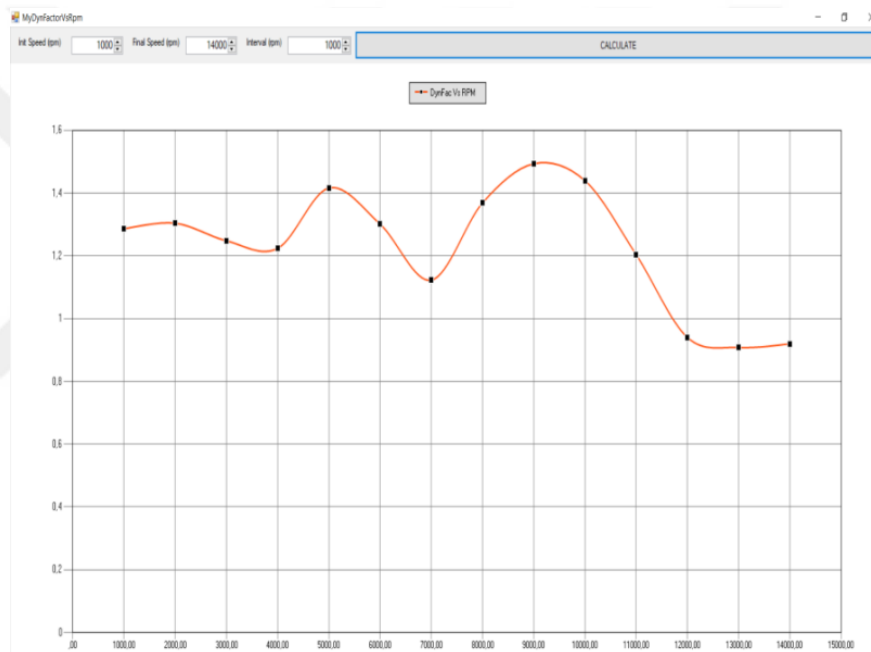


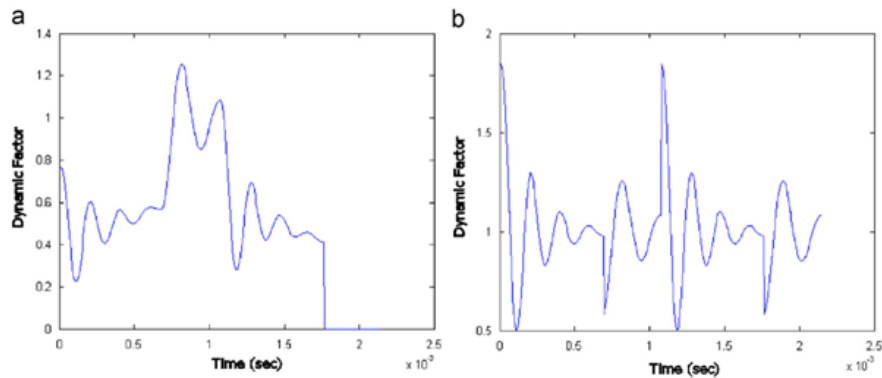
Figure 9.15 Dynamic factor vs rotational speed (contact ratio of 1.668) - Case S27

9.4.7 Case S28

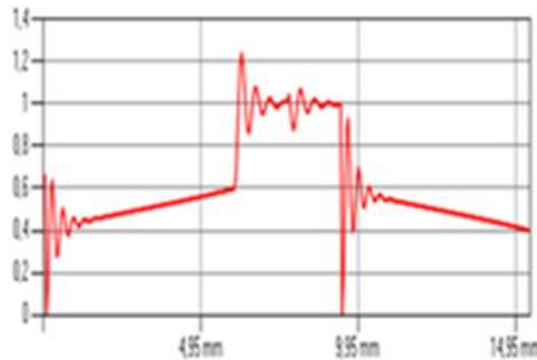
Symmetric tooth spur gear pair analyzed in Ref [127] was investigated in terms of dynamic factor and curves of dynamic load factor-rotational speed were illustrated in Figure 9.16 and 9.17. Figure 9.16 shows dynamic load factor for single tooth and double tooth contact regions separately while it is given in Figure 9.17 for whole contact region. Dynamic load factor were estimated lower than reference study. This can be expected because 2-DOF model ignores the effects and flexibilities of shaft and bearings.

Table 9.6 Gear pair parameters – Case S28

Parameter	Value	Unit
Normal module	3.18	mm
Teeth number	28	-
Pressure Angle	20	Degree
Load	1000	N
Face width	6.35	mm
Rotational speed	1500	rpm
Mesh damping	0.17	-
Mass of gear	0.3	kg



Dynamic factors for a single period at 1500 rpm: (a) single-tooth pair and (b) double-tooth pair.

Figure 9.16 Dynamic factors vs rotational speeds - Case S28 [127]**Figure 9.17** Dynamic factor versus rotational speed - Case S28

9.5 Case Studies of Asymmetric Spur Gears (2-DOF)

Some case studies were performed to verify our 2-DOF mathematical model's results with literature works in previous section. In addition to those, some case studies for asymmetric tooth spur gears were presented in this section.

9.5.1 Case A24

An asymmetrical tooth profile spur gear which has same parameters except pressure angle similar to symmetric one (Case S23) given in previous case study was analyzed

to obtain dynamic load. Gear pair parameters and operating conditions are given in Table 9.7. Dynamic load curves for 5000 and 10000 rpm are illustrated in Figure 9.18-9.19. The effect of rotational speed (by keeping all other parameters constant) was obtained from those figures. The maximum value of dynamic load is around 1300 Newton for both tools.

Table 9.7 Gear pair parameters of Case A24

Parameter	Value	Unit
Normal module	3	mm
Teeth number	20	-
Pressure angle (drive/coast)	30/20	Degree
Load	1000	N
Face width	25.4	mm
Rotational speed	5000-10000	rpm
Mesh damping	0.10	-

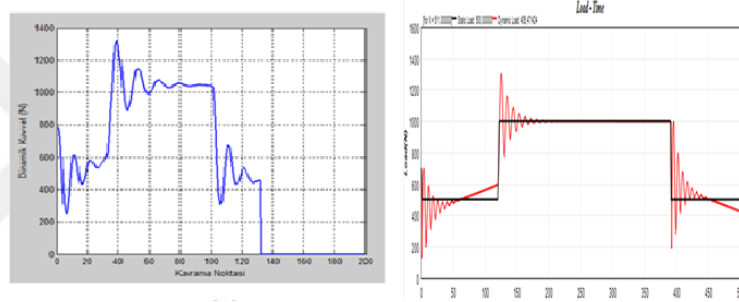


Figure 9.18 Dynamic load curves for 5000 rpm – Case A24

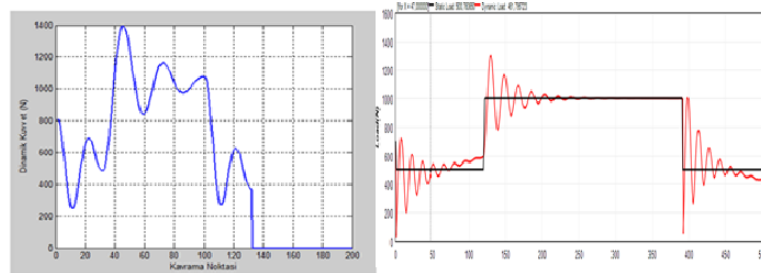


Figure 9.19 Dynamic load curves for 10000 rpm – Case A24

9.5.2 Case A25

An asymmetrical spur gear pair which is similar to Case A24 (except mesh damping coefficient) by keeping all parameters same was analyzed for mesh damping coefficient of 0.17. Dynamic load curves for two different rotational speed values of 5000 and 10000 rpm are generated (Figure 9.20 and 9.21). Greater rotational speed (10000 rpm) gives slightly higher dynamic load value.

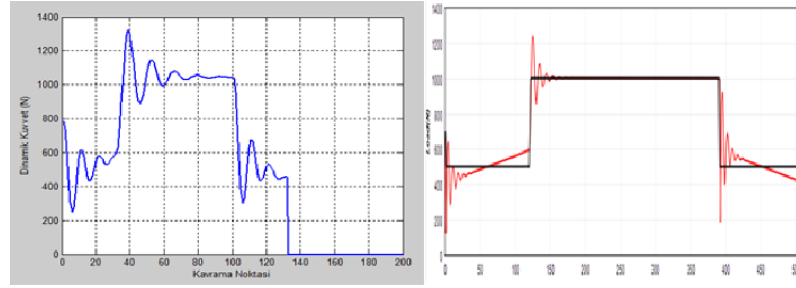


Figure 9.20 Dynamic load curves for 5000 rpm – Case A25

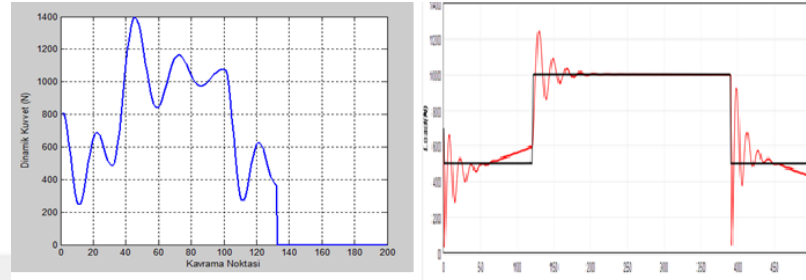


Figure 9.21 Dynamic load curves for 10000 rpm – Case A25

In this section, the efforts have been generally focused on guessing the dynamic characteristics of spur gear pairs by starting a 2-DOF torsional model of gear pair. 2-DOF model gives similar dynamic load and dynamic factor values with literature, but the model does not yield exactly same results with any published work. This can stem from the involved effects into models, each model include and exclude different effective factors such as friction between contacting gears, bearing stiffness, shaft stiffness, gear errors due to manufacturing and static transmission error.

9.6. Software for 6-DOF Mathematical Modeling

A 6-DOF torsional model for spur gear pair including flexible shafts and bearings has been presented (Figure 9.22). The equations of motion governing the angular displacements of the pinion and the gear, including torsional vibration of shafts and translational displacement of bearings were derived. Fourth order Runge-Kutta method has been used to solve nonlinear differential equations. An in-house software has been developed to solve nonlinear equations of motion based on multi degree of freedom (Figure 9.23-9.25). Backlash is taken into consideration to evaluate its effect on tooth separation and backside collision.

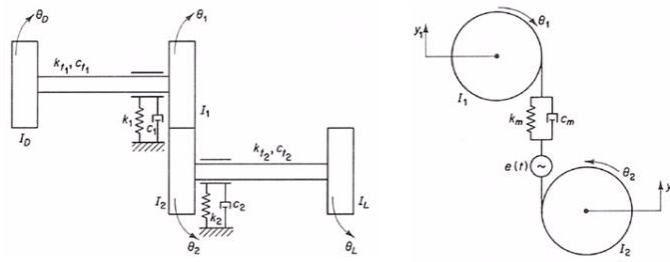


Figure 9.22 6-DOF model for spur gear pairs including shafts and bearing effects

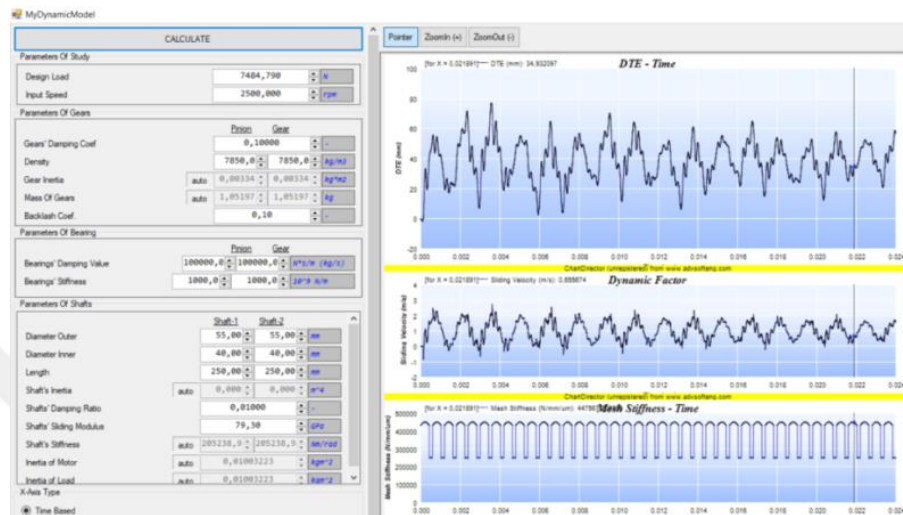


Figure 9.23 General view of software of mathematical modeling (6-DOF)

Figure 9.24 Input parameters software of mathematical modeling (6-DOF)

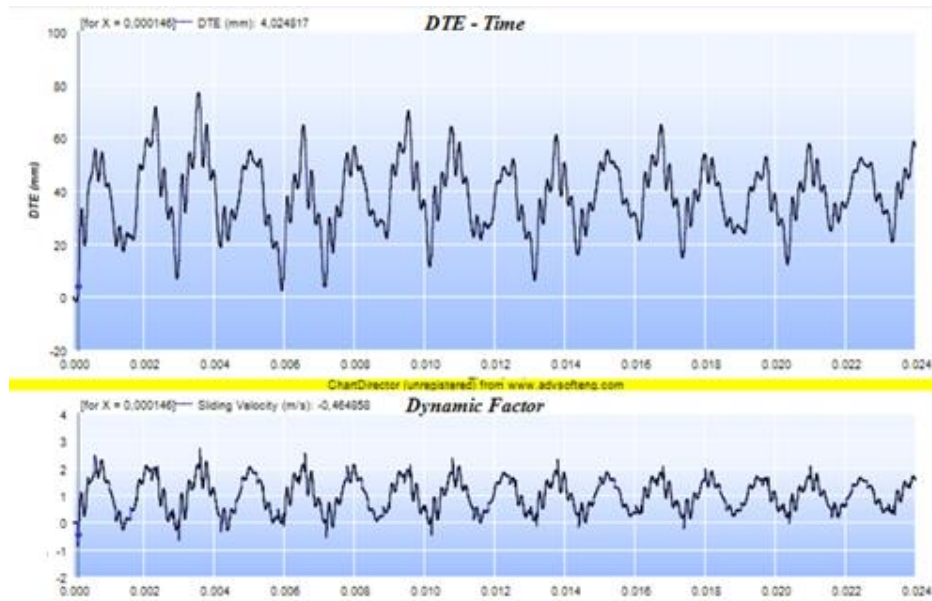


Figure 9.25 Software interface for dynamic transmission error and dynamic factor (6-DOF)

9.7 Case Studies of Spur Gears (6-DOF)

It is important to be aware of gear dynamics to be sure about NVH characteristics of gearboxes. Because vibration affects gear service life drastically and noise is source of disturbing during traveling by a vehicle. There are many literature works published about mathematical modeling and dynamic behaviors of gear pairs. These works based on single degree of freedom (SDOF) or multi degree of freedom (MDOF) dynamic models were deeply discussed in Chapter 2. MDOF studies include more effects because of taking surrounding and working conditions into consideration while SDOF studies are simplified models of multi degree of freedom systems as presented.

9.7.1 Case S29

Lin et al [157] developed spur gear transmission model to evaluate the effect of operating speed, damping, friction, applied load and shaft inertia on dynamic tooth load. Tooth stiffness has also been investigated in paper. Gear pair parameters, shaft, motor and load parameters used in related study are presented in Table 9.8. Peak to peak dynamic transmission error (PPDTE), dynamic transmission error (DTE) and dynamic factor (DF) versus rotational speed (rpm) curves have been constructed from in-house software and given in Figure 9.26. Dynamic factor (DF) versus rotational speed (rpm) curves are presented in Figure 9.27 and 9.28 for reference paper [157] and in-house software respectively.

Table 9.8 Gear and shaft parameters (Case S29) [157]

Parameter	Value	Unit
Diametral pitch	8	1/inch
Teeth number	36	-
Pressure angle	20	degree
Pitch diameter	114.3	mm
Face width	25.4	mm
Moment of inertia	3.3323×10^{-3}	kgm^2
Motor inertia	9.997×10^{-3}	kgm^2
Load inertia	9.997×10^{-3}	kgm^2
Shaft stiffness	1138.7	Nm/rad
Damping ratio	0.10	-

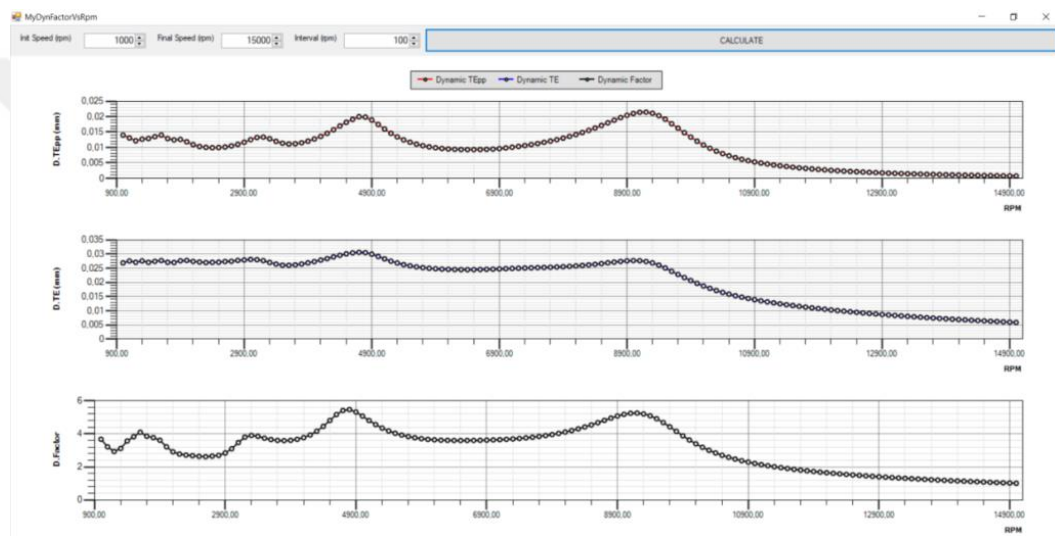


Figure 9.26 PPDTE, DTE and DF versus rpm (Case S29) (in-house software)

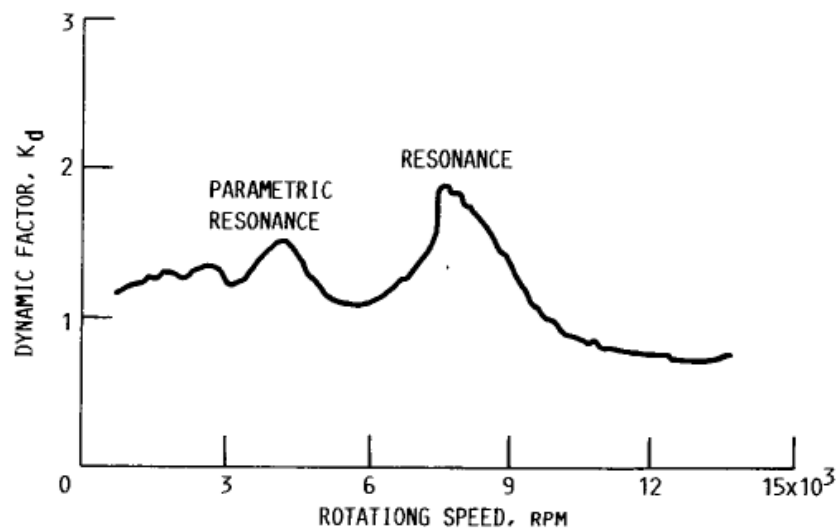


Figure 9.27 Dynamic factor versus rpm for Case S29 [157]

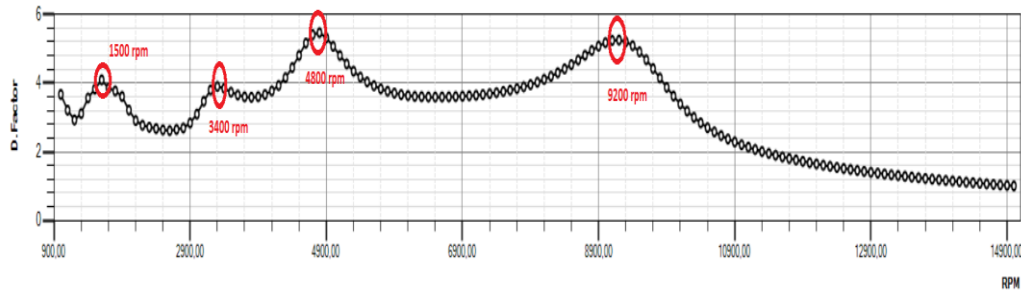


Figure 9.28 Dynamic factor versus rpm for Case S29 (in-house software)

As shown in Figure 9.27 resonance value can be read as around 8700 rpm for reference paper whereas it is slightly higher than 9000 rpm in our study. This difference may be due to mesh stiffness calculation methods used in reference (not clearly stated about stiffness calculation method) and current thesis (thin slice method for calculation of stiffness), because parametric excitation of time varying mesh stiffness affect significantly resonance frequency of gear pairs.

In contrast to resonance speeds proximity for reference and current studies, dynamic factor values are different; it slightly fluctuates around 2.0 for reference paper whereas it is around 5.0 for our study. This can be because of not including the effect of bearing stiffness and damping in reference study (4-DOF model used in reference paper).

When 2-DOF, 4-DOF and 6-DOF models are presented together in Figure 9.29. It is clear to say that 2-DOF models give lower values of dynamic factor (closer to reference paper values) whereas 4-DOF and 6-DOF models have approximate dynamic factor values but they differ from reference study's one and 2-DOF.



Figure 9.29 DF vs rpm for 2-DOF, 4-DOF and 6-DOF models

(Case S29, in-house software)

9.7.2 Case S30

Özgüven [134] stated that a mathematical model representing only the torsional vibrations of a gear pair controlled by mesh stiffness (as shown in Figure 9.30) may yield quite accurate results for practical cases. Author added that increasing degree of

freedom of the model by including inertias and compliances of other elements may not yield a considerable improvement in the dynamic transmission error and dynamic stress when the modes are almost uncoupled.

A six degree of freedom (6-DOF) non linear model was presented in paper [134] by including time varying mesh stiffness, mesh damping, torsional stiffnesses of shafts of pinion and gear, shafts' material damping, bearing stiffnesses and dampings, inertias, torques, tooth separation and profile modifications. Gear, shaft and bearings parameters used in this reference paper [134] are given in Table 9.9. Dynamic factors (DF) versus rotational speed (rpm) curves are illustrated in Figure 9.31 and 9.32 for reference paper and thesis's studies respectively. Peak values are recognized at 6500 rpm (DF of 1.5) and at 10500 rpm (DF of 2.4) in Figure 9.31 (for Ref [134]). For current study, at 6000 rpm and 13000 rpm are obtained as peak values; dynamic factors corresponding to these resonance speed values are recorded as 3.6 and 3.4.

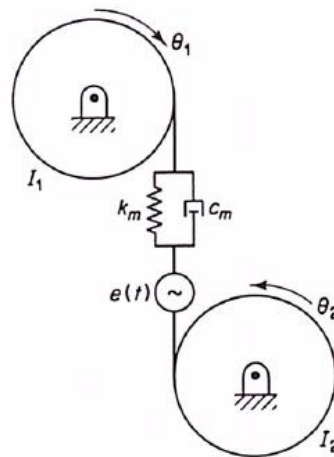


Figure 9.30 2-DOF semi definite dynamic model for a gear pair [134]

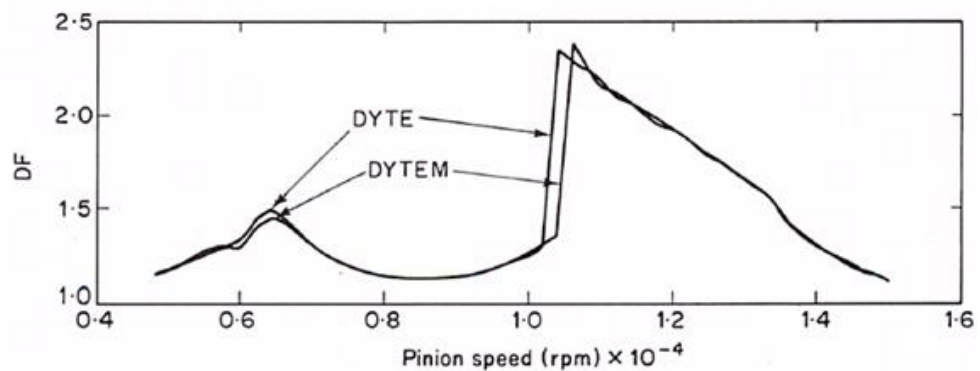
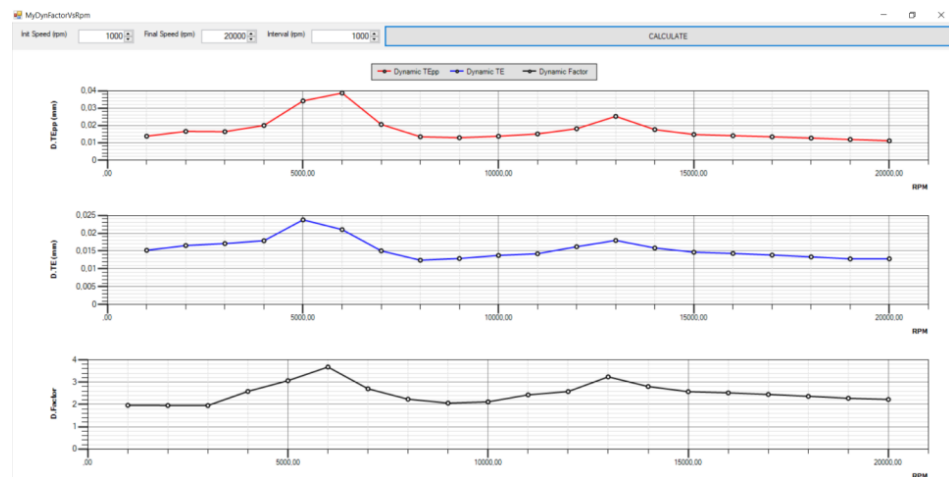


Figure 9.31 Dynamic factor versus rpm [134]

Table 9.9 Gear and shaft parameters (Case S30)

Parameter	Value	Unit
Module	4	mm
Teeth number	25	-
Pressure angle	20	degree
Pitch diameter	100	mm
Base diameter	94	mm
Tip diameter	108	mm
Root diameter	90	mm
Face width	15	mm
Backlash	0.1	mm
Load	2295	N
Moment of inertia (gears)	$1.152 \cdot 10^{-3}$	kgm^2
Motor inertia	$5.762 \cdot 10^{-3}$	kgm^2
Load inertia	$1.152 \cdot 10^{-3}$	kgm^2
Shaft stiffness	1138.7	Nm/rad
Damping ratio	0.10	-
Bearing stiffness	$3.503 \cdot 10^{12}$	N/m
Bearing damping	$3.503 \cdot 10^5$	Ns/m
Shaft stiffness (pinion)	$2.977 \cdot 10^6$	Nm/rad
Shaft stiffness (gear)	$5.253 \cdot 10^6$	Nm/rad

Gear profile studied thus far was generated without profile relief (no relief, NR). Although DTE values or graphs are not presented for gear pair with no relief in reference study [134], PPDTE and DTE curves are given in Figure 9.32 as output of software. To indicate the effect of long relief (LR) on gear dynamic parameters, PPDTE, DTE and DF curves are given in Figure 9.33. Dynamic factor peak value's speed increases from 10500 rpm (with NR) to 14000 rpm (with LR) whereas value decreases from 3.4 (with NR) to 3.0 (with LR).

**Figure 9.32** PPDTE, DTE and DF vs rpm (Case S30, no relief, in-house software)

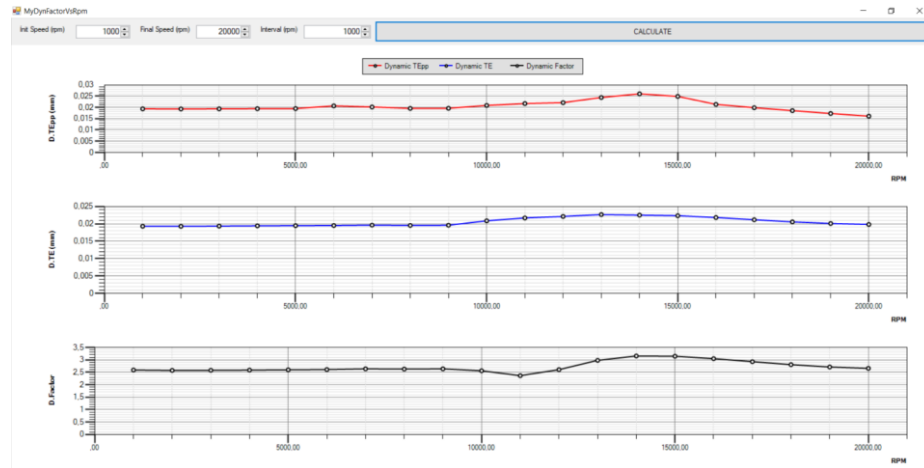


Figure 9.33 PPDTE, DTE and DF vs rpm (Case S30, long relief, in-house software)

9.7.3 Case S31

Hotait and Kahraman [119] carried out an experimental study, and measured dynamic factor and dynamic transmission error by using a back to back gear test rig shown in Figure 9.34. Gear and loading parameters are given in Table 9.10. Dynamic factor versus rpm curves obtained from reference paper [119] and current study are given in Figure 9.35-9.40 for different load values. Dynamic factor curves for 100Nm, 200Nm and 300Nm are presented in Figure 9.35, 9.36 and 9.37 respectively (for Ref [119]). These curves are derived for mesh frequency (f_m) and dynamic factor values are obtained for speed up and down. To make easy comparison of resonance speed and corresponding dynamic factor values, mesh frequency values are converted to rpm values (as given in Eq. 9.21).

$$f_m = \frac{TN}{60} \quad (9.21)$$

Whereas T and N are teeth number and rotational speed (in rpm) respectively.

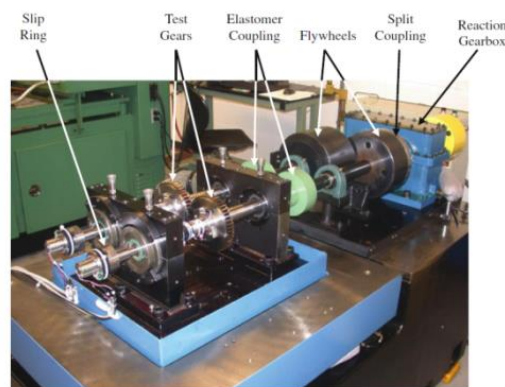


Figure 9.34 Gear test rig [119]

Resonance speed values can be read from Figure 9.35 as 600rpm (DF of 1.25), 840rpm (DF of 1.1), 1080rpm (DF of 1.5), 1500rpm (DF of 2.00), 2700rpm (DF of 1.8) and 3900rpm (DF of 1.25) [Ref 119] whereas 490rpm (DF of 3.3), 1500rpm (DF of 4.0), 1750rpm (DF of 4.2), 2000rpm (DF of 4.5) and 4000rpm (DF of 4.6) are observed as peak values in Figure 9.36 (current study) under torque of 100Nm. When speed up and speed down values are obtained from measurement, 2700rpm (DF of 1.8) shifts to 2400 rpm (DF of 2.2). For torque of 200Nm, dynamic factors of 1.10, 1.20, 1.40, 1.50, 1.25, 1.10, 1.40, 2.10 and 1.20 for 720rpm, 900rpm, 1200rpm, 1680rpm, 2000rpm, 2500rpm, 2880rpm, 3300rpm and 3900rpm respectively for Ref [119] as shown in Figure 9.37. Peak values 500rpm (DF of 3.0), 1750rpm (DF of 3.6), 2000rpm (DF of 3.8), 3750rpm (DF of 3.6) and 4000 rpm (DF of 3.8) are obtained from Figure 9.38.

Peak values of 720rpm, 840rpm, 960rpm, 1200rpm, 1800rpm, 2400rpm, 2700rpm, 2880rpm, 3000rpm, 3400 rpm and 3900 rpm are observed for dynamic factors of 1.1, 1.1, 1.2, 1.5, 1.5, 1.1, 1.2, 1.2, 1.4, 1.75 and 1.25 respectively for reference study [119] (Figure 9.39). 500rpm (DF of 2.8), 1750rpm (DF of 3.4), 3500 rpm (DF of 3.6) and 4000 rpm (DF of 3.4) values are obtained in current study (Figure 9.40).

Table 9.10 Gear and loading parameters (Case S31)

Parameter	Value	Unit
Module	3	mm
Teeth number	50	-
Pressure angle	20	degree
Base diameter	140.95	mm
Tip diameter	153.74	mm
Center distance	150	mm
Face width	15	mm
Tooth thickness	4.64	mm
Torque	100-200-300	Nm

Table 9.11 Mesh frequency values

Hz	rpm
500	600
1000	1200
1500	1800
2000	2400
2500	3000
3000	3600
3500	4200

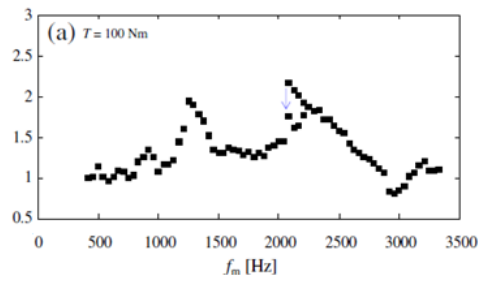


Figure 9.35 DF versus rpm under 100 Nm for unmodified gear [119]

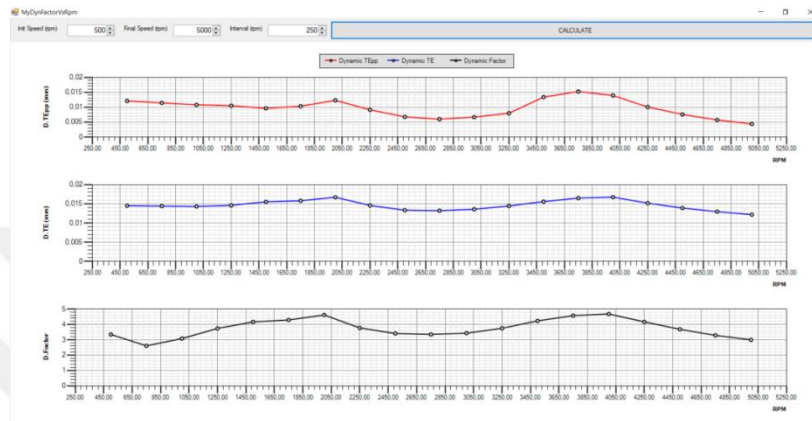


Figure 9.36 DF versus rpm under 100 Nm for unmodified gear (in-house software)

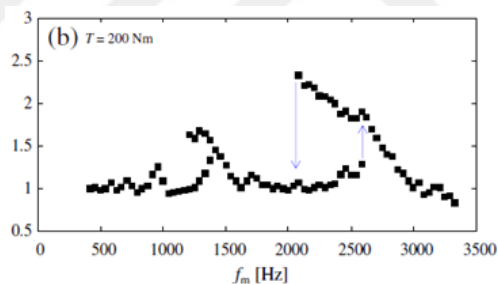


Figure 9.37 DF versus rpm under 200 Nm for unmodified gear [119]

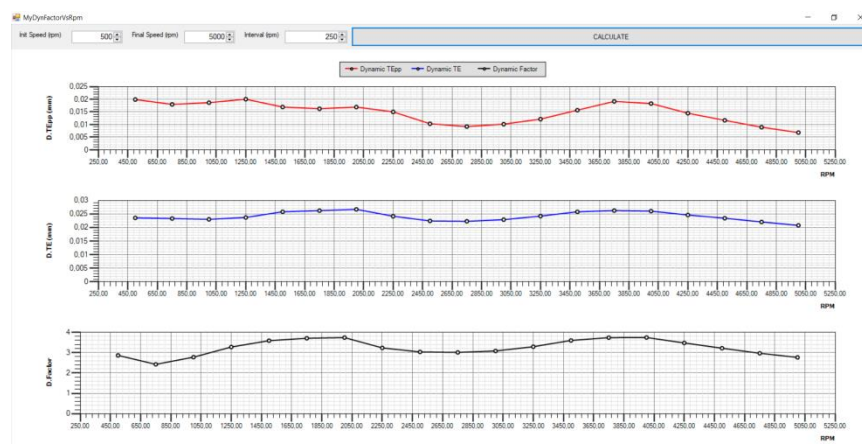


Figure 9.38 DF versus rpm under 200 Nm for unmodified gear (in-house software)

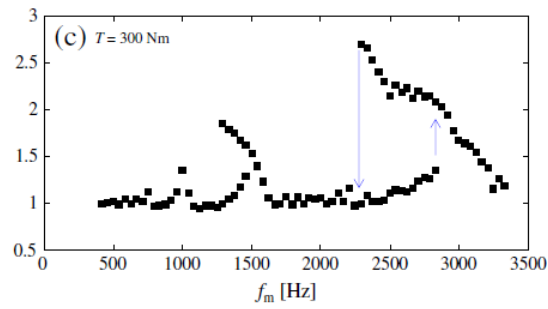


Figure 9.39 DF versus RPM under 300 Nm for unmodified gear [119]

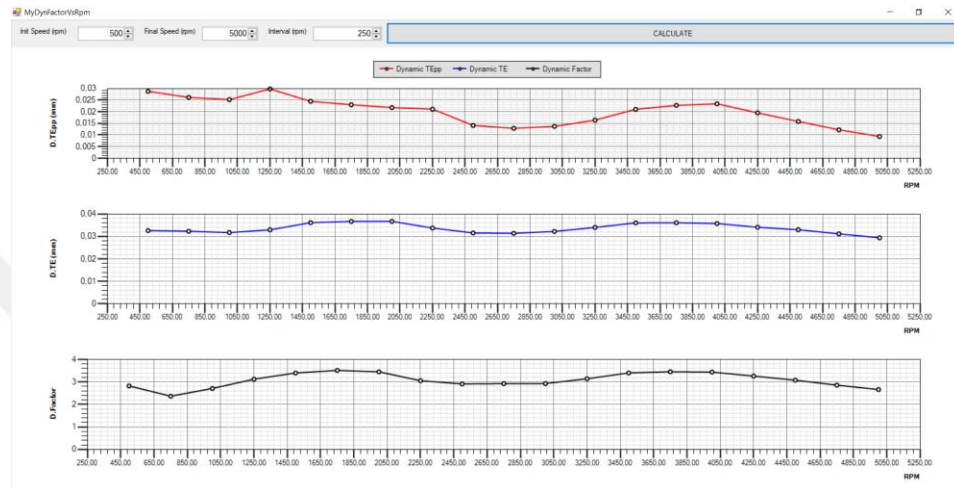


Figure 9.40 DF versus rpm under 300 Nm for unmodified gear (in-house software)

9.7.4 Case S32

Tamminana et al [120] searched the relationship between the dynamic factor and dynamic transmission error of spur gear pair whose parameters are given in Table 9.12. DTE values are obtained and curves are presented for spur gears with no relief and with relief in Figure 9.41-9.44.

Table 9.12 Gear parameters (Case S32)

Parameter	Value	Unit
Module	3	mm
Teeth number	50	-
Pressure angle	20	degree
Pitch diameter	150	mm
Base diameter	140.95	mm
Tip diameter	variable	mm
Center distance	150	mm

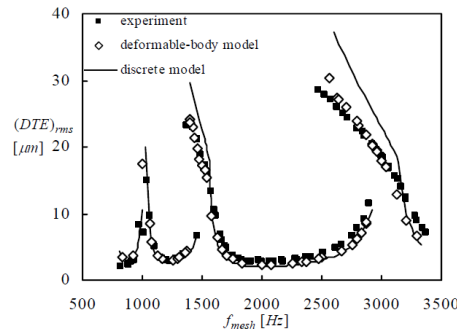


Figure 9.41 DTE versus rpm under 340 Nm for unmodified gear [120]

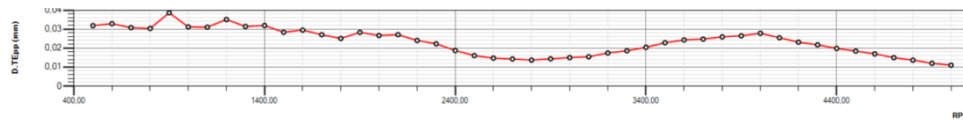


Figure 9.42 DTE versus rpm under 340 Nm for unmodified gear (in-house software)

DTE values of 40 μ m, 35 μ m, 32 μ m, 30 μ m, 30 μ m, 20 μ m and 28 μ m are obtained for 900rpm, 1200rpm, 1400rpm, 1600rpm, 1900rpm, 3400rpm and 4000rpm respectively in current study. From Ref [120], 1200rpm, 1800rpm and 3600rpm peak values are observed and corresponding DTE values are 20 μ m, 30 μ m and 18 μ m. These all values are for unmodified (no relief) spur gear pair. For modified (with relief) spur gear pair, 1200rpm (DTE of 13 μ m), 1800rpm (DTE of 16 μ m and 3600rpm (DTE of 14 μ m) are obtained as peak values in Ref [115] whereas 900rpm (DTE of 21 μ m), 1100rpm (DTE of 18 μ m), 1600rpm (DTE of 15 μ m), 2000rpm (DTE of 17.5 μ m), 3400rpm (DTE of 20 μ m) and 4000rpm (DTE of 22 μ m) are got in current study.

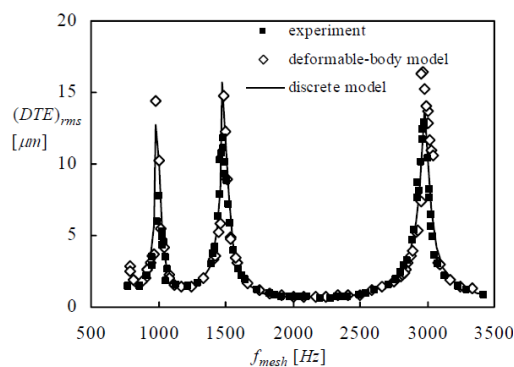


Figure 9.43 DTE versus rpm under 340 Nm for modified gear [120]

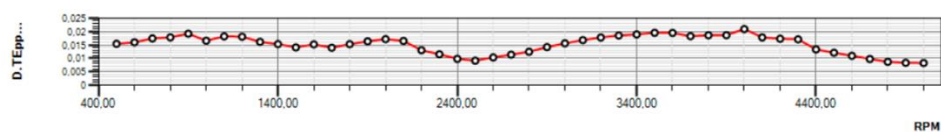


Figure 9.44 DTE versus rpm under 340 Nm for modified gear

9.7.5 Case S33

Wang et al [124] prepared a lumped dynamic model for spur gear pair (Figure 9.45) and parameters are presented in Table 9.13. A gear test rig is used in measurements of dynamic transmission error (Figure 9.46). As shown in Figure 9.47, 1200rpm (DTE of 140 μm), 1800 rpm (DTE of 120 μm) and 2700rpm (DTE of 60 μm) are got while 1100 rpm (DTE of 110 μm), 1300rpm (DTE of 105 μm), 1900rpm (DTE of 100 μm) and 2700rpm (DTE of 94 μm) are peak values in Figure 9.48 for in-house software.

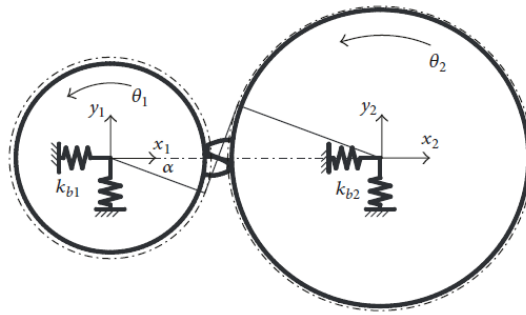


Figure 9.45 Mechanical model and gear test rig [124]

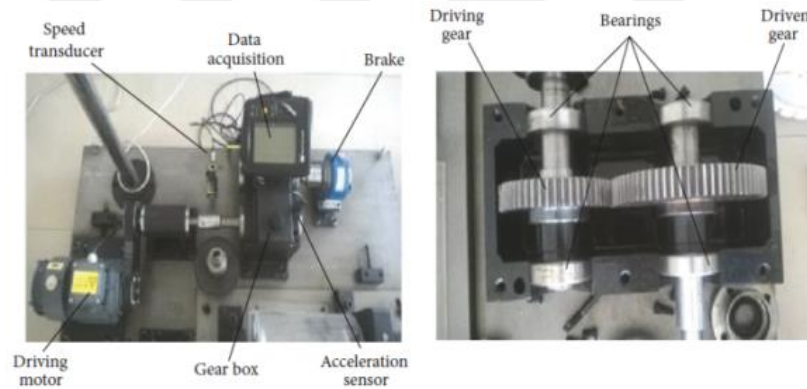


Figure 9.46 Gear test rig [124]

Table 9.13 Gear and bearing parameters (Case S33)

Parameter	Value	Unit
Module	2	mm
Teeth number	55/75	-
Pressure angle	22.5	degree
Face width	30	mm
Backlash	20	μm
Moment of inertia (pinion)	0.0033	kgm^2
Moment of inertia (gear)	0.0115	kgm^2
Bearing stiffness	2.6×10^8	N/m
Bearing damping	500	Ns/m

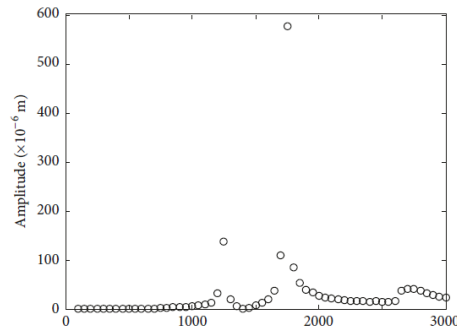


Figure 9.47 DTE vs rpm [124]

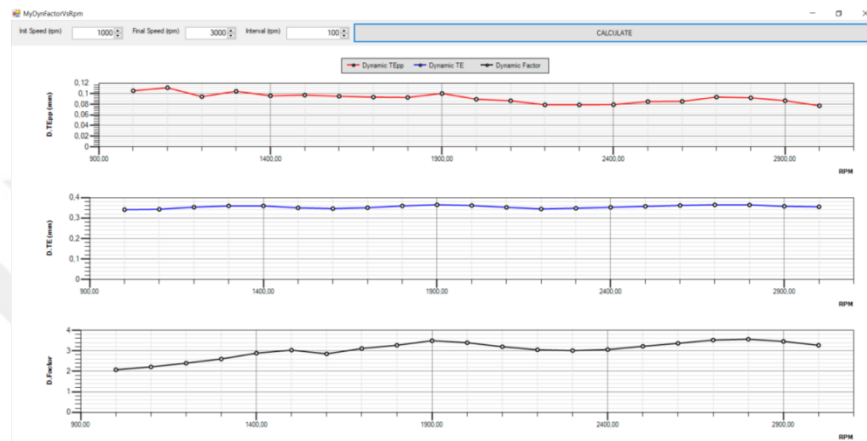


Figure 9.48 DTE vs rpm (in-house software)

9.8 Discussion and Conclusion

Mathematical modeling of gear pairs is very significant to investigate dynamic performance and vibration of gearboxes. In this study, the main aim is to minimize static and dynamic transmission error which is strongly related to gear dynamic performance and noise, vibration and harshness (NVH) characteristics of meshing gears.

In this chapter, the efforts has been generally focused on estimating the dynamic characteristics of spur gear pairs by starting a 2-DOF torsional model of gear pair and then improving model by increasing degree of freedom and taking shaft and bearing flexibilities into consideration.

2-DOF model gives similar dynamic load and dynamic factor values with literature, but the model does not yield exactly same results with any published work. This can stem from the involved effects into models, each model include and exclude different effective factors such as friction between contacting gears, bearing stiffness, shaft stiffness, gear errors due to manufacturing and static transmission error.

Then, 6-DOF model has been developed as torsional model of gear pair and module of in-house software has been prepared as a solver for nonlinear equations of motions. 6-DOF model gives similar dynamic factor and dynamic transmission error curves with literature in terms of resonance speed values and regime, but the model yield exactly higher results compared to any published work. This can stem from missing parameters namely shaft and bearing parameters (sometimes lack of profile relief type and amount parameters) and absence of information about used methods for calculation of stiffness.



CHAPTER 10

DISCUSSIONS AND CONCLUSIONS

10.1 Discussions and Conclusions on the Present Study

A theoretical and experimental study on asymmetric tooth profile spur gears has been conducted in this thesis. This study is a part of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) [1] which was launched at METU Technopark Facility within the scope of contract signed between Undersecretariat for Defence Industries (SSM) and Turkish Aerospace Industries, Inc. (TAI) on June 26, 2013.

The study mainly focuses on asymmetric tooth profile gears which can be an alternative to symmetric one especially used in helicopter transmission units because of their high load carrying capacity and low weight.

User friendly software for symmetric and asymmetric tooth spur gears has been developed in scope of this thesis and project of Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) [1]. The software has capabilities such as macro geometry generation, tooth root stress calculations based on ISO [2], stiffness calculations based on thin slice theory, and micro profile design for minimum transmission error based on Harris map and mathematical modeling simulation to obtain dynamic transmission error for symmetric and asymmetric tooth spur gears.

In addition to developed software, finite element analyses have been carried out for tooth root bending stress evaluations and micro profile analyses in terms of tooth deflection and static transmission error.

A back to back test rig has been designed, manufactured and assembled in the scope of reference project [1] and this thesis. Some experiments have been conducted to obtain static transmission error (STE) and dynamic transmission error (DTE) values. Curves of those values under different loads and at rotational speeds were constructed by using gear test rig.

10.2 About Macro Geometry Generation

Macro geometries of symmetric and asymmetric spur gears have been generated by using developed software. Generated macro geometries of symmetric gears were directly compared to outcomes of KissSoft [14] whereas comparable symmetric gear teeth were modeled for drive and coast sides of asymmetric tooth spur gears. Drive and coast sides of asymmetric tooth were compared by generating comparable teeth by KissSoft.

Generated macro geometries for symmetric and asymmetric tooth spur gears satisfy with each other for both tools. All points of involute and trochoidal root profiles coincide for outcomes of software and KissSoft.

Several spur gear pairs with symmetric and asymmetric tooth profile were generated for different module, teeth number, pressure angle and root fillet radii by using both tools. Macro geometries generated by in-house software and KissSoft do not differ and are in accordance with each other.

It can be deduced from these all findings, in-house software gives correct and accurate macro geometries for symmetric and asymmetric spur gears regardless of parameter values of module, teeth number, pressure angle and root fillet, symmetry or asymmetry for involute and root profiles.

10.3 About Tooth Root Bending Stress Evaluation

Tooth root bending stress values have been calculated based on ISO [2] by using developed in-house software for symmetric and asymmetric tooth spur gears. Bending stress values at tooth root sections and all stress related parameters were determined by using software. Parameters calculated by software were compared to those of literature works and commercial gear software, KissSoft [14].

Case studies S1, S2 and S3 were carried out for gear pairs studied in reference papers; Ristic [179], Spitas [83] and Sekar [42] respectively whereas Cases of S4 and S5 were based on KissSoft analyses for symmetric tooth spur gears. In-house software estimates tooth root bending stresses with almost same values compared to KissSoft whereas root stresses obtained by using software are higher than reference papers.

Case studies of A1, A2, A3 and A4 have been carried out for asymmetric tooth profile spur gears. Root bending stress values given in paper of Sekar [42] are lower than those of software (Case studies of A1, A2 and A3). For comparison of study of Kapelevich and Shekhtman [55] with current thesis, software gives higher stress than that of paper for asymmetric gear (Case A4). Root bending stress coming from software for comparable symmetric gear is exactly same as KissSoft's results.

Up to now, it has been mentioned about stress calculations based on gear standards and comparisons of them (Chapter 4). Finite element analyses have been conducted during this thesis for root bending stress evaluations of symmetric and asymmetric spur gears (Chapter 5). Several case studies for both tooth profiles based on finite element methods were presented in Chapter 6. Sekar and Muhtuveerappan [42] made a study for symmetric and asymmetric tooth spur gears for root bending stress evaluations (Case S6, S7, S8, A5 and A6).

Finite element analyses conducted in reference and current studies are very close to each other for Case S6 [42]. Two additional case studies of S7 and S8 based on same general gear parameters with Case S6 except pressure angle and root fillet radii values were added to make easy of comparisons of root stress for different tools. Results of in-house software and KissSoft are very close while finite element analyses carried out in this thesis are lower than values obtained by both tools.

In paper of Cavdar et al [43], root bending stress evaluations were conducted based on DIN [63] and finite element methods for Case S9. When same gear pair analyzed in current study, a stress value higher than result based on DIN [63] but lower than FEA of reference paper was obtained.

Case studies of S10, S11 and S12 were conducted by using in-house software, KissSoft and finite element analyses for root stress evaluations. Finite element works give lower stress values whereas other two tools' results are very close to each other.

When Cases of A5 and A6 of asymmetric tooth spur gears were compared, finite element analyses of this study give higher stress value than reference paper. For another asymmetric study of Case A7, finite element works of reference paper provides little bit higher stress level. When it comes to cases of A8, A9, A10 and A11,

finite element analyses estimate lower stress level in comparison with software's results.

10.4 About Tooth Root Bending Stress Evaluation for Asymmetric Root Fillet Profiles

As stated in Chapter 4 and 5, any increase in root fillet radius yields bending stress reduction at tooth root section. But this increase is not limitless because beyond a limit cutting tool starts to sharpen at the midpoint and a tool form disorder occurs. Due to the limitation of tool tip sharpening and form dis-order, it is not possible to make tool tip radius larger beyond the a specific limit for symmetric fillets by depending on gear tooth parameters such as teeth number, pressure angle, dedendum and clearance coefficients.

An alternative means of increasing the tip radius (without harming the tool) is to change the form of the cutter and fillet by shifting from symmetric tool tip to asymmetric tool tip shape.

Several case studies for symmetric tooth profile spur gears with asymmetric root fillet profiles have been conducted to evaluate the effect of asymmetry in root on tooth root bending stress [175-178]. In addition, dual asymmetry (stated also in reference papers [176, 177 and 178]) has been applied to spur gear pair to determine geometrical limits for relationship between pressure angle and root fillet radius coefficients, and obtain further decrease of root bending stress.

By grouping case studies based on pressure angles, it is easier to compare effects of root fillet type and coefficient of tool tip radius on bending stress. Therefore it is better to present stress values in different groups based on pressure angles of both symmetric and asymmetric tooth spur gears.

When symmetric tooth spur gear with circular fillet and trochoidal of symmetric and asymmetric root fillets are taken into account, maximum stress reductions of 15% and 10% were obtained for software and finite element works respectively.

Stress reduction levels reach to 16% and 11% for in-house software calculations and finite element analyses respectively while asymmetric tooth spur gears with symmetric and asymmetric root fillet profiles are considered. As seen in here and case studies

results presented previously, *dual asymmetry* yields higher stress reductions under constant torque transmission regardless of greater pressure angle for drive or coast side flanks.

10.5 About Micro Profile Design, Static Transmission Error and Back to Back Gear Test Rig

Transmission error theory and micro profile design of gears has been explained based on Harris map. Micro profile design has been performed for minimization of transmission error values and construction of smooth transmission error curves under desired load for symmetric and asymmetric tooth spur gear pairs.

A user friendly in-house software has been developed to fulfill functions abovementioned. Tooth pair and mesh stiffness, transmission error, load sharing, contact stress, sliding velocity and contact stress-sliding velocity ($P*V$) values can be determined and corresponding curves can be constructed by using the software.

Several case studies were performed for both symmetric and asymmetric spur gears to verify peak to peak transmission error (PPTE) values obtained by using in-house software. Some of them are based on published papers whereas remaining are compared to finite element analyses conducted within the scope of this thesis.

PPTE values estimated by in-house software are close to results of reference papers and finite element works. The differences for case studies are generally lower than 1.0 micron with some exceptions.

Within the scope of the thesis and the asymmetric gear project for helicopter transmission under the Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM) [1], a back to back test rig has been designed and manufactured. Quasi static transmission error values were measured for symmetric and asymmetric tooth spur gear pairs by this gear test rig.

Spur gear pairs with symmetrical and asymmetrical involute tooth profiles were designed and manufactured for desired macro geometry and micro profile in the scope of this thesis. Their design for high load carrying capacity with minimum transmission error was intended. These gear pairs were manufactured and mounted to test rig. Then

quasi static transmission error values were obtained and corresponding curves under different loads were constructed by measurements.

Comparisons for static transmission error values gotten from in-house software, finite element analyses and measurements conducted by gear test rig were done. Values are close to each other with small differences.

10.6 About Dynamic Transmission Error Measurements

In addition to measurements of quasi static transmission error, dynamic transmission error values for symmetric and asymmetric tooth spur gears under different loads and at several rotational speeds were achieved by using back to back gear test rig. Corresponding curves were also constructed for gear pairs. Some dynamic repeatability tests for the start and end of tests were presented to show safe operation of gearbox elements and accuracy of measurement devices.

Gear pairs were operated in CW and CCW directions regardless of symmetric and asymmetric profiles under different load values by applying torque on calibrated multi-hole couplings via torque wrench. After obtaining dynamic transmission error (DTE) values and curves, dynamic frequency response curves versus rotational speeds were attained. By examining these curves, resonance speed values were determined and presented. Some of these resonances originate from tooth pair stiffness under different loads and at different speeds. They were also presented and explained in this study. Tooth separation and contact lose were also stated by analyzing DTE curves.

10.7 About Mathematical Modeling

A mathematical modeling study has been performed to determine transmission error values for symmetric and asymmetric spur gears. Some case studies were carried out to verify obtained results of dynamic transmission error (DTE), dynamic load (DL) and dynamic factor (DF).

Different models were developed for 2-DOF, 4-DOF and 6-DOF to evaluate gear pairs' dynamic performance parameters. A module was added to software to solve governing equations of gear pair systems. Generated codes produce dynamic transmission error and dynamic load curves in a similar regime to reference studies but with greater values generally.

10.8 Further Studies

The followings may be considered as future works:

- These all efforts which have been made for symmetric and especially asymmetric spur gears during this study can be applied for helical gears. Asymmetric helical gears can be investigated for further stress reductions at tooth root and better dynamic performance with reduced transmission error values.
- Parallel axes helical gears with asymmetric tooth profiles can be evaluated by testing them using back to back gear test rig to measure static and dynamic transmission errors.
- Planet spur and helical gears with asymmetric tooth profiles can be studied for improvements of tooth root bending stress, static and dynamic transmission errors.
- Asymmetric root fillet applications can be searched for helical gear pair for higher load carrying capacity based on definite gear geometry and mass or lighter gears with a predefined load carrying capacity.
- *Dual asymmetry* (for both tooth and root fillet profiles) can be investigated for helical gears, planet spur and helical gears for better performance and safer operation.

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PUBLICATIONS

Papers Published in International Indexed Journals (sci-expanded)

- 1) **Şahin, B.**, Yıldırım, N., Akpolat, A. (2018). Gear Root Bending Stress Reduction by Employing Dual Asymmetry - Asymmetry in Tooth Profile Shape and Root Fillet Form. *Journal of the Chinese Society of Mechanical Engineers*. **39(6)**, 637-645.

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Papers Published in National Indexed Journals

- 1) Uçtu, Ö., Sevim, İ., Karataş, B., **Şahin, B.** (2017). Determination of Optimum Mesh Size to Measure Tooth Root Stress of Spur Gear Using Finite Element Analysis. *Turkish Journal of Engineering*. **1(1)**, 37-43.
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- 5) **Şahin, B.**, Yıldırım, N., Akpolat, A. (2017). Design and Analysis of Spur Gear Pairs Regarding Macro and Micro Profile Generation and Safety Standards. *International Conference on Engineering Technologies (ICENTE'17). 07-09 December 2017, 912-920.*
- 6) Akpolat, A., Ali, M., **Şahin, B.** (2017). Analysis of Contact Stresses in Spur Gears by Finite Element Method. *ICTEC 2017 - International Conference on Energy and Thermal Engineering. 25-28 April 2017.*
- 7) Qasim, B. M., **Şahin, B.**, Yaşar, M., Yıldırım, N. (2014). A Study on Comparison of Bolt Tensioning Applied By Conventional Torque Method And Hydraulic Tension Method. *The First International Symposium on Industrial Design Engineering (ISIDE14). May 8-9, 2014.*
- 8) **Şahin, B.**, Çelik, H. İ., Kaynak, H. K. (2018). Finite Element Analysis Applications to Textile Materials: A Case Study on Pet Yarn Mechanical Properties. *International Conference on Engineering Technology and Innovation, 07-11 March, Budapest.*
- 9) Çelik, H.İ., **Şahin B.**, Kaynak, H. K. (2017). Development of A Test Device Capable of Performing Static Loading Tests of Carpets Automatically, *International Conference on Engineering Technology and Innovation, 22-26 March, Sarajevo.*
- 10) Çelik, H.İ., **Şahin B.**, Kaynak H.K. (2016). Development of A Carpet Static Loading Test Device. *16th AUTEX World Textile Conference 2016, June 8–10, Ljubljana, SLOVENIA, 4-10.*
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- 1) Yıldırım, N., Akpolat, A., **Şahin, B.**, Karatas, B., Yıldırım, Ö. (2017). Asimetrik Uçlu Simetrik Kremayer Takımla İmal Edilen Dişli Çarklarda Diş Dibi Eğilme Gerilmelerinin Azaltılması. Bias Mühendislik Kullanıcılar Konferansı. 10-11.10.2017, İstanbul.

TASKS IN PROJECTS

- 1) **Participation as a researcher** in the project of Development of Specific Design Methods of High Performance Asymmetric Spur Gears conducted by Yıldırım, N within the scope of *Indigenous Helicopter Program of Rotary Wing Technology Center (DKTM)*. DKTM Project No DKTM/2014/01.
(Yıldırım, N. tarafından Özgün Helikopter Programı Kapsamında Döner Kanat Teknoloji Merkezi (DKTM), TAI-TUSAŞ, Savunma Sanayi Müsteşarlığı (SSM) ve Gaziantep Üniversitesi (GAÜN) işbirliğinde yürütülen “Yüksek Performanslı Asimetrik Düz Dişlilerin Özel Yüzey Tasarım ve Analiz Metotlarının Geliştirilmesi” adlı projede **Araştırmacı** olarak görev yapmıştır. Proje kapsamında doktora tezini büyük bir bölümünü tamamlanmıştır. DKTM Proje No: **DKTM/2014/01, TAI Sözleşme No: 201400511**).