PROJECTIVE LINEAR QUADRATIC CONTROL OF DIRECT CURRENT MOTOR UNDER DISTURBANCE TORQUES

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ISMAIL ZUGLAM

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PROJECTIVE LINEAR QUADRATIC CONTROL OF DIRECT CURRENT MOTOR UNDER DISTURBANCE TORQUES

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Approval of the Graduate School of Natural and Applied Sciences, Atilim University.

K. İbrahim AKMAN

Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Asst. Prof. Dr. Kemal Efe ESELLER

Head of Department

This is to certify that we have read the thesis " Projective Linear Quadratic Control of Direct Current Motor Under Disturbance Torques" submitted by "ISMAIL ZUGLAM" and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Reşat Özgür DORUK

Supervisor

Examining Committee Members

Assoc. Prof. Dr. Umut ORGUNER	METU, E.E.E. Dept.	________________
Asst. Prof. Dr. Mehmet Efe ÖZBEK	ATU, E.E.E. Dept.	________________
Assoc. Prof. Dr. Reşat Özgür DORUK	ATU, E.E.E. Dept	________________

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Name, Last name: Ismail Zuglam

Signature:

ABSTRACT

PROJECTIVE LINEAR QUADRATIC CONTROL OF DIRECT CURRENT MOTOR UNDER DISTURBANCE TORQUES

ISMAIL ZUGLAM

M.S., Electrical & Electronics Engineering Department

Supervisor: Assoc.Prof.Dr Reşat Özgür Doruk

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Projective control is a method of control based on output feedback to control a linear system. The main objective of this research is to design an output feedback controller for obtaining a DC Motor position and speed tracking controller, and analyze the stability of closed loop against exogenous disturbance torques using input-to-state stability theory. The control algorithm is implemented by using pole placement and Linear Quadratic Regulator as a reference full state feedback controller and one applies orthogonal projection of full state space to output state space to obtain the output feedback controller. The overall designs are demonstrated by MATLAB based simulations.

Keywords: DC motor, Projective control, Linear Quadratic regulator, Speed and Position Control, Input-to-State Stability, Disturbance Torque, MATLAB.

ÖZ

DOĞRU AKIM MOTORLARININ DOĞRUSAL KARESEL İZDÜŞÜMSEL DENETİMİ

ISMAIL ZUGLAM

Yüksek Lisans., Elektrik&ElektronikMühendisliğiBölümü

Danışman: Doç.Dr Reşat Özgür Doruk

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Bu araştırmanın amacı izdüşümsel kontrol yaklaşımından yola çıkarak doğru akım motorlareı için konum ve hız denetim yasaları geliştirmek ve elde edilen kapalı döngü denetleyicilerin bozucu etkilere karşı kararlılık ve performansını incelemektir. Kontrol algoritmalarında doğrusal karesel regülator ve kutup yerleştirme tekniği referans tam hal geri besleme denetleyici olarak tasarlanmakta ve çıktı geri beslemesi için tam hal uzayından çıktı uzayına dik izdüşüm alınmktadır. Elde edilen denetleyicilerin dış bozucu etki torklarına karşı performans ve kararlılık analizi teorik düzlemde girdiden-hale kararlılık yaklaşımıyla, sayısal düzlemde de rastgele sinyal olarak modelleme yapılarak benzetimler yoluyla yapılmaktadır. Hesaplama ortamı olarak MATLAB tercih edilmiştir.

Keywords: Doğru Akım motoru, İzdüşümseldenetim, Doğrusalkaresel regülatör, HızveKonumDenetimi, Girdiden-Halekararlılık, Bozucu etki torkları, MATLAB.

To My Parents

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And also, I must express my very profound gratitude to my family, especially to my parents, my wife, children, brothers and sisters for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

TABLE OF CONTENTS

List of Tables

LIST OF FIGURES

LIST OF ABBREVIATIONS

LQR -	Linear Quadratic Regulation
DC -	Direct Current
LQSF-	linear quadratic full state feedback design
ISS -	Input-to-State Stability

LIST OF SYMBOLS

x	general notation for a state vector
y	general output for a single input and output system
u	general input for a single input and output system
A,B	state space system and input matrices (determines property)
C,D	output matrices of a state space system definition (depends on the output definition)
J	Inertia of the Load
B	Viscous friction coefficient
Ra	Armature Resistance
La	Armature Inductance
Ki	Torque Constant
Kb	Back-EMF Constant
τ_L	disturbance torque
u	control signal {scalar}
y	output signal (scalar)
x	state vector {n-vector}
K	the state feedback gain
r	reference input signal
ω_d	is the desired angular velocity
ω	is the actual angular velocity
Q	is a matrix of weighting factors for the states which
R	positive definite matrix for a matrix weighting of control action.
ξ	output of the integrator
P	is the unique, positive semidefinite
Λ	is a diagonal matrix, which contains the eigenvalues of *[A-BK]*
$\boldsymbol{V}$	is a matrix containing the associated eigenvectors of *[A-BK]*
K_o	the output feedback gain

CHAPTER 1

INTRODUCTION

The Direct Current Motors may appear in almost all applications in our world. Practically every mechanical movement that you see around is a result of the action of a DC Motor. A DC engine is a machine that converts electrical energy into mechanical energy when supplied by a DC power source. The motors are accessible in various sizes. Large engines, which can carry loads more than 1,000 horse powers are used in industrial applications .There are several types of DC Motors The operation of which are basically based on the same principles.

A brushed Direct Current Motor consists of coil, called (armature winding) (or roter), inside permanent magnets, (called stator). Applying a voltage to the coils results a torque in the armature, producing motion.

DC Motor can be controlled directly by implementing analog circuitry (amplifiers, integrators, power drivers). The parts except power circuitry can be implemented as software routines on a microcontroller.

The performance requirements of DC motor control system design may require the implementation of linear quadratic control techniques where a specific control cost function is minimized. The usage of a full state feedback controller may not be applicable due to practical limitations. The most important limitation is the shortage of directly measurable variables. The linear quadratic method is known to provide a better performance and robustness than classical techniques. Its results can be used as a reference for the subsequent designs. However, every compensator design method does not provide low order products with acceptable results. In the last 30 years, there are researches conducted for solving the problem linked with the unavailability of states and approximating the linear quadratic full state feedback controller method by using only output feedback.

The linear quadratic full state feedback design (LQSF), is used to obtain a reference solution, which called eigenstructure, for projective control design technique. The quadratic design is in fact a simple optimization procedure in which the cost is defined as a function of state and input vectors.

1.1 Aim and Scope

In this study, the projective control method is utilized to develop speed and position controllers using linear quadratic and pole placement based full state feedback methods, as a reference.

The aim of this study is to design a control system that have possible uses in practical applications. The control system should just obtain feedback from the available information. It should be possible for the designer to use methodological design techniques without necessity of complex computational algorithms.

1.2. Outline of the Dissertation

Chapter 2 is the literature survey, which introduces what have been done so far in the same field. Chapter 3 introduces the dynamical model with mathematical equations. Chapter 4, presents the theoretical information about the mentioned full state feedback techniques, the projective control method and input-to-state stability theory together with its motivation in disturbance rejection analysis. Chapter 5 demonstrates the application of the theory presented in Chapter 4 to DC-Motors. Chapter 6 presents numerical and graphical results of the position and speed control of DC motors, presented in Chapter 5. Chapter 7 we will add comparison of DC motor controllers based on projective and backstepping controllers. In Chapter 8, we will have a summary and discussion of the overall results.

CHAPTER 2

BACGROUND INFORMATION AND LITERATURE SURVEY

Direct Current Machines are widely utilized torque transducers in mechanical systems the applications of which range from automotive, robotics, pneumatic and hydraulic systems and different biomedical engineering implementation. The simplest version of a DC motors involve a permanent magnet rotor and (stator) an armature winding which is often the case when one has a brushed DC motor [1,2]. These can be modelled according to the fundamental circuit theories and often available in control systems textbooks such as [3]. The brushless DC motors which have a permanent magnet stator but a wound rotor is actually an AC motor. It will require a dedicated driver circuitry to be operated from a DC supply [4]. The dynamical characteristics of brushed or brushless DC motors are similar [5]. A proper and beneficial DC motor application will require a position or speed controller so that the desired performances are obtained (constant or tracked speed and/or position). They are also a well established class of mechanical systems suited for control system development and numerous researches are available in literature.

Concerning the control approaches one can note that regardless of targeting position and speed most of the motor controller designs involves proportional+ integrator +derivative group (PID or PI) of controllers. Some related examples can be found in [6–14]. The PID group of controllers are structured control laws that can be tuned according to various methodologies such as Zeiger-Nichols charts [11], optimization [8, 13] and even neuroadaptive [12] techniques. Some other control related studies in the literature are about fuzzy logic based motor controls [9], a Kalman Filter based example [15] and another application using optimal state feedback [16].

Almost all control approaches need to implement a feedback from all or part of its state variables (position, angular velocity/speed, armature current etc.). Depending on the application some of the state variables may or may not be available for measurement. Factors affecting this availability may be the cost, the feasibility of the usage of certain instruments such as tachometers, encoders, current or torque sensors.

In the literature, there are sensorless control approaches such as [15, 17, 18]. These aim at the control of motor dynamics without the employment of a position or speed sensor. Such approaches generally require the utilization of an observer [3] such as a Kalman Filter [19–22]. Elimination of an observer/filter means employment of a static output feedback approach which is lack of a profound systematic knowledge. However thanks to [23–25], the flexibility of a full state feedback control can be reflected (or projected) to an output feedback by a simple orthogonal projection operation from the state space of the full state feedback to the state space of the output only feedback. This approach is formerly used in aerospace applications [26,27], some process control applications [28,29] and also as a Dynamic PI Control tuning helper [30].

In this research, we will utilize the approach presented in [23] to Dc motor control assuming that the feedback from the armature current is not available. This is a practically possible situation as the measurement of a current through a sense resistor followed by an signal amplification might lead to accumulation of unwanted noise. In this study, we will present a position and a speed control application which have a feedback only from the speed/position tracking errors.

Motors are often subject to disturbance torques when operated at harsh environments such as non-uniform fluid flows over the propeller or bad lubrication of the bearings etc. The level of disturbance torques might be a threat to the stability of the closed loop control system. The last part of this research is to deal with the effects of the available disturbance torques. The disturbance torques can be considered as an exogenous input to the closed loop system and thus the notion of input-to-state stability [31, 32] can be considered as a useful analysis approach. The disturbance decoupling concept [33–35] is an extension of input-to-state stability property which analyses the disturbance quenching capability of the closed loop control system. In this research, we will try to assess the conditions of input-to-state stability treating the disturbances as inputs to the closed loop.

The main contribution of this research to the literature can be summarized as follows:

- Application of Projective Control Approach to the design of electric motor control systems.
- Analysis of the disturbance handling capabilities of a closed loop output feedback motor control system through the utilization of input-to-state stability.

The demonstration of the results will be performed by numerical simulation of the designs. MATLAB is the main computational environment in this study.

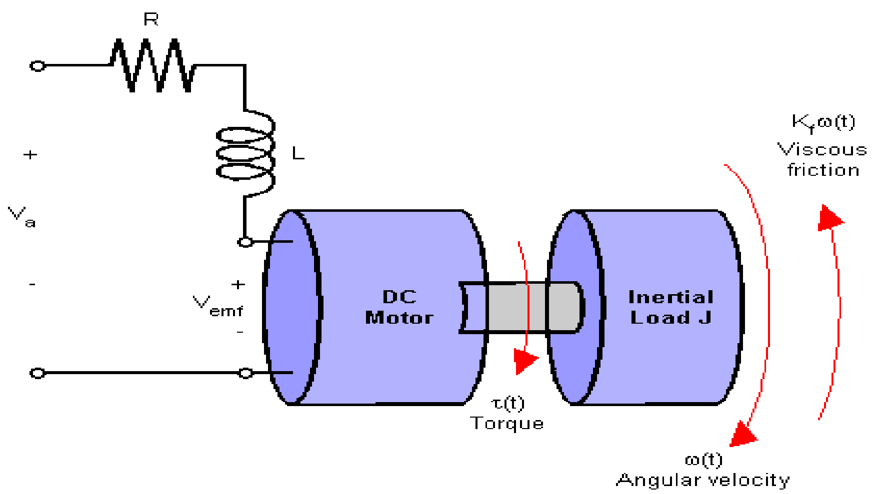
CHAPTER 3

MATHEMATICAL MODEL OF THE DIRECT CURRENT MOTOR

In this Chapter, we will give a summary of how a DC Motor is modelled using linear differential equations. The order of the models will be different for position and speed controllers.

3.1.Mathematical Model of DC Motor

The equivalent electrical circuit of a DC motor is shown in Figure (3.1) . It is driven by a voltage source (Va) across the coil of the armature. The electrical equivalent of the armature coil can be described by a resistance (R) with an inductance (L) in series with a voltage (Vc) which opposes the voltage source. The voltage is produced by the rotation of the electrical coil through the fixed flux lines of the permanent magnets. This voltage is regularly referred to as the back electromotive force [1].



Figure(3.1) courtesy of [38], Representation of a DC Motor.

The physical parameters of the DC motor.

Electric Armature Resistance, R (Ω)

Electric Armature Inductance, L (H)

Electromotive Force Constant, (V/rad/sec) (K_b)

Moment of Inertia of the Rotor, and the load. J (Kg $.m^2$)

(Viscous friction coefficient) of rotor bearings , B (N.m.s)

3.2.System equation

Generally, the torque produced by a DC motor is proportional to the armature current and the strength of the magnetic field [2]. In this case we will suppose that the magnetic field is constant and, thus, the motor torque is just proportional to the armature current I by a constant factor Ki as shown in the equation below. This is referred to as an armature-controlled motor[5].

$$T = K_i i$$

The back electromotive force , is proportional to the angular velocity of the shaft by a constant factor (K_b). $$e = K_b\dot{\theta} = K_b w$$

Where w is the angular velocity (rad/s) of motor and Q is its position(rad)

In the case of the motor torque and back of electromotive force (emf) constants are equal, Ki = K_b ; therefore, we will use K to represent both the back emf constant and motor torque constant .

From the figure above, we can find the following dynamical equations based on Kirchhoff's voltage law and Newton's 2nd law.

$$J\ddot{\theta} + b\dot{\theta} = ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

3.3. State-Space

The differential equation model of the problem from above can also be expressed in state-space form by choosing the motor speed, motor position and armature current as the state variables. Also, the rotational position or speed is chosen as the output and the armature voltage is treated as the input.

The direct current electrical machines generally involve the dynamics of the shaft position θ(t) (in degrees or radians) and armature current Ía(t) (in Amperes) and angular velocity ω(t) (in rad ⁄sec) or (degree / sec) [6].

$$\dot{\theta} = \omega \tag{1a}$$

$$\dot{\omega} = a_{22}\omega + a_{23} i_a + v\tau_L \tag{1b}$$

$$\dot{i}_a = a_{32}\omega + a_{33} i_a + b_3 V_a \tag{1c}$$

where

$$a_{22} = -\frac{B}{J}, a_{23} = \frac{K_i}{J}, a_{32} = \frac{K_b}{L_a}, a_{33} = \frac{R_a}{L_a}, b_3 = \frac{1}{La}$$

and

$$v = -\frac{1}{J}$$

In Table 3.1, one can see the definitions and nominal values of a particular DC motor. In state space form one will have the following equations:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} V_a + \begin{bmatrix} 0 \\ v \\ o \end{bmatrix} \tau_L \tag{2a}$$

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ b_3 \end{bmatrix} V_a + \begin{bmatrix} v \\ 0 \end{bmatrix} \tau_L \tag{2b}$$

In equation (2) there are two subsections The state space representation in (2a) is intended for position in control where as (2b) is intended for speed control [6].

Table 3.1:DC motor parameter.(Taken from [39])

Definition	Symbol	Value
Inertia of the Load	J	0.01 kg . m^2
Viscous friction coefficient	B	0.1 N . m . $^{sec}/_{rad}$
Armature Resistance	Ra	1 Ω
Armature Inductance	La	0.5 H
Torque Constant	K_i	0.01 N . m/A
Back-EMF Constant	K_b	0.01 V /rad/sec

The term τ_L symbolize the load or disturbance torques due to certain restrictive factors. These may be due to aerodynamic factors in a propeller or an unpredictable friction on the shaft etc. The closed loop of the motor model in (1) will still have the disturbance torque τ_L as input. Because of that we will make use of the input-to-state stability approaches may be reference [such as Sontag 1995] to analyze our motor controller under the disturbance torques.

Considering the control approaches there are various alternatives as stated in chapter 1. The main issue about the full state feedback techniques is that some state variables can not be measured or difficult to measure. In these cases either an observer [Luenberger and Kalman filters] should be used. In DC motor models such as (2), the armature current which determines the torque through the relation $\tau = K_I \times i_a$ may not be easy to sense continuously. Though devices such as low resistance sense resistors are often used in current measurement, their utilization in control requires amplification (such as OP-AMPS or Instrumentation-Amplifiers) which may bring noise and offset adjustment requirements. Thus, a control approach that does not need current feedback may benefit from being free of those issues. Apart from these, lower number of instruments will be required which is a cost reduction measure.

CHAPTER 4

CONTROL THEORIES APPLIED IN THIS WORK

In this chapter we will introduce a methodology called as projective control which can be applied to design an output feedback controller. Classical control techniques based on transfer function and compensation approaches will not be considered here as they are well established classical methodologies.

4.1: Pole placment.

4.1.1: General Discussion

Pole placement in general, is a technique used in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the complex plane{eigenspectrum}.Placing the poles to the desired locations are critical because they determine the characteristics of the closed loop [3].

In the closed loop input and an output transfer function can be displayed by a state space equation as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{3}$$

where :

C = 1 * n constant matrix {Row vector}
B = n * 1 constant matrix { column}
A = n * n constant matrix {square}
u = control signal {scalar}
y = output signal (scalar)
x = state vector {n-vector}

Then these poles of the system are definitely the roots of the characteristic equation given by

| sI - A |= 0

Full state feedback is utilized by feeding back by a signal which is a function of state vector x . This can be proportional {or a linear function } to x as shown below:

$$u = -Kx \tag{4}$$

Here the control signal u is determined by the instantaneous value of the state. The $K = [\, k_1 \;\; k_2 \ldots\ldots\ldots k_n \,]$ is (1xn) is named the state feedback gain .We suppose that all of state variables are available for feedback. In the next analysis we suppose that u is unconstrained. The block diagram associated with the pole placement full state feedback control is shown in Figure (4.1).

This closed-loop system does not have input. The states of a stable closed loop controller should converge to zero . However due to the disturbances that might be present, the output will deviate from zero .The nonzero output will be returned to the zero reference input due to feedback from states [3].A system where the reference input is every time zero is called as a regulator system.

substituting equation (4-4) to (4-3) yields :

$\dot{x} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{x}(\mathrm{t})$ $\quad\quad$ $\boldsymbol{y} = (\boldsymbol{C}\text{-DK})\,\boldsymbol{x}$

The solution of this equation is given by:

$$x(t) = e^{(A-BK)t}x(0) \tag{5}$$

where $x(0)$ is the initial values of state vector x { at time t=0 }.The transient and stability response characteristics are resolved by the eigenvalues of matrix A-BK.The roots of full state feedback system is given by: $|sI - (A - BK)|$

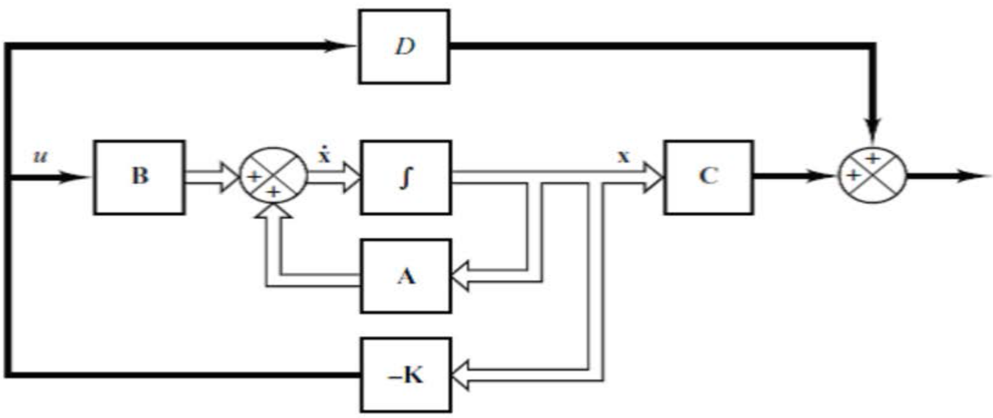


Figure.4.1. courtesy of Ogata[3]

Closed loop control system with $\boldsymbol{u} = -\boldsymbol{Kx}$

4.1.2 Pole placement design for servo systems:

The design procedure

suppose that the plant is defined by

$$\dot{x} = Ax + Bu \qquad (6)$$

$$y = Cx$$

Where: (A,B) are complete state controllable: $\begin{bmatrix} B & AB & A^2B & \dots A^{n-1}B \end{bmatrix}$ is of full rank.

As seen earlier, one can suppose that both the control signal (u) and the output signal (y) are scalar. Quite often one can see cases where output (y) equals to one of the states. If $y = x1$ one can write the following knowing that:

$$u = -kx$$

$$u = -[\, 0\; k_2 \;\; k_3 \;\; \dots .\, k_n \,] \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + k_1(r - x_1) \qquad (7)$$

$$= -k_x + k_1 r$$

$$k = [k_1 \; k_2 \; \dots .\, k_n]$$

Suppose that the step function (reference input) is applied at t=0. Then, for t > 0, the system dynamics become:

$$\dot{x} = Ax + Bu = (A - BK)x + Bk_1 r \qquad (8)$$

Now will design the type (1) servo system such that the closed-loop poles are located at desired positions. The designed system can be an asymptotically stable system, y(∞) will approach to r, u(∞) will approach to zero. where (r) is the reference input modelled as step function .

$$\dot{x}(\infty) = Ax + Bu = (A - BK)x(\infty) + Bk_1 r(\infty) \qquad (9)$$

Noting that r(t) is a step input, we have r(∞)= r(t) = r (constant) for t>0. By subtracting Equation (9) from Equation (8), we obtain

$$\dot{x}(t)-\dot{x}(\infty)=\left(A-BK\right)[x(t)-x(\infty)] \tag{10}$$

$$x(t)-x(\infty)=e(t)$$

Then Equation (10) becomes

$$\dot{e}=(A-BK)e \tag{11}$$

Which represent the error dynamics, of the system.

The design of the type 1 servo system here is changed to the design of an asymptotically stable regulator system such as e(t) approaches zero, starting from e(0). If the process defined by Equation (6) is absolutely state controllable, then, by appointing the desired eigenvalues $\mu_1\ \mu_2\ \ldots.\ \mu_n$ for the matrix A - BK, matrix K can be calculated by the pole placement method presented in Equation 4. The steady-state values of **x**(t) and u(t) can be found as follows:

$$u(\infty)=-Kx(\infty)+k_1 r=0 \tag{12}$$

x(∞) can be determined as

$$x(\infty)=-\left(A-BK\right)^{-1}Bk_1 r \tag{13}$$

Also, u(∞) can be obtained as

$$u(\infty)=-Kx(\infty)+k_1 r=0 \tag{14}$$

- Some time the plant $\dot{x}=Ax+Bu$ has no integration. In this case the plant shown by the general state equations

$$\dot{x}=Ax+Bu \tag{15}$$

$$y=Cx \tag{16}$$

should be augmented by the following:

$$\xi= r-y=r-Cx$$

and the control u can be defined by

$$u=-Kx+K_1\xi \tag{17}$$

Where output of the integrator ξ determined by

$$\xi = \int r - y = \int (r - Cx)dt \tag{18}$$

Where r = reference input signal:

Suppose that the step function (reference input) is applied at t=0. Then, for t>0, the system dynamics can be described by using equation (15) and (18):

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \tag{19}$$

We desire to obtain an asymptotically stable closed loop system so that, x (∞), $\boldsymbol{\xi}$(∞), and u(∞) approach constant values, respectively. Then, at steady state, $\dot{\boldsymbol{\xi}}$(t)=0 and we obtain y(∞)= r.

Notice that at steady state we have.

$$\begin{bmatrix} \dot{x}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(\infty) \tag{20}$$

Noting that $r(\infty) = r(t) = r$(constant) for t>0.where r(∞)is a step input. So when subtracting Equation (21) from Equation (20), we obtain

$$\begin{bmatrix} \dot{x}(t) - \dot{x}(\infty) \\ \dot{\xi}(t) - \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) - x(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} [u(t) - u(\infty)] \tag{21}$$

Define:

$$\boldsymbol{x(t) - x(\infty) = x_e(t)}$$
$$\xi(\boldsymbol{t}) - \xi(\infty) = \xi_{\boldsymbol{e}}(\boldsymbol{t})$$
$$\boldsymbol{u(t) - u(\infty) = u_e(t)}$$

If u(∞) = 0 → $u_e = u$

if u(∞) is not known one cannot calculate u, so no control action is available.

Then Equation (21) can be written as

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{\xi}_e(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \tag{22}$$

Where $$u_e(t) = -Kx_e(t) + k_1\xi_e(t) \tag{23}$$

By define a new (n+1)th-order error vector **e**(t) ,the Equation (22) becomes

$$\dot{e} = \hat{A}e + \hat{B}u_e \tag{24}$$

where

$$\widehat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \qquad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

and Equation (23) becomes

$$u_e = -\widehat{K}_e \tag{25}$$

where

$$\widehat{K} = [K \ \vdots -k_1]$$

The state error equation can be obtained by substituting Equation (25) into Equation (24):

$$\dot{e} = (\hat{A} - \hat{B}\widehat{K})e$$

Calculation of the gain matrix $\widehat{K}$ is similar to that of (11) but we obtain a $\widehat{K}$ having (n+1) elements instead of (n)

4.2. Linear Quadratic Regulation (LQR).

Linear Quadratic Regulator methods yields a linear feedback control law through a simple optimization with equality constraints. In other words, the method minimizes a quadratic performance index along the trajectories of the linear system {plant} in consideration. This is often an advantage, because one will get rid of pre-determination of closed loop eigenvalues. This will help one to implement a faster design process when the system order is too large which makes the determination of desired eigenvalues a cumbersome task. In addition, the resultant eigenspectrum will usually be better than the most manually planned set of desired eigenvalues .

Optimal design of the controllers are also an advantage as the proper selection of the cost function might lead to robust controllers.

Mathematical formulation.

Assume that the space model is

$$\dot{x} = Ax + Bu$$

with $X \in R^n$, $u \in R$, $A \in R^{nxn}$ *and* $B \in R^n$. The initial condition is x (0).

Assume that we have sensors to measure the entire state and that we use a controller (regulator) $u = -K\,x$

that aim to drive the state to zero.

we use the LQR method to evaluate the gain K . In this case, Let the cost function be defined as

$$J= \int_0^\infty (X^t \text{ Q x} + u^T \text{ R u})\, d\, t \quad (26)$$

Where Q is a matrix of weighting factors for the states which should be a positive semidefinite matrix, and R, (is also positive definite matrix) (or scalar), for a matrix weighting of control action.

To design the LQR controller, the first step is to choose the weighting matrices Q and R. The value R weight inputs more than the states while the value of Q weight the state more than the inputs. Then the feedback K can be computed. The closed loop system responses can be found by simulation.

The LQR controller is given by:

$$u = -Kx \quad (27)$$

where K is a constant feedback gain matrix /vector obtained from solution of the continuous algebraic Riccati equation. The gain matrix K which solve the LQR problem is given by:

$$K = R^{-1} B^T P \tag{28}$$

Where P is the unique, positive semidefinite solution to the Riccati equation defined by:

$$A^T P + PA - PBR^{(-1)} B^T P + Q = 0 \tag{29}$$

4.3. Projective control.

Projective control is a technique of retaining the most dominant eigenstructure of a full state feedback design using output feedback, where the optimal full state feedback control is used to form the reference eigenstructure. The controller might be in form of a static gain full state closed loop (static projective control) or a dynamic compensator (dynamic projective control). The number of eigenvalues and eigenvectors retained from the reference eigenstructure depends upon the number of outputs available, for feedback.

Static projective control is an output feedback design technique used to obtain a partial eigenstructure of a state feedback controller $(u = -K_x x, x \in R^{n_x}, u \in R^{n_u})$ using dynamic and/ or static output feedback. For static output feedback $(u = -K_y y, y \in R^{n_y}), n_y$ eigenvalues and associated eigenvectors of a full state feedback closed loop can be retained.

Consider the problem of designing output feedback regulators for a linear-time invariant system described by

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

The basic advantage of the projective feedback controller is the use of an output feedback instead of the state feedback, also the projective controller has other advantage of being a low-cost feedback controller, which uses the output measurements without requiring a state estimation.

$$\dot{x} = (A - BK)x \tag{30}$$

The spectral decomposition of *(A-BK)* is described by

$$(A - BK)V = V\Lambda \tag{31}$$

Where Λ is a diagonal matrix, which contains the eigenvalues of *[A-BK]* and ***V*** is a matrix containing the associated eigenvectors of *[A-BK]*.
When one applies the following output feedback .

$$u = -K^o y \qquad \text{Where} \quad y = Cx$$

$$\text{Then :-} \quad u = -K^o Cx \tag{32}$$

$$\left[\dot{x} = \left(A - BK^o C\right)x \right] \tag{33}$$

the spectral decomposition of ($\mathrm{A - BK^oC}$) is
given by,

$$[\mathrm{A - BK^oC}]V_d = V_d \Lambda_d \tag{34}$$

If the output feedback controller is to retain the dominant eigenvalues from Λ as Λ_d and if its corresponding eigenvectors are (from V as V_d) .
one should also be able to write the following:

$$[\mathrm{A - BK}]V_d = V_d \Lambda_d$$

as the V_d and Λ_d are subsets of V and Λ respectively.

So one can equate the above to (34) as:

$$[\mathrm{A - BK}]V_d = [\mathrm{A\text{-}BK}^o C]V_d$$

It will be easy to solve the above equation for K^o using basic matrix inversion. So one can obtain the output feedback gain K^O as:

$$\mathrm{K}^o = \mathrm{KV}_d \; (\mathrm{CV}_d)^{-1} \tag{35}$$

The number of eigenvalues to be retained is equal to the dimension of output feedback which is r : { $y \in R^r$ }.

4.4. INPUT-TO-STATE STABILITY

In this part, we will illustrate the input-to-state stability concept and present its applicability in the analysis of the comprehensive stability of the closed loop motor controller against disturbance torques. Before continuance it will be beneficial to provide some definitions [40].

Definition 1 (Class K Functions) These are a class of all functions μ: $\mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ satisfying the following conditions:

1. $\boldsymbol{\mu}(0)=0$
2. $\boldsymbol{\mu}(.)$ is continuous
3. $\boldsymbol{\mu}(.)$ is strictly increasing.

Definition 2 (Class K_∞ Functions) $\xi(P)$ will be of class K∞ if $\boldsymbol{\xi}(.)$ is of class K and $\xi(P) \rightarrow \infty$ when $p \rightarrow \infty$.

Definition 3 (Storage Functions) The function $\mathrm{W}(x)$:

$\mathfrak{R}^n \rightarrow \mathfrak{R}_+$ with $x \in \mathfrak{R}^n$ is said to be a storage Lyapunov function if it satisfies the following conditions:

1. W is continuously differentiable
2. W is radially unbounded $i.e$ $\mathrm{W}(x) \rightarrow \infty$ when $x \rightarrow \infty$.
3. W is a positive definite function $i.e.$ $W(0) = 0$ and $W(x) > 0$ $when$ $x \neq 0$.

Theorem 1 (Stability of an autonomous system)

Suppose that a system defined by the following differential equation:

$$\dot{x} = f(x) \tag{36}$$

and also suppose that $f(0) = 0$. The equilibrium point $x = 0$ will be stable in the sense of Lyapunov, if a function W(x) satisfying the properties given in Definition 3 and:

$$\frac{\partial w(x)}{\partial x} f(x) \leq -\mu(|x|) \tag{37}$$

where $\mu(|x|)$ is a class K∞ function.

Theorem 1 is valid when the system has no exogenous inputs. When the system in (36) has exogenous or normal inputs (u) one needs to take it into consideration. In this case we will need to define an ISS-Lyapunov function. See the definition below:

Definition 4 (ISS-Lyapunov Functions) An ISS - Lyapunov function W(x) is a type of storage function which satisfies the properties given in Definition 3 and the one shown below:

$$\frac{\partial w(x)}{\partial x} f(x,u) \leq -\mu(|x|) + \theta(|u|) \tag{38}$$

where $\mu(.)$ and $\theta(.)$ are class $K\infty$ functions.

Theorem 2 (Input-to-State Stability (ISS)) A system defined by the following general differential equation:

$$\dot{x} = f(x,u) \tag{39}$$

will be input to state stable (ISS) if there exist a W(x) satisfying the properties in Definitions 3 and 4 for the system in (39).

In general, the above case can be associated with the dissipation concept as (38) is a dissipation inequality with the storage function W(x) and the supply function $\sigma(x,u) = \theta(|u|) - \mu(|x|)$.

Input-to-state stability can be viewed as a disturbance to state stability when the closed loop formed by applying a feedback of the form $u = -k(x)$ to a general system with a exogenous disturbance input $n(t)$:

$$\dot{x} = f(x, u, n) \tag{40}$$

and when the feedback is applied the above system will become $\dot{x} = f(x, -k(x), n)$. Thus, the closed loop can be thought of a system with input n. Then the input to state stability condition in (38) can be rewritten as:

$$\frac{\partial w(x)}{\partial x} f(x, n) \leq -\mu(|x|) + \theta(|n|) \tag{41}$$

with the definitions of α, θ are same as that of (38). The above condition is called as Disturbance-to-State condition.

CHAPTER 5

CONTROL OF DIRECT CURRENT MOTORS

An appropriate and beneficial DC motor application will need speed and position controller. This is required to attain both the desired operation and performance. Position and speed control of direct current motors may require different configurations. For example position dynamics in (2a) involve a natural integration which helps in the elimination of any steady state error. The speed (angular velocity) dynamics (2b) on the contrary does not have any natural integrator which may lead to a steady state error. In order to overcome this issue one will need to add an artificial integrator to (2b). In this section, we will present the speed and position control of the DC motors with the aid of projective control approach.

5.1. Speed Control of DC Motors by Projective Control

In speed control, the aim is to hold the speed at the desired value and keep it at the desired value even if there are disturbing effects.

As noted in equation (2b) we do not have any natural integration in speed dynamics. This means that we have to integrate the tracking error as shown below:

$$\dot{\varepsilon} = \omega - \omega_d \tag{42}$$

where ω_d is the desired angular velocity and ω is the actual angular velocity of the motor shaft. So the state space representation of the motor speed control will be obtained from the combination of the above equation

and (2b) as:

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} V_a + \begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix} \tau_L + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \omega_d \tag{43}$$

According to [3] if the reference velocity ω_d is constant or slowly variable the difference between the state variables ($\varepsilon(t)$, $\omega(t)$, i_a (t)) and their steady state values $\varepsilon(\infty), \omega(\infty), i(\infty)$ will not involve reference speed (ω_d) .This is also a known fact from some mechanism theory.

So we can write the following:

$$\begin{bmatrix} \dot{e}_{\varepsilon} \\ \dot{e}_{\omega} \\ \dot{e}_{i_a} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} e_{\varepsilon} \\ e_{\omega} \\ e_{i_a} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} V_a \tag{44}$$

Where $e_{\varepsilon} = \varepsilon(t) - \varepsilon(\infty), e_{\omega} = \omega(t) - \omega(\infty), e_{i_a} = i_a(t) - i_a(\infty).$

Then, The control law can be written as:

$$V_a = -K^F x = -\left[k_{\varepsilon}{}^F k_{\omega}{}^F k_{i_a}^F \right] \begin{bmatrix} e_{\varepsilon} \\ e_{\omega} \\ e_{i_a} \end{bmatrix} \tag{45}$$

for the full state feedback,

$$V_a = -K^0 C x = -\left[k_{\varepsilon}{}^0 k_{\omega}{}^0 \right] \begin{bmatrix} e_{\varepsilon} \\ e_{\omega} \end{bmatrix} \tag{46}$$

for output feedback control. In (45) and (46) e=[e_{ε} e_{ω} e_{i_a}]^T

In (46) the output matrix C can be written as:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{47}$$

This means that, we are having a feedback from e_{ε} and e_{ω} .

From equation (44) and (45) the closed loop dynamics when full state feedback in (45) is employed, can be written as:

$$\dot{e} = (A - BK^F)e \tag{48a}$$

$$A - BK^F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ -_{b_3}k_\theta^F & a_{32} - {}_{b_3}k_\omega^F & a_{33} - {}_{b_3}k_{i_a}^F \end{bmatrix} \tag{48b}$$

The next stage is to apply projective control using equation (35). In the speed control of DC motors by projective control approach, we only will have two feedback variables, as understood from (47). This means that we can only retain two eigenvalues from the closed loop spectrum of (48b). Due to the fact that the size of (49b) is an odd number. if there are two complex eigenvalues one has to choose them as the retained eigenvalues. If all the eigenvalues are real the two dominant ones should be preferred as retained eigenvalues. After the application of (35) one have to check the closed loop eigenvalues of the output feedback evaluate the following:

$$\dot{e} = (A - BK^oC)e \tag{49a}$$

$$A - BK^oC = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ -_{b_3}k_\varepsilon^o & a_{32} - {}_{b_3}k_\omega^o & a_{33} \end{bmatrix} \tag{49b}$$

To check the overall spectrum of closed loop formed by output feedback.

5.2. Position Control of DC Motors by Projective Controls.

In (2a), the position dynamics has a natural integrator as position θ(t) is the integration of the angular velocity ω(t). So one does not have to add any sort of artificial integrators. One should rewrite the dynamical model for position control to proceed:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} V_a + \begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix} \tau_L + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \theta_d \tag{50}$$

where θ_d is the desired reference position. The other variables in the above equation are the same as that of (43) . One should observe that, (50) and (43) are totally the same except the error integral variable ε which is substitute by position θ and the reference velocity variable ω_d that is replaced by θ_d . Because of that, the details given in Section 5.1 can be directly applied to the problem, in the condition that the designer is careful about the state variables. The error dynamics of the position control problem will have the same matrices [A,B]. Only the state vectors should be modified properly:

$$\begin{bmatrix} \dot{e}_\theta \\ \dot{e}_\omega \\ \dot{e}_{i_a} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} e_\theta \\ e_\omega \\ e_{i_a} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} V_a \tag{51}$$

Thus the control equations (45) and (46) will be:

$$V_a = -K^F e = -\left[k_\theta{}^F k_\omega{}^F k_{i_a}^F \right] \begin{bmatrix} e_\theta \\ e_\omega \\ e_{i_a} \end{bmatrix} \tag{52}$$

with regard to situation of full state feedback. As one has to have the feedback from the speed and position deviation (from steady state values) e_θ and e_ω the feedback matrix should be similar to that of (47) (first two elements of the state vector).

$$V_a = -K^o C e = -\left[k_\theta{}^o k_\omega{}^o \right] \begin{bmatrix} e_\theta \\ e_\omega \end{bmatrix} \tag{53}$$

So the closed loop dynamics for full state feedback position control can be rewritten as:

$$\dot{e} = (A - BK^F)e \tag{54a}$$

$$A - BK^F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ -_{b_3}k_\theta^F & a_{32} - _{b_3}k_\omega^F & a_{33} - _{b_3}k_{i_a}^F \end{bmatrix} \tag{54b}$$

Similarly for output feedback.

$$\dot{e} = (A - BK^oC)e \tag{55a}$$

$$A - BK^oC = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ -_{b_3}k_\theta^o & a_{32} - _{b_3}k_\omega^o & a_{33} \end{bmatrix} \tag{55b}$$

5.3. Disturbance-to-State Stability for DC Motor Control.

In this section, one will be able to see how the theory developed in Section 4.5 is applied to the analysis of stability against the disturbance torques exerted on the DC motor shaft and load. To achieve this goal one should first take the closed loop dynamics in (49) or (55). However, referring to equation (43) or (50) one should modify the closed loop dynamics to include the disturbance torque τ_L.That is:

$$\dot{e} = (A - BK^O C)e + G\tau_L \tag{56}$$

where $G = [0 \;\; v \;\; 0]^T$. Before proceeding, some additional information should be presented.

Definition 5 (Quadratic Forms) For any symmetric matrix $P \in \Re^{n\times n}$, the form $x^T \; Px$ will be called as a quadratic form.

Theorem 3 (Lower and Upper Bounds) For any quadratic form defined in Definition [5] one can define the following lower and upper bounds:

$$\lambda_{\min}(\mathrm{P}) \; \mathrm{x^T x} \le \mathrm{x^T P x} \le \lambda_{\max}(\mathrm{P}) \, \mathrm{x^T x} \tag{57}$$

where $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ are the minimum and maximum eigenvalues of the matrix P respectively.

Now considering the storage function concept in Section 4.4 one can define the following as a quadratic storage function:

$$W(e) = \frac{1}{2} e^T e \tag{58}$$

and one can also write the rate of change of W(e) along the trajectories of e as:

$$\dot{W}(e) = \frac{\partial W(e)}{\partial e} \dot{e} \tag{59}$$

$$\dot{W} = \frac{1}{2}\ \dot{e}^T + \frac{1}{2}\ e^T \dot{e}$$

and substituting from (56) one will be able to obtain:

$$\begin{aligned} \dot{W}(e) &= e^T\{(A - BK^{\,0}C)e + G\tau_L\} \\ &= e^T(A - BK^{\,0}C)e + e^T G\tau_L \end{aligned} \tag{60}$$

$$\dot{W}(e) = \frac{1}{2}[\ e^T(A - BK^O C)^T e + \frac{1}{2}\ e^T(A - BK^O C)e + \frac{1}{2}\tau_L\ \ G^T e + \frac{1}{2}\ G^T \tau_L$$

Notice that transposition of a matrix does not change the eigenvalues $\lambda\left(\frac{1}{2}\ (P + P^T)\right) = \lambda(P)$ .Knowing this fact and using (57) one can obtain the following

Using Theorem 3, one can convert the above to an inequality as:

$$\dot{W}(e) \leq \lambda_{max}(A - BK^0 C)\left|\left|e\right|\right|^2 + e^T G\tau_L \tag{61}$$

In order to go further, we will need to deal with the term $e^T G\tau_L$ It is pretty obvious that

$(e - G\tau_L\,)^T\ (e - G\tau_L) \geq 0.$

we can expand this term as shown below:

$$\begin{aligned} &(e - G\tau_L\,)^T\ (e - G\tau_L) = \\ &\qquad e^T e - e^T\ G\tau L - \tau_L^T G^T e + \tau_L^T G^T\ G\tau_L\ \geq 0 \end{aligned} \tag{62}$$

Compiling the right side of the equation:

$$e^T e +\ \tau_L^T G^T\ G\tau_L\ \geq\ e^T\ G\tau_L + \tau_L^T G^T e \tag{63}$$

when τ_L is a scalar as in equation (2) $e^T G\tau_L = \tau_L^T G^T e$

So one can write the following:

$$e^T e + \tau_L^T G^T G\tau_L \geq 2\, e^T G\tau_L \tag{64}$$

and

$$\frac{1}{2}(e^T e + \tau_L^T G^T G\tau_L) \geq e^T G\tau_L \tag{65}$$

By using equation (65) one can rewrite equation (61) as:

$$\dot{W}(e) \leq \lambda_{max}(A - BK^0C)\left||e|\right|^2 + \frac{1}{2} e^T e + \frac{1}{2}\tau_L^T G^T G\tau_L \tag{66}$$

As it is known that $e^T e = \left||e|\right|^2$ in the sense of $\boldsymbol{\mathcal{L}_2}$ norms, the above inequality can be rewritten as:

$$\dot{W}(e) \leq \lambda_{max}(A - BK^0C)\left||e|\right|^2 + \frac{1}{2}\left||e|\right|^2 + \frac{1}{2}\tau_L^T G^T G\tau_L \tag{67}$$

The first two terms of the above can be combined as:

$$\dot{W}(e) \leq \left[\frac{1}{2} + \lambda_{max}(A - BK^0C)\right]\left||e|\right|^2 + \frac{1}{2}\tau_L^T G^T G\tau_L \tag{68}$$

Finally using Definition 3 once again, the above inequality is finalized as:

$$\dot{W}(e) \leq \left[\frac{1}{2} + \lambda_{max}(A - BK^0C)\right]\left||e|\right|^2 + \frac{1}{2}\lambda_{max}(G^T G)\left||\tau_L|\right|^2 \tag{69}$$

Looking at (69), one will easily note that it resembles (41). The input-to-state stability theorem (Theorem 2) dictates that $\frac{1}{2} + \lambda_{max}(A - BK^0C) < 0 \; and \; \lambda_{max}(G^T G) > 0$.

One can now state the following theorem:

Theorem 4 (Stability Against Disturbances) The closed loop controlled DC motor modelled by equations (43) or (50) with the control law presented in (46) or (53) will be disturbance-to-state ($\tau_L - to - e$) stable if the following conditions are satisfied:

$$\lambda_{max}(A - BK^0C) < -\frac{1}{2} \tag{70a}$$

$$\lambda_{max}(G^T G) > 0 \tag{70b}$$

One should note that, Theorem 4 is a sufficient condition not a necessary one. The $G^T G$ matrix is evaluated as

$$G^T G = v^2 \tag{71}$$

which is a scalar and $\lambda_{max}(G^T G) = v^2 > 0$ So condition (70b) is always satisfied. The condition in (70a) requires numerical analysis which is to be done in the next section.

CHAPTER6

NUMERICAL RESULTS AND ANALYSIS

In this chapter we will demonstrate our derived control laws for a DC motor by using the parameters given in Table (3.1).

6.1. Speed Control.

When the parametric values in table (3.1) are substituted to equation (43) one will obtain:

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -10 & 1 \\ 0 & -0.02 & -2 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} V_a + \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix} \tau_L + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \omega_d \tag{72}$$

When we suppose that the disturbance torque is negligible ($\tau_L = 0$) and the reference speed is a constant (ω_d is a step function). The equation will be similar to (44) with the system matrices similar as in above. The linear quadratic full state feedback control can be obtained by implementing in the "MATLAB "command lqr(A,B,Q,R) with Q, R being the matrices in the quadratic performance index (26).

In this model they are taken as $Q = qI_{3\times 3}$ and R = 1. When one invokes the MATLAB's "lqr " command for the given system in (73) with q = 50, the resultant full state feedback control gain K^F in $V_a = -K^F$ e (where e is defined in Section .5.1) is found as:

$$K^F = [\, 7.071 \quad 0.903 \quad 6.204 \,] \tag{73}$$

The above will yield the following closed loop spectrum:

$$A_C = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0 & -10.0000 & 1.0000 \\ -14.142 & -1.8269 & -14.409 \end{bmatrix} \tag{74a}$$

$$\Lambda_C = \begin{bmatrix} -0.098538 & 0 & 0 \\ 0 & -14.211 & 0 \\ 0 & 0 & -10.099 \end{bmatrix} \tag{74b}$$

$$V_C = \begin{bmatrix} -0.71399 & -0.016255 & -0.098064 \\ 0.070355 & 0.231 & 0.99034 \\ 0.69662 & -0.97282 & -0.098014 \end{bmatrix} \tag{74c}$$

Where $A_C = A - BK_f , \Lambda_C = \lambda(A_C)$ and V_C is the eigenvectors corresponding to each element of Λ_C given in the order. From (72) we will realize that the available state variables are the steady state errors of the integral of the velocity tracking error $e_\varepsilon(t) = \varepsilon(t) - \varepsilon(\infty)$ and the velocity itself $e_\omega(t) = \omega(t) - \omega(\infty)$. This means that the output feedback matrix C is:

$$C=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{75}$$

We have to also notice from the above equation that the number of available feedback lines is equal to two therefore the number of eigenvalues that are to be retained from the closed loop full state feedback spectrum in (74) is also equal to two. One has no control over the location of the third eigenvalue. In order to minimize the risk of an unstable mode, the desired retained eigenvalues amongst (74b) should be the two dominant ones in C . Looking at (74b), we can easily notice that the dominant poles of the full state feedback closed loop are:

$$\Lambda_d=\begin{bmatrix} -0.098538 & 0 \\ 0 & -10.099 \end{bmatrix} \tag{76}$$

and their corresponding eigenvectors are:

$$V_d = \begin{bmatrix} -0.71399 & -0.098064 \\ 0.070355 & 0.99034 \\ 0.69662 & -0.098014 \end{bmatrix} \tag{77}$$

So when one applies (35), the output feedback gain Ko will be found as:

$$K^o = [0.89686 \quad -0.32197] \tag{78}$$

The above gain will yield an output feedback closed loop eigenvalues as shown below:

$$\Lambda_C = \begin{bmatrix} -0.098538 & 0 & 0 \\ 0 & -1.8025 & 0 \\ 0 & 0 & -10.099 \end{bmatrix} \tag{79}$$

So only the second eigenvalue is different from the full state feedback equivalent which is -14.211 . However, we obtained a stable output feedback based DC motor speed control system. In Figures (6.1 ,6.2 ,6.3) one can see the simulation results obtained when the control law with gain (78) is applied.

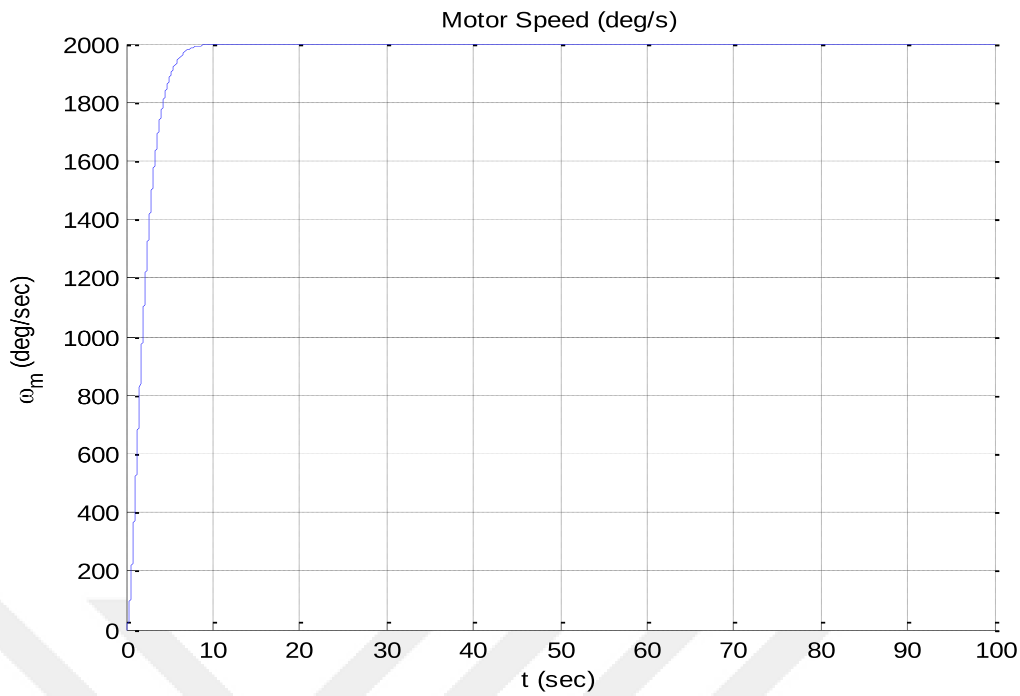


Figure 6.1 . Speed variation of the DC Motor parametrized in Table (3.1) under the control law defined by the output feedback gain in (78). The reference speed is $\boldsymbol{\omega_d}$ *= 2000 deg/sec*

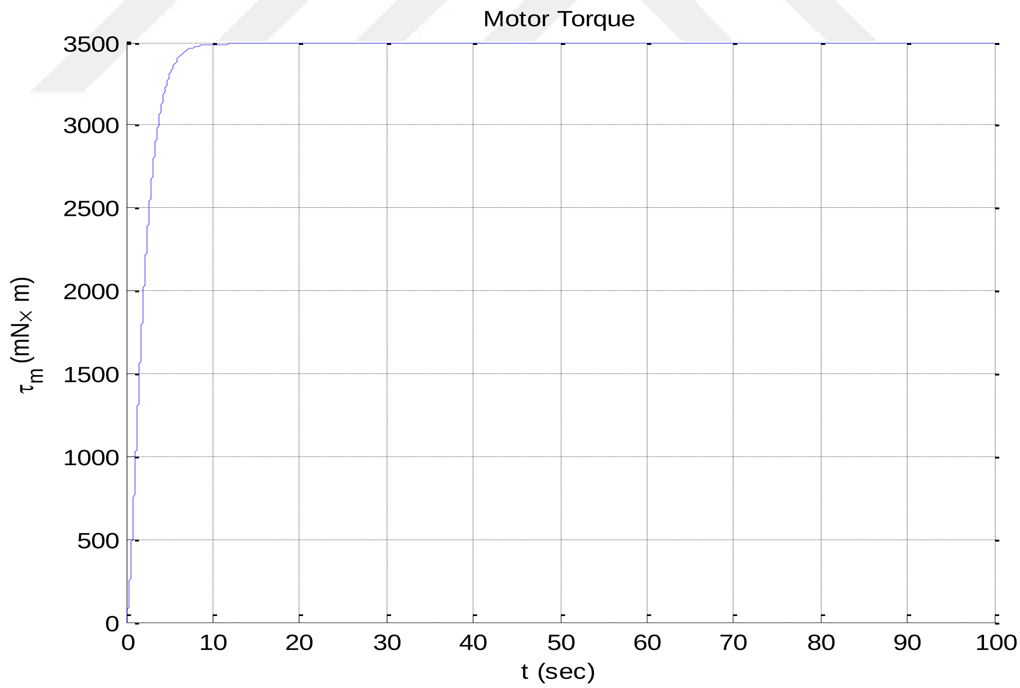


Figure.6.2.Variation of the torque generated by the DC Motor parametrized in Table (3.1) under the control law defined by the output feedback gain in (78). The reference speed is $\boldsymbol{\omega_d}$*= = 2000 deg/sec*

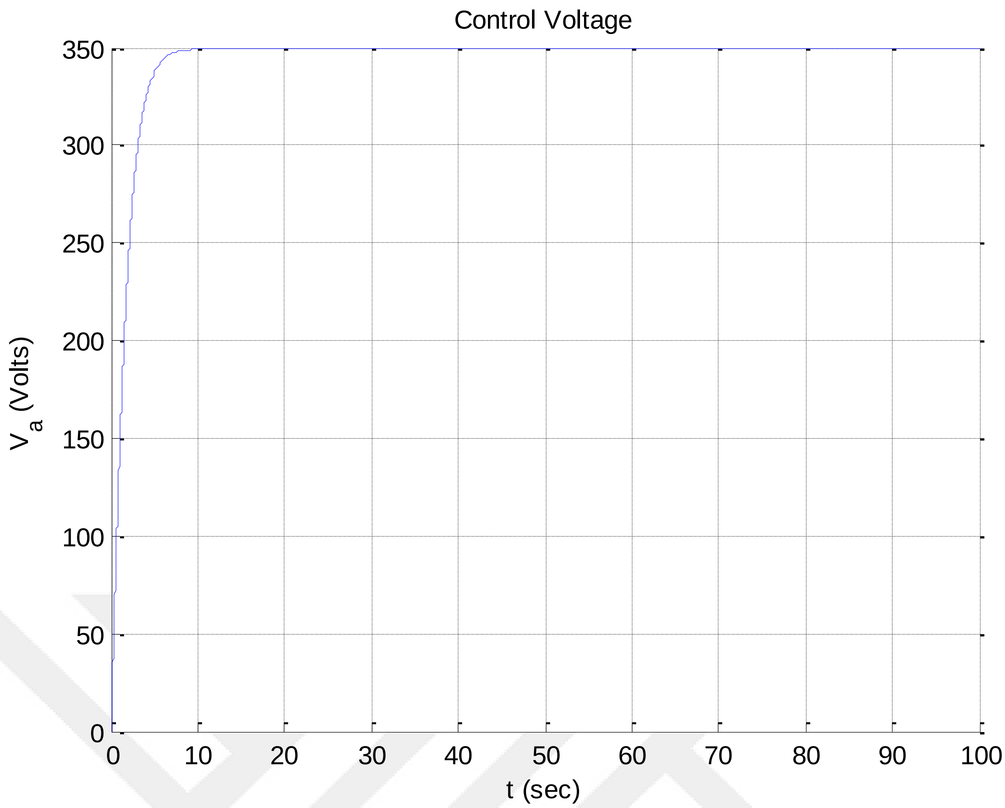


Figure.6.3.Variation of the armature voltage required by the DC Motor parametrized in Table (3.1) under the control law defined by the output feedback gain in (78). The reference speed is is $\boldsymbol{\omega_d}$*= 2000 deg/sec*

When one has a disturbance torque effective on the motor shaft (τ_L), one will note the results presented in Figures (6.4, 6.5 , 6.6) . Analysis of the results show that, the closed loop is working stable against the modelled disturbance torques. This might be seen as a violation of the Theorem 4. However, we have here to stress that Theorem 4 is a sufficient not necessary condition. It is also a conservative inequality as it is transformed from Lyapunov equation (60) by using upper/lower bound theorems (57). Because of that, choosing (78) which satisfies Theorem 4 will guarantee disturbance-to-state stability but this does not mean that poles positioned near to $j\omega$ axis will lead to instability under disturbance. One can see the simulation results in Figures when the eigenvalue nearest to $j\omega$ axis is shifted to the position $\lambda = -0.8$. These results are obtained when K^o is replaced by:

$$K^o = [4.4476 \quad 0.029499] \tag{80}$$

The above will yield the closed loop eigenvalues as:

$$\Lambda_o = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -1.101 & 0 \\ 0 & 0 & -10.099 \end{bmatrix} \tag{81}$$

The second eigenvalue is not placed as expected but it does not violate Theorem 4. The evaluation of the gain in (80) is performed by applying the orthogonal projection equation (35) to the full state feedback spectrum obtained from a pole placement design which replaces the eigenvalue violating Theorem 4 by a suitable one (i.e. $\lambda = -0.8$). Here pole placement is applied to change the location of one pole only. The other two are at the same locations obtained from LQR method. However this will result in the loss of optimality provided by the linear quadratic regulator (suboptimal/near optimal controller). It should also be noted that, there is a very little change in the simulation results.

The simulation based analysis of the disturbance torque effects are based on the repeated runs of the closed loop controlled model with a normally distributed random disturbance input. The disturbance torque τ_L(t) is considered as a random variable with zero mean and a certain level of variance which is chosen to be less than the 10% of the maximum value of the torque obtained. This exogenous input will affect the angular velocity of the motor and thus its position. In order to see the actual situation, the randomness of the disturbance will force one to repeat the simulations several times with the normally distributed disturbance torque active on the model. In this study, the number of repeats is chosen as N_{tst} = 200 (200 times repeating). This approach called as Monte Carlo methodologies. The numerical details of the disturbance torques for each group of simulation are either written in the figure captions or in the parts of the text referring to the illustrations.

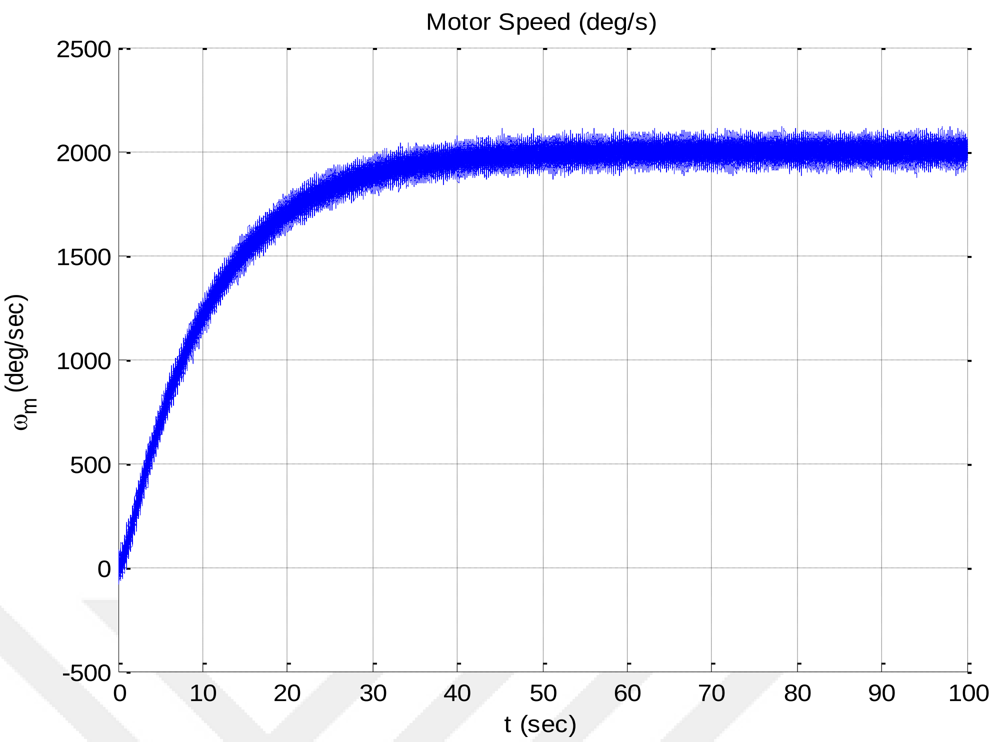


Figure .6.4.Speed variation of the DC Motor parameterized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d= 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\mu = 0$ and variance σ^2 = 200 mN.m

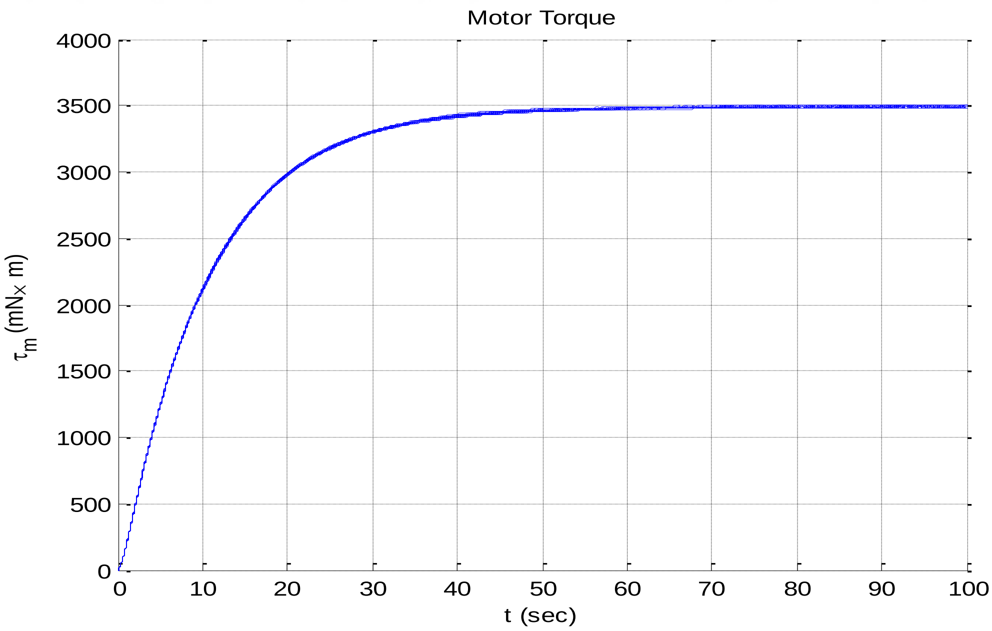


Figure 6.5.Variation of the torque generated by the DC Motor parameterized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d = 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian *distributed random variable with mean $\mu = 0$ and variance σ^2 = 200 mN*

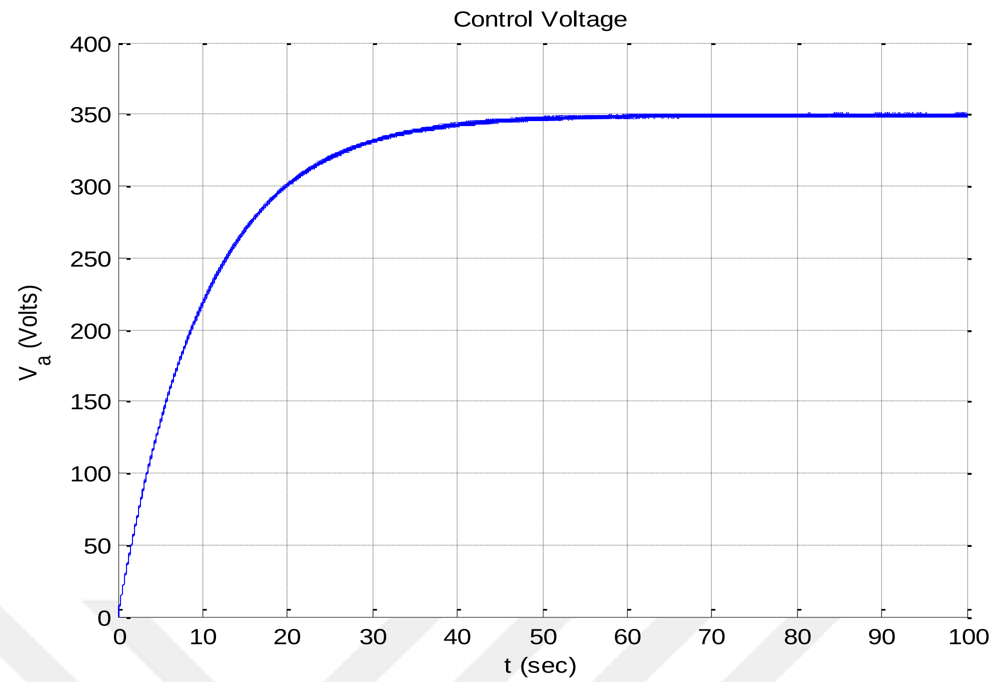


Figure.6.6.Variation of the armature voltage required by the DC Motor parametrized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d= 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\mu = 0$ and variance σ^2 = 200 mN

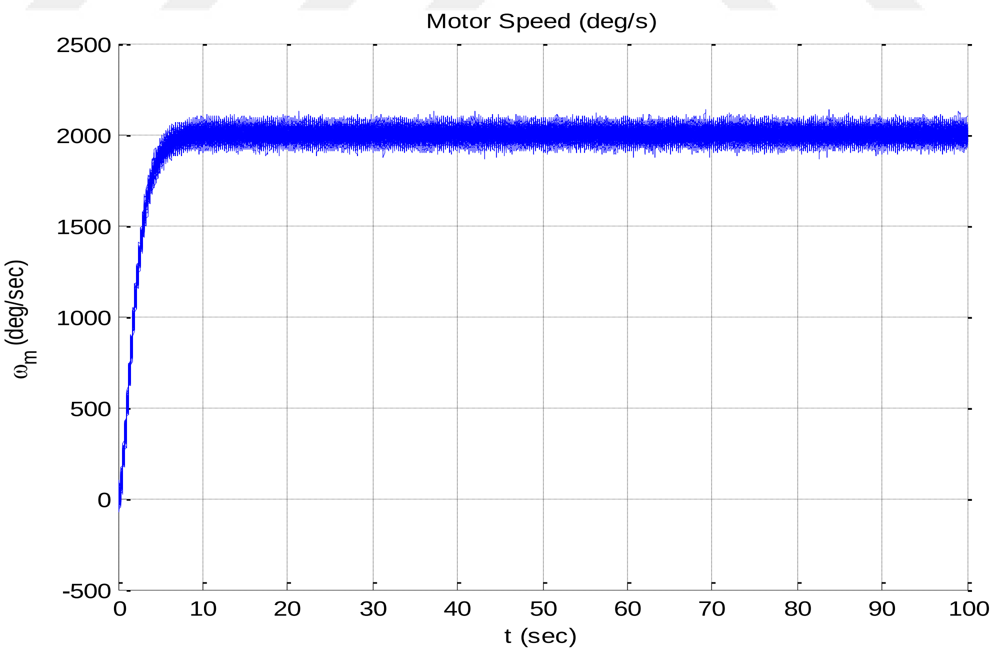


Figure.6.7.Speed variation of the DC Motor parametrized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d = 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\boldsymbol{\mu} = \mathbf{0}$ and variance $\boldsymbol{\sigma}^2$ = 200 mN. Here the control gain at (80) is generating the control law.

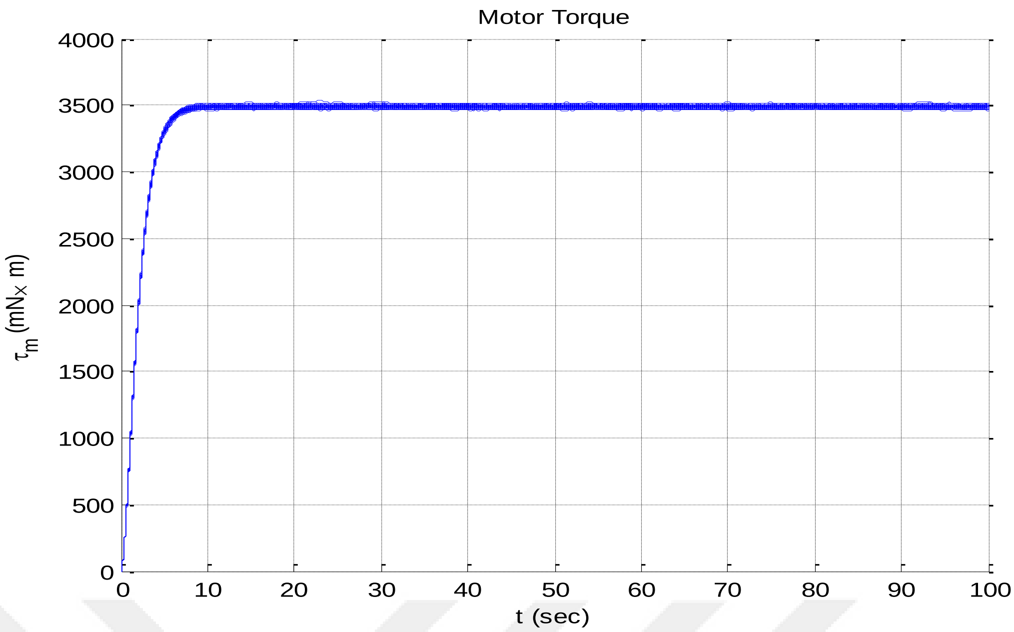


Figure .6.8.Variation of the torque generated by the DC Motor parametrized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d= 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\mu = 0$ and variance σ^2 = 200 mN. Here the control gain at (80) is generating the control law.

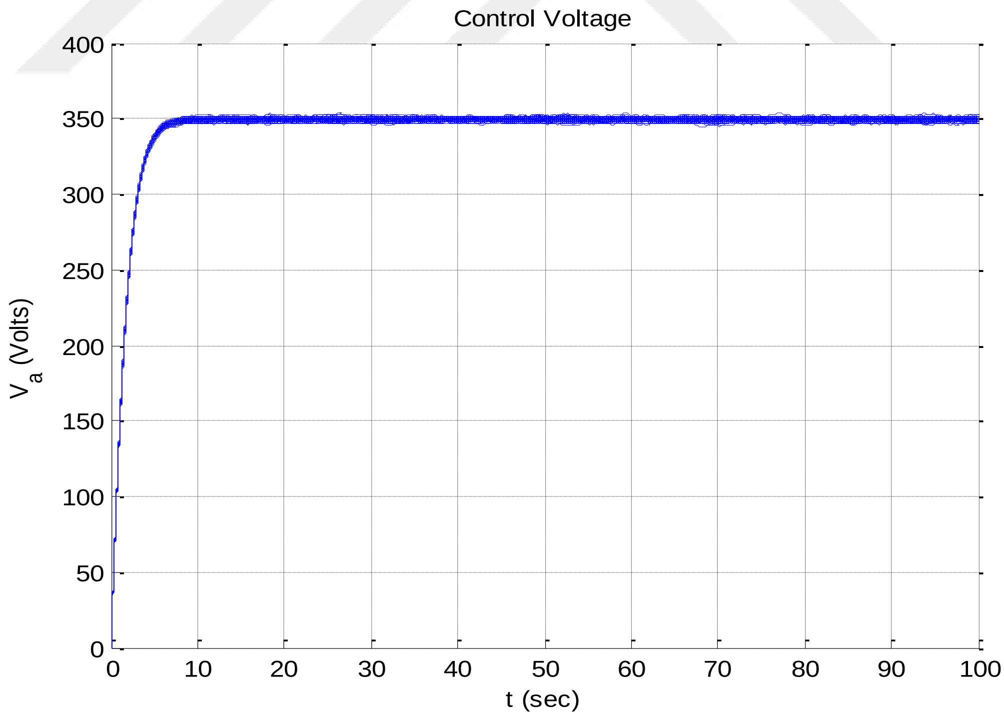


Figure.6.9.Variation of the armature voltage required by the DC Motor parametrized in Table 3.1 under the control law defined by the output feedback gain in (78). The reference speed is ω_d = 2000 deg/sec . In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\mu = 0$ and variance σ^2 = 200 mN. Here the control gain at (80) is generating the control law.

6.2.Position Control

In position control the numerics are mostly the same. Only some of the details on the state equations will differ. First of all, (72) will be replaced by:

$$\begin{bmatrix} \dot{e}_\theta \\ \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -10 & 1 \\ 0 & -0.02 & -2 \end{bmatrix} \begin{bmatrix} e_\theta \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} V_a + \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix} \tau_L \qquad (82)$$

where $e_{\theta=}\ \theta - \theta_d$ with θ_d being the desired/reference position of the DC motor. The measured state variables in the above configuration are e_θ and ω Thus the output feedback mapping matrix is the same as that of (47). In addition, as the system matrices of (82) are numerically the same as that of (72), the controller gains (74), (78), (80), closed loop spectrum (74), (76), (77), (79) and (81) will be same for position control problem provided that the quadratic performance coefficients are same $Q = qI_{3\times3}$ with q = 50 and R = 1.

In Figures (6.10 , 6.12 , 6.13) one will be able to see the position tracking simulation under noise free environment when the control law defined by the gain Ko in (78) is applied as:

$$V_a = -K^o[e_\theta \quad \omega]^T \qquad (83)$$

Using the same configuration that resulted Figures (6.10 ,6.11, 6.12 ,6.13) the simulation in a noisy environment (with disturbance torque) yields the results shown in Figures (6.14 , 6.15 , 6.16 , 6.17) In this case, a disturbance torque is effective on the motor shaft and it is modelled by a Gaussian distributed source with zero mean and variance $\sigma^2 = 0.01\ N \cdot m$. As we have done in the case of speed control, we will present the results of the simulation when the smallest eigenvalue at $\lambda = -0.098538$ is moved to $\lambda = -0.8$ in Figures (6.18, 6.19, 6.20, 6.21).

The examples given in this section are to demonstrate the approaches presented in chapter 4 which is the linear quadratic projective control approach. The purpose is to present the methodology such that, interested readers can replicate the procedure. Thus, given a single reference position or speed (final target position/speed) we presented the simulation results. For testing our controllers under noise due to the disturbance torques we presented repeated trials where the disturbance torque appears as a normally distributed random variable. In each run the disturbance profile will be different due to its randomness so one can reflect those analyses as Monte Carlo methods [36] which rely on repeated samples of random data to obtain the performance of an algorithm when there are parameters with uncertainty. With this view, one can also treat this approach as a robust stability test. Nevertheless, the theoretical stability discussion (Theorem 4) a better approach which is considered a general methodology regardless of the type of the disturbance torques.

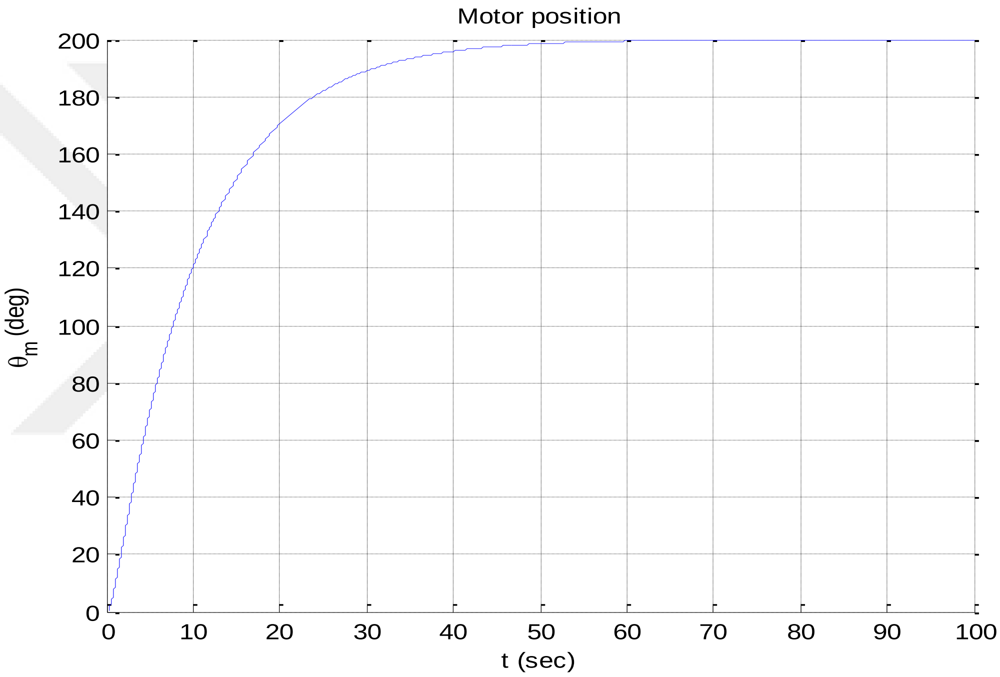


Figure.6.10.Variation of the position of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is $\theta_d = 200°$

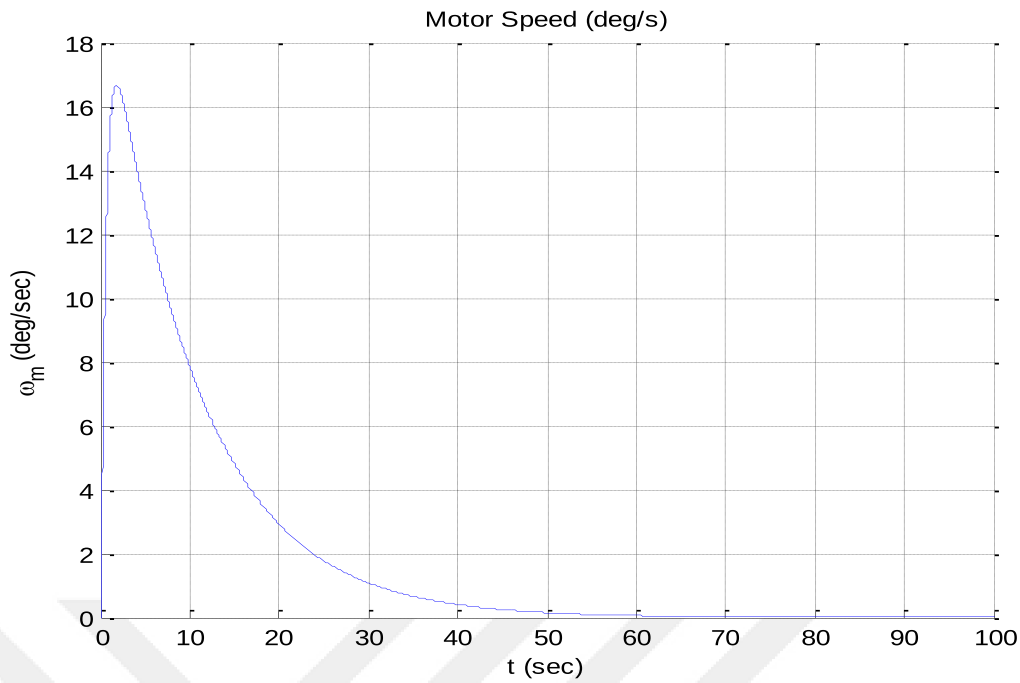


Figure 6.11.Variation of the speed of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is $\boldsymbol{\theta_d}$*=200°*

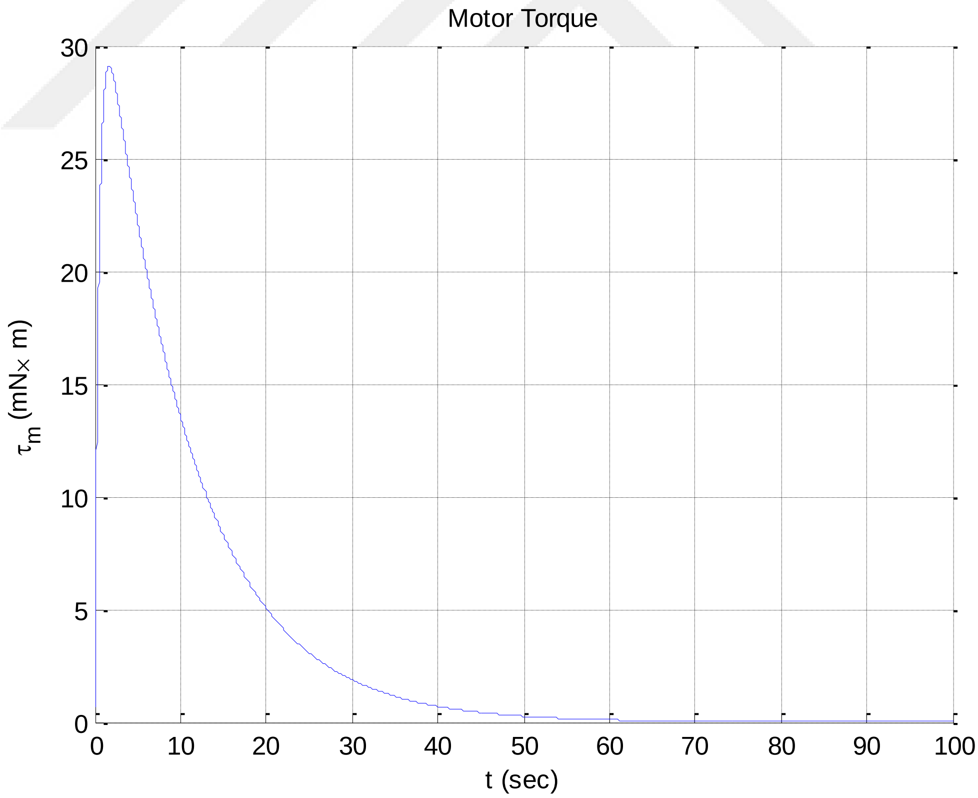


Figure 6.12.Variation of the torque generated by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d *=200°*

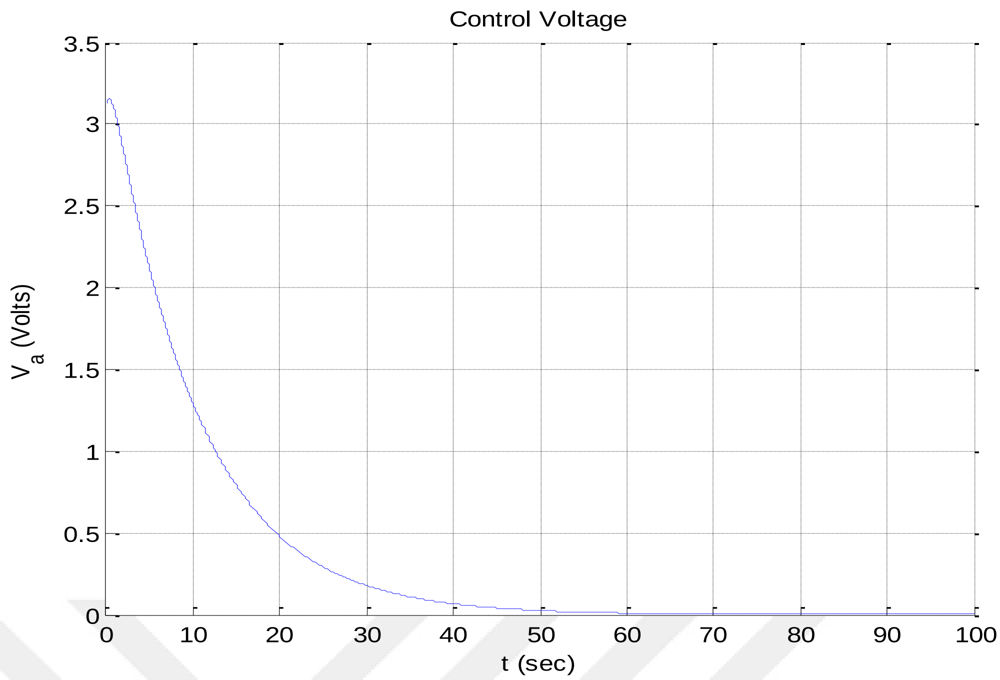


Figure.6.13.Variation of the armature voltage required by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d*=200°*

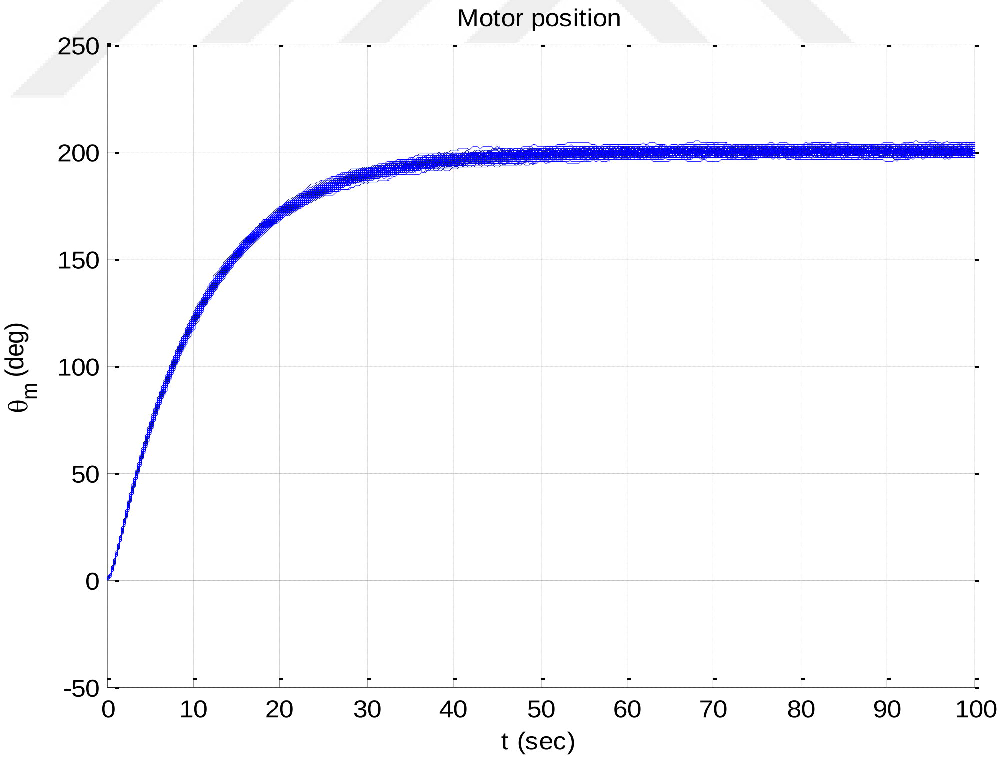


Figure .6.14.Variation of the position of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is $\boldsymbol{\theta_d}$ *=200° Here, the simulation is performed under the applied disturbance torques*

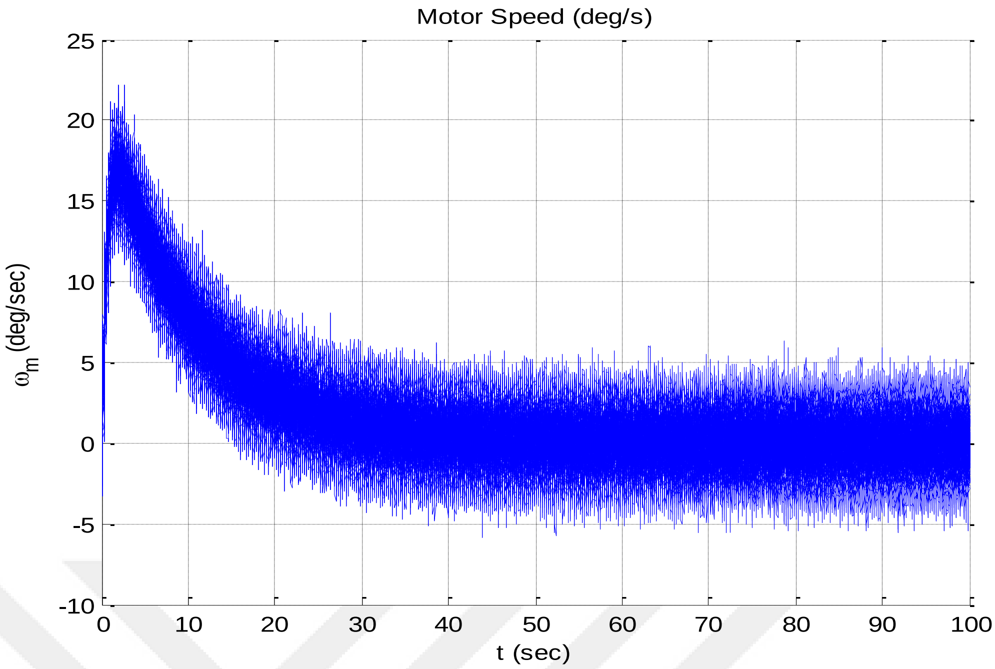


Figure 1.Variation of the speed of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques.

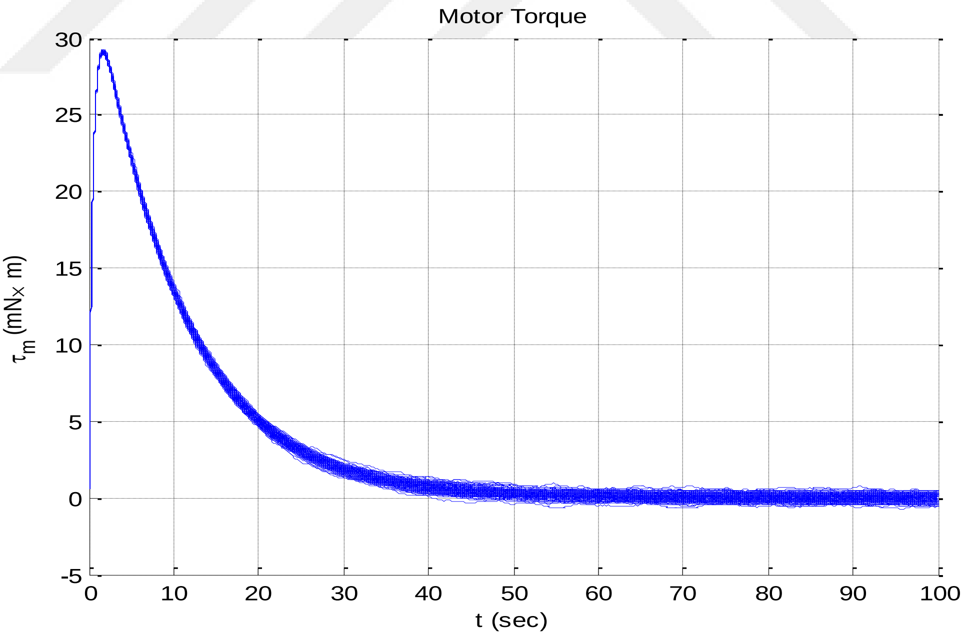


Figure.6.16.Variation of the torque generated by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d =200° Here, the simulation is performed under the applied disturbance torques

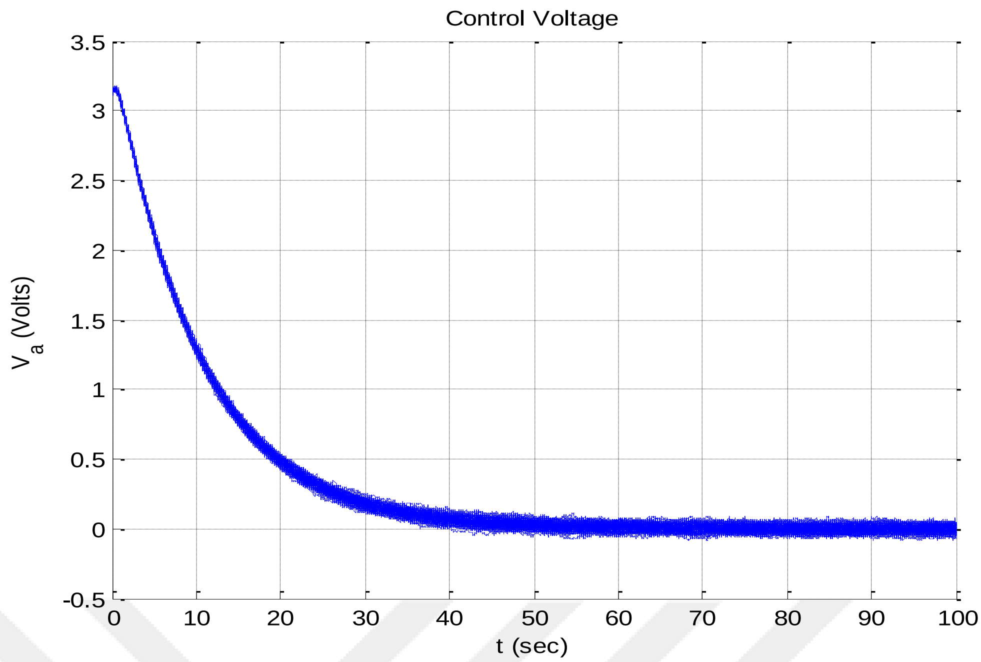


Figure.6.17.Variation of the armature voltage required by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques

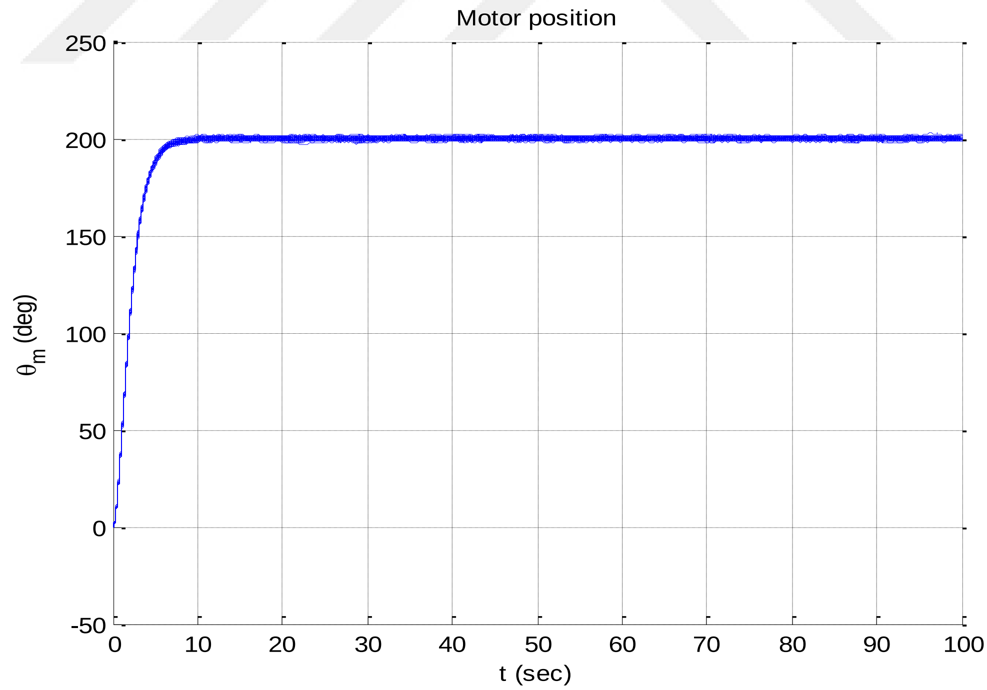


Figure 6.18.Variation of the position of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques. All the poles are satisfying Theorem 4.

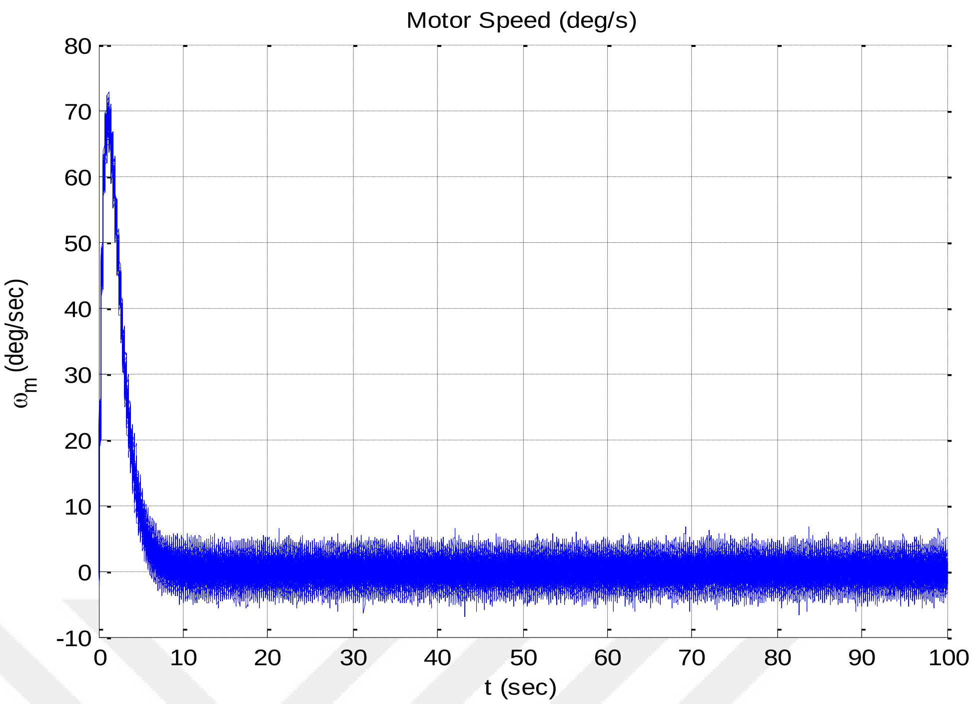


Figure 6.19.Variation of the speed of the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques. All the poles are satisfying Theorem 4.

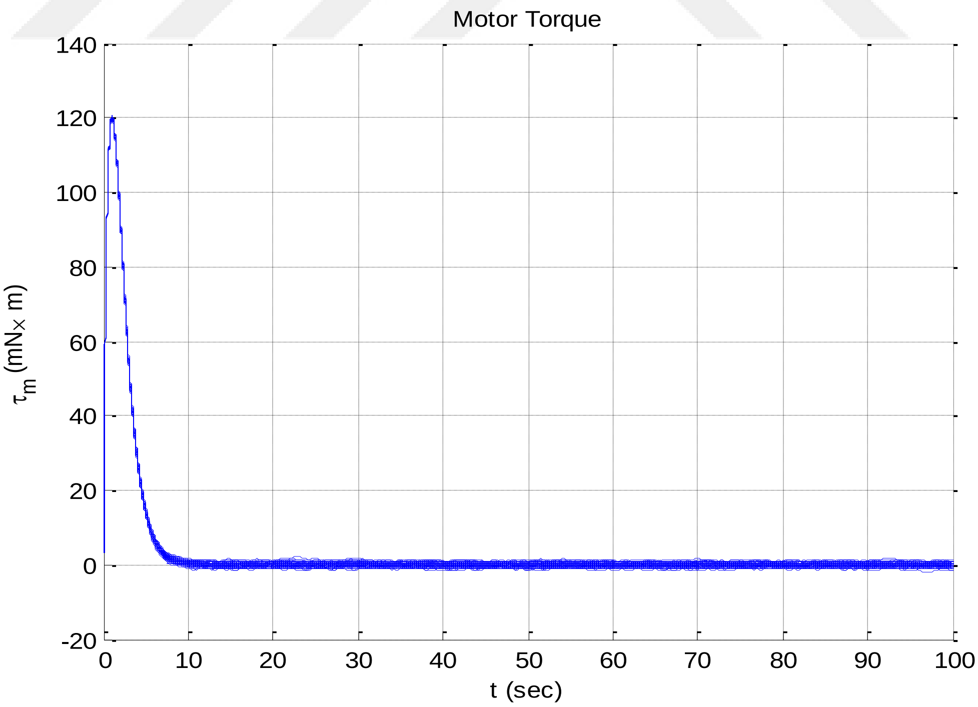


Figure.6.20.Variation of the torque generated by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques. All the poles are satisfying Theorem 4.

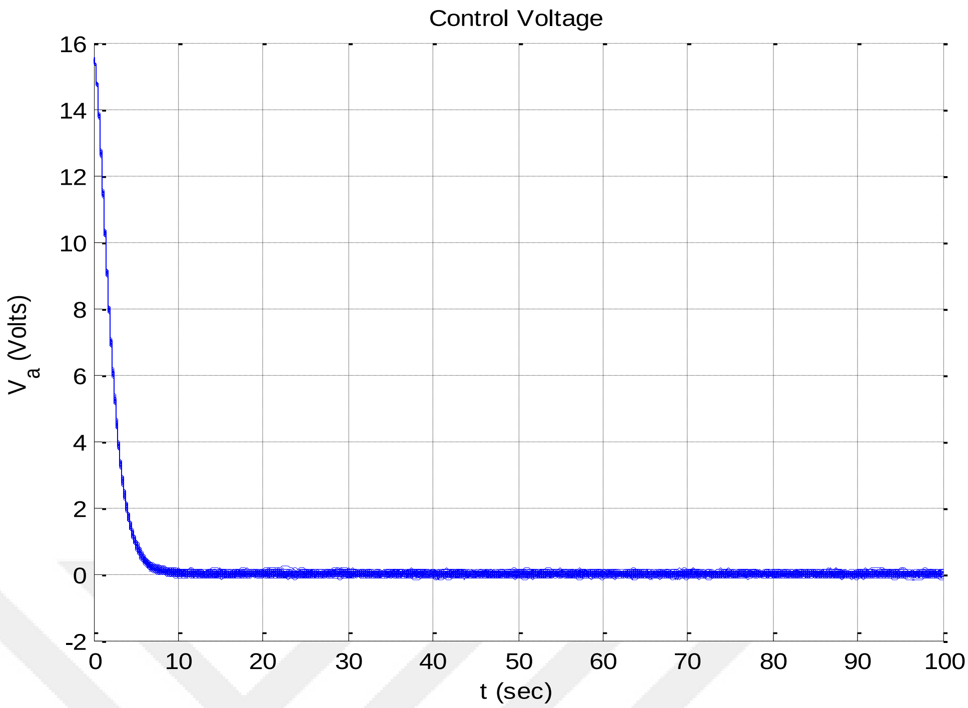


Figure.6.21.Variation of the armature voltage required by the DC Motor parameterized in Table 3.1 under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). The reference position is θ_d=200° Here, the simulation is performed under the applied disturbance torques. All the poles are satisfying Theorem 4.

CHAPTER 7

COMPARE WITH OTHER DESIGN METHODS

In this chapter ,a comparison between different DC motor control algorithms is performed. In this section we don't need to show all of the results , but we will present some of them as an example in order to compare them with the corresponding results from back stepping based study by [41]. Both of control methods was concentrated on speed and position control.

7.1- Speed control.

7.1.1 Comparison of DC motor controllers based on projective and backstepping controllers (no disturbance torque)

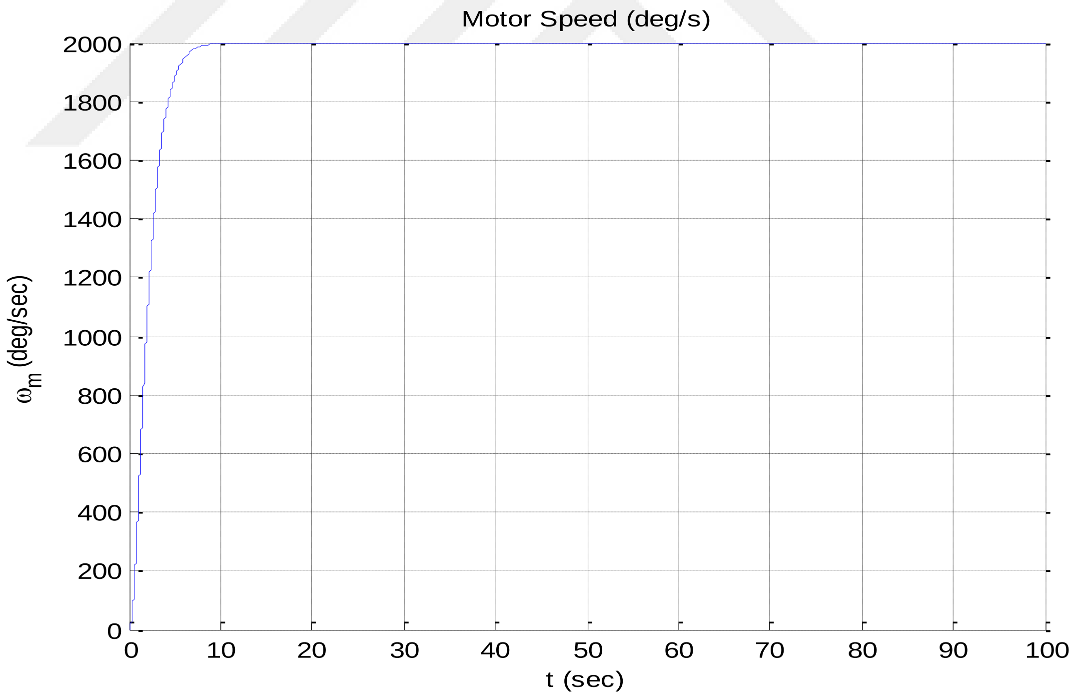


Figure.7.1. Speed variation of the DC Motor under the control law defined by the output feedback gain in (78 by using projective control with $(\tau_L = 0)$*.*

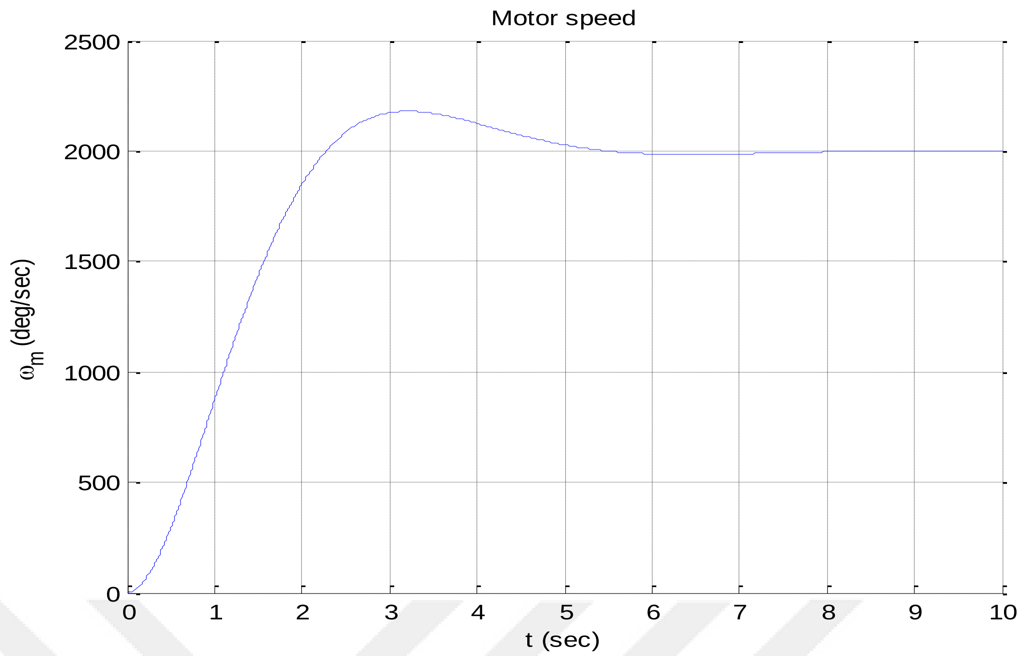


Figure.7.2. Speed control of DC Motor by *using backstepping control where* ($\boldsymbol{\tau_L} = \mathbf{0}$).

In the projective control in figure 7.1, one can see the simulation results obtained when the control law with output feedback gain (78) is applied $K^o = [0.89686 \quad -0.32197]$. without disturbance torque and the reference speed is $\omega_d = 2000$ deg⁄sec.

In backstepping control the reference speed same as in projective control but the output feedback gain $K_i = 1, \ K_\omega = 0.5$

As we see in the speed control figures of backstepping control method show that the control system suffers from problems related to a slight level of overshoot when the control gains have small values as in this case. However projective control based design converges to the desired value without overshot.

7.1.2 Comparison of DC motor controllers based on projective and backstepping controllers (with disturbance torque)

When we have a disturbance torque τ_L effective on the motor shaft , we will note the results presented in figure (7.3) for projective control and in figure (7.4) for backstepping control.

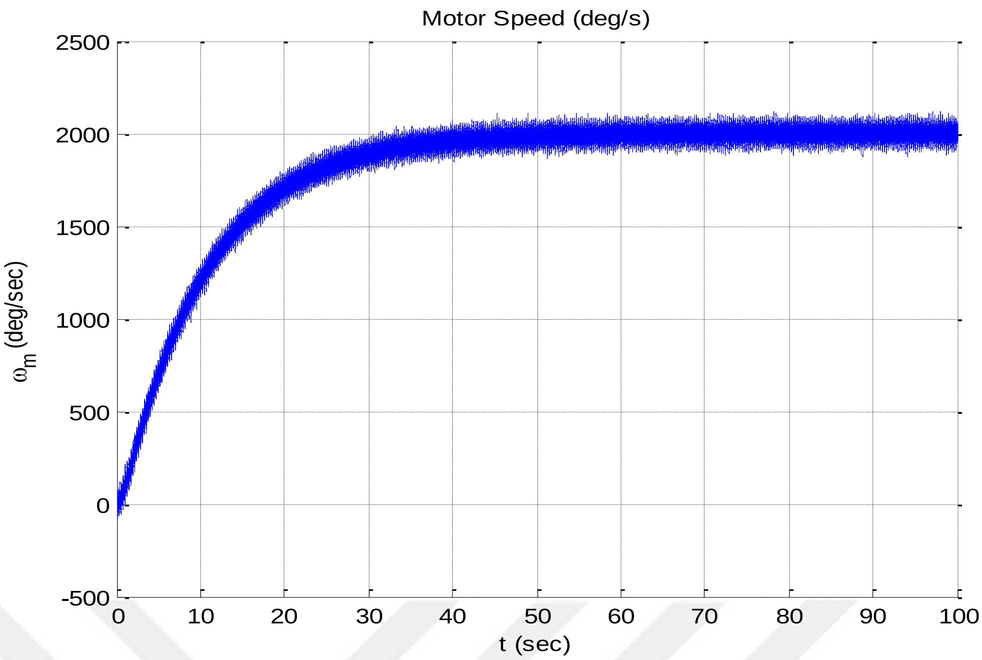


Figure.7.3. Speed variation of the DC Motor under the control law defined by the output feedback gain in (78) by using projective control where $(\tau_L \neq 0)$.

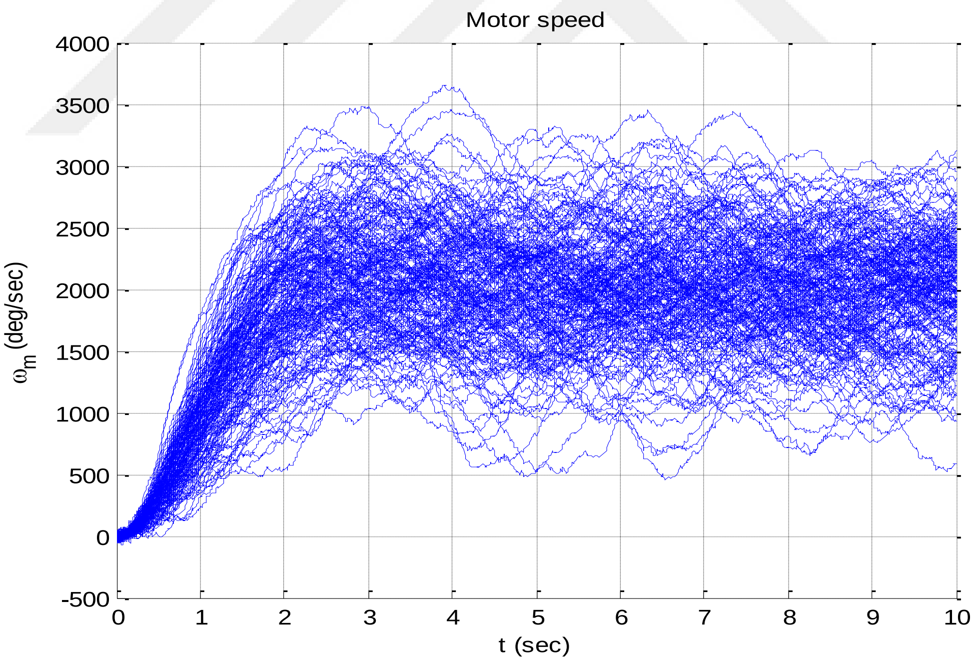


Figure.7.4. . Speed control of DC Motor by *using backstepping control defined by the output feedback gain* $K_i = 1$ *and* $K_\omega = 0.5$ *with disturbance torque* $\tau_L \neq 0$

In this simulation, disturbance torque is present as a Gaussian distributed random variable with mean $\mu = 0$ and variance $\sigma^2 = 200$ mN.m which is the same in both methods.

As you see in the last figure when we have disturbance torque, in projective control the design go to the desired value after 50 seconds and we don't have any overshot.

However, in the design of backstepping control we see a larger variation due to the disturbance torque.

7.1.3 .How to improve the analysis of Speed Controller for both techniques.

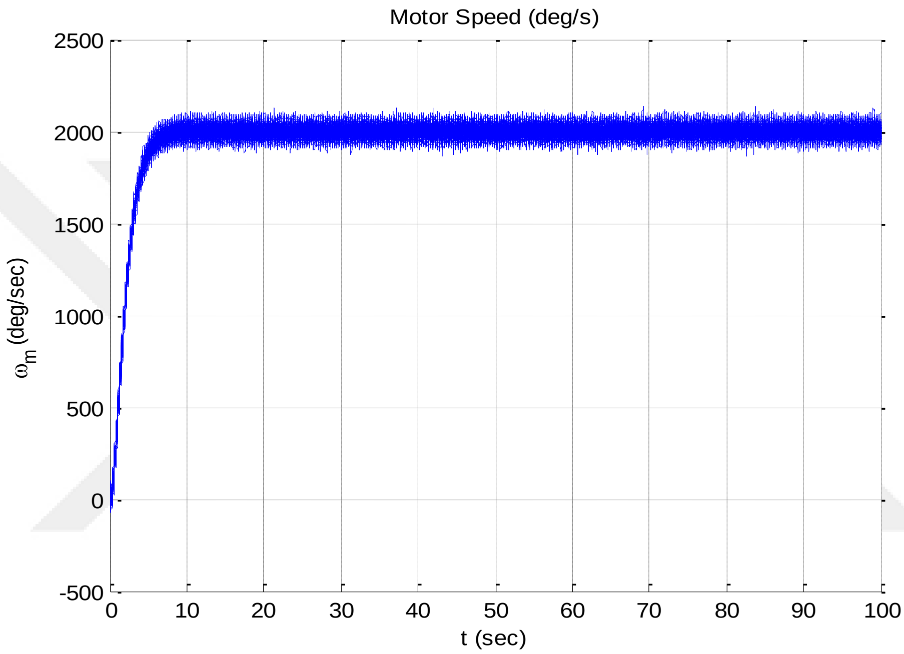


Figure7.5.*Speed variation of the DC Motor under the control law defined by the output feedback gain in (80) by using projective control.*

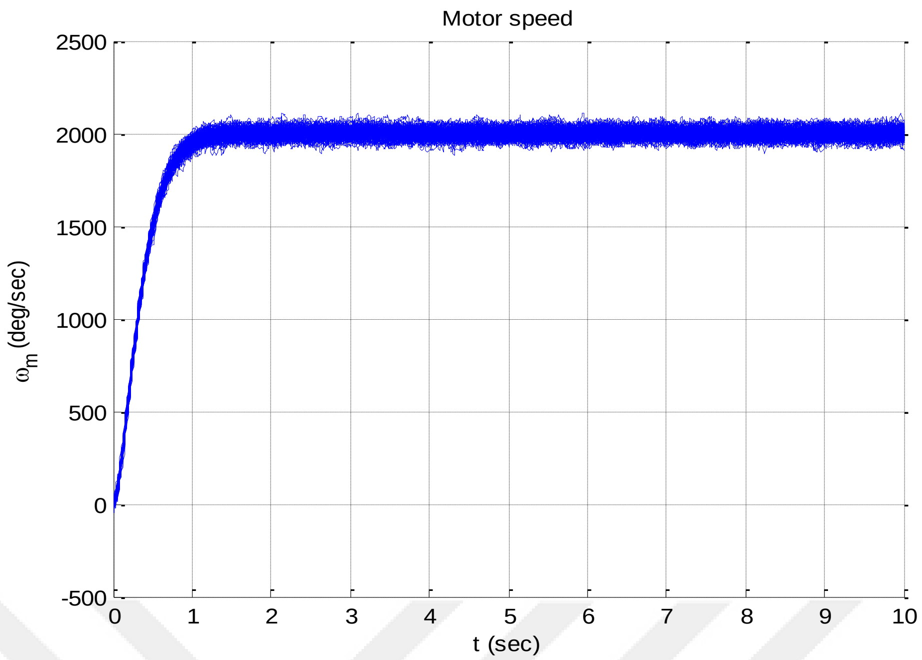


Figure.7.6. . *Speed* control of DC Motor by *using backstepping control defined by the output feedback gain* $K_i = 5$ *and* $K_\omega = 5$.

7.1.4. Findings of the Comparison for speed control

- For projective control Figure number (7.5), when the eigenvalue nearest to j$\boldsymbol{\omega}$ axis this mean the pole shifted to λ =(-0.8) in order to make the pole satisfying the disturbance-to-state stability .

- In figure (7.6) show that there is a significant improvement for speed control when we increase the control gains which became equal to ($K_\omega = 5$ *and* $K_i = 5$) instead of (0.5 and 1) in the backstepping control.

- In speed control backstepping control seems to have a batter noise attenuation, as is easier to tune the controller.

- In projective control this can be done by tuning the in LQR gains, but one may not guarantee a better result due to output feedback established in the projective control and necessity of a interim pole placement.

- Backstepping seems to have an overshoot but it can eliminated by increasing the gains K_i , K_w . In projective control the requires different steps, such as change in the LQR coefficient or change pole location .

7.2- Position control.

7.2.1. Comparison of DC motor controllers based on projective and backstepping controllers (no disturbance torque)

Here, there is no load disturbance torque ($\tau_L = 0$) and The reference position is θ_d=200° for both techniques, Where the control gain for backstepping equal $K_\theta = 0.5, K_\omega = 1, K_i = 2$, while the control gain for projective control equal $K^o = [0.89686 \quad -0.32197]$ which is utilized by $(V_a = -K^o\,[e_\theta \quad \omega]^T$

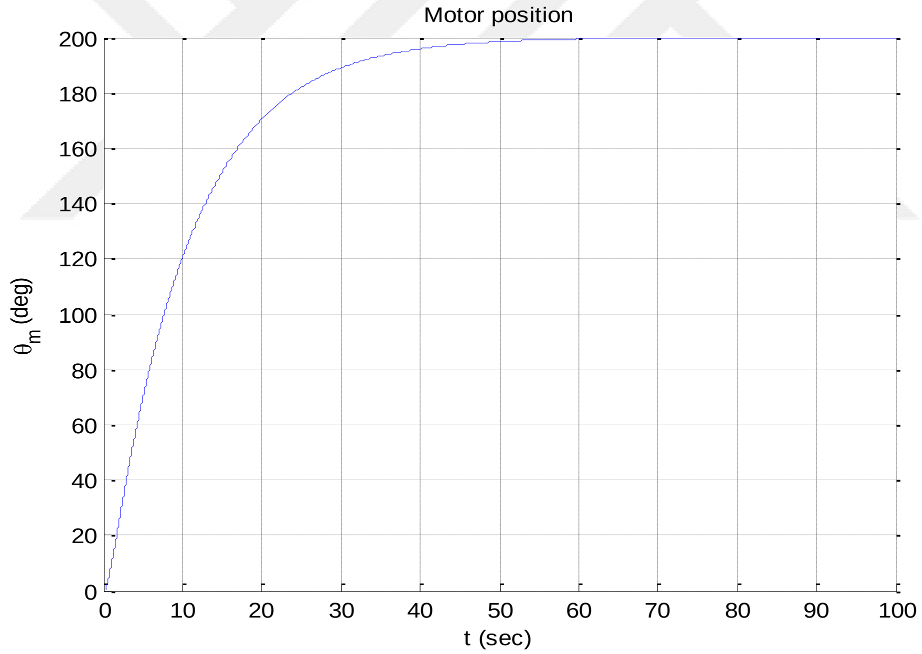


Figure 7.7.Variation of the position of the DC Motor under the position control law defined by the output feedback gain in (78) which is utilized as given in (83).

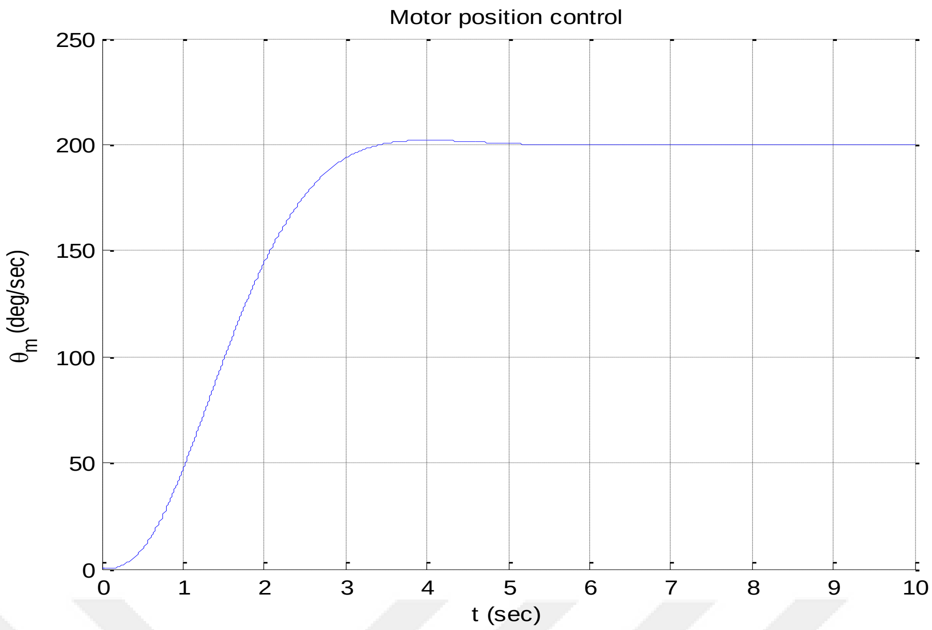


Figure 7.8 : The angular Position of the DC Motor by using backstepping control defined by the output feedback gain, $K_\theta = 0.5, K_\omega = 1, K_i = 2.$

In the cases above, the backstepping controller appeared to be much faster than the application based on the projective control. Though there is a small overshoot in backstepping, the level of that is negligible.

7.2.2. Comparison of DC motor controllers based on projective and backstepping controllers (with disturbance torque)

Here, the simulation is performed under the applied disturbance torques and here the values of standard deviation (σ) should be equal in both which will become (σ = 0.01 N.m), also the number of repeated trials will be equal for both methods ($200 = Ntst$).

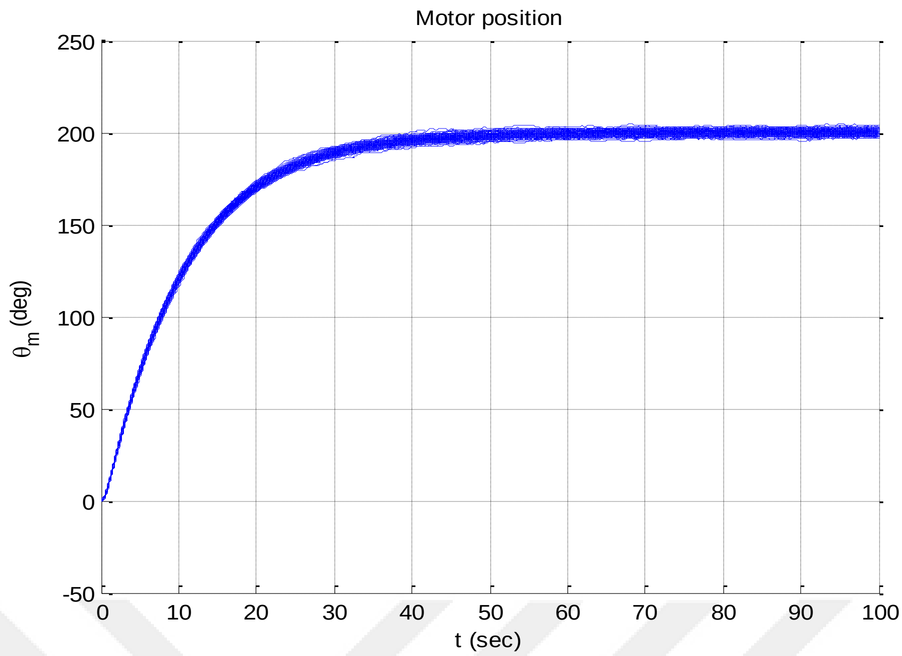


Figure. 7.9. Variation of the position of the DC Motor under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). by using projective control under the position control law where $\tau_L \neq 0$

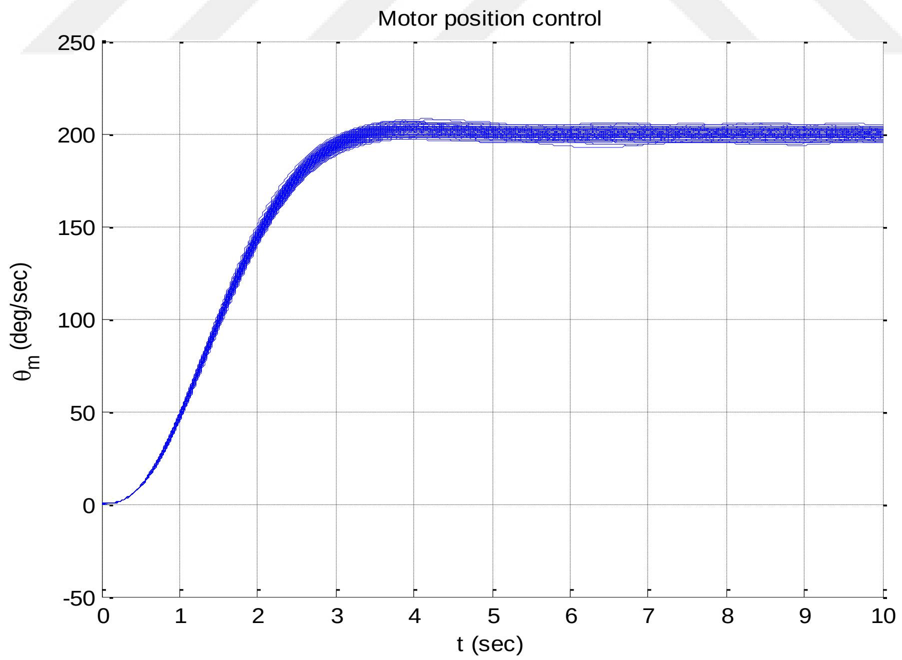


Figure.7.10. The angular Position of the DC Motor by using backstepping control defined by the output feedback gain, $K_\theta = 0.5, K_\omega = 1, K_i = 2$ *where* $\boldsymbol{\tau_L \neq 0}$

7.2.3. How to improve the analysis of Position Controller for both techniques

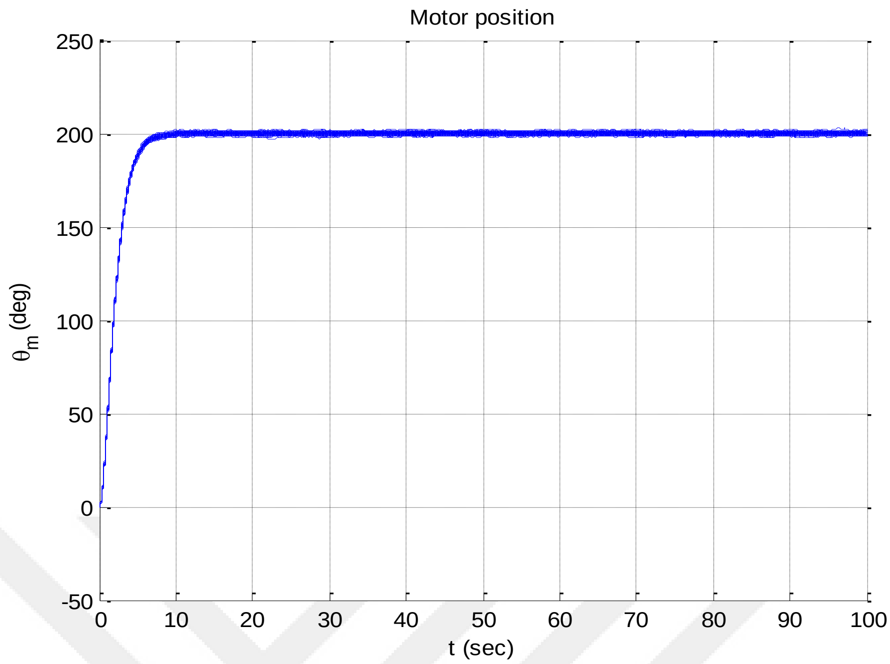


Figure.7.11 Variation of the position of the DC Motor under the position control law defined by the output feedback gain in (78) which is utilized as given in (83). by using projective.

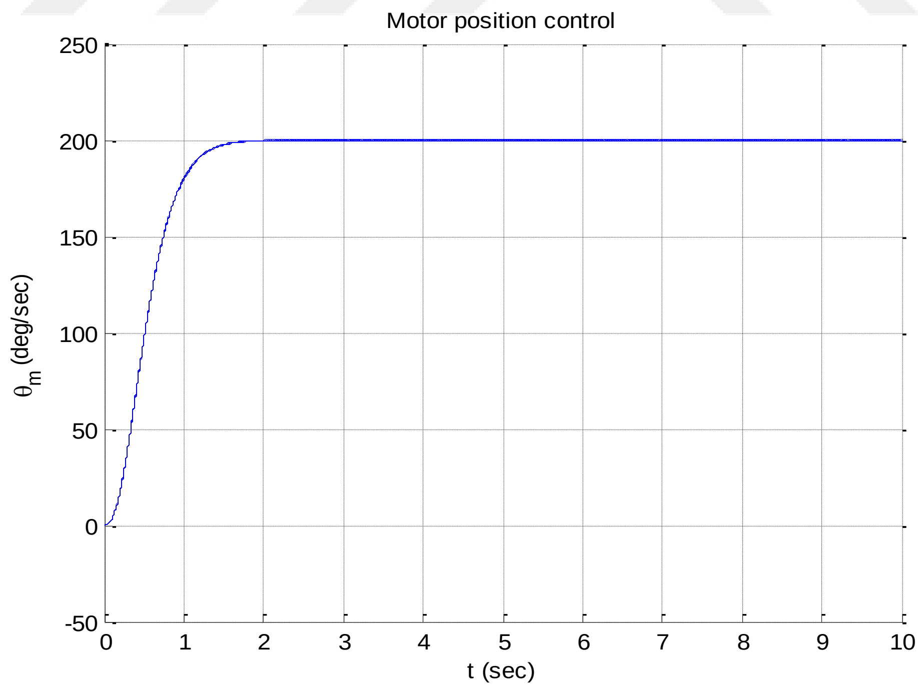


Figure 7.12. The angular Position of DC Motor by Backstepping *Control defined by the output feedback gain* $K_\theta = 5, K_\omega = 5, K_i = 5$

7.2.4. Findings of the comparison for position control

- In position c1ase, the projective control when one apply the input to state stability theory in section (4.4) in addition all the poles are satisfying Theorem 4 in the section (5.3) Disturbance-to-State Stability. We can see major improvement for the disturbance torque and the design go faster to the desired value without any overshot, as shown above in figure(7.11).

- In position control when we also apply the input to state stability theory in the backstepping control we will see development for decreasing the disturbance torque and the Design was able to get rid of overshoot which happened before increasing the gain into (5), as we have shown above in figure(7.12).

- In addition, concerning the disturbance attenuation the backstepping control becomes better than projective control when its gains are increased to $K_\theta = 5,\ K_\omega = 5,\ K_i = 5$.

CHAPTER 8
CONCLUSION

In this work, The design of DC Electrical motor control by using linear projective quadratic control approach has been demonstrated. The projective control technique has been established to be highly effective at designing of the output feedback controllers.

We have chosen these methods because they have proven to be perfect design techniques, and more importantly for the researcher, they demonstrate the insight required to develop control system in real practice. And also the chosen methodology can help the designer to eliminate the feedback from armature current which will increase cost of feedback and increase the possibility of noise accumulation due to its amplification in the signal conditioning circuitry.

The simulations reveal that under both ideal conditions and noisy environment (due to the disturbance torques on the motor shaft), the controller can handle its operation very good and works stable. In addition to simulations, a theoretical discussion on the stability of the closed loop against the disturbance torques that is based on the input-to-state stability concept is given. The theoretical development is fairly conservative as it is developed from the conversion of the equations related to the Lyapunov's second method to inequalities through its manipulation by upper and lower bound lemmas. This result is also seen from the simulations. The designs which does not satisfy the disturbance to state stability theorem (Theorem 4) can handle the noises without going into instability.

However, one cannot simulate all kinds of disturbance torques as disturbances are a particular category of stochastic methods as their name implies. So a design satisfying the disturbance to state stability theorem is expected to guarantee the closed loop's stability against great disturbance torques. In addition to that, in both speed and position controllers the great eigenvalues provide a closed loop with faster response times.

As well the larger gains lead to smaller variations due to disturbance torques in the steady state response of the closed loop.

A future study based on this work can be the application of various linear control methods to the same issue and repeat the theoretical and numerical analysis performed in this task on them.

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