A JOINT POLICY FOR PREVENTIVE MAINTENANCE AND SPARE PART INVENTORY CONTROL

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ABSTRACT

A JOINT POLICY FOR PREVENTIVE MAINTENANCE AND SPARE PART INVENTORY CONTROL

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Machine failures are the primary cause of production disruption in manufacturing systems, leading to significant profit loss. Preventive maintenance activities can adjust the frequency of failures to a certain point. Both failure and preventive maintenance activities are conducted by using spare parts. It is reasonable to stock spare parts in the inventory since procurement lead times of spare parts are common, and economies of scale are available. Maintenance and inventory control are usually planned by different functional units in the industry, although jointly managing them can change the profits of the companies. In this study, we propose a joint preventive maintenance and inventory management policy. The objective is to maximize the expected long-run profit rate, characterized by utilizing Renewal Reward Theory. We also introduce five decentralized decision-making strategies, which are usually applied in practice. Through computational analyses, performances of the proposed joint policy and decentralized decision-making strategies are investigated under different levels of problem parameters.

Keywords: Preventive maintenance, Spare part inventory control, Renewal reward theory

ÖZ

ÖNLEYİCİ BAKIM VE YEDEK PARÇA ENVANTER KONTROLÜ İÇİN BİR POLİTİKA

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Makine arızaları, üretim sistemlerinde üretim kesintisinin başlıca nedenidir ve önemli kar kayıplarına yol açar. Önleyici bakım faaliyetleri, arıza sıklığını belirli bir noktaya kadar ayarlayabilir. Hem arıza hem de önleyici bakım faaliyetleri yedek parça kullanılarak yapılmaktadır. Yedek parçaların tedarik süreleri yaygın olduğundan ve ölçek ekonomileri mevcut olduğundan, stokta yedek parça bulundurmak mantıklıdır. Bakım ve envanter kontrolü genellikle sektördeki farklı fonksiyonel birimler tarafından planlanır, ancak bunları birlikte yönetmek şirketlerin karlarını değiştirebilir. Bu çalışmada, ortak bir önleyici bakım ve envanter yönetimi politikası öneriyoruz. Amaç, Yenileme Ödül Teorisi kullanılarak karakterize edilen beklenen uzun vadeli kar oranını maksimize etmektir. Ayrıca, genellikle pratikte uygulanan beş merkezi olmayan karar verme stratejisini de tanıtıyoruz. Hesaplamalı analizler yoluyla, önerilen ortak politika ve merkezi olmayan karar verme stratejilerinin performansları, farklı problem parametreleri seviyeleri altında araştırılmaktadır.

Anahtar Kelimeler: Önleyici bakım, Yedek parça envanter kontrolü, Yenileme ödül teorisi

To the ones who believed in me

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CHAPTER 1

INTRODUCTION

Production systems rely on maintenance activities to keep the equipment up and running as production equipment may be subject to failure in nature. Maintenance/ repair activities performed upon failure are called corrective maintenance, whereas preventive maintenance is conducted before equipment failure occurs. Preventive maintenance is a planned activity that aims to keep the components in their operating state.

Both preventive and corrective maintenance activities usually require spare parts. So, conducting any type of maintenance that requires component replacement is constrained by the availability of spare parts. Shortages of spare parts cause delays in maintenance activities. Furthermore, production may stop due to shortages when a failure occurs. It is reasonable to stock spare parts in the inventory since procurement of spare parts has a corresponding lead time, and economies of scale are available in the system. So, there is a trade-off between maintenance-related and inventory-related costs. Therefore, decisions on maintenance planning and inventory management must be considered together and optimized jointly.

In this study, we aim to propose a joint policy for preventive maintenance and spare parts provisioning problem that maximizes the expected long-run profit rate of the system. We consider a manufacturing system that generates revenue as long as it is operational. We concentrate on a single critical component that is vital for the system to operate. The critical component is prone to failure. When the critical component fails, the system fails. A malfunctioned component is non-repairable, so it should be replaced with its spare part. Conducting preventive maintenance is cheaper than conducting corrective maintenance since it is a planned activity and prevents adverse effects of failure on the system. Preventive and corrective maintenance can only be done if a spare part is available. For each spare part order, a fixed ordering cost is incurred. Also, procurement cost is incurred for each ordered spare part. Spare parts are stocked in the inventory for upcoming maintenance activities. Inventory holding cost is incurred for each stocked spare part in the inventory.

We propose a joint age-based preventive maintenance and continuous review inventory control policy to address the problem. Age-based preventive maintenance is a threshold policy that components are preventively maintained when their age reaches the predetermined threshold. Spare parts are ordered in batches of the same size, and ordering time is set as a function of the preventive maintenance threshold and the supplier lead time. It is reasonable to give the orders in batches since economies of scale are available in the system. The expected long-run profit rate under our proposed policy is characterized by utilizing Renewal Reward Theory. The objective is to maximize the expected long-run profit rate. The decisions are age-based preventive maintenance threshold and order quantity of spare parts. Although we characterize our proposed policy's objective function, its calculation is computationally hard. Therefore, we introduce two approximations.

Furthermore, preventive maintenance and spare parts inventory management are usually planned by different functional units in the industry. These decentralized decision-making strategies are called sequential approaches. They decide on preventive maintenance threshold by using maintenance-related costs and then decide order quantity using the predetermined preventive maintenance threshold value and inventory-related costs. In our study, five sequential approaches are presented. These decentralized decision-making strategies in our study differ from each other in terms of considered parameters in their objective function. We want to show that sequential approaches can be improved by considering all of the system parameters.

To the best of our knowledge, the objective function of a given policy for any age-based preventive maintenance and continuous review inventory control policy is never characterized before, when there is supplier lead time and emergency orders are not allowed. It is hard to characterize the spare part demand during the supplier lead time when emergency orders are not allowed. However, there may be cases in which emergency orders are not an option under supplier lead time. So, we propose a joint policy that the objection function can be analytically characterized, easy to apply, and appropriate for comparison with different decentralized approaches. Furthermore, decentralized decision-making strategies (sequentially optimized decision variables) are employed for comparing our joint policy. One of our aims is to clarify the benefits of utilizing a joint approach instead of a sequential one, as in the literature. Additionally, we also want to investigate the possible improvements that can be made within the scope of sequentially optimized decision variables. Although the decisions on maintenance and inventory control are sequentially made, one can utilize different objective functions for sequential approaches by using different parameters.

Although we characterize the long-run average profit rate under our proposed policy, its calculation is complicated. So, we provide two approximations to handle this issue. Also, we want to show that, while the decision on order size is made, besides using inventory-related costs, using the rest of the parameters may increase the company's profits. We want to provide managerial insights to the firms that using a joint approach is not an option under their environment.

The rest of the study is organized as follows. In Chapter 2, we give an overview of maintenance and inventory policies, studies focusing on joint maintenance and inventory control problem in the literature and contributions of this study. In Chapter 3, we describe the problem environment and introduce our proposed joint policy for maintenance and inventory control problem with its analytical derivations under different settings. Furthermore, two approximations of the proposed policy and five decentralized decision-making strategies are presented. In Chapter 4, we carry out computational experiments. We provide a simulation optimization procedure of our proposed policy. The comparison of approximations and sequential approaches is made, and we present our findings. In Chapter 5, we conclude the study by providing a summary of our findings and suggesting future research directions.

CHAPTER 2

LITERATURE REVIEW

Preventive maintenance is the set of scheduled activities implemented on equipment to keep them in working condition and prevent costly unplanned downtime due to equipment breakdowns. Conducting maintenance activities that require component replacements is constrained by the availability of the associated component's spare parts. Therefore, as long as the component is in working condition, spare part demand occurs due to preventive and corrective maintenance. Maintenance activities are delayed in the unavailability of spare parts. Delayed maintenances for failed components possibly cause additional costs due to disruption of production.

Spare parts are usually procured from outside suppliers, possibly by a lead time. In addition, there are economies of scale in the ordering. These all favor keeping spare parts inventory. Stocking high inventory levels of spare parts decreases the proportion of delayed maintenance activities on one side, and on the other side causes inventory holding cost. The trade-off between maintenance-related and inventory-related costs is pointed out in the literature ([1], [2]). Hence, decisions on maintenance activities and inventory management of associated spare parts should be made jointly.

In this study, we focus on the joint problem of preventive maintenance and spare part inventory control which is first studied by Falkner [3]. In the rest of this chapter, we introduce maintenance and inventory policies used in the joint maintenance and inventory control literature. Then, studies that focus on the joint problem of preventive maintenance and spare part inventory control are reviewed. We restrict our review to the studies which consider independently working critical components that are prone to failure. The critical components' lifetimes are independent and identically distributed. Also, the critical parts are non-repairable in all of the reviewed studies. Our contributions to the existing literature are presented at the end of this chapter.

Van Horenbeek et al. [4] provides a review of studies on joint maintenance and inventory optimization. The most frequently used maintenance policies are:

- Age-based preventive maintenance: Spare part is replaced either at failure time or when the age of the component reaches a predetermined threshold [5].
- Block replacement: Maintenance is conducted periodically at prearranged points in time. This policy is not concerned with the failure history of the component.
- Group maintenance: In a multi-component environment, due to dependencies between the components, maintenance activity is conducted for all components at a fixed time [6].
- Condition-based maintenance: It is a predictive strategy in which the component's state is monitored. Maintenance is conducted when the state of the component reaches a predetermined threshold level.

Inventory control policies can be examined under two categories, which are continuous and periodic review inventory control policies. The inventory level is continuously monitored in the continuous review policies, and order is placed when an inventory level is reached. Often used continuous review policies are:

- (s, S) policy: Whenever the inventory level becomes s or below, an order of size $S - s$ is placed.
- (s, Q) policy: Whenever the inventory level becomes s or below, an order of size Q is placed.

Note that (s, S) and (s, Q) are identical when $S - s = Q$ and demand is a unit.

In the periodic review policies, the inventory level is periodically monitored, and an order is placed when a predetermined inventory position is reached. The most commonly used continuous review policies are:

- (R, s, S) policy: In every R time period, the inventory position is checked. If the inventory level is at s or below, an order of size $S - s$ is placed.

- (R, S) policy: In this policy, an order is placed every R period to raise the inventory position to S.

- (R, Q) policy: In every R time period, an order of size Q is placed.

Next, we review the studies which are most aligned with our research direction.

Acharya et al. [1] consider a joint maintenance and inventory provisioning problem in a multi-component environment. The components are independent and identical. The probability of a single component failure increases with its age. They utilize block replacement as the preventive maintenance policy. For inventory replenishment, they consider an (R, S) policy where the orders can be only placed at the preventive maintenance times of the components. Therefore, periodic review interval of the inventory becomes a decision variable for their policy. They consider both single-period and multi-period models. In the single-period model, spare parts are procured at every maintenance point, whereas in the multi-period model, an order is placed once in every $k > 1$ maintenance point. The objective of the models is to minimize the long-run expected total cost rate. The considered cost terms in the study are inventory holding, fixed ordering, backordering, corrective, and preventive maintenance costs. They derive their objective function and use a search algorithm for finding the optimal decision variables. For the single-period model, they compare two cases; (i) only the maintenance-related costs are considered, (ii), in addition to maintenance-related costs, inventory-related costs are considered. It is shown that block replacement interval is much larger when only maintenance-related costs are considered in the single-period model. Results show that optimal replacement interval is much shorter under multi-period setting than under single-period setting. Also, the objective function is improved in the multi-period setting.

Armstrong and Atkins [7] study joint optimization of maintenance and inventory management problem for a single component setting. The failure rate of the component increases in its age. They employ age-based replacement as the preventive maintenance policy. In the study, periodic review is considered in which review interval is a decision variable of the problem. Only a single spare part can be ordered at review points if the inventory is out of stock and there is no outstanding order. The supplier lead time is constant. The objective is to minimize the long-run expected total cost rate. The considered cost terms are inventory holding, shortage, corrective maintenance, and preventive maintenance costs. They employ an analytic approach and show that the cost function is pseudoconvex. They compare the results of the joint approach with sequentially optimized decision variables. Results show that the joint approach outperforms the sequentially optimized decision variables. Armstrong and Atkins [8] extend their study by adding age-dependent operating and maintenance cost as well as a constant lead time for the maintenance action. Also, they introduce emergency orders. In addition, they propose service constraints on fill rate and waiting time and characterize their objective function.

Kabir and Al-Olayan [9] consider a joint optimization of age-based maintenance and inventory provisioning policy for a single component. They use the (s, S) policy for inventory management. Supplier lead time and lifetime of the component are random and follow Weibull Distribution in their setting. Emergency orders are not allowed. The objective is to find optimal preventive maintenance age of the component, reorder and order up to inventory level by minimizing total expected cost in a planning horizon. Simulation optimization is employed as a solution approach. Forty simulation runs are taken for each problem instance. Common random numbers are used in simulation runs to draw consistent conclusions between alternative policy parameters. They compare their joint policy with the Barlow and Proschan [10] policy that policy parameters are optimized sequentially. Joint policy performs better than sequential policy almost in all problem instances. In their follow-up work, Kabir and Al-Olayan [11] extend their study by considering a multi-component environment. They reach the same conclusions regarding the joint approach and sequential approach. The same environment excluding emergency orders is studied by Kabir and Farrash [12] under the same policy. The simulation is used for the solution approach. Similar to previous work, the joint policy is compared with sequential policy. The joint approach is still superior to the sequential approach when emergency orders are not an option. Kabir and Farrash [13] study the same environment considered by Kabir and Farrash [12]. Instead of continuous inventory review, they consider periodic inventory review. They use age-based preventive maintenance, and a periodic (R, Q) policy, review interval R is a parameter of the problem, and Q is the decision variable representing the order quantity. They compare their proposed periodic review policy with Kabir and Farrash's continuous review policy. Periodic review policy performs worse than continuous review policy when levels of the shape parameter of Weibull distribution and the number of components in the system are low. An increase in both of these parameters results in a better performing periodic review policy than the continuous review policy. Although this result seems counter-intuitive at first glance, recall that the (s, Q) policy can only reorder at inventory level changes. Whereas the (R, Q) policy places the orders independently from the inventory level. In reality, orders can be placed anytime, independent of the inventory position. One can modify the (s, Q) policy by introducing postponement time, p, for orders as follows. In a (s, Q, p) policy, whenever inventory position becomes s, wait for p times and place the order of size Q. So, suppose the optimal value of postponement time is not zero for a given problem instance. In that case, there is a possibility that the (R, Q) policy performs better than the (s, Q) policy.

Hu et al. [14], extend Kabir and Al-Olayan [11]'s study by not allowing emergency orders. Therefore, instead of emergency ordering cost, shortage cost is added to the model. The same inventory control policy is considered. When the inventory runs out of stock and the preventive maintenance threshold is reached for the component, maintenance activity is delayed until the order is received. Since emergency orders are not allowed, it is reasonable to keep the component in working condition until failure occurs if no spares are available for preventive maintenance activity. For numerical analysis, they use Kabir and Al-Olayan [11]'s dataset. Comparison is made between the proposed policy and policy reported by Kabir and Al-Olayan [11]'s study. Hu et al. [14]'s policy performs better than Kabir and Al-Olayan [11]'s policy in almost all problem instances.

Sarker and Haque [15] study preventive maintenance and spare provisioning problem in a multi-component environment. They consider a manufacturing system that machines are used in production lines. They utilize block replacement as maintenance policy, and (s, S) policy is used for inventory control. They assume random lifetime and supplier lead times which follow Weibull Distribution. Emergency ordering is allowed with a cost that is three times of regular ordering cost. Also, they consider random replacement times for maintenance activities. Simulation is used for optimizing policy variables that minimizing total cost in a finite planning horizon. They consider cost and statistical parameters, which are used by Kabir and Al-Olayan [11]. They compare their joint policy with a sequentially optimized block replacement policy and (s, S) policy. In all problem instances, the joint approach yields better results than the sequential one.

Mardin and Dekker [16] modify Sarker and Haque [15]'s policy by considering the separate spare part ordering for corrective replacement and block replacement. They used simulation optimization as the solution approach. Numerical analysis is conducted for five different policies, which are only conducting corrective replacement with (s, S) inventory control policy, sequential block replacement with (s, S) inventory control policy, Sarker and Haque's policy, Mardin and Dekker [16]'s policy, and age-based maintenance with (s, S) inventory control policy. Results show that the proposed policy and age-based policy are superior to the rest. Proposed policy performs worse than age-based policy in most of the problem instances. The exceptional problem instances that proposed policy perform better than age-based policy have longer supplier lead time, high shortage cost, and high ordering cost.

Brezavscek and Hudoklin [2] consider a block replacement and periodic review of inventory provisioning policy in a multi-component environment. Replenishment cycle length, T, and order up-to level, S, are the decision variables of the policy. Supplier lead time, τ, is deterministic and assumed to be shorter than the replenishment cycle length. Therefore, there is at most one outstanding order at any time. Reorder point is a function of replenishment cycle length and supplier lead time. After $T - \tau$ units of time in a replenishment cycle, an order is placed. Block replacement is conducted immediately at the time of order arrival. They approach the problem analytically by deriving the objective function under their proposed policy by utilizing the renewal process. The objective is to minimize the long-run expected total cost rate. A comparison is made between the joint approach and sequential approach. An average of 13% of cost savings is obtained by utilizing the joint approach instead of the sequential approach.

Huang et al. [17] extend Brezavscek and Hudoklin [2]'s study by considering random supplier lead times. The maintenance policy remains the same except when there are not enough spares in the inventory for conducting preventive maintenance to all components, then block replacement time is postponed to a time when enough spare parts become available. Also, introducing random supplier lead time causes replenishment cycle length to be random. Their numerical analysis shows that optimality conditions are hard to find even when the order up-to level is the only decision variable.

Bulbul et al. [18] study joint optimization of preventive replacement and spare parts inventory planning in a manufacturing system that consists of multiple identical and independently working machines. The failure probability of the critical part increases in its age. There is no supplier lead time. They do not impose any maintenance or inventory policy; instead, they propose a dynamic programming formulation for the exact solution. The objective is to minimize total cost in a finite planning horizon. They state that exact dynamic programming formulation provides the optimal policy in state and stage-dependent. Therefore, they introduce three heuristic approaches which are Steady-State Approximation, which approximates the finite horizon problem to an infinite horizon problem for a single unit, Stationary Policy, which considers an age-based preventive maintenance policy, and a (R, S) inventory control policy and Myopic Approach which considers only the state variables (does not consider stage). The numerical analysis shows that, generally, the Myopic Approach performs worse than the other two approaches. Stationary Policy performs better than rest when the number of machines is relatively large. In the opposite case, when the number of machines is relatively small, Steady State Approximation performs better than the rest.

Panagiotidou [19] study joint optimization of spare parts ordering and age-based preventive replacement problem for a multi-component environment. The components' lifetimes follow Weibull Distribution. Preventive maintenance is conducted when the age of the component reaches the preventive maintenance threshold. Continuous review (s, S) is used as the inventory management policy. Corrective maintenance, preventive maintenance, fixed ordering, shortage, and inventory holding costs are considered. The objective is to minimize expected long-run costs by optimizing preventive maintenance threshold, reorder level, and order up-to level of spare parts. The problem is approached analytically by utilizing Renewal Reward Theorem. The objective function is derived under the proposed policy. Instead of observing every age of component, inter-replacement time distribution is derived. After any maintenance point of any component, it is known that a component is as good as new. The remaining ages of the components are considered distributed randomly according to the preventive maintenance threshold and probability density function of time to failure of a component in the long run. The optimal decision variables are found through a search algorithm. In the numerical analysis, the proposed policy is compared with sequential optimization of decision variables. It is shown that the proposed policy is superior to the sequential approach. Also, exponential approximation of replacement time is studied. The exponential approximation of replacement time is costly, especially in problem instances, including a small number of components and short lead times.

The reviewed studies in the literature that are most relevant to our research scope are summarized in Table 2.1.

Table 2.1: Summary of Reviewed Studies

Study	# of Components	PM Policy	IC Review	Supplier Lead Time	Emergency Orders	Approach
Acharya et al. [1]	Multi-unit	Block	Periodic	No	No	Analytic
Armstrong and Atkins [7]	Single-unit	Age-based	Periodic	Constant	No	Analytic
Armstrong and Atkins [8]	Single-unit	Age-based	Periodic	Random	No	Analytic
Brezavscek and Hudoklin [2]	Multi-unit	Block	Periodic	Constant	No	Analytic
Bulbul et al. [18]	Multi-unit	-	Periodic	No	No	Analytic
Hu et al. [14]	Multi-unit	Age-based	Continuous	Random	No	Simulation
Huang et al. [17]	Multi-unit	Block	Periodic	Random	No	Simulation
Kabir and Al-Olayan [9]	Single-unit	Age-based	Continuous	Random	Yes	Simulation
Kabir and Al-Olayan [11]	Multi-unit	Age-based	Continuous	Random	Yes	Simulation
Kabir and Farrash [12]	Multi-unit	Age-based	Continuous	Random	No	Simulation
Kabir and Farrash [13]	Multi-unit	Age-based	Periodic	Random	No	Simulation
Mardin and Dekker [16]	Multi-unit	Block	Continuous	Constant	Yes	Simulation
Panagiotidou [19]	Multi-unit	Age-based	Continuous	Constant	Yes	Analytic
Sarker and Haque [15]	Multi-unit	Block	Continuous	Random	Yes	Simulation
Current Study	Single-unit	Age-based	Continuous	Constant	No	Analytic/ Simulation

We study the joint preventive maintenance and spare parts inventory control problem in a production system that is continuously monitored. Revenue is earned as long as the system is operational. The functionality of the system depends on a single critical component. The critical component is prone to failure. Corrective maintenance should be conducted upon failure. Maintenance activities use spare parts. Spare parts are outsourced and may be held in the inventory for the smoothness of the manufacturing process. Emergency orders are not allowed. Considered cost terms are fixed ordering, unit procurement, inventory holding, maintenance, and failure costs.

The objective is to maximize the expected long-run profit rate since the considered system generates revenue as long as it operates. We propose a joint age-based preventive maintenance and continuous review inventory control policy. We characterize the objective function under our proposed policy. Two approximations are used since the proposed policy requires complicated computation. We compare our joint policy with five different sequential approaches. Those five sequential policy differs from each other by their objective function for order quantity decision.

To the best of our knowledge, the studies that propose a joint age-based preventive maintenance and continuous review inventory control policy that focus on an environment where there is supplier lead time and emergency orders are not allowed never provide the derivation of their objective function under their proposed policy. We want to fill the gap in the literature by proposing a joint age-based preventive maintenance and continuous review inventory policy that can be analytically characterized, easy to understand, and appropriate for comparison with the sequential approaches. In addition, as in the studies in the literature, we compare the results of our joint policy with the sequential approaches to show the critical problem parameters that yield significant profit gaps between joint and sequential approaches. Furthermore, we compare sequential approaches in themselves to provide managerial insights to the decision-makers in the different functional units of the same organization where a joint approach is not an option. In the literature, sequential approaches consider maintenance-related costs while deciding preventive maintenance threshold and inventory-related costs while deciding order size. We characterize the objective functions of ordering decisions of sequential approaches by using inventory-related costs and using some of the parameters defined in the problem.

CHAPTER 3

PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATIONS

This chapter discusses the preventive maintenance and spare parts inventory planning problem. In Section 3.1, we describe the problem environment. Then, we explain our proposed policy in Section 3.2. Special cases of the proposed policy are discussed in Section 3.3 and 3.4. Section 3.5 presents two approximation methods for our proposed policy. Later, to measure the inefficiency of sequential optimization compared to joint optimization, we introduce five sequential optimization methods in Section 3.6.

3.1 Problem Environment

We consider a joint preventive maintenance and spare parts inventory planning problem in a manufacturing system. The system generates a revenue of r (see Table 3.1) per unit time when it is operational. We concentrate on a continuously monitored critical component that is vital for the system to operate. Let X be a non-negative random variable representing the lifetime of the critical component with a known probability density function, $f(x)$. The critical component is prone to failure, which is attributed to wear-outs. When the critical component fails, the system fails. A malfunctioned component is non-repairable, so it should be replaced with its spare part. *"Corrective maintenance"*, is conducted upon failure of the critical component, with a cost of C_c. The system is assumed to be as good as new after the replacement of the critical part. The critical component can also be replaced before it fails. This operation is known as *"preventive maintenance"* and can be conducted at a cost of C_p when the system is operating. The replacement time is negligible for both corrective and preventive maintenance. Preventive maintenance is less costly than corrective maintenance since it is a scheduled activity and prevents possible adverse effects of a failure to the system. All of these aspects of the replacement activities are indirectly covered by C_p and C_c.

Preventive and corrective maintenances can only be done if a spare part is available for the critical component. Spare parts are ordered from an outside supplier. The supplier's lead time is constant and L time units. When an order is placed, a fixed ordering cost of C_k and a unit procurement cost of C_u for each ordered unit is incurred. At the time of maintenance activity, if there are no spare parts in the inventory, the system has to wait until a spare part becomes available. Emergency ordering is not possible. An inventory holding cost of C_h per unit per time is charged for the spare parts in the inventory.

In this environment, we propose a preventive maintenance and spare part inventory policy that maximizes the long-run average profit rate. The parameters used throughout the study is summarized in Table 3.1.

Table 3.1: Notation

Parameters	
X	Random variable representing the lifetime of a critical part.
X_i	Random variable representing i^{th} installed critical part's lifetime.
$f(x)$	Probability density function (pdf) of X.
$F(x)$	Cumulative distribution function (cdf) of X.
L	Supplier lead time.
r	Revenue per unit time obtained when the system is in operating condition.
C_c	Unit corrective maintenance cost.
C_p	Unit preventive maintenance cost.
C_h	Unit inventory holding cost per unit time.
C_u	Unit procurement cost.
C_k	Fixed cost of ordering.

3.2 Proposed Policy

We employ an age-based preventive maintenance approach since the critical component has an increasing failure rate. In this approach, the critical component is replaced by a new one when its age reaches the preventive maintenance threshold, τ, if a spare part is available. If no spare part is available, the preventive maintenance is postponed to the earliest time when an item becomes available in inventory. If a failure occurs before the age of the critical component reaches the preventive maintenance threshold, then corrective maintenance action is taken. Similarly, if spare parts are not available at the time of the failure, the corrective replacement must wait until a spare part becomes available.

To replenish the spare part inventory, we propose batch ordering due to fixed ordering cost, where Q is the order quantity. To hold minimum inventory level, orders are placed such that zero inventory level is ensured when the order is received. To maintain this condition, we propose setting the order time as if all the remaining maintenance activities are preventive maintenance until the order receipt. The timing of the orders is discussed in detail in Sections 3.3 and 3.4 for the exceptional cases, which are absence and existence of supplier lead time, respectively.

In such a system, the objective is to maximize the long-run average profit rate. In order to derive long-run average profit rate, one can model the system describing it by two-dimensional state space, $(A(t), I(t))$, for $t > 0$ where $A(t)$ be the status of the critical component which is in use in the system at time t and $I(t)$ be the inventory level at time t. Non-negative continuous values that $A(t)$ takes correspond to the critical component's age at time t. Whereas $A(t) = -1$ represents the case that the critical component in use has failed at time t. So, the state space of $A(t)$ is defined as $A(t) \in \{-1\} \cup [0, \infty)$. Under our proposed policy, a spare part is used for maintenance activities every time an order is received. Therefore, state-space of $I(t)$ becomes $I(t) \in \{0, 1, ..., Q-1\}$. The additional notation used for describing the proposed policy is summarized in Table 3.2.

Table 3.2: Notation Used for Proposed Policy

Indices	
t	Time. $t \in [0, \infty)$.
State Variables	
$A(t)$	Age of the critical component used in the system at time t. Age -1 means that the critical component is failed and it is not replaced yet. $A(t) \in \{-1\} \cup [0, \infty)$.
$I(t)$	On-hand spare parts inventory level at time t. $I(t) \in \{0, 1, ..., Q-1\}$.
Decision Variables	
τ	Preventive maintenance threshold of the critical component in use.
Q	Order quantity for spare parts.

We propose a reasonable policy that is easy to understand and apply. Maintenance activity is conducted either at the time of failure or critical component's age reaches the preventive maintenance threshold, whichever happens first. The order time is scheduled when the inventory level and age of the critical component become predetermined levels. An order at a size of Q is placed at the scheduled time or failure time of the component, whichever happens first.

Under our proposed policy, we can define stochastically identical cycles between successive order arrivals. It is ensured that each time an order arrives, either preventive or corrective maintenance is conducted immediately. Therefore, the age of the critical component in use is always at age zero, $A(t) = 0$, where t is the time just after the order receipt. Also, a spare part in the newly arrived order is used for the maintenance activity at the time of order receipt. So, the on-hand spare parts inventory level is always $I(t) = Q - 1$, where t is the time just after the order receipt. Recall that on-hand spare parts inventory is zero just before the order receipt. This means that Q many maintenance activities are conducted between successive order arrivals. Also, the critical components' lifetimes are independent and identically distributed with a common probability density function. Therefore, the maintenance events between the order arrivals are stationary. So, it can be said that the system states regenerate themselves at the time of each order receipt to the $(A(t) = 0, I(t) = Q - 1)$. Therefore, under our proposed policy, we consider a regenerative stochastic process with the renewal points at the time of order arrivals.

In order to derive the long-run average profit of the system, the Renewal Reward Theory is utilized. Expected profit per unit time in a single regeneration cycle represents the long-run behavior of the expected profit per unit time in the whole regenerative stochastic process. Let T_n be the sequence of non-negative random variables, independent and identically distributed with a common probability density function. $\{T_n, n \geq 1\}$ denote random variable representing the time between $(n-1)^{st}$ and n^{th} order arrivals. Let $\{R_n, n \geq 1\}$ be the random variable representing the reward gained in the T_n. Let $R(t), t \geq 0$ be the total reward earned up to time t. R_n and $R(t)$ include all of the cost items and revenue of the system. Then we have the following equations:

$$P\left\{\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[R_1]}{E[T_1]}\right\} = 1, \tag{3.1}$$

$$P\left\{\lim_{t\to\infty} \frac{E[R(t)]}{t} = \frac{E[R_1]}{E[T_1]}\right\} = 1. \tag{3.2}$$

In other words, only by analyzing a single renewal cycle, the long-run average reward rate can be found. $G(\tau, Q)$ is used for the long-run average reward rate throughout the thesis. Let R as the rewards earned in a single renewal cycle and T as the length of a single renewal cycle. Therefore $G(\tau, Q)$ is:

$$G(\tau, Q) = \frac{E[R(\tau, Q)]}{E[T(\tau, Q)]}. \tag{3.3}$$

The expected reward and cycle length per renewal cycle calculations are derived under the special case and general case in Sections 3.3 and 3.4, respectively.

3.3 Proposed Policy with No Supplier Lead Time

When there is no supplier lead time, order timing becomes trivial. After the inventory level drops to zero, an order should be placed when a spare part is needed by either preventive or corrective maintenance. The derivation of long-run average profit rates when $Q = 1, L = 0$ and $Q > 1, L = 0$ are given in Section 3.3.1 and Section 3.3.2, respectively.

3.3.1 Derivation of Long-Run Average Profit Rate When Q=1, L=0

Notice that when $Q = 1$ and $L = 0$, the system state $I(t) = 0\ \forall t$. Therefore, one needs only $A(t)$ for $t \geq 0$ to derive the objective function under our policy. Notice that inventory holding cost is not included in this case since $I(t) = 0\ \forall t$.

We define *"maintenance cycle"* term that corresponds to the time between any two successive maintenance activities. Let X_i be the random variable represents the i^{th} lifetime of the critical component in use. Therefore, i^{th} maintenance cycle has a length of $min(X_i, \tau)$. If the critical component fails before the preventive maintenance threshold, then the maintenance cycle has the length equivalent to the spare part's failure time. Otherwise, maintenance cycle has the length equivalent to the preventive maintenance threshold. In Figure 3.1 and the rest of the manuscript, S_n be the replacement time of the n^{th} spare part, where:

$$S_0 = 0,\, S_n = \sum_{i=1}^{n} min\{X_i, \tau\},\, n = 1, 2, ... \tag{3.4}$$

The sample path of the system state under the proposed policy, when there is no supplier lead time and spare part order quantity is equal to one, is given in Figure 3.1.

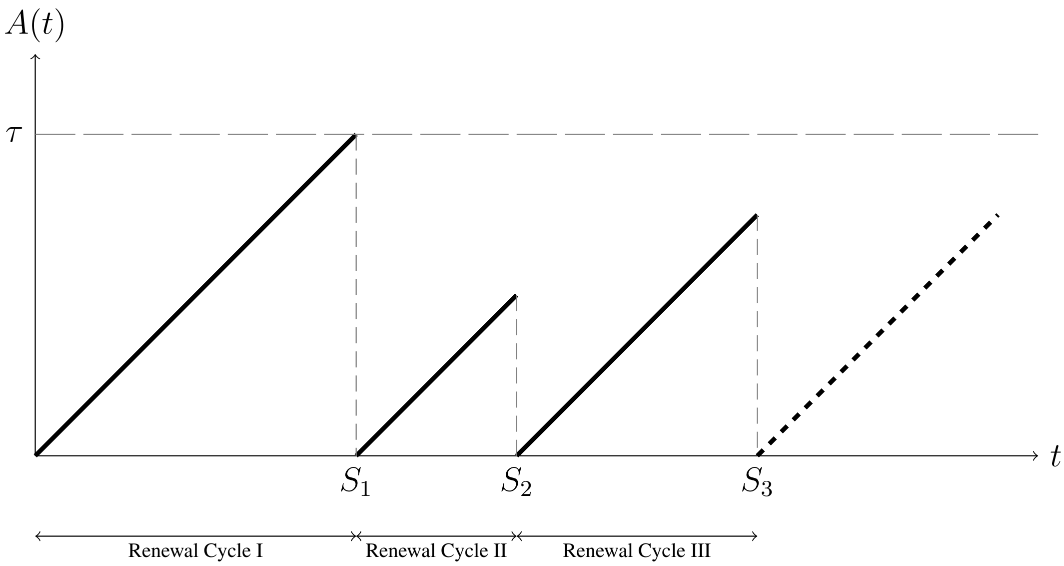

Figure 3.1: Sample Path of Age of the Critical Component in Use When $L = 0$, $Q = 1$

As it can be seen in Figure 3.1, at the time of every maintenance action, ordering and order receipt at the order size of one is observed simultaneously. The system state regenerates itself at each maintenance point since, at any maintenance action, the age of the critical component becomes zero, $A(t) = 0$. The maintenance cycle and renewal cycle terms are equivalent to each other in this case. The renewal cycles given in Figure 3.1 differ from each other according to the type of maintenance conducted at the end of the renewal cycle. In the first renewal cycle, the critical component works until its age is τ, and preventive maintenance is conducted. In the second and third renewal cycles, critical components fail before the preventive maintenance threshold. Hence, corrective maintenance action is taken at the time of the failure.

The realizations of reward and renewal cycle length with given random variable X in a single renewal cycle are:

$$R(\tau|X) = \begin{cases} -C_k - C_u + \tau r - C_p, & \text{if } X \geq \tau \\ -C_k - C_u + Xr - C_c, & \text{otherwise.} \end{cases} \tag{3.5}$$

$$T(\tau|X) = \begin{cases} \tau, & \text{if } X \geq \tau \\ X, & \text{otherwise.} \end{cases} \tag{3.6}$$

Expected reward and expected renewal cycle length of the system under the proposed policy become:

$$E[R(\tau)] = -C_k - C_u + \int_0^{\tau} (xr - C_c) f(x) dx + \int_{\tau}^{\infty} (\tau r - C_p) f(x) dx, \tag{3.7}$$

$$E[(T(\tau)] = \int_0^{\tau} x f(x) dx + \int_{\tau}^{\infty} \tau f(x) dx. \tag{3.8}$$

In (3.7), fixed ordering cost, C_k, is only incurred once since the renewal cycle is defined between successive order arrivals. Total procurement cost in a renewal cycle is C_u since order quantity is one.

The third term gives the expected total revenue and maintenance cost when corrective maintenance action is taken at the end of the renewal cycle. In this case, revenue is gained until the failure of the critical part. The fourth term gives the expected total revenue and maintenance cost when preventive maintenance is conducted at the end of the renewal cycle. This time, revenue is earned for τ units of time. Similarly, in (3.8), renewal cycle length depends only on the failure time and preventive maintenance threshold of the critical part. The long-run average profit rate can be calculated as:

$$G(\tau) = \frac{E[R(\tau)]}{E[(T(\tau)]}. \tag{3.9}$$

3.3.2 Derivation of Long-Run Average Profit Rate When Q>1, L=0

In this case, since $Q > 1$, we need two dimensional state space, $(A(t), I(t))$, for $t > 0$ to derive the objective function. The sample path of the system states, when there is no supplier lead time, and spare part order quantity is greater than one, is given in Figure 3.2.

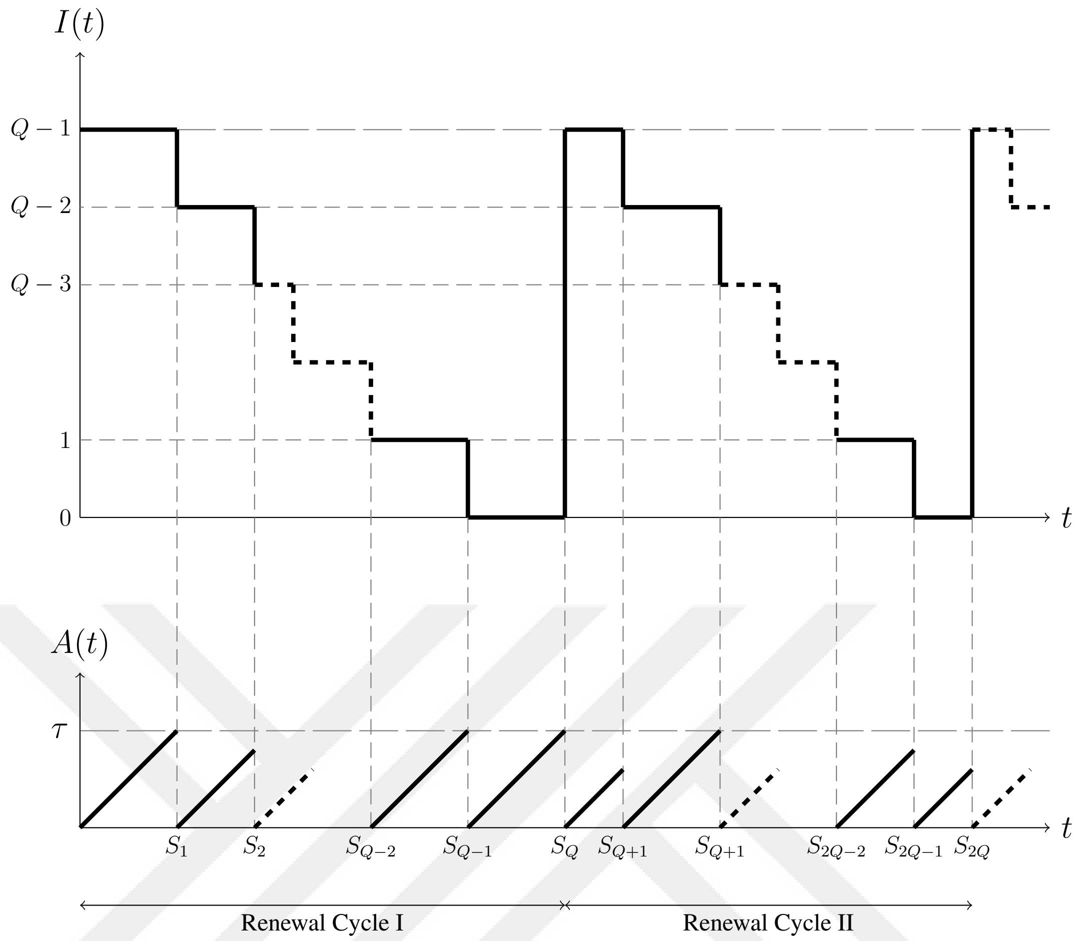

Figure 3.2: The Sample Path of Inventory Level and Age of Critical Component in Use When $L = 0$, $Q > 1$

As it can be observed in Figure 3.2, the time between successive ordering and order receipt times constitute renewal cycles. Notice that within each renewal cycle, there are always Q many maintenance cycles, and the maintenance that is performed when an order is received is the last maintenance of a renewal cycle, at times S_Q, S_{2Q}, ..., S_{nQ}. Therefore, at the time of ordering/ order receipt, the renewal point is observed, and the system states regenerate themselves to $(A(t) = 0, I(t) = Q-1)$.

The lengths of maintenance cycles are independent and identically distributed random variables. As the inventory is depleted with either preventive or corrective maintenance, the number of spare parts kept in the inventory stay at the same inventory level within each maintenance cycle. In this setting, the realizations of reward and renewal cycle length with given random variables of X's in a single renewal cycle are:

$$R(\tau, Q|X_1, X_2, ..., X_Q)$$
$$= -C_k - QC_u + \sum_{i=1}^{Q} \Big[min\{X_i, \tau\}\big(r - C_h(Q-i)\big) - C_p\phi_i - C_c(1-\phi_i)\Big], \tag{3.10}$$

$$T(\tau, Q|X_1, X_2, ..., X_Q) = \sum_{i=1}^{Q} min\{X_i, \tau\}, \tag{3.11}$$

$$\phi_i = \begin{cases} 1, & \text{if } X_i \geq \tau \\ 0, & \text{otherwise.} \end{cases}, i = 1, 2, ..., Q$$

where ϕ_i is the indicator variable representing the type of maintenance activity conducted at the end of i^{th} maintenance cycle. If ϕ_i is one, then preventive maintenance action is taken at the end of i^{th} maintenance cycle. Otherwise, corrective maintenance is conducted at the end of i^{th} maintenance cycle.

The expected reward and expected renewal cycle length of the system under the proposed policy are:

$$E[R(\tau, Q)] = -C_k - QC_u + \sum_{i=1}^{Q} \Bigg[\int_0^{\tau} \Big(x\big(r - C_h(Q-i)\big) - C_c\Big) f(x)dx + \int_{\tau}^{\infty} \Big(\tau\big(r - C_h(Q-i)\big) - C_p\Big) f(x)dx \Bigg], \tag{3.12}$$

$$E[T(\tau, Q)] = Q\Bigg(\int_0^{\tau} xf(x)dx + \int_{\tau}^{\infty} \tau f(x)dx\Bigg). \tag{3.13}$$

In (3.12), fixed ordering cost, C_k, is only incurred once as in the (3.7). In this case, while total procurement cost in the renewal cycle is calculated, C_u is multiplied with the order quantity at the size of Q.

Expected total reward and expected cycle length calculations are done as in the (3.7) and (3.8), except for this time Q many maintenance cycles are observed in a renewal cycle. The long-run average profit rate calculated as:

$$G(\tau, Q) = \frac{E[R(\tau, Q)]}{E[T(\tau, Q)]}. \tag{3.14}$$

3.4 Proposed Policy with Supplier Lead Time

When $L > 0$, order timing is more complicated. In our proposed policy, assuming that spare parts are used by preventive maintenance only, orders are placed to be received when the inventory level is zero, and it is time for the next preventive maintenance. Recall that the maximum length of a maintenance cycle is equal to the preventive maintenance threshold of the critical component. Let $\lceil.\rceil$ and $\lfloor.\rfloor$ be the operators which round up and round down a given fraction to the nearest integer, respectively. The maximum number of preventive maintenance activities that are possible during supplier lead time is $\lceil\frac{L}{\tau}\rceil$. Then, $\lfloor\frac{L}{\tau}\rfloor$ many of those preventive maintenance activities use the spare parts in the inventory. In contrast, the last preventive maintenance activity uses the spare part at the upcoming batch since the last preventive maintenance activity is conducted at the time of order receipt. Therefore, an order is placed when inventory level is $\lceil\frac{L}{\tau}\rceil - 1$ or equivalently $\lfloor\frac{L}{\tau}\rfloor$. However, placing the order immediately after inventory level drops to $\lfloor\frac{L}{\tau}\rfloor$ may not ensure zero inventory level at the time of order receipt. If all the maintenance activities are preventive maintenance after inventory level drops down to $\lfloor\frac{L}{\tau}\rfloor$, the duration of time of conducting all of those preventive maintenance activities is $\tau\lceil\frac{L}{\tau}\rceil$. Therefore, after inventory level drops to $\lfloor\frac{L}{\tau}\rfloor$, an order is placed after waiting for $\tau\lceil\frac{L}{\tau}\rceil - L$ units of time provided that the component in use does not fail until that time. If the failure is observed before the scheduled time, the order is placed immediately.

Let T_W be the random variable representing the time between ordering and maintenance point prior to ordering. T_W can be defined as an expression of waiting time of scheduled order time and lifetime of the critical component in use at the maintenance cycle that ordering occurs:

$$T_W = min\left\{\tau\left\lceil\frac{L}{\tau}\right\rceil - L, X_{Q-\lfloor\frac{L}{\tau}\rfloor}\right\}, \tag{3.15}$$

where, $X_{Q-\lfloor\frac{L}{\tau}\rfloor}$ is the lifetime of the critical component in use when the inventory level is $\lfloor\frac{L}{\tau}\rfloor$. Scheduling procedure for order timing is illustrated in Figure 3.3.

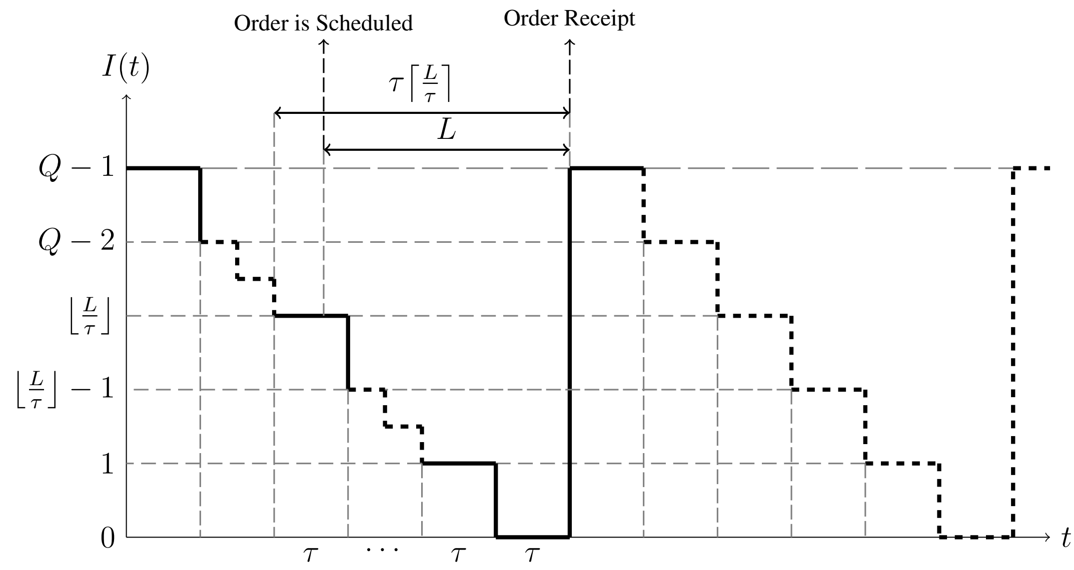

Figure 3.3: Illustration of Ordering Time

We restrict ourselves to having at most one outstanding spare part order, which is a common assumption in the literature [2], [14], [20]. In order to ensure the restriction, the following inequality should be satisfied:

$$\tau Q \geq L, \tag{3.16}$$

which means that the maximum possible renewal cycle length should be at least equal to supplier lead time. This restriction allows modeling the system as a renewal reward process under the proposed policy. First, let us restrict the order size equal to one, $Q = 1$. The derivation of long-run average profit rates when $Q = 1, L > 0$ and $Q > 1, L > 0$ are given in Section 3.4.1 and Section 3.4.2, respectively.

3.4.1 Derivation of Long-Run Average Profit Rate When Q=1, L>0

As in the Section 3.3.1, $A(t)$ for $t > 0$ is sufficient to represent the system state and to derive the objective function. Also, there is no inventory holding cost since $I(t) = 0$ $\forall t$.

As we allow at most one outstanding order, we have $\tau > L$ from (3.16). So, while placing the order, we propose adjusting the ordering time to receive the order when preventive maintenance demands a spare part. Therefore, an order is placed when the critical component in use reaches the age of $\tau - L$. If the critical component in use fails before the age of $\tau - L$, an order is placed immediately when it fails. The sample path of the system states, when there is supplier lead time and spare part order quantity is equal to one is given in Figure 3.4.

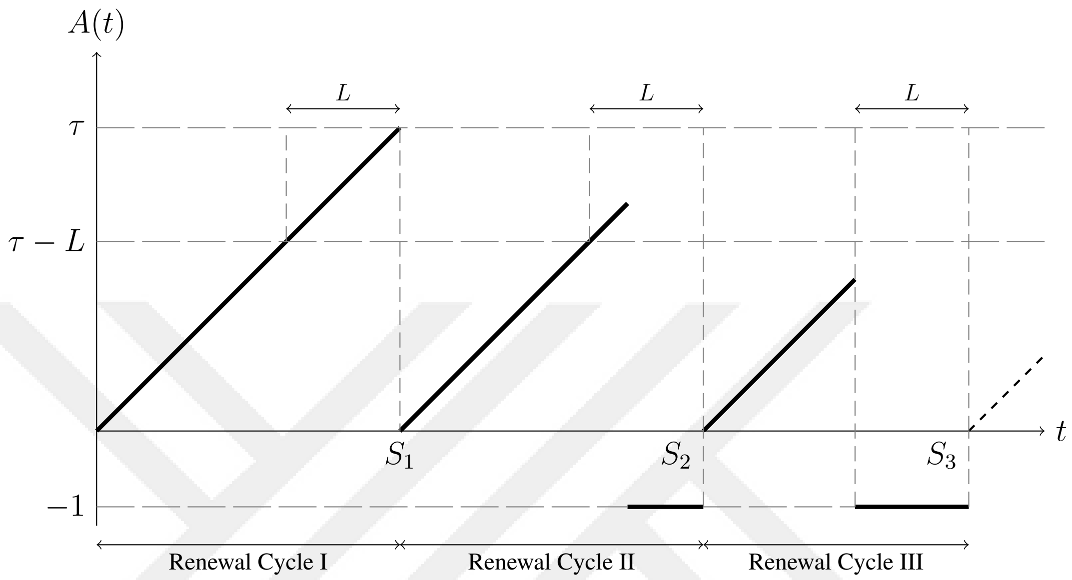

Figure 3.4: The Sample Path of Age of the Critical Component in Use When $L \geq 0$, $Q = 1$

Renewal cycles are defined between successive order receipts. In Figure 3.4, three realizations of renewal cycles, which are the only possible realizations, are shown. In renewal cycle I, critical component failure is not observed until τ. Therefore, the spare part order is placed at the scheduled time, $\tau - L$. Also, the system operates throughout the entire cycle, and the renewal cycle ends with a preventive maintenance action since the age of the critical component in use reaches the preventive maintenance threshold parameter, τ. In renewal cycle II, the order is placed at the scheduled time, $\tau - L$ as in renewal cycle I. However, the component in use fails before its preventive maintenance time, which is the same time as the order receipt. Therefore, there is a downtime in this renewal cycle. A corrective maintenance action is taken immediately at the time of order receipt. This renewal cycle's length is also τ. In renewal cycle III, the critical component in use fails before $\tau - L$. Therefore, an order is placed at the time of failure. Downtime is observed, which is equal to the duration of lead time. Corrective maintenance action is taken when the order arrives. The cycle length is the sum of the lifetime of the critical component in use and supplier lead time. The realization of reward and renewal cycle with given random variable X in a single renewal cycle are:

$$R(\tau|X) = \begin{cases} -C_k - C_u + \tau r - C_p, & \text{if } X \geq \tau \\ -C_k - C_u + Xr - C_c, & \text{otherwise.} \end{cases} \tag{3.17}$$

$$T(\tau|X) = \begin{cases} \tau, & \text{if } X \geq \tau - L \\ X + L, & \text{otherwise.} \end{cases} \tag{3.18}$$

As it can be seen, random reward realization stays the same as in Equation (3.5), in Section 3.3. The change is at cycle length since downtime can be realized in a given renewal cycle. Cycle length differs from Equation (3.6) since the cycle ends with the receipt of the order, which takes L units of time. The expected reward and expected renewal cycle length of the system under the proposed policy are:

$$E[R(\tau)] = -C_k - C_u + \int_0^{\tau} (xr - C_c) f(x) dx + \int_{\tau}^{\infty} (\tau r - C_p) f(x) dx, \tag{3.19}$$

$$E[T(\tau)] = L + \int_0^{\tau - L} x f(x) dx + \int_{\tau - L}^{\infty} (\tau - L) f(x) dx. \tag{3.20}$$

Equation (3.19) is same as Equation (3.7). In (3.20), expected renewal cycle length is sum of supplier lead time, and expected time until an order is placed. The long-run average profit rate calculated as:

$$G(\tau) = \frac{E[R(\tau)]}{E[T(\tau)]}. \tag{3.21}$$

3.4.2 Derivation of Long-Run Average Profit Rate When Q>1, L>0

When we consider order quantities greater than one, the policy becomes as stated in Section 3.2. We need a two dimensional state space, $A(t), I(t)$,to characterize the system's objective function under the proposed policy. In order to ensure that there is at most one outstanding order, inequality introduced in (3.16) should be enforced.

When the inventory level drops to $\left\lfloor \frac{L}{\tau} \right\rfloor$ units, the system waits for $\tau \left\lceil \frac{L}{\tau} \right\rceil - L$ units of time to place an order of size Q provided that the component in use does not fail by that time. If the critical component in use fails before the age $\tau \left\lceil \frac{L}{\tau} \right\rceil - L$, then the order is placed immediately. Note that the inventory level is definitely zero when the order arrives, and the proposed policy calls for a maintenance as soon as the order arrives. If the component installed in the system has failed before the order receipt, then corrective maintenance is conducted at the time of order arrival.

We introduce a new random variable, T_E, to simplify the realizations in the renewal cycles. Let T_E be the random variable representing the time between the order receipt and maintenance point prior to the order receipt. In other words, it is the length of the last maintenance cycle in a renewal cycle. T_E can be defined as:

$$T_E = L + T_W - min\Big\{X_{Q-\left\lfloor \frac{L}{\tau} \right\rfloor}, \tau\Big\} - \sum_{i=Q-\left(\left\lfloor \frac{L}{\tau} \right\rfloor - 1\right)}^{Q-1} min\Big\{X_i, \tau\Big\} \tag{3.22}$$

$L + T_W$ is the time between the maintenance event that decreases $I(t)$ to $\left\lfloor \frac{L}{\tau} \right\rfloor$ and the arrival of the order. The third term is the length of the maintenance cycle that ordering occurs. Recall that T_W is the minimum of $\tau \left\lceil \frac{L}{\tau} \right\rceil - L$ and X, where X is the $(Q - \left\lfloor \frac{L}{\tau} \right\rfloor)^{th}$ lifetime of the critical component in use. So, if the critical component in use at the maintenance cycle that ordering occurs fails before the scheduled time of order, then the second and third terms cancel out each other. Otherwise, the time between ordering and maintenance event just after the ordering is calculated by the second and third term and deducted from the remaining time of the order arrival. The last term is the total time between the maintenance that drops $I(t)$ to $\left\lfloor \frac{L}{\tau} \right\rfloor - 1$ and the maintenance at before the order arrival.

T_E realization depends on the lengths of maintenance cycles, which are observed after the ordering until inventory becomes zero. Since the ordering time is set in a way that ensures zero inventory level at the time of order receipt, every failure that occurs after the inventory level drops to $I(t)$ to $\left\lfloor \frac{L}{\tau} \right\rfloor$ increases the length of T_E. T_E could take any value between $[\tau, L]$. If all the maintenance actions are preventive maintenance after an order is placed, then T_E realization takes its lower bound value, which is τ. On the other hand, if all the lifetimes of the critical components in use after the ordering are equal to zero, $X_{Q-\lfloor \frac{L}{\tau} \rfloor+1} = 0$, $X_{Q-\lfloor \frac{L}{\tau} \rfloor+2} = 0$, ..., $X_{Q-1} = 0$, and the lifetime of the critical component in use at the maintenance cycle in which ordering occurs is less than the waiting time, $T_W = X_{Q-\lfloor \frac{L}{\tau} \rfloor}$, then T_E realization becomes L, which is the upper bound value of it. The sample path of the system states when supplier lead time exists, and spare part order quantity is greater than one is given in Figure 3.5.

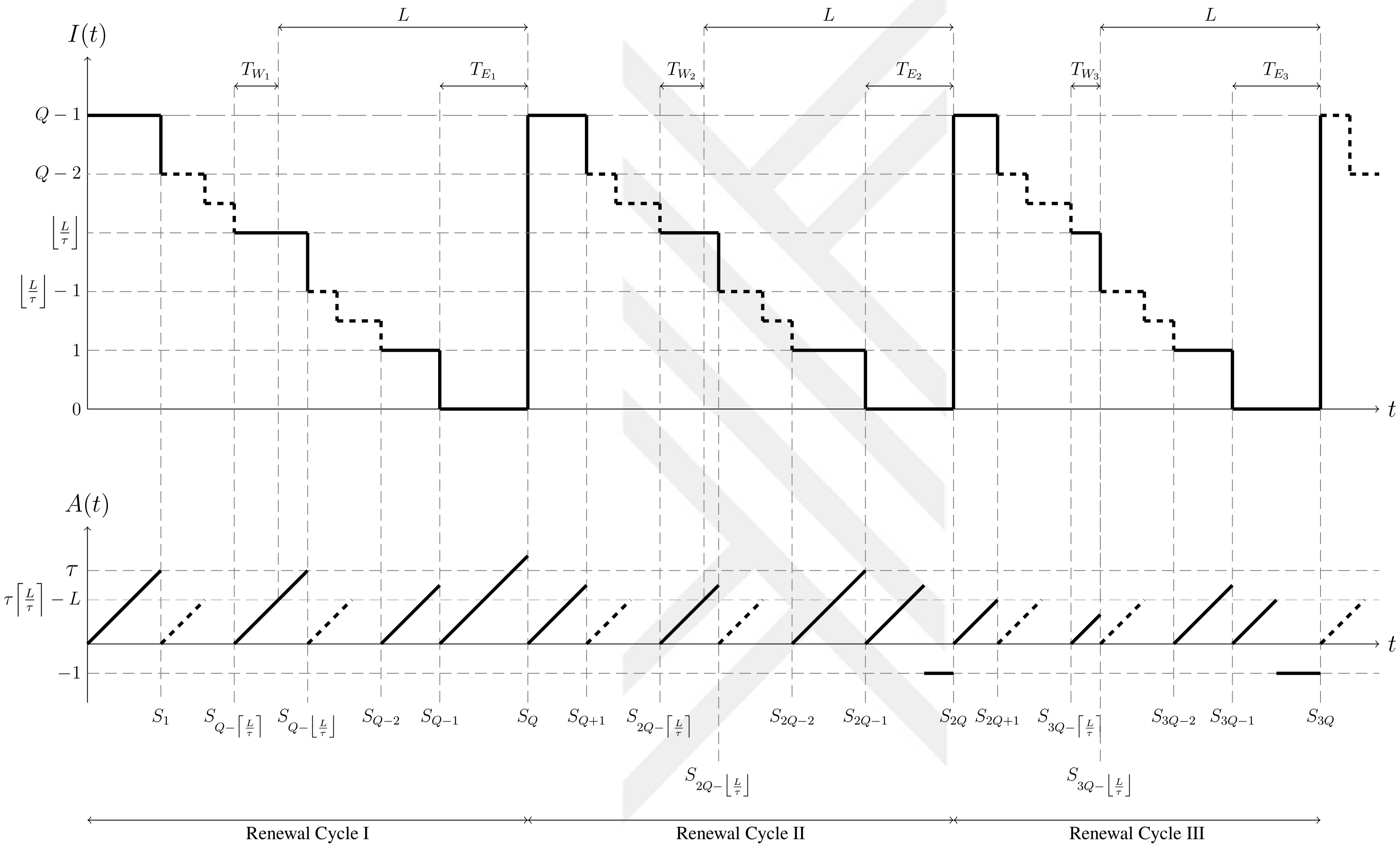

Figure 3.5: The Sample Path of Inventory Level and Age of Critical Component in Use When $L \geq 0$, $Q \geq 1$

In Figure 3.5, each renewal cycle corresponds to one of the three different characteristics that can be observed in the renewal cycles. Those renewal cycles differ from each other with the lifetime of the critical component in use at the maintenance cycle in which ordering occurs. In renewal cycle I, the next maintenance subsequent to ordering is of preventive, so the order is placed at its scheduled time. Therefore, $T_W = \tau\lceil \frac{L}{\tau} \rceil - L$. In renewal cycle II, the next maintenance subsequent to ordering is corrective. However, since failure time is after the scheduled time of order, again T_W realization becomes, $T_W = \tau\lceil \frac{L}{\tau} \rceil - L$. In renewal cycle III, since failure is observed during the waiting time of ordering, ordering and corrective maintenance occur together at the time of failure immediately. Therefore, T_W realization becomes equal to the lifetime of the critical component, $T_W = X_{Q-\lfloor \frac{L}{\tau} \rfloor}$. T_W realization is important for our proposed policy since the ordering time directly affects the expected reward and expected renewal cycle length calculations. The realization of renewal cycle length with given random variables of X's in single a renewal cycle is:

$$T(\tau, Q|X_1, X_2, ..., X_{Q-\lceil \frac{L}{\tau} \rceil}) = T_W + L + \sum_{i=1}^{Q-\lceil \frac{L}{\tau} \rceil} min\{X_i, \tau\}. \tag{3.23}$$

It can be seen in (3.23) that cycle length depends on the maintenance points before the ordering as well as the maintenance time at the maintenance cycle in which the ordering occurs.

Recall that $T_W + L$ is the time between the maintenance event that decreases $I(t)$ to $\lfloor \frac{L}{\tau} \rfloor$ and the arrival of the order. The third term is the total time between start of the renewal cycle and maintenance event that drops $I(t)$ to $\lfloor \frac{L}{\tau} \rfloor$, which is the inventory level at the time of ordering. Maintenance points before the ordering are identical to each other, the length of each is $min(X, \tau)$. After the ordering, supplier lead time is observed until the end of the renewal cycle. We can reorganize Equation (3.23) by plugging (3.15) into it, which yields:

$$\begin{aligned} &T(\tau, Q|X_1, X_2, ..., X_{Q-\lfloor \frac{L}{\tau} \rfloor}) \\ &= min\left\{\tau\left\lceil \frac{L}{\tau} \right\rceil - L, X_{Q-\lfloor \frac{L}{\tau} \rfloor}\right\} + L + \sum_{i=1}^{Q-\lceil \frac{L}{\tau} \rceil} min\{X_i, \tau\}. \end{aligned} \tag{3.24}$$

Therefore, expected renewal cycle length of the system under proposed policy is:

$$E[T(\tau,Q)] = \left[\int_{0}^{\tau\left\lceil \frac{L}{\tau}\right\rceil - L} x f(x)dx + \int_{\tau\left\lceil \frac{L}{\tau}\right\rceil - L}^{\infty} \left(\tau\left\lceil \frac{L}{\tau}\right\rceil - L\right) f(x)dx \right] + L$$
$$+ \left(Q - \left\lceil \frac{L}{\tau}\right\rceil\right) \left[\int_{0}^{\tau} x f(x)dx + \int_{\tau}^{\infty} \tau f(x)dx \right]. \tag{3.25}$$

In (3.25), the first term calculates the expected time between the ordering and the maintenance point before the ordering. The second term is the deterministic supplier lead time observed from the ordering to order receipt. The third term calculates the expected times of maintenance cycles before the maintenance cycle in which ordering occurs.

The realization of reward with given random variables of X's in a single renewal cycle is:

$$\begin{aligned} &R(\tau, Q|X_1, X_2, ..., X_Q) = -C_k - QC_u \\ &+ \sum_{i=1}^{Q-1} \Big[-C_p\phi_i - C_c(1-\phi_i) + min\{X_i,\tau\}\big(r - C_h(Q-i)\big)\Big] \\ &+ \Big[-C_p\phi_Q - C_c(1-\phi_Q) + min\{X_Q, T_E\}r\Big], \end{aligned} \tag{3.26}$$

$$\phi_i = \begin{cases} 1, & \text{if } X_i \geq \tau \\ 0, & \text{otherwise.} \end{cases}, i = 1, 2, ..., Q-1$$

$$\phi_Q = \begin{cases} 1, & \text{if } X_Q \geq T_E \\ 0, & \text{otherwise.} \end{cases}$$

where ϕ_i and ϕ_Q are the indicator variables representing the type of maintenance activity conducted at the end of i^{th} and Q^{th} maintenance cycles, respectively. If ϕ_i and ϕ_Q are one, then preventive maintenance action is taken at the end of i^{th} and Q^{th} maintenance cycles, respectively. Otherwise, corrective maintenance is conducted at the end of the corresponding maintenance cycles.

In (3.26), since the maintenance cycles lengths are independent and identically distributed except for the maintenance cycle in which order receipt occurs, the summation term considers the reward gained within the first $Q-1$ many maintenance cycles.

When we consider reward generated at the maintenance cycle in which order receipt occurs, we need to compare X_Q and T_E realizations. If the critical component does not fail until the order receipt, for example, $X_Q \geq T_E$, the system is operational through the entire renewal cycle. In this case, preventive maintenance is conducted when order receipt occurs, $\phi_Q = 1$. On the other hand, when the critical component fails prior to the order receipt, for example, $X_Q \leq T_E$, the system is not operational from failure to time of order receipt. This time, corrective maintenance is conducted at the time that order receipt occurs, $\phi_Q = 0$.

Let R_E be the random variable representing the reward earned during T_E,

$$R_E = -C_p\phi_Q - C_c(1-\phi_Q) + min\{X_Q, T_E\}r. \tag{3.27}$$

Then, the expected reward of the system under the proposed policy is:

$$\begin{aligned} E[R(\tau,Q)] = \sum_{i=1}^{Q-1}\Bigg[&\int_0^{\tau}\Big(x\big(r - C_h(Q-i)\big) - C_c\Big)f(x)dx \\ &+\int_{\tau}^{\infty}\Big(\tau\big(r - C_h(Q-i)\big) - C_p\Big)f(x)dx\Bigg] \\ &+E[R_E] - C_k - C_uQ. \end{aligned} \tag{3.28}$$

$E[R_E]$ depends on the lifetime of the critical component in use at $I(t) = 0$, as well as $\lceil \frac{L}{\tau} \rceil - 1$ many maintenance points' time just before the inventory level drops to zero. The special case of $E[R_E]$, $\lceil \frac{L}{\tau} \rceil = 1$, depends on only the lifetime of the critical part in use at $I(t) = 0$, since the ordering and order receipt occur in the same maintenance cycle. This case can be expressed as follows:

$$E\left[R_E \mid \left\lceil \frac{L}{\tau} \right\rceil = 1\right] = \int_0^{\tau}\big(xr - C_c\big)f(x)dx + \int_{\tau}^{\infty}\big(\tau r - C_p\big)f(x)dx \tag{3.29}$$

It can be observed that (3.29) is nothing but Equation (3.7) in Section 3.3.1, except for the exclusion of fixed cost and procurement cost, since these costs are considered in (3.28). For the rest of the cases, we can characterize $E[R_E]$ with the $\lceil \frac{L}{\tau} \rceil - 1$ many maintenance points' time and the lifetime of the critical component in use at $I(t) = 0$.

To simplify the following characterizations, let Y_m be the random variable representing the critical component's maintenance time at the m^{th} maintenance cycle, where

$m = 1$ corresponds to the maintenance cycle that ordering occurs and m is incremented by one for the subsequent maintenance cycles. Y_m can be defined as:

$$Y_m = min\{X_m, \tau\}. \tag{3.30}$$

Y_m is independent and identically distributed with the probability density function $h(y)$. Notice that the following equality holds for T_W:

$$T_W = min\left\{\tau\left\lceil\frac{L}{\tau}\right\rceil - L, Y_1\right\}. \tag{3.31}$$

Also, we can characterize T_E alternatively as follows:

$$T_E = L + min\left\{\tau\left\lceil\frac{L}{\tau}\right\rceil - L, Y_1\right\} - Y_1 - \sum_{m=2}^{\left\lceil\frac{L}{\tau}\right\rceil - 1} Y_m \tag{3.32}$$

Let $n = \left\lceil\frac{L}{\tau}\right\rceil$. Therefore, $E[R_E]$ for the cases, in which $n > 1$ is:

$$\begin{aligned}
&E\left[R_E \mid n = \left\lceil\frac{L}{\tau}\right\rceil, n > 1\right] \\
&= \int_0^{n\tau - L} h(y_1) \int_0^{L} h(y_2) \int_0^{L-y_2} h(y_3) \cdots \int_0^{L-y_2-y_3-\ldots-y_{n-2}} h(y_{n-1}) \\
&\left[\int_0^{L-y_2-y_3-\ldots-y_{n-1}} (x_n r - C_c) f(x_n) dx_n dy_{n-1} \ldots dy_3 dy_2 dy_1\right] \\
&+ \int_0^{n\tau - L} h(y_1) \int_0^{L} h(y_2) \int_0^{L-y_2} h(y_3) \cdots \int_0^{L-y_2-y_3-\ldots-y_{n-2}} h(y_{n-1}) \\
&\left[\int_{L-y_2-y_3-\ldots-y_{n-1}}^{\infty} \left(\left(L - \sum_{m=2}^{n-1} y_m\right) r - C_p\right) f(x_n) dx_n dy_{n-1} \ldots dy_3 dy_2 dy_1\right] \\
&+ \int_{n\tau - L}^{\tau} h(y_1) \int_0^{n\tau - y_1} h(y_2) \int_0^{n\tau - y_1 - y_2} h(y_3) \cdots \int_0^{n\tau - y_1 - y_2 - \ldots - - y_{n-2}} h(y_{n-1}) \\
&\left[\int_0^{n\tau - y_1 - y_2 - y_3 - \ldots - y_{n-1}} (x_n r - C_c) f(x_n) dx_n dy_{n-1} \ldots dy_3 dy_2 dy_1\right] \\
&+ \int_{n\tau - L}^{\tau} h(y_1) \int_0^{n\tau - y_1} h(y_2) \int_0^{n\tau - y_1 - y_2} h(y_3) \cdots \int_0^{n\tau - y_1 - y_2 - \ldots - - y_{n-2}} h(y_{n-1}) \\
&\left[\int_{n\tau - y_1 - y_2 - y_3 - \ldots - y_{n-1}}^{\infty} \left(\left(n\tau - \sum_{m=1}^{n-1} y_m\right) r - C_p\right) f(x_n) dx_n dy_{n-1} \ldots dy_3 dy_2 dy_1\right]
\end{aligned} \tag{3.33}$$

In (3.33), the first n fold integrals correspond for the case that ordering is triggered by component failure, and the last installed critical component in use in the renewal cycle fails before the order receipt. Maintenance time, Y_1, occurs prior to the scheduled time of order. During supplier lead time, $n-1$ many maintenance activities are conducted, which constitute $n-2$ many i.i.d. maintenance cycles and the last maintenance cycle in a renewal cycle. $n-2$ many maintenance times corresponds for $Y_2, Y_3, ..., Y_{n-1}$. At the last maintenance cycle, the critical component fails prior to the order receipt. The second n fold integrals are the same as the first n fold integrals except that at the last maintenance cycle, the critical component does not fail until the order receipt, and preventive maintenance is conducted at the time of order arrival. The third n fold integrals correspond to the case that ordering occurs at its scheduled time, and the last installed critical component in use in the renewal cycle fails before the order receipt. So, at the start of the next maintenance cycle after the ordering, the remaining time of the order arrival is $n\tau - y_1$. At the last maintenance cycle, the critical component fails before order arrival time. The last n fold integrals are identical to the third term, except the critical component in use at the last maintenance cycle survives until the time of order receipt. The long-run average profit rate is:

$$G(\tau, Q) = \frac{E[R(\tau, Q)]}{E[T(\tau, Q)]}. \tag{3.34}$$

3.5 Approximations

Calculating the expected reward between the order receipt and prior maintenance point, $E[R_E]$, requires evaluating $\lceil \frac{L}{\tau} \rceil$ fold integrals. So, evaluating the objective function for given τ and Q values under the proposed policy requires complicated computation. Therefore, two approximations for $E[R_E]$ are proposed.

Approximating $E[R_E]$ needs evaluating the time between the order receipt and prior maintenance point, T_E, (or equivalently the time period where inventory level is zero). A renewal cycle length of the system under the proposed policy can be characterized by utilizing T_E:

$$T(\tau, Q|X_1, ..., X_{Q-1}) = \sum_{i=1}^{Q-1} \Big[min\{X_i, \tau\}\Big] + T_E. \tag{3.35}$$

Since the maintenance points are identically distributed in terms of their duration, except for the maintenance point at order receipt, simply summing all of the maintenance cycles' durations gives us the renewal cycle length. The expected cycle length is then given by:

$$E[T(\tau,Q)] = (Q-1)\left[\int_0^{\tau} xf(x)dx + \int_{\tau}^{\infty} \tau f(x)dx\right] + E[T_E]. \tag{3.36}$$

Since Equations (3.25) and (3.36) are equal to each other, we can get the expected time between order receipt and prior maintenance point as:

$$\begin{aligned} E[T_E] = &\left[\int_0^{\tau\left\lceil \frac{L}{\tau}\right\rceil - L} xf(x)dx + \int_{\tau\left\lceil \frac{L}{\tau}\right\rceil - L}^{\infty} \left(\tau\left\lceil \frac{L}{\tau}\right\rceil - L\right) f(x)dx\right] + L \\ &-\left(\left\lceil \frac{L}{\tau}\right\rceil - 1\right)\left[\int_0^{\tau} xf(x)dx + \int_{\tau}^{\infty} \tau f(x)dx\right]. \end{aligned} \tag{3.37}$$

In (3.37), the summation of the first term and L stands for the expected time between the maintenance point prior to the ordering and the end of the renewal cycle. This duration consist of $\left\lceil \frac{L}{\tau}\right\rceil$ many maintenance cycles where $\left\lceil \frac{L}{\tau}\right\rceil - 1$ of them identically distributed and the remaining one is T_E. So, in the second term of the equation, simply subtracting the sum of identical maintenance cycles' expected times gives us the $E[T_E]$.

By knowing the exact value of $E[T_E]$, we also need the expected operational time of the system between order receipt and prior maintenance point to find $E[R_E]$. Two approximations tackle the problem of finding the expected operational time of the system between order receipt and prior maintenance point, or in other words, expected up-time of the system between order receipt and prior maintenance point. The exact expression of expected up-time between order receipt and prior maintenance point is:

$$E\Big[\text{Up-time}\Big] = E\Big[min\Big\{X, T_E\Big\}\Big]. \tag{3.38}$$

3.5.1 Approximation 1

The first approximation method considers the expected time of the system between order receipt and prior maintenance point, $E[T_E]$, as well as the lifetime of the critical part in use while approximating the expected operational time of the system between order receipt and prior maintenance point, or in other words, expected up-time of the system between order receipt and prior maintenance point accordingly:

$$E\Big[\text{Up-time}\Big] = E\Big[min\Big\{X, E[T_E]\Big\}\Big]. \tag{3.39}$$

In (3.39), Approximation 1 approximates up-time as the minimum of the lifetime of the critical component in use and expected time of the system between order receipt and prior maintenance point, $E[T_E]$. Approximating up-time means approximating generated revenue in the system between order receipt and prior maintenance point. Maintenance costs are also affected by this approximation. If the lifetime of the critical component in use is less than $E[T_E]$, then we assume that the last maintenance is corrective. Otherwise, preventive maintenance cost incurs. Consequently, Approximation 1 approximates the expected reward earned between the order receipt and prior maintenance point, $E[R_{E_1}]$, as:

$$E[R_{E_1}] = \Bigg[\int_0^{E[T_E]} (xr - C_c)f(x)dx + \int_{E[T_E]}^{\infty} (E[T_E]r - C_p)f(x)dx\Bigg]. \tag{3.40}$$

3.5.2 Approximation 2

The second approximation method considers the expected time of the system between order receipt and prior maintenance point, $E[T_E]$, as well as the expected lifetime of the critical component in use, $E[X]$, while approximating the expected up-time of the system between order receipt and prior maintenance point accordingly:

$$E\Big[\text{Up-time}\Big] = min\Big\{E[X], E[T_E]\Big\}. \tag{3.41}$$

In (3.41), Approximation 2 approximates up-time as the minimum of expected lifetime of the critical component in use, $E[X]$, or expected time of the system between order receipt and prior maintenance point, $E[T_E]$. According to the Approximation 2, the expected reward between the order receipt and prior maintenance point, $E[R_{E_2}]$, is approximated as:

$$E[R_{E_2}] = \begin{cases} E[T_E]r - C_p, & \text{if } E[X] \geq E[T_E] \\ E[X]r - C_c, & \text{otherwise.} \end{cases} \tag{3.42}$$

3.5.3 Search Algorithm for Approximations

We propose a search algorithm for approximation methods to find τ^* and Q^* values where $G(\tau, Q)$ is maximized. In the search algorithm, while the $G(\tau, Q)$ is evaluated, $E[R_{E_i}]$ is used if i^{th} approximation is considered. Let q be the index for representing batch size quantity, and τ_q be the preventive maintenance threshold value which maximizes $G(\tau, Q)$ when $Q = q$.The search algorithm is:

1. $q = 1$.
2. Given $Q = q$, search for τ_q that maximizes $G(\tau, Q)$ from $\frac{L}{Q}$ to $F^{-1}(0.9999)$ with step increments of 0.1.
3. Set $q = q + 1$.
4. Given $Q = q$, search for τ_q that maximizes $G(\tau, Q)$ from $\frac{L}{Q}$ to $F^{-1}(0.9999)$ with step increments of 0.1.
5. If $G(\tau_q, Q = q) > G(\tau_{q-1}, Q = q - 1)$, go to step 3.
6. Set $Q^* = q - 1$ and $\tau^* = \tau_{q-1}$

The search algorithm is used for determining optimal Q^* and τ^* values under the corresponding approximation's expected up-time evaluation. Therefore, $G(\tau^*, Q^*)$ is the approximation of the long-run average profit rate under the proposed policy.

3.6 Sequential Approaches

Preventive maintenance and spare parts inventory management are usually planned by different functional units in the industry [21], although joint planning provides globally less costly solutions [9], [12], [11], [15], [2], [22], [23], [24], [16], [19], [25]. In order to quantify the effect of decentralized planning in our environment, we consider five different sequential optimization approaches. The general framework of the sequential approaches are as follows:

1. Preventive maintenance threshold, τ^*, is determined considering only maintenance-related costs. It is assumed that a spare part is always available.

2. Batch size, Q^*, is determined considering all or some of the costs and revenue terms.

3. Ordering time is determined by our proposed policy.

We first characterize how τ^* is obtained in step 1. Assuming that inventory is always available, we can define independent and identically distributed cycles between each maintenance point since the lifetimes of the critical components in use are independent and identically distributed. So, at the start of each maintenance cycle, a critical component in use is at the age zero. Therefore, the optimization problem can be formulated by the renewal reward theorem. The realization of reward with a given random variable X in a single maintenance cycle, R_m, is:

$$R_m(\tau|X=x) = \begin{cases} -C_u - C_p, & \text{if } x \geq \tau \\ -C_u - C_c, & \text{otherwise.} \end{cases} \tag{3.43}$$

The realization of renewal cycle length with a given random variable X in a single maintenance cycle, T_m, is:

$$T_m(\tau|X=x) = \begin{cases} \tau, & \text{if } x \geq \tau \\ x, & \text{otherwise.} \end{cases} \tag{3.44}$$

The expected reward and expected renewal cycle length of the system in a single maintenance cycle are:

$$E[R_m(\tau)] = -C_u + \int_0^{\tau} -C_c f(x)dx + \int_{\tau}^{\infty} -C_p f(x)dx, \tag{3.45}$$

$$E[T_m(\tau)] = \int_0^{\tau} x f(x)dx + \int_{\tau}^{\infty} \tau f(x)dx. \tag{3.46}$$

Furthermore, τ^* is found where the long-run average maintenance cost rate, G_m, is minimized:

$$G_m(\tau) = \frac{E[R_m(\tau)]}{E[T_m(\tau)]}. \tag{3.47}$$

Sequential approaches differ from each other in how they determine the order quantity, Q^*. The differences can be examined under two categories; considered revenue and costs terms, and assumption made on time between two subsequent spare time demand occurrences. These sequential approaches are named as S1, S2, S3, S4, and S5.

3.6.1 Derivation of Objective Function When S1 is Employed

Let Λ be the inter-event time between two subsequent spare part demands. In S1, we solve the order quantity decision by assuming that unit demand occurs every $\Lambda = \tau$ period. Also, S1 considers only inventory-related costs, C_h and C_k. Since the revenue is not considered in this approach, supplier lead time also becomes obsolete. The sample path of system states realization becomes as in Figure 3.6.

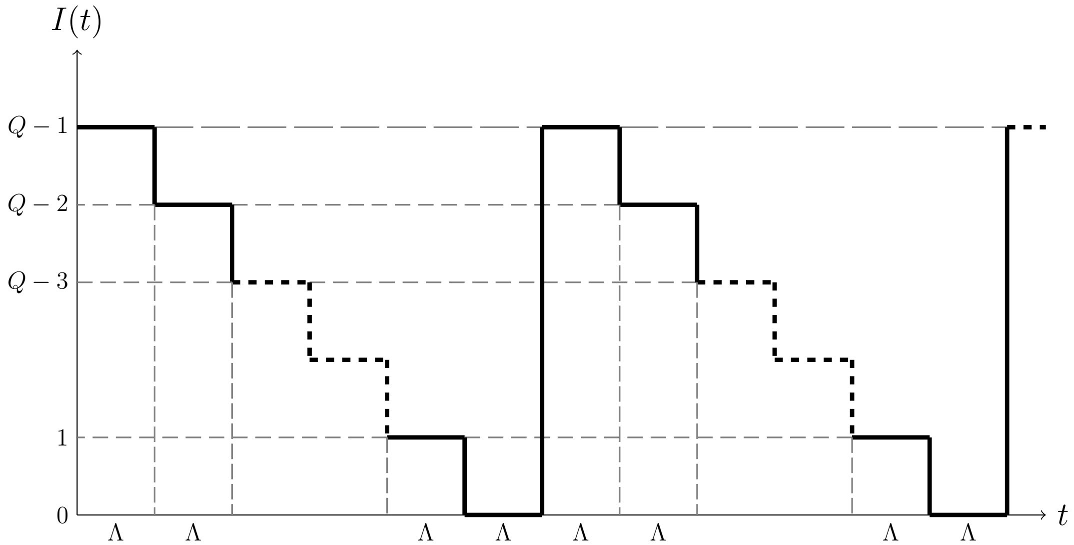

Figure 3.6: Sample Path of Sequential Approaches with Deterministic Demand

Since the revenue is not considered in this approach, up-time of the system does not considered. Therefore, total cost over a replenishment cycle, $R_{S1}(\tau^*, Q)$ is:

$$R_{S1}(\tau^*, Q) = C_k + \tau^* C_h \frac{(Q-1)Q}{2}, \tag{3.48}$$

and length of a replenishment cycle, $T_{S1}(\tau^*, Q)$, is:

$$T_{S1}(\tau^*, Q) = \tau^* Q. \tag{3.49}$$

Since no random aspects are taken into account, Q^* can be found by minimizing total cost over a single replenishment cycle length as follows:

$$G_{S_1}(\tau^*, Q) = \frac{R_{S1}(\tau^*, Q)}{T_{S1}(\tau^*, Q)}. \tag{3.50}$$

3.6.2 Derivation of Objective Function When S2 is Employed

In S2, the order quantity is determined based on the assumption that unit demand occurs every $\Lambda = E[X]$ periods. Also, S2 considers inventory-related costs, C_h and C_k. Since the revenue is not considered in this approach, supplier lead time also becomes obsolete. The sample path of system states realization becomes as in Figure 3.6. Therefore, total cost over a replenishment cycle, $R_{S2}(\tau^*, Q)$ is:

$$R_{S2}(\tau^*, Q) = C_k + E[X] C_h \frac{(Q-1)Q}{2}, \tag{3.51}$$

and length of a replenishment cycle, $T_{S2}(\tau^*, Q)$, is:

$$T_{S2}(\tau^*, Q) = E[X] Q. \tag{3.52}$$

Again, since no random aspects are taken into account, Q^* can be found by minimizing total cost over a single replenishment cycle length as follows:

$$G_{S_2}(\tau^*, Q) = \frac{R_{S2}(\tau^*, Q)}{T_{S2}(\tau^*, Q)}. \tag{3.53}$$

3.6.3 Derivation of Objective Function When S3 is Employed

In S3, we solve the order quantity decision by assuming that unit demand occurs every $\Lambda = E[min(X, \tau)]$ periods. Also, S3 considers inventory-related costs, C_h and C_k. Since the revenue is not considered in this approach, supplier lead time also becomes obsolete. The sample path of system states realization becomes as in Figure 3.6. Therefore, total cost over a replenishment cycle, $R_{S3}(\tau^*, Q)$ is:

$$R_{S3}(\tau^*, Q) = C_k + E[min\{X, \tau^*\}]C_h \frac{(Q-1)Q}{2}. \tag{3.54}$$

and length of a replenishment cycle, $T_{S3}(\tau^*, Q)$, is:

$$T_{S3}(\tau^*, Q) = E[min\{X, \tau^*\}]Q. \tag{3.55}$$

Again, since no random aspects are taken into account, Q^* can be found by minimizing total cost over a single replenishment cycle length as follows:

$$G_{S_3}(\tau^*, Q) = \frac{R_{S3}(\tau^*, Q)}{T_{S3}(\tau^*, Q)}. \tag{3.56}$$

3.6.4 Derivation of Objective Function When S4 is Employed

In S4, when determining the order quantity, inventory-related costs, C_h and C_k, and revenue are taken into account. So, replenishment cycle length calculations include the supplier lead time's effect this time. The proposed policy's objective function is employed for determining order quantity. However, maintenance-related costs are not included in the objective function. Also, predetermined τ^* is a parameter of the objective function of S4. Therefore, the realization of reward with given random variables of X's in a single replenishment cycle, $R_{S4}(\tau^*, Q)$ is:

$$\begin{aligned} R_{S4}(\tau^*, Q|X_1, ..., X_Q) = \sum_{i=1}^{Q-1} \Bigg[& min\{X_i, \tau^*\}r - \big(C_h(Q-i)\big) \Bigg] \\ & + \Bigg[min\{X_Q, T_E\}r \Bigg] - C_k. \end{aligned} \tag{3.57}$$

Equation (3.57) is nothing but Equation (3.26) where the maintenance-related costs are excluded, which are C_p, C_c and C_u. Therefore, the expected reward of the system under S4 is:

$$\begin{aligned} E[R_{S4}] = & \sum_{i=1}^{Q-1} \Bigg[\int_0^{\tau^*} \Big(x\big(r - C_h(Q-i)\big) \Big) f(x)dx \\ & + \int_{\tau^*}^{\infty} \Big(\tau^*\big(r - C_h(Q-i)\big) \Big) f(x)dx \Bigg] + E[R_E] - C_k, \end{aligned} \tag{3.58}$$

In Equation (3.58), $E[R_E]$ term is determined as it is explained in the approximations. When we consider the realization of the replenishment cycle length, it can be seen that it is the same as in Equation (3.23). Therefore, the expected length in a replenishment cycle is the same as in Equation (3.25), except for the usage of predetermined τ^*. Q^* can be found by maximizing expected reward over expected replenishment cycle length as follows:

$$G_{S_4}(\tau^*, Q) = \frac{E[R_{S4}(\tau^*, Q)]}{E[T(\tau^*, Q)]}. \tag{3.59}$$

3.6.5 Derivation of Objective Function When S5 is Employed

In this approach, while determining the Q^* value of the batch size, inventory-related costs, which are C_h and C_k, revenue gained per unit time r and maintenance-related costs, which are C_c, C_p and C_u, are considered. This approach is similar to the proposed policy. The only difference is that instead of an integrated solution approach, a sequential solution approach is used. So, the expected reward and cycle length for a single renewal cycle in the proposed policy determine the batch size. Since this is a sequential approach, predetermined τ^* age is used while deciding the spare part order quantity. As in S4, S5 also employs approximations for determining $E[R_E]$. Therefore, Q^* can be found by maximizing expected reward over expected replenishment cycle length as follows:

$$G_{S_5}(\tau^*, Q) = \frac{E[R(\tau^*, Q)]}{E[T(\tau^*, Q)]}. \tag{3.60}$$

Notice that the decision on order quantity represents the level that the inventory management function captures the systemic impact of inventory control. Besides investigating the trade-off utilizing a joint approach instead of a sequential approach, we also want to show the importance of capturing the system behavior as a whole, even in a decentralized decision-making system. The summary of sequential approaches is provided in Table 3.3.

Table 3.3: The Summary of Sequential Approaches

Methods	Cost Terms Considered While Deciding Q^*	Demand for the Spare Part
S1	C_h, C_k	τ^*
S2	C_h, C_k	$E[X]$
S3	C_h, C_k	$E[min(X, \tau^*)]$
S4	C_h, C_k, r	As in the Proposed Policy
S5	C_h, C_k, r, C_c, C_p, C_u	As in the Proposed Policy

3.6.6 Search Algorithm for Sequential Approaches

We use search algorithms for sequential approaches to find τ^* and Q^* values where $G_{S_j}(\tau^*, Q)$ is minimized for $j = 1, 2, 3$ and maximized $j = 4, 5$ sequential approach. The general structure of the search algorithm is:

1. Search for τ^* that minimizes $G_m(\tau)$ from 0 to $F^{-1}(0.9999)$ with step increments of 0.1.
2. Set $q = \left\lceil \frac{L}{\tau^*} \right\rceil$.
3. Given $Q = q$ evaluate $G_{S_j}(\tau^*, Q)$.
4. Set $q = q + 1$.
5. Given $Q = q$ evaluate $G_{S_j}(\tau^*, Q)$.
6. Return step 4 until $G_{S_j}(\tau^*, Q = q) \geq G(\tau^*, Q = q - 1)$ for $j = 1, 2, 3$, and $G_{S_j}(\tau^*, Q = q) \leq G(\tau^*, Q = q - 1)$ for $j = 4, 5$.
7. Set $Q^* = q - 1$.

The search algorithm is used for determining optimal Q^* and τ^* values under the corresponding sequential approach.

CHAPTER 4

COMPUTATIONAL STUDIES

In this chapter, we aim to answer several research questions. First, we want to investigate the performances of approximations. We have two approximation methods that differ in how they approximate the system's down-time (or equivalently up-time during the inventory level is zero). Approximations are reasonable since they are practical to use. However, the approximations' performances need to be checked either they result in similar performances with the best policy parameters. Also, by proposing two approximations (approximation 2 utilize a rougher approximate of up-time than approximation 1), we want to point out the importance of approximating the down-time in the system. Second, we want to examine the performances of sequential approaches. We have five sequential approaches that differ in how they decide order quantity. The decision is made by different costs and revenue parameters under each sequential approach. We want to show that awareness of all system parameters may yield better performances under sequential approaches. This is important since one can characterize his/her objective function by also considering the parameters in other functional units if the joint decision is not a viable option. Third, we want to examine the performances of the proposed joint approach and decentralized decision-making strategy. By doing so, the importance of realizing the trade-off between maintenance-related and inventory-related costs is clarified. Also, the parameter levels that affect the long-run average profit rate deviation from the best policy are examined to show the percentage of losses acquired using a sequential approach.

In order to clarify the above research questions, firstly, we need to find the optimal decision variables under the proposed policy as well as for the approximations and sequential approaches. In Chapter 3, search procedures for determining the preventive maintenance threshold of the critical component and order quantity are introduced for approximations and sequential approaches. However, we need to obtain the optimal policy parameters under our proposed policy as well. We can characterize the proposed policy's long-run average profit rate function analytically. However, computing the long-run average profit rate function is hard to obtain. Therefore, we use simulation optimization to obtain the optimal policy parameters. We searched τ and Q values by brute force in the simulation. The policy parameters which yield the maximum long-run average profit rate are selected as the best solution.

We conduct a computational study to analyze the policy variables found in approximations and the sequential approaches. We simulate given policy parameters found by approximations and sequential approaches to obtain long-run average profit rates of the policies. We use the percentage of long-run average profit rate deviation of a given policy from the best solution as the primary performance measure. For a given problem instance, the percent deviation of a given policy from the best policy, $\Delta\%$, can be expressed as:

$$\Delta\% = \frac{G^{*} - G}{G^{*}}, \tag{4.1}$$

where G^{*} and G are the long-run average profit rates under the best policy and the given policy, respectively.

The rest of the chapter is organized as follows: In Section 4.1, we introduce the simulation optimization procedure. In Section 4.2, the full factorial experiment is demonstrated. In Section 4.3 and Section 4.4, analysis of approximations and sequential approaches are discussed, respectively. In Section 4.5, remarks on the approaches are delivered.

4.1 Simulation Optimization

In order to obtain the best policy parameters under our proposed policy, we use a simulation optimization algorithm. The simulation optimization algorithm is implemented in MATLAB R2021a on an Intel(R) Core(TM) i7-10750H CPU @ 2.60GHz processor and 32 GB of RAMS computer. The codes are available upon request.

The algorithm consists of three main parts: the inner loop, outer loop, and simulation subroutine. The inner loop of the algorithm searches for the τ values. The outer loop of the algorithm searches for the Q values. The simulation subroutine simulates the system under our proposed policy for a given pair of τ and Q. It returns the long-run average profit rate for a given policy. Let q be the searched order quantity at any level of the algorithm. Let τ_q^* be the τ value which yields the maximum long-run average profit rate, given that $Q = q$. The general procedure we used for the simulation optimization algorithm can be seen in Figure 4.1.

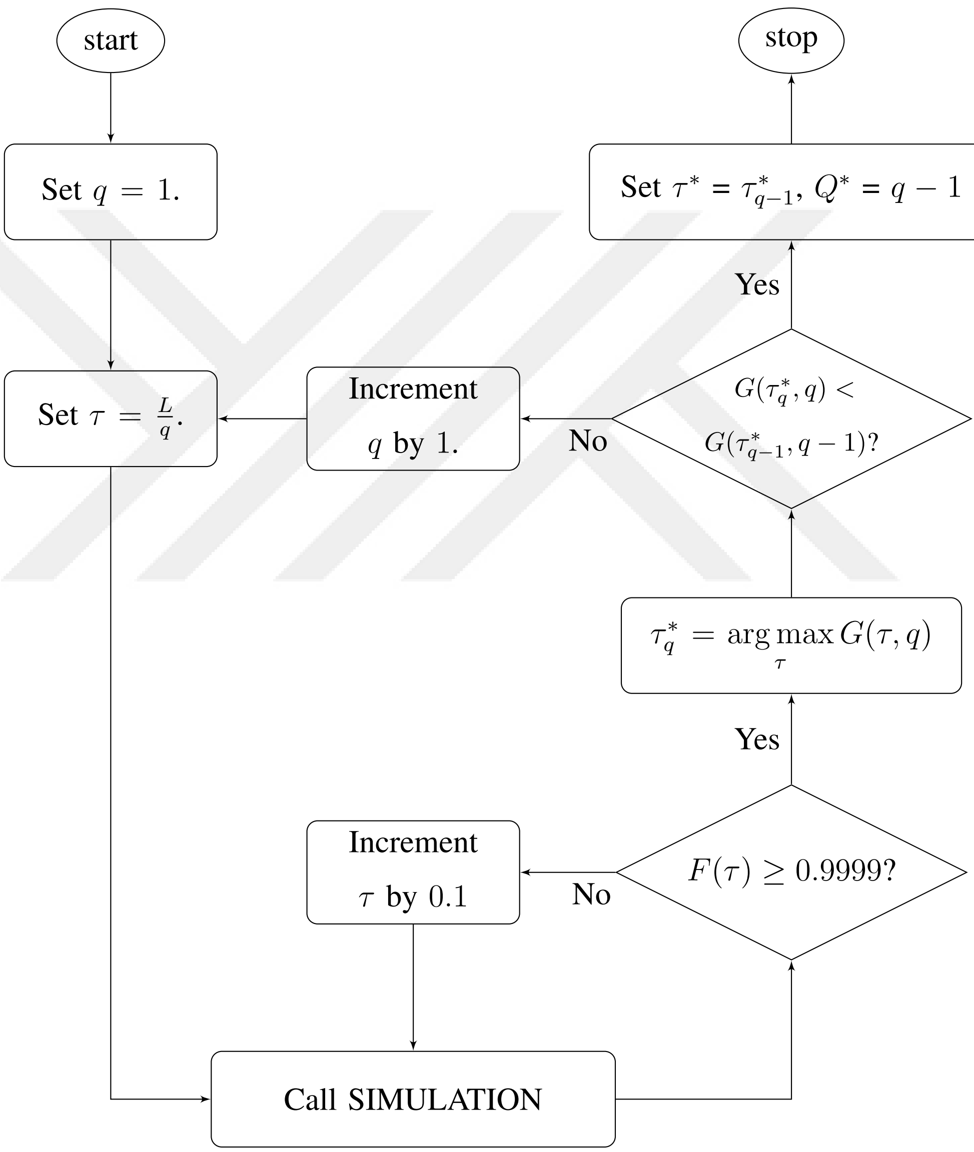

Figure 4.1: Main Algorithm of Simulation Optimization

In Figure 4.1, the main algorithm of our simulation optimization procedure is presented. The inner loop of the algorithm searches τ values. We use 0.1 step size while searching for τ values starting from $\frac{L}{q}$, until $F(\tau)$ exceeds 0.9999. The lower bound for τ search is nothing but the policy restriction, $\tau \geq \frac{L}{Q}$, introduced in Chapter 3, which is used for not allowing more than one outstanding spare part order. The upper bound for the search, $F(\tau) = 0.9999$, is the limit when the probability of corrective maintenance is practically 100%. When the search in the inner loop is completed for a q value, the τ value that yields the maximum long-run average profit rate, τ_q^*, is selected and reported.

The outer loop of the algorithm searches Q values to vary starting from one and using unit increments. The search for Q continues as long as the long-run average profit rate keeps increasing in Q. The search stops at the time when the long-run average profit rate starts decreasing, $\frac{R(\tau_q^*,q)}{T(\tau_q^*,q)} < \frac{R(\tau_{q-1}^*,q-1)}{T(\tau_{q-1}^*,q-1)}$. The reason for choosing such a termination condition for searching Q values is as follows; Although we could not prove concavity or unimodularity of the long-run average profit rate function, when we employ numerical analysis, the function seems to be unimodal in Q. A typical long-run average profit rate function is provided in Figure 4.2.

Figure 4.2: Long-run average Profit Rate Function: $r = 150$, $C_h = 1$, $C_c = 200$, $C_p = 25$, $C_k = 50$, $C_u = 200$, $L = 30$, $\lambda = 25$, $\alpha = 2$.

In Figure 4.3, we examine maximum long-run average profit rates for each Q value

$(G(\tau^*, Q)$, for $Q = 1, 2, ...)$ for the problem instance provided in Figure 4.2 except for $Q = 1$, which yields a very low long-run average profit rate and causes poor precision in the graph. The graph of the maximum long-run average profit rates, including $Q = 1$, can be seen in Appendix A. The long-run average profit rate is unimodal in Q.

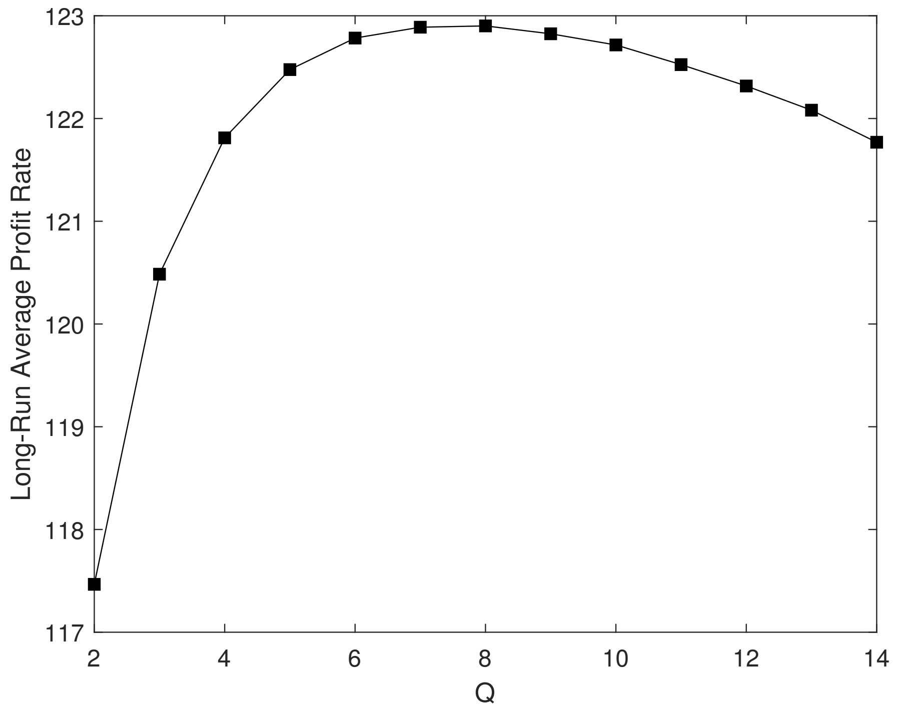

Figure 4.3: Unimodal Behaviour of Long-run Average Profit Rate Function in Q, Excluding $Q = 1$.

When the termination condition of the outer loop is satisfied, it is known that the maximum long-run average profit rate is observed when $Q^* = q-1$ and the corresponding preventive maintenance threshold of the critical component, $\tau^* = \tau^*_{q-1}$.

We also examine maximum long-run average profit rates for each τ value $(G(\tau, Q^*))$ for the problem instances provided in Figure 4.2 except for the interval $\tau = [0, 7.5]$, which yields a very low long-run average profit rate and causes poor precision in the graph. The graph of the maximum long-run average profit rates, including $\tau = [0, 7.5]$, can be seen in Appendix A.

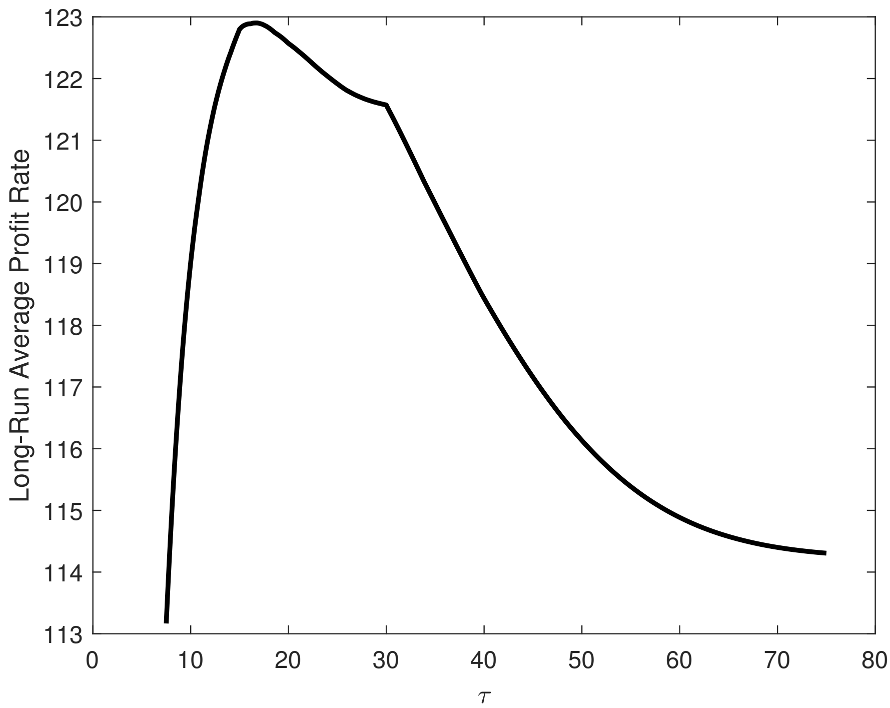

Figure 4.4: Unimodal Behaviour of Long-run Average Profit Rate Function in τ, Excluding $\tau = [0, 7.5]$.

Although τ also seems unimodal, we do not want to enforce any stopping condition that concerns long-run average profit rate while searching for τ values since τ is a continuous variable and fluctuation in simulation may results with a premature result.

The simulation subroutine is a discrete event simulation algorithm. Four different events can occur in the system. Each event has its corresponding subroutine as follows:

- PREV subroutine used for preventive maintenance events,
- FAIL subroutine used for component failure events,
- PLACE subroutine used for ordering events,
- RECE subroutine used for order receipt events.

The termination condition for the simulation is the number of order receipt events that occurred throughout the simulation run. Let j be the counter for order receipt events, and let J be the maximum number of order receipt events allowed in a simulation run. When j reaches J, the termination condition is satisfied, and the simulation subroutine returns to the main algorithm. We use $J = 10000$ for all of the problem instances. Our preliminary runs show that $J = 10000$ is satisfactory enough for the precision of the long-run average profit rates. The maximum relative half-length observed is 0.018.

In order to interpret the performances of the policies, we also define secondary performance measures. Let $TNOW$ be the simulation clock. The performance measures evaluated at the end of the simulation run and the statistical variables used for calculating them can be seen in Table 4.1.

Table 4.1: Secondary Performance Measures and Their Statistical Variables

Notation	
δ_{up}	Cumulative amount of time that the system is in operating condition at time TNOW.
δ_{spa}	Cumulative amount of time that at least one spare part is available at time TNOW.
δ_{pm}	Cumulative number of conducted preventive maintenance activities. at time TNOW.
$\%\Delta_{up}$	Percentage of time that the system is in operating condition, $\frac{\delta_{up}}{TNOW}$.
$\%\Delta_{spa}$	Percentage of time that at least one spare part is available, $\frac{\delta_{spa}}{TNOW}$.
$\%\Delta_{pm}$	Percentage of maintenances that are preventive maintenance, $\frac{\delta_{pm}}{QJ}$.

The flowchart of the discrete event simulation can be seen in Figure 4.5.

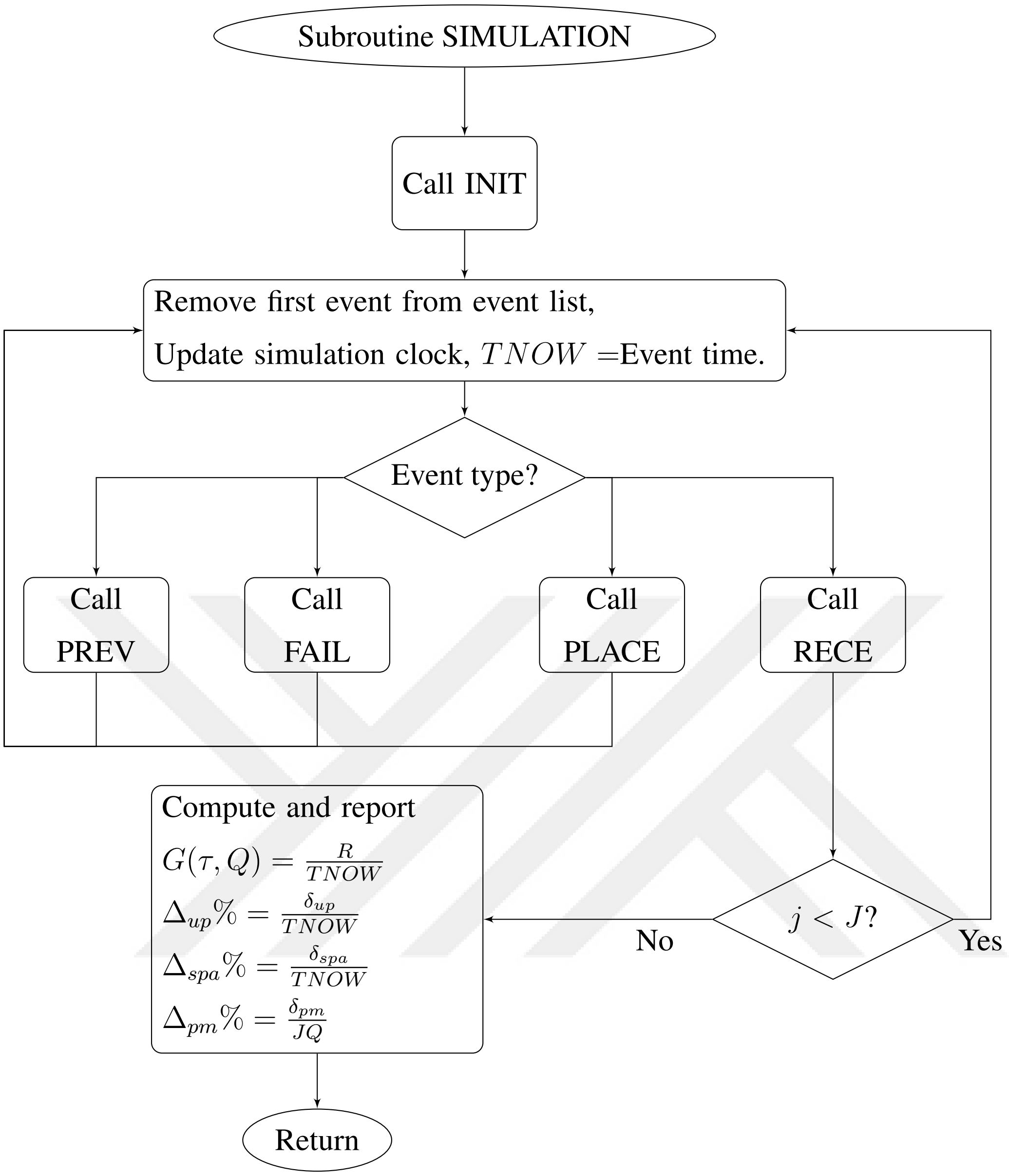

Figure 4.5: Discrete Event Simulation of the System under Proposed Policy

The simulation algorithm starts with the initialization step (see Figure 4.6), which is called INIT. In this step, the simulation clock, $TNOW$, is set to zero. The problem parameters r, C_h, C_c, C_p, C_k, C_u, L and lifetime distribution of the critical component is defined according to corresponding problem instance. The system states, which are inventory level, I, and age of the critical component, A, are initialized to $I = Q - 1$ and $A = 0$. A is only used for determining whether the system is in up or down condition. Simply, if the system is in working condition, $A = 0$. Otherwise, $A = -1$.

The statistics that are used in the simulation are set to zero. The lifetime of the critical component in use, x, is generated. If the lifetime of the critical component has a value that is less than the preventive maintenance threshold of the critical component, then the failure event is scheduled at time $TNOW + x$. In the opposite case, preventive maintenance is scheduled at time $TNOW + \tau$. After scheduling the first event and inserting it in the event list, the initialization step finishes.

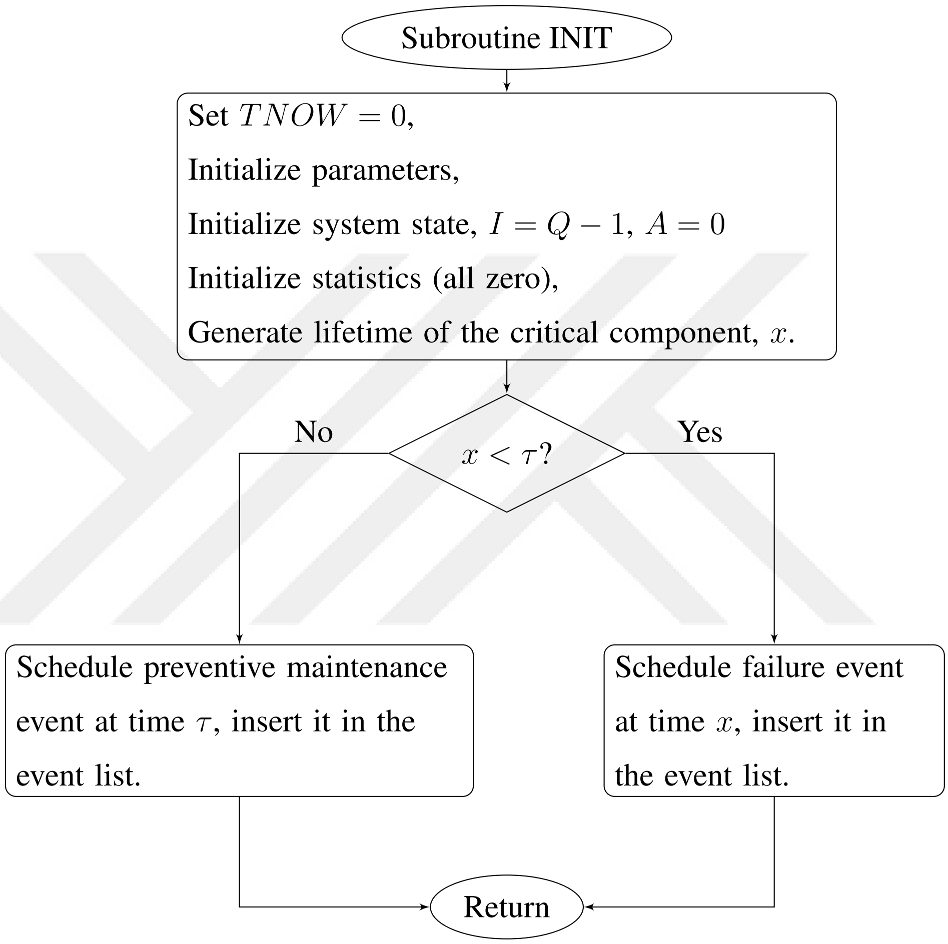

Figure 4.6: Subroutine Initialization

After the initialization step, the scheduled event is removed from the event list, and the simulation clock is updated to the event time. According to the removed event, the corresponding subroutine is called.

First, we examine the preventive maintenance event (see Figure 4.7). Let υ be the time of the inventory level change in the previous event. $(TNOW - \upsilon)$ gives us the time between the current event and the most recent event that inventory level is changed. $(TNOW - \upsilon)$ is needed for updating the statistics which are R, δ_{up} and δ_{spa}. At the start of subroutine PREV, statistics are updated, as can be seen in Figure 4.7. I is decreased by one since the spare part is installed in the system. Also, υ is set to TNOW after updating the other statistics.

Scheduling the upcoming events is similar to the same process in INIT. The difference is that if the inventory level is zero, regardless of the lifetime of the critical component in use, a failure event is scheduled. Since there are no spare parts in the inventory, preventive maintenance can not be scheduled. However, preventive maintenance can be conduct at the time of order receipt, and this event is considered in the RECE subroutine. Therefore, it is known that if the simulation entered into the PREV subroutine, then the system is in operating condition, and at least a spare part is available in the inventory. Also, if $I = \lfloor \frac{L}{\tau} \rfloor$, then ordering event is scheduled. Let Φ be an indicator variable representing whether the ordering event is in the event list or not. If $\Phi = 1$, then ordering is in the event list, and $\Phi = 0$ otherwise. In the end, PREV subroutine returns to the simulation subroutine.

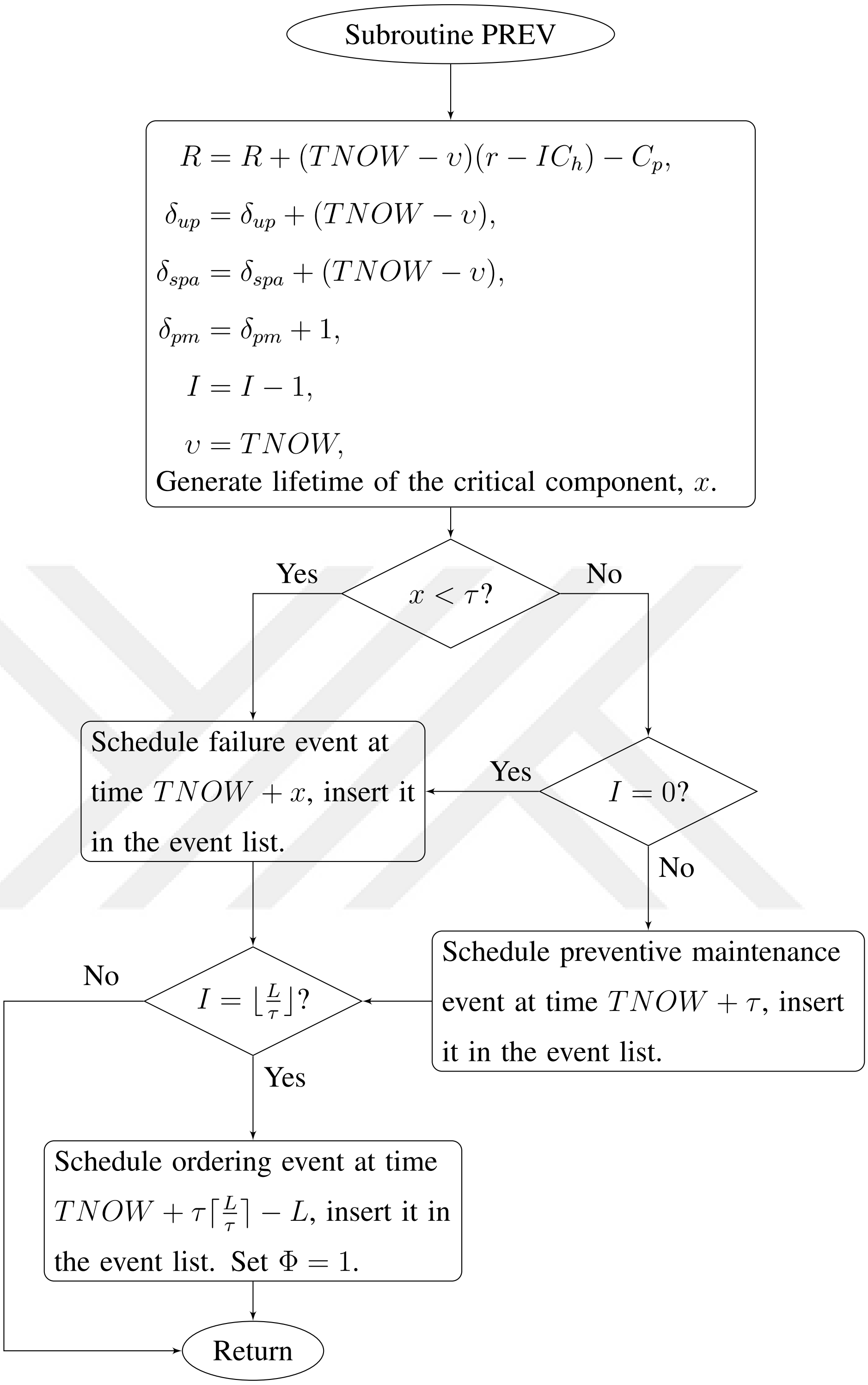

Figure 4.7: Subroutine Preventive Maintenance

Second, we examine the failure event, (see Figure 4.8). The procedure for updating statistics is similar to the same process in PREV. However, if there are no spare parts in the inventory, only the reward, and cumulative up-time is updated. A is set to -1, and the FAIL subroutine returns to the main algorithm since events can not be scheduled until the order receipt. If there is a spare part in the inventory, statistics are updated, as can be seen in Figure 4.8.

Scheduling the upcoming events is similar to the same process in PREV. The only difference is that a failure can trigger ordering. So, if the ordering is in the event list, $\Phi = 1$, then ordering event is removed from event list and subroutine PLACE is called.

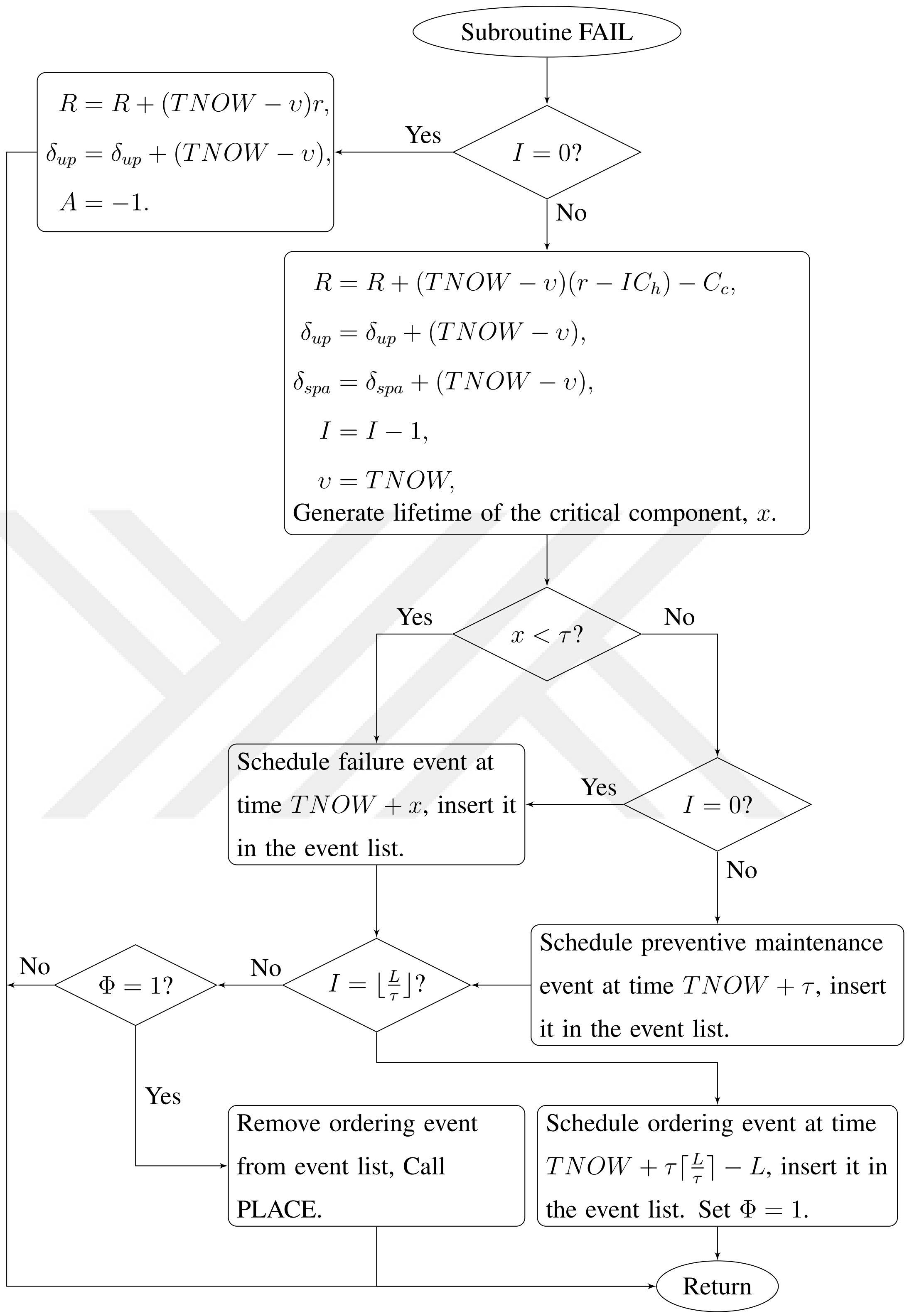

Figure 4.8: Subroutine Failure

The ordering subroutine, PLACE (see Figure 4.9), only schedules the order receipt event and inserts it into the event list. Also, since the ordering event is removed from the event list, Φ is set to zero.

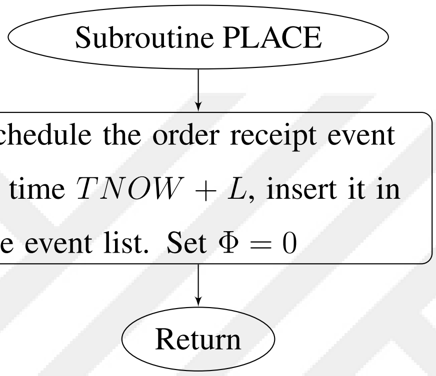

Figure 4.9: Subroutine ordering

Lastly, RECE subroutine (see Figure 4.10) is called if an order receipt event occurs. At the start of the RECE, the system is checked whether it is in working condition or not. A is set to zero if a failure occurred before the order arrival. According to A, statistics are updated. Then, the inventory level is updated to $Q - 1$ since the order has arrived and one of the spare parts is installed in the system. Counter of the order receipt, j, is increased by one. The process of scheduling the next event is identical to the process in INIT.

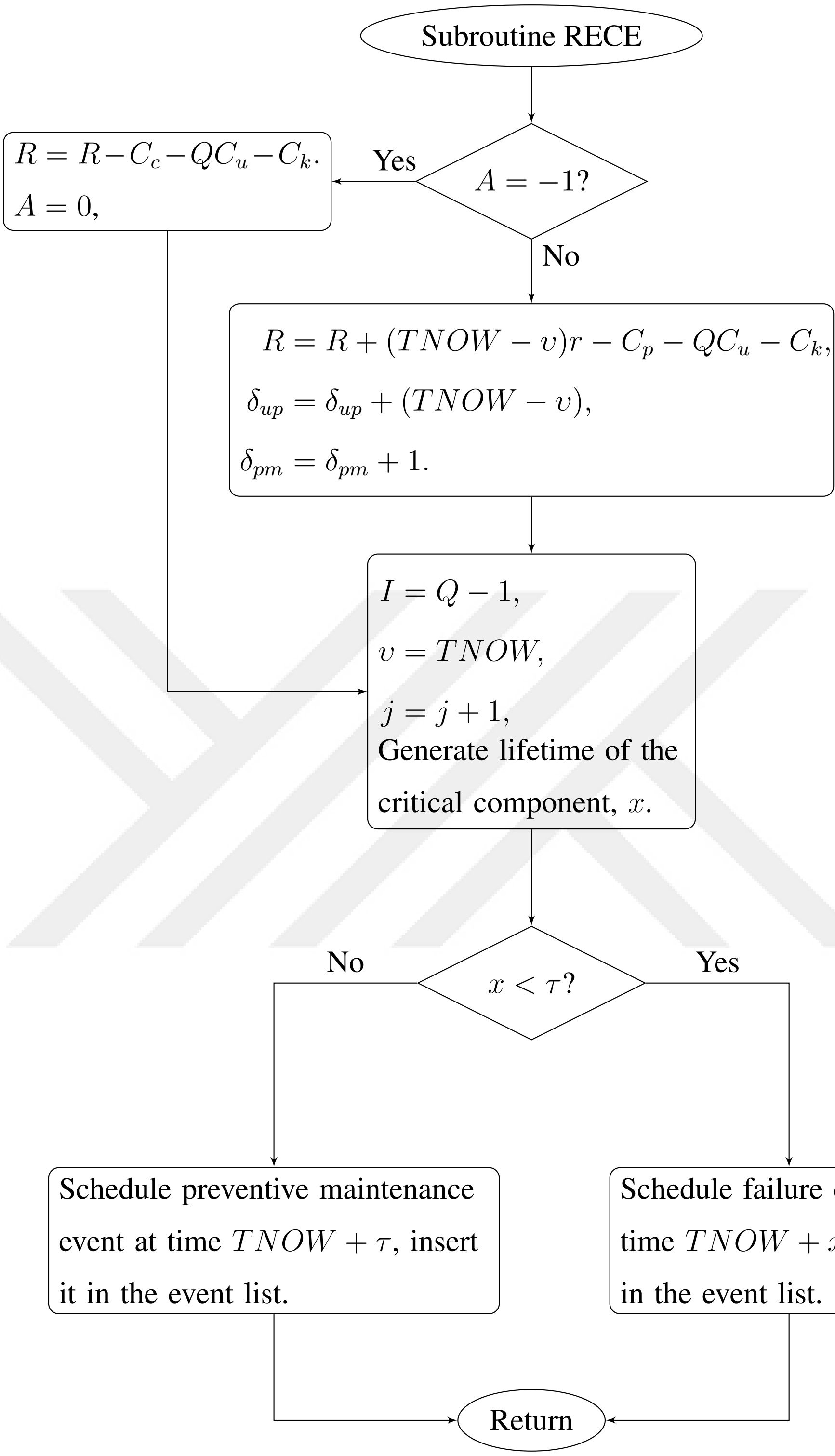

Figure 4.10: Subroutine Order Receipt

When the order counter reaches J, the simulation is terminated and performances are calculated.

After determining the preventive maintenance threshold and order quantities, five independent replications ($n = 5$) are made for each problem instance under each approach. The termination condition for a single simulation run is determined as 2000 order receipts. With these five independent replications, we constructed 95% confidence intervals to the long-run average profit rates. Let $\overline{G}$ and σ as the mean and standard deviation of the long-run average profit rate of the five replications, respectively. Since population variation is not known, we use t-distribution while constructing the confidence intervals. Therefore, the confidence intervals become:

$$\overline{G} \pm t_{(.975,n-1)} \frac{\sigma}{\sqrt{n}} \tag{4.2}$$

We use common random numbers as a variance reduction technique under each problem instance under each approach. At the start of each simulation run, the Mersenne Twister generator with seed zero is initiated to generate failure times of the critical components. We employ common random numbers to increase statistical efficiency and to compare alternatives under similar experimental conditions [9]. Using variance reduction results in obtaining confidence intervals with higher precision than those constructed without implementing any variance reduction technique under the same amount of computational effort. Therefore, differences in each performance of approaches are more likely caused by policy parameters instead of fluctuations due to random numbers used in the simulation runs.

4.2 Full Factorial Experiment

A full factorial experiment is implemented to investigate the effects of the problem parameters on the resulting policies and observe the percent long-run average rate deviation of each approach from best policy under different problem instances.

We assume that the lifetime of the critical components follows the Weibull Distribution. The pdf of the Weibull distribution is:

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, \tag{4.3}$$

where λ is the scale parameter, and k is the shape parameter. In the literature, Weibull distribution is widely used as the lifetime distribution of deteriorating systems ([3],

[9], [11], [12],[13], [15], [21], [14], [22], [26], [23], [27], [28], [16], [29], [19]). The parameter levels of factors that are used in the full factorial experiment can be seen in the Table 4.2.

Table 4.2: List of Parameter values Used for Full Factorial Design

Parameters		Levels
L	Supplier lead time	$20, 30, 60$
r	Revenue per unit time obtained when the system is in operating condition	$50, 75, 150$
C_c	Unit corrective maintenance cost	$100, 200, 400$
C_p	Unit preventive maintenance cost	25
C_h	Unit Inventory holding cost per unit time	$0.5, 1, 2$
C_u	Unit procurement cost	200
C_k	Fixed cost of ordering	$25, 50, 200$
(λ, k)	Scale and shape parameter of Weibull distribution	$(24.54, 1.5)$, $(25, 2)$, $(24.13, 5)$

In total, 729 problem instances are considered in the full factorial experiment. For L, r, C_c, C_h and C_k parameters, three levels are considered. We select a single value for C_p since only the difference between C_c and C_p levels matters. We also consider three levels for the shape parameter of the Weibull distribution. Scale parameter of the Weibull distribution is selected so that the expected lifetime of the critical component remains the same.

Scale parameters are selected to ensure an increasing failure rate exists, $(k > 1)$. Probability density functions of the Weibull distributions that are considered in the full factorial experiment can be seen in the Figure 4.11.

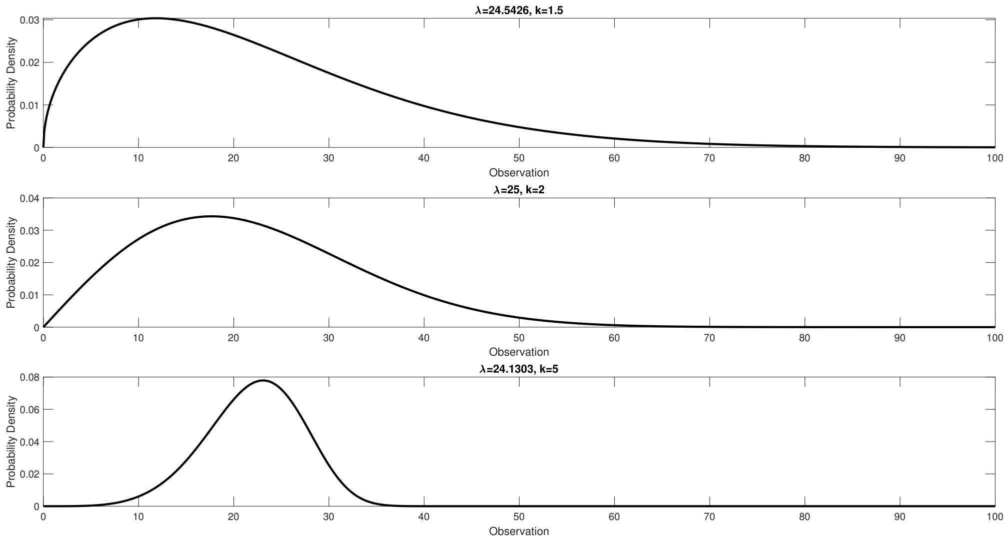

Figure 4.11: Probability Density Functions of Weibull Distribution.

Descriptive statistics of the considered probability density functions of the Weibull distribution are provided in the Table 4.3.

Table 4.3: Descriptive Statistics of Considered Probability Density Functions

(λ, k)	Mean	Median	Mode	Variance	Skewness
$(1.5, 24.54)$	22.15	19.22	11.80	226.29	1.07
$(2, 25)$	22.15	20.81	17.68	134.13	0.63
$(5, 24.13)$	22.15	22.42	23.07	25.75	-0.25

From Figure 4.11 and Table 4.3, it can be seen that, as the shape parameter increases, the variance of the distribution decreases drastically.

4.3 Comparison of Approximations

In this section, we investigate the overall performances of the approximations and the effects of utilizing approximations on the policy parameters. A1 and A2 corresponds for Approximation 1 and Approximation 2 in the rest of the manuscript, respectively.

In 672 out of 729 problem instances, A1 statistically performs better than A2. On the other hand, there are no problem instances where A2 statistically performs better than A1. In more than 99% of the problem instances, A1 results in statistically insignificant long-run average profit rate deviation from the best policy. In contrarily, in more than 92% problem instances, A2 results in statistically significant long-run average profit rate deviation from the best policy. Overall performances of the approximations in terms of percentage deviation from the best policy are summarized in Table 4.4.

Table 4.4: Performances of the Approximations

	Number of instances				
	$\Delta\% < 0.1$	$0.1 < \Delta\% < 1$	$1 < \Delta\% < 10$	$10 < \Delta\% < 20$	$\Delta\% > 20$
A1	655	74	0	0	0
A2	18	114	390	153	54

	Worst $\Delta\%$	Average $\Delta\%$	Best $\Delta\%$
A1	0.88	0.04	0.00
A2	50.89	7.50	0.00

A1 results in a much lower percentage deviation from the best policy compared to A2. In almost 90% of the problem instances, A1 results in less than 0.1 percent deviation from the best policy. In comparison, A2 results with a more than 1 percent deviation from the best policy in more than 80% problem instances.

For 169 problem instances, A1 could find the best policy parameters. The problem instances in which A1 results with the best policy parameters can be seen in Appendix B. On average, 0.04 percent profit deviation is observed, and even in the worst result, 0.88 percent profit deviation is obtained. The problem parameters, in which the worst result of A1 is observed, are $r = 50$, $C_h = 2$, $C_c = 100$, $C_k = 25$, $L = 60$, $k = 1.5$. In contrarily, A2 only achieves the best policy parameters in 7 problem instances. It should be noted that A1 can also find the best policy parameters in those problem instances. On the average, A2 performs with a 7.5 percent deviation, far beyond the worst result of A1. Worst performance of A2 results in the 50.89 percent deviation from the best policy's long-run average profit rate. The best and worst percent profit deviations observed for A2 can be seen in Table 4.5.

Table 4.5: Problem Instances in which Approximation 2 Performs at Best and Worst

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
50	1	100	25	60	$(24.13, 5)$	0	Best
50	2	100	25	60	$(24.13, 5)$	0	Best
50	2	100	50	60	$(24.13, 5)$	0	Best
50	2	200	25	60	$(24.13, 5)$	0	Best
50	2	200	50	60	$(24.13, 5)$	0	Best
75	2	100	25	60	$(24.13, 5)$	0	Best
75	2	100	50	60	$(24.13, 5)$	0	Best
75	2	100	50	60	$(24.13, 5)$	0	Best
50	2	400	200	20	$(24.54, 1.5)$	50.89	Worst

The common parameter levels among the problem instances in which A2 performs best are $L = 60$ and $(\lambda, k) = (24.13, 5)$. Those are the highest levels of supplier lead time and shape parameter of the Weibull Distribution. When we look at the problem instance in which A2 performs worst, it is seen that supplier lead time and shape parameter of the Weibull distributions take their lowest levels, $L = 20$ and $(\lambda, k) = (24.54, 1.5)$. In order to further investigate the effects of the problem instances on the performances of the approximations, the average $\Delta\%$ values and the average of the policy parameters with respect to each parameter level are given in Table 4.6. The average of the secondary performance measures with respect to each parameter level is given in Table 4.7.

Table 4.6: Average $\Delta\%$ and Policies of Approximations

Parameter Levels	Approximation 1 $\Delta\%$	Approximation 2 $\Delta\%$
$r = 50$	0.06	8.21
$r = 75$	0.04	7.01
$r = 150$	0.03	7.29
$C_h = 0.5$	0.03	8.41
$C_h = 1$	0.03	7.49
$C_h = 2$	0.06	6.61
$C_c = 100$	0.03	7.48
$C_c = 200$	0.03	7.70
$C_c = 400$	0.06	7.34
$C_k = 25$	0.04	7.73
$C_k = 50$	0.04	8.33
$C_k = 200$	0.04	6.45
$L = 20$	0.03	11.73
$L = 30$	0.04	7.25
$L = 60$	0.05	3.53
$k = 1.5$	0.06	12.75
$k = 2$	0.04	7.76
$k = 5$	0.02	2.01

Table 4.7: Secondary Performance Measures of the Approximations

	Best Policy			Approximation 1			Approximation 2		
	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$
$r = 50$	93.57	76.71	48.03	93.63	76.34	48.67	89.19	65.04	56.11
$r = 75$	95.34	78.89	53.68	95.41	78.75	54.10	89.21	65.09	56.17
$r = 150$	97.20	81.74	62.51	97.21	81.51	62.90	89.27	65.39	56.30
$C_h = 0.5$	96.54	85.96	51.02	96.61	86.03	51.45	89.92	67.81	56.71
$C_h = 1$	95.48	79.71	54.95	95.49	79.73	55.20	89.26	65.21	56.21
$C_h = 2$	94.09	71.65	58.25	94.14	70.84	59.02	88.49	62.49	55.67
$C_c = 100$	94.90	81.87	46.00	94.97	81.84	46.72	89.07	66.52	53.32
$C_c = 200$	95.29	79.10	54.12	95.37	78.94	54.46	88.93	66.39	53.89
$C_c = 400$	95.92	76.36	64.10	95.90	75.83	64.49	89.66	62.60	61.38
$C_k = 25$	95.20	75.55	57.15	95.22	75.17	57.46	87.68	60.98	55.78
$C_k = 50$	95.26	77.81	55.84	95.33	77.58	56.43	87.91	62.19	55.99
$C_k = 200$	95.65	83.96	51.24	95.69	83.85	51.79	92.07	72.35	56.81
$L = 20$	96.04	84.43	52.71	96.13	84.01	53.53	85.52	80.65	50.37
$L = 30$	95.54	75.88	53.01	95.56	75.82	53.38	89.43	48.61	55.21
$L = 60$	94.53	77.02	58.50	94.55	76.77	58.76	92.72	66.24	63.00
$k = 1.5$	92.46	83.73	34.17	92.66	83.41	35.76	83.41	60.84	49.10
$k = 2$	94.98	81.02	51.09	94.98	80.74	51.62	87.85	65.47	52.67
$k = 5$	98.68	72.58	78.96	98.60	72.46	78.30	96.40	69.21	66.82
Average	95.37	79.11	54.74	95.41	78.87	55.22	89.22	65.17	56.19

It seems that the poor performance of A2 is mainly caused by setting order quantity lower. In all problem instances, the order quantity found by A2 highly deviates from the best policy's order quantity. In the high levels of supplier lead time, to obtain a specific preventive maintenance threshold value, A2 is forced to increase its order quantity. Therefore, A2's order quantity comes closer to the best policy's order quantity, which causes a decrease in percent profit deviation while supplier lead time increases. While the shape parameter increases, both approximations perform better. Since the variation of the lifetime of the critical part decreases, there is an increase in the accuracy of downtime approximations.

When we examine the rest of the parameters, policy parameters in best policy and policy parameters in A1 are similar to each other. While revenue increases, policies tend to increase their percentage up-time as expected. An increase in the percentage of up-time can be achieved by decreasing the preventive maintenance threshold and/or increasing the order quantity. Decreasing the preventive maintenance threshold is reasonable because the system becomes more likely to do preventive maintenance, resulting in a decrease in the system's downtime. Conducting preventive maintenance at the early ages of the critical component also causes an increase in the percentage of conducted preventive maintenance activities. Recall that downtime can only be observed when the system runs out of stock. So, increasing order quantity means adding a maintenance cycle to the renewal cycle, which has 100% up-time, resulting in an increase in the percentage of spare part availability. When inventory holding cost increases, the system favors decreasing their percentage of spare part availability. So, both policy parameters are decreased. Decreasing the preventive maintenance threshold means a faster turnover of spare parts. Decreasing the preventive maintenance threshold also causes an increase in the percentage of conducted preventive maintenance activities. Decreasing order quantity means avoiding higher levels of inventory. When corrective maintenance cost increases, A1 and A2 tend to raise their percentage of conducted preventive maintenance activities. Therefore, A1 and A2 decrease preventive maintenance threshold values, which also causes an increase in the percentage of up-time. While fixed cost increases, A1 and A2 seek to increase their renewal cycle lengths. Therefore an increase is observed in both policy parameters. This behavior increases the percentage of spare part availability and reduces the percentage of conducted preventive maintenance activities.

It can be concluded that A1 performs better than A2. It is not surprising since A2 employs a rough approximation for the system's downtime, while A1 uses a more accurate approximation. However, the results show that precision is crucial while approximating the downtime since even a small proportion of downtime may significantly affect the system's long-run average profit rate.

4.4 Comparison of Sequential Approaches

In this section, we investigate the overall performances of the sequential approaches. We aim to observe the percentage of loss due to considering decentralized decision-making rather than considering a joint policy. Also, we intend to investigate the improvement in the long-run average profit rate due to considered parameters in the objective function utilized by sequential approaches. In the previous section, it is concluded that A1 performs better than A2. Therefore, $E[R_E]$ is approximated by utilizing A1 in S4 and S5. Comparative performances of the sequential approaches can be seen in Table 4.8, ($(i, j)^{th}$ entry of the table corresponds for the number of problem instances that i^{th} approach performs statistically better than j^{th} approach in terms of long-run average profit rate).

Table 4.8: Comparative Performances of the Sequential Approaches

						Deviation from the Best Policy's $\Delta\%$	
	S1	S2	S3	S4	S5	Insignificant	Significant
S1	-	36	0	0	0	69	660
S2	246	-	0	0	0	40	689
S3	308	98	-	0	0	83	646
S4	602	636	580	-	2	115	614
S5	607	643	588	75	–	148	581

Recall that the hierarchical structure between sequential approaches that S1, S2, and S3 only consider inventory-related costs, S4 considers the inventory-related costs and revenue, and S5 considers all problem parameters. It is observed that, generally, when a sequential approach considers an additional parameter, it results in a better performance than the rest of the sequential approaches which do not consider that parameter. The gap between the performances of the first three sequential approaches and the last two sequential approaches can be seen clearly. Also, a minor increase in the performances is observed between S5 and S4 due to considering maintenance-related costs while determining order quantity. At 36 of the problem instances, S1 statistically performs better than S2. These problem instances' common property is the high levels of corrective maintenance cost values, which causes low preventive maintenance threshold values. S1 assumes that the replenishment cycle length is τQ, whereas S2 assumes the replenishment cycle length as $E[X]Q$. Since the preventive maintenance threshold takes lower values on those problem instances, S1 uses a more accurate approximation of the replenishment cycle length. In contrast, S2 approximates the replenishment cycle length relatively poorly. Also, all of the approaches' long-run average profits significantly differ from the best policy's long-run average profits in most of the problem instances. The best-performing approach, S5, is indifferent from the best policy only for 20% of the problem instances. Overall performances of the sequential approaches in terms of percentage profit deviation from the best policy are summarized in Table 4.9.

Table 4.9: Performances of the Sequential Approaches

	Number of instances				
	$\Delta\% < 0.1$	$0.1 < \Delta\% < 1$	$1 < \Delta\% < 10$	$10 < \Delta\% < 20$	$\Delta\% > 20$
S1	35	98	225	77	294
S2	2	128	253	135	211
S3	41	102	246	129	211
S4	49	194	465	21	0
S5	52	214	446	17	0

	Worst $\Delta\%$	Average $\Delta\%$	Best $\Delta\%$
S1	70.17	19.28	0.00
S2	67.93	13.91	0.08
S3	67.93	13.67	0.00
S4	14.69	2.80	0.00
S5	14.07	2.54	0.00

S1, S2, and S3 result in more than 20% deviation from the best policy in the most of the problem instances. S4 and S5 performed better than the rest of the approaches by considering the revenue generated in the system. In most of the problem instances, S4 and S5 deviate from the best policy less than 2%.

On average, S1 is the worst performing approach with the 19.28 percent deviation from the best policy. S2 and S3 do not differ from each other drastically. There is a slight improvement when the expected replacement time is assumed instead of the expected lifetime of the critical component while computing a single maintenance cycle length. S4 and S5 are far better than the rest of the approaches since they utilize downtime information. Also, there is a slight improvement when the maintenance-related cost of the system is considered.

The best and worst percentage deviations observed for sequential approaches can be seen in Table 4.10 below.

Table 4.10: Problem Instances in which Sequential Approaches Performed at Best and Worst

	r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
S1	75	2	400	200	20	$(24.13, 5)$	0	Best
	150	0.5	100	50	60	$(24.54, 1.5)$	70.17	Worst
S2	50	2	400	200	60	$(24.13, 5)$	0.08	Best
	150	2	100	25	60	$(24.54, 1.5)$	67.93	Worst
S3	75	2	400	200	20	$(24.13, 5)$	0	Best
	150	2	100	25	60	$(24.54, 1.5)$	67.93	Worst
S4	75	2	400	200	20	$(24.13, 5)$	0	Best
	50	2	100	25	60	$(24.54, 1.5)$	14.69	Worst
S5	75	2	400	200	20	$(24.13, 5)$	0	Best
	50	2	100	25	60	$(24.54, 1.5)$	14.07	Worst

It is observed that all sequential approaches present their worst performance when corrective maintenance cost and shape parameter of the Weibull distribution are at their lowest levels, $C_c = 100$, $(\lambda, k) = (24.54, 1.5)$, supplier lead time is at its highest level $L = 60$, and fixed cost is at its low levels, $C_k = 25$. Also, since S1, S2, and S3 do not concern revenue generated in the system while deciding the policy parameters, they perform their worst when level of revenue is at its highest, $r = 150$. As expected, sequential approaches achieve their best performance in the opposite case, when $C_c = 400$, $(\lambda, k) = (24.13, 5)$, $L = 20$ and $C_k = 200$. In order to further study the consequences of the problem instances on the performances of the sequential approaches, the average percent profit deviation values, $\Delta\%$, and the average of the policy parameters with respect to each parameter level are given in Table 4.11. The average of the secondary performance measures with respect to each parameter level is given in Table 4.12.

Table 4.11: Average $\Delta\%$ and Policies of Sequential Approaches

Parameter Levels	S1 $\Delta\%$	S2 $\Delta\%$	S3 $\Delta\%$	S4 $\Delta\%$	S5 $\Delta\%$
$r = 50$	17.83	12.11	11.85	3.26	2.67
$r = 75$	19.19	13.83	13.59	2.78	2.60
$r = 150$	20.84	15.80	15.57	2.37	2.35
$C_h = 0.5$	17.15	11.97	11.70	1.60	1.42
$C_h = 1$	19.92	13.82	13.45	2.64	2.38
$C_h = 2$	20.78	15.95	15.86	4.16	3.81
$C_c = 100$	29.48	19.93	19.93	4.13	3.98
$C_c = 200$	20.03	14.32	14.29	2.75	2.51
$C_c = 400$	8.35	7.49	6.79	1.53	1.13
$C_k = 25$	23.50	19.03	18.91	3.15	2.87
$C_k = 50$	21.10	14.62	14.40	2.97	2.70
$C_k = 200$	13.26	8.09	7.69	2.28	2.05
$L = 20$	15.55	10.00	9.78	1.70	1.47
$L = 30$	19.33	13.30	13.00	2.64	2.42
$L = 60$	22.97	18.44	18.23	4.06	3.72
$k = 1.5$	36.62	24.35	24.22	4.30	3.76
$k = 2$	19.45	15.52	15.02	3.29	3.04
$k = 5$	1.78	1.88	1.77	0.82	0.82

Table 4.12: Secondary Performance Measures of the Sequential Approaches

	Best Policy			Sequential 1			Sequential 2			Sequential 3			Sequential 4			Sequential 5		
	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$	$\Delta_{up}\%$	$\Delta_{spa}\%$	$\Delta_{pm}\%$
$r = 50$	93.57	76.71	48.03	74.60	59.60	31.44	79.40	63.95	31.42	79.61	65.03	31.49	92.69	82.66	32.13	91.24	80.39	32.00
$r = 75$	95.34	78.89	53.68	74.60	59.60	31.44	79.40	63.95	31.42	79.61	65.03	31.49	93.90	85.18	32.25	93.10	83.60	32.17
$r = 150$	97.20	81.74	62.51	74.60	59.60	31.44	79.40	63.95	31.42	79.61	65.03	31.49	95.56	88.39	32.39	95.29	87.86	32.36
$C_h = 0.5$	96.54	85.96	51.02	77.92	64.99	31.64	82.75	69.32	31.64	82.99	70.72	31.73	95.93	89.85	32.45	95.33	88.75	32.41
$C_h = 1$	95.48	79.71	54.95	74.10	58.83	31.39	79.57	63.58	31.35	79.88	64.95	31.44	94.26	85.77	32.27	93.44	84.39	32.21
$C_h = 2$	94.09	71.65	58.25	71.78	55.00	31.27	75.88	58.96	31.27	75.97	59.43	31.30	91.95	80.61	32.04	90.85	78.71	31.91
$C_c = 100$	94.90	81.87	46.00	63.72	53.55	13.76	72.33	62.40	13.76	72.33	62.40	13.76	92.45	88.28	14.54	91.63	87.03	14.47
$C_c = 200$	95.29	79.10	54.12	73.75	59.31	29.78	78.82	64.23	29.76	78.84	64.52	29.78	93.99	85.22	30.34	93.16	83.83	30.30
$C_c = 400$	95.92	76.36	64.10	86.32	65.95	50.77	87.05	65.23	50.74	87.67	68.18	50.93	95.71	82.73	51.88	94.83	80.99	51.76
$C_k = 25$	95.20	75.55	57.15	69.98	52.10	31.14	74.06	55.78	31.13	74.17	56.44	31.16	93.84	83.96	32.19	92.93	82.34	32.09
$C_k = 50$	95.26	77.81	55.84	72.62	56.24	31.28	78.42	61.50	31.26	78.61	62.63	31.33	93.94	84.81	32.22	93.04	83.15	32.13
$C_k = 200$	95.65	83.96	51.24	81.19	70.47	31.89	85.72	74.58	31.86	86.06	76.03	31.98	94.36	87.46	32.35	93.65	86.37	32.30
$L = 20$	96.04	84.43	52.71	78.21	70.20	32.84	83.49	74.73	32.82	83.69	75.72	32.84	95.24	89.05	32.89	94.59	88.19	32.88
$L = 30$	95.54	75.88	53.01	74.72	58.54	31.55	80.13	63.20	31.52	80.39	64.81	31.62	94.34	84.86	32.30	93.52	83.33	32.23
$L = 60$	94.53	77.02	58.50	70.87	50.07	29.92	74.57	53.93	29.92	74.75	54.57	30.00	92.56	82.32	31.58	91.51	80.33	31.42
$k = 1.5$	92.46	83.73	34.17	53.84	50.80	6.83	64.71	61.70	6.83	64.82	61.85	6.84	90.91	90.05	7.17	89.46	88.39	7.14
$k = 2$	94.98	81.02	51.09	73.53	60.23	22.42	77.12	64.10	22.42	77.58	65.37	22.56	93.28	88.73	23.52	92.36	87.01	23.40
$k = 5$	98.68	72.58	78.96	96.43	67.78	65.06	96.36	66.05	65.01	96.44	67.87	65.07	97.95	77.45	66.08	97.80	76.46	65.98
Average	95.37	79.11	54.74	74.60	59.60	31.44	79.40	63.95	31.42	79.61	65.03	31.49	94.05	85.41	32.25	93.21	83.95	32.18

Recall that in sequential approaches, we first decide the critical component's preventive maintenance threshold and then decide the order quantity of spare parts. At determining the preventive maintenance threshold step, it seems that setting the preventive maintenance threshold lower is not as beneficial as in the joint policy. In the absence of downtime information, high values of preventive maintenance threshold are selected in the sequential approaches. This is the main problem of the sequential approaches that they decide on a much higher preventive maintenance threshold than the preventive maintenance threshold decided in the best policy. Also, since the policy parameters are decided sequentially, and the preventive maintenance threshold is set according to only considering maintenance-related costs, the preventive maintenance threshold can not react to most of the problem parameters. Therefore, sequential approaches have stationary preventive maintenance threshold values when revenue, inventory holding cost, fixed cost, and supplier lead time levels are changed. In addition, since revenue is not concerned with S1, S2, and S3, order quantity is also stationary in those policies when the revenue level is changed.

The first three approaches and the last two approaches mostly have the same behaviors according to a parameter level change. Firstly, we examine the first three approaches. These approaches become even worse as the revenue level increases. As it is said in Section 4.3, order quantity can be used to increase up-time. By ignoring the system's revenue, those approaches develop low order quantity values. So, the absence of revenue information affects both of the policy parameters in those approaches. This explains observing $r = 150$ at the worst-performing problem instance of the first three sequential approaches.

S4 and S5 can react to the increase in revenue level by increasing their order quantities. The improvement is significant. The percentage of up-time values are increased at least 12% from the first three approaches to the last two approaches. As a result of an increase in order quantity, the system's percentage of spare part availability is increased. For the percentage of conducted preventive maintenance activities, it is counter-intuitive to observe an increase in the first place since this performance measure is related to the preventive maintenance threshold. However, an increase in order quantity also triggers an increase in the percentage of preventive maintenance activities. Recall that maintenance cycles are identically distributed except for the last maintenance cycle in a renewal cycle. The probability of conducting preventive maintenance at the last maintenance is less or equal to the probability of conducting preventive maintenance at the rest of the maintenance cycles. In addition, the expected length of the last maintenance cycle is greater or equal to the expected length of any remaining maintenance cycles. Therefore, introducing an additional maintenance cycle at the start of the renewal cycle increases the frequency of conducted preventive maintenance activities. The percent profit deviation is improved for the last two approaches at the high values of revenue level.

As inventory holding cost increases, all of the approaches' performances decline. The approaches react to the change by lowering their order quantity values. However, since they can not change their preventive maintenance threshold values, which should be done in this case, the percent profit deviation from the best policy increases.

When corrective maintenance cost increases, the changes in the preventive maintenance threshold values are drastic. The same behavior is observed while fixed cost increases. The preventive maintenance threshold is greatly sensitive to changes in the levels of corrective maintenance cost and fixed cost in the sequential approaches. That is why the best and worst-performing problem instances observed at $C_c = 400$, $k = 5$ and $C_c = 100$, $k = 1.5$ respectively. Also, in the 81 problem instances, in which $C_c = 100$, $k = 1.5$, the preventive maintenance threshold is found at its upper limit, which means never conduct preventive maintenance in practice.

While supplier lead time increases, S4 and S5 increase their order quantity values to lower the decrease in the percentage of up-time. Order quantity increase is also observed in the first three approaches. However, the increase is related to the policy constraint, $\tau Q \geq L$, instead of the percentage of up-time performance. In order to ensure the restriction, an increase in order size is observed.

In addition, we also compare the best approximation, A1, and sequential approach, S5, with each other to see the differences between a joint policy and a sequential policy. The results show that A1 performs statistically better than S5 in 573 out of 729 of the problem instances. On the other hand, there is no problem instance in which S5 performs statistically better than A1. So in the rest of the 156 problem instances, A1 and S5 find policy variables that result in the statistically indifferent long-run average profit rates. The parameter levels observed for these problem instances can be seen in Table 4.13.

Table 4.13: Parameter Level's Occurrence in the Problem Instances Result with Indifferent for A1 and S5

Parameter level	Number of problem instances
$r = 50$	86
$r = 75$	49
$r = 150$	21
$C_h = 0.5$	64
$C_h = 1$	51
$C_h = 2$	41
$C_c = 100$	4
$C_c = 200$	33
$C_c = 400$	119
$C_k = 25$	38
$C_k = 50$	45
$C_k = 200$	73
$L = 20$	79
$L = 30$	51
$L = 60$	26
$k = 1.5$	24
$k = 2$	39
$k = 5$	93

The most occurred parameter levels are $C_c = 400$, $k = 5$ and $r = 50$ in order. It is not surprising to see the highest levels of corrective maintenance cost and shape parameter of the Weibull distribution since these are the S5's best performing parameter settings. The main cause for 86 problem instances resulted indifferent for A1 and S5 because confidence intervals' half-length proportion to the mean is highest at the parameter level $r = 50$.

It can be concluded that the performance of the sequential approaches is positively correlated with the number of problem parameters defined in the decision maker's objective function. The revenue information dramatically affects the performances of the sequential approaches. Therefore, S1, S2, and S3 are inferior to S4 and S5 in all problem instances. Considering the maintenance-related costs while deciding order quantity also slightly impacts the quality of the policies. This is because the last maintenance cycle observed in a replenishment cycle is not identical to the rest of the maintenance cycles. Also, when we compare A1 with the best sequential approach, S5, the results show that considering a joint policy yields better results than considering a sequential approach.

4.5 Summary of Results

We observe several specific features of the approximations and sequential approaches according to the full factorial experiment outcomes. Our conclusions are as follows:

- A1 performs better than A2 under all 729 problem instances. Recall that A2 uses a rougher approximation for approximating up-time between order receipt and prior maintenance epoch than A1. This finding indicates the importance of precision in approximation.
- A1 results in almost the same long-run average profit rate of best policy parameters in all problem instances. Therefore, it is reasonable to use A1 instead of searching for the best policy by brute force.
- When the contribution of maintenance-related costs to the objective function increases, all sequential approaches perform better since the preventive maintenance threshold is prioritized in the optimization process of sequential approaches.
- S1, S2, and S3 perform worse than S4 and S5. Since the first three sequential approaches do not include revenue term in their objective function, this finding shows us the importance of realizing the down-time of the system.
- S5 performs slightly better than S4. Although it may be seen as counter-intuitive in the first place, maintenance-related costs could also affect the deci-

sion on order quantity since the last maintenance cycle in a renewal cycle is not identically distributed with the rest of the maintenance cycles.

- S4 and S5 utilize changing order quantity more drastically than A1 to response parameter level changes since sequential approaches are not capable of fine-tuning their preventive maintenance threshold parameter to the parameter level changes except for the maintenance-related costs.

- The comparison between a joint policy and a decentralized policy shows that joint policy results in far more better than decentralized policy. Especially when the variance of considered critical component's lifetime is high, supplier lead time is high, the difference between the corrective and preventive maintenance costs is low, inventory holding cost is high, and fixed ordering cost is low.

In summary, one can utilize a joint policy to improve their earnings instead of sequentially decide on maintenance and inventory decisions. The trade-off between maintenance-related and inventory-related costs does exist, and this trade-off significantly affects the overall performances of the policies. In such situations where the joint decision on maintenance and inventory policies is not a feasible option, one can still increase his/her profits by considering the characteristics of the different functional units in a manufacturing system.

CHAPTER 5

CONCLUSION

The equipment in any manufacturing system is prone to failure. Equipment failure results in downtime in the system and causes significant profit losses. It is impossible to eliminate the failures entirely. However, one can utilize preventive maintenance activities to maintain the continuity of production. We consider a single critical component of an equipment that must be replaced both in failure and preventive maintenance activities. Hence, the availability of spare parts affects the operating condition of the production process. It is reasonable to hold spare part inventory on hand since economies of scale are available in the system. Therefore, the decisions on preventive maintenance activities and spare part inventory must be made jointly.

This study focuses on the joint problem of preventive maintenance planning and spare part inventory control. We consider a system with a single equipment that includes a single critical component. If the critical component fails, the system fails. There is supplier lead time for orders, and emergency orders are not allowed. We propose an age-based preventive maintenance policy together with a fixed batch size order. The proposed policy's objective function is to maximize the expected long-run profit rate that consists of revenue generated, inventory holding cost, procurement cost, fixed ordering cost, preventive maintenance cost, and corrective maintenance cost terms. We characterize the objective function of the proposed policy by utilizing Renewal Reward Theorem. For the ease of application of our proposed policy, we introduce two approximation methods. In the literature, the trade-off between maintenance-related and inventory-related costs is represented by comparing sequential approaches and joint approaches. However, it is not the primary purpose of these studies. We also utilize sequential optimization by introducing five sequential approaches that differ in how they construct their objective function while deciding on order quantity. We want to further investigate the behavior of sequentially made decisions to provide managerial insights to the firms that joint policy is not viable in their environment.

The major characteristics of our environment are continuously monitored system states, constant supplier lead time, and not allowing emergency orders. Several studies consider a similar environment. However, the objective functions of the proposed policies are never characterized in these studies since the demand during the supplier lead time, when emergency orders are not allowed, is hard to characterize. We want to fill the gap in the literature by characterizing the objective function of our proposed policy under the environment mentioned above. In addition, we compare the results of joint policy with sequential approaches to show the critical environmental parameters that yield significant profit losses due to using decentralized decision-making strategies. Lastly, we compare sequential approaches in themselves. This comparison is made for providing insights to the decision-makers that are employed in separate functional units in the same organization. Considering the rest of the parameters besides inventory-related cost terms while deciding order quantity may yield significant improvements in the objective function of the whole system.

Through computational study, we investigate performances of joint policy, approximations, and sequential approaches under different problem instances concerning the expected long-run profit rate. Our analyses reveal the following:

- Approximation 1 performs far better than Approximation 2, which indicates the importance of approximating the system's up-time.
- Approximation 1 and best policy result in almost the same expected long-run profit rate under all problem instances. Therefore, it can be concluded that utilizing approximation 1 is reasonable in practice.
- The performance gap between joint policy and sequential approaches expands with a longer supplier lead time, relatively low-cost difference between preventive and corrective maintenance activities, and high variation in considered critical component's lifetime.
- Introducing revenue term to the sequential approaches yields a drastic improve-

ment in terms of the long-run profit rate of the system. S4-S5 perform far better than the rest of the sequential approaches by considering revenue term.

- In S5, introducing maintenance-related costs to the ordering decision's objective function yields a slight improvement in terms of expected long-run profit rate. Therefore, it can be concluded that even the joint approach is not an option for such systems; communication between different functional units can result in better results than fully separated decision-making strategies.

As a future research direction, one can consider ordering time as a decision variable of the policy. However, it should be noted that the expected reward calculation is computationally challenging even with our ordering setting. Moreover, the single equipment environment of our study can be relaxed to a multi equipment environment; in that case, state-space and problem complexity increase. However, introduced approximations of our proposed policy can be utilized for multi equipment environments.

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Appendix A

UNIMODAL BEHAVIOUR OF EXPECTED LONG-RUN PROFIT RATE FUNCTION

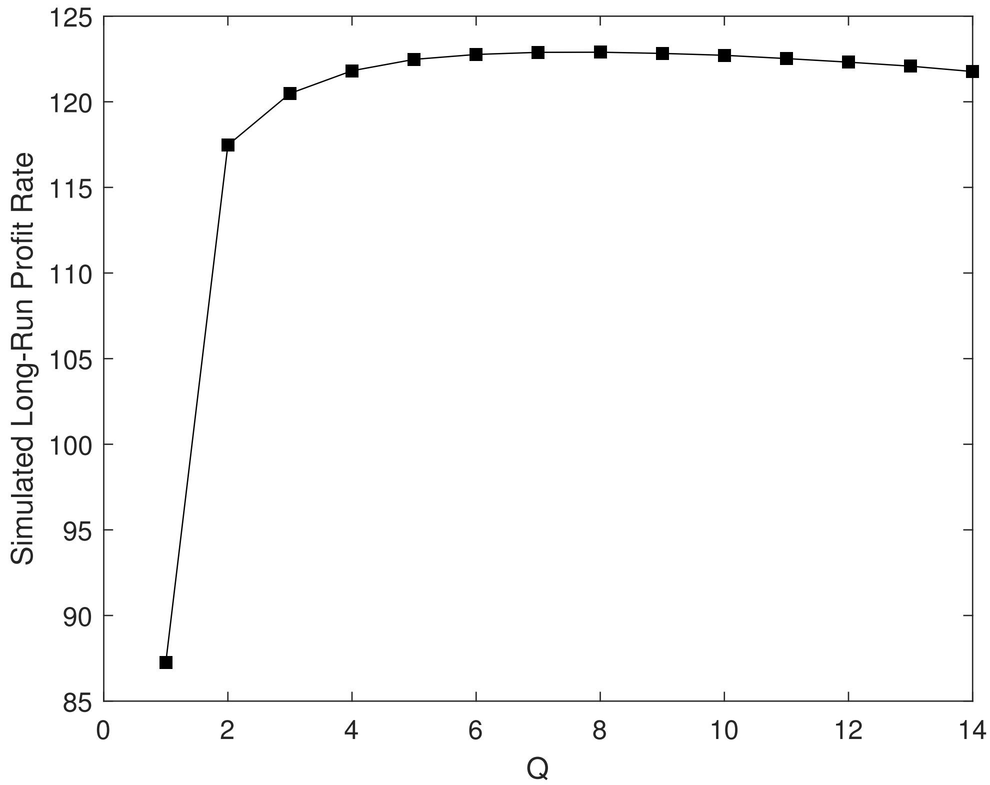

Figure A.1: Unimodal behaviour of Average Long-Run Profit Rate function in Q.

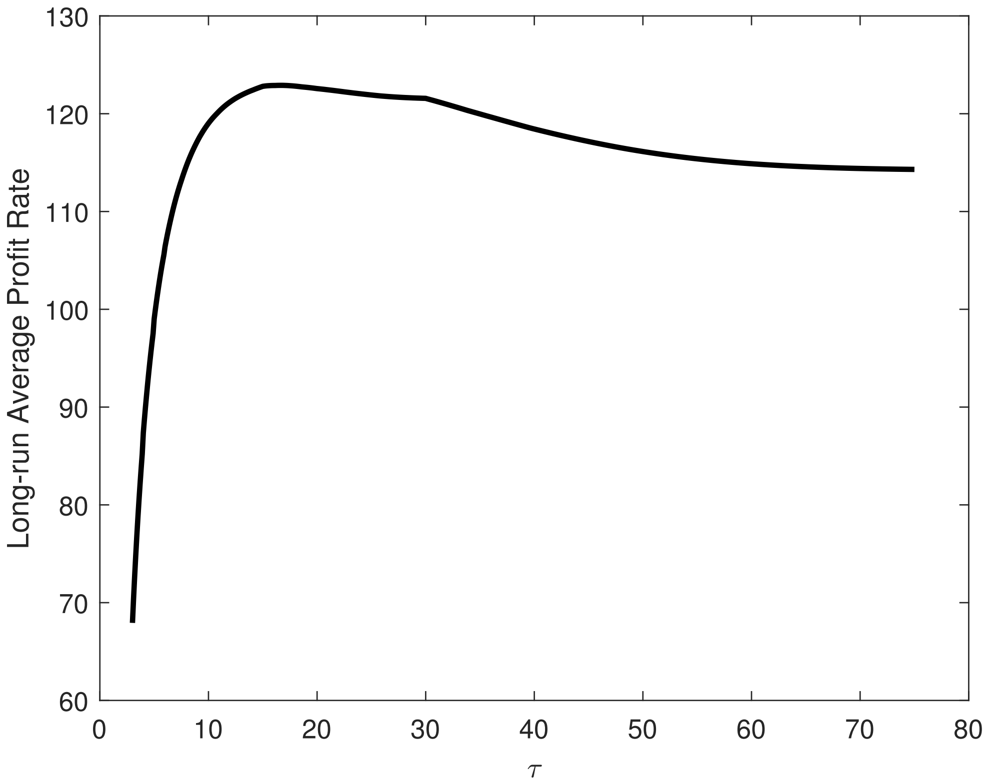

Figure A.2: Unimodal behaviour of Average Long-Run Profit Rate function in τ.

Appendix B

PROBLEM INSTANCES THAT A1 PERFOMS AT BEST AND WORST

Table B.1: Problem Instances which Approximation 1 Performed at Best and Worst

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
50	0.5	100	50	60	(24.54, 1.5)	0	Best
50	0.5	200	25	30	(24.54, 1.5)	0	Best
50	0.5	200	200	60	(24.54, 1.5)	0	Best
50	1	100	25	30	(24.54, 1.5)	0	Best
50	1	100	50	60	(24.54, 1.5)	0	Best
50	1	200	25	30	(24.54, 1.5)	0	Best
50	1	200	25	60	(24.54, 1.5)	0	Best
50	1	200	50	30	(24.54, 1.5)	0	Best
50	1	200	50	60	(24.54, 1.5)	0	Best
50	1	200	200	60	(24.54, 1.5)	0	Best
50	2	100	25	30	(24.54, 1.5)	0	Best
50	2	100	50	60	(24.54, 1.5)	0	Best
50	2	100	200	30	(24.54, 1.5)	0	Best
50	2	100	200	60	(24.54, 1.5)	0	Best
50	2	200	25	30	(24.54, 1.5)	0	Best
50	2	200	25	60	(24.54, 1.5)	0	Best
50	2	200	50	30	(24.54, 1.5)	0	Best
50	2	200	200	30	(24.54, 1.5)	0	Best
50	2	400	25	60	(24.54, 1.5)	0	Best
50	2	400	50	20	(24.54, 1.5)	0	Best
50	2	400	50	60	(24.54, 1.5)	0	Best

Table B.2: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
75	0.5	100	25	60	(24.54, 1.5)	0	Best
75	0.5	100	50	60	(24.54, 1.5)	0	Best
75	0.5	100	200	60	(24.54, 1.5)	0	Best
75	0.5	200	25	30	(24.54, 1.5)	0	Best
75	0.5	200	25	60	(24.54, 1.5)	0	Best
75	0.5	200	50	30	(24.54, 1.5)	0	Best
75	0.5	200	50	60	(24.54, 1.5)	0	Best
75	1	100	25	30	(24.54, 1.5)	0	Best
75	1	100	25	60	(24.54, 1.5)	0	Best
75	1	100	50	30	(24.54, 1.5)	0	Best
75	1	100	50	60	(24.54, 1.5)	0	Best
75	1	100	200	60	(24.54, 1.5)	0	Best
75	1	200	25	30	(24.54, 1.5)	0	Best
75	1	200	25	60	(24.54, 1.5)	0	Best
75	1	200	50	30	(24.54, 1.5)	0	Best
75	1	200	50	60	(24.54, 1.5)	0	Best
75	1	400	25	20	(24.54, 1.5)	0	Best
75	1	400	25	60	(24.54, 1.5)	0	Best
75	1	400	50	60	(24.54, 1.5)	0	Best
75	2	100	200	30	(24.54, 1.5)	0	Best
75	2	200	25	30	(24.54, 1.5)	0	Best
75	2	200	25	60	(24.54, 1.5)	0	Best
75	2	200	50	30	(24.54, 1.5)	0	Best
75	2	200	50	60	(24.54, 1.5)	0	Best
75	2	200	200	30	(24.54, 1.5)	0	Best
150	0.5	100	25	30	(24.54, 1.5)	0	Best
150	0.5	100	50	30	(24.54, 1.5)	0	Best
150	0.5	100	200	30	(24.54, 1.5)	0	Best
150	0.5	200	200	30	(24.54, 1.5)	0	Best

Table B.3: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
150	1	100	200	60	(24.54, 1.5)	0	Best
150	1	400	25	60	(24.54, 1.5)	0	Best
150	1	400	50	20	(24.54, 1.5)	0	Best
150	1	400	50	60	(24.54, 1.5)	0	Best
150	2	100	25	60	(24.54, 1.5)	0	Best
150	2	100	50	60	(24.54, 1.5)	0	Best
150	2	200	25	20	(24.54, 1.5)	0	Best
150	2	200	25	60	(24.54, 1.5)	0	Best
150	2	200	50	60	(24.54, 1.5)	0	Best
150	2	200	200	60	(24.54, 1.5)	0	Best
150	2	400	25	20	(24.54, 1.5)	0	Best
150	2	400	50	20	(24.54, 1.5)	0	Best
150	2	400	200	20	(24.54, 1.5)	0	Best
150	2	400	200	60	(24.54, 1.5)	0	Best
50	0.5	100	25	30	(25.00, 2.0)	0	Best
50	0.5	100	200	30	(25.00, 2.0)	0	Best
50	0.5	100	200	60	(25.00, 2.0)	0	Best
50	0.5	200	200	30	(25.00, 2.0)	0	Best
50	0.5	400	200	20	(25.00, 2.0)	0	Best
50	1	100	25	30	(25.00, 2.0)	0	Best
50	1	100	25	60	(25.00, 2.0)	0	Best
50	1	100	50	30	(25.00, 2.0)	0	Best
50	1	100	50	60	(25.00, 2.0)	0	Best
50	1	100	200	30	(25.00, 2.0)	0	Best
50	1	100	200	60	(25.00, 2.0)	0	Best
50	1	200	25	60	(25.00, 2.0)	0	Best
50	1	200	50	60	(25.00, 2.0)	0	Best
50	1	200	200	60	(25.00, 2.0)	0	Best
50	1	400	200	20	(25.00, 2.0)	0	Best

Table B.4: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
50	2	100	25	30	(25.00, 2.0)	0	Best
50	2	100	50	30	(25.00, 2.0)	0	Best
50	2	100	50	60	(25.00, 2.0)	0	Best
50	2	100	200	30	(25.00, 2.0)	0	Best
50	2	200	50	60	(25.00, 2.0)	0	Best
50	2	200	200	60	(25.00, 2.0)	0	Best
50	2	400	25	60	(25.00, 2.0)	0	Best
75	0.5	100	25	60	(25.00, 2.0)	0	Best
75	0.5	100	50	20	(25.00, 2.0)	0	Best
75	0.5	100	50	30	(25.00, 2.0)	0	Best
75	0.5	100	200	60	(25.00, 2.0)	0	Best
75	0.5	200	25	60	(25.00, 2.0)	0	Best
75	0.5	200	50	60	(25.00, 2.0)	0	Best
75	0.5	400	200	30	(25.00, 2.0)	0	Best
75	1	100	25	30	(25.00, 2.0)	0	Best
75	1	100	25	60	(25.00, 2.0)	0	Best
75	1	100	50	20	(25.00, 2.0)	0	Best
75	1	100	50	60	(25.00, 2.0)	0	Best
75	1	100	200	30	(25.00, 2.0)	0	Best
75	1	100	200	60	(25.00, 2.0)	0	Best
75	1	200	25	30	(25.00, 2.0)	0	Best
75	1	200	50	60	(25.00, 2.0)	0	Best
75	1	200	200	60	(25.00, 2.0)	0	Best
75	1	400	25	60	(25.00, 2.0)	0	Best
75	1	400	50	60	(25.00, 2.0)	0	Best
75	1	400	200	30	(25.00, 2.0)	0	Best
75	2	100	50	60	(25.00, 2.0)	0	Best
75	2	100	200	30	(25.00, 2.0)	0	Best
75	2	400	25	30	(25.00, 2.0)	0	Best

Table B.5: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
75	2	400	25	60	$(25.00, 2.0)$	0	Best
75	2	400	50	30	$(25.00, 2.0)$	0	Best
75	2	400	50	60	$(25.00, 2.0)$	0	Best
75	2	400	200	60	$(25.00, 2.0)$	0	Best
150	0.5	100	25	30	$(25.00, 2.0)$	0	Best
150	0.5	100	50	30	$(25.00, 2.0)$	0	Best
150	0.5	100	200	60	$(25.00, 2.0)$	0	Best
150	0.5	400	200	30	$(25.00, 2.0)$	0	Best
150	1	100	200	60	$(25.00, 2.0)$	0	Best
150	1	200	50	60	$(25.00, 2.0)$	0	Best
150	1	400	50	30	$(25.00, 2.0)$	0	Best
150	2	100	25	20	$(25.00, 2.0)$	0	Best
150	2	100	25	60	$(25.00, 2.0)$	0	Best
150	2	100	50	20	$(25.00, 2.0)$	0	Best
150	2	100	200	20	$(25.00, 2.0)$	0	Best
150	2	200	25	60	$(25.00, 2.0)$	0	Best
150	2	200	200	20	$(25.00, 2.0)$	0	Best
150	2	400	25	60	$(25.00, 2.0)$	0	Best
150	2	400	50	30	$(25.00, 2.0)$	0	Best
50	0.5	100	50	60	$(24.13, 5.0)$	0	Best
50	0.5	400	50	20	$(24.13, 5.0)$	0	Best
50	0.5	400	50	30	$(24.13, 5.0)$	0	Best
50	1	100	25	30	$(24.13, 5.0)$	0	Best
50	1	100	25	60	$(24.13, 5.0)$	0	Best
50	1	100	50	20	$(24.13, 5.0)$	0	Best
50	1	100	200	20	$(24.13, 5.0)$	0	Best
50	1	100	200	30	$(24.13, 5.0)$	0	Best
50	1	200	50	20	$(24.13, 5.0)$	0	Best
50	1	400	200	30	$(24.13, 5.0)$	0	Best

Table B.6: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
50	2	100	25	60	$(24.13, 5.0)$	0	Best
50	2	100	50	20	$(24.13, 5.0)$	0	Best
50	2	100	50	60	$(24.13, 5.0)$	0	Best
50	2	100	200	60	$(24.13, 5.0)$	0	Best
50	2	200	25	60	$(24.13, 5.0)$	0	Best
50	2	200	50	60	$(24.13, 5.0)$	0	Best
50	2	200	200	30	$(24.13, 5.0)$	0	Best
50	2	400	200	20	$(24.13, 5.0)$	0	Best
50	2	400	200	30	$(24.13, 5.0)$	0	Best
75	0.5	100	50	30	$(24.13, 5.0)$	0	Best
75	0.5	100	200	20	$(24.13, 5.0)$	0	Best
75	0.5	100	200	60	$(24.13, 5.0)$	0	Best
75	0.5	200	25	20	$(24.13, 5.0)$	0	Best
75	0.5	200	50	60	$(24.13, 5.0)$	0	Best
75	0.5	200	200	20	$(24.13, 5.0)$	0	Best
75	0.5	400	25	20	$(24.13, 5.0)$	0	Best
75	1	100	200	20	$(24.13, 5.0)$	0	Best
75	1	100	200	60	$(24.13, 5.0)$	0	Best
75	1	400	25	20	$(24.13, 5.0)$	0	Best
75	1	400	200	20	$(24.13, 5.0)$	0	Best
75	1	400	200	60	$(24.13, 5.0)$	0	Best
75	2	100	25	20	$(24.13, 5.0)$	0	Best
75	2	100	25	60	$(24.13, 5.0)$	0	Best
75	2	100	50	20	$(24.13, 5.0)$	0	Best
75	2	100	50	60	$(24.13, 5.0)$	0	Best
75	2	100	200	20	$(24.13, 5.0)$	0	Best
75	2	400	25	20	$(24.13, 5.0)$	0	Best
150	0.5	100	50	20	$(24.13, 5.0)$	0	Best
150	0.5	100	200	20	$(24.13, 5.0)$	0	Best

Table B.7: Problem Instances which Approximation 1 Performed at Best and Worst Cont'

r	C_h	C_c	C_k	L	(λ, k)	$\Delta\%$	
150	1	100	200	20	$(24.13, 5.0)$	0	Best
150	1	200	25	20	$(24.13, 5.0)$	0	Best
150	1	400	50	20	$(24.13, 5.0)$	0	Best
50	2	100	25	60	$(24.54, 1.5)$	0.88	Worst