THE INTERACTION BETWEEN MIDDLE SCHOOL MATHEMATICS TEACHERS' BELIEFS AND PEDAGOGICAL CONTENT KNOWLEDGE REGARDING RATIONAL NUMBERS

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ABSTRACT

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The aim of this study was to investigate middle school mathematics teachers' PCK regarding rational numbers and their mathematics-related beliefs, and to understand the nature of the possible interaction between their PCK and beliefs. Data of the study were collected by (i) observation of six middle school mathematics teachers' lessons about fractions and/or rational numbers content, (ii) two rounds of semi structured interviews, and (iii) vignettes. Data about their PCK were analyzed qualitatively with the Teacher Education Study in Mathematics (TEDS-M) Framework whereas belief data were analyzed in light of the related literature.

The analyses revealed that, participants mostly tended to teach with direct instruction method, they provided verbal explanations and rules, and they seemed to have sufficient mathematical knowledge most of the time. They connected the previous topics to the rational numbers and fractions in their teaching. They believed that this way of teaching was effective in teaching or that they had to use it because that was what suited the classroom context and the topics. Almost none of participants wanted to change their teaching even when students had better knowledge and skills, but they tried to change the level of questions they asked. Participants' beliefs and their PCK were mostly consistent in the interview and observation data. Analysis revealed that there was an interaction between their beliefs and PCK in direct and non-direct ways. Almost all of PCK dimensions of teachers interacted with their beliefs except for the Mathematical Curricular Knowledge.

Keywords: Pedagogical Content Knowledge, Beliefs, Rational Number, Fractions, Interaction

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN RASYONEL SAYILAR KONUSUNDAKİ PEDAGOJİK ALAN BİLGİLERİ İLE İNANIŞLARI ARASINDAKİ ETKİLEŞİMİ

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Bu çalışmanın amacı ortaokul matematik öğretmeninin rasyonel sayılar konusundaki pedagojik alan bilgilerini (PAB), matematikle ilgili inanışlarını ve pedagojik alan bilgileri ile inanışları arasındaki olabilecek etkileşimi araştırmaktır. Çalışmanın verileri (i) 6 ortaokul matematik öğretmeninin rasyonel sayılar ve/veya kesirler konusunu anlattıkları dersleri gözlemlenerek, (ii) iki farklı zamanda yarı yapılandırılmış görüşmeler ve (iii) oluşturulan örnek durumlar ile toplanmıştır. Ayrıca bu çalışmada araştırmacı tarafından notlar alınmıştır. PAB verileri nitel yöntemle Matematikte Öğretmen Eğitimi Çalışması (TEDS-M) modeli kullanılarak, inanış verileri ise alanyazın gözönüne alınarak araştırmacı tarafından analiz edilmiştir.

Yapılan analizler katılımcıların çoğunlukla düz anlatım yoluyla öğretim yapma eğiliminde olduklarını, sözlü açıklama ve kuralları kullandıkları ve çoğu zaman yeterli matematiksel bilgiye sahip olduklarını göstermiştir. Ayrıca rasyonel sayılar ve kesirler öğretimlerinde bağlantı kurmak için önceki konuları kullanmışlardır. Öğretmenler kullandıkları öğretim yolunun etkili olduğuna veya sınıf ortamına ve konuya uygun olduğu için kullanmak zorunda olduğuna inanmaktadır. Sınıf seviyesi farklı olsa bile neredeyse hiç bir katılımcı öğretim yöntemlerini değiştirmemişler ama sordukları soruların seviyesini değiştirmeye çalışmışlardır. Yarı yapılandırılmış görüşmeler ve gözlemlerden elde edilen veriler göz önüne aldığında, katılımcıların inanışlarının ve pedagojik alan bilgilerinin söyledikleriyle tutarlı olduklarını söyleyebiliriz. Analiz sonuçlarına göre inanışlar ile PAB arasında direk veya dolaylı bir etkileşim olduğu bulunmuştur. Matematiksel müfredat bilgisi hariç neredeyse bütün pedagojik alan bilgilerini oluşturan boyutlar ile inanışlarının etkileşimi bulunmuştur.

Anahtar Kelimeler: Pedagojik Alan Bilgisi, İnanışlar, Rasyonel Sayılar, Kesirler, Etkileşim

To my wife and my son

For patience and love

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LIST OF ABBREVIATIONS

CCK	Common Content Knowledge
CK	Content Knowledge
HCK	Horizon Content Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
MONE	Ministry of National Education
MPCK	Mathematics Pedagogical Content Knowledge
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics

CHAPTER 1

INTRODUCTION

Teaching mathematics is a complex action; therefore, it needs well-prepared teachers with different kinds of knowledge (The National Council of Teachers of Mathematics [NCTM], 2000). Teaching a new topic, employing new materials, navigating challenging issues of change, and performing new teaching practices depend on teachers' knowledge of mathematics (Ball, Lubienski, & Mewborn, 2001) and beliefs (Shulman, 1987). Qualified teachers know not only the subject matter, but also how to organize and teach their lessons in order to help students to learn more effectively (Berry, 2002). Teachers' capability to understand and use subject matter in their teaching is a key issue in students' achievement (Ball, 1990a; Ma, 1999; Shulman, 1986, 1987). They should also know how and why their students learn. Thus, teachers' knowledge is important to build up and enhance students' learning (NCTM, 2000). One of the most important concepts explaining teachers' effective teaching has been pedagogical content knowledge (PCK) (Shulman, 1986). Another one is teachers' beliefs which have influence on teachers' teaching related decisions and actions (Philipp, 2007).

Findings of different studies indicated that students have limited understanding in the rational number content (e.g., Clarke & Roche, 2009; Cramer, Post, & delMas, 2002; Depaepe et al., 2015; Mack, 1990). It is also known that rational numbers are one of the most difficult topics for middle grade students. Thus, teaching rational numbers requires well-grounded teacher knowledge in order to deal with students' difficulties (Depaepe et al., 2015). The aim of this study is to investigate middle school mathematics teachers' PCK, beliefs and the possible interaction between them in the context of teaching rational numbers. The concepts are explained in detail below.

1.1 Teachers' Knowledge and Beliefs

In the 1950s and 1960s, the focus of the researchers was mostly on problems of learning and curriculum rather than teaching. The focus moved to instruction and teaching in the 1970s and 1980s. Researchers looked for ways to understand teacher knowledge and how it is developed (Shulman, 2000). A number of constructs to understand the development of teacher knowledge were used by different researchers. In the last thirty years, researchers have combined teachers' knowledge with beliefs, attitudes, classroom instruction, technology, and other factors that affect teaching and learning. Meanwhile, a special type of teacher knowledge, PCK, was proposed by Shulman (1986). He divided teachers' knowledge into three categories; subject matter knowledge, curricular knowledge, and PCK (Shulman, 1986, 1987). In his definition, PCK means a combination of subject matter knowledge and pedagogical knowledge that allows teachers to provide the most essential learning experiences for their students. His framework has received broad acceptance among researchers. Many researchers built their framework mostly based on PCK definitions and moved towards defining PCK in specific subject areas, including mathematics (such as Ball, Thames, & Phelps, 2008; Kilpatrick et al., 2001; Tatto et al., 2008). Ball and her colleagues (2008) divided mathematics teachers' knowledge into two main parts; subject matter knowledge and PCK. Both main parts constituted three different components. Subject matter knowledge includes common content knowledge, horizon content knowledge and specialized content knowledge whereas PCK includes knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Ball et al. (2008) argue that there is a connection between teachers' mathematical knowledge and teaching practice in PCK construct, which is essential for effective teaching. Effective mathematics teaching addresses teachers' knowledge as important when teachers make the final decisions in the teaching process (Leinhardt & Smith, 1985). Effective teaching also depends on how well teachers transform their knowledge into pedagogical representations (Crespo & Nicol, 2006; Niess, 2005). Therefore, PCK and teacher knowledge becomes a powerful indicator of quality instruction (Baumert et al., 2010) and student learning (Charalambous, 2015).

While Ball et al.'s (2008) model provides a tool to explore mathematics teachers' knowledge in-depth, Teacher Education Study in Mathematics (TEDS-M) (Tatto et al., 2008) suggested a more specific framework to understand pre-service and inservice mathematics teacher's PCK based on cross-national studies. The study has examined teacher education policies, teacher educators, future mathematics teachers' knowledge and beliefs, and possible relationships (Tatto et al., 2008). The analysis of data collected with different tools such as questionnaires, interviews, and surveys in TEDS-M study have resulted in a framework. The framework was composed of two dimensions to assess teachers' knowledge; Mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). MPCK comprises of "Mathematical Curricular Knowledge", "Knowledge of Planning for Mathematics Teaching and Learning (pre-active)", and "Enacting Mathematics for Teaching and Learning" (Tatto, 2013).

The frameworks of mathematics teachers' knowledge briefly mentioned here show that PCK is categorized differently by different researchers with overlapping parts. In the current study, to understand mathematics teachers' PCK, TEDS-M framework was used because it was developed for both mathematics teachers and future teachers, and has more elaborative structure. In addition, TEDS-M was conducted in different countries which provided a cross-national validity for the framework.

Another reason was that TEDS-M study investigated both teachers' knowledge and their beliefs. Similar to PCK, teachers' beliefs are important aspects for teachers' knowledge because they affect the classroom practices (Schoenfeld, 1998; Wilson & Cooney, 2002). Beliefs are defined and studied by many researchers, but there is no common definition for them; as it is difficult to define beliefs precisely because they are not directly observable, but are inferred (Leder & Forgasz, 2002). Yet, these definitions address in common that beliefs are individuals' decisions and are reflected in their behaviors (Pajares, 1992; Thompson, 1992). Beliefs are one of the important indicators of decision making and therefore, they affect teachers' classroom actions (Bandura, 1986; Pajares, 1992; Richardson, 1996). Teachers' beliefs should be considered to understand teachers' actions during the teaching and learning process of mathematics (Cady, 2002) such as what knowledge is relevant, which teaching methods are appropriate and what subjects should be accomplished in teaching (Speer, 2005). Teachers' beliefs may also act as barriers against curriculum changes or the way of teaching mathematics (Drageset, 2010). For example, even when teachers have high mathematics content knowledge, they might not use inquiry-based learning as instructional approach if they do not believe it is effective (Wilkins, 2008). Therefore, it can be expressed that teachers' knowledge, and especially PCK, and teachers' beliefs are important for education because they are centrally located in teaching (Beswick, Callingham, & Watson, 2012; Döhrmann, Kaiser, & Blömeke, 2012; Drageset, 2010; Wilkins, 2008).

There are many studies about categorizations and classifications of beliefs in the literature, and for the characteristics of beliefs. There is also a considerable debate that the study of beliefs is important for teaching because measures of teacher knowledge cannot fully explain the nature of teachers' instruction (Ball et al., 2001; Speer 2005). On the other hand, the subject area or what teachers teach affects teachers' beliefs (Bonner, 2001; Calderhead, 1996). In the case of mathematics teaching, mathematical and pedagogical ideas of teachers have an impact on their beliefs (Wilson & Cooney, 2002). Therefore, in this study, beliefs are considered as an integral part of the mathematics teaching process that guides teachers in their decisions, and are explored along with PCK.

1.2 Rational Numbers

The subject, Rational numbers, is one of the most difficult topics for students and their teachers (Behr, Harel, Post, & Lesh, 1992). It is also one of the essential parts of school mathematics and mathematics curriculum in Turkey (MONE, 2013; 2018). Knowledge of rational numbers is one of the foundations for more advanced mathematics, particularly algebra and probability (Clarke & Roche, 2009; Lamon, 2005) and essential for students to succeed in their further studies in mathematics especially in algebra (NCTM, 2000). Students' struggle for understanding the rational numbers has been widely documented (see, for example, Behr, Wachsmuth, Post, & Lesh, 1984; Clarke & Roche, 2009; Mack, 1990) where they cannot "internalize a workable concept of rational number" (Behr et al., 1984, p. 323) and have incorrect generalization from their prior knowledge about natural numbers to rational numbers (Pesek, Gray, & Golding, 1997; Vamvakoussi, Van Dooren, & Verschaffel, 2012). In order to reduce students' difficulties, and to prevent misconceptions and incorrect generalization regarding rational numbers, it is important that teachers should have a well-developed PCK in relation to teaching rational numbers.

Teaching rational numbers requires well equipped teachers who have the appropriate knowledge base (Depaepe et al., 2015). In addition, they should know how and why their students learn. However, mathematics teachers' PCK about rational numbers, reasoning, and recognizing student difficulties and misconceptions can be limited (Depaepe et al., 2015; Izsak, Orrill, Cohen, & Brown, 2010; Pesek, Gray, & Golding, 1997) which might cause poor performance of rational numbers for students. Moreover, teachers can have misconceptions regarding multiplication and division of rational numbers and their meanings (Simon & Blume 1994; Tirosh 2000). Preservice mathematics teachers are also found to have misconceptions about multiplication of fractions due to different factors such as rote memorization, knowing primitive models, insufficient mathematical knowledge and anxiety (Isıksal & Cakiroglu, 2010).

Previous studies have concluded that teachers' mathematics teaching related decisions and actions are highly influenced by their PCK and beliefs (Sherin, 2002). These two important constructs do not influence teachers' teaching decisions and actions in isolation. Considering that teacher beliefs influence their decision making, it might be the case that their beliefs influence their use of PCK for specific instances, such as teaching decisions for the students who will take a national examination soon (Arslan, 2018). Similarly, it might be the case that a teacher who does not believe that not all students are good at mathematics may choose to address rational number concepts in the simplest way, reducing the content to a set of rules to be memorized. Thus, not only identifying teachers' PCK and their beliefs about the rational numbers, but also exploring the interaction between the two is important in order to provide more effective learning opportunities for students (Philipp, 2007).

1.3 Significance of the Study

Teaching and learning mathematics in the classroom depend upon several key factors. Teachers' knowledge and beliefs, both relate and contribute to teaching quality, are among those key factors (Wilson & Cooney, 2002). Teachers' knowledge is crucial for teaching quality, student learning and effective mathematics teaching (Charalambous, 2015; Leinhardt & Smith, 1985). Mathematics teachers' beliefs have a powerful impact on the practice of teaching (Ernest, 1989) and they are "critically important determinants of what teachers do and why they do it" (Schoenfeld, 1998, p. 2). Therefore, teachers' knowledge and beliefs affect their decisions, and eventually what is taught and ultimately learned in the classroom instruction. In the light of this information, the following questions appear: How are teachers' beliefs affected when their PCK is improved? Or, how do teachers' beliefs impact their PCK? In other words, how do knowledge and beliefs interact each other in making decisions for teaching? This remains an issue to explore in the field of mathematics education (Philipp, 2007).

Although a considerable number of researchers put particular emphasis on teachers' knowledge types and several studies have been conducted about how teachers' beliefs influence their thinking and behaviors, including instructional decision making and use of curriculum materials, the interaction between the two constructs has not been explored much. Two studies with preservice teachers found out that knowledge and beliefs interaction is key to understand teachers' behaviors, and teachers should be equipped with the most availing knowledge and beliefs to create mathematically rich environments (Blömeke, Buchholtz, Suhl, & Kaiser, 2014; Charalambous, 2015). Yet, how beliefs and knowledge interact when inservice teachers teach is not explored in detail. Additionally, in the literature, teachers' belief and their PCK have been mostly studied quantitatively. The relationship or interaction between them was investigated mostly by quantitative methods, such as Pearson product-moment correlation coefficient, which revealed a particular score in the studies. However, these scores did not give any detail about the relationship but only the existence of the relationshipand the strength of association. Therefore, there is a need to explain how belief and PCK relate each other especially in the field of mathematics education. In the light of these information, in current study, the interaction between teachers' mathematics related beliefs and their PCK was investigated qualitatively in order to respond how and what questions about the interaction. Moreover, it is expected that the findings of the present study will contribute to the field to address the possible interactions and will try to shed light on the gap in belief and PCK studies in terms of qualitative methods.

Students' learning of mathematics concepts, and the potential misconceptions and lack of understanding could be traced back to teachers' PCK and beliefs to a great extent (Campbell et al., 2014; Philipp, 2007). Considering that rational number concepts are important for further mathematics learning (Clarke & Roche, 2009; Lamon, 2005), are one of the important and most difficult topics for both students and their teachers (Behr, Harel, Post, & Lesh, 1992), and have been one of the essential components of mathematics curriculum in Turkey (MONE, 2013; 2018), how mathematics teachers teach these concepts and how they believe about teaching these concepts become important for students' effective learning. Inservice (Walters, 2009) and preservice (Tirosh, 2000; Türnüklü & Yeşildere, 2007) mathematics teachers have been reported to have difficulties and misconceptions in rational numbers, and handling students' misconceptions regarding rational numbers. These findings accordingly address ineffective PCK because teachers' PCK is the key to handle students' misconceptions and difficulties about rational numbers (Depaepe et al., 2015; Izsak et al., 2010; Pesek et al., 1997). They also address teachers' beliefs because they influence their choice of teaching methods (Leder, Pekhonen, & Törner, 2002). Campbell and colleagues (2014) indicated that although teachers might hold similar beliefs regarding mathematics teaching and had similar levels of mathematical knowledge, they might possess different interpretations of classroom interactions or the capabilities of their students. Therefore, teachers' difficulties in both understanding and teaching rational numbers and their teaching decisions, and accordingly their students' learning of rational numbers could be related to a possible interaction between their PCK and beliefs. Exploring this possible interaction will provide a strong basis for effective mathematics teaching and increased professional development opportunities for preservice and inservice teachers.

1.4 My Motivation for the Study

I wanted to explore PCK and belief together with different insights becacuse most of the prior studies found a relationship between PCK and belief, but they did not explain how they were related to each other. I believe that providing some clarification to this possible interaction will contribute to teachers' education at all levels. Besides, working with teachers is exciting for me because teachers shoulder students' education and they can give valuable information. As I have mostly studied with pre-service teachers, the current study with the inservice teachers provided me with an important experience.

1.5 Research Questions

In line with the purposes of the present study, the following are the main research question and underlying subquestions:

How do middle school mathematics teachers' beliefs and their PCK interact in teaching rational numbers?

1. What is the nature of middle school mathematics teachers' PCK regarding rational numbers?
2. What is the nature of middle school mathematics teachers' mathematics-related beliefs regarding rational numbers?
3. What is the interaction between middle school mathematics teachers' beliefs and their PCK for teaching rational numbers?

1.6 Definition of Terms

Pedagogical Content Knowledge (PCK): Pedagogical content knowledge is defined in the literature as the intersection of content knowledge (CK) and pedagogical knowledge (PK). Shulman's (1986) model has been studied and sometimes modified to analyze mathematics knowledge for teaching with other researchers. In this study, it refers to middle school mathematics teachers' pedagogical content knowledge about rational numbers. MPCK in TEDS-M (2008) teachers' knowledge framework was used to assess mathematics teachers' PCK in this study. It includes three subdimensions: (a) Mathematical Curricular Knowledge, which is mostly about teachers' knowledge for curriculum, learning programs and assessment formats; (b) Knowledge of Planning for Mathematics Teaching and Learning, which addresses about teachers' knowledge for planning their teaching process; and (c) Enacting Mathematics for Teaching and Learning which includes mostly teachers' actions in their lessons (Tatto, 2013). These dimensions were assessed by vignettes, interviews and observations in the current study. Therefore, PCK in the present study refers to MPCK used in the TEDS-M study.

Beliefs: According to Speer (2005), beliefs are conceptions, personal ideas or decisions which include how teachers decide the relevant knowledge, appropriate design, important features and goals for teaching the specific mathematics context. Hannula (2011) proposed beliefs as a subset of cognitive domain, which consists of mental representations, and thoughts, concepts and facts in mind. It is difficult to identify teachers' beliefs because beliefs cannot be directly observed or measured (Bonner, 2001). Therefore, teachers' beliefs can be traced through teachers' expressions, intentions and actions (Pajares, 1992).

Mathematics related beliefs regarding rational numbers: In this study, teachers' mathematics-related beliefs refer to middle school mathematics teachers' thoughts, ideologies or views regarding rational numbers, and its teaching and learning. Their beliefs were inferred from their reactions to vignettes (intentions), responses to interview questions (expressions) and observations of their teaching rational number practices (actions).

Rational numbers: It is one of the essential parts of school mathematics and mathematics curriculum (Clarke & Roche, 2009; NCTM, 2000). Fractions come before rational numbers in the curriculum (MONE, 2018). Fractions are taught until the 6th grade. Then, rational numbers are started and used throughout the school life (MONE, 2018). Therefore, it can be said that fractions set ground for rational numbers. In this study, fractions and rational numbers are addressed together. It refers to the contents of fractions concepts in the 6th grade and rational numbers concepts in the 7th grade in the middle school mathematics curriculum in Turkey at the time of the study (MONE, 2018).

Middle school mathematics teachers: In the present study, middle school mathematics teachers refer to in-service mathematics teachers who graduated from the Elementary Mathematics Education Programs and work in public middle schools (grades 5 to 8) in Turkey.

CHAPTER 2

LITERATURE REVIEW

The current study aimed to investigate middle school mathematics teachers' PCK and beliefs in the context of teaching rational numbers concepts. Furthermore, possible interactions between their PCK and their beliefs regarding rational numbers were explored. In order to provide a basis for this study, I conducted a literature review in this chapter which explains: (1) frameworks and related studies about PCK; (2) conceptual understanding about teachers' beliefs; (3) rational numbers in mathematics; and (4) studies about teachers' PCK and beliefs.

2.1 PCK Frameworks and Studies

Shulman (1986) suggested analyzing teachers' knowledge in several categories (Shulman, 1986, 1987). He indicated that teachers' knowledge was mostly considered as content knowledge (CK) in the 19^{th} century and as pedagogical knowledge (PK) in the 20^{th} century (Shulman, 1986). Because of an absence of balance between CK and PK, he put forward a special type of teacher knowledge, pedagogical content knowledge (PCK), which strengthened the difference between doing mathematics and teaching mathematics. Subject matter knowledge or content knowledge is "the amount and organization of knowledge per se in the mind of teacher" (Shulman, 1986, p. 9). It includes the basic ideas, concepts, and principles of the discipline, as well as what is considered as truth or false in the discipline and how validity or invalidity of knowledge is determined (Shulman, 1986). A teacher with fundamental subject matter knowledge is not only capable of defining truths of a domain, but he or she is also able to explain why a certain knowledge is true and worth knowing, and how it relates to other domains (Shulman, 1986).

Curriculum means the full range of education programs, and it is designed to teach particular subjects or topics at a given level. It also includes several instructional materials of particular subjects (Shulman, 1986). Therefore, curricular knowledge means teachers' knowledge about education programs and instructional materials regarding particular subjects or topics at a given level.

The other type of teacher knowledge in Shulman's framework is pedagogical content knowledge (PCK). Shulman's (1987) definition of PCK is as follows:

> for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others… Pedagogical content knowledge also includes an understanding of what makes learning of specific topics easy or difficult… (p. 9).

PCK is a blend of teachers' knowledge about what they teach (subject matter knowledge) with their knowledge about teaching (pedagogical knowledge) (Shulman, 1987). It is a unique type of knowledge, since it connects content with the aspects of teaching and learning (Ball et. al, 2001). After Shulman's work about teacher knowledge, researchers have continued to work on its domains to provide comprehensible relationships between these knowledge domains. In general, researchers have studied teacher's knowledge in a general way but subject specific versions of PCK were also proposed. In this manner, several models of mathematical knowledge have been proposed such as the "content knowledge for teaching" (Ball, Thames, & Phelps, 2008), "proficient teaching of mathematics" (Kilpatrick et al., 2001), and "knowledge of teaching mathematics" (Tatto et al., 2008) by several researchers.

Although there is a clear theoretical distinction between CK and PCK, research studies have not been able to separate them clearly (Kleickmann et al., 2013). CK and PCK represented two correlated dimensions, and CK might be a prerequisite for PCK development (Krauss et al., 2008). However, strong CK did not certainly lead to strong PCK (Lee, Brown, Luft, & Roehrig, 2007). Baumert and colleagues (2010) also noticed that CK had lower predictive power than PCK for student progress and learning.

2.1.1 Ball et al.'s Framework of Mathematics Teachers' Knowledge

Ball and her colleagues (2008) have studied with mathematics teachers and suggested a framework related to mathematics teachers' knowledge. They developed mathematics-specific subdomains of Shulman's (1986) pedagogical content knowledge to provide an in-depth analysis of mathematics knowledge for teaching (Ball et al., 2008). Moreover, Ball and colleagues (2008) indicated that PCK mostly had been generalized by researchers and it lacked specifications for different subjects. Their framework, as illustrated in Figure 1, divides mathematical knowledge for teaching into two parts: Subject Matter Knowledge (SMK) and PCK.

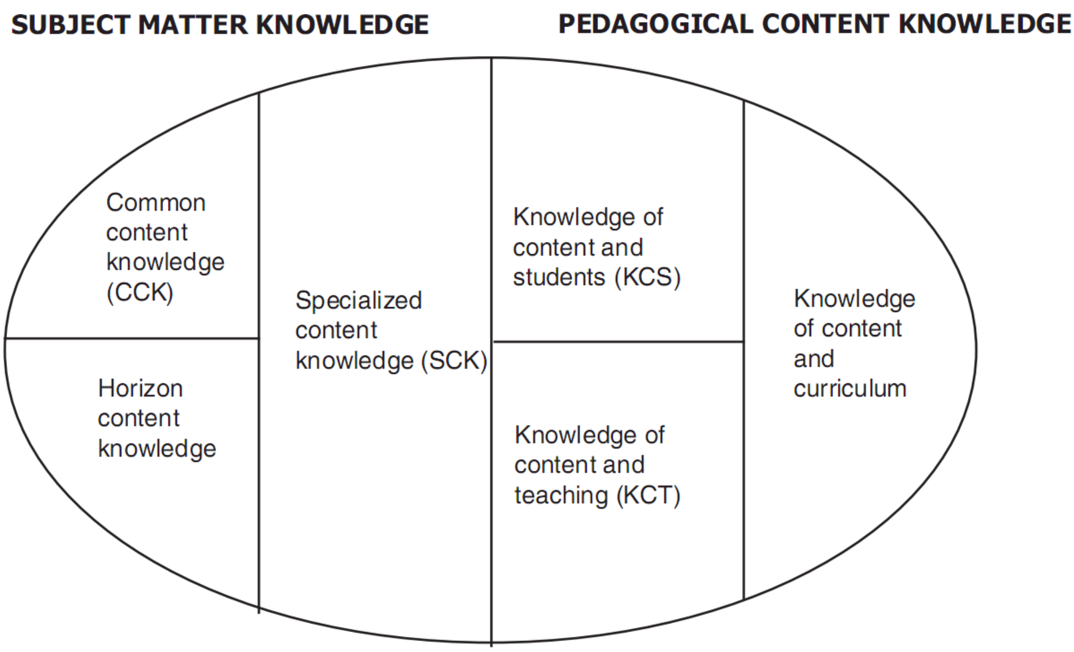


Figure 1. Ball et al.'s Model for mathematics teachers' knowledge. Adapted from "Content knowledge for teaching, what makes it special?" by Ball et al., 2008, *Journal of Teacher Education, 59*(5), p. 403.

In Ball et al.'s (2008) model, SMK is divided into three parts; Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialized Content Knowledge (SCK). CCK is defined as mathematical knowledge and skills that are used in a wide variety of settings, and therefore it is not unique to teaching. It includes teachers' knowledge about the material they teach such as, how to recognize a wrong answer in the class and an inaccurate definition in textbook (Ball et al., 2008). On the other hand, SCK is defined as mathematical knowledge and skills including mathematical ideas and explanations for common rules and procedures and it is unique to teaching (Ball et al., 2008). Both CCK and SCK correspond to Shulman's (1986) subject matter knowledge. HCK is about teacher's knowledge of the span of mathematical topics over the mathematics curriculum (Ball et al., 2008).

Ball et al. (2008) divided PCK into three components: (a) Knowledge of Content and Students (KCS), (b) Knowledge of Content and Teaching (KCT), and (c) Knowledge of Content and Curriculum (KCC). KCS consists of knowledge about students and knowledge about mathematics including predicting what students will find interesting and motivating, and will find it easy or hard (Ball et al., 2008). It has also been defined as "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill et al., 2008, p. 375). Briefly, teachers' understanding about how students learn specific topics has been emphasized in KCS. Shulman (1986) also states that the foundation of research on students' thinking and ideas is important for pedagogical knowledge; therefore, KCS corresponds to Shulman's (1986) definition of PCK. KCT includes knowing about teaching and knowing about mathematics including knowing how to design instruction for specific topics and evaluating the advantages and disadvantages of the designs (Ball et al., 2008). The third component, KCC is the subset of curricular knowledge dimension of Shulman's (1986) model. It is the knowledge about specific curriculum which includes teaching of particular content and topics at a particular level, and knowledge about curriculum or instructional materials (Ball et al., 2008). Ball and her colleagues (2008) provided an in-depth analysis of mathematics knowledge for teaching in their models.

Ball and colleagues' (2008) PCK model has given a new impulse to the mathematics-specific research in PCK studies because most of PCK studies before their framework were related to the general knowledge type that was not subject specific. Their model has been used as a starting point by many researchers (Thanheiser et al., 2009). This framework has been widely used in the field of mathematics education and teacher knowledge to explore knowledge development and circumstances for preservice and inservice teachers such as learning mathematics instruction (Charalambous, Hill, & Ball, 2011), identifying middle school mathematics teachers mathematical knowledge for teaching algebra (Girit, 2016), examining CK and PCK (Wilkie, 2014), exploring the relation between PCK and instructional practice (Sorto, Marshall, Luschei, & Carnoy, 2009), understanding the development of in-service teachers' PCK (Vale, McAndrew, & Krishnan, 2011) and investigating pre-service teachers' PCK (Jenkins, 2010; Karp, 2010).

However, those studies did not focus on teachers' beliefs and the interaction between their beliefs and the knowledge they used to teach mathematics. Thus, it can be said that while the Ball and colleagues' (2008) framework has provided a basis for looking at mathematics teachers' knowledge, it seems incomplete in terms of the interaction to their beliefs. Petrou and Goulding (2011) also asserted that the framework ignores mathematics teachers' beliefs especially about teaching and that it tested teachers' knowledge independently from the context.

On the other hand, determining which knowledge fit in the exact knowledge type of Ball et al.'s framework is a challenge (Thanheiser et al., 2009). Thanheiser and her colleagues (2010) indicated that distinctions among the different types of knowledge in the framework appeared to have blurred boundaries. Thus, it can be said that Ball and colleagues' framework cannot respond to the more specific needs of the present study to address teacher's PCK regarding rational numbers.

2.1.2 The Framework of Teacher Education Study in Mathematics (TEDS-M)

As stated above, many researchers have a considerable interest on understanding teacher knowledge and determining what kinds of knowledge are needed to be an effective teacher. However, there is a worldwide controversial issue about how to categorize the mathematics knowledge for teaching (Even & Loewenberg Ball, 2009). This scope brought researchers from different countries in cross-national studies to examine teacher education and mathematics teaching. One of these studies is Teacher Education Study in Mathematics (TEDS-M) which was conducted in 17 countries across continents. The general goal of the study was to understand the relationships between teacher education policies, institutional practices, and outcomes of primary and lower secondary school mathematics teacher's education programs. It also examined the nature of teacher education programs within and across countries and possible relationships between the beliefs about mathematics of teacher educators and those of future teachers (Tatto et al., 2008). Case study country reports, interviews, and surveys were used in TEDS-M. Surveys, which were implemented over 15,000 primary and over 9,000 lower-secondary future teachers and close to 5,000 teacher educators, comprised questions to explore respondents' mathematics content knowledge, the mathematics pedagogical content knowledge, and beliefs about teaching and learning mathematics (Tatto, 2013). Researchers developed instruments of the study and their assessment framework in two dimensions: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) (Tatto, 2013). MCK in TEDS-M, mostly based on trends in International Mathematics and Science Study (TIMSS) 2007 and TIMSS 2008 frameworks, comprised of three domains: content domain (number and operations, geometry and measurement, algebra and functions, data and chance), cognitive domain (knowing, applying, reasoning), and curricular-level domain (novice, intermediate, advanced) (Tatto, 2013). MPCK was classified into three subdomains; "Mathematical Curricular Knowledge", "Knowledge of Planning for Mathematics Teaching and Learning (pre-active)", "Enacting Mathematics for Teaching and Learning". Table 1 presented the subdomains of the MPCK and their explanations. MPCK framework provides descriptions of teacher actions that address the nature of knowledge in the subdomains.

Table 1.

Mathematics Pedagogical Content Knowledge (MPCK) Framework in TEDS-M (Tatto et al., 2008).

MPCK Subdomain	**Elaboration**
Mathematical Curricular Knowledge	• Establishing appropriate learning goals • Knowing different assessment formats • Selecting possible pathways and seeing connections within the curriculum • Identifying the key ideas in learning programs • Knowledge of mathematics curriculum
Knowledge of Planning for Mathematics Teaching and Learning (pre-active)	• Planning or selecting appropriate activities • Choosing assessment formats • Predicting typical students' responses, including misconceptions • Planning appropriate methods for representing mathematical ideas • Linking the didactical methods and the instructional designs • Identifying different approaches for solving mathematical problems • Planning mathematical lessons
Enacting Mathematics for Teaching and Learning (interactive)	• Analyzing or evaluating students' mathematical solutions or arguments • Analyzing the content of students' questions • Diagnosing typical students' responses, including misconceptions • Explaining or representing mathematical concepts or procedures • Generating fruitful questions • Responding to unexpected mathematical issues • Providing appropriate feedback

Tatto (2013) explains the content of the domains as follows: The mathematical curricular knowledge subdomain comprises the teachers' knowledge about the curriculum, learning goals, assessment formats, and knowledge of key concepts in curriculum and their interconnections. The second subdomain refers to the teachers' knowledge which are used to plan their mathematics teaching and learning, and preparatory work. This subdomain also includes selecting appropriate instructional methods and designs, and assessments, predicting students' responses and possible misconceptions. The enactment of mathematics subdomain indicates teachers' actions during the lessons such as analyzing students' responses and misconceptions, explaining mathematical concepts, developing questions, and giving feedback (Tatto, 2013). However, in TEDS-M study, "mathematical curricular knowledge" and "knowledge of planning for mathematics teaching and learning (pre-active)" subdomains were explored with their data. "Enacting mathematics for teaching and learning" subdomain was only propounded because participants of the TEDS-M study were selected from preservice teachers who could not teach. Besides, reaching inservice teachers were not feasible because of time limitations (Tatto et al., 2008). TEDS-M framework provides a more specific and wide range of dimensions and sub-dimensions that describe the MPCK more clearly based on a broad and comprehensive study. The descriptions of the dimensions and subdimensions match the purposes of the present study. Therefore, in the present study TEDS-M MPCK structure and its subdomains were used to investigate mathematics teachers' PCK.

In this study, middle school mathematics teachers' knowledge of common conceptions and misconceptions held by the middle school students in the content of rational numbers; their knowledge of the possible sources of these conceptions, misconceptions, and of the different assessment formats, the strategies and representations that they use to overcome these misconceptions; the strategies that middle school mathematics teachers use to explain the key facts, mathematical concepts; and other teachers' knowledge dimensions will be investigated in order to portray their MPCK in rational numbers.

TEDS-M examined PCK and beliefs together, and conceptualized PCK with MPCK framework (Tatto, 2013). Beliefs were investigated by a scale in five areas: the nature of mathematics, learning mathematics, mathematics achievement, preparedness for teaching mathematics, and program effectiveness in TEDS-M study (Tatto, 2013). Tatto (2013) explained that questions of the belief scale included questions about perceptions and purposes of mathematics as a subject, the appropriateness of particular instructional activities, various teaching strategies, and preparedness to teach in different areas. It can be said that the belief scale was parallel with the MPCK subdomains, however, it was broader in terms of mathematics content. On the other hand, in the present study, data were collected in a particular content and with semi sctructured interviews. Thus, belief scale was not used directly, but was used as a guide.

2.2 Beliefs about Teacher and Teaching

Beliefs are personal strong predictors of human behavior (Pajares, 1992; Thompson, 1992). Several researchers have defined beliefs relatively in similar ways with different words. For instance, Richardson (1996) defined beliefs as “psychologically held understandings, premises, or propositions about the world that are felt to be true” (p. 103). Goldin (2002) defined beliefs as “multiple encoded cognitive/affective configurations, to which the holder attributes some kind of truth value” (p. 59). According to Pehkonen (1997), beliefs are individual and subjective knowledge, and include a person's feelings or care. In the context of mathematics, Schoenfeld (1992) also stated that beliefs could be considered “as an individual’s understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior” (p. 358). On the other hand, Artzt (1999) defined mathematics teachers’ beliefs as “assumptions regarding the nature of mathematics, of students, and of ways of learning and teaching” (p. 145). As can be seen, there is no common definition of beliefs in the education literature and in mathematics education literature (Cross Francis, Rapacki, & Eker, 2015; Fennema & Franke, 1992; Furinghetti & Pehkonen, 2002; Pajares, 1992; Philipp, 2007). However, there is a common understanding that beliefs shape individuals’ current and future decisions and behaviors (Pajares, 1992; Thompson, 1984; 1992).

Beliefs are personal and strong predictors of human behavior, and very influential to determine the way individuals handle problems and organize tasks. (Thompson, 1992; Torff & Warburton, 2005). They are mostly based on past experiences (Pajares, 1992) and they develop over a long time (Emenaker, 1995). Beliefs influence individuals’ decisions and serve as one of the powerful indicators of their decisions (Goldin, Rösken, & Törner, 2009). In the same way, teachers’ beliefs affect their classroom practices (Pajares, 1992; Schoenfeld, 1998; Wilson & Cooney, 2002). Therefore, studies regarding beliefs of teachers can give valuable information for educational issues and determining teachers' beliefs would be beneficial to organize teacher education and in-service programs for effective teaching (Pajares, 1992).

There are many categorizations and classifications of beliefs which have attempted to characterize different beliefs. However, teachers' beliefs can change in terms of the subject area or what they teach. Educational beliefs were affected by the conceptions of the curriculum and the content area (Bonner, 2001). Beliefs about epistemological issues could be associated with what the subject is about and what teachers know about the subject (Calderhead, 1996). Furthermore, how teachers cope with the challenges of teaching process were influenced by their beliefs and knowledge about the mathematical subject matter (Fennema & Franke, 1992; Thompson, 1992).

Mathematics teachers' beliefs were characterized by their mathematical and pedagogical ideas, and their views or preference for mathematics learning (Wilson & Cooney, 2002). Handal (2003) proposed that teachers' mathematical beliefs "act as a filter through which teachers make their decisions rather than just relying on their pedagogical knowledge or curriculum guidelines" (p. 47). Similarly, Buehl and Beck (2015) noticed that teachers might use their beliefs as a filter and interpret information or frame a specific task, such as lesson planning. Considering this situation, different researchers have investigated relationships between teacher's beliefs and their practices. It was found that there were both consistencies and inconsistencies between teachers' expressed beliefs and their behaviors (Cohen, 1990; Thompson, 1984). For instance, Wilkins (2008) examined 481 American elementary teachers' beliefs regarding the effectiveness of inquiry, and pointed out that they were the strongest predictor of employing inquiry instructional practices. However, Lim and Chai (2008) studied six teachers who planned computer-mediated lessons in mathematics, science, and English, and found that although five of them expressed a constructivist orientation for teaching, they taught predominately traditional lesson. Niesche and Lerman (2010) also found the same inconsistency between 25 teachers' beliefs and their practices.

Haser (2006) investigated 20 preservice and 12 inservice first-year mathematics teachers' beliefs and explored the relationship among their beliefs, teacher education program courses and first-year teaching experiences via one-on-one interviews in two years. The findings indicated that although the courses did not influence their mathematics related beliefs as much as expected, the school contexts affected inservice participants' beliefs and practices (Haser, 2006). According to Haser (2006) inservice teachers tended to use teacher-centered teaching even though they had both student-centered and teacher-centered beliefs. In the present study, the interview protocols were developed benefiting from interview protocols in Haser's (2006).

Leatham (2006) pointed out that teacher actions might not always be in line with their beliefs. Their actions and beliefs should be explored in depth in order to understand the possible relationship. Buehl and Beck (2015) compiled beliefs and teaching practices studies, and noticed that beliefs and practices could influence each other. The strength of the relationship depended on the type of beliefs and practices being assessed as well as contexts. Other researchers also indicated that mathematics beliefs could influence teachers' pedagogical decisions and teaching practices (Beswick, 2012; Bray, 2011). On the other hand, the result of Woods' (1996) study indicated that it was difficult to differentiate between teachers' beliefs and knowledge, and teacher beliefs is one of the key parts in their cognition processes. In this regard, the study of teachers' beliefs and knowledge becomes quite important in the mathematics education field. Thus, to make better sense of the teacher's actions during the teaching process, their beliefs and MPCK were examined together in the current study.

2.3 Rational Numbers in Mathematics

Rational numbers are real numbers which consist of all the numbers in a number line. Baroody and Coslick (1998, p. 9-02) defined rational number as "a real number that can be put in the form of a common fraction a/b where a and b are integers and b is different from 0." Integers consist of negative integers and whole numbers; therefore, rational numbers set also include whole numbers. Due to its inclusiveness and that it provides a relevant basis for other contexts in mathematics, rational number concept is one of the most important concepts for students (Behr & Post, 1992).

The review of the literature indicated that one of the bases for difficulty of teaching and learning rational numbers is the complexity of its construct. Many researchers have focused on to explain rational numbers construct starting from 1970's. The pioneer of these studies was Kieren's (1976) study, which defined the rational numbers construct in several perspectives: Fractions, decimals, equivalence classes, ratio numbers *(p/q, p, q* integers, *q not 0*), operators, points on a number line, and elements of a quotient field. Other studies of rational numbers (such as Behr, Lesh, Post, & Silver, 1983; Ohlsson, 1988) have used Kieren's (1976; 1980) definition as baseline. Then, Kieren (1980, pp. 134–136) interpreted rational numbers into the five subconstructs: Part-whole, quotient, measure, ratio and operator. However, a controversial issue was emerged about whether *part-whole* was distinct from the *measure*, or not (Lester, 2007). After that, Kieren (1988) expressed that *measure, quotient, ratio,* and *operator* were distinct subconstructs. Moreover, *part-whole* was under these four subconstructs (Kieren, 1993). On the other hand, Behr, Harel, Post and Lesh (1993) distinguished "fractions as a part-whole relationship, rational numbers as the result of the division of two numbers, as a ratio, as an operator, and as a probability" (p. 997). Indeed, in the literature, each of the rational number sub-constructs share many of the same characteristics.

Part-whole means that the whole is broken up into equal "parts" so that the fractions reflect the relationship between the whole and the number of parts (Kieren, 1980). For instance, consider a pizza or cake cut into 8 equal pieces and one piece was eaten (see Figure 2.). In fractional form of the relationship between the remaining pieces and the whole is $\frac{7}{8}$. It means that there are 7 pieces left of the whole 8 pieces.

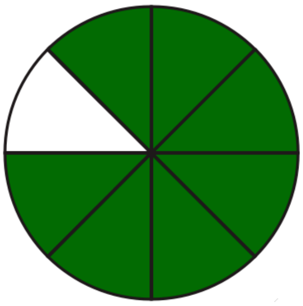


Figure 2. A pizza or cake cut into eight pieces missing one piece

The *quotient* subconstruct closely relates to the *part-whole*. However, they are applied in a different context. In *quotient,* a given quantity is divided into a given number of parts (Kieren, 1980). *Part-whole* represents the selection equal parts from a unit. To clarify this distinction, an example of *part-whole* is dividing a unit into fifths and choosing three of the parts $\left(\frac{3}{5}\right)$. An example of *quotient is* dividing three units into five parts $\left(\frac{3}{5}\right)$. Although the *quotient* and *part-whole* examples lead to $\frac{3}{5}$, these are different problems and they represent different ideas.

The subconstruct of *measure* also relates to the *part-whole* relationship. The difference is that the idea of unit is explicit in the *measure*. Measurement task means assigning a number to a region (one-, two-, or three-dimensional), which is done by co*unting* the number of whole units that cover the region (Kieren, 1980). To give an example, in Figure 3, each shaded region is measured by the unit, and covered region would be considered 4 units long. However, the same figure would be different in the *part-whole* interpretation, which is that the entire region would represent one whole, and each unit would represent $\frac{1}{4}$ of the whole.

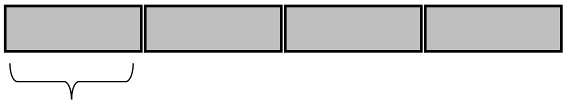


Figure 3. The measurement of 4 units

The *operator* "portrays rational numbers as mechanisms which map a set (or region) multiplicatively onto another set" (Kieren, 1980, p. 136). For instance, $\frac{3}{5}$ as an operator, expresses one to be multiplied by 3 and then divided by 5, or it expresses one to be divided by 5 and then be taken 3 times of that quantity. In other words, the *operator* subconstruct of rational numbers can be seen as elements in the algebra of functions (Kieren, 1980).

Kieren (1980) defined that the *ratio* subconstruct is the quantitative comparison of two quantities. To set an example, if there are 5 boys and 9 girls in a class (14 students in all), then the *ratio* of girls to boys in the class is $\frac{5}{9}$. The *part-whole* interpretation of $\frac{5}{9}$ is different from the *ratio*. However, Kieren (1980, p. 135) states that "the part-whole number relationships are a special case of ratio relationships." Later, Kieren (1988) suggested that *ratio* subconstruct would be included the *part-whole* subconstruct.

Kieren's (1980) theory about rational number subconstructs has had important implications to understand how students might develop the concept of rational number. However, students have difficulty to understand rational number concepts more than whole numbers because of the multiple representations and uses of rational numbers (Kilpatrick et al., 2001; Lamon, 2007; Moss, 2005). For instance, the rules of whole numbers can make transition to rational numbers, so students can generalize the whole numbers' rules. To make it clear, students assume that the greater the denominator, the greater the amount, such as $\frac{1}{6} > \frac{1}{5}$ because 6 > 5.

Teaching fractions has been considered more complex and difficult than teaching whole numbers (Cramer, Post, & delMas, 2002; Mack, 2001; Miller, 2005). Students work with whole numbers also outside of the school and they have fewer opportunities to use rational numbers and fractions in everyday situations. This makes rational number concepts more complex in terms of students' learning. Indeed, students' resist to learn fraction concepts because fractions are not similar to the pre-existing whole number knowledge (Dehaene, 1997). The studies of rational number concepts and usage reveal that understanding fractions takes time. On the other hand, insufficient instruction in the various interpretations of rational numbers and fractions can result in ineffective rational number and fractions teaching (Miller, 2005). Research studies have shown that teachers bring rather procedural knowledge to the classroom, which has the potential to cause misconceptions (Ball, 1990; Ma, 1999; Tirosh, 2000). Thus, teachers are very important actors in the process of teaching rational number concepts, which is difficult to be conceptualized by its nature.

Fractions and rational numbers are two important concepts among the several baseline mathematics concepts in the mathematics curricula of the middle schools. Introducing fractions starts in primary education and continues until the 6th grade. Rational numbers are introduced in the 7th grade and are used throughout the school life. In high school and university education, in order to success in mathematics, the understanding of fractions and rational numbers are essential. For instance, ratios, proportionality, equivalence among fractions and decimals are vital in Algebra, Geometry, Statistics, and many other mathematics concepts (Dopico, 2017; Kieren, 1993). Several studies have also indicated that understanding the structure of operations with rational numbers is essential to understand the structure of algebraic thinking and operations (Booth & Newton, 2012; Lee & Hackenberg, 2014). In a similar manner, in order to understand rational numbers, fractions should be understood. It is the most critical step because students encounter with abstraction substantially in fractions (Wu, 2005).

2.4 Rational Numbers and Teacher Knowledge Studies

In fraction instruction, Cramer, Post, and delMas (2002) expressed that teachers frequently focus on syntactic knowledge or rules. Other researches also indicated that teachers' knowledge used in the classroom was mostly procedurally-based and could be misunderstood (Kajander, 2005; Ma, 1999; Tirosh, 2000). While moving from whole numbers to fractions, teachers may make an over-generalization (Mack, 1995). Therefore, it is important to look into the studies of teacher knowledge regarding rational numbers.

Several researchers studied rational numbers with teachers in knowledge aspects. For instance, Golding (1994) found that middle school teachers had difficulty to respond the questions in the Rational Number Project test about part-whole, ordering fractions, operations, fraction equivalence, and unit conception. Besides, most of teachers could not explain their solutions adequately in a pedagogical way (Golding, 1994). Golding (1994) stated that teachers' inadequate conceptual knowledge about rational numbers caused weak understanding for students.

Walters (2009) worked with three teachers to understand their teaching about rational number. Similar to the present study, he used interview and classroom observation data in this case study. He found that teachers had difficulty about the core rational number content, in understanding the ratio and proportion concepts, and making connection among fractions and decimals, and ratios and proportions. Moreover, participants of Walters's (2009) study thought that ratio and proportion content was more complex.

Millsaps (2005) also investigated teachers' CK about rational numbers, their instructional practices and the interrelationship between them. Two teachers were examined with observations, interviews, and a test of rational number knowledge. It was found that interrelationships and teachers' CK contributed to their PCK which they used to design the instructional environment (Millsaps, 2005).

Ma (1999) made cross-cultural study that investigated American and Chinese elementary school teachers' knowledge regarding division and multiplication of fractions. She reported that only 43% of American elementary teachers completed division computations and finding the correct answer, but all the Chinese teachers reached the correct answer. American teachers knew various representations thanks to their PK; however, their representations were not proper. Thus, they had incomplete subject matter knowledge (Ma, 1999). She also stated that division in fractions could be thought as one of the most complicated operation in numbers. Ma (1999) also explained the meaning of division by fractions as the inverse operation of multiplication.

In Turkish context, different researchers examined both preservice and inservice mathematics teachers' knowledge in terms of fractions or rational numbers. To give an example, Can (2019) looked into 20 secondary school mathematics teachers' PCK about fraction operations. She explored teachers' CK and conceptual and procedural knowledge regarding students' difficulties and misconceptions. According to Can (2019), the findings revealed that teachers had insufficient procedural and conceptual knowledge about fraction operations. Furthermore, they had difficulties in posing word problems with real life aspect (Can, 2019).

In the same manner, Doruk (2020) also investigated mathematics teachers' suggestions when they meet with students' learning difficulties about ordering, addition, multiplication and division of rational numbers. He collected data from six mathematics teachers in public middle schools with semi-structured interviews. It was found that most of the teachers used the rule reminder method and repeated the instruction in order to eliminate students' difficulties regarding operations in rational numbers. For ordering the content, they mostly wanted to use models (Doruk, 2020). Moreover, Doruk (2020) asserted that the reason for using rules could be having deficient conceptual knowledge.

Another qualitative case study was Işıksal's (2006) study which examined subject matter knowledge and PCK about multiplication and division of fractions. She collected data from 28 senior preservice mathematics teachers with questionnaires, and semi-structured interviews with 17 of them. The findings revealed that they could easily symbolize and solve basic questions on multiplication and division of fractions. However, she found that preservice mathematics teachers did not have sufficient subject matter knowledge to represent and explain conceptually although they had strong beliefs to teach it conceptually (Işıksal, 2006). She developed a questionnaire in order to understand the subject matter knowledge and PCK of preservice teachers. Therefore, in the present study, Işıksal's (2006) and Haser's (2006) studies were considered in developing vignette questions and interview protocols.

2.5 PCK and Beliefs Studies

The literature addressing teachers' PCK and their beliefs separately is very broad and a representation of the studies were provided above. The study also focused on the possible interaction between mathematics teachers' PCK and their beliefs. Therefore, the research studies, which focused on the interactions or relationships between PCK and beliefs, take part in this section. The studies that guided this study were reported in detail. Firstly, studies in mathematics education and then, other areas were mentioned. On the other hand, even if PCK is the scope of this study, CK is a component of PCK and they both affect student progress (Kleickmann et al., 2013). Thus, studies focusing on CK were also taken into consideration in the following literature review.

2.5.1 Studies with Preservice and Inservice Mathematics Teachers

Most of the studies about preservice and inservice teachers' knowledge and beliefs found that both aspects were related and contributed to teaching quality (e.g. Blömeke et al., 2014; Campbell et al., 2014; Charalambous, 2015; Drageset, 2010; Strawhecker, 2004). The common findings of these studies were that knowledge and beliefs can ensure mathematically rich environments, and they should be examined together to understand teachers' behaviors. Mathematics teachers' beliefs also are influential on the teachers' pedagogical decisions and their classroom practices (Cross, 2009; Richardson, 1996; Wilson & Cooney, 2002). Wilkins (2008) indicated that beliefs partially intervened in content knowledge on instructional practice. In this regard, studies that are relevant to the present research were summarized below.

Drageset (2010) examined the interplay between knowledge constructs and beliefs constructs of 356 Norwegian mathematics teachers. He selected Ball et al.'s (2008) subject matter knowledge as knowledge construct, and rules and reasoning as belief constructs (Drageset, 2010). His study showed that beliefs have an impact on what one learns and learning Specialised Content Knowledge might change or influence teachers' beliefs. Thus, it can be said that knowledge and beliefs influence each other. Drageset (2010) concluded that it was necessary to consider teachers' beliefs and knowledge not only as linked, but also as elements that strengthen each other.

Different from the Drageset (2010), Campbell and her friends (2014) added student achievement and awareness, and they investigated the relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. They defined perceptions as teachers' beliefs and awareness. A total of 259 upper-elementary and 184 middle-grades mathematics teachers, who teach to students in grades 4–8, participated in their study. Quantitative data were collected from different surveys: 120 multiple-choice items (80 CK and 40 PCK items) in knowledge survey, 40-item beliefs and awareness survey, professional background, and instructional context surveys. They found significant interactions between teachers' perceptions and knowledge, which influenced student achievement (Campbell et al., 2014). Statistically significant interactions were found between upper-elementary teachers' mathematical knowledge and awareness scale, and between middle-grades teachers' knowledge and belief scores, and their students' mathematics achievement. However, Campbell and her friends (2014) recommended that in order to clarify the implication of this statistically significant interaction, further research was needed.

In addition to inservice teachers, preservice teachers were examined in different studies. Strawhecker (2004) explored the impact of the newly implemented design which included four groups of preservice teachers. One group of preservice teachers were involved in a content course, method course and fieldwork, and second group was enrolled in a mathematics methods course only after completing the content course and fieldwork. In the third group, preservice teachers, who completed content courses, took method course and fieldwork. Last group of preservice teachers involved in only content courses and did not take other courses. The reason employing four different groups was that former and new undergraduate programs with different courses were taught at the same time. She explored the impact of the design and relationships on preservice teachers' beliefs about teaching mathematics, CK, and PCK. Quantitative research methods with pre and post instruments were conducted with 96 preservice teachers. To determine the relationships between the variables, she used a Pearson Correlation test and found all relationships to be statistically significant (Strawhecker, 2004). Moreover, it was found that the methods course held positive impact one's beliefs about teaching mathematics. She also found that the relationship between beliefs and CK was stronger than the relationship between beliefs and PCK. Although there were relationships between all variables, the strength of them was minimal and remained complex (Strawhecker, 2004). Blömeke and her friends (2014) worked with 183 preservice mathematics teachers, examined the relationship between mathematics pedagogical content knowledge (MPCK) and beliefs, and determined the possible cause-and-effect relationship. Data were collected from the first, second, and third year mathematics teacher education students. Data analysis revealed that MPCK was significantly linked to beliefs identified in the analysis of the third data collection. In addition, higher MPCK at the first measurement had a causal effect on constructivist beliefs, whereas beliefs did not predict MPCK's third measurement, but this did not imply that beliefs function as filters (Blömeke et al., 2014).

Chen (2010) examined mainly preservice mathematics teachers' knowledge development and belief change about fraction division through the methods course. Moreover, whether special content knowledge development influenced belief change or not, and whether one participant's beliefs influenced the development of PCK or not were investigated. Chen (2010) used a qualitative methodology with interviews, surveys, concept mapping and the written assignment. The data were collected from 27 participants in two steps. In the first step, Chen (2010) traced the development process of knowledge and beliefs during the method course by the tests, surveys, concept mapping and the writing assignment. In the second step, six participants were selected based on different mathematics achievement, and they were observed and interviewed. Then, as a follow up analysis, classroom observations were continued with one preservice teacher (Chen, 2010). The result of Chen's (2010) study pointed out that the development of special content knowledge triggered rethinking about beliefs. In addition, it was implied that beliefs about fraction division and teaching it influenced what preservice teachers would teach and how they would teach it.

Ives (2009) investigated beliefs, orientations, CK, and PCK of probability, and the relationships among these three aspects with preservice secondary mathematics teachers through qualitative methodologies. Data were collected from individual interviews, test items and different tasks. The researcher found relationships between the preservice teachers' orientations and their CK, as well as their PCK. The most important finding was that orientations could affect CK and PCK, yet at the same time CK and PCK can influence orientations. These relationships were based on identifying basic concepts, misconceptions, approaches to finding probability, and the contexts (Ives, 2009).

There is considerable research on the interplay between preservice or inservice teachers' content knowledge, beliefs and their practices in the literature. To give an example, Barraugh (2011) examined seven upper elementary grade teachers, and used surveys to measure teachers' knowledge and beliefs about teaching and learning mathematics. Additionally, observations were conducted to characterize the mathematical quality of teachers' instruction, and interviews were conducted to reveal their interpretations of policies and to triangulate knowledge and beliefs data. Data analyses revealed mediational relationships between teachers' beliefs, knowledge, policy interpretation, and context-specific factors. According to Barraugh (2011), every factor could influence instruction, and priority was given in some circumstances to certain factors and in other circumstances to other factors. Barraugh (2011) also pointed out that to ensure higher quality instruction, teachers' knowledge, beliefs, and interpretation of policies should receive equal attention.

Wilkins (2008) investigated 481 in-service elementary teachers' level of mathematical content knowledge, attitudes toward mathematics, beliefs about the effectiveness of inquiry-based instruction, and employing it in teaching. Wilkins (2008) expressed that teachers' beliefs partially mediated the effects of content knowledge and attitudes on instructional practice, and have the strongest effect on teachers' practice. Moreover, it was found that content knowledge was negatively related to beliefs in the effectiveness of inquiry-based instruction and teachers' usage. However, overall, teachers with more positive attitudes and beliefs in the effectiveness of inquiry-based instruction used it more frequently (Wilkins, 2008).

Bray (2011) analyzed four mathematics teachers' beliefs and knowledge and found that beliefs and knowledge influenced their error-handling practices during the class discussion. Teacher beliefs were mostly related to the structure of class discussions, whereas their knowledge appeared to be the determinant of the mathematical and pedagogical quality of their responses (Bray, 2011).

2.5.2 Studies in Other Fields

It is important to indicate that other disciplines have investigated the relationships between teacher beliefs and their PCK, especially in science. For instance, Bonner (2001) focused on the interplay of educational beliefs, PCK and perceptions of a science curriculum of five science teachers for ten months through observations, formal and informal interviews, and documents such as lesson plans and course syllabus. It was found that participants' beliefs about students and the structure of the discipline were vital to determine what to teach and how to teach it. Perceptions of the difficulty of the topic and time constraints also affected the selection of the course material and instructional approach (Bonner, 2001). According to Bonner (2001), beliefs, PCK and perceptions of the curriculum worked together and the interplay between them determined the implementation of the curriculum. Moreover, educational beliefs of the participants influenced their pedagogical knowledge.

Bahcivan and Cobern (2016) examined science teaching belief systems and their relation to PCK and teaching practices by conducting a multiple case study design with three in-service science teachers. Data, which were collected through interviews and classroom observations, pointed out that beliefs shaped participants' conceptions, knowledge and practice of teaching and learning science. Moreover, science teachers' beliefs had a systematic coherence with their PCK and practices (Bahcivan & Cobern, 2016). In a similar manner, Anderson (2015) conducted a multiple qualitative case study to investigate the nature and influence of beliefs on the science teaching practice and the knowledge development of three primary teachers. Analysis of observations and multiple interviews data showed that beliefs had strong influence on the development of teacher knowledge, the nature of both subject matter knowledge and PCK for science developed by the teachers (Anderson, 2015).

Mavhunga and Rollnick (2016) investigated the relationship between topic-specific pedagogical content knowledge and underlying beliefs of science teachers. This study was more special than other studies because it investigated topic-specific PCK, not general PCK. Data were collected from audio-recorded discussions and written responses from open-ended questions in pre- and post-tests with sixteen pre-service chemistry teachers. Results of the Mavhunga and Rollnick's (2016) study indicated that the relationship between topic-specific PCK and teacher beliefs is more complex than reflected in other studies.

2.6 Summary of the Literature Review

In the literature, prior studies have mostly documented the individual effect of teacher knowledge and teacher beliefs. Some studies handle knowledge and beliefs together, and add other dimension (e.g. practice, policy, perception). However, they have examined the general context, mathematics teaching, and were conducted mostly with quantitative designs. Thus, statistically significant relations between the investigated dimensions were found, and researchers have warned that further research was needed to clarify it. As stated above, there is scarcity of studies that analyzed how teachers' PCK and their beliefs interact each other. Therefore, in this study, in order to provide a deep understanding, interactions between PCK and beliefs were investigated explicitly with qualitative methods in a subject specific context, the rational numbers.

The literature also focused on tracing changes in teachers' beliefs and knowledge as a result of participation in a training program or a course. Such studies mostly reported positive developments. Although they have contributed to the present study, the current study focused on the existing situation and the circumstances. On the other hand, many studies about the relationship between PCK and beliefs targeted preservice teachers, which addressed that there is a gap in the literature in terms of in-service teachers. Thus, the current study intends to contribute to the literature by deeply investigating the mathematics teachers' PCK and beliefs in the context of teaching rational numbers.

CHAPTER 3

METHODOLOGY

The purposes of this study were to investigate the nature of middle school mathematics teachers' PCK and beliefs regarding rational numbers and how these beliefs and PCK interact with each other. As explained in the literature review, TEDS-M's (2008) framework was employed in this study because it provides a comprehensive perspective for teachers' PCK and how it might be related to teachers' beliefs. This chapter gives detailed information about methodological approach of the study. In order to have a better understanding of the methodology, the research design, participants, data collection, pilot study, data analysis procedures, and trustworthiness of data and findings are clarified in this part of the study.

3.1 Research Design of the Study

In this study, in order to reveal middle school mathematics teachers' PCK and their beliefs, qualitative research design was employed. According to Creswell (2009), qualitative research is conducted in a natural setting by the researcher who has the opportunity for a face-to-face interaction with participants. Furthermore, multiple data sources such as interviews, observations and documents can be used, and inductive data analysis are conducted to set comprehensive themes about participants' acts, behaviors, and opinions in context (Creswell, 2009). According to Merriam (1999), qualitative study is "intensive holistic description and analysis of a single instance, phenomenon, or a social unit" (p. 21). A theoretical lens or a combination of theoretical lenses can be used to interpret the obtained data (Merriam, 1999). Based on the interpretive inquiry, researcher tries to establish holistic picture about the problem or issue (Creswell, 2009). If a problem or issue needs to be explored and needs to be understood in detail, or the interpretations of the participants are placed in the study, a qualitative research design can be conducted (Creswell, 2009). Therefore, qualitative design is appropriate to investigate teachers' PCK components and beliefs regarding rational numbers.

One of the approaches of qualitative research design is case study. Yin (2003) expresses that in order to answer "why" and "how" questions about a phenomenon in real-life, case studies are used. Case study is defined as "an empirical inquiry that investigates phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2003, p. 13). According to Merriam (2009) a phenomenon, a program, or a person should be selected in case study to investigate a particular reason for understanding better. In addition, in case study approach, researcher explores a case or multiple cases and collects detailed and in-depth data from multiple sources such as observations and interviews. Then, he reports the themes for the case or cases (Creswell, 2009).

In the literature, case studies were categorized by different researchers. Stake (1995) proposed three categories based on the focus of the research as 'intrinsic case study', 'instrumental case study' and 'multiple (collective) case study'. According to Stake (1995), 'intrinsic case study' concentrates on the specific individual or situation. The researcher describes the case or the particulars of the case in detail. In the 'instrumental case study', the researcher examines the case in order to understand an issue in the case (Stake, 1995). In the 'multiple (collective) case study', on the other hand, the researcher deals with multiple cases at the same time in order to explore the issue or the phenomenon with different perspectives (Stake, 1995). In the instrumental case study, one of the primary goals was to understand a theory, question, or problem. Using this approach, a particular issue was explained in detail (Hancock & Algozzine, 2006) to inform other similar issues (State, 1995). Therefore, in order to have a better insight on the interaction between the middle school mathematics teachers' PCK and beliefs in rational numbers, which was the case in this study, an instrumental case design was employed. This case was investigated with six middle school mathematics teachers. In the following section, details of the participant selection process are explained.

3.2 Participants

In the current study, three criteria were defined to select the volunteering participants. First, at least five-year experienced middle school mathematics teachers were sought because it was assumed that their beliefs and PCK would be well-established. Second criterion was teachers' availability to participate through the whole study. Third, volunteering middle school mathematics teachers should be teaching rational numbers in grades 6 and 7 in schools in Ankara, which was conveniently accessible for the researcher, because the study required long periods of lesson observations with possible overlaps across participants' weekly course schedules. Therefore, the observations and the interviews required commitment to the study and convenience for the researcher especially in terms of physical accessibility to the schools. Consequently, those teachers who were teaching in inner-city Ankara schools were reached through personal contacts for the study. Among the contacted teachers, six middle school mathematics teachers indicated their interests and accepted to participate. Table 2 provides an overview of the participants.

Table 2.

Participants of the Study

Part	Gender	Experience in teaching	Grades taught	Educational Level	School Neighborhood and Family Education Level	Class Student Achievement
T1	Female	11 year	6th 7th	Ongoing Ph.D.	Medium - Low	Below Average - Good
T2	Female	8 year	6th	Ongoing M.S.	Medium	Average
T3	Female	8 year	7th	B.S.	Low	Poor
T4	Female	10 year	7th	Ongoing Ph.D.	Medium - High	Good
T5	Female	5 year	7th	Ongoing M.S.	Very Low	Below Poor
T6	Male	10 year	7th	B.S.	High	Good

Note: There is no numerical data for the school neighborhood and students' family education level. This section was categorized based on how the participants described their school neighborhood and their students' family education level. Thus, information in this table depended on participants' subjective judgments.

As can be seen in Table 2., two of the participants were 6th grade teachers; four of them were 7th grade teachers; and one of them was teaching both 6th and 7th grades. Two of the teachers were doctoral students in the field of mathematics education; and two of teachers were conducting Master's studies. The schools of T1, T3 and T6 were located in same district. T4 and T5 were also located in same (other) district. T2 and T7 were located in other districts. All participants were experienced teachers in teaching at public schools, which were centrally located in four different and neighboring districts in Ankara. Thus, transportation in between and accessibility of schools did not affect the observation processes and interviews. All teachers, except for one teacher who had five-year experience, had a minimum of eight-year experience. Table 2 does not include one participant who had to leave the study due to the health problems which required her to stay home. Her data set was not complete and hence, she was not included in the study. Therefore, the participants of the study consisted the six teachers given in Table 2. One of participant is male whereas five of them were female, a situation representing the Turkish school contexts where most teachers are women (MONE, 2019). The teachers are presented below in detail in order to provide in-depth information about the study context.

Teacher 1 (T1) had been a teacher for eleven years at the time of the study and at the same school in the last 5 years. She got her bachelor and master's degrees from a public university. She continued with doctoral studies at the same university and was working on her dissertation. She was teaching to both 6th and 7th grade students. She described her classes as crowded because there were 33 students in the 6th grade and 40 students in the 7th grade. Students in her classes sit school desk individually, but they were very close to each other because of limited space. Classroom sizes were the same for 6th and 7th grade, so 7th grade class has lower free space than 6th grade. Both classrooms had smart board, internet connection and two white boards.

According to T1, the levels of 6th grade students were below-average and 7th grade students were average. She also states that school neighborhood is mixed with medium and low-income families because the neighborhood was under an urban transformation project.

Teacher 2 (T2) had been a teacher for eight years and last 4 years were in the same school at the time of the study. She graduated from a public university with bachelor degree. She continued with Master's degree at the same university. She was teaching to 6th grade students. She described that this class included 35 students where two of them were taking individualized education program and one of them was a hyperactive student. Observation data revealed that two students sit on the school desk together, but the classroom had sufficient free space for students. Classroom size seemed to be good for teaching and airflow. The classroom also had a smart board, internet connection and two white boards.

Based on the T2's description, her 6th grade students had average mathematics knowledge and skills. She also stated that the school neighborhood was mixed with medium and low-income families with mixed education levels because the neighborhood was under an urban transformation project.

Teacher 3 (T3) had been teaching for eight years, but it was her first year in the school at the time of the study. She graduated from a public university with bachelor degree. She was teaching to the 7th grade students. Her class had 22 students, two of whom were Syrian immigrants and three of whom were from an orphanage. Two students sit on the school desk together in her class, but the classroom had enough free space for students. The classroom had a smart board, internet connection and two white boards.

T3 expressed that the 7th grade class included students with average and poor level mathematics knowledge and skills. She also stated that school neighborhood had low-income families. Her students' family education level was also low.

Teacher 4 (T4) was in her 10th year in the profession and her last 3 years were in the same school. She had B.S. and M.S degrees from public universities and she finished the courses in her doctoral program at the time of the study. She was teaching to the 7th grade students. She described her class, which included 29 students, as crowded. Two students sit on the school desk together in her class and the classroom was narrower and longer than the other classrooms in this study. Thus, there was inadequate free space for students. The classroom did not have a smart board, but there was rather an old computer, projector and internet connection, and one white board.

According to T4, the mathematics knowledge and skill levels of 7th grade students were variable; half of them were above the average and quarter of them was below the average. She claimed that her school was the most successful school in that neighborhood. She also stated that school neighborhood was mixed with medium and upper medium income families. Most of the parents had professions such as doctor, teacher, engineer and tradesmen.

Teacher 5 (T5) had been teaching for five years and she had been teaching in the same school since she began teaching at the time of the study. She graduated from a public university with bachelor degree and continued with Master's degree in the same university. She was teaching 7th grade students. She stated that her class had 14 students where a small number of students had behavior disorder and a few of them were inclusive students. Two students sit on the school desk together in her class, but the classroom had considerable free space for students. Classroom size was good for teaching and airflow because the number of students was not high. The classroom had two white boards and a smart board which was not working because there was no internet connection and electrical connection.

T5 express that the 7th grade students had poor level of mathematics knowledge and skills. She also stated that the school neighborhood consisted of very low-income families and families where parents were lot living together. Most of the students were raised by grandparents. The neighborhood was under an urban transformation project.

Teacher 6 (T6) has been teaching for 10 years where the last 5 years were at the same school. He graduated from a public university with bachelor degree. He was teaching 7th grade students, and described his class crowded because there were 30 students in a limited space. Two students were sitting on the school desk together in his class and the classroom had limited free space for students. The classroom had a smart board, internet connection and two white boards.

According to T6's description, 7th grade students' mathematics knowledge and skill level were good. He also stated that there were 10 students in the school who gave correct answers to all questions in the national exam in the previous year and his school was one of the most successful schools in that neighborhood. He also expressed that there were high-income families in the school neighborhood. Most of the parents had professions such as doctor, judge, public prosecutor, and engineer.

3.3 Data Collection Tools and Procedures

Qualitative data are generally collected by observation, interviews, documents, and audiovisual materials (Cresswell, 2007). Interviews are the crucial data collection tools in case studies and using multiple data sources enable to explore the issue or case deeply (Cresswell, 2007; Yin, 2003). Due to the complex structure of PCK and beliefs, it is difficult to assess them (Baxter & Lederman, 1999) and collecting data only through one way may give incomplete or inaccurate information. Therefore, three types of data collection strategies were employed to answer the research questions as two rounds of semi-structured interviews, interviews based on vignettes and observations of participants. First, semi-structured interviews were conducted before the academic year. One week later, the second semi-structured interviews were conducted. Then, a few days later, interviews based on vignettes were conducted with participants. Observations started at the beginning of the fall semester and continued during the semester.

3.3.1 Interviews

Researchers unfold participants' thinking about the phenomenon with the interviews (Ritchie & Lewis, 2003) because interviews can provide a deep understanding of what is in people's mind (Creswell, 2012). Merriam (2009) categorized interviews into three types: (a) highly-structured in which questions and their order are predetermined; (b) semi-structured in which all questions are used in flexible order and wording; and (c) unstructured, which is more like a conversation, and uses to more formulate questions for later interviews. In semi-structured and unstructured interviews, the researcher uses more open-ended questions (Merriam, 2009). In the current study, semi-structured interviews were conducted to provide a more open-ended and flexible process. All questions in the interviews were asked to all participants, and then to understand or explain the responses, articulated questions were asked in this study. All the interview processes were conducted in one-to-one settings. Before having interviews, the place where participants preferred and felt comfortable were asked, and all of them selected an empty class or quiet place in their school. The duration of each interview ranged from 45 minutes to 1 hour 10 minutes. Interviews and vignettes were finished one week before the fall semester of 2017 – 2018 academic year.

The first interview (Interview I) questions were developed in line with the related literature and by considering Haser's (2006), Işıksal's (2006) and TEDS-M (2008) studies closely. The first interview started with the questions about demographic information of teachers and their experiences. Then, questions mostly about teachers' beliefs, opinions, thoughts and purposes about mathematics, curriculum and influencing factors were asked. The last part of Interview I was related to teachers' beliefs and opinions about mathematical problems, problem solving and strategies. This interview was conducted before the school year started. In Table 3, sample interview questions are presented, and all questions in Interview I are given in Appendix A.

Table 3.

Sample Questions from Interview I

Related to	**Interview Questions**
Background	How many years have you been teaching? Which department have you graduated from?
About Mathematics	What does it mean to know mathematics? What is your purpose when you are teaching mathematics?
Curriculum	When middle school curriculum is considered, are there any concepts or topics that are the most important, fundamental or central in the curriculum? Why are these important, fundamental or central?
Teaching	What are the factors which influence your mathematics teaching? How do you feel when you are teaching, learning and engaging in mathematics? Can students learn mathematics without teacher? (If yes,) How?
Other factors	Are there any pressures or supports that affect your teaching? (such as from students, parents, school administrators) (If yes,) How?
Problem Solving	What does a "mathematical problem" mean for you? What is the importance of problem solving in mathematics? What do students gain? How do you know that your students have understood a problem?

The second interviews (Interview II) were conducted one week later. Interview II questions were developed based on Haser's (2006), Işıksal's (2006), and TEDS-M (2008) studies and in line with the related literature and the national curriculum. Interview II had questions about teachers' opinions and preferences of teaching mathematics, fractions and rational numbers. This interview started with their practices, general preferences and understandings about their lessons. Then, questions related to teaching fractions and rational numbers were asked. Lastly, how they imagined their ideal classroom was described by the teachers. Sample questions of Interview II are presented in Table 4, and all questions are given in Appendix B.

Table 4.

Sample Questions from Interview II

Related to	Interview Questions
Lessons	Do you make a lesson plan before your lessons? (If yes,) Do you use additional resources? (books, printed paper, etc) (If yes,) What are those resources? Which topic do students understand better? Why do they understand this topic better?
Teaching Fractions (same questions were also asked about rational numbers)	How do you teach fractions? (If demonstration / using material / link to other subjects / link to real life) How? Do your students have difficulty in understanding any case/concept about fractions? (If yes,) What is that? Which measurement and assessment instruments/methods do you use to assess students' knowledge about fractions? Why do you choose this instrument/method?
Ideal Class	What do you think about your ideal class? Can you define it? Do you think that you are teaching ideal class?

3.3.2 Observations of the Teaching Process

Observations are also one of the primary sources of data in qualitative research (Merriam, 2009). Marshall and Rossman (2011) defined observation as it "entails the systematic noting and recording of events, behaviors, and artifacts (objects) in the social setting" (p. 139). If a researcher wants to reach a fresh perspective about an activity, event, or situation, then observation is the best technique to use (Merriam, 2009). Therefore, in addition to the interviews with six teachers, observations were conducted during teachers' entire instruction of rational numbers. What to observe depends on several factors, such as researcher's purpose or the theoretical framework or the problem (Merriam, 2009). The main purpose of the observation in this study was finding examples of their use of PCK and how the beliefs they expressed in the interviews might be interacting with their PCK when they were teaching, planning, and assessing. Merriam (2009) states that observations can be used to triangulate emerging findings. Therefore, it is used to substantiate the teachers' responses to interview questions in this study.

All observations were conducted during the Fall semester of 2017 – 2018 academic year. Participants were observed for five lessons per week, and notes were taken by writing down during the observations. The duration of the observations were: T1 for 28 hours in the 7th grade and 24 hours in 6th grade classrooms; T2 for 24 hours in the 6th grade classrooms; T3 for 20 hours in the 7th grade classrooms; T4 for 17 hours in the 7th grade classrooms; T5 for 15 hours in the 7th grade classrooms; and finally T6 for 21 hours in the 7th grade classrooms. Most of the notes were about teachers' teaching practices because I focused my attention on the teachers. I was continuously writing down while they were teaching in the class. I sat on the desk at the back of the class during the observations and did not have any interactions with the students and/or teachers in order not to distract them, and did not participate in the classroom activities. Based on Merriam (2009), in the current study, I was the "observer as participant" which means that researchers observe the group or person to gather the information. Participation in the group is the secondary role. After the observed lessons, if necessary, I noted down my personal interpretations of the observations in the observation notes.

3.3.3 Vignettes

Observations provide useful data to make sense of teachers' beliefs, PCK and their instruction. However, repeated observations over time can be difficult to conduct, or result in misjudgements or errors. On the other hand, less frequent observations may not provide an accurate measurement (Stecher, Le, Hamilton, Ryan, Robyn, & Lockwood, 2006). Thus, only observation or interview can be inadequate to understand teachers' belief and PCK. In that case, vignettes can lend assistance to gather accurate data (Campbell, 1996; Jeffries & Maeder, 2005) because vignettes are more realistic or closer to classroom context (Stecher et al., 2006). Vignettes are specific scenarios, which can include students' comments, questions, solutions, and misconceptions. Vignettes also consist of a case describing a part of a lesson, and/or a confusion in the class (Campbell, 1996; Ebert, 1994; Jeffries & Maeder, 2005), so that teachers respond each vignette in that case. Vignettes should give an opportunity to bring out the specific ideas (Campbell, 1996). Jeffries and Maeder (2005) found that vignettes were an effective assessment tool in teacher education, especially to assess pedagogical understanding. In the literature, vignettes have been used for different purposes in education, such as to identify beliefs and attitudes and to assess knowledge and effective teaching (Jeffries & Maeder, 2005). Therefore, in order to understand the teachers' beliefs and PCK, vignettes were used in this study.

There are eight questions in vignettes. To prepare or generate each question, cases, or tasks in Van de Walle, Karp and Bay-Williams (2013), TEDS-M (2008), Haser (2006), Işıksal (2006), and Tekin-Sitrava (2014), the related literature, national curriculum and textbooks were used as resources. They were checked by two different experts for the face and the content validity, to ensure that all problem situations in the vignettes were carefully defined. Vignettes were conducted one week after the interviews. Teachers responded vignettes individually and, in the meantime, meetings were recorded in an interview fashion. A table of specifications for vignettes in terms of PCK components and Kieren's (1988) sub dimensions are presented in Table 5, and as his definition, Part-Whole component is subsumed under the ratio construct. Questions in vignettes can be seen in Appendix C.

Table 5.

A Table of Specifications for Vignettes

	Mathematical Curricular Knowledge	**Knowledge of Planning for Mathematics Teaching and Learning**	**Enacting Mathematics for Teaching and Learning**
Measurement	Q3		
Quotient	Q3	Q2	Q2, Q7
Operator	Q3, Q6	Q1, Q7	Q1, Q6, Q7, Q8
Ratio		Q4, Q5	Q4, Q5

3.4 Pilot Study

Before conducting the actual study, a pilot study was conducted to ensure the clarity and suitability of interview protocols and vignettes. Clarity and comprehensiveness of the interview protocols and vignettes were also checked. Pilot study was conducted with two teachers three months before the main study. These teachers in pilot study have several characteristics and background in common with the participants in terms of teaching experience, educational level and school neighborhood, but they did not participate in the main study. Table 6 presents pilot study teachers' characteristics. Two interviews and interviews based on vignettes were done with them in three different times in the same week.

Table 6.

Participants of the Pilot Study

Parts.	**Gender**	**Year in the Profession**	**Educational level**	**School Neighborhood and Family Education Level**	**Student Achievement**
P1	Female	5th year	M.S.	Low	Poor
P2	Male	10th year	B.S.	Medium - Low	Poor

Teacher 1 (P1) had been a teacher for five years at the time of the study and at the same school in the last four years. She graduated from a public university with bachelor degree and she was teaching 5^{th}, 6^{th} and 7^{th} grade students. She described her classrooms as crowded because they ranged from 38 to 43 students. Her class had a smart board, internet connection and two white boards. P1 claimed that the levels of her students were poor. She also stated that school neighborhood was composed of low-income and low-education level families.

Teacher 2 (P2) had been a teacher for ten years at the time of the study and at the same school in the last six years. He graduated from a public university with bachelor and master's degree. He was mostly teaching 8^{th} grade students. In his first year, he taught 5^{th} grade students. As P1, he described his classrooms as crowded because they ranged from 40 to 45 students. His class had also smart board, internet connection and two white boards. P2 claimed that his students' levels were poor and school neighborhood consisted of low-income families. According to P2, his students' family education level was mixed with medium and low level.

All interviews and vignettes in the pilot study were recorded and transcribed. Findings of the pilot study addressed that interview protocols were applicable in terms of clarity and comprehensiveness. Three questions in the vignettes were improved. To increase clarity, little changes in questions or sub questions were done after taking the expert opinions.

3.5 Data Analysis Procedure

Qualitative data analysis starts from the first interaction with participants and goes on throughout the entire study (Gay et al., 2009). In qualitative research, one of the purposes of data analysis is to describe the case holistically and in depth (Yin, 2003). Moreover, in data analysis process, the aim is to get meaningful parts by reducing them, so that the research questions can be answered (Merriam, 2009). The data of the current study were gathered from multiple sources. Merriam (2009) suggests that first of all, a researcher should prepare data for analysis. Creswell (2009) also explains six steps to follow in qualitative data analysis procedure: i) arrange and organize the data for analysis; ii) read all the obtained data; iii) code the data; iv) produce the themes and/or the descriptions from the data; v) interrelate the themes and/or the descriptions; and vi) interpret the meaning of the themes and/or the descriptions. These steps were followed for data analysis in this study. Therefore, I began analysis by producing verbatim transcripts of each interview recording. Then, the questions and responses of vignettes were transcribed verbatim. Observation notes were also scanned and prepared for analysis. All data then were imported to MAXQDA 2018 qualitative data analysis software. Before the coding process, reading all transcriptions is suggested in order to get general sense of data (Creswell, 2009; Merriam, 2009). Thus, I read all the transcriptions and observation notes without an analytical perspective to have a general sense of the data. Then, I began the coding process.

In the coding process, there are two common methods to analyze: (a) open coding method in which researchers develop codes based on their conceptual understanding; (b) using the pre-determined codes in which researchers use the developed codes that are based on the phenomenon or findings in the literature (Creswell, 2007; Miles &

Huberman, 1994). In the current study, pre-determined codes were used. TEDS-M (2008) PCK framework was used as PCK codes, and belief statements were coded under the guidance of the TEDS-M framework with some room for additional emerging codes. Deciding on what is being coded is important in the coding process (Creswell, 2009; Merriam, 2009). Thus, any meaningful chunk of statement (e.g., a sentence, several sentences, or a paragraph) about the PCK and beliefs was coded. In the open coding part, “meaning” was also used as the unit of data, and similarities and the relationships among codes were examined. After my coding process ended and open coding process seemed to be saturated, a second coder coded one participant’s interviews and vignettes. The second coder is an expert who has PhD in the mathematics education field and has studies about teachers and teaching. Therefore, it is possible to claim that she has a sufficient knowledge of mathematics related PCK and beliefs. Before she started the coding process, the TEDS-M (2008) MPCK framework and PCK codes were introduced to her and a detailed discussion took place. In her initial analysis, there was an 80% agreement on the coding statements and codes. In addition, there was a 70% agreement in open coding part of beliefs codes. In order to ensure inter-coder agreement, at least 80% agreement should exist (Miles & Huberman, 1994). Although there was a fairly sufficient agreement on the codes, disagreements were discussed until the agreement was reached to at least 90%. For the remaining data of other participants, the second coder did not individually code because of the time issues. However, it was decided that the codes were appropriate for the understanding of teachers’ PCK and beliefs. Then, I continued to the coding process of the rest of the data.

3.6 Trustworthiness

Trustworthiness is used for the issues of reliability and validity in qualitative research (Lincoln & Guba, 1985; Merriam, 2009). Moreover, in qualitative terminology, credibility, transferability, dependability, and confirmability were used instead of internal validity, external validity, reliability and objectivity (Creswell, 2009). For qualitative methodology, trustworthiness should be ensured because findings and interpretations are needed to be accurate (Creswell, 2012; Yin, 2003). Merriam

(2009) states that credibility is necessary to conceptualize the study based on the findings. Therefore, I used the term “trustworthiness” in this study.

In order to provide the trustworthiness of a qualitative research, several methods have been used in the literature. One of the common and suggested strategies is thick descriptions (Creswell, 2009). In thick descriptions, researchers give detailed explanations about the context and process of the study, how the participants are selected, how the data are analyzed, and how the findings are revealed (Merriam, 2009). Thus, to provide rich and thick descriptions, detailed explanations about each procedure followed in the current study were given. Employed theories and frameworks from the literature were presented, and detailed explanations of participant selection and data analysis procedures were given. The explored findings were also presented with detailed quotations. On the other hand, the rich and thick description can be used to transfer this research study to other studies. Transferability of the qualitative design to other situations refers to external validity or generalizability (Merriam, 1999). In case studies, representativeness for similar cases is important for generalizability (Stake, 1995). Merriam (1999) suggests thick description, modal comparison, and sampling within in order to strengthen the external validity of a study. Besides, all the interview protocols in this study were checked by two experts in order to ensure the face and the content validity.

Another common strategy is a triangulation. Creswell (2012) define triangulation as “the process of corroborating evidence from different individuals (e.g. a principal and a student), types of data (e.g. observational field notes and interviews), or methods of data collection (e.g. documents and interviews) in descriptions and themes in qualitative research” (p. 259). In the current study, the three different data collection tools, which were the interviews, observations and vignettes, provided the triangulation of data. Moreover, a second coder who coded some part of the data contributed the triangulation.

In qualitative research, clarifying the researcher’s role is considered as a crucial step in the trustworthiness of the study (Creswell, 2007; Merriam, 2009). As the researcher of this study, I received a bachelor degree in elementary mathematics education in

2009. After this, I enrolled in a Master of Elementary Science and Mathematics Education degree program and studied pre-service teachers' technological pedagogical content knowledge, geometry knowledge, and effects of instructional technologies on these. After I completed the Master's degree, I enrolled in a Mathematics Education PhD program. In the current study, I used to know two participants who graduated from the same university that I did, but I worked with all participants for the first time in their teaching profession. Thus, I did not have any previous knowledge about their professional experiences and ideas. On the other hand, when participants share their views, there is a possibility of respondent bias. It means that participants share the desired responses rather than their actual views (Creswell, 2007). In order to eliminate this threat, I used vignettes and observed their teaching process. In addition, the findings of the current study indicated that all participants shared honest views with me throughout the data collection process. Their interview responses about teaching were consistent with their actions. Hence, I assumed that respondent bias was not a treat for this study.

CHAPTER 4

FINDINGS

In this chapter, the results of the analysis to respond to the main research question and subquestions are presented:

How do middle school mathematics teachers' beliefs and their PCK interact in teaching rational numbers?

1. What is the nature of middle school mathematics teachers' PCK regarding rational numbers?
2. What is the nature of middle school mathematics teachers' beliefs regarding rational numbers?
3. What is the interaction between middle school mathematics teachers' beliefs and their PCK for teaching rational numbers?

The chapter's sections are organized in the order of research questions. Participants' responses from instrument, vignettes and observation notes and their comparisons are presented under these sections. In order to understand their PCK and beliefs, findings are presented based on the TEDS-M framework for the PCK but not for each participant. Yet, all different findings are given for each component. For the beliefs, first, participants' beliefs are presented for each participant. Then, participants' common and different beliefs are presented for three different types of beliefs.

In the abbreviations in the quotations below, T and a number addresses the participants. "O" refers to observation data, "I1" refers to the data from first interview and "I2" refers to the data from second interview. For instance, T3_I2 means Teacher 3's responses from the second interview. Besides, "VQ" stands for vignette questions and the number is the question number. To give an example, *T2_VQ4* means the data from Teacher 2's responses to 4^{th} vignette question.

4.1 The Nature of Middle School Mathematics Teachers' PCK Regarding Rational Numbers

In this part, the findings of the interviews, vignettes and observation notes are presented through the TEDS-M framework in order to document participants' PCK.

4.1.1 Mathematical Curricular Knowledge in TEDS-M

The mathematical curricular knowledge subdomain comprises establishing appropriate learning goals, knowing different assessment formats, selecting possible pathways and seeing connections within the curriculum, identifying the key ideas in learning programs, and knowledge of mathematics curriculum. Nonetheless, there was not any evidence to in relation to establishing appropriate learning goals in the data.

For knowledge regarding *assessment formats*, interviews and observation notes indicated that participants knew about open-ended, multiple choice, ordering, short answer, and true-false questions. However, they mostly used open-ended questions in their exams. T1 said that she asked questions that required multiple operations in solving them. She expressed that she asked about operations in word problems, however, she mostly used problems as suggested in the curriculum, not in exams, as other teachers. In lessons, participants mostly asked questions to understand whether students learned properly or not. Besides, T2 and T3 stated that they sometimes used pop-quizzes, but they did not include their results in their students' final score. They just checked whether their students learned the concept or not.

For *selecting possible pathways and seeing connections within the curriculum* aspect, all participants made connections between rational numbers and previous topics in their teaching. For instance, they reminded the whole numbers when they taught rational numbers, especially negative parts. In practical terms, participants also made connections between old and new knowledge. To give an example, when T1 taught the improper fractions, she expressed that it was learned in the last semester and she just reminded. In rational numbers lessons, if the subject were related to the past learned subjects, all participants reminded the learned knowledge to students. T4, for example, reminded how to equalize the denominator which was in the fraction's topic in the last semester. Besides, they mostly reminded the previous lesson at the beginning of the new lesson. Then, they started the new subject.

When participants' *identification about the key ideas in learning programs* was investigated, it appeared that they mostly focused on the curriculum. They mentioned the key ideas of learning programs a few times in their interviews, vignettes and lessons. For instance, T1 said in her lesson that: *"…It adds on every year. Our curriculum has a spiral structure, so if you don't understand it this year, you'll have difficulties the next year" (T1_O).* T4 and T6 mentioned the same idea in the interview. In teaching multiplication of rational numbers, T1 and T3 started with adding rational numbers. Then, they said that multiplication was the short cut for "*adding the same number*". T3 also expressed that division was the shortcut for the subtraction. In participants' general idea, numbers and algebra were the basic, central content in mathematics because, they claimed, most of the other contents and topics built on them, and tied up the next issues.

Participants' *knowledge of mathematics rational numbers curriculum* seems to be adequate. Ordering fractions and operations of fractions were instructed in 6^{th} grade curriculum. Moreover, teachers should introduce and compare rational numbers, make operations and solve problems in rational numbers in 7^{th} grade curriculum (MEB, 2018). The guidebook by the Ministry, which included the learning outcomes respectively and some examples about how to teach particular concepts, was helpful for the teachers to remember the learning outcomes. They were also aware of what they had to teach in curriculum, but they acted differently when they taught about the content. They pointed out part-whole relationship, meaning of addition, whole-rational number relationship, negative rational numbers, and the idea about exponential numbers in their interviews and observation data. Moreover, they knew what they taught to reach rational numbers learning outcomes. To give an example, the in 6^{th} grade curriculum, multiplication of whole number and fractions was taught with repetitive addition. T1 and T2 also explained to students in this way. In addition, they explained the meaning of multiplication such as multiplication of $\frac{1}{2}$ and a number means to half of the number. All teachers were able to draw a multiplication model of rational numbers readily. For instance, in vignette question 1, it was asked to draw a visual model of 5 x $\frac{3}{10}$, and they drew the model of $\frac{3}{10}$ 5 times. Then, they stated that they added 5 times, which meant multiplying, and it was the way of teaching multiplication with models. It was seen T4's visual model of the multiplication of rational numbers in Figure 4.

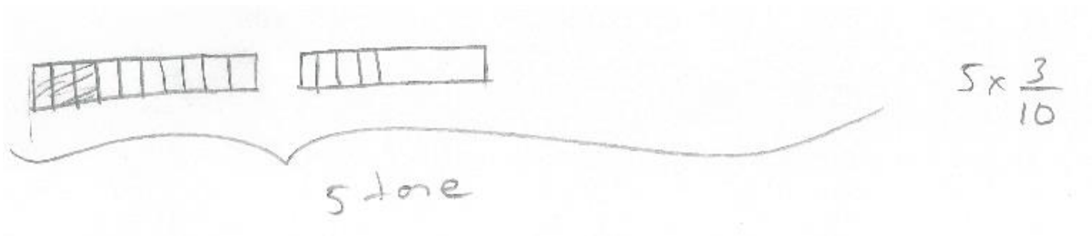


Figure 4. Sample visual model of 5 × $\frac{3}{10}$.

In vignette question 6, how to respond to a student who wanted to apply the multiplication rules on addition of rational numbers was asked. Most of the participants showed the operations correctly, but procedurally. T6 tried to explain why the same denominator had to be used in addition operations whereas it was not needed in multiplication. He gave an example of an addition operation of $\frac{1}{2}+\frac{1}{3}$ with two results; $\frac{2}{5}$ (incorrect) and $\frac{5}{6}$ (correct) to show the difference. He also drew addition models (Figure 5) to show the meaning of and difference between addition and multiplication of rational numbers. Moreover, T6 stated that without understanding addition models, giving multiplication models was meaningless.

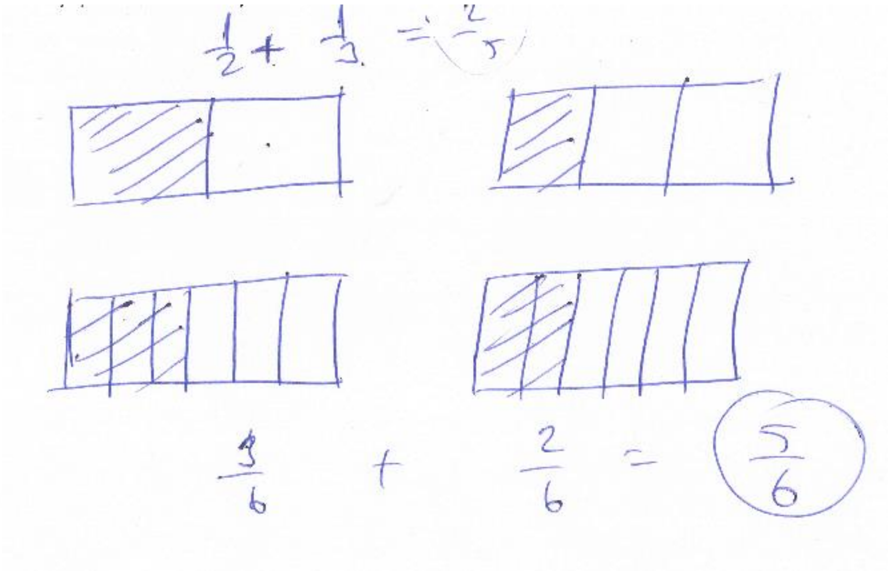


Figure 5. Addition and multiplication models

Although the participants seemed to have sufficient knowledge regarding multiplication and addition of rational numbers, they did not know reasons or background of all rational numbers content. For example, almost none of the participants could draw a division model of rational numbers although the 6th grade curriculum had a learning outcome about the use of models in teaching division of fractions. In question 1 of the vignette, drawing a visual model of $5 \div \frac{3}{10}$ was asked, as it was in the curriculum. Only T1 strove the drawing afterwards of the vignettes. However, she did not show the visual model of division when she saw for the first time (see Figure 6). She strove at the end of the vignette, but again, she could not do it. After the vignette process, she tried to do it and sent me the photos of the division model. Participants said that they taught the division of rational numbers with rules, even though they used some different methods in their teaching. It seemed that participants mostly focused on the procedural knowledge about division operation of rational numbers.

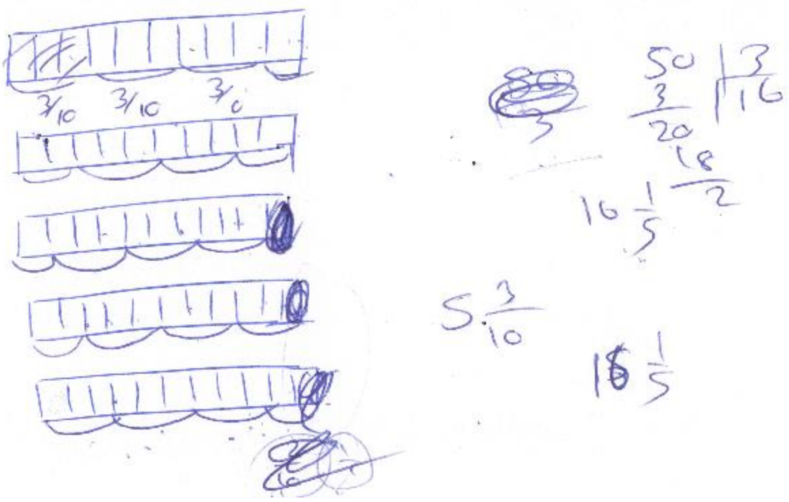


Figure 6. Sample visual model of $5 \div \frac{3}{10}$

Similarly, question 7 of vignette asked teachers to respond the student's claim with drawing the model of $\frac{5}{12} \div \frac{1}{3}$. However, they could not draw it properly even though they tried for several times and with different rational numbers. Figure 7 shows T1's attempts to draw. She turned back to the model of $5 \div \frac{3}{10}$ in question 1 and tried to draw it, but could not reach the right model.

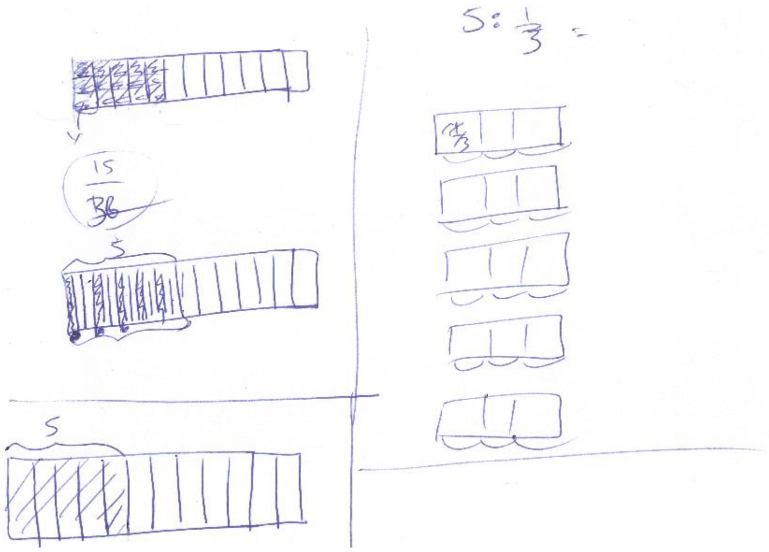


Figure 7. T1's drawings for division models

She gave up that moment to try again later. She said that she forgot or disregarded the model even though she learned it during her graduate education. Four weeks later, she handed me a paper on which she drew several division models as given in Figure 8.

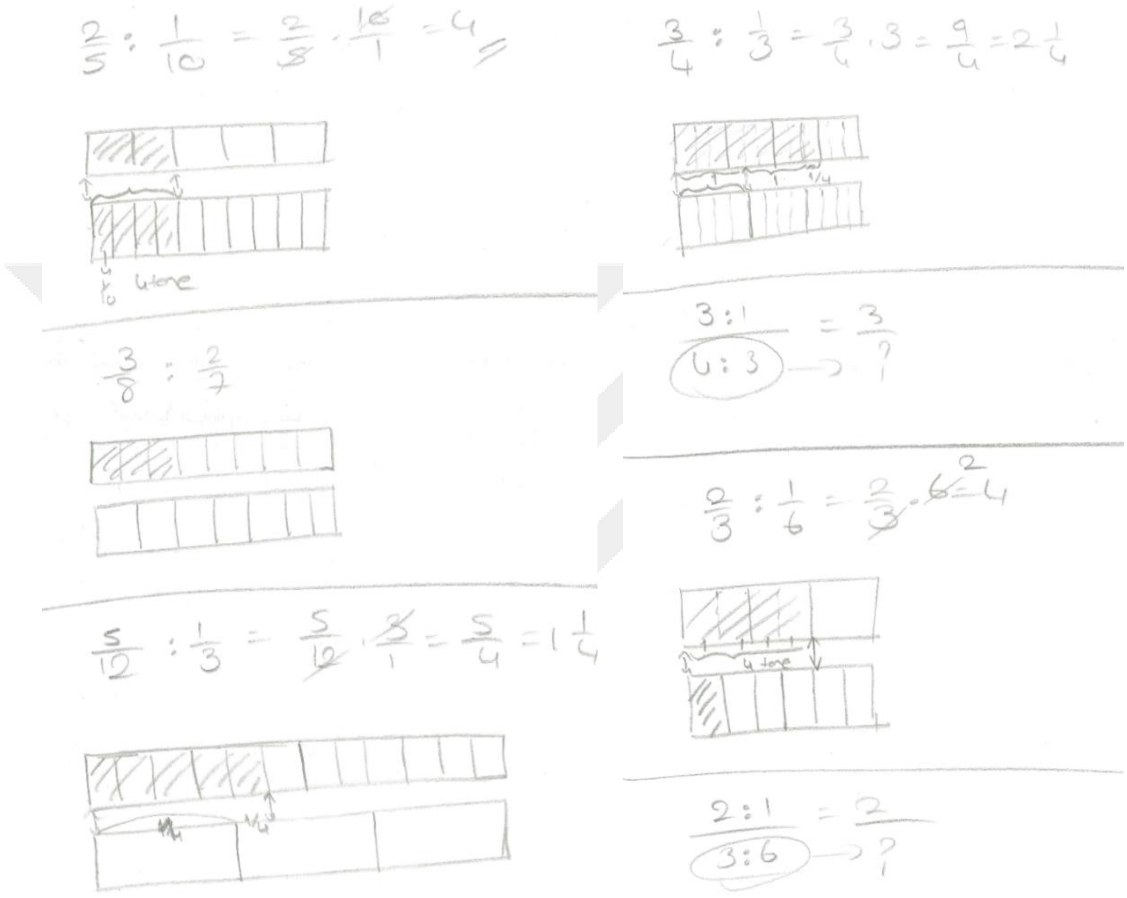


Figure 8. T1's subsequent drawings for division models

Question 3 of the vignette asked the comparison in four story problems in terms of difficultiness. The first and the fourth problems were related to the fractions in the 6th grade curriculum whereas the second and the third problems were related to the rational numbers in the 7th grade curriculum. Participants were expected to respond in terms of curriculum knowledge. Although, T1, T3 and T4 pointed properly that the second and the third problems were more difficult, their reasons were different. They stated that their students would not be able to do multiplication, proportioning, and operations of improper-proper rational numbers that the problems required. T4 defined the basis of difficulties correctly that there were negative numbers in the 3rd problem and a division issue in the 2nd problem, but she did not mention the curriculum. On the other hand, T5, T2 and T6 did not select the 6th grade and 7th grade problems correctly. They struggled with numerals in problems. The interesting finding was that T6 defined which curriculum the four problems belonged properly. However, he selected the first and the second problems as more difficult, and claimed that students did not handle ordering fractions in 6th grade. That is to say, none of the teachers in this study noticed the content of the curriculum, even though they showed indicators of such knowledge and knew what they taught. In short, they seemed to have adequate knowledge about the key ideas in curriculum and could see the connections within the curriculum especially in terms of learning outcomes and procedural knowledge. However, they could not state the rationale of the key ideas and how to show and explain to the students properly. They had deficiency about the conceptual knowledge of particular concepts in rational numbers.

4.1.2 Knowledge of Planning for Mathematics Teaching and Learning (pre-active) in TEDS-M

Based on the interviews and observation notes, all participants used both textbook and additional sources in their lessons. T3 used mostly additional sources, especially smart notebook on smartboard in her lessons, and other teachers used textbook and additional sources together. They adopted some exercises and worksheets they found in the Internet. T1 and T3 also used interactive websites in some lessons such as Morpakampüs, Mathplayground. T6 tried to use morpakampüs, but he could not connect to the website and internet.

T1 used supplementary sources which were *"more visual, rich in content" (T1_I1)* because *"students understand better. They understand and enjoy the lesson better" (T1_I1)*. In general, participants expressed that textbook was inadequate for teaching, especially because there were not sufficient number of exercises and examples: *"…I don't think the number of examples or exercises in the textbook are enough. Yes, the examples may explain the subject well, but there could have been more exercises"*

(T3_I1). T6 had the same opinion that *"the content is very empty, that is, one on each topic, that is ... there is one leaf of homework. You know, [the book] is very insufficient in terms of homework" (T6_I1).* T5 stated that *"There are very few questions in the textbooks" (T5_I1).*

Teachers expressed that they did not make any daily lesson plan, but they did preparations before the lesson. They did not write anything but just roughly revised the subjects except for T2. The reason could be that they were experienced teachers:

> *after a certain amount of time, that is, the job becomes automatic, that is. You know more or less what to explain. You are looking at the content of the book. You know, I was preparing for lessons in the first years, That is… Now you know what to explain to the students after a certain period of time (T6_I1).*

For the written preparation, T3 stated that

> *I don't make a preparation every night, but especially when I'm going to start a new topic or at the end of the topic… Since there are not enough exercises for the students, there are also tasks that I write myself or combine questions from the internet (T3_I1).*

T4 also mentioned same idea that *"Usually, if I've never taught the subject in any class, that's the first time I'm going to explain it. I'll write it down at home on a piece of paper and figure out what questions I'll solve" (T4_I1).* Only T2 expressed that she made a worksheet and preparation one night before her lessons.

Planning or selecting appropriate activities and *planning appropriate methods for representing mathematical ideas:* For this aspect in the framework, how they taught fractions and rational numbers was asked in interview. They described the introductions of fractions and rational numbers in their lessons. T1, T2 and T3 wanted to use concrete materials, whereas T5 and T6 underlined the real-life examples such as cake, paper modelling, and bread when they introduced the fractions. For instance, T1 expressed that:

> *We are bringing fruit. I have it brought [by the students]. I start by dividing them. You know, so they understand what a fraction means. After that... well, I've never used a fraction card or anything. Then, there is an apple and so on. In fact, we [make them] bring tangerines and peel them. (T1_I2)*

The reason for bringing these concrete materials was to introduce the fractions from the whole to the piece. T3 said that she used a mandarin (to represent the whole and the pieces) because it drew the students' attention. After using concrete materials, T1 switched to the number line, and continued with it. She explained the purpose of using number line as *"I think students understand the number line well. That's to say, when they see it on the number line ... they actually understand that there is a fraction and also division. I mean, I can say that they make it more concrete in their minds" (T1_I2).* She also said that she did not use fraction cards because she did not prefer and they were unnecessary. However, T2 asserted that

> *If I can't find anything, [I show] with paper in my hand, it's like I divide this and spared this.. ... I use fraction blocks as much as I can… And then in fractions, for example, when I explain the multiplication in fractions, colored things in these fraction blocks work for me on fraction cards.*

On the other hand, T6 gave an example with bread, he claimed that

> *I generally use bread a lot. It's bread, because in everyday life, everyone cuts, slices, divides. It's something that [the students] divide, such as one whole bread, half a whole bread, a quarter whole bread. I try to explain through [bread] all the time.*

however, bread is not a good example for fractions because it is hard to divide into equal parts. Then, he continued teaching with direct instruction through examples and operations. In the same way, T4 used direct instruction in her teaching. She stated that she used fraction cards for the first time to introduce fractions, but she gave up this method because "*The fraction cards on the acetate work at first, but they don't work at the next stages. At first… [they are] good for seeing that the whole is of equal size…"* She also taught the rational numbers in the same way.

In rational numbers, T1 and T5 planned to use the number line for the transition from natural numbers to integers. T1 stated that

> *…as we fill in between integers on the number line, the number line also has a negative part. Well, I will ask the students what we're going to do there. I'll start [the subject] it that way, by questioning whether [the same idea] is here or not (T1_I2).*

The reason for the using the number line was that she could not concretize negative rational numbers and her students were familiar with the number line. On the other hand, T6 and T3 connected the negative numbers with fractions. T3 also included whole numbers within rational numbers. She wanted to use direct instruction method and said that: *"…I say think like natural numbers.. which appear as normal positive numbers. I say that now we see the fractions with minus. Yet, I say, we give the fractions a different name. I'm making an introduction like that."* In addition, T2 and T4 expressed that they used direct instruction method with examples and exercises as they did in fractions. It seemed that all participants used more abstract concepts in teaching rational numbers whereas they used more concrete materials, examples and methods in fractions.

Predicting typical students' responses, including misconceptions: In order to understand all participants' knowledge about this aspect of the framework, responses in interviews and vignettes were examined. T1, T3, T6 stated that their students generally had difficulty in understanding meaning of numerator and denominator, especially how to read rational numbers for T1 and T3's students. In addition, T1 stated that they had difficulty in addition of rational numbers and they added denominators in the addition process. T3 also stated that students made additions and subtractions without equalizing the denominator. For the sources, T1 said that she did not identify the sources of these difficulties in full measure, T2 and T3 pointed out the same reason: *"I mean I guess the source of the difficulties is that the concept is not understood totally" (T2_I2).* Ultimately, according to T1, students had deficiency regarding the meaning of part-whole relationship. Besides, T3 pointed out that students had deficiency in doing multi-step operations in rational numbers and the reason can be more abstract. T4 mentioned that students confused the rules and they had difficulty to solve problems. According to T4, the reason could be that memorizing many rules was difficult for students. Similarly, T6 and T5 said that their students mixed negative rational numbers when taught as "reverse the rules of positive ones."

In question 1 of the vignette, students' misconceptions and possible sources were asked in multiplication and division of rational numbers. Participants addressed them adequately. According to them, students might generalize division and multiplication rules in whole numbers to rational numbers such as multiplication increased the result and division decreased the result. Teachers' general opinions about the possible sources of these misconceptions were deficient teaching. If students learned the meaning of multiplication and division of fractions very well, such a misconception would not occur. To remedy and prevent the occurrence of their misconceptions, they thought that examples from division of fractions and whole numbers should be given together and compared with each other.

For the question 2 in vignette, participants predicted the possible reasons of students' selections regarding the half of the pizza A (smaller in size) or a third of the pizza B. According to them, the students who selected pizza A thought that they ate a half of the pizza because $\frac{1}{2}$ was bigger than $\frac{1}{3}$, and thus, they ate more pizza. On the other hand, the students who selected pizza B thought that pizza B was bigger than pizza A, and one could eat more pizza even when it was $\frac{1}{3}$ of the pizza. They mostly expressed that pizza A and B were different wholes, so two different wholes could not be compared to determine the sizes.

In question 4 of the vignette, participants interpreted 6^{th} grade students' answers for 3 questions in the exam (Figure 9). For option (a), they stated that the students could not comprehend how to divide a whole on the number line, and students could not understand the whole number concept. The students accepted the interval between 0 and 5 as a whole. T1, T4, and T6 also emphasized that they had never seen such a mistake. They mostly asked the number line question in similar way, but they gave the number lines as written with wholes (0, 1, 2, 3,..) or only 0 and 1. T4 and T6 also blamed these students' teacher for giving only 0 and 5 and leading to confusion in students' mind.

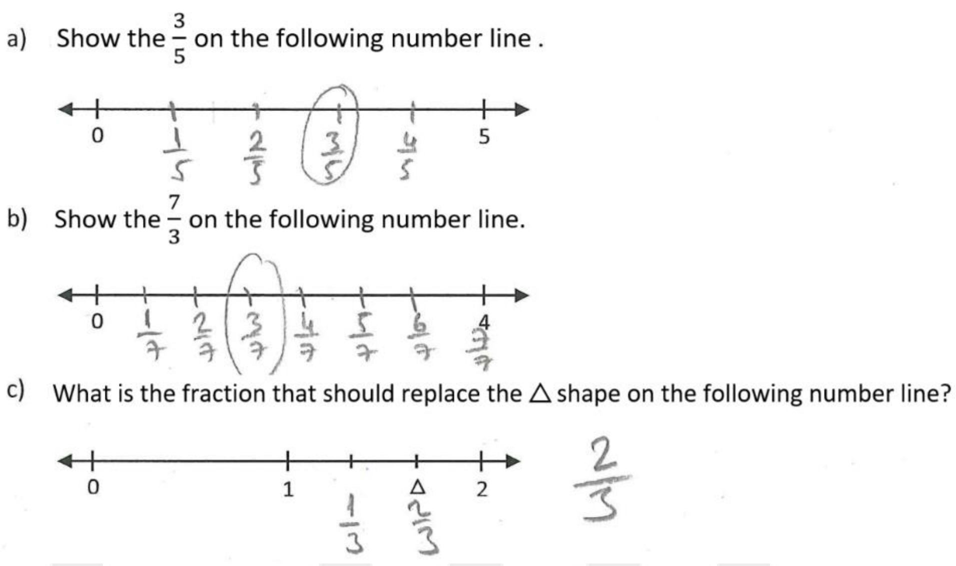


Figure 9. Question 4 of vignette - Sample answers from exam

Moreover, giving number line as in option (a) was interesting for T1, and she wanted to use the item in her lessons:

> *…I never thought about this, it seemed very interesting to me. Actually, it's a good thing, so it can be used. You know, to measure. It can be used to make it difficult. I've never used something like this… I wonder what they will do. Now I will try this in the… I'll try at least once in the class and see what they will do. (T1_VQ4)*

After that she asked the number line as an option (a) in her lessons and students divided the wholes correctly. Moreover, T1 encountered the case in option (b) in her lessons. In her opinion, it was based on that students could not distinguish between proper and improper rational numbers and not to make a part-whole relationship. T3 and T6 argued that the student in (b) mixed the denominator and numerator. They said that the student assumed that the smaller number was the numerator. Likewise, option (b), they met many times with option (c), and the reason could be the same for the teachers: Students could not understand the improper rational numbers well. In addition, they could not see the 0-1 range in number line or did not notice the whole, 1. For question 4, teachers generally expressed that students had a problem about comprehending the whole concept, and they did not understand it very well.

For the *Identifying different approaches for solving mathematical problems* aspect of framework, teachers mostly stated that they wanted to show different solutions if students solved with different solutions properly. For instance, T3 said that if students reached the solution just by chance with a way that would not work for other, then she would find a new example and indicate that the way would not work the new example. T4 stated that she liked different solutions and encouraged students to use them. On the other hand, T6 did not show all solutions because he acted based on students' level:

> *...class level, that is if you teach it, it will cause other things that will confuse students' mind, so you never teach it. You say [to the student with a different solution] that it is correct individually. It is true that you did but... let's not explain this here. Because it can go in different directions (T6_I2).*

Based on the vignette responses, teachers mostly did not prefer to draw a visual model of division and multiplication of rational numbers. Moreover, all participants drew the multiplication model, but they did not indicate a visual model of division except for T1. She also could not draw the division model at the first time. She tried and checked the model several times, but did not reach the right one. She gave up drawing it during the vignette interview; however, she said that she would look at this model again. After three days, she sent the right model.

In the vignette, question 8 relates to teachers' actions when they encounter a different solution method:

$$1\frac{5}{6} \div \frac{1}{2} = \left(1 + \frac{5}{6}\right) \div \frac{1}{2}$$
$$= \left(1 \div \frac{1}{2}\right) + \left(\frac{5}{6} \div \frac{1}{2}\right)$$
$$= 2 + 1\frac{2}{3}$$
$$= 3\frac{2}{3}$$

When they saw the question, participants first checked the results and found same result with the student. Then, they thought that there was nothing to say about operations and the result as it was reasonable. When T6 thought that commutative property of multiplication should work not only on the right side but also on the leftside. Furthermore, he noticed that the solution was not correct in reverse. However, the question was about distributive property and making operations on both side should gave the same result. Although T6 reached the correct result, the way he reached that result was not completely correct. He said that this solution could not be generalized for all questions:

> *[I explain that] commutative property does not have reverse… And here, for example, it's only from the right side. On the left side or when I get it at the beginning, it does not happen. I mean it can be applied here, but it's not a general rule, so you know, division isn't an operation [with commutative property] anyway and so on. I teach that it is not happen from there. (T6_VQ8)*

and he solved this question on the other side by himself and used this model (Figure 10).

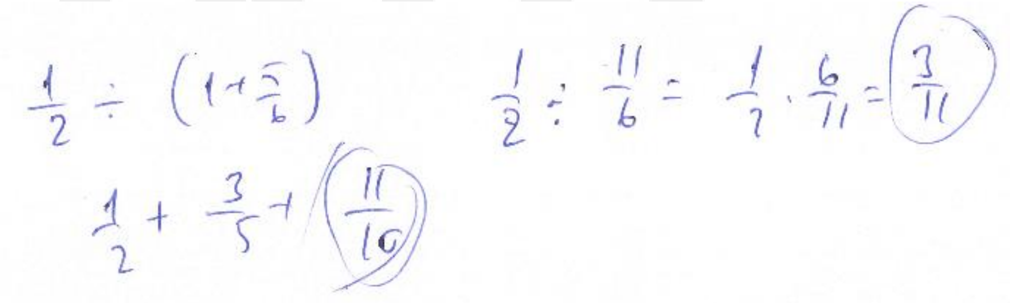


Figure 10. T6's solution for question 8 in vignette

On the other hand, T6 accepted the student's solution, but he showed the other side and expressed the student that it would not work all time, so it was not acceptable. T3 strove to find a way to refute student's solution, but she could not make it. She said that she accepted the solution in exams, she was not convinced that it could be generalized. Similarly, T4 refused the solution at the first view because she stated that "*In other words, since there is no commutative property in the division operation, nor there is the distributive property" (T4_VQ8).* However, she tried to do it and found the same result. Then, she had to accept the solution. T1 also did not accept this solution as correct in the first view. However, she did not have proper reason for rejection. In order to affirm herself, she tried this solution method with different rational numbers and found the same result. After that, she changed her idea and accepted the student’s solution. T2 and T5 also tried the solution themselves, confirmed the result, and accepted it. T5 explained her reason that

> *Because we say that multiplication has a distribution property on addition-subtraction. Division is the opposite of multiplication already. You know, there is only one operation, multiplication. You know, we don't do anything like division (T5_VQ8).*

T2 indicated that students used this method with distributive property, and said that *“If the solution is correct, then the student doesn't have to use my method” (T5_VQ8).*

All participants accepted different solutions in their lessons as revealed in the observations. They generally showed all possible solutions on the board and explained to their students. To give an example, for T1’s students, if they made different solution during the lesson either correct or not, they presented on the board and T1 gave feedback to them. On the other hand, the responses from question 8 of vignette indicated that participants did not notice the right distributive property of division except for T6. This might mean that (a) teachers were not familiar with the case of division in distribution property or (b) they did not develop an in-depth perspective about this property, ignored the meaning of division, and focused on the correctness of this case but not the case itself.

Choosing assessment formats aspect of the framework showed that all teachers tended to use different types of questions together in their assessments. They expressed that they mostly used open-ended questions in their exams, but it was not monotype questions. Different types of questions were also asked by teachers, such as true-false, fill in the blanks, word problems, short answers, matching, ordering and combined operations in rational numbers, and multiple-choice questions. T1 did not use the multiple-choice questions because

> *I think the test does not measure knowledge in the school exam. I mean, the student who does it does it, the student who does not do it does by guessing [because it is a multiple-choice] test. It doesn't really measure students’ knowledge (T1_I2).*

In order to eliminate guessing, T6 accepted multiple-choice responses as true, if students wrote the solutions, making them almost open-ended questions. To determine and make assessment whether students understood rational numbers or not during the lesson, the teachers asked questions from simple to complex regarding present and previous lessons, and made verbal assessments. T6 also stated that he checked the homework. Teachers' expressions were in line with the observation notes and their exams. T2 stated that she used pop quizzes to assess students in her lessons. Only T5 claimed that she also considered assessments from the other courses:

> *For example, a student learns something in math, which is also in science lesson. He/she needs to use it. I mean, the topic is ratio and proportion, here we do cross product or something. For instance, length measures, units are converted to each other, cm, mm and so on. Actually, when the student meets these in different lessons, if he/she can do it, at least if he/she can relate, then we understand that the student have comprehended. Here we get feedback from the other teachers. (T5_I2)*

Teachers in this study chose and used different assessment formats for the following reasons: (i) to eliminate the guessing in multiple-choice questions, (ii) to increase the ratio of right answers in the examinations, and (iii) to ask more questions in limited time because examinations took one lesson hour. When their examinations and questions were observed, it was seen that although participants used different types of questions, word problems were less than other ones. They mostly tended to ask basic and mixed operations in rational numbers similar to drill and practice exercises.

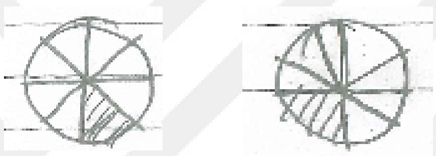
4.1.3 Enacting Mathematics for Teaching and Learning (interactive) in TEDS-M

Explaining or representing mathematical concepts or procedures aspect of the TEDS-M framework, how participants taught rational numbers in their lessons were investigated through the observations. T1 tried to use discussion method and concretization in her teaching. She also lectured both the 6th and 7th grade classes. To give an example for the 6th grade:

> *T1: What comes to [your] mind when [I]say fractions?*
> *Student (S): Half of a square's boxes are colored, 2 of 5 squares are colored, 3 are not colored.*

Students (S): The ratio comes [to my mind].
S: Numerator, denominator, and fraction line come [to my mind].
S: A whole is broken down to pieces comes [to my mind].
T1: What do numerator and denominator mean?
S: The numerator is my share, and the denominator is the total, everything.
T1: What are fractions as your friends say; [they] are representations that express the parts of a whole. How should those parts be?
S: They should be the same size.
S: [The whole] should be divided equally.
T1: It is a representation showing the equal parts of a whole.

After starting the fractions, she drew the pizzas on the board. The first pizza had equal pieces, but pieces in second pizza was not equal (see Figure 11). Students objected to second pizza and the following dialog took place:

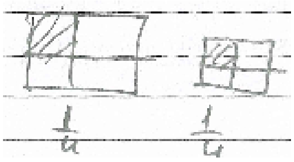


First pizza Second pizza

Figure 11. T1's representative drawings

S: That [does not show fraction], teacher.
T1: Why [not]?
S: Because you said that all slices would be equal.
S: The shapes are not the same.
T1: Why can't I show it? I also divided it into 8 parts, why can't I say $\frac{1}{8}$?
S: Because it is not divided equally.
T1: What happens when it's divided equally, what happens when it's not divided [equally]?
S: Then, we cannot divide.
T1: But I just divided it.
S: (By showing different parts) He said that if different people took [different slices], one would be full and the other would not be full.
T1: In order for parts of a whole to mean something, the parts must be equal (by showing the first pizza). When I say $\frac{1}{8}$, no matter which part you take, you get the same part. (By showing the second pizza) As you say, when I say $\frac{1}{8}$, not every part means the same thing. Therefore, the parts should be equal when showing the fraction.

Another example was seen at the beginning of comparing fractions content:

T1: What can we do when comparing fractions?
S: We can show it on the number line.
S: We equalize the denominators.
T1: Why do we equalize the denominators? Can't we just equalize the numerators?
Class (C): (Students did not respond with an exact answer). No [we can't]. It should be equal whole.
T1: Why equal whole?
Class (C):… (No answer)
T1: (Draw on board).  *Which one do these represent?*
C: Both of them are $\frac{1}{4}$
T1: Are they equal?
C: No, one of them is big, one is small.
T1: Then, it must be the same whole to compare the two fractions. There are two breads in the market. One is a big normal bread, the other one is a sandwich bread. You eat half of it. Which one of you eat more? You don't eat equal because they are not the same whole, arethey?
C: We got it.

There were some differences between her 6th grade and 7th grade teaching. First of all, she stated that she used discussion method more effectively in the 7th grade because there was more participation to the lesson and 7th grade students' level, interest, and preparedness were more than 6th grade students. When she asked something about the rational numbers to the 7th grade students, she received more correct responses than what she received from the 6th grade students. The following passage is an excerpt from the 7th grade class.

T1: Consider natural numbers. What were the properties of the addition operations in natural numbers?
C: Commutative property or interchange.
T1: What was that about?
C: Although the numbers change places, the result remains the same.
T1: The same property applies to rational numbers. Changing the places of numbers does not change the result in the addition operation.
S: Is there no expansion?
T1: It is not considered a property because you do not change the number when you expand it.
T1: What else were there in the natural numbers?
S: Distributive property.
T1: What do you mean? What is the explanation?

S: We distributed what was outside the parenthesis into the parenthesis.
T1: There was a distributive property of multiplication on addition. There was a number in front of the parenthesis and two numbers inside in the case of addition.
T1: What was the identity element of the addition operation?
C: It was 0.

Sometimes, if students did not give answers, she explained it. For example, in addition and subtractioncontent in the 6th grade class, she struggled to use discussion method because class level was lower than 7th grade and students did not give response her questions:

T1: What are you saying?
C: Denominators are equalized, operations are made...
T1: Why are they equalized?
S: The denominator should be a single number, it should be the same.
T1: Why should it be the same? (When she could not get the full answer, she explained it herself). Since it should be in the same whole, and it is shown on a single whole.

After starting the new subject with using discussion, she generally gave some time for the students to take notes at the end of the discussion. These notes could be definitions or explanations. After this, she wrote the examples to facilitate students' understanding of the subjects.

T2 wanted to use discussion method, and started with questions and drew models of fractions. If one of the students responded her questions, she moved on the lesson. T2 was the teacher who used drawings, models, real life examples, and concrete materails in lessons the most among the other teachers in this study. T2 made two different activities to help the students' understanding. She employed peer teaching and group teaching method in her lesson. In one of T2's lesson, students who solved questions were directed to other students who could not solve them in order to help with the unsolved questions. In group teaching method, she gave two fraction cards to every desk with two students in each. T2 asked each group to give an example of the multiplication of fractions and show how they multiplied them. In order to find the example, students worked together in their groups.

T3 described herself as a teacher who used technology in her teaching. However, she only turned on additional electronic sources on the smart board and taught rational numbers with direct instruction method. Almost all examples she solved on the blackboard were taken from the electronic resources. Moreover, she used metaphors to teach rational numbers such as set of rational numbers as can be seen in Figure 12. T3's set of rational numbers drawings below.

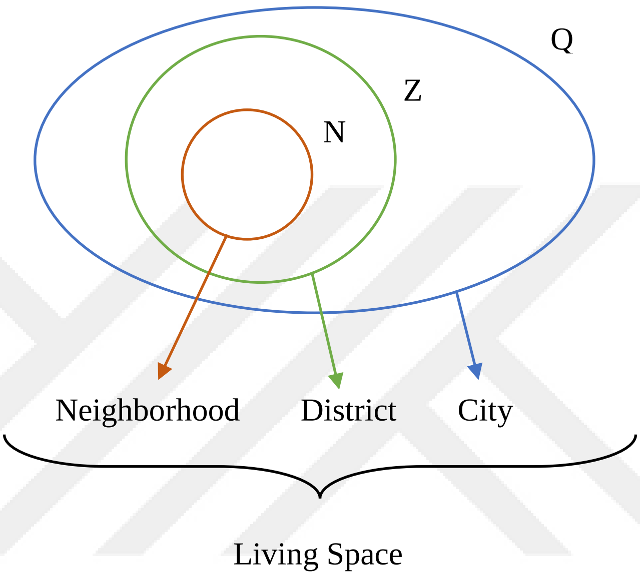


Figure 12. T3's set of rational numbers drawings

Her representation of division is in Figure 13.

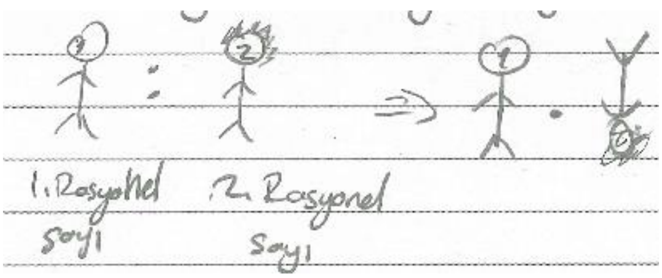


Figure 13. Division rule in rational numbers

T3 generally wrote rules and explanations, or made verbal explanations, then wrote on the board or showed examples. If there were models or number lines in the examples of smart notebook, T3 used them in the class in topics, such as addition and subtraction in rational numbers, as seen in Figure 14.

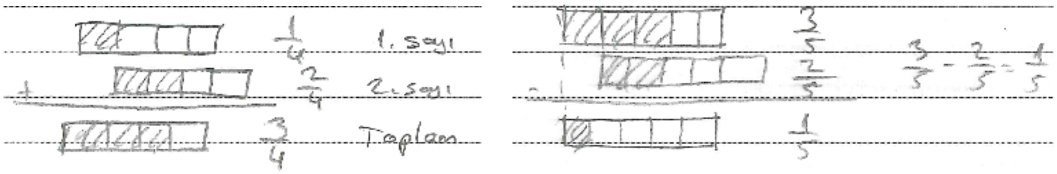


Figure 14. Examples of addition and subtraction

T4, T5, and T6 also used direct instruction method and made verbal explanations. They asked questions to students about the content, and if at least one student responded to the questions correctly, then the teachers continued the lessons. Another common point was that these teachers gave rules and explanations. T4 also illustrated the rational numbers and operations with models and the number line more than T5 and T6 did. She did not pass directly when one of the students had understood the model. She gave more time to the students for understanding more than the other teachers. Moreover, when T4 drew a model, her students applied it on the new questions faster probably because T4's questions and expressions were clearer than T3, T5 and T6.

In the interview, T1, T2, and T3 expressed that they wanted to use concrete materials, whereas T5 and T6 underlined the real-life examples when they introduced the fractions. However, in their lessons, only T1 and T2 did what they described and used concrete materials.

Generating fruitful questions aspect, the examples about rational numbers, which were given by participants were examined. Most of the questions that teachers asked were drill and practice of operations or showing rational numbers (see Figure 15). They used word problems as much as it appeared in the curriculum.

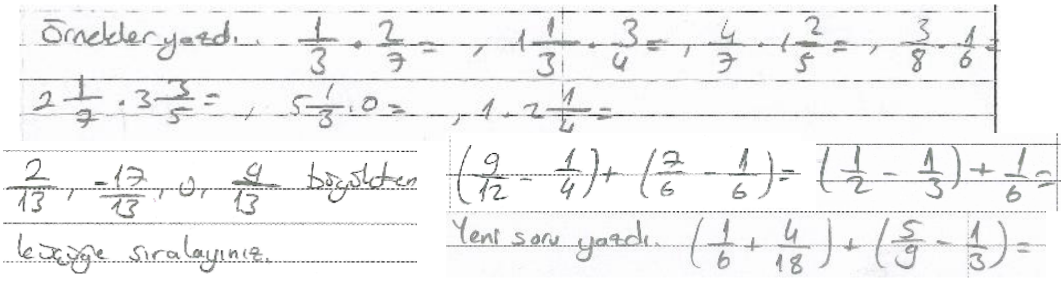


Figure 15. Examples of sample questions from rational numbers context

In order to make concretization, T1, T2, T3, and T4 first presented models, and then asked fractions or vice versa, as seen in the examples in Figure 16.

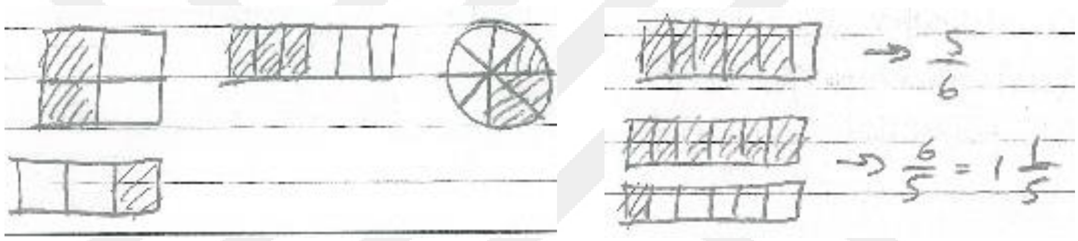


Figure 16. Fraction models of participants

T1 and T2 used models more than other teachers when they taught the subject and made an explanation. For instance, as can be seen in Figure 17, T1 and T2 taught the equivalent fractions with models and asked the fraction in each model to the students.

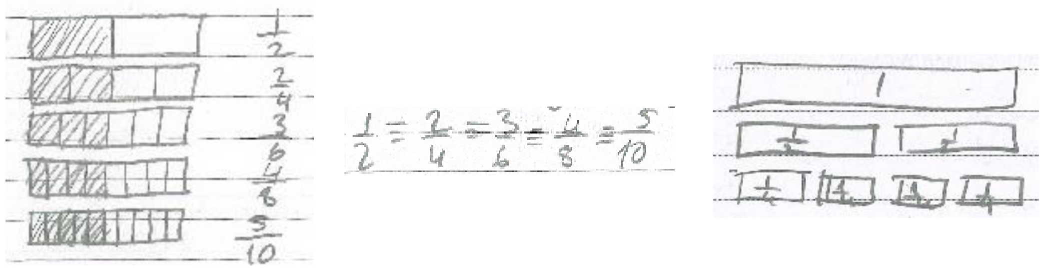


Figure 17. Equivalent fractions' model

Then, they defined the equivalent fractions. T1, T3, and T5 used number lines in examples (see Figure 18). T1 was the teacher who used the number line the most.

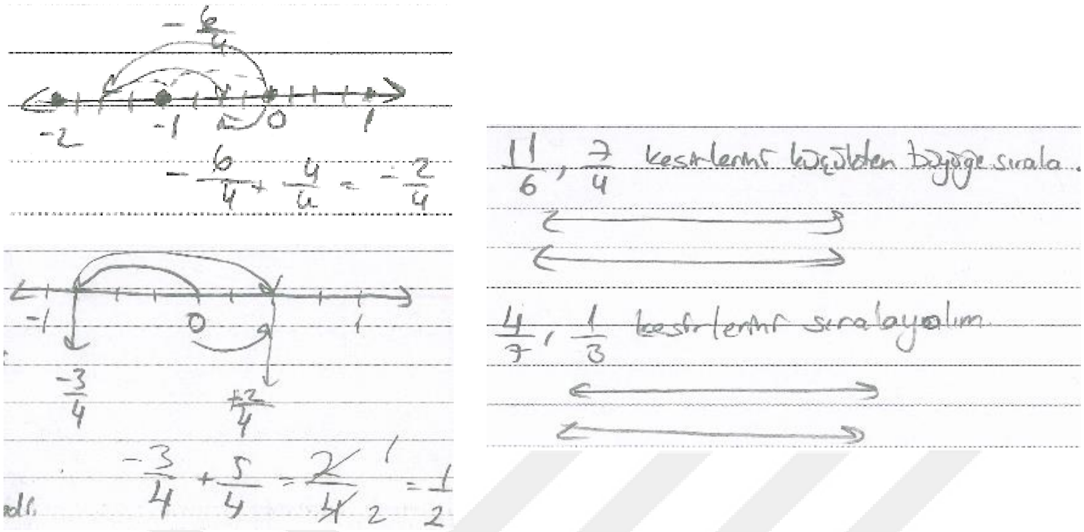


Figure 18. Number lines in examples

For the word problems, participants followed the curriculum and asked problems from the textbook or the supplementary resources. Sample problems were given in Table 7.

Table 7.

Word Problems Asked in the Lessons

Participants	**Problems**
T1	A water tank is filled by a $7\frac{1}{2}$ liter bucket by carrying water 16 times to be fully filled. After the tank was filled, $\frac{5}{6}$ of the water in the tank was used. According to this, how much water is left in this tank?
T2	$\frac{2}{5}$ of the apples in a basket is red, $\frac{1}{3}$ of the red apples is rotten. According to this, what fraction of apples in the basket are rotten and red?
T4	Half of a milk can is filled with milk. When 9 lt of milk is added to this can, $\frac{4}{5}$ of the can is full, so how many liters is the entire can?

Additionally, T6 mostly asked the problems in the textbook. One of the common points of participants' questions or problems was that they asked from easy to difficult and mostly drill and practice questions. Teachers seemed to consider students' level when they generated questions.

Analyzing or evaluating students' mathematical solutions or arguments, in order to analyze this aspect, many questions from participants' lessons were observed. T1, T2, and T4 followed the following path in their lessons while solving questions: First, they wrote questions and gave some time for solving it. Then, they gathered most of the students' responses. Only T1 asked why questions. T2 asked questions to students, but she pointed the solution way with her follow-up questions. If students told the right answer, then these teachers selected one of the volunteer students to solve the example on the board. Then, they explained the solution for the students who did not understand; or if there was a different solution, then they solved the example in that way. If students could not explain why questions correctly, then T1 explained it and one of the students solved on the board at the same time. The following scripts form T1's lesson in a 7th grade classroom illustrated this process:

(on the board) $A = \frac{5}{6} + 1\frac{1}{3}$, $B = \left(-\frac{5}{2}\right) + \frac{5}{9}$, A+B=?

S: $A = \underset{(1)}{\frac{5}{6}} + \underset{(2)}{\frac{4}{3}} = \frac{5+8}{6} = \frac{13}{6}$

$B = \underset{(9)}{\left(-\frac{5}{2}\right)} + \underset{(2)}{\frac{5}{9}} = \frac{-45+10}{18} = \frac{35}{18}$

$A + B = \underset{(3)}{\frac{13}{6}} + \underset{(1)}{\frac{35}{18}} = \frac{39+35}{18} = \frac{74}{18}$

T1: What do you think?
C: We found different results.
C: It's very complicated.
T1: (The student who solved it) There is no problem in the beginning, check it again.
C: B is wrong.
S: (The student realized his mistake, solved it again).

$B = \underset{(9)}{\left(-\frac{5}{2}\right)} + \underset{(2)}{\frac{5}{9}} = \frac{-45+10}{18} = -\frac{35}{18}$

$$A + B = \underset{(3)}{\frac{13}{6}} + \left(-\underset{(1)}{\frac{35}{18}}\right) = \frac{39 - 35}{18} = \frac{4}{18}$$

C: Teacher, we don't understand.
T1: (She explained the solution step by step). In the addition operation, the whole of one [addend) is 6 parts and the other one [addend] is 18 parts, so we need to make [denominators] equal.
T1: We also simplfy the result $\frac{\cancel{4}}{\cancel{18}} = \frac{2}{9}$.

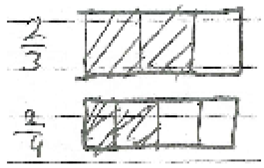
Another example from the 6th grade class is as follows:

S: $1\frac{1}{3} \cdot \frac{3}{4} = 1\frac{3}{12}$
T1: (To the student and the class) Is it correct?
C: Yes
T1: $1\frac{1}{3} = \frac{4}{3} \rightarrow \frac{4}{3} \cdot \frac{3}{4} = \frac{12}{12} = 1$.*There's one out of here. Then, if I don't process the whole part, it's like I've multiply two simple fractions. Pay attention to it.*
(After this example, students were careful when they made multiplication of improper fractions.)
S: $1\frac{1}{7} \cdot 3\frac{3}{5} = 6\frac{3}{35}$
C: No. You did wrong. It is $\frac{15}{7} \cdot \frac{18}{5}$

On the other hand, if a student thought or found a solution or arguments that no one thought of, she rewarded this student and assigned a plus point as seen here:

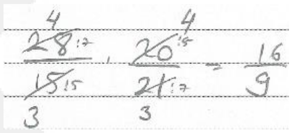
T1: Can we compare $\frac{2}{3}$ *and* $\frac{1}{2}$ *differently? What comes to your mind? (Except for the number line).*
S: Can't we just equalize their numerators?
T1: Let's try. (She expanded it) $\frac{2}{3}$, $\frac{1}{2}$ x $\frac{2}{2} = \frac{2}{4}$

S: We divide a pizza into 3 parts, we take 2 pieces. We divide the other one into 4 pieces. We take 2 pieces, but we get less. Although we take two pieces, we get less from one of them.
T1:(She gave + to the student and explained it by drawing it herself). There are two chocolates, we eat two pieces in both, but we eat more from one of them. What happens when the numerators are made equal?
C: The [part with the] smaller denominator gets bigger.



T2 asked $\frac{28}{15} \cdot \frac{20}{21}$ =? in the 6^{th} grade lesson and gave some time to the students to understand. Then,

T2: How do we do this?
C: We will multiply. Then we will simplify.
T2: So, what if we simplify it without multiplying?
C: Ok.
T2: How do we simplify without multiplying? Normally, the numerator and the denominator simplify together. Here, the numerator and denominator are simplified, both the top and diagonals. Yet, it does not simplify side by side. Then, what does 28 simplify with?
S_1: With 20.
T2: But they are side by side.
S_1: With 21.
T2: By which number does it simplify?
C: With 7.
T2: Is there any other?
S_2: Let's simplify 20 and 15.
S_3: (The teacher guided, the student did it on the board.)



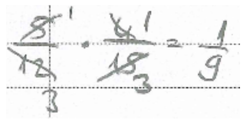
C: Well, if we do it without simplification and then simplify it, will we find the same result?
T2: Yes, but we simplify it first in order not to deal with large numbers.

After teaching the subject or solving examples or drawing something on the board, T1, T2 and T4 gave some time to the students to write. When they finished their explanations, they always asked if there was anything unclear or not understood.

T6, T5 and T3 followed this path when they solved examples in their classes: first they wrote examples on the blackboard or for T3, she mostly showed examples on smartboard, and gave some time for solving. If one student answered correctly, then the teachers selected him/her to solve the example on the board. Then, they explained the solution for students who did not understand or could not solve it. For example, T3 asked $\frac{5}{12} : a = \frac{15}{4}$, then asked another question; 30 : □ = 5, what divides 30 and it becomes 5?

S: 6.
T3: You get 30:5=6, right? (Made students write a note) If the dividend is given in the division, we can divide the quotient in order to find the divisor. If

there is no dividend, the divisor and the quotient are multiplied. Then, what do we do to find a?
S: We equalize the denominators.
T3: Remember the note I made you write. Which one is dividend, which one is divisor?
S: ... (There was no response from the students)
T3: (She waited for a while, and when no student could solve it, she solved it herself) $\frac{5}{12}:\frac{15}{4}$ *When we reverse, multiply and then simplify, we get*



If there was a different solution, then T3, T5 and T6 gave the opportunity to the students to show their solutions. They mostly selected the same students in the class because generally these students volunteered and raised hands to solve examples. T6 mostly selected students who sat at the front seats in the class. T6 also used leading questions when he explained the solutions.

In solving examples or explaining the subject on the board, T3, T5 and T6 gave some time to the students to write. When they finished their explanations, they always made a connection between the solution and the rules, and reminded these rules.

In the word problems and problem solving process, T1 explained to students that first they should understand the problem and write the given information in the problem. He expressed that when they wrote the given information of the problem, they solved the half of it. After dictating the problem, T1 generally wrote the given information on the blackboard or let students re-explain the problem. She continued the solution with the volunteer student in a question-and-answer context, in line with her discussion method. However, in the problem solving part, almost none of the students participated in the discussion. At the end, she explained it to all students. If there were different solutions or approaches about the problem, then she mentioned them later on. If no one solved the problem, then T1 explained it by asking questions to the students. To give an example, in the 6th grade class T1 asked, “A runner ran first $\frac{2}{5}$ and then $\frac{1}{3}$ of the path he planned to run. Now that there's 400 m of road left, how many meters did he run in the first place?” Then, the following interaction took place:

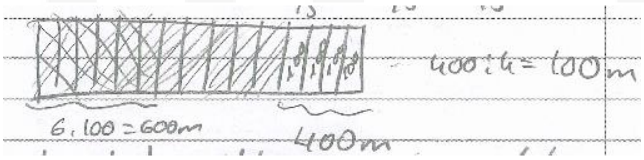
T1: Do you understand the question?
C: Yes.
T1:. Somebody explain it.
S_1: A runner ran $\frac{2}{5}$ and then $\frac{1}{3}$ of the path.
T1: Does $\frac{1}{3}$ ... refer to the rest of it?
S_1: Hayır. No.
T1: Where's the first part?

Students struggled with the problem, and showed their solution to T1. She gave feedback as correct-wrong or yes-no. Students could not solve it correctly. Then, she exphained more.

T1: You know the whole distance he runs. The rest is obvious. You can find it accordingly.
T1: When the students could not solve it, the teacher explained). What should we do when it is the same whole?
S_2: We make addition.
T1: We should add to find the total path he travels on the same road.

$\underset{(3)}{\frac{2}{5}} + \underset{(5)}{\frac{1}{3}} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$ *is the total path he goes. The remaining path*

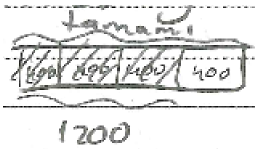
is $\frac{15}{15} - \frac{11}{15} = \frac{4}{15}$ *After that she drew a figure.*



How many pieces did he go to in the first place?
C: 6.
T1: Then, 6.100 = 600 m.

In the problem solving process, T2 and T4 first asked the problem and wrote it on the board. Then, they gave some time to the students for thinking and solving it. Students, who solved the problem, showed their solution to T2 and T4 individually. If it was correct, then the teachers approved. If it was false, then they said no. In order to check the students' solution, T2 walked around the students' desks. Next, they selected one of the students who solved the problem to show the solution on the board. At the end, they explained it to the other students. If there was a modeling approach in the problem, they showed this later on. If no one solved the problem, they explained it or directed students to solve it by asking questions. Sometimes, T2 changed the rational numbers with the whole numbers in the problem to simplify it. Thus, students were able to understand how to solve it this way. However, T2 and T4 did not mention, teach or apply steps of the problem solving process.

T2 asked word problems in the lessons the most among the participants. She also used word problems that were not illustrated in the 6th grade curriculum. For example, T2 asked, *"Mr. Yılmaz, whose* $\frac{3}{4}$ *of salary is 1200TL, pays his bills with* $\frac{5}{8}$ *of his salary. Accordingly, how many TL has Mr. Yılmaz paid for the bills?"* and the following took place:

> *T2: (He checked the desks one by one for students who had solved it. He first explained the problem to the students on the board.) Do we know his entire salary?*
> *C: No.*
> *T2: What do we know?*
> *C:* $\frac{3}{4}$ *[of it]*
> *T2: Then if we want to find his salary, 3 pieces are 1200, what about 4 pieces?*
> *C: One piece is 400, and 1600 TL is entire salary.*
> *T2: (She showed it with a model on the board with the guidance of the students.)*
> *1200 : 3 = 400 TL, one piece*
> *400 . 4 = 1600 TL entire salary*
> *T2: How much did he pay for the bills?*
>
> 
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>
> *C:* $\frac{5}{8}$
> *S:(One of the students did it on the board). Then,* $\frac{1600}{1} \cdot \frac{5}{8} = 1000$ *TL to bills.*
> *T2: How much money does he have left?*
> *C: 600 TL*
> *T2: Do you understand?*
> *C: Yes.*

T3 expressed to the students that they should be careful in executing the steps of the problem solving process and make the correct decisions about which steps should be done. Then, she mentioned the terms that whole as 1, half as $\frac{1}{2}$, quarter as $\frac{1}{4}$. However, she did not mention what these steps were or applied them in problem solving. T3 and T5 first asked the problem and gave some time to students for thinking and solving it. In T3's and T5's classes, most of the students could not solve it correctly, so they showed the solution step by step or gave a necessary rule for the solution. If there were different solutions or approaches about the problem such as modelling, then they mentioned them later on. Unlike others, T6 mentioned and taught the steps of problem solving process in his lessons. Then, he explained the rationale of the solved sample problems in the textbook. He showed problems in smart board as T3 did, but if one student answered correctly, T6 selected him/her to solve the example on the board. Then, he explained the solution for students who did not understand. If there were different solutions, he also gave the opportunity to the students who produced them to show. Athough T6 explained the steps of problem solving process at the beginning of the lesson, he did not apply the process in his lesson.

Diagnosing typical students' responses including misconceptions aspect, participants' acts, feedbacks and reponses to the students' questions in their lessons were examined. T1 and T2 sometimes asked questions to determine the students' misconceptions:

T1: Is $\frac{5}{0}$ *a rational number?*
C: It is not.
T1: Why not?
S: Denominator is zero.
T1: Why can't the denominator be zero?
T1: (When there was no response from the students, she explained herself.) The equal parts of a whole are fractions, but 0 also does not have a whole.

In T2's class during the addition-subtraction of improper fraction content;

T2: If there are mixed fractions, how do we do it?
S_1: We turn it into an improper fraction.
S_2: We make operation of integers separately.
T2: $1\frac{1}{5} + 2\frac{3}{5} = ?$

C: $3\frac{4}{5}$

T2: $2\frac{1}{5} - 1\frac{3}{5} = ?$

C:(They could not do the subtraction).
T2: Then converting to the improper fraction is the most guaranteed way.

T1 mostly explained the concepts to prevent misconception, however after explanation, she did not give any other examples. For instance, T1 asked that

T1: What is the sum of $\frac{1}{5}$ *and* $\frac{1}{7}$ *of 1400?*

S: (One of the students solved it on the board).

$$1400 : \frac{1}{5} = 280 , \quad 1400 : \frac{1}{7} = 200; \quad \begin{array}{r} 280 \\ +\,200 \\ \hline 480 \end{array}$$

T1: What do you think?
C: It is correct.
T1: (When the students did not notice, the teacher explained). For example, in $1400 : \frac{1}{7}$ *we invert and multiply, so it is* $1400 \,.\, \frac{7}{1} = \frac{9800}{1} = 9800$. *You make this mistake a lot. Thus, you can't make sense of the fractions of fractions. The right answer is like this:*

$$1400 \,.\, \frac{1}{7} = \frac{1400}{7} = 200 \;\; or \;\; 1400 : 7 = 200.$$

T1: What you write means how many $\frac{1}{7}$ *there are in 1400. Notice that this is a significant mistake.*

In general, participants could not always make an inference about the students' misconceptions based on their responses when they first saw these responses. In general, they corrected students' false responses. If false responses were queried by teachers, then they were able to understand the reasons of students' mistakes. Indeed, when possible false responses were asked in vignette questions, participants gave correct responses about students' misconceptions. According to observations, T1 asked questions and queried the most in her teaching as she employed discussion most of the time. Hence, she was aware of her students' misconception more than others. T2 and T4 also understood most of their students' misconception from their questions and responses. T2 and T4 tried to cope with these misconceptions by explaining one more time. In T5's class, students' level was very low; therefore, she had difficulty to teach rational numbers properly. T3 and T6 could not infer the reason of students' misconceptions from their responses. For example;

T3: $\frac{1}{3} \cdot \frac{1}{4} - \frac{1}{2} \cdot \frac{1}{3} = ?$

S_1: We can simplify $\frac{1}{3}$

T3: No. You cannot simplify $\frac{1}{3}$ *in this operation. Side-by-side rational numbers cannot be simplified, there is subtraction in between.*

S_2: I multiply by equalizing the numerators.

T3: $\frac{1}{12} - \frac{1}{6}$*. Why do you equalize the numerators? The numerators are not equalized in the multiplication operation. You make the operation difficult. (Then she continued with the solution).*

S_3: Equalise at 24.

T3: Why, isn't 12 a multiple of 6? (Then she completed the operation by equalizing at 12.)

T6 sometimes misdirected students which could cause the misconceptions. To give an example, a student wrote $2\frac{34}{105}$ on the blackboard in response to the question. Then, T6 reacted that *"This notation is in fractions. We will not use this notation in rational numbers (By showing the mixed fraction). Either we write directly, or we convert it into decimal notation and write it down. You should not use this."*

Providing appropriate feedback aspect, all participants mostly explained the content, question or solution again, if students could not understand or solve it. Participants' actions were parallel with their responses in interviews. T1, T2 and T4 changed the example, question, teaching method or material, if students still did not to understand it even after the re-explanation. To illustrate;

T4: $\square \div \left(-\frac{1}{4}\right) = \left(-\frac{2}{5}\right) \qquad \frac{12}{5} \div \triangle = -\frac{1}{3} \qquad \square + \triangle = ?$

T4: (She gave the students time to figure it out. When there was no answer, she explained). The dividend in the square is unknown. The divisor in the triangle is unknown. (She gave a simple example.) 10 ÷ 2 = 5. What would you do if 10 were not known? Inverse operation. What would you do if 2 were not known? You'd divide 10 by 5. It's the same here.
Two of the students, who solved it, solved it on the board. T4 explained the solution. She explained to the student, who said he/she did not understand, one more time.

T4: If the divisor is unknown, divide the dividend into the quotient. If you don't remember it, make a simple operation like 10 ÷ 2 = 5 and look.

To exemplify the case in T2's class;

T2: A person bought potatoes for $3\frac{1}{2}$ TL, tomatoes for $4\frac{3}{5}$ TL from the market and gave 9 TL. How much change should he get back?
T2: (She read the question to the student). Shouldn't we find out how much they cost first?
C: Yes.
T2: What is the solution way?
C: [They] will be added and subtracted from 9 TL.

S_1: $3\frac{1}{2} + 4\frac{3}{5} = 7\frac{11}{10}$
(x5) (x2)

(One of the students solved it on the board.)

T2: There is a problem with $7\frac{11}{10}$ and $\frac{11}{10}$. What should it be?

C: That is $1\frac{1}{10}$. Then, they cost $8\frac{1}{10}$.

T2: (She solved on the board with the students) $\frac{9}{1} - 8\frac{1}{10} = \frac{90}{10} - \frac{81}{10} = \frac{9}{10}$ *TL is the change*

S_2: If we subtract $8\frac{1}{10}$ from 9 wholes and subtract 8 from 9, wouldn't the result be $1\frac{1}{10}$?

T2: There is $\frac{1}{10}$. Don't you subtract it, too?

S_2: I don't understand.

T2: (Draw a model). How many wholes are there here?

C: 8.

T2: We are subtracting 8 of them first. After subtracting 8 wholes, shouldn't you need to subtract $\frac{1}{10}$ from the remaining one whole?

C: Yes.

T2:  *What's left? $\frac{9}{10}$. (After the model, the student understood it).*

Besides, sometimes the student who solved the example, explained the solution to other students in the class. To give an example in T1's class;

S_1: $\frac{4}{\frac{3}{5}} - \frac{\frac{4}{3}}{5}$ These two operations are the same.

C: Yes, the same, the result is zero.

S_2: No, it divides 4 into $\frac{3}{5}$ at first. In the other, divides $\frac{4}{3}$ into 5.

T1: Pay attention to the division, fraction lines. At first, look for $\frac{3}{5}$ inside 4. In the other, look for5 inside $\frac{4}{3}$. Pay attention to the large one on the fraction lines.

In T1's, T2's, and T4's classes, most of the students showed their responses to the teachers individually, or in order to give feedback, these participants walked around the desks when students engaged in the questions, problems, or activity. T3, T5, and T6 insisted on explaining the same solution or the same content in the same way. Because of students' low level, T5 mostly solved questions and problems with the students together. Sometimes, T6 responded to his students in rather a discouraging way when he gave feedback. For instance;

S_1: I don't understand multi-step operations.
T6: What are multi-step operations?
S_1: …(There is no answer)
T6:. You don't know what you don't understand.
S_2: Don't we start at the bottom?
T6: No. You start at the shortest fraction line.

T1 and T2 rewarded students who responded correctly to the questions. For example, T1 made an examination of the 6^{th} grade contents to the 7^{th} grade students in order to evaluate their readiness. She gave a book to the successful students in this examination as a reward. Besides, if a student gave a correct and unique answer, then T1 and T2 gave a plus mark to that student as a reward.

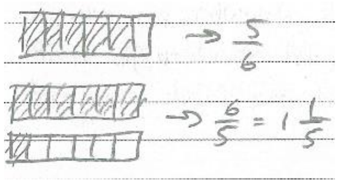
Analyzing the content of students' questions, when we consider that aspect, it seemed that participants' actions were parallel with the teaching methods and feedback types. T1 generally asked the student's questions to the other students:

S_1: If we change the numerator and denominator in fractions, will the result change?

T1: Your friend asked a question. Do you think it will change? What do you think?
C: It will change.
T1: Why?

S_2: (Writing on the board) For example, if we change $\frac{6}{5}$, it is $\frac{5}{6}$. Improper fraction, if we change it, took 5 out of 6 pieces, but the other fraction is one whole $\frac{1}{5}$. The fraction is completely changing.

T1: Yes. (Draw the model).



or students queried themselves;

T1: $2\frac{1}{5} \cdot \left(-1\frac{4}{11}\right) = ?$

C: Can we multiply the mixed number without inverting it?
T1: (She had students try solving on the board and in the classroom. Students found different results.) Why did this happen? (When there was no answer, she reminded the model she had shown before.)

Then it should be $\left(2+\frac{1}{5}\right) \cdot \left(-1+\frac{4}{11}\right)$.

T1: From here, the distributive property should be used and done. This would have lenghtened the work. Converting mixed fractions to improper fractions makes it easier to do an operation.

T2, T3, T4, T5, and T6 generally explained the content themselves. In their explanation, mostly rules were used or teachers reminded the past contents. T3, T5 and T6 suggested students to memorize necessary rules. However, participants mostly did not evaluate students' questions in terms of their understanding. They only focused on the explaining or responding to the students' question. They ignored the possible reasons for students' questions. To exemplify,

S: (Describing the addition operation in rational numbers) Why do we equalize the denominator and the other denominator?
T2: We can't add two different parts, they have to be equivalent.

In T3's class:

> *S: (Describing the addition operation in rational numbers) Are we adding the denominators?*
> *T3: We write the denominator common. We add the numerators.*

Responding to unexpected mathematical issues, in this aspect, it appeared that the teachers answered students' unexpected questions mostly by focusing on results, not on the reasons and how questions, and their explanations were based on rules. T1 tried to explain almost all students' questions about mathematical issues meaningfully. For example, in T1's class;

> *S: (Describing the properties of addition in rational numbers) Doesn't the expansion also count as a property?*
> *T1: It is not considered a property because you do not change the number when you expand it.*

Students responded before T1 to their friend's comments or answers:

> *T1: Why do we equalize the denominator when doing subtraction?*
> *C: To make an operation in the same whole.*
> *S_1: We subtract it even though the numerator is equal.*
> *S_2: No, the numerator represents the taken parts, not the whole.*

or T1 asked to the other students;

> *S_1: What is $\frac{4}{4}$ on the number line?*
> *T1: (She asked to the class) What do you think?*
> *S_2: We show it at one on the number line.*
> *T1: Why?*
> *S_2: Because, $\frac{4}{4} = 1$*

T2 also tried to explain most of the unexpected issues and responded students' questions. Students sometimes did not understand her explanations. For instance, when T2 used fraction cards in order to multiply the fractions, she gave an example by using blue and pink colored cards. T2 superposed them and showed the purple color pieces which consisted of the result of the example. Then, students asked about the superposition of fraction cards:

> *S_1: What if we put the blue on top?*
> *T2: Ok. Let's do it. (She put the blue card on top, and the result was the same.)*

S_2: Why don't we count pinks or blues?
T2: We take the places where it overlaps in two fractions, the common places, because it is multiplication.
S_3: What if we put them both on top of each other the same way?
T2: (She put the two cards on top of each other as rows and columns, and showed them). As you can see, the whole is not divided into equal parts in this case. If we put one row and one column to form a column (overlapped) in order to divide the whole into equal parts. As you can see, it is divided into equal parts.

Students understood the multiplication with fraction cards after this explanation. Then, T2 distrubuted two fraction cards to each desk and wanted students to illustrate the multiplication by these cards.

However, all participants gave a rule regarding how to made division operation in rational numbers and they did not explain the reason of multiplying the reciprocal of the second number. Almost all students accepted the rule except for one student in T2’s class:

S_1: Why do we convert and multiply?
T2: Because, it is rule.

After the lesson, I asked this situation to T2. She responded that she would explain to him later especially because if she stated the rationale of the rule, other students could be confused. They could not know reverse with respect to multiplication. She expressed that she did not know what they learned previously. She said she would make a statement to the specific student in the following way;

T2: $\frac{\frac{1}{3}}{\frac{2}{5}} \cdot 1 = \frac{\frac{1}{3}}{\frac{2}{5}} \cdot \frac{\frac{5}{2}}{\frac{5}{2}} = \frac{\frac{1}{3} \cdot \frac{5}{2}}{\frac{\not{2}}{\not{5}} \cdot \frac{\not{5}}{\not{2}}} = \frac{1}{3} \cdot \frac{5}{2}$

T3 and T6 could not understand students’ unexpected questions, therefore, they could not react properly. Sometimes they ignored the questions or gave deficient explanations, and other times their responses were related to the idle question. To illustrate for T3;

T3: (When teaching the division operation in rational numbers) Keep the first rational number the same, we write the reciprocal of the second rational number and multiply it.
S: What is the reciprocal of it?

T3: Change the numerator's and denominator's place.

In T6's lesson, when he taught rational numbers, he said that:

T6: There is no such thing as subtraction, actually it is addition.
S_1: Then, why are we learning the subtraction?
S_2: Because we didn't learn the negatives in elementary school.
T6: Yes, it is true.

When T6 was was teaching how to compare rational numbers and divide into negative and positive rational numbers, one of the students asked:

S: Why does the rational number get smaller when denominator is larger?
T6: The number actually gets bigger and smaller as it gets closer to "0" from number line. For example, when you're underwater, the closer you get to the top, the higher it gets.

T4 also joined T3 and T6 in terms of her responses which were related to idle question. For instance, when T4 taught repeating decimals in her class, she said that

T4: The curriculum has changed, but you cannot see this topic in 8th grade. Normally, [it is] not in the curriculum.
S: If it's [additional conent], why are we learning it?
T4: Because, if it will come back again, then you [would be in a position that you did] not learn it. Eventually you will learn. That is why I am teaching.

T4 asked a problem to class, then one volunteer student solved on blackboard, but it was incorrect. T4 could not notice at first. When she explained the solution with models, she noticed that the student's solution was wrong. After that, she checked again and stated that the student made a mistake. Then, she illustrated the correct solution with a model. On the other hand, in T5's class, students did not ask unexpected questions or there was not any unexpected event. Mostly, they claimed that they did not understand the content, questions, or solution. After that, T5 explained one more time.

4.2 The Nature of Middle School Mathematics Teachers' Beliefs Regarding Rational Numbers

In this part, the findings from interviews are presented. Participants' ideas about themselves (in relation to teaching mathematics), teaching, mathematics, classroom, and students. Their ideas were given seperately first, to provide in-depth information about their beliefs an inform the case, then a summary of their beliefs was given.

4.2.1 T1's Beliefs about Teaching Mathematics

T1 described herself as a disciplined teacher: *"I am a disciplined teacher. I don't like much movement in the classroom. I try to silence the students too much. I mean, I don't like voice in the classroom" (T1_I1).* T1 said that she felt happy when she was teaching, learning, and engaging in mathematics. For T1, an effective teacher should know the classroom management very well. Her claims could be seen in her teaching. T1 generally acted authoritatively during the lesson. She never started the lesson unless the students were silent. She also did not allow students disturb the lesson and intervened with students. In fact, when two 7th grade students damaged the top of their desks during a lesson, T1 saw it and got angry. She did not continue the lesson but gave advice to the whole class.

When T1 defined effective teacher's knowledge and skills, she expressed that effective teachers should know mathematics but not memorize, know its real life use, transfer this to the students, establish effective classroom management, know the readiness level of students very well, and reach the student level. However, she did not mention PCK dimensions directly.

According to T1, effective mathematics teaching ensured that students *"learn to think. A student learns to think analytically. He/she learns to solve problems. In fact, he/she learns to solve problems in his normal daily life" (T1_I1).* Her purpose in teaching mathematics was as follows:

> *…my goal is to give students a perspective. I mean ... mathematics is not just as a lesson, but how can I say it ... as a qualification, I teach them to make them being equipped with [mathematics] ... they can absorb it so that their mathematics knowledge will be useful in their lives in a way that works [for them] in the future. (T1_I1)*

However, she expressed that not every student could learn mathematics very well because they were not willing to learn. She also blamed herself for acting impatient and keeping up the subjects for the national examination. Because in her opinion, everybody could learn mathematics at a specific level.

T1 divided her teaching experience as before the graduate study and after the graduate study. She stated that before doing a master's degree, she used direct instruction method because she learned this by heart when she was educated. However, after the Master's degree, she started to use discussion method, as illustrated previously. According to her, the graduate study created awareness and contributed to her teaching in a considerable extent. T1 also gave an example:

> *… I conducted the study of whole numbers not last year, but the year before that, in my class, and I've learned most of the things about the whole numbers there by myself, notice that how many years.. I was in my 8th year as a teacher. I learned most of the things by memorizing and I understood that I was teaching them by memorizing. […] the study was about profit and loss. (T1_I1)*

She thought that using technology in mathematics teaching especially in geometry was very effective because students could configure the shapes in their minds. Besides, in her idea, she wanted to teach geometry in laboratory where she wished her students work alone. Since I conducted observations in the rational numbers and fractions concepts, I did not observe that she used technology to teach mathematics. She rather used technology for supplementary resources. T1 stated that she gave up using concrete teaching materials and group work because the classes were crowded and it could not be handled.

T1 expressed that whole numbers, rational numbers and fractions were the most important, fundamental or central topics in the middle school mathematics curriculum, because *"From now on, everything that's coming is built on all these things. Equations, systems, or university, everything including education depends on them" (T1_I1)*. She believed that problem solving skills and four operations skills were the most important, fundamental or central skills in the curriculum because *"And most of what they have earned here is the basis for future because the curriculum is structured that way. If students can't gain those skills, because I've seen the effect of it, a gap in one year reflects a lot on the other year" (T1_I1)*. She defined 'mathematical problem' as an obscurity to be solved. To lay emphasis on the importance of problem solving, she said: *"I think problem solving is fundamental in mathematics… includes thinking skills to solve problems… you know, it involves analytical thinking, thinking from different directions, and using the operations at the same time" (T1_I1)*. She believed that problem solving brought students multi-dimensional thinking.

Despite the emphasis she expressed, T1 did not spare specific time for problem solving activities, more than it was suggested in the curriculum. Her approach in these activities was:

> *I write the question ... I give 5 minutes to work in pairs, the friend next to them, depending on the length of the question. Then ... again, according to the [level] of the class and the problem, I gather their ideas first. Here's what you found, why did you follow such a method... I make them explain. If there are students who follow more than one method, we write their methods on the board. They discuss whether it is right or wrong, who thinks what. Then, at the end, I gather all. (T1_I1)*

T1 expressed that when there are different solutions than her solutions, she showed every different solution to students. She stated that she could not understand whether the students understood the problem or not at that time. Therefore, she would ask another problem:

> *If students can solve problems that can be solved in one way or another, or in different ways, then it means they have understood. Yet, this is not in the sense that they will use the same route. If they can apply it to different things and to a different problem using the information there, then they have understood. (T1_I1)*

T1 stated that her students liked problem solving and understood rational numbers better. This is because *"…you know, because the problems are constructed in the context of everyday life, you know, students make more connections, maybe that is what they like..." (T1_I2)*. Whereas, according to her, students had difficulty to understand exponential numbers. However, she did not understand why they had difficulty about it. Even though she started giving exponential numbers examples with 3 or 4 so as to avoid any misunderstanding, such as numbers are multiplied by exponents, she could not understand why exponential numbers would be misunderstood. Moreover, she could not find or define common points among the topics that students understood or did not understand. Problem solving was also a procedure that she could understand whether students had learned a concept or a topic: *"In class… for example, if he/she can use this concept when we create a classroom discussion or if he/she can use it correctly while explaining the problem solution, then she/he has learned" (T1_I2).*

She stated that when the class level was different, she taught mathematics based on this specific level:

> *… so the level can be low. In a situation like that, no matter how much you talk, no matter how much you try, the student doesn't get it because she/he doesn't have a background… Then, you have to keep both your teaching level and the standard low. I mean, as much as possible… after using concrete materials, such as here we have started the fractions. With very simple examples, you know, with enough things for them … I can leave it and leave it without going into more detail, without making the question level difficult. (T1_I1)*

T1 defined the ideal class as *"It's a class that can discuss. I mean, the class that's willing to learn" (T1_I2).* In the meantime, she also expressed her teaching strategies:

> *I always want to get an answer to the question I ask. I try to get him/her to talk, and I'm usually angry at those who don't talk. I mean, even if it's wrong, the student should be able to express his/her opinion in my class. I mean, here is what your friend says, what do you say? I do it a lot (T1_I2).*

She wanted to overcome students' anxiety with her teaching:

> *Usually, students start with a fear, you know, they start with a fear of getting to the board, standing up and speaking. I'm trying to overcome it as much as I can... the student feels comfortable and she/he can speak her/his minds so that we can reach something (T1_I2).*

She had taught mathematics occasionally in her ideal class some time in the past. If she had her ideal class at the moment, she wanted to use more concrete materials such as fraction cards, and more smart board. Actually, she thought that concrete materials were important for teaching. She also believed that these materials made concretization of the content easier, and group works and discussion settings were made easier in this environment. Yet, because of the physical condition and facilities of her class, she did not try to use the materials.

4.2.2 T2's Beliefs about Teaching Mathematics

T2 had effective classroom management even though the number of students was high. She played games, made activities and group work with concrete materials in her lessons. T2 expressed that she took pleasure in teaching, learning, and engaging in mathematics. If students did not understand, then she sometimes felt desperate. She stated that students would not learn mathematics properly without a teacher. There were two disadvantageous students, who had individualized education program in T2's class, and the number of the students was between 40 and 50. Therefore, she had to spend effort with them during the lessons. This situation interrupted her teaching many times. However, she never complained about the circumstances.

T2 believed that question-and-answer sequence during teaching was effective in mathematics teaching. She also said that she used this method, and activities, smart board activities, animations-games, and mostly direct instruction. However, she believed that direct instruction method was more effective than other methods because it was difficult to control crowded classes when using other methods. If classroom size was low, using other methods was easier. On the other hand, T2 believed that using technology in mathematics teaching was not as effective as it was said to be because there was not any progress when using technology in a crowded class and technology did not increase students' interest al the time: *"It doesn't make any difference for students to do something on a smart board all the time. It doesn't get their attention. But when I do this from time to time, they can listen to it more carefully."(T2_I2).* Thus, she gave up using technology and continued to use direct instruction. Observation notes approved T2's expressions.

T2 stated that knowing mathematics makes real life easier, develops thinking skills, and ensures real life skills, such as shopping. She also defined that effective teachers and effective mathematis teaching should make students question and think. She thought that if students made more questioning, they could understand much better.

T2 expressed that she preferred students write the content on their notebooks because it was more recallable this way. For T2, if students made themselves materials, such as fraction models with paperboard, it was clearer and more understandable for them. Her purpose in teaching mathematics was

> *... I teach the subject first, but let the student think and use his/her head while teaching the subject. May it work for him/her in his/her daily life. That's why I try to pick my examples especially from there. Make his/her life easier (T2_I1).*

T2 gave particular importance to using daily life examples and contexts in her lessons. However, students' bias against mathematics and their interests, and school principal affected her teaching. For school principal, she asserted that the principal interfered in her lessons and activities, such as he did not permit making activities in school garden. T2 claimed that she wanted to apply what they taught in real life, but she could not do it because physical conditions and high number of students in the classroom prevented it.

T2 thought that last changes in curriculum decreased the intensity of content. It was good for both students and learners. For her, sets content in curriculum should be returned. T2 also expressed that equations, rational numbers and fractions were the most important, fundamental or central topics in the middle school mathematics curriculum because these concepts were the bases for the future concepts and connected to them. On the other hand, T2 did not mention any fundamental or central skills in the curriculum.

According to T2, mathematical problem was a problem that could be encountered in daily life. To laid emphasis on the importance of problem solving, she said: *"I think that a student who can solve mathematical problems can also better analyze his/her problems in daily life and produce better solutions" (T2_I1)*. T2 expressed that she understood from students' eyes whether they understood the problem or not at that time, and she would ask other questions, such as "*what do you understand in the first sentence of problem?*" For problem solving activities, T2 spared more time then it was allocated in the curriculum. She claimed that every lesson or after the instruction of topic, she solved and asked problems. Indeed, she solved at least one problem in almost all observed lessons. Her approach in these activities was as follows:

> *Now I ask about the problem. I give time... I expect the student to solve it, but I do not wait for it myself at the time, I'm wandering around, looking at what the students are doing. If there are things that I see missing or need to intervene, I intervene while they are solving them in their notebooks, and then if I think it's generally understood, you know, if I think everyone can solve it, I take someone from the class on the board. But if I see that they cannot solve it in their notebooks, then I'm taking it back again and explain it myself. Then, I go back to the student (T2_I1).*

She claimed that students had difficulty to understand problems generally because they did not have reading habit and did not understand what they read. She expressed that when there were different solutions than her solutions that she did not explain in her class, she showed it and also every different correct solution from the students.

T2 did not think teaching she was teaching in an ideal class and she defined this class in terms of physical features, such as:

> *Class size is not crowded. For example, a class that does not exceed 20-22-23, 25. I mean, mathematics class. I mean, it's my special class. It's a class where I have all my materials, my ruler, this and that, my blocks, everything. I mean... For example, I should be able to control the classroom layout as I want. You know, in the seating arrangements, I should do 'u' if I want, I should do the normal desk, lines of two desks, if I want. (T2_I2)*

She also expressed that she would not change her teaching strategies, but she would do more practice about daily life and would want to use more concrete materials in her ideal class. However, there were not enough materials in her school for this.

4.2.3 T3's Beliefs about Teaching Mathematics

T3 defined herself as a person who was willing to teach and loved teaching and engaging in mathematics, but sometimes she did not feel the same energy from her students. For instance, if a student disturbed the others in the lesson, she got angry and spoke with a high volume. Yet, her behavior was more positive to the willing students. Besides, in her class there were immigrant students, who had also adaptation problems, and she complained about this situation. T3 was generally authoritative during the lesson and she never started the lesson before she established silence of the students.

Knowing mathematics for T3 was finding solution for the problems. T3 defined that effective teachers should make communication with their students because better communication ensured more successful teacher. She also said that even though mathematics teachers were not teachers of language, they should with speak proper sentences and should be careful in terms of grammatical rules when teaching. These rules *"[may increase] the student's interest in the lesson .. it attracts more. Maybe it increases success, you know, their success. For example, when I say the simplest spelling, they can start writing more properly" (T3_I1).* According to T3, effective mathematics teaching *"is also very helpful in exams ... apart from just exams, I think it affects ... In a daily problem, really... if it increases analytical thinking, mathematics -- I think it does – she/he can find solutions faster. If it's really strong (T3_I1)."* She said that the purpose of teaching mathematics was to ensure students' learning of mathematics. The reason for not learning mathematics adequately was the deficiency in students' fundamental concepts. T3 believed that teachers were important for students' learning and it would be difficult to learn mathematics properly through watching videos or reading books, without the teacher.

She believed that concretization, visualization and storifying in mathematics teaching was very effective because these strategies drew her students' attention more. T3 said that when the content was abstract, it was difficult to understand for students, but if there was visualization, then students understood more easily. She also indicated that giving more examples and exercises was important to help understanding as it was observed in her lessons. She also used direct instruction method, emphasized rules and progressed with it. She claimed that she sometimes used smartboard with "Eğitim Bilişim Ağı (EBA)" sources and videos that included qualified educational electronic contents. She used visualization and storifying in the observed lessons. However, she mostly used smartboard with the supplementary book, and solved the examples and problems from this book, but she did not use EBA sources or videos in the observed lessons. T3 said that she gave up making activities because activities needed paying attention to each student and the classes were crowded for this.

T3 thought that the intensity of content in curriculum was decreased in the process of time but it was not good for students because students would not study more. She said that *"To be honest, I'm one of those people who thinks that children learned more in former curricula, intensive curricula" (T3_I1).* In addition, T3 expressed that probability, permutation and combination should be returned to curriculum because she enjoyed these topics. She told that students' background and readiness affected her teaching. T3 complained that students found the solution of given homework in the internet, and they copied and submitted. She wanted to make one to one teaching and give special attention to students, but there was not enough time to do it. T3 expressed that numbers such as whole numbers, rational numbers, exponential numbers, and root numbers were the most important, fundamental or central topics in the middle school mathematics curriculum because

> *They come across [with the numbers] in all exams throughout their lives starting from middle school and they appear in all exams all their lives... They're dealing with numbers everywhere, especially the integers. Integers never come out of their lives. That's why it's important (T3_I1).*

The most important, fundamental or central skills in the curriculum were four operations skills. For preparing lessons herself, she expressed that *"I give more importance to some subjects .... sometimes I sit down with pen and paper and prepare something ... especially when I first introduce a new topic or at the end of the topic" (T3_I2).* As stated above, she expressed rational numbers as a central concept; however, her observed lessons did not reflect this importance in the way she decsribed. She did not have any preparation before the rational numbers class. She only used electronic sources and solved the problems, and did practice and drill on the smart board. She also did not do anything special at the end of the rational numbers.

T3 defined 'mathematical problem' as *"sorting out something complex" (T3_I1).* For the importance of problem solving, she said that mathematics is a problem and the importance is reaching the result. If students solved the problem, then they could apply it in daily life. In order to find out whether students understood the problem or not, she looked at their eyes. If they looked blank at the problem, then that meant they did not understand it. In addition to this, she asserted that verbal and written assessments helped to understand it. She did not spare special time for problem solving activities, but spent the time suggested in the curriculum. If there was remaining time from her lessons, then she asked problems. However, this claim was not observed in her lessons.

In problem solving activities, she followed the following path:

> *I ensure that they think about it first. You know, is there anyone who can do it ... Then sometimes in the first examples they cannot produce any ideas. I lead the way first. In the second and third examples, they start to say something (T3_I1).*

When there were different solutions than T3's solutions, first she checked it. Then, she showed it on the blackboard, but mostly she solved the problems in different ways because students did not generally come up with different solutions.

According to T3, the ideal class consisted of responsible students who were prepared for the topic before the lesson by watching videos from the internet and reading about the topic. She wanted to see students who had high readiness level. Additionally, the number of students should not be over the 20 in her ideal class. However, she indicated that her current classrooms only fitted her class size criteria for the ideal class. In the meantime, T3 expressed that although she would not change her teaching strategies in her ideal class, she would increase the level of questions.

4.2.4 T4's Beliefs about Teaching Mathematics

T4 indicated that she felt happy when she was learning and teaching mathematics. If students wanted to learn mathematics, she felt much better. Moreover, she expressed that parents of students were supportive to her in terms of guidance for the students, such as they bought additional resources and books that she asked. She stated that students could learn mathematics without a teacher in peer learning, but she did not seem to put it into practice. She emphasized the rules of rational numbers content in the interviews and used these rules sometimes in her lessons. T4 also thought that she had to use direct instruction method in mathematics teaching because there were several topics and learning outcomes, but there was limited time to teach, and the class was very crowded. She stated that she used models sometimes in her lessons, such as counters in whole numbers, solid models and paper folding, but she gave up using them except for the counters because of the crowded classroom context.

T4 asserted that in order to know mathematics, first, one should like mathematics, and then have the computation skills. She added that students should not have bias against mathematics because if they had it, they gave up studying it. Students should also like the teacher. After that, they could cope with mathematics. Students' knowledge of mathematics from the elementary school was also important for knowing mathematics. According to T4, students could learn mathematics without teacher in peer learning, but she did not seem to put it into practice. Besides, T4 defined that effective teachers should have content knowledge, pedagogical knowledge, and experience. For the pedagogy, teachers should establish communication and have classroom management. If the teacher did not have adequate content knowledge, students could have bias against the teacher because the teacher did not know anything. If there was no classroom management, the class would be very noisy, and it would prevent the students follow the lesson. According to T4, effective mathematics teaching ensured permanent learning. Her purpose in teaching mathematics was to make students love mathematics, not have bias against mathematics, and not be bored in the lessons. T4 thought that she accomplished the loving status.

T4 thought that the last changes in curriculum decreased the intensity of the content. The dispersing was also good except for dividing the content. For instance, she said that she had taught addition and subtraction in integers in last year, but she taught division and multiplication in integers in next year. Thus, she stated that there was a long time between these topics. T4 told that students' background, readiness, noise in lessons, and her mood affected her teaching. In addition to this, she wanted to make peer to peer teaching, but there was not enough time to do it.

T4 expressed that the most important, fundamental or central topics in the middle school mathematics curriculum were order of operations, equations and algebraic expressions because these concepts were connected to following concepts. For fundamental or central skills in the curriculum, T4 mentioned knowing the multiplication table. She defined ‘mathematical problem’ as an obscurity which students did not know and it was formed in students’ daily life. However, she claimed that problems in the textbook were far from reality and they did not fit the daily life.

To lay emphasis on the importance of problem solving, T4 claimed that if students could solve the problems, then they could handle other topics in mathematics. She also mentioned the steps of problem solving process as *“first she/he needs to reason, she/he needs to think about the problem. Then ... she/he needs to understand the problem thoroughly and analyze it. Finally, she/he must reach a solution” (T4_I1).* Nonetheless, she did not apply these steps in her lessons. T4 expressed that in order to check whether students understood the problem, she asked another similar problem. If the students solved that problem, then it meant they understood them. She spared time as suggested in the curriculum for the problem solving activities. She also said that if there were any abstract concepts in the problems, then she would concretize them and solve with models, as observed in her lessons. In the opinion of T4, when there were different solutions than her solutions, she liked and showed them on the board. She showed diffirent solutions for the most of the problems on the board in the observed lessons.

T4 defined ideal class as *“a class that loves mathematics, listens well to the lesson, does not pay too much attention on other things in the lesson ... A class without prejudice against mathematics” (T4_I2).* She was satisfied with her class, yet if the class was not crowded and did not have more than 5 students, she would be more satisfied and that would be an ideal class. Because of easy concretization, she claimed that she used real models. In her teaching, she showed models on the board with drawings.

4.2.5 T5’s Beliefs about Teaching Mathematics

T5 liked both learning and teaching mathematics. When students did not learn or understand the content, she claimed that she would get angry, but this did not happen during the observed lessons. T5 also mentioned that the class size in her school did not exceed 20, but most of the students needed particular attention from the teachers. Thus, after writing even a short script on the board, she had to wait for the students.

T5 thought that direct instruction, extensive practice and using models were effective in mathematics teaching because students would learn the topic directly from teacher. She also asserted that students would learn mathematics by using books without the teacher, yet that would not form a proper mathematical knowledge. For modelling, she said that students could see and learn together, but she taught rational numbers with direct instruction in almost all lessons. T5 expressed that she used group work and project-based learning in the past, but she gave up them because students did not adapt to these methods. She said that she gave homework from the internet sources and an instructional materials website, or assigned tests and copied materials; however, this was observed only in a few lessons. It was observed that she used supplementary book almost in her all teaching. There was no smart board in her class. If there was a smart board in the class, T5 expressed that she would solve more examples on the smart board because writing on the board took a lot of time. She generally gave rules or concretized these rules in her lessons. In addition to this, she tried to make students memorize these rules. She praised herself that if students understood very well, then that meant she she taught well.

According to T5, knowing mathematics meant not only solving mathematical problems, but also having a quick mind, generating an easy solution for the problem situation, and looking at it from a different aspect. T5 defined that effective teachers should reach students' level, and know the area of their interests. Effective teachers also should be able to give examples from students' life and interests. T5 believed that effective mathematics teaching ensured increasing students' enjoyment of mathematics, reducing the fear of mathematics and overcoming a negative bias. With effective teaching, students could also use mathematics in daily life, such as in shopping, and to have a different point of view. According to T5, mathematical problems had one correct result, but there were many ways to solve them. Her purpose in teaching mathematics was to make students score high in the national examinations because examination was important in students' and teachers' lives and mathematics was the most important lesson in the examination. However, she indicated that she could not fulfil her purpose because of careless parents of her students.

T5 thought that the last changes in the curriculum decreased the intensity of the content and made it weaker. Nevertheless, she did not propose any content or topic to add to the current curriculum. She just offered to assemble contents such as addition – subtraction and multiplication – division topics. T5 expressed that equations, rational numbers and whole numbers were the most important, fundamental or central topics in the middle school mathematics curriculum because students met these concepts for the first time and they were the basic concepts for next ones. T5 stated that performing mental operations was the fundamental or central skill in the curriculum because these operations facilitated students' work at the high school and the university.

According to T5, ‘mathematical problem’ was a problem that had one correct result, but there were more than one way to solve it. She did not define the importance of problem solving much. She only expressed that the problem solving could ensure students’ reasoning. T5 expressed that she asked problems in different ways to realise whether students understood the problem or not at that time. If students solved those problems, then that meant they understood them. To give an example, she said that she taught multiplication of whole numbers with models and number line. Then, she gave other model and asked writing a problem for the model. If students could write a problem, it meant that they understood the contexts and related problems. Additionally, if her students used the contexts in different lessons, such as science lesson, then it meant that students understood it. T5 described problem solving process that she gave easy problems and clues to solve it. She did not apply any steps. In problem solving activities, T5 claimed that she spared time as long as it was suggested in the curriculum. She stated that she used problem solving in every lesson about 20 – 25 minutes, and used it to make students find common rules for solving other problems. However, this was not seen during the observations. Her definition of problem solving process or activities was different as she defined all examples, finding rules in content, and word problems as problem solving. She expressed that she showed every different correct solution from the students on the board when there were different solutions than her solutions.

T5 did not think that she was teaching in an ideal class, which she defined as

> *(including) students who bring course materials, bring them completely, and listen to the lesson in a way that they are hungry for knowledge. Students who are eager to solve such questions... There may be things that encourage for doing mathematics in terms of physical properties. Or we can hang posters to present mathematical knowledge (T5_I2).*

In the meantime, she also expressed that she used concrete materials. However, she could not use materials in the lessons at the time of the study because she had a negative experience in using compass in the classroom. When she taught drawing circle with compass, one student darted the compass to his friend's head and he was injured.

4.2.6 T6's Beliefs about Teaching Mathematics

T6 stated that although he liked teaching and learning mathematics, he liked teaching science more than teaching mathematics. T6 taught science in the first years of the teaching profession, and he stated that he understood whether students comprehended the content or not in science better than in mathematics. He could not observe students' understanding completely in the mathematics lessons. Sometimes in mathematics, if students did not understand, he felt desperate and this situation was a torture for him. In spite of all these things, T6 was self-confident and said that he did not make a preparation before the lesson because he was an experienced teacher and remembered the topics.

When classroom and time management of his teaching were examined from the observation notes, it was seen that T6 did not have proper classroom management and most of the students did not listen to him. According to him, the reason for students' inattentive attitutes was that they mostly focused on the examinations, scoring high, and they attended private teaching institutions. However, during his teaching, he recognized and looked at only to the students who were sitting in front of the class, and other students were ignored. He checked the homeworks that he assigned at least twice a week. He also asked students to clean the classroom in the last five minutes of the lessons. These things were time consuming for his teaching process.

T6 thought that direct instruction and using technology together was effective in mathematics teaching. Although, his class had a smart board and the internet connection, he could not integrate them in his lessons actively because it was time consuming for him especially when the internet connection was lost. The existing technological infrastructure was not effective as observed in the lesson. He took a course about using GeoGebra in the past, but he said that he sometimes used it in geometry lessons. Yet, he could not manage the internet resources at a short time during the observations. He also said that even though he used real concrete materials and models to show fractions in the 6th grade, such as bread and cake, he did not want to use concrete materials in the 7th grade rational numbers, because the content was more abstract and there were negative numbers. He gave up distributing activity sheet and worksheet to students because he stated that the class was crowded and hard to control, thus he could not achieve classroom management. He also said *"maybe using manipulatives or concrete materials can be very effective in 10-15-student classrooms, but you cannot teach it to 30-student class (T6_I1)"*. Moreover, T6 asserted that students should be divided into two parts as those who learned mathematics and those who did not learn mathematics in their further education. He believed that

> *…those who will learn mathematics and those who will not learn, that is, after receiving compulsory education, [those who] will not study, the student will be a tradesman […]. Mathematics is unnecessary for him, it is cruelty for both [the teacher and the student]. It's bad if you teach him too much, and if you teach him less, then he [will have less] (T6_I1).*

After this kind of separation, he wanted to enrich the content and teach the students who wanted to learn more mathematics.

Observation notes indicated that T6 used direct instruction method and taught mathematics with rules. To give an example, when he showed the denominator in the number line, he divided the whole as marking one less number times of the denominator. T6 tried to make students memorize this procedure as a rule. He mostly dictated shortcuts, rules, and definitions as a note. He expressed that he wanted them to memorize these notes and rules. In addition to this, he stated that teachers had important role in teaching and students would not learn mathematics properly without teacher. Even if students had individual differences, teachers should exist and guide them.

T6 expressed that knowing mathematics meant sorting out the relations such as relations with numbers, meaning of numbers, or explaining the meaning of numbers with daily life. Effective teachers should be good with students and make good communication with her/his students. Effective teachers also should teach mathematics in students' way, or the way that students could understand. T6 believed that in order to be effective teachers, there was no need to be a mathematics expert because teaching mathematics in middle school did not require high level mathematics. It started with four operations and continued with little details. If teaching mathematics was equivalent to real life and included in activities, concrete materials, then it would be more effective. However, T6 said that he could not do it because of high number of students in the classroom and physical conditions. Although he thought that teaching through experience was more effective on students, he used direct instruction in his lesson. Additionally, based on T6's responses, effective mathematics teaching ensured that students could build cause and effect relationship and could feel confident in daily life and shopping. His purpose in teaching mathematics was to teach students mathematics *"enough to meet students' daily needs in their life... that is, to be able to teach mathematics that is sufficient for work environments, future professions, so that they do not have any problems" (T6_I1).* Despite the fact that T6 gave particular importance to connecting mathematics to daily life, this was not observed in his lessons. He asserted that he reached his purpose in more than half of the students. Because of not having essential communication, he did not reach the rest of students.

T6 thought that the latest changes in the curriculum decreased the intensity of content, which was not effective. He believed that *"We do not teach mathematics, so we try* *to convince them that mathematics is a lesson that can be learned. As I said, we are trying to teach four operations properly" (T6_I1).* For that matter, he expressed that he completed the 7th grade curriculum during the first semester last year because there were not many concepts to teach: *"You're passing, you're passing, there's nothing to teach. You always give and pass" (T6_I1).* He emphasized being successful in the national exam, TEOG, as observed in his lessons. T6 expressed that three factors, the content, the students, and the physical conditions, were affecting his teaching. He said that first, the content was important, and sometimes he did not understand fully the mathematics that he would teach. Understanding the content took considerable time for him. Second, students' preparedness and expectations affected his teaching. Third, physical conditions such as the class size, the number of the students, and available concrete materials affected his teaching.

T6 expressed that algebra, algebraic expression, 3D shapes' area-volume, rational numbers, fractions and four operations were the most important, fundamental or central topics in the middle school mathematics curriculum because middle school mathematics was the transition stage to the high school and to a more complex content. The thing that he wanted to do was opening up students' horizon in terms of mathematics. He also expressed that he was feeling like teaching real mathematics in the 8th grade lessons. On the other hand, T6 mentioned only using four operations as fundamental or central skills in the curriculum.

According to T6, mathematical problem was a problem that one could encounter in daily life. It was an obstacle that had to be solved by students. To lay emphasis on the importance of problem solving, he said that students applied what they learned in mathematics. Thus, he could see these applications in problem solving process. Students should also figure out that mathematics that they learned was functional, it was not empty, and it could be used. T6 expressed that he understood whether students understood the problem or not from their eyes. If a student looked blank at the problem, then he realized that the student did not understand the problem. For problem solving activities, T6 spared time as suggested in the curriculum, and his way in these activities was as follows:

You write down the problem, you get feedback from the students about the problem. ... We figure out what can be done or we solve it together at first. After that, there is the problem of time when you ask the problem and solve the problem. You can solve a maximum of 2 or 3 problems properly in a lesson hour. It takes a lot of your time. (T6_I1)

Observations showed that T6 instructed all steps of problem solving process, but he did not apply them. He gave some time to the students for solving the problems. If students could not solve it, then he helped them. When there were different solutions than his solutions, he expressed that he acted based on the students' level. If a different solution was higher than students' level or could confuse students' mind, then he did not want to show the different solution. He also accepted every different correct solution from students.

T6 did not think that he was teaching in an ideal class and he defined this class in terms of students, teachers, and physical conditions:

It is the classroom you know what to teach. It is the classroom where students know what to learn. ... I mean, students do not know what to learn in the classroom system in our current education system. The teacher also doesn't know exactly what to teach... a class where I have a lot of materials, that is, different teaching methods, time, low class size... enough to devote time to students, a wide range of opportunities. (T6_I2).

In the meantime, although T6 mentioned about employing different teaching strategies and concrete materials, he expressed that he did not change her teaching strategies. He wanted to be a teacher of students who had high mathematics level and eagerness to learn.

4.3 Middle School Mathematics Teachers' Beliefs Regarding Rational Numbers

The participants' beliefs were presented with the themes for this study, which included the recurring patterns of the participants' beliefs (Merriam & Tisdell, 2016). Moreover, in order to build an understanding, developing themes are necessary and make it easier (Maxwell, 2013). Hence, in this study, the themes were identified through a holistic analysis of participants' responses. The narrative and interpretative descriptions of each theme also were explained with examples from participants.

4.3.1 Beliefs about Students and Their Learning

All participants except T4 believed that students would not learn mathematics well without a teacher. For instance, T3 said that only watching videos or reading books were not enough to learn the mathematics concepts properly. However, T4 expressed that students could learn mathematics with peer learning. T1 and T3 expressed that audio-visual materials enriched educational environment and enhanced students' understanding. Participants believed that in order to draw the students' attention, concrete materials or real-life examples could be useful. Yet, T1 and T4 claimed that fraction cards were needless except for at the first glance. T1 and T2 claimed that they rewarded students who responded correctly to the special questions in order to increase students' attention to the lessons. T3 mentioned that visualization and storifying could draw students' attention more.

According to the participants, the reasons of students' misunderstanding or misconceptions about rational numbers were mislearning the content or confusing the rules. In this regard, T2 and T6 did not show all proofs or solutions because they stated that students could be confused. T1 proposed that if students were not willing to learn, they could not learn it very well. T2 believed that when students wrote the content on their notebooks, they could easily remember that content. T6 did not seem to believe that every student can learn mathematics. He rather believed that students should be separated based on their mathematics level and future job intentions, and should be taught mathematics at different levels.

Participants wanted to use different types of questions in their exams such as true-false, word problems, short answers, and multiple choice questions, in order to see indicators of students' learning. They believed that monotype questions could be inadequate for this. T2 and T3 claimed that they looked at students' eyes in the lesson in order to find out whether students have understood the content, problem or not. If students looked blank, teachers would interpret that students did not understand it.

For T5, when students used what they have learned in different lessons, such as science lesson, then this would be an indicator of their learning.

4.3.2 Beliefs about Mathematics Teaching

Most of the teachers in this study believed that direct instruction was more effective in teaching mathematics. T5 believed that students could learn the topic directly from the teacher easily. T3, T4, T5 and T6 expressed that students should memorize the necessary rules, definitions, and shortcuts. On the other hand, T1 believed that effective teachers should not make students memorize and they could transfer the content to students without memorization. T1, unlike other teachers, chose to employ discussion method in the lessons. T1 and T3 indicated that their teaching was affected by the students' background and readiness which would result in changing examples and teaching methodes.

Teachers were not much willing to use technology. T2 believed using technology in a crowded classroom was not effective. However, her using of smartboard seemed to be targeting students' attention because she believed that if smartboard would be used all the time, it would not attract students. T6 also believed that using activity sheets and worksheets would not be suitable in crowded classrooms as it would be difficult to control the class.

Except for T2, participants did not make preparation before their lessons generally because they believed that they were experienced teachers and a preparation was not necessary. For example, T6 stated that he remembered the topics because he was an experienced teacher and that teaching mathematics in middle school did not require high level mathematics, and therefore, there was no need to be an expert in order to teach the content. Teachers claimed that mathematics textbook was inadequate for teaching in terms of examples. Nonetheless, they mostly used supplementary sources.

Almost all teachers described their ideal class in terms of physical features where they focused on the number of students and existence of concrete materials. They also wanted to teach students who had high readiness level and were willing to learn mathematics, especially T3 and T5. Teachers did not think about changing their teaching strategies. T1 and T3 wanted to work with the students one to one in a mathematics laboratory, but there were not enough time or available place for this. T5 also stated that she wanted to use the smart board and solve more examples on it.

4.3.3 Beliefs about Mathematics

About fundamental topics of the middle school mathematics curriculum, all teachers agreed that those were the whole numbers, rational numbers, fractions, equations and algebra because these were the basis for the other concepts or connected to future ones. Additionally, for the most essential skills addressed in the curriculum, they claimed that those were problem solving skills and four operations skills. Only T4 reduced these skills on particular one, knowing the multiplication table.

Most of participants pointed out that if students knew mathematics, they could adapt real life easier and develop thinking skills. Indicators of these beliefs were visible in their teaching. For instance, T2 used daily life examples extensively and gave particular importance to them. T6's purpose was teaching students as much as possible to meet basic needs in daily life. On the other hand, mathematics was important for the participants due to its role in the national examination. T5's purpose was ensuring students to score high in the national examination because it was a significant indicator of achivement for both students and teachers.

At the end of the topics, teachers asked and solved problems in line with the learning outcome identified in the curriculum. Hence, they followed up the curriculum and gave a limited time for problem solving. Observations showed that they generally did not exceed the allocated time for problem solving activities in the curriculum. Some of them claimed that they asked word problems to students most of the lessons. T2 used problem solving the most among the participants as she expressed that if students solved problems, they would make better analyses in real life problems and solve them easily. T1 and T4 also stated that problem solving was important for mathematics because it included both analytical thinking and using operations together. On the other hand, T3, T5 and T6 gave more vague reply to view of problem solving, such as if students solved the problems, it would reflect to their daily life positively.

Based on the observation, participants' problem solving process was as same as the other exercises. They generally asked the question and wrote on the board, then gave some time. Students, who solved the problem, showed their solution to teachers individually, especially in T1's, T2's and T4's class. Teachers selected one of them to show it on the board. At the end, they explained it. If there was any different solution, then teachers showed it on the board. They mostly welcomed the different solutions. For instance, T6 said that sometimes he honored students for solving in a different way. Besides, it can be said that teachers acted parallel with their teaching method in problem solving. For instance, T1 used discussion method and students re-explained problems with own words, and other teachers, especially T3, T5 and T6, explained the solutions with rules. Furthermore, participants believed that problems were more complex than practice and drill. Hence, they changed the difficulty level of problems based on their students.

4.4 The Interaction between Middle School Mathematics Teachers' Beliefs and Their PCK about Rational Numbers

The analysis of participants' interviews and their observations revealed mostly coherency between the way teachers taught and the way teachers described their teaching as they believed they did. There seemed to be an interaction between their beliefs and their MPCK. The interaction between all teachers' beliefs and their MPCK in terms of rational numbers content can be seen in Figure 19.

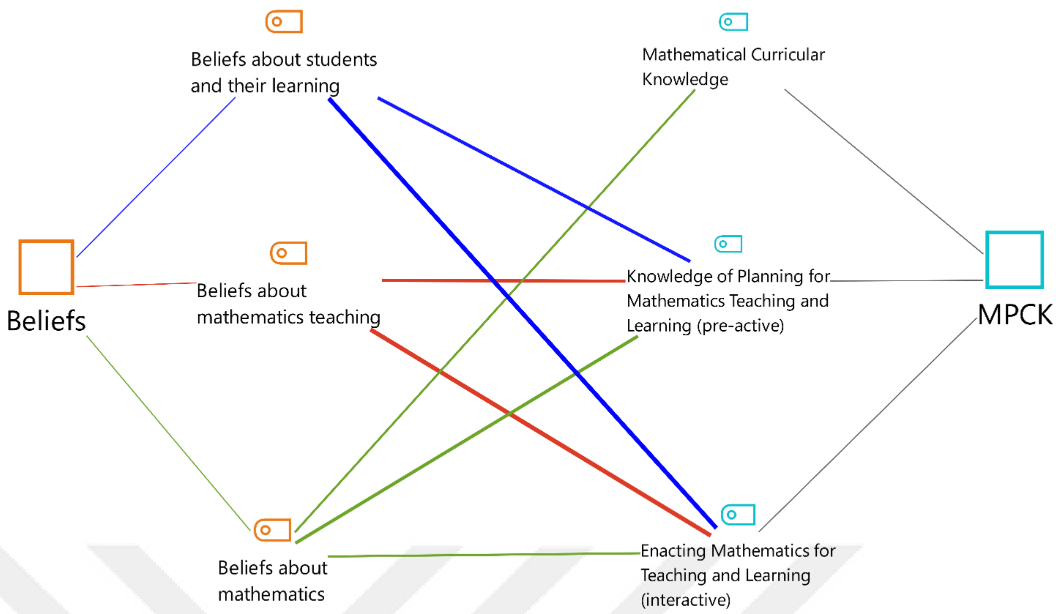


Figure 19. Interaction between participants' beliefs and MPCK

In *Mathematical Curricular Knowledge and Beliefs* aspect, teachers believed that certain ideas/concepts/skills were the key or central because these were connected the other content in the curriculum. They generally had very similar ideas in terms of topics in middle school mathematics curriculum. These beliefs about mathematics were interacted to the *identifying the key ideas in learning programs* component of the *Mathematical Curricular Knowledge*. Besides, half of participants' opinions about the simplification of curriculum were positive, but other half of teachers asserted that it was negative in terms of mathematics because the latter ones believed that more mathematics content in curriculum urged students to study hard. Hence, they determined the content of the curriculum based on these beliefs. Although the data set in this study was limited in this sense, it can be said that participants might prioritize key concepts in their teaching based on their beliefs and emphasize them further in teaching. Moreover, they were aware of the spiral structure of the curriculum. Thus, they believed that if the present content could not be taught properly, it would affect the following content. This was related to the component, *selecting possible pathways and seeing connections within the curriculum*.

Establishing appropriate learning goals, *knowing different assessment formats* and *Knowledge of mathematics curriculum* components did not interact their beliefs. The reason might be that (i) teachers did not determine learning goals because they are identified by the Ministry in the curriculum; they indicated and used different assessment formats in all mathematics topics, without much preference for specific situations; and they already had a considerable knowledge of curriculum, which they did not decide and only implement. In brief, it is most probably the case that they learned about these knowledge types during the teacher education program and their beliefs in relation to these were rather invisible because they simply used the knowledge regardless of the situation and without much decision making.

In *Knowledge of Planning for Mathematics Teaching and Learning and Beliefs* aspect, teachers' *Knowledge of Planning for Mathematics Teaching and Learning (pre-active)* were related to all the mathematics related beliefs found in this study. The interaction between this knowledge and their *beliefs about students and their learning* was traced in their planning of assessment and use of technology. Teachers preferred to employ different type of questions in their assessment because they believed that it was more effective in understanding the extent of students' learning than one type of question (*Choosing assessment formats*). Some teachers did not prefer to include technology in their planning of the lesson because they believed that use of technology would not be effective for students' learning. This was also related to *Planning or selecting appropriate activities* component of pre-active dimension.

Participants' planning action was in interaction with their beliefs that they did not need to plan because they were experienced teachers, and they believed that they did not have to prepare for the lesson. Besides, teachers in the current study seemed to choose direct instruction or discussion method because of their beliefs. These beliefs intereacted *Planning mathematical lessons* and *Planning appropriate methods for representing mathematical ideas* components. They did not seem to pay attention to *predicting typical students' responses* and *including misconceptions*. This was evident in the observations and the responses to the vignette's questions. On the other hand, they seem to think about real life examples in their informal planning of the lessons. This might be related to their beliefs that when students had knowledge about mathematics, they could adapt themselves to real life and have analytical thinking skills. Moreover, they asserted in interviews that they used word problems related to real life in problem solving parts of the lesson in line with their beliefs that problem solving process ensured analytical thinking and operational skills addressing the *Identifying different approaches for solving mathematical problems* component.

Enacting Mathematics for Teaching and Learning and Beliefs (interactive) seemed to be in interaction with all the belief aspects. What they *believed about students' and their learning* seemed to be related to their teaching. Participants believed that visual materials, real life examples, concrete materials, and rewarding them drew students' attention and enhanced their learning, and they employed these in their teaching. Similarly, beliefs about the discipline in the class was visible in their teaching when teachers responded to students' misbehavior. Most of the participants claimed that memorizing the necessary rules, definitions and shortcuts were more effective for teaching students because they would not be confused by the rationale of the rules. Then, they directed students to memorize these rules in rational numbers. None of the teachers showed the model of division of rational numbers and almost all of them taught divisions of rational numbers by memorizing it as a rule, reverse second rational numbers and multiply them. When students asked the reason of the reversing in division, teachers said that it was a rule and it was accepted in this way.

It seemed that the interaction between *beliefs about mathematics teaching* and *Enacting Mathematics for Teaching and Learning (interactive)* might be reciprocal. To illustrate, a limited set of data in this study suggested that when teachers saw a new example or a strategy, and believed that it could be beneficial for students, they may tend to use it in the classroom and possibly include it in their further practices. This was more evident in T1's beliefs and teaching. When T1 responded the vignette questions, she liked question 2 (different pizzas) and question 4 (number lines). Then, she said that she would use these examples in her lesson and used them both in the 6^{th} grade and 7^{th} grade classes in the observed lessons. Similarly, she realized during her graduate studies that she learned most mathematics content with memorizing and that discussion would be an effective method in the classroom instead of making students memorize. Then, she employed discussion method in her teaching more often as observed during the study. It is also possible that she realized more that discussion was a better method to teach mathematics concepts as she employed it more in her teaching, addressing a possible reciprocal relationship. Another example for the possible interrelationship was the differences in T2 and T6's beliefs and teaching in similar classrooms. T6 did not apply activities or worksheets because he believed that it was difficult to control the crowded class. However, T2 conducted activities and used worksheets in an equally crowded class with similar student level and two students who had individualized education program. Their beliefs seemed to affect the ways they taught mathematics although they were teaching in similar classroom contexts. T6 did not have effective classroom management in the observed lessons. Therefore, he might believe that employing any different form of teaching would result in more problems in the classroom and did not attempt to conduct activities. On the other hand, T6 might have seen the effectiveness of her implementations and come to believe that these were more effective for students' learning.

In the present study, participants believed that problems were related to daily life and that their students felt that problems were more complex than other exercises. Because of that, they changed the difficulty level of problems based on their students' understanding. For instance, T1's problems in the 6^{th} grade class were easier (with respect to the average 6th grade problems) than the 7^{th} grade class (with respect to the average 7th grade problems) because she believed that 7^{th} grade students' understanding were higher than 6^{th} grade ones.

CHAPTER 5

DISCUSSION

The purposes of the current study were threefold: 1) to explore the nature of middle school mathematics teachers' PCK regarding rational numbers, 2) to explore their mathematics-related beliefs regarding rational numbers, 3) to describe any interaction that may exist between middle school mathematics teachers' beliefs and their PCK for teaching rational numbers. The TEDS-M framework was used to analyze participants' responses which were presented the previous chapter.

The purpose of this chapter is to provide conclusions for the findings of this study and offer implications. In more detail, first, middle school mathematics teachers' PCK and mathematics-related beliefs regarding rational numbers, and the interaction between them are discussed. Although there are interpretations for the strenght or weight of their PCK below, it should be reminded that they are cautious speculations based on the data. Second, both the limitations of the current study and recommendations, suggestions and concluding remarks for the further research are presented.

5.1 Middle School Mathematics Teachers' PCK

The first research question which was addressed in the present study was about the nature of middle school mathematics teachers' PCK regarding rational numbers. Participant teachers' responses to the interview and vignettes questions, and teaching practices were examined. Knowledge of students' (mis)conceptions, instructional strategies, representations and curriculum are the core components of PCK (Depaepe, Verschaffel & Kelchtermans, 2013) and PCK has an important role on teachers' decisions about instruction such as selecting mathematical tasks, instructional methods, and types of assessments (Ball, 2000; Shulman, 1986). There were findings that indicated some of the teachers had more of a complete understanding than others about certain parts of rational numbers and teaching this concept. Knowledge of models or representations affect teaching mathematics (Tchoshanov, 2011). Almost all participants could draw a model for multiplication of rational numbers, but they did not show division model of rational numbers at the first glance. Only T1 wanted to show that model after the interviews and vignettes and she drew it after some struggle. Other teachers did not show the division models. Consequently, they did not teach division with a model but focused on 'invert and multiply' algorithm and did not explain why the algorithm works, as found in other studies with teachers and preservice teachers (Ball, 1990; Borko et al., 1992; Eisenhart et al., 1993; Ma, 1999; Singmuang, 2002; Tirosh, 2000). Similar results were also stated by Işıksal (2006) where she found that in order to symbolize and solve multiplication and division problems preservice mathematics teachers had sufficient content knowledge without conceptual understanding. Likewise, in the present study participants had sufficient procedural knowledge in terms of rational numbers contents, especially division and multiplication. In general, even though the participants had deficiency about how rational numbers were represented differently, they had sufficient knowledge to teach this concept with direct instruction and rules. Thus, most of them used direct instruction, verbal explanations, and rules. It means that participants might have the components of *mathematical curricular knowledge* aspects except for *establishing appropriate learning goals* which was not observed in their lessons and interviews. The reason was most possibly that the middle school curriculum is developed by MONE and it defines the objectives and contents, which cannot be changed by teachers. Teachers can only decide how they teach the content. Hence, the learning goals are certain and they are not established by teachers who learn about these goals from the curriculum. Therefore, the data set in this study was not informative in terms of *establishing appropriate learning goal* aspect.

Findings indicated that teacher education programs should emphasize the conceptual knowledge on some of the rational number topics, such as division operations, and relationships between representations. Other studies also mentioned that topics in elementary or secondary curriculum should be included explicitly in teacher education programs (Even & Tirosh, 1995; Singmuang, 2002). For the case of inservice teachers, particular training programs should be initiated for opportunities to develop the deep knowledge about concepts and relationships. Colleagues could discuss, interpret and share their ideas and experiences in order to enhance the knowledge structures and eventually teaching.

The content was particular in the current study, the rational numbers. The reason for selecting this topic was to understand teachers' teaching and PCK in order to inform similar contents in mathematics. To give an example, in geometry, teachers could use mostly visual images, concrete materials, or technology because geometry is more suitable and almost requires using these materials and methodologies. In algebra, teachers may use direct instruction more and materials and visuals less. However, teaching rational numbers are suitable almost all different methodologies, where teachers can use manipulatives, visuals, technology, and a number of different methods such as group work, direct instruction, and discussion.

Although participants used concrete materials or real-life examples in the introduction process of the rational numbers, they continued mostly with direct instruction apart from T1, who used discussion method. They did not employ technology in order to enhance students' learning and to illustrate fractions or rational numbers effectively, either. In the literature, researchers stated that many elementary mathematics teachers taught in a traditional manner where the teacher showed and explained examples and students practiced the shown examples. The teacher or the textbook was the source for authority in traditional classes (Ball et al., 2001; Philipp, 2007; Putnam & Borko, 2000; Steele, 2001). Findings of this study were in line with the previous studies. Although there were attempts of using concrete materials, such attempts had the potential to cause misconceptions, such as T6's bread example in rational number or slice of mandarin, which did not have equal parts. When teachers considered the misconceptions of rational numbers, they thought that the reason could be deficient teaching, and mislearning or not memorizing the rules, but did not seem to consider their own teaching as a reason. These findings indicated some kind of weakness in participants' *enacting mahematics for teaching and learning (interactive)* aspect especially for *explaining or representing mathematical concepts* *or procedures* component. Similar findings were also stated in other studies. For instance, Işıksal (2006) identified that inadequate formal knowledge, limited conceptions on the notion of fractions, and overgeneralization of the properties of natural numbers were important sources for the mistakes.

Participants did not make a lesson plan before their lesson and they generally used the textbook and supplementary sources in their teaching. It might be the case that they did not plan because they hardly employed any other method for teaching and believed that they did not need to plan for teaching the content which mostly included rules. These findings were also indicators of some weakness in the *knowledge of planning for mathematics teaching and learning* aspect. They were not prepared for making alternative explanations of the concepts and the rules. Teachers gave a time to students for writing the board, and if someone could not understand an example or content, they mostly intended to explain it again without much change in the content of the explanation. After this explanation, they always asked to students whether anything was unclear or not understood. Yet, when there was a different solution for questions, teachers gave opportunity to the students to show it. On the other hand, if students gave wrong answers to the questions, most of the teachers explained the correct solution in the correct way. Similarly, most of the students showed or expressed their responses to teachers individually, and teachers gave feedback to their students as correct or false. Only some of the teachers tried to understand the reason or misconception of students' incorrect answers during whole class teaching or while giving feedback individually in the lesson. Furthermore, teachers followed the same path in problem solving process even tough they knew or taught problem solving steps. For instance, T6 taught problem solving steps, but he did not apply these steps in the solution of problems.

These findings showed that teachers' PCK did not seem to include strong components for identifying and responding to students' misconceptions. They neither planned for identifying misconceptions, nor for responding to them. In this sense, they did not seem to have or prioritize *predicting typical students' responses, including misconceptions* component as a type of *knowledge of planning for mathematics teaching and learning*.

Baumert et al. (2010) found a statistically significant relationship between student achievement and teachers' PCK for mathematics such that if teachers had stronger PCK, they could assess their students' understanding at a high level. In the current study, it was explored that participants knew the different question types, such as open-ended, multiple choice, ordering, short answer, true-false questions, and intended to apply them together in their assessment. In terms of mathematical approach to assessment, they employed the word problems the least and basic algorithm questions the most both in their lessons and in the exams. This showed that participants had knowledge about the components of *knowing different assessment* formats and *choosing assessment formats*.

There seemed to be some confusion of the rational numbers and fractions content in curriculum. In the vignettes, participants could not determine which word problem was included in the 6th grade fractions or 7th grade rational numbers curricula. However, they were able to manage this during teaching. They also connected rational numbers, fractions and previous topics in their teaching. In this regard, they seem to have sufficient knowledge about *selecting possible pathways and seeing connections within the curriculum* component of the framework.

The findings of the study showed that participants' PCK seemed to be dominated by the use of direct instruction and related actions most of the time. This practice led to others such as lack of planning and not considering students' misconceptions. Although representations of concepts were crucial in mathematics classrooms and understanding mathematical concepts (Akkuş-Çıkla, 2004; Ball, 1990; Kurt, 2006), teachers preferred to use direct instruction in the present study. Thus, the findings revealed that the reason for participants' limited usage about models could be their choice or their PCK. They did not generally plan problem solving more than it was suggested in the curriculum. However, where there were different solutions to the problems and the questions, they valued these different solutions and provided students with the opportunity to present them to the rest of the class.

5.2 Middle School Mathematics Teachers' Beliefs

The second research question focused on participants' beliefs and in previous chapters teachers' beliefs were explained both separately and together. Beliefs influence individuals' decisions and are they are one of the most important indicators of individuals' decisions (Goldin et al., 2009). Teachers may use their beliefs as a filter and a guide to make their decisions (Handal, 2003; Fives & Buehl, 2012). It was seen that most of the participating teachers believed that direct instruction was effective in teaching, or they believed that it was the best option for teaching in crowded classrooms and keeping up with curriculum. Teachers in this study also believed that effective teaching provided analytical thinking and adaptability to daily life. In line with this, they prioritized concretization and visualization because they believed that these could help students' understanding and increase their attention. Most of the participants believed that every student could learn mathematics and if they were willing to learn, then they could learn. Yet, for some of them not every student needed to learn mathematics very well. In order to understand whether students learned or not, they believed that different types of questions in exams should be used. Drageset (2010) asserted that teachers held some beliefs which were more strongly held than others and they were harder to change. In the current study, almost all participants stated that they did not change their teaching method when the class level was different, but they tried to change the level of questions. This might indicate that teachers' beliefs about the effectiveness of the direct instruction method was rather strong. On the other hand, part of teachers' beliefs was specific to rational numbers content. It could be speculated that beliefs about mathematics was not affected by the topic, but if the content would change, teachers' beliefs could change too. For instance, *"Beliefs about mathematics teaching"* and *"Beliefs about students and their learning"* aspects are likely to depend on the content. Findings from these aspects could vary by contents, such as geometry or statistics.

Another common point was participants' beliefs about the most important, fundamental topics in the middle school mathematics curriculum; whole numbers, equations, algebra, rational numbers and fractions because these contents were connected to future contents. Moreover, for skills they stated that problem solving and four operations skills were important.

5.3 The Interaction between Beliefs and PCK for Mathematics Teachers

Teachers' beliefs are important and have an impact on their knowledge (Ernest, 1989; Fennema & Franke, 1992). In present study, it was seen that there was evidence for the interaction between middle school mathematics teachers' beliefs and their PCK regarding rational numbers, contributing to the proposals for this interaction in the literature (Hofer & Pintrich, 1997). Findings also confirmed that beliefs and knowledge should be considered connected and as elements that strengthen each other (Drageset, 2010) when exploring them. Although our findings might suggest a one-way effect from beliefs to knowledge dimensions explored in the study except for *Mathematical Curricular Knowledge* component, the direction of the effect remains to be further investigated. National curriculum was determined by Ministry of National Education and it was implemented in all schools. Thus, participants did not have to think about the curriculum, but they thought about the key ideas of it. It means that teachers could learn about middle school curriculum in their undergraduate education or in their first years of the teaching profession. Therefore, *establishing appropriate learning goals* and *knowledge of mathematics curriculum* components did not vary by teachers. Besides, thanks to particular middle school mathematics curriculum, beliefs about mathematics teaching and students learning interact pre-active and interactive dimensions of MPCK framework. On the other hand, participants can determine importance of topics based on their beliefs and experiences. Thus, their beliefs seemed to affect the selection of fundamental contents in rational numbers curriculum.

Teachers who have more traditional beliefs tended to employ more traditional practices (Stipek, Givvin, Salmon, & MacGyvers, 2001). Their beliefs and their knowledge have a relationship in terms of teaching and structuring the content (Turner, Christensen, & Meyer, 2009; Walshaw, 2012). In the present study, there was involved in an interaction between *planning and enacting for mathematics teaching and learning* component and beliefs. Participants selected and employed teaching methods in line with what they believed. The findings of the study might indicate that beliefs could be affecting teachers' practices through guiding them for what knowledge they would use in planning their lessons and teaching those lessons. T1 used discussion method, and others used direct instruction method. Visualization and concretization were also added to some parts of their lesson. T6 had a geogebra training, but he did not apply it because he believed that it was not appropriate for his students. T2 believed that technology was not effective in crowded classrooms and she gave up using it. Bray (2011) expressed that teachers' beliefs had a considerable role in their intentions about conceptual understanding. In the present study, teachers believed that ineffective teaching was one of the reasons for students' misunderstandings or misconceptions. T2 and T6 believed that students' mind could be confused by proofs and they did not choose to explain these to the students. These findings illustrated that teachers' beliefs filtered what kind of knowledge they would teach and how they would teach it. They also contribute to the findings in the literature that teachers' beliefs influence their practice (Chai, 2010; Cross, 2009; Richardson, 1996; Stipek et al., 2001; Torff, 2005; Wilkins, 2008; Wilson & Cooney 2002).

Teachers' PCK also has an importance in shaping teachers' beliefs and their decisions in mathematics teaching (An, Kulm, Wu, Ma & Wang, 2002). In the current study, there were evidences of this interaction between participant teachers' beliefs about mathematics teaching and interactive dimension of MPCK framework. The findings of the present study were particular to the rational numbers content. The interaction between PCK and mathematics related beliefs may change based on different topics. Yet, the findings have the potential to inform other content. Handal (2003) also expressed that teachers' mathematical beliefs were mostly a result of their learning experiences and reproduced in their classroom teaching. When T1 saw the vignette questions, she evaluated them in terms of their potential for students' learning (using her PCK) and used them in the 6^{th} and 7^{th} grade classrooms. Moreover, when she learned about discussion method in a graduate course, she changed her teaching to include the discussion method, she saw the positive results, and then she employed discussion method in her further lessons. This illustrated that even when teachers have short-duration experiences with a specific teaching idea, they may adopt this idea when they question their beliefs and practices, and develop new beliefs in line with the new experiences.

5.4 Limitations, Implications and Recommendations

The current study focused on the possible interaction between the middle school mathematics teachers' beliefs and their PCK regarding rational numbers. In line with this aim, six middle school mathematics teachers' beliefs and their PCK were explored via interviews and observations process. This study provided insight on how middle school mathematics teachers' beliefs interacted their PCK. However, there are a number of limitations.

The current study is a qualitative research study and it did not aim to generalize the findings. All participant teachers worked on public schools. It could be beneficial to see the findings when the participant teachers are chosen from private schools where the class sizes are low and technological facilities are generally better. Besides, six teachers' beliefs and PCK was explored in the current study. Even if findings showed an interaction between beliefs and PCK, it might be beneficial to examine more teachers to have a better understanding of that interaction. For instance, Simmons et al. (1999) mentioned that the first years of teaching are important for beginning teachers' beliefs because novice teachers can adapt classroom experiences and culture of their school in the first years, and their beliefs are shaped in these years. Therefore, studying with novice teachers could show different perspectives about that interaction. Participants self reported data were taken in the present study, especially in two interviews, with the assumption that they expressed their honest ideas. Interviews could not be conducted by the teachers right after the observed lessons to clarify their teaching. These were the limitations of this study which should be taken into consideration in further studies.

In this study, rational numbers concept was explored to understand the interaction between participant teachers' beliefs and PCK. As mentioned above, findings of mathematics related beliefs and PCK were based on rational numbers content. Different content may result in different findings. Therefore, other topics such as whole numbers, algebra, and geometry might be useful in order to have a better understanding of that interaction. Moreover, if other subject areas are explored as well, it can be possible to compare different subjects in terms of the interaction between beliefs and PCK. On the other hand, the future research can focus on other aspects, such as administrators, environmental factors, or longitudinal studies from the participants' undergraduate education until their first few years, to provide a comprehensive picture of their beliefs, PCK and the interaction between the two.

Mathematics teacher educators can take into consideration the interaction between the preservice teachers' beliefs and their knowledge to improve the teacher training programs. They can design their methods courses to address both beliefs and knowledge in order to support each other to lead to effective mathematics teaching and learning. These courses can focus more on how to use representations, models, and different instructional methods and tools in crowded classrooms and how teachers' knowledge (and beliefs) can provide better learning opportunities for students. Similarly, when in-service mathematics teacher training programs can be developed, their beliefs and PCK should be considered together. As it was seen in the findings, changing teachers' practice is related to understanding and changing teachers' beliefs as well as improving their PCK in relation to their beliefs.

I believe that the entire study process contributed me as an early career researcher and to other mathematics researchers. Before starting the present study, I realized that there is no simple answer about how teachers' beliefs interacted with their PCK. This connection seems to be complicated because understanding and interpreting teachers' beliefs is difficult and indicators of PCK are not easy to identify. Furthermore, teachers' beliefs and PCK can change based on the context or topic. All mathematics teachers also have different life histories and have different professional needs to develop beliefs and PCK. Thus, the present study has only took a snapshot of a specific part of their beliefs and PCK.

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APPENDICES

A. FIRST INTERVIEW QUESTIONS

What is your name?
How many years have you been teaching?
How many years have you been teaching in the current school?
Which department have you graduated from?
Do you describe in your classroom environment? (How many students?, What are their level?, school neighborhood?, what are their family characteristics? Etc.)
Which grade do you teach predominantly in your teaching profession?

1. What does it mean to know mathematics?
2. Which knowledge and skill should have effective mathematics teacher?
    - (If say mathematics knowledge, pedagogical knowledge) How should be? Could you describe that?
    - What are their roles in effective mathematics teaching?
3. What do students gain thanks to the effective mathematics teaching?
4. What is your purpose when you are teaching?
    - Do you think that you reach your purposes? / or how much do you reach your purposes?
    - (If participant reach some part) What is the reason for not reaching all part?
5. Which teaching methods are efficient in mathematics teaching? Why? Do you use this method?
    - Are there any teaching methods that you used, but gave up using it? Why?

6. What do you think about the strengths and weaknesses of the mathematics curriculum? If you are doing the mathematics curriculum, is there anything you add or remove to the current curriculum?

7. What are the factors which influence your mathematics teaching? Is there anything you want to do in mathematics teaching, but you cannot? If yes, what are these things?

8. How do you feel when you are teaching, learning and engaging in mathematics?

9. Are there any pressures or supports that affect your teaching? (such as students, parents, school administrators)

10. When middle school curriculum is considered, is there any concept or topic that is the most important, fundamental or central in the curriculum?

- Why are these important, fundamental or central? (If students are mentioned; What will students gain after learning these concepts or topics?)

11. When middle school curriculum is considered, is there any skill that is the most important, fundamental or central in the curriculum?

- Why are these important, fundamental or central?

12. Can students learn mathematics without teacher? (If yes,) How?

13. What does a "mathematical problem" imply for you?

14. What is your understanding about mathematical procedure?

15. What is the importance of problem solving in mathematics? What is gained to students?

16. How do you know that one of your students has understood a problem?

17. Do you have problem solving activities? (If yes)

- How do you follow a route in these activities in class?
- What are you doing when different solutions are occurring more than your solutions?

18. How much time do you reserve the problem solving activities?

19. Last thing, what would you want me to ask in this interview, that is important to you, but I did not ask? (If any, why is this question important to you?)

B. SECOND INTERVIEW QUESTIONS

- What are you using for a source in lessons? (Supplementary source?)
- Do you make a lesson plan before your lessons?
  - (If yes,) Do you use additional resources? (books, printed paper, etc)
    (If yes,) What are those resources?
- Which topic do students understand better? Why do they understand this topic better?
- Which topic do students have difficulty to understand?
  - Why do they have difficulty?
  - What are doing about this situation?
- When a topic that is understood and a topic that is not understood are compared, which specifications are appeared?
- How do you understand that students learned a concept or a topic?

- How do you teach fractions? /) How?
  - (If demonstration) Which demonstration do you use?
  - (If using material) Which material do you use?
    - Why do you choose this material?
    - How do you use it?
  - (If link to other subjects) Which subject/s do you link for teaching?
  - (If link to real life) How do you link it?
  - When you teach it, your students do not understand. What are you doing?
- Do your students have difficulty in understanding any case/concept about fractions? (If yes,) What is that?
  - What can be resource of these difficulties?
  - What are you doing to overcome these difficulties? (How?)

- Which questions do you ask your students during the lessons in order to determine whether they understood fractions or not? Why?
    - Which questions do you ask in exam?
- Which measurement and assessment instruments/methods do you use to assess students' knowledge about fractions? Why do you choose this instrument/method?
- How do you teach rational numbers?
    - (If say same as fractions) What are doing as distinct from fractions' teaching?
    - (If using material) Which material do you use?
        - Why do you choose this material?
        - How do you use it?
    - (If link to other subjects) Which subject/s do you link for teaching?
    - (If link to real life) How do you link it?
    - When you teach it, your students do not understand. What are you doing?
- Do your students have difficulty in understanding any case/concept about rational numbers? (If yes,) What is that?
    - What can be resource of these difficulties?
    - What are you doing to overcome these difficulties? (How?)
- Which questions do you ask your students during the lessons in order to determine whether they understood rational numbers or not? Why?
    - Which questions do you ask in exam?
- Which measurement and assessment instruments/methods do you use to assess students' knowledge about rational numbers? Why do you choose this instrument/method?

- What do you think about your ideal class? Can you define it?
- Do you think that you are teaching ideal class?
- (If it is not ideal class) Let's suppose that you have an ideal class, How do you teach fractions and rational numbers? Same as now or different? (Ask other questions)

- (If have not mentioned about material) You have not mentioned about material. What do you think for using material?

C. VIGNETTE QUESTIONS

1. A teacher was writing operations of 5 x $\frac{3}{10}$ ve 5 ÷ $\frac{3}{10}$ on the blackboard during the teaching operations of fractions, and found the results respectively smaller than five and bigger than five. A number of students warned the teacher that s/he confused the results and wrote wrong place.

a) What is the students' misconceptions that caused this situation?

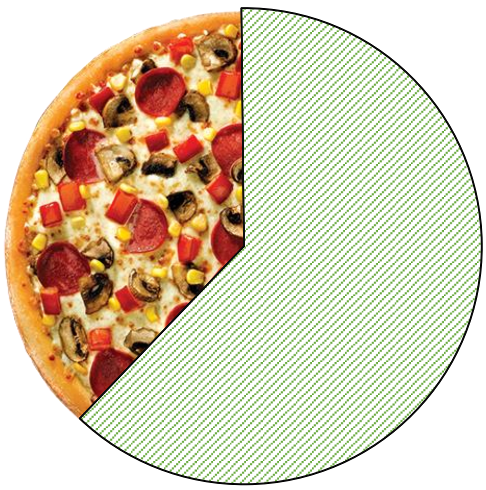
b) What is the possible sources of this misconception?

c) If you encounter with the same situation, what will you do to fix their misconceptions? If you have one lesson, how route will you follow?

d) Can you draw a visual model of 5 x $\frac{3}{10}$ and 5 ÷ $\frac{3}{10}$ operations to help student understanding?

2.

A pizzası



B pizzası

A teacher asks to students: “You are hungry and want to eat pizza. Do you choose a half of the pizza A or a third of the pizza B? Which pizza do you select if you want to eat more pizza?

A part of students selected pizza A and other part of students selected pizza B.

a) Which reasoning did students make when they selected pizza A?
b) Which reasoning did students make when they selected pizza B?
c) In such a question, after taking responses and reasons from students, how route will you follow?

3. A grade 6 mathematics teacher asked the following four story problems;

Problem 1: Merve, Ahmet and Tuğçe had started to read Atatürk’s book, name is Nutuk. In 3 weeks, Merve read $\frac{2}{3}$ of the book, Ahmet read $\frac{4}{5}$ of the book and Tuğçe read $\frac{5}{6}$ of the book. Who did read the most of the book?

Problem 2: A runner can run a specific distance in $\frac{21}{2}$ seconds. How many does s/he run this distance in 12 seconds?

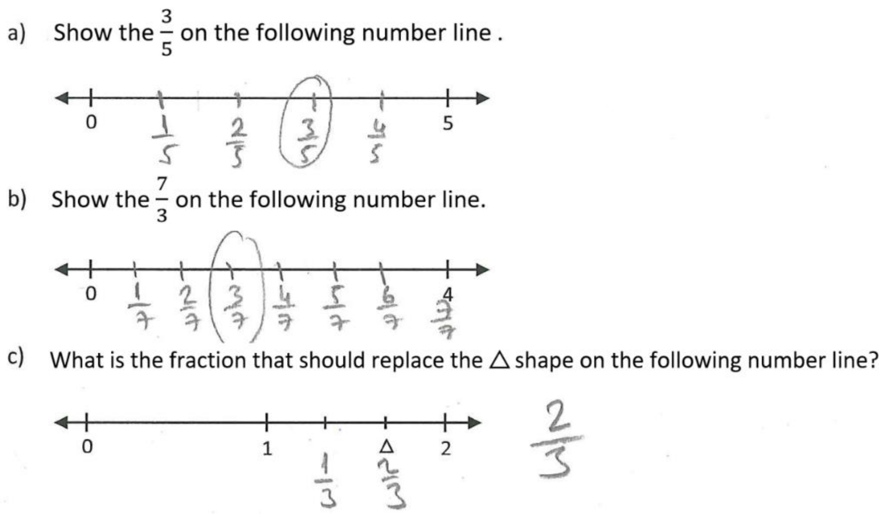
Problem 3: A swimmer jumped into the pool from a height of $3\frac{2}{5}$ meters and dived $\frac{5}{3}$ meters deep. How many meters is the distance between the point where the swimmer jumped and the deepest point s/he reached?

Problem 4: In physical education lesson, $\frac{1}{6}$ of the students in the class play basketball, $\frac{3}{5}$ of them play football, and the rest play volleyball. According to this, how many students are play football more than play basketball?

After the teacher asked questions, s/he noticed that two of the problems were more difficult for students than the other two.

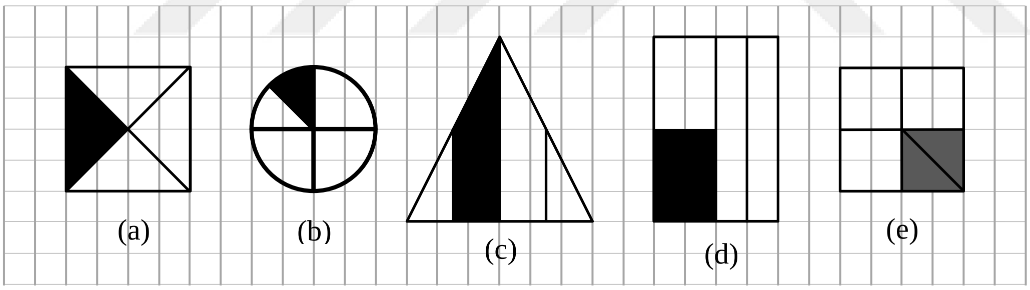
a) What are the two problems that the teacher describes as difficult?
b) Explain the reasons why these 2 problems are more difficult than others.

4. The answers given by a 6th grade student to the 3 questions in the exam are shown below.



- **a)** In your opinion, what could be the reasons why this student answered the questions incorrectly?
- **b)** If you encounter this situation, what do you do to correct the student's mistake? If you have one lesson, how route will you follow?

5.



The teacher who drew the above figures on the board asked which ones could be a model for $\frac{1}{4}$. When the answers were different from the students, the teacher grouped these answers and these results emerged:

Students in the first group selected (a), (d) and (e) models,

in the second group selected (a) and (e) models,

in the third group selected (a), (b), (c) and (d) models.

a) In your opinion, what kind of logic did these students make when deciding on models?
b) What could be the source of the mistakes of the students who made the wrong choice?
c) What do you do to correct the mistake of students who make a wrong choice?

6. While a teacher was teaching the subject of multiplication in fractions, he said "We multiply the numerator of the first fraction by the numerator of the second fraction and multiply the denominator of the first fraction by the denominator of the second fraction", and wrote $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ on the blackboard. One of the students wanted to speak and asked "Why don't we do it in addition and subtraction? We equalize the denominator. Should not we express them as $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$ and $\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$"

If you encounter this situation, what is your explanation to the students? How route will you follow when you making this explanation?

7. While the teacher explained the division in fractions, one of the students claimed that the division can be done like multiplication in fractions and gave the following example:

$$\frac{5}{12} \div \frac{1}{3} = \frac{5 \div 1}{12 \div 3} = \frac{5}{4}$$

If you encounter this situation, what is your explanation to the student? How route will you follow when you making this explanation?

8. During the lesson, one of the students claimed that distributive property can also be used in division of fractios and gave the following example:

$$1\frac{5}{6} \div \frac{1}{2} = \left(1 + \frac{5}{6}\right) \div \frac{1}{2}$$
$$= \left(1 \div \frac{1}{2}\right) + \left(\frac{5}{6} \div \frac{1}{2}\right)$$
$$= 2 + 1\frac{2}{3}$$
$$= 3\frac{2}{3}$$

If you encounter this situation, would you accept the student's explanation? Explain the reasons. If you do not agree, what is your explanation to the student? How route will you follow when you making this explanation?

C. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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Sayı: 28620816 / 445

06 Eylül 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Doç. Dr. Çiğdem HASER ;

Danışmanlığını yaptığınız doktora öğrencisi Aykut BULUT'un ***"Ortaokul Matematik Öğretmenlerinin Rasyonel Sayılar Konusundaki İnançları ile Pedagojik Alan Bilgileri Arasındaki İlişkileri"*** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-157** protokol numarası ile **18.09.2017 – 30.04.2018** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Yrd. Doç. Dr. Pınar KAYGAN
Üye

Yrd. Doç. Dr. Emre SELÇUK
Üye

D. PERMISSION FROM HEAD OF NATIONAL EDUCATION DEPARTMENT

ÖİDB

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481-605.99-E.18031050 30.10.2017
Konu : Araştırma İzni

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) 10/10/2017 Tarihli ve 54850036-300-4797 sayılı yazınız.

Matematik ve Fen Bilimleri Anabilim Dalı doktora öğrencisi Aykut BULUT'un **"Ortaokul Matematik Öğretmenlerinin Rasyonel Sayılar Konusundaki İnançları ile Pedagojik Alan Bilgileri Arasındaki İlişkileri"** kapsamında uygulama talebi Müdürlüğümüzce uygun görülmüş ve uygulamanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Görüşme formunun (7 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde bir örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme (1) Şubesine gönderilmesini rica ederim.

Vefa BARDAKCI
Vali a.
Milli Eğitim Müdürü

02-11-2017-17119

Güvenli Elektronik İmzalı
Aslı İle Aynıdır.
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Elektronik Ağ:
e-posta:

Bilgi için:
Tel:
Faks:

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E. CURRICULUM VITAE

Personal Information

Surname, Name: Bulut, Aykut

Nationality: Turkish

Date and Place of Birth: 28 September 1987, İstanbul

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aykut.bulut@ahievran.edu.tr

Educational Background

Degree	Year of Graduation	Institution
M.S.	2012	Middle East Technical University, Elementary Science and Mathematics Education, Ankara
B.S.	2009	Gazi University, Elementary Mathematics Education, Ankara
High School	2005	Pursaklar High School, Ankara

Work Experience

Year	Enrollment	Place
2018 - Present	Research Assistant	Kırşehir Ahi Evran University, Department of Elementary Mathematics Education
2010 – 2018	Research Assistant	Middle East Technical University, Department of Elementary Education

Areas of Research Interest

Teacher knowledge, Pedogogical Content Knowledge (PCK), Technology in Mathematics Education, Preservice Mathematics Teachers Education, Computer in Mathematics.

Awards and Scholarships

2010 – 2012 TÜBİTAK 2210 – The Domestic Scholarship Program for Master's Degree

2012 – 2017 TÜBİTAK 2211 – The Domestic Scholarship Program for PhD.

Master Thesis

Bulut, A. (2012). *Investigating perceptions of preservice elementary mathematics teachers on their Technological Pedagogical Content Knowledge (TPACK) regarding geometry*. Unpublished master's thesis, Middle East Technical University, Ankara.

Academic Publishing

Articles

Bulut, M, Bulut, N, **Bulut, A.** (2013). Öğretmen eğitiminde değer eğitimi fırsatı olarak topluma hizmet uygulamaları dersi/ service learning course as an opportunity for value education in teacher education. *Mustafa Kemal Üniversitesi Sosyal Bilimler Enstitüsü Dergisi, 9*(17), 347-357. Retrieved from https://dergipark.org.tr/tr/pub/mkusbed/issue/19553/208302.

Bulut, A. & Işıksal, M. (2019). Perceptions of pre-service elementary mathematics teachers on their technological pedagogical content knowledge (TPACK) regarding geometry. *Journal of Computers in Mathematics and Science Teaching, 38*(2), 153-176. Waynesville, NC USA: Association for the Advancement of Computing in Education (AACE). Retrieved October 6, 2019 from https://www.learntechlib.org/primary/p/173761/.

Presentations

Bulut, M., Bulut, N. & **Bulut, A.** (2011). *Öğretmen eğitiminde değer eğitimi firsati olarak topluma hizmet uygulamalari dersi*. Paper presented at the Değerler Eğitimi Sempozyumu. Eskişehir, Ankara, TURKEY.

Bulut, A. & Işıksal, M. (2012). *Developing technological pedagogical content knowledge (TPACK) scale regarding geometry*. Paper presented at the European Conference on Educational Research (ECER) 2012. Cadiz, Spain.

Bulut, A. (2013). *"Fatih" project in turkish national education system*. Paper Presented at the European Conference on Educational Research (ECER) 2013. Istanbul, Turkey.

Arslan, O & **Bulut, A.** (2015). Turkish prospective middle grades mathematics teachers' teaching efficacy beliefs and sources of these beliefs. In K. Krainer & N. Vondrova (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1116-1123). Charles University, Prague-Czech Republic.

Bulut, A. & Arslan, O. (2016). *Öğretim teknolojileri ve materyal geliştirme dersinin öğretmen adayları üzerindeki etkileri*. In T. Özsevgeç, N. Sönmez, Z. Özer, S. Toros, M. Doğan, D. Taşkın, O. Güven, & A. Kılınç (Eds.), Proceedings of the 12. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (UFBMEK) (pp. 48), Trabzon, Turkey.

Bulut, A. & Haser, Ç. (2018). *Ortaokul matematik öğretmenlerinin rasyonel sayılar konusundaki inanışları ile pedagojik alan bilgileri arasındaki ilişkiler*. Paper Presented at the 13. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (UFBMEK), October, 2018, Denizli, Turkey.

Bulut, A. & Haser, Ç. (2021). *Ortaokul matematik öğretmenlerinin kesirlerde çarpma ve bölme işlemlerinin öğretimi*. In Erduran Avcı, D., Korur, F., Genç, H., Ural, A., Taşlıdere, E., … & Bademli, N. (Eds.), Proceedings of the 14. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (UFBMEK) (pp. 241), Burdur, Turkey.

F. TURKISH SUMMARY / TÜRKÇE ÖZET

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN RASYONEL SAYILAR VE KESİRLER KONUSUNDAKİ İNANIŞLARI İLE PEDAGOJİK ALAN BİLGİLERİ ARASINDAKİ ETKİLEŞİMİ

1. GİRİŞ

Matematik öğretimi karmaşık bir eylemdir ve farklı bilgi türlerine sahip iyi yetişmiş, nitelikli öğretmenlere ihtiyaç duyar (The National Council of Teachers of Mathematics [NCTM], 2000). Nitelikli öğretmenler sadece konuyu değil, aynı zamanda öğrencilerin daha etkili öğrenmelerine yardımcı olmak için derslerini nasıl organize edeceklerini ve öğreteceklerini de bilirler (Berry, 2002). Öğretmenlerin konuyu öğretimlerinde anlama ve kullanma yetenekleri, öğrencilerin başarısında da önemli bir konudur (Ball, 1990a; Ma, 1999; Shulman, 1987). Ayrıca öğrencilerinin nasıl ve neden öğrendiklerini de bilmelidirler. Bu nedenle, öğretmenlerin bilgisi, öğrencilerin öğrenmesini oluşturmak ve geliştirmek için önemlidir (NCTM, 2000).

Matematik öğretmenlerinin inanışlarının öğretimleri üzerinde güçlü bir etkisi vardır (Ernest, 1989) ve bunlar "öğretmenlerin ne yaptıklarının ve neden yaptıklarının kritik öneme sahip belirleyicileridir" (Schoenfeld, 1998, p. 2). Bu nedenle, öğretmenlerin bilgi ve inanışları, kararlarını, sınıfta öğretilen ve öğrenilenleri etkiler. Bütün bu bilgiler ışığında şu sorular akla geliyor: Öğretmenlerin inanışları, pedagojik alan bilgilerini (PAB) gelişirken nasıl etkiler?, Diğer bir deyişle, öğretmenlerin bilgileri ve inanışları öğretim sırasında birbirlerini nasıl etkiler?

Araştırmacıların önemli bir kısmı öğretmenlerin bilgi türleri üzerinde çalışmalarına ve birçok çalışmada öğretmenlerin inanışlarının, düşünce ve davranışlarını nasıl etkilediklerini araştırmalarına rağmen bu iki yapı arasındaki ilişki pek incelenmemiştir. Öğretmen adaylarıyla yapılan iki çalışma bilgi ve inanış arasında ilişkinin öğretmen davranışlarını anlamada önemli bir faktör olduğunu ortaya koymuştur (Blömeke, Buchholtz, Suhl, & Kaiser, 2014; Charalambous, 2015). Ancak inanış ve bilginin özellikle öğretmenlerde nasıl etkileşime girdiği detaylı bir şekilde ortaya tam olarak koyulmamıştır. Dolayısıyla bu çalışmanın amacı ortaokul matematik öğretmenlerinin PAB'lerini, matematikle ilgili inanışlarını ve bu iki yapı arasında olabilecek etkileşimi araştırmaktır.

Alanyazın incelendiğinde PAB öğretmenlerin sahip olması gereken bilgi olarak öncelikle Shulman'ın (1986) çalışmasında ve daha sonra diğer araştırmacıların çalışmalarında yer edinmiştir. Shulman (1987) PAB'yi öğretmenlerin ne öğreteceklerinin bilgisi ile öğretim ile ilgili bilgilerinin karışımıyla ortaya çıkan öğretmen bilgisi olarak ifade etmiştir. Shulmandan sonra birçok araştırmacı hem öğretmenlerin sahip olması gereken genel bilgi, hemde özel alanlarda (matematik, fizik, ingilizce) öğretmenlerin sahip olması gereken bilgiler üzerine çalışmalar yapmışlardır. Matematik alanı açısından adından en çok söz edilen ve kendisinden sonra en çok kullanılan çalışmalardan birisi de Ball ve arkadaşlarının 2008 yılında yaptığı çalışmadır. Bu çalışmaya kadar birçok araştırmacı PAB'yi genel açıdan incelerken belirli bir alan üzerinde yoğunlaşmış çalışmalar az sayıdaydı. Ancak Ball ve arkadaşları (2008) PAB'yi incelerken matematik öğretmenlerinin matematik bilgileri açısından incelemişlerdir ve iki alt boyutta olduğunu belirtmişlerdir; Konu Bilgisi (SMK) ve Pedagojik Alan Bilgisi (PAB). PAB ise Ball ve arkadaşlarının modelinde (2008) üç boyuta ayrılmıştır; (a) İçerik ve öğrenciler bilgisi (KCS), (b) İçerik ve Öğretim Bilgisi (KCT), and (c) İçerik ve Müfredat Bilgisi (KCC). Bu modelle birlikte Ball ve arkadaşları (2008) öğretim için matematik bilgisinin derinlemesine bir analizini sağlamışlardır ve matematik eğitimi için yeni bir soluk getirmişlerdir. Birçok araştırmacı bu modeli başlangıç noktaları olarak kabul edip, matematik öğretmenlerinin ya da öğretmenlerin matematik eğitimlerinin gelişimini araştırmışlar ve modeli kullanmışlar.

Ancak yapılan çalışmaların çoğunda, Ball ve arkadaşlarının (2008) yaptığı çalışmada ve sonrası da dahil olmak üzere, öğretmenlerin inanışları göz önüne alınmamıştır. Dolayısıyla öğretmenlerin öğretirken kullandıkları bilgi ile inanışlar arasındaki etkileşimi de çalışmalarda pek yer almamıştır. Diğer taraftan öğretim için matematik bilgisinin nasıl kategorize edileceği konusunda da alanyazında tartışmalı bir durum mevcuttur (Even ve Loewenberg Ball, 2009). Bu durumu değerlendiren farklı ülkelerden araştırmacılar uluslararası araştırmalarla öğretmen eğitimi ve matematik öğretimini incelemeye yönelmiştir. Bu çalışmalardan bir tanesi de Matematikte Öğretmen Eğitimi Çalışması (TEDS-M) topluluğunun 17 ülkede yapmış oldukları çalışmadır. TEDS-M çalışmasınna baktığımızda; matematik öğretmen yetiştirme programları çıktıları ile öğretmen yetiştirme politikaları ve, kurumsal uygulamaları arasındaki ilişkileri incelemek, ülkelerin öğretmen yetiştirme programlarını incelemek ve karşılaştırmak şeklinde çeşitli amaçları mevcuttur. Mülakatlar ve anketler kullanılarak yaklaşık 15000 ilkokul ve 9000 ortaokul öğretmen adayına ve 5000'e yakın öğretmen eğitmenine ulaşılmış ve katılımcıların matematik bilgileri, matematik pedagojik alan bilgileri ve, matematik öğretimi ve öğrenimi hakkındaki inanışları incelenmiştir (Tatto, 2013).

Araştırmacılar, TEDS-M çalışmasının araçlarını ve değerlendirme çerçevesini iki boyutta geliştirmişlerdir: matematik alan bilgisi (MCK) ve matematik pedagojik alan bilgisi (MPCK) (Tatto, 2013). MCK üç alandan oluşur; içerik (sayılar ve işlemler, geometri ve ölçme, cebir ve fonksiyonlar, veri ve olasılık), bilişsel alan (bilme, uygulama, akıl yürütme) ve müfredat düzeyi (acemi, orta, ileri düzey). MPCK çerçevesi de 3 temel kısımdan oluşur; "Matematik Müfredat Bilgisi", "Matematik Öğretimi ve Öğrenimi İçin Planlama Bilgisi (Preaktif)", "Öğretme ve Öğrenme İçin Matematik Kullanma (Etkileşimli)" (Tatto, 2013). MPCK yapısının içerisinde yer alan alt boyutlar öğretmenlerin davranışlarını tanımlar ve bilginin doğasını içerir. "Matematik Müfredat Bilgisi" boyutu öğretmenin müfredat, öğrenme amaçları, değerlendirme çeşitleri bilgileri, müfredattaki temel kavramları ve bunların bağlantıları hakkındaki bilgileri içerir. İkinci boyut olan "Matematik Öğretimi ve Öğrenimi İçin Planlama Bilgisi (Preaktif)" boyutunda ise öğretmenin matematik öğretme ve öğrenmesini planlama bilgisi, hazırlık çalışmaları ile ilgili bilgileri, uygun öğretim, ve değerlendirme yöntemi seçme bilgileri, öğrencinin verebilecekleri cevapları ve kavram yanılgılarını tahmin etme bilgileri yer alır. Son boyutu olan "Öğretme ve Öğrenme İçin Matematik Kullanma (Etkileşimli)" boyutunda ise öğretmenlerin ders sırasındaki davranışları bulunmaktadır. Örneğin, öğrencilerinin cevaplarını ve kavram yanılgılarını analiz etme, matematiksel kavramları açıklama, sorular oluşturma ve geri dönüt verme bu boyutta yer almaktadır (Tatto, 2013). TEDS-M çalışmasında kullanılan çerçeve, MPCK, geniş ve kapsamlı bir çalışmaya dayalı olarak daha spesifik ve geniş bir boyut ve alt boyut yelpazesi sunar (Tatto, 2013). MPCK'in boyutlarının ve alt boyutlarının tanımları ve içerikleri bu çalışmanın amaçlarıyla örtüşmektedir. Bu sebeple bu çalışmada özellikle öğretmenlerin PAB'isini ortaya koymak için MPCK çerçevesi kullanılmıştır.

TEDS-M, PAB ve inanışları birlikte incelemiş ve PAB'yi kavramsallaşmıştır. İnanışlar beş alanda incelenmiştir; matematiğin doğası, matematik öğrenimi, matematik başarısı, matematik öğretimi için hazırlık ve program etkinliği. İnanışları ölçmek için kullanılan ölçek MPCK çerçevesinin alt boyutlarıyla uyumlu olmasına rağmen geniş bir yelpazede matematik içeriğini incelemiştir (Tatto, 2013). Ancak bu çalışmada belirli bir içerik kullanıldığı için inanış ölçeği direk kullanılmamış, yarı yapılandırılmış mülakatlar hazırlanırken bir rehber olarak göz önüne alınmıştır.

PAB'deki A, yani alan bilgisi, için bu çalışma rasyonel sayılar ve kesirler konusuyla sınırlandırılmıştır ve karışıklık olmaması açısından hepsine rasyonel sayılar denilmiştir. 1970'lerden bu yana birçok araştırmacı rasyonel sayılar konusunu araştırmıştır. Rasyonel sayılardaki en temel tanımlama Kieren'ın (1976; 1980) yaptığı çalışmalar neticesinde ortaya konmuş ve birçok açıdan incelenmiştir. Rasyonel sayılar, bir sayı doğrusundaki tüm sayılardan oluşan gerçek sayılardır ve alan yazında yapılan çalışmalar incelendiğinde, rasyonel sayıları öğretme ve öğrenme konusundaki zorlukların başında rasyonel sayıların yapılarının karmaşık olması gelmektedir. Aynı zamanda kesirlerin öğretimi tam sayıların öğretiminden daha karmaşık ve zordur (Cramer, Post, ve delMas, 2002; Mack, 2001; Miller, 2005). Öğrenciler okul dışı ortamlarda tam sayıları daha çok kullanırken rasyonel sayıları kullanmak için günlük hayatta daha az fırsatları olmaktadır. Bu durum, öğrencilerin rasyonel sayıları içselleştirmelerini zorlaştırmaktadır. Dehaene'e (1997) göre de öğrencilerin beyinleri kesirler kavramına karşı bir direnç gösterme eğilimindedirler çünkü kesirler daha önce var olan doğal sayılar bilgilerine benzememektedir. Miller'a (2005) göre ise rasyonel sayıların yorumlarındaki yetersizlik eğitim ve öğretim sürecinin de yetersiz olmasına sebep olmaktadır. Yapılan araştırmalara göre öğretmenler sınıfa genellikle prosedür bilgisiyle geldikleri için ve bu durum da kavram yanılgılarına sebep olabileceği için (Ball, 1990; Ma, 1999; Tirosh, 2000) öğretmenler ve bilgileri rasyonel sayıların öğretimi sürecinde çok önemli bir aktör konumundadır. Kesirler ve rasyonel sayılar ortaokul müfredatının iki önemli ve temel kavramlarındandır. Örneğin oran, orantı ve denklik gibi kesirler ve ondalık sayılardaki konular cebir, geometri, istatistik ve birçok matematik konuları için çok önemli ve zaruridir (Dopico, 2017; Kieren, 1993). Bu bilgiler ışığında bu çalışmada öğretmenlerin inanışları ile PAB'leri rasyonel sayılar konusu açısından incelenmiştir.

Araştırma Soruları:

Bu çalışmada şu üç soruya cevap aranmaktadır:

1. Ortaokul matematik öğretmenlerinin rasyonel sayılar konusundaki PAB'lerinin doğası nedir?
2. Ortaokul matematik öğretmenlerinin rasyonel sayılar konusundaki matematikle ilişkili inanışlarının doğası nedir?
3. Ortaokul matematik öğretmenlerinin matematikle ilişkili inanışları ile rasyonel sayılar konusundaki PAB'leri arasındaki etkileşim nedir?

2. YÖNTEM

2.1. Araştırma Deseni

Bu çalışmada yer alan araştırma sorularına yanıt verebilmek için nitel araştırma yöntemi kullanılmıştır. Creswell'e (2009) göre nitel çalışma araştırmacı tarafından doğal ortamda ve katılımcılarla yüzyüze olacak şekilde yapılmaktadır. Aynı zamanda mülakat, gözlem, döküman gibi birçok veri kaynakları kullanılabilir ve katılımcıların davranışları ve görüşleri hakkında kapsamlı temalar belirlemek için tümevarımsal veri analizi yapılır (Creswell, 2009). Elde edilen verileri yorumlamak için de teorik bir bakış açısı veya teorilerin kombinasyonu kullanılabilir (Merriam, 1999). Bu çalışmada da MPCK çerçevesinde öğretmenlerin PAB'leri ve inanışları açıklanmaya çalışılmıştır. Araştırmacı aynı zamanda problem veya konu hakkında bütüncül bir resim oluşturmaya çalışır (Creswell, 2009). Bu çalışmada da araştırmacının odağı öğretmenlerin PAB'sinin ve matematiksel inanışlarının ne olduğunu ortaya koymaktır. Merriam'ın (2009) söylediği gibi çalışmada gözlem sırasında sürece müdahale etmeden öğretmenin öğretimini nasıl yaptığını anlamaya çalışmıştır. Diğer bir taraftan nitel araştırmalarda bir olgu ya da bir olay, durumla ilgili betimsel bir ürün ortaya koyulur. Bu çalışmada da araştırmacı yazıya dökülen görüşmeleri, belirli senaryo durumlarını, ve ders gözlem notlarını öğretmenlerin bilgi ve inanışlarını detaylı bir şekilde tanımlamak için kullanmışlardır.

Nitel araştırma deseninin yaklaşımlarından biri de durum çalışmasıdır. Yin'e (2003) göre "niçin" ve "nasıl" sorularına cevap verebilmek için durum çalışmaları kullanılır. Merriam'da (2009) belirli bir nedeni araştırmak ve daha iyi anlamak için durum çalışmasında bir fenomen, bir program veya bir kişi seçilmesi gerektiğini söylemiştir. Creswell'de (2007) durum çalışmasında araştırmacı farklı kaynaklardan detaylı ve derinlemesine detaylı veriler topladığı ve temalandırarak durum veya durumları rapor ettiğini belirtmiştir. Bu çalışmada araştırılan PAB ve inanışları detaylandırmak için durum çalışma seçilmiştir.

2.2. Katılımcılar

Bu çalışmada katılımcı olarak Ankara iline bağlı devlet okulunda çalışan altı ortaokul matematik öğretmeni ile çalışılmıştır. Araştırmacı bu öğretmenleri seçerken uygun örnekleme yöntemini kullanmıştır. Üç tane kriter göz önüne alınarak öğretmenler belirlenmeye çalışılmıştır. İlk olarak öğretmenlerin en az 5 yıllık deneyim sahibi olması göz önüne alınmıştır çünkü deneyimli öğretmenlerin inanış ve PAB'lerinin daha iyi yerleşmiş oldukları ve daha iyi ifade edecekleri düşünülmüştür. İkinci olarak öğretmenlerin uygunluğu önemlidir çünkü bu çalışma rasyonel sayılar veya kesirler ünitesi boyunca devam edeceği için uzun bir süreç gerektirmektedir. Aynı zamanda öğretmenlerin derslerinin olası üst üste veya peşpeşe gelmesi durumunda okulun fiziksel olarak yakın ve merkezde olması da gerekmektedir. Üçüncü olarak öğretmenlerin bu çalışmada yer almaya gönüllü olması da bir etken olmuştur. Bu şartları sağlayan 6 ortaokul öğretmeni çalışmaya katılmayı kabul etmiştir ve katılımcıların özellikleri Tablo 1 de görülebilir.

Tablo 1.

Çalışmadaki katılımcılar

Katılımcı No	Cinsiyet	Öğretmenlik Deneyimi	Sınıf	Eğitim Seviyesi	Okul Çevresi ve Ailenin Eğitim Seviyesi	Sınıftaki Öğrencilerin Seviyesi
T1	Kadın	11 yıl	6 - 7	Doktora	Orta - Düşük	Ortalama Altı - İyi
T2	Kadın	8 yıl	6	Yüksek Lisans	Orta	Ortalama
T3	Kadın	8 yıl	7	Lisans	Düşük	Düşük
T4	Kadın	10 yıl	7	Doktora	Orta - Yüksek	İyi
T5	Kadın	5 yıl	7	Yüksek Lisans	Çok Düşük	Çok Düşük
T6	Erkek	10 yıl	7	Lisans	Yüksek	İyi

Not: Okul Çevresi ve Ailenin Eğitim Seviyesi için herhangi bir sayısal veri bulunmamaktadır. Bu bölüm katılımcı öğretmenlerin tanımlamalarıyla ve verdikleri subjektif bilgiler doğrultusunda oluşturulmuştur.

Bu çalışmada ayrıca ana çalışmadan üç ay önce iki öğretmen ile pilot bir çalışma da yapılmıştır. Görüşme protokollerinin ve örnek olayların netliği ve kapsamlılığı da kontrol edilmiştir.

2.3. Veri Toplama Araçları ve Analiz Süreci

Nitel araştırmalarda veriler genellikle gözlemler, mülakatlar, dökümanlar ve sesli-görsel materyaller ile toplanır (Cresswell, 2007). PAB'nin ve inanışların karmaşık yapısından dolayı bu olguları ölçmek zordur (Baxter and Lederman, 1999) ve tek bir yoldan veri toplamak yetersiz veya eksik bilgi elde edilmesine sebep olabilir. Bu yüzden çalışmanın verileri üç farklı araçla toplanmıştır; (i) iki farklı zamanda yapılmış yarı yapılandırılmış görüşmeler, (ii) daha önceden oluşturulan örnek olaylar ve (iii) katılımcıların rasyonel sayılar ve/veya kesirler konusunu anlattıkları derslerin gözlemleri.

Eğitim öğretim dönemi başlamadan önce yarı yapılandırılmış görüşmenin birincisi yapılmıştır. Bir hafta sonra ikincisi yapılmıştır. Görüşmeler ya da diğer adıyla mülakatlar insanların aklında ne olduğuna dair derin bir anlayış sağlayabilir (Creswell, 2012). Yarı yapılandırılmış görüşme ise araştırmacıya esneklik sağlar ve katılımcının cevabını anlamak için açıklama yapmasını ve devam sorularını sorabilmesini sağlar. Bu sebeple bu çalışmada da görüşmeler, katılımcıların kendilerini rahat hissettiği bir ortamda birebir şekilde gerçekleştirilmiştir. Görüşmelerde yer alan bütün sorular alanyazında yer alan; Haser (2006), Işıksal (2006) ve TEDS-M (2008), çalışmaları göz önüne alınarak hazırlanmıştır. İlk görüşmede öğretmenlerin demografik bilgileri, deneyimleri, matematikle ve müfredatla ilgili inanışları, düşünceleri, görüşleri ve bunları etkileyen faktörleri öğrenmek için hazırlanan sorular sorulmuştur. İkinci görüşmede sorulan sorularla katılımcıların matematik ve rasyonel sayılar öğretimi ve bununla ilgili görüşleri, ve ideal sınıfla ilgili düşünceleri anlaşılmaya çalışılmıştır. Görüşmelerden bir hafta sonra da 8 adet örnek olayla verilen yanıtları içeren veriler toplanmıştır. Bu örnek olaylar hazırlanırken alanyazında yer alan çalışmalar, güncel müfredat ve ders kitabı göz önüne alınmış ve iki uzman tarafından geçerliliği kontrol edilmiştir. Ayrıca araştırmacı altı öğretmenin derslerini rasyonel sayılar ve/veya kesirler ünitesi boyunca gözlemiştir ve gözlem notları tutulmuştur. Gözlemlerin amacı, öğretmenlerin bilgilerini, onları nasıl kullandıklarını, görüşmelerde verdikleri bilgilerin tutarlı olup olmadığını, öğrencilerle etkileşimini ve sınıf ortamını araştırmaktır.

Veri analizinde ilk olarak katılımcı öğretmenler ile yapılan görüşmeler ve örnek olay sürecinde öğretmenlerin verdiği cevaplar yazıya dökülmüştür. Daha sonra bu veriler araştırmacı tarafından okunmuş ve öğretmenlerin derslerinden elde edilen gözlem notları ile birlikte MAXQDA programına aktarılmıştır. Amaç, bütüncül bir bakış açısını elde etmek ve birlikte analiz ederek yorumlamaktır. Verileri analiz ederken anlamlı olduğu düşünülen her cümle, kelime veya diyalog analiz edilmiş ve kodlanmıştır. Kodlama için özellikle PAB verilerinde TEDS-M çalışmasındaki MPCK modeli ve alt boyutlarını kullanılmış, inanış verilerini kodlarken ise hem MPCK hem de alanyazın doğrultusunda araştırmacı tarafından analiz edilmiştir. Eğer herhangi bir model veya alt boyutlarına uymayan veri varsa açık kodlama yöntemi ile kodlanmış ve kavramsal çerçeve doğrultusunda kod kitapçığında yer almıştır. Sonraki süreçte veriler ile birlikte kod kitapçığı da matematik eğitimi alanında doktora derecesine sahip bir uzmana verilerek tartışılmıştır. Araştırma ve uzman kullanılan kodların ve kavramsal çerçevenin uyumlu olduğu sonucuna varılmıştır. Kalan öğretmenlerin görüşmeleri hazırlanan kod kitapçığından faydalanarak analiz edilmiştir.

3. BULGULAR

Bu çalışmanın bulguları araştırma sorularına cevap verecek şekilde açıklanmıştır. Katılımcı öğretmenlerden elde edilen veriler bütüncül olarak ele alınmıştır ve PAB'leri MPCK çerçevesi içerisinde açıklanmıştır. İnançlarla ilgili kısımda ise her bir katılımcının inanışları incelendikten sonra ortak ve farklı olanlar belirlenip oluşturulan yapı verilmiştir.

Bulgular kısmında kullanılan kısaltmalarda ise "T" çalışmaya katılan öğretmen yanındaki sayı ise numarasını belirtmektedir. Örneğin T2, 2 numaralı öğretmeni belirtmektedir. "O" gözlem verisini, "I1" birinci görüşmeden elde edilen veriyi, "I2" ikinci görüşmeden elde edilen verileri, "VQ" ise örnek olay sorularını ifade eder. Örneğin, T4_I2, 4 numaralı öğretmenin 2. görüşmesini ifade ederken, T3_VQ5 ise 3 numaralı öğretmeninin 5. örnek olay sorusuna verdiği yanıtı ifade etmektedir.

3.1. Ortaokul Matematik Öğretmenlerinin Rasyonel Sayılar Konusundaki PAB'leri

3.1.1. Matematik Müfredat Bilgisi

MPCK çerçevesinin ilk alt boyutu olan müfredat bilgisi boyutunu oluşturan ilk madde olan *"uygun öğrenme hedeflerinin belirlenmesi"* maddesi ile ilgili veri katılımcı öğretmenlerden elde edilemedi. Diğer madde olan *"farklı değerlendirme formatlarını bilmek"* maddesinde bütün katılımcılardan hem görüşmelerde hem de gözlem verilerinde farklı değerlendirme formatlarını bildikleri ve kısmen uyguladıkları görüldü. *"Olası yolları seçme ve müfredat içindeki bağlantıları görme"* maddesinde ise öğretmenlerin rasyonel sayılar konusunu anlatırken bir önceki konuyla ve/veya eski bilgileriyle bağlantı kurdukları görüldü. Örneğin öğretmenler konuyu anlatırken tam sayılar konusuyla bağlantı kurup özellikle negatif kısmı için eski konuları hatırladılar.

"Öğrenme programlarında temel fikirlerin belirlenmesi" maddesinde ise katılımcı öğretmenler çoğunlukla matematik müfredatına odaklanıp onun üzerinden anahtar kavramları belirttiler. Örneğin, T1 dersinde *"…Her sene üstüne koyarak gidiyor. Müfredatımız sarmal yapıda o yüzden bu sene anlamazsanız seneye zorlanırsınız."* şeklinde bir bilgi verdi öğrencilere. Katılımcı öğretmenlerin rasyonel sayılar konusundaki *"Matematik müfredatı bilgisi"* maddesine baktığımızda ise öğretmenlerin neler öğreteceğinin farkında olduğu görülmektedir. Milli Eğitim Bakanlığı'nın hazırladığı müfredatı anlatan kitabın içinde kazanımlar, öğrenme çıktıları ve öğretime yardımcı olması için hazırlanan bazı örnekler yer aldığı için öğretmenlerin bu konuda bütün bilgilere erişimi kolay olmaktadır. Ayrıca öğretmenler görüşmelerde parça-bütün ilişkisi, toplamanın anlamı, tam-rasyonel sayılar arasındaki ilişki, negatif rasyonel sayılar ve üslü sayılar konularına vurgu yaptılar. Ancak elde edilen verilerde öğretmenlerin rasyonel sayılardaki işlemler konusunda yöntemsel olarak bilgilerinin olduğu ama kavramsal olarak eksik oldukları görülmektedir. Örneğin, öğretmenler rasyonel sayılardaki çarpma ve bölme işlerimini doğru bir şekilde yapmalarına rağmen bu işlemlerin neden bu şekilde olduğu konusunda eksikleri olduğu tespit edildi. Çarpmayı tekrarlı toplama olarak belirtip model kullanarak gösterebilirlerken bölmeyi model kullanarak gösteremediler ve ders sırasında açıklarken her zaman 2. sayıyı ters çevir-çarp kuralını kullanarak ve uygulayarak anlattılar. Sadece bir katılımcı öğretmen örnek olay kısmında görüşmeden sonra ayrı olarak bu konuda uğraşmış ve ilgili sorunun çözümünü daha sonra araştırmacıya ulaştırmıştır. Bunların yanında öğretmenler müfredatta yer alan kazanımlarla ilgili problemlerle karşılaştığında, problemlerin kesirler ünitesinde mi yoksa rasyonel sayılar ünitesinde mi olduğunu da kısmen doğru olarak belirleyebildiler.

3.1.2. Matematik Öğretimi ve Öğrenimi İçin Planlama Bilgisi (Preaktif)

Elde edilen verilere göre bütün katılımcı öğretmenler derslerinde hem ders kitaplarını hem de yardımcı kaynakları kullanıyorlardı. T3 özellikle en çok yardımcı kaynak kullanan öğretmendi. Sınıfta akıllı tahtaya öğrencilere aldırdığı akıllı defteri yansıtıyordu. T1 ise derslerinde morpakampüs, mathplayground gibi interaktif internet sitelerini kullanarak öğrencilerle etkinlik veya soru çözümü yapıyordu. Öğretmenler ayrıca günlük ders planı yapmadıklarını ifade ettiler. Sebep olarak ise kendilerinin deneyimli olduklarını ve gerek duymadıklarını belirttiler. Örneğin, T6,

> *belli süre sonra iş şeye biniyor yani otomatiğe biniyor yani. Hani ne anlatacağını biliyorsun az çok. İçeriğine bakıyorsun kitabın. Hani ilk seneler ben hazırlanıyordum da yani, derslere ama... artık belli bir süreden sonra ne anlatacağını biliyorsun çocuklara (T6_I1).*

Sadece T2 herhangi bir yazılı bir plan yazmadığını ama derse girmeden önce konuyu ve anlatacaklarını gözden geçirdiğini belirtti.

"Uygun etkinlikleri planlamak veya seçmek" ile *"Matematiksel fikirleri göstermek için uygun metotları planlama"* boyutlarına bakmak için katılımcıların rasyonel sayılar konusunu nasıl öğrettikleri görüşmelerde soruldu. Öğretmenlerin cevaplarına göre T1, T2 ve T3 öğretim yaparken somut materyalleri kullanmak isterken, T5 ve T6 günlük yaşam örnekleri (pasta, kağıt katlama modeli, ekmek gibi) üzerinden anlatmayı düşündüler. T1 somut material kullandıktan sonra sayı doğrusuna geçiş yaptığını belirtti. Çünkü T1'e göre *"Çocuklar sayı doğrusunu iyi anladıklarını düşünüyorum. Yani şey sayı doğrusunda gördükleri zaman... kesri oradan da bölme olduğunu aslında anlıyorlar. Yani kafalarında daha da somutlaştırıyorlar diyebilirim" (T1_I2).* T6 ise ekmek üzerinden örnek verdiğini belirtti; *"Ben genelde ekmeği çok kullanıyorum. Ekmekten işte çünkü günlük hayatta herkes parçalıyor, dilimliyor, bölüyor yani. Böldüğü bişey o yüzden ekmek"*. Ancak, T6 aslında ekmeğin kesir örneği vermek için tam uymadığını farketmedi. T6 daha sonra düz anlatımla devam ettiğini belirtti. T4'de benzer şekilde önce kesir kartlarıyla başladığını ve sonraki aşamalarda işe yaramadığı için daha sonra düz anlatımla devam ettiğini belirtti. T1 ve T5 de negatif kısma geçmek için sayı doğrusundan yararlandıklarını söylediler. T6 ve T3 ise negatif kısımda tam sayılarla rasyonel sayılar arasında

bağlantı kurup aynı tam sayılar gibi düşünmek şeklinde konuyu anlattıklarını belirttiler.

Diğer bir madde olan *"Kavram yanılgıları da dahil olmak üzere öğrencilerin klasik yanıtlarını tahmin etme"* ile ilgili veriler incelendiğinde T1, T3, ve T6 öğrencilerin pay ve paydanın anlamını kavramakta zorlandığını belirttiler. Özellikle toplama ve çıkarma yaparken payda eşitlemenin mantığını anlama, çok adımlı işlemler ve parça-bütün ilişkisi kısmında eksikleri olduğunu ve bunların sebebinin kavramın tam olarak anlaşılmaması olarak belirttiler. T4, T5 ve T6 da öğrencilerin kuralları karıştırdıkları ve ezberlemekte zorlandıkları için kavram yanılgılarının olduğunu söylediler. Olası öğrenci cevapları ve kavram yanılgılarının sorulduğu 1., 2. ve 4. örnek olay sorularında öğretmenler öğrencilerin olası cevaplarını ve kavram yanılgılarını doğru tahmin ettiler. Hatta T1, 4. örnek olay sorusunda sadece 0 ile 5 işaretlenerek verilen sayı doğrusunu çok ilginç bularak kendi dersinde kullanmak istediğini belirtti:

> *...Hiç hiç düşünmedim çok ilginç geldi bana. Aslında güzel bir şey yani kullanılabilir. Hani ölçmek için. Zorlaştırmak için kullanılabilir. Hiç kullanmadım ben böyle bir şey... Acaba ne yapacaklar merak ediyorum. Şimdi bunu deneyeceğim bunu şeyde... Sınıfta en azından bir deneyeceğim bakalım ne yapacaklar. (T1_VQ4)*

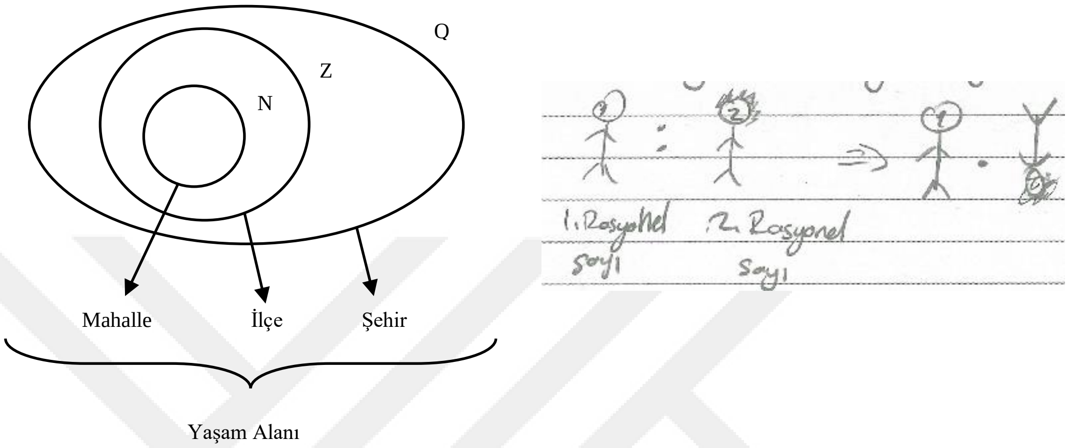
"Matematik problemlerini çözmek için farklı yaklaşımların belirlenmesi" maddesinde öğretmenler, eğer öğrenciler düzgün ve doğru bir şekilde farklı çözümler yaparlarsa çoğunlukla tahtada bu farklı çözümleri gösterdiklerini belirttiler. T3, eğer öğrencilerin farklı çözümlere şans yoluyla ulaştıklarını düşünürse yeni bir örnek soru üzerinde buldukları yöntemi uygulamalarını söylediğini belirtti. T4 farklı çözümleri sevdiğini ve teşvik ettiğini, T6 ise öğrenci seviyesini göz önüne alarak hareket ettiğini söyledi; *"...sınıf seviyesi hani anlatırsa çocukların kafasının karışacağı başka şeylere de neden olacağı için hiç anlatmıyorsun. Doğru diyorsun hani ona özel söylüyorsun." (T6_I2)*. Ancak 8. örnek olay sorusunda öğretmenlerin çoğu öğrencinin farklı yanıtını bir başka benzer örnekte de uygulayarak doğru sonuca ulaştığı için doğru kabul etti. Oysa ki T6 değişme özelliğinden dolayı kural olarak kabul edilmemesi gerektiğini belirtti; *"değişme özelliğinin tersten olmadığını... burada da mesela sadece mesela sağ taraftan oluyor. Sol taraftan, başa aldığımda olmadığını yani burada bura için uygulanır ama genel geçer bir kural olmadığını"*

(T6_VQ8). Yine de öğrencinin yanıtını doğru kabul edip her zaman olmayacağını göstereceğini söyledi. T1 ve T4 de öğrencinin çözümünü ilk başta doğru olarak kabul etmeyeceklerini ancak kendileri de farklı sayılarla uyguladıktan sonra aynı sonuca ulaştıklarını görünce öğrencilerin çözümlerini doğru olarak kabul edeceklerini belirttiler. Bu durum ise aslında öğretmenlerin bölme işlemindeki dağılma özelliği olup olmadığı hakkında bilgilerinin eksik olduğunu göstermektedir.

"Değerlendirme formatlarını seçme" boyutunda ise bütün katılımcı öğretmenler değerlendirmelerinde farklı soru türlerini bir arada kullanma eğiliminde olduklarını belirttiler. Sınavlarında çoğunlukla açık uçlu sorular kullandıklarını söyleseler de tek tip soru kullanmadıklarını, farklı tipte sorular (doğru-yanlış, boşluk doldurma, problemler, kısa cevaplar, eşleştirme, sıralama ve çoktan seçmeli gibi) kullandıklarını da söylediler. Bunun sebeplerini ise doğru yanıt alabilme oranını artırma, 1 saatlik zaman diliminde mümkün olduğunca çok soru sorabilme ve hepsini çoktan seçmeli sorunca oluşan şans ve tahmin olasığını önleme şeklinde belirttiler. Yine hazırladıkları sınavlar incelendiğinde çoğunlukla derste çözdükleri alıştırmalar gibi, temel ve karmaşık işlemler içeren sorular sordukları görüldü.

3.1.3. Öğretme ve Öğrenme İçin Matematik Kullanma (Etkileşimli)

Bu boyutta yer alan alt boyutlardan olan *"Matematiksel kavramları veya prosedürleri açıklamak veya tasvir etmek"* boyutu katılımcıların rasyonel sayıları nasıl öğrettiklerini içeren gözlem verileriyle açıklanmıştır. Örneğin T1 öğretiminde tartışma metotunu ve somutlaştırmayı kullanmaya çalışıyordu. 6. ve 7. sınıfa da girdiği için T1 her iki sınıfta da tartışma metotunu kullanmaya çalıştı ancak 7. sınıfta daha başarılı olduğunu söyledi. Çünkü 7. sınıftaki öğrencilerin seviyesinin, ilgilerinin ve hazırbulunuşluklarının 6. sınıftakilerden daha fazla olduğunu söyledi. Gözlem sonucunda da bu veriler T1'in derslerinde gözlendi. T2 de aynı şekilde tartışma yönetimini derslerinde kullanmaya çalışıyordu ama sorduğu sorulara bir öğrencinin cevap vermesi onun için yeterliydi. Ancak çizimleri, modelleri, günlük hayat örneklerini ve somut materyalleri diğer öğretmenlerden daha fazla ve daha sık kullanıyordu derslerinde. T2 aynı zamanda akran öğretimi ve grupla öğretim de kullandı. T3 görüşmelerde öğretiminde teknolojiyi kullandığını söyledi ancak teknolojiyi sadece akıllı tahtada kitap dosyalarını açarak kullandığı gözlem sırasında not edildi. Aslında sadece teknolojiyi tahtaya soru ya da not yazmayı kolaylaştırmak için kullanıyordu. Rasyonel sayıların anlatımı sırasında sıklıkla metaforlar, mecazi anlatımlar kullanmıştır T3. Örneğin T3'ün Şekil 1 de sırasıyla rasyonel sayılar kümesinin anlatımında kullandığı çizimler ve rasyonel sayılarda çarpma kuralı ile ilgili yaptığı çizimi görebilirsiniz.



Şekil 1. T3'ün rasyonel sayılar konusunda kullandığı bazı benzetimler

T3, T4, T5 ve T6 genellikle kurallar ve açıklamalar ile konuyu öğretme yoluna gittiler. Çoğunlukla da düz anlatım yöntemini kullandılar. Dört öğretmen de öğrencilere soru sordukları zaman, bir öğrenci doğru yanıtladığında derse devam etme eğiliminde oldular. Aralarındaki model ve sayı doğrusunu en fazla kullanan öğretmen ise T4 idi. Aynı zamanda öğrenciler T4'ün çizdiği bir modeli yeni sorulara ve örneklere daha çabuk uygulayabiliyorlardı çünkü T4'ün açıklamaları ve soruları daha net idi ve sınıfın çoğunluğu tarafından anlaşılıyordu.

Dersten önce yapılan görüşmelerde T1, T2 ve T3 rasyonel sayıyı anlatırken daha çok somut materyal kullandıklarını söylerken T5 ve T6 günlük yaşam örnekleri üzerinden gittiklerini belirttiler. Bunun yanında gözlem sırasında elde edilen veriler de T1 ve T2 nin anlatımlarını diğer katılımcılardan daha fazla doğrulamaktadır.

Alt boyutlardan birisi olan *"Yararlı ve verimli sorular üretmek"* boyutunda öğretmenlerin çoğu derslerinde alıştırmalar ve örnekler kullandılar. Sözlü problemleri ise müfredatta yer aldığı saat kadar kullandılar. Diğer derslerinde çok fazla kullanmadılar. T6 çoğunlukla ders kitabında yer alan problemleri sorarken, diğer öğretmenler farklı kaynaklardan da yararlandı. Problemler ve sordukları sorularla ilgili diğer bir ortak nokta ise öğretmenlerin kolay sorudan zor soruya doğru ilerleyecek şekilde sormaları ve soruları sorarken öğrencilerin seviyelerini göz önünde bulundurmaları oldu. Sorularda ve çözümlerde model kullanımını en fazla yapan öğretmenler T1 ve T2 idi. Sayı doğrusunu ise T1, T3, ve T5 diğer öğretmenlere göre daha fazla kullandılar.

Diğer bir alt boyut olan *"Öğrencilerin matematiksel çözümlerini veya argümanlarını analiz etme veya değerlendirme"* boyutunda T1, T2 ve T4 ilk önce öğrencilere soruyu yazdırıp daha sonra çözmeleri için bir süre verdiler. Sonra öğrencilerin çoğundan cevapları alıp; T1 tartışma yöntemine uygun olarak neden sorusunu sorarken, T2 ve T4 çözüme nasıl ulaştıklarını soruyordu. Eğer öğrenciler doğru cevaplar verirlerse aralarından bir tane gönüllü öğrenciyi çözümü tahtada göstermeleri için seçiyorlardı. Daha sonra anlamayan ya da farklı çözüm yoluyla çözen öğrenciler varsa, üç öğretmen de çözümü tahtada gösteriyordu. T1 ise öğrenciler neden sorusuna cevap veremezlerse kendisi tahtada açıklıyordu. Öğrencilere yazmaları için süre veriyordu ve anlamadığınız yer var mı sorusunu yöneltiyordu. T3, T5 ve T6 da benzer şekilde süreci takip ediyorlardı ancak T3 çoğunlukla soruları akıllı tahtada gösteriyordu. Eğer bir tane öğrenci doğru cevap verirse hemen tahtada çözümünü anlattırıp daha sonra anlamayan öğrenciler için kendileri anlatıyorlardı. Farklı çözümler olduğu zaman bunları da tahtada gösteriyorlardı. Sözel problemler içinde benzer süreci takip ediyorlardı. T1 tahtaya problemde verilenleri yazdırıp öğrencilerden problemi kendi ifadeleriyle anlatmalarını istiyordu. Sorunun çözümünde de öğrencilerle yine soru-cevap şeklinde ilerliyordu. Problem ve soru çözümünde bütün öğrenciler sürece dahil olmadığı için, öğretmenler anlamayan öğrenciler için tekrar anlatıyorlardı. Hem soru hemde problemin çözümlerinde öğrenciler sonuçlarını, özellikle T1, T2 ve T4'ün sınıfları için, öğretmenlere tek tek gösteriyorlardı. Problem çözme basamaklarını düzgün bir şekilde T6 sınıfında anlatttı. Diğer öğretmenler bu basamaklardan bahsetmedi. Ancak T6 da bahsetmesine rağmen soru çözümünde ne kendisi ne de sınıfta öğrenciler problem çözme basamaklarını uygulamıyordu. T2 bütün hocalar arasında en fazla sözlü problem soran ve çözdüren öğretmendir. Bazen normal konu anlatırken bile örnek çözümü için sözlü problem sormuştur T2. T3 ve T5 de aynı şekilde benzer süreci ilerletmeye çalıştı ancak her iki sınıftada seviye çok düşük olduğu için çoğu problemi öğrencileri çözemediler. Daha sonra kendileri soruyu ve çözümü açıkladılar.

"Kavram yanılgıları da dahil olmak üzere öğrencilerin klasik yanıtlarını tanımlayabilme" alt boyutunda katılımcı öğretmenlerin derslerinde davranışları, geridönütleri ve öğrencilerin sorularına verdikleri yanıtları gözlemlendi. T1 ve T2 bazen öğrencilerin kavram yanılgılarını belirlemek için derslerinde sorular sordular. Örneğin;

> *T1:* $\frac{5}{0}$ *rasyonel sayı mıdır?*
> *C: Değildir.*
> *T1: Neden?*
> *S: Payda sıfır.*
> *T1: Payda neden sıfır olamaz?*
> *T1: (Öğrencilerden cevap gelmeyince kendi açıkladı.) Bir bütünün eş parçaları kesir ama 0 da bir bütün yok.*

T1 çoğunlukla kavram yanılgılarını engellemek için açıklama yapıyordu, açıklamadan sonra örnek çözmüyordu. Genel olarak öğretmenler, öğrencilerin yanlış cevaplarını ilk kez gördüklerinde kavram yangıları olup olmadıklarını anlamıyorlardı. O yanlış cevapların üstüne sorular sorduklarında kavram yanılgılarını tespit edebiliyorlardı. Ancak örnek olay görüşmesinde yanlış cevaplardaki kavram yanılgıları direkt sorulduğunda doğru cevap verdiler. T1 tartışma yöntemi kullandığı için öğrencilerin yanıtlarının sebeplerini en çok sorgulayan öğretmendi. T2 ve T4 de ancak öğrencilerin soruları ve yanıtlarını sorguladıklarında öğrencilerde oluşmuş olan kavram yanılgılarını anlıyorlardı. Diğer öğretmenler ise öğrenciler yanlış yanıt verdiğinde onlara doğrusunu açıklamaya çalıştıkları için öğrencilerde oluşmuş olan kavram yanılgılarına hâkim değillerdi. Öğrencilerin yanlış cevap verdiklerini düşünüyorlardı. Örneğin; T3 dersinde,

> *T3:* $\frac{1}{3}\cdot\frac{1}{4}-\frac{1}{2}\cdot\frac{1}{3}=?$
>
> *S_1:* $\frac{1}{3}$ *leri sadeleştirebiliriz.*

T3: Hayır. Bu işlemde $\frac{1}{3}$ *leri sadeleştiremezsiniz. Yan yana olanlar sadeleşmez arada çıkarma var.*

S_2: Payları eşitleyerek çarpma yaparım.

T3: Neden payları eşitliyorsun. Çarpma işleminde paylar eşitlenmez ki. Işlemi zorlaştırmış olursun. (Daha sonra çözüme devam etti) $\frac{1}{12} - \frac{1}{6}$

S_3: 24 de eşitlerim.

T3: Neden 6'nın katı değil mi 12? (Daha sonra 12'de eşitleyerek işlemi tamamladı.)

"Uygun geri bildirim sağlamak" alt boyutunda ise çalışmada yer alan bütün öğretmenlerin, öğrencilerin anlamadığı kavram veya çözmediği soru olduğunda çoğunlukla tekrar açıklama yaptıkları ve tekrar anlattıkları gözlemlendi. Bu açıklamaların hepsi öğretmenlerin kullandıkları öğretim yönetimine paralel şekilde gerçekleşti. Öğrenciler tekrar açıklamaya rağmen anlamadığında örneği, soruyu ve anlatırken kullandığı yönetimi değiştiren öğretmenler T1, T2 ve T4 olmuştur. Örneğin;

T4: $\square \div \left(-\frac{1}{4}\right) = \left(-\frac{2}{5}\right) \qquad \frac{12}{5} \div \triangle = -\frac{1}{3} \qquad \square + \triangle = ?$

T4: (Öğrencilere çözmeleri için zaman verdi. Cevap gelmeyince açıklama yaptı.) Karede bölünen bilinmiyor. Üçgende bölen bilinmiyor. (Basit bir örnek verdi.) $10 \div 2 = 5$. *10 bilinmeseydi ne yapardınız? Ters işlem. 2 bilinmeseydi ne yapardınız? 10'nu 5'e bölerdiniz. Burada da aynısı. (Öğrencilerden çözenlerden 2 kişi tahtaya kalkıp çözdü. T4 çözümü anlattı. Anlamadık diyen öğrenciye tekrar anlattı.)*

T4: Eğer bölen sayı bilinmiyorsa bölünen sayıyı sonuca bölün. Eğer aklınıza gelmezse $10 \div 2 = 5$ *gibi basit bir işlem yapıp bakın.*

Eğer soruyu çözmek için tahtaya çıkan öğrenciler olursa, öğretmenler çoğunlukla öğrencinin, arkadaşlarına çözümü anlatmasını isterdi. T1, T2 ve T4 soru sorduklarında sınıf içinde dolaşıyorlar ve öğrenciler soru çözümlerini çoğunlukla öğretmenin yanına gelerek bireysel olarak gösteriyorlardı. Öğretmenler de bu sırada doğru, yanlış veya şu şekilde ilerle diyerek geridönüt veriyordu. T1 ve T2 bazen öğrencilere gösterdikleri başarılardan dolayı ödül de veriyorlardı. T3 ve T5 ise sınıf seviyesi düşük olduğundan sorularına cevap almıyorlar ve öğrencilerle birlikte açıklama yapıyorlardı.

Diğer iki alt boyut olan *"Öğrencilerin sorularının içeriğini analiz etme"* ve *"Beklenmeyen matematiksel sorunlara yanıt verme"* boyutunda öğretmenler dersi anlatırken kullandıkları öğretim yönetimi ve geri dönüt yöntemine paralel hareket ediyorlardı. Örneğin T1 öğrencilerin sorduğu soruları tekrar sınıfa diğer öğrencilere soruyordu. Diğer öğretmenler ise kendileri açıklıyordu ve çoğunlukla bu açıklamalarda kuralları ve geçmiş öğrenmelerini hatırlatıyorlardı. T3, T5 ve T6 bu kuralları ezberlemeleri gerektiğini vurguluyorlardı. Öğrencilerin sorularının içeriğini analiz etmeyi çoğunlukla sorularını doğru cevaplandırma olarak yapıyorlardı. Soruların arkasında yatan sebebi araştırmıyorlardı. Bazı durumlarda öğrencilerin sorularını anlayamadıkları için görmezden geliyor veya eksik açıklama yapıyorlardı. Örneğin T6'nın sınıfında bir öğrenciyle şöyle bir diyalog yaşamıştı;

T6: Çıkarma işlemi diye bir işlem yoktur aslında o da toplama işlemidir.
S_1: O zaman neden çıkarma işlemi öğreniyoruz?
S_2: Çünkü ilkokulda negatifleri öğrenmiyorduk.
T6: Evet doğru.

3.2. Ortaokul Matematik Öğretmenlerinin Rasyonel Sayılar Bağlamında Oluşan İnançları

Öğretmenlerle yapılan görüşmeler ve gözlemler sonucunda çalışmaya katılan öğretmenlerin matematiksel inanışları 3 temel temada toplanmıştır. Bunları sırasıyla açıklanmıştır.

3.2.1. Öğrenciler ve Öğrenmeleri ile İlgili İnanışlar

T4 hariç bütün öğretmenler matematiğin öğretmen olmadan öğretilebiceğini düşünmüyordu. T4 ise akran öğrenmesiyle mümkün olacağını söyledi. Öğretmenler ayrıca öğrencilerin dikkatini çekmek için somut materyal veya günlük hayat örneklerinin etkili olduğuna inandıklarını söylediler. Öğrencilerin kavram yanılgılarının sebepleri öğretmenlere göre yanlış veya eksik öğrenmeleri ve konuyla ilgili kuralları karıştırmalarıdır. Bu bağlamda T2 ve T6 bütün ispatların ve çözümlerin gösterilmemesi gerektiğini ve öğrencilerin kafasının karışabileceğini söylediler. Örneğin T2 bir dersinde rasyonel sayılarda bölme işleminde 2. kesri neden ters çevirip çarpıyoruz diye soran öğrenciye kural olduğu için şeklinde açıklayıp, ders sonrasında araştırmacıya öğrenciye bu kuralın ispatını bilerek göstermediğini söyledi. Çünkü söylediği zaman sınıftaki diğer öğrencilerin kafasının karışabileceği belirtti. T1, eğer öğrenciler öğrenmeye istekli değillerse tam olarak iyi öğrenme gerçekleşmediğini iddia etti. T6 da her öğrencinin matematiği öğrenmemesi gerektiğini, sadece kullanmaya ihityacı olanların belirli bir seviyeden sonrasını öğrenmesi gerektiğini belirtti.

Çalışmada yer alan öğretmenler hem öğrencilerin öğrenip öğrenmediğini hemde doğru yanıt oranını artırmak için farklı türde soruları sınavlarında kullanmak istediklerini ve kullandıklarını belirttiler. Tek tip soruların yetersiz kaldığını düşünüyorlardı. Sınavın dışında öğrencilerin anlayıp anlamadığını T2 ve T3 öğrencilerin gözünün içine bakarak anladıklarını, T5 ise başka derslerdeki benzer konulardaki görevleri yapıp yapamadıklarından anladığını söylediler.

3.2.2. Matematik Öğretimi ile İlgili İnanışlar

Katılımcıların çoğu düz anlatım yoluyla öğretimin daha etkili olduğuna inandıklarını belirttiler. T1 ise tartışma yönteminin daha etkili olduğunu gördüğünü ve inandığını belirtti. T3, T4, T5 ve T6, öğrencilerin gerekli kuralları, tanımları ve kısayolları ezberlemesi gerektiğine inanmaktadırlar. Bunların aksine T1 etkili öğretmenlerin öğrencilere ezberlemeye teşvik etmemesini, onun yerine içeriği anlatması gerektiğini belirtti. Aynı zamanda T1 ve T3 de öğrencilerin geçmiş öğrenmelerinin ve hazırbulunuşluklarının kendi öğretimlerini etkilediklerini söylüyorlardı. Bunların dışında çalışmada yer alan öğretmenler teknoloji kullanmaya çok fazla istekli değillerdi T2 teknolojinin kalabalık sınıflarda etkili olmadığına inanıyordu. T6 da çalışma kağıdının kalabalık sınıflarda kontrolü zorlaştırdığını ve uygun olmadığını düşünüyordu.

T2 hariç diğer öğretmenler derse girmeden önce belirli bir hazırlık yapmadıklarını çünkü deneyimli öğretmen olduklarını düşündükleri için hazırlık yapmaya ihtiyaç duymadıklarını söylüyorlardı. Ayrıca bütün öğretmenler ders kitabının özellikle alıştırma ve örnek bakımından yetersiz olduğunu düşünüyorlardı ve çoğunlukla ekstra yardımcı kaynak kullanıyorlardı. İdeal sınıf tanımlarında ise nerdeyse bütün öğretmenler sınıfın fiziksel özelliklerinin iyileştirilmesi açısından bahsetmişlerdir. Herhangi bir öğretim yöntemi değişikliğini tercih etmediler.

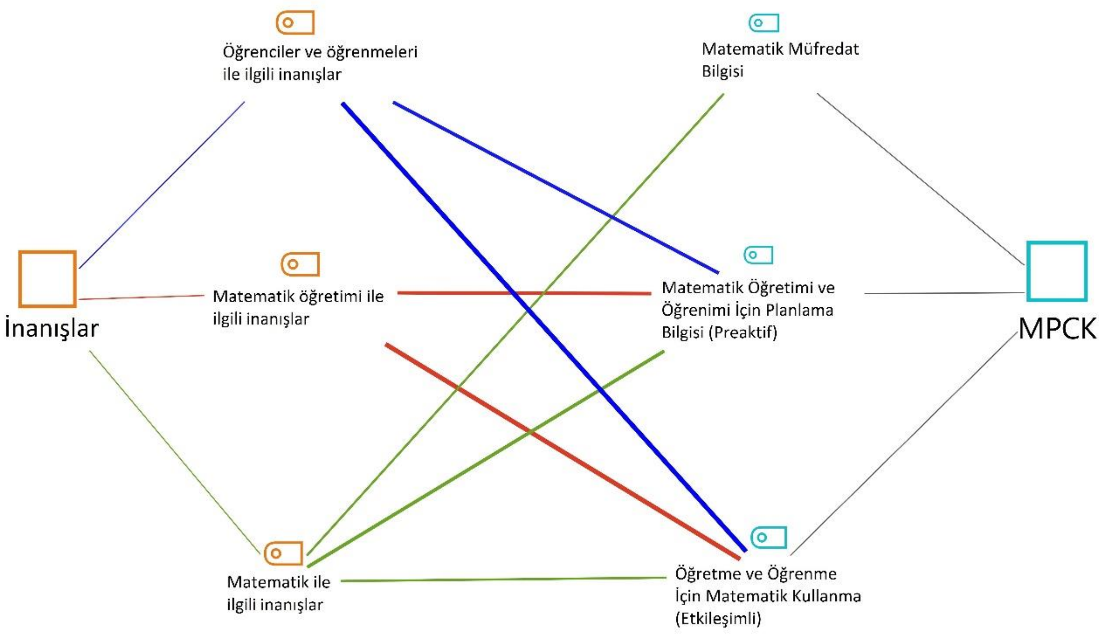
3.2.3. Matematik ile İlgili İnanışlar

Çalışmada yer alan öğretmenler matematik müfredatında yer alan temel konular konusunda benzer konuları söylediler; tam sayılar, rasyonel sayılar, denklemler, cebir. Çünkü bu konuların sonraki konulara temel teşkil ettiğini belirttiler. Diğer taraftan temel beceri olarak problem çözme ve dört işlem becerisi olduğunu söylediler. Öğretmenlere göre eğer öğrenciler matematik bilirlerse günlük hayata daha kolay adapte olurlar ve düşünme becerilerini geliştirebilirler.

Bütün bunların dışında katılımcılar için önemli olan bir diğer konu da ulusal sınav yani liselere giriş sınavı idi. Çalışmaya katılan öğretmenler, öğrenciler için önemli olan bu sınavın öğretmenler içinde önemli olduğunu, öğretmen ve öğrenci başarısı için bir gösterge olduğunu düşünüyorlardı. Matematikte problem çözmenin önemli olduğunu görüşmelerde vurguladı bütün öğretmenler ancak müfredatta yer aldığı kadarıyla kullandılar. Hatta problemlerin alıştırmalardan daha karmaşık ve zor olduğunu belirtmelerine rağmen, diğer alıştırma ve sorularda izlediği yöntemle aynı yöntemi izlediler. Farklı çözüm yollarına da bütün öğretmenler olumlu yaklaştılar. Ayrıca öğrenci seviyesine göre sorunun seviyesini belirlediklerini söylediler.

3.3. Ortaokul Matematik Öğretmenlerinin İnançları ile PAB'lerinin Rasyonel Sayılar Konusundaki Etkileşimi

Elde edilen veriler ışığında öğretmenlerin inanışlar ile PAB'lerinin arasında bir etkileşim olduğu görüldü. Şekil 2'de bu etkileşim görülebilir. MPCK deki *"Matematik müfredat bilgisi"* alt boyutunda yer alan *"Öğrenme programlarında temel fikirlerin belirlenmesi"* ile matematikle ilgili inanışların etkileşimi vardır. Öğretmenler inanışlarına dayalı olarak öğretimlerinde anahtar veya temel kavramlara öncelik verebilir ve öğretimde daha fazla vurgulayabilirler. Ayrıca öğretmenlere göre müfredattaki sarmal yapıdan dolayı bir konunun diğerini etkilediği için *"Olası yolları seçme ve müfredat içindeki bağlantıları görme"* boyutunda da etkileşimi vardır.



Şekil 2. İnanışlar ile PAB arasındaki etkileşim

MPCK çerçevesindeki diğer iki altboyutun ise inanışlardaki 3 alt boyutla da etkileşimi bulunmuştur. *"Değerlendirme formatlarını seçme", "Uygun etkinlikleri planlamak veya seçmek", "Kavram yanılgıları da dahil olmak üzere öğrencilerin klasik yanıtlarını tahmin etme"* gibi alt boyutlar ile inanışla arasında bir etkileşim bulunmuştur. MPCK çerçevesinin etkileşimli kısmında ise *"Matematiksel kavramları veya prosedürleri açıklamak veya tasvir etmek"*, *"Öğrencilerin matematiksel çözümlerini veya argümanlarını analiz etme veya değerlendirme"*, *"Beklenmeyen matematiksel sorunlara yanıt verme"*, *"Uygun geri bildirim sağlamak"* gibi alt boyutlarda bir etkileşim bulunmuştur.

4. TARTIŞMA VE ÖNERİLER

Bu çalışmada elde edilen bulgular amaç ve araştırma soruları doğrultusunda bu bölümde alan yazındaki çalışmalarla ilişkilendirilmiştir. İlk araştırma sorusu olan öğretmenlerin PAB lerine baktığımızda bazı öğretmenlerin diğerlerinden daha farklı öğretime sahip olduğu görülmektedir. Model veya gösterim ile ilgili bilginin matematik öğretimini etkilediği bilinmektedir (Tchoshanov, 2011). Çalışmada bütün öğretmenler rasyonel sayılarda çarpma modelini çizebilirken ancak bölme işleminin modelini ilk bakışta çizememişlerdir. Sadece T1 görüşmeden sonra deneyip uğraşıp doğru cevabı verebilmiştir. Sonuç olarak çalışmaya katılan öğretmenler ters çevir çarp kuralıyla öğretip bu kuralın nerden geldiğini açıklamamışlardır. Bu konuda çalışma yapan araştırmacılar da benzer sonuçlar elde etmiştir (Ball, 1990; Işıksal, 2006; Ma, 1999; Singmuang, 2002; Tirosh, 2000). Işıksal (2006) yaptığı çalışmasında öğretmen adaylarının kavramsal anlama olmadan yeterli bilgiye sahip olduklarını belirtmiştir. Bu çalışmada da benzer sonuçlara ulaşılmış ve öğretmenlerin kavramsaldan çok yöntemsel bilgiye sahip oldukları görülmüştür. Kavramsal anlamaları artırmak için öğretmen eğitimi programlarında düzenlemeler yapılabilir.

Yapılan çalışmalar bir çok öğretmenin klasik yöntem olan düz anlatım yoluyla öğretim yaptığı, öğretmen veya ders kitabının ana kaynak olarak yer aldığını söylemiştir (Philipp, 2007; Putnam ve Borko, 2000; Steele, 2001). Bu çalışma da da benzer sonuçlara ulaşılmaktadır. Ayrıca öğretmenler, öğrencilerde yer alan kavram yanılgılarının sebeplerini öğretimin tam yapılmaması, yada kuralların yanlış uygulanması, ezberlenmemesi olarak görürken kendi öğretimlerinde hiçbir şekilde eksiklik olduğunu düşünmediler. Işıksal da (2006) çalışmasında benzer sonuçlar elde etmiştir. Çalışmada elde edilen bulgulara göre öğretmenlerin anlatırken modelleri sınırlı kullanımı öğretmenlerin tercihlerinden veya PAB'lerinden kaynaklı olduğu düşünülmektedir.

İkinci araştırma sorusundan öğretmenlerin inanışlarıyla ilgili elde edilen bulgulara göre öğretmenler kalabalık sınıflarda müfredatı da yetiştirebilmek için en etkili öğretimin düz anlatım olduğuna inanıyorlar. Somutlaştırma ve görselleştirmenin ise dersteki ilgiyi artırdığını düşünüyorlar. Yapılan araştırmalarda da inanışların kişilerin kararlarından en önemli faktörlerden biri olduğunu ve öğretmenlerin bu inanışları karar verme sürecinde bir rehber gibi kullandıklarını ortaya koymuştur (Handal, 2003; Fives ve Buehl, 2012). Bu durumda çalışmadaki öğretmenlerin kararlarını inanışlarının da etkilediğini söyleyebiliriz.

Öğretmenlerin inanışları ile PAB'lerinin etkileşime bakacak olursak, alanyazında yapılan bazı araştırmalar inanışların öğretmenlerin bilgileri üzerinde bir etkisi olduğunu, bağlantılı olarak düşünülmesi gerektiğini ve birbirlerini etkilediğini ortaya koymuştur (Drageset, 2010; Fennema ve Franke, 1992; Hofer ve Pintrich, 1997). İnanç ve PAB arasındaki etkileşimin en az olduğu kısım *"Matematik müfredat bilgisi"* olduğu görülmüştür. Bunun en önemli sebeplerinden birisi müfredatın Milli Eğitim Bakanlığı tarafından hazırlanarak çoğu sürecin örneklerle anlatılmış şekilde öğretmenlere verilmesidir. Bu yüzden öğretmenler müfredatla ilgili çok fazla bir düşünceye sahip olamıyorlar. Daha geleneksel inanışlara sahip olan öğretmenlerin daha geleneksel uygulamalar yaptıkları sonucunu bazı araştırmacılar çalışmalarında elde etmiştir (Stipek, Givvin, Salmon, ve MacGyvers, 2001). Bu çalışmada daha çok öğretmenler düz anlatım yolunu tercih ettiklerini belirttiler. T1 öğretmenliğe başlarken düz anlatımla başlasa da tartışma yöntemini öğrenince daha başarılı olacağını düşünerek uyguladığını ve bunun olumlu sonucunu görünce öğretim yöntemini değiştirdiğini belirtti. T2, T6 gibi bazı öğretmenler de teknoloji kullanımı yönteminin özellikle kalabalık sınıflar için uygun ve etkili olmadığını belirttiler. T6, geogebra program ile ilgili bir eğitim almasına rağmen kullanmadığını söyledi. Bu çalışmada sonuçları şunu göstermiştir ki öğretmenlerin inanışları neyi nasıl öğretmek istediklerini etkilemektedir. Yapılan çalışmalar aynı zamanda öğretmenlerin PAB'lerinin de öğretmenlerin inanışlarını ve öğretimle ilgili kararlarını şekillendirmede önemli bir faktör olduğunu belirtmiştir (An, Kulm, Wu, Ma ve Wang, 2002). T1'in örnek olayda gördüklerini uygulaması ve yüksek lisanstan sonra öğretim yöntemini değiştirmesi de yapılan çalışmaların bulgularına örnek oluşturmaktadır.

Bu çalışmada da bazı sınırlılıklar bulunmaktadır. Örneğin nitel veri analizi ile yapıldığı için genelleme amacı güdülmemiştir ve bütün öğretmenler devlet okulunda çalıştığı için özel okulda çalışan öğretmenlerle bu çalışmayı yapmak ve sonucunu görmek inanış ve PAB arasındaki etkileşimi görmek açısından katkı sağlar. Ayrıca rasyonel sayılar yerine diğer konularda da çalışma yapmak bu etkileşimi netleştirmek için önemlidir.

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<u>**TEZİN ADI /** TITLE OF THE THESIS</u> (**İngilizce** / English): THE INTERACTION BETWEEN MIDDLE SCHOOL MATHEMATICS TEACHERS' BELIEFS AND PEDAGOGICAL CONTENT KNOWLEDGE REGARDING RATIONAL NUMBERS

<u>**TEZİN TÜRÜ**</u> **/** <u>DEGREE:</u> **Yüksek Lisans** / Master ☐ **Doktora** / PhD ☒

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