

USING TECHNOLOGY-ENHANCED STEP-BY-STEP UNROLLED STRATEGY  
TO IMPROVE NINTH-GRADE STUDENTS' GEOMETRIC REASONING

by

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## ABSTRACT

### USING TECHNOLOGY-ENHANCED STEP-BY-STEP UNROLLED STRATEGY TO IMPROVE NINTH-GRADE STUDENTS' GEOMETRIC REASONING

The purpose of this action research is to examine and compare the geometric reasoning skills, perceptions about geometric proofs, and success in geometry of the students before and after the geometry course integrated with technology -enhanced applications and step-by-step strategy. In line with these purposes, a lesson plan and materials related to the Pythagorean theorem were prepared. The sample group of the study consisted of 34 students in the 9th grade studying at a state high school in the Esenler district of Istanbul. The lesson plan application lasted for 2 weeks. In order to investigate the effectiveness of the developed materials, the geometric reasoning levels of the students were evaluated with the Van Hiele test before and after the course, the students' perceptions about geometric proofs and perceptions about the implementation of the geometry unit with the answers they gave to the reflection questions asked before, during, and after the course, and the students' geometry success was evaluated with the unit achievement test created. According to the results obtained, it was concluded that the geometry course integrated with these strategies made a positive contribution to the students' geometric reasoning skills and course success. It was determined that there was a significant difference between the students' pre-Van Hiele levels and post-Van Hiele levels. In addition, it was determined that there was a significant difference between the pre-test and post-test scores in the unit achievement test. According to the findings obtained from the qualitative data of the research, it was determined that the students had positive perceptions about the geometry course and this strategy, and that they found the use of this strategy instructive and entertaining.

## ÖZET

# DOKUZUNCU SINIF ÖĞRENCİLERİNİN GEOMETRİK MUHAKEMELERİNİ GELİŞTİRMEK İÇİN TEKNOLOJİ DESTEKLİ ADIM ADIM ÇÖZÜMLEME STRATEJİSİNİN KULLANILMASI

Bu eylem araştırmasının amacı, teknoloji destekli uygulamalar ve adım adım çözümleme stratejisiyle bütünleştirilmiş geometri dersi öncesi ve sonrası öğrencilerin geometrik akıl yürütme becerilerini, geometrik ispat hakkındaki düşüncelerini ve geometrideki başarılarını incelemek ve karşılaştırmaktır. Bu amaçlar doğrultusunda, Pisagor teoremi konusu ile ilgili ders planı ve materyalleri hazırlanmıştır. Araştırmanın örneklem grubunu, İstanbul ili Esenler ilçesinde bulunan bir devlet lisesinde öğrenim gören 9. Sınıftaki 34 öğrenci oluşturmaktadır. Ders planı uygulaması 2 hafta sürmüştür. Geliştirilen materyallerin etkililiğini araştırmak için, ders öncesi ve sonrası Van Hiele testi ile öğrencilerin geometrik akıl yürütme seviyeleri, öğrencilerin ders öncesinde, sırasında ve sonrasında sorulan açık uçlu sorulara verdikleri cevaplarla geometrik ispatla ilgili algıları ve geometri ünitesinin uygulanmasına ilişkin düşünceleri, oluşturulan ünite başarı testi ile ise öğrencilerin geometri başarıları değerlendirilmiştir. Elde edilen sonuçlara göre bu stratejilerle bütünleştirilmiş geometri dersinin, öğrencilerin geometrik akıl yürütme becerilerine ve ders başarılarına olumlu katkı yaptığı sonucuna varılmıştır. Öğrencilerin ön Van hiele seviyeleri ile son Van hiele seviyeleri arasında anlamlı farklılık olduğu belirlenmiştir. Ayrıca, ünite başarı testinde ön test ve son test puanları arasında anlamlı farklılık olduğu belirlenmiştir. Araştırmanın nitel verilerinden elde edilen bulgulara göre ise, öğrencilerin geometri dersi ve bu strateji ile ilgili olumlu düşünceye sahip oldukları, bu stratejinin kullanılmasını öğretici ve eğlenceli buldukları tespit edilmiştir.

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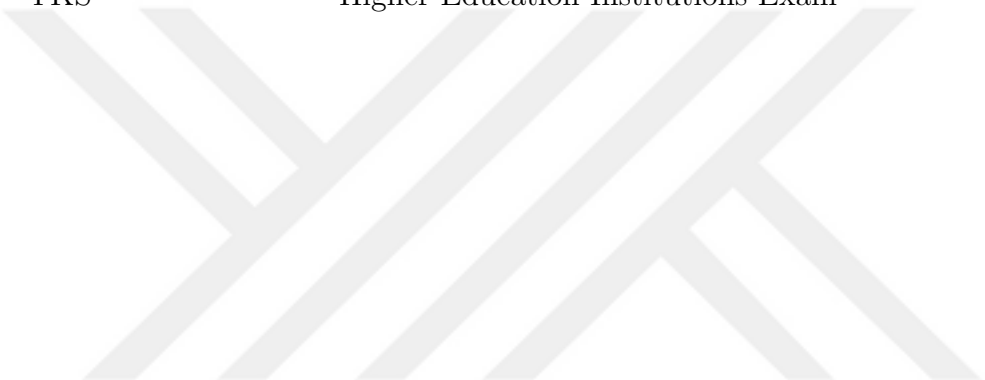
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## LIST OF ACRONYMS/ABBREVIATIONS

DGS	Dynamic Geometry Software
MoNE	Ministry of National Education
SPSS	Statistical Package for the Social Sciences
SU	Step-By-Step Unrolled
VHGT	Van Hiele Geometry Test
YKS	Higher Education Institutions Exam



## 1. INTRODUCTION

Developing students' ability to understand geometric proofs is a critical component of a balanced mathematics education and can benefit students. Making sense of proofs can help students develop their critical thinking skills and problem-solving abilities by teaching them to analyze and reason about complex geometric situations. By gaining a deeper understanding of proofs, students can build a strong foundation in mathematical reasoning, assisting them in their various academic and professional ventures. By reflecting on the concepts on which the proofs are based, students can see the inner logic and structure of geometric relationships and gain a better understanding of related mathematical principles. Skillfully constructing geometric proofs can benefit students' academic and professional lives. According to Mansi (2003), numerous advantages of geometric proof in mathematics teaching have been demonstrated. Students benefit from geometric proof because it teaches them to build logical arguments and support their findings with established axioms and theorems. Due to the fact that students must evaluate and use their knowledge to resolve issues and support hypotheses, this encourages a better comprehension of mathematical ideas.

The ability to prove and understand proof is vital in fields such as science, engineering, and mathematics. As a mathematics teacher, I witnessed that the students at the school where I work had difficulties in subjects involving high-level mathematical reasoning, especially in geometry. I clearly experienced that students need to be stronger in terms of course success. I understood that they were having difficulty applying some basic algorithms because they could not answer the questions I asked in class and made mistakes. The topics I wanted to teach took longer than usual because I had to constantly remind them of their prior knowledge. For example, when they encountered a radical or exponential expression in problem solving, students did not know what to do. Therefore, while the problem-solving lesson needed to be understood, the lesson suddenly turned into a lesson on radical numbers. My first goal was to teach students geometric proof. However, when the geometry topic was brought up, or they

saw a shaped question, some students stated that this topic was difficult due to their prejudices, while others stated that it was difficult because they could not do it.

To give an example, especially from the problem topic, when they saw the circular track shape in the speed problems, the students thought that this problem was difficult and said that they could not do it without reading the question. I also observed various difficulties in geometric proof. These difficulties include not being able to produce examples that clearly illustrate a point, not understanding the importance of definitions when writing proofs, difficulties with mathematical language and notation, and not knowing where to begin when writing a proof. In addition, Cheng and Lin (2009), noted that students needed help in constructing formal geometry proofs that were valid or acceptable. When faced with a problem, students were content to simply state memorized information and had difficulty putting events in a logical order. Despite these shortcomings, I have seen people have a perspective that low-achieving students will never understand anything, no matter what is done. Not caring about students leads them to more failure. I developed a lesson plan to help them develop a skill, keeping in mind that should not be given up on them, that every student is special, and that everyone can learn something.

These experiences highlight the importance of integrating students' prior knowledge more effectively in lesson planning and teaching strategies and making them more equipped to teach geometric proofs. Therefore, I decided to conduct this study with the aim of investigating how to improve my students' competencies related to proof in order to help them develop their geometric reasoning skills.

As Wong *et al.* (2011), stated hard-to-learn skills are required to prove geometry theorems. According to the authors, there are several challenges. According to the authors, these challenges are as follows.

- Need to complete understanding of the problem and mathematical symbols.
- The tendency to produce proofs that rely solely on direct visual observations.

- Lack of strategic knowledge in constructing proofs.
- Need help comprehending complex geometry instruction.
- Difficulty in grasping abstract concepts and their relationships to other geometric objects.
- Difficulty in applying geometric concepts to real-world problems.

Instead of dealing with abstract and complex representations, students can start working with concrete, graphic representations. Healy and Hoyles (2000) also state that the act of proving requires a variety of student abilities, each of which is by no means simple, including detecting assumptions, isolating specified qualities and structures, and arranging logical arguments. These difficulties may be made much more difficult by the ambiguity of the word “proof” and the fact that, outside of mathematics, proof and evidence can be used interchangeably. Proving mathematical expressions, formulas, or theorems by giving reasons further simplifies the understanding of mathematical concepts (Kaplan and Öztürk, 2017).

In addition, the proof can advance people’s cognitive processes and improve their mathematical reasoning and analytical thinking capacities (Kaplan and Öztürk, 2017). Kaplan and Öztürk (2017) stated that rich thinking skills are required for the act of proof, which is defined as not only saying a given example is true but also explaining why it is true. This requires advanced cognitive abilities such as reasoning, deduction, and a deep grasp and understanding of mathematical relationships.

Chang and Lin (2009) developed a strategy to improve students’ ability to understand geometric proofs. Cheng and Lin (2009) introduced the step-by-step unrolled (SU) strategy, which is based on scaffolding. Scaffolding is used when a new skill or concept is taken up for learners to master. In this case, the opening of each condition in the proof awaits a scaffolding task for students to build on their prior knowledge and understanding. This strategy breaks down complex proofs into smaller and more manageable steps. Students can focus on each piece and gradually develop their overall understanding. In addition, Boero (1999) suggested that below-average students

could benefit from a more structured approach to proof construction, and this can be achieved with the help of the SU strategy. By providing clear guidance and support throughout the proof-building process, students can be better equipped to tackle complex problems and develop mathematical reasoning skills. Using these concepts and principles, educators can gain effective support from their lower-level goals in mastering multi-step geometry proofs (Cheng and Lin, 2009). In addition, I knew that technology has an important place in our lives today. I also believe that students' knowledge of technological tools can contribute to their learning process. For this reason, I combined technology with this strategy. In this study, a 9th-grade geometry unit was implemented using the SU strategy supported with technology to teach geometric proof.

## 2. REVIEW OF RELATED LITERATURE

### 2.1. Geometric Reasoning

According to Duval (1998), geometric reasoning is one of the three kinds of cognitive processes involved in learning geometry, along with visualization and construction. Geometric reasoning involves using mathematical language and logic to make deductions and prove theorems. It is a crucial component of proficiency in geometry, as it allows learners to understand the relationships between geometric objects and apply their knowledge to solve problems.

Geometric reasoning is a crucial component of proficiency in geometry. Van Hiele levels are considered a framework used to assess geometric thinking skills. The model was developed in the Netherlands in the late 1950s to characterize and forecast students' geometric thinking. This model consists of five levels. Level 0 is visualization, level 1 is analysis, level 2 is informal deduction, level 3 is formal deduction, and level 4 is rigor (Crowley, 1987). According to the Van Hiele model, students progress through the levels in a hierarchical order, supported by appropriate instructional experiences. This process begins with recognizing shapes as a whole (level 0), continues with exploring the properties of shapes and informal reasoning about these shapes and their properties (levels 1 and 2), and finally ends with a formal inferential and literal examination of axiomatic geometry (levels 3 and 4) (Baffoe and Mereku, 2011). Van Hiele levels are widely used in geometry teaching to assess students' spatial skills and understanding of geometry (Hock *et al.*, 2015). These levels are considered an important criterion for determining students' ability to understand geometry topics and solve problems. It is stated that students' Van Hiele geometry levels are strongly related to geometry success and participation rate in the online education process (Chen *et al.*, 2019).

The Van Hiele Geometry Test is a valuable tool for assessing students' geometric reasoning levels. According to Senk *et al.* (2022), an evaluation instrument called

the Van Hiele Geometry Test was created to examine the descriptive and predictive validity of the Van Hiele model of geometry education. Van Hiele Test is used in research studies all over the world to determine the geometric thinking levels of students and instructors from lower secondary grades through post secondary education. It has been translated into more than a dozen languages. Although many studies have used it with various demographics, it is used to evaluate secondary school students' geometric comprehension.

Geometrical reasoning, in Zhang's view (2020), is essential to the teaching of mathematics. Dynamic geometry software, for example, has been acknowledged as a useful technology -enhanced tool for assisting students' geometric thinking (Zhang, 2020). According to Zhang (2020), technology may be used to enhance students' knowledge of geometric ideas through interactive activities and visual representations. It has been discovered that using technology in the classroom can help students develop their mental knowledge of geometry and improve their geometric reasoning.

According to Jayathirtha (2018), in studies on geometric reasoning, the van Hiele model of geometric comprehension has been extensively employed (Jayathirtha, 2018). According to this approach, there are five different degrees of geometric comprehension, from visual perception through inductive reasoning (Jayathirtha, 2018). This approach has been utilized by researchers to construct instructional techniques that support the growth of deeper levels of geometric comprehension in children as well as to analyze students' capacities for reasoning (Jayathirtha, 2018).

In terms of curriculum design, an examination of the national geometry curriculum's planned structure showed that it needed to be rearranged for students to gradually move from visual to deduction-based reasoning (Jayathirtha, 2018). The results also highlighted the significance of conceptual representations for helping students learn relational geometry (Jayathirtha, 2018).

## 2.2. Geometric Proof

For a long time, scholars in cognitive science and mathematics education have been interested in geometric proof. An early example of research in this field was Harold Fawcett's 1938 study on the nature of proof (Fawcett, 1938). Since then, many academics have looked at the use of proof in mathematics instruction, especially in teaching geometry. The knowledge and use of geometric proof by students have garnered much attention. According to Herbst and Brach (2006), students frequently believe that a single example or diagram is sufficient to prove a claim, according to research on common misunderstandings regarding proofs.

The connection between geometric evidence and visual reasoning has also attracted attention. Arcavi (2003) made the case that using visual representations can help understand and build proofs. In contrast, Weber and Alcock (2004) claimed that relying too much on visual reasoning might result in mistakes or insufficient reasons.

According to Kaplan and Öztürk (2017), mathematical proof is an essential skill that distinguishes mathematics from other sciences. Because in other branches of science, the accuracy of data is obtained through observation, experiment, or sense, while the accuracy of mathematical information is obtained through logical inferences and proofs. In general, geometric proof research has increased our understanding of how mathematicians think and how it is taught to students (Cheng and Lin, 2009). By identifying common misconceptions and helpful teaching strategies, researchers have contributed to improving mathematics instruction for students at all levels.

Healy and Hoyles (2000) defined geometric proof as the process of developing a logical justification for a mathematical assertion, which is that students can grasp the information they have learned in a logical framework instead of memorizing it. A conclusion is reached by the use of axioms, definitions, and theorems that have already been demonstrated. Because it enables mathematicians to establish the veracity of mathematical claims with certainty, geometric proof is a crucial part of mathemati-

cal thinking and deductive reasoning. Building formal geometric proofs is a critical component of mathematics education in many nations (Healy and Hoyles, 2000). According to Herbst (2006), geometric proof is different from proof in other mathematical domains in that it often relies on visual representations, such as diagrams.

Cheng and Lin (2009) found that many junior high students need help creating valid or acceptable formal geometry proofs. This emphasizes the requirement for efficient learning techniques to improve students' capacity to create multi-step geometry proofs. The step-by-step unrolled technique created by Cheng and Lin (2009) is one such approach. By breaking down significant issues into manageable parts and offering thorough explanations for each step, this technique seeks to assist struggling students in strengthening their proficiency in geometry proof. Numerous studies have proven the efficacy of this tactic (Cheng and Lin, 2009; Cheng *et al.*, 2011).

To sum up, geometric proof is a crucial idea in mathematics that enables mathematicians to verify the veracity of mathematical claims firmly. However, many students need help formulating legitimate or acceptable formal geometry proofs. The step-by-step unrolled method is an effective learning technique to improve students' aptitude in this field.

### **2.3. Learning and Teaching Proof**

A key idea in mathematics, proof has generated a lot of research and controversy in mathematics education. Growing interest has been shown in recent years in both how teachers may help students build and comprehend mathematical proofs as well as how students themselves learn to do so (Hanna and De Villiers, 2018). The many sorts of reasoning that go into creating and assessing mathematical proofs have been the subject of significant investigation. For instance, Schoenfeld (1992) distinguished between deductive reasoning (using logical rules to draw conclusions), heuristic reasoning (using problem-solving techniques to generate ideas), and persuasive reasoning (using rhetorical techniques to persuade others) as the three types of reasoning that

are used in the proof. By examining various forms of proof-related reasoning, such as visual reasoning (Arcavi, 2003) and analogical reasoning (Leron and Hazzan, 2006), other scholars have built upon this paradigm.

Mathematical proof serves further goals in mathematics and mathematics education in addition to these. The proof involves more than just verification; it also involves articulating, justifying, and comprehending mathematical ideas (Uğurel, 2012). Proof lays the groundwork for more mathematical thinking and discovery while also confirming the accuracy and validity of mathematical knowledge (Uğurel, 2012).

The importance of language and communication in proof has been the subject of more research. For instance, Reid (2005) contends that for students to succeed at proof, they must have both verbal and mathematical abilities. Language, they claim, plays a significant role in the construction of mathematical arguments. Overall, the literature on proof in mathematics education emphasizes the difficulty of this idea and the difficulties that instructors and students have when using it. However, it also offers insightful information about how teachers can aid this process and how students learn about proof.

Mathematics education uses geometry and mathematical proof for a variety of reasons. Geometry's significance may have been overshadowed in recent years by the rising emphasis on other subjects in mathematics education (Jablonski and Ludwig, 2023). However, according to Jablonski and Ludwig (2023), geometry continues to be an essential part of research in mathematics education.

There are various important components that make up the definition of students' ability to understand geometric proofs. Understanding the form and reasoning of deductive arguments is the first need, as noted in the research by (Miyazaki *et al.*, 2016). For students to participate in the process of creating and evaluating geometric proofs, they must have this knowledge. Additionally, the capacity to engage in rigorous geometric deduction from intuitive conceptions to formal proofs, as highlighted by

(Zhang and Wong, 2021), is a vital component of students' skill in comprehending geometric proofs. Furthermore, Dintarini's (2022) study emphasizes the value of spatial thinking in geometric problem-solving, which is directly related to the comprehension of geometric arguments.

To sum up, the definition of students' comprehension of geometric proofs includes their ability to apply geometric and visuospatial thinking skills, build legitimate proofs, engage in rigorous geometric deduction, understand the structure of deductive proofs, and recognize the significance of proofs in mathematics.

The development of students' geometrical practices and thought processes is one goal of geometric proof (Jablonski, 2023). Understanding and using geometrical principles and qualities includes geometric thinking (Altakhayneh, 2021). Students can improve their grasp of geometric ideas and their capacity for mathematical reasoning by participating in proof (Altakhayneh, 2021). In addition to assisting students in comprehending and applying geometric theories, geometric proof also aids in problem-solving (Altakhayneh, 2021).

Supporting argumentation and reasoning in geometry is another goal of geometric proof (Jablonski, 2023). According to Cervantes-Barraza *et al.* (2020), proof entails giving logical reasons for mathematical assertions and arguments. Students build strong arguments and proofs in geometry class by using mathematical definitions and properties (Cervantes-Barraza *et al.*, 2020). Students improve their capacity to reason mathematically and effectively convey their mathematical reasoning by engaging in debate and proof (Cervantes-Barraza *et al.*, 2020). Geometric evidence is also essential for teacher preparation and advancement (Jablonski, 2023). To effectively teach geometric ideas and proof to their students, teachers need to have a solid grasp of geometry (Jablonski, 2023). The development of teachers' knowledge and pedagogical techniques for teaching geometry and proof is a common topic of coursework and experiences in teacher education programs (Jablonski, 2023).

Another area of study and application pertaining to geometric proof is the use of digital technologies for teaching and learning geometry (Jablonski, 2023). Students' involvement and comprehension of geometric ideas and evidence can be improved by using digital tools, such as dynamic geometry software (Abdelfatah, 2011). These resources provide students the chance to study and manipulate geometric objects, form hypotheses, and create and verify geometric proofs (Abdelfatah, 2011).

Overall, geometric and mathematical proof has many uses in the teaching of mathematics. They aid in the growth of students' geometrical reasoning, thinking, and argumentation abilities. Along with the use of digital tools for teaching and learning geometry, they also contribute to the professional development and education of teachers. Additionally, proof serves more general purposes in mathematics and mathematics education, including idea clarification, justification, and understanding.

#### **2.4. Difficulties in Learning Geometric Proof**

According to Harel and Sowder (2007), students frequently view proof as a ritualistic and authoritarian task where the main goal is to get the greatest scores possible. Therefore, when pursuing mathematical validation, they frequently choose more formal and algebraic arguments over their original choices. Furthermore, Harel and Sowder (2007) observe that students typically need a deeper comprehension of the meaning and purpose of evidence, believing that proofs just serve as a way to corroborate already established truths.

Maarif *et al.* (2019) have done several studies on the difficulties students have in understanding the notion of proof. These issues are caused by a number of things, such as a lack of understanding of definitions, a poor understanding of the concepts that need to be proved, problems building proofs, an inability to produce examples that clearly illustrate a point, a failure to recognize the importance of definitions in writing proofs, difficulties with mathematical language and notation, and not knowing where to start when verifying a proof.

Furthermore, Selden and Selden (2008) point out that a lot of students, even those who are academically gifted, have trouble understanding the fundamentals of evidence and creating even the most basic proofs. Selden and Selden (2008) propose that some students may need help to discern between mathematical proof and empirical reasoning. Thus, it is important to be aware of this distinction.

According to Harel and Sowder (2007), students face several challenges in their understanding of mathematical proofs. First, a lot of students need help understanding the function of counterexamples and frequently fail to appreciate that only one counterexample is enough to refute a hypothesis. Even in courses where one would anticipate a concentration on proof, these problems with proof understanding may be made worse by a lack of opportunity to participate in activities relevant to proof (Harel and Sowder, 2007).

According to Maarif *et al.* (2019), it can be inferred from the data analysis and discussion that some of the challenges faced by mathematics teaching students in developing the geometrical proof include:

- Difficulty in creating a diagrammatic sketch of a conjecture made entirely with the proper geometrical notation.
- Difficulty in determining the cause-and-effect of geometrical difficulties when conditional clauses were included in the proof.
- The inability to express a conjecture using axiomatic deduction, formulae, or geometrical symbols.
- Lack of understanding and expertise with axiomatic systems and geometrical knowledge.
- Difficulty selecting a manufactured hypothesis that is true.
- Struggled to begin writing proof when they encountered the issue of indirect proof since the assumption of geometrical issues can only be verified by direct evidence.
- Challenging in formal proof writing.

Selden and Selden (2008) underline that the challenges students encounter are made worse by the lack of time spent teaching them how to build proofs. They also draw attention to how some college students adopt unusual conceptions of evidence, turning to imitation tactics rather than understanding the logical foundations of arguments. In addition, students could have trouble deciphering the logical structure of statements, particularly when it comes to understanding the order of quantifiers and figuring out if a certain theorem is proved by an argument (Selden and Selden, 2008).

### **2.5. Technology - Enhanced Applications**

In today's educational landscape, the widespread use of technology among students has opened up new avenues for enhancing learning experiences. The integration of technology in geometry education has shown significant promise in improving students' understanding and engagement. Oliver (2010) stated that student inquiry is a fundamental goal of contemporary science, and there are many Web 2.0 tools that support this goal. "A number of Web 2.0 tools are available to assist students in developing conceptual understanding. Graphic organizers have been used in science for decades both as a process to help learners develop their understanding and as an assessment tool to help instructors identify student misconceptions." (Oliver, 2010, p.1). Web 2.0 technologies are important in geometry education because they provide interactive learning environments. Students can share, discuss, and gain a deeper understanding of geometry content on online platforms and social media tools. In addition, these technologies provide teachers with valuable tools to track student progress and adapt teaching methods to individual needs. The interactive learning environments offered by Web 2.0 enable students to learn geometry topics more effectively and enjoyably.

According to Oliver (2010), math teachers and students can represent mathematical concepts and solve problems thanks to Web 2.0 tools. "To assist with representing mathematical concepts, Sketchcast is a Web-based tool that allows users to record a virtual sketch with or without voice narration, then save or share their sketch with others. This tool would allow a student or teacher to diagram a problem, much as they

would on a chalkboard. Students could turn in their worked-out solutions to problems for analysis by the instructor, who could diagnose misconceptions by reviewing the step-by-step production of the student's solution.” (Oliver, 2010, p.4)

In mathematics instruction, GeoGebra is a dynamic geometry program that is often utilized (Doğan and İçel, 2011). Doğan and İçel (2011) assert that GeoGebra can help students do better in geometry. After two weeks of instruction with and without GeoGebra, they gave two groups of eighth-grade students a post-test as part of their study. According to Doğan and İçel (2011), the usage of GeoGebra as dynamic mathematics software is expanding globally. As a crucial component of learning geometry, it offers visualization, estimation, conjecture, construction, discovery, proof, and more in addition to dynamic geometry construction. An interactive geometry system is called GeoGebra. With points, vectors, segments, lines, conic sections, and functions, you can build things and then change them dynamically. An algebra window plus a graphics window make up GeoGebra's user interface.

Overall, it has been demonstrated that the usage of dynamic geometry software, such as GeoGebra, is beneficial in the teaching of mathematics. It is crucial to remember that processing particular talents, emotional characteristics, psycho motor abilities, and self-regulation requires extra consideration (MoNE, 2007).

Numerous studies have examined the use of GeoGebra in mathematics instruction and have shown that it is beneficial in enhancing students' learning outcomes and conceptual grasp of mathematics. Arbain and Shukor (2015) did research in Malaysia to look at how well GeoGebra software affected the learning of mathematics among 62 students. The findings demonstrated that students' attitudes regarding using GeoGebra for learning were favorable, and they outperformed their peers in typical classroom settings in terms of learning outcomes.

After examining 36 impact sizes from 29 main studies, Juandi *et al.* (2021) discovered that, on average, students who were exposed to GeoGebra-based learning

outperformed those in conventional schools. The use of GeoGebra software generally had an effect size of 0.96, which is a considerable improvement in students' mathematical skills. According to Jelatu *et al.* (2018), GeoGebra can balance students with strong and low spatial abilities' grasp of geometry ideas. When taught with GeoGebra as opposed to conventional techniques, students with limited spatial ability demonstrated higher understanding. According to Weinhandl *et al.* (2020), employing GeoGebra has a beneficial impact on students' mathematical learning outcomes and can boost participation and interaction among students in math classrooms. Students who studied utilizing the GeoGebra-assisted problem-based learning technique had superior problem-solving skills than those who studied with conventional learning, according to experimental research by Septian *et al.*(2020). The study also revealed a link between students' views regarding using GeoGebra to learn and their development of mathematical problem-solving skills.

Septian and Monariska (2021) investigated how GeoGebra-based game teaching may increase students' mathematical comprehension of a system of linear equations. The findings indicated that GeoGebra-based gaming teaching improved students' mathematics reasoning abilities more effectively than traditional instruction. Putra *et al.* (2021) investigated how potential primary teachers used GeoGebra to create 3-D forms. Incorporating GeoGebra into primary beginning teacher training was seen well by prospective teachers, who also felt that it helped them comprehend the concepts of volume and surface area.

These studies collectively offer compelling evidence that GeoGebra software may enhance students' learning outcomes, conceptual comprehension, problem-solving skills, and learning attitudes when used in mathematics instruction. GeoGebra is a useful tool for increasing mathematics education because of its attributes, including its mix of geometry, algebra, and calculus, dynamic visualization capabilities, and interactive learning experiences.

## 2.6. Enhancing Student Learning in Geometry

There are several methods and tactics that may be used to improve students' grasp of geometry. The use of dynamic geometry software (DGS) such as GeoGebra in mathematics classes is one of the effective tools that has been found to improve mathematical ability (Chan and Leung, 2014). According to Chan and Leung's (2014) study, using DGS in elementary school had a larger impact on students' knowledge of geometry. This is in line with the Van Hiele paradigm, which contends that children should concentrate on achieving the first two stages of geometric comprehension throughout elementary school (Chan and Leung, 2014).

A contextual teaching and learning technique called REACT (Relating, Experiencing, Applying, Cooperating, and Transferring) was created by CORD (The Center for Occupational Research and Development) in the United States (Crawford, 2001). These strategies emphasize contextualized teaching and learning, which is a cornerstone of constructivism. The finest teachers employ REACT, and it is backed by studies on how individuals learn most effectively (Crawford, 2001). The GeoGebra-assisted REACT technique is another method that has been shown to improve students' knowledge of geometry. Jelatu *et al.*'s (2018) study looked into how this tactic affected Grade 8 students' comprehension of geometry subjects. According to the findings, as compared to traditional teaching strategies, the GeoGebra-assisted REACT technique led to greater accomplishment in learning geometry topics. Using technology, like GeoGebra, in geometry lessons might help students comprehend concepts more conceptually.

The use of ethnomathematics methodologies can improve students' comprehension of geometry in addition to technology-based teaching methods. However, due to a lack of education and experience, teachers may find it challenging to include ethnomathematics in geometry lessons (Sunzuma and Maharaj, 2019). Sunzuma and Maharaj (2019) stated that stress the significance of instructors receiving training in the integration of ethnomathematics to enable its successful use in geometry education. For instructors to successfully integrate cultural contexts into geometry education, they

must possess the knowledge and abilities to integrate ethnomathematics methodologies (Sunzuma and Maharaj, 2019).

It has also been demonstrated that practical exercises help students become more adept at solving geometrical puzzles. Functional near-infrared spectroscopy (fNIRS) was utilized in a study by Shi *et al.* (2023) to look at the brain activity patterns of middle school children during practical geometry instruction. The findings demonstrated that practical application facilitated the growth of more intricate understandings and enhanced geometric problem-solving (Shi *et al.*, 2023). This shows that giving students practical learning opportunities might improve their spatial thinking and geometry problem-solving abilities.

Additionally, meta cognitive techniques can significantly help students' comprehension of geometry. In a research published in 2021, Pasandaran *et al.* investigated students' comprehension of geometry concepts and suggested effective learning techniques for problem-solving. The results showed how crucial it is to provide educational opportunities that emphasize conceptual understanding over procedural knowledge (Pasandaran *et al.*, 2021). Teachers may aid students in comprehending geometry ideas by relating prior knowledge to new information and employing a variety of representations, examples, and manipulations (Pasandaran *et al.*, 2021). In conclusion, several methods may be used to improve students' knowledge of geometry, including the use of dynamic geometry software, the integration of ethnomathematics, practical experiences, and meta cognitive tactics. These methods provide students the chance to participate actively in the learning process, link ideas, and get a better comprehension of geometry. Teachers may support students' mental grasp of geometry and mathematical achievement by including these tactics in their geometry lessons.

### 3. SIGNIFICANCE OF THE STUDY

Solving geometric proofs skillfully can provide practical benefits to students in their academic and professional lives. Making and understanding proofs is vital in fields such as science, engineering, and mathematics. Improving students' ability to understand geometric proofs is a critical component of a balanced mathematics education and can provide many benefits to students. My aim as a teacher was to investigate how I could increase my students' geometric reasoning skills and success by overcoming their prejudices against geometry.

Wong, Yin, Yang, and Cheng (2011) noted that proving geometry theorems requires complex skills. Students can start with concrete, graphical representations instead of struggling with abstract and complex representations. Healy and Hoyles (2000) also noted that proving requires various student abilities.

I thought that this research would achieve its purpose and increase my students' interest and success in class. Also, when I talked to other teachers in the location of my school, I saw that they had similar problems in their classes. For this reason, I think it will be a guide for teachers in similar situations. In addition, proving geometric expressions, formulas, or theorems by stating the reason makes it easier to understand the concepts. In addition, proof will contribute positively to the mental development of people and will improve their mathematical reasoning and analytical thinking capacities (Kaplan and Öztürk, 2017). In addition, while teaching this proof, I applied the step-by-step strategy of Cheng and Lin (2009) with technology. This strategy is important because students who cannot answer questions that require two-step solutions or use only one theorem to support their answers are below average. Their ability to prove geometry is limited to the first level of proof only (Cheng and Lin, 2009). In other words, below-average students are students who cannot complete a task that requires several steps (Cheng and Lin, 2009)

This study combines the SU strategy with technology -enhanced teaching and creates an effective teaching method suitable for the age requirements. I have witnessed that teachers generally prefer the presentation method due to the intensive nature of the MoNE curriculum and that they give importance to memorization rather than the logic and meaning of the concepts. With this research, both teachers will save time, and students will be able to give importance to the concept content they need rather than memorize it.



## 4. STATEMENT OF THE PROBLEM

The purpose of this study is to investigate and compare the students' Van Hiele geometric reasoning level, their achievement in geometry, and their perceptions about geometric proof before and after the geometry lessons integrated with technology-enhanced applications and step-by-step unrolled strategy.

### 4.1. Variables

The variables of the study are students Van Hiele's geometric reasoning level, students' perceptions about geometric proof and students' achievement in geometry. The descriptions and operational definitions of the variables are stated below.

According to Duval (1998), geometric reasoning is one of the three kinds of cognitive processes involved in learning geometry, along with visualization and construction. Geometric reasoning involves the process of utilizing visual representation, construction, and reasoning processes to interpret and manipulate geometric objects and statements (Pittalis & Christou, 2010). A student's Van Hiele geometric reasoning level was determined according to the levels created by obtaining the results of the Van Hiele Test.

The operational definition of student perception in educational research encompasses a wide range of factors that influence students' attitudes, behaviors, and academic outcomes. Studies have examined various dimensions of student perception, including teacher closeness, peer engagement, motivation, critical thinking engagement, and classroom culture (Jung, 2006). These perceptions are of vital importance because they affect students' interactions in the learning environment and their overall learning experience (Sawada *et al.*, 2002). A student's perceptions about geometric proof in the course were determined by the reflection questions asked of the students before, during, and after the course.

Academic success is the measure of the knowledge and skills a student acquires during the education process. This success is evaluated by the student's performance in classes, exam results, project work, and overall progress in their education life. According to Giofrè *et al.* (2013), it is crucial to consider both the academic performance and the cognitive processes that underpin geometry-related learning to define students' geometric achievement. Knowledge of students' success in geometry requires a knowledge of the cognitive processes that underlie geometry-related scholastic accomplishment, including the impact of working memory and intelligence in children (Giofrè *et al.*, 2014). A student's geometry achievement is determined according to the score obtained from the Unit Achievement Test prepared by the researcher.

#### 4.2. Research Questions

According to Sagor (2010), research questions are generally examined in three dimensions in action research. The first is the action dimension, that is, it emphasizes the action and implementation that is the focus of the research. What the researcher plans to do and what kind of action researcher will take is addressed in this dimension. The second is the change dimension, that is, it examines the change targeted by the research and how this change will be measured. The changes that occur in the process of reaching the success goals targeted by the researcher are evaluated in this dimension. The third is the relationship dimension, that is, it examines the relationship between the researcher's actions and the changes that occur. Which actions lead to which changes and how this relationship can be understood are addressed in this dimension. These three dimensions ensure that the research questions are addressed comprehensively and guide the researcher in both creating the action plan and evaluating the results obtained.

The first of the following research questions examines the action dimension, the questions from the second to the fifth question examine the change dimension, and the sixth question examines the relationship dimension.

- (i) What are the specific features of the geometry unit plan developed to support students with low academic achievement?
- (ii) What are students' Van Hiele geometric reasoning levels before and after the implementation of the geometry unit based on the technology-enhanced step-by-step unrolled strategy?
- (iii) What are the students' perceptions related to geometric proof before and after the implementation of the geometry unit based on the technology-enhanced step-by-step unrolled strategy?
- (iv) What are the students' reflections on the geometry unit based on the technology-enhanced step-by-step unrolled strategy during the implementation of the unit?
- (v) Is there a statistically significant difference between the geometry achievements of the students before and after the geometry unit based on the technology-enhanced step-by-step unrolled strategy?
- (vi) What is the relationship between students' engagement with technology-enhanced step-by-step unrolled strategy and their improvement in geometric reasoning skills?

## 5. METHODOLOGY

This chapter provides detailed information about the methodology. It contains the research design, participants, settings, procedure, instruments, data collection, and data analysis in this context.

### 5.1. Research Design

Kurt Lewin introduced action research methods in 1946 as a social psychology research tool. Action research is the process by which practitioners examine their own work to address their own practical issues (Corey, 1953).

The research fits the reasons given in the definition. I used the action research method because I am doing this research on the difficulties experienced by my students at the school where I work, and I conducted this research with my students. Action research provides teacher researchers with a method of producing solutions to the problems they encounter in the teaching-learning environment (Gay *et al.*, 2009). Action research is a strategy that seeks to find realistic solutions to challenges and problems. It is similar to applied research. Action research is basically learning by doing.

According to Metteta (2001), step one of action research is to define a question or problem. In 2022, I was appointed to a public school by the Ministry of National Education and started working. I took applied courses during my university education. At the same time, I completed my internship training at two different private schools. The school where I started working was different from these schools. The difficulties I encountered were that the economic conditions of the students and the school were average and below.

I clearly experienced that students needed to be stronger in terms of course success. I understood that they had difficulty in performing some basic algorithms,

as they could not answer the questions I asked during the lesson and from the mistakes they made. The topics I wanted to teach took longer than usual because I always had to remind them of their prior knowledge. For example, when a radical or exponential expression came up in the Problem topic, the children did not know what to do. For this reason, while the problem topic needed to be understood, the lesson suddenly turned into a lesson on radical numbers.

My first aim was to teach geometric proof to students. When the subject of geometry was brought up or when they saw a shaped question, some said it was difficult because of prejudice, and some said it was difficult because they could not do it. I can give an example from the subject of problems. When they saw the shape in a circular track question in speed problems, they said it was difficult, and they could not do it without reading the question. Besides, I observed some difficulties in geometric proof. Some of these challenges are the inability to produce examples that clearly demonstrate a point, failure to understand the importance of definitions in writing proofs, difficulties with mathematical language and notation, and not knowing where to start when writing a proof. Additionally, Cheng and Lin (2009) stated that students need help creating valid or acceptable formal geometry proofs.

In addition, in case of a problem, students only expressed the information they memorized and could not put the events in a logical order. For example, I asked two different types of questions. The first one was given a triangle, and all the information was given on it; the other question consisted of verbal expressions, and they had to draw the shape themselves. I observed that they had more difficulty with the second question. At the same time, students stated that they could not visualize the question. When I asked how they found the answers and the reason why they did these operations, they could not explain the reason. They replied that it should be like that. I realized that the main reason for this might be that students cannot establish cause-effect relationships. According to Maarif *et al.* (2019), it is difficult for students to determine the cause-and-effect conditional clauses included in the proof.

Since the purpose of action research is to inform decision-making, the question or problem should look at something under teacher control, such as instructional strategies, student assignments, and classroom activities (Metteta, 2001). My goal was to find out how to solve the problem I described above.

According to Metteta (2001), the second step of action research is to review the literature. This review helped me find a new method. Because of the large number of topics in the curriculum, I could not see any change in the averages of the students in the courses I taught with the direct instruction method in the first semester. This situation led me to try new methods. Cheng and Lin (2009) emphasize the need for effective learning techniques to develop students' capacity to create multi-step geometry proofs, as students need assistance. This method is the step-by-step technique created by Cheng and Lin (2009). By breaking down important topics into manageable parts and providing comprehensive explanations for each step, this technique aims to help struggling students strengthen their proficiency in geometry proofs. Numerous studies have proven the effectiveness of this tactic (Cheng and Lin, 2009; Cheng *et al.*, 2011).

The students' achievement levels were low, and for this reason, I observed that their perspectives on mathematics and geometry were not positive and that they had prejudices. I observed that trying to learn by rote and direct teaching prevented them from learning. They could not explain the operations performed, what the operations meant, or why they did this. In order to solve these problems, I decided to conduct this study to investigate how students' ability to understand geometric proofs, their success in geometry, and their perceptions about geometric proofs can improve if they learn the course in a way other than direct teaching.

According to Metteta (2001), step three of action research is planning a research strategy. In addition, according to Kyrö (2004), action research is a methodical procedure that involves problem-solving and investigation through a cycle of planning, doing, observing, and reflecting. It is a transformational strategy that seeks to raise educational achievements while also enhancing living quality (Schoen and Nolen, 2004).

According to Schoen and Nolen (2004), action research is defined by its dedication to critical thought, inquiry, and the active engagement of practitioners. Individuals are empowered to make adjustments and improve results through this collaborative and participatory method (Coker, 2020).

According to Kyrö (2004), action research starts when an issue or a need for improvement is identified. The planning step comes next, where researchers create an action plan to deal with the discovered issue. After research, I found the step-by-step unrolled technique that was created by Cheng and Lin (2009). I learned that this technique strengthened students' proficiency in geometry proof by breaking down important topics into manageable parts and providing comprehensive explanations for each step, and I began to integrate this technique into my lesson plan. At the same time, knowing that the use of technology -enhanced applications in education has a positive effect on students' success, as stated in the literature, I developed lesson plans using technology -enhanced applications with the SU strategy. This lesson plan is provided in Appendix C.

Kyrö (2004) assumed that the intended interventions or adjustments are put into practice during the action phase. According to Moch *et al.* (2016), data is gathered in this phase using a variety of techniques, including surveys, interviews, observations, and document analysis. I implemented the lesson plans I prepared for this stage, and I planned lessons according to this plan for two weeks. Before the lessons, the Van Hiele test was applied to the students to measure their geometry reasoning levels. In addition, reflection questions (Appendix A) were asked to learn their perceptions about geometry proofs. While the course process was continuing, data was collected with reflection questions. At the end of the lessons, the same tests applied before the study were repeated as a post-test, and a Unit Achievement Test was performed. At the same time, students' exam scores were also considered. In order to understand the efficacy of the lessons, the data obtained during the observation phase were analyzed and interpreted (Kyrö, 2004).

Finally, the reflection phase entails critically evaluating the findings and the research process's experiences (Schoen and Nolen, 2004). This stage enables researchers to assess their findings, pinpoint their key takeaways, and change their plans for further steps (Kyrö, 2004). According to Gebhard (2005), the opportunity to pose and address difficulties in practice, the development of reflective abilities, and the formation of a forum for debating teaching concerns and beliefs are all advantages of action research. Zhang *et al.* (2014) stated that, as stakeholders are involved in the research process and the design of interventions, it also enables a direct connection between research and organizational results. Action research does, however, have several drawbacks, including a limited emphasis on issues and a linear procedure that could not adequately reflect the complexity of real-world circumstances (Gebhard, 2005).

## 5.2. Participants and the Settings

This study was conducted in a state high school in Istanbul. The high school is not a high school that accepts students by central exam. For this reason, the success of the students in the high school entrance exam is low. The majority of the students come from low or middle-socioeconomic-status groups. I have been working as a mathematics teacher in this high school for two years. The age range of the students is between 14 and 16, 2 participants are 13 years old, and 1 participant is 17 years old. Since the school is a girls' Anatolian Imam Hatip high school, all the students are girls. The grade level of the students is 9th grade. The class size is 34. The reason for the different age ranges of the students is that there are students who are late to school or fail the grade. I conducted the research in the school where I work and in the classes where I teach. Since the students who participated in the research were under the age of 18, a consent form was prepared, and consent was obtained from their parents. Since the school is affiliated with the Ministry of National Education, permission was obtained from the ministry for the research to be conducted in the school. In addition, ethics committee approval was received from Boğaziçi University Science and Engineering Human Research Board with the application number 2023/15. These permissions are provided in Appendix G and Appendix H, respectively.

At the beginning of the lesson, the purpose of the study was clearly explained to the students. It was stated that the procedures that students who did not want to participate would not be used as data within the scope of the research. Students were informed that no sanctions would be applied if they did not allow their data to be used for the research. It was explained that participation in the study was completely voluntary and that they would not receive any financial income or a grade. The research data and the names of the volunteers participating in the study are kept in a locked cabinet. Codes were assigned to the students in the research, and expressions such as student one were used. There is a security guard and camera system in the building where the data is kept. Only the researcher can access these files.

### **5.3. Procedure**

The geometry unit is included in the curriculum and the curriculum of the Ministry of Education on triangles. The unit lasted for two weeks. Before the study, students were given the Van Hiele test to measure their geometry reasoning level and reflection questions to learn their perceptions about geometry proofs (Appendix A). During the instruction of the unit, reflection questions were asked to the students in order to get feedback about lessons. In addition, a Unit Achievement Test was applied. At the end of the lessons, the same tests given before the study were repeated as the post-test.

This lesson plan is based on a technology-integrated and step-by-step unrolled teaching strategy to develop the thinking of students with below-average achievement in geometric proof, increase their geometric reasoning levels, and increase their success. The plan includes teaching objectives, learning materials, teaching methods, and assessment tools. The process of creating the lesson plan is explained in Table 5.1 below.

Table 5.1. Lesson plan implementation process.

<b>Date</b>	<b>Process</b>	<b>Technological Tools</b>
02.05.2023	Van Hiele Pre-Test	
05.05.2023	Unit Achievement Pre-Test	
17.05.2023	Worksheet 1	Video Menti
17.05.2023	Worksheet 2	Geogebra Proof Desmos
24.05.2023	Worksheet 3	Desmos
24.05.2023	Worksheet 4	Desmos
26.05.2023	Worksheet 5	Geogebra Pythagorean Inequality
26.05.2023	Worksheet 6	Desmos
30.05.2023	Worksheet 7	Desmos
02.06.2023	Unit Achievement Post-Test	
07.06.2023	Van Hiele Post Test	

Table 5.2. Technological application links.

<b>Technological Tool</b>	<b>Link</b>
Menti	<a href="https://www.menti.com">https://www.menti.com</a>
Geogebra Proof	<a href="https://www.geogebra.org/m/bgndedjt">https://www.geogebra.org/m/bgndedjt</a>
Geogebra Pythagoras inequality	<a href="https://www.geogebra.org/m/sxbrktn6">https://www.geogebra.org/m/sxbrktn6</a>
Desmos	<a href="https://teacher.desmos.com/activitybuilder/custom/646e22183598fd219bd46ea1?lang=tr">https://teacher.desmos.com/activitybuilder/custom/646e22183598fd219bd46ea1?lang=tr</a>

The need to prepare a lesson plan arose from the problems I mentioned earlier. My search for solutions to the problems in my own classroom led me to this action

research. This study was carried out in the second semester of the academic year. For this reason, I had the opportunity to observe my students a lot in the first semester. My students and I had six hours of lessons per week. Although it was the course they took with the most hours, they frequently stated that the course they had the most difficulty with and were afraid of was mathematics. I clearly saw from the exam results and the semester average that my students were having difficulty, and their success was low. Then, I set goals for my students in the first stage of the lesson plan. The students started to say that they definitely did not understand and would not be able to succeed before moving on to the geometry section. The subject of triangles in the 9th grade is a very important subject for the high school curriculum because when I examined the curriculum and according to my experiences in the previous year, the subject of triangles constantly came up in new subjects in the later grades. For example, in the 10th grade, on quadrilaterals, or in the 11th grade, on trigonometry, I had to constantly return to topics such as the Pythagorean and Euclid theorems because students were lacking in previous topics. I discovered here that students' memorization of formulas did not stay in their minds for a long time. Because they both forgot the formulas they memorized and did not know how and where to use which formula.

Based on these experiences, I set new goals for my students. In the first step, the general and specific goals of the course were determined. These goals were created to improve the students' geometric reasoning levels, to ensure that they think positively about geometric proofs, and to increase their success. Later, the Van Hiele and Achievement tests were applied as pre-tests to measure the students' geometric reasoning levels and their existing knowledge about the topics. I started to prepare a lesson plan in line with these results. The content of the lesson plan was structured to cover the Pythagorean theorem on triangles, and the mathematical big idea of the lesson plan is "Corresponding sides of similar triangles prove the Pythagorean Theorem is true for all right triangles. Use the structure of a set of equations from the Pythagorean Theorem and the structure of a set of right triangles to make matches between the equations and the right triangles."

Seven worksheets were prepared, each covering a specific topic and including step-by-step explanations. These working papers are included in Appendix E. It was planned to make the lessons more effective and attractive by using Geogebra, video and online resources. With these, students' active participation in the lessons was encouraged. In addition, this lesson plan was transferred to the Desmos application to increase interactivity with technology. This application is compatible with the worksheets.

First, worksheet 1 was prepared. I started the lesson by first learning how they felt. For this, I used a start screen on Desmos. In order to both entertain and motivate the students, a video containing a joke about the Pythagorean theorem and to ensure that it would be memorable was shown. Then, I asked them what came to their minds when they heard the Pythagorean theorem. To see the general perceptions of this question, students answer the question by logging into the Menti application with the help of their phones. This application creates a word chart on the smart board, and the more words are written, the bigger they get. The purpose of this question was to see how much the students knew about this theorem, which was mentioned a little in the 8th grade. According to the feedback I received from the students, I saw that they had forgotten the Pythagorean theorem or only remembered some memorized information.

In the second question, I explained a problem situation to the students. Again, I made the question interesting for them by using the names of their teachers at school. I aimed to find the sides of the triangle by asking the students what was needed to find the area of the triangle. Then, I told them to complete the problem by paying attention to the areas and sides. With this worksheet, the students learned the basic part of the Pythagorean theorem, which is the sum of the squares of the perpendicular sides equals the square of the hypotenuse. However, since I did not want them to memorize this, I prepared the second worksheet, which included proof of the Pythagorean theorem. In this part, I asked the students how to make a proof, and then I gave them some directions so that they could make a proof in accordance with the SU Strategy I used. The students used a certain measurement while making the proof. I created a GeoGebra link so they could see that this proof is valid for all measurements. Here, they changed

the measurements and generalized the proof.

The third and fourth worksheets were prepared by me to have students find situations where the hypotenuse is an integer. Again, here, with the SU strategy, I asked them questions that could both provide clues and make them think and find the answer. In addition, in the fourth worksheet, they discovered that the hypotenuse length of a right triangle with an integer length is an integer multiple of a similar triangle, and they experienced it through Geogebra. In the fifth worksheet, they discovered the Pythagorean inequality theorem by changing the angles of the triangles in the application I prepared in Geogebra. Again, I helped the students with the questions I asked. Then, in the sixth worksheet, I aimed to practice and reinforce what they learned. In the last worksheet, I gave them the opportunity to apply the SU strategy on their own. At the end of the term, I used a fun exit card to learn about their feelings.

Various assessment tools have been developed to measure student progress and course impact. The Van Hiele test, reflection questions and achievement test administered before the lesson were repeated. Additionally, reflection questions were asked during the two-week training period.

The validity of the lesson plan was ensured by clearly and explicitly determining the objectives and preparing the content and materials in accordance with these objectives. In addition, valid teaching strategies and methods in the literature were taken into consideration while creating the lesson plan. The academic and pedagogical accuracy of the plan was confirmed by consulting the opinions of a doctoral student, an assistant professor, and three other mathematics teachers in my school who are experts in this field.

The reliability of the lesson plan was ensured by the consistency and repeatability of the assessment tools used. Although I had to change some sections when the lesson plan was used for the second time, it can be adjusted according to the student's

prior knowledge and achievement levels since it is a flexible plan. Pre-tests and post-tests, as well as regular observations, were designed to measure students' performance objectively and consistently. In addition, detailed guidelines were provided on how teachers should implement the lesson plan so that different teachers could reach the same results.

All of these processes ensure that the lesson plan is both valid and reliable. In this way, it is aimed to develop the perceptions of students with below-average achievement on geometric proof, to increase their geometric reasoning levels and to increase their success.

## 5.4. Instruments

### 5.4.1. Van Hiele Test

According to Senk *et al.* (2022), The van Hiele Geometry Test (VHGT) was developed by researchers working on the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project, directed by Usiskin, in the early 1980s. The test is based on the van Hiele model of geometric thinking, which proposes that individuals progress through five levels of geometric understanding: visualization, analysis, abstraction, deduction, and rigor. The test measures a student's level of geometric thinking by asking multiple-choice questions that require them to identify and apply geometric concepts at different levels of understanding. It consists of a 25-item multiple-choice exam with five items designed to measure reasoning at each of the five van Hiele levels in ascending order. Questions 1 to 5 testing Level 1, questions 6 to 10 testing Level 2, questions 11 to 15 testing Level 3, questions 16 to 20 testing Level 4, and questions 21 to 25 testing Level 5.

The validity and reliability of the Van Hiele Geometry Test have been studied extensively. According to Senk *et al.* (2022), several studies have examined the reliability of the Van Hiele Geometry Test (VHGT). For instance, the findings of Senk's (1989)

study linking Van Hiele levels to proof-writing proficiency demonstrate the predictive validity of the VHGT. In a study of Czech secondary students, Haviger and Vojtková (2015) examined the validity and reliability of the first 20 items on the VHGT test. The study's findings imply that these items have high levels of reliability and validity.

Duatepe (2000) translated the Van Hiele geometric thinking levels test created by Usiskin (1982) into Turkish and conducted validity and reliability evaluations. According to the levels, the translated test's validity and reliability coefficients came out to be .82, .51, .70, .72, and .59, respectively. Additionally, the results revealed that the test used in the thesis's Cronbach Alpha reliability coefficient was .44 according to the levels. Permission was obtained from Professor Asuman Duatepe Paksu and Professor Zalman Usiskin to use the test. Permissions are included in Appendix F. In addition, the Cronbach Alpha reliability coefficient used in this research was found to be .387 in the pre-test and .495 in the post-test, according to the levels.. Pearson correlation value was .69 in the pre-test and .65 in the post-test.

#### **5.4.2. Reflection Questions**

The aim of the study was also to investigate the participants' perceptions about the concept of geometric proof and the applied method in the geometry course integrated with technology -enhanced applications. In order to evaluate their perceptions, the written responses of the students to the reflection questions were collected before, during and after the application. These questions are given in Appendix A. Among these questions, there are questions that question the students' experiences. Before coding, the names of the students were hidden and numbers were given instead of their names. The students' answers were itemized under the relevant questions. Accordingly, I collected the students' answers under certain codes. Later, I asked a geography teacher who I worked with at the school and who also worked on coding while doing her master's degree to code. As a result, we reached a consensus on which categories the answers would be placed in as a result of the discussions we held together. We first determined a category for each question, then divided the categories into codes. I did

this to increase reliability because, according to Galura *et al.* (2022), it is important for more than one researcher to independently code the data and then evaluate the coding compatibility in order to increase the reliability of the coding process. Additionally, the coding framework and categories may need to be determined in advance to increase the validity of the coding process (Galura *et al.*, 2022).

### 5.4.3. Unit Achievement Test

A Unit Achievement Test was created to collect data on the geometry achievement of ninth-grade students on the subject of triangles. For content validity, questions were prepared to cover the relevant objectives on the subject of triangles determined by the Ministry of National Education (MoNE) for the ninth-grade curriculum of the 2022-2023 academic year. Unit Achievement Test was prepared by the researcher by examining the questions prepared by the Ministry of National Education, General Directorate of Secondary Education. This test is given in Appendix B.

As the mathematics department at my school, one of the issues we pay the most attention to is that students in different classes or in the same class do not receive significantly different grades despite giving the same answers. To prevent this, we prepare a detailed answer sheet. The exam is graded out of 100 points and the answer key includes step-by-step scoring for each operation. In addition, when there are different answers, we discuss how many points we will give and reach a common decision. In order to be able to evaluate what each student does in the same way, I prefer to read the exams question by question without looking at the names. The achievement test used in this research was also scored in the same way. In order to ensure the reliability of the results, I asked my teacher friend at school to read the same exams. Since we were used to the exams we did at school, we generally gave the student the same score, even though there were very slight differences in some results. We discussed why the scores were different and took the average of the given scores.

### 5.5. Data Collection

Four data sources were used to answer the research questions: (i) Van Hiele Geometry Test, (ii) Reflection questionnaires about participants' perceptions, (iii) Unit Achievement Test, (iv) teacher researcher's field notes . Van Hiele Geometry Test, Achievement test, and reflection questionnaire were administered before starting the instruction and after completing the instruction. The lessons lasted for two weeks. Participants' written works during instruction were collected. The following Table 5.3 shows the process of data collection.

Table 5.3. Process of data collection.

Before Instruction	Instruction Period	After Instruction
Van Hiele Geometry Test	Lesson-related	Van Hiele Geometry Test
Reflection questions about proof	feedback questions	Reflection questions about proof
Unit Achievement Test	Teacher researcher's field notes	Unit Achievement Test

### 5.6. Data Analysis

Van Hiele and Unit Achievement Test results were measured quantitatively. Ninth-grade participants took the Van Hiele test to determine their level of geometry reasoning before and after attending classes based on the Step-By-Step Unrolled strategy.

Depending on the normality test results, non-parametric Wilcoxon signed-rank test was used to compare the pre-test and post-test scores of geometry achievement and Van Hiele test results of the participants. For small sample sizes, according to Privitera (2017), the normality assumption is most crucial. Since the standard error decreases with bigger samples ( $n > 30$ ), this assumption is less critical as a result. (Privitera, 2017). The maximum number of students is 34. The number of students

participating in the achievement test is 28. In addition, when the distribution of the results was examined in SPSS, it was seen that it was not normal. The distribution of the population's data does not have to be normal for non-parametric tests to work (Privitera, 2017).

Since the data do not meet parametric assumptions, which is normality, a non-parametric test was required. As a non-parametric substitute for the repeated measures t-test, the Wilcoxon signed-ranks test compares results at times 1 and 2 by converting them to ranks rather than averages (Pallant, 2010). The Wilcoxon signed-ranks test was used to analyze the Van Hiele and Unit Achievement Test results.

Reflection questions were asked before and after the lesson, and the answers to the questions were analyzed qualitatively. First, code words were assigned to their answers to reflection questions, and then themes were created from the codes.

## 6. RESULTS

In this section, findings regarding how the student group who were taught with technology-enhanced step-by-step unrolled strategy, geometry reasoning levels, their perceptions about this teaching method, and their geometry achievements included. Data obtained from the Van Hiele test, reflection questions, and Unit Achievement Test were used to express the findings. The findings obtained from analyzing the data collected for the research questions are presented with tables and explanations in accordance with the order of the research questions.

### 6.1. Features of the Geometry Unit Plan

As a teacher researcher, I constantly observed my students. In this section, I included findings regarding the functioning of the lesson based on secondary data source. Special features of a geometry unit plan developed to support students with low academic achievement are designed to make the learning process more effective and increase their achievement level. Using various strategies and tools, this plan makes geometry more understandable and interesting to students.

First, using a step-by-step strategy, complex concepts are explained to students from simple to complex. When teaching the Pythagorean theorem, I first introduced the sides of a right triangle and then went through an example problem step by step to ensure that they understood each step. At each stage, I asked students questions and received feedback. For example, I checked students' understanding with questions such as "How can we find the hypotenuse of a right triangle?" Students stated that they understood the topic better and learned complex concepts by digesting them step by step. This method immediately corrected misunderstandings, increased students' self-confidence, and ensured their participation in the lesson.

The use of technology-supported learning tools also plays an important role in my lesson plan. I supported students' visual and auditory learning channels by using interactive software and applications. Thanks to dynamic geometry software, students had the opportunity to visualize concepts they had difficulty with. These tools make learning more fun by allowing students to learn by trial and error. For example, using software to visually explain a student's question of "How does the Pythagorean theorem work?" makes the concept more understandable.

Planning individual and small group work is also important for a good lesson plan. Small group work develops students' social skills and allows them to learn from each other. A collaborative environment helps students support and trust each other. In the group work we did with students, I encouraged students from different levels to come together and help each other.

Action-based and applied activities are an important part of the lesson plan to attract students' attention. Applied activities based on real-life problems attract students' attention and increase their learning motivation. Students have the opportunity to apply the theoretical knowledge they have learned in practice. For example, there were three male teachers who entered the students' lesson, and they compared us to a triangle because we were good friends. They were surprised when they saw our names and information about us on the worksheet, and this attracted their attention. In addition, I provided regular feedback to the students to determine their progress and deficiencies in their learning process. I presented the feedback in a motivating and guiding way. Since the students were no longer afraid of the mistakes they made and could respond bravely, they participated more in the lesson and increased their interaction with the lesson. I tried to create a positive and encouraging learning environment where students felt safe and supported. I encouraged them to make mistakes as a natural part of the learning process and to always try again.

Additional study materials, resources, and guidance are important for low-achieving students. Extra resources and practices are provided for students to use during their

individual study time. Students practiced a lot, consolidated what they learned, and did not forget the subject as easily as they did when they memorized it.

Also, this lesson plan is based on student-centered teaching. Student-centered teaching methods are applied to encourage students' active participation and ensure that they take responsibility. Students are given opportunities to follow and manage their own learning processes. Since some students had difficulty following the lesson and were not aware of what they needed to learn, the teaching objectives were clearly determined in the lesson plan and shared with the students. At the beginning and end of each lesson, the objectives and how much of them were achieved were discussed.

These features are some of the strategies used to help low-achieving students improve their understanding of geometry. This type of unit plan is designed to increase student achievement and improve their understanding of mathematical concepts.

## 6.2. Geometric Reasoning

Participants' pre-test and post-test scores of the Van Hiele Test, which assesses participants' geometric reasoning levels, were compared. Since the data do not meet parametric assumptions, which is normality, a non-parametric test was required. The Wilcoxon signed-ranks test is a more widely used non-parametric substitute for the related-samples t-test (Privitera, 2017). The purpose of this test is to ascertain whether there is a significant difference between the pre-test and post-test scores of the Van Hiele Test.

Table 6.1. Descriptive statistics related to pre-test and post-test levels of Van Hiele Test.

	<b>N</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>Minimum</b>	<b>Maximum</b>
<b>Pre-test</b>	34	0.85	0.558	0	3
<b>Post-test</b>	34	2.38	0.739	0	4

Table 6.1 indicates that there has been an increase in the mean post-test score of the participants in comparison to the mean pre-test score. On the other hand, the significance of the difference in pre and post-test scores is shown in the following Table 6.2. The pre-test and post-test participants' scores on the Van Hiele test differ statistically significantly, as indicated by the Wilcoxon signed-rank test results ( $Z = 5.045$ ,  $p < .001$ ). The Wilcoxon signed-rank test ranks and test statistics are displayed in Table 6.3 below.

Table 6.2. Ranks of Wilcoxon signed-rank test.

<b>Ranks</b>				
		N	Mean Rank	Sum of Ranks
post-level - pre-level	Negative Ranks	0 <sup>a</sup>	.00	.00
	Positive Ranks	32 <sup>b</sup>	16.50	528.50
	Ties	2 <sup>c</sup>		
	Total	34		
<i>a</i> post-level <pre-level, <i>b</i> post-level >pre-level, <i>c</i> post-level = pre-level				

Table 6.3. Test statistics of Wilcoxon signed-rank test.

<b>Test Statistics</b>	
	post-level - pre-level
Z	-5.045 <sup>a</sup>
Asymp. Sig. (2-tailed)	0,000
<i>a</i> Based on negative ranks	

### **6.3. Students' Perceptions Related to Proof and Their Reflections on the Implementation of the Geometry Unit**

#### **6.3.1. Learning Geometric Proof**

Participants were asked, "How do you think geometric proof is best learned?". The codes and categories for the answers given by the students to the question are presented in Table 6.4. It is noteworthy that the responses from the participants overlap with many themes. Descriptive data regarding the participants' responses before and after the practical lesson regarding their perceptions of learning geometric proof are presented in Table 6.4 below.

According to Table 6.4, we can examine all students who answered the questions under four categories. These categories can be expressed as active learning, getting information, connection, and strategies.

As seen from the pre-and post-reflection questions, participants do not tend to change their perceptions about learning geometric proof as follows: "Geometric proof is best learned by memorizing." and "Geometric proof is best learned from experts and teachers." before and after attending the instruction. Although there seems to be a numerical equality, the students differ. In other words, students who were not in that category at first, later entered the category. For example, when we look at the Learning from expert category, students numbered 2, 19, 22 and 25 changed their minds, students numbered 5 and 12 remained in the same opinion, and students numbered 7, 9, 13 and 16 were added to the category later.

However, in the pre-questionnaire, no participant mentioned that geometric proof is learned with a step-by-step strategy. In addition, in the post-questionnaire, none of the participants mentioned that geometric proof is learned by making connections to real life, geometric proof is learned by making connections to mathematics, and geometric proof is learned by doing experiments and observation. Although few people

mentioned these categories at the beginning, they later looked at the strategy they had just learned.

Table 6.4. Students' responses regarding their perceptions of learning geometric proof.

Category	Codes	Students (pre-question)	f	Students (post-question)	f
Active Learning	Solving Questions	S3, S5, S6	3	S10	1
	Experiment and Observation	S7, S8, S9, S16, S17	5	-	0
Getting Information	Research	S1, S11, S13, S16, S18, S20	6	S8	1
	Learning from the Experts	S2, S5, S12, S19, S20, S22, S24, S25	8	S5, S7, S9, S12, S13, S16, S20, S22	8
Connection	Making Connections to Real-Life	S10, S14, S23	3	-	0
	Making Connections to Mathematics	S4	1	-	0
Strategies	Understanding the Logic	S23	1	S2, S3, S6, S8, S10, S23, S24, S25	8
	Memorizing	S3, S15, S25, S26	4	S4, S8, S23	3
	Visualization	S8, S14, S21	2	-	0
	Step by Step	-	0	S1, S11, S13, S14, S15, S17, S18, S19, S21, S26	10

As can be seen from Table 6.4 above, the number of students who stated that they learned geometric proof with the step-by-step strategy in the final answers increased compared to the number of students in the preliminary question.

There is a decrease in the number of students who say that geometric proof is learned by solving questions, researching, and memorizing formulas.

For each code, sample participant responses from the pre-questionnaire are given below.

*S4: With mathematical operations. (Making Connections to Mathematics)*

*S6: Geometric proof is learned as the questions are solved. (Solving questions)*

*S8: Using the expansion of the cube, we can prove it by seeing that a cube emerges from that expansion, or we can prove that when we add the interior angles of a triangle, we get 180 degrees. (Visualization)*

*S9: With experiments and observations in math class and with experiments that have been done before. (Experiment and observation)*

*S12: It is learned by examining the studies done by experts. (Learning from the Experts)*

*S14: We can tell what shape objects are with shape toys. (Making Connections to Real-Life, visualization)*

*S16: It can be learned by using shapes, observing, researching, and trying to reach a conclusion. (Experiment and observation, research)*

*S23: We can learn with examples, or we can prove it from real life if we know how provable formulas are formed and understand the logic of it. (Understanding the Logic)*

*S25: I think we can understand the lesson well by listening, studying, and learning. By memorizing the formulas and studying. (Memorizing)*

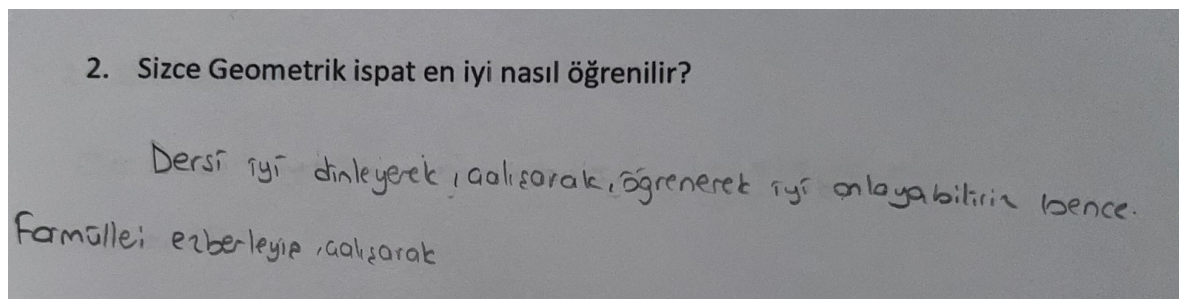


Figure 6.1. Geometry proof learning question student pre-answer example.

For each code, sample participant responses from the post-questionnaire are given below.

*S7: By listening carefully to the explanation of the subject and the teacher. (Learning from the Experts)*

*S8: It can be learned if it is researched and curious. By knowing the geometric proof, I wrote above. By working. By trying to understand the geometric proof and proving it ourselves, it can be learned better. (Research, memorizing)*

*S10: By applying shapes and understanding logic and solving questions (Solving questions, Understanding the logic)*

*S11: Breaking the topic into pieces (Step by Step)*

*S17: Proving with operations. Proceeding one by one. (Step by Step)*

*S21: We can prove geometry through shapes by doing the steps given step by step. (Step by Step)*

*S22: It is learned from what the teachers tell. (Learning from the Experts)*

*S23: By grasping the formulas and understanding the logic ( Understanding the Logic)*

*S24: It is learned by understanding the logic of the operation. ( Understanding the Logic)*

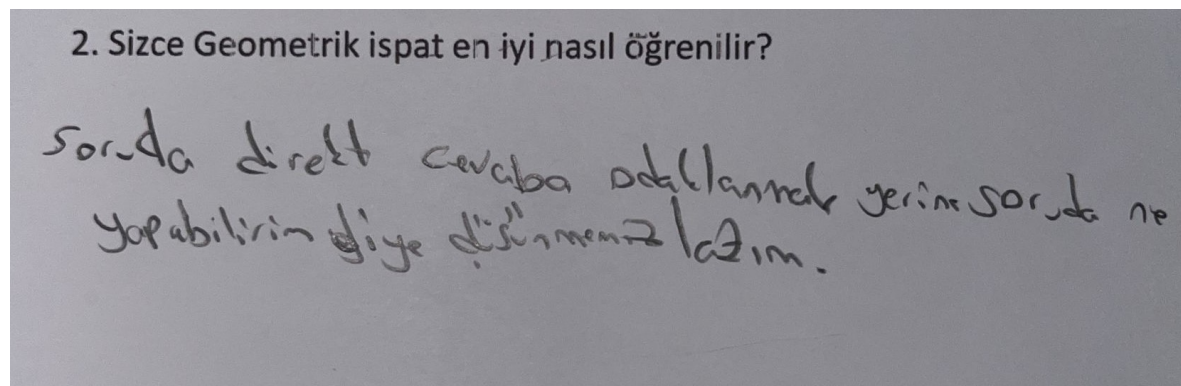


Figure 6.2. Geometry proof learning question student post-answer example.

### 6.3.2. Definition of Geometric Proof

Participants were asked, “How would you define geometric proof?”. The codes and categories for the answers given by the students to the question are presented in Table 6.5. It is noteworthy that the responses from the participants overlap with many themes.

Descriptive data regarding the participants' responses before and after the practical lesson regarding their perceptions of the definition of geometric proof are presented in Table 6.5 below.

Table 6.5. Students' responses regarding their perceptions of learning geometric proof.

Category	Codes	Students (pre-question)	f	Students (post-question)	f
Passive learning	Experts	S2, S3, S5, S9, S11, S22	6	-	0
	Real-Life	S19, S20	2	S11	1
	Formulas and mathematical operations	S4, S6, S15, S25, S26	4	S4, S5, S16, S17, S24, S26	6
Active learning	Focus and Understanding the Logic	-	0	S2, S10, S12, S19	4
	Exemplification	S1, S23, S24	3	S6, S23, S24	3
	Visualization	S7, S8, S10, S12, S13, S14, S16, S17, S18, S19, S21, S26	11	S3, S7, S8, S15, S18, S22,	6
	Step by Step	-	0	S1, S13, S14, S20, S21, S25	6

According to Table 6.5, we can examine all students who answered the questions under two categories. These categories can be expressed as active learning and passive learning.

As seen from the pre-and post-reflection questions, no significant change was observed in the number of participants' perceptions of the definition of geometric proof, namely those who think that geometric proof is related to real life and those who think that geometric proof is about giving examples. However, in the preliminary question, there is a significant decrease in the number of those who think that the proof was made by experts and those who think that the proof is related to visualization. The number of those who think that proof is about focusing and grasping logic, those who define proof as a formula with a mathematical process, and those who say that proof is done step by step has increased significantly.

For each code, sample participant responses from the pre-questionnaire are given below.

*S2: Geometric proof is a science that scientists have proven through various studies. (Experts)*

*S8: Geometric shapes, such as triangles, squares, rectangles, etc. For example, the expansion of a cube. The sum of the interior angles of a triangle is 180 degrees. (Visualization)*

*S15: A topic covered in mathematics class. Trying to prove geometry with formulas. (Formulas and mathematical operations)*

*S19: There are many examples around us that can be defined through shapes. (Real-Life, Visualization)*

*S21: For example, the sum of the interior angles of a triangle is 180 degrees. To prove this, the proof can be found by giving the degrees on the triangle shape. (Visualization)*

*S23: Proving geometry can be proven with examples. (Exemplification)*

*S26: I can define the geometric proof as follows: everything has a shape, and I think geometry is used for this, which is defined with formulas. (Visualization, Formulas, and mathematical operations)*

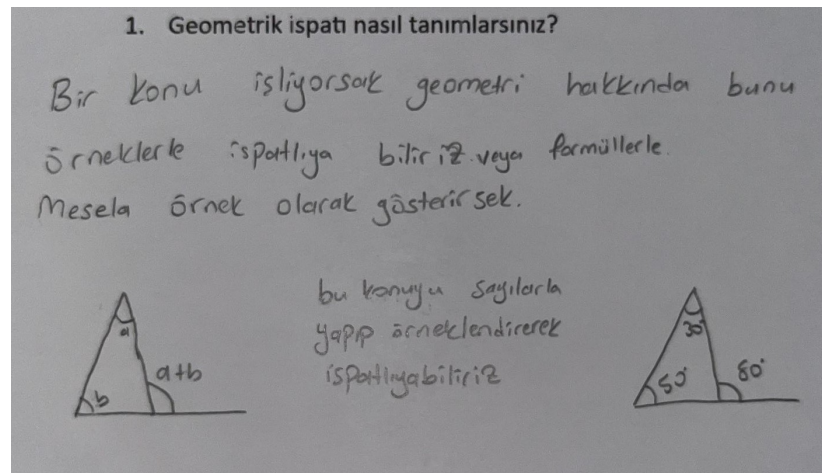


Figure 6.3. Definition of geometric proof question student pre-answer example.

For each code, sample participant responses from the post-questionnaire are given below.

*S7: For example, why are the interior angles of a triangle 180 degrees? I think the*

*explanation of how to prove this is called geometric proof. (Visualization)*

*S11: By showing examples from daily life (Real-life)*

*S12: About where the formulas come from and how they emerged (Understanding the Logic)*

*S14: By doing the steps shown in order (Step by Step)*

*S16: With formulas and solutions (Formulas and mathematical operations)*

*S17: We prove by doing operations (Formulas and mathematical operations)*

*S23: We can explain with examples. (Exemplification)*

*S25: Breaking the whole into parts and moving forward (Step by Step)*

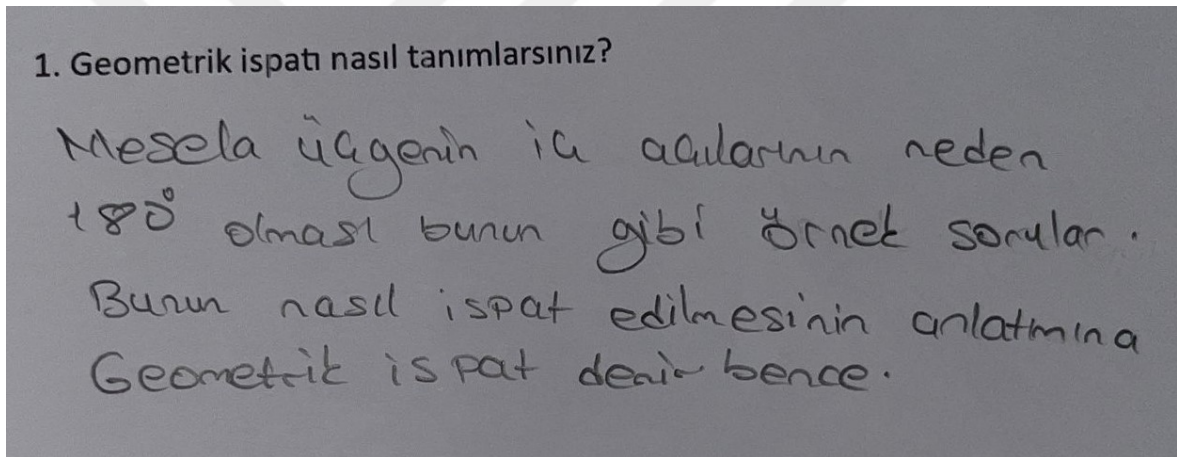


Figure 6.4. Definition of geometric proof question student post-answer example.

### 6.3.3. Step-By-Step Unrolled Strategy

Participants were asked, “What do you think about the methods we use while learning geometric proof?”. The codes and categories for the answers given by the students to the question are presented in Table 6.6. It is noteworthy that the responses from the participants overlap with many themes. Descriptive data regarding the participants’ responses after the practical lesson regarding their perceptions of learning geometric proof based on a step-by-step unrolled strategy.

Table 6.6. Students' perceptions of learning geometric proof based on a step-by-step unrolled strategy.

Category	Codes	Students	f
Positive	Easy	S1, S2, S6, S13, S17, S20, S21	7
	Instructive	S3, S4, S5, S6, S7, S15	6
	Step by step	S7, S8, S13, S15, S17, S19, S20, S21	8
	Useful	S8	1
	Good (nice)	S10, S16	2
	Logical	S12, S25	2
	Fun	S12, S24	2
Negative	Time	S14	1
	Difficult	S14, S23	2
	Nothing	S9	1

When Table 6.6 is examined, ten different codes were created depending on the students' perceptions about the methods they applied while learning geometric proof. These codes are shaped around "positive" and "negative" categories. These codes are; "Easy", "Instructive", "Step by step", "Useful", "Good(Nice)", "Logical", "Fun", "Time", "Difficult", "Nothing".

For each code, sample participant responses from the questionnaire are given below.

*S3: I think it is a good thing because it would be more understandable and permanent to see the mistakes at first and then learn the subject and solve it again (Instructive)*

*S7: Frankly, after reading this question, special triangles came to my mind directly, we go step by step in the methods. For example, we do 1 item first and then we move on to 2 items, in other words, we proceed step by step, we understand the formulas and apply them correctly. (Step by step)*

*S8: We learned the methods step by step while doing the proof. I think these methods are also useful (Step by step, Useful)*

*S9: Nothing(Nothing)*

*S10: Very good (Good)*

*S12: Very logical and fun (Logical, Fun)*

*S13: It would be easier to find by going in order and following the rules (Easy, step by step)*

*S14: If it is memorized, we can easily apply it and save more time. (Time, Difficult)*

*S17: It can be learned more easily by learning step by step. (Step by step)*

*S23: The methods are a bit difficult and not memorable, but if I think about it a bit, it will be fine. (Difficult)*

*S24: It was different and fun because it was out of the ordinary. (Fun)*

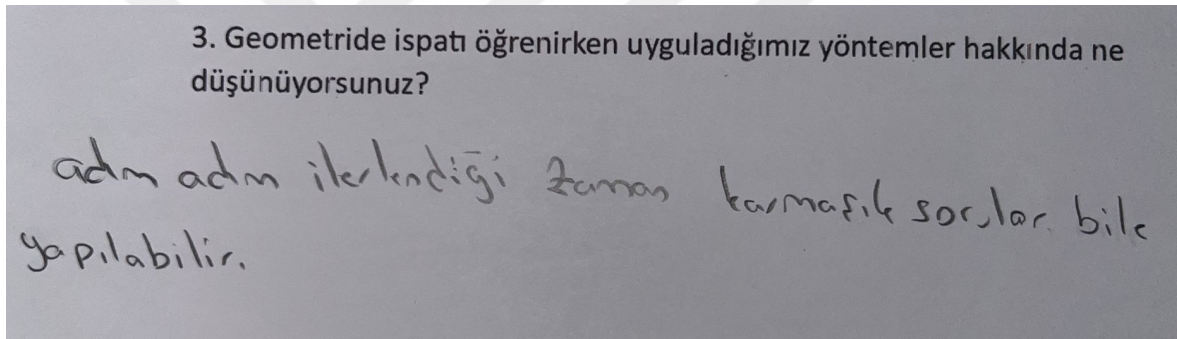


Figure 6.5. Student answer example about strategy.

#### 6.3.4. Topics Students Remember After Class

Participants were asked, “What do you remember from what you learned this week?”. The codes and categories for the answers given by the students to the question are presented in Table 6.7. There are students who do not answer the questions. For this reason, the numbers of these students are missing.

When Table 6.7 was examined, four different codes were created depending on what the students remembered from what they learned. These codes are shaped around “Learners” and “Developing Learners” categories. These codes are; “Triangles,” “Proof,” “All,” and “Incomplete.”

Table 6.7. Topics students remember after class.

Category	Codes	Students	f
Learners	Triangles	S1, S3, S4, S5, S8, S9, S14, S16, S17, S20, S23, S26	12
	Proof	S7, S13, S15, S21	4
	All	S2, S19	2
Developing learners	Incomplete	S12	1

For each code, sample participant responses from the questionnaire are given below.

*S2: Since it is exam week, most of the things are on my mind (All)*

*S3: Special triangles, concepts like Pythagoras, hypotenuse (Triangles)*

*S7: I learned why the interior angles of a triangle are 180 degrees, I learned the proof (proof)*

*S12: The topic about Pythagoras remained in my mind, but I forgot the others. If I don't review a topic every day, I forget the topic. (incomplete)*

*S15: How to prove the sum of the interior angles of a triangle. We learned the term proof. (proof)*

*S21: We learned to find the hypotenuse using special triangles. We learned to prove geometry step by step. (proof)*

*S23: Congruence in triangles, special triangles (Triangles)*

### 6.3.5. Difficulties in Learning Geometric Proofs

Participants were asked, "Did you have difficulty learning geometric proof? What issues did you encounter difficulties with?". It is noteworthy that the responses from the participants overlap with many themes. The codes and categories for the answers given by the students to the question are presented in Table 6.8.

Table 6.8. Difficulties in learning geometric proofs.

Category	Codes	Students	f
Straightforward	Easy	S2, S4, S10, S19, S20, S23, S26	7
Struggled	Little	S5, S7, S12, S13, S15, S16, S17	7
	Overcome	S1, S3, S8, S21	4
	Incomplete	S9, S13	2

When Table 6.8 was examined, four different codes were created depending on whether the students had difficulty in doing geometric proofs and what difficulties they encountered. These codes are shaped around “Straightforward” and “Struggled” categories. These codes are; “Easy”, “Little”, “Overcome”, and “Incomplete”.

For each code, sample participant responses from the questionnaire are given below.

*S2: I didn't have any difficulty. (Easy)*

*S5: I had some difficulty. wide angle, narrow-angle etc. (Little)*

*S7: I didn't have much difficulty but of course I got a little confused in some parts. (Little)*

*S8: Since it was the first time we saw geometric proof being handled in such a proven way, it seemed a little complicated and challenging to me at first but then I completed the deficiencies by learning. (Overcome)*

*S9: I couldn't learn it completely. (Incomplete)*

*S13: Yes I had difficulty and I still do. (Incomplete)*

*S21: I had difficulty at first but then I understood better by solving examples with our teacher. (Overcome)*

*S23: No I didn't have any difficulty (Easy)*

*S26: No I think the subject was very easy (Easy)*

### 6.3.6. Factors Facilitating Learning of Geometric Proof

Participants were asked, “Explain what factors make learning easier.”. It is noteworthy that the responses from the participants overlap with many themes. The codes and categories for the answers given by the students to the question are presented in Table 6.9.

Table 6.9. Factors facilitating learning of geometric proof.

Category	Codes	Students	f
Strategies	Step By Step	S1, S8, S12, S13, S14, S15, S16, S17	8
	Focus And Study	S2, S3, S5, S9, S19, S20, S23	7
	Logic	S10	1
	Formulas	S4	1
External Factors	Quiet Classroom	S8, S12, S14, S23	4
	Teacher	S7, S12, S13, S14, S20, S21, S26	7
	Materials	S9	1

When Table 6.9 is examined, seven different codes were created depending on whether the students had difficulty in doing geometric proofs and what difficulties they encountered. These codes are shaped around “Strategies” and “External Factors” categories. These codes are; “Step By Step”, “Focus And Study”, “Logic”, “Formulas”, “Quiet Classroom”, “Teacher”, “Materials”. For each code, sample participant responses from the questionnaire are given below.

*S4: Formulas (Formulas)*

*S8: I think our teacher will make it easier for us to learn by repeating the explanation. (Teacher)*

*S9: Explaining step by step and ensuring silence in the classroom. (Step By Step, Quiet Classroom)*

*S10: Listening to the lesson carefully and doing the worksheets. (Materials, Focus, and Study)*

*S12: Understanding logic without memorizing theorems. (Logic)*

*S13: Ensuring silence in the classroom and the teacher explaining the lesson in parts.*

*(Step By Step, Quiet Classroom)*

*S14: Our teacher explains in an understandable language and goes slowly. (Step By*

*Step, Teacher)*

*S21: Our lecturer explained the solution to the problem to us in a simple and easy way.*

*(Step By Step, Teacher)*

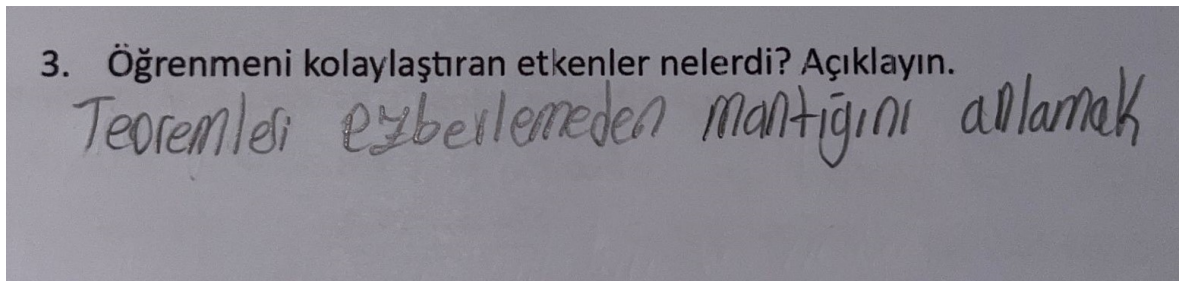


Figure 6.6. Factors make learning easier question student answer example.

### 6.3.7. Evaluation of Class Participation

Participants were asked, “How would you rate your participation in class this week? Explain with an example.”. It is noteworthy that the responses from the participants overlap with many themes. The codes and categories for the answers given by the students to the question are presented in Table 6.10.

Table 6.10. Evaluation of class participation.

Category	Codes	Students	f
Positive	Interesting	S1, S7, S12	3
	Supportive Of Learning	S2, S3, S4, S8, S14, S15, S19, S20, S21, S26	10
	Fun	S5, S10	2
Neative	Low Participation	S9, S17, S23	3
	Uninterested	S13, S16	2

When Table 6.10 is examined, five different codes were created depending on whether the students had difficulty in doing geometric proofs and what difficulties they encountered. These codes are shaped around “Positive” and “Negative” categories. These codes are; “Interesting”, “Supportive Of Learning”, “Fun”, “Low Participation”, “Uninterested”.

For each code, sample participant responses from the questionnaire are given below.

*S1: Good. I listened because I couldn't focus before, but this topic seemed interesting. (Interesting)*

*S5: Very good; we didn't have much fun before, but this week we both had fun and learned. (Fun)*

*S13: I was relieved because the exams were over, so I didn't listen much. (Uninterested)*

*S21: We usually participated in the lesson together as a class. It made it easier for me to learn. (Supportive Of Learning)*

*S23: Well, I can't say I participated in the lessons a lot, maybe 1-2 times a week. (Low Participation)*

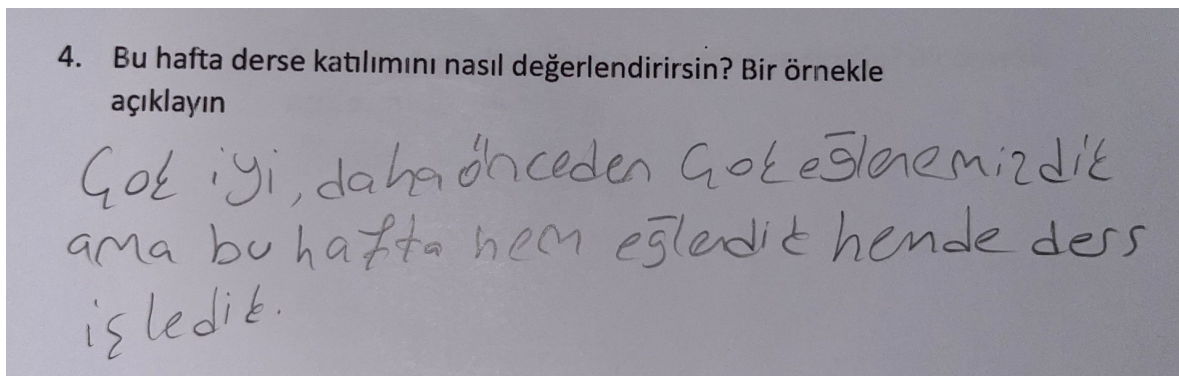


Figure 6.7. Class participation question student answer example.

#### 6.4. Students' Achievement

The participant's knowledge of the Pythagorean Theorem was assessed using the Unit Achievement Test, and the pre-and post-test results were compared. Non-parametric testing was required for group comparison because the sample size of par-

ticipants was 28 and less than 30. The Wilcoxon signed-rank test was used to examine if there was a significant change in the individuals' pre-and post-test results (Privitera, 2012). Descriptive statistics regarding the Unit Achievement Test pretest and post-test scores are shown in Table 6.11 below.

Table 6.11. Descriptive statistics related to pre-test and post-test scores of unit achievement test.

	<b>N</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>Minimum</b>	<b>Maximum</b>
<b>Pre-test</b>	28	24.6429	18.78379	0	54
<b>Post-test</b>	28	40.8571	22.26393	3	64

Table 6.11 indicates that there has been an increase in the mean post-test score of the participants in comparison to the mean pre-test score. On the other hand, the significance of the difference in pre and post-test scores is shown in the following Table 6.12. The pre-test and post-test participants' scores on the Unit Achievement Test differ statistically significantly, as indicated by the Wilcoxon signed-rank test results ( $Z = 4.410$ ,  $p < .001$ ). The Wilcoxon signed-rank test ranks and test statistics are displayed in the Table 6.12 and Table 6.13 .

Table 6.12. Ranks of Wilcoxon signed-rank test.

<b>Ranks</b>				
		<b>N</b>	<b>Mean Rank</b>	<b>Sum of Ranks</b>
post-test - pre-test	Negative Ranks	1 <sup>a</sup>	5.50	5.50
	Positive Ranks	26 <sup>b</sup>	14.33	372.50
	Ties	1 <sup>c</sup>		
	Total	28		
<i>a</i> post-test <pre-test, <i>b</i> post-test >pre-test, <i>c</i> post-test = pre-test				

Table 6.13. Test statistics of Wilcoxon signed-rank test.

Test Statistics	
	post-test - pre-test
Z	-4.410 <sup>a</sup>
Asymp. Sig. (2-tailed)	0,000
<i>a</i> Based on negative ranks.	

### 6.5. Impact of Technology-Enhanced Step-by-Step Strategies on Students' Geometric Reasoning Skills

In conclusion, this study investigated the relationship between students' engagement with a technology-enhanced step-by-step unrolled strategy and their improvement in geometric reasoning skills. The findings indicate that employing a structured, incremental approach, supported by dynamic geometry software and interactive applications, significantly enhances students' understanding and retention of complex geometric concepts. By breaking down the learning process into manageable steps and providing continuous feedback, students are able to build confidence and actively participate in the learning process.

The use of technology to visualize geometric principles facilitates deeper comprehension and makes learning more engaging. The results from the Van Hiele test demonstrated a statistically significant improvement in geometric reasoning skills among the students who were taught using this method, as indicated by the Wilcoxon signed-rank test results ( $Z = 5.045$ ,  $p < .001$ ). Specifically, the data showed that out of the 34 participants, 32 students exhibited an increase in their geometric reasoning levels, while none of the students experienced a decline, and 2 students remained at the same level. These results underscore the effectiveness of the step-by-step unrolled strategy in improving geometric reasoning skills.

Moreover, the study highlighted the positive emotional impact on students. Many students reported finding the lessons enjoyable and engaging, which contributed to their active participation and sustained interest in the subject. The technology-enhanced approach not only made learning more interactive and fun but also helped to reduce the anxiety often associated with complex geometric concepts. This positive emotional response is crucial, as students who enjoy their learning experiences are more likely to engage deeply and persist through challenges.

The integration of interactive software and applications allowed students to visualize and interact with geometric concepts, making the learning experience more tangible and less abstract. The ability to manipulate geometric shapes and see immediate feedback helped students understand and internalize the principles being taught. The collaborative environment fostered by small group work and real-life problem-based activities further enhanced students' engagement and motivation, leading to a more robust learning experience.

Furthermore, students' perceptions of learning geometric proof evolved throughout the study. Initially, no participants mentioned learning geometric proof through a step-by-step strategy. However, by the end of the study, a significant number of students recognized the effectiveness of this approach. This shift highlights the positive impact of the step-by-step unrolled strategy on students' learning processes. Since I had the opportunity to apply the plan twice, I made some changes the second time. I observed that some of the activities were very easy even for the most difficult students. Therefore, I rearranged the worksheets. The part with Pythagorean triples can be given as an example. Since some had memorized triangles such as 3-4-5, this example was easy for the students. For this reason, I asked the students to find triangles such as 20-21-29, which were more unknown. In addition, in the second application, I took the students to the computer room and ensured that everyone interacted in order to increase the effectiveness of the lesson plan. What I observed was that the students participated in the lesson more actively and said that they did not realize how time passed.

## 7. DISCUSSION AND CONCLUSION

The design of this study is an action research. Therefore, it is essential to think about the results of the study and make discussions based on the literature in this section. After being assigned to a state school, I observed the difference between the school I was assigned to and the private schools where I worked and did my internship. I observed that the students' social, economic, and success levels were below average. I aimed to guide my students in order to break their prejudices against geometry, increase their level of knowledge, and make them think systematically, even if they had difficulty with simple mathematical operations. My aim as a teacher was to investigate how I could increase my students' geometric reasoning skills and success by overcoming their prejudices against geometry.

For this reason, I reviewed the literature and the current curriculum and created a lesson plan using the step-by-step method. Then, I implemented this lesson plan and stated the results of the data I collected in the previous section. These results were discussed in this section. At the same time, the lesson plan was prepared for the 2022-2023 term, and since my thesis process continues in the 2023-2024 academic year, I had the opportunity to use the same plan again. As I will explain in the discussion section, I had to make some changes in the implementation of the plan. Thus, I revised my plan according to feedback. Since the dynamics and success levels of each class differ, the plan can be applied again and again because it is flexible. The discussion section consists of three parts. These parts are Geometric reasoning, students' perceptions, and geometry achievement.

### 7.1. Geometric Reasoning

The action research conducted includes teaching the subject of the Pythagorean theorem in the 9th-grade Mathematics course with the step-by-step method and teaching practices supported by Web 2.0 tools. It was aimed to measure the geometric

reasoning levels of the students before and after the geometry unit integrated with technology-enhanced practices based on the SU strategy. In this direction, the Van Hiele test was applied and used as a pre-test and post-test to investigate whether there was a significant difference between the results before and after the lesson. As a result of the mathematical analysis and the findings obtained, there is a significant difference between the Van Hiele levels of the students before and after the lesson.

Since I was newly assigned to the school, we had not covered the triangles unit with the students before the pre-test at this school. The triangles unit is among the last units in the 9th-grade curriculum. Generally, according to my previous experiences, this unit was not covered completely due to the intensity of the 9th-grade curriculum and the unit being at the end. When I applied the test, only one of the students was at the 3rd level, deduction level. Most students remained at the visualization and analysis level.

As it is understood from these results, the method I applied seemed to have a positive effect on the student's level. As a teacher-researcher, I was happy to see that they were positively affected in the classroom. In the first test, I was scared that there were students who could not even recognize basic shapes such as triangles and squares. However, I already believed that a style of expression that could attract attention and make them think instead of monotonous, routinized expressions could be effective. However, I observed that such applications were not used very often in public schools. For this reason, I determined some deficiencies in the lesson plan I developed. For example, I had difficulty in applying it in the classroom because the internet connections in the lesson plan were broken or some smart boards did not have internet. When I used the lesson plan for the second time this year, I downloaded the Geogebra program to my computer and connected it to the smart board. I also took the students to the computer class and ensured that everyone was active.

When the literature is examined, many studies have shown that educational technologies, especially the use of dynamic geometric software, enable students to under-

stand geometric concepts more deeply. For example, Jones (2000) states that dynamic geometry software improves students' conceptual understanding and reasoning skills in geometry. Such software allows students to manipulate geometric shapes and visually examine various situations, thus enabling them to concretize concepts.

Educational technologies can increase students' participation in geometric reasoning processes by creating interactive and collaborative learning environments. In particular, web-based learning platforms and digital games can make geometry learning processes more effective by increasing students' motivation. For example, Clements and Battista (1992) showed that technology-enhanced educational environments significantly improved students' geometric thinking and problem-solving skills.

Van Hiele's theory of levels of learning in geometry states that students go through certain stages in the process of understanding geometric concepts. These stages are recognition, analysis, ordering, logical inference, and rigor levels. The step-by-step unrolled strategy can help students develop higher-level geometric reasoning skills by supporting these stages. Burger and Shaughnessy (1986) have shown that step-by-step explanations are effective in understanding Van Hiele levels and helping students progress through them.

According to Zhang (2020), technology may be used to enhance students' knowledge of geometric ideas through interactive activities and visual representations. It has been discovered that using technology in the classroom can help students develop their geometry knowledge and improve their geometric reasoning.

## **7.2. Students' Perceptions**

The participants in this study participated in a step-by-step strategy and technology-based lessons aimed at understanding geometry with the help of geometric proof. Since I am a teacher-researcher, I used different resources to investigate the effects of the step-by-step strategy and technology in order to break the prejudices of the students

about geometry, to increase the memorability of the content by learning the underlying reason without memorizing the formulas with logical and sequential thinking, and to ensure that they have positive perceptions about geometry. The participants answered and completed reflection questions about their understanding of geometric proof and their perceptions of learning proof before and after the training. Students were also asked about their learning experiences during the lesson. In addition, the worksheets distributed during the lesson were also filled out by the students.

### **7.2.1. Perceptions of Learning Geometric Proof**

The answers given to the reflection questions about learning and perceptions of geometric proofs provide information about the application of the step-by-step approach in terms of teaching principles. The students were asked how geometric proofs can be learned best. According to the pre-test results, most of the students stated that they could learn with the help of an external source. The students gave answers such as “It is learned by examining the studies done by experts.”, “I think we can understand the lesson well by listening, studying, and learning” and “By memorizing the formulas and studying.” In the post-test, the step-by-step strategy came to the fore. In addition to giving answers such as “Breaking the topic into pieces”, and “Proceeding one by one”, the students also stated that the main task was to grasp the logic of the formula. Although no student mentioned the step-by-step strategy as a result of the pre-test, ten students mentioned this method in the post-test. This is expected because they did not know about this strategy at the beginning. In addition, only one student stated that he could learn geometry well by grasping its logic in the pre-test, while this number increased to 8 in the post-test. According to the post-test result, the number of students who attributed learning to an external source decreased visibly. However, the number of students in the category of “It is best learned from an expert” has not changed. The reason for this is that I conducted the research in my own class as a teacher-researcher. Although I insisted that there would be no grade, I think they may have used expressions such as “It is best learned from İbrahim Hoca” in order to impress me.

When I asked them to define geometric proof, they said in the pre-test that it could be defined with the help of experts' definitions. While six students mentioned that it could be done with the help of experts' definitions in the pre-test, no student used this definition in the post-test. When the pre-test and post-test were compared, there was not much difference in the number of those who thought that geometric proof consisted of mathematical operations and formulas and that proof would be done by giving examples. In the post-test, the step-by-step method term was used by six students. When students were asked about their perceptions about the method, nineteen students expressed positive opinions due to being educational, easy, useful, and fun. Three students expressed negative opinions due to reasons such as it being time-consuming and difficult to use. The reason for this is that students take the YKS exam in the Turkish education system. Since there is a time limit for this exam and the exam does not directly include the subject of proof, they may want to study only for the exam. Since the students are in the 9th grade, they have experienced the exam stress one year before and have experienced the necessity of making fast and correct solutions.

Students' preference for memorization in mathematics may be influenced by various factors identified in the literature. Yıldız (2016) emphasized that students perceive memorization and repetition as important elements for success in mathematics. This tendency towards memorization is further supported by (Salas, 2024), who found that students believe that mathematics is best learned through memorization. In addition, Koçak and Soylu (2017) stated that pre-service mathematics teachers who have difficulty understanding the relationship between mathematical rules and concepts tend to rely on memorization, which can affect their teaching methods. When I started teaching, I told some students to memorize as a last resort when they did not understand the subject, but I realized that this does not have long-term effects. After the study, I think that students' understanding that formulas do not come out of nowhere and that they have logic makes it easier for them to learn. I also witnessed how an applicable and useful lesson plan makes a teacher's job easier with the help of today's technology and step-by-step strategies. The necessity of memorization has also been mentioned in

the literature. As discussed by Hartman *et al.* (2023), the historical context of mathematics education has often emphasized the necessity of memorizing basic facts and algorithms. This traditional approach to learning mathematics may have ingrained the habit of memorization in students. Furthermore, the study by Vanayan *et al.* (1997) revealed that a significant percentage of students perceive learning mathematics as primarily based on memorization. This perception may lead students to prioritize memorization over developing a deeper understanding of mathematical concepts.

### **7.2.2. Perceptions of Lesson Integrated With Step-by-step Unrolled Strategy and Technology**

Reflection questions were also asked to the students during the class period. Data were collected to learn how they understood geometry and their perceptions about the course. Only one of the students stated that he did not fully understand and had deficiencies. The student responded as follows: “The topic of Pythagoras remained in my mind, but I forgot the others. If I don’t review a topic every day, I forget the topic.” While the number of students who stated that the topic of triangles remained in their minds was 12, the number of people who mentioned proof in their answers was 4. The reason for the high number of people mentioning triangles is that the students learned the topic of triangles in the 8th grade. In the pretests, the students could not remember this prior knowledge, but when they repeated what they had learned before during the class period, they learned better. The students mostly stated that they had difficulty with the proof. I think that the fact that there were people who said it was easy was due to those who remembered the 8th-grade information better. After lessons, I noticed that some parts of the lesson plan were missing. One of the easiest topics for students is right triangles with integer side lengths. There are four special triangles here, and the aim was to have students find them by practicing and also find that the multiples of these sides are special triangles by using the similarity logic in triangles. However, I concluded that the effectiveness of the worksheet was reduced because some students wrote the triangles they had memorized. Since this study was conducted in the 2022-2023 academic year, I used the study again this year. I did not

collect data this year, but I filled this gap. This year, I gave these familiar triangles and generally investigated unknown triangles. For this reason, the study helped me develop my lesson plan.

Although the students had difficulty in proving, seven students stated that the teacher and eight students stated that the method used made their learning easier. Fifteen students stated that their participation in the lesson increased. I was positively affected by the participation of students who did not participate in the lesson compared to the previous lessons. The reasons for this were that the students found the lesson interesting, had fun, and supported their learning. The fact that the students interacted with tools such as worksheets and technology in the lesson was beneficial for me both in terms of ensuring classroom management and in terms of being able to interact more with the students. Even students who had fallen asleep and disrupted the lesson in previous lessons participated.

### **7.3. Achievement**

Another aim of the study was to increase the success of the students in geometry class. The students' achievements were measured with the Unit Achievement Test. As a result of this test, which was conducted as a pre-test and post-test, 26 students increased their success while one student remained at the same level, and one student decreased his/her grade. In this exam, which was evaluated out of 100, the average in the first test was 24.64, while the average in the second test was 40.85. The reason for their increase in success may have been that they learned a subject they did not know, but the Pythagorean theorem is mentioned in the 8th grade. The important thing here was to give the students an idea through geometric proof and to increase their success. In addition, the increase in students' participation in the class, their correct answers to the questions asked, and the increase in their grades in the mathematics course convinced me that this method was beneficial for my course. Moreover, when the literature on this subject is examined, it is understood that technology and the step-by-step unrolled method are beneficial for below-average students. According to

Chang and Lin (2009), students who perform below average demonstrate that the SU can assist students in becoming competent hypothetical bridge builders. Using the step-by-step unrolled technique, they may effectively begin the hypothetical reasoning process and proceed to provide a response to the issue (Chang and Lin, 2009).

Additionally, with technology, students were able to visualize easily and increased their success through active participation in the lesson. In numerous studies conducted in Turkey, it has been highlighted that technology-supported geometry instruction improves student achievement (Uğur *et al.*, 2016). Furthermore, it has been noted that the use of technology in geometry teaching makes a significant difference in student success. Çavuş and Deniz (2022) conducted a meta-analysis study stating that technology-supported teaching is more effective than traditional learning methods in increasing mathematics and geometry achievement. This study emphasizes the role of technology in teaching mathematics and geometry and reveals that technology is an important tool in increasing students' success.

Overall, this study highlights the effectiveness of integrating technology and step-by-step strategies in teaching geometry, especially for students with low academic achievement, ultimately fostering a more supportive and effective learning environment. The significant improvement in students' geometric reasoning skills, coupled with their positive feedback on the teaching methods and their enjoyment of the learning process, suggests that this approach can be a valuable tool in geometry education. By creating an engaging, supportive, and interactive learning environment, educators can significantly enhance students' geometric reasoning skills and their overall learning experience.

## 8. SUGGESTIONS

First of all, I have prepared a user guide for teachers who want to use this study in their classes in Turkey. This guide is included in Appendix D. In the study, it was seen that the use of the technology-supported step-by-step strategy was generally effective in developing the geometric reasoning of ninth-grade students. However, there are some points that need further attention. This section includes suggestions based on my experience as a teacher-researcher during the implementation.

- (i) In order to use technological tools effectively, shortcomings in the technical infrastructure, such as internet access, should be identified in advance, and attempts should be made to remedy them. Increasing the number of computers in computer classes and workshops in schools may be important for more effective results..
- (ii) Lesson plans can be developed using SU strategy targeting different topics in other grade levels, and similar research can be conducted to examine the effectiveness of those lesson plans.
- (iii) In this research, activities were conducted in the form of group work due to the physical and technical infrastructure of the school where the research was conducted. This situation may have negatively affected students who like to learn with activities based on individual learning. Therefore, in future studies, activities can be conducted in schools with a physical and technical infrastructure that can also allow individual learning.
- (iv) Paying attention to the writing style, point size, and resolution of the images used while preparing the materials in the application processes carried out with technology is recommended.
- (v) More student participation can be ensured in the study. The number of students may increase as the teacher enters different classes at the same grade level.
- (vi) The number or variety of the technological tools used can be increased.

- (vii) Due to the curriculum's intensity, part of the research was conducted after the semester's last exam. This situation may have affected the student's participation in the lesson. For this reason, the study period can be changed.
- (viii) It was observed that some activities were too easy, even for the most struggling students. Therefore, worksheets can be revised to balance the activities' complexity while carefully considering the students' level. The part where Pythagorean triples are found can be given as an example. Since there are those who memorize triangles like 3-4-5, this example remained simple for the students.



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## APPENDIX A: REFLECTION QUESTIONS

This section contains the reflection questions given to the students.

Before Instruction:

- (i) How do you define geometric proof?
- (ii) How do you think the geometric proof is best learned?

After Instruction:

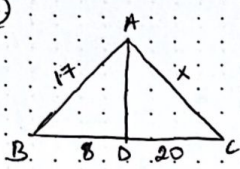
- (i) How do you define geometric proof?
- (ii) How do you think the geometric proof is best learned?
- (iii) What do you think about the methods we use to learn proof in geometry?

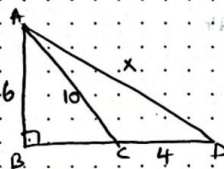
Instruction period:

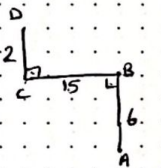
- (i) What do you remember from what you learned this week? Explain.
- (ii) Did you have difficulty learning geometric proof? Which topics did you have difficulty with? Explain.
- (iii) What were the factors that made it easier for you to learn? Explain.
- (iv) How would you evaluate your participation in class this week? Explain with an example.

## APPENDIX B: UNIT ACHIEVEMENT TEST

This page contains front page of unit achievement test questions given to students.

1)   $ABC$  bir üçgendir.  
 $[AD] \perp [BC]$   
 $|AB| = 17 \text{ cm}$   
 $|BD| = 8 \text{ cm}$   
 $|DC| = 20 \text{ cm}$   
 Verilen bilgilere göre  $|AC| = x$  kaçtır?

2)   $ABC$  bir üçgendir.  
 $|AB| = 6 \text{ cm}$   
 $|AC| = 10 \text{ cm}$   
 $|CD| = 4 \text{ cm}$   
 Yukarıda verilere göre  $|AD| = x$  kaçtır?

3)   $[DC] \perp [AB]$   
 $[CE] \perp [AB]$   
 $|DC| = 2 \text{ cm}$   
 $|AD| = 6 \text{ cm}$   
 $|CB| = 15 \text{ cm}$   
 Verilen bilgilere göre A ile D noktaları arasındaki en kısa uzaklık nedir?

4) Bir dik üçgenin kenar uzunlukları kareleri toplamı  $162 \text{ cm}^2$  olduğuna göre, bu üçgenin hipotenüsünün uzunluğu kaç santimetredir?

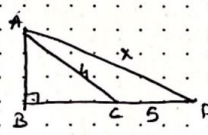
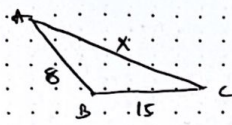
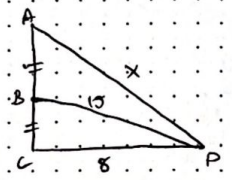
5)   $ABC$  bir dik üçgendir.  
 $|AC| = 4 \text{ cm}$   
 $|CD| = 5 \text{ cm}$   
 Verilenlere göre  $|AD| = x$  in alabileceği en küçük tam sayı değeri kaç cm'dir.

Figure B.1. Unit achievement test front page.

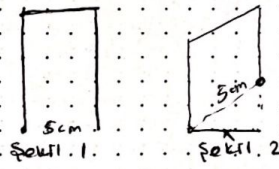
This page contain back page of unit achievement test questions given to students.

⑥  ABC bir üçgendir.  
 $|AB| = 8 \text{ cm}$   
 $|BC| = 15 \text{ cm}$   
 $m(\angle B) > 90^\circ$   
 $x$  bir tön sayıdır.

Verilen bilgilere göre  $x$ 'in alabileceği değerler toplamı nedir?

⑦  ACD bir dik üçgendir.  
 $|AB| = |BC|$   
 $|BD| = 10 \text{ cm}$   
 $|CD| = 8 \text{ cm}$

Verilen bilgilere göre  $x$  kaç santimdir?

⑧  Şekil 1. Şekil 2.

Şekil 1 de kapalı durumda bir kâğıt verilmiştir. Bu kâğıt daha sonra açılmıştır. Şekil 2 de açık hali verilmiştir.

Kâğıdın eni 5 cm'dir. Kâğıt 3 cm açıldığına göre Şekil 2 de verilen  $x$  uzunluğu kaç santimdir?

Figure B.2. Unit achievement test back page

## APPENDIX C: SAMPLE LESSON PLAN

Topic: Pythagoras was a Greek mathematician and philosopher who discovered one of the most famous rules in mathematics. In mathematics, a rule is called a theorem. So, the rule that Pythagoras discovered is called the Pythagorean Theorem.

Objectives In The Turkish Education Curriculum:

### 9.4.4. Right Triangle and Trigonometry

Terms and Concepts: Pythagorean theorem, Euclid's theorem, Trigonometric ratio

Symbols and Notations:  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\cot x$

9.4.4.1. Solves problems by obtaining the Pythagorean theorem in right triangles.

- a) Model diversity is included while obtaining the theorem.
- b) Real life problems are included.
- c) Pythagoras' works are included.

Mathematical Big Ideas: Corresponding sides of similar triangles prove the Pythagorean Theorem is true for all right triangles. Use the structure of a set of equations from the Pythagorean Theorem and the structure of a set of right triangles to make matches between the equations and the right triangles.

Materials:

- Pen
- Paper
- Activity Sheet
- GeoGebra
- Desmos
- Menti

Prerequisite Knowledge Needed:

9.3.4.1. Solve the equations that require exponential expressions.

Instructional Sequence:

First, the teacher makes the students watch a funny video about Pythagoras to motivate them to the lesson. Then worksheet one is distributed, and the first question, “What do you know about Pythagoras?” is asked. In the second question, the problem situation is explained to the students. The Step-By-Step Unrolled Strategy aims to find the sides of the triangle by asking the students what it takes to find the area of the triangle. They are then told to complete the problem, paying attention to the areas and edges. Then, students follow the steps.

Worksheet 2 is distributed and questioned how the proof of the Pythagorean theorem can be done. Students are given a blank square sheet of paper to use in the exploration part and are asked to follow the steps on the worksheet. Then, students follow the steps.

Worksheet number 3 is distributed. The students are given a challenging situation and a problem related to daily life. The following steps can be followed in order. The first step is to ask what the Pythagorean theorem is. Ask what the diagonal lines represent. Ask if the hypotenuse is an integer. Then, students followed the steps and answered the questions.

The teacher asks students if a right triangle might be formed by the 8, 15, and 17 side lengths. If not, why not? The teacher asks students to explain why they believe that these three lengths can form a right triangle. Because 8, 15, and 17 form a right triangle and all three side lengths are whole integers, this triangle is referred to as a Pythagorean triple. Ask students to look for examples of Pythagorean triples that are not merely multiples of another Pythagorean triple and encourage them

to use this information to their advantage if they learn it to lessen their burden. Students investigate various combinations of  $a$ ,  $b$ , and  $c$ . To get useful numbers more quickly, they can utilize the table of perfect squares. They must demonstrate that each value they discover produces a Pythagorean triple using the example from Part A of the student worksheet 4. While doing this, students could realize that a Pythagorean triple is a multiple of a Pythagorean triple. Students collaborate with a partner to pool their sample Pythagorean triples and then discuss the values they discovered to be effective. After that, they begin collaborating on Part B to demonstrate why a Pythagorean Triple's multiple is always another Pythagorean Triple. Look for students who utilize triangle similarity for their reasons while they work on this one.

Worksheet number 5 is distributed. Students use the applet and record their observations on the first page of this handout as they move points A and B around. Students work with a partner and first share their observations with each other and then use those observations to determine when the sum of the square of  $a$  and the square of  $b$  is greater than, equal to, or less than the square of  $c$ .

Worksheet number 6 is distributed to students so that they can practice. Students answer the questions given on the student worksheet based on what they learned about the relationship between  $a^2$ ,  $b^2$ ,  $c^2$ , and the angle between the two legs  $a$  and  $b$ . Worksheet 7 is distributed, and students are provided with the steps to follow.

## APPENDIX D: LESSON PLAN GUIDE FOR TEACHERS

Bu kılavuz, dokuzuncu sınıf öğrencilerinin geometrik muhakeme becerilerini geliştirmek amacıyla oluşturulmuş bir ders planının nasıl uygulanacağını detaylı bir şekilde açıklamaktadır. Teknoloji destekli ve adım adım çözümleme stratejisi ile öğrencilerin geometriyi daha iyi anlamaları hedeflenmiştir.

Chang ve Lin (2009), öğrencilerin geometrik ispatları anlama becerilerini geliştirmek amacıyla adım adım çözümleme stratejisini geliştirmişlerdir. Bu stratejinin temelinde “yapı iskelesi” veya “destekleme” (scaffolding) olarak adlandırabileceğimiz bir yaklaşım vardır. “Yapı iskelesi” öğrencilerin yeni bir beceri veya bilgi alanında ustalaşmalarını kolaylaştırmak için öğretmen veya daha yetkin bir birey tarafından sağlanan geçici desteği ifade eder. Bu strateji, karmaşık kanıtları daha küçük ve daha yönetilebilir adımlara ayırır. Kanıtta açılan her bir adım, öğrencilerin önceki bilgi ve anlayışlarını inşa etmeleri için bir iskele görevi görür. Böylece öğrenciler tek tek parçalara odaklanabilir ve genel anlayışlarını kademeli olarak geliştirebilirler. Strateji, her adım için kapsamlı açıklamalar sunarak, zorluk çeken öğrencilerin geometrik ispat yeterliliklerini güçlendirmelerine yardımcı olmayı amaçlamaktadır.

Hedefler:

- Öğrencilerin Pisagor teoremi konusundaki bilgilerini derinleştirmek.
- Teknolojik araçlar kullanarak öğrencilerin geometriye olan ilgisini artırmak.
- Öğrencilerin adım adım problem çözme becerilerini geliştirmek.
- Öğrencilerin geometrik muhakeme düzeylerini geliştirmek.

Kullanılacak Teknolojik Araçlar ve Yazılımlar:

- GeoGebra: Geometrik şekillerin dinamik olarak oluşturulmasını ve manipüle edilmesini sağlar. Öğrencilerin ispatları genelleştirmesi için kullanılmıştır.

- Desmos: Ders materyalleri ile online ortamda etkileşimde olup öğrencilerin cevaplarının takibi için kullanılabilir.
- Menti: Öğrencilerin fikirlerini paylaşabilecekleri bir platform.

Ders Planı Adımları:

Hazırlık Aşaması:

- Materyal Hazırlığı: GeoGebra ve diğer teknolojik araçlarla ilgili gerekli döküman ve çalışma kağıtlarını hazırlayın.
- Öğrencileri Bilgilendirme: Dersin amacı ve kullanılacak araçlar hakkında öğrencileri bilgilendirin.
- Teknolojik Altyapı Kontrolü: Ders sırasında kullanılacak cihazların (bilgisayar, tablet vb.) ve internet bağlantısının çalıştığından emin olun.

Uygulama Aşaması

- Giriş Aktivitesi: Menti ve eğlenceli bir video kullanarak öğrencilerin dikkatini çekmek ve konuya ilgi uyandırmak.
- Ana Aktivite: GeoGebra ile adım adım açıklanan geometrik problemler üzerinde çalışmak. Öğrenciler, verilen adımları takip ederek problemlerin çözümlerine ulaşacaklar.
- İşbirliği ve Paylaşım: Desmos üzerinde öğrenciler çalışmalarını ve düşüncelerini paylaşacaklar.

Değerlendirme Aşaması:

- Ünite başarı testi: Öğrencilerin anladıklarını değerlendirmek için ünite başarı testini kullanabilirsiniz.
- Geri Bildirim: Öğrencilerden geri bildirim alarak ders planının etkinliğini değerlen-

dirin ve gerekli düzeltmeleri yapın.

#### Çalışma Kağıtlarının Kullanımı:

Çalışma kağıtları, ders sırasında ve sonrasında öğrencilerin öğrendiklerini pekiştirmeleri için kullanılacaktır. Her çalışma kağıdının başında, öğrencilerin yapması gereken adımlar ve amaçları açıkça belirtilmelidir.

Çalışma kağıdı 1: İlk olarak, öğretmen öğrencilere derse motive etmek için Pisagor hakkında komik bir video izletir. Ardından çalışma kağıdı 1 dağıtılır ve ilk soru sorulur. Sorunun cevabı Menti uygulaması ile toplam sınıfın genel düşünce dağılımı gözlenir. İkinci soruda, problem durumu öğrencilere açıklanır. Adım adım çözümleme stratejisine göre, öğrencilere üçgenin alanını bulmak için ne gerektiğini sorarak üçgenin kenarlarını bulmaları amaçlanır. Daha sonra, alanlara ve kenarlara dikkat ederek problemi tamamlamaları söylenir.

Çalışma Kağıdı 2: Çalışma sayfası 2 dağıtılır ve Pisagor teoreminin ispatının nasıl yapılabileceği sorulur. Öğrencilere araştırmada kullanmaları için boş bir kare kağıt verilir ve çalışma sayfasındaki adımları izlemeleri istenir. Geogebra İspat adındaki link ile ispat genelleştirilir.

Çalışma Kağıdı 3: Çalışma kağıdı numarası 3 dağıtılır. Öğrencilere zorlayıcı bir durum ve günlük yaşamla ilgili bir problem verilir. Aşağıdaki adımlar sırayla takip edilebilir. İlk adım, nedir sorusunun sorulmasıyla atılır. Öğrenciler adımları takip eder. Çapraz çizgilerin neyi temsil ettiğini sorabilirsiniz. Hipotenüsün bir tam sayı olup olmadığını sorun.

Çalışma Kağıdı 4: Öğretmen öğrencilere 8, 15 ve 17 kenar uzunluklarıyla dik bir üçgen oluşturulabilir mi diye sorar. Eğer oluşturulamazsa neden oluşturamayacağını sorar. Öğretmen öğrencilerden bu üç uzunluğun dik bir üçgen oluşturabileceğine neden inandıklarını açıklamalarını ister. Öğrencilerden başka Pisagor üçlülerine dair

örnekler bulmalarını isteyin ve yüklerini azaltmak için bu bazı sayıların karesini verebilirsiniz..

Çalışma Kağıdı 5: Sorulara yanıt vermek için öğrencilerden pisagor eşitsizliği linkini açmalarını isteyin.

Çalışma Kağıdı 6: Bu çalışma kağıdını değerlendirme yapmak için kullanabilirsiniz.

Çalışma Kağıdı 7: Bu çalışma kağıdı adım adım verilen soruları cevaplandırarak öğrencilerin ispat yapmalarını sağlamayı amaçlamaktadır. Değerlendirme yapmak için kullanabilirsiniz. Öğrencilerin adımları takip etmesi sağlayın.

Öneriler ve İpuçları:

- Zaman Yönetimi: Ders planını uygularken zaman yönetimine dikkat edin. Her bir aktivite için yeterli zaman ayırın. Her bir çalışma kağıdı ve sonunda öğrencilere geri bildirim verilmesi için bir ders süresi kadar zaman yeterlidir.
- Teknolojik Sorunlar: Olası teknolojik aksaklıklar için yedek planlar hazırlayın. Öğrenciler bilgisayar laboratuvarına götürülebilir.
- Etkileşim: Öğrencilerin aktif katılımını teşvik edin ve sorular sormalarını teşvik edin.

## APPENDIX E: WORKSHEETS

This section contains the worksheets given to students.

(i) Pisagor ismini duyunca aklınıza gelenler nelerdir?

(ii) İbrahim, Erdi ve Samet Bey emekliliklerini geçirmek için Edirne'nin bir sahil kasabasında birer arsa almaya karar verirler. Aşağıdaki şekilde şekli verilen kare şeklindeki arsaların alanların  $81\text{m}^2$ ,  $144\text{m}^2$  ve  $225\text{m}^2$  olduğu bilinmektedir. Şekilde görüldüğü gibi 3 tarlanın arasında kalan bölgeye ise çardak yapmaya karar verirler. Arsaların alanından yola çıkarak çardak yapılması planlanan bölgenin alanını bulunuz. Çardak yapılan bölgenin kenarları arasında bir ilişki var mıdır?

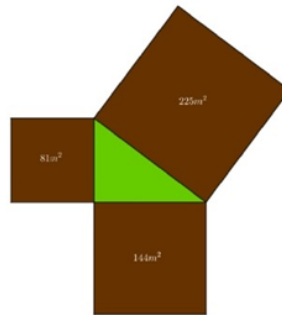


Figure E.1. Worksheet 1.

Worksheet 2, distributed to students, is as follows.

TEMEL SORU:

(i) Pisagor Teoremini nasıl ispatlayabilirsiniz?

(ii) KEŞİF: Pisagor Teoremini Sözcükler Olmadan Kanıtlamak

(a) Dik kenarları  $a$  ve  $b$  ve hipotenüsü  $c$  olan bir dik üçgen çizin ve kesin.

(b) Dik üçgeninizin üç kopyasını yapın. Gösterildiği gibi büyük bir kare oluşturmak için dört üçgeni de düzenleyin.

(c) Üçgenin ve küçük karenin alanlarını toplayarak büyük karenin alanını  $a$ ,  $b$  ve  $c$  cinsinden bulun.

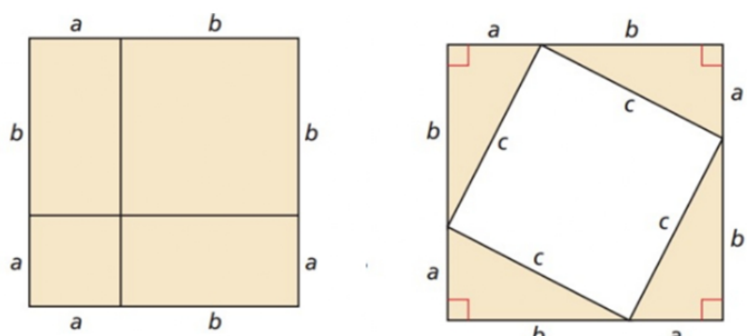


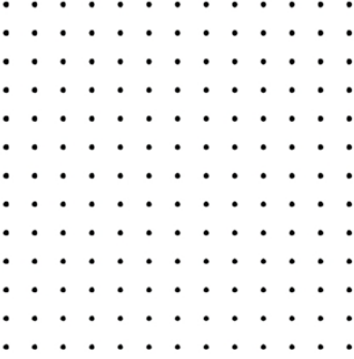
Figure E.2. Worksheet 2.

Worksheet 3, distributed to students, is as follows.

İDDİA DURUMU

İrem, A ve B arasındaki mesafenin 1 olduğunu söylüyor. İrem ızgara üzerine ne yatay ne de dikey olan bir doğru parçası çizer. Tam sayı uzunluğunda böyle bir doğru parçası çizebileceğini iddia ediyor.

İrem haklı mı? Eğer öyleyse, İrem kaç tane olası çizgi parçası çizebilir?



The grid consists of 10 rows and 10 columns of dots, forming a square lattice. The dots are arranged in a regular pattern, with equal spacing between them both horizontally and vertically.

Figure E.3. Worksheet 3.

Worksheet 4 part a, distributed to students, is as follows.

Pisagor Üçlülerini Keşfetmek

Bölüm A: Pisagor üçlülerinin örneklerini bulma

Hesap makinenizi kullanın ve a, b ve c'nin farklı olası değerlerini araştırın. Aşağıdaki tabloyu en az 5 Pisagor üçlüsü örneği ile doldurunuz. Size bir örnek verilmiştir.

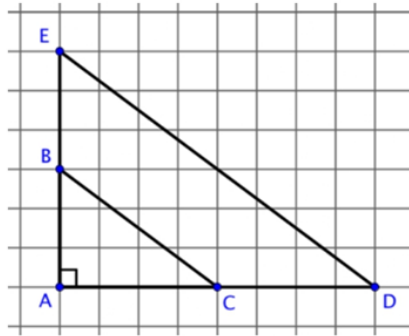
a	b	c	KONTROL ET $a^2 + b^2 = c^2$
8	15	17	$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

Figure E.4. Worksheet 4 part a.

Worksheet 4 part b, distributed to students, is as follows.

Bölüm B: Daha fazla Pisagor Üçlüsü bulmak için Pisagor Üçlülerini kullanma

- (i) Aşağıda iki dik üçgen çizilmiştir. Hem ABC hem de AED için tüm kenarların uzunluklarını bulun.



- (ii) ABC'nin kenar uzunlukları ile AED'nin kenar uzunlukları arasındaki ilişki hakkında ne fark ettiniz?
- (iii) Bu ilişkinin neden var olduğunu gerekçelendirin ve bu ilişkiyi diğer Pisagor Üçlülerini bulmak için nasıl kullanabileceğinizi açıklayın.

Figure E.5. Worksheet 4 part b.

Worksheet 5, distributed to students, is as follows.

Pisagor Eşitsizliği Teoremlerini İncelemek

Bölüm A ve B'ye yanıt vermek için verilen UYGULAMAYI kullanın

**BÖLÜM A:** Bireysel çalışma

A ve B noktaları etrafında hareket etmeyi deneyin. Gözlemlerinizi aşağıya kaydedin.

**BÖLÜM B:** Sıra arkadaşınızla, aşağıdaki soruları yanıtlamak için gözlemlerinizi kullanın.

Hangi durumda	Hangi durumda	Hangi durumda
$a^2 + b^2 = c^2$	$a^2 + b^2 < c^2$	$a^2 + b^2 > c^2$

Figure E.6. Worksheet 5.

Worksheet 6, part 1, distributed to students, is as follows.

Pisagor Eşitsizliği Teoremlerini Uygulamak

Bölüm 1: Aşağıdaki üçgenlerin her birini dar, geniş veya dik olarak sınıflandırın ve nasıl bildiğinizi açıklayın. (Diyagramlar ölçekli çizilmemiştir)

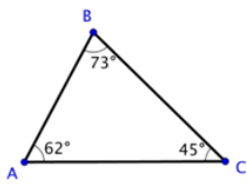
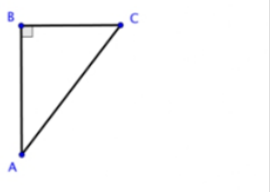
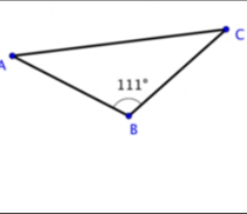
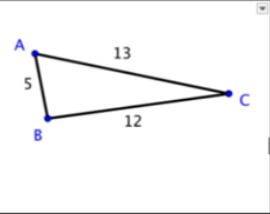
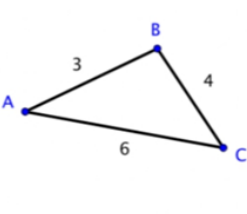
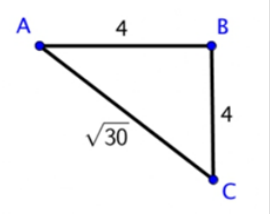
	
	
	

Figure E.7. Worksheet 6.

Worksheet 6, part 2, distributed to students, is as follows.

**Bölüm 2: Her üçgen için, üçgeni dar, geniş veya dik açılı yapan bir  $x$  değeri yazın.**

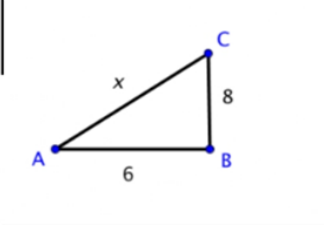
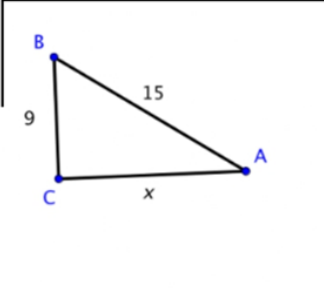
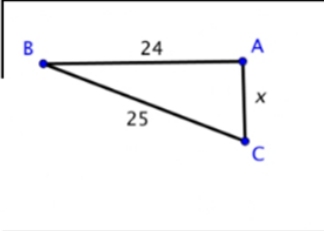
	$\triangle ABC$ 'yi dar açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi geniş açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi dik açılı yapan bir $x$ değeri yazın.
<p>Üçgen 2</p> 	$\triangle ABC$ 'yi dar açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi geniş açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi dik açılı yapan bir $x$ değeri yazın.
<p>Üçgen 3</p> 	$\triangle ABC$ 'yi dar açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi geniş açılı yapan bir $x$ değeri yazın. <hr/> $\triangle ABC$ 'yi dik açılı yapan bir $x$ değeri yazın.

Figure E.8. Worksheet 6 part 2.

Worksheet 7, distributed to students, is as follows.

**PİSAGOR EŞİTSİZLİKLER TEOREMİNİN KANITI**

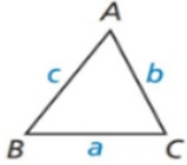
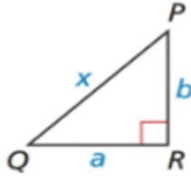
**Teorem:**

**Verilen :**

- ABC üçgeninde,  $c^2 < a^2 + b^2$  (burada c en uzun kenardır)
- PQR üçgeninin a, b ve x kenar uzunlukları vardır, burada x hipotenüsün uzunluğudur
- R açısı bir dik açıdır.

**İspat:**

- ABC üçgeni dar açılı bir üçgendir.

Aşağıdaki sorulara cevap vererek bu teoremi ispatlayacaksınız.

1. Elimizde bir dik üçgen olduğuna göre, dik üçgenlerde kenarlar arasında ilişki kuran teorem hangi teoremdir?
2. Bu teoreme göre kenarlar arasındaki ilişkiyi bulunuz.
3. Bulduğumuz ilişki ile verilen  $c^2 < a^2 + b^2$  ilişkilerini karşılaştırsak ne gibi bir sonuç çıkarabiliriz?
4. Eşitsizlikler konusunu hatırlarsak, 3. Madde ile birlikte bulduğun sonuçta kenarlar hakkında ne söyleyebilirsin?
5. Açı ölçüleri ile kenar uzunlukları arasında bir ilişki olduğunu öğrenmiştin. Buna göre ABC üçgeninin açılarının ölçüleri hakkında ne söyleyebilirsin?
6. Üçgenlerin açılarına göre sınıflandırılması neye göre yapılmaktadır? Buna göre ABC üçgeni açısına göre hangi çeşit bir üçgendir?
7. Eğer bir üçgende  $c^2 > a^2 + b^2$  olsaydı bu üçgen açı ölçülerine göre hangi çeşit bir üçgen olurdu?

Figure E.9. Worksheet 7.

## APPENDIX F: VAN HIELE TEST PERMISSION

Permission was obtained from Professor Asuman Duatepe Paksu and Professor Zalman Usiskin to use the test.

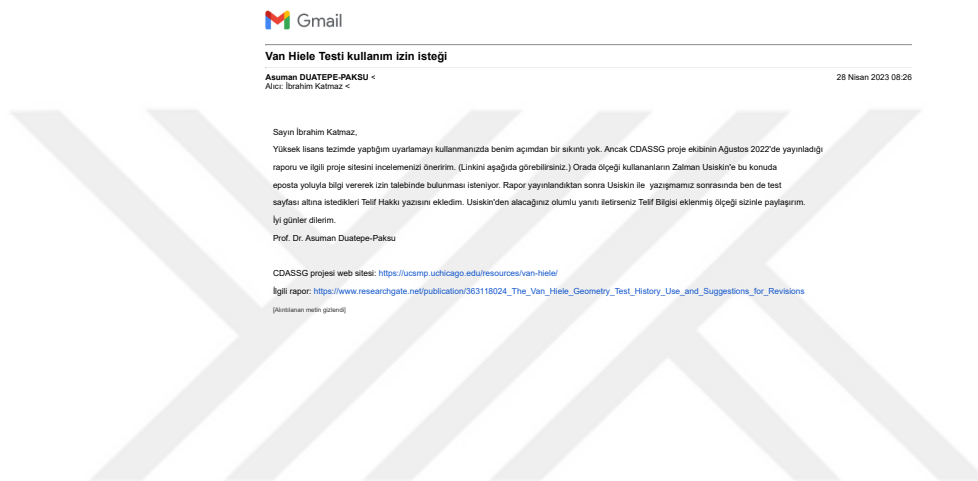


Figure F.1. Van hiele permission.

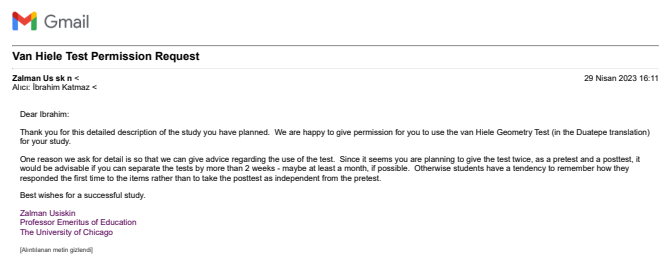


Figure F.2. Translated van hiele permission.

## APPENDIX G: MONE PERMISSION

Permission was obtained from the Ministry of National Education for the research to be conducted in the school.



T.C.  
İSTANBUL VALİLİĞİ  
İl Millî Eğitim Müdürlüğü

Sayı : E-59090411-20-76948019  
Konu : Anket ve Araştırma İzni (İbrahim KATMAZ)

26/05/2023

VALİLİK MAKAMINA

İlgi : a) Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 21.01.2020 tarihli ve 2020/2 sayılı genelgesi.  
b) Boğaziçi Üniversitesinin 08.05.2023 tarihli ve 125138 sayılı yazısı.  
c) Müdürlüğümüz Araştırma ve Anket Komisyonunun 18.05.2023 tarihli tutanağı.

Araştırma Konusu : Bilgisayar Destekli Uygulamalarla Bütünleştirilmiş Geometri Dersi ile Öğrencilerin Geometrik İspatlarını Anlama Becerisini ve Geometri Başarısını Geliştirmek Üzerine Eylem Araştırılması  
Araştırma Türü : Anket  
Araştırma Yeri : Esenler Şehit Muhammed Aksu Kız Anadolu İmam Hatip Lisesi  
Araştırma Kişiler : Öğrenci  
Araştırmanın Süresi : 2022 - 2023 Eğitim ve Öğretim Yılı

Yukarıda bilgileri verilen araştırmanın; 6698 sayılı Kişisel Verilerin Korunması Kanununa aykırı olarak kişisel veri istenmemesi, öğrenci velilerinden açık rıza onayı alınması, bir örneği Müdürlüğümüzde muhafaza edilen mühürlü ve imzalı veri toplama araçlarının kurumlarımıza araştırmacı tarafından ulaştırılarak uygulanması, katılımcıların gönüllülük esasına göre seçilmesi, araştırma sonuç raporunun kamuoyuyla paylaşılmaması ve araştırma bittikten sonra 2 (iki) hafta içerisinde Müdürlüğümüze gönderilmesi, okul idarelerinin denetim, gözetim ve sorumluluğunda, eğitim ve öğretimi aksatmayacak şekilde, ilgi (a) genelge esasları dâhilinde uygulanması kaydıyla Müdürlüğümüze uygun görülmektedir.

Makamınızca da uygun görüldüğü takdirde olurlarınıza arz ederim.

Levent YAZICI  
İl Millî Eğitim Müdürü

OLUR  
26/05/2023  
Dr. Hasan Hüseyin CAN  
Vali a.  
Vali Yardımcısı

Ek:  
1- İlgi (b) Yazı ve Ekleri (14 Sayfa)  
2- İlgi (c) Tutanak (1 Sayfa)

Bu belge ölümlü elektronik imza ile imzalanmıştır.

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Telefon : : Bilgi İçin : Aydın BALTA  
E-posta : : Unvan : VHKİ  
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Figure G.1. Mone permission.

## APPENDIX H: ETHICS COMMITTEE APPROVAL

Ethics committee approval was received from Boğaziçi University Science and Engineering Human Research Board with the application number 2023/15.

Evrak Tarih ve Sayısı: 13.04.2023-122568



T.C.  
BOĞAZIÇI ÜNİVERSİTESİ REKTÖRLÜĞÜ  
Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu  
(FMINAREK)



Sayı : E-84391427-050.01.04-122568  
Konu : 2023/15 Kayıt no'lu başvurunuz hakkında

13.04.2023

Sayın Dr. Öğr. Üyesi Yeşim İMAMOĞLU  
Matematik ve Fen Bilimleri Eğitimi Bölüm Başkanlığı - Öğretim Üyesi

"Bilgisayar destekli uygulamalarla bütünleştirilmiş geometri dersi ile öğrencilerin geometrik ispatlarını anlama becerisini ve geometri başarısını geliştirmek üzerine eylem araştırması" başlıklı projeniz ile Boğaziçi Üniversitesi Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu (FMINAREK)'e yaptığımız 2023/15 kayıt numaralı başvuru 03.04.2023 tarihli ve 2023/04 No.lu kurul toplantısında incelenerek etik onay verilmesi uygun bulunmuştur.

Bu karar tüm üyelerin toplantıya on-line olarak katılımıyla ve oybirliği ile alınmıştır. Onay mektubu tüm üyeler adına Komisyon Başkanı tarafından e-imzalanmıştır.

Saygılarımızla bilginize sunarız.

Prof. Dr. Tınaz EKİM AŞICI  
Başkan

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Doğrulama Kodu :BSM4B8JUM2 Pin Kodu :61952 Belge Takip Adresi : <https://turkiye.gov.tr/ebd?eK=4787&eD=BSM4B8JUM2&eS=122568>

Bilgi için: Nürşen MUNAR  
Unvan: Mühendis



Bu belge, güvenli elektronik imza ile imzalanmıştır.

Figure H.1. Ethics committee approval.