Rüzgar Türbini İçeren Güç Sistemlerinde Ekonomik Güç Dağıtım Probleminin Genetik Algoritma İle Çözümü

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Solution To Economic Power Dispatch Problem In Power Systems Including Wind Turbines With Genetic Algorithm

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ETİK BEYAN

Eskişehir Osmangazi Üniversitesi Fen Bilimleri Enstitüsü tez yazım kılavuzuna göre, Dr. Öğr. Üyesi Burak Urazel danışmanlığında hazırlamış olduğum “Rüzgar Türbini İçeren Güç Sistemlerinde Ekonomik Güç Dağıtım Probleminin Genetik Algoritma İle Çözümü” başlıklı YÜKSEK LİSANS tezimin özgün bir çalışma olduğunu; tez çalışmamın tüm aşamalarında bilimsel etik ilke ve kurallara uygun davrandığımı; tezimde verdiğim bilgileri, verileri akademik ve bilimsel etik ilke ve kurallara uygun olarak elde ettiğimi; tez çalışmamda yararlandığım eserlerin tümüne atıf yaptığımı ve kaynak gösterdiğimi ve bilgi, belge ve sonuçları bilimsel etik ilke ve kurallara göre sunduğumu beyan ederim. 30/06/2021

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İmza

ÖZET

Bu çalışmada Genetik Algoritma kullanarak rüzgâr türbini içeren güç sistemlerinde ekonomik güç dağıtım problemi çözülmüştür. Ekonomik güç dağıtım problemi, matematiksel olarak doğrusal olmayan bir kısıtlı optimizasyon problemi şeklinde modellenebilir. Elektriksel ve fiziksel kısıtlar altında, termik birimlerin toplam yakıt maliyetlerinin en küçüklenmektedir. Rüzgâr türbininin güç üretimi modellenirken Weibull dağılımından yararlanılmıştır. Güç sisteminin iletim kayıpları ekonomik güç dağıtım problemine "*Kron's Formula*" yöntemi kullanılarak dâhil edilmiştir. Benzetimler,15 üretim birimi içeren bir güç sistemi üzerinde gerçekleştirilmiş ve rüzgar gücünün sistemin toplam yakıt maliyeti üzerine etkileri tartışılmıştır.

Anahtar Kelimeler: Ekonomik güç dağıtım problemi, Rüzgar gücü, Yenilenebilir enerji, Genetik algoritma.

SUMMARY

In this thesis, the economic power distribution problem in power systems including wind turbines is solved by using Genetic Algorithm. The economic power distribution problem can be mathematically modeled as a nonlinear constrained optimization problem. Under electrical and physical constraints, the total fuel costs of thermal units are minimized. Power generation of the wind turbine is modelled by using Weibull distribution. The transmission losses of the power system are included in the economic power distribution problem by means of Kron's Formula. The simulations are performed on a power system with 15 generating units and the effects of wind power on the total fuel cost of the system are discussed.

Keywords: Economic power dispatch problem, Wind power, Renewable energy, Genetic algorithm.

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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviation	**Statement**
ELD	Economic load dispatch
ED	Economic dispatch
OPF	Optimal power flow
IGF	Incomplete gamma function
PDF	Probability density function
CDF	Cumulative distribution function
DED	Dynamic economic dispatch
GA	Genetic Algorithm

1. INTRODUCTION

Electrical power is the backbone of modern life like water does. Enabling all these life-saving technological equipment used by hospitals, those data servers running different fields whether it is industrial, commercial, or residential, coming down to agricultural watering serve on behalf of the society in daily routine. These variety of electrical systems which are at the core of everything humanity continues to achieve. The necessity for electric dependent systems will continuously develop in the upcoming years. In economic point of view a reliable electrical power system is has to operate constantly without interruption, also minimizing operational faults to maintain the service it provides to the community disregard to overall system demand. Not only Proper design but matching the consumption demand to the generation quantity through optimizing the system grant a power system to perform silently out of the public scenery, causing less operational interruptions to day and night operations. Also, community have strong expectations for their electrical power systems utilization and direction it goes next decades. To maintain currently existing high-tech dependent standard of living, quality boosting, service maintenance and price minimization has to be the desired goal. These advanced power systems must be able to adapt a new and innovative technology that could make power-output demands grow more secure as well as reliable.

In this study, a solution to the economic dispatch problem where the optimal power generation values are tried to be achieved in the purpose to meet the energy demand with the lowest fuel cost in a power system containing thermal and renewable energy units. First of all, information about the studies in the literature related to the problem discussed is given. Afterwards, a detailed mathematical model of the problem was created. Then, the methods to be used in solving the problem were proposed and these methods were tested on various problems. Finally, the results obtained were presented and comments and suggestions were made for these results.

2. LITERATURE REVIEW

Last decade's world encounters increasing power demand with technological development and growing environmental awareness towards conventional energy usage, since fossil fuel gases causes pollution and global warming crises. The fossil fuel based electric power plants was and still important power source to entire world electrification. It has been a key source since the beginning of electric generation and was preferred due to its availability on earth surface and having capacity to produce significant quantity of electricity at any place (Chen et al., 2016). Consideration of the environment while generating conventional power become sensitive topic last couple of years, since fossil fuel generated electricity led to air pollution and global warming threats. Plants and any generator that use fossil resources release dangerous gases, carbon oxide, sulfur oxide, and nitrogen oxide included to the air. To resolve the problem of environmental concern, clean and renewable energy resources is to be promoted.

Recently, the utilization of renewable energy has been going forward and become attractive to different entities including power industries, researchers due to the growing demand for electricity. To be noted the most power generated by renewables are limited quantity and not big enough to meet the demand that fossil fuels could easily maintained. Also, among the available renewable energy resources wind is preferable due to rapid expansion in the field of power electronics and other associated technologies (Jadhav et al., 2012).

Wind power is not only superior to other renewables but one of a kind among renewable energy popular resources. Wind power is environmental friend, and sustainable energy due to its lack of harmful byproduct or any effect that give arise to pollution or gaseous emission to our atmosphere. In addition, comparing the wind and thermal power generation in consideration of economical point of view wind generation is costly-effective, it saves thermal fuel cost regardless of initial and capital cost (Güvenç and Kaymaz, 2019). Wind power farms are becoming more invested in many counties as a private sector or as a government owned, however the stochastic nature, variability and difficult predictability of wind parameters increase uncertainty and creates extreme challenges in operating integrated power systems. The security and dependability of power grid may be simply ruined if no reliable strategy is employed to schedule the power generated by wind turbines (Liu, 2010). However, the stochastic characteristics of wind parameters in nature at any forthcoming period of time is one of the existing drawbacks of wind power generation. To overcome these challenges various efforts have been made over the years in this direction, The Weibull distribution function (pdf) is being used more in forecasting and fairly predict the nature of the wind and its parameters.

An incredible optimization approach called economic dispatch problem was used last decades to tackle with fuel cost minimization problem. According to different research interest in solving economic load dispatch (ELD) problem in the electric power systems has extremely increased. Without further a due let us see what that economic dispatch problem is, its main objective function and how it deals with cost minimization problems.

Economic dispatch problem is defined as short-term determination to power output optimality of number of generating units in the meantime satisfying the power demand at the lowest probable while considering transmission loss, and operational system constraints. When it comes to solving economic dispatch problem specialized computer aided software is modelled the intended problem and it has to fulfill the operational and system constraints of the available resources and related transmission capabilities (Anonymous, 2021). In the Us Energy Policy Act of 2005, the dispatch problem is described as the operation of generation facilities to produce energy at the least cost to constantly provide quality serve consumers, acknowledging any operational limits of generation and transmission facilities (Anonymous, 2005). In other words, The Economic dispatch problem is a nonlinear programming optimization technique where key purpose is to produce the optimal schedule of available generating units in order to reach the demand power at minimum operating cost under different system and operating constraints. The goal is to get maximum usable power applying minimum resources. Traditionally the Economic power dispatch of a power system is solved in the unit commitment environment and real time operation plants by taking into account that each of the dispatchable on-line units can be regulated continuously between its minimum generation limit and its maximum generation limit (Anonymous, 2021).

According to Akkaş et al. (2017) to operate power system components economically and generate the energy with cheapest cost was considered very important. Economic load dispatch problem reduces the generation cost as well as matching the load demand by satisfying the equality and inequality constraints. Here to schedule the outputs of all available generation units in the power system such that the fuel cost is decreased while system constraints are satisfied is the goal of economic power dispatch. It can be also described as the procedure of assigning generation among the designated units such that the constraints imposed are satisfied and the energy requirements are minimized (Al Farsi et al., 2015).

To achieve the required demand from the generation site with the consideration of transmission loss at the lowest production cost, economic load dispatch (ELD) is the best option to deliver the expected generation schedule. Different inquiries on ELD have been taken to the search of significant better solution to accomplish economical gains (Bhattacharya and Chattopadhyay, 2010). The chief intention of fulfilling objectives of Economic dispatch problem is to determine the optimal schedule of available generating components to meet the power demand at lowest operating cost under various system and operating constraints like ramp rate limits and prohibited zones (Naama et al., 2013)

According to recently published optimization work mostly related with economic load dispatch problem is concentrated on heuristic methods including the algorithms like particle swarm optimization, genetic algorithms, neural network, and so on. Economic dispatch handles the problem of dynamic resource Energy allocation, where a set of generation units must meet the energy demand in specific time frame at the most economical solution (Mejia and Patiño, 2016). Wulandhari et al. (2018) used bat algorithm to solve ELD problem, and the experimental results indicate that method saves 1.23% of generating cost hourly compared to the actual condition, in the meantime firefly algorithm saves around 1.16%. So, bat algorithm can deliver generators output efficiently while minimum cost and transmission loss is achieved.

Tholath (2014) successfully employed bat algorithm on ELD problem not only solving thermal units but also considering wind power units. The objective function of the method takes different costs into account. To confirm the feasibility of the presented Bat algorithm obtained results of different test systems are compared with particle swarm optimization (PSO) algorithm and the proposed method is capable of getting higher quality solutions. In Al-Nahhal et al. (2019), wind speed data was used to integrate ELD problem thermal units with wind power unit, by applying PSO algorithm to the thermal and wind power penetration combined for load data of 96 hours on behalf of the whole year. The simulation result shows that the wind power has a powerful and fast influence in minimizing total fuel cost.

Generation of power from different sources including thermal and renewables in electrical network are to be optimally arranged for economical and efficient operation system. To do so optimal power flow (OPF) was formulated to solve the problem combining stochastic wind and solar with conventional thermal power units in the system. The objective function of the method considers all the various system costs including reserve cost for overestimation, penalty cost for underestimation of random renewable sources. The OPF provides optimum result satisfying all the network constraints (Partha et al., 2017).

According to Basu (2019), dynamic economic dispatch problem considering solar, wind, pumped storage hydro, and thermal power system was solved using chaotic fast convergence evolutionary programming (CFCEP). Executing three different test systems for three different problems by utilizing the suggested CFCEP, differential evolution (DE) and PSO. It was noticed that the recommended CFCEP approach performs better than DE and PSO. This technique can be making more interesting by utilizing tent chaotic mapping for population generation, alternative to random initialization.

In Modiri-Delshad and Nasrudin Abd Rahim (2014), backtracking search algorithm (BSA) is proposed to solve the non-convex economic dispatch (ED) problem in a way it considers transmission loss a and valve point effects, using four varied size and complexity case studies, the result gained by BSA were compared with other classical evolutionary methods and it converged to the same optimal solution in all trails. BSA proved robustness, lower generation cost, and high-quality solutions through all the test trails demonstrated, results confirmed BSA is capable of solving ED problems well and robustly. Many academic studies have presented different evolutionary algorithms applied to ED problems, In Mandal et al. (2014), a new stochastic search technique called krill herd algorithm (KHA) was used to find the solution of various forms of non-convex ELD problems. Developing the convergence behavior of the proposed technique different types of non-convex and non-smooth ELD problems with non-linearities of valve point effects, prohibited operating zones, ramp rate constraints, and then fuel option is used to estimate the performance of the (KHA). To clarify the rigidness of the proposed KHA based methods, statistical analysis was fulfilled on all the test systems. Also, the simulated result presents that KHA approach performs better than other reported methods in terms of solution quality, computational efficiency, robustness, and stability. And that implies the recommended KHA method is promising and encouraging for further research work.

As Lee Gaing (2003), two decades ago successfully used particle swarm optimization (PSO) method to solve the ED problem with the generator constraints. The method has been demonstrated to have better characteristics such as high-quality solution, stable convergence features and good computation efficiency. In addition, for practical generator operation consideration nonlinear characteristics of the unit including ramp rate limits, valve point and non-smooth cost function is regarded. The proposed method’s result shows that it indeed capable of obtaining quality solution efficiency in ED problems.

According to Banerjee et al. (2015), a method called teaching learning based optimization (TLBO) is implemented to solve ELD problem by valve point loading effect consideration in a way transmission loss and other constraints were not included. The proposed method proved its superiority after TLBO is efficiently and effectively applied on different test systems and compared with other existing methods. So, this method has the ability to explore the solution space to obtain the global optimal solution. That means TLBO is a promising method for ELD calculations in power systems.

In order to solve dynamic economic dispatch (DED) problem considering the valve point effects while regarding different complex constraints a method with the name chaotic differential bee colony algorithm (CDBCO) based on bee hunt behavior and differential evolution strategy is effectively modeled. To verify the feasibility and effeteness of the proposed method, various test cases has been employed while considering valve point effects and transmission loss. After comparing test results with other reported methods, it has been revealed that the proposed CDCBO method is able to get higher quality solutions with faster convergence speed and stronger local search strength (Lu et al., 2014).

In Güvenç and Kaymaz (2019), ED problem incorporated with wind power is implemented using coyote optimization algorithm (COA). Weibull probability density function (pdf) and incomplete gamma function (IGF) was employed because of wind power nature uncertainty. The approach is tested with numerous test cases consisting thermal and wind units and achieved simulation results proved that COA has minimum fuel cost and better performance. An ED problem with and without wind power plant simulation result illustrate that wind power have strong performance in total cost production and can minimize total cost in overall power system. In the meantime, if the wind farm owned by system operator, total cost can be reduced better than when the farm owned by private sector (Dozein et al., 2012).

A brain storm optimization (BSO) called algorithm is studied for wind power integrated ELD problem, the uncertainty of the wind characteristics was calculated through Weibull distribution. Besides the regular fuel cost other costs including direct, under estimation, and over estimation costs brought by wind power units was considered in the method. Since the actual wind power output is randomly changing and different from the predicted capacity redundant power should be maintained for reliable and secure power system operation. The BSO’s ability to search global optimal solution is validated after overall cost operation result was compared with other test results of the existing methods (Jadhav et al., 2012).

The heuristic method’s goal is to develop and increase the solution quality of the proposed method. Here, in Arai et al. (2018), targeting optimal operation planning of energy production through formulating nonlinear optimization problem conducted using modified brain storm optimization (MBSO). The method was compared with other conventional techniques particularly the original BSO, and it is validated that the total energy cost by this method is lower than all those comparative conventional ones.

Transmission loss coefficient based genetic algorithm called (GALCs) is proposed by Jethmalani et al. (2018). Since estimation of transmission loss is very important in planning, scheduling, and optimization in power system. It has to keep in mind ignoring or inaccurate transmission loss effects the revenue of many utilities. The submitted method uses real power generation samples, consumption, and collected losses from different operating conditions. The technique extracts loss coefficients from that data by minimizing the mean absolute error between actual and collected loss values employing real coded GA. The method was used to evaluate the dynamic economic dispatch problem and its performance was compared with other traditional loss estimation procedures. The investigation result uncovers that the proposed GALCs is well performed in loss estimation compared to B-loss coefficient method.

In the search for optimal solution determined on dynamic economic dispatch problem by GA, an optimization method with the name Hybrid genetic algorithm and bacterial foraging (HGABF) is proposed. The practical constraints of generators, such as transmission loss, valve point effects and ramp rate were considered. The method was demonstrated applying the commonly used test systems including 5, 10, and 30 test systems. After comparing numerical results with the recently reported approaches, the output dispatch solution obtained by HGABF method led to a smaller operational cost compared to those found by other methods. Which makes the algorithm a promising tool capable of determining the global or near global optimal solution (Elattar,2015).

3. PROBLEM FORMULATION

The major objective of the economic dispatch problem in electric power systems and other fields is to determine the optimal cost operation of each generating unit. The proposed economic dispatch problem is formulated considering different cost including fuel cost of the thermal units, direct, overestimation and underestimation costs of the available wind power units.

3.1. Objective Function

The cost function of integrated thermal and wind turbine economic dispatch problem is given by:

$$C(p,w) = \sum_{i=1}^{N} f_i(p_i) + \sum_{j=1}^{M} g_j(w_j) \tag{3.1}$$

Here the above equation, p_i and w_j are active power outputs of ith thermal and jth wind power units, respectively. N and M are the total number of thermal generators and wind turbines, accordingly. The fuel cost function of the ith thermal generator, $f_i(p_i)$, and the cost of the jth wind turbine, $g_j(w_j)$, can be described as:

$$f_i(p_i) = a_i p_i^2 + b_i p_i + c_i + \left| e_i \sin\left(f_i\left(p_{i,min} - p_i\right)\right)\right| \tag{3.2}$$

$$g_j(w_j) = q_j w_j + C_{rw,j} E\left(Y_{oe,j}\right) + C_{pw,j} E\left(Y_{ue,j}\right) \tag{3.3}$$

Where a_i, b_i, c_i, e_i and f_i are the cost coefficients of ith thermal generator unit. $p_{i,min}$ is the lower limit for the active power generation of ith thermal generator unit. Besides, q_j is the cost coefficient of the wind unit which sometimes called direct cost, $C_{rw,j}$ represents the cost coefficient of the remainder energy quantity of the jth available wind power, which is sometimes called underestimation cost coefficient. Also, $C_{pw,j}$ represents the cost coefficient of purchasing extra electric from other reserve to back up the shortage of the predicted wind power, and it is called overestimation cost. The above mentioned,

$E(Y_{oe,j})$, and $E(Y_{ue,j})$ are the expected energy value of wind power overestimation and underestimation for jth wind generator unit. The derivates of these values is referred to Liu and Xu, (2010) and illustrated the below lines.

$$\begin{aligned} E(Y_{oe}) = \omega \left[1 - \exp\left(-\frac{v_{in}^k}{c^k}\right) + \exp\left(-\frac{v_{out}^k}{c^k}\right)\right] \\ + \left(\frac{\omega_r v_{in}}{v_r - v_{in}} + \omega\right)\left[\exp\left(-\frac{v_{in}^k}{c^k}\right) - \exp\left(-\frac{v_1^k}{c^k}\right)\right] \\ + \frac{w_r c}{v_r - v_{in}}\left\{\Gamma\left[1 + \frac{1}{k}, \left(\frac{v_1}{c}\right)^k\right] - \Gamma\left[1 + \frac{1}{k}, \left(\frac{v_{in}}{c}\right)^k\right]\right\} \end{aligned} \tag{3.4}$$

$$\begin{aligned} E(Y_{ue}) = (\omega_r - \omega)\left[\exp\left(-\frac{v_r^k}{c^k}\right) - \exp\left(-\frac{v_{out}^k}{c^k}\right)\right] \\ + \left(\frac{\omega_r v_{in}}{v_r - v_{in}} + \omega\right)\left[\exp\left(-\frac{v_r^k}{c^k}\right) - \exp\left(-\frac{v_1^k}{c^k}\right)\right] \\ + \frac{w_r c}{v_r - v_{in}}\left\{\Gamma\left[1 + \frac{1}{k}, \left(\frac{v_1}{c}\right)^k\right] - \Gamma\left[1 + \frac{1}{k}, \left(\frac{v_r}{c}\right)^k\right]\right\} \end{aligned} \tag{3.5}$$

Here, c and k are Weibull distribution parameters for dealing with wind speed estimation, and it is called scale and shape factor of the wind generator unit accordingly. Also V_{in}, V_r and V_{out} represents the value of cut-in, rated, and cut-out wind speed for the wind turbine, respectively. Furthermore, ω_r is the wind power generated at the rated speed of the turbine. v_1 can be considered as an intermediate value which can be calculated as follows:

$$v_1 = v_{in} + (v_r - v_{in})\omega_1/\omega_r \tag{3.6}$$

Γ in (3.5) incomplete gamma function (IGF) respectively and can be computed as follows (Arai et al., 2018):

$$\Gamma(\alpha, x) = \int_x^\infty y^{\alpha-1} \exp(-y)\, dy \tag{3.7}$$

3.2. Wind Power Calculation

Since wind data randomly changes in any moment it is difficult to convert wind energy to electricity instantly. The most familiar statistical functions applied to wind power availability representation are Weibull and Rayleigh distribution.

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v^{(k-1)}}{c}\right)e^{-(v/c)^K}, \qquad 0 < v < \infty \tag{3.8}$$

The statistical function, $f(v)$ stand for the probability or portion of time for which the velocity of wind v at a given site. In the other hand cumulative distribution function (CDF), here $f(v)$ represents the probability that the wind velocity is equal or less than v and is given by the following equation.

$$f(v) = 1 - e^{-(v/c)^{\mathrm{k}}} \tag{3.9}$$

The cumulative distribution function (cdf) can be found by integrating the probability density function (pdf). The output power generated by wind turbine in watt unit is given by:

$$P_w = \frac{1}{2}\rho A\pi V^3 \tag{3.10}$$

Where, ρ is the air density, A is the cross-sectional area which air passes through, V is the wind speed that changes randomly. The following formulas is a simplified model representing the relation between different wind power outputs and corresponding wind speeds.

$$\begin{aligned} \omega &= 0 && if \quad v < v_{in}\ and\ v > v_{out} \\ \omega &= \omega_r \frac{v - v_{in}}{v_r - v_{in}} && if \quad v_{in} < v < v_r \\ \omega &= \omega_r && if \quad v_r < v < v_{out} \end{aligned} \tag{3.11}$$

3.2. System Constraints

The ED problem is subject to the system constraints including active power output limit for the ith thermal generator units, jth wind turbine unit, and supply demand balance are given in the following equations:

$$p_{i,min} \leq p_i \leq p_{i,max} \tag{3.12}$$

$$w_{j,min} \leq w_j \leq w_{j,max} \tag{3.13}$$

$$\sum_{i=1}^{N} p_i + \sum_{j=1}^{M} w_j = P_D + P_{Loss} \tag{3.14}$$

Where $p_{i,min}$ and $p_{i,max}$ symbolize the minimum and maximum active power output of ith thermal generator units, while also $w_{i,min}$ and $w_{i,max}$ denotes the minimum and maximum of the available jth wind power output unit, respectively. P_D is the total power load demand of the system in (MW) scale, P_{Loss} is the system transmission loss in (MW) scale represented by B-matrix formula know as Kron's loss formula which is described as below.

$$P_{Loss} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{N_g} B_{0i} P_{Gi} + B_{00} \tag{3.15}$$

In (3.15), N_g is the total number of generation units in the considered power system and P_{Gi} is the active power generation of the ith unit either it is thermal unit or wind turbine. B_{ij}, B_{0i} and B_{00} are B-matrix loss formula coefficients.

4. METHOD

Genetic algorithm (GA) is global optimization technique emerged as a contender to other optimization applications due to its flexibility and efficiency in order to achieve the finest potential solution on a target population. It's one of those stochastic search algorithms and categorized as global search metaheuristic techniques. The method was found by John Holland and his colleague students in 1975 (Khosa et al., 2015). Since then, the technique was used different fields in science and Engineering world.

The genetic algorithm conceptual is based on a principle that allows the fittest gene solution to be survived in a given population. The approach starts with approximation to converge the best solution. Each and every generation selected, a new population is created, and the individual optimum solution is discovered based on its fitness value. The procedure is done differently many times, until all elements of a given generation share the same genetic tradition. After that, there are nearly no dissimilarity between individuals. All individual members of these final generations, which are often quite different from their descendants, possess genetic evidence that corresponds to the best solution to the optimization problem.

The advantage GA approach in terms of flexibility, problem reduction, and solution targeting methodology is presented below lines (Garg et al., 2011).

i. Simply to understand and also to implement, and its way of giving early good solution approach.
ii. It is ability to solve problems with multiple results.
iii. Since the GA implementation technique does not reliant on the error surface, multi-dimensional, non-differential, non-continuous, and even non-parametrical problems can be solved.
iv. It is suitable for parallel computers.
v. Optimizes variables with exceptionally complicated cost surfaces (they can jump out of a local minimum).

4.1. Operators of Genetic Algorithm

In this work, The GA main operators are the decision variables to execute a given problem and reach the best solution, and it is based on the following routine.

Selection sometimes called as reproduction is the first genetic algorithm operator with the aim of searching the optimum solutions in the given population for the chromosome production procedure. The healthiest which means progressed individual in the first trail has the better probability to take part in the next generation for reproduction. Sometimes fitness is described to as an ability of an individual within a certain environment with available resources. In genetic algorithm, members of population compete each other and are selected to the next level based on their performance within given problem statement, the crucial objective of the selection operator is not only to pick the best individual and remove the bad solutions in population but also keeping the population size fixed. In selection process, there are various methods used as selection operator including roulette wheel selection, tournament selection, Boltzmann selection, rank selection, and steady state selection.

Crossover or recombination is the most significant factor in Genetic algorithm. The crossover operator produces a new offspring by swapping one or more gene traces of two parents. The easiest form of crossover is that of single point crossover. And that means two chromosome strings are selected randomly from the matting pool, then the crossover site is also selected randomly along the string length. in crossover, a sequence of two chromosomes are exchanged.

Mutation is the essential element of genetic algorithm. The mutation procedure appears after the completion of crossover of two parents. The last operator, which is mutation helps constructing the new generation. Mutation is a process where genes of some chromosome randomly modify it self.

4.2. Parameters of Genetic Algorithm

The flowchart of Genetic algorithm execution process is illustrated in Figure 4.1. The performance presented by the GA simulation depends on choice given to following GA parameters

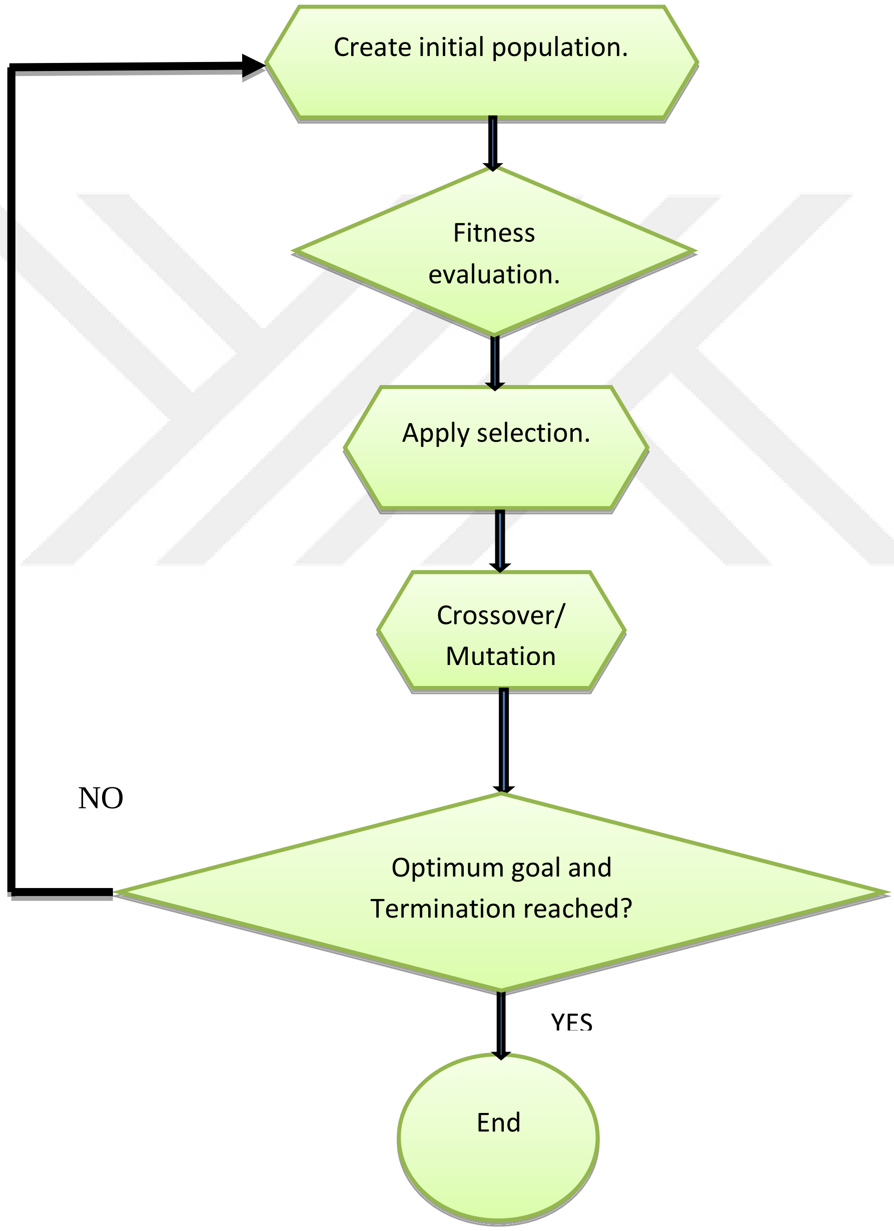


Figure 4.1. Flowchart of GA

Population size: The size of the population has direct impact to the performance and efficiency of the genetic algorithm. When it comes to the comparison between higher population size and lower population size it’s always a matter of practical and trail precision, but higher population size increases its diversity and reduces the chance of premature converge to the local optimum. Although the time to converge to the optimal region will increase. However, lower population size may lead to poor performance to the algorithm due to lack of ability to cover the entire problem space. Overall, best population size reported as between 50-100 and more (Mansour et al., 2013).

Crossover rate: The crossover rate is the parameter that effect the rate at which the procedure of crossover is employed. This rate should usually be higher value, about 80-95%.

Mutation rate: The mutation rate I is a secondary search operator that improves the diversity of the population. So low-rate mutation helps to prevent any bit position been trapped at a single value. This rate must be very low.

Termination of the GA: At the end after different iterations have been applied to the problem until termination condition has been fulfilled. The common terminating terms are as follow: fixed number of generations reached, a best solution is not changed after different iterations made, or the lowest cost that could be reached.

5. RESULTS AND DISCUSSION

The proposed economic dispatch problem is solved by genetic algorithm simulation using MATLAB R2020a on a core i7, 2.70GH processor with 8GB RAM PC. Different case studies have been reviewed and this approach only focuses on 15 thermal generating units but in dissimilar scenarios unlike other similar test systems implemented on metaheuristic techniques before. The projected simulated results are classified into three parts. In part 1 the fuel cost minimization of the system generating units is found without regarding the transmission loss. Part 2 the total fuel cost is uncovered by considering transmission loss effect. Part 3 the total fuel cost calculation was little bit hybrid by integrating wind generating unit with the other thermal generating units. In all cases, GA is applied to the considered dispatch problem by using the parameters given in table 5.1.

Table 5.1. GA parameters used in all cases.

Parameter	Value
Population Size	850
Number of Generations	450
Number of Elite Count	30
Crossover Ratio	0.925
Mutation Ratio	0.075
Crossover Type	Arithmetic Crossover

5.1. Lossless Case without Wind Power Generation

This study case test system contains 15 thermal generating units and transmission loss was not considered toward the fuel cost calculation. According to Cai et at. (2007), the load demand capacity of this system is 2630 MW. The fuel cost coefficients of all generating units and also considering valve point effects are given in Table 5.2. In this case different trails have been conducted to the algorithm in order to find the optimal fuel cost value. After exactly 30 trails with the following GA parameters: Population number 850, Generation size 450, Elite count 30, Crossover value 0.925, and Mutation ratio of 7.5%. With those operators used, the best, the worst and the average results among 30 trials are given in Table 5.3. For the loseless case, among 30 trials, the lowest total generation cost is found to be 33625 $/h. For the best solution, all 15-unit real power generation in MW and Total fuel cost in $/h have been presented in Table 5.5.

Table 5.2. Coefficients related with power generating units.

Units (i)	$P_{min,\,i}$ (MW)	$P_{max,\,i}$ (MW)	α_i ($)	b_i $(\$/MW)$	c_i $(\$/MW2)$	e_i	f_i
1	150	455	671	10.1	0.000299	100	0.0840
2	150	455	574	10.2	0.000183	100	0.0840
3	20	130	374	8.8	0.001126	100	0.0840
4	20	130	374	8.8	0.001126	150	0.0630
5	150	470	461	10.4	0.000205	120	0.0770
6	135	460	630	10.1	0.000301	100	0.0840
7	135	465	548	9.8	0.000364	200	0.0420
8	60	300	227	11.2	0.000338	200	0.0420
9	25	162	173	11.2	0.000807	200	0.0420
10	25	160	175	10.7	0.001203	200	0.0420
11	20	80	186	10.2	0.003586	200	0.0420
12	20	80	230	9.9	0.005513	200	0.0420
13	25	85	225	13.1	0.000371	300	0.0350
14	15	55	309	12.1	0.001929	300	0.0350
15	15	55	323	12.4	0.004447	300	0.0350
16 (w)	0	80	0	0	0	0	0

5.2. Lossy Case without Wind Power Generation

In this case the simulation is employed on the 15 thermal generating units while considering the transmission loss. The B loss coefficients applied in this test system is taken from Gaing (2003). This case and previous case are similar for not including wind power generation into the system generation list. It is worth to be mentioned that utilized GA parameters like crossover value, mutation ratio are kept unchanged. As it is given in

Case 5.1., the best, the worst and the average results among 30 trials are given in Table 5.3. For the lossy case, among 30 trials, the optimal fuel generation cost output came as 34002 $/h which is higher compared to previous lossless case since the transmission losses are included with the problem and thus the total demand is increased. In order to supply the load demand and the transmission losses, the total power generation of the system is increased. Therefore we have an increment on the total fuel cost of the thermal units. For the best solution, all 15-unit real power generation in MW and Total fuel cost in $/h have been presented in Table 5.5.

Table 5.3. For three different cases, the best, worst and average results for 15-unit test system among 30 trials.

	Lossless Case without wind	Lossy case without Wind	Lossy case with wind
Best Fuel Cost	33625.09 $/h	34002.57 $/h	32878.63 $/h
Worst Fuel Cost	34120.25 $/h	34666.20 $/h	33631.04 $/h
Average Fuel Cost	33876.75 $/h	34334.15 $/h	33299.70 $/h
Best Solution Time	11.98 sec	5.08 sec	7.74 sec
Worst Solution Time	48.27 sec	25.90 sec	40.84 sec
Average Solution Time	15.67 sec	10.60 sec	13.66 sec

5.3. Lossy Case with Wind Power Generation

This case has its similarity with the second case in one way and has its difference in another way. Considering transmission loss shown in Table 5.2. is what the two cases share but considering wind turbine generation unit as one of the 15-unit test system is what the two cases are dissimilar. Furthermore, this case consists of 14-unit thermal generation unit and one wind turbine generation unit. In other words, 15'th thermal unit is replaced by a wind turbine. The considered wind turbine has the following parameters, rated power 80 MW, rated wind speed 12.5 m/s, cut-in speed 4 m/s, and cut-out speed 20 m/s as given ,n table 5.4.

Table 5.4. Parameters of the selected wind turbine.

Parameter	Value
Rated Power (MW)	80
Rated Wind Speed (m/s)	12.5
Cut-in Speed (m/s)	4
Cut-out Speed (m/s)	20

The wind distribution is assumed to be a Weibull distribution with the following parameters: shape factor (k) as 2, and scale factor (c) as 10 m/s. Selected Weibull distribution plotted as shown in Figure 5.1.

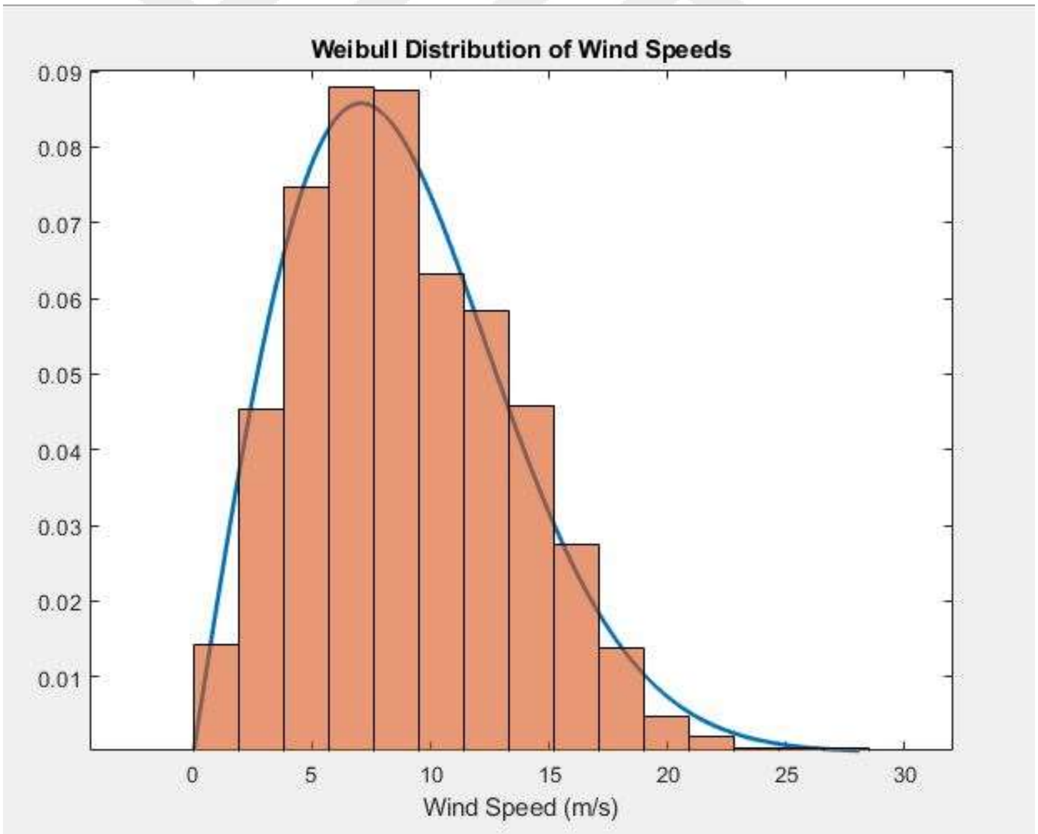


Figure 5.1. Weibull distribution of the selected wind parameters

Extra costs brought by stochastic wind turbine is also introduced. Direct cost (WC), cost due to overestimation (RC), and cost due to underestimation (PC) are included by the coefficients those have the values of 7 $/MW, 10 $/MW, and 6 $/MW, respectively.

Table 5.5. The best results for 15-unit test system for three different cases.

	Lossless Case without wind	Lossy case without Wind	Lossy case with wind
p_1 (MW)	337.2950	450.2662	403.4030
p_2 (MW)	335.2440	403.5286	337.0180
p_3 (MW)	100.9460	66.8459	96.5610
p_4 (MW)	104.2280	125.7217	114.4800
p_5 (MW)	432.8680	389.2969	394.8020
p_6 (MW)	395.5450	299.8021	397.7260
p_7 (MW)	434.2090	432.5936	359.4540
p_8 (MW)	135.1040	149.7793	209.6000
p_9 (MW)	80.4120	110.3017	99.8000
p_{10} (MW)	98.7710	104.8513	99.8000
p_{11} (MW)	50.3900	44.2615	47.4590
p_{12} (MW)	39.8160	33.0766	35.5170
p_{13} (MW)	39.3220	27.9106	25.8590
p_{14} (MW)	26.2970	16.7900	19.2450
p_{15} (MW)	19.5530	17.9278	34.6830
Total Power Generation (MW)	2630.00	2672.95	2675.40
P_{Loss} (MW)	0.0000	42.95	45.41
Total Fuel Cost ($/h)	33625.09	34002.57	32878.63

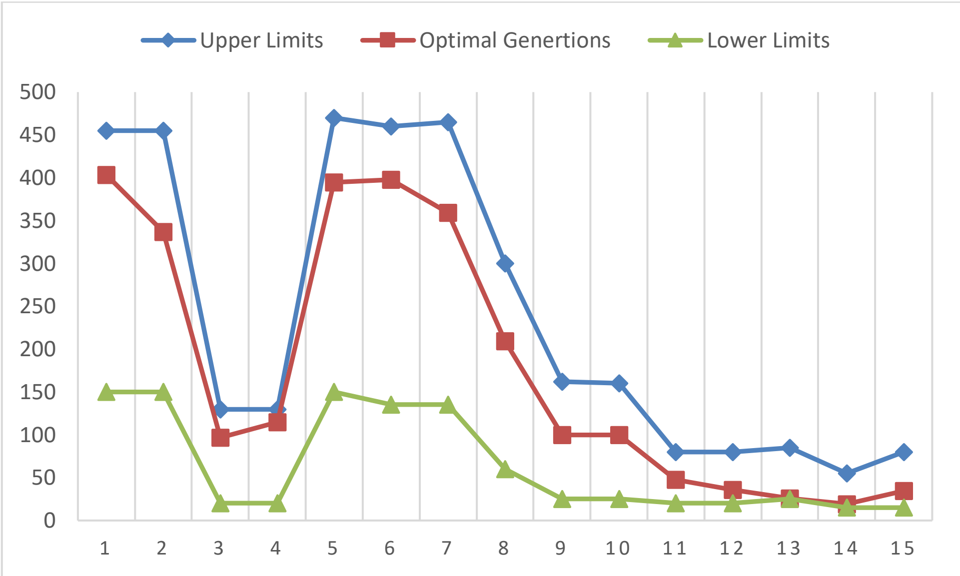
5.4. Discussions

With considering wind power generation, the lowest fuel cost of the thermal units is obtained as 32878.63$/h which is lower compared to previous test case output since the wind turbine also supplies the load demand and the system loss and reduces the total thermal power generation. The best, the worst and the average results among 30 trials are given in Table 5.3. For the best solution, all 15-unit real power generation in MW and Total fuel cost in $/h have been presented in Table 5.5.

As it can be seen from Table 5.5, the total fuel cost of the thermal units is reduced from 34002.57 $/h to 32878.63 $/h when the fifteenth thermal unit is replaced by a wind turbine even the transmission loss is increased from 42.95 MW up to 45.41 MW. That 3.30 percent saving on the fuel cost is obvious since the wind turbine also supplies the total demand, which is equal to the sum of load demand and transmission losses, and results a decrement on the thermal power generation.

When the active power generations are eamined for the lossy case with wind power generation, it can be seen that wind turbine is generated 34.6830 MW. By using (3.11), the wind speed required to generate this amount of active power can be found as 7.6850 m/s. So, it is obvious that the wind turbine is operating on a wind speed which has higher probability to occur as it is given in Figure 5.1., according to the selected Weibull parameters. This is because the cost coefficient of the wind unit such as direct cost coefficient, q_j , underestimation cost coefficient, $C_{rw,j}$, and overestimation cost coefficients, $C_{pw,j}$, used in (3.3) are forced the wind turbine to operate with wind speeds around the ones that are highly expected according to the selected distribution functions.

Furthermore, the total power generations are equal to the sum of load demand and transmission losses. Also it can be also seen in Table 5.5. and Figure 5.2, the active power generations obtained for the best solutions among 30 trials are within the upper and lower generation limits and there is not any violation occurred in the solutions. Therefore it can be proven that all constraints defined in section 3.2. are satisfied in the solution points for all three cases considered in this study.


Upper Limits
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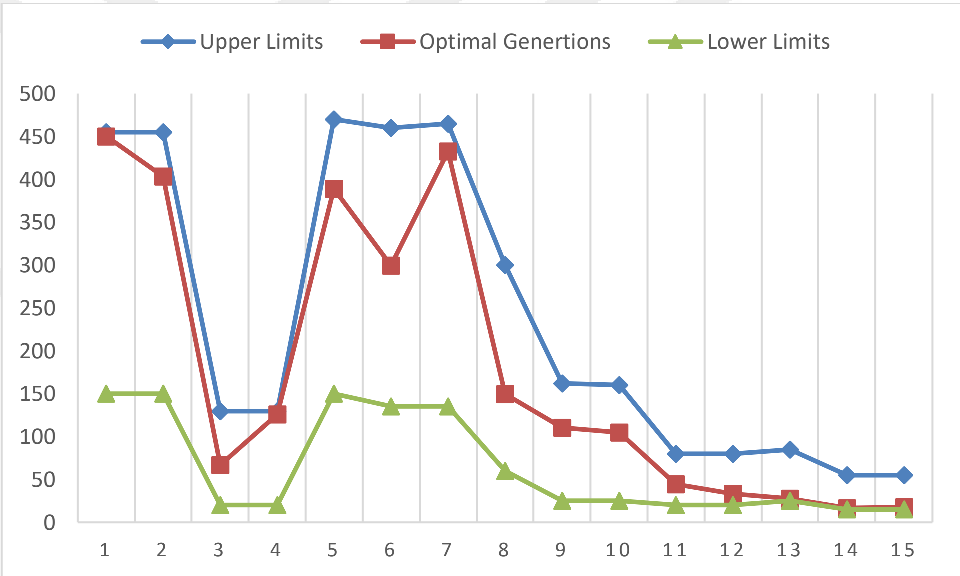
(a)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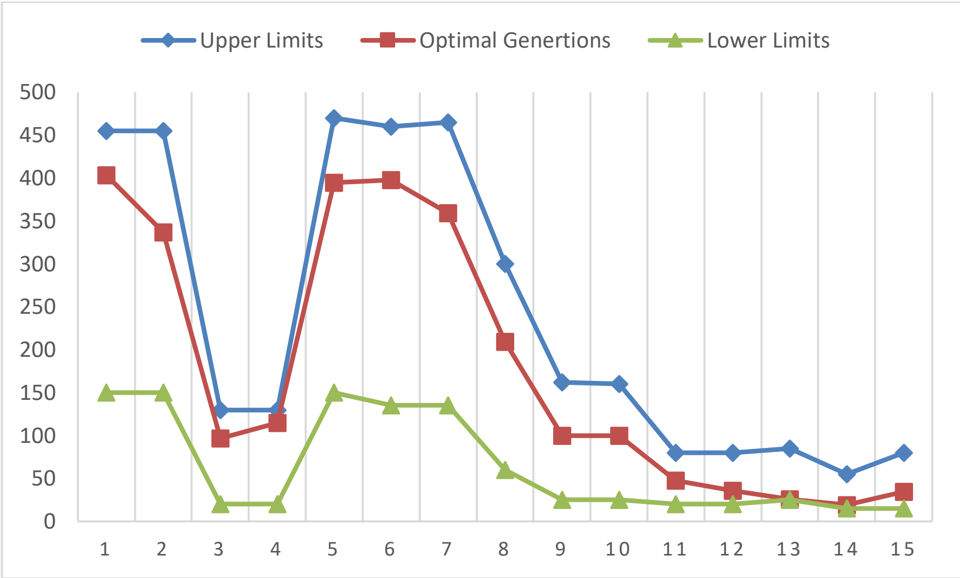
(b)



(c)

Figure 5.2. Optimal active power generations and generation limits related with (a) loseless case, (b) lossy case without wind power generation and (c) lossy case with wind power generation.

6. CONCLUSION AND RECOMMENDATIONS

In this thesis, economic dispatch problem in power systems including wind turbine is solved by means of genetic algorithm. Genetic algorithm is one of the well-known stochastic search algorithms to solve optimization problems existing in different fields in science and engineering world.

Economic dispatch problem is mathematically modelled as a constrained optimization problem. Total fuel cost of the thermal units are minimized under electrical and physical constraints of the power system. Weibull distribution is used to represent the wind power availability. Besides, the uncertainty in the wind power generation is inserted to the problem with underestimation and overestimation costs where ıncomplete gamma functions are used.

To demonstrate the effects of transmission losses and wind power injection, considered dispatch problem is solved in three different cases: (i) lossless case without wind power, (ii) lossy case without wind power, (iii) lossy caes with wind power. Transmission losses are calculated by Kron's Loss Formula. Results show that total fuel are reduced when the wind turbine is committed to the system.

For the future works, additional renewable energy sources such as solar systems, hydro units, etc. can be also considered in the dispatch problem. Furthermore, power transmission line limits should be inserted in order to control the security constraints. Moreover, not only the effects of the renewable energy sources on the fuel cost, but also the effects on the carbon emissions should be examined.

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APPENDIXES

Appendix –A: B-Matrix Loss formula coefficients of 15 unit test system.

Appendix –A: B-Matrix Loss formula coefficients of 15 unit test system.

B-Matrix loss formula coefficients of 15 unit test system is given in Figure A.1.

$$B_{ij} = \begin{bmatrix}
0.0014 & 0.0012 & 0.0007 & -0.0001 & -0.0003 & -0.0001 & -0.0001 & -0.0001 & -0.0003 & 0.0005 & -0.0003 & -0.0002 & 0.0004 & 0.0003 & -0.0001 \\
0.0012 & 0.0015 & 0.0013 & 0.0000 & -0.0005 & -0.0002 & 0.0000 & 0.0001 & -0.0002 & -0.0004 & -0.0004 & -0.0000 & 0.0004 & 0.0010 & -0.0002 \\
0.0007 & 0.0013 & 0.0076 & -0.0001 & -0.0013 & -0.0009 & -0.0001 & 0.0000 & -0.0008 & -0.0012 & -0.0017 & -0.0000 & -0.0026 & 0.0111 & -0.0028 \\
-0.0001 & 0.0000 & -0.0001 & 0.0034 & -0.0007 & -0.0004 & 0.0011 & 0.0050 & 0.0029 & 0.0032 & -0.0011 & -0.0000 & 0.0001 & 0.0001 & -0.0026 \\
-0.0003 & -0.0005 & -0.0013 & -0.0007 & 0.0090 & 0.0014 & -0.0003 & -0.0012 & -0.0010 & -0.0013 & 0.0007 & -0.0002 & -0.0002 & -0.0024 & -0.0003 \\
-0.0001 & -0.0002 & -0.0009 & -0.0004 & 0.0014 & 0.0016 & -0.0000 & -0.0006 & -0.0005 & -0.0008 & 0.0011 & -0.0001 & -0.0002 & -0.0017 & 0.0003 \\
-0.0001 & 0.0000 & -0.0001 & 0.0011 & -0.0003 & -0.0000 & 0.0015 & 0.0017 & 0.0015 & 0.0009 & -0.0005 & 0.0007 & -0.0000 & -0.0002 & -0.0008 \\
-0.0001 & 0.0001 & 0.0000 & 0.0050 & -0.0012 & -0.0006 & 0.0017 & 0.0168 & 0.0082 & 0.0079 & -0.0023 & -0.0036 & 0.0001 & 0.0005 & -0.0078 \\
-0.0003 & -0.0002 & -0.0008 & 0.0029 & -0.0010 & -0.0005 & 0.0015 & 0.0082 & 0.0129 & 0.0116 & -0.0021 & -0.0025 & 0.0007 & -0.0012 & -0.0072 \\
-0.0005 & -0.0004 & -0.0012 & 0.0032 & -0.0013 & -0.0008 & 0.0009 & 0.0079 & 0.0116 & 0.0200 & -0.0027 & -0.0034 & 0.0009 & -0.0011 & -0.0088 \\
-0.0003 & -0.0004 & -0.0017 & -0.0011 & 0.0007 & 0.0011 & -0.0005 & -0.0023 & -0.0021 & -0.0027 & 0.0140 & 0.0001 & 0.0004 & -0.0038 & 0.0168 \\
-0.0002 & -0.0000 & -0.0000 & -0.0000 & -0.0002 & -0.0001 & 0.0007 & -0.0036 & -0.0025 & -0.0034 & 0.0001 & 0.0054 & -0.0001 & -0.0004 & 0.0028 \\
0.0004 & 0.0004 & -0.0026 & 0.0001 & -0.0002 & -0.0002 & -0.0000 & 0.0001 & 0.0007 & 0.0009 & 0.0004 & -0.0001 & 0.0103 & -0.0101 & 0.0028 \\
0.0003 & 0.0010 & 0.0111 & 0.0001 & -0.0024 & -0.0017 & -0.0002 & 0.0005 & -0.0012 & -0.0011 & -0.0038 & -0.0004 & -0.0101 & 0.0578 & -0.0094 \\
-0.0001 & -0.0002 & -0.0028 & -0.0026 & -0.0003 & 0.0003 & -0.0008 & -0.0078 & -0.0072 & -0.0088 & 0.0168 & 0.0028 & 0.0028 & -0.0094 & 0.1283
\end{bmatrix}$$

$$B_{0i} = [-0.0001, -0.0002, 0.0028, -0.0001, 0.0001, -0.0003, -0.0002, -0.0002, 0.0006, 0.0039, -0.0017, -0.0000, -0.0032, 0.0067, -0.0064]$$

$$B_{00} = 0.0055$$

Figure A.1. B-Matrix loss formula coefficients of 15 unit test system.