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Department of Electrical-Electronic Engineering

**DESIGN AND IMPLEMENTATION OF A HYBRID  
ALGORITHM FOR COMMUNICATION NETWORK  
RELIABILITY CALCULATION**



Master Thesis

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Supervisor

Assist. Prof. Dr. Musaria Karim MAHMOOD

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### Turkish Abstract

İletişim ağının güvenilirliği, işletim ağlarının kullanılabilirliğini ve dayanıklılığını tanımlayan hizmet faktörlerinin kalitesi faktörlerinden biri olarak kullanılır. Pek çok klasik algoritma mevcuttur, ancak bunların çoğu, hesaplama yükünün karmaşıklığının bir işlevi olarak arttığı karmaşık ağlarda gerçek zamanlı güvenilirlik değerlendirmesi için uygulanamaz. Bu çalışmada, her tür ağ için (basit ve karmaşık) güvenilirlik değerlendirmesi sorununu çözmek için yeni bir hibrit algoritma önerilmiş ve kesin bir çözüm sunulmuştur. Algoritma, verimli bir hibrit algoritma sağlayan iki klasik güvenilirlik yönteminin kombinasyonuna dayanmaktadır. Grafik dönüştürme yöntemi ve bağ setleri yöntemleri, çok aşamalı algoritma ile birleştirilmiştir. Ağ, karmaşık bir ağı daha basit bir ağa dönüştürmek için seri, paralel, kenar faktörleme ve delta-yıldız grafik dönüşümlerine dayalı birçok basitleştirme katmanından geçer. Basitleştirilmiş topoloji daha sonra klasik bir algoritma için girdi olarak kullanılır; burada iki uçlu güvenilirlik değerlendirmesi için bağlantı setleri. Algoritmayı doğrulamak ve hesaplama süresinde yapılan

iyileştirme hakkında net bir vizyon vermek için birçok vaka çalışmasının simülasyonu gerçekleştirilir. 6 düğümlü 9 bağlantılı, 10 düğümlü 15 bağlantılı ve 30 düğümlü 41 bağlantılı ağlar değerlendirme aşamasındadır. Bu ağlar, küçük, basitten büyük karmaşık topoloji ağlarına kadar çeşitlilik göstermektedir. Önerilen algoritmanın sonuçları, hesaplama süresi açısından bağ kümeleri algoritmasının (standart klasik algoritma) kullanımından kaynaklananlarla karşılaştırılmıştır. Algoritmaların MATLAB tarafından uygulanması, güvenilirlik hesaplama süresinde gözle görülür bir gelişme olduğunu göstermektedir. İyileştirme, kaynak-hedef çifti olarak seçilen ürüne, ağ topolojisi karmaşıklığına ve düğüm ve bağlantıların sayısına bağlıdır. Örneğin, simülasyon altındaki 6 düğümlü ağ, düğümler (2-4) için klasik bağ seti algoritmasına kıyasla zaman hesaplamasında% 379'luk bir gelişme gösterirken, 10 düğümlü ağ için gelişme çok büyüktür. 30 düğümlü ağ durumunda olduğu gibi karmaşık büyük ağlar için, klasik algoritmalar çözümü bulmada başarısız olurken, önerilen algoritma küçük basit ağlar için olanla karşılaştırılabilir hesaplama süresi ile güvenilirliği bulmayı başarır. Ayrıca, ağın boyutu ve karmaşıklığı konusunda herhangi bir sınırlama yoktur.

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## **DECLARATION**

I hereby declare that in the preparation of this thesis, scientific ethical rules have been followed, the works of other persons have been referenced in accordance with the scientific norms if used, there is no falsification in the used data, any part of the thesis has not been submitted to this university or any other university as another thesis.

Nazik Kadhim Obaid AL-JEBUR

.../.../2021



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**THE DIRECTORATE OF SOCIAL SCIENCES INSTITUTE**

The thesis study of Nazik Kadhim Obaid AL-JEBUR titled as DESIGN AND IMPLEMENTATION OF A HYBRID ALGORITHM FOR COMMUNICATION NETWORK RELIABILITY CALCULATION has been accepted as MASTER THESIS in the department of ELECTRICAL-ELECTRONIC ENGINEERING by out jury

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**APPROVAL**

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Director of the Institute

## SUMMARY

The reliability of the communication network is used as one of the quality of service factors defining the availability and resilience of operating networks. Many classical algorithms exist but most of them fail to be applied for real-time reliability assessment in complex networks where the calculation load is increased in the function of the complexity. In this work, a new hybrid algorithm is proposed to resolve the problem of the two-terminal reliability evaluation for all kinds of networks (simple and complex), giving an exact solution. The algorithm is based on the combination of two classical reliability methods giving an efficient hybrid algorithm. Graph transformation method and tie sets methods are fused into multi stages algorithm. The network passes through many simplification layers based on series, parallel, edge factoring, and delta-star graph transformations to transform a complex network into a simpler network. The simplified topology is then used as input for a classical algorithm; here the tie sets for the two-terminal reliability evaluation. The simulation of many case studies are performed to validate the algorithm and to give a clear vision about the improvement made in computing time. A 6-node 9-link, 10-node 15-link, and 30-node 41-link networks are under evaluation. The results of the proposed algorithm are compared to those resulting from the use of tie sets algorithm (standard classical algorithm) in terms of computing time. The implementation of the algorithms by MATLAB shows a noticeable improvement in the time for reliability calculation. The improvement depends on the selected commodity as source-destination pair, the network topology complexity, and the number of nodes and links. For example, the 6-node network shows an improvement in time computing compared to classical tie-set algorithm of 379% for the commodity (2-4) while for 10-node network the improvement is high. In the case of 30-node network, the classical algorithms fail to find the solution, while the proposed algorithm succeeds to find the reliability with computing time comparable to that for small simple networks.

**Key Words:** Network Reliability, Graph Reduction, Tie-Set

## ÖZET

İletişim ağının güvenilirliği, işletim ağlarının kullanılabilirliğini ve esnekliğini tanımlayan hizmet faktörlerinin kalitesi faktörlerinden biri olarak kullanılır. Birçok klasik algoritma mevcuttur, ancak bunların çoğu, hesaplama yükünün karmaşıklığın bir işlevi olarak arttığı karmaşık ağlarda gerçek zamanlı güvenilirlik değerlendirmesi için uygulanamaz. Mevcut çalışmada, her tür ağ için (basit ve karmaşık) iki uçlu güvenilirlik değerlendirmesi sorununu çözmek için yeni bir hibrit algoritma önerilmiş ve tam bir çözüm sağlanmıştır. Algoritma, verimli bir hibrit algoritma sağlayan iki klasik güvenilirlik yönteminin kombinasyonuna dayanmaktadır. Grafik dönüştürme yöntemi ve bağ setleri yöntemleri, çok aşamalı algoritma ile birleştirilmiştir. Ağ, karmaşık bir ağı daha basit bir ağa dönüştürmek için seri, paralel, kenar faktörleme ve delta-yıldız grafik dönüşümlerine dayalı birçok basitleştirme katmanından geçer. Basitleştirilmiş topoloji daha sonra klasik bir algoritma için girdi olarak kullanılır; burada iki uçlu güvenilirlik değerlendirmesi için bağlantı setleri. Algoritmayı doğrulamak ve hesaplama süresinde yapılan iyileştirme hakkında net bir vizyon vermek için birçok vaka çalışmasının simülasyonu gerçekleştirilir. 6 düğümlü 9 bağlantılı, 10 düğümlü 15 bağlantılı ve 30 düğümlü 41 bağlantılı ağlar değerlendirme aşamasındadır. Önerilen algoritmanın sonuçları, hesaplama süresi açısından bağ setleri algoritmasının (standart klasik algoritma) kullanımından kaynaklananlarla karşılaştırılmıştır. Algoritmaların MATLAB tarafından uygulanması, güvenilirlik hesaplama süresinde gözle görülür bir gelişme olduğunu göstermektedir. İyileştirme, kaynak-hedef çifti olarak seçilen ürüne, ağ topolojisi karmaşıklığına ve düğüm ve bağlantıların sayısına bağlıdır. Örneğin, 6 düğümlü ağ, emtia (2-4) için klasik bağ set algoritmasına kıyasla zaman hesaplamasında% 379'luk bir gelişme gösterirken, 10 düğümlü ağ için gelişme çok büyük. 30 düğümlü ağ durumunda, klasik algoritmalar çözümü bulmada başarısız olurken, önerilen algoritma, küçük basit ağlar için karşılaştırılabilir hesaplama süresi ile güvenilirliği bulmayı başarır.

**Anahtar kelimeler:** Ağ Güvenilirliği, Grafik Azaltma, Tie-Set

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## ABBREVIATION

<b>ANNs</b>	:	artificial neural networks
<b>ATR</b>	:	All-Terminal Reliability
<b>CNN</b>	:	Convolutional Neural Networks
<b>CS</b>	:	Cut-Set
<b>GRM</b>	:	Graph Reduction Method
<b>INCN</b>	:	Iraqi National communication network
<b>MTSA</b>	:	The proposed multi-stage algorithm
<b>MTTF</b>	:	The mean Time to Fail
<b>QoS</b>	:	Quality of Service
<b>RTA</b>	:	A Recursive Truncation Algorithm
<b>SA</b>	:	Subset Approximations
<b>SCS</b>	:	Sensor-Cloud System
<b>SGCS</b>	:	Smart Grid Communication System
<b>SSD</b>	:	State Space Decomposition
<b>SSEM</b>	:	State Space Enumeration Method
<b>TA</b>	:	The Truncation Approximations
<b>TS</b>	:	Tie-Set

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## **PREFACE**

First, I want to thank my supervisor Dr. Musaria K. MAHMOOD for his help and support throughout this period until I reach this stage, also I thank Assoc. Prof. Dr. Indrit MYDERRIZI for his critical feedback. Finally, I thank my husband and my family for their support, and encouragement.



## INTRODUCTION

The reliability is the probability that a system will function properly within a specified period under many constraints like failure probability, system complexity, and operating conditions (Anumaka & Chukwukadibia ,2011). This means reliability is a quality measurement over time, affected by the system performances and its environment. It describes the system capability of being able to perform the intended function under specified conditions for a specified interval.

Recently, computer communication network has experienced a huge increase in its widespread dominance, and high complexity. Thus high reliability has become a need and requirement for various applications controlled by a computer network such as military, aircraft control systems, banking systems, control of nuclear reactors and distributed systems in general (Guimaraes, A. P. et al., 2011). Network reliability evaluation plays a major role in the functionality assessment of the communication network. For example, in natural disasters situation such as earthquakes and hurricanes, the physical failures, software-hardware unavailability, as well as inability of the network to survive due to electronic devices failure can damaged lives (Hui, K. P. et al., 2005). Therefore, the first stage of the structure design of a network; the network topological connectivity problem should be carefully analyzed to build a survivable, robust, and functional network. This stage is an initial factor in determining network reliability; the probability of failure of network components often directly affects the reliability of the network. The communication networks reliability is an important Quality of Service (QoS) factor in the study of the system performances especially those based on heterogeneous networks. It checks how long the infrastructure of a network is operational without interruption providing a functionality capability indication. Many technologies rely on reliability calculations in communication networks to ensure that information is sent and received with good reliability. A reliable network contains redundant path between the source node and destination node, which means that in the case of any link failure, it can be replaced by a backup link giving an alternate source-destination path.

Considering the specific graph elements (nodes and edges), two assumptions should be considered. First, nodes may be considered either perfect, which is entirely accurate with a probability of failure equal to 0 helping the minimization of calculation complexity, or imperfect with a probability of failure value greater than 0. Secondly, the independence of edge failure meaning that the success of its operation and failure may not affect the operation of the remaining edges in a network. However, they are simply tractable conditions, as the edges can be subjected to the same event that affects all of them in the same position. Due to the difficulty and complexity of the task, edge failure is often treated as an independent occurrence, even though it is really dependent on the task (Brecht, T. B., & Colbourn, C. J., 1988).

Network topological presentation using graph theory is the main factor in the network reliability studies. Communications networks are becoming more and more complex because of the development of modern technologies, applications, and services. The reliability assessment for complex networks requires a complicated computing operation due to the complexity and availability of many possible paths between source-destination. Therefore, original computational methods should evaluate the complex network reliability under multiple operational conditions. Various classical probabilistic methods are used to estimate network reliability based on different techniques such as Graph Reduction Method (GRM) (Konak, A., 2007), State Space Enumeration Method (SSEM) (Gaun, A et al., 2010) , Tie-set (Mahmood, M. K et al., 2014) , Cut-set (Al-Muhaini, M., & Heydt, G. T. 2012), and approximation methods (Hayashi, M. 2012),( Cancela, H., et al., 2013). The reliability assessment for complex networks requires a complicated computing operation due to the complexity and availability of many possible paths between various nodes. Therefore, the classical computational methods fail to evaluate the reliability under multiple operational conditions. Several hybrid algorithms can be used to find network reliability based on one or more of the previous methods to be applied for complex communication reliability assessment (Zhu, P. et al., 2016). New studies based on the use of Artificial Neural Network (ANN) (Puri, M. 2015), clustering technique (Saxena, A. 2017) and (Mahmood, M. K., & Myderrizi, I. 2020). The network reliability can be computed for a pair of nodes (source-destination) defining the notion of two-terminal reliability. It checks the probability of having

a good path for data transmission between the source node and the destination node (Jin, W., Yu, P., 2018). The all-terminal reliability is the probability that every node in the network stays linked to all others. Two-terminal algorithm is modified to consider all terminal pairs such that a set of operational edges provides communication paths between every pair of nodes (Brecht, T. B., & Colbourn, 1988). The problem of k-terminal reliability, where only part of the network is considered, is presented in (Shooman, A. M., & Kershenbaum, A. 1991). The k-terminal reliability is the general method varying between two-terminal to all-terminal methods.



# CHAPTER ONE

## STATE OF THE ART

### 1.1. Literature Review

There is an important research literature body published about system reliability and especially communication network reliability. Interest in system reliability began in the 1970s and saw a marked increase with the development of networks and communications in general.

(Gupta, H., & Sharma, J. 1978) have introduced to simplify complex reliability block diagrams consisting of 2-case or 3-case devices, by using new detailed expressions for delta-star transformation. The conditions are given under which the transformation applies. To find an equivalent star configuration, should be the expressions are interrelated and require less computation time. It is also possible to derive expressions for star-delta transformation in this method.

(Shooman, A. M., & Kershenbaum, A. 1991) have presented an exact algorithm for graph-reduction to solve the problem of k-terminal reliability with node failures on an arbitrary network. K-terminal reliability means that a particular group of k terminal nodes must be able to interact with each other by modeling an undirected probabilistic graph of the network whose vertices represent the nodes and whose edges represent the connections. Two contributions are a version of the delta-y transformation for k-terminal reliability and an extension of Satyanarayana and Wood's polygon to chain transformations to handle graphs with imperfect, vertices. This algorithm is faster than or equal to the Satyanarayana and Wood, and, the algorithm without delta-y and polygon to chain transformations to solve every problem.

An improvement of the previous work has been published by (Shooman, A. M., & A. Kershenbaum 1992). The work presents a graph-reduction algorithm for computing a random network of probably unstable nodes with k-terminal reliability. The proposed algorithm is faster than or equal to the Satya Narayana and Wood easy algorithm without delta-y and polygon to chain transformations and runs on series-parallel graphs in linear time and approximate algorithms faster than other algorithms reduce the computation time for the problem of network

reliability by 2 to 3 orders of magnitude for large problems, thus providing relatively accurate answers (less than in most cases relative error).

(Murray, K et al 1993) have been discussing the realistic network design, and the different estimated techniques whose complexity results in uptime short enough. Three possible approximation methods are discussed in theory in this work. The first strategy uses all network segments but cuts off the extension of reliability that leads to the upper and lower limits. The second method subset of the tie sets and a subset of the cut sets like the linking sets and shorter cutting sets (fewer components) are then chosen by another process and the expansion is fully performed. Finally, the third approximation method includes the characteristics of both the first two methods. It has been shown that the combined method is polynomial it is faster than the reduced tie and cut-set methods.

(Li, H., & Zhao, Q., 2005) have put under focus the problem of reliability evaluation for control systems where an approach is introduced to evaluate the reliability by searching for the equivalent tie or cut sets based on the control system performance. The dynamical and feedback relations among the elements in control systems have been considered in this research. Because it is hard to obtain a valid reliability assessment tool by traditional reliability engineering approaches, the reliability is reevaluated by updating the cut or tie sets where the faults are detected and reported online and/or the control objective is modified.

(Gebre, B. A., & Ramirez-Marquez, J. E, 2007) have presented a comprehensive algorithm for analyzing networks that follow two-tailed logic. The general algorithm was developed as a comprehensive technology that relies on two sub-algorithms that when combined provide a fast computation of the minimum clusters of complex networks. The algorithm is based on a pre-matrix and element substitution technology that allows computing the sets of minimum pieces and the immediate inclusion of a node failure without any changes. Thus, reducing the computational time, it takes to filter out unlimited cuts.

The estimate of the all-terminal reliability of a given random network with a pre-specified precision has been studied in (Sharafat, A. R., & Ma'rouzi, O. R., 2009). A Recursive Truncation Algorithm (RTA) is proposed for bounding an approximation algorithm for reliability evaluation. RTA searches all the minimum cut sets of the network representing the graph, and identifies the poor cut sets of the graph by comparing cut set failure probabilities to an adaptive threshold that

depends on the accuracy of the approximation. Calculation of the network unreliability versus the probability of failure occurring in the weak cut sets, in addition to the all-terminal reliability by estimating the probabilities of the union of intersection events.

(Rebaiaia, M. L., et al., 2009) have presented an algorithm to reduce the number of shorthand and by decomposition rules that expected to reduce the execution time to assess network efficiency. The algorithm operates according to the accuracy of the complex distributed systems regardless of the complexity of the corresponding graph and distinguishes between ideal and incomplete nodes.

(Won, J. M., & Karray, F., 2011) have introduced a new approach based on greedy network factoring that produces the sequential lower and upper limits of the All-Terminal Reliability (ATR) by finding the most reliable spanning tree in the given network and the most unreliable cut collection. To change the lower and upper boundaries of the ATR, their operative and failing probabilities are used. Sub-networks are then generated and this procedure is applied recursive, until either the lower or upper bound exceeds the preset ATR requirement. Because of the rapid convergence of the ATR boundaries, it indicates a relationship between performance of each implementation and the network's characteristics, such as layout and edge operating probabilities.

(Mahmood, M. K., et al., 2014) have presented A new algorithm for reliability calculation based on a two methods by combining the Graph Reduction (GR) and Tie-Set (TS) algorithms in a hybrid one. The algorithm is examined for all possible scenarios, such as homogenous, heterogeneous networks, unidirectional links, and bidirectional links, and applied to a random complex topology. Compared to the classical algorithms, the simulation results indicate the efficiency of the proposed algorithm in terms of computing time.

Another similar improved version of this algorithm has been published by the same other in (Mahmood, M. K., et al., 2015). The algorithm is a generalization of the previous taking in account the imperfection nodes in a network with a probability strictly less than 1. The reduction method for performance-reliability-preserving centered on methods of network graph transformation, which can decrease the complexity of performance reliability assessment by scaling down networks have been proposed. The algorithm is mapped into many layers (stages) of simplification where the first stage simplifies

the topologies by eliminating all series-parallel links and solving the problem of node imperfection, which reduces the number of tie-set and the time needed to measure reliability.

A method of network performance-reliability-preserving reduction has been introduced by (Zhang, H., Huang, N., & Liu, H., 2014). The propose method is used for evaluating network performance reliability that combines the simulation of Monte Carlo reliability with a performance model of the network. The performance indicators are likely to retain their values within expected ranges under a certain traffic flow.

Park, J. H. (2015) has introduced a new algorithm with two terminals modified to account for all pairs of terminals so that a set of operational edges provides communication paths between each pair of nodes ( $n_i, n_j$  with  $\neq$ ). Any two nodes are connected by a wireless link consisting of a pair of radio modules as well as analyzing the reliability of all ends of the wireless network and the random network assuming that the module fails all the cut-off radio links. To improve reliability, fault tolerance can give redundant radio modules at each node. The mean Time to Fail (MTTF) is calculated for the wireless networks, which illustrates the random mesh reliable network more than any other network.

(Jeyaraj, J. P., & Haenggi, M., 2017) have presented an orthogonal street network with transmission and reception of vehicles on each street forming a Poisson bipolar network. It has been studied using instruments from stochastic geometry to analyze vehicle-to-vehicle communications modeled using 1-dimension point processes. Derive analytical expressions for the success probabilities of two types of users—the typical general user and the typical intersection user. It demonstrates that certain properties of both 1-dimensional and 2-dimensional Poisson networks are shared by the orthogonal street structure. The orthogonal street system works like a 2-dimensional Poisson bipolar network in the low-reliability method. The deduction that the success probability general/intersection user is upper bounded by the minimum of the success probabilities of the 1-dimension and 2-dimension Poisson networks.

(Daibo, M. 2017) has been presenting the reliability evaluation of the n-tuple bridge by repeatedly transforming the delta star. It converts the complex structure of the bridge into a simple one whose reliability can be easily determined by the reliable or inaccurate rules of the product. By comparing it with

the exponential scale in three other approaches, the scaling power of the modeling is calculated and the numerical assessment of system reliability for the transformation and linear measurement.

An algorithm of large-scale segmentation of space for Smart Grid Communication System (SGCS) and segmentation of the entire network into communal groups is introduced by (W. Jin et al., 2018). Assuming that each community's all-terminal reliability is independent, and calculated in parallel, then, the effect of all-terminal reliability is roughly equal to the total large-scale all-terminal reliability SGCS. This yields to reduce algorithm complexity with a final result close to the exact value.

(Mahmood, M. K., et al., 2018) have studied the design of a reliable Communication Network by taking the Iraqi National communication network (INCEN) as a case study. The INCEN links Baghdad with the major cities in Iraq as the central node, by simplifying the original network by transforming the graph before applying a second classic algorithm to the simplified analogous network. The backbone was designed by Prim's algorithm that has spaces between cities as input data. INCEN is subject to Improve reliability by adding links to the primary backbone. An algorithm was developed based on the link cluster method to calculate network reliability, represented by three specific networks (Net1, Net2 and Net3) for INCEN. One of the proposed networks is selected as the best network for the INCEN case study. The proposed multi-stage algorithm (MTSA) is based on the classical tie-set algorithm, with a new proposed method. Combinations will simplify the reliability assessment by reducing the number tie groups, and thus complicate the calculation.

(Wang, X. D. et al., 2019) have proposed a new concept of operational reliability for production systems based on system-engineering theory and multistate system theory. The multi-state production network is used to simplify the production system's production process, taking consumer demand, machine efficiency and product quality into account. The operational reliability assessment approach for multi-state production system is given. The proposed reliability assessment method will reliably diagnose the state of operation of the production system, provide a scientific basis for decisions on production scheduling and maintenance, and further ensure the reliable functioning of the production system.

(Bai, G., Liu, T., et al., 2020) have presented the improvement of the existing State Space Decomposition (SSD) algorithm, so that the decomposition process can work in parallel. With select a proper minimal path vector (d-MP), this is used to decompose the sets of unspecified cases. The results showed that this method outperforms other algorithms, as well as it can improve the evaluation efficiency significantly.

(A. D. Frias, & O. P. Yadav, 2020) have proposed an efficient algorithm for estimating all terminal network reliability using Artificial Neural Networks (ANNs). Addressing the problem of estimating the total network reliability by Convolutional Neural Networks (CNN) without an upper limit of reliability as an input. By proposing a resultant regression layer preceded by a x-layer to significantly reduce the computational time and to achieve predictions within the range of reliability characteristics. An average 1.18 msec/net has been achieved by CNN for backtracking exact algorithm took around 500 sec/net previous algorithms.

A new algorithm for reliability evaluation is presented by (Musaria K. Mahmood, & Myderrizi, I. 2020). It is based on the segmentation of complex networks into smaller partitions where any classical reliability algorithm can be applied within the formed cluster-partition. The method can be used efficiently for two-terminal reliability evaluation by simplifying network complexity to many smaller networks with limited links and nodes. Partitions are classified into three types. The first is a cluster that not contain any source-destination pair node. The second type includes the source or destination, while both are included in the third type. The algorithm is validated by a random network where the calculated two-terminal reliability manifests its exactness compared to results collected from classical algorithm.

(Mo, Y., et al., 2020) have presented a research focus on the K-terminal, which is interested with efficient communication between all pairs of network nodes that belong to a subset pre-specified. To determine their K-terminal reliability, the increased complexity and size of real-life Sensor-Cloud System (SCS) require new efficient techniques. By proposes a method of network simplification that can eliminate all redundant network edges and vertices efficiently, resulting in a substantially reduced network model for accurate and efficient study of K-terminal reliability. The technique is based on the

decomposition of graphs and reconstruction by articulation vertices. Empirical research shows that the proposed simplification approach implemented with the evaluation algorithm based on binary-decision-diagrams will greatly accelerate the K-terminal reliability analysis of large real-life SCSs.

The development of an algorithm based on the search for all minimal paths in a graph has introduced by (Lamalem, Y., & Housni, K. 2020). By using a new approach to enumerate all minimal network paths (oriented or not, has a loop or parallel links). This new technique is based on exploring the networks from two nodes, one from the source and another from the target. Starting from the two terminal nodes, at each iteration, check if there is a connection between the last traverse nodes from the source side and from the target side. This algorithm reduces the execution time for finding all the minimum paths, and allowed the number of tests needed to find a given path to be reduced by half compared to the latest and fastest algorithms in the literature.

## **1.2. Research Motivation**

Network reliability is a crucial issue for network and electronic system designers because a good reliability study will lead to:

- (a) Increase the independence of the system on network failures.
- (b) Good design and analysis of networks led to increasing network vulnerability due to component failures.
- (c) The study of reliability will affect the choice of communication network protocols, and topology.
- (d) Choice of an appropriate network model is highly application dependent.

The current work designs a new algorithm to calculate the exact reliability with a faster time for complex communication networks. This algorithm can be used to find the reliability of all systems that can be modeled in graph theory such as wired communication networks, electronic systems, computer networks, and mobile communication systems. The new algorithm is based on two stages of simplification based on two classical methods that are; tie-set and GRT. The proposed algorithm can be viewed as a multi-stage algorithm by a first

initialization stage followed by the graph form recognition procedures such as parallel, series reduction, edge-factoring and delta star. The algorithm continues by applying the tie-set method to the simplified ‘but analogous’ version of the network to find the exact reliability value.

### **1.3. Objectives**

The proposed algorithm is implemented using various simplification stages based on the graph reduction technique followed by the application of tie set algorithm. Four variations of GRT are used; series, parallel, edge, and delta-star transformations. The implementation of GRT sub algorithms contribute to the simplification of any complex network to another simpler with a restricted number of nodes and links. The original and simplified networks are analogous in terms of reliability. The simplified version of the network is then used for reliability evaluation by tie-set algorithm.

### **1.4. Thesis Organization**

This thesis contains five chapters and has been organized as follows: Chapter one introduces the Literature Review, Research Motivation, Objectives and Thesis organization.

Chapter two; the main publications in the field are presented, analyzed, and evaluated the theoretical background of communication network reliability methods is given. GRT and tie-set methods are presented with the communication network modeling in graph theory.

Chapter three consists of the development of the proposed hybrid algorithm. Two random communication networks of increased complexity are implemented using the hybrid algorithm.

Chapter four: Simulation results are presented, analyzed, and the performance is compared to the existing classical algorithm; the tie-set.

Chapter five: concludes the work by giving main results and improvements. Proposed future development ideas are given as future research.

## CHAPTER TWO

### RELIABILITY THEORETICAL BACKGROUND

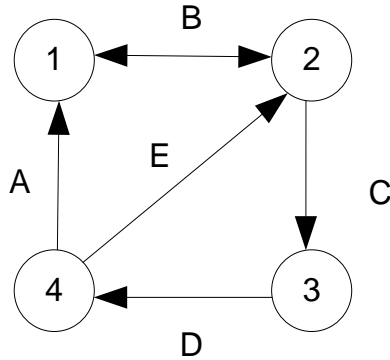
#### 2.1. Network Modeling

##### 2.1.1. General Network Model

Communication network problems can be graphically represented during the design and optimization phases (Shooman, M. L., 2002). It is modeled by a graph  $G = (N, L)$ , where  $N$  is the set of communication nodes, while  $L$  is the set of undirected/ bi-directed links connecting the network (Lee, H., & Kim, J., 2004). Node and link individual reliability obeys to the binomial distribution law where each element has two possible states, either success (working) with probability  $p$  or failure with the complement probability  $q = 1 - p$ . The event-state of a network component (working or failed) is considered independent from the other components event-state. A 3-dimension matrix  $M [N, N, K]$  describing the network topology where  $K$  is the maximum number of parallel links between two nodes in the network. The matrix element  $M [i, j, x]$  represents the probability of the  $x^{ieme}$  unidirectional parallel link connecting node-pair  $(ni, nj)$ .  $M$  is a multilayer matrix where the first layer shows the basic network connectivity and the  $x^{ieme}$  layer describes the network topology connectivity between node-pair having  $x$  links parallel between them with  $1 \leq x \leq K$ . If no link between two nodes, the corresponding element in  $M$  is set to zero.

Consider  $P_n$  and  $P_e$  are the probability of working correctly for nodes and links. When two nodes can communicate via the same link in two ways, this link is to be bi-directed link, while only one node can receipt data from the other sender and vice versa in the case of unidirectional link. If we consider a small network contains four nodes connected separately with each other by five unidirectional and bi-directional links, as shown in Figure 1. explains the links (1) and (2) are connected by the bidirectional link (B), in two directions, whereas (C) is an undirected link connected nodes (2) and (3) in one direction. The path from a source node  $n_s$  to a destination node  $n_d$  is composed by many links that trace a continuous path from the two nodes. A loop is a path that starts and ends at the same node making a closed form while link-disjoint paths are paths with perfectly different links. Two nodes connected by two links at the same time form a

parallel-link form. A simple graph is a graph without any loop or any parallel connection. Commonly large networks are much more connected so as these networks are more complicated.



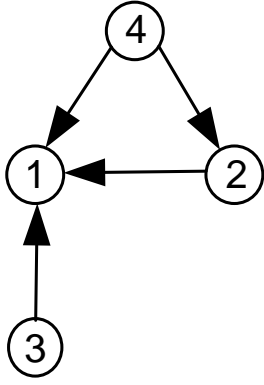
**Figure 1.** Directed & undirected links network.

### 2.1.2. Matrix Representation of Graphs

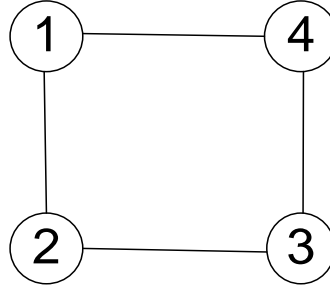
A matrix is used to represent mathematically a connected graph, where rows and columns represent network nodes. This leads to a  $N \times N$  square matrix, where  $N$  is the number of nodes. An undirected graph describes a graph with all links as bi-directional with the same characteristics in both directions. It can be represented by a square symmetric matrix where a link  $(i, j)$  is exactly same than the link  $(j, i)$ . Examples of connectivity as undirected and directed graph topology are given in Figure 2.a, and 2.b respectively. The matrices following describing the general forms to represent graphs by mathematical equations. Matrix A describes the undirected graph while B matrix is presenting the directed graph. These square matrices are called connectivity matrices, with element having two possible values; (1) if a connection between the two nodes in the row and column otherwise (0) is assigned for no connection. The diagonal elements are set to be (1) and represent node probabilities, assumed to be connected to itself.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad (1)$$

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (2)$$



A. directed graph.



B. Undirected graph.

**Figure 2.** Graph representation.

Another important matrix is the probability matrix  $P$ , given in equation (3), presenting the connectivity of the graph but instead of using (1) for connected link the probability is used.  $p_{ij}$ , is the probability of the link between nodes (i) and (j).

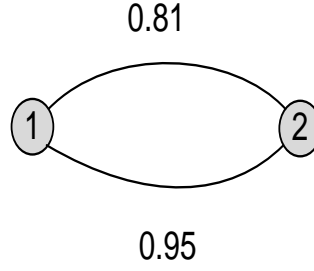
$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ p_{r1} & p_{r2} & p_{r3} & p_{rr} \end{bmatrix} \quad (3)$$

The probability is equal to:

$$p_{ij} = \begin{cases} p_{ij} & \text{if there is a link between } i \text{ and } j \\ 0 & \text{no link between } i \text{ and } j \end{cases} \quad (4)$$

The probability of a link is an operation characteristic provided by the manufacture company and depends also of the installation and operating environments. Diagonal are represented by the elements  $p_{kk}$ , which reflect the node probability, usually equal to 1 by using redundant materials in communication nodes.

Based on the maximum number of parallel links between two communicated nodes, a communication network can be represented graphically by a multilayer three-dimensional adjacency matrix,  $M(N \times N \times k)$ . For example, if maximum there are four parallel links, k is 4 and each matrix layer is representative of one parallel case. If the network has no parallel links, the elements of  $M_{N,N,1}$  represents the real links of the network, while all elements of the second one  $M_{N,N,2}$  are set to be zeros. For example, the two-node network shown in Figure 3. Is represented by a matrix  $M(2 \times 2 \times 2)$  where k=2 indicates



**Figure 3.** Parallel links

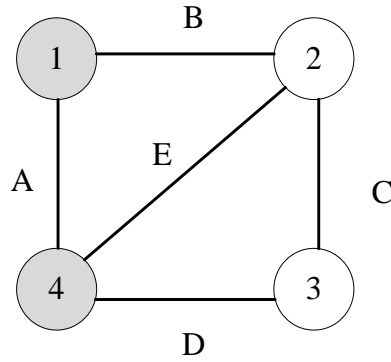
that there are two layers representing the parallel links as presented by the two matrices:

$$PM_1 = \begin{bmatrix} 1 & 0.81 \\ 0.81 & 1 \end{bmatrix} \quad (5)$$

$$PM_2 = \begin{bmatrix} 0 & 0.95 \\ 0.95 & 0 \end{bmatrix} \quad (6)$$

## 2.2. Network Reliability Evaluation Strategies

There are three reliability evaluation strategies, which are the two-terminal reliability, the all-terminal reliability, and the K-terminal reliability. The two-terminal reliability is the probability that there is at least one path connecting the source node ( $n_s$ ) to the destination node ( $n_d$ ). The two terminal reliability can also be defined as the probability of good and successful operation between a pair of nodes (Musaria K. Mahmood et al., 2021). As for other reliability methods, the increase in the number of nodes and links increases the complexity of the network and then the computing complexity. In the graph of the network in Figure 4. The two terminal reliability between the node (1) and (4), is the measure of the probability that a path exists between these two nodes. Three paths connecting the two nodes, which are {A}, or {B-C-D}, or {B-E}. The reliability evaluation using various methods will be based on the failure or success of the links composing these paths (Suri, P. K., & Bhushan, B., 2008).



**Figure 4.** Simple network

All-terminal reliability is the probability that every node in a network can communicate with other nodes. (Jin, W., Yu, P., 2018). Two ways for the all-terminal reliability calculation that the details will be given in next sections. For example, there are four possible pairs of nodes that should be able to communicate each with other for solving all terminal connection problems as the small network shown in Figure 4. Nodes (1) can communicate with (2), (3) and (4). All terminal reliability connects each node with other nodes, that's mean each node in a network should be able to send-receive data to other nodes (Karger, D. R., & Tai, R. P. 1997).

K-terminal reliability is defined as the probability that K nodes are connected by a path. It is considered as the general case where the two-terminal reliability is found for  $K=2$ , while the all-terminal reliability is found for  $K=N$  (Yeh, F. M., Lu, S. K., & Kuo, S. Y. 2002), (Hui, K. P., 2005).

### 2.3. Reliability Evaluation Methods

The reliability calculation is based on mathematical development of graph theory and probability theory. Various methods are used for reliability evaluation, including exact and approximate methods. The most important methods are using probabilistic calculation is so called classical methods.

#### 2.3.1 State Space Enumeration Method

This method is used to evaluate the two-terminal reliability and can be extended for all-terminal reliability for simple networks that are composed of less

than 15 nodes (Musaria K. Mahmood et al., 2021). The State-Space Enumeration Method (SSEM) is the simplest method compared to other classical methods because it is based on enumerating all possible combinations of possible paths. The state of links (good or bad) is enumerated yielding a complexity of  $(2^e)$  where  $(e)$  is the number of the total links in the network. The computation of the reliability is based on good Events that result path between the source node and the destination node. These events are disjoint events (Konak, A., 2007), either good operational events, or bad events and the reliability of two-terminal nodes is essentially the union process of good events as in:

$$R_{sd} = P(E_1 + E_2 + \dots + E_N) \quad (7)$$

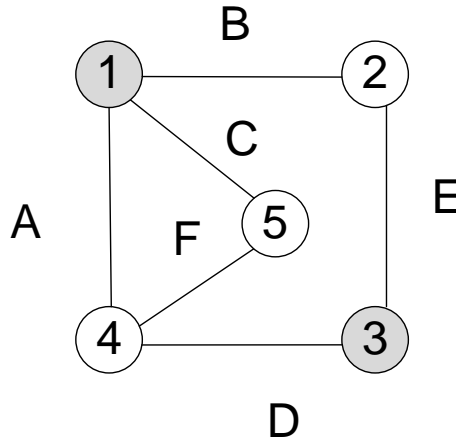
Each of these events is a mutually exclusive event based on a set of individual link-event as in:

$$R_{sd} = P(E_1) + P(E_2) + \dots + P(E_N) \quad (8)$$

Consider the network in Figure 5. As an example to evaluate the reliability of two terminals between nodes (1) as  $n_s$  and (3) as  $n_d$ . There are six links in this network, resulting  $2^e = 2^6 = 64$  (good and bad) mutually exclusive events. If the link probability is considered as equal to (0.9) for all links, then Table 1 gives an enumeration of all events.

The application of equation (8) on the good events where there is at least one path from  $n_s$  to  $n_d$  gives

$$R_{13} = 0.9^6 + 6 \times 0.9^5 \times 0.1 + 13 \times 0.9^4 \times 0.1^2 + 9 \times 0.9^3 \times 0.1^3 + 2 \times 0.9^2 \times 0.1^4 = R_{13} = 0.97776$$



**Figure 5.** State space enumerations.

**Table 1.** The event-space enumeration

NO.	EVENTS	STATE
No failure $\binom{6}{0} = 1$	$E_1 = ABCDEF$	Good
	$E_2 = \bar{A}BCDEF$	Good
One failure $\binom{6}{1} = 6$ events	$E_3 = A\bar{B}CDEF$	Good
	$E_4 = ABC\bar{D}EF$	Good
	$E_5 = ABCD\bar{E}F$	Good
	$E_6 = ABCDE\bar{F}$	Good
	$E_7 = ABCDEF\bar{F}$	Good
	$E_8 = \bar{A}\bar{B}CDEF$	Good
Two failures $\binom{6}{2} = 15$ events	$E_9 = \bar{A}B\bar{C}DEF$	Good
	$E_{10} = \bar{A}BC\bar{D}EF$	Good
	$E_{11} = \bar{A}BCDE\bar{F}$	Good
	$E_{12} = \bar{A}BCDEF\bar{F}$	Good
	$E_{13} = A\bar{B}\bar{C}DEF$	Good
	$E_{14} = A\bar{B}C\bar{D}EF$	Bad
	$E_{15} = A\bar{B}CDE\bar{F}$	Good
	$E_{16} = A\bar{B}CDEF\bar{F}$	Good
	$E_{17} = AB\bar{C}\bar{D}EF$	Good
	$E_{18} = AB\bar{C}DE\bar{F}$	Good
	$E_{19} = AB\bar{C}DEF\bar{F}$	Good
	$E_{20} = ABC\bar{D}\bar{E}F$	Bad
	$E_{21} = ABC\bar{D}E\bar{F}$	Good
	$E_{22} = ABCDE\bar{F}\bar{F}$	Good
	$E_{23} = \bar{A}\bar{B}\bar{C}DEF$	Bad

Three failures events	$\binom{6}{3} = 20$	$E_{24} = \bar{A} \bar{B} C \bar{D} E F$	Bad
		$E_{25} = \bar{A} C \bar{B} D \bar{E} F$	Good
		$E_{26} = \bar{A} \bar{B} C D E \bar{F}$	Bad
		$E_{27} = \bar{A} B C \bar{D} E F$	Good
		$E_{28} = \bar{A} B C \bar{D} \bar{E} F$	Bad
		$E_{29} = \bar{A} B C D E \bar{F}$	Good
		$E_{30} = \bar{A} B C \bar{D} \bar{E} F$	Bad
		$E_{31} = \bar{A} B C D E \bar{F}$	Good
		$E_{32} = \bar{A} B C D \bar{E} \bar{F}$	Bad
		$E_{33} = A \bar{B} \bar{C} \bar{D} E F$	Bad
		$E_{34} = A \bar{B} \bar{C} D \bar{E} F$	Good
		$E_{35} = A \bar{B} \bar{C} D E \bar{F}$	Good
		$E_{36} = A B \bar{C} \bar{D} \bar{E} F$	Bad
		$E_{37} = A B C \bar{D} E \bar{F}$	Good
		$E_{38} = A B C \bar{D} \bar{E} \bar{F}$	Bad
		$E_{39} = A B C D \bar{E} \bar{F}$	Good
		$E_{40} = A \bar{B} C \bar{D} E \bar{F}$	Bad
		$E_{41} = A \bar{B} C \bar{D} \bar{E} F$	Bad
		$E_{42} = A \bar{B} C D \bar{E} \bar{F}$	Good

---

Four failures events		$E_{43} = \bar{A} \bar{B} \bar{C} \bar{D} E F$	Bad
	$\binom{6}{4} = 15$	$E_{44} = \bar{A} \bar{B} \bar{C} D \bar{E} F$	Bad
		$E_{45} = \bar{A} \bar{B} \bar{C} D E \bar{F}$	Bad
		$E_{46} = \bar{A} \bar{B} C \bar{D} \bar{E} F$	Bad
		$E_{47} = \bar{A} \bar{B} C D E \bar{F}$	Bad
		$E_{48} = \bar{A} \bar{B} C D \bar{E} \bar{F}$	Bad
		$E_{49} = A \bar{B} \bar{C} \bar{D} \bar{E} F$	Bad

	$E_{50} = A\bar{B} \bar{C} \bar{D} E\bar{F}$	Bad
	$E_{51} = A\bar{B} C\bar{D} \bar{E} \bar{F}$	Bad
	$E_{52} = ABC \bar{D} \bar{E} \bar{F}$	Bad
	$E_{53} = \bar{A} B\bar{C} \bar{D} \bar{E} F$	Bad
	$E_{54} = \bar{A} B\bar{C} \bar{D} E \bar{F}$	Good
	$E_{55} = \bar{A} BC \bar{D} \bar{E} \bar{F}$	Bad
	$E_{56} = A\bar{B} \bar{C} D\bar{E} \bar{F}$	Good
	$E_{57} = \bar{A} B \bar{C} D \bar{E} \bar{F}$	Bad
	$E_{58} = \bar{A} \bar{B} \bar{C} \bar{D} \bar{E} F$	Bad
Five failures $\binom{6}{5} = 6$	$E_{59} = \bar{A} \bar{B} \bar{C} \bar{D} E\bar{F}$	Bad
	$E_{60} = \bar{A} B\bar{C} \bar{D} \bar{E} \bar{F}$	Bad
	$E_{61} = \bar{A} \bar{B} C \bar{D} \bar{E} \bar{F}$	Bad
	$E_{62} = \bar{A} \bar{B} \bar{C} D\bar{E} \bar{F}$	Bad
	$E_{63} = A\bar{B} \bar{C} \bar{D} \bar{E} \bar{F}$	Bad
Six failures events = $C_6^6 = 1$	$E_{64} = \bar{A} \bar{B} \bar{C} \bar{D} \bar{E} \bar{F}$	Bad

### 2.3.2. Cut-Set and Tie-Set Methods

The tie-set technique is based on enumerating all the links groups forming a loop-free path between source nodes  $n_s$  and the destination node  $n_d$  called tie sets (Li, H., & Zhao, Q., 2005). Therefore, this method is applied only to simple and medium networks because the enumeration process increases exponentially with the network complexity. Tie-set algorithm is an exact solution used to evaluate two-terminal, K-terminal, and all-terminal network reliability. It starts by finding all tie sets  $(T_1, T_2, \dots, T_i)$  between a source-destination pair and applying the expansion equation (Poincare):

$$R_{sd} = P(T_1 + T_2 + \dots + T_i) \quad (9)$$

Where  $R_{sd}$  is the reliability between the source node and the destination node.

Tie sets  $(T_1, T_2, \dots, T_i)$  are not disjoint link groups because they have common links. This requires the expansion of equation (9):

$$\begin{aligned}
R_{sd} = & P[(T_1) + P(T_2) + \dots P(T_i)] \\
& - [P(T_1 T_2) + P(T_1 T_3) + \dots + P(T_r T_k)_{r \neq k}] \\
& + [P(T_1 T_2 T_3) + P(T_1 T_2 T_4) + \dots + P(T_r T_k T_j)_{r \neq k \neq j}] \\
& + \dots + (-1)^{i-1} [P(T_1 T_2 T_3 \dots T_i)] \quad (10)
\end{aligned}$$

The cut-set is a set of links, those if removed or failed, the network will have failed to connect source-destination pair (Soh, S., & Rai, S., 2005). Cut-set and tie-set are two analogical methods that use the same procedures (Cancela, H., et al., 2013). To find the reliability expression between s and d using cut-set group, one should start by enumerating all cut-set  $[C_1, C_2, \dots, C_j]$  and write:

$$R_{sd} = 1 - P(C_1 + C_2 + \dots + C_j) \quad (11)$$

Consider the network in the previous example given in Figure 2, which consists of five nodes and six links. The reliability between nodes (1) and (3) is calculated after enumerating all minimal tie-set and cut-set, as shown in Table 2. Using tie-set  $(T_1, T_2, T_3)$ , the reliability by application of the equation (10) yields:

$$\begin{aligned}
R_{13} &= [P(T_1) + P(T_2) + P(T_3)] - [P(T_1 T_2) + P(T_1 T_3) + P(T_2 T_3)] + [P(T_1 T_2 T_3)] \\
R_{13} &= P(AD + BE + CFD) - [P(ADBE + ACFD + BECFD)] + [P(ABCDEF)] \\
R_{13} &= [2(0.9^2) + (0.9^3)] - [2(0.9^4) + (0.9^5)] + [(0.9^6)] = 2.349 - 1.90269 + 0.531441 \\
&= 0.97776
\end{aligned}$$

Which confirm the result found using the enumeration method.

**Table 2.** Minimal tie sets & cut set

Minimal Tie sets	Minimal cut sets
$T_1 = AD$	$C_1 = BD$
$T_2 = BE$	$C_2 = DE$
$T_3 = CFD$	$C_3 = BAF$
	$C_4 = BAC$
	$C_5 = EAC$
	$C_6 = EAF$

Using minimal cut-set method between  $n_s = 1$  and  $n_d = 3$  to evaluate the reliability for the same example requires more computation compared to the tie-set method because there are (6) cut sets, while only (3) tie set are present.

$$\begin{aligned}
R_{13} = 1 - & [P(C_1) + P(C_2) + P(C_3) + P(C_4) + P(C_5) + P(C_6)] \\
& - [P(C_1C_2) + P(C_1C_3) + P(C_1C_4) + P(C_1C_5) + P(C_1C_6) \\
& + P(C_2C_3) + P(C_2C_4) + P(C_2C_5) + P(C_2C_6) + P(C_3C_4) \\
& + P(C_3C_5) + P(C_3C_6) + P(C_4C_5) + P(C_4C_6) + P(C_5C_6)] \\
& + [P(C_1C_2C_3) + P(C_1C_2C_4) + P(C_1C_2C_5) + P(C_1C_2C_6) \\
& + P(C_1C_3C_4) + P(C_1C_3C_5) + P(C_1C_3C_6) + P(C_1C_4C_5) \\
& + P(C_1C_4C_6) + P(C_1C_5C_6) + P(C_2C_3C_4) + P(C_2C_3C_5) \\
& + P(C_2C_3C_6) + P(C_2C_4C_5) + P(C_2C_4C_6) + P(C_2C_5C_6) \\
& + P(C_3C_4C_5) + P(C_3C_4C_6) + P(C_3C_5C_6) + P(C_4C_5C_6)] \\
& - [P(C_1C_2C_3C_4) + P(C_1C_2C_4C_5) + P(C_1C_2C_5C_6) + P(C_1C_2C_3C_5) \\
& + P(C_1C_2C_3C_6) + P(C_1C_3C_4C_5) + P(C_1C_3C_4C_6) + P(C_3C_4C_5C_6) \\
& + P(C_1C_4C_5C_6) + P(C_2C_3C_4C_5) + P(C_2C_3C_4C_6) + P(C_1C_2C_4C_6) \\
& + P(C_2C_3C_5C_6) + P(C_1C_3C_5C_6) + P(C_2C_4C_5C_6)] \\
& + [P(C_1C_2C_3C_4C_5) + P(C_1C_2C_3C_4C_6) + P(C_1C_2C_3C_5C_6) \\
& + P(C_1C_2C_4C_5C_6) + P(C_1C_3C_4C_5C_6) + P(C_2C_3C_4C_5C_6) \\
& - [P(C_1C_2C_3C_4C_5C_6)]] = R_{13} = 0.97776
\end{aligned}$$

The complexity of tie-set and cut-set method depends on two variables: the first is the enumeration of tie sets/or cut sets and the second is the implementation of the inclusion-exclusion expansion equation, which represents the union of all event based on probability principles.

### 2.3.3. Graph Transformation

Complex networks can be simplified into a simpler analogous network by a series of graph transformations (graph reduction). Series/Parallel (Musaria K. Mahmood et al., 2021), delta-to-star (Gadani, J. P. ,1981), and link factoring (Carrier, J., & Lucet, C.,1996) are considered among the most popular transformations.

A. Series transformation

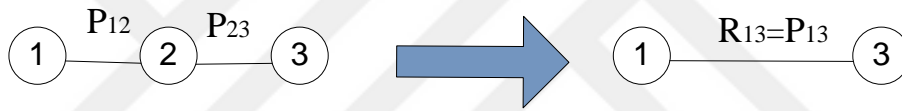
It is the most widely used due to its frequent presence in the functional networks. If three nodes are connected in series as shown in Figure 6-a. the node 2 can be removed, and the new link reliability is calculated by:

$$R_{13} = P_{12} \times P_{23} \quad (12)$$

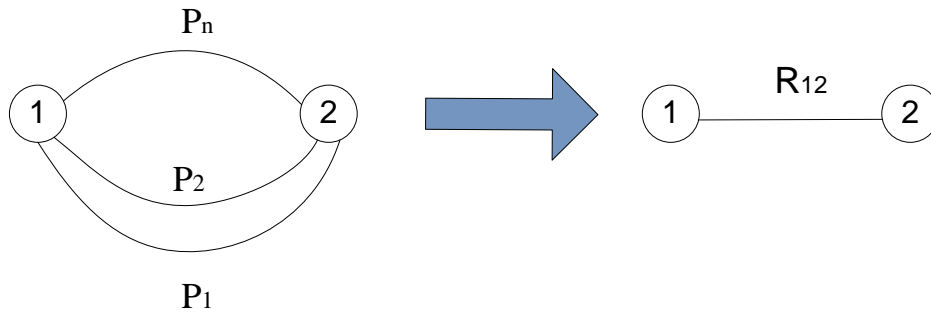
The parallel simplification is also a frequent topological transformation when there are two or more links in parallels between two nodes as in Figure 6-b. These links are replaced by one link with reliability equal to:

$$R_{12} = 1 - \prod_{i=1}^n (1 - p_i) = 1 - \prod_{i=1}^n q_i \quad (13)$$

Where  $q_i = 1 - p_i$



A. Series simplification



B. Parallel simplification

**Figure 6.** Series-parallel simplification

Using these two graph simplifications, the reliability between  $n_s = 1$  and  $n_d = 3$  of the same example given in Figure 5. Can be evaluated, the step of simplifications is given in Figure 7.

- Removing node (2), and node (5) by series simplification

$$R = P_1 P_3 = 0.9 \times 0.9 = 0.81$$

$$R = P_1 P_4 = 0.9 \times 0.9 = 0.81$$

- Removing the parallel links between nodes (1) and (4), and replace them with one link

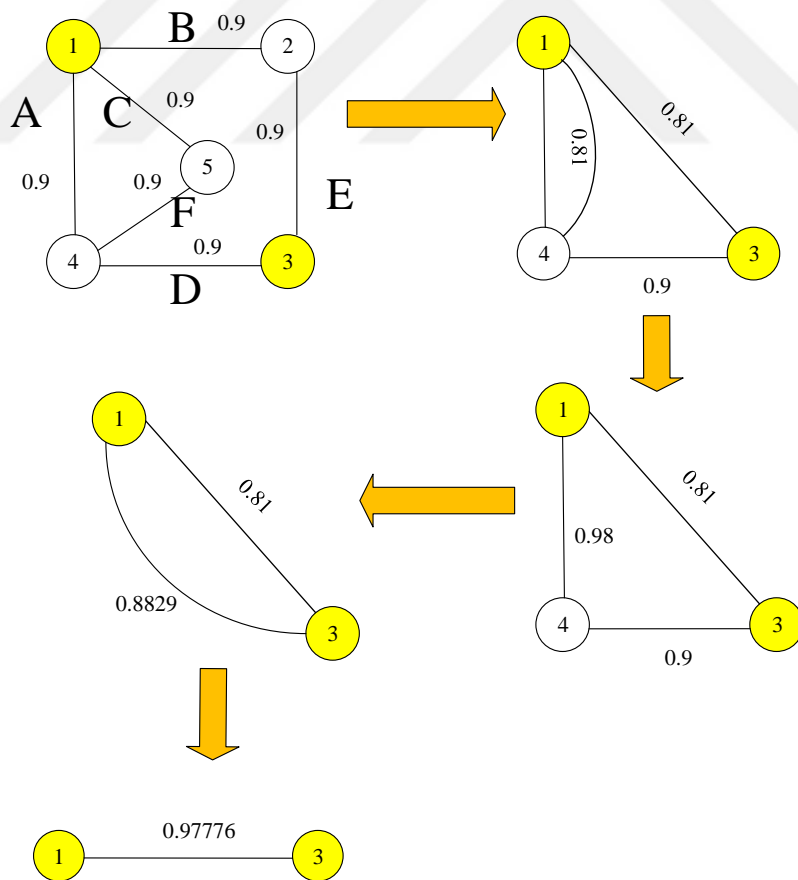
$$R = 1 - (1 - q_1)(1 - q_4) = 1 - (1 - 0.9)(1 - 0.81) = 0.98$$

- Removing node (4) by series simplification

$$R = P_1 P_3 = 0.98 \times 0.9 = 0.8829$$

- Finally, the reliability between the source and destination is found as the probability of the link between nodes (1) and (3) by parallel simplification. The result naturally is the same found using the previous method.

$$R_{13} = 1 - (1 - q_1)(1 - q_3) = 1 - (1 - 0.81)(1 - 0.8829) = 0.97776$$



**Figure 7.** Simplification steps

### B. The Edge-factoring

It is more complex simplification than the previous ones since it demands to consider two cases then summing them in the final equation to evaluate analogous network reliability. This graph is partitioned into two sub graphs as shown in Figure 8, according to the state of the link between the nodes  $i_1$  and  $i_2$ . The application of the law of probability twice for the G1-graph and G2-graph and integrating the solutions in one equation giving the final solution. If the link  $i_1 - i_2$  is up ( $p_5 = 1$ ), then the topology is reduced to G1-graph:

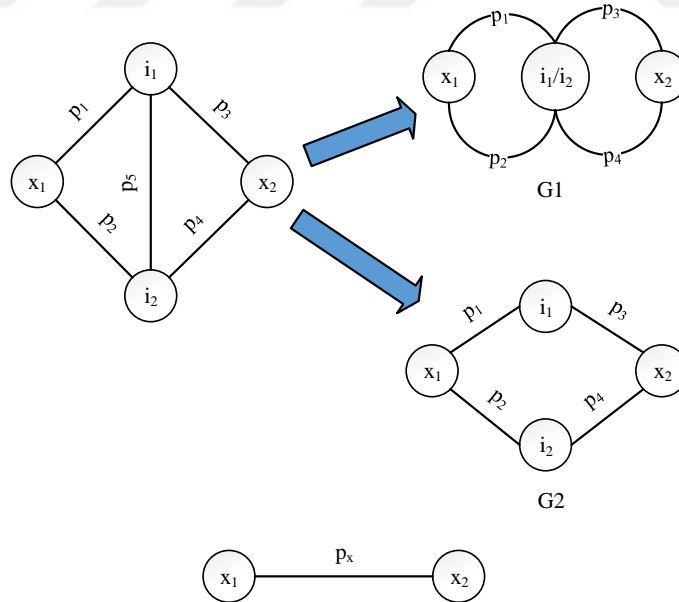
$$P(G_1) = (1 - q_1q_2)(1 - (1 - q_3q_4)) \quad (14)$$

Considering that the link  $i_1 - i_2$  is down ( $p_5 = 0$ ), G2-graph gives:

$$P(G_2) = 1 - (1 - p_1p_3)(1 - p_2p_4) \quad (15)$$

By using the probability theory, and by combining 14 and 15, the equivalent reliability between nodes  $x_1 - x_2$  becomes:

$$R_{x_1-x_2} = p_x = p_5 P(G_1) + q_5 P(G_2) \quad (16)$$



**Figure 8.** Links factoring simplification

To illustrate this technique, first, considering (P5) as an open link with probability of successful equal to 1 (perfect link) yielding to the case ( $G_1$ ). Second, the link P5 is considered as a failed link with probability of successful equal to 0 giving rise of the case ( $G_2$ ). The two cases are disjoining events then the reliability is simply given by equation 16. Suppose all the links are with probability of success equal to  $p = 0.9$ , then:

$$P(G_1) = [1 - (1 - p_1)(1 - p_2)][1 - (1 - p_3)(1 - p_4)] = 0.9801$$

$$P(G_2) = (1 - p_1p_3)(1 - p_2p_4) = 0.0361$$

Using equation 16 we have:  $p_x = 0.9 \times 0.9801 + 0.1 \times 0.0361 = 0.8857$

### C. Delta-Star Simplification

Considering a triangular (delta) network containing three nodes represented by A, B, and C, as shown in Figure 9a. Values q, r, w, represent the probability of links connecting nodes A, B, and C. The goal of the delta star simplification is to transform delta form into star form as shown in Figure 9b. Delta-star transformation mathematical basics are similar to using the delta-star transformation in network analysis except that here a new node N is used to simplify the theoretical analysis. A delta form with three nodes and three links is then converted to a star form with four nodes and three links. All nodes are considered perfect with probability equal to 1.

The idea of this simplification is based on the fact of reliability conservation between any two nodes, in that  $R_{AB}$ ,  $R_{AC}$ ,  $R_{BC}$ , are the same for both forms. To find the unknown values of the newly introduced link probabilities x, y, z, in function of given values q, r, w, a calculation is performed keeping in mind that both forms are analogous in terms of reliability.

- Delta form:

$$P_{AB} = q$$

$$P_{BC} = r$$

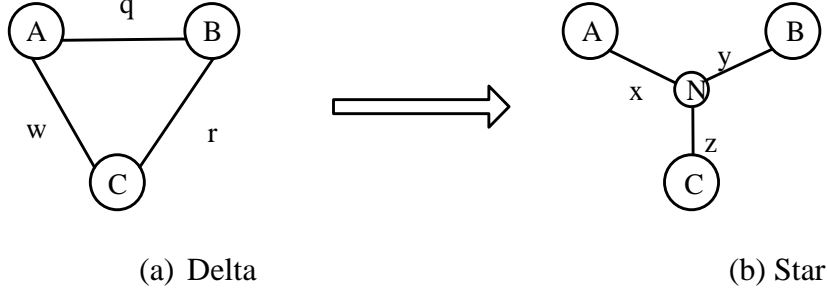
$$P_{AC} = w,$$

Using series and parallel simplification, the two-terminal reliability is calculated by

$$R_{AB} = 1 - (1 - q)(1 - rw) \quad (17)$$

$$R_{AC} = 1 - (1 - w)(1 - qr) \quad (18)$$

$$R_{BC} = 1 - (1 - r)(1 - qw) \quad (19)$$



**Figure 9.** Delta – star transformation.

- Star Form

$$P_{AN} = x$$

$$P_{BN} = y$$

$$P_{CN} = z$$

Using series simplification, the two-terminal reliability is calculated by:

$$R_{AB} = xy \quad (20)$$

$$R_{AC} = xz \quad (21)$$

$$R_{BC} = zy \quad (22)$$

Combining equations 17 and 20 yields:

$$R_{AB} = xy = 1 - (1 - q)(1 - rw)$$

$$x = \frac{1 - (1 - q)(1 - rw)}{y} = \frac{q + rw - qrw}{y} \quad (23)$$

Combining equations 18 and 21 yields:

$$R_{AC} = xz = 1 - (1 - w)(1 - qr) = w + qr - qrw \quad (24)$$

Combining equations 19 and 22 yields:

$$R_{BC} = zy = 1 - (1 - r)(1 - qw)$$

$$z = \frac{1 - (1 - r)(1 - qw)}{y} = \frac{r + qw - qrw}{y} \quad (25)$$

Replace equations 23 and 25 in 24, gives:

$$\frac{q + rw - qrw}{y} \times \frac{r + qw - qrw}{y} = w + qr - qrw$$

$$y = \sqrt{\frac{(q + rw - qrw) \times (r + qw - qrw)}{w + qr - qrw}} \quad (26)$$

After evaluating the value of  $y$ ,  $x$  and  $z$  will be found by (23) and (25). To illustrate that an example is given by  $q = w = 0.9$ , and  $r = 0.95$ .

Using equations (17), (18), and (19), the solutions of the delta form are:

$$R_{AB} = q + rw - qrw = 0.9855$$

$$R_{AC} = r + qw - qrw = 0.9905$$

$$R_{BC} = w + qr - qrw = 0.9855$$

$$\text{Then } y = \sqrt{\frac{0.9855 \times 0.9905}{0.9855}} = \sqrt{0.9905} = 0.99524$$

$$x = 0.99021$$

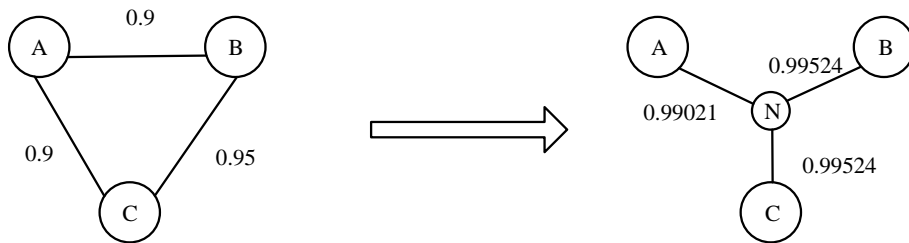
$$z = 0.99524$$

Using equations (20), (21), and (22) for the star form, the same solutions are found as shown in Figure 10.

$$R_{AB} = 0.99524 \times 0.999021 = 0.9855$$

$$R_{AC} = 0.99524 \times 0.99524 = 0.9905$$

$$R_{BC} = 0.99524 \times 0.999021 = 0.9855$$



**Figure 10.** Delta-Star example

#### 2.3.4. Approximation Methods

Reliability estimation has become more concerned with large engineering systems, which operate in record time. However, complex networks are one of the most common problems encountered in the reliability calculation process, and therefore, resorting to the approximation method is the best solution. Depending on their theoretical foundations, methods for evaluating reliability can be classified into: precise or specific, and they may be approximately correct (Caşcaval, P. 2018).

The Truncation Approximations (TA) is one of the practical methods used to evaluate the reliability as closer as possible. The result of TA is included between an upper and a lower bound. The set of joint events make the exact calculation as a difficult task, thus the difficulty can be reduced by neglecting many of the higher-order terms in the sequence, resulting in a simpler approximation formula as (Paredes, R., 2019):

$$\text{higher bounds} \geq R_{ab} \geq \text{lower bounds}$$

Another way of simplification is the Subset Approximations (SA) by reducing the complexity of the "inclusion-exclusion expansion equation" by dropping the cut sets or the tie sets of higher order. For example, if there are four tie-set ( $T_1, T_2$ ) of two hops and ( $T_3$ ) of three hops, where ( $T_4$ ) of four jumps. By eliminating ( $T_4$ ), which is the higher-order set, to get the highest bounds, this greatly reduces the difficulty of finding cut and tie sets. Must compute all the terms in the expansion equation. We can also calculate higher-order cut-set from the reliability equation, considering the same cases of the mentioned tie-set.

## CHAPTER THREE

### METHODOLOGY AND ALGORITHM DEVELOPMENT

#### 3.1. General

The reliability calculation complexity radically changes when the network size and topology complexity are changed by increasing computing time. Many methods haven't succeeded efficiently assessing the reliability of complex networks with varying size and increased complexity. This chapter introduces the development of effective methods based on hybridize two algorithms to calculate the reliability for communication networks of any complexity and size. The proposed hybrid algorithm is based on two classical reliability evaluation methods that are the graph GRT and the tie-set method. It consists of three stages, the first one is the initialization stage followed by two other stages where each one is composed of many sub-stages.

After the initialization stage, the application of multi-stage GRT results a simpler network by reducing the number of nodes and links by applying series, parallel, edge-factoring and delta star simplifications. Finally, the third stage evaluates the reliability of the simplified network by applying tie-set method based on the inclusion exclusion expansion equation. All the sub-algorithms are developed and programmed using MATLAB for validation of the proposed method.

#### 3.2. Algorithm Structure

##### 3.2.1. Initialization

The network topology is presented as 3D-matrix ( $M$ ) describing the connectivity state of the network. The elements of  $M$  are the working probability of a link or a node defined by their corresponding reliability:

$$M_{ijk} = \begin{cases} M[i, j, k] & \text{links between } i - j \\ 0 & \text{no link between } i - j \\ M[i, i] & \text{probability of node } (i) \end{cases}$$

Where the third dimension of  $M[i, j, k]$ , is defining the link in parallel between  $i - j$  nodes.

The network topology and connectivity are the main inputs of the algorithm describing the problem under consideration.

### 3.2.2. Parallel Reduction Algorithm

The second stage concerning the application of the GRT starts by removing links in parallel as first sub-stage as presented in Figure 11. This step begins with a consideration of all links in parallel by checking the values of the element  $M [i, j, k]$  for  $k > 1$  only. After the localization of parallel form, equation (13) is applied for simplification. The simplification continues until all links in parallel are removed.  $M$  becomes with all elements null for  $k > 1$  (no more parallel links), but it is kept as a 3D matrix for future simplifications (new born parallel links after series simplification).

The algorithm for parallel simplification starts after the network topology matrix initialization. It checks for parallel links between any two nodes  $n_i$  and  $n_j$ . If all elements  $M [i, j, k]$  for  $k > 1$  are nulls, then no parallel links between these nodes are presents. If one (or more) of these elements is not null, then a parallel simplification is required. After recognizing the parallel form, equation (13) is applied to replace all parallel links with one link and updating the reliability (probability) of the new link accordingly. This process is executed for all possible node pairs and end by a virtual step A announcing the end of parallel simplification procedure.

### 3.2.3. Series Reduction

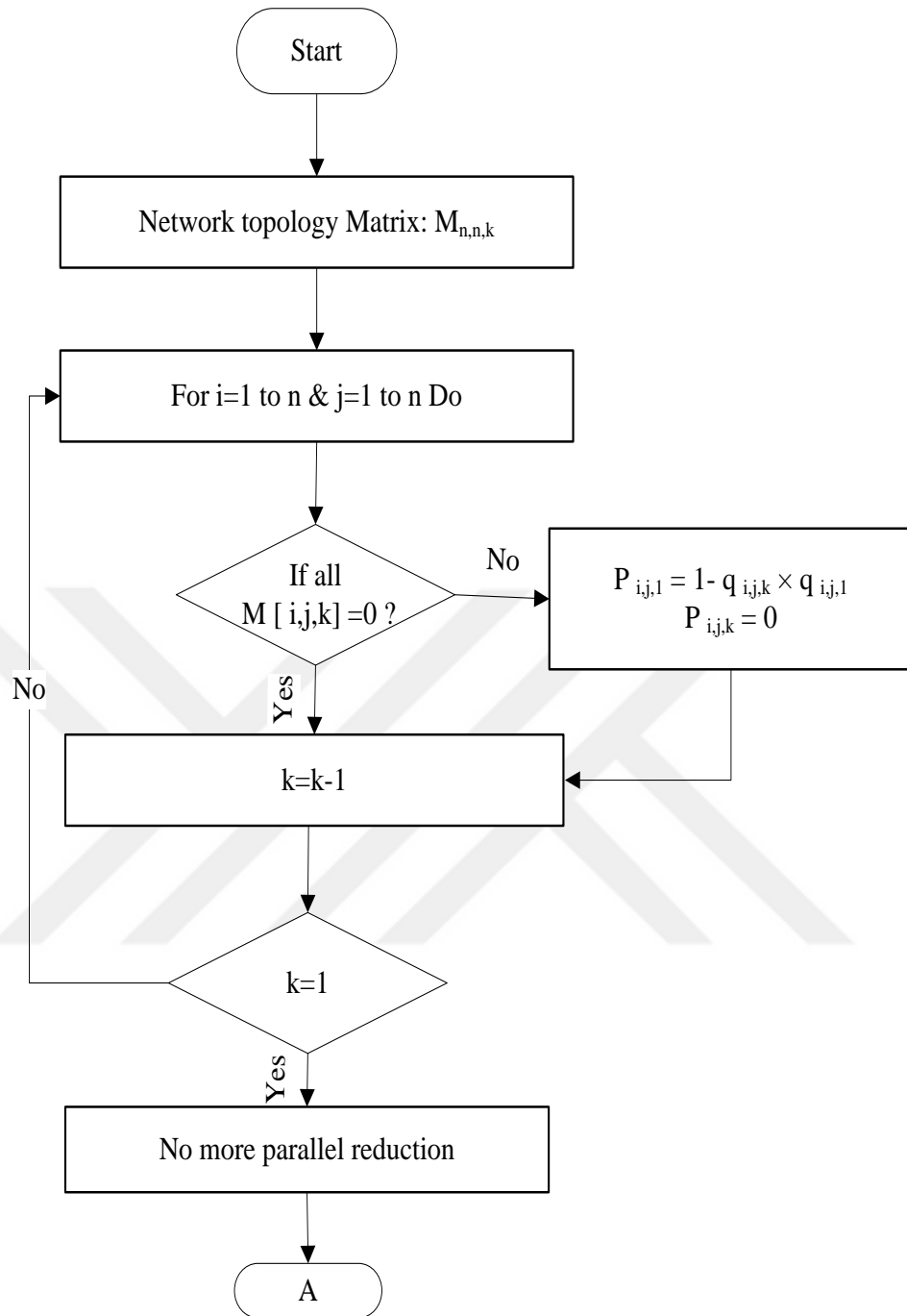
The series reduction sub-stage begins with the recognition of nodes in series by considering cases where a node is connected to only two other nodes as depicted in Figure 12. This is carried out by check node connectivity from  $M$ , and then the application of the equation (12) for removing the redundant nodes in the middle. This process is applied to all nodes except the case where the source node  $n_s$  or the destination node  $n_d$  are parts of the simplification process (as node in the middle), because they are unreducible.

The series reduction sub-stage starts after the application of parallel sub-algorithm from step A. The matrix  $M_{N,N,1}$  present the new connectivity of the

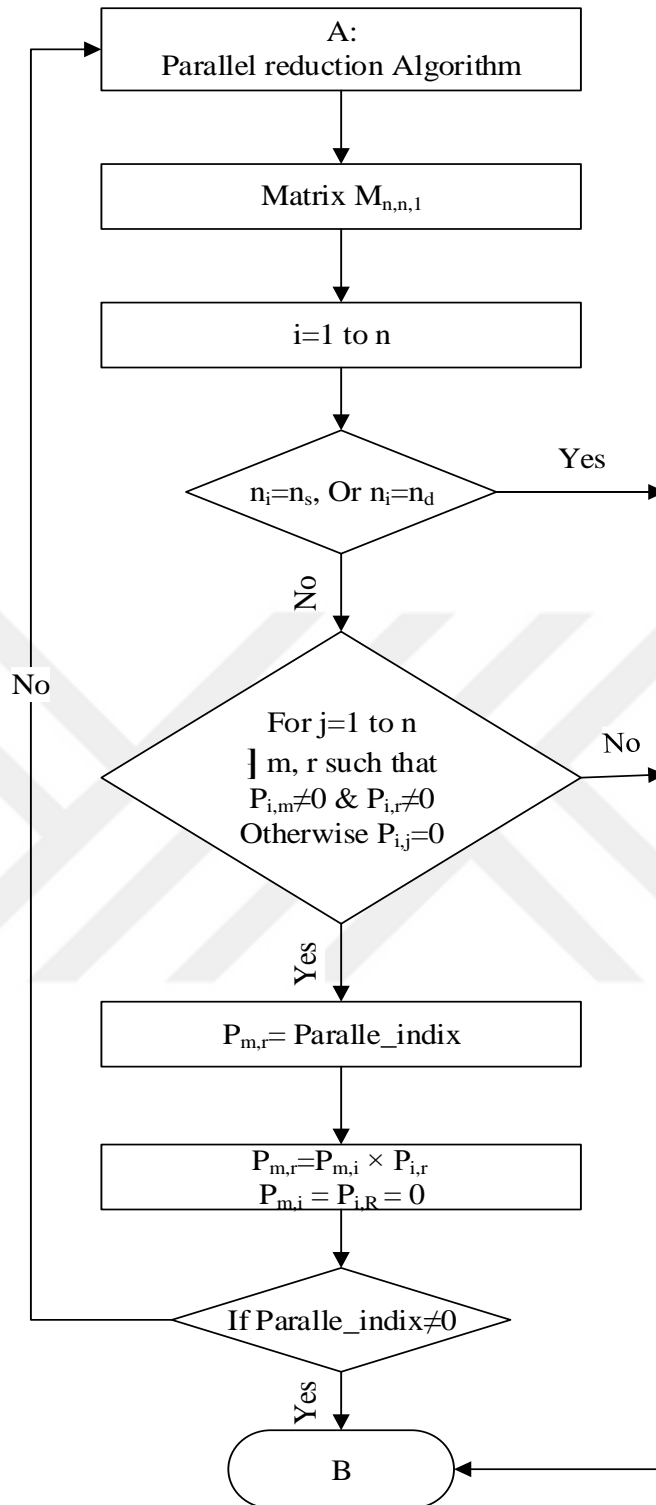
network topology with no parallel links. The series simplification process excludes the commodity nodes, which are the source-destination, namely,  $n_s$  and  $n_d$ . The recognition process is accomplished by scanning for all nodes  $n_i$  connectivity. If the node  $n_i$  is only connected to two other nodes  $n_m$  and  $n_r$ , then a series form is identified, otherwise a new node connectivity is tested. By removing the node  $n_i$  by series simplification (node in the middle), a birth of new parallel links is possible in there is a direct connection between nodes  $n_m$  and  $n_r$  before this simplification. To resolve this problem, the algorithm checks for possible new born parallel links between two nodes after removing the node in the middle by introducing the binary variable `parallel_index` to describe this prior connectivity. If the adjoining nodes  $n_m$  and  $n_r$  are checked for possible parallel links between them. If `parallel_index=1`, then there is a connection appearing as new parallel connections, and this problem must be solved by reactivating the parallel reduction sub-stager as in Figure 11.

#### 3.2.4. Edge Factoring

This method can solve many problems in reliability calculations by a noticeable simplification of the graph. As in the parallel and series sub-algorithm, the edge-factoring algorithm starts by recognition of the topological 4-node form as presented in Figure 13-a, and then applying equation (16) for simplification. The recognition of the edge form is accomplished through many sub-steps by verifying the connectivity in  $M$ . This process is the most difficult in the edge-factoring sub-stage. After confirmation of the topology-shape searched for, the adjoining nodes are subject to node simplification procedures by removing two mid nodes  $n_{i_1}$ , and  $n_{i_2}$  updating the  $M$  elements values and setting the diagonal elements  $M_{i_1,i_1}$ , and  $M_{i_2,i_2}$  to zero. Usually this sub-stage is located after the parallel and series simplification sub-stages. The edge-factoring algorithm is presented in Figure 14. Starting from the simplified matrix after parallel, and series simplification, and ending by the removal of the edge forms from the network. The source node  $n_s$ , and destination node  $n_d$  are excluded from this simplification as in the series simplification.



**Figure 11.** Parallel reduction algorithm.



**Figure 12.** Series reduction algorithm

The simplification results in the removal of two nodes (the mid nodes:  $n_{i_1}$ , and  $n_{i_2}$ ) keeping the two others as reduction survivor nodes ( $n_{x_1}$ , and  $n_{x_2}$ ). The process of edge-factoring form recognition is shown step-by-step in Figure 13.

Step 1: Edge form recognition:

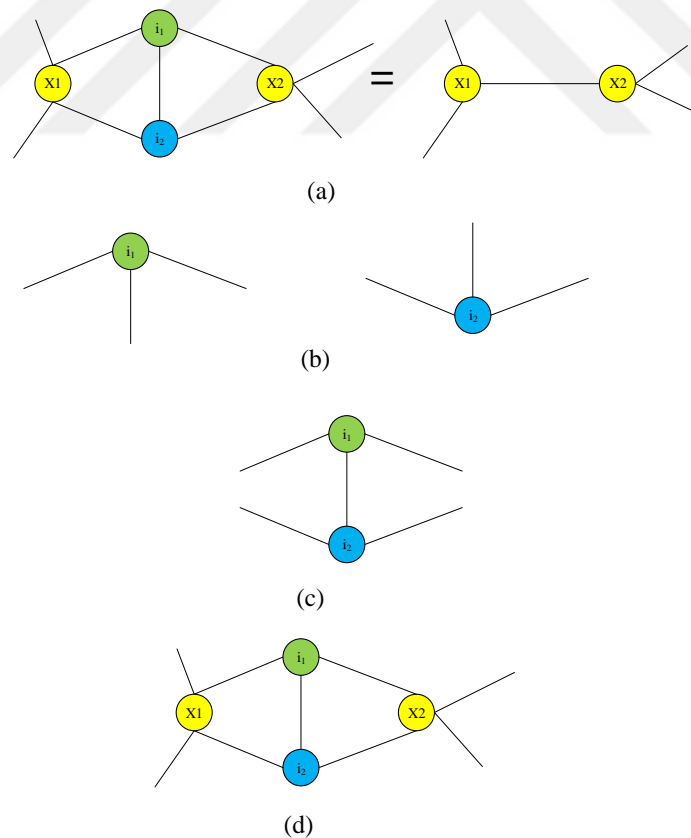
- Recognition of all nodes having the form in Figure 13b. By using the index G in the flowchart. These nodes have three connections with three nodes in the network. Recognized nodes are the first round candidates for simplification.

- Check the connectivity between these nodes two-by-two. If there is direct connection between the selected pair, then it is candidate for the second round of simplification, as shown in Figure 13c.

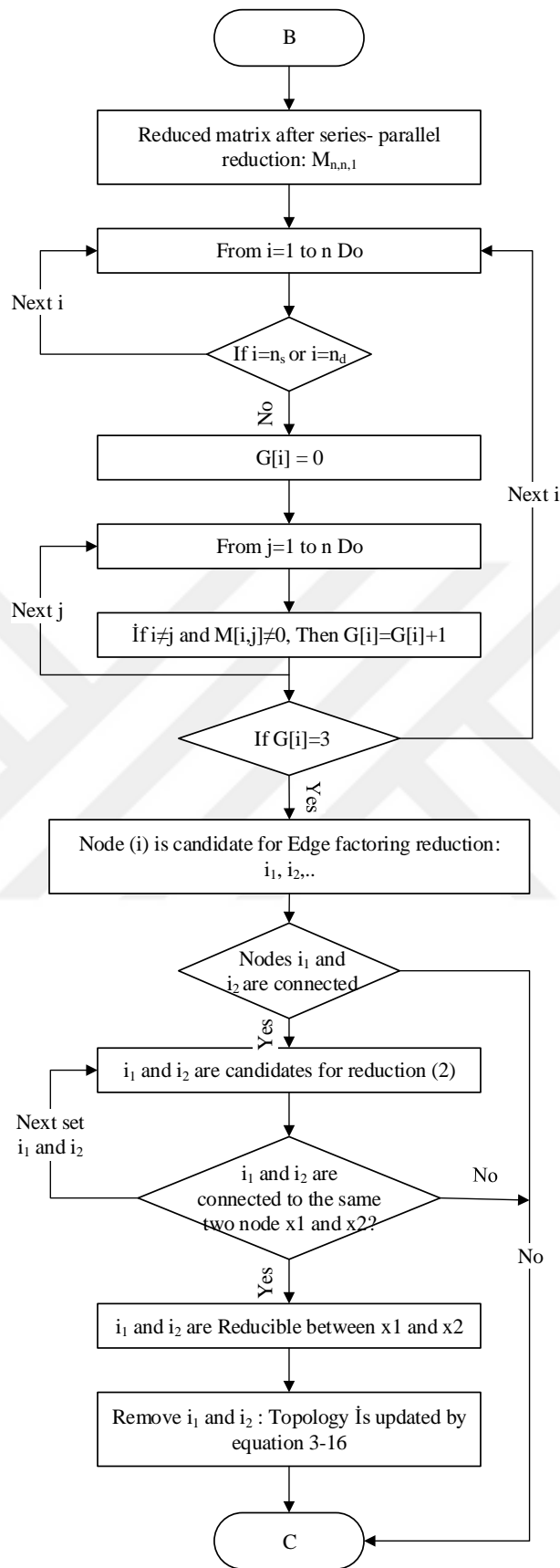
- The pair candidates for the second simplification round are checked if they are connected to same nodes as in figure 13d. Then, the form is recognized as edge form.

Step 2: Graph simplification:

- Edge forms resulting from step 1 are subject to simplification by applying equation 16 by removing nodes  $n_{i_1}$ , and  $n_{i_2}$ , keeping nodes  $n_{x_1}$ , and  $n_{x_2}$ , and updating link reliability according to this equation.



**Figure 13.** Edge-factoring reduction steps



**Figure 14.** Edge-factoring reduction algorithm.

### 3.2.5. Delta-Star

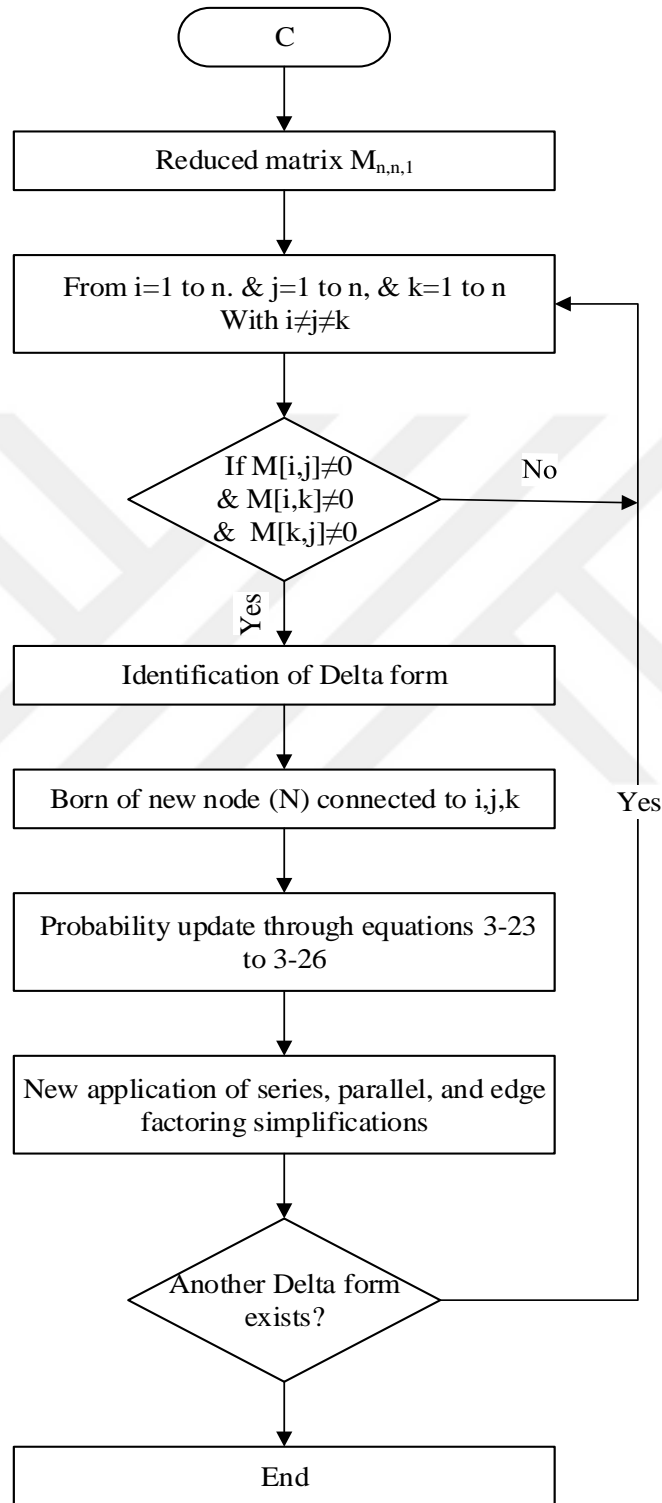
For Delta-star simplification, an original algorithm is developed as shown in Figure 15. The algorithm starts by the recognition of the delta form and then the application of the simplification equations as has been given through (23) to (26). Delta-star simplification is applied after series, parallel, and edge-factoring stages. The reduced matrix  $M_{n,n,1}$ , which is the output of the previous simplification stage (step C) is used as input for the algorithm. The algorithm starts by recognizing the delta form where three nodes ( $i, j, k$ ) are interconnected together in delta topology. According to the theory presented in section 3-3-3-C, the next step is the expansion of the 3-node delta form into 4-node star form, generating a new node (N) as a star center. Although the expansion of the topology through the birth of a single node (N) can appear as the counter-nature of the required network reduction, several simplifications could be the result of this temporary expansion of the network, which ultimately results in a marked decrease in the numbers of nodes and links. The calculation of the probability of the links connecting node N with nodes  $i, j$ , and  $k$  is accomplished through equations (23) to (26), and precedes the network final simplification. Many new series, parallel, and edge-factoring forms is expected to result from this operation requiring a new application of corresponding sub-algorithms in the usual order. A new check for possible delta forms is performed till such they are removed by simplification.

### 3.2.6. Tie-set Algorithm

The application of reduction stages results a reduced matrix  $R_{Nr,Nr}$  that is used to generate tie-set matrix according to the theory presented in the previous chapter section 3.3.2. The tie-set generation starts by indicating the  $n_s$  and the  $n_d$  nodes as the main target for the two-terminal reliability evaluation, and then searching for all possible paths connecting these pairs of nodes. All loop-free paths between the  $n_s$  and the  $n_d$  are represented by a set of links, representing tie-set.

The tie-set method can be regarded as two main sub-stages composed of the tie set enumeration and the Poincare equation application as shown in equation 10. The tie-set enumeration process is the hard sub-stage in the application of this

algorithm, while the application of equation 10 is a matter of mathematical programming. Although, the time of Poincare



**Figure 15.** Delta-star algorithm

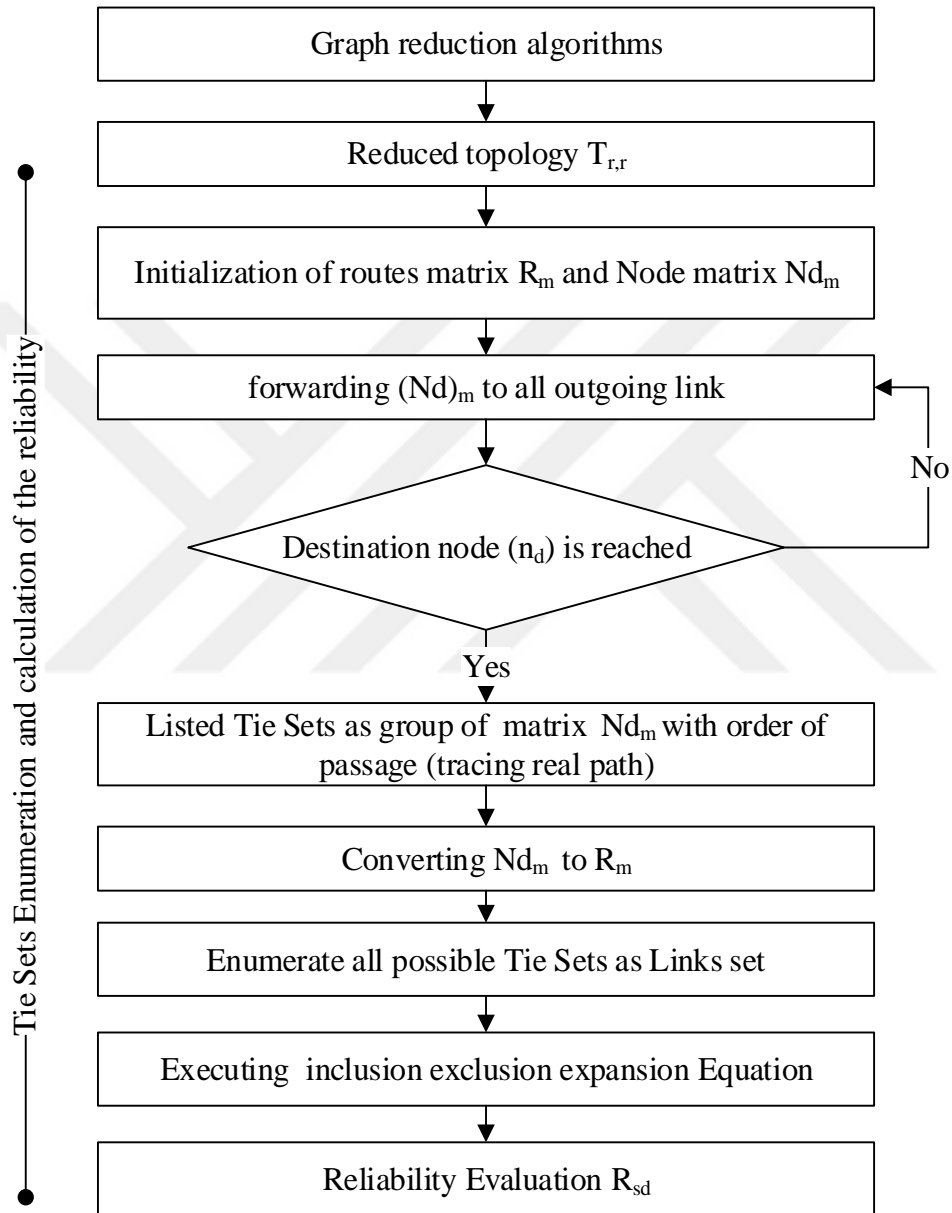
The equation evaluation is increased with tie-set number, making the application of this algorithm impossible for large tie-set number. An original procedure is proposed for tie-set enumeration where matrix manipulation is adopted for simplification of the process. The path-matrix  $R_m$  and node-matrix  $Nd_m$ , are used to simplify the enumeration process of all tie sets from  $n_s$  to  $n_d$  passing through all intermediary nodes. The matrix  $Nd_m$  presents the group of nodes between  $n_s$  to  $n_d$  for one the tie-set while  $R_m$  rows give the links included in paths. The matrix  $Nd_m$  is a vector matrix with dimensions  $[1 \times N_r]$ , that indicating all reduced network nodes, whereas,  $R_m$ , is a square matrix of dimensions  $[N_r \times N_r]$ , representing network links. At the start of the tie-set algorithm application,  $Nd_m$  and  $R_m$ , elements are set to '0'. Every tie-set between  $n_s$  and  $n_d$  is represented by a set of links (loop free), and then by different matrix set  $Nd_m, R_m$ .

The developed algorithm starts by forwarding an empty  $Nd_m$  corresponding to the tie-set (m) from  $n_s$  by a flooding process until it reaches  $n_s$ . The matrix  $Nd_m$  presents the group of nodes between  $n_s$  to  $n_d$  for a select the tie set where elements corresponding to nodes 'in the route' are set to '1'. The corresponding link matrix  $R_m$  is then extracted from  $Nd_m$ . A total of 'm' different matrix  $R_m$  evaluated for 'm' tie set. These  $R_m$  are used in the inclusion expansion TS in equation (10) to evaluate the reliability, as shown in Figure 16.

### 3.3. Algorithm Application

A step-by-step application of the GRT starts from the parallel reduction, series, checking again for possible newly born parallel form, edge-factoring and delta star. The process is repeated until a simplified analogous network topology is reached (fixed by program as links or nodes number). The matrix M is reduced in dimensions beginning from its original 3D form with size  $(N \times N \times K)$  to a 2D matrix with size  $(R \times R)$  with  $R \ll N$ . R represents the largest number of network nodes enabling smooth and efficient application of tie-set method with acceptable computation time. In This Thesis R is considered to be equal to 6 nodes. The original M is transferred by preserving its main ingredients, which make the two matrices exactly

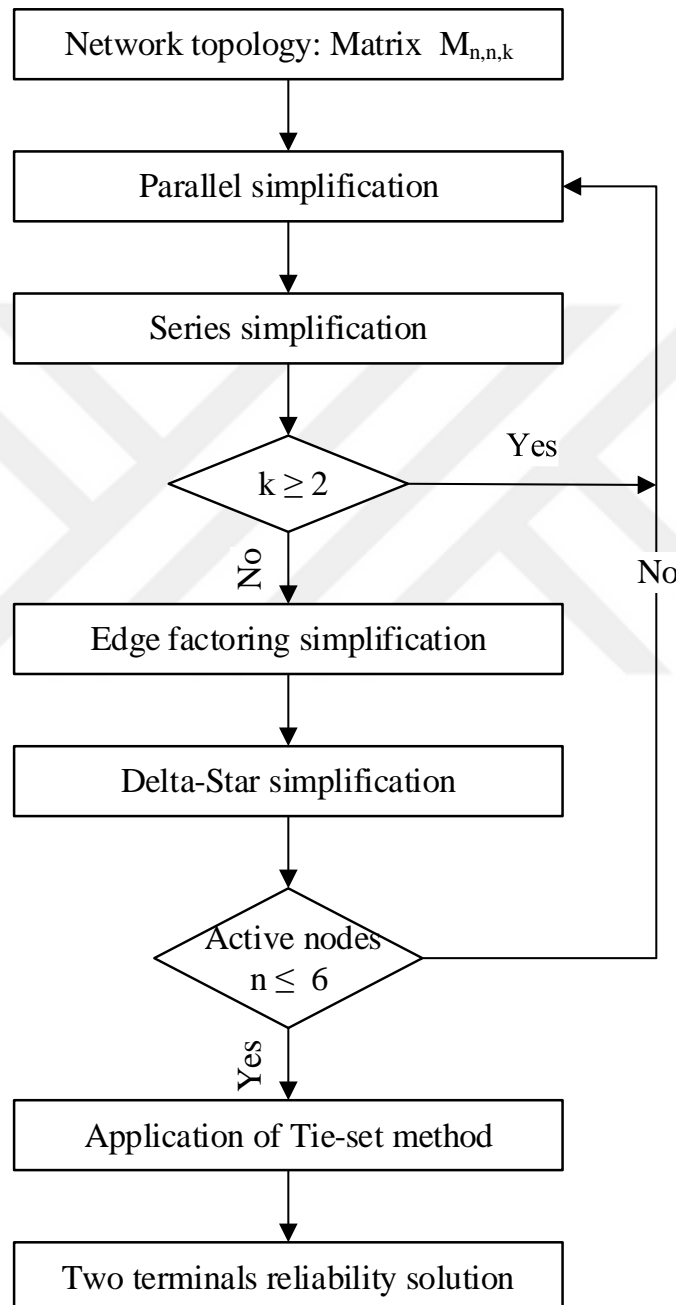
analogous but given in two forms. The well-known tie-set algorithm is then applied to the two-terminal reliability calculation. All loop-free paths between  $n_s$  and  $n_d$  are found, which represent the tie set link-group, and equation (10) is used for exact reliability value evaluation.



**Figure 16.** Tie sets generation

This algorithm starts with input the network topology  $M_{N,N,K}$  after applying parallel simplification to remove all parallel links, series simplification is applied

as presented in Figure 17. Applying the edge-factoring and delta star simplifications as more advanced reductions follow the series-parallel reduction. If the active nodes are less than 6 directly applying tie-set technique and find the reliability, otherwise repeat the simplification process.



**Figure 17.** Algorithm Application flowchart

## CHAPTER FOUR

### RESULTS AND DISCUSSION

#### 4.1. General

The proposed hybrid algorithm is validated by three random networks of increased complexities. A comparison is made with a classical reliability evaluation algorithm, which is the tie-set method to show the noticeable improvement in computing time of the proposed algorithm. Results of the two-terminal reliability are 'if course' the same using classical or hybrid algorithm but differ by their computing time as the main contribution of the new algorithm. The computing time is the main factor that leads the newly proposed algorithm to be used for real-time applications. Added to that, when the network becomes complex, like the 30-node network in example 3, the classical tie-set fails to find the solution.

The proposed algorithm is a collection of many sub-algorithms organized in efficient method leading to apply to all kinds of complex networks without any limitation in links and node number. Case studies are based on a 6-node 9-link simple network, 10-node 15-link medium sized, complex network, and finally a 30-node 41-link complex large network. Simulations are performed on core i5 computer using MATLAB programing.

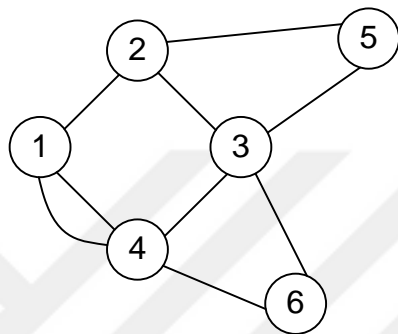
#### 4.2. Simulation Results

##### 4.2.1. 6-Node Network

The new hybrid two-terminal algorithm and the classical tie-set algorithm are simulated for a randomly generated simple network containing 6-nodes and 9 links as in figure 18. Nodes are considered perfects (reliability= 100%), and all links are bidirectional with probability ( $P_{ik} = 0.9$ ).

Results collected from the application of the new algorithm are compared with results from the application of the classical tie-set algorithm as in Table 3. The new hybrid algorithm outperforms the tie-set algorithm by computing time

(less time required). For example, for the reliability evaluation between nodes 2 and 4 ( $R_{2,4}$ ), the computing time using the hybrid method is about 0.079701 second while it is 0.302577 second for the classical tie-set algorithm. This result shows an improvement of 379% in reliability evaluation time. Results on the table also indicate that the reliability between and pair of nodes is the same using the two methods, which verify the exactness of our developed algorithm. The decrease in computing time is expected to be more important for complex big networks.



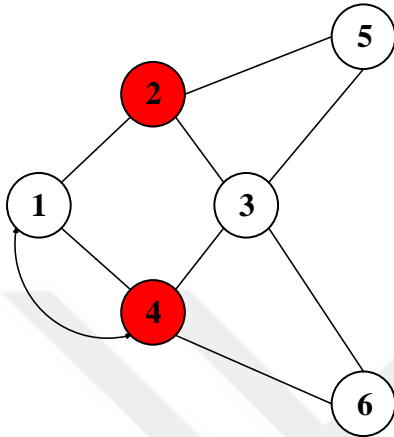
**Figure 18.** Random 6-node network

**Table 3.** Results 6 nodes

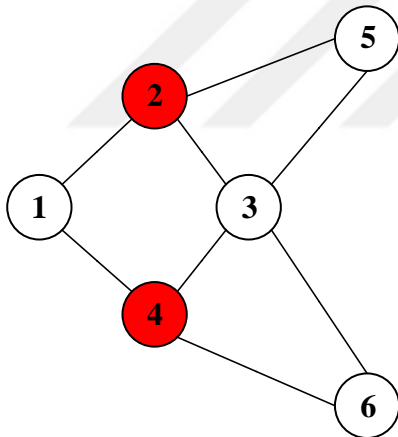
S-D		New Hybrid Algorithm		Classical tie-set Algorithm		S-D		New Hybrid Algorithm		Classical tie-set Algorithm	
$n_s$	$n_d$	$R_{sd}$	$T(sec)$	$R_{sd}$	$T(sec)$	$n_s$	$n_d$	$R_{sd}$	$T(sec)$	$R_{sd}$	$T(sec)$
1	2	0.9953	0.085470	0.9953	0.413018	2	6	0.9869	0.086006	0.9868	0.349450
1	3	0.9966	0.076725	0.9966	0.284243	3	4	0.9976	0.087623	0.9976	0.246814
1	4	0.9987	0.081634	0.9987	0.318014	3	5	0.9889	0.093021	0.9889	0.229976
1	5	0.9861	0.107216	0.9860	0.346667	3	6	0.9889	0.080047	0.9889	0.252020
1	6	0.9878	0.091325	0.9877	0.338489	4	5	0.9869	0.078463	0.9868	0.306615
2	3	0.9976	0.085582	0.9976	0.270703	4	6	0.9889	0.072876	0.9889	0.292480
2	4	0.9959	0.079701	0.9959	0.302577	5	6	0.9781	0.079294	0.9780	0.377231
2	5	0.9889	0.098894	0.9889	0.306986						

To give an idea about the step-by-step simplification performed by the new proposed algorithm on this network, below the transformations reported for the commodity  $n_s = 2, n_d = 4$ .

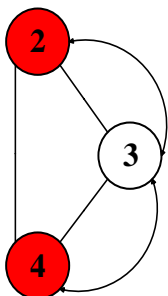
- Original network with commodity in red



- Removing parallel links



- Removing series nodes.

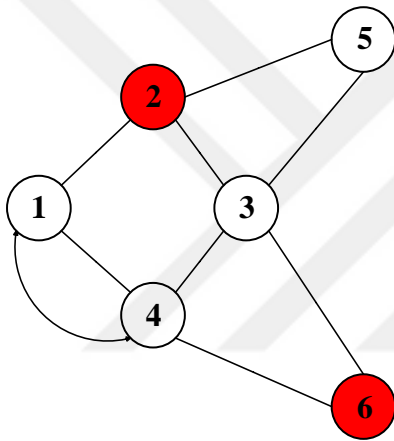


- Parallel and series simplification, and then the final reliability is found

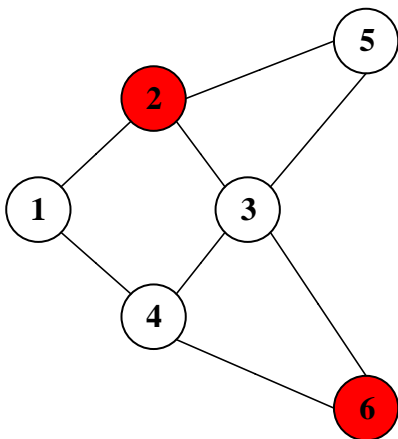


These simplifications depend on the topology and the selected commodity as source-destination. For example, for the same network but with the commodity  $n_s = 2, n_d = 6$ , the simplifications are:

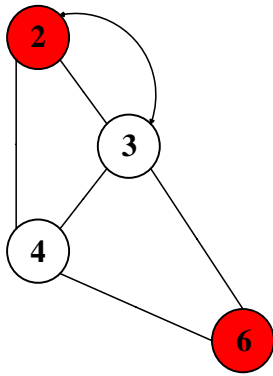
- Original network with commodity in red



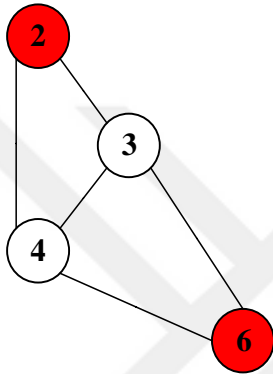
- Removing parallel links



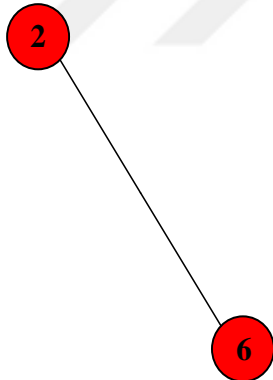
- Removing series nodes



- Parallel and series simplification



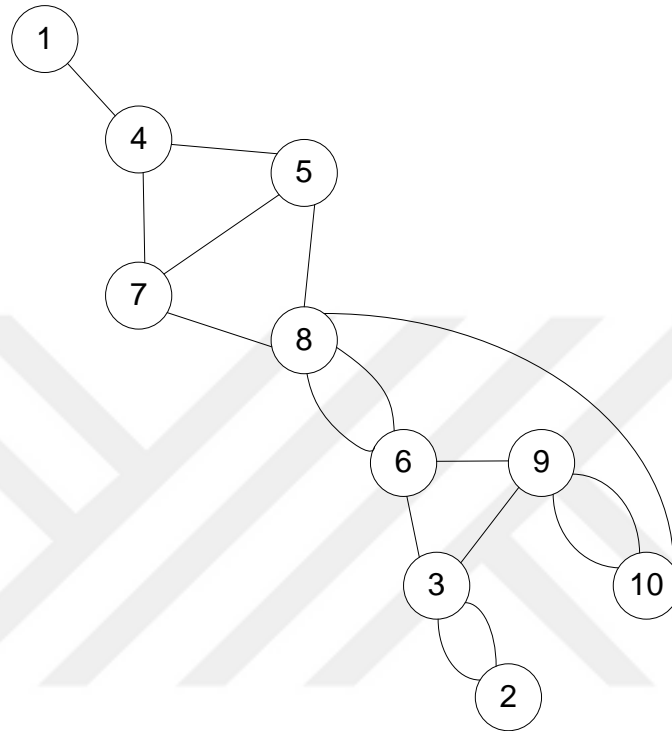
- Edge-factoring form simplification



#### 4.2.2. 10-Node Network

A case study of 10 nodes and 15 link networks, as presented in figure 19. is simulated. The graph transformation procedures are applied in sequence to simplify the network before the application of the tie-set method. In the present example ‘as in the previous one’, the graph transformation technique starts by parallel, series, edge factoring, and end delta star transformation where parallel-series simplification is used in various stages for topology simplification. The graph reduction stops when the number of nodes reaches 6 nodes fixed by the main algorithm given in Figure 17. where the classical tie-set algorithm is applied

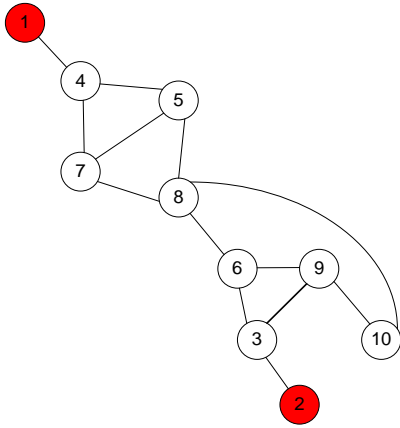
on the simplified form of the network. Results collected from the application of the new hybrid algorithm and the tie-set algorithm (alone) are collected and projected in Table 4. Data show that the improvement in computing time accomplished by the new algorithm is high compared to that of the classical tie-set algorithm.



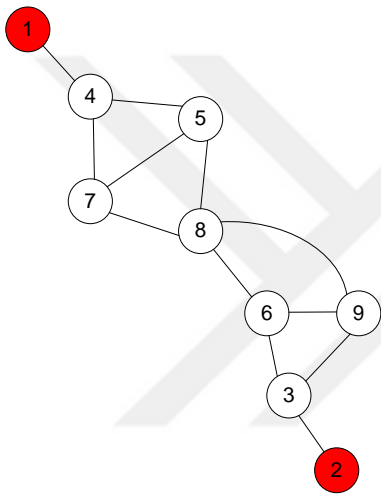
**Figure 19.** 10-Node network

For example, the two-terminal reliability of the commodity (1, 2) is equal to 0.861 for both algorithm application (the new and the classical alone). This is normal because the change of the algorithm for reliability evaluation is not reflected to the reliability values but just to the computing time. For this commodity, the computing time is of 0.768069 second for the new hybrid algorithm, while is equal to 99.815256 seconds for the tie-set algorithm. As expected, more nodes, and links in the network result a larger relative improvement in computing time. As an example for the graph transformation before applying the second stage in the new algorithm, the network simplification for the commodity  $n_s = 1, n_d = 2$ , is given below:

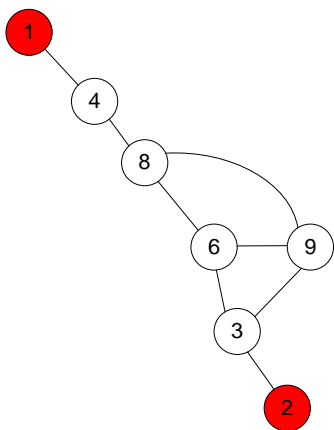
1. Removing parallel links



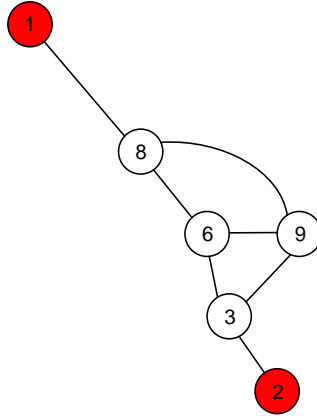
2. Removing series nodes.



3. Edge-factoring simplification results



4. Application the series reduction yields a 6-node network ready to be evaluated through tie-set sub-algorithm stage:

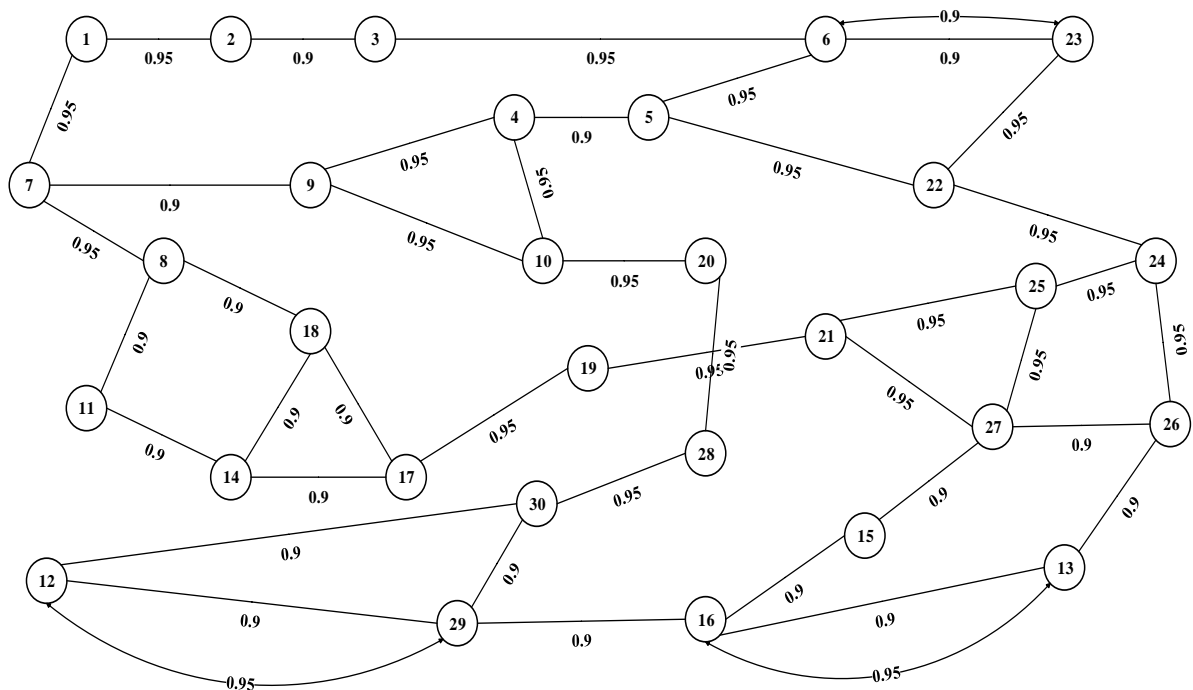


**Table 4.** 10-nodes network results

S-D		New Hybrid Algorithm		Classical tie-set Algorithm		S-D		New Hybrid Algorithm		Classical tie-set Algorithm	
$n_s$	$n_d$	$R_{sd}$	$T(sec)$	$R_{sd}$	$T(sec)$	$n_s$	$n_d$	$R_{sd}$	$T(sec)$	$R_{sd}$	$T(sec)$
1	2	0.8613	0.768069	0.8613	99.815256	3	10	0.9882	0.230928	0.9882	38.704346
1	3	0.8699	0.097397	0.8699	93.408054	4	5	0.9883	0.228849	0.9883	64.206468
1	4	0.9000	0.104673	0.9000	25.474119	4	6	0.9772	0.229305	0.9772	73.490794
1	5	0.8895	0.360740	0.8895	21.339873	4	7	0.9883	0.231325	0.9883	79.612763
1	6	0.8795	0.193422	0.8795	21.797562	4	8	0.9785	0.115998	0.9785	77.823526
1	7	0.8895	0.184674	0.8895	21.181596	4	9	0.9754	0.233591	0.9754	80.233674
1	8	0.8806	0.094613	0.8806	20.443827	4	10	0.9747	0.235277	0.9747	72.400296
1	9	0.8779	0.185500	0.8779	23.058189	5	6	0.9870	0.197984	0.9870	48.121599
1	10	0.8772	0.133696	0.8772	22.570570	5	7	0.9964	0.259024	0.9964	48.222930
2	3	0.9900	0.308988	0.9900	12.060503	5	8	0.9883	0.248405	0.9883	48.033562
2	4	0.9569	0.108141	0.9569	67.909281	5	9	0.9852	0.199140	0.9852	48.869167
2	5	0.9665	0.188774	0.9665	14.115008	5	10	0.9845	0.268268	0.9845	43.940942
2	6	0.9790	0.188425	0.9790	12.139499	6	7	0.9870	0.207302	0.9870	35.831310
2	7	0.9665	0.195201	0.9665	13.287393	6	8	0.9987	0.240635	0.9987	36.243997
2	8	0.9780	0.102962	0.9780	12.011086	6	9	0.9978	0.252932	0.9978	35.751226
2	9	0.9790	0.187281	0.9790	11.757940	6	10	0.9969	0.224477	0.9969	35.888998
2	10	0.9783	0.134570	0.9783	11.887915	7	8	0.9883	0.255088	0.9883	43.570795
3	4	0.9666	0.096956	0.9666	91.303666	7	9	0.9852	0.214799	0.9852	43.715735
3	5	0.9763	0.192700	0.9763	42.730678	7	10	0.9845	0.197735	0.9845	43.743550
3	6	0.9889	0.184044	0.9889	39.155001	8	9	0.9969	0.234009	0.9969	26.738554
3	7	0.9763	0.179979	0.9763	40.585935	8	10	0.9969	0.243761	0.9969	26.605802
3	8	0.9879	0.121310	0.9879	38.646236	9	10	0.9987	0.234044	0.9987	28.204465
3	9	0.9889	0.237610	0.9889	38.564093						

### 4.2.3. 30-Node Complicated Network

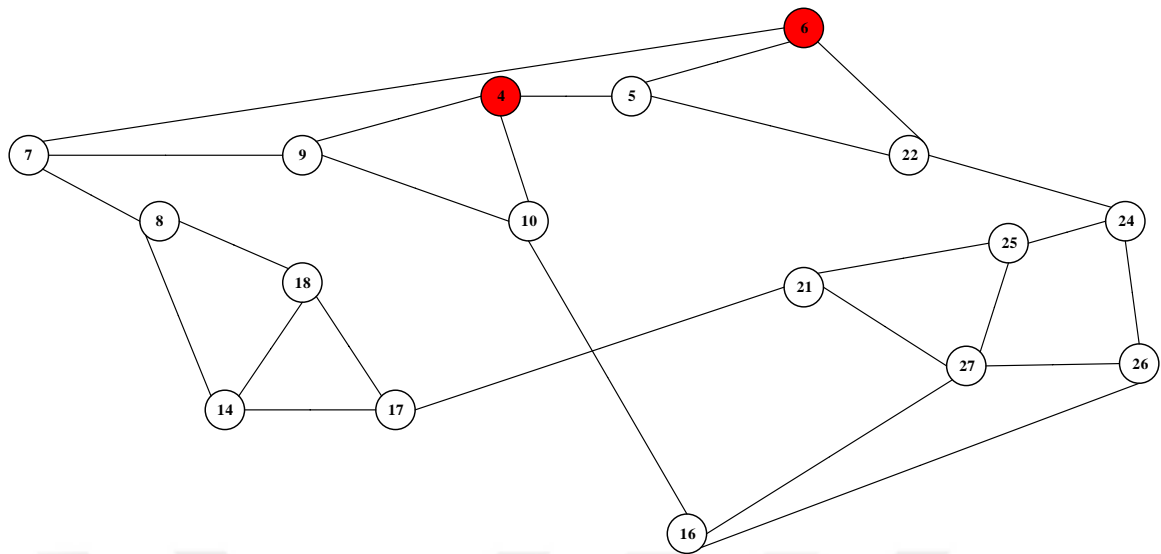
A case study of 30 nodes with 41 links random generated network is given for simulation as shown in figure 20. The application of the tie-set algorithm is practically impossible within acceptable time due to the large number of tie sets generated between any pair of nodes (commodity). The simulation is restricted to the case of the new hybrid algorithm application where a new output is collected concerning the number of tie sets for the simplified networks (one network for one commodity). Because of the large set of data, only a random sample is presented in the Table 5. This case study demonstrates the importance of the developed hybrid algorithm as the application of the classical reliability algorithm demonstrates that it is not productive. The application of the new algorithm gives indication of the efficiency of the proposed method by evaluating network reliability with minimum time. The computing time depends on the initial topology, simplification, simplified topology, and especially on the number of tie-set in the second stage (after simplification). The computing time can be very small as in the case of the commodity (6, 23) because after simplification the number of tie-set is only 5. A greater computing time is found if the number of the remaining tie-set is as high as in the commodity (1, 26) where there are still 15 pegs after simplification.



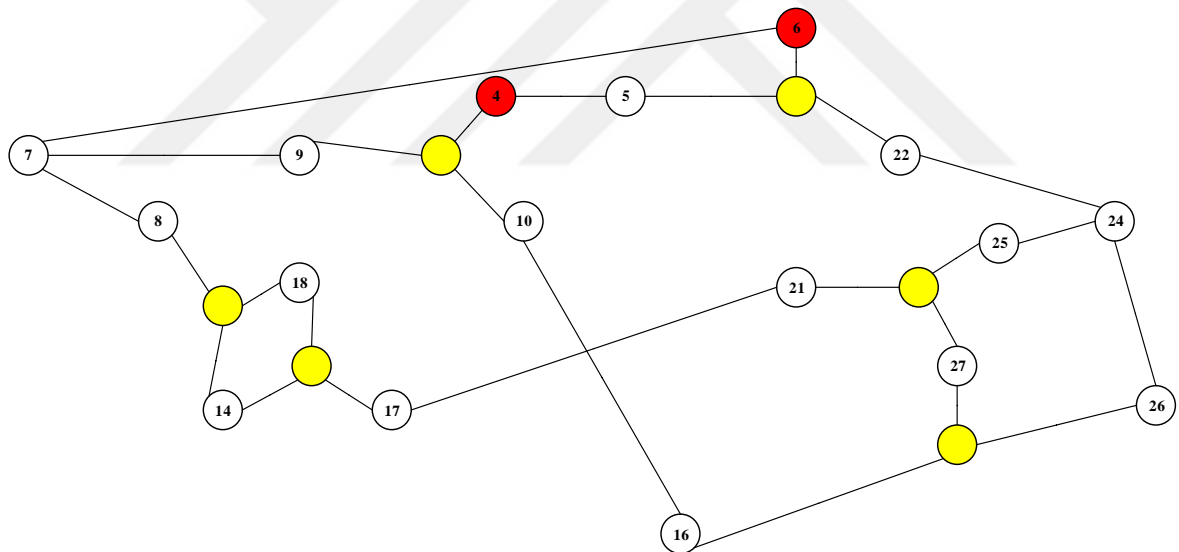
**Figure 20.** 30-Node complicated network



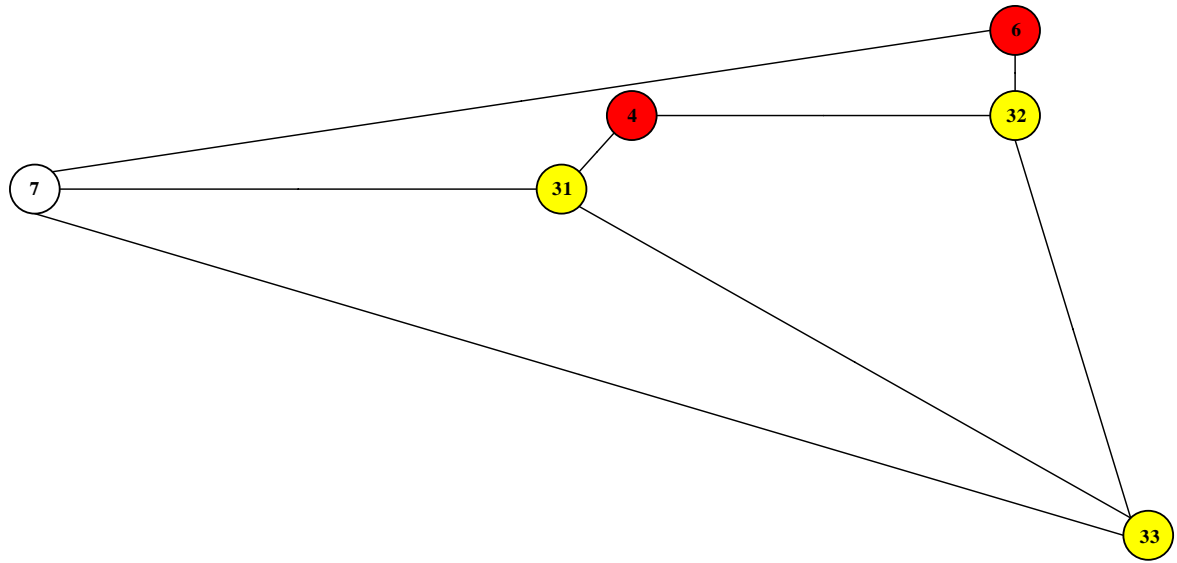
3. Application of parallel and then series reduction a second time



4. Delta-star transformation



5. Application of parallel and then series reduction a third time. The resulting simplified network is considered as applicable with the second stage because it has only 6 nodes



**Table 5.** Results of 30 nodes network

$n_s$	$n_d$	$R_{n_s n_d}$	Time (Sec)	Tie-set	$n_s$	$n_d$	$R_{n_s n_d}$	Time (Sec)	Tie-set
1	2	0.989351	0.1802660	5	12	18	0.968141	0.0486501	4
1	3	0.981798	0.0696030	5	12	19	0.971260	0.0402143	4
1	4	0.983871	0.1035941	7	12	20	0.979852	0.0280179	1
1	5	0.985126	0.0876691	7	12	21	0.978298	0.0407773	4
1	6	0.985072	0.0480382	5	12	22	0.977206	0.0400557	4
1	7	0.989351	0.0376265	5	12	23	0.976114	47.789545	15
1	8	0.981644	0.0860336	7	12	24	0.978702	0.0431515	4
1	9	0.983969	0.0867641	7	12	25	0.978621	0.2811640	8
1	10	0.983482	0.1515736	8	12	26	0.978525	0.0460728	4
1	26	0.983204	33.153783	15	12	27	0.978758	0.0473633	4
1	27	0.983721	35.237370	15	13	11	0.972705	42.914238	15
1	28	0.967399	0.1502042	8	13	12	0.979823	0.0409728	4
1	29	0.966287	0.1466714	8	13	14	0.981050	42.596097	15
1	30	0.966022	0.1466310	8	13	15	0.986867	0.0974111	7
2	1	0.989351	0.0475082	5	13	16	0.999256	0.0435497	5
2	3	0.983074	0.0477479	5	13	17	0.982756	37.947746	15
2	4	0.979489	0.0869360	7	13	18	0.981911	42.766520	15
2	5	0.981575	0.0870214	7	13	19	0.985877	37.676562	15
2	6	0.981798	0.0478363	5	13	20	0.981132	0.0981140	7
2	7	0.983273	0.0376092	5	13	21	0.993667	37.400776	15
2	19	0.974345	0.0867407	7	13	27	0.994176	0.0541688	6
2	20	0.968907	0.1461318	8	13	28	0.978064	0.0968297	7

2	21	0.979287	0.0872774	7	13	29	0.980330	0.0975239	7
2	22	0.981611	0.0876060	7	13	30	0.979485	0.0971218	7
2	23	0.981410	0.0889902	7	14	1	0.978258	0.0377314	4
2	24	0.979749	0.0882927	7	14	2	0.972979	0.0373120	4
2	25	0.979521	14.028114	14	14	3	0.976279	0.0372194	4
3	1	0.981798	0.0479437	5	14	4	0.983969	0.2344924	8
3	2	0.983074	0.0479234	5	14	5	0.985037	0.2421094	8
3	4	0.984730	0.0877142	7	14	6	0.984782	0.0385722	4
3	5	0.988556	0.0884793	7	14	7	0.988076	0.0257063	1
3	6	0.989351	0.0489902	5	14	18	0.996798	0.0335623	4
3	16	0.980796	0.1399841	8	14	19	0.988902	0.0270454	1
3	17	0.977528	0.0867380	7	14	20	0.973890	0.0383234	4
3	18	0.977165	0.0339284	4	14	21	0.986337	0.0260796	1
3	19	0.978924	0.0862366	7	14	22	0.985283	0.0368171	4
3	20	0.974151	0.1456662	8	14	23	0.984270	16.557891	14
3	21	0.984962	0.0865660	7	14	24	0.986128	0.0369575	4
3	22	0.988567	0.0872957	7	15	13	0.986867	0.0967008	7
3	23	0.988771	0.0883396	7	15	14	0.974069	18.394826	14
3	24	0.985566	0.0875019	7	15	16	0.987374	0.0525529	5
3	29	0.967529	0.1699633	8	15	17	0.975784	17.740353	14
3	30	0.967234	0.1692054	8	15	18	0.974923	19.954758	14
4	1	0.983871	0.1000319	7	15	19	0.978984	17.591616	14
4	2	0.979489	0.098546	7	15	20	0.972960	0.114250	7
4	3	0.984730	0.099414	7	16	27	0.994611	0.093068	7
4	5	0.994784	0.055532	5	16	28	0.978588	0.025703	1
4	6	0.994230	0.100548	7	16	29	0.980931	0.025369	1
4	7	0.992806	0.054967	6	16	30	0.980073	0.025337	1
4	8	0.986735	0.166071	8	17	1	0.979213	0.093374	7
4	9	0.999255	0.101194	7	17	2	0.974025	0.092767	7
4	10	0.998988	0.100190	7	17	3	0.977528	0.090771	7
4	18	0.984861	0.223569	8	17	4	0.985325	0.154368	8
4	19	0.986808	0.166328	8	17	5	0.986494	0.157767	8
4	20	0.987547	0.100231	7	18	1	0.979173	0.038173	4
4	21	0.992968	0.169008	8	18	2	0.973880	0.038332	4
4	22	0.994689	0.103915	7	18	3	0.977165	0.038137	4
4	23	0.993894	0.177798	8	18	4	0.984861	0.238280	8
4	24	0.993475	0.102503	7	18	27	0.987149	0.039615	4
4	25	0.993259	38.45633	15	18	28	0.969300	0.036634	4
4	26	0.992844	14.55406	14	18	29	0.968621	0.036734	4
4	27	0.993320	36.36405	15	18	30	0.968281	0.036733	4
4	28	0.980620	0.091441	7	19	1	0.979014	0.092841	7
4	29	0.978245	0.091162	7	19	2	0.974345	0.092915	7

4	30	0.978189	0.089028	7	19	3	0.978924	0.092480	7
5	1	0.985126	0.088593	7	19	4	0.986808	0.156675	8
5	2	0.981575	0.088576	7	20	26	0.983419	0.095332	7
5	3	0.988556	0.088469	7	20	27	0.983815	0.094163	7
5	4	0.994784	0.048699	5	20	28	0.988335	0.026113	1
5	6	0.998922	0.088628	7	20	29	0.980281	0.025312	1
5	7	0.993237	0.048408	6	20	30	0.981189	0.025461	1
5	8	0.987622	0.148851	8	21	1	0.983458	0.093185	7
5	9	0.994633	0.089459	7	21	2	0.979287	0.092672	7
5	18	0.985922	0.193719	8	21	3	0.984962	0.092353	7
5	19	0.988440	0.149337	8	21	27	0.999391	0.094128	7
5	20	0.984087	0.088547	7	21	28	0.978625	0.031896	4
5	21	0.995070	0.149327	8	21	29	0.978789	0.031901	4
5	22	0.999299	0.089138	7	21	30	0.978302	0.031886	4
5	23	0.998477	0.050156	6	22	1	0.985174	0.092815	7
5	24	0.995747	0.089565	7	22	2	0.981611	0.093830	7
5	25	0.995389	35.48666	15	22	3	0.988567	0.093765	7
5	26	0.994975	14.55950	14	22	4	0.994689	0.092913	7
5	27	0.995442	36.57019	15	22	5	0.999299	0.092694	7
5	28	0.978371	0.089877	7	22	6	0.998923	0.091889	7
5	29	0.977444	0.090531	7	22	29	0.977689	0.032895	4
5	30	0.977142	0.090755	7	22	30	0.977377	0.032857	4
6	1	0.985072	0.049892	5	23	1	0.984778	0.097540	7
6	2	0.981798	0.048898	5	23	2	0.981410	0.095165	7
6	3	0.989351	0.048907	5	23	3	0.988771	0.094726	7
6	4	0.994230	0.089271	7	23	4	0.993894	0.157507	8
6	5	0.998922	0.089880	7	23	5	0.998477	0.053489	6
6	23	0.999326	0.040069	5	24	14	0.986128	0.032936	4
6	24	0.995350	0.089047	7	24	15	0.986991	0.156029	8
6	25	0.994991	14.509826	14	24	16	0.994468	0.031285	4
6	26	0.994576	36.753214	15	24	17	0.987854	0.031297	4
6	27	0.995044	37.820807	15	24	18	0.986993	0.034048	4
6	28	0.977888	0.159542	8	24	19	0.991042	0.031336	4
6	29	0.976992	0.158985	8	24	20	0.983908	0.031432	4
6	30	0.976685	0.157924	8	24	21	0.998923	0.031239	4
7	1	0.989351	0.039289	5	24	22	0.996138	0.025127	1
7	2	0.983273	0.039338	5	24	23	0.995016	0.094193	7
7	3	0.985072	0.039245	5	24	25	0.999317	0.038969	5
7	4	0.992806	0.049903	6	24	26	0.998943	0.039867	5
7	5	0.993237	0.049811	6	24	27	0.999355	0.050744	6
7	6	0.992907	0.039407	5	24	28	0.979063	0.031667	4
7	8	0.991725	0.029309	1	24	29	0.979193	0.031943	4

7	9	0.992960	0.040733	5	24	30	0.978711	0.031957	4
7	10	0.992671	0.031272	4	25	1	0.983666	14.843653	14
7	11	0.980411	0.026207	1	25	2	0.979521	15.404780	14
7	12	0.974605	0.033727	4	25	3	0.985251	15.498565	14
7	13	0.987564	37.55220	15	25	4	0.993259	37.630363	15
7	14	0.988076	0.025860	1	25	5	0.995389	37.501486	15
7	15	0.980395	14.69624	14	25	6	0.994991	15.393548	14
7	16	0.988014	0.031691	4	25	7	0.992454	0.099662	7
7	17	0.988968	0.025528	1	26	12	0.978525	0.034530	4
8	11	0.987647	0.026334	1	26	27	0.998986	0.092424	7
8	12	0.969611	0.034590	4	26	28	0.978705	0.092306	7
8	13	0.983152	36.27322	15	26	29	0.978992	0.092004	7
8	14	0.994460	0.025830	1	26	30	0.978483	0.092985	7
8	15	0.976116	14.71384	14	27	1	0.983721	36.940406	15
8	16	0.983599	0.032745	4	27	2	0.979575	37.113121	15
8	17	0.994350	0.026787	1	27	3	0.985304	36.579763	15
8	18	0.995479	0.027184	1	27	4	0.993320	37.719180	15
8	19	0.988968	0.026875	1	27	5	0.995442	37.471046	15
8	20	0.976514	0.033941	4	27	28	0.979025	0.089595	7
8	21	0.988275	0.026506	1	27	29	0.979222	0.089934	7
8	22	0.987848	0.033025	4	27	30	0.978729	0.090917	7
8	23	0.986854	15.31409	14	28	1	0.967399	0.150038	8
8	24	0.988221	0.034240	4	28	2	0.963164	0.150205	8
8	25	0.988230	0.099787	7	28	3	0.968471	0.151542	8
8	26	0.987648	0.162727	8	28	4	0.980620	0.090406	7
8	27	0.988282	0.097062	7	28	5	0.978371	0.092136	7
8	28	0.970917	0.032937	4	28	6	0.977888	0.155173	8
8	29	0.970091	0.0328104	4	28	7	0.976294	0.031303	4
8	30	0.969775	0.0328147	4	29	7	0.975084	0.033045	4
9	1	0.983969	0.0963685	7	29	8	0.970091	0.033532	4
9	2	0.979529	0.0933738	7	29	9	0.978191	0.095734	7
9	3	0.984654	0.0968429	7	29	10	0.978554	0.026475	1
9	4	0.999255	0.1003910	7	29	11	0.959652	0.035938	4
9	5	0.994633	0.1051768	7	29	12	0.999396	0.026115	1
9	6	0.994096	0.1095133	7	29	13	0.980330	0.099105	7
9	7	0.992960	0.0469617	5	29	14	0.967760	0.035272	4
9	21	0.992905	0.0924743	7	29	15	0.970334	0.099499	7
9	22	0.994556	0.1567738	8	29	16	0.980931	0.027030	1
9	23	0.993667	37.779719	15	29	17	0.969305	0.034006	4
9	24	0.993403	0.1576910	8	29	18	0.968621	0.034995	4
9	25	0.993192	37.267066	15	29	19	0.971744	0.032165	4
9	26	0.992777	37.944023	15	30	5	0.977142	0.090616	7

9	27	0.993254	15.090757	14	30	6	0.976685	0.152644	8
9	28	0.980571	0.0973672	7	30	7	0.974832	0.030737	4
9	29	0.978191	0.0950044	7	30	8	0.969775	0.030682	4
9	30	0.978136	0.0948777	7	30	9	0.978136	0.092154	7
10	1	0.983482	0.1558551	8	30	10	0.978542	0.026228	1
10	2	0.979084	0.1551419	8	30	11	0.959327	0.035389	4
10	13	0.988697	0.0935217	7	30	12	0.996888	0.025703	1
10	14	0.983806	0.0332756	4	30	13	0.979485	0.096590	7
10	15	0.981290	0.0930954	7	30	14	0.967420	0.034172	4
10	16	0.989103	0.0258860	1	30	15	0.969667	0.095463	7
11	9	0.976043	0.0375651	4	30	16	0.980073	0.025625	1
11	10	0.975759	0.0351128	4	30	17	0.968950	0.032277	4
11	12	0.959177	0.0415392	4	30	18	0.968281	0.034050	4
11	13	0.972705	41.685692	15	30	19	0.971324	0.031206	4
11	14	0.987305	0.0286339	2	30	20	0.981189	0.025491	1
11	15	0.965764	16.787844	14	30	21	0.978302	0.032625	4
11	16	0.973147	0.0369037	4	30	22	0.977377	0.031409	4
11	17	0.985779	0.0256733	1	30	23	0.976288	36.72074	15
11	18	0.986216	0.0317816	4	30	24	0.978711	0.032666	4
11	19	0.979499	0.025630	1	30	25	0.978623	0.160734	8
11	20	0.965865	0.036666	4	30	26	0.978483	0.098598	7
11	21	0.977862	0.025761	1	30	27	0.978729	0.097103	7
11	22	0.977121	0.036855	4	30	28	0.988335	0.026682	1
11	23	0.976127	18.96898	14	30	29	0.997241	0.026301	1

### 4.3. Discussion

In all previous cases, and for every commodity, the same reliability values are found regardless of the technique used for evaluation. The difference is on the computing time required for the evaluation which depends on the nodes number, link number, and the topology complexity.

A shorter computing time is found when applying the hybrid algorithm to resolve the problem of the two-terminal reliability evaluation. The improvement in computing time is clearer when the network becomes more complicated. If the number of the tie-set generated from the original network (before transformation) is high, classical method as tie-set or cut-set will fail to find the reliability value. The proposed algorithm is applicable with short time to all kinds of algorithm ranging from simple to complex without any limitation in terms of nodes and

links number. Some points are outlined from the simulation of the three random networks:

- In Example 1, a simple network is simulated where both the hybrid and tie-set algorithms were successful in evaluating the reliability of all pairs. However, the computing time of the proposed algorithm shows an improvement that can be as high as 300% compared to the classical tie-set algorithm.

- In example 2, a medium network is simulated and more improvement is observed in terms of computing time, which means that more complicated network leads to more computing time differences between the classical and the proposed algorithm.

- In example 3, the network is very complicated with large nodes and links number making the application of the tie-set alone impossible. Data collected from the application of the proposed hybrid algorithm shows that this algorithm is not be capable within this range of time for (2 sec or 5 sec) to evaluating reliability for this complicated algorithm with acceptable computing time.

- The proposed hybrid algorithm can be used to improve the required computing time for reliability evaluation compared to classical algorithms for small to medium networks. Also, it can be applied for complex networks where the classical algorithms fail to find the solution.

- Finally, the tie-set method is used in this research in two ways, the first as second stage of the hybrid algorithm and also as comparison algorithm representing classical algorithms class. No restriction of using other algorithms as cut-set, enumeration, or even approximated technique at the place of tie-set algorithm.

# CHAPTER FIVE

## CONCLUSION AND FUTURE WORK

### 5.1. Conclusion

The difficulty of network reliability evaluation depends directly on the complexity and size. Classical evaluation algorithms like tie-set give exact solution but can require long computing time for complex network. In this work, a hybrid technique for reliability evaluation is proposed. It is based on combined successive procedures using graph transformation techniques and the classical tie-set algorithm as a final evaluation stage. Graph transformation starts by simplifying the topology by a structured and sequential application of parallel, series, edge factoring, and delta-to-star techniques. Network topology is transformed to a simpler one regardless of complexity of the original network. Finally, an exact solution is found by the application of the tie-set algorithm to the simplified topology. As a final stage of calculation, any classical exact method can be used as part of this hybrid algorithm at the place of tie-set algorithm. Using the proposed hybrid algorithm, three case studies of randomly generated networks of 6-node, 10-node, and 30-node, are simulated and compared to the results collected from the application of classical tie-set algorithm on the same networks.

The correctness of the developed algorithm is validated by the reliability which are the same values resulting from the hybrid and the classical approved algorithm. The perfect match of the reliability values indicates the validity and correctness of the proposed algorithm.

The efficiency of the hybrid algorithm is approved by comparing the computing delay for all commodities in various networks resulting from the application of both classical tie-set and the proposed hybrid algorithm. Depending on the source-destination nodes, and the network topology, the computing time is decreased in the case of the proposed hybrid algorithm to small portions of that of the classical tie-set algorithm. As important result, the hybrid algorithm performs well of any kind of networks regardless its size and complexity while classical algorithms fail to find the solution in acceptable time. Thus the proposed

algorithm can be applied for real time reliability evaluation required in many applications.

## **5.2. Future Work**

- 1- Evaluate the possibility and the efficiency of applying the proposed algorithm for k-mean and all-terminal reliability evaluation problem.
- 2- The use of new techniques for simplifying the reliability evaluation problem such as clustering nodes into many subnetworks by using neural network or any other technique.



## REFERENCES

- Al-Muhaini, M., & Heydt, G. T. (2012, May). Minimal cut sets, Petri nets, and prime number encoding in distribution system reliability evaluation. In *PES T&D 2012* (pp. 1-8). IEEE.
- Anumaka, M., & Chukwukadibia, M. (2011). Fundamentals of Reliability of Electric Power System and Equipment. *Int. J. Eng. Sci. Technol.*, 3.
- Bai, G., Liu, T., Zhang, Y. A., & Tao, J. (2020). An Improved Method for Reliability Evaluation of Two-Terminal Multistate Networks Based on State Space Decomposition. *IEEE Transactions on Reliability*.
- Brecht, T. B., & Colbourn, C. J. (1988). Lower bounds on two-terminal network reliability. *Discrete Applied Mathematics*, 21(3), 185-198.
- Cancela, H., Robledo, F., Rubino, G., & Sartor, P. (2013). Monte Carlo estimation of diameter-constrained network reliability conditioned by path sets and cut sets. *Computer Communications*, 36(6), 611-620.
- Carrier, J., & Lucet, C. (1996). A decomposition algorithm for network reliability evaluation. *Discrete Applied Mathematics*, 65(1-3), 141-156.
- Caşcaval, P. (2018). Approximate method to evaluate reliability of complex networks. *Complexity*, 2018.
- Daibo, M. (2017, December). Toroidal vector-potential transformer. In 2017 Eleventh International Conference on Sensing Technology (ICST) (pp. 1-4). IEEE.
- Davila-Frias, A., & Yadav, O. P. (2020). All-terminal network reliability estimation using convolutional neural networks. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 1748006X20969465.
- Gadani, J. P. (1981). System effectiveness evaluation using star and delta transformations. *IEEE Transactions on Reliability*, 30(1), 43-47.
- Gaun, A., Rechberger, G., Renner, H., & Lehtonen, M. (2010, July). Enumeration based reliability assessment algorithm considering nodal uncertainties. In *IEEE PES General Meeting* (pp. 1-8). IEEE.
- Gebre, B. A., & Ramirez-Marquez, J. E. (2007). Element substitution algorithm for general two-terminal network reliability analyses. *IIE transactions*, 39(3), 265-275.
- Guimaraes, A. P., Oliveira, H. M. N., Barros, R., & Maciel, P. R. (2011, May). Availability analysis of redundant computer networks: A strategy based on

- reliability importance. In *2011 IEEE 3rd International Conference on Communication Software and*
- Gupta, H., & Sharma, J. (1978). A delta-star transformation approach for reliability evaluation. *IEEE Transactions on Reliability*, 27(3), 212-214.
- Hayashi, M. (2012, May). Approximation method for evaluating frequency of long service outages in telecommunications networks. In *2012 IEEE International Workshop Technical Committee on Communications Quality and Reliability (CQR)* (pp. 1-5). IEEE
- Hui, K. P. (2005). *Network reliability estimation* (Doctoral dissertation).
- Jeyaraj, J. P., & Haenggi, M. (2017, December). Reliability analysis of V2V communications on orthogonal street systems. In *GLOBECOM 2017-2017 IEEE Global Communications Conference* (pp. 1-6). IEEE.
- Jin, W., Yu, P., Xiong, A., Zhang, Q., Jin, D., Zhang, G., & Wang, Y. (2018, April). An approximate all-terminal reliability evaluation method for large-scale smart grid communication systems. In *NOMS 2018-2018 IEEE/IFIP Network Operations and Management Symposium* (pp. 1-5). IEEE.
- Karger, D. R., & Tai, R. P. (1997, January). Implementing a fully polynomial time approximation scheme for all terminal network reliability. In *SODA* (Vol. 97, pp. 334-343).
- Konak, A. (2007, December). Combining network reductions and simulation to estimate network reliability. In *2007 Winter Simulation Conference* (pp. 2301-2305). IEEE.
- Lamalem, Y., & Housni, K. (2020, September). New and efficient method to find all minimal paths. In *2020 3rd International Conference on Advanced Communication Technologies and Networking (CommNet)* (pp. 1-4). IEEE.
- Li, H., & Zhao, Q. (2005, June). A cut/tie set method for reliability evaluation of control systems. In *Proceedings of the 2005, American Control Conference, 2005.* (pp. 1048-1053). IEEE.
- Mahmood, M. K., & Myderrizi, I. (2020, July). Reliability Evaluation Using a Clustering Technique Based on Tie-set Method. In *2020 43rd International Conference on Telecommunications and Signal Processing (TSP)* (pp. 139-142). IEEE.
- Mahmood, M. K., Abdulla, L. S., & Al-Naima, F. M. (2014). Hybrid Algorithm for Complex Communication Networks Reliability Evaluation.
- Mahmood, M. K., Al-Naima, F. M. M., & Zaidan, Z. (2018). Reliability Assessment of the Iraqi National Communication Network. *Indonesian Journal of Electrical Engineering and Informatics (IJEI)*, 6(4), 448-457.
- Mahmood, M. K., Al-Naima, F. M., & Abdulla, L. S. (2015). An efficient multi-stages algorithm for the determination of communication network

- reliability. *International Journal of Computers and Communications*, 9, 36-43.
- Mahmood, M. K., Ucan, O., Zaidan, Z., & Karim, S. M. (2021, February). Hybrid algorithm for two-terminal reliability evaluation in communication networks. In *2021 Indonesian Journal of Electrical Engineering and Computer Science (ISSN)* (pp.1185-1192).
- Mo, Y., Liang, M., Xing, L., Liao, J., & Liu, X. (2020). Network Simplification and K-Terminal Reliability Evaluation of Sensor-Cloud Systems. *IEEE Access*, 8, 177206-177218.
- Murray, K., Kershenbaum, A., & Shooman, M. L. (1993, January). Communications network reliability analysis approximations and bounds. In *Annual Reliability and Maintainability Symposium 1993 Proceedings* (pp. 268-275). IEEE.
- Paredes, R., Dueñas-Osorio, L., Meel, K. S., & Vardi, M. Y. (2019). Principled network reliability approximation: A counting-based approach. *Reliability Engineering & System Safety*, 191, 106472.
- Park, J. H. (2015). All-terminal reliability analysis of wireless networks of redundant radio modules. *IEEE Internet of Things Journal*, 3(2), 219-230.
- Rebaiaia, M. L., Ait-Kadi, D., & Merlano, A. (2009). A practical algorithm for network reliability evaluation based on the factoring theorem-a case study of a generic radiocommunication system. *品質學報*, 16(5), 323-336.
- Saxena, A., Prasad, M., Gupta, A., Bharill, N., Patel, O. P., Tiwari, A., ... & Lin, C. T. (2017). A review of clustering techniques and developments. *Neurocomputing*, 267, 664-681.
- Sharafat, A. R., & Ma'rouzi, O. R. (2009). All-terminal network reliability using recursive truncation algorithm. *IEEE Transactions on Reliability*, 58(2), 338-347.
- Shooman, A. M., & Kershenbaum, A. (1991, December). Exact graph-reduction algorithms for network reliability analysis. In *IEEE Global Telecommunications Conference GLOBECOM'91: Countdown to the New Millennium. Conference Record* (pp. 1412-1420). IEEE.
- Shooman, A. M., & Kershenbaum, A. (1992, January). Methods for communication-network reliability analysis: probabilistic graph reduction. In *Annual Reliability and Maintainability Symposium 1992 Proceedings* (pp. 441-448). IEEE.
- Shooman, M. L. (2002). *Reliability of computer systems and networks*. John Wiley & Sons, Incorporated.
- Soh, S., & Rai, S. (2005). An efficient cutset approach for evaluating communication-network reliability with heterogeneous link-capacities. *IEEE Transactions on Reliability*, 54(1), 133-144.

- Wang, X. D., Wang, J. C., Ma, T. F., Wang, H. X., & Hao, Z. J. (2019, August). Operational Reliability Evaluation Method of Production Systems Based on Multistate Production Network. In *2019 International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE)* (pp. 768-773). IEEE.
- Won, J. M., & Karray, F. (2011). A greedy algorithm for faster feasibility evaluation of all-terminal-reliable networks. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *41*(6), 1600-1611.
- Yeh, F. M., Lu, S. K., & Kuo, S. Y. (2002). OBDD-based evaluation of k-terminal network reliability. *IEEE Transactions on Reliability*, *51*(4), 443-451.
- Zhang, H., Huang, N., & Liu, H. (2014, January). Network performance reliability evaluation based on network reduction. In *2014 Reliability and Maintainability Symposium* (pp. 1-6). IEEE.
- Zhu, P., Han, J., Guo, Y., & Lombardi, F. (2016). Reliability and criticality analysis of communication networks by stochastic computation. *IEEE Network*, *30*(6), 70-76.

# RESUME

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Master	Electrical-Electronic engineering	2021
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## Publications

- 1- Combination of Graph Reduction and Tie-set Techniques for Network Reliability Assessment*

