MONETARY POLICY EFFECTIVENESS DURING PANDEMICS

GRADUATE SCHOOL OF SOCIAL SCIENCES

TOBB UNIVERSITY OF ECONOMICS AND TECHNOLOGY

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MASTER OF SCIENCE

AUGUST 2021

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I hereby declare that all information in this document has been obtained and presented in accordance with academic traditions and the rules of ethical conduct. I also declare that, I have fully cited and referenced all material and results that are not original to this work, as required by these traditions and the rules of ethical conduct.

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ABSTRACT

MONETARY POLICY EFFECTIVENESS DURING PANDEMICS

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This thesis uses a calibrated New-Keynesian model to compare different monetary policy rules' effectiveness during in the context of the COVID-19 outbreak. It uses Taylor-type monetary policy rules which differ in degree of monetary policy inertia. It is found that both price stickiness and monetary policy sluggishness increase the loss of the monetary authority. The important point is that their effect is amplified when these two frictions are employed at once. We also find that a very high policy rate inertia in the central bank's policy function leads very significant losses in its loss function even if the central bank has the interest rate smoothing incentive. Hence, we conclude that when the economy experiences sudden turbulences like in the case of the pandemic, it gets even more important to actively respond to economic conditions.

Keywords: Pandemic, Taylor Rules, Price Stickiness, Inertia

ÖZ

PANDEMİ DÖNEMİNDE PARA POLİTİKASI ETKİNLİĞİ

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Bu tez, COVID-19 salgını sırasında farklı para politikası kurallarının etkinliğini karşılaştırmak için kalibre edilmiş bir New-Keynesyen model kullanır. Tezde para politikasının atalet derecesi açısından farklılık gösteren Taylor tipi para politikası kuralları kullanılmaktadır. Hem fiyat yapışkanlığının hem de para politikasındaki durgunluğun merkez bankasının zararını arttırdığı tespit edilmiştir. Önemli olan nokta ise bahsedilen iki sürtünmenin aynı anda uygulandığında etkilerinin artmasıdır. Merkez bankasının politika fonksiyonundaki çok yüksek politika faiz ataleti, merkez bankası faiz oranı yumuşatma teşvikine sahip olsa bile, kayıp fonksiyonunda çok önemli kayıplara yol açtığı da bulunmuştur. Dolayısıyla, pandemi durumunda olduğu gibi ekonomide ani sallantılar yaşandığı zaman, ekonomik koşullara aktif olarak yanıt vermenin daha da önemli hale geldiği sonucuna varılmaktadır.

Anahtar Kelimeler: Pandemi, Taylor Kuralları, Fiyat Yapışkanlığı, Atalet

To my family, friends and cat

ACKNOWLEDGMENTS

Primarily, I would like to express my deepest gratitude to my supervisor, Assoc. Prof. Ozan EKŞİ for his invaluable advices, support, inspiration and patience at all stages of my thesis.

I offer my sincere graditude to the members of my thesis committee, Prof. Selahattin TOĞAY and Prof. Turalay KENÇ for my thesis defense and their constructive comments. My very special gratitude extends to the Department of Economics and the Graduate School of Social Sciences for their contribution and guideliness.

I would like to offer my deepest appreciate to my mother, father and sisters, Aybike and Ayça, for supporting and belief in me over time. Last, I thank my friends, Aybüke, Batuhan, Nesrin and Merve for their endless support and patience.

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ABBREVIATION LIST

ATUS	: American Time Use Survey
BoC	: Bank of Canada
BoJ	: Bank of Japan
CBRT	: Central Bank of the Republic of Turkey
CDS	: Credit Default Swap
CFR	: Case-Fatality Rate
COVID-19	: Coronavirus Disease-2019
DSGE	: Dynamic Stochastic General Equilibrium
ECB	: European Central Bank
EM	: Emerging Market
FED	: Federal Reserve System
FOMC	: Federal Open Market Committee
GDP	: Gross Domestic Product
LSAP	: Large Scale Asset Purchases
QE	: Quantitative Easing
RBI	: Reserve Bank of India
SIR	: Susceptible, Infectious and Recovered
US	: United States
WHO	: World Health Organization
ZLB	: Zero Lower Bound

CHAPTER I

INTRODUCTION

With the onset of the pandemic, a variety of policy measures have been taken to protect the public from the COVID-19 outbreak, such as the obligation to wear masks, social distancing restrictions and quarantines. These protective measures, especially quarantine policies, have worsened many markets and businesses. In consequence of the destructive impacts of the pandemic, the global economy contracted; economic activities have declined dramatically and unemployment rates have increased across many countries. To alleviate the effects of the COVID-19 outbreak, governments have implemented expansionary fiscal policies. Moreover, central banks have used both conventional and unconventional monetary tools. In advance economies, most central banks cut their policy rates to the zero lower bounds (ZLB). Moreover, these central banks have implemented asset purchasing programs: purchases of government securities (e.g. the Federal Reserve (Fed)), buying private assets (e.g. the European Central Bank (ECB) and Bank of Canada (BoC)) and purchasing subnational bonds (e.g. the Fed). Besides, liquidity provisions and credit supports are among the tools of the central banks (such as the ECB and Bank of Japan (BoJ)). The emerging market (EM) central banks also have lowered their rates (e.g. the Reserve Bank of India (RBI)), and have adopted asset purchasing programs (such as the Central Bank of the Republic of Turkey (CBRT)), liquidity provisions and credit support.

Even though different vaccines against the virus have developed, countries have not been able to return to their pre-pandemic economic situations. Moreover, the conduct and effectiveness of monetary policy, which has been heavily discussed since 2007-2008 global financial crises, continued to play a crucial role in improving the welfare of countries during pandemics.

The thesis aims to analyze the effects of two different Taylor-type rules on a central bank's objective function. We compare the optimality of these rules under varying degrees of the price stickiness and the spread of the pandemic in an economy. We use a New-Keynesian model where intermediate goods producers are monopolistically competitive.

This thesis borrows the New-Keynesian model from Eichenbaum et al. (2020), which extends the model of Kermack and McKendrick (1927). In the latter paper, the population members are categorized as three groups: susceptible, infected and recovered, and the paper aims to estimate the spread of the disease with calculating the change in the size of these groups of people over time. In Eichenbaum et al. (2020), the transmission of the virus is related to economic activities, as well. It demonstrates that agents face a trade-off between the recession and rate of dying. If people reduce their economic activities, which are consumption and working, the number of died people due to the virus declines. However, if people avoid to these activities, the decline in output increases. Hence, the model is appropriate to understand the economic dynamics during pandemics.

Initially, we look at the flexible price economy with and without the pandemic. Second, we compare the sticky price and flexible price versions of the model during the outbreak to show how price stickiness affects output and inflation. Then, we examine the effects of monetary policy on the central bank's loss function during the pandemic using Taylor rules which are the baseline (Taylor, 1993) rule and inertia rule. The motivation for using inertia rule is that it implies sluggish policy rates adjustment by central bank. Thus, we examine the welfare implications of the aggressive or non-aggressive policy reactions to changes in the macroeconomic variables (e.g. inflation or output) due to the pandemic. Indeed, we use two inertia rules that differ in the degree in terms of policy inertia to show the gradual monetary policy's effects in more detail. The United States (US) data are used to calibrate the model.

We find that the pandemic induces a large recession and the declines in prices. Nominal price rigidities increase the depth of the recession with quite small impact for all monetary policy rules. The change in prices is smoother relative to the scenario with flexible prices since the stickiness in prices creates a friction in the economy. For both scenarios with flexible prices and sticky prices, very sluggish partial adjustment of short term interest rate leads to larger deviations in inflation. More importantly, the inability of the monetary authority to react aggressively to changes in the economy further increases its losses, even though the central bank aims to smooth its policy rate.

We contribute to the literature as follows: This thesis shows that monetary policy sluggishness leads very important losses in the central bank's loss function even if the central bank has the interest rate smoothing incentive. It also highlights that it gets even more important to actively respond to changes in economic conditions during extreme times like the pandemic.

This thesis is organized as follows. The next subsection discusses the related literature. Chapter 2 presents the New-Keynesian model. In chapter 3, the model is calibrated. Chapter 4 provides the results of this thesis. Chapter 5 concludes. The appendix involves the derivation of the model and the equilibrium conditions.

1.1. Related Literature

This study is related to the emerging literature on monetary policy effectiveness during the coronavirus pandemic. Costa et al. (2020) find that expansionary monetary policy and increase in government expenditures are the most effective policies to mitigate the impacts of the pandemics on economic variables. Bianchi et al. (2020) propose a coordination policy among fiscal and monetary policies to alleviate the effects of pandemic. Wei and Han (2021) show that the transmission of monetary policy to financial markets has diminished and both unconventional and conventional monetary policy tools have not significantly affected the credit default swap (CDS), government bond, exchange rate and stock markets in the COVID-19 crisis as compared with the non-pandemic period. Caballero and Simsek (2020a) state that the financial market and the real economy are temporarily disconnected during the pandemic due to boosted asset prices by the central bank. Pellegrino et al. (2020) find that monetary policy which is aggressively implemented and focuses on output stabilization is more effective during the extreme times triggered by large uncertainty shocks when considering an anticipated uncertainty with different sizes in the third quarter of 2020 as a second wave of the pandemic. Also, many studies in this literature examine the effectiveness of unconventional monetary policy in the context of the pandemic.[1]

This study considers different Taylor rules. Taylor (1993) provides an interest rate rule to explain the policy rate decisions made by the Federal Open Market Committee (FOMC). According to this rule, the policy rate reacts to the deviations of inflation rate and real GDP from their targets. A number of papers examine different versions of the original Taylor rule (1993). One of the extentions covers the interest rate smoothing or monetary policy inertia (see e.g. Woodford (1999), Rudebusch (2002) and Rudebusch (2005)). [2] Another common extention covers the expected future values of variables (e.g. inflation) in the rule since central banks are forward-looking (such as Clarida et al. (1998)).[3] We are not interested in forward-looking type of rules but in the rules implying monetary policy inertia. The reason is that the pandemic implies sudden and unexpected changes in the economic conditions and the sluggishness of monetary policy forms a barrier in front of the regulatory role of the central banks.

Finally, since the price stickiness is at the core of New-Keynesian models, we examine the implications of our setting for a varying degrees of price stickiness, which is created in our model using the Calvo (1983) approach.

[1]Rebucci et al. (2020), Sims and Wu (2020), Caballero and Simsek (2020b) and Cortes et al.(2021) analyze the effects of LSAP or Quantitative Easing (QE) on macroeconomic variables. Most studies find that the effects of forward guidance are limited (see e.g. Levin and Sinha (2020), Lepetit and Fuentes-Albero (2020) and Coibion et al. (2020).

[2]It is also referred to as gradualism, or partial adjustment.

[3]There are also limitations which affect the performance of Taylor-type rules. One of the limitations is the measurment error for the variables (the equilibrium real interest rate and output gap) in policy rules (see e.g. Orphanides (2003), Rudebusch (2001) and Smets (2002)). This error arises since policymakers are not able to observe these variables and they estimate with uncertainty. The second limitation is zero lower bound which constrains monetary policy (see e.g. Reifschneider and Williams (2000), Belke and Klose (2013) and Bernanke et. al (2019)). Asso and Leeson (2012) explain the historical progress of monetary policy rules since 1776 in detailed and the strengths of Taylor rules.

CHAPTER II

MODEL

The baseline model is the New-Keynesian model of Eichenbaum et al. (2020). In the last period before the pandemics, all individuals are identical and the economy is in the steady state. Due to the pandemic, the family (or population) members are divided into four groups: susceptible, infected, recovered and deceased. At time zero, the fractions of infected, i, and susceptible, s, individuals within the family are given by:

$$i_0 = \varepsilon, \tag{1}$$

$$s_0 = 1 - \varepsilon, \tag{2}$$

that is, the total number of people is normalized to 1.

(1) and (2) imply that at time 0, whole population except infected people is susceptible to the virus. Additionally, r_t, d_t and τ_t are the fractions of recovered, deceased and newly infected people within the family at time t, respectively. The transmission function governing the fraction of the newly infected people:

$$\tau_t = \pi_1(s_t c_t^s)(i_t c_t^i) + \pi_2(s_t n_t^s)(i_t n_t^i) + \pi_3(s_t i_t), \tag{3}$$

where c_t^s and n_t^s denote the consumption and hours worked of a susceptible individual and c_t^i and n_t^i are the consumption and hours worked of an infected individual. Total consumption and hours worked of susceptible people are respectively reflected by the terms $s_t c_t^s$ and $s_t n_t^s$ whereas total consumption and hours worked of infected people are denoted by the terms $i_t c_t^i$ and $i_t n_t^i$, respectively.

According to (3), there are three channels through which the health situation of a susceptible person turns into being infected. First, susceptible people can interact with infected people in the consumption-related activities and the term $\pi_1(s_t c_t^s)(i_t c_t^i)$ in the transmission function denotes the fraction of the newly infected individuals due to the consumption-related activities. Another interaction between infected and susceptible people is the interection at work, given by the term $\pi_2(s_t n_t^s)(i_t n_t^i)$. The last term in the transmission fuction, $\pi_3(s_t i_t)$, shows the fraction of the newly infected individuals due to random meetings between infected and susceptible people that are not related with economic activities. π_1, π_2 and π_3 are the probabilities of being infected in consequence of consumption-related, work-related and other activities.

Susceptible individuals are inclined to decline their consumption and hours worked to reduce their probabilities of being infected, π_1 and π_2 (Note that there is always positive transmission of virus in the society due to non-economic activities, which is π_3.). So that, π_1 and π_2 arise as negative demand for consumption and labor supply shocks. These probabilites are known by agents.

At time t+1, the fraction of susceptible individuals within the family:

$$s_{t+1} = s_t - \tau_t, \tag{4}$$

and the fraction of infected individuals within the family:

$$i_{t+1} = i_t + \tau_t - (\pi_r + \pi_d)i_t, \tag{5}$$

where π_r and π_d denote the probability infected people recover and the probability infected people die from the virus, in order. As a result, the fractions of recovered and deceased people within the family are given by:

$$r_{t+1} = r_t + \pi_r i_t, \tag{6}$$

$$d_{t+1} = d_t + \pi_d i_t. \tag{7}$$

2.1. Households

A representative household's objective function:

$$U = E\sum_{n=0}^{\infty}\beta^t\left\{s_t\left[log(c_t^s) - \frac{\theta}{2}(n_t^s)^2\right] + i_t\left[log(c_t^i) - \frac{\theta}{2}(n_t^i)^2\right] + r_t\left[log(c_t^r) - \frac{\theta}{2}(n_t^r)^2\right]\right\}. \tag{8}$$

The budget constraint is given by:

$$B_{t+1} + P_t(s_t c_t^s + i_t c_t^i + r_t c_t^r + x_t) + \psi_t = R_{t-1}^b B_t + W_t(s_t n_t^s + i_t n_t^i + r_t n_t^r) + R_t^k k_t + \Phi_t, \tag{9}$$

Here, c_t^r and n_t^r represent the consumption and hours worked of a recovered person within the family at time t. The nominal bond holdings are denoted by B_t and x_t is the investment of a household. In the equation (9), ψ_t and Φ_t are the lump-sum taxes and the profits from the firms which are monopolistically competitive at time t. The consumer price index, the nominal wage rate, the interest rate on nominal bonds and the nominal rental price at time t are denoted by P_t, W_t, R_t^b and R_t^k, respectively. R_t^b is also the policy rate of the central bank. The capital accumulation:

$$k_{t+1} = x_t + (1-\delta)k_t. \tag{10}$$

Households maximize the objective function, (8), subject to the equations (9), (10), (3), (4), (5) and (6). The first-order conditions of the households' problem with respect to c_t^s, c_t^i, c_t^r,n_t^s, n_t^i, n_t^r, k_{t+1}, s_{t+1}, i_{t+1}, r_{t+1}, τ_t and B_{t+1} are given into the appendix.

2.2. Firms

There are two types of firms: a perfectly competitive final goods producer and monopolistically competitive intermediate goods producers. The firm producing final goods maximizes its profit:

$$\Pi = P_t y_t - P_{i,t} y_{i,t}, \tag{11}$$

where $y_t = \left(\int_0^1 y_{i,t}^{\frac{1}{\gamma}} \mathrm{d}i \right)^{\gamma}$, $\gamma > 1$. y_t and $y_{i,t}$ represent the final output and the quantity of input i which intermediate goods firms produce. P_t and $P_{i,t}$ are the prices of final output and intermediate input i in units of the final good, respectively. The first order condition for $y_{i,t}$:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\gamma}{\gamma-1}} y_t.$$

Then, the price of final output:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\gamma-1}} \mathrm{d}i \right)^{-(\gamma-1)}.$$

A monopolist intermediate firm i uses labor and capital of a household, $n_{i,t}$ and $k_{i,t}$, in accordance with the technology to produce intermediate good i.

$$y_{i,t} = A k_{i,t}^{1-\alpha} n_{i,t}^{\alpha}.$$

Monopolist firms maximize their profits:

$$\pi_{i,t} = P_{i,t} y_{i,t} - P_t mc_t y_{i,t}. \tag{12}$$

Here, the real marginal cost at time t is represented by mc_t. (12) is subject to the demand equation given by:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\gamma}{\gamma-1}} y_t. \tag{13}$$

According to Eichenbaum et al. (2020), intermediate producers face Calvo-price frictions, and they reoptimize their prices with the probability 1 - ξ. The monopolist firms maximize their profits with determining their optimal price, $\tilde{P}_t$, at time t:

$$\max_{\tilde{P}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j}^b \left(\tilde{P}_t y_{i,t+j} - P_{t+j} mc_{t+j} y_{i,t+j}\right), \tag{14}$$

subject to:

$$y_{i,t} = \left(\frac{\tilde{P}_t}{P_t}\right)^{-\frac{\gamma}{\gamma-1}} y_t. \tag{15}$$

Thus, the real marginal cost at time t, mc_t:

$$mc_t = \frac{W_t^{\alpha}(R_t^k)^{1-\alpha}}{P_t A \alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{16}$$

The real marginal cost depends on the real wage, $w_t = W_t/P_t$, the real rental rate, $r_t^k = R_t^k/P_t$, productivity, A, and the labor share, α.

2.3. Fiscal Policy

The government uses lump-sum taxes, ψ_t, to finance government spending, G, which is constant over time. The budget constraint of government is:

$$\psi_t = P_t G. \tag{17}$$

2.4. Monetary Policy and Taylor-Type Rules

The central bank aims to minimize output gap, the deviations of actual inflation from its target and the volatility in its policy rate. Its loss function (Rudebusch and Svensson (1999)) is given by:

$$E_t \sum_{n=0}^{\infty} \zeta^n \left\{(\pi_{t+n} - \pi)^2 + \upsilon_x (y_{t+n} - y_{t+n}^f)^2 + \upsilon_R (R_{t+n}^b - R_{t+n-1}^b)^2\right\}. \tag{18}$$

Here, ζ, $0 < \zeta < 1$, denotes the discount factor of the central bank. The coefficients of the central bank's preferences, υ_x and υ_R, are the weights on the output gap minimization and interest rate smoothing relative to inflation stabilization. The real interest rate is given by:

$$rr_t = \frac{R_t^b}{\pi_{t+1}}.$$

The central bank follows Taylor rule and uses the nominal interest rate as a policy rate. We examine two different Taylor rules. First, the baseline policy rule is in accordance with Taylor (1993) rule:

$$logR_t^b = logR^b + \theta_\pi log(\pi_t/\pi) + \theta_x log(y_t/y_t^f), \tag{19a}$$

where R^b is the policy rate in the equilibrium and the denominator terms, π and y_t^f, are the inflation target and the potential output, respectively. Also, θ_π and θ_x denote the response coefficients on deviations in inflation and output gap. The baseline rule implies that the policy rate of the central bank reacts by the movements in the inflation and output gap and at the steady state, the policy rate equals to its value in the equilibrium. The second Taylor-type rule (Sack (1998b)) is given by:

$$logR_t^b = (1-\theta_R)\left(logR^b + \theta_\pi log(\pi_t/\pi) + \theta_x log(y_t/y_t^f)\right) + \theta_R logR_{t-1}^b. \tag{19b}$$

According to (19b), the adjustment of the central bank would be sluggish which depends on the response coefficient θ_R. The parameter, θ_R, addresses the degree of monetary policy inertia and is within the range (0,1). The higher values of θ_R imply that the central bank responds to the movements in inflation and output gap gradually. Note that $\theta_R = 0$ means no interest rate smoothing.

2.5. Equilibrium

The goods market clearing condition is given by:

$$Ak_t^{1-\alpha}n_t^{\alpha} = c_t + x_t + g.$$

Here, c_t denotes the fraction of total consumption in the family and is given by:

$$c_t = s_t c_t^s + i_t c_t^i + r_t c_t^r.$$

The labor market clearing condition is given by:

$$s_t n_t^s + i_t n_t^i + r_t n_t^r = n_t,$$

where n_t is the fraction of total hours worked in the family. Also, (10) is one of the equilibrium conditions.

In equilibrium, nominal bonds are given by:

$$B_t = 0.$$

The appendix contains all equilibrium conditions.

CHAPTER III

CALIBRATION AND THE STEADY STATE OF THE MODEL

The model is calibrated at a weekly frequency to take changes in the health status of people due to the outbreak into account for the US. In Table 3.1, the marginal product of labor, α, is 2/3 and the weekly depreciation rate of capital, δ, is set to 0.06/52. In the pre-pandemic steady state, according to the Bureau of Labor Statistics 2019 ATUS, the representative household works at 39.5 hours on weekdays at its workplace. Besides, the weekly per capita income in 2019, y, is \$1,255.4. [4] So, we calculate that A and θ are 1.9236 and 0.00048, respectively. We choose to set the weekly utility discount rate, β, to $0.97^{1/52}$, thus the economic value of a life, VoL, would be \$8,916,459 which is within the range of the value of statistical life calculated by Kniesner et al. (2012). The gross price markup rate, γ, is set to 1.35 due to monopolistic competition of intermediate firms. The value of Calvo price stickiness is 0.98 (it is 0 in the flexible price economy). The rate of government spending to output in the pre-pandemic steady state is 0.18 which implies that the share of consumption and investment to output are 0.66 and 0.16, respectively. [5]

Eichenbaum et al. (2020) normalize the initial population to 1 and set the number of people initially infected from the infection, ε, to 0.001. We assume that the duration of being recovered or died from the virus is approximately 15 days and we use weekly data, so that the sum of the weekly probability of dying and recovering, $\pi_d + \pi_r$, equals to 7/15. According to WHO (World Health Organization), the crude CFR (case-fatality rate) in USA is 1.78% implying $\pi_d = 0.0178 \times 7/15$ and $\pi_r = 7/15 - \pi_d$.

To calculate the values of the Susceptible, Infectious and Recovered (SIR) model

[4] The gross domestic product in 2019 is \$ 21,427,690 (millions) obtained from the U.S. Bureau of Economic Analysis (March 2020) and the 2019 population estimate is 328,239,523 obtained from the U.S Census Bureau.

[5] According to the U.S. Bureau of Economic Analysis (March 2020), the government consumption expenditures and gross investment in 2019 are \$3,752,950 (millions).

parameters π_1, π_2 and π_3, we consider the assumptions below. Eichenbaum et al. (2020) state that in the onset of the pandemic, 2/3 of the virus transmissions is related to random non-economic activities. Since we only consider negative demand for consumption shock as a pandemic shock, consumption-related activities induce approximately 1/3 of the virus transmissions. As in Eichenbaum et al. (2020), we assume that the ratio of the family which will be infected as long as the pandemic continues is 60% .[6] Besides, the paramerters π_1, π_2 and π_3 are calculated with respect to initial infection rate, ε, π_d and π_r. The values of π_1, π_2 and π_3 are 2.58×10^{-7}, 3.74×10^{-5} and 0.467, respectively.

The coefficient of discount rate in loss function, ζ, is $0.99^{1/52}$ which is close to unity, and the weights on the output gap stabilization and interest rate smoothing relative to inflation stabilization, υ_x and υ_R, are 0.5 and 0.1, respectively (Leitemo and Söderström (2005)). The values of θ_π, θ_x and θ_R are shown at Table 3.2. We set the response coefficients on inflation and output gap to 1.5 and 0.5/52. These values are taken from Taylor (1993) and the response coefficient on output gap is divided by 52 since the time period of this thesis is a week. In the baseline rule, the degree of monetary policy inertia, θ_R, is 0. This response coefficient in the inertia rule 2 is set to 0.8 which is preferred in the literature, on average. Also, to understand the impact of the policy inertia, we choose θ_R as 0.2 in the inertia rule 1.

[6]This ratio was mentioned by Chancellor Angela Merkel on March 11, 2020.

Definition	Parameter	Numerical value
The labor share	α	2/3
Utility discount rate (weekly)	β	$0.97^{1/52}$
Depreciation rate of capital (weekly)	δ	0.06/52
Gross price markup	γ	1.35
Initial infection	ε	0.001
Probability of dying (weekly)	π_d	0.0083
Probability of recovering (weekly)	π_r	0.4584
Prob. of infected from consumption	π_1	2.58×10^{-7}
Prob. of infected from works	π_2	3.74×10^{-5}
Prob. of infected from other activities	π_3	0.467
Calvo price stickiness (weekly)	ξ	$\{0, 0.98\}$
Loss function discount rate	ζ	$0.99^{1/52}$
The weight on output gap	υ_x	0.5
The weight on interest rate smoothing	υ_R	0.1
Government share of output	η	0.18
Hours worked (weekly)	n	39.5
Income (weekly)	y	1255.4
Consumption (weekly)	c	823.9
Investment (weekly)	x	205.6
Government spending (weekly)	g	226
Real wage (weekly)	w	15.7
Capital (weekly)	k	178170
Real rental price	r^k	0.0017
The economic value of a life	VoL	8.9×10^6
Productivity	A	1.9236
—	θ	0.00048
Marginal cost	mc	0.74
Inflation	π	1
Nominal interest rate	R^b	1.0006
Real interest rate	rr	1.0006
Lagrange multiplier on bud. const.	λ^b	0.0012
Lagrange multiplier on trans. func.	λ^τ	-184.8

Table 3.1. Parameters and Steady - State Calibration Targets

Policy Rules	θ_π	θ_x	θ_R
Baseline rules (19a)	1.5	0.5/52	0
Inertia Rule 1 (19b)	1.5	0.5/52	0.2
Inertia Rule 2 (19b)	1.5	0.5/52	0.8

Table 3.2. Parameters in the Monetary Policy Rules

CHAPTER IV

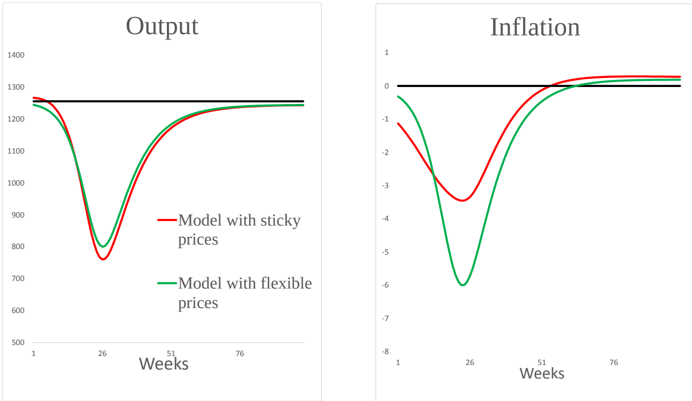
RESULTS

We now analyze the effects of the pandemic, price stickiness and monetary policy inertia. We compare the output and the inflation under different scenarios. As emphasized before, the pandemic diffuses through the society through consumption activities, which produce a large recession. Then the pandemic disappears when 60% of the family (population) members is infected since the benchmark value in the thesis is 60%. As shown in Figure 4.1, the depth of the pandemic reaches its highest level in the middle of the year. Due to the negative demand for consumption shock, we observe decline in prices during the recession.

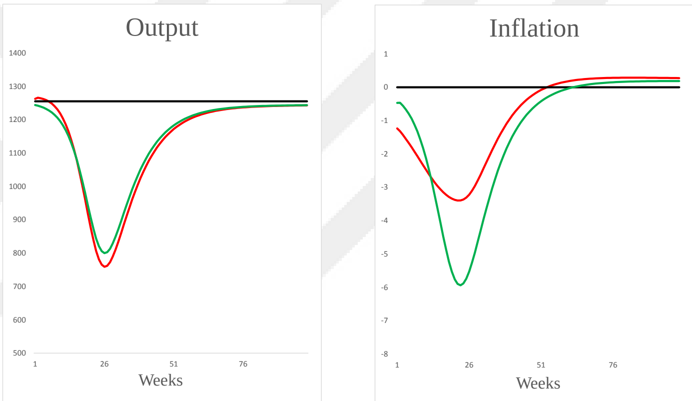
In Figure 4.1, the green lines show the flexible price scenario and the red lines show the scenario where intermediate firms face nominal price rigidities. Panel (a) examines the case with no monetary policy inertia. Panel (b) examines the case where the coefficient on previous period's policy rate in the policy function is 0.2. Finally, Panel (c) examines the case where the coefficient on previous period's policy rate is 0.8, which implies where sluggish response of central bank to the changes in the inflation and output.

Panel (a) of Figure 4.1 shows that with nominal price rigidities, we observe smooth changes in the inflation, which, by creating friction in the economy, results in higher output loss compared to the flexible price scenario. Yet, as in Eichenbaum et al. (2020), the impacts of nominal price rigidities on output are quite small for all the rules.

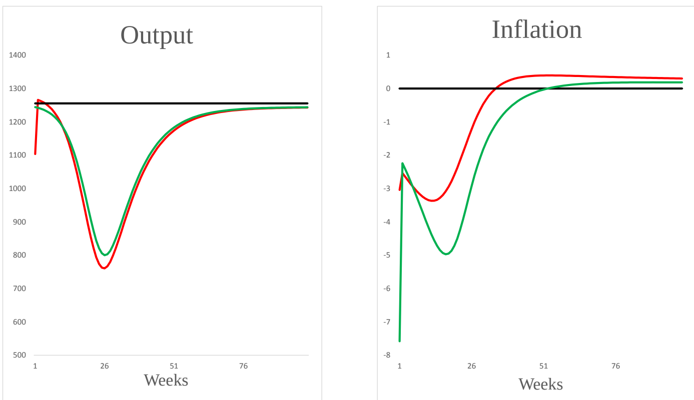
Panel (b) of Figure 4.1 shows slight changes compared to Panel (a), so we move to Panel (c). In the Panel (c), the change in inflation in the flexible price scenario is higher compared to Panel (a) and Panel (b). This is because the central bank gives sluggish response in Panel (c), which leads to larger deviations in inflation. The case is similar with the inflation under sluggish prices. This time, the change in prices in Panel (c) for



(a) Baseline Rule (θ_R=0)



(b) Inertia Rule 1 (θ_R=0.2)



(c) Inertia Rule 2 (θ_R=0.8)

Figure 4.1. Output and Inflation under Three Cases

this case is higher those in Panel (a) and Panel (b).

Finally, Panel (c) shows that the inflation deviates in smaller amounts with sluggish prices compared to the scenario with flexible prices. As with Panel (a) and Panel (b), the price friction in the economy results in higher output loss compared to the flexible price scenario.

θ_R	Loss
0 (Baseline Rule)	0.0153
0.2	0.0156
0.8	0.0243

Table 4.1. Losses in the Sticky Price Economy during the Pandemic

The results on Figure 4.1 are summarized in Table 4.1, which shows the central bank's loss function. The loss function is calculated for different levels of inertia. In all the cases, we use the price stickiness. Consistent with Figure 4.1, we observe that the loss gets higher with higher monetary policy inertia. Note finally that this result is obtained even though the central bank cares with smoothing the interest rate in all the cases. Though this loss between different degrees of interest rate inertia gets smaller if we use flexible prices instead of sticky prices.

CHAPTER V

CONCLUSION

Governments and central banks have tried to handle the contractions of their economies during the pandemic. Moreover, it is common to assume price stickiness in the markets. Therefore, we combine these two cases and investigate the impacts of monetary policy rules under the pandemic and price stickiness. We use Taylor-type monetary rules which differ in terms of the degree of monetary policy inertia, and we compare the losses of the central bank under these rules. The central bank's loss function includes the fluctuations in inflation, output, and the policy rate.

We find that both price stickiness and the monetary policy sluggishness generate the losses of the central bank. The crucial point is that their effect is amplified when we use these two frictions at once. We also find that even though the monetary authority has the policy rate smoothing incentive, a very high interest rate inertia leads very significant losses in its loss function. Hence, we conclude that when the economy experiences sudden turbulences like in the case of pandemic, it gets even more important to actively respond to economic conditions.

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APPENDIX

Households seek to maximize their lifetime utility subject to the budget constraint, the capital accumulation, the transmission function and the fractions of susceptible, infected and recovered people in the family. The first-order condition of the households' problem for c_t^s:

$$\frac{1}{c_t^s} = \tilde{\lambda}_t^b - \lambda_t^\tau \pi_1 (i_t c_t^i),$$

where $\tilde{\lambda}_t^b = \lambda_t^b P_t$. λ_t^b and λ_t^τ denote the Lagrange multipliers on the budget constraint and the transmission function. The first-order condition for c_t^i:

$$\frac{1}{c_t^i} = \tilde{\lambda}_t^b.$$

The first-order condition for c_t^r:

$$\frac{1}{c_t^r} = \tilde{\lambda}_t^b.$$

The first-order condition for n_t^s:

$$\theta n_t^s = \tilde{\lambda}_t^b w_t + \lambda_t^\tau \pi_2 (i_t n_t^i).$$

Here, $w_t = W_t / P_t$ which is real wage. The first-order condition for n_t^i:

$$\theta n_t^i = \tilde{\lambda}_t^b w_t.$$

The first-order condition for n_t^r:

$$\theta n_t^r = \tilde{\lambda}_t^b w_t.$$

The first-order condition for k_{t+1}:

$$\tilde{\lambda}_t^b = \beta (r_{t+1}^k + 1 - \delta) \tilde{\lambda}_{t+1}^b.$$

Here, $r_t^k = R_t^k / P_t$ which denotes the real rental price. The first-order condition for s_{t+1}:

$$log(c_{t+1}^s) - \frac{\theta}{2}(n_{t+1}^s)^2 + \lambda_{t+1}^\tau \left[\pi_1 c_{t+1}^s (i_{t+1} c_{t+1}^i) + \pi_2 n_{t+1}^s (i_{t+1} n_{t+1}^i) + \pi_3 i_{t+1}\right] + \tilde{\lambda}_{t+1}^b [w_{t+1} n_{t+1}^s - c_{t+1}^s] - \lambda_t^s / \beta + \lambda_{t+1}^s = 0.$$

Here, λ_t^s is the Lagrange multiplier on the fraction of susceptible people within the family. The first-order condition for i_{t+1}:

$$log(c_{t+1}^i)-\frac{\theta}{2}(n_{t+1}^i)^2+\tilde{\lambda}_{t+1}^b[w_{t+1}n_{t+1}^i-c_{t+1}^i]-\lambda_t^i/\beta+\lambda_{t+1}^i(1-\pi_r-\pi_d)+\lambda_{t+1}^r\pi_r=0,$$

where λ_t^i and λ_t^r are the Lagrange multipliers on the fractions of infected and recovered people within the family. The first-order condition for r_{t+1}:

$$log(c_{t+1}^r)-\frac{\theta}{2}(n_{t+1}^r)^2+\tilde{\lambda}_{t+1}^b(w_{t+1}n_{t+1}^r-c_{t+1}^r)-\lambda_t^r/\beta+\lambda_{t+1}^r=0.$$

The first-order condition for τ_t:

$$\lambda_t^i=\lambda_t^\tau+\lambda_t^s.$$

The first-order condition for B_{t+1}:

$$\lambda_t^b=\beta R_t^b\lambda_{t+1}^b,$$

which implies that:

$$\tilde{\lambda}_t^b=\beta rr_t\tilde{\lambda}_{t+1}^b.$$

The final goods producer maximizes its profits:

$$\Pi^F=P_ty_t-P_{i,t}y_{i,t},$$

where $y_t=\left(\int_0^1 y_{i,t}^{\frac{1}{\gamma}}\mathrm{d}i\right)^\gamma$, $\gamma>1$.

Thus, the modified profit of the final good producer is:

$$\Pi^F=P_t\left(\int_0^1 y_{i,t}^{\frac{1}{\gamma}}\mathrm{d}i\right)^\gamma-P_{i,t}y_{i,t}.$$

The first-order condition for $y_{i,t}$:

$$\frac{\partial\Pi^F}{\partial y_{i,t}}=P_t\gamma y_t^{\frac{\gamma-1}{\gamma}}\frac{1}{\gamma}y_{i,t}^{\frac{1-\gamma}{\gamma}}-P_{i,t}=0,$$

which can be reduced to

$$\frac{y_t}{y_{i,t}}=\left(\frac{P_{i,t}}{P_t}\right)^{\frac{\gamma}{\gamma-1}},$$

which implies that

$$y_{i,t}=\left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\gamma}{\gamma-1}}y_t.$$

Substituting the demand equation into $y_t = \left(\int_0^1 y_{i,t}^{\frac{1}{\gamma}} \mathrm{d}i\right)^{\gamma}$ yields the price of output:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\gamma-1}} \mathrm{d}i\right)^{-(\gamma-1)}.$$

Intermediate goods producers minimize their costs. The cost functions of the firms:

$$Cost^{int} = R_t^k k_{i,t} + W_t n_{i,t},$$

subject to:

$$y_{i,t} = A k_{i,t}^{1-\alpha} n_{i,t}^{\alpha}.$$

The Lagrangian can be written as:

$$\mathscr{L} = R_t^k k_{i,t} + W_t n_{i,t} + \lambda \left[y_{i,t} - A k_{i,t}^{1-\alpha} n_{i,t}^{\alpha}\right].$$

The first-order condition for $k_{i,t}$:

$$\frac{\partial \mathscr{L}}{\partial k_{i,t}} = R_t^k - \lambda A(1-\alpha) k_{i,t}^{-\alpha} n_{i,t}^{\alpha} = 0,$$

$$\frac{\partial \mathscr{L}}{\partial k_{i,t}} = R_t^k - \lambda (1-\alpha) \frac{y_{i,t}}{k_{i,t}} = 0,$$

which can be reduced to

$$\lambda = \frac{R_t^k k_{i,t}}{(1-\alpha) y_{i,t}}. \tag{A1}$$

The first-order condition for $n_{i,t}$:

$$\frac{\partial \mathscr{L}}{\partial n_{i,t}} = W_t - \lambda A \alpha k_{i,t}^{1-\alpha} n_{i,t}^{\alpha-1} = 0,$$

$$\frac{\partial \mathscr{L}}{\partial n_{i,t}} = W_t - \lambda \alpha \frac{y_{i,t}}{n_{i,t}} = 0,$$

which can be reduced to

$$\lambda = \frac{W_t n_{i,t}}{\alpha y_{i,t}}, \tag{A2}$$

when A1 and A2 are combined:

$$\lambda = \frac{R_t^k k_{i,t}}{(1-\alpha) y_{i,t}} = \frac{W_t n_{i,t}}{\alpha y_{i,t}},$$

which implies that

$$R_t^k k_{i,t} = \frac{1-\alpha}{\alpha} W_t n_{i,t}. \tag{A3}$$

Intermediate good producers maximize their profits:

$$\Pi^I = P_{i,t}y_{i,t} - R_t^k k_{i,t} - W_t n_{i,t},$$

subject to:

$$y_{i,t} = Ak_{i,t}^{1-\alpha} n_{i,t}^{\alpha} \text{ and } R_t^k k_{i,t} = \frac{1-\alpha}{\alpha} W_t n_{i,t}.$$

When the profit maximization problem of intermediate good producers is reduced to a single equation:

$$\Pi^I = P_{i,t}\left[Ak_{i,t}^{1-\alpha} n_{i,t}^{\alpha}\right] - R_t^k k_{i,t} - W_t n_{i,t},$$

when combined with (A3)

$$\Pi^I = P_{i,t}A\left[\frac{(1-\alpha)W_t}{\alpha R_t^k}\right]^{1-\alpha} n_{i,t} - \frac{1-\alpha}{\alpha} W_t n_{i,t} - W_t n_{i,t},$$

which reduces to

$$\Pi^I = P_{i,t}A\left[\frac{(1-\alpha)W_t}{\alpha R_t^k}\right]^{1-\alpha} n_{i,t} - \frac{1}{\alpha} W_t n_{i,t}.$$

The first-order condition for $n_{i,t}$:

$$\frac{\partial \Pi^I}{\partial n_{i,t}} = P_{i,t}A\left[\frac{(1-\alpha)W_t}{\alpha R_t^k}\right]^{1-\alpha} - \frac{1}{\alpha} W_t = 0,$$

which can be reduced to

$$P_{i,t} = \frac{W_t^{\alpha}}{(R_t^k)^{\alpha-1}} \frac{1}{A\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha-1},$$

then

$$P_{i,t} = \frac{W_t^{\alpha}(R_t^k)^{1-\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{A4}$$

Since $P_{i,t} = P_t\, mc_t$, the real marginal cost is:

$$mc_t = \frac{W_t^{\alpha}(R_t^k)^{1-\alpha}}{P_t A\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$

which equals to:

$$mc_t = \frac{w_t^{\alpha}(r_t^k)^{1-\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}.$$

These monopolist producers choose their optimal price with maximizing their profits under the condition of price frictions proposed by Calvo (1983).

$$\max_{\tilde{P}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j}^b \left(\tilde{P}_t y_{i,t+j} - P_{t+j} mc_{t+j} y_{i,t+j}\right),$$

subject to:

$$y_{i,t+j} = \left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{-\frac{\gamma}{\gamma-1}} y_{t+j}.$$

The modified profits of intermediate good producers are:

$$\max_{\tilde{P}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j}^b \left(\tilde{P}_t \left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{-\frac{\gamma}{\gamma-1}} y_{t+j} - P_{t+j} mc_{t+j} \left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{-\frac{\gamma}{\gamma-1}} y_{t+j}\right),$$

or,

$$\max_{\tilde{P}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j}^b P_{t+j} y_{t+j} \left(\left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{1-\frac{\gamma}{\gamma-1}} - mc_{t+j} \left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{-\frac{\gamma}{\gamma-1}}\right),$$

so that:

$$\max_{\tilde{p}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}_{t+j}^b y_{t+j} \left(\left(\tilde{p}_t X_{t,j}\right)^{1-\frac{\gamma}{\gamma-1}} - mc_{t+j} \left(\tilde{p}_t X_{t,j}\right)^{-\frac{\gamma}{\gamma-1}}\right),$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t},$$

and

$$X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\pi_{t+j-1}\ldots\pi_{t+1}}, & j > 0 \\ 1, & j = 0 \end{cases}$$

Henceforth, the intermediate good producer i chooses $\tilde{p}_t$ to maximize its profit. The first order condition for $\tilde{p}_t$:

$$\max_{\tilde{p}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}_{t+j}^b y_{t+j} X_{t,j}^{-\frac{\gamma}{\gamma-1}} \left(\left(1 - \frac{\gamma}{\gamma-1}\right) \tilde{p}_t^{-\frac{\gamma}{\gamma-1}} X_{t,j} + \frac{\gamma}{\gamma-1} mc_{t+j} \left(\tilde{p}_t\right)^{-\frac{\gamma}{\gamma-1}-1}\right) = 0,$$

then,

$$\max_{\tilde{p}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}_{t+j}^b y_{t+j} X_{t,j}^{-\frac{\gamma}{\gamma-1}} \left(-\tilde{p}_t^{-\frac{\gamma}{\gamma-1}} X_{t,j} + \gamma mc_{t+j} \left(\tilde{p}_t\right)^{-\frac{\gamma}{\gamma-1}-1}\right) = 0.$$

Multiplying that by $\tilde{p}_t^{1+\frac{\gamma}{\gamma-1}}$:

$$\max_{\tilde{p}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}_{t+j}^b y_{t+j} X_{t,j}^{-\frac{\gamma}{\gamma-1}} \frac{1}{\tilde{p}_t^{1+\frac{\gamma}{\gamma-1}}} \left(-\tilde{p}_t X_{t,j} + \gamma mc_{t+j}\right) = 0.$$

Rearranging the first order condition for $\tilde{p}_t$:

$$\max_{\tilde{p}_t} \sum_{j=0}^{\infty} (\xi\beta)^j W_{t+j} \left(\tilde{p}_t X_{t,j} - \gamma mc_{t+j}\right) = 0,$$

where

$$W_{t+j} = \tilde{\lambda}_{t+j}^b \left(\tilde{p}_t X_{t,j}\right)^{-(1+\frac{\gamma}{\gamma-1})} y_{t+j} X_{t,j},$$

then:

$$\tilde{p}_t = \frac{\sum_{j=0}^{\infty} (\xi\beta)^j W_{t+j} \gamma mc_{t+j}}{\sum_{j=0}^{\infty} (\xi\beta)^j W_{t+j} X_{t,j}} = \frac{K_t^f}{F_t},$$

where

$$K_t^f = \sum_{j=0}^{\infty} (\xi\beta)^j W_{t+j} \gamma mc_{t+j},$$

$$F_t = \sum_{j=0}^{\infty} (\xi\beta)^j W_{t+j} X_{t,j}.$$

The numerator, K_t^f, and the denominator, F_t, in recursive form are:

$$K_t^f = \gamma mc_t \tilde{\lambda}_t^b y_t + \xi\beta \pi_{t+1}^{\frac{\gamma}{\gamma-1}} K_{t+1}^f, \tag{A5}$$

$$F_t = \tilde{\lambda}_t^b y_t + \xi\beta \pi_{t+1}^{\frac{1}{\gamma-1}} F_{t+1}. \tag{A6}$$

Using the aggregate price level definition under Calvo-price frictions:

$$P_t = \left[\int_0^1 P_{i,t-1}^{-\frac{1}{\gamma-1}} di + (1-\xi)(\tilde{P}_t)^{-\frac{1}{\gamma-1}}\right]^{-(\gamma-1)}$$

$$= \left[\xi(P_{t-1})^{-\frac{1}{\gamma-1}} + (1-\xi)(\tilde{P}_t)^{-\frac{1}{\gamma-1}}\right]^{-(\gamma-1)}.$$

Dividing both side by P_{t-1}:

$$\pi_t = \left[\xi + (1-\xi)\left(\frac{\tilde{P}_t}{P_{t-1}}\right)^{-\frac{1}{\gamma-1}}\right]^{-(\gamma-1)},$$

which reduces to:

$$\pi_t^{-\frac{1}{\gamma-1}} = \xi + (1-\xi)\left(\tilde{p}_t\pi_t\right)^{-\frac{1}{\gamma-1}}.$$

Then,

$$\tilde{p}_t = \left(\frac{1-\xi\pi_t^{\frac{1}{\gamma-1}}}{1-\xi}\right)^{-(\gamma-1)}. \tag{A7}$$

Also, the equation (A7) can be written as:

$$K_t^f = F_t\left(\frac{1-\xi\pi_t^{\frac{1}{\gamma-1}}}{1-\xi}\right)^{-(\gamma-1)}. \tag{A8}$$

The equations (A5), (A6) and (A8) are the optimality conditions for optimal price setting. $\breve{y}_t$ is defined with using the demand curve (13):

$$\breve{y}_t = y_t\int_0^1\left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\gamma}{\gamma-1}}di = y_tP_t^{\frac{\gamma}{\gamma-1}}\int_0^1\left(P_{i,t}\right)^{-\frac{\gamma}{\gamma-1}}di = y_tP_t^{\frac{\gamma}{\gamma-1}}\breve{P}_t^{-\frac{\gamma}{\gamma-1}}. \tag{A9}$$

Hence, the aggregate output is given by:

$$y_t = \breve{p}_tAk_{i,t}^{1-\alpha}n_{i,t}^{\alpha},$$

where

$$\breve{p}_t = \left(\frac{\breve{P}_t}{P_t}\right)^{\frac{\gamma}{\gamma-1}}. \tag{A10}$$

From the equation (A9):

$$\breve{P}_t = \left(\int_0^1 P_{i,t}^{-\frac{\gamma}{\gamma-1}}\mathrm{d}i\right)^{-\frac{(\gamma-1)}{\gamma}}. \tag{A11}$$

After (A11) is divided by P_t and (A10) is taken into account, the price dispersion term is:

$$\breve{p}_t = \left[(1-\xi) \left(\frac{1-\xi \pi_t^{\frac{1}{\gamma-1}}}{1-\xi} \right)^{\gamma} + \xi \frac{\pi^{\frac{\gamma}{\gamma-1}}}{\breve{p_{t-1}}} \right]^{-1}.$$