APPLICATION OF STRUCTURAL MODIFICATION METHOD TO NONLINEAR VIBRATION ANALYSIS OF BLADED DISKS

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ABSTRACT

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High cycle fatigue failure of turbine blades is one of the most important problems in the design of gas turbine engines; hence, bladed-disk assemblies have been studied extensively for more than half a century. Damping design becomes an important issue in order to attenuate the blade vibration. For bladed-disk systems, friction damping concept is a common strategy to decrease vibration levels. There are different strategies in order to add friction damping to the system: blade-to-blade dampers such as shrouds and underplatform dampers such as wedge dampers. In the design of friction dampers, geometry of the friction contact becomes an important issue for maximizing energy dissipation. In order to determine the optimum damper geometry, i.e. shroud angle for shroud contacts and wedge surface angles for wedge dampers, nonlinear forced response analysis should be repeated by changing the finite element models in order to incorporate these geometry changes. Repeating the finite element analysis for each geometry change requires significant amount of time in the case of large finite element models used in real life examples. Moreover, in order to accurately model frictional contacts, high number of friction elements are required and utilizing modal superposition approach the number of nonlinear equations can be decreased significantly compared to receptance methods. Therefore, in this thesis, in order to decrease the computational time required for the finite element analysis, a new structural modification method with additional degrees of freedom is developed. The developed method is capable of determining the modal data of the modified structure by using modal data of the original structure and system matrices of the modifying part. The developed structural modification method is compared with the available modification methods in terms of computational time required and it is observed that the developed modification method is computationally more efficient than the existing methods. As a case study, bladed disk systems with integrally inserted shrouds are studied. Using the developed structural modification method and the nonlinear forced response prediction method, the effect of change of shroud angle is studied.

Keywords: Bladed Disks, Structural Modification, Nonlinear Vibration Analysis, Friction Damper Geometry, Friction Damping

ÖZ

YAPISAL DEĞİŞİKLİK YÖNTEMİNİN KANATÇIKLI DİSKLERİN DOĞRUSAL OLMAYAN TİTREŞİM ANALİZİNDE UYGULANMASI

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Türbin kanatçıklarındaki en önemli kırılım problemi yüksek çevrim yorulmasından kaynaklanmaktadır. Bu nedenle kanatçıklı disk sistemleri üzerinde detaylı olarak yarım asırdan fazla süredir çalışılmaktadır. Kanatçık titreşimlerini azaltmada için sönümleyici tasarımı önemli bir konudur. Sürtünmeli sönümleyici konsepti, kanatçıklı disklerdeki titreşim seviyelerini azaltmak için genellikle kullanılan bir konsept haline gelmiştir. Sönümleyiciyi sisteme eklemek için değişik yöntemler kullanılmaktadır: kanatçıklar arası sönümleyici (kanatçıklardan çıkıntı şeklinde) ve kanatçıkların altına konan sönümleyiciler (üçgen şeklindeki platform yapılar). Sürtünmeli sönümleyici tasarımında enerji kaybını en yüksek seviyelerde tutmak için, sürtünme yüzeyinin geometrisi önemli bir konu haline gelmiştir. Örnek olarak, çıkıntı sönümleyiciler için çıkıntı açısı ya da üçgen platform sönümleyiciler için üçgen yüzey açısı verilebilir. Her geometri değişimi için sonlu elemanlar modeli tekrar güncellenmeli ve doğrusal olmayan titreşim analizi tekrarlanmalıdır. Her değişim için yapılan model güncelleme işi, özellikle gerçek hayatta bulunan yapılar için yapılan büyük sonlu elemanlar modellerinde ciddi oranda zaman kaybına neden olmaktadır. Ayrıca, sürtünme yüzeylerini daha doğru modelleyebilmek için, yüzeyde fazla sayıda sürtünme elemanı tanımlanmalıdır ve modal süper pozisyon yönteminin kullanılmasıyla, elde edilen doğrusal olmayan denklem sayısı frekans tepki fonksiyonu yöntemine göre ciddi oramda azalmaktadır. Diğer taraftan, bu tezde, sonlu elemanlar analizi işlem süresini azaltmak için, serbestlik derecesi artan sistemlere uygulanabilen, yeni bir yapısal modifikasyon yöntemi geliştirilmiştir. Geliştirilen yöntem, orijinal sistemin modal bilgisi ve eklenen sistemin sistem matrislerini kullanarak, oluşan sistemin modal bilgisini elde etmektedir. Geliştirilen yapısal modifikasyon yöntemi, hâlihazırda bulunan modifikasyon yöntemleriyle işlem süresi açısından karşılaştırılmakta ve geliştirilen yöntemin çok daha verimli olduğu gözlenmektedir. Örnek olay incelemesi olarak, entegre olarak yerleştirilmiş çıkıntılı kanatçıklı disk sistemleri ele alınmıştır. Geliştirilmiş yapısal değişiklik yöntemi ve doğrusal olmayan zorlanmış tepki analiz yöntemi kullanılarak, sönümleyici açısındaki değişimin neden olduğu etkiler gözlenmektedir.

Anahtar Kelimeler: Kanatçıklı Diskler, Yapısal Değişiklikler, Doğrusal Olmayan Titreşim Analizi, Sürtünmeli Sönümleyici Geometrisi, Sürtünmeli Sönümleyici

To my family, my little niece and Onur…

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

High level of reliability, efficiency, and low failure rates are desired in the design of modern gas turbine engines. In turbo machinery applications, operating conditions are severe due to aerodynamic loads which may result in high vibratory stresses, resonance and flutter. Two main reasons of blade failures are: fatigue failures and creep failure. Fatigue failures can be divided in to two as high cycle fatigue (HCF) [1] and low cycle fatigue (LCF) failures. High cycle fatigue (HCF) is the most common reason of component failures in gas turbine engines causing wide range of damage, since it affects all engine sections. Sources of HCF failure are: aerodynamic excitation, mechanical vibrations, airfoil flutter [2].

Friction dampers are often used in the design of gas turbine engines in order to attenuate blade vibrations; hence, to prevent blade failures. For obtaining the dynamic behavior of bladed disk systems, many works have been done in the past.

The main aim of this work is to study the effect of different damper geometries in order to maximize vibration reduction. In order to accurately model the frictional contact, the number of contact points used should be high; hence, for these systems, harmonic balance method with modal superposition approach is the most suitable one [3, 5], since the number of nonlinear algebraic equations is significantly decreased compared to receptance methods. In the modal superposition approach, the number of nonlinear equations is a factor of the number of modes used and independent of the number of contact points. Therefore, a new structural modification method with additional degrees of freedom, which will be detailed in Chapter 2, is developed in order to obtain modal information of a modified system by using the modal data of the original structure and the system matrices of the modifying part. The developed methodology is applicable for all types of linear and nonlinear systems; however, as a case study, in this work bladed disks with shroud contacts are studied.

1.1. Literature Survey

The first subject of this section is structural modification. Structural modification methods, their history and application areas will be mentioned. The second main subject is the nonlinear response analyses of bladed disk systems. This includes a wide range of subjects, namely; friction modeling, methods of solving nonlinear algebraic equations, and response analyses.

1.1.1. Structural Modification

When analyzing different design alternatives, structural modification techniques are useful at any time in design stage which shortens the analysis time by a considerable amount. Especially for experimental studies or working with large scale Finite Element Models (FEM), it is difficult and time consuming to adapt every modification to the system in order to reanalyze it accurately. Direct structural modification, also called as reanalysis, is applied

using original system information and dynamic structural modification matrix of the modifying part in order to predict the dynamic characteristics of modified systems. Structural modification techniques can be adopted for both systems with or without additional degrees of freedoms. These techniques predict the behavior of the modified system very accurately, when all information of the original system is included in the analysis. Structural modification techniques are divided into two as direct structural modification and inverse structural modification techniques. Direct modification is based on determining the dynamic characteristics of the modified system by modifying the original system. On the other hand, inverse structural modification is considered as an optimization process by determining the modifying structure in order to obtain the desired modified system characteristics. Direct structural modification (reanalysis) is applicable for cases in need of vibration optimization or FEM updating. On the other hand, inverse structural modification is mostly used in order to eliminate or reduce the effects of vibration related problems of an already constructed structure. Structural modification methods include experimental and analytical methods. There are exact and approximate methods for both. The analytical approach for structural modification began in 1960s. Weissenburger [6] worked on a simple undamped lumped mass model by changing mass and stiffnesses. Pomazal and Synder [7] extended this work for obtaining eigenvalues and eigenvectors of the locally modified linear systems which can be considered as direct modification problem. Another early work on direct structural modification is done by Lancaster [8, 9] which is the basis of an analytical approach. Crowley et al. [10] applied direct modification by using experimentally obtained FRFs of the original structure. Tsuei et al. [11] worked on inverse structural modification by using modal data. Bucher et al. [12] used frequency-response function (FRF) function data for the inverse problem. Kyprianou, et al. [13, 14], Mottershead et al. [15, 16] and Park et al. [17] also worked on inverse structural modifications. When necessary, modal reduction techniques are also applicable for large scale systems where the most dominant modes are selected for the analysis in this method resulting in a decrease in the computational time [18, 19]. For systems without additional degrees of freedom, Sherman-Morrison [20] and Woodbury [21] formulas were presented in the past. These methods find dynamic matrix of modified systems by updating the dynamic matrix of the original systems. Sanliturk presented a new method [22] based on Sherman-Morrison formula, which determined the FRFs and the response of the modified structures without using matrix inversion or a solution of a new eigenvalue problem. Özgüven [23] proposed a general method for structural modification problems with or without additional degrees of freedoms in the modified structures. This method is an FRF based method which obtains FRF of the modified structure by using FRF of the original structure and dynamic structural modification matrix of the modifying structure. Tahtali [24] applied this method to a plate and a propeller. Koksal et al. [25] used this method by using power series expansion in order to avoid matrix inversion. Distributed modifications were studied by D'Ambrogio and Sestieri [26] for systems without additional degrees of freedom. Hang et al. [27, 29], worked on distributed systems with additional degrees of freedom.

1.1.2. Nonlinear Response Analysis of Bladed Disks

In turbo machinery applications, due to operating conditions and material defects, bladed disk systems encounter fatal problems because of stress concentrations on blades. In order to prevent failure and increase the blades' life, damping is added to the system in the form of dry friction. This type of damping can be added to the blade-disk interfaces [30], between

blades by shroud contacts [31] or by means of under platform dampers [32, 33]. Also, friction rings are studied as an alternative way of friction damping [34]. Since friction damping makes the problem nonlinear, the analysis of such systems is complicated. In order to predict the forced vibrations of bladed disk systems with friction dampers, modeling of dry friction is first addressed. Using the friction models it is possible to obtain the nonlinear contacts forces which are used in the dynamic analysis of the structure resulting in a set of nonlinear algebraic equations which are to be solved by iterative methods.

1.1.2.1. Modeling of Friction Contact

In the past, many applications are done to investigate the dynamic behavior of the structures having friction nonlinearities. Two approaches, micro-slip and macro-slip approaches, are used to define the contact behavior. Macro-slip approach [35, 37] is widely used due to its simplicity. Since the friction interface is considered as a rigid body; it either entirely slips or sticks. This is the case when the normal load is small as a result of which, gross slip occurs. This approach provides mathematical simplicity to the problem. In micro-slip approach, friction surface is modeled as an elastic body where partial slip and stick may occur. If the normal load acting on the friction interface is high or the contact area is large, micro-slip approach is required to accurately model the friction contact. Menq et al. worked on modeling micro-slip friction [38, 40]. In addition to this, authors also performed an experiment [41] which showed that micro-slip model is superior to macro-slip model especially when the contact pressure is high. Cigeroglu et al. [42] developed a one dimensional dynamic micro-slip friction model using a continuous bar element with shear layer. For three different normal load distributions; stick slip regions, hysteresis loops and fourier coefficients are compared. An equivalent point contact model is as well established, the results of which are compared with the micro-slip model. Cigeroglu et al. [3] later developed a micro-slip model having normal load variation. Cigeroglu and Özgüven [43] presented a model including the effects of both micro-slip and macro-slip approaches which has simplicity of a macro-slip model and reflects the realistic features of a micro-slip model.

Griffin [44] worked on the early one-dimensional friction modeling. In this model, normal load is constant and has a simple hysteresis loop. Cameron et al. [45] worked on an optimum damping design considering constant normal load and made verifications via experiments. While analyzing vibratory response of bladed disk systems, it is also important to consider the flexural and torsional modes of the system as a result of which, one-dimensional modeling becomes insufficient. Menq et al. [46] developed 2-D frictional model from the idea that "Even the system is uncoupled and subjected to in-phase forcing in two orthogonal directions; the system response can be two-dimensional." Later, Sanliturk and Ewins [47] extended the work of Menq and came up with a mathematically simpler and more accurate modeling which was not restricted by Coulomb friction. Yang et al. [48, 49] developed a 1-D model having normal load variation and the analytical transitions between stick, slip, and separation were defined. Hysteresis loops were obtained and nonlinear response analysis with Harmonic Balance Method (HBM) was performed considering the model as 2-D. Later, Yang et al. [50] improved this model by modeling the tangential motion as 2-D, so the contact motion can be considered as a 2-D with normal load variation or a 3-D motion.

1.1.2.2. Nonlinear Dynamic Response Analysis

Numerous works have been performed about nonlinear response analysis of bladed disk systems. Menq and Griffin [35] used one-term harmonic balance method (HBM) to obtain the harmonic forced response of a blade with friction damper. A modified harmonic balance method, so called incremental harmonic balance method (IHM), was studied by Pierre et al. [51]. It is observed that IHM results in very accurate results and having advantages over time integration methods. However using multiple harmonics decreases the efficiency of the method. Later, alternating frequency-time (AFT) method was investigated by Cameron and Griffin [52] which can also include multiple harmonics and calculation switches between frequency and time domains. Wang et al. [53] worked on the response analysis of a blade-to-ground damper, where the normal load was constant. He compared the effect of using different number of harmonics in HBM solution and also compared these results with direct time integration method. According to the results, one-term approximation predicts the response accurately when the system is in fully stuck and fully slipping conditions and HBM solves the problem in a very short time compared to direct time integration method. Yang et al. [49] investigated the transition angles and hysteresis loops for the 1-D system having normal load variation. The authors observed softening and hardening effects in addition to the jump phenomena. For the nonlinear dynamic response analysis, authors used HBM with one-term approximation and compared results obtained to time integration results. Yang et al. [50] later worked on the same concepts on a 3-D contact model (2-D with normal load variation). Cigeroglu et al. [3] developed a modal superposition method which is used to obtain forced response of nonlinear system by utilizing the free mode shapes of the linear structure. In this method, the number of nonlinear algebraic equations is dependent on the number of modes included only; hence, not on the number of nonlinear DOFs. Authors applied the method to a one-dimensional bar like damper and a more realistic blade to ground damper using a 1-D micro-slip contact model having normal load variation. Later authors [4] applied the modal superposition method for multiple harmonics case where a bladed-disk system having wedge dampers is considered. It is observed that for the case when the normal load is low and jump occurs, multiple harmonics should be included in the solution while for high normal load cases, using single harmonic results in accurate response levels.

Petrov and Ewins [54] developed a special area contact friction element between blade and root interfaces which allows variation of contact areas during vibration. Multiharmonic balance method is applied by choosing the necessary number of harmonics for time reduction and accuracy. Method is applicable for large scale finite element models that modal synthesis method is used for reducing the size of the model. Chen et al. [55] worked on 3-D contact model of bladed disk system having shroud constraints. Authors defined the transition criteria and possible response states like separation, fully stuck, stick-slip and jump phenomena. They compared the response results to time integration methods and proved that using single harmonic in HBM resulted in overestimated response levels for low preload values.

Sanliturk et al. [56] defined the nonlinear friction damper behavior as an equivalent amplitude-dependent complex stiffness and performed the response analysis using one-term HBM. Method was validated by numerical test cases. Authors also worked on the theoretical modeling of an underplatform dampers (or wedge dampers) to analyze the dynamic

characteristics without conducting experiments [57]. They compared the results by conducting two experiments; one with real turbine blades, real underplatform dampers and another one with two cantilever beams simulating blades and real underplatform dampers. Experiment results showed that optimum damping condition, frequency shift, and response reduction can be well defined by the proposed method. Petrov et al. [58] developed a method by which resonance frequencies and forced response levels at resonance peaks can be directly obtained as functions of design parameters or excitation levels.

Petrov [31] used cyclic symmetry property of bladed disk system in order to perform multiharmonic response analysis for two cases of system formulation; using sector finite element matrices and using sector modal data. Szwedowicz et al. [59] considered a tuned bladed disk system and numerical static, free, and forced vibration analyses in the spin pit environment were conducted. Also, damping performances for different contact configurations, such as slant and zigzag, are compared. Results show that for small angles (0-30 degrees) between the circumferential direction and the normal vector to the contact surface, higher damping performance is achieved. Schwingshackl et al. [60] studied the parameters that defines the friction contact; friction coefficient, tangential contact stiffness, and normal load distribution in the contact. The accuracy of the measured values of these parameters is studied to further model the contact interfaces. Later authors [61] studied the similar aspects on underplatform dampers. This parametric study is based on following parameters: friction contact interface, nonlinear model setup, and analysis parameters.

1.2. Objective

In this thesis, a new structural modification method is presented for systems having additional degrees of freedoms. This method provides natural frequencies and mode shapes of the modified structure, when modal data of the original structure and mass and stiffness matrices of the modifying structure exist. The first aim of this work is to show the accuracy and advantages of the developed method. For accuracy considerations, results are compared to modal analysis results of the modified structures either as lumped parameter models or FEM models that are modeled in ANSYS. In order to demonstrate the computational advantage of the proposed method, computation times of the developed method, Özgüven's matrix inversion method, which is a structural modification method with additional degrees of freedom, and direct solution of the eigenvalue problem by using a FE software and Matlab are compared.

In the second part of this thesis, nonlinear forced response analyses of bladed disk systems having shroud constraints are studied under different shroud geometries. Friction contact between shrouds are modeled by using a 1-D contact model with normal load variation and harmonic balance method with modal superposition is used for forced response prediction which takes the modal data of the linear structure as an input. Using the results of the forced response analysis, optimum shroud geometry resulting in minimum vibration amplitude is sought in terms of shroud angles. Since remodeling the whole system for each interested angle is time consuming and computationally expensive, the developed structural modification method is applied for this purpose.

1.3. Outline of the Thesis

In Chapter 2, structural modification methods are explained. Firstly, the developed structural modification method is formulized which is a modal information based method. Formulations are explained in detail and an application on a simple lumped parameter model is performed and compared with the exact solution. Later, being an FRF based method, Ozguven's method is explained and two methods are compared by means of accuracy and solution time for varying system sizes.

Chapter 3 is about dynamic response analysis of nonlinear systems. Firstly, equation of motion of a nonlinear system is described. Harmonic balance method with modal superposition approach is explained which is used to obtain the nonlinear algebraic equations. Later, Newton's method and arc-length continuation method, solution techniques for nonlinear algebraic equations, are described. Nonlinearity types used in the analyses; cubic stiffness nonlinearity, friction nonlinearity with constant normal load and varying normal load, are detailed.

In Chapter 4, applications of the developed structural modification method to nonlinear systems are presented. Initially, the method is presented on a lumped parameter model. Later, FEM applications are given. Finally, dynamic response analysis of a bladed disk system with shroud contacts is performed. Different shroud angles are considered by the application of the developed structural modification method which resulted in a significant decrease in the computational time required to obtain response levels.

In Chapter 5, discussions and comments are made and possible future studies are offered. Finally, the study is concluded.

CHAPTER 2

STRUCTURAL MODIFICATION

2.1. A New Structural Modification

Structural modification techniques are commonly used during design and optimization processes as well as studying the dynamic behavior of already constructed structures. These methods are very reliable and accurate even when working with large scale finite element models (FEM). Since it is time consuming to adapt every change to a system and reanalyze for each change during design process, structural modification methods are widely preferred due to their high accuracy and time shortening capabilities.

A new formulation is developed that is useful if the modal data (i.e. natural frequencies and mode shapes) of the modified structure is desired to be obtained. Modal data of the original structure and mass and stiffness matrices of the modifying structure are used to obtain the modal data of the modified structure. The method is significantly useful if the size of the finite element model is large.

Mass and stiffness matrices of the original and modifying systems and their addition are presented in the equation of motion of the modified system as given by Equation (2.1). Structural and/or viscous damping matrices can also be included in the equation of motion if they are known in a similar manner. Since the amount of damping is specified instead of damping matrices they are not included to the formulation. Structural coupling is assumed as rigid coupling as shown in Figure 2.1. Subscript a corresponds to DOFs which belong only to the original structure, subscript b corresponds to DOFs which belong to both structures and subscript c corresponds to DOFs which belong only to the modifying structure.

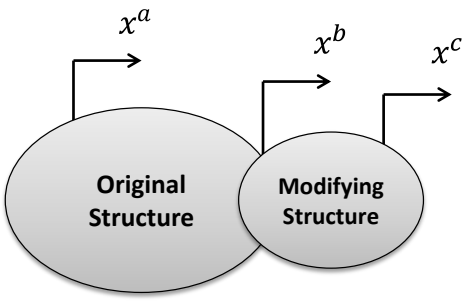


Figure 2.1 Modified Structure (Rigid Coupling)

$$\left[\begin{bmatrix} M_0^{aa} & M_0^{ab} & 0 \\ M_0^{ba} & M_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix}\right]\{\ddot{x}\}$$
$$+\left[\begin{bmatrix} K_0^{aa} & K_0^{ab} & 0 \\ K_0^{ba} & K_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix}\right]\{x\} = \{F\} \tag{2.1}$$

$$[\Phi]^T\left[\begin{bmatrix} M_0^{aa} & M_0^{ab} & 0 \\ M_0^{ba} & M_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix}\right][\Phi]\{\ddot{q}\}$$
$$+[\Phi]^T\left[\begin{bmatrix} K_0^{aa} & K_0^{ab} & 0 \\ K_0^{ba} & K_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix}\right][\Phi]\{q\} = [\Phi]^T\{F\} \tag{2.2}$$

where $[M_0]$ and $[K_0]$ matrices represent mass and stiffness matrices of the original system while $[M_m]$ and $[K_m]$matrices represent mass and stiffness matrices of the modifying part. Let the response of the modified system be written as

$$\{x\} = \begin{Bmatrix} x^a \\ x^b \\ x^c \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q_0 \\ x^c \end{Bmatrix} = [\Phi]\{q\}, \tag{2.3}$$

where $[\phi_0]$ is the mass normalized mode shape matrix of the original system. Identity matrix is added to the right bottom corner of the matrix in order to keep the information coming from the modifying part. Inserting this expression into Equation (2.1) and pre-multiplying it by $[\Phi]^T$ gives the following mass and stiffness matrices of the modified structure as

$$[M_{\text{mod}}] = \begin{bmatrix} [I] & 0 \\ 0\ \ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}, \tag{2.4}$$

$$[K_{\text{mod}}] = \begin{bmatrix} [\Omega_0] & 0 \\ 0\ \ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}, \tag{2.5}$$

The equation of motion becomes:

$$[M_{\text{mod}}]\{\ddot{q}\} + [K_{\text{mod}}]\{q\} = [\Phi]^T\{F\}, \tag{2.6}$$

where $[\Omega_0]$ is a diagonal matrix composed of squares of the natural frequencies of the original structure. For free vibrations, eigenvalue problem (EVP) is defined as

$$\left(\left[K_{\text{mod}}\right]-\lambda\left[M_{\text{mod}}\right]\right)\{q\}=\{0\}, \tag{2.7}$$

Solving the eigenvalue problem defined by Equation (2.7) , the modal data of the modified structure can be obtained. However, it should be noted that eigenvectors obtained define $\{q\}$ rather than $\{x\}$; hence, they should be pre-multiplied by $[\Phi]$ which gives the mode shapes of the modified structure in $\{x\}$.

2.2. Matrix Inversion Method (Özgüven's Method)

Matrix inversion method is firstly developed by Özgüven [23] for systems with or without additional degrees of freedom. It is an FRF based technique in which FRF of the original system and dynamic structural modification matrix of the modifying system are used to obtain the FRF of the modified system. In this study, the one with additional degrees of freedom is considered.

Equation of motion of the original system is given as follows

$$\left[M_0\right]\{\ddot{x}\}+i\left[H_0\right]\{x\}+\left[K_0\right]\{x\}=\{F\}, \tag{2.8}$$

from which receptance matrix of the original system can be obtained as

$$[\alpha_0]=\begin{bmatrix}\alpha_0^{aa} & \alpha_0^{ab}\\ \alpha_0^{ba} & \alpha_0^{bb}\end{bmatrix}=\left[\left[K_0\right]-\omega^2\left[M_0\right]+i\left[H_0\right]\right]^{-1}. \tag{2.9}$$

Dynamic structural modification matrix of the modifying system is defined as

$$\left[D_m\right]=\left[K_m\right]-\omega^2\left[M_m\right]+i\left[H_m\right], \tag{2.10}$$

as a result of which receptance matrix of the modified system becomes

$$[\gamma]=\left[\left[\left[K_0\right]+\left[K_m\right]\right]-\omega^2\left[\left[M_0\right]+\left[M_m\right]\right]+i\left[\left[H_0\right]+\left[H_m\right]\right]\right]^{-1}. \tag{2.11}$$

Receptance matrix of the modified system defined in Eq. (2.11) can be written in terms of receptance matrix of the original system and the dynamic structural modification matrix of the modifying structure as follows

$$\begin{bmatrix}\gamma^{aa} & \gamma^{ab} & \gamma^{ac}\\ \gamma^{ba} & \gamma^{bb} & \gamma^{bc}\\ \gamma^{ca} & \gamma^{cb} & \gamma^{cc}\end{bmatrix}^{-1}=\begin{bmatrix}\left[\alpha_0\right]^{-1} & 0\\ 0\ \ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & \left[D_m\right] & \\ 0 & & \end{bmatrix}, \tag{2.12}$$

$$\begin{bmatrix}\gamma^{ba}\\ \gamma^{ca}\end{bmatrix}=\left[\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}\alpha_0^{bb} & 0\\ 0 & I\end{bmatrix}\left[D_m\right]\right]^{-1}\begin{bmatrix}\alpha_0^{ba}\\ 0\end{bmatrix}, \tag{2.13}$$

$$\begin{bmatrix}\gamma^{bb} & \gamma^{bc}\\ \gamma^{cb} & \gamma^{cc}\end{bmatrix}=\left[\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}\alpha_0^{bb} & 0\\ 0 & I\end{bmatrix}\left[D_m\right]\right]^{-1}\begin{bmatrix}\alpha_0^{bb} & 0\\ 0 & I\end{bmatrix}, \tag{2.14}$$

$$\left[\gamma^{aa}\right]=\left[\alpha_0^{aa}\right]-\left[\alpha_0^{ab}\ \vdots\ 0\right]\left[D_m\right]\begin{bmatrix}\gamma^{ba}\\ \gamma^{ca}\end{bmatrix}, \tag{2.15}$$

$$\left[\gamma^{ab}\ \vdots\ \gamma^{ac}\right]=\left[\alpha_0^{ab}\ \vdots\ 0\right]\left[\left[I\right]-\left[D_m\right]\begin{bmatrix}\gamma^{bb} & \gamma^{bc}\\ \gamma^{bb} & \gamma^{bb}\end{bmatrix}\right]. \tag{2.16}$$

2.3. Case Study 1: Application on a Lumped Parameter Model

In order to validate the developed structural modification method, it is applied on a simple lumped parameter model given in Figure 2.2. As seen from the figure, the 4-DOF system, original structure, given at the top is modified by the 3-DOF structure, modifying part, given at the bottom by rigid connection. System parameters of the original structure are $M_1 = M_2 = M_3 = M_4 = 1\text{kg}$, $K_1 = K_2 = K_3 = K_4 = 10000\text{N/mm}$; whereas, for the modifying part they are $M_5 = M_6 = M_7 = 2\text{kg}$, $K_5 = K_6 = K_7 = 20000\text{N/mm}$.

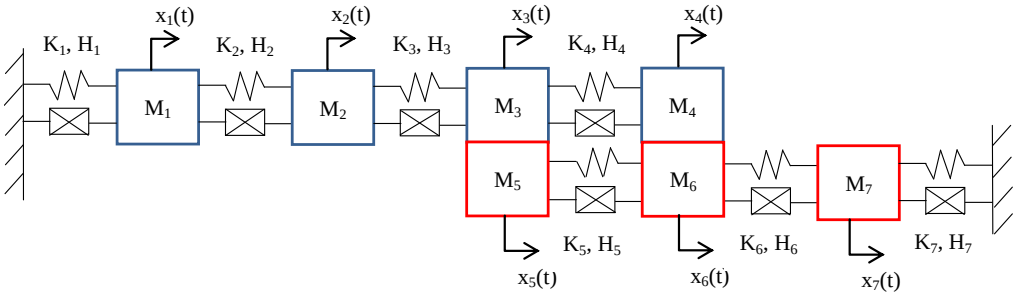


Figure 2.2 Simple lumped parameter model: Blue-original structure, red-modifying part

Developed structural modification method is applied to this problem and the results obtained are presented in which are compared with the exact ones obtained by solving the 5-DOF system. In the comparison, for natural frequencies error between the obtained and the exact one and for the mode shapes, the modal assurance criterion (MAC) is used which can be defined as

$$MAC\left(\{\psi\}_r,\{\psi\}_s\right)=\frac{\left(\{\psi\}_r^T,\{\psi\}_s\right)^2}{\left(\{\psi\}_r^T,\{\psi\}_r\right)\left(\{\psi\}_s^T,\{\psi\}_s\right)}, \tag{2.17}$$

which takes a value between 0 and 1, where 1 denotes that the compared vectors $\{\psi\}_r$ and $\{\psi\}_s$ are identical.

Table 2.1 Comparison of Results Structural Modification Results to the Exact Results

Mode #	Natural frequencies found by the proposed method [rad/s]	Natural frequencies found by modal analysis of the modified system [rad/s]	Error %	MAC
1	42.34	42.34	0%	1.00
2	100.00	100.00	0%	1.00
3	132.75	132.75	0%	1.00
4	168.77	168.77	0%	1.00
5	179.17	179.17	0%	1.00

2.4. Case Study 2: Application on a FEM

2.4.1. Reduction of the Original Structure

For very large scale systems, reduction of the system is required in order to keep computational expenses at minimum or at a practical value. This can be achieved in the developed method by using a certain number of mode shapes which is much less than the number of total DOFs. In order to present the developed method, a cantilever beam modeled in a commercial finite element software is considered. FEM of the cantilever beam and the modifying part are shown in Figure 2.3, where total numbers of DOFs on the original structure, modifying structure and the modified system are 216, 81 and 270, respectively. Assuming only the first 20 modes are of interest for the modified system, natural frequencies and mode shapes of the first 20 modes of the modified system are obtained by utilizing 20, 30 and 40 modes of the original structure. For the comparison of mode shapes obtained, modal assurance criterion (MAC) is used.

Comparison of the natural frequencies and the mode shapes of the modified structure for different number of modes retained in the original system are given in Table 2.2 and Table 2.3 respectively.

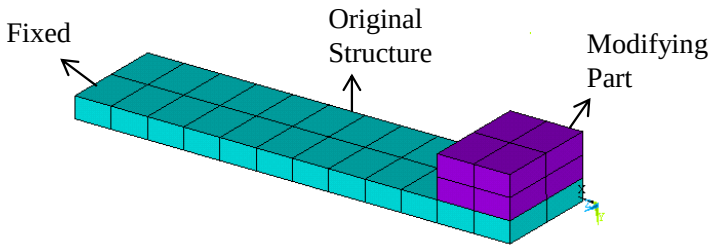


Figure 2.3 FEM of the cantilever beam

Table 2.2 Natural frequencies [Hz]

Developed Method							
Mode #	Exact results	Using 20 modes		Using 30 modes		Using 40 modes	
		Nat. Freq.	Err %	Nat. Freq.	Err %	Nat. Freq.	Err %
1	130.14	130.02	0.09	130.00	0.11	129.99	0.12
2	604.54	605.62	0.18	605.43	0.15	605.10	0.09
3	988.99	996.14	0.72	991.79	0.28	990.44	0.15
4	1104.65	1110.54	0.53	1109.59	0.45	1107.76	0.28
5	2799.18	2895.71	3.45	2838.28	1.40	2820.46	0.76
6	4158.96	4190.02	0.75	4185.60	0.64	4176.36	0.42
7	4409.29	4482.54	1.66	4469.44	1.36	4446.96	0.85
8	4793.14	4849.86	1.18	4821.67	0.60	4811.72	0.39
9	6292.99	6561.00	4.26	6390.09	1.54	6344.17	0.81
10	8803.87	9040.56	2.69	8998.08	2.21	8929.63	1.43
11	10058.67	10187.86	1.28	10143.49	0.84	10118.95	0.60
12	10877.85	11804.89	8.52	11206.78	3.02	11057.74	1.65
13	14492.40	15020.50	3.64	14898.24	2.80	14752.55	1.80
14	16458.13	16633.87	1.07	16563.70	0.64	16544.13	0.52
15	16562.25	17128.54	3.42	16841.34	1.69	16770.67	1.26
16	18108.22	21359.44	17.95	18735.07	3.46	18422.54	1.74
17	21922.63	23193.94	5.80	22835.13	4.16	22444.45	2.38
18	24152.82	24860.07	2.93	24344.07	0.79	24318.58	0.69
19	24288.33	37521.24	54.48	24630.32	1.41	24502.37	0.88
20	27322.41	44340.93	62.29	27570.93	0.91	27446.95	0.46

Table 2.3 MAC numbers

Developed Method			
Mode #	Using 20 modes	Using 30 modes	Using 40 modes
1	1.00	1.00	1.00
2	1.00	1.00	1.00
3	1.00	1.00	1.00
4	1.00	1.00	1.00
5	1.00	1.00	1.00
6	0.96	0.97	0.99
7	0.98	0.99	0.99
8	0.99	1.00	1.00
9	0.98	1.00	1.00
10	0.99	0.99	1.00
11	0.99	0.99	1.00

Table 2.3 MAC numbers (continued)

12	0.94	0.99	1.00
13	0.98	0.98	0.99
14	0.95	0.98	0.99
15	0.96	0.98	0.99
16	0.85	0.97	0.99
17	0.94	0.95	0.98
18	0.98	1.00	1.00
19	0.10	0.95	0.99
20	0.05	0.98	0.99

2.4.2. Comparison of Computational Time Required

Developed structural modification method is compared with Özgüven's Method by means of computational time required to obtain receptance of the modified structure. For this comparison, three cases are considered. In the first case, size of the original system and the number of common DOFs are kept constant in both methods. In the second case, sizes of the original and the modifying parts are kept constant while the number of common DOFs is changed. In the third case, sizes of the original structure, the modifying part and the number of common DOFs are changed in order to keep the size of the modified system constant. Following steps are included for the calculation of the computational time for the developed method and Özgüven's method.

For the developed method:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. Calculation of mass and stiffness matrices of the modified system
4. Solving the EVP for the modified system.
5. Calculation of the receptance of the defined DOF of the modified system in the range of frequency of interest.

For Özgüven's Method:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. For the frequency range of interest:
    a. Calculation of the receptance of the original system.
    b. Calculation of the dynamic structural modification matrix.
    c. Calculate the receptance of the desired DOF of the modified system.

The calculations are done on a computer having a processor Intel Core i3-2310M CPU @ 2.10 GHz with 4,00 GB of RAM.

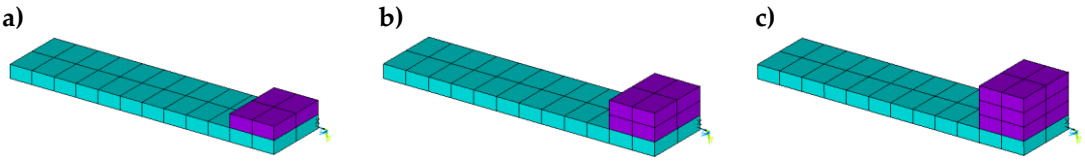


Figure 2.4 Finite element models used in Case 1: Green-original structure, purple-modifying part, size of the modifying part: a) 54, b) 81, c) 108

In this case, the effect of modifying system size (hence, the size of the modified system) on the computational time is studied. Common number of DOFs is 27 in all cases. The sizes of the FEM considered in this comparison are presented in Table 2.4. For the cases when first 20-40-80 modes of the original system are included in the calculations. In the computations, the frequency range of interest is considered as 500-1200 rad/s utilizing 351 frequency points. From the results (See Table 2.5) it can be observed that as the number of DOFs in the modifying part; hence, the modified structure, increases, time required to determine the receptance increases as well. However, it can be observed that proposed method decreases the computational time significantly compared to Özgüven's Method. On the other hand, it should be noted that developed method cannot be used if the experimental receptance of the original structure is at hand. In that case, Özgüven's Method is the fastest choice as indicated by [62] for systems without additional DOFs.

Table 2.4 Sizes of the models: Case 1

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	54	243	27
b	216	81	270	27
c	216	108	297	27

Table 2.5 Comparison of calculation times: Case 1

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.05	0.07	0.11	0.02	0.02	0.04
b	0.10	0.12	0.18	0.02	0.03	0.06
c	0.17	0.19	0.27	0.04	0.05	0.09
Model	Özgüven's Method [s]					
	including the FRF calculation of the original structure			excluding the FRF calculation of the original structure		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	3.31	3.84	4.57	3.04	3.19	3.26
b	5.81	6.25	6.90	5.44	5.62	5.82
c	8.70	9.59	10.01	8.31	8.58	8.68

Also, for FEM b) in Figure 2.11, modal analysis computational times of the modified system are compared by performing the modal analysis by ANSYS and by MATLAB. For both solutions, only modal analysis is included in computational times. It takes 0.50 seconds to obtain 40 modes by ANSYS, while it takes 0.10 seconds by MATLAB.

Table 2.6 represents the computational times of modal analysis for obtaining 20 modes of the modified finite element models a) to c) which are also plotted in Figure 2.5 and Figure 2.6. It is observed that solving the eigenvalue problem for a constant number of modes results in a linear increase in computational time, both in ANSYS and MATLAB, if system size increases. Therefore, it is possible to predict the computational time required for the solution of the eigenvalue problem for any system size.

Table 2.6 Modal analysis computational times: Case 1

Model	ANSYS [s]	MATLAB [s]
	20 modes	20 modes
a	0.44	0.03
b	0.50	0.08
c	0.56	0.13

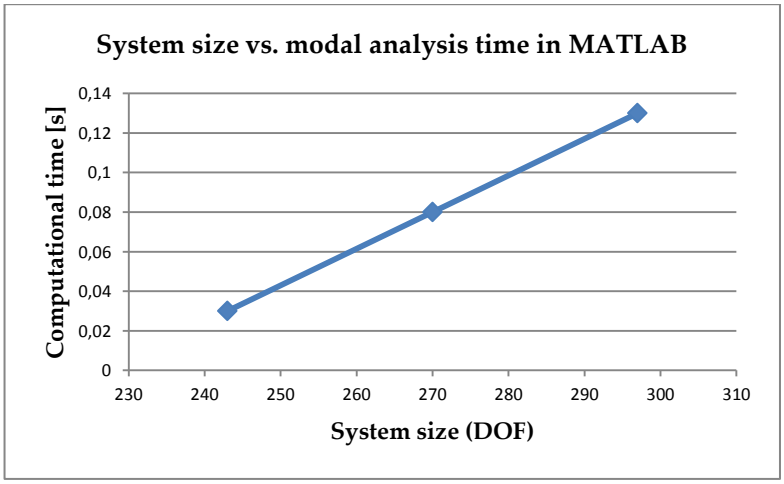


Figure 2.5 Modal analysis time vs. system size obtained in MATLAB

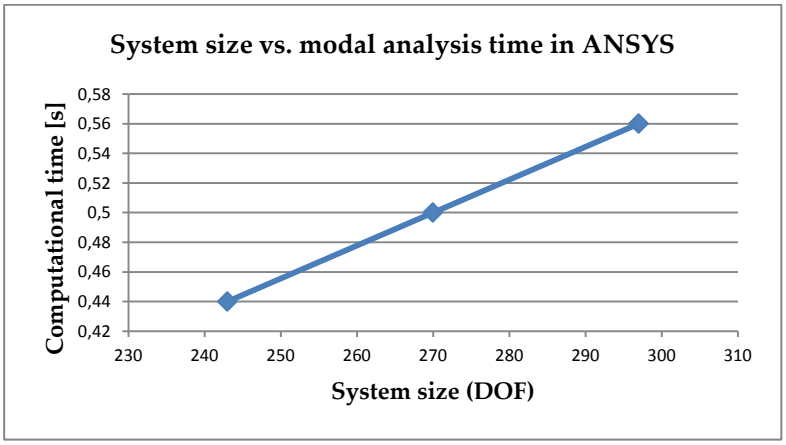


Figure 2.6 Modal analysis time system size obtained in ANSYS

In the second case, sizes of modifying system and the original systems are kept constant, where the number of common DOFs is changed as 18, 27, and 36 as shown in Figure 2.7. Table 2.7 gives the sizes of the models used; whereas, Table 2.8 presents the calculation time considering the first 20, 40 and 80 modes of the original system in both methods. The frequency range of interest is the same as the first case utilizing 351 frequency points. It can be seen that as the number of common DOFs increases, resulting in decrease in the total number of DOFs of the modified system, computation time required decreases slightly for both methods. However, if both methods are compared, the proposed method results in a drastic decrease in the computational time.

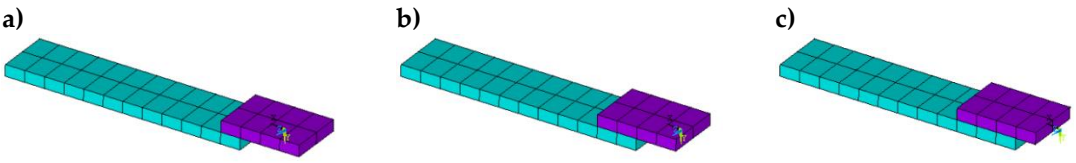


Figure 2.7 Finite element models used in Case 2: Blue-original Structure, purple-modifying part, size of the common DOFs: a) 18, b) 27, c) 36

Table 2.7 Sizes of the models: Case 2

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	90	288	18
b	216	90	279	27
c	216	90	270	36

Table 2.8 Comparison of calculation times: Case 2

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.13	0.16	0.22	0.04	0.05	0.08
b	0.12	0.14	0.20	0.03	0.05	0.07
c	0.11	0.14	0.20	0.02	0.04	0.06
Model	Özgüven's Method [s]					
	including the FRF calculation of the original structure			excluding the FRF calculation of the original structure		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	6.82	7.18	8.14	6.43	6.60	6.79
b	6.58	7.05	7.81	6.22	6.50	6.59
c	6.25	6.63	7.49	5.82	6.15	6.32

In the third case, the number of total DOFs of the modified system is kept constant by changing the sizes of the, original structure, modifying part and the common DOF. The finite element models used are presented in Figure 2.8; whereas, the sizes of part are presented in Table 2.9. Results obtained by utilizing the first 20, 40 and 80 modes of the original structure are given in Table 2.10. Identical frequency range of interest is used as in the previous cases. It can be observed from the results that as the number of DOFs in the modifying structure increases, solution time also increases for both methods, even though the number of common DOFs or the number of DOFs in the original structure decreases. Similar to the previous cases, the proposed requires significantly less computational time compared to Özgüven's method.

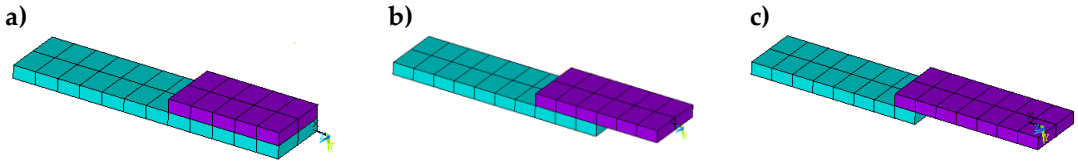


Figure 2.8 Finite element models used in Case 3: Green-original structure, purple-modifying part, size of the modifying part: a) 108, b) 126, c) 144

Table 2.9 Sizes of the models: Case 3

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	108	270	54
b	180	126	270	36
c	144	144	270	18

Table 2.10 Comparison of calculation times: Case 3

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.14	0.17	0.22	0.02	0.03	0.07
b	0.22	0.24	0.32	0.05	0.06	0.09
c	0.30	0.33	0.44	0.08	0.09	0.14
Model	Özgüven's Method [s]					
	including the FRF calculation of the original structure			excluding the FRF calculation of the original structure		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	7.75	8.27	8.94	7.40	7.47	7.52
b	9.84	10.14	10.67	9.44	9.77	9.88
c	12.03	12.33	12.86	11.73	12.15	12.21

In order to show the decrease in computational time more clearly, similar comparisons are performed on larger finite element models. In the previous case studies, analyses are performed for using different number of modes of the original system (20, 40, 80 modes) and for the proposed method, all modes of the modified system are obtained and the same number of modes (5 modes) of the modified system is included in the solutions. However, in the proposed method, not all the modes of the modified system have to be obtained and obtaining only the desired number of modes in the eigenvalues solution also decreases the computational time significantly. In Figure 2.9, FEM of a cantilever beam and the node for which FRF is calculated are presented. System sizes of the cantilever beam are given in Table 2.11. In Figure 2.10, FRFs of the node, shown in Figure 2.9, obtained by the proposed and Özgüven's methods utilizing 20 and 40 modes of the original system together with the exact FRF around the 5^{th} mode of the system are plotted. The figure includes the exact result, results obtained by Özgüven's method using 20 and 40 modes of the original system and the results obtained by the proposed method by using different number of modes of the original and modified systems. For the case utilizing 20 modes of the original system; 10 and all modes of the modified system are used. For the case utilizing 40 modes of the original system; 10, 20 and all modes of the modified system are used. Results obtained from the proposed method have a good agreement with Özgüven's method when all modes of the original system are included. However, it is seen that using less number of modes of the modified system for the proposed method also gives accurate results compared to the Özgüven's method and it requires less computational time.

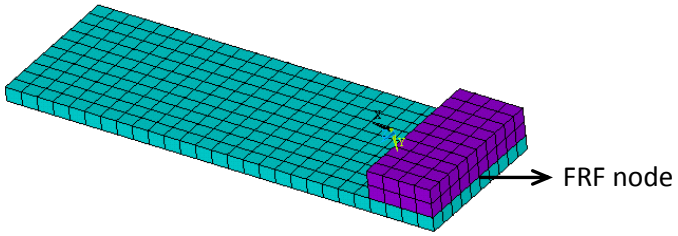


Figure 2.9 FEM of the cantilever beam

Table 2.11 Properties of the finite element cantilever beam model

Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
1782	495	2112	165

In order to compare the results with the exact solution, normalized root mean square error is used as an error criterion.

Root mean square error is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(FRF_{estimated,i} - FRF_{exact,i}\right)^2}{n}}. \tag{2.18}$$

where n is the total number of frequency points, $FRF_{estimated,i}$ is the FRF found for the i^{th} point and $FRF_{exact,i}$ is the exact value of the FRF at the i^{th} point. Root mean square error is normalized to the average of the exact FRF values, $\overline{FRF_{exact}}$, throughout the frequency region.

Normalized root mean square error:

$$NRMSE = \frac{RMSE}{\overline{FRF_{exact}}}. \tag{2.19}$$

NRMSE results for the current case are presented in Table 2.12. It is seen both in this table and in Figure 2.10 that for the same accuracy with Özgüven's method, for the proposed method, not all the modes of the modified system is needed to be obtained and/or included in the analysis which makes the solution time much shorter. It can be seen that, increasing the number of modes of the original system increases the accuracy of the FRF calculation; whereas, obtaining and using a limited number of modes of the modified structure does not affect the accuracy. Therefore, obtaining a limited number of modes of the modified structure it is possible to decrease computational effort significantly.

Table 2.12 NRMSE of the Cases

CASE	**NRMSE**
Özgüven 20 modes	2.068
Developed method 20/10 modes	2.073
Developed method 20/20 modes	2.068
Developed method 20/all modes	2.068

Table 2.12 NRMSE of the Cases (continued)

Özgüven 40 modes	1.015
Developed method 40/20 modes	1.016
Developed method 40/all modes	1.015

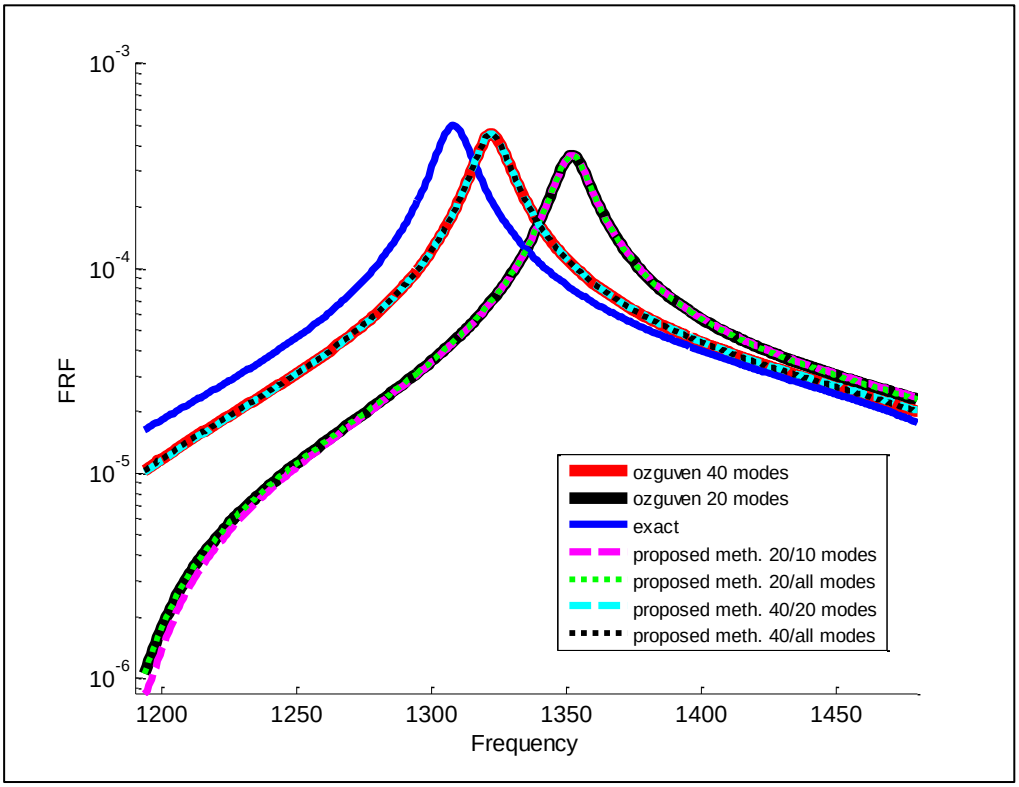


Figure 2.10 FRFs obtained by the developed method and Özgüven's method using different number of modes

Comparisons of computational times are performed by a similar matter defined before on larger DOF systems. In the first case, the effect of modifying system size (hence, the size of the modified system) on the computational time is studied. Common number of DOFs is 165 in all cases. The sizes of the FEM considered in this comparison are presented in Table 2.13. In the computations, the frequency range of interest considered is 7500-9300 rad/s utilizing 361 frequency points. For both methods, the number of modes of the original system included in the analysis is 20 and 40. For the developed method, when 20 modes of the original system is utilized, for FRF calculation, 10, 20 and all modes of the modified system are used; whereas, for the case of 40 modes of, for FRF calculation, 20 and all modes of the modified system are used. It can be observed that proposed method decreases the computational time significantly compared to Özgüven's Method. It is seen that the very good accuracy in FRF calculations can be obtained by the proposed method utilizing some of the modes of the modified structure which decreases the computational times required as well. Computational time increases by increasing the size of the modifying part and

increasing the number of modes used for both methods. However, comparing to Özgüven's method, proposed method provides significantly less computational times resulting in the same accuracy.

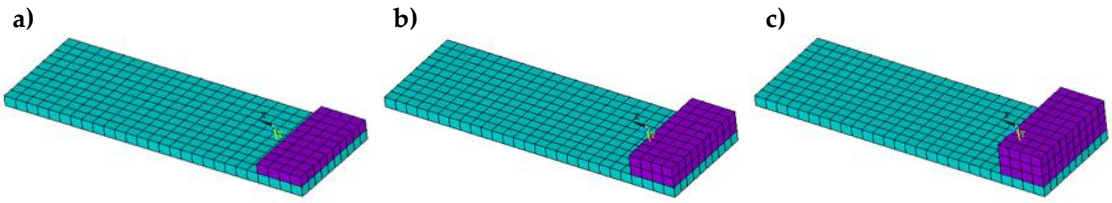


Figure 2.11 Finite element models used in Case 1: Green-original structure, purple-modifying part, size of the modifying part: a) 330, b) 495, c) 660

Table 2.13 Sizes of the models: Case 1

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	1782	330	1947	165
b	1782	495	2112	165
c	1782	660	2277	165

Table 2.14 Comparison of calculation times: Case 1

Model	Developed Method [s]				
	including the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	1.23	1.23	1.45	1.43	1.71
b	2.49	2.51	3.80	2.74	4.23
c	4.32	4.43	7.7	4.67	8.39
Model	excluding the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	0.02	0.02	0.16	0.02	0.24
b	0.02	0.02	1.04	0.02	1.17
c	0.02	0.02	2.47	0.02	2.63

Model	Özgüven's Method [s]			
	including the FRF calculation of the original structure		excluding the FRF calculation of the original structure	
	20 modes	40 modes	20 modes	40 modes
a	724.46	739.74	690.64	693.34
b	1349.72	1385.66	1315.36	1337.70
c	2273.74	2425.24	2237.98	2376.32

In the second case, sizes of the modifying structure and the original structure are kept constant, where the number of common DOFs is changed as 99, 165 and 231 as shown in Figure 2.12 and given in Table 2.15. Table 2.15 gives the sizes of the models used; whereas, Table 2.16 presents the calculation times. The frequency range of interest is the same as the previous case utilizing 361 frequency points. It can be seen that as the number of common DOFs increases, resulting in decrease in the total number of DOFs of the modified system, computational time required decreases for both methods. However, similar to the previous case, developed method is significantly faster than Özgüven's method, since it is a modal based method.

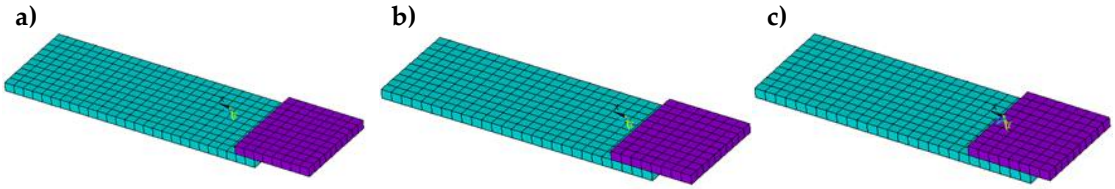


Figure 2.12 Finite element models used in Case 2: Blue-original Structure, purple-modifying part, size of the common DOFs: a) 99, b) 165, c) 231

Table 2.15 Sizes of the models: Case 2

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	1782	594	2277	99
b	1782	594	2211	165
c	1782	594	2145	231

Table 2.16 Comparison of calculation times: Case 2

Model	Developed Method [s]				
	including the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	3.76	3.73	7.05	3.98	7.63
b	3.59	3.62	6.07	3.88	6.69
c	3.57	3.58	5.29	3.87	5.66
Model	excluding the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	0.02	0.02	2.51	0.02	2.92
b	0.02	0.02	2.08	0.02	2.31
c	0.02	0.02	1.38	0.02	1.55

Table 2.16 Comparison of calculation times: Case 2 (continued)

Model	Özgüven's Method [s]			
	including the FRF calculation of the original structure		excluding the FRF calculation of the original structure	
	20 modes	40 modes	20 modes	40 modes
a	1893.12	1906.44	1858.02	1860.99
b	1807.13	1816.38	1771.91	1773.58
c	1725.94	1739.92	1690.66	1691.90

In the third case, the number of total DOFs of the modified system is kept constant by changing the sizes of the, original structure, modifying part and the common DOFs. The finite element models used are presented in Figure 2.13; whereas, the sizes of the structures are given in Table 2.17. Identical frequency range of interest and number of modes are used as in the previous cases. It can be observed from the results that as the number of DOFs in the modifying structure increases, solution time also increases for both methods, even though the number of common DOFs or the number of DOFs in the original structure decreases. Similar to the previous cases, the proposed method requires significantly less computational time compared to Özgüven's method.

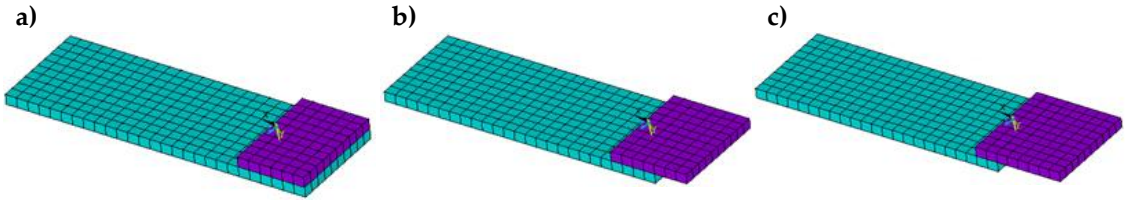


Figure 2.13 Finite element models used in Case 3: Green-original structure, purple-modifying part, size of the modifying part: a) 462, b) 528, c) 594

Table 2.17 Sizes of the models: Case 3

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	1782	462	2013	231
b	1650	528	2013	165
c	1518	594	2013	99

Table 2.18 Comparison of calculation times: Case 3

Model	Developed Method [s]				
	including the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	2.248	2.356	2.696	2.501	3.329
b	2.841	2.825	4.646	3.026	5.083
c	3.565	3.575	6.928	3.845	7.441
Model	excluding the modal solution				
	20/10 modes	20/20 modes	20/all modes	40/20 modes	40/all modes
a	0.015	0.016	0.389	0.016	0.604
b	0.016	0.016	1.449	0.016	1.616
c	0.016	0.016	2.545	0.016	2.771

Model	Özgüven's Method [s]			
	including the FRF calculation of the original structure		excluding the FRF calculation of the original structure	
	20 modes	40 modes	20 modes	40 modes
a	1130.8	1143.7	1095.72	1096.37
b	1345.2	1380.3	1154.55	1156.58
c	1596.8	1621.1	1388.44	1402.32

CHAPTER 3

NONLINEAR DYNAMIC RESPONSE ANALYSIS

3.1. Nonlinear Systems

Equation of motion of a nonlinear system is given in Equation (3.1) where $[M]$, $[C]$, $[H]$, and $[K]$ matrices are the mass, viscous damping, structural damping and stiffness matrices and i is the unit imaginary number. $\{x(t)\}$ is the response vector and $\{f(t)\}$ is the external forcing acting on the system. $\{f_N(t)\}$ is the nonlinear internal forcing

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+i[H]\{x(t)\}+[K]\{x(t)\}+\{f_N(t)\}=\{f(t)\} \tag{3.1}$$

If the external forcing is harmonic, then the response of the system is also assumed to be harmonic. External forcing and response of the structure can be defined as follows

$$\{f(t)\}=\mathrm{Im}\left(\sum_{m=0}^{\infty}\{F\}_m e^{im\omega t}\right), \tag{3.2}$$

$$\{x(t)\}=\mathrm{Im}\left(\sum_{m=0}^{\infty}\{X\}_m e^{im\omega t}\right), \tag{3.3}$$

where m is the harmonic index. $\{F\}_m$ and $\{X\}_m$ are the complex amplitudes of the external forcing and response of the structure for the m^{th} harmonic. According to Kuran and Özgüven [63], for systems having harmonic response, nonlinear internal forcing can also be assumed as harmonic as follows

$$\{f_N(t)\}=\mathrm{Im}\left(\sum_{m=0}^{\infty}\{F_N\}_m e^{im\omega t}\right), \tag{3.4}$$

where $\{F_N\}_m$ is the complex amplitude of the nonlinear internal forcing of the corresponding harmonic. Here, it is important to note that $\{F_N\}_m$ depends on the relative displacement of the DOFs that nonlinearities are attached. Since external forcing and response are defined utilizing multiple harmonics, nonlinear internal forcing should also be expressed in terms of multiple harmonics.

Substituting Equations (3.4), (3.2), and (3.3) into (3.1) results in the equation for the m^{th} harmonic as

$$\begin{aligned}[K]\{X\}_0+\{F_N\}_0&=\{F\}_0,\\ \left[-(m\omega)^2[M]+i[H]+[K]\right]\{X\}_m+\{F_N\}_m&=\{F\}_m.\end{aligned} \tag{3.5}$$

In this thesis, harmonic balance method with modal superposition approach is employed for determination of the nonlinear algebraic equations by considering only the first harmonic term. Therefore, Equation (3.5) becomes

$$\begin{aligned} [K]\{X\}_0 + \{F_N\}_0 &= \{F\}_0, \\ \left[-\omega^2[M] + i[H] + [K]\right]\{X\}_1 + \{F_N\}_1 &= \{F\}_1. \end{aligned} \tag{3.6}$$

3.2. Harmonic Balance Method

For the solutions of nonlinear systems, there exist time domain methods as well as frequency domain methods. Harmonic balance method (HBM) is a frequency domain method which is used in this work in order to quasi-linearize the nonlinearities to be used in the frequency domain analysis. For the application of HBM, it is important to decide the number of harmonics to be used. For strong nonlinearities, using many harmonics may be required; however, in this study, HBM with a single harmonic is employed. In reference [49], the same type of nonlinearity, 1-D contact model with normal load variation (See 3.4.2.2), is employed as in this study. It is seen that using single harmonic in HBM provides accurate results comparing to time integration results.

Considering the equation of motion (EOM) defined by Equation (3.1), external forcing and response of the system are expressed as Fourier series

$$\{f\} = F_0 + \sum_{m=1}^{n}\left[(F_s)_m \sin(m\theta) + (F_c)_m \cos(m\theta)\right], \tag{3.7}$$

$$\{x\} = X_0 + \sum_{m=1}^{n}\left[(X_s)_m \sin(m\theta) + (X_c)_m \cos(m\theta)\right], \tag{3.8}$$

where

$$\theta = \omega t, \tag{3.9}$$

m is the harmonic index and n is the number of harmonics used. Similarly, the nonlinear internal forces can be expressed as Fourier series

$$\{f_N\} = F_{N0} + \sum_{m=1}^{n}\left[(F_{Ns})_m \sin(m\theta) + (F_{Nc})_m \cos(m\theta)\right], \tag{3.10}$$

where

$$\begin{aligned} F_{N0} &= \frac{1}{2\pi}\int_0^{2\pi} f_N \, d\theta, \\ (F_{Ns})_m &= \frac{1}{\pi}\int_0^{2\pi} f_N \sin(m\theta)\, d\theta, \\ (F_{Nc})_m &= \frac{1}{\pi}\int_0^{2\pi} f_N \cos(m\theta)\, d\theta. \end{aligned} \tag{3.11}$$

Using only the first harmonic term, i.e $n = 1$, and omitting the subscript m

$$
\begin{aligned}
F_{N0} &= \frac{1}{2\pi}\int_0^{2\pi} f_N d\theta, \\
F_{Ns} &= \frac{1}{\pi}\int_0^{2\pi} f_N \sin(\theta) d\theta, \\
F_{Nc} &= \frac{1}{\pi}\int_0^{2\pi} f_N \cos(\theta) d\theta.
\end{aligned}
\tag{3.12}
$$

3.2.1. Modal Superposition Method

Cigeroglu et al. [3], [4], [5] proposed a modal superposition method coupled with harmonic balance method, which becomes significantly useful if the number of nonlinear elements is large. Unlike receptance methods, nonlinear algebraic equations to be solved depend on the number of modes used in the modal superposition approach, not the number of nonlinear DOFs. In this method, the response of the system is written in terms of the mode shapes of the linear system by assuming a single harmonic motion in complex form as follows

$$
\{x(t)\} = \sum_{r=1}^{N_m} a_r \{\phi_r\} e^{i\omega t} = [\phi]\{a\} e^{i\omega t}, \tag{3.13}
$$

where N_m is the number of modes used in the modal superposition approach, $[\phi]$ is the mode shape matrix of the linear system. a_r is the complex coefficient of the r^{th} mode shape vector, $\{\phi_r\}$.

Substituting Equation (3.13) into Equation (3.1) and pre-multiplying by $[\phi]^T$ results in

$$
\begin{aligned}
&-\omega^2 [\phi]^T [M][\phi]\{a\} + i\omega [\phi]^T [C][\phi]\{a\} \\
&+[\phi]^T \left(i[H]+[K]\right)[\phi]\{a\} + [\phi]^T \{F_N\} = [\phi]^T \{F\},
\end{aligned}
\tag{3.14}
$$

Assuming proportional damping and $[\phi]$ to be mass normalized, Equation (3.14) becomes

$$
\left[(1+i\gamma)[\Omega] - \omega^2 [I] + i\omega [C_r]\right]\{a\} + [\phi]^T \{F_N\} = [\phi]^T \{F\}, \tag{3.15}
$$

where γ is the loss factor, $[I]$ is the identity matrix and

$$
[\Omega] = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_r^2 \end{bmatrix}, \quad [C_r] = \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 2\xi_r\omega_r \end{bmatrix}. \tag{3.16}
$$

where ω_r and ξ_r are the r^{th} natural frequency and the damping ratio of the linear system.

3.3. Solution of Nonlinear Algebraic Equations

After the nonlinear differential equations are converted into a set of nonlinear algebraic equations, an iterative process is needed to solve these equations. In this thesis, in order to obtain nonlinear response Newton's method with arc-length continuation is used. In the cases where jump phenomena occurs [49], solution of the nonlinear equation set may give more than one result for a single frequency, i.e. solution path may turn back; hence, for those cases application of a path following method is needed.

3.3.1. Newton's Method

Nonlinear algebraic equation set which is to be solved can be expressed as follows

$$\left\{R\left(\{x\},\omega\right)\right\}=\{0\}, \tag{3.17}$$

$$\left\{R\left(\{x\},\omega\right)\right\}=\left[(1+i\gamma)[\Omega]-\omega^2[I]+i\omega[C_r]\right]\{a\}+[\phi]^T\{F_N\}-[\phi]^T\{F\}=\{0\}. \tag{3.18}$$

Using Newton's method, solution of a set of nonlinear algebraic equations can be obtained iteratively as follows

$$\{x\}_{new}=\{x\}_{old}+\{\Delta x\}, \tag{3.19}$$

$$\{x\}^{j+1}=\{x\}^{j}-\left[J\left(\{x\}^{j},\omega\right)\right]^{-1}\cdot\left\{R\left(\{x\}^{j},\omega\right)\right\}. \tag{3.20}$$

where j is the iteration number, $\{x\}^j$ is the solution vector at the j^{th} iteration and $\left[J\left(\{x\}^j,\omega\right)\right]$ is the Jacobian matrix. Error criterion is given in Equation (3.21). When the criterion is achieved, solution vector $\{x\}$ is decided for the corresponding frequency ω and it is used as the initial guess for the next frequency point.

$$\max\left(abs\left\{R\left(\{x\},\omega\right)\right\}\right)\le 10^{-8}. \tag{3.21}$$

3.3.2. Arc-Length Continuation Method

In some cases frequency response curve turns back, for which increasing the frequency results in a jump up or down in the frequency response and solution cannot be obtained if Newton's method is used. Determinant of the Jacobian matrix becomes zero or very small at the turning points; hence, the inverse of the Jacobian cannot be calculated, i.e. the Jacobian is singular. Path following method should be used for such problems. Arc-length continuation method is a path following method for which a new parameter, i.e the arc length parameter, is added to the set of nonlinear algebraic equations which results in a nonsingular Jacobian. Arc-length parameter, s is defined as the radius of a hypothetical sphere on which the next solution point is to be obtained [64].

For this new system, frequency, ω is considered as an unknown with the addition of the arc-length parameter. The new vector of unknowns becomes

$$\{q\} = \begin{Bmatrix} \{x\} \\ \omega \end{Bmatrix}, \tag{3.22}$$

The required additional equation is the equation of the n-dimensional sphere which can be expressed as follows [65], [66].

$$h\left(\{x\}_k, \omega_k\right) = \{\Delta q\}_k^T \{\Delta q\}_k - s^2 = 0, \tag{3.23}$$

where k is the index of the data point and

$$\{\Delta q\}_k = \left\{\{q\}_k - \{q\}_{k-1}\right\}. \tag{3.24}$$

Iteration step using arc-length continuation method can be written as follows

$$\{q\}_k^{i+1} = \{q\}_k^i - \left[\begin{matrix} \dfrac{\partial\left\{R\left(\{x\}_k, \omega_k\right)\right\}}{\partial\{x\}} & \dfrac{\partial\left\{R\left(\{x\}_k, \omega_k\right)\right\}}{\partial\omega} \\ \dfrac{\partial h\left(\{x\}_k, \omega_k\right)}{\partial\{x\}} & \dfrac{\partial h\left(\{x\}_k, \omega_k\right)}{\partial\omega} \end{matrix}\right]^{-1}\Bigg|_{\{x\}_k^i, \omega_k^i} \begin{Bmatrix} \left\{R\left(\{x\}_k^i, \omega_k^i\right)\right\} \\ h\left(\{x\}_k^i, \omega_k^i\right) \end{Bmatrix}, \tag{3.25}$$

where i is the iteration step, and

$$\left[\begin{matrix} \dfrac{\partial h\left(\{x\}, \omega\right)}{\partial\{x\}} & \dfrac{\partial h\left(\{x\}, \omega\right)}{\partial\omega} \end{matrix}\right]\Bigg|_{\{x\}_k^i, \omega_k^i} = \left[2\{\Delta q\}_k^i\right]^T. \tag{3.26}$$

During path following, there may exist sharp corners that the initially assigned path following parameter, i.e. step length, may not let the solution follow the path. Therefore, it is necessary to check the iteration number for each frequency point and update the steps size. Crisfield [67] proposed a formula given in Equation (3.27) that adjusts the step length by comparing the number of iterations of the previous step to the desired number of iterations n_{norm}^{iter} which is not necessarily an integer as follows

$$s_k = s_{k-1}\sqrt{\frac{n_{norm}^{iter}}{n_{k-1}^{iter}}} \tag{3.27}$$

3.4. Nonlinearity Types

For nonlinear forced response analyses, two types of nonlinearities are used in this work. These are cubic stiffness nonlinearity and dry friction nonlinearity. Two different types of dry friction nonlinearity are considered: constant normal load and variable normal load. In the following subsections, nonlinear forcing formulations by using harmonic balance method are presented.

3.4.1. Cubic Stiffness Nonlinearity

Nonlinear force in the case of cubic stiffness can be written as

$$f_N = k_c \cdot x^3 \tag{3.28}$$

where k_c is the coefficient of cubic stiffness nonlinearity. Assuming a single harmonic motion as

$$x = X_s \sin(\omega t) + X_c \cos(\omega t) \tag{3.29}$$

Nonlinear forcing can be written as follows

$$F_N = F_{Ns} \sin(\omega t) + F_{Nc} \cos(\omega t) \tag{3.30}$$

$$\begin{aligned} F_{Ns} &= \frac{1}{\pi} \int_0^{2\pi} k_c \left(X_s \sin(\theta) + X_c \cos(\theta) \right)^3 \sin(\theta) d\theta = \frac{3}{4} k_c \left({X_s}^3 + {X_c}^2 X_s \right) \\ F_{Nc} &= \frac{1}{\pi} \int_0^{2\pi} k_c \left(X_s \sin(\theta) + X_c \cos(\theta) \right)^3 \cos(\theta) d\theta = \frac{3}{4} k_c \left({X_c}^3 + {X_s}^2 X_c \right) \end{aligned} \tag{3.31}$$

where $\theta = \omega t$. Hence, the nonlinear internal forcing becomes

$$F_N = \frac{3}{4} k_c \left(\left({X_s}^3 + {X_c}^2 X_s \right) \sin(\omega t) + \left({X_c}^3 + {X_s}^2 X_c \right) \cos(\omega t) \right) \tag{3.32}$$

3.4.2. Dry Friction Nonlinearity

Adaption of dry friction damping concept to bladed disk assemblies has been studied by numerous authors throughout the history. In Figure 3.1, a general 3-D contact model can be seen. For the contact plane between mating surfaces, there are many contact pairs for each of which, contact analysis should be performed individually. Tangential relative motion can be 1-D or 2-D depending on the path of the relative motion. When the path is linear, it means that the tangential relative motion is one-dimensional. When it follows an elliptical path on contact plane, tangential relative motion is considered as two-dimensional. Normal relative motion of the mating surfaces is either considered as constant or variable. In case of constant normal load, a much simpler hysteresis loop (force-displacement relationship) can be obtained compared with the variable normal load case.

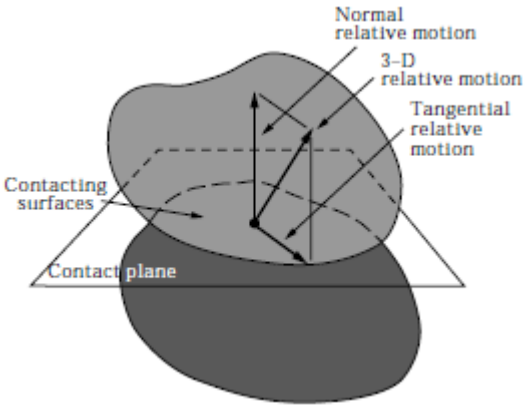


Figure 3.1 Contact Model [49]

In this thesis, as a contact model for bladed disk assembly, 1-D contact model with variable normal load is used. Contact models and friction force formulations of 1-D with constant normal load and 1-D with variable normal load cases are presented in the upcoming sections.

3.4.2.1. 1-D Contact Model with Constant Normal Load

Hysteresis loop of this simplest contact model for single harmonic motion is presented in Figure 3.2. F_N is the friction force, k is the tangential contact stiffness, A is the vibration amplitude, and μN is the slip load. Inclined lines indicates the stick state while the constant friction force lines indicate positive slip state for $F_N = \mu N$ and negative slip state for $F_N = -\mu N$. In this type of friction contact, stick to positive/negative slip and positive/negative slip to stick transitions can be easily predicted for single harmonic motion. For instance, during positive slip, when the contact point reverses its direction, stick state occurs which is followed by negative slip state and the hysteresis loop in Figure 3.2 is obtained.

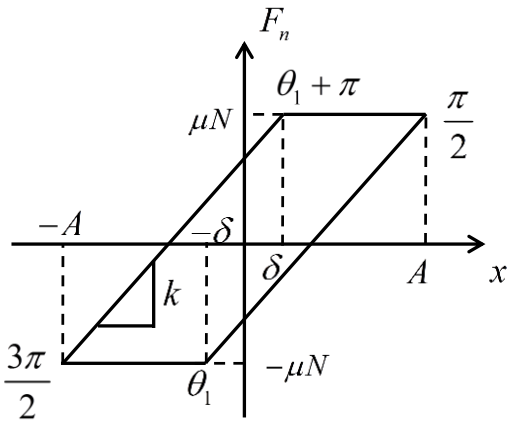


Figure 3.2 Hysteresis Loop of a 1-D Friction Element Having Constant Normal Load

Define displacement as:

$$x(t) = A\sin(\theta), \tag{3.33}$$

Friction force:

$$f_N(x) = \begin{cases} -\mu N + k(x+\delta) & \frac{\pi}{2} \le \theta \le \theta_1 \\ -\mu N & \theta_1 \le \theta \le \frac{3\pi}{2} \end{cases}, \tag{3.34}$$

where

$$\delta = \frac{2\mu N - kA}{k}, \tag{3.35}$$

$$\theta_1 = a\sin\left(\frac{kA - 2\mu N}{kA}\right) = a\sin\left(\frac{\delta}{A}\right). \tag{3.36}$$

Using HBM with one-term approximation, friction force is defined as:

$$F_N = F_{Ns}\sin(\theta) + F_{Nc}\cos(\theta), \tag{3.37}$$

where

$$F_{Ns} = \frac{1}{\pi}\int_0^{2\pi} f_N\left(A\sin(\theta)\right)\sin(\theta)\,d\theta, \tag{3.38}$$

$$F_{Ns} = \frac{2}{\pi}\left(\int_{\pi/2}^{\theta_1}\left(-\mu N + k\left(A\sin(\theta) + \frac{2\mu N - kA}{k}\right)\right)\sin(\theta)\,d\theta + \int_{\theta_1}^{3\pi/2} -\mu N\sin(\theta)\,d\theta\right), \tag{3.39}$$

$$F_{Ns} = \frac{2}{\pi}(kA - 2\mu N)\cos(\theta_1) + \frac{kA}{2\pi}(2\theta_1 - \sin(2\theta_1) - \pi), \tag{3.40}$$

and

$$F_{Nc} = \frac{1}{\pi}\int_0^{2\pi} F_N(x)\cos(\theta)\,d\theta, \tag{3.41}$$

$$F_{Nc} = \frac{2}{\pi}\left(\int_{\pi/2}^{\theta_1}\left(-\mu N + k\left(A\sin(\theta) + \frac{2\mu N - kA}{k}\right)\right)\cos(\theta)\,d\theta + \int_{\theta_1}^{3\pi/2} -\mu N\cos(\theta)\,d\theta\right), \tag{3.42}$$

$$F_{Nc} = -\frac{4\mu N - 2kA}{\pi k}(1 + \sin(\theta_1)) + \frac{4\mu N}{\pi} - \frac{kA\cos^2(\theta_1)}{\pi}. \tag{3.43}$$

3.4.2.2. 1-D Contact Model with Normal Load Variation

1-D model having normal load variation is developed by Yang et al. [48], [49]. Authors defined the analytical transitions between stick, slip, and separation and formulized the corresponding transition angles between these states. Contact model is presented in Figure 3.3 where Body 2 is considered as ground. u and v are contact coordinates defining tangential and normal relative motions and w is the relative slip motion of the contact point. k_u and k_v are the contact stiffnesses in tangential and normal directions, n_0 is the initial pre-load if it is defined as positive and initial gap if it is defined as negative, which is equal to $-k_v e$ for a gap of e. Friction contact point is assumed to obey Coulomb's friction law having a coefficient of friction of μ.

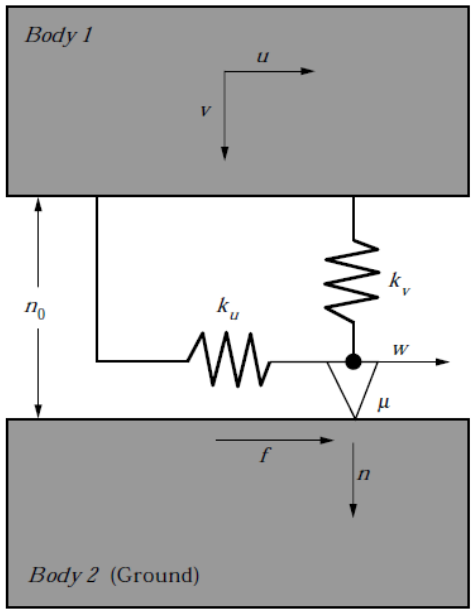


Figure 3.3 1-D Contact Model Having Variable Normal Load [49]

Variable normal load n and fiction force f can be expressed as

$$n(t)=\begin{cases} n_0 + k_v v(t), & v(t) \geq -n_0 / k_v, \\ 0, & v(t) < -n_0 / k_v, \end{cases} \tag{3.44}$$

$$f(t) = k_u \left(u(t) - w(t) \right). \tag{3.45}$$

where

$$u = a\sin(\theta), \tag{3.46}$$

$$v = b\sin(\theta + \phi). \tag{3.47}$$

Stick, slip, separation states and their transition conditions are presented in [49], which are detailed by Cigeroglu et al. [3]. For the following parameters: $a = b = 1$, $k_u = k_v = 1$, $\mu = 0.4$;

and $n_0 = 3.2$ for fully stick case, $n_0 = 1.2$ for stick-slip case, and $n_0 = 0.8$ for separation case the following friction force vs. theta and hysteresis loop plots given by Figures 3.4, to 3.9 are created.

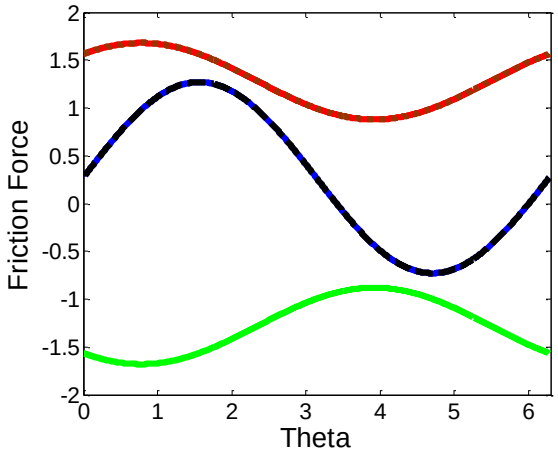


Figure 3.4 Fully Stick Case ($n_0 = 3.2, \phi = 45°$); red: μN , green: $-\mu N$, blue: exact friction force, black: friction force with HBM, brown: μN with HBM

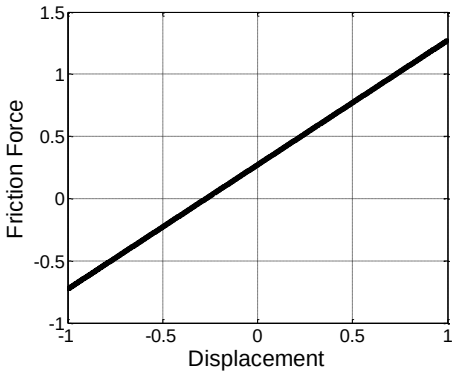


Figure 3.5 Hysteresis loop for fully stick case ($n_0 = 3.2, \phi = 45°$)

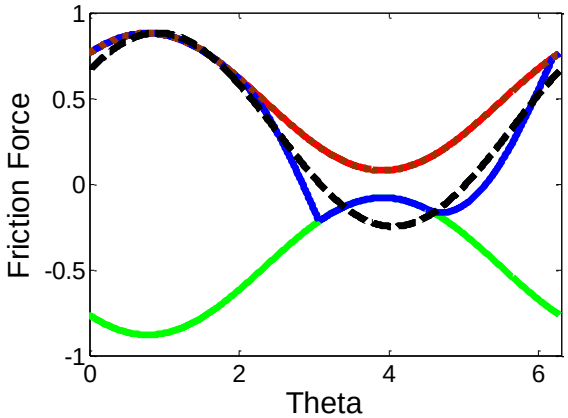


Figure 3.6 Stick-Slip Case ($n_0 = 1.2$, $\phi = 45°$); red: μN , green: $-\mu N$, blue: exact friction force, black: friction force with HBM, brown: μN with HBM

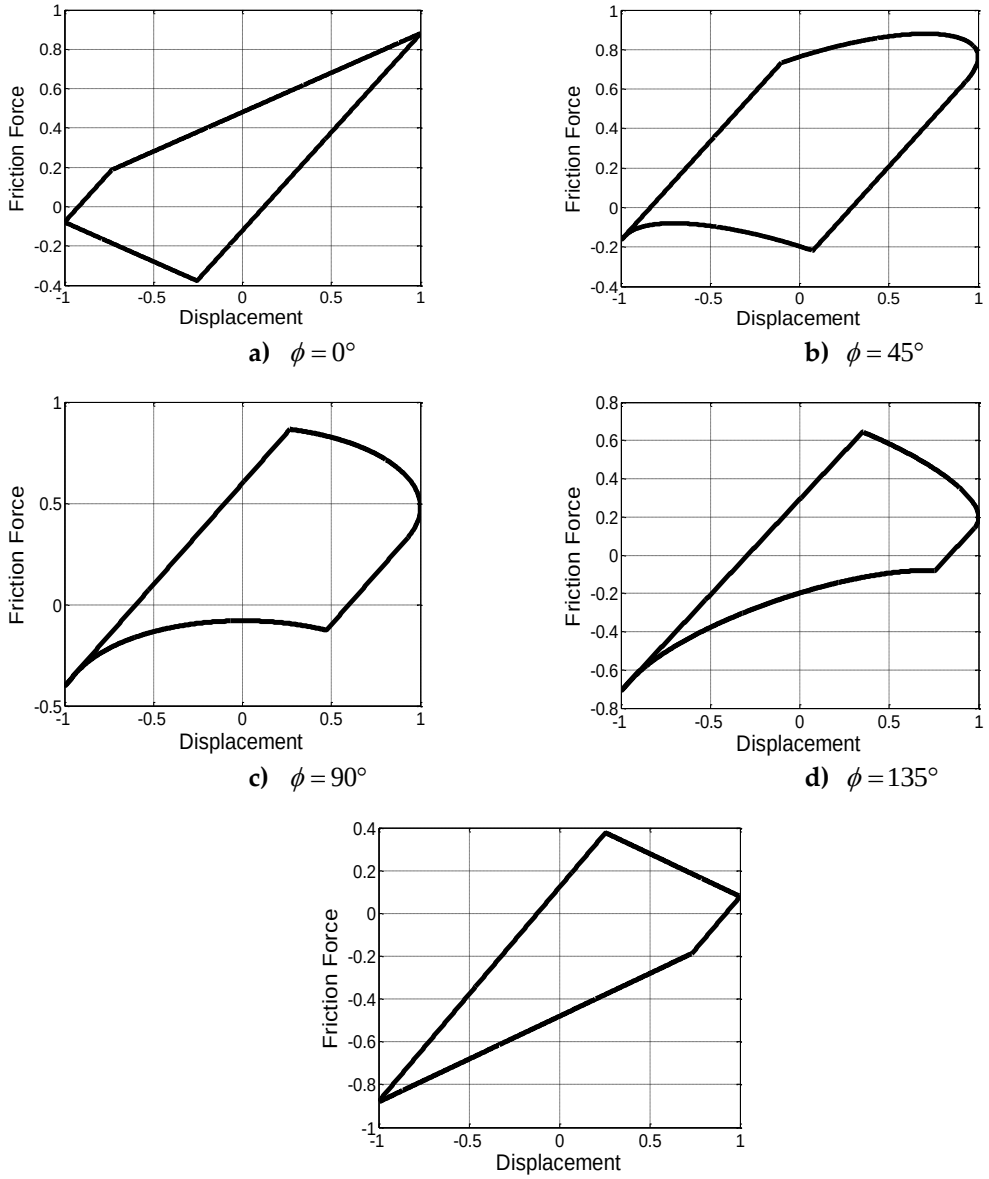


Figure 3.7 Hysteresis loops for stick-slip cases for different phase angles ($n_0 = 1.2$)

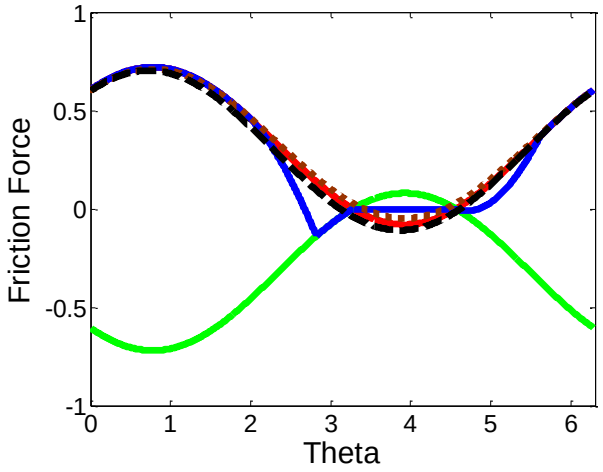


Figure 3.8 Separation Case ($n_0 = 0.8$, $\phi = 45°$); red: μN , green: $-\mu N$, blue: exact friction force, black: friction force with HBM, brown: μN with HBM

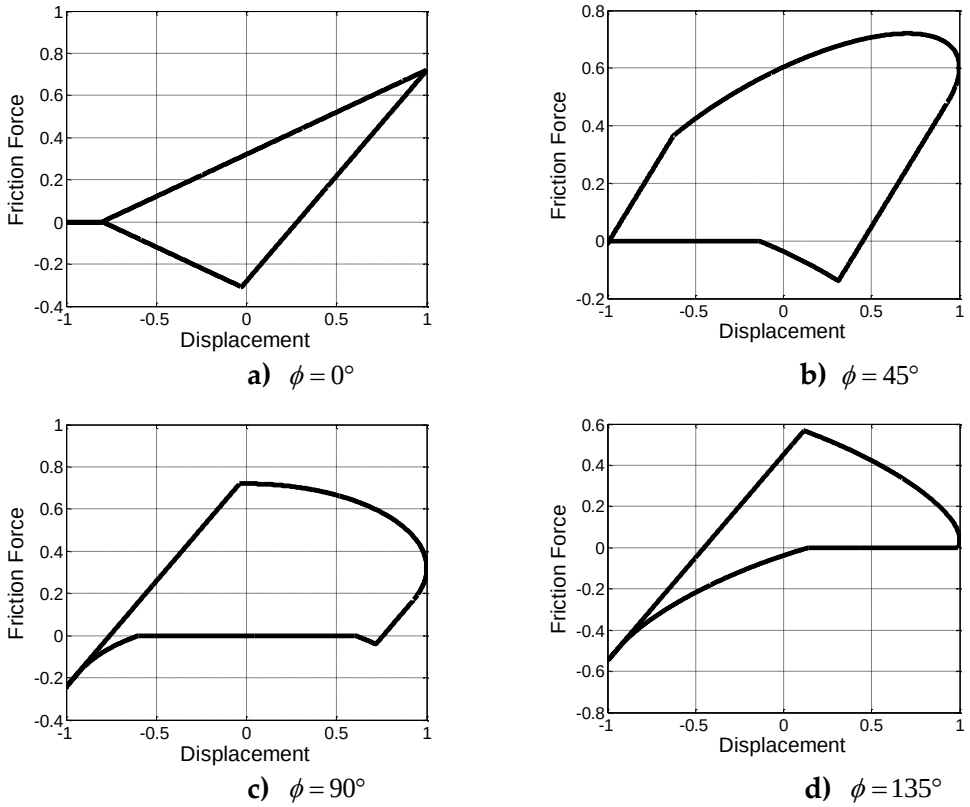


Figure 3.9 Hysteresis loops for stick-slip cases for different phase angles ($n_0 = 0.8$)

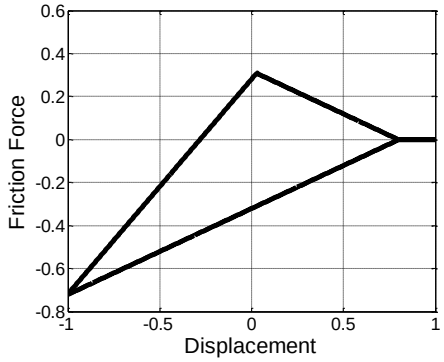


Figure 3.9 Hysteresis loops for stick-slip cases for different phase angles ($n_0 = 0.8$) (continued)

In order to validate the nonlinear response analysis code for friction nonlinearity having normal load variation, response analysis of a two DOF system given in reference [49] is repeated, for which the system parameters are given as: modal masses $m_1 = 1.0$, $m_2 = 1.0$; modal frequencies $\omega_{n1} = 1.0$, $\omega_{n2} = 1.0$; modal damping ratio $\zeta_1 = 0.01$, $\zeta_2 = 0.01$ and mass normalized mode shapes $\phi_1 = \begin{bmatrix} 0.707 & 0.707 \end{bmatrix}^T$, $\phi_2 = \begin{bmatrix} 1.0 & -0.5 \end{bmatrix}^T$; friction coefficient $\mu = 0.4$, tangential and normal stiffnesses $k_u = k_v = 1.0$ and external harmonic excitation in shear and normal directions $f_u = f_v = 1.0$. Amplitudes of displacement in tangential direction for different initial preload values are presented in Figure 3.10. Results are identical to the ones presented in [49].

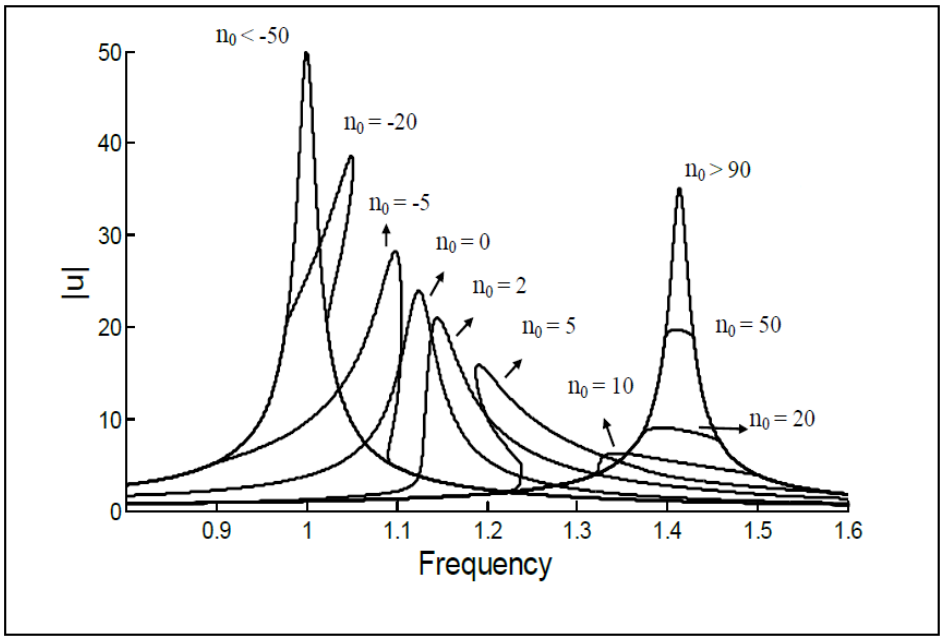


Figure 3.10 Responses of a two DOF system for different preload values

CHAPTER 4

APPLICATION OF STRUCTURAL MODIFICATION METHOD TO NONLINEAR SYSTEMS

4.1. Lumped Parameter Model

A simple lumped parameter model (See Figure 4.1) is constructed representing the modified system. 3-DOF-system is added to the original (unmodified) 4-DOF system by rigid coupling resulting in a 5-DOF-system. Modal data of the modified system is obtained from directly modeling the 5-DOF system using the new structural modification method utilizing *all information* of the unmodified system. Nonlinear forced response analysis of the modified structure is performed by using these modal data and the results obtained are compared. The system parameters used for the lumped parameter model; whereas, for the modifying part they are $M_1 = M_2 = M_3 = M_4 = 1$kg, $K_1 = K_2 = K_3 = K_4 = 10000$N/mm, $F = 50$N, $\gamma = 0.01$ and $M_5 = M_6 = M_7 = 2$kg, $K_5 = K_6 = K_7 = 20000$N/mm, $F = 50$N, $\gamma = 0.01$. Lumped model including nonlinearities in the unmodified system is presented in Figure 4.1 and corresponding response plots of the first for different k_c values are given in Figure 4.2. Another lumped model is presented for which nonlinearities are included in modifying system (See Figure 4.3) and corresponding response plots of the seventh mass for different k_c values are given in Figure 4.4.

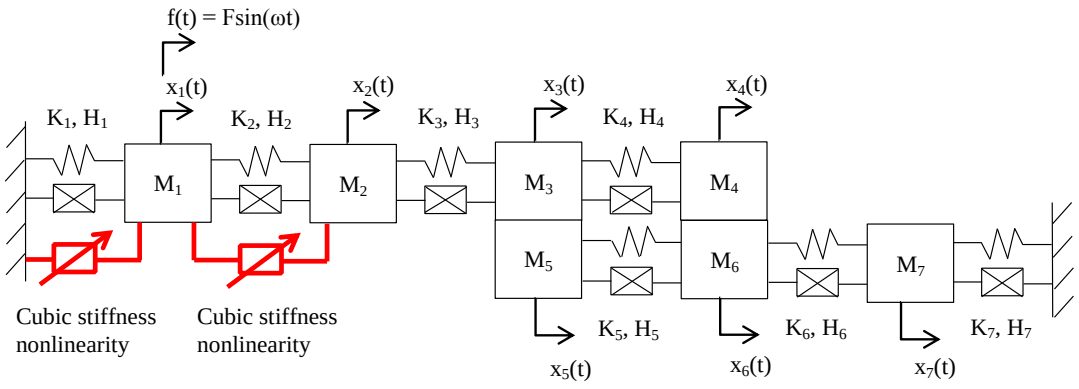


Figure 4.1 Lumped parameter model (modified system) – Case 1

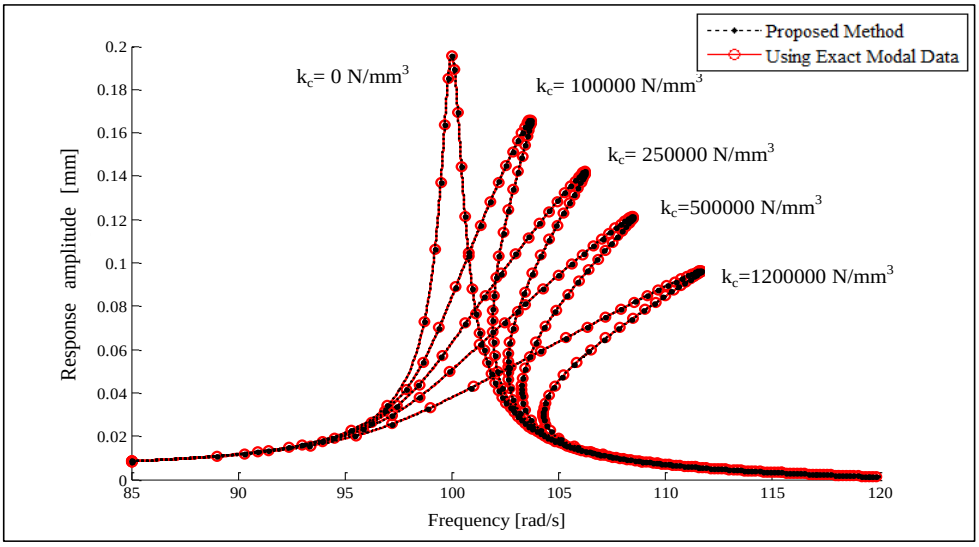


Figure 4.2 Responses of the 1st mass - Case 1

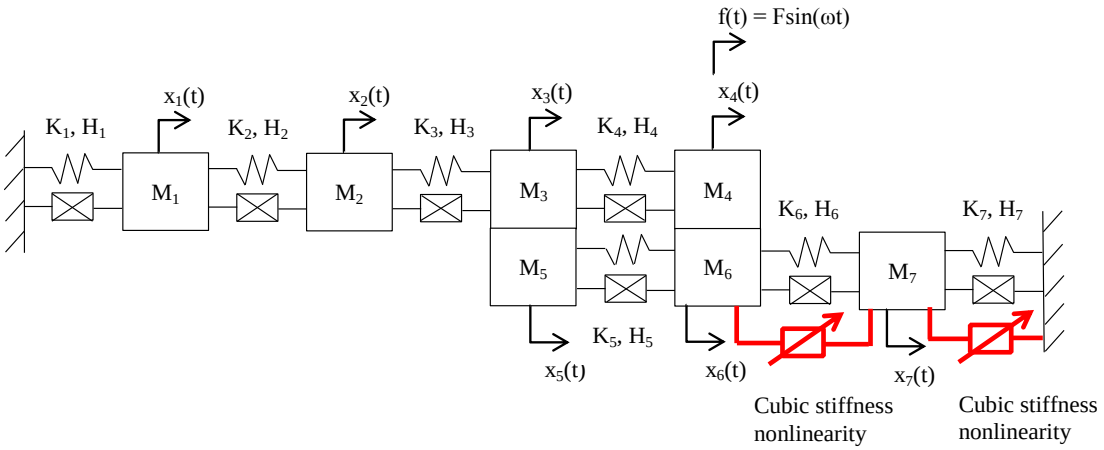


Figure 4.3 Lumped parameter model (modified system) – Case 2

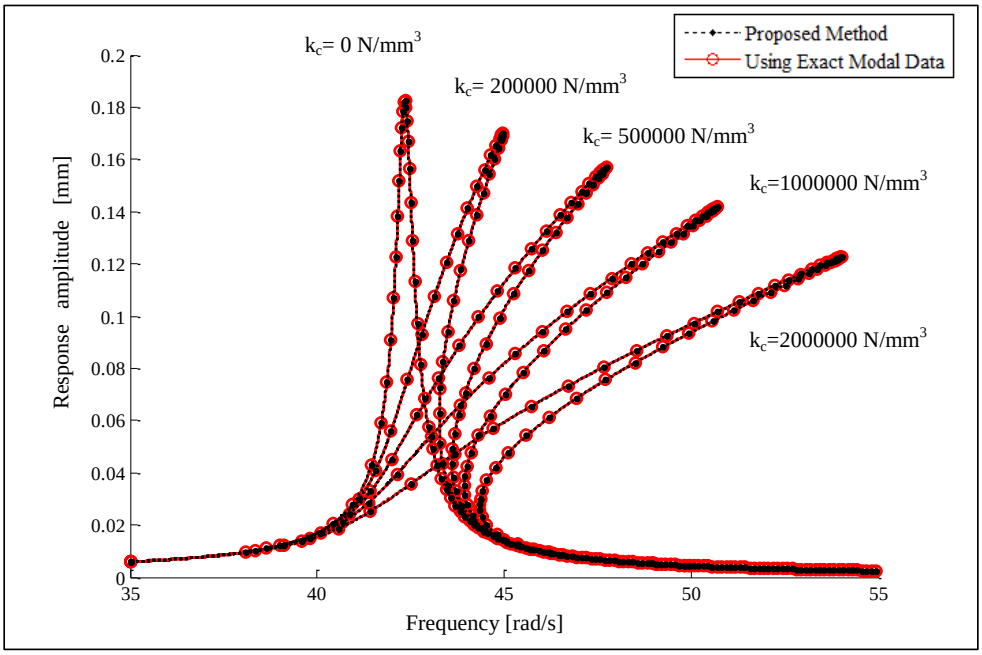


Figure 4.4 Responses of the 7th mass - Case 2

It can be seen from Figure 4.2 and Figure 4.4 that results obtained the proposed method are in exact agreement with the ones obtained by utilizing the exact modal data of the modified system.

4.2. Finite Element Model

Same concept as in the lumped mass parameter model is applied to a cantilever beam modeled in ANSYS. Since it is a large DOF system, all information of the original system is not included in the analysis. First, different number of modes is used for structural modification and modal data of the modified system is obtained. Then, using the obtained modal data, HBM with modal superposition is applied to obtain the nonlinear response of the modified beam. For using exact modal data in the nonlinear solution, modified beam is also modeled in ANSYS and nonlinear solution is repeated for modal data obtained from modal analysis in ANSYS. *5 modes* of the modified beam are included in all nonlinear response solutions.

Modified system can be seen in Figure 4.5 in which modifications are shown in purple. Cubic stiffness nonlinearities are shown in the figure defined in y direction. In Case 1, they are in the original system, while in Case 2, they are in the modifying system. External loading is also shown in Figure 4.5 in y direction. Response plots are presented in Figure 4.6 and Figure 4.7 for the DOF shown in Figure 4.5 by using k_c = 0, 10, 20, 40N/m^3; F = 50N and γ = 0.1.

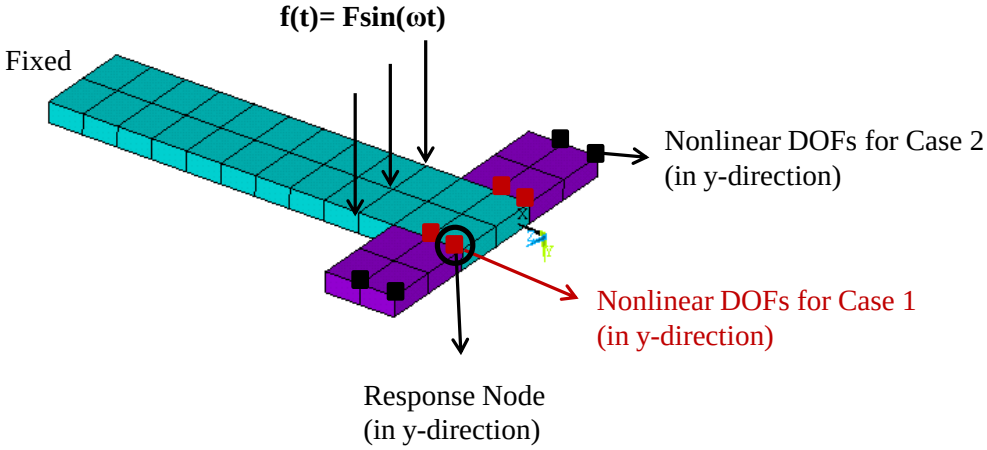


Figure 4.5 Modified system

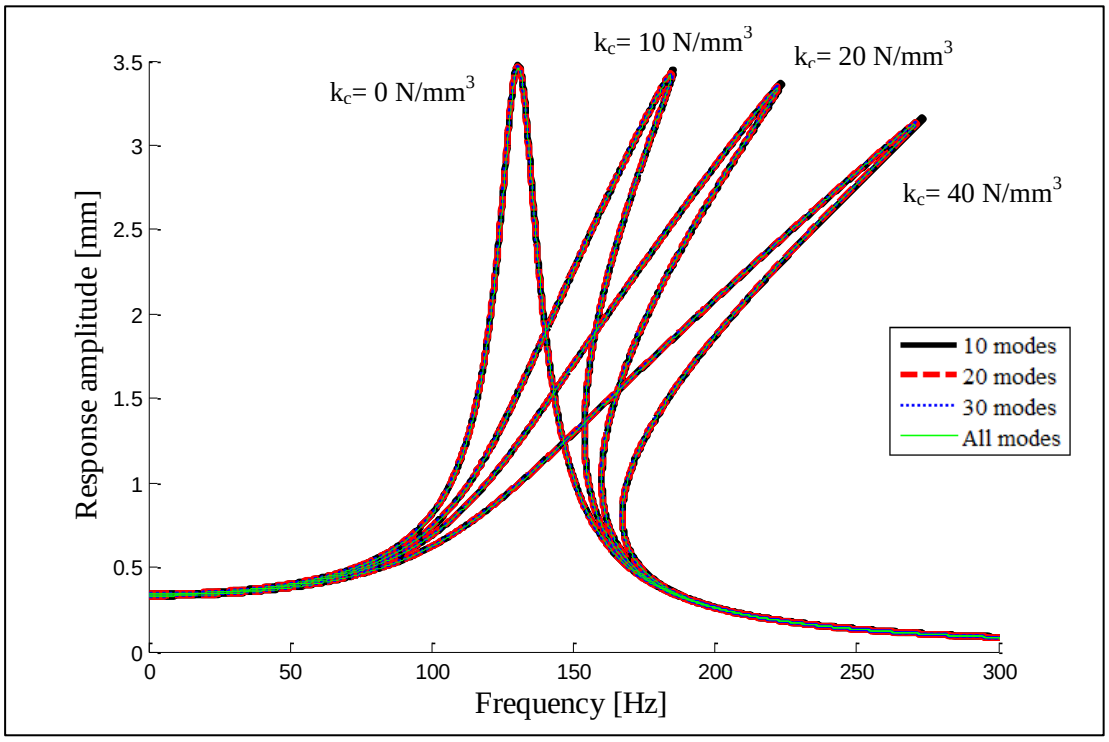


Figure 4.6 Case 1 - Nonlinear responses using 10, 20, 30 modes in formulation and using exact modal data (5 modes are used in all cases)

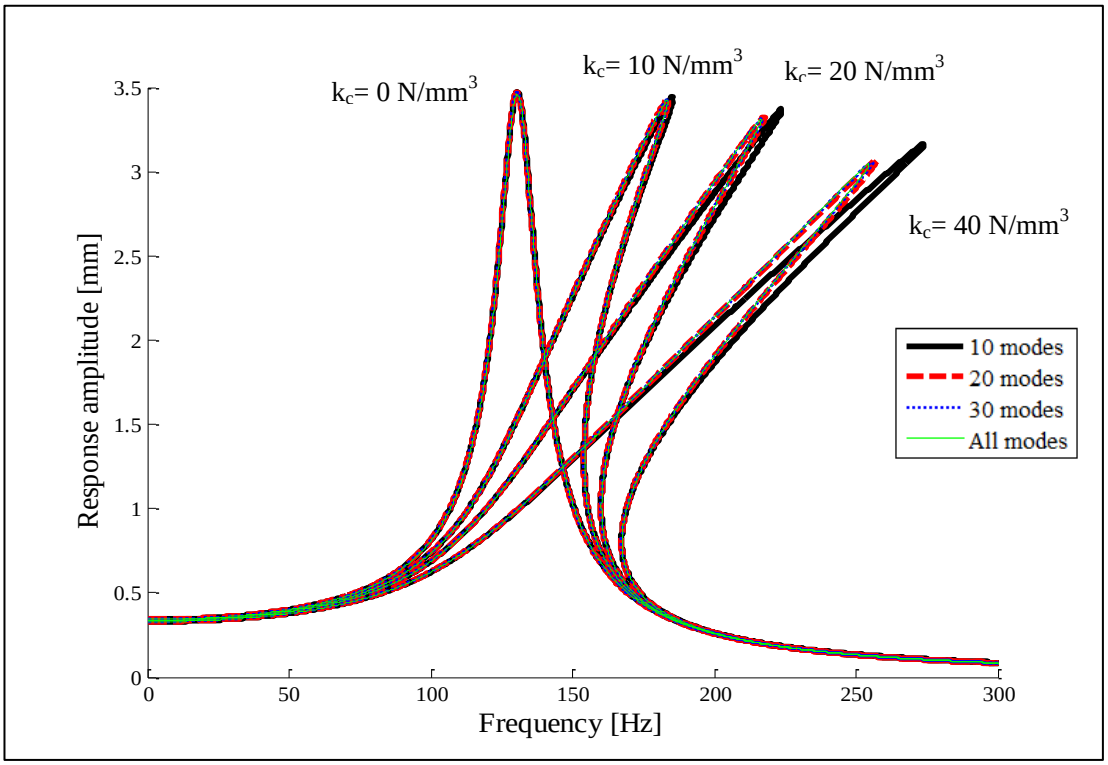


Figure 4.7 Case 2 - Nonlinear responses using 10, 20, 30 modes in formulation and using exact modal data (5 modes are used in all cases)

Results in Figure 4.6 and Figure 4.7 show that accuracy of the method increases if more number of modes are included in the analyses. Moreover, for this particular case, the difference between the results is more visible if the nonlinearities are added to the modifying part.

4.3. Bladed Disk Model

Model of a bladed-disk system with 24 blades and shrouds is presented in Figure 4.8 and Figure 4.9. A tuned bladed disk system is composed of cyclically repeated sections, in this case 24 sections, which are referred as sectors (See Figure 4.10). For a tuned bladed disk system, nonlinear vibration analyses of the whole structure can be performed by considering only one sector.

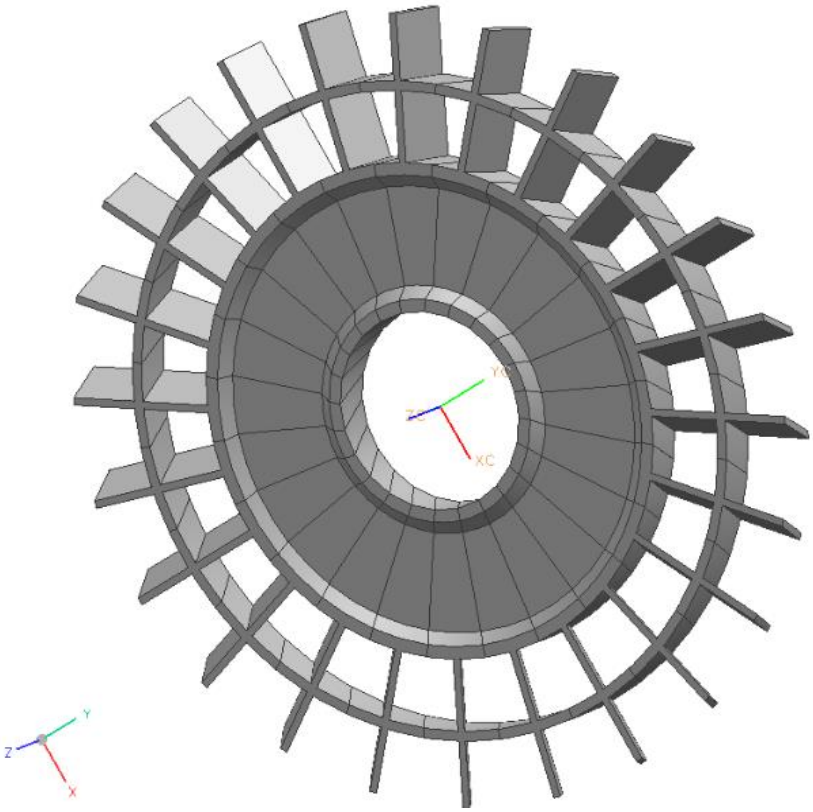


Figure 4.8 Bladed Disk Model (isometric view)

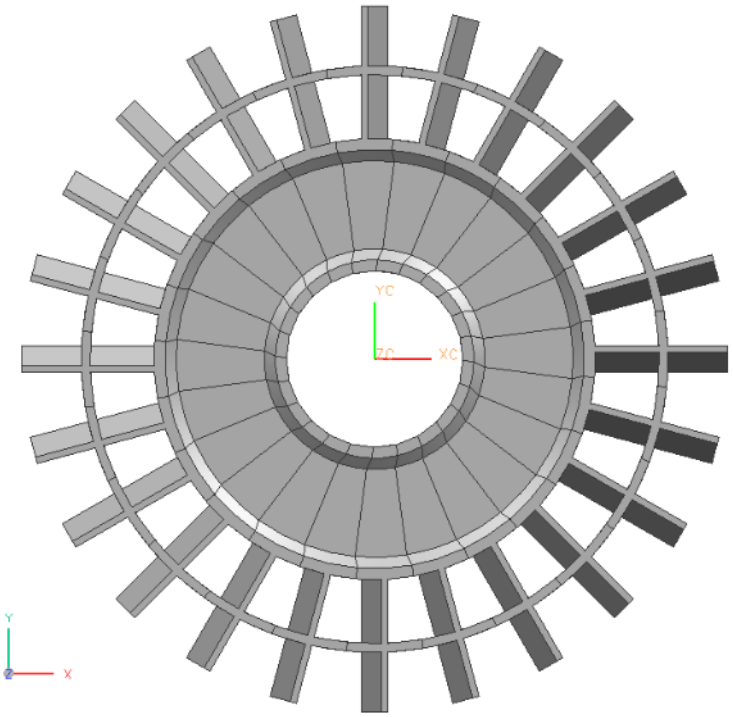


Figure 4.9 Bladed Disk Model (front view)

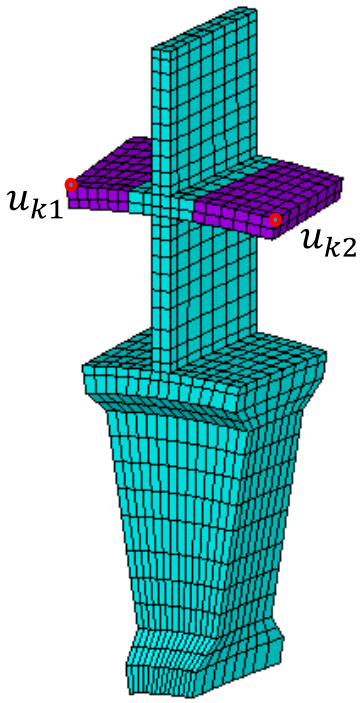


Figure 4.10 Sector Model

For tuned bladed disk systems, each blade vibrates in the same manner having an inter-blade phase angle, φ, when excited with engine order excitation. Due to the pressure variation of the working fluid in circumferential direction, when the disk rotates in this flow field, each blade experiences a periodic excitation having a rotational speed of Ω. Excitation frequency is defined as:

$$\omega = n \cdot \Omega, \tag{4.1}$$

where n is the engine order (EO). Excitation force on the k^{th} blade (See Figure 4.11) can be defined as:

$$f_k = F \cdot e^{(j\omega t + (k-1)\varphi)}. \tag{4.2}$$

Interblade phase angle is defined as follows

$$\varphi = \frac{2\pi n}{N}, \tag{4.3}$$

where N is the number of blades. If the response of the k^{th} blade is u_k, then the response of the $(k-1)^{th}$ and $(k+1)^{th}$ blades are:

$$\begin{aligned} u_{k-1} &= u_k \cdot e^{-j\varphi} \\ u_{k+1} &= u_k \cdot e^{j\varphi} \end{aligned} \tag{4.4}$$

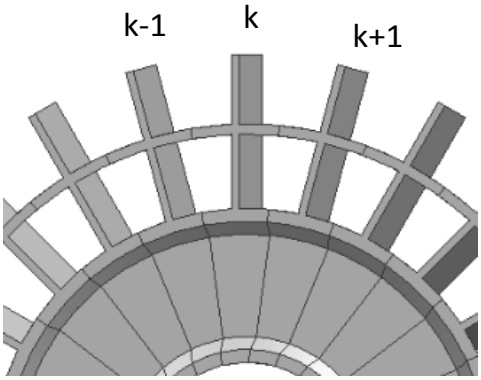


Figure 4.11 Blade Numbering

Using Eqn. (4.4), response of the nodes on the adjacent contact surfaces can be obtained. In Figure 4.10, 2 nodes are selected and response of the left node is assigned as u_{k1}, and right node is assigned as u_{k2}. In order to calculate contact forces, relative response of the contact nodes are needed that can be obtained using the response of the k^{th} blade. Response of the node that is in contact with the node on the left shroud can be found as

$$u_{k2}^{L} = u_{k2} \cdot e^{-j\varphi} \tag{4.5}$$

Response of the node that is in contact with the node on the right can be found in the same manner

$$u_{k1}^{R} = u_{k1} \cdot e^{-j\varphi} \tag{4.6}$$

Relative responses become:

$$\begin{aligned} u_{k1\_rel} &= u_{k2}^{L} - u_{k1} \\ u_{k2\_rel} &= u_{k2} - u_{k1}^{R} \end{aligned} \tag{4.7}$$

It is important to note that sign considerations should be checked while using Eqn. (4.7) in nonlinear analysis according to the analysis coordinate frame.

Since the contact surfaces are not aligned to global coordinate system, nonlinear forces should be obtained in the local coordinate system defining the tangential and normal planes on the contact surfaces. First, the responses are defined on the local coordinate systems shown in Figure 4.12. They are obtained by rotating the global coordinate system by 7.5° and -7.5° about z-axis which is parallel to the rotation axis.

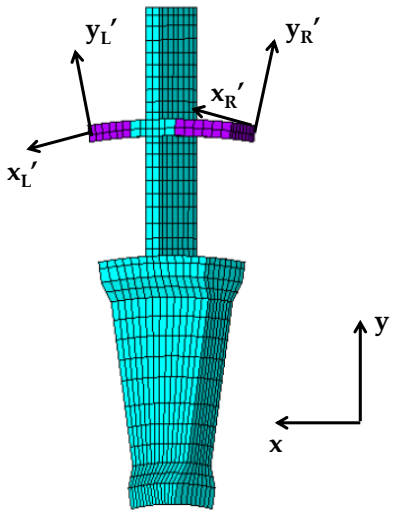


Figure 4.12 Local coordinates for the first coordinate transformation

First transformation matrix:

$$R_z = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.8}$$

For the bladed disk model considered in this work, for the right contact surface, $\alpha = 7.5$; for the left contact surface, $\alpha = -7.5$. After the first transformation is achieved, obtained values are used for the second transformation. The second coordinate transformation is related to the angle of the shroud surface about y-axis. In Figure 4.13, definition of shroud angle, β can be seen. In this figure, blades have 30° angle with respect to z-axis whereas shroud angle is 20°. In Figure 4.13, local coordinate system on the left contact surface is only shown.

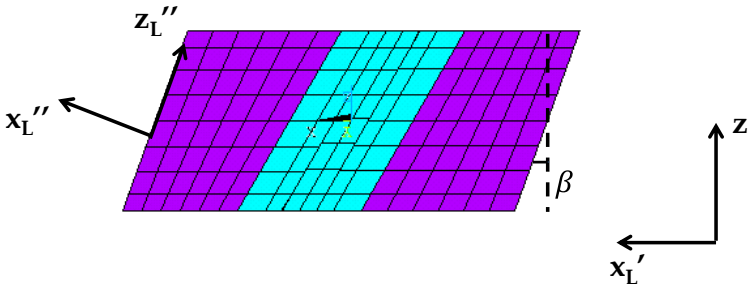


Figure 4.13 Local coordinate for the left side for the second coordinate transformation

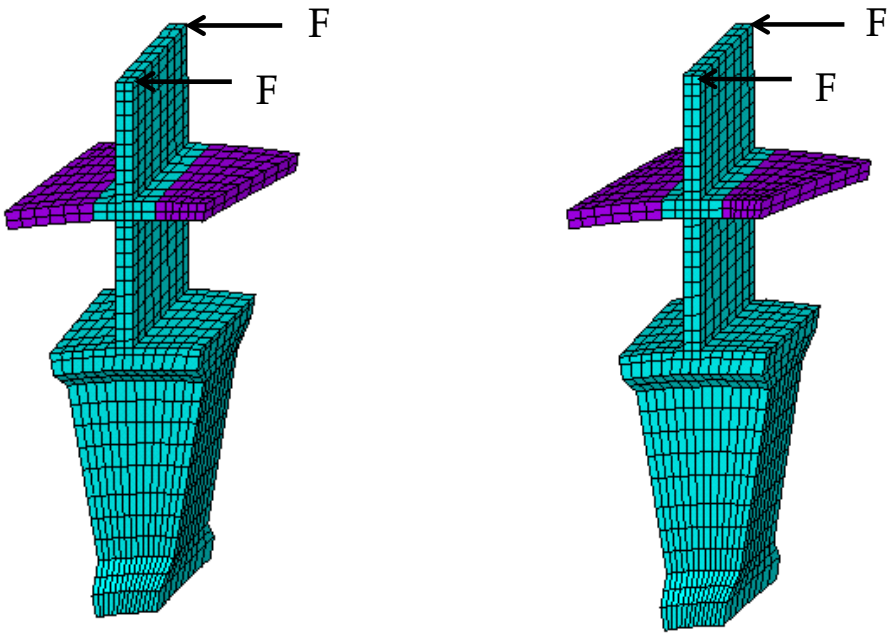
Second transformation matrix:

$$R_y = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{4.9}$$

After responses of the nodes on the contact surfaces are written for local coordinate system defined on contact surfaces, nonlinear forces are obtained. It is important to note that, obtained forces are needed to be written in the global coordinate system by reversing the mentioned coordinate transformations, after which, they can be inserted into the EOM.

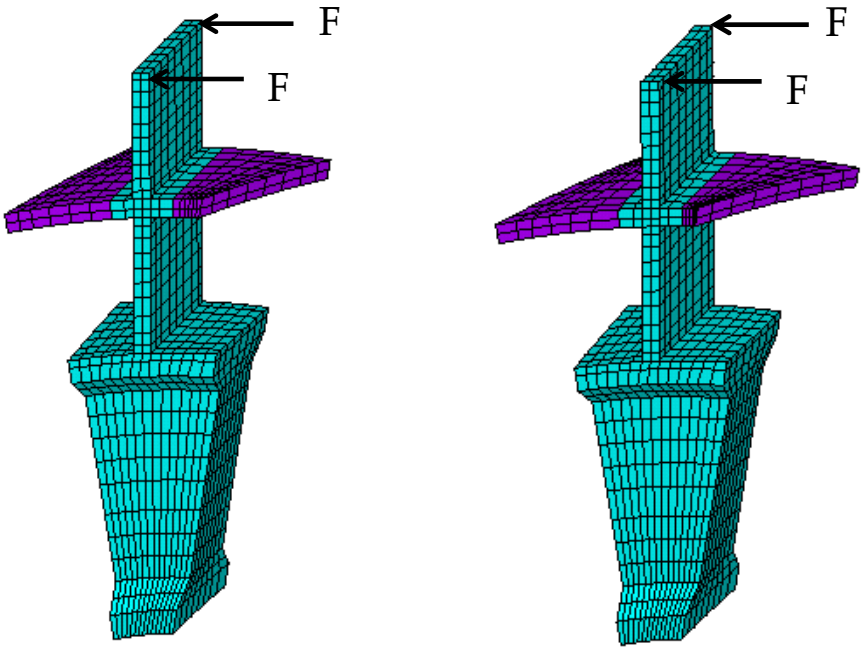
4.3.1. Case Studies

Case studies are applied on a tuned bladed disk system with shroud contacts by modeling only one sector of it as explained in the previous section. Different shroud angles are studied by updating the system information using the developed structural modification explained in Chapter 2. Models are presented in Figure 4.14 in which forcing can also be seen. In the figure, original structure is presented as green while modifying parts are presented in purple. Material properties of the system are: density, ρ = 7850kg/m^3; modulus of elasticity, E = 210GPa and Poisson's ratio, ν = 0.3.



a) β = 45° **b)** β = 50°

Figure 4.14 Sector finite element models for a) β = 45°, b) β = 50°, c) β = 55° and d) β = 60° shroud angles, F = 1N, γ = 0.02



c) β = 55° **d)** β = 60°

Figure 4.14 Sector finite element models for a) β = 45°, b) β = 50°, c) β = 55° and d) β = 60° shroud angles, F = 1N, γ = 0.02 (continued)

Nonlinear response analysis is performed for initial preload values of, n_0 = -10, -5, 0, 1, 2, 5, 10, 20N and for engine orders of EO = 1, 2 and 3. For structural modification, 50 modes of the original system is used to obtain the modal data of the modified system given in Figure 4.14 and 10 modes of the modified system are used for nonlinear analysis using HBM with modal superposition. Contact elements on the shroud surfaces are modeled by the 1-D friction contact model with normal load variation. Response plots are given in the direction of loading for the front forcing node shown in Figure 4.14 around the first mode which is the first cantilever blade mode. For each angle configuration and for each EO, individual optimum curves and frequency shift curves are given. Also for each EO, optimum curves are given on the same plot in order to see the effect of shroud angle.

4.3.1.1. 45° Shroud Case

Response plots, optimum curves and frequency shift curves for the 45° shroud case for different engine orders are presented in Figures 4.15 to 4.23 for the loading presented in Figure 4.14.

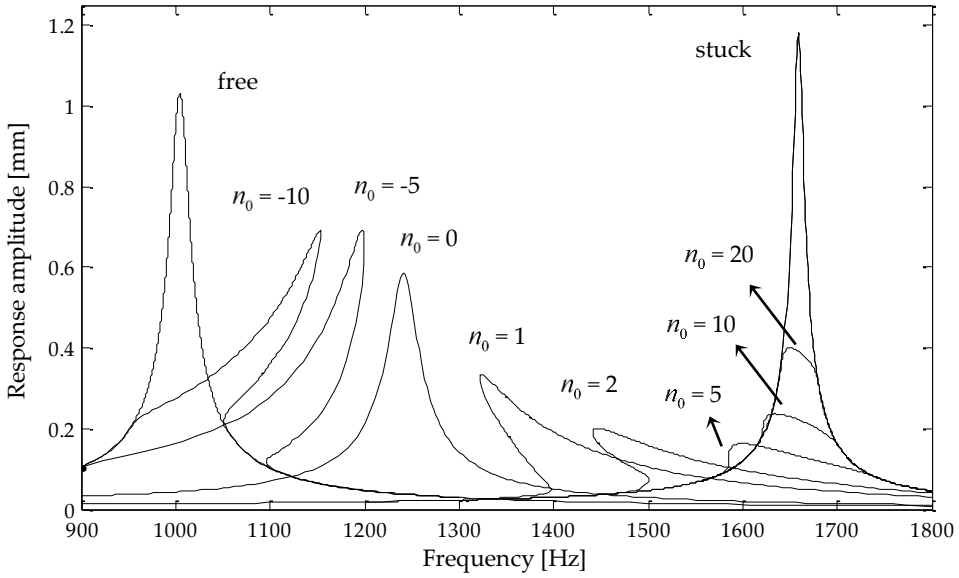


Figure 4.15 Response amplitude vs. frequency for β = 45°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

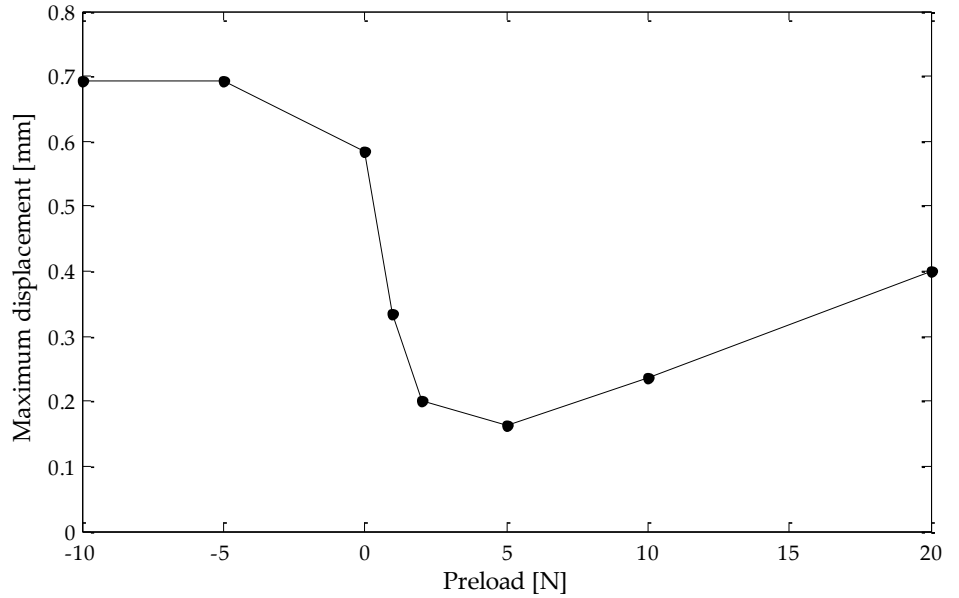


Figure 4.16 Optimal curve for β = 45°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

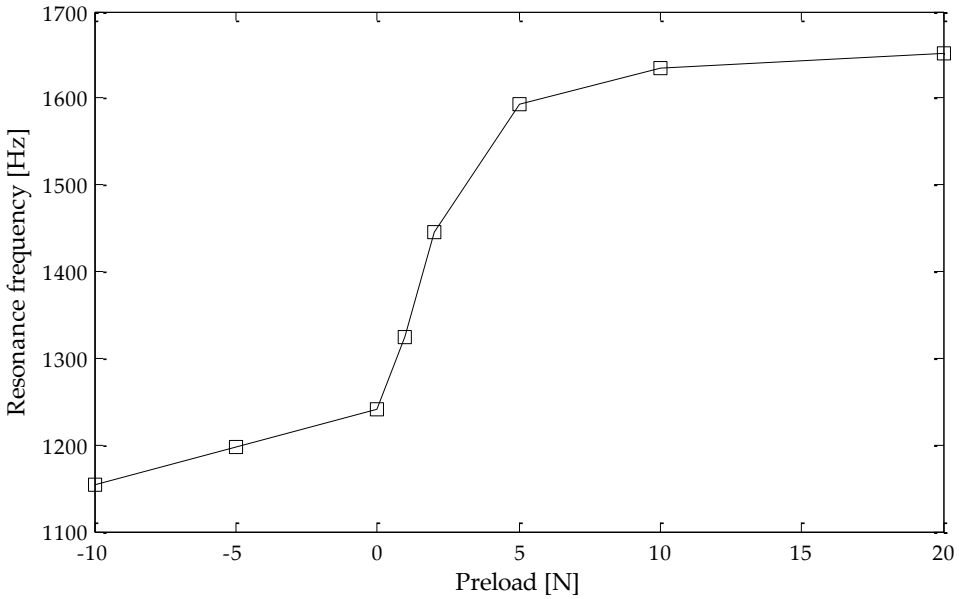


Figure 4.17 Frequency shift curve for β = 45°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

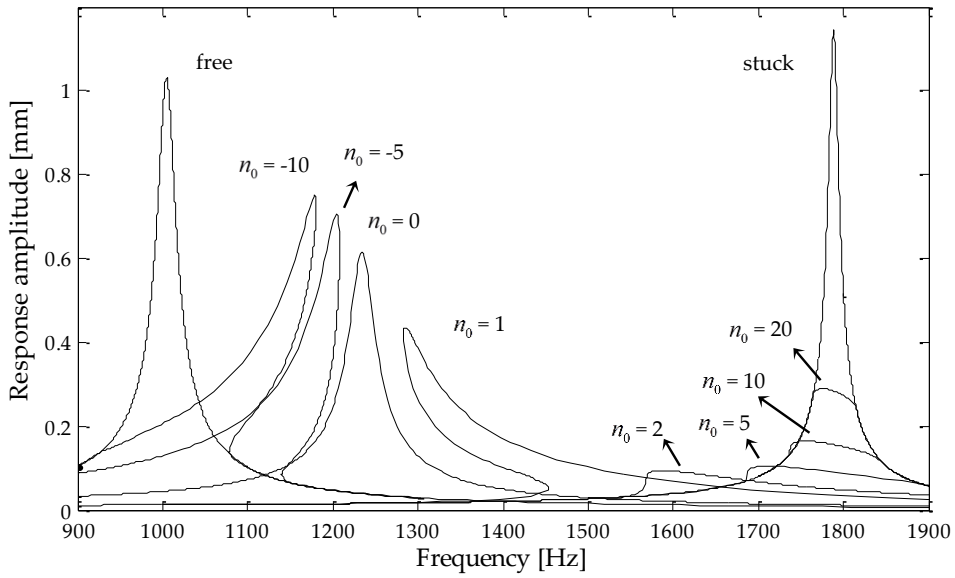


Figure 4.18 Response amplitude vs. frequency for β = 45°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

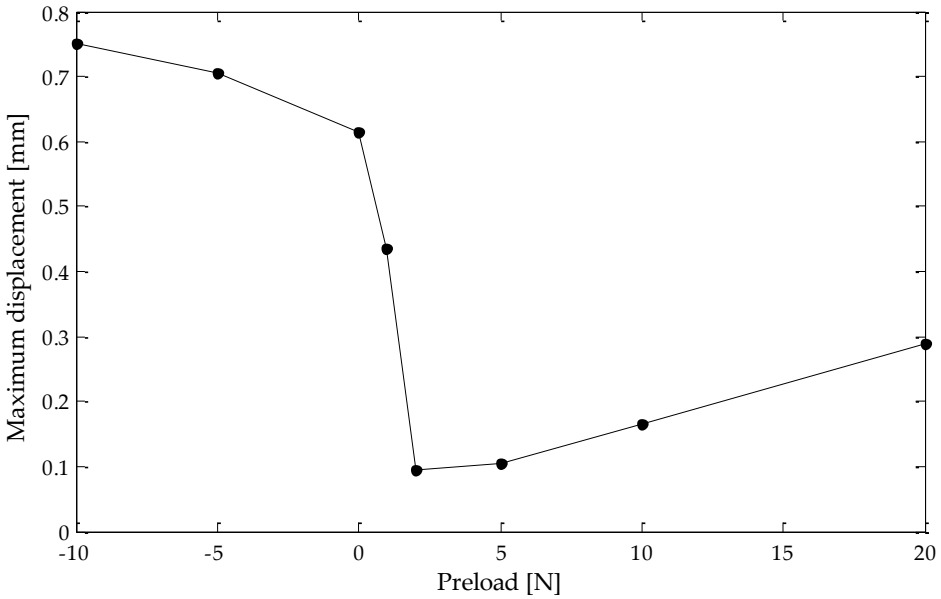


Figure 4.19 Optimal curve for β = 45°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

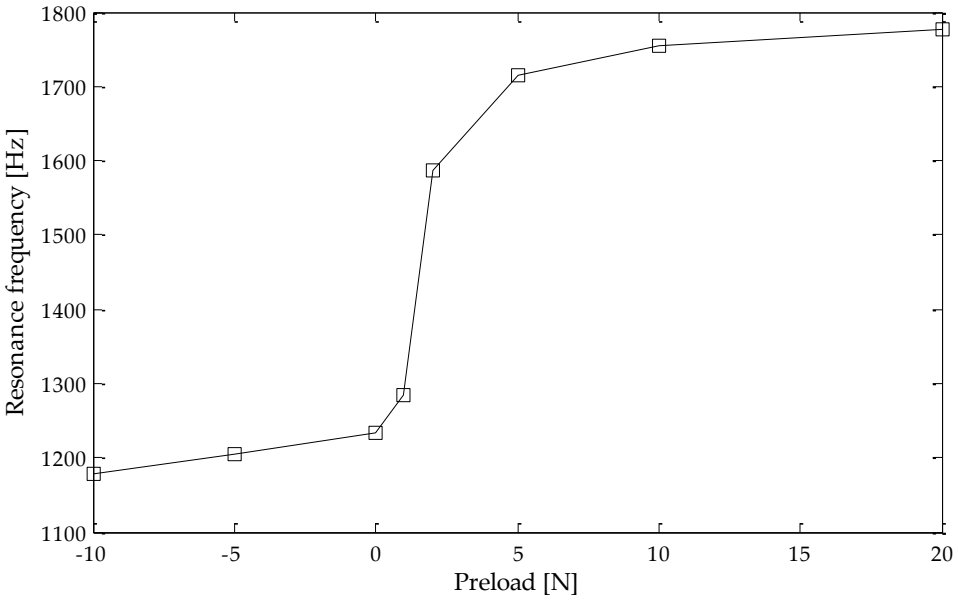


Figure 4.20 Frequency shift curve for β = 45°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

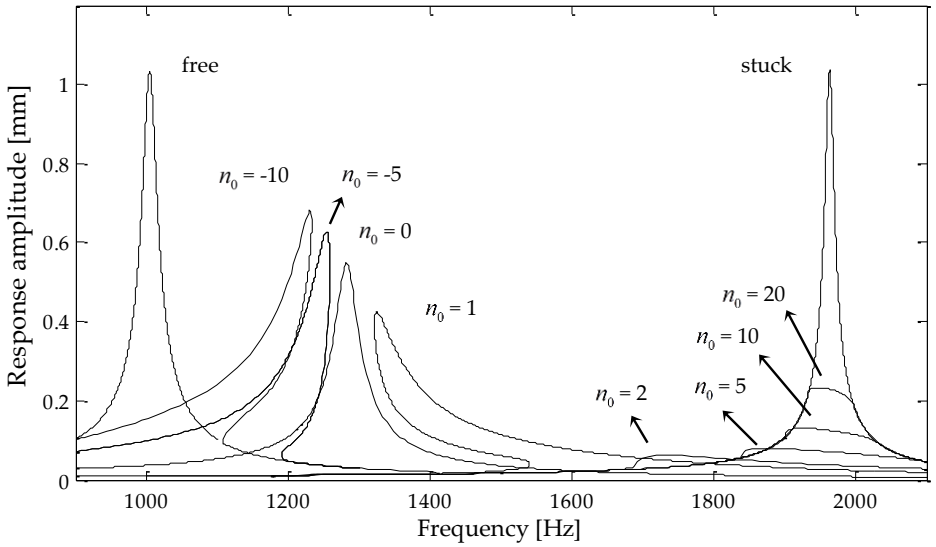


Figure 4.21 Response amplitude vs. frequency for β = 45°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

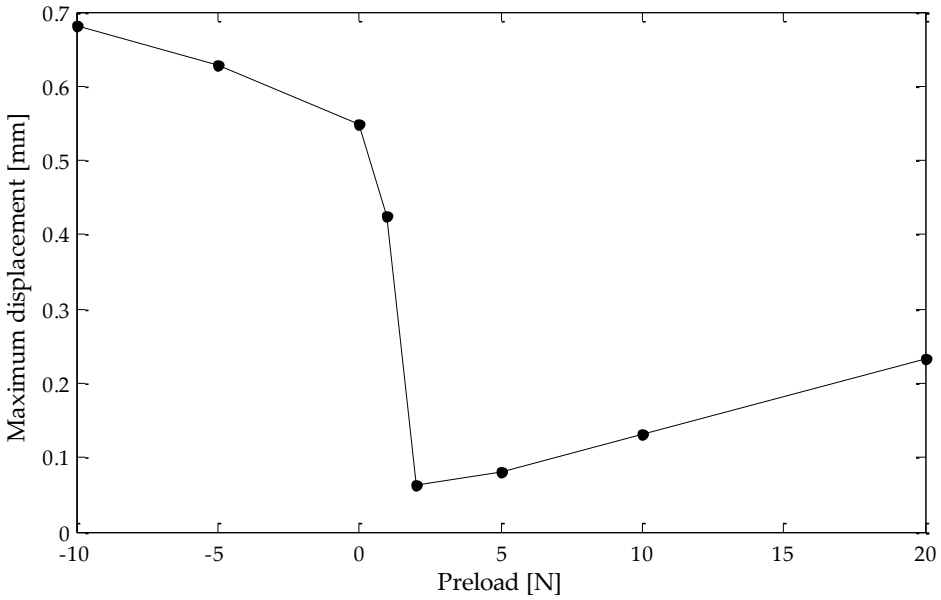


Figure 4.22 Optimal curve for β = 45°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

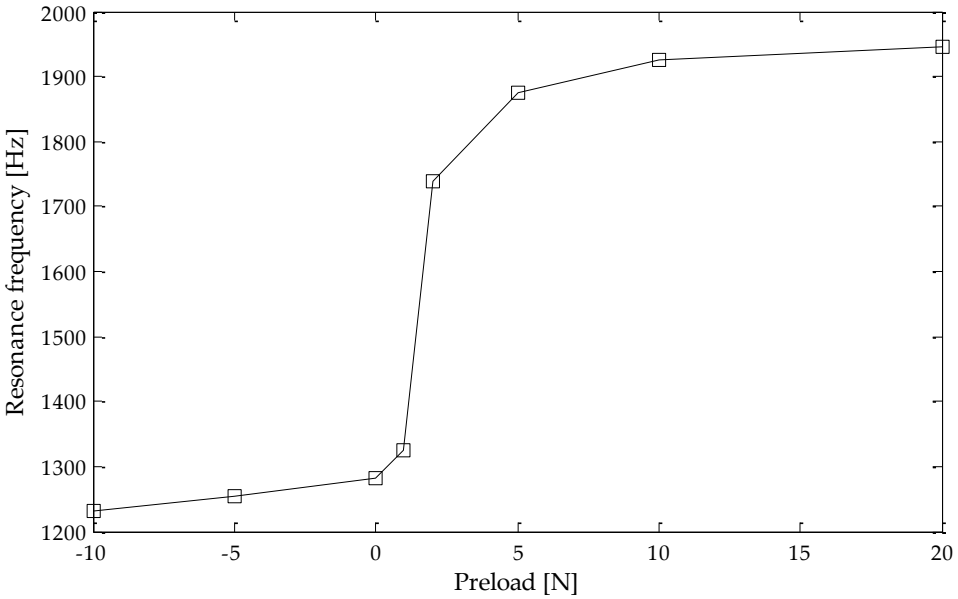


Figure 4.23 Response amplitude vs. frequency for β = 45°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

4.3.1.2. 50° Shroud Case

Response plots, optimum curves and frequency shift curves for the 50° shroud case for different engine orders are presented in Figures 4.24 to 4.32 for the loading presented in Figure 4.14.

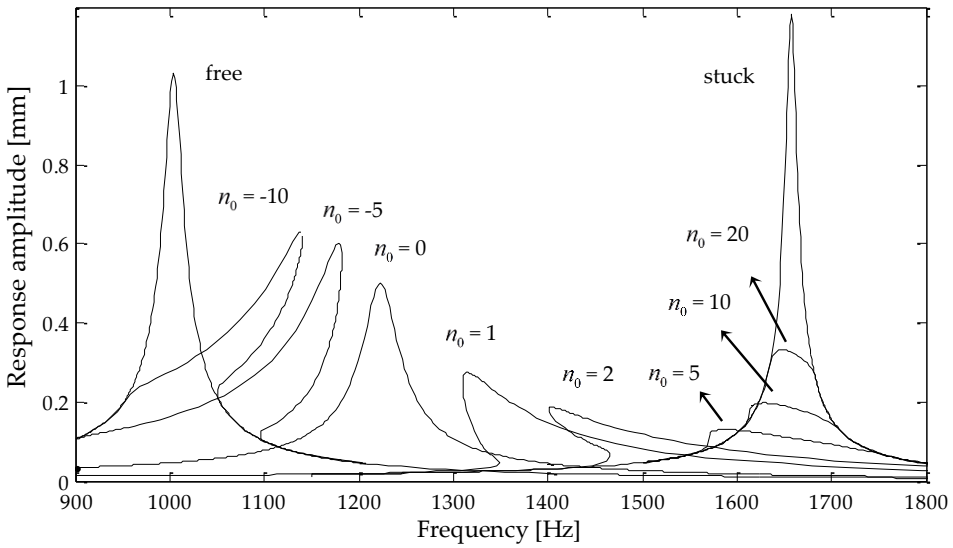


Figure 4.24 Response amplitude vs. frequency for β = 50°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

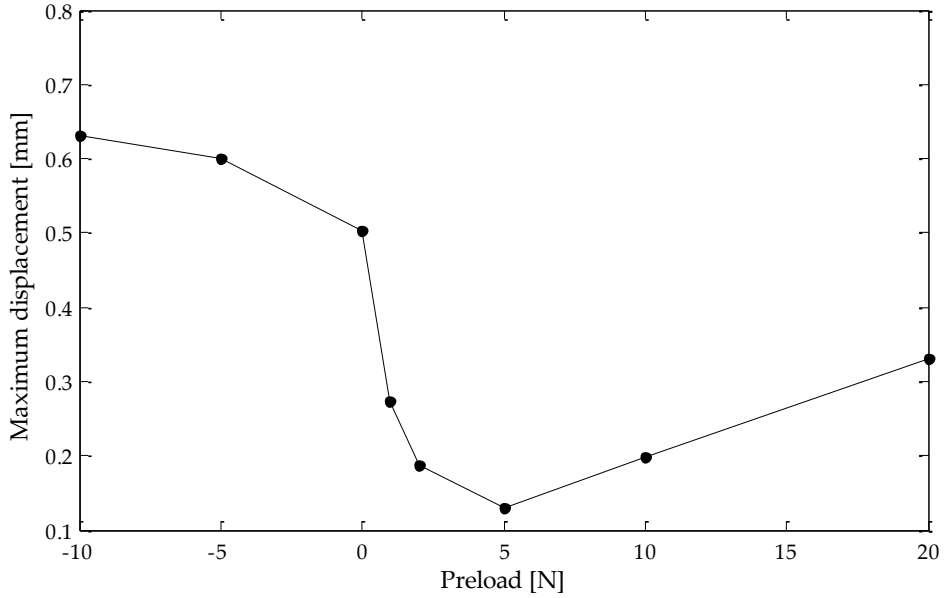


Figure 4.25 Optimal curve for β = 50°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

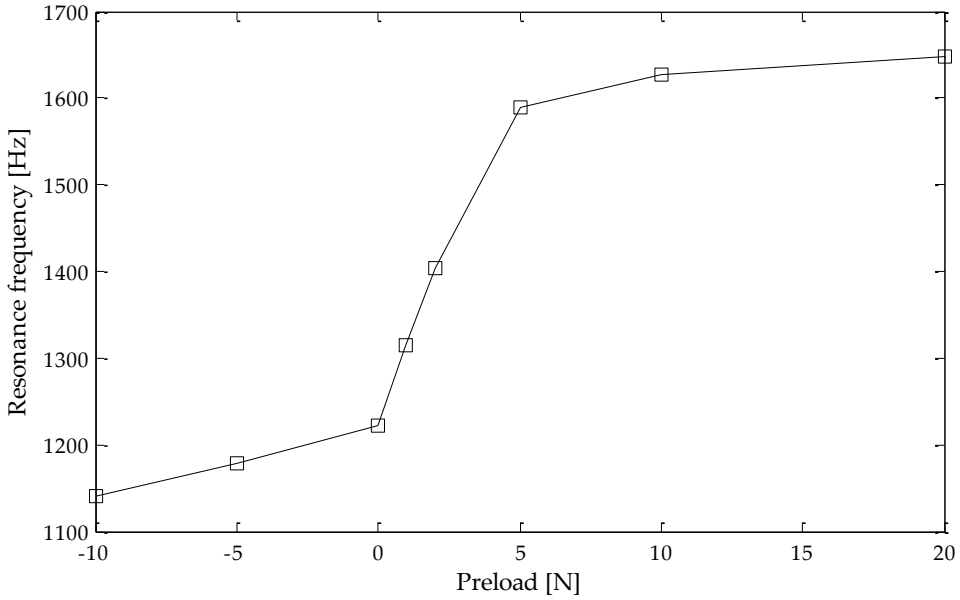


Figure 4.26 Frequency shift curve for β = 50°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

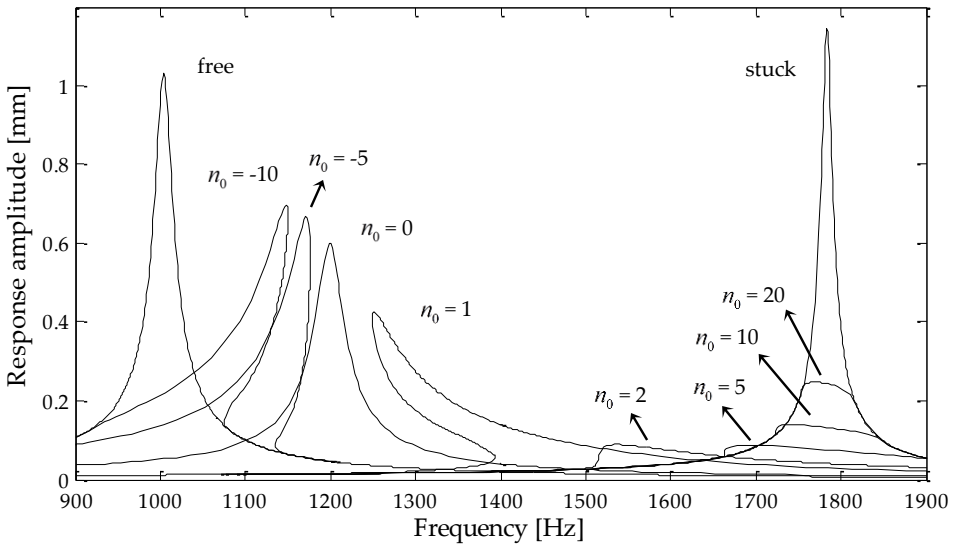


Figure 4.27 Response amplitude vs. frequency for β = 50°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

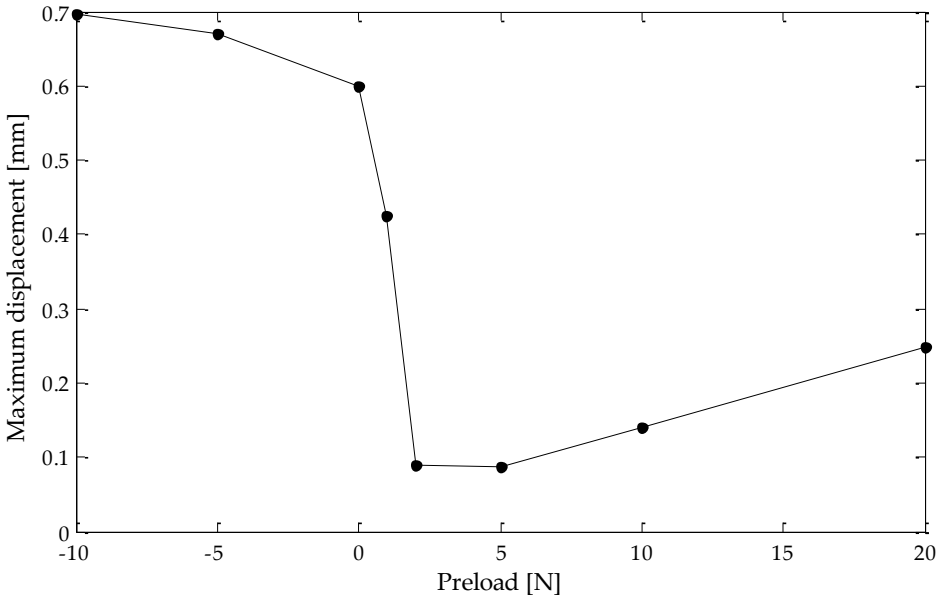


Figure 4.28 Optimal curve for β = 50°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

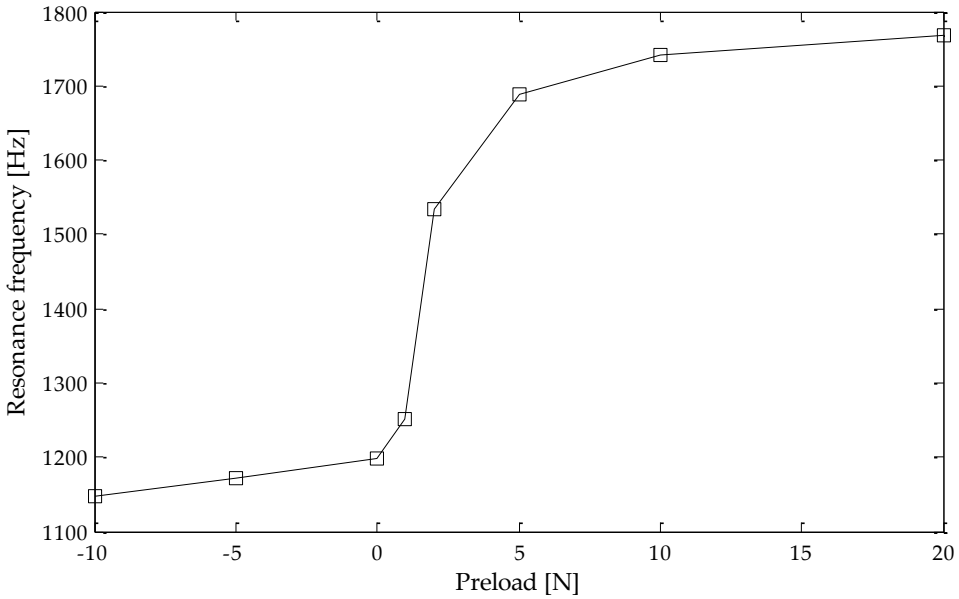


Figure 4.29 Frequency shift curve for β = 50°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

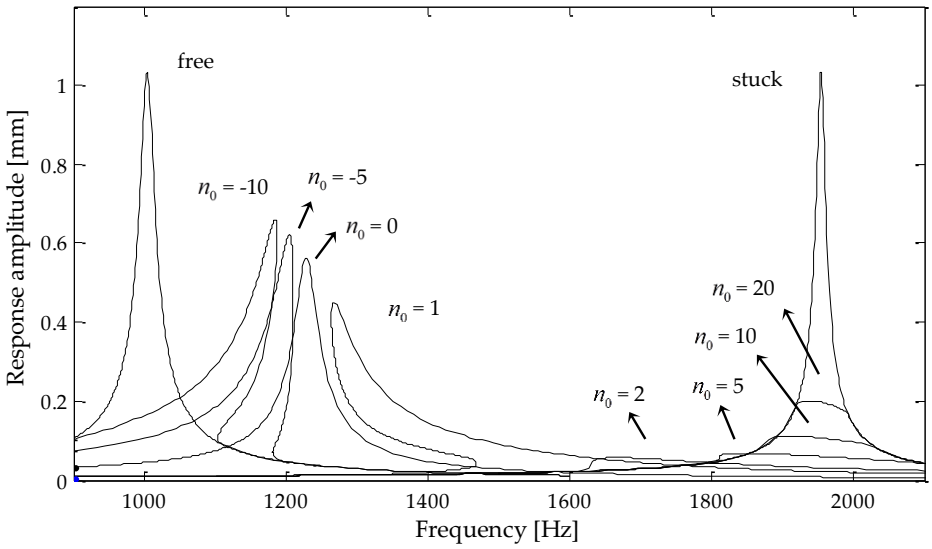


Figure 4.30 Response amplitude vs. frequency for β = 50°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

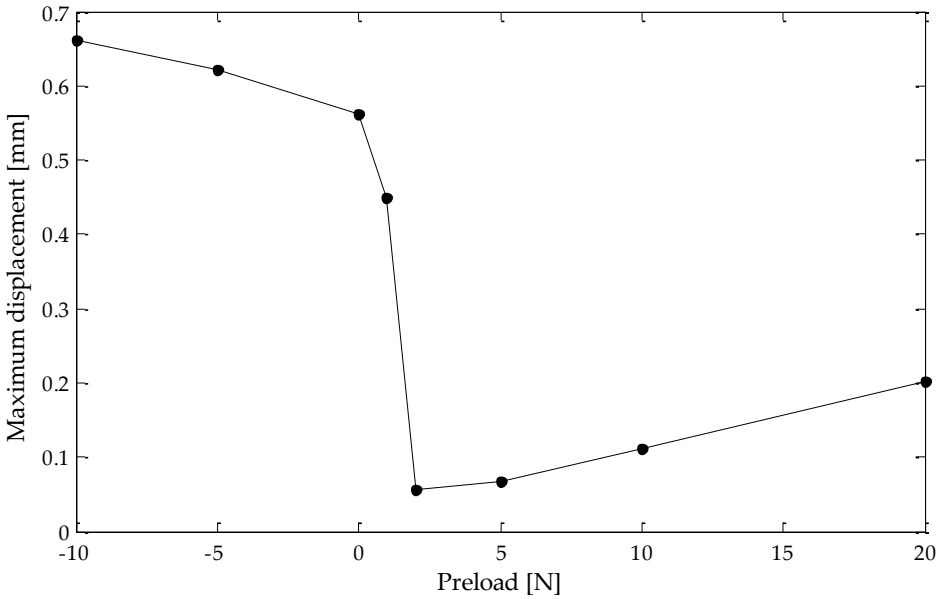


Figure 4.31 Optimal curve for β = 50°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

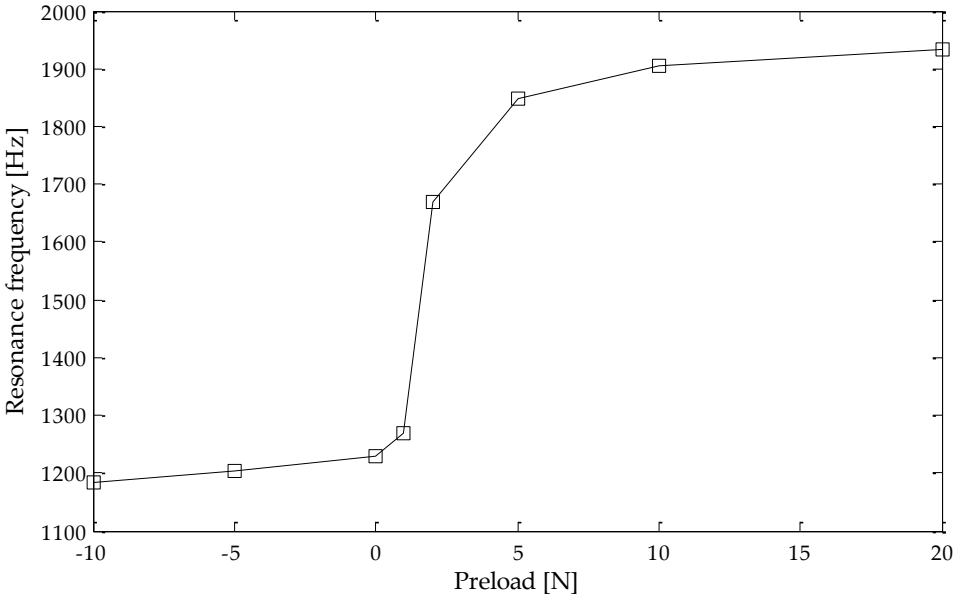


Figure 4.32 Frequency shift curve for β = 50°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

4.3.1.3. 55° Shroud Case

Response plots, optimum curves and frequency shift curves for the 55° shroud case for different engine orders are presented in Figures 4.33 to 4.41 for the loading presented in Figure 4.14.

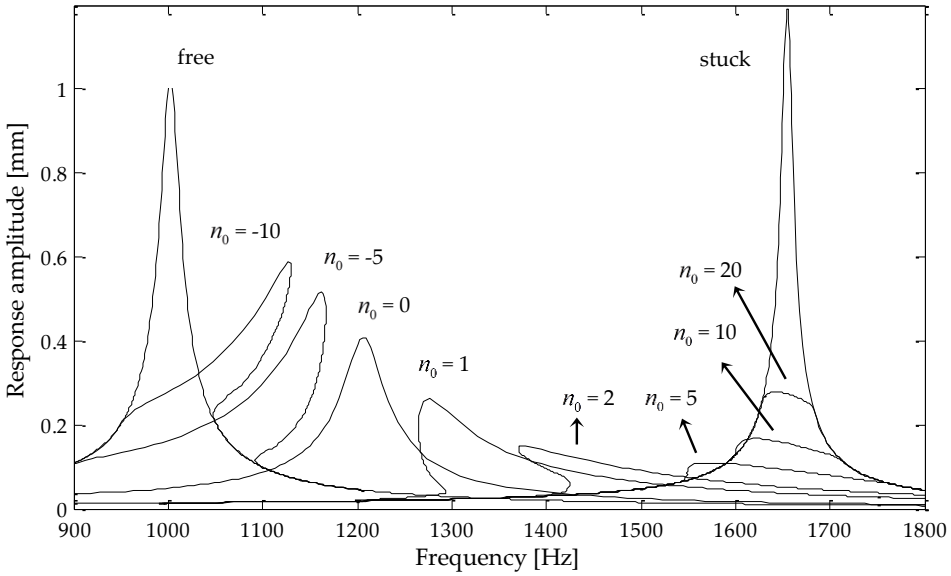


Figure 4.33 Response amplitude vs. frequency for β = 55°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

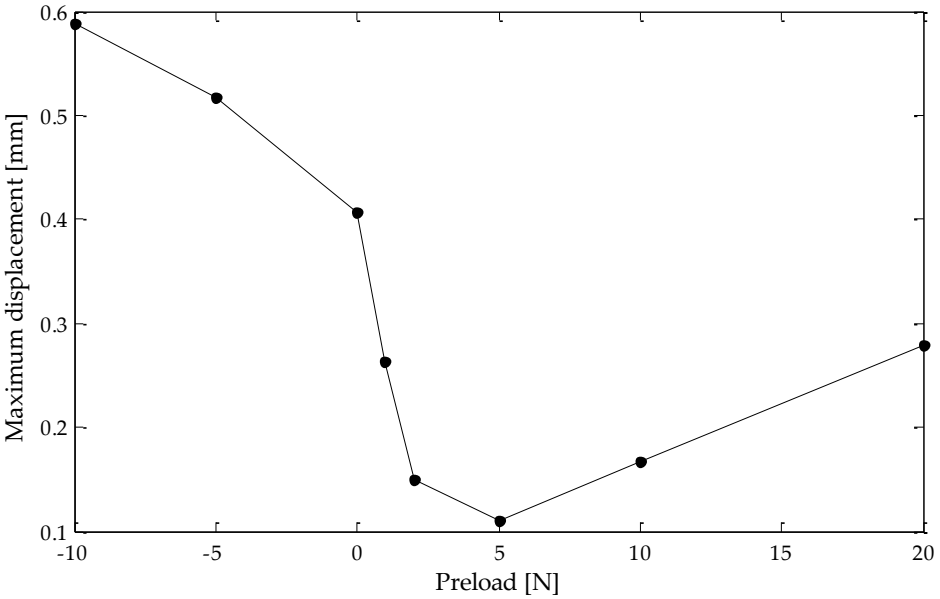


Figure 4.34 Optimal curve for β = 55°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

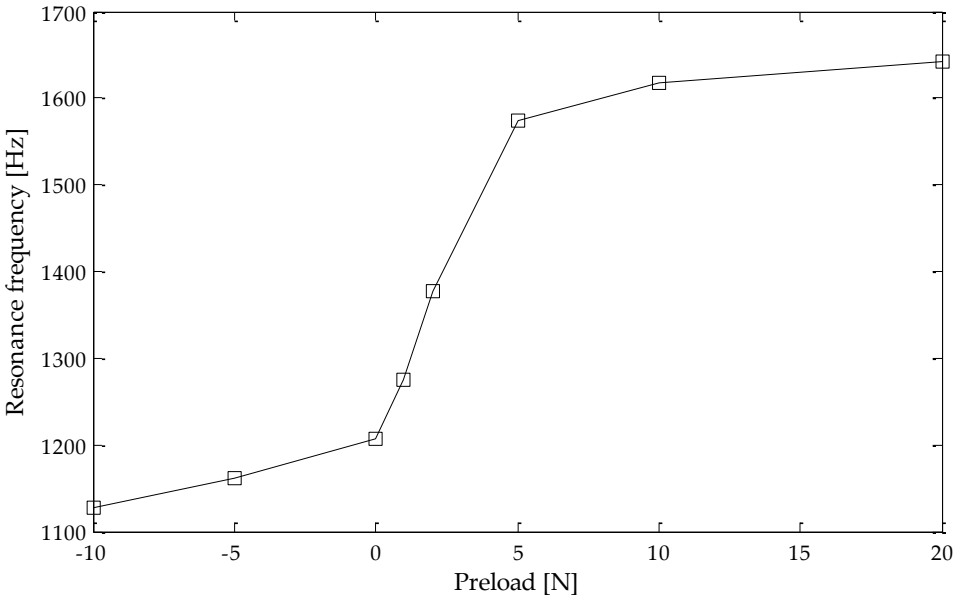


Figure 4.35 Frequency shift curve for β = 55°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

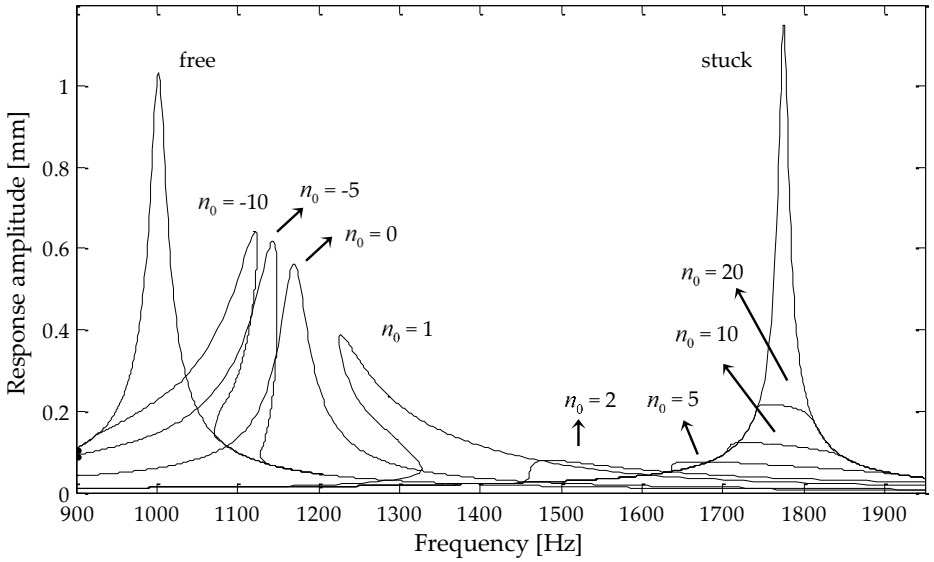


Figure 4.36 Response amplitude vs. frequency for β = 55°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

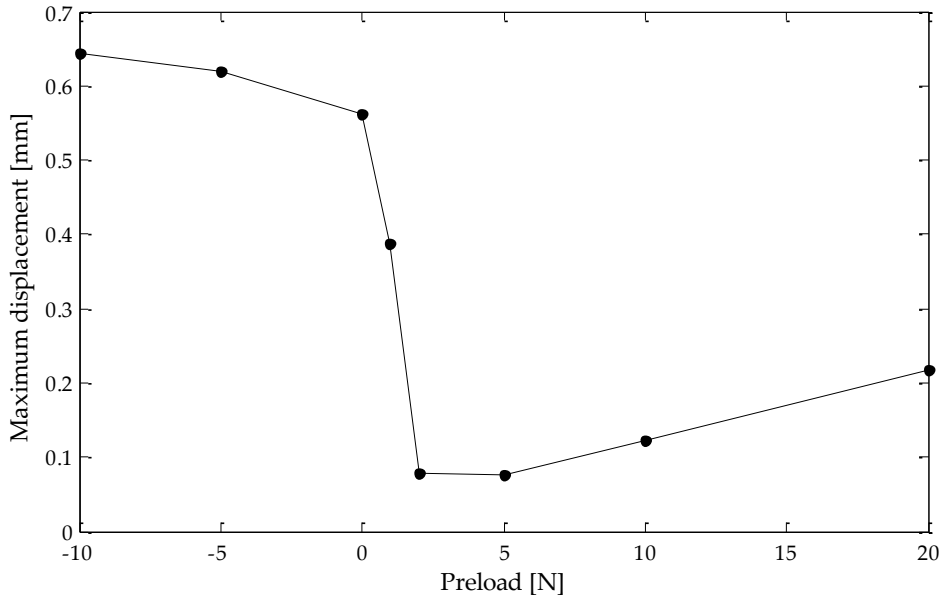


Figure 4.37 Optimal curve for β = 55°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

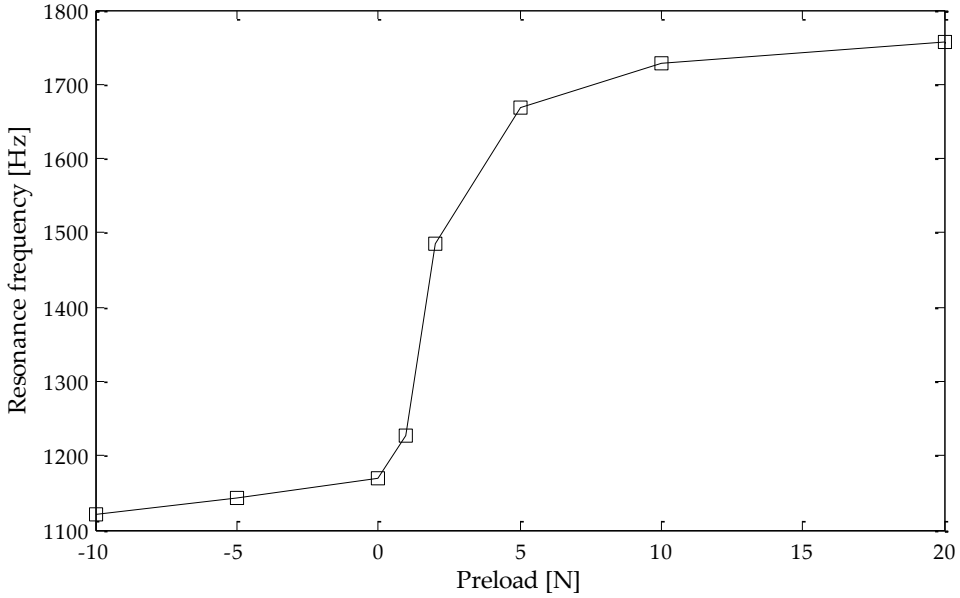


Figure 4.38 Frequency shift curve for β = 55°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

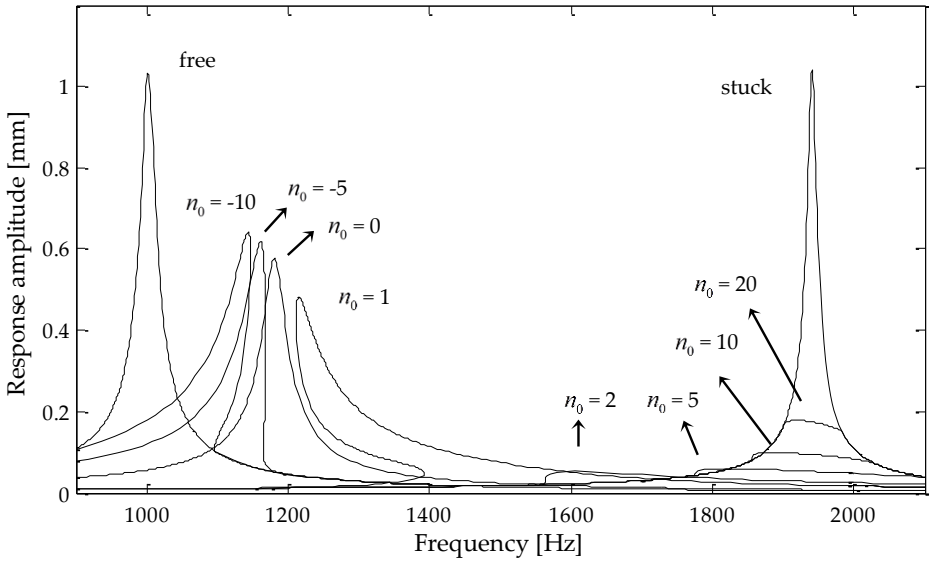


Figure 4.39 Response amplitude vs. frequency for β = 55°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

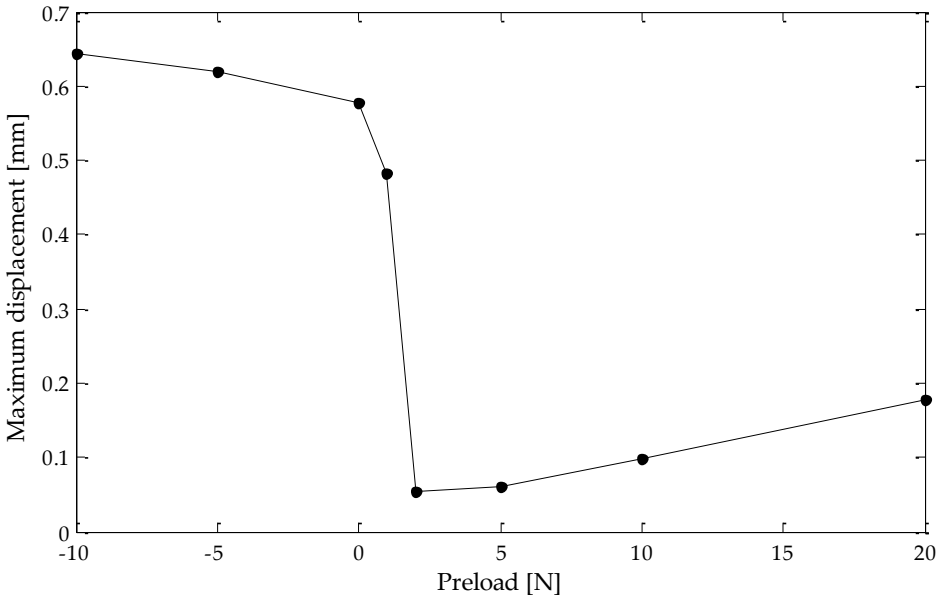


Figure 4.40 Optimal curve for β = 55°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

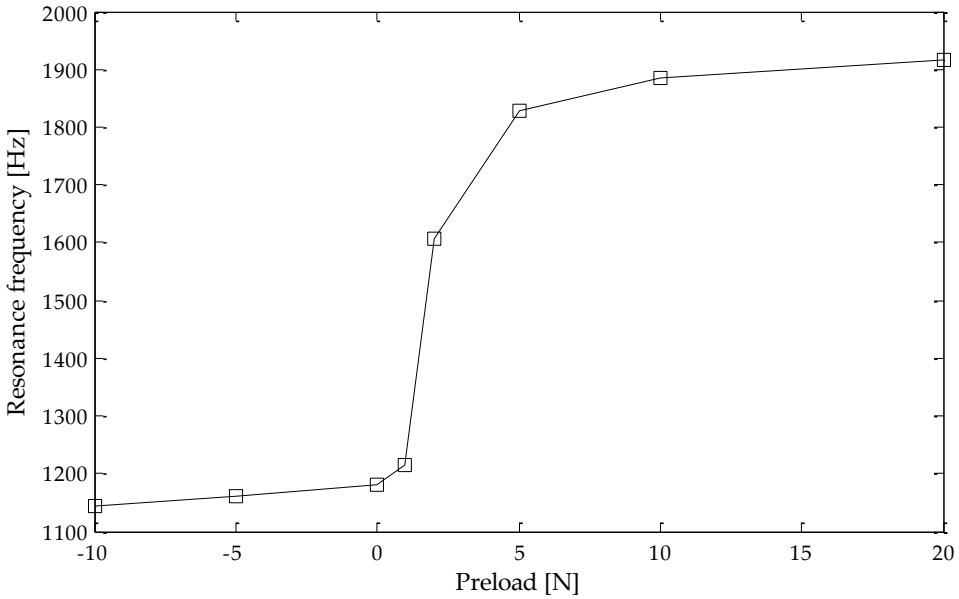


Figure 4.41 Frequency shift curve for β = 55°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

4.3.1.4. 60° Shroud Case

Response plots, optimum curves and frequency shift curves for the 60° shroud case for different engine orders are presented in Figures 4.42 to 4.50 for the loading presented in Figure 4.14.

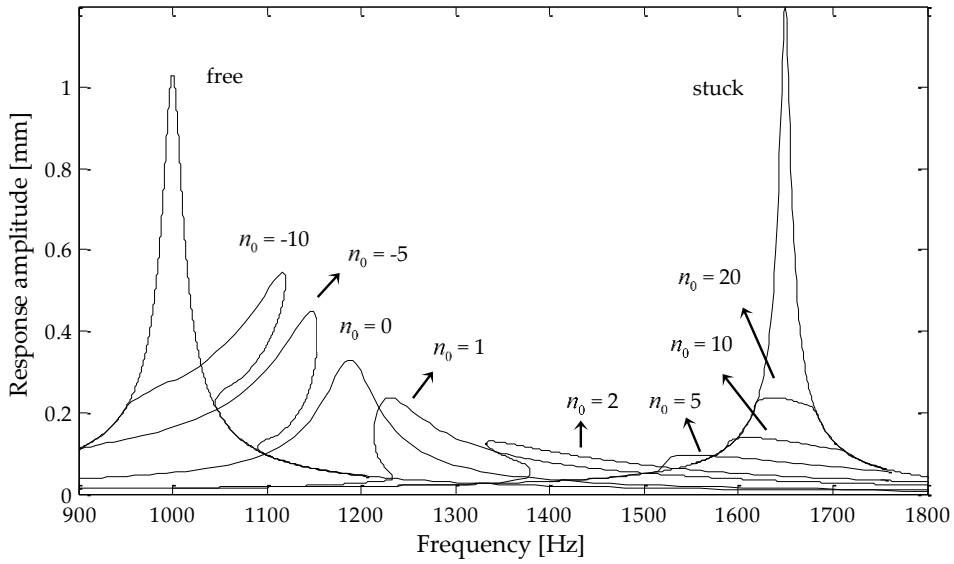


Figure 4.42 Response amplitude vs. frequency for β = 60°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

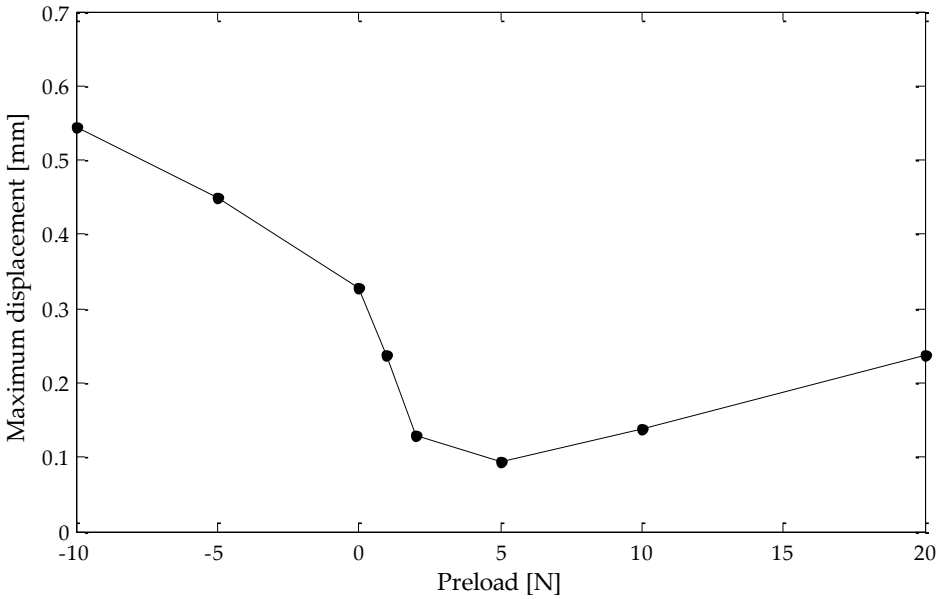


Figure 4.43 Optimal curve for β = 60°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

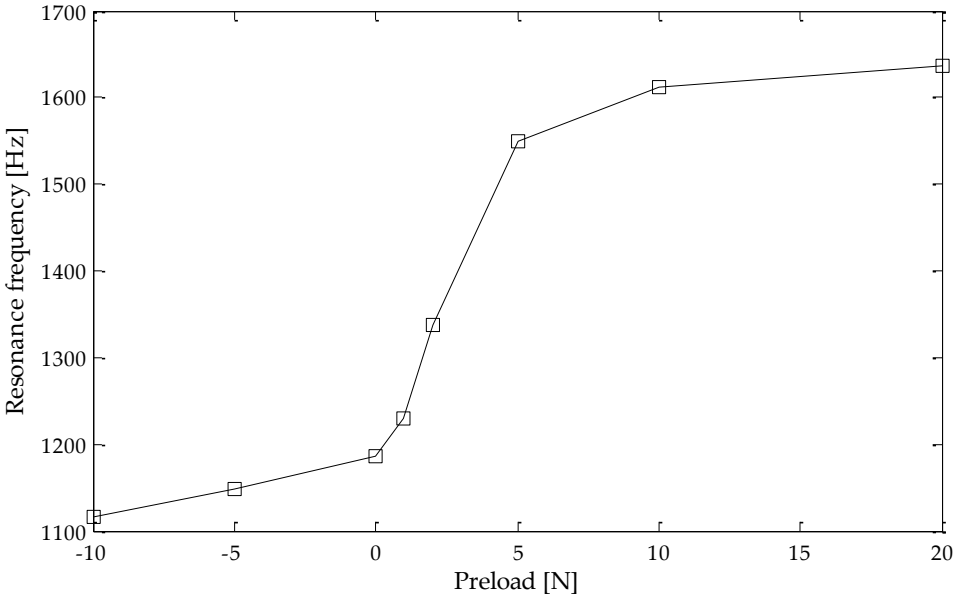


Figure 4.44 Frequency shift curve for β = 60°, EO = 1 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

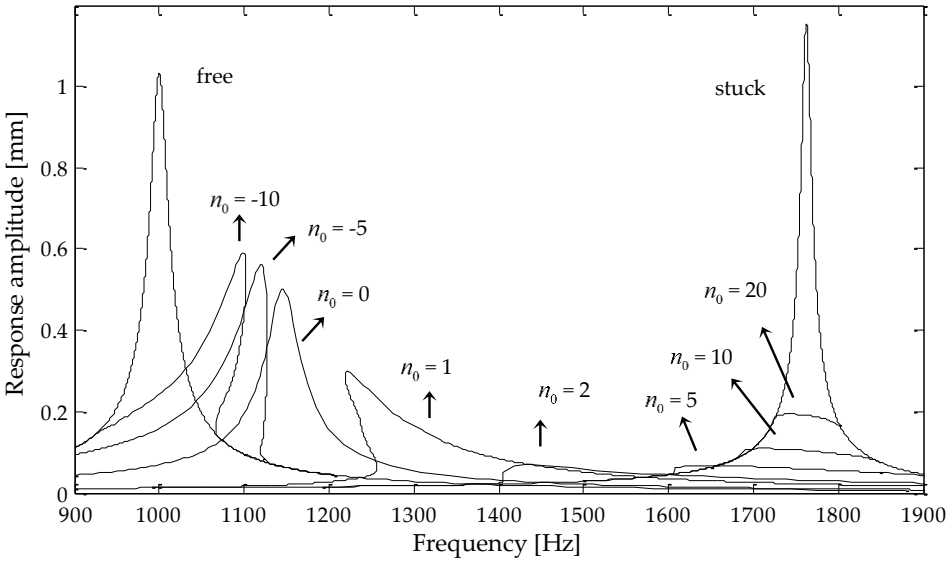


Figure 4.45 Response amplitude vs. frequency for β = 60°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

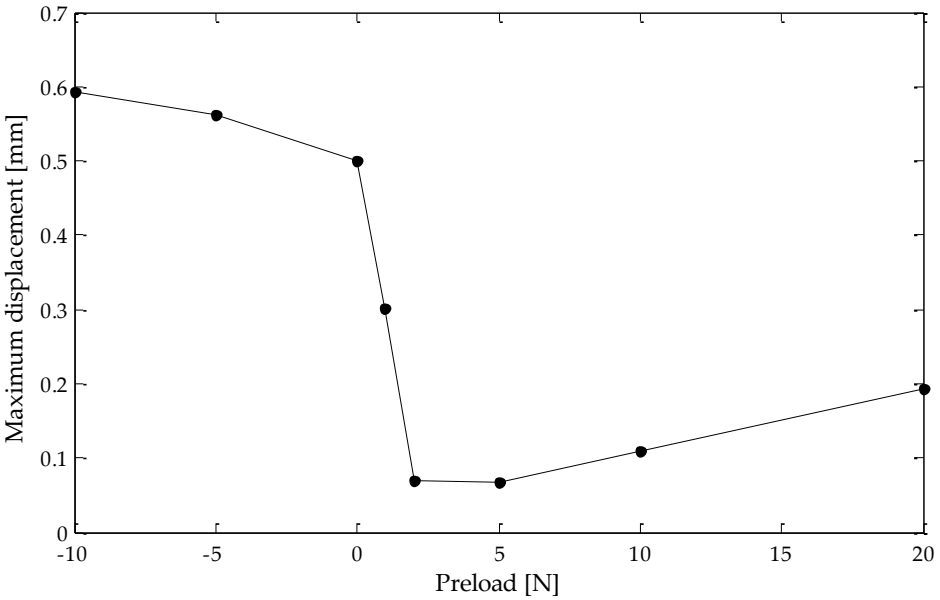


Figure 4.46 Optimal curve for β = 60°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

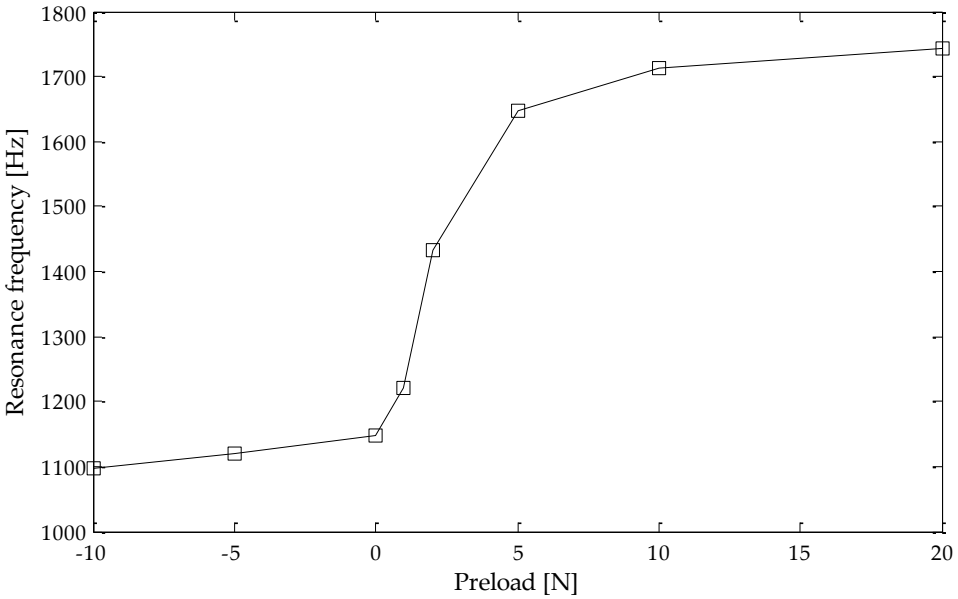


Figure 4.47 Frequency shift curve for β = 60°, EO = 2 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

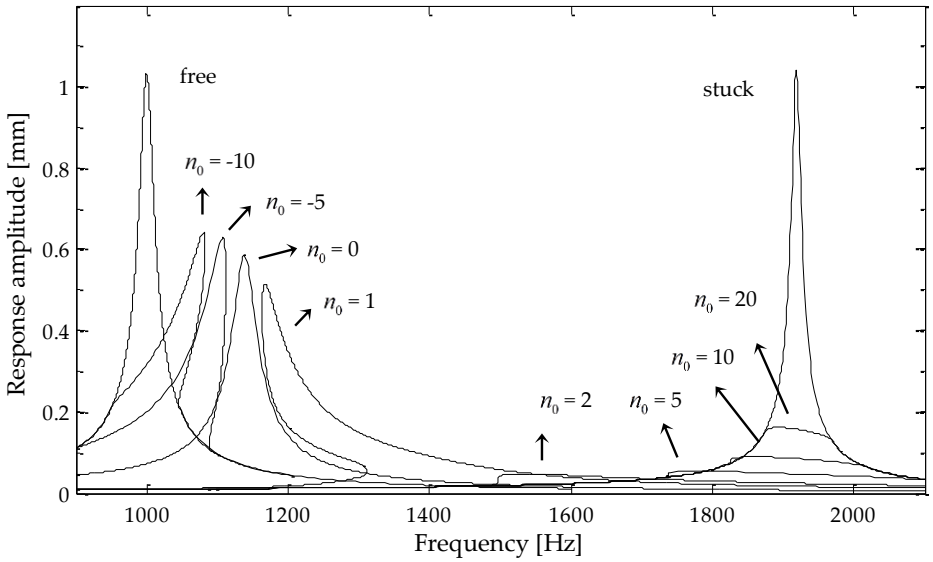


Figure 4.48 Response amplitude vs. frequency for β = 60°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

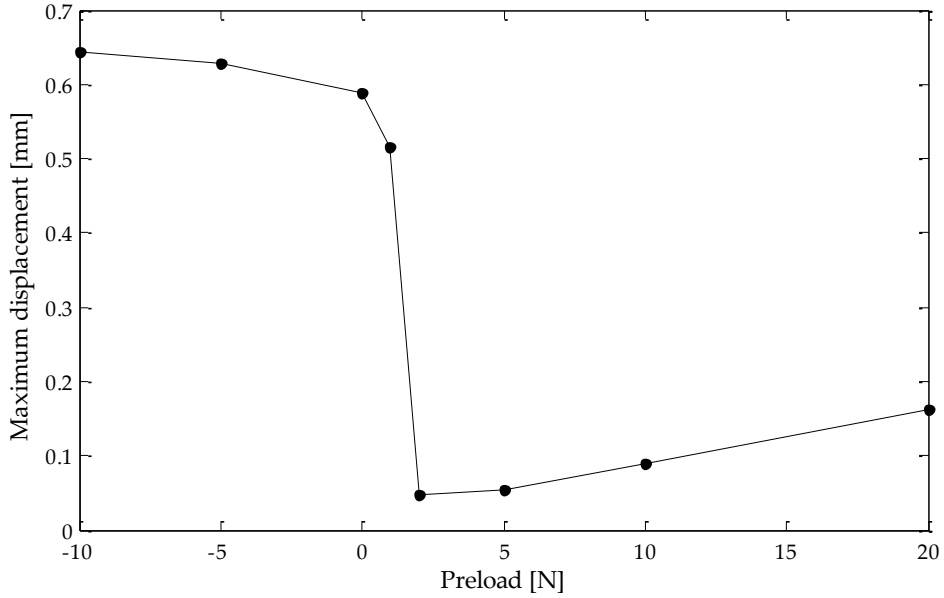


Figure 4.49 Optimal curve for β = 60°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

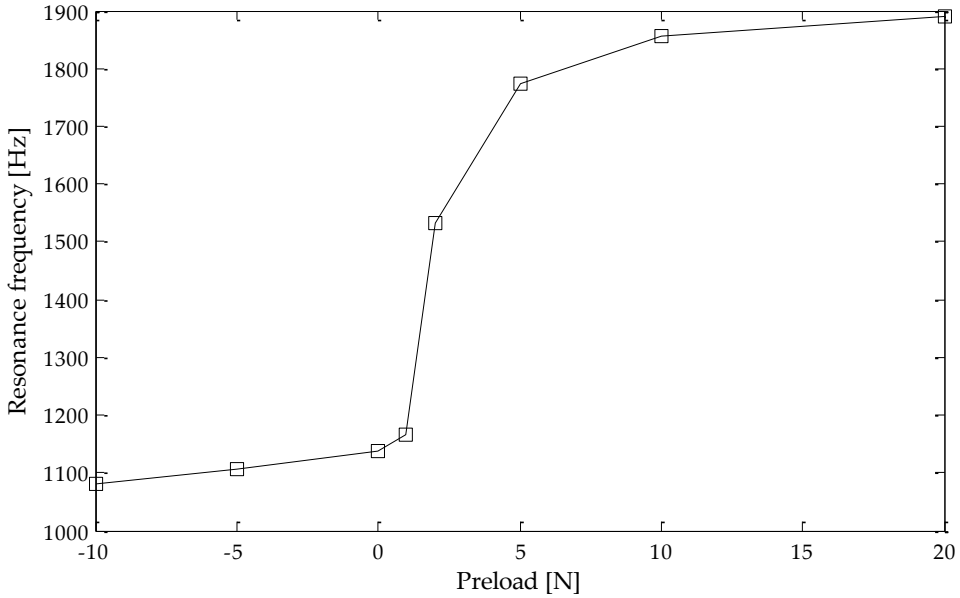


Figure 4.50 Frequency shift curve for β = 60°, EO = 3 and n_0 = -10, -5, 0, 1, 2, 5, 10, 20N

For each engine order, optimal curves are also plotted on the same figure for different shroud angles. Figures 4.51 to 4.53 present the optimal curves for shroud angles: β=45°, 50°, 55° and 60° when EO=1, 2 and 3.

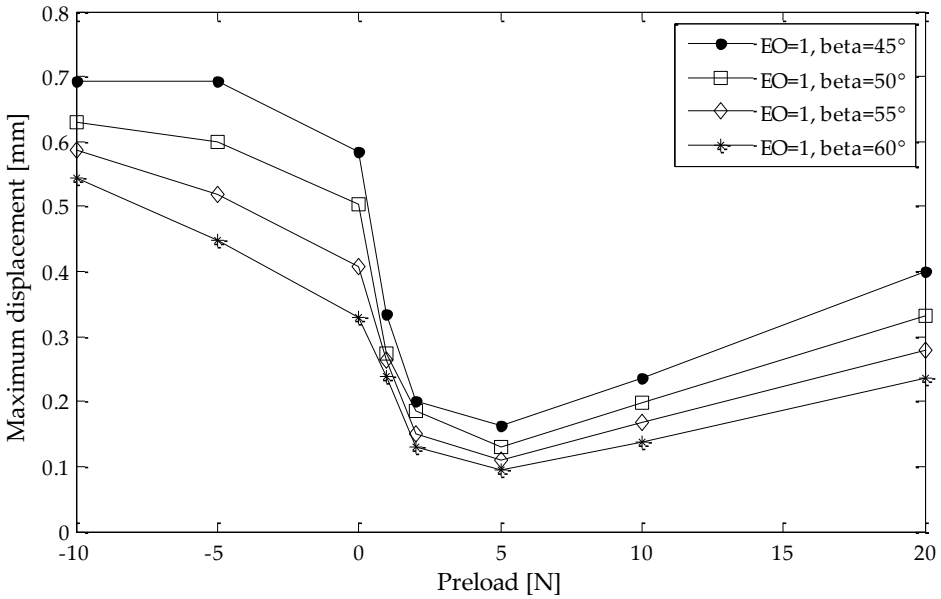


Figure 4.51 Optimal curves for shroud angles: β=45°, 50°, 55°, 60° when EO=1

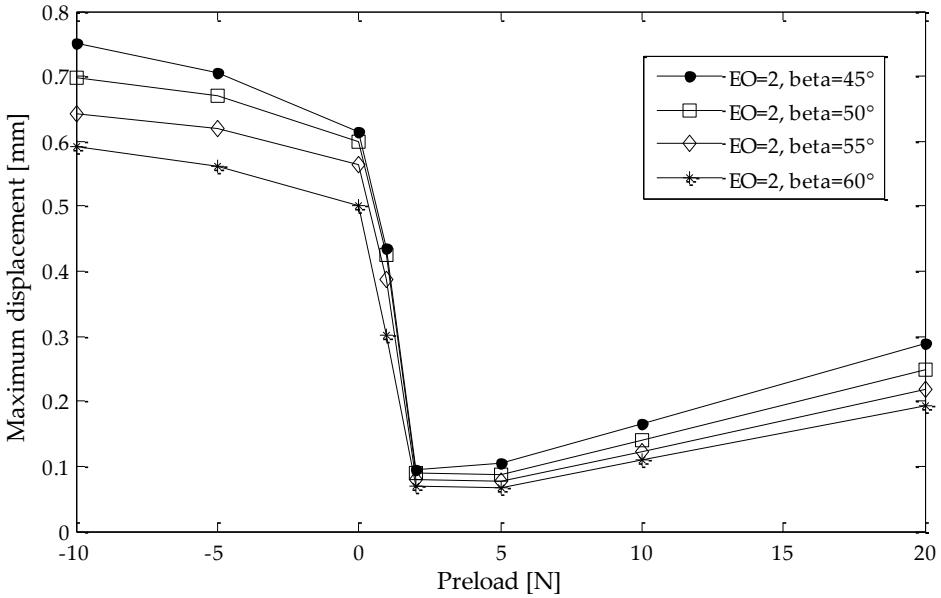


Figure 4.52 Optimal curves for shroud angles: β=45°, 50°, 55°, 60° when EO=2

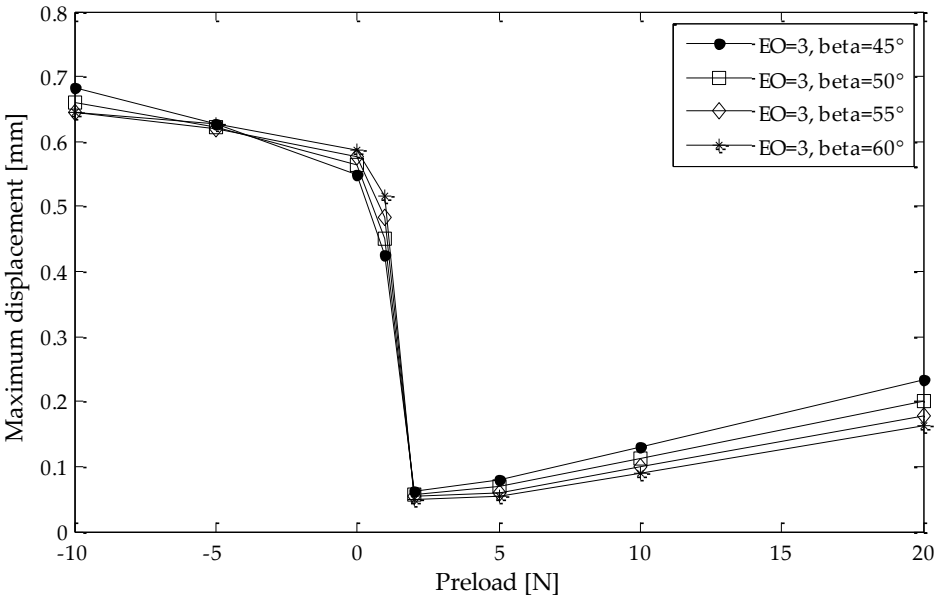


Figure 4.53 Optimal curves for shroud angles: β=45°, 50°, 55°, 60° when EO=3

It is observed from the results that for the first EO excitation, shroud dampers are not working properly. However, for the second and third EO excitations, shroud dampers are satisfactory. It should be noted that increasing shroud angle decreases the response of the no damper case as well. Nevertheless, due to the increased angle it is possible to dissipate more energy; hence, better performance is observed for the higher shroud angle.

For each engine order, frequency shift curves are also plotted on the same figure for different shroud angles. Figures 4.54 to 4.56 present the frequency shift curves for shroud angles: β=45°, 50°, 55°, 60° when EO=1, 2 and 3.

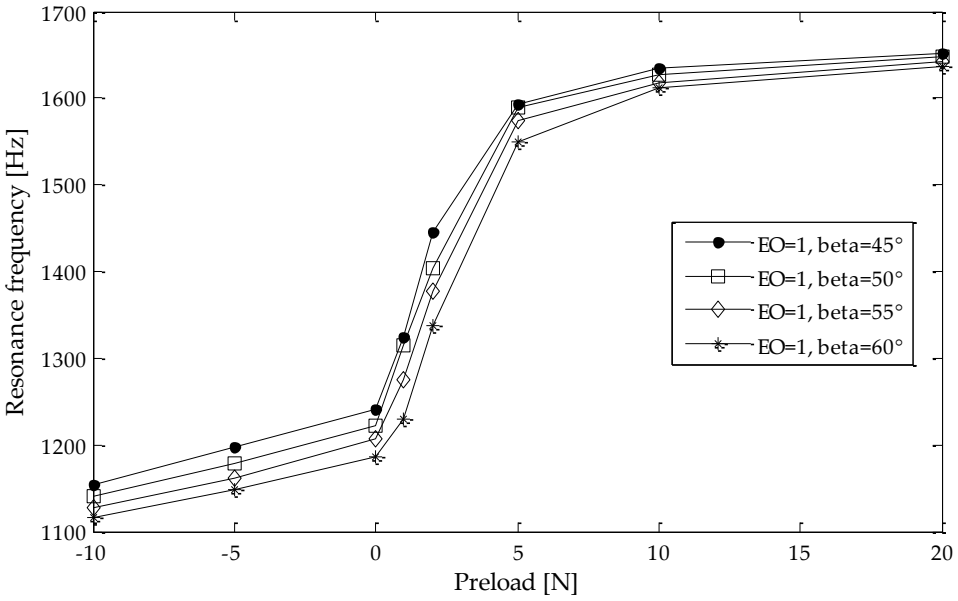


Figure 4.54 Frequency shift curves for shroud angles: β=45°, 50°, 55°, 60° when EO=1

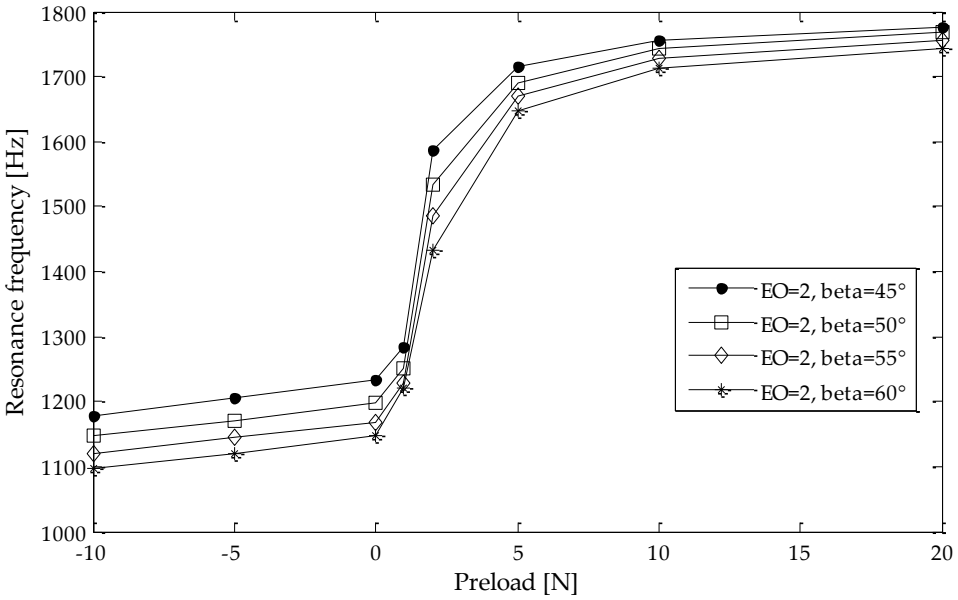


Figure 4.55 Frequency shift curves for shroud angles: β=45°, 50°, 55°, 60° when EO=2

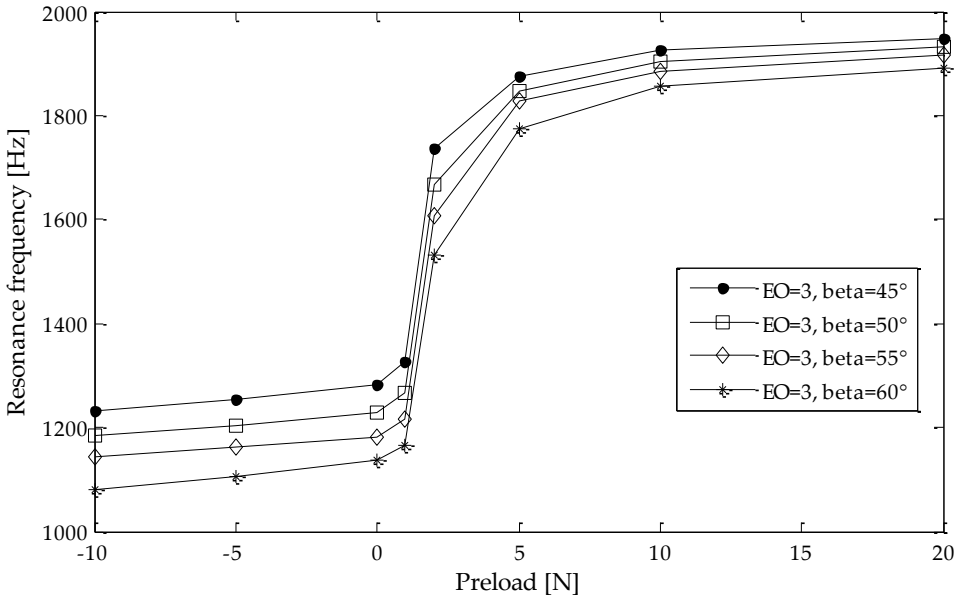


Figure 4.56 Frequency shift curves for shroud angles: β=45°, 50°, 55°, 60° when EO=3

CHAPTER 5

DISCUSSION AND CONCLUSION

In this study, main aim is to apply structural changes to a nonlinear system using a structural modification method which is more efficient than performing a FEA for each change that occur in the system. Modal superposition method is superior to receptance methods if the number of nonlinear DOFs in the system is large, since the number of nonlinear algebraic equations obtained are only related to the number of modes included in the analysis and also, if the modal data of the system is available. In order to use modal superposition method for solving nonlinear algebraic equations, modal data of the linear system is required which, in this study, is obtained by the developed structural modification method. In this thesis, a new structural modification method is presented which determines the natural frequencies and mode shapes of the modified structure by using the modal data of the original structure and mass and stiffness matrices of the modifying part.

In order to demonstrate the method, a simple lumped parameter model is considered first. If all modal information of the original structure is used in the analysis, it is seen that the exact mode shapes and natural frequencies of the modified structure can be obtained. For large scale models, using all information is not practical due to high computational time requirements. Therefore, reduction of the original structure is studied by considering FEM of a cantilever beam retaining 20, 40, and 80 modes of the original structure in the calculations. It is seen that, using more information provides more accurate results as expected. Furthermore, the proposed method is compared to the Özgüven's method, which is a structural modification method with additional degrees of freedom, in terms of calculation time. Three different cases utilizing different DOFs in the original structure, modifying structure and modified system are studied. Also, similar comparisons are performed for beam models having relatively more DOFs in order to make the time comparison more remarkable. For those models, 20 and 40 modes of the original system are used for both methods while different number of modes of the modified system is used in the proposed method. It is concluded that in all cases, proposed method provides a significant decrease in the computational time required compared to Özgüven's method. Therefore, the developed structural modification method can be employed in reanalysis of large finite element models, especially for design optimization purposes.

For an application of the developed method to nonlinear systems, first, a simple lumped parameter model is considered utilizing all information of the unmodified structure. It is observed that identical results are obtained by using the proposed approach and directly solving the nonlinear system. Later, in order to show the capability of the proposed method, a FEM of a cantilever beam is considered. Instead of using all information of the unmodified system; 10, 20, and 30 modes are included in the analysis by utilizing the proposed method and response of the system is studied. It is observed from the results that increasing the number of modes used in structural modification method provides more accurate results; however, for the selected range of frequency, utilizing limited number of modes, it is

possible to obtain the same results as if the all modal information of the modified structure is employed.

For a real life example, sector of a bladed disk system is considered. For engine order excitation and assuming the system as tuned, response levels of all blades can be obtained. Different shroud angles are considered by utilizing the developed structural modification method. For shroud angles of 45°, 50°, 55° and 60°, nonlinear response analyses are performed and for different preload values, response plots are obtained for the frequency interval that normal motion for shroud surfaces are dominant. Response plots are obtained for engine orders (EO); 1, 2 and 3. Furthermore, optimal and frequency shift curves are obtained and they are compared with each other for all angles for each EO. Since increasing the damper angle increases slip motion, i.e. energy dissipation, vibration amplitudes decrease. This can be seen in the optimal curve figures for all engine orders. Also, it is seen from the frequency shift plots that, for the corresponding EO, increasing the shroud angle decreases resonance frequency for all preload cases.

As a future work, developed structural modification method can be improved for inverse modification problems. Furthermore, for studies on finite elements models, residual vectors can be obtained during modal analysis of the original system and included in the structural modification formulation. Since the effect of higher modes is included by this way, modal information of the modified system can be more accurately obtained.

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APPENDIX

IMAC XXXI A CONFERENCE AND EXPOSITION ON STRUCTURAL DYNAMICS
PAPER #171

A New Structural Modification Method with Additional Degrees of Freedom for Dynamic Analysis of Large Systems

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ABSTRACT

Structural modification techniques are widely used to predict the dynamic characteristics of modified systems by using the information of the original and modifying systems. Nowadays, due to the increased computational power, large finite element models are regularly used. During design optimization, modifications are done on the original structure in order to meet the design requirements which require reevaluation of the dynamic response of the structure. This is a time consuming process since for each modification, the whole structure needs to be analyzed. In this study, a new structural modification method is proposed in order to decrease the computational effort during the design optimization. Proposed method uses the modal data of the original system and system matrices of the modifying structure in order to predict the modal data of the modified system which can be used for the dynamic analysis. Initially, the application of the method is presented on a lumped parameter model. Later, the proposed method is applied on a finite element model of a cantilever beam which is modified by additional systems having different number of total degrees of freedom (DOFs) and different number of connection DOFs. Modal data of the original structure and the system matrices of the modifying part are extracted from a commercial finite element software. The modal information obtained from the proposed method and the one obtained from finite element software are compared and it is observed that they are in perfect agreement if all information of the original structure is used. The effect of reduction of the original structure is as well investigated and the performance of the developed method is studied for varying the size of the modifying system. The computation time required for the determination of frequency response function is compared with other structural modification methods where significant improvement in computation time is observed.
Keywords: Structural modification, Özgüven's Method, Additional degrees of freedom, Reanalysis, Direct modification

1. INTRODUCTION

When analyzing different design alternatives, structural modification techniques are useful at any time in the design stage which shorten the analysis time by a considerable amount. Especially for experimental studies or working with large scale Finite Element Models (FEM), it is difficult and time consuming to adapt every modification to the system in order to reanalyze it accurately. Direct structural modification, also called as reanalysis, is applied using original system information and dynamic structural modification matrix of the modifying part in order to predict the dynamic characteristics of modified systems. Structural modification techniques can be adopted for both systems when additional degrees of freedoms are present or not. These techniques predict the behavior of the modified system very accurately, when all information of the original system is included in the analysis. Structural modification techniques are divided into two as direct structural modification and inverse structural modification techniques [1]. Direct modification is based on determination of the dynamic characteristics of the modified system by modifying the original system. On the other hand, inverse structural modification is considered as an optimization process where the modifying structure

is determined in order to obtain the desired modified system characteristics. Direct structural modification is studied by Crowley et al. [2] using experimentally obtained frequency response functions (FRFs) of the original structure; whereas, inverse structural modifications are studied by Kyprianou, et al. [3, 4], Mottershead et al. [5, 6] and Park et al. [7].

Modifications may or may not contribute additional DOFs to the original system. For structural modification without additional degrees of freedom, Sherman-Morrison [8] and Woodberry [9] formulas are employed. These methods find the dynamic matrix of the modified system by updating the dynamic matrix of the original system. Özgüven [10] proposed a general method for structural modification problems with or without additional degrees of freedom in the modified structures. This method is an FRF based method which obtains frequency response function of the modified structure by using the FRF of the original structure and dynamic structural modification matrix of the modifying structure. Sanliturk [11] presented a new method based on matrix inversion method combined with Sherman-Morrison formula and Woodberry formula for systems without additional degrees of freedoms. Tahtali et al. [12] applied this method to a plate and a propeller; whereas, Koksal et al. [13] used this method by using power series expansion in order to avoid matrix inversion. In addition to these, distributed modifications studied by D'Ambrogio and Sestieri [14] for systems without additional degrees of freedom and Hang et al. [15, 16, 17] worked on distributed modification for systems that their DOFs have changed after modification.

This paper presents a new structural modification method which can be applied to large systems having additional degrees of freedoms. This method uses the modal data of the original structure and system matrices of the modifying part in order to determine the modal data of the modified structure. Mode shapes and natural frequencies of the modified structure can be obtained exactly if all information of the original part is used. Computational efficiency of the method becomes much significant especially working with large scale models. In order to increase the computational efficiency of the method, largest possible system should be considered in design optimizations for the determination of the original structure; whereas, for the modifying part, system matrices can be extracted from a finite element software. Firstly, the developed method is presented on a lumped parameter model. Later, since reduction of the original structure is highly required for large scale systems, accuracy of the reduction process is investigated on a cantilever beam model. Developed method is compared to the Matrix Inversion Method, also called as ÖZGÜVEN's Method [10], by comparing the computational time required for FRF calculation. Methods are applied on a simple cantilever beam model and comparisons are done for different sizes of the modifying system and different sizes of coupling DOFs.

2. New Structural Modification Method

Fig. 1 shows two structures coupled by rigid connection, where subscript a corresponds to DOFs which belong only to the original structure, subscript b corresponds to DOFs which belong to both structures and subscript c corresponds to DOFs which belong only to the modifying part. For the coupled structure, system matrices of the original and modifying parts are added to obtain the equation of motion of the modified structure as seen in Eq. (1). Similarly, structural and/or viscous damping can also be included in the equation of motion, which is not considered here.

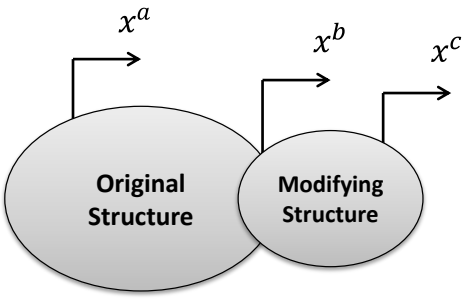

Fig. 1 – Modified structure (rigid connection)

$$\left[\begin{bmatrix} M_0^{aa} & M_0^{ab} & 0 \\ M_0^{ba} & M_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix}\right]\{\ddot{x}\} + \left[\begin{bmatrix} K_0^{aa} & K_0^{ab} & 0 \\ K_0^{ba} & K_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix}\right]\{x\} = \{F\} \tag{1}$$

where $[M_o]$ and $[K_o]$ represent mass and stiffness matrices of the original structure while $[M_m]$ and $[K_m]$ represent mass and stiffness matrices of the modifying part. Let the response of the modified system be written as

$$\{x\} = \begin{Bmatrix} x^a \\ x^b \\ x^c \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q_0 \\ x^c \end{Bmatrix} = [\Phi]\{q\} \tag{2}$$

where $[\phi_o]$ is the mass normalized mode shape matrix of the original system. Identity matrix is added to the right bottom corner of the matrix in order to keep the information coming from the modifying part. Inserting this expression into Eq. (1) and pre-multiplying it by $[\Phi]^T$ gives the following mass and stiffness matrices of the modified structure as

$$[M_{\text{mod}}] = \begin{bmatrix} [I] & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \tag{3}$$

$$[K_{\text{mod}}] = \begin{bmatrix} [\Omega_0] & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \tag{4}$$

The equation of motion becomes:

$$[M_{\text{mod}}]\{\ddot{q}\} + [K_{\text{mod}}]\{q\} = [\Phi]^T \{F\} \tag{5}$$

where $[\Omega_0]$ is a diagonal matrix composed of squares of the natural frequencies of the original structure. For free vibrations, eigenvalue problem is defined as

$$([K_{\text{mod}}] - \lambda[M_{\text{mod}}])\{q\} = \{0\} \tag{6}$$

Solving the eigenvalue problem defined by Eq. (6) the modal data of the modified structure can be obtained. However, it should be noted eigenvectors obtained define $\{q\}$ rather than $\{x\}$; hence, they should be pre-multiplied by $[\Phi]$ which gives the mode shapes of the modified structure in $\{x\}$.

3. CASE STUDY 1: APPLICATION ON A SIMPLE LUMPED PARAMETER MODEL

In order to validate the developed structural modification method, it is applied on a simple lumped parameter model given in Fig. 2. As seen from the figure, the 4-DOF system, original structure, given at the top is modified by the 3-DOF structure, modifying part, given at the bottom by rigid connection. System parameters of the original structure are $M_1 = M_2 = M_3 = M_4 = 1$, $K_1 = K_2 = K_3 = K_4 = 10000$; whereas, for the modifying part they are $M_5 = M_6 = M_7 = 2$, $K_5 = K_6 = K_7 = 20000$.

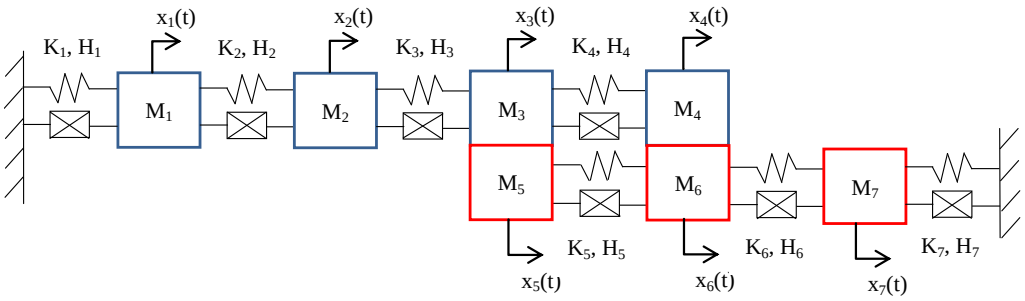

Fig. 2 – Simple lumped parameter model: Blue-original structure, red-modifying part

Developed structural modification method is applied to this problem and the results obtained are presented in Table which are compared with the exact ones obtained by solving the 5-DOF system. In the comparison, for natural frequencies error between the obtained and the exact one and for the mode shapes, the modal assurance criterion (MAC) is used which can be defined as

$$MAC\left(\{\psi\}_r,\{\psi\}_s\right)=\frac{\left(\{\psi\}_r^T,\{\psi\}_s\right)^2}{\left(\{\psi\}_r^T,\{\psi\}_r\right)\left(\{\psi\}_s^T,\{\psi\}_s\right)} \tag{7}$$

which takes a value between 0 and 1, where 1 denotes that the compared vectors $\{\psi\}_r$ and $\{\psi\}_s$ are identical.

Table 1

Mode #	Natural frequencies found by the proposed method [rad/s]	Natural frequencies found by modal analysis of the modified system [rad/s]	Error %	MAC
1	42.34	42.34	0%	1.00
2	100.00	100.00	0%	1.00
3	132.75	132.75	0%	1.00
4	168.77	168.77	0%	1.00
5	179.17	179.17	0%	1.00

4. CASE STUDY 2: APPLICATION ON A FEM

4.1. REDUCTION OF THE ORIGINAL STRUCTURE

For very large scale systems, reduction of the system is required in order to keep computational expenses at minimum or at a practical value. This can be achieved in the developed method by using a certain number of mode shapes which is much less than the number of total DOFs. In order to present the developed method, a cantilever beam modeled in a commercial finite element software is considered. FEM of the cantilever beam and the modifying part are shown in Fig. 3, where total numbers of DOFs on the original structure, modifying structure and the modified system are 216, 81 and 270, respectively. Assuming only the first 20 modes are of interest for the modified system, natural frequencies and mode shapes of the first 20 modes of the modified system are obtained by utilizing 20, 30 and 40 modes of the original structure. For the comparison of mode shapes obtained, modal assurance criterion (MAC) is used.

Comparison of the natural frequencies and the mode shapes of the modified structure for different number of modes retained in the original system are given in Table 2 and Table 3, respectively.

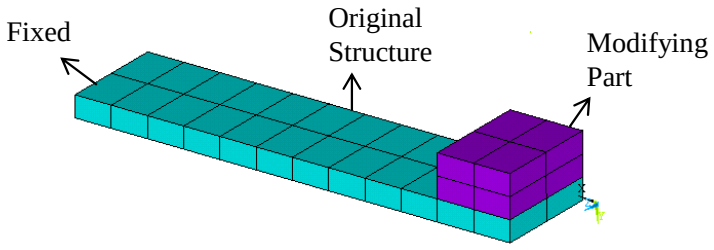

Fig. 3 – FEM of the cantilever beam

Table 2 - Natural frequencies [Hz]

Developed Method							
Mode #	Exact results	Using 20 modes		Using 30 modes		Using 40 modes	
		Nat. Freq.	Err %	Nat. Freq.	Err %	Nat. Freq.	Err %
1	130.14	130.02	0.09	130.00	0.11	129.99	0.12
2	604.54	605.62	0.18	605.43	0.15	605.10	0.09
3	988.99	996.14	0.72	991.79	0.28	990.44	0.15
4	1104.65	1110.54	0.53	1109.59	0.45	1107.76	0.28
5	2799.18	2895.71	3.45	2838.28	1.40	2820.46	0.76
6	4158.96	4190.02	0.75	4185.60	0.64	4176.36	0.42
7	4409.29	4482.54	1.66	4469.44	1.36	4446.96	0.85
8	4793.14	4849.86	1.18	4821.67	0.60	4811.72	0.39
9	6292.99	6561.00	4.26	6390.09	1.54	6344.17	0.81
10	8803.87	9040.56	2.69	8998.08	2.21	8929.63	1.43
11	10058.67	10187.86	1.28	10143.49	0.84	10118.95	0.60
12	10877.85	11804.89	8.52	11206.78	3.02	11057.74	1.65
13	14492.40	15020.50	3.64	14898.24	2.80	14752.55	1.80
14	16458.13	16633.87	1.07	16563.70	0.64	16544.13	0.52
15	16562.25	17128.54	3.42	16841.34	1.69	16770.67	1.26
16	18108.22	21359.44	17.95	18735.07	3.46	18422.54	1.74
17	21922.63	23193.94	5.80	22835.13	4.16	22444.45	2.38
18	24152.82	24860.07	2.93	24344.07	0.79	24318.58	0.69
19	24288.33	37521.24	54.48	24630.32	1.41	24502.37	0.88
20	27322.41	44340.93	62.29	27570.93	0.91	27446,95	0.46

Table 3 – MAC numbers

Developed Method			
Mode #	Using 20 modes	Using 30 modes	Using 40 modes
1	1.00	1.00	1.00
2	1.00	1.00	1.00
3	1.00	1.00	1.00
4	1.00	1.00	1.00
5	1.00	1.00	1.00
6	0.96	0.97	0.99
7	0.98	0.99	0.99
8	0.99	1.00	1.00
9	0.98	1.00	1.00
10	0.99	0.99	1.00
11	0.99	0.99	1.00
12	0.94	0.99	1.00
13	0.98	0.98	0.99
14	0.95	0.98	0.99
15	0.96	0.98	0.99
16	0.85	0.97	0.99
17	0.94	0.95	0.98
18	0.98	1.00	1.00
19	0.10	0.95	0.99
20	0.05	0.98	0.99

4.2. COMPARISON OF COMPUTATIONAL TIME REQUIRED

Developed structural modification method is compared with Özgüven's Method by means of computational time required to obtain receptance of the modified structure. For this comparison three cases are considered. In the first case, size of the original system and the number of common DOFs are kept constant in both methods. In the second case, sizes of the original and the modifying parts are kept constant while the number of common DOFs are changed. In the third case, sizes of the original structure, the modifying part and the number of common DOFs are changed in order to keep the size of the modified system constant. For the calculation of the computational time, two different procedures as described below are used:

For the developed model:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. Calculation of mass and stiffness matrices of the modified system
4. Solving the EVP for the modified system.
5. Calculation of the receptance of the defined DOF of the modified system in the range of frequency of interest.

For Özgüven's Method:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. For the frequency range of interest:
   a. Calculation of the receptance of the original system.

b. Calculation of the dynamic structural modification matrix.
c. Calculate the receptance of the desired DOF of the modified system.

The calculations are done on a computer having a processor Intel Core i3-2310M CPU @ 2.10 GHz with 4,00 GB of RAM. Finite element models of the original and modifying structures for the first case is given Fig.4.

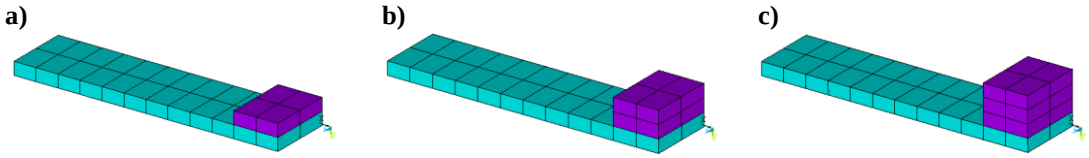

Fig. 4 – Finite element models used in Case 1: Green-original structure, purple-modifying part, size of the modifying part: a) 54, b) 81, c) 108

In this case, the effect of modifying system size (hence, the size of the modified system) on the computational time is studied. Common number of DOFs is 27 in all cases. The sizes of the FEM considered in this comparison are presented in Table 4. For the cases when first *20-40-80 modes* of the original system are included in the calculations. In the computations, the frequency range of interest is considered as 500-1200 rad/s utilizing 351 frequency points. From the results (See Table 5) it can be observed that as the number of DOFs in the modifying part; hence, the modified structure, increases, time required to determine the receptance increases as well. However, it can be observed that proposed method decreases the computational time significantly compared to Özgüven's Method. On the other hand, it should be noted that developed method cannot be used if the experimental receptance of the original structure is at hand. In that case, Özgüven's Method is the fastest choice as indicated by [18] for systems without additional DOFs.

Table 4– Sizes of the models: Case 1

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	54	243	27
b	216	81	270	27
c	216	108	297	27

Table 5 – Comparison of calculation time: Case 1

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.05	0.07	0.11	0.02	0.02	0.04
b	0.10	0.12	0.18	0.02	0.04	0.06
c	0.17	0.19	0.27	0.04	0.05	0.09
Model	Özgüven's Method [s]					
	including the FRF of the original system			excluding the FRF of the original system		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	3.31	3.84	4.57	3.04	3.19	3.26
b	5.81	6.25	6.90	5.44	5.62	5.82
c	8.70	9.59	10.01	8.31	8.58	8.68

In the second case, sizes of modifying system and the original systems are kept constant, where the number of common DOFs is changed as 18, 27, and 36 as shown in Fig. 5. Table 6 gives the sizes of the models used; whereas, Table 7 presents the calculation time considering the first *20*, *40* and *80 modes* of the original system in both methods. The frequency range of interest is the same as the first case utilizing 351 frequency points. It can be seen that as the number of common DOFs increases, resulting in decrease in the total number of DOFs of the modified system, computation time required decreases slightly for both methods. However, if both methods are compared, the proposed method results in a drastic decrease in the computational time.

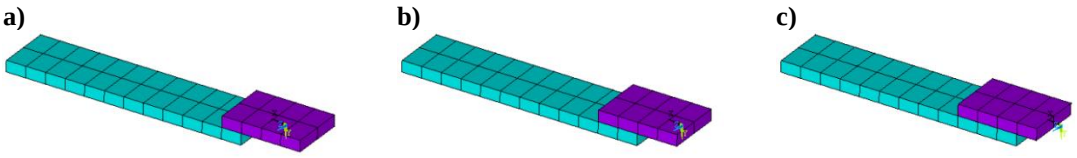

Fig. 5 – Finite element models used in Case 2: Blue-original Structure, purple-modifying part, size of the common DOFs: a) 18, b) 27, c) 36

Table 6 – Sizes of the models: Case 2

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	90	288	18
b	216	90	279	27
c	216	90	270	36

Table 7 – Comparison of calculation time: Case 2

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.13	0.16	0.22	0.04	0.05	0.08
b	0.12	0.14	0.20	0.03	0.05	0.07
c	0.11	0.14	0.20	0.02	0.04	0.06
Model	Özgüven's Method [s]					
	including the FRF of the original system			excluding the FRF of the original system		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	6.82	7.18	8.14	6.43	6.60	6.79
b	6.58	7.05	7.81	6.22	6.50	6.59
c	6.25	6.63	7.49	5.82	6.15	6.32

In the third case, the number of total DOFs of the modified system is kept constant by changing the sizes of the, original structure, modifying part and the common DOF. The finite element models used are presented in Fig. 6; whereas, the sizes of part are presented in Table 8. Results obtained by utilizing the first 20, 40 and 80 modes of the original structure are given in Table Table 2.10 9. Identical frequency range of interest is used as in the previous cases. It can be observed from the results that as the number of DOFs in the modifying structure increases, solution time also increases for both methods, even though the number of common DOFs or the number of DOFs in the original structure decreases. Similar to the previous cases, the proposed requires significantly less computational time compared to Özgüven's method.

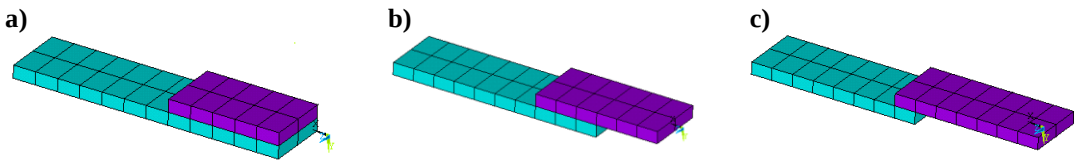

Fig. 6 – Finite element models used in Case 3: Green-original structure, purple-modifying part, size of the modifying part: a) 108, b) 126, c) 144

Table 8 - Sizes of the models: Case 3

Model	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
a	216	108	270	54
b	180	126	270	36
c	144	144	270	18

Table 9 - Comparison of calculation time: Case 3

Model	Developed Method [s]					
	including the modal solution			excluding the modal solution		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	0.14	0.17	0.22	0.02	0.03	0.07
b	0.22	0.24	0.32	0.05	0.06	0.09
c	0.30	0.33	0.44	0.08	0.09	0.14
Model	Özgüven's Method [s]					
	including the FRF of the original system			excluding the FRF of the original system		
	20 modes	40 modes	80 modes	20 modes	40 modes	80 modes
a	7.75	8.27	8.94	7.40	7.47	7.52
b	9.84	10.14	10.67	9.44	9.77	9.88
c	12.03	12.33	12.86	11.73	12.15	12.21

5. CONCLUSION

In this paper, a new structural modification method is presented which determines the natural frequencies and mode shapes of the modified structure using modal data of the original structure and mass and stiffness matrices of the modifying part. In order to demonstrate the method, a simple lumped parameter model is considered. If all modal information of the original structure is used in the analysis, it is seen that the exact mode shapes and natural frequencies of the modified structure can be obtained. For large scale models, using all information is not practical due to high computational time requirements. Therefore, reduction of the original structure is studied by considering FEM of a cantilever beam retaining 20, 40, and 80 modes of the original structure in the calculations. It is seen that, using more information provides more accurate results. Furthermore, the proposed method is compared to the Özgüven's method in terms of calculation time. Three different cases utilizing different DOFs in the original structure, modifying structure and modified system are studied. It is concluded that in all cases, proposed method provides a significant decrease in the computational time required compared to Özgüven's method. Therefore, the developed structural modification method can be employed in reanalysis of large finite element models, especially for design optimization purposes.

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IMAC XXXI A CONFERENCE AND EXPOSITION ON STRUCTURAL DYNAMICS
PAPER #225

Reanalysis of Large Finite Element Models with Structural Nonlinearities

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ABSTRACT

Structural modification methods are widely used for design optimization purposes, especially for systems having large number of degrees of freedoms. By the use of these methods, modifications can be adapted to the system by avoiding reevaluation of dynamic characteristics of the modified system. In this paper, a method is proposed for the reanalysis of large finite element models having structural nonlinearities in order to reflect design variations. The finite element model (FEM) of the structure is divided into two parts representing the modified and unmodified sections. The modified part characterizes the sections where design variations are executed. Harmonic balance method (HBM) with modal superposition approach is used to convert the resulting nonlinear differential equations of motion into nonlinear algebraic equations. The modal information used in the nonlinear analysis is obtained from a new structural modification method with additional degrees of freedom details of which are given in another paper. Since the proposed method uses a model superposition approach, it can be applied to systems having large number of nonlinear elements. The proposed method is demonstrated on an example cantilever beam with nonlinear elements attached and the effect of design variations on the nonlinear response of the system is studied.

Keywords: Structural modification, Reanalysis of Nonlinear systems, Harmonic balance method, Modal superposition, Cubic stiffness nonlinearity.

1. INTRODUCTION

Structural modification techniques are commonly used in the design optimization of an already constructed structure. These methods are very reliable and accurate even when working with large scale finite element models (FEM). Since it is time consuming to adapt every change to the system and reanalysis for each step during design process; due to their high accuracy and time shortening capabilities, structural modification methods are widely preferred. If the modification is based on determination of the dynamic characteristics of a modified system, it is referred as reanalysis or direct structural modification. Direct structural modification is studied by Crowley et al. [1] using experimentally obtained FRFs of the original structure. On the other hand, it is called inverse structural modification when the modification on an already constructed structure is to be decided, which is mostly applied in order to eliminate vibration problems. Kyprianou, et al. [2, 3], Mottershead et al. [4, 5] and Park et al. [6] work on inverse structural modifications. Structural modification methods can be applied either on systems having additional degrees of freedoms or not. Sherman-Morrison [7] and Woodberry [8] formulas are utilized for systems without additional degrees of freedoms. These methods find dynamic matrix of the modified system by updating the dynamic matrix of the original system. Özgüven [9] proposed a general method, matrix inversion method, for structural modification problems with or without additional degrees of freedoms in the modified structures. This method is an FRF based method which obtains FRF of the modified structure by using FRF of the original structure and dynamic structural modification matrix of the modifying structure. Sanliturk [10] presented a new method based on matrix inversion method combined with Sherman-Morrison formula and Woodberry formula for systems without additional degrees of freedoms. Matrix inversion method is also studied by Koksal et al. [11] where power series expansion is utilized in order to avoid matrix inversion. For distributed modifications, D'Ambrogio and Sestieri [12] developed a modeling approach for systems without additional degrees of freedoms.

Modifications can be applied to linear systems as well as nonlinear systems. In this paper, structures having nonlinearities either in the original or in the modifying parts are considered. In order the perform reanalysis of the structure, a new structural modification method developed by Sayin and Cigeroglu [13] is used. With the use of this method, modal information of the modified structure can be obtained by considering the modal information of the unmodified part and system matrices of the modifying part, which are later used in the nonlinear reanalysis for design optimizations.

In this study, the proposed method is first applied to a lumped mass parameter model, where the nonlinearities are considered either in the unmodified part or in the modifying part. After demonstrating the proposed method on a simple lumped parameter model, it is applied on a FEM. For this purpose, FEM of a cantilever beam with attached nonlinear elements is utilized. Effect of number of modes used in the structural modification method on the nonlinear response of the structure is studied and the accuracy of the results is investigated by utilizing the modal information obtained from a commercial FEM software, ANSYS.

2. HARMONIC BALANCE METHOD WITH MODAL SUPERPOSITION

For the solutions of nonlinear systems, there exist time domain methods as well as frequency domain methods. Harmonic balance method (HBM) is a frequency domain method which is used in this work in order to linearize the nonlinearities. Cigeroglu et al. [14, 15, 16] proposed a model superposition method coupled with harmonic balance method, which becomes significantly useful if the number of nonlinear elements is large. Equation of motion of a nonlinear system with structural damping can be written as follows

$$[M]\{\ddot{x}(t)\}+i[H]\{x(t)\}+[K]\{x(t)\}+f_N\left(\{x(t)\}\right)=\{f(t)\} \tag{1}$$

where $[M]$, $[H]$, and $[K]$ are the mass, structural damping and stiffness matrices and $\{x(t)\}$ is the response vector of the system. $f_N\left(\{x(t)\}\right)$ is the nonlinear forcing matrix.

In this method, the response is written in terms of the mode shapes of the linear system as follows by assuming harmonic motion

$$\{x(t)\}=\sum_{r=1}^{N_m} a_r\{\phi_r\}e^{i\omega t}=[\phi]\{a\}e^{i\omega t} \tag{2}$$

where N_m is the number of modes, $[\phi]$ is the mode shape matrix of the linear system. a_r is the complex coefficient of the r^{th} mode shape, $\{\phi_r\}$.

Substituting Eq (2) into (1) and pre-multiplying by $[\phi]^T$ results in

$$-\omega^2[\phi]^T[M][\phi]\{a\}+[\phi]^T\left(i[H]+[K]\right)[\phi]\{a\}+[\phi]^T\{F_N\}=[\phi]^T\{F\} \tag{3}$$

Assuming proportional damping and $[\phi]$ to be mass normalized, Eq. (3) becomes:

$$\left[(1+i\gamma)[\Omega]-\omega^2[I]\right]\{a\}+[\phi]^T\{F_N\}=[\phi]^T\{F\} \tag{4}$$

where γ is the loss factor, $[I]$ is the identity matrix and

$$[\Omega]=\begin{bmatrix}\omega_1^2 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \omega_r^2\end{bmatrix} \tag{5}$$

where ω_r is the r^{th} natural frequency of the linear system.

2.1. CUBIC STIFFNESS NONLINEARITY

In this study, as a nonlinear element cubic stiffness is considered, for which the nonlinear force can be written as

$$F = k_c \cdot x^3 \tag{6}$$

where k_c is the coefficient of cubic stiffness nonlinearity. Assuming a single harmonic motion as

$$x = X_s \sin(\omega t) + X_c \cos(\omega t) \tag{7}$$

Nonlinear forcing can be written as follows

$$F_N = F_{Ns} \sin(\omega t) + F_{Nc} \cos(\omega t) \tag{8}$$

$$\begin{aligned} F_{Ns} &= \frac{1}{\pi} \int_0^{2\pi} k_c \left(X_s \sin(\theta) + X_c \cos(\theta) \right)^3 \sin(\theta) d\theta = \frac{3}{4} k_c \left({X_s}^3 + {X_c}^2 X_s \right) \\ F_{Nc} &= \frac{1}{\pi} \int_0^{2\pi} k_c \left(X_s \sin(\theta) + X_c \cos(\theta) \right)^3 \cos(\theta) d\theta = \frac{3}{4} k_c \left({X_c}^3 + {X_s}^2 X_c \right) \end{aligned} \tag{9}$$

where $\theta = \omega t$. Hence, the nonlinear internal forcing becomes

$$F_N = \frac{3}{4} k_c \left(\left({X_s}^3 + {X_c}^2 X_s \right) \sin(\omega t) + \left({X_c}^3 + {X_s}^2 X_c \right) \cos(\omega t) \right) \tag{10}$$

3. STRUCTURAL MODIFICATION METHOD

It is possible to consider the nonlinearities either included in the unmodified structure or in the modifying part. Structural modification method developed in [13] provides the modal data of the modified structure without employing the modal analysis to the whole (modified) structure. It uses the modal data of the original structure and mass and stiffness matrices of the modifying part as inputs. After finding the modal data of the modified structure, they are used in the modal superposition method as described in Section 2. Application of the new structural modification method, results in the following system matrices for the modified structure

$$\left[M_{\text{mod}}\right] = \begin{bmatrix} [I] & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_{um}^a \\ \phi_{um}^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_{um}^a \\ \phi_u^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \tag{11}$$

$$\left[K_{\text{mod}}\right] = \begin{bmatrix} [\Omega_{um}] & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_{um}^a \\ \phi_{um}^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_{um}^a \\ \phi_{um}^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \tag{12}$$

where $[M_m]$ and $[K_m]$ represent mass and stiffness matrices of the modifying part, $[\phi_{um}]$ is the mass normalized mode shape matrix of the unmodified system and $[I]$ is the identity matrix. Subscript a corresponds to DOFs which belong only to the unmodified structure, subscript b corresponds to DOFs which belong to modified structure and subscript c corresponds to DOFs which belong only to the modifying part. $[\Omega_{um}]$ is a diagonal matrix composed of squares of natural frequencies of the unmodified structure.

Solving the eigenvalue problem defined by Eqs. (11) and (12), natural frequencies and mode shapes of the modified structure can be obtained. It should be noted that, in order to use the mode shapes obtained, they should be pre-multiplied by

$$\begin{bmatrix} \begin{bmatrix} \phi_{um}^{a} \\ \phi_{um}^{b} \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^{T} \tag{13}$$

4. CASE STUDIES

4.1. LUMPED PARAMETER MODEL

A simple lumped parameter model (Fig. 1) is constructed representing the modified system. 3-DOF-system is added to the original (unmodified) 4-DOF system by rigid coupling resulting in a 5-DOF-system. Modal data of the modified system is obtained from directly modeling the 5-DOF system using the new structural modification method utilizing *all information* of the unmodified system. Nonlinear forced response analysis of the modified structure is performed by using these modal data and the results obtained are compared. The system parameters used for the lumped parameter model are $M_1 = M_2 = M_3 = M_4 = 1$, $K_1 = K_2 = K_3 = K_4 = 10000$, $F = 50$, $\gamma = 0.01$ and $M_5 = M_6 = M_7 = 2$, $K_5 = K_6 = K_7 = 20000$, $F = 50$, $\gamma = 0.01$. Lumped model including nonlinearities in the unmodified system is presented in Fig. 1 and corresponding response plots for the first mass for different k_c values are given in Fig. 2. Another lumped model is presented for which nonlinearities are included in modifying system (See Fig. 3) and corresponding response plots of the seventh mass for different k_c values are given in Fig. 4.

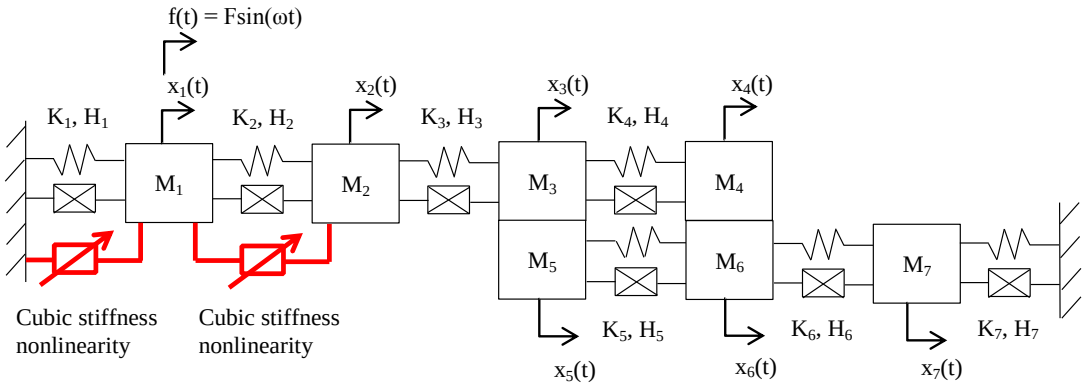

Fig. 1 – Lumped parameter model (modified system) – Case 1

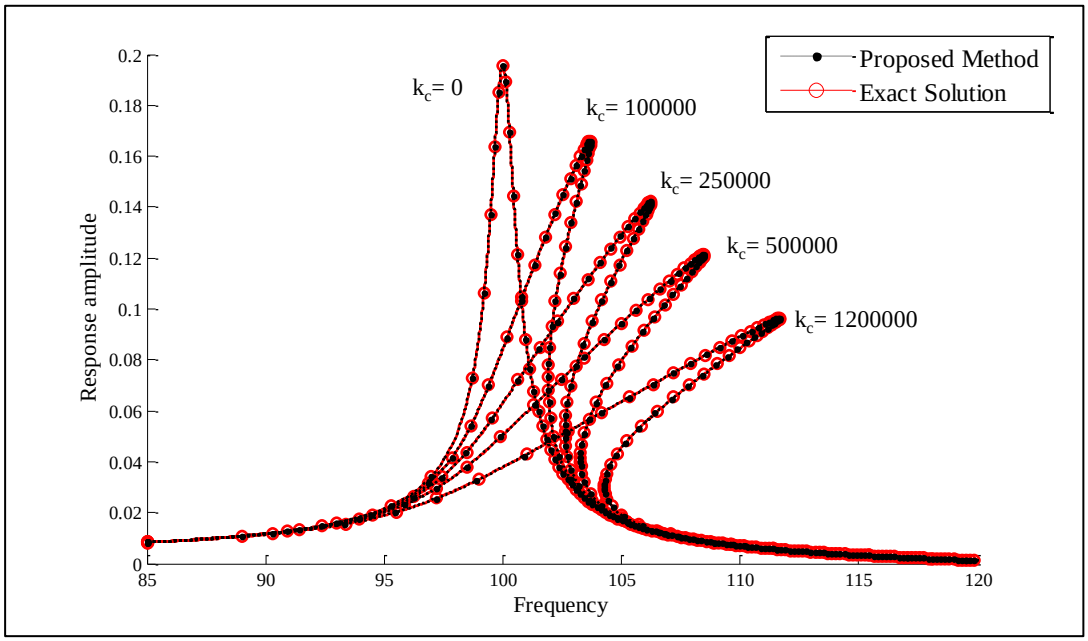

Fig. 2 – Responses of the first mass - Case 1

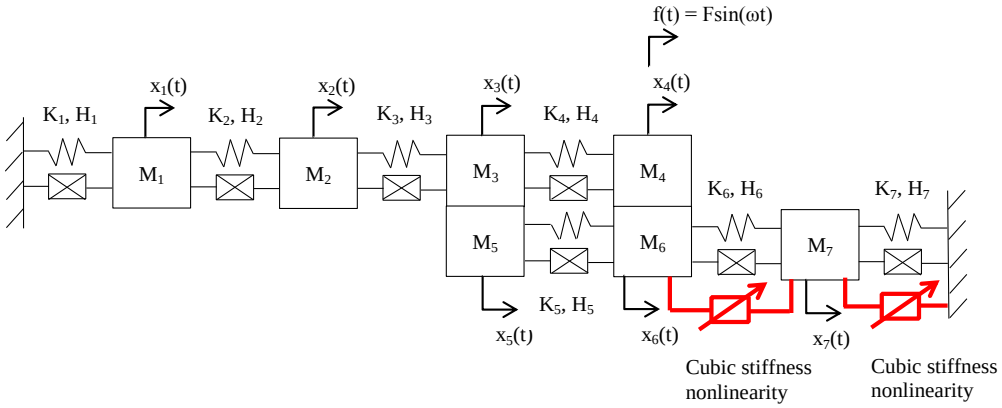

Fig. 3 – Lumped parameter model – Case 2

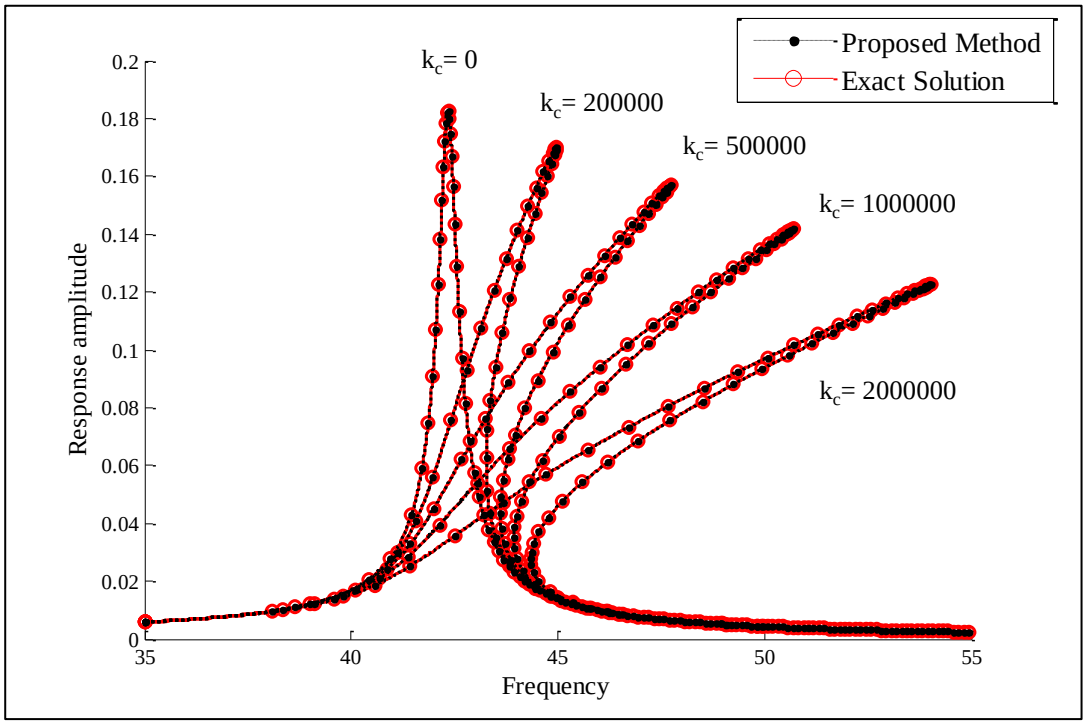

Fig. 4 – Responses of the seventh mass - Case 2

It can be seen from Fig. 2 and Fig. 4 that results obtained the proposed method are in exact agreement with the ones obtained by utilizing the exact modal data of the modified system.

4.2. FINITE ELEMENT MODEL

Same concept as in the lumped mass parameter model is applied to a cantilever beam modeled in ANSYS. Since it is a large DOF system, all information of the original system is not included in the analysis. First, different number of modes is used for structural modification and modal data of the modified system is obtained. Then, using the obtained modal data, HBM with modal superposition is applied to obtain the nonlinear response of the modified beam. For an exact solution, modified beam is also modeled in ANSYS and nonlinear solution is repeated for modal data obtained from modal analysis in ANSYS. *5 modes* of the modified beam are included in all nonlinear response solutions.

Modified system can be seen in Fig. 5 in which modifications are shown in purple. Cubic stiffness nonlinearities are shown in the figure defined in y direction. In Case 1, they are in the original system, while in Case 2, they are in the modifying system. External loading is also shown in Fig. 5 in y direction. Response plots are presented in Fig. 6 and Fig. 7 for the DOF shown in Fig. 5 by using k_c = 0, 10, 20, 40 N/m^3; F = 50 N and γ = 0.1.

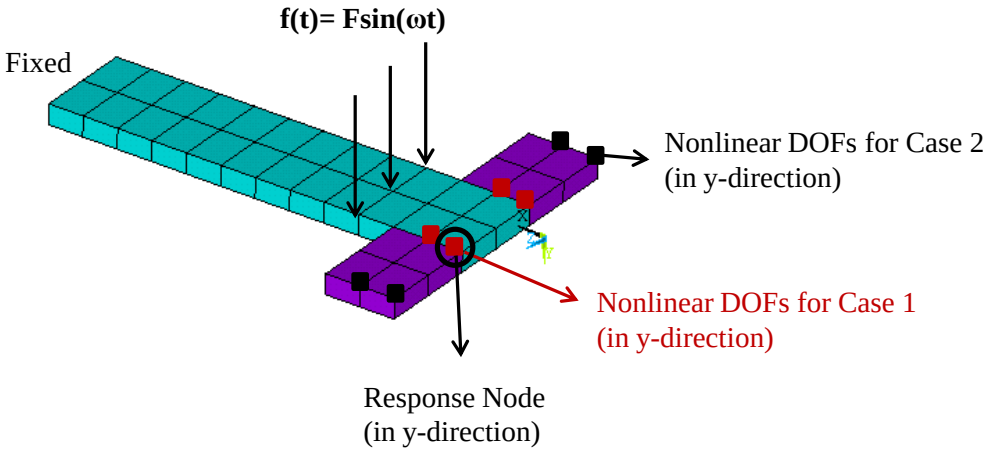

Fig. 5 – Modified system

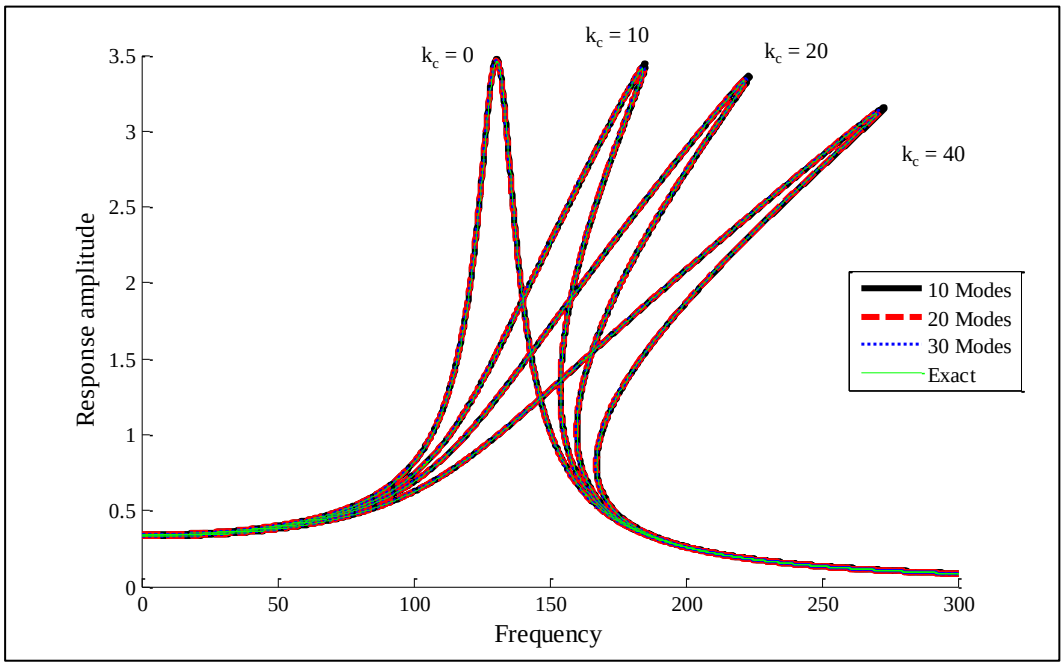

Fig. 6 – Case 1 – Nonlinear responses using 10, 20, 30 modes in formulation and exact solutions (5 modes are used in all cases)

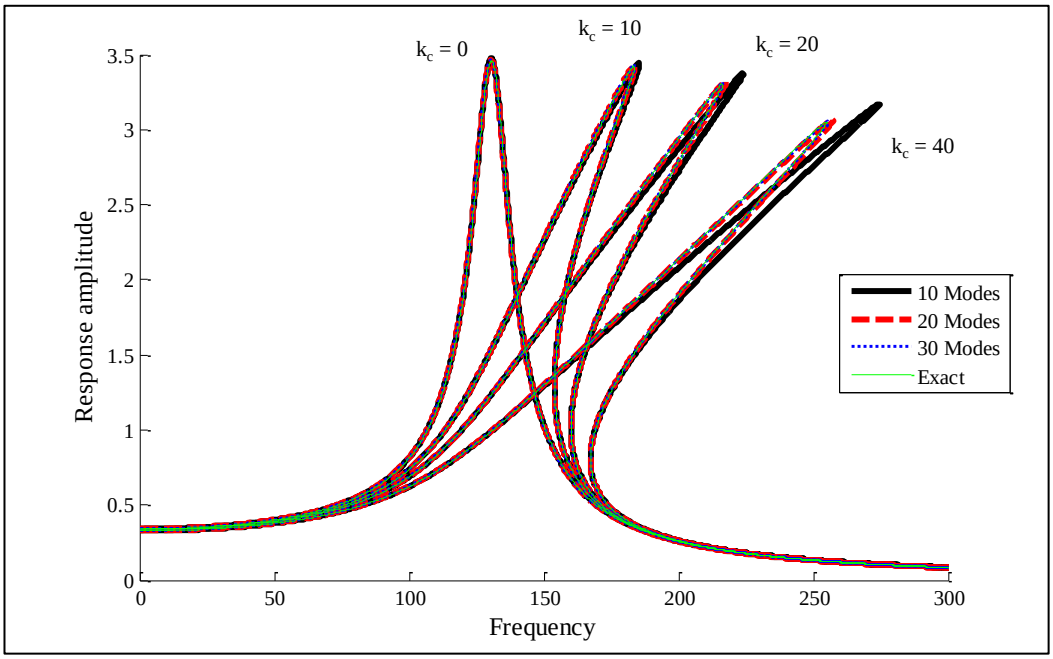

Fig. 7 - Case 2 – Nonlinear responses using 10, 20, 30 modes in formulation and exact solutions (5 modes are used in all cases)

Results in Fig. 6 and Fig. 7 show that accuracy of the method increases if more number of modes are included in the analyses. Moreover, for this particular case, the difference between the results is more visible if the nonlinearities are added to the modifying part.

5. CONCLUSIONS

In this study, a nonlinear reanalysis method is presented by using a new structural modification method developed by the authors. First, in order to verify the method, a simple lumped parameter model is considered utilizing all information of the unmodified structure. It is observed that identical results are obtained by using the proposed approach and directly solving the nonlinear system. Later, in order to show the capability of the proposed method, a FEM of a cantilever beam is considered. Instead of using all information of the unmodified system; 10, 20, and 30 modes are included in the analysis by utilizing the proposed method and response of the system is studied. It is observed from the results that increasing the number of modes used in structural modification method provides more accurate results; however, for the selected range of frequency, utilizing limited number of modes, it is possible to obtain the same results as if the exact modal information of the modified structure is employed.

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