REPUBLIC OF TURKEY
ADANA ALPARSLAN TÜRKEŞ SCIENCE AND TECHNOLOGY UNIVERSITY

INSTITUTE OF GRADUATE SCHOOL
DEPARTMENT OF NANOTECHNOLOGY AND ENGINEERING SCIENCES

EFFECTS OF GEOSTATISTICAL METHODS ON MODELLING OF DIFFERENT MINE RESERVE TYPES

UĞUR YİĞİT ZİHİNLİ
MASTER OF SCIENCE

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[Signature]

Uğur Yiğit ZİHİNLİ

ABSTRACT

EFFECTS OF GEOSTATISTICAL METHODS ON MODELLING OF DIFFERENT MINE RESERVE TYPES

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06.2022, 46 pages

The development and progress of a country depends on the proper evaluation of underground and open pit resources, their analyses and production by scientific and economic methods. Because, 99% of the tools and materials that make life active and functional are provided from natural resources, especially from mines. In short, raw material potentials are the most important power of countries and are the real sources on which development and development will be based.

In this study, two different mining reserve bedding type are analysed. One of them is coal bed. Coal beds are found in the form of basin type bedding. In other words, this type of bedding exhibits a layer type distribution type over large scale area and do not show sharp changes in terms of quality parameters and thickness (close to uniform distribution). The second data set consists of gold (Au), copper (Cu), lead (Pb) and zinc(Zn) mine which presents a porphyric bedding types exhibiting a non-uniform distribution of data.

The study covers the geostatistical analyses and mapping of two different mining types using kriging and simulation techniques. As a results of this study, the application results of the geostatistical tools in different type bedded mining is assessed. The calculations are carried out by an open-source software GSLIB. Results reflected that kriging based methods are rather suitable in mining beds showing no sharp changes whereas simulation is giving much healthier results in where suddenly local changes are observed.

Keywords: mining, geostatistics, coal, gold, kriging, simulation, GSLIB

ÖZET

JEOİSTATİSTİKSEL METODLARIN FARKLI MADEN YATAĞI TİPLERİ MODELLEMELERİ ÜZERİNE ETKİLERİ

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Bir ülkenin kalkınması ve ilerlemesi, yeraltı ve açık ocak kaynaklarının doğru değerlendirilmesine, bunların bilimsel ve ekonomik yöntemlerle analizi ve üretilmesine bağlıdır. Çünkü yaşamı aktif ve işlevsel kılan araç ve malzemelerin %99'u doğal kaynaklardan, özellikle madenlerden sağlanmaktadır. Kısacası hammadde potansiyelleri ülkelerin en önemli güçleridir ve kalkınma ve kalkınmanın dayanacağı gerçek kaynaklardır.

Bu çalışmada iki farklı maden rezerv yatak tipi analiz edilmiştir. Bunlardan biri kömür yatağıdır. Kömür yatakları havza tipi yatak şeklinde bulunur. Başka bir deyişle, bu tip yataklar, geniş ölçekli alan üzerinde bir katman tipi dağılım tipi sergiler ve kalite parametreleri ve kalınlık açısından (eşdeğer dağılıma yakın) keskin değişiklikler göstermezler. İkinci veri seti ise altın (Au), bakır (Cu), kurşun (Pb) ve çinko (Zn) madenlerinden oluşmakta olup, homojen olmayan bir veri dağılımı sergileyen porfiri bir yatak tipi sunmaktadır.

Çalışma, kriging ve simülasyon tekniklerini kullanarak iki farklı madencilik türünün jeoistatistik analizlerini ve haritalamasını kapsamaktadır. Bu çalışmanın bir sonucu olarak, farklı tip yataklı madencilikte jeoistatistik araçların uygulama sonuçları değerlendirilmiştir. Hesaplamalar açık kaynak kodlu yazılım olan GSLIB tarafından gerçekleştirilmiştir. Sonuçlar, kriging temelli yöntemlerin, keskin değişiklik göstermeyen maden yataklarında oldukça uygun olduğunu, buna karşın simülasyonun ise keskin yerel değişikliklerin gözlemlendiği yerlerde çok daha sağlıklı sonuçlar verdiğini göstermiştir.

Anahtar Kelimeler: Madencilik, jeoistatistik, kömür, altın, kriging, simülasyon, GSLIB

to my Family

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

The development and progress of a country depends on the proper evaluation of underground and open pit resources, their analyses and production by scientific and economic methods. Because, 99% of the tools and materials that make life active and functional are provided from the natural resources, especially from mines.

Mining, which holds an indispensable place in human and social life, is the factor that plays the most effective role in the attainment of technology and welfare levels of developed countries. In short, raw material potentials are the most important forces of countries and are the real sources on which development and development will be based. Classical reserve analysis and modelling methods such as polygon, triangulation are very often used in the reserve and tenor evaluation of mineral deposits. However, these methods do not show the spatial dependence structure of the mineral deposit quality and reserve changes, the continuity of variable and the rate of error as a result of the calculation. Therefore, it is inadequate for mine planning. Using these methods, the accuracy and reliability of the prediction decreases. Modern geostatistical methods model the structural change and spatial dependency structure in a mineral deposit. It provides mine production planning by giving error rates as a result of prediction or simulation. The distinction between these two different techniques is made as linear (kriging) and non-linear (simulation) techniques. In addition, these methods are not also limited to mining, but are used in all areas where spatial data analysis is available, such as hydrology, environment, oil reservoir, air-water pollution analysis.

Among these methods, the kriging technique is the most used method. This method has a simple algorithm and computer package programs can overcome exhaustive matrix calculations in a short time. It has a wide usage area with its solution. It is the objective method that has the highest numerical accuracy, especially since the calculation results in obtaining the smallest error variance theoretically. So, it has quite a wide range of using area.

Another geostatistical method for reserve modelling is simulations. In geostatistical simulation, the spatial continuity of the data set is modelled. If the simulation model uses values at the sampled points and produces back the same distribution of data set properties, such as mean, variance, histogram, and semi-variogram, this is called conditional simulation. The effectiveness of the simulation model depends on the quality of data entered and the accuracy and continuity of the semi-variogram model. Nowadays, simulation techniques are quite common in the mining and oil fields.

In this study, “Ordinary Kriging” and “sequential Gaussian simulation (SGS)” techniques were applied to the two different data set exhibiting different distribution behaviour and structure. The data sets are obtained from two different mining reserves bedding type. One of them is coal bed. Coal beds are found in the form of basin type bedding. In other words, this type of bedding exhibits a layer type distribution over large-scale area and do not show sharp changes in terms of quality parameters and thickness (close to uniform distribution).

On the other hand, the second data set consists of a metallic ore mine including the gold (Au), copper (Cu), lead (Pb) and zinc (Zn) mine which is a type of porphyric bedding. This type of beddings is a result of magmatic-hydrothermal activity of earth crust. Local and different size and quality sub-beddings are observed in whole reserve body. The data set of these two types of bedding consist of 3-dimensional drilling log data.

The study will cover the geostatistical analyses and mapping of two mining types with two type geostatistical methods. As a results of this study, the application results of the geostatistical tools in different type bedded mining will be assessed and the most suitable method will be revealed in accordance with a specific bedding type. The calculations are carried out by a public free software GSLib (C. V. Deutsch & Journel, 1998) and the with the aid of MS Excel in some parts of study. The GSLib software has found many application and study areas in the literature and gained a wide scale scientific acceptance (literature add about GSLib).

2. LITERATURE REVIEW

This study basically consists of analysing the structure of two different mining deposit in terms of quality distribution using two different geostatistical methods and comparing the calculation results as per the mine bedding type to reveal which model is suitable for which type of bedding. Thus, Ordinary Kriging (OK) and the Sequential Gaussian Simulation (SGS) methods are used in the calculations. Because, the Kriging (BLUE: Best Linear Unbiassed Estimator) provides more of an advantage in calculating the actual parameters of reserves of the land, while the simulation technique provides greater advantage in determining the best distribution structure of the relevant variable in a mine deposit.

Steps of this study may be summarised as follows:

1. Data processing (Accumulation (kriging)/Normalisation (simulation) of the data set),
2. Exploratory data analysis,
3. Variogram modelling,
4. OK / SGS in two dimensions for both bedding type,
5. Mapping,
6. Comparing results in accordance with the bedding types.

2.1. Data processing (accumulation and normalization)

The first step of a geostatistical analysis is a well data organisation and data management of the interested region. Basically, geostatistical analyses are spatial analyses including values, their coordinates and relation among them. Thus, geostatistical analyses are greatly vulnerable to input erratic data and the input data should be prepared carefully. Also, some analyses require data preparation before starting calculations relevant data processing tools are given in next topics.

2.2. Data Accumulation

When analysing a locally sharp changing data values such as metallic or thin-layer bedding mineralisation, direct analyses have highly potential to calculate incorrect results not actually reflect the true spatial distribution especially for the kriging estimation. Because, in near localisation, generally smoothing effect causes estimating high values as lower and vice versa. But mineral grade importance (support) can also be effected by two other variables including thickness and accumulation (grade multiplied by thickness), which are amenable to direct kriging (Chilès & Delfiner, 2012). Also, the 3-dimensional (wellbore) data is transformed to 2-dimensional data (point) in this way.

The problem of support is of critical importance and motivates the 2D approach. On variable support, the grade of unequal sample lengths cannot be linearly averaged to obtain a correct result because the outcome of the linear averaging is not a grade. Therefore, accumulated data (weighted average in a drilling location as taking into account the thickness and related grade) can be used as input data for analyses. The simple expression of the accumulated data at one drilling location is:

$$\boldsymbol{V_x} = \frac{\sum_{i=1}^{n}(\boldsymbol{V_{xi}} * \boldsymbol{t_{xi}})}{\sum \boldsymbol{t_{xi}}}$$

Where:

V_x **:** The accumulated data value of the drilling log "X",

n **:** Count of the sample thickness having a grade in the drilling log "X",

t **:** Thickness of a sample having a grade in the drilling log "X",

Finally, accumulated data at whole drilling location can be calculated and analysis is carried out over accumulated data. In addition, when data is accumulated, the data is transformed to 2-D data from 3-D.

2.3. Data normalisation and transformations

Since the simulation calculation methods is applied in a Gaussian distribution framework, the data is required to be transformed into normal distribution to ensure the gaussianity. Thus, data normalization is applied prior to implementation of the simulation methods.

When a mathematical modification is applied to the values of a variable, this application shows that Data Transformation is occurred. Possible data transformations are applied on from adding constants to multiplying, squaring, or raising to a power, converting to logarithmic scales, inverting and reflecting, taking the square root of the values, and even applying trigonometric transformations such as sine wave transformations (Clark, Judd, & McClelland, 1990; Cleveland, 1984; Micceri, 1989). There are a few transformation methods such as normal score transformation, log-normal distribution, Gaussian anamorphosis modelling.

On a statistical or geostatistical calculations the normal distribution is widely used because of available normality tests and many natural data exhibits a normal distribution (such as height of students in a class). The normal distribution behaviour gives an opportunity to detect any outlier or a trend in a normal distribution. On the contrary a non-normal distributed data (skewed) tends to be produced incorrect or biased results. There are plenty of normality transformations. A few of them are normal score, lognormal, Gaussian anamorphosis. Statistics based analyses –kriging and simulations in this case- mostly requires normalisation of the skewed data to increase the accuracy and the test of results. In this study, the kriging does not need for normalisation because of local estimation, but simulations require normalisation of input data to ensure the gaussianity because that the simulations algorithm require normal distribution.

2.4. Normal Score transformation

Sometimes the distribution of the input data may require the normalisation because of the analyses applied such as simulations. The mostly used and the simplest method is the normal score transformation (NST). Using these method, basic normality assumption is provided. Steps of this method may be summarised as follows:

1. The input data values are ranked from lowest to highest.
2. Each rank matched to their equivalent rank obtained from normal distribution.
3. Relevant normal distribution values equivalent to these ranks are fitted to the transformed dataset.

The ranking of the values may be assigned using either frequency distribution or the cumulative distribution of input data.

Examples are showed in Figure 1. as histograms and cumulative distributions before and after a normal score transformation was applied.

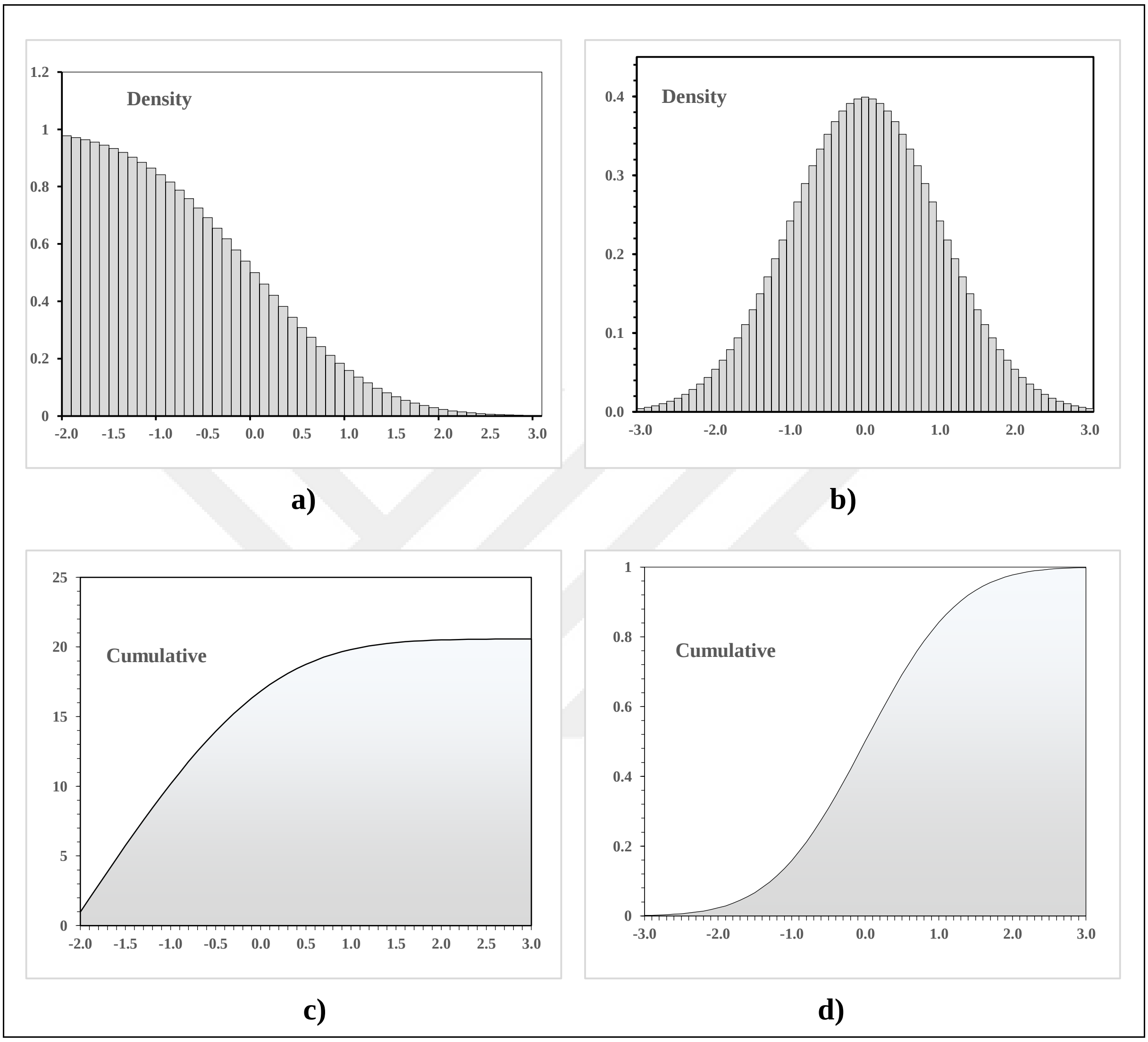


Figure 1. Normalisation of a positive skewed data, **a)** Histogram of skewed data, **b)** Normal score transformation result of skewed data, **c)** Cumulative distribution of the skewed data, **d)** Cumulative distribution of the skewed data after normal score transformation.

Although, the ordinary kriging procedure is carried out in a Gaussian base, does not require transform input data into Gaussian distribution because of local estimation. On the other hand, simulations require input data to be transformed to Gaussian distribution, because every simulation process requires cumulative probability density function. Therefore, data transformation or other process is applied on data before implementing calculations.

Multivariate Gaussian data is constructed in the context of many geostatistical techniques. Rarely are data multivariate Gaussian; data transformation is required. The widely used univariate quantile transform or profit enforces univariate Gaussianity. Linear rotations, such as principal component analysis and minimum/maximum autocorrelation factors, may be applied to decorrelate variables.

2.5. Declustering Techniques:

Declustering is well documented and widely practiced ((C. V. Deutsch & Journel, 1998); (Schloeder, Zimmerman, & Jacobs, 2001); (Goovaerts, 1997)). Cell, polygonal and kriging weight declustering are types of this method. All of these method depends on to account for spatial representability on the weight of the sample data. The assumptions of the declustering techniques divided in two groups:

1. True distribution range is sampled or the data is not spatially biased,

2. Declustering may not perform well without these assumptions. The first assumption is necessary because weighting only sets the effect of each sample on distribution and does not change the actual sample value so clustering nature is understood.

Without these assumptions declustering may not perform completely. The first assumption is that weighting is only necessary because it adjusts the effect of each sample on distribution and does not change the actual sample value.

The second assumption is for understanding of the clustering nature. When the data has no spatial correlation, there is no reason to use declustering method. From the underlying distribution of the data would be draw. Declustering applied incorrect way when spatial nature of the data is not understood. Therefore, there are kind of methods to estimate the declustering weights. Polygonal declustering, cell and kriging weight declustering are available. The most used one is the cell declustering.

2.5.1. Cell Declustering

In geostatistics the cell declustering technique is the most used technique. Cell declustering was proposed by (Journel, 1983) and the first widely used public code was made available by the author (Clayton V Deutsch, 1999). A version of that code (declus) is available in GSLIB (C. V. Deutsch & Journel, 1998). Cell size is the spacing of the data rarely sampled areas. The number of cells that are full counted (n_{occ}) and all of these fulled cells get the same weight. It is a formula to get weight of only one data ($1/n_{occ}$) which is in the cell alone. If data is not alone in the cell they divide their weights assigned to the cell (Figure 2). In Figure 2, there are six cells with data value where every data in each cell receive weight. So that the cell weight is one sixth. Multiple data share the weights equally. As a result, a cell declustering analysis is carried out, but not included here because of no important change observation.

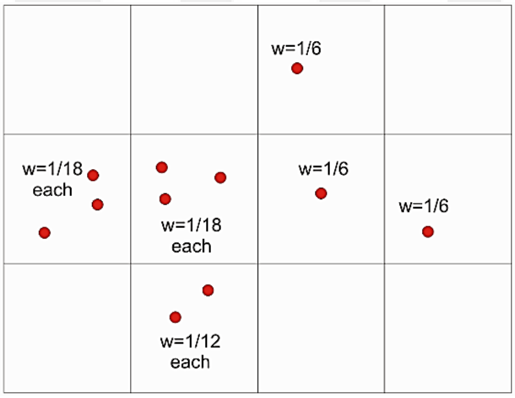


Figure 2. An example of cell declustering weighting.

3. GEOSTATISTICS

3.1. Exploratory data analysis

Descriptive statistics are showed descriptive coefficients that summarize a given data set, which can be either a representation of the entire or a sample of a population. Descriptive statistics are broken down into measures of central tendency and measures of variability (spread). Measures of central tendency include the mean, median, and mode, while measures of variability include the standard deviation, variance, the minimum and maximum variables, and the kurtosis and skewness. Thus, exploratory data includes:

1. **Mean:** Determined by summing X_i values and dividing the sum by n. $\mu = \{x_i/n\}$
2. **Median:** Central data value corresponding the middle data in an ordered data set (from high to low values, or vice versa)
3. **Mode:** Highest frequency value. Modes are local peaks on histogram.
4. **Skewness:** Indication of departure of tails of distribution from symmetry about the mean.
5. **Positively skewed distribution:** Excess of values extending as a tail toward high values.
6. **Negatively skewed distribution:** Excess values have a tail extending toward low values.
7. **Skewness:** $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^3}{n\sigma^3}$,
8. **Kurtosis:** $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^4}{n\sigma^4}$,
9. **Coefficient of variation (CV): ($\frac{\sigma}{\mu}$)** is indication of skewness. If it is greater than %80 (0.8), it can be said that distribution is not a normal distribution. Values of CV less than 0.5 likely to approach a normal distribution.

Where:

x_i= i_{th} value in a dataset,

μ= Mean of the dataset,

n= total value number of dataset,

σ= Standard deviation of the dataset, In a standard normal distribution, mean, standard deviation, skewness and kurtosis are 0, 1, 0, 3, respectively. Thus, a data distribution can be assessed using these constants after data transformation. For example, the sum of the following data set is 20: (2, 3, 4, 5, 6). The mean is 4 (20/5). The mode of a data set is the value appearing most often, and the median is the figure situated in the middle of the data set (4). It is the figure separating the higher figures from the lower figures within a data set. However, there are less-common types of descriptive statistics that are still very important.

3.2. Variogram analysis

Spatial statistics assumes that the sample values ore realizations of random function. Sample values are the function of deposit positions; their relative position is taken into account in the analysis. Therefore, it should be known that the structural function is represented by the variogram. A variogram shows a description of the spatial continuity of the data. The experimental variogram is a discrete function calculated using a measure of variability between pairs of points at various direction and distances. Thus, variogram analyses are used in both kriging and simulation processes. Elements of a variogram are in Figure 3 (https://vsp.pnnl.gov/help/vsample/Kriging_Variogram.htm). The exact measure used depends on the variogram type selected (C. V. Deutsch & Journel, 1998).

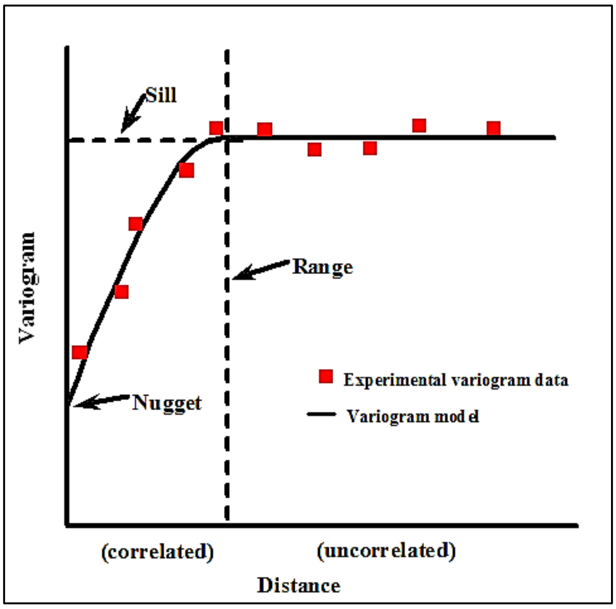


Figure 3. Experimental and model variogram elements.

Sill is the value at which the model first flattens out, nugget is the value at which the semi-variogram (almost) intercepts the y-value. Additionally, geostatistical term used to describe the variability seen between samples that are closely spaced is called as the nugget effect. The nugget effect is composed of a geological component, which can be thought of as inherent, and a sampling component, which is not fixed. Range is the maximum distance that the variables are interrelated.

There are different types of variogram models such as Spherical Models, Exponential Models, Gaussian Models and Linear Models. Spherical Models are the most used model which show linear behaviour at small separation distances near the origin but flattening out at larger distances and reaching a sill limit. Different type of variogram models can be seen in Figure 4.

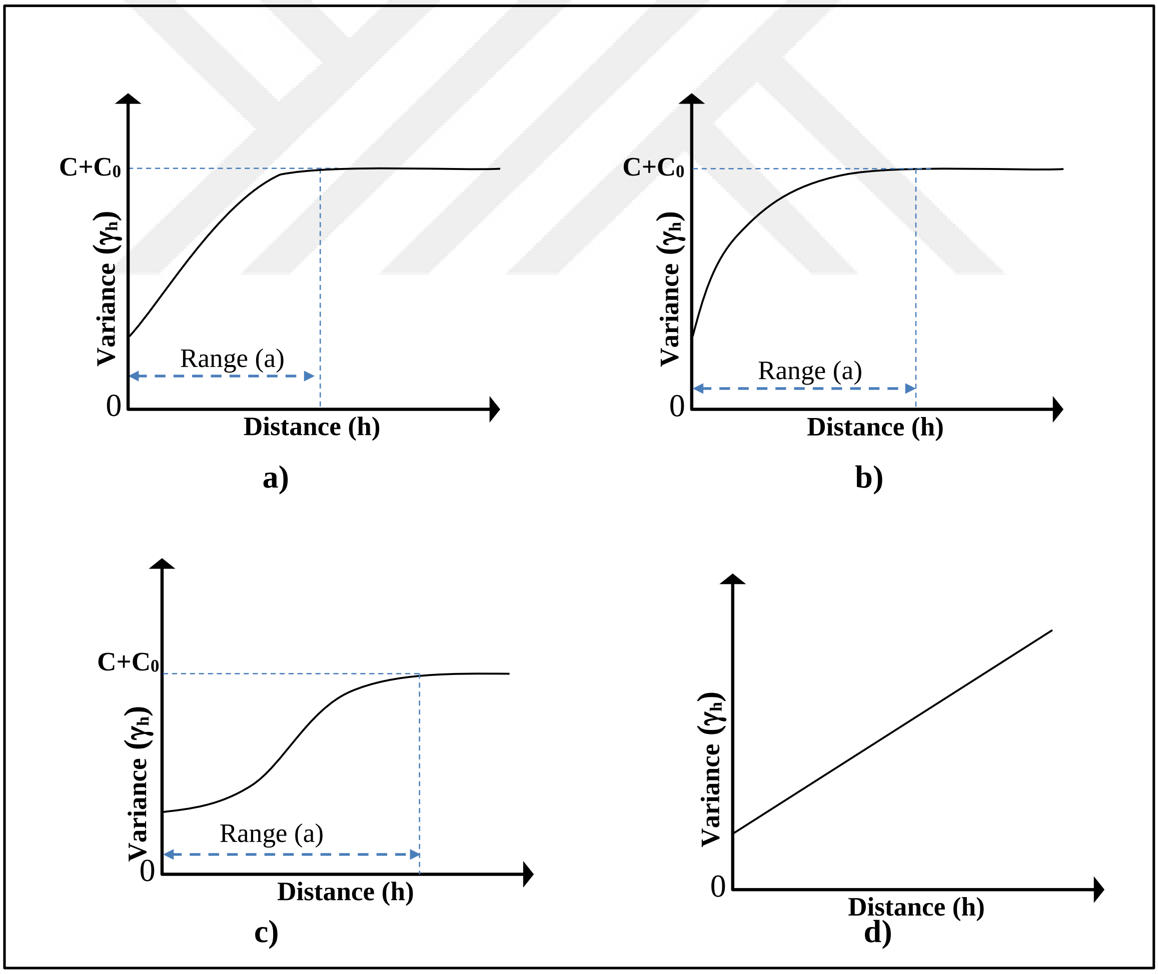


Figure 4. Different variogram models: **a)** Spherical, **b)** Exponential, **c)** Gaussian, **d)** Linear type

3.3. Ordinary kriging

A lot of kriging technique are available for particular or different data types including ordinary, lognormal, indicator, disjunctive, co-kriging. Ordinary kriging calculates the grid point data from neighbouring values multiplying a weight as per the position of the neighbouring data. But, differently from the other weighting estimation methods (invers-distance) the weights are assigned in accordance to a semi-variogram that is a function of direction and distance among data. Also, this method ensures the minimum estimation variance as the algorithm below:

Once the semi-variogram models constructed, a jack-knifing technique is applied to estimation. In an estimation area, the unsampled points can be estimated by kriging technique on a grid basis. The kriging algorithm can be simplified as below:

The kriging equation is:

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i), \tag{1}$$

Where:

$\hat{Z}(x_0)$= Estimation value of the x at the point 0,

$\lambda_i =$Estimation weight of neighbouring data (i),

$Z(x_i)$= Real data value including into estimation at the point i,

Here the weighting value (λ_i) is found by the following procedure:

$$\sum_{i=1}^{n} \lambda_i \gamma\left(d_{ij}\right) + \mu = \gamma\left(d_{0j}\right), \quad i, j = 1, \dots, n\,, \tag{2}$$

Here in equation (2) the μ is the Lagrangian multiplier which is necessary to solve that an equation with n-unknown value requires n+1 equation. In addition, d_{ij} and d_{0j} are the distances between their location and estimation points as per designation.

$$\sum_{i=1}^{n} \lambda_i = 1, \tag{3}$$

As an algorithmic result, the sum of weight equals to 1 (Eq.3). Also, this equation ensures minimising estimation variance and the unbiasedness.

As a result of kriging estimation, the estimation error also can be found by the equation (4). It can be clearly seen that the kriging error is mainly affected by Lagrangian multiplier, weights and variogram values at the estimation distance.

$$S_{KE} = \sqrt{\mu + \sum_{i=1}^{n} \lambda_i \, . \, \gamma(d_{0i})}, \tag{4}$$

In the Kishné et al., (2003), the application of kriging in environmental pollution analysis used. In this study, the distribution of the total cadmium value, which is taken from the soil and has a distorted distribution, was compared with lognormal kriging and ordinary kriging due to the distorted data distribution. As a result of the study, it was seen that the accuracy of the kriging maps is not only dependent on the accuracy of semi-variogram modelling, but also on the number of neighbouring samples used in the kriging process. In addition, it was determined that Ordinary Kriging performed better in predicting high values and Lognormal Kriging in predicting average and low values.

Sampling design, which is an important parameter in the kriging estimation method. It was made by (Van Groenigen, 2000). In the study the estimation of the sampling design has been stated how it affects the results. The study is a good tool for a good kriging prediction. The necessity of sampling and the sampling design of kriging variance states that it can be used to evaluate its quality. As a result of the study regular grid sampling to obtain minimum kriging variance has shown that it provides an advantage.

(Diko, Vervoort, & Vergauwen, 2001) made an application in the lateritic bauxite deposit and a different path was followed with the preparation of the data, which is as important as the estimation made.

Yünsel (2012) applied an ordinary kriging technique to a cement raw material deposit. In this study different mineral occurrences and their chemical distribution across the study area are modelled.

(Hohn & McDowell, 2001) examined how declustering, which is a method applied when the data is not properly distributed, changes the semi-variogram structure, which is very important in both kriging and simulation techniques. Conditional simulation technique has been used in metal mining by many researchers (for example: Abichequer et al., 2011; Dowd, 1984; Gambin et al., 2005) applied conditional simulation for the quality classification and evaluation of coal deposits.

3.4. Simulations

Geostatistical simulations (also conditional simulations since the simulated value added to simulation process) getting wider application area although it was used for oil reservoir modelling. This geostatistical technique creates different models of a deposit resembling the original data distribution spatially and statistically. The number simulated models can be generated as parallel to the power of the computer. Thus, the generated simulation models are equally probable models.

In the simulation process the roughly the following steps are followed:

1. Input data histogram is generated,
2. Since the simulation process is carried out in a Gaussian framework, if data is normally distributed, then transformed into Gaussian distribution using one of normalisation procedure,
3. Variogram models are plotted on transformed data
4. Cumulative probability density function is calculated,
5. A random value is drawn from the distribution values,
6. This value is added to conditioning data set,
7. The above steps are repeated starting from step 4 until all grid nodes are visited
8. Again, a new simulation model is generated starting from the step 1.

A case study on the application of stochastic simulation in a coal mine was conducted by (Abichequer et al., 2011). This study done by focusing the coal pile (accumulation) technique and the sulphur content, and analysed risk in the coal beds.

Another study on risk analysis in a coal deposit is again by (De Souza, C.L. Costa, & Koppe, 2004). This study shows that Sequential Gaussian Simulation (SGS) demonstrates how reserve quality parameters can be used in uncertainty analysis, and how simulated models are applied to the mine site. In the study, the distribution of quality parameters in the field was determined by AGS in order to achieve optimum production in coal deposits. The fluctuations in the coal bed were calculated by taking into account the production of different coal layers belonging to the same bed and uncertainty maps were produced.

Definition of simulation and geostatistical simulation methods and their detailed descriptions are given in the work of (J Vann, S Jackson, & O Bertoli, 2003). The main simulation methods, simulation approaches and application paths are stated here. This study contains more literature information and does not include an application related to simulation. Detailed explanations about the algorithms of the simulation techniques, their differences with each other and which of these methods may be more suitable under which conditions are given.

The spatial distribution of groundwater nitrate concentration has been simulated by the application of successive Gaussian simulation (Cinnirella, Buttafuoco, & Pirrone, 2005). With this study, simulation techniques were applied in a wide range of application areas and simulation, mean simulation, probability, and standard deviation maps of nitrate content were produced in the field. According to the maps made, contaminated and uncontaminated areas of the entire site have been identified. The difference between kriging and simulation techniques in a field is shown in (Goovaerts, 1999) study.

3.5. Cross validation

Cross-validation is a technique that is used for the assessment of how the results of statistical analysis generalize to an independent data set. Cross-validation is largely used in settings where the target is prediction, and it is necessary to estimate the accuracy of the performance of a predictive model.

In the kriging procedure basically, cross validation is carried out by removing a real data in the input data set and it is estimated by the suggested predictive model. Thus, every input data point has a real and estimated data. The difference between them indicates the accuracy of predictive model. Actually, in a kriging estimation, the mean and standard deviation of the normal distribution of the estimated values are expected to be 0 and 1 respectively.

As for the simulations, the geostatistical simulations model is validated by comparing input and output data comparison. The simulations do not minimise the local error but generates equally probable models resembling input data distribution paramters. Thus, generally histogram and variogram models of input and output data are compared for validation stage.

4. MATERIALS AND METHODS

In this study, two different mining reserve bedding type will be analysed. One of them is coal bed. Coal beds are found in the form of basin type bedding. In other words, this type of bedding exhibits a layer type distribution over large-scale area and do not show sharp changes in terms of quality parameters and thickness (close to uniform distribution).

The second data set consists of Gold (Au), Copper (Cu), Lead (Pb) and Zinc (Zn) mine which presents a porphyric bedding types. These types of beddings are a result of magmatic-hydrothermal activity of earth crust. Local and different size and quality sub-beddings are observed.

The data set of these two types of bedding consist of 2 (coal) and 3-dimensional (gold) drilling log data. The study covers the geostatistical analyses and mapping of two mining types with different methods. As a results of this study, the application results of the geostatistical tools in different type bedded mining are assessed and the most suitable method will be revealed for a specific bedding type. The study is carried out by a public free software GSLIB (C. V. Deutsch & Journel, 1998) with a minor aid of MS-Excel office software. This software (GSLIB) covers many applications and studies in the literature and gained a wide scale scientific acceptance.

4.1. Study areas

4.1.1. Coal deposit

The coal deposit analyses of the study take place on Ankara province having a distance 25km Beypazarı, and 5km to Çayırhan (Figure 5). Approximately 5 million tons lignite is produced yearly in the Çayırhan lignite deposit. This is region is a keystone and energy intensive region in providing electrical energy in Turkey. Because the produced coal is delivered to a thermic power plant of 620MW. The Cayırhan coal deposit consists of seven production panel including A, B, C, D, E, F and G. A, B, C panels have already been operated, and lignite production is undergone in the D, E and F panels. G panel is being designed and planed for coal production. The G panel was selected for the study area, which has approximately 5.5 million m^2 (550 hectare). The lignite is extracted by underground methods, which involves a fully mechanised longwall. The Cayırhan lignite deposit contains two coal seams (lower and upper).

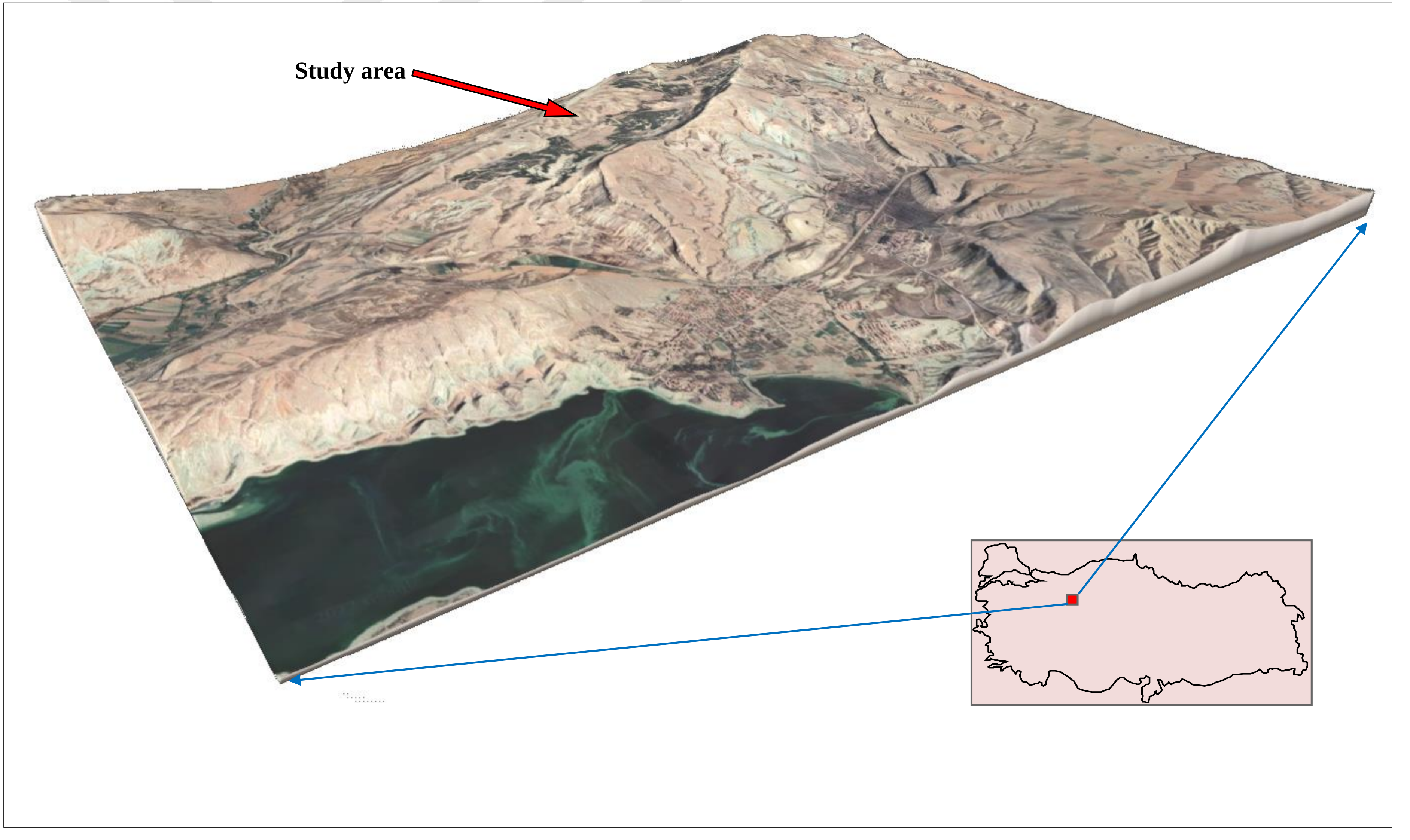


Figure 5. Location map of the coal study region (Çayırhan G-region).

The lower lignite seam is a really restricted and laterally discontinuous, having numerous seam partings and varying from 1- to 11-m in thickness. It is a low-quality seam with high ash (50%) and sulphur (4%) contents. This seam is not exploited at present and not discussed further here. There are up to 150 m of parting sediments between the lower seam and upper seam. These sediments fine upwards, from sandy braided river channel fill facies above the lower seam through interbedded sandstone, siltstone and claystone to shale and carbonaceous shale, deposited in a lacustrine environment, below the upper seam. The upper lignite seam is being laterally extensive and varies from 0.65 m to 5 m in thickness, averaging 1.65 m and 0.65 m thick average intercalation or parting, composed of siltstone and claystone with chert nodules, splits the upper seam into two lignite beds, referred to locally as the upper (Tv) and lower (Tb) seams. Quality data on the seams were obtained from core samples from 53 coal exploration boreholes drilled in the study area, but only 45 suitable data point is used for the analysis. The locations of the boreholes are shown in Figure 6. A grid design was not applied for the drill locations. The data base contains the thickness, calorific value, sulphur and ash contents, moisture, and elevation for each intersection of each seam. However, only the calorific value, is studied.

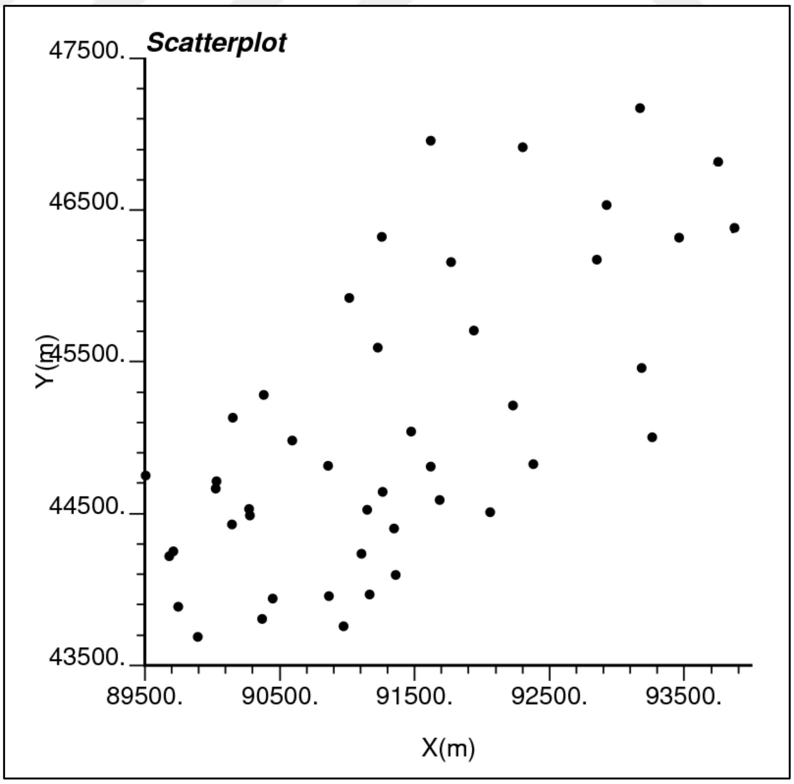


Figure 6. Drilling bore holes in the field (for the coal data).

4.1.2. Gold deposit

The gold deposit is located in the Eastern Pontides mineral belt, within the province of Sivas in north eastern, Turkey (Figure 7). The geology of the area is dominated by, Cretaceous age basalt flows and pyroclastics overlain by porphyritic andesite to dacide tuffs and flours. The volcanic rocks are intruded by stocks of granodiorite composition. Locally, two sets of faults appear to be 'some of the main controls to alteration and mineralization to varied degrees at different structural levels in the area, one oriented northwest and the other northeast. Both structures are steeply dipped. Northeast structures are prominent controls on phyllic alteration while northwest structures appear to be the major control on the silica cap. The geology of the deposit area has been exposed partly by intense weathering and leaching conditions. Sulphide content (mostly pyrite) of broadly altered volcanic rocks produces acidic waters and resultant.

The data for the gold analysis is obtained from core samples of 58 diamond drill holes, but only suitable 49 data location is used. Drill holes' locations for the study area are shown in Figure 8. A grid design has not been established for the drill locations. The drilling logs contain the thickness, assays, geological description of gold mineralization. Nine drill holes are excluded from the study area due to long distance. The distance between the well bore locations is approximately 175 m on the basis of the topographical conditions and geological explorations of the area. Approximately. a total of 8 584 m drilling has been made in the study area. Most of the wells are not vertical. and the azimuth and inclination of each well are taken into account for the data. Core samples were taken on an approximately 1 m spacing assays for Au, Ag, As, Cu, Mo, Pb and Zn. Values of above 10-5 Au (only two values) were excluded from the data due to outlier problems and missing assays. Thus, 6711 good samples were used for the analyses. Only the accumulated Au data is used for the analyses.

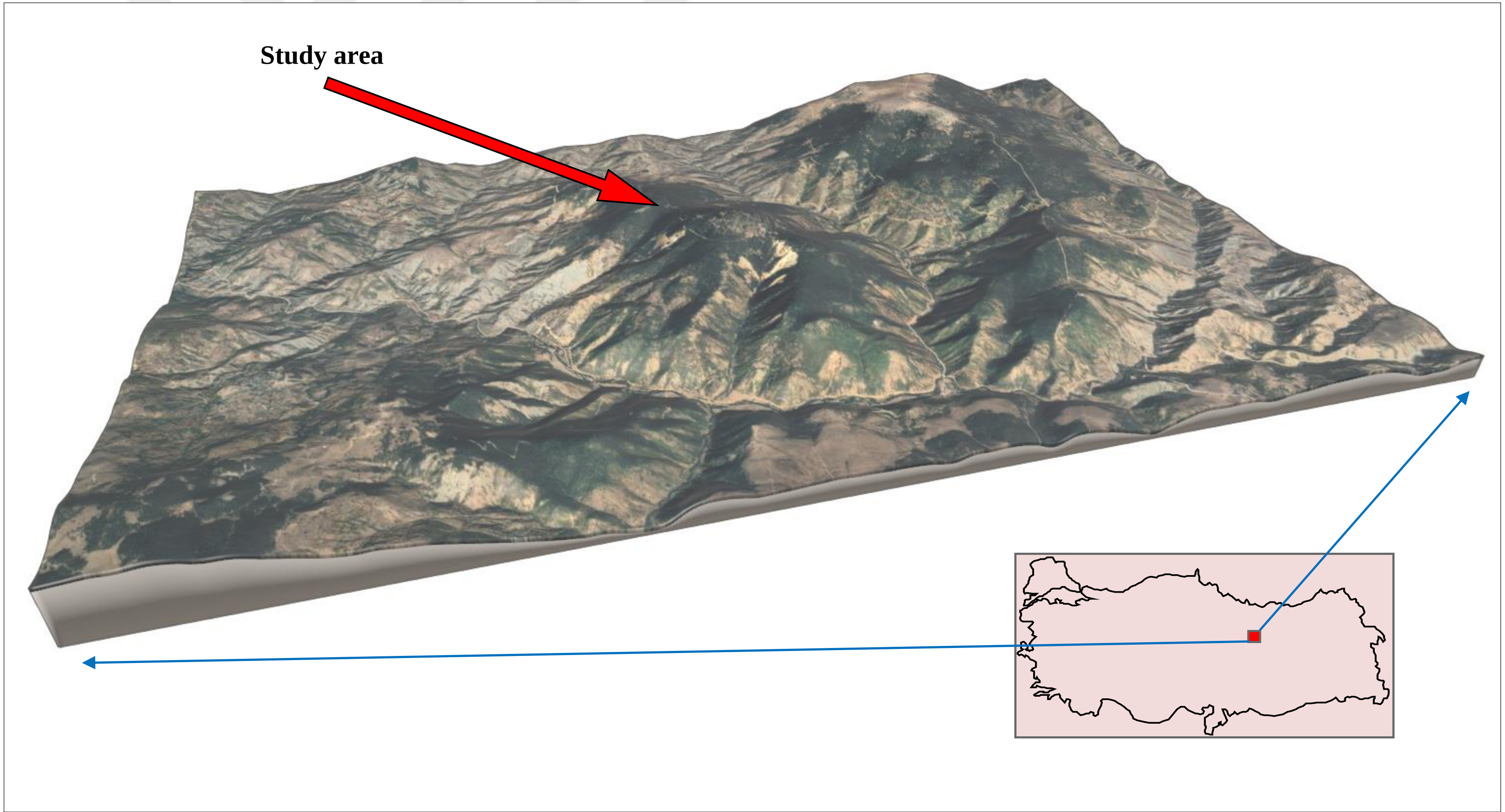


Figure 7. Location map of the gold study area (Sivas region).

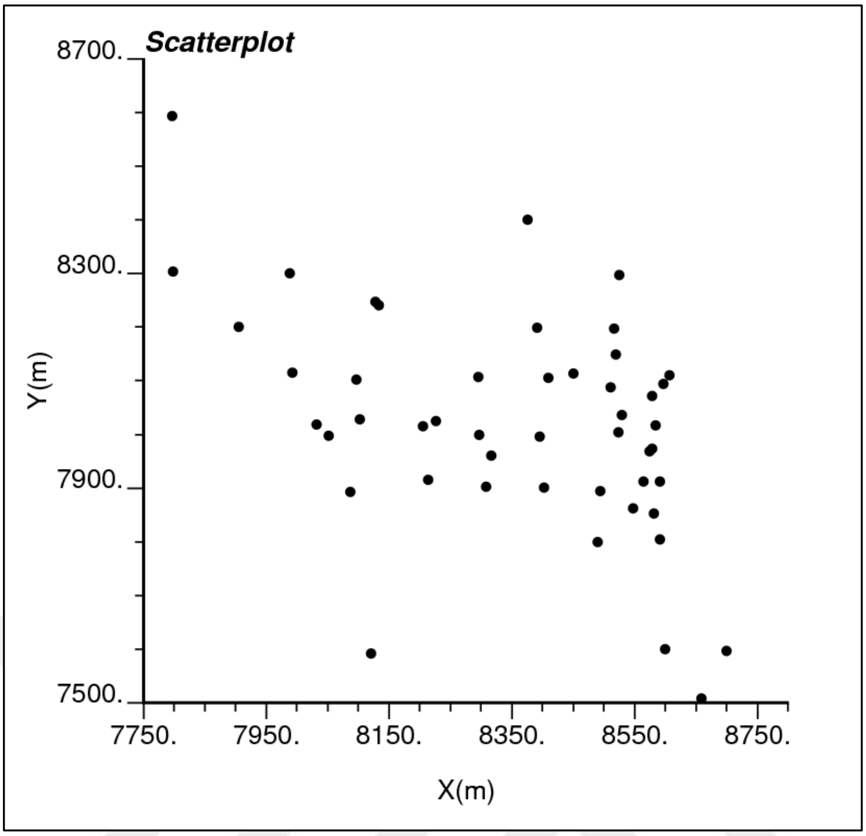


Figure 8. Location of drillings (for gold data).

4.2. GSLIB software

The entire study -from data preparation to the mapping- is carried out by the software “GSLIB”. GSLIB software is developed by the Deutsch & Journel, (1998) at the Stanford University. The software is coded in FORTAN code as open source, and it is established on the basis of three important areas geostatistics. They are “Variogram” (spatial variability analyses), “Kriging” (linear regression modelling with a trend) and “Simulations” (reproduction of spatial trend on the basis of probabilities). The GSLIB software is a rather old and simple software when it compared to the up-to-date softwares. But, it’s efficiency and performance for a general geostatistical analysis is indisputable. Therefore, the GSLIB is accepted as the predecessor of the geostatistical softwares and in addition it is public-free. As it can be seen in Figure 9, the software user-interface (UI) presents a lot of tools in a simple drop-down menu. When this development of the software is thought in 1998, it is clear that it was quite innovative for that time. Besides, the manual of the software is still a handbook of geostatistical analyses and algorithm in general concept.

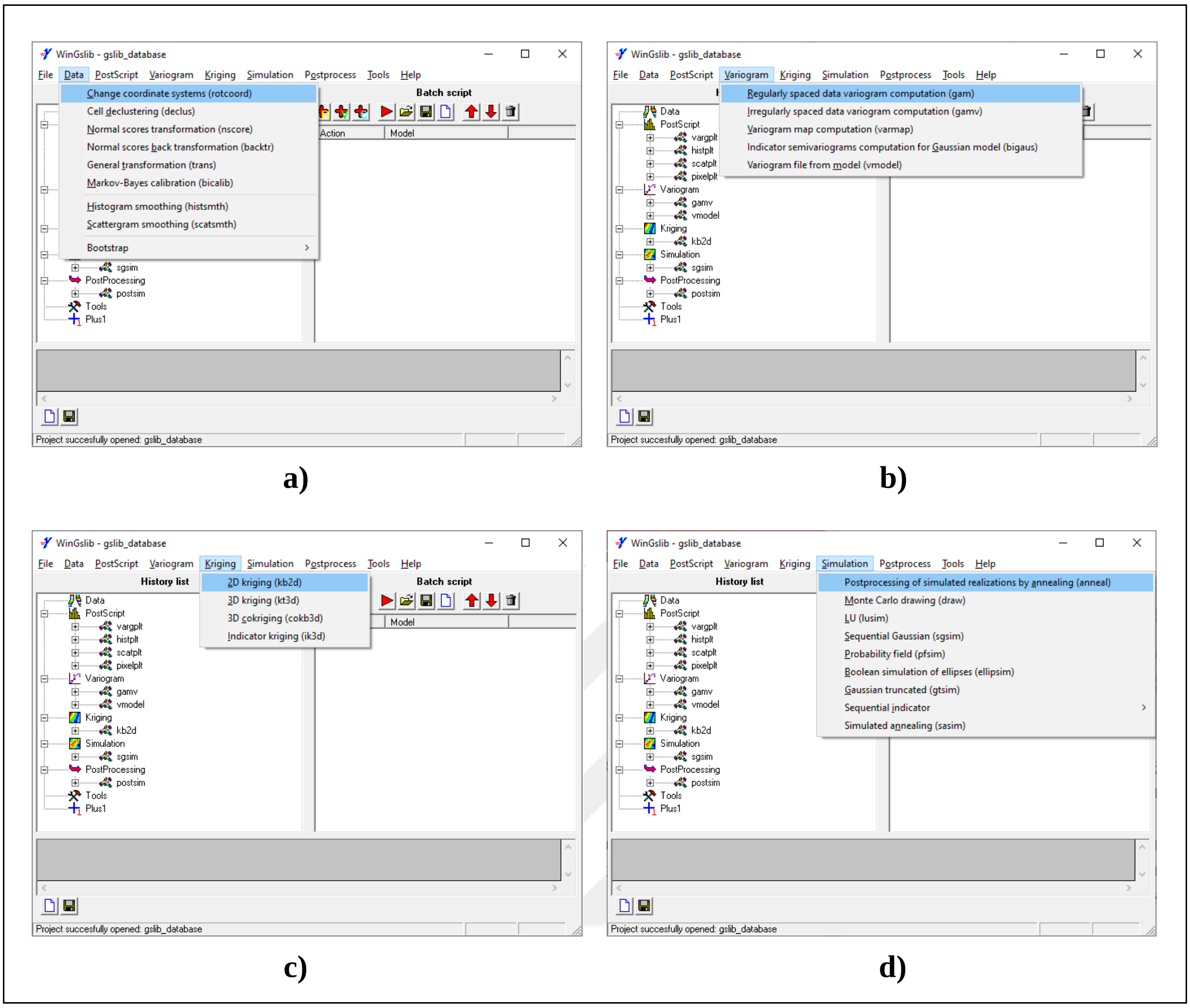


Figure 9. GSLIB user interface and a few of relevant analysis tools dropdown menu list.

5. RESULTS AND DISCUSSIONS

5.1 Exploratory data analysis

The data used in the thesis may be splitted into two parts. The first data set consist of the coal data standing for a real data, and the second data set includes the gold chemical analysis results which implying the locally distributed variable. Both data set will be used by two different geostatistical methods to characterise and their behaviours in accordance with calculation methods are expected to unveil the differences and performance of the methods to find out which model is more suitable in a specific mineralisation type. The coal data and the gold data summary statistics are presented in Table 1. The Table shows that calorific value exhibits approximately normal distribution. Because the mean and median values are merely same, in addition the coefficient of variation is as low as 12%. On the contrary, the gold data presents a clear skewed data since the mean and median values departed from each other and skewness about 1.3. Also, the skewness value implies that there is a positive skewness in other words right-tailed distribution. In a statistical distribution, the skewness larger than 80% (0.80) is assumed as skewed distribution. Finally, the third column includes of the accumulated (3D to 2D space conversion and normalisation for simulations) gold data. Again, this column presents a clear normal distribution because of normal score transformation.

Table 1. The summary statistics of the coal and the gold data.

Variable	Calorific Value (kcal/kg)	Gold data (accumulated, ppm)	Coal data (normalised)	Gold data (normalised)
Count	45	49	45	49
Minimum	1929	18.2	-2.11	-2.19
Maximum	3529	881.9	2.11	1.94
Mean	2808.8	281.33	0.01	-0.07
Median	2776	204.3	0	-0.09
Std. Dev	364.71	227.09	0.971	0.96
Variance	133013.4	51569.35	0.943	0.91
Coeff. of Var.	0.129845	0.81	0.01	-13.02
Skewness	-0.16343	1.32	0.01	-0.03
Kurtosis	-0.29972	3.76	-0.366	2.51

In Figure 10, the probability plot of the calorific value of the coal data is shown. Another way to analyse a data distribution is the probability plot. In a normal distribution, it is expected that the data values are distributed on diagonal line on a probability plot. Figure 10 suggest that this condition is met with normality assumption on a probability plot for coal data. As a validation of the normal distribution, the histogram of the coal data is presented in Figure 12. In the Figure 12, the histogram of the coal data exhibits a clear normal distribution pattern as expected.

As for the gold data, the gold data is not distributed regularly on the diagonal line of the probability plot as expected (Figure 11). Also, the skewing intervals can be distinguished as per departure from diagonal line. Again, the skewness mentioned in both summary statistics and the probability plot can be seen on the histogram of the gold data in Figure 13.

Finally, since the simulation analyses requires the transformation of raw data to normal distribution, histogram of transformed coal data is shown Figure 14. Figure 14 exhibits a normal distribution as expected. The histogram of the accumulated and normalised gold data after normal score transformation is also shown in Figure 15. Figure 15 indicates a clear normal distribution as a result of normal data transformation, again.

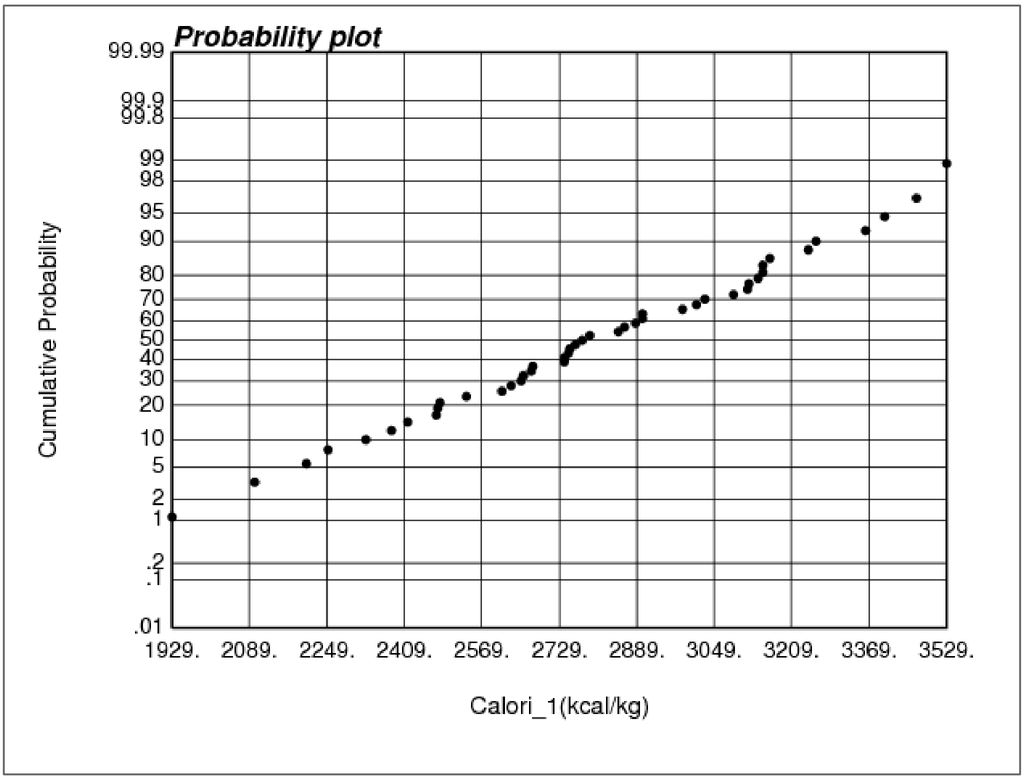


Figure 10. Probability plot of the coal data.

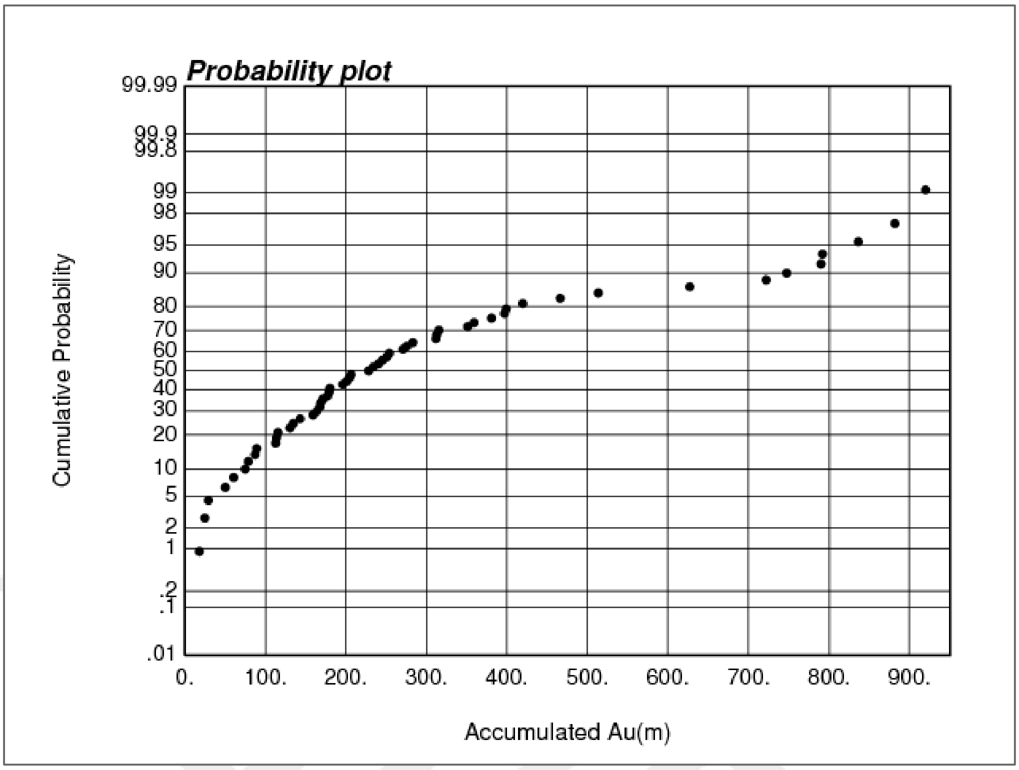


Figure 11. Probability plot of the gold data.

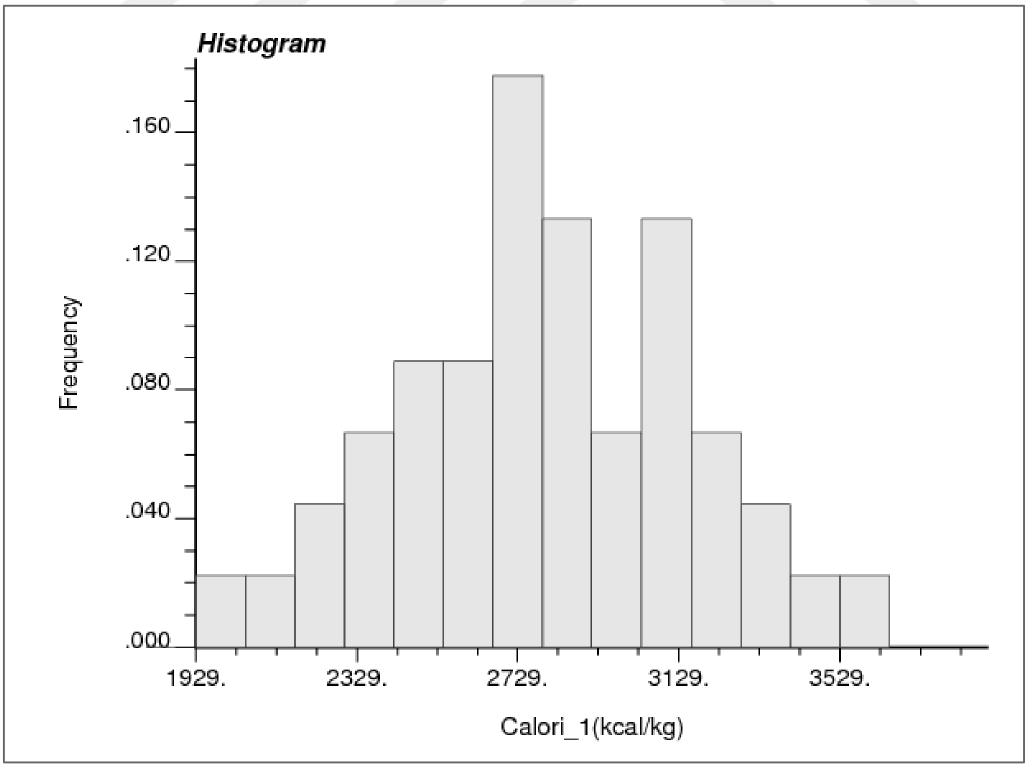


Figure 12. Histograms of coal data.

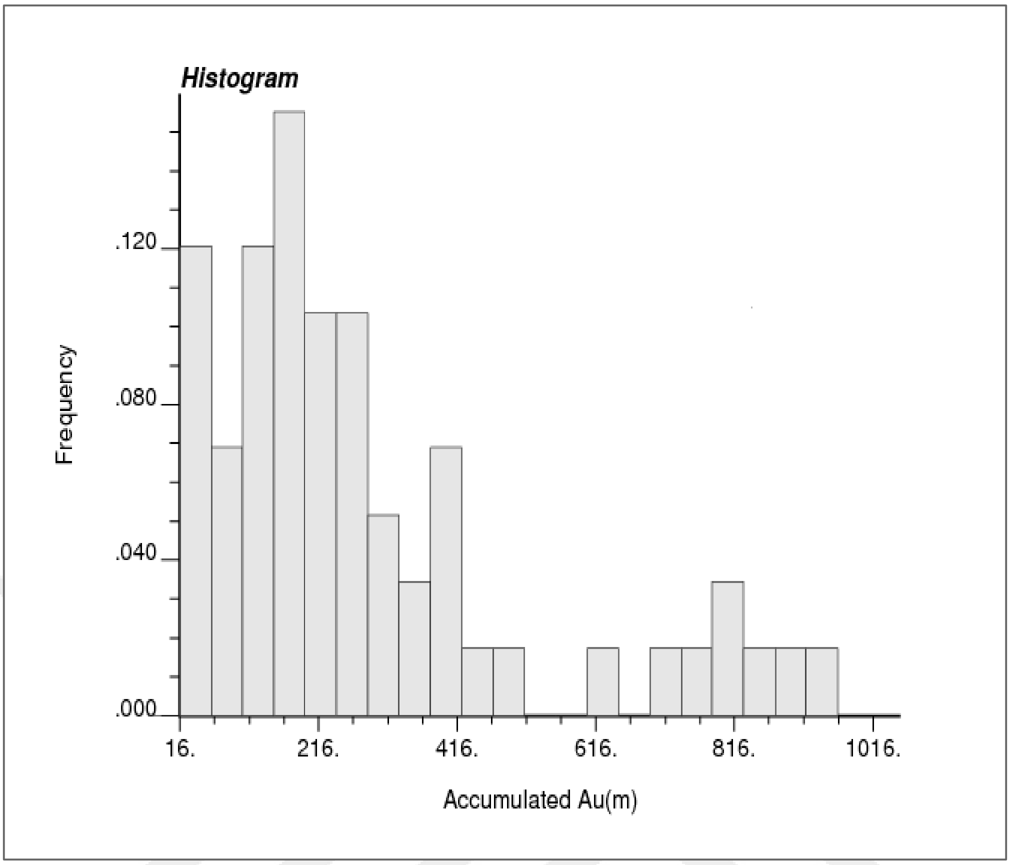


Figure 13. Histogram of the gold data.

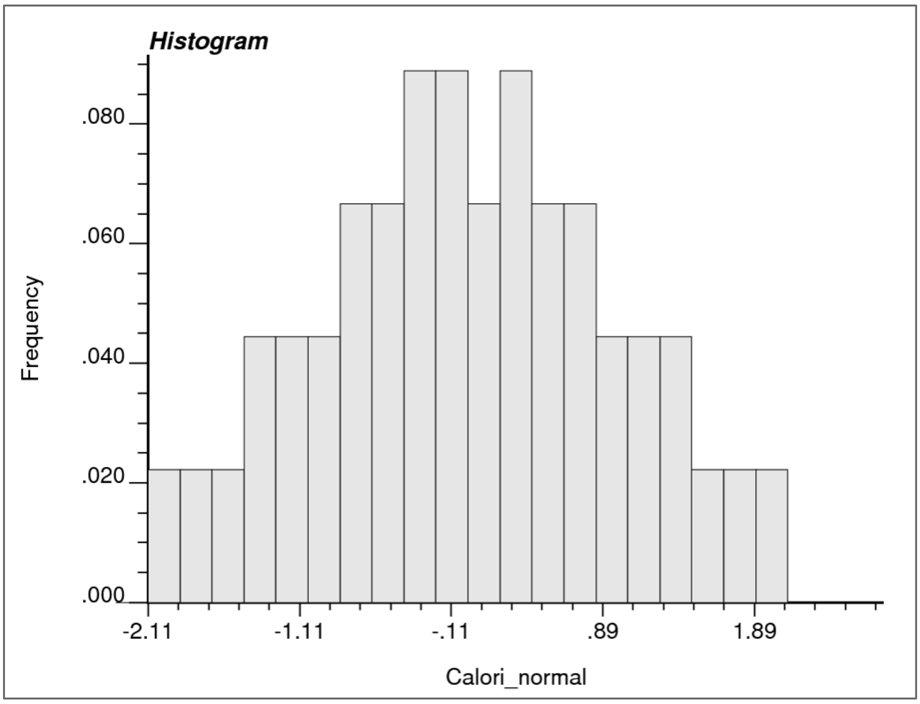


Figure 14. Normal score transformed coal data.

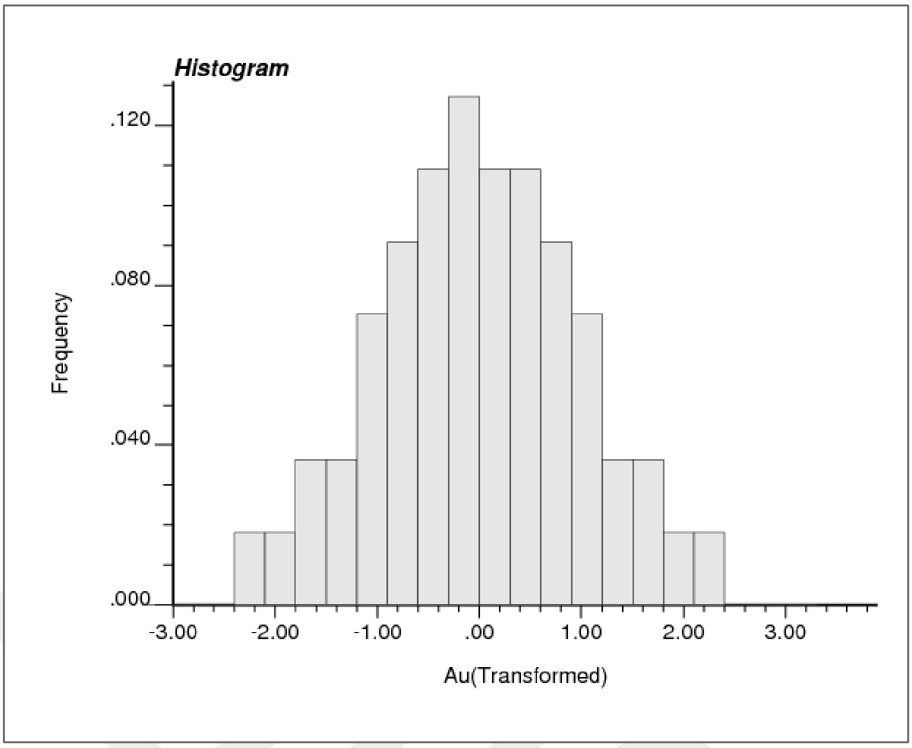


Figure 15. Normal score transformed gold data.

5.2. Variogram analyses

Variogram analyses are carried out over the normal and normalised data for both coal and gold data. The variogram graph of the coal and normalised coal data types are presented in Figure 16 and 17 respectively. Since the analyses were carried out by both kriging and simulation process, the raw and normalised data variograms are constructed.

Main important results can be drawn from the coal data variogram plots. In the regular data there is high nugget variance which implies there may be a sampling problem. It is not a desired situation having a high nugget variance. It means, there may be a problem about sampling analyses or sampling design. But if we look at location maps of the coal data, we can see un-sampled locations north-west and south-east region of the area. Thus, un-sampled areas are the one of the main obstruct of the spatial structure modelling. Also, it is good that sill is reached around variance. In addition, normalised coal data exhibits approximately same parameters apart from nugget (Figure 16). Nugget variance is more suitable here as a result of normalisation.

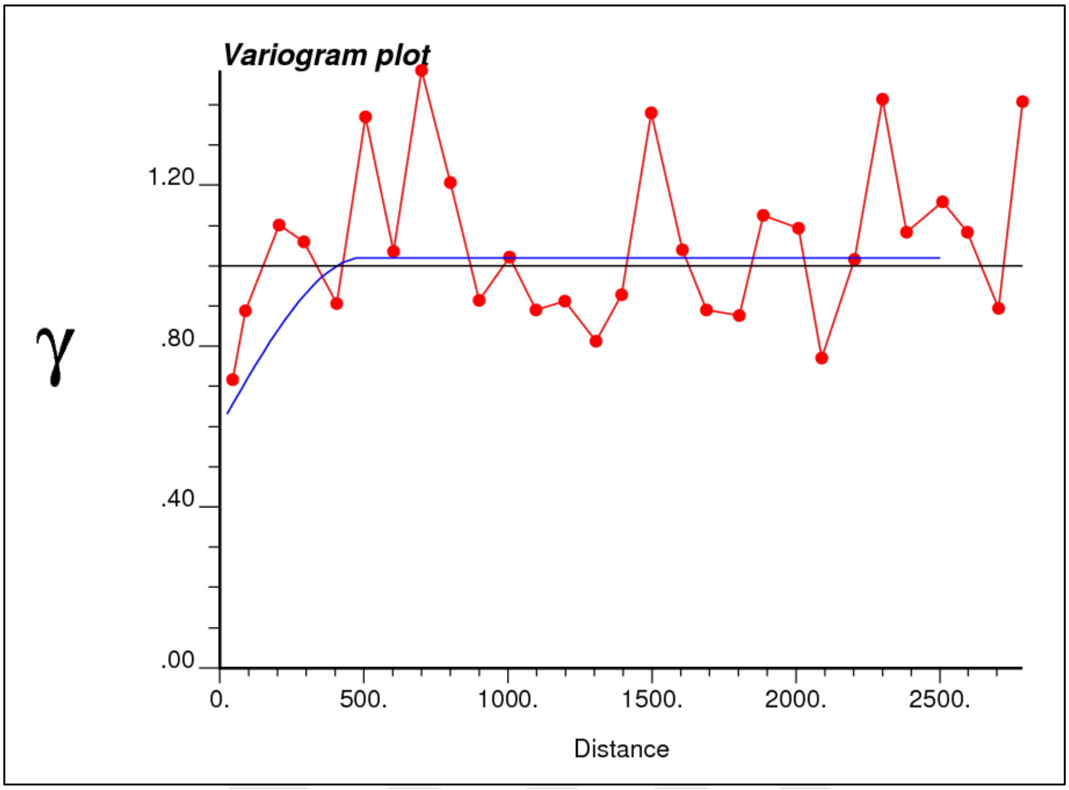


Figure 16. Variogram of the coal data.

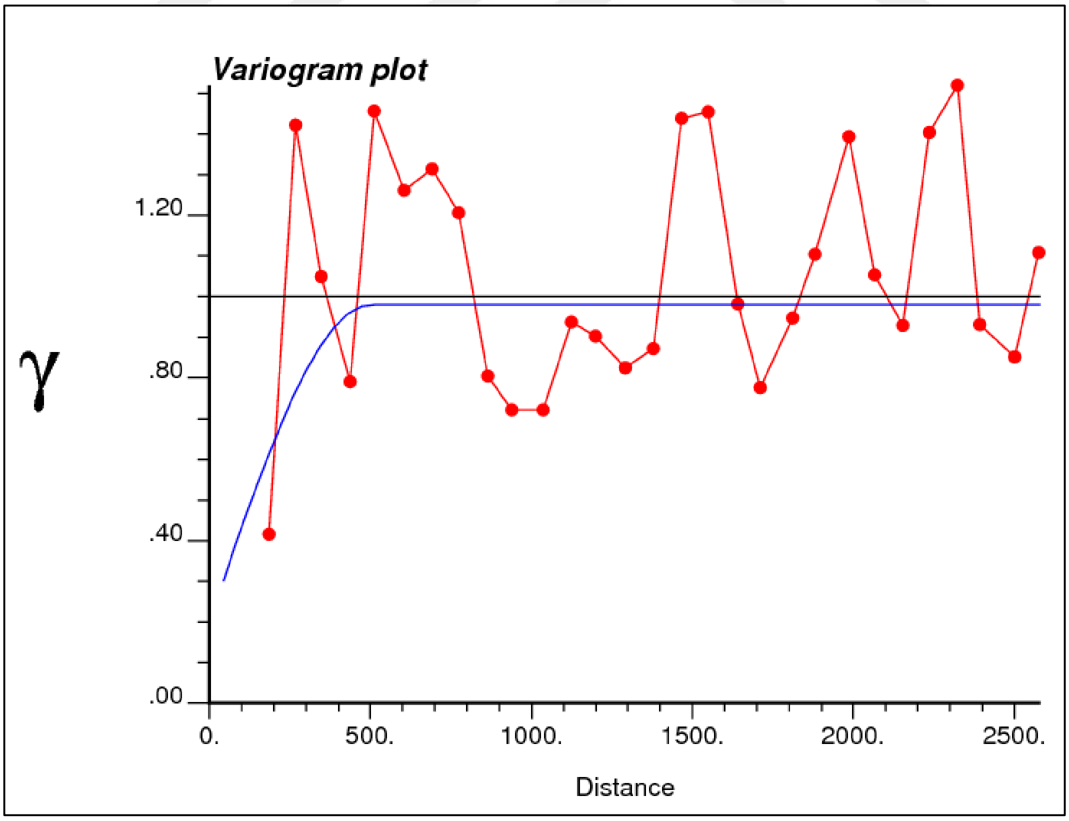


Figure 17. Variogram of the coal data for simulation (normalised).

The variogram graph of the gold and normalised gold data types are presented in Figure 18 and 19 respectively. Variogram graph of the gold data values are presents different spatial structure in comparison to coal data. Range of the gold data is change around 100m for both regular and normalised data. Also, it can be observed again large nugget effect in regular data and low in normalised data variogram plots. But the most important discrimination from the coal data is range. Coal data present a longer range (about 500m) than the gold data (about 100m) when study area dimensions are taking into account. Main reason for this difference is bedding type. As it is previously indicated, the coal data exhibits very slight changes across the large distances, whereas gold data presents sharp changes in local areas across the study area. Thus, the range is smaller in gold deposit experimental variogram model than coal deposit experimental variogram model. As a result of variogram analyses, the theoretical variogram models are used for the kriging estimation and normalised data are used for the simulations.

The theoretical variogram model parameters obtained from the experimental variogram models (spatial distribution behaviour) parameters are given in Table 2.

Table 2. The variogram parameters of the coal and the gold data.

Variable	**Variogram model**	**Nugget (C_0)**	**Sill (C_0+C_1)**	**Range (a)**
Coal	**Spherical**	0.4	0.60	500
Coal (normalised)	**Spherical**	0.2	0.78	500
Gold	**Spherical**	0.62	0.40	210
Gold (normalised)	**Spherical**	0.1	0.92	100

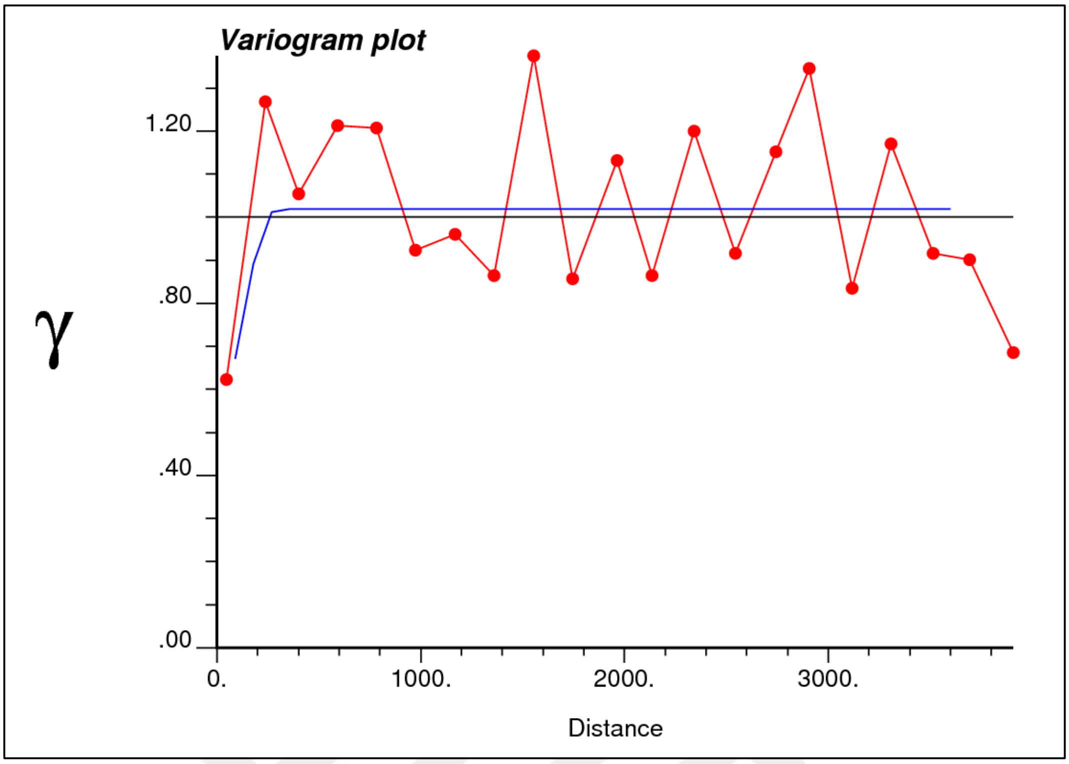


Figure 18. Variogram of the gold data.

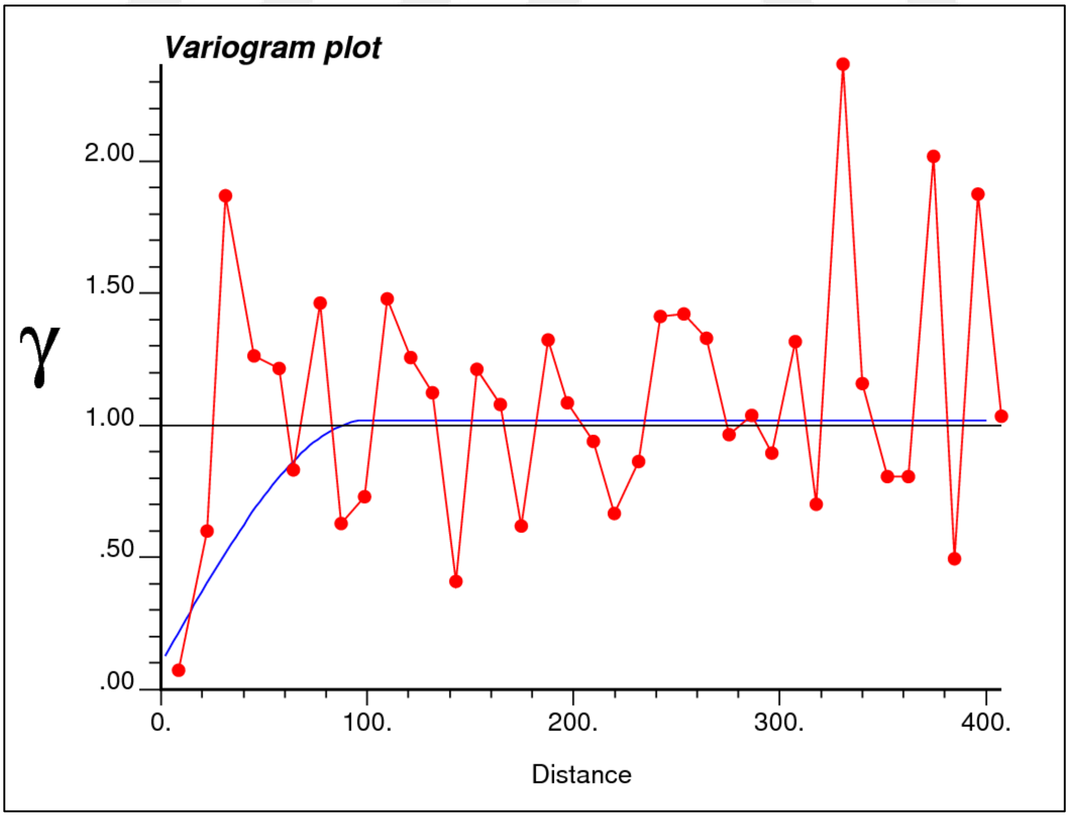


Figure 19. Variogram of the gold data for simulation (normalised).

5.3. Kriging estimation

In the kriging estimation the variogram model for the regular coal data is used. The kriging estimation is applied on a grid system covering the real data on the field. Thus, a grid design including the real data set on the study area is required. A few important parameters in grid design are study area dimension, the distances among the samples and desired resolution of the kriging estimation maps. According to these parameters the best grid design is made suitable for the study area that the estimation process is taken place on it. The grid design for the coal deposit is given in Table 3.

Table 3. Grid design parameters for coal data on the study field.

	Grid origin	Grid cell size	Number of cells
X (Easting, m)	89000	20	226
Y (Northing, m)	43500	20	201

The kriging estimation result on the grid system (Table 3) is presented in Figure 20. In the Figure 20, the coal having high calorific value can be observed south-west region of the study field. Generally, low quality coals are spread across to north-west region of the study field. If it is taken into account the real data locations in Figure 5, there is a high estimation error and artificial calorific values at unsampled locations, and they should be discarded or ignored.

The gold data distribution map obtained from kriging estimation is given in Figure 21. The grid design for the kriging estimation for the gold is also given in Table 4. As it can be seen on the Figure 21, the high gold concentration promising regions are located around in the middle of horizontal line of the study area. On the contrary to the coal kriging estimation maps, gold data kriging maps exhibit point like spotting (bull’s eye effect) at the sampling locations. It is originated from that the gold data has sampled very narrow area, whereas the nearby coal data sampling locations have strong relation among them. Thus, the smoothing effect is very low in gold data kriging estimation. If the kriging map of the gold is analyses together with gold data location map (Figure 8), north and south-west regions of the study area are seldom sampled or no sampling at all. Thus, estimation error is high at these regions and should be disregarded or not to take into consideration.

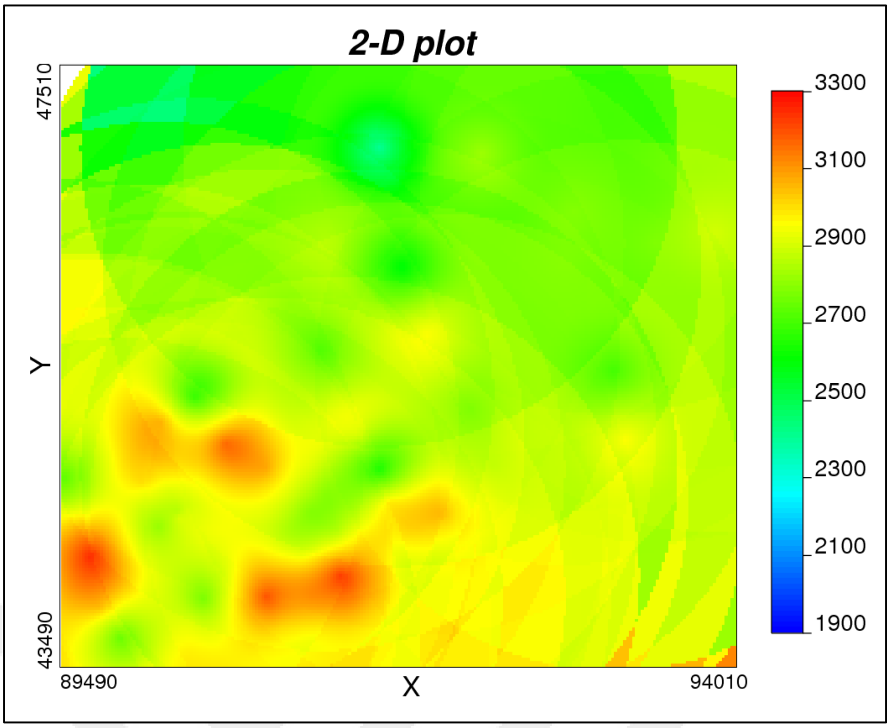


Figure 20. Kriging estimation result for the coal data.

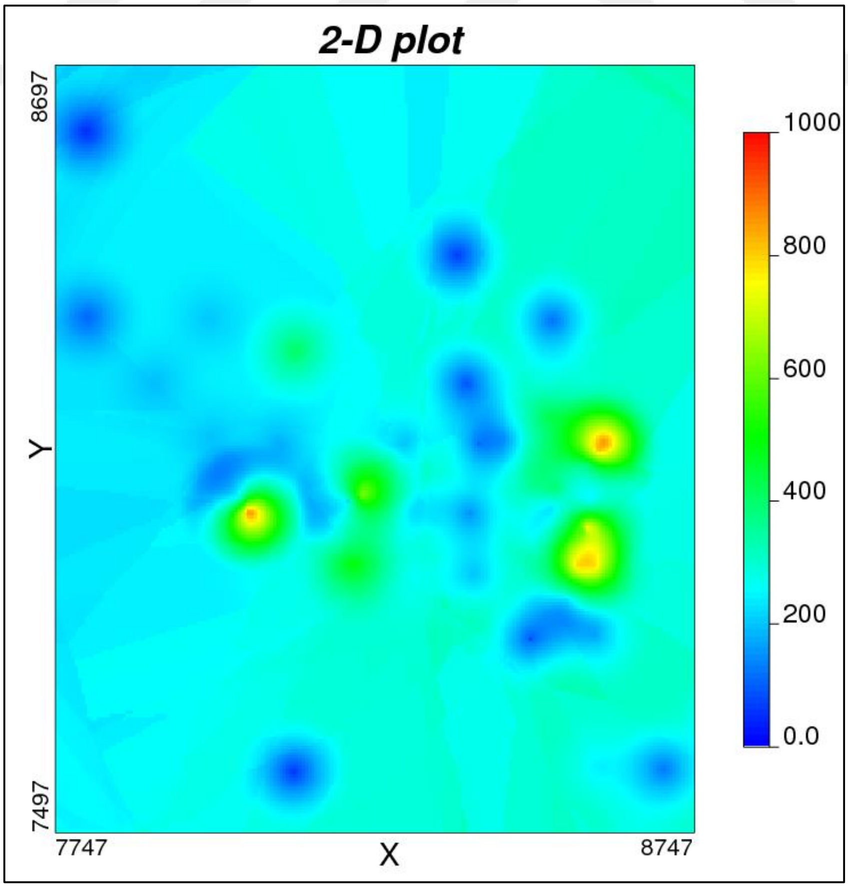


Figure 21. Kriging estimation result for the gold data.

Table 4. Grid design parameters for gold data on the study field.

	Grid origin	Grid cell size	Number of cells
X (Easting, m)	7750	5	200
Y (Northing, m)	7500	5	240

5.4. Sequential Gaussian Simulations

Since all generated simulation maps are equally probable, in other words, every simulated map is identical to the original data set retaining variability, spread and statistical measures theoretically, the number of simulation is limitless. But in this study, only five simulations are generated for coal data, and their maps are presented in Figure 22.

On the contrary to the kriging maps, there is no smooth distribution of the coal values across the area. The distribution pattern does not exhibit a smoothing effect. This is due to local probability conditioning of the data. In other words, the local distribution probabilities are taking into account (simulations) instead of minimising the local estimation variance (kriging). The simulation maps tell us that although the main input data locations reflect the expected coal data value distribution, there are some regions that should not have high calorific values. For example, high values are distributed south, south-west region of the coal field. But, in the simulations maps indicated some high calorific value containing region (Figure 22c,d). In Figure 22c,d there are high calorific valued regions on the south part of the study area. It is thought that these erroneous simulations may be sourced from taking into consideration the unsampled locations because of more sparsely distributed sampling at these zones. For the sake of comparing analysis methods, it is thought that the kriging estimation results give reasonable good maps if the coal data distribution behaviour is taking into account.

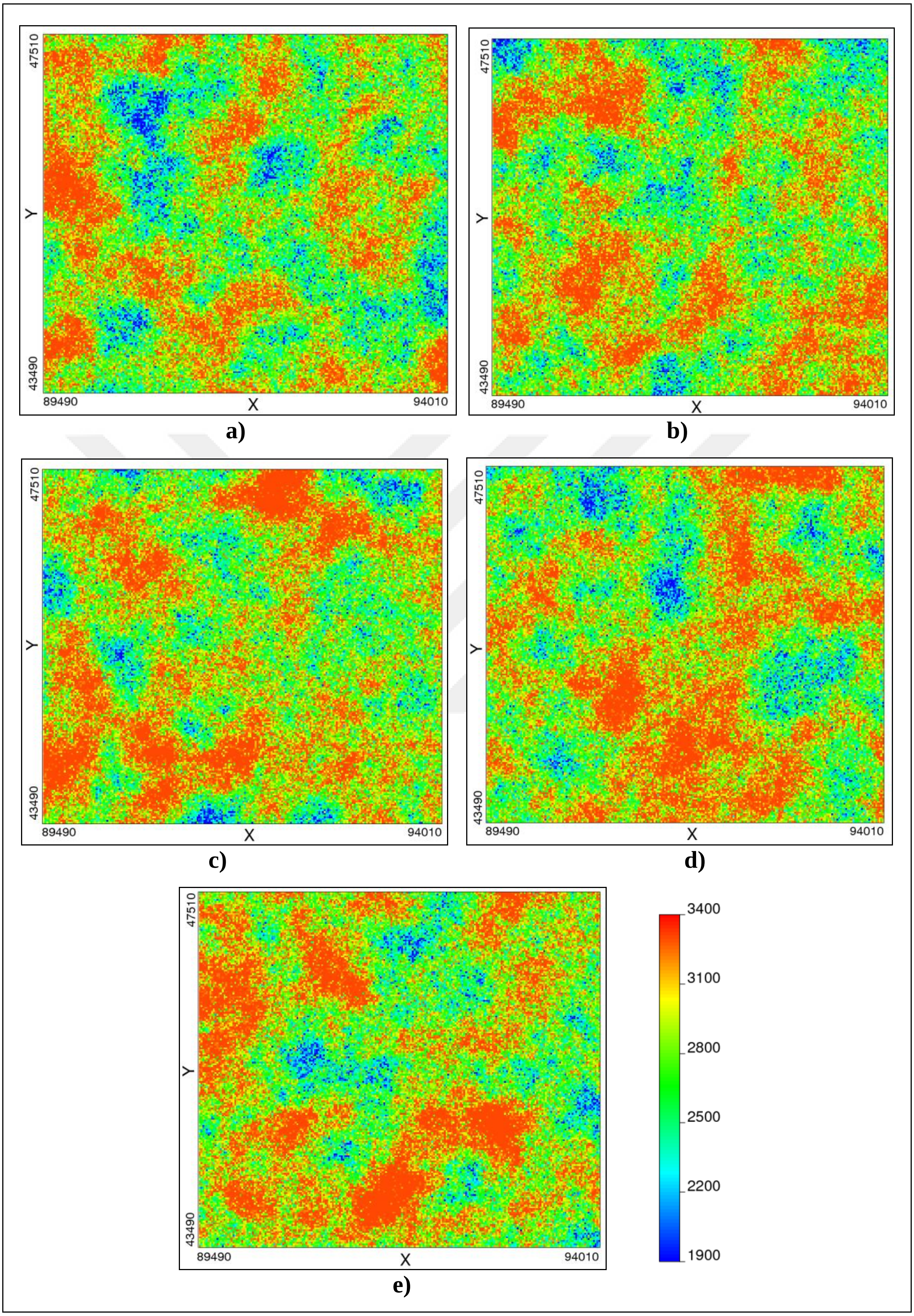


Figure 22. Equally probable SGS maps of the coal data.

In the Figure 23, sequential Gaussian simulation maps belong to gold data is exhibited. 5 gold data simulations are generated based on grid design used in the kriging estimations. As it can be seen on the Figure 23 that mainly all conditional images reflect the real data distribution more accurately in accordance to the location map and kriging estimation map (Figure 21) of the gold data. Again here, there are more or less simulation errors are observed in these maps. In Figure 23c and 23e have high values on the northern part of the gold study area, but it is not as severe as coal simulation maps. These results suggest us that simulation technique reflects best the data distribution over the kriging technique and has superiorities over the kriging technique where there are sharp changes in local data values. Because local sharp changes are mainly masked on kriging technique as a result of smoothing effect. In the kriging procedure smoothing is works as lowering high values and increasing low values locally according to the kriging estimation parameters. Yet, already it should be strongly stressed that for both kriging estimation and the simulations should be implemented in a region limited by a polygon area including sampled locations and excluding unsampled locations. In this way overall estimation or simulation errors could be reduced.

5.5. Validation of the simulation results

Validation of the simulation results can be carried out different ways. The most used technic is judging of reproduction statistical and spread properties of input data. Generally, comparison of summary statistics reproduction, histogram and variogram reproduction are used for this purpose. In addition, theoretically averages of the simulation results equals to the kriging estimations (reference). Therefore, comparing the simulation averages maps and kriging maps is another testing method. This study covers the comparison of the statistical and histogram reproduction, and averages of the simulation with kriging estimation images for the simulation validation. When one equally probable coal simulation histogram and its statistics (Figure 24) are compared to input coal data (Table 1) it can be said basically two distributions are identical to each other in terms of min/max/median and standard deviation. But, there is a slight difference between standard deviations which is a result of the erratic high valued simulations (right-hand side of the histograms) at unsampled locations. However, the simulations are acceptable under these circumstances.

Regarding to gold data simulations, comparison of the Figure 13 together with Table 1 to Figure 25, it is observed that input gold data reproduced merely same statistical structure. Also, approximately the same histogram is reproduced. Thus, it is clear that sequential Gaussian simulations gives better performance on distributions showing locally sharp changes such as metallic or porphyric mineralisation.

Finally, the averages of the simulations for coal and gold data is presented in Figure 26 and Figure 27 for coal and gold data respectively. In Figure 26 and 27, the kriging and images of averaged simulations are given side by side for better distinguish the differences. Figures 26 and 27 tell us that main data locations are simulated successfully, in general. But there are still tolerable errors are available on the simulation images because of not excluding the unsampled locations from the calculations.

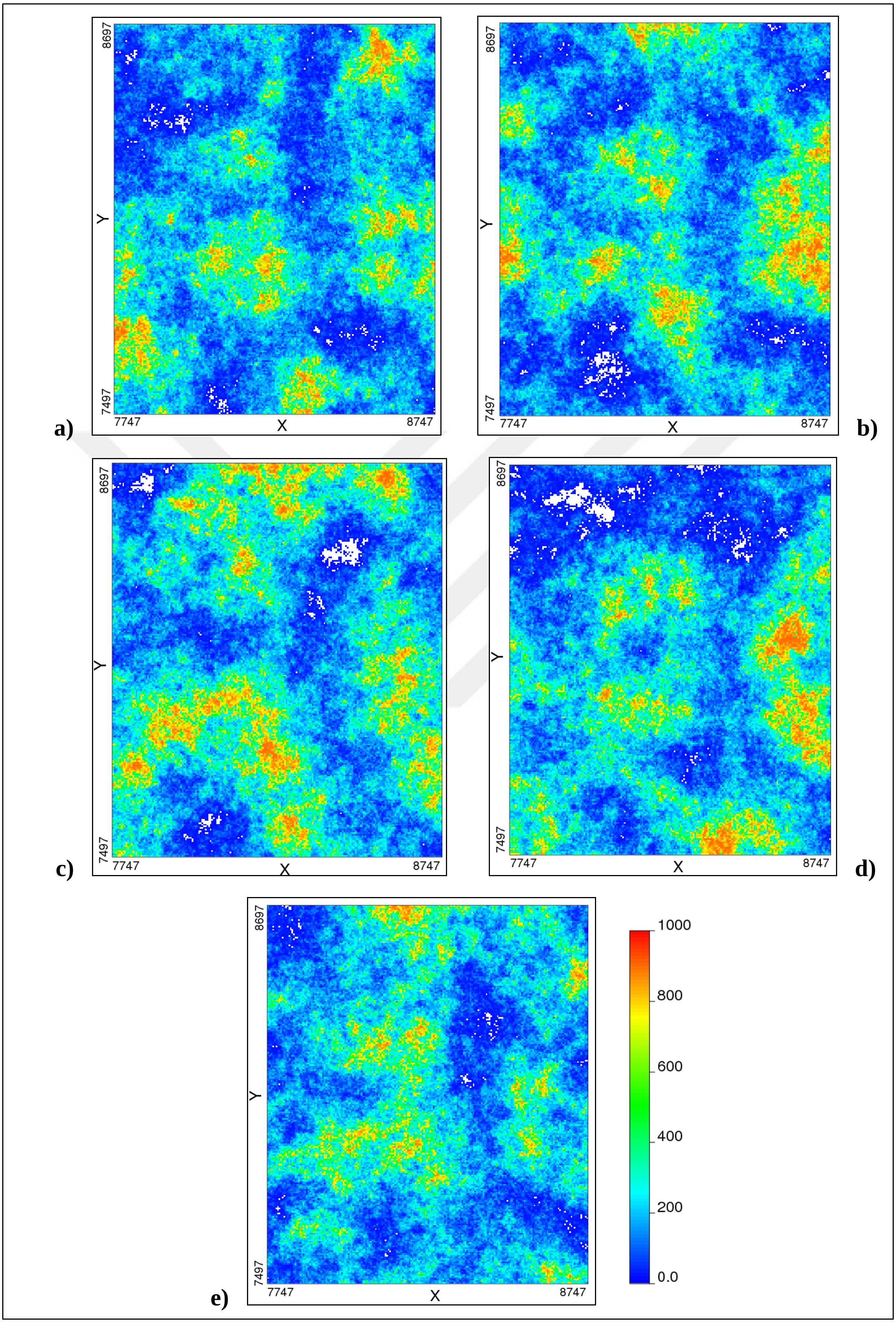


Figure 23. Equally probable SGS maps of the gold data.

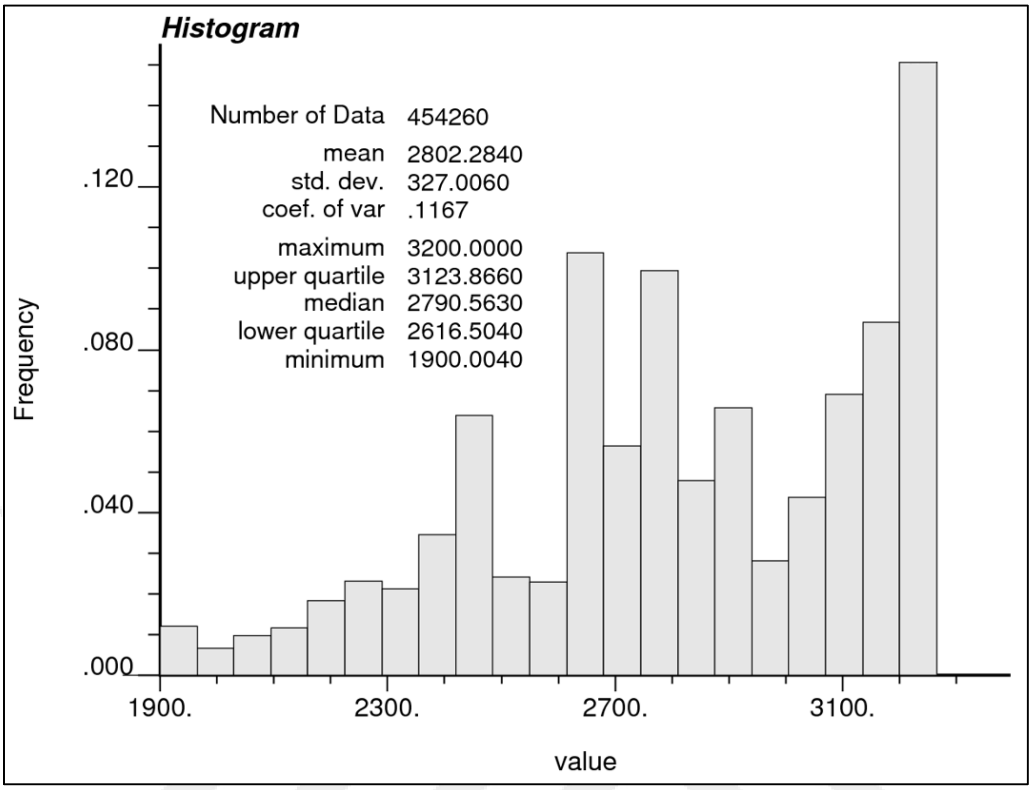


Figure 24. Simulation reproduction histogram of the input coal data.

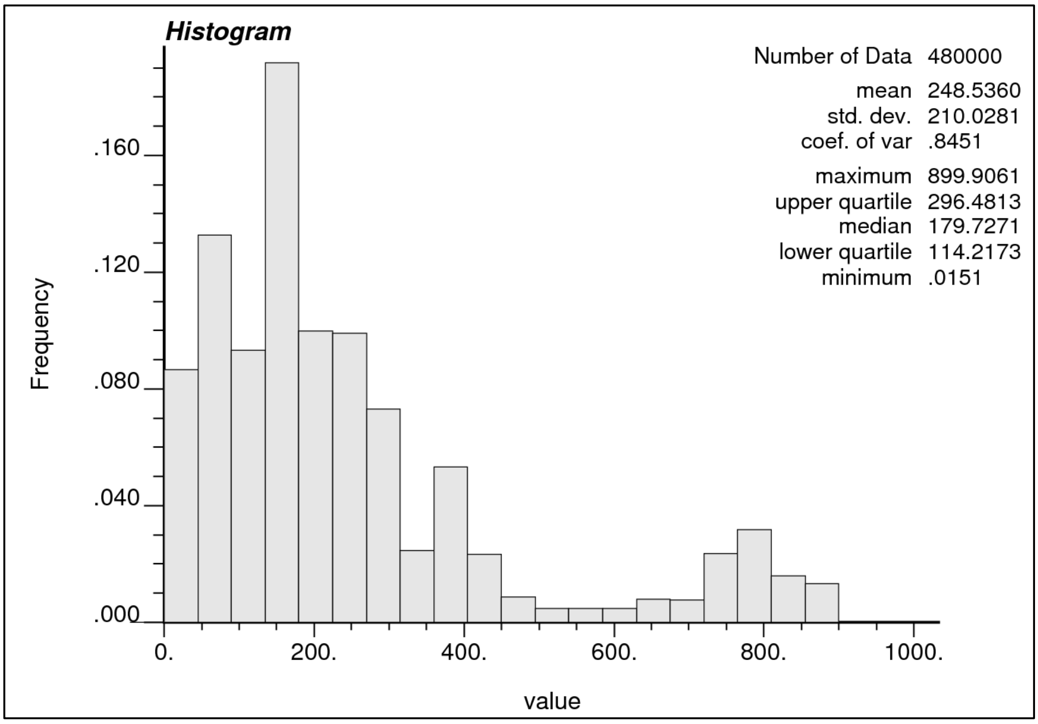
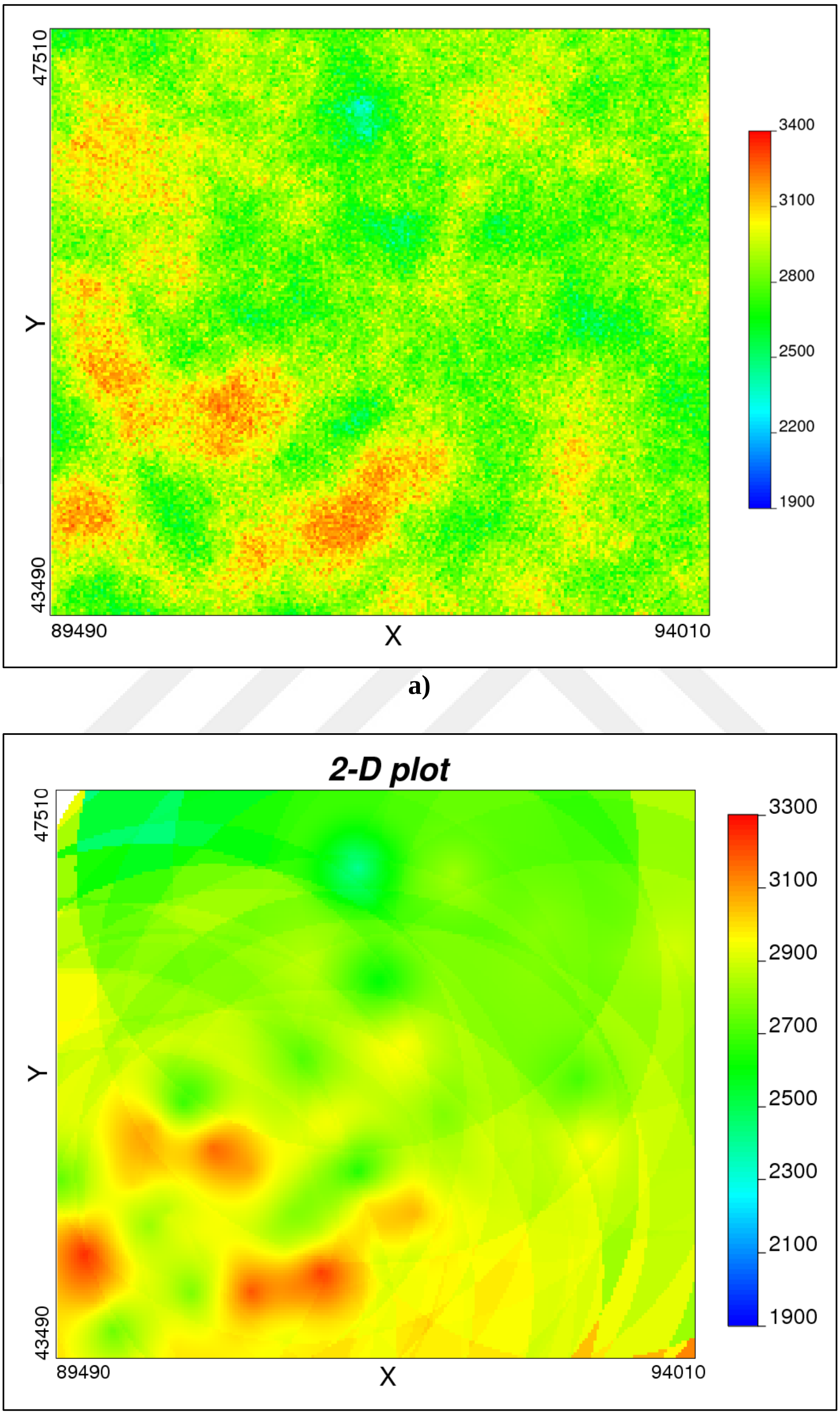


Figure 25. Simulation reproduction histogram of the input gold data.



a)

b)

Figure 26. SGS average map for the coal data.

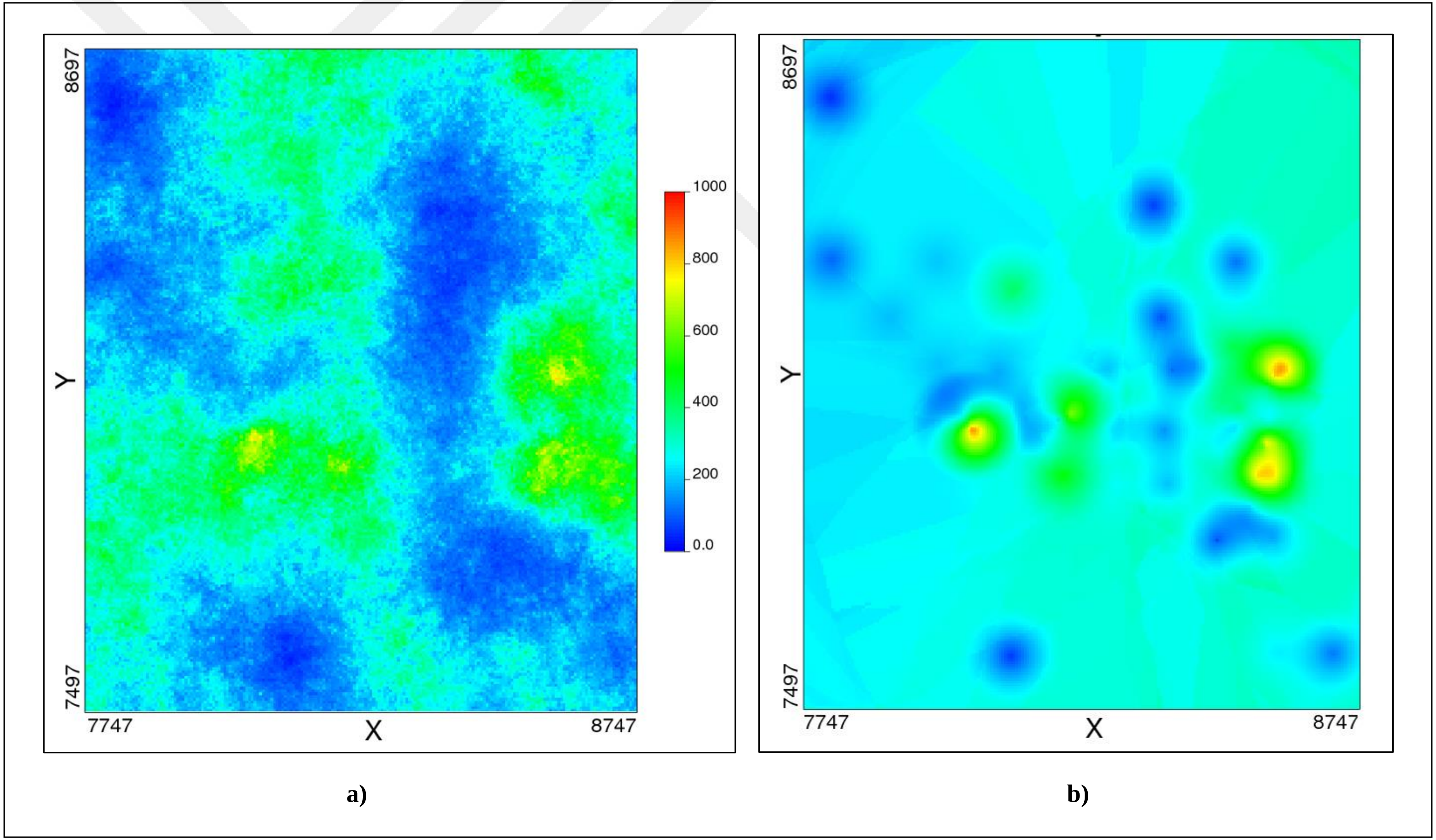


Figure 27. SGS average map for the gold data.

6. CONCLUSIONS

Today a wide range of geostatistical analysis methods are developed, and they are extensively being used with relatively derived softwares. In other words, geostatistical analyses cannot be thought without an application software. Because geostatistics requires processing exhaustive data sets, extensive calculation steps in accordance to the method type which cannot be done by hand calculations. Thus, there are a lot of geostatistical software tools as well as geostatistical methods. In addition, with the expansion of the geostatistical methods and softwares some part of the geostatistical analyses and softwares are specialised. Underground, open cast mining, special and individual mine occurrence reserve analyses can be carried out separately. For example, Isatis, Vulcan, Surpac are specialised softwares and they include relevant geostatistical analysis tools for 2D, 3D mining reserve analyses, environmental analyses, too.

In this study, two different occurrence type of the mining reserve is analysed by a public software “GSLIB” which is very simple but effective software. The GSLIB (C. V. Deutsch & Journel, 1998) is the predecessor of the geostatistical softwares and is still public free. Main geostatistical analyses in 2D and 3D dimensional data set can be carried out with a wide range of methods from statistical analyses to simulations using this software. It is well known that new geostatistical analysis methods are emerged as a need for different conditions such as occurrence type, sampling strategy and type, data distribution space. Thus, a coal mine and a gold reserve representing sedimentary bedding and local mineralisation type respectively are analysed with kriging and sequential Gaussian simulation methods. These two methods differ from each other that kriging minimises local estimation error while sGs mimics the local distribution behaviour on the basis of local probability density distribution. These two analyses methods are applied on different known bedding type to reveal which method is suitable for individual mine.

The results drawn from this study are as follows:

1. Two type of the mine bedding is analysed. One of them is coal bed exhibiting a sedimentary distribution behaviour (low data variability in both lateral and vertical scale). Since the low distribution across the field individual data can be thought as

identical for wide range of neighbouring unknown location. Thus, minimising local estimation is in favour of acceptable. Second mineralisation is gold mine. This gold mine shows sharp and drastic variabilities locally. Therefore, analysing in the concept of kriging will result in erroneous images. Instead, sequential Gaussian simulation is expected more suitable due to reproduce local distribution probabilities.

2. The coal data has a two-dimensional (point) and the gold data has a three-dimensional (borehole) data. The calculations are carried out based on 2-dimensional space. Thus, gold data is converted 3-D to 2-D data using data accumulation method and analyses are carried out over accumulated data. Data distributions are analysed, and normal-score transformations are made to ensure the Gaussianity for simulation process.

3. On the two different bedding type, two geostatistical methods including kriging and sequential Gaussian simulation are applied to reveal the effects of the methods on bedding type. Kriging estimation contour maps exhibited very smooth distribution pattern for both coal and gold data, as expected. Since the variations are tolerable for the coal data, the kriging maps with low estimation errors are acceptable. On the other hand, smoothed contour map pattern of the gold data seen misbehaved. Because kriging procedure estimates low values as high and vice versa as a result of smoothing effect. Thus, local high or low value changes and their local effects are disregarded. It resulted in erroneous kriging map. On the contrary, the sequential Gaussian simulation method generated more realistic images of the local gold data because of taking into account the local distribution probabilities.

4. The kriging estimation and sequential Gaussian simulation results also validated by comparing input and output results.

5. The results showed that kriging estimation method is suitable for sedimentary manner bedding type mines. Conversely, the sequential Gaussian simulation exhibited better performance on gold mine showing local severe local data variability.

7. REFERENCES

Abichequer, L. A., Costa, J. F. C. L., Pasti, H. A., & Koppe, J. C. (2011). Design of blending piles by geostatistically simulated models - A real case reconciliation. *International Journal of Mineral Processing*, *99*(1–4), 21–26.

Chilès, J. P., & Delfiner, P. (2012). *Geostatistics: Modeling Spatial Uncertainty* (Second). New York: John Wiley and Sons doi:10.1002/9781118136188.

Cinnirella, S., Buttafuoco, G., & Pirrone, N. (2005). Stochastic analysis to assess the spatial distribution of groundwater nitrate concentrations in the Po catchment (Italy). *Environmental Pollution* doi:10.1016/j.envpol.2004.06.020.

Clark, V., Judd, C. M., & McClelland, G. H. (1990). Data Analysis: A Model-Comparison Approach. *Journal of the American Statistical Association* doi:10.2307/2289821.

Cleveland, W. S. (1984). Graphical methods for data presentation: Full scale breaks, dot charts, and multibased logging. *American Statistician* doi:10.1080/00031305.1984.10483224.

De Souza, L. E., C.L. Costa, J. F., & Koppe, J. C. (2004). Uncertainty estimate in resources assessment: A geostatistical contribution. *Natural Resources Research*, *13*(1), 1–15.

Deutsch, C. V., & Journel, A. G. (1998). *GSLIB: geostatistical software library and user's guide. Second edition*. *GSLIB: geostatistical software library and user's guide. Second edition* (Second). New York: Oxford University Press doi:10.1080/00401706.1995.10485913.

Deutsch, Clayton V. (1999). A Short Note on Cross Validation of Facies Simulation Methods. *Centre for Computational Geostatistics*, 1–12.

Diko, L., Vervoort, A., & Vergauwen, I. (2001). Geostatistical modelling of lateritic bauxite orebodies in Surinam: Effect of the vertical dimension. *Journal of Geochemical Exploration* doi:10.1016/S0375-6742(01)00172-8.

Dowd, P. A. (1984). The Variogram and Kriging: Robust and Resistant Estimators. In G. Verly, M. David, A. G. Journel, & A. Marechal (Eds.), *Geostatistics for Natural Resources Characterization* (pp. 91–106). Dordrecht.

Gambin, F., Costa, J. F. C. L., & Koppe, J. C. (2005). Forecasting fluctuations in coal quality delivered to a power plant via stochastic simulation. *Mining Technology*, *114*(3), 167–175.

Goovaerts, P. (1997). *Geostatistics for natural resources evaluation*. *Geostatistics for natural* *resources evaluation* Retrieved from http://www.amazon.com/dp/0195115384.

Goovaerts, P. (1999). Geostatistics in soil science: state-of-the-art andperspectives. *Geoderma*, *89*, 1–45.

Hohn, M. E., & McDowell, R. R. (2001). Uncertainty in coal property valuation in West Virginia: A case study. *Mathematical Geology* doi:10.1023/A:1007535301969.

J Vann, S Jackson, & O Bertoli. (2003). Quantitative Kriging Neighbourhood Analysis for the Mining Geologist , A Description of the Method With Worked Case Examples. The Australasian Institute of Mining and Metallurgy.

Journel, A. G. (1983). Nonparametric estimation of spatial distributions. *Journal of the International Association for Mathematical Geology*, *15*(3), 445–468.

Kishné, A. S., Bringmark, E., Bringmark, L., & Alriksson, A. (2003). Comparison of ordinary and lognormal kriging on skewed data of total cadmium in forest soils of Sweden. *Environmental Monitoring and Assessment* doi:10.1023/A:1023326314184.

Micceri, T. (1989). The Unicorn, The Normal Curve, and Other Improbable Creatures. *Psychological Bulletin* doi:10.1037/0033-2909.105.1.156.

Schloeder, C. A., Zimmerman, N. E., & Jacobs, M. J. (2001). Comparison of Methods for Interpolating Soil Properties Using Limited Data. *Soil Science Society of America Journal*, *65*, 470–479.

Van Groenigen, J. W. (2000). The influence of variogram parameters on optimal sampling schemes for mapping by kriging. *Geoderma* doi:10.1016/S0016-7061(00)00040-9.

Yunsel, T. Y. (2012). A practical application of geostatistical methods to quality and mineral reserve modelling of cement raw materials. *Journal of the Southern African Institute of Mining and Metallurgy*, *112*(3), 239–249.