

T.R.
GEBZE TECHNICAL UNIVERSITY
INSTITUTE OF ENERGY TECHNOLOGIES

**VERSATILE ROTOR VIBRATION ANALYSIS TOOL FOR
SINGLE ROTATING ELEMENTS**

ALİCAN KILIÇASLAN
**A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
IN APPLIED PROPULSION SYSTEM DESIGN &
ENGINEERING FOR AEROSPACE TECHNOLOGIES**

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THESIS SUPERVISOR
ASSIST. PROF. SEDAT TOKGÖZ

GEBZE
2022

GEBZE TEKNİK ÜNİVERSİTESİ	YÜKSEK LİSANS JÜRİ ONAY FORMU
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GTÜ Enerji Teknolojileri Enstitüsü Yönetim Kurulu’nun 13/06/2022 tarih ve 2022/08 sayılı kararıyla oluşturulan jüri tarafından 01/07/2022 tarihinde tez savunma sınavı yapılan Alican KILIÇASLAN’ın tez çalışması Enerji Teknolojileri Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

JÜRİ

ÜYE

(TEZ DANIŞMANI) : DR.ÖĞR. ÜYESİ SEDAT TOKGÖZ

ÜYE

: PROF.DR. İLYAS KANDEMİR

ÜYE

: DR.ÖĞR. ÜYESİ SEHER EKEN

ONAY

Gebze Teknik Üniversitesi Enerji Teknolojileri Enstitüsü Yönetim Kurulu’nun
...../...../..... tarih ve/..... sayılı kararı.

İMZA/MÜHÜR

SUMMARY

A turbofan engine development requires a complex, well-organized plan and sufficient budget to meet every requirement and time target. Therefore, a turbofan engine needs to be designed and verified to be sufficiently durable for the loads it encounters within its operating envelope. Verification tests are also carried out in order to ensure this condition and meet the requirements in every loading condition.

During the conceptual and preliminary design phases, there should be a quick iteration step to observe the loading of a turbofan engine. To overcome this, Joe Solver, an in-house and open source tool built directly for engine dynamics and load calculations, was created using the Transfer Matrix Method.

In this study, Fan-Blade Out (FBO), which is one of the extreme and possible events during the duty cycle of a turbofan engine (the event of a fan blade breaking while the engine is running), was examined with a numerical model on MATLAB (Joe Solver) and the results showing that it can be used in the design stages are based on the same geometry and literature models, it has been demonstrated by validation studies.

Key Words: Turbofan Engine, Fan-Blade Out, Load Engineering, Rotor Dynamics

ÖZET

Bir turbofan motor geliřtirmesi, karmařık, iyi organize edilmiř bir plan ve her gereksinimi ve zaman hedefini karřılamak için yeterli bütçeyi gerektirmektedir. Bu nedenle bir turbofan motorunun, çalıřma zarfı içinde karřılařtıęı yükler için yeterince dayanıklı olarak tasarlanması ve bu yükler altında doęrulanması gerekir. Her yükleme kořulunda bu durumun saęlanabilmesi ve gereksinimlerin karřılanması için doęrulama testleri de yapılmaktadır.

Kavramsal ve ön tasarım ařamalarında, bir turbofan motorunun yüklenmesini gözlemek için hızlı bir iterasyon adımı olmalıdır. Bunun üstesinden gelmek için, Transfer Matrisi Metodu kullanılarak doğrudan motor dinamięi ve yük hesaplamaları için oluřturulmuř řirket içi ve açık kaynaklı bir araç olan Joe Solver oluřturulmuřtur.

Bu çalıřmada, bir turbofan motorunun çalıřma çevrimi sırasındaki ekstrem ve olası olaylardan biri olan Fan-Blade Out (FBO), (motor çalıřırken bir fan kanadının kopması olayı) MATLAB (Joe Solver) üzerinde sayısal model ile incelenmiř ve tasarım ařamalarında kullanılabileceęini gösteren sonuçlar aynı geometriye sahip model içinde ve literatür modelleri kullanılarak validasyon çalıřmaları ile gösterilmiřtir.

Anahtar Kelimeler: Turbofan Motor, Fan Kanadı Kopması, Yük Mühendislięi, Rotor Dinamięi

ACKNOWLEDGEMENTS

Firstly, I would like to sincerely thank my Supervisor, Assistant Professor Sedat TOKGÖZ for his significant directions and worthy contribution.

I am greatly appreciated to attend to master program in Applied Propulsion System Design Engineering in Aviation and Space Technologies in Gebze Technical University, Turkey. Engine Dynamics and Load Calculation Code (Joe Solver) studies have started with this program and my master thesis topic.

I would like to thank my Company Mentors İlker BAŞOL and Umut KARAHAN from TEI for their valuable supports.

I would like to thank my colleague Hüseyin Tarık NİŞ, without his motivation and amazing contribution, Joe Solver could not be grown up.

Finally, I am appreciated my wife, for her patience and support.

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LIST of SYMBOLS and ABBREVIATIONS

<u>Symbols and</u> <u>Abbreviations</u>	<u>Explanations</u>
<i>FBO</i>	: Fan Blade Out
<i>FAA</i>	: Federal Aviation Administration
<i>FEM</i>	: Finite Element Method
<i>TMM</i>	: Transfer Matrix Method
<i>LP – Shaft</i>	: Low Pressure Shaft
<i>HP – Shaft</i>	: High Pressure Shaft

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1. INTRODUCTION

1.1. Jet Engines

According to Newton's third law, when a force is applied to an object, the same level of reaction force occurs. The jet engine uses this principle by forcing a fluid, which creates a reaction in the required direction; hence, the thrust that moves the platform is achieved. A rotating garden sprinkler is a basic example of a jet propulsion mechanism. There is a sprinkler nozzle where the water jet is forced and the rotational momentum is obtained. Gas turbines are mostly modern jet engines and they burn fuel to convert it into heat energy. The simplest gas turbine can be described as a turbojet, which is basically an open-ended tube with air easily flowing through it. On modern aircraft, gas turbine engines become in various operational forms, such as turbojet, turboprop, turbofan, and turboshaft. All of these engines have a core engine and its basic components such as a compressor, combustion chamber, and gas turbine [1]. Air enters the engine, is compressed, mixed with fuel and burned in combustion. Combustion gases are run through the rear nozzle to provide thrust. At the same time, the turbine generates power for the compressor. This is a basic cycle of the turbojet engine and is called the Brayton cycle [1].

Brayton Thermodynamic Cycle given in Figure-1 shows a T-S diagram is used for all gas turbine engines [2]. The cycle begins with free-flow conditions at station 0 using the turbine engine station numbering system. In cruise flight, the intake slows the airflow as it is brought to the compressor at station 2. As the flow slows, some of the energy associated with the speed of the aircraft increases the static pressure of the air and the flow is compressed. Ideally, the compression is isentropic. The compressor raises the pressure and temperature isentropically to station 3 at the compressor outlet. Since compression is ideally isentropic, a vertical line in the T-S diagram defines the process. Combustion in the burner takes place at constant pressure from station 3 to station 4. The hot exhaust then passes through the power turbine where work is done by the flow from station 4 to station 5. Since the turbine and compressor are on the same shaft, the work done on the turbine is exactly equal to the work done by the compressor and ideally, the temperature difference is the same. Then returns the flow

isentropically from station 5 to station 8 to free flow pressure. From the outside, the flow conditions revert to free-flow conditions, which completes the cycle [2].

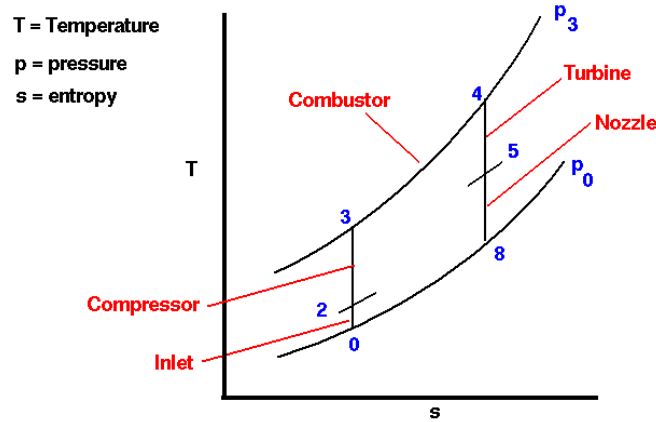


Figure 1: Brayton Thermodynamic Cycle [2]

1.2. Turbojet and Turbofan Engines

There is various type of shaft configuration used in jet engines. There are many combinations of solutions, depending on design requirements and constraints. Single and dual shaft systems are generally used in aviation engines. It appears that both systems can work together smoothly and efficiently, as the turbojet and turbofan are well-matched in thermodynamic behavior [3].

Although the single shaft system has been studied in this thesis, turbofan engines usually have two shafts, which are low-pressure shaft and high-pressure shaft systems. A low-pressure shaft belonging to a turbofan engine is modeled scope of this thesis.

1.2.1. Main Components of a Turbofan Engine

The modern military and civil aviation industry mostly prefer turbofan engines. Although these engines come in various shapes and sizes, all turbofan engines have common parts. The main parts are inlet, fan, compressor, combustion chamber, low-pressure and high-pressure turbines, and exhaust nozzle.

The air inlet is a part of the air entrance area before the compressor within minimum drag or energy loss. The compressor is the section that, supplies compressed air to combustion chamber. The combustion chamber is where the combustion process takes place with the compressed air coming from the compressor. The turbine converts some of the kinetic energy of the exhaust gases into mechanical energy to drive the

compressor and fan. The exhaust is the part that directs the flow of hot gases backward to prevent turbulence [4]. A representative section view of turbofan engine components is given in Figure 2.

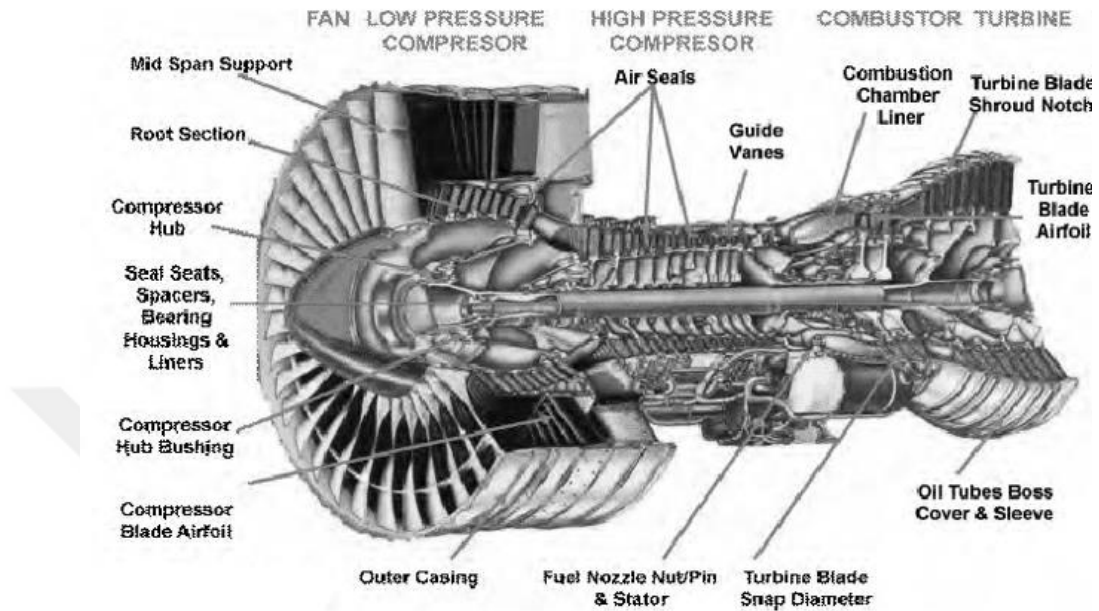


Figure 2: Turbofan Engine Components [5]

Other types of jet engines used in aviation, both one and two shaft systems are described also in the following section to understand difference between single rotating and two-spool rotor systems.

- **Single-Shaft Turbojet**

This configuration contains a compressor and a gas turbine on the same drive shaft given in Figure 3. It is an engine for earlier platforms chosen turbojet thrust [3].

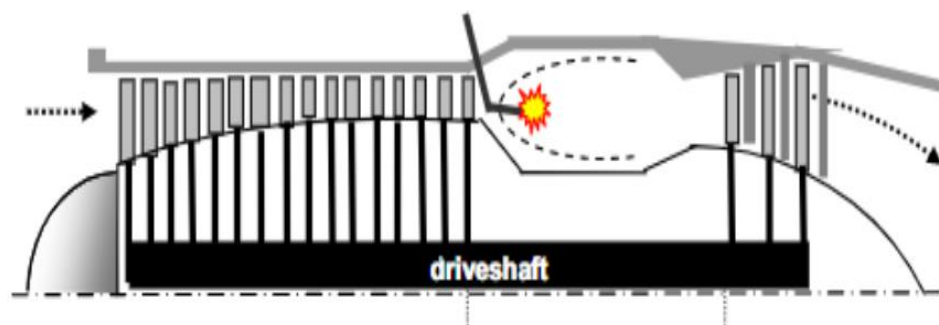


Figure 3: An Example of Single-Shaft Turbojet [3]

- **Dual-Shaft Turbojet**

In this application, an example is given in Figure 4, commonly there are separately two spools, which are the outer driveshaft and inner driveshaft. A low-pressure compressor works on the inner driveshaft, while, a high-pressure compressor runs on the outer driveshaft. Due to a separate shaft group, each shaft can rotate at different rotational speeds and directions. It is an engine used on North American F-100 Super Sabre, Vought F8U Crusader, Boeing 707, and DC-8 [3].

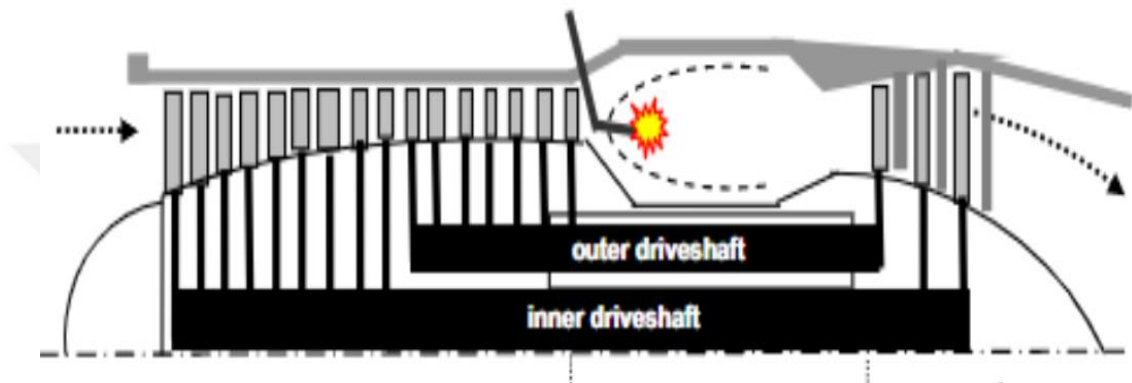


Figure 4: An Example of Dual-Shaft Turbojet [3]

- **Dual-Shaft Internally Mixed Turbofan**

A low-pressure compressor with an integral fan works on the same driveshaft. A high-pressure compressor and its turbine runs on another driveshaft. An example of this combination is given in Figure 5. It is an engine used on Boeing 737 and McDonnell-Douglas MD80 [3].

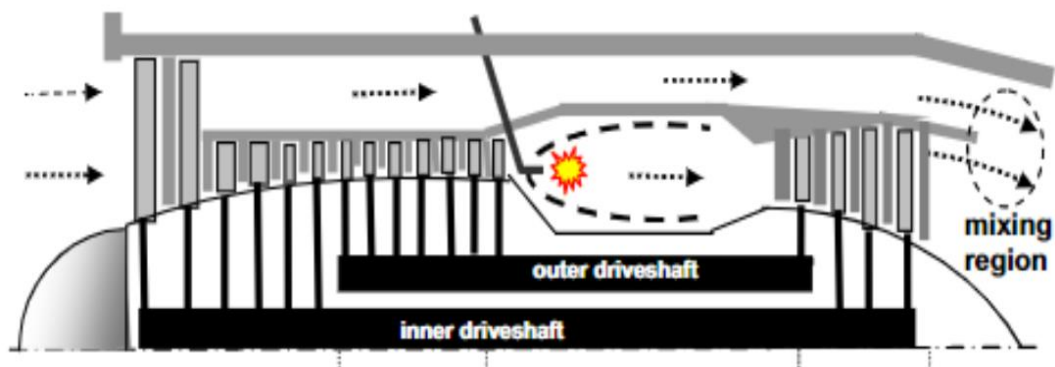


Figure 5: An Example of Dual-Shaft Internally Mixed Turbofan [3]

- **Dual-Shaft Low Bypass Turbofan**

This configuration looks similar to a dual-shaft turbofan engine arrangement, an example given in Figure 6, but in this layout, the fan flow is not mixed internally, flows around the outside of the compressor casing. It is an engine used on Boeing 747 and Airbus A300 [3].

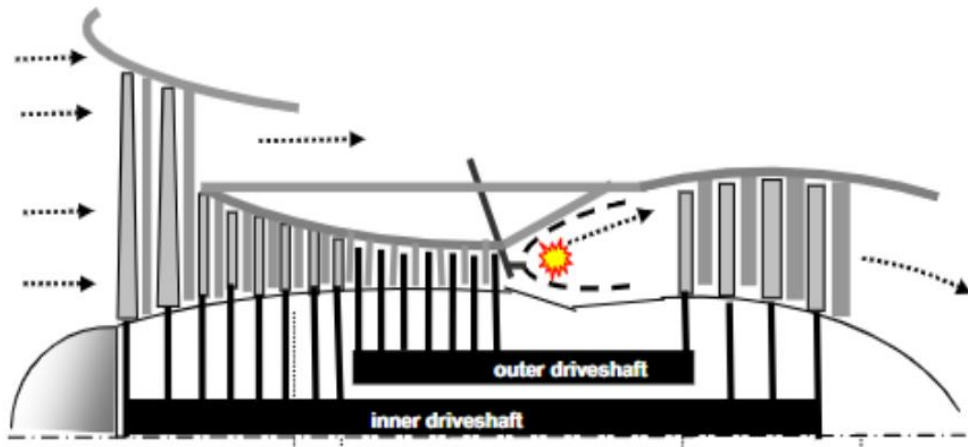


Figure 6: An Example of Dual-Shaft Low Bypass Turbofan [3]

- **Dual-Shaft High Bypass Turbofan**

It is commonly similar to a dual-shaft combination but a high bypass turbofan owns a large fan diameter. There is an externally mixing of fan flows. An example is given in Figure 7. It is an engine used commonly on commercial airline planes [3].

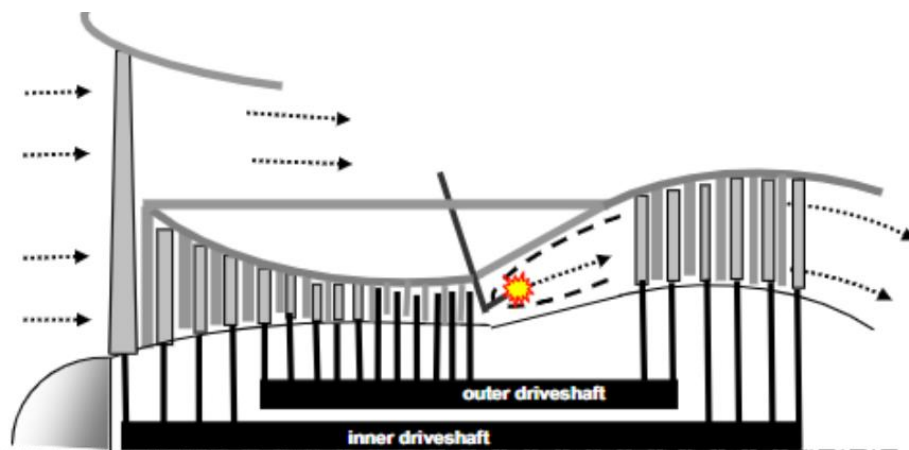


Figure 7: An Example of Dual-Shaft High Bypass Turbofan [3]

- **Dual-Shaft Afterburning Turbofan**

This configuration is similar to the low bypass turbofan layout which has two separate driveshafts but is empowered afterburning end of the flow after mixing to obtain temporary higher thrust because of combat maneuvers. It is an engine commonly on fighter jets such as the F16 Fighting Falcon or Eurofighter Typhoon, an example given in Figure 8 [3].

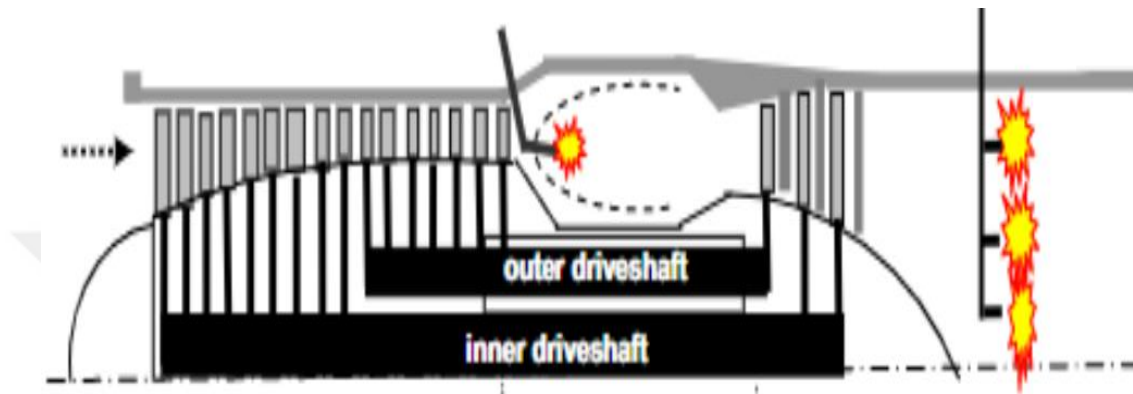


Figure 8: An Example of Dual-Shaft Afterburning Turbofan [3]

1.3. Fan Blade-Out Event

A turbofan engine, both commercial and military, must be sufficiently resistant to loads under operating conditions. Each loading condition must be demonstrated by verification tests that it meets the aircraft manufacturers' requirements. On the other hand, certification, which is a very conservative approach to the design process, remains vital for an engine development program for the aviation industry. Limit, ultimate and catastrophic events must be simulated and certified so that the engine remains in one piece during flight under these loads. For instance, according to the regulation of Federal Aviation Administration (FAA), an event of Fan Blade Out (FBO) shall not cause a fracture on engine structures, especially cases, mounts, and not cause an engine fire due to rupture fuel or oil supply lines [6]. Therefore overcoming a FBO and Blade Containment Test is a major design checkpoint in the engine development program. At the same time, this test is costly and requires investment. An example image is given in Figure 9 for preparation for a fan blade-out test. During the test, a colored blade is going to release by an explosive suddenly but under controlled conditions [7].



Figure 9: Preparation of Fan Blade-Out Test on Fan Blades [6]

Due to the high risk of failing the FBO test, in most engine development programs, the engine structural components in the main load path are designed conservatively, which means adding extra thickness or ribs, to resist heavy loads. As a result, this design approach adds to the weight of the engine. For this reason, it is very important to obtain the load level correctly, especially during the design phase of an engine development program [8].

On a Turbofan engine, FBO event may be seen because of several reasons - foreign object damage to the flow zone or blade could fly out due to high cycle fatigue on operating loads. During FBO events, loads mainly affect structural frames on the primary load path on the engine such as bearings, mounts, and frame supports [9]. When one or more blade ruptures occur on a rotating shaft, the structural components must resist this impact load. That load causes an excessive deformation on rotating parts, which causes rub between rotor and stator parts. [10]. Event sequences each by each during FBO event are given below in Figure 10. An example of structural failure after FBO event and uncontained blade or blades is given in Figure 11.

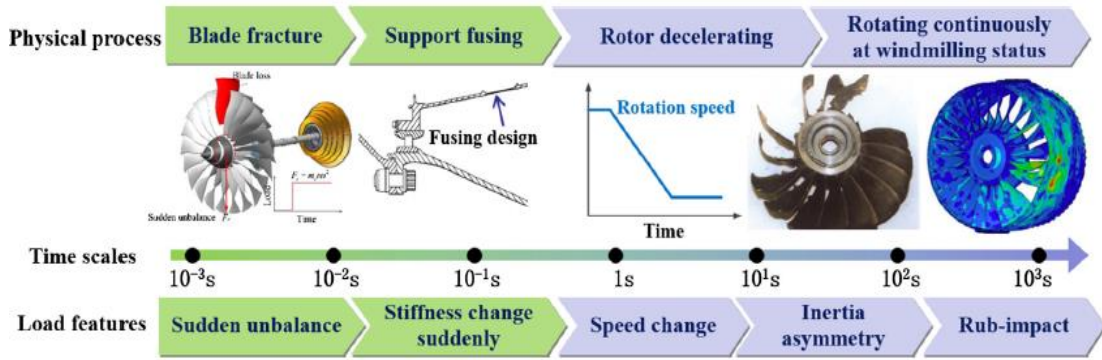


Figure 10: Physical Process of FBO Event at Different Time Scales [11]



Figure 11: Example Fan Blade-Out Event on Commercial Turbofan Engine during Flight [12]

1.4. Definition of Rotor Dynamics

Rotor dynamic name could be taken as a part of a rotating machine that is "rotor is a body suspended through a set of cylindrical hinges or bearings that allows it to rotate freely about an axis fixed in space" [15].

Rotor dynamic is a discipline of dealing with the dynamic behavior of turbomachinery. It could be defined as the discipline of engineering that is interested in lateral, torsional, and axial vibration of rotating systems. It aims to predict vibration levels and contain them under safe limits [13].

Rotor dynamics is where structural vibrations due to gyroscopic moments and combined forces are calculated. In rotating machinery, the power density source mostly is its rotor dynamic behavior.

The scope of the rotor dynamics can be expressed below,

- Estimating the natural frequency of the system and observing mode shapes of rotors at that detected frequencies.
- Noticing critical speeds close to or near to operating range of rotors.
- Estimating rotor displacements and extracting forces in the operating cycle [14].

1.4.1. Vibration Sources on Rotor

The vibration types seen in a rotor are as follows;

- Axial
- Torsional
- Lateral

Because of lateral vibration, on the normal rotation axis, displacements may occur. These movements combine to produce an orbital behavior of the rotor. For this reason, rotor lateral vibration often makes the highest effect in a turbomachinery design. [14].

Unbalance mass of the rotor is a well-known factor for vibration of a rotor. This is defined as the uneven distribution of rotor weight or the non-coincidence of the rotor center of mass with the axis of rotation. Therefore, unbalanced forces are seen. A very small amount of unbalance can cause serious problems at high speeds [14].

1.4.2. Single Mass Jeffcott Rotor

The behavior of simple rotor systems basically can be expressed using single mass Jeffcott Rotor calculations. Jeffcott rotor was analyzed in 1919. An example of Jeffcott rotor is given in Figure 12 [16].

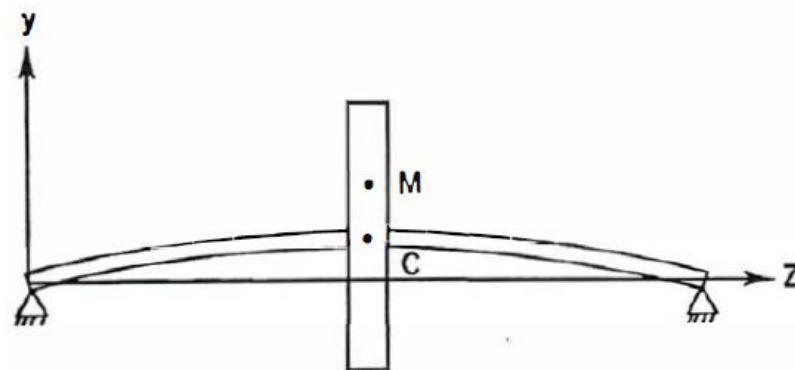


Figure-12: An Example of Jeffcott Rotor [16]

A Jeffcott Rotor system contains an elastic but massless shaft. In center of the shaft, a disc is located, which is a lumped mass. Bearings are located in beginning of the shaft and at the end of the shaft. Due to disc location, center of the shaft, there is no gyroscopic effect on 1st critical speed [16].

1.4.3 Motion Equations of Jeffcott Rotor

A system basically has a mass, a damper, and a spring. The governing equation of motion is defined as [17],

$$F = m\ddot{X} + c\dot{X} + kX \quad (1.1)$$

The equation of motion belongs to the single mass Jeffcott rotor, illustrated below for Cartesian coordinates X and Y to represent the motion of the rotor on center C, given in Figure 13 [16].

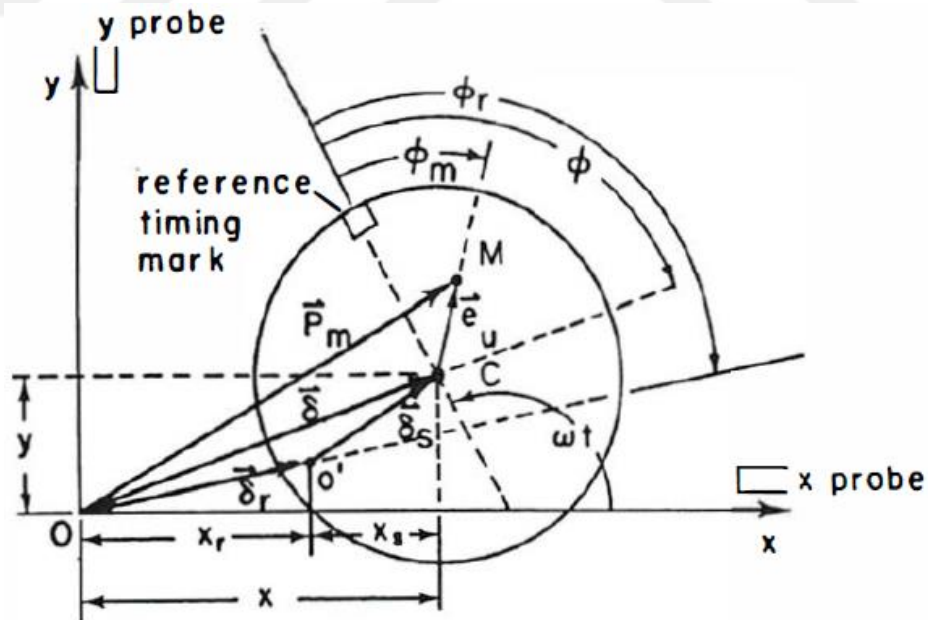


Figure 13: Reference Frame of Motion [16]

In a rotating element, the forcing function is related to rotor system mass, stiffness, and damping of supports. In the Jeffcott rotor, the mass of the system is the

center of the elastic shaft within an offset on centricity that creates an unbalance. Rotating unbalance force of a rotating shaft at speed defined below [16],

$$F_u = M x e_u x \omega^2 \quad (1.2)$$

Stiffness of the shaft defined below,

$$F_{shaft} = -K_s x \delta_s \quad (1.3)$$

Damping force of the shaft defined below,

$$F_{damping} = -C_s x V_c \quad (1.4)$$

In X and Y coordinates for center axis of shaft motion are given below

$$M \frac{d^2 X}{dt^2} + C_s \frac{dX}{dt} + K_s X = M e_u \omega^2 \cos(\omega t - \phi_m) \quad (1.5)$$

$$M \frac{d^2 Y}{dt^2} + C_s \frac{dY}{dt} + K_s Y = M e_u \omega^2 \cos(\omega t - \phi_m) \quad (1.6)$$

In X axis, system motion equation without damping of unbalance, are given below

$$F_{damping} = -C_s x V_c \quad (1.7)$$

In equation-7 ω_{cr} is critical speed of rotor system and equals to,

$$\omega_{cr} = \sqrt{\frac{K_s}{M_{modal}}} \quad (1.8)$$

Where M_{modal} in Equation 2.8 is rotor modal mass.

1.5. Purpose of the Thesis

Aerodynamic, performance and mechanical design inputs are constantly changing during the conceptual design phases. As a result, rotor dynamic design inputs, which are the load path of the entire rotating system, rotor and stator geometries, disc and blade mass and inertia, load path of bearing clearance, velocity envelope and velocity ratio, frame stiffness values, damping needs, and levels, change day to day. Therefore, it is important to determine the load sources and load levels on the rotor and frame structural parts when making rapid design changes, especially investigating the dynamic behavior such as a catastrophic event.

Computation technologies and, accordingly, structural analyzes support design activities more nowadays. Especially the finite element method is one of the most popular ones and can simulate any loading condition. On the other hand, these finite element models usually contain a larger number of nodes. That means higher degree of freedom and requires more time to solve the problems or relatively expensive and faster computational processors. On the other hand, to obtain adequate and fast results from turbofan dynamic models especially new design architecture phases mostly in earlier phases are crucial to decide where bearings, discs, or mountings are located. An example of design flow for turbomachinery design at the conceptual design phase is given in Figure 14.

Rapid solving models serve that, the structural frames are not going to be heavier, because conservative approaches are not followed during determining the thickness of frames at ultimate and catastrophic loading events. Rapid models can estimate loads more precisely even during conceptual design phases. In this thesis, a rapid solver for a single rotating element which is a model of low bypass turbofan engine drive shaft group is built by Transfer Matrix Method. In addition, in the thesis, the Transfer Matrix Method and Finite Element Method used to same numerical model to make a comparison of results. Finally, results are compared and examined for when FBO events on different modeling techniques. Transfer Matrix Model is built in MATLAB and Finite Element Model is built by ANSYS Mechanical.

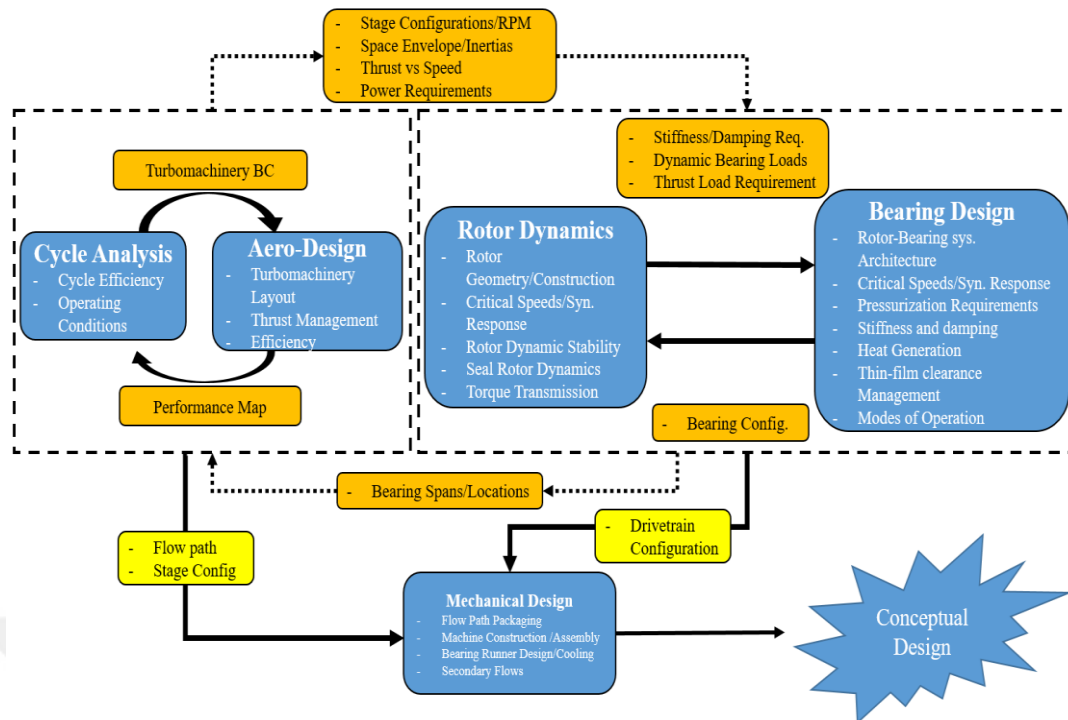


Figure 14: Design and Iteration Flow at Conceptual Design Phase for a Turbofan Engine [17]

2. LITERATURE REVIEW

When fan blade out studies are examined in the literature, numerical and analytical rotor dynamic models are encountered. The shaft group modeled in most of these studies has been simplified, provided that it remains in the perspective of rotor dynamics. As a common working method, dynamic modeling is done with the finite element method or transfer matrix method of the simplified shaft group model. At the same time, studies were found in which solutions were obtained by analytical and numerical methods. While there are studies in which shaft groups are modelled separately, there are also studies in which separate shafts groups are modeled. In this section, studies related to rotor dynamic and blade out modelling are examined.

As a first literature example [11], modeling of a dual rotor system in ANSYS mechanical, which is a finite element solution, can be given by simplified enough to meet rotor dynamic analysis expectations. As given Figure 15, a sketch of dual rotor system is simplified to a dynamic model. Shaft parts are converted to beam elements. Blade and disks are modelled by mass element, and supporting stiffness added to model by spring elements, given in Figure 16.

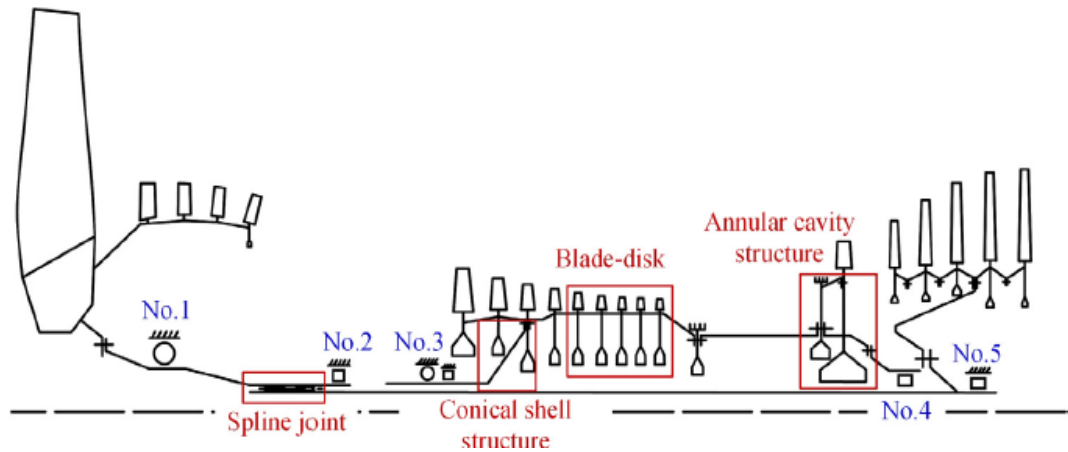


Figure 15: Schematic View of the Dual Rotor System [11]

When the study [11] and its results are examined, the sudden imbalance creates a shock effect in the rotor system. The most dangerous thing for a typical turbofan engine is that the No.1 bearing has the greatest reaction force during FBO. On the other hand, if schematic and simplified model are deeply examined, these are from detail design phase. There are very detail blades and shaft layout. It is important to

learn of the load level for FBO event to decide enough and well-required thickness of static structures.

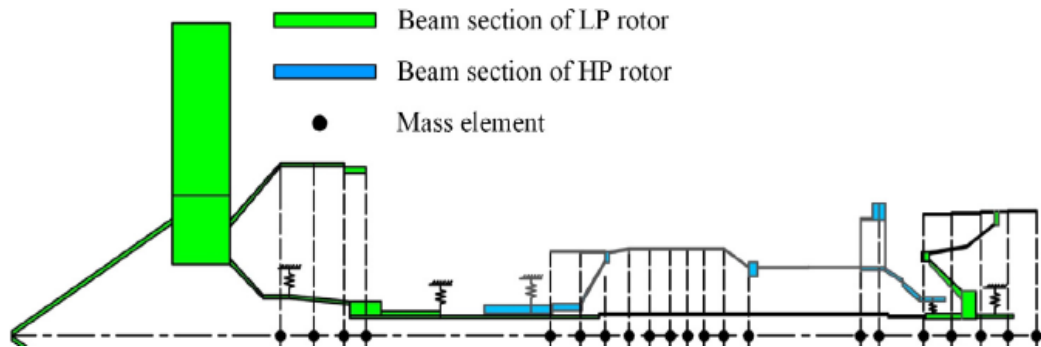


Figure 16: Simplified Diagram of the Dual Rotor System [11]

In another study [18] in which the dual rotor system was modeled, the rotor system was simplified and modeled in ANSYS Mechanical. As seen in this study, while the rotor model is simplified, the expectations of rotor dynamics have not deviated. Rotor geometry schema is given in Figure 17. Disc and shaft elements are modeled with flexible elements and suspended mass elements. Bearing connections are modeled with spring elements. The simplified model is given in Figure 18. Also in this study, transient responses of sudden unbalance changing are examined. The most important finding from this study is that a sudden unbalanced impact load worsens the dynamic response of the system. The peak value of the transient response is much larger than the amplitude of steady-state response.

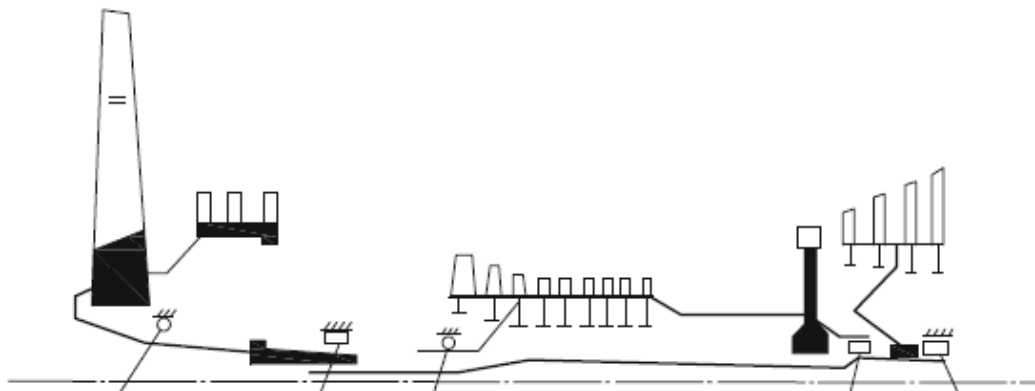


Figure 17: Schematic View of Rotor System [18]

Although the simplified model is suitable for fast enough solutions, when the bearing layout and design of the discs are examined in the study, an example of analysis made in the detail design phase is encountered. The important thing is to establish a model that can predict these loads in the concept design phase, and to direct the mechanical design at critical points such as the weight of the discs and bearing placement.

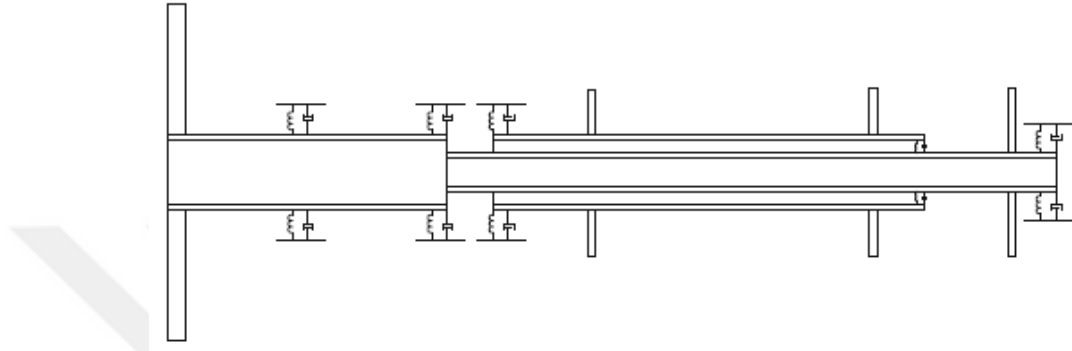


Figure 18: Simplified Model View of Rotor System [18]

When the results of the study [19] in which the FBO load was tried to be obtained with LS-DYNA explicit solver with advanced modeling details, are examined, it is seen that the FBO load has a serious effect on the engine. When the loads that the bearings are exposed to during the FBO event are examined, it is seen that the bearing carrier structures must be designed to carry this load. The graph of the reaction forces calculated in the study is given in Figure 19, below.

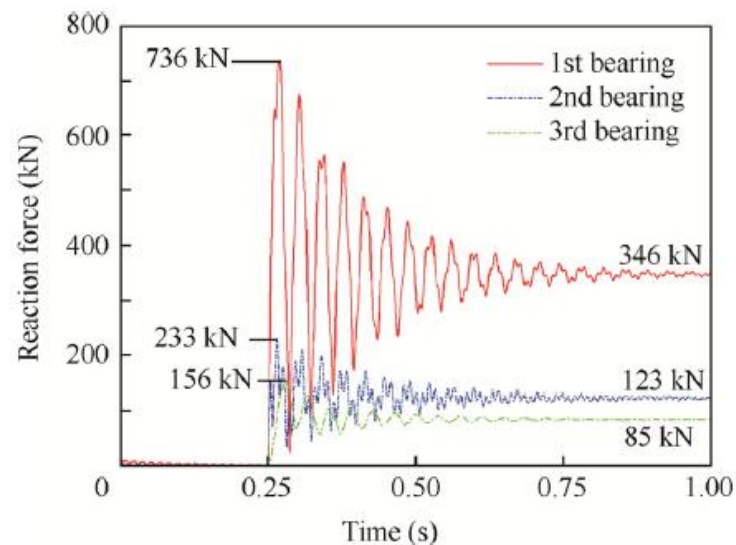


Figure 19: Reaction Forces Example during FBO Event [19]

When the whole studies are examined, it is seen that a detailed finite element model has been established with NASTRAN, ANSYS, or other related package programs. Alternatively, the detailed physical model is simplified from the perspective of rotor dynamics, and it is generally solved with the finite element method. The common feature of these softwares is that they are not specialized for rotor dynamics solutions only; they are developed for all finite element solutions. The disadvantage of this is that it is difficult to obtain specific solutions to the rotor dynamics problem. In addition, there are programs such as DYROBES and MADYN, which have been specially developed to obtain rotor dynamic solutions. However, the disadvantage of these programs is that they are not open source and are expensive.

Another study is AMrotor tool [20], which takes a solution with one-dimensional Timoshenko beam theory under the Finite Element Method. It works on MATLAB. Visualization of the rotor system are mentioned in the study is given in Figure 20. This tool calculates the Campbell diagram, mode shapes, displacement, and reaction force results of the rotor system it analyzes. The aim of the development of the tool is to understand the active magnetic bearing characteristics and verify it in the test setup, as mentioned in the study.

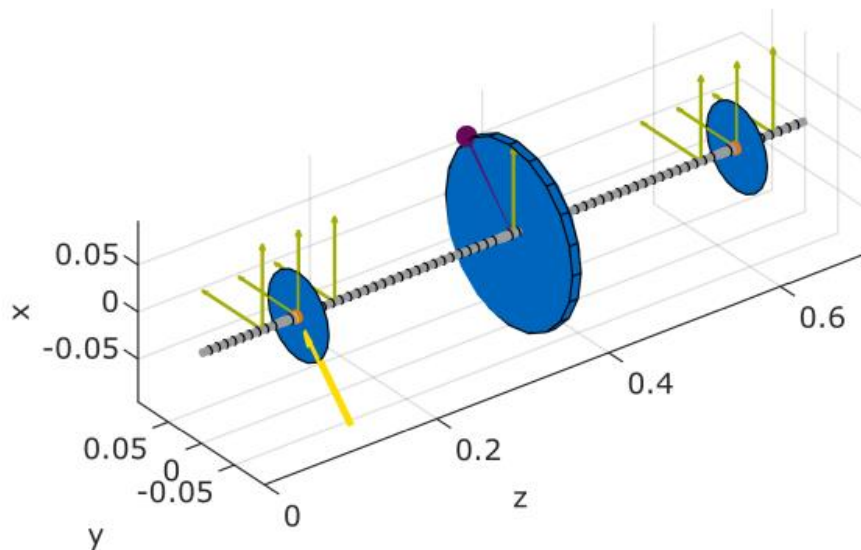


Figure 20: Visualization of the Rotor System [20]

When evaluated in this context, a model that takes faster solutions provides more flexible design capabilities and is easier to model mathematically is foreseen with the

Transfer Matrix Method compared to the Finite Element Method. In addition, Transfer Matrix Method is also described in the following section.

2.1. Transfer Matrix Method

In the early 1940s, vibration problems were solved with very simple models. After these years, with the development of computer technology, more complex vibration problems began to be solved with computers. One of the first methods that made the application of the theory easier by the computer was the transfer matrix method. The method of Transfer Matrix (TMM) used for torsion was called the Holzer method, and the method used for bending was called the Myklestad method. [21]

In the following years, with the increase in the computational power of computers, the use of the Finite Element Method (FEM) became a standard for rotor-bearing systems as well as for static and dynamic analysis [22]. The fact that Transfer Matrix Method has a coarser modeling power compared to Finite Element Method, where complex structures can be modeled in 3D or 2D, highlighted the use and development of the Finite Element Method. Rather than a stress analysis, the vibration problem should be based on simplicity because of its essence. For this reason, the creation of a simple 1D-beam model instead of a complex 3D model of a rotor vibration model is considered sufficient for the analysis of the rotor [23]. Because the more complex the model, the more problems we face. Singularity points and the length of time to get results in complex modeling with the Finite Element Method are some of these problems.

The Transfer Matrix Method, on the other hand, provides a solution to these problems by providing fewer operations and faster results, since the size of the matrix does not increase as the degree of freedom of the system increases in the simple modeling and calculation process. Ultimately, it can be said that Transfer Matrix Method is a fast and effective approach for the rotor dynamics problem. Due to these advantages, Transfer Matrix Method is widely used in the dynamic analysis of complex rotating machines. The fact that it allows parametric studies with its fast response clearly shows that this method is also suitable for optimization problems. [24]

3. NUMERICAL MODEL

Modern turbofan engines have mostly 2-spool configurations. That can be named as usually High-Pressure Shaft (HP- Shaft) and Low-Pressure Shaft (LP-Shaft). A basic cross-section is given in Figure 21. In this thesis, a Low-Pressure Shaft belonging to a Low-bypass turbofan engine is modeled. Typical Low-Pressure shafts have two or three fan discs and have usually bearings by two or three flexible supports. In this model, there are three flexible supports and two fan discs.

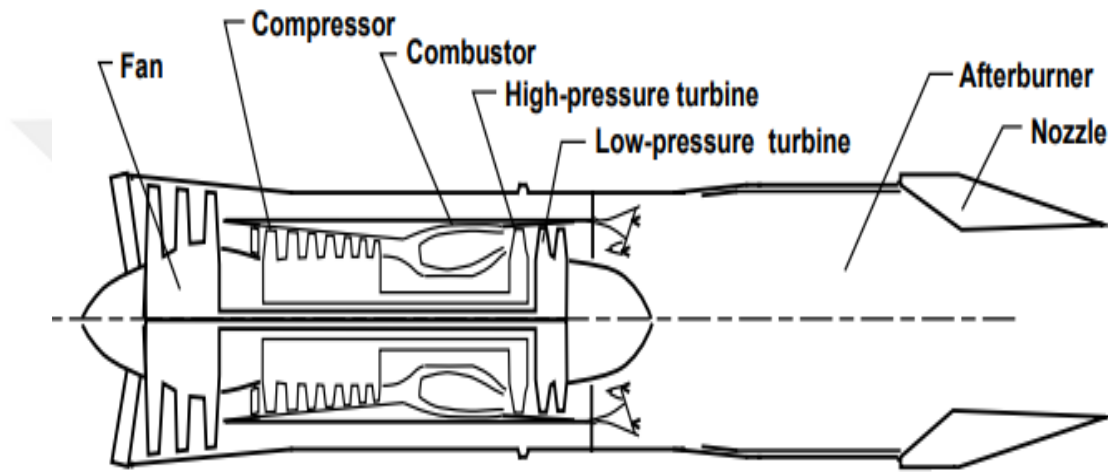


Figure 21: Cross-section of F110-GE-129 Engine [25]

3.1. Numerical Model Details

In this study, representative of a typical Low Pressure Shaft system that contains a hollow shaft in different inner and outer radiuses, overhung discs of different sizes and masses and, three flexible supports. The rotor group is simplified, covering physical parameters, which are mass, the center of gravity, and the moment of inertia.

Disc-1 and **Disc-2** are representative of fan stages and **Disc-3** mimics to Low-Pressure Turbine Three flexible supports have stiffness and damping values.

In addition, the schematic view shows details in Figure 22 and further parameters belong to Low-Pressure Shaft system for a Turbofan Engine given in Table 1. In the Chapter on Transfer Matrix Model Details, a parametric model is going to be built as defined parameters.

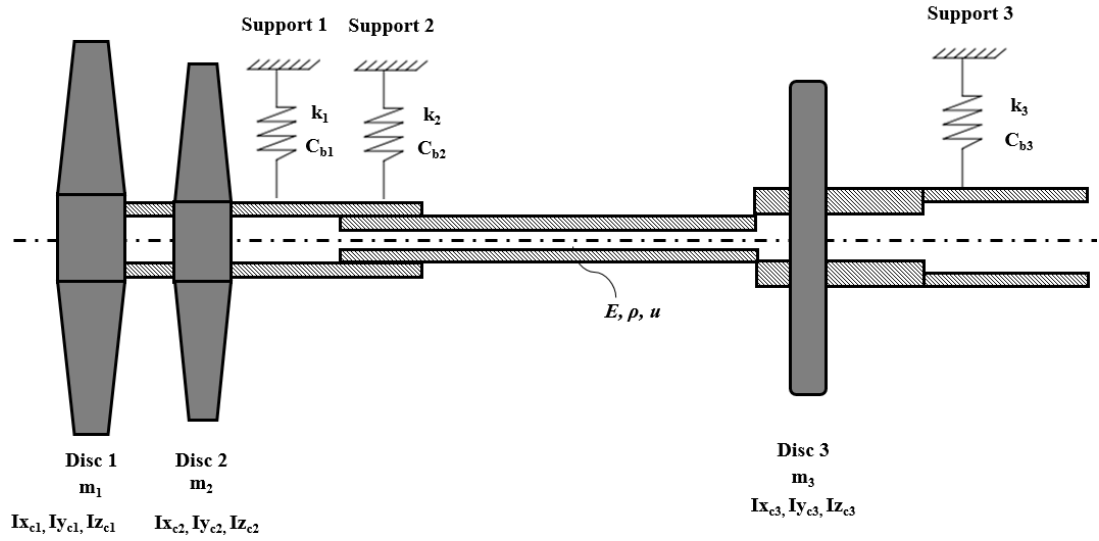


Figure 22: Schematic View of Low Pressure Shaft

Mass of the discs, stiffness, damping of bearings, main length properties, and mechanical properties of the material are listed in Table 1, to present a generic Low-Pressure Shaft System, used in this study.

Table 1: Main Parameters of LP Shaft System

m_1	Mass of the disc-1	10 kg
m_2	Mass of the disc-2	8 kg
m_3	Mass of the disc-3	30 kg
E	Young Modulus	2×10^5 MPa
ρ	Density of Hollow Shaft	7850 kg/m^3
u	Poisson Ratio	0.3
L_1	Length of the Shaft	~ 1500 mm
K_1	Stiffness of the Support-1	100,000 N/mm
K_2	Stiffness of the Support-2	100,000 N/mm
K_3	Stiffness of the Support-3	50,000 N/mm
C_1	Damping Ratio of the Support-1	0.2 to 2 Ns/mm
C_2	Damping Ratio of the Support-2	0.2 to 2 Ns/mm
C_3	Damping Ratio of the Support-3	0.2 to 2 Ns/mm

3.2. Transfer Matrix Model Details

The low-Pressure Shaft is divided into 10 pieces of elements in following the geometry. As given in schema in Figure 15, there is four main inner radius changing along the shaft length. Therefore at L_1 , L_2 , L_3 , at L_4 , L_5 , at L_6 , at L_7 , L_8 , and at L_9 , L_{10} same inner radiuses are applied to Transfer Matrix Model, respectively.

Two fan discs are combined into one mass covering to fan discs' mass, Center of Gravity, and Inertia values. As a result of superposing, m_1 is representing to whole fan disc in one element at the end of the L_1 element. Low-Pressure Turbine is located at end of L_7 .

There is three bearing configuration in this layout. So three spring elements are used for bearing modeling at end of L_2 , at end of and end of L_9 . Stiffness and damping values are added to the model. The shaft is modeled as an elastic beam element with steel material.

Element numbering, bearing and mass locations are illustrated in Figure 23.

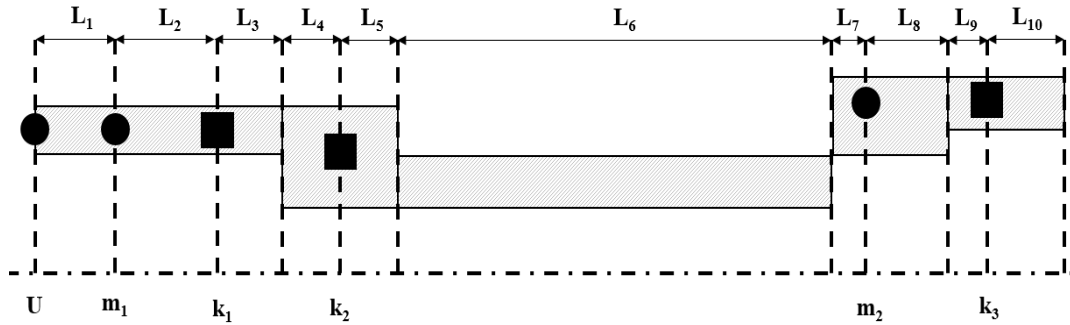


Figure 23: Transfer Matrix Model of LP Shaft

Detailed parameters of the Transfer Matrix Model belong to the illustration given in Figure 23, and are listed in detail in Table 2.

Table 2: Detail Parameters of LP Shaft System Transfer Matrix Model

m_1	Mass	18 kg
m_2	Mass	30 kg
$I_{d1} - I_x$	Diametral Moment of inertia –Mass-1	0.5 kgm ²
$I_{d1} - I_y$	Diametral Moment of inertia –Mass-1	0.5 kgm ²
$I_{p1} - I_z$	Polar Moment of inertia – Mass-1	1.0 kgm ²

$I_{d2} - I_x$	Diametral Moment of inertia –Mass-1	0.6 kgm ²
$I_{d2} - I_y$	Diametral Moment of inertia –Mass-1	0.6 kgm ²
$I_{p2} - I_z$	Polar Moment of inertia – Mass-1	1.2 kgm ²
L_1	Length of the Element	30.0 mm
R_{1i}	Inner Radius of Element	50.0 mm
R_{1o}	Outer Radius of Element	55.0 mm
L_2	Length of the Element	50.0 mm
R_{2i}	Inner Radius of Element	50.0 mm
R_{2o}	Outer Radius of Element	55.0 mm
L_3	Length of the Element	150.0 mm
R_{3i}	Inner Radius of Element	50.0 mm
R_{3o}	Outer Radius of Element	55.0 mm
L_4	Length of the Element	75.0 mm
R_{4i}	Inner Radius of Element	25.0 mm
R_{4o}	Outer Radius of Element	55.0 mm
L_5	Length of the Element	40.0 mm
R_{5i}	Inner Radius of Element	25.0 mm
R_{5o}	Outer Radius of Element	55.0 mm
L_6	Length of the Element	1000.0 mm
R_{6i}	Inner Radius of Element	25.0 mm
R_{6o}	Outer Radius of Element	45.0 mm
L_7	Length of the Element	20.0 mm
R_{7i}	Inner Radius of Element	50.0 mm
R_{7o}	Outer Radius of Element	70.0 mm
L_8	Length of the Element	50.0 mm
R_{8i}	Inner Radius of Element	50.0 mm
R_{8o}	Outer Radius of Element	70.0 mm
L_9	Length of the Element	75.0 mm
R_{9i}	Inner Radius of Element	52.5 mm
R_{9o}	Outer Radius of Element	70.0 mm
L_{10}	Length of the Element	25.0 mm
R_{10i}	Inner Radius of Element	52.5 mm
R_{10o}	Outer Radius of Element	70.0 mm

K_1	Stiffness of the Support-1	100,000 N/mm
K_2	Stiffness of the Support-2	100,000 N/mm
K_3	Stiffness of the Support-3	50,000 N/mm
E	Young Modulus	2×10^5 MPa
ρ	Density of Hollow Shaft	7850 kg/m ³
u	Poisson Ratio	0.3
C_1	Damping Ratio of the Support-1	0.2 to 2 Ns/mm
C_2	Damping Ratio of the Support-2	0.2 to 2 Ns/mm
C_3	Damping Ratio of the Support-3	0.2 to 2 Ns/mm
U	Unbalance due to FBO Event	765 gr-mm

3.3. Building Transfer Matrix Model

The transfer matrix method is a fast and effective approach for rotor dynamic and loads calculations. Because, in the transfer matrix calculation process, the size of the matrix does not increase as the degree of freedom of the system increases. However, it is suitable for programming, as it requires a small storage unit. Therefore, the transfer matrix method is widely used in the dynamic analysis of complex rotary machines. In this section, the each main components of the Transfer Matrix Method are mentioned [26].

3.3.1. State Vector

In the system i. (the state vector) representing the point tells the mechanical state of that point. This mechanical state consists of displacement and internal forces called state variables. The same letter can be used for the physical coordinate and the modal coordinate to represent the state vectors and can be written in upper or lower case depending on where it will be used. Z represents the physical coordinate while z represents the modal coordinate. Of the sub-index given as i, j, the first one represents the rigid element and the second one represents the elastic element [26].

- **Axial System**

In this system, which is given as a lumped mass connected to the spring, the first and sixth elements can be assumed as the root and end of the system, given in Figure 24 respectively. For the state vector at any P_{ij} point, [26]

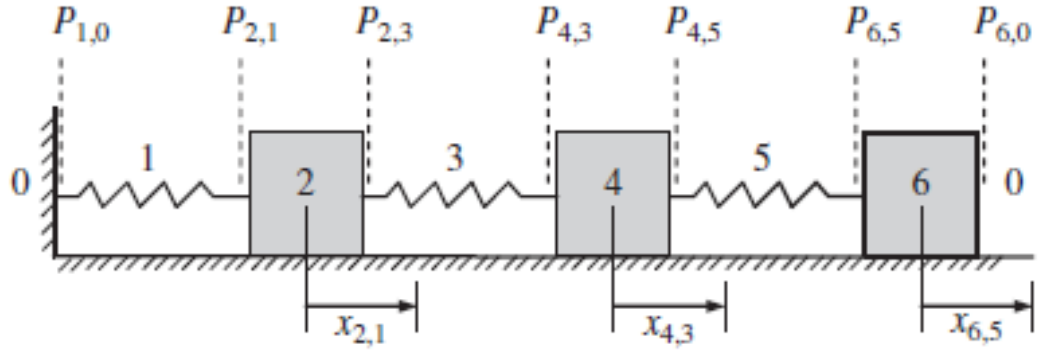


Figure 24: Spring-Mass Chain System [26]

In Equation 3.1, X represents the coordinates, q represents the internal force and T represents the transpose [26].

$$[Z_{i,j}] = [X, q_x]_{i,j}^T \quad (3.1)$$

- **Radial System**

In this system, a torsion vibration system covers an elastic shaft and disks. An example of a torsion vibration system is given in Figure 25. In Equation 3.2, the position is expressed as θ , which is the angular expression, and the moment of the system is expressed as m [26].

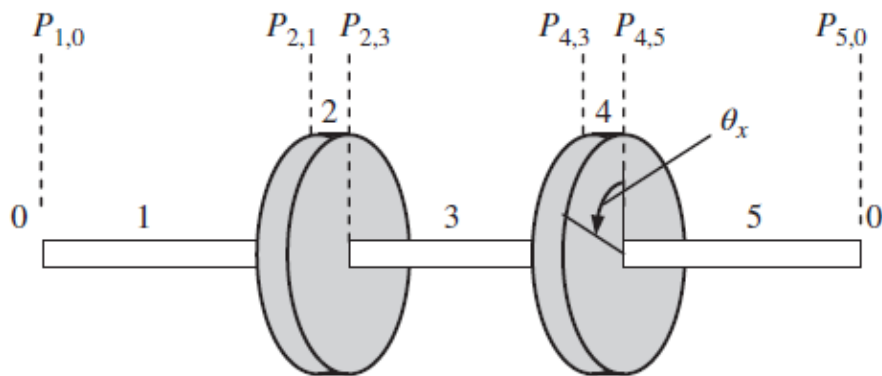


Figure 25: Torsion Vibration System of Shaft Disks [26]

$$[Z_{i,j}] = [\theta_x, m_x]_{i,j}^T \quad (3.2)$$

- **Step Beam**

State Vector of step beams transversely vibrating in the plane, an example given in Figure 26, defined in Equation 3.3. For this system, both radial and axial positions and corresponding internal forces are involved. [26]

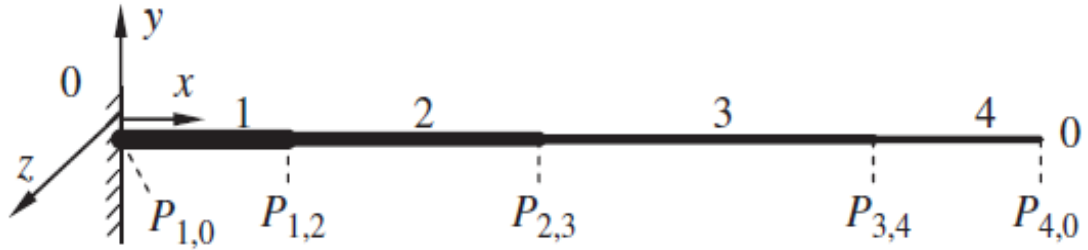


Figure 26: Step Beam System [26]

$$[Z_{i,j}] = [y, \theta_z, m_z, q_y]_{i,j}^T \quad (3.3)$$

- **Rigid Body**

Rigid body for a spatial small vibration labelled i with one input end and one output end is given in Figure 27. State Vectors for input and output ends are defined in Equation 3.4. [26].

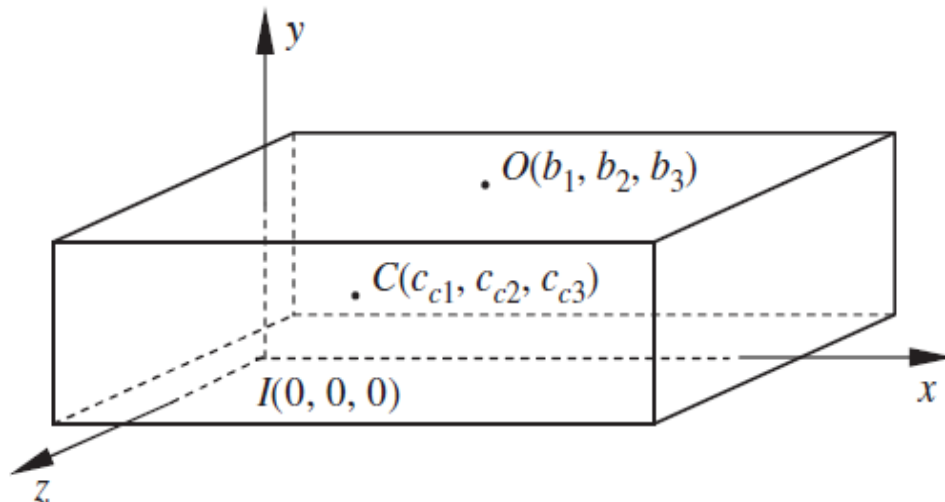


Figure 27: Rigid Body [26]

$$\begin{aligned} [Z_{I,i}] &= [x, y, z, \theta_x, \theta_y, \theta_z, m_x, m_y, m_z, q_x, q_y, q_z]_{I,i}^T \\ [Z_{O,i}] &= [x, y, z, \theta_x, \theta_y, \theta_z, m_x, m_y, m_z, q_x, q_y, q_z]_{O,i}^T \end{aligned} \quad (3.4)$$

3.3.2. State Vector Under Modal Coordinates

The undamped free vibration of the system can be obtained by superposition of the principal modes. The state vector of the system corresponding to the eigenvalue ω_k under the k th modal is written as in Equation 3.5. [26].

$$\begin{aligned} z &= Z^k e^{i\omega_k t} \\ z &= Z e^{i\omega t} \end{aligned} \quad (3.5)$$

In this equation, I is the imaginary unit, and Z^k represents for state vector under k th modal coordinates, whose variables are displacements and internal forces for eigenfrequency ω_k [26].

3.3.3. Transfer Equation and Transfer Matrix

In this section, the transfer matrices of different elements are discussed. In general, the principle is similar to the establishment of the equations of motion static relation expressions and the creation of the transfer matrix over them. Several transfer matrixes for each kind of elements can be defined but in this study only required elements that are explained [26].

- Transfer Matrix of Longitudinal Vibration Lumped Mass
- Transfer Matrix of a Disk
- Transfer Matrix of a Longitudinal Spring Element
- Transfer Matrix of a Planar Transverse Vibration Massless Elastic Beam

- **Transfer Matrix of Longitudinal Vibration Lumped Mass**

In this system, lumped mass is represented by m_i in longitudinally, an example given in Figure 28. Mass element are assumed as rigid and displacements in the system are equal at both sides, left and right according to Newton's second rule. Explained in Equation 3.6. [26].

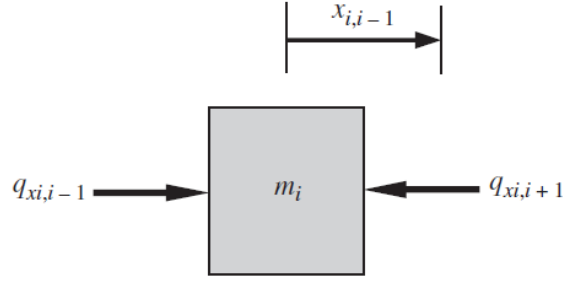


Figure 28: Lumped Mass [26]

$$\begin{cases} X_{i,i+1} = X_{i,i-1} \\ q_{xi,i+1} = -m_i \ddot{X}_{i,i-1} + q_{xi,i-1} \end{cases} \quad (3.6)$$

Compacting matrix form can be expressed in Equation 3.7. [26].

$$\begin{bmatrix} X \\ Q_x \end{bmatrix}_{i,i+1} = \begin{bmatrix} 1 & 0 \\ m_i \omega^2 & 1 \end{bmatrix} \begin{bmatrix} X \\ Q_x \end{bmatrix}_{i,i-1} \quad (3.7)$$

Transfer matrix of the system covers lumped mass can be defined in Equation 3.8. [26].

$$U_i = \begin{bmatrix} 1 & 0 \\ m_i \omega^2 & 1 \end{bmatrix} \quad (3.8)$$

- **Transfer Matrix of a Disk**

In this system, rigid disks, an example given in Figure 29, within its rotational inertia representing by J_i , assumed twist angles of both ends are equal. This equilibrium explained in Equation 3.9. [26].

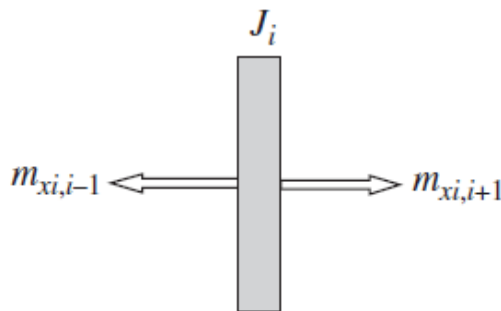


Figure 29: Rigid Disk [26]

$$\theta_{xi,i+1} = \theta_{xi,i-1} \quad (3.9)$$

Rotation expression is added and explained in Equation 3.10. [26].

$$m_{xi,i+1} = m_{xi,i-1} + j_i \ddot{\theta}_{x,i-1} \quad (3.10)$$

Transfer matrix of the system covers rigid disk can be defined in Equation 3.11. [26].

$$U_i = \begin{bmatrix} 1 & 0 \\ -\omega^2 J_i & 1 \end{bmatrix} \quad (3.11)$$

- **Transfer Matrix of a Longitudinal Spring Element**

In this system, a spring-mass element is used. An example given in Figure 30. It is assumed that, forces at both directions are equal. There is a stiffness of spring system and representing K_i [26].

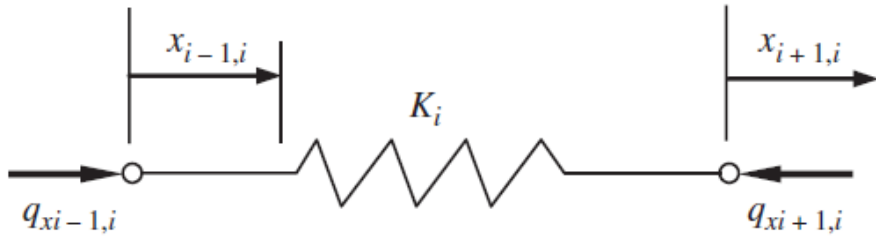


Figure 30: Spring Element [26]

Equilibrium of forces on both end for spring element and stiffness properties can be expressed in Equation 3.12. [26]

$$\begin{cases} q_{xi+1,i} = q_{xi-1,i} \\ -q_{xi+1,i} - K_i(x_{i+1,i} - x_{i-1,i}) = 0 \end{cases} \quad (3.12)$$

Compacting matrix form can be expressed in Equation 3.13. [26]

$$\begin{bmatrix} X \\ Q_x \end{bmatrix}_{i+1,i} = \begin{bmatrix} 1 & -\frac{1}{K_i} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Q_x \end{bmatrix}_{i-1,i} \quad (3.13)$$

Transfer matrix of the system covers spring element can be defined in Equation 3.14. [26].

$$U_i = \begin{bmatrix} 1 & -\frac{1}{K_i} \\ 0 & 1 \end{bmatrix} \quad (3.14)$$

- **Transfer Matrix of Elastic Beam Element**

In this system, massless elastic beam element covering length and bending stiffness representing by l_i and EI_i . An example of system given in Figure 31. Assumptions and equilibrium for displacement and forces are given in Equation 3.15. [26].

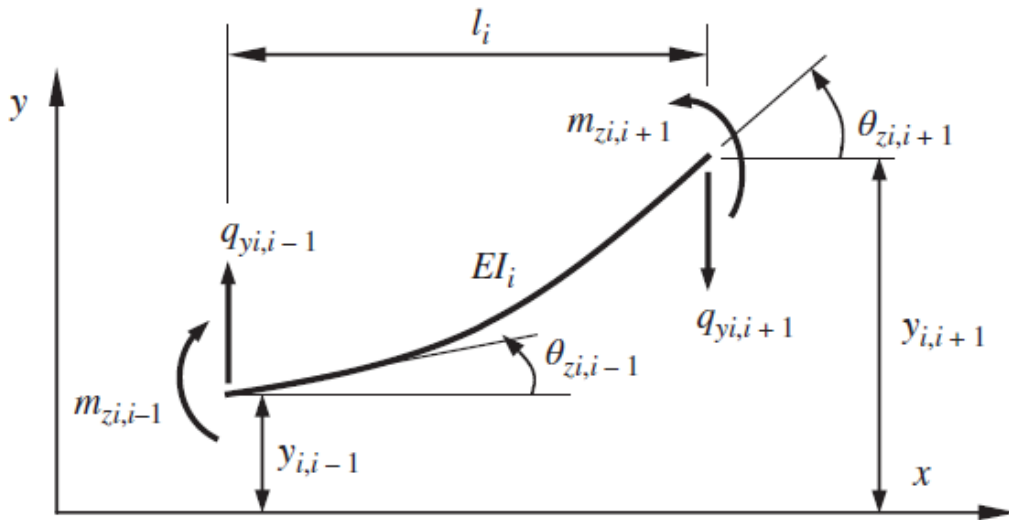


Figure 31: Elastic Beam Element [26]

$$\begin{cases} q_{yi,i-1} - q_{yi,i+1} = 0 \\ -m_{zi,i-1} + m_{zi,i+1} - q_{yi,i+1} \times l_i = 0 \end{cases} \quad (3.15)$$

Displacement and slope of output can be defined as equation 3.16, according to elementary theory of a beam. EI represents flexural rigidity and l represents length [26].

$$\begin{aligned}
y &= \frac{m_z l^2}{2EI} - \frac{q_y l^3}{3EI} \\
\theta_z &= \frac{m_z l}{EI} - \frac{q_y l^2}{2EI}
\end{aligned}
\tag{3.16}$$

Transfer matrix of the system covers elastic beam can be defined in Equation 3.17. [26].

$$U_i = \begin{bmatrix} 1 & l_i & \frac{l_i^2}{2EI_i} & \frac{l_i^3}{6EI_i} \\ 0 & 1 & \frac{l_i}{EI_i} & \frac{l_i^2}{2EI_i} \\ 0 & 0 & 1 & l_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\tag{3.17}$$

3.3.4. Overall Transfer Equation and Transfer Matrix

For a system includes n elements, transfer equations can be obtained as shown in Equation 3.18. [26].

$$\begin{cases} Z_{2,1} = U_1 Z_{1,0} \\ Z_{2,3} = U_2 Z_{2,1} \\ \vdots \\ \vdots \\ Z_{n,0} = U_n Z_{n-1,n} \end{cases}
\tag{3.18}$$

Overall transfer equation can be expressed in equation 3.19. [26].

$$Z_{n,0} = U_{1-n} Z_{1,0}
\tag{3.19}$$

To obtain overall transfer matrix of system, transfer matrixes of each element is to be multiplied, given in equation 3.20. [26].

$$U_{1-n} = U_n U_{n-1} \dots U_{i+1} U_i U_{i-1} \dots U_3 U_2 U_1
\tag{3.20}$$

3.4. Workflow

The main idea in the workflow used in this thesis is to use transfer matrices from the library and easily build complex models based on the boundary and dynamic conditions. Firstly, state vectors are defined one by one according to the element properties modeling approach. After obtaining every state vector and transfer equations, the overall transfer matrix, represented by U_{all} , is written. Adding boundary conditions to the system equation, due to model behavior, The Eigen equation can be obtained. Finally solving this equation gives roots that are the results are desired. An example workflow is given in Figure 32 [26].

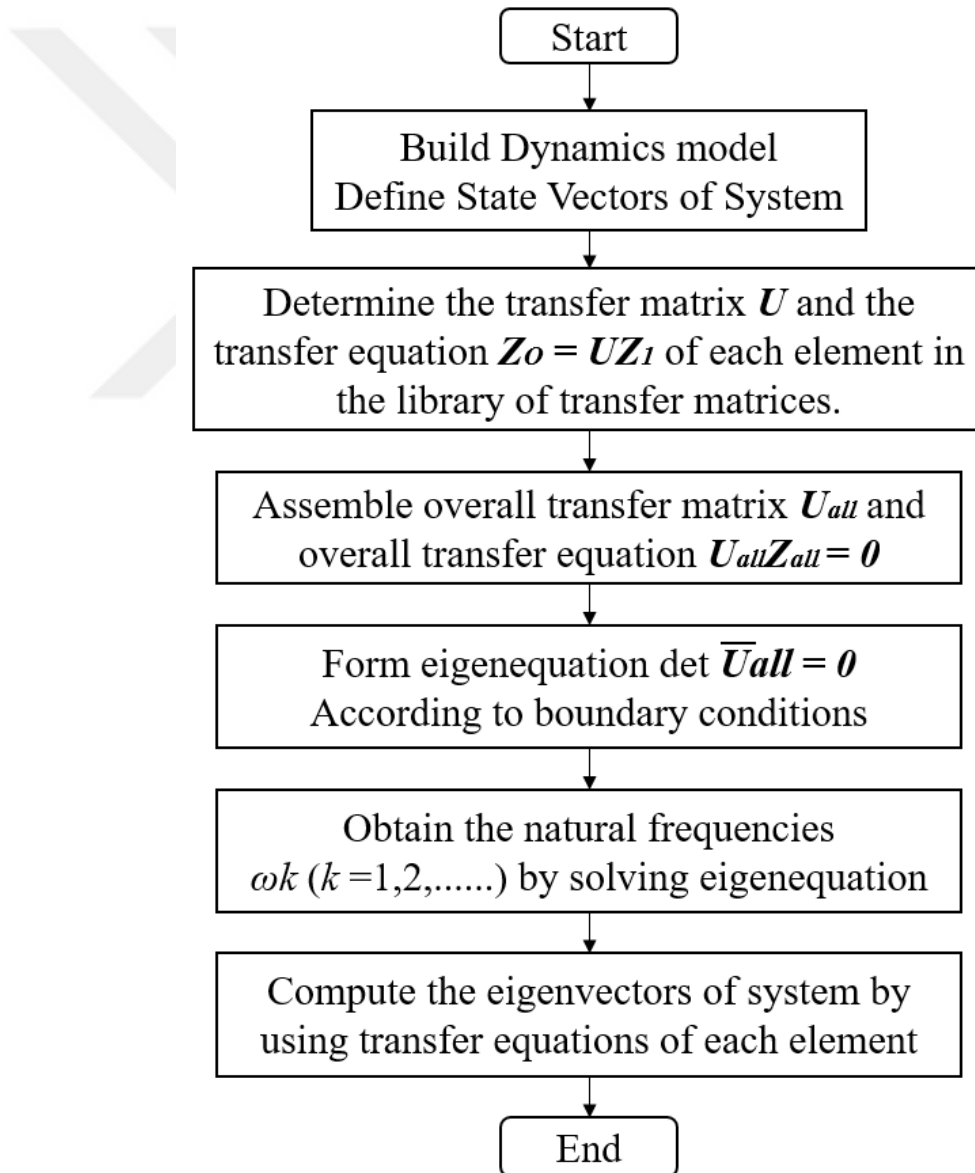


Figure 32: An Example Workflow of Transfer Matrix Method [26]

3.5. Unbalance Load Calculation

Before FBO event thanks to well-balanced fan stages in the manufacturing process and due to aviation assembly procedures, eccentricity can be neglected. During the event, if a whole fan blade is assumed to be flown out to the radial direction of the engine, a sudden unbalanced load occurs. An example illustration for FBO event is given in Figure 33 [27].

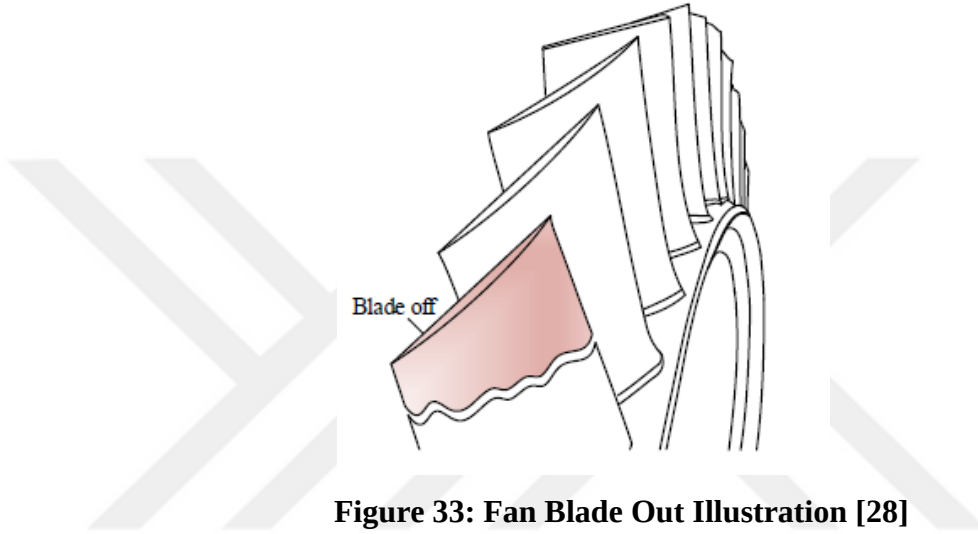


Figure 33: Fan Blade Out Illustration [28]

This load of magnitude can be calculated as below, given in Equation 3.21. [27].

$$\begin{cases} \{0 \ 0 \ 0 \ 0\}^T t < t_i \\ m_i e \omega^2 \{\cos \omega t \ \sin \omega t \ 0 \ 0\}^T t \geq t_i \end{cases} \quad (3.21)$$

Imbalance weight can be found as given in Equation 3.22. [27].

$$W_u = N_{bo} \times P_{of} \times \left(\frac{M_{fb}}{N_{fb}} \right) \quad (3.22)$$

The eccentricity value that causes unbalance loads to the center of the disc can be found in Equation 3.23. [27].

$$e = \frac{\left(\frac{D_f}{2} + L_f \times (CM_{pro} - 1) \times W_u \right)}{D_w + W_u} \quad (3.23)$$

Detail parameters are for calculating unbalance load given in Table-3.

Table 3: Detail Parameters for Unbalance Loading Calculation

<i>Nbo</i>	Number of Blade Out	1 blade
<i>Pof</i>	Proportion of Weight of an Fan Blade Airfoil	%100
<i>Dw</i>	Total Disc Weight	10 kg
<i>Mfb</i>	Mass of Total Fan Blades	4 kg
<i>Nfb</i>	Number of Fan Blades	24 blades
<i>Df</i>	Diameter of Fan	800 mm
<i>Lf</i>	Length of a Fan Blade	200 mm
<i>CMpro</i>	Percentage of the length of where blade CoG is located	%40
<i>e</i>	Eccentricity of CM	4.59 mm
<i>U</i>	Unbalance Load	765 gr-mm

In this study, it is assumed that a fan blade has broken off. According to this acceptance, unbalanced forces are calculated. At the end of the calculation, 765 gr-mm unbalance value is added to the model to obtain an unbalance due to FBO event.

3.6. Validation using Finite Element Model

In this study, the numerical model of the Low-Pressure Shaft system was built by Transfer Matrix Method, also built by the Finite Element Model to compare results and validate the model. On ANSYS 2021R1, a beam and nodal mass-based model is established. Like same as Transfer Matrix Model, geometrical and mechanical properties are kept the same. The cross-section of beams is covered by the inner and outer radius of tubular elements that belong to the Transfer Matrix Model.

ANSYS Beam189 element is used for beam-type elements. Beam 189 is based on Timoshenko Theory that includes shear-deformation effects. Suitable for thick beam elements [29].

Bearing connections are modeled by the Combi214 element. That is specialized for bearing on ANSYS Mechanical. Stiffness and damping values of bearings are added to elements [30].

In the Finite Element Method, it is known that the result changes as the mesh size and solution interval change. Therefore, a sensitivity study was carried out to determine the mesh size and the solution interval value. Mesh values ranging from 50mm to 0.1 mm were scanned. When the results were examined in Figure 34, there is a slight change in modal results. At the mesh size of 0.1 mm, the most accurate results are seen. Besides, solution interval sensitivity for 0.1 mm mesh size is given in Figure 35. According to the results, 1000 steps of solution is enough for getting correct displacement results.

It was seen that the 1000 solution range and 0.1mm mesh size was sufficient for validation work.

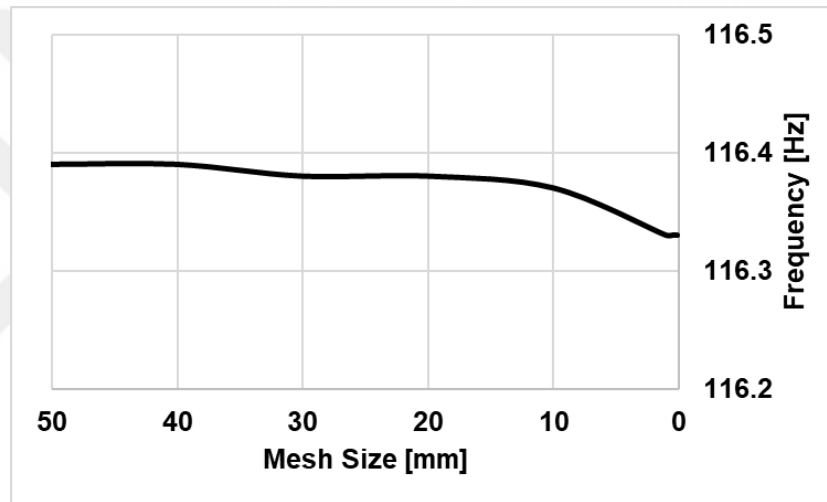


Figure 34: Mesh Size – 1st Mode Frequency Sensitivity

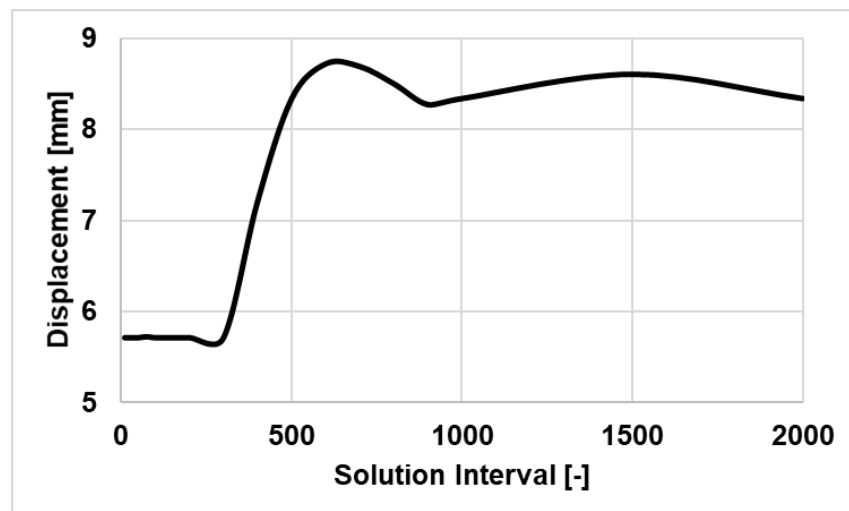


Figure 35: Solution Interval - Displacement Sensitivity

As a result of sensitivity study on mesh size, quadratic mesh is applied to the model. Generally, a 0.1 mm mesh size is selected to capture accurate results. Therefore, there are 45,451 nodes and 15,150 elements in the FE Model. Thick beam view of FE model on ANSYS Mechanical Workbench given in Figure 36.

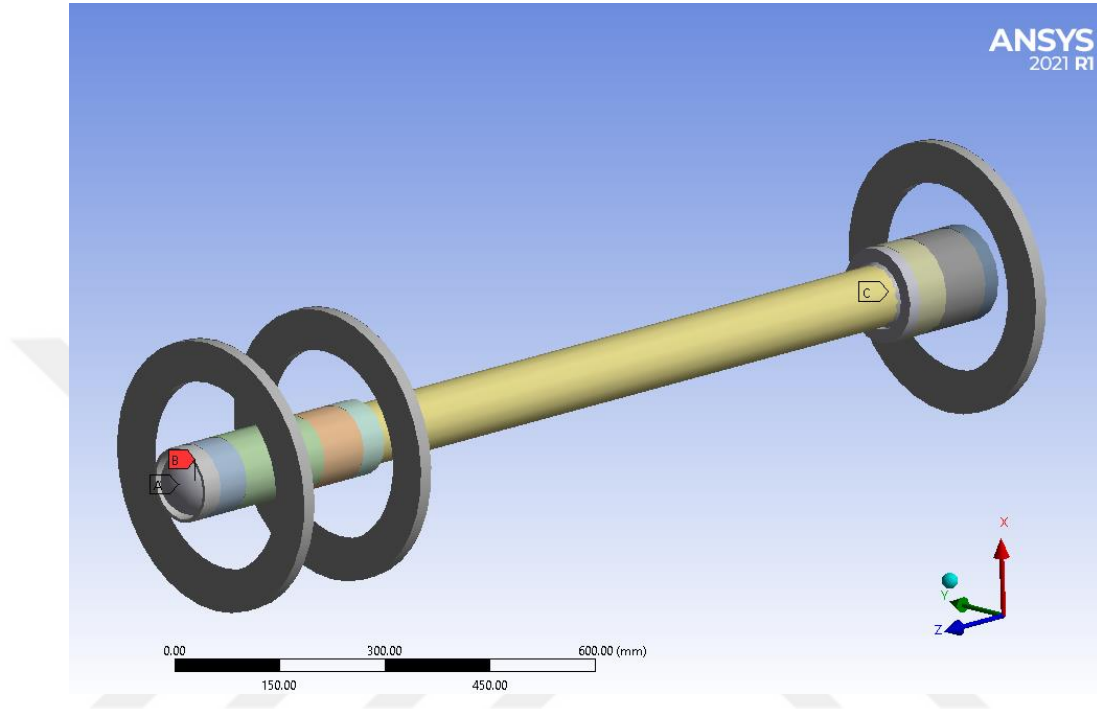


Figure 36: FE Model View

Firstly, undamped modal analysis is conducted on the finite element model. In order to build a Campbell diagram, different rotational velocities are applied. From 0 rpm to 15000 rpm individually reacted to the model to see gyroscopic effects on results. On 0 rpm result, eigenvalue Results comparison given in Table 4.

Table 4: Eigenvalue Result Comparison

#	Transfer Matrix Model	Finite Element Model	Error %
1st Mode	116.90	116.33	0.49
2nd Mode	221.90	221.05	0.38
3rd Mode	274.73	273.17	0.57
4th Mode	390.10	387.67	0.62

Below in Figure 37, the Campbell diagram shows a comparison between Transfer Matrix Model results by MATLAB and Finite Element Model Results by ANSYS. Forward and Backward modes of the system for every solution method is printed on the diagram. Black lines represent the Transfer Matrix Model and Blue dots are for results of the Finite Element Model.

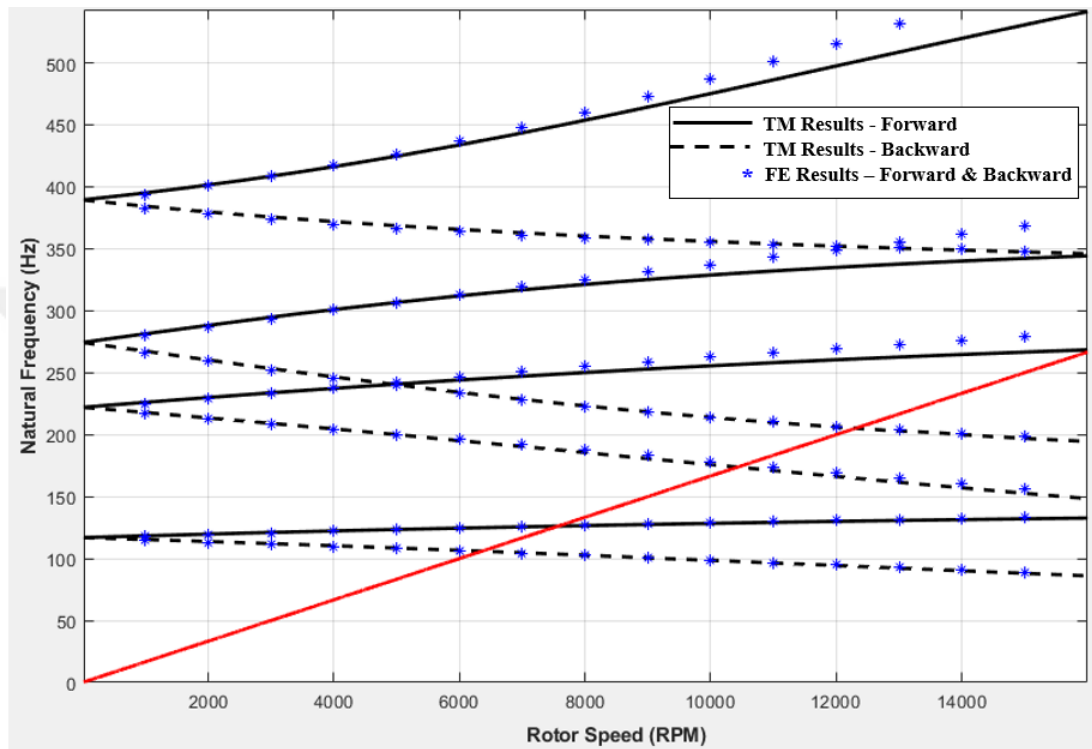


Figure 37: Campbell Diagram for Result Comparison

On the Campbell diagram, when the results are examined, it is seen that consistent results are obtained at natural frequencies and critical speeds for the first four natural frequencies. In this study, gyroscopic effects are added to the numerical model in the Transfer Matrix Method. Therefore, from 0 rpm to 14,000 rpm there is changing along with shaft speed on the lines, which show forward modes and backward modes.

The vibrational modes of a structure are the shapes that the structure vibrates when excited. To estimate reactions and displacements correctly at critical speeds, mode shapes have to be validated. Four bending modes are seen, when the comparison views are seen in Figure 38, Figure 39, Figure 40, and Figure 41. Bending modes also known as flexure modes is the typical description of dynamic behavior. Mode shape results are given for both Transfer Matrix and Finite Element Methods in the comparison view printed. According to comparison views, in Figure 35 and Figure

36, these are first bending mode shapes. As known on a numerical model of LP-Shaft, bearing stiffness values are also given in Table 2, there is no soft bearing support, which can be expressed like below 20,000 – 30,000 N/mm stiffness in the rotor system. Because of that, it is not observed rigid mode shapes. In Figures 37 and 38, It is seen that these are second bending mode shapes.

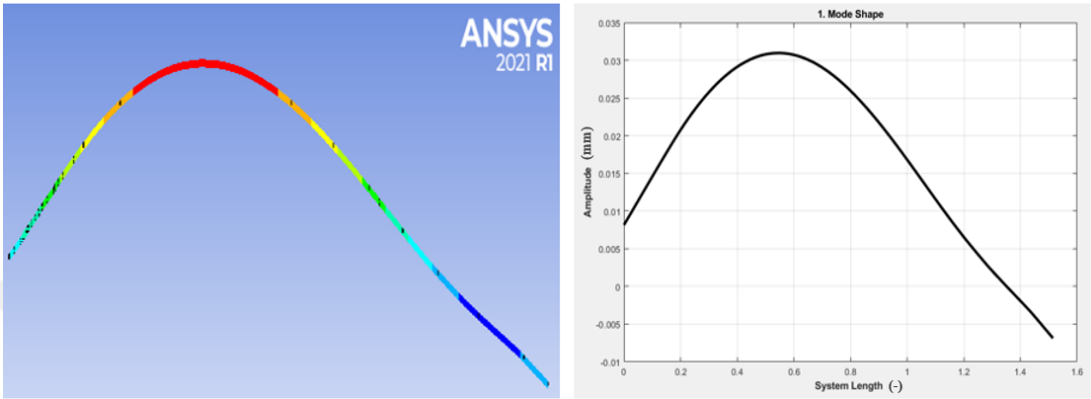


Figure 38: Mode Shape-01 Comparison

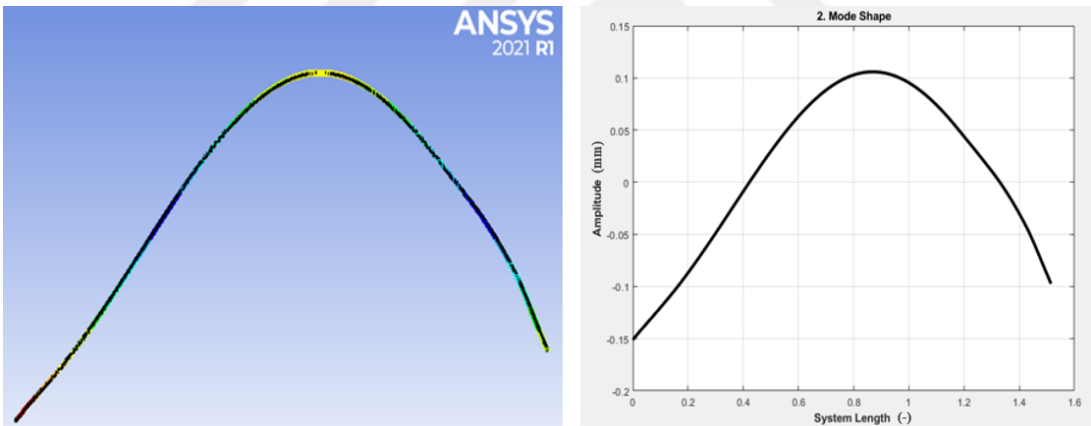


Figure 39: Mode Shape-02 Comparison

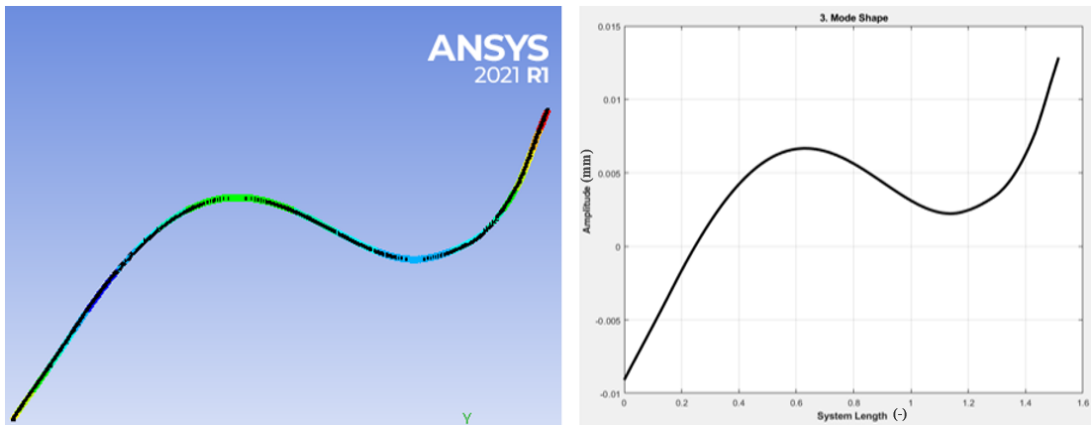


Figure 40: Mode Shape-03 Comparison

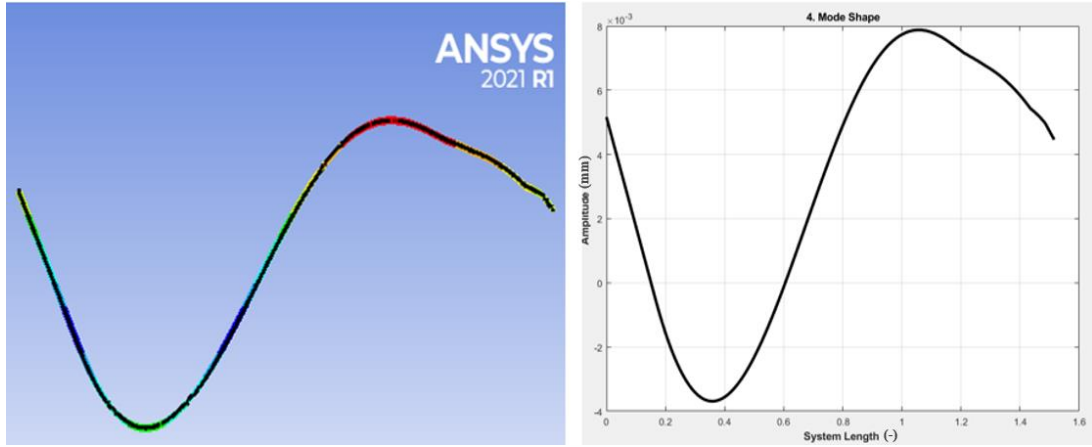


Figure 41: Mode Shape-04 Comparison

At FBO event, due to sudden unbalance change and passing over critical speeds, displacements and reaction forces are to be calculated very consistently at frequency and magnitude levels.

When the displacement results are analyzed for the first bearing support in Figure 43, frequency peaks were determined for all three peaks with an error rate of approximately 1% when compared with the finite element method. In Figure 44 and Figure 45, the same model results for both Transfer Matrix and Finite Element Method are also given for Bearing Support-2 and Bearing Support-3. For this reason, frequency results with the same consistency are seen.

However, when the obtained deformation and reaction results are examined, the inconsistency rate between the two methods grows. However, the reason for this issue is related to the number of solution steps in the Finite Element Method. Especially at low damping values such as 0.2 Ns/mm, more consistent results are obtained when results are obtained from more points. This returns as solution time. On the other hand, In the solutions obtained by the Transfer Matrix Method, there is no change in the solution time even if the damping or solution step changes. It can get a solution in 10-15 seconds.

In this study, it is observed that, mostly at low damping values, to achieve higher precision, Finite Element Frequency Response Analysis is to be run in a very small step. 0 Hz to 500 Hz are swept in 0.5 Hz frequency incline. Within 28 CPU run, 41 minutes and 54 seconds are required to get the FRF results. At higher damping values, error level at percentage, getting lower, and sensitivity graph for this study given in Figure 42.

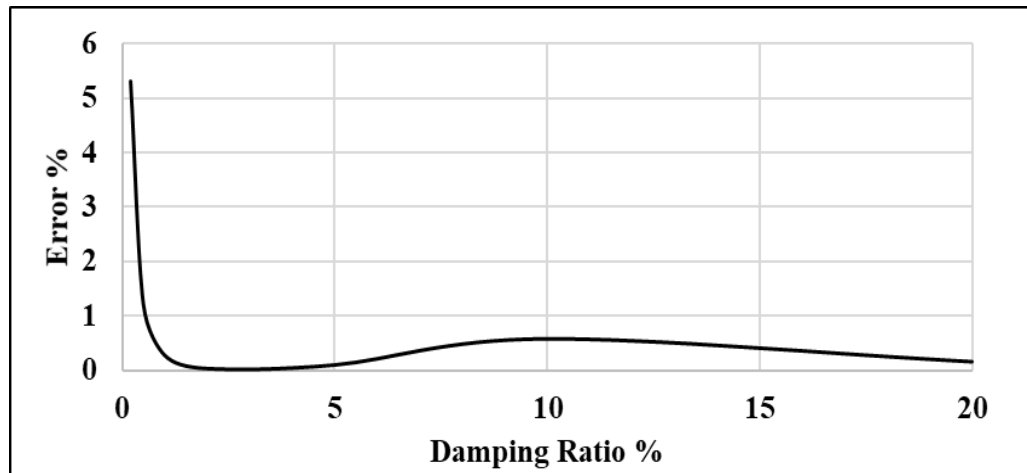


Figure 42: Damping Ratio Changing Sensitivity on Results

Displacement result comparison both for two methods printed together for Bearing Support-01, Bearing Support-02, and Bearing Support-03 in Figure 39, Figure 40, and Figure 41, respectively. The Red line shows FE Results and The Black line is for Transfer Matrix Model results.

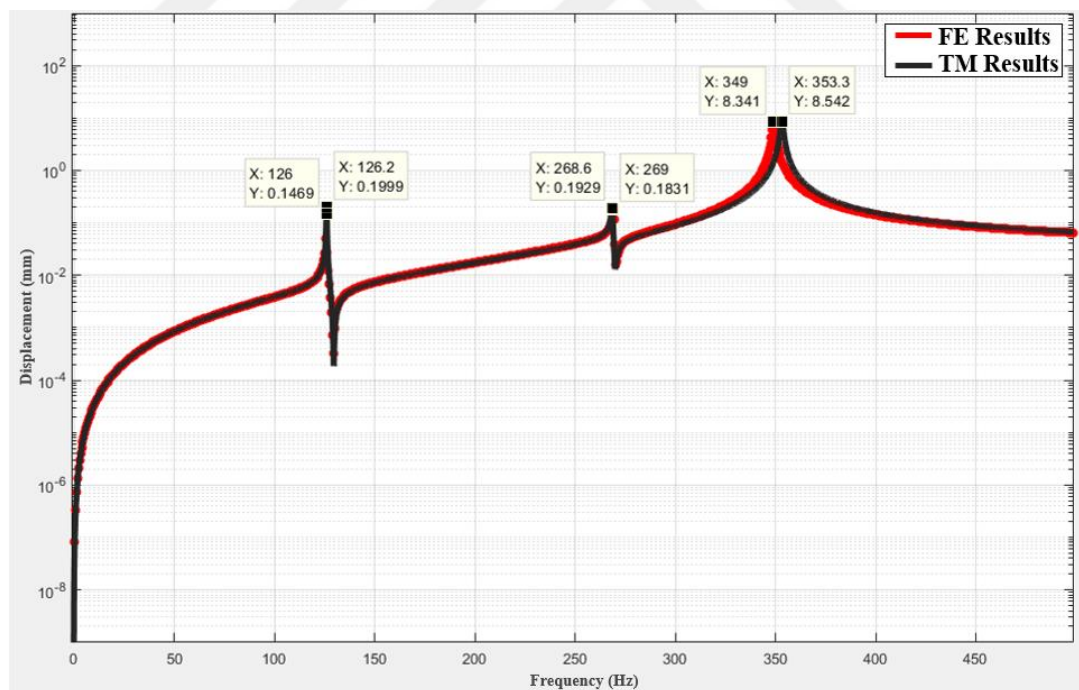


**Figure 43: Displacement Result Comparison for Bearing Support-01
@ 0.2 Ns/mm Damping Value**

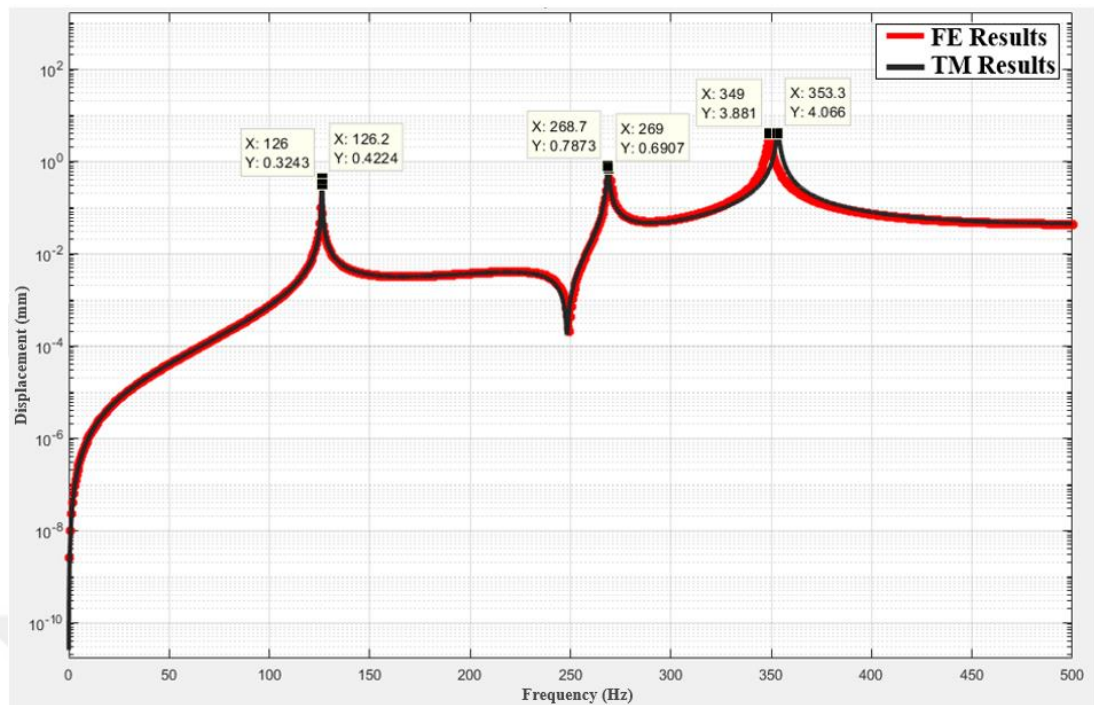


**Figure 44: Displacement Result Comparison for Bearing Support-02
@ 0.2 Ns/mm Damping Value**

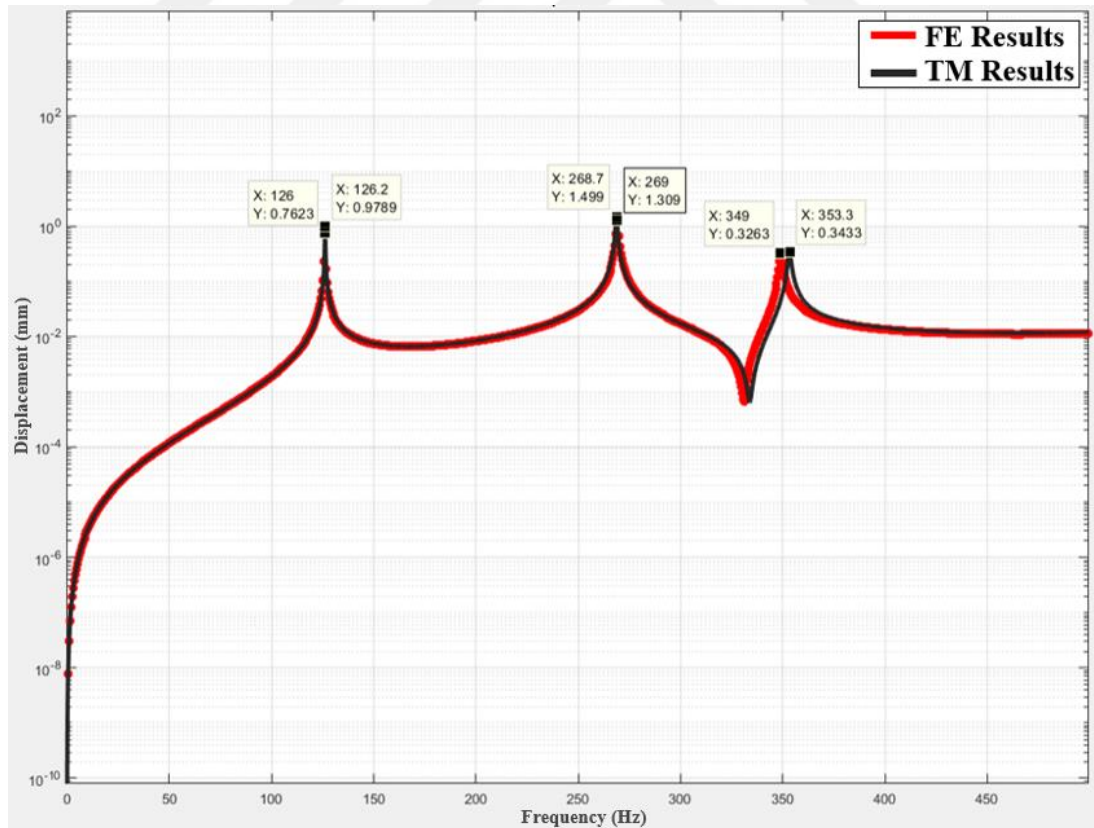


**Figure 45: Displacement Result Comparison for Bearing Support-03
@ 0.2 Ns/mm Damping Value**

Reaction Force result comparison at peak load magnitude frequency both for two methods printed together for Bearing Support-01, Bearing Support-02, and Bearing Support-03 in Figure 46, Figure 47, and Figure 48, respectively. The Red line shows FE Results and The Black line is for Transfer Matrix Model results.

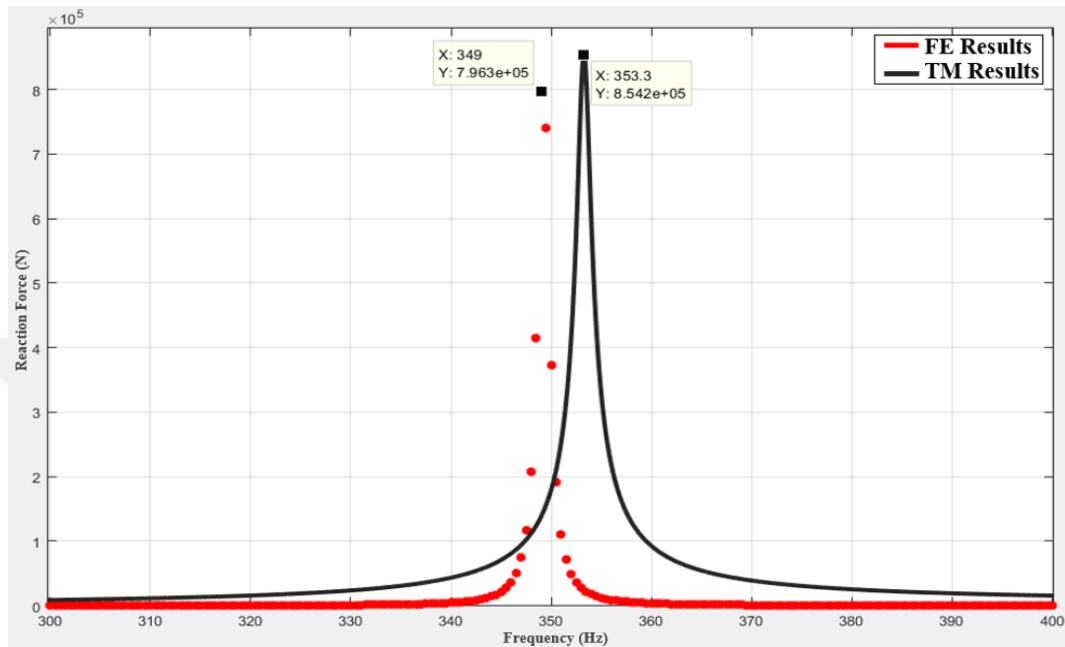


**Figure 46: Reaction Force Result Comparison for Bearing Support-01
@ 0.2 Ns/mm Damping Value**

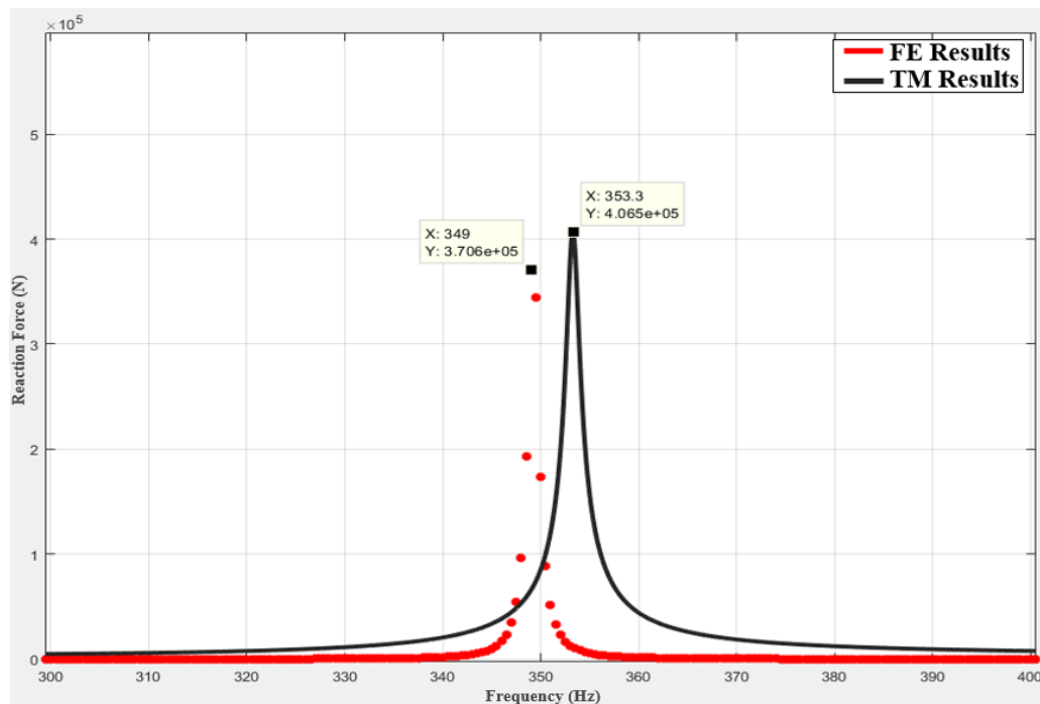


**Figure 47: Reaction Force Result Comparison for Bearing Support-02
@ 0.2 Ns/mm Damping Value**

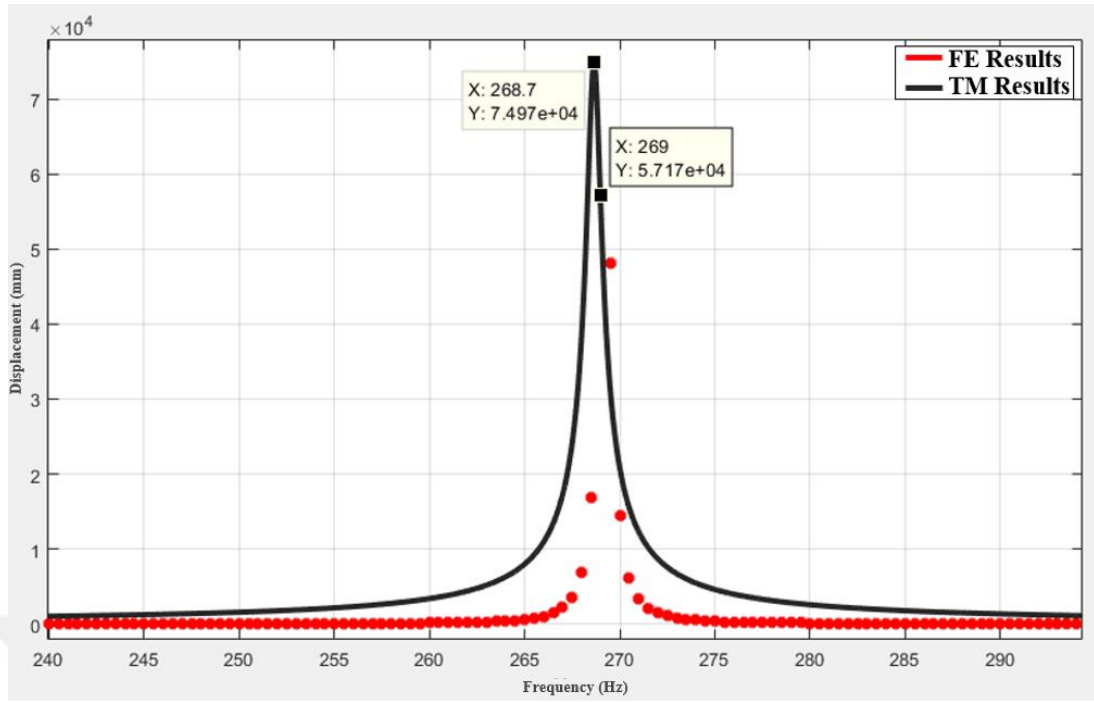


**Figure 48: Reaction Force Result Comparison for Bearing Support-03
@ 0.2 Ns/mm Damping Value**

3.7. Validation Using Models in the Literature

In this chapter, there are two different validation studies from the literature-based are expressed.

3.7.1 Two-Disk, Two-Bearing Rotor Model

In this chapter, although a validation study was accomplished between the Transfer Matrix model and its Finite Element model in the previous section, a validation runs that model results are published in a book covering rotor dynamics. According to a published example, “a 1.5m long-shaft, has a diameter 0.05m. The disk is 0.07 m thick with a diameter of 0.28m; the right disk is 0.07m thick with a diameter of 0.35m. For the shaft, $E= 211\text{GN/m}^2$ and $G=81.2 \text{ GN/m}^2$. There is no internal shaft damping. For both the shaft and the disks, $p=7.810 \text{ kg/m}^3$. The shaft is supported by identical bearings at its ends.” [31].

Illustration of the example given in Figure 49.



Figure 49: Illustration of two-disk, two bearing rotor [31]

Results are investigated at 0 rpm and 4000 rpm separately, given in Table 5 and Table 6, respectively.

Table 5: at 0 Rpm Result Comparison for Literature Model

#	Transfer Matrix Model	Literature Model[31]	Error %
1st Mode	86.64	86.66	0.02
2nd Mode	274.82	274.31	0.18
3rd Mode	726.78	716.78	1.423

Table 6: at 4000 Rpm Result Comparison for Literature Model

#	Transfer Matrix Model	Literature Model[31]	Error %
1st Mode	87.71	87.80	0.1
2nd Mode	294.15	294.71	0.19
3rd Mode	831.19	827.08	0.49

3.7.2 NSA 1-HPT Engine Model

In this chapter, named NSA 1-HPT engine model results from that study and Transfer Matrix Model results are compared. This model is from a master thesis shared results for Fan Blade Out event calculations on an analytical approach [32].

Campbell diagram given in Figure 50, critical speed of the model is at 135.9801 rad/s (1298.51 rpm), shown with red circle on the diagram.

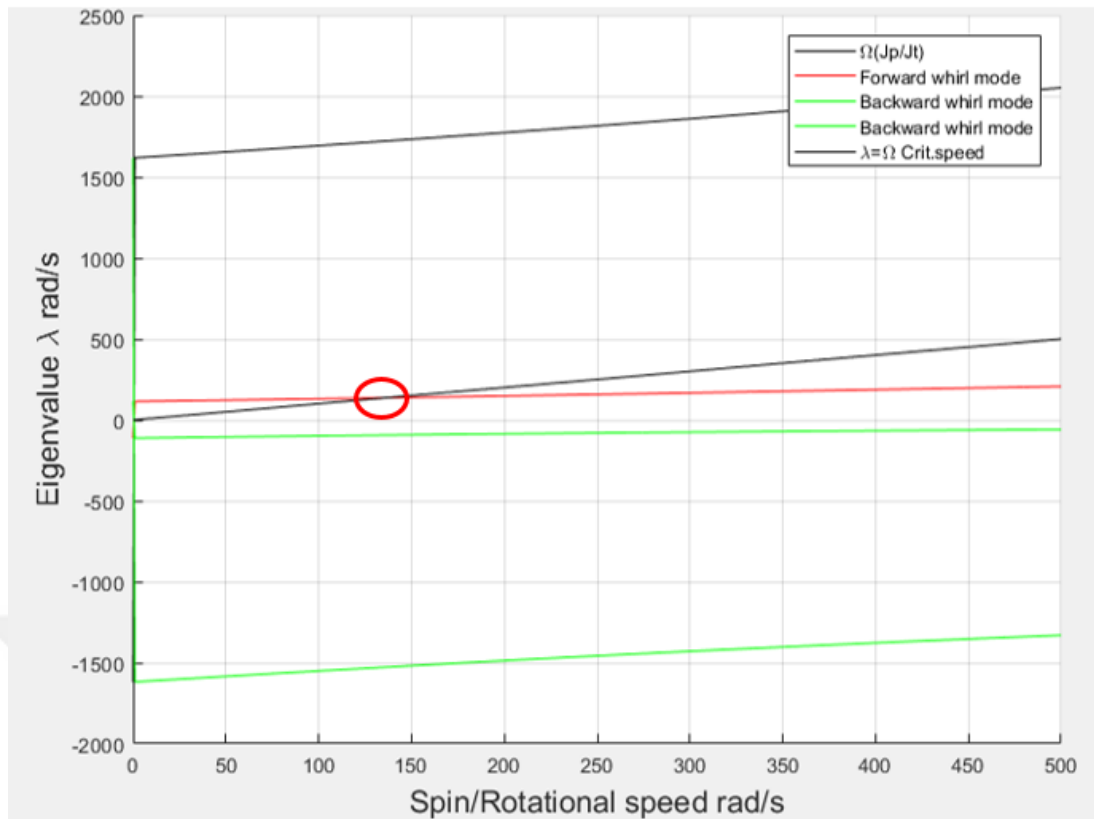


Figure 50: Campbell Diagram Results for NSA 1-HPT Engine Model [32]

The same model is built at Transfer Matrix Method, covering whole geometrical and mechanical properties are kept as same as published study. Critical speed for the model is found at 124.68 rad/s (1190.7 rpm).

The critical Speed comparison is given in Table 7.

Table 7: Critical Speed Results Comparison Table

#	Transfer Matrix Model	NSA-1 HPT Engine Model [32]	Error %
Critical Speed	1190.7 rpm	1298.51 rpm	8.30

Campbell Diagram is from Transfer Matrix Model given in Figure 51.

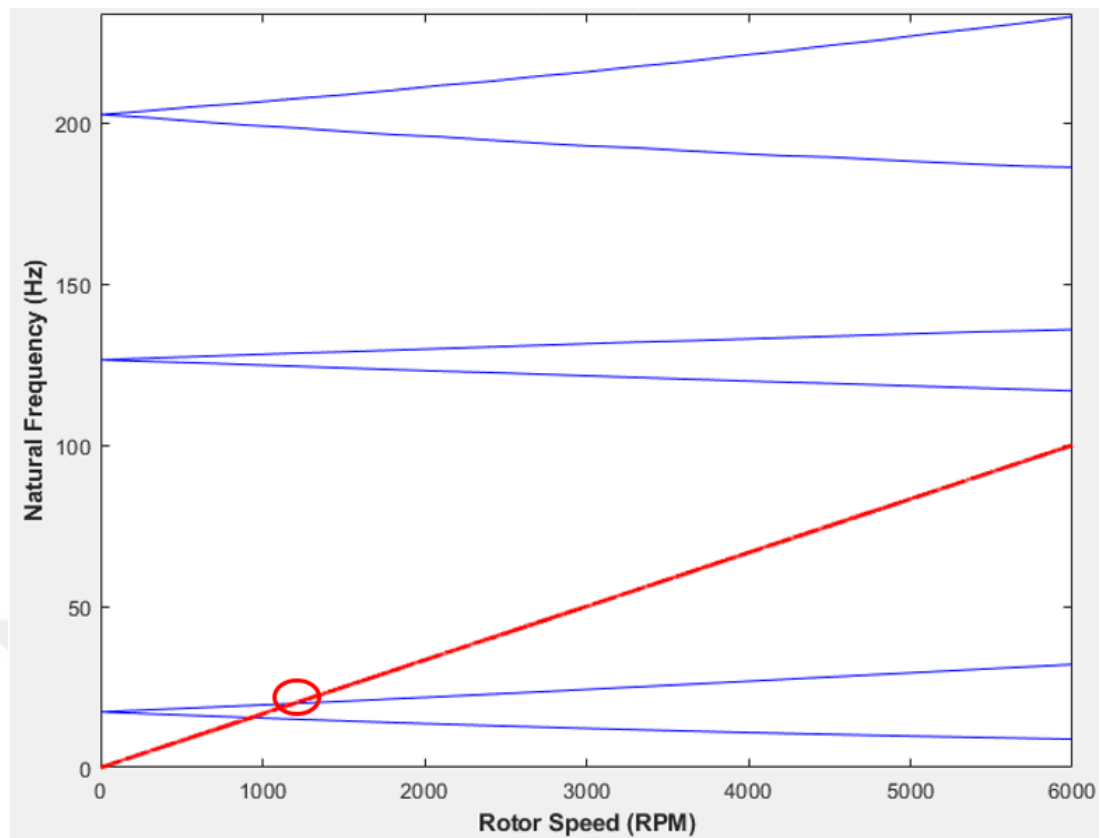


Figure 51: Campbell Diagram Results for Transfer Matrix Model of NSA-1 HPT Engine

4. RESULTS

Results are from the Transfer Matrix Model, built by details given in Table 2. Models are solved on MATLAB 2017Rb and all results are plotted in this chapter.

Different bearing damping values from 0.2 to 1.0 Ns/mm are run in the model to see the effects of damping changes on the forces and amplitudes level. In this chapter, firstly, 0.2 Ns/mm damping results are plotted but at further, every damping value result are also given in this study.

4.1. Campbell Diagrams

On the Campbell diagram given in Figure 52, the results are numerical models printed. The first four eigenvalues are of the rotor system captured at 116.90 Hz (7,014 rpm), 221.90 Hz (13,314 rpm), 274.73 Hz (16,483.8 rpm), and 390.1 Hz (23,406.0 rpm). As mentioned in Validation Chapter, in this study, gyroscopic effects are added to numerical mode.. Therefore, on Campbell Diagram, there are results at from 0 rpm to 14,000 rpm, changing along shaft speed.

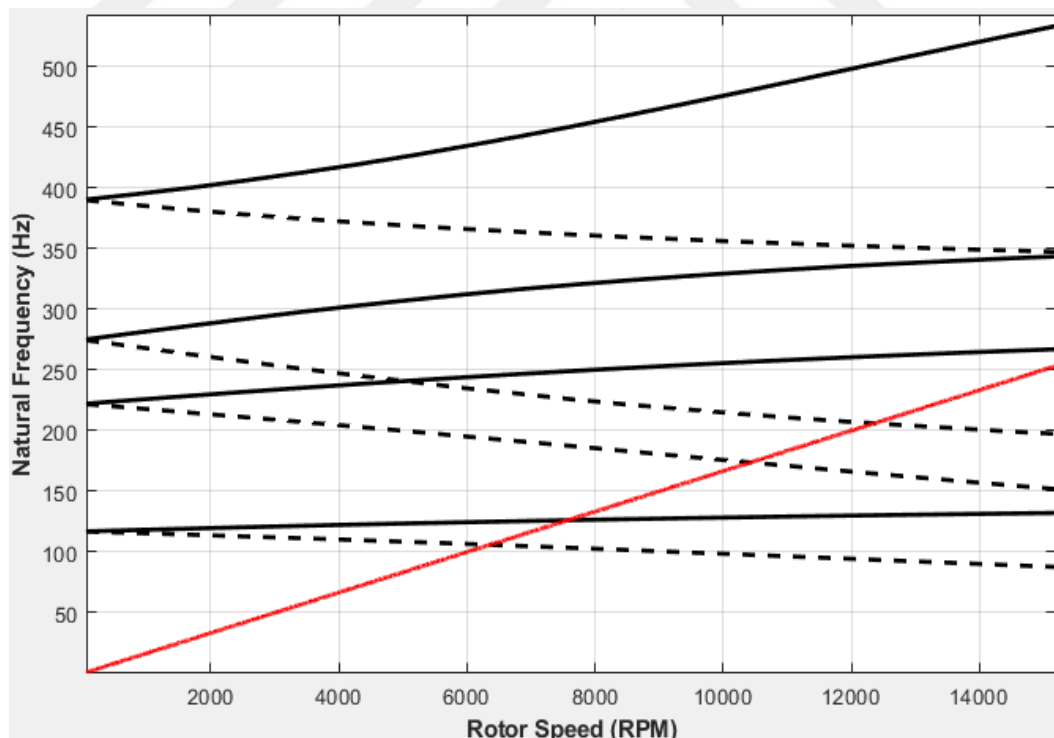


Figure 52: Campbell Diagram of LP Shaft

4.2. Mode Shapes

When the relevant frequency results and the corresponding mode shapes are examined, as can be seen, no rigid modes were found. This is due to the relatively high stiffness of static structures. If the modeling detail is examined in Table 2, the lowest stiffness value are seen as 50000 N/mm. First four mode shapes is also given in Figure 53, Figure 54, Figure 55, and Figure 56.

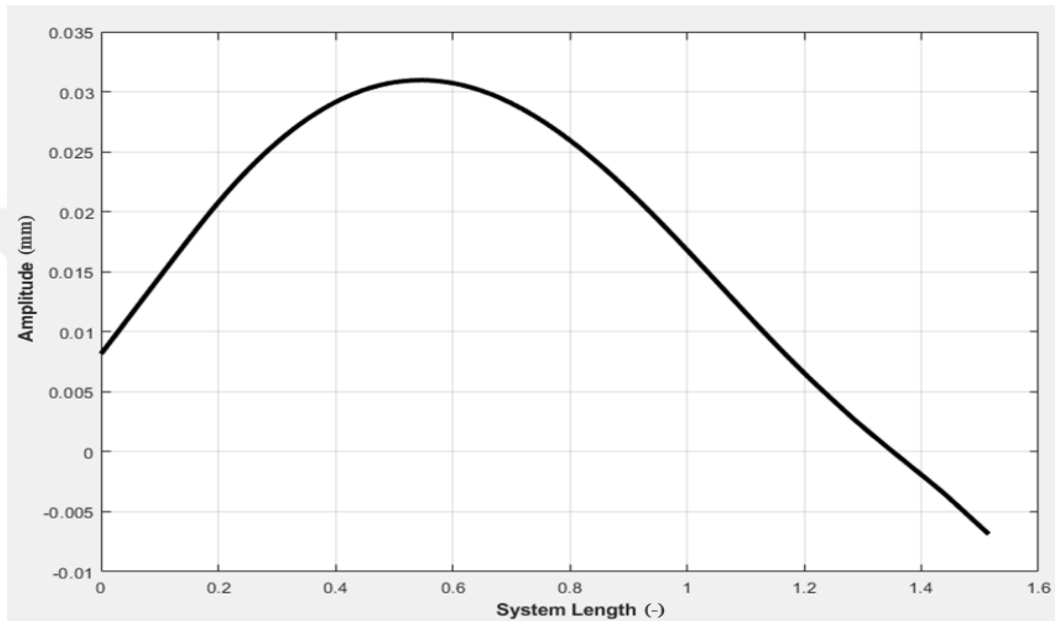


Figure 53: Mode Shape-1

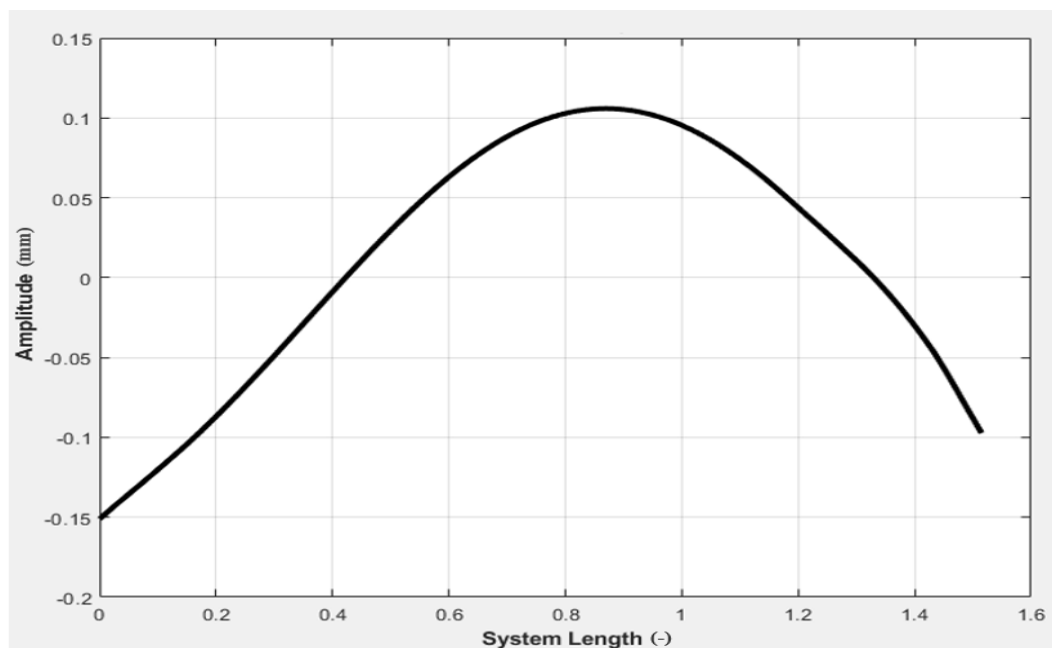


Figure 54: Mode Shape-2

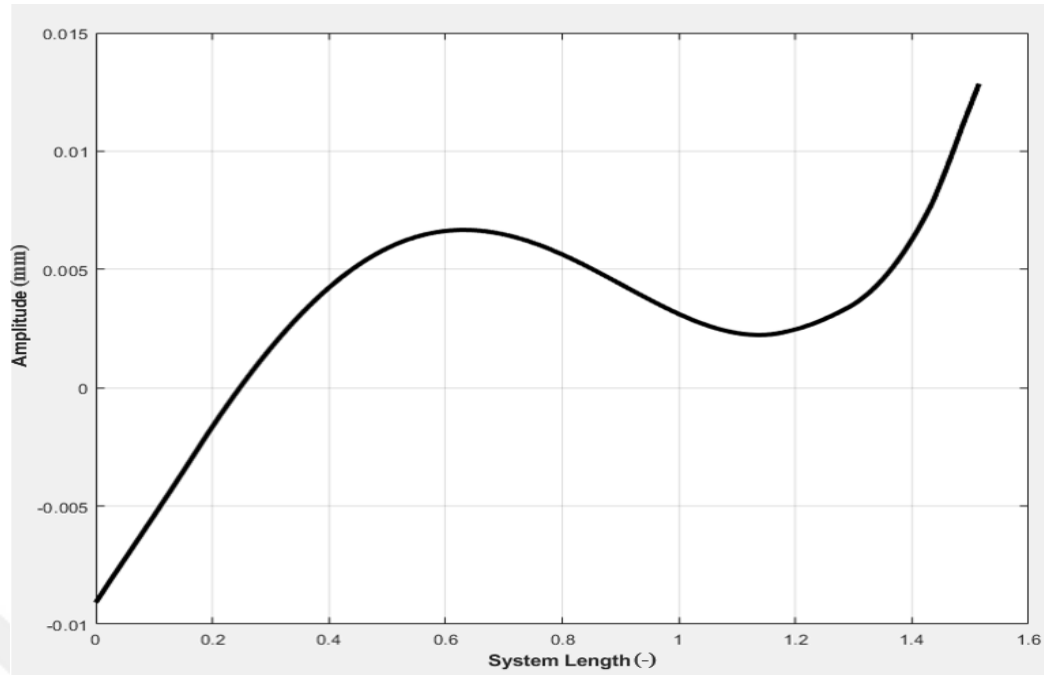


Figure 55: Mode Shape-3

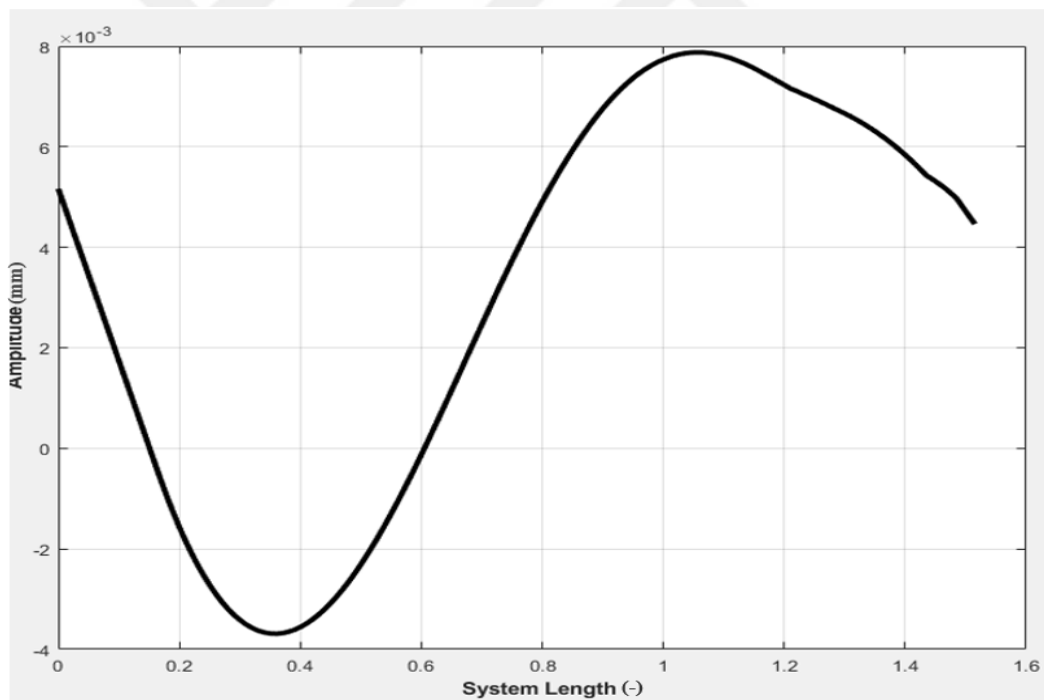


Figure 56: Mode Shape-4

4.3. Displacement and Reaction Force

During FBO event, a very small magnitude of damping ratio, 0.2 Ns/mm, is applied to the supporting structure and bearings. Because tight clearances on special dampers for aero engines on oil films are accepted to be closed. For 0.2 damping ratio,

reaction forces and displacement results on Support-1, Support-2, and Support-3 are given below. Displacement results are given in Figure 57, Figure 58, and Figure 59.

When the results are examined, very high deformation values are seen. The radial deformation values seen are well above the rotor-stator clearances when a turbofan engine is operating under normal conditions. During the FBO event, friction will also occur between the rotor and the stator. However, for the motor to shut down safely and not cause a different catastrophic situation after the FBO event, the clearances between the rotor and the stator should be evaluated against the possibility of a FBO event. In this model, frictional loads are not modeled.

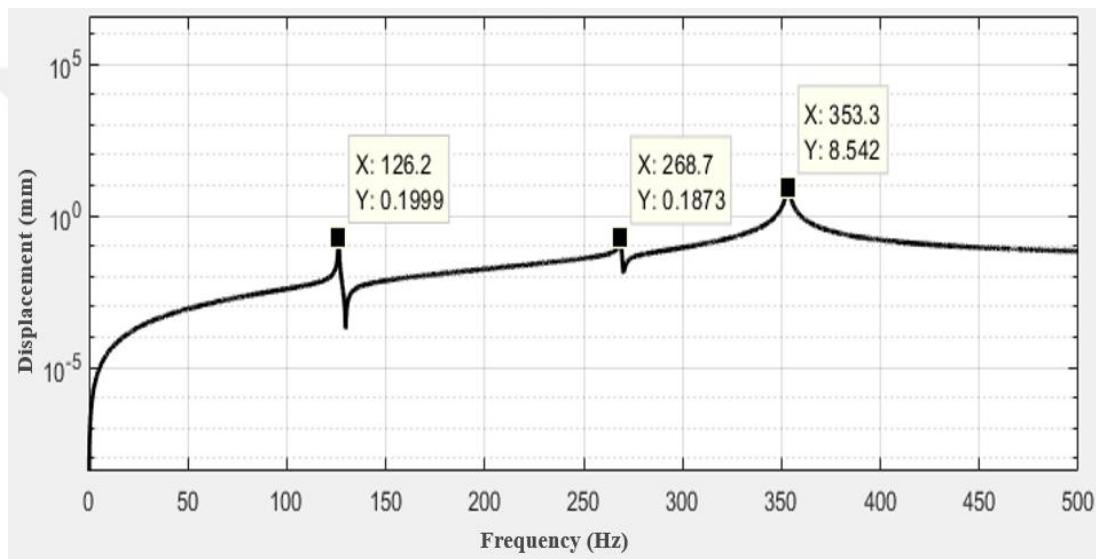


**Figure 57: Displacement Result at Bearing Support-1
For 0.2 Ns/mm Damping Value**

When the first support results were examined in Figure 54, it was observed that the maximum deformation occurred in the 3rd mode transition at 353.3 Hz. The radial deformation was calculated as 8,542 mm. Since it is known that the clearances between the rotor and the stator are at the level of microns, rubbing will most likely be seen during the FBO event.

If the second support results were examined in Figure 55, it was observed that the maximum deformation occurred in the 3rd mode transition at 353.3 Hz. The radial deformation was calculated as 4,065 mm. It seems lower than the result of first bearing suffers but radial displacement value is still too high and rubbing may be seen during FBO event.

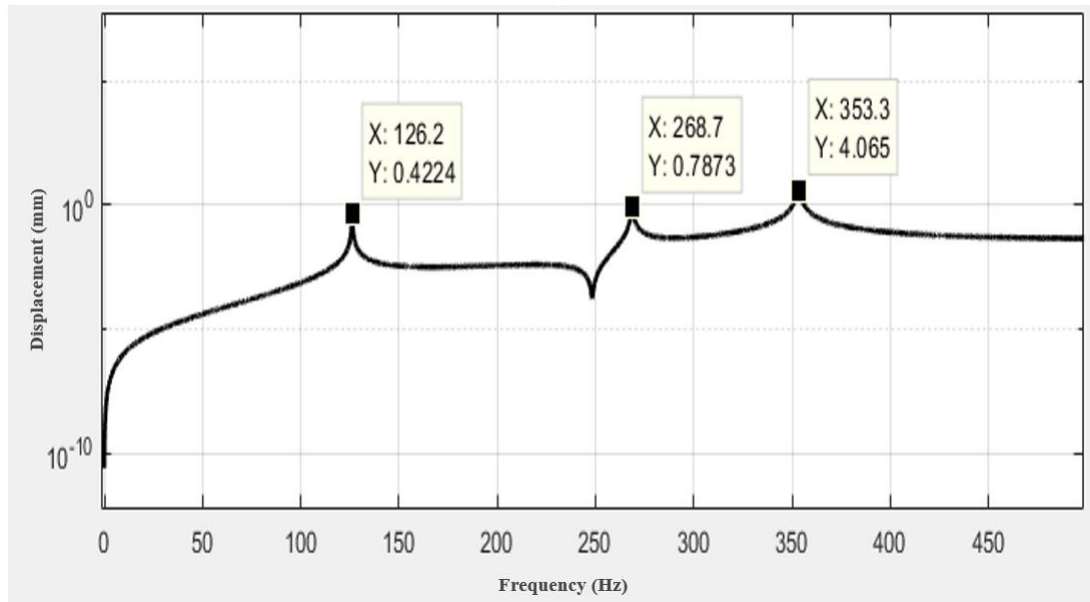


**Figure 58: Displacement Result at Bearing Support-2
For 0.2 Ns/mm Damping Value**

If the results of the third bearing region are examined in Figure 56, very small radial deformation results are seen compared to the results of the other two bearing regions. The reason for this can be explained as the third bearing region being away from the fan discs. The peak value is seen at the second mode transition at 268.7 Hz.

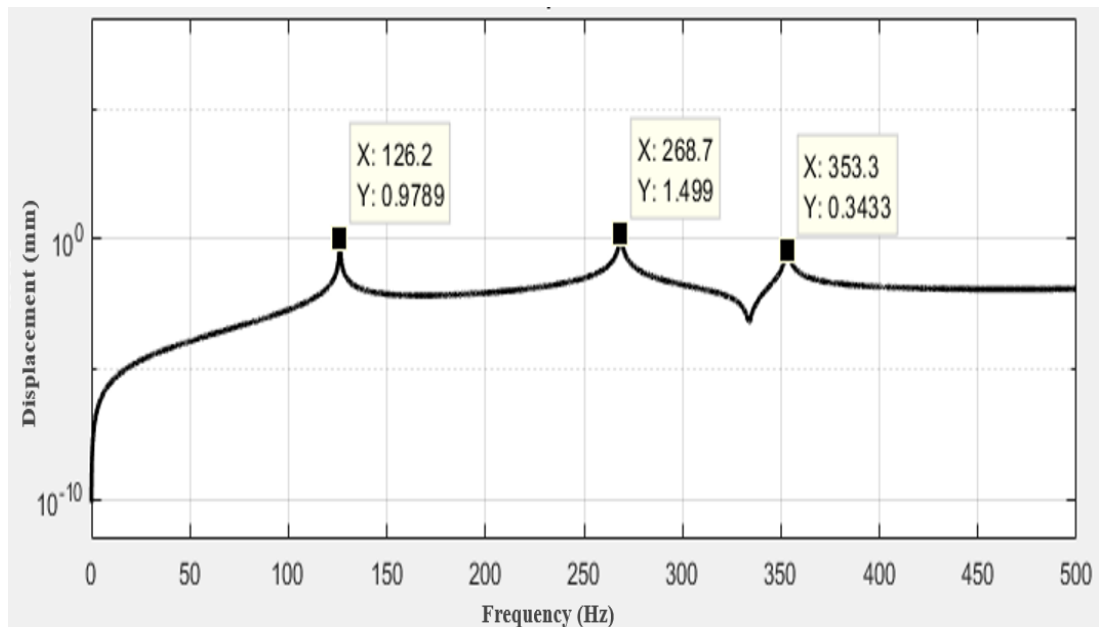


**Figure 59: Displacement Result at Bearing Support-3
For 0.2 Ns/mm Damping Value**

When the reaction force results are examined, as seen in the literature research, very high loads are applied to the structural parts carrying the rotor, especially at mode transitions. It is important to consider these loads while designing the motor static parts and to create designs to withstand these loads. Especially if these loads cannot be calculated or known during the concept design, this important detail will be ignored while making the engine layout. Because engine layout study is a subject that many disciplines decide together such as aero, performance, lubrication, and cooling. After the rotor dynamics models are established during the detail design phase, making the design changes that can withstand these high loads in the static parts may force them to return to the engine layout studies.

In the design of a turbofan engine, clearances between stator and rotor are tight to keep performance and aerodynamic requirements. After FBO event, according to results on deflections and reaction forces on supports, it is so hard to avoid rubbing between rotor and stator. On the other hand, there are huge peaks at critical speed zones. After a FBO event, the turbofan engine has to be shut down in the safe, which means the LP shaft has to continue wind-milling. On calculations, from 0 to 30,000 rpm are swept to check the whole operation area. Especially, at first support, there are very high reaction forces and deflections. It is not easy to stand for bearings and static structure. Therefore, these loads have to be managed for the safe design philosophy that requires rapid load and design iterations on architecture even in earlier stages of turbofan engine design.

In this study, even during a fan blade out event, a very small magnitude of damping ratio was applied to supporting structures, to check and determine rotor system behavior at higher damping ratios. Because of that, in that content, 0.2 Ns/mm, 0.5 Ns/mm, 1.0 Ns/mm and 2.0 Ns/mm damping values results are also given in appendix.

When the loads that the first bearing support is exposed to are examined in Figure 60, very high loads such as 850 kN are seen at the third mode transition. As seen in the literature research, it was the place exposed to the highest load during the FBO event in a turbofan motor.

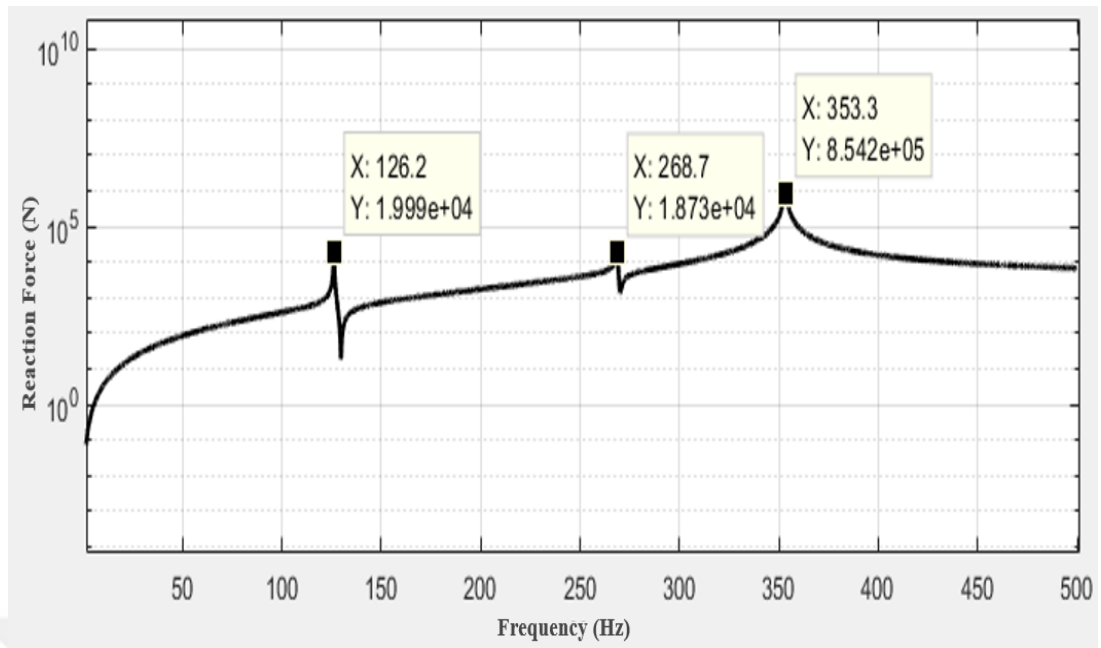


**Figure 60: Reaction Force Result at Bearing Support-1
For 0.2 Ns/mm Damping Value**

If the results of the second bearing support are examined in Figure 61, the highest load value of the order of 406 kN was calculated in the third mode transition.

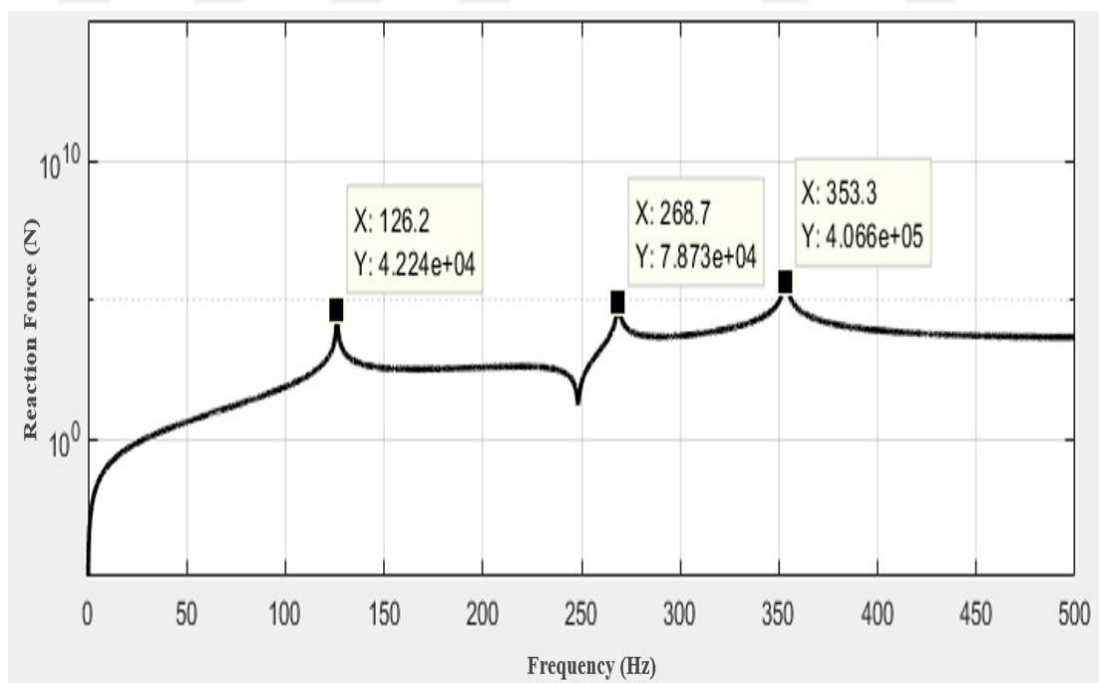


**Figure 61: Reaction Force Result at Bearing Support-2
For 0.2 Ns/mm Damping Value**

Considering the results of the third bearing support in Figure 62, the highest load value is observed in the different mode transitions compared to the other two bearing regions. The reason for the high load value to be seen in the second mode transition is that the maximum deformation in the second mode shape is close to the investigated mode shape.

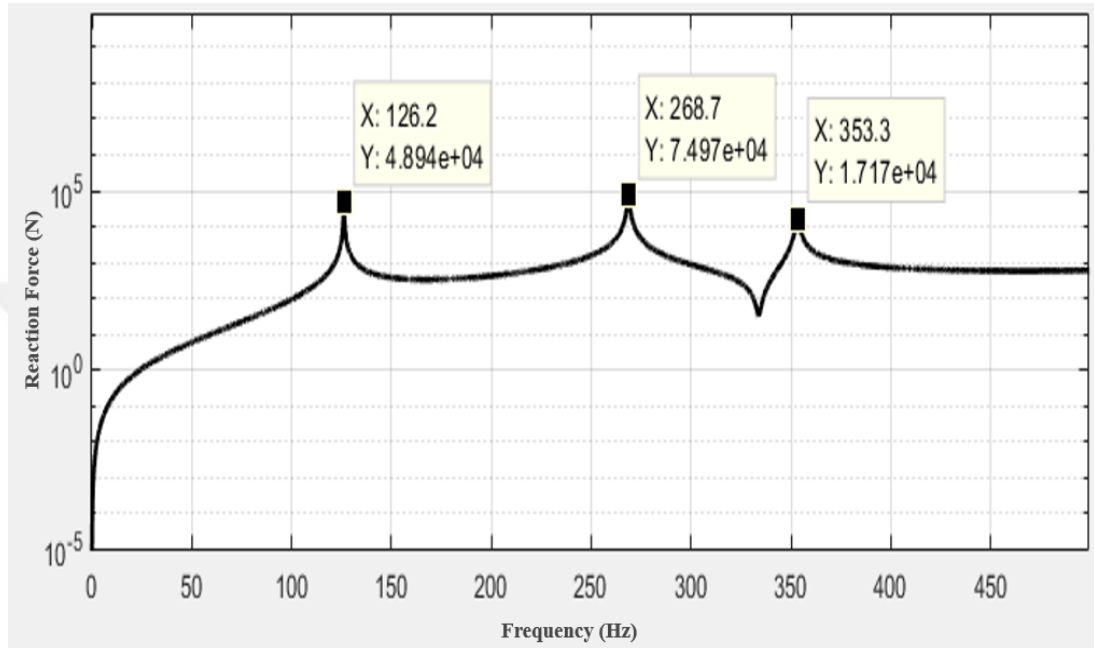


**Figure 62: Reaction Force Result at Bearing Support-3
For 0.2 Ns/mm Damping Value**

4.4 Damping Sensitivity

Besides initial result comparison with deflection and force reaction on supports, Transfer Matrix Method Model is examined according to damping ratio changing sensitivity in this study. From 0 to 0.2% damping values, on steady-state solutions passing through critical speeds, It is observed that there is an asymptote behavior for estimating peak values. Because of that, transient solving has to be run regarding coast up or down speed.

While the FBO event is happening, it is assumed that the squeeze film damper tolerances are closed to see the worst-case scenario. This means that the damping value is close to zero, that is, minimum. However, if the damper continues to operate during this event, the results in reaction forces and deformation values decrease. In fact, this decreased rate does not behave linearly with the dumping increase rate but

decreases dramatically. These results show that the damping elements should still be operable if possible in terms of design while the FBO event is happening.

As seen also in the results section, the highest deformation and load are observed in the first bearing region during the FBO event. For this reason, as the damping ratio in the bearings changes, the results of the displacement and reaction force occurring in the same bearing region are examined. As the damping rate increases, there is a decrease in the calculated loads and deformations at a non-linear rate. Damping Changing – Displacement and Reaction Force peak value results for Bearing Support-01 and peak status re given in Figure 63 and Figure 64 respectively.

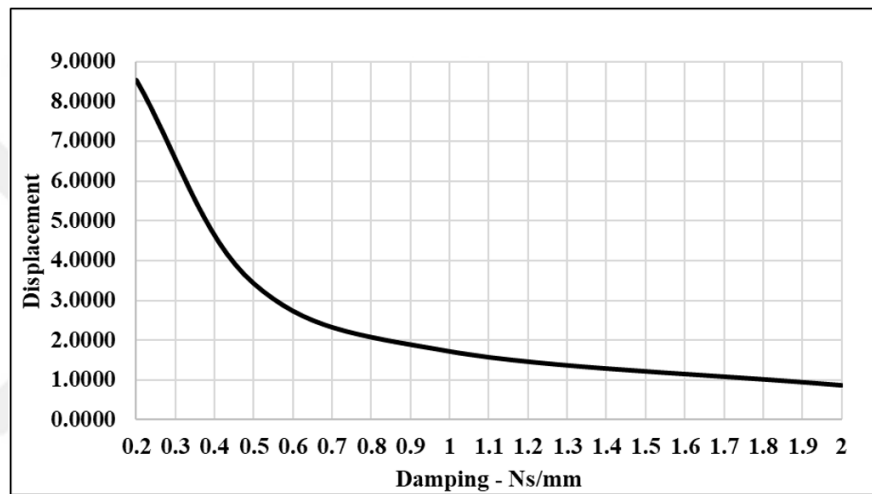


Figure 63: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-01 @ Peak-3 ~ 353 Hz

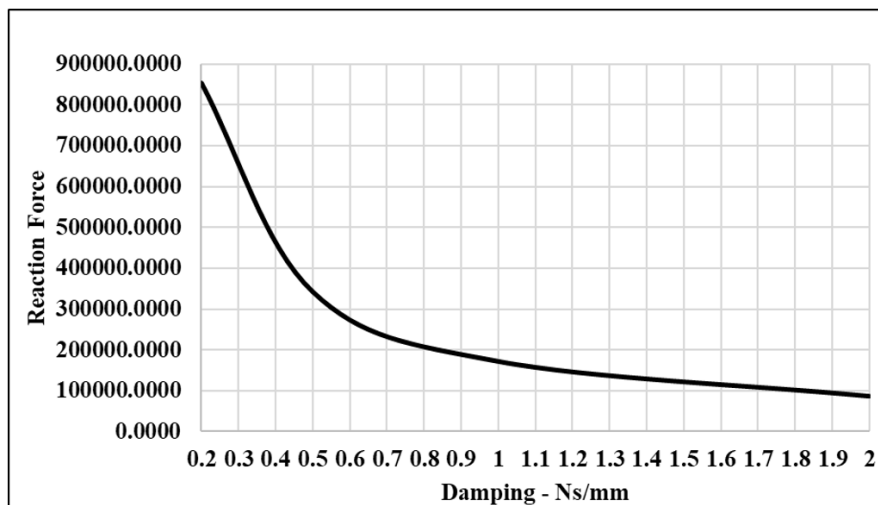


Figure 64: Damping Ratio Changing Sensitivity on Reaction Force Results for Bearing Support-01 @ Peak-3 ~ 353 Hz

5. CONCLUSION

This study aims to show that it can be applicable to use a simplified model in earlier phases of a turbofan development engine to estimate behavior on specific load scenarios such as Fan-Blade Out. Therefore, an in-house code (Joe Solver) using the Transfer Matrix Method is built for rotor dynamic design, according to the layout of the turbofan engine, bearing, and support responses at ultimate load events.

In this study, the rotor system of a low-pressure shaft rotor of a turbofan engine with a three-bearing configuration is simplified and modeled according to rotor dynamics criteria. Analytical solutions are solved by Transfer Matrix Method on MATLAB. On the other hand, there is also a validation model on ANSYS Mechanical, which is solved by the Finite Element Method. The results have been compared. Consistency was good enough to examine the FBO event.

Damping sensitivity to displacement and reaction forces are also examined in this study. It is seen that, even in low damping values, Transfer Matrix Model gives accurate results compared to the Finite Element Model. Validations show that deflection and reaction force results are found closely.

The displacement and reaction force results show that the highest force is seen in the first bearing support when FBO event occurs in a turbofan engine. In addition, although relatively lower load values are observed in other bearing supports, the highest load values that a turbofan engine can see during its working life are observed. In the design of a turbofan engine, considering the possibility of the FBO event, these load and deformation calculations should be taken into account in the concept phase. In this way, major design changes such as bearing position, stiffness, material and thickness of the structures are made.

In summary, in-house code (Joe Solver) for rotor dynamics and load calculations for aero engines have been built and in this study is examined for a specific event of Fan-Blade Out by a typical Low-Pressure Shaft and well validated by finite element method on ANSYS. Numerical solution and finite element results from ANSYS show that the results are so close and the model is ready to use for rapidly changing design activities.

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BIOGRAPHY

Alican KILIÇASLAN was born on February 1989. His bachelor's degree from Yıldız Technical University between 2008-2012, was completed in Naval Architecture and Marine Engineering. After that, he completed his master's in Naval Architecture and Marine Engineering at Istanbul Technical University in 2018. He started his career as a Research and Development Engineer in MILPER Propeller Technologies in 2013. Between 2014 to 2019, he worked in Automotive Industry as Structural Analysis Engineer at TOFAŞ FIAT and BMC Automotive. From 2019 to now, he is in Aviation Industry, in TUSAŞ Engine Industry (TEI) he is working as an Advanced Lead Structural Analysis Engineer.

APPENDICES

Appendix A: Displacement and Reaction Force Results for 0.5 Ns/mm Damping Value

For 0.5 Ns/mm, reaction forces and displacement results on Support-1, Support-2, and Support-3 are printed below. In Figure 65, Figure 66 and Figure 67, Displacements are given for every bearing. In Figure 68, Figure 69, and Figure 70, Reaction Forces are given for every bearing, respectively.

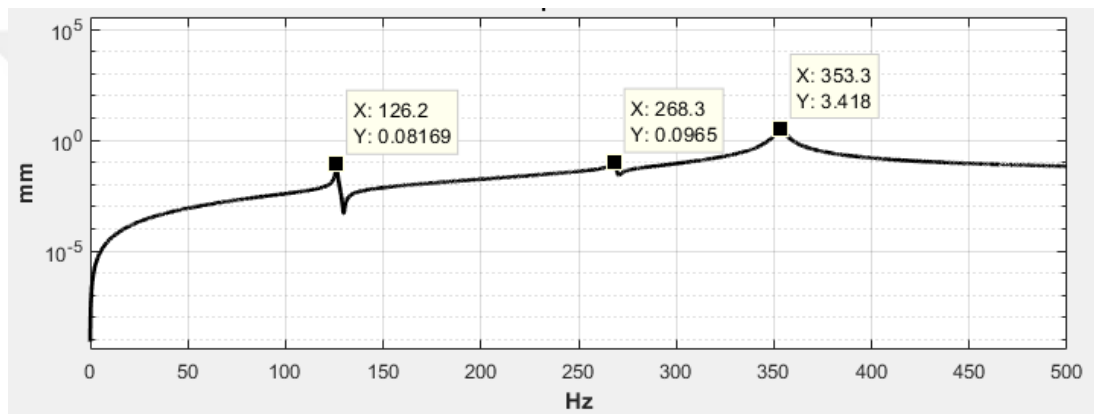


**Figure 65: Displacement Result at Bearing Support-1
For 0.5 Ns/mm Damping Value**

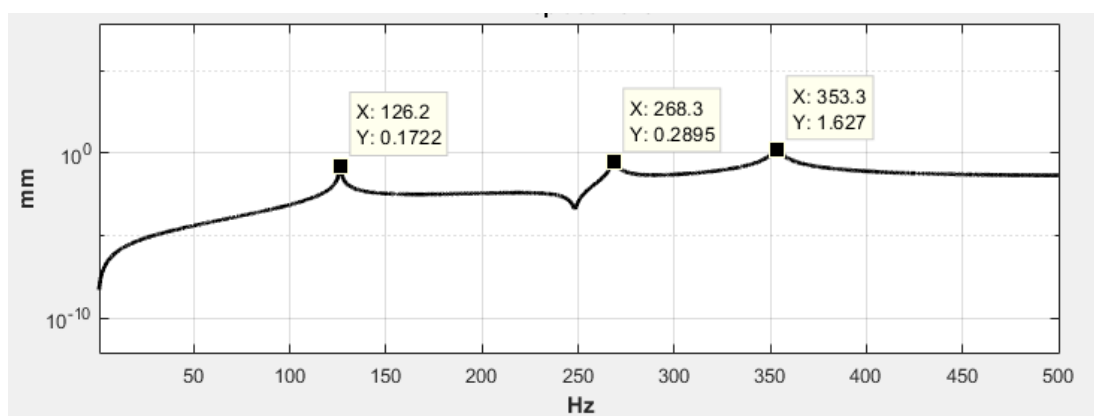


**Figure 66: Displacement Result at Bearing Support-2
For 0.5 Ns/mm Damping Value**

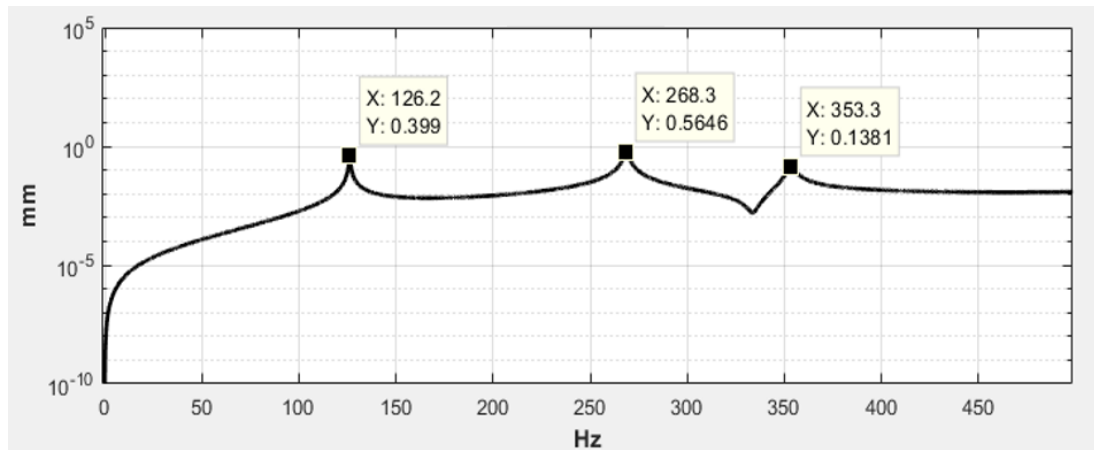


**Figure 67: Displacement Result at Bearing Support-3
For 0.5 Ns/mm Damping Value**

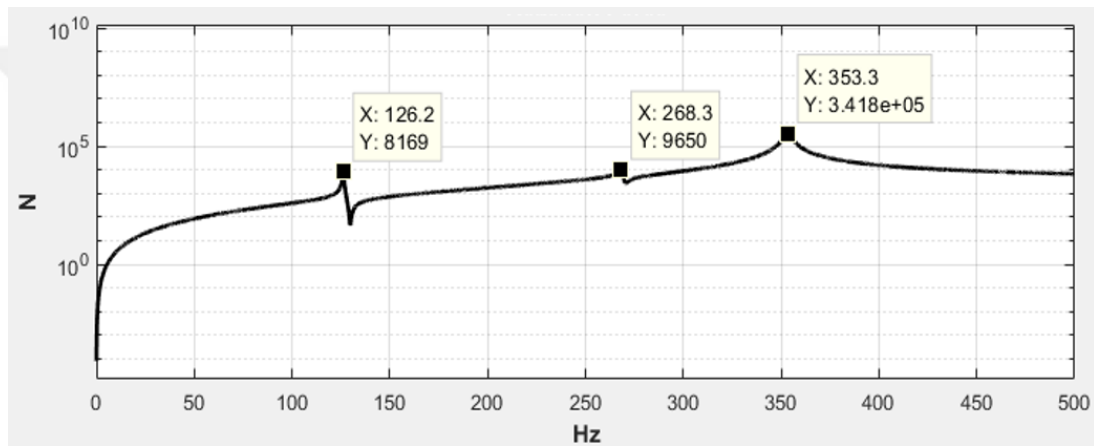


**Figure 68: Reaction Force Result at Bearing Support-1
For 0.5 Ns/mm Damping Value**

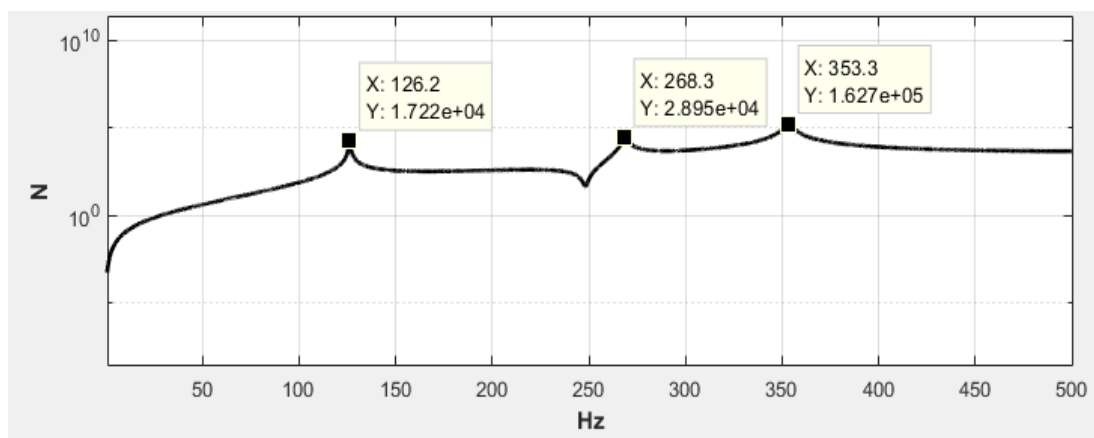


**Figure 69: Reaction Force Result at Bearing Support-2
For 0.5 Ns/mm Damping Value**

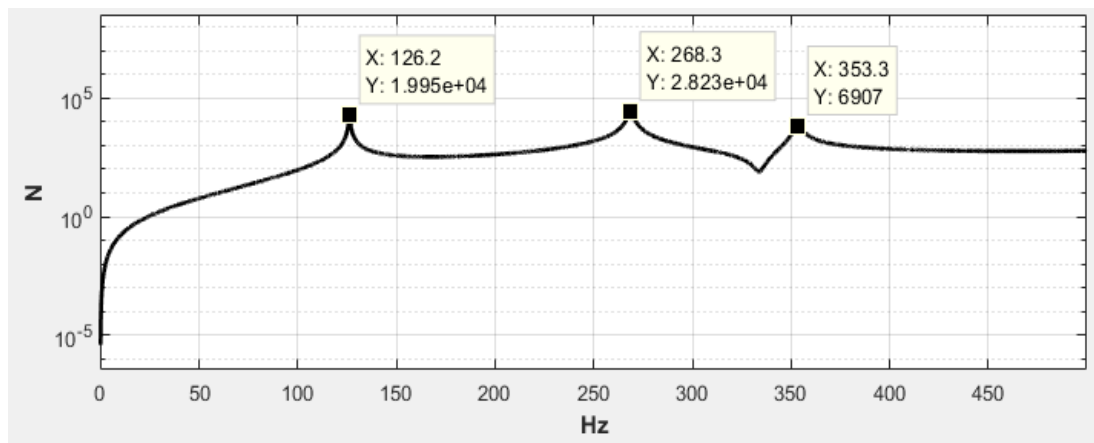


Figure 70: Reaction Force Result at Bearing Support-3



Appendix B: Displacement and Reaction Force Results for 1.0 Ns/mm Damping Value

For 1.0 Ns/mm, reaction forces and displacement results on Support-1, Support-2 and Support-3 are printed below. In Figure 71, Figure 72, and Figure 73, Displacements are given for every bearing. In Figure 74, Figure 75 and Figure 76, Reaction Forces are given for every bearing, respectively.

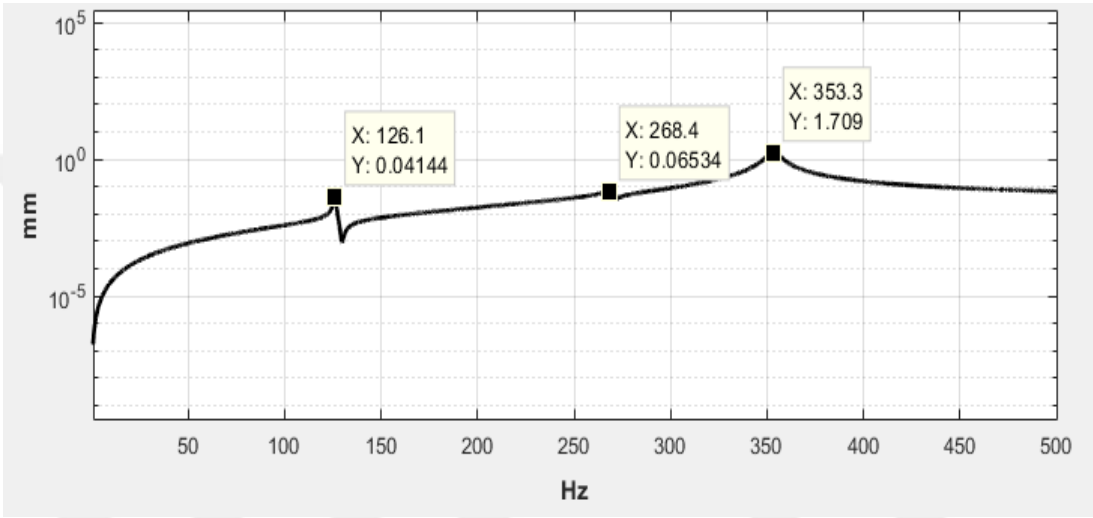


**Figure 71: Displacement Result at Bearing Support-1
For 1.0 Ns/mm Damping Value**

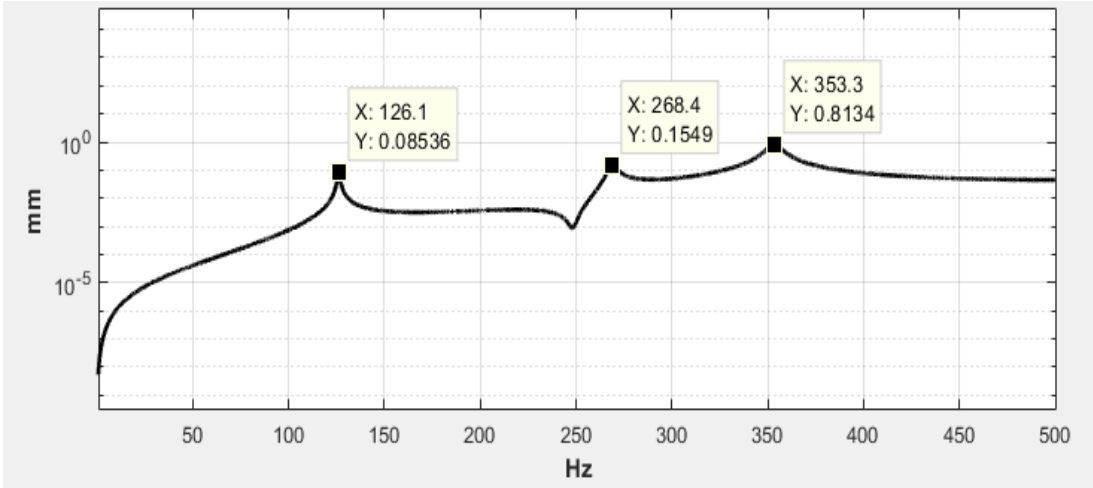


**Figure 72: Displacement Result at Bearing Support-2
For 1.0 Ns/mm Damping Value**

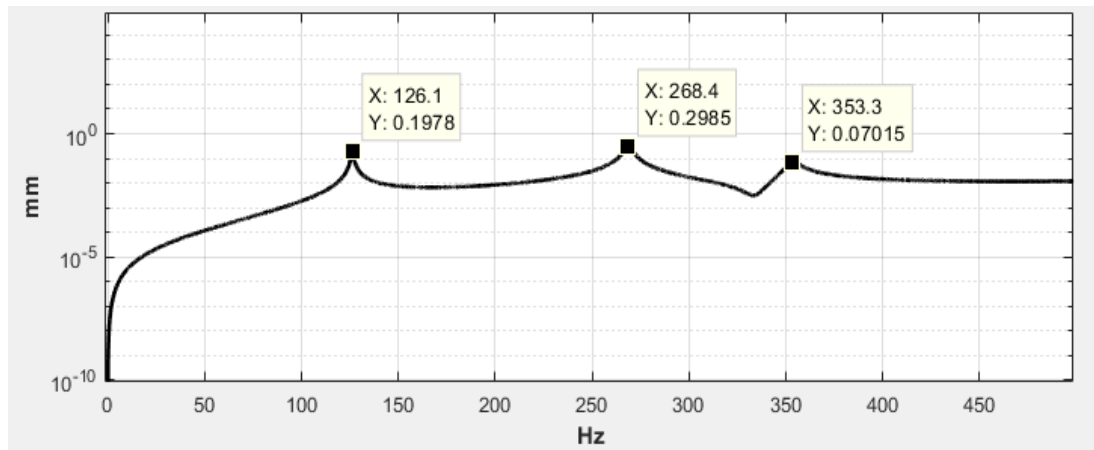


Figure 73: Displacement Result at Bearing Support-3
For 1.0 Ns/mm Damping Value

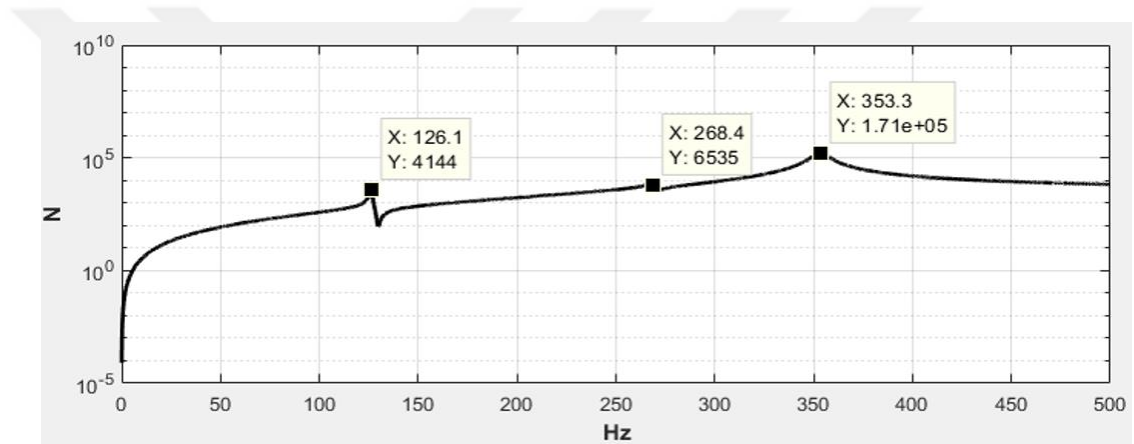


Figure 74: Reaction Force Result at Bearing Support-1
For 1.0 Ns/mm Damping Value

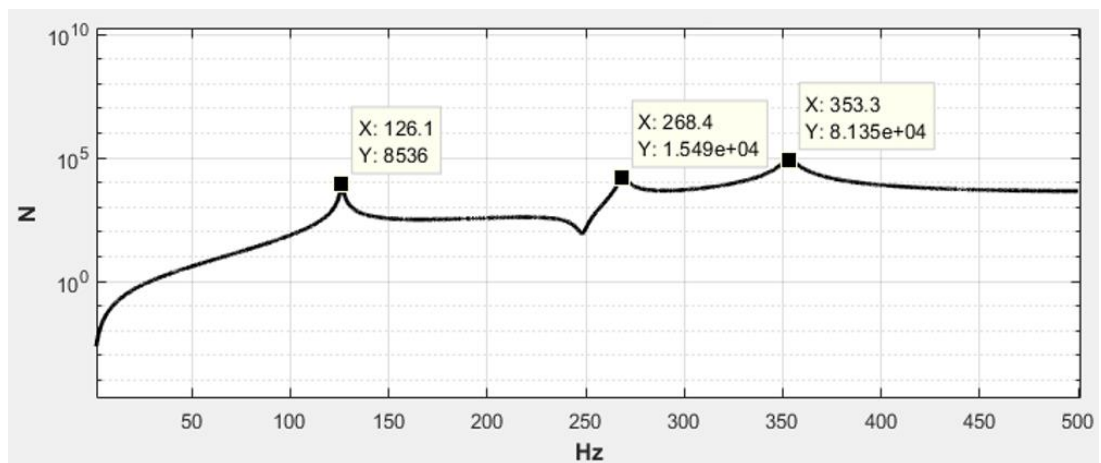


Figure 75: Reaction Force Result at Bearing Support-2
For 1.0 Ns/mm Damping Value

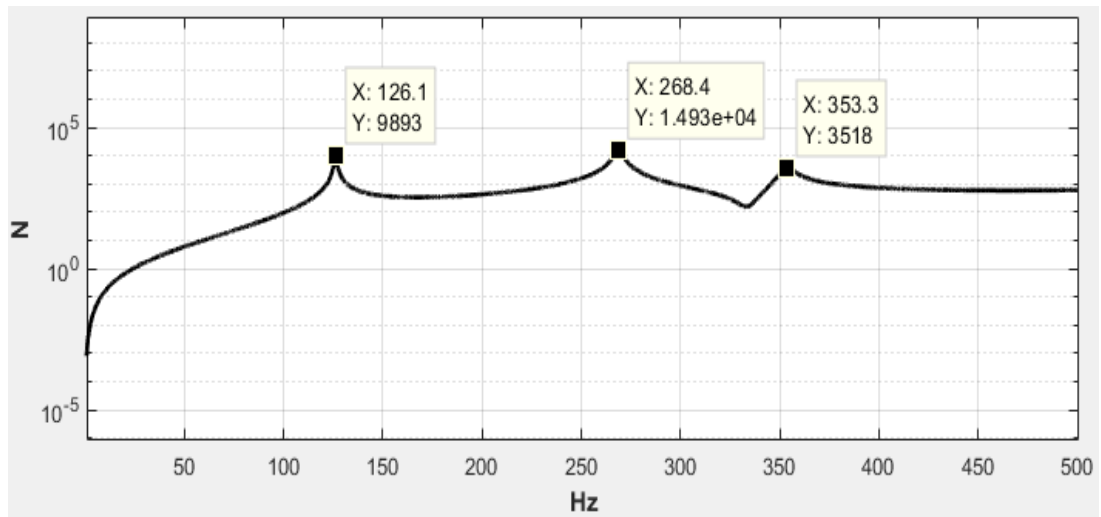


**Figure 76: Reaction Force Result at Bearing Support-3
For 1.0 Ns/mm Damping Value**

Appendix C: Displacement and Reaction Force Results for 2.0 Ns/mm Damping Value

For 2.0 Ns/mm, reaction forces and displacement results on Support-1, Support-2, and Support-3 are printed below. In Figure 77, Figure 78, and Figure 79, Displacements are given for every bearing. In Figure 80, Figure 81 and Figure 82, Reaction Forces are given for every bearing, respectively.

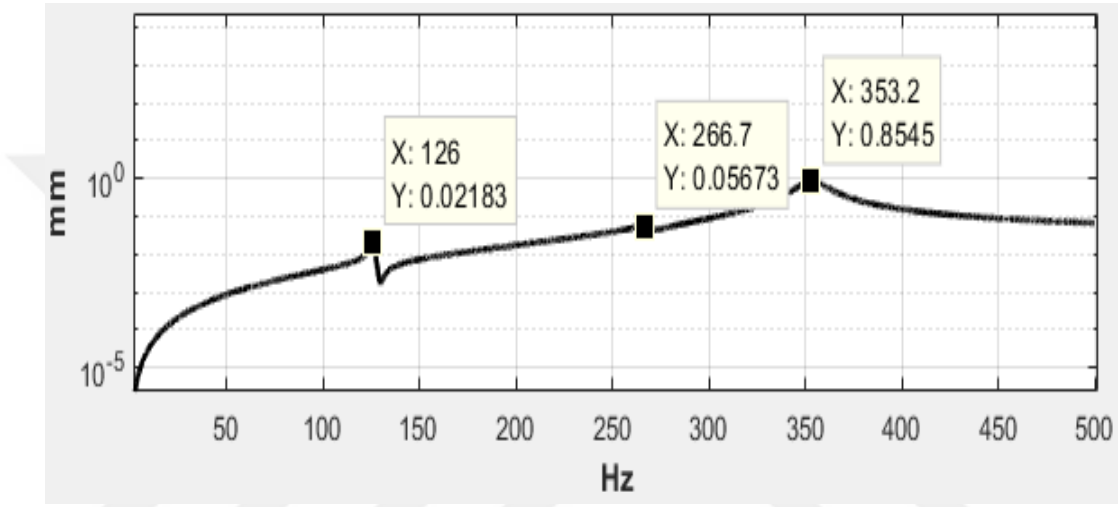


Figure 77: Displacement Result at Bearing Support-1
For 2.0 Ns/mm Damping Value

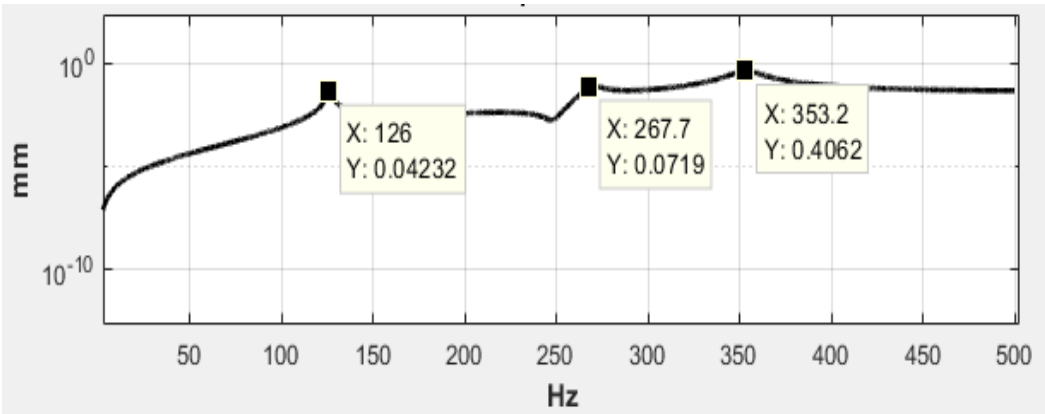


Figure 78: Displacement Result at Bearing Support-2
For 2.0 Ns/mm Damping Value

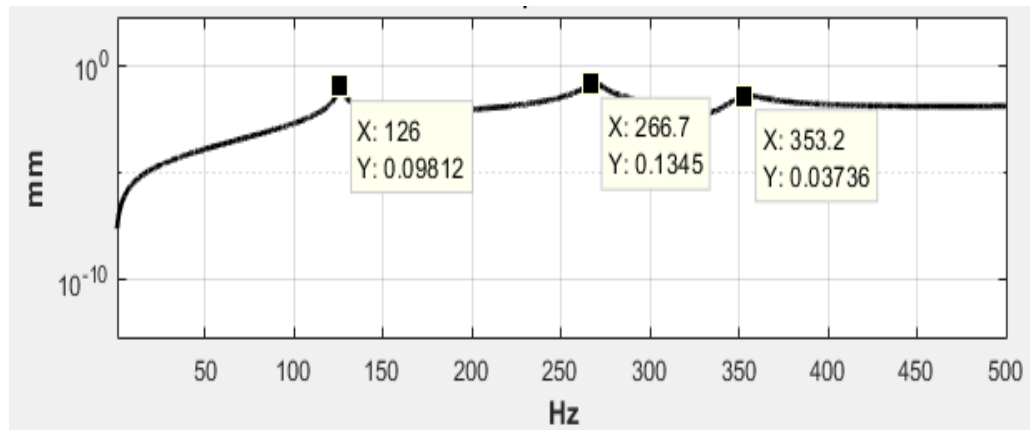


**Figure 79: Displacement Result at Bearing Support-3
For 2.0 Ns/mm Damping Value**

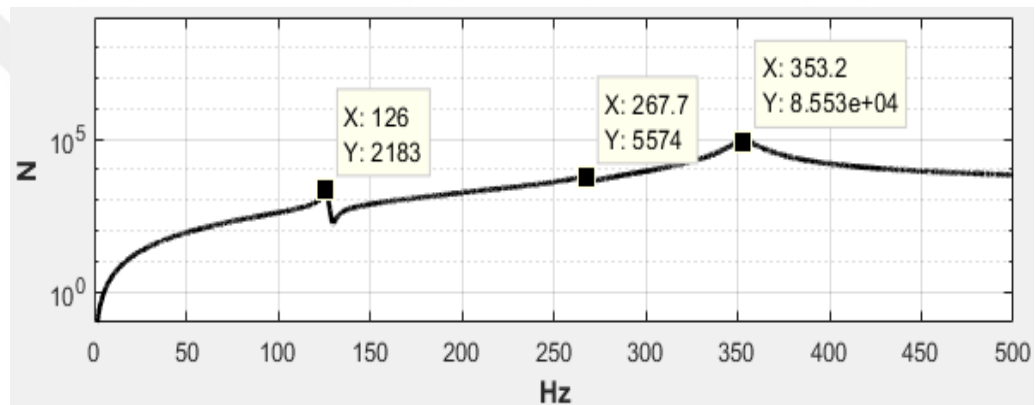


**Figure 80: Reaction Force Result at Bearing Support-1
For 2.0 Ns/mm Damping Value**

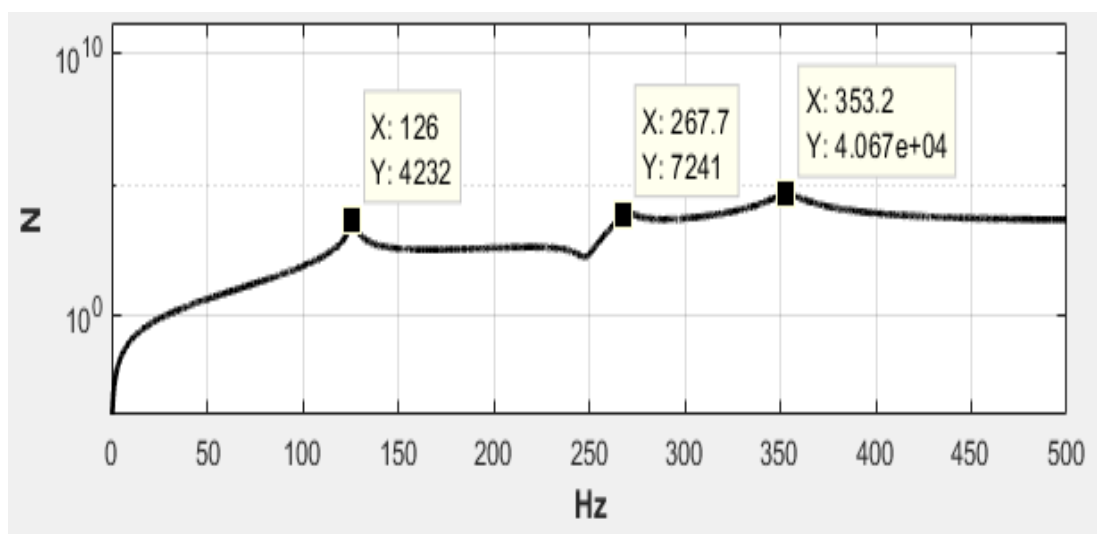


**Figure 81: Reaction Force Result at Bearing Support-2
For 2.0 Ns/mm Damping Value**

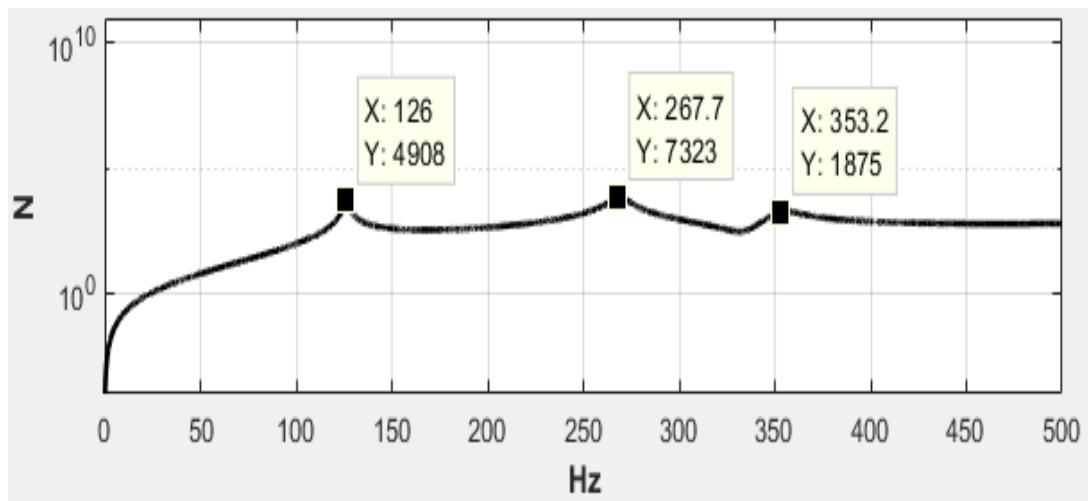


**Figure 82: Reaction Force Result at Bearing Support-3
For 2.0 Ns/mm Damping Value**

Appendix D: Damping Sensitivity on Displacement Result for Bearing Support-01

Damping Changing – Displacement peak value results for Bearing Support-01 and peak status are given in Figure 83 and Figure 84 respectively.

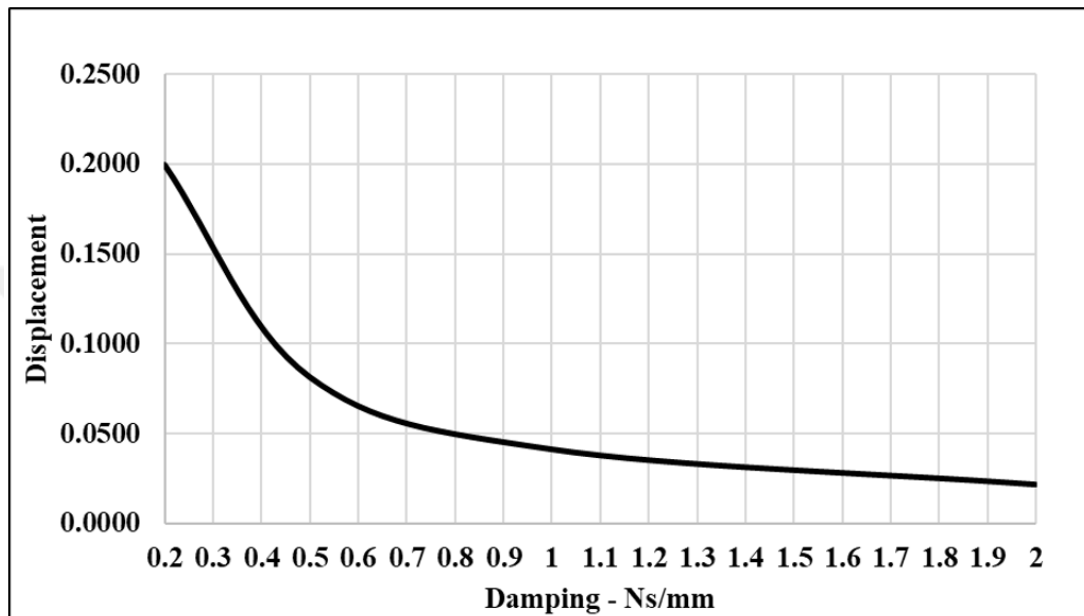


Figure 83: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-01 @ Peak-1 ~ 126Hz

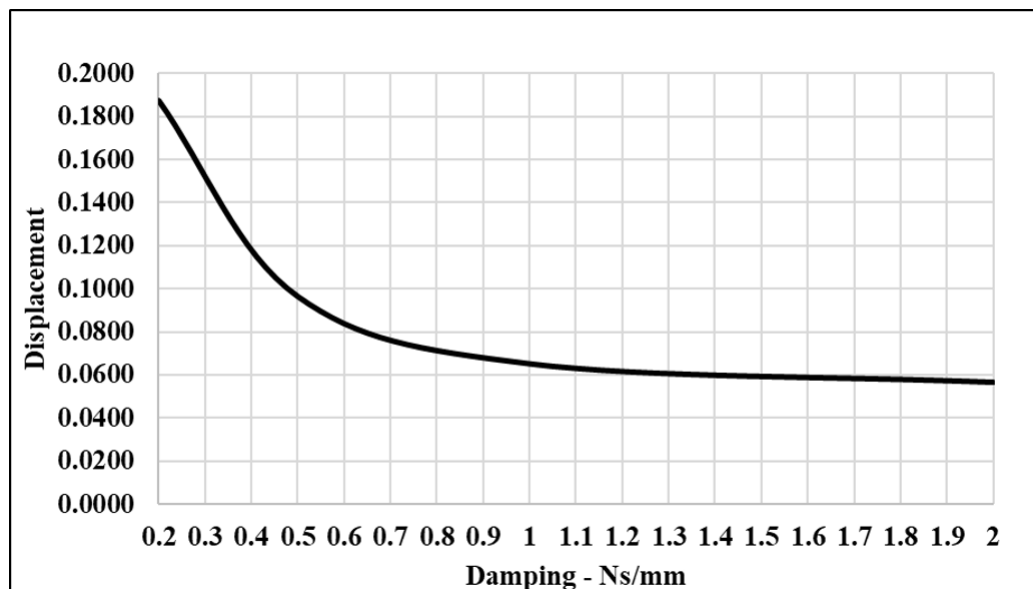


Figure 84: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-01 @ Peak-2 ~ 268 Hz

Appendix E: Damping Sensitivity on Displacement Result for Bearing Support-02

Damping Changing – Displacement peak value results for Bearing Support-02 and peak status are given in Figure 85, Figure 86 and Figure 87 respectively.

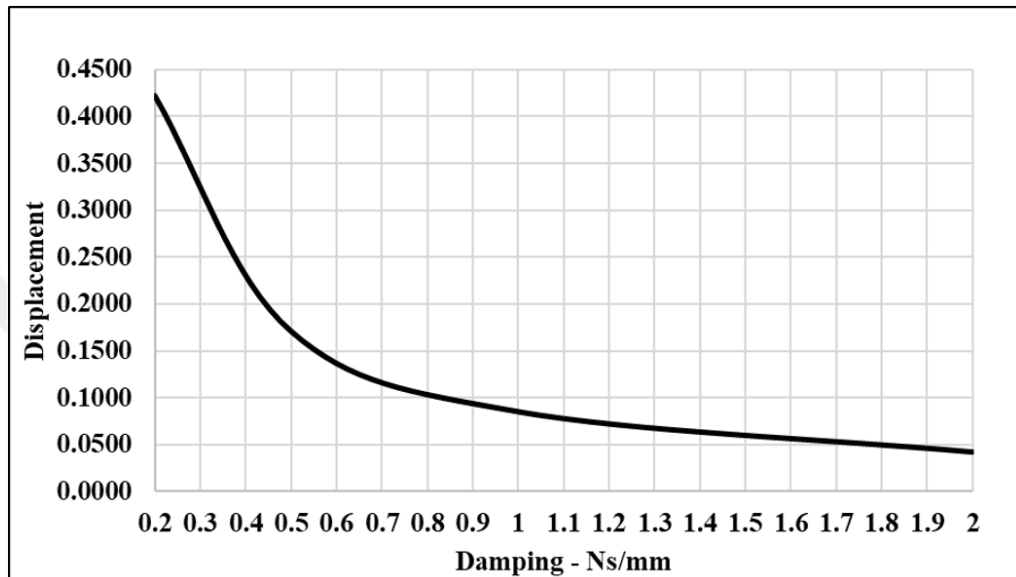


Figure 85: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-02 @ Peak-1 ~ 126Hz

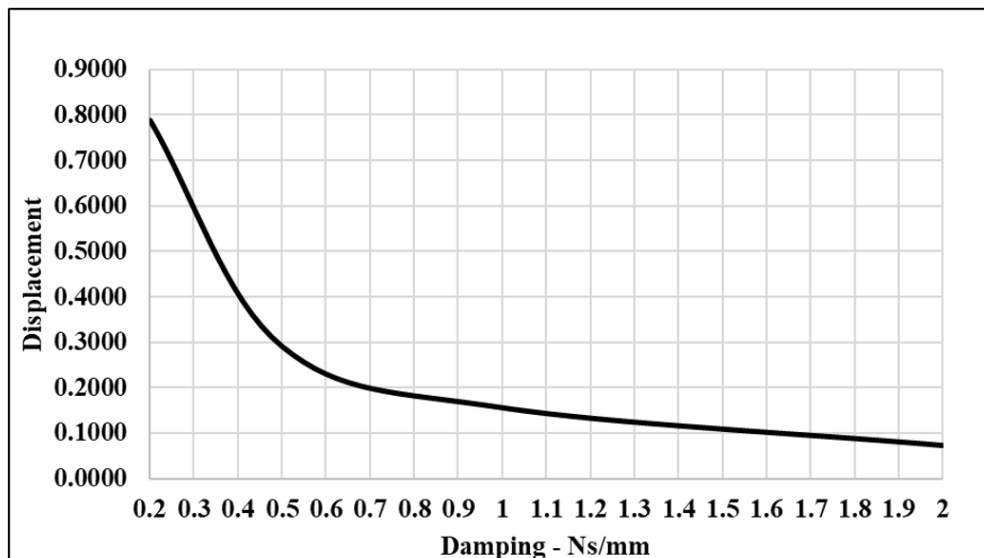


Figure 86: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-02 @ Peak-2 ~ 268 Hz

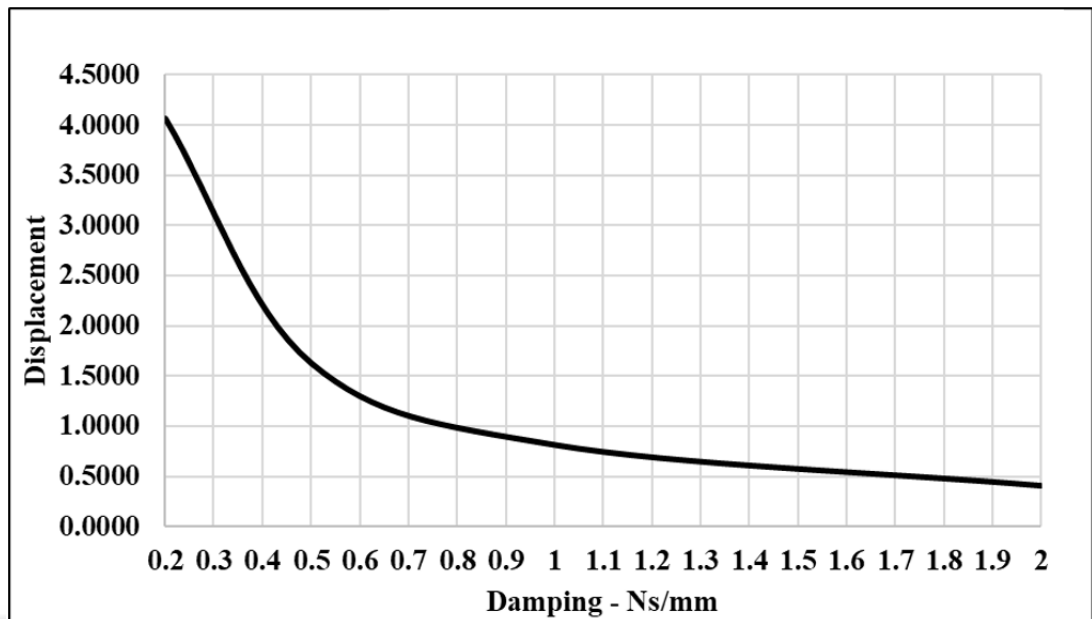


**Figure 87: Damping Ratio Changing Sensitivity on Displacement Results
for Bearing Support-02 @ Peak-3 ~ 353 Hz**

Appendix F: Damping Sensitivity on Displacement Result for Bearing Support-03

Damping Changing – Displacement peak value results for Bearing Support-03 and peak status are given in Figure 88, Figure 89 and Figure 90, respectively.

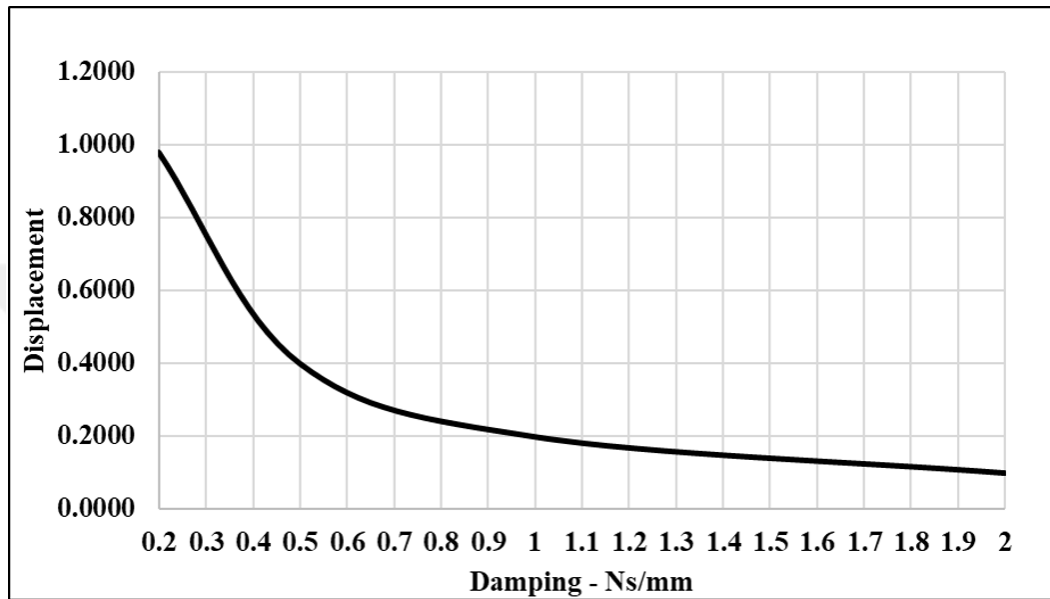


Figure 88: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-03 @ Peak-1 ~ 126Hz

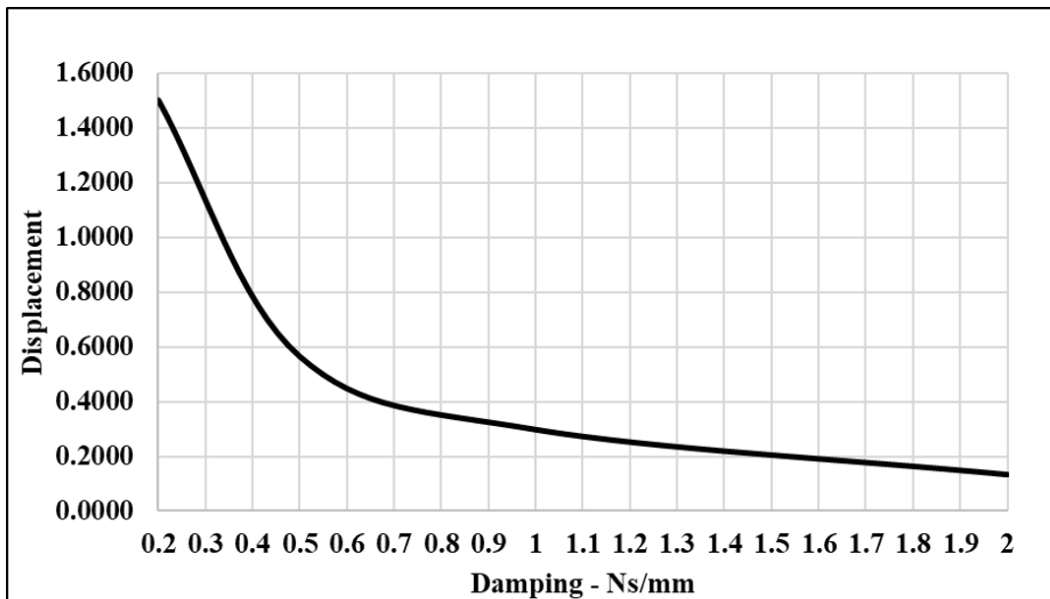


Figure 89: Damping Ratio Changing Sensitivity on Displacement Results for Bearing Support-03 @ Peak-2 ~ 268 Hz

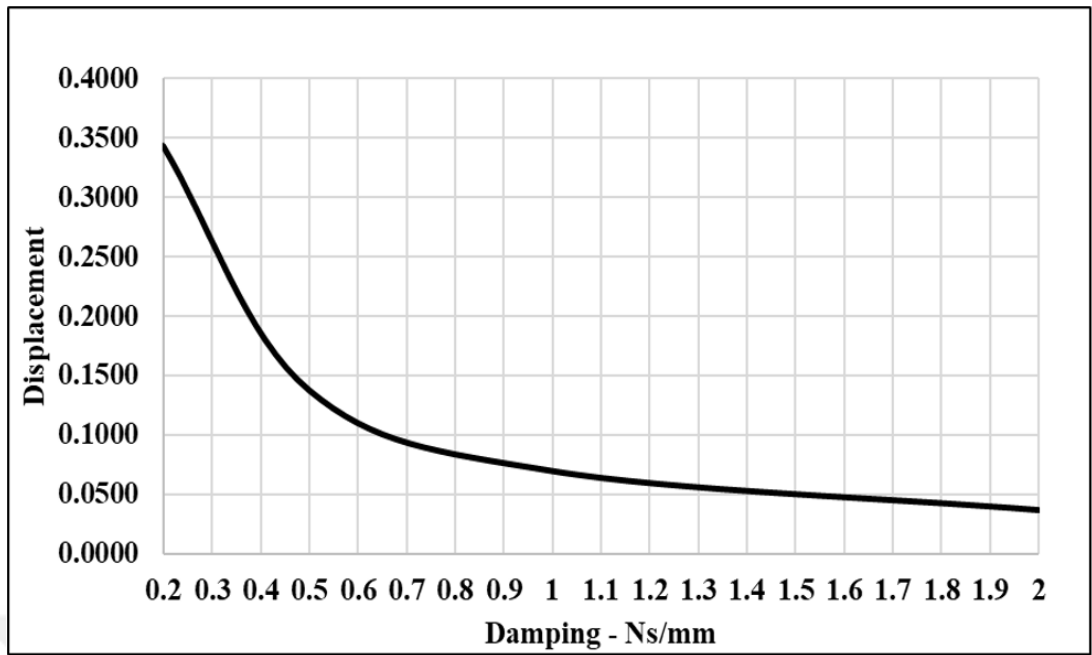


**Figure 90: Damping Ratio Changing Sensitivity on Displacement Results
for Bearing Support-03 @ Peak-3 ~ 353 Hz**

Appendix G: Damping Sensitivity on Reaction Forces Result for Bearing Support-01

Damping Changing – Reaction Force peak value results for Bearing Support-01 and peak status are given in Figure 91 and Figure 92 respectively.

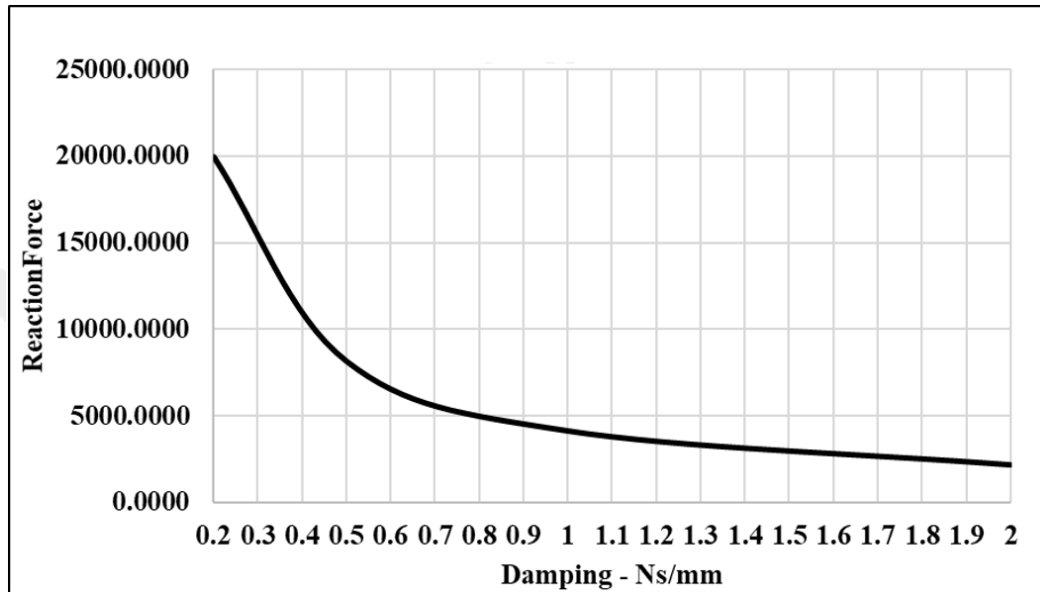


**Figure 91: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-01 @ Peak-1 ~ 126Hz**

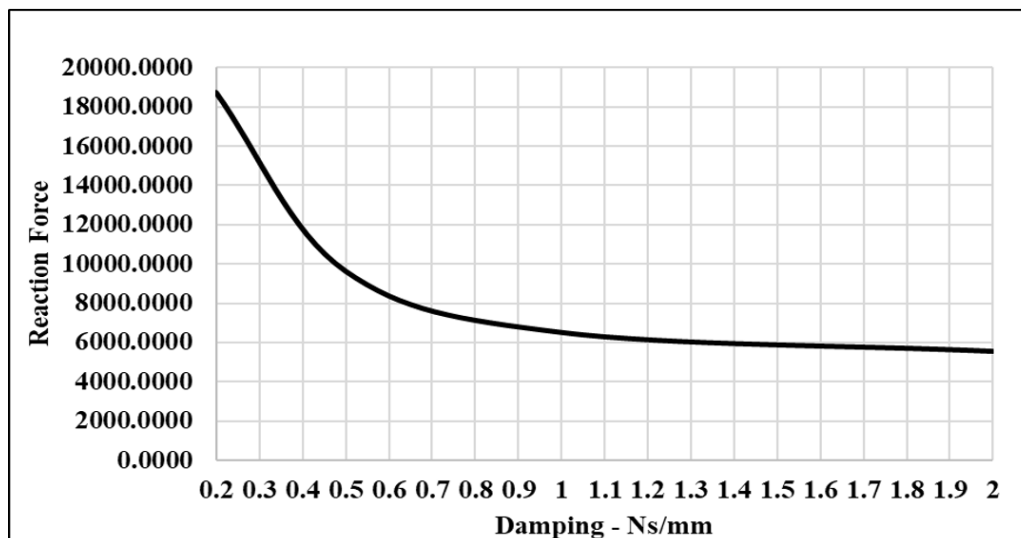


**Figure 92: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-01 @ Peak-2 ~ 268 Hz**

Appendix H: Damping Sensitivity on Reaction Forces Result for Bearing Support-02

Damping Changing – Reaction Force peak value results for Bearing Support-02 and peak status are given in Figure 93, Figure 94 and Figure 95, respectively.

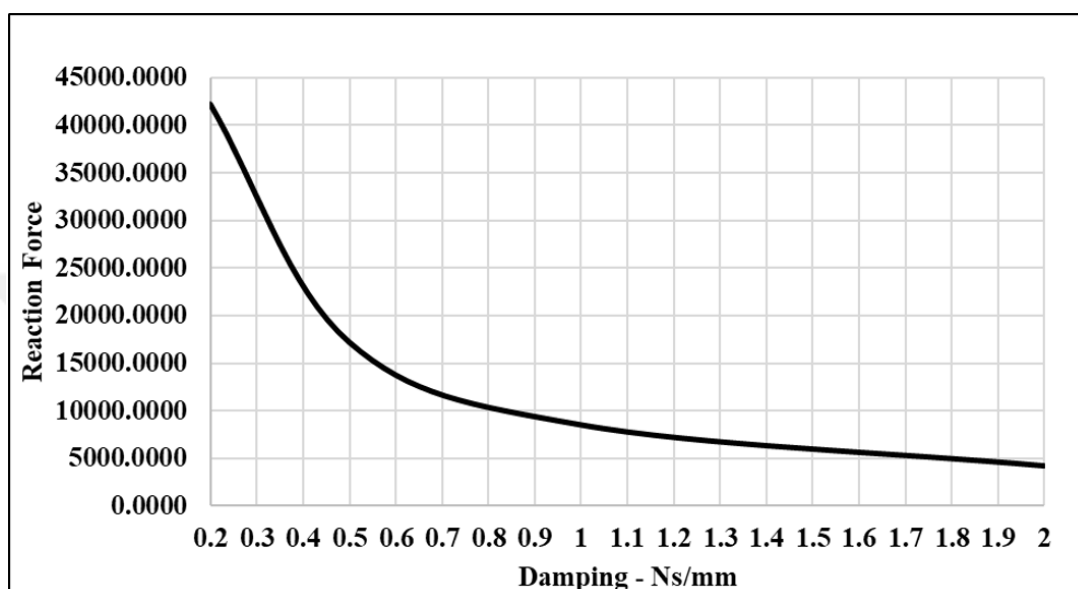


**Figure 93: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-02 @ Peak-1 ~ 126Hz**

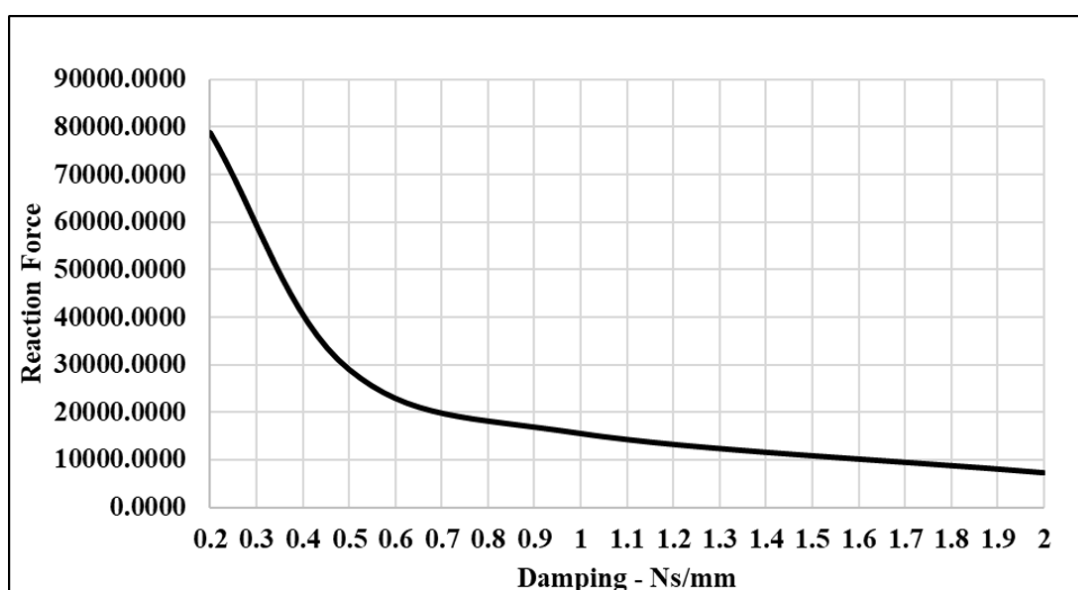


**Figure 94: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-02 @ Peak-2 ~ 268 Hz**

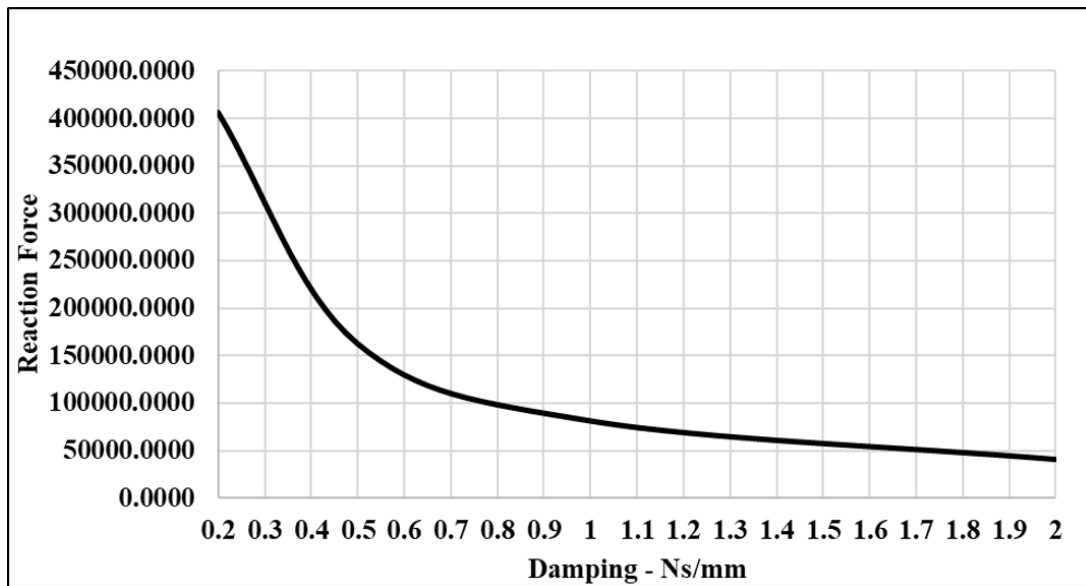


Figure 95: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-02 @ Peak-3 ~ 353 Hz

Appendix I: Damping Sensitivity on Reaction Forces Result for Bearing Support-02

Damping Changing – Reaction Force peak value results for Bearing Support-03 and peak status are given in Figure 96, Figure 97 and Figure 98, respectively.

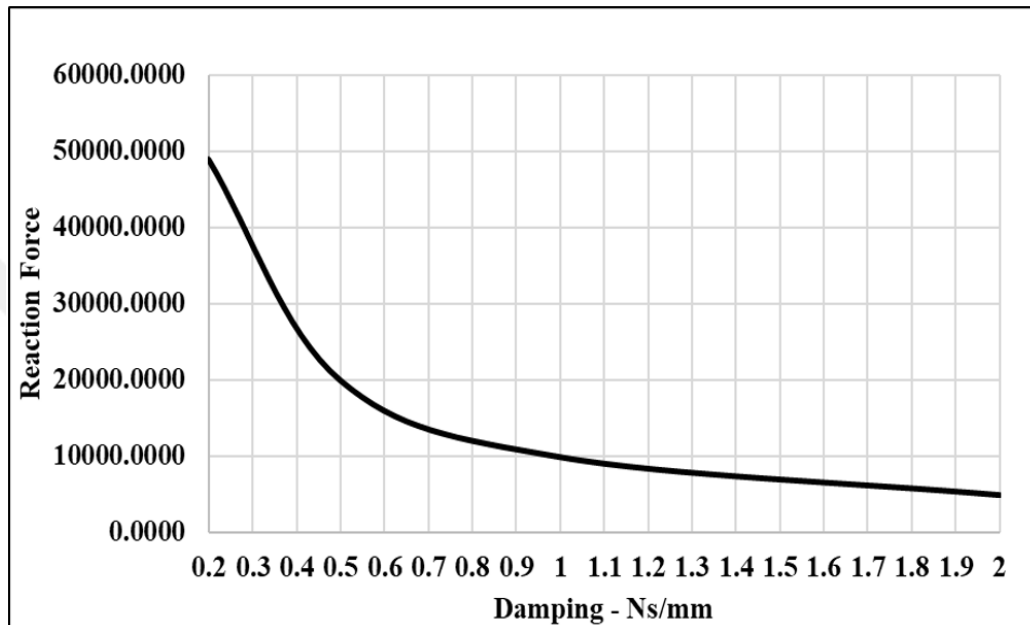


**Figure 96: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-03 @ Peak-1 ~ 126Hz**

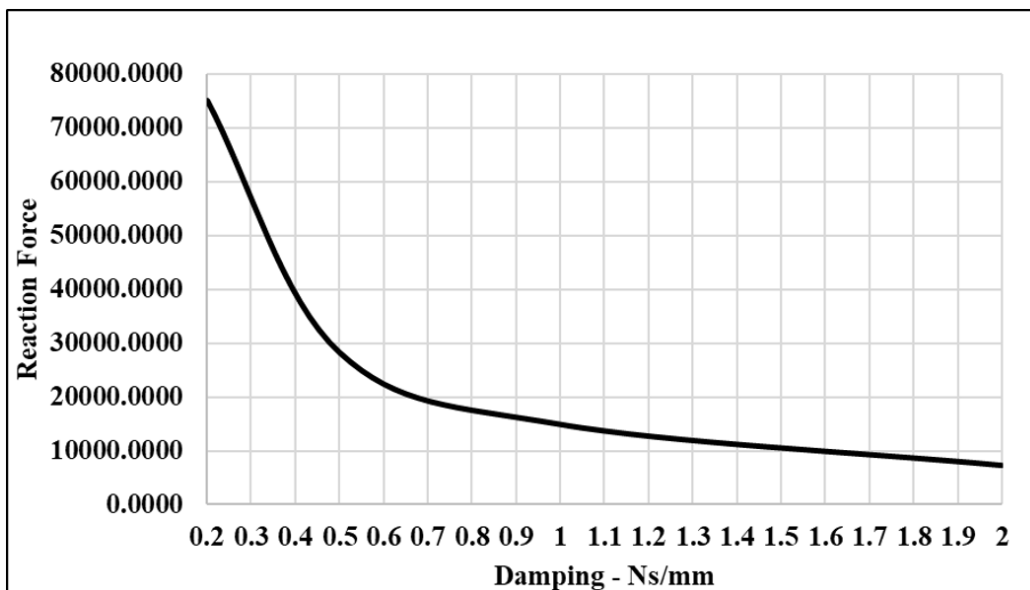


**Figure 97: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-03 @ Peak-2 ~ 268 Hz**

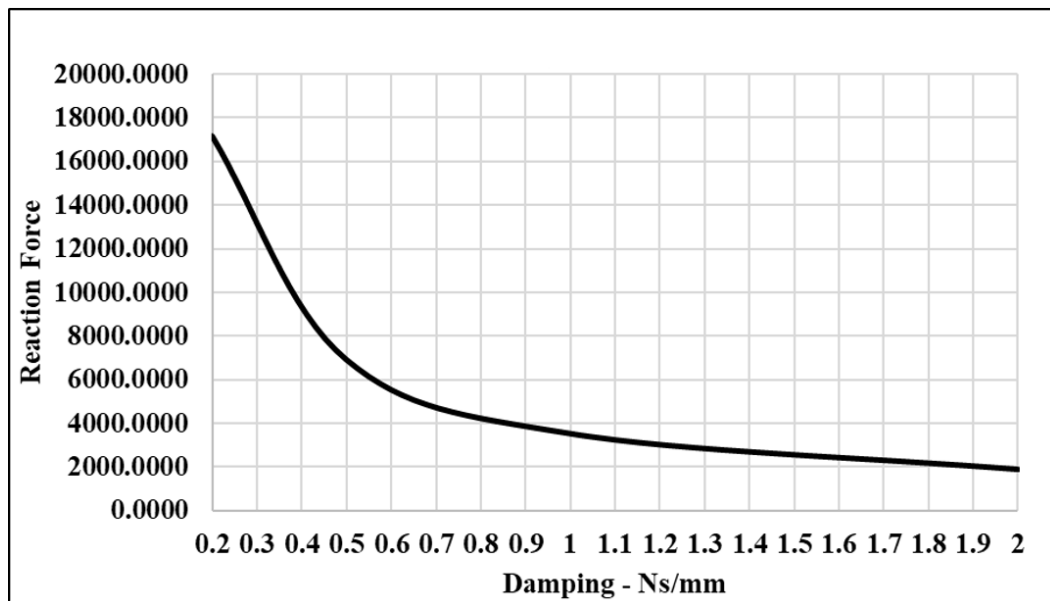


Figure 98: Damping Ratio Changing Sensitivity on Reaction Force
Results for Bearing Support-03 @ Peak-3 ~ 353 Hz

Appendix J: Publications in the Scope of Thesis Study

Kılıçaslan A., Niş H.T., (2022), “Load Calculation on Fan Blade Out (FBO) Event for a Turbofan Engine by Transfer Matrix Method and Validation” GTU 1st International Aerospace Propulsion System Congress, Gebze, Turkey, 20 April 2022

