MODELING PRICE AND DEMAND INTERACTION IN INVENTORY SYSTEMS

by

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ABSTRACT

MODELING PRICE AND DEMAND INTERACTION IN INVENTORY SYSTEMS

In this thesis, we examine the impact of price on demand in inventory systems with one and two suppliers. We present two models. In the first model, price determines the probability of demand occurrence. In case demand occurs, its amount is random and independent of the price. In the second model, price determines the distribution of the amount of demand by governing the purchasing probability of each potential customer. In both models, we try to find the order amount and the price that maximize the average one-period profit. We discuss how the optimal policy and the profit change with respect to the parameters related to cost, demand and the relationship between price and demand. Under each of the models, we present a variation in order to account for supply uncertainty. We discuss the benefit of taking the capacity constraints into account, and demonstrate the effect of parameters related to the capacity constraint on the optimal policy and profit.

ÖZET

ENVANTER SİSTEMLERİNDE FİYAT VE TALEP İLİŞKİSİNİN MODELLENMESİ

Bu tezde, fiyatın talep üzerindeki etkisi bir ve iki tedarikçi ile çalışılan envanter sistemlerinde incelenmiştir. Bu amaçla iki model sunulmuştur. İlk modelde fiyat, talebin varolma olasılığını belirlemektedir. Talep varolduğunda fiyattan bağımsız bir olasılık dağılımını takip emektedir. İkinci modelde ise fiyat, her potansiyel müşterinin satın alma olasılığını belirlemek suretiyle toplam talep miktarını etkilemektedir. Her iki modelde de, ortalama dönem karını enbüyükleyen sipariş miktarı ve fiyat belirlenmeye çalışılmıştır. En iyi kararın maliyet, talep ve fiyat talep ilişkisi parametrelerine göre değişimi de ayrıca incelenmiştir. Ayrıca, her iki model için birer alt model sunularak tedarik konusundaki belirsizliğin etkisi sorgulanmıştır. Kapasite kısıtını dikkate almanın sağladığı fayda gösterilmiş ve kapasite kısıtına ilişkin parametrelerin en iyi kararı ve karı nasıl etkilediği tartışılmıştır.

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LIST OF SYMBOLS

b	Lost sales cost per one unit of product
$B^i(.)$	Lost sales cost for the individual choice model
c	Ordering cost per one unit of product in one supplier settings
c_a	Ordering cost per one unit of product for the low-cost supplier in two suppliers settings
c_b	Ordering cost per one unit of product for the high-cost supplier in two suppliers settings
$CV(.)$	Coefficient of variation
D_n	Demand for period n for the model in Chapter 3
D_n^i	Demand for period n for the the individual choice model
$f(.)$	Probability density function of Δ
$F(.)$	Cumulative distribution function of Δ
$f^i(.\|\Pi(p_n))$	Probability density function of demand for the individual choice model given that the price is p_n
$F^i(.\|\Pi(p_n))$	Cumulative distribution function of demand for the individual choice model given that the price is p_n
h	Holding cost per one unit of product per unit time
$H(.)$	One-period profit for the model in Chapter 3
$H^i(.)$	Holding cost for the individual choice model
I_n	Inventory level at the pegining of period n before placing order
j_n	Actualized value of the random variable $J_n(p_n)$
k_D	Scale parameter for the Gamma distribution of D_n^i
k_Δ	Scale parameter for the Gamma distribution of Δ
$k^{i,II}(.)$	Ordering cost for the two suppliers setting of the individual choice model
$k^{II}(.)$	Ordering cost for the two suppliers setting of the model in Chapter 3
$\mathbf{J}(.)$	Vector of J(.)$values for all periods$
$J_n(p_n)$	Binary random variable indicating if there is demand in period n

$L(.)$	Sum of holding and lost sales costs
n	Period index
$\mathbf{p}$	Vector of prices for all periods
p_n	Unit sales price for period n
p^r	Revenue maximizing price
$P(.)$	Total profit for the one supplier setting of the model in Chapter 3
$P^{II}(.)$	Total profit for the two suppliers setting of the model in Chapter 3
$P^i(.)$	Total profit for the one supplier setting of the individual choice model
$P^{i,II}(.)$	Total profit for the two suppliers setting of the individual choice model
Q	Capacity of the low-cost supplier
$R(.)$	Revenue for the model in Chapter 3
$R^i(.)$	Revenue for the individual choice model
T_n	Duration of period n for the individual choice model
u_n	Order amount for period n
$\mathbf{y}$	Vector of order-up-to levels for all periods
y_n	Order-up-to level for period n
y^r	Order-up-to level that maximizes the profit given that the price is p^r
α	Discount rate
β	Coefficient to define the price sensitivity of the demand occuring probability
Δ	Random number denoting the volume of the demand when it is given that demand is to occur
λ	Scale parameter for the Gamma distribution of Q
μ	Scale parameter for the Gamma distribution of demand
$\mathbf{\Pi}(.)$	Vector of $\Pi(.)$ values for all periods
$\Pi(p_n)$	Probability that an individual customer makes a purchase in the individual choice model given that the price is p_n

$\Pi_0(p_n)$	Probability that demand does not occur as a result of p_n for the model in Chapter 3
$\Pi_1(p_n)$	Probability that demand occurs as a result of p_n for the model in Chapter 3

LIST OF ACRONYMS/ABBREVIATIONS

SQP	Sequential Quadratic Programming
WTP	Willingness to Pay

1. INTRODUCTION

Inventory systems have been thoroughly studied in the literature. Decision making on volume and frequency of orders, and the rules that drive these decision are analyzed. The impact of cost structure, including inventory holding cost, fixed and variable ordering costs, lost sales or backorder costs on optimal ordering policy are examined. The effect of the nature of demand on inventory decisions has also been investigated.

In another area of research, the effect of price on customer behavior is studied. The literature on pricing is quite wide as it involves researches from various disciplines including operations research, management, economics and psychology. The researches on this subject that are relevant to our study include studies on reservation price and willingness to pay, and discrete choice models.

Reservation price and willingness to pay refer to the maximum amount that a person would pay for a good or a service. They are widely studied in the context of auctions. Another lines of study on these two concentrate on methods to measure willingness to pay and willingness of people to pay for public goods. Discrete choice models, on the other hand, are used to examine settings where customers are to make choices out of a finite set of alternatives in order to maximize the utility they acquire from their choices. The structures of the models in this area differ from each other in whether there exists a correlation between alternatives, and how to model the correlation, if any.

Separate researches on inventory and pricing have been useful to a certain extent. However, starting in 1950s, a new line of research arose. Joint consideration of the decisions on inventory and price is offered in order to outperform the policies driven by separate consideration of these two. Papers regarding joint decision making on inventory and price typically suppose that price affects the volume of the demand.

These papers vary in how they model the way price defines the volume, and whether they assume deterministic or probabilistic demand as a function of price.

In this thesis, we study the interaction of price and demand in inventory systems. Our first model differs from the literature in that we model not the volume of the demand, but the probability of demand occurrence as a function of price. In case demand occurs, its amount follows a certain probability distribution independent of price. The contribution of our second model is that we model the demand volume as a function of the purchasing decision of each potential customer, and this decision is probabilistically determined by price. In other words, we incorporate a price-driven choice model into an inventory system.

Our first and second models allow adjustment of price elasticity of probability of demand occurrence and the volume of demand, respectively. Therefore, we study the impact of this elasticity on both pricing and ordering decisions. We also examine how cost parameters, such as holding, lost sales and ordering costs change the optimal policy of price and ordering.

Another aspect of inventory systems that this thesis features is uncertainty in supply. We first present a review of the literature on supply uncertainty. We mainly discuss three groups of papers. First group include papers that focus on yield uncertainty. Second group of studies examine the uncertainty of availability of supplier, some of them regarding the randomness of the length of the duration the supplier is available. Finally, the last group analyzes the multiple supplier settings.

We offer two sub-models for each of our two settings. In the first sub-models for each setting, we examine a retailer working with a single supplier, and assume that the capacity of the supplier is infinite. In the second sub-models, however, we impose an uncertainty in the supply process. The retailer works with two suppliers where one of them offers a low unit cost and the other one offers a higher unit cost. The low-cost supplier has a finite capacity following a probability distribution, therefore when the order the retailer places exceeds that capacity, the excess amount is met by the high-cost supplier, which has an infinite capacity. In this way, we incorporate supply uncertainty into an inventory system in which demand is probabilistically determined by price.

We investigate the effect of supply uncertainty on the decisions of the retailer. We examine the change in the optimal ordering and price decisions with respect to three aspects of the underlying uncertainty. First factor is how much more expensive the high-cost supplier compared to the low-cost one. Second factor is the expectation of capacity of the low-cost supplier compared to the expected demand. Last one is the variability of the capacity. After observing the effects of these three parameters on the decision variables, we continue the analysis by comparing the profit earned in optimal policy to the profit of other two approaches. First approach is to assume infinite capacity for the low-cost supplier, and making inventory and price decisions accordingly. By comparing the profit made in this approach to the profit in optimal policy, the benefit of accounting for the supply uncertainty is observed. Second approach is to determine the price on revenue-maximizing basis and to decide of order amount given that price. Comparison of this method to the optimal strategy enables us to observe the impact of cost structure on the profit in an inventory system with price-driven demand and supply uncertainty.

The thesis is structured as follows. In Chapter 2, we review the literature on pricing including reservation price and willingness to pay, discrete choice models, price effect of demand volume. We also review the literature on supply uncertainty types including multiple supplier settings and uncertainties on availability and amount of supply. In Chapter 3, we present our model that regards the impact of price on probability of demand occurrence in an inventory system, and then discuss the results. There are two sub-models in this chapter, and the second one incorporates supply uncertainty into the current setting. In Chapter 4, we present our model in which price probabilistically determines the purchase decision of each potential customer in an inventory system, and then discuss the results. This chapter as well includes two sub-models second one of which is designed to account for supply uncertainty. Finally, conclusions are drawn and directions for future research are offered in Chapter 5.

2. LITERATURE REVIEW

The focus of this study is on the interaction between price and demand. We first present paper regarding this subject. We examine these papers in three groups. First group regards the papers that investigate the impact of price on the volume of the demand in inventory systems. The second group consists of the papers that study reservation price and willingness to pay. Finally, the third group of papers examine discrete choice models.

We also incorporate supply uncertainty into the models that we establish in examining the relation between price and demand. The papers regarding supply uncertainty also consist of three groups. These groups examine yield uncertainty, availability of supply, and multiple supplier settings, respectively.

2.1. Pricing

2.1.1. Price Determining Demand Volume

In the first effort in the literature to analyze pricing and inventory decisions jointly, Whitin (1955) states that economic theorists study price-demand relationship in order to maximize profit, and businessmen focus on inventory control to minimize cost, but a model to incorporate these two concerns would be more realistic and therefore useful. To this end, he adds price as a variable to existing inventory models and offers solutions to the problem of jointly determining inventory and price for various settings.

Elmaghraby and Keskinocak (2003) state that dynamic pricing has been used in industries with perishable products or services for a long time; but recently, its usage in industries with storable products emerged as well because demand data became more available, price changes became easier with new technologies and opportunity to use decision-support tools to utilize demand information for pricing arose. Along with an extensive review of literature and practices, they offer three criteria to classify the settings in pricing with inventory concerns.

First criterion is whether there is a chance to replenish inventory during planning horizon. Some products with short life cycles may not be possible to replenish within the season because of long supply lead times whereas some other types can be replenished periodically. Second measure to consider is the time dependency of demand. Total demand over time for a durable good may be fixed, either being known or unknown, or repeated purchases may be limited. If this is the case, then sales in a period implies less sales in following periods. Another factor causing time-dependent demand may be accumulated customer knowledge on product, which results in differing demand patterns through the life cycle. On the other hand, products that are to be purchased repeatedly, or products with selling periods not long enough to let accumulation of customer knowledge can be assumed to have time-independent demand. Last criterion in the classification is whether the customers of the product of interest tend to make myopic or strategic decisions. Myopic customers consider only current price, however strategic customers regard the future path of the price thus forcing the seller to take into account the impact of current and future prices on customer behavior.

There are two widely used types of random demand functions relating price to demand. Additive demand function models randomness of the demand as a noise added to mean demand, and therefore variance of demand is constant in mean. Multiplicative demand, on the other hand, models randomness as a multiplicative factor thus making the variance increasing in mean.

Chen and Simchi-Levi (2004) consider a model involving both pricing and inventory decisions for a finite horizon. Objective function is the expected profit over the finite horizon, and setup cost is positive. They conclude that for additive demand models, the profit-to-go functions are k-concave, making (s, S, p) policy optimal where (s, S) has its standard meaning in the contexts with only inventory decision, and p is determined regarding the inventory position at the beginning of each period. However, this conclusion does not apply to demand functions consisting of both additive and multiplicative parts as the profit-to-go function is no more k-concave. Also, they point out that it is not possible to include any capacity constraints into this solution because (s, S) policy is not optimal when capacity is finite.

Federgruen and Heching (1999) incorporate the additive and multiplicative demand functions. Under full backlogging, they examine both finite and infinite horizon models. They offer two variations with the first one allowing increasing and decreasing the price, and the second one allowing only decreasing. They offer a value iteration method to find optimal inventory levels and prices.

There is also a line of research trying to incorporate capacity constraint into the joint decision on price and inventory. Yao *et al.* (2006) try to unify existing demand models to get rid of the necessity of using some specific demand functions to solve the problems involving inventory and price decisions, and come up with an easily interpretable general demand model. Considering demand functions consisting of the mean and random demand, they conclude that as long as the mean demand has increasing price elasticity and the random demand has generalized strict increasing failure rate, then the expected profit function is unimodal or quasi-concave.

In Klemperer and Meyer (1989), an oligopoly is studied under supply uncertainty. The authors come up with a 'supply function' to relate quantity to price, which, they state, helps the firm to act better than the cases where either quantity or price is fixed. Uncertainty significantly reduces the number of equilibria, and the paper shows the existence of a Nash equilibrium under a symmetric oligopoly with a homogeneous product, and finds the conditions that guarantee uniqueness. As the number of firms decrease, or the differentiation between products increase, the supply functions in equilibria becomes steeper. As the steepness of the supply functions in equilibria increases, the competition gets closer to the case with fixed quantities whereas decreasing steepness brings the competition closer to the case with fixed prices.

In Tang and Yin (2007), no responsive pricing and responsive pricing is compared for a retailer where supply yield is uncertain and product demand is linear in price. In the former policy, the retailer decides on order quantity and the retail price before observing the supply yield. In the latter one, however, order quantity is determined first, and the retail price is set after the supply yield is realized. The conclusion is that the responsive pricing policy always creates higher expected profit as it enables the retailer to utilize pricing in responding to the uncertainty in supply. They also analyze two extensions with the first one involving a chance for the retailer to place an emergency order after observing the yield, and the second one obliging the retailer to distribute the order between more than one suppliers.

In Deng and Yano (2006), under deterministic demand functions and capacity, there is a positive setup cost associated with a production run, in addition to the cost per unit produced. Both the demand function and the capacity are allowed to change over periods. They draw four main conclusions. First, they state that optimal prices are not necessarily decreasing in capacity even for constant capacity settings. Second, the marginal returns sometimes increase in increasing capacity. In addition, the authors compare their method to a sequential procedure in which price is determined first and then decision on production is made accordingly. It turns out that in order for the sequential method to create a near-optimal solution, the price decision-makers should be informed on details much better than they usually are; otherwise the procedure with simultaneous decision making significantly outperforms it. Last, the authors suggest firms to use pricing more actively than common practice to manage demand in the following fashion: When demand patterns vary over periods, firms should aggressively suppress demand by increasing price, and produce at capacity to increase supply for periods with higher incremental profit.

In this subsection, we covered papers that examine the interaction between price and demand in inventory systems and offer various ways of modeling this interaction. The aim, in general, is to figure out the optimal policy of ordering and pricing. They typically relate the price to the volume of the demand. Some of them use a deterministic model to represent this relation whereas the others model this interaction probabilistically.

2.1.2. Reservation Price and Willingness to Pay

Reservation price and willingness to pay (WTP) refer to the maximum amount that a person would pay for a good or a service. They are relevant to this study as they certainly impact the probability of a potential customer to make a purchase for a given price.

Reservation price is studied in the literature in the context of auctions. Levin and Smith (1996) examine optimal reservation prices in auctions. Under risk-neutral independent private values, seller is better off by puzzling bidders by announcing a reservation price higher than the true value, no matter how many bidders are involved in. However, when information is correlated, optimal reservation price to announce converges to true value as the number of bidders increase.

Elyakime *et al.* (1994) focus on first-price sealed-bid auctions and show that strategy of public reserve price is better for seller than strategy of secret reserve price, and propose a method to evaluate the gain from moving from secret to public reserve price.

Lizzeri and Persico (2000) study first price auction, the combination of first and second price auctions, war of attrition, and the all pay auction. They prove the uniqueness and existence of equilibrium in auctions with a reserve price.

Wertenbroch and Skiera (2002) categorize methods to measure WTP regarding whether they give the customers an incentive to tell their true WTP and whether they 'simulate actual point-of-purchase contexts'. They test the incentive-compatible method proposed by Becker *et al.* (1964) and show that it gives lower WTP estimates than methods that are not incentive-compatible, such as open-ended contingent valuation. In the end, they conclude that the differences in WTP estimates are caused by whether the facilitator provides an incentive to reveal the true WTP, rather than how much effort the respondents spend in responding.

Another group of papers focuses on WTP for public goods. Kahneman *et al.* (1993) question the assumptions of the contingent valuation method. In a survey, people state how much they are willing to pay to avoid a problem related to public health or environment, and WTP turns out to be strongly correlated with the other measures of attitudes, e.g. rating of the importance of the issue. Authors conclude that the results of a survey on people's WTP for public goods reflect their attitudes rather than economic valuations.

Flores and Carson (1997) examine the income elasticity of WTP for environmental and public goods and show that it is not necessarily in accordance with income elasticity of demand. They state that the reason is that the income elasticity of WTP is affected remarkably by some unobservable factors.

In this subsection, we presented papers that regard reservation price and WTP. The papers from the perspective of the seller try to figure out how the seller should make the pricing decisions. Other papers investigate what impacts the decisions of the customers or how to measure their WTP.

2.1.3. Discrete Choice Models

In this thesis, we examine systems where potential customers are to make choices between purchasing and not purchasing given the price of the product offered. Therefore, in this subsection, we present papers that regard situations where the customers make choices depending on the attributes of the products offered.

In random utility models, a customer is to make choice out of a set of alternatives. She desires to maximize the utility she acquires from this decision, and this utility consists of a part that is observable to the researcher and the other part that is unobservable.

Observable part of the utility gathered by a customer from a particular alternative can be computed given the attributes of the alternative and the customer. The unobserved part, on the other hand, is considered a random variable. Categorization of random utility models is based on the choice situation of interest, and they are specified by which distribution they attribute to the random part of the utility.

Multinomial logit is the most commonly used model because of its simplicity. It took its current form with the contribution from number of researchers. A study by Thurstone (1927) in a psychological context was utilized in economics by Marchak (1960), and a fundamental property called "independence from irrelevant alternatives" was developed by Luce (1959). The model assumes that the random part follows independent and identical extreme value distribution. Therefore, it is suitable for choice situations where alternatives have uncorrelated unobservable parts with equal variances.

In order to account for the dependence between the unobservable parts of the utilities of the alternatives, models that are more complex are developed. Nested logit, for instance, groups alternatives such that correlation of unobserved factors is the same for the alternatives within a group (nest) and is zero for alternatives in different nests. Forinash and Koppelman (1993), for instance, apply this model to a transportation problem. Generalized nested logit allows alternatives to belong to more than one nest, and they can belong to these nests to different degrees, as instructed by Train (2009).

Probit models dependence in a different fashion. Unobserved factors are considered to have a joint normal distribution, thus allowing the researcher to specify the correlation between each pair of alternatives. Hausman and Wise (1978) and Daganzo (1979) contributed to the earlier works of Thurstone and Marchak for the probit model to take its current form, as recognized by Train (2009). The main restriction of the probit model is that the normal distribution assumption may not be always appropriate.

Mixed logit is the most general discrete choice model and was used first by Boyd and Mellman (1980) and Cardell and Dunbar (1980) for automobile industry. In mixed logit, unobserved utility consists of a part that has independent and identical extreme value distribution, and the other part with any distribution, to account for correlation.

In this subsection, we covered papers that study discrete choice models. The way the models of this subsection are established depends mainly on whether there exists a correlation between the alternatives, and how this correlation is represented, if any. Researcher is to select the appropriate discrete choice model for the situation of interest under the usual tradeoff between generality and tractability.

2.2. Supply Uncertainty

There are three important surveys on supply uncertainty. Tajbakhsh *et al.* (2007) review works on both lead-time and yield uncertainty. Minner (2003), on the other hand, focuses on multi-supplier settings with lead-time uncertainty. Yano and Lee (1995) present an extensive overview of works on yield uncertainty. They categorize the types of uncertainties in yield into five groups.

The first type of models assume that production of a conforming unit of product is a Bernoulli process, thus making the distribution of number of conforming units in a batch a Binomial distribution with parameters Q and p. This approach is advantageous in that it is enough to specify p. However, it is realistic only when it makes sense to assume that the production process is stationary and qualities of the units in a batch are not autocorrelated.

The second approach is to specify the fraction conforming; therefore, it allows one to specify the variance of the fraction conforming as well as the mean, as opposed to the first approach. However, the assumption that the fraction good is independent of the batch size remains. This model is reasonable when batches are large and relatively stable from production run to production run.

The third approach allows modeling the dependence of the distribution of the fraction good on the batch size. It is appropriate to use this approach to model processes in which the fraction conforming changes over production runs. It may decrease as in the case of increasing failure rate caused by tool wear, breakage etc., or increase as in the cases with a startup processes. Henig and Gerchak (1994), cited by Yano and Lee, introduce the interrupted geometric model in which the process produces only conforming units until it goes "out of control", i.e. the state where it produces only nonconforming units; and the time to go out of control is distributed geometrically. Yano and Lee offer reversing this approach to model the cases with decreasing failure rate.

The fourth type of model involves the uncertainty of the capacity. The output quantity is the minimum of input quantity and the random capacity; and the decision-makers are to determine the input quantity.

The last type of model is the most general one. It involves specifying the probabilities of each possible output quantity to occur for each possible batch size. Obviously, it necessitates a great effort in collecting data for a wide spectrum of batch sizes, including the ones that are not frequently used.

Papers dealing with yield uncertainty will be classified as ones with continuous and discrete time models like Yano and Lee do. Regarding continuous models, Silver (1976), models yield uncertainty as a result of defective units, administrative errors, damage etc. Expected amount received is proportional to lot size, and there is a positive probability of amount received being larger than amount ordered. Two variations are the one with amount received having standard deviation independent of lot size, and with a standard deviation proportional to lot size. Shih (1980) narrows the reasons of uncertainty to only defections; and both Silver and Shih end up with optimal order expressions slightly different from standard EOQ formula.

Kalro and Gohil (1982) offer an extension to Silver's model by allowing backlogging of demand either completely or partially. Mak (1985) relaxes the assumption of Kalro and Gohil that the fraction that can be backlogged is constant, and makes it a random variable. The optimal lot size and optimal duration to stay stocked out turns out to be the functions of the first two moments of fraction conforming and fraction of demand that can be backlogged.

Under Poisson demand, Moinzadeh and Lee (1987) examine yield uncertainty under continuous review. They approximate the operating characteristic to find optimal or near-optimal solutions for ordering policy.

In discrete time, Karlin (1958) initiates the research by a single period model. He proves for a model allowing only a fixed amount of order that if holding and shortage costs are increasing and convex, then there is a certain inventory level such that an order should be placed if on-hand inventory is below that level, and otherwise it is better not to order. In a more general model allowing several ordering levels, under the assumption that the likelihood of distribution of delivery amount is monotone, he shows that there are intervals of on-hand inventory levels corresponding to intervals where a particular order level is optimal.

Parlar and Berkin (1991) focus on uncertainty regarding the length of the duration of the availability and unavailability of supply. They try to evaluate the best quantity to order. They conduct a numerical study for cases for two special distribution of length of availability and unavailability periods.

Güllü *et al.* (1999) examine an inventory system in which supply is either available or completely unavailable. Under deterministic dynamic demand, they prove the optimality of order-up-to level policy and provide a newsboy-like formula for optimal order-up-to levels. For a model that also allows partial availability of supply, they present a numerical study.

In Anupindi and Akella (1993), inventory policies are examined for a buyer with two uncertain suppliers. The authors mention three different models as follows. In Model I, contracts oblige the full amount of order to be delivered at once. It is delivered in the current period with a certain probability and in the next period with the remaining probability. In Model II, however, uncertainty is related to the fraction of the order to be delivered rather than delivery time. A random fraction of the order is delivered in the current period, and the remainder is cancelled. Model III is rather an extension of Model II as the remainder is delivered in the next period.

In Kouvelis and Milner (2002), supply uncertainty is studied along with demand uncertainty to determine how they affect capacity and outsourcing decisions. The impact of supply uncertainty on investment levels is investigated for both single and multi-period settings in cases where the market is affected by that particular firm's investments. They conclude that the need for vertical integration increases in increasing uncertainty in supply process.

Burke *et al.* (2007) study the decision on working with single or multiple suppliers. In a single period, single product setting, they involve supplier reliabilities and firm-specific inventory costs; and conclude that the single supplier strategy is better only when the capacity of the supplier is large compared to the demand and there is no benefit from diversification to the firm. In all other cases, multiple sourcing performs better.

In Serel (2007), a risk-reducing approach to handle supply uncertainty is offered. With uncertainty of the quantity of input available in the spot market, the paper studies a capacity reservation contract in a multi-period setting. The contract obliges the supplier to make available a predetermined amount of input, and the manufacturer pays a fixed amount to the supplier at each period. The manufacturer is also allowed to purchase from the spot market. The paper offers an analytical solution to the optimal policy for the manufacturer. It concludes that the uncertainty in the market increases the percentage of input bought in advance, and the capacity reservation contracts increase the capacity utilization of the supplier.

We covered three types of papers in this section. The first group of papers focus on yield uncertainty. The second group examines the uncertainty of availability of supplier, some of them regarding the randomness of the length of the duration the supplier is available. Finally, the last group analyzes the multiple supplier settings.

The first model we present in this thesis differs from the literature in that in the inventory systems we present, we model not the volume of the demand but the probability of demand occurrence as a function of price. Demand follows a probability distribution independent of the price, in case it occurs. Our second model, on the other hand, represents the volume of the demand as a result of a choice model for potential customers. In the extensions of both of the models, we represent the uncertainty in supply in settings with two suppliers where the capacity of the lower-cost supplier is finite and random.

3. AN INVENTORY MODEL WITH PRICE EFFECT ON THE PROBABILITY OF DEMAND OCCURRENCE WITH ONE AND TWO SUPPLIERS

3.1. Model

In this section, we present an inventory system in which the price determines the probability of demand occurrence. In case demand occurs, it follows a probability distribution independent of price. In Section 3.1.1, the retailer of interest works with a single supplier who has an infinite capacity. In Section 3.1.2, on the other hand, there are two suppliers. First one of these suppliers offers a low unit cost and has a finite random capacity whereas the second one offers a higher unit cost and has an infinite capacity.

3.1.1. One Supplier Setting

In this model, an inventory system is considered. A retailer holds inventory for a single product for which there is demand D_n that occurs only in some periods. When it occurs, it is represented by Δ, and Δ follows a known probability distribution with probability density and cumulative distribution functions $f(.)$ and $F(.)$, respectively.

For each unit of inventory hold, the retailer incurs a holding cost h per unit time. Any unsatisfied demand is lost, and costs the retailer a given shortage cost b. Also, there is a given cost c for ordering one unit of product. Capacity of the supplier the retailer works with is assumed to be infinite.

At the beginning of each period, the retailer has the information on his inventory I_n, price p_n for the current period, and j_n, indicating whether there will be demand in the current period. Then she is to decide on what price to announce for the next period, and how much to order for the current period.

Inventory models in the literature that examine the interaction of price and demand typically suppose that price determines the volume of the demand, either deterministically or probabilistically. In this study, however, we present a model in which the price determines the probability of demand occurence. In case the demand occurs, its volume is random and independent of price. The price determines the probability of demand occurrence by the following function.

$$\Pi_1(p_n) = e^{-\beta p_n} \tag{3.1}$$

where β is a parameter to account for the price elacticity of the probability of demand occurence. The probability of not having demand is represented by $\Pi_0(p_n)$, and computed as

$$\Pi_0(p_n) = 1 - e^{-\beta p_n} \tag{3.2}$$

Lead time is assumed to be zero. That is, inventory increases right at the period the order is placed. No setup cost is incurred for ordering. The system is examined for infinite horizon.

Since inventory level at the beginning of period n is I_n, by ordering u_n units, the retailer reaches the level y_n before the demand for that period is observed. Therefore,

$$y_n = I_n + u_n \tag{3.3}$$

Denoting the demand for period n by D_n, the number of units sold in a period is the minimum of y_n and D_n. Obviously,

$$I_n = (y_{n-1} - D_{n-1})^+ \tag{3.4}$$

$J_n(p_n)$ represents the binary random variable indicating whether or not there is demand for period n. The probability mass fucntion of this random variable is determined by

p_n via Equation 3.1. j_n is the actualized value of this random variable.

$$J_n\left(p_n\right) \in \{0,1\} \tag{3.5}$$

$$j_n \in \{0,1\} \tag{3.6}$$

Δ represents the volume of the demand in case demand occurs. $D_n(.)$. on the other hand, represents the demand whether or not it occurs. Therefore,

$$D_n\left(0\right) = 0 \tag{3.7}$$

$$D_n\left(1\right) = \Delta \tag{3.8}$$

Since the retailer knows at the beginning of a period whether demand is to occur for that period, j_n is used instead of $J_n(p_n)$ for the formulations that regard the calculations for the current period.

$L(y_n, j_n)$ is defined as the total cost of holding and lost sales for period n. Then taking the expectations over both the probability of demand occurrence and the volume of the demand,

$$L\left(y_n, j_n\right) = hE\left[\left(y_n - D_n\left(j_n\right)\right)^+\right] + bE\left[\left(D_n\left(j_n\right) - y_n\right)^+\right] \tag{3.9}$$

$R(y_n, j_n, p_n)$ is the one-period revenue, calculated as

$$R\left(y_n, j_n, p_n\right) = p_n E\left[\min\left(y_n, D_n\left(j_n\right)\right)\right] \tag{3.10}$$

$$= p_n\left(y_n - E\left[\left(y_n - D_n\left(j_n\right)\right)^+\right]\right) \tag{3.11}$$

Then, $V(y_n, j_n, p_n)$ is defined to be

$$V(y_n, j_n, p_n) = R(y_n, j_n, p_n) - L(y_n, j_n) - cy_n \tag{3.12}$$

$$= (p_n - c)\, y_n - (p_n + h)\, E\left[(y_n - D_n(j_n))^+\right] - bE\left[(D_n(j_n) - y_n)^+\right] \tag{3.13}$$

Profit for period 1 is

$$P_1(y_1, j_1, p_1) = E\left[V(y_1, j_1, p_1)\right] + cI_1 \tag{3.14}$$

Profit for period 2, discounted to period 1, is

$$P_2(y_1, j_1, p_1, y_2, J_2(p_2), p_2) = \alpha\left(E\left[V(y_2, J_2(p_2), p_2)\right] + cE\left[(y_1 - D_1(j_1))^+\right]\right) \tag{3.15}$$

Then, sum of the discounted profits for all periods is

$$P(\mathbf{y}, \mathbf{J}(\mathbf{p}), \mathbf{p}) = cI_1 + \sum_{n=1}^{\infty} \alpha^{n-1}\left\{E\left[V(y_n, J_n(p_n), p_n)\right] + \alpha cE\left[(y_n - D_n(J_n(p_n)))^+\right]\right\} \tag{3.16}$$

Under the assumption that the retailer applies a base stock policy with a fixed price, y_n and p_n are replaced by y and p. Then the expression in the curly brackets in Equation 3.16 can be considered a fixed one-period profit whose maximization maximizes the total profit as well. That function is called $H(y, p)$, therefore,

$$H(y, p) = E\left[V(y, J(p), p)\right] + \alpha cE\left[(y - D(J(p)))^+\right] \tag{3.17}$$

That expression can be expanded as

$$H\left(y,p\right)=-y\left(h+c-\alpha c\right) \tag{3.18}$$

$$+\Pi_1\left(p\right)\left(\left(p+b+h-\alpha c\right)\left(y\left(1-F\left(y\right)\right)+\int_0^y xf\left(x\right)dx\right)-bE\left[\Delta\right]\right) \tag{3.19}$$

The first derivative of $H(y,p)$ with respect to y is

$$\frac{\partial H\left(y,p\right)}{\partial y}=-\left(h+c-\alpha c\right)+e^{-\beta p}\left(p+b+h-\alpha c\right)\left(1-F\left(y\right)\right) \tag{3.20}$$

$$\tag{3.21}$$

Then, the second derivative is

$$\frac{\partial^2 H\left(y,p\right)}{\partial y^2}=-e^{-\beta p}\left(p+b+h-\alpha c\right)f\left(y\right) \tag{3.22}$$

Since the second derivative is negative as long as $p+b+h>\alpha c$, y value that makes the first derivative 0 for the current p maximizes profit.

$$y^*=F^{-1}\left(-\frac{\left(\left(h+c\left(1-\alpha\right)\right)e^{\beta p}\right)}{p+b+h-\alpha c}+1\right) \tag{3.23}$$

Derivative of $H(y,p)$ with respect to p is

$$\frac{\partial H\left(y,p\right)}{\partial p}=\Pi_0{}'\left(p\right)\left[y\left(1-\alpha\right)\left(-h-c\right)\right]+\Pi_1\left(p\right)\left\{y-E\left[\left(y-\Delta\right)^+\right]\right\} \tag{3.24}$$

$$+\Pi_1{}'\left(p\right)\left\{\left(p-c\right)y-\left(p+h-\alpha c\right)E\left[\left(y-\Delta\right)^+\right]-bE\left[\left(\Delta-y\right)^+\right]\right\} \tag{3.25}$$

Considering

$$\Pi_1{}'(p) = -\beta e^{-\beta p} \tag{3.26}$$

$$= -\beta \Pi_1(p) \tag{3.27}$$

$$\Pi_0{}'(p) = \beta e^{-\beta p} \tag{3.28}$$

$$= \beta \Pi_1(p) \tag{3.29}$$

$$E\left[(\Delta - y)^+\right] = E\left[(y - \Delta)^+\right] + E[\Delta] - y \tag{3.30}$$

the derivative turns out to be

$$\frac{\partial H(y,p)}{\partial p} = \Pi_1(p)\left\{[1 - \beta(h + b + p - \alpha c)]\left\{y - E\left[(y-\Delta)^+\right]\right\} + \beta b E[\Delta]\right\} \tag{3.31}$$

Then, the second derivative is

$$\frac{\partial^2 H(y,p)}{\partial p^2} = -\beta \Pi_1(p)\left\{[2 - \beta(h + b + p - \alpha c)]\left\{y - E\left[(y-\Delta)^+\right]\right\} + \beta b E[\Delta]\right\} \tag{3.32}$$

Since the second derivative of $H(y,p)$ with respect to p is not necessarily negative, $H(y,p)$ is not necessarily concave in p but its concavity depends on parameters. Then, it is not jointly concave, either.

In this subsection, we considered a price-driven demand model with no restriction on supply. In the next subsection, we will consider a model with supply uncertainty.

3.1.2. Two Supplier Setting

In this subsection, the inventory described in the previous subsection is examined under supply uncertainty. In order to account for possible capacity constraints in the supply process, this model differs from the previous one in the structure and the cost of procurement. In this model, there are two suppliers. The low-cost supplier has a finite capacity Q and charges c_a per unit whereas the high-cost supplier has infinite capacity and has unit cost c_b with $c_b > c_a$. Before observing the demand, retailer places an order. If the size of this order is less than Q, then it is completely fulfilled by the low-cost supplier. Otherwise, the remainder is met by the high-cost supplier.

To establish the profit function for the two-supplier case, a similar procedure to the one-supplier setting is followed. $R(y_n, j_n, p_n)$ and $L(y_n, j_n)$ representing revenue and cost of holding and lost sales for period n, as before, $V^{II}(y_n, j_n, p_n)$ is defined as

$$V^{II}\left(y_n, j_n, p_n\right) = R\left(y_n, j_n, p_n\right) - L\left(y_n, j_n\right) \tag{3.33}$$

Then profit for the first period is computed as

$$P_1^{II}\left(y_1, j_1, p_1\right) = V^{II}\left(y_1, j_1, p_1\right) - k^{II}\left(y_1 - I_1\right) \tag{3.34}$$

where the function $k^{II}(.)$ represents the total cost of ordering.

Similarly, profits for following periods discounted to the first period are

$$P_2^{II}\left(y_1, j_1, p_1, y_2, J_2(p_2), p_2\right) \tag{3.35}$$

$$= \alpha\left(E\left[V^{II}\left(y_2, J_2\left(p_2\right), p_2\right)\right] - k^{II}\left(\min\left(D_1\left(J_1\left(p_1\right)\right), y_1\right)\right)\right) \tag{3.36}$$

$$P_3^{II}\left(y_1, j_1, p_1, y_2, J_2(p_2), p_2, y_3, J_3(p_3), p_3\right) \tag{3.37}$$

$$= \alpha^2\left(E\left[V^{II}\left(y_3, J_3\left(p_3\right), p_3\right)\right] - k^{II}\left(\min\left(D_2\left(J_2\left(p_2\right)\right), y_2\right)\right)\right) \tag{3.38}$$

Therefore, total profit over all periods is

$$P^{II}(\mathbf{y}, \mathbf{J}(\mathbf{p}), \mathbf{p}) = -k^{II}(y_1 - I_1) \tag{3.39}$$

$$+\sum_{n=1}^{\infty} \alpha^{n-1} \left\{ E\left[V^{II}(y_n, J_n(p_n), p_n)\right] - \alpha k^{II}\left(E\left[\min\left(D_n\left(J_n(p_n)\right), y_n\right)\right]\right)\right\} \tag{3.40}$$

which equals, under the assumption that inventory and price is hold constant over time

$$P^{II}(\mathbf{y}, \mathbf{J}(\mathbf{p}), \mathbf{p}) = -k^{II}(y - I_1) \tag{3.41}$$

$$+\frac{1}{1-\alpha} \left\{ E\left[R(y, j, p) - L(y, j)\right] - \alpha k^{II}\left(E\left[\min\left(D\left(J(p)\right), y\right)\right]\right)\right\} \tag{3.42}$$

where

$$k^{II}(y - I_1) = c_a E_Q\left[\min(y - I_1, Q)\right] + c_b E_Q\left[(y - I_1 - Q)^+\right] \tag{3.43}$$

$$= c_a\left(y - I_1 - E_Q\left[(y - I_1 - Q)^+\right]\right) + c_b E_Q\left[(y - I_1 - Q)^+\right] \tag{3.44}$$

$$= (c_b - c_a) E_Q\left[(y - I_1 - Q)^+\right] + c_a(y - I_1) \tag{3.45}$$

and $k^{II}(\min(D(J(p)), y))$ is computed as follows.

$$k^{II}(\min(D(J(p)), y)) = E\left[c_a E_Q\left[\min\left(\min\left(D(J(p)), y\right), Q\right)\right]\right] \tag{3.46}$$

$$+c_b E_Q\left[\left(\min\left(D(J(p)), y\right) - Q\right)^+\right] \tag{3.47}$$

Expanding the outside min() function,

$$k^{II}(\min(D(J(p)), y)) = E\left[c_a E_Q\left[\min\left(y - (y - D(J(p)))^+, Q\right)\right]\right] \tag{3.48}$$

$$+c_b E_Q\left[\left(\min\left(D(J(p)), y\right) - Q\right)^+\right] \tag{3.49}$$

Expanding the inside min() functions,

$$k^{II}\left(\min\left(D\left(J\left(p\right)\right),y\right)\right)=E\left[c_a\left\{y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-E_Q\left[\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-Q\right)^{+}\right]\right\}\right.$$
$$\left.+c_bE_Q\left[\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-Q\right)^{+}\right]\right.$$

Rearranging terms,

$$k^{II}\left(\min\left(D\left(J\left(p\right)\right),y\right)\right)=E\left[\left(c_b-c_a\right)E_Q\left[\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-Q\right)^{+}\right]\right.$$
$$\left.+c_a\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}\right)\right.$$

Taking the constants out of the expectations,

$$k^{II}\left(\min\left(D\left(J\left(p\right)\right),y\right)\right)=\left(c_b-c_a\right)E\left[E_Q\left[\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-Q\right)^{+}\right]\right] \quad (3.50)$$
$$+c_a\left(y-E\left[\left(y-D\left(J\left(p\right)\right)\right)^{+}\right]\right) \quad (3.51)$$

Therefore, the overall profit function is

$$P^{II}\left(\mathbf{y},\mathbf{J}(\mathbf{p}),\mathbf{p}\right)=-\left(c_b-c_a\right)E_Q\left[\left(y-I_1-Q\right)^{+}\right]-c_a\left(y-I_1\right)$$
$$+\frac{1}{1-\alpha}\left\{E\left[R\left(y,J\left(p\right),p\right)-L\left(y,J\left(p\right)\right)\right]-\alpha\left\{\left(c_b-c_a\right)E\left[E_Q\left[\left(y-\left(y-D\left(J\left(p\right)\right)\right)^{+}-Q\right)^{+}\right]\right]\right.\right.$$
$$\left.+c_a\left(y-E\left[\left(y-D\left(J\left(p\right)\right)\right)^{+}\right]\right)\right.$$

which can be expanded as

$$P^{II}\left(\mathbf{y},\mathbf{J}(\mathbf{p}),\mathbf{p}\right)=-\left(c_b-c_a\right)\left(\left(y-I_1\right)F\left(y-I_1\right)-\int_0^{y-I_1}xf\left(x\right)dx\right)-c_a\left(y-I_1\right)$$
$$+\frac{1}{1-\alpha}\left(-\left(1-\Pi_1\left(p\right)\right)hy+\Pi_1\left(p\right)\left(-\left(p+b+h-\alpha c_a\right)\left(yF\left(y\right)-\int_0^{y}xf\left(x\right)dx\right)\right.\right.$$
$$+y\left(p+b-\alpha c_a\right)-bE\left[\Delta\right]-\alpha\left(c_b-c_a\right)\left(\left(1-\Pi_1\left(p\right)\right)\left(yF_Q\left(y\right)-\int_0^{y}xf_Q\left(x\right)dx\right)\right.$$
$$\left.+\Pi_1\left(p\right)\left(\int_0^{y}xF_Q\left(x\right)f\left(x\right)dx+yF_Q\left(x\right)\left(1-F\left(y\right)\right)-\int_0^{y}\int_0^{z}xf_Q\left(x\right)f\left(z\right)dxdz\right)\right.$$

The price that maximizes not the profit but the revenue is calculated in order to find out how much difference it creates to account for the cost structure compared to focusing only on revenue maximization. The revenue-maximizing price p^r is calculated as follows, under the assumption that all of the demand can be met. Expected revenue is found by multiplying the price with the demand as a function of price.

$$E\left[pD\left(J\left(p\right)\right)\right] = p\Pi_1\left(p\right)\int\limits_0^\infty xf\left(x\right)dx \tag{3.52}$$

$$= pe^{-\beta p}E[\Delta] \tag{3.53}$$

To maximize the revenue, the first derivative of expectation with respect to p is equated to 0,

$$\frac{\partial E\left[pD\left(J\left(p\right)\right)\right]}{\partial p} = \left(e^{-\beta p} - \beta pe^{-\beta p}\right)E[\Delta] \tag{3.54}$$

$$= \left(e^{-\beta p}\left(1-\beta p\right)\right)E[\Delta] \tag{3.55}$$

$$= 0 \tag{3.56}$$

$$\Rightarrow p^r = \frac{1}{\beta} \tag{3.57}$$

Checking out the second derivative,

$$\frac{\partial^2 E\left[pD\left(J\left(p\right)\right)\right]}{\partial p^2} = -\beta e^{-\beta p}\left(1-\beta p\right) - \beta e^{-\beta p} \tag{3.58}$$

$$= \beta e^{-\beta p}\left(\beta p - 2\right) \tag{3.59}$$

It turns out that $1/\beta$ maximizes revenue as long as $\beta p < 2$, which holds by definition.

In this section, we incorporated a capacity constraint into an inventory system with price-driven demand probability. In the next section, we present the way the computations are conducted and discuss the results for our models with and without supply uncertainty.

3.2. Computation and Numerical Results

In this section, we present the procedure followed in computations in MATLAB for both models presented so far. The first subsection regards the setting without a capacity constraint whereas in the second one the capacity costraint is involved as well. After explaining the procedure in each subsection, we discuss the interpretation of the numerical results.

3.2.1. One Supplier Setting

Since the concavity of $H(y, p)$ with respect to (y, p) is not guaranteed, numerical methods are employed to find y^* and p^*. Two different methods are utilized to this end. First one is simultaneous optimization and the second one is iterative optimization.

<u>3.2.1.1. Simultaneous Optimization.</u> In simultaneous optimization of (y, p), the following procedure is employed. In order to compute the profit, Equation 3.19 is coded as in Figure 3.1.

```
function result = profit1(xx,h,b,c,alpha,beta,k,mu)
result=-((-xx(1))*(h+c-alpha*c)+PI(xx(2),beta)*((xx(2)+b+h
-alpha*c)*(xx(1)*(1-gamcdf(xx(1),k,mu))+quad(@(x)
(x.*gampdf(x,k,mu)),0,xx(1)))-b*(k*mu)));
end
```

Figure 3.1. Fixed one-period profit function.

Note that the value computed is actually the negative of the profit as the optimization function *fmincon* in MATLAB minimizes its argument. Having the profit function, optimization is performed using the function *fmincon* as follows. First, the constraints are defined. Both decision variables are constrained to be non-negative. Then come the equality constraints that the function *fmincon* requires. There is no equality constraint in our model, therefore the corresponding matrices are defined to be null. Upper and lower bound are also defined to be null. The reason is that although all of the decision variables actually have the lower bound 0, we already define them via the matrices A and bb. There is no nonlinear constraint, either. Therefore, it is null as well. In MATLAB, all these constraints are expressed in matrix form as in Figure 3.2.

```
A=[-1 0;0 -1];
bb=[0;0];
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
```

Figure 3.2. Constraints.

Two different starting points for each decision variable is used in order to check whether the solution found depends on the starting point. For all instances that this comparison is carried out, no significant difference is spotted.

There are two alternative algorithms that can be used by $fmincon$ for our study. They are called Sequential Quadratic Programming(SQP) and Interior Point. For all the parameter sets that the outputs given by these algorithms are compared, it turned out that they practically give the same results. Therefore, one of them, Interior Point, is arbitrarily chosen and all calculations are performed based on it for the sake of consistency.

The parameters are assigned with their respective values. Then, $fmincon$ is used as in Figure 3.3. Execution of that code assigns the optimal values of the decision variables found in the solution to the variable $xx1$, and the corresponding profit function to the varible $fval1$.

```
[xx1,fval1]= fmincon(@(xx)profit1(xx,h,b,c,alpha,beta,k,mu),
xx01,A,bb,Aeq,beq, lb,ub,nonlcon,options);
```

Figure 3.3. Use of $fmincon$.

3.2.1.2. Iterative Optimization. In iterative optimization of (y, p), the profit is maximized over y and p in the following way. For a fixed p, $y^*_{(1)}$ is found by derivation as follows.

$$\frac{\partial H(y,p)}{\partial y} = -(h + c - \alpha c) + e^{-\beta p}(p + b + h - \alpha c)(1 - F(y)) \tag{3.60}$$

$$\frac{\partial^2 H(y,p)}{\partial y^2} = -e^{-\beta p}(p + b + h - \alpha c) f(y) \tag{3.61}$$

Since the second derivative is negative as long as $p + b + h > \alpha c$, y value that makes the first derivative 0 for the current p maximizes profit.

$$y^*_{(1)} = F^{-1}\left(-\frac{((h + c(1-\alpha)) e^{\beta p})}{p + b + h - \alpha c} + 1\right) \tag{3.62}$$

Then, using this equation, $y^*_{(1)}$ is computed as in Figure 3.4.

```
function result = find_y_for_fixed_p(h,b,c,alpha,beta,k,mu,p)
result = gaminv(-(((h+c-alpha*c)*(exp(beta*p)))
/(p+b+h-alpha*c))+1,k,mu); end
```

Figure 3.4. Finding y for a fixed p.

Then fixing $y^*_{(1)}$, value of $p^*_{(1)}$ is found as follows. First, the function to calculate the negative of the profit for the current values of the decision variables is defined as in Figure 3.5.

Then, this function is minimized by the code in Figure 3.6.

```
function result = profit_for_fixed_y(h,b,c,alpha,beta,k,mu,
y,pprice)
result = -(((-y))*(h+c-alpha*c)+PI(pprice,beta)*((pprice+b
+h-alpha*c)
*(y*(1-gamcdf(y,k,mu))+quad(@(x)(x.*gampdf(x,k,mu)),0,y))
-b*(k*mu))); end
```

Figure 3.5. Profit for a fixed y.

```
[p1,fval1]= fmincon(@(pprice)profit_for_fixed_y(h,b,c,alpha,
beta,k,mu,y,pprice),p01,A,bb,Aeq,beq,lb,ub,nonlcon,options)
```

Figure 3.6. Use of $fmincon$ to find p.

Continuing iteratively, convergence is sought for y^* and p^* values.

The simultaneous and iterative optimizations gave practically the same results for all the parameter sets they were run. Therefore, we present the results of only one procedure, the simultaneous optimization, which is chosen arbitrarily.

3.2.1.3. Numerical Results and Interpretation. The behaviors of the optimal values of decision variables y and p, and the one-period profit function $H(y, p)$ are observed under different values of parameters h, b, c, α and β. To this end, the default values for the parameters are defined as follows.

$$h = 1 \tag{3.63}$$

$$b = 5 \tag{3.64}$$

$$c = 2 \tag{3.65}$$

$$\alpha = 0.8 \tag{3.66}$$

$$\beta = 0.05 \tag{3.67}$$

$$\Delta \sim \text{Gamma}\,(k_\Delta, \mu) \tag{3.68}$$

$$k_\Delta = 2 \tag{3.69}$$

$$\mu = 10 \tag{3.70}$$

Then, in order to observe the impact of a particular parameter on y, p, and $H(y,p)$, the value of that parameter is changed keeping all other parameters constant. Tables 3.1-6 present sample outputs for this setting.

An increase in h decreases y^* in a decreasing rate. The reason is that ending up with leftover inventory becomes more costly, and this fact drives the retailer to stock less. As h increases further, the marginal effect it creates reduces, therefore the decrease in y slows down as well. A higher h brings about a lower p^* since a lower p^* means a higher chance of observing demand, thus a lower expectation for unsold inventory. However, p^* responds to a change in h in a different way for high values of b. Following an initial decrease, p^* starts to rise as h increases further. Because there is another factor that acts in favor of a higher p^*. High costs imply reduced profitability, and the retailer needs to increase p^* in order to compensate for this effect. Thus, when b is high as well as h, the need for this compensation becomes strong enough to dominate the necessity of lowering the probability of demand occurring, therefore causes a higher p^*. In both cases, the retailer gains less as h increases, and the marginal decrease in profit diminishes as h increases further.

Table 3.1. The impact of h on y^*, p^* and $H(y^*, p^*)$.

h	y^*	p^*	$H(y^*, p^*)$
1.0	33.070	21.138	79.203
1.5	29.336	20.855	66.748
2.0	26.466	20.586	55.848
2.5	24.134	20.331	46.161
3.0	22.169	20.091	37.460
3.5	20.470	19.868	29.589
4.0	18.971	19.665	22.428
4.5	17.628	19.483	15.886
5.0	16.407	19.326	9.891
5.5	15.284	19.199	4.386

A higher b yields a higher y^* because it implies that it is more costly to fail to meet the demand. Therefore, the retailer tends to stock more in order to reduce that risk. Since the probability of demand occurring can be lowered by increasing p^*, an increase in b increases p^* as well, aiming a lower risk of experiencing shortfall. Obviously, a higher cost for lost sales causes the retailer to gain less. Changes in y^*, p^* and profit with respect to b all slow down for further increase in b.

An increase in c implies a less profitable environment for the retailer to operate in, therefore she reduces y^* to trade in smaller volumes. p^*, on the other hand, increases in an increasing rate for the following two reasons. First, the same motive as in the case of y^* applies: The retailer tends to decrease the volume she trades in. This motive results in the decision to increase p^* to end up with a lower probability of observing demand. Second, p^* needs to be raised in order to compensate for the reduction in profitability caused by an increasing c. In this more costly conditions, profit reduces.

β determines how strongly the probability of demand occurrence depends on price.

Table 3.2. The impact of b on y^*, p^* and $H(y^*, p^*)$.

b	y^*	p^*	$H(y^*, p^*)$
5	33.070	21.138	79.203
6	33.557	21.218	78.542
7	34.022	21.292	77.909
8	34.467	21.360	77.300
9	34.894	21.423	76.715
10	35.304	21.481	76.152
11	35.700	21.536	75.608
12	36.081	21.587	75.083
13	36.450	21.635	74.575
14	36.806	21.680	74.083
15	37.151	21.722	73.606

Table 3.3. The impact of c on y^*, p^* and $H(y^*, p^*)$.

c	y^*	p^*	$H(y^*, p^*)$
2.0	33.070	21.138	79.203
2.5	31.898	21.598	73.487
3.0	30.774	22.062	67.968
3.5	29.690	22.531	62.642
4.0	28.643	23.005	57.504
4.5	27.627	23.485	52.550
5.0	26.638	23.970	47.776
5.5	25.672	24.463	43.179
6.0	24.727	24.963	38.757
6.5	23.798	25.471	34.507
7.0	22.884	25.990	30.427

Therefore, a higher β results in a lower p^* as the retailer needs to care more about the effect of p^* on the probability of observing demand. Since p^* is lower, profitability is lower as well, and this drives the retailer to reduce the volume of the business by lowering y^*. Working with a smaller volume under a decreased price, profit decreases as well. The changes in y^*, p^* and profit with respect to β all slow down for further increase in β.

Table 3.4. The impact of β on y^*, p^* and $H(y^*, p^*)$.

β	y^*	p^*	$H(y^*, p^*)$
0.02	46.754	51.111	299.512
0.03	42.207	34.523	181.527
0.04	39.019	26.267	123.456
0.05	36.569	21.344	89.147
0.06	34.579	18.087	66.623
0.07	32.901	15.782	50.783
0.08	31.443	14.073	39.088
0.09	30.147	12.762	30.139
0.10	28.972	11.731	23.100
0.11	27.888	10.904	17.442
0.12	26.872	10.232	12.815

The parameter α is employed in order to account for the time value of money, and it is inversely related to the interest rate. An increase in α implies a smaller interest rate, which reduces the loss caused by ordering and paying in a period to sell in a later period. This means that there is now less to hesitate about ending up with leftover inventory, which will be sold in following periods, therefore the retailer decreases the probability of demand occurrence by increasing p^*. For the same reason, starting a period with a high level of inventory becomes less risky, therefore the base stock level y^* increases as well. As a higher α means a lower interest rate, the environment becomes more stable and thus safe, and this fact increases profit. All the changes in

y^*, p^* and profit with respect to α occur with an increasing rate, which corresponds to a decreasing rate with respect to the interest rate.

Table 3.5. The impact of α on y^*, p^* and $H(y^*, p^*)$.

α	y^*	p^*	$H(y^*, p^*)$
0.725	31.828	21.052	75.275
0.75	32.228	21.081	76.564
0.775	32.642	21.109	77.874
0.8	33.070	21.138	79.203
0.825	33.514	21.167	80.553
0.85	33.974	21.196	81.926
0.875	34.451	21.226	83.320
0.9	34.948	21.255	84.739
0.925	35.465	21.284	86.182
0.95	36.004	21.314	87.651
0.975	36.569	21.344	89.147

When μ increases, it is expected to have a higher demand, in case demand occurs. Therefore, y^* linearly increases in order to meet the increasing demand. p^*, on the other hand, remains the same because price determines the probability of demand occurrence, and not the amount of demand. As a result, profit linearly increases as well.

Table 3.6. The impact of μ on y^*, p^* and $H(y^*, p^*)$.

μ	y^*	p^*	$H(y^*, p^*)$
10	46.754	51.111	299.512
12	56.105	51.111	359.415
14	65.455	51.111	419.317
16	74.806	51.111	479.220
18	84.157	51.111	539.122
20	93.508	51.111	599.024
22	102.858	51.111	658.927
24	112.209	51.111	718.829
26	121.560	51.111	778.732
28	130.911	51.111	838.634
30	140.262	51.111	898.537

In this subsection, we discussed the method in computations and the meaning of outputs for the price-driven demand model with no supply uncertainty.

3.2.2. Two Suppliers Setting

3.2.2.1. Simultaneous Optimization. The profit function for the two suppliers setting is not concave with respect to y or p. Therefore, in simultaneous optimization with respect to (y, p), the following procedure is employed. In order to compute the profit, $func1$ and $func2$ defined as in Figure 3.7.

Then the negative of the profit function of the two suppliers setting is coded as in Figure 3.8.

This function is minimized over y and p simultaneously by the code in Figure 3.9.

```
function result = func1(I1,kQ,lambda,x)
result=(x(1)-I1)*gamcdf(x(1)-I1,kQ,lambda)-quad(@(t)
t.*gampdf(t,kQ,lambda),0,x(1)-I1); end
function result = func2(beta,k,mu,kQ,lambda,x)
result=PI(x(2),beta)*((1-gamcdf(x(1),k,mu))*(x(1)*gamcdf
(x(1),kQ,lambda)
-quad(@(t)t.*gampdf(t,kQ,lambda),0,x(1)))
+gamcdf(x(1),k,mu) *(quadl(@(t)
t.*gamcdf(t,kQ,lambda).*gampdf(t,k,mu),0,x(1))
-dblquad(@(t,z)t.*gampdf(t,kQ,lambda).*gampdf(z,k,mu),
0,x(1),0,z))); end
```

Figure 3.7. Defining $func1$ and $func2$.

As in the case with one supplier, different starting points are used, and solutions are confirmed to be the same, in order to make sure that the solution found does not depend on the starting point.

```
function result = profit_two_sup(h,b,ca,cb,I1,alpha,beta,k,
mu,kQ,lambda,x)
result=-(-(cb-ca)*func1(I1,kQ,lambda,x)-ca*(x(1)-I1)
+(1/(1-alpha)) *(-(1-PI(x(2),beta))*h*x(1)+PI(x(2),beta)
*(-(x(2) +b+h-alpha*ca) *(x(1)*gamcdf(x(1),k,mu)-quad(@(t)
t.*gampdf(t,k,mu), 0,x(1)))+x(1)*(x(2)+b-alpha*ca)
-b*(k*mu) -alpha*(cb-ca)*func2(beta,k,mu,
kQ,lambda,x)))); end
```

Figure 3.8. One-period profit for two supplier setting.

```
[xx1,fval1] = fmincon(@(x)profit_two_sup(h,b,ca,cb,I1,alpha,
beta,k,mu,kQ,lambda,x),xx01,A,bb,Aeq,beq,lb,ub,nonlcon,
options);
```

Figure 3.9. Use of $fmincon$ in two suppliers setting.

<u>3.2.2.2. Iterative Optimization.</u> The other method employed in optimization over (y, p) is as follows. First, using $func1$ and $func2$ presented previously, the profit function is defined as in Figure 3.10.

Then, $y^*_{(1)}$ is computed by optimizing this following function in $fmincon$ for an arbitrary initial p as in Figure 3.11. Then using $y^*_{(1)}$, the corresponding value of p, namely $p^*_{(1)}$ is found by optimizing the profit function over p in $fmincon$ as in Figure 3.12. Continuing iteratively, convergence is sought for y^* and p^* values.

The simultaneous and iterative optimizations gave practically the same results for all the parameter sets they were run. Therefore, we present the results of only one procedure, the simultaneous optimization, which is chosen arbitrarily.

```
function result = profit_two_sup_fixed_p(h,b,ca,cb,I1,alpha,
beta,k,mu,kQ,lambda,y,pprice)
x(1)=y;
x(2)=pprice;
result=-(-(cb-ca)*func1(I1,kQ,lambda,x)-ca*(y-I1)
+(1/(1-alpha))*(-(1-PI(pprice,beta))
*h*y+PI(pprice,beta)*(-(pprice+b+h-alpha*ca)*(y*gamcdf(y,k,
mu) -quad(@(t)t.*gampdf(t,k,mu),0,y))+y*(pprice+b-alpha*ca)
-b*(k*mu) -alpha*(cb-ca)*func2(beta,k,mu,kQ,lambda,x))));
end
```

Figure 3.10. Profit for a fixed price in two suppliers setting.

```
[y1,fval1]= fmincon(@(y)profit_two_sup_fixed_p(h,b,ca,cb,I1,
alpha,beta,k,mu,kQ,
lambda,y,pprice),y01,A,bb,Aeq,beq,lb,ub,nonlcon,
options);
```

Figure 3.11. Use of $fmincon$ to find p in two suppliers setting.

3.2.2.3. Numerical Results and Interpretation. The changes in optimal inventory level, price and profit are examined for different values of c_b/c_a, $E[Q]/E[\Delta]$, and coefficient of variation of capacity $CV(Q)$. The y^* and p^* values and corresponding profit for certain values of c_b/c_a, $E[Q]/E[\Delta]$ and $CV(Q)$ are then compared to those of two other approaches. Calling the original method Approach 1 and the corresponding profit Profit 1,

- Approach 2: y^* and p^* values are computed under the assumption that there is only one supplier with infinite capacity as in one supplier setting, charging c_a per unit. Then these values are used to calculate the actual profit (Profit 2).
- Approach 3: Fixing the price to the revenue maximizing price p^r found in 3.57, and assuming that all the demand can be met, the inventory level that maximizes

```
[p1,fval1]= fmincon(@(pprice)profit_two_sup_fixed_y(h,b,ca,
cb,I1,alpha,beta,k,mu,kQ,lambda,y,pprice),p01,A,bb,Aeq,beq,
lb,ub,nonlcon,options);
```

Figure 3.12. Maximizing profit over p.

the profit for this price is found and called y^r. Using these values, the actual profit is then computed (Profit 3).

Percentage deviation from the optimal solution are calculated for both approaches, and called Ratio 1 and Ratio 2, respectively. The default values for the parameters for all computations of this subsection are

$$h = 1 \tag{3.71}$$

$$b = 5 \tag{3.72}$$

$$c_a = 2 \tag{3.73}$$

$$c_b = 3 \tag{3.74}$$

$$\alpha = 0.8 \tag{3.75}$$

$$\beta = 0.05 \tag{3.76}$$

$$\Delta \sim \text{Gamma}\,(k_\Delta, \mu) \tag{3.77}$$

$$k_\Delta = 2 \tag{3.78}$$

$$\mu = 10 \tag{3.79}$$

$$Q \sim \text{Gamma}\,(k_Q, \lambda) \tag{3.80}$$

$$k_Q = 2 \tag{3.81}$$

$$\lambda = 7.5 \tag{3.82}$$

The impact of a particular parameter is observed by optimizing the profit for different values of that parameter while keeping all the other parameters constant. Tables 3.7-9 present the corresponding numerical results.

Note that the optimal inventory level and price in Approach 1 are independent of c_b/c_a, $E[Q]/E[\Delta]$ and $(CV(Q))$ as they all are to define capacity of the low-cost supplier, and Approach 1 assumes infinite capacity for it.

A higher c_b/c_a ratio lowers y^* while increasing p^*. Because a higher c_b/c_a ratio implies that it is more costly to procure when order exceeds the capacity of the low-cost supplier, which drives the retailer to be less confident about ordering in high volumes. Therefore, he lowers the order, and increases price to end up with a lower demand. In these more costly conditions, profit decreases as well, in a decreasing rate. Profit 2 also decreases, but linearly, unlike Profit 1. The reason is that the order and price decisions in Approach 2 cannot be modified, therefore the impact of increasing c_b/c_a ratio does not diminish, instead, Profit 2 keeps decreasing proportionally to the increase in this ratio. As a result, Ratio 1, a measure of difference between Profit 1 and Profit 2, increases. As for Approach 3, even though the price is fixed, order amount can be adjusted according to the new conditions, and therefore y^r decreases in c_b/c_a, as y^* does. Profit 3 decreases in a decreasing rate, not linearly, as this approach allows at least a partial adjustment of decisions by modifying stock level. Ratio 2, as can be expected, increases since the cost structure now has a greater effect, implying that making decisions aiming only revenue maximization is bound to perform worse.

An increase in $E[Q]/E[\Delta]$ ratio increases y^* while decreasing p^*. Because a higher $E[Q]/E[\Delta]$ ratio implies that a larger proportion of demand is expected to be met by the low-cost supplier, thus driving the retailer to trade in larger volume by increasing demand by a lower p^*, and keeping a higher level of inventory. Profit increases as a result of this higher volume of business caused by a more advantageous environment. Though inventory and price decisons are fixed for Approach 2, Profit 2 increases due to the shift of orders from high-cost to low-cost supplier. Ratio 1 decreases as $E[Q]/E[\Delta]$

Table 3.7. The impact of c_b/c_a ratio on the decision variables, profits and ratios.

c_b/c_a	y^*	p^*	Profit 1	Profit 2	Ratio 1	y^r	Profit 3	Ratio 2
1	33.070	21.138	396.014	396.014	0.000	33.218	394.965	0.265
1.1	32.674	21.204	391.596	391.552	0.011	32.826	390.424	0.299
1.2	32.290	21.268	387.265	387.089	0.045	32.445	385.967	0.335
1.3	31.917	21.332	383.017	382.626	0.102	32.074	381.589	0.373
1.4	31.554	21.394	378.851	378.163	0.182	31.714	377.290	0.412
1.5	31.201	21.456	374.765	373.701	0.284	31.364	373.066	0.453
1.6	30.858	21.517	370.755	369.238	0.409	31.022	368.916	0.496
1.7	30.524	21.576	366.819	364.775	0.557	30.690	364.837	0.540
1.8	30.198	21.635	362.956	360.312	0.728	30.366	360.829	0.586
1.9	29.881	21.693	359.163	355.850	0.922	30.050	356.887	0.634
2	29.573	21.750	355.438	351.387	1.140	29.742	353.012	0.682

increases since the the impact of the high-cost supplier is now less observable. The value of y^r behaves in parallel to y^*, The profit of the revenue-maximizing approach, Profit 3, increases as well. Ratio 2 decreases, reflecting the fact that the cost structure is now less significant and focusing only on revenue maximization hurts less.

An increase in $CV(Q)$ increases p^* since the variability in the cost of procurement becomes higher, and the retailer desires to reduce the probability of demand occurence by increasing the price. She also tends to stock more as she needs to be more cautious about the high ordering prices in the future, and prefers guaranteeing the supply in an earlier time. Profit decreases as as result of these more volatile conditions. y^r increases for the same reason as y^*. Profits of the Approaches 2 and 3 decrease acting parallel to Profit 1. Since the capacity is more volatile now, the impact of taking the capacity into account deteriorates. Therefore, the gap between optimal decision and the decision based on one supplier assumption, represented by Ratio 1, decreases.

Table 3.8. The impact of $E[Q]/E[\Delta]$ ratio on the decision variables, profits and ratios.

$E[Q]/E[\Delta]$	y^*	p^*	Profit 1	Profit 2	Ratio 1	y^r	Profit 3	Ratio 2
0.50	31.041	21.549	369.105	367.829	0.346	31.212	367.179	0.522
0.55	31.063	21.529	370.328	369.085	0.336	31.232	368.452	0.507
0.60	31.091	21.510	371.510	370.306	0.324	31.258	369.682	0.492
0.65	31.124	21.491	372.645	371.485	0.311	31.290	370.863	0.478
0.70	31.161	21.473	373.731	372.618	0.298	31.325	371.992	0.465
0.75	31.201	21.456	374.765	373.701	0.284	31.364	373.066	0.453
0.80	31.243	21.440	375.746	374.733	0.270	31.405	374.085	0.442
0.85	31.287	21.424	376.676	375.713	0.256	31.448	375.051	0.432
0.90	31.332	21.410	377.556	376.643	0.242	31.492	375.963	0.422
0.95	31.378	21.396	378.387	377.524	0.228	31.537	376.824	0.413
1.00	31.424	21.384	379.171	378.356	0.215	31.582	377.637	0.405

Table 3.9. The impact of $CV(Q)$ on the decision variables, profits and ratios.

$CV(Q)$	y^*	p^*	Profit 1	Profit 2	Ratio 1	y^r	Profit 3	Ratio 2
0.50	31.096	21.444	375.569	374.391	0.314	31.253	373.900	0.444
0.55	31.124	21.446	375.416	374.268	0.306	31.282	373.741	0.446
0.60	31.150	21.449	375.233	374.115	0.298	31.310	373.552	0.448
0.65	31.175	21.452	375.026	373.935	0.291	31.337	373.337	0.450
0.70	31.198	21.455	374.798	373.731	0.285	31.360	373.101	0.453
0.75	31.218	21.459	374.554	373.508	0.279	31.382	372.848	0.456
0.80	31.237	21.463	374.296	373.270	0.274	31.401	372.580	0.458
0.85	31.253	21.467	374.028	373.018	0.270	31.419	372.302	0.461
0.90	31.267	21.471	373.752	372.755	0.267	31.434	372.016	0.465
0.95	31.279	21.475	373.469	372.485	0.264	31.447	371.723	0.468
1.00	31.290	21.479	373.183	372.209	0.261	31.459	371.426	0.471

In this subsection, we covered the method employed in the computations for the capacity constrained inventory system where the price determines the probability of demand occurrence.

4. AN INVENTORY MODEL INVESTIGATING THE PRICE EFFECT ON INDIVIDUAL CUSTOMERS WITH ONE AND TWO SUPPLIERS

4.1. Model

In this chapter, we present an inventory model in which the price determines the volume of the demand by governing the probability of each potential customer to make a purchase. In Section 4.1.1, we present this model with no restriction on supply. In Section 4.1.2, on the other hand, the retailer works with two suppliers, where the low-cost supplier has a finite random capacity and the high-cost supplier has an infinite capacity.

4.1.1. One Supplier Setting

In this model, a single product inventory system is examined. A retailer holds inventory for a product for which holding cost h incurs per a unit of product per unit time. Any unsatisfied demand is lost, costing the retailer b per unit. Cost of ordering one unit of product is c.

The system is investigated for two periods. At the beginning of period n, the on-hand inventory I_n is known to the retailer. Then, she makes two decisions simultaneously. First decision is the order amount, denoted by u_n, to reach the inventory level y_n.

$$y_n = I_n + u_n \tag{4.1}$$

The other decision she faces is the price to charge for that period, denoted by p_n. The decision on price impacts the demand to be realized in the following fashion. Potential sales (or number of potential customers) has a certain probability distribution, and each potential customer independently decides whether or not to make a purchase. Price determines the probability that a potential customer will make a purchase. Therefore, it determines the parameter or parameters of the distribution of demand realized. In this way, we integrate a choice model based on price into the inventory system of interest. In our model, the probability of a potential customer making a purchase is

$$\Pi\left(p_n\right) = e^{-\beta p_n} \tag{4.2}$$

Then, the demand is denoted as $D^i(\Pi(p_n))$. It is assumed that the demand in a period is distributed uniformly over time. That is, for a given total demand in a period, purchases occur at a constant rate through the period. Lead time for order arrival is assumed to be zero. It is also assumed that there is no fixed cost for ordering. The retailer works with a supplier with infinite capacity.

As a result of the uniform distribution of demand over a period, the average level of inventory durind the period is the average of the inventory levels at the beginning and end. Therefore, denoting the length of period n by T_n, the holding cost for period n is found as

$$H^i\left(y_n, \Pi\left(p_n\right)\right) = hT_n \frac{y_n + E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right]}{2} \tag{4.3}$$

The lost sales cost is

$$\begin{aligned} B^i\left(y_n, \Pi\left(p_n\right)\right) &= bE\left[\left(\left(D_n^i\left(\Pi\left(p_n\right)\right)\right) - y_n\right)^+\right] && (4.4) \\ &= b\left(E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right] + E\left[D_n^i\left(\Pi\left(p_n\right)\right)\right] - y_n\right) && (4.5) \end{aligned}$$

Revenue is the price multiplied by demand, which is

$$\begin{aligned} R^i\left(y_n, p_n, \Pi\left(p_n\right)\right) &= p_n E\left[\min\left(y_n, D_n^i\left(\Pi\left(p_n\right)\right)\right)\right] && (4.6) \\ &= p_n\left(y_n - E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right]\right) && (4.7) \end{aligned}$$

The inventory level at the beginning of the period 2 before ordering is

$$I_2 = E\left[\left(y_1 - D_1^i\left(\Pi\left(p_1\right)\right)\right)^+\right] \tag{4.8}$$

which equals

$$I_2 = y_1 F^i\left(y_1|\Pi(p_n)\right) - \int_0^{y_1} x f^i\left(x|\Pi(p_n)\right) dx \tag{4.9}$$

Then, the profit function is written as

$$P_n^i\left(y_n, p_n, \Pi\left(p_n\right)\right) = R^i\left(y_n, p_n, \Pi\left(p_n\right)\right) - H^i\left(y_n, \Pi\left(p_n\right)\right) - B^i\left(y_n, \Pi\left(p_n\right)\right) - c\left(y_n - I_n\right) \tag{4.10}$$

which can be expanded as

$$P_n^i\left(y_n, p_n, \Pi\left(p_n\right)\right) = p_n\left(y_n - E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right]\right) \tag{4.11}$$

$$- hT_n \frac{y_n + E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right]}{2} \tag{4.12}$$

$$- b\left(E\left[\left(y_n - D_n^i\left(\Pi\left(p_n\right)\right)\right)^+\right] + E\left[D_n^i\left(\Pi\left(p_n\right)\right)\right] - y_n\right) - c\left(y_n - I_n\right) \tag{4.13}$$

which equals

$$P_n^i\left(y_n, p_n, \Pi\left(p_n\right)\right) = y_n\left(p_n + b - \frac{hT_n}{2} - c\right) - bE\left[D_n^i\left(\Pi\left(p_n\right)\right)\right] \tag{4.14}$$

$$- \left(p_n + b + \frac{hT_n}{2}\right)\left(y_n F^i\left(y_n|\Pi\left(p_n\right)\right) - \int_0^{y_n} x f^i\left(x|\Pi\left(p_n\right)\right) dx\right) + cI_n \tag{4.15}$$

The total profit for two periods is

$$P^i(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}(\mathbf{p})) = P_1^i(y_1, p_1, \Pi(p_1)) + P_2^i(y_2, p_2, \Pi(p_2)) \tag{4.16}$$

The first derivative of the total profit, $P^i(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}(\mathbf{p}))$ with respect to y_n is

$$\frac{\partial P^i(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}(\mathbf{p}))}{\partial y_n} = p_n + b - \frac{hT_n}{2} - \left(p_n + b + \frac{hT_n}{2}\right) F^i(y_n) \tag{4.17}$$

The second derivative is

$$\frac{\partial^2 P^i(y, p, \Pi(p))}{\partial y_n^2} = -\left(p_n + b + \frac{hT_n}{2}\right) f^i(y_n) < 0 \tag{4.18}$$

Therefore, $P^i(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}(\mathbf{p}))$ is concave with respect to y^n. Then, for a fixed p_n, y_n^* can be found by equating the first derivative to 0. Therefore,

$$y_n^* = F^{-1}\left(1 - \frac{hT_n}{p_n + b + \frac{hT_n}{2}}\right) \tag{4.19}$$

However, the concavity of $P^i(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}(\mathbf{p}))$ with respect to p_n is not guaranteed. The reason is that p_n is involved in the distribution function of the demand. Therefore, no generalization on the concavity with respect to p_n can be made, instead, the concavity depends on the type of the distribution. As a result, joint concavity with respect to y_n and p_n is not guaranteed, either.

In this subsection, we covered an inventory system where the price probabilistically determines the volume of the demand by governing the purchasing probability of each potential customer, without a restriction on supply. In the next subsection, we present our model that incorporates a capacity constraint into this model.

4.1.2. Two Suppliers Setting

In two suppliers setting, there is a supplier with a finite random capacity Q, charging c_a per unit, and another supplier with infinite capacity, charging c_b per unit where $c_b > c_a$. Naturally, when the amount of order is less than Q, it is met solely by the low-cost supplier. Otherwise, the remainder is met by the high-cost supplier. Therefore, ordering cost can be written as

$$k^{i,II}\left(y_n, \Pi\left(p_n\right)\right) = c_a E_Q\left[\min\left(y_n - I_n, Q_n\right)\right] + c_b E_Q\left[\left(y_n - I_n - Q_n\right)^+\right] \tag{4.20}$$

Rearranging the terms,

$$k^{i,II}\left(y_n, \Pi\left(p_n\right)\right) = c_a\left(y_n - I_n\right) + \left(c_b - c_a\right) E_Q\left[\left(y_n - I_n - Q_n\right)^+\right] \tag{4.21}$$

Expressing the one-period profit as

$$P_n^{i,II}\left(y_n, p_n, \Pi\left(p_n\right)\right) = R\left(y_n, p_n, \Pi\left(p_n\right)\right) - L\left(y_n, p_n, \Pi\left(p_n\right)\right) - k^{i,II}\left(y_n, \Pi\left(p_n\right)\right) \tag{4.22}$$

Expanding the cost and revenue functions, we get

$$P_n^{i,II}\left(y_n, p_n, \Pi\left(p_n\right)\right) = p_n\left(y_n - E\left[\left(y_n - D_n\left(\Pi\left(p_n\right)\right)\right)^+\right]\right)$$

$$-hT_n\frac{y_n + E\left[\left(y_n - D_n\left(\Pi\left(p_n\right)\right)\right)^+\right]}{2} - b\left(E\left[\left(y_n - D_n\left(\Pi\left(p_n\right)\right)\right)^+\right]\right.$$

$$\left. + E\left[D_n\left(\Pi\left(p_n\right)\right)\right] - y_n - c_a\left(y_n - I_n\right) - \left(c_b - c_a\right) E_Q\left[\left(y_n - I_n - Q_n\right)^+\right]\right.$$

which equals

$$P_n^{i,II}\left(y_n, p_n, \Pi\left(p_n\right)\right) = y_n\left(p_n + b - \frac{hT_n}{2} - c_a\right) - bE\left[D_n\left(\Pi\left(p_n\right)\right)\right] \tag{4.23}$$

$$-\left(p_n + b + \frac{hT_n}{2}\right)\left(y_n F\left(y_n\right) + \int_0^{y_n} xf\left(x\right)dx\right) \tag{4.24}$$

$$+ c_a I_n - \left(c_b - c_a\right)\left[\left(y_n - I_n\right)F_Q\left(y_n - I_n\right) - \int_0^{y_n - I_n} xf_Q\left(x\right)dx\right] \tag{4.25}$$

The total profit for two periods is

$$P^{i,II}\left(\mathbf{y}, \mathbf{p}, \mathbf{\Pi}\left(\mathbf{p}\right)\right) = P_1^{i,II}\left(y_1, p_1, \Pi\left(p_1\right)\right) + P_2^{i,II}\left(y_2, p_2, \Pi\left(p_2\right)\right) \tag{4.26}$$

The price that maximizes not the profit but the revenue is calculated in order to find out how much difference it creates to account for the cost structure compared to focusing only on revenue maximization. The revenue-maximizing price p^r is calculated as follows, under the assumption that all of the demand can be met. Expected revenue is found by multiplying the price with the demand as a function of price.

$$E\left[pD^i\left(\Pi\left(p\right)\right)\right] = p\int_0^{\infty} xf^i\left(x|\Pi(p)\right)dx \tag{4.27}$$

Regardless of the distribution of the demand, it can be stated that since the price determines the purchasing probability of a potential customer, it impacts the *expected* demand in the following way.

$$E\left[pD^i\left(\Pi\left(p\right)\right)\right] = p\Pi\left(p\right)\int_0^{\infty} xf^i\left(x|\Pi(0)\right)dx \tag{4.28}$$

To maximize the revenue, the first derivative of expectation with respect to p is equated to 0,

$$\frac{\partial E\left[pD\left(J\left(p\right)\right)\right]}{\partial p} = \left(e^{-\beta p} - \beta p e^{-\beta p}\right) E[\Delta] \tag{4.29}$$

$$= \left(e^{-\beta p}\left(1 - \beta p\right)\right) E[\Delta] \tag{4.30}$$

$$= 0 \tag{4.31}$$

$$\Rightarrow p^r = \frac{1}{\beta} \tag{4.32}$$

Checking out the second derivative,

$$\frac{\partial^2 E\left[pD\left(J\left(p\right)\right)\right]}{\partial p^2} = -\beta e^{-\beta p}\left(1 - \beta p\right) - \beta e^{-\beta p} \tag{4.33}$$

$$= \beta e^{-\beta p}\left(\beta p - 2\right) \tag{4.34}$$

It turns out that $1/\beta$ maximizes revenue as long as $\beta p < 2$, which holds by definition.

In this subsection, we integrated a capacity constraint into the inventory system of Section 4.1.1 with price-driven demand.

4.2. Computation and Numerical Results

As the profit is not necessarily jointly concave in y_n and p_n, numerical methods are employed in order to maximize the profit. As iterative optimization of the order-up-to level and the price did not give any better results than simultaneous optimization in Chapter 3, only simultaneous optimization is performed in this Chapter.

In this section, we present the procedure followed in computations in MATLAB for both models presented in this chapter. First, we present the part of the procedure that is used in both of the models with and without the capcity constraint.

Revenue and the sum of holding and lost sales costs are computed as in Figure 4.1.

```
function result=R(parameters,y,p)
result=p*(y*(1-cumulative(parameters,y))+integ(parameters,
(y))); end
function result=Holding(h,parameters,y,T)
result=T*h*((y+y*cumulative(parameters,y)-integ(parameters,
y))/2); end
function result=LostSales(b,parameters,y)
result=b*(mean(parameters)+y*(cumulative(parameters,y)-1)
-integ(parameters,y)); end
function result=L(h,b,parameters,y,T)
result=Holding(h,parameters,y,T)+LostSales(b,parameters,y);
end
```

Figure 4.1. Revenue and costs.

```
function result=integ(parameters,y)
result=quad(@(t)t.*gampdf(t,parameters(1),parameters(2)),
0,y);end
```

Figure 4.2. Defining the function *integ*.

In order to represent the expression $\int\limits_0^{y_1} xf(x)\,dx$, the function *integ* is defined as in Figure 4.2.

Before using the minimization function $fmincon$, the constraints are defined. All four decision variables are constrained to be non-negative. Then comes the equality constraint that the function $fmincon$ requires. There is no equality constraint in our model, therefore the corresponding matrices are defined to be null. Upper and lower bound are also defined to be null. The reason is that although all of the decision variables actually have the lower bound 0, we already define them via the matrices A and *bb*. There is no nonlinear constraint, either. Therefore, those matrices are null as well. In MATLAB, all these constraints are expressed in matrix form as in Figure 4.3.

```
A=[-1 0 0 0;0 -1 0 0;0 0 -1 0;0 0 0 -1];
bb=[0;0;0;0];
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
```

Figure 4.3. Constraints.

Two different starting points for each decision variable is used in order to check whether the solution found depends on the starting point. For all instances that this comparison is carried out, no significant difference is spotted.

Now that the procedure common for the models with and without the capacity constraint is presented, will explain the details specific to the respective models. The first subsection regards the setting without a capacity constraint whereas in the second one the capacity costraint is involved as well. After explaining the procedure in each subsection, we discuss the interpretation of the numerical results. Note that for both models, we use the average profit unit time as the objective function.

4.2.1. One Supplier Setting

4.2.1.1. Optimization Procedure. Based on the common framework presented for two models of this chapter, the following procedure is employed for the numerical study for the model without the capacity constraint. Ordering cost for this setting is coded as in Figure 4.4. Therefore, the negative of the profit can be computed by the function in Figure 4.5.

Having the profit function, optimization is performed using the function $fmincon$ as follows. Out of two possible algorithms that MATLAB can use, Interior Point is

```
function result=ordering_one_sup(c,y,I)
result=c*(y-I); end
```

Figure 4.4. Ordering cost for one supplier setting.

```
function result=profit_one_sup(xx,h,b,c,beta,T1,T2,kD)
lambda=10;
I1=0;
parameters=[kD*T1,PI(beta,xx(2))*lambda];
I2=xx(1)*cumulative(parameters,xx(1))-integ(parameters,
xx(1));
profit1=-(R(parameters,xx(1),xx(2))-L(h,b,parameters,xx(1),
T1)
-ordering_one_sup(c,xx(1),I1));
parameters=[kD*T2,PI(beta,xx(4))*lambda];
profit2=-(R(parameters,xx(3),xx(4))-L(h,b,parameters,xx(3),
T2)
-ordering_one_sup(c,xx(3),I2));
result=(profit1+profit2)/2; end
```

Figure 4.5. Profit for one supplier setting.

selected arbitrarily as the results are not influenced by this selection.

For practical purposes, a maximum number of iterations for $fmincon$ is defined in options in order to let the code break in case it takes too long to converge. After all the parameters are assigned with their respective values, $fmincon$ is used as in Figure 4.6. Execution of that code assigns the optimal values found in the solution to the variable $xx1$, and the corresponding profit function to the varible $fval1$.

```
[xx1,fval1]= fmincon(@(xx)profit_one_sup(xx,h,b,c,beta,T1,
T2,kD), xx01,A,bb,Aeq,beq,lb,ub,nonlcon,options);
```

Figure 4.6. Use of $fmincon$ for one supplier setting.

4.2.1.2. Numerical Results and Interpretations. The behaviors of the optimal values of decision variables y_1^*, p_1^*, y_2^* and p_2^*, and the profit are observed under different values of parameters h, b, c, α, β and T_1. To this end, the default values for the parameters are defined as follows.

$$h = 1 \tag{4.35}$$

$$b = 5 \tag{4.36}$$

$$c = 2 \tag{4.37}$$

$$\beta = 0.05 \tag{4.38}$$

$$D^i(\Pi(p_n)) \sim \text{Gamma}\,(k_\Delta, \mu\Pi(p_n)) \tag{4.39}$$

$$k_\Delta = 2 \tag{4.40}$$

$$\mu = 10 \tag{4.41}$$

$$T_1 = 1 \tag{4.42}$$

$$T_2 = 1 \tag{4.43}$$

The impact of a particular parameter is observed by optimizing the profit for different values of that parameter while keeping all the other parameters constant. Tables 4.1-5 present the corresponding numerical results.

As unit cost of leftover inventory increases, the retailer tends to stock less to avoid that cost. Therefore, y_1^* and y_2^* decrease in h. The values of p_1^* and p_2^*, on the other hand, increase in order to compensate for the decrease in profitability caused by a higher h. As a result of the more costly conditions, profit decreases.

Table 4.1. The impact of h on the decision variables and profit.

h	y_1^*	p_1^*	y_2^*	p_2^*	Profit
1.000	15.295	24.231	10.727	26.094	109.456
1.500	13.422	25.040	9.985	26.702	104.633
2.000	12.084	25.773	9.347	27.284	100.319
2.500	11.047	26.453	8.791	27.844	96.395
3.000	10.204	27.094	8.300	28.386	92.789
3.500	9.498	27.702	7.861	28.911	89.449
4.000	8.891	28.284	7.465	29.422	86.335
4.500	8.363	28.844	7.106	29.921	83.420
5.000	7.895	29.386	6.778	30.408	80.679
5.500	7.478	29.911	6.477	30.885	78.095
6.000	7.101	30.422	6.199	31.352	75.650

A higher b yields higher y_1^* and y_2^* values since failing to meet demand becomes more costly and the retailer stocks more to reduce that risk. p_1^* and p_2^* also increase in b because a reduced demand by higher prices decreases the risk of experiencing shortfall. Since the business environment is now more costly due to higher b, profit decreases. All these changes occur at a decreasing rate.

An increase in c decreases y_1^* and y_2^* since it implies a less profitable setting, and this fact drives the retailer to reduce the volume she trades in. p_1^* and p_2^* both increase in c, but in different ways because of the reason that follows. Any leftover inventory from period 1 can be carried to and sold in period 2, therefore a decreased demand in period 1 does not hurt as much as a decreased demand in period 2. Therefore, the retailer does not hesitate to decrease demand by increasing price for period 1. As a result, p_1^* increases linearly in order to compensate for the effect of increasing c on profitability. For p_2^*, on the other hand, decresaing demand by increasing p_2^* may result in more leftover inventory that cannot be sold in the future, either. Therefore, the

Table 4.2. The impact of b on the decision variables and profit.

b	y_1^*	p_1^*	y_2^*	p_2^*	Profit
5	15.295	24.231	10.727	26.094	109.456
6	15.397	24.267	10.789	26.195	109.243
7	15.495	24.302	10.847	26.292	109.039
8	15.590	24.335	10.904	26.384	108.842
9	15.681	24.367	10.958	26.471	108.652
10	15.769	24.397	11.010	26.555	108.469
11	15.854	24.426	11.062	26.634	108.295
12	15.936	24.453	11.108	26.711	108.120
13	16.016	24.480	11.155	26.784	107.954
14	16.093	24.505	11.200	26.855	107.794
15	16.168	24.529	11.244	26.923	107.638

increase in p_2^* with respect to c is not as fast as that of p_1^*, but it occurs in a decreasing rate. Under these more costly conditions, profit decreases.

An increase in β decreases both p_1^* and p_2^* since a higher dependence of demand on price forces the retailer to be more cautious on pricing. In this less profitable setting, the retailer is less eager to operate, therefore y_1^* and y_2^* decrease as well. As a result of operating with a smaller volume under a lower price, profit also decreases. All the mentioned changes occur at a decreasing rate.

An increase in the length of a period results in a higher total demand for that period. Therefore, at the beginning of the period, the retailer stocks more to meet this demand. On the tradeoff between a higher volume of demand and higher profitability, she shifts towards higher profitability since the volume of the demand is already high. Therefore, the price increases as well. Both of the changes in inventory level and price occur at a decreasing rate. Since the inventory and price decisions can be made only

Table 4.3. The impact of c on the decision variables and profit.

c	y_1^*	p_1^*	y_2^*	p_2^*	Profit
2.00	15.295	24.231	10.727	26.094	109.456
2.50	14.917	24.731	9.860	26.952	105.446
3.00	14.549	25.231	9.116	27.784	101.672
3.50	14.189	25.731	8.468	28.594	98.107
4.00	13.839	26.231	7.895	29.386	94.728
4.50	13.497	26.731	7.385	30.161	91.516
5.00	13.164	27.231	6.926	30.922	88.458
5.50	12.839	27.731	6.511	31.671	85.541
6.00	12.522	28.231	6.133	32.408	82.752
6.50	12.213	28.731	5.788	33.135	80.084
7.00	11.911	29.231	5.471	33.852	77.528

Table 4.4. The impact of β on the decision variables and profit.

β	y_1^*	p_1^*	y_2^*	p_2^*	Profit
0.02	20.170	54.584	15.319	57.044	321.451
0.03	18.071	37.748	13.301	39.913	202.638
0.04	16.527	29.306	11.855	31.291	144.078
0.05	15.295	24.231	10.727	26.094	109.456
0.06	14.263	20.841	9.803	22.617	86.725
0.07	13.374	18.417	9.020	20.127	70.743
0.08	12.590	16.597	8.342	18.256	58.951
0.09	11.889	15.180	7.745	16.800	49.933
0.10	11.254	14.046	7.212	15.634	42.844
0.11	10.674	13.118	6.732	14.679	37.147
0.12	10.140	12.344	6.297	13.884	32.488

at the beginning of a period, a longer period implies less frequent time points in which the decisions can be updated. As a result, policies of the retailer becomes less flexible, and profit is reduced by this fact.

Table 4.5. The impact of T_1 on the decision variables and profit.

T_1	y_1^*	p_1^*	y_2^*	p_2^*	Profit
1	15.295	24.231	10.727	26.094	109.456
2	20.947	24.850	10.727	26.094	109.151
3	26.010	25.416	10.727	26.094	107.368
4	30.662	25.955	10.727	26.094	105.136
5	34.983	26.478	10.727	26.094	102.757
6	39.020	26.990	10.727	26.094	100.343
7	42.803	27.494	10.727	26.094	97.944
8	46.356	27.993	10.727	26.094	95.581
9	49.694	28.489	10.727	26.094	93.265
10	52.830	28.982	10.727	26.094	91.000
11	55.778	29.473	10.727	26.094	88.790

This subsection covered the method employed for the computations of the model with price and demand interaction with no uncertainty in supply, and then a discussion of the numerical results. In the next subsection, we present the procedure for computational study and a discussion of the results for the model with a capacity constraint.

4.2.2. Two Suppliers Setting

4.2.2.1. Optimization Procedure. Based on the common framework presented for two models of this chapter, the following procedure is employed for the numerical study for the model with the capacity constraint. Ordering cost for the two suppliers setting is calculated as in Figure 4.7.

```
function result=ordering_two_sup(ca,cb,y,I,kQ,mu)
result=(cb-ca)*((y-I)*gamcdf(y-I,kQ,mu)-quad(@(t)
t.*gampdf(t,kQ,mu),0,y-I))+ca*(y-I); end
```

Figure 4.7. Ordering cost for two suppliers setting.

Therefore, the profit can be computed by the function in Figure 4.8.

```
function result=profit_two_sup(xx,h,b,ca,cb,beta,T1,T2,kQ,
mu,kD)
lambda=10;
I1=0;
parameters=[kD,PI(beta,xx(2))*lambda*T1];
I2=xx(1)*cumulative(parameters,xx(1))-integ(parameters,
xx(1));
profit1=-(R(parameters,xx(1),xx(2))-L(h,b,parameters,xx(1),
T1)
-ordering_two_sup(ca,cb,xx(1),I1,kQ,mu));
parameters=[kD,PI(beta,xx(4))*lambda*T2];
profit2=-(R(parameters,xx(3),xx(4))-L(h,b,parameters,xx(3),
T2)
-ordering_two_sup(ca,cb,xx(3),I2,kQ,mu));
result=(profit1+profit2)/(T1+T2); end
```

Figure 4.8. Profit for two suppliers setting.

4.2.2.2. Numerical Results and Interpretations. The changes in optimal inventory level, price and profit are examined for different values of c_b/c_a, $E[Q]/E[D^i]$, and coefficient of variation of capacity $CV(Q)$. The y_1^*, y_2^*, p_1^* and p_2^* values and the corresponding profit for certain values of c_b/c_a, $E[Q]/E[D^i]$ and $CV(Q)$ are then compared to those of two other approaches. Calling the original method Approach 1 and the corresponding profit Profit 1,

- Approach 2: y_1^*, y_2^*, p_1^* and p_2^* values are computed under the assumption that there is only one supplier with infinite capacity as in one supplier setting, charging c_a per unit. Then these values are used to calculate the actual profit (Profit 2).
- Approach 3: Fixing the price for both periods to the revenue maximizing price p^r, and assuming that all of the demand can be met, the inventory levels that maximizes the profit for this price is found and called y_1^r and y_2^r. Using these values, the actual profit is then computed (Profit 3).

Note that since the volume of the demand depends on price as opposed to the case in Chapter 3, the following convention is adopted. The value of p^r is used in the calculation of expected demand. That is, the quantity represented by the ratio of expected capacity to the expected demand, $E[Q]/E[D^i]$, is actually $E[Q]/E[D^i(\Pi(p^r))]$, but shortened for convenience.

Percentage deviation from the optimal solution are calculated for both approaches, and called Ratio 1 and Ratio 2, respectively. The default values for the parameters for all computations of this subsection are as follows.

$$h = 1 \tag{4.44}$$

$$b = 5 \tag{4.45}$$

$$c_a = 2 \tag{4.46}$$

$$c_b = 3 \tag{4.47}$$

$$\beta = 0.05 \tag{4.48}$$

$$D^i(\Pi(p_n)) \sim \text{Gamma}\,(k_\Delta, \mu\Pi(p_n)) \tag{4.49}$$

$$k_D = 2 \tag{4.50}$$

$$\mu = 10 \tag{4.51}$$

$$Q \sim \text{Gamma}\,(k_Q, \lambda) \tag{4.52}$$

$$k_Q = 2 \tag{4.53}$$

$$\lambda = 7.5 \tag{4.54}$$

$$T_1 = 1 \tag{4.55}$$

$$T_2 = 1 \tag{4.56}$$

The impact of a particular parameter is observed by optimizing the profit for different values of that parameter while keeping all the other parameters constant. Tables 4.6-8 present the corresponding numerical results.

Note that the optimal inventory level and price in Approach 1 are independent of c_b/c_a, $E[Q]/E[D^i]$ and $CV(Q)$ as they all are to define capacity of the low-cost supplier, and Approach 1 assumes infinite capacity for it.

Since leftover inventory of period 1 can be sold in period 2, y_1^* is higher than y_2^*. However, an increase in $E[Q]/E[D^i]$ reduces this advantage of period 1. The reason is that a higher $E[Q]/E[D^i]$ implies that a greater proportion of order is to be met by the low-cost supplier, and as this ratio increases, the operating conditions become safer. Fearing less about experiencing high ordering costs in the future, the need for ordering high in period 1 diminishes. Therefore, the relative advantage of operating in period 1 over operating in period 2 also deteriorates. This results in a shift of business towards period 2. This shift is carried out by increasing p_1^* to lower the first period demand and decreasing y_1^*, and doing the opposite for p_2^* and y_2^*. Profit increases in $E[Q]/E[D^i]$ since a higher value of this ratio implies better operating conditions. Though in Approach 2 inventory and price are fixed and there is no chance to adapt to the change in $E[Q]/E[D^i]$, Profit 2 also increases due to the improved conditions. As for Approach 2, prices are fixed, however, y_1^r and y_2^r behave in the similar fashion as y_1^* and y_2^* to the same end of shifting the operations from period 1 to period 2. Ratio 2 decreases in $E[Q]/E[D^i]$ since the cost structure becomes less significant, and therefore focusing only on maximizing revenue becomes less disadvantageous.

c_b/c_a ratio reflects how much more costly it is to order beyond the capacity of the low-cost supplier. An increase in this ratio implies more disadvantageous conditions, and therefore the retailer decides to reduce the volume of the business by increasing p_1^* and p_2^* to lower the demand, and decrease y_1^* and y_2^*. As a result, profit decreases in a decreasing rate. Profit 2, on the other hand, decreases linearly since in Approach 2 there is no chance to adjust order amount or price, and therefore the change in ordering cost is directly reflected to the profit. Increasing c_b/c_a ratio means that the impact of having to work with a high-cost supplier is greater, therefore working under one low-cost supplier assumption performs even worse, resulting in a higher Ratio 1. y_1^r, y_2^r and Profit 3 behave in parallel with y_1^*, y_2^* and Profit 1, respectively. As the cost structure is more considerable with a higher c_b/c_a, performance of the approach of focusing only on revenue now performs worse, and this fact yields a higher Ratio 2.

Table 4.6. The impact of $E[Q]/E[D^i]$ ratio on the decision variables, profit and ratios.

$E[Q]/E[D^i]$	y_1^*	p_1^*	y_2^*	p_2^*	Profit 1	Profit 2	Ratio 1	y_1^r	y_2^r	Profit 3	Ratio 2
0.50	12.724	25.864	10.004	26.802	104.231	103.610	0.596	16.421	12.755	98.114	5.869
0.55	12.671	25.881	10.051	26.753	104.437	103.797	0.613	16.333	12.798	98.346	5.832
0.60	12.631	25.891	10.096	26.708	104.635	103.980	0.626	16.257	12.838	98.571	5.795
0.65	12.602	25.895	10.137	26.666	104.826	104.161	0.634	16.191	12.876	98.790	5.757
0.70	12.584	25.894	10.175	26.627	105.008	104.337	0.639	16.134	12.911	99.003	5.718
0.75	12.575	25.887	10.212	26.591	105.182	104.510	0.639	16.087	12.944	99.209	5.679
0.80	12.574	25.876	10.246	26.557	105.348	104.678	0.637	16.048	12.976	99.408	5.638
0.85	12.579	25.862	10.277	26.525	105.506	104.841	0.631	16.016	13.006	99.601	5.598
0.90	12.590	25.844	10.307	26.496	105.657	104.999	0.622	15.991	13.034	99.786	5.556
0.95	12.607	25.825	10.334	26.469	105.800	105.152	0.612	15.973	13.061	99.964	5.515
1.00	12.627	25.803	10.360	26.444	105.935	105.300	0.600	15.960	13.086	100.136	5.474

Table 4.7. The impact of c_b/c_a ratio on the decision variables, profit and ratios.

c_b/c_a	y_1^*	p_1^*	y_2^*	p_2^*	Profit 1	Profit 2	Ratio 1	y_1^r	y_2^r	Profit 3	Ratio 2
1.0	15.295	24.231	10.727	26.094	109.456	109.456	0.000	18.168	13.554	105.213	3.876
1.1	14.514	24.618	10.673	26.145	108.503	108.467	0.034	17.584	13.461	103.945	4.201
1.2	13.905	24.967	10.580	26.232	107.610	107.478	0.123	17.123	13.345	102.717	4.547
1.3	13.399	25.290	10.467	26.340	106.764	106.488	0.258	16.734	13.216	101.520	4.911
1.4	12.962	25.596	10.342	26.461	105.956	105.499	0.431	16.393	13.082	100.352	5.289
1.5	12.575	25.887	10.212	26.591	105.182	104.510	0.639	16.087	12.944	99.209	5.679
1.6	12.225	26.167	10.079	26.725	104.439	103.520	0.879	15.807	12.806	98.089	6.080
1.7	11.906	26.438	9.946	26.862	103.723	102.531	1.149	15.547	12.669	96.990	6.491
1.8	11.612	26.701	9.814	27.001	103.032	101.542	1.447	15.305	12.532	95.912	6.911
1.9	11.339	26.956	9.684	27.140	102.366	100.552	1.771	15.077	12.398	94.853	7.339
2.0	11.084	27.204	9.557	27.280	101.721	99.563	2.121	14.861	12.265	93.813	7.774

An increase in $CV(Q)$ increases y_1^* and decreases p_1^* in order to shift the business towards period 1. The motivation is similar to that of the case of $E[Q]/E[D^i]$. y_1^* is higher than y_2^* due to the the advantage of the period 1 created by the fact that carrying inventory to period 2 is possible. A higher $CV(Q)$ implies less stable conditions to operate in, therefore avoiding higher ordering cost in the future becomes more important, resulting in an even greater advantage for period 1. Therefore, the retailer decides to conduct an even larger proportion of the business in period 1, and achieves this objective by increasing order amount and decreasing price for period 1, and acting in the opposite way for period 2. Profit decreases due to this more volatile environment. Taking the capacity constraint into account becomes less beneficial since the capacity is more variable, causing a decrease in the quality of the decisions made. Therefore, the percentage difference between profits with and without capacity consideration, Ratio 1, decreases. The inventory levels for Approach 3, y_1^r and y_2^r move in parallel with y_1^* and y_2^*.

In this subsection, we presented the way the computations are carried out for the capacity constrained inventory model with a price-driven demand, and then discussed the interpretation of the numerical results.

Table 4.8. The impact of $CV(Q)$ on the decision variables, profit and ratios.

$CV(Q)$	y_1^*	p_1^*	y_2^*	p_2^*	Profit 1	Profit 2	Ratio 1	y_1^r	y_2^r	Profit 3	Ratio 2
0.50	12.304	26.027	10.334	26.469	105.374	104.562	0.770	15.892	13.022	99.402	5.667
0.55	12.371	25.992	10.301	26.502	105.333	104.555	0.738	15.939	13.001	99.359	5.672
0.60	12.438	25.957	10.271	26.532	105.288	104.545	0.706	15.986	12.982	99.313	5.675
0.65	12.503	25.924	10.242	26.560	105.240	104.531	0.674	16.033	12.964	99.266	5.677
0.70	12.566	25.892	10.215	26.587	105.190	104.513	0.644	16.080	12.947	99.216	5.679
0.75	12.627	25.861	10.190	26.613	105.136	104.490	0.614	16.127	12.931	99.165	5.680
0.80	12.685	25.831	10.165	26.637	105.081	104.464	0.587	16.174	12.915	99.112	5.681
0.85	12.742	25.804	10.142	26.661	105.024	104.435	0.561	16.220	12.900	99.057	5.682
0.90	12.796	25.778	10.119	26.684	104.966	104.402	0.538	16.266	12.886	99.001	5.683
0.95	12.847	25.753	10.097	26.706	104.908	104.366	0.516	16.310	12.872	98.943	5.685
1.00	12.897	25.730	10.076	26.728	104.848	104.328	0.496	16.354	12.859	98.885	5.688

5. CONCLUSION

In this thesis, we study the effect of price on demand in inventory systems. We develop two models. In the first model, price determines the probability of demand occurrence. When demand occurs, its volume is random and independent of the price. In the second model, price determines the distribution of the amount of demand by governing purchasing probability of each potential customer. In both models, we try to find the order amount and the price that maximize the average one-period profit. We examine how the optimal policy and the profit change in response to the parameters related to cost, demand and the relationship between price and demand.

For each of our models, we present a variation in order to consider supply uncertainty. We discuss the benefit of taking the capacity constraints into account, and demonstrate the effect of parameters related to the capacity constraint on the optimal policy and profit. We also investigate the performance of the decision based on revenue maximization compared to the profit earned in the optimal policy.

We take one step further from the existing literature on the price and demand interactions in inventory systems. We use a choice model based on price. We conclude that the incorporation of this choice model into an inventory system provides remarkably useful insights on how the joint decision on inventory and price should be made.

We observe that an increase in holding cost drives the retailer to stock less in order to avoid the cost caused by leftover inventory, and decrease the price so as to have a higher demand amount or probability, which results in a decreased leftover stock. Increasing lost sales cost, on the other hand, brings about decisions on higher inventory levels which reduces the risk of a shortfall, and a higher price, which reduces the amount or probability of demand to the same end.

A higher ordering cost implies a less profitable environment; therefore the retailer reduces the amount she orders. She increases the price for two reasons. First, as she tends the decrease the volume of the business, she reduces the volume or the probability of demand by increasing the price. Second, a higher ordering cost results in a lower profitability, and therefore she increases the price to compensate for that decrease.

As the dependence of the amount or probability of demand on the price becomes stronger, the retailer tends to make pricing decisions more cautiously, and this results in a lower price. Since a lower price decreases the profitability, she starts reducing the volume of investment, which is the volume of orders placed. Under a higher interest rate, the disadvantage caused by ordering in a period to sell in the following periods increases, therefore the retailer decides to order less. She decreases the price to boost the demand, with the same aim of ending up with a smaller amount of leftover inventory.

In the sub-models with supply uncertainty, we examine the effect of three factors on the optimal decision and the corresponding profit. We also investigate the performance of the decision based on a low-cost supplier with an infinite capacity, and decision with the aim of revenue maximization, compared to the profit earned in the optimal policy. The impact of an increased ratio of ordering cost for the high-cost supplier to the ordering cost for the low-cost supplier affects the decisions of the retailer in a similar way to the case of a higher ordering cost in the one supplier case. She reduces the order amount and increases the price. As the impact of the capacity constraint of the low-cost supplier is now higher, the quality of the decision based on the assumption that the low-cost supplier has infinite capacity deteriorates. The performance of the inventory and price decisions based on the revenue maximization also diminishes as the cost structure is now more relevant.

An increase in the capacity of the low-cost supplier results in a higher ordering amount and a lower price. The reason is that knowing the operating conditions are now safer; the retailer increases the volume of the business by increasing the order amount and boosting the volume or the probability of the demand by lowering the price. The advantage of taking the capacity constraint into account decreases since a larger proportion of the demand is actually to be met by the low-cost supplier. As the overall cost is expected to decrease as that proportion increases, the impact of the overall cost structure decreases. Therefore, the decision based on revenue maximization becomes performing better.

As the capacity of the low-cost supplier becomes more volatile, the retailer becomes acting more cautiously. This results in a higher order amount in order to avoid a possible high ordering cost in the future. It also causes the optimal price to increase as the retailer is not confident about the supply, and therefore she needs to reduce the amount or the probability of demand. The benefit of taking the capacity constraint into account deteriorates since the estimations on the capacity is now less reliable. The relative performance of the revenue maximization compared to the optimal solution, on the other hand, decreases as the overall cost structure is now more relevant.

Future research based on our study may be conducted in several areas. First, the way the price determines the purchasing probability of the customers can be elaborated. A model to regard the choice out of multiple alternatives can be incorporated into an inventory system.

A second possible way to go beyond this study by making use of the framework provided here is examine the behavior of the optimal policy and the profit under a finite horizon for the first model that would create an opportunity to observe the difference of the optimal policies around the beginning and the end of the working horizon.

Another direction of research based on this study may feature a different aspect in supply uncertainty. The impact of a random lead time may be incorporated into the current model so as to make the model even more realistic.

APPENDIX A: MATLAB CODE FOR THE INVENTORY MODEL WITH PRICE EFFECT ON THE PROBABILITY OF DEMAND OCCURRENCE WITH ONE AND TWO SUPPLIERS

```
function result = PI(p,beta) result = exp(-beta*p);
end
```

A.1. One Supplier Setting

A.1.1. Simultaneous Optimization of y and p

For simultaneous optimization of y and p.

```
function result = profit1(xx,h,b,c,alpha,beta,k,mu)
result = -((-xx(1))*(h+c-alpha*c)+PI(xx(2),beta)*((xx(2)
+b+h-alpha*c) *(xx(1)*(1-gamcdf(xx(1),k,mu))+quad(@(x)
(x.*gampdf(x,k,mu)),0,xx(1)))-b*(k*mu)));
end
MAIN
clear all;
A=[-1 0;0 -1];
bb=[0;0];
xx01=[10;10];
xx02=[100;100];
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
```

```
options=optimset('Algorithm','interior-point');
hvector=[1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6];
bvector=(5:15);
cvector=[2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7];
alphavector=[0.975,0.95,0.925,0.90,0.875,0.85,0.825,0.80,
0.775,0.75,0.725];
betavector=[0.02,0.03,0.04,0.05,0.06,0.07,0.08,0.09,
0.10,0.11,0.12];
CVDvector=[0.50,0.55,0.60,0.65,0.70,0.75,0.80,0.85,
0.90,0.95,1];
kvector=CVDvector.  ̂(-2);
muvector=[10,12,14,16,18,20,22,24,26,28,30];
maxitvector=[1,1,1,1,1,10,10];
counter8=1;
for counter1=1:maxitvector(1)
h=hvector(counter1);
for counter2=1:maxitvector(2)
b=bvector(counter2);
for counter3=1:maxitvector(3)
c=cvector(counter3);
for counter4=1:maxitvector(4)
alpha=alphavector(counter4);
for counter5=1:maxitvector(5) beta=betavector(counter5);
for counter6=1:maxitvector(6)
k=kvector(counter6);
for counter7=1:maxitvector(7)
mu=muvector(counter7);

    xx1,fval1
= fmincon(@(xx)profit1(xx,h,b,c,alpha,beta,k,mu),xx01,A,bb,
Aeq,beq,lb,ub, nonlcon,options);
```

```
    xx2,fval2
= fmincon(@(xx)profit1(xx,h,b,c,alpha,beta,k,mu),xx02,A,bb,
Aeq,beq,lb,ub, nonlcon,options);
if (-fval1>-fval2) xx=xx1;
fval=-fval1;
else
xx=xx2;
fval=-fval2;
end
resultmatrix1=[h,b,c,alpha,beta];
resultmatrix(1:2)=xx;
resultmatrix(3)=fval;
xlswrite('output.xlsx',resultmatrix1,sprintf('A%d:E%d',
counter8,counter8));
xlswrite('output.xlsx',resultmatrix,sprintf('F%d:H%d',
counter8,counter8));
counter8=counter8+1;
end
end
end
end
end
end
end
```

A.1.2. Iterative Optimization of y and p

```
    function result = findyforfixedp(h,b,c,alpha,beta,k,mu,p)
result = gaminv(-(((h+c-alpha*c)*(exp(beta*p)))/(p+b+h-alpha*c))
+1,k,mu); end
```

```
function result = profitforfixedy(h,b,c,alpha,beta,k,mu,y,pprice)
result = -(((-y))*(h+c-alpha*c)+PI(pprice,beta)*((pprice+b+h
-alpha*c) *(y*(1-gamcdf(y,k,mu))+quad(@(x)
(x.*gampdf(x,k,mu)),0,y))-b*(k*mu)));
end
clear all;
A=-1;
bb=0;
p01=10;
p02=100;
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
hvector=[2,3,4,5,6];
bvector=[5,7.5,10,12.5,15];
cvector=[2,3,4,5,6];
alphavector=[0.90,0.90,0.85,0.80,0.75];
betavector=[0.05,0.05,0.075,0.1,0.125];
kvector=[2,4,10,20];
muvector=[10,15,20,25,30];
maxitvector=[1,1,1,1,1,10,10];
critgapp=0.001;
critgapy=0.001;
MaxIt=100;
options=optimset('Algorithm','interior-point');
counter8=1;
for counter1=1:maxitvector(1)
h=hvector(counter1);
for counter2=1:maxitvector(2)
```

```
b=bvector(counter2);
for counter3=1:maxitvector(3)
c=cvector(counter3);
for counter4=1:maxitvector(4)
alpha=alphavector(counter4);
for counter5=1:maxitvector(5)
beta=betavector(counter5);
for counter6=1:maxitvector(6)
k=kvector(counter6);
for counter7=1:maxitvector(7)
mu=muvector(counter7);
p=1/beta;
y=PI(p,beta)*k*mu;
for NumIt = 1:MaxIt
pprev=p;
yprev=y;
    p1,fval1
= fmincon(@(pprice)profitforfixedy(h,b,c,alpha,beta,k,mu,y,
pprice),p01,A,bb,Aeq,beq, lb,ub,nonlcon,options);
    p2,fval2
= fmincon(@(pprice)profitforfixedy(h,b,c,alpha,beta,k,mu,y,
pprice),p02,A,bb,Aeq,beq, lb,ub,nonlcon,options);
if(abs(p1-p2)<0.1)
p=p1;
fval=fval1;
else
message6=sprintf('Inconsistent p values');
disp(message6);
break;
end
```

```
y=findyforfixedp(h,b,c,alpha,beta,k,mu,p);
message0=sprintf('Iteration %d',NumIt);
message2 = sprintf('y = %f',y);
message4 = sprintf('p = %f',p);
message5=sprintf('***');
disp(message0);
disp(message2);
disp(message4);
disp(message5);
gapp=abs(p-pprev);
gapy=abs(y-yprev);
if ((gapp<critgapp)  (gapy<critgapy))
break;
end
end
profitfinal=-profitforfixedy(h,b,c,alpha,beta,k,mu,y,p);
if profitfinal<0
y=0;
p=1000;
profitfinal=0;
end
resultmatrix1=[h,b,c,alpha,beta];
resultmatrix(1:2)=[y,p];
resultmatrix(3)=profitfinal;
xlswrite('output.xlsx',resultmatrix1,sprintf('A%d:E%d',
counter8,counter8));
xlswrite('output.xlsx',resultmatrix,sprintf('F%d:H%d',
counter8,counter8));
counter8=counter8+1;
end
end
```

```
end
end
end
end
end
```

A.2. Two Suppliers Setting

```
    function result = func1(I1,kQ,lambda,x)
result=(x(1)-I1)*gamcdf(x(1)-I1,kQ,lambda)
-quad(@(t)t.*gampdf(t,kQ,lambda),0,x(1)-I1); end
function result = func2(beta,k,mu,kQ,lambda,x)
result=PI(x(2),beta)*( (1-gamcdf(x(1),k,mu))*(x(1)*gamcdf(x(1),
kQ,lambda) -quad(@(t)t.*gampdf(t,kQ,lambda),0,x(1)))
+ gamcdf(x(1),k,mu)*(quadl(@(t)t.*gamcdf(t,kQ,lambda)
.*gampdf(t,k,mu),0,x(1)) -dblquad(@(t,z)t.*gampdf(t,kQ,lambda).
*gampdf(z,k,mu),0,x(1),0,z)) );
end
```

A.2.1. Simultaneous Optimization of y and p

```
    function result = profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,x)
result=-( -(cb-ca)*func1(I1,kQ,lambda,x) -ca*(x(1)-I1)
+(1/(1-alpha)) *(-(1-PI(x(2),beta))*h*x(1) + PI(x(2),beta)
*(-(x(2)+b+h-alpha*ca) *(x(1)*gamcdf(x(1),k,mu)-
quad(@(t)t.*gampdf(t,k,mu),0,x(1))) +x(1)*(x(2)+b-alpha*ca)
-b*(k*mu) -alpha*(cb-ca)
*func2(beta,k,mu,kQ,lambda,x) ) ) );
```

```
end
MAIN
clear all;
A=[-1 0;0 -1];
bb=[0;0];
xx01=[10;10];
xx02=[100;100];
xx=[0 0];
A1=-1;
bb1=0;
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
h=1;
b=5;
ca=2;
cbvector=[3,2.2,2.4,2.6,2.8,3,3.2,3.4,3.6,3.8,4];
I1=0;
alpha=0.80;
beta=0.05;
kvector=[2,5];
CVQvector=[0.50,0.55,0.60,0.65,0.70,0.75,0.80,0.85,0.90,0.95,1];
kQvector=CVQvector.^(-2);
Qmeanvector=(10:20);
y=10;
pprice=10;
options=optimset(’Algorithm’,’sqp’);
options.MaxFunEvals=500;
counter5=1;
```

```
for counter6 = 1:1
for counter1 = 1:1
for counter2 = 1:1
for counter3 = 1:11
k=kvector(counter6);
mu=20/k;
cb = cbvector(counter1);
kQ=kQvector(counter2);
lambda=Qmeanvector(counter3)/kQvector(counter2);

    xx1,fval1
= fmincon(@(x)profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,kQ,
lambda,x),xx01,A,bb,
Aeq,beq,lb,ub,nonlcon,options);

    xx2,fval2
= fmincon(@(x)profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,kQ,


    lambda,x),xx02,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
if((fval1>0.1)  (fval2>0.1))
xx(1)=0;
xx(2)=1000;
fval=0;
else if(-profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,kQ,lambda,xx1)
>=-profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,kQ,lambda,xx2))
xx=xx1;
fval=fval1;
else
xx=xx2;
fval=fval2;
```

```
end
end
profit1=fval;
resultmatrix(1)=xx(1);
resultmatrix(2)=xx(2);
resultmatrix(3)=-profit1;
    xx3,fval3
= fmincon(@(x)profittwosup(h,b,ca,ca,I1,alpha,beta,k,mu,kQ,
lambda,x),xx01,A,bb, Aeq,beq,lb,ub,nonlcon,options);
    xx4,fval4
= fmincon(@(x)profittwosup(h,b,ca,ca,I1,alpha,beta,k,mu,kQ,
lambda,x),xx02,A,bb, Aeq,beq,lb,ub,nonlcon,options);
if((fval3>0.1)  (fval4>0.1)) xx(1)=0;
xx(2)=1000;
fval=0;
else if(-profittwosup(h,b,ca,ca,I1,alpha,beta,k,mu,kQ,lambda,xx3)
>=-profittwosup(h,b,ca,ca,I1,alpha,beta,k,mu,kQ,lambda,xx4))
xx=xx3;
fval=fval3;
else
xx=xx4;
fval=fval4;
end
end
profit2=profittwosup(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,xx);
if(profit2>0.1) profit2=0;
end
resultmatrix(4)=xx(1);
resultmatrix(5)=xx(2);
```

```
resultmatrix(6)=-profit2;
ratio1=(profit1-profit2)/profit1;
resultmatrix(7)=100*ratio1;
pprice=1/beta;

    xxa,fval1
= fmincon(@(y)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,y,pprice),
xx01(1),A1,bb1,Aeq,beq,lb,ub,nonlcon,options);

    xxb,fval2
= fmincon(@(y)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,y,pprice),
xx02(1),A1,bb1,Aeq,beq,lb,ub,nonlcon,options);
if((fval1>0.1)  (fval2>0.1)) xxc=0;
fval=0;
else if(-profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,xxa,pprice)
>=-profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,xxb,pprice)) xxc=xxa;
fval=fval1;
else xxc=xxb;
fval=fval2;
end
end
profit3=profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,xxc,pprice);
if(profit3>0.1) profit3=0;
end
resultmatrix(8)=xxc;
resultmatrix(9)=pprice;
resultmatrix(10)=-profit3;
```

```
ratio2=(profit1-profit3)/profit1;
resultmatrix(11)=100*ratio2;
cboverca=cb/ca;
EQoverED=Qmeanvector(counter3)/(k*mu);
CV=1/sqrt(kQ);
xlswrite('output.xlsx',k,sprintf('A%d:A%d',counter5,counter5));
xlswrite('output.xlsx',kQ,sprintf('B%d:B%d',counter5,counter5));
xlswrite('output.xlsx',Qmeanvector(counter3),sprintf('C%d:C%d',
counter5,counter5));
xlswrite('output.xlsx',cboverca,sprintf('D%d:D%d',counter5,
counter5));
xlswrite('output.xlsx',CV,sprintf('E%d:E%d',counter5,
counter5));
xlswrite('output.xlsx',EQoverED,sprintf('F%d:F%d',counter5,
counter5));
xlswrite('output.xlsx',resultmatrix,sprintf('G%d:Q%d',counter5,
counter5));
counter5=counter5+1;
end
end
end
end
```

A.2.2. Iterative Optimization of y and p

```
    function result = profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,
k,mu,kQ,lambda,y,pprice)
x(1)=y;
x(2)=pprice;
result=-( -(cb-ca)*func1(I1,kQ,lambda,x) -ca*(y-I1) +(1/(1-alpha))*
```

```
(-(1-PI(pprice,beta))*h*y + PI(pprice,beta)*(-(pprice+b+h-alpha*ca)
*(y*gamcdf(y,k,mu)
-quad(@(t)t.*gampdf(t,k,mu),0,y))
+y*(pprice+b-alpha*ca)-b*(k*mu)
-alpha*(cb-ca)*func2(beta,k,mu,kQ,lambda,x) ) ) );
end
clear all;
A=-1;
bb=0;
p01=10;
p02=100;
y01=10;
y02=100;
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
h=1;
b=5;
ca=1;
cbvector=[1,1.25,1.5,1.75,2];
I1=0;
alpha=0.9;
beta=0.05;
kvector=[2,5];
kQvector=[100,4,2];
Qmeanvector=[4,10,20];
critgapp=0.001;
critgapy=0.001;
MaxIt=100;
```

```
pprice=20;
options=optimset('Algorithm','interior-point');
options.MaxFunEvals=500;
counter5=1;
for counter6 = 1:1
for counter1 = 2:5
for counter2 = 1:3
for counter3 = 1:3
k=kvector(counter6);
mu=20/k;
cb = cbvector(counter1);
kQ=kQvector(counter2);
lambda=Qmeanvector(counter3)/kQvector(counter2);
p=1/beta;
y=PI(p,beta)*k*mu;
for NumIt = 1:MaxIt pprev=p;
yprev=y;

    p1,fval1
= fmincon(@(pprice)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,
k,mu,kQ,lambda,y,pprice),
p01,A,bb,Aeq,beq,lb,ub,nonlcon,options);

    p2,fval2
= fmincon(@(pprice)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,
k,mu,kQ,lambda,y,pprice),
p02,A,bb,Aeq,beq,lb,ub,nonlcon,options);
if(abs(p1-p2)<0.1) p=p1;
fval=fval1;
else message6=sprintf('Inconsistent p values');
disp(message6);
break;
```

```
end
    y1,fval1
= fmincon(@(y)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,y,pprice),
y01,A,bb,Aeq,beq,lb,ub,nonlcon,options);
    y2,fval2
= fmincon(@(y)profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,y,pprice),
y02,A,bb,Aeq,beq,lb,ub,nonlcon,options);
if(abs(y1-y2)<0.1) y=y1;
fval=fval1;
else message6=sprintf('Inconsistent p values');
disp(message6);
break;
end
message0=sprintf('Iteration %d',NumIt);
message2 = sprintf('y = %f',y);
message4 = sprintf('p = %f',p);
message5=sprintf('***');
disp(message0);
disp(message2);
disp(message4);
disp(message5);
gapp=abs(p-pprev);
gapy=abs(y-yprev);
if ((gapp<critgapp)  (gapy<critgapy)) break;
end
end
profitfinal=-profittwosupfixedp(h,b,ca,cb,I1,alpha,beta,k,mu,
kQ,lambda,y,p);
```

```
if profitfinal<0 y=0;
p=1000;
profitfinal=0;
end
cboverca=cb/ca;
EQoverED=Qmeanvector(counter3)/(k*mu);
CV=1/sqrt(kQ);
resultmatrix=[y,p,profitfinal];
xlswrite('output.xlsx',k,sprintf('A%d:A%d',counter5,counter5));
xlswrite('output.xlsx',kQ,sprintf('B%d:B%d',counter5,counter5));
xlswrite('output.xlsx',Qmeanvector(counter3),sprintf('C%d:C%d',
counter5,counter5));
xlswrite('output.xlsx',cboverca,sprintf('D%d:D%d',counter5,
counter5));
xlswrite('output.xlsx',CV,sprintf('E%d:E%d',counter5,
counter5));
xlswrite('output.xlsx',EQoverED,sprintf('F%d:F%d',counter5,
counter5));
xlswrite('output.xlsx',resultmatrix,sprintf('G%d:I%d',counter5,
counter5));
counter5=counter5+1;
end
end
end
end
```

APPENDIX B: MATLAB CODE FOR THE INVENTORY MODEL INVESTIGATING THE PRICE EFFECT ON INDIVIDUAL CUSTOMERS WITH ONE AND TWO SUPPLIERS

```
    function result = PI(beta,p)
result = exp(-beta*p);
end
function result=mean(parameters)
result=parameters(1)*parameters(2);
end
function result=density(parameters,x)
result=gampdf(x,parameters(1),parameters(2));
end
function result=cumulative(parameters,x)
result=gamcdf(x,parameters(1),parameters(2));
end
function result=sigma(parameters,y)
result=quad(@(t)t.*gampdf(t,parameters(1),parameters(2)),0,y);
end
function result=Holding(h,parameters,y,T)
result=T*h*((y+y*cumulative(parameters,y)-sigma(parameters,y))/2);
end
function result=LostSales(b,parameters,y)
result=b*(mean(parameters)+y*(cumulative(parameters,y)-1)
-sigma(parameters,y));
end
function result=L(h,b,parameters,y,T)
result=Holding(h,parameters,y,T)+LostSales(b,parameters,y);
```

```
end
function result=R(parameters,y,p)
result=p*(y*(1-cumulative(parameters,y))+sigma(parameters,(y)));
end
```

B.1. One Supplier Setting

```
    function result=orderingonesup(c,y,I)
result=c*(y-I);
end
function result=profitwopar(xx,h,b,c,beta,T1,T2,kD)
lambda=10;
I1=0;
parameters=[kD*T1,PI(beta,xx(2))*lambda];
I2=xx(1)*cumulative(parameters,xx(1))-sigma(parameters,xx(1));
profit1=-(R(parameters,xx(1),xx(2))-L(h,b,parameters,xx(1),T1)
-orderingonesup(c,xx(1),I1));
parameters=[kD*T2,PI(beta,xx(4))*lambda];
profit2=-(R(parameters,xx(3),xx(4))-L(h,b,parameters,xx(3),T2)
-orderingonesup(c,xx(3),I2));
result=profit1+profit2;
end
```

```
    MAIN
clear all;
A=[-1 0 0 0;0 -1 0 0;0 0 -1 0;0 0 0 -1];
bb=[0;0;0;0];
xx01=[10;10;10;10];
xx02=[100;100;100;100];
```

```
Aeq=[];
beq=[];
lb=[];
ub=[];
nonlcon=[];
options=optimset('Algorithm','interior-point');
options.MaxFunEvals=500;
hvector=[1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6];
bvector=(5:15);
cvector=[2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7];
betavector=[0.05,0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.10,
0.11,0.12];
T1vector=(5:11);
T2vector=(1:11);
CVDvector=[0.50,0.55,0.60,0.65,0.70,0.75,0.80,0.85,0.90,0.95,1];
kDvector=CVDvector. ˆ(-2);
maxitvector=[1,1,1,1,1,10,10];
counter9=1;
for counter1 ¯1:maxitvector(1)
h=hvector(counter1);
for counter2=1:maxitvector(2)
b=bvector(counter2);
for counter3=1:maxitvector(3)
c=cvector(counter3);
for counter4=1:maxitvector(4)
beta=betavector(counter4);
for counter5=1:maxitvector(5)
T1=T1vector(counter5);
for counter6=1:maxitvector(6)
T2=T2vector(counter6);
for counter7=1:maxitvector(7)
```

```
kD=kDvector(counter7);

    xx1,fval1
= fmincon(@(xx)profitwopar(xx,h,b,c,beta,T1,T2,kD),xx01,A,bb,
Aeq,beq,lb,ub,nonlcon,options);

    xx2,fval2
= fmincon(@(xx)profitwopar(xx,h,b,c,beta,T1,T2,kD),xx02,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
if (-fval1>-fval2)
xx=xx1;
fval=-fval1;
else
xx=xx2;
fval=-fval2;
end
resultmatrix(1:4)=xx;
avgprofit=fval/(T1+T2);
resultmatrix(5)=avgprofit;
xlswrite('output.xlsx',h,sprintf('A%d:A%d',counter9,counter9));
xlswrite('output.xlsx',b,sprintf('B%d:B%d',counter9,counter9));
xlswrite('output.xlsx',c,sprintf('C%d:C%d',counter9,counter9));
xlswrite('output.xlsx',beta,sprintf('D%d:D%d',counter9,counter9));
xlswrite('output.xlsx',T1,sprintf('E%d:E%d',counter9,counter9));
xlswrite('output.xlsx',T2,sprintf('F%d:F%d',counter9,counter9));
xlswrite('output.xlsx',kD,sprintf('G%d:G%d',counter9,counter9));
xlswrite('output.xlsx',resultmatrix,sprintf('H%d:L%d',counter9,
counter9));
counter9=counter9+1;
end
end
end
```

```
end
end
end
end
```

B.2. Two Suppliers Setting

```
    function result=orderingtwosup(ca,cb,y,I,kQ,mu)
result=(cb-ca)*((y-I)*gamcdf(y-I,kQ,mu)-quad(@(t)
t.*gampdf(t,kQ,mu),0,y-I))
+ca*(y-I);
end
function result=profitwopartwosup(xx,h,b,ca,cb,beta,T1,T2,
kQ,mu,kD) lambda=10;
I1=0;
parameters=[kD,PI(beta,xx(2))*lambda*T1];
I2=xx(1)*cumulative(parameters,xx(1))-sigma(parameters,xx(1));
profit1=-(R(parameters,xx(1),xx(2))-L(h,b,parameters,xx(1),T1)
-orderingtwosup(ca,cb,xx(1),I1,kQ,mu));
parameters=[kD,PI(beta,xx(4))*lambda*T2];
profit2=-(R(parameters,xx(3),xx(4))-L(h,b,parameters,xx(3),T2)
-orderingtwosup(ca,cb,xx(3),I2,kQ,mu));
result=(profit1+profit2)/(T1+T2);
end
MAIN
clear all;
A=[-1 0 0 0;0 -1 0 0;0 0 -1 0;0 0 0 -1]; bb=[0;0;0;0];
xx01=[10;10;10;10];
xx02=[100;100;100;100];
Aeq=[];
```

```
beq=[];
lb=[];
ub=[];
nonlcon=[];
options=optimset(’Algorithm’,’sqp’);
options.MaxFunEvals=500;
hvector=[1,2,3,4,5,6];
bvector=[5,7,9,11,13];
cavector=[2,3,4,5,6];
cbvector=[3,2.2,2.4,2.6,2.8,3,3.2,3.4,3.6,3.8,4];
betavector=[0.05,0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.10,
0.11,0.12];
T1vector=(1:10);
T2vector=(1:10);
CVQvector=[0.50,0.55,0.60,0.65,0.70,0.75,0.80,0.85,0.90,0.95,1];
kQvector=CVQvector. ˆ(-2);
kD=2;
lambda=10;
avgdemand=kD*lambda;
avgcapvector=avgdemand*[0.50,0.55,0.60,0.65,0.70,0.75,0.80,
0.85,0.90,0.95,1];
maxitvector=[1,1,1,1,1,1,1,10,10];
counter10=1;
for counter1=1:maxitvector(1)
h=hvector(counter1);
for counter2=1:maxitvector(2)
b=bvector(counter2);
for counter3=1:maxitvector(3)
ca=cavector(counter3);
for counter4=1:maxitvector(4)
cb=cbvector(counter4);
```

```
for counter5=1:maxitvector(5)
beta=betavector(counter5);
for counter6=1:maxitvector(6)
T1=T1vector(counter6);
for counter7=1:maxitvector(7)
T2=T2vector(counter7);
for counter8=1:maxitvector(8)
kQ=kQvector(counter8);
for counter9=1:maxitvector(9) mu=(avgcapvector(counter9)
*exp(-1))/kQ;

    xx1,fval1
= fmincon(@(xx)profitwopartwosup(xx,h,b,ca,cb,beta,T1,T2,
kQ,mu,kD),xx01,A,bb,
Aeq,beq,lb,ub,nonlcon,options);

    xx2,fval2
= fmincon(@(xx)profitwopartwosup(xx,h,b,ca,cb,beta,T1,T2,
kQ,mu,kD),xx02,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
if (-fval1>-fval2)
xx=xx1;
fval=-fval1;
else
xx=xx2;
fval=-fval2;
end
profit1=fval;
resultmatrix(1:4)=xx;
resultmatrix(5)=profit1;

    xx1,fval1
= fmincon(@(xx)profitwopartwosup(xx,h,b,ca,ca,beta,T1,T2,
```

```
kQ,mu,kD),xx01,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
    xx2,fval2
= fmincon(@(xx)profitwopartwosup(xx,h,b,ca,ca,beta,T1,T2,
kQ,mu,kD),xx02,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
if((fval1>0.1)  (fval2>0.1))
xx(1)=0;
xx(3)=0;
xx(2)=1000;
xx(4)=1000;
fval=0;
else if(-profitwopartwosup(xx1,h,b,ca,ca,beta,T1,T2,
kQ,mu,kD)
>=-profitwopartwosup(xx2,h,b,ca,ca,beta,T1,T2,
kQ,mu,kD))
xx=xx1;
fval=-fval1;
else
xx=xx2;
fval=-fval2;
end
end
profit2=-profitwopartwosup(xx,h,b,ca,cb,beta,T1,T2,kQ,mu,kD);
if(profit2<0.1)
profit2=0;
end
resultmatrix(6:9)=xx;
resultmatrix(10)=profit2;
ratio1=(profit1-profit2)/profit1;
resultmatrix(11)=100*ratio1;
```

```
xx03=[10;1/beta;10;1/beta];
xx04=[100;1/beta;100;1/beta];
    xx1,fval1
= fmincon(@(xx)profitwopartwosupfixedp(xx,h,b,ca,cb,beta,T1,T2,
kQ,mu,kD),xx03,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
    xx2,fval2
= fmincon(@(xx)profitwopartwosupfixedp(xx,h,b,ca,cb,beta,T1,T2,
kQ,mu,kD),xx04,A,bb,
Aeq,beq,lb,ub,nonlcon,options);
if((fval1>0.1)  (fval2>0.1))
xx(1)=0;
xx(3)=0;
xx(2)=1000;
xx(4)=1000;
fval=0;
else if(-profitwopartwosupfixedp(xx1,h,b,ca,cb,beta,T1,T2,kQ,mu,kD)
>=-profitwopartwosupfixedp(xx2,h,b,ca,cb,beta,T1,T2,kQ,mu,kD))
xx=xx1;
fval=-fval1;
else
xx=xx2;
fval=-fval2;
end
end
ddd=fval;
profit3=-profitwopartwosup(xx,h,b,ca,cb,beta,T1,T2,kQ,mu,kD);
if(profit3<0.1)
profit3=0;
end
```

```
resultmatrix(12:15)=xx;
resultmatrix(16)=profit3;
ratio2=(profit1-profit3)/profit1;
resultmatrix(17)=100*ratio2;
cratio=cb/ca;
CV=1/sqrt(kQ);
EQoverED=(kQ*mu)/(avgdemand*exp(-1));
resultmatrix1=[h,b,ca,cb,beta,T1,T2,kQ,mu,cratio,CV,EQoverED];
xlswrite('output.xlsx',resultmatrix1,sprintf('A%d:L%d',
counter10,counter10));
xlswrite('output.xlsx',resultmatrix,sprintf('M%d:AC%d',
counter10,counter10));
counter10=counter10+1;
end
end
end
end
end
end
end
end
```

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