

APPLICATIONS OF ROBUST OPTIMIZATION IN LOGISTICS AND PRODUCTION PLANNING

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Farzad Avishan

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APPLICATIONS OF ROBUST OPTIMIZATION IN LOGISTICS AND PRODUCTION PLANNING

Advisor: Assistant Professor Dr. İhsan Yanıkoğlu
Coadvisor: Associate Professor Dr. Mehmet Soysal

Approved by:

Assistant Professor Dr. İhsan
Yanıkoğlu, Advisor
Department of Industrial Engineering
Özyeğin University

Associate Professor Dr. Ertan Yakıcı
Department of Industrial Engineering
NDU Turkish Naval Academy

Professor Dr. Okan Örsan Özener
Department of Industrial Engineering
Özyeğin University

Assistant Professor Dr. Tonguç Yavuz
Department of Industrial Engineering
Istanbul Bilgi University

Date Approved: February 28, 2023

Assistant Professor Dr. Başak Altan
Faculty of Business
Özyeğin University



To my family...

ABSTRACT

We analyze three applications of the robust optimization approach in this thesis. The initial part is dedicated to planning a dairy production and distribution problem considering uncertain demand. The second part presents an adjustable robust optimization approach for relief distribution in a post-disaster scenario where travel times are uncertain. Lastly, in the third part, we tackle the electric bus scheduling problem that incorporates uncertainty in both travel times and energy consumption.

The first part of this thesis investigates a robust dairy production and distribution planning problem that considers the complexities of dairy production, including perishability, sequence dependence, and demand uncertainty. To tackle this uncertainty, we introduce an adjustable robust optimization approach that generates a robust and Pareto-efficient production and distribution management plan. This approach provides decision-making flexibility by allowing for adjustments based on the actual demand observed over a multi-period planning horizon. The effectiveness of the proposed method is evaluated through extensive Monte Carlo simulation experiments. Additionally, we conduct a case study to demonstrate how the adjustable approach outperforms the static robust approach in terms of the objective function value and solution performance.

In the second part, we present an adjustable robust optimization approach for relief supply distribution in the aftermath of a disaster. The approach generates routes for relief logistics teams and determines the service times for visited sites to distribute supplies, taking into account the uncertainty of travel times. The model allows for adjustments to service decisions based on real-time information, resulting in robust solutions for the worst-case scenario of travel times but also more flexible and less

conservative than those of static robust optimization. Due to the computational complexity of solving resulting models, we propose heuristic algorithms as an alternative solution approach. Using 2011 earthquake data from the Van province of Turkey, we have also demonstrated the effectiveness of our approach.

In the last part, we investigate a scheduling problem for electric fleets faced with uncertain travel times and energy consumption. We propose a mixed-integer linear programming model to optimize electric fleet purchasing and charging operation costs, utilizing robust optimization to address uncertainty. A new uncertainty set is introduced to control the robustness of the solution. The model determines the required number of buses to cover all trips, schedules the trips, and designs charging plans for the buses. We evaluate the effectiveness of the model through extensive Monte Carlo simulations. Additionally, we present a case study on the off-campus transport network at Binghamton University to demonstrate the real-world applicability of the model and solution approach.

ÖZETÇE

Bu tezde gürbüz optimizasyon yaklaşımının üç uygulamasını analiz ediyoruz. İlk bölüm, belirsiz talep ortamında süt üretimi ve dağıtımının planlanmasına ayrılmıştır. İkinci bölüm, seyahat sürelerinin belirsiz olduğu bir afet sonrası senaryoda yardım dağıtımı için ayarlanabilir gürbüz bir optimizasyon yaklaşımı sunar. Son olarak, üçüncü bölümde, hem seyahat süreleri hem de enerji tüketimindeki belirsizliği içeren elektrikli otobüs çizelgeleme problemini ele alıyoruz.

Bu tezin ilk kısmı, bozulabilirlik, sıra bağımlılığı ve talep belirsizliği dahil olmak üzere süt üretiminin karmaşıklıklarını dikkate alan gürbüz bir süt ürünleri üretimi ve dağıtım planlama problemini araştırmaktadır. Bu belirsizliğin üstesinden gelmek için, gürbüz ve Pareto açısından verimli bir üretim ve dağıtım yönetim planı oluşturan, ayarlanabilir gürbüz bir optimizasyon yaklaşımı sunuyoruz. Bu yaklaşım, çok dönemli bir planlama ufkunda gözlemlenen gerçek talebe dayalı ayarlamalara izin vererek karar verme esnekliği sağlar. Önerilen yöntemin etkinliği, kapsamlı Monte Carlo simülasyon deneyleri ile değerlendirilmiştir. Ek olarak, ayarlanabilir yaklaşımın, amaç fonksiyonu değeri ve çözüm performansı açısından statik gürbüz yaklaşımdan nasıl daha iyi performans gösterdiğini göstermek için bir vaka çalışması yürütüyoruz.

İkinci bölümde, bir afet sonrasında yardım malzemesi dağıtımı için ayarlanabilir gürbüz bir optimizasyon yaklaşımı sunuyoruz. Yaklaşım, yardım lojistiği ekipleri için rotalar oluşturur ve seyahat sürelerinin belirsizliğini hesaba katarak malzemeleri dağıtmak için ziyaret edilen sitelerin hizmet sürelerini belirler. Model, gerçek zamanlı bilgilere dayalı hizmet kararlarında ayarlamalara izin vererek, seyahat sürelerinin en kötü senaryosu için gürbüz çözümler sağlar, ancak aynı zamanda statik gürbüz optimizasyondan daha esnek ve daha az ihtiyatlıdır. Ortaya çıkan modelleri çözmenin

hesaplama karmaşıklığı nedeniyle, alternatif bir çözüm olarak sezgisel algoritmalar öneriyoruz. Türkiye'nin Van ilinden 2011 deprem verilerini kullanarak, yaklaşımımızın etkinliğini de gösterdik.

Son bölümde, belirsiz seyahat süreleri ve enerji tüketimi ile karşı karşıya kalan elektrik filoları için bir çizelgeleme problemi inceliyoruz. Belirsizliği ele almak için gürbüz optimizasyondan yararlanarak, elektrik filosu satın alma ve şarj işletme maliyetlerini optimize etmek için yeni bir karma tamsayılı doğrusal programlama modeli öneriyoruz. Çözümün gürbüz lığını kontrol etmek için yeni bir belirsizlik seti tanıtıldı. Model, tüm yolculukları kapsayacak şekilde gerekli otobüs sayısını belirler, yolculukları planlar ve otobüsler için ücretlendirme planları tasarlar. Kapsamlı Monte Carlo simülasyonları aracılığıyla modelin etkinliğini değerlendiriyoruz. Ek olarak, model ve çözüm yaklaşımının gerçek dünyada uygulanabilirliğini göstermek için Binghamton Üniversitesi'ndeki kampüs dışı üniversite ulaşım ağına ilişkin bir vaka çalışması sunuyoruz.

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TABLE OF CONTENTS

DEDICATION	iii
ABSTRACT	iv
ÖZETÇE	vi
ACKNOWLEDGEMENTS	viii
LIST OF TABLES	xiv
LIST OF FIGURES	xvi
I INTRODUCTION	1
II AN ADJUSTABLE ROBUST OPTIMIZATION APPROACH FOR DAIRY PRODUCTION AND DISTRIBUTION PLANNING UN- DER DEMAND UNCERTAINTY	4
2.1 Introduction	5
2.2 Literature review	9
2.3 Robust dairy production and distribution planning problem	14
2.3.1 Problem definition and deterministic model	14
2.3.2 Robust reformulation	18
2.3.3 Adjustable robust reformulation	20
2.4 Solution methods	24
2.4.1 Re-optimization approach	24
2.4.2 Heuristic approaches	26
2.4.3 Monte Carlo simulation procedure	29
2.5 Numerical experiments	33
2.5.1 Data structure	33
2.5.2 Evaluating CPU time performances of proposed approaches .	34
2.5.3 Evaluating OFV performances of static and adjustable robust approaches	36
2.5.4 Case study	37

2.5.4.1	Evaluating OFV performance of static robust approach	38
2.5.4.2	Evaluating feasibility performance of deterministic approach	40
2.5.4.3	Evaluating OFV performances of static and adjustable robust approaches	42
2.6	Managerial insights	43
2.6.1	Importance of production planning in dairy industry and challenges	43
2.6.2	Flaw of using nominal demand	44
2.6.3	Decreasing the level of disposal via adjustable robust approach	44
2.7	Conclusion and future research	45
III	HUMANITARIAN RELIEF DISTRIBUTION PROBLEM: AN ADJUSTABLE ROBUST OPTIMIZATION APPROACH	46
3.1	Introduction	47
3.2	Literature Review	50
3.2.1	Effectiveness and Equity	51
3.2.2	Modeling and Solution Approaches with Equity	51
3.2.3	Relief Distribution under Uncertainty	52
3.2.4	Contributions	54
3.3	Problem Definition and Formulation	55
3.3.1	Adjustable Robust Reformulation	60
3.3.2	Pareto Efficient Reformulation and Re-optimization Method .	65
3.4	Heuristic Algorithms	69
3.4.1	Clustering-based Heuristic Approach for Routing (H1.1) . . .	70
3.4.2	Clustering-based Heuristic Approach Utilizing Lower Bounds (H1.2)	72
3.4.3	Heuristic Algorithm to Find Decision Rule Coefficients (H2) .	72
3.5	Numerical Experiments	73
3.5.1	Evaluating the Performances of Heuristic Algorithms	76
3.5.2	Comparison of Adjustable Robust versus Static Robust Approaches	78

3.5.3	Optimality performance of LDRs	79
3.5.4	Case Study	80
3.6	Conclusion and Managerial Implications	84
3.6.1	Effects of Number of RLTs and Service Time Limit	84
3.6.2	Effect of Uncertainty Level	85
3.6.3	Advantages of Adjustable Decision Making under Uncertainty	86
IV	AN OPTIMIZATION APPROACH FOR ELECTRIC BUS SCHEDULING UNDER TRAVEL TIME AND ENERGY CONSUMPTION UNCERTAINTY	87
4.1	Introduction	88
4.2	Literature review	92
4.2.1	Homogeneous EB optimization models	94
4.2.2	Mixed EB Optimization Models	95
4.3	Problem definition and deterministic model	96
4.4	Robust reformulation and uncertainty sets	104
4.5	Heuristic algorithm	108
4.6	Numerical experiments	112
4.6.1	Data structure	112
4.6.2	Heuristic algorithm performance	116
4.6.3	Deterministic and robust solutions comparison	117
4.6.4	Monte Carlo simulation procedure	120
4.6.5	Effect of charging station selection on OFV	125
4.7	Conclusion and Managerial implications	127
V	CONCLUSIONS	130
	REFERENCES	132
	APPENDIX A —	148
A.1	Robust mathematical optimization model [RPDP]	148
A.2	Applying three step of RO procedure for inventory capacity constraint	148
A.3	Tractable reformulation of [ARPDP]	150

A.4	Tractable reformulation of [PR](ARPD)	154
A.5	Tractable reformulation of [PR](RPDP)	155
APPENDIX B	—	156
B.1	Parameters of case study instance	156
B.2	Parameter Generation Schemes	156
B.3	Properties of [RRDP]	158
B.4	Deriving RC of Time Limit Constraints	159
B.5	Tractable Reformulation of [ARDP] and [PR]	160
B.6	Suboptimality of Adopting Linear Decision Rules for Auxiliary Variable	162
B.7	Step 2 of H1.2	163
B.8	Steps of H2	164
B.9	Extra Discussions on Numerical Results	165
B.10	Model Extensions and Discussions	168
APPENDIX C	—	171
C.1	Literature review table	171
C.2	Robust counterpart	172
C.3	Summary of the heuristic algorithm and parameters	172
C.4	Simulation nomenclature and pseudo-code	174
VITA		176

LIST OF TABLES

1	Summary of studies on food logistics network management that respect product perishability	10
2	Nomenclature for PDP	16
3	Re-optimization algorithm	25
4	Number constraints and decision variables for different-sized problems	26
5	Benchmark heuristic	27
6	Pareto efficient heuristic for solving [ARPDP]	28
7	Model dimensions	34
8	Sampling distributions of parameters	34
9	Average CPU times of mathematical optimization models (seconds) .	34
10	Average cost improvement of heuristic models with respect to [RPDP]	36
11	Robuts OFV for different uncertainty levels	39
12	Monte Carlo simulation on feasibility of deterministic approach . . .	41
13	Average cost improvement of heuristic approaches for [RPDP]	42
14	Nomenclature of the mathematical optimization model	58
15	Parameters of utility functions	66
16	Instance size classification	75
17	Algorithms	76
18	Performances of the complete heuristic algorithms	77
19	Comparison of adjustable robust versus static robust solutions	78
20	Optimality performance of LDRs	79
21	Comparison of adjustable robust versus static robust approaches . . .	82
22	Equity of relief supply distribution – Gini coefficient ($\rho = 0.45$) . . .	83
23	Nomenclature of the mathematical optimization model	99
24	Trip information	114
25	Bus specification	114
26	Nomenclature and values of parameters in equation (88)	115

27	Comparison of heuristic and standard approach	117
28	Schedule for deterministic solution	118
29	Breakdown of costs	119
30	In-sample simulation results	122
31	Out-of-sample-simulation results	124
32	Costs analysis for different charging station	126
33	Product information	156
34	Warehouse information	156
35	Retailers' nominal demand	156
36	Summary statistics of data set	157
37	Gini coefficient ($\rho = \mathbf{0.45}$)	158
38	Performance of the algorithms for Phase I	165
39	Performance of the algorithms for Phase II	166
40	Out-of-sample optimality performance of LDRs	167
41	Comparison of OFV and CPU time performances	167
42	Some of the studies for homogeneous EB systems	171
43	Comparison of heuristic and standard approach for deterministic problem	173
44	Inputs and variables of the Monte Carlo simulation procedure	174

LIST OF FIGURES

1	The flow chart of the addressed network	15
2	Manufacturer and retailers' locations	37
3	Robust OFV for different uncertainty levels	39
4	Utility function with two breakpoints	56
5	Illustrative example	61
6	The flow chart of the proposed solution approaches	69
7	A sample graph to represent [EBSP] solution	98
8	Overview of Step 1	109
9	Overview of Step 2	110
10	WS out route and stations	113
11	Breakdown of costs	120
12	Case study solution representation ($\rho = 0.25, V = 10$)	158

CHAPTER I

INTRODUCTION

Problems in the real world often have uncertainties that can impact the outcome of a decision or system. *Robust optimization* (RO) is an approach to optimize problems while considering uncertainty in the input data, creating resilient solutions against any realization of the uncertainty parameter residing in a predefined bounded uncertainty set. RO is a worst-case-oriented approach and yields solutions that are less sensitive to changes in the inputs and, therefore, more reliable in real-world applications. One way to address the potential over-conservatism of using a static RO approach is to adopt the *adjustable* RO (ARO) paradigm. ARO allows for updating certain or all decisions throughout the planning process based on the realized information of the uncertain parameters at a given time. In this thesis, we study three different real-life problems that contain uncertain parameters and adopt RO and ARO to address the uncertainties.

As the opening part of the thesis, we propose *dairy production and distribution planning problem* (PDP) in Chapter 2. The research takes into account various factors, such as temperature tracking for multiple dairy products, temperature-dependent setup costs for operating facilities, limited inventory capacities in production and warehousing, short lead times for retailer deliveries, and uncertain demand. The goal is to determine a production plan that optimizes setup costs and to develop efficient production and transportation plans that meet lead times while minimizing overall costs. To address the uncertainty in demand, we use the RO approach and formulate the robust PDP (RPDP), which does not require any information about the demand distribution. Instead, we assume that the demand values realize within

intervals defined by the decision-maker based on historical demand data. We apply ARO to avoid overproduction and excess inventory costs when actual demand is lower than the worst-case scenario. ARO allows for more flexible solutions that can be adjusted based on new information as it becomes available throughout the planning process, resulting in the proposed *adjustable* RPDP (ARPD).

In chapter 3 of the thesis, we propose an optimization problem for deciding on routes and service times for *relief logistics teams* (RLT) to distribute relief supplies in a post-disaster setting. This problem, known as the *post-disaster relief supply distribution problem* (RDP), takes into account the uncertainty of travel times that may be caused due to debris and traffic caused by panicked people. To address this uncertainty, we propose an RO approach that can be used in a data-scarce environment during the response phase. This is referred to as the *robust* reformulation of RDP (RRDP). It addresses the travel time uncertainty to provide a robust solution feasible for any realization of travel times within a given interval. However, this approach may yield over-conservative solutions when non-worst-case data is realized. To address this issue, we propose an *adjustable* reformulation of RDP (ARDP) that allows RLTs to adjust their service time decisions based on realized information over time. The service times are modeled as wait-and-see decisions that depend on uncertain travel times.

Finally, in Chapter 4, we try to find a solution for scheduling and charging *electric buses* (EBs) in a transportation system to minimize the cost of purchasing EBs and operating them within a defined period. We refer to the problem as the *electric bus scheduling problem* (EBSP) and aim to determine the number of EB needed of different types to cover a set of trips. When scheduling an EB network, the uncertainty of travel times must be considered since it affects the scheduling of the EBs and their energy consumption. Additionally, EB weight is often uncertain due to unpredictable passenger demand. The uncertainty of the weight and travel time will likely influence

the energy consumption of EBs. We formulate the energy consumption of EBs and propose RO to address the uncertainties. Comparing the solution performance of deterministic and robust models may lead to misleading conclusions since they are solved for different data sets. We propose a Monte Carlo simulation to compare the average solution performance of the deterministic and robust models. Through this method, we evaluate the benefits of using a robust approach regarding the reliability and feasibility of the scheduling and charging plans for EBs. Our results can guide the development and deployment of EB networks in other transportation systems to optimize costs and make strategic decisions, such as the number of EBs that should be purchased and the location of the charging stations.

CHAPTER II

AN ADJUSTABLE ROBUST OPTIMIZATION APPROACH FOR DAIRY PRODUCTION AND DISTRIBUTION PLANNING UNDER DEMAND UNCERTAINTY

Comprehensive management of production and distribution planning of dairy products is of high importance due to the low-profit margin of the dairy industry and the increased environmental costs of manufacturing. The planning problem in the dairy industry is relatively more complicated than that in other industries as it combines multiple challenging factors such as perishability, temperature tracking of products, sequence-dependent facility setup cost, and uncertain demand in one setting. This paper studies a robust dairy production and distribution planning problem that unifies the factors above in an optimization framework. We propose an (adjustable) robust optimization method to address the uncertainty in demand. The method generates a resilient and Pareto efficient production and distribution management plan against demand uncertainty and yields a flexible optimization approach that may update some decisions according to the realized demand in a multi-period planning horizon. We assess the model efficiency through extensive Monte Carlo simulation experiments. According to the case study results, the adjustable approach improves the objective function value by 7–11% for that of the static robust. Numerical results also show that the adjustable robust approach is always resilient against uncertainty, while the percentage of unmet demand for the deterministic plan may get as high as 17%.

2.1 Introduction

World consumption of processed and fresh dairy products is predicted to increase by the *food and agriculture organization* (FAO). Even though the positive outlook of the dairy industry shows that some key players such as India, European Union, the United States, and Russia are assumed to increase their production year-on-year, and the overall world production has the potential to reach up to 860 million tones in 2022 (FAO Outlook Report, 2020), the production deficit of some countries in Asia and Africa may not be fulfilled by these core producers. When the production deficit of dairy products is added to the environmental costs of manufacturing these products, the dairy industry needs to ensure that not even a single liter of milk is thrown away. Recent environmental studies show that 628 liters of water are required to produce one liter of milk [1], while the daily consumption of a person should not exceed 25 liters, in the long run, to sustain valuable freshwater resources in some countries such as South Africa [2]. Moreover, livestock that supplies milk and other animal products is responsible for more than 50% of the worldwide greenhouse emissions [3]. This is why a dairy production network should be configured to keep dairy products fresh and prepared for customers exactly when demanded and with the minimum environmental cost. Advanced quantitative models for dairy production and distribution optimization can help decision-makers create efficient dairy supply chains in practice to reach to this ultimate goal.

Matching supply and demand is one of the main challenges of food supply chains, as is the case for others, and demand uncertainty is one of the prominent factors that complicates the production and distribution planning process [4]. Notice that demand is an inherently uncertain parameter, and the dependence of demand on changing consumer preferences increases the associated uncertainty in dairy products. Namely, the presence of various types of products in a competitive market may lure even a loyal customer into changing her/his choice and buying a different product. Furthermore,

some external factors, such as cultural traditions and diseases, may cause demand uncertainty (e.g., [5, 6]). When the actual demand does not match the planned amount, there are two possible negative consequences for the producer: i) customer dissatisfaction or loss, or ii) extra disposal costs and waste of valuable (environmental) resources due to the perishability of products, respectively. Notice that the second consequence is overproduction and keeping extra inventory to compensate for the risk of losing customers' demand. Therefore, considering the demand uncertainty in dairy distribution networks is crucial to provide resilient solutions against the challenges mentioned above.

Perishable products exist not only in food but also in other supply chain networks such as donation tailoring [7, 8], high-tech electronics [9], and drugs [10]. However, dairy products distinguish from the other ones in three ways: i) dairy products are relatively more perishable than similar products because milk is an appropriate medium for creating microorganisms that cause early spoilage of the product and may result in deadly diseases (e.g., *Brucella*); ii) the quality of a dairy product may be affected at each phase of the supply chain, including production, warehousing, and distribution; and iii) a particular dairy product often needs to be processed and kept at different temperature levels at each supply chain stage. For example, white cheese should be stored and transported between 4–5C°, while the temperature processed milk may be produced at ambient temperature, and there is no need to decrease the temperature and expose extra cost to the network. These limits force producers to produce, warehouse, transport some products separately in different periods or facilities because of varying temperature requirements. To overcome the perishability-related challenges in a dairy supply chain, producers aim to reduce the time between production and consumption as much as possible. Additionally, dairy companies invest in temperature tracking technologies and operational planning at each phase of the supply chain. However, traditional supply chain models in the operations research

field lag behind to provide efficient dairy distribution management because the associated models often do not take perishability into account and ignore the temperature tracking of products at each phase of the supply chain, as non-perishable goods do not necessitate quality and temperature tracking-related operations.

In this study, we propose a *dairy production and distribution planning problem* (PDP) that includes temperature tracking of multi-dairy products, temperature sequence-dependent setup cost of operating the facilities, limited inventory capacities in production and warehousing, short lead times on retailer deliveries, and uncertain demand. We propose a mathematical optimization framework for the addressed problem to minimize total production, distribution, and disposal costs, yielding environmentally friendly solutions for the dairy industry. Accordingly, it is critical to determine production sequences that minimize setup costs and obtain efficient production and transportation plans that meet lead times while minimizing the total cost. To address the demand uncertainty, we adopt *robust and t optimization* (RO) paradigm and formulate *robust* PDP (RPDP) that does not need any distribution information. Instead, we assume that the demand values reside in intervals with known lower and upper bounds defined by the decision-maker based on the historical data of demand confronted by retailers. These intervals are called box uncertainty sets, and they may be easily generated from the historical data of demand even when the associated data is scarce.

Given the above-mentioned in providing of the RO model RPDP, the model may result in overproduction and extra inventory when the demand is realized less than the worst-case value, leading to unnecessary holding and disposal costs. To avoid the associated over-conservatism, we apply *adjustable robust optimization* (ARO) to RPDP to get more flexible solutions that can be adjusted according to the revealed portion of data at different points of the planning horizon; this problem is referred to

as *adjustable* RPDP (ARPD). ARPD provides the flexibility of adjusting production and distribution decisions from warehouses to retailers according to the realized portion of the demand in a multi-period planning horizon, and the producer is allowed to use the information of realized demand in the previous periods and decide on production, warehousing, and backorder amount accordingly for the upcoming periods. On the negative side, the ARO reformulation may yield alternative optimal solutions that yield significantly different *objective function value* (OFV) performances for non-worst-case data (e.g., the nominal) even though the alternative solutions yield the same worst-case OFV performance. To tackle the related Pareto efficiency issue, we propose a re-optimization approach that maintains optimality for the worst-case realization while selecting the best ARO solution for the nominal case. Moreover, we propose a dedicated Monte Carlo simulation approach to compare robust, adjustable robust, and Pareto adjustable robust reformulations on equal ground. We conduct the proposed Monte Carlo experiment to a realistic data set inspired by a real-life dairy industry company; the numerical results show that (Pareto) adjustable robust approach improves the OFV performance of the static robust model by 7-11% on average and obtains solutions that are feasible and adjustable for different realizations of the uncertainty parameter while the nominal solution cannot satisfy as high as 17% of the demand when demand realizes different than the expected.

The remainder of the paper is organized as follows. Section 2.2 presents a brief literature review. Section 2.3 gives the problem definition and the proposed deterministic and (adjustable) robust mathematical optimization models. Section 2.4 provides the re-optimization approach for Pareto efficiency, the developed heuristic to solve the problem, and the Monte Carlo simulation procedure. The numerical experiments and the case study are provided in Section 2.5. Section 2.6 gives the managerial insights of the proposed approach and the numerical results. Finally, Section 2.7 presents conclusions and future research directions.

2.2 Literature review

In this section, we first provide a general overview of the studies that propose quantitative models for food logistics network management. Then, among those studies, special attention is given to the ones that employ robust approaches for food logistics management to reveal the contribution of our research to the related literature.

To have a general overview of food logistics problems, we have first searched for the *articles* indexed by Web of Science Core Collection database with the keyword, “*supply chain*” in the “*topic*” field. This search has been narrowed down by using keywords “*network*”, “*food*”, “*model*”, and “*perishability*”, respectively. The references of the obtained studies are searched to identify further relevant studies as well. Lastly, the obtained studies on the topic have been reviewed in detail. Table 1 presents the summary of the reviewed studies in terms of several dimensions: (i) number of periods, (ii) sequence dependency in setup cost, (iii) uncertainty, (iv) temperature tracking, (v) model employed, (vi) uncertainty handling, and (vi) the case study addressed. Our scope here is on the main features of the addressed food production planning and distribution problem. An interested reader who would like to have more information on other food logistics problems is referred to the reviews conducted in [11, 12, 13, 14, 15].

Our review results show that studies on food logistics network management often consider problems with planning horizon involving multiple periods; whereas sequence-dependent setup cost, demand uncertainty and temperature tracking are relatively less tackled dimensions. Mixed-integer programming is the prominent modeling approach used, while formulating the food logistics problems confronted in several sectors. The current study contributes to the literature by simultaneously considering the aforementioned features, i.e., multi-period, sequence-dependent setup cost, demand uncertainty and temperature tracking (see Table 1). In the remainder, we will only focus on food logistics problems under uncertainty and elaborate on

Table 1: Summary of studies on food logistics network management that respect product perishability

#	Studies	M*	SD	Uncertainty	T	Model	Uncertainty handling	Case study
1	[16]	✓				MILP		Hypothetical
2	[17]	✓			✓	MILP		Fruit & vegetable chain
3	[18]	✓	✓		✓	MONLP		Hypothetical
4	[19]		✓		✓	MILP		Hypothetical
5	[9]	✓		D, PC		MIP	RO	Hypothetical
6	[20]	✓	✓			MILP		Dairy industry
7	[21]	✓	✓			MILP		Dairy industry
8	[22]	✓				MILP		Catering chain
9	[23]	✓		S,D		MIP	FLPA	Dairy industry
10	[24]	✓				Analytical		Agri-food chain
11	[25]	✓				MOLP		Hypothetical
12	[26]	✓		SC		MILP	FP	Seafood chain
13	[27]	✓				MILP		Hypothetical
14	[28]	✓				MOLP		Meat chain
15	[29]	✓		D, LT, PC, S		MIP	SP	Hypothetical
16	[30]					MOLP		Fruit & vegetable chain
17	[31]	✓		D		MIP	RO	Dairy industry
18	[32]	✓				MOLP		Fruit & vegetable chain
19	[33]				✓	MILP		Flower supply chain
20	[34]					MILP		Hypothetical
21	[35]	✓		D	✓	Analytical		Supermarket chain
22	[36]					MCDM		Meat chain
23	[37]					MOLP		Hypothetical
24	[38]	✓				Analytical		Hypothetical
25	[39]			D		MILP	SP	Catering chain
26	[40]					MILP		Fruit & vegetable chain
27	[41]	✓				MILP		Fruit & vegetable chain
28	[10]	✓		D, SC, FC		MIP	RO	Hypothetical
29	[42]	✓				MILP		Agri-food chain
30	[43]			R		MIP	RO	Dairy industry
31	[44]	✓		TT, FT		MILP	RO	Hypothetical
32	[45]					MOLP		Fruit & vegetable chain
33	[46]	✓		D, RR, QR		MILP	RO	Dairy industry
34	[47]	✓		LFT		MIP	CCSP	Dairy industry
35	[48]	✓		D, SC, FC		MIP	CCSP	Dairy industry
36	[49]	✓		D, SC, FC		MILP	RO	Dairy industry
37	[50]	✓		SC		MILP	RO	Dairy industry
38	[51]	✓		D, PC		MILP	RO	Flavour industry
This study		✓	✓	D	✓	MILP	(A)RO	Dairy industry

M: Multi-period, SD: Sequence-dependent cost, T: Temperature tracking

* D: Demand, SC: Several costs, LT: Lead time, PC: Purchasing cost, S: Supply, WC: Facility capacity R: Return

PC: process time, RR: Rate of return QR: Quality of return LFT: Life time TT: Travel time FT: Freshness-life

** (A)RO: (Adjustable) Robust Optimization, CCSP: Chance-constrained stochastic programming, FP: Fuzzy programming

similarities and differences of our work from the associated studies in handling the uncertainty.

In [52], a comprehensive survey is done on supply chain network design to review potential uncertainties along with the approaches to deal with them. The authors' review reveals that demand is one of the main uncertain parameters in supply chain problems, and RO has been proposed as a useful method to deal with uncertainty. In line with this finding, it has been observed that the RO is the most used approach to handle uncertainties in food logistics network problems as well (also see Table 1). Another finding of [52] is that few papers address real-life problems, and it would be valuable to do investigations based on real-life case studies. In that respect, our study contributes to the relevant field as we propose an RO model for a real-life production and distribution problem with demand uncertainty confronted in the dairy industry. Similar to our research, several other studies [9, 10, 31, 46, 44, 43] develop RO models as well to handle uncertainties in food logistics network problems. In the following, we provide an overview of those studies that propose robust food logistics models; subsequently, the distinctive features of the proposed method will be explained.

In [9], a multi-product closed-loop supply chain problem is addressed with demand and purchasing cost uncertainty. The uncertain parameters reside in a known interval, and RO is employed to handle the uncertainty. They show how the proposed model is compatible with the decision-making context of high-tech electronic and food industries, and the model can handle the uncertainty without using extensive historical data. Numerical results indicate that applying RO yields a solution that outperforms the deterministic one with respect to feasibility. In [31], authors propose an RO model for a location and allocation problem with demand uncertainty. The authors solve the location problem using a commercial solver and adopt a metaheuristic for the allocation problem. In [10], a supply chain network problem

is addressed to minimize cost and greenhouse gas emissions. RO is employed to handle uncertainties in demand, operational cost, transportation cost, and distribution center capacity. Another work is [43], where a bi-objective optimization model is proposed for product portfolio problems in the dairy industry with product return uncertainty, and RO is employed to deal with the associated uncertainty. The first objective function minimizes the investment risk, and the second one maximizes the total return of the product portfolio. In [44], an MILP model is proposed for the routing problem with cross-docking centers. The model has two objective functions, i.e., minimizing the penalty cost of earliness and tardiness, inventory holding and transportation costs, and maximizing the total freshness of the products. The associated bi-objective model is formulated as a single objective via L_1 metric method. The authors have considered the travel time and freshness-life of products as uncertain parameters by implementing RO. The numerical results show that compared to the product perishability, the effect of travel time uncertainty is more significant in OFV. In [48], a multi-objective RO is formulated for a dairy company under uncertainties observed in demand, facility capacities, and supply chain main costs. The authors handle the perishability of products by considering the FIFO policy for managing the inventory. The model leads a decision-maker to determine the optimal flows in the network under the associated uncertainties. RO is implemented in [46], to handle demand, return rate, and the quality of returned products uncertainties in a supply chain problem that aims to minimize total cost and environmental pollution. The authors provide heuristics to solve the problem and examine the performances of the proposed approaches by instances of a real-life case study in a dairy company. Apart from the reviewed RO attempts that respect food product perishability, some other studies employ RO for the food industry but do not consider product perishability. RO is presented in [53] for a grain supply chain considering crop yields as the uncertain parameter, [54] develops a robust model for an agri-food supply chain by taking

demand and supply uncertainty into account, and [55] proposes a multi-objective mathematical optimization model for the wheat supply chain considering supply and demand uncertainties.

Lastly, a few studies develop ARO models for practical optimization problems, e.g., project scheduling [56], flood protection [57], and supply chain management [58, 59], while there is no implementation to the dairy industry in particular. As pointed out in the recent ARO survey [60], practical applications of ARO in the industry are lagging behind.

Based on the literature review, the *contributions* of this paper to the related literature can be summarized as the following.

1. To the best of our knowledge, this is the first attempt that combines the critical components of dairy production and distribution management, such as perishability, temperature tracking, sequence-dependent setup cost, and demand uncertainty in a single model. The proposed ARO model does not need any probability distribution information on demand and gives flexibility to the decision-maker to adjust production and product distribution decisions according to the realized demand values over time in a multi-period planning horizon in a realistic setting.
2. The Pareto efficiency issue in ARO is only investigated in a theoretical setting for a class of optimization problems in [61] and [62] or for toy-sized instances in integer programming in [63]. While only one study considers a Pareto efficiency in static RO for a real-life machine scheduling problem [64], no study proposes a Pareto efficient ARO approach for a realistically-sized mixed-integer production planning problem. This is one of the first attempts that considers Pareto efficiency in ARO for a realistically-sized MILP.

3. Lastly, we propose a dedicated Monte Carlo simulation approach for the proposed mathematical optimization models and solution methods, which is extensively discussed in the numerical experiments. The simulation experiment results of the case study show that the proposed ARO methodology may reduce the overall production and distribution cost (on average) between 7–11% compared with that of the static RO approach.

2.3 Robust dairy production and distribution planning problem

In this section, we first give the problem definition and propose the deterministic formulation of PDP in Section 2.3.1. The robust reformulation of PDP (RPDP) is presented in Section 2.3.2. Finally, Section 2.3.3 provides a less conservative adjustable robust reformulation of the PDP (ARPDP).

2.3.1 Problem definition and deterministic model

Given a dairy production and distribution network with sets of production lines ($\mathcal{P}$), warehouses ($\mathcal{W}$), and retailers ($\mathcal{R}$), PDP aims to find the optimal production ($\mathbf{Y}$), inventory ($\mathbf{I}$), backorder ($\mathbf{B}$), and distribution ($\mathbf{X}$) amounts of dairy products at each period $t \in \mathcal{T}$ of a multi-period planning horizon to meet the retailers' demand. Furthermore, the model decides on optimal temperature level of warehouses ($\mathbf{Z}$) at each period. Each dairy product $j \in \mathcal{J}$ is manufactured in a production line, and either is stored in a warehouse or is directly shipped to a retailer. The retailers are assumed to have a limited dairy storage capacity. The related inventory has a certain capacity determined according to the demand in a predetermined time window to avoid product perishability. Production lines and warehouses are located in the same facility. Figure 1 demonstrates possible product flows in the network.

In the related network, the temperature tracking is required to avoid quality degradation of products during the warehousing phase. Proper temperature levels for each

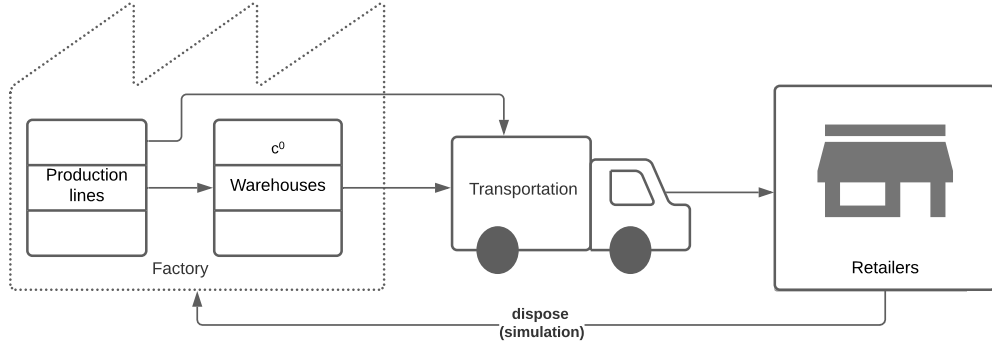


Figure 1: The flow chart of the addressed network

product are predefined. Accordingly, different types of products can be stored together in the same warehouse as long as they have the same temperature requirement. Some products are flexible and can be stored at different temperature levels, such as UHT milk can be in the temperature range $4-20^{\circ}\text{C}$, while others are stored at a unique temperature level, e.g., cheese can only be processed at 4°C . A warehouse can operate at a single temperature level at each period, the associated temperature can be changed by bearing a sequence-dependent setup cost. Therefore, one needs to optimize the temperature selection decisions to minimize operational and set-up related costs while sustaining a fresh product flow. The warehouses located in the factory have limited capacities and distributing a product from the factory to a retailer $r \in \mathcal{R}$ takes ℓ_r period. Backordering is allowed and results in an additional cost for the producer. However, there is a time limitation for backordering and the later deliveries than the demand period plus the lead time (ℓ_r) are not allowed. Lastly, retailers' demand are assumed to be known in advance but this assumption will be relaxed in the upcoming subsections. All vehicles used to transport products are the same and must have refrigeration units installed to maintain the necessary temperature. For the transportation of dairy products, the temperature should be kept at $4-5^{\circ}\text{C}$. This ensures that the products remain fresh and in good condition during transportation. The temperature in all the vehicles is kept at the same level.

The nomenclature and the deterministic mathematical optimization model are given below.

Table 2: Nomenclature for PDP

Sets	
$\mathcal{W}$	set of warehouses ($w = 1, \dots, W$)
$\mathcal{R}$	set of retailers ($r = 1, \dots, R$)
$\mathcal{J}$	set of product types ($j = 1, \dots, J$)
$\mathcal{T}$	set of periods ($t = 1, \dots, T$) where $\mathcal{T}_a \equiv \mathcal{T} - \{1, \dots, a\}$
$\mathcal{K}$	set of temperatures ($k = 1, \dots, K$)
$\mathcal{K}_1^j$	set of temperatures that product j can be stored $\mathcal{K}_1^j \subseteq \mathcal{K}$
Parameters	
$Q_{jt}^{(p)}$	capacity of producing product j in period t
$Q_{wt}^{(w)}$	capacity of warehouse w in period t
$\bar{D}_j$	total number of product j that can be stored in warehouse
d_{jrt}	demand of retailer r of product j in period t
$\sigma_{kk'}$	setup cost of a warehouse for changing temperature level from k to k'
$C_{jt}^{(p)}$	production cost of product j in period t
$C_k^{(w)}$	warehouse opening cost at temperature k
$C_{jk}^{(h)}$	holding cost of product j at temperature level k
$C_{jrt}^{(b)}$	backorder cost of product j in period t for retailer r
C_{jr}	unit distribution cost of product j to retailer r
ℓ_r	transportation lead time from the factory to retailer r
I_{jkw0}	initial inventory level of product j in warehouse w at temperature level k
Decision variables	
$I_{jkw t}$	inventory level of product j in warehouse w at temperature level k in period t
Y_{jt}	amount of product j manufactured in period t
X_{jrt}	amount of product j distributed to retailer r in period t
B_{jrt}	amount of backorder for product j in retailer r in period t
$Z_{kw t}$	1, if warehouse w is active at temperature level k in period t; 0, o.w

[PDP]:

$$\begin{aligned}
\min_{\mathbf{B}, \mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}} \quad & \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} Y_{jt} + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkw t} + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} X_{jrt} + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt}
\end{aligned}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} I_{jkw t} \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (1)$$

$$Y_{jt} \leq Q_{jt}^{(p)} \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (2)$$

$$Y_{jt} = \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw t} - I_{jkw(t-1)} \right) + \sum_{r \in \mathcal{R}} X_{jrt} \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (3)$$

$$X_{jr(t-\ell_r)} \geq d_{jrt} - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \quad (4)$$

$$B_{jrt} = \sum_{t'=1}^t d_{jrt'} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \quad (5)$$

$$\sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} B_{jrt} = 0 \quad (6)$$

$$\sum_{t'=1}^t X_{jrt'} \geq \sum_{t'=1}^t d_{jrt'} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (7)$$

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} I_{jkw t} \leq \bar{D}_j \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (8)$$

$$\sum_{j \in \mathcal{J}} I_{jkw t} \leq M Z_{kw t} \quad \forall k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T} \quad (9)$$

$$\sum_{k \in \mathcal{K}} Z_{kw t} = 1 \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (10)$$

$$I_{jkw T} = I_{jkw 0} \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W} \quad (11)$$

$$\mathbf{Z} \in \{0, 1\}. \quad (12)$$

$$\mathbf{B}, \mathbf{I}, \mathbf{X}, \mathbf{Y} \geq 0 \quad (13)$$

The objective function comprises seven components: (i) the production cost, (ii) the setup cost of warehouses in the first period, (iii) the setup cost of the warehouses in

each period, (iv) the warehouse opening and (v) operating costs, (vi) the distribution cost, and (vii) the backorder cost. The third cost component contains products of binary variables, which can be linearized by using standard linearization techniques. Constraints (1) and (2) ensure that inventory and production amounts in each period do not exceed the capacities. Constraints (3) and (4) balance the inventory and backorders. Constraints (5) guarantee that products that are ordered in the first ℓ_r periods are backordered due to the lead time restriction; constraints (6) ensure that the planning horizon finishes without backordering. Constraints (7) ensure that backorders cannot be delayed more than ℓ_r periods. Constraints (8) ensure that the inventory level of product j cannot exceed a certain amount since products are perishable. Constraints (9) ensure that a warehouse is either idle or active in one temperature level, where M is a large enough number. Constraints (10) ensure that a warehouse should be idle or active at a single temperature level. Constraints (11) ensure that the inventory level at the last period should be equal with the initial inventory, and finally, (12) and (13) are 0-1 and non-negativity constraints, respectively.

2.3.2 Robust reformulation

A production plan that is designed for the expected (or nominal) demand may result in significant drawbacks to the system when the demand realizes differently, namely, the associated plan will result either in overproduction or in lost sales. Hence, having a solution that is resilient against the demand uncertainty is essential in planning. In this setting, we do not have any distributional information about the uncertain dairy demand. Instead, we only know certain lower and upper bounds for the demand of each retailer during the planning horizon. We aim to propose a tractable approach that is capable to address the uncertain demand. RO is the state of the art approach that models/meets all of the aforementioned specifications/requirements. RO capitalizes on computational efficiency and tractability, and yields solutions that

are resilient against uncertainty. However, such benefits come with an additional cost as the approach yields the optimal solution for the worst-case realization of the uncertain data. For further details on RO, we refer the reader to [65, 66, 67].

The robust problem aims to find the optimal solution that is feasible for all realizations of the demands within the defined bounded set with known lower and upper bounds for each retailer. In [RPDP], the retailers' demand in each period for each product is reformulated as follows:

$$\mathcal{U} := \{\mathbf{d} \in \mathcal{R}_+ : d_{jrt} = (1 + \rho\zeta_{jrt})d_{jrt}^0, -1 \leq \zeta_{jrt} \leq 1 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}\}. \quad (14)$$

The set is symmetric around the nominal value of the demand, where $\mathbf{d}^0$ is the nominal demand value, $\rho\mathbf{d}^0$ denotes the half-length of the interval for demand, ρ is the uncertainty level, and $\boldsymbol{\zeta}$ denotes the primitive uncertainty vector residing in $[-1, 1]$. We reformulate the problem presuming that the worst-case of the uncertainty parameters are realized; the reformulation is called as the *robust counterpart* (RC). Since the uncertainty parameter takes value in an interval, the tractable robust reformulation of the problem as [RPDP] can be obtained by substituting the upper-bound of retailers' demand ($d_{jrt} = (1 + \rho)d_{jrt}^0$) in the deterministic problem data. In other words, to get the [RPDP], we replace constraints (4),(5), and (7) in [PDP] with the following constraints:

$$X_{jr(t-\ell_r)} \geq (1 + \rho)d_{jrt}^0 - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \quad (15)$$

$$B_{jrt} = \sum_{t'=1}^t (1 + \rho)d_{jrt'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \quad (16)$$

$$\sum_{t'=1}^t X_{jrt'} \geq \sum_{t'=1}^t (1 + \rho)d_{jrt'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (17)$$

Notice that [PDP] and [RPDP] have the same mathematical optimization complexity. The complete model of [RPDP] is given in Appendix A.1.

2.3.3 Adjustable robust reformulation

In [RPDP], we solve the addressed problem by considering retailers' worst-case demands, i.e., resulting in an over-conservative solution. This over-conservatism causes excessive production, inventory, and distribution costs for the system when the worst-case demand is not realized. To overcome the associated over-conservatism issue, in the remainder, we consider warehouse opening decisions as here-and-now that are decided at the beginning of the planning horizon; and production, inventory, backorder, and distribution decisions as wait-and-see decision variables that allow the decision-maker to wait until a part of the demand is realized over time and update decisions accordingly. E.g., if a retailers' demand is realized less than its worst-case, the producer has a road map to update related wait-and-see decisions and benefit from the realized information. The corresponding road map is referred to as the *decision rule* in the realm of ARO [68, 60].

The decision rule used for a wait-and-see (or adjustable) decision at time $t \in \mathcal{T}$ is a linear parametric function of demands realized until time t . The realized part of the uncertainty parameter is often referred to as the *information base*. The associated decision rules for inventory and distribution decisions are given below:

$$I_{jkw t} = \bar{I}_{jkw t} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkw t} (1 + \rho \zeta_{j'rt'}) d_{j'rt'}^0 \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T} \quad (18)$$

$$X_{jrt} = \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} (1 + \rho \zeta_{j'r't'}) d_{j'r't'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}, \quad (19)$$

where $\bar{\mathbf{I}}$, $\bar{\mathbf{X}}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ denote the coefficients that will be optimized in the decision rules. The third summations (over time period) in (18)–(19) determine the information base (from period 1 to t) used in the adjustable inventory and distribution decisions. Notice that the decision-maker will only know the exact transportation and inventory amounts when the uncertain parameter is realized and substituted in

the decision rule. Unlike distribution and inventory decisions, we use the full information base ($t' \in \mathcal{T}$) for backorder variables ($\mathbf{B}$) since they are auxiliary. The related decision rule for $\mathbf{B}$ is given as the following:

$$B_{jrt} = \bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} (1 + \rho \zeta_{j'r't'}) d_{j'r't'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}, \quad (20)$$

where $\bar{\mathbf{B}}$ and γ are the coefficients that will be optimized in the associated decision rule. Notice that for the simplicity of notation, the decision rules of variables are shortly referred to as $\mathbf{I}(\zeta)$, $\mathbf{B}(\zeta)$ and $\mathbf{X}(\zeta)$ in the mathematical optimization models. Lastly, the decision rule for the production ($\mathbf{Y}$) may be obtained from the inventory balance constraints (3), i.e., one may substitute the following expression:

$$Y_{jt}(\zeta) = \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw t}(\zeta) - I_{jkw(t-1)}(\zeta) \right) + \sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (\text{re-3})$$

as the decision rule of $\mathbf{Y}$ after substituting the decision rules $\mathbf{I}(\zeta)$ and $\mathbf{X}(\zeta)$ in the inventory balance constraint (3). Next, one may remove constraints (3) from the model after substituting the decision rule $\mathbf{Y}(\zeta)$ for each $\mathbf{Y}$ in [PDP]. The related reformulation technique is also suggested in [66, Section 7]. Note that binary decision variables ($\mathbf{Z}$) in our problem are considered as here-and-now, and they are decided at the beginning of the planning horizon. The resulting semi-infinite adjustable robust reformulation of [PDP] ([ARPD]) after the substitutions mentioned above are made given as the following:

[ARPD]:

$$\begin{aligned} & \min_{\substack{\bar{\mathbf{B}}, \bar{\mathbf{I}}, \bar{\mathbf{X}}, \mathbf{Z} \\ \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}}} \max_{\zeta \in \mathcal{U}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw t}(\zeta) - I_{jkw(t-1)}(\zeta) \right) + \sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \right) + \\ & \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \\ & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkw t}(\zeta) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} X_{jrt}(\zeta) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt}(\zeta) \end{aligned}$$

s.t. (9), (12)

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}} I_{jkw t}(\zeta) \leq Q_{wt}^{(w)} \quad \forall t \in \mathcal{T}, w \in \mathcal{W}, \zeta \in \mathcal{U} \quad (21)$$

$$\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw t}(\zeta) - I_{jkw(t-1)}(\zeta) \right) +$$

$$\sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \leq Q_{jt}^{(p)} \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, \zeta \in \mathcal{U} \quad (22)$$

$$X_{jr(t-\ell_r)}(\zeta) \geq d_{jrt}(\zeta) - B_{jrt}(\zeta) + B_{jr(t-1)}(\zeta) \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r}, \zeta \in \mathcal{U} \quad (23)$$

$$B_{jrt}(\zeta) \geq \sum_{t'=1}^t d_{jrt'}(\zeta) \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\}, \zeta \in \mathcal{U} \quad (24)$$

$$\sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} B_{jrt}(\zeta) = 0 \quad \forall \zeta \in \mathcal{U} \quad (25)$$

$$\sum_{t'=1}^t X_{jrt'}(\zeta) \geq \sum_{t'=1}^t d_{jrt'}(\zeta) \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}, \zeta \in \mathcal{U} \quad (26)$$

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} I_{jkw t}(\zeta) \leq \bar{D}_j \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, \zeta \in \mathcal{U} \quad (27)$$

$$\sum_{j \in \mathcal{J}} I_{jkw t}(\zeta) \leq M Z_{kw t} \quad \forall k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T}, \zeta \in \mathcal{U} \quad (28)$$

$$I_{jkw T}(\zeta) = I_{jkw 0} \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W}, \zeta \in \mathcal{U} \quad (29)$$

$$\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw t}(\zeta) - I_{jkw(t-1)}(\zeta) \right) + \sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \geq 0 \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, \zeta \in \mathcal{U} \quad (30)$$

$$\mathbf{B}(\zeta), \mathbf{I}(\zeta), \mathbf{X}(\zeta) \geq \mathbf{0} \quad \forall \zeta \in \mathcal{U} \quad (31)$$

In the adjustable reformulation [ARPD], we minimize OFV over decision rule coefficients $(\bar{\mathbf{B}}, \bar{\mathbf{I}}, \bar{\mathbf{X}}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ and facility opening decision $(\mathbf{Z})$ with respect to the worst-case realization of the (primitive) uncertainty parameter ζ that resides in the uncertainty set $\mathcal{U}$ where $d_{jrt}(\zeta)$ denotes the uncertain demand. To this end, we maximize OFV over the uncertainty parameter to obtain the worst-case OFV for the given values of the decision rule coefficients. Notice that when we fix the decision rule coefficients, the objective function is linear over ζ , i.e., the minimax reformulation may be

translated to a minimization problem when we take dual of the inner maximization problem over ζ . Similar to the objective function, we substitute the decision rules of the adjustable variables to time constraints as the following. Where we aim to satisfy the constraints for all realizations of the uncertainty parameter $\zeta \in \mathcal{U}$. The above given robust reformulation is a semi-infinite optimization problem due to the universal quantifier ($\forall \zeta \in \mathcal{U}$) in the constraints, i.e., intractable in its current form. To derive the tractable RC, we adopt the three step procedure of the RO paradigm. For the sake of brevity, we demonstrate deriving the tractable robust counterpart over (21).

First, we derive an equivalent reformulation of (21) by maximizing the left-hand side (LHS):

$$\max_{\zeta \in \mathcal{U}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkw} d_{j'rt'}^0 \left(1 + \rho \zeta_{j'rt'} \right) \right) \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T}. \quad (32)$$

Notice that (32) implies (21) because a solution that satisfies (21) for all realizations of the uncertainty parameter ζ is also feasible for the value of ζ that maximizes LHS of (32). It can be observed that if the optimal decision rule coefficient $\alpha_{j'rt'}^{jkw}$ is non-negative, then the corresponding value of the uncertainty parameter $\zeta_{j'rt'}$ that maximizes the LHS is +1; and $\zeta_{j'rt'}$ takes -1 , when the coefficient is negative. Namely, (32) may be equivalently reformulated as

$$\begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw} \right) + \\ & \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\left| \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw} \right| \rho d_{j'rt'}^0 \right) \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T}, \end{aligned} \quad (33)$$

which can be linearized by introducing an auxiliary decision variable. Notice that another way to obtain the tractable RC of (32) is by taking the dual of the maximization in (32) and then deleting the minimization from the resulting dual objective since the

dual problem yields an upper-bound for the maximization problem by the weak duality theorem; both (33) and the duality based reformulation technique will yield the same RC. Deriving the RC of (21) by duality is provided in detail in Appendix A.2, and the complete tractable reformulation of [ARPDP] is given in Appendix A.3.

2.4 *Solution methods*

This section presents methods to solve the robust reformulations [RPDP] and [ARPDP]. In Section 2.4.1, we discuss the re-optimization procedure of [ARPDP] to yield Pareto efficient solutions. In Section 2.4.2, we develop a heuristic to solve proposed models. Finally, in Section 2.4.3, we propose a Monte Carlo simulation approach to compare OFV performances of the proposed models.

2.4.1 Re-optimization approach

[ARPDP] often has alternative optimal solutions that may yield significantly different OFVs when non-worst-case (or nominal) data is realized. Notice that the commercial solvers are indifferent in selecting one of the alternative optimal decision rules with respect to the worst-case data. We aim to find the *Pareto efficient* decision rule coefficients that outperform the OFV performances of alternative solutions when the nominal data is realized. To this end, we initially solve [ARPDP] and record its optimal OFV as $(\text{obj}^*[\text{ARPDP}])$; next, we re-optimize the coefficients of the decision rules for the nominal data, ensuring that the selected coefficients are still optimal for the worst-case realization of the uncertain parameter. The related re-optimization algorithm is presented as follows.

The re-optimization model is presented by $[\text{PR}](\text{ARPDP})$, where (ARPDP) denotes the input OFV that is used in the first constraint. We present the semi-infinite reformulation of $[\text{PR}](\text{ARPDP})$ above, and the explicit tractable reformulation of $[\text{PR}](\text{ARPDP})$ is given in Appendix A.4. An illustrative example on Pareto efficiency is given in the following.

Table 3: Re-optimization algorithm

Step 1 Solve [ARPDP] and **record** the optimal OFV ($\text{obj}^*[\text{ARPDP}]$)

Step 2 Solve the following optimization problem:

[PR](ARPDP):

$$\begin{aligned} & \min_{\substack{\overline{B}, \overline{I}, \overline{X}, \mathbf{Z} \\ \alpha, \beta, \gamma}} \text{obj}[\text{ARPDP}] \left(\mathbf{B}(\zeta^0), \mathbf{I}(\zeta^0), \mathbf{X}(\zeta^0), \mathbf{Z} \right) \\ & \text{s.t. } \text{obj}[\text{ARPDP}] \left(\mathbf{B}(\zeta), \mathbf{I}(\zeta), \mathbf{X}(\zeta), \mathbf{Z} \right) \leq \text{obj}^*[\text{ARPDP}] \quad \forall \zeta \in \mathcal{U} \\ & \quad (9), (12), (21) - (31) \end{aligned}$$

where ζ denotes the uncertain parameter, ζ^0 the nominal data, and $\mathcal{U}$ is the uncertainty set (see equation(14))

Record the optimal solution $(\overline{B}^*, \overline{I}^*, \overline{X}^*, \mathbf{Z}^*, \alpha^*, \beta^*, \gamma^*)$ of [PR] as the optimal solution.

An illustrative example. Suppose a network with a single retailer that demands a product in two periods; demands are uncertain and reside in the interval uncertainty set $D \in [12, 18]$; and the lead time is 1. Next, suppose we adopt the following linear decision rules for the variables in the first period: $Y_1 = \overline{Y}_1 + \alpha D_1$, $X_1 = \overline{X}_1 + \beta D_1$ and $B_1 = \overline{B}_1 + \gamma D_1$ where $(\alpha, \beta, \gamma, \overline{X}_1, \overline{Y}_1, \overline{B}_1)$ denote the intercepts and coefficients of the decision rules. Notice that demand information for the second period is not included in the information base of production and distribution decisions of the first period because the associated demand value is not realized at that time. Moreover, the production and distribution decisions in the second period are equivalent to zero at optimality since backordering is not allowed in the last period and the lead time is 1 (i.e., $X_2^* = Y_2^* = B_2^* = 0$). When the decision rules of the first period are optimized by [PR](ARPDP), the resulting optimal coefficients are $\alpha^* = \beta^* = \gamma^* = 1$ and intercepts are $\overline{Y}_1^* = \overline{X}_1^* = 18$, and $\overline{B}_1^* = 0$.

Now, suppose that the nominal demand is realized, i.e., $D_1 = D_2 = 15$, then the

resulting production, distribution and backorder decisions for the first period are:

$$Y_1 = \bar{Y}_1^* + \alpha^* D_1 = 18 + 15 = 33$$

$$X_1 = \bar{X}_1^* + \beta^* D_1 = 18 + 15 = 33$$

$$B_1 = \bar{B}_1^* + \gamma^* D_1 = 0 + 15 = 15$$

It is important to point out that the improved nominal performance of [ARPDP] is not guaranteed when the re-optimization algorithm is not used. For example, $\alpha^* = \beta^* = \gamma^* = 0$, $\bar{Y}_1^* = \bar{X}_1^* = 36$, $\bar{B}_1^* = 18$, $X_2^* = Y_2^* = B_2^* = 0$ is an alternative adjustable robust optimal solution for [ARPDP]. Nevertheless, when the nominal data is realized, the former solution yields a better OFV since it produces, transports and backorders less than the latter solution. As mentioned before, the commercial solver is indifferent in selecting one of the solutions since both yields the same OFV in the worst-case, even though they yield different performances for the nominal demand.

2.4.2 Heuristic approaches

[ARPDP] is a computationally expensive MILP, and a standard personal computer usually cannot solve realistically sized instances to optimality due to huge memory requirement; [RPDP], on the other hand, is a relatively more tractable mathematical optimization model than [ARPDP], and it can be solved for practically sized instances. Table 4 compares the number of constraints and decision variables of both [RPDP] and [ARPDP] for different sized instances.

Table 4: Number constraints and decision variables for different-sized problems

instance size $J \times K \times R \times T \times W$	# constraints		# decision variables			
	[RPDP]	[ARPDP]	continuous		binary	
			[RPDP]	[ARPDP]	[RPDP]	[ARPDP]
$5 \times 3 \times 5 \times 5 \times 2$	1.2×10^3	3.1×10^5	5.4×10^2	2.2×10^5	30	30
$6 \times 3 \times 8 \times 8 \times 2$	2.8×10^3	3.6×10^6	1.2×10^3	2.3×10^6	48	48
$8 \times 4 \times 8 \times 10 \times 3$	6.2×10^3	12×10^6	2.9×10^3	8.0×10^6	120	120

* #of product types (J), temperature levels (K), retailers (R), periods (T), warehouses (W)

When the binary warehouse opening decisions in [RPDP] and [ARPDP] are fixed, the associated mathematical optimization models become *linear optimization* (LO)

models that are more tractable than the original models. To this end, we combine the two complexity reductions that are mentioned above, i.e., [RPDP] being more tractable than [ARPDP], and [ARPDP] becomes LO problem when the binary decisions are fixed to yield high-quality solutions for [ARPDP] with improved CPU time performance. In the following, we present two heuristics that exploit the complexity reductions. As explained above, [RPDP] can be solved computationally more efficiently than [ARPDP], and both models are LO problems when binary decisions are fixed. To this end, we propose first solving [RPDP] and implementing the optimal binary decision variables of [RPDP] to [ARPDP], and solving the resulting LO problem to optimize [ARPDP] with improved CPU time performance. We conjecture that [RPDP] and [ARPDP] yield the same optimal OFV and the optimal binary decision variables even though their (average) performances may be significantly different. Therefore, the benchmark heuristic may not yield Pareto efficient solutions for [ARPDP]. The algorithm is given as follows.

Table 5: Benchmark heuristic

Step 1	Solve [RPDP], record optimal binary variables ($\mathbf{Z}^{\text{RPDP}}$)
Step 2	Solve the following optimization problem:
	[ARPDP](RPDP):
	$\min_{\substack{\mathbf{B}, \mathbf{I}, \mathbf{X} \\ \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}}} \max_{\boldsymbol{\zeta} \in \mathcal{U}} \text{obj}[\text{ARPDP}] \left(\mathbf{B}(\boldsymbol{\zeta}), \mathbf{I}(\boldsymbol{\zeta}), \mathbf{X}(\boldsymbol{\zeta}), \mathbf{Z}^{\text{RPDP}} \right)$
	$\text{s.t.} \quad \sum_{j \in \mathcal{J}} I_{jkw}(\boldsymbol{\zeta}) \leq M Z_{kw}^{\text{RPDP}} \quad \forall k \in K, w \in W, t \in T, \boldsymbol{\zeta} \in \mathcal{U}$
	(21)–(27), (29)–(31)

In the sequel, we propose a new heuristic that yields Pareto efficient adjustable robust solutions and combines the complexity reductions utilized in the benchmark algorithm with the conjecture on equivalence of $\text{obj}^*[\text{RPDP}]$ and $\text{obj}^*[\text{ARPDP}]$. More precisely, we first solve $[\text{RPPD}]$ and record the optimal binary decisions $\mathbf{Z}^{\text{RPDP}}$ and optimal OFV $\text{obj}^*[\text{RPDP}]$ as before. Then we adapt the re-optimization approach that is presented in Section 2.4.1 with two modifications. First, we substitute $\text{obj}^*[\text{RPDP}]$ for $\text{obj}^*[\text{ARPDP}]$; this modification improves the performance of the algorithm further since $[\text{RPDP}]$ is computationally more tractable than $[\text{ARPDP}]$. Second, we fix the binary decision variables to the robust optimal solution of $[\text{RPDP}]$. The tractable reformulation of the resulting model $[\text{PR}](\text{RPDP})$ used in the heuristic is given in Appendix A.5. The numerical results show that the heuristic tackles realistically sized instances that the regular re-optimization model $[\text{PR}](\text{ARPDP})$ cannot solve within the given time limit and yields optimal solutions for medium-sized instances; further details on optimality and CPU time performances of the proposed approach will be provided in Section 2.5.2. The algorithm is given as follows. In the remainder, we refer to the main algorithm with the model id $[\text{PR}](\text{RPDP})$ used in the heuristic.

Table 6: Pareto efficient heuristic for solving $[\text{ARPDP}]$

Step 1 **Solve** $[\text{RPDP}]$, **record** the optimal OFV ($\text{obj}^*[\text{RPDP}]$) and binary variables $\mathbf{Z}^{\text{RPDP}}$

Step 2 **Solve** the following optimization problem:

$[\text{PR}](\text{RPDP})$:

$$\begin{aligned}
& \min_{\substack{\mathbf{B}, \mathbf{I}, \mathbf{X}, \\ \boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\beta}}} \text{obj}[\text{ARPDP}] \left(\mathbf{B}(\boldsymbol{\zeta}^0), \mathbf{I}(\boldsymbol{\zeta}^0), \mathbf{X}(\boldsymbol{\zeta}^0), \mathbf{Z}^{\text{RPDP}} \right) \\
& \text{s.t.} \quad \text{obj}[\text{ARPDP}] \left(\mathbf{B}(\boldsymbol{\zeta}), \mathbf{I}(\boldsymbol{\zeta}), \mathbf{X}(\boldsymbol{\zeta}), \mathbf{Z}^{\text{RPDP}} \right) \leq \text{obj}^*[\text{RPDP}] \quad \forall \boldsymbol{\zeta} \in \mathcal{U} \\
& \quad \sum_{j \in \mathcal{J}} I_{jkw}(\boldsymbol{\zeta}) \leq M Z_{kw}^{\text{RPDP}} \quad \forall k \in K, w \in W, t \in T, \boldsymbol{\zeta} \in \mathcal{U} \\
& \quad (21)–(27), (29)–(31)
\end{aligned}$$

2.4.3 Monte Carlo simulation procedure

Comparing the optimal OFVs of the proposed models may lead to false conclusions because each model yields an optimal solution for a different data set, i.e., while [PDP] is solved for the nominal demand, [RPDP] and [ARPD] are solved for the worst-case. In this section, we propose a Monte Carlo simulation approach to analyze the OFV performances of [PDP], [RPDP] and [ARPD] on equal grounds. To this end, we uniformly sample the demand from the uncertainty set; next, if the solution at hand is infeasible for the sampled demand, we regain feasibility of the solution with redemptive decisions by incurring penalty costs. On the contrary, if the resulting solution is feasible, we re-optimize auxiliary decisions to improve the optimality performance of the solution at hand according to the sampled data. Ultimately, we calculate OFVs of feasible models (after the corrective decisions are taken) over sampled demands and report the average OFV performance of each model. In the remainder, we delve into details of the Monte Carlo simulation design for different models.

Monte Carlo simulation for deterministic model. We first solve [PDP] and record the optimal production decision ($\mathbf{Y}^D$). We assume that inventory, backorder, and distribution variables may be updated according to the realized demand given that production decision is fixed to $\mathbf{Y}^D$. Moreover, the surplus production is disposed of with extra cost and the disposed quantities are denoted by the decision variable $\mathbf{S}$. On the other hand, if the nominal production plan cannot satisfy the realized demand, we regain the feasibility with the newly introduced auxiliary variable $\mathbf{F}$ that denotes the amount of unmet demand. In the following, we propose the related optimization model ([S1]) where $\mathbf{P}^{(S)}$ is the unit disposal cost and $\mathbf{P}^{(F)}$ is the unit penalty cost of unmet demand.

[S1]:

$$\begin{aligned}
& \min_{\mathbf{B}, \mathbf{F}, \mathbf{I}, \mathbf{S}, \mathbf{X}, \mathbf{Z}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} P_j^{(F)} F_{jt} + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkwt} + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} \left(C_{jr} X_{jrt} + P_j^{(S)} S_{jrt} \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt} \\
\text{s.t. } & Y_{jt}^D + F_{jt} = \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} I_{jkwt} - I_{jkw(t-1)} + \sum_{r \in \mathcal{R}} \left(X_{jrt} + S_{jrt} \right) \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (34) \\
& X_{jr(t-\ell_r)} = d_{jrt}^s - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \quad (35) \\
& B_{jrt} = \sum_{t'=1}^t d_{jrt'}^s \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \quad (36) \\
& \sum_{t'=1}^t X_{jrt'} \geq \sum_{t'=1}^t d_{jrt'}^s \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (37) \\
& \mathbf{B}, \mathbf{F}, \mathbf{I}, \mathbf{S}, \mathbf{X} \geq 0 \quad (38) \\
& (1), (6), (8) - (12),
\end{aligned}$$

where $\mathbf{d}^s$ denotes the sampled demand. Constraints (34)–(36) ensure the inventory balance, and constraints (37) re-optimize the transportation decisions according to the sampled demand $\mathbf{d}^s$. The remaining constraints are the same with [PDP].

The corrective actions of robust ([RPDP]) and adjustable robust ([ARPDP]) models are taken similar to the procedure mentioned above. The related re-optimization models for [RPDP] and [ARPDP] are provided below.

Monte Carlo procedure for robust model. Unlike the nominal production plan, the robust plan that comes out of [RPDP] does not result in unmet demand since it is already resilient against the worst-case demand. In the Monte Carlo experiment, only backorder decisions are re-optimized according to the realized demand, and the surplus production of the robust plan is disposed of at the end of the planning horizon

with an additional cost. Lastly, the average OFV performance of the robust plan is recorded according to the sampled demand data. The resulting mathematical optimization model [S2] is given as follows:

[S2] :

$$\begin{aligned}
\min_{\mathbf{B}} \quad & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt} \\
\text{s.t.} \quad & X_{jrt(t-\ell_r)}^R \geq d_{jrt}^s - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \\
& B_{jrt} = \sum_{t'=1}^t d_{jrt'}^s \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} B_{jrT} = 0 \\
& \mathbf{B} \geq \mathbf{0},
\end{aligned}$$

where $\mathbf{d}^s$ denotes the sampled demand, $\mathbf{X}^R$ denote the optimal distribution value taken from [RPDP]. To obtain the right OFV of the problem for the sampled demand, we include the fixed cost terms to the optimal OFV of [S2] using the following expression:

[M1] :

$$\begin{aligned}
& \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} P_j^{(S)} \left(Y_{jt}^R - \sum_{r \in \mathcal{R}} d_{jrt}^s \right) + \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} Y_{jt}^R + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1}^R + \\
& \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt}^R Z_{k'w(t+1)}^R + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt}^R + \\
& \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkw}^R + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} X_{jrt}^R + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt}^{S2} \quad ,
\end{aligned}$$

where $\mathbf{I}^R$, $\mathbf{Y}^R$, and $\mathbf{Z}^R$ are the optimal values of inventory level, production, and opening decisions that are taken from [RPDP]; $\mathbf{B}^{S2}$ is the backorder that is re-optimized for the sampled demand. The first term of [M1] is the disposal cost; the second one is the production cost; from the third to the sixth term are warehouse opening and inventory holding costs; the seventh one is the distribution cost; and the

last one is the backorder cost.

Monte Carlo procedure for adjustable robust model. Similar approach, the back-order decisions are re-optimized for the adjustable robust model in the Monte Carlo experiment. OFV is calculated according to the sampled demand and the optimal decision rule coefficients are gathered from [ARPD]. Ultimately, the average OFV performance is recorded over the sampled demand using the mathematical optimization model [S3] that is given as follow:

[S3] :

$$\begin{aligned} \min_{\mathbf{B}} \quad & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt} \\ \text{s.t.} \quad & X_{jr(t-\ell_r)}^A(\zeta) \geq d_{jrt}^s - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, \mathcal{T}_{\ell_r}, \zeta \in \mathcal{U} \end{aligned} \quad (39)$$

$$B_{jrt} = \sum_{t'=1}^t d_{jrt'}^s \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \quad (40)$$

$$\sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} B_{jrT} = 0 \quad (41)$$

$$\mathbf{B} \geq \mathbf{0}, \quad (42)$$

where $\mathbf{d}^s$ denotes the sampled demand, $\mathbf{X}^A$ is the decision rule of the distribution taken from [ARPD]. To find the final OFV for the sampled demand, the use following expression:

[M2] :

$$\begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} P_j^{(S)} \left(\left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw}^A(\zeta) - I_{jkw(t-1)}^A(\zeta) \right) + \sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \right) - \sum_{r \in \mathcal{R}} d_{jrt}^s \right) + \\ & \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(I_{jkw}^A(\zeta) - I_{jkw(t-1)}^A(\zeta) \right) + \sum_{r \in \mathcal{R}} X_{jrt}(\zeta) \right) \right) + \\ & \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1}^A + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kw}^A + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kw}^A Z_{k'w(t+1)}^A + \\ & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkw}^A(\zeta) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} X_{jrt}^A(\zeta) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt}^{S3} \quad , \end{aligned}$$

where $\mathbf{I}^A$ is the decision rule for production and $\mathbf{Z}^A$ is the opening decisions that are taken from [ARPD $\bar{P}$]; $\mathbf{B}^{S3}$ is the backorder value is calculated by [S3] for sampled demand.

Monte Carlo procedure for re-optimization algorithm. Notice that the Monte Carlo simulation of the re-optimization algorithm is similar to the procedure of [ARPD $\bar{P}$] that is mentioned above, but we use the re-optimized coefficients that come out of Step 2 of the algorithm.

2.5 Numerical experiments

This section presents CPU time, OFV, and feasibility performances of the proposed solution methods over several numerical experiments. Section 2.5.1 explains the data structure used in the numerical experiments. We evaluate CPU time performances of the heuristic approaches in Section 2.5.2. The Monte Carlo simulation experiment that compares the average OFV performances of proposed methods is presented in Section 2.5.3. Finally, we present a case study in Section 2.5.4 and test the proposed methods with realistic data.

2.5.1 Data structure

We define three different problem types for the numerical experiments that use synthetic data (see Table 7 for dimensions). We randomly generate five data sets for each instance type (see Table 8 for the lower and upper bounds of the uniform distributions that are used in data generation). The intervals used in the data generation process are obtained by randomly perturbing the realistic data set used in the case study. All models and algorithms are coded in Python and solved by CPLEX 20.1 with Concert Technology using a 64-bit Windows computer equipped with a AMD 64-Core, 2.00 GHz Processor and 32GB of RAM.

Random sampling of demand. The uncertain demand that is residing in the uncertainty set $\mathcal{U}$ is given by $d^0(1 + \rho\zeta)$, and we sample the uncertain component of

Table 7: Model dimensions

instance type	J	K	R	T	W
1	5	3	5	5	2
2	6	3	8	8	2
3	8	4	8	10	3

Table 8: Sampling distributions of parameters

parameter	value	parameter	value
d_{jrt}^0	$U_d(5, 25)$	$Q_{wt}^{(w)}$	$U_c(1, 2) \times \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}} d_{jrt}^0$
$Q_{jt}^{(p)}$	$U_c(1, 2) \times \sum_{r \in \mathcal{R}} d_{jrt}^0$	P_j^s	$U_d(10, 15)$
$C_{jt}^{(p)}$	$U_d(20, 50)$	$C_{jk}^{(h)}$	$U_c(0.10, 0.20) \times C_{jt}^{(p)}$
$C_{jrt}^{(b)}$	$U_c(0.25, 0.35) \times C_{jt}^{(p)}$	$\bar{D}_j$	$2 \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} d_{jrt}^0 T^{-1}$
$C_k^{(w)}$	$U_d(800, 1000)$	C_{jr}	$C_{jt}^{(p)} \times \ell_r \times 0.1$
$\sigma_{kk'}$	$U_d(250, 500)$	I_{j0}	$\sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} d_{jrt}^0 T^{-1}$
ℓ_r	$U_d(1, 2)$		

$U_{\{c,d\}}(a, b)$: denotes a continuous "c" or discrete "d" uniform distribution between a and b

the demand ($\rho\zeta$) from three different distributions. More precisely, we use two uniform distributions $U(-\rho, +\rho)$ and $U(0, +\rho)$ where the terms in parenthesis denote the lower and upper bounds; and the triangular distribution $T(-\rho, +\rho, \frac{\rho}{2})$ where $\frac{\rho}{2}$ is the distribution mode, $-\rho$ and $+\rho$ are the distribution bounds.

2.5.2 Evaluating CPU time performances of proposed approaches

This subsection evaluates the CPU time performances of the proposed approaches for solving [ARPD]. Table 9 presents the required CPU times to get the final solution in seconds. The first column denotes the instance type, the other columns give the CPU times of the solution approaches; "*" is used when the solver cannot solve the instance within the given time limit.

Table 9: Average CPU times of mathematical optimization models (seconds)

instance type	[ARPD]		[PR](ARPD)		[ARPD](RPDP)	
1	496	+	520		23	28
2	81312	+	10712		747	797
3	*		*		5941	9418

* cannot be solved in 24 hours and no optimality gap is reported

+ to solve [PR](ARPD), first [ARPD] is solved; CPU times are reported individually

By implementing the heuristic (PR[RPDP]), we yield Pareto efficient solutions for the first instance within the 5% of CPU time required by the standard re-optimization

algorithm ([PR](ARPD)). For the second (larger-sized) test instance, the heuristic needs less than 1% of the time required for the re-optimization algorithm. Moreover, the standard re-optimization algorithm cannot yield a solution within 24 hours for any data set of the third instance type. We want to highlight that for [PR](RPDP) and [ARPD](RPDP), one should first solve [RPDP]; however, the CPU time for solving [RPDP] is ignored because it is less than five seconds for all instances. On the other hand, for solving [PR](ARPD), one should first solve [ARPD]; as a result, for this problem, we should consider the CPU time of [ARPD] as well. The numerical results show that adopting the standard re-optimization algorithm [PR](ARPD) to realistically sized instances is computationally expensive, while the heuristic [PR](RPDP) solves all instances in less than three hours. When we compare the CPU time performances of non-Pareto efficient approaches, namely, standard model [ARPD] and the benchmark heuristic [ARPD](RPDP) that uses optimal binary decisions of the static robust model as its input, the numerical results show that the benchmark heuristic significantly outperforms the CPU time performance of the standard model solved by CPLEX. It is worthwhile to mention that both approaches yield the same solutions for all the instances used in this experiment and this is why OFV comparison is not provided.

Ultimately, the numerical results show that the proposed heuristic approaches are computationally more efficient than the standard approaches. We use the benchmark heuristic [ARPD](RPDP) to yield non-Pareto efficient adjustable solutions, and the main heuristic [PR](RPDP) is employed to yield Pareto efficient adjustable solutions. Numerical results show that the benchmark heuristic [ARPD](RPDP) outperforms the CPU time performance of the main approach [PR](RPDP). Nevertheless, in the following subsection, we will show that [PR](RPDP) significantly outperforms the (average) OFV performance of the benchmark since it is Pareto efficient.

2.5.3 Evaluating OFV performances of static and adjustable robust approaches

In this subsection, we evaluate the OFV performances of the adjustable approach (before and after re-optimization) and the static robust model by adopting the proposed Monte Carlo simulation experiment explained in Section 2.4.3. To solve [ARPDP], we use the benchmark heuristic [ARPDP](RPDP); to yield Pareto efficient solutions for the same model, we use the main heuristic [PR](RPDP) in the experiments, which are shown to be computationally more efficient than other methods in the above experiment. To solve [RPDP], we solve the standard mathematical optimization model.

The results are summarized in Table 10, the percentages in the third and fourth columns show the improvement of the adjustable model with respect to the static robust model before and after re-optimizing decision rule coefficients, respectively. The first element in the parenthesis is the disposal cost improvement, and the second one is the total cost improvement. Notice that for numeric instances the uncertainty level is assumed to be 0.2.

Table 10: Average cost improvement of heuristic models with respect to [RPDP]

sampling	instance	(disposal cost, total cost) improvement%	
		[ARPDP](RPDP)	[PR](RPDP)
$U(-0.2, +0.2)$	1	(3%, 8%)	(37%, 14%)
	2	(2%, 7%)	(21%, 10%)
	3	(2%, 7%)	(16%, 10%)
$U(0, +0.2)$	1	(3%, 5%)	(37%, 8%)
	2	(2%, 4%)	(21%, 6%)
	3	(2%, 4%)	(16%, 6%)
$T(-0.2, +0.2, 0.1)$	1	(3%, 7%)	(37%, 12%)
	2	(2%, 6%)	(21%, 9%)
	3	(2%, 6%)	(16%, 9%)
average		(2.3%, 4.5%)	(24.6%, 7.0%)

According to the simulation results, using the adjustable robust approach improves the penalty cost and the total cost on average by 2.3% and 4.5%; respectively, compared with those of the static robust. On the other hand, re-optimizing the coefficient of the decision rules by solving [PR](RPDP) yields further cost improvement such that the disposal cost and the total cost decrease on average by 24.6% and

7.0%. This comparison shows the advantages of adjustable solutions and indicates the importance of Pareto efficiency. Ultimately, we can conclude that even though the CPU time performance of the solution approach deteriorates when we aim for Pareto efficiency, the numerical results in this section show that the Pareto efficient heuristic [PR](RPDP) yields significant improvements over the non-Pareto efficient benchmark [ARPDP](RPDP).

2.5.4 Case study

In this section, we solve deterministic, robust, and adjustable robust models for a real data set inspired by a dairy production facility located in the Middle East. The company's primary marketing strategy is to produce and distribute milk products to retailers located in the populated cities of the domestic market or cities that are close to borders and distributing to neighboring countries. Figure 2 presents the locations of the manufacturer and the eight retailers on the actual map of the region.

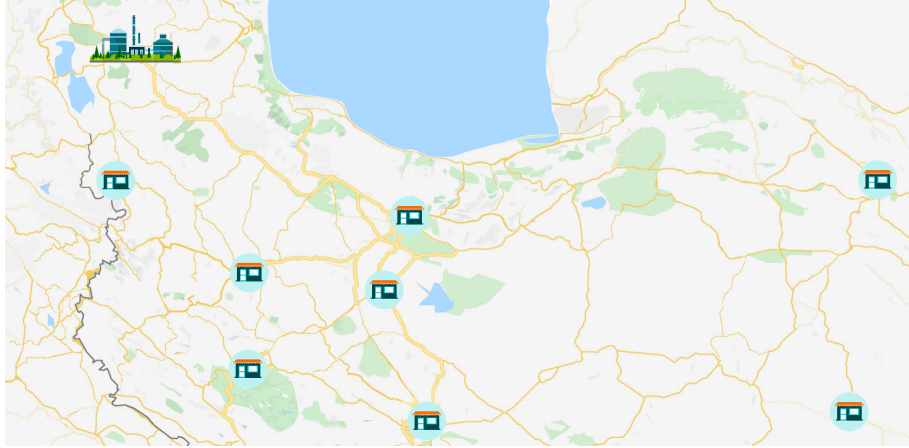


Figure 2: Manufacturer and retailers' locations

The data and other details required for the addressed problem are obtained by communicating with the production planning expert of the company; due to confidentiality reasons, we scale the data. The major products are different types of UHT and daily milk, cheese, yogurt, cream, and doogh; each product is manufactured in a separate production line and can be stored in two warehouses located in the same

facility with the production lines. The warehouses may operate in two different temperatures, i.e., $4-5^{\circ}\text{C}$ and $20-22^{\circ}\text{C}$. UHT milk and doogh could be stored at both temperature levels, while cheese, yogurt, and cream can only be stored in $4-5^{\circ}\text{C}$. The per-unit production cost consists of labor, machinery, and electricity usage costs of the related production line. The transportation cost depends on the distance of the factory from the retailers and the temperature requirement of the product in transportation, which is approximated to be 10% of the production cost multiplied by the delivery lead time of a given retailer. The holding cost of each product depends on the temperature setting of the related warehouse and the product type. It is approximated to be 15% of the production cost if the product is kept in $20-22^{\circ}\text{C}$ and 20% of production cost if kept in $4-5^{\circ}\text{C}$. The backorder cost of each product is determined according to the customer's loss of goodwill and approximated to be 25% of the production cost. The problem is solved every week for the six active days of the week; the factory has two shifts in the first five days of the week, and on the sixth day, the factory has one shift, and machines must be deactivated 5-6 hours earlier for sanitation. The capacities of the production lines are fixed during the weekdays; nevertheless, on the first day of the week, they work with 60% capacity because of the setup requirements. Lastly, daily milk, cheese, yogurt, cream are produced with half capacity in the night shifts. The initial inventory for each product is equal to the average daily demand of the product. The parameter values used in the case study are given in Appendix B.1.

2.5.4.1 Evaluating OFV performance of static robust approach

In this subsection, we evaluate the OFV performance of the robust approach under different uncertainty levels and present the percentage of each cost component for the total cost. The results are given in Table 11 and Figure 3. In the first row of the table, we report OFV of the deterministic problem, which is used as a benchmark

for OFVs of robust models reported in the other rows. The first column shows the uncertainty level, and the second one shows how much OFV of the robust approach is higher than that of the deterministic for the given uncertainty level. The percentages of production, warehouse (including the sum of setup, warehouse opening, and operating), backorder, and transportation costs with respect to the total cost are reported in columns three to six.

Table 11: Robusts OFV for different uncertainty levels

uncertainty level(ρ)	OFV	cost type percentages			
		p	w	b	t
0.0	bench.	81%	6%	3%	10%
0.1	+11%	80%	5%	4%	11%
0.2	+21%	80%	4%	5%	11%
0.3	+33%	79%	4%	6%	11%
0.4	+47%	78%	3%	7%	10%
0.5	*	*	*	*	*

p: production; w: warehouse – setup, opening, holding
b: backorder; t: transportation
“*” denotes infeasibility

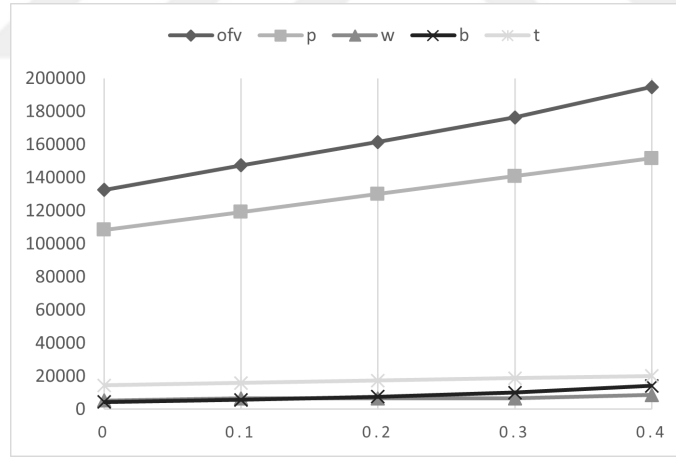


Figure 3: Robust OFV for different uncertainty levels

As it may be anticipated, we get higher OFVs as the uncertainty level increases because RO is a worst-case-oriented approach. Indeed, the producer should produce more to compensate for more conservative demand realizations. The optimal OFV of the robust model is 47% higher than that of the deterministic model when the uncertainty level is 0.4. Nevertheless, as we mentioned before, one needs to adopt a Monte Carlo simulation experiment to compare the OFV performances of the two

approaches on equal grounds. Notice that the robust model becomes infeasible when the uncertainty level is 0.5 because the production capacity cannot meet the worst-case demand. It is essential to highlight that a significant part of the cost comes from production, almost 80% of the total cost. As anticipated, the most significant amount of cost increase arising from uncertainty is realized in the production. The numerical results also show that the percentage of warehouse costs gets lower as the uncertainty level increases. The warehouse cost is not affected by the uncertainty because warehouse opening and setup decisions, which constitute a significant part of the related cost, do not significantly change for different uncertainty levels; nevertheless, the relative importance of the cost in OFV decreases since other costs increase more with the uncertainty level.

2.5.4.2 Evaluating feasibility performance of deterministic approach

In this subsection, we adopt a Monte Carlo simulation by randomly sampling demand from predefined distributions to check the average feasibility performance of the deterministic (or nominal) optimal solution. In the Monte Carlo simulation, we give flexibility to the deterministic approach to re-optimize some of the decisions, including inventory holding, backorder, and transportation for each sampled demand while fixing the production amounts to the nominal solution as explained in Section 2.4.3. Notice that this approach favors the deterministic OFV and it may not be applicable in practice. The numerical results of the Monte Carlo simulation in Table 12 report the percentage of unmet demand for the deterministic plan for three uncertainty levels $\rho \in \{0.1, 0.2, 0.3, 0.4\}$ and distributions $\{U(-\rho, +\rho), U(0, +\rho), T(-\rho, +\rho, \frac{\rho}{2})\}$. The first column shows the uncertainty level. The values reported in columns three to five denote the percentage of unmet demand averaged over 100 demand scenarios sampled from the distributions.

Increasing the uncertainty level increases the percentage of unmet demand for

Table 12: Monte Carlo simulation on feasibility of deterministic approach

uncertainty level (ρ)	sampling procedure		
	$U(-\rho, +\rho)$	$U(0, +\rho)$	$T(-\rho, +\rho, \frac{\rho}{2})$
0.1	0%	5%	2%
0.2	1%	9%	3%
0.3	1%	13%	5%
0.4	2%	17%	6%
average	1%	11%	4%

the deterministic production plan. The highest unsatisfied demand that gets as high as 17% is obtained when data is sampled from $U(0, +\rho)$ with $\rho = 0.4$ uncertainty level. Moreover, sampling demand from more conservative sets leads to higher unsatisfied demand; such that, when we sample the demand from $U(0, +\rho)$, we have the highest unsatisfied demand percentage by 11%; however, sampling from $U(-\rho, +\rho)$ yields the lowest unsatisfied demand percentage by 1% because in this case, the deterministic plan may compensate (using [S1]) the supply shortage in periods that it under-produces with periods having over-production when demand realizes less than the nominal. Having discussed the feasibility performance, it is also worth comparing the disposal performance of the deterministic approach. To begin with, when demands are sampled from $U(0, +\rho)$, products are not disposed of since the realized demand is always higher than the nominal, and all products are distributed to retailers to meet the increased demand. The same holds for the triangular distribution $T(-\rho, +\rho, \frac{\rho}{2})$. On the other hand, when demand is sampled from a relatively more progressive distribution, i.e., $U(-\rho, +\rho)$, then the nominal production plan may still result in some surplus inventory at retailers that are not sold and sent back to the producer for disposal. Nevertheless, the corrective actions yielded by [S1] avoid such disposals as much as possible. Lastly, among all the probability distributions, $U(-\rho, +\rho)$ is the closest to the nominal demand, and we make significant improvements to the deterministic solution with the corrective actions we take; however, even in this case, the deterministic solution is infeasible (although it is 1-2%) for all the sampled scenarios (except for $\rho = 0.1$).

2.5.4.3 Evaluating OFV performances of static and adjustable robust approaches

As the last numerical experiment, we compare the OFV performances of static and adjustable robust approaches via Monte Carlo simulation. In the sampling procedure, we adopt the experimental design that was used in the previous experiment. Nevertheless, unlike the Monte Carlo simulation of the deterministic approach, we only re-optimize backorder decisions of static robust and adjustable robust methods according to the realized demand. For each sampled demand, we use OFV of the robust solution as the benchmark for that of non-Pareto efficient and Pareto efficient adjustable robust solutions obtained from the state of the art solution methods [ARPD](RPDP) and [PR](RPDP), respectively. The results are summarized in Table 13 where the third column shows the cost improvement made by [ARPD](RPDP) over [RPDP], and the last column shows that for [PR](RPDP). The first element in the parenthesis denotes the disposal cost improvement, and the second element is the total cost improvement for those of the [RPDP] model.

Table 13: Average cost improvement of heuristic approaches for [RPDP]

sampling	uncertainty level(ρ)	(disposal cost, total cost) improvement %	
		[ARPD](RPDP)	[PR](RPDP)
$U(-\rho, +\rho)$	0.1	(0%, 4%)	(23%, 7%)
	0.2	(0%, 6%)	(29%, 11%)
	0.3	(2%, 8%)	(33%, 16%)
	0.4	(7%, 10%)	(42%, 21%)
$U(0, +\rho)$	0.1	(0%, 4%)	(23%, 3%)
	0.2	(0%, 4%)	(29%, 7%)
	0.3	(2%, 5%)	(32%, 9%)
	0.4	(7%, 6%)	(42%, 11%)
$T(-\rho, +\rho, \frac{\rho}{2})$	0.1	(0%, 4%)	(23%, 6%)
	0.2	(0%, 5%)	(29%, 10%)
	0.3	(2%, 7%)	(34%, 14%)
	0.4	(7%, 9%)	(42%, 18%)
average		(2.25%, 6.0%)	(31.5%, 11.0%)

Notice that both static and adjustable robust approaches are resilient against uncertainty, and this is why the feasibility performances of methods are not included in this experiment. According to the simulation results reported in Table 13, obtaining Pareto efficient solutions with [PR](RPDP) enables us to decrease the disposal cost by 31.5% and the total cost by 11% compared with the static robust model [RPDP].

When the non-Pareto efficient adjustable robust approach ([ARPDP](RPDP)) is used, the improvements with respect to the robust approach are 2.25% for the disposal cost and 6% for the total cost. Therefore, one of the main findings of this experiment is that adopting Pareto efficiency via the proposed heuristic [PR](RPDP) significantly improves the average performance of the standard adjustable robust approach. Moreover, the associated improvement becomes even more significant when the uncertainty level increases. Numerical results also show that the disposal and total cost improvements made by the Pareto efficient method become more significant for the non-Pareto adjustable robust approach as the uncertainty level increases. Lastly, the relative importance of the adjustable approach decreases for $U(0, +\rho)$ because the sampled demands become more conservative, resulting in the adjustable robust approach having small room for improvement with respect to the static robust approach.

2.6 *Managerial insights*

This section provides the managerial insights of this research.

2.6.1 Importance of production planning in dairy industry and challenges

In this study, we have proposed a production and distribution planning model that combines multiple challenging factors that are often faced in the dairy industry in one setting. These factors include perishability, temperature tracking of products, sequence-dependent facility setup cost, and uncertain demand. To model the uncertainty, we adopted the RO paradigm; moreover, we have employed adjustable RO to decrease conservatism in the robust models. Considering the low profit margins and environmental costs of producing dairy products, it becomes crucial to optimizing production, warehouse, and transportation decisions for the dairy industry. Nevertheless, the proposed robust models yield challenging MILPs that cannot be solved efficiently for realistically sized instances. To this end, we have proposed heuristic approaches that yield high-quality solutions for the associated models in less than

three hours for realistically sized instances. Notice that the decision-maker has to invest the related CPU time only once at the beginning of the planning horizon. The adjustable decisions are instantly made when the required data in the information base is realized. Hence, the proposed solution approaches may be used as a decision support tool that can be efficiently used in practice. Moreover, it is essential to mention that different industries that show similarities to the dairy industry, such as health-care and catering, may benefit from the presented research.

2.6.2 Flaw of using nominal demand

The Monte Carlo simulation results show that the deterministic production plan becomes obsolete when demand realizes differently than the nominal demand. The percentage of unmet demand may get as high as 17% even when we take corrective actions according to the realized demand. Therefore, a decision-maker needs to take demand uncertainty into account in order to yield resilient production and distribution plans.

2.6.3 Decreasing the level of disposal via adjustable robust approach

Even though the robust approach hedge against the demand uncertainty, it may result in overproduction and hence extra disposals that have significant environmental externalities. As a remedy for the associated over-conservatism issue, we propose an adjustable robust approach that significantly avoids disposals compared with the robust approach. The numerical results show that the associated improvement can get as high as 29% and the proposed heuristic approach solves realistically sized instances in less than three hours. It is essential to emphasize it once more that the proposed solution approach not only improves the producer's cost of manufacturing but also decreases the negative environmental effects of surplus dairy production.

2.7 Conclusion and future research

This paper addressed a *dairy production and distribution planning problem* (PDP) that decides on the production and distribution plan of dairy products to minimize the total cost of production, warehousing, backorder, and distribution. We adopted *robust optimization* (RO) and proposed *robust PDP* (RPDP) to deal with the uncertainty of retailers' demand. RPDP may yield over-conservative solutions as it is a worst-case-oriented approach; therefore, the *adjustable* RO was employed, and an adjustable variant of RPDP was formulated. The approach allows the producer to adjust production, inventory, backorder, and distribution decisions according to the realized part of demands. The OFV and CPU time performances of the developed algorithms for solving the adjustable model were compared over different sized test instances and a real-life case study. Results have shown that the proposed heuristic enables us to save a considerable amount of time. Moreover, studying the OFV performance of the adjustable approach has shown that the adjustable robust model improved the disposal cost by 2.25% and the total cost by 6% over the static RO approach. Furthermore, we observed that adopting the Pareto efficient approach made the OFV improvement even more significant such that disposal cost was improved by 31.5% and the total cost was improved by 11% for the static RO.

In *future research*, our goal is to show the formal conditions when the proposed robust and adjustable robust models yield the same objective function value for the worst-case data and consider the adjustability of warehouse opening decisions according to the realized demand.

CHAPTER III

HUMANITARIAN RELIEF DISTRIBUTION PROBLEM: AN ADJUSTABLE ROBUST OPTIMIZATION APPROACH

Management of humanitarian logistics operations is one of the most critical planning problems to be addressed immediately after a disaster. The response phase covers the first 12 hours after the disaster and is prone to uncertainties because of debris and gridlock traffic influencing the dispatching operations of relief logistics teams in the areas affected. Moreover, the teams have limited time and resources, and they must provide equitable distribution of supplies to affected people. This paper proposes an adjustable robust optimization approach for the associated humanitarian logistics problem. The approach creates routes for relief logistics teams and decides the service times of the visited sites to distribute relief supplies by taking the uncertainty in travel times into account. The associated model allows relief logistics teams to adjust their service decisions according to the revealed information during the process. Hence, our solutions are robust for the worst-case realization of travel times but still more flexible and less conservative than those of static robust optimization. We propose novel reformulation techniques to model these adjustable decisions. The resulting models are computationally challenging optimization problems to be solved by exact methods, and hence we propose heuristic algorithms. The state-of-the-art heuristic, which is based on clustering and a dedicated decision rule algorithm, yields near-optimal results for medium-sized instances and is scalable even for large-sized instances. We have also shown the effectiveness of our approach in a case study using a data set obtained from an earthquake that hit the Van province of Turkey in 2011.

3.1 Introduction

The first 12 hours after an earthquake are the most critical hours in which humanitarian organizations carry out critical operations in the response phase including providing medical aids service, temporary shelter, food, and drinking water to the affected and vulnerable people [69]. Even though the critical operations must be done swiftly and efficiently, humanitarian organizations often face several difficulties and uncertainties while executing these operations. Firstly, it is challenging to evaluate the status of the transportation routes in an affected area promptly because of unknown debris amount that may partly or fully block some of the roads. For example, the Great East Japan Earthquake in March 2011 affected the transportation system of North Japan drastically. Based on the Japan Ministry of the Environment’s estimation, about 25 million tons of debris were created by the earthquake and the tsunami that clogged the transportation routes; the debris blocked and damaged coastal roads and reduced the efficiency of service operations and relief teams’ pace [70, 71, 72]. Secondly, the effects of a disaster in different regions cannot be immediately assessed in the response phase due to delays in the communication systems. The unavailability of such information is one of the main challenges for humanitarian operations after a disaster happens. Response teams usually have more information about the actual situation as they start visiting the sites, and more information unfolds at each visit. Therefore, the dynamic nature of humanitarian operations must be taken into account in the decision-making process, and *relief logistics teams* (RLT)s must be flexible in adjusting their decisions according to the information that realizes during the process. Last but not least, practice shows that the RLTs must prioritize affected people’s critical needs and manage their service times to deliver and distribute essential supplies and limited resources to as many affected people as possible within 12 hours after the disaster. For instance, when the twin earthquakes hit the East

Azerbaijan province of Iran, RLTs failed to have a comprehensive analysis of the situation due to insufficient information; the far-flung villages were deprived of the very basic needs such as medical supplies, water, and food, while easily accessible sites got an excessive amount of supplies. The associated failure in the disaster response planning resulted in a waste of resources, and more importantly, caused damage to the survivors who could normally be serviced on time [73].

After an RLT is dispatched to an affected area, it is essential to distribute the critical supplies, e.g., water, meal, temporary shelters, and medicine, equitably across the affected regions. Equity in the humanitarian logistics context refers to a fair distribution of services or products to disaster victims. As mentioned in several other studies, such as [74] and [75], defining equity via quantitative models is generally not straightforward. Nevertheless, to avoid problems like the ones in the East Azerbaijan, diminishing returns are considered in the utility function as an RLT stays more in the same site to favor equitable distribution. The problem we formulate in this study is referred to as the *post-disaster relief supply distribution problem* (RDP). In RDP, we determine the routes to be followed by each RLT to distribute the relief supplies to the affected sites and the service time at each visited site. The second important goal is ensuring the distribution within a short time window in the post-disaster response phase. Therefore, the two key characteristics of the problem are equity and the time limit. RLTs are responsible for distributing and transporting relief supplies in the corresponding setting. Ultimately, the service times for distributing supplies are important decisions since equitable distribution is based on effective time management within the given time limit.

The uncertain factors, such as debris, gridlock traffic, and dynamic information flow, mainly affect transportation times. To address the uncertainty in the RLTs' travel times, we propose a *robust optimization* (RO) approach that can be effectively

used in a data-scarce environment like the response phase. The proposed robust reformulation of RDP, referred to as RRDP, focuses on travel time uncertainty and yields a robust solution for any realization of the travel times residing in the interval uncertainty set that defines bounds for the travel time of each road segment in the given area. The proposed framework aims to maximize the total utility of the distributed relief supplies and decides on the sites to be visited, the service times that will be spent in the selected sites to distribute relief supplies, and the optimal ordering of the visits such that the predetermined time limit is not exceeded for the worst-case realizations of the travel times. Even though the worst-case data realizations are more likely to occur in humanitarian logistics than in standard logistics networks due to the chaotic nature of a disaster, adopting a static RO paradigm based solely on the worst-case data may still result in over-conservative solutions. When the RLTs' services are planned for the worst-case realization of travel times by ignoring the realized portions of data that dynamically unfold as teams visit more sites, the teams may not manage their time effectively according to the realized information. Notice that the total travel and service times of the robust solution will always be feasible for any realization of the uncertain travel times; nevertheless, the associated solution may be suboptimal when non-worst-case data is realized. To address the associated over-conservatism of RRDP, we propose an *adjustable* reformulation of RRDP (ARDP) that allows RLTs to dynamically adjust their service time decisions according to the realized information over time. More precisely, the service times are modeled as “wait-and-see” decisions that are parametric functions of the uncertain travel times; as the uncertainty is realized, i.e., when a team traverses a road segment, the team may update the next service decision according to the revealed data. In other words, if a non-worst-case travel time is realized for a road segment, then adjustable robust optimization gives a kind of road map, i.e., referred to as the decision rule, on how to utilize this information in the following site(s) to be visited. Consequently, the extra

time realized in a road segment may be utilized in the sites that are succeeding the related segment to distribute more humanitarian relief supplies.

The numerical results show that ARDP that is solved by efficient heuristic algorithms proposed in this study yields *objective function values* (OFV)s that are (on average) 3–4% better than RRDP; moreover, the total given service time is improved (on average) 5–6%. The OFV and CPU time performances of the proposed heuristic algorithms are also tested via an extensive Monte Carlo simulation experiment using a data set obtained from the Van earthquake.

The remainder of the paper is organized as follows. In §3.2, we provide an overview of the relevant literature. In §3.3, we define the problem and present the mathematical formulations. The proposed solution approaches are provided in §3.4 and evaluated computationally in §3.5. Finally, the concluding remarks, the managerial implications of our approach, and future research directions are given in §3.6.

3.2 Literature Review

The significance of humanitarian operations has inspired many researchers to heed disaster management problems, and it has triggered increased attention on the effectiveness and efficiency of humanitarian aid for those in need as quickly as possible [76]. This fact has resulted in an attentive focus on improving humanitarian relief logistics [77, 78, 75]. The studies by [79], [80], and [81] are among the studies that consider the distribution of relief supplies from a center to affected regions. The presented work in this study belongs to the practical and equitable supply relief distribution stream of the humanitarian logistics literature; moreover, it is also related to distinct streams, including adjustable robust optimization. For the sake of brevity, in the remainder, the emphasis of the literature survey will be on studies considering relief supply distribution; we characterize the related literature under three subcategories.

3.2.1 Effectiveness and Equity

In order to consider the implicit convolutions of an effective relief supply distribution, followed-up studies encompass different settings such as multiple commodities [82, 83, 84], heterogeneous service teams (vehicles) [85, 86], multiple periods [87, 88, 89], uncertainty in roads and travel times [90, 91], and debris clearance to connect supply to demand under uncertain debris volumes [92]. Like effectiveness and efficiency, equity is also one of the primary aspects for decision-makers [74].

In the literature, there are different definitions and formulations for the fair distribution of supplies and services in nonprofit systems. [80] minimizes the summation of the arrival times, [93] maximizes the minimal satisfaction rate, [94] considers the discrepancy of satisfaction rate between sites. Besides, [95] and [96] consider the Gini coefficient of deprivation cost and coverage rate as equity metrics in their models. In [97], authors formulate a model to realize envy-free allocation of food. In envy-free allocation, all sites receive the same service, or the minimum demands of all sites are satisfied. For more information about modeling equity, we refer readers to the study by [98] that has surveyed the equity-related challenges in nonprofit and public organizations. The authors also focus on the challenges of equity in *vehicle routing problem* (VRP) and state that the problems with disaster relief distribution applications are mainly deciding on routing and supply allocation.

3.2.2 Modeling and Solution Approaches with Equity

Relief supply distribution problems are too complex to be solved to optimality since even the most fundamental VRP is proven to be NP-hard [99, 100]. Moreover, since in humanitarian operations making unhesitating decisions are of high importance, the scholars seek quality and, at the same time, computationally-not-complex solution approaches that may not necessarily be exact [101]. [102] develops a heuristic algorithm that combines ant colony optimization with greedy randomized adaptive

search procedure for the relief plannings in real-life problems. In [97], greedy and local search heuristic algorithms are proposed for large-size instances of the their food delivery and rescue problem with objective of minimizing the total travel times. In [103], a selective routing problem is formulated for distributing critical supplies while considering travel time uncertainties. The proposed problem is solved with an iterative greedy algorithm, and the performance of the algorithm is studied via benchmark instances. Three selective routing models are formulated as fuzzy, robust, and reliable models in [104], for humanitarian settings, and the models are solved with three parallel genetic algorithms. For more information on employing algorithms in routing and distribution in humanitarian problems, we refer readers to [101] and [105].

Piecewise linear functions can be used for modeling the equity in humanitarian logistics, e.g., [75] and [74]. [74] propose a method to evaluate the efficiency of delivered aids using five attributes for humanitarian aid distribution problems. The utility of each attribute is formulated as a piecewise function with a single breakpoint. E.g. for the quality of total cargo delivery, they have defined a metric as the ratio of total delivery to total requested cargo, and the utility is modeled via a piecewise function in which the second segment yields a smaller benefit than the first segment. Three different metrics are proposed in [75], to measure equity. The two metrics, which are based on service spread levels among sites, are only used for evaluation purposes. The third metric captures the time and equity by using a piecewise function such that the disutility gets larger as the fraction of the unsatisfied demands increases.

3.2.3 Relief Distribution under Uncertainty

The deterministic relief supply distribution problem has been considered in the literature from different perspectives (see, for example, [106], [107], [89], and [88]). Although the success of RLTs depends so much on the swift and equitable distribution of the supplies, in practice, a relief operation may fail due to several decisive

factors, such as a lack of proper planning for uncertainties in travel and service times for the affected people [88]. Broadly speaking, uncertainty can be considered in different ways in any routing problem. While some studies focus on the uncertainty in travel times [108, 109, 66, 110], service times [111], and demand [112, 66, 110] are also among the parameters that are considered as uncertain. [113] provides a comprehensive review of uncertainty in transportation networks. To address uncertainties in problems, there are two optimization methods that are often used in practice: *stochastic programming* (SP) and *robust optimization* (RO). For more details on RO and SP, we refer the readers to [114] and [115], respectively, and references therein. As it is mentioned in [116], there exists a very exhaustive range of studies considering SP models that consider uncertainties in different humanitarian settings, while only a few studies consider RO methods to model humanitarian logistics problems under uncertainty. In the remainder, we primarily focus on the papers associated with the RO line of optimization under uncertainty research that is closely related to the methods that will be proposed in this study.

A mathematical model is provided in [117], for a disaster relief routing problem considering demands and travel times as uncertain parameters. They consider five different objective functions, namely, minimization of utilized vehicles, minimization of total travel time, minimization of summation of arrival times, minimization of summation of demand-weighted arrival times, and minimization of latest arrival times. By using a robust optimization approach and a two-stage heuristic method, they solve a large-scale relief routing problem. In [118], a robust approach is proposed to manage the logistics of relief commodities and affected people in the response phase. Their model minimizes the total waiting time of unserved affected people, total lead time of commodity delivery, and total vehicle utilization. A needs assessment operation under travel time uncertainty is studied by [119], adopting a robust approximation of the ambiguous chance constraints and proposing an efficient local-search algorithm

to solve the problem. In [120], the authors propose RO for a relief supply distribution problem with uncertainty in travel times and demands. They conduct sensitivity analysis in order to study the trade-off between optimization and robustness. To the best of the authors' knowledge, there are a few studies that consider adjustable robust reformulations of humanitarian logistics problems. [91] represents an application of *adjustable robust optimization* (ARO) in emergency logistics. The authors study evacuation and transportation plans by proposing a robust model against the demand uncertainty and show how advantageous it is to apply the ARO for their problem compared to the deterministic model. In [121], ARO is adopted for distributing relief commodities after a disaster happens, considering demands as uncertain parameters. They also consider transported amount of commodity as a wait-and-see decision to decrease the over-conservatism of the model.

3.2.4 Contributions

The contributions of this research to the existing literature can be summarized as the following.

- As mentioned in the literature survey, equity–efficiency trade-offs and optimization under uncertainty frameworks are established in the realm of humanitarian logistics. However, this paper is the first that combines the essential properties of the ARO models, such as Pareto efficiency, decision-dependent uncertainty, and optimality of decision rules, with a humanitarian logistics problem inspired by practice. Innovative reformulation techniques and comprehensive experimental designs have been proposed to add these properties to the related humanitarian logistics model and to quantify our findings in the numerical results.
- The resulting mathematical optimization model is a computationally expensive *mixed-integer linear programming* (MILP) problem that cannot be solved to

optimality for realistically-sized instances due to a large memory requirement. On the other hand, decision-makers need rapid humanitarian relief distribution decisions that are high-quality but not necessarily optimal since the timely response is the most critical criterion. This paper proposes heuristic algorithms that yield near-optimal solutions for real-life instances in less than two hours.

3.3 *Problem Definition and Formulation*

RDP is defined on a network that consists of I affected areas (sites) (denoted by indices $i, j = 1, \dots, I$) and a *disaster coordination center* (DCC) as the departure point of RLTs. RLTs visit the affected sites to deliver relief supplies, and the service time that RLT v spent on site i is denoted by T_{iv} . We represent relief supply distribution via service time decisions because RLTs must distribute humanitarian aid supplies to affected sites within a given time limit ($T_{\max}$), which is one of the most critical constraints for the given problem. Therefore, the sum of the total travel time of a given RLT route and the service times of distributing relief supplies cannot exceed $T_{\max}$. The problem aims for equitable distribution of supplies among the sites in the given RLT route, and to achieve that, a piecewise linear concave utility function is adopted for each site to express utility as a function of RLT's service time. The related piecewise function, to favor the equitable distribution and visiting more sites, decreases the utility gained per unit time at each site when more aid is distributed than the critical aid requirement of the site. In the remainder, we demonstrate our approach over utility functions containing two breakpoints and three time segments (see Figure 4), which are also used in the numerical results and the case study. Similar to our approach [74] also suggests using a piecewise linear function with one breakpoint for each attribute since such functions are general enough to model different situations in practice.

To begin with, the total utility of the relief supplies distributed to site i by RLT

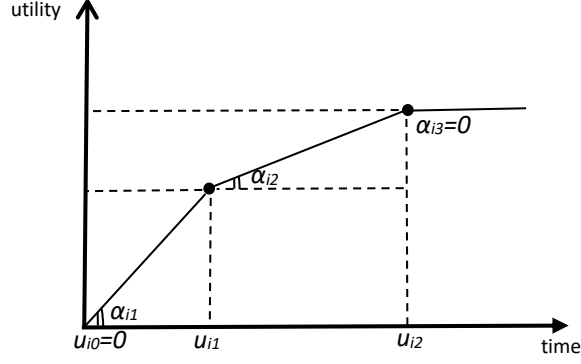


Figure 4: Utility function with two breakpoints

$v(S_{iv})$ is represented by the following expression:

$$S_{iv} = \alpha_{i1} \min\{u_{i1}, T_{iv}\} + \max\{0, \alpha_{i2} (\min\{u_{i2}, T_{iv}\} - u_{i1})\},$$

where u_{i1} and u_{i2} are the joint points and α_{i1} , α_{i2} and α_{i3} are the slopes of the time segments in the utility function as in Figure 4.

We determine the prioritization of sites via a score-based mechanism that determines the slopes (α) used in the utility functions. Namely, the R_i parameter determines the severity score of site i :

$$R_i = \sigma_1 \times \text{distance score of site } i + \sigma_2 \times \text{population score of site } i \quad (43)$$

after a disaster has occurred. The main score is a weighted sum of the two critical metrics, namely, the distance to the epicenter and the population, to determine the severity of a natural disaster for site i ; the higher the score in both metrics, the higher the priority of servicing the site. The slopes (α) used in the utility function, which are related to the severity score of each site i are determined by using R_i and the procedure given in Appendix B.2.

The supply amount distributed to a site is equivalent to the product of the related service time and the distribution rate of an RLT. The content of the distributed packages/pallets may be generic according to the population needs and the severity

of the disaster, and RLTs often use *Federal Emergency Management Agency* (FEMA) planning factors in distributing supplies, e.g., three liters of water and two meals per person each day. The critical supply requirements are fed into the utility function after the initial prompt assessment of the disaster and before RLTs start their operations. The humanitarian supplies are divided into critical, such as food, water, shelter, and less critical supportive aid in the related utility structure. The supportive aid can be characterized in two ways: 1) the number of supplies that are distributed more than a certain threshold, i.e., associated with the critical supply requirement of a given site or 2) the supportive operations such as health services held by the RLT after the required critical supply distribution is made. The breakpoints of the utility function (u_i) for each site i are given by the following formulas:

$$u_{i0} = 0, \quad u_{i1} = T_{\max} \times \min \left\{ \beta_1 \frac{\text{population of site } i}{\text{total population}}, 1 \right\}, \quad u_{i2} = \min \{ \beta_2 u_{i1}, T_{\max} \}, \quad (44)$$

where β_1 and β_2 denote the coefficients that ensure the relationship between the service time limits and the supply amounts. The β_1 parameter is relatively more important than β_2 because it is associated with the critical supply requirements of the sites. This parameter can be interpreted in two different ways: 1) if the given time limit constraint, which includes both service and travel times, is binding, then the decision-maker sets the β_1 parameter according to the critical needs of the population in the affected site, e.g., the site with the minimum population must receive service for at least 10 minutes or 2) if the total humanitarian supply is at a critical level so that it is known that shortage will occur even for supplying the most critical needs of the sites, to handle this situation the decision-maker may adjust β_1 such that $\beta_1 T_{\max} = T_{\text{supply}}$ in the u_{i1} formula and set $u_{i2} = u_{i1}$, where T_{supply} denotes the limited service time given by the amount of limited supply divided by the distribution rate of an RLT. The β_2 parameter is set to relatively larger value than β_1 because RLTs usually do not exploit the second time segment due to the supply shortage and

time limit constraints. Ultimately, we propose a generic approach that may handle different cases with mild modifications in the problem parameters.

The goal of the proposed model is to distribute the related humanitarian relief supplies as equitably as possible without exceeding the given time limit. The objective is to maximize the total utility using the aforementioned piecewise utility functions by deciding on the routes and service time decisions of RLTs under travel time uncertainty. Next, we formulate the robust optimization model of RDP denoted by [RRDP]; the mathematical optimization model will be given for the general case containing n time segments in the utility function. The nomenclature of the model is given below in Table 14.

Table 14: Nomenclature of the mathematical optimization model

Sets	
$\mathcal{I}$	the set of sites ($i, i', j, j' = 1, \dots, I$)
$\mathcal{I}_0$	$\mathcal{I} \cup \{0\}$ (0 index denotes the DCC)
$\mathcal{V}$	the set of RLTs ($v = 1, \dots, V$)
$\mathcal{N}$	the set of time segments indices ($n = 0, \dots, N$)
$\mathcal{K}$	the set of visiting orders ($k = 1, \dots, I$)
$\mathcal{U}$	the uncertainty set
Parameters	
$T_{\max}$	the service time limit
t_{ij}^0	the nominal travel time from site $i \in \mathcal{I}_0$ to site $j \in \mathcal{I}_0$
ζ_{ij}	the primitive uncertainty parameter for travel time (i, j) ($\in [-1, 1]$)
ρ	the coefficient of worst-case dispersion percentage from nominal data (i.e., the uncertainty level)
u_{in}	the upper bound of time segment $n \in \mathcal{N}$ in the utility function of site $i \in \mathcal{I}$
α_{in}	the slope of segment $n \in \mathcal{N}$ in the utility function of site $i \in \mathcal{I}$
Decision variables	
X_{ijkv}	1, if site $j \in \mathcal{I}$ is visited immediately after site $i \in \mathcal{I}_0$ in the k^{th} order by vehicle $v \in \mathcal{V}$; 0, o.w.
T_{iv}	the time spent in site $i \in \mathcal{I}$ by RLT $v \in \mathcal{V}$
Z_{inv}	the time spent in site $i \in \mathcal{I}$ over breakpoint u_{in} by RLT $v \in \mathcal{V}$

[RRDP] :

$$\begin{aligned}
& \max_{\mathbf{X}, \mathbf{T}, \mathbf{Z}} \quad \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left[\alpha_{i1} T_{iv} - \sum_{n \in \mathcal{N}} (\alpha_{in} - \alpha_{i(n+1)}) Z_{inv} \right] \\
& \text{s.t.} \quad \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} (1 + \rho) t_{ij}^0 X_{ijkv} + \sum_{i \in \mathcal{I}} T_{iv} \leq T_{\max} \quad \forall v \in \mathcal{V} \quad (45)
\end{aligned}$$

$$\sum_{v \in \mathcal{V}} (T_{iv} - Z_{inv}) \leq u_{in} \quad \forall i \in \mathcal{I}, n \in \mathcal{N} \quad (46)$$

$$T_{iv} - M \sum_{j \in \mathcal{I}_0} \sum_{k \in \mathcal{K}} X_{jikv} \leq 0 \quad \forall i \in \mathcal{I}, v \in \mathcal{V} \quad (47)$$

$$\sum_{i \in \mathcal{I}_0} \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}} X_{ijkv} \leq 1 \quad \forall j \in \mathcal{I} \quad (48)$$

$$\sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}} X_{ijkv} \leq 1 \quad \forall i \in \mathcal{I} \quad (49)$$

$$\sum_{j \in \mathcal{I}} X_{0j1v} = 1 \quad \forall v \in \mathcal{V} \quad (50)$$

$$\sum_{\substack{k' \in \mathcal{K} \\ k' \neq k}} \sum_{j \in \mathcal{I}} X_{ijk'v} + \sum_{j \in \mathcal{I}_0} X_{ji(k-1)v} \leq 1 \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \setminus \{1\}, v \in \mathcal{V} \quad (51)$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} X_{ijkv} - \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} X_{ij(k-1)v} \leq 0 \quad \forall k \in \mathcal{K} \setminus \{1\}, v \in \mathcal{V} \quad (52)$$

$$\sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} X_{ijkv} - \sum_{j \in \mathcal{I}_0} \sum_{k \in \mathcal{K}} X_{jikv} \leq 0 \quad \forall i \in \mathcal{I}, v \in \mathcal{V} \quad (53)$$

$$\mathbf{X} \in \{0, 1\} \quad (54)$$

$$\mathbf{T}, \mathbf{Z} \geq \mathbf{0}. \quad (55)$$

The objective function aims to maximize the total utility gained by RLTs by adopting the aforementioned piecewise linear structure. Constraints (45) ensure that total travel and service times do not exceed the given time limit $T_{\max}$. Constraints (46) and the auxiliary variable ($\mathbf{Z}$) ensure the excess service times spent after each breakpoint, which is used in the utility calculation in the objective function $[\sum_{n \in \mathcal{N}} (\alpha_{in} - \alpha_{i(n+1)}) Z_{inv}]$. Constraints (47) ensure that only the sites that are visited must be served. Constraints (48) and (49) guarantee that a site can be visited at most once. All RLTs must start from the coordination center, which is set by constraints (50). Constraints (51) and (52) determine the visiting orders of sites that are visited in a given tour of an RLT; such that the first one ensures that if an RLT visits site i in order $k - 1$ then the team may only leave site i in order k but not in the other orders, and the latter one assures that if a vehicle visits a site in

any order, it must visit another site in the previous order. Constraints (53) ensure that an RLT can leave site i if the site is visited. Constraints (54) and (55) are 0-1 and non-negativity constraints, respectively. Ultimately, [RRDP] yields robust distribution plans for RLTs that are robust for any uncertainty defined in the associated interval uncertainty set; [RRDP] is equivalent to the deterministic model when $\rho = 0$.

To model the uncertainty in travel times, we use the following box (or interval) uncertainty set:

$$\mathcal{U} := \{\mathbf{t} \in \mathbb{R}_+^N : t_{ij} = (1 + \rho\zeta_{ij})t_{ij}^0, -1 \leq \zeta_{ij} \leq 1 \quad \forall i, j \in \mathcal{I}_0\},$$

where $\mathbf{t}^0$ is the vector of nominal travel times, $\zeta_{ij} \in [-1, 1]$ is the primitive uncertainty parameter, and $\rho \in [0, 1)$ is the scalar that denotes the uncertainty level. The robust reformulation of the travel time constraints is equivalent to substituting the upper bound of each travel time interval yielding the RC in (45). For further details on properties of [RRDP] we refer the reader to Appendix B.3.

3.3.1 Adjustable Robust Reformulation

In the static robust reformulation [RRDP], the travel times are replaced by their upper bounds to immunize the optimal solution for all realizations of the uncertain travel times. Moreover, the service time decisions are made at the beginning of the planning horizon, i.e., referred to as “here-and-now”; therefore, the decision-maker cannot update service time decisions over time according to the realized data. Due to the reasons mentioned above, the static approach has the disadvantage of being overly conservative. As a remedy for the related over-conservatism, we propose adjustability of the service times via ARO. To this end, we model the service time of each site as a wait-and-see decision variable; henceforth, as an RLT traverses a road segment, it may update its future service time decision(s) based on the realized travel time information. In other words, if a non-worst-case travel time is realized for a road segment, then ARO gives a kind of road map, i.e., referred to as the decision rule, on

how to utilize this information in the following site(s) to be visited.

To discuss the advantages of ARO better, let us consider the following toy-sized network in Figure 5 that consists of two sites and a DCC. For the given route, when the RLT traverses the first arc (0,1), the travel time t_{01} is realized. If the related travel time is less than the worst-case realization, then the RLT may utilize the related extra time as additional service time in site one and/or site two.

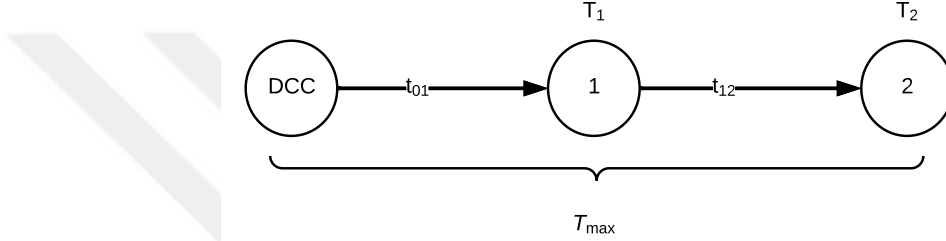


Figure 5: Illustrative example

Similarly, if the actual travel time of arc (1,2) is realized less than the worst-case, then the extra time may be utilized in the second site, and hence the service time for site two may be prolonged more than that of the static robust approach. It is worth mentioning that the travel time information of the second arc (t_{12}) cannot be used in the first site because the related data reveals itself only after the RLT leaves the first site and traverses (1,2). The relationship between the realized data and the associated wait-and-see service time decisions is set by the information base, namely, the non-zero coefficients of the decision rule determines which uncertainty parameters are active in the decision rule; the decision rule is a parametric function of the uncertainty parameters. In our setting, the information base of each site i is given by all the arcs in the related RLT route, which are traversed before i ; therefore, the coefficients of the related parameters are active in the decision rule, while the others are set to zero. More precisely, the decision rules for sites one and two are

given as follows:

$$T_1(\mathbf{t}) = \bar{T}_1 + \gamma_{(0,1)}^1 t_{01}, \quad T_2(\mathbf{t}) = \bar{T}_2 + \gamma_{(0,1)}^2 t_{01} + \gamma_{(1,2)}^2 t_{12},$$

where we adopt *linear decision rules* (LDR)s to service times, and $(\bar{\mathbf{T}}, \boldsymbol{\gamma})$ denote the coefficients of the decision rules to be optimized. E.g., when the worst-case travel time t_{01}^* is realized for (0,1), the service time decision for the first site is obtained by substituting t_{01}^* in the related LDR, i.e., $\bar{T}_1 + \gamma_{(0,1)}^1 t_{01}^*$. If we take the service time decision of the first site for the worst-case data as the benchmark, the extra service time when a non-worst-case data $\tilde{t}$ is realized is given by $T_1(\tilde{t}) - T_1(t^*) = -\gamma_{(0,1)}^1(t_{01}^* - \tilde{t}_{01})$ for the above given illustrative example. E.g., when $\gamma_{(0,1)}^1 = -0.25$, it may be interpreted as 25% of the residual time realized at road segment (0,1) is utilized as additional service time for the first site, where the residual time is equivalent to the difference between the worst-case and the realized travel times of (0,1) given by $(t_{01}^* - \tilde{t}_{01})$. As it may be anticipated, the remaining 75% will be utilized by the decision rule $T_2(\mathbf{t})$ of the second site in this case since the goal is to maximize the total utility of supplies distributed to the affected sites. Therefore, the decision rule coefficient $\gamma_{(i',j)}^i$ denotes the percentage of the residual time realized at (i', j) that is utilized as an additional service time at site i .

Ultimately, RLTs gather more information about the related network as they visit more sites over time and update their service time decisions according to the realized portion of travel times in the ARO approach. In the remainder, we extend the related wait-and-see structure that is demonstrated over one RLT and an illustrative network to multiple RLTs and a general humanitarian logistics network. We also introduce LDRs to auxiliary variables ($\mathbf{Z}$) as introducing a decision rule does not affect the robust optimal solution but increases the degrees-of-freedom of the model [63]. The

resulting LDRs are given as the following:

$$T_{iv} = \bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \underbrace{(1 + \rho \zeta_{i'j}) t_{i'j}^0}_{t_{i'j}} \quad \forall i \in \mathcal{I}, v \in \mathcal{V} \quad (56)$$

$$Z_{inv} = \bar{Z}_{in} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \theta_{i'jk}^{ivn} (1 + \rho \zeta_{i'j}) t_{i'j}^0 \quad \forall i \in \mathcal{I}, v \in \mathcal{V}, n \in \mathcal{N} \quad (57)$$

$$\bar{\mathbf{T}}, \bar{\mathbf{Z}}, \boldsymbol{\gamma}, \boldsymbol{\theta} \in \mathbb{R}, \quad (58)$$

where $\bar{\mathbf{T}}, \bar{\mathbf{Z}}, \boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ are the coefficients of the decision rules that are analogous to those in the above given illustrative example, and the travel time uncertainty is scaled between -1 (lower bound) and 1 (upper bound) using $\rho \in [0, 1)$ and the primitive uncertainty parameter $\boldsymbol{\zeta} \in [-1, 1]$.

In the orthodox ARO approach [68], the information base used in the decision rule is predetermined. Nevertheless, the information base in our setting is determined by the first-stage here-and-now routing decisions ($\mathbf{X}$). Namely, the realized portion of uncertain travel times is determined according to the visit order of sites given by the decided RLT route, which is known only after the problem is solved. More precisely, the information base of the decision rule is decision-dependent in our setting. To formulate such a decision rule, we relate routing decisions ($\mathbf{X}$) to the coefficients of the decision rule ($\boldsymbol{\gamma}$) via additional constraints given as follows:

$$-1 + \sum_{j' \in \mathcal{I}_0} X_{j'ikv} \leq \gamma_{i'jk'}^{iv} \quad \forall i, j \in \mathcal{I}, i' \in \mathcal{I}_0, k \in \mathcal{K}, v \in \mathcal{V}, k' \in [k+1 : K] \quad (59)$$

$$1 - \sum_{j' \in \mathcal{I}_0} X_{j'ikv} \geq \gamma_{i'jk'}^{iv} \quad \forall i, j \in \mathcal{I}, i' \in \mathcal{I}_0, k \in \mathcal{K}, v \in \mathcal{V}, k' \in [k+1 : K] \quad (60)$$

$$-X_{i'jkv} \leq \gamma_{i'jk}^{iv} \quad \forall i, j \in \mathcal{I}, i' \in \mathcal{I}_0, k \in \mathcal{K}, v \in \mathcal{V} \quad (61)$$

$$X_{i'jkv} \geq \gamma_{i'jk}^{iv} \quad \forall i, j \in \mathcal{I}, i' \in \mathcal{I}_0, k \in \mathcal{K}, v \in \mathcal{V}. \quad (62)$$

Constraints (59) and (60) ensure that if site i is visited in the k^{th} order, the sites that are visited after i , i.e., in order $\{k+1, \dots, K\}$, are not included in the information base of the wait-and-see service time decision of site j ; constraints (61) and (62) ensure

that only the coefficients of the road segments that are traversed by an RLT may be included in the information base of the sites visited by the RLT, i.e., the related coefficients may take values different than zero.

To derive the adjustable robust reformulation that is called the *adjustable robust counterpart* (ARC), we substitute the LDRs for variables $(\mathbf{T}, \mathbf{Z})$ in [RRDP]. Ultimately, the semi-infinite representation of the ARC is given as the following.

[ARDP] :

$$\max_{\mathbf{X}, \overline{\mathbf{T}}, \overline{\mathbf{Z}}, \boldsymbol{\theta}, \boldsymbol{\gamma}} \min_{\boldsymbol{\zeta} \in \mathcal{U}} \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left[\alpha_{i1} T_{iv}(\boldsymbol{\zeta}) - \sum_{n \in \mathcal{N}} (\alpha_{in} - \alpha_{i(n+1)}) Z_{inv}(\boldsymbol{\zeta}) \right]$$

$$\text{s.t. (48) - (54), (58) - (62)}$$

$$\sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} (1 + \rho \zeta_{ij}) t_{ij}^0 X_{ijkv} + \sum_{i \in \mathcal{I}} T_{iv}(\boldsymbol{\zeta}) \leq T_{\max} \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, v \in \mathcal{V} \quad (63)$$

$$\sum_{v \in \mathcal{V}} (T_{iv}(\boldsymbol{\zeta}) - Z_{inv}(\boldsymbol{\zeta})) \leq u_{in} \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, i \in \mathcal{I}, n \in \mathcal{N} \quad (64)$$

$$T_{iv}(\boldsymbol{\zeta}) - T_{\max} \sum_{j \in \mathcal{I}_0} \sum_{k \in \mathcal{K}} X_{jikv} \leq 0 \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, i \in \mathcal{I}, v \in \mathcal{V}. \quad (65)$$

Tractable robust reformulation of (63) is derived by adopting the three step procedure of the RO paradigm (see Appendix B.4), and the tractable reformulation of [ARDP] is provided in Appendix B.5. Notice that [ARDP] is reduced to [RRDP] when the coefficients of decision rules are set to zero ($\boldsymbol{\gamma}, \boldsymbol{\theta} = \mathbf{0}$), and hence [ARDP] always yields an optimal solution that is at least as good as [RRDP].

Discussions on the optimality of LDRs. 1) Giving theoretical support to the optimality of LDRs may be possible under special conditions. E.g., LDRs yield the optimal decision rule when RLTs make point-to-point transportation to the affected sites; each RLT serves a single site under the given conditions mentioned before. When a point-to-point distribution is made, the optimal decision rule is equivalent to fully utilizing extra time realized at each road segment in the connected site, which is represented by a univariate LDR for each site, where the optimal γ coefficient is -1.2) For the general case, we conjecture that *piecewise linear decision rules* (PLDR)s are

optimal for [ARDP]. The related conjecture is inspired by the second property of the re-optimization approach that will be presented in the last subheading of the next subsection and the results given in [122]. Nevertheless, it is essential to point out that the number and locations of breakpoints cannot be known from the outset because they are affected by the parameter setting. Moreover, PLDR yields a computationally more challenging ARC than that with LDR. Therefore, PLDRs are often approximated by LDRs; for further details, we refer the reader to [123] and [124]. 3) Due to the reasons given in the second item, we cannot propose a theoretical bound on the optimality performances of LDR; instead, we offer a Monte Carlo simulation approach to yield safe simulation-based optimality bound of LDR from the best scenario-based decisions; for further details, we refer the reader to §3.5.3.

3.3.2 Pareto Efficient Reformulation and Re-optimization Method

Although [ARDP] is flexible in adjusting decisions according to the realized information of travel times, it is still a worst-case-oriented approach. More often than not, an adjustable robust reformulation yields alternative optimal decision rules for adjustable decision variables. Nevertheless, different decision rules that yield the same OFV for the worst-case travel time realizations may yield significantly different OFV performances for other data realizations, e.g., when the nominal data is realized. The commercial solvers such as GUROBI and CPLEX are indifferent in selecting one of the alternative optimal decision rules unless the Pareto efficiency of a decision rule is considered in the reformulation. The related Pareto efficiency issue has been diagnosed both in RO and ARO for practical operations research problems, including applications from production planning and machine scheduling as shown by [61], [62], [63] and [64]. The following toy-sized example illustrates a similar Pareto efficiency issue in our case.

Illustrative example. We use the toy-sized instance in Figure 5; the travel times

of both road segments (0,1) and (1,2) are assumed to reside in the box uncertainty set $[80, 120]$ and $T_{\max} = 325$. The parameters used in the utility functions of the two sites are given in Table 15.

Table 15: Parameters of utility functions

site	breakpoints		slope		
	u_{i1}	u_{i2}	α_{i1}	α_{i2}	α_{i3}
1	40	90	5	1	0
2	50	110	6	4	0

As before, the LDRs for the service time decisions of the two sites are: $T_1 = \bar{T}_1 + \gamma_{(0,1)}^1 t_{01}$ and $T_2 = \bar{T}_2 + \gamma_{(0,1)}^2 t_{01} + \gamma_{(1,2)}^2 t_{12}$. Now, suppose the coefficients are equivalent to $\gamma_{(0,1)}^1 = -0.25$, $\gamma_{(0,1)}^2 = -0.75$, and $\gamma_{(1,2)}^2 = -1$ and the intercepts are $\bar{T}_1 = 65$ and $\bar{T}_2 = 260$ in the decision rules, i.e., 25% of the extra time realized at arc (0,1) is used in the first site, and the remainder (75%) is used in the second site in the illustrative example. It is worth mentioning that since the second site yields a higher utility, the decision rule utilizes the residual time realized in (0,1) more in the second site. Notice that if the model fully uses the residual time in the second site, the service time might exceed the aid requirement of site two and brings no benefit given that the extra time realized at arc (1,2) is also fully used in the service time of the second site.

The associated decision rules are feasible for the time limit constraint:

$$t_{01} + t_{12} + \underbrace{\bar{T}_1 - 0.25t_{01}}_{T_1} + \underbrace{\bar{T}_2 - 0.75t_{01} - t_{12}}_{T_2} \leq T_{\max} \quad \forall t_{01}, t_{12} \in [80, 120],$$

and when the worst-case data is realized, i.e., $t_{01}^* = 120$ and $t_{12}^* = 120$, the decision rules yield the following service times:

$$T_1 = 65 - 0.25t_{01}^* = 35, \quad T_2 = 260 - 0.75t_{01}^* - t_{12}^* = 50.$$

Next, let us consider alternative LDRs with coefficients $\gamma_{(0,1)}^1 = -1$, $\gamma_{(0,1)}^2 = 0$, $\gamma_{(1,2)}^2 = -1$, $\bar{T}_1 = 155$, and $\bar{T}_2 = 170$ that yield the same service times ($T_1 = 35$, $T_2 = 50$) with the former LDRs when the worst-case data ($t_{01}^*, t_{12}^* = 120$) is realized. Nevertheless,

it may be easy to see that one should still prefer implementing the former LDRs because they perform better than the latter when the nominal data ($t_{01}, t_{12} = 100$) is realized, i.e., yielding about 7% improved utility (OFV=640) for that of the latter LDRs yielding OFV=595.

As shown in the illustrative example, alternative LDRs that yield the same performance for the worst-case data may yield significantly different performances when a non-worst-case data is realized. To avoid such cases, we propose a re-optimization method to find a Pareto efficient adjustable robust solution yielding improved performance when non-worst-case data is realized. The method first solves [RRDP] and records the optimal solution. Then, it re-optimizes the decision rule coefficients for the nominal data while yielding the optimality of the decision rule for the worst-case data in the second stage. The resulting mathematical optimization model [PR] that yields the Pareto adjustable robust optimal solution is presented below.

[PR] :

$$\max_{\mathbf{X}, \mathbf{T}, \mathbf{Z}^0, \gamma} \min_{\boldsymbol{\zeta} \in \mathcal{U}} \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left[\alpha_{i1} T_{iv}(\boldsymbol{\zeta}^0) - \sum_{n \in \mathcal{N}} (\alpha_{in} - \alpha_{i(n+1)}) Z_{inv}^0 \right]$$

$$\text{s.t. } [\text{ARDP}(\mathbf{X}^*)](59) - (62), (65)$$

$$\sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} (1 + \rho_{\zeta_{ij}}) t_{ij}^0 X_{ijkv}^* + \sum_{i \in \mathcal{I}} T_{iv}(\boldsymbol{\zeta}) \leq T_{\max} \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, v \in \mathcal{V} \quad (\text{re-63})$$

$$\sum_{v \in \mathcal{V}} (T_{iv}(\boldsymbol{\zeta}^0) - Z_{inv}^0) \leq u_{in} \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, i \in \mathcal{I}, n \in \mathcal{N} \quad (66)$$

$$T_{iv}(\boldsymbol{\zeta}) \geq T_{iv}^* \quad \forall \boldsymbol{\zeta} \in \mathcal{U}, i \in \mathcal{I}, v \in \mathcal{V}, \quad (67)$$

where $\boldsymbol{\zeta}^0 (= \mathbf{0})$ is the nominal realization of the uncertain parameter, $\mathbf{T}^*$ and $\mathbf{X}^*$ denote the optimal service time and routing decisions of [RRDP]; respectively, and analogous to $\mathbf{Z}_{inv}$, $\mathbf{Z}_{inv}^0$ denotes the time spent at site $i \in \mathcal{I}$ after each breakpoint u_{in} by vehicle $v \in \mathcal{V}$ but for the nominal data. In the Pareto efficient reformulation, we replace the worst-case OFV with the nominal since the goal is to find the LDRs

that yield the best nominal performance among the alternatives that yield the same worst-case performance. We include all constraints in [ARDP] (excluding (64)) to ensure robust feasibility of decision rules in [PR] when routing decisions are fixed to $\mathbf{X}^*$, i.e., the optimal routing decisions of [RRDP], which is denoted by notation $([\text{ARDP}(\mathbf{X}^*)](59)-(62),(65))$. Two new set of constraints (66) and (67) ensure the residual time ($\mathbf{Z}_{iv}^0$) calculations of the service times ($T_{iv}(\boldsymbol{\zeta}^0)$) for the nominal data $\boldsymbol{\zeta}^0$, and the optimality of decision rules when the worst-case data is realized, respectively. The explicit tractable ARC of the above-given semi-infinite model is presented in Appendix B.5.

Important properties of re-optimization approach. 1) Solving the adjustable reformulation in the first stage, and re-optimizing the decision rule coefficients for nominal data while constraining the worst-case OFV to be larger than equal to the optimal first-stage OFV [63, §5], may result in incorrect calculations of adjustable auxiliary variable $\mathbf{Z}$ for the nominal data due to suboptimality of LDRs (see Appendix B.6 for an illustrative example). Therefore, substituting LDRs for $\mathbf{Z}$ in [PR] may result in suboptimal decision rules for the service time decisions ($\mathbf{T}$) when the worst-case data is realized. The proposed [PR] reformulation resolves the associated issue via $\mathbf{Z}^0$, namely, while the robust optimality is ensured, LDRs yielding the best nominal performance for $\mathbf{T}(\boldsymbol{\zeta}^0)$ are selected. 2) [125] show that when an RC problem satisfies specific properties, i.e., given by Assumption 1 in the related paper, and the problem has constraint-wise uncertainty, then RC and ARC of a given model yield the same optimal OFV. It is essential to point out that [RRDP] satisfies the associated properties, and the uncertainty parameters are constraint-wise in (45). Hence, [RRDP] and [ARDP] yield the same optimal OFV for the worst-case data. It is also worthwhile to mention that solving [ARDP] is significantly more complex than [RRDP]. Ultimately, the proposed re-optimization approach enables us to solve a more efficient formulation [RRDP] in the first stage and decompose [ARDP] in the

latter stage so that the CPU time performance of solving the adjustable reformulation has improved. 3) More precisely, when routing decisions are known, the decision rule problem of each RLT may be solved individually. Hence we may decompose the decision rule problem in the second phase of the aforementioned re-optimization approach. As we have mentioned before, exploiting the associated structure will lead us to decrease the computational complexity of solving [PR].

3.4 Heuristic Algorithms

[RRDP], [ARDP], and [PR] include computationally expensive MILP problems that cannot be solved to optimality for realistically-sized instances because they require a large amount of memory or the commercial solver terminates with more than %50 optimality gap within the given time limit, i.e., adjusted between six to twelve hours in the numerical experiments.

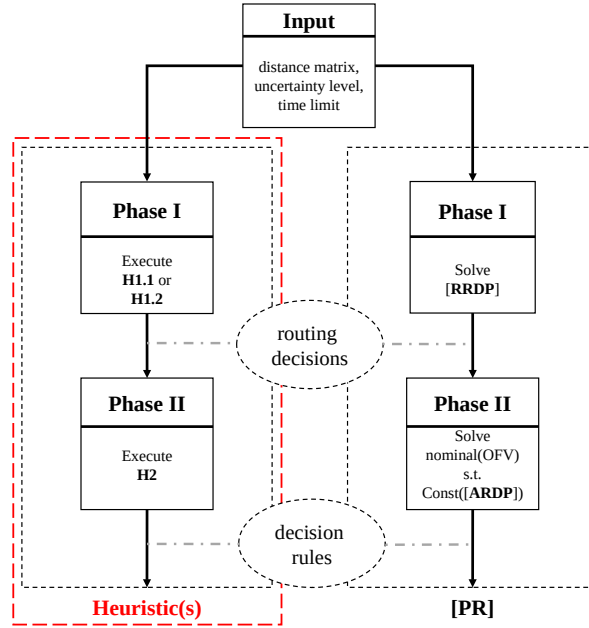


Figure 6: The flow chart of the proposed solution approaches

In this section, we propose heuristic algorithms that yield near-optimal solutions while improving CPU time performance considering the challenges mentioned earlier.

The proposed algorithms deal with the considered problem in two phases; in Phase I, we solve the routing problem and select the sites to be visited and the service time decisions for the worst-case data as in [RRDP]. In Phase II, we find near-optimal decision rules for the wait-and-see service time variables in [ARDP] for a given route. We propose two algorithms for Phase I, namely, two clustering-based heuristics (H1.1, H1.2), and a dedicated heuristic algorithm (H2) to find decision rule coefficients in Phase II. The solution approaches are represented using a flow chart in Figure 6.

In the remainder, we provide the heuristic algorithms for Phase I in §3.4.1 and §3.4.2; and we present the heuristic algorithm for optimizing the decision rule coefficients in §3.4.3.

3.4.1 Clustering-based Heuristic Approach for Routing (H1.1)

In this section, we propose a clustering-based algorithm also denoted by H1.1 that adopts clustering in Step 1, calculates the utility of each cluster in Step 2, and chooses the best solution among the candidates in Step 3. The inputs of the algorithm are the total number of sites (I), the total number of RLTs (V), the time limit ($T_{\max}$), distance matrix, the slope of the segments in the utility function of site i (α_i), the breakpoints of the time segments in the utility function of each site i (u_i), and the uncertainty level (ρ); and the outputs are the routes, service times, and utilities of chosen routes for RLTs. The steps of the algorithm are as follows:

[Step 1] Iterative cluster generation. We generate different routing patterns for RLTs by using an iterative clustering algorithm. In each iteration, a random point is chosen on the map as the base point, and a probability is assigned for each site with respect to its distance from the base point. The sites on each cluster are then chosen using a roulette wheel selection approach. Since we seek to generate unique clusters when a repetitive cluster is generated, such a generation is denoted as a failure. The associated clustering approach and the probability calculation used

in the roulette wheel selection are inspired by [126]. It should be noted that we limit the upper bound for the number of sites in each cluster to $(\frac{I}{V} + 2)$ and the number of unsuccessful trials used in terminating the cluster generation to $25000 \times V$. We refer the reader to Appendix B.2 for further details on the selected parameter setting.

[Step 2] Determining the total utility gained by each cluster. After the clusters are generated, we calculate the utility (ν) gained by RLT(s) solving [RRDP] for each cluster.

[Step 3] Determining the best solution. By solving a set packing problem, we select high utility clusters among the ones generated in Step 1. We use Δ to denote the set of clusters and ν_δ to denote the total utility of cluster δ , which is calculated in Step 2. We define the binary parameter $\psi_{i\delta}$ which takes value 1 if site i is visited in cluster δ ; 0, otherwise. Moreover, we use the following decision variable:

$$H_\delta = \begin{cases} 1, & \text{if cluster } \delta \text{ is selected} \\ 0, & \text{otherwise} \end{cases}, \quad \delta \in \Delta$$

to determine the selected clusters, and the resulting set packing problem [SPP] is given as the following.

[SPP] :

$$\begin{aligned} \max_{\mathbf{H}} \quad & \sum_{\delta \in \Delta} \nu_\delta H_\delta \\ \text{s.t.} \quad & \sum_{\delta \in \Delta} H_\delta \leq V \end{aligned} \tag{68}$$

$$\sum_{\delta \in \Delta} \psi_{i\delta} H_\delta \leq 1 \quad \forall i \in \mathcal{I} \tag{69}$$

$$\mathbf{H} \in \{0, 1\}, \tag{70}$$

where the objective is to maximize the total utility; constraints (68) ensure that the number of selected clusters must be at most equal to the number of vehicles; constraints (69) guarantee that each site is at most included in one of the selected clusters; and constraints (70) are 0-1 restrictions.

3.4.2 Clustering-based Heuristic Approach Utilizing Lower Bounds (H1.2)

In this subsection, we propose a heuristic algorithm that modifies Step 2 of H1.1 by proposing a lower bound for each cluster instead of solving [RRDP].

The new algorithm is denoted as H1.2 in the remainder and is designed to decrease the runtime of H1.1. In this algorithm, after the clusters are generated, we calculate a lower bound for the total utility gained by RLT(s) using an efficient constructive algorithm that sorts the sites in the clusters according to a weighted function that considers the distances of nodes from the lately visited site (depot in the beginning of the algorithm) and their slope in the first segment of piecewise function. The nodes are added to the set of visited sites in every step while checking the feasibility of the solution (not violating $T_{\max}$). The modified version of Step 2 utilized in the algorithm is given Appendix B.7.

Lastly, it must be noted that in H1.2, there must be a post-processing stage as the algorithm utilizes a lower bound for each cluster and solves [SPP] accordingly; therefore, we solve [RRDP] for all the selected clusters after Step 3 to yield the exact utility of the selected routes, and then the algorithm is terminated.

3.4.3 Heuristic Algorithm to Find Decision Rule Coefficients (H2)

This subsection proposes a polynomial-time heuristic algorithm that yields near-optimal solutions in reasonable CPU times without using a commercial solver. The inputs of the algorithm are the given route ($\mathbf{R}$), the robust service time decisions ($\mathbf{T}$), distance matrix, the slope of the segments in the utility function of each site (α_i), the upper bound of the time segments in the utility function of each site ($\mathbf{u}_i$), and the uncertainty level (ρ); and the outputs are the decision rule coefficients ($\bar{\mathbf{T}}, \gamma$). The algorithm mimics the properties of the coefficients in the optimal LDRs. More

precisely, the LDR of a given site i may be reformulated as follows:

$$T_i(\zeta) = \bar{T}_i + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \gamma_{i'j}^i (t_{i'j}^* - \underbrace{(1 + \rho)\zeta_{i'j} t_{i'j}^0}_{t_{i'j}}) \quad \forall i \in \mathcal{I},$$

where $t_{i'j}^*$ denotes the worst-case travel time realization for edge (i', j) , $\gamma_{i'j}^i$ determines the percentage of extra time that is utilized at site i when the actual travel time of edge (i', j) is less than the worst-case. Therefore, $\gamma_{i'j}^i$ takes values between 0 and -1 since the algorithm uses the LDR representation in (14), and the extra time realized at each edge must be fully utilized as additional service time since our goal is to maximize the total service time. Ultimately, the associated relationship inspired by the optimal LDR structure is exploited in the proposed heuristic, and the algorithm can be applied for each RLT separately since LDR used in each RLT's route may be individually analyzed. Hence H2 does not use the vehicle index v . Moreover, the order index k is not used because the route information is given as an input, and the orders of arcs are known. The notation and the main steps of the proposed algorithm are given in Appendix B.8.

Notice that H2 is used in conjunction with a Phase I (H1.1 or H1.2) algorithm to yield a solution for [ARDP]. Like in [PR], the related heuristic approach also aims to find the adjustable robust solution that yields the best nominal performance.

3.5 Numerical Experiments

In this section, we present the results of the extensive numerical experiments designed to test the performances of the proposed algorithms. In §3.5.1, the complete heuristic algorithms that include both phases of the problem are tested via synthetic instances. In §3.5.2, we compare OFV performances of [RRDP] and [ARDP]. In §3.5.3, we evaluate the optimality performance of LDRs using a Monte Carlo simulation. Finally, we present the Van Earthquake case study in §3.5.4 using the state-of-the-art heuristic

algorithm. We compare OFV and CPU time performances of the proposed algorithms for Phase I and II in Appendix B.9; these results are based on the individual performances of the proposed algorithms in the related phase of ARDP.

Computer specifications. The computational experiments are carried out on a 64-bit Windows server with AMD EPYC 2.00 GHz CPU, and 32 GB RAM. The algorithms are implemented using Python and CPLEX 20.1 with Concert Technology.

Data set and parameter generation. The instance used in the case study consists of 94 (including DCC) sites and is associated with an earthquake disaster that occurred in 2011 (October, 23) in Van Province; the same data set is used in other studies such as [127] and [119]. We will discuss the Van earthquake data set more in the case study (§3.5.4). We also use the associated data to generate the test instances presented in the sequel. More precisely, in lower-dimensional instances, we randomly sample sites from the given network, and use the parameter setting of the selected sites calculated for the case study. The nominal travel times are generated according to the proximity of the sites from the epicenter as in [119]. To generate the parameters used in the utility functions, we have used the data generation scheme that is described in Section 3. The uncertainty in travel times, namely, the lower and upper bounds of the travel time interval for each edge, are generated according to the given uncertainty level $\rho \in \{0.25, 0.35, 0.45\}$ and the related nominal travel time. The breakpoints of the utility function for each site in the case study are calculated by using equations (44), and we set $\beta_1=8$ such that the smallest u_{i1} take the value of five minutes (for the population with size 112) and $\beta_2=5$ is set as a relatively large value. Finally, we calculate the final score R_i for each site i using equation (43) and Appendix B.2 where $\sigma_1 = 0.9$ and $\sigma_2 = 0.1$ are the associated weights for distance and population scores, respectively. The proximity to the epicenter is weighted more in the numerical experiments since it is more important in determining an earthquake's severity. The data set that shows the proximity of sites to the epicenter, the populations of the

sites, and priority scores of the sites are available online (see RRDP Data Set on page www.ihsanyanikoglu.com/data-sets). Summary statistics of the data set used in our numerical experiments are given in Appendix B.2.

Remark 1 *It is important to point out that a decision-maker may implement a different generation scheme to score the importance of sites based on the geographical and demographical requirements of the affected area and the community group. Our optimization models and solution algorithms may be similarly adapted for different utility schemes as long as the piecewise linear structure of the utility function holds.*

Instance size. In the remainder, we may refer to the size of an instance as “small”, “medium” or “large”. The related classification is presented in Table 16. The number of RLTs (V) is obtained by setting the $\frac{I}{V}$ ratio to $\{5, 10\}$ (rounded to nearest integer) in all the numerical experiments.

Table 16: Instance size classification

I	V	Size
6-9	1	small
10	1,2	
20	2,4	medium
40	4,8	
60	6,12	large
80	8,16	
94	10,19	case study

Monte Carlo simulation. We adopt Monte Carlo simulation experiments to compare the average optimality performances of the proposed algorithms and evaluate the quality of the adjustable robust approach. To do that, we randomly sample five networks from the Van data set for each instance size and record the optimal decision rule coefficients of [ARDP] for each network. Next, we randomly sample 1000 data sets for the travel times for each network and report the average OFV performance of the [ARDP] solution over the sampled data. Finally, we report the average performance over five networks. The same approach is also used in testing the benchmark approaches, CPU time performances, and the feasibility of the time limit constraint in out-of-sample tests.

3.5.1 Evaluating the Performances of Heuristic Algorithms

This subsection compares the OFV and CPU time performances of exact and heuristic solution algorithms proposed to solve [ARDP]. Table 17 provides an overview of all the algorithms used in the numerical experiments. It is essential to point out that we have four algorithms to solve the problem at hand, consisting of one exact three heuristic algorithms. As the exact algorithm, we use EM that solves [RRDP] in Phase I and then solves the re-optimization problem [PR] in Phase II for each route separately. Notice that we decompose [PR] and solve the decision rule problem of each RLT separately, as it is described in Section 3.3. The first heuristic algorithm (HA1)

Table 17: Algorithms

Algorithm		Phase I	Phase II
ID	Type		
EM	Exact	[RRDP]	[PR]
HA1	Heuristic	[RRDP]	H2
HA2	Heuristic	H1.1	H2
HA3	Heuristic	H1.2	H2

solves [RRDP] in Phase I and then applies the decision rule algorithm H2 for Phase II. The second heuristic algorithm HA2 uses the clustering-based heuristic approach H1.1 to solve Phase I and applies the decision rule algorithm H2 to solve Phase II. Finally, the third heuristic HA3 utilizes the clustering-based heuristic approach with lower bounds (H1.2) and decision rule algorithm H2 to solve Phase I and Phase II, respectively.

The numerical results and algorithm comparison are presented in Table 18. The first three columns represent the instance sizes, and the OFV performance of the algorithms is presented in columns four to seven. CPU times are presented in columns eight to eleven. Notice that Phase I of EM and HA1 is solved by [RRDP], and the optimality gap is reported for instances that cannot be solved in the given time limit (6 hours for medium-sized instances, 12 hours for large-sized instances) in the last column.

Table 18: Performances of the complete heuristic algorithms

Size	Instance		OFV performance				CPU				[RRDP] gap %
	<i>I</i>	<i>V</i>	EM	HA1	HA2	HA3	EM	HA1	HA2	HA3	
small	6	1	*	*	*	*	2	2	16	11	0%
	7	1	*	*	*	*	2	2	24	12	0%
	8	1	*	*	*	*	2	2	35	13	0%
	9	1	*	*	*	*	2	2	56	14	0%
	10	1	*	*	*	*	4	3	84	15	0%
		2	*	*	*	1%	3	3	69	26	0%
medium	20	2	*	*	*	1%	1189	1182	762	42	0%
		4	*	*	*	4%	21608	21604	231	75	2%
	40	4	1%	1%	*	4%	21682	21683	2280	238	7%
		8	8%	8%	*	6%	21771	21770	668	445	13%
large	60	6	15%	15%	*	5%	44583	44580	5861	469	28%
		12	43%	43%	*	11%	46903	46901	1283	865	56%
	80	8	25%	25%	*	3%	46906	46899	6313	808	50%
		16	m	m	*	7%	m	m	1980	1488	-

(*) denotes the best OFV performance; (m) denotes the memory error

The numerical results show that EM and HA1 are applicable for the instances with less than 40 sites, and the optimality gap gets larger as the number of the sites increases and the instances ($I=80, V=16$) are not solved because of memory error. In other words, for the instances up to 20 sites, optimal solutions are obtained, while for instances with more than 20 sites, the results are obtained with optimality gaps. However, HA2 not only can solve all the instances with different sizes, but it also outperforms the other algorithms when the number of sites is larger than 10. Finally, HA3 can solve the small instances without optimality gap except for ($I=10, V=2$), and by increasing the instance size, it losses on average 3% in the OFV. Regarding the CPU times, comparing the performance of algorithms shows that even though EM and HA1 perform better for instances up to 10 sites, HA2 and HA3 can obtain solutions more efficiently for medium and large-sized instances. As EM and HA1 fail to solve the instances ($I=80, V=16$), the CPU times are not represented. All in all, comparing the results shows that HA2 is the most efficient algorithm since it yields a better OFV than other algorithms and results in practicable CPU times less than two hours. It is worth mentioning that HA3 yields solutions in less than half an hour but losses in OFV (on average 3%), i.e., one may also accept the loss in utility function for the sake of CPU time performance according to the situation in a post-disaster

setting.

3.5.2 Comparison of Adjustable Robust versus Static Robust Approaches

In this subsection, we aim to show the advantages of applying ARO and present how adopting the adjustable approach may increase the average OFV performance of the static robust approach. Table 19 summarizes the result where the first three columns show the instance size, the columns from four to six show OFV and the total service time improvements of the adjustable robust approach for the static robust, respectively. We refer the reader to Appendix B.9 that presents the replication of the same experiment for out-of-sample data.

Table 19: Comparison of adjustable robust versus static robust solutions

Size	Instance		$\frac{\text{OFV}(\text{adjustable}) - \text{OFV}(\text{robust})}{\text{OFV}(\text{robust})}$	$\frac{\text{TS}(\text{adjustable}) - \text{TS}(\text{robust})}{\text{TS}(\text{robust})}$	TS improvement (hr)
	I	V			
small	6	1	4.9%	7.4%	0.42
	7	1	4.7%	7.5%	0.42
	8	1	4.3%	6.2%	0.37
	9	1	4.2%	4.7%	0.28
	10	1	4.2%	6.1%	0.37
medium		2	4.0%	4.7%	0.49
	20	2	3.9%	5.3%	0.52
		4	3.6%	4.3%	1.13
	40	4	3.3%	4.5%	1.16
		8	3.7%	4.7%	2.11
large	60	6	3.5%	4.8%	1.81
		12	3.3%	4.3%	3.21
	80	8	3.8%	5.7%	3.45
		16	3.6%	4.9%	8.80
Average			3.9%	5.3%	

OFV: Objective Function Value; $\text{TS (Total Service time)} = \sum_{i \in I} \sum_{v \in V} T_{iv}$

The numerical results show that by applying ARO (and re-optimizing the decision rule coefficients via [PR]), the OFV is improved by (on average) 3.8% where the total service time is improved by (on average) 5.3%. We would like to mention that the associated OFV improvement may get as high as 4.9% for $(I=6, V=1)$ and the largest total service time improvement is 7.5% for $(I=7, V=1)$. The improvement of the total service time can get as high as 8.8 hours overall RLTs when $(I=80, V=16)$.

3.5.3 Optimality performance of LDRs

In this subsection, we assess the optimality performances of LDRs using a Monte Carlo simulation experiment. The routing decisions are given, and the service decisions of LDRs are calculated for the sampled travel times using the coefficients that come out of H2. A *scenario-based model* (SbM) is used as a benchmark; SbM is equivalent to solving [RRDP] when the sampled travel times (t_{ij}^s) substitute for $(1 + \rho)t_{ij}^0$ and the routing decisions ($\mathbf{X}$) are fixed; therefore, the resulting problem is an LP. SbM is resolved for each sampled data s of the simulation experiment. Notice that SbM works with complete information, i.e., the travel times do not unfold over time, and yields a specific solution (T_i^s) for each scenario realization s . Therefore, SbM yields a safe upper bound on the optimal OFV of the ARO approach. Nevertheless, it is worthwhile to mention that SbM cannot be adopted in practice because the actual values of travel times cannot be known at the beginning of the planning horizon, and there are infinitely many scenarios to be optimized. Therefore, SbM is utilized for yielding safe simulation-based optimality bounds on the adjustable robust approach in this experiment. Ultimately, our goal is to present the optimality performance of LDRs. To this end, the total utility and service time performances of the SbM and LDR approaches are compared over the sampled data in Table 20.

Table 20: Optimality performance of LDRs

route length	sampling	OFV(SbM)−OFV(adjustable) OFV(adjustable)			TS(SbM)−TS(adjustable) TS(adjustable)		
		$\rho = 0.25$	$\rho = 0.35$	$\rho = 0.45$	$\rho = 0.25$	$\rho = 0.35$	$\rho = 0.45$
6	$\mathbb{U}(-\rho, \rho)$	2.6%	3.6%	4.3%	4.1%	5.5%	6.9%
7		2.1%	2.8%	4.3%	3.2%	4.3%	6.7%
8		1.6%	2.5%	3.3%	2.7%	4.1%	5.4%
9		2.1%	2.8%	3.1%	3.7%	4.8%	5.3%
10		1.7%	2.4%	2.9%	2.9%	3.9%	4.7%
average		2.0%	2.8%	3.6%	3.3%	4.5%	5.8%
6	$\mathbb{T}(-\rho, \rho, \frac{\rho}{2})$	2.3%	3.2%	3.7%	3.6%	5.0%	6.2%
7		1.7%	2.7%	3.9%	2.7%	4.0%	6.0%
8		1.3%	2.1%	2.6%	2.0%	3.3%	4.2%
9		1.8%	2.3%	2.6%	3.1%	3.9%	4.5%
10		1.4%	2.2%	2.5%	2.3%	3.5%	4.1%
average		1.7%	2.5%	3.1%	2.7%	3.9%	5.0%

The first column of the table denotes the route length of an RLT. The second

column gives the sampling information where $\mathbb{U}(-\rho, \rho)$ denotes a uniform distribution between $[-\rho, \rho]$ and $\mathbb{T}(-\rho, \rho, \frac{\rho}{2})$ denotes a triangular distribution with bounds $-\rho$ and ρ , and mode $\frac{\rho}{2}$. The rest of the columns show the average improvement made over the utility and service time metrics of the adjustable approach by SbM. The numerical results validate close to optimal performance of LDRs as the average optimality gap of LDR for SbM is less than 3.6% (see the “average” rows in Table 20).

3.5.4 Case Study

In October 2011, an earthquake with a Richter scale magnitude of 7.2 hit the Van Province of Turkey and damaged 60% of the housing, caused 644 fatalities and 4152 casualties, and left about 200,000 people in need. In this subsection, we evaluate the performances of the proposed algorithms over a realistic data set taken from the related natural disaster. In the case study, we have focused on 93 ($=I$) sites in Van that suffered from the earthquake, and the number of RLTs is set to $V \in \{10, 19\}$; the uncertainty level to $\rho \in \{0.25, 0.35, 0.45\}$; the time limit to $T_{\max} \in \{8, 12\}$. We have generated uncertain travel times and utility functions using the data generation process described in §3.3.

Comparison of proposed approaches. In this subsection, we discuss the OFV and CPU time performance of HA2 and HA3 algorithms. Notice that EM and HA1 algorithms cannot solve the corresponding instances because [RRDP] exceeds the memory limits of the server. Therefore, two state-of-the-art approaches adopted in the case study are HA2 and HA3 algorithms. The numerical results show that HA2 outperforms the OFV performance of HA3 (on average) by 4%. Regarding the CPU time, HA3 solves all instances in less than 1075 seconds and performs better than HA2, translating into (on average) 3100 seconds improvement for HA2. Adopting both algorithms is applicable in practice; however, the decision-maker must consider the trade-off between the OFV and CPU time performances of HA2 and HA3 to decide

on the final approach to be implemented. More precisely, a humanitarian organization must use HA3 if the relief distribution plan is required in less than an hour for a region that contains 90–100 sites; nevertheless, we suggest implementing HA2 because the associated algorithm outperforms the OFV performance of the HA3 up to 8% and yields practicable CPU times. We refer the reader to the final subheading of Appendix B.9 for the complete numerical results.

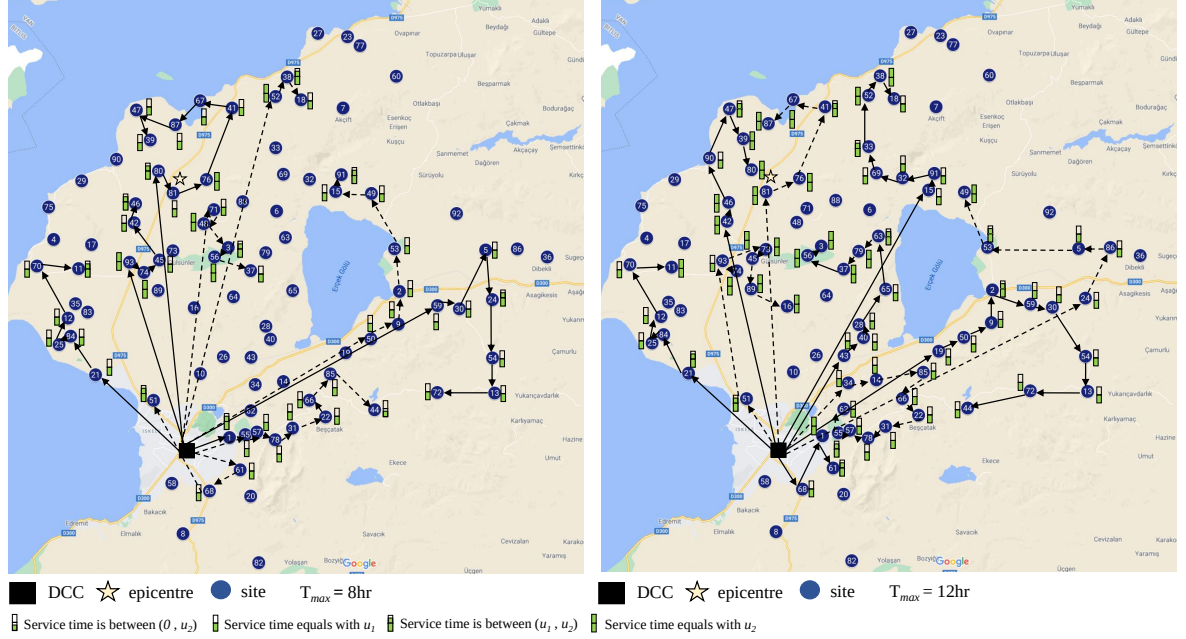
Comparison of adjustable robust and static robust approaches. The numerical results in Table 21 show that the OFV improvement made by the adjustable robust approach for the static robust is between 2.0 to 4.8% while the grand average is 2.9%. Furthermore, the total service time is increased by (on average) 4.5% by adopting the adjustable robust approach. The last row of the table shows that the OFV improvement increases from (on average) 2.3% to 3.4% and the total service time improvement increases from 3.4% to 5.7% when the uncertainty level increases from 25% to 45%. What is more striking is that the total service time improvement ranges from 2.8 to 7.2 hours overall RLTs (see columns 9 through 11) when the adjustable approach is used, which are significant improvements considering the importance of every minute for distributing more humanitarian relief supplies. Similar to the previous results, the total service time improvement becomes more significant as the uncertainty level increases. Lastly, as the number of RLTs decreases, the relative importance of utilizing RLTs better increases since we aim to serve the population with fewer teams, which is also reflected in the numerical results because the algorithm yields improved solutions when the number of RLTs decreases from 19 to 10. Ultimately, the numerical results validate that the humanitarian organization benefits from ARO by utilizing their limited resources much more efficiently using the information of travel times realized over time.

We present the RLT routes generated by HA2 in Figure 3.5.4 for the case study with the parameter setting: $\rho = 0.25, V = 10$ and $T_{\max} = \{8, 12\}$; the routes of

Table 21: Comparison of adjustable robust versus static robust approaches

$T_{\max}$	V	$\frac{\text{OFV}[\text{ARDP}] - \text{OFV}[\text{RRDP}]}{\text{OFV}[\text{RRDP}]}$			$\frac{\text{TS}[\text{ARDP}] - \text{TS}[\text{RRDP}]}{\text{TS}[\text{RRDP}]}$			TS improvement (hr)		
		$\rho = 0.25$	$\rho = 0.35$	$\rho = 0.45$	$\rho = 0.25$	$\rho = 0.35$	$\rho = 0.45$	$\rho = 0.25$	$\rho = 0.35$	$\rho = 0.45$
8 hr	10	3.0%	4.0%	4.8%	4.8%	6.3%	8.0%	2.8	4.0	5.0
	19	2.0%	2.8%	3.8%	3.0%	4.2%	5.9%	3.8	5.2	7.2
12 hr	10	2.2%	2.8%	2.9%	3.5%	4.5%	5.5%	3.5	4.4	5.3
	19	2.0%	2.0%	2.3%	2.4%	2.6%	3.3%	4.6	4.8	6.3
Average		2.3%	2.9%	3.4%	3.4%	4.4%	5.7%			

OFV: objective function value; TS: total service time = $\sum_{i \in I} \sum_{v \in V} T_{iv}$



Solution representation for $(\rho = 0.25, V = 10)$

RLTs are presented by directed arcs and the service time of each site is presented by the closest bar chart to the associated site in the figure. As it may be anticipated, when the time limit increases from 8 to 12 hours, RLTs visit more sites, i.e., 53 versus 69 sites, respectively, and distribute more humanitarian aid packages to the affected area. The numerical results show that the proposed solution algorithm and the data generation process yield high-quality solutions for the case study since the distributions plans presented below visit all the critical sites at the periphery of the epicenter, the critical needs of the visited sites are prioritized, and more than 56% and 73% of the sites are visited for the given time limits 8 and 12 hours, respectively.

Equity. To measure whether the proposed state-of-the-art method HA2 and HA3 yield an equitable relief supply distribution or not, we calculate the Gini coefficients [128] of the proposed solutions for the case study. The Gini coefficient takes values between zero and one, where one shows the maximum inequality. We set the performance metric (x_i) used in the Gini coefficient calculation to the percentage of critical relief supply requirement satisfied for site i , where $x_i = 1$ denotes the critical supply requirement of the site is fully satisfied (i.e., $T_{iv} \geq u_{i1}$), and $x_i = 0$ denotes no service is provided; the related metric is also used in [74] as one of the main attributes to measure equity. Table 22 presents the Gini coefficients for instances of [ARDP] with $\rho = 0.45$, (the highest uncertainty level) solved by HA2 and HA3.

Table 22: Equity of relief supply distribution – Gini coefficient ($\rho = 0.45$)

$T_{\max}$	V	ARDP	
		HA2	HA3
8	10	0.46	0.48
	19	0.25	0.26
12	10	0.31	0.29
	19	0.12	0.15

Notice that in Table 22, we base our results on all the sites in the affected region; as anticipated, the sites which cannot be served due to the given time limit constraint are also included in the Gini coefficient calculations. The numerical results in Table 22 show that the proposed solution method HA2 yields equitable relief supply distribution since the related Gini coefficient ranges between 0.12 and 0.46 (e.g., if we consider the Gini coefficient of the world income distribution as a benchmark, even the country with the minimum inequality could not archive a Gini score less than 0.24 in the last 25 years). Moreover, as the number of RLTs and $T_{\max}$ increase, more sites are visited, and hence more critical relief supplies are distributed; therefore, we obtain the lowest Gini coefficient of 0.12 when $V = 19$ and $T_{\max} = 12$; and the highest Gini coefficient 0.458 at $V = 10$ and $T_{\max} = 8$ for HA2. It is worth mentioning that when the related Gini coefficients are calculated among only the visited sites, the proposed solutions yield Gini coefficients that are less than 0.1 (which may be considered

as almost perfect equality) even for $V = 10$ and $T_{\max} = 8$. Even though the time limit constraint is a significant barrier to equitable distribution, we can conclude that the proposed reformulation and the solution method achieve equitable relief supply distribution according to the numerical results presented in this subsection. We have also calculated and reported the Gini coefficient for solutions yielded by HA3 and the numerical results show that the coefficient ranges between 0.15 and 0.48, and the two approaches produce almost similar results.

3.6 Conclusion and Managerial Implications

This paper adapted the adjustable robust optimization paradigm to gain resilience against travel time uncertainty in a humanitarian relief distribution problem. The numerical results showed the effectiveness of the proposed model and the solution approach. This section discusses how humanitarian organizations can benefit from the presented research.

3.6.1 Effects of Number of RLTs and Service Time Limit

After a calamity hits, it is essential to dispatch RLTs as soon as possible. One of the major decisions to make is determining the number of RLTs beforehand to achieve an equitable distribution of aid. As it may be anticipated, whenever the number of RLTs increases, more sites are visited, and more utility is gained, which results in improved OFV performance and more equitable distribution. The numerical results in the case study showed that as the number of RLTs increased from 10 to 19, the Gini scores achieved by the proposed approach improved significantly; an average Gini score that is as low as 0.12 was achieved for $V = 19$ and $T_{\max} = 12$. Therefore, it is clear that increasing the number of RLTs will have a positive impact on humanitarian logistics; however, humanitarian organizations are often compelled to work with a limited number of RLTs in real life due to the highly uncertain nature of a disaster and the difficulties in dispatching operations. In such cases, the benefits of implementing

the adjustable robust approach were even more profound. The numerical results showed that the improvement made by the adjustable approach over the static robust increased more than 40% both in OFV and the total service time when the number of RLTs decreased from 19 to 10.

The lesson learned from the practice shows that ignoring the importance of optimization in post-disaster distribution planning may cause significant damage and casualties for the affected communities due to the time limit. The numerical results of the case study showed that RLTs visited all the critical sites at the periphery of the epicenter, and the critical needs of the visited sites were prioritized. Moreover, the percentage of sites that received the critical service increased from 56% to 73% as the service time limit increased from 8 to 12 hours. On the other hand, similar to the effect of the number of RLTs, the relative importance of the adjustable approach increased as the service time limit decreased from 12 to 8 hours. Lastly, The highest OFV and the total service time improvements for the robust approach were achieved when $T_{\max} = 8$ and $V = 10$, where the resources were more scarce, and the need for adjustability became more important. Therefore, humanitarian organizations must aim for adjustable robust approaches since the time limit is often a hard constraint.

3.6.2 Effect of Uncertainty Level

As debris and gridlock traffic affect the dispatch of teams in the affected areas, the response phase is subject to travel time uncertainty. RLTs can use the proposed approach to adjust their service decisions based on information gained during the process. Besides being robust for the worst-case scenario of travel times, our solutions continue to be more flexible and less conservative than those of static robust optimization. The case study results showed that when the uncertainty level increased from 25% ($\rho = 0.25$) to 45%, the (average) OFV improvement of the adjustable approach over the robust increased from 2.3% to 3.4%, and the total service time improvement

increased from 3.4% to 5.7%, respectively. As it may be anticipated, when the uncertainty level increases, the cost of being robust for the worst-case increases; therefore, the resilience of the adjustable robust approach pays off in highly uncertain settings like humanitarian logistics.

3.6.3 Advantages of Adjustable Decision Making under Uncertainty

It is challenging to assess the status of the transportation routes and the effects of a disaster in different regions in the response phase immediately due to delays in the communication systems. RLTs get more information about the actual situation as they visit sites. Therefore, the dynamic nature of the problem must be taken into consideration, and RLTs must be flexible in adjusting their decisions according to the information realized during the process. Employing LDRs from the Pareto efficient ARO approach helps RLTs adjust their decisions based on the realized data. The numerical results showed that the ARO approach improved the total service time, which improved more than (on average) 4.5 hours of service time over the static RO approach in the case study instance. RLTs may easily interpret what percentages of the realized residual time at each visit will be utilized as extra service times in the following sites by checking the decision rule coefficients. The coefficients are known at the beginning of the planning horizon and may be efficiently passed to RLTs as a guideline before the teams leave DCC or via remote team management tools.

In *future research*, our goal is to consider the adjustability of binary decision variables associated with site selections according to the realized travel times. We also aim to extend our approach via distributionally robust optimization to yield probabilistic guarantees for the selected distribution plan. Lastly, we aim to further investigate the structure of optimal decision rules for the related problem.

CHAPTER IV

AN OPTIMIZATION APPROACH FOR ELECTRIC BUS SCHEDULING UNDER TRAVEL TIME AND ENERGY CONSUMPTION UNCERTAINTY

The public transportation system is undergoing a significant transformation due to the rapid growth of electromobility infrastructure and operations. This transformation is expected to significantly contribute to the decarbonization and sustainability of the environment. One of the most critical planning problems in this context is the optimization of electric fleet scheduling and charging operations, which has recently gained the attention of researchers. In this paper, we investigate the scheduling problem of electric fleets under travel time and energy consumption uncertainty. We propose a new mixed-integer linear programming model to optimize electric fleet purchasing and charging operation costs. In this study, robust optimization is used to address uncertainty, and a novel uncertainty set is introduced to control the robustness of the solution. It determines the number of buses required to cover all trips, yields the schedule of the trips, and creates bus charging plans. The efficiency of the model is evaluated through an extensive Monte Carlo simulation. In addition, we conduct a case study on the off-campus college transport network at Binghamton University to study the real-world applicability of the model. The case study results show that ignoring the uncertainty of parameters can lead to impractical plans, while the robust solution is resilient against the uncertainty even though it may impose some extra cost. In this study, we sought to provide a plan that would enable a decision-maker to make trade-offs by considering both the costs and benefits of robust optimization. Decision-makers can use this information to determine the most appropriate approach

in their specific circumstances.

4.1 *Introduction*

Air pollution and greenhouse gas emissions from public and non-public vehicles have always been a concern for governments and scholars. The *world health organization* (WHO) estimates that about 7 million people lose their lives every year due to exposure to air pollution, and these pollutants affect 99% of the world's population [129]. According to the European environment agency, 30% of the total greenhouse gas emissions in the *European union* (EU) is caused by transportation, and it consumes almost two-thirds of the total oil used in the EU [130]. The presence of smog in industrial cities and air pollution caused by excessive consumption of fossil fuels increase the death rate from diseases such as stroke, heart disease, chronic obstructive pulmonary disease, lung cancer, and acute respiratory infections. This high level of conflict with diseases caused by air pollution imposes costs on the public health of society and exorbitant medical costs for families and health systems. To mitigate the negative impacts of transportation on public health and the environment, governments and policymakers have been promoting sustainable transportation solutions. One such solution that has gained popularity in recent years is the adoption of electric buses (EBs). EBs have become an important means to increase the quality of the transportation fleet, reduce air pollution, and decrease fossil fuel consumption. In June 2022, the European Parliament voted to ban the sale of new *combustion engine vehicles* (CEV) from 2035. This order will mean a ban on the sale of these cars in 27 members of the EU. The legislators also approved a 55% reduction in carbon dioxide emissions from cars by 2030 compared to 2021 [131].

The public transportation system is required to contribute to decreasing greenhouse gas emissions and using fossil fuels. Governments are planning to replace CEVs with electric vehicles that are powered by renewable energy sources [132]. China has

already deployed over 420,000 EBs as of May 2020, which accounts for 99% of the global EB fleet and 60% of China's transit bus fleet. Meanwhile, in the United States, 3,364 EBs have been deployed as of 2021, representing approximately 5% of the transit bus fleet of the country [133]. Electric vehicles boast lower running and operating costs offering significant fuel savings compared to their combustion counterparts [134]. While there are many advantages of using EBs, the expansion of the electric-based transportation system has challenges. The growth of EBs has been hindered by several drawbacks, including the higher cost of EBs compared to CEVs, the lack of charging infrastructure, and the limited range of EBs. Additionally, the availability of materials used to make batteries is limited, and the production and disposal or recycling of batteries may cause additional challenges. These issues highlight the importance of optimizing the usage of EBs, such as through careful planning of fleet size, scheduling, and charging plans. While EBs have the potential to contribute significantly to reducing greenhouse gas emissions and promoting sustainable transportation, it is critical to address these challenges to ensure the efficient and effective implementation of EBs in public transportation systems.

In order to facilitate the use of EBs, it is necessary to provide adequate charging facilities for EBs. There are different charging technologies as a) plug-in chargers in depots, b) overhead or inductive chargers during driving, c) pantograph chargers at terminals or bus stops, and d) battery swapping [135]. The first method only requires the installation of charging stations at the beginning and end of terminals which allows for more flexibility in terms of route changes compared to using overhead power grids. The use of overhead power grids restricts the EB to a specific route and limits its flexibility. Additionally, adding new routes would be costly as it would require installing charging infrastructure along the new route since EBs are charging while moving [135]. The third method involves installing overhead chargers at designated

terminals or bus stops. Buses can charge their batteries while stopped at these locations. The installation cost of plug-in chargers in depots is lower than pantograph chargers; furthermore, some local planning regulations prohibit pantographs in urban areas [136]. Finally, the battery swapping method involves replacing the depleted battery with a fully charged one at designated swap stations. This can be a convenient option for buses operating along routes with limited charging infrastructure. [137] has studied opportunities and challenges of battery swapping and ponied out infrastructure complexity, battery degradation, and ownership as challenges. Providing adequate charging facilities is crucial for the effective operation of EBs. While different charging technologies are available, plug-in chargers in depots offer several advantages, including lower installation costs, more flexibility in route planning, and minimal visual impact on urban areas. To fully leverage the benefits of this method, it is important to optimize the scheduling of recharging activities to minimize downtime and ensure the timely and efficient deployment of EBs. Our study aims to develop a strategic plan for employing EBs and optimizing their recharging schedule to maximize the benefits of this technology for sustainable and efficient public transportation.

Another challenge with employing EBs is the uncertainty of travel time and energy consumption. Travel time of EBs can be uncertain due to various factors, such as traffic and weather conditions. These factors can contribute to delays and increase the time it takes an EB to reach its destination. When scheduling an EB network, the uncertainty of travel times must be taken into consideration since it affects not only the scheduling of the EBs but also their energy consumption. On the other hand, passenger demand is often unpredictable, leading to EB weight uncertainty. The energy consumption of an EB is a function of the weight and travel time and is prone to be influenced by their uncertainties. Ignoring these uncertainties can result in solutions that may result in missed or delayed trips due to the realization of uncertain

parameters different than the preplanned values. This study provides an approach to formulate the uncertainty of the energy consumption of EBs and adopts the *robust optimization* (RO) approach to provide a robust solution against travel time and energy consumption uncertainties. We investigate how ignoring uncertainties can lead to scheduling and charging plans that cannot be implemented in real-world situations. We also show that the RO approach can provide more reliable scheduling and charging solutions. In this study, we propose a Monte Carlo simulation to compare the average solution performance of the deterministic and robust models. We evaluate the potential benefits of using a robust approach in terms of the reliability and feasibility of the scheduling and charging plans for EBs.

Our research aims to address the mentioned challenge of scheduling and charging EBs in a transportation system, minimizing the cost of purchasing new EBs and operating them within a defined time period. We define *electric bus scheduling problem* (EBSP) to decide on the number of required EBs of different types to cover a set of trips. Usually, energy suppliers offer tariffs to encourage users to consume electricity during off-peak hours. To this end, we take into account the discounts on electricity tariffs, which may vary depending on the time of day. Off-peak hours are usually discounted, meaning that charging EBs during these times is less expensive. Accordingly, we assume EBs will be discounted during off-peak hours. The formulated optimization model poses computational challenges that can be difficult to solve exactly by commercial solvers. Hence, we propose a heuristic algorithm to address the challenge. The algorithm provides near-optimal solutions for medium-sized instances and can be scaled to accommodate large-sized instances. To show the effectiveness of the proposed model and the robust approach, we use a real-world case study obtained from the off-campus college transportation system at Binghamton University, New York, United States. In this study, we investigate the properties of a possible EB network for the university under different uncertainty sets. We also study the

effect of off-peak hour charging and suggest a potential depot location among available candidates. By considering these factors, we aim to provide a comprehensive analysis of the properties of a possible EB network and inform the decision-making process for its implementation. Our results can be used to guide the development and deployment of an EB network at other transportation networks to optimize costs.

The remainder of the paper is structured as follows. Section 4.2 provides a review of relevant literature. In Section 4.3, the problem is defined, and the deterministic optimization model is provided. The RO approach and uncertainty formulations are provided in Section 4.4. Section 4.5 provides the solution approach and the heuristic algorithm to solve the problem. In Section 4.6, computational results and the case study is discussed. Finally, in Section 4.7, managerial implications and conclusion are discussed, and future research directions are provided.

4.2 *Literature review*

The scheduling of EBs has received increasing attention in recent years due to their potential to reduce greenhouse gas emissions and improve air quality. The efficient scheduling of EBs is critical for maximizing their environmental and economic benefits. Nevertheless, it is a complex task due to the inherent uncertainty in travel times, passenger demand, and energy consumption. In this literature review, we survey the existing work on EB scheduling and highlight the various approaches and techniques that have been proposed to address these challenges. We also discuss the limitations and potential future directions of this research area. *vehicle scheduling problem* (VSP) refers to the optimal assignment of vehicles to the corresponding timetable of the system. A chain of trips will be assigned to one vehicle such that there is no one trip can be assigned to two vehicles or more in parallel [138]. VSP has two categories, *single depot* VSP (SD-VSP) and *multiple-depot* VSP (MD-VSP), see e.g., [138, 139, 140].

The SD-VSP can be described as a minimum cost flow problem [141] or an approximate assignment problem [142]. MD-VSP models can be categorized into connection network models, including the multi-commodity network flow model [143, 144] or the set partitioning model [145, 146, 147]. MD-VSP is proven to be NP-hard [148], thus alternative approaches such as time-space diagram and *Column Generation* (CG) have been utilized to break down the computational time obstacle [147, 141]. The strategy used for vehicle scheduling relies on a connection-based network, each of two vertices is connected by one arc [138, 149, 150]. Generally, the connection-based model problem is hard to solve by optimization software (e.g., GUROBI or CPLEX) when thousands of trips must be scheduled. To this end, connection-based networks could lead to an immense number of deadheading trips. As an alternative solution, a space-time network modeling concept has been commonly used [150, 151]. [152, 153] formulated the VSP as a set partitioning problem which solvable by CG.

VSP can be solved by exact methods such as *branch & bound* (BB), CG, or by heuristic methods taking *genetic algorithm* (GA) and *particle swarm optimization* (PSO) as examples. BB algorithm can solve small to medium VSP with total trips up to 600 and three depots [154, 155] depending on the nature of the problem. CG algorithm can solve medium to large-scale reaches to 10,000 trips [144]. Besides, heuristic methods are utilized to solve large-scale VSPs [156, 157].

Most developed countries, including the USA and China, are moving toward cleaner energy by utilizing natural renewable energy sources such as solar energy and wind energy in addition to replacing current fuel-based public transportation buses with environmentally-friendly EBs [158]. In 2021, the United States of America deployed 3,364 EBs while China approached half a million EBs [159, 133]. Electrifying public buses can reduce deaths number by more than 4,000 each year and create 960,000 direct and indirect jobs in the USA [160]. Electrification of the public transport system is still one of the contemporary challenges to alleviating pollution and

improving public health [161, 162]. Despite the tangible advantages of EBs for the healthcare system and environment, driving range and frequent charging hinder their propagation [163].

EBSP is rather sophisticated due to battery charging frequency that poses additional complications on problem formulation. EBSP can be categorized into two main categories, single-depot EB scheduling problem (SD-EBSP) and multiple-depot EB scheduling problem (MD-EBSP). We refer the reader to [163, 161, 162, 164, 165, 166, 167, 168, 141, 169, 170] for SD-EBSP and [171, 172, 173, 174, 175, 176, 177]

It is important to know that pure or homogeneous EB fleet systems are different than mixed systems. In homogeneous EB systems, it is assumed that all EBs have the same energy consumption on a particular path/route as illustrated in [161] and [141]. However, the energy consumption for mixed buses varies based on the bus type or size. In this study, we propose a generic model for formulating a multi-depot scheduling problem. It is worth mentioning that the flexibility of this model allows for its application to single-depot scheduling problems as well. We present a comprehensive literature review of homogeneous and mixed EB scheduling problems in the following.

4.2.1 Homogeneous EB optimization models

Studying the literature on homogeneous EB systems shows the lack of research that simultaneously finds the optimal solution for charging planning and fleet scheduling in an integrated fashion with considering uncertainty in energy consumption and travel time. [133] is the first who developed an integrated model to find the optimal solution for the integrated model of charging planning, battery size, and fleet size. Peak-on electricity price and five different chargers were tested in this study. The rolling horizon approach was used to build a robust model. Some of the recent research developments on pure EB systems are summarized in Table 42. Overall, the

development of robust charging planning and fleet scheduling strategies considering against uncertain nature of the problems is crucial. For the successful deployment and operation of electric bus systems, further research in this area has the potential to make a significant impact on the EB field.

4.2.2 Mixed EB Optimization Models

In this section, more attention will be placed on the recent research advances in mixed EB optimization models. The mixed fleet EB system was proven to be more cost-effective than homogeneous EB systems as shown by [177, 178] for the short term. Li and his colleagues addressed the MD-EPSP for the mixed fleet under range and refueling constraints [172]. [179] developed SD-EBSP for the mixed fleet system. [180] presented a MILP model for the mixed bus fleet and used a time-based decomposition framework. Zhou and his colleagues established a multi-objective bi-level programming model to optimize the EB scheduling and charging planning of the mixed bus fleet and solved it with an integrated heuristic algorithm [178]. [181] presented a study on the electric bus fleet composition and scheduling, taking into account various factors such as multiple depots, different types of vehicles, and recharging technologies. The authors developed an algorithm based on the CG method to solve the real-world problem and also conducted several experiments to gain a deeper understanding of the electrification of public transit. In 2021, [182] referred in their article that most mixed EB optimization models in the literature lack addressing the uncertainty in energy consumption and travel time for their studies. Therefore, more research is needed to enhance the current models since uncertainty is naturally inherited in public transportation systems.

Based on our review of the literature on homogeneous and mixed EB scheduling problems, we have identified a lack of attention to the following two issues.

- The existing literature lacks integration models for optimizing the charging

planning, battery capacity, and operations in an integrated fashion to achieve optimal solutions. For example, the first attempt to address this issue has been conducted by [133]. However, the uncertainty of energy consumption and travel times are ignored, and there are no details about the robustness of the model against the uncertainty. In this study, a novel uncertainty set is formulated for the uncertain parameters, and the tractable robust formulation of the problem is solved to get resilient solutions against the uncertainty.

- To the best of our knowledge, no single study has been developed to handle charging planning and operations planning for mixed EB systems under energy consumption and travel time uncertainty. Moreover, [135] has identified robust scheduling of Electric fleets as a future area of research. Our study is the first of its kind to build a general mathematical model that optimizes the schedule of trips, charging planning, and fleet size for mixed EB systems, taking into account uncertainties in travel time and energy consumption. The proposed model can also handle multiple-depot.
- In our review of the literature, we found that many studies that adopt robust optimization do not conduct a simulation to compare the performance of the deterministic solution and the robust counterpart for the same set of uncertain parameters. In contrast, our study takes this extra step by conducting a simulation to evaluate the effectiveness of using robust optimization and to understand the trade-off between solution optimality and robustness. This additional analysis allows us to draw more robust conclusions about the benefits and limitations of using robust optimization in our problem context.

4.3 Problem definition and deterministic model

Given a set of charging stations C , a set of EB types K , and scheduled trips T with specified start times and travel times, [EBSP] aims to minimize the total cost of

purchasing EBs, energy consumption, and operational costs. Each trip is assigned to an EB, and all trips should be covered without delay. EBs depart from charging stations with a fully charged battery at the beginning of the planning horizon and can charge their batteries at charging stations during the day. An EB equipped with a larger battery can cover a greater number of trips without the need for recharging. However, larger EBs have higher operational costs, and the increased weight of the EB results in a higher energy consumption rate. In [EBSP], a detailed cost structure of charging batteries is considered such that the daily schedule is divided into different time periods, and the cost of charging during these periods may vary depending on the electric supplier's policy. For example, the cost of charging may be lower during off-peak hours compared to peak hours. The optimization model has been designed to consider the variation in charging costs, aiming to generate more precise and cost-efficient plans. To formulate the associated problem, a source and a sink nodes are defined for each charging station, and the daily schedule of each EB starts from a source node and ends at a sink node. The number of EB of each type starting from the source node of a charging station is equal to the number of EB of the same type that finish their daily schedule at the sink node of the same charging station. In other words, the schedule of each EB starts and ends at a charging station and charging stations are denoted as source nodes for the start of the schedule and sink nodes for the end of the schedule. A decision variable is defined as X_{ij}^k , which takes the value 1 if an EB of type k covers trip j after finishing trip i , and 0 otherwise. Moreover, Y_{ic} decision variable is defined that takes value 1 if charging station $c \in C$ is visited after trip $i \in T$, and 0 otherwise. Each EB can visit any charging station more than once during its daily schedule. In this formulation, G_{icn} denotes the charging amount an EB receives after finishing trip i in charging station c in period n . Figure 7 sketches a sample network of [EBSP]. In this network, two EBs are used to cover trips. . An EB of type $k = 1$ (black EB) departs from charging station 1 and starts the daily

schedule by covering trip 1 ($X_{s_1 1}^1 = 1$). After finishing the first trip, the EB visits charging station 1 to recharge the battery ($Y_{11} = 1$) and then covers the 3rd trip ($X_{13}^1 = 1$). The EB gets to charge station 1 after finishing trip number 3 ($Y_{31} = 1$) to recharge the battery and finish the daily schedule ($X_{3f_1}^1 = 1$).

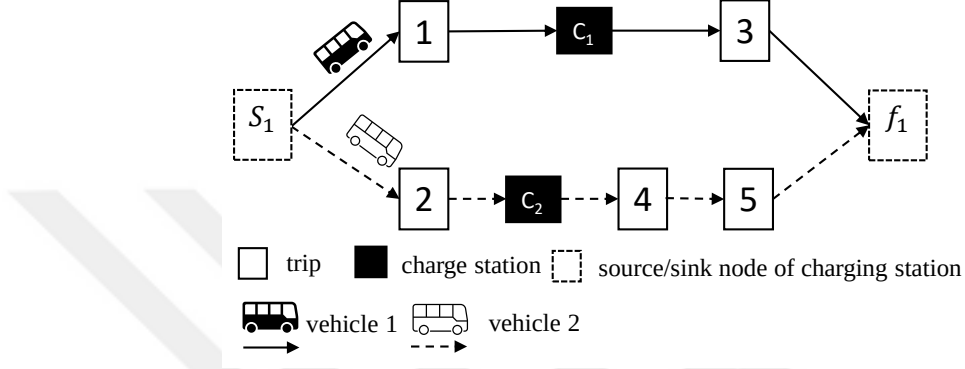


Figure 7: A sample graph to represent [EBSP] solution

In this model, the travel times from the endpoints of trips to charging stations and from charging stations to the start points of trips are known. These trips without passengers on board are called deadhead trips. An EB may start a trip immediately after finishing the previous trip if the battery level is enough to cover the trip by having a deadhead trip from the endpoint of the finished trip to the start point of the next trip. The assumption can be relaxed to consider the time needed to start a new trip after finishing the previous one. This extra time may be necessary for reasons such as the need to clean an EB before beginning a new trip. The travel times from the endpoint of trips to the start points of the trips are also known. If the endpoint of the finished trip and the start point of the next trip are the same, and the battery level is enough to cover the trip, the EB can start the trip immediately, according to the schedule. It should be highlighted that EBs are finishing the planning horizon with fully charged batteries to start the service for the next planning horizon. During a trip, the energy consumption of an EB may be affected by different parameters (such as EB weight and trip travel time). In this problem, we assume that all parameters

are known, such as the energy consumption and travel time of each EB type for each trip. In Section 4.4, we will address the uncertainty of travel times and energy consumption and provide a model to formulate this uncertainty and an approach to handle it. All parameters and decision variables of the problem are presented in Table 23.

Table 23: Nomenclature of the mathematical optimization model

Sets	
C	set of charging stations ($c = 1, 2, \dots, C$)
S	set of source nodes ($s_c = s_1, s_2, \dots, s_C$)
F	set of sink nodes ($f_c = f_1, f_2, \dots, f_C$)
T	set of trips ($i, i', j = 1, 2, \dots, \mathcal{I}$), $T^s = T \cup S, T^f = T \cup F$
T_i	set of trips ($i, i', j = 1, 2, \dots, \mathcal{I}$) that can be covered after trip $i \in T$, $T_i^s = T_i \cup S, T_i^f = T_i \cup F$
T'_i	union of the set of trips that can be covered before trip i with source nodes, ($T'_i = \{T^s \setminus \{T_i\}\}$)
K	set of EB type ($k = 1, 2, \dots, \mathcal{K}$)
K_i	set of EB type ($k = 1, 2, \dots, \mathcal{K}$) that can cover trip $i \in T$, $K_i \subseteq K$
N	set of periods,
Parameters	
l_k	amortized purchasing cost of an EB type $k \in K$
h_k	traveling cost per unit time for EB type $k \in K$
p_n	charging cost per KW in period $n \in N$
r_c	fixed cost of opening charging station $c \in C$
Ω	fixed charging cost
σ_k	charging rate for EB type $k \in K$
D_k	battery capacity of EB type $k \in K$, $D_{max} = \max_{k \in K} \{D_k\}$
b_i	start time of the trip $i \in T$
d_i	duration of the trip $i \in T$
ts_{ic}	travel time between start point of trip $i \in T$ and charging station $c \in C$
te_{ic}	travel time between end point of trip $i \in T$ and charging station $c \in C$
u_{ij}	travel time between end point of trip $i \in T$ and start point of trip $j \in T$
q_n	start time of period $n \in N$
es_{ick}	energy consumption from start point of trip $i \in T$ to charging station $c \in C$ for EB type $k \in K$
ee_{ick}	energy consumption from end point of trip $i \in T$ to charging station $c \in C$ for EB type $k \in K$
eu_{ijk}	energy consumption from end point of trip $i \in T$ to start point of trip $j \in T$ for EB type $k \in K$
ed_{ik}	energy consumption of EB type $k \in K$ in trip $i \in T$
a^{low}	the lower limit for battery level
a^{upp}	the upper limit for battery level
λ	required time for parking and connecting an EB to charging infrastructure
Decision variables	
X_{ij}^k	1, if trip $j \in T_f$ is visited immediately after trip $i \in T_s$ by EB type $k \in K$; 0, o.w.
Y_{ic}	1, charge station $c \in C$ is visited after trip $i \in T$; 0, o.w.
T_{ijk}	time duration of deadhead trips to start trip $j \in J$ after $i \in T$ with vehicle $k \in K$
G_{icn}	charging amount of EB after trip $i \in T$ on charge station $c \in C$ in period $n \in N$
B_i	battery level at the beginning of trip $i \in T$
Z_c	1, if charging station $c \in C$ is opened
W_{in}	1, if charging after trip $i \in T$ starts in period $n \in N$

Notation. In this study, boldface letters are used to represent vectors and matrices, and their dimensions are typically clear from the context. We use brackets to refer to the mathematical optimization formulation of a problem. For example, EBSP refers to the problem, while [EBSP] refers to its formulation.

Infeasible arcs. The [EBSP] may become computationally heavy even for small-sized instances due to generating all possible arcs. We can avoid defining constraints for the arcs that are known to be infeasible, i.e., if the starting time of trip j is smaller than the ending time of trip i , then we are sure that the trip j can not be covered after trip i . Therefore, we define set $T_i \subset T$ as the set of trips that can be covered after trip $i \in T$ by the same EB, and we determine the set for all i as follows:

$$T_i := \{j \in T : b_j > b_i + d_i, \quad \forall i \in T\}$$

In the following resulting deterministic optimization problem is formulated.

Objective function

[EBSP] :

$$\begin{aligned} \min \quad & \sum_{j \in T} \sum_{k \in K} \sum_{c \in C} l_k X_{scj}^k + \sum_{i \in T} \sum_{c \in C} \sum_{n \in N} p_n G_{icn} + \sum_{i \in T} \sum_{c \in C} \Omega Y_{ic} + \sum_{i \in T} \sum_{j \in T_i^f} \sum_{k \in K} h_k d_i X_{ij}^k \\ & + \sum_{i \in T} \sum_{j \in T} \sum_{k \in K} h_k T_{ijk} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{sci}^k t_{sic} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{ifc}^k t_{eic} \end{aligned}$$

The objective function has seven terms. The first term is the cost of purchasing EBs based on the total EBs leaving the charging stations. The second and third terms are the charging cost and calculating the total fixed cost of charging and total energy consumption cost by all EB to cover all trips. The last four terms are the total operating and in-service costs, such as bus driver costs. They are calculated for a specific period, typically a daily schedule, based on a time-dependent cost factor. The optimization problem at hand involves minimizing both daily operational costs and strategic purchase costs, where the latter are paid once for a period of 10 or 15 years. This presents a challenge because the purchase costs are not a daily expense, unlike operational costs. Therefore, the straightforward approach of summing up all costs on a daily basis may not be appropriate. To address this issue, we have amortized the strategic costs over their life span, which means that we have spread them out over their expected lifespan. By doing so, we can incorporate the purchase

costs into the daily cost calculations, allowing us to optimize the overall costs of the system effectively.

EBs network Constraints

$$\sum_{i \in T'_j} \sum_{k \in K_i} X_{ij}^k = 1 \quad \forall j \in T \quad (71)$$

$$\sum_{j \in T_i^f} \sum_{k \in K_j} X_{ij}^k = 1 \quad \forall i \in T \quad (72)$$

$$\sum_{i \in T'_j} X_{ij}^k = \sum_{i \in T_i^f} X_{ji}^k \quad \forall j \in T, k \in K \quad (73)$$

$$\sum_{i \in T} X_{s_c i}^k = \sum_{i \in T} X_{i f_c}^k \quad \forall k \in K, c \in C \quad (74)$$

$$b_i + d_i + T_{ijk} + \sum_{c \in C} \sum_{n \in N} \frac{1}{\sigma_k} G_{icn} + \sum_{c \in C} \lambda Y_{ic} \leq b_j + M(1 - X_{ij}^k) \quad \forall i \in T, j \in T_i, k \in K \quad (75)$$

$$T_{ijk} \geq \sum_{c \in C} Y_{ic}(te_{ic} + ts_{jc}) + (1 - \sum_{c \in C} Y_{ic})u_{ij} + M(X_{ij}^k - 1) \quad \forall i \in T, j \in T_i, k \in K \quad (76)$$

Constraints (71) ensure that each trip must be covered, and (72) ensure that an EB should start only one trip after finishing the previous trip. Constraints (73) ensure that if trip j is covered after trip i , the EB type for these trips must be the same. Constraints (74) ensure that the number of EBs of each type that leaves the source node and enters the sink node of a charging station should be the same. Constraints (75) ensure that if trip j is started after trip i by EB type k , the start time of trip j is greater or equal to the sum of the start time and duration of trip i , plus charging time after trip i and the duration of deadhead trips. Constraints (76) derive the travel time of deadhead trips between connected trips.

Battery level and energy consumption constraints

$$B_i \leq \sum_{j \in T'_i} \sum_{k \in K} a^{upp} D_k X_{ji}^k \quad \forall i \in T \quad (77)$$

$$B_i - \left(ed_{ik} + \sum_{c \in C} ee_{ick} Y_{ic} \right) \geq \sum_{j \in T_i^s} a^{low} D_k X_{ij}^k \quad \forall i \in T, k \in K \quad (78)$$

$$\begin{aligned} \sum_{c \in C} \sum_{n \in N} G_{icn} &\geq D_k - B_i + (ed_{ik} + \sum_{c \in C} Y_{ic} ee_{ick}) \\ &+ M \left(\sum_{f_c \in F} X_{if_c}^k - 1 \right) \end{aligned} \quad \forall i \in T, k \in K \quad (79)$$

$$\begin{aligned} B_i - \left(ed_{ik} + \sum_{c \in C} Y_{ic} (ee_{ick} + es_{jck}) + (1 - \sum_{c \in C} Y_{ic}) eu_{ijk} \right) \\ + \sum_{c \in C} \sum_{n \in N} G_{icn} + M(1 - X_{ij}^k) &\geq B_j \end{aligned} \quad \forall i \in T, j \in T_i, k \in K \quad (80)$$

$$\sum_{n \in N} G_{icn} \leq D_{max} Y_{ic} \quad \forall i \in T, c \in C \quad (81)$$

To ensure the longevity and proper functioning of the battery, the upper and lower limit for battery charge levels are considered in this problem. Constraints (77) ensure that the battery level of an EB can not exceed its maximum capacity. Constraints (78) prevent the battery level of EBs from dropping below their allowed lower battery level. These limits prevent damage to the battery caused by excessive charging or discharging. Constraints (79) guarantee that EBs must finish each day fully charged to their upper limit. Constraints (80) determine the battery level at the beginning of each trip, ensuring that if trip j is covered after trip i , then the battery level at the beginning of trip j will be less than the battery level at the beginning of trip i plus the charge amount that the EB receives after trip i minus the battery consumption of trip i and required deadhead trips. Constraints (81) ensure that a charge station must be visited if EB needs to be charged after trip i .

Peak and off-peak charging constraints

$$\sum_{n \in N} W_{in} q_n \leq \sum_{c \in C} Y_{ic} (b_i + d_i + te_{ic}) \leq \sum_{n \in N} W_{in} q_{(n+1)} \quad \forall i \in T \quad (82)$$

$$\sum_{c \in C} G_{icn} \leq D_{max} W_{in} \quad \forall i \in T, n \in N \quad (83)$$

$$\sum_{n \in N} W_{in} \leq 1 \quad \forall i \in T \quad (84)$$

$$\mathbf{X}, \mathbf{Y}, \mathbf{W} \in \{0, 1\} \quad (85)$$

$$\mathbf{T}, \mathbf{G}, \mathbf{B} \geq \mathbf{0} \quad (86)$$

As part of the proposed model, we take into account off-peak charging discounts. Constraints (82) are formulated to determine the period that charging of EB starts in after covering trip i . Constraints (83) ensure that if charging after trip i is not started in a period, the charging amount in that period after the trip should be zero. Constraints (84) ensure that an EB can be charged after a trip only in one segment. Constraints (85) and (86) are 0-1 and non-negativity constraints, respectively.

Deciding on charging stations. This study mainly focuses on formulating a problem to optimize daily costs. However, deciding on charging stations may be a concern for the decision-maker. It is important to note that [EBSP] can be easily adapted to determine the optimal number and location of charging stations. To accomplish this, a decision variable Z_c can be incorporated, which takes a value of 1 if charging station c is opened and 0 otherwise. The objective function would then include the amortized cost of opening the station and a constraint ensuring that if there is an entrance to a station, the associated Z_c value must be set to 1. However, the decision on the number of charging stations is a strategic one that requires consideration of factors such as amortized opening and maintenance costs.

The associated extended formulation of the problem is given in as follows:

$$\begin{aligned}
\min \quad & \sum_{j \in T} \sum_{k \in K} \sum_{c \in C} l_k X_{scj}^k + \sum_{i \in T} \sum_{c \in C} \sum_{n \in N} p_n G_{icn} + \sum_{i \in T} \sum_{c \in C} \Omega Y_{ic} + \sum_{i \in T} \sum_{j \in T_i^f} \sum_{k \in K} h_k d_i X_{ij}^k \\
& + \sum_{i \in T} \sum_{j \in T} \sum_{k \in K} h_k T_{ijk} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{sci}^k t_{sic} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{ifc}^k t_{eic} + \sum_{c \in C} r_c Z_c \\
\text{s.t} \quad & (71 - 86) \\
& Z_c \leq M \sum_{i \in T} Y_{ic} \quad \forall c \in C \\
& \mathbf{Z} \geq \mathbf{0}.
\end{aligned}$$

4.4 Robust reformulation and uncertainty sets

The two uncertain factors that can impact the energy consumption of an EB during a trip are the number of passengers on the bus and the traffic conditions along the route. These uncertainties can affect not only the battery usage of an EB but also the punctuality of the planned trips. While the uncertain factors jointly influence battery consumption, the earliness and lateness of trips are mainly affected by the travel time uncertainty. Travel times and energy consumption of trips are typically calculated based on the nominal values in practice. Nevertheless, if the actual energy consumption or the travel time differs from the nominal value, it may result in inefficiencies such as insufficient battery level or delays in arrival. To yield robust solutions immunized against the uncertainties, we propose *robust* EB scheduling problem (RBSP), a tractable robust optimization model. The robust model produces feasible solutions for any possible realization of the data residing in predefined uncertainty sets. In the following, we will first discuss the formulation of the uncertainty sets ($\mathbb{U}^t, \mathbb{U}^m$) and then present the mathematical optimization model for the [RBSP].

To better understand the structure of the uncertainty in the related problem, we express the uncertainty formulations in the two key components mentioned above, i.e., travel time and energy consumption.

Travel time and EB weight uncertainty sets. The interval or box uncertainty sets are often used in practice when no distributional information but certain lower and upper bounds are known about the uncertainty parameter [63]. The following interval uncertainty set is used to model the uncertainty associated with travel times in trip i :

$$\mathbb{U}_i^t := \{\mathbf{t} \in \mathbb{R}_+^{N_i} : t_{ri} = (1 + \rho_{ri}^t \zeta_r) t_{ri}^0, \quad -1 \leq \zeta_r \leq 1 \quad \forall r \in S_i\}$$

where S_i is the set of segments in trip i (indexed by r), $\mathbf{t}^0$ represents the vector of nominal travel times, and the uncertainty set is symmetric around it, i.e., $\hat{\mathbf{t}} = \boldsymbol{\rho}^t \mathbf{t}^0$ is the half-length of the interval where $\rho \in [0, 1)$ denotes the uncertainty level, and $\boldsymbol{\zeta} \in [-1, 1]$ is the primitive uncertainty vector. Analogous to the travel time, the uncertainty in the EB weight may also be represented using an interval uncertainty set:

$$\mathbb{U}_{ik}^m := \{\mathbf{m} \in \mathbb{R}_+^{N_i} : m_{rik} = (1 + \rho_{rik}^m \xi_r) m_{rik}^0, \quad -1 \leq \xi_r \leq 1 \quad \forall r \in S_i\}$$

where $\boldsymbol{\xi} \in [-1, 1]$ is the primitive uncertainty vector for the EB weight. Notice that all uncertainties are formulated over trips since the uncertainty at each trip may be individually analyzed.

Robust travel time parameters. Each trip $i \in I$ is divided into segments ($r \in S_i$), and the robust duration of each trip i is given by the following expression.

$$\bar{d}_i := \max_{\mathbf{t} \in \mathbb{U}_i^t} \sum_{r \in S_i} t_{ri} \equiv \sum_{r \in S_i} (1 + \rho_{ri}^t) t_{ri}^0 \quad \forall i \in I \quad (87)$$

In the robust reformulation, the robust travel times presented above are substituted for d_i in the deterministic formulation. Similarly, the upper bound for $(u_{ij}, te_{ic}, ts_{ic})$ are calculated using equation 87, and the resulting parameters are denoted by an overline symbol (see Appendix C.2).

Robust energy consumption parameters. Each trip i is divided into segments ($r \in S_i$) to increase the granularity of the calculations. The total battery consumption (et_{ik}) of EB type k in trip i is equal to the sum of the energy consumption in all

segments of the associated trip. The energy consumption formula that is also used in other papers such as [183] is expressed as follows:

$$et_{ik} := \sum_{r \in S_i} \left[\frac{v_{ri} t_{ri}}{\delta} \left(\underbrace{\frac{P}{2} \kappa A_k v_{ri}^2}_{\text{aerodynamics}} + \underbrace{\mu m_{rik} g \cos \theta_{ri}}_{\text{rolling resistance}} + \underbrace{v_{ri} g \sin \theta_{ri}}_{\text{slope resistance}} \right) + O t_{ri} \right] \quad \forall i \in I, k \in K$$

The first term in the parenthesis is related to the effect of a vehicle's aerodynamic design on energy consumption, the second one is the rolling resistance which is the energy required to overcome resistance between tires and the road surface, and the third one is the effect of gravitational force on energy consumption that is needed to run an EB uphill or downhill. The equation also considers the energy for running accessories such as air conditioners in the last term. Notice that the travel time of segment r in trip i is denoted by t_{ri} and the weight of type k EB in segment r of trip i is denoted by m_{rik} as before; therefore, the combined effect of travel time and weight on energy consumption may be interpreted from the above-given expression. The nomenclature for the rest of the parameters is given in Numerical experiments (see Section 4.6.1).

For the sake of notation simplicity, we aim to re-express the above-given formulation of et_{ik} over the primitive uncertainties (ζ, ξ) of the two uncertainty parameters $(\mathbf{t}_i, \mathbf{m}_{ik})$ by simplifying the constant terms as the following.

$$\begin{aligned} \delta_{rik} &= \left(\left(\frac{P}{2} \kappa A_k v_{ri}^3 + O \right) t_{ri}^0 + \mu (\cos \theta_{ri} + \sin \theta_{ri}) g v_{ri} t_{ri}^0 m_{rik}^0 \right) & \forall r \in S_i, k \in K \\ \phi_{rik} &= \frac{1}{\delta} \left[\left(\left(\frac{P}{2} \kappa A_k v_{ri}^3 + O \right) \hat{t}_{ri} + \mu (\cos \theta_{ri} + \sin \theta_{ri}) g v_{ri} \hat{t}_{ri} m_{rik}^0 \right) \right] & \forall r \in S_i, k \in K \\ \lambda_{rik} &= \mu (\cos \theta_{ri} + \sin \theta_{ri}) g v_{ri} t_{ri}^0 \hat{m}_{rik} & \forall r \in S_i, k \in K \\ \omega_{rik} &= \mu (\cos \theta_{ri} + \sin \theta_{ri}) g v_{ri} \hat{t}_{ri} \hat{m}_{rik} & \forall r \in S_i, k \in K \end{aligned}$$

Consequently, the final formulation of et_{ik} may re-express (88) as

$$et_{ik} := \sum_{r \in S_i} \delta_{rik} + \sum_{r \in S_i} \phi_{rik} \zeta_r + \sum_{r \in S_i} \lambda_{rik} \xi_r + \sum_{r \in S_i} \omega_{rik} \zeta_r \xi_r \quad \forall i \in I, k \in K. \quad (88)$$

Budget Uncertainty set. Adopting the interval uncertainty sets to yield robust energy consumption estimations using (88) may result in overly conservative solutions since each parameter may independently take its worst-case realization in the robust reformulation. To this end, in this subsection, we propose a budget parameter Γ_i to control the level of conservatism in the robust reformulation. More precisely, we limit the number of times the two primitive parameters take their worst-case realizations in the same segment (i.e., $(\zeta_r, \xi_r) \in \{-1, 1\} \times \{-1, 1\}$) to Γ controlling the conservatism level with a pre-defined budget. The resulting uncertainty is given as the following.

$$\mathbb{U}_{ik} := \{\boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\psi} \in \mathbb{R}_+^{N_i} : \|\boldsymbol{\zeta}\|_1 \leq 1, \|\boldsymbol{\xi}\|_1 \leq 1, \|\boldsymbol{\psi}\|_\infty \leq \Gamma, \psi_r = \zeta_r \xi_r \quad \forall r \in S_i\}$$

where $\mathbf{t}^0$ and $\mathbf{m}^0$ are the vector of nominal travel times and weight of the bus, $\boldsymbol{\zeta}, \boldsymbol{\xi} \in [-1, 1]$ are the vectors of the primitive uncertainty parameters, and $\boldsymbol{\rho} \in [0, 1]$ is the scaling vector that denotes the half-lengths of the intervals for the travel times ($\hat{\mathbf{t}} = \boldsymbol{\rho} \mathbf{t}^0$) and weights ($\hat{\mathbf{m}} = \boldsymbol{\rho} \mathbf{m}^0$). Ultimately, the robust estimation of the energy consumption of trip i with EB type k is expressed as

$$\overline{et}_{ik} := \left(\max_{\boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\psi}} et_{ik}(\boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\psi}) \quad \text{s.t.} \quad (\boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\psi}) \in \mathbb{U}_{ik} \right) \quad \forall i \in T, \forall k \in K.$$

Yanikoğlu and Avishan (2023) has shown that the sum of block separable bilinear expressions like (88) attains their maximum at boundary solutions of the budgeted constraints of type $\mathbb{U}_i$ (i.e., $(\zeta_r^*, \xi_r^*) \in \{-1, 0, 1\} \times \{-1, 0, 1\}$). Therefore, using the associated property, we may calculate all $\overline{et}_{ik}$ using the following value function for a dynamic program that can be computed in polynomial time using a standard bottom-up or top-down approach.

$$\begin{aligned} V_{ik}(r, \Gamma) = & \left\{ \max\{|\phi_{rik}|, |\lambda_{rik}|\} + V_{ik}(r-1, \Gamma), V_{ik}(r-1, \Gamma-1) + \dots \right. \\ & \max\{\phi_{rik} + \lambda_{rik} + \omega_{rik}, \phi_{rik} - \lambda_{rik} - \omega_{rik}, -\psi_{rik} - \lambda_{rik} + \omega_{rik}, \dots \\ & \left. - \psi_{rik} + \lambda_{rik} - \omega_{rik}\} \right\} \end{aligned}$$

where i denotes trip, r denotes segment, and Γ_i remaining budget, cost of trip $i \equiv V(n, \Gamma_i)$ where $n = |S_i|$ and Γ_i is the budget.

It is worth noting that the methodology for robust energy consumption, as demonstrated through the use of et_{ik} , will be adopted to all energy consumption parameters ($ed_{ik}, ec_{ick}, es_{jck}, eu_{ijk}$) in the same way. The resulting robust calculations for these parameters will be denoted by an overline symbol, for instance, $\overline{eu}_{ijk}$. The definitive representation of the robust counterpart can be found in Appendix C.2.

4.5 *Heuristic algorithm*

The formulated problem in this study is a computationally expensive MILP that requires a significant amount of memory to achieve an optimal solution for large-sized instances. A heuristic algorithm is developed to solve problems that the standard approach fails to provide useful solutions. This section presents the heuristic algorithm that produces near-optimal solutions while enhancing CPU time performance to address this issue. The algorithm steps are summarized as follows:

[Step 1] Initialization and generating sub-schedules. We sort the trips based on their start time and divide them into n blocks. For each block, we solve [EBSP] to get a solution. Solving [EBSP] for trips in blocks may take a while and may increase the CPU time of the algorithm. To avoid the issue, we can set a time limit of (e.g., 30 or 60 seconds) to terminate the solving process. The solution of each block consists of sub-schedules, and each sub-schedule is covered by an EB. We name the set of all sub-schedules in all blocks as a division. This procedure is iterated for d iterations considering different dividing points to generate d individual divisions. Figure 8 shows a sample division created in Step 1.

[Step 2] Generating schedules. In this step, we aim to generate schedules by merging sub-schedules and schedules. Notice that while merging the sub-schedules and schedules, the type and size of the EBs should be taken into consideration, and

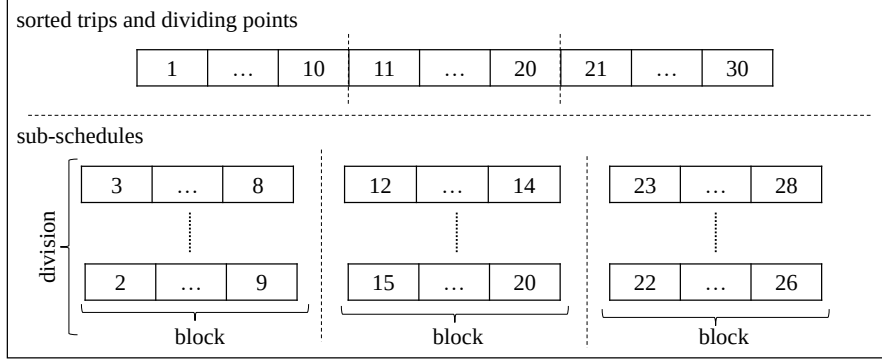


Figure 8: Overview of Step 1

if the types of EBs that are covering them are different, they can not be merged. [Step 2.1] As shown in Figure 9, we merge the sub-schedules obtained in the first step considering each division separately. To this end, we randomly select a sub-schedule from each block, and by merging them, we get a schedule. All candidates in a block have an equal chance of being selected by random selection. We repeat the procedure to generate s schedules. We repeat the procedure for each division. At the end of this step, we will have $s \times d$ schedules.

[Step 2.2] We randomly select two schedules from the ones generated in Step 2.1, while all candidates have the same chance of being selected. Then we merge them to get a new schedule and remove the repeated trips.

[Step 3] Feasibility check and calculating objective function. Merging sub-schedules and schedules may yield new schedules that are infeasible to be covered by an EB. In this step, we regain the feasibility of the schedules and get their *objective function value* (OFV). To this end, we solve the modified version of the [EBSP] as [M-EBSP] for each schedule. In this problem, the objective function is reformulated to cover the maximum number of trips.

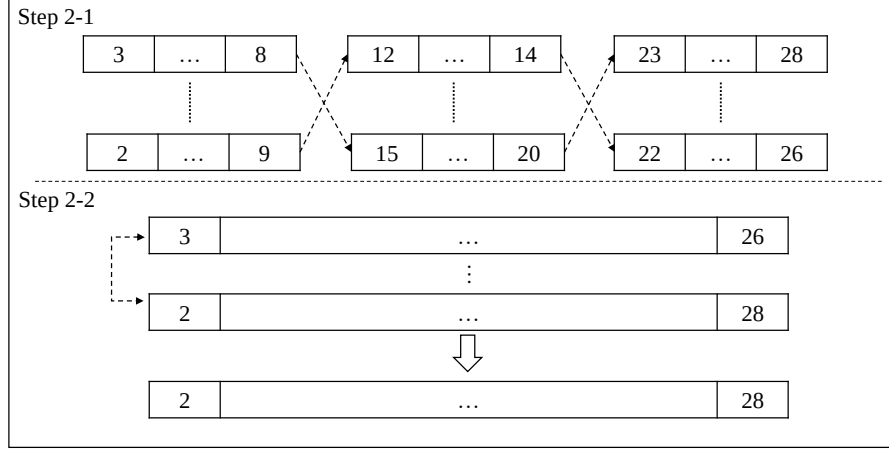


Figure 9: Overview of Step 2

[M-EBSP]:

$$\begin{aligned}
 \min \quad & -M \sum_{i \in T} \sum_{k \in K} \sum_{k \in K} X_{ij}^k + \sum_{i \in T} \sum_{c \in C} \sum_{n \in N} p_n G_{icn} + \sum_{i \in T} \sum_{c \in C} \Omega Y_{ic} + \sum_{i \in T} \sum_{j \in T_i^f} \sum_{k \in K} h_k d_i X_{ij}^k \\
 & + \sum_{i \in T} \sum_{j \in T} \sum_{k \in K} h_k T_{ijk} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{sc i}^k t s_{ic} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{if c}^k t e_{ic}
 \end{aligned}$$

s.t (73 – 86)

$$\sum_{i \in T_j^f} \sum_{k \in K_i} X_{ij}^k \leq 1 \quad \forall j \in T \quad (89)$$

$$\sum_{j \in T_i^f} \sum_{k \in K_j} X_{ij}^k \leq 1 \quad \forall i \in T \quad (90)$$

$$\sum_{j \in T_i^f} \sum_{k \in K_j} X_{ij}^k = \sum_{j \in T_j^f} \sum_{k \in K_i} X_{ji}^k \quad \forall i \in T \quad (91)$$

$$\sum_{j \in T} \sum_{k \in K_j} \sum_{c \in C} X_{sc j}^k = 1 \quad (92)$$

In this problem, the T set is defined as the set of trips in the schedule, and the K set is defined as the set of EB(s) that can cover the schedule. Constraints (71) and (72) are modified as (89) and (90) since covering all of the trips is not necessary. constraints (91) ensure that if a trip is covered with an EB, then the EB should cover another trip or go to a sink node after covering the trip. Constraints (92) are added to ensure that a single EB will cover the trips.

[Step 4] Schedule selection. In this step, a set covering problem is solved to select the best schedules, ensuring each trip is covered at least once. We aim to select the schedules with lower costs among those generated in Step 2 by solving a set covering problem. We use Υ to denote the set of generated schedules in Steps 2 and 3 and a_v to denote the cost of schedule $v \in \Upsilon$, which is calculated in Step 3. The binary parameter b_{iv} is defined where it takes the value 1 if trip i is covered in cluster v ; 0, otherwise. To determine the selected clusters, decision variables are defined as H_v and take value 1 if schedule v is selected: 0, otherwise. The resulting set covering the problem [SCP] is formulated follows:

$$\begin{aligned}
& \text{[SCP]} : \\
& \min_{\mathbf{H}} \quad \sum_{v \in \Upsilon} a_v H_v \\
& \text{s.t.} \quad \sum_{v \in \Upsilon} b_{iv} H_v \geq 1 \quad \forall i \in \mathcal{T} \\
& \mathbf{H} \in \{0, 1\}.
\end{aligned} \tag{93}$$

In this problem, the objective is minimizing the total cost while constraints (93) guarantee that each site is included in at least one of the selected schedules.

[Step 5] Iterative schedule generation and selection. Repeat steps from 2 to 4 for predefined iterations and record the selected schedules of each iteration. Subsequently, solve the set covering problem for the recorded schedules to obtain the final solution. **[Step 6] Final solution.** As a result of constraints (93), there might be trips that appear in multiple schedules and are covered more than once in the final solution. To address this issue, we eliminate the repeated trips from the selected schedules, giving priority to removing them from the EB with higher operating costs. Next, we obtain the final solution and its OFV for the set of schedules by solving [EBSP] individually for each schedule and its given trips. At this stage, the trips and their sequence in each schedule are predetermined, and [EBSP] is solved for the given

$\mathbf{X}$ values. As a result, the problem is solved quickly to obtain optimal values of other decision variables for the given schedule.

The algorithm's summary is provided in Appendix C.3.

4.6 *Numerical experiments*

In this section, we discuss the data and evaluate CPU time, objective function, and the feasibility performances of deterministic and robust solutions. First, we describe the case study data structure used in the numerical experiments in Section 4.6.1. In Section 4.6.2, the performance of the heuristic algorithm is tested and discussed via synthetic instances. We evaluate the solution performances of deterministic and robust approaches in Section 4.6.3. The Monte Carlo simulation procedure and experiment that compares the average solution performances of deterministic and robust solutions is provided in Sections 4.6.4. Finally, we compare the solution performance of the model for different charging station numbers and locations in Section 4.6.5. In this research, computations are run on a 64-bit Windows server with AMD EPYC 2.00 GHz CPU and 32 GB RAM.

4.6.1 Data structure

In this section, we present the data used in this study.

Trip information. For this study, we utilized data from the Binghamton University of New York shuttle network. Specifically, we focused on the Monday-Thursday schedule, which comprises 237 trips across ten lines. We selected this schedule as our case study and solved the problem of obtaining the daily schedule. Figure 4.6.1 shows the locations of the charging stations and a sample trip with the stations indicated by squares on the route. Table 24 contains information about trips, where the first column is the line name, and the second column indicates the line code used in solution representations. The third column is the number of trips with the associated line. The routes can be classified as a) starting from c_1 (union) and ending at the

same point: b) starting from c_1 and ending at c_2 (university downtown center); c) starting from c_2 and ending at c_1 . The number of segments, duration of trips, and trip start and end points are given in columns four through six, e.g., the *WS out* line is repeated 38 times with different start times, and each route of the line has 20 segments. An EB takes twenty minutes to complete each trip. The trips for this line start from c_1 and end in c_2 . It is important to note that the given durations in the table are the nominal values of the trips. Detailed trip information is available online (see EBSP *Cast study* data set on page www.ihsanyanikoglu.com/data-sets). In this study, it is assumed that WS, DCL, and UDC trips during the rush hours (8:00–10:00 and 15:00–20:00) could be covered only with the K9MD vehicle.

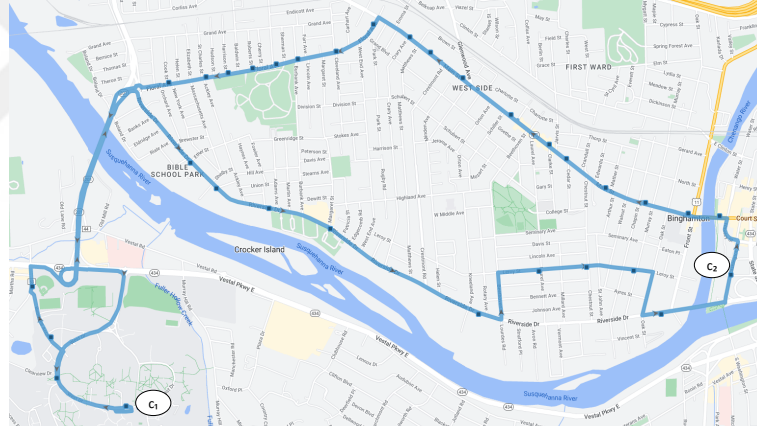


Figure 10: WS out route and stations

EB specifications. The specifications of the available EBs are summarized in Table 25. The curb weight refers to the weight of the vehicle without passengers, cargo, or optional equipment, and the gross weight is the vehicle weight with the maximum load (passengers) an EB is designed to take.

In this study, purchasing costs of EBs are amortized over their expected lifetimes to spread them out. Since the operation and charging costs are calculated for a daily schedule, the amortized cost of purchasing EBs is calculated per day using the following formula, which is a widely recognized financial formula and is commonly

Table 24: Trip information

line	code	number of trips	number of segments	duration (min)	start point	end point
WS out	101-138	38	20	20	c_1	c_2
WS in	201-239	39	20	23	c_2	c_1
DCL in	301-340	40	11	19	c_2	c_1
DCL out	401-440	30	15	12	c_1	c_2
UDC out	501-528	28	6	10	c_1	c_2
UDC in	601-616	16	6	10	c_2	c_1
ITC	701-729	29	22	9	c_1	c_1
LRS	801-805	5	41	45	c_1	c_1
RRT	901-907	7	12	25	c_1	c_1
OAK	1001-1004	4	7	22	c_1	c_1

c_1 :Union, c_2 :University downtown center

Table 25: Bus specification

EB type	index	price(\$)	(curb , gross) weight(kg)	frontal area(m^2)	battery capacity(kWh)	charging rate(kW)
BYD K7M	1	450000	(10679, 14500)	7.9	212	90
BYD K9MD	2	350000	(15940, 20300)	8.8	446	90

<https://en.byd.com/bus/k9md/>, <https://en.byd.com/bus/k9m/>

used in financial and accounting applications.

$$\text{amortized cost} = (\text{purchase cost-salvage value}) \frac{r(1+r)^n}{((1+r)^n - 1)365}$$

According to the literature, we assume that EBs have an economic life of 12 years and salvage values are 15% percent of the purchasing cost [181]. In our calculations, the interest rate is considered as 3%.

Parameters of energy consumption model. Table 26 summarizes the parameters used in the energy consumption model and their values. In this study, We assume that the slope of all segments is zero and that all segments have the same friction factor. We refer readers to see [184, 162] for detailed information on obtaining the parameters.

Charging and operating cost. The highest electricity prices occur during the

Table 26: Nomenclature and values of parameters in equation (88)

symbol	description	unit	value
et_i	energy consumption in trip i	Ws	variant
v_{ri}	velocity of EB in segment r of trip i	m/s	40
A_k	frontal area of EB type k	m^2	2.27
g	gravitation constant	m/s^2	9.81
δ	energy consumption efficiency rate	scalar	0.5
μ	friction coefficient	scalar	0.012
ρ	air density	kg/m^3	1.2
θ_{ri}	the slope of block r of trip i	radians	0
κ	coefficient of drag	scalar	0.29
O	required power to run accessories	W	0
m_{rik}	weight of EB type $k \in K$ in segment r of trip i	kg	variant
t_{ri}	travel time of segment r of trip i	s	variant

morning rush hours (8:00–10:00) and the evening rush hours (15:00–20:00). The price during these peak hours is 0.223\$ per kWh, while the price during off-peak hours is 0.068\$; the same policy is adopted in [133]. The operating cost per hour for EB service is 25\$ for K7M and 30\$ for K9MD. This includes a wide range of expenses, such as bus driver costs, maintenance and repair expenses, and other costs associated with providing reliable and efficient bus service.

Synthetic instance generation. We generated a random data set to evaluate the performance of the heuristic algorithm in different scenarios and conditions. The new data set was created by taking inspiration from the main data set, consisting of trip information. The random data set is available online (see EBSP *Synthetic instance* data set on page www.ihsanyanikoglu.com/data-sets).

Uncertainty levels. In numerical experiments, we solve the [EBSP] and [RBSP] for different scaling vector values and budget parameters and compare their solution performances. For the travel times, the uncertainty levels were assumed to be identical for all segments of all trips in each instance and were set to $\rho_{ri}^t \in 0.25, 0.35$. To determine the uncertainty level of the EB weights, the capacity of the EBs should be taken into account. The nominal values of the weights were assumed to be the average

curb and gross weight, and the half-lengths of the EB weights for trips that can be covered with both EBs are calculated as $\rho_{rik}^m \times m_{rik}^0 = 1910.5$ and $\rho_{rik}^m \times m_{rik}^0 = 2180$, respectively. Based on these values, the uncertainty levels for the weights should be set to $\rho_{ri1}^m \leq 0.15$ and $\rho_{ri2}^m \leq 0.904$ for trips that can be covered with both EBs, and $\rho_{ri2}^m \leq 0.907$ for trips that K9MD can only cover. For the deterministic problem, the weights of the EBs were assumed to be equal to their nominal value, and the robust problems were solved for $\rho_{ri1}^m \in 0.07, 0.15$ and $\rho_{ri2}^m \in 0.045, 0.09$. It was also assumed that the scaling vectors of weights of each EB were identical for all trip segments in each instance. To detect budget parameters of trips, we use the following formula where $\bar{\Gamma}$ is the parameter to control the total conservatism level of the problem and calculate Γ_i values individually.

$$\bar{\Gamma} = \frac{\sum_{r \in S_i}^{\Gamma_i} t_{ri}}{\sum_{r \in S_i} t_{ri}} \quad \forall i \in T \quad (94)$$

The equation is used to calculate the Γ_i values for each trip for given $\bar{\Gamma}$. In this equation, S_i is the set of segments in trip i , and t_{ri} is the length of each segment r in trip i . In this study, the robust problems are solved for $\bar{\Gamma} \in \{0.3, 0.8\}$.

4.6.2 Heuristic algorithm performance

In this subsection, we discuss the OFV and CPU time performances of the heuristic algorithm compared to the standard approach and present the numerical results in Table 27. We randomly generate five data sets for each size and report the average performances. The first column represents the number of trips, columns two and three show the OFV performance, and columns four and five show CPU times. Optimality gaps are presented in the last column for instances that cannot be solved within the given time limit (6 hours) with the standard approach that uses a commercial solver.

The results demonstrate that the heuristic algorithm is an effective solution approach for instances with 50 and 100 trips, achieving results with a gap of less than

Table 27: Comparison of heuristic and standard approach

number of trips (I)	OFV		CPU		gap
	SA	Heuristic	SA	Heuristic	
50	*	*	21600	470	2%
100	*	1%	21600	616	3%
250	*	5%	21600	4525	8%
500	2%	*	21600	5636	17%
1000	19%	*	21600	10277	52%
1500	m	*	m	20687	m
237 (case study)	*	3%	21600	4755	6%

SA: standard approach where [EBSP] is solved by solver

(*) denotes the best OFV performance; (m) denotes the memory error

1% compared to the standard approach in less than an hour. As the number of trips increases, the optimality gap of the standard approach increases. For example, the commercial solver terminates with a more than 17% optimality gap for instances with 500 trips. In contrast, the heuristic algorithm outperforms the standard approach in terms of both CPU time and OFV for $I = 500$ by 2%. Additionally, the benefits of the heuristic approach become even more apparent when the number of trips increases to $I = 1000$ or 1500, where the standard approach cannot provide an efficient solution due to memory constraints. The heuristic algorithm, however, is capable of producing a solution in less than 6 hours. The performance of the heuristic algorithm for the case study is analyzed in more detail in Appendix C.3.

4.6.3 Deterministic and robust solutions comparison

In this section, we will examine the details of the deterministic solution, reviewing schedules and analyzing cost breakdowns next, we will compare the deterministic and robust solutions. The summary of the deterministic is presented in 28. Five EBs of type K9MD and three EBs of type K7M should be purchased to cover the trips. Trips displayed in boldface indicate that the EB must recharge after covering the trip, and the second number in parentheses indicates the charging station to visit after completing the trip. All schedules start and end at a charging station, and the

first and last numbers in each schedule show the associated charging station. For instance, in the first schedule, EB type K7M leaves the first charging station to start the daily schedule with trip 401. After finishing trips 603 and 528, it visits the first charging station to recharge its battery. After completing trip 339, the schedule ends by fully recharging the battery in the first charging station.

Table 28: Schedule for deterministic solution

EB type	covered trips
K7M	[1, 401, 201, 504, 204, 510, (603, 1) , 515, 212, 519, 310, 521, 311, 108, 313, 411, 220, 413, 222, 714, 415, 224, (528, 1) , 720, 903, 1002, 1004, 906, 430, 239, 137, (339, 1) , 1]
K7M	[1, 501, 202, 506, 602, 703, 704, 705, 706, 707, 708, 107, 312, 410, 219, 412, 221, (526, 2) , 320, 716, 901, 718, 719, (801, 1) , 802, 1003, 803, (427, 2) , 335, 804, (730, 1) , 1]
K7M	[1, 503, 301, 702, 511, 604, 105, 214, 408, 216, 409, 218, 110, 315, 112, 317, 414, 223, 116, 613, (717, 1) , 902, 1001, 721, 722, (724, 1) , 131, 236, 134, 336, 907, 138, (340, 1) , 1]
K9MD	[2, 601, 402, 302, 508, 304, 103, 307, 406, 213, 520, 217, 710, 711, 525, 611, 117, 614, 418, 227, 420, 229, 422, 231, 424, 233, 726, 727, 429, 238, (729, 1) , 1]
K9MD	[1, 101, 303, 102, 305, 513, 210, 516, 606, 709, 109, 609, 111, 316, 114, 319, 527, 321, 416, 225, 120, 324, 122, 615, 124, 235, (135, 2) , 2]
K9MD	[1, 701, 509, 207, 512, 306, 104, 605, 407, 215, 522, 608, 524, 610, 113, 318, 115, 612, 118, 323, 121, 228, 421, 616, 126, 232, 129, 331, 426, 333, 133, (337, 1) , 1]
K9MD	[1, 505, 205, 403, 208, 405, 211, 517, 322, 417, 226, 419, 326, 125, 327, 127, 329, 425, 234, 428, 237, 136, 338, (805, 1) , 1]
K9MD	[1, 502, 203, 507, 206, 404, 209, 514, 308, 518, 309, 106, 607, 523, 314, 712, 713, 715, 119, 325, 123, 230, 423, 328, 128, 330, 905, 132, 334, (728, 1) , 1]

The starting and ending points of a schedule don't have to be the same, but the number of EB of each type that leaves a charging station should match the number of EBs of the same type that ends the schedule in that associated charging station. In this solution, three K7M and three K9MD EBs leave the first charging station, and the same number of each EB returns to that charging station at the end of the day. However, one K9MD EB starts the daily schedule from the second charging station, and at the end of the day, one K9MD will be in that charging station. In total, the EBs are charged 14 times, and the first charging station is used more frequently. This leads us to examine the impact of having one charging station instead of two in Section 4.6.5. The solution demonstrates that the EBs with a smaller battery capacity (K7M) need to visit charging stations more frequently, whereas the EBs with a larger battery capacity (K9MD) only require recharging at the end of the daily schedule in order to fully refill the battery and prepare it for the next day.

We evaluate the OFVs of the deterministic and robust solutions for different uncertainty levels and budget parameters. The results are shown in Table 29 and Figure 11. The first row displays the results of the deterministic problem, which serves as a benchmark for the robust models listed in the following rows. Column two shows the uncertainty levels, and column three displays the difference between the robust and deterministic objective functions. Columns four to six show the percentage of purchasing, charging, and operational costs with respect to the total cost of the associated solution. The last column shows the number of EBs required for each type. Operating costs comprise the most considerable portion of the total cost, with 2028\$ for the deterministic solution. As the RO approach is worst-case oriented, the OFVs increase as the uncertainty level and budget parameter increase. This means we will need more EBs and energy to account for more conservative travel time and energy consumption scenarios. When the uncertainty level is 0.35%, and the budget is 0.8, the total cost for the robust model is 41% higher than the deterministic model.

Table 29: Breakdown of costs

instance number	uncertainty levels ($\rho^t, \rho_1^m, \rho_2^m, \bar{\Gamma}$)	OFV	cost type percentages			
			P	C	O	NV
1	(0.0, 0.0, 0.0, 0.0)	bench.	24%	10%	66%	(3,5)
2	(0.25, 0.075, 0.045, 0.3)	25%	27%	8%	65%	(5,6)
3	(0.25, 0.075, 0.045, 0.8)	26%	26%	8%	66%	(5,6)
4	(0.35, 0.15, 0.09, 0.3)	40%	27%	9%	64%	(4,8)
5	(0.35, 0.15, 0.09, 0.8)	41%	29%	8%	63%	(5,8)

OFV: objective function value; P: purchase; C: charging; O: operational
NV: number of EB (type one, type two)

Figure 11 illustrates the cost breakdown for each component of the objective function. The cost increases are displayed in the figure, and all cost components increase as we get more conservative solutions. Checking the charging cost shows an exception to the cost increase. It was expected that the charging cost would increase as the uncertainty levels increased; however, when the uncertainty is 0.35 and $\bar{\Gamma}$ increases from 0.3 to 0.8, the charging cost decreases despite the overall cost

increase. A detailed survey and study show that this is due to buying more EBs when $\bar{\Gamma} = 0.8$, leading to a more flexible charging schedule and taking advantage of discounted charges during off-peak hours.

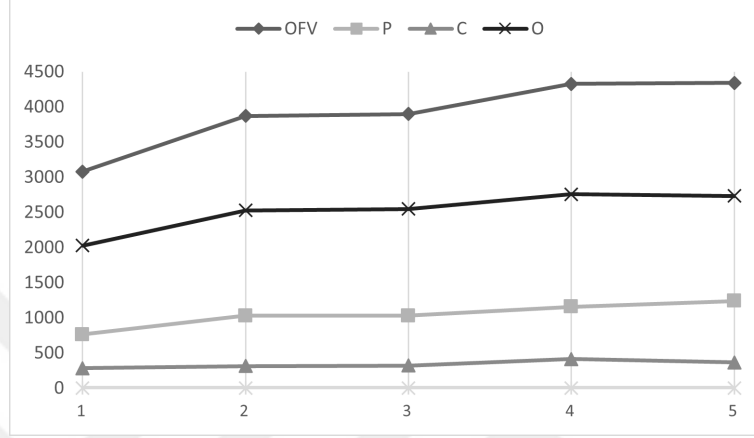


Figure 11: Breakdown of costs

4.6.4 Monte Carlo simulation procedure

Comparing robust and deterministic solutions can lead to incorrect conclusions because each model is solved for a different data set. Specifically, the deterministic model is solved for their nominal travel times and EB weights, and the robust model is solved for the worst-case realizations. To this end, we propose a Monte Carlo simulation approach to evaluate the optimality and solution performance of the models fairly. To accomplish this, we will randomly sample travel times and EB weight from an uncertainty set for different segments, then use equation (88) and the given procedure in Section 4.4 to determine the energy consumption for each trip. Next, we determine the arrival time of EBs at the starting point of each trip by taking into account the travel time of the previous trip, the duration of deadhead trips before the trip, and the necessary charging time. We also update the battery level based on the sampled parameters. If the battery level is insufficient to complete the trip, the bus must be charged, and the charging time must be included in the calculation. It is essential to check the battery level of EBs before starting the trip and ensure that

it has enough energy to complete the trip and travel from the ending point of the trip to the starting point of the next trip or to a charging station. Failure to address this issue can result in the EB battery becoming below the allowed minimum level during the trip or in a location far from a charging station. Due to realized travel times and charging requirements that may deviate from the pre-plan, EB may start trips with a delay and not be able to adhere to the proposed solution. The delay in starting the trip is recorded, and if it exceeds a certain threshold, the trip will be considered as missed. It should be noted that when the proposed solution is infeasible for the realized parameters, we regain feasibility by allowing the EB to begin the trip with a delay. When a trip is missed, the EB should move to the start point of the next trip to cover that without covering the current trip. The EB may also arrive at the starting point of a trip earlier than its scheduled starting time, which can occur when travel times are realized shorter than the nominal value. This can happen in both deterministic and robust solutions. Additionally, this can happen for a trip if the previous trip is missed. In such cases, the EB should wait at the starting point until the scheduled start time and cannot begin the trip earlier than planned. We repeat the procedure for each EB schedule in a solution and record the total number of missed trips and delays. The whole procedure is repeated for a predefined number of iterations, and we report the average number of missed trips and delay time. The input parameters, the variables obtained through the simulation, and the pseudo-code of the simulation algorithm are given in Appendix C.4. The simulation is iterated for 100 iterations in this study, and the average solutions are reported.

In-sample performance of models. We aim to evaluate the effectiveness of using robust optimization to address uncertainty, as well we aim to calculate the price of robustness and discuss the trade-off between the cost and robustness while uncertain parameters are realized *within* the associated uncertainty sets. The results of the simulation are summarized in Table 30, where the first column shows the uncertainty

set used to sample uncertain parameters, and the second column shows the specific uncertainty parameters for the solved instance. In this simulation, EBs may start trips with delays if they cannot arrive at the start point on time. This delay may be due to travel times being longer than the nominal value or the need to recharge the battery due to increased consumption. In columns three and four, we assumed that the maximum allowed delay for starting a trip is 10 minutes, and if the EB arrives at the start point more than 10 minutes late, the trip will be missed. Column three presents the performance of the deterministic solution, while column four presents the performance of the robust solution. Columns five and six show the simulation results when the EBs are allowed to start trips with a 20-minute delay.

Table 30: In-sample simulation results

sampling distribution	$(\rho^t, \rho_1^m, \rho_2^m, \bar{\Gamma})$	mt,dt,tdt, delay =10		mt,dt,tdt, delay =20	
		[EBSP]	[RBSP]	[EBSP]	[RBSP]
$\mathbb{U}(-\rho^t, \rho^t)$ $\mathbb{U}(-\rho^m, \rho^m)$	(0.25, 0.075, 0.045, 0.3)	(0, 11, 7)	(0, 0, 0)	(0, 11, 7)	(0, 0, 0)
	(0.25, 0.075, 0.045, 0.8)	(0, 11, 8)	(0, 0, 0)	(0, 11, 8)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.3)	(0, 12, 11)	(0, 0, 0)	(0, 12, 11)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(0, 13, 13)	(0, 0, 0)	(0, 13, 13)	(0, 0, 0)
$\mathbb{U}(0, \rho^t)$ $\mathbb{U}(0, \rho^m)$	(0.25, 0.075, 0.045, 0.3)	(1, 34, 82)	(0, 0, 0)	(0, 36, 110)	(0, 0, 0)
	(0.25, 0.075, 0.045, 0.8)	(2, 30, 68)	(0, 0, 0)	(0, 36, 118)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.3)	(3, 49, 125)	(0, 0, 0)	(1, 54, 178)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(5, 48, 121)	(0, 0, 0)	(2, 55, 205)	(0, 0, 0)
$\mathbb{T}(\rho^t, +\rho^t, \frac{\rho^t}{2})$ $\mathbb{T}(\rho^m, +\rho^m, \frac{\rho^m}{2})$	(0.25, 0.075, 0.045, 0.3)	(0, 24, 25)	(0, 0, 0)	(0, 24, 25)	(0, 0, 0)
	(0.25, 0.075, 0.045, 0.8)	(0, 24, 27)	(0, 0, 0)	(0, 24, 27)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.3)	(1, 22, 42)	(0, 0, 0)	(0, 23, 47)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(1, 22, 45)	(0, 0, 0)	(0, 23, 58)	(0, 0, 0)

As may be anticipated, the solutions yielded by the robust model are resilient for any realization of the uncertain parameters within the associated uncertainty set. The results show that even if the travel times are realized within a symmetric uniform set, allowing parameters to take values less than the nominal values, deterministic solutions may not be feasible however, the infeasibility level is not significant, e.g., in the first row, 11 trips are delayed where the delay is minutes, which is less than one minute per trip and can be ignored. It should be highlighted that whenever trip times are realized within the uniform set, any larger realizations of travel times in segments can be offset by smaller realizations, resulting in the total trip time converging to its nominal value. However, in real-life situations, this may not always be the case, and

the total trip time may converge to a value that is larger than the nominal value. To address this, we repeated the simulation by sampling uncertain parameters from more conservative sets.

When parameters are sampled from $\mathbb{U}(0, +\rho)$, the model exhibits the highest level of infeasibility. As demonstrated in the table, setting uncertainty levels to $(\rho^t = 0.35, \rho_1^m = 0.15, \rho_2^m = 0.09, \bar{\Gamma} = 0.8)$ to sample uncertain parameters and setting allowed delay to 10 minutes leads the deterministic solution to miss, on average, five trips and start 48 with delay. Increasing the allowed delay threshold will lead to fewer missed trips but more overall delays. This occurs because when a trip is missed, the EB will proceed to the next trip without covering the missed one, which can compensate for some part of the accumulated delays and prevent the delay from cascading into a 'bullwhip effect. Sampling the travel times of the trip segments from a less conservative set will decrease the infeasibility of the solutions. As shown in the table, the simulation is iterated by sampling travel times within triangular distribution, and the infeasibility decreases as the set is less conservative than $\mathbb{U}(0, +\rho)$. When the uncertainty levels are fixed, and $\bar{\Gamma}$ is increased, we can expect a higher level of infeasibility in the deterministic solution. This fact is reflected in the results. For instance, in rows seven and eight, the uncertainty levels are the same, but the performance of the deterministic solution is evaluated with different $\bar{\Gamma}$ values used in generating the energy consumption parameters. The results demonstrate that as $\bar{\Gamma}$ increases from 0.3 to 0.8, the number of missed trips increases from three to five. It should be highlighted that decreasing the average number of delayed trips is caused because when a trip is missed, the EB can proceed directly to the starting point of the next trip, compensating for some part of the total cumulated delays in the schedule. In brief, the feasibility performance of the deterministic solution decreases as $\bar{\Gamma}$ increases and travel times and EB weights are realized within more conservative sets.

Out-of-sample performance of the models. The simulation is repeated by sampling uncertain parameters *out* of the uncertainty sets. This allows for comparing the average solution performance between deterministic and robust solutions when the uncertain parameters are outside of the given uncertainty set. We sample the travel times from uniform $0.1 + \mathbb{U}(-\rho^t, \rho^t)$ and triangular $0.1 + \mathbb{T}(-\rho^t, \rho^t, \frac{\rho^t}{2})$ distributions for $\rho^t \in \{0.25, 0.35\}$. In this procedure for generating uncertainty parameters, $\bar{\Gamma} = 1$, $\rho_1^m = 0.15$, and $\rho_2^m = 0.09$. The results of the simulation are summarized in Table 31.

Table 31: Out-of-sample-simulation results

sampling distribution for (ρ^t)	$(\rho^t, \rho_1^m, \rho_2^m, \bar{\Gamma})$	mt,dt,tdt	
		delay=10	delay=20
$\mathbb{U}(0, 0.35)$	(0.0, 0.0, 0.0, 0.0)	(4, 80, 214)	(3, 83, 270)
	(0.25, 0.075, 0.045, 0.3)	(0, 1, 1)	(0, 1, 1)
	(0.25, 0.075, 0.045, 0.8)	(0, 1, 1)	(0, 1, 1)
	(0.35, 0.15, 0.09, 0.3)	(0, 0, 0)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(0, 0, 0)	(0, 0, 0)
$\mathbb{U}(0, 0.45)$	(0.0, 0.0, 0.0, 0.0)	(12, 92, 320)	(6, 103, 475)
	(0.25, 0.075, 0.045, 0.3)	(0, 16, 10)	(0, 16, 10)
	(0.25, 0.075, 0.045, 0.8)	(0, 14, 7)	(0, 14, 7)
	(0.35, 0.15, 0.09, 0.3)	(0, 1, 1)	(0, 1, 2)
	(0.35, 0.15, 0.09, 0.8)	(0, 0, 0)	(0, 0, 0)
$\mathbb{T}(-0.15, 0.35, \frac{22.5}{2})$	(0.0, 0.0, 0.0, 0.0)	(2, 38, 125)	(0, 42, 148)
	(0.25, 0.075, 0.045, 0.3)	(0, 0, 0)	(0, 0, 0)
	(0.25, 0.075, 0.045, 0.8)	(0, 0, 0)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.3)	(0, 0, 0)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(0, 0, 0)	(0, 0, 0)
$\mathbb{T}(-0.25, 0.45, \frac{27.5}{2})$	(0.0, 0.0, 0.0, 0.0)	(4, 48, 123)	(2, 54, 207)
	(0.25, 0.075, 0.045, 0.3)	(0, 1, 1)	(0, 1, 1)
	(0.25, 0.075, 0.045, 0.8)	(0, 1, 1)	(0, 1, 1)
	(0.35, 0.15, 0.09, 0.3)	(0, 0, 0)	(0, 0, 0)
	(0.35, 0.15, 0.09, 0.8)	(0, 0, 0)	(0, 0, 0)

According to the simulation results, the deterministic model may generate infeasible solutions with significant infeasibility levels when the parameters are realized within more conservative uncertainty sets than those discussed in the previous section. The simulation results indicate that the highest level of infeasibility for the deterministic solution occurs when travel times are generated with $\rho^t \in \mathbb{U}(0, 0.45)$. On average, 12 trips are missed, and 92 trips are delayed when the allowed delay is 10 minutes. If trips are allowed to be delayed by 20 minutes, the number of missed trips reduces to 6. However, on average, 103 trips will be delayed, resulting in a total delay of 475 minutes. The performance of more conservative solutions shows that they are

usually feasible even when parameters are realized out of the sample. When the robust model is solved for $(\rho^t = 0.35, \rho_1^m = 0.15, \rho_2^m = 0.09, \bar{\Gamma} \in \{0.3, 0.8\})$, infeasibility is minimal, occurring only when ρ^t realizes within the $\mathbb{U}(0, 0.45)$ range. Reducing conservativeness to $(\rho^t = 0.25, \rho_1^m = 0.075, \rho_2^m = 0.045, \bar{\Gamma} \in \{0.3, 0.8\})$ will result in lower costs but may increase infeasibility and delayed trips. However, it is important to note that the level of infeasibility and overall daily schedule delay is relatively minor. The total delay for the daily schedule is between 14 and 16 minutes when ρ^t realizes within the $\mathbb{U}(0, 0.45)$ range. Furthermore, we observed negligible infeasibility when using $0.1 + \mathbb{T}(-\rho^t, \rho^t, \frac{\rho^t}{2})$ distributions to sample uncertain parameters. When making a decision, a trade-off between infeasibility and the cost of purchasing additional EBs should be considered. Based on the cost of robustness presented in Table 29 and the infeasibility levels shown in Tables 30 and 31, purchasing 4 K7M and 8 K9MD EBs appears to be a reasonable choice for our problem. While higher conservatism levels and purchasing additional EBs would increase the cost, they may not have considerable benefits.

4.6.5 Effect of charging station selection on OFV

The analysis in this section provides a strategic decision-making perspective for a decision-maker to determine the worthiness of investing in a charging station. The [EBSP] considers multi-charge stations and can be solved for a single charging station without any modification. In this subsection, we solve the deterministic and robust model $(\rho^t = 0.25, \rho_1^m = 0.075, \rho_2^m = 0.045, \bar{\Gamma} = 0.3)$ for a single charging station in two different locations and compare the solution with the problem solved for multi-charge stations. The objective is to assess whether having multiple charging stations would lead to higher profitability or whether having only one charging station would impact the daily costs of the schedule. The results are summarized in Table 32 and show the conservatism level, active charging station, the OFV, and the number of

the required EBs for each model. It is important for the decision-maker to carefully

Table 32: Costs analysis for different charging station

uncertainty level ($\rho^t, \rho_1^m, \rho_2^m, \bar{\Gamma}$)	available CS	OFV	cost type percentages			NV
			P	C	O	
deterministic (0.0, 0.0, 0.0, 0.0)	(c_1, c_2)	3079 ¹ \$	24%	10%	66%	(3,5)
	(c_1)	1%	27%	8%	65%	(4,5)
	(c_2)	4%	26%	8%	66%	(4,5)
robust (0.25, 0.075, 0.045, 0.3)	(c_1, c_2)	3872 ² \$	27%	8%	65%	(5,6)
	(c_1)	1%	26%	8%	66%	(5,6)
	(c_2)	3%	26%	8%	66 %	(5,6)

OFV: objective function value; P: purchase; C: charging; O: operational

CS: (charging station) NV: number of EB (type one, type two)

1,2: benchmarks for deterministic and robust solutions respectively

consider the cost and benefits of opening charging stations at each candidate location. Opening a single charging station at Union (c_1) is expected to increase the total cost by 1% for both deterministic and robust models. This translates to an increase of approximately 30\$ for the deterministic model and 38\$ for the robust model. In addition, for the deterministic model, an extra K7M EB is required. On the other hand, opening a single charging station at the downtown center of the university (c_2) is expected to increase the total cost by 4% for the deterministic model and 3% for the robust model. The decision-maker should consider the trade-off between paying a higher daily cost and opening two charging stations and make the final decision accordingly. However, if both candidate locations have the same opening costs, it is rational to choose location c_1 if only one charging station is going to be opened.

In this study, due to the limited number of charging station candidates and to keep the model simple, the problem was formulated in its current form without considering these additional costs. Instead of determining the optimal number and location of charging stations in the model, we focused on analyzing the impact of existing charging stations on the scheduling and charging plans. We provided a perspective to the decision-makers on how opening charging stations in different locations may

affect the daily schedule.

4.7 *Conclusion and Managerial implications*

This study aims to address the critical problem of scheduling and charging *electric buses* (EBs) in a transportation system, taking into account travel time and energy consumption uncertainties. A mixed-integer linear model was proposed, and *robust optimization* (RO) approach was employed to provide resilient solutions against parameter uncertainty. The model determines the number of EBs required to cover a set of trips, schedules the trips, and creates charging plans while minimizing the cost of purchasing, charging, and operating EBs. A Monte Carlo simulation was conducted to compare the average solution performance of the deterministic model and the robust ones. The case study results at Binghamton University showed the effectiveness of the proposed model and the robust approach in optimizing the cost of operating an EB network. According to the results, the deterministic problem may yield infeasible solutions for real data realization. The total delay in a daily schedule may increase to 121 minutes, and five trips may be missed because EBs can not start the trips even with the allowed delay. The benefits of RO become more apparent when we conduct the simulation for more conservative uncertainty levels and the out-of-sample realization of uncertain parameters. The deterministic solution may miss 12 trips and face 92 delayed trips with 320 minutes delay in total, while robust models are still resilient, and the solution yielded by the robust model with conservatism level of $(\rho^t = 0.25, \rho_1^m = 0.075, \rho_2^m = 0.045, \bar{\Gamma} = 0.8)$ may be the best choice for the decision-maker. This model is resilient for any in-sample realization of the uncertain parameters and for out-of-sample realizations, shows acceptable performance, and in the worst case, 16 trips are delayed while the average delay for each trip is less than a minute. Notice that the deterministic solution suggests purchasing 8 EBs in total; however, the associated robust model suggests purchasing 11 EBs. The results of

this study can guide the development and deployment of EB networks in other transportation systems and inform the decision-making process for their implementation and the trade-off between investing more and being penalized for delayed and missed trips.

The *managerial implications* of this study are significant. The proposed model can be used by decision-makers in transportation systems to optimize the cost of operating an EB network and decide on the EB types and also the location of the charging stations. The results suggest that considering uncertainty is important, as ignoring it may result in infeasible solutions. On the other hand, increasing the level of conservatism may yield more robust solutions but also impose additional costs that may not be worth it. The proposed model in this research is versatile in considering different types of EBs available in the market.

The model can be relatively easy to implement in practice as long as the necessary data and information are available. However, there may be some obstacles, such as increasing the size of the problem and the difficulty in obtaining accurate data on travel time and energy consumption, as well as the computational challenges posed by the optimization model. These limitations should be considered when implementing the model in a real-world scenario. In this regard, the RO method is suggested to address the parameter uncertainty, and we solve the problem when we do not have information on the distributional information of the uncertain parameters, and we can just estimate the upper and the lower bounds. Moreover, the heuristic algorithm is proposed to solve large-sized problems such as the urban transportation network. The proposed heuristic algorithm provides near-optimal solutions for small and medium-sized instances and yields solutions for large-sized instances where the standard approach fails to solve the problem efficiently. If the decision-maker wants to use the model to determine the number of charging stations needed, an extended formulation can be used without affecting the solution approach. However, it should

be noted that estimating the amortized costs of strategic decision variables can be challenging, and the costs of installing and maintaining the charging stations should be considered as well. Another factor that should be taken into account is the potential degradation of battery capacity over time. To address this issue, upper and lower limits are considered, and it is assumed that batteries are not charged to their full capacity. This approach ensures that the EBs can fulfill the schedule efficiently even as batteries age. It is worth noting that many of the parameters in this model are dependent on the vehicle type, such as energy consumption and charging rate. Over time, these parameters may vary, and two electric EBs of the same type may perform differently. To address this issue, EB of the same type that may perform differently may be considered as different types of EBs, and the decision-maker can solve the problem again for the given number of EBs and updated parameters.

In comparison with other models in the literature, this study provides a comprehensive analysis of the properties of a possible EB network, taking into account the uncertainty in travel time and energy consumption, and provides a Monte Carlo simulation to conduct a fair comparison between robust and deterministic solutions. The case study results at Binghamton University provide a real-world example of the effectiveness of the proposed model and the robust approach.

In conclusion, the proposed model can result in significant cost savings for transportation systems by optimizing the cost of operating an EB network while considering the travel time and energy consumption uncertainties. This model benefits decision-makers in selecting electric vehicles for school, university, and airport shuttles and converting *combustion engine vehicles* (CEVs) to electric ones. Additionally, the proposed heuristic algorithm can solve larger instances that are not solvable with standard approaches using commercial solvers, making this model applicable to urban transportation systems with large numbers of trips.

CHAPTER V

CONCLUSIONS

In this thesis, we investigate three optimization problems, and for each problem, we propose an optimization model. To handle the uncertainty associated with each problem, we adopt the *robust optimization* (RO) approach.

In Chapter 2, we addressed a *dairy production and distribution planning problem* (PDP) that decides on the production and distribution plan of dairy products to minimize the total cost of production, warehousing, backorder, and distribution. We adopted RO and proposed *robust* PDP (RPDP) to deal with the uncertainty of retailers' demand. Since RPDP is a worst-case-oriented approach, it may yield over-conservative solutions; therefore, the *adjustable* RO was employed, and an adjustable variant of RPDP was formulated. The approach allows the producer to adjust production, inventory, backorder, and distribution decisions according to the realized part of demands. A Monte Carlo simulation was developed to compare and evaluate the average objective function value and CPU time performance of models. We proposed a heuristic approach that yields high-quality solutions for the associated models in less than three hours for realistically sized instances. The performance of the developed algorithm was compared for different-sized test instances and a real-life case study from a dairy factory located in the Middle East. The proposed solution approaches may be used as a decision support tool that can be efficiently used in practice. Finally, it is essential to mention that different industries that show similarities to the dairy industry, such as health care and catering, may benefit from the presented research.

In Chapter 3, we studied the post-disaster relief supply distribution problem that

decides the visiting order of sites and the service time of distributing relief supplies to the affected community at each site in a post-disaster setting. The problem aims to maximize the total utility of supplies distributed by achieving equitable distribution among the visited sites. We adopted the adjustable robust optimization paradigm to gain resilience against travel time uncertainty in the problem data. According to the results, the highest objective function value and the total service time improvements for the robust approach were achieved where the resources were more scarce, and the need for adjustability became more important. Therefore, humanitarian organizations must aim for adjustable robust approaches since the time limit is often a hard constraint. It is also shown that the improvement made by the adjustable approach over the static robust increased when the number of RLTs decreased. All in all, the numerical results showed the effectiveness of the proposed model and the solution approach.

A study is conducted in Chapter 4 to find a solution to the problem of scheduling and charging *electric buses* (EB)s in a transportation system. This involves considering uncertainties in travel time and energy consumption. To achieve this, we developed a mixed-integer linear model and used a RO approach to obtain resilient and feasible schedules against uncertainties. The model determines the required number of EBs for a set of trips, schedules the trips, and creates charging plans while minimizing the cost of purchasing, charging, and operating the buses. A Monte Carlo simulation was conducted to compare the performance of the deterministic model and the robust ones. The results of the case study at Binghamton University showed that the proposed model and the robust approach were effective in optimizing the cost of operating an EB network. However, the deterministic model may yield infeasible solutions for real data realization. On the other hand, the robust model is resilient and provides a better solution, especially for more conservative uncertainty sets and out-of-sample realization of uncertain parameters.

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APPENDIX A

A.1 Robust mathematical optimization model [RPDP]

[RPDP] :

$$\begin{aligned}
 \min_{\mathbf{B}, \mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}} & \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} Y_{jt} + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1}^{(w)} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)}^{(w)} + \\
 & \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} I_{jkwt} + \\
 & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} X_{jrt} + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} B_{jrt} \\
 \text{s.t.} \quad & X_{jr(t-\ell_r)} \geq (1 + \rho) d_{jrt}^0 - B_{jrt} + B_{jr(t-1)} \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \quad (95) \\
 & B_{jrt} = \sum_{t'=1}^t (1 + \rho) d_{jrt'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \{1, \dots, \ell_r\} \quad (96) \\
 & \sum_{t'=1}^t X_{jrt'} \geq \sum_{t'=1}^t (1 + \rho) d_{jrt'}^0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (97) \\
 & (1) - (3), (6), (8) - (55).
 \end{aligned}$$

A.2 Applying three step of RO procedure for inventory capacity constraint

We present three main steps of deriving robust reformulation that is often used in static RO for constraint (21). Since RO approach is a constraint-wise approach, hence we take one of the constraints in the remainder and show all derivations for that. Indices of the constraint is denoted by w^* and t^* . As mentioned in Section 2.3.2 we can formulate demand as $d_{jrt} = d_{jrt}^0(1 + \rho\zeta_{jrt})$. Then we reformulate a single constraint (21) as follow:

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw^*t^*} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(d_{j'rt'}^0 \left(1 + \rho\zeta_{j'rt'} \right) \beta_{j'rt'}^{jkw^*t^*} \right) \right) \leq Q_{w^*t^*}^{(w)} \quad \forall \zeta \in \mathcal{U}.$$

Step 1. According to the RO definition, if the constraint is robust for the worst-case realization of the uncertainty parameter, then any realization of the uncertainty set

will not violate the constraint. Thus, we reformulate the constraint as follow:

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw^*t^*} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw^*t^*} \right) + \max_{-1 \leq \zeta \leq 1} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\rho d_{j'rt'}^0 \alpha_{j'rt'}^{jkw^*t^*} \zeta_{j'rt'} \right) \leq Q_{w^*t^*}^{(w)}. \quad (98)$$

Step 2. Next, we take the dual of the inner maximization problem in (98). According to strong duality, the primal and dual problems yield the same optimal OFV since the inner maximization problem is a linear programming problem over the uncertainty parameter ζ .

$$\text{fixed terms} \min_{\mathbf{u} \geq 0, \mathbf{v} \leq 0} \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(u_{j'rt'}^{w^*t^*} - v_{j'rt'}^{w^*t^*} \right) \leq Q_{w^*t^*}^{(w)} \quad (99)$$

$$\text{s.t.} \quad u_{j'rt'}^{w^*t^*} + v_{j'rt'}^{w^*t^*} = \left(\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw^*t^*} \right) \rho d_{j'rt'}^0 \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, t' \in [1, t^*], \quad (100)$$

where $[\text{fixed terms}] = \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw^*t^*} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw^*t^*} \right)$ and $\mathbf{u}$ and $\mathbf{v}$ are the dual variables.

Step 3. By weak duality, the dual minimization problem yields an upper bound for the optimal OFV of the primal problem. Therefore, we can remove the minimization operator in (99) and get rid of nested minimization in the constraint. The resulting constraint is presented below:

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw^*t^*} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw^*t^*} \right) + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(u_{j'rt'}^{w^*t^*} - v_{j'rt'}^{w^*t^*} \right) \leq Q_{w^*t^*}^{(w)}$$

$$u_{j'rt'}^{w^*t^*} + v_{j'rt'}^{w^*t^*} = \left(\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw^*t^*} \right) \rho d_{j'rt'}^0 \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, t' \in [1, t^*]$$

$$\mathbf{u} \geq 0, \mathbf{v} \leq 0.$$

The associated three steps for the single constraint, will be repeated for all the constraints ($\forall w^* \in \mathcal{W}, t^* \in \mathcal{T}$) and the resulting the tractable reformulation of constraint (21) is given as follow:

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw^*t^*} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw^*t^*} \right) + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(u_{j'rt'}^{wt} - v_{j'rt'}^{wt} \right) \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (101)$$

$$u_{j'rt'}^{wt} + v_{j'rt'}^{wt} = \left(\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \beta_{j'rt'}^{jkw^*t^*} \right) \rho d_{j'rt'}^0 \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (102)$$

$$\mathbf{u} \geq 0, \mathbf{v} \leq 0. \quad (103)$$

Remark 2 The following formulation:

$$\begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t d_{j'rt'}^0 \alpha_{j'rt'}^{jkw} \right) + \\ & \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\left| \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw} \right| \rho d_{j'rt'}^0 \right) \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (\text{re-35}) \end{aligned}$$

is equivalent to (101)–(103) after the absolute value expression is linearized. By substituting the maximum value of ζ for the inner maximization problem in Step 1, we can get the same robust reformulation in Step 3 and linearize the resulting absolute value expression in Remark 2.

A.3 Tractable reformulation of [ARPD P]

$ax_{jj'rr'tt'}^1$: is 1, if $j = j', r = r', t = t'$; o.w. 0
 $ax_{jj'rr'tt't''}^2$: is 1, if $j = j', r = r', t = t' = t''$; o.w. 0
 $ax_{jj'rr'tt't''}^3$: is 1, if $j = j', r = r', t = t' = t''$; o.w. 0
 AX^v : Auxiliary decision variable $v \in \{0, \dots, 11\}$

[ARPD P]:

$$\begin{aligned} & \min_{\substack{\bar{B}, \bar{I}, \bar{X}, \mathbf{Z} \\ \alpha, \gamma, \beta}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 \right) + \right. \\ & \left. \left(\sum_{r \in \mathcal{R}} \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \right) + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{k' \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \\ & \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkw} d_{j'rt'}^0 \right) + \\ & \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \left(\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \\ & + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} AX_{jrt}^0 \rho d_{jrt}^0 \end{aligned}$$

s.t.

$$\begin{aligned}
AX_{j'r't'}^0 &\geq \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\alpha_{j'r't'}^{jkw t} - \alpha_{j'r't'}^{jkw(t-1)} \right) + \sum_{r \in \mathcal{R}} \beta_{j'r't'}^{jrt} \right) + \\
&\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \alpha_{j'r't'}^{jkw t} + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \beta_{j'r't'}^{jrt} + \\
&\sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \gamma_{j'r't'}^{jrt} \quad \forall j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in \mathcal{T} \quad (104)
\end{aligned}$$

$$\begin{aligned}
AX_{j'r't'}^0 &\geq - \left(\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\alpha_{j'r't'}^{jkw t} - \alpha_{j'r't'}^{jkw(t-1)} \right) + \sum_{r \in \mathcal{R}} \beta_{j'r't'}^{jrt} \right) + \right. \\
&\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \alpha_{j'r't'}^{jkw t} + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \beta_{j'r't'}^{jrt} + \\
&\left. \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \gamma_{j'r't'}^{jrt} \right) \quad \forall j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in \mathcal{T} \quad (105)
\end{aligned}$$

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(\bar{I}_{jkw t} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'r't'}^{jkw t} d_{j'r't'}^0 + AX_{j'rw t t'}^1 \rho d_{j'r't'}^0 \right) \right) \leq Q_{wt}^{(w)} \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (106)$$

$$AX_{j'rw t t'}^1 \geq \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'r't'}^{jkw t} \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (107)$$

$$AX_{j'rw t t'}^1 \geq - \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \alpha_{j'r't'}^{jkw t} \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (108)$$

$$\begin{aligned}
&\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw t} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'r't'}^{jkw t} - \alpha_{j'r't'}^{jkw(t-1)} \right) d_{j'r't'}^0 + \right. \\
&\sum_{r \in \mathcal{R}} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \Bigg) + \\
&\sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} AX_{jj'r't't'}^2 \rho d_{j'r't'}^0 \leq Q_{jt}^{(p)} \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (109)
\end{aligned}$$

$$AX_{jj'r't't'}^2 \geq \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \alpha_{j'r't'}^{jkw t} - \alpha_{j'r't'}^{jkw(t-1)} + \sum_{r \in \mathcal{R}} \beta_{j'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (110)$$

$$AX_{jj'r't't'}^2 \geq - \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \alpha_{j'r't'}^{jkw t} - \alpha_{j'r't'}^{jkw(t-1)} + \sum_{r \in \mathcal{R}} \beta_{j'r't'}^{jrt} \right) \quad \forall j, j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (111)$$

$$\bar{X}_{jr(t-\ell_r)} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^{(t-\ell_r)} \beta_{jj'r't'}^{jr(t-\ell_r)} d_{j'r't'}^0 + \bar{B}_{jrt} - \bar{B}_{jr(t-1)} - d_{jrt}^0 +$$

$$\sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \left(\sum_{t' \in \mathcal{T}} \left(\gamma_{jj'r't'}^{jrt} - \gamma_{jj'r't'}^{jrt-1} \right) d_{j'r't'}^0 - \sum_{t'=1}^t AX_{jj'rr'tt'}^3 \rho d_{j'r't'}^0 \right) \geq 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T}_{\ell_r} \quad (112)$$

$$AX_{jj'rr'tt'}^3 \geq \beta_{jj'r't'}^{jr(t-\ell_r)} + \gamma_{jj'r't'}^{jrt} - \gamma_{jj'r't'}^{jr(t-1)} - ax_{jj'rr'tt'}^1 \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T}_{\ell_r} \quad (113)$$

$$AX_{jj'rr'tt'}^3 \geq -\beta_{jj'r't'}^{jr(t-\ell_r)} - \gamma_{jj'r't'}^{jrt} + \gamma_{jj'r't'}^{jr(t-1)} + ax_{jj'rr'tt'}^1 \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T}_{\ell_r} \quad (114)$$

$$\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'' \in \mathcal{T}} \gamma_{jj'r't''}^{jrt} d_{j'r't''}^0 -$$

$$\sum_{t'=1}^t d_{jrt'}^0 - \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t''=1}^{t'} AX_{jj'rr'tt't''}^4 \rho d_{j'r't''}^0 \geq 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in [1, \ell_r] \quad (115)$$

$$AX_{jj'rr'tt't''}^4 \geq \gamma_{jj'r't''}^{jrt} - ax_{jj'rr'tt't''}^2 \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t'' \in [1, t'], t, t' \in [1, \ell_r] \quad (116)$$

$$AX_{jj'rr'tt't''}^4 \geq -\left(\gamma_{jj'r't''}^{jrt} - ax_{jj'rr'tt't''}^2 \right) \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t'' \in [1, t'], t, t' \in [1, \ell_r] \quad (117)$$

$$\bar{B}_{jrT} = 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R} \quad (118)$$

$$\gamma_{jj'r't'}^{jrT} = 0 \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t' \in \mathcal{T} \quad (119)$$

$$\sum_{t'=1}^t \left(\bar{X}_{jrt'} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t''=1}^{t'} \beta_{jj'r't''}^{jrt'} d_{j'r't''}^0 \right) -$$

$$\sum_{t'=1}^t d_{jrt'}^0 - \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t''=1}^{t'} AX_{jj'rr'tt't''}^5 \rho d_{j'r't''}^0 \geq 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (120)$$

$$AX_{jj'rr'tt't''}^5 \geq \beta_{jj'r't''}^{jrt'} - ax_{jj'rr'tt't''}^3 \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t'' \in [1, t'], t, t' \in \mathcal{T} \quad (121)$$

$$AX_{jj'rr'tt't''}^5 \geq -\left(\beta_{jj'r't''}^{jrt'} - ax_{jj'rr'tt't''}^3 \right) \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t'' \in [1, t'], t, t' \in \mathcal{T} \quad (122)$$

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw t'} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \alpha_{jj'rt'}^{jkw t} d_{j'rt'}^0 \right) +$$

$$\sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t' \in \mathcal{T}} AX_{jj'rtt'}^6 \rho d_{j'rt'}^0 \leq \bar{D}_j \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (123)$$

$$AX_{jj'rtt'}^6 \geq \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \alpha_{jj'rt'}^{jkw t} \quad \forall j, j' \in \mathcal{J}, r \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (124)$$

$$AX_{jj'rtt'}^6 \geq -\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \alpha_{jj'rt'}^{jkw t} \quad \forall j, j' \in \mathcal{J}, r \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (125)$$

$$\sum_{j \in \mathcal{J}} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} d_{j'rt'}^0 + AX_{jj'krwtt'}^7 \rho d_{j'rt'}^0 \right) \right) \leq MZ_{kw} \quad \forall k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T} \quad (126)$$

$$AX_{jj'krwtt'}^7 \geq \sum_{j \in \mathcal{J}} \alpha_{j'rt'}^{jkw} \quad \forall j' \in \mathcal{J}, k \in \mathcal{K}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (127)$$

$$AX_{jj'krwtt'}^7 \geq - \sum_{j \in \mathcal{J}} \alpha_{j'rt'}^{jkw} \quad \forall j' \in \mathcal{J}, k \in \mathcal{K}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (128)$$

$$\sum_{k \in \mathcal{K}} Z_{kw} = 1 \quad \forall w \in \mathcal{W}, t \in \mathcal{T} \quad (129)$$

$$\bar{I}_{wjkt} = I_{wjkt} \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W} \quad (130)$$

$$\alpha_{jj'r't'}^{wjkT} = 0 \quad \forall j, j' \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W}, r' \in \mathcal{R}, t' \in \mathcal{T} \quad (131)$$

$$\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \left(\gamma_{jj'r't'}^{jrt} d_{j'r't'}^0 - AX_{jj'r'r'tt'}^8 \rho d_{j'r't'}^0 \right) \geq 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (132)$$

$$AX_{jj'r'r'tt'}^8 \geq \gamma_{jj'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t', t \in \mathcal{T} \quad (133)$$

$$AX_{jj'r'r'tt'}^8 \geq -\gamma_{jj'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t', t \in \mathcal{T} \quad (134)$$

$$\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \left(\beta_{jj'r't'}^{jrt} d_{j'r't'}^0 - AX_{jj'r'r'tt'}^9 \rho d_{j'r't'}^0 \right) \geq 0 \quad \forall j \in \mathcal{J}, r \in \mathcal{R}, t \in \mathcal{T} \quad (135)$$

$$AX_{jj'r'r'tt'}^9 \geq \beta_{jj'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (136)$$

$$AX_{jj'r'r'tt'}^9 \geq -\beta_{jj'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (137)$$

$$\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} d_{j'rt'}^0 - AX_{jj'krwtt'}^{10} \rho d_{j'rt'}^0 \right) \geq 0 \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T} \quad (138)$$

$$AX_{jj'krwtt'}^{10} \geq \alpha_{j'rt'}^{jkw} \quad \forall j, j' \in \mathcal{J}, k \in \mathcal{K}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (139)$$

$$AX_{jj'krwtt'}^{10} \geq -\alpha_{j'rt'}^{jkw} \quad \forall j, j' \in \mathcal{J}, k \in \mathcal{K}, r \in \mathcal{R}, w \in \mathcal{W}, t' \in [1, t], t \in \mathcal{T} \quad (140)$$

$$\begin{aligned} & \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 + \right. \\ & \left. \sum_{r \in \mathcal{R}} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{jj'r't'}^{jrt} d_{j'r't'}^0 \right) - \right. \\ & \left. \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} AX_{jj'r'r'tt'}^{11} \rho d_{j'r't'}^0 \right) \geq 0 \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (141) \end{aligned}$$

$$AX_{jj'r'r'tt'}^{11} \geq \sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} + \sum_{r \in \mathcal{R}} \beta_{jj'r't'}^{jrt} \quad \forall j, j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (142)$$

$$AX_{jj'r'r'tt'}^{11} \geq - \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} + \sum_{r \in \mathcal{R}} \beta_{jj'r't'}^{jrt} \right) \quad \forall j, j' \in \mathcal{J}, r' \in \mathcal{R}, t' \in [1, t], t \in \mathcal{T} \quad (143)$$

$$\mathbf{Z} \in \{0, 1\} \quad (144)$$

A.4 Tractable reformulation of [PR](ARPD)

[PR](ARPD):

$$\begin{aligned}
& \min_{\substack{\mathbf{B}, \mathbf{I}, \mathbf{X}, \mathbf{Z} \\ \boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\beta}}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkwt} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkwt} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 \right) + \right. \\
& \left. \left(\sum_{r \in \mathcal{R}} \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \right) + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{k' \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \left(\bar{I}_{jkwt} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkwt} d_{j'rt'}^0 \right) + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \left(\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \\
& \text{s.t.} \quad (104)-(144) \\
& \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkwt} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkwt} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 \right) + \right. \\
& \left. \left(\sum_{r \in \mathcal{R}} \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \right) + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1} + \sum_{k \in \mathcal{K}} \sum_{k' \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kwt} Z_{k'w(t+1)} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kwt} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \left(\bar{I}_{jkwt} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkwt} d_{j'rt'}^0 \right) + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \left(\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \\
& + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} AX_{jrt}^0 \rho d_{jrt}^0 \leq \text{obj}^*[\text{ARPD}]
\end{aligned}$$

A.5 Tractable reformulation of [PR](RPDP)

[PR](RPDP):

$$\begin{aligned}
& \min_{\substack{\bar{B}, \bar{I}, \bar{X} \\ \alpha, \gamma, \beta}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 \right) \right) + \\
& \left(\sum_{r \in \mathcal{R}} \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1}^{\text{RPDP}} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kw}^{\text{RPDP}} Z_{k'w(t+1)}^{\text{RPDP}} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kw}^{\text{RPDP}} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkw} d_{j'rt'}^0 \right) + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \left(\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} d_{j'r't'}^0 \right)
\end{aligned}$$

s.t. (104)–(125), (130)–(143)

$$\begin{aligned}
& \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} C_{jt}^{(p)} \left(\sum_{k \in \mathcal{K}_1^j} \sum_{w \in \mathcal{W}} \left(\bar{I}_{jkw} - \bar{I}_{jkw(t-1)} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} - \alpha_{j'rt'}^{jkw(t-1)} \right) d_{j'rt'}^0 \right) \right) + \\
& \left(\sum_{r \in \mathcal{R}} \bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sigma_{1k} Z_{kw1}^{\text{RPDP}} + \sum_{k \in \mathcal{K}} \sum_{\substack{k' \in \mathcal{K} \\ k \neq k'}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \sigma_{kk'} Z_{kw}^{\text{RPDP}} Z_{k'w(t+1)}^{\text{RPDP}} + \\
& \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_k^{(w)} Z_{kw}^{\text{RPDP}} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} C_{jk}^{(h)} \left(\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \alpha_{j'rt'}^{jkw} d_{j'rt'}^0 \right) + \\
& \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jr} \left(\bar{X}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t'=1}^t \beta_{j'r't'}^{jrt} d_{j'r't'}^0 \right) + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} C_{jrt}^{(b)} \left(\bar{B}_{jrt} + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}} \sum_{t' \in \mathcal{T}} \gamma_{j'r't'}^{jrt} d_{j'r't'}^0 \right) \\
& + \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} AX_{jrt}^0 \rho d_{jrt} \leq \text{obj}^*[\text{RPDP}]
\end{aligned}$$

$$\bar{I}_{jkw} + \sum_{j' \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{t'=1}^t \left(\alpha_{j'rt'}^{jkw} d_{j'rt'}^0 + AX_{j'kwtt'}^1 \rho d_{j'rt'}^0 \right) \leq M Z_{kw}^{\text{RPDP}} \quad \forall k \in \mathcal{K}, w \in \mathcal{W}, t \in \mathcal{T}$$

$$AX_{j'kwtt'}^1 \geq \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw} \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, w \in \mathcal{W}, t, t' \in \mathcal{T}$$

$$AX_{j'kwtt'}^1 \geq - \sum_{k \in \mathcal{K}} \alpha_{j'rt'}^{jkw} \quad \forall j' \in \mathcal{J}, r \in \mathcal{R}, w \in \mathcal{W}, t, t' \in \mathcal{T}$$

APPENDIX B

B.1 Parameters of case study instance

Table 33: Product information

product name line	production capacity	cost			holding cost	
		production	backorder	transportation	level 1	level 2
milk (UHT)	180	14	3.5	$1.4 \times \ell_r$	2.1	2.8
milk (daily)	130	12	3.0	$1.2 \times \ell_r$	-	2.4
cheese	130	50	12.5	$5.0 \times \ell_r$	-	10
yoghurt	130	30	7.5	$3.0 \times \ell_r$	-	6
cream	130	45	11.25	$4.5 \times \ell_r$	-	9
dough	130	12	3.0	$1.2 \times \ell_r$	1.8	2.4

capacity of the production lines are constant for all periods

all costs are per unit and constant for all periods

(-) denotes that the product can not be hold in that temperature level

Table 34: Warehouse information

temperature level	capacity	opening cost	set up cost		
			idle	1	2
idle	-	-	0	400	500
1	500	800	250	0	300
2	500	1000	280	260	0

capacity of the production lines are constant for all periods

all costs are per unit and constant for all periods

Table 35: Retailers' nominal demand

Demand	product 1	product 2	product 3	product 4	product 5	product 6
retailer 1	15	20	5	7	3	8
retailer 2	9	12	8	6	8	8
retailer 3	25	0	13	10	7	8
retailer 4	20	0	10	12	6	14
retailer 5	15	15	10	10	8	12
retailer 6	10	0	5	5	3	9
retailer 7	8	8	4	6	3	9
retailer 8	8	8	3	5	4	10

B.2 Parameter Generation Schemes

Generation of utility function parameters. In the distance score, the closest site is scored as I (the highest visit order), and the furthest site is scored as 1. Similarly, in the population score, the site with the highest population is scored as I , and that with the lowest is scored as 1; the scores are decremented by one for all the sites in between in the same order. The first slope of site i^* (a_{i^*1}), which has the highest score ($\bar{R} = \max_{i=1,\dots,I}\{R_i\}$), is set to I , and we adopt the following formula:

Table 36: Summary statistics of data set

Criteria	Min	Max	Average
travel time from DCC (second)	358	5126	2841
travel time from epicenter (second)	261	6010	2601
population	112	5073	772
upper bound of first time segment ($\mathbf{u}_1$)	283	28800	2207
upper bound of second time segment ($\mathbf{u}_2$)	1418	28800	8973
slope of first segment (α_1)	60	100	79.4
slope of second segment (α_2)	59.9	19.9	39.3

$\alpha_{i1} = I - 0.5(\bar{R} - R_i)$ for the other sites. We assume that the highest utility of the second segments cannot be larger than the lowest utility of the first segments to prioritize distributing critical supplies within the given time limit. Consequently, the second slope of each site i is given by $\alpha_{i2} = \min_{i=1,\dots,I} \{\alpha_{i1}\} - 0.5(\bar{R} - R_i)$.

Parameters for cluster generation. In H1.1 and H1.2 algorithms, we limit the upper bound for the number of sites in each cluster to $(\frac{I}{V} + a)$. Although increasing a will yield a better solution, it may increase the CPU time of the algorithm. Moreover, having no limit for cluster size will provide us with large clusters while the RLT cannot visit all of them due to $T_{\max}$ limit. In our study, we set $a = 2$ based on a sensitivity analysis. Needless to say, if all the possible clusters are generated, the optimal assignments of sites to RLTs would be achieved. However, generating all the possible clusters is computationally challenging due to the time and memory limits. This is why we terminate cluster generation after $(b \times I \times (\frac{I}{V})^{-1} = bV)$ unsuccessful trials, where b represents a scalar and is fixed to 25000 based on a sensitivity analysis. Notice that different terminating conditions can be adopted in the algorithm, but we prefer related measure to represent the positive correlation of the number of generated clusters with the instance size I and the average number of sites in each cluster $(\frac{V}{I})$.

Considering different weights for σ in severity score. This study focuses on the aftermath of an earthquake in which RLTs serve the sites after earthquake occurrence. Therefore, the sites close to the epicenter are likely to be affected more seriously by the disaster. We aim to cover the maximum number of sites and give priority to the sites close to epicenter, which are likely to be affected more from the disaster. As a result, in Section 5, the parameters are generated considering $(\sigma_1 = 0.9, \sigma_2 = 0.1)$ where the weight of the distance from the epicenter metric is significantly more important than the population in the case study. Our goal was to represent the associated relationship by giving more weight to σ_1 . It is worthwhile to mention that there was no significant update in the related routes when σ_1 and σ_2 are defined in ranges $[0.6, 1]$ and $[0.1, 0.4]$, respectively; similarly, the computation time is not affected. Nevertheless, as it may be anticipated, when the second weight σ_2 that denotes the importance of population becomes more significant, this will impact the resulting routes. We also solved the case study instance by assigning $(\sigma_1 = 0.5, \sigma_2 = 0.5)$ for the two metrics to compare the differences with the solutions presented in the case

study. As it is shown in Figure 5, when the weights are set to $(\sigma_1 = 0.5, \sigma_2 = 0.5)$ and $(\rho = 0.25, T_{\max} = 8)$, sites 76, 80 and 81 are not served even though they are very close to the epicenter, while they are visited when $(\sigma_1 = 0.9, \sigma_2 = 0.1)$ (Figure 4). Moreover, further analysis shows that the Gini coefficient increases when weights are set to $(\sigma_1 = 0.5, \sigma_2 = 0.5)$ since RLTs visit fewer sites, e.g., 61 sites were visited in the case study instance $(\rho = 0.25, V = 10)$; however, 47 sites are visited when weights are $(\sigma_1 = 0.5, \sigma_2 = 0.5)$.

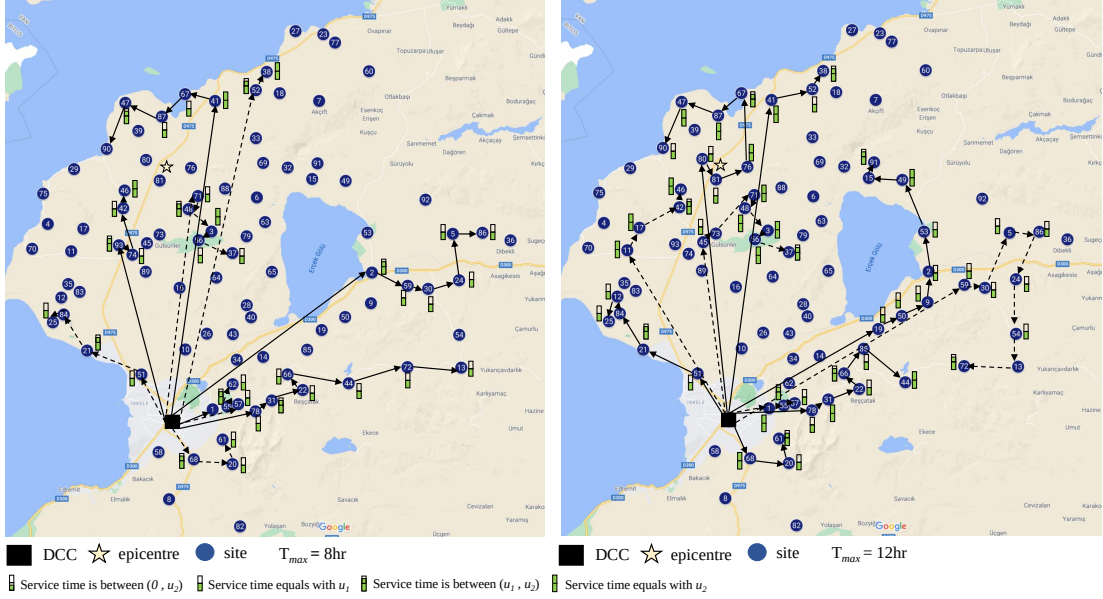


Figure 12: Case study solution representation ($\rho = 0.25, V = 10$)

Table 37: Gini coefficient ($\rho = 0.45$)

$T_{\max}$	V	RRDP		ARDP	
		HA2	HA3	HA2	HA3
8	10	0.56	0.61	0.56	0.61
	19	0.39	0.40	0.39	0.40
12	10	0.47	0.42	0.47	0.42
	19	0.18	0.17	0.18	0.17

B.3 Properties of [RRDP]

1) *Subtour elimination constraints*: [RRDP] may be modeled without using the order index k by introducing subtour elimination constraints. Nevertheless, we prefer the order-based model because it helps in reformulating the ARO model. 2) *Number of time segments*: We demonstrated the utility functions over three segments in the problem definition because it is generic enough to model many different practical cases. It is worthwhile to mention that the model can also be adopted for more time

segments when needed as in [RRDP]. The sensitivity analysis showed that CPU time performances of the proposed solution approaches are not significantly affected by having more segments in the utility function, even though it increases the number of auxiliary variables. 3) *Point-to-point distribution*: The model proposed in this paper may also be adapted to point-to-point distribution in three ways: i) if the number of the RLTs is considered to be greater or equal to the number of the sites; ii) if upper bound of the first time segment in the utility function of site i is relatively a large number (i.e., $u_{i1} \geq T_{\max}$), in other words, the critical needs of sites are excessive; iii) if we add a constraint forcing the RLTs to visit at most one site. In all three cases, the proposed model will yield a point-to-point distribution plan as these settings will ensure the RLTs visit one site at most. The proposed solution approach can also be extended to this setting with minor modifications. 4) *Selecting the uncertainty set*: Two main criteria are considered in the uncertainty set selection: i) tractability of the resulting robust counterpart, and ii) practicability of the parameter selection process for the uncertainty set. The box uncertainty set yields an RC that does not add additional mathematical optimization complexity to that of the deterministic model. Moreover, as mentioned before, the parameter selection process of the related set is practically relevant. We refer the reader to Appendix B.10 for advantages of the box uncertainty set over the other tractable uncertainty set classes, such as the budget.

B.4 Deriving RC of Time Limit Constraints

Since RO is a constraint-wise approach, we focus on one of the constraints in this subsection. The index of the constraint is denoted by v^* .

Notation. We can reformulate the semi-infinite ARC of the time limit constraint (63) as follows:

$$\begin{aligned} & \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} t_{ij}^0 X_{ijkv} + \sum_{i' \in \mathcal{I}_0} \sum_{i, j \in \mathcal{I}} \sum_{k \in \mathcal{K}} t_{i'j}^0 \gamma_{i'jk}^{iv} + \sum_{i \in \mathcal{I}} \bar{T}_{iv} + \\ & \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv} + \sum_{i' \in \mathcal{I}} \gamma_{i'jk}^{i'v} \right) \rho t_{ij}^0 \zeta_{ij} \right) \leq T_{\max} \quad \forall v \in \mathcal{V}, \zeta \in \mathbb{U} \end{aligned}$$

In the reminder we show the constraint as:

$$[\text{fixed term}] + \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{i'jk}^{i'v^*} \right) \rho t_{ij}^0 \zeta_{ij} \right) \leq T_{\max} \quad \forall \zeta \in \mathbb{U}$$

where $[\text{fixed term}] = \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} t_{ij}^0 X_{ijkv^*} + \sum_{i' \in \mathcal{I}_0} \sum_{i, j \in \mathcal{I}} \sum_{k \in \mathcal{K}} t_{i'j}^0 \gamma_{i'jk}^{i'v^*} + \sum_{i \in \mathcal{I}} \bar{T}_{iv^*}$.

Step 1. If the problem is robust for the worst-case realization of the uncertainty parameter, then a feasible solution for this problem is also feasible for any realization of the uncertainty set. Thus, we reformulate the constraint as follows:

$$[\text{fixed term}] + \max_{-1 \leq \zeta \leq 1} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{i'jk}^{i'v^*} \right) \rho t_{ij}^0 \zeta_{ij} \right) \leq T_{\max} \quad (145)$$

Step 2. Next, we take the dual of the inner maximization problem in (145). Notice that the primal and dual problems yield the same optimal OFV since the inner maximization problem is a linear programming problem over the uncertainty parameter ζ .

$$[\text{fixed term}] + \min_{\mathbf{g} \geq \mathbf{0}, \mathbf{s} \leq \mathbf{0}} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} (g_{ij}^{v*} - s_{ij}^{v*}) \leq T_{\max} \quad (146)$$

$$\text{s.t. } g_{ij}^{v*} + s_{ij}^{v*} = \sum_{k \in \mathcal{K}} \left(X_{ijkv*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v*} \right) \rho t_{ij}^0 \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (147)$$

where $\mathbf{g}$ and $\mathbf{s}$ are the dual variables.

Step 3. The dual minimization problem yields an upper bound for the optimal OFV of the primal problem in Step 1 by weak duality. Therefore, we can remove the minimization operator in (146) and break the nested structure of the constraint. The resulting constraint is presented below.

$$[\text{fixed term}] + \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} (g_{ij}^{v*} - s_{ij}^{v*}) \leq T_{\max} \quad (148)$$

$$g_{ij}^{v*} + s_{ij}^{v*} = \sum_{k \in \mathcal{K}} \left(X_{ijkv*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v*} \right) \rho t_{ij}^0 \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (149)$$

$$\mathbf{g} \geq \mathbf{0}, \mathbf{s} \leq \mathbf{0} \quad (150)$$

Remark 3 The following formulation is equivalent to (148)–(150) after the absolute value expression is linearized:

$$[\text{fixed term}] + \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(\left| \sum_{k \in \mathcal{K}} \left(X_{ijkv*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v*} \right) \rho t_{ij}^0 \right| \right) \leq T_{\max}$$

Remark 4 One can retrieve the same robust reformulation in Step 3 by directly substituting the maximum values of ζ for the inner maximization problem in Step 1, and linearize the resulting absolute value expression in Remark 3.

B.5 Tractable Reformulation of [ARDP] and [PR]

AX: auxiliary decision variables

[ARDP]:

$$\begin{aligned} \max_{\mathbf{X}, \mathbf{AX}, \bar{\mathbf{T}}, \bar{\mathbf{Z}}, \boldsymbol{\theta}, \boldsymbol{\gamma}} \quad & \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left[\alpha_{i1} \left(\bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} t_{ij}^0 \right) + \right. \\ & \left. \sum_{n \in \mathcal{N}} \left[\left(\alpha_{in} - \alpha_{i(n+1)} \right) \left(\bar{Z}_{inv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \theta_{i'jk}^{inv} t_{ij}^0 \right) \right] \right] - \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} AX_{ij}^0 \rho t_{ij}^0 \end{aligned}$$

s.t. (48) – (12), (58) – (62)

$$AX_{i'j'}^0 \geq \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left(\sum_{k \in \mathcal{K}} \alpha_{i1} \lambda_{i'j'k}^{iv} + \sum_{n \in \mathcal{N}} \left(\alpha_{in} - \alpha_{i(n+1)} \right) \theta_{i'j'k}^{inv} \right) \quad \forall i' \in \mathcal{I}_0, j' \in \mathcal{I}$$

$$AX_{i'j'}^0 \geq - \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left(\sum_{k \in \mathcal{K}} \alpha_{i1} \lambda_{i'j'k}^{iv} + \sum_{n \in \mathcal{N}} \left(\alpha_{in} - \alpha_{i(n+1)} \right) \theta_{i'j'k}^{inv} \right) \quad \forall i' \in \mathcal{I}_0, j' \in \mathcal{I}$$

$$\sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) + \rho AX_{ijv}^1 \right) \right) + \sum_{i \in \mathcal{I}} \bar{T}_{iv} \leq T_{\max} \quad \forall v \in \mathcal{V}$$

$$AX_{ijv}^1 \geq \sum_{k \in \mathcal{K}} \left(X_{ijkv} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I}, v \in \mathcal{V}$$

$$AX_{ijv}^1 \geq \sum_{k \in \mathcal{K}} \left(-X_{ijkv} - \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I}, v \in \mathcal{V}$$

$$\sum_{v \in \mathcal{V}} \left(\bar{T}_{iv} - \bar{Z}_{inv} + \left(\sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \left(\gamma_{i'jk}^{iv} - \theta_{i'jk}^{inv} \right) + \rho AX_{ii'jn}^2 \right) \right) \right) \leq u_{in} \quad \forall i \in \mathcal{I}, n \in \mathcal{N}$$

$$AX_{ii'jn}^2 \geq \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}} \left(\gamma_{i'jk}^{iv} - \theta_{i'jk}^{inv} \right) \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, n \in \mathcal{N}$$

$$AX_{ii'jn}^2 \geq - \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}} \left(\gamma_{i'jk}^{iv} - \theta_{i'jk}^{inv} \right) \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, n \in \mathcal{N}$$

$$\bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} + \rho AX_{ivi'j}^3 \right) \leq M \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} X_{jikv} \quad \forall i \in \mathcal{I}, v \in \mathcal{V}$$

$$AX_{ivi'j}^3 \geq \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, v \in \mathcal{V}$$

$$AX_{ivi'j}^3 \geq - \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, v \in \mathcal{V}$$

$$\bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} - \rho AX_{ivi'j}^4 \right) \geq 0 \quad \forall i \in \mathcal{I}, v \in \mathcal{V}$$

$$AX_{ivi'j}^4 \geq \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, v \in \mathcal{V}$$

$$AX_{ivi'j}^4 \geq - \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, v \in \mathcal{V}$$

$$\bar{Z}_{inv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \theta_{i'jk}^{inv} - \rho AX_{iji'nv}^5 \right) \geq 0 \quad \forall i \in \mathcal{I}, n \in \mathcal{N}, v \in \mathcal{V}$$

$$AX_{iji'nv}^5 \geq \sum_{k \in \mathcal{K}} \theta_{i'jk}^{inv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, n \in \mathcal{N}, v \in \mathcal{V}$$

$$AX_{iji'nv}^5 \geq - \sum_{k \in \mathcal{K}} \theta_{i'jk}^{inv} \quad \forall i' \in \mathcal{I}_0, i, j \in \mathcal{I}, n \in \mathcal{N}, v \in \mathcal{V}$$

$$\bar{T}, \bar{Z}, \gamma, \theta \in \mathbb{R}, \mathbf{AX} \geq \mathbf{0}$$

[PR] :

$$\begin{aligned}
& \max_{\mathbf{X}, \bar{\mathbf{T}}, \mathbf{Z}^0, \gamma} \min_{\zeta \in \mathbb{U}} \sum_{i \in \mathcal{I}} \sum_{v \in \mathcal{V}} \left[\alpha_{i1} \left(\bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} t_{ij}^0 \right) - \sum_{n \in \mathcal{N}} \left(\alpha_{in} - \alpha_{i(n+1)} \right) Z_{inv}^0 \right] \\
& \text{s.t. [ARDP}(X^*)](59) - (62), (65) \\
& \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(t_{ij}^0 \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) + \rho A X_{ijv}^1 \right) \right) + \sum_{i \in \mathcal{I}} \bar{T}_{iv} \leq T_{\max} \quad \forall v \in \mathcal{V} \\
& A X_{ijv}^1 \geq \sum_{k \in \mathcal{K}} \left(X_{ijkv} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I}, v \in \mathcal{V} \\
& A X_{ijv}^1 \geq \sum_{k \in \mathcal{K}} \left(-X_{ijkv} - \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v} \right) \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I}, v \in \mathcal{V} \\
& \sum_{v \in \mathcal{V}} \left(\left(\bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} t_{ij}^0 \right) - Z_{inv}^0 \right) \leq u_{in} \quad \forall i \in \mathcal{I}, n \in \mathcal{N} \\
& \bar{T}_{iv} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{i'j}^0 \left(\sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} - \rho A X_{ijv}^5 \right) \geq T_{iv}^* \quad \forall i \in \mathcal{I}, v \in \mathcal{V} \\
& A X_{iji'v}^5 \geq \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i \in \mathcal{I}_0, i', j \in \mathcal{I}, v \in \mathcal{V} \\
& A X_{iji'v}^5 \geq - \sum_{k \in \mathcal{K}} \gamma_{i'jk}^{iv} \quad \forall i \in \mathcal{I}_0, i', j \in \mathcal{I}, v \in \mathcal{V} \\
& \bar{\mathbf{T}}, \bar{\mathbf{Z}}, \gamma \in \mathbb{R}, \quad \mathbf{Z}^0 \geq \mathbf{0}
\end{aligned}$$

B.6 Suboptimality of Adopting Linear Decision Rules for Auxiliary Variable

In this section, we show that the Pareto efficient reformulation method suggested in the practical guide on RO [63] that is based on constraining the worst-case OFV to be larger than or equal to the robust optimal OFV (for minimization) and reoptimizing the nominal OFV may result in incorrect calculations of the adjustable auxiliary variable $\mathbf{Z}$ for the nominal data due to suboptimality of LDRs, which are also used in the auxiliary variable.

We use the toy-sized illustrative example and the data set provided in §3.3.2. The service time decisions rules of the two sites are: $T_1 = \bar{T}_1 + \gamma_{(0,1)}^1 t_{01}$ and $T_2 = \bar{T}_2 + \gamma_{(0,1)}^2 t_{01} + \gamma_{(1,2)}^2 t_{12}$. Similarly, the decision rules of the auxiliary variables are: $Z_1 = \bar{Z}_1 + \theta_{(0,1)}^1 t_{01} + \theta_{(1,2)}^1 t_{12}$ and $Z_2 = \bar{Z}_2 + \theta_{(0,1)}^2 t_{01} + \theta_{(1,2)}^2 t_{12}$. When the conventional method that is described above is used, the optimal coefficients are equivalent to $\gamma_{(0,1)}^1 = -0.5$; $\gamma_{(0,1)}^2, \gamma_{(1,2)}^2 = -0.5, -1$ and the intercepts are $\bar{T}_1 = 95$ and $\bar{T}_2 = 230$. For the auxiliary variables, coefficients are $\theta_{(0,1)}^1, \theta_{(1,2)}^1 = -0.375, 0$; $\theta_{(0,1)}^2, \theta_{(1,2)}^2 = -0.5, -1$ and the intercepts are $\bar{Z}_1 = 45$ and $\bar{Z}_2 = 180$. When we substitute the nominal realizations of the uncertainty parameters, the resulting service time and the excess time decisions for site one become $T_1 = 45$ and $Z_1 = 7.5$. However, the actual time RLT spends

after u_{11} must be 5 (from $T_1 - u_{11} = 45 - 40 = 5$) which is not equal to what is calculated by the decision rule of Z_1 . Notice that [PR] avoids such inconsistencies.

B.7 Step 2 of H1.2

[Step 2] **Determining the lower bound for each cluster.** Assuming I_c stands for the set of sites on the cluster c , $siteService_j$ stands for utility of site j , $SpentTime_j$ stands for the total time the RLT spends in site j is serving, cs stands for the ‘current site’ at the related iteration of the algorithm, $CurrentTime$ stands for the time the RLT has spent so far, $TotalUtility$ stands for the total utility of the RLT, and finally, $VisitedSites$ stands for the set of the sites the RLT visits, the steps of the constructive algorithm that decides on the routes and the services of an RLT for each cluster are as follows:

[Step 2.1] Set the $SiteService_j$ and $SpentTime_j$ for each site, cs , $CurrentTime$, $TotalUtility$ as 0 and the set of $VisitedSites$ as $\{0\}$.

[Step 2.2] Using the following equation, calculate the score of choosing each site (s_i):

$$s_i = \eta_1 \frac{\alpha_{i1}}{\sum_{j \in I_c} \alpha_{j1}} - \eta_2 \frac{t_{cs,i}}{\sum_{j \in I_c} t_{cs,j}} \quad \forall i \in I_c \quad (151)$$

[Step 2.3] Sort the sites in I_c in a descending order of scores and call this array I'_c .

[Step 2.4] **For** each the site j in I'_c **do**:

If $CurrentTime + t_{cs,j} \leq T_{\max}$, do the following updates:

$$SpentTime_j = \min\{T_{\max} - CurrentTime, u_{1j}\}$$

$$SiteService_j = \alpha_{j1} SpentTime_j$$

$$CurrentTime = CurrentTime + t_{cs,j} + SpentTime_j$$

$$TotalUtility = TotalUtility + SiteService_j$$

$$VisitedSites \leftarrow \{j\}$$

$$cs = j.$$

The first term in the weighted function (151) stands for the ratio of the potential utility that can be gained from the site to the summation of the potential utilities of the nodes, and the second term stands for the ratio of the distance of the node to the summation of the distances of the nodes from the current node.

B.8 Steps of H2

Notation. $E(R)$ denotes the edges in route R , $S(R)$ denotes the set of sites visited in route R but not fully served, and $S_{(i,j)}(R) \subseteq S(R)$ denotes the set of sites that include (i, j) in the information base for route R . The time segment of the utility function that the service of a given site i is finalized is referred to as the *active segment* denoted by n_i^* , and the slope of the associated segment is given by α_{in^*} . The decision rule coefficients are given by parameter γ ; $\omega_{i'j}$ parameter determines the unutilized percentage of the extra time that is realized at edge (i', j) , which is initialized as one in the algorithm. Our goal is to set $\omega_{i'j}$'s as low as possible for utilizing the information gathered from edge (i', j) in the decision rules of relevant sites in R ; as it may be anticipated the ultimate goal is reaching zero when the algorithm is terminated.

[**Step 1**] Set $\gamma_{i'j}^i = 0$ and $\omega_{i'j} = 1$ for all $(i', j) \in E(R)$ and $i \in S(R)$.

[**Step 2**] **If** $S(R)$ is empty, then **go to** Step 7. **Else**, find site $i \in S(R)$ that has the largest slope in the active segments: $i^* = \arg \max_{i \in S(R)} \{a_{in_i^*}\}$.

[**Step 3**] **If** $E(R)$ is empty, then terminate the algorithm. **Else**, sort the edges that are in $E(R)$ and traversed before site i^* , and assign them to list $Q(i^*, R)$ according to their distances to i^* in the ascending order.

[**Step 4.1**] Select the first edge (i'_1, j_1) in $Q(i^*, R)$, set $\gamma_{i'_1j_1}^{i^*} = -\omega_{i'_1j_1}$ and

$$T_{i^*} = \bar{T}_{i^*} - (1 + \rho) \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{i'j}^0 \gamma_{i'j}^{i^*}, \quad (152)$$

where T_{i^*} is the robust service time decision taken out of a Phase I algorithm or [RRDP].

[**Step 4.2**] Check the feasibility of the following inequality:

$$\bar{T}_{i^*} + \sum_{i' \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} t_{i'j}^0 \gamma_{i'j}^{i^*} \leq u_{i^*n_{i^*}^*}. \quad (153)$$

If the inequality is satisfied, Update $\omega_{i'_1j_1} = \omega_{i'_1j_1} - \gamma_{i'_1j_1}^{i^*}$, remove (i'_1, j_1) from $Q(i^*, R)$ and $E(R)$ if $\omega_{i'_1j_1} = 0$), and **go to** Step 5. **Else**, decrease $\gamma_{i'_1j_1}^{i^*}$ with step size δ , update $\bar{T}_{i^*}$ as in (152), and **go to** Step 4.2.

[**Step 5**] **If** $Q(i^*, R)$ is empty, then **go to** Step 6. **Else**, go to Step 4.1

[**Step 6**] **If** the active segment $n_{i^*}^*$ is the last segment, then remove i^* from $S(R)$ and go to Step 2. **Else**, update the active segment to $n^* = n^* + 1$ and **go to** Step 2.

[**Step 7**] Find the site index $i_{(i,j)}^* = \arg \max_{i' \in S_{(i,j)}(R)} \{a_{in_i^*}\}$ for each $(i, j) \in E(R)$ such that $\omega_{ij} \neq 0$, and update $\gamma_{ij}^{i_{(i,j)}^*}$ and $\bar{T}_{i_{(i,j)}^*}$ as follows:

$$\gamma_{ij}^{i_{(i,j)}^*} = \gamma_{ij}^{i_{(i,j)}^*} - \omega_{ij}, \quad T_{i_{(i,j)}^*} = \bar{T}_{i_{(i,j)}^*} - (1 + \rho) \sum_{i' \in \mathcal{I}_0} \sum_{j' \in \mathcal{I}} t_{i'j'}^0 \gamma_{i'j'}^{i_{(i,j)}^*}.$$

Remark 5 *The Pareto efficiency of the resulting heuristic solution is mainly ensured in Steps 4.1 and 4.2 by (152) and (153). Namely, the robust optimality of the decision rule is ensured by employing (152) and the aforementioned property $OFV([RRDP]) = OFV([ARDP])$ that is satisfied at optimality. Ultimately, a high-quality nominal solution is ensured because the algorithm decrements γ coefficients by a small constant δ in Step 4.2 in order to yield the highest utility by taking into account slope updates of utility functions via (153).*

B.9 Extra Discussions on Numerical Results

Comparison of Phase I algorithms. We evaluate the OFV and CPU time performances of the heuristic algorithms proposed for Phase I and present the results in Table 38. The first three columns represent the instance sizes, the OFV performances are presented in columns four to six, where the percentages represent the OFV gap from the best performing algorithm denoted by “*”. From column seven to nine the CPU times (in seconds) are presented.

Table 38: Performance of the algorithms for Phase I

Size	Instance		Robust OFV performance			CPU (%gap)			safe gap
	I	V	[RRDP]	H1.1	H1.2	[RRDP]	H1.1	H1.2	
Small	6	1	*	*	*	1	15	10	0
	7	1	*	*	*	1	23	11	0
	8	1	*	*	*	1	34	12	0
	9	1	*	*	*	1	55	13	0
	10	1	*	*	*	1	83	14	0
		2	*	*	1%	2	68	25	0
Medium	20	2	*	*	1%	1180	760	40	0
		4	*	*	4%	(2%)	229	73	2%
	40	4	1%	*	4%	(7%)	2277	235	6%
		8	8%	*	6%	(13%)	663	440	7%
Large	60	6	15%	*	5%	(28%)	5858	463	8%
		12	43%	*	11%	(56%)	1281	861	8%
	80	8	25%	*	3%	(50%)	6311	805	8%
		16	m	*	7%	m	1977	1484	-

(*) denotes the best OFV performance (safe gap) = $\frac{OFV(RRDP)(1+gap\%) - OFV(H1.1)}{OFV(RRDP)(1+gap\%)} \times 100$
(m) denotes the memory error

As illustrated in the table, the [RRDP] solves small-sized instances in few seconds, while medium-sized instances are not solved to optimality within the given time limit (6 hours for medium-sized instances, 12 hours for large-sized instances); the numbers in parenthesis show the optimality gaps for instances that cannot be solved to optimality within the given time limit. Finally, we compare the OFV of the best performing algorithm with the upper bound of the problem that is calculated

by the solver for [RRDP] and report it as the “safe gap” of the best performing algorithm in the last column. Regarding the OFV performance, the clustering-based heuristic H1.1 outperforms [RRDP] and the clustering-based heuristic H1.2 with a lower bound approach. It may be easy to see that H1.1 outperforms H1.2 in medium and large-sized instances by (on average) 4% and 6% respectively, and the associated improvement gets as high as 11% for $(I=60, V=12)$ instance. Therefore, we can conclude that H1.1 is the best Phase I algorithm in terms of the OFV performance when the number of sites is larger than 10. The numerical results show that the CPU time of solving [RRDP] significantly increases as the numbers of sites and RLTs increase, e.g., compare CPU times at rows $(I=10, 20, 40)$ and the [RRDP] column in Table 38. The H1.2 algorithm yields the best CPU time performance among the other algorithms. All in all, the decision-maker must prefer H1.1 or H1.2 for realistically-sized instances by considering the trade-off between the CPU time and the OFV performance.

Comparison of Phase II algorithms. We discuss the OFV and CPU time performances of [PR] and decision rule algorithm H2 proposed to solve Phase II and present the numerical results in Table 39. The numerical results show that the effect of out-of-sampling on feasibility of LDRs is negligible and again LDRs and SbM show very close OFV performances. Notice that both H2 and [PR] use the route taken from Phase I as an input of the problem, and the decision rules of a given RLT route are solved by [PR]. Since [PR] can be decomposed over RLTs, we generate instances with the given number of sites in the first column of Table 39 and one RLT in this experiment. To this end, we use [RRDP] in Phase I to generate the route. The second and third columns present the OFV performances, and the last two columns show CPU times (in seconds). The numerical results show that if an RLT visits (on average) 21–25 sites, the CPU time of [PR] gets as high as 1442 while H2 solves the problem in (on average) 45 seconds. Ultimately, H2 is significantly better than [PR] in the CPU time performance, and it yields the same OFV performance; therefore, we suggest using H2 in Phase II.

Table 39: Performance of the algorithms for Phase II

number of visited sites	OFV performance		CPU	
	[PR]	H2	[PR]	H2
1-5	*	*	1	1
6-10	*	*	12	4
11-15	*	*	13	14
16-20	*	*	587	21
21-25	*	*	1442	45

(*) denotes the best OFV performance

Out-of-sample optimality performance of LDRs. We repeat the simulation in §3.5.3, by sampling travel times out of the uncertainty set, where we sample the primitive uncertainty parameter from uniform $0.1 + \mathbb{U}(-\rho, \rho)$ and triangular $0.1 + \mathbb{T}(-\rho, \rho, \frac{\rho}{2})$ distributions. In this experiment, we compare the average OFVs of [ARDP] and SbM with that of the RRDP that is solved for the worst-case realization of the travel times and report the results in Table 40. We may get infeasible solutions while out-of-sampling the travel times and replacing them in the decision rules of the

service times. However, since the number of infeasible scenarios is negligible for all instances, we exclude them and do not report them in the table. Comparing the results in the table shows that the scenario-based problem yields better OFV performances. Lastly, the OFV performances of the two approaches get closer when data is sampled from a more conservative triangular distribution because in this case both [ARDP] and SbM start yielding conservative solutions; see, the rows that are associated with $\mathbb{T}(-\rho, \rho, \frac{\rho}{2})$ in Table 20. It is worth mentioning that the decision rule coefficients are obtained separately for each RLT, and this is why we have done these simulations only for a single RLT.

Table 40: Out-of-sample optimality performance of LDRs

route length	sampling	$\rho=0.25$		$\rho=0.35$		$\rho=0.45$	
		SbM	ARDP	SbM	ARDP	SbM	ARDP
6	$0.1 + \mathbb{U}(-\rho, \rho)$	4.2%	3.4%	6.7%	5.8%	8.1%	4.5%
7		4.7%	3.3%	6.3%	4.0%	7.5%	3.4%
8		3.8%	3.0%	5.9%	4.5%	6.9%	4.3%
9		4.9%	4.2%	6.6%	5.2%	6.1%	5.3%
10		3.6%	1.8%	5.2%	2.7%	5.4%	2.8%
average		4.2%	3.1%	6.1%	4.4%	6.8%	4.0%
6	$0.1 + \mathbb{T}(-\rho, \rho, \frac{\rho}{2})$	3.4%	2.6%	5.6%	4.7%	7.6%	4.1%
7		3.8%	2.7%	5.0%	3.3%	6.2%	2.2%
8		3.2%	2.5%	5.0%	3.7%	6.1%	4.2%
9		4.0%	3.5%	5.1%	4.4%	5.0%	4.4%
10		2.8%	1.3%	4.3%	2.2%	4.6%	2.3%
average		3.4%	2.5%	5.0%	3.7%	5.9%	3.4%

Comparison of OFV and CPU time performances of HA2 and HA3 for case study. Table 41 shows the result of comparing the OFV and CPU performance of HA2 and HA3 algorithms for the case study. The best performing algorithm in regard to the OFV performance is denoted by “*” in Table 41, and we report the optimality gaps of other algorithms with respect to the best performing one at each row.

Table 41: Comparison of OFV and CPU time performances

$T_{\max}$	V	OFV						CPU	
		$\rho = 0.25$		$\rho = 0.35$		$\rho = 0.45$		HA2	HA3
		HA2	HA3	HA2	HA3	HA2	HA3		
8hr	10	*	2%	*	2%	*	3%	5487	630
	19	*	5%	*	4%	*	6%	1846	1074
12hr	10	*	2%	*	1%	*	3%	6555	708
	19	*	8%	*	4%	*	7%	1847	1075

(*) denotes the best OFV performance

B.10 Model Extensions and Discussions

This section gives alternative reformulations of the time limit constraints (45) for the budget and ellipsoidal uncertainty sets. We also present the connection between RO and SP via safe approximations of chance constraints.

Before delving into details of the reformulations, we first clarify why we selected the box uncertainty set in our research. Two main criteria are considered in the *uncertainty set selection*: 1) tractability of the resulting robust counterpart and 2) practicability of the parameter selection process for the uncertainty set. Notice that even the deterministic version of RDP is computationally challenging. The related mathematical optimization complexity limited our uncertainty selection process to polyhedral uncertainty sets, e.g., box, budget, or a general polyhedron. Box and budget are practically relevant uncertainty sets since we have only limited information about uncertain travel times such as lower and upper bounds; additionally, these uncertainty sets yield tractable robust counterparts. While it is true that the box uncertainty set is relatively more conservative than the budget uncertainty set because the “budget” (generally represented by Γ) may control the level conservatism, the selection of the budget brings some practical inconsistencies to the problem at hand. More precisely, the number of uncertainty parameters (i.e., edges) in an RLT route cannot be known before the problem is solved, and hence the budget of the uncertainty set cannot be fixed in advance. Consequently, the level of conservatism is affected for each route differently because the number of sites visited may differ. Therefore, selecting the related parameter requires a serious investigation due to the reason mentioned above. Ultimately, we have chosen the interval uncertainty set that is practically more relevant to the problem than the budget uncertainty set.

Another uncertainty class often used in practice is the ellipsoidal or ball uncertainty set. Such uncertainty sets yield tractable RCs for linear uncertain optimization problems and may be associated with safe approximations of ambiguous chance constraints, i.e., probabilistic guarantees on the feasibility of the resulting RC solutions may be provided. Nevertheless, it is worth mentioning that even though the RC of the ellipsoidal uncertainty set satisfies the chance constraint properties in our setting, it results in a computationally expensive mathematical optimization model presented in the sequel.

In the remainder, we first present ARC for budget and ellipsoidal uncertainty sets. Later, we discuss how one can relate these ARCs to chance-constrained travel time constraints by selecting the parameters of the uncertainty sets. As mentioned in Appendix B.4, RO is a constraint-wise approach and similarly we focus on one of the constraints which denoted by v^* .

Budget uncertainty set. We derive the tractable reformulation of constraint (63) with respect to the budget uncertainty set by applying the three step procedure of RO.

$$\mathbb{U}_{\text{budget}} := \{\mathbf{t} \in \mathbb{R}_+^N : t_{ij} = (1 + \rho\zeta_{ij})t_{ij}^0, \quad -1 \leq \zeta_{ij} \leq 1 \quad \forall i, j \in \mathcal{I}_0, \quad \sum_{i,j \in \mathcal{I}_0} |\zeta_{ij}| \leq \Gamma\},$$

where Γ parameter is defined as the maximum number of the uncertain parameters that can deviate from their nominal value. It can take values between zero and the

total number of uncertain parameters, and called as the budget of the uncertainty [185, 186]:

Step 1.

$$[\text{fixed term}] + \max_{\zeta} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left(\sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v^*} \right) \rho t_{ij}^0 \zeta_{ij} \right) \leq T_{\max} \quad (154)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} |\zeta_{ij}| \leq \Gamma \quad (155)$$

$$-1 \leq \zeta_{ij} \leq 1 \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (156)$$

Next, we reformulate (154)–(156) as:

$$[\text{fixed term}] + \max_{\zeta} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \left| \sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v^*} \right) \rho t_{ij}^0 \zeta_{ij} \right| \leq T_{\max} \quad (157)$$

$$\text{s.t.} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \zeta_{ij} \leq \Gamma \quad (158)$$

$$0 \leq \zeta_{ij} \leq 1 \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (159)$$

Step 2.

$$[\text{fixed term}] + \min_{\mathbf{p}, \mathbf{q} \geq 0} \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} q_{ij}^{v^*} + \Gamma p^{v^*} \leq T_{\max} \quad (160)$$

$$\text{s.t.} \quad q_{ij}^{v^*} + p^{v^*} \geq \sum_{k \in \mathcal{K}} \rho t_{ij}^0 \left| \sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v^*} \right) \right| \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (161)$$

where $\mathbf{p}$ and $\mathbf{q}$ are the dual variables.

Step 3.

$$[\text{fixed term}] + \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} q_{ij}^{v^*} + \Gamma p^{v^*} \leq T_{\max} \quad (162)$$

$$q_{ij}^{v^*} + p^{v^*} \geq \sum_{k \in \mathcal{K}} \rho t_{ij}^0 \left| \sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v^*} \right) \right| \quad \forall i \in \mathcal{I}_0, j \in \mathcal{I} \quad (163)$$

It is important to point out that the procedures will be repeated for all the constraints ($\forall v \in \mathcal{V}$)

Ellipsoidal uncertainty set. Next, we derive the tractable reformulation of constraint (63) with respect to the ellipsoidal uncertainty set by applying the RO paradigm mentioned above.

$$\mathbb{U}_{\text{ellip}} := \{\mathbf{t} \in \mathbb{R}_+^N : t_{ij} = (1 + \rho \zeta_{ij}) t_{ij}^0, \quad \|\zeta_{ij}\|_2 \leq \Omega\}$$

Considering constraint (63) as :

$$[\text{fixed term}] + \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \underbrace{\left(\sum_{k \in \mathcal{K}} \left(X_{ijkv^*} + \sum_{i' \in \mathcal{I}} \gamma_{ijk}^{i'v^*} \right) \rho t_{ij}^0 \right)}_{\psi_{ij}} \zeta_{ij} \leq T_{\max} \quad \forall \zeta \in \mathbb{U}$$

The robust counterpart of the time limit constraint for the ellipsoidal uncertainty set is given by the following expression.

$$[\text{fixed term}] + \Omega \sqrt{\sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \psi_{ij}^2} \leq T_{\max}$$

Safe approximations of ambiguous chance constraints via RO. If the primitive uncertainty parameter satisfies the following assumptions:

$$\mathbb{E}[\zeta_{ij}] = 0, \quad -1 \leq \zeta_{ij} \leq 1 \quad \forall i \in I_0, \forall j \in I \text{ and } \zeta_{ij}'\text{'s are independent,}$$

then the feasible solutions ($\mathbf{X}^*$) of the ARCs proposed above to satisfy the chance-constrained reformulation of the time limit constraint:

$$\Pr \left(\zeta : \sum_{i \in \mathcal{I}_0} \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} (1 + \rho \zeta_{ij}) t_{ij}^0 X_{ijkv}^* + \sum_{i \in \mathcal{I}} T_{iv}(\zeta) \leq T_{\max} \right) \geq 1 - \epsilon \quad \forall v \in \mathcal{V}$$

when $\Omega = \sqrt{2 \ln(1/\epsilon)}$ for the ellipsoid and $\Gamma = \sqrt{2I(I+1) \ln(1/\epsilon)}$ for the budget uncertainty set. For further details, we refer the reader to Propositions 2.2.3 and 2.2.4 in [114]. Lastly, because of the joint structure of the chance constraints, one needs to select small values for ϵ and consider an approximation scheme such as Bonferroni (e.g., [187]) in order to satisfy joint feasibility over all RLTs.

Remark 6 *The ARC for the box uncertainty set proposed in the paper satisfies the associated chance constraint almost surely for all realizations of the uncertainty parameter because it is extra robust.*

APPENDIX C

C.1 Literature review table

Table 42: Some of the studies for homogeneous EB systems

Ref.	Objective (minimize)	Charging type	Depots	Method	Battery variable
[188]	total deadheading distance	S, Partial/full options	MD	ALNS	Constant
[189]	number of EBs	Ignored	MD	GA & CG	Constant
[141]	number of EBs	S	SD	B&P	Constant
[190]	number of EBs and CC.	—	SD	MOPSO	Constant
[162]	number of EBs and CC.	DWC	SD	Gurobi	Variable
[161]	number of EBs and CC.	DWC	SD	Gurobi	Variable
[182]	delays in departure and TC.	S	SD	NGSA-II	Constant
[163]	number of EBs and CC.	DWC	SD	Gurobi	Variable
[133]	number of EBs and CC.	S	SD	GAMS	Variable

S: stationary, DWC: Dynamic Wireless Charging ALNS: Adaptive Large Neighborhood Search
GA: genetic algorithm, CG: column generation, B&P: branch & price, CC: charging cost, TC, total cost
MOPSO: Multi-objective Particle Swarm Optimization, MIQP: Mixed-integer quadratic programming

C.2 Robust counterpart

In order to obtain the robust counterpart of the problem, we replace the travel time parameters $(u_{ij}, te_{ic}, ts_{ic}, d_i)$ with their upper bound and calculate energy consumption parameters $(ed_{ik}, ec_{ick}, es_{jck}, eu_{ijk})$ as explained in Section 4.4 and denote them using overline symbole. The definitive representation of the robust counterpart is given as follows.

[RBSP] :

$$\begin{aligned}
\min \quad & \sum_{j \in T} \sum_{k \in K} \sum_{c \in C} l_k X_{scj}^k + \sum_{i \in T} \sum_{c \in C} \sum_{n \in N} p_n G_{icn} + \sum_{i \in T} \sum_{c \in C} \Omega Y_{ic} + \sum_{i \in T} \sum_{j \in T_i^f} \sum_{k \in K} h_k \bar{d}_i X_{ij}^k \\
& + \sum_{i \in T} \sum_{j \in T} \sum_{k \in K} h_k T_{ijk} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{sci}^k \bar{ts}_{ic} + \sum_{i \in T} \sum_{k \in K} \sum_{c \in C} h_k X_{ifc}^k \bar{te}_{ic} \\
\text{s.t} \quad & (71 - 74, 77, 81, 83 - 54) \\
& b + \bar{d}_i + T_{ijk} + \sum_{c \in C} \sum_{n \in N} \frac{1}{\sigma_k} G_{icn} + \sum_{c \in C} \lambda Y_{ic} \leq b_j + M(1 - X_{ij}^k) \quad \forall i \in T, j \in T_i, k \in K \\
& T_{ijk} \geq \sum_{c \in C} Y_{ic} (\bar{te}_{ic} + \bar{ts}_{jc}) + (1 - \sum_{c \in C} Y_{ic}) \bar{u}_{ij} + M(X_{ij}^k - 1) \quad \forall i \in T, j \in T_i, k \in K \\
& B_i - \left(\bar{ed}_{ik} + \sum_{c \in C} \bar{ec}_{ick} Y_{ic} \right) \geq \sum_{j \in T_i^s} a^{low} D_k X_{ij}^k \quad \forall i \in T, k \in K \\
& \sum_{c \in C} \sum_{n \in N} G_{icn} \geq D_k - B_i + (\bar{ed}_{ik} + \sum_{c \in C} Y_{ic} \bar{ec}_{ick}) \\
& + M \left(\sum_{fc \in F} X_{ifc}^k - 1 \right) \quad \forall i \in T, k \in K \\
& B_i - \left(\bar{ed}_{ik} + \sum_{c \in C} Y_{ic} (\bar{ec}_{ick} + \bar{es}_{jck}) + (1 - \sum_{c \in C} Y_{ic}) \bar{eu}_{ijk} \right) \\
& + \sum_{c \in C} \sum_{n \in N} G_{icn} + M(1 - X_{ij}^k) \geq B_j \quad \forall i \in T, j \in T_i, k \in K \\
& \sum_{n \in N} W_{inq_n} \leq \sum_{c \in C} Y_{ic} (b_i + \bar{d}_i + te_{ic}) \leq \sum_{n \in N} W_{inq_{(n+1)}} \quad \forall i \in T
\end{aligned}$$

C.3 Summary of the heuristic algorithm and parameters

Parameters used in the algorithm. For the instances solved in this study, in the first step of the algorithm, we generate three divisions. For instances with 50 and 100 trips, the dividing points are set at [20, 25, 30]; for the instances with 250, 500, 1000 trips, and the case study, the dividing points are [25, 30, 35]; and for instances with 1500 trips, they are [40, 50, 60]. In the second step, we generate 1000 schedules for each division. Then, in step 2.2, we add an additional 1000 schedules, resulting in a total of 4000 clusters at the end of step two. The algorithm iterates steps 2 to 4 three times for all instances in the study, and the final solution is obtained by selecting the

Algorithm 1: Heuristic summary

Step 1: Initialization and generating sub-schedules

Sort the trips based on start time
Divide the trips into blocks
Solve [EBSP] for each block to generate feasible sub-schedules
Iterate the step for different dividing points and generate divisions

Step 2: Generating schedules

Combine the sub-schedules of individual divisions to create schedules
Select two schedules randomly and generate a new schedule
Repeat the process until a desired number of schedules are generated

Step 3: Feasibility check and calculating objective function

Check feasibility and get the objective function for all schedules

Step 4: Schedule selection

Solve the [SCP] to select the best schedules

Step 5: Iterative schedule generation and selection

Repeat steps from 2 to 4 and record the schedules
Solve the [SCP] for the recorded schedules and find the final solution

Step 6: Final solution

Remove repeated trips from the selected schedules
Calculate the final OFV for the set of schedules
Output the final solution and the final OFV

best among the three distinctive schedules generated in step 5. Note that increasing the number of divisions, generated schedules in step 2, and iterations in step 5 may improve the quality of the solution, but it also increases the computational time. For the instances in this study, a comprehensive sensitivity analysis was conducted to determine the mentioned parameters.

Heuristic algorithm performance for the case study. To assess the performance of the heuristic algorithm for the case study instance, we resolved the problem using both the standard approach and the heuristic algorithm and then compared their solution performance. Table 43 provides an overview of our results. According to the comparison, the standard solver produced a solution with a 6% optimality gap within the given time limit. Meanwhile, the heuristic algorithm produced a solution with an OFV that is 3% worse than the standard solver. However, the heuristic algorithm still outperformed the standard approach in terms of CPU time.

Table 43: Comparison of heuristic and standard approach for deterministic problem

solution approach	OFV	CPU time (second)	NV(type 1, type 2)
SA	3079\$	21600*	(3,5)
Heuristic	3170\$	4755	(3,5)

SA: standard approach where [EBSP] is solved by solver

* the solver terminated with 6% optimality gap

NV: number of required vehicle of each type

C.4 Simulation nomenclature and pseudo-code

In the following, the nomenclature and pseudo-code of the simulation procedure are given. The inputs of the algorithm are the given routes, charging decisions, and the sampled travel times. The energy consumption of each trip is calculated for the sampled parameters using equation (88). The procedure is implemented separately for deterministic and robust solutions, and the calculation procedure is discussed in the following. In this research, the simulation is iterated 100 times for each solution, and the average performances are reported.

Table 44: Inputs and variables of the Monte Carlo simulation procedure

Extracted solutions	
X_{ij}^{k*}	1, if trip $j \in T_f$ is visited immediately after trip $i \in T_s$ by EB type $k \in K$; 0, o.w.
Y_{ic}^*	1, charge station $c \in C$ is visited after trip $i \in T$; 0, o.w.
G_{icn}^*	charging amount of EB after trip $i \in T$ on charge station $c \in C$ in period $n \in N$
B_i^*	battery level at the beginning of trip $i \in T$
Sampled Parameters	
ds_i	duration of the trip $i \in T$
tss_{ic}	travel time between start point of trip $i \in T$ and charging station $c \in C$
tes_{ic}	travel time between end point of trip $i \in T$ and charging station $c \in C$
us_{ij}	travel time between end point of trip $i \in T$ and start point of trip $j \in T$
ous_{ij}	travel time between start point of trip $i \in T$ and start point of trip $j \in T$
Calculated parameters according sampled data	
ess_{ick}	energy consumption from start point of trip $i \in T$ to charging station $c \in C$ for EB type $k \in K$
$eess_{ick}$	energy consumption from end point of trip $i \in T$ to charging station $c \in C$ for EB type $k \in K$
eus_{ijk}	energy consumption from end point of trip $i \in T$ to start point of trip $j \in T$ for EB type $k \in K$
$oeus_{ijk}$	energy consumption from start point of trip $i \in T$ to start point of trip $j \in T$ for EB type $k \in K$
eds_{ik}	energy consumption of EB type $k \in K$ in trip $i \in T$
r	the maximum time a trip can be delayed (delay threshold)
Variables	
R_i	requires energy to cover trip $i \in T$
mt	total number of missed trips
dt	total number of delayed trips
tdt	total delay (min)
bl	battery level of EB

Algorithm 2: Simulation pseudo-code

initialization parameters and solution of the model, $mt=dt=tdt=0$
for all generated schedules
 $bl = a^{low} D_k^{Max}$, *current time*: $ct=0$, $delay=0$
 for i in trips of the schedule
 $mt_{temp} = 0$
 #calculate required energy to cover next trip
 if i is the last trip of the schedule
 $R_i = eds_{ik} + \sum_{c \in C} Y_{ic}^* ees_{ick}$
 else
 $R_i = eds_{ik} + \sum_{c \in C} Y_{ic}^* (ees_{ick} + ess_{(i+1)ck}) + (1 - \sum_{c \in C} Y_{ic}^*) eus_{i(i+1)k}$
 #check if EB has enough energy to complete the next trip
 if $bl \leq R_i$
 #if the battery level is not enough, charge the EB sufficiently to cover the trip and update the current time and battery level
 $ct = ct + \frac{bl-R_i}{\sigma}$, $bl = R_i$
 #check if trips could be started on time
 if $ct \geq b[i]$
 #check if the trip is missed or started with delay
 if $ct \geq b[i] + r$
 $mt = mt + 1$, $mt_{temp}=1$
 else
 $delay = ct - b[i]$, $dt = dt + 1$, $tdt = tdt + delay$
 #update current time and battery level
 if i is the last trip of the schedule
 $ct = b_i + \sum_{c \in C} \sum_{n \in N} \frac{1}{\sigma_k} G_{icn}^* + delay + (\sum_{c \in C} Y_{ic}^* tss_{ick})(mt_{temp}) +$
 $(ds_{ik} + \sum_{c \in C} Y_{ic}^* tes_{ick}) + (1 - mt_{temp})$
 $bl = bl + \sum_{c \in C} \sum_{n \in N} G_{icn}^* -$
 $(\sum_{c \in C} Y_{ic}^* ess_{ick})(mt_{temp}) + (eds_{ik} + \sum_{c \in C} Y_{ic}^* ees_{ick})(1 - mt_{temp})$
 else
 $ct = b_i + \sum_{c \in C} \sum_{n \in N} \frac{1}{\sigma_k} G_{icn}^* + delay +$
 $(\sum_{c \in C} Y_{ic}^* (tss_{ick} + tss_{jck}) + (1 - \sum_{c \in C} Y_{ic}^*) ous_{ijk})(mt_{temp})$
 $(ds_{ik} + \sum_{c \in C} Y_{ic}^* (tes_{ick} + tss_{jck}) + (1 - \sum_{c \in C} Y_{ic}^*) us_{ijk})(1 - mt_{temp})$
 $bl = bl + \sum_{c \in C} \sum_{n \in N} G_{icn}^* -$
 $(\sum_{c \in C} Y_{ic}^* (ess_{ick} + ess_{jck}) + (1 - \sum_{c \in C} Y_{ic}^*) oeus_{ijk})(mt_{temp})$
 $(eds_{ik} + \sum_{c \in C} Y_{ic}^* (ees_{ick} + ess_{jck}) + (1 - \sum_{c \in C} Y_{ic}^*) eus_{ijk})(1 - mt_{temp})$

VITA

Farzad Avishan obtained his B.Sc. degree from the University of Tabriz in Iran and later pursued his M.Sc. in Industrial Engineering at Mazandaran University of Science and Technology. In 2018, he began his Ph.D. studies in the Industrial Engineering Department at Özyeğin University, where he worked under the supervision of Dr. İhsan Yanıkoğlu. During his time at Özyeğin University, he served as a Teaching Assistant and a Research Assistant in the Industrial Engineering Department. As a Ph.D. student, he collaborated with scholars from Binghamton University in New York for research, in addition to his work with his advisor. Farzad's research interests include optimization under uncertainty, logistics and public transportation, and production planning.