

**TWO PLAYER ZERO SUM FUZZY GAMES FOR PLAYERS
WITH DIFFERENT RISK LEVELS**

**FARKLI RİSK SEVİYELERİNE SAHİP İKİ OYUNCULU
SIFIR TOPLAMLI BULANIK OYUN MODELLERİ**

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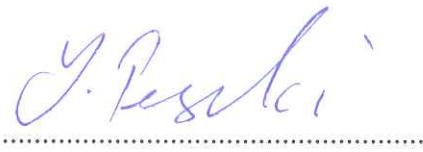
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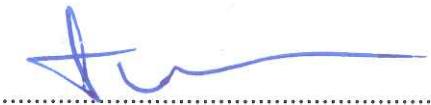
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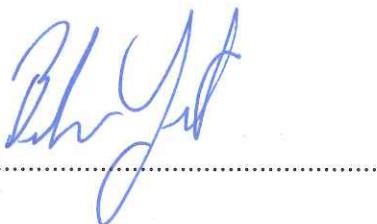
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29/05/2017



Yesim KOCA

*I dedicate this thesis to
my wonderful parents Gülsüm and Güner,
for their endless support,
encouragement
and love.*



ETHICS

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Yeşim KOCA

ABSTRACT

TWO PLAYER ZERO SUM FUZZY GAMES FOR PLAYERS WITH DIFFERENT RISK LEVELS

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Two-player zero sum games, or in short matrix games, are useful models in game theory that models total conflict between the players. Since it is unrealistic for players to know exact payoff values in advance, in recent years fuzzy logic is implemented into matrix games to model payoff matrices. Various solution methods have been proposed in literature for matrix games with fuzzy payoffs, in which the solutions of the game are mostly presented as mixed strategies that make the game optimal and their respective α -cut values. Players' different risk levels have not been considered in these methods. However, in real life players' risk levels may differ since importance of the game can change for each player. Therefore main purpose of this thesis is to provide a solution method for two-player zero sum games with fuzzy payoffs, that considers each player's different risk levels. In the method, α - cut concept is used in order to model risk levels for players. Mixed strategies that give the optimal game value are found by solving the game for specific α values.

The proposed model is implemented to a real world problem to show its applicability and use. In the problem important marketing activities are aimed to be determined for two types of online shopping sites in Turkey, to maximize their return. The problem is modeled as two-player zero (constant) sum game by considering shopping sites as players and their pre-determined store attributes as game strategies. Expert opinions are used to determine

payoff values and the fuzzy payoff matrix is created by converting linguistic expert data into triangular fuzzy numbers. Different risk levels are utilized in the problem since the importance of the game tend to be different for each e-store due to competitive environment. The solution of the game is found by the proposed model. Important marketing attributes for these two online store types are presented and discussed in terms of Turkish customers' online shopping behavior and strategy definitions. The proposed model is found superior to the models proposed in the previous literature on matrix games with fuzzy payoffs, in terms of performance and computational easiness when players' different risk levels are taken into account in the game.

This application provides a novel approach to related literature for determining important marketing activities for online stores. Also, results will be important for Turkish practitioners since there are limited studies on determining important store attributes for Turkish customers.

Key words: Matrix games, Fuzzy logic, Multi-objective linear programming, Online shopping, Marketing.

ÖZET

FARKLI RİSK SEVİYELERİNE SAHİP İKİ OYUNCULU SIFIR TOPLAMLI BULANIK OYUN MODELLERİ

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İki oyunculu sıfır toplamlı oyunlar, ya da kısaca matris oyunları, oyuncular arasında çatışma bulunduğu durumları modellemede kullanışlı bir yöntemdir. Son yıllarda yapılan çalışmalarda oyuncuların kazanç matrislerinin bulanık mantık yöntemi ile elde edilmesi önerilmektedir, çünkü oyuncuların kazanç değerlerini önceden kesin ve tam olarak bilmeleri her zaman mümkün olmamaktadır. Literatürde bulanık kazanç matrisli oyunlar için farklı çözüm yöntemleri önerilmiştir. Bu yöntemlerde çözümler oyunu optimal yapan karma strateji olasılıkları ve bunların α kesim değerleri cinsinden verilmiş, oyuncuların birbirinden farklı olabilecek risk seviyeleri hesaba katılmamıştır. Ancak pratikte farklı oyuncular için oyunun önemi değiştirmektedir, bu yüzden oyuncuların risk seviyeleri farklı olabilmektedir. Bu tezin asıl amacı bulanık kazanç matrisli matris oyunları için oyuncuların farklı risk seviyelerini de hesaba katan bir çözüm yöntemi önermektedir. Bu yöntemde risk seviyelerini modellemek için α kesim konsepti kullanılmıştır. Oyuna optimal değerini veren karma stratejiler oyunun sadece belli α kesim değerleri için çözülmesi ile bulunmuştur.

Önerilen yöntemin kullanılabilirliği bir gerçek hayat problemine uygulanarak gösterilmiştir. Problemde kazançlarını maksimize etmek isteyen Türkiye'deki iki tip online alışveriş

sitesinin önemli pazarlama aktivitelerinin araştırılması amaçlanmıştır. Problem alışveriş sitelerinin oyuncu olarak ve sitelerin önem verdikleri özelliklerin oyun stratejileri olarak düşünülmesi ile iki oyunculu sıfır (sabit) toplamlı oyun olarak modellenmiştir. Kazanç değerlerinin belirlenmesinde uzman görüşleri kullanılmış, uzmanlardan toplanan sözel verinin üçgensel bulanık sayılarla dönüştürülmesi ile oyunun bulanık kazanç matrisi elde edilmiştir. Rekabet ortamında her bir alışveriş sitesinin risk seviyesi farklı olabileceğinden problemin çözümünde farklı risk seviyeleri kullanılmıştır. Problemin önerilen çözüm yöntemi kullanılarak çözülmesi ile, ele alınan iki tip online alışveriş sitesi için önemli pazarlama aktiviteleri sunulmuş, bu sonuçlar Türk müşterilerin online alışveriş davranışları ve sitelerin uygulamaları gereken stratejiler üzerinden tartışılmıştır. Tezde önerilen model, oyuncuların farklı risk seviyelerinin hesaba katıldığı oyunlar için performans ve hesaplama kolaylığı açısından, bulanık kazanç matrisli matris oyunları için önceki çalışmalarda önerilen modellerden daha iyi bir yöntem olarak görülmüştür.

Bu uygulama önemli pazarlama aktivitelerinin belirlenmesi için literatüre farklı bir yaklaşım getirmektedir. Ayrıca Türkiye'deki online alışveriş sitelerinin pazarlama uygulamalarında baz alınabilecek bir uygulama olacağı düşünülmektedir.

Anahtar Kelimeler: Matris oyunları, Bulanık mantık, Çok amaçlı Doğrusal programlama, Online alışveriş, Pazarlama.

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SYMBOLS AND ABBREVIATIONS

Symbols

G	Game model
m	Number of game strategies for player I
n	Number of game strategies for player II
A	Payoff matrix
a_{ij}	Element in i^{th} row j^{th} column of payoff matrix
S_m	Strategy set for player I
S_n	Strategy set for player II
x_i	Mixed strategies for player I
y_j	Mixed strategies for player II
v	Game value for player I
w	Game value for player II
FG	Fuzzy game model
\tilde{A}	Fuzzy number, Fuzzy payoff matrix
$\mu_{\tilde{A}}$	Membership function for \tilde{A}
(a^l, a^m, a^r)	Triangular fuzzy number with lower, medium and upper values
λ	A scalar
p	Number of strategies for player I in fuzzy game
\tilde{v}, \tilde{w}	Fuzzy game values for players I and II respectively
\tilde{A}_α	α -cut of \tilde{A}
\tilde{A}_α^j	j^{th} limit for α -cut of \tilde{A}
α_i	α level for player i
e	Vector of ones

Abbreviations

LP	Linear Program
SEM	Structural Equation Modeling
TFN	Triangular Fuzzy Number
ES	Expert Systems
MCDM	Multi-Criteria Decision Making
FAHP	Fuzzy Analytical Hierarchy Process

FMCDM	Fuzzy Multi-Criteria Decision Making
MOLP	Multi-Objective Linear Programming
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
ELECTRE	Elimination and Choice Expressing the Reality
SERVQUAL	Service quality measurement scale
SWOT	Strengths, Weaknesses, Opportunities and Threats



1. INTRODUCTION

Game theory is a mathematical modeling technique used for decision problems when there are two or more decision makers in conflict or cooperation with each other. Each decision maker plays the game to outsmart the others. By choosing their actions or strategies, they try to optimize their returns or payoffs [1], [2]. In game theory, decision makers are called as players and they are assumed to be rational and intelligent, so that they are consistent in their actions and they are able to see the consequences of each strategy combination for all players [3]. There are different game models that are determined by the number of players and the type of payoffs. Two-player zero sum game (matrix game) is a popular game model, and will be the focus of this thesis, in which two players play the game in total conflict. Loss of one player is equal to the gain of the other player in these game models [2], [4].

Fuzzy logic is an alternative view of set theory, introduced by Zadeh in 1965, allowing membership degrees to elements instead of stating exactly whether an element is a member of a set or not. It allows states to have gradual transitions, for that reason it is useful in mathematically modeling imprecision, vagueness and uncertainties caused by human commonsense and their imprecisely defined, approximate and subjective linguistic data [5], [6]. For instance in decision problems, decision makers' approximate opinions which are affected by their personal experiences, create subjectivity and vagueness and ignoring them may create misleading results. Since fuzzy logic keeps these uncertainties in model instead of ignoring or avoiding them as classical view of science does, it provides more realistic results [7]. Also α -cut concept of fuzzy logic proves useful in modeling optimism and pessimism levels of the decision makers and provides more flexible decision making environments [8]. As a technique for modeling vague, imprecise and uncertain concepts, fuzzy logic has been lately utilized in determining payoff values in game models [9].

In recent literature, payoff matrices are modeled with fuzzy numbers since the assumption of the players to know their exact, numeric payoff values in advance has been found unrealistic [10]–[12]. Therefore, fuzzy logic is implemented to game theory in late studies. Different solution methods for matrix games with fuzzy payoffs are suggested, such as using primal and dual relationship and defuzzification concept [10], [13], solving three linear programs (LP) for each player to optimize different values of the triangular fuzzy number (TFN) separately [11] or solving multi-objective linear programs (MOLP) for the

players by combining these three LPs [12]; and the results are mostly presented in terms of efficient strategy mixes and their respective α -cut values in these studies. Yet as far as it is seen, there is no study that takes different risk levels of players into account. However, in real life the importance of the game for two players may not be the same. For one player the game may be crucial and that player may have a little tolerance to lose, while for the other player the game may not be really important and that player may be more tolerant to lose the game. For example when a large company and small company play a game, the players' risk levels would probably be not the same. The large company would be more risk-tolerant than the small company since it invests only some small proportion of its money on the game and have the opportunity to compensate its losses from other investment (or games) it made (played). On the other hand the small company would have no tolerance to lose and wants his/her gain strictly maximum. It may also be possible for the players to see what their gains would be and their respective strategy mixes change as their risk levels differ and decide on how to play the game accordingly. Therefore, in this thesis, the aim is to propose a solution method for matrix games that consider players with different risk levels. For that, Chandra and Aggarwal's [12] MOLP based approach has been considered, but different α -cut values have been implemented for the players, representing their risk levels. Also the efficient solutions are found for different α -cut values and their respective mixed strategies, different than related literature. In other words, in the proposed model, the strategy mixes are found as making a specified α -cut level optimum.

The proposed model is implemented on a real life problem which aims to determine the important marketing activities (store attributes) for two online shopping sites in order to maximize their gains. The problem assumes the stores offer the same product for the same price in order to focus on the marketing activities only. The problem is modeled as two player zero (constant) sum game in which game strategies are defined as store attributes determined from related literature. Payoffs for a website are determined in terms of the degree they would prefer one of the stores to the other, under given strategy combinations for customers who are known to make the purchase from one of these two stores. Potential consumers' opinions are included in the analysis, if the particular customer has been doing online shopping and they are investigated through comparison questions, asking them to state in which degree they would prefer one of the stores to the other, for each strategy combination. That linguistic data is gathered and converted into fuzzy numbers and fuzzy

payoffs are defined. Once the fuzzy payoff matrix is obtained the proposed model is applied to problem by determining risk levels for players and solving respective MOLPs. The solutions provided a good understanding on the problem and when risk levels differ for different players, the proposed method yields superior performance to the models proposed in the previous literature on matrix games with fuzzy payoffs.

Besides development of a model for solving fuzzy payoff matrix games with different risk levels of players, this thesis contributes to literature as generating a novel approach for determining important marketing attributes for online stores as modeling the problem as a game with fuzzy payoff values. Fuzziness of the payoffs and risk level based solutions provides more flexible decision environment and more realistic results for online stores. This is different than previous literature, in which studies use Structural Equation Modeling (SEM), correlation coefficients and linear regression models mostly to define relationships among store attributes and purchase behavior [14]–[17]. Also these results will be a theoretical basis, especially for practitioners in Turkey, because online shopping is growing rapidly in last few years for Turkish internet users. This growth is expected to increase due to high young population of Turkey and increasing trend of online shopping among young people [17], [18]. Despite this growing pattern, literature on Turkish online shoppers' preferences is limited. Since importance of store attributes, such as customer trust or shopping behavior may show varieties for different cultures [19]. Studies on this topic may not be generable and may not represent Turkish users' behavior. Also internet shopping business need to make provisions about factors affecting customer behavior and understanding the online retail mechanism is crucial, in order to survive [14]. This thesis will contribute to literature by searching Turkish online shoppers' preferences through fuzzy game approach.

The rest of the thesis is organized as follows. In Chapter 2 and 3 game theory and fuzzy logic is defined respectively, some important concepts and literature review is provided on the topics. Then in 4th chapter fuzzy game models are investigated with an emphasis on matrix games with fuzzy payoffs. Proposed model is provided in 5th chapter for matrix games with fuzzy payoff values that considers players' different risk levels and in the following chapter implementation of the model is presented into online shopping problem, along with its results. In chapter 7 conclusion and discussion of the model are presented.

2. GAME THEORY

Game theory concept is initiated as treating the market competitions as a game by Neumann and Morgenstern in 1944. It is a useful mathematical modeling technique for decision-making problems when there are conflict and cooperation between rational and intelligent decision makers. It studies the evolution and overall results of the system, based on each strategic interaction of the players and illustrates the structure of these interactions for helping the decision makers in choosing strategies to optimize their return and predicts how players behave in certain situations like conflicts [20].

A game consists of players, strategies and payoffs. Players are the decision makers (individuals or groups), each player plays the game to outsmart the others and for maximizing their return. Game theory assumes the players are rational and intelligent. Rational means players being consistent in their actions for maximizing their own objectives and intelligent means each player understands the game structure, its rules and are able to see the results for each strategy combination chosen by each player [1], [3]. Strategies are the available set of options or moves for each player. The players choose one of these strategies for gaining more by considering their opponents' situations. Payoffs are the returns gained by players as a result of each combination of strategies the players play.

Usually two forms of game are encountered: extensive form and strategic form. Extensive forms are illustrated with a game tree structure. Nodes show the decision points for players and arcs show possible strategies. With a game tree, every possible alternative and their resulting payoffs can be seen. In extensive form games all sequence of choices are given from beginning to the end. In Figure 2.1, an extensive form game with two players with two strategies is presented. Strategy set for player I is $S = (A1, A2)$ and for player II is $S = (B1, B2)$. Payoffs of the game are given at the right-hand side of the tree in which first term in the parenthesis indicates the return of player I and the second term is the return of player II. If we suppose both players aim to maximize their payoffs, player II will choose B1 if player I chooses A1, and s/he will choose B2 if player I chooses A2. Given this situation, player I will choose A1 in order to gain more. The equilibrium point is indicated by a backwards arrow in Figure 2.1. Chess and checkers are examples for this type of games. Strategic form of game is the most common form in game theory research. It illustrates the players, their alternatives and payoffs for each strategy combination in a matrix form with dimension equal to number of players. For two-player games, in the payoff matrix the first number in each cell represents payoff values for Player I and the

second number is for Player II. Strategic games differ from extensive form in players simultaneous actions, in other words there is no sense of timing in this form of games. This means when players choose a strategy, they do not know the action of the others [3]. Examples of payoff matrices for strategic games are given later in this chapter (see Tables 1, 2 and 3).

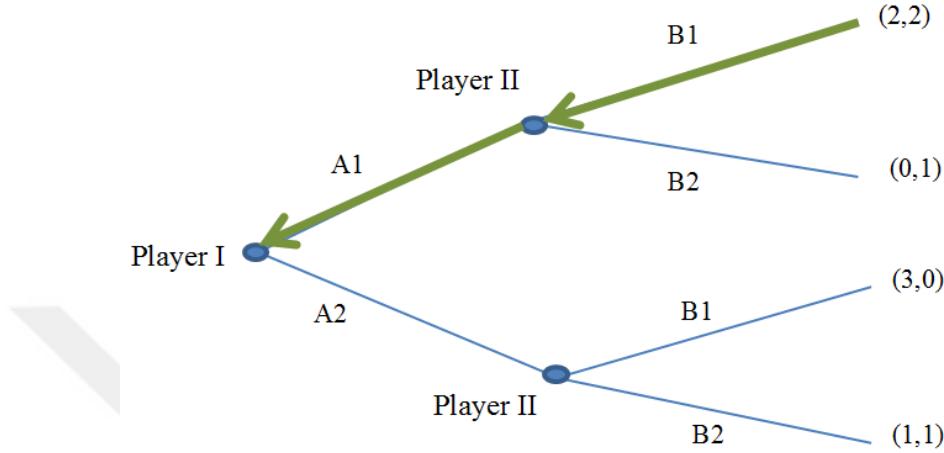


Figure 2.1. Illustration of a game tree for an extensive form game

In game theory, the solutions for players are determined by analyzing each player's decision problem together, since return for each player depends on the strategy the other player selects in the game. But they are different from multi objective decision making problems. Multi objective decision making supposes all players acting in perfect cooperation, for optimization of overall system, regardless of their own benefits; but in game theory each player assumed to optimize his own objectives, taking into account that their actions are affected by other players as well as theirs' effect the others' (the actions are interdependent). Therefore in game theory non-cooperative patterns are common, even though cooperation would yield better results for all players or the system. This means game solutions are not Pareto optimal in general [21]. In cooperative games players coordinate their strategies to achieve the best result, Pareto optimal solution. On the other hand in a non-cooperative game, individuals do not act together and coordinate their strategies; they try to optimize their own return, making the Nash equilibrium point of the game [22]. In short, Nash equilibrium is a state where individuals strive for their own good, regardless of the overall system and in a Pareto optimal solution overall system is optimized, without considering the individuals' good in the system [21].

One example can be given as one of the well-known game theory models: prisoners' dilemma game, which is an example of two player non-constant sum strategic game. In the

game, the police have no sufficient clues to convict two prisoners and for questioning, they are put in two different cells to prevent any communication. Each prisoner has two strategies; testify against the other criminal (betray) or remain silent (cooperate). If both prisoners betray, both of them will be convicted and sentenced to 5 year in prison, if both remain silent, both will sentenced to 1 year in jail due to lack of insufficient evidence. If one betrays while the other remains silent, the one testifies against the other will be free while the other one sentenced to 10 years in prison. The payoff table for prisoner's dilemma game can be seen in Table 2.1. Here, cooperation of both prisoners is the Pareto optimal point. It is the best result for both players (overall system). However cooperation is very risky. If one player chooses to remain silent and the other one betrays, cooperative prisoner gets the longest time in prison (worst result) while the other is set free. Hence assuming both of the players are rational, none of them tend to cooperate, especially when the trust among them is low. Therefore betrayal of both prisoners yielding 5 years in prison for each is the solution of the game in this example [1]. This point is called as "Nash equilibrium" or saddle point. In equilibrium, no player can have a better payoff value by changing their strategies, given the strategies of the other player. In this example a unique Nash equilibrium exists.

Table 2.1. Prisoner's Dilemma Game [1]

		Player II	
		Betray	Cooperate
Player I	Betray	5,5	0,10
	Cooperate	10,0	1,1

Number of players and type of payoffs determine the game model, which are two player zero sum games, two player constant sum games, two player non-constant sum games and n -player games. Two-player zero sum and two-player constant sum games are non-cooperative game models in which the players are in total conflict. In two player zero sum games (or matrix games) one player's gain is exactly as the amount of the other player's loss whereas in constant sum games, sum of the two players' payoffs are equal to a constant number for each strategy combination. It is obvious that zero sum games are a special version of constant sum games when the constant is equal to zero. In two player non-constant sum games (or bi-matrix games) the sum of the players' payoffs are not equal to some specific value. Prisoner's dilemma is an example for bi-matrix games. These games give the possibility of cooperation to the players. N player games investigate

interactions when there are more than two players. In this thesis two player zero (constant) sum games is the main concern.

2.1. Literature Review

Game theory has wide application areas, such as economics, social sciences, biology, healthcare, or simply whenever two or more agents' actions have influence on the others'.

Chew et al. [22] implemented game theory concept into wastewater saving inter-plant water integration schemes in which different companies of an eco-industrial park try to maximize their own benefits. They analyzed individual gains for each scheme and determined the ones that give the best return for each company. They investigated cooperative and non-cooperative approaches. For non-cooperative game Nash equilibrium is found as the solution of the game and Pareto optimal solution is found for cooperative approach which gives better results than Nash equilibrium by sharing water importer and exporter costs among cooperative companies. Madani [21] implemented game theory in management of water resources, where the members are in conflict. He claimed cooperation will improve the overall system by increasing each player's return and reducing environmental impact, and investigated how each player's attempt to improve his/her own return affects the development of the system. He modeled water resource management based on some common game models. In Prisoner's dilemma model the game will not optimize the system since the players do not tend to cooperate. However if the game will be played again and trust among the players improve, the results would change positively. In time as the same game repeated increased trust among players will change the model that suits the game. As the game model updates according to current conditions a better understanding of the system will be provided. Therefore the players will be able to determine their strategies in that way.

Ozkan and Vurus Akcagoz [23] used game theory approach to define a strategy for planting a field for optimizing income and risk values. The problem is modeled as a two-player zero sum game, where the players are taken as farmer and nature. The results are given in terms of crop patterns giving the highest income as mixed strategies for different risk levels. In a similar study of Şahin and Miran [24], the players are taken as the farmer and market.

Arthanari [25] used game theory approach for increasing robustness in process design. He suggested an interactive two player zero sum game in which one opponent using strategies

as design parameters and the other uses objectives and iteratively updating the weights of utility function.

Dowd [26] and Dowd and Root [27] claimed hospital managers who have knowledge of game theory will be more successful and explained importance of game theory to hospital managers. They stated a person who plays well a game like chess or poker will be a better manager, because being good in these games, requires characteristics like discipline, good memory, being able to understand the relationship between the players, which are also needed for being a good manager. Tarrant [20] explained doctor and patient relationship by different game models. Under favor of these game models they tried to give better understanding of doctor patient relationship, made some suggestions for improving patient satisfaction and service quality.

Li et al. [28] examined advertising expense shares for retailer and manufacturer where manufacturer aims to improve brand knowledge and retailer wishes to maximize local advertising. As a game model they provided Stackelberg equilibrium and proposed another approach by considering equal power for retailer and the manufacturer, rather than taking manufacturer as leader and the retailer as the follower in Stakelberg equilibrium. They suggested higher expenses on national brand knowledge and local advertising will yield Pareto optimal solution (higher return than Stackelberg equilibrium). Also they provided a method to determine fraction of reimbursement to that advertising expenses. Esmaeili et al. [29] modeled seller and buyer relationship in supply chains as cooperative and non-cooperative games and compared the results. They used Stackelberg strategy concept for non-cooperative games where one player is assumed as leader and determines the progress of the game. Stackelberg strategy is applied for both cases where the buyer and seller are considered as leaders differently. Also Pareto optimal solution is found for the case which buyer and seller cooperate in determining some parameters of the game such as lot size, selling price and etc. Through a numerical example they showed cooperation gives better outcome for the seller.

Li et al. [30] studied game theory in supply chains and shelf space allocation. They used game theory for modeling conflicts between members of the supply chain. They investigated the system's evolution through different game models used in modeling the game.

Yao [31] proposed a method for evaluating supply chain co-operators like logistics companies, payment platforms and etc. of online stores for integration. He modeled the integration decision as a cooperative game in which he considered different stages. The reason is that the subjective strategies and objective factors of the players can change in time for long term relationships which gives a dynamic nature to game. He used fuzzy memberships for the strategies for different game stages therefore evaluated the alternative partners in fuzzy membership functions. He claimed this method gives better results since it considers different game preferences and provides different results for them, rather than single constant solution given by other methods.

2.2. Two Player Zero Sum Games

Two-player zero sum games, or matrix games, model total conflicts among players. The amount of payoff gained by one player is equal to the amount lost by the other [4]. These game models are well represented by a matrix form, a payoff matrix A , where one player's strategies are illustrated in rows and the other's in columns. Let's say player I has m and player II has n strategies. Then dimension of payoff matrix A is $m \times n$. $S_m = (x_1, x_2, \dots, x_m)$ and $S_n = (y_1, y_2, \dots, y_n)$ denotes strategy sets for player I and player II respectively. In the payoff matrix A , a_{ij} represents the amount gained by player I when player I selects strategy i and player II selects strategy j . This means $-a_{ij}$ is the amount gained by player II, or in other words a_{ij} is the amount lost by player II for strategies i and j played by player I and player II respectively. A common representation of game G is $G = (S_m, S_n, A)$. Expected payoffs of player I are represented as $E = x^T A y$ [32].

In matrix games each player chooses the strategy that will give him/her the best result, given that the other player knows which strategy he/she will choose to maximize his/her return. Therefore the player I follows the strategy that will give the largest minimum of the rows since for each strategy player I will play, the player II will choose a strategy to minimize his/her loss or give player I the minimum in that row. This is similar for the player II, who will select the smallest maximum in a column since for each strategy he/she has, the player I will try to maximize his/her outcome, maximizing player II's loss for that strategy. In brief each player play to get best of the worst payoffs in a sense. See Table 2.2 for an illustration of a matrix game [4]. The player I chooses strategy 3, since s/he gets the largest minimum and the player II chooses strategy 2 for minimizing the maximum payoff. In this game 5 is the maximum value player I can get and the minimum value the player II can give, considering their opponents are also trying to optimize their returns. Since this

value is equal for both players, we say the game has a saddle point and the value (v) of the game is 5. 5 is the equilibrium point where no player can be better off by changing his/her strategy. The value of the game represents the average amount that player I gets from player II.

Table 2.2. Illustration of a Matrix Game with Saddle Point [4]

Player I's Strategies	Player II's Strategies			Row Minimum
	Column 1	Column 2	Column 3	
Row 1	4	4	10	4
Row 2	2	3	1	1
Row 3	6	5	7	5
Column Maximum	6	5	10	

Another approach for finding the solution of the game is eliminating dominated strategies since one will never pick a strategy, giving worse payoffs, regardless of the other player's selected strategy. For some matrix games the payoff matrix may be too complex and saddle point cannot be easily found. However eliminating dominated strategies may decrease the size of the game and saddle point may be found easier [3]. See Table 2.3 for an example. *Strategy 3* dominates *strategy 1* for player II whereas there is no dominated strategy for player I. Therefore *column 1* can be eliminated and the size of the problem is decreased.

Table 2.3. Payoff Matrix with Dominated Strategies [1]

Player I's Strategies	Player II's Strategies		
	Column 1	Column 2	Column 3
Row 1	19	0	1
Row 2	11	9	3
Row 3	23	7	-3

On the other hand, most of the matrix games have no saddle points. This means the value of the game v cannot be found by following one strategy (a pure strategy). Therefore players may choose their strategies by assigning probabilities to them. A mixed strategy is the probability distribution of one player's strategies. Notice that a pure strategy is a special version of mixed strategies, where one strategy has a probability of one and the others zero. These optimal strategy mix values and the value of the game can be found by graphical representation if at least one of the players has exactly two strategies or by LP approach [2]. Let (x_1, x_2, \dots, x_m) be the strategy mix for player I. Since player I aims to

maximize her gain when the player II tries to give her the least, LP for player I can be written as:

$$\begin{aligned} \max_{x_i} \{ & \min \left(\sum_{i=1}^m a_{i1}x_i, \sum_{i=1}^m a_{i2}x_i, \dots, \sum_{i=1}^m a_{in}x_i \right) \\ & x_1 + x_2 + \dots + x_m = 1 \\ & x_i \geq 0, i = 1, 2, \dots, m \end{aligned}$$

Let

$$\nu = \min \left\{ \sum_{i=1}^m a_{i1}x_i, \sum_{i=1}^m a_{i2}x_i, \dots, \sum_{i=1}^m a_{in}x_i \right\}$$

This implies

$$\sum_{i=1}^m a_{ij}x_i \geq \nu, j = 1, 2, \dots, n$$

Therefore player I's problem is

$$\text{Max } z = \nu$$

Subject to

$$\begin{aligned} \sum_{i=1}^m a_{ij}x_i & \geq \nu, j = 1, 2, \dots, n \\ x_1 + x_2 + \dots + x_m & = 1 \\ x_i & \geq 0, i = 1, 2, \dots, m \end{aligned}$$

ν urs

Likewise player II's optimal strategy mix (y_1, y_2, \dots, y_n) is computed by solving

$$\begin{aligned} \min_{y_j} \{ & \max \left(\sum_{j=1}^n a_{1j}y_j, \sum_{j=1}^n a_{2j}y_j, \dots, \sum_{j=1}^n a_{mj}y_j \right) \} \\ & y_1 + y_2 + \dots + y_n = 1 \\ & y_j \geq 0, j = 1, 2, \dots, n \end{aligned}$$

With similar procedure applied as player I's

$$\text{Min } w = \nu$$

Subject to

$$\begin{aligned} \sum_{j=1}^n a_{ij}y_j & \leq \nu, i = 1, 2, \dots, m \\ y_1 + y_2 + \dots + y_n & = 1 \\ y_j & \geq 0, j = 1, 2, \dots, n \end{aligned}$$

ν urs

Solving these LPs will give the value of the game and optimal strategy mix. Notice that the two player's LPs are dual for one another. This means optimal solution to one of them gives the optimal solution to the other due to complementary slackness in duality concept. The optimal results are equal for the two LPs and it is called the value of the game ν [2]. x_1, x_2, y_1 and y_2 gives strategy probabilities that give value of the game, in other words optimal

strategy mixes. The solution of the game represents the optimal mixed strategy and value of the game.

In summary for solving two-player zero sum games one should first look for dominated strategies to reduce the size of the game. After iteratively elimination of dominated strategies, saddle points should be found by finding row minimums and column maximums. If no saddle point exists and if in the reduced matrix is at least one player has exactly two strategies then the problem can be solved graphically, otherwise use LP for finding the solution of the game.



3. FUZZY LOGIC

Science has been subject to a change towards uncertainty in recent years. In classical view of science, precise mathematical representations of real world systems are required for solving problems. Therefore scientists and engineers avoided imprecision and uncertainty as much as they can. However, in reality situations are not always deterministic, imprecision and uncertainty are highly involved and ignoring this vagueness may be impossible due to high complexity of real world problems [5]. Even though computers can handle complexity to some degree, they still have a limited capacity for information processing. Therefore complexity must be traded with some other characteristics of the system, such as uncertainty. Allowing more uncertainty in models decreases complexity and improves credibility of the model [5]. Statistical methods and theory of probability are developed in order to handle a certain type of uncertainty; however they require high number of variables and randomness. Yet, real world problems mostly deal with components with a high level of interactions, where a gradual transition exists between the variables, rather than sharp differences. Also in practice, systems involve numerous elements like machines, humans and computers, thus various types of uncertainties exist in systems caused by linguistic variables, incomplete data and imprecisions. Imprecision is considered somehow associated with probability; however it may be caused by gradual transition between membership and nonmembership. As an example, probability can be utilized for modeling the term “probably” in the proposition “tomorrow will probably be very cloudy”, whereas it will not be enough to model the imprecision caused by the graded nature in the statement “very cloudy” [33]. As can be seen probability is not capable of representing all these distinct types of uncertainty [34]. Fuzzy logic provides opportunity to model these vagueness and ambiguity and provide generable models.

Fuzzy logic is introduced by Zadeh in 1965 as an alternative view of science for modeling vagueness and ambiguity in systems that represents uncertainties, human knowledge, human commonsense and their imprecisely defined, approximate and subjective linguistic data in mathematical terms. Zadeh [34] claims that human brain is able to reason approximately and when stating their opinions, humans tend to be possibilistic, instead of being probabilistic. Therefore he introduced possibility theory and possibility distributions by arguing their similarities and differences with probability theory. Thus he not only saved probability theory being the only concept for modeling uncertainty, but also he challenged classical set theory, where an element either belongs to a set or not. Meaning of

information, instead of its measure is an analysis of possibility theory, rather than probability theory. Probability measures the likelihood of a future event, however possibility theory of fuzzy logic defines how well (in which membership degree) an element belongs to a set.

Human brain is able to make reasonable decisions under vague conditions and judgments that are partially true and approximate such as tall, young, cool, fast and etc. [6]. In some cases linguistic data like expert opinions, need to be used in problems because ignoring them would yield significant information loss. These verbal data usually involves high subjectivity since they depend on personal experiences, expertise of people and personal preferences and these data must be converted into mathematical terms. Fuzzy logic allows uncertainties and vagueness in models and represents gradual membership transitions of variables instead of avoiding them as traditional science does. Hence it creates more realistic, useful and effective results by providing meaningful representations for the vagueness [7].

Fuzzy logic relaxes classical set theory by allowing gradual memberships of elements to fuzzy sets. In classical set theory, an element either belongs to a set or not, while fuzzy logic states the proposition “ x is an element of set \tilde{A} ” may be both true and false in some degrees. The function that assigns a value to each element in a set, which represents the elements’ membership to the set, is called a membership function, and the set defined by this membership function is called a fuzzy set. Membership degrees to fuzzy sets are represented in a closed interval of $[0,1]$. Success of a fuzzy application depends on how successfully the membership function is defined for a given concept. This makes generation of membership functions based on a concept under a specific context the cornerstone of fuzzy applications [35]. Membership function of a fuzzy set \tilde{A} is represented by $\mu_{\tilde{A}}$:

$$\mu_{\tilde{A}}: X \rightarrow [0,1]$$

The extreme values of this interval, namely 0 and 1, represent total non-membership and total membership in a fuzzy set, or in other words total falsity and truth of the given proposition. The higher this number for an element is, the stronger membership has this element to the set. For instance fuzzy set “*tall people*” might assign a membership value of 1 to a person with 190 cm height and membership degree of 0 to a person with 150 cm height. A person with a height of 160 cm may have a membership degree of 0.3 to a fuzzy set “*tall people*”, while s/he may be a member of a fuzzy set “*short people*” with a

membership degree of 0.8. These membership degrees represent how true is the proposition “ x is a tall person” or how an element approximates a subjective concept of “*tall*”. Also notice that membership functions of fuzzy sets are context-dependent, a person with 190 cm height would have different memberships to fuzzy sets “*tall men*”, “*tall women*” or “*tall basketball players*” [5].

Triangular and trapezoidal fuzzy numbers are the most popular types of fuzzy numbers, because of their computational easiness due to their linear membership functions. Also many applications demonstrated that shape of these membership functions does not overly affect the results, hence triangular and trapezoidal fuzzy numbers are usually convenient in most situations. Still, if an appropriate distribution can be determined, nonlinear membership functions like bell-shaped functions can be defined [5]. A variable whose states are defined as fuzzy linguistic concepts is called a fuzzy variable. In Figure 3.1 temperature is defined as a fuzzy variable. This is an example of trapezoidal membership functions. Notice that different temperature values belong to different fuzzy (linguistic) sets (very low, low, medium, high and very high) and how a specific temperature value may belong to two different sets with different membership values.

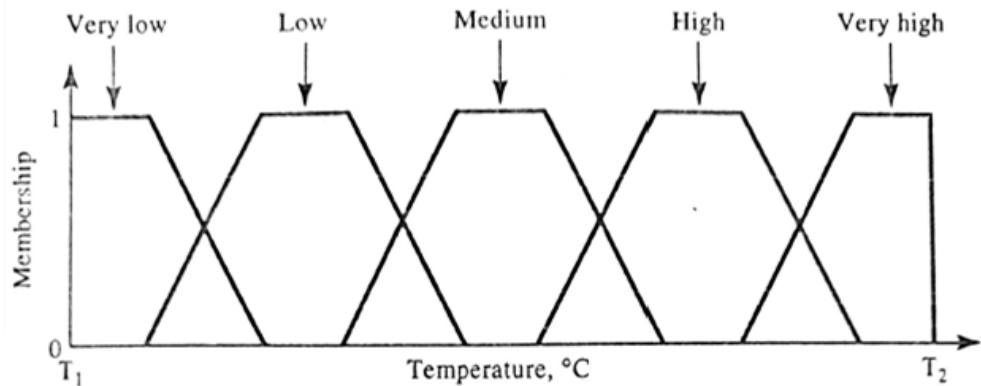


Figure 3.1. Temperature as a fuzzy variable [5]

Arithmetic of fuzzy numbers is generated under two equivalent approaches [32]. One is based on the extension principle of Zadeh and the other one is on α – cut concept of fuzzy numbers. Here latter approach will be presented, since in the following chapters fuzzy arithmetic will be based on α – cut concept. In addition, TFNs’ arithmetic is provided for computational easiness, since TFNs will be the focus, later in the thesis.

Definition: A α – cut set of a fuzzy number \tilde{A} is defined as $\tilde{A}_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$. This means for $\alpha \in [0,1]$, a α – cut set is a crisp interval and can be represented as $\tilde{A}_\alpha = [a_\alpha^1, a_\alpha^2]$.

Let \tilde{A} and \tilde{B} be two fuzzy numbers and $\tilde{A}_\alpha = [a_\alpha^1, a_\alpha^2]$, $\tilde{B}_\alpha = [b_\alpha^1, b_\alpha^2]$ be α – cuts for \tilde{A} and \tilde{B} respectively. Fuzzy summation, subtraction, multiplication and division of these fuzzy numbers $((+), (-), (\cdot), (:))$ and multiplication with a scalar λ is given below.

$$\begin{aligned}\tilde{A}_\alpha (+) \tilde{B}_\alpha &= (a_\alpha^1 + b_\alpha^1, a_\alpha^2 + b_\alpha^2) \\ \tilde{A}_\alpha (-) \tilde{B}_\alpha &= (a_\alpha^1 - b_\alpha^1, a_\alpha^2 - b_\alpha^2) \\ \tilde{A}_\alpha (\cdot) \tilde{B}_\alpha &= (a_\alpha^1 b_\alpha^1, a_\alpha^2 b_\alpha^2) \\ \tilde{A}_\alpha (:) \tilde{B}_\alpha &= \left(\frac{a_\alpha^1}{b_\alpha^2}, \frac{a_\alpha^2}{b_\alpha^1} \right), 0 \notin [b_\alpha^1, b_\alpha^2] \\ \lambda \tilde{A}_\alpha &= (\lambda a_\alpha^1, \lambda a_\alpha^2)\end{aligned}$$

A TFN has a membership function in the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a^l)}{(a^m-a^l)}, & a^l \leq x < a^m, \\ 1, & x = a^m, \\ \frac{(a^r-x)}{(a^r-a^m)}, & a^m < x \leq a^r \end{cases} \quad (1)$$

Where a^m is the mean value, a^l and a^r are the left and right limits of TFN \tilde{A} . Notice when $a^l = a^m = a^r$ the fuzzy number $\tilde{a} = (a^l, a^m, a^r)$ reduces to a real number. Let $\tilde{A} = (a^l, a^m, a^r)$ and $\tilde{B} = (b^l, b^m, b^r)$ be two TFNs. Then fuzzy arithmetic operators are as follows:

$$\begin{aligned}\tilde{A}(+) \tilde{B} &= (a^l + b^l, a^m + b^m, a^r + b^r) \\ \tilde{A}(-) \tilde{B} &= (a^l - b^l, a^m - b^m, a^r - b^r) \\ -\tilde{A} &= (-a^l, -a^m, -a^r) \\ \lambda \tilde{A} &= (\lambda a^l, \lambda a^m, \lambda a^r), \lambda > 0\end{aligned}$$

As stated before a α – cut set of a TFN $\tilde{A} = (a^l, a^m, a^r)$ is defined as $\tilde{A}_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$. This means for $\alpha \in [0,1]$ a α – cut set is a crisp interval represented as $\tilde{A}_\alpha = [a_\alpha^1, a_\alpha^2]$. From Eq. (1) $a_\alpha^1 = \alpha a^m + (1 - \alpha) a^l$ and $a_\alpha^2 = \alpha a^m + (1 - \alpha) a^r$. Obviously for a TFN, $\tilde{A}_1 = [a_1^1, a_1^2] = [a^m, a^m] = a^m$ and $\tilde{A}_0 = [a_0^1, a_0^2] = [a^l, a^r]$.

Figure 3.2 illustrates a TFN with a α – cut.

Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$ and $\tilde{B} = \{(x, \mu_{\tilde{B}}(x)) | x \in X\}$.

$$\mu_{a \cap b}(x) = \min(\mu_a(x), \mu_b(x)), \quad x \in X$$

$$\mu_{a \cup b}(x) = \max(\mu_a(x), \mu_b(x)), \quad x \in X$$

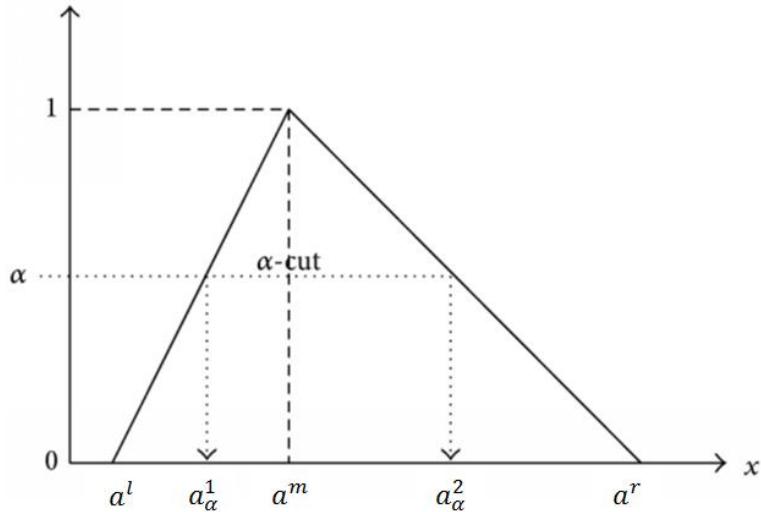


Figure 3.2. Illustration of a TFN and α – cut concept

There are several types of fuzzy sets introduced, in which the most common type in literature is found to be ordinary fuzzy sets that have been discussed so far. Different fuzzy set types relax the assumption of precise knowledge of membership functions. One such example is interval-valued fuzzy sets. On these types of fuzzy sets a membership function does not assign one real number to a given element, but it assigns a closed interval of real numbers. Type 2 fuzzy numbers assign fuzzy membership degrees to each element, rather than a closed interval or a real number. Level 2 fuzzy sets are introduced for the situations when elements of the universal set are unknown in precise terms. In these models elements (x values) are considered as fuzzy sets. Further generalizations and combinations of these fuzzy set types are also possible. These generalized models yield more appropriate results; however, they are computationally non-effective. Usually, applications represented the improvements do not provide over sensitive, much superior results, thus hard computations outweighs the advantages of these generalized models in some cases [5].

3.1. Literature Review

Fuzzy logic has been applied to many fields in literature like management science, statistics, operations research, control theory, human behavior, etc., or whenever vagueness exists in systems [35].

Human reason can model vague concepts that are partially true, so Kosko and Isaka [6] claimed when used in machine design, fuzzy yields user-friendlier machines and provided some fuzzy-ruled machine examples with better performances than conventional machines. Chien and Tsai [36] converted the linguistic answers of the customers to TFNs in their research for assessment of perceived quality in retail industry. In this way they overcame

the vagueness resulting from linguistic variables in the survey analysis. They defined thirteen store attributes for five types of stores. A questionnaire is applied to collect customer satisfaction data and importance level for each attribute. They converted the results to fuzzy numbers due to subjectivity of these linguistic data and analyzed these fuzzy numbers. S. Li et al. [37] proposed an approach for developing marketing strategies under specific circumstances. They combined group Delphi method, fuzzy logic and expert systems (ES) and provided a hybrid approach to overcome weaknesses of these methods with their strengths. In this approach, an expert group performs a SWOT analysis as a group Delphi application to determine business strengths and marketing attractiveness of the company in terms of weights and scores. These numbers are converted into fuzzy numbers since they are affected by individual judgment and their expertise, and through fuzzy rules ES is established. The performance of this method is found to be better through a survey, in which ES, fuzzy ES and proposed hybrid approach are compared by master students in marketing who performed all these methods in an example.

Aydin and Pakdil [38] used fuzzy logic for assessing service quality of an airline company through passenger questionnaires. Since sample was not distributed well in terms of demographic profiles and subjectivity of answers, they applied fuzzy logic for analyzing the results. They converted the linguistic answers into trapezoidal fuzzy numbers. By different α – cut values of the results, they obtained and interpreted results for managers with different risk levels and passengers with different optimism levels. Chou et al. [39] used weighted SERVQUAL method to evaluate airline service quality in Taiwan. They modeled passengers' service expectations and perceptions as fuzzy numbers due to uncertainties in linguistic answers of respondents. They claimed fuzzy application improves appropriateness of the method for considering vagueness of human judgments, instead of using exact numbers in the analysis. Aydin and Chouseinoglou [8] used fuzzy logic in survey analysis in investigating health information system security perception by the users. Based on the results the managers will be able to find which areas should be improved and the results may also work as a decision support system for hiring new staff, determining if the candidate is suitable or not for the security requirements of the managers. Vesely et al. [40] compared linear regression analysis and fuzzy logic model to predict paper recycling behavior of people. Vaguely known, imprecise predictors are modeled as fuzzy numbers and fuzzification and defuzzification steps are implemented into classical regression modeling. Fuzzy logic modeling gives better fitted results since it gives

more weight to observations that are more similar to predicted case, instead of equally weighting them as in linear regression.

3.2. Fuzzy Logic in Decision Making

In decision making, decision makers' opinions are used in finding the best possible alternative. These decision makers' linguistic variables create imprecision in problems since they are subjective judgments or depend on personal opinions and expertise of people in most cases. Accepting this uncertainty and modification of the analysis based on this vagueness provide the decision maker a flexible decision making environment and better results [7]. Also decision makers' optimism/pessimism levels are modeled with fuzzy logic. Therefore fuzzy logic is lately implemented in decision making tools for weighting the criteria, comparison values or whenever the data may yield subjectivity and imprecision. Some examples from literature are given below.

Tang et al. [41] implemented fuzzy multi-criteria decision making (FMCMD) for evaluating marketing strategies in online environment. They used fuzzy logic to convert decision makers' linguistic variables into performance values of different strategies under each criterion to rank them. Aydin [7] used fuzzy logic in finding the best location for a new hospital in Ankara, Turkey. In the study, fuzzy analytic hierarchy processes (FAHP) is used for ranking the alternatives in terms of their possibility to be the best alternative. Comparison matrices are modeled by TFNs, since the comparison is made by experts whose results will contain subjectivity. Arslan and Aydin [42] implemented fuzzy logic in MCDM tools and developed software for the methodology. To test the software they applied FMCMD into two real world military problems. For one problem they used ideal and anti-ideal concept algorithm, the weights and comparison values are modeled as fuzzy numbers and for the other problem they used outranking method and modeled discordance values as fuzzy numbers since these values are dependent on subjective terms of experts involved in the study. They solved the problems in different α – cut levels and displayed the different alternative results for different risk levels of the decision makers. Devi & Yadav [43] implemented triangular intuitionistic fuzzy sets to ELECTRE method for plant location problem. They modeled rates of the alternatives for each criterion and weights of the criteria as fuzzy numbers since they involve decision makers' subjective judgments. Gong [44] proposed a method, utilizing interval-valued type 2 fuzzy sets to determine weights of attributes in multi-attribute group decision making problem. The numerical example aimed to determine the superior supplier among three suppliers for a

manufacturing company. Three decision makers evaluated these three suppliers through four attributes by providing linguistic terms as “very high”, “high”, “medium high”, “medium”, “medium low”, “low” and “very low”. Then they converted these linguistic variables into interval-valued type 2 fuzzy numbers through a given scale and applied their approach. They claimed the approach provided a flexible manner for fuzzy multi-attribute group decision making problems. Kannan et al. [45] implemented TFNs into green supplier selection problem to handle subjective judgments of experts. They considered maximizing purchase amount while minimizing purchase cost. For determining weight of the criteria they used FAHP to use three experts’ linguistic importance data, fuzzy TOPSIS for ranking the suppliers based on expert opinions under five criteria and fuzzy multi-objective linear programming to determine optimal purchase quantity allocation among these suppliers given weights of the criteria and ranks of the suppliers.

Yao [31] developed a method to determine potential supply chain collaborators for online stores. He modeled the problem as a dynamic game, in which subjective strategies and objectives of the co-operators like reducing risks, increasing profits and etc. are considered and evaluated as fuzzy memberships to different game stages. The method is claimed to give more flexible and better results for the players, while other methods always consider the potential partners in online store’s view.

Fuzzy applications in game theory are also quite popular in literature. Classical game theory assumes players will know their outcome precisely for each strategy combination beforehand. However in real life this is not realistic due to the vast amount of uncertainties and complexities in systems and being able to know the exact amount of payoff values can be impossible in many situations. Sometimes players may only believe their payoffs will be “really high”, “high”, or “low”, as linguistic variables. These variables usually involve personal experiences, opinions, so they are subjective. These uncertainties, ambiguities and imprecision caused by subjectiveness can be well modeled by fuzzy logic and game models with fuzzy payoffs are quite popular in literature which will be investigated deeper in the next chapter [10]–[13], [32], [46]. Also in real life players’ risk levels may differ. For one player the game may be crucial, that player has no tolerance to lose and wish a higher membership degree to the value of the game, in other words one player may be pessimistic, while for the other player the game may not be that much important, s/he may have tolerance to lose to some degree or in other words may be satisfied with lower degrees of membership to optimal value of the game. Hence s/he may be more optimistic.

Or it may be possible for the players to see what their gains would be and their respective strategy mixes change as their risk levels differs and decide on how to play the game accordingly, in terms of visualizing the problem under different risk levels. These situations can be well modeled with fuzzy numbers, and α – cut concept in fuzzy logic. For these reasons fuzzy logic is found to be an appropriate and useful method in this thesis.



4. FUZZY GAME THEORY

As stated in detail in the previous chapter, fuzzy logic is used when there are uncertainties, unavailable information or imprecision of available information. Game theory assumes everything is known exactly in advance by each player, which is unrealistic in many cases in real life [13]. Therefore fuzzy logic is applied to game theory in late studies. Yuh-Wen et al. [47] used fuzzy objective functions for players. Game theory is utilized for modeling behaviors of different parties in supply chain. Each partner is assigned many objectives and suggestions made for system improvements by examining three different collaboration scenarios. Li & Hong [46] solved matrix games with TFN payoff values, when the players' strategies have constraints. They claimed in practice players may not be able to choose all strategies freely. There may be some constraints, like money invested for strategies. They suggested a model for solving constrained matrix games with fuzzy payoffs and applied their model into a numerical example to show its usefulness. Dang & Hong [48] suggested a Cournot production game model in determining strategic and operational plans for production facilities under fuzzy random environment. The model contains two stages; first production strategy is determined then production quantities are found, which are modeled as TFNs. Through a case study they showed the application of their model. Zhang et al. [49] used fuzzy logic in game constraints. In reality, all of the players are not able to choose their strategies in any way they want or they may accept different constraints in different levels, also human behavior may not be exactly the same as game theory assumes. These uncertainties in game nature modeled as fuzzy constrained game model and prisoners' dilemma and stag hunt games are solved with fuzzy constraints. The results are claimed to be more realistic. In some studies [10]–[12] fuzzy logic is used in determining payoff values of players for each strategy combination in two player zero sum games for players inability of knowing exact payoff values certainly in reality. They provided solution methods for fuzzy payoff matrix games, which will be investigated deeply in this chapter. In this thesis matrix games with fuzzy payoffs will be concerned.

4.1. Two Player Zero Sum Games with Fuzzy Payoffs

In traditional matrix games, payoffs are assumed to be known precisely and exactly for each strategy combination to each player. However this is not realistic in most cases. Usually players are not able to know exact payoff values due to unavailable or inadequate information. Therefore fuzzy logic is used in modeling the payoff matrix in recent studies [11].

Let \tilde{A} be fuzzy payoff matrix of any matrix game, and S_p and S_n be the crisp strategy sets for both players where p and n represents number of strategies for player I and player II respectively. Then in fuzzy game $FG = (S_p, S_n, \tilde{A})$ only the payoff matrix is fuzzy, strategy sets for both players are assumed as crisp.

Bector et al. and Vidyottama et al. [10], [13] developed a model for solving matrix games with fuzzy payoffs, which uses primal and dual relationship of linear problems. They used defuzzification in finding the value of the game and found different game values for the players. They explained the reason of this difference as one cannot expect these two defuzzified values, as crisp numbers, to be equal every the time since the equal game values hold in fuzzy logic. Li [11] proposed another model for solving matrix games with payoffs of TFNs, which is given below for player I. In the model he used three different LPs for each value of the TFN; in other words he maximized right (v^r), medium (v^m) and left (v^l) values separately for player I and minimized the values w^r, w^m and w^l separately for player II. Thus they solved three LPs for each player, which are primal and dual to each other. Expected payoff for player I is $\tilde{E} = y^T \tilde{A} x$ and the player II is $\tilde{E} = -y^T \tilde{A} x$ in this model. Therefore in the model both players' game values are found equal.

$$(LP - I)_1 \quad \text{Max } v^l$$

Subject to

$$\begin{aligned} \sum_{i=1}^p (a_{ij})^l x_i &\geq v^l, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p x_i &= 1 \\ x_i &\geq 0 \end{aligned}$$

$$(LP - I)_2 \quad \text{Max } v^m$$

Subject to

$$\begin{aligned} \sum_{i=1}^p (a_{ij})^m x_i &\geq v^m, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p x_i &= 1 \\ x_i &\geq 0 \end{aligned}$$

$$(LP - I)_3 \quad \text{Max } v^r$$

Subject to

$$\begin{aligned} \sum_{i=1}^p (a_{ij})^r x_i &\geq v^r, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p x_i &= 1 \\ x_i &\geq 0 \end{aligned}$$

Chandra & Aggarwal [12] proposed a model which treats the problem as multi-objective programming, optimizing the values of fuzzy number at once. Two multi-objective linear programs (MOLP) with payoffs of TFNs are defined. They opposed Li's[11] model since the optimal strategy mix values can be found differently by solving separate LPs for each value of the TFN. Also they claimed values for these two programs do not have to be equal since the payoffs are fuzzy numbers and various efficient points are obtained when they are solved.

$$(\text{MOP - I}) \quad \text{Max } (v^l, v^m, v^r)$$

Subject to

$$\begin{aligned} \sum_{i=1}^p (a_{ij})^l x_i &\geq v^l, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p (a_{ij})^m x_i &\geq v^m, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p (a_{ij})^r x_i &\geq v^r, \quad (j = 1, 2, \dots, n) \\ \sum_{i=1}^p x_i &= 1 \\ x_i &\geq 0 \end{aligned}$$

$$(\text{MOP - II}) \quad \text{Min } (w^l, w^m, w^r)$$

Subject to

$$\begin{aligned} \sum_{i=1}^n (a_{ij})^l y_j &\leq w^l, \quad (i = 1, 2, \dots, p) \\ \sum_{i=1}^n (a_{ij})^m y_j &\leq w^m, \quad (i = 1, 2, \dots, p) \\ \sum_{i=1}^n (a_{ij})^r y_j &\leq w^r, \quad (i = 1, 2, \dots, p) \\ \sum_{j=1}^n y_j &= 1 \\ y_i &\geq 0 \end{aligned}$$

Chandra & Aggarwal [12] proposed the following model for piecewise fuzzy numbers:

$$(\text{VP})_1 \quad \text{Max } (v_{\alpha_i}^j, (i = 1, 2, \dots, r, j = 1, 2))$$

Subject to

$$\begin{aligned} x^T \tilde{A}_{\alpha_i}^j &\geq (v_{\alpha_i}^j) e, \quad (i = 1, 2, \dots, r, j = 1, 2) \\ x &\in S^m \end{aligned}$$

$$(\text{VP})_2 \quad \text{Min } (w_{\alpha_i}^j, (i = 1, 2, \dots, r, j = 1, 2))$$

Subject to

$$\begin{aligned} \tilde{A}_{\alpha_i}^j y &\leq (w_{\alpha_i}^j) e, \quad (i = 1, 2, \dots, r, j = 1, 2) \\ y &\in S^n \end{aligned}$$

Here MOP is used for piecewise fuzzy numbers. First r is chosen as number of α – cut values that describe the fuzzy number in best way (it is logical to choose these numbers as break points of the membership function). In the model $e^T = (1, \dots, 1)$ represents the vector of ones with its context-dependent dimension. j represents if the value is upper or lower bound of that α – cut level, so $[(a_{lk})_{\alpha_i}^1, (a_{lk})_{\alpha_i}^2]$ denotes the α_i -cut of the fuzzy number \tilde{a}_{lk} . $\tilde{A}_{\alpha_i}^1$ and $\tilde{A}_{\alpha_i}^2$ denote the matrix $\tilde{A}_{\alpha_i}^1 = [(a_{lk})_{\alpha_i}^1]$ and $\tilde{A}_{\alpha_i}^2 = [(a_{lk})_{\alpha_i}^2]$ respectively and $[\tilde{v}_{\alpha_i}^1, \tilde{v}_{\alpha_i}^2]$ and $[\tilde{w}_{\alpha_i}^1, \tilde{w}_{\alpha_i}^2]$ denote the α_i -cut of \tilde{v} and \tilde{w} respectively. Different α – cut levels for fuzzy payoff matrix \tilde{A} is found by predefined membership functions for each element in the matrix. Then problems are solved to find the best value in terms of these α – cut levels, and their respective strategy mix values.



5. PROPOSED MODEL

As stated in earlier chapters, two-player zero sum games or matrix games are useful when there is total conflict between players. Fuzzy payoff matrices are recently being used since payoff values for each strategies selected by players are uncertain in many situations. However importance of the game may be different for players. For one player the game may be really important because s/he may be investing all his/her money on this game, s/he may have no tolerance to lose and may not want to take any risk. On the other hand, other player may be investing only some small amount of his/her money on the game, may have more tolerance to lose, the game may not be as important for that player as the other one. Thus acceptable game values for the players may differ. Risk-averse player may only accept optimization in high α – cut levels of the value, while the risk-tolerant player may be satisfied with smaller α – cut levels of the value of the game, and be interested in evaluating different results of the game, different strategy mixes and game values and select what is best for him/her. Optimizing one α – cut level selected based on the risks of players may yield better results and show the importance of the strategies for different situations.

Below can be seen proposed model that considers different risk levels of the players.

(MOLP-I)

$$\text{Max} (v_{\alpha_1}^j, (j = 1,2)) \quad (2)$$

Subject to

$$x^T \tilde{A}_{\alpha_1}^j \geq (v_{\alpha_1}^j)e, (j = 1,2) \quad (3)$$

$$\sum_{i=1}^p x_i = 1 \quad (4)$$

$$x_i \geq 0$$

(MOLP-II)

$$\text{Min} (w_{\alpha_2}^j, (j = 1,2)) \quad (5)$$

Subject to

$$\tilde{A}_{\alpha_2}^j y \leq (w_{\alpha_2}^j)e, (j = 1,2) \quad (6)$$

$$\sum_{t=1}^n y_t = 1 \quad (7)$$

$$y_t \geq 0$$

In the model α_1 denotes the α -cut level for the first player and α_2 is α -cut level for the second player, selected according to respective risk levels of the players. As player i becomes more risk averse, α_i should get higher values in the interval $0 < \alpha < 1$, since that

value represents, that player has at least α level risk. e^T represents the vector of ones. $j=1,2$ denotes if the value is upper or lower bound of the α -cut level. In other words, $[(a_{lk})_{\alpha_i}^1, (a_{lk})_{\alpha_i}^2]$ represents α_i -cut of the fuzzy number \tilde{a}_{lk} . \tilde{A} represents fuzzy payoff matrix of the game, with dimension $p \times n$, where p and n are the numbers of strategies for the first and second player respectively. \tilde{A}_{α_i} denotes α_i -cut of the payoff matrix for player i where, matrix $\tilde{A}_{\alpha_i}^1 = [(a_{lk})_{\alpha_i}^1]$ is composed of lower limits and matrix $\tilde{A}_{\alpha_i}^2 = [(a_{lk})_{\alpha_i}^2]$ is composed of upper limits of the matrix \tilde{A}_{α_i} . As for the decision variables, x in MOLP-I and y in MOLP-II represents the efficient probability distributions of the players' mixed strategies. $[v_{\alpha_1}^1, v_{\alpha_1}^2]$ and $[w_{\alpha_2}^1, w_{\alpha_2}^2]$ denote the α_i -cut of game solutions, \tilde{v} and \tilde{w} , respectively.

In MOLP-I, Eq. (2) and (3) maximize the minimum value player I can gain, since while player I tries to maximize his/her payoff, player II tries to give him/her the minimum amount s/he can give in matrix games. In Eq. (2) objective functions are maximizing the boundaries of α_1 -cut of game value v , namely $v_{\alpha_1}^1$ and $v_{\alpha_1}^2$ and Eq. (3) ensures these values are not higher than any of the strategy combination player I can play. Since the player will choose the optimal strategy mix probabilities among these predetermined strategies and the probabilities cannot be greater than one, Eq. (4) is included in the model.

On the other hand Eq. (5) and (6) of MOLP-II ensures the player II loses the minimum of the maximum amount s/he can lose, because player I aims to maximize player II's loss while player II is aiming to minimize it. Eq. (5) minimizes the boundaries of α_1 -cut of game value w , namely $w_{\alpha_2}^1$ and $w_{\alpha_2}^2$ and Eq. (6) ensures these values are not lower than any of the strategy combination player II can choose. For the same reason of MOLP-I's Eq. (4), Eq. (7) equates the mixed strategy sums of player II to one. Finally for both MOLP-I and MOLP-II the strategy probabilities should be greater than or equal to zero.

6. IMPLEMENTATION OF THE PROPOSED MODEL ON AN ONLINE MARKETING PROBLEM

In this section proposed model is implemented to a real-world problem to show its applicability and use. The problem aims to find the most important marketing activities for two online stores to maximize their return. In this section first online marketing is discussed through literature review. Then the problem is defined and the solution is represented through implementation of the proposed method.

6.1. Online Marketing

Online shopping is one of the most popular activities on the internet. Literature demonstrates online shopping marketing elements differ from traditional marketing strategies. Thus marketing frames and activities that stores should perform are different in order to attract consumers purchase behavior. Some marketing activity suggestions are made that affect consumer purchase behavior. Allen & Fjermestad [50] marked e-commerce marketing activities should be different than traditional marketing model and in their study they developed a new framework for e-store practitioners as means of traditional marketing mix model 4Ps (product, place, price and promotion). In online domain, information itself becomes a product and customers can obtain various information about different kinds of product simultaneously, unlike traditional models in which information gathering about the product takes time and money. In terms of place, online stores have profound effect on value chains and they reach everywhere with internet connection. Price element will also differ since through internet price comparisons are easier to make. Online stores also have advantage in terms of promotion since by data mining individual or customer profile based promotions could be provided. They also claimed marketing tradeoffs should be changed like dependencies between place and promotion will not mean the same on internet. All of these indicate e-commerce marketing activities are significantly different from traditional model.

Lee and Lin [51] modified the SERVQUAL model which is used in measuring service quality, to make it applicable to e-commerce services. Therefore in consideration of related literature, they aimed to determine what dimensions affect the service quality essentially, and how they influence the consumers. They built a model in which they defined the e-service quality dimensions as website design, reliability, responsiveness, trust and personalization, then tested the hypotheses if these dimensions have significant effect on overall service quality and customer satisfaction, and their relationship with purchase

intentions. The results demonstrated that all the dimension have significant relationships with customer satisfaction and service quality expect for personalization. The possible reason is stated as customers' fear of personal information revelation. Park and Kim [52] investigated customer purchase behavior and how consumer willingness can be affected in order to make them purchase from an online store. They defined some online store attributes as user interface quality, product information quality, service information quality, security perception and site awareness and examined if there are relationships with information satisfaction, relational benefit, site commitment and purchase behavior. A questionnaire is applied to consumers who are using a certain online bookstore and the results indicated information satisfaction is strongly related to production information quality and relational benefit is strongly related to service information quality. Information satisfaction and relational benefits both have significant effect on site commitment where information satisfaction has stronger effect and site commitment has significant effect on purchase behavior. Topaloglu [17] examined if hedonic and utilitarian value, security and privacy has positive influence on search intention and purchase intention of online customers in Turkey. Testing research hypotheses through regression analysis she found no significant relationship between privacy and search and purchase intentions. Also utilitarian value has no relationship with search intentions while the other hypotheses are supported in the study. Kim et al. [14] examined how customers' hedonic and utilitarian values influenced by system quality, information quality and service quality and investigated effects of e-purchase value on customer repurchase intention or loyalty. Results show variations for different customer characteristics. Importance of web-page design on customer loyalty to online stores is investigated by Bilgihan and Bujisic [15]. Relationship between hedonic and utilitarian attributes of the web-page design and customer commitment (which is divided into affective commitment and calculative commitment), trust and loyalty is examined. They used SEM in order to test the research hypothesis. The result of the analysis supported each hypothesis claiming positive relationship between the variables; except that insignificant relationship between calculative commitment and loyalty.

Literature review demonstrates purchase intention factors can change based on culture or nation of customers. Sakarya and Soyer [53] investigated cultural differences on online shopping behaviors and consumption values in terms of hedonic and utilitarian terms. They compared Turkish and British online shoppers. A survey study is applied and results

demonstrated that although consumption values showed no significant difference, online shopping behaviors differ for Turkish and British users. The reason was asserted as Turskikh users' higher tendency of risk aversion. Rouibah et al. [19] investigated the factors affecting customer trust to online payment and if those factors have positive or negative relationship with purchase intentions. They claimed trust is influenced by cultural differences. In the research they examined Kuwait, an Arab country with risk averse and collectivist people. Some results demonstrated conflicts with related literature which are explained by Arabic cultural differences and some strategies are suggested for practitioners in order to improve customer trust for online payment activities in Kuwait.

In some studies the marketing activities are categorized as pre- and post-purchase strategies and how their effects change on purchase behavior. Cao & Gruca [54] analyzed reasons for price differences in online book stores. They claimed better service quality and different competitive advantage of the brands may be the reasons, so they took pre- and post-purchase marketing strategies and brand name as factors. Pre-purchase involves information share on products and prices where post- purchase involves delivery of products, track of delivery and consumer support services. Through hypothesis testing they found high post-purchase service quality providers and popular brands charge more for their products and pre-purchase service quality has no direct significant effect on prices. A deeper analysis demonstrated popular brands actually give better services to their customers instead of only gaining more with their brand's name. Ha [55] examined risk perception of customers before an online purchase is made and how pre-purchase information like brand name, word-of-mouth and customized information influences customers in terms of risk reduction. Consumer experience-based attributes like word-of-mouth and providing customized information have found to affect purchase behavior and brand name has a significant effect on customers' perceived risk.

Customer characteristics also influence purchase attitude and they should not be ignored when considering marketing activities. Wu [56] states that consumer attitude is the easiest thing to be affected in online store marketing. Therefore they examined how customer characteristics change consumer attitudes toward purchase behavior and how they vary based on different customer characteristics. They advised practitioners that the customer types with higher purchase attitude score characteristics should be the target for marketing activities.

6.2. Problem Definition and Implementation of Proposed Model

Two competing online shopping sites are considered in a problem which aims to find best marketing activities (attributes) for each store to maximize their gain. *Store A* stands for the website for the brand of the product; this means *Store A* offers various products only from the same particular brand, whereas *Store B* represents a shopping site where different kinds of products and brands are offered simultaneously. This would probably affect competitive advantage of the stores and their risk levels. For instance *Store B* may have the ability to compensate loses by selling different brands' products and may be more tolerant to lose for that reason. This means *Store B* may be more risk-tolerant than *Store A*.

In the problem, stores are assumed to offer the same product for the same price in order to focus only on the marketing activities. Since the price is assumed equal, gain values for a store can be calculated with degree of preference of customers for a store, who are known to buy that specific product from one of these stores. In short, the problem aims to find important store attributes in order to maximize each store's preference degree.

The problem is modeled as a game. Here, *Stores A* and *B* represent the rational and intelligent players and the payoffs are determined for a site, as the degree of preference of customers for that site that are known to make the purchase from either *Store A* or *B*, for each strategy combination. Hence, when a customer prefers a store to purchase the product with a degree, that customer is won by that store with that preference level, while that preference level represents the degree that the other store is not chosen and that customer is lost by that store in that preference level. This creates total conflict between the two stores. Therefore two-player zero (constant) sum games would be appropriate for modeling this problem.

The store attributes (marketing activities) of *Store A* and *B* stand for game strategies. The literature review assisted in understanding online store dynamics and selecting game strategies for the problem, which are marketing activities or attributes that stores obtain in order to attract customers' purchase intentions.

Despite its advantages over traditional shopping like easy price comparison and information search, no loss of time and effort, online shopping generates various risks for customers since they are only able to know the information about the product/service that online store provides them, they are not able to physically experience the product mostly before payment [53]. According to Ha [55] these risks may include performance risks that

incur when a product does not function in a way as expected, financial risks representing the risk when no money-back warranty is offered or repair costs if needed, psychological risk described as the discomfort of customer about regretting the purchase or fear of information are not kept in safe and time risk. Due to these risk terms customers behave in a way to reduce their perceived risks and increase chance of a satisfactory purchase [55]. Considering related literature for determining factors that affect customers' online purchase intentions either by reducing their risks or by improving satisfaction, six game strategies are determined for both stores, as providing the following services well to the customers [15], [16], [19], [31], [52], [54].

1. ***Customer support:*** Giving prompt response to consumers' questions and complaints, providing frequently asked questions, paying attention to customer feedback to attract customers' purchase intentions [52].
2. ***Product information quality:*** Providing information abundance on the product such as previous buyer's comments, technical information about the product, detailed size information, photographs of the product (on a model if necessary) and etc., in order to assist consumers in predicting the product quality and their satisfaction from the product and reduce their perceived risk on making the purchase [52].
3. ***Delivery service:*** Fast and/or timely delivery, store's tracking of delivery process to improve customer satisfaction [31], [54].
4. ***Return policy:*** Supporting customers on return process, setting customer oriented return rules, providing money-back warranty in order to reduce their perceived financial risk of purchasing an unsatisfactory product [52].
5. ***Trust:*** Customers' ability to trust that the site keep personal information or payment information secure, store's wish to make good impression on customers so that gaining customer loyalty and making sure previous customers give positive recommendations (word-of-mouth) about the website to potential customers, advertisements to enable customers to know the store [16], [19].
6. ***User interface design:*** User interface design eases customer's shopping, without time loss, customers are able to find easily what they are looking for and interface provides understandable and attractive design [15].

After strategy sets for the players are identified as above, customers' preference degree is detected for *Store A*, for each of these strategies. Potential customers' opinions are

investigated by an opinion collection form, asking them to state in which degree they would prefer *Store A* under each strategy combination. Participants who do not shop online are not included in the analysis. The form is composed of two sections: in the first section 11 questions are asked to gather information about participants' demographics, their internet use and online shopping behaviors. In the second section, questions are built as comparison questions like "*When Store A is known to provide customer support while Store B is known to share good quality product information, in what degree would you prefer Store A to Store B?*" and the answers are taken as linguistic terms, in the scale of "*extremely preferable, really preferable, somewhat preferable, rarely preferable, equally preferable, rarely not preferable, somewhat not preferable, really not preferable, extremely not preferable*". In total 36 comparisons are made for each strategy combination as verbal statements of the experts (participants who do online shopping). A sample form can be seen in the Appendix.

For gathering opinions, 50 opinion collection forms were distributed to students and personnel in different departments of Hacettepe University, Beytepe Campus, Ankara, Turkey. Among 43 returned forms, 3 were discarded since the respondents do not shop online. In total 37 forms were found appropriate for the analysis from 22 female and 15 male participants. 20 of the respondents are students, which have been considered as participants with no income and 17 are employees of the university, mostly academic staff, which are considered as participants with income. On the average, respondents use internet for 12.5 years, 5.6 hours a day. 25 of the participants (67.5%) stated they shop online for 1-5 years and 6 of them (16.2%) stated they shop online for 0-1 year; regardless of their ages and years of internet usage. The average minimum percentage of satisfaction level, in order to make repurchase from a website is obtained as 83.2%, for which the range is between 60% minimum and is 100% maximum. 31 respondents (83.8%) stated they trust online shopping.

In the second section, median value of 37 respondents is found for each comparison question. They represent the crisp payoffs for *Store A* for the strategy combinations given in the respective question. The vagueness and uncertainty caused by the linguistic variables' nature of carrying personal experiences and subjective judgments of different respondents as well as unevenly distributed customer characteristics in the sample (such as higher number of female respondents than male respondents), are overcame by modeling them as TFNs [36]. All equivalent fuzzy numbers are given for each linguistic answer in

Table 6.1. The table is obtained from Aydin [7], who generated it by Chou et al. [39] and Kahraman et al.'s [57] studies by making necessary adjustments.

Table 6.1. Fuzzy preference scale and fuzzy opposite preference scale [7], [39], [57]

Crisp Preference Scale	Definition	Fuzzy Preference Scale	Fuzzy Opposite Preference Scale
1	Equally preferable	(1, 1, 1)	(1, 1, 1)
2		(1/2, 3/4, 1)	(1, 4/3, 2)
3	Rarely preferable	(2/3, 1, 3/2)	(2/3, 1, 3/2)
4		(1, 3/2, 2)	(1/2, 2/3, 1)
5	Somewhat preferable	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
6		(2, 5/2, 3)	(1/3, 2/5, 1/2)
7	Really preferable	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)
8		(3, 7/2, 4)	(1/4, 2/7, 1/3)
9	Extremely preferable	(7/2, 4, 9/2)	(2/9, 1/4, 2/7)

After transforming the linguistic data into TFNs, the payoff matrix \tilde{A} is obtained as in Table 6.2. The payoff matrix illustrates fuzzy preference levels for *Store A* under each strategy combination. For instance, when *Store A* chooses strategy 1 (*customer support*) and *Store B* chooses strategy 2 (*product information quality*), *Store A*'s fuzzy preference degree (payoff) is $\tilde{a}_{12} = (2/3, 1, 3/2)$.

Table 6.2 Fuzzy payoff table for the problem

		Store B					
		B1	B2	B3	B4	B5	B6
Store A	A1	(3/2, 2, 5/2)	(2/3, 1, 3/2)	(1,1,1)	(2/3, 1, 3/2)	(1,1,1)	(2/3, 1, 3/2)
	A2	(2/3, 1, 3/2)	(2/3, 1, 3/2)	(1,1,1)	(2/3, 1, 3/2)	(2/3,1,3/2)	(3/2, 2, 5/2)
	A3	(3/2, 2, 5/2)	(2/3, 1, 3/2)	(1,1,1)	(2/3, 1, 3/2)	(2/3,1,3/2)	(3/2, 2, 5/2)
	A4	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(2/3, 1, 3/2)	(2/3,1,3/2)	(5/2, 3, 7/2)
	A5	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(2/3, 1, 3/2)	(3/2, 2, 5/2)	(1,1,1)	(3/2, 2, 5/2)
	A6	(2/3, 1, 3/2)	(2/3, 1, 3/2)	(2/5, 1/2, 2/3)	(2/7, 1/3, 2/5)	(2/5,1/2 2/3)	(1,1,1)

Once the payoff matrix is obtained, solution method proposed in the previous chapter is implemented to the problem. First, α_1 and α_2 values are determined in accordance with risk levels for the players. Since *Store A* is the website of the brand and *Store B* is an online shopping site that offers different brands at the same time, competitive advantage and the risk levels of the stores may differ, meaning, importance of the problem can vary and each store may have different tolerances to lose the game. One store may be satisfied with smaller gains or the stores may only wish to see how their optimal solutions change for different risk levels and choose the best among those solutions in order to optimize their overall outcome. For example for *Store B* the problem may not be crucial and that store

may accept smaller gain values if the strategies cost less for him/her since s/he can compensate the loses by selling products from different brands. As mentioned before, different optimism, pessimism and risk levels can be well modeled with α – cut concept in fuzzy logic [8]. Since risk tolerance means the player tolerates some amount of loss, lower membership degrees of the optimal value may be used, on the other hand if the game has high importance to the player he/she would not have any tolerance to lose and wants to optimize his/her solution in higher membership degrees. When α equals to one, this means the player only accepts the optimal value as the solution of the game and respective mixed strategies giving 1-cut value of the game. Smaller α values provide the player a more flexible decision environment but smaller membership degrees to the optimum solution [58].

In the thesis, α values are taken as 0, 0.25, 0.5, 0.75 and 1, representing the risk levels gradually; where $\alpha = 0$ and $\alpha = 1$ stand for extreme optimism (risk tolerance) and pessimism (risk aversion) levels respectively, and $\alpha = 0.5$ represents moderate optimism (risk) level for the stores, when the risk level is at least 0.5 for a store. Respective α – cut values for each of the fuzzy preference level of Table 6.1 are calculated and the TFN payoff matrix in Table 6.2 is converted into respective α – cut payoff matrices accordingly. For an illustration, let's say both stores have the maximum optimism levels, which means $\alpha_1 = \alpha_2 = 0$. Table 6.3 demonstrates the 0 – cut payoff matrix of the problem \tilde{A}_0 , which is obtained by transforming \tilde{A} in Table 6.2 based on preference scales obtained for 0 - cut interval.

Table 6.3. Payoff matrix for $\alpha = 0$, \tilde{A}_0

		Store B					
		B1	B2	B3	B4	B5	B6
Store A	A1	(1.50, 2.50)	(0.67, 1.50)	(1, 1)	(0.67, 1.50)	(1, 1)	(0.67, 1.50)
	A2	(0.67, 1.50)	(0.67, 1.50)	(1, 1)	(0.67, 1.50)	(0.67, 1.50)	(1.50, 2.50)
	A3	(1.50, 2.50)	(0.67, 1.50)	(1, 1)	(0.67, 1.50)	(0.67, 1.50)	(1.50, 2.50)
	A4	(1.50, 2.50)	(1.50, 2.50)	(1.50, 2.50)	(0.67, 1.50)	(0.67, 1.50)	(2.50, 3.50)
	A5	(2.50, 3.50)	(1.50, 2.50)	(0.67, 1.50)	(1.50, 2.50)	(1, 1)	(1.50, 2.50)
	A6	(0.67, 1.50)	(0.67, 1.50)	(0.4, 0.67)	(0.29, 0.40)	(0.4, 0.67)	(1, 1)

Obviously from Table 6.3 \tilde{A}_0^1 and \tilde{A}_0^2 is:

		Store B					
		B1	B2	B3	B4	B5	B6
$\tilde{A}_0^1 = \text{Store A}$	A1	1.50	0.67	1	0.67	1	0.67
	A2	0.67	0.67	1	0.67	0.67	1.50
	A3	1.50	0.67	1	0.67	0.67	1.50
	A4	1.50	1.50	1.50	0.67	0.67	2.50
	A5	2.50	1.50	0.67	1.50	1	1.50
	A6	0.67	0.67	0.40	0.29	0.40	1

		Store B					
		B1	B2	B3	B4	B5	B6
$\tilde{A}_0^2 = \text{Store A}$	A1	2.50	1.50	1	1.50	1	1.50
	A2	1.50	1.50	1	1.50	1.50	2.50
	A3	2.50	1.50	1	1.50	1.50	2.50
	A4	2.50	2.50	2.50	1.50	1.50	3.50
	A5	3.50	2.50	1.50	2.50	1	2.50
	A6	1.50	1.50	0.67	0.40	0.67	1

Applying the proposed model on Chapter 5, respective MOLPs for both stores are constructed as follows:

(MOLP-I)

$$\text{Max } (v_0^1, v_0^2)$$

Subject to

$$\begin{aligned}
 1.50x_1 + 0.67x_2 + 1.50x_3 + 1.50x_4 + 2.50x_5 + 0.67x_6 &\geq v_0^1 \\
 0.67x_1 + 0.67x_2 + 0.67x_3 + 1.50x_4 + 1.50x_5 + 0.67x_6 &\geq v_0^1 \\
 1.00x_1 + 1.00x_2 + 1.00x_3 + 1.50x_4 + 0.67x_5 + 0.40x_6 &\geq v_0^1 \\
 0.67x_1 + 0.67x_2 + 0.67x_3 + 0.67x_4 + 1.50x_5 + 0.29x_6 &\geq v_0^1 \\
 1.00x_1 + 0.67x_2 + 0.67x_3 + 0.67x_4 + 1.00x_5 + 0.40x_6 &\geq v_0^1 \\
 0.67x_1 + 1.50x_2 + 1.50x_3 + 2.50x_4 + 1.50x_5 + 1.00x_6 &\geq v_0^1 \\
 2.50x_1 + 1.50x_2 + 2.50x_3 + 2.50x_4 + 3.50x_5 + 1.50x_6 &\geq v_0^2 \\
 1.50x_1 + 1.50x_2 + 1.50x_3 + 2.50x_4 + 2.50x_5 + 1.50x_6 &\geq v_0^2 \\
 1.00x_1 + 1.00x_2 + 1.00x_3 + 2.50x_4 + 1.50x_5 + 0.67x_6 &\geq v_0^2 \\
 1.50x_1 + 1.50x_2 + 1.50x_3 + 1.50x_4 + 2.50x_5 + 0.40x_6 &\geq v_0^2 \\
 1.00x_1 + 1.50x_2 + 1.50x_3 + 1.50x_4 + 1.00x_5 + 0.67x_6 &\geq v_0^2 \\
 1.50x_1 + 2.50x_2 + 2.50x_3 + 3.50x_4 + 2.50x_5 + 1.00x_6 &\geq v_0^2 \\
 1.00x_1 + 1.00x_2 + 1.00x_3 + 1.00x_4 + 1.00x_5 + 1.00x_6 &= 1 \\
 x_i &\geq 0, i = 1, 2, \dots, 6
 \end{aligned}$$

(MOLP – II)

$$\text{Min } (w_0^1, w_0^2)$$

Subject to

$$\begin{aligned}
 1.50y_1 + 0.67y_2 + 1.00y_3 + 0.67y_4 + 1.00y_5 + 0.67y_6 &\leq w_0^1 \\
 0.67y_1 + 0.67y_2 + 1.00y_3 + 0.67y_4 + 0.67y_5 + 1.50y_6 &\leq w_0^1 \\
 1.50y_1 + 0.67y_2 + 1.00y_3 + 0.67y_4 + 0.67y_5 + 1.50y_6 &\leq w_0^1 \\
 1.50y_1 + 1.50y_2 + 1.50y_3 + 0.67y_4 + 0.67y_5 + 2.50y_6 &\leq w_0^1 \\
 2.50y_1 + 1.50y_2 + 0.67y_3 + 1.50y_4 + 1.00y_5 + 1.50y_6 &\leq w_0^1 \\
 0.67y_1 + 0.67y_2 + 0.40y_3 + 0.29y_4 + 0.40y_5 + 1.00y_6 &\leq w_0^1 \\
 2.50y_1 + 1.50y_2 + 1.00y_3 + 1.50y_4 + 1.00y_5 + 1.50y_6 &\leq w_0^2 \\
 1.50y_1 + 1.50y_2 + 1.00y_3 + 1.50y_4 + 1.50y_5 + 2.50y_6 &\leq w_0^2 \\
 2.50y_1 + 1.50y_2 + 1.00y_3 + 1.50y_4 + 1.50y_5 + 2.50y_6 &\leq w_0^2 \\
 2.50y_1 + 2.50y_2 + 2.50y_3 + 1.50y_4 + 1.50y_5 + 3.50y_6 &\leq w_0^2 \\
 3.50y_1 + 2.50y_2 + 1.50y_3 + 2.50y_4 + 1.00y_5 + 2.50y_6 &\leq w_0^2 \\
 1.50y_1 + 1.50y_2 + 0.67y_3 + 0.40y_4 + 0.67y_5 + 1.00y_6 &\leq w_0^2 \\
 1.00y_1 + 1.00y_2 + 1.00y_3 + 1.00y_4 + 1.00y_5 + 1.00y_6 &= 1 \\
 y_t &\geq 0, t = 1, 2, \dots, 6
 \end{aligned}$$

6.3. Results

MOLPs are solved through Excel Solver by weighted sum method and some of the efficient solutions for *Store A* and *Store B* are given in Table 6.4 in terms of strategy mixes and game values for the players. These solutions represent the case when equal weights are utilized for upper and lower limits of v_α and w_α , since for this example giving different importance weights to these values has no proper explanation. Based on Table 6.4, one of the efficient solutions for optimistic *Stores A* and *B*, gives a game value for *Store A* as $[v_0^1 = 0.6667, v_0^2 = 1.5]$ which can be achieved by strategy mix of $(x_1 = 0, x_2 = 0.6667, x_3 = 0, x_4 = 0.3333, x_5 = 0, x_6 = 0)$. This may be thought either as *Store A* should choose to play strategy 2 (*product information quality*) with 0.6667 of the time and strategy 4 (*return policy*) with 0.333 of the time in the long run, or as *Store A* should invest 66.67% of its money into strategy 2 and 33.33% of its money into strategy 4, to gain an optimal preference degree from the customers, given that *Store B* for also tries to optimize his/her own preference level. The same game value can also be reached by mixed strategies $(x_1 = 0, x_2 = 0, x_3 = 0.6667, x_4 = 0.3333, x_5 = 0, x_6 = 0)$ or by $(x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 0)$. These different solution vectors are obtained as taking different starting points for decision variables in the Excel Solver. The reader should know that Table 6.4 represents the extreme probabilities that make the efficient game value feasible and any value between these values also yield the efficient solution. Some solutions that give close probabilities to efficient strategies are also given in the table to demonstrate this situation. This means, *Store A* can obtain the efficient game value $[v_0^1 = 0.6667, v_0^2 = 1.5]$ as long as $x_4 \geq 0.3333$ and $x_2, x_3 \leq 0.6667$ are satisfied. Efficient value of the game for *Store B* for $\alpha_2 = 0$ is obtained as $[w_0^1 = 1, w_0^2 = 1.5]$ by selecting strategy 5 (*customer trust*) as a pure strategy which is found as $(y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 0, y_5 = 1, y_6 = 0)$. Hence *Store B* should choose to play strategy 5 all the time or invest in strategy 5 with all of his/her money in order to optimize his/her preference level given the conflicting objectives with *Store A*'s. Similarly if the stores' are only willing to accept a small amount of risk $\alpha_1 = \alpha_2 = 0.75$ can be used. Therefore one of the efficient strategy mix for *Store A* when his/her risk level is at least 0.75 can be given as $(x_1 = 0, x_2 = 0.8889, x_3 = 0, x_4 = 0.1111, x_5 = 0, x_6 = 0)$ with game value of $[v_{0.75}^1 = 0.9167, v_{0.75}^2 = 1.125]$. For *Store B* when $\alpha_2 = 0.75$, the efficient strategy is again the pure strategy of strategy 5, $(y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 0, y_5 = 1, y_6 = 0)$, with a game value of $[w_{0.75}^1 = 1, w_{0.75}^2 = 1.5]$. If *Store A* is more risk-averse than *Store B*,

$\alpha_1 = 0.75$ and $\alpha_2 = 0.25$ can model this situation. This means *Store A* has at least 0.75 risk level and *Store B*'s risk level is at least 0.25. Then *A* should select between strategies 2, 3 and 4 by ensuring $x_4 \geq 0.1111$ and $x_2, x_3 \leq 0.8889$ and *B* should play strategy 5 as a pure strategy, at that case the two stores can balance their fuzzy gains of the game. Game values are $[v_{0.75}^1 = 0.9167, v_{0.75}^2 = 1.125]$ and $[w_{0.25}^1 = 1, w_{0.25}^2 = 1.375]$ for player I and player II respectively.

Table 6.4. Some of the efficient solutions for *Store A* and *Store B* for different α levels

STORE A								
α level	x1	x2	x3	x4	x5	x6	v1	v2
0	0	0	0	1	0	0	0.6667	1.5
	0	0.6667	0	0.3333	0	0	0.6667	1.5
	0	0	0.6667	0.3333	0	0	0.6667	1.5
	0	0.4452	0.1712	0.3836	0	0	0.6667	1.5
0.25	0	0	0	1	0	0	0.75	1.375
	0	0.7273	0	0.2727	0	0	0.75	1.375
	0	0	0.7273	0.2727	0	0	0.75	1.375
	0	0.3197	0.3197	0.3606	0	0	0.75	1.375
	0	0.4786	0.2214	0.3	0	0	0.75	1.375
0.5	0	0	0	1	0	0	0.8333	1.25
	0	0.8	0	0.2	0	0	0.8333	1.25
	0	0	0.8	0.2	0	0	0.8333	1.25
	0	0.5077	0.2923	0.2	0	0	0.8333	1.25
	0	0.3333	0.3333	0.3333	0	0	0.8333	1.25
0.75	0	0	0	1	0	0	0.9167	1.125
	0	0.8889	0	0.1111	0	0	0.9167	1.125
	0	0	0.8889	0.1111	0	0	0.9167	1.125
	0	0.4660	0.4229	0.1111	0	0	0.9167	1.125
	0	0.3333	0.3333	0.3333	0	0	0.9167	1.125
1	0	1	0	0	0	0	1	1
	0	0	0	1	0	0	1	1
	1	0	0	0	0	0	1	1
	0	0	1	0	0	0	1	1
	0	0	0	0	1	0	1	1
STORE B								
α level	y1	y2	y3	y4	y5	y6	w1	w2
0	0	0	0	0	1	0	1	1.5
0.25	0	0	0	0	1	0	1	1.375
0.5	0	0	0	0	1	0	1	1.25
0.75	0	0	0	0	1	0	1	1.125
1	0	0	0	0	1	0	1	1

Solution of MOLP-II yields an interesting result with strategy 5 as a pure strategy as efficient solution for all α levels as $(y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 0, y_5 = 1, y_6 = 0)$. This will be discussed later in detail.

As can be noticed, the solutions do not give the equal game values for the players, even for the same α – cuts ($\tilde{v}_\alpha \neq \tilde{w}_\alpha$). Bector et al. [10] explained this situation with nature of fuzzy numbers and Chandra and Aggarwal [12] explained this case with MOLPs create various efficient solutions for both players, so one cannot expect these two values to be equal.

The proposed method is applied for all α -cut levels, which were determined as 0, 0.25, 0.5, 0.75 and 1. The respective α -cut payoff matrices are obtained; MOLPs are built and solved in Excel Solver. Some of the efficient points for all α -cut levels are provided in Table 6.4, in terms of strategy mix probabilities and game values. In some cases these efficient solutions give different strategy mixes for the same game values. However a general understanding on the problem is obtained based on the findings.

The results state that, *Store A* should focus on strategy 4 for all risk levels. S/he may choose to play strategy 4 as a pure strategy, as well as s/he may combine strategy 4 with strategy 3 (*delivery service*), with strategy 2 or with both of these two strategies, in different mix probabilities. As α level increases, higher probabilities of strategy 2 and 3 are able to make the solution efficient, in other words *Store A* will be able to choose strategy 2 or 3 in higher percentage to obtain an efficient solution for more risk-averse states. This can provide a flexible decision environment for *Store A*, selecting a marketing policy for minimizing the cost of investment by alternating between these three strategies and get an efficient preference level from customers.

On the other hand *Store B* should choose strategy 5 (*customer trust*) as a pure strategy, in order to obtain an efficient solution, regardless of the α -cut level (risk level). This indicates customer trust is a critical attribute for an online shopping site, because the model gives no flexibility to *Store B* in choosing the other strategies for obtaining an effective solution.

These results mean *Store A* can obtain efficient solution by focusing on return policy, additionally s/he may invest in product information quality and delivery service to some level, since these strategies exist in efficient solutions for *Store A*. Choosing between customer friendlier return policy, improvement in product information quality and tracking the delivery service would provide *Store A* the maximum preference level s/he can get

from customers given that *Store B* also plays for maximizing his/her preference level. However *Store B* must choose to play on customer trust in all cases, otherwise s/he cannot obtain the effective solution. This may indicate a significant trust problem of customers towards shopping sites with different brands, because the model does not give flexibility to *Store B* in choosing the strategies for obtaining an effective solution. This may either be a security concern of the customer, regarding to personal and transactional data leakage due to past experiences or negative word-of-mouth, may be customers' negative perceptions about security issues even though the website operates in a secure way or it may be unfamiliarity of the website. Once these online shopping sites overcome this trust issue other strategies may be available for balancing the preference levels with *Store A*, and may yield *Store B* a better game value, however strategy 5 dominates other alternatives in the current case.

If we consider the findings in strategy levels, strategies 2, 3 and 4 are *Store A*'s effective strategies. Since the customers are not able to experience the products before the purchase in online shopping, information abundance on the product reduces the perceived risk of the customers by helping them evaluate the approximate satisfaction level they are likely to get from the product. Return policy also relieve customers' mind about making an unsatisfactory purchase. Once the customer believes s/he can take his/her money back when s/he is not satisfied with the purchase, reduced risk of financial loss could make him/her more willing to make the purchase [52]. Strategy 5 is found as the effective solution for *Store B*. It was explained in terms of security of personal and transaction data, and positive word-of-mouth as well as familiarity of the website. When the customers' have good perception about security concerns of the site their likelihood of facing any unwanted situation about personal and transactional data reduces. Positive word-of-mouth usually reduces risks about the performance of the website [19] since experiences of previous customers provide an opinion about the risk of the purchase from that website. Familiarity also improves customers believe in satisfactory performance of the website and reduces their risk [16], [19].

Strategy 6 (*user interface*) and strategy 1 (*customer support*) have no importance among other strategies, since they exist in none of the efficient solutions in any cases. These strategies may be considered mostly related with improving customer satisfaction rather than risk reduction.

The overall results imply Turkish customers' behavior is risk averse when shopping online. Both efficient strategies and non-efficient strategies support this claim. Therefore we may say Turkish online shoppers try to reduce their risks while shopping online. This complies with literature which states Turkish users have security concerns [17], [53]. Also respondents' minimum satisfaction level for making another purchase from the website was found to be 83.2% in the opinion collection form, which may be interpreted as customers are willing to accept only a small amount of risk in online shopping.



7. CONCLUSIONS AND DISCUSSION

In this thesis a solution method for matrix games with fuzzy payoffs, which considers players' different risk levels is proposed. Risk levels for the players are represented with α – cuts, where pessimistic (risk-averse) players are modeled with higher α values and optimistic (risk-tolerant) players are modeled with lower α values. The method is implemented to a marketing problem to demonstrate its applicability and use. The aim of this problem was to determine the important marketing activities for online shopping sites. The problem is modeled as a fuzzy matrix game and through proposed method the important marketing attributes are found for different websites, as well as providing understanding into Turkish customers behavior while shopping online. The thesis will provide a theoretical basis for online marketing practitioners and contribute to literature as providing a different approach to online shopping problems. However the main contribution of the thesis is the proposed solution method for fuzzy matrix games which considers players' different risk levels.

In the thesis Chandra and Aggarwal's [12] model motivated the construction of the proposed method, and a model is developed to consider different pessimism and optimism levels of the players, by utilizing α – cut concept. The proposed method provided a broader insight on the problem, by solving it for all α – cuts, rather than solving the LPs and providing optimal mixed strategies, along with α – cuts of the optimal game value. The proposed method solves the MOLPs for obtaining an optimal value for a specific α – cut level individually, and provides respective strategy mixes. If Chandra and Aggarwal's [12] model would be implemented to the marketing problem considered in the thesis, and the same α levels (as 0, 0.25, 0.5, 0.75 and 1) are taken, it would have 9 objective functions and 55 constraints for each player. Hence, if various α levels are included in the problem, the size of this model increases dramatically. Also that model solves the game for all α levels simultaneously; therefore if an additional α level is added to the model later, the model should be constructed from the beginning. On the other hand the proposed model gives a smaller model than that of Chandra and Aggarwal's [12], with 2 objectives and 13 constraints for each player, since it considers a specified α – cut value only, rather than including all selected α levels in one program. Optimal results provide solutions for specific risk levels of the players. Also the game can be solved for various α levels to gain insight on the problem, to determine the optimal strategy selection policy.

When Chandra and Aggarwal's [12] model is used to solve the marketing problem discussed in the previous chapter, the efficient solutions are found to be feasible for $\alpha = 0$ case in the proposed model where the efficient solutions give close weights (probabilities) to strategies 2,3 and 4 for *Store A*. It is not able to find the extreme mixed strategies the proposed model reaches, as α increases. For instance, efficient game value for *Store A* $[v_0^1 = 0.6667, v_0^2 = 1.5, v_{0.25}^1 = 0.75, v_{0.25}^2 = 1.375, v_{0.5}^1 = 0.8333, v_{0.5}^2 = 1.25, v_{0.75}^1 = 0.9167, v_{0.75}^2 = 1.125, v_1^1 = 1]$ with mixed strategies $(x_1 = 0, x_2 = 0.6667, x_3 = 0, x_4 = 0.3333, x_5 = 0, x_6 = 0)$ was found as an efficient solution in Chandra and Aggarwal's [12] model, which was also one of the efficient points in the proposed model for $\alpha = 0$ case. On the other hand, one of the efficient mixed strategies of the proposed model for $\alpha = 0.75$ case $(x_1 = 0, x_2 = 0.8889, x_3 = 0, x_4 = 0.1111, x_5 = 0, x_6 = 0)$ was infeasible in Chandra and Aggarwal's [12] method. The proposed model gave different solutions for different α levels, when solved individually, and more flexible decision environment for the player.

The implementation considered in the thesis study has some limitations. First the sample is limited, since it does not contain various professions or age groups. Customer types are not well distributed in the sample such as female respondents are much higher than the male respondents. Also the sample includes highly educated people, for which the lowest education level is composed of university students. Although this is overcome by fuzzy modeling, generalizability of the results would be limited for different professions, and age groups and education levels.

For future studies, payoff matrices may be modeled with different fuzzy membership functions to see how the solutions will be affected and how the proposed models' performance would change.

Also the solution of the problem yields strategy 5 as effective strategy for all α levels. This may be the cause of an important trust problem of customers, for online shopping websites with different brands. If this trust issue is overcome, either by taking actions or improving customer perceptions about store's security policies, brand name and etc., efficient strategies are expected to diversify for *Store B*, like *Store A*'s solution. Therefore other important marketing activities may then be investigated for shopping websites in various α levels.

In addition, important strategies may be investigated further for future work. For instance dimensions of customer trust may be examined for different online store types. Customer perceptions on security may be evaluated and improvement methods may be suggested. Also price and product quality are not considered as factors in this thesis, as a future work effects of product quality and price differences may be included for website preferences and results may be analyzed.



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APPENDIX

OPINION COLLECTION FORM

In this survey study, attributes that customers wish to have an online store, is aimed to be investigated. Please answer all questions in the form.

Section I

1. Gender F [.....] M [.....]
2. Age
3. Profession
4. Department/Grade you are studying at (Skip this question if you are not a student.)
.....

5. For how many years you have been using internet?
6. On average how many hours per day you are using internet?
7. For how many years you are using online shopping?
[.....] I don't [.....] 0-1 years [.....] 1-5 years [.....] >5 years
8. How many shopping sites you use regularly?
9. What should be your minimum satisfaction level (percentage of your expectations satisfied by the shopping site) in order for you to make shopping from that website again?
10. Select the device/s you use for your online sopping
[.....] Computer [.....] Tablet Computer [.....] Smart Phone
11. Do you think online shopping is safe?
[.....] Yes [.....] No

Section II

STARTING FROM THE NEXT PAGE WEBSITE COMPARISON QUESTIONS ARE GIVEN.

WHILE ANSWERING THE QUESTIONS ASSUME YOU WANT TO BUY A PRODUCT ONLINE, WHICH IS OFFERED ONLY IN TWO WEBSITES FOR THE SAME PRICE. ONE OF THEM IS THE WEBSITE OF THAT PARTICULAR BRAND (**STORE A**) WHILE THE OTHER IS AN ONLINE SHOPPING SITE WHICH OFFERS VARIOUS BRANDS AT THE SAME TIME (**STORE B**).

WHILE ANSWERING THE QUESTIONS ASSUME THAT YOU KNOW **FOR SURE** THAT THE SERVICES GIVEN IN THE QUESTION ARE PROVIDED BY THAT STORE AND YOU HAVE NO INFORMATION ABOUT ANY WHICH ADDITIONAL SERVICES ARE PROVIDED. PLEASE STATE IN WHAT DEGREE YOU WOULD PREFER THE WEBSITE OF THE BRAND (**STORE A**) TO THE ONLINE SHOPPING SITE THAT OFFERS VARIOUS BRANDS (**STORE B**) ACCORDINGLY.

For example, at Q1 assume you know that both sites provide good quality customer support services but you don't have any information about what additional services (if any) are offered from any of the stores. In that case mark the table below in which degree you would prefer Store A to Store B.

1. **Customer support:** Giving prompt response to consumers' questions and complaints, providing frequently asked questions, paying attention to customer feedback.
2. **Product information quality:** Providing information abundance on the product such as previous buyer's comments, technical information about the product, detailed size information, photographs of the product (on a model if necessary) and etc.
3. **Delivery service:** Fast and/or timely delivery, store's tracking of delivery process.
4. **Return policy:** Supporting customers on return process, setting customer oriented return rules, providing money-back warranty.
5. **Trust:** Customers' ability to trust that the site keep personal information or payment information in safe, store's wish to make good impression on customers so that gaining customer loyalty and making sure previous customers give positive recommendations (word-of-mouth) about the website to potential customers, advertisements to enable customers to know the store.
6. **User interface design:** User interface design eases customer's shopping, without time loss, customers are able to find easily what they are looking for and interface provides understandable and attractive design.

Question	In which degree you would prefer Store A to Store B ?										
Question	Option (Brand's website)	Extremely preferable	Really preferable	Somewhat preferable	Rarely preferable	Equally preferable	Rarely not preferable	Somewhat not preferable	Really not preferable	Extremely not preferable	Option (Shopping site that offers various brands)
1	A										B
2	A										B
3	A										B
4	A										B
5	A										B
6	A										B
7	A										B
8	A										B
9	A										B
10	A										B
11	A										B
12	A										B

1. **Customer support:** Giving prompt response to consumers' questions and complaints, providing frequently asked questions, paying attention to customer feedback.
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13	A										B
14	A										B
15	A										B
16	A										B
17	A										B
18	A										B
19	A										B
20	A										B
21	A										B
22	A										B
23	A										B
24	A										B

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Question	In which degree you would prefer Store A to Store B ?											
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	25	A										B
	26	A										B
	27	A										B
	28	A										B
	29	A										B
	30	A										B
	31	A										B
	32	A										B
	33	A										B
	34	A										B
	35	A										B
	36	A										B

CURRICULUM VITAE

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Projects and Budgets

Publications

Oral and Poster Presentations

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- Koca, Y., Testik, Ö.M., “Solving Two-Player Zero Sum Games with Fuzzy Payoffs for Two Online Shopping Sites with Different Risk Levels”, *ENBIS 17*, Naples, 9-14 September 2017.





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GRADUATE SCHOOL OF SCIENCE AND ENGINEERING
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