YAŞAR UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

APPROXIMATE FORMULA FOR THE PERFORMANCE MEASURES OF BUFFERLESS PRODUCTION LINES

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Thesis Advisor: Prof. Sencer YERALAN

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Bornova-İZMİR

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This study titled "APPROXIMATE FORMULA FOR THE PERFORMANCE MEASURES OF BUFFERLESS PRODUCTION LINES" and presented as M.Sc Thesis by Shohreh ROSHANI has been evaluated in compliance with the relevant provisions of Y.U Graduate Education and Training Regulation and Y.U Institute of Science Education and Training Direction and jury members written below have decided for the defense of this thesis and it has been declared by consensus ~~/ majority~~ of votes that the candidate has succeeded in thesis defense examination dated...11.07.2014

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ABSTRACT

APPROXIMATE FORMULA FOR THE PERFORMANCE MEASURES OF BUFFERLESS PRODUCTION LINES

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Production lines with unreliable machines have received a great amount of attention in the literature. This research presents an analytical method for approximating formula for the performance measures of bufferless production lines. It is essential to analyze the expected production rate as a function of line characteristics because this performance measure is one of the most important and effective system behavior indicators. Approximating formula is useful to find the relation between the production rates of production lines and the failure and repair probabilities. In this study, curve-fit method is used to establish simple and useful formulas for production rate over failure and repair probabilities for lines consisting three, four and five identical stations.

Before approximating formula, Markovian analysis of production lines is presented using steady-state probability matrix, for two-station production line. Markov chain analysis produces an exact analysis of such lines. We conducted non-linear regression method in the computational environment MATLAB. Numerical analysis and error analysis show that the approximated formula is effective and useful to estimate production rate of production lines.

Keywords: Multi-Station Production Lines, Curve-fitting Method, Approximation, Discrete-Time Markov Chain, Non-linear Regression.

ÖZET

STOKSUZ ÜRETİM HATLARINDA PERFORMANS ÖLÇÜMÜ İÇİN YAKLAŞIK FORMÜLLER

Shohreh ROSHANI

Yüksek Lisans Tezi Endüstri Mühendisliği Bölümü

Danışman: Prof. Sencer yeralan

Temmuz 2014, 74 Sayfa

Üretim hatlarında güvenilmez makineler ile ilgili literatürde çok çalışma yapilmiştir. Bu araştırma tampon stoksuz üretim hatlarında performansı ölçmek amaçlı yaklaşık formül için analitik bir yöntem sunuyor. Ortalama üretim oranını hat özelliklerin bir fonksiyonu olarak analiz etmek, bu performans ölçüsü sistem davranış göstergelerinin en önemli ve etkili ölçütlerinden birisi olduğu için önemlidir. Yaklaştırma formülü üretim hatlarının üretim oranları ile makine bozulma ve onarım olasılıkları arasındaki ilişkiyi bulmak için yararlıdır. Bu çalışmada, curve-fit yöntemi ile, üç, dört ve beş istasyonları için makine bozulma ve onarım olasılıkları kullanılarak, üretim oranını yaklaşık hesaplamak için kullanışlı formüller oluşturulmuştur.

Yaklaşım formülü sunulmadan önce, üretim hatlarında, iki istasyonlu üretim hatları için Markov zinciri analizi, kararlı durum olasılık matrisi kullanılarak sunulmaktadır. Markov zincir yöntemi bu tür hatların tam bir analizini üretir. Bilgisayar ortamında MATLAB aracılığı ile doğrusal olmayan regresyon yöntemi kullanılmıştır. Yapılan sayısal analiz ve hata analizi, önerilen yaklaşım formülünün tampon stoksuz üretim hatlarının üretim oranını tahmin etmek için yararlı olduğunu göstermektedir.

Anahtar Kelimeler: Çok- İstasyonlu Üretim Hatları, Eğri uydurma yöntemi, Ayrı-Zaman Markov Zinciri, Doğrusal Olmayan Regresyon.

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I would like to express my enduring love to my parents, who are always supportive, loving and caring to me in every possible way in my life.

TEXT OF OATH

I declare and honestly confirm that my study titled "APPROXIMATE FORMULA FOR THE PERFORMANCE MEASURES OF BUFFERLESS PRODUCTION LINES", and presented as Master's Thesis has been written without applying to any assistance inconsistent with scientific ethics and traditions and all sources I have benefited from are listed in bibliography and I have benefited from these sources by means of making references.

01/06/2014

Shohreh ROSHANI

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INDEX OF SYMBOLS AND ABBREVIATIONS

Symbols	Explanations
P	Production Rate
v	Steady-state Probability Vector
q_i	Failure probability of a working machine (i)
r_i	Repair probability of a failed machine (i)
N	Number of Station
a_i	Coefficient of Formula
∇	Gradient

Abbreviations

GEM	Generalized Expansion Method
DTMC	Discrete-Time Markov Chains
CTMC	Continuous-Time Markov Chains
RMS	Root-Mean-Square
SAA	Stand Alone Availability
MAPE	Mean Absolute Percent Error

CHAPTER 1: INTRODUCTION

Production lines are sets of machines or workstations arranged in a serial structure to produce finished products or components. A workstation is a group of machines or operators, performing one or more operations on the jobs. In another word, a production line is a materials handling and processing device in industry. Production lines with unreliable machines have received a great amount of attention in the literature. Much of the past work has involved the development of analytical models, empirical formulas, and simulation programs to evaluate several performance measures in order to improve the design and planning of production lines. The production rate is one of the main performance measures of production lines.

It is essential to predict the expected production rate as a function of line characteristics. One method to predict the production rate is by approximation. Approximating formulas are useful to find the relation between the production rate of production lines and the failure and repair probabilities. Curve-fit analysis is used to develop formulas to find the production rate. Curve-fitting is the process of constructing a curve, or mathematical function, that has the best fit to a series of data points.

1.1 SCOPE OF THE THESIS

In this thesis the aim is to develop approximate formula for performance measures of production lines. Simple and useful empirical formulas for lines consisting of three, four and five identical stations are developed. First a general structure for a suitable model is sought. The general model contains parameters which will be chosen through a curve-fitting process. The general model is inspired by the exact analytical solution of the two-station case. The model is in the form of a ratio of two multinomial involving the breakdown and repair probabilities. Afterwards the model is generalized for N-station production lines. Numerical analysis will show that the suggested and approximate formulas give acceptable results for N-station production lines. After all, error analysis for the production rate function, is conducted to analyze the estimates and results. The purpose is to have a simple analytical result that shows the mathematical relationships between the performance measures of production lines. In another words, the aim is detecting equations and hidden mathematical relationships in a set of data, and identifying the simplest mathematical formulas. A formula with a simple structure is helpful in several ways. It helps provide an intuitive understanding of the underlying model, so that analysts and decision makers can use the model with confidence. This study starts with two station production line then generalizes the formula to a line of any length.

The focus here is on discrete part production lines where each part produced is distinct. From here on, when reference is made to production lines, discrete part production lines will be understood. In a production or flow line, all jobs are required to pass through each station in the same sequence once. These production lines are usually associated with scale rather than scope, and a major advantage of production lines is the associated simple materials handling requirements. A production line consists of work-stations, materials, human resources, and inter-work-station storage facilities. Randomness is introduced due to random processing times and the random behavior of work-stations in relation to failure and repair. In terms of classical queuing theory, production lines would be described as finite buffer tandem queuing systems where the work-stations are the servers, storage facilities are the buffers or the waiting lines, and the jobs are the customers.

In this study, curve-fitting methods are used. There are many statistical packages such as R, Minitab and numerical software such as the GNU Scientific Library, MATLAB, SciPy and OpenOpt which include commands for curve-fitting in a variety of scenarios.

The production rate is the most important performance measure in production system. This project uses MATLAB software to show the relation between the expected production rates of production lines and the failure and repair probabilities and also, it is used for approximation, numerical analysis and curve fitting. MATLAB is a high-performance language for technical computing. MATLAB is a multi-paradigm numerical computing environment and fourthgeneration programming language. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

CHAPTER 2: LITERATURE REVIEW

Production lines and queuing systems have been the object of much study in the past. As a literature review, we refer to some related works. To begin with, the production rate of lines with and without intermediate buffers has been studied starting from the mid-fifties. Kerbache and Smith is the important study conducted for Generalized Expansion Method (GEM), for queuing systems. GEM was used as the prime performance evaluation tool. GEM transforms the queuing network in to an equivalent Jackson network, which can be decomposed so that each node can be solved independently of each other. This method is similar to a product form solution approach. GEM is an effective and robust approximation technique to measure the performance of open finite queuing systems. The generalized expansion method is prevalent in most systems, such as production systems and manufacturing, transportation and other similar systems. This approximation method has become an appealing approximation technique for performance evaluation of queuing networks due to its accuracy and relative simplicity. GEM is basically a combination of two approximation methods. The name of two approximation methods are: the "repeated trials" and the "node by node" decomposition. In order to evaluate the performance of a queuing network, the method first divides the network into single nodes with revised service and arrival parameters. Blocked customers are registered into an artificial "holding node" and are repeatedly sent to this node until they are serviced. The addition of the holding node expands the network and transforms the network into an equivalent Jackson network in which each node can be solved independently. The effectiveness of GEM as a performance evaluation tool has been presented in many papers, such as Kerbache and Smith.

We can divide production lines in to two categories: production lines with intermediate buffers and production lines without buffering. A part of manufacturing systems that employ long lines typically contain several automated workstations tightly coupled without intermediate buffers. Cetinay analyzed the system performance measures of production lines with no intermediate buffers. In that research, the common approach to model tightly coupled lines is to consider each workstation as an individual machine. The characteristics of these composite machines need to be derived from the characteristics of the individual stations that comprise the workstation. This study evolves with the purpose of developing the composite workstation characteristics from the individual station characteristics. Note that the composite workstation will have the same processing time as the other stations in the line. Cetinay developed software to calculate the transition probability matrices to allow the analysis of system behavior. Analyzing the system behavior in manufacturing environments is very important for production efficiency. The purpose of the project was to analyze the system performance measures such as starvation and blockage times of stations, production rate and work-in-process. The starvation and blocking conditions on stations are idle times, which are defined and studied by many researches. When one machine in the system fails then the rest of the line is prone to stop, especially if there is no intermediate buffer present. A failure may cause the preceding machine to be blocked, while at the same time, the downstream machines may be starved because there is no input available. Furthermore the production rate and the work-in-process measures over failure and repair probabilities are approximated for lines consisting three, four and five stations, Cetinay.

Blumenfeld, analyzed the system throughput of a line dependent on the size of the buffers. As we know, an important measure of performance for a production line is the system throughput, or the average number of jobs produced per hour. In the mentioned study, they derived a simple formula for the throughput (jobs produced per unit time) of a serial production line with workstations that are subject to random failures. The derivation is based on equations developed for a line flow model that takes into account the impact of finite buffers between the workstations. The obtained formula applies in the special case of a line with identical workstations and equal size of buffers. In that research they expressed the mathematical relationships between the system parameters, that can be used to gain basic insight into system behavior at the initial design. The aim of the Blumenfeld research is to obtain a simple formula for throughput from general equations. The model considers a two-station line and provides a building block for modeling longer lines. It analyzes the flow of jobs through a line of stations and derives analytical equations for line performance. In general, in the model, which stations can have different speeds and reliabilities, and the buffers can have different sizes. The developed model, can be used to compute throughput for general serial lines very efficiently, and allows quick and fast comparisons. The approximated equations are suited for conveniently computing numerical results rather than providing insight from their functional form. The purpose was to have a simple formula that shows the mathematical relationships between the key system parameters. The result of this research is useful in the initial design stage, when basic insight into system behavior is needed before detailed numerical analysis are performed. The paper starts with the basic model for a general two-station line developed and uses the model to derive a throughput formula in the special case of identical machines. After that, the paper extends the formula to apply to a line of any length. The extended formula is compared with numerical results obtained from simulation studies.

Most studies that are mentioned above are based on operation-dependent failures. Yeralan and Muth approximated formula for production line with finite N intermediate buffer size. They found a simple representation of production rate as a function of buffer size that would approximately hold over a substantial portion of the space of variables. They offered a general model for a class of production lines with two unreliable stations, a finite capacity inter-station buffer, discrete items, constant cycle time, and synchronous transfer. A production line consisting of two work stations in series and intermediate buffer of size m is modeled. The mentioned system is modeled as a discrete parameter Markov chain. In this research, the steady-state probabilities are obtained by the successive solution of systems of four simultaneous equations. This study is presented with a model and its solution, of a two-station line with an intermediate buffer of arbitrary size. The results of this research, that they are for two-station line, can form the basis for approximations of the production rate of lines comprising more than two stations.

One method which has proved practically robust for production lines with finite buffers and unreliable machines is decomposition which was introduced by Gershwin. Under the same assumption, a decomposition method was introduced by many researchers. Greshwin considered a two-machine and one buffer, production line with discrete time, which is a generalization of earlier models. The machines in the system have multiple up and down states. When a machine is not blocked or starved, the transitions among its up and down states are described by a Markov chain. The system operates in discrete time and produces discrete material. The buffer in the system is finite. An analytical solution of the transition equations is formulated and numerical results are shown in this research. They have briefly summarized a solution method for two-machine lines whose machines states are described by arbitrary Markov chains. Finally they have demonstrated that the method works for buffers of size up to 100, even though the line's state space is large and complex.

Dallery and David presented an approximate method for the analysis of transfer lines with unreliable machines and finite buffers. In the system blocking and starvation states, which occur as a consequence of machine failures, are important phenomena. They first considered homogeneous lines, which means lines for all machines have the same processing times. The behavior of the line was approximated by a continuous flow model. Then they used a decomposition technique which enables one to decompose the analysis of the line into the analysis of a set of two-machine lines. This leads to a simple and fast algorithm which provides performance parameters such as production rate and average buffer levels. Final results showed that this approximate technique is very accurate. Then they considered the transfer lines with machines that having different processing times. A simple transformation was introduced which replaced the line by a homogeneous line. In this study, the approximate transformation provided good results for a large class of systems.

Senanayake developed an approximate analytical method to evaluate the performance of production lines that can manufacture multiple part-types. The manufacture of multiple products in a common production facility allows one to share resources, such as processing machines, among the part-types. However, the sharing of resources often hinders the efficient management and planning of production due to the difficulty of evaluating the many production policies and system configurations that are possible. Production policies determine when to switch processing of part-types on shared machines and which part-type to switch to the next. The performance of the manufacturing system will vary depending on the policy used. Random disturbances, for example, machinery failures, inherent in manufacturing systems further impedes the accurate performance evaluation of production systems. In addition, one needs to properly account for the phenomena specifically observed in the manufacture of multiple part-types, such as machine setups, routing with bypass, shared machines, part-type dependent machine processing times etc. In this research study, to emulate real manufacturing systems, it is assumed that machines are unreliable and buffers are finite. The main contribution of this research is the incorporation of key system phenomena observed in industry including machine setups, routing with bypass, and stations with parallel machines. Recently, the authors proposed a decomposition based analytical approach to evaluate the performance of these complex systems. A two machine building block was first constructed based on the continuous material approximation, and decomposition equations were then developed to approximately capture the dynamics of multiple part-type flow behavior. In this paper, the authors show the simplicity of extending the model to analyze systems with part-type dependent machine processing times and cyclic production policies. Numerical comparison with simulation results shows the good accuracy of the method in evaluating the performance of several example manufacturing systems including a system based on a real production line. This paper analyzed the systems with part-type dependent (unequal) machine processing times by applying the principle of homogenization that is mentioned in Dallery and David. The processing times of machines are not all equal. The simple extension to systems with unequal and part-type dependent machine processing times and the ability to model different production policies increases the flexibility of the model and its applicability to real manufacturing systems. In addition, they showed the ease of incorporating different production policies into the model by extending the methodology to systems operating on a cyclic production policy.

Several researchers have attempted to evaluate the performance of multiple part-type manufacturing systems using approximate analytical methods. Nemec extended the large body of literature that focused on the decomposition analysis of single part-type production lines to systems producing two part-types. He presents the analysis of two separate manufacturing systems, using two different approximation procedures, contained in two stand-alone parts. In this study he presents an analysis of the system by decomposing it into smaller sub-systems. He formulated a deterministic single failure multi-part type line. He assumed that machines were unreliable and intermediate buffer space was finite. However, he was only able to investigate two part-type production lines with up to six processing stations connected in series. This formulation worked only for small two-part lines, and there is no specific way of generalizing his equations for longer lines. Jang extended this work study to multiple part-type systems and reported satisfactory accuracy in the prediction of production rates of systems with up to six stations and three part-types. Both authors considered systems where each station consisted of only one shared machine and assumed a production policy where switching between part-types was based on a fixed priority policy.

Tolio proposed a way of analyzing two-part type lines with multiple failure modes with Markov model. He present an analytical method for evaluating the performance of production lines with a finite buffer and two unreliable machines. The model that they present, evaluates the steady state probabilities of the states of the system with a computational effort that depends only on the number of failure modes considered and not on the capacity of the buffer. The expression 'multiple failure modes' means that each machine of the line can fail in different ways. Each mode of failure is characterized by a specific MTTF (mean time between two successive failures) and MTTR (mean time to repair a failure). In earlier papers, each machine can fail in more than one way. For each failure mode, geometrically distributed times to failure and times to repair are specified. In this research, the method evaluates the steady-state probabilities of the states of the system with a computational effort that depends only on the number of failure modes considered and not on the capacity of the buffer. A comparison of performance of the method with those obtained with existing techniques that consider only one failure mode is reported.

Cetinay established reliable forecasts with simple formulas. In this study the relation of the production rate of an N-station production line with the failure and the repair probabilities was found with curve fitting method. Curve-fitting was realized to develop a reasonably accurate equation in order to establish reliable forecasts with simple formulas. The formula for the production rate and work-in-process is represented as rational function. The same curve-fitting methodology is performed for three, four and five station production lines to calculate the specific parameters.

Least Squares approximation is used to determine approximate solutions for a system of equation or to fit an approximate function to a set of data points. As the nonlinear least square will be used in this study to fit an approximate formula for three, four and five station production line, we will show different methods in this part. Seber, and Wild introduced the list of computational methods for nonlinear least squares.

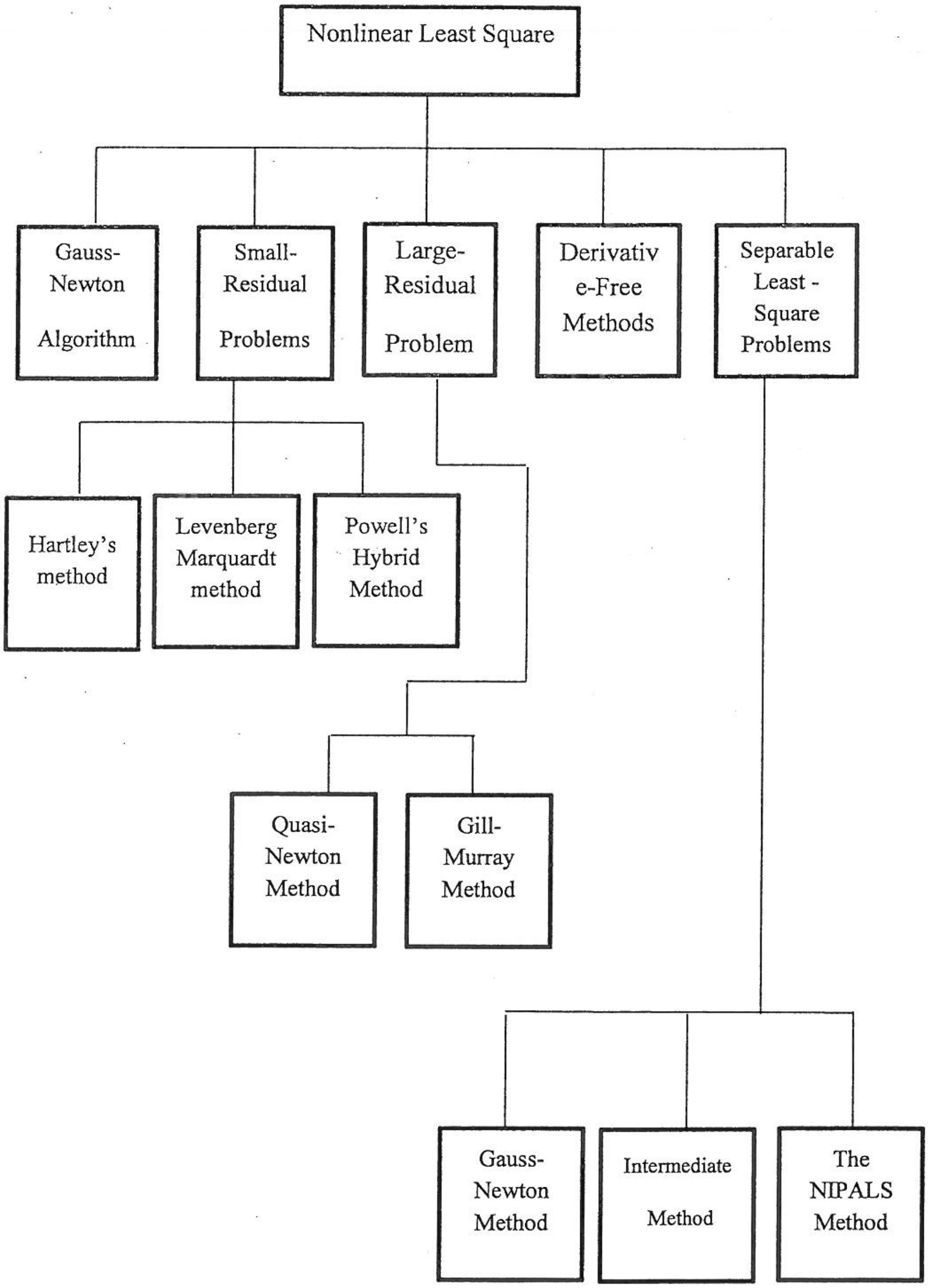

Figure 2.1 Computational Methods for Nonlinear Least Squares

Most studies are restricted to two or three station production lines. Production line with two stations are easy to analyze and model rather than more stations.

Complexity increases for the evaluation and analysis of production lines longer than two stations. This study brings a novelty for analyzing the performance measures of bufferless production lines.

CHAPTER 3: THE MARKOV CHAIN MODEL

Markov chains are well-known in engineering. There are several basic, direct and iterative methods for steady-state analysis of Markov chains, such as: Gaussian Elimination method and Grassman method, as well as Power, Jacobi.s and Gauss-Seidel.s methods. Algorithms for computation of steady-state probability vector for finite Markov chains must be developed. Performance and dependability measures for systems can be derived and evaluated with steady-state analysis of Discrete-Time Markov Chains (DTMC) and Continuous-Time Markov Chains (CTMC). For a review see Markov chains book is written by Norris J. R.

3.1 SYSTEM PARAMETERS

As machines are unreliable, in the stochastic model, each machine has a unique failure probability. Each machine if it is operating during the current period, has a unique probability of fail at the end of the periods. Also, the machine failures occur at the end of periods, after the station completes its operation on the work-piece. As Schick and Gershwin (1978) assumed, the repair of a failed machine starts at the beginning of the next period after the failure happened. The probability of repair for failed machines are constant during the periods. For machine (i), where $1 \leq i \leq N$, the failure and repair parameters are defined as:

q_i : The failure probability of a working machine (i)

r_i : The repair probability of a failed machine (i)

($\bar{q}_i = 1 - q_i$ and $\bar{r}_i = 1 - r_i$)

3.2 STATION STATES

In the production line each station can be in one of the five station states. Each state represents the condition of the station throughout a period from the start of the period. All the station states remain the same during the period. The change in the station states happens at the end of the periods because station state transitions occur at the end of the periods. The time that a unit is processed by the stations, is defined as a period. These are the five possible station states:

1) Up (U)

In Up state at the beginning of the period, the machine operates and processes the work-piece during the period and completes processing the piece at the end of the period. By definition, as machine breakdowns occur at the end of periods, the machine can either fail or can be in good condition after it completes processing the work piece at the end of the period. In another words, end of the period is at the beginning of the next period.

2) Down (D)

In Down state at the beginning of the period, the machine can either be repaired or not be repaired during the period.

3) Blocked (B)

The machine becomes blocked when a machine in up condition in the beginning of the period, finishes processing a work-piece at the end of the period and does not fail. If the downstream machine is down or blocked, then the machine cannot pass the item to the next station . If a machine is blocked at the beginning of the period, for the next period it can still be blocked or not depending on the downstream station state. Note that a blocked machine does not fail.

4) Down-Blocked (DB)

The machine becomes down and blocked when a machine with up condition in the beginning of the period, finishes processing a work piece and then fails at the end of the period, while the downstream machine is full, then the machine cannot pass the item that is completed. So the machine needs repair to operate again. A down-blocked machine at the beginning of the period can either be repaired or not during the period and it can either pass the finished item or not to the downstream, which demonstrates the possible station states of the machine at the beginning of the next period.

5) **Starved (S)**

In starved state, a machine in good working condition is not fed by the upstream. A starved machine does not fail.

Station N can never be blocked or down-blocked. In addition, station 1 is never starved due to infinite supply. All possible states for the stations are summarized in Figure3.1. As the figure shows, all five station states are valid for stations in the line except the first and the last one.

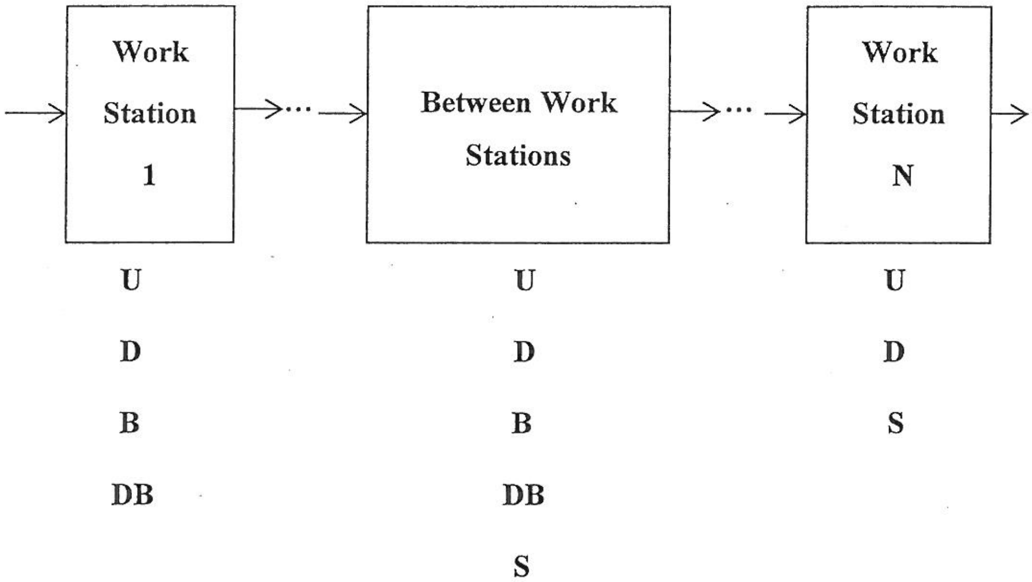

Figure 3.1 Station States

We can define K as a number of station state. Table 3.1, shows different station states of the production line.

Table 3.1 Different Station States

Station State	Number of Station State (k)
Up	1
Down	2
Blocked	3
Down-Blocked	4
Starved	5

Table 3.2, shows the possible number of system states in the production line.

Table 3.2 System states in the production line

Station 1	**Station 2**	**...**	**Station N**	**System State (J)**
Up	Up	...	Up	1
Up	Down	...	Down	2
Up	Starved	...	Starved	...
Down	Up	...	Starved	...
...	...	...	...	...
Down-Blocked	Down		Starved	2^{2N-1}

The goal to evaluative the models is to calculate a performance measure of the system under study, such as the production rate.

3.3 ASSUMPTIONS

First we make some assumptions for the discrete-time Markov chain model. Gershwin and Schick, studied discrete-time Markov chains in modeling manufacturing lines. In their model they defined some assumptions, the relevant assumptions are given by:

1. The processing time for stations is constant and equal for all stations, so the model considers homogenous lines.

2. Transportation time between stations is too small and negligible.

3. Only one unit can be processed at a time, so machine capacities are limited to one.

4. Time is scaled so that the station periods take one time unit.

5. An infinite supply of material is available to the station 1, and an unlimited storage area is considered for the station N. As a result, station 1 is never starved, and station N is never blocked.

6. The failure probability of a starved or blocked machine is zero, because only operation dependent failures of machines are considered, which means that machines can only fail while they are processing a work-piece.

7. We have no intermediate buffer between machines in production line.

8. Whenever a machine is processing a work-piece, there is a unique probability (q_i) that the machine fails. By convention, machine breakdowns occur at the end of periods after machines complete their operations on the work-piece.

9. There is a unique probability (r_i) that given a failed machine at the beginning of any period can be repaired during the period.

We adopt the same assumptions.

3.4 PRODUCTION RATE

Production rate is one of the most important performance measure in production line. The system under study, with serial-connected stations without intermediate buffer is efficient only when all stations are up and operating.

We can define sets of system states as below:

$$\Omega_P = \{ \text{System states } j \ni \text{The last station N is in state } k = 1 \text{ (Up)} \} \qquad (1)$$

Where:

N: Number of stations

k : State of station (i), for i: 1, 2, 3, …, N

j : State of the system

The production rate will be equal to:

$$\text{Production rate} = \sum_{j \in \Omega_P} \pi_j = \sum_{J \in \Omega_P^i} \pi_j \qquad (2)$$

Where:

π_j : The probability of the system in state J , $\pi_j = P[X = j]$

X: System state

$$\Omega_P^i = \{ \text{System states } J \ni \text{station i is in state } k = 1 \text{ (Up)} \} \qquad (3)$$

3.5 ALGEBRAIC SOLUTION OF THE TWO-STATION CASE

The two-station production line which is the minimum number of stations in a production line is shown in Figure 3.2.

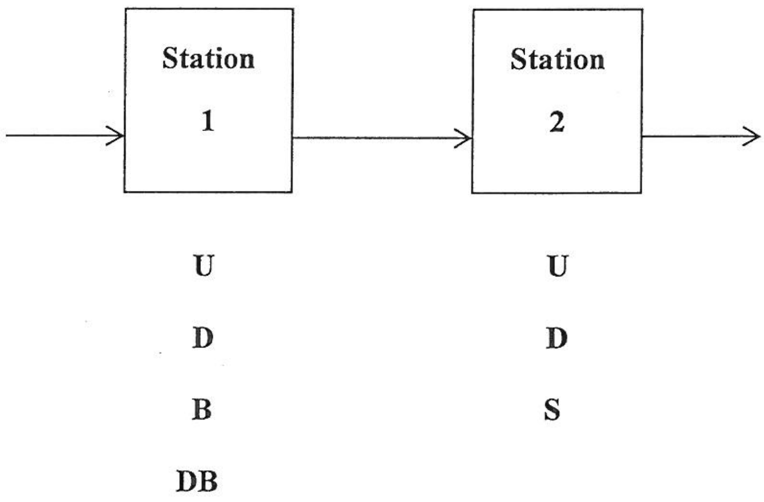

Figure 3.2 Two-Station Line

The system states consists of the various allowable combinations of the station states. All possible system states of the two-station production line are enumerated in Table 3.3.

Table 3.3 System States of the Two-Station Production Line

Station 1	Station 2	System States (J)
Up	Up	1
Up	Down	2
Up	Starved	3
Down	Up	4
Down	Down	5
Down	Starved	6
Blocked	Down	7
Down-Blocked	Down	8

In this section, Markovian analysis of production lines is presented using the underlying queuing system structure of production lines. It produces an exact analysis of such lines.

In this chapter a two-station production line is considered. We calculate the production rate in closed form as an analytical expression in model parameters.

To calculate the exact relation between performance measures for two-station production line with Markov chain method, we construct transition probability matrix. In total, dimensions of the steady-state probability matrices is obtained with this formula (Cenitay):

Dimensions of the steady-state probability matrix = $2^{2N-1} \times 2^{2N-1}$

N: Number of stations

2^{2N-1} : Total number of system states

Considering the above formula, for two-station production line we will have 8×8 transition probability matrix. The resulting transition probability matrix is presented in Table 3.4.

Table 3.4 Transition Probability Matrix

To / From	UU	UD	DU	DD	BD	DS	US	D-B D
UU	$(1-q)^2$	0	q.(1-q)	0	(1-q).q	0	0	q^2
UD	(1-q).r	0	q.r	0	(1-q). (1-r)	0	0	q.(1-r)
DU	0	r.q	0	(1-r).q	0	(1-r). (1-q)	r.(1-q)	0
DD	0	r.(1-r)	0	$(1-r)^2$	0	(1-r).r	r^2	0
BD	r	0	0	0	(1-r)	0	0	0
DS	0	0	0	0	0	(1-r)	r	0
US	(1-q)	0	q	0	0	0	0	0
D-B D	r^2	0	(1-r).r	0	r.(1-r)	0	0	$(1-r)^2$

With considering two conditions in Markov chain method, we can find the relationships between failure probability (q), repair probability (r) and production rate (P).

The steady-state probability vector v, is a row vector, whose entries are non-negative and sum to 1. The row vector is unchanged by the operation of transition probability matrix M.

The solution in our case is shown in Table 3.5, using the transition probability matrix given in Table 3.4.

Table 3.5 Steady-state Probability Vector

System State	Steady-state Probability
UU	$\frac{-qr + qr^2 - r^2 + 2r}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
UD	$\frac{q^2r}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
DU	$\frac{qr(2-r)}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
DD	$\frac{q^2(1-r)}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
BD	$\frac{q(2-r-q)}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
DS	$\frac{q(2-r-qr)}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
US	$\frac{qr(2-r-q)}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$
D-B D	$\frac{q^2}{qr - r^2 + 2r + 4q - 2q^2r - qr^2 + q^2}$

The production rate is calculated with summation of some probabilities from steady-state probability vector. Specifically, Production rate: $P[UU] + P[DU] = P[UU] + P[US] + r.P[DB\,D]$ (4)

Production rate for two-station production line with identical stations is simplified to be:

$$P = \frac{qr+2r-r^2}{qr-r^2+2r+4q-2q^2r-qr^2+q^2} \quad (5)$$

The analysis of the system behavior is an important issue to find the relation between different performance measures in the system. In the next chapter, analysis of the system behavior will be conducted. The relation between production rate (P) and repair probability (r) in specific failure probability (q) for different number of stations in production line will be shown in the next chapter.

CHAPTER 4: ANALYSIS OF THE SYSTEM BEHAVIOR

4.1 ANALYSIS OF THE DATA

The performance measures were available (Cetinay), for specific failure and repair probabilities for three, four and five station production lines. A sample of the data is provided in Appendix A. For the failure probability, the interval of [0.01, 0.1] and for the repair probability, the interval of [0.05, 1] were taken as the intervals of interest.

4.2.1 ANALYSIS OF PRODUCTION RATE

One of the most important performance measure for production line efficiency, is the production rate. Figure 4.1 illustrates the relationship between the production rate and the repair probability and the failure probability for different failure probabilities.

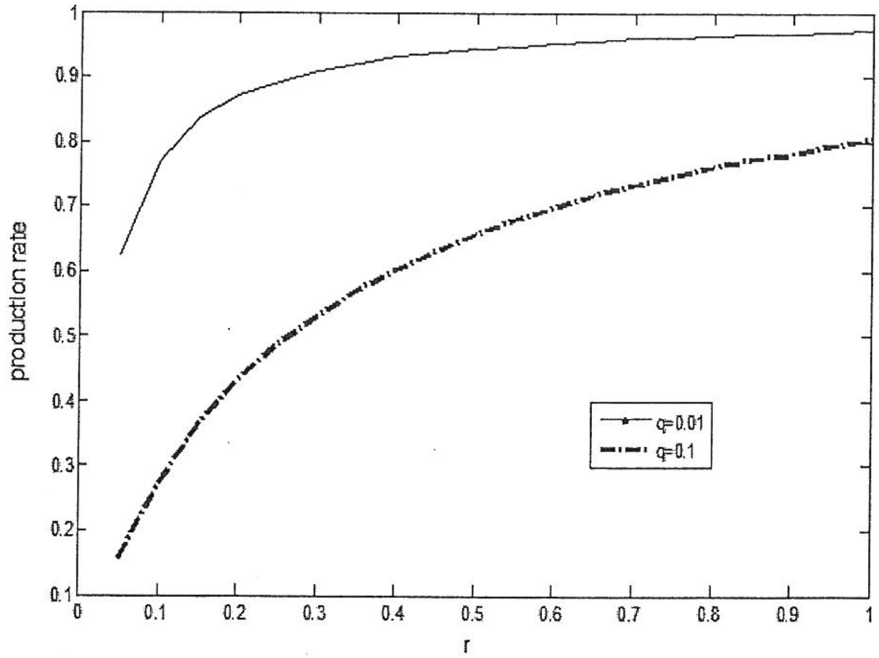

Figure 4.1 **Production Rate versus Repair Probability (3-station)**

Figure 4.1 shows that the production rate increases with increasing the repair probability, while, the production rate decreases with increasing the failure probability.

Analyzing the system behavior shows that in a production line as the number of stations increases, the production rate decreases relatively. Figure 4.2 illustrates the relation between production rate and repair probability for the failure probability of 0.1. The figure depicts that the production rate in two-station production line is more than the production rate in three-station production line and so on.

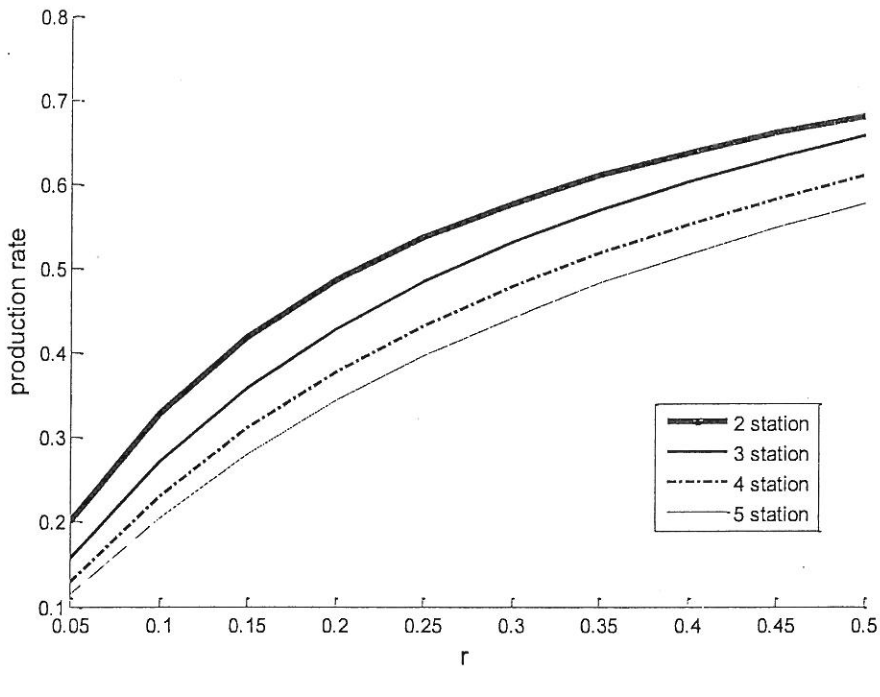

Figure 4.2 Production Rate versus Repair Probability (2, 3, 4, 5-station)

In the next chapter, numerical analysis for three, four and five station production lines will be shown, according to the formula that was obtained for two-station production lines. In another words, the formula for two-station production lines will be generalized for other lengths of lines and then curve-fitting will be conducted to develop a reasonably accurate equation in order to establish reliable forecasts with the formula. After that approximated formula will be suggested and the numerical analysis for two, three, four and five station production lines will be conducted with an approximated formula.

CHAPTER 5: NUMERICAL ANALYSIS

5.1 METHODOLOGY

A curve-fit analysis is conducted in order to develop useful formulas. The aim of curve-fit analysis is to formulate simple functions that provide forecasts of the important performance measures with a reasonable accuracy. Although linear regression is an important tool for statistical analysis, but it also has limitations that must always be considered. In this study according to our data, we cannot use linear regression. By conducting linear regression our results will not be good. The error analysis will show that linear regression is not appropriate method for this research. So non-linear regression is selected as a solution method for this study.

There are many cases in engineering and science where nonlinear models must be fit to data. In the present context, these models are defined as those that have a nonlinear dependence on their parameters. For example in this study the relation between the production rate (P), failure probability (q) and repair probability (r) is defined as a quadratic ratio formula (5).

As with linear least-squares, nonlinear regression is based on determining the values of the parameters that minimize the sum of the squares of the residuals. However, for the nonlinear case, the solution must proceed in an iterative fashion.

There are techniques expressly designed for nonlinear regression. For example, the Gauss-Newton method uses a Taylor series expansion to express the original nonlinear equation in an approximate, linear form. Then least-squares theory can be used to obtain new estimates of the parameters that move in the direction of minimizing the residual.

An alternative is to use optimization techniques to directly determine the least-squares fit. The approximation formula, can be expressed as an objective function to compute the sum of the squares of the errors:

$$f(a) = \sum_{i=1}^{m} [f(q_i, r_i) - P_i]^2 \qquad (6)$$

m : Number of data points

P_i : The real output

We can also define f(a) as:

$$f(a) = r(a)^T r(a) \quad (7)$$

That r is a vector-valued function:

$$r(a) = [\, r_1(a), r_2(a), \ldots, r_3(a)]^T \quad (8)$$

The gradient of f is defined as:

$$\nabla f(a) = \nabla r(a) r(a) = J(a)^T r(a) \quad (9)$$

Where J(a) is the Jacobian of r(a).

The Hessian of a least-squares objective function is a sum of two terms:

$$\nabla^2 f(a) = \nabla r(a) \nabla r(a)^T + \sum_{i=1}^{m} r_i(a) \nabla^2 r_i(a) = J(a)^T J(a) + Q(a) \quad (10)$$

For unconstrained nonlinear optimization problems, MATLAB software has a command that is called *fminsearch.* This command uses the Nelder-Mead simplex algorithm. The Nelder-Mead algorithm is used to determine the values of the parameters that minimize the function.

MATLAB code for non-linear regression method with *fminsearch* command is shown in appendix 5, also the Nelder-Mead algorithm procedure is shown in appendix 6.

5.2 A QUADRATIC RATIO FORMULA

In chapter 3, we obtained the exact formula (5) for two-station production line with identical stations. In this section we want to assume the formula that we obtained for two-station production line, works for the three, four and five station production lines. So we must generalize the formula for different number of stations of production lines.

If we insert parameters in a quadratic ratio formula (5), the equation will be defined as:

$$P(q,r) = \frac{a_0 qr + a_1 r + a_2 r^2}{a_3 qr + a_4 r^2 + a_5 r + a_6 q + a_7 q^2 r + a_8 qr^2 + a_9 q^2} \quad (11)$$

where a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9 are the parameters that need to be solved.

We want to show how the above formula (11), fits the data for three, four and five station production lines. In the quadratic ratio formula (11), we have 10 parameters as a constants that must be calculated. After several runs in MATLAB, the best constants for three, four and five station production lines is calculated. The below table shows the constants of the formula for the production rate, failure and repair probabilities for different number of stations of production line.

Table 5.1 Constants for a Quadratic Ratio Formula

Constants	3-Station	4-Station	5-Station
a_0	10.69	25.29	51.36
a_1	6.117	9.6	15.47
a_2	-1.546	-1.913	-3.694
a_3	6.283	16.33	30.92
a_4	-1.507	-1.857	-3.559
a_5	6.096	9.583	15.45
a_6	18.56	38.8	77.69
a_7	3.278	1.362	2.43
a_8	-1.383	-0.7172	-4.96
a_9	5.846	17.58	24.57

As we can see in the table 5.1, when the number of stations increases, the amount of positive constants also increases. But with increasing the number of stations, the amount of negative constants will decrease.

To depict the relation between the production rates of production lines and the failure and repair probabilities, three-dimensional plot was generated with MATLAB software. Figure 5.1 illustrates the relation between the production rate (p), the failure (q) and repair probabilities (r), for three-station production lines according to the exact formula and quadratic ratio formula.

Exact Approximated

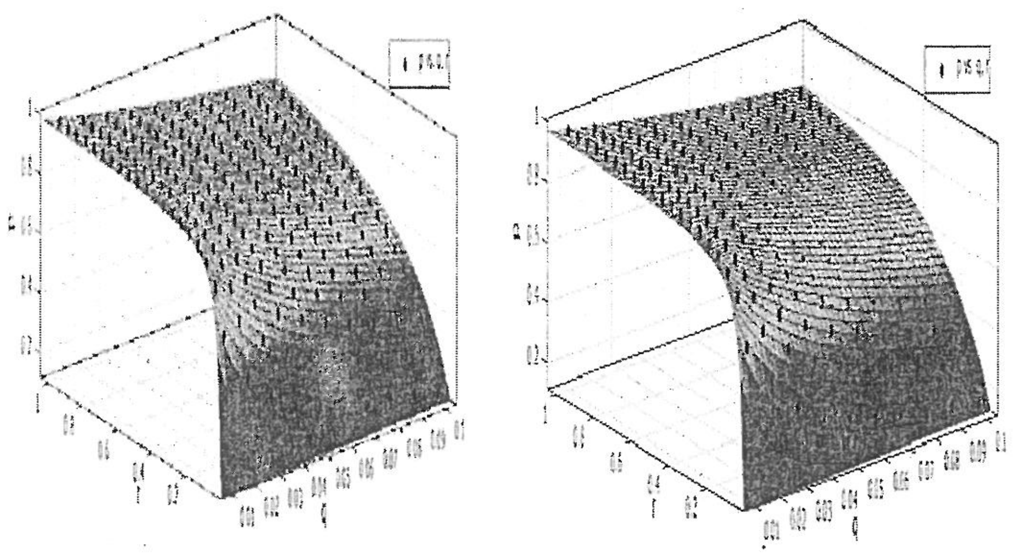

Figure 5.1 Production Rate versus q and r (3-station)

Figure 5.2, illustrates the production rate versus failure and repair probabilities for four-station production lines.

Exact Approximated

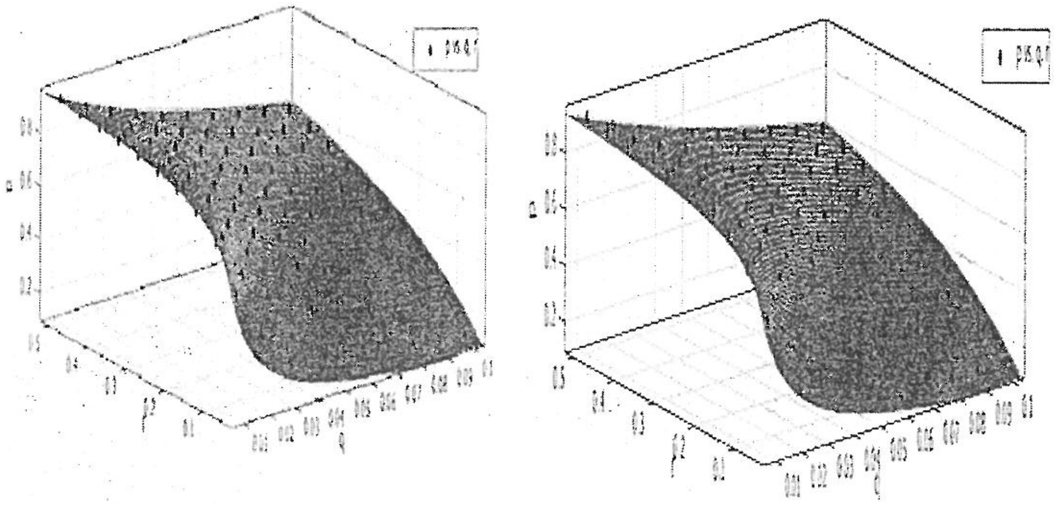

Figure 5.2 **Production Rate versus q and r (4-station)**

According to the quadratic ratio formula (11), the relation between production rate, failure and repair probabilities for five-station production line is shown in the three-dimensional plot. Figure 5.3 illustrates the production rate versus failure and repair probability for five-station production lines according to the exact formula and quadratic ratio formula.

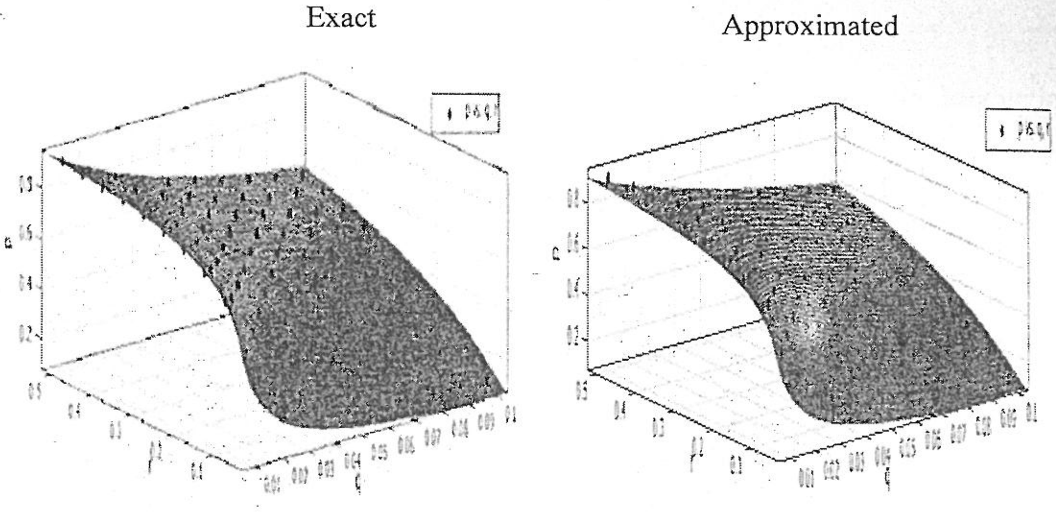

Figure 5.3 **Production Rate versus q and r (5-station)**

The table 5.2 shows the mean absolute error for three, four and five station production lines. Mean absolute errors is obtained with the summation of absolute differences between real production rate and approximated production rate over the number of data.

The below equation shows the mean absolute error:

$$\text{Mean Absolute Error} = \frac{\sum |P_{real} - P_{approximate}|}{N} \quad (12)$$

Table 5.2 Mean Absolute Error

No. of Station / Error	3-Station	4-Station	5-Station
Mean Absolute Error	0.00043	0.00035	0.00024

5.3 THE SIMPLIFIED FORMULA

In this part, we derive a simple formula for the throughput of a serial production line with work-stations that are subject to random failures. In a quadratic ratio formula (11), we can eliminate some terms, because the amounts that will obtain are too small. Although, with eliminating some terms of formula, the errors maybe increase a little bit, but analyzing the system will become more comfortable. It is clear that for production lines with a large number of stations, it is not possible to develop exact numerical results due to the complexity of the numerical calculations involved. As a result of this restriction, approximate solutions were conducted.

Here we suggest a simplified formula:

$$P(q,r) = \frac{a_0 qr + a_1 r}{a_2 qr + a_3 r + a_4 q} \qquad (13)$$

Coefficients of the simplified formula (13), is optimized with nonlinear least-square method in MATLAB.

We analyzed the approximated production rates according to the formula (13), for two, three, four and five station production lines.

Constants of a simplified formula (13) were calculated with nonlinear least-square method in MATLAB. The determined constants for two-station production line are represented in the Table 5.3.

Table 5.3 Constants for the simplified Formula (2-Station)

Constants	2-Station
a_0	3.774
a_1	11.33
a_2	12.36
a_3	11.36
a_4	22.96

To depict the relation between the expected production rates of production lines and the failure and repair probabilities, three-dimensional plot was generated with MATLAB software. Figure 5.4 illustrates the relation between the expected production rate and the failure and repair probabilities for two-station production lines according to the exact formula and the simplified formula (13).

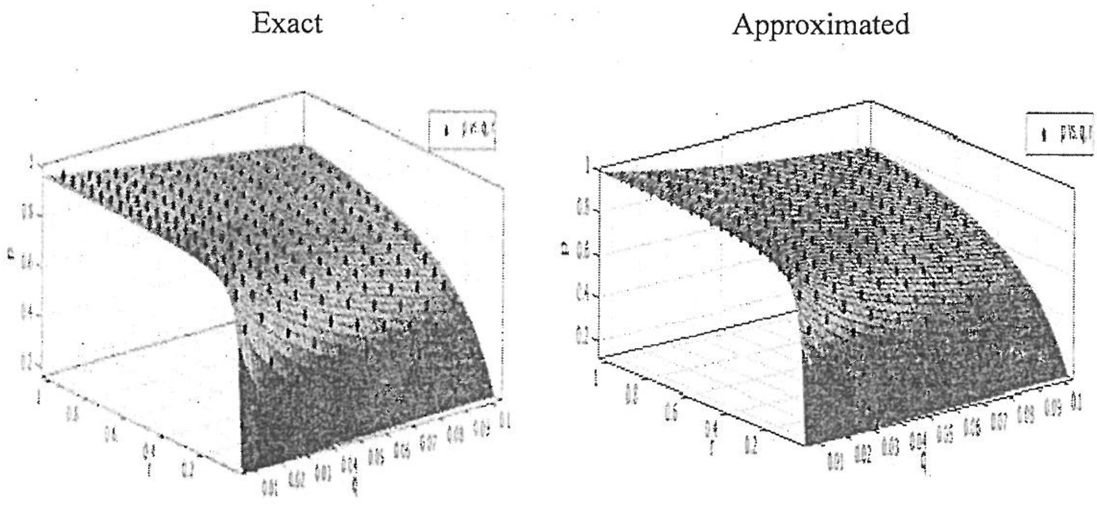

Figure 5.4 **Production Rate versus q and r (2-station)**

Coefficients of the production rate functions was calculated with nonlinear least-square method in MATLAB. The solution of the determined constants are explained in the part of Methodology (5.1). The determined constants are represented in the Table 5.4.

Table 5.4 Constants of the simplified Formula (3, 4 and 5 station)

Constants	3-Station	4-Station	5-Station
a_0	6.902	9.585	13.71
a_1	6.016	4.994	4.969
a_2	5.328	7.721	10.63
a_3	6.038	5.017	5.004
a_4	18.15	20	24.84

Based on the results of the three-dimensional plots, curve-fitting was realized to develop a reasonably accurate equation in order to establish reliable forecasts with simple formula.

The relation between the expected production rate and the failure and repair probabilities for three-station production lines is shown in Figure 5.5.

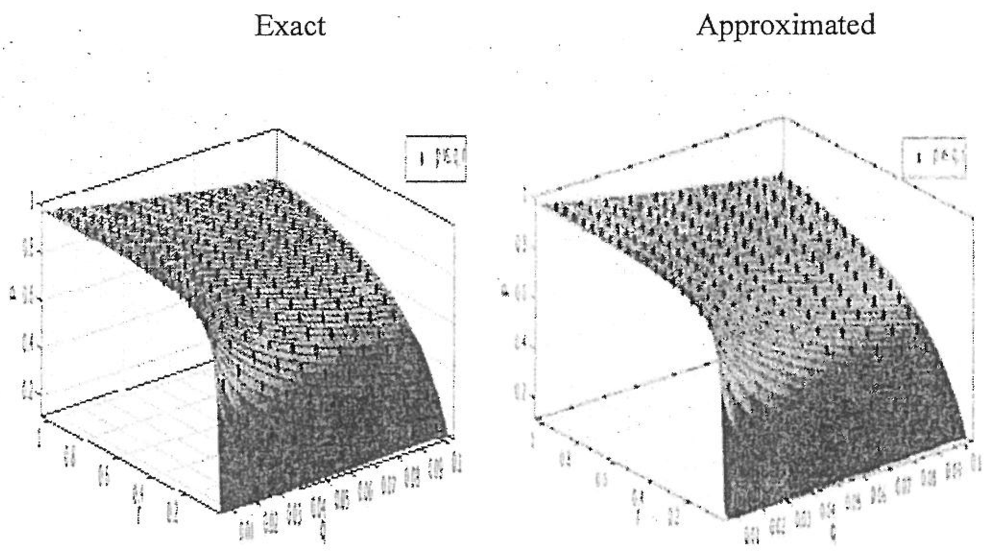

Figure 5.5 Production Rate versus q and r (3-station)

Figure 5.6 shows the relation between the expected production rate and the failure and repair probabilities for four-station production lines based on the exact and the simplified formula (13).

Exact Approximated

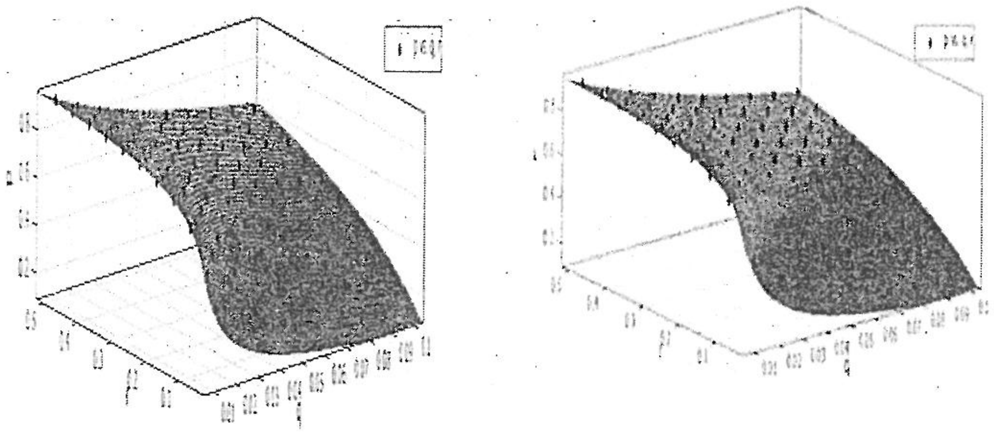

Figure 5.6 **Production Rate versus q and r (4-station)**

Figure 5.7, illustrates the Production Rate versus failure and repair probabilities for five-station production lines.

Exact Approximated

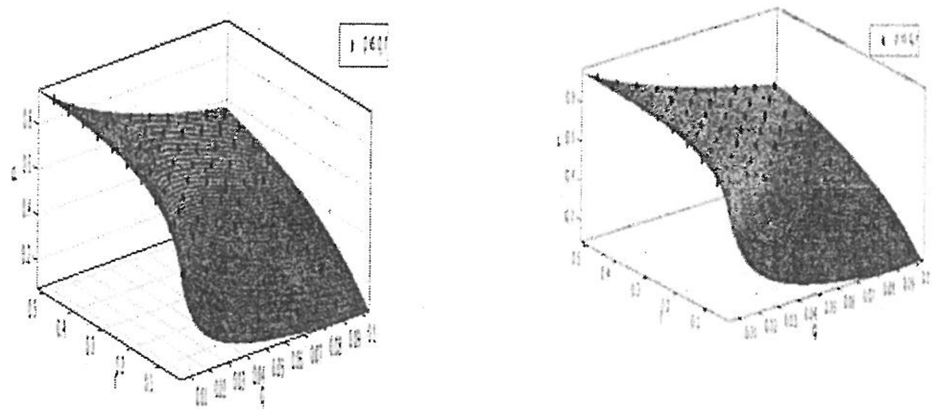

Figure 5.7 Production Rate versus q and r (5-station)

In the next chapter, error analysis will show according to the quadratic ratio formula and the approximated formula for the different number of stations.

CHAPTER 6: ERROR ANALYSIS

6.1 ERROR ANALYSIS FOR THE PRODUCTION RATE FUNCTION

In order to measure the accuracy of the approximated equation for the production rate the error statistics have been analyzed.

In addition to minimum and maximum absolute error statistics, mean absolute error, the mean of error, and variance of error were collected. Furthermore, the mean absolute percent error (MAPE) and mean standard error percent (MSEP) were calculated for further insight in terms of accuracy and precision of the fit.

In this section first we analyze the error statistics for the production rate according to the quadratic ratio formula (5). The error statistics was conducted for three, four and five station production lines.

Table 6.1 summarizes the error statistics collected for the production rate based on the quadratic ratio formula (5).

Table 6.1 Production rate Error Statistics According to the Quadratic Ratio Formula

Stations / Errors	3-Station Line	4-Station Line	5-Station Line
Min. Absolute Error	$1.5 \cdot 10^{-6}$	$1.0 \cdot 10^{-6}$	$2.6 \cdot 10^{-6}$
Max. Absolute Error	0.0019	0.0020	0.0010
Mean Absolute Error	0.0004	0.0003	0.0002
Mean of Error	0.0061	0.0035	0.0024
Variance of Error	$3.72 \cdot 10^{-5}$	$1.25 \cdot 10^{-5}$	$5.79 \cdot 10^{-6}$
Min.Abs.Percent Error	0.0001	0.0002	0.0011
Max.Abs.Percent Error	0.0014	0.0006	0.0023
Mean Percent Error	-0.0016	-0.0023	-0.0011
MAPE (%)	$8 \cdot 10^{-4}$	$2.3 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$
MSEP (%)	0.020	0.048	0.033

Table 6.2 shows the error statistics collected for the production rate forecasts based on the approximated formula (13). The statistics, collected in Table 6.2, show that the approximations perform well when compared to the real outputs.

Table 6.2 Production rate Error Statistics According to Simplified Approximation

Errors \ Stations	2-Station Line	3-Station Line	4-Station Line	5-Station Line
Min. Absolute Error	$1.7\cdot 10^{-5}$	$4.02\cdot 10^{-7}$	$1.7\cdot 10^{-6}$	$4.9\cdot 10^{-6}$
Max. Absolute Error	0.0052	0.0039	0.0019	0.0021
Mean Absolute Error	0.0010	0.0007	0.0005	0.0006
Mean of Error	0.0146	0.0113	0.0016	0.0063
Variance of Error	$2.1\cdot 10^{-5}$	$1.2\cdot 10^{-5}$	$2.69\cdot 10^{-5}$	$4.01\cdot 10^{-5}$
Min.Abs.Percent Error	0.0036	0.0033	0.0009	0.0025
Max.Abs.Percent Error	0.039	0.090	0.068	0.022
Mean Percent Error	-0.0018	-0.0014	-0.0015	-0.0017
MAPE (%)	$1.6\cdot 10^{-3}$	$1.5\cdot 10^{-3}$	$9.8\cdot 10^{-3}$	$2.2\cdot 10^{-3}$
MSEP (%)	0.027	0.013	0.073	0.081

As we can see in Table 6.2, the amount of errors are too small and the approximated numbers are very close to real ones. Error analysis shows that the approximated function provides reliable estimates.

In the above table, the Mean Absolute Percent Error (MAPE) is calculated as:

$$\text{MAPE} = \frac{1}{n}\sum \left| \frac{(True\ value) - (Forecast\ value)}{(True\ value)} .\ 100\% \right| \quad (14)$$

In Table 6.2, the Mean Standard Error Percent (MSEP) is calculated as:

$$\text{MSEP} = \frac{1}{n}\sum \frac{RMS\ (True\ value) - (Forecast\ value)}{(True\ value)} .\ 100\% \quad (15)$$

Where the Root-Mean-Square (RMS) of the forecast error is equal to the square root of the variance (or standard deviation) of the error as presented in the above tables.

Although by comparing Table 6.1 and Table 6.2, we will realize that the error statistics of a quadratic ratio formula is less than the error statistics of a linear approximation formula but in this study, we suggest a linear approximated formula, it is more simple than the first one and calculations will more comfortable. The error statistics of Table 6.2 are small and good enough, so we can use the linear approximation formula that is suggested in this research.

6.2 COMPARISON OF ERROR STATISTICS

After calculating the error statistics for different number of stations of production lines, in this part we want to show the errors in a plot. Error analysis for two types of formula was conducted. First the error analysis was calculated according to the quadratic ratio formula. After that the error statistic analysis was conducted for the approximated formula.

Figure 6.1, illustrates the error statistics for three, four and five station production line according to a quadratic ratio formula (5).

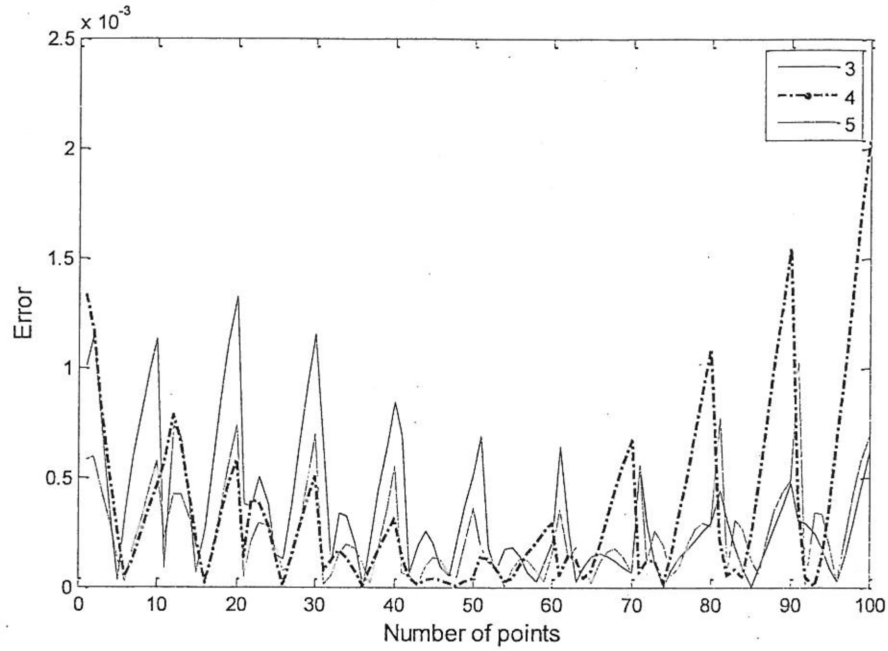

Figure 6.1 Errors according to a Quadratic Ratio Formula

Figure 6.2, shows the error statistics for three, four and five station production line according to the approximated formula that is proposed in this research.

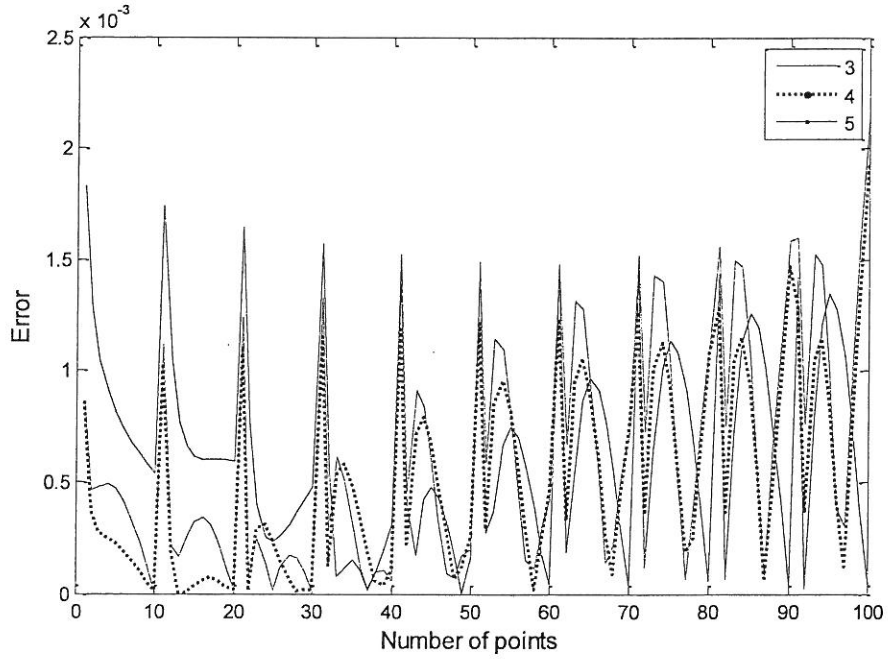

Figure 6.2 Errors according to the Approximated Formula

6.3 ERROR PLOT FOR THE PRODUCTION RATE FUNCTION

In this part, we will show the error sequences according to the approximated function. The error sequence for every station is illustrated separately. Error sequence for a three-station production line versus number of points, is shown in Figure 6.3. The amount of samples in three-station production line is equal to 200. Figure 6.3 illustrates the amount of errors are between 0 to 0.004.

Every point relates to a specific combination of failure and repair probability values. The failure probability is varied over the whole sample range, whereas the repair probability is variable over every value of failure probability.

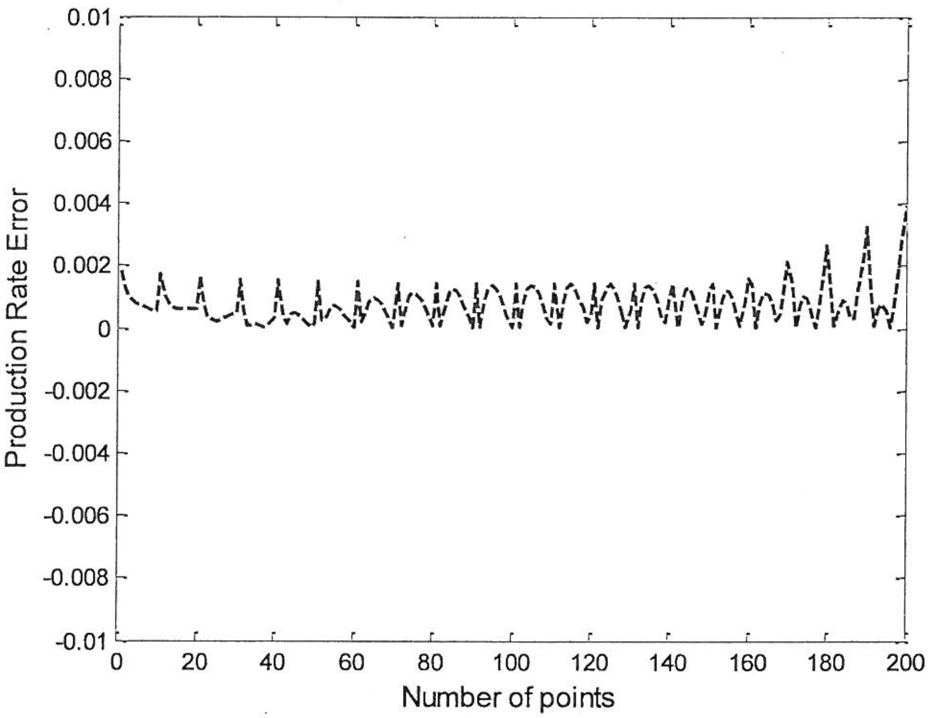

Figure 6.3 Error Plot for the Production Rate Function (3-station)

Figure 6.4 illustrates the error sequence for four-station production line versus number of points. The amount of data points in four-station production line is equal to 100. Figure 6.4 shows, the amount of errors are between 0 to 0.002.

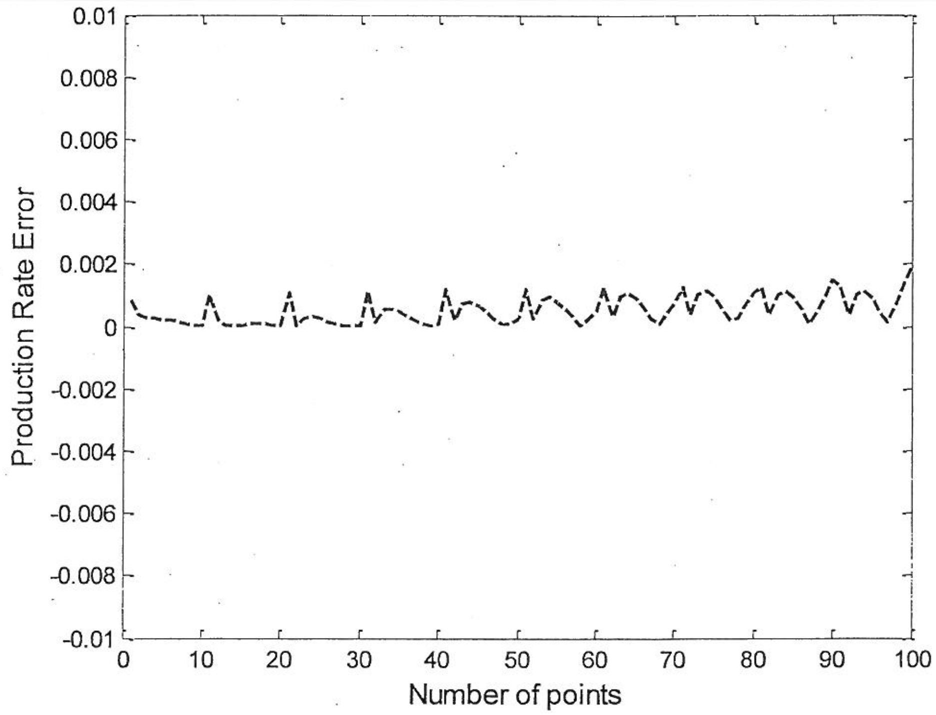

Figure 6.4 Error Plot for the Production Rate Function (4-station)

The sequence of errors for the production rate function for five-station production line is shown in Figure 6.5. The amount of data points in five-station production line is equal to 100. Figure 6.5 illustrates, the amount of errors are between 0 to 0.002.

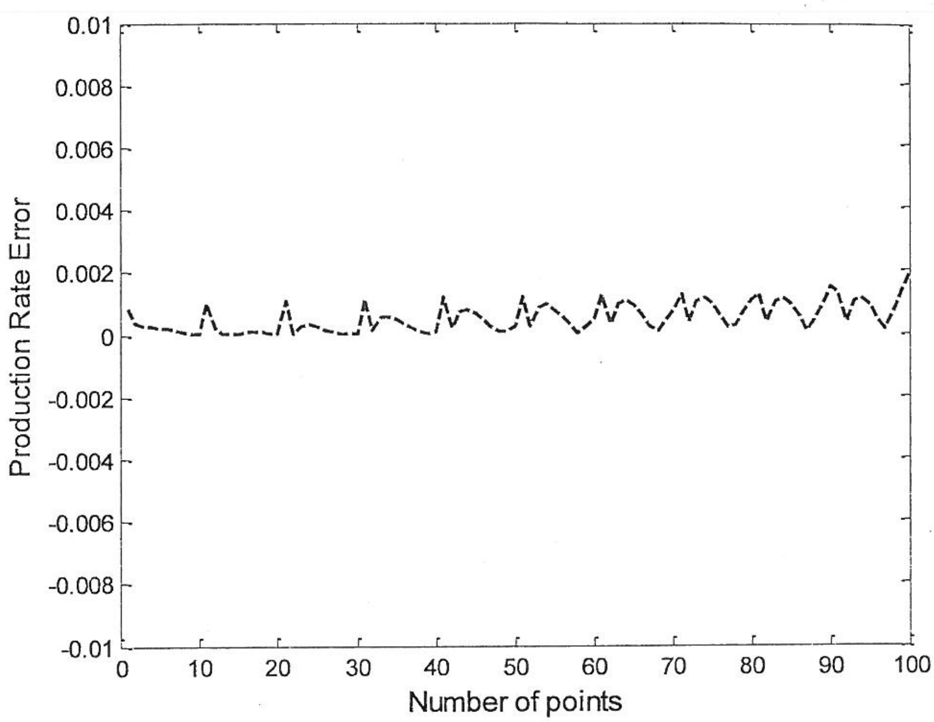

Figure 6.5 Error Plot for the Production Rate Function (5-station)

Above figures demonstrate the forecast error sequence for a three, four and five station production line versus number of points. Each sample relates to a specific combination of failure and repair probability values.

Figure 6.6 demonstrates that the failure probability is varied over the whole number of points, also the repair probability is variable over every value of failure probability.

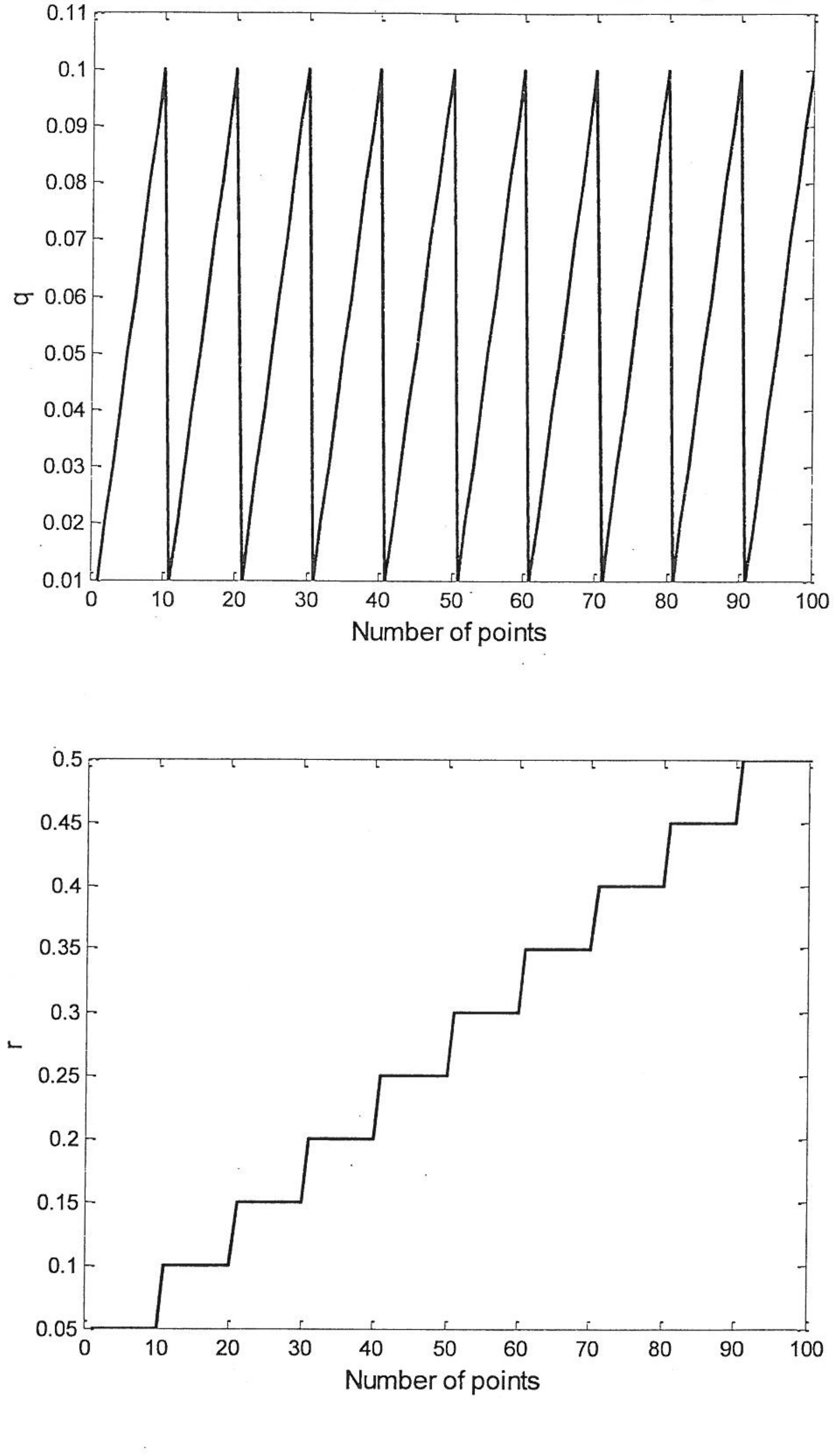

Figure 6.6 Generation of Data points

The accuracy of the obtained rational fit is considered more than adequate. As a result, the error analysis showed that the approximated formula yield very effective approximations.

6.4 RATIONAL-FIT FORECASTING

Conducting analysis on exact results that were gathered, shows that the important performance measures can also be estimated by relatively easy rational equations as a function of particular line characteristics, namely the failure probabilities and repair probabilities. In this research, all stations have the same failure and repair probability because, only lines with identical machines are considered. Consequently, it is shown that the approximated function to forecast the expected production rate is effective. As the error analysis shows, the approximate formula can be used most adequately and computationally more effective instead of the exact solutions for lines with identical machines.

CHAPTER 7: CONCLUSION

7.1 CONCLUDING REMARKS

This research study is focused on approximating formula for the performance measures of bufferless production lines. Discrete-time Markov chain method is conducted to find exact formula for two-station production line. In order to find exact formula, transition probability for two-station production line is found.

To find the production line system behavior, the results for three, four and five-station production lines were analyzed in detail. Production rate that is one of critical performance measures of production line were analyzed to establish valid formulas in accordance with the system parameters.

In this research, we approximated a linear formula for different number of stations. Also a curve-fit analysis was conducted to forecast the production rate a reasonable accuracy.

Error analysis showed that linear regression is not an appropriate method for such data, so nonlinear least-square method is conducted. Results of the error analysis for nonlinear least-square method was so good. In this study all steps such as: curve-fitting, 3D plots, nonlinear least-square method and etc... is conducted in MATLAB software.

7.2 FUTURE WORK

This study provided exact formula for two-station production line by constructing transition probability matrix of Markov chain method. In addition, formula for three, four and five station production line is approximated. The formula shows the relation between production rate, failure and repair probability. Lines with identical machines are curve-fitted to obtain useful approximate equations for the production rate measures.

Further research can be devoted to include intermediate storage (buffers) between the stations. Approximating formula for the production lines with non-identical machines also can be conducted. Moreover, according to the analysis of the system behavior of three, four and five station production lines, the model for more than five station production lines can be simulated with appropriate simulation methods.

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APPENDIX 1

SAMPLE OF ANALYSIS RESULTS FOR TWO-STATION

Failure Probability (q)	Repair Probability (r)	Production Rate (real)	Production Rate (Linear Approximation Formula)	Error
0.01	0.05	0.709810596	0.707142502	0.002668095
0.02	0.05	0.550864046	0.548572473	0.002291573
0.03	0.05	0.450542699	0.44863409	0.001908609
0.04	0.05	0.381459899	0.379882471	0.001577428
0.05	0.05	0.330988829	0.329689736	0.001299093
0.06	0.05	0.292499782	0.291434826	0.001064956
0.07	0.05	0.262177632	0.26131145	0.000866182
0.08	0.05	0.237671522	0.236976009	0.000695513
0.09	0.05	0.217453871	0.216906566	0.000547305
0.1	0.05	0.200488998	0.200071775	0.000417222
0.01	0.1	0.826911421	0.824968794	0.001942626
0.02	0.1	0.705571072	0.704076745	0.001494327
0.03	0.1	0.615787123	0.614605667	0.001181455
0.04	0.1	0.546663661	0.545714503	0.000949158
0.05	0.1	0.491803279	0.491035041	0.000768237
0.06	0.1	0.447202701	0.446580827	0.000621875
0.07	0.1	0.410228645	0.409728878	0.000499767
0.08	0.1	0.37907796	0.378682675	0.000395285
0.09	0.1	0.352474406	0.352170423	0.000303983
0.1	0.1	0.329489292	0.329266509	0.000222782
0.01	0.15	0.875033323	0.873482947	0.001550376
0.02	0.15	0.778453085	0.777547403	0.000905682
0.03	0.15	0.701571071	0.701057417	0.000513655
0.04	0.15	0.638916434	0.638644792	0.000271642
0.05	0.15	0.586872587	0.586750773	0.000121814

APPENDIX 2

SAMPLE OF ANALYSIS RESULTS FOR THREE-STATION

Failure Probability (q)	Repair Probability (r)	Production Rate (real)	Production Rate (Quadratic Formula)	Error (Quadratic Formula)	Production Rate (Linear Approximation Formula)	Error (Linear Approximation Formula)
0.01	0.05	0.627778	0.62676865	0.00100935	0.625948435	0.001829565
0.02	0.05	0.46039	0.459241915	0.001148085	0.459100485	0.001289515
0.03	0.05	0.365228	0.36446686	0.00076114	0.36418061	0.00104739
0.04	0.05	0.303833	0.303493451	0.000339549	0.302924445	0.000908555
0.05	0.05	0.260936	0.260965481	2.94814E-05	0.260120878	0.000815122
0.06	0.05	0.229266	0.229605317	0.000339317	0.22852346	0.00074254
0.07	0.05	0.204924	0.205518917	0.000594917	0.204240853	0.000683147
0.08	0.05	0.185627	0.186434181	0.000807181	0.184996496	0.000630504
0.09	0.05	0.169952	0.170936122	0.000984122	0.169369738	0.000582262
0.1	0.05	0.156966	0.158097536	0.001131536	0.156428152	0.000537848
0.01	0.1	0.771386	0.771300176	8.58237E-05	0.769643878	0.001742122
0.02	0.1	0.630645	0.629910229	0.000734771	0.629597649	0.001047351
0.03	0.1	0.535267	0.534586089	0.000680911	0.534496738	0.000770262
0.04	0.1	0.466356	0.465951665	0.000404335	0.465696404	0.000659596
0.05	0.1	0.414229	0.414161388	6.76116E-05	0.413611714	0.000617286
0.06	0.1	0.373415	0.373682139	0.000267139	0.37281072	0.00060428
0.07	0.1	0.340588	0.34116516	0.00057716	0.339985018	0.000602982
0.08	0.1	0.313608	0.314464956	0.000856956	0.313004426	0.000603574
0.09	0.1	0.291037	0.292143402	0.001106402	0.290435366	0.000601634
0.1	0.1	0.271875	0.273200413	0.001325413	0.271277629	0.000597371
0.01	0.15	0.835063	0.835440529	0.000377529	0.83341825	0.00164475
0.02	0.15	0.71932	0.718953168	0.000366832	0.718547129	0.000772871
0.03	0.15	0.633606	0.633100015	0.000505985	0.633207297	0.000398703
0.04	0.15	0.567566	0.567183512	0.000382488	0.5673079	0.0002581
0.05	0.15	0.515117	0.514967785	0.000149215	0.514885385	0.000231615

APPENDIX 3

SAMPLE OF ANALYSIS RESULTS FOR FOUR-STATION

Failure Probab ility (q)	Repair Probabili ty (r)	Production Rate (real)	Production Rate (Quadratic Formula)	Error (Quadratic Formula)	Production Rate (Linear Approximation Formula)	Error (Linear Approximation Formula)
0.01	0.4	0.911099	0.911032787	6.62132E-05	0.909842498	0.001256502
0.02	0.4	0.839921	0.840043416	0.000122416	0.84027663	0.00035563
0.03	0.4	0.781602	0.781721439	0.000119439	0.782610693	0.001008693
0.04	0.4	0.732907	0.732923123	1.61235E-05	0.734031865	0.001124865
0.05	0.4	0.691607	0.691465952	0.000141048	0.692549079	0.000942079
0.06	0.4	0.656111	0.655787328	0.000323672	0.656713355	0.000602355
0.07	0.4	0.625256	0.624739061	0.000516939	0.625445075	0.000189075
0.08	0.4	0.598169	0.597458527	0.000710473	0.597923335	0.000245665
0.09	0.4	0.574187	0.573285077	0.000901923	0.573512837	0.000674163
0.1	0.4	0.552791	0.551704196	0.001086804	0.55171423	0.00107677
0.01	0.45	0.920246	0.920026328	0.000219672	0.918968687	0.001277313
0.02	0.45	0.855318	0.85537148	5.34803E-05	0.855682457	0.000364457
0.03	0.45	0.801388	0.801464378	7.63779E-05	0.802427746	0.001039746
0.04	0.45	0.755844	0.755800898	4.3102E-05	0.756994558	0.001150558
0.05	0.45	0.716841	0.71659919	0.00024181	0.717777653	0.000936653
0.06	0.45	0.68304	0.682556772	0.000483228	0.68358293	0.00054293

0.07	0.45	0.653444	0.652699482	0.000744518	0.65350372	5.97196E-05
0.08	0.45	0.627297	0.626284153	0.001012847	0.626839211	0.000457789
0.09	0.45	0.604015	0.602733972	0.001281028	0.603039161	0.000975839
0.1	0.45	0.583139	0.581594396	0.001544604	0.581665525	0.001473475
0.01	0.5	0.9277	0.927300821	0.000399179	0.926402522	0.001297478
0.02	0.5	0.868058	0.868010821	4.71791E-05	0.868419907	0.000361907
0.03	0.5	0.817974	0.817972974	1.02615E-06	0.819018923	0.001044923
0.04	0.5	0.775284	0.775149474	0.000134526	0.776425675	0.001141675
0.05	0.5	0.738434	0.738060431	0.000373569	0.739323657	0.000889657

APPENDIX 4

SAMPLE OF ANALYSIS RESULTS FOR FIVE-STATION

Failure Probability (q)	Repair Probability (r)	Production Rate (real)	Production Rate (Quadratic Formula)	Error (Quadratic Formula)	Production Rate (Linear Approximation Formula)	Error (Linear Approximation Formula)
0.01	0.4	0.892435	0.891886821	0.000548179	0.890914801	0.001520199
0.02	0.4	0.811092	0.811205433	0.000113433	0.811816802	0.000724802
0.03	0.4	0.747299	0.747554004	0.000255004	0.748730152	0.001431152
0.04	0.4	0.695836	0.696022313	0.000186313	0.697240055	0.001404055
0.05	0.4	0.653373	0.653422618	4.96178E-05	0.654418147	0.001045147
0.06	0.4	0.617682	0.61759525	8.67497E-05	0.618245479	0.000563479
0.07	0.4	0.58722	0.587025085	0.000194915	0.58728492	6.49204E-05
0.08	0.4	0.560881	0.560617781	0.000263219	0.560485659	0.000395341
0.09	0.4	0.537853	0.537563183	0.000289817	0.537061569	0.000791431
0.1	0.4	0.517524	0.517248699	0.000275301	0.516412804	0.001111196
0.01	0.45	0.90333	0.902556821	0.000773179	0.901771365	0.001558635
0.02	0.45	0.82879	0.8288621	7.20997E-05	0.829541499	0.000751499
0.03	0.45	0.769438	0.76974384	0.00030584	0.770938113	0.001500113
0.04	0.45	0.720969	0.721234167	0.000265167	0.722437543	0.001468543
0.05	0.45	0.680569	0.680685848	0.000116848	0.681634621	0.001065621

0.06	0.45	0.646323	0.646265052	5.79482E-05	0.646831513	0.000508513
0.07	0.45	0.616879	0.616661222	0.000217778	0.616795497	8.35027E-05
0.08	0.45	0.591258	0.590912856	0.000345144	0.590609908	0.000648092
0.09	0.45	0.568733	0.568298555	0.000434445	0.567578944	0.001154056
0.1	0.45	0.54875	0.548266469	0.000483531	0.547164926	0.001585074
0.01	0.5	0.912246	0.911224075	0.001021925	0.910648999	0.001597001
0.02	0.5	0.843533	0.843539737	6.73692E-06	0.844288429	0.000755429
0.03	0.5	0.788148	0.788487104	0.000339104	0.789676075	0.001528075
0.04	0.5	0.742466	0.742800135	0.000334135	0.74394585	0.00147985
0.05	0.5	0.70407	0.704249704	0.000179704	0.705093834	0.001023834

APPENDIX 5

NON-LINEAR REGRESSION WITH MATLAB

```
function f= fSSR(a,q,r,p)

yp = (a(1)*q*r'+a(2)*r)/(a(3)*q*r'+a(4)*r+a(5)*q);

f = sum((p-yp).^2);


clc

clear all

st3 = xlsread('P3.xlsx');

q3=st3(:,1)';

r3=st3(:,2)';

p3=st3(:,3)';


a=fminsearch(@fSSR, [1, 1, 1, 1, 1], [], q3,r3, p3)

%a=fminsearch(@fSSR, [7, 6, 5, 6, 20], [], q3,r3, p3)

%a=fminsearch(@fSSR, [a1, a2, a3, a4, a5], [], q3,r3, p3)

yp = (a(1)*q3*r3'+a(2)*r3)/(a(3)*q3*r3'+a(4)*r3+a(5)*q3);

ee = sum((p3-yp).^2)

e_mean=ee/200
```

APPENDIX 6

NELDER-MEAD ALGORITHM PROCEDURE

Nelder-Mead algorithm uses a simplex of n+1 points for n-dimensional vectors x. The algorithm first makes a simplex around the initial guess x_0 by adding 5% of each component $x_0(i)$ to x_0, and using these n vectors as elements of the simplex in addition to x_0. Then, the algorithm modifies the simplex repeatedly according to the following procedure.

1. Let x(i) for: i = 1,...,n+1.
2. Order the points in the simplex from lowest function value f(x(1)) to highest f(x(n+1)). At each step in the iteration, the algorithm discards the current worst point x(n+1), and accepts another point into the simplex.
3. Generate the reflected point

   $$r = 2k - x(n+1),$$

   Where

   $$k = \sum x(i) \quad i: 1, 2, \ldots, n,$$

   And calculate f(r).
4. If $f(x(1)) \leq f(r) < f(x(n))$, accept r and terminate this iteration. Reflect
5. If $f(r) < f(x(1))$, calculate the expansion point s

   $$s = k + 2(k - x(n+1)),$$

   and calculate f(s).

   5.1. If $f(s) < f(r)$, accept s and terminate the iteration. Expand

   5.2. Otherwise, accept r and terminate the iteration. Reflect

6. If $f(r) \geq f(x(n))$, perform a contraction between m and the better of x(n+1) and r:

   6.1 If $f(r) < f(x(n+1))$, calculate

   $$c = k + (r - k)/2$$

   and calculate f(c). If $f(c) < f(r)$, accept c and terminate the iteration. Contract outside Otherwise, continue with Step 7 (Shrink).

6.2 If $f(r) \geq f(x(n+1))$, calculate

$$cc = m + (x(n+1) - k)/2$$

and calculate f(cc). If $f(cc) < f(x(n+1))$, accept cc and terminate the iteration. Contract inside Otherwise, continue with Step 7 (Shrink).

7. Calculate the n points

$$v(i) = x(1) + (x(i) - x(1))/2$$

and calculate f(v(i)), $i = 2,\ldots,n+1$. The simplex at the next iteration is x(1), v(2),...,v(n+1). Shrink.

APPENDIX 7

MATLAB CODE FOR PRODUCTION RATE VERSUS REPAIR PROBABILITY PLOT FOR THREE STATION

```
clc

clear all

r3_01=[0.05;0.1;0.15;0.2;0.25;0.3;0.35;0.4;0.45;0.5;0.55;0.6;0.65;0.7;0.75;0.8;0.8
5;0.9;0.95;1];



p3_01=[0.627778;0.771386;0.835063;0.871016;0.894115;0.910209;0.922065;0.9
31164;0.938368;0.944213;...

0.949051;0.953123;0.956597;0.959598;0.962215;0.964519;0.966563;0.968389;0.
970032;0.971517];



p3_1=[0.156966;0.271875;0.359663;0.428944;0.485038;0.531403;0.57039;0.603
65;0.632379;0.657461;...

0.679569;0.699221;0.716825;0.732705;0.747123;0.760294;0.772396;0.78358;0.7
93971;0.803681];



plot(r3_01,p3_01,'k--','LineWidth',2);

hold on

plot(r3_01,p3_1,'k-.','LineWidth',2);
```

```
%axis([0 20 -1 1])

%grid on

legend('q=0.01','q=0.1','n=5');%, 'Non-optimal Solution','Equation (35)');


xlabel('r ','fontsize',12)

ylabel('production rate','fontsize',12)
```

APPENDIX 8

MATLAB CODE FOR PRODUCTION RATE VERSUS REPAIR PROBABILITY PLOT FOR 2, 3, 4, AND 5 STATION

```
clc

clear all

r2_01=[0.05;0.1;0.15;0.2;0.25;0.3;0.35;0.4;0.45;0.5];

p2_01=[0.709810596;0.826911421;0.875033323;0.901259772;0.917766074;0.92
9112182;0.937391599;0.94370036;0.948667878;0.952681388];

p2_1=[0.200488998;0.329489292;0.419505199;0.485933504;0.53701016;0.5775
40107;0.610515824;0.637898687;0.661028266;0.680851064];

r3_01=[0.05;0.1;0.15;0.2;0.25;0.3;0.35;0.4;0.45;0.5];

p3_01=[0.627778;0.771386;0.835063;0.871016;0.894115;0.910209;0.922065;0.9
31164;0.938368;0.944213];

p3_1=[0.156966;0.271875;0.359663;0.428944;0.485038;0.531403;0.57039;0.603
65;0.632379;0.657461];
```

```
r4_01=[0.05;0.1;0.15;0.2;0.25;0.3;0.35;0.4;0.45;0.5];

p4_01=[0.560537;0.718496;0.792988;0.836347;0.864719;0.884732;0.899607;0.9
11099;0.920246;0.9277];

p5_01=[0.507507;0.673476;0.75588;0.805144;0.837917;0.861295;0.878815;0.89
2435;0.90333;0.912246];

p4_1=[0.129988;0.230797;0.3113;0.377106;0.431938;0.478359;0.518198;0.5527
91;0.583139;0.610007];

p5_1=[0.113733;0.205114;0.280187;0.343003;0.396376;0.442326;0.482336;0.51
7524;0.54875;0.576681];

figure (1)

%plot(r2_01,p2_01,'b-','LineWidth',2);

hold on

plot(r2_01,p2_1,'b-.','LineWidth',2);

hold on

%plot(r3_01,p3_01,'k-','LineWidth',2);
```

```
hold on

plot(r3_01,p3_1,'k-','LineWidth',2);

hold on

%plot(r4_01,p4_01,'k-.','LineWidth',2);

hold on

plot(r4_01,p4_1,'k-.','LineWidth',2);

hold on

%plot(r4_01,p5_01,'k--','LineWidth',2);

hold on

plot(r4_01,p5_1,'k--','LineWidth',2);


%axis([0 20 -1 1])

%grid on

legend('q=2 station','q=3 station','q=4 station','q=5 station');%, 'Non-optimal
Solution','Equation (35)');


xlabel('r','fontsize',12)

ylabel('production rate','fontsize',12)


% figure (2)

% plot(r4_01,p4_1,'k--','LineWidth',2);
```

```
% hold on

% plot(r4_01,p5_1,'k-.','LineWidth',2);


%axis([0 20 -1 1])

%grid on

%legend('q=4 station','q=5 station');%, 'Non-optimal Solution','Equation (35)');
```

APPENDIX 9

MATLAB CODE FOR ERROR PLOT FOR THE PRODUCTION RATE FUNCTION (THREE-STATION)

```
clc

clear all


st3 = xlsread('P3.xls');

err=st3(:,7);

s=1:length(err);

plot(s,err,'k--','LineWidth',2);


axis([0 200 -0.01 0.01])

%grid on

%legend('');%,


xlabel('Samples ','fontsize',12)

ylabel('Production Rate Error','fontsize',12)
```

APPENDIX 10

MATLAB CODE FOR ERROR PLOT FOR THE PRODUCTION RATE FUNCTION (FOUR-STATION)

```
clc

clear all



st4 = xlsread('P4.xls');

err=st4(:,7);

s=1:length(err);

plot(s,err,'k--','LineWidth',2);

axis([0 100 -0.01 0.01])

%grid on

%legend('');%,



xlabel('Samples ','fontsize',12)

ylabel('Production Rate Error','fontsize',12)
```

APPENDIX 11

MATLAB CODE FOR GENERATION OF DATA SAMPLES

```
clc

clear all


st3 = xlsread('P3.xls');

q3=st3(:,1);

%st4 = xlsread('P4.xlsx');

%q4=st4(:,1);

%st5 = xlsread('P5.xls');

%q5=st5(:,1);

s=1:100;


plot(s,q3(1:100),'k-','LineWidth',2);

% hold on

% plot(s,q4,'k-.','LineWidth',2);

% hold on

% plot(s,q5,'k--','LineWidth',2);
```

```
plot(s,q3(1:100),'k-','LineWidth',2);

% hold on

% plot(s,q4,'k-.','LineWidth',2);

% hold on

% plot(s,q5,'k--','LineWidth',2);

xlabel('Samples ','fontsize',12)

ylabel('q','fontsize',12)


st3 = xlsread('P3.xlsx');

r3=st3(:,2);

st4 = xlsread('P4.xlsx');

r4=st4(:,2);

st5 = xlsread('P5.xlsx');

r5=st5(:,2);

s=1:100;

plot(s,r3(1:100),'k-','LineWidth',2);

% hold on

% plot(s,r4,'k-.','LineWidth',2);

% hold on

% plot(s,r5,'k--','LineWidth',2);
```

```
xlabel('Samples ','fontsize',12)

ylabel('r','fontsize',12)
```

APPENDIX 12

MATLAB CODE FOR ERROR PLOT FOR THE PRODUCTION RATE ACCORDING TO A QUADRATIC APPROXIMATION FORMULA

```
clc

clear all


st3 = xlsread('p3 Markov.xls');

err3=st3(:,7);


st4 = xlsread('p4 Markov.xlsx');

err4=st4(:,7);


st5 = xlsread('p5 Markov.xlsx');

err5=st5(:,7);

s=1:100;


plot(s,err3(1:100),'k-','LineWidth',2);

hold on

plot(s,err4,'k-.','LineWidth',2);

hold on

plot(s,err5,'k--','LineWidth',2);
```

```
%axis([0 200 -0.1 0.1])

%grid on

legend('3','4','5');%,


xlabel('Samples ','fontsize',12)

ylabel('Error','fontsize',12)
```

APPENDIX 13

MATLAB CODE FOR ERROR PLOT FOR THE PRODUCTION RATE ACCORDING TO A LINEAR APPROXIMATION FORMULA

```
clc
clear all

%st3 = xlsread('P3.xls');
%err3=st3(:,8);
 err3= importdata('P3.txt');

st4 = xlsread('P4.xlsx');
err4=st4(:,7);

st5 = xlsread('P5.xlsx');
err5=st5(:,7);
s=1:100;

plot(s,err3(1:100),'k-','LineWidth',2);
hold on
plot(s,err4,'k-.','LineWidth',2);
hold on
plot(s,err5,'k--','LineWidth',2);

%axis([0 200 -0.1 0.1])
%grid on
legend('3','4','5');%,

xlabel('Samples ','fontsize',12)
ylabel('Error','fontsize',12)
```