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Synthetic Aperture Radar Image Formation Using Time Domain Algorithm

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Zaman Alanı Algoritmasını Kullanarak Sentetik Açıklıklı Radar Görüntüsü Oluşturma

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Abstract— The aim of this paper is to achieve a basic knowledge about the Synthetic Aperture Radar (SAR) and its image formation using a time domain algorithm. Firstly, the theoretical information about SAR is given as its principles and mathematical model. After that, the backprojection algorithm as one of time domain algorithms is applied on the raw SAR data in the simulation part. In this practical side, there are initially three targets in a scene specified. Moreover, the spotlight mode SAR method is preferred in order to sense this objects. The collecting data is recorded, thus, the image formation is started to obtain the image of the targets. For this purpose, the backprojection algorithm is fulfilled in order for range compression, upsampling and backprojection. The results are analysed by comparison with the mathematical data. As a result, the resolution, which is found in the range, is 3 meter, on the other hand, the first sidelobe in the spectrum is 13 dB lower than the main peak as it is expected.

Index Terms— SAR image formation, time domain algorithm, reconstruction, backprojection, synthetic aperture radar

I. INTRODUCTION

RADAR is a highly treasurable invention in communication systems in terms of the military purposes during World War II [1]. Originally, radar as an acronym comes from radio detection and ranging which explains what the purpose is. This is detecting by using radio signals and processing of achieved, therefore information can be achieved about targets [2]. Although the military purposes are the source of inspiration of the radar systems, it has been used in a wide range of areas such as air traffic control, weather forecasting, radio astronomy, navigation etc.[3, 4]. In addition, there are many types of radar which are used for different purposes or by different techniques, for instance, monostatic radars, bistatic radars, pulsed radars, continuous-wave (CW) radars, secondary surveillance radars, synthetic aperture radars (SAR) etc.[5].

This paper initially has based on a basic understanding of one of those radars which is SAR. This understanding covers the general meaning of the SAR, it's mathematical aspects and signal-image processing aspects. Then, it is also aimed to perform SAR signal that is mentioned in the previous section by using MATLAB. As specifically for this study, the backprojection algorithm has been introduced and applied to gain a fine resolution SAR image.

Section 2 has given a brief information about SAR. In its subsections, firstly, it has been examined how SAR works as a system. Then, Linear Frequency Modulation (LFM) signal has been introduced. Finally, the mathematical model of the SAR has been presented in detail in Section 2.c.

As an image processing step, the backprojection algorithm has been detailed in Section 3 and its subsections. After that, the algorithm has been simulated in Matlab. Then, the results of this simulation have been discussed.

II. SYNTHETIC APERTURE RADAR

In early of 1950s SAR was invented, which was a technological advance for imaging radar systems. SAR was offered increasing the image resolution.[6, 7] It replaced using a long aperture real radar because it enabled to create synthetic aperture using a small size real radar. Therefore, it is called as synthetic aperture radar [8].

In the SAR principle, there are a moving platform, on which the radar is mounted, and a stationary target. However, inverse version of this is called as Inverse Synthetic Aperture Radar (ISAR). The platform can be airborne or spaceborne which is called satellite[9]. The feature that makes SAR more advantageous is that it operates at day/night and in all weather conditions[10].

There are some types of SAR such as spotlight mode, stripmap mode and scanSAR mode. Their differences result from sensing and observation method. For example, while the radar illustrates a specific target in each aperture position in the spotlight mode, however, the stripmap mode SAR is performed for different parts along the radar track [11-13]. The mean of aperture positions corresponds to signal transmitting and receiving position where the radar exists. For some cases, the start-stop approximation can be used to make the scenario simple [14]. Fig 1 shows the basic SAR model for spotlight SAR on an aircraft. In some case, there are studies on hybrid spotlight/stripmap mode SAR [15-17]

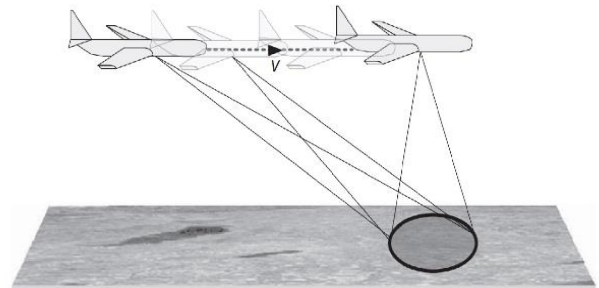


Fig. 1. Basic SAR model for spotlight SAR [8]

Following subsections have explained how SAR works, what Linear Frequency Modulation (LFM) signal is and how the mathematical model of SAR can be achieved.

A. How SAR Works

SAR uses electromagnetic waves as a signal. At each aperture position, a signal is sent to a target, then some of the signals is reflected back into radar, others are scattered and lost. The receiving signals are recorded with time and location information, which are used in processing sections. End of the data collection from each aperture position, all data is stored and raw SAR is achieved.

B. LFM (Chirp) Signal

In SAR applications, different signal types can be performed such as Continuous Wave, Linear Frequency modulated CW, short pulse, and LFM signals. LFM signals are the preferable signal types in SAR applications. The cause of this preference is to create a short pulse which is put the more energy into. It is known that the bandwidth is reciprocal to pulse time duration which is $B=1/T$. Therefore, it is used to obtain a high energy from the signal, in other words, to achieve high resolution for image [8].

The signal is used in two types: pulsed and CW. However, the pulsed LFM signal is commonly operated in SAR systems. This signal's principle is that transmitting signal in a quick way, then waiting for receiving signal and this process is repeated. The time duration between each transmit signal is called as pulse repetition interval (PRI). On the other hand, the amount of the transmitted pulse in a second is known as pulse repetition frequency (PRF) which is $1/PRI$ [18].

Essentially, LFM signal has instant frequency related to a linear function of time, to put it in a different way the frequency changes in each different time [1]. The chirp signal frequency is shown in Equation 1 below.

$$f(t) = f_0 + Kt \quad (1)$$

In the equation, K is referred to frequency change rate. If K is positive, the signal is called upchirp signal, on the other hand, it becomes downchirp signal.

Chirp signal has a constant amplitude. Fig 2 illustrates a chirp signal that is firstly upchirp and then downchirp.

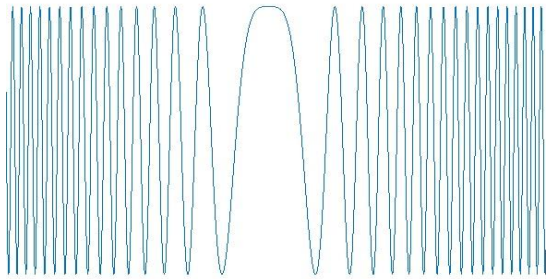


Fig. 2. An example for a chirp signal that has plotted on Matlab.

On the other hand, the spectrum of the chirp signal is illustrated in Figure 3.

When the center or starting frequency, f_0 and the initial phase is set to zero, the chirp pulse signal can be shown as

$$s(t) = \text{rect}\left(\frac{t}{T}\right) \cdot \exp(j\pi Kt^2) \quad (2)$$

where T is the time duration of the signal. The code which produces the Figure 2 and 3 is in Appendix 1.

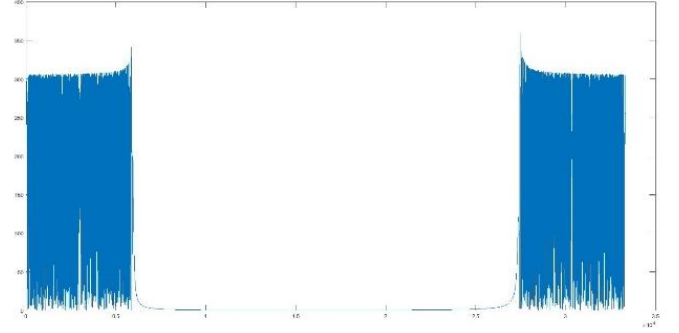


Fig. 3. An example of a spectrum of the chirp signal that has plotted on Matlab.

C. Mathematical Model of SAR

In radar principles, a signal is transmitted and desired information is achieved by processing of the received signal or reflected signal back to the radar, which created the interaction between transmitted signal and target. The transmitted signal, which is shown in Equation 3, is a modulated signal and it contains two features as amplitude and phase.

$$P_t(t) = P(t) \cdot \exp(j2\pi f_c t) \quad (3)$$

where P_t , $P(t)$ and exponential term denote transmitted signal, the amplitude of the signal and the phase of the signal respectively.

Then, the echoed signal has different amplitude and phase and the effect of time delay are seen on the signal. The received or reflected signal is

$$P_r(t) = P(t - t_d) \cdot \exp(-j2\pi f_c (t - t_d)) \quad (4)$$

where t_d refers time delay.

After the receiving the signal, I/Q modulation is applied and then the SAR signal is achieved at the end of this pre-processing. The SAR signal is shown in Equation 5.

$$P_{sar}(t) = P(t - t_d) \cdot \exp(-j2\pi f_c t_d) \quad (5)$$

Here, the time delay is a function of distance. In other words, it can be recognized by using the value of the locations, which is the place of the radar and target. Hence, the SAR equation is converted to the Equation 6.

$$P_{sar}(t, u) = P(t - t_d(u)) \cdot \exp(-j2\pi f_c t_d(u)) \quad (6)$$

where u represents the time of the radar along the radar path, azimuth. It has values at each PRI between $-T/2$ to $T/2$.

The time delay can be calculated by using the distance, R (u), which is double of the distance between radar and target, and the velocity that is the speed of light. In Equation 7, the time delay is shown;

$$t_d(u) = \frac{2R(u)}{c} \quad (7)$$

R (u) has two components as x and y. The component of x axis identifies the distance in the range. On the other hand, the component of y axis is varied related to the position of the radar. The distance along the radar path is found by using y axis component of the location of the radar and it is $y - y_u$. Here, y is a position of the target in the cross-range, v is the velocity

of the radar and u is related to time in azimuth. Hence, $R(u)$ can be defined as in Equation 8.

$$R(u) = \sqrt{x^2 + (y - vu)^2} \quad (8)$$

Substituting Equation 8 into 6, final SAR received signal has been achieved like in Equation 9. This signal shows that it has a quadratic feature that is like LFM – chirp signal.

$$P_{sar}(t, u) = P(t - t_d(u)) \cdot \exp\left(-j2\pi f_c \left(\frac{2R(u)}{c}\right)\right) \quad (9)$$

III. BACKPROJECTION ALGORITHM

There are different algorithms to process the raw SAR data to get an image. Each algorithm may have an advantage on others regarding their specialty [19-21]. Typically, those algorithms are classified as FFT based and time domain based algorithms. Mostly, the algorithms based on FFT are popular due to its efficiency for SAR image processing; however, it is the drawback that those have limitations. On the other hand, the computational load has been found as a disadvantage for time domain algorithms, however, a recent technology allows to perform it [22].

Time domain backprojection algorithm is based on reconstruction as related to projection. It forms the data by using the information of the existing data and target. It mainly consists of three steps which are pulse compression, upsampling (or interpolation) and backprojection [23]. The block diagram in Figure 3 is summarized the backprojection algorithm.

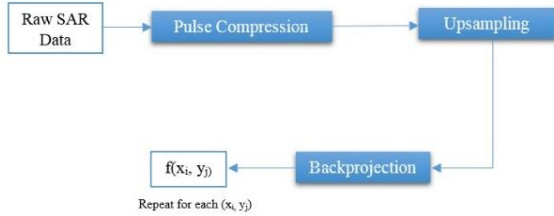


Fig. 3. The backprojection algorithm block diagram

In the following subtitles, the steps of the backprojection algorithm has been examined in detail.

A. Pulse Compression – Matched Filtering

This part is the point that the SAR processing starts. The SAR signal has information about transmitted signal and target. Here, matched filtering provides to separate the information about the target from the echoed signal by using deconvolution process [24]. Finally, the range compressed waveform is achieved. Matched filter signal is equal to the complex conjugate of $P_t(t)$. Following model in Figure 4 and equations show what the matched filter model is and how to utilize the matched filtering and what is range compressed SAR signal is respectively.

$$P_{sar\ out}(t, u) = P_{sar\ in}(t, u) * h(t) \quad (10)$$

$$P_{sar\ out}(t, u) = P_{sar\ in}(t, u) * P_t^*(-t) \quad (11)$$

$$P_{sar\ out}(t, u) = F^{-1}\{F\{P_{sar\ in}(t, u)\} \cdot F\{P_t^*(-t)\}\} \quad (12)$$

Matched filtering can be performed via two approaches; fast-time, t and slow time, u . Three equations above based on fast time matched filtering. [23].

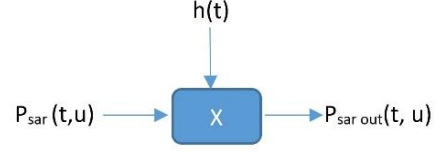


Fig. 4 The matched filter model

B. Upsampling

Upsampling is a transition phase in the backprojection algorithm. It apparently increases the amount of the current sample by zero-padding into the pulse compressed SAR data[23]. It provides to recover all the range profile. Hence, the point, which corresponds to the nearest neighbor of the range, can be found easily. Upsampling is performed for range dimension by obeying Nyquist rate, therefore it does not cause high computational load [25]. End of the upsampling stage, the data is achieved to use the backprojection section.

C. Backprojection

In this stage, the contribution of each echo is performed on the desired location. For this purpose, two main things are needed; SAR motion history and the pulse compressed data which is discussed in the Section 3.a. There are two approaches for backprojection algorithm: pulse by pulse and pixel by pixel [22]. This paper has focused on a pixel by pixel approaches. The scene is divided into grids and each grid corresponds to a pixel. The scene can be called by multiplexing a number of range samples and azimuth samples, for instance, N by M . Figure 4 is referred to an example of the aperture and the N by M pixel scene.

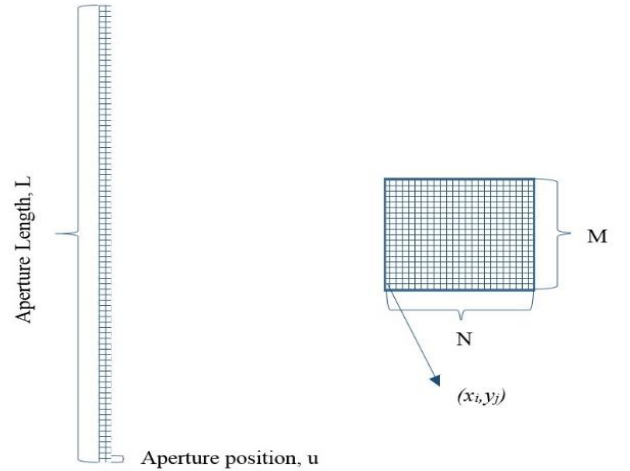


Fig. 5. The N by M pixel scene and aperture length

To find each pixel's value, integration is performed over the aperture for the pixel, therefore, all the values of the pixels are specified and an image is achieved [25]. The effect of phase on the pixel is applied by multiplexing the data, which is found in the nearest neighbor of the pixel with respect to range.

$$f(x_i, y_j) = \int P_{sar} out(t, u) \cdot \exp\left(j2\pi f_c \left(\frac{2R(u)}{c}\right)\right) du \quad (13)$$

IV. SIMULATIONS

In this study, the backprojection algorithm that has mentioned in Section 3 has simulated in Matlab. The simulation has been based on the raw data that is in Appendix 2.a.

The scenario is that an aircraft on the constant height is moving along a path, in the meantime, it is collecting data from the desired locations by using SAR. A scene and have been selected and the scene is in Figure 5 as a sketch. It is seen that the range is in between 5000 and 5050 meters, on the other hand, the cross-range thresholds are -30 and 30 meters.

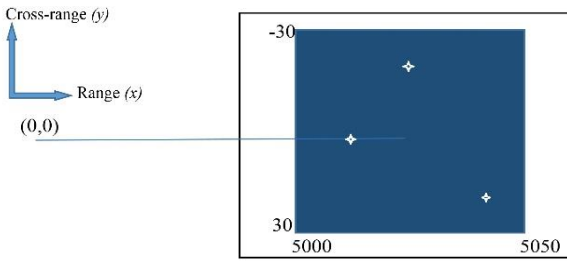


Fig. 6. The appearance of the scene

The locations of three targets in the scene have been specified as two dimensions, x and y. Table 1 shows the target locations in detail.

TABLE 1
THE CO-ORDINATES OF TARGETS

Target	x(m)	y(m)
1	5010	0
2	5025	-20
3	5040	25

Next, I/Q demodulation has been performed, then, the raw SAR data and phase history have stored. By this step, data acquisition process and signal processing have been completed.

After that, the backprojection algorithm as image processing has applied following the pulse compression, upsampling and backprojection. At the end of the process, the image about the targets has been achieved.

The Matlab code, which is related to this simulation, is in the Appendix 2.b

In addition, other constant values of the parameters such as carrier frequency, the aircraft velocity, the speed of light etc. is shown in Table 2.

V. RESULTS

The simulation is based on the raw data which is in Appendix 2.b. In this data, there are n samples for range and m samples for azimuth.

In the beginning of the simulation, backprojection algorithm stages have been taken into account. These are range compression, upsampling and backprojection. Firstly, matched filtering has been applied to the raw data, it has provided to maximize the SNR. Due to fact that noiseless channel is assumed, a quite clear range compressed data has been achieved in Figure 6. The figure illustrates that the data is compressed in range and it has reflectivity along the cross-range for three different objects that are referred to the targets specified.

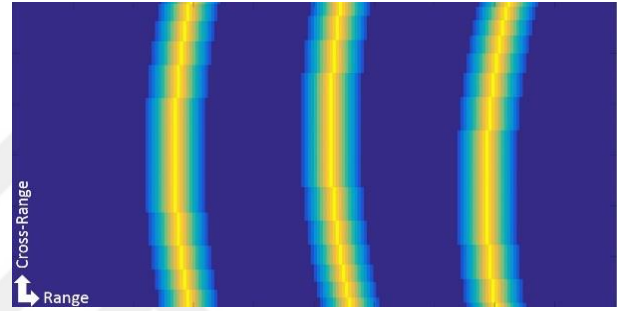


Fig. 6. The image after the range compression

In addition, the data in the spectrum is shown like in Figure 7. It shows that it is similarly a spectrum of the chirp signal. As mentioned in Section 2.b, the chirp signal helps to carry more energy by using short pulses.

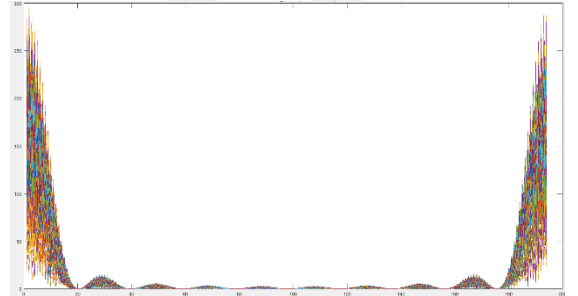


Fig. 7. The spectrum of the range compressed data

Next, the upsampling has processed on the range compressed data. In this stage, zeros as four times the number of sample in the range are added in the middle of the data in the range. Therefore, the amount of the samples has become $5 \times n$. It has enabled to have more accurate values to use at the backprojection section. In Figure 7, there is a comparison between the image of the range compressed data and the upsampling data. It is obvious that the image of range compressed data has a hard-edge energy level, however, the upsampling data has more smooth layers than it. The values between the layers for the upsampling data can be determined, thereby, the nearest neighbor for pixels can find more accurately.

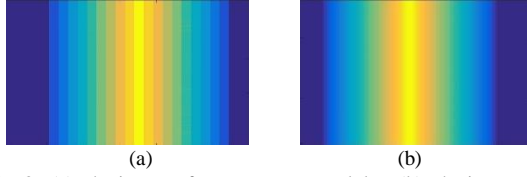


Fig. 8. (a) The image of range compressed data (b) The image of upsampling data.

The effect of upsampling on the data can be clearly seen in Figure 9 when comparing with the spectrum of the range compressed data in Figure 6. The value of frequencies of the data has become more explicit by means of adding zeros. Hence, it enables to see image smoothly.

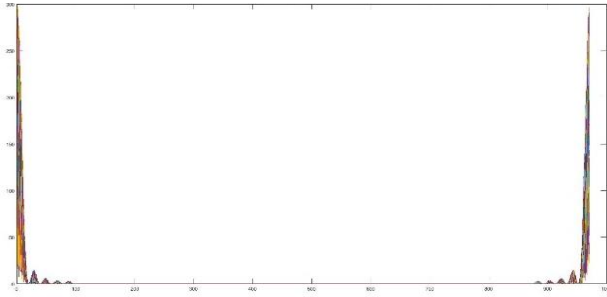


Fig. 9. The spectrum of the upsampling data

Finally, the backprojection has performed and the backprojection algorithm has accomplished. In a result, an image has been viewed. In this image, exact three targets have been seen. It is illustrated in Figure 10.

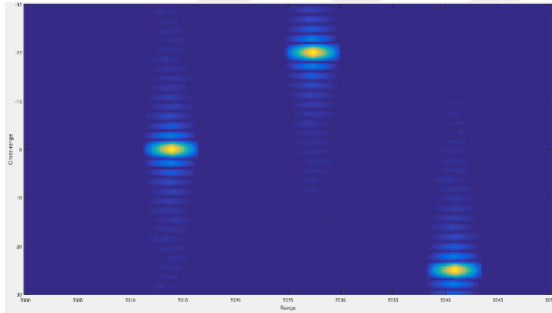


Fig. 10. The image at the end of the processing

Backprojection has provided a compression in cross-range, in other words, the contribution of each aperture position for each (x_i, y_j) has been summed. Therefore, the specific locations have been acquired in each coordinate. The locations of these three targets appear in Table 3.

TABLE 3
THE CO-ORDINATES OF TARGETS AFTER THE BACKPROJECTION ALGORITHM

Target	x(m)	y(m)
1	5014	0
2	5027	-20
3	5041	25

In addition, some values in Table 4 have been calculated and used in the simulation.

TABLE 4
THE PARAMETERS CALCULATED IN THE SIMULATION

Symbol	Parameter	Value
L	Aperture Length	240.65 (m)
Rmin	Minimum Range	5000 (m)
Rmax	Maximum Range	5052 (m)
n	Sample in Range	194
m	Sample in Cross-range	231
PRF	Pulse Repetition Frequency	96.1671 (Hz)

VI. DISCUSSION

The results show that the locations of the targets in y-axis have been accomplished accurately, however, the values in the x-axis are slightly different from the real values. These differences are 0.08%, 0.04%, and 0.02% respectively in terms of looking from the aperture positions.

Furthermore, the simulation has been repeated for one target to see the efficiency matching filtering in cross-range and the resolution in range. Firstly, it is clearly seen that the spectrum in Figure 11 shows the sinc function. Apparently, the transmitted signal is a rectangular signal which has been transmitted in the simulation. On the other hand, the main peak is approximately 13 dB above the first sidelobe and this is an expected value.

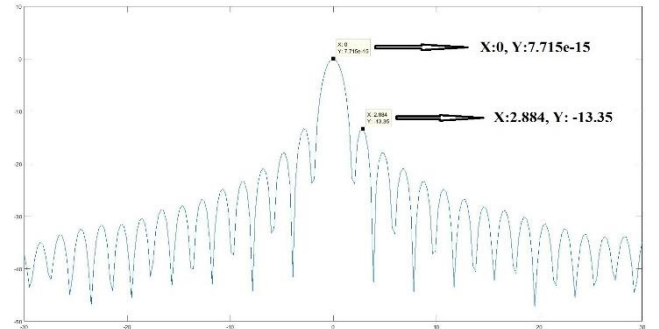


Fig. 11. The sinc function distribution of the image in azimuth

In other respects, the range resolution specified in Table 2 has been confirmed with Figure 12.

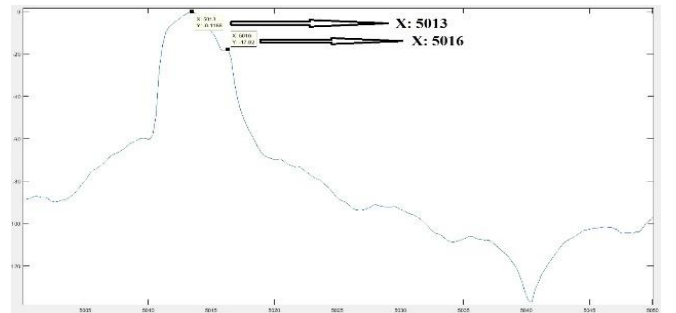


Fig. 12. The resolution of the image in range

Due to changing to one target, a small shift has been realized in the range, nevertheless, the resolution has been seen as 3 m in the range.

VII. CONCLUSION

In this study, as a starting point, the synthetic aperture radar has been taken into account from its basics to an achievement of an image from a target using performing signal and image processing. After mentioning about radar and SAR systems, the details of SAR have been examined such as its working principles and mathematical model. In addition, LFM signal has been introduced and supported by the Matlab simulation. Then, the backprojection algorithm as a time domain algorithm has been stated in detail. In the simulation part, three targets have been designated and the image formation has been fulfilled using the backprojection algorithm. In Section 5, the results have been released and it has suggested that the image is successfully achieved about three targets. Finally, the results have been compared with the initial values, thus, it has been seen that these values coincide with each other.

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APPENDICES

- Appendix 1

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               LFM CHIRP Signal                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fc=9*10^9;           %The carrier frequency
c=3*10^8;           %The speed of light
Rx=5000;           %The location of the radar in x dimension
Ry=0;              %The location of the radar in y dimension
L=100;            %Synthetic aperture length
v=30;             %The velocity of the radar
PRI=1e-3;         %Pulse repetetion interval

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
***PARAMETERS***
T=L/v;
%Time duration of the radar along the synthetic aperture
Nmax=round(T/PRI);
%The total amount of the sample in the a synthetic aperture
N=(1:Nmax+1);
u=[(-T/2)+(N-1)*PRI];
%The location time when the radar is located there, slow time
R=sqrt((Rx^2)+((Ry-v.*u).^2));
%The distance between radar and target,slant Range
td=2.*R./c           %Time Delay

%%% FIGURE 2 %%%
figure
y=exp((1i*2*pi*fc).*(td)); %The phase of the Received Signal
plot((real((y))))

%%% FIGURE 3 %%%
figure
y=exp((1i*2*pi*fc).*(td)); %The phase of the Received Signal
plot(abs(real(fft(y))))
```

- Appendix 2.a

(The raw data provided by supervisor)

```
clear
clc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               CONSTANTS                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cj = sqrt(-1);
c = 3e8;           % Speed of light
fc = 1.6e9;       % Carrier frequency
lambda = c/fc;    % Wavelength at carrier
B = 50e6;         % Signal's bandwidth (Hz)
fmin = fc-B/2;    % Minimum frequency
fmax = fc+B/2;    % Maximum frequency
```

```

lambda_max = c/fmin;           % Maximum wavelength (NOT wavelength at
maximum frequency)
lambda_min = c/fmax;           % Minimum wavelength
Tp = 1/B;                       % Signal's time duration (sec)
DR = c/(2*B);                   % Range resolution (m)
DY = 2;                          % Cross-range resolution (m)
V = 100;                         % Aircraft velocity (m/sec)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               %
%          SCENE LIMITS          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Xmin = 5000;                     % Minimum ground range of observation
Xmax = 5050;                     % Maximum ground range of observation
Xc = 5025;                       % Range scene centre
Ymin = -30;                      % Minimum cross-range
Ymax = 30;                       % Maximum cross-range
Yc = 0;                          % Cross-range scene centre
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          TARGET CO-ORDINATES  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ntarget = 1;
xn = zeros(1,ntarget);          % Ground range coordinate (m)
yn = xn;                        % Cross-range coordinate (m)
fn = xn;                        % Reflectivity
% xn          % un          % fn
xn(1) = 5010;   yn(1) = 0;     fn(1) = 1;
xn(2) = 5025;   yn(2) = -20;   fn(2) = 1;
xn(3) = 5040;   yn(3) = 25;    fn(3) = 1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          SLOW-TIME PARAMETERS  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L = 240.6581; % Total aperture length required to achieve resolution DY (m)
Tobs = 2.4066; % Observation time
fumax = 32.0557;
fumin = -32.0557;
Bfu = 64.1114; % Doppler Bandwidth
prf=96.1671; % PRF (azimuth sampling frequency)
du =0.0104; % Azimuth sample spacing
u = (-1.2033:0.0104:1.1929); % Azimuth-time array
m = 231; % The amount of sampling in azimuth
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          FAST-TIME PARAMETERS  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Rmin = 5000; % Minimum range
Rmax = 5052; % Maximum range
or=10; % Oversampling ratio
dt = 2.0000e-09; % Fast-time sample spacing
Ts = 3.3313e-05; % Start time of observation
t = [3.3313e-05:2.0000e-09:3.36993e-05]; % Fast-time array
n = 194; % The amount of sampling in range
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          SIMULATION          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tx = [ones(1,or) zeros(1,n-or)]; % Transmitted signal (rectangle pulse in
this case)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          ECHOED SIGNAL          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

s = zeros(n,m); % Initialise echo data
for j = 1:m
    for i = 1:ntarget

```

```

temp = zeros(1,n);
td = 2*sqrt((xn(i)^2)+(yn(i)-V*u(j))^2)/c;
% Target time delay at position u(j)
sft = floor((td-Ts)/dt);
temp(sft+1:sft+or) = fn(i)*ones(1,or)*exp(-cj*2*pi*fc*td);
% Target echo at u(j)

s(:,j) = s(:,j)+temp.';
% Echo signal (after quadrature)
clear temp

end;
end;

```

- Appendix 2.b

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           RANGE COMPRESSION           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ftx=fft(tx); %The Fourier of the transmit signal
SF=fft(s,[ ],1); %The Fourier of the echoed signal
hl=zeros(n,m);

for j=1:m
    hl(:,j)=SF(:,j).*conj(ftx'); %Matched Filtering
end

i_hl=ifft(hl);
%The inverse fourier of the range compressed data

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           UPSAMPLING in RANGE           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

kl=floor(n/2);
%Finding the point correspond to half of the samples
hl_up=[hl(1:kl,1:m);zeros(4*n,m);hl((kl+1):n,1:m)]; %Upsampling
i_hl_up=ifft(hl_up);
%The inverse fourier transform of the upsampling data

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           BACKPROJECTION           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

xx=linspace(Xmin,Xmax,n); %Sampling between Xmin and Xmax
yy=linspace(Ymin,Ymax,m); %Sampling between Ymin and Ymax

bak=linspace(Rmin,Rmax,5*n); %Sampling between Rmin and Rmax

back=zeros(n,m);

for j=1:m
    for i=1:n
        for k=1:m

```

```

        R=(sqrt (xx (i) ^2+(yy (j) -V*u (k)) ^2));           %Current range value
        def=(exp ((cj*4*pi*fc*R)/c));                       %Current phase
        [~, idx] = min(abs(bak-(R)));                       %Finding nearest
neighbor in the range domain
        temp1=(i_hl_up(idx,k).*def);                       %Remove the phase effect
on the pixel
        back(i,j) = back(i,j)+temp1';                     %The value of the pixel
    end
end
end

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Figures                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

figure 6
imagesc(xx,yy,abs(iffthl));
title('The Display of the Data After Range Compression')
ylabel('Cross-range')
xlabel('Range')

```

```

figure 7
plot(abs(hl));
title('The Data After Range Compression in Spectrum')

```

```

figure 8
imagesc(xx,yy,abs(iffthl_up));
title('The Display of the Data After Upsampling')

```

```

figure 9
plot(abs(hl_up));
title('The Data Performed Upsampling in Spectrum')

```

```

figure 10
imagesc(xx,yy,abs(back));
title('The Display of the Data After Backprojection')
ylabel('Cross-range')
xlabel('Range')

```

```

figure 11
[ka,b]=(max(20*log10(abs(back)./max(max(abs(back))))));
plot(yy,20*log10(abs(back(b,:))/max(abs(back(b,:)))));

```

```

figure 12
[a,c]=(max(20*log10(abs(back)/max(max(abs(back))))));
plot(xx,20*log10(abs(back(:,c))/max(abs(back(:,c)))));

```