MEAN HOMOGENEITY TESTS UNDER THE VIOLATION OF THE ASSUMPTIONS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

MEAN HOMOGENEITY TESTS UNDER THE VIOLATION OF THE ASSUMPTIONS

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In social, educational and health sciences, obtained data are generally non-normal and heterogeneous although many methods expect one or more assumptions to be satisfied. However, there is still a need for an extensive Monte Carlo comparison and a new way for testing the means for populations that have both non-normality and heterogeneity. So, the first aim of the current study is to compare several means for both under non-normality and variance heterogeneity. The second aim is to compare several means that are normally distributed with heterogeneity of variance. The B = 1000 bootstrap tests are performed for α = .05 from the selected distributions with two different plans in the current study. To estimate the type 1 error rates and powers, M = 1000 Monte Carlo samples are used. Nominal significance level is α = .05. The results show that, for non-normal distributions, whereas ANOVA, Welch F test, and Brown-Forsythe test with plan 1 bootstrap are almost comparable for two groups, Welch F test with bootstrap shows more or less powerful properties in some scenarios for three and six groups. On the other hand, Welch F test is more powerful rather than ANOVA with plan 1 bootstrap and Brown-Forsythe test for normal distribution. However, type 1 error rates of tests with plan 2 do not maintain the nominal significance level.

Keywords: ANOVA, Welch, Brown-Forsythe, Bootstrap

ÖZ

VARSAYIMLARIN İHLALİ DURUMUNDA ORTALAMA HOMOJENLİK TESTLERİ

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Sosyal, eğitim ve sağlık bilimlerinde, elde edilen veriler genellikle normal ve heterojen değildir, ancak birçok yöntem bir veya daha fazla varsayımın yerine getirilmesini bekler. Bununla birlikte, kapsamlı bir Monte Carlo karşılaştırmasına ve hem normal olmayan hem de heterojenliğe sahip olan popülasyonlar için ortalama karşılaştırmalarını test etmenin yeni bir yoluna hala ihtiyaç vardır. Dolayısıyla, bu çalışmanın ilk amacı, hem normal olmayan hem de varyans heterojenliği altındaki popülasyon ortalamalarını karşılaştırmaktır. İkinci amaç, varyans heterojenliği ile normal dağılan çeşitli popülasyon ortalamalarını karşılaştırmaktır. Bu çalışmada seçilen dağılımlardan B = 1000 bootstrap testleri α = 0,05 için iki farklı planla yapılmıştır. Tip 1 hata oranlarını ve güçlerini tahmin etmek için M = 1000 Monte Carlo örnekleri kullanılmıştır. Nominal anlamlılık düzeyi α = 0,05'tir. Sonuçlar, normal olmayan dağılımlar için, ANOVA, Welch F testi ve Brown-Forsythe'ın bootstrap testinin birinci plan iki grupla neredeyse karşılaştırılabilir olduğunu gösterirken, bootstraplı Welch F testinin üç ve altı grup için bazı senaryolarda az ya da çok güçlü özellikler gösterdiğini ortaya koymuştur. Diğer yandan, sonuçlar normal dağılımda, Welch F testinin ANOVA’nın plan 1 ile uygulanan bootstrap testinden ve Brown-Forsythe testinden daha güçlü olduğunu göstermektedir. Öte yandan, plan 2 ile yapılan testlerin tip 1 hata oranları nominal önem seviyesini koruyamamıştır.

Anahtar Kelimeler: ANOVA, Welch, Brown-Forsythe, Bootstrap

To my beloved husband and my lovely parents and sister…

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CHAPTER 1

INTRODUCTION

1.1. Motivation of the Study

In social, educational and health sciences, data are generally non-normal (e.g. Bono, Blanca, Arnau, & Gomez-Benito, 2017; Micceri, 1989; Olsen, 2003) and heterogeneous in terms of the variance (e.g. Grissom, 2000; Erceg-Hurn & Mirosevich, 2008). Our focus is comparing the means of different groups when data are non-normal and group variances are unequal. When comparing means across groups, Analysis of Variance (ANOVA) is a widely used procedure in many disciplines (Erceg-Hurn & Mirosevich, 2008; Kim & Cribbie, 2018). Besides, Welch's F test and Brown-Forsythe test are the most highly suggested options to ANOVA (Tomarken & Serlin, 1986). These parametric methods require one or more assumptions to be satisfied. However, in general, the features of real data obtained from social, educational and health sciences are not matched with the expectations of these assumptions. As a result of this, there is a risky situation that inaccurate findings may be reached and spread to literature.

Several studies show that data with normal distribution seldom occur in social, educational and health sciences. It is indicated that typical data in social and behavioral sciences are often heavy-tailed (Yuan, Bentler, & Chan, 2004). Blanca et al. (2013) examine 693 distributional shapes of real-life data involving evaluations of cognitive ability and other psychological variables such as personality, anxiety, and depression. The results indicate that 74.4% of distributions are slightly or moderately non-normal and 20% are extremely or very extremely non-normal. Several variables covered in biomedical studies are positively skewed such as cell counts and colony forming units counts (Olsen, 2003). For instance, age at onset of a disease in human medicine studies and income in social sciences are fit with log-normal distribution (Limpert, Stahel, & Abbt, 2001). Bono et al. (2017) review 279 distributions in the social, educational and health sciences to understand which distribution is commonly seen. The findings demonstrate that gamma, log-normal, and exponential distributions are most common continuous distributions. Micceri (1989) states that many aspects, such as the existence of subpopulations, treatment effects, may have a contribution to non-normal distributions.

Heterogeneity is also common in real data (Erceg-Hurn & Mirosevich, 2008). Keppel and Wickens (2004, p. 147) argue that basically three factors lead to heterogeneity. Firstly, subpopulations are characterized using an intrinsic factor, such as gender, age, or anxiety level, have diverse variances. For example, Der and Deary (2006) demonstrate that women have more variability than men on reaction time in cognitive functioning. Also, they show that reaction time becomes more variable with age. Secondly, variance heterogeneity may be because of experimental manipulations. For instance, a treatment group getting therapy in an experimental design may be more helpful or harmful for some patients than others (Grissom, 2000). At the beginning, because of the nature of experimental design, it is expected that treatment and control groups vary similarly. However, as explained, the treatment responses may vary, so heterogeneity may be the case. Thirdly, the width of the scores in measurement may affect differences in variances.

The assumptions that underlie the statistical mean comparison methods such as ANOVA are based on normality of the distribution of the data and homogeneity of variances. Nevertheless, researchers often do not explain whether these assumptions are checked. Keselman et al. (1998) review 61 articles related to educational and behavioral science based on between-subjects univariate designs. The results indicate that 11.5% of articles consider normality but without no specific tests for violation and only 8.2% of them mention homogeneity of variance. Only 1.6% of articles have information about both assumptions. Similarly, Osborne (2008) show that only few articles report assumption checking published in high-quality educational psychology journals, such as Journal of Educational Psychology, Contemporary Educational Psychology, and the British Journal of Educational Psychology. The findings reveal that only 7.3% of 55 articles published between 1969 and 1996, and only 8.3% of 96 articles published between 1998 and 1999 included information about assumption testing.

Hoekstra, Kiers, and Johnson (2012) conduct a study to comprehend what researchers understand about the assumptions and how they inspect these assumptions. The study includes an analyzing and reporting part with a given data, and a questionnaire involving descriptions for why an assumption is/is not inspected. The results yield that testing for assumptions is problematic. Only 12% of the researchers test the normality correctly and 23% of them test homogeneity of variance correctly. Additionally, it is reported that there is a common information deficiency about the notice of assumptions, the robustness of the methods considering the assumptions, and how assumptions must be tested. Osborne and Waters (2002) state that the accuracy of the results and arguments are dubious, because of the inadequate information about the assumptions checking.

A series of essential assumptions exist in the mean comparison methods. Unless the data at hand satisfy the assumptions, the method is likely lead to serious problems in terms of hypothesis testing particularly with the calculation of p-values. Main assumptions of mean comparison across groups are independence of samples, normal distribution for populations, and homogeneity of variance. Independence assumption is common to all the mean comparison methods considered here, namely ANOVA, Welch's F test and Brown-Forsythe test. The other two assumptions are needed for ANOVA.

The assumptions of normality and homogeneity can be tested with some formal analysis, such as Shapiro-Wilks, Kolmogorov-Smirnov test and Levene’s test. However, although some researchers suggest that these tests are valid, the others argue that when there is a violation of assumptions, the validity of these tests may be risky

(Erceg-Hurn & Mirosevich, 2008). For example, it is stated that Levene's test used for testing homogeneity of variance may have liberal Type 1 error rates under the non-normal distributions (Delacre, Lakens, & Leys, 2017; Weston & Hopkins, 1998, as cited in Grissom, 2000). Additionally, the assumptions are reciprocally connected (Keppel & Wickens, 2004) which means that violation of one assumption is generally related to violation of one or more other assumptions. However, there is some contradictory information about severity of violation of assumptions. Many researchers assume that parametric tests are valid under the violation of assumption in psychology, education, and social sciences (Zimmerman & Zumbo, 1992). Nevertheless, recent perspectives accept that it is a common mistake to use parametric tests under the violence of assumptions (Choi, 2005; Moder, 2007). The violation of the assumptions may lead to increased or decreased Type I error or reduced power (Erceg-Hurn & Mirosevich, 2008; Moder, 2007; Osborne & Waters, 2002). For instance, it is shown that the results of ANOVA with normally distributed data and with heterogeneity of variance tend to have inflated Type 1 error rate (Liu, 2015; Moder, 2007; Tomarken & Serlin, 1986). Additionally, when the data distributed non-normally with homogeneity of variance, both ANOVA and Welch's F test have liberal Type 1 error rates (Liu, 2015).

If there is only the violation of equality of variance, Welch's F test and Brown-Forsythe test are suggested (Grissom, 2000). On the other hand, though Welch's F test is robust when variance heterogeneity is satisfied under normal distribution, it has inflated Type 1 error rate and reduced power under non-normal distribution (Harwell, Rubinstein, Hayes, & Olds, 1992; Lix, Keselman, & Keselman, 1996). There are some other suggestions about what can be done when faced with the violation of assumptions for parametric mean comparison methods. The first one is to transform the data to meet the assumptions. If transforming the data end up with reaching the normal distribution or with stabilizing the variance homogeneity, the parametric options may be used with these transformed data. However, it is emphasized that interpretation of the results from the analyses with transformed data can be challenging (Grissom, 2000; Judd, McClelland, & Culhane, 1995; Lix et al., 1996). Because results do not belong to the raw data, researchers should warn the readers that the findings are limited to transformed data. Still, it is difficult for the readers to understand the sense of results like "logarithms of IQ scores" (Tabachnick & Fidell, 2012). Furthermore, some studies indicate that transformation may have little effect on the Type 1 error rate and power or lowered them (Grissom, 2000).

The second suggestion when faced with the violation of assumptions is to use nonparametric methods such as Kruskal Wallis test instead of ANOVA, Welch's F test, or Brown-Forsythe test. Kruskal Wallis test is based on the ranks of the data (Liu, 2015) and does not need normality (Keselman et al., 1998). However, it is criticized that ranking the data may cause information loss (Liu, 2015). Moreover, variance homogeneity/heterogeneity may still have an impact on the results. It is stated that Kruskal Wallis test is highly sensitive to the heterogeneity of variance (Lix et al., 1996; Tomarken & Serlin, 1986; Zimmerman & Zumbo, 1992) and the violation of homogeneity effects Type 1 error and power of the test. So, performing Kruskal Wallis test is not a good option for the cases where groups under study have unequal variances. Furthermore, it should be specified that Kruskal Wallis test is performed to compare groups' medians, not groups' means. Therefore, if the aim of the analysis is to compare means, Kruskal Wallis test is not an alternative to ANOVA, Welch's F test, or Brown-Forsythe test because of the different null hypothesis.

The last one is to use modern techniques such as bootstrap (Efron & Tibshirani, 1993), permutation (Good, 1995), or randomization tests (Edgington, 1980). Bootstrap methods basically have more advantage than classical methods. Bootstrap methods give the opportunity to the researchers to generate empirical sampling distributions of their data instead of considering normality and variance homogeneity to generate hypothetical sampling distributions (Ruscio & Roche, 2012).

It is known that to make mistakes in statistical method used result in misreading of the data and incorrect consequences (Olsen, 2003). Keselman et al. (1998) suggest that

"The applied researcher who routinely adopts a traditional procedure without giving thought to its associated assumptions may unwittingly be filling the literature with non-replicable results" (p.351). Therefore, the current study has two main aims. First, comparing several means for both under non-normality and variance heterogeneity is considered. Because there is no test that is valid in the condition of both, developing a valid method under the violation of these assumptions is the first aim of the study. Second, comparing several means that are normally distributed with heterogeneity of variance is taken into account. So, investigating the performance of the existing statistical methods and comparing those with the new procedure is the second aim of the study.

1.2. Contributions of the Study

In social, educational and health sciences, the real data are usually non-normal and group variances are heterogeneous. There is no robust test for comparing the expectations of populations when both non-normality and heterogeneity are simultaneously the case. In the literature, there are several studies discussing robustness of mean comparison methods. Some of them test the performance of the existing statistical methods generally such as ANOVA, Welch's F test, Brown-Forsythe test (e.g. Delacre, Lakens, & Leys, 2017; Delacre, Leys, Mora, & Lakens, 2018; Ekiz & Gamgam, 2002; Liu, 2015; Tomarken & Serlin, 1986; Zimmerman & Zumbo, 1992). However, most of these studies conduct their analysis with the normally distributed data. Some of these studies develop a new statistical method but again with normally distributed data (e.g. Chang, Pal, Lim, & Lin, 2010; Krishnamoorthy, Lu, & Mathew, 2007; Lu, 2007; Xu, Mei, Chen, Guo, & Wang, 2014). On the other hand, there are limited studies to produce a robust method for the conditions of the violation of both normality and heterogeneity (e.g. Konietschke, Bathke, Harrar, & Pauly, 2015; Krishnamoorthy & León-Nevole, 2014; Krishnamoorthy & Mathew, 2003; Zhou, Gao, & Hui, 1997). However, the results of these studies do not have enough robustness to generalize. Therefore, we believe that there is still a need for an extensive Monte Carlo comparison and a new way for comparing the means for populations that have both non-normality and heterogeneity.

CHAPTER 2

METHODS

2.1. Statistical Methods

One-Way ANOVA (F-test), Welch's F-test (W-test), and Brown-Forsythe test (F^*-test), are considered in this study. The F- test, W- test, and F^*-test are the most common methods that are used to determine the difference between the means of at least two independent groups.

The null hypothesis and alternative hypothesis of interest are;

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_i$$

$$H_A: \text{At least two group means are different from each other}$$

where μ_i is the population mean of group *i*. The exact distributions of these test statistics and their bootstrap approximations are considered.

2.1.1. One-Way ANOVA

One-Way ANOVA test is a method that is used to compare means of several independent populations if the groups are normally distributed and have equal variances (Fisher, 1925).

Let x_{ij} be the value of j^{th}observation for the i^{th} population for $j = 1, \dots, n_i$ and $i = 1, \dots, g$ and $x_{ij} \sim iid\ N(\mu_i, \sigma_i^2)$.

Then F statistic is;

$$F = \frac{\sum_i n_i (x_{i.} - x_{..})^2/(g-1)}{\sum_i (n_i - 1)s_i^2/(N-g)}$$

where,

$N = \sum_i n_i,\ x_{i.} = \sum_i x_{ij}/n_i,\ x_{..} = \sum_i \sum_j x_{ij}/N$, and $s_i^2 = \sum_i (x_{ij} - x_{i.})^2/(n_i - 1)$

Let $F_{(1-\alpha;g-1,N-g)}$ be a critical point obtained from *F* distribution. Then the H_0 is rejected if *F* statistic is higher than the critical point and not rejected if *F* statistic is equal or lower than the critical point.

2.1.2. Welch's *F* test

Welch's *F* test is a method that is used to compare means of several independent populations under the normality where group variances are heterogeneous (Welch, 1951). The rationale behind Welch's *F* test is to down-weight the influence of heterogeneity by using a weight function defined below.

$$W = \frac{\sum_i w_i (x_{i.} - \tilde{x}_{..})^2/(g-1)}{[1 + \frac{2(g-2)}{g^2-1} \sum_i (1 - w_i/u)^2/(n_i - 1)]}$$

where,

$w_i = n_i/s_i^2$, $u = \sum_i w_i$, and $\tilde{x}_{..} = \sum_i w_i x_{i.}/u$. *W* has an approximate *F* (*g* – 1, *f*) distribution where $1/f = (3/(g^2 - 1)) \sum_i (1 - w_i/u)^2/(n_i - 1)$.

Let $F_{(1-\alpha;\, g-1,1/f)}$ be a critical point obtained from *F* distribution, $(1 - \alpha)^{th}$ percentile of $F_{(g-1,1/f)}$ distribution. Then the H_0 is rejected if *W* statistic is higher than the critical point and not rejected if *W* statistic is equal or lower than the critical point.

2.1.3. Brown-Forsythe Test

Brown-Forsythe test is another method that is used to compare means of several independent populations under the normality with heterogeneity of variances (Brown & Forsythe, 1974). The main idea of Brown-Forsythe test is to use degrees of freedom for denominator which has the equal expected value as degrees of freedom for numerator when means are equal across groups.

$$F^* = \frac{\sum_i n_i (x_{i.} - x_{..})^2}{\sum_i \left(1 - \frac{n_i}{N}\right) s_i^2}$$

F^* has an approximate F (g – 1, f) distribution, and $1/f = \sum_i {c_i}^2/(n_i - 1)$ where, $c_i = (1 - n_i/N)s_i^2/[\sum_i(1 - n_i/N)s_i^2]$. Let $F_{(1-\alpha;\, g-1,1/f)}$ be a critical point obtained from F distribution. Then the H_0 is rejected if F^* statistic is higher than the critical point and not rejected if F^* statistic is equal or lower than the critical point.

2.2. Bootstrap Methods

Bootstrap is a resampling method that is used to approximate the exact distributions of the test statistics (Efron & Tibshirani, 1993). In most cases, e.g. for complex test statistics or test statistics in which sampling variance formulations do not have closed form expressions, it is not analytically possible to obtain the exact distribution of the test statistic. For these cases bootstrap methods can be employed at least to obtain it approximately. It is based on drawing several random datasets with replacement from the sample at hand and recalculate the test statistic for each sampled dataset. The values from several resamples are used to reach the bootstrap distribution of a statistic. The knowledge about the real sampling distribution of the test statistic is yielded via this bootstrap distribution. The main advantage of bootstrap is that it relies on the empirical sampling distributions of existing data rather than assuming a specific distribution (Ruscio & Roche, 2012).

One of the common area that bootstrap is used in the literature is hypothesis testing (or significance test). Paired sample t-test (e.g. Konietschke & Pauly, 2014), one-sample t-test (e.g. Baklizi & Kibria, 2009), two-samples t-test (e.g. Baklizi & Kibria, 2009), one-way ANOVA (e.g. Krishnamoorthy et al., 2007; Zhou & Wong, 2011), two-way ANOVA (Xu, Yang, Abula, & Qin, 2013; Zhou & Wong, 2011), general MANOVA (Krishnamoorthy & Lu, 2010; Konietschke et al., 2015), two-way MANOVA (Xu, 2015), and regression (Li & Wang, 1998) are some hypothesis testing procedures for which bootstrap solutions recommended. At least B = 1000 samples, the number of bootstrap samples, are suggested for hypothesis testing (Chernick, 2008, p. 10).

For comparing the means of several groups, the following sampling plans for the Bootstrap based analysis is used:

Plan 1: For each group, generate $(X_1^*, \ldots, X_{n_1}^*)$ with samples replacement from $\{X_1 - \bar{X} + G, \ldots, X_{n_1} - \bar{X} + G\}$.

Plan 2: For each group, generate $(X_1^*, \ldots, X_{n_1}^*)$ with samples replacement from $\{(X_1 - \bar{X})/s_x, \ldots, (X_{n_1} - \bar{X})/s_x\}$.

where,

$j = 1, \ldots, n_1$, $\bar{X} = \sum X_j/n_1$, $s_x = \sqrt{\sum_j (X_j - \bar{X})^2/(n_1 - 1)}$, and G is grand mean.

The algorithm of the bootstrap is given below;

For a given *i* number of groups $(n_1, \ldots, n_i)$;

- Compute the test statistics for ANOVA, Welch's *F* test and Brown-Forsythe test for the observed data, call it F_{obs}, W_{obs}, and F_{obs}^*.
- For each bootstrap replicate, indexed *j* = 1, …, B:
    - Generate sample $X^{*(j)} = X_1^*, \ldots, X_{n_1}^*$ with samples replacement from $\{X_1 - \bar{X} + G, \ldots, X_{n_1} - \bar{X} + G\}$ for plan 1 and $\{(X_1 - \bar{X})/s_x, \ldots, (X_{n_1} - \bar{X})/s_x\}$ for plan 2.
    - Compute the test statistics.
    - Obtain the $(1 - \alpha)^{th}$ percentile of F_js, W_js, and F_j^*s and compare the F_{obs}, W_{obs}, and F_{obs}^* with those percentiles.
- Compute the *p*-value as;

$$p = \frac{\#\ of\ (F_j > F_{obs})}{B}$$

$$p = \frac{\#\ of\ (W_j > W_{obs})}{B}$$

$$p = \frac{\# \, of \; (F_j^* > F_{obs}^*)}{B}$$

CHAPTER 3

SIMULATION STUDIES

3.1. Simulation Methods

In this section, simulation experiments are designed to study the behavior of the tests of interest in balanced/unbalanced groups and under different distributions. According to Bono et al. (2017), gamma and lognormal are among the most common distributions in health, educational, and social sciences research. Therefore, current study is conducted for these non-normal distributions. Using Monte Carlo simulations, type I errors and powers are estimated. In the balanced (unbalanced) designs, group sample sizes are equal (unequal). For Type I error analysis, groups data are generated with equal means across the groups: for normal distribution, the data are generated with ($\mu_1 = \ldots = \mu_i = 0$), for gamma distribution ($\mu_1 = \ldots = \mu_i = 1$), and for lognormal distribution ($\mu_1 = \ldots = \mu_i = 1.3$). For power analysis, the groups are generated with different means. Krishnamoorthy, Lu, and Mathew (2007), Krishnamoorthy and León-Nevole (2014), and Krishnamoorthy and Mathew (2003) simulation studies are used to set the true means for different groups. Accordingly, for comparing 2, 3, and 6 groups, the true parameter vectors are given below respectively. Each parameter vector represents a different simulation scenario.

For gamma distribution,

(μ_1, μ_2) = (1, 1), (1, 1.3), (1, 1.5), (1, 1.7), (1, 2), (1, 2.3)

(μ_1, μ_2, μ_3) = (1, 1, 1), (1, 1, 1.3), (1, 1, 1.5), (1, 1, 1.7), (1, 1.3, 2), (1, 1.3, 2.3)

$(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$ = (1, 1, 1, 1, 1, 1), (1, 1, 1, 1, 1, 1.3), (1, 1, 1, 1, 1, 1.5), (1, 1, 1, 1, 1, 1.7), (1, 1, 1, 1, 1.3, 2), (1, 1, 1, 1, 1.3, 2.3)

For lognormal distribution,

(μ_1, μ_2) = (1.3, 1.3), (1.3, 1.6), (1.3, 1.9), (1.3, 2.2), (1.3, 2.5), (1.3, 2.8)

(μ_1, μ_2, μ_3) = (1.3, 1.3, 1.3), (1.3, 1.3, 1.6), (1.3, 1.3, 1.9), (1.3, 1.3, 2.2), (1.3, 1.6, 2.2), (1.3, 1.6, 2.8)

$(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$ = (1.3, 1.3, 1.3, 1.3, 1.3, 1.3), (1.3, 1.3, 1.3, 1.3, 1.3, 1.6), (1.3, 1.3, 1.3, 1.3, 1.3, 1.9), (1.3, 1.3, 1.3, 1.3, 1.3, 2.2), (1.3, 1.3, 1.3, 1.3, 1.6, 2.2), (1.3, 1.3, 1.3, 1.3, 1.6, 2.8)

For normal distribution;

(μ_1, μ_2) = (0, 0), (0, 0.1), (0, 0.2), (0, 0.5), (0, 0.7), (0, 1)

(μ_1, μ_2, μ_3) = (0, 0, 0), (0, 0, 0.2), (0, 0, 0.5), (0, 0, 0.7), (0, 0.5, 1), (0, 0, 1), (0, 1.5, 1)

$(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$ = (0, 0, 0, 0, 0, 0), (0, 0, 0, 0, 0, 0.2), (0, 0, 0, 0, 0, 0.5), (0, 0, 0, 0, 0, 0.7), (0, 0, 0, 0, 0.5, 1), (0, 0, 0, 0, 0, 1), (0, 0, 0, 0, 1.5, 1)

The variances of the distributions, which are formed based on Krishnamoorthy, Lu, and Mathew (2007) simulation study, are set as;

For i = 2,

(σ_1^2, σ_2^2) = (1, 0.01); (1, 0.05); (1, 0.10); (1, 0.20); (1, 0.30); (1, 0.40); (1, 0.50); (1, 0.60); (1, 0.70); (1, 0.80); (1, 0.90); or (1, 1)

For i = 3,

$(\sigma_1^2, \sigma_2^2, \sigma_3^2)$ = (1, 1, 1); (1, 1, 0.5); (1, 1, 0.1); (1, 0.5, 0.5); (1, 0.5, 0.7); (1, 0.1, 0.1); (1, 0.1, 0.9); (1, 0.5, 0.9); (1, 0.3, 0.9); (1, 0.3, 0.6); (1, 0.1, 0.3); or (1, 0.05, 0.05)

For i = 6,

$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2)$ = (1, 1, 1, 1, 1, 1); (1, 0.1, 0.1, 0.5, 0.5, 0.5); (1, 0.1, 0.2, 0.3, 0.4, 0.5); (1, 0.1, 1, 1, 1, 1); (1, 0.2, 0.4, 0.4, 0.2, 0.1); (1, 0.5, 0.5, 0.5, 0.5, 1); (1, 0.3, 0.9, 0.4, 0.7, 0.1); or (1, 0.01, 0.01, 0.06, 0.1, 0.1)

Shape and scale parameters of gamma distribution as well as mean and variance parameters of lognormal distributions are selected to satisfy the means and variances given above.

3.2. Comparison Methods

In the current study, different statistical methods used to test mean homogeneity are compared in terms of their Type I error rates and powers. To estimate the type 1 error rates and powers, M = 1000 Monte Carlo samples are used. Nominal significance level is α = .05.

Empirical type 1 error rate for each test is calculated as the rejection rate of the null hypothesis when the null hypothesis is true. The range of empirical type I error rates should be between 0.025 and 0.050 for the nominal significance level α = 0.05 (Bradley, 1978). Power of each test is measured as the rejection rate of the null hypothesis when the alternative hypothesis is true.

The algorithm of the analysis is;

For a given *i* number of groups $(n_1, \ldots, n_i)$;

- Generate random samples
- Compute the test statistics for ANOVA, Welch's F test and Brown-Forsythe test of the given samples, called it F_{obs}, W_{obs}, and F^*_{obs}.
- Generate random sample from the null distribution for B times.
- For j = 1, …, B, compute the test statistics.

- For each bootstrap of the data, compare the F_{obs} with the F_j, W_{obs} with the W_j, and F^*_{obs} with the F^*_j.
- Compute the p-value as;

$$p = \frac{\#\ of\ (F_j > F_{obs})}{B}$$

$$p = \frac{\#\ of\ (W_j > W_{obs})}{B}$$

$$p = \frac{\#\ of\ (F^*_j > F^*_{obs})}{B}$$

- For m = 1, …, M; if p-value is less than the alpha level, set it as $Q_m = 1$
- Compute the proportion $\frac{1}{M}\,\Sigma_{m=1}^{M} Q_m$ to obtain a Monte Carlo based estimate for Type I error.

Algorithm is similar for power estimation except data are generated following the alternative hypothesis.

3.3. The Results of Type 1 Error Rates

3.3.1. Type 1 Error Rates for Gamma Distribution

Type 1 error rates are evaluated for the gamma distributed data with a specific mean ($\mu_1 = \ldots = \mu_i = 1$) and variance ranging between 0.01 and 1 for i = 2, 3, and 6 and different sample sizes, which is equal or unequal, using Monte Carlo Simulation as explained in Section 3.1. For gamma distribution, the simulations are generated based on the different bootstrap scenarios (i.e. plan 1 and plan 2 bootstraps) for one-way ANOVA (F_{P1}-*test* & F_{P2}-*test*), Welch's F test (W_{P1}-*test* & W_{P2}-*test*), and Brown-Forsythe test (F^*_{P1}-*test* & F^*_{P2}-*test*) because of the violation of assumptions for all tests.

Table 3.1. shows the estimated type 1 error rates for i = 2 and different sample sizes. The results indicate that all plan 1 bootstrap tests have type 1 error rates that are close to the nominal level, but they have a tendency of over-rejecting when small sample

size group has high variance and large sample size group has small variance. Examination of the estimated type 1 error rates for equal and unequal sample sizes also represents that tests used plan 2 bootstrap are inclined to be liberal when the difference between variance is high.

Table 3.2. demonstrates the empirical type 1 error rates for $i = 3$ and different sample sizes. Accordingly, type 1 error rates are close to nominal level except ($n_1 = 15, n_2 = 20, n_3 = 25$) sample size with the variances ($\sigma_1^2 = 1$, $\sigma_2^2 = 0.1$, $\sigma_3^2 = 0.1$ and $\sigma_1^2 = 1$, $\sigma_2^2 = 0.1$, $\sigma_3^2 = 0.3$) for F_{P1}-test and F_{P1}^*-test. Tests with plan 2 have higher type 1 error rates than the nominal level .05 for different situations except sample size changing from large to small size for F_{P2}-test and W_{P2}-test.

Table 3.1. *Type 1 Error Rates for Gamma Distribution in i = 2*

$i = 2$, $\sigma_1^2 = 1$
$\mu_1 = \mu_2 = 1$

σ_2^2	$n_1 = n_2 = 15$					
	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.06	0.06	0.06	0.09	0.09	0.09
0.05	0.06	0.05	0.05	0.15	0.08	0.08
0.10	0.06	0.06	0.06	0.08	0.08	0.08
0.20	0.06	0.06	0.06	0.06	0.06	0.06
0.30	0.06	0.06	0.06	0.06	0.06	0.06
0.40	0.05	0.05	0.05	0.05	0.05	0.05
0.50	0.04	0.04	0.04	0.05	0.05	0.05
0.60	0.04	0.04	0.04	0.05	0.05	0.05
0.70	0.04	0.04	0.04	0.05	0.05	0.05
0.80	0.05	0.05	0.05	0.05	0.05	0.05
0.90	0.04	0.04	0.04	0.05	0.05	0.05
1.00	0.04	0.04	0.04	0.05	0.05	0.05

σ_2^2	$n_1 = 15, n_2 = 25$					
	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.08	0.08	0.08	0.17	0.10	0.10
0.05	0.07	0.06	0.06	0.15	0.08	0.08
0.10	0.08	0.07	0.07	0.14	0.09	0.09
0.20	0.06	0.05	0.05	0.11	0.06	0.06
0.30	0.05	0.05	0.05	0.08	0.06	0.06
0.40	0.06	0.06	0.06	0.08	0.06	0.06
0.50	0.06	0.06	0.06	0.08	0.06	0.06
0.60	0.05	0.05	0.05	0.06	0.05	0.05
0.70	0.05	0.05	0.05	0.06	0.05	0.05
0.80	0.04	0.04	0.04	0.05	0.04	0.04
0.90	0.05	0.05	0.05	0.06	0.06	0.06
1.00	0.05	0.05	0.05	0.06	0.06	0.06

σ_2^2	$n_1 = 25, n_2 = 15$					
	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.06	0.06	0.06	0.04	0.08	0.08
0.05	0.06	0.06	0.06	0.04	0.08	0.08
0.10	0.04	0.04	0.04	0.03	0.06	0.06
0.20	0.05	0.05	0.05	0.04	0.05	0.05
0.30	0.05	0.05	0.05	0.04	0.05	0.05
0.40	0.04	0.04	0.04	0.03	0.04	0.04
0.50	0.04	0.04	0.04	0.05	0.05	0.05
0.60	0.04	0.03	0.03	0.04	0.04	0.04
0.70	0.05	0.05	0.05	0.06	0.05	0.05
0.80	0.05	0.05	0.05	0.05	0.06	0.06
0.90	0.04	0.04	0.04	0.04	0.04	0.04
1.00	0.05	0.05	0.05	0.05	0.05	0.05

Table 3.2. *Type 1 Error Rates for Gamma Distribution in i = 3*

$i = 3$, $\sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = 1$

	$n_1 = n_2 = n_3 = 15$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.06	0.05	0.06
1, 0.5	0.04	0.04	0.04	0.06	0.05	0.06
1, 0.1	0.04	0.06	0.04	0.06	0.08	0.06
0.5, 0.5	0.04	0.04	0.04	0.05	0.05	0.05
0.5, 0.7	0.05	0.05	0.05	0.07	0.06	0.07
0.1, 0.1	0.07	0.05	0.07	0.09	0.06	0.09
0.1, 0.9	0.04	0.06	0.04	0.06	0.08	0.06
0.5, 0.9	0.04	0.04	0.04	0.05	0.05	0.05
0.3, 0.9	0.05	0.06	0.05	0.06	0.07	0.06
0.3, 0.6	0.05	0.04	0.05	0.06	0.05	0.06
0.1, 0.3	0.06	0.05	0.06	0.09	0.06	0.09
0.05, 0.05	0.07	0.05	0.07	0.11	0.07	0.11

	$n_1 = 15, n_2 = 15, n_3 = 30$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.05	0.03	0.05	0.06	0.05
1, 0.5	0.05	0.06	0.04	0.07	0.07	0.06
1, 0.1	0.04	0.07	0.04	0.13	0.10	0.06
0.5, 0.5	0.05	0.05	0.05	0.07	0.05	0.05
0.5, 0.7	0.05	0.05	0.05	0.06	0.05	0.05
0.1, 0.1	0.07	0.06	0.06	0.12	0.07	0.09
0.1, 0.9	0.04	0.06	0.04	0.04	0.06	0.06
0.5, 0.9	0.04	0.04	0.04	0.04	0.05	0.05
0.3, 0.9	0.05	0.05	0.04	0.05	0.06	0.06
0.3, 0.6	0.06	0.05	0.06	0.06	0.06	0.07
0.1, 0.3	0.06	0.04	0.06	0.08	0.06	0.07
0.05, 0.05	0.07	0.07	0.07	0.17	0.09	0.12

	$n_1 = 30, n_2 = 15, n_3 = 15$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.05	0.05	0.05
1, 0.5	0.05	0.05	0.05	0.04	0.06	0.06
1, 0.1	0.04	0.07	0.04	0.03	0.07	0.06
0.5, 0.5	0.04	0.04	0.04	0.04	0.05	0.05
0.5, 0.7	0.04	0.05	0.04	0.04	0.05	0.05
0.1, 0.1	0.06	0.06	0.06	0.04	0.05	0.08
0.1, 0.9	0.04	0.05	0.04	0.03	0.07	0.06
0.5, 0.9	0.04	0.04	0.03	0.03	0.05	0.04
0.3, 0.9	0.04	0.04	0.04	0.03	0.04	0.05
0.3, 0.6	0.04	0.05	0.05	0.04	0.06	0.06
0.1, 0.3	0.04	0.04	0.04	0.03	0.04	0.06
0.05, 0.05	0.04	0.04	0.04	0.03	0.04	0.07

Table 3.2. *Type 1 Error Rates for Gamma Distribution in i = 3 (continued)*

$i = 3, \sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = 1$

	$n_1 = 15, n_2 = 20, n_3 = 25$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.06	0.06	0.05
1, 0.5	0.04	0.06	0.04	0.06	0.06	0.06
1, 0.1	0.04	0.07	0.04	0.10	0.10	0.07
0.5, 0.5	0.05	0.05	0.06	0.07	0.06	0.06
0.5, 0.7	0.06	0.06	0.06	0.07	0.07	0.07
0.1, 0.1	0.08	0.07	0.08	0.14	0.08	0.10
0.1, 0.9	0.06	0.07	0.06	0.08	0.08	0.07
0.5, 0.9	0.04	0.05	0.04	0.06	0.06	0.06
0.3, 0.9	0.04	0.04	0.03	0.06	0.05	0.04
0.3, 0.6	0.06	0.07	0.06	0.07	0.07	0.07
0.1, 0.3	0.08	0.06	0.08	0.12	0.08	0.09
0.05, 0.05	0.07	0.04	0.06	0.16	0.06	0.11

	$n_1 = 25, n_2 = 20, n_3 = 15$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.05	0.05	0.05
1, 0.5	0.05	0.05	0.05	0.06	0.06	0.06
1, 0.1	0.04	0.05	0.04	0.03	0.07	0.05
0.5, 0.5	0.04	0.04	0.04	0.05	0.05	0.05
0.5, 0.7	0.04	0.04	0.03	0.05	0.05	0.05
0.1, 0.1	0.06	0.05	0.07	0.06	0.06	0.09
0.1, 0.9	0.05	0.07	0.05	0.07	0.09	0.07
0.5, 0.9	0.04	0.04	0.04	0.05	0.05	0.05
0.3, 0.9	0.04	0.06	0.04	0.05	0.07	0.06
0.3, 0.6	0.05	0.04	0.05	0.06	0.06	0.06
0.1, 0.3	0.06	0.06	0.06	0.06	0.07	0.07
0.05, 0.05	0.06	0.06	0.06	0.06	0.06	0.10

Table 3.3. *Type 1 Error Rates for Gamma Distribution in* i *= 6*

$i = 6, \sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 1$

	$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 15$					
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.03	0.02	0.03	0	0.001	0
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.04	0.04	0.002	0.003	0.002
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.05	0.05	0.001	0.002	0.001
0.1, 1, 1, 1, 1	0.04	0.06	0.04	0	0.013	0
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.03	0.04	0.002	0.003	0.002
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.04	0	0.001	0
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.04	0.04	0	0.002	0
0.01, 0.01, 0.06, 0.1, 0.1	0.07	0.04	0.07	0	0.002	0

	$n_1 = 15, n_2 = 20, n_3 = 25, n_4 = 30, n_5 = 35, n_6 = 40$					
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.05	0.04	0.001	0.002	0.002
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.001	0.001	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.05	0.05	0	0	0
0.1, 1, 1, 1, 1	0.04	0.07	0.04	0	0.007	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.06	0.06	0.001	0.002	0.001
0.5, 0.5, 0.5, 0.5, 1	0.05	0.05	0.05	0	0.002	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.06	0.04	0	0.001	0
0.01, 0.01, 0.06, 0.1, 0.1	0.08	0.04	0.07	0	0.001	0

	$n_1 = 40, n_2 = 35, n_3 = 30, n_4 = 25, n_5 = 20, n_6 = 15$					
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.05	0.04	0.002	0.001	0.001
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.003	0.001	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.06	0.05	0.003	0.004	0.003
0.1, 1, 1, 1, 1	0.04	0.06	0.03	0	0.004	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0.04	0.05	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.04	0.001	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.05	0.05	0	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0.05	0.06	0	0	0

	$n_1 = 15, n_2 = 15, n_3 = 15, n_4 = 30, n_5 = 30, n_6 = 30$					
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.03	0.03	0.001	0.003	0.003
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.04	0.04	0.001	0.003	0.002
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0.05	0.06	0	0.002	0
0.1, 1, 1, 1, 1	0.04	0.06	0.04	0	0.006	0
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.05	0.06	0.002	0.002	0.002
0.5, 0.5, 0.5, 0.5, 1	0.05	0.05	0.04	0	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.05	0.04	0	0.002	0
0.01, 0.01, 0.06, 0.1, 0.1	0.08	0.04	0.08	0	0	0

Table 3.3. *Type 1 Error Rates for Gamma Distribution in i = 6 (continued)*

i = 6, σ_1^2 = 1

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 1$

$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-test	W_{P1}-test	F_{P1}^*-test	F_{P2}-test	W_{P2}-test	F_{P2}^*-test
	$n_1 = 30, n_2 = 30, n_3 = 30, n_4 = 15, n_5 = 15, n_6 = 15$					
1, 1, 1, 1, 1	0.04	0.05	0.04	0	0	0.001
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.003	0.003	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.04	0.05	0.05	0.002	0.001	0.001
0.1, 1, 1, 1, 1	0.04	0.07	0.04	0.002	0.005	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.05	0.04	0.001	0.002	0.002
0.5, 0.5, 0.5, 0.5, 1	0.05	0.05	0.05	0.001	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.06	0.04	0	0.002	0
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0.05	0.07	0	0.001	0.001

$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-test	W_{P1}-test	F_{P1}^*-test	F_{P2}-test	W_{P2}-test	F_{P2}^*-test
	$n_1 = 15, n_2 = 15, n_3 = 30, n_4 = 30, n_5 = 45, n_6 = 45$					
1, 1, 1, 1, 1	0.04	0.05	0.03	0	0	0
0.1, 0.1, 0.5, 0.5, 0.5	0.06	0.05	0.05	0.001	0.002	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0.05	0.06	0	0.001	0
0.1, 1, 1, 1, 1	0.04	0.05	0.04	0.001	0.003	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.05	0.06	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.06	0.03	0	0.004	0
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.06	0.04	0	0.001	0
0.01, 0.01, 0.06, 0.1, 0.1	0.09	0.06	0.07	0	0	0

$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-test	W_{P1}-test	F_{P1}^*-test	F_{P2}-test	W_{P2}-test	F_{P2}^*-test
	$n_1 = 45, n_2 = 45, n_3 = 30, n_4 = 30, n_5 = 15, n_6 = 15$					
1, 1, 1, 1, 1	0.04	0.04	0.03	0.002	0	0
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.002	0.001	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.06	0.04	0.001	0.001	0
0.1, 1, 1, 1, 1	0.04	0.07	0.04	0.001	0.002	0
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.03	0.05	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.05	0.04	0.003	0.003	0.001
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.04	0.04	0	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0.05	0.07	0	0	0

Table 3.3. displays the empirical type 1 error rates for k = 6 and different sample sizes. For unequal sample sizes changing from small to large size, the type 1 error rates of F_{P1}-test have a tendency of inflation for the situation (σ_1^2 = 1, σ_2^2 = 0.01, σ_3^2 = 0.01, σ_4^2 = 0.06, σ_5^2 = 0.1, σ_6^2 = 0.1), though it is close to the nominal level .05 for the other scenarios. Besides, the type 1 error rates of W_{P1}-test are conservative when variances are equal, although W_{P1}-test and F_{P1}^*-test are close to the intended level .05 for the

other scenarios. Unexpectedly, the analysis of plan 2 tests reach inconclusive findings such as “0” or close to zero for simulations with gamma distributions.

3.3.2. Type 1 Error Rates for Lognormal Distribution

Type 1 error rates are evaluated for the log-normally distributed data with a specific mean ($\mu_1 = \ldots = \mu_i = 1.3$) and variance changing from 0.01 to 1 for *i* = 2, 3, and 6 and different sample sizes ranging from equal to unequal using Monte Carlo Simulation as explained in Section 3.1. For lognormal distribution, the simulations are generated based on the different bootstrap scenarios (i.e. plan 1 and plan 2 bootstraps) for F_{P1}-*test*, F_{P2}-*test*, W_{P1}-*test*, W_{P2}-*test*, F^*_{P1}-*test*, and F^*_{P2}-*test* because of the violation of assumptions for all tests similar to simulations with gamma distribution.

Table 3.4. indicates the estimated type 1 error rates for *i* = 2 and different sample sizes when the data is log-normally distributed. The findings point out that all plan 1 bootstrap tests perform sufficiently for type 1 error rates. Moreover, for equal and unequal sample sizes changing from small to large size, tests used plan 2 bootstrap have a tendency of being liberal when the difference of variances are high.

Table 3.5. demonstrated the estimated type 1 error rates for *i* = 3 and different sample sizes. The estimated type 1 error rates of plan 1 tests indicate that type 1 error rates are close to nominal level, a situation similar to the simulations with gamma distribution, except equal sample size with the variances ($\sigma_1^2 = 1$, $\sigma_2^2 = 0.05$, $\sigma_3^2 = 0.05$) and ($n_1 = 15$, $n_2 = 20$, $n_3 = 25$) sample size with the variances ($\sigma_1^2 = 1$, $\sigma_2^2 = 0.1$, $\sigma_3^2 = 0.1$) for F_{P1}-test and F^*_{P1}-test. Tests with plan 2 have higher type 1 error rates than the nominal level .05 for different situations except sample size changing from large to small size for F_{P2}-test and W_{P2}-test.

Table 3.4. *Type 1 Error Rates for Lognormal Distribution in i = 2*

$i = 2$, $\sigma_1^2 = 1$
$\mu_1 = \mu_2 = 1.3$

	$n_1 = n_2 = 15$					
σ_2^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.06	0.06	0.06	0.09	0.09	0.09
0.05	0.06	0.06	0.06	0.07	0.07	0.07
0.10	0.06	0.06	0.06	0.07	0.07	0.07
0.20	0.06	0.06	0.06	0.07	0.07	0.07
0.30	0.06	0.06	0.06	0.07	0.07	0.07
0.40	0.04	0.04	0.04	0.04	0.04	0.04
0.50	0.04	0.04	0.04	0.05	0.05	0.05
0.60	0.04	0.04	0.04	0.05	0.05	0.05
0.70	0.04	0.04	0.04	0.05	0.05	0.05
0.80	0.04	0.04	0.04	0.05	0.05	0.05
0.90	0.04	0.04	0.04	0.04	0.04	0.04
1.00	0.04	0.04	0.04	0.04	0.04	0.04

	$n_1 = 15, n_2 = 25$					
σ_2^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.06	0.06	0.06	0.15	0.08	0.08
0.05	0.07	0.07	0.07	0.15	0.08	0.08
0.10	0.06	0.06	0.06	0.12	0.07	0.07
0.20	0.07	0.06	0.06	0.09	0.07	0.07
0.30	0.06	0.05	0.05	0.07	0.06	0.06
0.40	0.06	0.05	0.05	0.07	0.06	0.06
0.50	0.05	0.05	0.05	0.05	0.05	0.05
0.60	0.05	0.05	0.05	0.06	0.06	0.06
0.70	0.05	0.05	0.05	0.05	0.05	0.05
0.80	0.04	0.04	0.04	0.04	0.05	0.05
0.90	0.04	0.03	0.03	0.04	0.04	0.04
1.00	0.04	0.04	0.04	0.04	0.04	0.04

	$n_1 = 25, n_2 = 15$					
σ_2^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
0.01	0.06	0.06	0.06	0.04	0.07	0.07
0.05	0.06	0.05	0.05	0.03	0.07	0.07
0.10	0.04	0.05	0.05	0.03	0.06	0.06
0.20	0.05	0.05	0.05	0.04	0.06	0.06
0.30	0.04	0.05	0.05	0.04	0.05	0.05
0.40	0.05	0.05	0.05	0.04	0.05	0.05
0.50	0.04	0.04	0.04	0.04	0.04	0.04
0.60	0.05	0.04	0.04	0.05	0.05	0.05
0.70	0.04	0.04	0.04	0.06	0.05	0.05
0.80	0.04	0.04	0.04	0.04	0.05	0.05
0.90	0.04	0.04	0.04	0.05	0.05	0.05
1.00	0.05	0.05	0.05	0.05	0.06	0.06

Table 3.5. *Type 1 Error Rates for Lognormal Distribution in i = 3*

$i = 3$, $\sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = 1.3$

	$n_1 = n_2 = n_3 = 15$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.03	0.03	0.03	0.04	0.05	0.04
1, 0.5	0.04	0.04	0.04	0.05	0.05	0.05
1, 0.1	0.04	0.06	0.04	0.06	0.08	0.06
0.5, 0.5	0.04	0.04	0.04	0.05	0.04	0.05
0.5, 0.7	0.04	0.04	0.04	0.06	0.06	0.06
0.1, 0.1	0.07	0.06	0.07	0.09	0.07	0.09
0.1, 0.9	0.04	0.07	0.04	0.07	0.09	0.07
0.5, 0.9	0.04	0.04	0.04	0.06	0.05	0.06
0.3, 0.9	0.04	0.05	0.04	0.06	0.05	0.06
0.3, 0.6	0.04	0.03	0.04	0.04	0.05	0.04
0.1, 0.3	0.06	0.06	0.06	0.07	0.07	0.07
0.05, 0.05	0.08	0.06	0.08	0.11	0.08	0.11

	$n_1 = 15, n_2 = 15, n_3 = 30$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.05	0.04	0.05
1, 0.5	0.05	0.06	0.05	0.07	0.07	0.06
1, 0.1	0.04	0.06	0.04	0.14	0.08	0.05
0.5, 0.5	0.05	0.04	0.04	0.06	0.05	0.05
0.5, 0.7	0.04	0.05	0.04	0.06	0.07	0.05
0.1, 0.1	0.07	0.05	0.06	0.14	0.07	0.09
0.1, 0.9	0.04	0.07	0.04	0.05	0.08	0.07
0.5, 0.9	0.04	0.04	0.04	0.04	0.04	0.05
0.3, 0.9	0.04	0.04	0.04	0.04	0.04	0.05
0.3, 0.6	0.05	0.06	0.06	0.06	0.07	0.07
0.1, 0.3	0.06	0.06	0.06	0.09	0.06	0.08
0.05, 0.05	0.07	0.05	0.06	0.17	0.06	0.10

	$n_1 = 30, n_2 = 15, n_3 = 15$					
σ_2^2, σ_3^2	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1	0.04	0.04	0.04	0.04	0.05	0.05
1, 0.5	0.04	0.05	0.04	0.07	0.06	0.05
1, 0.1	0.04	0.06	0.04	0.04	0.07	0.06
0.5, 0.5	0.05	0.05	0.05	0.06	0.05	0.06
0.5, 0.7	0.04	0.03	0.04	0.04	0.04	0.05
0.1, 0.1	0.06	0.05	0.07	0.04	0.06	0.09
0.1, 0.9	0.05	0.07	0.05	0.05	0.09	0.07
0.5, 0.9	0.05	0.04	0.05	0.05	0.05	0.06
0.3, 0.9	0.04	0.04	0.03	0.03	0.04	0.05
0.3, 0.6	0.04	0.04	0.04	0.04	0.04	0.05
0.1, 0.3	0.05	0.04	0.04	0.04	0.05	0.06
0.05, 0.05	0.05	0.04	0.06	0.03	0.05	0.09

Table 3.5. *Type 1 Error Rates for Lognormal Distribution in i = 3 (continued)*

$i = 3$, $\sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = 1.3$

σ_2^2, σ_3^2	$n_1 = 15, n_2 = 20, n_3 = 25$					
	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1	0.04	0.04	0.04	0.05	0.05	0.05
1, 0.5	0.04	0.04	0.03	0.07	0.05	0.06
1, 0.1	0.04	0.06	0.03	0.09	0.09	0.06
0.5, 0.5	0.06	0.06	0.06	0.08	0.07	0.08
0.5, 0.7	0.04	0.04	0.03	0.05	0.04	0.05
0.1, 0.1	0.08	0.06	0.08	0.15	0.08	0.12
0.1, 0.9	0.04	0.07	0.04	0.07	0.09	0.06
0.5, 0.9	0.04	0.05	0.04	0.05	0.06	0.06
0.3, 0.9	0.04	0.04	0.04	0.05	0.05	0.05
0.3, 0.6	0.04	0.05	0.04	0.07	0.06	0.06
0.1, 0.3	0.06	0.05	0.06	0.09	0.07	0.07
0.05, 0.05	0.07	0.06	0.07	0.15	0.07	0.11

σ_2^2, σ_3^2	$n_1 = 25, n_2 = 20, n_3 = 15$					
	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1	0.04	0.04	0.04	0.05	0.05	0.05
1, 0.5	0.04	0.04	0.04	0.05	0.05	0.06
1, 0.1	0.04	0.06	0.04	0.04	0.08	0.07
0.5, 0.5	0.04	0.04	0.04	0.04	0.04	0.05
0.5, 0.7	0.04	0.04	0.04	0.04	0.05	0.05
0.1, 0.1	0.06	0.05	0.06	0.06	0.06	0.08
0.1, 0.9	0.04	0.07	0.04	0.05	0.08	0.06
0.5, 0.9	0.04	0.05	0.04	0.05	0.06	0.05
0.3, 0.9	0.04	0.06	0.04	0.05	0.07	0.05
0.3, 0.6	0.04	0.05	0.04	0.05	0.05	0.05
0.1, 0.3	0.06	0.06	0.06	0.06	0.08	0.07
0.05, 0.05	0.06	0.05	0.06	0.06	0.06	0.09

Table 3.6. displays the empirical type 1 error rates for *i* = 6 and different sample sizes. In this scenario, Type 1 error rates of lognormal distribution show similar pattern with gamma distribution. For unequal sample sizes changing from small to large size, the type 1 error rates of F_{P1}-test have a tendency of inflation for the situation ($\sigma_1^2 = 1$, $\sigma_2^2 = 0.01$, $\sigma_3^2 = 0.01$, $\sigma_4^2 = 0.06$, $\sigma_5^2 = 0.1$, $\sigma_6^2 = 0.1$), though it is close to the nominal level .05 for the other scenarios. Besides, the type 1 error rates of W_{P1}-test are conservative when variances are equal, although W_{P1}-test and F^*_{P1}-test are close to the intended level .05 for other conditions. Furthermore, similar to simulations with gamma distribution, the analysis of plan 2 tests reach inconclusive findings such as “0” or close to zero for simulations with lognormal distributions.

Table 3.6. *Type 1 Error Rates for Lognormal Distribution in i = 6*

i = 6, σ_1^2 = 1

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 1.3$

	$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 15$					
σ_2^2, ..., σ_6^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1, 1, 1, 1	0.03	0.02	0.03	0.001	0	0.001
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0	0.001	0
0.1, 0.2, 0.3, 0.4, 0.5	0.04	0.06	0.04	0	0.003	0
0.1, 1, 1, 1, 1	0.03	0.06	0.03	0	0.009	0
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.03	0.04	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.04	0	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.05	0.04	0.001	0	0.001
0.01, 0.01, 0.06, 0.1, 0.1	0.07	0.05	0.07	0.001	0	0.001

	$n_1 = 15, n_2 = 20, n_3 = 25, n_4 = 30, n_5 = 35, n_6 = 40$					
σ_2^2, ..., σ_6^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1, 1, 1, 1	0.04	0.04	0.03	0	0	0
0.1, 0.1, 0.5, 0.5, 0.5	0.06	0.06	0.06	0.002	0.003	0.003
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0.05	0.06	0.001	0.001	0.001
0.1, 1, 1, 1, 1	0.04	0.05	0.04	0.002	0.006	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.05	0.06	0.001	0.001	0.001
0.5, 0.5, 0.5, 0.5, 1	0.05	0.04	0.05	0	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.06	0.05	0.001	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.08	0.05	0.07	0	0.001	0

	$n_1 = 40, n_2 = 35, n_3 = 30, n_4 = 25, n_5 = 20, n_6 = 15$					
σ_2^2, ..., σ_6^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1, 1, 1, 1	0.04	0.03	0.04	0	0	0
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.04	0.04	0.002	0	0.001
0.1, 0.2, 0.3, 0.4, 0.5	0.04	0.06	0.05	0.001	0.002	0
0.1, 1, 1, 1, 1	0.03	0.06	0.03	0.004	0.007	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0.05	0.05	0.002	0.003	0.003
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.04	0.002	0.001	0.001
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.04	0.04	0	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.05	0.05	0.06	0	0	0

	$n_1 = 15, n_2 = 15, n_3 = 15, n_4 = 30, n_5 = 30, n_6 = 30$					
σ_2^2, ..., σ_6^2	F_{P1}-*test*	W_{P1}-*test*	F^*_{P1}-*test*	F_{P2}-*test*	W_{P2}-*test*	F^*_{P2}-*test*
1, 1, 1, 1, 1	0.03	0.03	0.03	0	0	0
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0.06	0.05	0	0	0
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.04	0.05	0	0.002	0
0.1, 1, 1, 1, 1	0.04	0.06	0.03	0.002	0.006	0.003
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.06	0.06	0.001	0.002	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.03	0.03	0	0	0.002
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.06	0.04	0	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.10	0.06	0.08	0	0	0

Table 3.6. *Type 1 Error Rates for Lognormal Distribution in* i *= 6 (continued)*

$i = 6$, $\sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 1.3$

	$n_1 = 30, n_2 = 30, n_3 = 30, n_4 = 15, n_5 = 15, n_6 = 15$					
$\sigma_2^2, ..., \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.03	0.04	0.001	0	0.001
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.007	0.007	0.002
0.1, 0.2, 0.3, 0.4, 0.5	0.04	0.05	0.04	0.002	0.003	0.002
0.1, 1, 1, 1, 1	0.04	0.05	0.03	0.002	0.003	0.001
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.04	0.04	0	0.001	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.03	0	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.06	0.05	0	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0.05	0.07	0.002	0.002	0.002

	$n_1 = 15, n_2 = 15, n_3 = 30, n_4 = 30, n_5 = 45, n_6 = 45$					
$\sigma_2^2, ..., \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.05	0.03	0.002	0.001	0
0.1, 0.1, 0.5, 0.5, 0.5	0.06	0.05	0.05	0.002	0.004	0.003
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.05	0.05	0	0	0
0.1, 1, 1, 1, 1	0.04	0.04	0.03	0.001	0.003	0
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0.04	0.06	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.04	0.04	0	0.002	0.001
0.3, 0.9, 0.4, 0.7, 0.1	0.04	0.05	0.03	0	0.003	0
0.01, 0.01, 0.06, 0.1, 0.1	0.08	0.04	0.07	0	0	0

	$n_1 = 45, n_2 = 45, n_3 = 30, n_4 = 30, n_5 = 15, n_6 = 15$					
$\sigma_2^2, ..., \sigma_6^2$	F_{P1}-*test*	W_{P1}-*test*	F_{P1}^*-*test*	F_{P2}-*test*	W_{P2}-*test*	F_{P2}^*-*test*
1, 1, 1, 1, 1	0.04	0.05	0.04	0.002	0.001	0.001
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.05	0.04	0.003	0.002	0
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.06	0.05	0.003	0.002	0.003
0.1, 1, 1, 1, 1	0.04	0.06	0.04	0.008	0.007	0.002
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0.04	0.04	0	0	0
0.5, 0.5, 0.5, 0.5, 1	0.04	0.06	0.04	0.001	0	0
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0.06	0.05	0.001	0	0
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0.05	0.07	0	0	0

3.3.3. Type 1 Error Rates for Normal Distribution

Type 1 error rates are estimated for the normally distributed data with a specific mean ($\mu_1 = \ldots = \mu_i = 0$) and variance changing from 0.01 to 1 for i = 2, 3, and 6 and different sample sizes ranging from equal to unequal using Monte Carlo Simulation as explained in Section 3.1. For normal distribution, the simulations are generated for F_{P1}-test, F_{P2}-test, W-test, and F^*-test.

Table 3.7. shows the empirical type 1 error rates for i = 2 and different sample sizes for normally distributed data. Empirical type I error rates demonstrate that F_{P1}-test, W-test, and F^*-test have similar and satisfactory results for type 1 error rates for i = 2. In other words, type 1 error rates are all around the nominal level .05 for these three methods. However, although F_{P2}-test also has type 1 error rates close to the nominal level .05 for equal sample sizes, it has lower or higher type 1 error rates than the nominal level .05 for unequal sample sizes. Specifically, F_{P2}-test is liberal when small sample size group has high variance and large sample size group has small variance. On the other hand, F_{P2}-test is more conservative when large sample size group has high variance and small sample size group has small variance. Therefore, if the variance heterogeneity gets larger for unequal sample sizes, F_{P2}-test tend to show more conservative or liberal type 1 error rates.

Table 3.8. demonstrates the estimated type 1 error rates for i = 3 and different sample sizes for normally distributed data. Detection of the estimated type 1 error rates for equal and unequal sample sizes displays that F_{P1}-test, W-test, and F^*-test provide a better approximation to intended level. Besides, F_{P2}-test tends to behave over-reject when the difference among variances are large for equal sample size and for unequal sample sizes which consist of decreasing sample sizes are connected with increasing variance. Conversely, the type 1 errors of F_{P2}-test are conservative when decreasing sample sizes are connected with decreasing variance.

Table 3.7. *Type 1 Error Rates for Normal Distribution in i = 2*

$i = 2, \sigma_1^2 = 1$
$\mu_1 = \mu_2 = 0$

σ_2^2	$n_1 = n_2 = 15$			
	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
0.01	0.04	0.06	0.05	0.05
0.05	0.05	0.06	0.06	0.06
0.10	0.04	0.06	0.05	0.05
0.20	0.04	0.04	0.04	0.04
0.30	0.04	0.05	0.05	0.05
0.40	0.06	0.05	0.05	0.05
0.50	0.05	0.05	0.05	0.05
0.60	0.05	0.04	0.04	0.04
0.70	0.05	0.04	0.04	0.04
0.80	0.04	0.04	0.04	0.04
0.90	0.05	0.05	0.05	0.05
1.00	0.05	0.05	0.05	0.05

σ_2^2	$n_1 = 15, n_2 = 25$			
	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
0.01	0.05	0.14	0.05	0.05
0.05	0.06	0.14	0.06	0.06
0.10	0.04	0.12	0.04	0.04
0.20	0.06	0.11	0.05	0.05
0.30	0.06	0.09	0.05	0.05
0.40	0.05	0.07	0.04	0.04
0.50	0.05	0.08	0.05	0.05
0.60	0.05	0.05	0.04	0.04
0.70	0.05	0.06	0.05	0.05
0.80	0.05	0.06	0.05	0.05
0.90	0.06	0.05	0.05	0.05
1.00	0.05	0.05	0.05	0.05

σ_2^2	$n_1 = 25, n_2 = 15$			
	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
0.01	0.04	0.01	0.05	0.05
0.05	0.05	0.02	0.05	0.05
0.10	0.05	0.02	0.05	0.05
0.20	0.04	0.02	0.04	0.04
0.30	0.04	0.03	0.05	0.05
0.40	0.05	0.02	0.05	0.05
0.50	0.05	0.03	0.05	0.05
0.60	0.06	0.04	0.05	0.05
0.70	0.05	0.04	0.05	0.05
0.80	0.05	0.04	0.05	0.05
0.90	0.05	0.04	0.05	0.05
1.00	0.05	0.05	0.04	0.04

Table 3.8. *Type 1 Error Rates for Normal Distribution in i = 3*

$i = 3$, $\sigma_1^2 = 1$ $\mu_1 = \mu_2 = \mu_3 = 0$				
	$n_1 = n_2 = n_3 = 15$			
σ_2^2, σ_3^2	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1	0.04	0.04	0.04	0.04
1, 0.5	0.05	0.05	0.06	0.05
1, 0.1	0.06	0.06	0.06	0.06
0.5, 0.5	0.05	0.06	0.06	0.06
0.5, 0.7	0.04	0.05	0.05	0.04
0.1, 0.1	0.04	0.06	0.05	0.06
0.1, 0.9	0.05	0.06	0.04	0.06
0.5, 0.9	0.04	0.05	0.06	0.05
0.3, 0.9	0.04	0.06	0.05	0.05
0.3, 0.6	0.05	0.06	0.05	0.06
0.1, 0.3	0.05	0.06	0.05	0.06
0.05, 0.05	0.05	0.08	0.05	0.07

	$n_1 = 15, n_2 = 15, n_3 = 30$			
σ_2^2, σ_3^2	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1	0.06	0.06	0.06	0.06
1, 0.5	0.06	0.08	0.06	0.06
1, 0.1	0.05	0.12	0.05	0.05
0.5, 0.5	0.05	0.06	0.05	0.05
0.5, 0.7	0.05	0.05	0.05	0.05
0.1, 0.1	0.05	0.13	0.05	0.06
0.1, 0.9	0.05	0.05	0.05	0.06
0.5, 0.9	0.06	0.06	0.06	0.06
0.3, 0.9	0.05	0.04	0.05	0.05
0.3, 0.6	0.04	0.05	0.04	0.04
0.1, 0.3	0.06	0.09	0.06	0.06
0.05, 0.05	0.05	0.14	0.04	0.07

	$n_1 = 30, n_2 = 15, n_3 = 15$			
σ_2^2, σ_3^2	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1	0.06	0.05	0.06	0.05
1, 0.5	0.05	0.04	0.04	0.05
1, 0.1	0.04	0.04	0.06	0.05
0.5, 0.5	0.05	0.03	0.06	0.06
0.5, 0.7	0.05	0.03	0.05	0.05
0.1, 0.1	0.05	0.02	0.05	0.06
0.1, 0.9	0.05	0.04	0.05	0.06
0.5, 0.9	0.06	0.05	0.05	0.06
0.3, 0.9	0.06	0.04	0.06	0.06
0.3, 0.6	0.05	0.03	0.04	0.06
0.1, 0.3	0.04	0.02	0.04	0.05
0.05, 0.05	0.05	0.02	0.05	0.07

Table 3.8. *Type 1 Error Rates for Normal Distribution in i = 3 (continued)*

$i = 3$, $\sigma_1^2 = 1$ $\mu_1 = \mu_2 = \mu_3 = 0$				
	$n_1 = 15, n_2 = 20, n_3 = 25$			
σ_2^2, σ_3^2	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1	0.05	0.05	0.05	0.06
1, 0.5	0.04	0.06	0.04	0.04
1, 0.1	0.05	0.08	0.05	0.05
0.5, 0.5	0.04	0.05	0.04	0.04
0.5, 0.7	0.05	0.06	0.05	0.05
0.1, 0.1	0.04	0.10	0.04	0.05
0.1, 0.9	0.05	0.06	0.06	0.06
0.5, 0.9	0.04	0.05	0.04	0.04
0.3, 0.9	0.04	0.05	0.04	0.04
0.3, 0.6	0.06	0.06	0.04	0.06
0.1, 0.3	0.05	0.09	0.05	0.05
0.05, 0.05	0.05	0.15	0.05	0.07
	$n_1 = 25, n_2 = 20, n_3 = 15$			
σ_2^2, σ_3^2	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1	0.04	0.04	0.04	0.04
1, 0.5	0.05	0.05	0.05	0.06
1, 0.1	0.05	0.03	0.05	0.05
0.5, 0.5	0.05	0.04	0.05	0.05
0.5, 0.7	0.05	0.04	0.05	0.05
0.1, 0.1	0.04	0.03	0.05	0.06
0.1, 0.9	0.05	0.06	0.05	0.05
0.5, 0.9	0.05	0.05	0.05	0.05
0.3, 0.9	0.05	0.05	0.04	0.05
0.3, 0.6	0.05	0.05	0.06	0.06
0.1, 0.3	0.05	0.04	0.06	0.06
0.05, 0.05	0.05	0.04	0.04	0.07

Table 3.9. displays the empirical type 1 error rates for *i* = 6 and different sample sizes for normally distributed data. The estimates of type 1 error rates, which are close to the nominal level .05 for all conditions, are similar for F_{P1}-test and W-test. However, F^*-test tends to show liberal results especially for the scenarios of ($\sigma_1^2 = 1$, $\sigma_2^2 = 0.01$, $\sigma_3^2 = 0.01$, $\sigma_4^2 = 0.06$, $\sigma_5^2 = 0.1$, $\sigma_6^2 = 0.1$). Further, similar to simulations with gamma distribution, the analysis of F_{P2}-test reaches inconclusive findings such as "0" or close to zero.

Table 3.9. *Type 1 Error Rates for Normal Distribution in i = 6*

$i = 6$, $\sigma_1^2 = 1$

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 0$

	$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 15$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.05	0	0.05	0.05
0.1, 0.1, 0.5, 0.5, 0.5	0.06	0.002	0.06	0.07
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0	0.06	0.07
0.1, 1, 1, 1, 1	0.04	0.002	0.04	0.04
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0	0.05	0.07
0.5, 0.5, 0.5, 0.5, 1	0.04	0	0.05	0.05
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0	0.05	0.07
0.01, 0.01, 0.06, 0.1, 0.1	0.05	0	0.05	0.08

	$n_1 = 15, n_2 = 20, n_3 = 25, n_4 = 30, n_5 = 35, n_6 = 40$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.04	0	0.05	0.05
0.1, 0.1, 0.5, 0.5, 0.5	0.04	0.002	0.04	0.06
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0	0.05	0.07
0.1, 1, 1, 1, 1	0.04	0	0.06	0.05
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0.001	0.04	0.06
0.5, 0.5, 0.5, 0.5, 1	0.05	0	0.05	0.05
0.3, 0.9, 0.4, 0.7, 0.1	0.06	0	0.06	0.07
0.01, 0.01, 0.06, 0.1, 0.1	0.05	0	0.06	0.09

	$n_1 = 40, n_2 = 35, n_3 = 30, n_4 = 25, n_5 = 20, n_6 = 15$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.04	0	0.05	0.05
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0.002	0.04	0.06
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0.001	0.06	0.07
0.1, 1, 1, 1, 1	0.05	0.001	0.05	0.06
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0	0.06	0.07
0.5, 0.5, 0.5, 0.5, 1	0.05	0.002	0.06	0.06
0.3, 0.9, 0.4, 0.7, 0.1	0.06	0.001	0.06	0.07
0.01, 0.01, 0.06, 0.1, 0.1	0.04	0	0.05	0.12

	$n_1 = 15, n_2 = 15, n_3 = 15, n_4 = 30, n_5 = 30, n_6 = 30$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.04	0	0.04	0.04
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0	0.06	0.06
0.1, 0.2, 0.3, 0.4, 0.5	0.04	0	0.05	0.05
0.1, 1, 1, 1, 1	0.06	0	0.08	0.07
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0	0.05	0.07
0.5, 0.5, 0.5, 0.5, 1	0.04	0	0.04	0.05
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0	0.05	0.06
0.01, 0.01, 0.06, 0.1, 0.1	0.05	0	0.05	0.10

Table 3.9. *Type 1 Error Rates for Normal Distribution in i = 6 (continued)*

$i = 6$, $\sigma_1^2 = 1$				
	$n_1 = 30, n_2 = 30, n_3 = 30, n_4 = 15, n_5 = 15, n_6 = 15$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.05	0	0.05	0.05
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0.002	0.06	0.06
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0.002	0.04	0.07
0.1, 1, 1, 1, 1	0.04	0	0.05	0.04
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.05	0.003	0.06	0.07
0.5, 0.5, 0.5, 0.5, 1	0.06	0	0.05	0.06
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0	0.05	0.06
0.01, 0.01, 0.06, 0.1, 0.1	0.05	0	0.06	0.10

	$n_1 = 15, n_2 = 15, n_3 = 30, n_4 = 30, n_5 = 45, n_6 = 45$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.04	0	0.04	0.04
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0.001	0.05	0.07
0.1, 0.2, 0.3, 0.4, 0.5	0.06	0	0.05	0.07
0.1, 1, 1, 1, 1	0.04	0	0.05	0.06
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.06	0	0.05	0.08
0.5, 0.5, 0.5, 0.5, 1	0.04	0	0.05	0.04
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0	0.06	0.06
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0	0.04	0.10

	$n_1 = 45, n_2 = 45, n_3 = 30, n_4 = 30, n_5 = 15, n_6 = 15$			
$\sigma_2^2, \ldots, \sigma_6^2$	F_{P1}-*test*	F_{P2}-*test*	W-*test*	F^*-*test*
1, 1, 1, 1, 1	0.05	0.001	0.05	0.05
0.1, 0.1, 0.5, 0.5, 0.5	0.05	0	0.06	0.07
0.1, 0.2, 0.3, 0.4, 0.5	0.05	0	0.05	0.06
0.1, 1, 1, 1, 1	0.05	0.002	0.05	0.05
0.2, 0.4, 0.4, 0.2, 0.2, 0.1	0.04	0	0.05	0.07
0.5, 0.5, 0.5, 0.5, 1	0.06	0	0.06	0.06
0.3, 0.9, 0.4, 0.7, 0.1	0.05	0	0.05	0.06
0.01, 0.01, 0.06, 0.1, 0.1	0.06	0	0.05	0.11

3.4. The Results of Power Studies

In the power studies, tests with plan 2, which are F_{P2}-test, W_{P2}- test, and F_{P2}^*-test, are not investigated for their powers because type 1 error rates do not maintain the nominal significance level. Moreover, power studies are restricted to balanced cases and selected unbalanced sample sized ($n_1 = 15$, $n_2 = 20$, $n_3 = 25$), ($n_1 = 25$, $n_2 = 20$, $n_3 = 15$) for *i* = 3 and ($n_1 = 15$, $n_2 = 15$, $n_3 = 30$, $n_4 = 30$, $n_5 = 45$, $n_6 = 45$), ($n_1 = 45$, $n_2 = 45$, $n_3 = 30$, $n_4 = 30$, $n_5 = 15$, $n_6 = 15$) for *i* = 6. For *i* = 3 and *i* =

6, each distribution showed a similar type 1 error rates pattern in itself as the number of samples increased or decreased. Therefore, one sample example is chosen for decreasing and increasing sample sizes.

3.4.1. Power Studies for Gamma Distribution

Powers are evaluated for the gamma distributed data with a specific mean of first group (μ_1= 1) and the others ranging from 1 to 2.3, and variance changing from 0.01 to 1 for i = 2, 3, and 6 and different sample sizes ranging from equal to unequal. For gamma distribution, the simulations are generated for F_{P1}-test, W_{P1}- test, and F^*_{P1}-test.

The findings demonstrate that powers of all tests that are F_{P1}-test, W_{P1}- test, and F^*_{P1}-test are approximately similar for i = 2 with all sample size. For i = 3, W_{P1}-test seems to be more powerful than the other tests where (σ_1^2 = 1, σ_2^2 = 1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.3), and (σ_1^2 = 1, σ_2^2 = 0.05, σ_3^2 = 0.05). In the other cases, all tests display comparable power features. Similar to the results for i = 3, for i = 6, W_{P1}-test reveals more powerful characteristics than the other tests in some cases that are (σ_1^2 = 1, σ_2^2 = 0.2, σ_3^2 = 0.4, σ_4^2 = 0.4, σ_5^2 = 0.2, σ_6^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.3, σ_3^2 = 0.9, σ_4^2 = 0.4, σ_5^2 = 0.7, σ_6^2 = 0.1), and (σ_1^2 = 1, σ_2^2 = 0.01, σ_3^2 = 0.01, σ_4^2 = 0.06, σ_5^2 = 0.1, σ_6^2 = 0.1). Conversely, W_{P1}-test exhibits less powerful properties than the other tests in two cases that are (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 1, σ_4^2 = 1, σ_5^2 = 1, σ_6^2 = 1) and (σ_1^2 = 1, σ_2^2 = 0.5, σ_3^2 = 0.5, σ_4^2 = 0.5, σ_5^2 = 0.5, σ_6^2 = 1). As explained in the "2.1.2. Welch' F test" part, W-test consists of weight function which is calculated using group sample size and variance. Obviously, the estimated power of W-test also depends on this weight function. Therefore, for a specified sample sizes, if the ratio of group variances is high, like our simulations, this may have a positive effect on the power of W_{P1}-test for i = 3. However, increased number of groups may lead to instability for power of W_{P1}-test for non-normal distributions.

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1$
$n_1 = n_2 = 15$

σ_2^2	Tests	(μ_2) (1)	(1.3)	(1.5)	(1.7)	(2)	(2.3)
0.01	F_{P1}-*test*	0.06	0.24	0.44	0.60	0.76	0.90
	W_{P1}-*test*	0.06	0.24	0.44	0.60	0.76	0.90
	F_{P1}^{*}-*test*	0.06	0.24	0.44	0.60	0.76	0.90
0.05	F_{P1}-*test*	0.06	0.25	0.42	0.60	0.77	0.88
	W_{P1}-*test*	0.05	0.25	0.42	0.60	0.77	0.88
	F_{P1}^{*}-*test*	0.05	0.25	0.42	0.60	0.77	0.88
0.10	F_{P1}-*test*	0.06	0.25	0.44	0.61	0.80	0.90
	W_{P1}-*test*	0.06	0.25	0.44	0.61	0.80	0.90
	F_{P1}^{*}-*test*	0.06	0.25	0.44	0.61	0.80	0.90
0.20	F_{P1}-*test*	0.06	0.26	0.41	0.58	0.80	0.91
	W_{P1}-*test*	0.06	0.26	0.41	0.58	0.80	0.91
	F_{P1}^{*}-*test*	0.06	0.26	0.41	0.58	0.80	0.91
0.30	F_{P1}-*test*	0.06	0.23	0.43	0.60	0.78	0.92
	W_{P1}-*test*	0.06	0.23	0.43	0.60	0.78	0.92
	F_{P1}^{*}-*test*	0.06	0.23	0.43	0.60	0.78	0.92
0.40	F_{P1}-*test*	0.05	0.19	0.41	0.56	0.78	0.91
	W_{P1}-*test*	0.05	0.19	0.41	0.56	0.78	0.91
	F_{P1}^{*}-*test*	0.05	0.19	0.41	0.56	0.78	0.91
0.50	F_{P1}-*test*	0.04	0.18	0.40	0.56	0.80	0.91
	W_{P1}-*test*	0.04	0.18	0.40	0.56	0.80	0.91
	F_{P1}^{*}-*test*	0.04	0.18	0.40	0.56	0.80	0.91
0.60	F_{P1}-*test*	0.04	0.18	0.35	0.55	0.81	0.89
	W_{P1}-*test*	0.04	0.18	0.35	0.55	0.81	0.89
	F_{P1}^{*}-*test*	0.04	0.18	0.35	0.55	0.81	0.89
0.70	F_{P1}-*test*	0.04	0.16	0.33	0.56	0.78	0.90
	W_{P1}-*test*	0.04	0.16	0.33	0.56	0.78	0.90
	F_{P1}^{*}-*test*	0.04	0.16	0.33	0.56	0.78	0.90
0.80	F_{P1}-*test*	0.05	0.15	0.32	0.50	0.75	0.90
	W_{P1}-*test*	0.05	0.15	0.32	0.50	0.75	0.90
	F_{P1}^{*}-*test*	0.05	0.15	0.32	0.50	0.75	0.90
0.90	F_{P1}-*test*	0.04	0.14	0.31	0.51	0.76	0.90
	W_{P1}-*test*	0.04	0.14	0.31	0.51	0.76	0.90
	F_{P1}^{*}-*test*	0.04	0.14	0.31	0.51	0.76	0.90
1	F_{P1}-*test*	0.04	0.13	0.28	0.48	0.77	0.88
	W_{P1}-*test*	0.04	0.13	0.28	0.48	0.77	0.88
	F_{P1}^{*}-*test*	0.04	0.13	0.28	0.48	0.77	0.88

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1$ $n_1 = 15$, $n_2 = 25$		(μ_2)					
σ_2^2	Tests	(1)	(1.3)	(1.5)	(1.7)	(2)	(2.3)
0.01	F_{P1}-test	0.08	0.25	0.41	0.60	0.76	0.86
	W_{P1}-test	0.08	0.25	0.40	0.60	0.76	0.86
	F_{P1}^{*}-test	0.08	0.25	0.40	0.60	0.76	0.86
0.05	F_{P1}-test	0.07	0.26	0.42	0.60	0.80	0.88
	W_{P1}-test	0.06	0.24	0.40	0.57	0.79	0.87
	F_{P1}^{*}-test	0.06	0.24	0.40	0.57	0.79	0.87
0.10	F_{P1}-test	0.08	0.31	0.47	0.63	0.80	0.91
	W_{P1}-test	0.07	0.30	0.44	0.58	0.78	0.90
	F_{P1}^{*}-test	0.07	0.30	0.44	0.58	0.78	0.90
0.20	F_{P1}-test	0.06	0.29	0.46	0.64	0.84	0.92
	W_{P1}-test	0.05	0.26	0.43	0.60	0.80	0.90
	F_{P1}^{*}-test	0.05	0.26	0.43	0.60	0.80	0.90
0.30	F_{P1}-test	0.05	0.26	0.46	0.64	0.81	0.94
	W_{P1}-test	0.05	0.24	0.42	0.59	0.78	0.91
	F_{P1}^{*}-test	0.05	0.24	0.42	0.59	0.78	0.91
0.40	F_{P1}-test	0.06	0.29	0.46	0.61	0.82	0.94
	W_{P1}-test	0.06	0.27	0.43	0.58	0.78	0.91
	F_{P1}^{*}-test	0.06	0.27	0.43	0.58	0.78	0.91
0.50	F_{P1}-test	0.06	0.25	0.47	0.60	0.84	0.94
	W_{P1}-test	0.06	0.24	0.44	0.57	0.81	0.91
	F_{P1}^{*}-test	0.06	0.24	0.44	0.57	0.81	0.91
0.60	F_{P1}-test	0.05	0.25	0.42	0.62	0.82	0.95
	W_{P1}-test	0.05	0.24	0.40	0.59	0.78	0.92
	F_{P1}^{*}-test	0.05	0.24	0.40	0.59	0.78	0.92
0.70	F_{P1}-test	0.05	0.23	0.44	0.59	0.84	0.93
	W_{P1}-test	0.05	0.22	0.42	0.55	0.80	0.90
	F_{P1}^{*}-test	0.05	0.22	0.42	0.55	0.80	0.90
0.80	F_{P1}-test	0.04	0.22	0.41	0.59	0.83	0.93
	W_{P1}-test	0.04	0.22	0.40	0.56	0.81	0.90
	F_{P1}^{*}-test	0.04	0.22	0.40	0.56	0.81	0.90
0.90	F_{P1}-test	0.05	0.22	0.37	0.58	0.83	0.94
	W_{P1}-test	0.05	0.23	0.36	0.56	0.80	0.92
	F_{P1}^{*}-test	0.05	0.23	0.36	0.56	0.80	0.92
1	F_{P1}-test	0.05	0.18	0.40	0.58	0.78	0.93
	W_{P1}-test	0.05	0.18	0.40	0.55	0.77	0.91
	F_{P1}^{*}-test	0.05	0.18	0.40	0.55	0.77	0.91

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1$ $n_1 = 25$, $n_2 = 15$		(μ_2)					
σ_2^2	Tests	(1)	(1.3)	(1.5)	(1.7)	(2)	(2.3)
0.01	F_{P1}-*test*	0.06	0.32	0.58	0.79	0.94	0.97
	W_{P1}-*test*	0.06	0.32	0.58	0.79	0.94	0.97
	F_{P1}^{*}-*test*	0.06	0.32	0.58	0.79	0.94	0.97
0.05	F_{P1}-*test*	0.06	0.28	0.55	0.78	0.94	0.98
	W_{P1}-*test*	0.06	0.29	0.56	0.79	0.94	0.98
	F_{P1}^{*}-*test*	0.06	0.29	0.56	0.79	0.94	0.98
0.10	F_{P1}-*test*	0.04	0.28	0.53	0.74	0.93	0.98
	W_{P1}-*test*	0.04	0.30	0.55	0.76	0.94	0.98
	F_{P1}^{*}-*test*	0.04	0.30	0.55	0.76	0.94	0.98
0.20	F_{P1}-*test*	0.05	0.27	0.51	0.74	0.92	0.99
	W_{P1}-*test*	0.05	0.28	0.53	0.76	0.94	0.99
	F_{P1}^{*}-*test*	0.05	0.28	0.53	0.76	0.94	0.99
0.30	F_{P1}-*test*	0.05	0.27	0.51	0.73	0.93	0.97
	W_{P1}-*test*	0.05	0.29	0.53	0.75	0.94	0.98
	F_{P1}^{*}-*test*	0.05	0.29	0.53	0.75	0.94	0.98
0.40	F_{P1}-*test*	0.04	0.23	0.47	0.72	0.91	0.97
	W_{P1}-*test*	0.04	0.24	0.48	0.73	0.92	0.98
	F_{P1}^{*}-*test*	0.04	0.24	0.48	0.73	0.92	0.98
0.50	F_{P1}-*test*	0.04	0.19	0.46	0.67	0.89	0.97
	W_{P1}-*test*	0.04	0.20	0.46	0.69	0.91	0.98
	F_{P1}^{*}-*test*	0.04	0.20	0.46	0.69	0.91	0.98
0.60	F_{P1}-*test*	0.04	0.18	0.43	0.65	0.88	0.96
	W_{P1}-*test*	0.03	0.18	0.44	0.66	0.89	0.98
	F_{P1}^{*}-*test*	0.03	0.18	0.44	0.66	0.89	0.98
0.70	F_{P1}-*test*	0.05	0.17	0.40	0.63	0.88	0.97
	W_{P1}-*test*	0.05	0.17	0.38	0.64	0.90	0.98
	F_{P1}^{*}-*test*	0.05	0.17	0.38	0.64	0.90	0.98
0.80	F_{P1}-*test*	0.05	0.15	0.36	0.58	0.87	0.96
	W_{P1}-*test*	0.05	0.14	0.36	0.59	0.88	0.97
	F_{P1}^{*}-*test*	0.05	0.14	0.36	0.59	0.88	0.97
0.90	F_{P1}-*test*	0.04	0.32	0.84	0.99	1	1
	W_{P1}-*test*	0.04	0.31	0.85	0.99	1	1
	F_{P1}^{*}-*test*	0.04	0.31	0.85	0.99	1	1
1	F_{P1}-*test*	0.05	0.12	0.30	0.56	0.83	0.95
	W_{P1}-*test*	0.05	0.11	0.29	0.55	0.84	0.97
	F_{P1}^{*}-*test*	0.05	0.11	0.29	0.55	0.84	0.97

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1$ $n_1 = n_2 = n_3 = 15$		(μ_2, μ_3)					
σ_2^2, σ_3^2	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
1, 1	F_{P1}-*test*	0.04	0.10	0.27	0.45	0.66	0.86
	W_{P1}-*test*	0.04	0.10	0.25	0.44	0.66	0.88
	F_{P1}^*-*test*	0.04	0.10	0.27	0.45	0.66	0.86
1, 0.5	F_{P1}-*test*	0.04	0.15	0.30	0.54	0.73	0.90
	W_{P1}-*test*	0.04	0.18	0.37	0.65	0.79	0.94
	F_{P1}^*-*test*	0.04	0.15	0.30	0.54	0.73	0.90
1, 0.1	F_{P1}-*test*	0.04	0.16	0.36	0.57	0.76	0.91
	W_{P1}-*test*	0.06	0.28	0.53	0.72	0.86	0.97
	F_{P1}^*-*test*	0.04	0.16	0.36	0.57	0.76	0.91
0.5, 0.5	F_{P1}-*test*	0.04	0.16	0.36	0.62	0.78	0.92
	W_{P1}-*test*	0.04	0.19	0.41	0.70	0.81	0.96
	F_{P1}^*-*test*	0.04	0.16	0.36	0.62	0.78	0.92
0.5, 0.7	F_{P1}-*test*	0.05	0.14	0.35	0.55	0.76	0.92
	W_{P1}-*test*	0.05	0.13	0.36	0.58	0.76	0.94
	F_{P1}^*-*test*	0.05	0.14	0.35	0.55	0.76	0.92
0.1, 0.1	F_{P1}-*test*	0.07	0.24	0.46	0.66	0.79	0.92
	W_{P1}-*test*	0.05	0.55	0.92	0.99	1	1
	F_{P1}^*-*test*	0.07	0.24	0.46	0.66	0.79	0.92
0.1, 0.9	F_{P1}-*test*	0.04	0.12	0.28	0.55	0.75	0.91
	W_{P1}-*test*	0.06	0.10	0.22	0.49	0.62	0.90
	F_{P1}^*-*test*	0.04	0.12	0.28	0.55	0.75	0.91
0.5, 0.9	F_{P1}-*test*	0.04	0.13	0.30	0.53	0.70	0.90
	W_{P1}-*test*	0.04	0.11	0.27	0.50	0.67	0.91
	F_{P1}^*-*test*	0.04	0.13	0.30	0.53	0.70	0.90
0.3, 0.9	F_{P1}-*test*	0.05	0.11	0.31	0.57	0.76	0.90
	W_{P1}-*test*	0.06	0.09	0.26	0.52	0.70	0.91
	F_{P1}^*-*test*	0.05	0.11	0.31	0.57	0.76	0.90
0.3, 0.6	F_{P1}-*test*	0.05	0.18	0.37	0.62	0.77	0.93
	W_{P1}-*test*	0.04	0.15	0.38	0.69	0.79	0.96
	F_{P1}^*-*test*	0.05	0.18	0.37	0.62	0.77	0.93
0.1, 0.3	F_{P1}-*test*	0.06	0.21	0.44	0.69	0.79	0.91
	W_{P1}-*test*	0.05	0.26	0.67	0.92	0.96	1
	F_{P1}^*-*test*	0.06	0.21	0.44	0.69	0.79	0.91
0.05,0.05	F_{P1}-*test*	0.07	0.22	0.46	0.67	0.78	0.89
	W_{P1}-*test*	0.05	0.80	0.99	1	1	1
	F_{P1}^*-*test*	0.07	0.22	0.46	0.67	0.78	0.89

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1$ $n_1 = 15, n_2 = 20, n_3 = 25$							
		(μ_2, μ_3)					
σ_2^2, σ_3^2	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
1, 1	F_{P1}-*test*	0.04	0.18	0.40	0.64	0.78	0.93
	W_{P1}-*test*	0.04	0.20	0.41	0.66	0.78	0.94
	F_{P1}^*-*test*	0.04	0.18	0.39	0.63	0.77	0.92
1, 0.5	F_{P1}-*test*	0.04	0.21	0.44	0.67	0.81	0.96
	W_{P1}-*test*	0.06	0.26	0.52	0.74	0.84	0.98
	F_{P1}^*-*test*	0.04	0.21	0.44	0.66	0.80	0.96
1, 0.1	F_{P1}-*test*	0.04	0.21	0.49	0.69	0.82	0.97
	W_{P1}-*test*	0.07	0.33	0.61	0.79	0.87	0.97
	F_{P1}^*-*test*	0.04	0.20	0.48	0.68	0.81	0.96
0.5, 0.5	F_{P1}-*test*	0.05	0.24	0.51	0.77	0.86	0.96
	W_{P1}-*test*	0.05	0.26	0.58	0.84	0.89	0.99
	F_{P1}^*-*test*	0.06	0.23	0.48	0.76	0.84	0.95
0.5, 0.7	F_{P1}-*test*	0.06	0.22	0.50	0.71	0.84	0.97
	W_{P1}-*test*	0.06	0.20	0.49	0.76	0.86	0.98
	F_{P1}^*-*test*	0.06	0.22	0.48	0.68	0.83	0.96
0.1, 0.1	F_{P1}-*test*	0.08	0.29	0.56	0.80	0.88	0.96
	W_{P1}-*test*	0.07	0.67	0.97	1	1	1
	F_{P1}^*-*test*	0.08	0.27	0.53	0.77	0.84	0.94
0.1, 0.9	F_{P1}-*test*	0.06	0.18	0.46	0.76	0.87	0.97
	W_{P1}-*test*	0.07	0.13	0.43	0.79	0.83	0.98
	F_{P1}^*-*test*	0.06	0.19	0.45	0.74	0.84	0.95
0.5, 0.9	F_{P1}-*test*	0.04	0.19	0.45	0.70	0.85	0.96
	W_{P1}-*test*	0.05	0.17	0.43	0.70	0.84	0.97
	F_{P1}^*-*test*	0.04	0.19	0.44	0.69	0.83	0.94
0.3, 0.9	F_{P1}-*test*	0.04	0.20	0.45	0.74	0.85	0.98
	W_{P1}-*test*	0.04	0.17	0.42	0.76	0.84	0.98
	F_{P1}^*-*test*	0.03	0.20	0.44	0.72	0.83	0.96
0.3, 0.6	F_{P1}-*test*	0.06	0.25	0.52	0.79	0.88	0.97
	W_{P1}-*test*	0.07	0.24	0.55	0.88	0.91	0.99
	F_{P1}^*-*test*	0.06	0.24	0.50	0.76	0.85	0.96
0.1, 0.3	F_{P1}-*test*	0.08	0.26	0.56	0.82	0.88	0.97
	W_{P1}-*test*	0.06	0.39	0.85	0.98	0.99	1
	F_{P1}^*-*test*	0.08	0.24	0.51	0.78	0.84	0.96
0.05,0.05	F_{P1}-*test*	0.07	0.30	0.51	0.77	0.85	0.95
	W_{P1}-*test*	0.04	0.92	1	1	1	1
	F_{P1}^*-*test*	0.06	0.27	0.49	0.74	0.84	0.94

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1$
$n_1 = 25$, $n_2 = 20$, $n_3 = 15$

		(μ_2, μ_3)					
σ_2^2, σ_3^2	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
1, 1	F_{P1}-*test*	0.04	0.10	0.26	0.51	0.77	0.94
	W_{P1}-*test*	0.04	0.09	0.24	0.50	0.77	0.95
	F_{P1}^*-*test*	0.04	0.10	0.25	0.51	0.77	0.95
1, 0.5	F_{P1}-*test*	0.05	0.16	0.35	0.61	0.82	0.96
	W_{P1}-*test*	0.05	0.21	0.47	0.73	0.88	0.98
	F_{P1}^*-*test*	0.05	0.16	0.36	0.62	0.83	0.97
1, 0.1	F_{P1}-*test*	0.04	0.18	0.40	0.65	0.87	0.98
	W_{P1}-*test*	0.05	0.37	0.66	0.86	0.97	1
	F_{P1}^*-*test*	0.04	0.19	0.41	0.66	0.88	0.98
0.5, 0.5	F_{P1}-*test*	0.04	0.17	0.43	0.69	0.87	0.97
	W_{P1}-*test*	0.04	0.17	0.49	0.80	0.91	1
	F_{P1}^*-*test*	0.04	0.17	0.44	0.71	0.88	0.98
0.5, 0.7	F_{P1}-*test*	0.04	0.15	0.36	0.63	0.85	0.96
	W_{P1}-*test*	0.04	0.14	0.38	0.67	0.86	0.98
	F_{P1}^*-*test*	0.03	0.15	0.37	0.64	0.86	0.97
0.1, 0.1	F_{P1}-*test*	0.06	0.24	0.54	0.81	0.89	0.90
	W_{P1}-*test*	0.05	0.64	0.98	1	1	1
	F_{P1}^*-*test*	0.07	0.25	0.56	0.83	0.90	0.98
0.1, 0.9	F_{P1}-*test*	0.05	0.14	0.35	0.61	0.82	0.95
	W_{P1}-*test*	0.07	0.09	0.28	0.54	0.72	0.94
	F_{P1}^*-*test*	0.05	0.13	0.34	0.61	0.83	0.96
0.5, 0.9	F_{P1}-*test*	0.04	0.12	0.33	0.56	0.82	0.97
	W_{P1}-*test*	0.04	0.11	0.28	0.55	0.79	0.97
	F_{P1}^*-*test*	0.04	0.11	0.33	0.56	0.82	0.97
0.3, 0.9	F_{P1}-*test*	0.04	0.12	0.33	0.59	0.82	0.96
	W_{P1}-*test*	0.06	0.08	0.27	0.56	0.78	0.96
	F_{P1}^*-*test*	0.04	0.11	0.33	0.60	0.83	0.96
0.3, 0.6	F_{P1}-*test*	0.05	0.18	0.43	0.69	0.87	0.97
	W_{P1}-*test*	0.04	0.16	0.45	0.75	0.89	0.99
	F_{P1}^*-*test*	0.05	0.17	0.43	0.70	0.87	0.97
0.1, 0.3	F_{P1}-*test*	0.06	0.23	0.49	0.77	0.89	0.98
	W_{P1}-*test*	0.06	0.29	0.74	0.98	0.99	1
	F_{P1}^*-*test*	0.06	0.24	0.52	0.79	0.91	0.98
0.05,0.05	F_{P1}-*test*	0.06	0.25	0.52	0.77	0.89	0.96
	W_{P1}-*test*	0.06	0.91	1	1	1	1
	F_{P1}^*-*test*	0.06	0.26	0.54	0.79	0.90	0.97

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 6, \sigma_1^2 = 1, \mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$ $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 15$		(μ_5, μ_6)					
$\sigma_2^2, \ldots, \sigma_6^2$	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
(1)	F_{P1}-*test*	0.03	0.06	0.19	0.38	0.68	0.91
	W_{P1}-*test*	0.02	0.05	0.15	0.28	0.64	0.88
	F_{P1}^*-*test*	0.03	0.06	0.19	0.38	0.68	0.91
(2)	F_{P1}-*test*	0.04	0.14	0.36	0.65	0.92	0.98
	W_{P1}-*test*	0.04	0.12	0.28	0.59	0.94	0.99
	F_{P1}^*-*test*	0.04	0.14	0.36	0.65	0.92	0.98
(3)	F_{P1}-*test*	0.05	0.16	0.38	0.67	0.93	0.98
	W_{P1}-*test*	0.05	0.10	0.31	0.62	0.96	1
	F_{P1}^*-*test*	0.05	0.16	0.38	0.67	0.93	0.98
(4)	F_{P1}-*test*	0.04	0.09	0.18	0.42	0.70	0.91
	W_{P1}-*test*	0.06	0.06	0.12	0.25	0.57	0.86
	F_{P1}^*-*test*	0.04	0.09	0.18	0.42	0.70	0.91
(5)	F_{P1}-*test*	0.04	0.18	0.43	0.76	0.94	0.99
	W_{P1}-*test*	0.03	0.47	0.92	1	1	1
	F_{P1}^*-*test*	0.04	0.18	0.43	0.76	0.94	0.99
(6)	F_{P1}-*test*	0.04	0.09	0.24	0.53	0.83	0.96
	W_{P1}-*test*	0.04	0.06	0.13	0.31	0.70	0.94
	F_{P1}^*-*test*	0.04	0.09	0.24	0.53	0.83	0.96
(7)	F_{P1}-*test*	0.04	0.11	0.28	0.59	0.90	0.99
	W_{P1}-*test*	0.04	0.40	0.81	0.96	1	1
	F_{P1}^*-*test*	0.04	0.11	0.28	0.59	0.90	0.99
(8)	F_{P1}-*test*	0.07	0.24	0.48	0.75	0.90	0.97
	W_{P1}-*test*	0.04	0.58	0.97	1	1	1
	F_{P1}^*-*test*	0.07	0.24	0.48	0.75	0.90	0.97

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

i = 6, σ_1^2 = 1, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$ $n_1 = 15, n_2 = 15, n_3 = 30, n_4 = 30, n_5 = 45, n_6 = 45$							
		(μ_5, μ_6)					
$\sigma_2^2, \ldots, \sigma_6^2$	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
(1)	F_{P1}-*test*	0.04	0.19	0.55	0.86	0.98	1
	W_{P1}-*test*	0.05	0.16	0.49	0.83	0.98	1
	F_{P1}^*-*test*	0.03	0.18	0.52	0.84	0.98	1
(2)	F_{P1}-*test*	0.06	0.43	0.85	0.98	1	1
	W_{P1}-*test*	0.05	0.36	0.89	0.99	1	1
	F_{P1}^*-*test*	0.05	0.40	0.82	0.96	1	1
(3)	F_{P1}-*test*	0.06	0.40	0.87	0.99	1	1
	W_{P1}-*test*	0.05	0.33	0.90	1	1	1
	F_{P1}^*-*test*	0.06	0.36	0.84	0.97	1	1
(4)	F_{P1}-*test*	0.04	0.92	0.94	0.95	0.98	0.99
	W_{P1}-*test*	0.05	0.84	0.82	0.87	0.91	0.94
	F_{P1}^*-*test*	0.04	0.91	0.93	0.94	0.97	0.98
(5)	F_{P1}-*test*	0.06	0.47	0.93	0.99	1	1
	W_{P1}-*test*	0.05	0.88	1	1	1	1
	F_{P1}^*-*test*	0.06	0.41	0.89	0.97	0.99	1
(6)	F_{P1}-*test*	0.04	0.29	0.74	0.94	1	1
	W_{P1}-*test*	0.06	0.17	0.52	0.88	0.99	1
	F_{P1}^*-*test*	0.03	0.28	0.72	0.93	0.99	1
(7)	F_{P1}-*test*	0.04	0.32	0.84	0.99	1	1
	W_{P1}-*test*	0.06	0.71	0.99	1	1	1
	F_{P1}^*-*test*	0.04	0.30	0.82	0.97	1	1
(8)	F_{P1}-*test*	0.09	0.51	0.90	0.96	1	1
	W_{P1}-*test*	0.06	0.99	1	1	1	1
	F_{P1}^*-*test*	0.07	0.43	0.83	0.93	0.98	0.99

Table 3.10. *Powers for Gamma Distribution in i = 2, 3, and 6 (continued)*

$i = 6$, $\sigma_1^2 = 1$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$ $n_1 = 45, n_2 = 45, n_3 = 30, n_4 = 30, n_5 = 15, n_6 = 15$							
		(μ_5, μ_6)					
$\sigma_2^2, \ldots, \sigma_6^2$	Tests	(1, 1)	(1, 1.3)	(1, 1.5)	(1, 1.7)	(1.3, 2)	(1.3, 2.3)
(1)	F_{P1}-*test*	0.04	0.08	0.20	0.43	0.80	0.97
	W_{P1}-*test*	0.04	0.07	0.16	0.37	0.78	0.95
	F_{P1}^*-*test*	0.03	0.08	0.19	0.41	0.80	0.97
(2)	F_{P1}-*test*	0.04	0.15	0.43	0.75	0.98	1
	W_{P1}-*test*	0.05	0.10	0.38	0.74	0.99	1
	F_{P1}^*-*test*	0.04	0.15	0.43	0.75	0.98	1
(3)	F_{P1}-*test*	0.05	0.18	0.45	0.77	0.99	1
	W_{P1}-*test*	0.06	0.12	0.38	0.74	0.99	1
	F_{P1}^*-*test*	0.04	0.17	0.46	0.79	0.99	1
(4)	F_{P1}-*test*	0.04	0.09	0.22	0.47	0.86	0.98
	W_{P1}-*test*	0.07	0.07	0.12	0.27	0.70	0.94
	F_{P1}^*-*test*	0.04	0.09	0.21	0.45	0.86	0.98
(5)	F_{P1}-*test*	0.04	0.17	0.54	0.91	1	1
	W_{P1}-*test*	0.03	0.62	0.99	1	1	1
	F_{P1}^*-*test*	0.05	0.17	0.56	0.92	1	1
(6)	F_{P1}-*test*	0.04	0.11	0.30	0.59	0.92	0.99
	W_{P1}-*test*	0.05	0.07	0.17	0.37	0.83	0.98
	F_{P1}^*-*test*	0.04	0.11	0.29	0.57	0.92	0.99
(7)	F_{P1}-*test*	0.04	0.11	0.35	0.73	0.99	1
	W_{P1}-*test*	0.04	0.51	0.95	1	1	1
	F_{P1}^*-*test*	0.04	0.11	0.34	0.72	1	1
(8)	F_{P1}-*test*	0.06	0.19	0.59	0.90	1	1
	W_{P1}-*test*	0.05	0.73	1	1	1	1
	F_{P1}^*-*test*	0.07	0.20	0.62	0.93	1	1

Note. (1)1, 1, 1, 1, 1, 1; (2)1, 0.1, 0.1, 0.5, 0.5, 0.5; (3)1, 0.1, 0.2, 0.3, 0.4, 0.5; (4)1, 0.1, 1, 1, 1, 1; (5)1, 0.2, 0.4, 0.4, 0.2, 0.2, 0.1; (6)1, 0.5, 0.5, 0.5, 0.5, 1; (7)1, 0.3, 0.9, 0.4, 0.7, 0.1; (8)1, 0.01, 0.01, 0.06, 0.1, 0.1

3.4.2. Power Studies for Lognormal Distribution

Powers are evaluated for the log-normally distributed data with a specific mean of first group (μ_1= 1.3) and the others ranging from 1.3 to 2.8, and variance changing from 0.01 to 1 for *i* = 2, 3, and 6 and different sample sizes ranging from equal to unequal using Monte Carlo Simulation as explained in Section 3.1. For lognormal distribution, the simulations are generated for F_{P1}-test, W_{P1}- test, and F^*_{P1}-test.

Simulations reveal similar results with gamma distribution. It is seen that powers of all tests that are F_{P1}-test, W_{P1}- test, and F^*_{P1}-test are almost comparable for i = 2 with all sample size. For i = 3, W_{P1}-test shows more powerful attributes than the other tests where (σ_1^2 = 1, σ_2^2 = 1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.3), and (σ_1^2 = 1, σ_2^2 = 0.05, σ_3^2 = 0.05). In the other cases, all tests appear to have comparable power features. Like i = 3, for i = 6, W_{P1}-test reveals more powerful features than the other tests in some cases, specifically (σ_1^2 = 1, σ_2^2 = 0.2, σ_3^2 = 0.4, σ_4^2 = 0.4, σ_5^2 = 0.2, σ_6^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.3, σ_3^2 = 0.9, σ_4^2 = 0.4, σ_5^2 = 0.7, σ_6^2 = 0.1), and (σ_1^2 = 1, σ_2^2 = 0.01, σ_3^2 = 0.01, σ_4^2 = 0.06, σ_5^2 = 0.1, σ_6^2 = 0.1). On the other hand, W_{P1}-test exhibits less powerful properties than the other tests in two cases that are (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 1, σ_4^2 = 1, σ_5^2 = 1, σ_6^2 = 1) and (σ_1^2 = 1, σ_2^2 = 0.5, σ_3^2 = 0.5, σ_4^2 = 0.5, σ_5^2 = 0.5, σ_6^2 = 1). Similar to gamma distribution, the ratio of group variances may have a positive effect on the power of W_{P1}-test for i = 3. However, increased number of groups may lead to instability for power of W_{P1}-test for non-normal distributions.

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$ $n_1 = n_2 = 15$							
		(μ_2)					
σ_2^2	Tests	(1.3)	(1.6)	(1.9)	(2.2)	(2.5)	(2.8)
0.01	F_{P1}-*test*	0.06	0.28	0.55	0.73	0.83	0.90
	W_{P1}-*test*	0.06	0.28	0.55	0.73	0.83	0.90
	F^*_{P1}-*test*	0.06	0.28	0.55	0.73	0.83	0.90
0.05	F_{P1}-*test*	0.06	0.28	0.55	0.75	0.86	0.90
	W_{P1}-*test*	0.06	0.28	0.55	0.75	0.86	0.90
	F^*_{P1}-*test*	0.06	0.28	0.55	0.75	0.86	0.90
0.10	F_{P1}-*test*	0.06	0.29	0.59	0.75	0.87	0.90
	W_{P1}-*test*	0.06	0.29	0.59	0.75	0.87	0.90
	F^*_{P1}-*test*	0.06	0.29	0.59	0.75	0.87	0.90
0.20	F_{P1}-*test*	0.06	0.25	0.58	0.74	0.85	0.92
	W_{P1}-*test*	0.06	0.25	0.58	0.74	0.85	0.92
	F^*_{P1}-*test*	0.06	0.25	0.58	0.74	0.85	0.92
0.30	F_{P1}-*test*	0.06	0.22	0.55	0.74	0.87	0.93
	W_{P1}-*test*	0.06	0.22	0.55	0.74	0.87	0.93
	F^*_{P1}-*test*	0.06	0.22	0.55	0.74	0.87	0.93
0.40	F_{P1}-*test*	0.04	0.24	0.53	0.76	0.88	0.93
	W_{P1}-*test*	0.04	0.24	0.53	0.76	0.88	0.93
	F^*_{P1}-*test*	0.04	0.24	0.53	0.76	0.88	0.93
0.50	F_{P1}-*test*	0.04	0.19	0.50	0.71	0.88	0.94
	W_{P1}-*test*	0.04	0.19	0.50	0.71	0.88	0.94
	F^*_{P1}-*test*	0.04	0.19	0.50	0.71	0.88	0.94
0.60	F_{P1}-*test*	0.04	0.21	0.48	0.73	0.87	0.92
	W_{P1}-*test*	0.04	0.21	0.48	0.73	0.87	0.92
	F^*_{P1}-*test*	0.04	0.21	0.48	0.73	0.87	0.92
0.70	F_{P1}-*test*	0.04	0.18	0.46	0.71	0.88	0.93
	W_{P1}-*test*	0.04	0.18	0.46	0.71	0.88	0.93
	F^*_{P1}-*test*	0.04	0.18	0.46	0.71	0.88	0.93
0.80	F_{P1}-*test*	0.04	0.17	0.45	0.71	0.86	0.92
	W_{P1}-*test*	0.04	0.17	0.45	0.71	0.86	0.92
	F^*_{P1}-*test*	0.04	0.17	0.45	0.71	0.86	0.92
0.90	F_{P1}-*test*	0.04	0.16	0.42	0.68	0.86	0.94
	W_{P1}-*test*	0.04	0.16	0.42	0.68	0.86	0.94
	F^*_{P1}-*test*	0.04	0.16	0.42	0.68	0.86	0.94
1	F_{P1}-*test*	0.04	0.12	0.40	0.68	0.85	0.92
	W_{P1}-*test*	0.04	0.12	0.40	0.68	0.85	0.92
	F^*_{P1}-*test*	0.04	0.12	0.40	0.68	0.85	0.92

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$
$n_1 = 15$, $n_2 = 25$

σ_2^2	Tests	(μ_2) (1.3)	(1.6)	(1.9)	(2.2)	(2.5)	(2.8)
0.01	F_{P1}-*test*	0.06	0.27	0.55	0.74	0.85	0.89
	W_{P1}-*test*	0.06	0.26	0.55	0.73	0.85	0.88
	F_{P1}^{*}-*test*	0.06	0.26	0.55	0.73	0.85	0.88
0.05	F_{P1}-*test*	0.07	0.32	0.55	0.73	0.85	0.92
	W_{P1}-*test*	0.07	0.31	0.53	0.72	0.83	0.91
	F_{P1}^{*}-*test*	0.07	0.31	0.53	0.72	0.83	0.91
0.10	F_{P1}-*test*	0.06	0.29	0.58	0.76	0.89	0.94
	W_{P1}-*test*	0.06	0.27	0.56	0.73	0.87	0.93
	F_{P1}^{*}-*test*	0.06	0.27	0.56	0.73	0.87	0.93
0.20	F_{P1}-*test*	0.07	0.32	0.60	0.77	0.88	0.93
	W_{P1}-*test*	0.06	0.29	0.56	0.73	0.85	0.91
	F_{P1}^{*}-*test*	0.06	0.29	0.56	0.73	0.85	0.91
0.30	F_{P1}-*test*	0.06	0.30	0.60	0.80	0.90	0.94
	W_{P1}-*test*	0.05	0.29	0.57	0.76	0.88	0.92
	F_{P1}^{*}-*test*	0.05	0.29	0.57	0.76	0.88	0.92
0.40	F_{P1}-*test*	0.06	0.29	0.59	0.78	0.90	0.94
	W_{P1}-*test*	0.05	0.27	0.55	0.74	0.88	0.91
	F_{P1}^{*}-*test*	0.05	0.27	0.55	0.74	0.88	0.91
0.50	F_{P1}-*test*	0.05	0.25	0.55	0.79	0.90	0.96
	W_{P1}-*test*	0.05	0.24	0.52	0.76	0.87	0.94
	F_{P1}^{*}-*test*	0.05	0.24	0.52	0.76	0.87	0.94
0.60	F_{P1}-*test*	0.05	0.28	0.56	0.80	0.91	0.93
	W_{P1}-*test*	0.05	0.26	0.53	0.77	0.88	0.91
	F_{P1}^{*}-*test*	0.05	0.26	0.53	0.77	0.88	0.91
0.70	F_{P1}-*test*	0.05	0.24	0.54	0.76	0.90	0.95
	W_{P1}-*test*	0.05	0.23	0.52	0.73	0.87	0.93
	F_{P1}^{*}-*test*	0.05	0.23	0.52	0.73	0.87	0.93
0.80	F_{P1}-*test*	0.04	0.22	0.53	0.77	0.90	0.95
	W_{P1}-*test*	0.04	0.22	0.51	0.74	0.87	0.92
	F_{P1}^{*}-*test*	0.04	0.22	0.51	0.74	0.87	0.92
0.90	F_{P1}-*test*	0.04	0.19	0.51	0.76	0.90	0.97
	W_{P1}-*test*	0.03	0.20	0.51	0.74	0.88	0.94
	F_{P1}^{*}-*test*	0.03	0.20	0.51	0.74	0.88	0.94
1	F_{P1}-*test*	0.04	0.19	0.51	0.76	0.88	0.95
	W_{P1}-*test*	0.04	0.20	0.51	0.75	0.86	0.93
	F_{P1}^{*}-*test*	0.04	0.20	0.51	0.75	0.86	0.93

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$
$n_1 = 25$, $n_2 = 15$

σ_2^2	Tests	(μ_2) (1.3)	(1.6)	(1.9)	(2.2)	(2.5)	(2.8)
0.01	F_{P1}-*test*	0.06	0.36	0.69	0.87	0.94	0.97
	W_{P1}-*test*	0.06	0.36	0.69	0.87	0.94	0.98
	F_{P1}^{*}-*test*	0.06	0.36	0.69	0.87	0.94	0.98
0.05	F_{P1}-*test*	0.06	0.36	0.65	0.88	0.95	0.98
	W_{P1}-*test*	0.05	0.37	0.67	0.89	0.95	0.98
	F_{P1}^{*}-*test*	0.05	0.37	0.67	0.89	0.95	0.98
0.10	F_{P1}-*test*	0.04	0.34	0.64	0.85	0.95	0.97
	W_{P1}-*test*	0.05	0.36	0.65	0.86	0.96	0.98
	F_{P1}^{*}-*test*	0.05	0.36	0.65	0.86	0.96	0.98
0.20	F_{P1}-*test*	0.05	0.27	0.66	0.86	0.94	0.97
	W_{P1}-*test*	0.05	0.29	0.68	0.88	0.96	0.98
	F_{P1}^{*}-*test*	0.05	0.29	0.68	0.88	0.96	0.98
0.30	F_{P1}-*test*	0.04	0.26	0.67	0.84	0.94	0.97
	W_{P1}-*test*	0.05	0.26	0.69	0.86	0.96	0.98
	F_{P1}^{*}-*test*	0.05	0.26	0.69	0.86	0.96	0.98
0.40	F_{P1}-*test*	0.05	0.24	0.62	0.84	0.94	0.97
	W_{P1}-*test*	0.05	0.24	0.64	0.86	0.96	0.98
	F_{P1}^{*}-*test*	0.05	0.24	0.64	0.86	0.96	0.98
0.50	F_{P1}-*test*	0.04	0.22	0.57	0.84	0.92	0.97
	W_{P1}-*test*	0.04	0.22	0.59	0.86	0.94	0.98
	F_{P1}^{*}-*test*	0.04	0.22	0.59	0.86	0.94	0.98
0.60	F_{P1}-*test*	0.05	0.20	0.56	0.84	0.93	0.97
	W_{P1}-*test*	0.04	0.18	0.57	0.86	0.95	0.99
	F_{P1}^{*}-*test*	0.04	0.18	0.57	0.86	0.95	0.99
0.70	F_{P1}-*test*	0.04	0.18	0.55	0.79	0.94	0.98
	W_{P1}-*test*	0.04	0.17	0.53	0.80	0.95	0.99
	F_{P1}^{*}-*test*	0.04	0.17	0.53	0.80	0.95	0.99
0.80	F_{P1}-*test*	0.04	0.16	0.50	0.79	0.94	0.97
	W_{P1}-*test*	0.04	0.14	0.49	0.80	0.96	0.98
	F_{P1}^{*}-*test*	0.04	0.14	0.49	0.80	0.96	0.98
0.90	F_{P1}-*test*	0.04	0.15	0.46	0.79	0.93	0.96
	W_{P1}-*test*	0.04	0.14	0.44	0.79	0.94	0.97
	F_{P1}^{*}-*test*	0.04	0.14	0.44	0.79	0.94	0.97
1	F_{P1}-*test*	0.05	0.14	0.47	0.76	0.92	0.96
	W_{P1}-*test*	0.05	0.11	0.44	0.75	0.92	0.97
	F_{P1}^{*}-*test*	0.05	0.11	0.44	0.75	0.92	0.97

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$
$n_1 = n_2 = n_3 = 15$

		(μ_2, μ_3)					
σ_2^2, σ_3^2	Tests	(1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
1, 1	F_{P1}-*test*	0.03	0.10	0.33	0.65	0.79	0.93
	W_{P1}-*test*	0.03	0.10	0.34	0.67	0.82	0.96
	F^*_{P1}-*test*	0.03	0.10	0.34	0.65	0.79	0.93
1, 0.5	F_{P1}-*test*	0.04	0.16	0.44	0.74	0.83	0.92
	W_{P1}-*test*	0.04	0.18	0.53	0.82	0.88	0.97
	F^*_{P1}-*test*	0.04	0.16	0.44	0.74	0.83	0.92
1, 0.1	F_{P1}-*test*	0.04	0.20	0.48	0.76	0.86	0.94
	W_{P1}-*test*	0.06	0.33	0.68	0.87	0.92	0.97
	F^*_{P1}-*test*	0.04	0.20	0.48	0.76	0.86	0.94
0.5, 0.5	F_{P1}-*test*	0.04	0.18	0.52	0.82	0.88	0.95
	W_{P1}-*test*	0.04	0.20	0.61	0.90	0.94	0.98
	F^*_{P1}-*test*	0.04	0.18	0.52	0.82	0.88	0.95
0.5, 0.7	F_{P1}-*test*	0.04	0.14	0.46	0.78	0.86	0.95
	W_{P1}-*test*	0.04	0.13	0.46	0.82	0.90	0.98
	F^*_{P1}-*test*	0.04	0.14	0.46	0.78	0.86	0.95
0.1, 0.1	F_{P1}-*test*	0.07	0.27	0.61	0.84	0.88	0.95
	W_{P1}-*test*	0.06	0.54	0.95	1	1	1
	F^*_{P1}-*test*	0.07	0.27	0.61	0.84	0.88	0.95
0.1, 0.9	F_{P1}-*test*	0.04	0.12	0.46	0.78	0.88	0.95
	W_{P1}-*test*	0.07	0.08	0.38	0.76	0.84	0.97
	F^*_{P1}-*test*	0.04	0.12	0.46	0.78	0.88	0.95
0.5, 0.9	F_{P1}-*test*	0.04	0.12	0.41	0.73	0.87	0.95
	W_{P1}-*test*	0.04	0.11	0.37	0.74	0.87	0.97
	F^*_{P1}-*test*	0.04	0.12	0.41	0.73	0.87	0.95
0.3, 0.9	F_{P1}-*test*	0.04	0.12	0.45	0.75	0.88	0.95
	W_{P1}-*test*	0.05	0.09	0.36	0.74	0.87	0.97
	F^*_{P1}-*test*	0.04	0.12	0.45	0.75	0.88	0.95
0.3, 0.6	F_{P1}-*test*	0.04	0.18	0.53	0.82	0.89	0.95
	W_{P1}-*test*	0.03	0.15	0.55	0.89	0.94	0.99
	F^*_{P1}-*test*	0.04	0.18	0.53	0.82	0.89	0.95
0.1, 0.3	F_{P1}-*test*	0.06	0.24	0.61	0.83	0.88	0.94
	W_{P1}-*test*	0.06	0.26	0.83	0.99	0.99	1
	F^*_{P1}-*test*	0.06	0.24	0.61	0.83	0.88	0.94
0.05,0.05	F_{P1}-*test*	0.08	0.27	0.62	0.84	0.85	0.94
	W_{P1}-*test*	0.06	0.79	1	1	1	1
	F^*_{P1}-*test*	0.08	0.27	0.62	0.84	0.85	0.94

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$
$n_1 = 15$, $n_2 = 20$, $n_3 = 25$

σ_2^2, σ_3^2	Tests	(μ_2, μ_3) (1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
1, 1	F_{P1}-test	0.04	0.16	0.53	0.81	0.90	0.98
	W_{P1}-test	0.04	0.17	0.53	0.85	0.92	0.98
	F_{P1}^*-test	0.04	0.16	0.52	0.80	0.89	0.97
1, 0.5	F_{P1}-test	0.04	0.21	0.59	0.87	0.91	0.97
	W_{P1}-test	0.04	0.26	0.67	0.92	0.94	0.98
	F_{P1}^*-test	0.03	0.21	0.58	0.86	0.91	0.96
1, 0.1	F_{P1}-test	0.04	0.22	0.62	0.82	0.93	0.96
	W_{P1}-test	0.06	0.36	0.73	0.90	0.95	0.97
	F_{P1}^*-test	0.03	0.22	0.61	0.82	0.93	0.96
0.5, 0.5	F_{P1}-test	0.06	0.26	0.64	0.92	0.93	0.98
	W_{P1}-test	0.06	0.27	0.71	0.95	0.97	1
	F_{P1}^*-test	0.06	0.24	0.62	0.90	0.91	0.97
0.5, 0.7	F_{P1}-test	0.04	0.23	0.66	0.89	0.94	0.98
	W_{P1}-test	0.04	0.22	0.68	0.93	0.97	0.99
	F_{P1}^*-test	0.03	0.22	0.64	0.87	0.92	0.97
0.1, 0.1	F_{P1}-test	0.08	0.35	0.71	0.91	0.91	0.97
	W_{P1}-test	0.06	0.71	0.99	1	1	1
	F_{P1}^*-test	0.08	0.32	0.68	0.89	0.89	0.95
0.1, 0.9	F_{P1}-test	0.04	0.20	0.65	0.89	0.93	0.97
	W_{P1}-test	0.07	0.14	0.62	0.95	0.96	0.98
	F_{P1}^*-test	0.04	0.21	0.63	0.87	0.90	0.96
0.5, 0.9	F_{P1}-test	0.04	0.18	0.60	0.89	0.92	0.97
	W_{P1}-test	0.05	0.17	0.60	0.92	0.94	0.99
	F_{P1}^*-test	0.04	0.18	0.59	0.88	0.90	0.96
0.3, 0.9	F_{P1}-test	0.04	0.21	0.63	0.90	0.93	0.98
	W_{P1}-test	0.04	0.17	0.61	0.93	0.95	0.99
	F_{P1}^*-test	0.04	0.22	0.62	0.88	0.91	0.97
0.3, 0.6	F_{P1}-test	0.04	0.24	0.66	0.92	0.94	0.97
	W_{P1}-test	0.05	0.22	0.74	0.97	0.98	0.99
	F_{P1}^*-test	0.04	0.23	0.63	0.90	0.92	0.96
0.1, 0.3	F_{P1}-test	0.06	0.31	0.71	0.92	0.93	0.97
	W_{P1}-test	0.05	0.39	0.94	1	1	1
	F_{P1}^*-test	0.06	0.30	0.69	0.89	0.90	0.96
0.05,0.05	F_{P1}-test	0.07	0.32	0.68	0.88	0.91	0.95
	W_{P1}-test	0.06	0.90	1	1	1	1
	F_{P1}^*-test	0.07	0.30	0.66	0.86	0.90	0.94

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = 1.3$ $n_1 = 25, n_2 = 20, n_3 = 15$		(μ_2, μ_3)					
σ_2^2, σ_3^2	Tests	(1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
1, 1	F_{P1}-*test*	0.04	0.10	0.39	0.72	0.86	0.96
	W_{P1}-*test*	0.04	0.08	0.37	0.73	0.90	0.98
	F_{P1}^*-*test*	0.04	0.10	0.39	0.72	0.88	0.96
1, 0.5	F_{P1}-*test*	0.04	0.14	0.50	0.79	0.90	0.97
	W_{P1}-*test*	0.04	0.19	0.62	0.90	0.96	1
	F_{P1}^*-*test*	0.04	0.15	0.51	0.81	0.91	0.97
1, 0.1	F_{P1}-*test*	0.04	0.20	0.56	0.82	0.94	0.97
	W_{P1}-*test*	0.06	0.40	0.80	0.96	0.99	0.99
	F_{P1}^*-*test*	0.04	0.19	0.56	0.83	0.94	0.97
0.5, 0.5	F_{P1}-*test*	0.04	0.15	0.54	0.86	0.93	0.98
	W_{P1}-*test*	0.04	0.17	0.65	0.95	0.98	1
	F_{P1}^*-*test*	0.04	0.15	0.56	0.88	0.93	0.98
0.5, 0.7	F_{P1}-*test*	0.04	0.16	0.52	0.83	0.92	0.97
	W_{P1}-*test*	0.04	0.13	0.52	0.88	0.95	1
	F_{P1}^*-*test*	0.04	0.16	0.52	0.85	0.94	0.98
0.1, 0.1	F_{P1}-*test*	0.06	0.28	0.71	0.90	0.92	0.96
	W_{P1}-*test*	0.05	0.64	1	1	1	1
	F_{P1}^*-*test*	0.06	0.29	0.73	0.91	0.93	0.97
0.1, 0.9	F_{P1}-*test*	0.04	0.12	0.50	0.84	0.93	0.97
	W_{P1}-*test*	0.07	0.09	0.40	0.82	0.91	0.98
	F_{P1}^*-*test*	0.04	0.11	0.49	0.85	0.93	0.98
0.5, 0.9	F_{P1}-*test*	0.04	0.14	0.45	0.81	0.90	0.98
	W_{P1}-*test*	0.05	0.10	0.43	0.82	0.91	0.99
	F_{P1}^*-*test*	0.04	0.13	0.45	0.82	0.91	0.98
0.3, 0.9	F_{P1}-*test*	0.04	0.12	0.48	0.83	0.94	0.97
	W_{P1}-*test*	0.06	0.10	0.41	0.85	0.94	0.99
	F_{P1}^*-*test*	0.04	0.12	0.47	0.84	0.95	0.98
0.3, 0.6	F_{P1}-*test*	0.04	0.17	0.55	0.88	0.93	0.98
	W_{P1}-*test*	0.05	0.16	0.58	0.94	0.96	1
	F_{P1}^*-*test*	0.04	0.17	0.56	0.90	0.94	0.98
0.1, 0.3	F_{P1}-*test*	0.06	0.21	0.66	0.91	0.96	0.97
	W_{P1}-*test*	0.06	0.27	0.88	1	1	1
	F_{P1}^*-*test*	0.06	0.22	0.69	0.93	0.96	0.98
0.05,0.05	F_{P1}-*test*	0.06	0.26	0.68	0.88	0.93	0.98
	W_{P1}-*test*	0.05	0.88	1	1	1	1
	F_{P1}^*-*test*	0.06	0.27	0.70	0.89	0.93	0.98

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

i = 6, σ_1^2 = 1, $\mu_1 = \mu_2 = \mu_3 = \mu_4 =$ 1.3 $n_1 = n_2 = n_3 = n_4 = n_5 = n_6$ =15		(μ_5, μ_6)					
$\sigma_2^2, \ldots, \sigma_6^2$	Tests	(1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
(1)	F_{P1}-*test*	0.03	0.07	0.25	0.57	0.81	0.95
	W_{P1}-*test*	0.02	0.06	0.18	0.49	0.81	0.94
	F_{P1}^*-*test*	0.03	0.07	0.25	0.57	0.81	0.95
(2)	F_{P1}-*test*	0.04	0.15	0.51	0.88	0.97	0.98
	W_{P1}-*test*	0.05	0.10	0.44	0.88	0.99	1
	F_{P1}^*-*test*	0.04	0.15	0.51	0.88	0.97	0.98
(3)	F_{P1}-*test*	0.04	0.15	0.56	0.88	0.96	0.99
	W_{P1}-*test*	0.06	0.09	0.47	0.90	0.99	1
	F_{P1}^*-*test*	0.04	0.15	0.56	0.88	0.96	0.99
(4)	F_{P1}-*test*	0.03	0.08	0.29	0.61	0.86	0.97
	W_{P1}-*test*	0.06	0.07	0.19	0.43	0.76	0.93
	F_{P1}^*-*test*	0.03	0.08	0.29	0.61	0.86	0.97
(5)	F_{P1}-*test*	0.04	0.19	0.63	0.92	0.96	0.98
	W_{P1}-*test*	0.03	0.50	0.98	1	1	1
	F_{P1}^*-*test*	0.04	0.19	0.63	0.92	0.96	0.98
(6)	F_{P1}-*test*	0.04	0.09	0.38	0.75	0.94	0.98
	W_{P1}-*test*	0.04	0.05	0.23	0.54	0.86	0.97
	F_{P1}^*-*test*	0.04	0.09	0.38	0.75	0.94	0.98
(7)	F_{P1}-*test*	0.04	0.12	0.47	0.82	0.95	0.99
	W_{P1}-*test*	0.05	0.42	0.93	1	1	1
	F_{P1}^*-*test*	0.04	0.12	0.47	0.82	0.95	0.99
(8)	F_{P1}-*test*	0.07	0.23	0.64	0.88	0.93	0.95
	W_{P1}-*test*	0.05	0.58	0.99	1	1	1
	F_{P1}^*-*test*	0.07	0.23	0.64	0.88	0.93	0.95

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

i = 6, σ_1^2 = 1, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1.3$ $n_1 = 15$, $n_2 = 15$, $n_3 = 30$, $n_4 = 30$, $n_5 = 45$, $n_6 = 45$		(μ_5, μ_6)					
σ_2^2, ..., σ_6^2	Tests	(1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
(1)	F_{P1}-*test*	0.04	0.22	0.72	0.96	1	1
	W_{P1}-*test*	0.05	0.17	0.69	0.96	1	1
	F_{P1}^*-*test*	0.03	0.20	0.70	0.95	0.99	1
(2)	F_{P1}-*test*	0.06	0.40	0.95	0.99	1	1
	W_{P1}-*test*	0.05	0.36	0.96	1	1	1
	F_{P1}^*-*test*	0.05	0.38	0.92	0.98	0.99	1
(3)	F_{P1}-*test*	0.05	0.44	0.95	0.99	1	1
	W_{P1}-*test*	0.05	0.36	0.98	1	1	1
	F_{P1}^*-*test*	0.05	0.41	0.93	0.97	0.99	1
(4)	F_{P1}-*test*	0.04	0.22	0.74	0.98	1	1
	W_{P1}-*test*	0.04	0.17	0.72	0.98	1	1
	F_{P1}^*-*test*	0.03	0.22	0.73	0.96	0.99	0.99
(5)	F_{P1}-*test*	0.06	0.49	0.95	0.99	0.99	1
	W_{P1}-*test*	0.04	0.86	1	1	1	1
	F_{P1}^*-*test*	0.06	0.46	0.93	0.99	0.99	1
(6)	F_{P1}-*test*	0.04	0.28	0.86	0.99	1	1
	W_{P1}-*test*	0.04	0.18	0.72	0.98	1	1
	F_{P1}^*-*test*	0.04	0.27	0.84	0.98	1	1
(7)	F_{P1}-*test*	0.04	0.34	0.92	0.99	1	1
	W_{P1}-*test*	0.05	0.73	0.99	1	1	1
	F_{P1}^*-*test*	0.03	0.32	0.90	0.99	0.99	1
(8)	F_{P1}-*test*	0.08	0.56	0.93	0.98	0.99	1
	W_{P1}-*test*	0.04	0.98	1	1	1	1
	F_{P1}^*-*test*	0.07	0.48	0.87	0.95	0.98	0.99

Table 3.11. *Powers for Lognormal Distribution in i = 2, 3, and 6 (continued)*

$i = 6$, $\sigma_1^2 = 1$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1.3$ $n_1 = 45$, $n_2 = 45$, $n_3 = 30$, $n_4 = 30$, $n_5 = 15$, $n_6 = 15$		(μ_5, μ_6)					
$\sigma_2^2, \ldots, \sigma_6^2$	Tests	(1.3, 1.3)	(1.3, 1.6)	(1.3, 1.9)	(1.3, 2.2)	(1.6, 2.2)	(1.6, 2.8)
(1)	F_{P1}-*test*	0.04	0.07	0.29	0.72	0.92	1
	W_{P1}-*test*	0.05	0.07	0.24	0.64	0.90	0.98
	F_{P1}^*-*test*	0.04	0.06	0.28	0.71	0.92	0.99
(2)	F_{P1}-*test*	0.04	0.15	0.59	0.93	0.99	1
	W_{P1}-*test*	0.05	0.11	0.54	0.94	1	1
	F_{P1}^*-*test*	0.04	0.15	0.60	0.95	0.99	1
(3)	F_{P1}-*test*	0.05	0.16	0.62	0.95	0.99	1
	W_{P1}-*test*	0.06	0.10	0.57	0.96	1	1
	F_{P1}^*-*test*	0.05	0.15	0.64	0.96	1	1
(4)	F_{P1}-*test*	0.04	0.07	0.35	0.73	0.93	0.98
	W_{P1}-*test*	0.06	0.06	0.20	0.51	0.86	0.97
	F_{P1}^*-*test*	0.04	0.07	0.33	0.72	0.94	0.98
(5)	F_{P1}-*test*	0.04	0.15	0.75	0.96	0.99	1
	W_{P1}-*test*	0.04	0.61	1	1	1	1
	F_{P1}^*-*test*	0.04	0.16	0.77	0.98	0.99	1
(6)	F_{P1}-*test*	0.04	0.08	0.43	0.82	0.98	1
	W_{P1}-*test*	0.06	0.04	0.26	0.69	0.95	0.99
	F_{P1}^*-*test*	0.04	0.07	0.41	0.82	0.98	1
(7)	F_{P1}-*test*	0.05	0.11	0.58	0.93	0.98	1
	W_{P1}-*test*	0.06	0.56	0.99	1	1	1
	F_{P1}^*-*test*	0.05	0.10	0.57	0.92	0.98	1
(8)	F_{P1}-*test*	0.06	0.22	0.75	0.94	0.98	0.99
	W_{P1}-*test*	0.05	0.70	1	1	1	1
	F_{P1}^*-*test*	0.07	0.24	0.78	0.96	0.99	0.99

Note. (1)1, 1, 1, 1, 1, 1; (2)1, 0.1, 0.1, 0.5, 0.5, 0.5; (3)1, 0.1, 0.2, 0.3, 0.4, 0.5; (4)1, 0.1, 1, 1, 1, 1; (5)1, 0.2, 0.4, 0.4, 0.2, 0.2, 0.1; (6)1, 0.5, 0.5, 0.5, 0.5, 1; (7)1, 0.3, 0.9, 0.4, 0.7, 0.1; (8)1, 0.01, 0.01, 0.06, 0.1, 0.1

3.4.3. Power Studies for Normal Distribution

Powers are estimated for the normally distributed data with a specific mean of first group ($\mu_1 = 0$) and the others ranging from zero to 1.5, and variance changing from 0.01 to 1 for *i* = 2, 3, and 6 and different sample sizes ranging from equal to unequal using Monte Carlo Simulation as explained in the method section. For normal distribution, the simulations are generated for F_{P1}-test, W-test, and F^*-test.

The results show that powers of all tests that are F_{P1}-test, W-test, and F^*-test are nearly identical for i = 2 with equal and unequal sample size. For i = 3, W-test seems to be more powerful than the other tests where (σ_1^2 = 1, σ_2^2 = 1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.1, σ_3^2 = 0.3), and (σ_1^2 = 1, σ_2^2 = 0.05, σ_3^2 = 0.05). In the other cases, all tests display similar power attributes. Similar to i = 3, for i = 6, W-test exhibits more powerful properties than the other tests in some situations that are (σ_1^2 = 1, σ_2^2 = 0.2, σ_3^2 = 0.4, σ_4^2 = 0.4, σ_5^2 = 0.2, σ_6^2 = 0.1), (σ_1^2 = 1, σ_2^2 = 0.3, σ_3^2 = 0.9, σ_4^2 = 0.4, σ_5^2 = 0.7, σ_6^2 = 0.1), and (σ_1^2 = 1, σ_2^2 = 0.01, σ_3^2 = 0.01, σ_4^2 = 0.06, σ_5^2 = 0.1, σ_6^2 = 0.1). As explained in the "2.1.2 Welch' F test" part, W-test consists of weight function which is calculated using group sample size and variance. Obviously, the estimated power of W-test also depends on this weight function. Therefore, for a specified sample sizes, if the ratio of group variances is high, like our simulations, this may have a positive effect on the power of W-test.

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 0$ $n_1 = n_2 = 15$		(μ_2)					
σ_2^2	Tests	(0)	(0.1)	(0.2)	(0.5)	(0.7)	(1)
0.01	F_{P1}-*test*	0.04	0.06	0.09	0.44	0.68	0.94
	W-*test*	0.05	0.06	0.10	0.47	0.71	0.95
	F^*-*test*	0.05	0.06	0.10	0.47	0.71	0.95
0.05	F_{P1}-*test*	0.05	0.05	0.10	0.38	0.68	0.94
	W-*test*	0.06	0.06	0.11	0.40	0.70	0.95
	F^*-*test*	0.06	0.06	0.11	0.40	0.70	0.95
0.10	F_{P1}-*test*	0.04	0.06	0.09	0.41	0.68	0.92
	W-*test*	0.05	0.07	0.11	0.42	0.71	0.93
	F^*-*test*	0.05	0.07	0.11	0.42	0.71	0.93
0.20	F_{P1}-*test*	0.04	0.07	0.10	0.41	0.64	0.92
	W-*test*	0.04	0.07	0.10	0.42	0.65	0.93
	F^*-*test*	0.04	0.07	0.10	0.42	0.65	0.93
0.30	F_{P1}-*test*	0.04	0.05	0.10	0.36	0.62	0.89
	W-*test*	0.05	0.06	0.10	0.36	0.63	0.90
	F^*-*test*	0.05	0.06	0.10	0.36	0.63	0.90
0.40	F_{P1}-*test*	0.06	0.06	0.10	0.32	0.57	0.87
	W-*test*	0.05	0.06	0.10	0.32	0.58	0.88
	F^*-*test*	0.05	0.06	0.10	0.32	0.58	0.88
0.50	F_{P1}-*test*	0.05	0.05	0.09	0.31	0.55	0.86
	W-*test*	0.05	0.05	0.09	0.33	0.56	0.86
	F^*-*test*	0.05	0.05	0.09	0.33	0.56	0.86
0.60	F_{P1}-*test*	0.05	0.04	0.10	0.30	0.50	0.85
	W-*test*	0.04	0.05	0.10	0.30	0.51	0.86
	F^*-*test*	0.04	0.05	0.10	0.30	0.51	0.86
0.70	F_{P1}-*test*	0.05	0.04	0.09	0.29	0.50	0.82
	W-*test*	0.04	0.04	0.09	0.30	0.50	0.83
	F^*-*test*	0.04	0.04	0.09	0.30	0.50	0.83
0.80	F_{P1}-*test*	0.04	0.06	0.08	0.29	0.52	0.78
	W-*test*	0.04	0.06	0.09	0.31	0.52	0.78
	F^*-*test*	0.04	0.06	0.09	0.31	0.52	0.78
0.90	F_{P1}-*test*	0.05	0.06	0.09	0.26	0.50	0.75
	W-*test*	0.05	0.06	0.10	0.27	0.51	0.75
	F^*-*test*	0.05	0.06	0.10	0.27	0.51	0.75
1	F_{P1}-*test*	0.05	0.06	0.08	0.25	0.44	0.75
	W-*test*	0.05	0.06	0.08	0.26	0.45	0.76
	F^*-*test*	0.05	0.06	0.08	0.26	0.45	0.76

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 0$
$n_1 = 15$, $n_2 = 25$

σ_2^2	Tests	(μ_2) (0)	(0.1)	(0.2)	(0.5)	(0.7)	(1)
0.01	F_{P1}-*test*	0.05	0.06	0.10	0.42	0.69	0.93
	W-*test*	0.05	0.07	0.11	0.44	0.70	0.93
	F^*-*test*	0.05	0.07	0.11	0.44	0.70	0.93
0.05	F_{P1}-*test*	0.06	0.05	0.11	0.43	0.69	0.95
	W-*test*	0.06	0.06	0.11	0.44	0.71	0.96
	F^*-*test*	0.06	0.06	0.11	0.44	0.71	0.96
0.10	F_{P1}-*test*	0.04	0.06	0.11	0.39	0.72	0.93
	W-*test*	0.04	0.06	0.11	0.39	0.72	0.93
	F^*-*test*	0.04	0.06	0.11	0.39	0.72	0.93
0.20	F_{P1}-*test*	0.06	0.06	0.10	0.41	0.65	0.94
	W-*test*	0.05	0.05	0.10	0.41	0.66	0.94
	F^*-*test*	0.05	0.05	0.10	0.41	0.66	0.94
0.30	F_{P1}-*test*	0.06	0.07	0.10	0.38	0.66	0.92
	W-*test*	0.05	0.07	0.10	0.38	0.65	0.91
	F^*-*test*	0.05	0.07	0.10	0.38	0.65	0.91
0.40	F_{P1}-*test*	0.05	0.07	0.09	0.38	0.66	0.91
	W-*test*	0.04	0.07	0.09	0.37	0.66	0.91
	F^*-*test*	0.04	0.07	0.09	0.37	0.66	0.91
0.50	F_{P1}-*test*	0.05	0.08	0.12	0.40	0.66	0.88
	W-*test*	0.05	0.07	0.11	0.39	0.64	0.88
	F^*-*test*	0.05	0.07	0.11	0.39	0.64	0.88
0.60	F_{P1}-*test*	0.05	0.06	0.09	0.38	0.62	0.91
	W-*test*	0.04	0.06	0.08	0.36	0.60	0.90
	F^*-*test*	0.04	0.06	0.08	0.36	0.60	0.90
0.70	F_{P1}-*test*	0.05	0.06	0.10	0.36	0.63	0.87
	W-*test*	0.05	0.06	0.09	0.35	0.62	0.86
	F^*-*test*	0.05	0.06	0.09	0.35	0.62	0.86
0.80	F_{P1}-*test*	0.05	0.06	0.12	0.32	0.58	0.88
	W-*test*	0.05	0.06	0.11	0.32	0.58	0.87
	F^*-*test*	0.05	0.06	0.11	0.32	0.58	0.87
0.90	F_{P1}-*test*	0.06	0.06	0.10	0.33	0.56	0.88
	W-*test*	0.05	0.05	0.10	0.32	0.56	0.87
	F^*-*test*	0.05	0.05	0.10	0.32	0.56	0.87
1	F_{P1}-*test*	0.05	0.08	0.08	0.33	0.54	0.83
	W-*test*	0.05	0.07	0.08	0.31	0.53	0.82
	F^*-*test*	0.05	0.07	0.08	0.31	0.53	0.82

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 2$, $\sigma_1^2 = 1$, $\mu_1 = 0$ $n_1 = 25$, $n_2 = 15$							
		(μ_2)					
σ_2^2	Tests	(0)	(0.1)	(0.2)	(0.5)	(0.7)	(1)
0.01	F_{P1}-*test*	0.04	0.06	0.16	0.64	0.91	1
	W-*test*	0.05	0.07	0.16	0.65	0.92	1
	F^*-*test*	0.05	0.07	0.16	0.65	0.92	1
0.05	F_{P1}-*test*	0.05	0.06	0.15	0.63	0.89	1
	W-*test*	0.05	0.06	0.16	0.64	0.90	1
	F^*-*test*	0.05	0.06	0.16	0.64	0.90	1
0.10	F_{P1}-*test*	0.05	0.08	0.10	0.61	0.88	0.99
	W-*test*	0.05	0.08	0.10	0.62	0.88	0.99
	F^*-*test*	0.05	0.08	0.10	0.62	0.88	0.99
0.20	F_{P1}-*test*	0.04	0.06	0.14	0.56	0.82	0.99
	W-*test*	0.04	0.06	0.14	0.57	0.83	0.99
	F^*-*test*	0.04	0.06	0.14	0.57	0.83	0.99
0.30	F_{P1}-*test*	0.04	0.07	0.14	0.49	0.79	0.97
	W-*test*	0.05	0.07	0.14	0.52	0.80	0.97
	F^*-*test*	0.05	0.07	0.14	0.52	0.80	0.97
0.40	F_{P1}-*test*	0.05	0.07	0.12	0.44	0.76	0.95
	W-*test*	0.05	0.07	0.13	0.45	0.76	0.95
	F^*-*test*	0.05	0.07	0.13	0.45	0.76	0.95
0.50	F_{P1}-*test*	0.05	0.07	0.10	0.45	0.71	0.97
	W-*test*	0.05	0.07	0.11	0.46	0.71	0.97
	F^*-*test*	0.05	0.07	0.11	0.46	0.71	0.97
0.60	F_{P1}-*test*	0.06	0.07	0.10	0.39	0.68	0.92
	W-*test*	0.05	0.07	0.11	0.40	0.68	0.92
	F^*-*test*	0.05	0.07	0.11	0.40	0.68	0.92
0.70	F_{P1}-*test*	0.05	0.07	0.09	0.39	0.65	0.90
	W-*test*	0.05	0.07	0.09	0.39	0.64	0.90
	F^*-*test*	0.05	0.07	0.09	0.39	0.64	0.90
0.80	F_{P1}-*test*	0.05	0.06	0.13	0.35	0.62	0.88
	W-*test*	0.05	0.06	0.12	0.34	0.62	0.88
	F^*-*test*	0.05	0.06	0.12	0.34	0.62	0.88
0.90	F_{P1}-*test*	0.05	0.06	0.10	0.34	0.57	0.86
	W-*test*	0.05	0.06	0.10	0.34	0.57	0.86
	F^*-*test*	0.05	0.06	0.10	0.34	0.57	0.86
1	F_{P1}-*test*	0.05	0.05	0.10	0.32	0.56	0.84
	W-*test*	0.04	0.05	0.09	0.32	0.56	0.83
	F^*-*test*	0.04	0.05	0.09	0.32	0.56	0.83

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

i = 3, σ_1^2 = 1, $\mu_1 = 0$ $n_1 = n_2 = n_3 = 15$								
		(μ_2, μ_3)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
1, 1	F_{P1}-*test*	0.04	0.08	0.22	0.44	0.63	0.77	0.95
	W-*test*	0.04	0.08	0.23	0.45	0.63	0.76	0.94
	F^*-*test*	0.04	0.08	0.23	0.46	0.64	0.77	0.95
1, 0.5	F_{P1}-*test*	0.05	0.07	0.25	0.53	0.72	0.86	0.98
	W-*test*	0.06	0.09	0.35	0.62	0.76	0.91	0.97
	F^*-*test*	0.05	0.08	0.28	0.54	0.74	0.87	0.98
1, 0.1	F_{P1}-*test*	0.06	0.07	0.30	0.59	0.79	0.94	0.98
	W-*test*	0.06	0.14	0.55	0.84	0.94	0.99	0.96
	F^*-*test*	0.06	0.09	0.34	0.63	0.83	0.95	0.98
0.5, 0.5	F_{P1}-*test*	0.05	0.09	0.34	0.64	0.80	0.93	0.99
	W-*test*	0.06	0.11	0.42	0.71	0.80	0.96	0.98
	F^*-*test*	0.06	0.09	0.37	0.66	0.81	0.94	0.99
0.5, 0.7	F_{P1}-*test*	0.04	0.08	0.30	0.60	0.76	0.90	0.98
	W-*test*	0.05	0.10	0.33	0.63	0.72	0.91	0.98
	F^*-*test*	0.04	0.09	0.32	0.62	0.77	0.91	0.98
0.1, 0.1	F_{P1}-*test*	0.04	0.06	0.24	0.46	0.64	0.76	0.96
	W-*test*	0.05	0.06	0.23	0.46	0.64	0.76	0.95
	F^*-*test*	0.06	0.07	0.25	0.47	0.65	0.77	0.96
0.1, 0.9	F_{P1}-*test*	0.05	0.07	0.29	0.61	0.74	0.90	1
	W-*test*	0.04	0.06	0.32	0.64	0.65	0.91	1
	F^*-*test*	0.06	0.09	0.32	0.64	0.77	0.91	1
0.5, 0.9	F_{P1}-*test*	0.04	0.08	0.28	0.51	0.70	0.84	0.99
	W-*test*	0.06	0.09	0.29	0.51	0.65	0.83	0.99
	F^*-*test*	0.05	0.09	0.29	0.52	0.71	0.85	0.99
0.3, 0.9	F_{P1}-*test*	0.04	0.08	0.32	0.57	0.70	0.86	0.99
	W-*test*	0.05	0.08	0.32	0.58	0.63	0.88	0.99
	F^*-*test*	0.05	0.09	0.34	0.59	0.73	0.88	0.99
0.3, 0.6	F_{P1}-*test*	0.05	0.10	0.35	0.63	0.80	0.93	1
	W-*test*	0.05	0.11	0.42	0.70	0.77	0.95	0.99
	F^*-*test*	0.06	0.11	0.38	0.65	0.82	0.94	1
0.1, 0.3	F_{P1}-*test*	0.05	0.10	0.41	0.77	0.84	0.98	0.99
	W-*test*	0.05	0.16	0.75	0.97	0.90	1	1
	F^*-*test*	0.06	0.12	0.46	0.82	0.88	0.99	1
0.05,0.05	F_{P1}-*test*	0.05	0.07	0.47	0.82	0.91	1	1
	W-*test*	0.05	0.51	1	1	1	1	1
	F^*-*test*	0.07	0.11	0.60	0.92	0.94	1	1

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 3, \sigma_1^2 = 1, \mu_1 = 0$ $n_1 = 15, n_2 = 20, n_3 = 25$								
		(μ_2, μ_3)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
1, 1	F_{P1}-*test*	0.05	0.10	0.36	0.65	0.78	0.92	0.98
	W-*test*	0.05	0.10	0.36	0.64	0.77	0.92	0.97
	F^*-*test*	0.06	0.10	0.37	0.66	0.79	0.92	0.98
1, 0.5	F_{P1}-*test*	0.04	0.09	0.42	0.69	0.83	0.97	0.99
	W-*test*	0.04	0.10	0.49	0.76	0.87	0.98	0.97
	F^*-*test*	0.04	0.09	0.43	0.70	0.83	0.97	0.99
1, 0.1	F_{P1}-*test*	0.05	0.10	0.44	0.82	0.88	0.98	0.99
	W-*test*	0.05	0.15	0.63	0.92	0.95	1	0.98
	F^*-*test*	0.05	0.11	0.46	0.83	0.89	0.99	0.99
0.5, 0.5	F_{P1}-*test*	0.04	0.12	0.48	0.82	0.89	0.99	0.99
	W-*test*	0.04	0.14	0.58	0.89	0.89	1	0.99
	F^*-*test*	0.04	0.12	0.49	0.83	0.90	0.99	0.99
0.5, 0.7	F_{P1}-*test*	0.05	0.10	0.47	0.74	0.84	0.99	0.99
	W-*test*	0.05	0.11	0.52	0.79	0.82	0.99	0.99
	F^*-*test*	0.05	0.11	0.48	0.75	0.86	0.99	0.99
0.1, 0.1	F_{P1}-*test*	0.04	0.10	0.64	0.95	0.95	1	1
	W-*test*	0.04	0.43	1	1	1	1	1
	F^*-*test*	0.05	0.13	0.74	0.98	0.97	1	1
0.1, 0.9	F_{P1}-*test*	0.05	0.11	0.47	0.78	0.87	0.98	1
	W-*test*	0.06	0.12	0.56	0.85	0.80	1	1
	F^*-*test*	0.06	0.12	0.49	0.78	0.88	0.99	1
0.5, 0.9	F_{P1}-*test*	0.04	0.09	0.42	0.73	0.83	0.96	0.99
	W-*test*	0.04	0.10	0.43	0.74	0.78	0.97	0.99
	F^*-*test*	0.04	0.10	0.42	0.74	0.83	0.96	0.99
0.3, 0.9	F_{P1}-*test*	0.04	0.10	0.45	0.74	0.84	0.97	0.99
	W-*test*	0.04	0.11	0.48	0.79	0.80	0.98	1
	F^*-*test*	0.04	0.11	0.46	0.76	0.86	0.97	1
0.3, 0.6	F_{P1}-*test*	0.06	0.11	0.51	0.81	0.89	0.99	1
	W-*test*	0.04	0.12	0.62	0.88	0.88	1	1
	F^*-*test*	0.06	0.11	0.53	0.82	0.90	0.99	1
0.1, 0.3	F_{P1}-*test*	0.05	0.13	0.58	0.90	0.93	1	1
	W-*test*	0.05	0.26	0.92	1	0.98	1	1
	F^*-*test*	0.05	0.14	0.63	0.93	0.95	1	1
0.05,0.05	F_{P1}-*test*	0.05	0.10	0.65	0.95	0.94	1	1
	W-*test*	0.05	0.74	1	1	1	1	1
	F^*-*test*	0.07	0.14	0.75	0.98	0.98	1	1

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 3, \sigma_1^2 = 1, \mu_1 = 0$ $n_1 = 25, n_2 = 20, n_3 = 15$								
		(μ_2, μ_3)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
1, 1	F_{P1}-*test*	0.04	0.08	0.28	0.51	0.76	0.83	1
	W-*test*	0.04	0.08	0.29	0.52	0.76	0.83	1
	F^*-*test*	0.04	0.08	0.29	0.52	0.77	0.83	1
1, 0.5	F_{P1}-*test*	0.05	0.07	0.33	0.63	0.86	0.93	1
	W-*test*	0.05	0.10	0.43	0.73	0.89	0.96	1
	F^*-*test*	0.06	0.08	0.35	0.64	0.87	0.94	1
1, 0.1	F_{P1}-*test*	0.05	0.08	0.37	0.75	0.94	0.99	1
	W-*test*	0.05	0.17	0.71	0.95	0.99	1	1
	F^*-*test*	0.05	0.10	0.42	0.78	0.96	0.99	1
0.5, 0.5	F_{P1}-*test*	0.05	0.10	0.42	0.72	0.91	0.96	1
	W-*test*	0.05	0.13	0.49	0.78	0.89	0.97	1
	F^*-*test*	0.05	0.11	0.44	0.74	0.92	0.97	1
0.5, 0.7	F_{P1}-*test*	0.05	0.08	0.36	0.65	0.85	0.93	1
	W-*test*	0.05	0.09	0.38	0.65	0.80	0.92	1
	F^*-*test*	0.05	0.08	0.37	0.66	0.85	0.93	1
0.1, 0.1	F_{P1}-*test*	0.04	0.11	0.63	0.94	0.98	1	1
	W-*test*	0.05	0.35	0.99	1	1	1	1
	F^*-*test*	0.06	0.15	0.72	0.98	0.99	1	1
0.1, 0.9	F_{P1}-*test*	0.05	0.08	0.37	0.64	0.85	0.94	1
	W-*test*	0.05	0.09	0.37	0.65	0.78	0.93	1
	F^*-*test*	0.05	0.10	0.38	0.67	0.87	0.94	1
0.5, 0.9	F_{P1}-*test*	0.05	0.09	0.35	0.59	0.82	0.89	1
	W-*test*	0.05	0.09	0.32	0.57	0.77	0.87	1
	F^*-*test*	0.05	0.10	0.35	0.60	0.82	0.90	1
0.3, 0.9	F_{P1}-*test*	0.05	0.10	0.37	0.62	0.86	0.92	1
	W-*test*	0.04	0.09	0.37	0.61	0.80	0.91	1
	F^*-*test*	0.05	0.11	0.39	0.63	0.86	0.92	1
0.3, 0.6	F_{P1}-*test*	0.05	0.10	0.40	0.70	0.92	0.96	1
	W-*test*	0.06	0.11	0.44	0.76	0.89	0.97	1
	F^*-*test*	0.06	0.11	0.42	0.72	0.92	0.97	1
0.1, 0.3	F_{P1}-*test*	0.05	0.12	0.52	0.87	0.96	1	1
	W-*test*	0.06	0.19	0.77	0.97	0.96	1	1
	F^*-*test*	0.06	0.14	0.58	0.91	0.97	1	1
0.05,0.05	F_{P1}-*test*	0.05	0.09	0.66	0.98	0.99	1	1
	W-*test*	0.04	0.62	1	1	1	1	1
	F^*-*test*	0.07	0.13	0.79	1	0.99	1	1

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 15$								
		(μ_5, μ_6)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
(1)	F_{P1}-*test*	0.05	0.05	0.22	0.39	0.76	0.75	1
	W-*test*	0.05	0.06	0.22	0.37	0.74	0.71	1
	F^*-*test*	0.05	0.06	0.23	0.41	0.78	0.77	1
(2)	F_{P1}-*test*	0.06	0.08	0.37	0.71	0.98	0.96	1
	W-*test*	0.06	0.10	0.44	0.76	1	0.97	1
	F^*-*test*	0.07	0.10	0.43	0.78	0.99	0.98	1
(3)	F_{P1}-*test*	0.05	0.09	0.40	0.72	0.98	0.97	1
	W-*test*	0.06	0.09	0.42	0.74	0.99	0.97	1
	F^*-*test*	0.07	0.12	0.46	0.79	0.99	0.99	1
(4)	F_{P1}-*test*	0.04	0.07	0.22	0.42	0.78	0.76	1
	W-*test*	0.04	0.07	0.22	0.43	0.82	0.75	1
	F^*-*test*	0.04	0.08	0.26	0.46	0.82	0.80	1
(5)	F_{P1}-*test*	0.05	0.08	0.40	0.87	1	1	1
	W-*test*	0.05	0.22	0.94	1	1	1	1
	F^*-*test*	0.07	0.11	0.51	0.94	1	1	1
(6)	F_{P1}-*test*	0.04	0.07	0.29	0.54	0.88	0.84	1
	W-*test*	0.05	0.06	0.24	0.43	0.85	0.74	1
	F^*-*test*	0.05	0.09	0.32	0.57	0.90	0.87	1
(7)	F_{P1}-*test*	0.05	0.06	0.28	0.65	0.98	0.99	1
	W-*test*	0.05	0.21	0.89	1	1	1	1
	F^*-*test*	0.07	0.08	0.34	0.75	0.99	0.99	1
(8)	F_{P1}-*test*	0.05	0.08	0.50	0.90	1	1	1
	W-*test*	0.05	0.35	0.99	1	1	1	1
	F^*-*test*	0.08	0.15	0.78	0.99	1	1	1

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 3$, $\sigma_1^2 = 1$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ $n_1 = 15, n_2 = 15, n_3 = 30, n_4 = 30, n_5 = 45, n_6 = 45$								
		(μ_5, μ_6)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
(1)	F_{P1}-*test*	0.04	0.11	0.55	0.89	0.99	1	1
	W-*test*	0.04	0.09	0.54	0.87	0.99	1	1
	F^*-*test*	0.04	0.10	0.55	0.89	1	1	1
(2)	F_{P1}-*test*	0.05	0.05	0.17	0.90	1	1	1
	W-*test*	0.05	0.04	0.20	0.93	1	1	1
	F^*-*test*	0.07	0.06	0.21	0.91	1	1	1
(3)	F_{P1}-*test*	0.06	0.21	0.89	1	1	1	1
	W-*test*	0.05	0.22	0.92	1	1	1	1
	F^*-*test*	0.07	0.24	0.91	1	1	1	1
(4)	F_{P1}-*test*	0.04	0.13	0.59	0.89	1	1	1
	W-*test*	0.05	0.12	0.59	0.90	1	1	1
	F^*-*test*	0.06	0.14	0.61	0.91	1	1	1
(5)	F_{P1}-*test*	0.06	0.07	0.44	0.85	1	1	1
	W-*test*	0.05	0.24	0.96	1	1	1	1
	F^*-*test*	0.08	0.10	0.56	0.93	1	1	1
(6)	F_{P1}-*test*	0.04	0.07	0.30	0.56	0.89	0.85	1
	W-*test*	0.05	0.07	0.25	0.44	0.86	0.74	1
	F^*-*test*	0.04	0.08	0.33	0.59	0.91	0.87	1
(7)	F_{P1}-*test*	0.05	0.12	0.87	1	1	1	1
	W-*test*	0.06	0.40	1	1	1	1	1
	F^*-*test*	0.06	0.13	0.91	1	1	1	1
(8)	F_{P1}-*test*	0.06	0.19	0.98	1	1	1	1
	W-*test*	0.04	0.86	1	1	1	1	1
	F^*-*test*	0.10	0.28	1	1	1	1	1

Table 3.12. *Powers for Normal Distribution in i = 2, 3, and 6 (continued)*

$i = 3, \sigma_1^2 = 1, \mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ $n_1 = 45, n_2 = 45, n_3 = 30, n_4 = 30, n_5 = 15, n_6 = 15$								
		(μ_2, μ_3)						
σ_2^2, σ_3^2	Tests	(0, 0)	(0, 0.2)	(0, 0.5)	(0, 0.7)	(0.5, 1)	(0, 1)	(1.5, 1)
(1)	F_{P1}-*test*	0.05	0.09	0.23	0.47	0.87	0.83	1
	W-*test*	0.05	0.09	0.23	0.46	0.85	0.79	1
	F^*-*test*	0.05	0.09	0.23	0.47	0.87	0.83	1
(2)	F_{P1}-*test*	0.05	0.10	0.44	0.79	1	0.99	1
	W-*test*	0.06	0.12	0.47	0.79	0.99	0.99	1
	F^*-*test*	0.07	0.13	0.49	0.83	1	0.99	1
(3)	F_{P1}-*test*	0.05	0.09	0.51	0.83	1	0.99	1
	W-*test*	0.05	0.09	0.50	0.80	1	0.98	1
	F^*-*test*	0.06	0.12	0.58	0.87	1	0.99	1
(4)	F_{P1}-*test*	0.05	0.07	0.22	0.50	0.88	0.83	1
	W-*test*	0.05	0.08	0.22	0.48	0.87	0.79	1
	F^*-*test*	0.05	0.08	0.24	0.52	0.89	0.84	1
(5)	F_{P1}-*test*	0.04	0.08	0.59	0.96	1	1	1
	W-*test*	0.05	0.32	0.98	1	1	1	1
	F^*-*test*	0.07	0.12	0.70	0.98	1	1	1
(6)	F_{P1}-*test*	0.06	0.07	0.36	0.61	0.95	0.90	1
	W-*test*	0.06	0.06	0.27	0.48	0.92	0.78	1
	F^*-*test*	0.06	0.08	0.36	0.62	0.96	0.91	1
(7)	F_{P1}-*test*	0.05	0.06	0.35	0.82	1	1	1
	W-*test*	0.05	0.27	0.97	1	1	1	1
	F^*-*test*	0.06	0.09	0.44	0.86	1	1	1
(8)	F_{P1}-*test*	0.06	0.08	0.71	0.99	1	1	1
	W-*test*	0.05	0.38	1	1	1	1	1
	F^*-*test*	0.11	0.19	0.94	1	1	1	1

Note. (1)1, 1, 1, 1, 1, 1; (2)1, 0.1, 0.1, 0.5, 0.5, 0.5; (3)1, 0.1, 0.2, 0.3, 0.4, 0.5; (4)1, 0.1, 1, 1, 1, 1; (5)1, 0.2, 0.4, 0.4, 0.2, 0.2, 0.1; (6)1, 0.5, 0.5, 0.5, 0.5, 1; (7)1, 0.3, 0.9, 0.4, 0.7, 0.1; (8)1, 0.01, 0.01, 0.06, 0.1, 0.1

CHAPTER 4

AN APPLICATION: ATTACHMENT STYLE AND BODY MASS INDEX

In this section, the knowledge obtained in the previous chapter is employed to analyze the hypothesis on the association among gender, adults' attachment style and body mass index (BMI). BMI is a regular measurement that is used to compute an individual's level of body fat. The National Institute of Health (1998) defines four groups based on the BMI score; scoring between 14.50 and 18.49 means underweight, between 18.50 and 24.99 means healthy weight, between 25.00 and 29.99 means overweight, and being higher than 30 means obese.

The BMI of individuals can be influenced by many factors such as gender and attachment style of individuals. The body image is formed based on some societal expectations. For example, women are expected to be slim (Frederick, Forbes, Grigorian, & Jarcho, 2007), on the other hand, men are expected to be muscular (McCreary, Saucier, & Courtenay, 2005). Therefore, these may lead to women try to remain in underweight and men over-try to keep healthy or over-weight. Consistent with this, in the literature, it is indicated that men have significantly higher BMI scores than women (e.g. Frederick et al., 2007; Galambos, Leadbeater, & Barker, 2004; Ostry, Radi, Louie, & LaMontagne, 2006; Yates, Edman, & Aruguete, 2004).

The adult attachment style is one of the psychological factors that is thought to be related to BMI. Attachment theory argues that the interaction between a child and his/her main caregiver during the early years of that child shape his/her reactions based on emotional and behavioral and future social interaction (Bowlby, 1971). Hazan and Shaver (1987) extend the theory to adult romantic relationships. Adult attachment generally has two central dimensions that are secure and insecure attachment; and insecure attachment consists of attachment avoidance and attachment anxiety

(Mikulincer & Shaver, 2007). Attachment anxiety is defined as the feelings of being rejected and deserted, and the attempts at excessive intimacy, whereas attachment avoidance is stated as a withdrawal from intimate relationships and dependency (Harma & Sümer, 2016). Secure attachment is explained as both low attachment anxiety and avoidance (Pietromonaco, Uchino, & Schetter, 2013) and individuals who are securely attached feel relax while relying on others. Bartholomew and Horowitz (1991) suggest that insecure attachment may be divided into three groups based on the dimensions of anxiety and avoidance. Individuals who are fearful, high on both anxiety and avoidance, show an effort to prevent from attachment figures because of their abandonment fear. Individuals, who are preoccupied have high levels of attachment anxiety and low levels of attachment avoidance, show the need for proximity but with abandonment fear. Individuals, who are dismissing have low attachment anxiety and high attachment avoidance, show an effort to prevent from attachment figures while dismissing their underlying abandonment fear. In the literature, it is shown that attachment anxiety and avoidance do not differ on BMI based on their levels that are low and high (Ditzen et al., 2008). Similarly, Kiesewetter et al. (2012) show that securely and insecurely attached individuals do not differ in terms of BMI.

4.1. Data Description

The data is obtained from the study of Sakmar-Balkan & Kuru (2019) and consist of information on gender, adults' attachment styles, and their BMI. The study includes N = 541 participants (n_{men} = 270 and n_{women} = 271) who are couple. The means and standard deviations of BMI based on gender and attachment styles is shown in the Table 4.1.

Table 4.1. *Means and Standard Deviations for BMI*

Variable	M	SD
Gender		
Men	27.06	3.28
Women	25.88	4.31
Attachment Style		
Fearful	26.37	3.87
Preoccupied	25.77	3.86
Dismissing	27.11	4.17
Secure	26.66	3.71

4.1.1. Experiences in Close Relationships-Revised (ECR-R)

Experiences in Close Relationships-Revised (ECR-R) is used to assess the attachment styles of adults as anxiety and avoidance (Fraley, Waller, & Brennan, 2000). The measurement comprehends 36 items, in which 18 of them evaluate the attachment anxiety, whereas the other 18 of them evaluate the attachment avoidance. Low scores of both styles are assumed as secure attachment, though high scores of one or two styles are supposed as insecure attachment. The Turkish version of ECR-R is reliable and valid. It has the reliability coefficients as .86 for the attachment anxiety and .90 for attachment avoidance, and the test-retest reliability as .82 for the attachment anxiety and .81 for the attachment avoidance (Selçuk, Günaydın, Sümer, & Uysal, 2005).

Fraley (2012) suggests that ECR-R scores are divided into 4 groups based on the median of each style. When high anxiety scores match with high avoidance scores, it is labelled as fearful group. When low anxiety scores match with high avoidance scores, it is labelled as dismissing group. When high anxiety scores match with low avoidance scores, it is labelled as preoccupied group. When low anxiety scores match with low avoidance scores, it is labelled as secure group.

4.1.2. Body Mass Index (BMI)

Body Mass Index (BMI) is calculated based on the participants' self-report. The information about weight is on kilogram and height is on meters. BMI formula is given below:

$$BMI = \frac{kg}{m^2}$$

4.2. Analysis

In the analysis, F_{P1}-test, W_{P1}-test, and F_{P1}^{*}-test are used to compare BMI scores based on gender (i.e. men and women), i = 2, and the attachment style (i.e. fearful, preoccupied, dismissing, and secure), i = 4. The p-values of the tests are calculated using simulations including B = 1000 runs. Symbolic demonstration of variables are seen in Table 4.2. and Table 4.3.

Table 4.2. *Symbolic Demonstration of Gender Variable*

Men	Women
x_{11}	x_{21}
x_{12}	x_{22}
$\cdots$	$\cdots$
x_{1n_1}	x_{2n_2}

where x_{ij} is the BMI of individual j in group i, n_i is the size of group i. The p-values are calculated with plan 1 bootstrap methods for comparing two groups at a time.

Table 4.3. *Symbolic Demonstration of Attachment Variable*

Fearful	Preoccupied	Dismissing	Secure
x_{11}	x_{21}	x_{31}	x_{41}
x_{12}	x_{22}	x_{32}	x_{42}
$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{1n_1}	x_{2n_2}	x_{3n_3}	x_{4n_4}

where x_{ij} is the BMI of individual j in group i, n_i is the size of group i. The p-values are calculated with plan 1 bootstrap methods for comparing four groups at a time.

4.3. Results

The results show that there is a gender difference on BMI scores (p = 0.002 for F_{P1}-test, p = 0.002 for W_{P1}-test, and p = 0.002 for F^*_{P1}-test). Consistent with the literature (e.g. Frederick et al., 2007; Galambos et al., 2004; Ostry et al., 2006; Yates et al., 2004), men have significantly higher BMI scores than women. It is observed that p-values of F_{P1}-test, W_{P1}-test, and F^*_{P1}-test are same for the application, similar to our simulation findings with equal sample sizes for two groups.

The findings indicate that there is no difference in terms of adult attachment on BMI scores (p = 0.14 for F_{P1}-test, p = 0.17 for W_{P1}-test, and p = 0.15 for F^*_{P1}-test). Consistent with the literature findings (e.g. Ditzen et al., 2008; Kiesewetter et al., 2012), individuals who are attached fearful, preoccupied, dismissing, or secure do not differ regarding as BMI. It is observed that W_{P1}-test produced the largest p-values when i = 4. This is consistent with our simulation results that W_{P1}-test is conservative when variances are equal for i = 6.

CHAPTER 5

CONCLUSION

In this study, it is investigated which bootstrap method that can be used under the violation of assumptions when comparing group means. Generally, type 1 error rates of W-test, F^*-test, and the proposed bootstrap method with plan 1 are robust to both non-normality and normality in terms of Type I error. However, whereas F_{P1}-test, W_{P1}- test, and F^*_{P1}-test are almost comparable for i = 2, W_{P1}- test shows more or less powerful properties in some scenarios for i = 3 and i = 6 for non-normal distributions. On the other hand, W-test seems to be more powerful than F_{P1}-test and F^*-test for normal distribution. Type 1 error rates of one of the proposed bootstrap method, which is plan 2, fail to maintain the nominal significance level for both normal and non-normal distributions. To sum up, based on the information about type 1 error rate and power in the current study, W-test may be a better choice when comparing group means for normal distribution under the violation of homogeneity assumption, though there is not an exact method that is valid for all scenarios when comparing group means for non-normal distribution.

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