AN INVESTIGATION OF YOUNG CHILDREN'S INFORMAL FRACTION KNOWLEDGE

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ABSTRACT

AN INVESTIGATION OF YOUNG CHILDREN'S INFORMAL FRACTION KNOWLEDGE

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The purpose of this study was to investigate five-year-old children's informal knowledge about basic fraction concept in terms of part-whole relationship, one-half, one-third and quarter concept. The sample of the study consisted of 27 preschool age children living in Çankaya district of Ankara. The research has been realized by qualitative design and data were collected through interviews in one-to-one settings. The interview questions were prepared by the researcher under the supervision of both early childhood education and mathematics education researchers. Appropriate materials were prepared and used to represent the concepts of halves, thirds and quarters during the interviews.

Findings of the study indicated that, most of the five-year-old children had their own understanding about part-whole relationship and half concept based on their informal knowledge which can be considered as meaningful. Furthermore, according to perception of preschool children, half concept was related with the division of whole into two parts. In particular, equality of the parts was not important for preschool children in the half concept.

Also, it was observed that, children's knowledge about quarters was highly related with the concept of half. In addition, preschool children were aware of equal sharing of four parts among four persons through one-to-one matching. On the other hand, results indicated that for-five-year old children, concept of one-third could be considered as difficult.

Keywords: preschool children, informal knowledge, part-whole, fraction.

ÖZ

OKUL ÖNCESİ ÇOCUKLARININ DOĞAL KESİR BİLGİLERİNİN İNCELENMESİ

UTKU, Burcu
Yüksek Lisans, İlköğretim Bölümü
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Bu çalışmanın amacı 5 yaş çocuklarının doğal kesir bilgilerini parça-bütün ilişkisi, yarım, üçte-bir ve çeyrek kavramları açısından incelemektir. Bu çalışmanın örneklemini, Ankara'nın Çankaya ilçesinde yaşayan 27 okul öncesi yaş dönemi çocuğu oluşturmaktadır. Çalışmada nitel araştırma yöntemleri kullanılmış olup, çalışmanın verileri bire bir görüşmeler aracılığı ile toplanmıştır. Görüşme soruları araştırmacı tarafından okul öncesi eğitimi ve matematik eğitimi araştırmacılarının görüşleri doğrultusunda hazırlanmıştır. Yarım, üçte-bir ve çeyrek kavramalarını gösteren uygun materyaller hazırlanmış ve bu materyaller görüşmeler sırasında kullanılmıştır.

Bu çalışmanın bulgularına göre, 5 yaşındaki çocukların birçoğunun parça-bütün ilişkisi ve yarım kavramı hakkındaki doğal bilgileri doğrultusunda anlamlı sayılabilecek kavramaya sahip oldukları anlaşılmıştır. Ayrıca, okul öncesi dönemdeki çocukların algılarına göre, yarım kavramı bir bütünün ikiye bölünmesi ile alakalıdır. Özellikle, okul öncesi dönemdeki çocuklara göre parçaların eşitliği yarım kavramında önemli değildir.

Aynı zamanda, çocukların çeyrek kavramı hakkındaki bilgilerinin yarım kavramı ile ilişkili olduğu anlaşılmıştır. Buna ek olarak, okul öncesi dönemdeki çocuklar dört eşit parçayı dört kişi arasında bire-bir eşleştirme

yaparak eşit olarak paylaştırabilmektedirler. Diğer taraftan çalışmanın sonuçlarına göre üçte-bir kavramının 5 yaş çocukları için zor bir kavram olduğu görülmüştür.

Anahtar kelimeler: okul öncesi dönemdeki çocuklar, doğal bilgi, parça-bütün, kesir.

To my dear family

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LIST OF ABBREVIATIONS

ACARA: Australian Curriculum, Assessment and Reporting Authority

MoNE: Ministry of National Education

NAEYC: National Association for the Education of Young Children

NC: National Curriculum for England

NCTM: The National Council of Teachers of Mathematics

CHAPTER 1

INTRODUCTION

Early childhood is defined as a period during which children gain fundamental concepts and learn fundamental process skills (Charlesworth, Lind & Fleege, 2003). In other words, early years address the period, which children's cognitive, social, emotional and psychomotor development should occur efficiently. The early years programs do not only improve children's social skills, but also develop their academic skills related to language, mathematics, and science (Epstein & Barnett, 2012).

According to Worthington and Carruthers (2006) "children are born into a mathematical world" (p.147). In other words, as Anthony and Walshaw (2009) stated "children's development of mathematical competencies begins at birth" (p.107). From the time that child comes into the world, they need to experience their world from the firsthand (Davis & Tu, 2008). Children's curiosity leads them explore new things for better understanding of their environments. In this exploration process, young children's concepts grow and develop parallel to their physical, social and mental development (Charlesworth, Lind & Fleege, 2003). Children's interactions with their environment and acquisition of new information through observations construct their first mathematical experiences and development in early years.

1.1 Early Years and Mathematics

In early years, children's achievements about mathematical concepts start before they enter the school and their mathematical ideas are shaped based upon their relations with their environment, from the relations with peers, adults and their daily observations (Copley, 2000). According to Worthington and Carruthers (2006), children's earliest interaction for learning was mathematical as they explored the spaces, shapes and patterns.

Mathematical development of children is parallel to their changes that take place due to growth and experience (Charlesworth, Lind & Fleege, 2003). Jean Piaget's four-stage theory of cognitive development provides a better understanding and explanation about the children's mathematical development in early years. Based on Piaget's theory, ages between 2 and 6 are considered as preoperational stage (Piaget, 1963). Children's thinking are focused on symbols like words, pictures and they need to organize their thinking all over again while they pass through the sensory motor intelligence stage to preoperational stage (Crain, 2004). In the stage of preoperational thought, children's action on environment becomes more conceptual and their active participation is necessary for their development (Wadsworth, 1996). In this active participation process, while children interact with environment, they are willing to make observations and acquire new information. Children use their schemes to acquire new information and informally learn new contents through their everyday life. When children observe their parents and peers, they will be able to construct and understand new concepts from their previous information (Sheffield & Cruikshank, 2005).

It appears that much of children's knowledge is intuitive rather than formal (Sophian, 1999). In addition to that, in the construction of informal knowledge, importance of the informal learning cannot be disregarded. Usually, informal learning is obtained during naturalistic activities when an adult, older child or peer provides a comment or a question (Charlesworth, 2005). Moreover, children's informal knowledge helps them to construct their own understandings about the mathematics.

When children start preschool, they already have some knowledge about numbers and shapes from their daily experiences, even if it is not correct. In school environment, a bridge is provided between the child's informal knowledge of mathematics and the formal school mathematics by the teachers (Copley, 2000).

In early childhood mathematics, children use patterns, measuring materials, tangrams and especially blocks in school settings. Playing with blocks is considered as a central activity both in school and at home (Varol &

Farran, 2004). With the use of concrete objects like blocks, children develop understanding about classification, counting and fraction (Wolfgang, Stannard & Janes, 2001). Furthermore, through the concrete objects such as blocks or composed materials like puzzles, children formally learn basic fraction terms, such as whole and part in a classroom area context.

According to Charlesworth (2005), mathematical concepts such as shapes, numbers and part-whole relationship develop in early years. For preschoolers, above mentioned mathematical concepts could be supported with everyday exploratory activities, mathematics games and through literature experiences (Charlesworth, 2005). Especially, through free play activities with appropriate materials, children's informal and formal knowledge may merge for better understanding about mathematical concepts such as fractions. This addresses a need for understanding their informal knowledge about mathematical concepts.

1.2 Fractional Understanding in Early Childhood Mathematics

Fractions concept could be considered as an advanced mathematical concept in the early year's education. Conaway and Midkiff (2004) stated that "because of its symbolic nature, it is rather a difficult concept for students in early grades to master" (p. 69). Fractions are considered as an expression of the relationship between the part and the whole (Pitkethly & Hunting, 1996; Van De Walle, Karp & Bay-Williams, 2008). Additionally, children's first introduction for fractions is the part-whole model (Sheffield and Cruikshank, 2005). In other words, children's knowledge about the parts and the wholes in the preschool support their fractional concept development in the primary grades.

Partitioning, equal sharing and the concept of the parts and wholes are closely related in the fractions concept. Fischer (1990) described that "the words whole and parts emphasize the achievement of partitioning relationship in the early school years" (p.2). Through the partitioning, children learn and understand that whole of an object can be divided into equal parts and still maintain the same amount (Charleswoth, Lind & Fleege, 2003). Moreover, according to Charlesworth, Lind and Fleege (2003) "children have

a natural understanding and interest in parts and wholes that can be used later as a bridge to understand fractions" (p.179). It means that, part-whole relations, partitioning, and equal sharing strategies support children's basic knowledge about fractions. Furthermore, children's informal and formal knowledge about partitioning and equal sharing serve them to understand basic fraction concepts like half, one-third, and quarters.

1.3 Problem, Purpose and Significance of the Study

Even though the concept of fraction is rather complicated for the early years, past studies have shown that children are already familiar with the basic fraction concepts such as part-whole relationship, division of a whole into half, and partitioning of the equal parts based on their informal knowledge (Briuzela, 2005; Hunting & Sharpley, 1988a; Tirosh, Tsamir, Tabach, Levenson & Barkai, 2011; Wing & Beal, 2004). Furthermore, according to the researchers, children's both informal and formal knowledge in the early years are highly important for the development of fractional understanding in primary grades (Empson, 1999; Mack, 1990; Mack, 1995).

Since the Turkish Early Childhood Curriculum (MoNE, 2006) does not specifically support the development of fraction concepts and it only includes the parts and wholes concepts, children's informal knowledge from their daily experiences and their knowledge about parts and wholes play an important role for the future achievements in fractional understanding in primary grades. However, there are no known studies about five-year old children's fraction knowledge in Turkey in the accessible literature. The studies that were carried out abroad have addressed that understanding five-year-old children's informal fraction knowledge is essential for early education programs and elementary school mathematics (Empson, 1999; Mack, 1990; Mack, 1995).

Investigation of preschool children's informal knowledge about fraction concept is significant since they first meet part-whole relationship and one-half concept informally through their experiences. In general, it is believed that, for young children mathematical understanding emerges from intuition and informal learning (Anthony & Walshaw, 2009; Maxim, 1989; Sophian,

1999). With the help of the daily experiences and naturalistic activities, this informal knowledge is used as a bridge for formal knowledge while learning mathematics in preschool classes. Particularly, preschool children acquire fundamental mathematical concepts and skills such as shapes, number sense, comparing, spatial relations and, parts and wholes through thousands of experience both in and out of school. For that reason, revealing of children's informal knowledge about part-whole relationship, one-half, one-third and quarters is important for formal school teaching about fractions. Therefore, the purpose of the present study is to investigate five-year-old children's informal knowledge about the part-whole relationship, one-half, one-third and quarters.

This study intended to investigate five-year-old children's informal knowledge about fraction concepts by using age appropriate tasks, which allowed them to reveal their informal knowledge. The focus of this study was on children's informal knowledge about basic fraction concept, such as halves, thirds, and quarters using the concepts of the parts and the wholes and equal sharing. In order to reveal children's informal knowledge about the fraction concepts, the study was guided by the following research questions:

How do five-year old children perceive fraction concepts through their informal knowledge?

1. What is five-year old children's informal knowledge about the part-whole relation?
2. What is five-year old children's informal knowledge about the half?
3. What is five year old children's informal knowledge about one-third and quarter?

It is believed that the findings of the current study will make a contribution to early childhood education program in terms of supporting young children's mathematics knowledge and skills.

1.4 Definition of Terms

Early Childhood Education: Early childhood education refers to a period that promotes children's physical, emotional, mental and social development

and helps children to gain good practice before compulsory primary education starts (MoNE, 2006).

Informal Knowledge: The knowledge that emerges from everyday naturalistic activities, before being subject to a specific instruction**.**

Formal Knowledge: The knowledge that is gained in a school environment through specific instruction and it refers to the school-taught knowledge (Baroody & Wilkins, 1999).

Fractions: Fractions are the expression of part-whole relationship (Pitkethly & Hunting, 1996) and it emphasizes the importance of partitioning strategies in early years.

Parts and Wholes: A whole is a quantity consisting of parts which can be decomposed into them and these decomposed parts can be combined to form the whole (Baroody, 2003).

Equal sharing: Partitioning of the material equally among the number of participants (Wing & Beal, 2004).

Partitioning: Subdividing a continuous whole into equal parts (Pothier & Sawada, 1983).

Half: Division of the initial whole into two equal parts (Piaget, 1965).

CHAPTER 2

REVIEW OF LITERATURE

In this chapter, review of literature that addressing young children's fraction concept was presented. Studies about part-whole relationship, equal sharing and partitioning strategies in preschool and primary grades were reviewed in order to explain and highlight the importance of the development of fraction concept in early ages. In the first section, importance of mathematics in early childhood was presented. The second section was about preschool children's fractional understanding. In this section, children's understandings about the part-whole relationship, one-half, one-third and quarters were given. Studies about partitioning and equal sharing were also presented under the second section. In the final section, studies about fractional understanding at early ages in Turkey were described.

2.1 Early Years and Mathematics

Importance of the mathematical experiences in early childhood has gained considerable attention in recent years. The National Council of Teachers of Mathematics (NCTM) and National Association for the Education of Young Children (NAEYC) (2002) stated that "Mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning" (p.1). Furthermore, mathematics that children learn in preschool years have important impact on children's later mathematical learning (Akman, 2002; Charlesworth, 2005; Clements, 2001; Linder, Powers-Costello & Stegelin, 2011; Sophian, 2002; Unutkan, 2007; Varol & Farran, 2004). Young children are capable of learning mathematics since they directly observe the mathematical concepts in the natural learning environment (Linder, Powers-Costello & Stegelin, 2011) and they build and develop explicit mathematical knowledge through the specific mathematical activities (Clements, 2001). Besides, rich learning environments that are combined with appropriate

educational play materials such as blocks, puzzles, Lego's and real objects and assessment of the mathematical learning have enhanced young children's mathematical competencies in early years for their future achievements (Charlesworth, 2005; Unutkan, 2007; Varol & Farran, 2004). In acquiring the mathematical knowledge in preschool, importance of the educational play cannot be disregarded. Educational play materials are designed to teach specific skills and concepts such as part-whole relations, recognizing and learning colors, arranging items by size and understanding one-to-one correspondence through the play (Johnson, Christie & Yawkey, 1999). With the help of these materials, teachers can properly combine children's informal knowledge about mathematics to more formal knowledge of mathematics (Copley, 2000).

Mathematics education in early childhood arises both from informal experiences and planned activities (Anthony & Walshaw, 2009). Informal experiences of a children usually develop from the discoveries while they play with the objects around them (Maxim, 1989) and many children develop skills about numbers, counting and shapes before they enter school (Clements, 2001). In the school settings, while children continue to acquire mathematical knowledge through naturalistic activities, integration of the planned activities such as literature, art, music and movement support children's skills and arise their interest (Charlesworth, 2005). From theoretical point of view, Piaget's constructivist theory provides a foundation for instruction that follows the children's interests (Charlesworth, Lind & Fleege, 2003).

Piaget's constructivist theory mainly describes children's developmental levels according to their ages, while they simultaneously construct their own knowledge (Edwards, 2005). According to constructivist theory proposed by Piaget, children are not blank slates; they are rather creators of their own learning (Van de Walle, Karp & Bay-Williams, 2008). In other words, children acquire and construct knowledge through the interaction with environment (Charlesworth, Lind & Fleege, 2003). Piaget divides a child's construction of knowledge into three areas; physical knowledge that includes the characteristics of objects in terms of color,

weight and size; logico-mathematical knowledge that includes the classification and number; and social knowledge that includes the behavioral rules for social situations (Charlesworth, Lind & Fleege, 2003). In the construction of mathematical knowledge, children's intuition and spontaneous discoveries have great influences (Anthony & Walshaw, 2009; Charlesworth, Lind & Fleege, 2003).

In the light of constructivism, researchers stated that, mathematical learning of children starts from the birth (Anthony & Walshaw, 2009) and they begin developing mathematical understanding in the first few years of their lives (Varol & Farran, 2004). Maxim (1989) expressed that mathematical concepts in young children are developed through many experiences and with the help of repeated experiences; children expand their understandings about the objects around them. In this exploration process, children first come across with counting skill as a mathematical concept with the help of language (Maxim, 1989) and develop this cognitive skill spontaneously with practicing (Pepper & Hunting, 1998). In this cognitive development process, children acquire mathematical concepts informally through discovery (Varol & Farran, 2004).

In early childhood years, children's initial mathematical thoughts are based on their intuition (Güven, 2000) and an important part of mathematical learning is also based on their intuition (Squire & Bryant, 2002). Children's intuition serves them to construct informal knowledge about mathematical skills and ideas well before formal school mathematics (Akman, 2002; Baroody & Wilkins, 1999). Children learn much about quantities and their behavior informally from their everyday experiences through their concrete knowledge (Baroody & Wilkins, 1999). In school environment, with the help of these informal discoveries, children learn about mathematical symbols and manipulation of these symbols (Baroody & Wilkins, 1999).

2.2 Fractional Understanding in Early Childhood

Fractions concept in early grades is rather complicated because of its representational nature (Conaway & Midkiff, 2004). For this reason, children's cognitions about part-whole relationship are the key point for

fractions concept (Wing & Beal, 2004). Dividing quantities is the starting point of understanding of part-whole relationship. By studying the division of quantities, children can naturally learn part-whole relationship and quantify this as a fraction (Piaget, Inhelder & Szeminska, 1960).

Children's perceptions about the part-whole relationship and sharing are based on their intuition when they are learning fraction concept at early years. In the naturalistic activities, children explored the ideas that related with the sharing and at the end they became familiar with sharing (Dacey & Eston, 1999). For example, they are already familiar with dividing objects into two or more sets even when the sets are not equal through their informal experiences (Tirosh, Tsamir, Tabach, Levenson & Barkai, 2011). Hence, this type of informal experience may serve as a base for instruction in fraction concepts (Wing & Beal, 2004).

While learning part-whole relationship, using discrete, continuous and composite objects for the understanding about one-half, one-thirds and quarters play an important role. However, children's perception of the use of discrete and continuous objects produced different difficulties for the representation of fractions. In most of the studies (Hiebert & Tonnessen, 1978; Hunting & Sharpley, 1988; Piaget, Inhelder & Szeminska, 1960; Pothier & Sawada, 1983; Tirosh, Tsamir, Tabach, Levenson & Barkai, 2011; Wing & Beal, 2004), it was observed and suggested that part-whole representation and sharing of discrete objects such as sharing crackers or a piece of cake were considerably easier than the division of continuous objects such as a sheet of paper or whole of a cake.

In order to define young children's understandings about the part-whole interpretations, Piaget, Inhelder and Szeminska (1960) conducted an experimental research with children of ages between 4 and 7. Their purpose was to investigate children's understanding about the notion of equal fractional parts such as one-half, one-third, quarters, fifths and sixths. The focus of the experiment was the sharing of the same amount among the number of dolls. In order to explore this, Piaget, Inhelder and Szeminska used circular slab of modeling clay (continuous object) together with different number of dolls. Researchers wanted from children to cut the cake with a

wooden knife to share exactly the same amount among the dolls. The procedure first started with sharing of one-half among two dolls equally and the same procedure was repeated for one-third, quarters, fifths and sixths. The results of the study showed that many of the younger children had considerable difficulty in dividing the continuous object “the cake” into halves, thirds and quarters. In detail, children perceived the sharing process as, sharing the parts whether the parts are equal or unequal. Furthermore, in the sharing process, they left sum of the parts unshared because they did not anticipate the relation that the sum of the parts were equal to the whole of it.

In accordance with the results of the study, Piaget, Inhelder and Szeminska asserted that children’s understanding about part-part and part-whole relationship developed through several stages and these stages were explained by the Piaget with so-called “anticipatory scheme”. According to this scheme, children’s understanding of part-whole and part-part relationships begins and then continues through some stages intuitively. The first stage of the anticipatory scheme is rather easy for children since it requires only dividing into two parts, which is called dichotomy. It then naturally extends to division into fourths, which can also be named as successive dichotomy. However, the situation involving thirds is considered to be significantly more difficult then dividing into halves because it requires both anticipating and comparison of the three individual parts with each other.

Based on anticipatory scheme, in order to provide a better understanding about fractional parts, Piaget, Inhelder and Szeminska (1960) identified seven sub-concepts for fractions: (i) whole is divisible and composed from separate parts, (ii) a fraction implies a determinate number of parts, (iii) subdivision of a whole must end with no remaining part, (iv) relation between the number of parts and number of division is stable, (v) the concept of arithmetical fraction implies that all of the parts are equal, (vi) the fractions are the parts of the original whole and they also subdivided further and (vii) conservation of a whole must exist that is sum of the parts be equal the original whole. Hence, Piaget, Inhelder and Szeminska used these sub-concepts to make a classification about children’s responses to part-whole

and part-part tasks to identify the children's developmental level in understanding the fraction concept.

In the literature, the research about the part-whole interpretation of fraction carried out by Piaget, Inhelder and Szeminska (1960) was the beginning point for many studies (such as Hiebert & Tonnessen, 1978; Hunting & Sharpley, 1988b; Pothier & Sawada, 1983; Wing & Beal, 2004). The preceding studies used nearly the same procedures in order to assess the preschooler's cognitions about the fraction concepts. In detail, researchers focused on the equal sharing of the quantities among the different number of persons.

Hiebert and Tonnessen (1978) replicated and extended the study of Piaget, Inhelder and Szeminska and reported similar results from similar experiments. In addition, Pothier and Sawada (1983) investigated the partitioning strategies of children in order to understand their knowledge about the emergence of rational number ideas. Sample was consisted of eight kindergarten, eight grade-1, twelve grade-2 and fifteen grade-3 children. Researchers also used the same "Cake Problem" as Piaget, Inhelder and Szeminska (1960) used for the kindergarteners' partitioning strategies. In this study, cake problem was introduced to children in two different shapes; circular and rectangular. In order to explain the results, researchers proposed five-level theory, which included sharing, algorithmic halving, evenness, oddness and logical explanations about the partitioning strategies. The fifth level was not included in this study, since this level was at a hypothetical stage. According to this theory, kindergarteners took the action of partitioning in the first level which was called as "sharing". Therefore, in the first level children learned to partition into two and then extended this into dividing into halves in powers of two. In the second level, children partition the circular and rectangular shapes not only into halves and fourths but also eighths and sixteenths. In this process child has no concern about equality of the parts. In the third level, which was named as evenness, meaning of half becomes more meaningful to children and parts in even number can be obtained. Therefore, children would obtain twelfths from sixths or tenths from eighths. In the fourth level, children dealt with the odd

numbers such as three and five. They realized that they had to make initial cut to half to obtain three parts. Researchers also concluded that their results showed significant agreement with the results of Piaget, Inhelder, and Szeminska (1960) and Hiebert and Tonnessen (1978).

In another experimental study, Hunting and Sharpley (1988a) focused on preschoolers' knowledge about partitioning with one-half, one-third and quarters. Researchers introduced five different problem tasks by using discrete and continuous objects to twenty-two preschool children in order to assess their fractional understanding. Results of the study showed that even though children had a little understanding about the partitioning strategies with continuous objects, almost all of them showed great success while sharing discrete objects equally among the dolls. Moreover, Hunting and Sharpley (1988b) implemented a follow up study to validate their results with 206 preschoolers whose ages ranged between 3 years to 5 years. Two sets of problems were prepared for the interviews. In the first set, sharing of different materials between two, three and four dolls were introduced. In the second set, representations of one-half, one-third and quarters were given. Results showed that, in this age group, children had little understanding about the thirds and quarters and only a few of them had knowledge about the one-half. In detail, most of the participating children had lacked understandings about the notion of equal when sharing the quantities. On the other hand, considering discrete items, researchers found that children were able to share equally 12 crackers between the dolls one-by-one correctly. Moreover, according to researchers, even though children had different understandings about the meaning of share, they could solve sharing problems well before they were taught this concept in school settings.

While above mentioned studies focused on only the sharing of continuous and discrete objects for the assessment of children's fractional understandings, Wing and Beal (2004), compared children's perceptions of fraction concept using continuous, discrete and composite objects. Researchers proposed three different problem tasks about sharing of materials into halves and thirds to assess children's judgments about the sharing of portions. The experiments were conducted with ninety-four

preschoolers and sixty-three first graders. Results showed that children's performance was poorest with the continuous objects and best with the composite materials.

Since the subdivision of quantities started with the division into two, the notion of half played an important role. Tirosh, Tsamir, Tabach, Levenson and Barkai, (2011) investigated kindergarten children's cognitions about one-half considering that they were familiar to half concept from their daily experiences. Researchers explored the difference between the finding the half of continuous items and finding the half of discrete items. Results indicated that even though children had successfully found the half of discrete items, they divided the continuous objects into two sets that were not equal in size.

As mentioned before, children's informal knowledge has a great influence on their learning process of fractional concepts. In order to highlight the importance of their informal fraction knowledge, Empson (1999) aimed to explore children's informal knowledge of fractions and also how teachers inferred this informal knowledge in the classroom area. Children's equal sharing strategies and their informal knowledge were considered as key components in the study. Teacher-implemented lessons about equal sharing were implemented for five weeks and the researcher conducted interviews with children before and after the instructional unit. All lessons were planned based on solving equal-sharing problems, comparing unit fractions and identifying part-whole representations. The results showed that children's informal knowledge of fraction concepts in the early years helped them while learning fraction concepts in a formal school setting in 1st grade. This study was different from the other studies because the emphasis was given to children's informal knowledge and its advantages while they were learning equal sharing and fractions relations in a 1st grade classroom.

Despite the fact about the challenge of the representational nature of fractions in early years, the study carried out by Brizuela (2005) investigated preschool children's fraction knowledge in a quiet different context. The focus of the study was five and six-year-old children's notations for fractional numbers, specifically the notion of half. While past studies pointed out the

importance of the concepts such as equal sharing and part-whole relationship, the researcher investigated children's understandings about half concept and equal sharing using their own representational figures. Children were asked to draw symbols on paper for representing their own understanding about half concept and equal sharing. Results revealed that a great majority of the kindergarten students were able to express their own understandings of fractions with their own symbols.

2.3 Fractional Understanding at Early Ages in Turkey and International Curricula

Young children's understanding of fractional concepts was presented in the previous section considering the part-whole relationship, one-half, one-third, quarters, equal sharing and partitioning. In this section, implementations about the young children's understanding about fractional concept in Turkey introduced.

In Turkey, there are no known studies in the accessible literature about preschool children's basic fraction knowledge. However, there are some studies about mathematics education in early childhood in Turkey. For instance, Alisinanoğlu, Güven and Kesicioğlu, (2009) and Umay, (2003) focused on preschool teacher candidates views about teaching in mathematics at preschool. Moreover, Akman (2002) summarized the importance of mathematics in early years, and Güven (2000) summarized the importance of intuition and intuitive knowledge in mathematical learning. In addition, few studies focused on preschool children's mathematical knowledge in terms of number sense (Olkun, Fidan & Özer, 2013) and readiness to first grade mathematics (Unutkan, 2007). In addition to these studies, few Turkish researchers have focused on the literature on children's learning of fractions. For example, Toluk (2001) summarized the views on equal sharing and part-whole representations to make suggestions for the development of fraction concept in the second and third grades. Toluk (2001) also pointed out the difficulty of fraction concept in the early years even though the children typically come across the basic terms like "half" and "quarter" from their daily life. In summary, Toluk (2001) stressed the

importance of part-whole relationship and representations, and how these concepts could be combined with sharing activities for second and third grades with suggestions to teachers.

Although there are no known studies about preschool children's fraction concepts, Turkish Early Childhood Curriculum (MoNE, 2006) considers objectives and acquisitions expected to be gained by children in five different domains as cognitive, language, social-emotional, self-care, and psychomotor domain for 36-72 months children, part-whole relationship and half concept were emphasized in the cognitive domain. The objectives and acquisitions that are related to the part-whole relationship and half concept can be summarized as being able to understand part-whole relationship through expressing parts of a whole, dividing shapes into two equal parts, obtaining a whole with combining two halves, showing half of materials, and expressing the relationship between half and whole (MoNE, 2006).

In addition to these objectives and acquisitions that mentioned above, national preschool curriculum had some recommendations for development of preschool children. In the curriculum, it is stated that children's daily experiences provides great contribution to their development. Through the different stimulants, different materials and different activities that teachers provide, children's development is supported gradually (MoNE, 2006). For instance, with the use of stories that emphasize the sharing of the parts or with the use of different reconstruction games and play-doughs, teachers can support the acquisitions about part-whole relationship in early years.

Similar to Turkish Early Childhood Curriuculum (MoNE, 2006), the standards of The National Council of Teachers of Mathematics (NCTM, 2000) in USA, highlighted the fundamental concepts and skills; one-to-one correspondence, number sense and counting, logic and classifying, comparing, geometry, spatial relations and parts and wholes during prekindergarten period. Regarding the parts and wholes, NCTM (2000) expectations include that, from prekindergarten to grade 2, children develop a sense of whole numbers and represent it by composing and decomposing. In addition, they will also understand and represent the commonly used fractions as one-half, one-third and quarters until grade 2.

Besides, according to the National Curriuculum for England (NC, 1999, 2011) fractions are highlighted in the Key Stage 1 at age of 6. During Key Stage 1, mathematical concepts and skills build on the early learning goals, which gained before Key Stage 1. Therefore, before the age of six, prior experience of mathematics includes, number sense and counting, shapes, comparing, logic and classifying are expected to gain. During the Key Stage 1, students are able to understand that halving is the inverse of doubling and able to find one-half and one-quarter of shapes (NC, 1999).

Similar to National Curriculum for England (1999, 2011), according to Australian Curriculum, Assessment and Reporting Authority (ACARA, 2013), fractions are not achieved in the foundation year. Recognizing and describing the "one-half as two equal parts of whole" (ACARA, 2013) is achieved in Year 1.

In the light of these standards and curriculum, it can be concluded that, fundamental concepts and skills such as; number sense and counting, classifying, spatial understanding and shapes are the common for all children in cognitive development. On the other hand, fraction concepts are considered differently in above-mentioned curriculums. In the MoNE (2006) curriculum, part-whole relationship, dividing shapes into two equal parts, obtaining a whole with combining two halves, showing half of materials, and expressing the relationship between half and whole considered as an objective in cognitive domain for five-year-old. Nearly same acquisitions are also given in the NCTM (2000) standards under the fundamental concepts and skills for mathematics in prekindergarteners. On the other hand, NC (1999, 2001) and ACARA (2013) considered acquisition of fraction concept in the 1st grade.

2.4 Summary

In summary, the importance of the children's understandings about one-half, one-third and quarter have been researched in terms of part-whole perception and equal sharing strategies of children regarding their informal knowledge.

As can be seen in the above literature review, informal knowledge of the preschool children has great contribution to their mathematical achievement. The importance of this type knowledge is pointed by Empson (1999), Tirosh, Tsamir, Tabach, Levenson and Barkai, (2011) and Wing and Beal (2004) while researchers explained the part-whole perception and equal sharing strategies of preschoolers and 1st graders. In addition, it was observed that informal experiences and naturalistic activities related with part-whole relationship helped children while learning fraction in primary grades (Empson, 1999; Wing & Beal, 2004). Thus, in the present study, the researcher focused on children's informal knowledge while investigating their understandings about fraction concept.

Furthermore, in the past studies part-whole perception was considered as a central concept for preschoolers in order to understand their knowledge about one-half, one-third and quarters. It was asserted that, children's understandings about equal parts and sharing was related with the type of the material that was given to the child (Hunting & Sharpley, 1988a, 1988b; Piaget, Inhelder & Szeminska, 1960; Pothier & Sawada, 1983; Wing & Beal, 2004). According to researchers, since five-year-old age group did not anticipate the relation that the sum of the parts is equal to the initial whole, sharing of the continuous materials among two, three or four persons in terms of equal share seemed to be challenging for five-year-old children.

On the other hand, studies (Hunting & Sharpley, 1988a, 1988b; Pothier & Sawada, 1983; Tirosh, Tsamir, Tabach, Levenson & Barkai , 2011; Wing & Beal, 2004) showed that, in order to understand children's perceptions about one-half, one-third, quarters with the help part-whole relationship, using of discrete and composite materials seemed to be more appropriate for this age group. Therefore, in this study, children's understandings about fractions concept were investigated with the composite and discrete materials in order to capture their informal knowledge more effectively.

In the literature, there are also studies (Brizuela, 2005; Tirosh, Tsamir, Tabach, Levenson & Barkai , 2011) that focused on fractional understanding of preschoolers in terms of half concept. Even though Brizuela (2005)

examined the children's understandings about half with their own representational figures, both of the studies emphasized that children had their own understandings about half concept, since they were familiar with the word "half" from their daily lives.

In the light of these studies, the current study intends to investigate children's informal knowledge about part-whole relationship, one-half, one-third and quarter concept, since children's intuitive knowledge has an important contribution to their learning of fractions in the primary school.

CHAPTER 3

METHODS

The purpose of this study was to investigate five-year-old children's informal knowledge about basic fraction concept. The content of this chapter presents the methodology employed in the study including the design of the study, participants, data collection procedures and methods of analysis.

3.1 Design of the Study

This study aimed to investigate five-year-old children's informal fraction knowledge by using age appropriate tasks prepared to explore their informal knowledge about basic fraction concept. The study was guided by the following research questions:

How do five-year old children perceive fractional concepts through their informal knowledge?

1. What is five-year old children's informal knowledge about the part-whole relation?
2. What is five-year old children's informal knowledge about the half?
3. What is five year old children's informal knowledge about one-third and quarter?

In order to respond to these questions, qualitative methods have been utilized for both the data collection and data analysis procedures.

3.2 Participants

In order to explore five-year-old children's informal knowledge about fractions, purposive and convenient sampling methods were utilized for choosing the participants of this study. Five-year old children in this study were the children who were 48-60 months old. The sample of this study

consisted of 28 five-year-old children. Fifteen girls and thirteen boys have participated in this study from two different primary schools in Çankaya district in Ankara.

The first school had 64 children in the five-year-old age group at the time of the study and 21 of these children participated in the study based on parents' consent. The second school had 28 children in the five-year-old age group and data were obtained from only one representative class for the second school. This class had 13 students and only 7 of them participated in the study after their parents consented. Parents of the participants belonged to middle or upper-middle socio-economical class.

Private schools were selected for this study based on their convenience for the researcher. The researcher was the graduate of one of the schools and for the other school she started working as a preschool teacher. First, administration of the both schools was informed about the research procedure. Ethical permission for the research which was taken from METU Ethical Commission was introduced to the administration of both schools. Then, five-year-old age preschool teachers were informed about the study and the parents' consent forms were delivered to parents with the help of teachers. After receiving parents' consents, participant's (children's) lists were constructed, and each interview appointment was planned with the help of preschool teachers in each school.

3.3 Data Collection Procedures

The data of this study were collected through structured interviews with twenty-eight five-year-old children in one-to-one settings. Details of the data collection procedures are presented below.

3.3.1 Pilot Study

Three pilot studies were conducted in order to finalize the interview questions for the main study. All of the pilot study participants were chosen from five-year old girls of a rhythmic gymnastic team. The researcher of this study was also a trainer of this sports club at the time of the study. However, the pilot study participants were chosen from a group trained by a different

person. Each interview took approximately 10 minutes. For each participant, an audio recorder recorded all interview sessions.

After the first pilot study, under the supervision of both early childhood education and mathematics education researchers, some modifications were made according to the children's answers. As a result, order of some of the questions was changed and new materials were added to the study. Similar interview procedures were implemented during the second and the third pilot studies, and the main interview questions and related sub-questions were finalized.

The final form of the interview protocol consisted of five general questions about parts, wholes, and also consisted of four tasks about halves, thirds, and quarters. In addition, each task consisted of different sub-questions. Therefore, first task consisted of seventeen questions about halves and equal concept, second task consisted of five questions about thirds, third task consisted of seven questions about quarters and finally fourth task consisted of four questions about conservation of half concept. Detailed information about the final version of the interview questions was presented in the following section.

3.3.2 Interview Protocol

In order to investigate children's informal fraction knowledge, based on the related literature (Hunting and Sharpley, 1988a, 1988b; Piaget, Inhelder & Szeminska, 1960; Tirosh, Tsamir, Tabach, Levenson & Barkai, 2011; Wing and Beal, 2004) and Turkish Early Childhood Curriculum (MoNE, 2006) the researcher prepared questions and tasks related to the basic fraction concepts. Furthermore, materials that can be composed and decomposed were prepared and used to represent the concepts of halves (½), quarters (¼), and thirds (1/3) during the interviews. In this study, fraction concept not only refered to the halves (½), quarters (¼) and thirds (1/3), but also to equal sharing and partitioning of equal parts. Parts and wholes were the key part of the all tasks and the half was the central part of this study while assessing children's informal fraction knowledge.

The interview process consisted of thirty-eight questions. Questions were arranged in the order from easy to difficult. At the beginning of the study, five general questions were asked to each participant. These questions covered the definitions of parts and wholes and identification of the objects which were related with the parts and wholes. These questions are as follows:

1. Can you tell me what a part is?
2. Can you tell me what a whole is?
3. Can you give any examples to parts and wholes?
4. Do you know these materials names and can you tell me which one of them consists of parts/wholes?
5. What comes to your mind when someone says half of something?

After the general questions, tasks about the halves, thirds and quarters were introduced. First task consisted of three parts. These parts were (i) circle part, (ii) fruits and vegetables part, and (iii) square part. Details of the parts are given below.

In the “circle part”, two circles were shown to children. One of the circles was equally divided into two parts and the other one was only divided into two unequal parts. Equal part concept was asked by using these objects. The questions are presented in Table 3.1.

In the second part, some preselected toys representing the parts, wholes and halves were given to each child. This set of toys consisted of one whole fruit (banana), four pieces of fruits and vegetables (lemon, orange, tomato) which could be divided into two equal parts, and some fruits which could not equally be divided into two parts (carrot, cucumber and pear). The cases here included the objects where the whole object was either divided into two equal parts or two unequal parts. The children were then asked to answer whether the parts were the halves of the whole object or not. The questions are presented in Table 3.1.

In the last part which was named as “square part”, square shapes were given to each child in order to introduce the relationship between halves and quarters. Then, they were asked to make a big square by using the given

four equal squares. Following this, they were asked to divide the big square into half. Lastly, it was asked whether they would separate this half into the half of it again. The questions are presented in Table 3.1.

In the second task, in order to give the 1/3 concept three rectangular shapes were shown to children. They were asked to make a big rectangular shape by using these three equal rectangles. Finally, they were asked to name this shape which consisted of three equal parts.

In the third task, the emphasis was given to the concept of dividing a whole into half and equal sharing of these four parts among four people. The quarter concept was introduced using the half concept. The questions are presented in Table 3.1.

In the final task (4^{th} task), two geometric shapes with different sizes were presented. These shapes were equally divided into two parts. It was asked whether each half of an object changed as the size of the object became larger. The questions are presented in Table 3.1.

Table 3.1 The tasks and the related questions about parts and wholes.

The Tasks	Questions
General Questions: Pre selected toys about parts and wholes are presented during the questions. One candle in square shape (whole), Salt-cellar set (consist of three parts and also a whole), one circular shape (whole), matryoshka doll (parts and wholes).  	**1.** Can you tell me what a half is? **2**. Can you tell me what a whole is? **3.** Can you give any examples to parts and wholes? **4.** Do you know these materials name and can you tell me which one of them consists of parts/wholes? **5.** What comes to your mind when someone says half of a something?

Table 3.2 The tasks and the related questions about halves.

1st Task	
The Tasks	**Questions**
Circle Part (Meaning of "Equal") Two circles are presented, one of the circles is equally divided into two parts and the other one is only divided into two parts. 	**1.** Do you think that there is a difference between these two shapes? **2.** How many parts are there in these circles? **3.** Which one is divided into two equal parts? **4.** What is an equal?
Fruits and Vegetables Part (Meaning of "Half") Set of toys consists of one whole fruit (banana), four pieces of fruits and vegetables (lemon, orange, tomato) which can be divided into two equal parts, and some fruits which can not equally be divided into two parts (carrot, cucumber and pear) are presented. 	**1.** Which one of them is part/whole? **2.** Do you think that the parts are the halves of the objects or do you think that they are divided into simply two parts? **3.** What is a half?

Table 3.2 Continued

1st Task	
The Tasks	**Questions**
Square Part (Half of an Object) Four squares, all of them are in the same size for halves and quarters. These squares are presented separately to each child. 	**1.** How many squares are there? Do you think that their sizes are equal? **2.** Can you make a big square by using these four squares? **3.** Can you divide them into half? **4.** Do you think that you can divide these two parts again into half?

Table 3.3 The task and the related questions about one-third.

2nd Task	
The Task	**Questions**
Three rectangular shapes, all of them in same size are presented for 1/3. 	**1.** How many pieces are there? **2.** Do you think that these pieces are equal to each other? **3.** Can you built up a rectangular shape by using these three rectangles? **4.** How can you name this rectangular shape consisting of three equal parts?

Table 3.4 The task and the related questions about quarters.

3rd Task	
The Task	**Questions**
Whole circular region, divided into four equal parts is presented. The quarter concept is given through the use of half concept.  	**1.** Can you tell me the number of the parts of this shape? **2.** Do you think that these parts are equal to each other? **3.** If you want to divide this shape into two equal parts, how do you separate it? **4.** Considering four persons, do you share these parts equally to everyone? **5.** What is the number of pieces for each person after you share with four people? **6.** How do you name this shape, which consists of four equal parts which can be shared equally among four persons?

Table 3.5 The task and the related question about conservation of half concept.

4th Task	
The Task	**Question**
Two geometric shapes with different size are presented. These shapes are equally divided into two parts. 	**1.** Do you think that half of an object changes as the size of the object increases?

3.3.3 Data Collection

The data collection procedure was completed in a two-month period from May 2012 to June 2012. Before carrying out the study, ethical permission of the study was obtained from Middle East Technical University Research Center for Applied Ethics (METU-RCAE). The plan of the study, related questions and the parents' consent forms were delivered for the permission. To protect the rights of the informants, parents' consent forms were delivered to the children's parents after permission of the study was obtained from the METU-RCAE. The study was conducted after parents consented for their children's participation.

The interviews were conducted in an empty classroom which was assigned by the school administrations. The child and the researcher were alone in the classroom and no interruptions occurred during the interview sessions. At the beginning of each interview, the researcher first introduced herself to the child. All children were asked whether they were willing to participate in the interview although their parents gave their permission. Additionally, they were told that they could stop or take a break if they were bored. During the data collection procedure, only one child stated that she did not want to answer the questions. Therefore, the child did not take part in this study.

The following explanation was stated before the general question part of the study for each child:

> Today we are going to make an enjoyable activity with some materials and shapes that you are already familiar. During this activity, I am going to ask you some questions. These questions do not have any right or wrong answers. I am just wondering about your thoughts and answers about these questions. There is no time limitation, you can observe and use every material as long as you need. I have to record our conversations, because I need your answers for my work.

After this statement, interviews started. Each child was asked the same question in the same task with the same order. In some cases, according to the children's answers, some additional questions were asked to children and they were given more time to think more about their responses. Each interview took approximately 15 minutes. Data obtained from the one-

to-one interviews were audio-taped and answers related with the materials representing the one-half, one-third and quarters which were photographed.

3.4 Data Analysis

The data obtained from the one-to-one interviews were transcribed from the audio recordings. Researcher transcribed every answer of the participants carefully. After this process, participants' answers read carefully a few times to form the codes of the data correctly. In order to reach a common understanding, the codes of the study were constructed based on the participant's answers for each question. Therefore, coding refers to the participants' informal knowledge and definitions about the part-whole, equal, half, one-third and quarters. Furthermore, codes of the data were revised under the supervision of both early childhood education and mathematics education researchers and took its final form.

3.5 Internal Validity and External Validity of the Study

3.5.1 Internal Validity

According to Fraenkel and Wallen (2006), internal validity refers to the difference which is observed on dependent variable caused by the independent variable, not because of any ambiguous variables. In the current study, the possible internal threats were subject characteristics, location, and instrumentation.

In order to control the subject characteristics threat, participant of the study were chosen from same age group from both schools. In addition, socioeconomic status of the children's families was close to each other. Consequently, it was assumed that there were no subject characteristics threats for the study.

As Fraenkel and Wallen (2006) stated, location of the study may create alternative explanations for the results. Therefore, location might be affect the results of the study. The researcher conducted all interviews in the same classroom, which was assigned by the school administrations, to avoid this threat. On the other hand, even though the researcher tried to provide the same conditions for all children, some of them took the interviews in the

morning and some of them took in their lunch break according to their lesson plans. Hence, this might have affected student's concentration and answers and should be considered as a limitation of the study.

At last, instrumentation can create some problems if the nature of the instrument is changed (Fraenkel & Wallen, 2006). Characteristics of the data collector might be considered as instrumentation threat. In the present study, researcher of the study was also the data collector of the study, therefore it can be said that characteristics of the data collector was same for all of the children. Lastly, to prevent the data collector bias threat, all questions were asked in the same order for all children and children were not directed to the right or expected answers.

3.5.2 External Validity

External validity of the study is defined as the degree to which the results of the study can be generalized to population (Fraenkel & Wallen, 2006). In order to generalize the results of the study to the population, both the nature of the sample and the environmental conditions of the sample should represent the population.

The target population of this study was all five-year-old children who attend to private primary schools pre-classes in Çankaya district in Ankara. However, accessible population of the study consisted of two private schools' pre-classes five-year-old students according to convenient sampling method. As stated, the sampling method of the current study was convenient and for that reason, it is not possible to generalize the results of the study to all five-year-old children. However, the results of this study might be generalized to private primary schools pre-classes in terms of ecological conditions. For instance, in the present study, pre-classes environment prepared for children's needs and teachers support children's development with different kinds of play tools, books, songs and rhymes. Hence, results of the current study might be generalized to five-year-old children who attend the private schools in Çankaya district and having the same conditions that were mentioned.

3.6 Assumptions and Limitations of the Study

The sampling method of the current study can be considered as the first and foremost limitation, since the sample of the study was determined by convenience sampling method. At the beginning of the study, it was assumed that all five-year-old age classes of both schools participated in the study. However, according to the parents' consent forms the sample size of this study was limited with twenty-seven five-year-old children, from two different private primary schools' preschool classes. Therefore, the results cannot reflect all five-year-old children's informal knowledge about the fraction concepts and the results cannot be generalized for all five-year-old children in Turkey. Moreover, since both schools were private schools in Çankaya, Ankara district, the results that were obtained from this study cannot be generalized to private schools and public schools in other districts or without similar conditions.

The study data consisted of only interviews conducted with five-year-old children. Therefore, the findings were limited to this type of data. Moreover, since this age group has limited ability in expressing their thoughts verbally on specific subjects, their responses to the interview questions are limited to their verbal ability.

CHAPTER 4

FINDINGS

This chapter gives the main findings of the study that has been carried out with twenty-seven, five-year-old preschool children about their informal fraction knowledge. In this study, children's informal fraction knowledge has been considered under three categories: (i) children's informal knowledge about part-whole relationship, (ii) children's informal knowledge about half, (iii) children's informal knowledge about one-third and quarter.

First category was defined as "children's informal knowledge about part-whole relationship. The meanings of part and whole were asked to each child to identify their definitions about parts and wholes. Moreover, in order to clarify their understanding of the part-whole concept of fractions, children were also asked to describe each component part in the material set (circle, candle, matryshko doll and salt-cellar) either as whole or part. Interpretations will be made about their understanding of part-whole relationship based on their responses.

Children's informal knowledge about the half concept and the way they divide the whole into half were documented in the second category. In order to provide a comprehensive documentation of children's informal knowledge about half, children's definition about the meaning of equal and meaning of half is given in this category. In addition conservation of half of an object is also documented under this category. Therefore, in the second category, children's answers were given under the four sub-categories: (i) meaning of "equal", (ii) meaning of "half", (iii) understanding of half of an object, and (iv) conservation of half concept.

Finally, the third category gives the definitions and explanations of the children about the one-thirds and the quarters. It also includes the equal sharing among four persons of an object, which already consists of four equal parts.

In the following sections, when a reference is made to a particular interview conducted with children, the symbol “C” (followed by a number) is used to describe answers or part of the answers given by that particular participant. For instance, C15 represents the answers or findings from the 15th child in that group. Moreover, the letter “R” represents the researcher in the following sections.

Unavoidably, some additional phrases were added during translation of the transcripts from Turkish to English in order to avoid the loss of meaning. In addition, square brackets [ ] have been used to clarify children’s responses and to help the readers understand their responses clearly.

4.1 Children’s Informal Knowledge about Parts and Wholes

Almost all of the children’s answers showed that they had informal knowledge for the meaning of whole. Only one of them (C17) did not produce a meaning for the whole. Participating children either focused on the quantity such as “Whole means a lot” (C22), “Many things” (C8), “A lot of things” (C5) or the content of a whole such as “It means all of them” (C13), “It means everything” (C15), and “Whole means that every part is in there” (C9).

Children also defined a “part” through their informal knowledge. Only one of them (C13) could not give any definition. Most of the children gave similar answers to the question asking the meaning of half. They gave answers such as “part of a something”, “dividing parts”, and “small parts”. Some children (n=3) stated that, “Part is part of a puzzle” and other children (n=4) explained part as “separation of a lot of things.”

Children also gave examples to objects in parts and wholes from their daily life. Most of the children (n=19) stated that the puzzles, toys and legos’ are examples for the parts and wholes.

While almost all of the children (25 out of 27 children) expressed their definitions about parts and wholes, they had difficulty in finding the objects in whole (candle and circle) and objects in parts (matryshko doll and salt-cellar). Eighteen of them identified correctly the objects in parts and the objects in whole correctly. In addition, 12 out of 18 children indicated that the matryshko doll was consisted of two parts and salt-cellar was consisted of three parts.

However, nine children were not very successful in recognizing whether the objects consisted of parts or consisted of an undivided whole. Some of these children (n=2) indicated that the objects matryshko doll and salt-cellar were whole and the other objects; candle and circle were parts. While two of them (C25, C27) indicated that all of the objects were undivided whole, one of them (C17) asserted that all of the objects consisted of parts. Other sample answers are as follows:

> **Researcher:** Do you know these materials' names? Can you tell me which one of them consists of parts, which one of them is whole?
>
> **C13:** (Pointed out the salt-cellar) Whole. I think it looks like a fish. Circle is a whole too. Candle is a whole. (Pointed out the matryshko doll) I think it has parts.
>
> **C10:** It is a square (candle) and it is part, circle is whole, doll is whole and this is whole (pointed out the salt-cellar)
>
> **C15:** This is a part (showed the matryshko doll), candle is not separated, circle is part, the other one is whole (showed salt-cellar).
>
> **C11:** Salt-cellar is part and others are whole.

Children's answers addressed whole as a quantity: "a lot of things" and also as something with content: "whole means everything." Moreover, children's answers showed that the part of an object was also related with the quantity and was defined as division and separation of lot of things. In addition, almost one-third of the children had some difficulties to find out which one of the materials was a whole and which of the materials consisted of parts.

4.2 Children's Informal Knowledge about Half

Children's informal knowledge about half was investigated through their knowledge about the "equal" concept and how they related equalness to half.

4.2.1 Meaning of “Equal”

In order to assess the children’s informal knowledge about the “equal” concept, two circles were shown to the children and then they were asked, “What is an equal?” One of the circles is divided into two equal parts and the other one consists of two unequal parts. Children were asked to compare the size of the circles’ parts and then they were asked the show the equally divided circle. Most of the participating children (n=23) correctly compared the size of the parts as “big” and “small” and correctly showed the circle which consisted of two equal parts.

Most of the children seemed to be aware of the meaning of the word “equal”. As a result, “same” and “equally divided” were the common answers for the question “What is an equal?” Thirteen of the children stated that equal meant “the same” and 4 of them defined equal as “equally divided.” Only 2 of the children (C4 and C27) stated that they did not know the answer.

Other children gave different answers for the meaning of equal. Participating children defined the meaning of equal either through composition of parts such as “Attach to each other” (C6 and C17), “They are all connected” (C7 and C10) and “Matching the parts” (C20) or through the separation of parts such as “One part for me and one part for you” (C8), “Separate equally” (C12) and “Parts from each other” (C21). On the other hand, two of the children (C19 and C26) defined the “equal” as “whole of an object.”

4.2.2 Meaning of “Half”

The children were asked to answer whether the parts were the halves of the whole object or simply two parts (unequal) of it. The test cases here included set of toys consisted of one whole fruit (banana), fruits and vegetables (lemon, orange, tomato) which could be divided into two equal parts, and some fruits which were divided simply into two parts (carrot, pear and cucumber).

According to children’s answers, 18 out of 27 children stated that orange, tomato and lemon were not the half of each other, although each of these objects were divided into two equal parts. On the other hand, 9 children

recognized that the parts of the orange, tomato and lemon were the half of each other.

Twenty-one children defined the carrot, pear and cucumber as “the objects divided into two parts”. However, 6 out of 27 children stated that the given fruit pieces were the half of each other, even though they were not.

It can be concluded that children’s informal knowledge of the parts seemed to be the same whether parts were equal or unequal. This was also evident in children’s definitions of half. They stated half as “Divided into parts” (C7), “Cut something into two” (C1), and “Divided into two parts” (C26). Children did not mention the equal nature of the two parts of the whole while expressing their knowledge about half.

In addition, only one child stated a definition which might indicate a correct informal knowledge about half: “Half means to cut something in the middle” (C18).

4.2.3 Half of an Object

For showing the half of an object, four equal squares were introduced to the each child and asked them to make a big square by using the four small squares. All participating children expressed that these squares were equal to each other and they made a big square by using these four equal squares.

When children were asked to divide a whole square into half, twenty-three out of twenty-seven children divided the whole square into half correctly. One of the children (C20) did not show the half of this square shape. On the other hand, three children showed (C14, C17 and C25) the half of the square by separating all of its four parts. Examples of the children’s rather irregular responses about the half of an object are illustrated below:

Researcher: Can you make a big square by using these four squares?

C11: 

R: Can you divide them into half?

C11: [Put two squares one side and put other two squares on the other side]

R: Do you think that you can divide these two parts again into half?

C11: Yes. I divide these two squares again into two parts.

(Only answered the question, did not show on the figure)

Researcher: Can you make a big square by using these four squares?

C10: 

R: Can you divide them into half?

C10: 

R: Do you think that you can divide these two parts again into half?

C17: No we cannot.

Researcher: Can you make a big square by using these four squares?

C17: (Correctly made).

R: Can you divide them into half?

C17: 

R: Do you think that you can divide these two parts again into half?

C17: No. But we can divide it if we make it small by cutting it.

Eleven children stated that they could cut these parts again into half, while 7 of them stated that they could not. Some children expressed how they could cut these parts again into half. Their expressions focused on rather how to cut the existing parts, often in the instrumental sense. Most of the children's expressions appeared to the more acting than thinking. Some sample answers of these children are as follows:

C12: Joined the pieces and cut it again.

C3: If there is a big part of it we can divide again into half.

C9: If we joined all the pieces we can divide it again.

C22: If we use a knife we can cut them into half again.

4.2.4 Conservation of Half Concept

Children's informal knowledge about the half of an object was also investigated with the conservation of half concept. Therefore, two geometric shapes of different size, each consisting of two equal parts, were presented to children. All of the participating children defined the size of the objects as "big" and "small". Moreover, twenty-one of them indicated that these parts were simply divided into two parts. On the other hand, 5 of them stated that the parts were the half of each other. Only one child indicated that the parts of the shape were simply divided into two parts but also these parts consisted of a whole.

Eleven out of 27 children answered the question "Do you think that the half of an object changes as the size of the object increases?" as "changes". They explained these changes through the changing size of the objects. Sample answers are as follows:

C2: It changes. Because, their sizes change.

C4: It changes. Because, one of them is big and the other one is small.

C23: When the parts consisted of a whole, it changes. Because, one of them is big.

4.3 Children's Informal Knowledge about One-Third and Quarter

In order to explore children's informal knowledge about one-third, three equal rectangles were introduced to the each child and they were asked to make a big rectangle by using the three small rectangles. Following this process, children were asked to name the shape consisting of three equal parts. Some of the children (n=3) named this rectangular shape which consisted of three equal parts as "square" and "triangle". Four of them (C2, C8, C16 and C27) answered this question as a "rectangle", and one of them (C7) named it as a "long rectangle". Only four of them stated that they did not know the answer.

On the other hand, six of the children expressed definitions for the rectangular shape that represents one-third from a quite different context. While four of the children focused on the parts of the rectangle such as "It has three parts and equal" (C1), "three parts" (C13), "consisted of three parts" (C17) and "divided into three parts" (C14), two other children named this rectangular shape as "separate". Furthermore, two of the participating children defined the rectangular shape, which represented the one-third as "half split" (C21) and "half of it" (C25). In addition, 25 out of 27 children confirmed that these three parts were equal to each other and they were of the same size.

When children's informal knowledge about quarter was investigated, almost all of them (25 out of 27 children) stated that each part of the circular shape (it consisted of four equal parts) were equal to each other. Following the equalness of the parts, children were asked to divide circular shape which consisted of four equal parts into its half. Twenty-one out of 27 children divided the circular shape into half correctly. However, 6 of them divided the circular shape into its four parts instead of two equal parts.

Children were asked to name the shape which consisted of four equal parts. Eleven of 27 five-year-old children expressed their definitions. Children's answers focused on defining the quarter concept through "half"

such as “consists of half of a circle” (C6), “half of a circle” (C14), “I think it can be a big half of a circle” (C21) and “it is divide into half” (C24). On the other hand, their definitions covered the part-whole relationship and division of quantities such as “circle which consists of four parts” (C5 and C8), “a round shape with four pieces” (C11 and C17), “it has a lot of parts” (C7), “divided into four” (C9) and “divided circle” (C15).

Furthermore, children were asked to determine what share each person would receive if an object of four equal pieces was shared among four people. Twenty-five children answered this question correctly. They stated that each person would receive one piece when the object equally shared among four people. Only 2 of them could not give answer to this question. Examples of the dialogue between the researcher and the children about 1/4 and equal sharing was illustrated below:

Researcher: Can you tell me the name of this shape?

C17: Circle.

R: How many parts does the circle consist of?

C17: 1, 2, 3, 4 (counts)…four parts

R: Can you divide this shape into half?

C17:

R: Now, could you tell me your best friends in school?

C17: I think everybody.

R: Assume that, you, I, Nehir and Mert would be in here. How do you share these parts equally among four of us?

C17: One is for Mert and the other parts are yours.

R: All right, how do you name this shape, which consists of four equal parts which can be shared equally among four persons?

C17: A round shape with four pieces.

Researcher: Can you tell me the name of this shape?

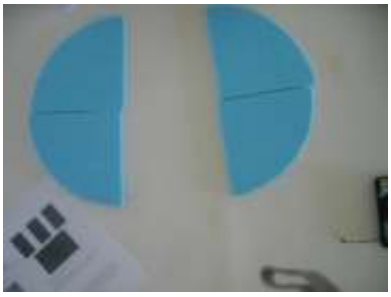
C13: Circle.

R: How many parts does the circle consist of?

C13: Four parts.

R: Can you divide this shape into half?

C13:



R: Could you tell me your best friends in school?

C13: Defne, Ekin.

R: Assume that, you, I, Defne and Ekin would be in here. How do you share these parts equally among four of us?

C13: One is for Defne, one is for Ekin, and one is for you. This is for me.

R: How many parts each person would receive?

C13: One part.

R: All right, how do you name this shape, which consists of four equal parts which can be shared equally among four persons?

C13: I do not know.

4.4 Summary of the Findings

Briefly, almost all the participating children were able to define parts and wholes meaningfully. However, almost one-third of the children had some difficulties to find out which one of the materials was a whole and which of the materials consisted of parts even though they defined the parts and wholes based on their informal knowledge.

Furthermore, most of the children seemed to be aware of the meaning of the word “equal”. Children were able to identify the equal parts of both rectangular and circular shapes. Less then one-third of the children had their own understanding about one-third, which cannot be considered as meaningful. On the other hand, over one-third of the children seemed to have

their own understanding about quarter concept, which can be considered as meaningful. In addition, children's actions on equal sharing seemed to be the same, since almost all of them shared four pieces equally among four persons correctly.

CHAPTER 5

DISCUSSION

In this chapter, the main findings of the study were discussed in the light of the related literature. Furthermore, implications and recommendations were presented for future research under this chapter.

5.1 Summary of the Study

The current study aimed to investigate five-year-old children's informal fraction knowledge by using age appropriate tasks prepared to explore their informal knowledge about basic fraction concepts.

The sample of this study consisted of 27 five-year-old children; fourteen girls and thirteen boys. In order to investigate children's informal knowledge about fraction concepts, qualitative methods have been utilized for both the data collection and data analysis procedures. Data of this study were collected through structured interviews in one-to-one settings by the researcher. The key findings of the current study were presented as follows.

5.2 Key Findings

Preschool children were able to define the parts and wholes through their informal knowledge. Children's examples about materials in parts and wholes were mainly related with reconstruction toys like puzzles and Lego's. They were able to count the parts of a whole as one, two and three and were able to tell that these parts constructed the whole object. Children also realized that there are big parts, small parts and also equal parts that composed the whole of an object. Most of the preschool children seemed to understand the composed and decomposed nature of wholes.

Furthermore, preschool children were able to show the equal parts in a whole. When they were given the definition of equal, they were able to define meaning of equal easily based on their informal experiences.

According to perception of preschool children, meaning of “half” was related with the division of a whole into two parts. In particular, equality of the parts was not important for preschool children in the half concept. Moreover, preschool children were able construct a whole by using the four equal-sized squares and were able divide the composed materials into two equal parts.

For the one-third concept, preschool children were able to construct a whole by using the three equal-sized rectangles. However, preschool children’s understanding about one-third was extremely limited. Moreover, some of them were able to define the name of equal-sized parts for quarters with using half concept through their informal knowledge. In addition, preschool children were aware of sharing of four parts among four persons equally.

5.3 Discussion

5.3.1 Children’s Informal Knowledge about Parts and Wholes

Findings of the current study indicated that almost all of the children provided meaningful definitions about parts and wholes based on their informal knowledge. For most of the children, the parts were the separate pieces of the whole and they could be combined to form the whole when the parts were equal or unequal as Piaget, Inhelder and Szeminska (1960) claimed. Their answers about “part” concept, such as “parts of a puzzle”, “dividing parts” or “separated parts” also supported this claim.

As Piaget, Inhelder and Szeminska (1960), highlighted, children’s perceptions about part-whole concept were constructed from their experiences intuitively. Therefore, even though five-year-old children did not learn the division of objects and part-whole relationship in formal settings, their definitions could be considered as insightful, since definitions included the terms “division” and “separation”.

A child learns intuitively that whole things can be constructed from its parts and can be divided into smaller parts or pieces (Charlesworth, Lind and Fleege, 2003). For example, children naturally learn that they have two hands, ears, eyes, legs and feet (Maxim, 1989; Varol & Farran, 2006)) and then learn that these hands, ears, eyes, legs and feet construct their body

parts (Charlesworth, Lind & Fleege, 2003). Moreover, children also observe that whole is combination of the sum of all its parts (Charlesworth, Lind & Fleege, 2003). In the present study, almost similar knowledge was presented by most of the children. For instance, in this study, the participating children were able to make a connection between the parts of a matryshko doll and salt-cellar in constructing the whole from its separate pieces.

Children's examples about parts and wholes showed that, they related the part and whole concept usually with puzzles and specific toys like Lego's. Since children are familiar with working with these manipulative toys in classroom, their part-whole perception develops simultaneously through these reconstruction activities as Montford and Readdick (2008) stated in their study, which investigated the relationship between preschoolers' puzzle-making strategies and part-whole perception. The findings related with parts and wholes in this study can be supported with the views of Montford and Readdick (2008). According to them, manipulatives developed children's perceptions about part-whole relationship and helped them to understand that the whole of the objects composed and decomposed to its parts.

The above-mentioned literature strongly indicated the inclusion of manipulative materials in preschool classes. The same points are also emphasized in MoNE (2006) curriculum in order to achieve objectives and acquisitions about parts and wholes. With the help of these reconstruction materials, teachers can properly combine children's informal knowledge about mathematics to more formal knowledge of mathematics.

5.3.2 Children's Informal Knowledge about Half

5.3.2.1 Meaning of "Equal" and Meaning of "Half"

The participating children seemed to have a meaningful conception of the word "equal". Children's definitions showed that they had related the "equal" concept with the division, separation, and matching the parts of a whole. Furthermore, their definitions also addressed the identification of two equal parts in a circular whole.

On the other hand, for most of the children, it seemed that, the concepts of "equally divided into two parts" and "simply divided into two

parts" did not make much difference. Mainly, their perceptions about "half" concept were related to the division of objects into two parts like Hunting and Davis (1991) and Sophian (2007) stated. The equality of the parts was not important for the children in this age group.

The same results were also indicated by Piaget, Inhelder and Szeminska (1960), Hunting and Sharpley (1988b), and Tirosh et al. (2011). According to researchers, children's perception about the half notion does not include the equality of the parts in the sharing activities. According to children's perception, it is important to share or divide the parts into two but less important to equally share and divide parts into two.

5.3.2.2 Showing the "Half of an Object"

Opposed to above mentioned findings, in showing of half of an object with composite material, almost all of the children correctly combined the decomposed material, constructed a big square and told that the parts of the square were equal to each other. Followed this, they correctly divided the whole into half when it was asked to divide the whole of the square into half. It can be concluded that, composed and decomposed materials facilitated the understanding and identification of the concept of equal and half for the five-year-old children. This result also confirmed previous research where children's performance was the best with the composed materials in opposed to continuous and discrete materials (Wing & Beal, 2004), where it was observed and suggested that part-whole representation and sharing of discrete objects such as sharing crackers, sharing piece of cake were considerably easier than the division of continuous objects such as a sheet of paper or whole of a cake. The underlying reason of the mentioned difficulty related with the division of continuous objects was that, children did not see the relation that the sum of parts were equal to the whole as Piaget, Inhelder and Szeminska (1960) and Hunting and Sharpley (1988b) explained.

5.3.2.3 Conservation of Half Concept

Although the conservation of the objects is considered as difficult concept for preschool children and is not achieved in preoperational stage

(Smith 2006), almost half of the participating children indicated that the half of the object would not change if the size of the object was increased. While they indicated the unchanged nature of half concept, they did not mention the reason of this situation. On the other hand, the other participants stated that the half did not remain as a half as the size of the object increased. They explained this situation by the changing size of the objects.

These findings can be explained by the Piaget's stage theory. Based on the Piaget's stage theory, the ages between two and six are considered as preoperational stage (Smith, 2006). In this stage, children can use symbols such as, language and drawing to represent their thinking (Smith, 2006). However, many mathematics tasks such as conservation are not achieved until the concrete stage, which includes the ages between 7 to 11 (Smith, 2006). In the preoperational stage, while children represent their thinking through their own perspectives, they are able to represent the static situations and are not able to understand the changing situations clearly, until the concrete stage (Siegler & Alibali, 2005). In other words, their logical explanations about the changing situations include only their own perspectives. Therefore, it can be concluded that, even though the children used words to describe relationship between the objects such as "one of them is big and the other one is small but half is not change" (C1, C6; C11), their reasoning was not sufficient to explain the conservation of the half concept.

5.3.3 Children's Informal Knowledge about One-Third

According to children's informal knowledge about one-third, results showed that this concept was rather complicated for this age group as Piaget, Inhelder and Szeminska (1960) and Pothier and Sawada (1983) indicated. Only four children expressed meaningful definitions for the rectangular shape that was combined by three equal parts. On the other hand, two of the children related this rectangular shape with the half concept and considered the equal parts of the rectangle as half-split. According to Piaget, Inhelder and Szeminska (1960), children first understand the part-whole and part-part relationship with the division into two. Then, it naturally

extends to division into fourths. However, the situations involving thirds are considered more difficult than division into halves and fourths because it requires both anticipating and comparison of three individual parts with each other.

5.3.4 Children's Informal Knowledge about Quarter

Children's perceptions about quarter concept seemed to be parallel with the claims of Piaget Inhelder and Szeminska (1960). Children's descriptions such as "big half of a circle", "it is divided into half" showed that, they perceived quarter concept through the half concept. Similarly, Pothier and Sawada (1983) observed that children learned to partition into two and then extended this into dividing into halves in powers of two.

According to Van de Walle, Karp and Bay-Williams (2008), sharing tasks are generally introduced to children in the form of a simple story as introduced in the current study: "Assume that me, you and two of your friends would be in here. How do you share these parts equally among four of us?" Results of sharing the parts showed that children easily grasped the process of equal sharing through matching the parts with each person. It can be concluded that, children seemed to be familiar to sharing of objects. The same one-by-one matching process in the sharing task was observed by Hunting and Sharpley (1988a, 1988b) and Pepper and Hunting (1998). The researchers stated that preschoolers were systematically able to share different number of items, for example 12 crackers, equally among three or four persons. In the light of these conclusions about equal sharing, it might be concluded from the current study that, children are able share the parts of the combined whole equally among the number of persons when the number of parts equals to the number of persons.

The sharing process of preschoolers was highly related with their understandings about matching as it can be seen above past studies. It was naturally related with children's fundamental concepts and skills such as, number sense and one-to-one correspondence. In particular, number sense makes the connection between quantities, such as parts and wholes (Charlesworth, Lind & Fleege, 2003) and when children share parts among

the number of persons one-by-one, their rational counting skill is supported gradually (Charlesworth, 2005). Since both counting and sharing skills seem to develop in early years (Pepper & Hunting, 1998) children's sharing skills might be supported with the sharing of larger numbers of items through counting and one-to-one correspondence.

5.4 Educational Implications

The current study described the five-year old children's perceptions about fraction concepts through their informal knowledge. Results of this study indicated that, children's perceptions about one-half, one-third and quarter are related with the part-whole relationship and it can be concluded that the part-whole relationship is the starting point of the fractional understanding in early years since the first introduction for fractions that children typically come across is the parts and wholes (Sheffield & Cruikshank, 2005).

Even though the current study investigated the children's informal knowledge about fraction concepts with the small sample size, some suggestions can be made based on the findings of this study and previous studies.

The findings of the current study revealed that, children had knowledge about part-whole relationship based on their informal experiences. However, it was observed in past studies (Hunting & Sharpley, 1988a, 1988b; Pothier & Sawada, 1983; Tirosh, Tsamir, Tabach, Levenson & Barkai, 2011; Wing & Beal, 2004) that composed and decomposed materials could facilitate children's understanding and identification about part-part, part-whole and half-whole concepts. Therefore, it can be suggested that, early childhood classrooms should be equipped with appropriate manipulative materials, which represent the part-whole and part-part relationships. In particular, teachers should be encouraged to use manipulative materials like blocks, puzzles or Lego's in mathematics activities in order to support children's development of part-whole perception and half-whole perception.

On the other hand, based on the conclusions of the past studies, children's perception of the use of discrete and continuous objects produced different difficulties for the representation of part-whole and half concepts (Hunting & Sharpley, 1988a, 1988b; Tirosh et al, 2011; Wing & Beal, 2004). In this regard, not only composite materials, but also continuous and discrete materials must be presented to the children in the classroom area to develop their understanding about the part-part, part-whole and half-whole concepts.

While teachers introduce part-whole and half-whole concept to children with different types of materials, they should clarify children's misconceptions about the meaning of equal, meaning of half and part-whole relationship. Since these concepts are related with each other and the objectives and acquisitions about these concepts are emphasized in MoNE (2006) curriculum, teachers should implement different activities and should provide rich environment in the classroom. In the classroom area, children must discover the mathematical relationship from the experiences visually, tactually and auditorily (Conaway & Midkiff, 2004). For instance, teachers can use story problems or storybooks in order to clarify children's understandings about fraction concepts. In addition to literature activities, dramatic play can be used since this type of play reveals how mathematics used in the children's home informally (Dacey & Eston, 1999). Furthermore, using of picture cards that represents the equally divided objects into two, three and four parts enhanced children's visual understandings about one-half, one-third and quarter. With the use of fractional phrases such as; equal parts, half of an apple, sharing of four candies, and the teaching of fraction concepts can easily be improved.

Besides, regarding the children's informal knowledge, in order to support children's formed ideas from their informal experiences, preschool teachers can use real world contexts and naturalistic activities to enhance children's informal knowledge. Since children's taughts about mathematics are usually limited with the classroom performance (Dacey & Eston, 1999), teachers should be encouraged to use outdoor activities in order to support children's informal knowledge. For example, equipments in playground such as climbing frame, merry-go-round (with its base divided into equal parts with

different colors) can be use for observing the basic part-whole relationship. In addition, during the daily routines such as breakfast, lunch time, children can easily grasp the idea that vegetables and fruits are cutting into two, three or more parts and they can transfer this knowledge while learning division formally. Briefly, teachers can be encouraged to use daily experiences and naturalistic activities as a learning bridge for more formal school mathematics in preschool classes.

5.5 Recommendations for Further Researches

Based on the results of this study, some recommendations can be made for future researches about children's fractional understanding in early childhood years.

Foremost, since the relevant literature presents a gap about the fractional understanding in early years, it is important to search children's understandings about fraction concept in preschool. Therefore, repeated studies about part-whole relationship, sharing of quantities, one-half, one-third and quarters can be conducted with different samples of preschool children. Moreover, these concepts can be investigated separately through both qualitative and quantitative research methods regarding gender and school type differences to make in-depth explanations about children's understandings about fractional concepts.

Besides, in the current study, equal sharing task included the sharing of four parts among four persons. For future studies, children's equal sharing strategies can be considered separately and should be focused on sharing of more parts (for example, sharing of 6 cookies among two or three persons or 12 coins among three of four persons) among two, three or four persons like presented in the past studies (Hunting & Sharpley, 1988a, 1988b; Pepper & Hunting, 1999; Wing & Beal, 2004). Moreover, children's conceptions about the sharing of quantities can be investigated through not only composite objects but also with continuous and discrete objects to make comparisons about children's understandings about fractional concepts; one-half, one-third and quarters.

Moreover, the current study focused on the children's informal knowledge through planned tasks. For in-depth understanding about the preschoolers' informal knowledge, the views of parents and teachers about how children obtain their informal knowledge might also be included by carrying out interviews with parents and teachers. In addition, children's formal knowledge about part-whole and half-whole relationship that they learn in school settings can be investigated through longitudinal studies with teachers and students.

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APPENDICES

APPENDIX A

INTERVIEW QUESTIONS

GENEL SORULAR

1. Sence parça ne demektir?
2. Sence bütün ne demektir?
3. Parça ve bütüne örnek verebilir misin?
4. Burada gösterdiğim materyallerin isimlerini biliyor musun? Hangilerinin parçadan oluştuğunu yada hangilerinin bütün olduğunu söyleyebilir misin?
5. Bir şeyin yarısı denildiğinde senin aklına ne geliyor?

I. ADIM

(Üç alt bölümden oluşmaktadır, bunlar: daire seti, meyve seti, kare seti)

DAİRE SETİ

(İki eşit parçadan ve iki parçadan oluşan daire seti)

1. Burada gördüğün geometrik şekillerimizin adlarını söyleyebilir misin?
2. Bu dairelerimiz kaç parçadan oluşuyor?
3. Sence bu şekillerin parçaları arasında bir farklılık var mı?
4. Bu şekillerden hangisi eşit bir şekilde ikiye ayrılmıştır?
5. Sence eşit ne demektir?

MEYVE SETİ

(Bütün muz, iki eşit parçadan oluşan elma, portakal, limon, iki parçadan oluşan armut, havuç, salatalık. 1-4 arasındaki sorular iki parçadan oluşan meyveler için tekrardan sorulur)

1. Burada gördüğün meyvelerin adlarını biliyor musun?
2. Burada parçalardan oluşan ve bütün olan meyveleri söyleyebilir misin?

3. Kaç parçadan oluşuyor bu şekillerimiz?
4. Sence bu meyveler birbirinin yarısı mı yoksa ikiye bölünmüş hali mi?
5. Sence yarım ne demektir?
6. Sence bu meyvelerin parçaları arasında bir farklılık var mı?

KARE SETİ

(Dört küçük ve eşit kareden oluşan kare seti)

1. Burada gördüğün şekillerimizin adlarını söyleyebilir misin?
2. Kaç tane şeklimiz var burada?
3. Sence bu şekillerin büyüklükleri aynı mı?
4. Bana bu şekilleri kullanarak büyük bir kare oluşturabilir misin?
5. Oluşturduğun bu şekli yarısına bölebilir misin?
6. Sence biz yarısına böldüğümüz bu şekli bi kere daha yarısına bölebilir miyiz?

II. ADIM

(Üç küçük ve eşit dikdörtgenden oluşan dikdörtgen seti, 1/3 kavramı)

1. Burada gördüğün şekillerin adlarını söyleyebilir misin?
2. Kaç tane şeklimiz var burada?
3. Sence bu şekillerin büyüklükleri aynı mı?
4. Bana bu şekilleri kullanarak büyük bir dikdörtgen oluşturabilir misin?
5. Bu üç eşit parçadan oluşan bu şeklin özel bir adı olabilir mi?

III. ADIM

(Dört eşit parçadan oluşan daire, ¼ kavramı)

1. Burada gördüğün şekillerin adlarını söyleyebilir misin?
2. Kaç tane şeklimiz var burada?
3. Sence bu şekillerin büyüklükleri aynı mı?
4. Bu şekli nasıl yarıya bölebiliriz?
5. Sen, ben ve bizimle birlikte iki tane arkadaşın daha burada olsa, dört kişiye bu dört parçayı eşit olarak nasıl paylaştırırsın?

6. Her kişi kaç parça alır?
7. Dört eşit parçadan oluşan ve dört kişiye eşit olarak paylaştırılabilen bu şeklin özel bir adı olabilir mi?

IV. ADIM

(Şekiller büyüdükçe yarım kavramının korunumu)

1. Sence bu şeklimiz ne olabilir?
2. Bu şekillerin parçalarının büyüklükleri hakkında ne söyleyebilirsin?
3. Sence bu parçalar birbirinin yarısı mı yoksa ikiye bölünmüş hali mi?
4. Bir şeklin yarısı şekil büyüdükçe değişiyor mu?

APPENDIX B

ETHICAL PERMISSION

ODTÜ ETİK KURULU
İnsan Araştırmaları
Veli Onay Mektubu

Sayın Veli,

Orta Doğu Teknik Üniversitesi, Eğitim Fakültesi, Okul Öncesi Öğretmenliği Bölümünde yüksek lisans öğrencisi olarak çalışmaktayım. Yrd. Doç. Dr. Refika Olgan ve Yrd. Doç. Dr. Çiğdem Haser danışmanlığında yürütmekte olduğum yüksek lisans tezim kapsamında okul öncesi dönemdeki çocukların kesir kavramı ile ilgili bilgilerini araştırmaktayım. Bu araştırma sonucunda, 5 yaş grubu okul öncesi dönem eğitim programlarında kesir bilgisinin daha iyi ve etkili bir şekilde ele alınması amaçlanmaktadır.

Yürüteceğim çalışmada, çocuğunuzla yaklaşık 20 dakika sürecek görüşmeler yapılması ve bu sürede araştırılan konu ile ilgili sorular sorulması ve konu ile ilgili materyallerin kullanılması amaçlanmaktadır. Görüşme sırasında ses kayıt cihazı kullanılacak olup, çalışmanın sonunda herhangi bir maddi veya manevi yarar sağlanmayacaktır. Çalışmaya katılımda sizin ve çocuğunuzun gönüllülük esası temel alınmaktadır. Bu bağlamda katılımcıların kimlikleri ve bilgileri gizli tutulup hiçbir nedenle kullanılmayacaktır.

Çalışmaya ya da çocuğunuzun katılımına yönelik daha fazla bilgi için başvurulacak kişilerin telefon numaraları ve e-posta adresleri:

Yrd. Doç. Dr. Refika Olgan, tel: 210 3671, e-mail: rolgan@metu.edu.tr
Yrd. Doç.Dr. Çiğdem Haser, tel: 210 6415, e-mail: chaser@metu.edu.tr
Burcu Utku: e-mail: burcu.utku@metu.edu.tr

Teşekkürler,
Burcu Utku

Yukarıda açıklamasını okuduğum çalışmaya, oğlum/kızım _____'nin katılımına izin veriyorum.

Ebeveynin:

Adı, soyadı: _____ İmzası: _____

Tarih: _____

Çocuğunuzun katılımı ya da haklarının korunmasına yönelik sorularınız varsa ya da çocuğunuz herhangi bir şekilde risk altında olabileceğine, strese maruz kalacağına inanıyorsanız Orta Doğu Teknik Üniversitesi Etik Kuruluna (312) 210-37 29 telefon numarasından ulaşabilirsiniz.

APPENDIX C

TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü	
Sosyal Bilimler Enstitüsü	X
Uygulamalı Matematik Enstitüsü	
Enformatik Enstitüsü	
Deniz Bilimleri Enstitüsü	

YAZARIN

Soyadı : UTKU
Adı : Burcu
Bölümü : İlköğretim Bölümü

TEZİN ADI : AN INVESTIGATION OF YOUNG CHILDREN'S INFORMAL FRACTION KNOWLEDGE

TEZİN TÜRÜ : Yüksek Lisans [X] Doktora [ ]

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. [ ]
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. [ ]
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. [X]

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: