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DANIŞMANI
YARD. DOÇ. DR. HASARİ ÇELEBİ

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SUMMARY

Node localization in wireless networks is crucial for supporting advanced location-based services (LBSs) and improving the performance of network algorithms such as routing schemes. In this thesis, we study the fundamental limits for time delay based location estimation in cooperative relay networks. The analyses are performed under both line-of-sight (LOS) and multipath environments in two chapters. In Chapter IV, the theoretical limits are investigated in single-path environment by obtaining Cramer-Rao Lower Bound (CRLB) expressions for the unknown source location under different relaying strategies when the location of the destination is known and unknown. More specifically, the effects of amplify-and-forward (AF) and decode-and-forward (DF) relaying strategies on the location estimation accuracy are studied. Furthermore, the CRLB expressions are derived for the cases where the location of only source as well as both source and destination nodes are unknown considering the relays as reference nodes. In addition, the effects of the node topology on the location estimation accuracy of the source node are investigated. The results reveal that the relaying strategy at relay nodes, the number of relays, and the node topology can have significant impacts on the location accuracy of the source node. Additionally, knowing the location of the destination node is crucial for achieving accurate source localization in cooperative relay networks. In Chapter V, the CRLB expressions are derived in a multipath environment under AF and DF strategies by considering the location of the destination is known. In the simulations, the effects of non-line-of-sight (NLOS) components on the location estimation accuracy are investigated. The results show that processing multipath components (MPCs) improves the location estimation accuracy.

Keywords: Time Delay; Location Estimation; Cooperative Relay Networks; Cramer-Rao Lower Bounds.

ÖZET

Konum tabanlı sistemlerin günlük hayatta birçok alanda geliştirilmesi ile kablosuz ağlarda düğümlerin konumlarının kestirilmesi büyük bir önem kazanmıştır. Bu çerçevede literatürde birçok çalışmalar yapılmıştır. Ancak işbirlikçi röle ağlarında konum kestiriminin incelenmesi hakkında çok az çalışma mevcuttur. Bu tezde işbirlikçi röle ağlarında konum kestirimi başarımı analiz edilmiş ve sonuçlar elde edilmiştir. Analizlerde sinyallerin tek yoldan ve çok yoldan iletilme durumları dikkate alınarak her iki farklı ortam için kestirim doğruluğu incelenmiştir. Bölüm IV' de; tek yol haberleşme ortamınd a işbirlikçi röle ağlarda haberleşme sinyallerinin zaman gecikmesi tabanlı konum kestirimi için Cramer –Rao düşük sınırları tanımlanarak kestirim performansı incelenmiştir. İşbirlikçi röle ağlarda birçok röle stratejisi bulunmaktadır. Bu tez kapsamında güçlendir-ve-ilet (AF) ve kodçöz-ve-ilet (DF) röle stratejilerinin konum kestirimi üzerindeki etkileri incelenmiştir. Ayrıca, işbirlikçi ağlarda – röle düğümlerinin referans olduklarını varsayarak - hedef düğümün konumunun bilinip bilinmemesinin kaynak düğümün konumunun kestirimindeki etkisi incelenmiştir. Elde edilen sonuçlara göre, DF röle stratejisinin AF'e göre daha iyi sonuçlar verdiği görülmüştür. Ayrıca, hedef düğümün konumunun bilinmesi; kestirim hatasının düşürülmesine çok büyük etkisi olmaktadır. Bölüm V' de ise; hedef düğümün konumunun bilindiği varsayımı altında çok yollu haberleşme ortamında AF ve DF stratejilerinin kullanılması durumlarında konum kestirimi başarımları ölçülmüştür. Benzetim sonuçlarına göre; alınan sinyallerdeki güçlü çok yol komponentlerinin konum kestiriminde kullanılması hata düşürülmesine katkı sağlamaktadır. Ayrıca, AF stratejide 7'den fazla komponentin kullanılmasının kestirim hatasını düşürmeye etkisinin olmadığı fark edilmiştir. Fakat DF stratejisinde daha fazla çok yol komponentinin kullanılmasıyla kestirimin hatasının daha fazla düştüğü saptanmıştır.

Anahtar Kelimeler: Zaman Gecikmesi; Lokasyon Kestirimi; İşbirlikçi Röle Ağlar; Cramer-Rao Düşük Sınırlar.

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LIST OF ACRONYMS AND ABBREVIATIONS

Acronyms and Abbreviations		**Descriptions**
AF	:	Amplify-and-forward
AOA	:	Angle of arrival
AP	:	Access point
AWGN	:	Additive white Gaussian noise
BS	:	Base station
CRLB	:	Cramer –Rao lower bound
DF	:	Decode-and-forward
E2E	:	End to end
FIE	:	Fisher information element
FIM	:	Fisher information matrix
G-CRLB	:	Generalized Cramer Rao lower bound
GPS	:	Global positioning system
LBS	:	Location based services
LOS	:	Line-of-sight
MAP	:	Maximum a posteriori
MF	:	Matched-filter
MHz	:	Mega Hertz
ML	:	Maximum likelihood
MPC	:	Multipath components
MS	:	Mobile station
NLOS	:	Non-line-of-sight
pdf	:	Probability density function
RMSE	:	Root mean square error
RSS	:	Received signal strength
RX	:	Receiver
RV	:	Random variable
SNR	:	Signal-to-noise ratio
TDOA	:	Time difference of arrival
TOA	:	Time of arrival
TX	:	Transmitter
UWB	:	Ultra wide band

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1. INTRODUCTION

Next generation wireless networks allow their nodes to communicate with each other without the need of an infrastructure [1], [2]. In this context, cooperative relay networks have such feature by allowing the relay nodes to help the transmission between source and destination nodes [3]-[6]. One of the main advantages of these networks is to provide large network coverage without increasing the transmit power of the source node. This is achieved by creating a virtual antenna array using multiple relay nodes equipped with single antenna [4]. In the context of localization, the architecture of cooperative relay networks is inherently suitable for supporting the node localization since the relay nodes can be considered as reference nodes. Likewise in the other types of wireless networks, the node localization is also a crucial process in the cooperative relay network [6], [7]. For instance, the location information of the nodes can be used for the network authentication, ranging, and cluster forming in these networks. Hence, the location estimation plays an important role for supporting the advanced LBSs as well as improving the performance of network algorithms in the cooperative relay networks. The node localization problem in cooperative relay networks has several unique aspects compared to the previous studies in the literature. The most important differences are the relaying strategies employed at the relay nodes and transmission over multiple hop links between the source and destination nodes. Hence, in this thesis, the fundamental limits of the location estimation of the source node problem in cooperative relay networks are studied considering different relaying strategies, i.e., amplify-and-forward (AF) and decode-and-forward (DF) over two hop links.

Location estimation in wireless networks can be performed by using different signal parameters, time delay (e.g., time-of-arrival (TOA), time-difference-of – arrival (TDOA)), signal strength (e.g., received signal strength (RSS)), and angle (e.g., angle-of-arrival (AOA)) metrics of received signals [8]-[10]. Location estimation methods based on these signal parameters also can be performed in cooperative relay networks by employing the received signal at the destination node. However, in this thesis, we consider time delay based approach due to its high accuracy feature compared to its counterparts.

In a realistic environment, the transmitted signals are often subject to multipath propagation. Therefore, the received signals at the relay nodes and the destination node have combined multipath components (MPCs) of the transmitted signals. These MPCs usually have distinct time delays and channel coefficients. We can divide these MPCs into two groups: line-of-sight (LOS) and non-line-of-sight (NLOS) components. It is obvious that there is one LOS MPC and the others are NLOS MPCs.

In this thesis, the location estimation in cooperative relay networks are investigated under two phases. In the first phase, the location estimation of the source node is employed in single-path environments; in other words, only the LOS component of the received signal is used to estimate the location estimation. While performing the location estimation, two different scenarios are considered. In the first scenario, it is assumed that the location of the destination node is known whereas the second scenario considers that the location of the destination node is unknown. In the second phase, the multipath components are employed when estimating the location of the source node by resolving all the MPCs at the destination node. For each phase, system and signal models and Cramer-Rao Lower Bounds (CRLBs) for location estimation of the source node are represented under AF and DF relaying strategies. The numerical results and discussions about the result are expressed at the end of the chapters.

2. RELATED WORKS

To the best of our knowledge, there are few works in the literature which consider cooperative relay networks for location estimation. TOA based localization method for a cooperative relay network consists of a single relay node is proposed in [7]. Additionally, an RSS based localization algorithm for cognitive cooperative relay networks consisting of a single cognitive relay node is proposed in [11]. The main differences of our studies from these studies are that we consider multiple relay nodes and different relaying strategies - AF and DF – whereas [7] considers one relay node under AF relaying strategy. In addition, the location of destination node is considered to be known in [7]. Furthermore, the multipath environment is considered in this thesis whereas it is not considered in [7], [11].

TOA estimation in multipath environment is investigated by deriving the CRLBs for the localization of a mobile station (MS) using the TOA estimation from base stations (BSs) in a cellular network in [12]. The TOA estimations are performed by employing only the first arriving (LOS) signals using maximum-likelihood (ML) estimator which is unbiased in this case. In addition, the CRLB analysis is also performed in that study when the prior NLOS propagation error statistics are available in the multipath environment. In such case, the generalized CRLB (G-CRLB) are defined to evaluate the accuracy of TOA estimation using maximum a posteriori (MAP) estimator.

TOA based location estimation in single-path channels is investigated in a cellular network assuming that the BSs receive either LOS or NLOS signals from a MS in [13]. Due to the absence of prior knowledge about NLOS error statistics, only the BSs which receive LOS signals contribute to the location estimation hence the CRLB is derived in terms of LOS signal statistics.

The difference between ML and MAP estimator is that the prior knowledge about the unknown parameters is available in MAP estimator. However, ML estimator does not use a prior knowledge when estimating the unknown parameter. Hence, only the BSs which receive LOS signals are used by ML estimator to perform location estimation due to the lack of prior knowledge about NLOS error statistics [13]. On the other hand, MAP estimator is used when the prior knowledge about NLOS error statistics are available. In this thesis, it is assumed that the prior knowledge about the NLOS errors statistics is available in the multipath environment, so that the G-CRLB for the location estimation of source node is derived in the second chapter.

3. AN OVERVIEW ON TIME DELAY ESTIMATION

In the literature, there are a lot of works on how to estimate time delay of the received signal in the wireless networks [8], [14]-[15]. Two conventional approaches are used in time delay estimation: Correlator and Matched-Filter (MF) [8], [17]. The time delay estimation is performed at the receiver using one of these approaches above.

We assume that two wireless nodes are communicating with each other - one of is transmitter (TX) and the other is receiver (RX) - which is illustrated in Figure 3.1.

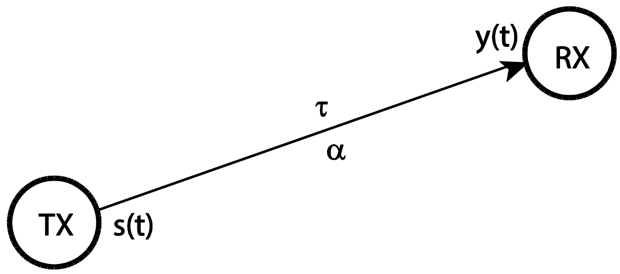

Figure 3.1: An illustration of basic communication between RX and TX.

We can model the baseband representation of the received signal $y(\mathrm{t})$ over a period $[0,\mathrm{T}]$ in the following form,

$$y(\mathrm{t}) = hs(\mathrm{t}-\tau) + \mathrm{n}(\mathrm{t}),\ \mathrm{t} \in [0,\mathrm{T}], \tag{3.1}$$

where h is channel coefficient, τ is the time delay of $s(\mathrm{t})$ and $n(\mathrm{t})$ is zero mean additive white Gaussian noise (AWGN) with variance σ^2, i.e. $N(0,\sigma^2)$ at the receiver. In the correlator based approach, the τ time delay can be estimated at the receiver by cross-correlating $y(\mathrm{t})$ with a local template $s(t-\tau)$ using different τ delays [14]. The τ value that maximizes the cross-correlation expression is the estimated time delay value $\hat{\tau}$, namely,

$$\hat{\tau} = \arg\max_{\tau}\{\mathrm{E}[\mathrm{y(t)}\,\mathrm{s(t}-\tau)]\}. \tag{3.2}$$

In the MF approach, a filter is employed which is matched to $s(\mathrm{t})$. This filter estimates the instant when the filter output gives the largest value [8]. This instant is the estimated time delay value $\hat{\tau}$. Both of these approaches are optimal in the maximum likelihood (ML) estimation considering the signal model in (3.2).

If $y(\mathrm{t})$ is noise-free, in other words σ^2 =0, then the estimated time delay value is equal to the exact time delay, i.e., $\hat{\tau} = \tau$. However, the received signal is generally corrupted by the thermal noise, $n(\mathrm{t})$. Due to the noise effect on $y(\mathrm{t})$, the peak value of the cross-correlation results may be shifted, so that the estimated time delay value includes estimation error, which can be modeled as [13], [18],

$$\hat{\tau} = \tau + \varepsilon, \tag{3.3}$$

where ε Gaussian distributed estimation error which can be modeled as $\varepsilon \sim N(0,\xi^2)$. The theoretical limit for estimation error variance can be set by CRLB defining the variance and/or covariance for the unbiased estimates of unknown parameters [18]-[21]. We assume that the h is a known parameters in this case, so that the τ is the only parameters which is unknown. In order to calculate the CRLB for time delay estimation – the unknown parameter is $\theta = [\tau]$ in this case- , the log-likelihood function of the received signal should be obtained and then the Fisher Information Matrix (FIM) should be stated. The inverse of the FIM gives the CRLBs for unknown parameters, i.e., CRLB= $[\mathrm{I}_{\theta}{}^{-1}]$.

Since $s(\mathrm{t}-\tau)$ is a deterministic signal and $n(\mathrm{t})$ is a random signal, we can represent the log-likelihood function of θ as follows [19],

$$\Lambda(\theta) = c - \frac{1}{2\sigma^2}\int_0^T |\,\mathrm{y(t)} - \mathrm{hs(t}-\tau)\,|^2 dt, \tag{3.4}$$

where c is a constant that is independent of θ. Since τ is the only parameter in θ, we can call FIM as FIE (Fisher Information Element) in this case which is defined as follows [20],

$$I_{\theta} = I_{\tau\tau}, \tag{3.5}$$

where

$$I_{\tau\tau} = E_{\theta}\left[\frac{\partial}{\partial\tau}\Lambda(\theta)\left(\frac{\partial}{\partial\tau}\Lambda(\theta)\right)^{T}\right], \tag{3.6}$$

where T stands for "transpose". Actually, we can represent $I_{\tau\tau}$ for this case as follows,

$$I_{\tau\tau} = E_{\theta}\left[\frac{\partial^{2}}{\partial\tau\partial\tau}\Lambda(\theta)\right]. \tag{3.7}$$

$I_{\tau\tau}$ is defined in [19] as follows,

$$I_{\tau\tau} = \gamma\tilde{E}, \tag{3.8}$$

where γ is the signal-to-noise ratio (SNR) and $\tilde{E} = \int_{0}^{T} |\, s^{'}(t-\tau)\,|^{2} dt.$ We can represent the γ as follows [19],

$$\gamma = \frac{h^{2}}{\sigma^{2}}. \tag{3.9}$$

Then, the CRLB for τ can be represented using (3.8) and (3.9) as,

$$CRLB_{\tau} = \frac{1}{\gamma\tilde{E}}. \tag{3.10}$$

$CRLB_\tau$ gives us the theoretical limit for variance of time delay estimation ξ^2. Using this bound, we can express the theoretical limits for location estimation of a node by using the relation between time delay and the distance. These analyses are performed in the next chapters.

4. LINE-OF-SIGHT ENVIRONMENT ANALYSIS

4.1. System Model

In this chapter, a system model that consists of a two-hop cooperative wireless relay network with multiple relay nodes $(R_i, i = 1, 2, 3, \ldots, M)$ is employed, which is shown in Figure 3.1. In this model, a source (S) node communicates with a destination (D) node through multiple relay nodes due to the challenging channel conditions between the S and D node. The relay nodes and the D nodes receive single-path (LOS) signals in this case. In this study, we employ the fixed relaying protocol where the relays either amplify, or fully decode and re-encode the received signals from the S node [4]. Then, the amplified or decoded- encoded signals are transmitted to the D node.

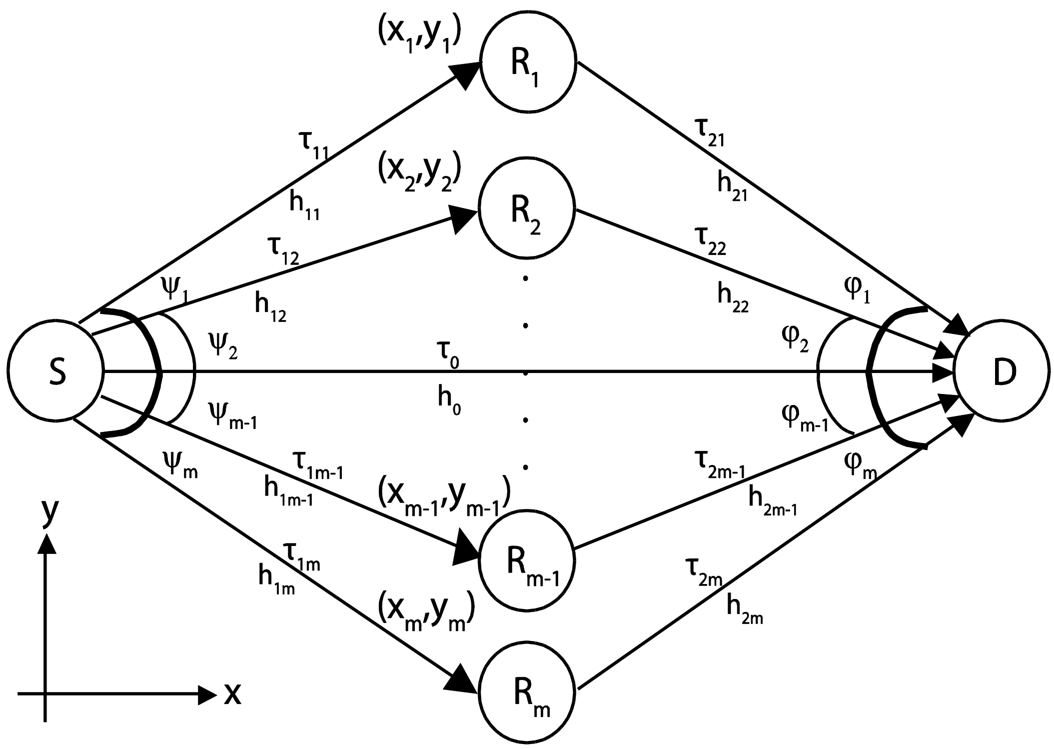

Figure 4.1: System model for cooperative relay networks.

In a realistic environment, $S \rightarrow R_i, R_i \rightarrow D$ and $S \rightarrow D$ channels are often subject to multipath propagation. As stated in the previous section that cooperative relay networks differ from the other wireless networks in the literature in terms of relaying strategies at the relay nodes and multihop links between the S and D node through the R_i nodes. Hence, time delay based location estimation problem in such cooperative relay networks with multiple relay nodes and multihop links is a great interest of research community. In this context, time delay based geolocation in cellular networks considering single-path propagation environment with LOS and NLOS scenarios is studied in [13]. This study is extended to multipath propagation environment with LOS and NLOS scenarios in [11]. Hence, in order to study the fundamental limits and challenges of time delay based location estimation problem specific to cooperative relay networks, we consider single-path LOS propagation channels for all the links of the cooperative relay network in this chapter.

The communication between the S and D node is performed over two time slots (*SlotI*, *SlotII*) using orthogonal time-division channel allocation protocol (see [4]). In *SlotI*, the S node broadcasts the signal *s(t)* and the R_i nodes as well as the D node receive this signal. In *SlotII*, the R_i nodes process the signal *s(t)* according to their relaying strategies and forward it to the D node. We assume that all the R_i nodes employ the same relaying strategies (either AF or DF) in this study. In addition, it is assumed that all the nodes transmit over mutually orthogonal subchannels (e.g., M time slots) during *SlotII* (see [1]). At the end of *SlotII*, the D node possesses multiple copies of the source signal obtained through relayed links and direct link, which is a form of cooperative diversity [18]**.** In the context of location estimation, these signals are used to estimate the location of source node or both source and destination nodes considering that the relay nodes are reference nodes with known locations. In wireless communication systems, the time delay of received signals can be estimated via correlator or matched-filter (MF) at the receivers [8], [17]**.**

In the correlator-based time delay estimation, the received signal is correlated with its template using different delay and the correlation peak is considered as the time delay of the received signal. In the MF-based estimation, a filter which is designed for matching to the received signals, estimates the time delay as the instant at which the filter output reaches its largest value. Further details on the correlator and MF based time delay estimation can be found in [1]. However, in our considered cooperative relay network, τ_{1i} and τ_{2i} can be estimated at the respective nodes using either MF-based or correlator-based time delay estimation techniques.

In the following sections, the signal models for the AF and DF relaying strategy based cooperative relay networks are presented.

4.2. AF Relaying Strategy Based Signal Model

In the AF relaying strategy based cooperative relay networks, the received signals from the S node are amplified and forwarded to the D node at the relay nodes. In this strategy, the amplification factor is determined to maintain the average power *P* of the source signal *s(t)* while amplifying the received signal. Assuming that the available power at the S node and the relay nodes are *P*, hence, the S node broadcasts *s(t)* with average power *P*. As mentioned previously, the communication between the S and D nodes is performed over two time slots where T is defined to be the duration of a time slot. In the $Slot\,I$, the S node broadcasts the signal *s(t)* and the D node and the relay nodes receive this signal.

The received signal $y_{d0}(t)$ at the D node through the direct link is given by,

$$y_{d0}(t) = \sqrt{P}h_0 s(t-\tau_0) + n_0(t), \;\; t \in [0,T], \tag{4.1}$$

where h_0 is the channel coefficient and τ_0 is the time delay of the direct link between the S and D nodes. Moreover, $n_0(t)$ is the noise, which is modeled as additive white Gaussian noise (AWGN) with zero mean and σ_0^2 variance. The received signal at the R_i relay node can be expressed as,

$$y_{ri}(t) = \sqrt{P}h_{1i} s(t-\tau_{1i}) + n_{1i}(t), \;\; t \in [0,T], \tag{4.2}$$

where h_{1i} is the channel coefficient and τ_{1i} is the time delay of the link between the S and R_i relay node and $n_{1i}(t)$ is the noise that is modeled as AWGN with zero mean and σ_{1i}^2 variance. As stated above, the received signal $y_{ri}(t)$ is amplified with an amplification factor and forwarded to the D node in $SlotII$. The amplification factor α_{ri} at the R_i relay node is given by [4],

$$\alpha_{ri} = \sqrt{\left(\frac{P}{P\left|h_{1i}\right|^2 + \sigma_{1i}^2}\right)}. \tag{4.3}$$

The R_i relay node forward the $y_{ri}(t)$ signal to the D node by amplifying it with α_{ri}. Notice that α_{ri} depends on the h_{1i} which is assumed to be known by the R_i [4]. As a result, the D node receives the signals that are forwarded by the relay nodes at the end of the $SlotII$ .Using equation (4.2) and equation (4.3), the end-to-end (E2E) signal received by the D node through R_i relay node can be represented as,

$$y_{di}(t) = \alpha_{ri} h_{2i} y_{ri}(t - \tau_{2i}) + n_{2i}(t), \ \ t \in [T, 2T], \tag{4.4}$$

$$y_{di}(t) = \sqrt{P}\alpha_{ri} h_{1i} h_{2i} s(t - \tau_i) + n_{Ti}(t), \ \ t \in [T, 2T], \tag{4.5}$$

where $\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc}$ is the total time delay of the signal between the S and D nodes through the R_i relay node. Note that τ_{proc} is the processing time used at the relay nodes for amplifying the signal. We assume that this processing time is negligible for all the relays with the AF relaying strategy (i.e., $\tau_{proc} \approx 0$). In equation (4.1), $n_{2i}(t)$ is the AWGN with zero mean and σ_{2i}^2 variance whereas $n_{Ti}(t)$ in equation (4.5) is the E2E AWGN with zero mean and $\sigma_{Ti}^2 = \alpha_{ri}^2 \left|h_{2i}\right|^2 \sigma_{1i}^2 + \sigma_{2i}^2$ variance through the R_i relay node.

4.3. DF Relaying Strategy Based Signal Model

In the DF relaying strategy based cooperative relay networks, the received signals from the S node are decoded and forwarded to the D node at the relay nodes. In this study, we assume that the transmitted signal *s(t)* is fully estimated and decoded by the relay nodes (see [4] and Theorem 1 in [13]). After the decoding, the relay nodes forward the decoded signal *s(t)* with the average signal power *P* to the D node. Similar to the AF based system model, the communication between the S and D node is conducted over two time slots. In the *SlotI* , the S node broadcasts *s(t)* and the relay nodes as well as the D node receive this signal. The received signals at the D node and the relay nodes are exactly same as equation (4.1) and equation (4.2), respectively, in the *SlotI*.

The R_i nodes fully decode the signal *s(t)* using the received signal $y_{ri}(t)$. In *SlotII*, the decoded signal at the R_i relay node is transmitted with the same average power *P* to the D node. The received signal at the D node through the R_i relay node is expressed as,

$$y_{di}(t) = \sqrt{P}h_{2i}s(t-\tau_i) + n_{Ti}(t), \;\; t \in [T, 2T], \tag{4.6}$$

where $\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc}$ is the total time delay of the signal between the S and D nodes through R_i relay node and τ_{proc} is the processing time for decoding s(t) at the relay nodes. We assume that τ_{proc} is a known parameter for the DF relaying strategy case. Note that $n_{Ti}(t)$ is the E2E AWGN with zero mean and $\sigma_{Ti}^2 = \sigma_{2i}^2$ variance through the R_i relay node. By comparing equation (4.5) and equation (4.6), it can be seen that $n_{1i}(t)$ term does not exist in $n_{Ti}(t)$ in equation (4.6), due to the regeneration of a clear copy of *s(t)* after fully decoding at the relay nodes in the DF relaying strategy case.

In the following section, the effects of the AF and DF relaying strategies on the accuracy of location estimation in terms of CRLB under different conditions are investigated.

4.4. Cramer-Rao Lower Bound Analysis for Line-of-Sight Environment

The scenarios that are considered in this chapter are tabulated in Table 4.1. According to these scenarios, the CRLB expressions are derived for the time delay based location estimation of the S node under the AF and DF relaying strategies when the location of the D node is known and unknown.

Note that the location estimation of the S node is performed at the D node by using the received signals through the relay links and the direct link. It is also assumed that the locations of all the relay nodes are known throughout in this study so that we can use the relay nodes as “reference” nodes.

Table 4.1: The scenarios that are considered in this section.

Location of S node	Location of Relay nodes	Location of D node	Relaying Strategies
Unknown	Known	Known	AF DF
Unknown	Known	Unknown	AF DF

4.4.1. Scenario I: The location of D node is known

In this section, the CRLB expressions for the location estimation of the S node are derived when the location of the D node is known. In this scenario, the vector of unknown parameters is defined as $\theta = [x_s, y_s]^T$, where x_s and y_s are the coordinates of the S node.

In order to estimate the unknown parameters in θ at the D node, time delay estimation technique is employed. Firstly, we relate time delay parameters to the unknown parameters in θ since our aim is to obtain the lower bounds for the estimation errors of x_s and y_s parameters. The relation between the time delay parameters τ_{1i} and (x_s, y_s) parameters can be established through $\tau_{1i} = d_{1i} / c$ [1], where d_{1i} is the distance between the S and R_i relay node that is given by,

$$d_{1i} = \sqrt{(x_s - x_{ri})^2 + (y_s - y_{ri})^2}. \tag{4.7}$$

Using equation (4.7), we can express τ_{1i} parameters as,

$$\tau_{1i} = \frac{1}{c}\sqrt{(x_s - x_{ri})^2 + (y_s - y_{ri})^2}. \tag{4.8}$$

[1] c is the speed of light.

Similarly, we can represent τ_{2i} and τ_0 , respectively, as,

$$\tau_{2i} = \frac{1}{c}\sqrt{(x_d - x_{ri})^2 + (y_d - y_{ri})^2}, \tag{4.9}$$

$$\tau_0 = \frac{1}{c}\sqrt{(x_s - x_d)^2 + (y_s - y_d)^2}. \tag{4.10}$$

In this study, we use the E2E time delay metric for estimating the location of the S node at the D node, which is defined in the following vector form,

$$\tau = [\tau_0, \tau_1, \tau_2, \ldots, \tau_M],\ i = 1, 2, 3, \ldots, M. \tag{4.11}$$

where $\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc}$. In this scenario, τ_{2i} are known parameters since the location of the relay and D nodes are known. However, τ_0 and τ_{1i} remain as unknown parameters which their estimates can be defined, respectively, as follows [13],

$$\tau_0 = \tau_0 + \epsilon_0, \tag{4.12}$$

$$\tau_{1i} = \tau_{1i} + \epsilon_{1i}, \tag{4.13}$$

where $\epsilon_0 \sim N(0, \xi_0^2)$ and $\epsilon_{1i} \sim N(0, \xi_{1i}^2)$ are the Gaussian distributed estimation errors of the time delays for the direct link and the first hops, respectively. Since τ_{1i} parameters are the only unknown parameters in τ_{1i} , “1” indices in τ_{1i} , ϵ_{1i} and ξ_{1i} can be dropped in this scenario, i.e.,

$$\epsilon_i = \epsilon_{1i},\ \ \xi i = \xi_{1i}. \tag{4.14}$$

Now, using equation (4.11), equation (4.12) and equation (4.13), the estimated time delay vector takes the following form,

$$\hat{\tau} = [\tau_0, \tau_1, \tau_2, \ldots, \tau_M], \tag{4.15}$$

where $\tau_i = \tau_i + \epsilon_i$. Since ϵ_i estimation errors are Gaussian distributed and τ_i are independent random variables (RVs), we can write the joint probability density function (pdf) of $\hat{\tau}$ for given θ in the following form,

$$p(\hat{\tau} / \theta) \propto \prod_{i=0}^{M} \exp(\frac{-(\tau_i - \tau_i)^2}{2\xi_i^2}). \tag{4.16}$$

In order to calculate the CRLBs for $\theta = [x_s, y_s]^T$, Fisher Information Matrix (FIM) for θ is obtained, which is given as follows,

$$I_\theta = \begin{bmatrix} I_{x_s x_s} & I_{x_s y_s} \\ I_{y_s x_s} & I_{y_s y_s} \end{bmatrix}, \tag{4.17}$$

where

$$CRLB_{x_s} = \left[I_\theta^{-1} \right]_{11}, \tag{4.18}$$

$$CRLB_{y_s} = \left[I_\theta^{-1} \right]_{22}. \tag{4.19}$$

The elements of the FIM is obtained by 0,

$$I_\theta = E_\theta \left[\frac{\partial}{\partial \theta} \ln p(\hat{\tau} / \theta) \left(\frac{\partial}{\partial \theta} \ln p(\hat{\tau} / \theta) \right)^T \right], \tag{4.20}$$

where E_θ is the expectation operator conditioned over θ. Since the parameters in τ are functions of θ according to equation (4.13) and equation (4.12), we can use chain rule for determining $\frac{\partial}{\partial \theta} \ln p(\hat{\tau} / \theta)$ term as follows [6],

$$\frac{\partial}{\partial\theta}\ln p(\hat{\tau}/\theta) = \frac{\partial\tau}{\partial\theta}\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau). \tag{4.21}$$

Replacing equation (4.21), into equation (4.20), the FIM takes the following form,

$$I_\theta = \frac{\partial\tau}{\partial\theta}\left[E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^T\right]\right]\left(\frac{\partial\tau}{\partial\theta}\right)^T. \tag{4.22}$$

Furthermore, $\frac{\partial\tau}{\partial\theta}$ is calculated and it is given by,

$$\frac{\partial\tau}{\partial\theta} = \frac{1}{c}\begin{bmatrix}\cos(\psi_0) & \cos(\psi_1) & \cos(\psi_2) & \dots & \cos(\psi_M)\\ \sin(\psi_0) & \sin(\psi_1) & \sin(\psi_2) & \dots & \sin(\psi_M)\end{bmatrix}, \tag{4.23}$$

where $\psi_i = \tan^{-1}\left(\frac{y_s - y_{ri}}{x_s - x_{ri}}\right)$. Note that ψ_0 is defined as $\psi_0 = \tan^{-1}\left(\frac{y_s - y_d}{x_s - x_d}\right)$. Then, the term with the expectation operator is obtained, which is given by,

$$E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^T\right] = \text{diag}\left[\frac{1}{\xi_0^2},\frac{1}{\xi_1^2},\frac{1}{\xi_2^2},\dots,\frac{1}{\xi_M^2}\right]. \tag{4.24}$$

Using equation (4.22), equation (4.23) and equation (4.24), the FIM is obtained,

$$I_\theta = \frac{1}{c^2}\begin{bmatrix}\sum_{i=0}^{M}\cos^2(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\psi_i)\sin(\psi_i)\xi_i^{-2}\\ \sum_{i=0}^{M}\sin(\psi_i)\cos(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin^2(\psi_i)\xi_i^{-2}\end{bmatrix}. \tag{4.25}$$

Note that I_θ^{-1} is determined after some straightforward manipulations and it has the following form,

$$I_\theta^{-1} = c^2 \frac{\begin{bmatrix} \sum_{i=0}^{M} \sin^2(\psi_i)\xi_i^{-2} & -\sum_{i=0}^{M} \cos(\psi_i)\sin(\psi_i)\xi_i^{-2} \\ -\sum_{i=0}^{M} \sin(\psi_i)\cos(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M} \cos^2(\psi_i)\xi_i^{-2} \end{bmatrix},}{[\det(I_\theta)]} \tag{4.26}$$

where $\det(I_\theta) = \sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right)\xi_i^{-2}\xi_j^{-2}$. Using equation (4.18), equation (4.19) and equation (4.26), we can express the CRLBs for x_s and y_s , respectively, as,

$$CRLB_{x_s} = c^2 \frac{\sum_{i=0}^{M} \sin^2(\psi_i)\xi_i^{-2}}{\sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right)\xi_i^{-2}\xi_j^{-2}}, \tag{4.27}$$

$$CRLB_{y_s} = c^2 \frac{\sum_{i=0}^{M} \cos^2(\psi_i)\xi_i^{-2}}{\sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right)\xi_i^{-2}\xi_j^{-2}}. \tag{4.28}$$

Furthermore, we can represent the CRLB for the location of S node $p_s = (x_s, y_s)$, i.e., $CRLB_{ps}$ using (4.27) and (4.28) as [26],

$$CRLB_{p_s} = CRLB_{x_s} + CRLB_{y_s}, \tag{4.29}$$

$$CRLB_{p_s} = c^2 \frac{\sum_{i=0}^{M} \xi_i^{-2}}{\sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right)\xi_i^{-2}\xi_j^{-2}}, \tag{4.30}$$

where ξ_i^2 is the variance (CRLB) for time delays error which is defined as (see Section 3.2.1 in [19]),

$$\xi_i^2 = \frac{1}{\gamma_i \tilde{E}_i}, \tag{4.31}$$

where γ_i is the E2E SNR through relay node and $\tilde{E}_i = \int_0^T \left[s'(t-\tau_i) \right]^2 dt$. Similarly, ξ_0^2 is defined as,

$$\xi_0^2 = \frac{1}{\gamma_0 \tilde{E}_0}, \tag{4.32}$$

where $\gamma_0 = \frac{P h_0^2}{\sigma_0^2}$ is the SNR for the direct link $\tilde{E}_0 = \int_0^T \left[s'(t-\tau_0) \right]^2 dt$. At this point, the effects of the AF and DF relay strategies on the CRLBs are differ in terms of the E2E SNR values. If the AF relaying strategy is employed, the E2E SNR through R_i node $\gamma_{i_{AF}}$ is given by [6],

$$\gamma_{i_{AF}} = \frac{P \alpha_{ri}^2 \left| h_{1i} \right|^2 \left| h_{2i} \right|^2}{\sigma_{Ti}^2}. \tag{4.33}$$

However, if the DF relaying strategy is employed at the R_i nodes, the E2E SNR through the R_i node $\gamma_{i_{DF}}$ takes the following form [1],

$$\gamma_{i_{DF}} = \frac{P \left| h_{2i} \right|^2}{\sigma_{2i}^2}. \tag{4.34}$$

As seen in equation (4.33) and equation (4.34), the E2E SNRs through R_i nodes take different values when different relay strategies are employed. Note that in the case of DF relaying strategy, the first hops do not contribute to the E2E SNR since the received signals at relay nodes are fully decoded, consequently, a clear of copy of source signal is generated. By using equation, (4.33) and equation (4.34), the $CRLB_{p_s}$ for the AF and DF relaying strategies are obtained and they are given as follows, respectively,

$$CRLB_{p_{s_{AF}}} = c^2 \frac{\sum_{i=0}^{M} \gamma_{i_{AF}} E_i}{\sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right) \gamma_{i_{AF}} \gamma_{j_{AF}} E_i E_j}, \tag{4.35}$$

$$CRLB_{p_{s_{DF}}} = c^2 \frac{\sum_{i=0}^{M} \gamma_{i_{DF}} E_i}{\sum_{i=0}^{M-1} \sum_{j=i+1}^{M} \sin^2\left(\psi_i - \psi_j\right) \gamma_{i_{DF}} \gamma_{j_{DF}} E_i E_j}. \tag{4.36}$$

4.4.2. Scenario II: The location of D node is unknown

In this section, the CRLB for the location estimation of the S node is derived when the D node has unknown location. Such case can be seen in many practical implementations such as a mobile data fusion center (D node is mobile) in a military environment. In addition, the S node and the D node may be mobile station equipped without any location estimation devices, such as GPS.

In this case, only the relay nodes with known locations are considered to be the "reference" nodes. Therefore, we calculate the CRLBs for x_s and y_s under these conditions. Since x_d and y_d are unknown parameters, τ_{2i} values are also unknown parameters in this scenario and the vector for unknown parameters is,

$$\theta = \left[x_s, y_s, x_d, y_d\right]^T . \tag{4.37}$$

The CRLB for the location of the S node is obtained using the estimates of the E2E time delays. Similar to the case where the location of the D node is known, the location estimation is also performed at the D node in this case. The equations (4.9) - (4.13) are also valid and used in this section. Besides these equations, time delay estimates for the second hops $(R_i \to D)$ τ_{2i} can be approximated as [13],

$$\tau_{2i} = \tau_{2i} + \epsilon_{2i}, \tag{4.38}$$

where $\epsilon_{2i} \sim N(0, \xi_{2i}^2)$ is the Gaussian distributed estimation error of the time delays of the second hop. Since ϵ_{1i} and ϵ_{2i} are Gaussian distributed RVs, the estimation error of the E2E time delays is,

$$\epsilon_i = \epsilon_{1i} + \epsilon_{2i}, \tag{4.39}$$

where $\epsilon_i \sim N(0, \xi_i^2)$ and ξ_i^2 is defined as,

$$\xi_i^2 = \xi_{1i}^2 + \xi_{2i}^2. \tag{4.40}$$

Then, the $\hat{\tau}$ vector in this case is defined as,

$$\hat{\tau} = [\tau_0, \tau_1, \tau_2, \ldots, \tau_M], \tag{4.41}$$

where

$$\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc} + \epsilon_i. \tag{4.42}$$

Also, we can represent the joint pdf of $\hat{\tau}$ conditioned on θ as,

$$p(\hat{\tau} / \theta) \propto \prod_{i=0}^{M} \exp(\frac{-(\tau_i - \tau_i)^2}{2\xi_i^2}). \tag{4.43}$$

The FIM for this case is given by,

$$I_\theta = \begin{bmatrix} I_{x_s x_s} & I_{x_s y_s} & I_{x_s x_d} & I_{x_s y_d} \\ I_{y_s x_s} & I_{y_s y_s} & I_{y_s x_d} & I_{y_s y_d} \\ I_{x_d x_s} & I_{x_d y_s} & I_{x_d x_d} & I_{x_d y_d} \\ I_{y_d x_s} & I_{y_d y_s} & I_{y_d x_d} & I_{y_d y_d} \end{bmatrix}, \tag{4.44}$$

where

$$CRLB_{x_s} = \left[I_\theta^{-1} \right]_{11}, \tag{4.45}$$

$$CRLB_{y_s} = \left[I_\theta^{-1} \right]_{22}. \tag{4.46}$$

Using equation (4.43), the elements of the FIM for θ can be obtained using the following equation,

$$I_\theta = E_\theta\left[\frac{\partial}{\partial\theta}\ln p(\hat{\tau}/\theta)\left(\frac{\partial}{\partial\theta}\ln p(\hat{\tau}/\theta)\right)^T\right]. \tag{4.47}$$

Similar to the previous case, I_θ can be calculated by applying the chain rule as follows,

$$I_\theta = \frac{\partial\tau}{\partial\theta}\left[E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^T\right]\right]\left(\frac{\partial\tau}{\partial\theta}\right)^T. \tag{4.48}$$

Furthermore, $\frac{\partial\tau}{\partial\theta}$ expression is calculated, which is given by,

$$\frac{\partial\tau}{\partial\theta} = \frac{1}{c}\begin{bmatrix} \cos(\psi_0) & \cos(\psi_1) & \cos(\psi_2) & \dots & \cos(\psi_M) \\ \sin(\psi_0) & \sin(\psi_1) & \sin(\psi_2) & \dots & \sin(\psi_M) \\ \cos(\phi_0) & \cos(\phi_1) & \cos(\phi_2) & \dots & \cos(\phi_M) \\ \sin(\phi_0) & \sin(\phi_1) & \sin(\phi_2) & \dots & \sin(\phi_M) \end{bmatrix}, \tag{4.49}$$

where $\phi_i = \tan^{-1}\left(\frac{y_d - y_{ri}}{x_d - x_{ri}}\right)$. As a special case, $\psi_0 = -\phi_0 = \tan^{-1}\left(\frac{y_s - y_d}{x_s - x_d}\right)$. Then, the following term with the expectation operator is calculated and the result is given as,

$$E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^T\right] = \mathrm{diag}\left[\frac{1}{\xi_0^2}, \frac{1}{\xi_1^2}, \frac{1}{\xi_2^2}, \dots, \frac{1}{\xi_M^2}\right]. \tag{4.50}$$

Using (4.48), (4.49) and (4.50), we obtain the FIM for this case, which is given by,

$$I_\theta = \frac{1}{c^2}\begin{bmatrix} \sum_{i=0}^{M}\cos^2(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\psi_i)\sin(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\psi_i)\cos(\phi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\psi_i)\sin(\phi_i)\xi_i^{-2} \\ \sum_{i=0}^{M}\sin(\psi_i)\cos(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin^2(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin(\psi_i)\cos(\phi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin(\psi_i)\sin(\phi_i)\xi_i^{-2} \\ \sum_{i=0}^{M}\cos(\phi_i)\cos(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\phi_i)\sin(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos^2(\phi_i)\xi_i^{-2} & \sum_{i=0}^{M}\cos(\phi_i)\sin(\phi_i)\xi_i^{-2} \\ \sum_{i=0}^{M}\sin(\phi_i)\cos(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin(\phi_i)\sin(\psi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin(\phi_i)\cos(\phi_i)\xi_i^{-2} & \sum_{i=0}^{M}\sin^2(\phi_i)\xi_i^{-2} \end{bmatrix}. \tag{4.51}$$

Since I_θ is a 4x4 square matrix, we write I_θ in the following form,

$$I_\theta = \frac{1}{c^2}\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}, \tag{4.52}$$

where

$$X = \begin{bmatrix} I_{x_s x_s} & I_{x_s y_s} \\ I_{y_s x_s} & I_{y_s y_s} \end{bmatrix},$$
$$Y = \begin{bmatrix} I_{x_d x_s} & I_{x_d y_s} \\ I_{y_d x_s} & I_{y_d y_s} \end{bmatrix}, \tag{4.53}$$
$$Z = \begin{bmatrix} I_{x_d x_d} & I_{x_d y_d} \\ I_{y_d x_d} & I_{y_d y_d} \end{bmatrix}.$$

Using the block matrix inversion, we can obtain the first 2x2 block of the I_θ^{-1} as follows [28],

$$I_\theta^{-1} = c^2\left[X - YZ^{-1}Y^T\right]^{-1}. \tag{4.54}$$

Then, we obtain $CRLB_{x_s}$ and $CRLB_{y_s}$ expression using these submatrices as follows,

$$CRLB_{x_s} = c^2\left[X - YZ^{-1}Y^T\right]_{11}^{-1}, \tag{4.55}$$

$$CRLB_{y_s} = c^2\left[X - YZ^{-1}Y^T\right]_{22}^{-1}. \tag{4.56}$$

Using equation (4.29), we can define the $CRLB_{p_s}$ as,

$$CRLB_{p_s} = c^2 \frac{-(KA + I_{y_s x_d} B + I_{y_s y_d} C + I_{x_s x_d} D + I_{x_s y_d} E)}{AL + F^2 + GB + HC - I_{y_s y_s}\left[I_{x_s x_d} D + I_{x_s y_d} E \right]}, \tag{4.57}$$

where

$A = I_{x_d y_d}^2 - I_{x_d x_d} I_{y_d y_d}$,
$B = I_{y_s x_d} * I_{y_d y_d} - I_{y_s y_d} I_{x_d y_d}$,
$C = I_{x_d x_d} * I_{y_s y_d} - I_{x_d x y_s} * I_{x_d y_d}$,
$D = I_{y_d y_d} * I_{x_s x_d} - I_{x_s y_d} * I_{x_d y_d}$,
$E = I_{x_d x_d} * I_{x_s y_d} - I_{x_s x_d} * I_{x_d y_d}$,
$F = I_{x_s x_d} * I_{y_s y_d} - I_{x_s y_d} * I_{y_s x_d}$,
$G = 2I_{x_s y_s} * I_{x_s x_d} - I_{x_s x_s} * I_{y_s x_d}$,
$H = 2I_{x_s y_s} * I_{x_s y_d} - I_{x_s x_s} * I_{y_s y_d}$,
$K = I_{x_s x_s} + I_{y_s y_s}$,
$L = I_{x_s y_s}^2 - I_{x_s x_s} I_{y_s y_s}$.

4.4.2.1. AF Relaying Strategy Based Location Estimation

Notice in equation (4.57) that the $CRLB_{p_s}$ expression consists of ξ_{1i}^2 and ξ_{2i}^2 terms. These time delay variances should be jointly estimated at the D node for each received signal. We can represent the log-likelihood function of τ_i for the signal at the D node through R_i relay node as follows,

$$\Lambda(\tau_i) = -\frac{1}{2\sigma_{Ti}^2}\int_0^T \left[y_{di}(t) - Gs(t-\tau_i) \right]^2 \mathrm{d}t, \tag{4.58}$$

where $\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc}$, $i = 1,2,3,\ldots,M$, and $G = \alpha_i h_{1i} h_{2i} \sqrt{P}$ when the AF relaying strategy is employed. Assume that α_i, h_{1i} and h_{2i} are known parameters. Then, the joint log-likelihood function at the D node is given by,

$$\Lambda(\tau) = \sum_{i=1}^{M} \Lambda(\tau_i). \tag{4.59}$$

In this case, the unknown parameters at the D node are

$$\theta = [\tau_{11}, \tau_{12}, \ldots, \tau_{1M}, \tau_{21}, \tau_{22}, \ldots, \tau_{2M}]. \tag{4.60}$$

In order to calculate the CRLBs for θ unknown parameters, the FIM can be obtained as follows,

$$I_\theta = E_\theta\left[\frac{\partial}{\partial\theta}(\Lambda(\tau)/\theta)\left(\frac{\partial}{\partial\theta}(\Lambda(\tau)/\theta)\right)^T\right]. \tag{4.61}$$

Using equation (4.61), the 2Mx2M FIM matrix is calculated, which is given by,

$$I_\theta = \begin{bmatrix}
\frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}^2} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}\partial\tau_{12}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}\partial\tau_{1M}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}\partial\tau_{21}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{11}\partial\tau_{2M}} \\
\frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}\partial\tau_{11}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}^2} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}\partial\tau_{1M}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}\partial\tau_{21}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{12}\partial\tau_{2M}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}\partial\tau_{11}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}\partial\tau_{12}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}^2} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}\partial\tau_{21}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{1M}\partial\tau_{2M}} \\
\frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}\partial\tau_{11}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}\partial\tau_{12}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}\partial\tau_{1M}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}^2} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{21}\partial\tau_{2M}} \\
\frac{\partial^2\Lambda(\tau)}{\partial\partial\tau_{22}\partial\tau_{11}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{22}\partial\tau_{12}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{22}\partial\tau_{1M}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{22}\partial\tau_{21}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{22}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{22}\partial\tau_{2M}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}\partial\tau_{11}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}\partial\tau_{12}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}\partial\tau_{1M}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}\partial\tau_{21}} & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}\partial\tau_{22}} & \cdots & \frac{\partial^2\Lambda(\tau)}{\partial\tau_{2M}^2}
\end{bmatrix}_{2Mx2M} \tag{4.62}$$

The elements of I_θ matrix can be obtained after straightforward manipulations as follows [19],

$$I_\theta = \begin{bmatrix} \gamma_1 \tilde{E}_1 & \cdots & 0 & \gamma_1 \tilde{E}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_M \tilde{E}_M & 0 & \cdots & \gamma_M \tilde{E}_M \\ \gamma_1 \tilde{E}_1 & \cdots & 0 & \gamma_1 \tilde{E}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_M \tilde{E}_M & 0 & \cdots & \gamma_M \tilde{E}_M \end{bmatrix}, \tag{4.63}$$

where $\gamma_i = \frac{G^2}{\sigma_{Ti}^2}$ and $\tilde{E}_i = \int_0^T [s'(t-\tau_i)]^2 \, dt$. We can divide I_θ matrix into MxM blocks such as,

$$I_\theta = \begin{bmatrix} Q & Q \\ Q & Q \end{bmatrix}, \tag{4.64}$$

where

$$Q = \begin{bmatrix} \gamma_1 \tilde{E}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_M \tilde{E}_M \end{bmatrix}. \tag{4.65}$$

If we apply the block matrix inversion to I_θ in order to find the CRLBs for θ parameters, the first MxM block of I_θ^{-1} matrix can be obtained as follows,

$$I_\theta^{-1} = \left(Q - QQ^{-1}Q \right)^{-1}. \tag{4.66}$$

By using equation (4.66), it can be seen by calculating that the elements of I_θ^{-1} get ∞ values. Therefore, the CRLBs for θ parameters are ∞ namely, $\xi_{1i}^2 = \xi_{2i}^2 = \infty, i = 1, 2, 3, \ldots, M$. In this scenario, the CRLB for the location estimation of the S node cannot be estimated at the D node. However, this can be overcome by designing new protocols for this case. For instance, a protocol can be designed that allows the localization of the S and D nodes at the relay nodes with known positions rather than the D node.

4.4.2.2. DF Relaying Strategy Based Location Estimation

In the DF relay strategy case, as stated in Section 2.2, the source signal *s(t)* is fully decoded without errors at the relay nodes [4]. This indicates that the τ_{1i} values can be estimated without errors at the relay nodes employing the DF relaying strategy.

The location of the S node can be estimated at the D node by sending the τ_{1i} values to this node. However, we want to estimate the location of the S node at the D node through the estimation of the E2E time delay which is modeled as,

$$\tau_i = \tau_{1i} + \tau_{2i} + \tau_{proc} + \epsilon_i. \tag{4.67}$$

Since τ_{1i} and τ_{proc} are known parameters, equation (4.67) becomes $\tau_i = \tau_{2i} + \epsilon_i$ where $\epsilon_i = \epsilon_{2i} \sim N(0, \xi_{2i})$ in this case [19]. The variance ξ_i^2 is calculated, which is given by,

$$\xi_i^2 = \frac{\sigma_{2i}^2}{P\left|h_{2i}\right|^2 \tilde{E}_i}. \tag{4.68}$$

By plugging equation (4.68) into equation (4.57), $CRLB_{p_s}$ for the DF relaying strategy can be obtained. In summary, the AF relaying strategy cannot be used for the location estimation of the S node at the D node when the location of the D node is unknown. However, the DF relaying strategy can be used for the location estimation of the S node even if the location of the D node is not known.

4.5. Simulation Results

In this section, the numerical results using the CRLB expressions derived in the previous sections are presented. Especially, the effects of the AF and DF relaying strategies on the location estimation accuracy are studied when the location of the D node is known and unknown. In addition, the effects of the node topology on the location estimation accuracy are investigated. Moreover, the impacts of the number of relay nodes on the location estimation accuracy are quantified.

The average signal power P is used to investigate the estimation accuracy under different SNR regime by selecting P =1mW … 1W. $\sigma_0^2, \sigma_{1i}^2$ and σ_{2i}^2 values are selected as 1mW. Since we are considering single-path LOS fading environment, the channel coefficients h_0, h_{1i} and h_{2i} are selected as Rician RVs. The simulation results are obtained in terms of root mean square error (RMSE), which is equivalent to $\sqrt{CRLB_{p_s}}$.

In the simulations, the $s(t)$ signal is generated using Gaussian second order derivative pulse shape which is given by [6],

$$s(t) = A(1 - \frac{4\pi t^2}{\varsigma^2})e^{-2\frac{\pi t^2}{\varsigma^2}}, \tag{4.69}$$

where A and ς are the parameters that are used to adjust the energy and pulse width of $s(t)$, respectively. A is selected as 1 to generate unit energy pulse. Also, $\varsigma = 1/B$ where B = 20MHz is the bandwidth of $s(t)$ without loss of generality. The pulse duration T_s is selected as $T_s = 2.5\varsigma$. When performing the simulations, the following specifications are considered. The relay nodes are uniformly distributed over a geographical region with the dimension of 1000m by 1000m. This is repeated 10 times in order to consider random node topologies. During the random distribution of the nodes, we make sure that the relay nodes are not colinear[1] with the S and D nodes. For each random topology, the $CRLB_p$ is calculated over 100 random channel realizations. It is assumed that the channels $(S \to R_i, S \to D$ and $R_i \to D)$ are identical.

[1] Colinearity occurs when a relay node is located on the line between S and D nodes.

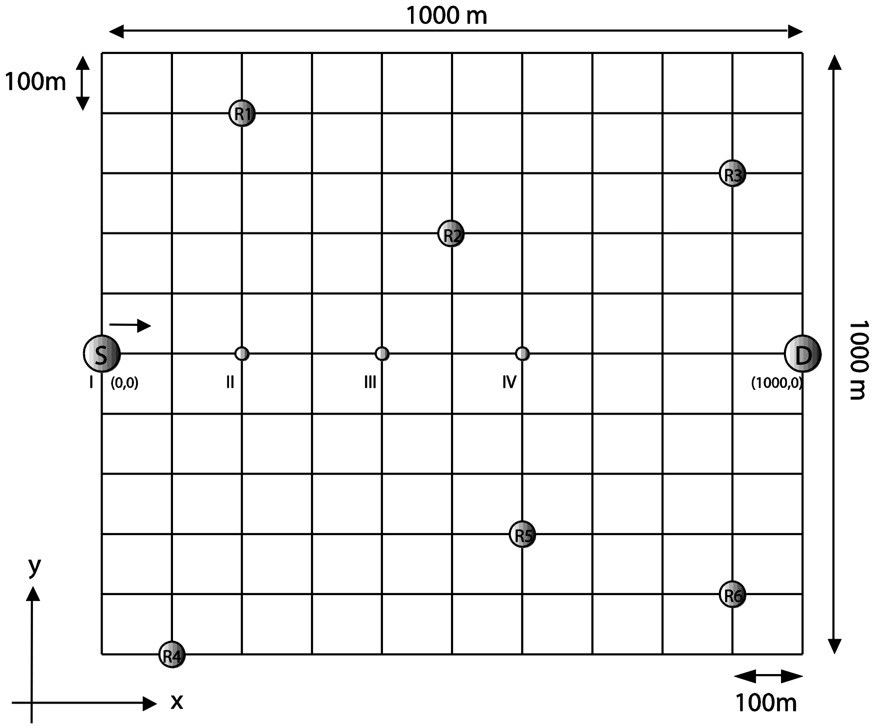

Figure 4.2: The node topology for the cooperative relay network with uniformly distributed relay nodes - S Node is mobile.

The illustration of the node topology when the location of the D node is known and the S node is mobile is shown in Figure 4.2. In this topology, we assume that there are 6 relay nodes that are distributed uniformly between S and D nodes as mentioned previously. The $CRLB_p$ results for this scenario are presented in Figure 4.3. This figure shows the effects of the AF and DF relaying strategies on $CRLB_p$ for different S node position shown in Figure 4.2 when the location of the D node is known. The results show that the $CRLB_p$ for the DF relaying strategy has lower values than those for the AF relaying strategy for all the position of the S node. The reason for this result is that the effects of the first hops' channel conditions is suppressed when the DF relaying strategy is employed, so that the location estimation accuracy is improved. On the other hand, the DF relaying strategy has higher process costs than that of the AF relaying strategy which is neglected in this paper. Furthermore, the S node is moved from I to III position which leads to the $CRLB_p$ decrease. However, when the S node is moved from III to IV position, the $CRLB_{p_s}$ relatively increases.

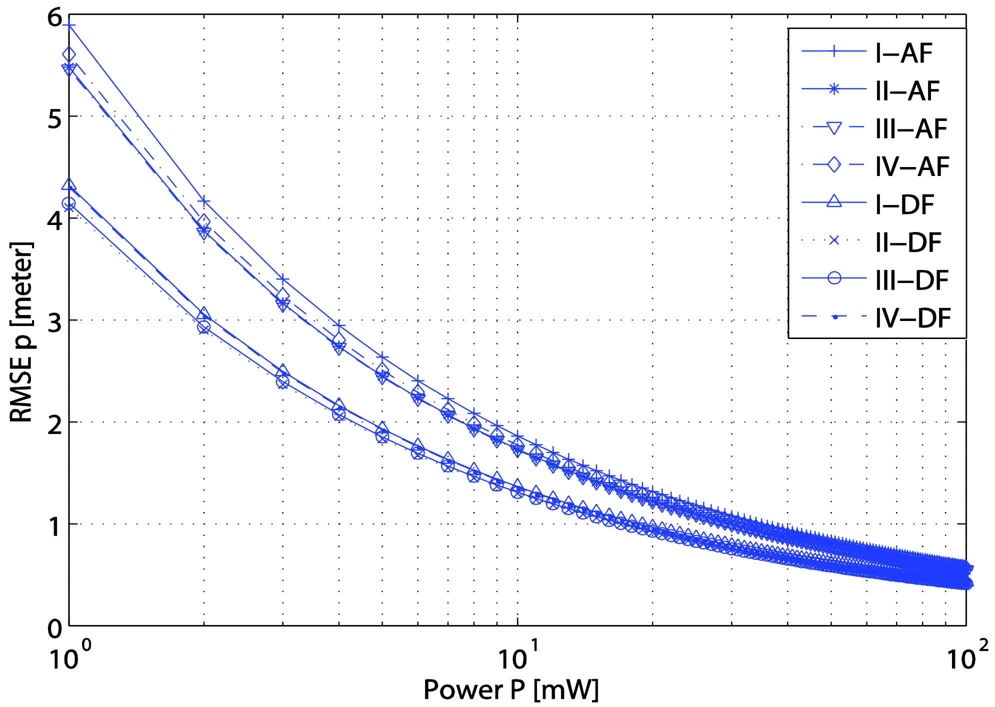

Figure 4.3: The performance comparison of the AF and DF relaying strategy when the location of D node is known.

The $CRLB_{p_s}$ is highly dependent on sine values of the angles (ϕ_i and ψ_i) and these angles take values between $\left[0, \frac{\pi}{2}\right]$ when moving from I to III. Therefore, the estimation accuracy increased when moving from I to III. On the other hand, due to the angles which take values between $\left[\frac{\pi}{2}, \pi\right]$ when moving from III to IV, the estimation accuracy decreases compared with the $I \to III$ movement.

In addition, approaching to the D node (moving from I to IV position) does not always necessarily decrease the $CRLB_{p_s}$, since the $CRLB_{p_s}$ expression is independent of the D node in the case of the location of the D node is known. The $CRLB_{p_s}$ values are also obtained under the DF relaying strategy, when the D node position is unknown (e.g. mobile), as illustrated in Figure 4.4.

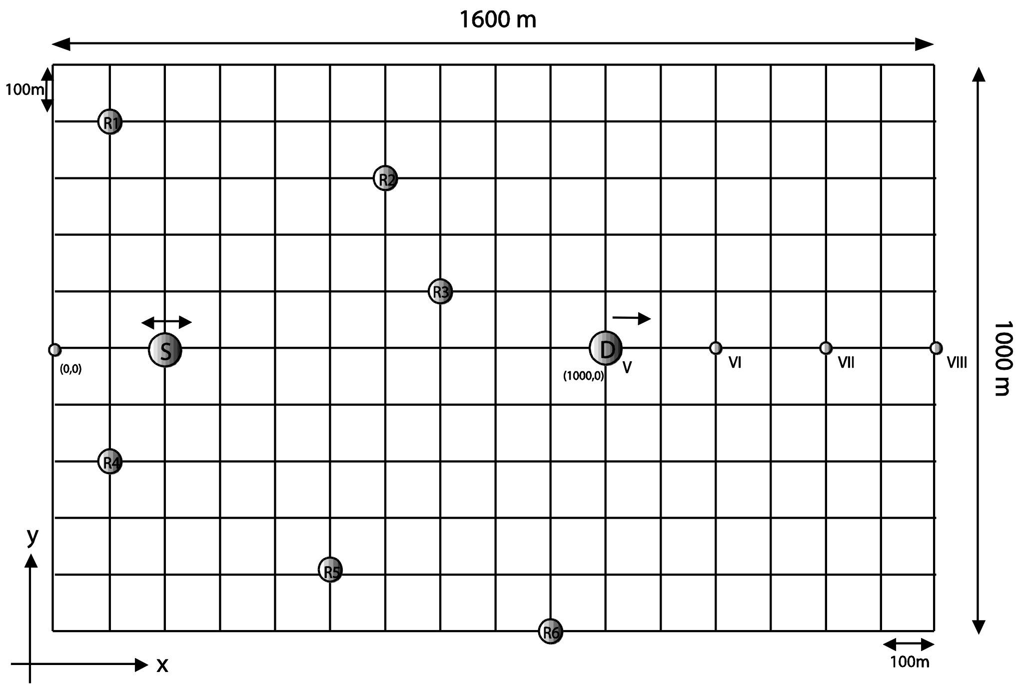

Figure 4.4: The node topology for the cooperative relay network with uniformly distributed relay nodes with the DF relaying strategy – S and D nodes are mobile.

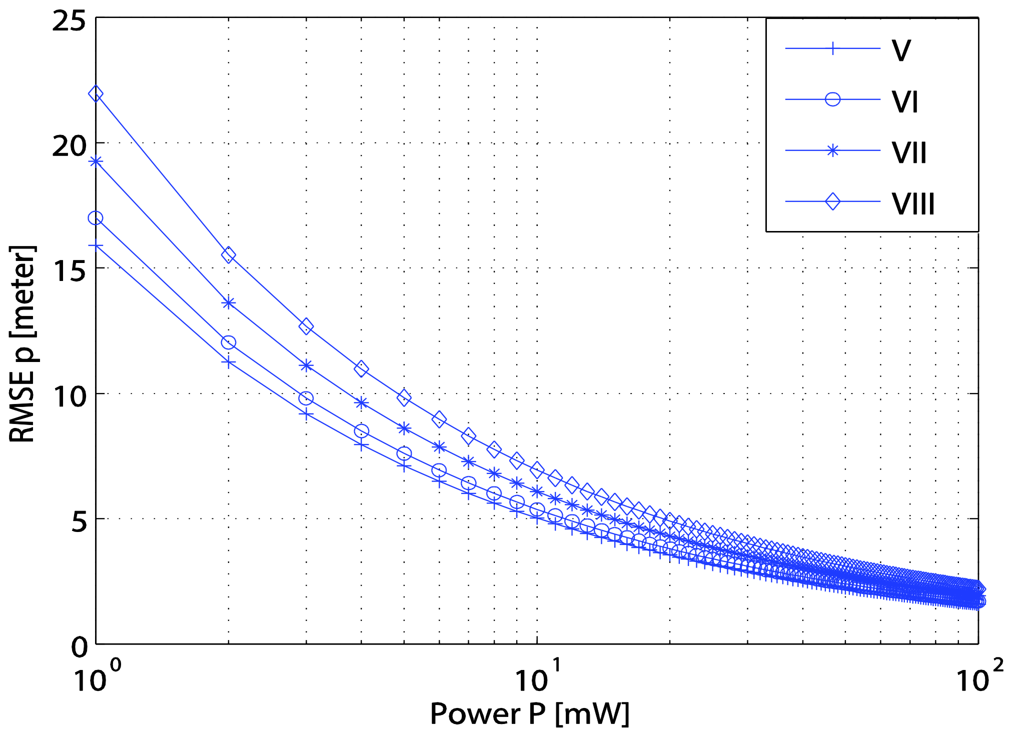

Figure 4.5: The performance of the DF strategy on the estimation accuracy when the location of the D node is unknown.

As stated above, τ_{1i} time delays can be estimated at the R_i nodes and are assumed to be known parameters when the D node is unknown under the DF relaying strategy. Under these conditions, when the D node moves from *V* to *VIII*, the location estimation error increases due to the decreasing SNR at the D node value and the change of angle differences between the nodes.

The effects of the number of relay nodes on the $CRLB_{p_s}$ are also investigated in this study. The S node is placed at II position which is stated in Figure 4.2 and the $CRLB_{p_s}$ is calculated with different number of relay nodes. At the beginning, the first three relay nodes are selected randomly. After calculating $CRLB_{p_s}$ for $M = 3$, the number of relay nodes is increased by adding a relay node into the network which is located randomly. Following this procedure, the $CRLB_{p_s}$ calculations are performed for $M = 3, 4, 5, 6$. $CRLB_{p_s}$ results for the AF and DF relaying strategies with different number of relay nodes are shown in Figure 4.6.

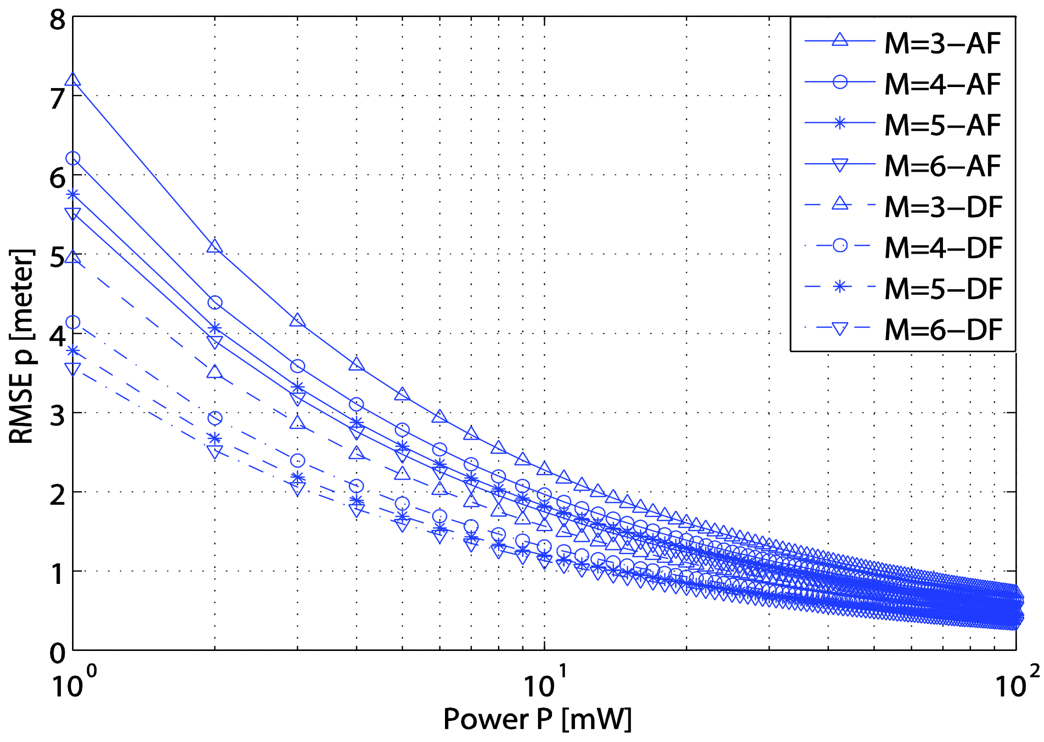

Figure 4.6: The performance comparison of the AF and DF relaying strategy with different number of relay nodes when the location of the D node is known.

Again, the DF relaying strategy provides lower estimation errors than the AF relaying strategy for all different number of relay nodes. According to the results, the $CRLB_{p_s}$ decreases while the number of relay nodes increases for both relaying strategies.

It can be inferred that the location estimation accuracy can be improved by increasing the number of relay nodes in the cooperative relay networks. Furthermore, the effects of the number of relay nodes are investigated when the location of the D node is unknown under the DF relaying strategy.

According to the results, using more relay nodes dramatically decreases the estimation error which is shown in Figure 4.7 .

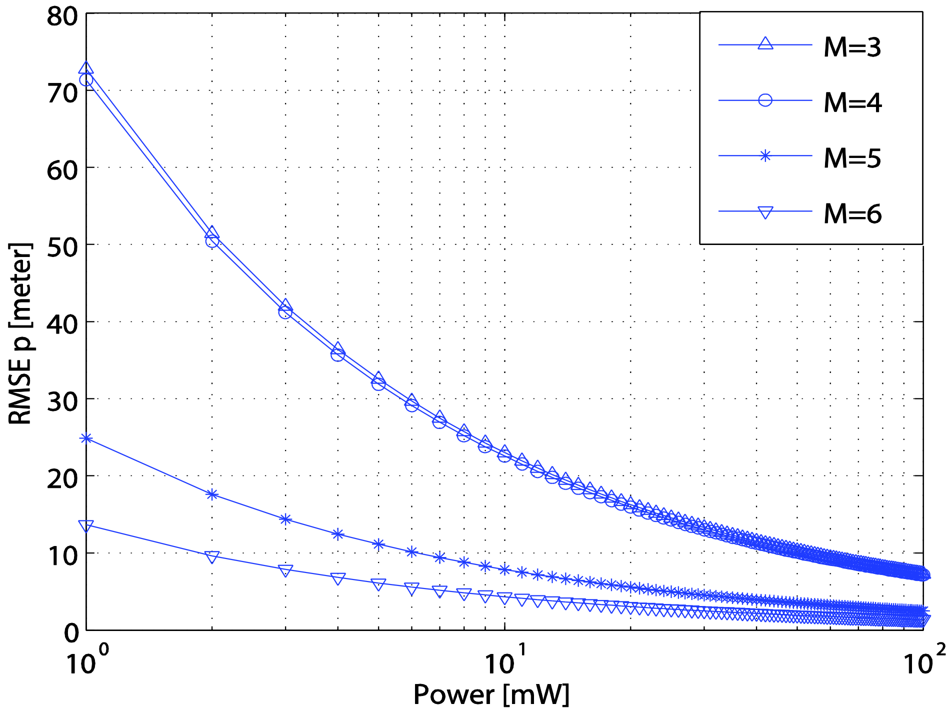

Figure 4.7: The performance comparison of the DF relaying strategy with different number of relay nodes when the location of the D node is unknown.

In addition, if we compare the results of Figure 4.6 with Figure 4.7 when the relay nodes employs the DF relaying strategy, it can be seen that the lack of information about the location of the D node increases the location estimation error, dramatically, even if the same number of relay nodes are used in both scenarios.

5. MULTIPATH ENVIRONMENT ANALYSIS

5.1. System Model

In this chapter, we investigate the theoretical limits for time delay based location estimation in cooperative relay networks in a multipath environment. Regarding the information about the cooperative relay networks which is stated in the previous chapter, we can illustrate the system model for a multipath environment as stated in Figure 5.1.

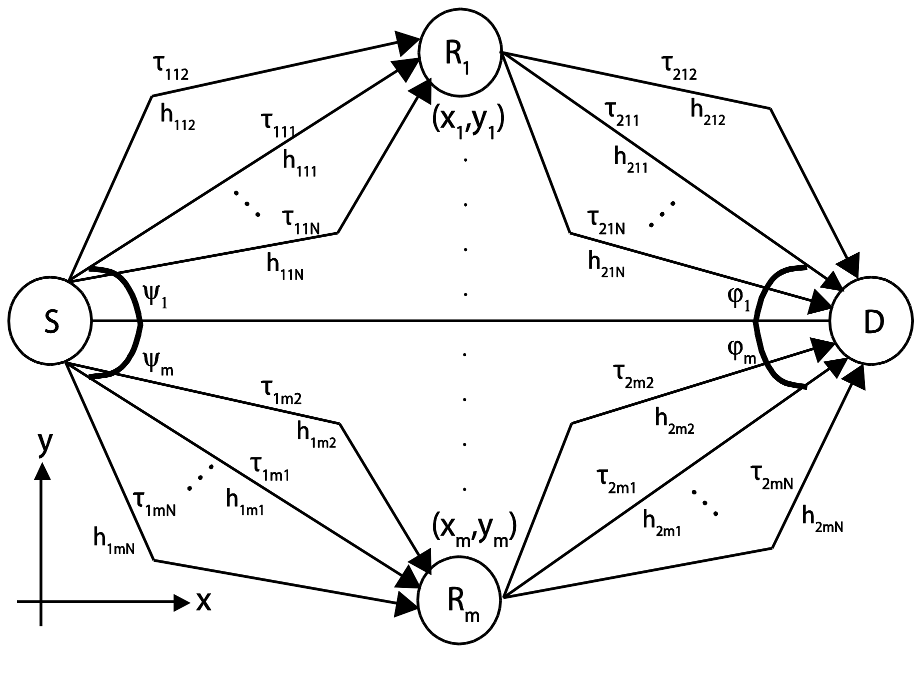

Figure 5.1: The system model for cooperative relay networks in a multipath environment.

In such a network, the relay nodes and the destination nodes receive multipath signals from the S node. We assume that the location of the relay nodes and the destination node are known in the analysis. Furthermore, it is assumed that the number of MPCs for each link is equal to N to simplify the analysis. τ_{1ik}, τ_{2ik} and h_{1ik}, h_{2ik} are the time delay values and the channel coefficients for first and second hops, respectively, where $i = 0,1,2,...,M$ and $k = 1,2,...,N$.

Since the D has a known location, the time delays of the MPCs at the second hops can be regarded as known parameters. Therefore, the time delay of the first hops' MPCs are the unknown time delay values in this chapter.

The signal models under AF and DF relaying strategies are stated in the following two sections, respectively. After these sections, the CRLB analysis for location estimation in multipath environment is presented. At the end, the simulation results of the CRLB are given under different condition.

5.1.1. AF Relaying Strategy Based Signal Model

As stated in previous chapter, the received signals are amplified and forwarded to the destination node at the relay nodes in AF relaying strategy. We can represent the received signals at the R_i node and the D node in the $SlotI$, respectively, in a multipath environment as follows,

$$y_{ri}(t) = \sqrt{P}\sum_{k=1}^{N} h_{1ik} s(t-\tau_{1ik}) + n_{1i}(t), \;\; t \in [0,T], \tag{5.1}$$

$$y_{d0}(t) = \sqrt{P}\sum_{k=1}^{N} h_{10k} s(t-\tau_{10k}) + n_{10}(t), \;\; t \in [0,T], \tag{5.2}$$

where $n_{1i}(t)$ is zero mean AWGN with variance σ_{1i}^2. The amplification factor α_{ri} of at the R_i node can be represented as follows [4],

$$\alpha_{ri} = \sqrt{\left(\frac{P}{P\sum_{k=1}^{N} |h_{1ik}|^2 + \sigma_{1i}^2} \right)}. \tag{5.3}$$

Since $y_{ri}(t)$ signal is amplified by α_{ri} and forwarded to the D node, we can represent the received signal at the D node in the $SlotII$ as follows,

$$y_{di}(t)=\alpha_{ri}\sum_{k=1}^{N}h_{2ik}y_{ri}(t-\tau_{2ik})+n_{2i}(t),\ \ t\in[T,2T], \tag{5.4}$$

$$y_{di}(t)=\sqrt{P}\alpha_{ri}\sum_{k=1}^{N}h_{2ik}\left\{\sum_{m=1}^{N}h_{1im}s(t-\tau_{1im}-\tau_{2ik})+n_{1i}(t-\tau_{2ik})\right\}+n_{2i}(t), \tag{5.5}$$

where $n_{2i}(t)$ is zero mean AWGN with variance σ_{2i}^2. As seen in equation (5.5), the noise n_{1i}(t) is also amplified by both α_{ri} and h_{2ik} multipath channel coefficients. Therefore, the variance of the E2E noise $n_{Ti}(t)$ at the D node can be represented as,

$$\sigma_{Ti}^2=\alpha_{ri}^2\sum_{k=1}^{N}|\mathrm{h}_{2ik}|^2\,\sigma_{1i}^2+\sigma_{2i}^2 \tag{5.6}$$

where $n_{Ti}(t)\sim \mathrm{N}(0,\sigma_{Ti}^2)$.

The transmitted signal s(t) is transmitted to the D node over two multipath hops, so that the MPCs of the first hop is multiplied by the MPCs of the second hop. Therefore, the received signal $y_{di}(t)$ includes N times of a MPC of the first hop .

5.1.2. DF Relaying Strategy Based Signal Model

In the DF relaying strategy, the received signal is decoded and forwarded to the D node as stated in [1],[4]. In this analysis, it is assumed that the MPCs of the received signal $y_{ri}(t)$ are fully decoded [5] by the R_i node and these decoded signals are forwarded to the D node over a multipath channel. We assume that the MPCs are not overlapping into each other, in other words; they are resolvable signals if an ultra wideband (UWB) technology is used.

The received signals at the R_i node and at the D node in the $SlotI$ are same as equation (5.1) and equation (5.2), respectively. However, the received signal at the D node through R_i node is different from equation (5.5) and is represented as follows,

$$y_{di}(t)=\sqrt{P}\sum_{k=1}^{N}\sum_{m=1}^{N}h_{2ik}s(t-\tau_{1im}-\tau_{2ik})+n_{2i}(t). \tag{5.7}$$

Since fully decode procedure is employed at the R_i node, the noise term $n_{1i}(t)$ is suppressed, so that the E2E noise is represented as $n_{Ti}(t) \sim \mathrm{N}(0, \sigma_{Ti}^2)$ where $\sigma_{Ti}^2 = \sigma_{2i}^2$. As a result, the MPCs of the first link are forwarded to the D node without noise which results into a better estimation accuracy.

5.2. Cramer-Rao Lower Bound Analysis for Multipath Environment

In the literature, there are some studies which investigate the location estimation in a multipath environment. TOA based positioning in a multipath environment is investigated in a cellular network in [12]. In that study, the analysis is divided into two groups: When the prior NLOS statistics are available or not. When the prior knowledge about the NLOS statistics of the MPCs is available, the MAP estimator can be used to estimate location of a node, so that the G-CRLB expressions are obtained. However, when there is no such statistics available, the ML estimation of the location of the S node can be performed by using only the LOS MPCs in the received signals. In our study, the location estimation using LOS signals are performed and the results are obtained in previous chapter. In order to investigate the multipath environment about location estimation, it is assumed that the prior NLOS statistics are available in this analysis, so that the G-CRLB expressions are obtained for location estimation of the S node.

As stated above, the τ_{2ik} time delay values can be considered as known parameters, because the location of the D node is known, i.e., D is fixed. Therefore, only the τ_{1ik} time delays are considered as unknown parameters to be estimated. The vector of the time delays of MPCs are represented in the vector form as,

$$\tau = [\tau_1, \tau_1, \tau_3, ..., \tau_M], \tag{5.8}$$

$$\tau_i = [\tau_{1i1}, \tau_{1i2}, \tau_{1i3}, ..., \tau_{1iN}]. \tag{5.9}$$

The time delays τ_{1ik} can be represented as follows,

$$\tau_{1ik} = \frac{1}{c}\left(\sqrt{(x_s - x_{ri})^2 + (y_s - y_{ri})^2} + l_{1ik}\right), \tag{5.10}$$

where l_{1ik} are the uniformly distributed, $U[0, \mathrm{b})$, NLOS-induced path-length errors. We assume that the relay nodes and the D node receive LOS signals besides the NLOS components. Therefore, the path-length errors of the LOS signals take zero, i.e., $l_{1i1} = 0$. Also, it is assumed that the path-length errors of the NLOS components can be represented with the order, $0 < l_{1i2} < l_{1i2} < \ldots < l_{1iN}$.

The unknown parameters to be estimated are the location of the S node, x_s and y_s, and the path-length errors, l, namely,

$$\theta = [\mathrm{x}_s, \mathrm{y}_s, l^{\mathrm{T}}]^{\mathrm{T}}, \tag{5.11}$$

where

$$l = [l_1, l_2, l_3, \ldots, l_M]^{\mathrm{T}}, \tag{5.12}$$

$$l_i = [l_{1i1}, l_{1i2}, l_{1i3}, \ldots, l_{1iN}]. \tag{5.13}$$

In addition, the vector of estimated time delay values of first hops' is represented as,

$$\hat{\tau} = [\hat{\tau}_1, \hat{\tau}_1, \hat{\tau}_3, \ldots, \hat{\tau}_M]^{\mathrm{T}}, \tag{5.14}$$

$$\hat{\tau}_i = [\hat{\tau}_{1i1}, \hat{\tau}_{1i2}, \hat{\tau}_{1i3}, \ldots, \hat{\tau}_{1iN}]. \tag{5.15}$$

In a multipath environment, the ML estimate of the time delays of MPCs can be approximated as,

$$\hat{\tau}_{1ik} = \tau_{1ik} + \epsilon_{1ik}, \tag{5.16}$$

where $\epsilon_{1ik} \sim N(0, \xi_{1ik}^2)$. How to estimate the time delay of MPCs is stated in [29]. We assume that the vectors l and $\hat{\tau}$ have independent elements.

Let ξ_i be the covariance matrix of the multipath time delays .The off-diagonal elements of the covariance matrix of time delays ξ_i takes zero values due to the independence and the diagonal elements have the estimation variances of the time delay which is defined as follows [18],

$$\xi_i = \mathrm{diag}(\xi_{1i1}^2, \xi_{1i2}^2, \xi_{1i3}^2, ..., \xi_{1iN}^2). \tag{5.17}$$

In addition, we assume that the ξ_i covariance matrices are also independent to each other, so that we can represent the general covariance matrix ξ for all the multipath delays as follows,

$$\xi = \mathrm{diag}(\xi_1^2, \xi_2^2, \xi_3^2, ..., \xi_N^2). \tag{5.18}$$

Let ψ be the inverse of the time delay estimation variances which is stated as,

$$\psi = \mathrm{diag}(\psi_1, \psi_2, \psi_3, ..., \psi_M), \tag{5.19}$$

where

$$\psi_i = \mathrm{diag}(\xi_{1i1}^{-2}, \xi_{1i2}^{-2}, \xi_{1i3}^{-2}, ..., \xi_{1iN}^{-2}). \tag{5.20}$$

The pdf for the time delay estimate $\hat{\tau}$ conditioned over θ is ,

$$p(\hat{\tau} / \theta) \propto \prod_{i=0}^{M} \exp\left\{-(\hat{\tau}_i - \tau_i)^{\mathrm{T}} \psi_i (\hat{\tau}_i - \tau_i)\right\}. \tag{5.21}$$

The FIM that is conditioned over θ can be represented as ,

$$I_\theta = E_\theta\left[\frac{\partial}{\partial\theta}\ln p(\hat{\tau}/\theta)\left(\frac{\partial}{\partial\theta}\ln p(\hat{\tau}/\theta)\right)^{\mathrm{T}}\right]. \tag{5.22}$$

Applying chain rule in (5.22) $\frac{\partial\tau}{\partial\theta}\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)$, we obtain I_θ as ,

$$I_\theta = \frac{\partial\tau}{\partial\theta}\left[E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^{\mathrm{T}}\right]\right]\left(\frac{\partial\tau}{\partial\theta}\right)^{\mathrm{T}}. \tag{5.23}$$

We can represent $\frac{\partial\tau}{\partial\theta}$ expression in terms of the angles as,

$$\frac{\partial\tau}{\partial\theta} = \frac{1}{c}\begin{bmatrix} G_1 & G_2 & G_3 & \dots & G_M \\ \mathrm{diag}\{D_1, & D_2, & D_3, & \dots & D_M\} \end{bmatrix}_{(MN+2)xMN}, \tag{5.24}$$

where

$$G_i = \begin{bmatrix}\cos\phi_i \\ \sin\phi_i\end{bmatrix}\begin{bmatrix}\overbrace{1 \quad 1 \quad 1 \quad \dots \quad 1}^{N}\end{bmatrix}. \tag{5.25}$$

$$D_i = \mathrm{diag}(\overbrace{0,1,1,\dots 1}^{N}). \tag{5.26}$$

The angle ϕ_i between the S node and the R_i node can be represented as $\phi_i = \tan^{-1}\left(\frac{y_s - y_r}{x_s - x_r}\right)$. Since the time delay estimates are independent, we can state the expectation operator E_τ in the following form,

$$E_\tau\left[\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\left(\frac{\partial}{\partial\tau}\ln p(\hat{\tau}/\tau)\right)^{\mathrm{T}}\right] = \mathrm{diag}[\psi_1, \psi_2, \psi_3, \dots, \psi_N]. \tag{5.27}$$

Using the equations between equation (5.22) and equation (5.27), we can express the FIM for this case as,

$$I_\theta = \frac{1}{c^2}\begin{bmatrix} Q & V_1 & V_2 & V_3 & \dots & V_M \\ V_1 & \psi_1 & 0 & 0 & \dots & 0 \\ V_2 & 0 & \psi_2 & 0 & \dots & 0 \\ V_3 & 0 & 0 & \psi_3 & \dots & 0 \\ \vdots & 0 & 0 & 0 & \ddots & 0 \\ V_M & 0 & 0 & 0 & \cdots & \psi_M \end{bmatrix}, \tag{5.28}$$

where

$$Q = \begin{bmatrix} \sum_{i=1}^{M}\sum_{k=1}^{N}\cos^2\phi_i \xi_{1ik}^{-2} & \sum_{i=1}^{M}\sum_{k=1}^{N}\cos\phi_i \sin\phi_i \xi_{1ik}^{-2} \\ \sum_{i=1}^{M}\sum_{k=1}^{N}\cos\phi_i \sin\phi_i \xi_{1ik}^{-2} & \sum_{i=1}^{M}\sum_{k=1}^{N}\sin^2\phi_i \xi_{1ik}^{-2} \end{bmatrix}, \tag{5.29}$$

$$V_i = \begin{bmatrix} \cos\phi_i \\ \sin\phi_i \end{bmatrix}\psi_i. \tag{5.30}$$

As stated above, we assume that the prior statistics about NLOS path-length errors are available. Based in this assumption, the pdfs of the NLOS path-length errors $p(\hat{l}_{1ik} / l_{1ik})$ are considered as known. Since the path-length errors are assumed to be independent, we can express the joint pdf $p(\hat{l}_i / l_i)$ as,

$$p(\hat{l}_i / l_i) \propto \prod_{k=1}^{N} p(\hat{l}_{1ik} / l_{1ik}). \tag{5.31}$$

The FIM for prior knowledge about NLOS statistics can be stated as,

$$I_P = \begin{bmatrix} 0 & 0 \\ 0 & \Omega \end{bmatrix}_{(MN+2)\,\mathrm{x}(\mathrm{MN}+2)}, \tag{5.32}$$

where

$$\Omega = \text{diag}(\Omega_1, \Omega_2, \Omega_3, ..., \Omega_M), \tag{5.33}$$

$$\Omega_i = E_l \left[\frac{\partial}{\partial\theta} \ln p(\hat{l}_i / l_i) \left(\frac{\partial}{\partial\theta} \ln p(\hat{l}_i / l_i) \right)^{\mathrm{T}} \right]_{NxN}. \tag{5.34}$$

We can state the elements of Ω_i as,

$$\Omega_i = \text{diag}(\omega_{1i1}^{-2}, \omega_{1i2}^{-2}, \omega_{1i3}^{-2}, ..., \omega_{1iN}^{-2}), \tag{5.35}$$

where

$$\omega_{1ik}^{-2} = -E_{l_{ik}} \left[\frac{d^2}{dl_{1ik}} \ln p(\hat{l}_{1ik} / l_{1ik}) \right]. \tag{5.36}$$

In [12], it is stated that the prior information about NLOS errors can be involved to the location estimation by summing the FIMs, I_θ and I_P to obtain the total FIM I_T for the location estimation, i.e.,

$$I_T = I_\theta + I_P. \tag{5.37}$$

The inverse of I_T gives the variance and the covariance for the unknown parameters θ. The CRLB for location estimation of the S node is obtained by using I_T^{-1} as,

$$CRLB_{p_s} = [I_T^{-1}]_{11} + [I_T^{-1}]_{22}. \tag{5.38}$$

As stated in previous chapter, the estimation error variance of time delays can be represented in terms of SNR γ_{1ik} and $\tilde{E}_{1ik} = \int_0^T [s'(t - \tau_{1ik})]^2 \, dt$ as,

$$\xi_{1ik}^2 = \frac{1}{\gamma_{1ik}\, \tilde{E}_{1ik}}. \tag{5.39}$$

The E2E SNR γ_{1ik} under AF relaying strategy can be represented as,

$$\gamma_{1ik} = \frac{P\alpha_{ri}^2 \sum_{m=1}^{N} | h_{2im} |^2 | h_{1ik} |^2}{\sigma_{Ti}^2}, \tag{5.40}$$

where $\sigma_{Ti}^2 = \alpha_{ri}^2 \sum_{k=1}^{N} | \mathrm{h}_{2ik} |^2 \sigma_{1i}^2 + \sigma_{2i}^2$. Since the noise term is amplified at the relay node, the total noise is increased at the D node. However, the E2E SNR under DF relaying strategy is stated as,

$$\gamma_{1ik} = \frac{P\sum_{m=1}^{N} | h_{2im} |^2}{\sigma_{2i}^2}, \tag{5.41}$$

It can be seen that the noise $n_{1i}(\mathrm{t})$ is suppressed by the relay nodes when DF relaying strategy is employed whereas the $n_{1i}(\mathrm{t})$ is amplified by the relay nodes when AF strategy is used. Replacing γ_{1ik} and $\tilde{E}_{1ik}$ into equation (5.39), ξ_{1ik}^2 values is obtained for AF and DF relaying strategy and the $CRLB_{p_s}$ for these strategies can be calculated.

5.3. Simulation Results

In this section, the $CRLB_{p_s}$ expressions are simulated under both relaying strategies. The system parameters and signal pulse shape which are stated in previous chapter are also used in this simulation. Differ from the previous chapter, the channels $S \to R_i, R_i \to D$ and $S \to D$ are assumed Rayleigh channels due to the multipath environment. Therefore, the channel coefficients h_{1ik} and h_{2ik} are selected as Rayleigh RVs.

In addition to these parameters, the MPCs of the first hops' are adjusted as follows,

$$\tau_{1ik} - \tau_{1ik-1} \geq \frac{2}{W} + \upsilon_{1ik}, \tag{5.42}$$

where W =20MHz and the υ_{1ik} is random time delay value uniformly distributed over $[0, 0.5/\mathrm{W}]$ which is caused by the NLOS path-error l_{1ik} . Equation (5.42) means that the adjacent MPCs are not overlapping and the they can be resolved easily [12].

The signal strengths of multipath components are set according to "exponential gain". Using this gain model, the strength of first NLOS component is set 6 dB below the corresponding LOS component and the adjacent NLOS components have -6 dB differences in strength.

Considering the simulation environment in Figure 4.4, $CRLB_{p_s}$ is simulated under AF and DF relaying strategy. When performing the simulations, the relay nodes are uniformly distributed over a geographical region with the dimension of 1000m by 1000m. This is repeated 10 times in order to consider random node topologies. During the random distribution of the nodes, we make sure that the relay nodes are not colinear with the S and D nodes. For each random topology, the $CRLB_{p_s}$ is calculated over 100 random channel realizations with different number of multipath components which is selected as $[3, 7, 20]$. In order to simplify the analysis, the channels ($S \rightarrow R_i$, $S \rightarrow D$ and $R_i \rightarrow D$) are assumed to be identical.

Figure 5.2 illustrated the location estimation accuracy under AF relaying strategy with different number of MPCs. The effects of the node position on the estimation accuracy are discussed in previous chapter and it is not stated here. If we compare Figure 5.2 with Figure 4.3, it can be seen that processing the multipath components besides the LOS signal decreases the estimation error, significantly. However, the increasing the number of processed MPCs does not always decreases the estimation accuracy as shown in Figure 5.2. Since the noise $n_{1i}(\mathrm{t})$ is amplified and is involved to the received signal at the D node, the NLOS components which have lower signal strengths do not introduce significant effects on the location estimation accuracy.

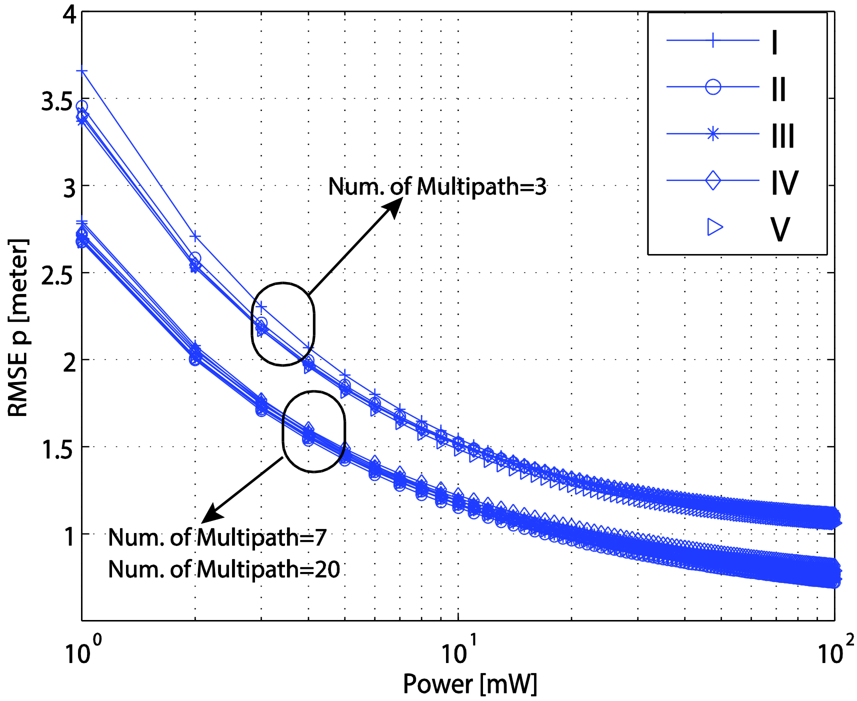

Figure 5.2: The location estimation accuracy under AF relaying strategy with different number of multipath components.

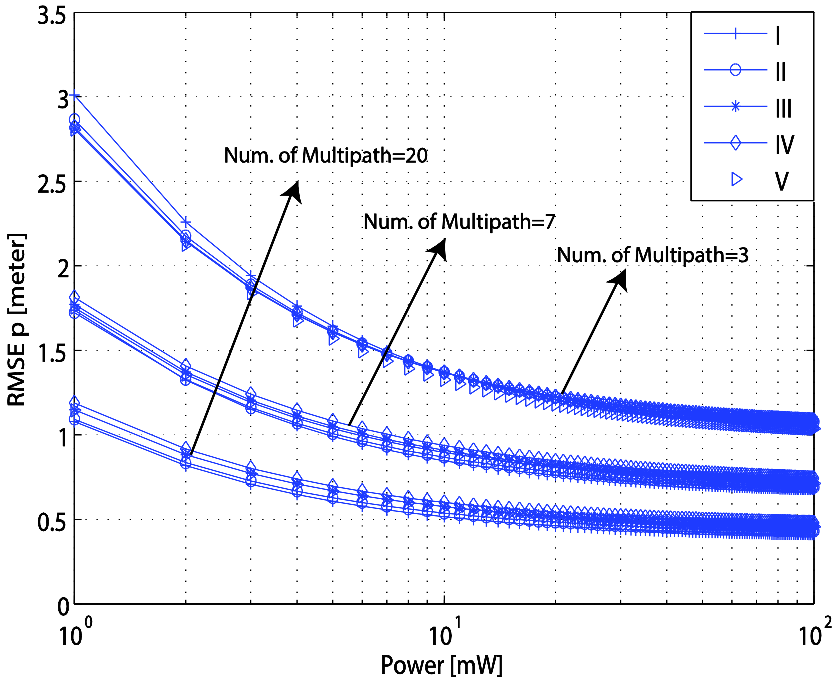

Figure 5.3: The location estimation accuracy under DF relaying strategy with different number of multipath components.

Converse to AF, DF relaying strategy gives better results on the estimation accuracy as depicted in Figure 5.3. Due to the noise cancellation under DF relaying strategy at the relay nodes, the location estimation accuracy is improved.

Comparing Figure 5.3 with Figure 4.3, it can be easily seen that employing the MPCs improves the estimation accuracy. In addition, it can be seen that processing more MPCs at the D node increases the estimation accuracy, dramatically.

$CRLB_{p_s}$ expression is also simulated with different number of relay nodes under both relaying strategy. The networks are established with different number of relay nodes and for each network, $CRLB_{p_s}$ expressions are calculated using different number of MPCs. AF strategy based estimation accuracy is shown in Figure 5.4.

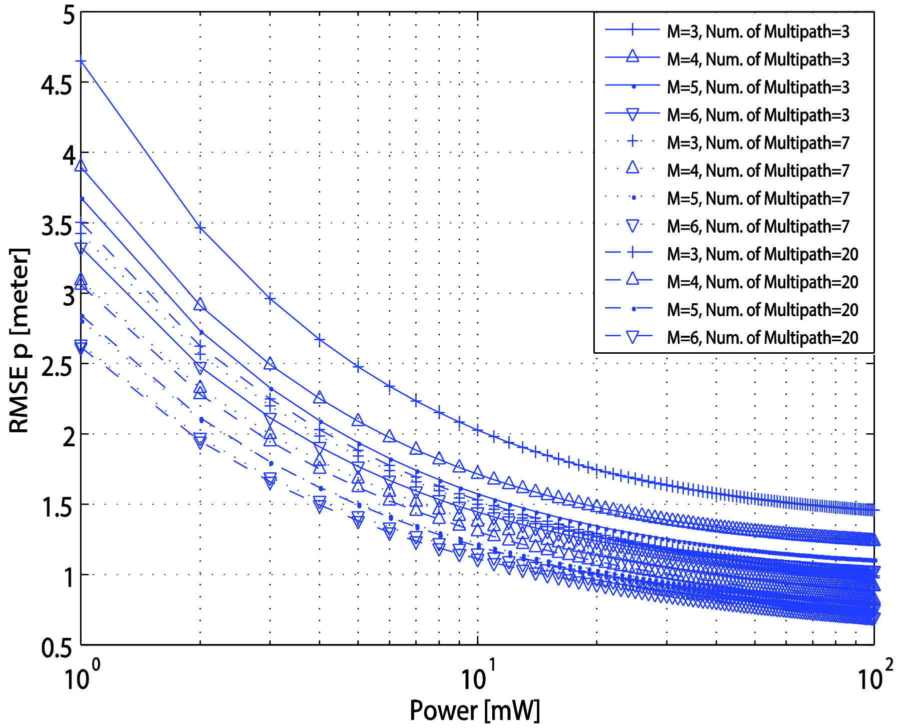

Figure 5.4: The location estimation accuracy with different number of multipath components and relay nodes under AF relaying strategy.

Similar to the results in Figure 5.2, processing more MPCs does not always increases the estimation accuracy due the reason which is stated above. However, estimation accuracy is improved when the number of relay node and the MPCs are increased under DF strategy which is shown in Figure 5.5. It can be inferred that increasing the number of MPCs and the relay nodes always increases the estimation accuracy in DF relay networks.

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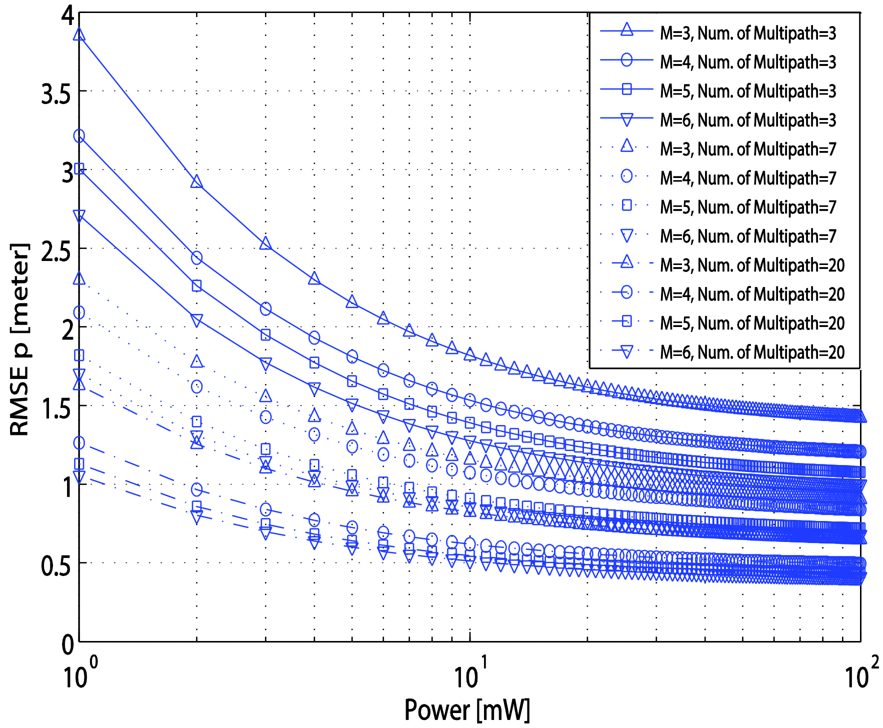

Figure 5.5: The location estimation accuracy with different number of multipaths and relay nodes under DF relaying strategy.

Considering the results in Figure 5.4 and Figure 5.5, it can be inferred that increasing the processed number of MPCs instead of the number of relay nodes provides the approximately same estimation accuracy. For example, using 4 relay nodes with 7 MPCs gives better accuracy than the network which has 6 relay nodes with 3 MPCs under AF strategy which is shown in Figure 5.4. In DF strategy, using 3 relay nodes with 20 MPCs gives approximately same bounds with a scenario with 6 relay nodes and 7 MPCs.

It should be noted that the processing MPCs may introduce more cost than using additional relay nodes in the network. In this case, using more relay nodes instead of employing more MPCs in the estimation may be efficient to decrease the estimate error. The complexities of these two method used to decrease the estimation error should be defined as prior information, so that the best way can be selected to achieve a given bound in the location estimation.

6. CONCLUDING REMARKS

In this thesis, we investigate the theoretical limits for the time delay based location estimation technique in cooperative relay networks. These limits are studied by the derivation of CRLBs under different relaying strategies such as the AF and DF mode and different assumption of node position information including the location of the source only and the location of the source and destination nodes are unknown.

The numerical results are presented to quantify these limits. The results show that the relaying strategies (AF and DF) can affect the location estimation accuracy of the time delay based techniques, significantly. More specifically, the DF relaying strategy performs better than the AF relaying strategy in terms of location estimation accuracy. Also, the node topology in cooperative relay networks can have significant impacts on the location estimation error. In addition, the number of relay nodes is inversely proportional to the location estimation error. Finally, the existence of the location information of the destination node is very important for lowering the location estimation error dramatically [30].

In this thesis, we consider both single path LOS fading and multipath environments to investigate the effects on the location estimation of the source node in cooperative relay networks. The results showed that processing NLOS components of the received signals generally introduces better localization in the cooperative relay networks. However, using more than 7 MPCs does not increase the estimation accuracy anymore. Due to the noise amplification, the weak NLOS components do not significant effects on location estimation accuracy. On the other hand, the MPCs which are processed in the location estimation always decrease the estimation error, but the accuracy improvement is not linear as shown in Figure 5.3.

Another conclusion about the multipath environment analysis can be given that the error bound obtained using higher number of relay nodes can be achieved by processing more MPCs in a network which has less relay nodes than the previous one. By this way, the network complexity may be reduced by decreasing the number of relay nodes used in the network. On the other hand, processing MPCs may introduce more complexity than using more relay nodes. In this case, the number of MPCs to be processed should be reduced and the number of relay nodes should be increased to achieve the given bound. As a result, the complexities of these operations should be stated at the beginning and the location estimation should be performed according to the selected way.

In this thesis, the DF method is assumed to fully decoding the received signals-without errors-which is explained in [5]. However, there may be a detecting and/or decoding error which can introduce time delay estimation error at the D node. Therefore, the DF error may be statistically modeled and the effects of this error on the location estimation can be investigated. This open issue may be investigated in the future works.

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RESUMÉ

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