EFFECTS OF COMPETITION ON PRODUCT RECOVERY DECISIONS

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ABSTRACT

EFFECTS OF COMPETITION ON PRODUCT RECOVERY DECISIONS

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In this study, we consider a hybrid manufacturing-remanufacturing environment consisting of an Original Equipment Manufacturer (OEM) and Independent Remanufacturer (IR). OEM can produce both new and remanufactured products, whereas IR is only capable of remanufacturing. Market price is deterministic and decreasing linearly with total quantity supplied to the market. Remanufacturing yield increases with remanufacturability investment that has a cost which increases with the level of remanufacturability. Remanufacturing cost for a successfully remanufactured product decreases with increase in remanufacturability. We analyze this system in a two-period setting. OEM determines the level of investment in remanufacturability and the quantity of new products in the first period, which determines the quantity of returns in the second period. OEM and IR simultaneously determine their remanufacturing input quantities considering opponent's action in the second period. Our objective is to investigate the effects of competition, sorting information availability and competition awareness on environmental and economic performances of this system. For this environment, models that differ in competition, sorting information availability and competition awareness are considered. Total system-wide profit and ratio of remanufacturing output to total returns are used to represent economical and environmental performances, respectively. We explore the effects of various settings via a computational study.

Keywords: Closed Loop Supply Chain, Remanufacturing, Sales Competition

ÖZ

REKABETİN ÜRÜN GERİ KAZANIM KARARLARINA ETKİSİ

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Çalışmamızda Özgün Donanım İmalatçısı (ÖDİ) ve Bağımsız Yeniden İmalatçı'dan (BYİ) oluşan bir melez imalat-yeniden imalat ortamı incelenmiştir. ÖDİ hem imalat hem de yeniden imalat yapabilir; ancak, BYİ sadece yeniden imalat yapabilir. Piyasa fiyatı belirlenimcidir ve piyasaya sunulan toplam arza bağlı olarak doğrusal azalır. Yeniden imalat verimi yeniden imal edilebilirlik yatırımı, yeniden imal edilebilirlik seviyesiyle birlikte artan, ile artar. Yeniden imalatla üretilmiş ürün başına düşen yenien imalat maliyeti yeniden imal edilebilirlikteki artışla azalır. Bu sistem iki dönemlik bir düzende incelenmiştir. ÖDİ yeniden birinci dönemde imal edilebilirlik yatırımına ve imalat miktarına karar verir, böylece ikinci dönemdeki geri dönüş miktarını belirler. İkinci dönemde ÖDİ ve BYİ rakibinin hamlesini göz önünde bulundurarak aynı anda yeniden imalat girdi miktarlarına karar verirler. Bizim amacımız, rekabetin, sıralama bilgisinin varlığının ve rekabet farkındalığının ekonomik ve çevresel performans ölçütlerine etkisini incelemektir. Bu ortam için, rekabet, sıralama bilgisi varlığı ve rekabet farkındalığı açılarından değişkenlik gösteren modeller kurulmuş ve incelenmiştir. Sırasıyla, toplam sistem karı ve değer geri kazanım oranı, ekonomik ve çevresel performans ölçütlerini temsil etmek için kullanılmıştır. Çeşitli

düzenlerin etkilerini incelemek için hesaplama çalışması yapılmıştır.

Anahtar Kelimeler: Kapalı Devre Tedarik Zinciri, Yeniden İmalat, Satış Rekabeti

To my family...

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LIST OF ABBREVIATIONS

MNS	Monopolistic model without sorting information
MYS	Monopolistic model with sorting information
DNS	Duopolistic model without sorting information
DOS	Duopolistic model with sorting information for only OEM
DYS	Duopolistic model with sorting information for both OEM and IR
DM	Duopolistic model with assumption of monopoly

CHAPTER 1

INRODUCTION

In traditional supply chains which are known as forward supply chains, goods are produced from raw materials and transferred to the downstream stages to meet the demand of final customers. In recent decades, manufacturing firms are forced to collect their products that reach their useful lives with environmental legislation. The products that have reached their useful life are called end-of-life product. The flow of used products from end customer to producers constitutes the reverse supply chain. The combination of forward and reverse supply chains is named as closed loop supply chain (Govindan et al. [1]).

There is a cost associated with collection, hence the firms search for the alternative ways to create value from these products. There are some alternative ways of value recovery such as direct reuse, repair, refurbishing, remanufacturing, cannibalization, and recycling, (Thierry et al. [2]). Direct reuse is reselling the used product returned from the end customer. Repair is mainly changing the condition of a product from broken to working. The damaged part of the product is fixed or replaced with a spare part. In refurbishing, the modules of the product are inspected and the damaged ones are replaced with higher or equal quality modules in order to increase the life time of the product. Even though the broken module of the product is replaced with a new one, the life time of the refurbished product is shorter than the new one. In remanufacturing, used product is completely disassembled to part level and all parts are inspected to be used later on producing a new product. Remanufacturing is the only value recovery activity that claims as good as new condition for its output. Cannibalization is using the functional parts of a product. These parts are reused in other type of value recovery activities. Lastly, recycling is reusing the material of the end-of-life product. Value recovered in recycling is the least of all alternative reuse activities.

One of the commonly implemented value recovery method is remanufacturing. In remanufacturing, the parts of returned products are inspected for their functionality and the functional parts of the products are used in production of a new product. By using the functional parts of the products, firms can reduce their production costs.

In order to decrease production cost of remanufacturing, products should be designed in a way that disassembly and reassembly are time and cost efficient, and yield loss of remanufacturing is small. We name remanufactuability for the suitability of returned items for remanufacturing. Together with specialized design for remanufacturing, investment on remanufacturing operations should be made. Remanufacturability of end-of-life product is proportional to its remanufacturability level, which is determined at the beginning of life cycle of the product. Since, remanufacturability level is determined at the beginning of life cycle, it is a decision of original equipment manufacturer (OEM).

Remanufacturing can be seen a cost reduction method for production. Since, the material cost usually is less than brand-new products. Main drawback of remanufacturing is limited supply. At the beginning of products life cycle, there are not used product to remanufacture. Product need to be mature enough for accumulation of used products in the market. As a result, firms can make remanufacturing at the later periods of product's life cycle. Another problem with remanufacturing is that not all used products, i.e. input supply, returned. Some of the products may not be collected, they can be sold as second hand by customers or they can be landfilled.

Material cost advantage may attract other firms to begin remanufacturing. When the original product is designed and produced to be remanufactured, it is easier for other firms to begin remanufacturing than manufacturing. Since other firms who makes only remanufacturing do not have to spend their capital investment of manufacturing or design of remanufacturable product, they prefer to enter remanufactured product market. The firms that engage only in remanufacturing used and returned products are called independent remanufacturers (IR). Since, independent remanufacturers do not design but only collect and remanufacture, the design decisions by OEM affect their remanufacturing process efficiency as well. As a result, competition arise in the remanufactured product market.

High level of remanufacturability does not solve all all shortcomings of remanufacturing. One of the potential problems about remanufacturing is not knowing which returned items are suitable for remanufacturing operations. We call that information as sorting information throughout this thesis. Firms might not know the exact state of the returns, i.e. sorting information, at the beginning of production. This cause remanufacturing process to have yield loss, i.e. only a certain portion of remanufacturing input is transformed into remanufactured product. Since, firms waste their resources for unsuitable returned items.

Even if the main objective of the firms is profit maximization, remanufacturing has some additional benefits for the environment as well. Implementing either of the alternative reuse methods decreases waste generated. Remanufacturing has an advantage over other alternatives, remanufactured product are considered identical to brand new products. As a result, together with the decrease in waste generated, decrease in energy and raw material use is another environmental benefit of remanufacturing. Thanks to its advantages, remanufacturing can be considered as a superior alternative of all reuse methods. Environmental performance of firms can be defined as proportion of remanufactured items to all returned items.

We have five main research questions and we build our models in order to answer them.

- What are the equilibrium manufacturing quantity, remanufacturability level and remanufacturing quantities of OEM and IR?
- How do the economical and environmental performances of OEM and IR change with different problem parameters?
- How do entry of a competitor in remanufactured product market affect the economical and environmental performances of OEM and IR?
- How do sorting information affect the economical and environmental performances of OEM and IR?
- What is the effect of not considering a potential entrant while doing initial deci-

sions of OEM to the economical and environmental performances of OEM and IR?

In order to gain insights on the above research questions, we introduce several stylized models. Namely, we consider a two-period model where an original equipment manufacturer (OEM) manufactures brand new products into the first period. In the second period, some of the products that are sold in the first period are returned and remanufactured. We consider

(i) The monopolistic setting where OEM is the only party that remanufactures

(ii) The duopolistic setting where there is also an independent remanufacturer

In the first period, the OEM determines the level of investment on the remanufacturability of the product product and the number of new products to produce. In the second period, a certain fraction of manufactured items by OEM are shared by OEM and IR and the OEM (and IR in the duopolistic setting) determines number of units to remanufacture. The yield of the remanufacturing process depends on the investment made by the OEM in the first period.

In order to model competition, a quantity sensitive price function is used. Price is a deterministic linear function of total quantity supplied to the market in that period. Customers may not perceive remanufactured products identical to manufactured product. Similiarly, customers may not perceive remanufactured products of IR as identical to the ones of OEM. We investigate the effect of perception differences via different price function parameters.

We initially assume that firms do not know which returned products are suitable for remanufacturing before starting the remanufacturing process and this situation causes total remanufacturing cost to increase. We create models for both monopolistic and duopolistic settings with perfect sorting information prior to remanufacturing process in order to investigate the effect of sorting information.

At the end of the remanufacturing process, there might be returned products on hand. There are two sources of these leftovers, items that are not used for remanufacturing and items that are not successfully remanufactured are the inputs of recycling. Firms uses recycling as a salvaging mechanism for leftover items.

We observe that competition has negative effects on OEM's profit and IR's profit does not compensate the decrease in the total system-wide profit. As a result, total system wide profit decreases when firms compete in remanufactured product market. Sorting information helps firms to increase their profits and total system-wide profit unless it is available for only OEM. Effects of competition and sorting information to environmental performance is found to be sensitive to the problem parameters. Lack of competition awareness of OEM always decreases OEM's profit and always increases remanufacturability investment. Environmental performance is better when OEM is not aware of competition.

This thesis study is organized is organized as follows; related literature is summarized and problem is defined in Chapter 2, monopolistic model without sorting information is created and analyzed in Chapter 3, duopolistic model without sorting information is created and analyzed in Chpater 4, monopolistic and duopolistic models with sorting information and duopolistic model with assumption of monopoly are created and analyzed in Chapter 5, results of computational study is presented in Chapter 6 and thesis study is concluded in Chapter 7.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM DEFINITION

This chapter is organized as follows: first, review of related literature is given in Section 2.1, then, the problem is defined in Section 2.2.

2.1 Literature Review

Closed loop supply chain management (CLSC) is a very rich and diverged research area. Review articles consider different criteria in order to classify the literature. One of such reviews, Atasu et al. [3], classify CLSC studies under four main research streams,

- Industrial Engineering/Operational Research
- Design
- Strategy
- Behavioral

IE/OR stream focuses on inventory and logistics decisions in CLSC. Design stream contains articles that investigate the decisions made about product design and about supply chain network structure like roles of players in supply chain, time value of product returns and durability choice. Strategy stream is composed of articles that examine the effect of competition in the market and market segmentation. The last stream defined in the article focuses on behavioral issues such as consumer perception about remanufactured products.

In this thesis, we mainly focus on remanufacturability investment of an original equipment manufacturer, pricing of products (indirectly by determining manufacturing and remanufacturing quantities) and the resulting economical and environmental performances. We have constructed different settings in order to investigate the effect of competition, the effect of remanufacturability information and the effect of being aware of competition. Hence, we limit our literature review with the articles that are related to those issues. The topics that we focus on are:

- Structural decisions
- Effect of competition on recovery decisions
- Pricing of brand-new and recovered products
- Value of sorting information

We classify the studies considering these issues as in Table 2.1.

2.1.1 Structural Decisions

In order for value recovery to be profitable, reverse channel operations have to be determined accordingly. Structural decisions in CLSCs include determining alternative actors to take collection responsibility, certain investment decisions like collection effort and remanufacturability, and determining the level of quality.

Savaskan et al. [4] investigate effects of alternative actors to take collection responsibility on product return rate, quantity demanded and total profit. Three alternative with alternative actors collecting the returns are considered:

- Manufacturer
- Retailer
- Third party collector who is subcontracted for collection

The returned fraction of sales is determined by collection effort. Total collection cost increases linearly by return quantity and quadratically by collection effort. Demand

Table 2.1: Classification of studies in literature

Studies	Structure	Pricing	Competition	Sorting
Savaskan et al. (2004)	+	+		
Hong et al. (2013)	+	+		
Savaskan and Van Wassenhove (2006)	+	+	+	
Ferguson and Toktay (2006)	+	+	+	
Oraiopoulos et al. (2012)	+	+	+	
Orsdemir et al. (2014)	+	+	+	
Zikopoulos and Tagaras (2006)	+			+
Majumder and Groenevelt (2001)		+	+	
Atasu et al. (2008)		+	+	
Atasu and Subramanian (2012)	+	+		
Wu (2012)	+	+		
Gan et al. (2017)	+	+	+	
Ketzenberg and Zuidwijk (2009)	+	+		
Subramanian et al. (2013)	+	+		
Chen and Chang (2013)		+		
Debo et al. (2005)	+	+		
Gu et al. (2005)	+			+
Ketzenberg et al. (2006)				+
Hosoda et al. (2015)				+

is a decreasing function of retail price. In all settings, the manufacturer is the supply chain leader. Customers perceive remanufactured products as identical to the manufactured ones. Hence, remanufacturing functions as a reduced cost alternative for manufacturing. When the manufacturer is not the collector, he pays a unit buy back price (a price to purchase used items) to the collector. Product return rate, quantity demanded and total profit are considered as environmental performance, consumer welfare and economical performance measures, respectively. Retailer collects case outperforms other alternatives in all performance measures.

Hong et al. [5] search for the best closed loop supply chain structure. The environment designed for the analysis is the as same as Savaskan et al. [4] except number of parties who determine collection rate. In each setting two of supply chain parties are responsible of collection activities. Supply chain alternative structures are as follows:

- MR model: manufacturer and retailer collect
- RT model: retailer and a third party collect
- MT model: manufacturer and a third party collect

Collectors determine their collection rates separately, total return quantity is proportional to the summation of collection rates. Retailer and third party collector are paid by the manufacturer for the items collected. In all settings, wholesale price is determined by the manufacturer and the retail price is determined by the retailer. Environmental performance measure (product return rate), economical performance measure (total profit of the supply chain) and consumer welfare (retail price of product) are evaluated in this study. MR gives the best results for all three performance measures. With the same problem parameters MR model dominates "Retail Collects Case" of Savaskan et al. [4].

Savaskan and van Wassenhove [6] investigate the effects of competition among retailers on the optimal supply chain structure. The environment defined by Savaskan et al. [4] is very similar. Third part collects alternative is omitted in this study. Five different settings are created and evaluated. Manufacturer is the supply chain leader in all settings. The settings differ from each other with respect to whether supply chain is centralized or not and whether used products are collected by the manufacturer or the retailer. Lastly, a model without remanufacturing is constructed as a benchmark. The following settings are considered,

- NR: no remanufacturing
- DD: decentralized direct collection
- DI: decentralized indirect collection
- CD: centralized direct collection
- CI: centralized indirect collection

Under indirect collection settings, retailers determine retail prices and product collection effort (a distinct product collection effort for each retailer), and manufacturer determines wholesale price. In direct collection settings, product collection effort becomes manufacturer's decision. When decision making is centralized, a central planner determines retail prices and product collection effort(s). Cost of collection effort increases quadratically with collection rate of that player. Acquisition fee is omitted in this study. Since, there is more than one retailer in the market, market price is a function of both retailers' supply quantity to the market. The article states that profits of supply chain parties in DD is higher than the profits in DI. Retailer's profits are always higher when they are the collectors and decisions are decentralized. In order for manufacturer to benefit increased profit of retailers, he can charge a fixed franchise fee by retailers. The articles that we review so far is similar to our study with investment decisions prior to remanufacturing operations.

Many governments are planning to legislate about end-of-life products to decrease waste generated by end-of-use products and to create incentives to increase product recovery. To increase product recovery, governments direct companies to implement strategies for ease of disassembly, limiting variety of parts/components and increasing recoverable components. Atasu and Subramanian [7] investigate the effects of individual (IPR) and collective producer responsibility (CPR) types of take back legislations. They consider a market with two manufacturers that are differentiated with their market position. One of them produces high end products and the other one produces low end products. Utility gained by customers for the two products is non-identical and each decreases in market price of the products. Marginal benefit of a recycled item decreases in quantity recycled and increases with investment on recyclability. Cost of manufacturing increases with the level of recyclability. There is also an unit collection cost of end-of-life products, which is higher in IPR setting. At first, firms determine design for recyclabilities (DfR) (design of a product to be easily recycled) of their products, simultaneously. Then, they determine price of their product (they determine quantity demanded indirectly), simultaneously. In IPR setting, firms are responsible of collecting their own product. In CPR setting, mixture of end-of-life products recycled and recycling cost is shared to all manufacturers in the market. During recycling in CPR setting, it may be possible that brands can be differentiated. Hence, total cost is shared by manufacturers proportional to their end-of-life product quantities. Under IPR model, competition has no effect on recyclability. With exogenous recycling cost sharing, firms DfR increases with its share of recycling cost. DfR of IPR is always superior to DfR of CPR when recycling cost is shared proportional to quantity recycled. When brand differentiation is high, demand of low end product increases. Hence, share of low end product in total recycling cost increases. As a result, DfR of low end product is high, whereas DfR of high end product is low. It is stated that with exogenous cost sharing, IPR is better for low end manufacturer, CPR is better for high end manufacturer when brand differentiation is low. With endogenous cost sharing, CPR is better for low end manufacturer, IPR is better for high end manufacturer. Customers benefit from brand differentiation, with the increase in brand differentiation consumer surplus increases. The article consider effects of different law enforcements to recoverability, we investigate the effect of competition, sorting and competition awareness to remanufacturability.

2.1.2 Effect of competition on recovery decisions

Another issue addressed in CLSC literature is the conditions under which original equipment manufacturer (OEM) remanufactures and how original equipment manufacturer behaves when competition exists. Ferguson and Toktay [8] investigate the conditions under which remanufacturing is profitable, the effect of competition among manufactured and remanufactured products of OEM and OEM's decisions when there is a threat of an independent remanufacturer to enter the market. The model is designed as a two period one. In the first period, there is only manufacturing. In the second period, together with manufacturing by OEM, OEM and independent remanufacturer (IR) produce goods via remanufacturing. OEM and IR makes their moves simultaneously in the second period. Remanufacturing quantity in the second period is bounded by manufacturing quantity in the first period and if IR enters the market they share returned products with a constant rate. Price is a decreasing function of quantity supplied to the market. Customer's willingness to pay for products remanufactured by IR is less than those of OEM. Remanufacturing and collection cost is represented with a function which is dependent on quantity collected and remanufactured. There is a fixed cost of remanufacturing. Remanufacturing (when it is not profitable) and collection to preempt entry of a competitor are two strategies that can be employed by OEM. They found that benefit of remanufacturing can compensate the negative effects of internal competition. The article states that when there is no fixed cost, remanufacturing is always profitable. When there is a positive fixed cost, remanufacturing decision is dependent on unit remanufacturing cost. It is possible that remanufacturing is not profitable for OEM for some problem parameters. However, OEM can implement remanufacturing and collection without remanufacturing in order to preempt entry of IR to the market. They investigate the behavior of OEM when there is a competitor without a structural decision, whereas we also consider investment needed for remanufacturability.

Oraiopoulos et al. [9] investigate the strategies to maximize OEM's profit when OEM charges relicensing fee for remanufactured products of other firms. OEM is a monopoly in manufactured product market. Customers perceive remanufactured products as inferior products. The product manufactured in the second period by OEM is equivalent or superior to manufactured product in the first period with technological improvement. Quantity demanded is inversely proportional to the price of product. Products depreciates with use, that is utility gained decreases. It is assumed that customers cannot sell products to each other. The sequence of events is as follows: Firstly, OEM determines the selling price in the first period, then selling price in the second period and relicensing fee. Then, independent remanufacturer decides acquisition fee for used products and remanufactured product selling price. For higher willingness to pay for remanufactured products, OEM chooses not to eliminate independent remanufacturer since the benefit of resale value and relicensing fee is higher than negative effect of competition. Even if relicensing fee is equal to zero, independent remanufacturer may not enter the market for very low consumer utility of remanufactured products. Relicensing fee increases with willingness to pay (WTP) for remanufactured products and decreases with durability, manufacturing cost and remanufacturing cost. For higher values of technological improvement (WTP increase in the second period for new product), relicensing fee is higher. As an extension, multiple independent remanufacturers are investigated. The results are counterintuitive, OEM benefits from independent remanufacturers to enter the market. Another extension of this study is vertically differentiated duopoly setting in which remanufacturers compete. The difference between relicensing fees increases with difference in brand premium. For higher than certain value of brand differentiation, low end products are remanufactured more than high end products.

Orsdemir et al. [10] considers determination of quality level to handle competition with independent remanufacturer. Not only profit but also total environmental impact and social welfare are questioned. The model is constructed as a single period model with steady state assumption. Price of products is dependent on quantity produced and quality level. Remanufactured products are perceived as inferior. Manufacturing and remanufacturing cost increases with increase in the quality level. Quantity remanufactured is constrained with manufacturing quantity. Sequence of events is as follows; first, quality level is determined by OEM. Then, OEM and IR simultaneously decide manufacturing and remanufacturing quantities. When the ratio of unit remanufacturing cost to value generated from remanufactured items (cost to value ratio) is high, OEM deters entry. For moderate levels of cost to value ratio IR remanufactures, but not all available returns. If cost to value ratio is too low then IR enters and remanufactures all available returns. Entry of independent remanufacturer to market is always beneficial for consumers. Social surplus is defined as summation of consumer surplus and summation of OEM's and IR's profits. Social surplus is higher when remanufacturing occurs. When IR cannot enter the market, environmental impact is less than or equal to the level with no remanufacturing. When IR enters but does not remanufacture all cores, environmental impact is less than no remanufacturing setting. When IR remanufactures all available cores environmental benefit depends on ratio of environmental impact of remanufacturing and manufacturing. As an additional competitive lever, remanufacturing by OEM is considered. For this setting, it is assumed that IR can only obtain the cores OEM does not prefer to remanufacture. For low values of cost to value ratio of remanufactured products, OEM uses all available cores. For moderate level of cost to value ratio, IR remanufactures some portion of cores. And, for some high enough level of the ratio, remanufacturing is not chosen by any of the agents. The least level of environmental impact can be obtained with parameter set under which OEM does not remanufacture but IR does. Another competitive lever is preemptive collection. For very low values of cost to value ratio, OEM collects and disposes all available cores. In this case, quality level is higher than no remanufacturing case. This study investigates the effects of quality level to handle competition. They defined remanufacturing cost to increase with quality, whereas we consider quality of products with remanufacturability and remanufacturing cost in our study decreases with increase in remanufacturability.

Majumder and Groenevelt [11] investigate effects of competition to market price and players' profits. A market with two actors is designed, an OEM and an IR. A two-period model is considered. In the first period, only products supplied to the market is OEM's manufactured ones. In the second, a fraction of manufactured items in the first period is returned and shared among OEM and IR. OEM and IR uses that returned items to remanufacture with a constant rate. OEM can increase its output via manufacturing. When a player does not use all of its available returns, the competitor can use those. Customers perceives remanufactured products of OEM as identical to brand new products. However, remanufactured products of IR is taken as inferior. Demand of each player is sensitive to price of remanufactured products of both players. Whether or not they use their share of returned items creates four different cases. For lower values of returns, both players uses all of their returned products. When return increases, IR does not use all its returns to remanufacture and only OEM makes remanufacturing. OEM increases manufacturing quantity in the first period if he remanufactures. Remanufacturing cost decreases remanufacturing activities of players. As a result, OEM and IR are willing to cooperate to decrease remanufacturing cost. It is observed that existence of competition makes OEM manufacture less in the first period and increase remanufacturing cost of IR. A dynamic model is constructed to represent real cases better such that manufacturing occurs in the first period and remanufacturing and manufacturing coexist in the second period. Supply of remanufacturing is constrained with the first periods manufacturing quantity and constant return rate. Results of this model coincide with the static model. Optimal price for remanufactured product depends on green segment size. Supply of remanufacturing is another issue to consider for firms. Increasing manufacturing quantity in the earlier stages of product life cycle, increasing return rate and waiting more before beginning remanufacturing are three alternative ways presented for increasing remanufacturing supply. Static competition is also considered. Brand image of competitor is inferior than the brand image of original manufacturer. Unit cost of remanufacturing should be sufficiently low in order for remanufacturing to be profitable when there is a competitor. The market is divided into two customer types, primary and green. Benefit of remanufacturing increases with green segment size. The profit difference created by remanufacturing is higher under competition than monopoly case. Benefit of remanufacturing increases with the brand power of competitor. The original manufacturer only uses high price strategy for remanufactured products (separating primary segment and green segment). This study investigates effects of competition but do not consider investment needed for remanufacturability.

Wu [12] considers pricing and disassemblability together. At first OEM determines disassemblability, which decreases both unit manufacturing and unit remanufacturing cost and has a fixed cost. OEM determines manufacturing quantity in the first period. In the second period, remanufacturer gets product returns with certain disassemblability. Then, OEM and remanufacturer determine manufacturing and remanufacturing quantity, respectively. There are two customer types primary and green customers. Primary customers perceive remanufactured product as inferior, whereas green customers see manufactured and remanufactured products as identical. OEM has two alternative strategies for disassemblability, high and low. Remanufacturer determines the pricing strategy, she determine whether she sells remanufactured products to primary customers by keeping prices low or not. There are cases in which high disassemblability is beneficial for both OEM and remanufacturer. This study considers disassemblability with competition of OEM's manufactured products and remanufacturer's remanufactured prodcuts, whereas we consider competition of OEM and remanufacturer in the remanufactured product market.

2.1.3 Pricing of Brand-new and Recovered Products

Atasu et al. [13] argue that there are three main drivers an OEM to start remanufacturing. It is possible to cannibalize competitor's new product sales with remanufacturing. For some industries, existence of green consumer segment helps firms to make extra profits via price discrimination. With remanufactured products, firms can expand their market share. This study explores the conditions under which benefits of remanufacturing are maximized. As a benchmark scenario, static monopoly is discussed. Remanufacturing supply is assumed to be unlimited. Customers are differentiated with respect to their perception for remanufactured products, primary customers and green customers. Primary customers perceive remanufactured products as inferior goods. Whereas, green customers perceive remanufactured products as identical to brand new products. It is shown that depending on green segment size pricing strategy changes. When green segment is small, it is better to keep price of remanufactured product low and attract primary customers. When green segment is larger than a threshold, higher prices for remanufactured products and creating two distinct markets maximize profit. We use a price function similar to price function of this study.

Gan et al. [14] investigate optimal pricing policy together with channeling decisions. Brand new products are sold to customers via a retailer and remanufactured products are sold directly to customers by the manufacturer. An independent collector collects remanufacturable used items and sold them to manufacturer. The manufacturer determines wholesale price for manufactured products sold through a retailer and market price for remanufactured products. Retailer determines market price for manufactured products. Collector determines acquisition fee for used products. Life cycle of a product is divided into four intervals such that at first, there is only manufactured products in the market, then remanufacturing starts, share of remanufactured products in the market becomes higher than manufactured ones, and lastly market consists of only remanufactured products. They consider pricing of manufactured and remanufactured products and there are intervals in which only manufactured and only remanufactured products are supplied to the market as we do.

Ketzenberg and Zuidwijk [15] investigate pricing of products considering effect of return policies. Demand is a function of both market price and the return policy. When return policy is more flexible, customers are encouraged to purchase the product. A two-period model is created. The first period's demand is satisfied with manufactured products in the first period. The second period's demand is satisfied with returns in the first period and the products that are manufactured in the first period and carried to the second period. The returns of the second period is salvaged. They also consider the case where the market size and returns are uncertain. They conclude that for several settings an intermediate return policy is optimal. This study focuses on pricing decisions together with return policies whereas we focus on pricing, competition and remanufacturability investment.

Customer perception toward low end products, high end products and remanufactured products is an important issue while making pricing decisions. Common parts and subassemblies of low and high end products help manufacturer to reduce manufacturing and inventory related costs. However, with the increase in common parts, customers' valuation of high end products decreases and customers' valuation of low end products increases. Subramanian et al. [16] investigate the effect of remanufacturing operation to commonality decision and the effect of not considering remanufacturing while determining commonality of a product. To perform this analysis, they create three models, without remanufacturing, with remanufacturing by OEM, and with remanufacturing by a third party remanufacturer. Production cost of high-end product decreases with commonality. Production cost of low end product may increase or decrease depending on problem parameters. Only high end products are used for remanufacturing. Customers' quality perception of remanufactured product is in between high-end and low-end products. They conclude that when a third party is the one who remanufactures, it is more important to take remanufacturing into account. When the third party remanufactures, commonality is less preferable. In this study, pricing decisions are investigated with commonality decision, whereas we determine pricing with remanufacturability investment.

Chen and Chang [17] investigate the pricing decision using dynamic programming technique. A two period model is constructed with one player. In the first period, manufacturer supply manufactured products to the market. In the second period, manufactured and remanufactured products are supplied to the market with different market price. Market potential is shared among two product types and quantity demanded is a deterministic function of both product types. The input of remanufacturing in the second period is limited with a fraction of manufacturing quantity in the first period. For comparison a single period model and a multi-period model are created. In the single period model, remanufacturing input is unlimited. They consider competition of OEM's manufactured and remanufactured products, whereas we consider competition of remanufactured products of OEM and IR.

Debo et al. [18] consider pricing decisions in an environment where remanufacturability of products is a decision variable. A model with an OEM is designed in which OEM makes manufacturing and remanufacturing. OEM chooses level of technology of the original product which determines cost of manufacturing and remanufacturing. Manufacturing cost increases with an increase in the technology level whereas remanufacturing cost is a non-increasing function of level of technology. In order to reach certain technology level, OEM should make an investment. They investigate the pricing decisions of manufactured and remanufactured products of OEM that are sold in the same market and compete with each other. They also consider the competition of independent remanufacturers in the remanufactured product market. Our study differ from this study with extensive analysis of effects of competition of OEM's and IR's remanufactured products.

2.1.4 Value of Sorting Information

One of the drawbacks of remanufacturing is not having information about availability of returned items for remanufacturing. This drawback can be eliminated by sorting returned products prior to remanufacturing process. Value of sorting information (whether benefits of sorting prior to remanufacturing is higher than its cost) is investigated in Zikopoulos and Tagaras [19]. Quick but not perfect sorting is investigated in this article. The supply chain consists of a collection site and a remanufacturing facility. Two quality classes are considered, remanufacturable and not remanufacturable. There are two types of classification error, eliminating remanufacturable returns and not eliminating not remanufacturable ones. Exact condition of returned product is appeared at remanufacturing facility. The proportion of remanufacturable items is assumed to be a continuous random variable. Price is assumed to be deterministic. A shortage cost is incurred for unsatisfied demand. Procurement quantity and remanufacturing quantity are the decisions of the manufacturer. Benefits of sorting is dependent on remanufacturing cost, sorting cost, disassembly cost and failure rates of sorting. When failure rates of sorting and sorting cost is sufficiently low sorting helps manufacturer to decrease total remanufacturing cost. It is stated that the rate of failure that is classifying remanufacturable ones as not remanufacturable have greater impact on efficiency of sorting operations. Gu and Tagaras [20] consider similar reverse supply chain problem such that used products are sorted imperfectly in order to eliminate not remanufacturable ones. However, there are errors classifying used products. Contribution of this study to literature is it considers effect of decentralization together with effect of imperfect sorting. A collector who is responsible of inspection of used products sends more than ordered remanufacturable products to remanufacturer. Actual conditions of the items are revealed after disassembly process by remanufacturer and the items that are not remanufacturable and that are not needed by remanufacturer are discarded. If the remanufacturable items are not sufficient to satisfy remanufacturer's order then collector pays a penalty per unit shortage. They create a centralized setting to observe effect of centralization. These two studies investigate the effects of sorting information in monopolistic setting. We consider the effect of sorting information together with competition.

One of the main difficulties in closed loop supply chains is uncertainties. Ketzenberg et al. [21] investigate effects of uncertainties in demand, in product returns and in product recovery rate. The uncertain demand is satisfied via procuring new product from an external supplier or recovering returned products. Shortage cost and inventory holding cost is charged for unsatisfied demand and excess supply, respectively. The firm determines order quantity from external supplier to minimize total cost. Demand and product returns are independent normally distributed random variables. Product recovery is not capacitated and a fraction of product returns becomes marketable goods. They create different cases in which information available is different. They consider the cases with no information, with number of demands, number of returns and two cases with combination additional information alternatives in order to investigate effects of uncertainties. Hosoda et al. [22] also consider value of information when demand, product return and yield rate is uncertain. In addition to that they consider effect of remanufacturing lead time and the effect of correlation between demand and product returns. The two study takes product return as uncertain, whereas we assume a deterministic return rate. They investigates effect of sorting information (uncertainty in yield rate). However, we differ from these two studies such that we also consider remanufacturability investment and competition in remanufactured product market.

The aim of this thesis study is to investigate the effects of competition in the remanufactured market, effects of sorting information of returned items and effect of being aware of competition on economical and environmental performance of the system. The study is differed from existing literature, the effects of competition is examined for the settings with or without sorting information. Moreover, effects of sorting information is examined for both monopolistic and duopolistic settings. The effects of competition awareness is investigated with remanufacturability investment.

2.2 Problem Definition

In this study, we consider a two-period manufacturing/remanufacturing problem. In the first period an OEM supplies new products to the market. In the second period, both OEM and IR remanufacture the returns. There is no new items produced in the second period. The level of remanufacturability of the returns is determined by the OEM's investment in the first period.

All information is common knowledge, which includes the manufacturing quantity and remanufacturability level at the beginning of the second period. It is assumed that both players and customers are rational.

At the beginning of product's life cycle, there is no returned product to be used for remanufacturing. As a result, manufactured products do not compete with remanufactured products. Intervals of manufacturing and remanufacturing do not overlap with each other. In other words, remanufacturing does not start until manufacturing ends.

Products sold in the first period is the only source of returned products. As a result, OEM can determine maximum remanufacturing input quantity by indirectly determining manufacturing quantity. Some of the used items cannot be collected, only a fraction of sold items returned. Remanufacuturing output also depends on remanufacturability level. Number of remanufacturable items, i.e. maximum remanufacturing output, is proportional to remanufacturability level as well as total returns. Remanufacturability level is another tool of OEM in order to deal with competition in remanufactured product market. OEM and IR serve the same market. As a result, they compete in the remanufactured product market in the second period.

In the very beginning of the product design stage, OEM should determine level of remanufacturability. Since, level of remanufacturability is modeled as the determinant of yield rate of remanufacturing process. The investment required to establish a specific remanufacturability level, $0 \leq e \leq 1$, is formulated as a quadratic function, ke^2.

In order to focus on our main research questions, we do not take the demand uncertainty into account. The utility gained by a customers is a linear decreasing function of market price of the product. Hence, quantity demanded is a linear decreasing function of price and can be manipulated by market price of product. By using this information, we can say that market price is a function of total quantity supplied to the market in that period. If Q is total quantity supplied to market in a period, price obtained by inverse demand function is in the following form: (please see table 2.2 for the notation used throughout the thesis)

$$P(Q) = (a - bQ)$$

where a and b are maximum selling price and coefficient of quantity sensitivity of inverse demand function, respectively.

The nature of market price for manufactured and remanufactured products (by OEM and IR) can be different, since the customers perception can be differed for these

Table 2.2: Table of Notation

	<u>Decision Variables</u>
Q_1	Quantity of new products manufactured in the first period
Q_{2M}	Quantity of remanufactured products by OEM in the second period
Q_{2R}	Quantity of remanufactured products by IR in the second period
e	Level of remanufacturability, i.e. desired remanufactured option yield fraction
	<u>Parameters</u>
$\pi_1(Q_1, e)$	OEM's profit in the first period for given values of Q_1 and e
$\pi_{2M}(Q_{2M}, Q_{2R})$	OEM's profit in the second period for given values of Q_{2M} and Q_{2R}
$\pi_{2R}(Q_{2R}, Q_{2M})$	IR's profit in the second period for given values of Q_{2R} and Q_{2M}
$P_1(Q_1)$	Inverse demand function for the first period for given values of Q_1
$P_{2M}(Q_{2M}, Q_{2R}\|e)$	Inverse demand function of OEM for the second period for given values of Q_{2R} and Q_{2M}
$P_{2R}(Q_{2R}, Q_{2M}\|e)$	Inverse demand function of IR for the second period for given values of Q_{2M} and Q_{2R}
c_1	unit manufacturing cost
c_{2M}	unit remanufacturing cost of OEM
c_{2R}	unit remanufacturing cost of IR
s	unit recycling revenue
k	investment cost coefficient of remanufacturability
τ	return rate of used products $(0 \leq \tau \leq 1)$
γ	OEM's share of the returned products $(0 \leq \gamma \leq 1)$

products. In order to capture these effects, we concentrate on price functions for manufactured items in the first period and remanufactured items by OEM & IR in the second period with different parameters. We specifically consider the following price functions for manufactured products in the period 1, remanufactured products by OEM in period 2 and remanufactured products by IR in the period as follows:

$$P_1(Q_1) = (a_1 - b_1 Q_1) \tag{2.1}$$

$$P_{2M}(Q_{2M}, Q_{2R}|e) = (a_{2M} - b_{2M} e(Q_{2M} + Q_{2R})) \tag{2.2}$$

$$P_{2R}(Q_{2M}, Q_{2R}|e) = (a_{2R} - b_{2R} e(Q_{2M} + Q_{2R})) \tag{2.3}$$

Notice that the parameters maximum selling prices, a_1, a_2 and a_3, and coefficient of quantity sensitivities of inverse demand function, b_1, b_{2M} and b_{2R} are allowed to be non-identical for 2.1, 2.2 and 2.3.

In order to exclude the competition between OEM and IR in collecting used products, we assume that customers return used products when they are no longer needed without any acquisition fee . There is no investment needed to collect used products either. In the second period, τ, a deterministic fraction of sold manufactured products are returned. The returns are split between the OEM and IR based on a deterministic rate, γ. Hence if Q_1 units is produced in the first period, the available returns for remanufacturing in the second period is τQ_1. OEM and IR take $\gamma\tau Q_1$ and $(1-\gamma)\tau Q_1$ of returns, respectively. In monopolistic settings, OEM gets all returned items. $\gamma\tau Q_1$ and $(1-\gamma)\tau Q_1$ are maximum remanufacturing quantities, OEM and IR do not have to use them to remanufacture.

Level of remanufacturability is the decision of OEM that directly changes the remanufacturing yield. That is, level of remanufacturability and remanufacturing input quantity determines remanufacturing output. Total output of remanufacturing process is equal to eQ_{2M} for OEM and eQ_{2R} for IR, respectively. Both parties are affected from remanufacturability level.

Unused returned products are recycled to the material level. Sources of recycling are unused returned items and scraps caused by low remanufacturing yield. Products that are sent to recycling is equal to $(\gamma\tau Q_1 - eQ_{2M})$ for OEM and $((1-\gamma)\tau Q_1 - eQ_{2R})$ for IR.

The sequence of events in our problem environment is as follows:

- **Period 1** OEM determines manufacturing quantity, $Q_1 \geq 0$, and level of remanufacturability, $0 \leq e \leq 1$, simultaneously and sells all manufactured products at price of $P_1(Q_1)$.
- **Period 2** OEM gets $\gamma\tau Q_1$ of returned products and IR gets $(1-\gamma)\tau Q_1$ of returned products with remanufacturability level e and they determine simultaneously remanufacturing input quantities, $0 \leq Q_{2M} \leq \gamma\tau Q_1$ and $0 \leq Q_{2R} \leq (1-\gamma)\tau Q_1$, without knowing which returned products are suitable for remanufacturing. All remanufactured products of OEM and IR are sold at prices $P_{2M}(Q_{2M}, Q_{2R}|e)$ and $P_{2R}(Q_{2M}, Q_{2R}|e)$, respectively.

The competition is modeled in a duopolistic setting where OEM and IR sells their remanufactured products in the same market. In order to see the effects of competition to our performance measures, we concentrate a monopolistic setting where there is only OEM that remanufactures as a benchmark. The equilibrium outcome of this model is compared to the optimal solution of the monopolistic market. Total value recovered, remanufacturability level and total system wide profit are considered as performance measures.

The ratio of successfully remanufactured items to remanufacturing input quantity is equal to the remanufacturability level. Remanufacturing cost per unit incurred is regardless of the result of remanufacturing process. That is, the cost is incurred for all items that enter the remanufacturing process.

We also investigate the effect of perfect sorting information prior to remanufacturing. To do so, we also created different settings. These settings differ from benchmark settings with remanufacturing cost structure. Total remanufacturing cost is proportional to succesfully remanufactured products instead of total remanufacturing input.

For the settings with perfect sorting information prior to remanufacturing process, only remanufacturable products enter the remanufacturing process and cost only incurs for remanufacturable products. It is assumed that there is no cost related with sorting process. In order to investigate the effects of the sorting information, we compare optimal solutions of monopolistic model without sorting information and monopolistic model with sorting information, and we compare equilibrium solutions of duopolistic setting without sorting information, duopolistic setting with sorting information for only OEM, and duopolistic setting with sorting information for both OEM and IR.

CHAPTER 3

MONOPOLISTIC SETTING

OEM is a monopoly in the manufactured product market as in remanufactured product market in this setting. In other words, there is no competitor in any of the markets. OEM determines manufacturing quantity and remanufacturability level in the first period and remanufacturing quantity for given sales of manufactured product and remanufacturability level in the second period. Since, there is only OEM in the market, OEM takes all returned products. OEM as a monopoly is our benchmark model in order to see the effect of competition to our performance measures by comparing it with duopolistic model.

Depending on the sorting information two different monopolistic models are created:

(i) Monopolistic model without sorting information.

(ii) Monopolistic model with sorting prior to remanufacturing.

The problem of OEM is analyzed in Section 3.2 and 3.3 when sorting information of returned items is not available at the beginning of remanufacturing process. The problem of OEM is analyzed in Subsections 5.1.1 and 5.1.2 when quality information of returned items is available prior to remanufacturing process.

3.1 Monopolistic Model without Sorting Information

In the monopolistic model without sorting information, OEM does not know which returned items are suitable for remanufacturing until the process ends. In order to obtain a desired remanufacturing output level, Q, OEM have to start remanufacturing with more than planned remanufacturing output quantity,Q/e, since $(1-e)$ portion of remanufacturing input fails to be remanufactured. When OEM starts remanufacturing with Q_{2M}, remanufactured product supplied to the market is eQ_{2M}. $(1-e)Q_{2M}$ is the quantity sent to recycling due to yield loss. Since success of remanufacturing is observed at the end of the remanufacturing process, cost of remanufacturing is incurred for all items, Q_{2M}, that enter remanufacturing process.

Sequence of events for monopolistic model without sorting information prior to remanufacturing is as follows:

- **Period 1** OEM determines manufacturing quantity, $Q_1 \geq 0$, and level of remanufacturability, $0 \leq e \leq 1$, by incurring total manufacturing cost of c_1Q_1 and making an investment of ke^2 for the level of remanufacturability, respectively.

  Unit market price for manufactured items become $P_1(Q_1) = a_1 - b_1Q_1$. OEM sells all Q_1 units at a unit price of $P_1(Q_1)$.

- **Period 2** OEM gets τQ_1 returns with remanufacturability level e.

  OEM determines remanufacturing input quantity Q_{2M} such that $0 \leq Q_{2M} \leq \tau Q_1$, incurring total remanufacturing cost of $c_{2M}Q_{2M}$ and obtains unit recycling revenue of s for returns that are not remanufactured, $(\tau Q_1 - Q_{2M})$, and items lost in remanufacturing process, $(1-e)Q_{2M}$.

  The market price becomes $P_{2M}(Q_{2M}) = a_{2M} - b_{2M}eQ_{2M}$. OEM sells all Q_{2M} at a unit price of $P_{2M}(Q_{2M})$.

Profit of OEM in the first and the second periods are represented by $\pi_1(Q_1, e)$ and $\pi_{2M}(Q_{2M}|Q_1, e)$, respectively.

In order to solve the problem of OEM, we use backward induction. First, for the second period problem, we characterize the optimal Q_{2M} for given values of $Q_1 \geq 0$ and $0 \leq e \leq 1$. Then, using the optimal solutions obtained, the first period problem where the sum of the first period & the second period profits under the optimal Q_{2M} decision is maximized. The analysis for the second and the first period problems are given in Section 3.2 and 3.3, respectively.

3.2 Analysis of OEM's Second Period Problem

OEM's second period problem can be stated as follows:

$$maximize \quad \pi_{2M}(Q_{2M}|Q_1, e) = (a_{2M} - b_{2M}eQ_{2M})eQ_{2M} - c_{2M}Q_{2M} + s(Q_1\tau - Q_{2M}e)$$

subject to

$$Q_{2M} \leq Q_1\tau \tag{3.1}$$

$$Q_{2M} \geq 0 \tag{3.2}$$

The objective function is composed of revenue generated by selling remanufactured products, cost of remanufacturing and revenue generated by the returned items that are not successfully remanufactured and not remanufactured. Constraint 3.2 represent non-negativity of remanufacturing quantity in the second period. Constraint 3.1 represents that maximum τ fraction of the first period manufacturing quantity can be remanufactured in the second period. Right hand side of 3.1 is dependent on manufacturing quantity determined in the first period.

Lemma 3.2.1. *$\pi_{2M}(Q_{2M})$ is a concave function of Q_{2M} .*

Proof. The second derivative with respect to Q_{2M} is as follows:

$$\frac{d^2\pi_{2M}(Q_{2M})}{dQ_{2M}^2} = -2b_{2M}e^2 \quad .$$

Since $b_{2M} > 0$, $-2b_{2M}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2M}(Q_{2M})$ is concave in Q_{2M}. □

Theorem 3.2.2. *If for a given value of $0 \leq e \leq 1$, $\frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} < 0$, then remanufacturing is not profitable for all given values of $Q_1 > 0$ in the second period, i.e., $Q_{2M}^* = 0$ under the optimal solution. Otherwise, for given $Q_1 \geq 0$ and $0 \leq e \leq 1$ values, the optimal input quantity to the remanufacturing process, Q_{2M}^*, in the second period is*

$$Q_{2M}^*(Q_1, e) = \begin{cases} \frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} & if \quad 0 \leq \frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2} \leq Q_1\tau \\ Q_1\tau & if \quad Q_1\tau < \frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2} \end{cases} . \tag{3.3}$$

Proof. The first derivative of $\pi_{2M}(Q_{2M})$ with respect to Q_{2M} is

$$\frac{d\pi_{2M}(Q_{2M})}{dQ_{2M}} = a_{2M}e - 2b_{2M}e^2 Q_{2M} - c_{2M} - se \quad . \tag{3.4}$$

If $a_{2M}e - c_{2M} - se < 0$, $\frac{d\pi_{2M}(Q_{2M})}{dQ_{2M}} < 0$ for all possible values of $Q_{2M} \geq 0$. Hence, the second period profit function is decreasing in Q_{2M} and $Q_{2M} = 0$ maximizes $\pi_{2M}(Q_{2M})$.

Otherwise, since $\pi_{2M}(Q_{2M})$ is a concave function of Q_{2M}, if the unconstrained solution is feasible with respect to constraint 3.1, it is optimal. Otherwise, under the optimal solution, constraint 3.1 is binding. Unconstrained solution can be found by setting 3.4 to zero. Equation 3.3 provides the result.

$$Q_{2M} = \frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} \tag{3.5}$$

□

When $a_{2M}e - c_{2M} - se < 0$, OEM does not remanufacture since profit margin is negative for for any given Q_1. $\frac{c_{2M}}{a_{2M}-s}$ is the critical ratio for OEM to begin remanufacturing activities. Any remanufacturability level, e, less than the critical ratio is dominated by no remanufacturing decision. Profit always decreases with an increase in the remanufacturability level until the critical ratio. As a result remanufacturability levels which is less than critical ratio is never implemented.

Following the result provided in Theorem 3.2.2, profit in the second period under the optimal solution can be characterized.

If $\frac{a_{2M}e-c_{2M}-se}{2b_{2M}e^2} < 0$, then the second period's profit only consists of recycling revenue as below.

$$\pi^*_{2M}(Q_1, e) = s\tau Q_1$$

When the optimal solution characterized in the Theorem 3.3 is plugged to the second period problem into the profit expression, we obtain the second period profit as a function of Q_1 and e is as follows:

$$\pi^*_{2M}(Q_1, e) = \begin{cases} \frac{(a_{2M}e-c_{2M}-se)^2}{4b_{2M}e^2} + s\tau Q_1 & if \quad 0 \leq \frac{(a_{2M}e-c_{2M}-se)}{2\tau b_{2M}e^2} \leq Q_1 \\ (a_{2M}e - c_{2M} - se)\tau Q_1 & if \quad Q_1 < \frac{(a_{2M}e-c_{2M}-se)}{2\tau b_{2M}e^2} \\ -b_{2M}(\tau e Q_1)^2 + s\tau Q_1 \end{cases} \tag{3.6}$$

3.3 Analysis of OEM's First Period Problem

We can now solve the first period problem where OEM determines Q_1 and e to maximize its total profit over two periods. Let $\pi_T(Q_1, e)$ stands for total profit function over two periods. In order to find Q_1 and e that maximizes total profit, joint concavity of it should be used.

For the sake of brevity, let $A_{2M}(e) = \frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2}$.

$$\pi_T(Q_1, e) = \pi_1(Q_1, e) + \begin{cases} \pi_{2M}^{i}(Q_1, e) & if \leq A_{2M}(e) \leq 0 \\ \pi_{2M}^{ii}(Q_1, e) & if \quad 0 \leq A_{2M}(e) \leq Q_1\tau \\ \pi_{2M}^{iii}(Q_1, e) & if \quad Q_1\tau \leq A_{2M}(e) \end{cases}$$

where

$$\pi_{2M}^{(i)}(Q_1, e) = s\tau Q_1$$

$$\pi_{2M}^{(ii)}(Q_1, e) = s\tau Q_1 + \frac{(a_{2M}e - c_{2M} - se)^2}{4b_{2M}e^2}$$

$$\pi_{2M}^{(iii)}(Q_1, e) = s\tau Q_1 + (a_{2M}e - c_{2M} - se)\tau Q_1 - b_{2M}(\tau e Q_1)^2$$

OEM's first period problem can be expressed as

$$maximize \quad \pi_T(Q_1, e) = \pi_1(Q_1, e) + \pi_{2M}^{*}(Q_1, e)$$

where

$$\pi_1(Q_1, e) = (a_1 - c_1 - b_1 Q_1)Q_1 - ke^2 \tag{3.7}$$

subject to

$$Q_1 \geq 0 \tag{3.8}$$

$$0 \leq e \leq 1 \tag{3.9}$$

Objective function of the first period consists of revenue generated by selling manufactured products, cost of manufacturing and investment cost for the remanufacturability level, e. Constraint 3.8 represent non-negativity of manufacturing in the first period. Constraint 3.9 represents that since remanufacturability is modeled as yield rate fraction, it should lie in the interval, $[0, 1]$.

We cannot show the joint concavity of the total profit function for Q_1 and e. Hence, we implement an algorithm to find optimal solution.

Below, we characterize optimal Q_1 for given e.

Proposition 1. *Given* $\frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2} < 0$, *optimal manufacturing quantity in the first period,* Q_1^*, *is*

$$Q_1^* = \frac{a_1 - c_1 + s\tau}{2b_1} \quad . \tag{3.10}$$

Proof. Profit function given $\frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2} < 0$, can be expressed as follows

$$\pi_T^{(i)}(Q_1|e) = (a_1 - c_1 - b_1 Q_1 + s\tau)Q_1 - ke^2$$

The second derivative of $\pi_T^{(i)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(i)}(Q_1|e)}{\partial Q_1^2} = -2b_1$$

Since $b_1 > 0$ is always positive, $\frac{\partial^2 \pi_T^{(i)}(Q_1|e)}{\partial Q_1^2} < 0$. Therefore, $\pi_T^{(i)}(Q_1|e)$ is concave in Q_1.

The first derivative of $\pi_T^{(i)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(i)}(Q_1|e)}{\partial Q_1} = a_1 - 2b_1 Q_1 - c_1 + s\tau \tag{3.11}$$

Optimal solution given in equation 3.10 is found by setting 3.11 to zero. □

Proposition 2. *Given* $\frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2} \geq 0$, *optimal manufacturing quantity in the first period,* Q_1^*, *is*

$$Q_1^* = \begin{cases} \frac{a_1 - c_1 + s\tau}{2b_1} & if \quad 0 \leq A_{2M}(e) \leq \tau \frac{a_1 - c_1 + s\tau}{2b_1} \\ \frac{(a_1 - c_1 + s\tau) + \tau(a_{2M}e - c_{2M} - se)}{2b_1 + 2b_{2M}e^2} & if \quad \tau \frac{a_1 - c_1 + s\tau}{2b_1} \leq A_{2M}(e) \end{cases}$$

Proof. Profit function given $0 \leq A_{2M}(e) \leq \tau Q_1$

$$\pi_T^{(ii)}(Q_1|e) = (a_1 - c_1 - b_1 Q_1 + s\tau)Q_1 - ke^2$$

The second derivative of $\pi_T^{(ii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(ii)}(Q_1|e)}{\partial Q_1^2} = -2b_1$$

Since $b_1 > 0$, $\frac{\partial^2 \pi_T^{(ii)}(Q_1|e)}{\partial Q_1^2} < 0$. Therefore, $\pi_T^{(ii)}(Q_1|e)$ is concave in Q_1.

The first derivative of $\pi_T^{(ii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(ii)}(Q_1|e)}{\partial Q_1} = a_1 - 2b_1Q_1 - c_1 + s\tau \tag{3.12}$$

Optimal solution can be found by setting 3.12 to zero given $0 \leq A_{2M}(e) \leq \tau Q_1$.

Profit function given $\tau \frac{a_1 - c_1 + s\tau}{2b_1} \leq A_{2M}(e)$

$\pi_T^{(iii)}(Q_1|e) = (a_1 - c_1 - b_1Q_1 + s\tau)Q_1 - ke^2 + (a_{2M}e - c_{2M} - se)\tau Q_1 - b_{2M}(\tau e Q_1)^2$

The second derivative with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(iii)}(Q_1|e)}{\partial Q_1^2} = -2b_1 - 2b_{2M}(e\tau)^2$$

Since $b_1 > 0$, $b_{2M} > 0$, $\tau > 0$ and e is defined in $0 \leq e \leq 1$. Therefore, $\pi_T^{(iii)}(Q_1|e)$ is concave in Q_1.

The first derivative with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(iii)}(Q_1|e)}{\partial Q_1} = a_1 - c_1 + s\tau + \tau(a_{2M}e - c_{2M} - se) - 2b_1Q_1 - 2b_{2M}Q_1(e\tau)^2 \tag{3.13}$$

Optimal solution can be found by setting 3.13 to zero given $\tau Q_1 \leq A_{2M}(e)$.

Profit function, $\pi_T(Q_1|e)$, is continuously differentiable at $Q_1 \leq A_{2M}(e)/\tau$.

$$\frac{\partial \pi_T^{(ii)}(A_{2M}(e)/\tau|e)}{\partial Q_1} = a - 2b_1(\frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2}) - c_1 + s\tau$$

$$\frac{\partial \pi_T^{(iii)}(A_{2M}(e)/\tau|e)}{\partial Q_1} = a_1 - c_1 + s\tau + \tau(a_{2M}e - c_{2M} - se)$$
$$-2b_1(\frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2}) - 2b_{2M}(\frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2})(e\tau)^2$$

$$\frac{\partial \pi_T^{(ii)}(A_{2M}(e)/\tau|e)}{\partial Q_1} = \frac{\partial \pi_T^{(iii)}(A_{2M}(e)/\tau|e)}{\partial Q_1}$$

If the first derivative of total profit function is positive in the region for all Q_1 in the region, optimal manufacturing quantity lies in the region $Q_1 \leq A_{2M}(e)/\tau$. Otherwise, manufacturing quantity that maximizes total profit lies in the region $0 \leq A_{2M}(e)/\tau \leq Q_1$. □

As a result of Theorem 3.2.2 and Proposition 2, optimal solution to overall problem for a given $0 \leq e \leq 1$ is as follows:

$$(Q_1^*, Q_{2M}^*) = \begin{cases} (\frac{a_1-c_1+s\tau}{2b_1}, 0) & if \quad A_{2M}(e) < 0 \\ (\frac{a_1-c_1+s\tau}{2b_1}, \frac{(a_{2M}e-c_{2M}-se)}{2b_{2M}e^2}) & if \quad 0 \leq A_{2M}(e) \leq \tau\frac{a_1-c_1+s\tau}{2b_1} \\ (Q_1', \tau Q_1') & if \quad \tau\frac{a_1-c_1+s\tau}{2b_1} \leq A_{2M}(e) \end{cases}$$

Where $Q_1' = \frac{(a_1-c_1+s\tau)+\tau(a_{2M}e-c_{2M}-se)}{2b_1+2b_{2M}e^2}$

The steps of the solution procedure in order to find the optimal level of remanufacturability, manufacturing quantity in the first period and remanufacturing quantity in the second period are as follows:

1. Optimal manufacturing quantity, Q_1, and remanufacturing quantity, Q_{2M}, are determined using 3.14 that maximizes total profit for increasing levels of remanufacturability, e .
2. We plug optimal Q_1, Q_{2M} and the given remanufacturability level used in the first step, e, to the total profit function.
3. We search for the optimal remanufacturability level that maximizes total profit in its range.

CHAPTER 4

DUOPOLISTIC SETTING

In this chapter, we focus on the setting where OEM is not the only actor in the market. IR competes with OEM in the remanufactured product market. Both players have perfect information about problem parameters. Manufacturing is carried out by only OEM in the first period. OEM can manipulate total returned items by controlling manufacturing quantity. OEM and IR shares returned items with a constant sharing ratio. OEM and IR remanufacture in the second period using returned items. Our aim is to characterize the solution to the duopolistic model and compare it to the optimal solution of the monopolistic model in order to investigate effect of competition to economical and environmental performance measures.

Depending on whether sorting information is available, three different duopolistic settings are considered:

(i) Duopolistic model without sorting information.

(ii) Duopolistic model with sorting information for only OEM.

(iii) Duopolistic model with sorting information for both OEM and IR.

We consider these settings in order to investigate the effect of sorting information on economical and environmental performance measures. Duopolistic model without sorting information is analyzed in Sections 4.2 and 4.3. Duopolistic model with sorting information with sorting information for only OEM is analyzed in Subsections 5.2.1 and 5.2.2. Duopolistic model with sorting information for both OEM and IR is analyzed in Section 5.3.

4.1 Duopolistic Model without Sorting Information

In the duopolistic model without sorting information, OEM and IR do not know which specific returned items are remanufacturable until the the remanufacturing process ends and yield loss occurs throughout remanufacturing process. Therefore, remanufacturing cost incurs due to inefficiency caused by yield loss. OEM (IR) starts remanufacturing with Q_{2M} (Q_{2R}). But, due to not remanufacturable returned items, successfully remanufactured product quantity is equal to eQ_{2M} (eQ_{2R}). Hence, both firms suffer from yield loss in remanufacturing.

Cost of remanufacturing for duopolistic model without sorting information prior to remanufacturing is the same as its monopolistic counterpart. When OEM (IR) starts remanufacturing with input quantity Q_{2M} (Q_{2R}), cost of remanufacturing is $eQ_{2M}c_{2M}$ $(eQ_{2R}c_{2R})$.

OEM does not get all returned items as in the monopolistic setting. OEM and IR shares returned products with constant sharing fraction. With total return quantity τQ_1, OEM gets $\gamma\tau Q_1$ and IR gets $(1-\gamma)\tau Q_1$. Both firms have the recycling option and recycling unit revenue is same for OEM and IR.

Sequence of events for duopolistic model without sorting information prior to remanufacturing is as follows:

- **Period 1**

  OEM determines manufacturing quantity, $Q_1 \geq 0$, and the level of remanufacturability, $0 \leq e \leq 1$, by incurring a total manufacturing cost of $c_1 Q_1$ and making an investment of ke^2 for level of remanufacturability, respectively.

  The market price for manufactured items become $P_1(Q_1) = a_1 - b_1 Q_1$. OEM sells all Q_1 units at a price of $P_1(Q_1)$.

- **Period 2**

  OEM and IR get $\gamma\tau Q_1$ and $(1-\gamma)\tau Q_1$ returns with remanufacturability level e, respectively.

  OEM and IR simultaneously determine remanufacturing input quantities Q_{2M}

and Q_{2R} such that $0 \leq Q_{2M} \leq \gamma\tau Q_1$ and $0 \leq Q_{2M} \leq (1-\gamma)\tau Q_1$, incurring total remanufacturing cost of $c_{2M}Q_{2M}$ and $c_{2R}Q_{2R}$ and obtain recycling revenue of s for returns that are not remanufactured, $(\gamma\tau Q_1 - Q_{2M})$ and $((1-\gamma)\tau Q_1 - Q_{2R})$, and items lost in remanufacturing process, $(1-e)Q_{2M}$ and $(1-e)Q_{2R}$, respectively .

The market price becomes $P_{2M}(Q_{2M}, Q_{2R}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of OEM. OEM sells all eQ_{2M} at an unit price of $P_{2M}(Q_{2M}, Q_{2R})$.

The market price becomes $P_{2R}(Q_{2R}, Q_{2M}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of IR. IR sells all eQ_{2R} at an unit price of $P_{2R}(Q_{2R}, Q_{2M})$.

Profit of OEM in the first and the second periods are represented by $\pi_1(Q_1, e)$ and $\pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e)$ respectively. Profit of IR in the second period is represented by $\pi_{2R}(Q_{2R}, Q_{2M}|Q_1, e)$.

In order to solve the problem of OEM and IR, we use backward induction. First, for the second period problem of two firms, we characterize the equilibrium Q_{2M} and Q_{2R} for given values of $Q_1 \geq 0$ and $0 \leq e \leq 1$. Then, using the equilibrium solution obtained, the first period problem where the sum of the first period and the second period profits of OEM under the equilibrium $Q_{2M}$and Q_{2R} is maximized. The analysis for the second and the first period problems are given in Sections 4.2 and 4.3, respectively.

4.2 Analysis of the Second Period Problem

OEM's second period problem can be stated as follows:

$$\underset{Q_{2M}}{\text{maximize}} \quad \pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e) \quad = \quad \begin{array}{l} (a_{2M} - b_{2M}e(Q_{2M} + Q_{2R}))eQ_{2M} \\ -c_{2M}Q_{2M} + s(\gamma\tau Q_1 - Q_{2M}e) \end{array}$$

subject to

$$Q_{2M} \leq \tau\gamma Q_1 \tag{4.1}$$

$$Q_{2M} \geq 0 \tag{4.2}$$

IR's second period problem can be stated as follows:

$$\begin{aligned} \underset{Q_{2R}}{\text{maximize}} \quad \pi_{2R}(Q_{2M}, Q_{2R}|Q_1, e) \quad = \quad & (a_{2R} - b_{2R}e(Q_{2R} + Q_{2M}))eQ_{2R} \\ & -c_{2R}Q_{2R} + s((1-\gamma)\tau Q_1 - Q_{2R}e) \end{aligned}$$

subject to

$$Q_{2R} \leq (1-\gamma)\tau Q_1 \tag{4.3}$$

$$Q_{2R} \geq 0 \tag{4.4}$$

Objective functions are composed of revenue generated by selling remanufactured products, cost of remanufacturing, and revenue generated by the returned items that are not successfully remanufactured and not remanufactured. Constraints 4.2 and 4.4 represent non-negativity of remanufacturing quantity of OEM and IR, respectively, in the second period. Constraints 4.1 and 4.3 represent that maximum $\gamma\tau$ and $(1-\gamma)\tau$ fractions of the first period manufacturing quantity can be remanufactured by OEM and IR, respectively, in the second period. Right hand sides of 4.1 and 4.3 are dependent on the manufacturing quantity.

We start our analysis with the profit functions of OEM and IR. For the sake of brevity, let $A_{2M}(e) = \frac{(a_{2M}e - c_{2M} - se)}{b_{2M}e^2}$ and $A_{2R}(e) = \frac{(a_{2R}e - c_{2R} - se)}{b_{2R}e^2}$.

Lemma 4.2.1. *$\pi_{2M}(Q_{2M}, Q_{2R})$ is a concave function of Q_{2M}.*

Proof. The second derivative with respect to Q_{2M} is as follows:

$$\frac{d^2\pi_{2M}(Q_{2M}, Q_{2R})}{dQ_{2M}^2} = -2b_{2M}e^2 \quad .$$

Since $b_{2M} > 0$, $-2b_{2M}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2M}(Q_{2M}, Q_{2R})$ is concave in Q_{2M} for given $Q_{2R} > 0$. □

Theorem 4.2.2. *For a given value of* $0 \leq e \leq 1$ *and* $Q_1 > Q$*, if* $\frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} - \frac{Q_{2R}}{2} < 0$*, then remanufacturing is not profitable for all given values of* $Q_1 > 0$ *in the second period, i.e.,* $Q_{2M} = 0$*. Otherwise, for given* $Q_1 \geq 0$ *and* $0 \leq e \leq 1$ *values, the best response function of OEM in the second period is*

$$Q_{2M}(Q_{2R}|Q_1, e) \quad = \quad \begin{cases} \frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} - \frac{Q_{2R}}{2} & if \quad \frac{A_{2M}(e) - Q_{2R}}{2\tau\gamma} \leq Q_1 \\ \tau\gamma Q_1 & if \quad Q_1 \leq \frac{A_{2M}(e) - Q_{2R}}{2\tau\gamma} \end{cases} \tag{4.5}$$

Proof. The first derivative of $\pi_{2M}(Q_{2M}, Q_{2R})$ with respect to Q_{2M} is

$$\frac{d\pi_{2M}(Q_{2M}, Q_{2R})}{dQ_{2M}} = a_{2M}e - 2b_{2M}e^2 Q_{2M} - b_{2M}e^2 Q_{2R} - c_{2M} - se \quad . \tag{4.6}$$

If $\frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} - \frac{Q_{2R}}{2} < 0$, $\frac{d\pi_{2M}(Q_{2M}, Q_{2R})}{dQ_{2M}} < 0$ for all possible values of $Q_{2M} \geq 0$. Hence, the second period profit function is decreasing in Q_{2M} and the $Q_{2M} = 0$ maximizes $\pi_{2M}(Q_{2M}, Q_{2R})$.

Otherwise, since $\pi_{2M}(Q_{2M}, Q_{2R})$ is a concave function of Q_{2M}, if the unconstrained solution is feasible with respect to constraint 4.1, it is the best response of OEM. Otherwise, under the equilibrium solution, constraint 4.1 is binding. Unconstrained solution can be found by setting 4.6 to zero. Equation 4.7 provides the result.

$$Q_{2M} = \frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} - \frac{Q_{2R}}{2} \tag{4.7}$$

□

Lemma 4.2.3. $\pi_{2R}(Q_{2R}, Q_{2M})$ *is a concave function of* Q_{2R}*.*

Proof. The second derivative with respect to Q_{2R} is as follows:

$$\frac{d^2\pi_{2R}(Q_{2R}, Q_{2M})}{dQ_{2R}^2} = -2b_{2R}e^2 \quad .$$

Since $b_{2R} > 0$, $-2b_{2R}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2R}(Q_{2R}, Q_{2M})$ is concave in Q_{2R} for given $Q_{2M} > 0$. □

Theorem 4.2.4. *If for a given value of* $0 \leq e \leq 1$, $\frac{a_{2R}e - c_{2R} - se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} < 0$*, then remanufacturing is not profitable for all given values of* $Q_1 > 0$ *in the second period, i.e.,* $Q_{2R} = 0$*. Otherwise, for given* $Q_1 \geq 0$ *and* $0 \leq e \leq 1$ *values, the best response function of IR in the second period is*

$$Q_{2R}(Q_{2M}|Q_1, e) \quad = \quad \begin{cases} \frac{a_{2R}e - c_{2R} - se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} & if \quad \frac{A_{2R}(e) - Q_{2M}}{2} \leq \tau(1-\gamma)Q_1 \\ \tau(1-\gamma)Q_1 & if \quad \tau(1-\gamma)Q_1 \leq \frac{A_{2R}(e) - Q_{2M}}{2} \end{cases} \tag{4.8}$$

Proof. The first derivative of $\pi_{2R}(Q_{2R}, Q_{2M})$ with respect to Q_{2R} is

$$\frac{d\pi_{2R}(Q_{2R}, Q_{2M})}{dQ_{2R}} = a_{2R}e - 2b_{2R}e^2 Q_{2R} - b_{2R}e^2 Q_{2M} - c_{2R} - se \quad . \tag{4.9}$$

If $\frac{a_{2R}e - c_{2R} - se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} < 0$, $\frac{d\pi_{2R}(Q_{2R}, Q_{2M})}{dQ_{2R}} < 0$ for all possible values of $Q_{2R} \geq 0$. Hence, the second period profit function is decreasing in Q_{2R} and the $Q_{2R} = 0$ maximizes $\pi_{2R}(Q_{2R}, Q_{2M})$.

Otherwise, since $\pi_{2R}(Q_{2R}, Q_{2M})$ is a concave function of Q_{2R}, if the unconstrained solution is feasible with respect to constraint 4.3, it is the best response of IR. Otherwise, under the equilibrium solution, constraint 4.3 is binding. Unconstrained solution can be found by setting 4.9 to zero. Equation 4.10 provides the result.

$$Q_{2R} = \frac{a_{2R}e - c_{2R} - se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} \tag{4.10}$$

□

When a player's monopolistic remanufacturing quantity is higher than twice of remanufacturing quantity of other player's remanufacturing quantity, remanufacturing quantity of the player with smaller market share is equal to zero.

We plug best response functions of OEM and IR into each other and obtain equilibrium solution in the second period. Equilibrium outcome for the second period game is as follows:

$$
(Q_{2M}^*, Q_{2R}^*) = \begin{cases}
(0,0) & if \quad A_{2M}(e) < 0, A_{2R}(e) < 0 \\
(\frac{A_{2M}(e)}{2}, 0) & if \quad A_{2R}(e) < A_{2M}(e)/2, \\
& \quad 0 \leq A_{2M}(e) < 2\gamma\tau Q_1 \\
(\gamma\tau Q_1, 0) & if \quad A_{2R}(e) \leq \gamma\tau Q_1, \\
& \quad 2\gamma\tau Q_1 \leq A_{2M}(e) \\
(0, \frac{A_{2R}(e)}{2}) & if \quad A_{2M}(e) < A_{2R}(e)/2, \\
& \quad 0 \leq A_{2R}(e) \\
& \quad A_{2R}(e) < 2(1-\gamma)\tau Q_1 \\
(\frac{2A_{2M}(e)-A_{2R}(e)}{3}, \frac{2A_{2R}(e)-A_{2M}(e)}{3}) & if \quad 0 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
& \quad \frac{2A_{2R}(e)-A_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\
& \quad 0 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
& \quad \frac{2A_{2M}(e)-A_{2R}(e)}{3} < \gamma\tau Q_1 \\
(\gamma\tau Q_1, \frac{A_{2R}(e)-\gamma\tau Q_1}{2}) & if \quad \gamma\tau Q_1 < A_{2R}(e) \\
& \quad A_{2R}(e) < (2-\gamma)\tau Q_1 \\
& \quad \gamma\tau Q_1 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
(0, (1-\gamma)\tau Q_1) & if \quad A_{2M}(e) < (1-\gamma)\tau Q_1, \\
& \quad (2-\gamma)\tau Q_1 < A_{2R}(e) \\
(\frac{A_{2M}(e)-(1-\gamma)\tau Q_1}{2}, (1-\gamma)\tau Q_1) & if \quad (1-\gamma)\tau Q_1 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
& \quad (1-\gamma)\tau Q_1 < A_{2M}(e) \\
& \quad A_{2M}(e) < (1+\gamma)\tau Q_1 \\
(\gamma\tau Q_1, (1-\gamma)\tau Q_1) & if \quad (2-\gamma)\tau Q_1 < A_{2R}(e), \\
& \quad (1+\gamma)\tau Q_1 < A_{2M}(e)
\end{cases} \tag{4.11}
$$

OEM's second period profit under the equilibrium solution is

$$\pi^*_{2M}(Q_1,e) = \begin{cases} \pi^{(i)}_{2M}(Q_1,e) & if \quad A_{2M}(e) < 0, A_{2R}(e) < 0 \\ \pi^{(ii)}_{2M}(Q_1,e) & if \quad A_{2R}(e) < A_{2M}(e)/2, \\ & \quad 0 \leq A_{2M}(e) < 2\gamma\tau Q_1 \\ \pi^{(iii)}_{2M}(Q_1,e) & if \quad A_{2R}(e) \leq \gamma\tau Q_1, \\ & \quad 2\gamma\tau Q_1 \leq A_{2M}(e) \\ \pi^{(iv)}_{2M}(Q_1,e) & if \quad A_{2M}(e) < A_{2R}(e)/2, \\ & \quad 0 \leq A_{2R}(e) \\ & \quad A_{2R}(e) < 2(1-\gamma)\tau Q_1 \\ \pi^{(v)}_{2M}(Q_1,e) & if \quad 0 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\ & \quad \frac{2A_{2R}(e)-A_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\ & \quad 0 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\ & \quad \frac{2A_{2M}(e)-A_{2R}(e)}{3} < \gamma\tau Q_1 \\ \pi^{(vi)}_{2M}(Q_1,e) & if \quad \gamma\tau Q_1 < A_{2R}(e) \\ & \quad A_{2R}(e) < (2-\gamma)\tau Q_1 \\ & \quad \gamma\tau Q_1 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\ \pi^{(vii)}_{2M}(Q_1,e) & if \quad A_{2M}(e) < (1-\gamma)\tau Q_1, \\ & \quad (2-\gamma)\tau Q_1 < A_{2R}(e) \\ \pi^{(viii)}_{2M}(Q_1,e) & if \quad (1-\gamma)\tau Q_1 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\ & \quad (1-\gamma)\tau Q_1 < A_{2M}(e) \\ & \quad A_{2M}(e) < (1+\gamma)\tau Q_1 \\ \pi^{(ix)}_{2M}(Q_1,e) & if \quad (2-\gamma)\tau Q_1 < A_{2R}(e), \\ & \quad (1+\gamma)\tau Q_1 < A_{2M}(e) \end{cases}$$

where

$$\pi_{2M}^{(i)}(Q_1,e) = Q_1 s\tau\gamma$$
$$\pi_{2M}^{(ii)}(Q_1,e) = \frac{(a_{2M}e-c_{2M}-se)^2}{4b_{2M}e^2} + Q_1 s\tau\gamma$$
$$\pi_{2M}^{(iii)}(Q_1,e) = (a_{2M}e-c_{2M}-se)\gamma\tau Q_1 - b_{2M}(e\gamma\tau Q_1)^2 + Q_1 s\tau\gamma$$
$$\pi_{2M}^{(iv)}(Q_1,e) = Q_1 s\tau\gamma$$
$$\pi_{2M}^{(v)}(Q_1,e) = \frac{2(a_{2M}e-c_{2M}-se)^2}{9b_{2M}e^2} - \frac{(a_{2R}e-c_{2R}-se)(a_{2M}e-c_{2M}-se)}{9b_{2R}e^2} + Q_1 s\tau\gamma$$
$$\pi_{2M}^{(vi)}(Q_1,e) = ((a_{2M}e-c_{2M}-se) - \frac{(a_{2R}e-c_{2R}-se)b_{2M}}{2b_{2R}})\tau\gamma Q_1 - b_{2M}(Q_1 e\tau\gamma)^2 + Q_1 s\tau\gamma$$
$$\pi_{2M}^{(vii)}(Q_1,e) = Q_1 s\tau\gamma$$
$$\pi_{2M}^{(viii)}(Q_1,e) = \frac{(b_{2M}e^2)(A_{2M}(e)-(1-\gamma)\tau Q_1)^2}{4} + Q_1 s\tau\gamma$$
$$\pi_{2M}^{(ix)}(Q_1,e) = (a_{2M}e-c_{2M}-se)\gamma\tau Q_1 - b_{2M}\gamma(e\tau Q_1)^2 + Q_1 s\tau\gamma$$

4.3 Analysis of the OEM's First Period Problem

We can now solve the first period problem where OEM determines Q_1 and e to maximize its total profit over two periods. $\pi_T(Q_1,e)$ stands for total profit function over two periods, which is given by

$$
\pi_T(Q_1,e) = \pi_1(Q_1,e) + \begin{cases}
\pi_{2M}^{(i)}(Q_{2M}(Q_1,e)) & if \quad A_{2M}(e) < 0, A_{2R}(e) < 0 \\
\pi_{2M}^{(ii)}(Q_{2M}(Q_1,e)) & if \quad A_{2R}(e) < A_{2M}(e)/2, \\
 & \quad\;\; 0 \le A_{2M}(e) < 2\gamma\tau Q_1 \\
\pi_{2M}^{(iii)}(Q_{2M}(Q_1,e)) & if \quad A_{2R}(e) \le \gamma\tau Q_1, \\
 & \quad\;\; 2\gamma\tau Q_1 \le A_{2M}(e) \\
\pi_{2M}^{(iv)}(Q_{2M}(Q_1,e)) & if \quad A_{2M}(e) < A_{2R}(e)/2, \\
 & \quad\;\; 0 \le A_{2R}(e) < 2(1-\gamma)\tau Q_1 \\
\pi_{2M}^{(v)}(Q_{2M}(Q_1,e)) & if \quad 0 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
 & \quad\;\; \frac{2A_{2R}(e)-A_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\
 & \quad\;\; 0 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
 & \quad\;\; \frac{2A_{2M}(e)-A_{2R}(e)}{3} < \gamma\tau Q_1 \\
\pi_{2M}^{(vi)}(Q_{2M}(Q_1,e)) & if \quad \gamma\tau Q_1 < A_{2R}(e), \\
 & \quad\;\; A_{2R}(e) < (2-\gamma)\tau Q_1 \\
 & \quad\;\; \gamma\tau Q_1 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
\pi_{2M}^{(vii)}(Q_{2M}(Q_1,e)) & if \quad A_{2M}(e) < (1-\gamma)\tau Q_1, \\
 & \quad\;\; (2-\gamma)\tau Q_1 < A_{2R}(e) \\
\pi_{2M}^{(viii)}(Q_{2M}(Q_1,e)) & if \quad (1-\gamma)\tau Q_1 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
 & \quad\;\; (1-\gamma)\tau Q_1 < A_{2M}(e), \\
 & \quad\;\; A_{2M}(e) < (1+\gamma)\tau Q_1 \\
\pi_{2M}^{(ix)}(Q_{2M}(Q_1,e)) & if \quad (2-\gamma)\tau Q_1 < A_{2R}(e), \\
 & \quad\;\; (1+\gamma)\tau Q_1 < A_{2M}(e)
\end{cases}
$$

$$\pi_1(Q_1,e) = (a_1 - c_1 - b_1 Q_1)Q_1 - ke^2 \tag{4.12}$$

$$Q_1 \ge 0 \tag{4.13}$$

$$0 \le e \le 1 \tag{4.14}$$

Objective function of the first period consists of revenue generated by selling manufactured products, cost of manufacturing and investment cost for the remanufacturability level, e. Constraint 4.13 represent non-negativity of manufacturing in the first period. Constraint 4.14 represents that remanufacturability is modeled as yield rate fraction, it lies in the interval, $[0, 1]$.

For given remanufacturability level, e, positive remanufacturing quantity of IR is possible when OEM's remanufacturing quantity is zero. However, OEM never invests on remanufacturability for that cases. As a result, those cases are not observed since OEM does not benefit from remanufacturability but incurs cost of remanufacturability investment.

We cannot prove the concavity of the total profit function. Hence, we continue our analysis with given remanufacturability level and manufacturing quantity. Then, we implement a solution procedure in order to find optimal solution. The steps of the algorithm is as follows:

1. Equilibrium remanufacturing quantity of OEM, Q_{2M} and remanufacturing quantity of IR, Q_{2R}, are determined by using 4.11 for various levels of remanufacturability, e and manufacturing quantities,Q_1, in the first period.

2. We plug equilibrium Q_{2M} and Q_{2R}, the given manufacturing quantity in the first step, Q_1, the given remanufacturability level used in the first step, e, to the total profit function.

3. We search for the profit maximizing manufacturing quantity and remanufacturability level that maximizes total profit in their range among all alternative Q_1 and e pair.

CHAPTER 5

VALUE OF INFORMATION

We have created and analyzed models without sorting information in the previous chapters and we have assumed OEM is aware of potential entry of a competitor. In this chapter, we create and analyze models with sorting information prior to remanufacturing in order to investigate effect of sorting information to our performance measures. Three alternative models are created with perfect sorting information prior to remanufacturing.

In this chapter, we extend the settings that we previously analyzed in two directions: (i) presence of sorting information and (ii) value of awareness of competition in the second period. By sorting information we mean that the party that has sorting information can perfectly sort returns into remanufacturable and nonremanufacturable items and feed only remanufacturables into the process.

To analyze the effects of sorting information and awareness of competition, we consider the following settings:

1. Monopolistic model with sorting information

2. Duopolistic model with sorting information for only OEM

3. Duopolistic model with sorting information for both OEM and IR

4. Duopolistic model with assumption of monopoly

5.1 Monopolistic Model with Sorting Information

In the monopolistic model with sorting information prior to remanufacturing, OEM knows which returned items are suitable for remanufacturing at the beginning of remanufacturing process and there is no error classifying returned items. As a result, OEM can avoid costs due to yield loss. As a result, a cost is incurred only for successfully remanufactured products by elimination of not remanufacturable items. There is no cost related with sorting operations.

Sequence of events for monopolistic model with sorting information prior to remanufacturing is as follows:

- **Period 1**

  OEM determines manufacturing quantity, $Q_1 \geq 0$, and level of remanufacturability, $0 \leq e \leq 1$, by incurring total manufacturing cost of $c_1 Q_1$ and making an investment of ke^2 for the level of remanufacturability, respectively.

  Unit market price for manufactured items become $P_1(Q_1) = a_1 - b_1 Q_1$. OEM sells all Q_1 units at an unit price of $P_1(Q_1)$.

- **Period 2**

  OEM gets τQ_1 returns with remanufacturability level e.

  OEM determines remanufacturing input quantity Q_{2M} such that $0 \leq Q_{2M} \leq \tau Q_1$ knowing which returned items are remanufacturable, incurring total remanufacturing cost of $c_{2M} e Q_{2M}$ and obtains unit recycling revenue of s for returns that are not remanufactured, $(\tau Q_1 - Q_{2M})$, and items lost in remanufacturing process, $(1-e)Q_{2M}$.

  The market price becomes $P_{2M}(Q_{2M}) = a_{2M} - b_{2M} e Q_{2M}$. OEM sells all Q_{2M} at an unit price of $P_{2M}(Q_{2M})$.

Profit of OEM in the first and the second periods are represented by $\pi_1(Q_1, e)$ and $\pi_{2M}(Q_{2M}|Q_1, e)$, respectively.

In order to solve the problem of OEM, we use backward induction. First, for the second period problem, we characterize the optimal Q_{2M} for given values of $Q_1 \geq 0$

and $0 \leq e \leq 1$. Then, using the optimal solutions obtained, the first period problem where the sum of the first period & the second period profits under the optimal Q_{2M} decision is maximized. The analysis for the second and the first period problems are given in Section 5.1.1 and 5.1.2, respectively.

5.1.1 Analysis of OEM's Second Period Problem

OEM's second period problem can be stated as follows:

$$\text{maximize} \quad \pi_{2M}(Q_{2M}|Q_1, e) = (a_{2M} - b_{2M}eQ_{2M})eQ_{2M} - c_{2M}eQ_{2M} + s(Q_1\tau - Q_{2M}e)$$

subject to

$$Q_{2M} \leq Q_1\tau \tag{5.1}$$

$$Q_{2M} \geq 0 \tag{5.2}$$

The objective function is composed of revenue generated by selling remanufactured products, cost of remanufacturing and revenue generated by the returned items that are not successfully remanufactured and not remanufactured. Constraint 5.2 represent non-negativity of remanufacturing quantity in the second period. Constraint 5.1 represents that maximum τ fraction of the first period manufacturing quantity can be remanufactured in the second period. Right hand side of 5.1 is dependent on manufacturing quantity in the first period.

Lemma 5.1.1. *$\pi_{2M}(Q_{2M})$ is a concave function of Q_{2M} .*

Proof. The second derivative with respect to Q_{2M} is as follows:

$$\frac{d^2\pi_{2M}(Q_{2M})}{dQ_{2M}^2} = -2b_{2M}e^2 \quad .$$

Since $b_{2M} > 0$, $-2b_{2M}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2M}(Q_{2M})$ is concave in Q_{2M}. □

Theorem 5.1.2. *If for a given value of $0 \leq e \leq 1$, $a_{2M} - c_{2M} - s < 0$, then remanufacturing is not profitable for all given values of $Q_1 > 0$ in the second period, i.e., $Q_{2M}^* = 0$ under the optimal solution. Otherwise, for given $Q_1 \geq 0$ and $0 \leq e \leq 1$*

values, the optimal input quantity to the remanufacturing process, Q^*_{2M}, in the second period is

$$Q^*_{2M}(Q_1, e) = \begin{cases} \frac{a_{2M}-c_{2M}-s}{2b_{2M}e} & if \quad 0 \leq \frac{(a_{2M}-c_{2M}-s)}{2\tau b_{2M}e} \leq Q_1 \\ Q_1\tau & if \quad Q_1 < \frac{(a_{2M}-c_{2M}-s)}{2\tau b_{2M}e} \end{cases} . \tag{5.3}$$

Proof. The first derivative of $\pi_{2M}(Q_{2M})$ with respect to Q_{2M} is

$$\frac{d\pi_{2M}(Q_{2M})}{dQ_{2M}} = a_{2M}e - 2b_{2M}e^2Q_{2M} - c_{2M}e - se \quad . \tag{5.4}$$

If $a_{2M} - c_{2M} - s < 0$, $\frac{d\pi_{2M}(Q_{2M})}{dQ_{2M}} < 0$ for all possible values of $Q_{2M} \geq 0$. Hence, the second period profit function is decreasing in Q_{2M} and the $Q_{2M} = 0$ maximizes $\pi_{2M}(Q_{2M})$.

Otherwise, since $\pi_{2M}(Q_{2M})$ is a concave function of Q_{2M}, if the unconstrained solution is feasible with respect to constraint 5.1, it is optimal. Otherwise, under optimal solution, constraint 5.1 is binding. Unconstrained solution can be found by setting 5.4 to zero. Equation 5.3 provides the result.

$$Q_{2M} = \frac{a_{2M}e - c_{2M} - se}{2b_{2M}e^2} \tag{5.5}$$

□

When $a_{2M}-c_{2M}-s < 0$ and the sorting information is available for OEM, OEM does not make remanufacturing since profit margin is negative for any given Q_1. However, the cases that does not satisfy these condition is out of our scope since not satisfying the condition implies that the supply chain consists of only forward chain activities. When the condition is satisfied remanufacturability level is always positive.

Following the result provided in Theorem 5.1.2, profit in the second period under the optimal solution can be characterized.

If $a_{2M} - c_{2M} - s < 0$, then the second period's profit only consists of recycling revenue as below.

$$\pi^*_{2M}(Q_1, e) = s\tau Q_1$$

When the optimal solution characterized in the theorem 5.3 is plugged to the second period problem into the profit expression, we obtain the second period profit as a function of Q_1 and e is as follows:

$$\pi^*_{2M}(Q_1,e) \;=\; \begin{cases} \frac{(a_{2M}-c_{2M}-s)^2}{4b_{2M}} + s\tau Q_1 & if \quad 0 \leq \frac{(a_{2M}-c_{2M}-s)}{2b_{2M}e} \leq Q_1\tau \\ (a_{2M}-c_{2M}-s)\tau e Q_1 & if \quad Q_1\tau < \frac{(a_{2M}-c_{2M}-s)}{2b_{2M}e} \\ -b_{2M}(\tau e Q_1)^2 + s\tau Q_1 & \end{cases} \tag{5.6}$$

5.1.2 Analysis of OEM's First Period Problem

We can now solve the first period problem where OEM determines Q_1 and e to maximize its total profit over two periods. Let $\pi_T(Q_1,e)$ stands for total profit function over two periods. In order to find Q_1 and e that maximizes total profit, joint concavity of it should be used.

For the sake of simplicity, let $B_{2M}(e) = \frac{(a_{2M}-c_{2M}-s)}{2b_{2M}e}$.

$$\pi_T(Q_1,e) = \pi_1(Q_1,e) + \begin{cases} \pi^i_{2M}(Q_1,e) & if \leq B_{2M}(e) \leq 0 \\ \pi^{ii}_{2M}(Q_1,e) & if \quad 0 \leq B_{2M}(e) \leq Q_1\tau \\ \pi^{iii}_{2M}(Q_1,e) & if \quad Q_1\tau \leq B_{2M}(e) \end{cases}$$

where

$$\pi^{(i)}_{2M}(Q_1,e) = s\tau Q_1$$

$$\pi^{(ii)}_{2M}(Q_1,e) = s\tau Q_1 + \frac{(a_{2M}-c_{2M}-s)^2}{4b_{2M}}$$

$$\pi^{(iii)}_{2M}(Q_1,e) = s\tau Q_1 + (a_{2M}-c_{2M}-s)\tau e Q_1 - b_{2M}(\tau e Q_1)^2$$

OEM's first period problem can be expressed as

$$\text{maximize} \quad \pi_T(Q_1,e) = \pi_1(Q_1,e) + \pi^*_{2M}(Q_1,e)$$

where

$$\pi_1(Q_1,e) = (a_1 - c_1 - b_1Q_1)Q_1 - ke^2 \tag{5.7}$$

subject to

$$Q_1 \geq 0 \tag{5.8}$$

$$0 \leq e \leq 1 \tag{5.9}$$

Objective function of the first period consists of revenue generated by selling manufactured products, cost of manufacturing and investment cost for the remanufacturability level, e. Constraint 5.8 represent non-negativity of manufacturing in the first period. Constraint 5.9 represents that remanufacturability is modeled as yield rate fraction, it lies in the interval, $[0, 1]$.

We cannot show the joint concavity of the total profit function for Q_1 and e. Hence, we implement an algorithm to find the optimal solution.

Below, we characterize optimal Q_1 for given e

Proposition 3. *Given $a_{2M} - c_{2M} - s < 0$, optimal manufacturing quantity in the first period, Q_1^*, is*

$$Q_1^* = \frac{a_1 - c_1 + s\tau}{2b_1} \quad . \tag{5.10}$$

Proof. Profit function given $\frac{(a_{2M}-c_{2M}-s)}{2b_{2M}e} < 0$

$$\pi_T^{(i)}(Q_1|e) = (a_1 - c_1 - b_1Q_1 + s\tau)Q_1 - ke^2$$

The second derivative of $\pi_T^{(i)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(i)}(Q_1|e)}{\partial Q_1^2} = -2b_1$$

Since $b_1 > 0$ is always positive, $\frac{\partial^2 \pi_T^{(i)}(Q_1|e)}{\partial Q_1^2} < 0$. Therefore, $\pi_T^{(i)}(Q_1|e)$ is concave in Q_1.

The first derivative of $\pi_T^{(i)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(i)}(Q_1|e)}{\partial Q_1} = a_1 - 2b_1Q_1 - c_1 + s\tau \tag{5.11}$$

Optimal solution given in equation 5.10 is found by setting 5.11 to zero. □

Proposition 4. *Given $\frac{(a_{2M}-c_{2M}-s)}{2b_{2M}e} \geq 0$, optimal manufacturing quantity in the first period, Q_1^*, is*

$$Q_1^* = \frac{(a_1 - c_1 + s\tau) + \tau e(a_{2M} - c_{2M} - s)}{2b_1 + 2b_{2M}e^2} \tag{5.12}$$

Proof. Profit function given $0 \leq B_{2M}(e) \leq \tau Q_1$

$$\pi_T^{(ii)}(Q_1|e) = (a_1 - c_1 - b_1Q_1 + s\tau)Q_1 - ke^2$$

The second derivative of $\pi_T^{(ii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(ii)}(Q_1|e)}{\partial Q_1^2} = -2b_1$$

Since $b_1 > 0$, $\frac{\partial^2 \pi_T^{(ii)}(Q_1|e)}{\partial Q_1^2} < 0$. Therefore, $\pi_T^{(ii)}(Q_1|e)$ is concave in Q_1.

The first derivative o f$\pi_T^{(ii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(ii)}(Q_1|e)}{\partial Q_1} = a_1 - 2b_1 Q_1 - c_1 + s\tau \tag{5.13}$$

Optimal solution can be found by setting 5.13 to zero given $0 \leq B_{2M}(e) \leq \tau Q_1$.

The optimal solution of $\pi_T(e|Q_1)$ in the region $0 \leq B_{2M}(e)/2 \leq \tau Q_1$ is never feasible. Optimal e and Q_1 cannot be in the region $0 \leq B_{2M}(e) \leq \tau Q_1$. If the remanufacturing is profitable ($B_{2M}(e) \geq 0$), the constraint 5.1 is always binding. The second derivative of $\pi_T(e|Q_1)$ with respect to e is

$$\frac{d\pi_T(e|Q_1)}{de} = -2ke \quad .$$

Since $k > 0$ and e is defined in $0 \leq e \leq 1$. Therefore, $\pi_T(e|Q_1)$ is concave in e.

The first derivative of $\pi_T(e|Q_1)$ with respect to e is

$$\frac{d\pi_{2M}(Q_{2M})}{de} = -2ke \quad .$$

$e = 0$ is the optimal remanufacturability level for that part of total profit function. Left hand side of the following constraint becomes infinity for $e = 0$, $\frac{(a_{2M} - c_{2M} - s)}{2b_{2M}e} \leq Q_1 \tau$ is never satisfied.

Profit function given $\tau \frac{a_1 - c_1 + s\tau}{2b_1} \leq B_{2M}(e)$

$\pi_T^{(iii)}(Q_1|e) = (a_1 - c_1 - b_1 Q_1 + s\tau)Q_1 - ke^2 + (a_{2M} - c_{2M} - s)\tau e Q_1 - b_{2M}(\tau e Q_1)^2$

The second derivative of $\pi_T^{(iii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial^2 \pi_T^{(iii)}(Q_1|e)}{\partial Q_1^2} = -2b_1 - 2b_{2M}(e\tau)^2$$

Since $b_1 > 0$, $b_{2M} > 0$, $\tau > 0$ and e is defined in $0 \leq e \leq 1$. Therefore, $\pi_T^{(iii)}(Q_1|e)$ is concave in Q_1.

The first derivative $\pi_T^{(iii)}(Q_1|e)$ with respect to Q_1 is as follows:

$$\frac{\partial \pi_T^{(iii)}(Q_1|e)}{\partial Q_1} = a_1 - c_1 + s\tau + \tau e(a_{2M} - c_{2M} - s) - 2b_1Q_1 - 2b_{2M}Q_1(e\tau)^2 \quad (5.14)$$

Optimal solution can be found by setting 5.14 to zero given $\tau Q_1 \leq B_{2M}(e)$. □

As a result of Theorem 5.1.2 and Proposition 4, optimal solution to overall problem for a given $0 \leq e \leq 1$ is as follows:

$$(Q_1^*, Q_{2M}^*) = \begin{cases} (\frac{a_1 - c_1 + s\tau}{2b_1Q_1}, 0) & if \quad B_{2M}(e) < 0 \\ (Q_1^{'}, \tau Q_1^{'}) & if \quad \tau B_{2M}(e) \geq 0 \end{cases}$$

Where $Q_1^{'} = \frac{(a_1 - c_1 + s\tau) + \tau(a_{2M}e - c_{2M} - se)}{2b_1 + 2b_{2M}e^2}$

The steps of the solution procedure in order to find optimal level of remanufacturability, manufacturing quantity in the first period and remanufacturing quantity in the second period are as follows:

1. Optimal manufacturing quantity, Q_1, and remanufacturing quantity, Q_{2M}, are determined using 5.15 that maximizes total profit for increasing remanufacturability, e.
2. We plug optimal Q_1, Q_{2M} and the remanufacturability level used in the first step, e, to the total profit function.
3. We search for the optimal remanufacturability level that maximizes total profit in its range.

5.2 Duopolistic Model with Sorting Information for only OEM

In the duopolistic model with sorting information prior to remanufacturing for only OEM, OEM benefits from being the manufacturer. Only OEM knows which specific returned items are suitable for remanufacturing at the beginning of remanufacturing process and benefits from sorting information in order to decrease its remanufacturing cost. OEM's cost to supply eQ_{2M} is $c_{2M}eQ_{2M}$. Total remanufacturing cost saving of OEM is equal to $c_{2M}(1-e)Q_{2M}$. On the other way, IR's cost to supply eQ_{2R} is $c_{2R}Q_{2R}$. IR makes loss of money trying to remanufacture not suitable returned items.

Yield rate is independent of sorting information. Yield rate of both OEM and IR is equal to remanufacturability level, e. When OEM (IR) begins remanufacturing with Q_{2M} (Q_{2R}), total remanufacturing output is equal to eQ_{2M} (eQ_{2R}).

There is not any cost related with sorting process for OEM.

Sequence of events for duopolistic model with sorting information prior to remanufacturing for only OEM is as follows:

- **Period 1**

  OEM determines manufacturing quantity, $Q_1 \geq 0$, and the level of remanufacturability, $0 \leq e \leq 1$, by incurring a total manufacturing cost of c_1Q_1 and making an investment of ke^2 for level of remanufacturability, respectively.

  The market price for manufactured items become $P_1(Q_1) = a_1 - b_1Q_1$. OEM sells all Q_1 units at a price of $P_1(Q_1)$.

- **Period 2**

  OEM and IR get $\gamma\tau Q_1$ and $(1-\gamma)\tau Q_1$ returns with remanufacturability level e, respectively.

  OEM and IR simultaneously determine remanufacturing input quantities Q_{2M} and Q_{2R} such that $0 \leq Q_{2M} \leq \gamma\tau Q_1$ and $0 \leq Q_{2M} \leq (1-\gamma)\tau Q_1$, incurring total remanufacturing cost of $c_{2M}eQ_{2M}$ and $c_{2R}Q_{2R}$ and obtain recycling revenue of s for returns that are not remanufactured, $(\gamma\tau Q_1 - Q_{2M})$ and $((1-\gamma)\tau Q_1 - Q_{2R})$, and items lost in remanufacturing process, $(1-e)Q_{2M}$ and $(1-e)Q_{2R}$, respectively.

  The market price becomes $P_{2M}(Q_{2M}, Q_{2R}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of OEM. OEM sells all Q_{2M} at an unit price of $P_{2M}(Q_{2M}, Q_{2R})$.

  The market price becomes $P_{2R}(Q_{2R}, Q_{2M}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of IR. IR sells all Q_{2R} at an unit price of $P_{2R}(Q_{2R}, Q_{2M})$.

Profit of OEM in the first and the second periods are represented by $\pi_1(Q_1, e)$ and $\pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e)$ respectively. Profit of IR in the second periods are represented by $\pi_{2R}(Q_{2R}, Q_{2M}|Q_1, e)$ respectively.

In order to solve the problem of OEM and IR, we use backward induction. First, for the second period problem of two firms, we characterize the equilibrium Q_{2M} and Q_{2R} for given values of $Q_1 \geq 0$ and $0 \leq e \leq 1$. Then, using the equilibrium solution obtained, the first period problem where the sum of the first period and the second period profits of OEM under the equilibrium Q_{2M} and Q_{2R} is maximized. The analysis for the second and the first period problems are given in 5.2.1 and 5.2.2, respectively.

5.2.1 Analysis of the Second Period Problem

OEM's second period problem can be stated as follows:

$$\underset{Q_{2M}}{\text{maximize}} \quad \pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e) \quad = \quad \begin{aligned} &(a_{2M} - b_{2M}e(Q_{2M} + Q_{2R}))eQ_{2M} \\ &-c_{2M}eQ_{2M} + s(\gamma\tau Q_1 - Q_{2M}e) \end{aligned}$$

subject to

$$Q_{2M} \leq \tau\gamma Q_1 \tag{5.15}$$

$$Q_{2M} \geq 0 \tag{5.16}$$

IR's second period problem can be stated as follows:

$$\underset{Q_{2R}}{\text{maximize}} \quad \pi_{2R}(Q_{2M}, Q_{2R}|Q_1, e) \quad = \quad \begin{aligned} &(a_{2R} - b_{2R}e(Q_{2R} + Q_{2M}))eQ_{2R} \\ &-c_{2R}Q_{2R} + s((1-\gamma)\tau Q_1 - Q_{2R}e) \end{aligned}$$

subject to

$$Q_{2R} \leq (1-\gamma)\tau Q_1 \tag{5.17}$$

$$Q_{2R} \geq 0 \tag{5.18}$$

Objective functions are composed of revenue generated by selling remanufactured products, cost of remanufacturing and revenue generated by the returned items that are not successfully remanufactured and not remanufactured. Constraints 5.16 and 5.18 represent non-negativity of remanufacturing quantity of OEM and IR, respectively, in the second period. Constraints 5.15 and 5.17 represent that maximum $\gamma\tau$

and $(1-\gamma)\tau$ fractions of the first period manufacturing quantity can be remanufactured by OEM and IR, respectively, in the second period. Right hand sides of 5.15 and 5.17 are dependent on manufacturing quantity.

For the sake of brevity, let $C_{2M}(e) = \frac{(a_{2M}-c_{2M}-s)}{b_{2M}e}$ and $C_{2R}(e) = \frac{(a_{2R}e-c_{2R}-se)}{b_{2R}e^2}$.

Lemma 5.2.1. *$\pi_{2M}(Q_{2M}, Q_{2R})$ is a concave function of Q_{2M} for given Q_{2R}.*

Proof. The second derivative with respect to Q_{2M} is as follows:

$$\frac{d^2\pi_{2M}(Q_{2M}, Q_{2R})}{dQ_{2M}^2} = -2b_{2M}e^2 \quad .$$

Since $b_{2M} > 0$, $-2b_{2M}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2M}(Q_{2M}, Q_{2R})$ is concave in Q_{2M} for given Q_{2R}. □

Theorem 5.2.2. *If for a given value of $0 \leq e \leq 1$, $\frac{a_{2M}-c_{2M}-s}{2b_{2M}e} - \frac{Q_{2R}}{2} < 0$, then remanufacturing is not profitable for all given values of $Q_1 > 0$ in the second period, i.e., $Q_{2M} = 0$. Otherwise, for given $Q_1 \geq 0$ and $0 \leq e \leq 1$ values, the best response function of OEM in the second period is*

$$Q_{2M}(Q_{2R}|Q_1, e) \quad = \quad \begin{cases} \frac{a_{2M}-c_{2M}-s}{2b_{2M}e} - \frac{Q_{2R}}{2} & if \quad \frac{C_{2M}(e)-Q_{2R}}{2\tau\gamma} \leq Q_1 \\ \tau\gamma Q_1 & if \quad Q_1 \leq \frac{C_{2M}(e)-Q_{2R}}{2\tau\gamma} \end{cases} \tag{5.19}$$

Proof. The first derivative of $\pi_{2M}(Q_{2M}, Q_{2R})$ with respect to Q_{2M} is

$$\frac{d\pi_{2M}(Q_{2M}, Q_{2R})}{dQ_{2M}} = a_{2M}e - 2b_{2M}e^2Q_{2M} - c_{2M}e - se \quad . \tag{5.20}$$

If $\frac{a_{2M}-c_{2M}-s}{2b_{2M}e} - \frac{Q_{2R}}{2} < 0$, $\frac{d\pi_{2M}(Q_{2M},Q_{2R})}{dQ_{2M}} < 0$ for all possible values of $Q_{2M} \geq 0$. Hence, the second period profit function is decreasing in Q_{2M} and the $Q_{2M} = 0$ maximizes $\pi_{2M}(Q_{2M}, Q_{2R})$.

Otherwise, since $\pi_{2M}(Q_{2M}, Q_{2R})$ is a concave function of Q_{2M}, if the unconstrained solution is feasible with respect to constraint 5.15, it is the best response of OEM. Otherwise, under the equilibrium solution, constraint 5.15 is binding. Unconstrained solution can be found by setting 5.20 to zero. Equation 5.21 provides the result.

$$Q_{2M} = \frac{a_{2M} - c_{2M} - s}{2b_{2M}e} - \frac{Q_{2R}}{2} \tag{5.21}$$

□

Lemma 5.2.3. *$\pi_{2R}(Q_{2R}, Q_{2M})$ is a concave function of Q_{2R} for given Q_{2M}.*

Proof. The second derivative with respect to Q_{2R} is as follows:

$$\frac{d^2\pi_{2R}(Q_{2R}, Q_{2M})}{dQ_{2R}^2} = -2b_{2R}e^2 \quad .$$

Since $b_{2R} > 0$ and $-2b_{2R}e^2 \leq 0$ for $0 \leq e \leq 1$. Therefore, $\pi_{2R}(Q_{2R}, Q_{2M})$ is concave in Q_{2R}. □

Theorem 5.2.4. *If for a given value of $0 \leq e \leq 1$, $\frac{a_{2R}e-c_{2R}-se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} < 0$, then remanufacturing is not profitable for all given values of $Q_1 > 0$ in the second period, i.e., $Q_{2R}^* = 0$. Otherwise, for given $Q_1 \geq 0$ and $0 \leq e \leq 1$ values, the equilibrium input quantity to the remanufacturing process, Q_{2R}^*, in the second period is*

$$Q_{2R}^*(Q_{2M}|Q_1, e) = \begin{cases} \frac{a_{2R}e-c_{2R}-se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} & if \quad \frac{C_{2R}(e)-Q_{2M}}{2\tau(1-\gamma)} \leq Q_1 \\ \tau(1-\gamma)Q_1 & if \quad Q_1 \leq \frac{C_{2R}(e)-Q_{2M}}{2\tau(1-\gamma)} \end{cases} \tag{5.22}$$

Proof. The first derivative of $\pi_{2R}(Q_{2R}, Q_{2M})$ with respect to Q_{2R} is

$$\frac{d\pi_{2R}(Q_{2R}, Q_{2M})}{dQ_{2R}} = a_{2R}e - 2b_{2R}e^2Q_{2R} - c_{2R} - se \quad . \tag{5.23}$$

If $\frac{a_{2R}e-c_{2R}-se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} < 0$, $\frac{d\pi_{2R}(Q_{2R},Q_{2M})}{dQ_{2R}} < 0$ for all possible values of $Q_{2R} \geq 0$. Hence, the second period profit function is decreasing in Q_{2R} and the $Q_{2R} = 0$ maximizes $\pi_{2R}(Q_{2R}, Q_{2M})$.

Otherwise, since $\pi_{2R}(Q_{2R}, Q_{2M})$ is a concave function of Q_{2R}, if the unconstrained solution is feasible with respect to constraint 5.17, it is the best response of OEM. Otherwise, under the equilibrium solution, constraint 5.17 is binding. Unconstrained solution can be found by setting 5.23 to zero. Equation 5.24 provides the result.

$$Q_{2R} = \frac{a_{2R}e - c_{2R} - se}{2b_{2R}e^2} - \frac{Q_{2M}}{2} \tag{5.24}$$

□

We plug best response functions of OEM and IR into each other and obtain equilbirum outcome. Equilibrium outcome for the second period game is as follows:

$$
(\mathrm{Q}^*_{2M}, Q^*_{2R}) = \begin{cases}
(0,0) & if \quad C_{2M}(e) < 0, C_{2R}(e) < 0 \\
(\frac{C_{2M}(e)}{2}, 0) & if \quad C_{2R}(e) < C_{2M}(e)/2, \\
 & \quad 0 \leq C_{2M}(e) < 2\gamma\tau Q_1 \\
(\gamma\tau Q_1, 0) & if \quad C_{2R}(e) \leq \gamma\tau Q_1, \\
\quad 2\gamma\tau Q_1 \leq C_{2M}(e) & \\
(0, \frac{C_{2R}(e)}{2}) & if \quad C_{2M}(e) < C_{2R}(e)/2, \\
 & \quad 0 \leq C_{2R}(e) < 2(1-\gamma)\tau Q_1 \\
\frac{2C_{2M}(e)-C_{2R}(e)}{3}, \frac{2C_{2R}(e)-C_{2M}(e)}{3} & if \quad 0 < \frac{2C_{2R}(e)-C_{2M}(e)}{3}, \\
 & \quad \frac{2C_{2R}(e)-C_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\
 & \quad 0 < \frac{2C_{2M}(e)-C_{2R}(e)}{3} \\
 & \quad \frac{2C_{2M}(e)-C_{2R}(e)}{3} < \gamma\tau Q_1 \\
(\gamma\tau Q_1, \frac{C_{2R}(e)-\gamma\tau Q_1}{2}) & if \quad \gamma\tau Q_1 < C_{2R}(e) < (2-\gamma)\tau Q_1, \\
 & \quad C_{2R}(e) < (2-\gamma)\tau Q_1, \\
 & \quad \gamma\tau Q_1 < \frac{2C_{2M}(e)-C_{2R}(e)}{3} \\
(0, (1-\gamma)\tau Q_1) & if \quad C_{2M}(e) < (1-\gamma)\tau Q_1, \\
 & \quad (2-\gamma)\tau Q_1 < C_{2R}(e) \\
(\frac{C_{2M}(e)-(1-\gamma)\tau Q_1}{2}, (1-\gamma)\tau Q_1) & if \quad (1-\gamma)\tau Q_1 < \frac{2C_{2R}(e)-C_{2M}(e)}{3}, \\
 & \quad (1-\gamma)\tau Q_1 < C_{2M}(e) \\
 & \quad C_{2M}(e) < (1+\gamma)\tau Q_1 \\
(\gamma\tau Q_1, (1-\gamma)\tau Q_1) & if \quad (2-\gamma)\tau Q_1 < C_{2R}(e), \\
 & \quad (1+\gamma)\tau Q_1 < C_{2M}(e)
\end{cases}
$$

OEM's second period profit under the equilibrium solution is

$$\pi^*_{2M}(Q_1,e) = \begin{cases} \pi^{(i)}_{2M}(Q_1,e) & if \quad C_{2M}(e)<0, C_{2R}(e)<0 \\ \pi^{(ii)}_{2M}(Q_1,e) & if \quad C_{2R}(e)<C_{2M}(e)/2, \\ & \quad 0\le C_{2M}(e)<2\gamma\tau Q_1 \\ \pi^{(iii)}_{2M}(Q_1,e) & if \quad C_{2R}(e)\le \gamma\tau Q_1, 2\gamma\tau Q_1 \le C_{2M}(e) \\ \pi^{(iv)}_{2M}(Q_1,e) & if \quad C_{2M}(e)<C_{2R}(e)/2, \\ & \quad 0\le C_{2R}(e)<2(1-\gamma)\tau Q_1 \\ \pi^{(v)}_{2M}(Q_1,e) & if \quad 0<\frac{2C_{2R}(e)-C_{2M}(e)}{3}<(1-\gamma)\tau Q_1, \\ & \quad 0<\frac{2C_{2M}(e)-C_{2R}(e)}{3}<\gamma\tau Q_1 \\ \pi^{(vi)}_{2M}(Q_1,e) & if \quad \gamma\tau Q_1<C_{2R}(e)<(2-\gamma)\tau Q_1 \\ & \quad \gamma\tau Q_1<\frac{2C_{2M}(e)-C_{2R}(e)}{3} \\ \pi^{(vii)}_{2M}(Q_1,e) & if \quad C_{2M}(e)<(1-\gamma)\tau Q_1, \\ & \quad (2-\gamma)\tau Q_1<C_{2R}(e) \\ \pi^{(viii)}_{2M}(Q_1,e) & if \quad (1-\gamma)\tau Q_1<\frac{2C_{2R}(e)-C_{2M}(e)}{3}, \\ & \quad (1-\gamma)\tau Q_1<C_{2M}(e)<(1+\gamma)\tau Q_1 \\ \pi^{(ix)}_{2M}(Q_1,e) & if \quad (2-\gamma)\tau Q_1<C_{2R}(e), \\ & \quad (1+\gamma)\tau Q_1<C_{2M}(e) \end{cases}$$

where

$\pi^{(i)}_{2M}(Q_1,e)=Q_1 s\tau\gamma$
$\pi^{(ii)}_{2M}(Q_1,e)=\frac{(a_{2M}-c_{2M}-s)^2}{4b_{2M}}+s\tau\gamma Q_1$
$\pi^{(iii)}_{2M}(Q_1,e)=(a_{2M}-c_{2M}-s)\gamma\tau e Q_1-b_{2M}(e\gamma\tau Q_1)^2+s\tau\gamma Q_1$
$\pi^{(iv)}_{2M}(Q_1,e)=Q_1 s\tau\gamma$
$\pi^{(v)}_{2M}(Q_1,e)=b_{2M}e^2(\frac{2(a_{2M}-c_{2M}-s)}{3b_{2M}e}-\frac{(a_{2R}e-c_{2R}-se)}{3b_{2R}e^2})^2+s\tau\gamma Q_1$
$\pi^{(vi)}_{2M}(Q_1,e)=((a_{2M}-c_{2M}-s)-\frac{(a_{2R}-c_{2R}-s)b_{2M}}{2b_{2R}})\tau\gamma e Q_1-\frac{(Q_1 e\tau\gamma)^2}{2}+s\tau\gamma Q_1$
$\pi^{(vii)}_{2M}(Q_1,e)=Q_1 s\tau\gamma$
$\pi^{(viii)}_{2M}(Q_1,e)=\frac{(b_{2M}e^2)(C_{2M}(e)-(1-\gamma)\tau Q_1)^2}{4}+s\tau\gamma Q_1$
$\pi^{(ix)}_{2M}(Q_1,e)=(a_{2M}-c_{2M}-s)\gamma\tau e Q_1-b_{2M}\gamma(e\tau Q_1)^2+s\tau\gamma Q_1$

5.2.2 Analysis of the OEM's First Period Problem

We can now solve the first period problem where OEM determines Q_1 and e to maximize its total profit over two periods. $\pi_T(Q_1,e)$ stands for total profit function over two periods. In order to find Q_1 and e that maximizes total profit, joint concavity of it should be used.

$$\pi_T(Q_1,e) = \pi_1(Q_1,e) + \begin{cases} \pi_{2M}^{(i)}(Q_{2M}(Q_1,e)) & if \quad C_{2M}(e) < 0, C_{2R}(e) < 0 \\ \pi_{2M}^{(ii)}(Q_{2M}(Q_1,e)) & if \quad C_{2R}(e) < C_{2M}(e)/2, \\ & \quad 0 \le C_{2M}(e) < 2\gamma\tau Q_1 \\ \pi_{2M}^{(iii)}(Q_{2M}(Q_1,e)) & if \quad C_{2R}(e) \le \gamma\tau Q_1, \\ & \quad 2\gamma\tau Q_1 \le C_{2M}(e) \\ \pi_{2M}^{(iv)}(Q_{2M}(Q_1,e)) & if \quad C_{2M}(e) < C_{2R}(e)/2, \\ & \quad 0 \le C_{2R}(e) < 2(1-\gamma)\tau Q_1 \\ \pi_{2M}^{(v)}(Q_{2M}(Q_1,e)) & if \quad 0 < \frac{2C_{2R}(e)-C_{2M}(e)}{3}, \\ & \quad \frac{2C_{2R}(e)-C_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\ & \quad 0 < \frac{2C_{2M}(e)-C_{2R}(e)}{3} \\ & \quad \frac{2C_{2M}(e)-C_{2R}(e)}{3} < \gamma\tau Q_1 \\ \pi_{2M}^{(vi)}(Q_{2M}(Q_1,e)) & if \quad \gamma\tau Q_1 < C_{2R}(e) < (2-\gamma)\tau Q_1 \\ & \quad \gamma\tau Q_1 < \frac{2C_{2M}(e)-C_{2R}(e)}{3} \\ \pi_{2M}^{(vii)}(Q_{2M}(Q_1,e)) & if \quad C_{2M}(e) < (1-\gamma)\tau Q_1, \\ & \quad (2-\gamma)\tau Q_1 < C_{2R}(e) \\ \pi_{2M}^{(viii)}(Q_{2M}(Q_1,e)) & if \quad (1-\gamma)\tau Q_1 < \frac{2C_{2R}(e)-C_{2M}(e)}{3}, \\ & \quad (1-\gamma)\tau Q_1 < C_{2M}(e), \\ & \quad C_{2M}(e) < (1+\gamma)\tau Q_1 \\ \pi_{2M}^{(ix)}(Q_{2M}(Q_1,e)) & if \quad (2-\gamma)\tau Q_1 < C_{2R}(e), \\ & \quad (1+\gamma)\tau Q_1 < C_{2M}(e) \end{cases}$$

where

$$\pi_1(Q_1,e) = (a_1 - c_1 - b_1 Q_1)Q_1 - ke^2 \tag{5.25}$$

subject to

$$Q_1 \ge 0 \tag{5.26}$$

$$0 \le e \le 1 \tag{5.27}$$

Objective function of the first period consists of revenue generated by selling manufactured products, cost of manufacturing and investment cost for the remanufacturability level, e. Constraint 5.26 represent non-negativity of manufacturing in the first period. Constraint 5.27 represents that remanufacturability is modeled as yield rate fraction, it lies in the interval, $[0, 1]$.

We cannot prove the concavity of the total profit function. Hence, we continue our analysis with given remanufacturability level and manufacturing quantity. Then, we implement a solution procedure in order to find optimal remanufacturability level. The steps of the solution procedure is as follows:

1. Equilibrium remanufacturing quantity of OEM, Q_{2M} and remanufacturing quantity of IR, Q_{2R}, are determined using 5.25 for various levels of remanufacturability, e and manufacturing quantities,Q_1, in the first period.

2. We plug equilibrium Q_{2M} and Q_{2R}, the manufacturing quantity in the first step, Q_1,the remanufacturability level used in the first step, e, to the total profit function.

3. We search for the profit maximizing manufacturing quantity and remanufacturability level in their range among all alternative Q_1 and e pair..

5.3 Duopolistic Model with Sorting Information for OEM and IR

In the duopolistic model with sorting information prior to remanufacturing for both OEM and IR, both OEM and IR can differentiate remanufacturable returned items at the beginning of remanufacturing process and can eliminate not remanufacturable ones. Both firms face remanufacturing cost for only successfully remanufactured ones. OEM start remanufacturing process with $Q_{2R}(Q_{2M})$ to produce $eQ_{2R}(eQ_{2M})$ units, cost incurred due to remanufacturing is $c_{2R}eQ_{2R}(c_{2R}eQ_{2R})$.

There is not any cost related with sorting process for OEM.

Profit of OEM in the first period is represented with $\pi_1(Q_1, e)$, profit of OEM in the second period is represented with $\pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e)$ and profit of IR in the second period is represented with $\pi_{2R}(Q_{2R}, Q_{2M}|Q_1, e)$.

For the sake of simplicity, let $D_{2M}(e) = \frac{(a_{2M}-c_{2M}-s)}{b_{2M}e}$ and $D_{2R}(e) = \frac{(a_{2R}-c_{2R}-s)}{b_{2R}e}$.

Solution procedure of duopolistic model with sorting information for both OEM and IR is as same as duopolistic model with sorting information for only OEM.

We plug best response functions of OEM and IR into each other and obtain equilibrium outcome. Equilibrium outcome for the second period game is as follows:

$$(Q_{2M}^*, Q_{2R}^*) = \begin{cases} (0,0) & if \quad D_{2M}(e) < 0, D_{2R}(e) < 0 \\ (\frac{D_{2M}(e)}{2}, 0) & if \quad D_{2R}(e) < D_{2M}(e)/2, \\ & \quad 0 \le D_{2M}(e) < 2\gamma\tau Q_1 \\ (\gamma\tau Q_1, 0) & if \quad D_{2R}(e) \le \gamma\tau Q_1, \\ & \quad 2\gamma\tau Q_1 \le D_{2M}(e) \\ (0, \frac{D_{2R}(e)}{2}) & if \quad D_{2M}(e) < D_{2R}(e)/2, \\ & \quad 0 \le D_{2R}(e) < 2(1-\gamma)\tau Q_1 \\ (\frac{2D_{2M}(e)-D_{2R}(e)}{3}, \frac{2D_{2R}(e)-D_{2M}(e)}{3}) & if \quad 0 < \frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\ & \quad \frac{2D_{2R}(e)-D_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\ & \quad 0 < \frac{2D_{2M}(e)-D_{2R}(e)}{3} \\ & \quad \frac{2D_{2M}(e)-D_{2R}(e)}{3} < \gamma\tau Q_1 \\ (\gamma\tau Q_1, \frac{D_{2R}(e)-\gamma\tau Q_1}{2}) & if \quad \gamma\tau Q_1 < D_{2R}(e), \\ & \quad D_{2R}(e) < (2-\gamma)\tau Q_1, \\ & \quad \gamma\tau Q_1 < \frac{2D_{2M}(e)-D_{2R}(e)}{3} \\ (0, (1-\gamma)\tau Q_1) & if \quad D_{2M}(e) < (1-\gamma)\tau Q_1, \\ & \quad (2-\gamma)\tau Q_1 < D_{2R}(e) \\ (\frac{D_{2M}(e)-(1-\gamma)\tau Q_1}{2}, (1-\gamma)\tau Q_1) & if \quad (1-\gamma)\tau Q_1 < \frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\ & \quad (1-\gamma)\tau Q_1 < D_{2M}(e), \\ & \quad D_{2M}(e) < (1+\gamma)\tau Q_1 \\ (\gamma\tau Q_1, (1-\gamma)\tau Q_1) & if \quad (2-\gamma)\tau Q_1 < D_{2R}(e), \\ & \quad (1+\gamma)\tau Q_1 < D_{2M}(e) \end{cases} \tag{5.28}$$

OEM's second period profit under the equilibrium solution is

$$\pi^*_{2M}(Q_1,e) = \begin{cases} \pi^{(i)}_{2M}(Q_1,e) & if \quad D_{2M}(e)<0, D_{2R}(e)<0 \\ \pi^{(ii)}_{2M}(Q_1,e) & if \quad D_{2R}(e)<D_{2M}(e)/2, \\ & \quad 0\le D_{2M}(e)<2\gamma\tau Q_1 \\ \pi^{(iii)}_{2M}(Q_1,e) & if \quad D_{2R}(e)\le \gamma\tau Q_1, \\ & \quad 2\gamma\tau Q_1 \le D_{2M}(e) \\ \pi^{(iv)}_{2M}(Q_1,e) & if \quad D_{2M}(e)<D_{2R}(e)/2, \\ & \quad 0\le D_{2R}(e)<2(1-\gamma)\tau Q_1 \\ \pi^{(v)}_{2M}(Q_1,e) & if \quad 0<\frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\ & \quad \frac{2D_{2R}(e)-D_{2M}(e)}{3}<(1-\gamma)\tau Q_1, \\ & \quad 0<\frac{2D_{2M}(e)-D_{2R}(e)}{3} \\ & \quad \frac{2D_{2M}(e)-D_{2R}(e)}{3}<\gamma\tau Q_1 \\ \pi^{(vi)}_{2M}(Q_1,e) & if \quad \gamma\tau Q_1<D_{2R}(e), \\ & \quad D_{2R}(e)<(2-\gamma)\tau Q_1, \\ & \quad \gamma\tau Q_1<\frac{2D_{2M}(e)-D_{2R}(e)}{3} \\ \pi^{(vii)}_{2M}(Q_1,e) & if \quad D_{2M}(e)<(1-\gamma)\tau Q_1, \\ & \quad (2-\gamma)\tau Q_1<D_{2R}(e) \\ \pi^{(viii)}_{2M}(Q_1,e) & if \quad (1-\gamma)\tau Q_1<\frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\ & \quad (1-\gamma)\tau Q_1<D_{2M}(e), \\ & \quad D_{2M}(e)<(1+\gamma)\tau Q_1 \\ \pi^{(ix)}_{2M}(Q_1,e) & if \quad (2-\gamma)\tau Q_1<D_{2R}(e), \\ & \quad (1+\gamma)\tau Q_1<D_{2M}(e) \end{cases}$$

where

$$\pi^{(i)}_{2M}(Q_1,e) = Q_1 s\tau\gamma$$
$$\pi^{(ii)}_{2M}(Q_1,e) = \frac{(a_{2M}-c_{2M}-s)^2}{4b_{2M}} + s\tau\gamma Q_1$$
$$\pi^{(iii)}_{2M}(Q_1,e) = (a_{2M}-c_{2M}-s)\gamma\tau e Q_1 - b_{2M}(e\gamma\tau Q_1)^2 + s\tau\gamma Q_1$$
$$\pi^{(iv)}_{2M}(Q_1,e) = Q_1 s\tau\gamma$$
$$\pi^{(v)}_{2M}(Q_1,e) = b_{2M}(\frac{2(a_{2M}-c_{2M}-s)}{3b_{2M}} - \frac{(a_{2R}-c_{2R}-s)}{3b_{2R}})^2 + s\tau\gamma Q_1$$
$$\pi^{(vi)}_{2M}(Q_1,e) = ((a_{2M}-c_{2M}-s) - \frac{(a_{2R}-c_{2R}-s)b_{2M}}{2b_{2R}})\tau\gamma e Q_1 - \frac{(Q_1 e\tau\gamma)^2}{2} + s\tau\gamma Q_1$$
$$\pi^{(vii)}_{2M}(Q_1,e) = s\tau\gamma Q_1$$
$$\pi^{(viii)}_{2M}(Q_1,e) = \frac{(b_{2M}e^2)(D_{2M}(e)-(1-\gamma)\tau Q_1)^2}{4} + s\tau\gamma Q_1$$
$$\pi^{(ix)}_{2M}(Q_1,e) = (a_{2M}-c_{2M}-s)\gamma\tau e Q_1 - b_{2M}\gamma(e\tau Q_1)^2 + s\tau\gamma Q_1$$

$\pi_T(Q_1, e)$ stands for total profit function over two periods. In order to find Q_1 and e that maximizes total profit, joint concavity of it should be used.

$$
\pi_T(Q_1,e) = \pi_1(Q_1,e) + \begin{cases}
\pi_{2M}^{(i)}(Q_{2M}(Q_1,e)) & if \quad D_{2M}(e) < 0, D_{2R}(e) < 0 \\
\pi_{2M}^{(ii)}(Q_{2M}(Q_1,e)) & if \quad D_{2R}(e) < D_{2M}(e)/2, \\
 & \quad 0 \le D_{2M}(e) < 2\gamma\tau Q_1 \\
\pi_{2M}^{(iii)}(Q_{2M}(Q_1,e)) & if \quad D_{2R}(e) \le \gamma\tau Q_1, \\
 & \quad 2\gamma\tau Q_1 \le D_{2M}(e) \\
\pi_{2M}^{(iv)}(Q_{2M}(Q_1,e)) & if \quad D_{2M}(e) < D_{2R}(e)/2, \\
 & \quad 0 \le D_{2R}(e) < 2(1-\gamma)\tau Q_1 \\
\pi_{2M}^{(v)}(Q_{2M}(Q_1,e)) & if \quad 0 < \frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\
 & \quad \frac{2D_{2R}(e)-D_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\
 & \quad 0 < \frac{2D_{2M}(e)-D_{2R}(e)}{3} \\
 & \quad \frac{2D_{2M}(e)-D_{2R}(e)}{3} < \gamma\tau Q_1 \\
\pi_{2M}^{(vi)}(Q_{2M}(Q_1,e)) & if \quad \gamma\tau Q_1 < D_{2R}(e) \\
 & \quad D_{2R}(e) < (2-\gamma)\tau Q_1 \\
 & \quad \gamma\tau Q_1 < \frac{2D_{2M}(e)-D_{2R}(e)}{3} \\
\pi_{2M}^{(vii)}(Q_{2M}(Q_1,e)) & if \quad D_{2M}(e) < (1-\gamma)\tau Q_1, \\
 & \quad (2-\gamma)\tau Q_1 < C_{2R}(e) \\
\pi_{2M}^{(viii)}(Q_{2M}(Q_1,e)) & if \quad (1-\gamma)\tau Q_1 < \frac{2D_{2R}(e)-D_{2M}(e)}{3}, \\
 & \quad (1-\gamma)\tau Q_1 < D_{2M}(e), \\
 & \quad D_{2M}(e) < (1+\gamma)\tau Q_1 \\
\pi_{2M}^{(ix)}(Q_{2M}(Q_1,e)) & if \quad (2-\gamma)\tau Q_1 < D_{2R}(e), \\
 & \quad (1+\gamma)\tau Q_1 < D_{2M}(e)
\end{cases}
$$

where

$$\pi_1(Q_1, e) = (a_1 - c_1 - b_1 Q_1)Q_1 - ke^2 \tag{5.29}$$

5.4 Duopolistic Model with Assumption of Monopoly

In order to study the effects of competition awareness, we create a benchmark setting to the duopoly environment we considered in Chapter 4. In this setting OEM is not aware of the competition in the second period while determining level of remanufacturability and manufacturing quantity in the first period.

Sequence of events is as follows:

- **Period 1**

  OEM determines manufacturing quantity, $Q_1 \geq 0$, and level of remanufacturability, $0 \leq e \leq 1$, by incurring total manufacturing cost of $c_1 Q_1$ and making an investment of ke^2 for level of remanufacturability, respectively without considering a competitor to enter the remanufactured product market.

  The market price for manufactured items become $P_1(Q_1) = a_1 - b_1 Q_1$. OEM sells all Q_1 units at a price of $P_1(Q_1)$.

- **Period 2**

  OEM and IR get $\gamma\tau Q_1$ and (1 $-\gamma$tau Q$_1$ returns with remanufacturability level e, respectively.

  OEM and IR simultaneously determine remanufacturing input quantities Q_{2M} and Q_{2R} such that $0 \leq Q_{2M} \leq \gamma\tau Q_1$ and $0 \leq Q_{2M} \leq (1-\gamma)\tau Q_1$, incurring total remanufacturing cost of $c_{2M}Q_{2M}$ and $c_{2R}Q_{2R}$ and obtain recycling revenue of s for returns that are not remanufactured, $(\gamma\tau Q_1 - Q_{2M})$ and $((1-\gamma)\tau Q_1 - Q_{2R})$, and items lost in remanufacturing process, $(1-e)Q_{2M}$ and $(1-e)Q_{2R}$, respectively .

  The market price becomes $P_{2M}(Q_{2M}, Q_{2R}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of OEM. OEM sells all eQ_{2M} at a price of $P_{2M}(Q_{2M}, Q_{2R})$.

  The market price becomes $P_{2R}(Q_{2R}, Q_{2M}) = a_{2M} - b_{2M}e(Q_{2M} + Q_{2R})$ for remanufactured products of IR. IR sells all eQ_{2R} at a price of $P_{2R}(Q_{2R}, Q_{2M})$.

Profit of OEM in the first period is represented with $\pi_1(Q_1, e)$, profit of OEM in the second period is represented with $\pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e)$ and profit of IR in the second period is represented with $\pi_{2R}(Q_{2R}, Q_{2M}|Q_1, e)$.

The first period problem of OEM can be expressed as ;

$$\text{maximize} \quad \pi_1(Q_1, e) = (a_1 - b_1 Q_1)Q_1 - c_1 Q_1 - ke^2$$

subject to

$$Q_1 \geq 0 \tag{5.30}$$

$$0 \leq e \leq 1 \tag{5.31}$$

The second period problem of OEM can be expressed as ;

$$\underset{Q_{2M}}{\text{maximize}} \quad \pi_{2M}(Q_{2M}, Q_{2R}|Q_1, e) \;=\; \begin{array}{l}(a_{2M} - b_{2M}e(Q_{2M} + Q_{2R}))eQ_{2M} \\ -c_{2M}Q_{2M} + s(\gamma\tau Q_1 - Q_{2M}e)\end{array}$$

subject to

$$Q_{2M} \leq Q_1\gamma\tau \tag{5.32}$$

$$Q_{2M} \geq 0 \tag{5.33}$$

The second period problem of IR can be expressed as ;

$$\underset{Q_{2R}}{\text{maximize}} \quad \pi_{2R}(Q_{2M}, Q_{2R}|Q_1, e) \;=\; \begin{array}{l}(a_{2R} - b_{2R}e(Q_{2R} + Q_{2M}))eQ_{2R} \\ -c_{2R}Q_{2R} + s((1-\gamma)\tau Q_1 - Q_{2R}e)\end{array}$$

subject to

$$Q_{2R} \leq Q_1(1-\gamma)\tau \tag{5.34}$$

$$Q_{2R} \geq 0 \tag{5.35}$$

Constraints 5.30, 5.33 and 5.35 represent non-negativity of manufacturing quantity, remanufacturing quantity of OEM and remanufacturing quantity of IR. Constraint 5.32 and 5.34 stand for input capacity of remanufacturing for OEM and IR. Right hand sides of 5.32 and 5.34 are dependent on manufacturing quantity in the first period. Constraint 5.31 represents the lower and upper limit of variable. Since, remanufacturability is a ratio, it lies in the interval, $0 \leq e \leq 1$.

5.4.1 Analysis of Duopolistic Model with Assumption of Monopoly

We implement an solution procedure in order to find equilibrium remanufacturability level, manufacturing quantity, remanufacturing quantity of OEM and remanufacturing quantity of IR. The solution procedure composed of two parts. First, in order to find optimal remanufacturability level and manufacturing quantity, the solution procedure in Section 3.3 is implemented. Then, equilibrium remanufacturing quantities are found as in Section 4.1. The steps of the solution procedure is as follows:

1. Optimal manufacturing quantity, Q_1, and remanufacturing quantity, Q_{2M}, are determined using 3.14 that maximizes total profit for increasing values of remanufacturability level, e. .

   We plug optimal Q_1, Q_{2M} and the given remanufacturability level used in the first step, e, to the total profit function.

   We search for the optimal remanufacturability level that maximizes total profit in its range considering remanufactured product market is monopoly.

2. Equilibrium solution in Section 4.2 is implemented in order to find Q_{2M} and Q_{2R} for the manufacturing quantity and the level of remanufacturability found in the previos step.

For the sake of simplicity, let $A_{2M}(e) = \frac{(a_{2M}e - c_{2M} - se)}{b_{2M}e^2}$ and $A_{2R}(e) = \frac{(a_{2R}e - c_{2R} - se)}{b_{2R}e^2}$.

Optimal manufacturing quantity for given remanufacturability level is as follows:

$$Q_1^* = \begin{cases} (\frac{a_1 - c_1 + s\tau}{2b_1}, \frac{(a_{2M}e - c_{2M} - se)}{2b_{2M}e^2}) & if \quad A_{2M}(e)/2 \leq \tau \frac{a_1 - c_1 + s\tau}{2b_1} \\ (Q_1^{'}, \tau Q_1^{'}) & if \quad \tau \frac{a_1 - c_1 + s\tau}{2b_1} \leq A_{2M}(e)/2 \end{cases}$$

Where $Q_1^{'} = \frac{(a_1 - c_1 + s\tau) + \tau(a_{2M}e - c_{2M} - se)}{2b_1 + 2b_{2M}e^2}$

The second period equilibrium point is as same as in section 4.2

$$
(Q^*_{2M}, Q^*_{2R}) = \begin{cases}
(0,0) & if \quad A_{2M}(e) < 0, A_{2R}(e) < 0 \\
(\frac{A_{2M}(e)}{2}, 0) & if \quad A_{2R}(e) < A_{2M}(e)/2, \\
 & \quad 0 \leq A_{2M}(e) < 2\gamma\tau Q_1 \\
(\gamma\tau Q_1, 0) & if \quad A_{2R}(e) \leq \gamma\tau Q_1, \\
 & \quad 2\gamma\tau Q_1 \leq A_{2M}(e) \\
(0, \frac{A_{2R}(e)}{2}) & if \quad A_{2M}(e) < A_{2R}(e)/2, \\
 & \quad 0 \leq A_{2R}(e) \\
 & \quad A_{2R}(e) < 2(1-\gamma)\tau Q_1 \\
(\frac{2A_{2M}(e)-A_{2R}(e)}{3}, \frac{2A_{2R}(e)-A_{2M}(e)}{3}) & if \quad 0 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
 & \quad \frac{2A_{2R}(e)-A_{2M}(e)}{3} < (1-\gamma)\tau Q_1, \\
 & \quad 0 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
 & \quad \frac{2A_{2M}(e)-A_{2R}(e)}{3} < \gamma\tau Q_1 \\
(\gamma\tau Q_1, \frac{A_{2R}(e)-\gamma\tau Q_1}{2}) & if \quad \gamma\tau Q_1 < A_{2R}(e) \\
 & \quad A_{2R}(e) < (2-\gamma)\tau Q_1 \\
 & \quad \gamma\tau Q_1 < \frac{2A_{2M}(e)-A_{2R}(e)}{3} \\
(0, (1-\gamma)\tau Q_1) & if \quad A_{2M}(e) < (1-\gamma)\tau Q_1, \\
 & \quad (2-\gamma)\tau Q_1 < A_{2R}(e) \\
(\frac{A_{2M}(e)-(1-\gamma)\tau Q_1}{2}, (1-\gamma)\tau Q_1) & if \quad (1-\gamma)\tau Q_1 < \frac{2A_{2R}(e)-A_{2M}(e)}{3}, \\
 & \quad (1-\gamma)\tau Q_1 < A_{2M}(e) \\
 & \quad A_{2M}(e) < (1+\gamma)\tau Q_1 \\
(\gamma\tau Q_1, (1-\gamma)\tau Q_1) & if \quad (2-\gamma)\tau Q_1 < A_{2R}(e), \\
 & \quad (1+\gamma)\tau Q_1 < A_{2M}(e)
\end{cases} \tag{5.36}
$$

CHAPTER 6

COMPUTATIONAL STUDY

In this chapter, we present the results of an extensive computational study that we conduct in order to identify, (i) how optimal decisions change under various problem settings, (ii) effects of competition in remanufacturing, (iii) effects of sorting information and (iv) effects of competition awareness. For this purpose, we implement the solution procedures presented in related sections in Matlab R2015b.

In Section 6.1, research questions that we answer and the performance measures that we create in order to evaluate outcomes of models are presented. In Section 6.2, effects of problem parameters are investigated. In Section 6.3, effects of competition are investigated. In Section 6.4, effects of sorting information are investigated. In Section 6.5, effects of competition awareness are investigated.

6.1 Research Questions and Performance Measures

The research questions that we address through computational study are as follows:

1. What are the equilibrium manufacturing quantity, remanufacturability level and remanufacturing quantities of OEM and IR?
2. How does the economical and environmental performances of OEM and IR change with different problem parameters?
3. How does entry of a competitor in the remanufactured product market affect the economical and environmental performances of OEM and IR?
4. How does sorting information affect the economical and environmental perfor-

mances of OEM and IR?

5. What are the effects of not considering a potential entrant while determining manufacturing quantity and remanufacturabiliy level of OEM on the economical and environmental performances of OEM and IR?

In order to answer the research questions stated above, we use two performance measures. Our main performance measures that we used throughout the computational study are (i) the total system-wide profit for the economical performance, and (ii) value recovery ratio for the environmental performance. Total system-wide profit is equal to summation of OEM's profit and IR's profit, $\pi_1(Q_1, e) + \pi_{2M}(Q_{2M}, Q_{2R}) + \pi_{2R}(Q_{2R}, Q_{2M})$. Value recovery ratio is equal to ratio of total successfully remanufactured products to total returned products, $\frac{e(Q_{2M}+Q_{2R})}{\tau Q_1}$. We start with sensitivity analysis in order to answer questions 1 and 2 and continue the computational study with the comparison of outcomes of different models.

For simplicity, OP_i, TP_i and VRR_i are used for OEM's total profit, total system-wide profit and value recovery ratio of model i where $i \in \{MNS, MYS, DNS, DOS, DYS, DM\}$, respectively. Meaning of abbreviations are as follows:

- MNS: Monopolistic model without sorting information
- MYS: Monopolistic model with sorting information
- DNS: Duopolistic model without sorting information
- DOS: Duopolistic model with sorting information for only OEM
- DOS: Duopolistic model with sorting information for both OEM and IR
- DM: Duopolistic model with assumption of monopoly

In order to analyze effects of competition, sorting information and competition awareness, we compared optimal solutions and equilibrium outcomes of different models.

- To analyze the effects of competition (question 3), we compare total profits and value recovery ratios of monopolistic settings to those of duopolistic settings.

- To understand the significance of sorting information (question 4), we compare the performance measures under settings with sorting information to those without sorting information.
- To understand the effects of competition awareness (question 5), we compare economical and environmental performances under duopolistic setting without sorting information to duopolistic setting with assumption of monopoly.

6.2 Effects of Problem Parameters to the Performance Measures

In order to investigate the effect of parameters on economical and environmental performance, a sensitivity analyses are conducted. Total system-wide profit and value recovery ratio is evaluated under the optimal or equilibrium solutions under various values of one parameter while the rest is kept unchanged. The evaluation is performed for all six alternative models; MNS, MYS, DNS, DOS, DYS and DM.

To perform the sensitivity analysis, we use the base parameter set given in Table 6.1. Range and step size of the parameters considered are given in Table 6.2

Table 6.1: Base parameter set

a_1	a_{2M}	a_{2R}	b_1	b_{2M}	b_{2R}	c_1	c_{2M}	c_{2R}	s	k	τ	γ
200	100	100	0.5	0.5	0.5	30	20	20	20	1000	0.7	0.5

Table 6.2: Range and step size of the parameters

Parameter	a_{2M}	a_{2R}	c_1	k	τ	γ
Range	10-200	10-200	10-200	0-3800	0.05-1.00	0.05-1.00
Step Size	10	10	10	200	0.05	0.05

For simplicity, we use up arrow and down arrow to represent changes in the performance measures. Up arrow (↑) represents the increase in related performance measure when the parameter in concern increases. Down arrow (↓) represents the decrease in related performance measure when the parameter in concern increases.

Table 6.3 summarizes how economical and environmental performance of monopolistic models change with respect to change in a parameter. The tables that demonstrate the numerical results of parameter sensitivity for monopolistic models can be found in Appendices A and B.

Table 6.3: Parameter sensitivity for monopolistic models

Increase in		a_{2M}	c_1	τ	k
MNS	**TP**	↑	↓	↑	↓
	VRR	↑	↑	↑↓	↓
MYS	**TP**	↑	↓	↑	↓
	VRR	↑	↑	↑↓	↓

Table 6.4 summarizes economical and environmental performance of duopolistic models. The tables that demonstrate the numerical results of parameter sensitivity for monopolistic models can be found in Appendices C, D, E and F.

Table 6.4: Parameter sensitivity for duopolistic models

Increase in		a_{2M}	a_{2R}	γ	τ	k	c_1
DNS	**OP**	↑	↓	↑	↑	↓	↓
	TP	↑	↓	↑	↑	↓	↓
	VRR	↑↓	↑↓	↑↓	↑↓	↓	↑↓
DOS	**OP**	↑	↓	↑	↑	↓	↓
	TP	↑	↑	↑	↑	↓	↓
	VRR	↑↓	↑↓	↑↓↑	↑↓	↓	↑↓
DYS	**OP**	↑	↓	↑	↑	↓	↓
	TP	↑	↑	↑	↑	↓	↓
	VRR	↑↓	↓↑	↑	↑↓	↓	↑↓
DM	**OP**	↓↑	↓	↑↓↑	↑	↓	↓
	TP	↑	↑	↑	↑	↓	↓
	VRR	↑↓	↑	↑↓	↑↓	↓	↑↓

Effects of maximum selling price of remanufactured products of OEM, a_{2M}

- In both monopolistic and duopolistic models, there exists a threshold a_{2M} where OEM starts investing in remanufacturability. Figure 6.1 shows that this threshold is larger in the duopolistic setting compared to the monopolistic setting.

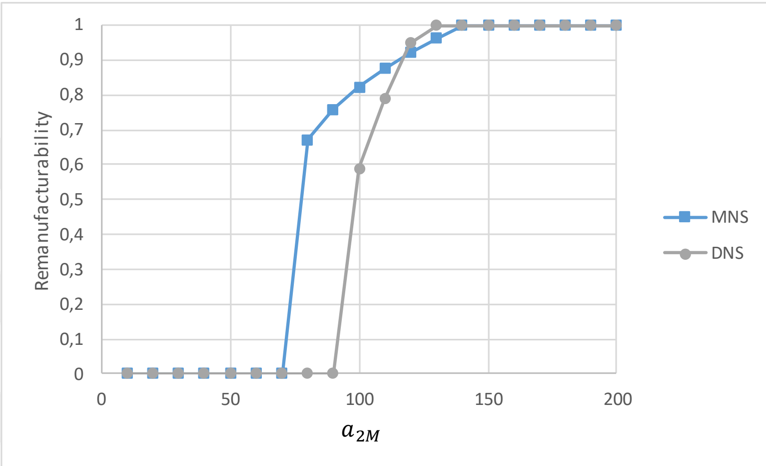

Figure 6.1: Effect of a_{2M} to remanufacturability level

- In the duopolistic settings, when OEM makes remanufacturability investment, IR always makes remanufacturing. IR's remanufacturing quantity decreases with an increase in a_{2M}.

- With an increase in a_{2M}, remanufacturing input quantity of OEM always increases.

- Total remanufactured product supply always increases with an increase in a_{2M} in all settings. (Figure 6.2)

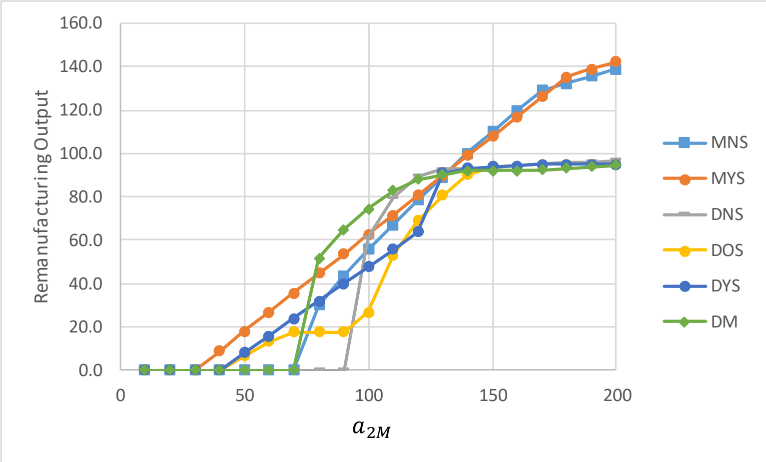

Figure 6.2: Effect of a_{2M} to total remanufacturing output

- OEM's profit always increases with an increase in a_{2M} if OEM is aware of competition in the remanufactured product market. If OEM is not aware of competition in the remanufactured product market, OEM's profit decreases at the a_{2M} level that OEM starts remanufacturing. (Figure 6.3)

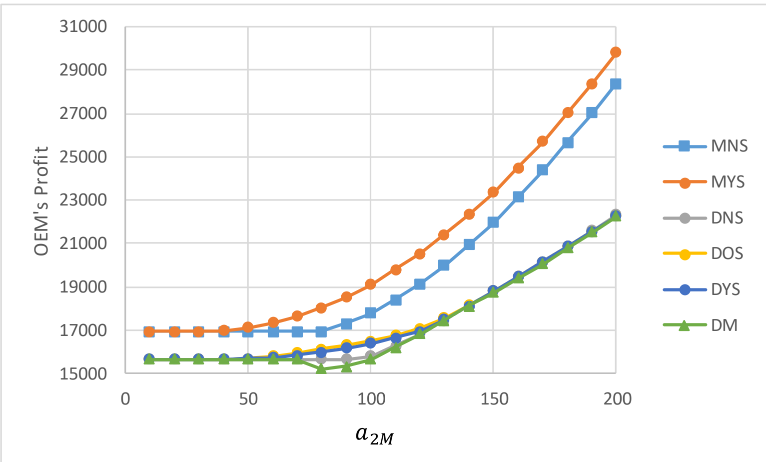

Figure 6.3: Effect of a_{2M} to OEM's profit

Effects of manufacturing unit cost, c_1

- OEM always makes remanufacturability investment except duopolistic model without sorting information for studied values of parameter c_1. Remanufacturability investment is first non-decreasing with an increase in c_1 since return quantity in the second period decreases, then decreases since remanufacturability investment becomes more costly for a unit remanufactured. (Figure 6.4)

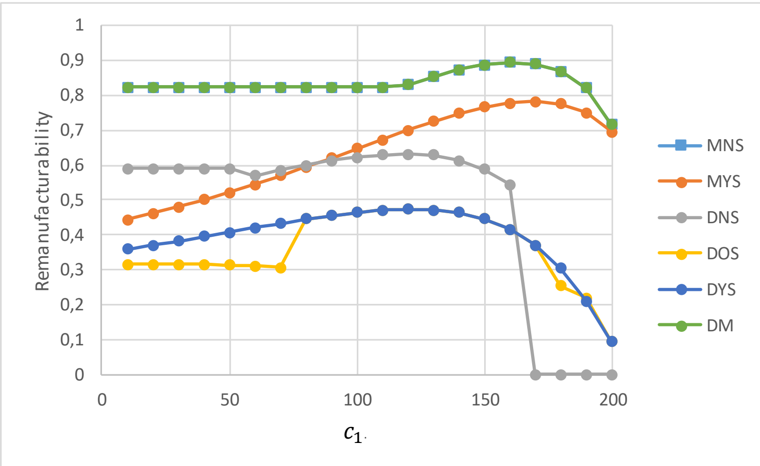

Figure 6.4: Effect of c_1 to remanufacturability level

- IR's remanufacturing input quantity is always the same as the OEM's except for DOS. When sorting information is only available for OEM, IR's remanufacturing input quantity is lower than OEM's for very high and very low c_1.

- With an increase in c_1, remanufacturing input quantity of OEM always decreases.

- Total remanufactured product supply always decreases with an increase in the c_1 except for DOS. When sorting information is only available for OEM, total remanufactured product supply for moderate values of c_1 is higher than very high and very low values of c_1. (Figure 6.5)

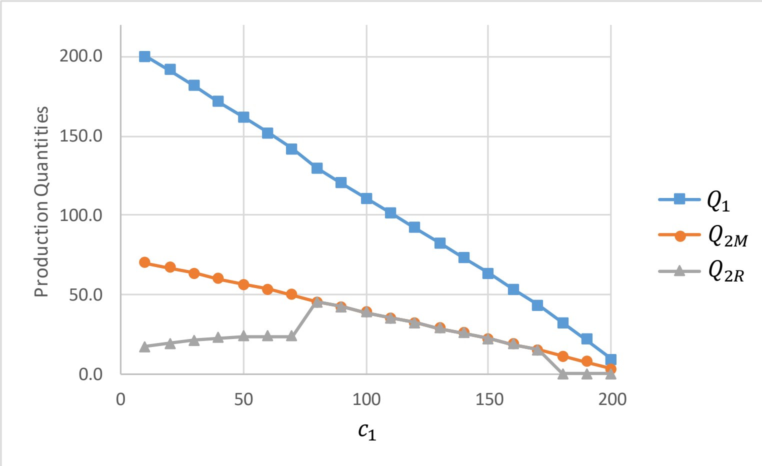

Figure 6.5: Effect of c_1 to total remanufacturing output

- OEM's profit always decreases with an increase in c_1 and becomes negative when OEM is not aware of competition in the remanufactured product market as it can be seen at Figure 6.4.

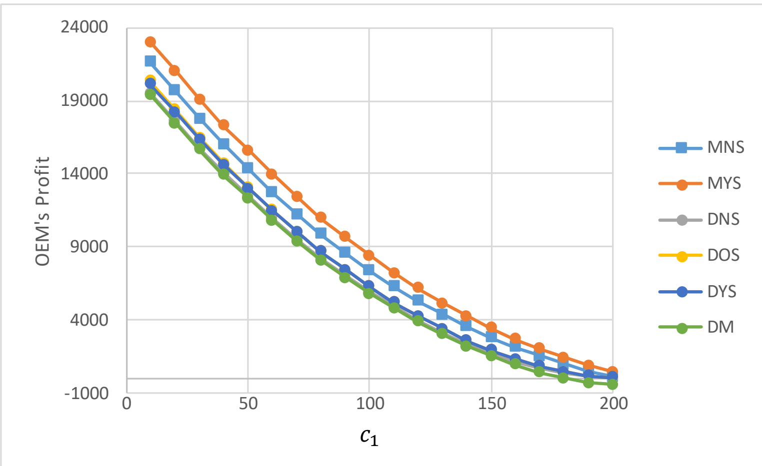

Figure 6.6: Effect of c_1 to OEM's profit

Effects of return rate, τ

- Since investment cost of remanufacturability for a successfully remanufactured item is very high for low τ when sorting information is not available, OEM might not make investment on remanufacturability for lower τ. Otherwise, OEM always makes remanufacturability investment for all τ values considered. (Figure 6.7)

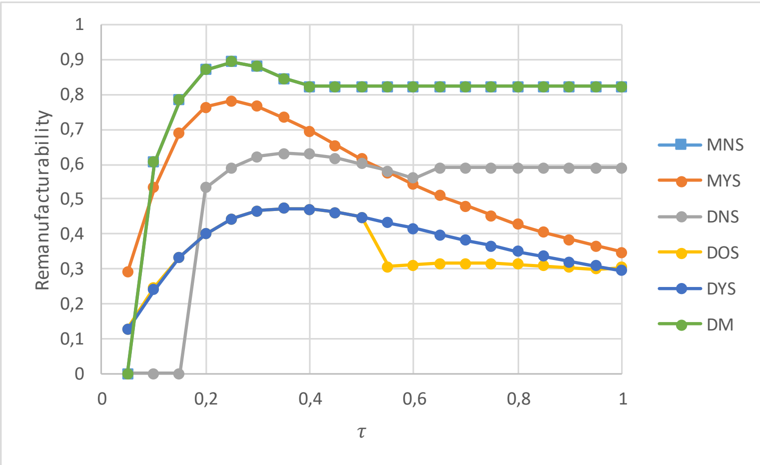

Figure 6.7: Effect of τ to remanufacturability level

- IR's remanufacturing input quantity is always the same as the OEM's except for duopolistic model when sorting information is only available for OEM. In this case, IR's remanufacturing input quantity is lower than OEM's for very high and very low τ values considered.

- With an increase in τ, remanufacturing input quantity of OEM is always non-decreasing. Figure 6.8 exemplifies this.

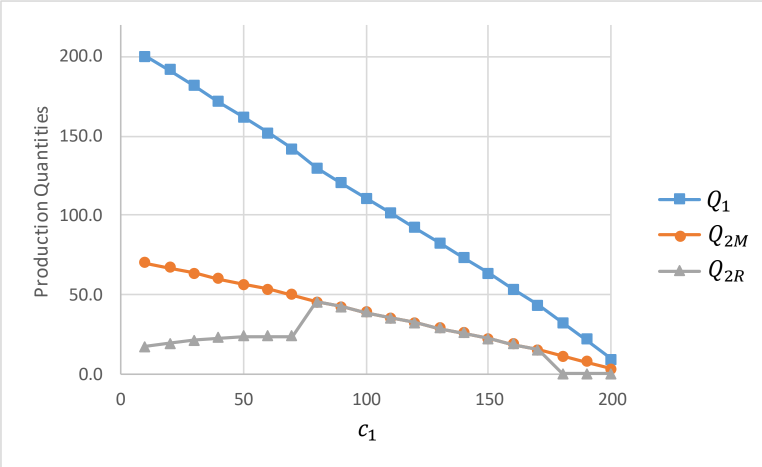

Figure 6.8: Effect of τ to production quantities in MNS

- Total remanufactured product supply always decreases with an increase in τ except for duopolistic setting with sorting information for only OEM. When sorting information is only available for OEM, total remanufactured product supply decreases at some τ such that OEM has sufficient input to decrease remanufacturability level to decrease IR's remanufacturing output then increases again. (Figure 6.9)

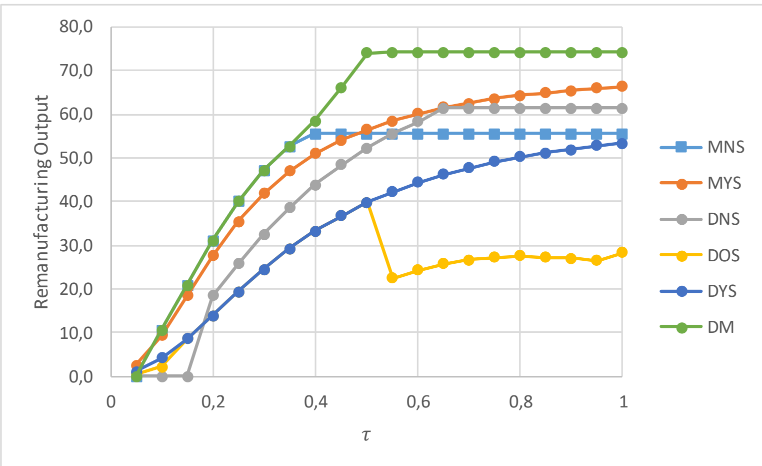

Figure 6.9: Effect of τ to remanufactured product supply

- OEM's profit always increases with an increase in τ.

Effects of investment cost coefficient of remanufacturability, k

- OEM always makes remanufacturability investment when sorting information is available. When sorting information is not available, OEM does not make remanufacturability investment for high values of k. Remanufacturability investment always decreases with an increase in k. (Figure 6.10)

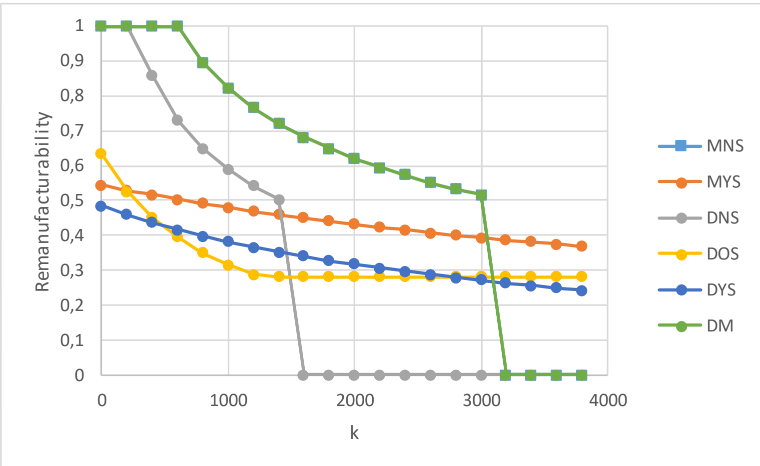

Figure 6.10: Effect of k to remanufacturability level

- IR's remanufacturing input quantity is always the same as the OEM's except for duopolistic model with sorting information for only OEM. When sorting information is only available for OEM, IR's remanufacturing input quantity is lower than OEM's.

- With an increase in k, remanufacturing input quantity of OEM increases when remanufacturability level is positive and sorting information is not available. When sorting information is not available, remanufacturing input quantity of OEM increases with an increase in k.

- Figure 6.11 shows that total remanufactured product supply always decreases with an increase in the k.

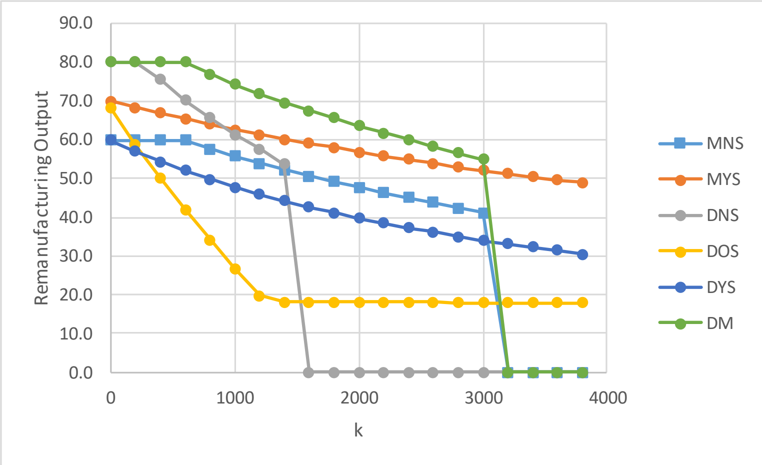

Figure 6.11: Effect of k to remanufactured product supply

- OEM's profit always decreases with an increase in k.

Effects of OEM's share of return, γ

- OEM always makes remanufacturability investment when sorting information is available or when he is not aware of competition. When sorting information is not available and OEM is aware of competition, OEM does not make remanufacturability investment for low values of γ.

- When OEM invests on remanufacturability, IR's remanufacturing input quantity always decreases except for duopolistic model with sorting information for only OEM. When sorting information is only available for OEM, IR needs OEM's share of return to be sufficiently high for remanufacturing, IR's remanufacturing quantity first increases then decreases with γ. (Figure 6.12)

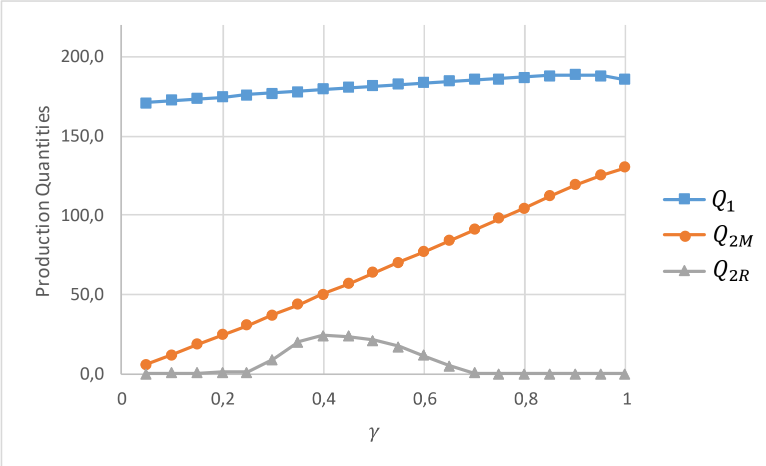

Figure 6.12: Effect of γ to production quantities in DOS

- With an increase in γ, remanufacturing input quantity of OEM always increases.
- Total remanufactured product supply always increases with an increase in γ for the models with sorting information for OEM and for both OEM and IR. When sorting information is not available, with an increase in γ, total remanufactured product supply first increases then decreases since IR's input quantity is limited.
- OEM's profit always increases with an increase in γ.

Effects of maximum selling price of remanufactured products of IR, a_{2R}

- OEM always makes remanufacturability investment for considered values of a_{2R}.
- IR's remanufacturing input quantity always increases except for duopolistic model with sorting information for only OEM when a_{2R} increases. When sorting information is only available for OEM, decreases at a point remanufacturability level is very low.
- With an increase in a_{2R}, remanufacturing input quantity of OEM always decreases.

- Total remanufactured product supply always increases with an increase in the a_{2R} when OEM is not aware of competition. When sorting information is available for both and IR, and not available, total remanufactured product supply first increases, then decreases with an increase in the a_{2R}. When sorting information is available for only OEM total remanufactured product supply first decreases, then increases with an increase in the a_{2R}. (Figure 6.13)

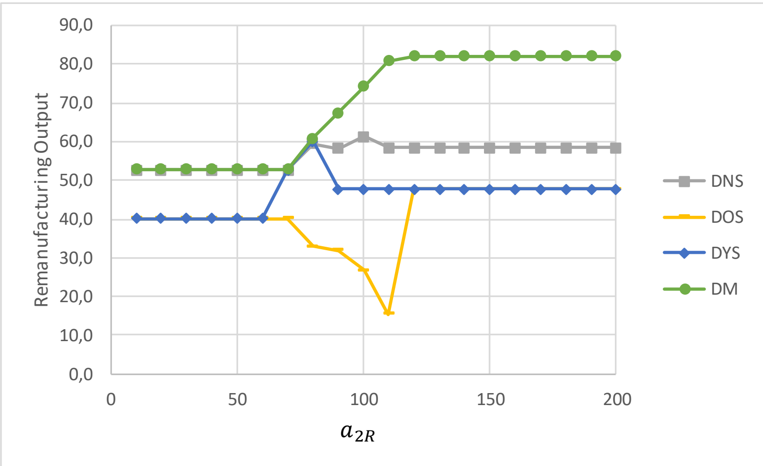

Figure 6.13: Effect of a_{2R} to remanufactured product supply

- OEM's profit always decreases with an increase in a_{2R}.

6.3 Effects of Competition to the Performance Measures

In order to observe the effects of competition on economical and environmental performance measures, we compare the outcomes of monopolistic and duopolistic models with the same availability of sorting information when only one of the problem parameters changes.

- When maximum selling price of OEM's remanufactured product increases and there is competition in the remanufactured product market, the value under which $e > 0$, is higher under competition.

- OEM's profit in the monopolistic settings are always higher than those in the duopolistic settings. When IR enters the market OEM decreases its remanufactured product supply and shares the remanufactured product market potential with IR. Since IR's profit does not compensate the decrease in the OEM's profit, total system-wide profit decreases when a competitor enters the market. Figure 6.14 exemplifies the effect of competition to OEM's profit.

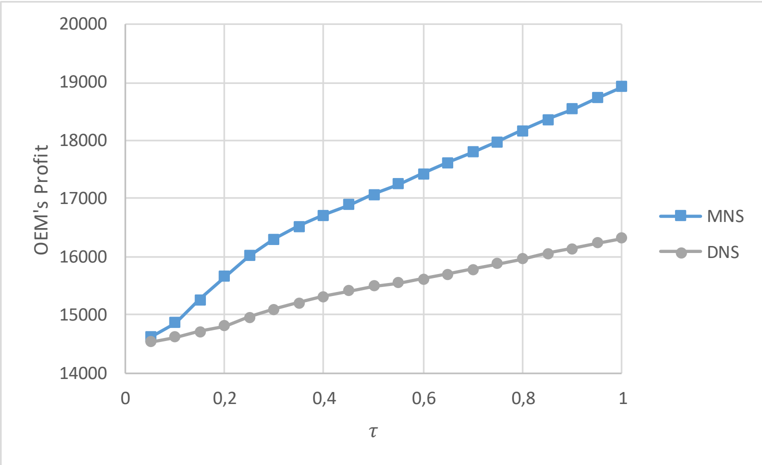

Figure 6.14: Effect of competition to OEM's profit without sorting information when τ increases

- Figure 6.15 shows that when maximum selling prices of OEM is very high or very low and sorting information availability is same for both players (sorting information is not available or available for both OEM and IR), remanufactured product output and value recovery ratio is lower in the duopolistic settings than monopolistic settings. Otherwise, remanufactured product output and value recovery ratio is higher in the duopolistic settings than monopolistic settings.

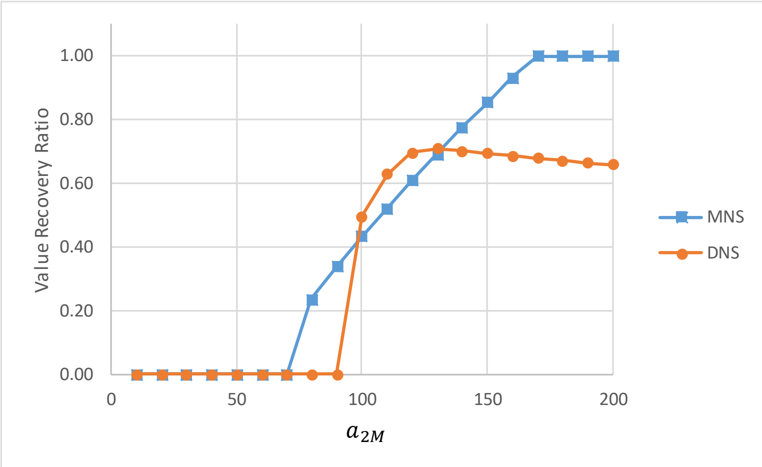

Figure 6.15: Effect of competition to value recovery ratio without sorting information when a_{2M} increases

- Figure 6.12 shows that when sorting information is available for only OEM and OEM gets most of the returned items, OEM pushes IR out of the remanufactured product market by choosing low remanufacturability level with the advantage of lower effective remanufacturing cost.
- When sorting information is not available and OEM's market power in the remanufactured product market is higher than IR's market power, OEM invests more on remanufacturability in the duopolistic setting than in the monopolistic setting since OEM cannot get all returns in the duopolistic setting and increases his remanufactured output with high remanufacturability level.

6.4 Effects of Sorting Information to the Performance Measures

We search for the effect of sorting information to economical and environmental performance measures in this section. In order to investigate these effects, we compare the outcome of the settings without sorting information with their with sorting counterparts.

- Since effective remanufacturing cost of OEM is lower when sorting information is available, OEM makes remanufacturability investments in the cases that he does not when sorting information is not available. When OEM makes remanufacturability investment in the settings without sorting information, remanufacturability level is higher than those with sorting information.

- Figure 6.16 shows that OEM's profit is higher when sorting information is available since sorting information decreases remanufacturing cost and. OEM's profit in the setting with sorting information for only OEM is higher than OEM's profit in the setting with sorting information for both OEM and IR.

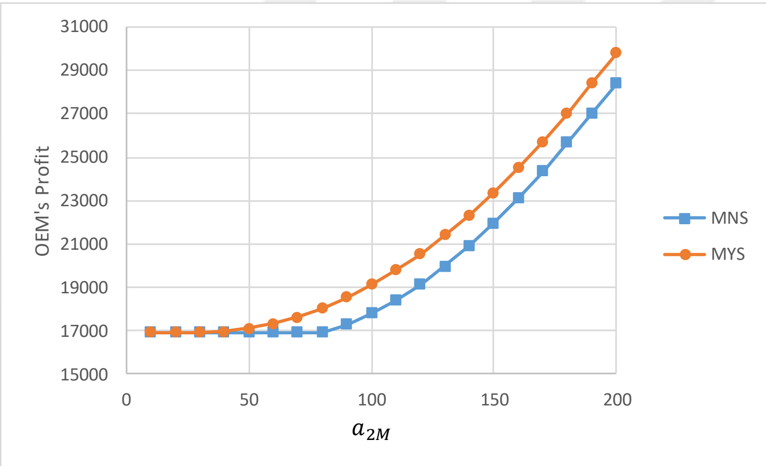

Figure 6.16: Effect of sorting information to OEM's profit when a_{2M} increases

- When sorting information is available, OEM's remanufacturing input quantity always increases with an increase in returns (increase in the return rate and increase in OEM's share of returns).

- Remanufactured product supply of the settings without sorting information is always higher than remanufactured product supply of the settings with sorting information, if OEM makes remanufacturability investment. Figure 6.17 exemplifies the effect of sorting information to remanufactured product supply.

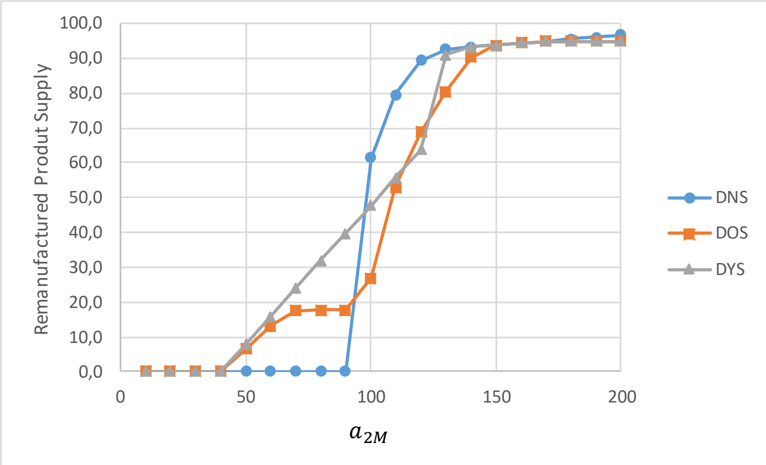

Figure 6.17: Effect of sorting information to remanufactured product supply when a_{2M} increases

6.5 Effect of Competition Awareness to the Performance Measures

In this section, we investigate effect of competition awareness by comparing duopolistic setting without sorting information with the duopolistic setting where OEM is not aware of competition.

- When OEM is not aware of competition, OEM might make remanufacturability investment in some cases that he would not do if he is aware of competition as can be seen from Figure 6.18. As a result of lack of competition awareness of OEM, IR can enter remanufactured product market in that cases. For most of cases, remanufacturability investment of the setting without competition awareness is higher than those with competition awareness. When OEM is aware of competition and IR's maximum selling price is low, OEM may invest more on remanufacturability because of limited number of returns in the duopolistic setting. remanufacturability level is not always higher when OEM is not aware of competition.

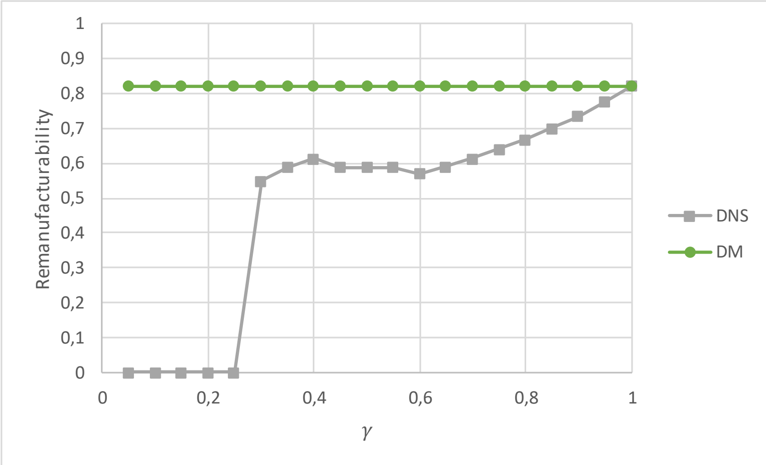

Figure 6.18: Effect of competition awareness to remanufacturability level when γ increases

- OEM's profit is always less when he is not aware of competition in the remanufactured product market. The loss of OEM is higher when maximum selling price of IR's remanufactured products is higher. Figure 6.19 exemplifies the effect of competition awareness to OEM's profit for changing values of a_{2R}.

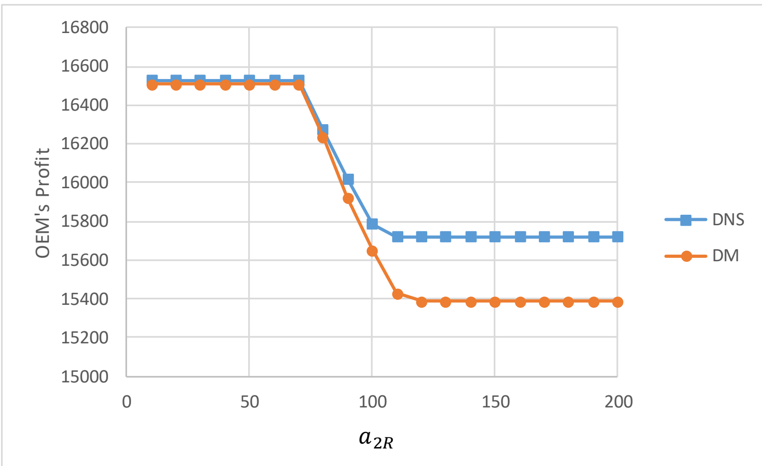

Figure 6.19: Effect of competition awareness to OEM's profit when a_{2R} increases

- Value recovery ratio in DM is always higher than value recovery ratio in DNS.

However, remanufactured product supply is not always higher since when OEM is aware of competition, he might increase manufacturing quantity in order to increase return quantity.

CHAPTER 7

CONCLUSION

With the help of increased environmental consciousness, governments begin to enforce firms to be responsible of their products when they are no longer needed by the customers. Companies turns this enforcement into an opportunity to make profit by using end-of-life products. Remanufacturing is one of the commonly implemented method of value recovery from end-of-life products. However, firms should design their original product to be suitable for remanufacturing, i.e. remanufacturable. Original equipment manufacturers need to consider fixed cost remanufacturability investment to reach certain level of remanufacturability and determine the level of remanufacturability before the beginning of the manufacturing operations.

One of the important issues to consider about remanufacturing is limited input. Original products are the only source of remanufacturing. Since independent remanufacturers collect end-of-life products and also use them for remanufacturing and , entrance of an independent remanufacturer to the remanufactured product market decreases available input quantity for OEM. Another issue to consider is that OEM is no longer a monopoly in remanufactured product market and should consider IR's actions while determining his own.

Another issue for OEM is that it is usually not certain whether a return is remanufacturable until it is remanufactured. Not knowing which returned items are remanufacturable cause remanufacturing cost to incur for every item that begins the remanufacturing operation.

Lastly, OEM may not be aware of the competition in the remanufactured product market while determining level of remanufacturability and original product manufacturing quantity.

In this study, in order to address the issues summarized above, we study several settings and investigate the effects of competition, sorting information and competition awareness. Specifically, we consider a two-period environment where manufacturing quantity and level of remanufacturability is determined in the first period, and remanufacturing quantities of OEM and IR are determined in the second.

After characterizing the optimal levels of manufacturing quantity and remanufacturability, we run a detailed computational study to fully address our research questions. We observe that competition decreases total system-wide profit in all sorting information settings and effect of competition to environmental performance is found to be sensitive to the problem parameters. Total system-wide profit is generally higher in settings with sorting information is available for more firms. When OEM is not aware of competition in the remanufactured product market, OEM's profit is always less than the setting with awareness of competition. Effect of competition awareness to economical performance is found to be sensitive to the problem parameters. Environmental performance is always higher when OEM is not aware of competition in remanufactured product market.

In future studies, acquisition process can be included in the model, firms can determine their return quantity with acquisition fee. Demand is considered as deterministic, market with stochastic demand can be investigated. Problem parameters are known by both players, this assumption can be removed. In order to investigate internal competition, manufactured and remanufactured products can be sold at the same period.

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APPENDIX A

OUTCOME OF MONOPOLISTIC SETTING WITHOUT SORTING INFORMATION

Optimal solution of monopolistic model without sorting information and the performance measures related with it is presented in this chapter.

Table A.1: Outcome of monopolistic model without sorting information for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0	0	0	0	0.671	0.758	0.823	0.876	0.922	0.963	1	1	1	1	1	1	1
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.6	189.3	194.0	198.7
Q_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	45.0	57.5	67.7	76.7	84.9	92.7	100.0	110.0	120.0	129.2	132.5	135.8	139.1
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	30.2	43.6	55.7	67.2	78.3	89.2	100.0	110.0	120.0	129.2	132.5	135.8	139.1
π_T	16928	16928	16928	16928	16928	16928	16928	16934	17305	17802	18416	19144	19982	20928	21978	23128	24378	25686	27027	28401
$\pi_T + \pi_{2R}$	16928	16928	16928	16928	16928	16928	16928	16934	17305	17802	18416	19144	19982	20928	21978	23128	24378	25686	27027	28401
VRR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.34	0.43	0.52	0.61	0.69	0.78	0.85	0.93	1.00	1.00	1.00	1.00
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	107.7	105.4	103.0	100.7
P_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	64.9	68.2	72.2	76.4	80.8	85.4	90.0	95.0	100.0	105.4	113.8	122.1	130.5
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A.2: Outcome of monopolistic model without sorting information for increasing c_1

c_1	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.83	0.854	0.874	0.888	0.894	0.89	0.869	0.822	0.717
Q_1	204.0	194.0	184.0	174.0	164.0	154.0	144.0	134.0	124.0	114.0	104.0	94.6	86.8	79.3	71.9	64.7	57.5	50.1	42.1	32.1
Q_{2M}	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	66.2	60.8	55.5	50.4	45.3	40.3	35.1	29.5	22.5
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	54.9	51.9	48.5	44.7	40.5	35.8	30.5	24.2	16.1
π_T	21682	19692	17802	16012	14322	12732	11242	9852	8562	7372	6282	5291	4384	3554	2798	2115	1504	966	504	130
$\pi_T + \pi_{2R}$	21682	19692	17802	16012	14322	12732	11242	9852	8562	7372	6282	5291	4384	3554	2798	2115	1504	966	504	130
VRR	0.39	0.41	0.43	0.46	0.49	0.52	0.55	0.59	0.64	0.70	0.77	0.83	0.85	0.87	0.89	0.89	0.89	0.87	0.82	0.72
P_1	98.0	103.0	108.0	113.0	118.0	123.0	128.0	133.0	138.0	143.0	148.0	152.7	156.6	160.4	164.0	167.6	171.2	174.9	179.0	184.0
P_{2M}	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.5	74.1	75.8	77.6	79.7	82.1	84.8	87.9	92.0
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A.3: Outcome of monopolistic model without sorting information for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	1	1	1	1	0.897	0.823	0.766	0.72	0.682	0.649	0.62	0.595	0.573	0.552	0.533	0.516	0	0	0	0
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0
Q_{2M}	60.0	60.0	60.0	60.0	64.3	67.7	70.4	72.5	74.3	75.8	77.0	78.0	78.7	79.3	79.7	79.9	0.0	0.0	0.0	0.0
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	60.0	60.0	60.0	60.0	57.7	55.7	53.9	52.2	50.7	49.2	47.7	46.4	45.1	43.8	42.5	41.2	0.0	0.0	0.0	0.0
π_T	18728	18528	18328	18128	17949	17802	17676	17566	17468	17379	17299	17225	17157	17094	17035	16980	16928	16928	16928	16928
$\pi_T + \pi_{2R}$	18728	18528	18328	18128	17949	17802	17676	17566	17468	17379	17299	17225	17157	17094	17035	16980	16928	16928	16928	16928
VRR	0.47	0.47	0.47	0.47	0.45	0.43	0.42	0.41	0.39	0.38	0.37	0.36	0.35	0.34	0.33	0.32	0.00	0.00	0.00	0.00
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0
P_{2M}	70.0	70.0	70.0	70.0	71.1	72.2	73.1	73.9	74.7	75.4	76.1	76.8	77.5	78.1	78.8	79.4	0.0	0.0	0.0	0.0
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A.4: Outcome of monopolistic model without sorting information for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0	0.605	0.785	0.872	0.894	0.88	0.847	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823
Q_1	171.0	174.2	177.0	178.5	178.9	178.7	178.1	178.0	179.0	180.0	181.0	182.0	183.0	184.0	185.0	186.0	187.0	188.0	189.0	190.0
Q_{2M}	0.0	17.4	26.5	35.7	44.7	53.6	62.3	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7	67.7
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	0.0	10.5	20.8	31.1	40.0	47.2	52.8	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7
π_T	14621	14863	15259	15659	16011	16299	16530	16716	16894	17074	17254	17436	17618	17802	17986	18172	18358	18546	18734	18924
$\pi_T + \pi_{2R}$	14621	14863	15259	15659	16011	16299	16530	16716	16894	17074	17254	17436	17618	17802	17986	18172	18358	18546	18734	18924
VRR	0.00	0.61	0.79	0.87	0.89	0.88	0.85	0.78	0.69	0.62	0.56	0.51	0.47	0.43	0.40	0.37	0.35	0.33	0.31	0.29
P_1	114.5	112.9	111.5	110.7	110.5	110.7	111.0	111.0	110.5	110.0	109.5	109.0	108.5	108.0	107.5	107.0	106.5	106.0	105.5	105.0
P_{2M}	0.0	94.7	89.6	84.4	80.0	76.4	73.6	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2	72.2
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

APPENDIX B

OUTCOME OF MONOPOLISTIC SETTING WITH SORTING INFORMATION

Optimal solution of monopolistic model with sorting information and the performance measures related with it is presented in this chapter.

Table B.1: Outcome of monopolistic model with sorting information for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0.069	0.138	0.207	0.276	0.345	0.412	0.48	0.547	0.613	0.679	0.744	0.808	0.871	0.934	0.996	1	1
Q_1	184.0	184.0	184.0	184.1	184.2	184.5	184.8	185.3	185.8	186.5	187.2	188.0	188.9	189.8	190.8	191.9	193.0	194.2	198.7	203.4
Q_{2M}	0.0	0.0	0.0	128.8	128.9	129.1	129.4	129.7	130.1	130.5	131.0	131.6	132.2	132.9	133.6	134.3	135.1	135.9	139.1	142.3
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	0.0	0.0	0.0	8.9	17.8	26.7	35.7	44.7	53.6	62.7	71.7	80.7	89.8	98.9	107.9	117.0	126.2	135.4	139.1	142.3
π_T	16928	16928	16928	16973	17107	17330	17642	18044	18536	19118	19789	20551	21403	22345	23379	24504	25720	27027	28401	29808
$\pi_T + \pi_{2R}$	16928	16928	16928	16973	17107	17330	17642	18044	18536	19118	19789	20551	21403	22345	23379	24504	25720	27027	28401	29808
VRR	0.00	0.00	0.00	0.07	0.14	0.21	0.28	0.35	0.41	0.48	0.55	0.61	0.68	0.74	0.81	0.87	0.93	1.00	1.00	1.00
P_1	108.0	108.0	108.0	108.0	107.9	107.8	107.6	107.4	107.1	106.8	106.4	106.0	105.6	105.1	104.6	104.0	103.5	102.9	100.7	98.3
P_{2M}	0.0	0.0	0.0	35.6	41.1	46.6	52.1	57.6	63.2	68.7	74.2	79.7	85.1	90.6	96.0	101.5	106.9	112.3	120.5	128.8
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table B.2: Outcome of monopolistic model with sorting information for increasing c_1

c_1	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.443	0.461	0.48	0.5	0.522	0.545	0.569	0.594	0.62	0.647	0.673	0.7	0.725	0.747	0.766	0.778	0.783	0.775	0.75	0.695
Q_1	205.9	196.2	186.5	176.8	167.2	157.7	148.3	139.1	129.9	120.9	112.1	103.5	95.0	86.9	78.9	71.0	63.3	55.6	47.6	38.9
Q_{2M}	144.1	137.3	130.5	123.8	117.1	110.4	103.8	97.3	90.9	84.6	78.5	72.4	66.5	60.8	55.2	49.7	44.3	38.9	33.3	27.2
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	63.9	63.3	62.7	61.9	61.1	60.2	59.1	57.8	56.4	54.8	52.8	50.7	48.2	45.4	42.3	38.7	34.7	30.2	25.0	18.9
π_T	23041	21031	19118	17301	15581	13956	12425	10988	9643	8389	7225	6147	5155	4245	3417	2667	1995	1401	884	451
$\pi_T + \pi_{2R}$	23041	21031	19118	17301	15581	13956	12425	10988	9643	8389	7225	6147	5155	4245	3417	2667	1995	1401	884	451
VRR	0.44	0.46	0.48	0.50	0.52	0.55	0.57	0.59	0.62	0.65	0.67	0.70	0.73	0.75	0.77	0.78	0.78	0.78	0.75	0.70
P_1	97.0	101.9	106.8	111.6	116.4	121.1	125.8	130.5	135.0	139.5	144.0	148.3	152.5	156.6	160.6	164.5	168.3	172.2	176.2	180.6
P_{2M}	68.1	68.3	68.7	69.1	69.4	69.9	70.5	71.1	71.8	72.6	73.6	74.7	75.9	77.3	78.9	80.7	82.6	84.9	87.5	90.5
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table B.3: Outcome of monopolistic model with sorting information for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	0.544	0.529	0.516	0.503	0.491	0.48	0.469	0.459	0.45	0.441	0.432	0.424	0.416	0.408	0.401	0.394	0.387	0.381	0.375	0.369
Q_1	184.0	184.6	185.1	185.6	186.1	186.5	186.8	187.2	187.5	187.7	188.0	188.2	188.4	188.6	188.8	188.9	189.1	189.2	189.3	189.4
Q_{2M}	128.7	129.2	129.6	129.9	130.3	130.5	130.8	131.0	131.2	131.4	131.6	131.7	131.9	132.0	132.1	132.3	132.4	132.4	132.5	132.6
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	70.0	68.4	66.9	65.4	64.0	62.7	61.3	60.1	59.0	57.9	56.8	55.9	54.9	53.9	53.0	52.1	51.2	50.5	49.7	48.9
π_T	19378	19321	19266	19214	19165	19118	19073	19029	18988	18948	18910	18874	18838	18804	18772	18740	18710	18680	18651	18624
$\pi_T + \pi_{2R}$	19378	19321	19266	19214	19165	19118	19073	19029	18988	18948	18910	18874	18838	18804	18772	18740	18710	18680	18651	18624
VRR	0.54	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.43	0.42	0.42	0.41	0.40	0.39	0.39	0.38	0.38	0.37
P_1	108.0	107.7	107.4	107.2	107.0	106.8	106.6	106.4	106.3	106.1	106.0	105.9	105.8	105.7	105.6	105.5	105.5	105.4	105.3	105.3
P_{2M}	65.0	65.8	66.6	67.3	68.0	68.7	69.3	69.9	70.5	71.0	71.6	72.1	72.6	73.1	73.5	73.9	74.4	74.8	75.2	75.5
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table B.4: Outcome of monopolistic model with sorting information for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.29	0.532	0.69	0.765	0.783	0.767	0.735	0.696	0.655	0.615	0.577	0.542	0.51	0.48	0.453	0.428	0.405	0.384	0.365	0.347
Q_1	172.0	175.2	178.3	180.5	181.7	182.4	182.9	183.3	183.7	184.1	184.6	185.2	185.8	186.5	187.2	187.9	188.7	189.6	190.4	191.3
Q_{2M}	8.6	17.5	26.8	36.1	45.4	54.7	64.0	73.3	82.7	92.1	101.5	111.1	120.8	130.5	140.4	150.3	160.4	170.6	180.9	191.3
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	2.5	9.3	18.5	27.6	35.6	42.0	47.1	51.0	54.1	56.6	58.6	60.2	61.6	62.7	63.6	64.3	65.0	65.5	66.0	66.4
π_T	14707	15113	15596	16083	16534	16936	17294	17614	17905	18174	18426	18665	18895	19118	19334	19547	19756	19963	20168	20372
$\pi_T + \pi_{2R}$	14707	15113	15596	16083	16534	16936	17294	17614	17905	18174	18426	18665	18895	19118	19334	19547	19756	19963	20168	20372
VRR	0.29	0.53	0.69	0.77	0.78	0.77	0.74	0.70	0.66	0.62	0.58	0.54	0.51	0.48	0.45	0.43	0.41	0.38	0.37	0.35
P_1	114.0	112.4	110.8	109.8	109.1	108.8	108.5	108.4	108.2	107.9	107.7	107.4	107.1	106.8	106.4	106.0	105.6	105.2	104.8	104.4
P_{2M}	98.8	95.3	90.8	86.2	82.2	79.0	76.5	74.5	72.9	71.7	70.7	69.9	69.2	68.7	68.2	67.8	67.5	67.2	67.0	66.8
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

APPENDIX C

OUTCOME OF DUOPOLISTIC SETTING WITHOUT SORTING INFORMATION

Equilibrium solution of duopolistic model without sorting information and the performance measures related with it is presented in this chapter.

Table C.1: Outcome of duopolistic model without sorting information for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0	0	0	0	0	0	0.589	0.788	0.949	1	1	1	1	1	1	1	1
Q_1	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	180.3	183.3	186.6	189.9	193.2	196.5	199.8	203.1	206.4	209.7
Q_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	52.1	63.1	64.2	65.3	66.5	67.6	68.8	69.9	71.1	72.2	73.4
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	52.1	37.8	30.0	27.3	26.8	26.2	25.6	25.0	24.5	23.9	23.3
$Remanuf.Out$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	61.4	79.5	89.4	92.7	93.2	93.8	94.4	95.0	95.5	96.1	96.7
π_T	15665	15665	15665	15665	15665	15665	15665	15665	15665	15789	16275	16829	17471	18129	18800	19482	20175	20880	21596	22324
$\pi_T + \pi_{2R}$	16904	16904	16904	16904	16904	16904	16904	16904	16904	17499	17980	18518	19151	19817	20495	21185	21887	22601	23326	24064
VRR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.63	0.70	0.71	0.70	0.69	0.69	0.68	0.67	0.67	0.66
P_1	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	109.9	108.4	106.7	105.1	103.4	101.8	100.1	98.5	96.8	95.2
P_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	69.3	70.3	75.3	83.7	93.4	103.1	112.8	122.5	132.2	141.9	151.7
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	69.3	60.3	55.3	53.7	53.4	53.1	52.8	52.5	52.2	51.9	51.7

Table C.2: Outcome of duopolistic model without sorting information for increasing c_1

c_1	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
e	0.589	0.589	0.589	0.589	0.589	0.569	0.585	0.599	0.613	0.623	0.63	0.632	0.628	0.614	0.588	0.544	0	0	0	0
Q_1	197.0	187.0	177.0	167.0	157.0	144.5	135.1	125.7	116.4	107.2	98.1	89.0	79.9	70.7	61.3	51.5	37.0	27.0	17.0	7.0
Q_{2M}	52.1	52.1	52.1	52.1	52.1	50.6	47.3	44.0	40.7	37.5	34.3	31.2	28.0	24.7	21.5	18.0	0.0	0.0	0.0	0.0
Q_{2R}	52.1	52.1	52.1	52.1	52.1	50.6	47.3	44.0	40.7	37.5	34.3	31.2	28.0	24.7	21.5	18.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	61.4	61.4	61.4	61.4	61.4	57.6	55.3	52.7	49.9	46.7	43.3	39.4	35.1	30.4	25.2	19.6	0.0	0.0	0.0	0.0
π_T	19529	17609	15789	14069	12449	10940	9543	8239	7028	5910	4883	3947	3103	2350	1691	1126	685	365	145	25
$\pi_T + \pi_{2R}$	21379	19389	17499	15709	14019	12414	10990	9652	8402	7233	6145	5135	4200	3335	2541	1815	944	554	264	74
VRR	0.45	0.47	0.50	0.53	0.56	0.57	0.59	0.60	0.61	0.62	0.63	0.63	0.63	0.61	0.59	0.54	0.00	0.00	0.00	0.00
P_1	101.5	106.5	111.5	116.5	121.5	127.8	132.5	137.2	141.8	146.4	151.0	155.5	160.1	164.7	169.4	174.3	181.5	186.5	191.5	196.5
P_{2M}	69.3	69.3	69.3	69.3	69.3	71.2	72.3	73.6	75.0	76.6	78.4	80.3	82.4	84.8	87.4	90.2	0.0	0.0	0.0	0.0
P_{2R}	69.3	69.3	69.3	69.3	69.3	71.2	72.3	73.6	75.0	76.6	78.4	80.3	82.4	84.8	87.4	90.2	0.0	0.0	0.0	0.0

Table C.3: Outcome of duopolistic model without sorting information for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0	0	0	0.533	0.591	0.621	0.632	0.629	0.618	0.601	0.582	0.561	0.589	0.589	0.589	0.589	0.589	0.589	0.589	0.589
Q_1	170.5	171.0	171.5	173.3	174.0	174.4	174.6	174.5	174.4	174.2	173.9	173.6	176.5	177.0	177.5	178.0	178.5	179.0	179.5	180.0
Q_{2M}	0.0	0.0	0.0	17.3	21.8	26.2	30.6	34.9	39.2	43.6	47.8	52.1	52.1	52.1	52.1	52.1	52.1	52.1	52.1	52.1
Q_{2R}	0.0	0.0	0.0	17.3	21.8	26.2	30.6	34.9	39.2	43.6	47.8	52.1	52.1	52.1	52.1	52.1	52.1	52.1	52.1	52.1
$Remanuf.Out$	0.0	0.0	0.0	18.5	25.7	32.5	38.6	43.9	48.5	52.3	55.7	58.4	61.4	61.4	61.4	61.4	61.4	61.4	61.4	61.4
π_T	14535	14621	14706	14814	14956	15090	15212	15319	15410	15489	15556	15613	15700	15789	15877	15966	16055	16145	16234	16324
$\pi_T + \pi_{2R}$	14620	14792	14963	15468	15819	16126	16384	16593	16762	16898	17008	17096	17319	17499	17680	17861	18044	18227	18411	18595
VRR	0.00	0.00	0.00	0.53	0.59	0.62	0.63	0.63	0.62	0.60	0.58	0.56	0.54	0.50	0.46	0.43	0.40	0.38	0.36	0.34
P_1	114.8	114.5	114.3	113.4	113.0	112.8	112.7	112.8	112.8	112.9	113.1	113.2	111.8	111.5	111.3	111.0	110.8	110.5	110.3	110.0
P_{2M}	0.0	0.0	0.0	90.8	87.1	83.8	80.7	78.0	75.7	73.8	72.2	70.8	69.3	69.3	69.3	69.3	69.3	69.3	69.3	69.3
P_{2R}	0.0	0.0	0.0	90.8	87.1	83.8	80.7	78.0	75.7	73.8	72.2	70.8	69.3	69.3	69.3	69.3	69.3	69.3	69.3	69.3

Table C.4: Outcome of duopolistic model without sorting information for increasing γ

γ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0	0	0	0	0	0.55	0.589	0.614	0.589	0.589	0.589	0.57	0.592	0.615	0.641	0.669	0.701	0.736	0.776	0.823
Q_1	170.7	171.4	172.1	172.8	173.5	175.5	176.4	177.1	176.3	177.0	177.7	175.9	176.7	177.5	178.3	179.2	180.2	181.4	182.6	184.0
Q_{2M}	0.0	0.0	0.0	0.0	0.0	36.9	43.2	49.6	52.1	52.1	52.1	54.2	56.4	58.6	60.5	62.4	64.0	65.4	66.7	67.7
Q_{2R}	0.0	0.0	0.0	0.0	0.0	60.9	56.6	52.4	52.1	52.1	52.1	49.3	43.3	37.3	31.2	25.1	18.9	12.7	6.4	0.0
$Remanuf.Out$	0.0	0.0	0.0	0.0	0.0	53.8	58.8	62.7	61.4	61.4	61.4	58.9	59.0	58.9	58.8	58.5	58.1	57.5	56.7	55.7
π_T	14569	14689	14809	14930	15051	15209	15371	15530	15665	15789	15913	16062	16243	16432	16629	16837	17056	17288	17536	17802
$\pi_T + \pi_{2R}$	16840	16849	16857	16865	16873	17490	17531	17536	17494	17499	17503	17480	17537	17590	17642	17689	17732	17767	17792	17802
VRR	0.00	0.00	0.00	0.00	0.00	0.44	0.48	0.51	0.50	0.50	0.49	0.48	0.48	0.47	0.47	0.47	0.46	0.45	0.44	0.43
P_1	114.7	114.3	114.0	113.6	113.3	112.3	111.8	111.5	111.9	111.5	111.2	112.1	111.7	111.3	110.9	110.4	109.9	109.3	108.7	108.0
P_{2M}	0.0	0.0	0.0	0.0	0.0	73.1	70.6	68.7	69.3	69.3	69.3	70.5	70.5	70.5	70.6	70.8	70.9	71.3	71.6	72.2
P_{2R}	0.0	0.0	0.0	0.0	0.0	73.1	70.6	68.7	69.3	69.3	69.3	70.5	70.5	70.5	70.6	70.8	70.9	71.3	71.6	0.0

Table C.5: Outcome of duopolistic model without sorting information for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	1	1	0.857	0.73	0.649	0.589	0.543	0.504	0	0	0	0	0	0	0	0	0	0	0	0
Q_1	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0	177.0
Q_{2M}	40.0	40.0	44.1	48.0	50.5	52.1	53.0	53.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Q_{2R}	40.0	40.0	44.1	48.0	50.5	52.1	53.0	53.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	80.0	80.0	75.6	70.1	65.6	61.4	57.6	53.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π_T	16465	16265	16084	15960	15865	15789	15725	15670	15665	15665	15665	15665	15665	15665	15665	15665	15665	15665	15665	15665
$\pi_T + \pi_{2R}$	18504	18304	18037	17814	17642	17499	17378	17270	16904	16904	16904	16904	16904	16904	16904	16904	16904	16904	16904	16904
VRR	0.65	0.65	0.61	0.57	0.53	0.50	0.46	0.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P_1	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5	111.5
P_{2M}	60.0	60.0	62.2	64.9	67.2	69.3	71.2	73.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
P_{2R}	60.0	60.0	62.2	64.9	67.2	69.3	71.2	73.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table C.6: Outcome of duopolistic model without sorting information for increasing a_{2R}

a_{2R}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.788	0.639	0.589	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
Q_1	178.1	178.1	178.1	178.1	178.1	178.1	178.1	180.3	177.0	177.0	174.6	174.6	174.6	174.6	174.6	174.6	174.6	174.6	174.6	174.6
Q_{2M}	62.3	62.3	62.3	62.3	62.3	62.3	62.3	63.1	61.2	52.1	49.2	49.2	49.2	49.2	49.2	49.2	49.2	49.2	49.2	49.2
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.4	29.9	52.1	61.1	61.1	61.1	61.1	61.1	61.1	61.1	61.1	61.1	61.1
$Remanuf.Out$	52.8	52.8	52.8	52.8	52.8	52.8	52.8	59.5	58.3	61.4	58.5	58.5	58.5	58.5	58.5	58.5	58.5	58.5	58.5	58.5
π_T	16530	16530	16530	16530	16530	16530	16530	16275	16022	15789	15721	15721	15721	15721	15721	15721	15721	15721	15721	15721
$\pi_T + \pi_{2R}$	17777	17777	17777	17777	17777	17777	17777	17585	17444	17499	17689	18013	18337	18660	18984	19308	19632	19956	20280	20604
VRR	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.47	0.47	0.50	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
P_1	111.0	111.0	111.0	111.0	111.0	111.0	111.0	109.9	111.5	111.5	112.7	112.7	112.7	112.7	112.7	112.7	112.7	112.7	112.7	112.7
P_{2M}	73.6	73.6	73.6	73.6	73.6	73.6	73.6	70.3	70.9	69.3	70.8	70.8	70.8	70.8	70.8	70.8	70.8	70.8	70.8	70.8
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.3	60.9	69.3	80.8	90.8	100.8	110.8	120.8	130.8	140.8	150.8	160.8	170.8

APPENDIX D

OUTCOME OF DUOPOLISTIC SETTING WITH SORTING INFORMATION FOR ONLY OEM

Equilibrium solution of duopolistic model with sorting information for only OEM and the performance measures related with it is presented in this chapter.

Table D.1: Outcome of duopolistic model with sorting information for only OEM for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0.001	0.106	0.212	0.28	0.281	0.281	0.316	0.474	0.632	0.79	0.947	1	1	1	1	1	1
Q_1	177.0	177.0	177.0	177.0	177.1	177.5	178.2	179.5	180.2	181.6	183.0	184.8	187.2	190.0	193.2	196.5	199.8	200.0	200.0	200.0
Q_{2M}	0.0	0.0	0.0	0.0	62.0	62.1	62.4	62.8	63.1	63.6	64.1	64.7	65.5	66.5	67.6	68.8	69.9	70.0	70.0	70.0
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.1	47.7	44.2	36.5	28.9	26.2	25.6	25.0	25.0	25.0	25.0
$Remanuf.Out$	0.0	0.0	0.0	0.0	6.6	13.2	17.5	17.7	17.7	26.8	53.0	68.8	80.6	90.4	93.8	94.4	95.0	95.0	95.0	95.0
π_T	15665	15665	15665	15664	15697	15796	15957	16133	16310	16491	16743	17099	17562	18135	18800	19482	20175	20875	21575	22275
$\pi_T + \pi_{2R}$	16904	16904	16904	16903	16937	17039	17204	17389	17571	17784	18280	18782	19287	19840	20495	21185	21887	22588	23288	23988
VRR	0.00	0.00	0.00	0.00	0.05	0.11	0.14	0.14	0.14	0.21	0.41	0.53	0.61	0.68	0.69	0.69	0.68	0.68	0.68	0.68
P_1	111.5	111.5	111.5	111.5	111.5	111.3	110.9	110.3	109.9	109.2	108.5	107.6	106.4	105.0	103.4	101.8	100.1	100.0	100.0	100.0
P_{2M}	0.0	0.0	0.0	0.0	46.7	53.4	61.3	71.2	81.1	86.6	83.5	85.6	89.7	94.8	103.1	112.8	122.5	132.5	142.5	152.5
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	86.6	73.5	65.6	59.7	54.8	53.1	52.8	52.5	52.5	52.5	52.5

Table D.2: Outcome of duopolistic model with sorting information for only OEM for increasing c_1

c_1	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.315	0.316	0.316	0.316	0.314	0.311	0.307	0.445	0.455	0.465	0.471	0.474	0.472	0.464	0.446	0.416	0.37	0.254	0.22	0.094
Q_1	200.0	191.5	181.6	171.7	161.7	151.8	141.8	130.0	120.4	110.9	101.4	91.9	82.4	72.9	63.3	53.5	43.3	32.1	21.5	9.0
Q_{2M}	70.0	67.0	63.6	60.1	56.6	53.1	49.6	45.5	42.1	38.8	35.5	32.2	28.8	25.5	22.2	18.7	15.2	11.2	7.5	3.2
Q_{2R}	17.4	19.4	21.1	22.8	23.6	23.9	23.6	45.5	42.1	38.8	35.5	32.2	28.8	25.5	22.2	18.7	15.2	0.0	0.0	0.0
$Remanuf.Out$	27.5	27.3	26.8	26.2	25.2	24.0	22.5	40.5	38.3	36.1	33.4	30.5	27.2	23.7	19.8	15.6	11.2	2.9	1.7	0.3
π_T	20320	18356	16491	14724	13057	11490	10022	8667	7414	6258	5197	4230	3359	2582	1901	1317	833	454	184	31
$\pi_T + \pi_{2R}$	21735	19715	17784	15952	14217	12580	11040	9877	8581	7376	6254	5217	4262	3389	2594	1880	1250	679	334	94
VRR	0.20	0.20	0.21	0.22	0.22	0.23	0.23	0.45	0.46	0.47	0.47	0.47	0.47	0.46	0.45	0.42	0.37	0.13	0.11	0.05
P_1	100.0	104.3	109.2	114.2	119.2	124.1	129.1	135.0	139.8	144.6	149.3	154.1	158.8	163.6	168.4	173.3	178.4	184.0	189.3	195.5
P_{2M}	86.2	86.4	86.6	86.9	87.4	88.0	88.8	79.8	80.8	82.0	83.3	84.8	86.4	88.2	90.1	92.2	94.4	98.6	99.2	99.9
P_{2R}	86.2	86.4	86.6	86.9	87.4	88.0	88.8	79.8	80.8	82.0	83.3	84.8	86.4	88.2	90.1	92.2	94.4	0.0	0.0	0.0

Table D.3: Outcome of duopolistic model with sorting information for only OEM for increasing γ

γ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.176	0.255	0.258	0.261	0.264	0.276	0.296	0.308	0.314	0.316	0.314	0.31	0.304	0.302	0.308	0.315	0.323	0.332	0.341	0.413
Q_1	171.1	172.4	173.6	174.8	176.0	177.2	178.3	179.5	180.5	181.6	182.6	183.6	184.6	185.6	186.3	187.1	188.1	188.9	188.3	185.8
Q_{2M}	6.0	12.1	18.2	24.5	30.8	37.2	43.7	50.3	56.9	63.6	70.3	77.1	84.0	90.9	97.8	104.8	111.9	119.0	125.2	130.1
Q_{2R}	0.0	0.1	0.5	0.7	0.7	8.7	20.2	23.8	23.5	21.1	16.8	11.4	4.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	1.1	3.1	4.8	6.6	8.3	12.7	18.9	22.8	25.2	26.8	27.3	27.4	27.0	27.5	30.1	33.0	36.2	39.5	42.7	53.7
π_T	14601	14803	15012	15222	15432	15643	15855	16068	16279	16491	16701	16910	17118	17326	17532	17736	17936	18131	18321	18536
$\pi_T + \pi_{2R}$	16877	16976	17078	17180	17280	17383	17496	17602	17696	17784	17865	17944	18024	18106	18184	18259	18331	18396	18453	18536
VRR	0.01	0.03	0.04	0.05	0.07	0.10	0.15	0.18	0.20	0.21	0.21	0.21	0.21	0.21	0.23	0.25	0.27	0.30	0.32	0.41
P_1	114.5	113.8	113.2	112.6	112.0	111.4	110.9	110.3	109.8	109.2	108.7	108.2	107.7	107.2	106.9	106.5	106.0	105.6	105.9	107.1
P_{2M}	99.5	98.4	97.6	96.7	95.8	93.7	90.6	88.6	87.4	86.6	86.3	86.3	86.5	86.2	84.9	83.5	81.9	80.2	78.7	73.1
P_{2R}	0.0	98.4	97.6	96.7	95.8	93.7	90.6	88.6	87.4	86.6	86.3	86.3	86.5	86.2	84.9	83.5	81.9	80.2	0.0	0.0

Table D.4: Outcome of duopolistic model with sorting information for only OEM for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.127	0.248	0.333	0.4	0.444	0.466	0.474	0.471	0.462	0.449	0.307	0.312	0.315	0.316	0.315	0.313	0.309	0.305	0.3	0.304
Q_1	170.7	171.7	172.8	173.8	174.8	175.5	176.1	176.5	176.9	177.3	179.3	180.1	180.8	181.6	182.3	183.1	183.8	184.5	185.2	185.9
Q_{2M}	4.3	8.6	13.0	17.4	21.9	26.3	30.8	35.3	39.8	44.3	49.3	54.0	58.8	63.6	68.4	73.2	78.1	83.0	88.0	93.0
Q_{2R}	0.0	0.0	13.0	17.4	21.9	26.3	30.8	35.3	39.8	44.3	23.7	23.9	23.0	21.1	18.2	14.8	10.4	5.8	0.5	0.3
$Remanuf.Out$	0.5	2.1	8.6	13.9	19.4	24.5	29.2	33.3	36.8	39.8	22.4	24.3	25.8	26.8	27.3	27.6	27.3	27.1	26.5	28.3
π_T	14551	14684	14835	14999	15166	15330	15486	15634	15774	15906	16037	16189	16340	16491	16641	16791	16940	17089	17237	17385
$\pi_T + \pi_{2R}$	14637	14856	15161	15507	15848	16161	16441	16688	16907	17102	17050	17297	17541	17784	18024	18266	18507	18751	18997	19244
VRR	0.06	0.12	0.33	0.40	0.44	0.47	0.47	0.47	0.46	0.45	0.23	0.23	0.22	0.21	0.20	0.19	0.18	0.16	0.15	0.15
P_1	114.7	114.2	113.6	113.1	112.6	112.3	112.0	111.8	111.6	111.4	110.4	110.0	109.6	109.2	108.9	108.5	108.1	107.8	107.4	107.1
P_{2M}	99.7	98.9	95.7	93.0	90.3	87.7	85.4	83.4	81.6	80.1	88.8	87.8	87.1	86.6	86.4	86.2	86.3	86.5	86.7	85.8
P_{2R}	0.0	0.0	95.7	93.0	90.3	87.7	85.4	83.4	81.6	80.1	88.8	87.8	87.1	86.6	86.4	86.2	86.3	86.5	86.7	85.8

Table D.5: Outcome of duopolistic model with sorting information for only OEM for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	0.633	0.526	0.451	0.395	0.351	0.316	0.288	0.282	0.282	0.282	0.282	0.282	0.282	0.282	0.281	0.281	0.281	0.281	0.281	0.281
Q_1	180.5	181.1	181.4	181.5	181.6	181.6	181.6	181.6	181.6	181.6	181.6	181.6	181.6	181.6	181.1	181.1	181.1	181.1	181.1	181.1
Q_{2M}	63.2	63.4	63.5	63.5	63.6	63.6	63.6	63.6	63.6	63.6	63.6	63.6	63.6	63.6	63.4	63.4	63.4	63.4	63.4	63.4
Q_{2R}	44.9	48.1	47.3	42.6	33.8	21.1	4.9	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0
$Remanuf.Out$	68.4	58.6	50.0	41.9	34.2	26.8	19.7	18.0	18.0	18.0	18.0	18.0	18.0	18.0	17.8	17.8	17.8	17.8	17.8	17.8
π_T	16690	16624	16576	16540	16513	16491	16472	16456	16440	16425	16409	16393	16377	16361	16345	16329	16313	16298	16282	16266
$\pi_T + \pi_{2R}$	18357	18211	18073	17952	17854	17784	17745	17728	17712	17696	17680	17664	17648	17632	17613	17597	17581	17565	17550	17534
VRR	0.54	0.46	0.39	0.33	0.27	0.21	0.16	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
P_1	109.8	109.5	109.3	109.3	109.2	109.2	109.2	109.2	109.2	109.2	109.2	109.2	109.2	109.2	109.5	109.5	109.5	109.5	109.5	109.5
P_{2M}	65.8	70.7	75.0	79.0	82.9	86.6	90.1	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.1	91.1	91.1	91.1	91.1	91.1
P_{2R}	65.8	70.7	75.0	79.0	82.9	86.6	90.1	91.0	91.0	91.0	91.0	91.0	91.0	91.0	0.0	0.0	0.0	0.0	0.0	0.0

Table D.6: Outcome of duopolistic model with sorting information for only OEM for increasing a_{2R}

a_{2R}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.632	0.632	0.632	0.632	0.632	0.632	0.632	0.474	0.395	0.316	0.243	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382
Q_1	181.4	181.4	181.4	181.4	181.4	181.4	181.4	183.0	182.2	181.6	181.0	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6
Q_{2M}	63.5	63.5	63.5	63.5	63.5	63.5	63.5	64.1	63.8	63.6	63.4	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.5	17.1	21.1	0.0	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5
$Remanuf.Out$	40.1	40.1	40.1	40.1	40.1	40.1	40.1	33.0	32.0	26.8	15.4	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8
π_T	16858	16858	16858	16858	16858	16858	16858	16743	16604	16491	16403	16380	16380	16380	16380	16380	16380	16380	16380	16380
$\pi_T + \pi_{2R}$	18128	18128	18128	18128	18128	18128	18128	18027	17902	17784	17670	18198	18436	18675	18914	19153	19391	19630	19869	20108
VRR	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.26	0.25	0.21	0.12	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
P_1	109.3	109.3	109.3	109.3	109.3	109.3	109.3	108.5	108.9	109.2	109.5	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7
P_{2M}	79.9	79.9	79.9	79.9	79.9	79.9	79.9	83.5	84.0	86.6	92.3	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	63.5	74.0	86.6	0.0	96.1	106.1	116.1	126.1	136.1	146.1	156.1	166.1	176.1

APPENDIX E

OUTCOME OF DUOPOLISTIC SETTING WITH SORTING INFORMATION FOR BOTH OEM AND IR

Equilibrium solution of duopolistic model with sorting information for both OEM and IR, and the performance measures related with it is presented in this chapter.

Table E.1: Outcome of duopolistic model with sorting information for both OEM and IR for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0.001	0.064	0.128	0.192	0.256	0.319	0.382	0.445	0.507	0.948	1	1	1	1	1	1	1
Q_1	177.0	177.0	177.0	177.0	177.0	177.2	177.4	177.7	178.1	178.6	179.2	179.9	186.6	189.9	193.2	196.5	199.8	200.0	200.0	200.0
Q_{2M}	0.0	0.0	0.0	0.0	62.0	62.0	62.1	62.2	62.3	62.5	62.7	63.0	65.3	66.5	67.6	68.8	69.9	70.0	70.0	70.0
Q_{2R}	0.0	0.0	0.0	0.0	62.0	62.0	62.1	62.2	62.3	62.5	62.7	63.0	30.6	26.8	26.2	25.6	25.0	25.0	25.0	25.0
$Remanuf.Out$	0.0	0.0	0.0	0.0	7.9	15.9	23.8	31.8	39.8	47.8	55.8	63.8	91.0	93.2	93.8	94.4	95.0	95.0	95.0	95.0
π_T	15665	15665	15665	15664	15684	15744	15843	15982	16161	16380	16639	16938	17476	18129	18800	19482	20175	20875	21575	22275
$\pi_T + \pi_{2R}$	16904	16904	16904	16903	17146	17398	17658	17928	18205	18493	18789	19094	19204	19817	20495	21185	21887	22588	23288	23988
VRR	0.00	0.00	0.00	0.00	0.06	0.13	0.19	0.26	0.32	0.38	0.45	0.51	0.70	0.70	0.69	0.69	0.68	0.68	0.68	0.68
P_1	111.5	111.5	111.5	111.5	111.5	111.4	111.3	111.2	111.0	110.7	110.4	110.1	106.7	105.1	103.4	101.8	100.1	100.0	100.0	100.0
P_{2M}	0.0	0.0	0.0	0.0	46.0	52.1	58.1	64.1	70.1	76.1	82.1	88.1	84.5	93.4	103.1	112.8	122.5	132.5	142.5	152.5
P_{2R}	0.0	0.0	0.0	0.0	96.0	92.1	88.1	84.1	80.1	76.1	72.1	68.1	54.5	53.4	53.1	52.8	52.5	52.5	52.5	52.5

Table E.2: Outcome of duopolistic model with sorting information for both OEM and IR for increasing c_1

c_1	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.358	0.37	0.382	0.395	0.407	0.42	0.433	0.445	0.455	0.465	0.471	0.474	0.472	0.464	0.446	0.416	0.37	0.303	0.211	0.093
Q_1	198.3	188.4	178.6	168.8	159.1	149.4	139.7	130.0	120.4	110.9	101.4	91.9	82.4	72.9	63.3	53.5	43.3	32.6	21.2	8.9
Q_{2M}	69.4	65.9	62.5	59.1	55.7	52.3	48.9	45.5	42.1	38.8	35.5	32.2	28.8	25.5	22.2	18.7	15.2	11.4	7.4	3.1
Q_{2R}	69.4	65.9	62.5	59.1	55.7	52.3	48.9	45.5	42.1	38.8	35.5	32.2	28.8	25.5	22.2	18.7	15.2	11.4	7.4	3.1
$Remanuf.Out$	49.7	48.8	47.8	46.7	45.3	43.9	42.3	40.5	38.3	36.1	33.4	30.5	27.2	23.7	19.8	15.6	11.2	6.9	3.1	0.6
π_T	20149	18215	16380	14642	13003	11461	10015	8667	7414	6258	5197	4230	3359	2582	1901	1317	833	452	183	31
$\pi_T + \pi_{2R}$	22410	20403	18493	16680	14963	13342	11815	10382	9040	7791	6630	5556	4567	3662	2839	2098	1441	876	423	111
VRR	0.36	0.37	0.38	0.40	0.41	0.42	0.43	0.45	0.46	0.47	0.47	0.47	0.47	0.46	0.45	0.42	0.37	0.30	0.21	0.09
P_1	100.9	105.8	110.7	115.6	120.5	125.3	130.2	135.0	139.8	144.6	149.3	154.1	158.8	163.6	168.4	173.3	178.4	183.7	189.4	195.6
P_{2M}	75.2	75.6	76.1	76.7	77.3	78.0	78.8	79.8	80.8	82.0	83.3	84.8	86.4	88.2	90.1	92.2	94.4	96.5	98.4	99.7
P_{2R}	75.2	75.6	76.1	76.7	77.3	78.0	78.8	79.8	80.8	82.0	83.3	84.8	86.4	88.2	90.1	92.2	94.4	96.5	98.4	99.7

Table E.3: Outcome of duopolistic model with sorting information for both OEM and IR for increasing γ

γ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.132	0.209	0.26	0.294	0.32	0.339	0.354	0.365	0.374	0.382	0.388	0.393	0.398	0.401	0.404	0.406	0.408	0.41	0.411	0.413
Q_1	170.9	171.9	172.9	173.8	174.7	175.5	176.3	177.1	177.9	178.6	179.4	180.1	180.8	181.6	182.3	183.0	183.7	184.4	185.1	185.8
Q_{2M}	6.0	12.0	18.2	24.3	30.6	36.9	43.2	49.6	56.0	62.5	69.1	75.6	82.3	89.0	95.7	102.5	109.3	116.2	123.1	130.1
Q_{2R}	113.6	108.3	102.9	97.3	91.7	86.0	80.2	74.4	68.5	62.5	56.5	50.4	44.3	38.1	31.9	25.6	19.3	12.9	6.5	0.0
$Remanuf.Out$	15.8	25.1	31.5	35.8	39.1	41.6	43.7	45.2	46.6	47.8	48.7	49.5	50.4	51.0	51.6	52.0	52.5	52.9	53.3	53.7
π_T	14593	14764	14950	15144	15344	15547	15752	15960	16169	16380	16592	16805	17018	17233	17449	17665	17882	18099	18317	18536
$\pi_T + \pi_{2R}$	17648	18004	18192	18296	18365	18409	18440	18462	18479	18493	18503	18511	18518	18524	18528	18531	18533	18535	18536	18536
VRR	0.13	0.21	0.26	0.29	0.32	0.34	0.35	0.37	0.37	0.38	0.39	0.39	0.40	0.40	0.40	0.41	0.41	0.41	0.41	0.41
P_1	114.6	114.1	113.6	113.1	112.7	112.3	111.9	111.5	111.1	110.7	110.3	110.0	109.6	109.2	108.9	108.5	108.2	107.8	107.5	107.1
P_{2M}	92.1	87.4	84.3	82.1	80.4	79.2	78.2	77.4	76.7	76.1	75.6	75.2	74.8	74.5	74.2	74.0	73.8	73.5	73.4	73.1
P_{2R}	92.1	87.4	84.3	82.1	80.4	79.2	78.2	77.4	76.7	76.1	75.6	75.2	74.8	74.5	74.2	74.0	73.8	73.5	73.4	0.0

Table E.4: Outcome of duopolistic model with sorting information for both OEM and IR for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.126	0.24	0.333	0.4	0.444	0.466	0.474	0.471	0.462	0.449	0.433	0.416	0.399	0.382	0.366	0.35	0.335	0.321	0.308	0.295
Q_1	170.7	171.7	172.8	173.8	174.8	175.5	176.1	176.5	176.9	177.3	177.6	177.9	178.3	178.6	179.0	179.4	179.8	180.1	180.5	181.0
Q_{2M}	4.3	8.6	13.0	17.4	21.9	26.3	30.8	35.3	39.8	44.3	48.8	53.4	57.9	62.5	67.1	71.8	76.4	81.0	85.7	90.5
Q_{2R}	4.3	8.6	13.0	17.4	21.9	26.3	30.8	35.3	39.8	44.3	48.8	53.4	57.9	62.5	67.1	71.8	76.4	81.0	85.7	90.5
$Remanuf.Out$	1.1	4.1	8.6	13.9	19.4	24.5	29.2	33.3	36.8	39.8	42.3	44.4	46.2	47.8	49.1	50.2	51.2	52.0	52.8	53.4
π_T	14551	14682	14835	14999	15166	15330	15486	15634	15774	15906	16032	16152	16268	16380	16489	16595	16699	16801	16902	17002
$\pi_T + \pi_{2R}$	14669	14973	15334	15716	16091	16442	16766	17061	17335	17591	17831	18059	18280	18493	18702	18906	19108	19306	19504	19701
VRR	0.13	0.24	0.33	0.40	0.44	0.47	0.47	0.47	0.46	0.45	0.43	0.42	0.40	0.38	0.37	0.35	0.34	0.32	0.31	0.30
P_1	114.7	114.2	113.6	113.1	112.6	112.3	112.0	111.8	111.6	111.4	111.2	111.1	110.9	110.7	110.5	110.3	110.1	110.0	109.8	109.5
P_{2M}	99.5	97.9	95.7	93.0	90.3	87.7	85.4	83.4	81.6	80.1	78.9	77.8	76.9	76.1	75.4	74.9	74.4	74.0	73.6	73.3
P_{2R}	99.5	97.9	95.7	93.0	90.3	87.7	85.4	83.4	81.6	80.1	78.9	77.8	76.9	76.1	75.4	74.9	74.4	74.0	73.6	73.3

Table E.5: Outcome of duopolistic model with sorting information for both OEM and IR for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	0.484	0.459	0.437	0.417	0.399	0.382	0.367	0.353	0.34	0.328	0.317	0.307	0.297	0.288	0.279	0.271	0.264	0.257	0.25	0.243
Q_1	177.0	177.5	177.9	178.2	178.4	178.6	178.8	178.9	179.1	179.2	179.2	179.3	179.4	179.4	179.4	179.5	179.5	179.5	179.5	179.5
Q_{2M}	62.0	62.1	62.3	62.4	62.4	62.5	62.6	62.6	62.7	62.7	62.7	62.8	62.8	62.8	62.8	62.8	62.8	62.8	62.8	62.8
Q_{2R}	62.0	62.1	62.3	62.4	62.4	62.5	62.6	62.6	62.7	62.7	62.7	62.8	62.8	62.8	62.8	62.8	62.8	62.8	62.8	62.8
$Remanuf.Out$	60.0	57.0	54.4	52.0	49.8	47.8	45.9	44.2	42.6	41.1	39.8	38.5	37.3	36.2	35.0	34.1	33.2	32.3	31.4	30.5
π_T	16564	16520	16480	16444	16410	16380	16352	16326	16302	16280	16259	16239	16221	16204	16188	16173	16158	16145	16132	16120
$\pi_T + \pi_{2R}$	18703	18660	18617	18575	18533	18493	18454	18416	18380	18345	18311	18279	18248	18218	18188	18161	18135	18109	18084	18059
VRR	0.48	0.46	0.44	0.42	0.40	0.38	0.37	0.35	0.34	0.33	0.32	0.31	0.30	0.29	0.28	0.27	0.26	0.26	0.25	0.24
P_1	111.5	111.3	111.1	110.9	110.8	110.7	110.6	110.6	110.5	110.4	110.4	110.4	110.3	110.3	110.3	110.3	110.3	110.3	110.3	110.3
P_{2M}	70.0	71.5	72.8	74.0	75.1	76.1	77.0	77.9	78.7	79.4	80.1	80.7	81.4	81.9	82.5	83.0	83.4	83.9	84.3	84.7
P_{2R}	70.0	71.5	72.8	74.0	75.1	76.1	77.0	77.9	78.7	79.4	80.1	80.7	81.4	81.9	82.5	83.0	83.4	83.9	84.3	84.7

Table E.6: Outcome of duopolistic model with sorting information for both OEM and IR for increasing a_{2R}

a_{2R}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.632	0.632	0.632	0.632	0.632	0.632	0.711	0.632	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382
Q_1	181.4	181.4	181.4	181.4	181.4	181.4	182.5	181.4	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6	178.6
Q_{2M}	63.5	63.5	63.5	63.5	63.5	63.5	63.9	63.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	10.3	31.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5	62.5
$Remanuf.Out$	40.1	40.1	40.1	40.1	40.1	40.1	52.7	60.1	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8	47.8
π_T	16858	16858	16858	16858	16858	16858	16672	16458	16380	16380	16380	16380	16380	16380	16380	16380	16380	16380	16380	16380
$\pi_T + \pi_{2R}$	18128	18128	18128	18128	18128	18128	17976	17926	18254	18493	18731	18970	19209	19448	19686	19925	20164	20403	20642	20880
VRR	0.32	0.32	0.32	0.32	0.32	0.32	0.41	0.47	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
P_1	109.3	109.3	109.3	109.3	109.3	109.3	108.8	109.3	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7	110.7
P_{2M}	79.9	79.9	79.9	79.9	79.9	79.9	73.6	70.0	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1	76.1
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	43.6	50.0	66.1	76.1	86.1	96.1	106.1	116.1	126.1	136.1	146.1	156.1	166.1	176.1

APPENDIX F

OUTCOME OF DUOPOLISTIC SETTING WITH ASSUMPTION OF MONOPOLY

Equilibrium solution of duopolistic model with assumption of monopoly and the performance measures related with it is presented in this chapter.

Table F.1: Outcome of duopolistic model with assumption of monopoly for increasing a_{2M}

a_{2M}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0	0	0	0	0	0	0	0.671	0.758	0.823	0.876	0.922	0.963	1	1	1	1	1	1	1
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.6	189.3	194.0	198.7
Q_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.8	29.6	45.1	58.7	64.4	64.4	64.4	64.4	64.4	64.6	66.2	67.9	69.5
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	64.4	55.9	45.1	35.9	31.0	29.3	27.8	27.8	27.8	27.7	26.9	26.1	25.2
$Remanuf.Out$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	51.8	64.8	74.3	82.9	88.0	90.2	92.2	92.2	92.2	92.3	93.1	93.9	94.8
π_T	15640	15640	15640	15640	15640	15640	15640	15227	15317	15652	16196	16827	17448	18111	18755	19399	20052	20779	21515	22260
$\pi_T + \pi_{2R}$	16928	16928	16928	16928	16928	16928	16928	17564	17504	17629	17978	18525	19135	19786	20430	21074	21728	22465	23212	23969
VRR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.50	0.58	0.64	0.68	0.70	0.72	0.72	0.72	0.71	0.70	0.69	0.68
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	107.7	105.4	103.0	100.7
P_{2M}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	54.1	57.6	62.9	68.6	76.0	84.9	93.9	103.9	113.9	123.9	133.4	143.0	152.6
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	74.1	67.6	62.9	58.6	56.0	54.9	53.9	53.9	53.9	53.9	53.4	53.0	52.6

Table F.2: Outcome of duopolistic model with assumption of monopoly for increasing c_1

c_1	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.83	0.854	0.874	0.888	0.894	0.89	0.869	0.822	0.717
Q_1	204.0	194.0	184.0	174.0	164.0	154.0	144.0	134.0	124.0	114.0	104.0	94.6	86.8	79.3	71.9	64.7	57.5	50.1	42.1	32.1
Q_{2M}	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	43.4	39.9	36.4	33.1	30.4	27.7	25.2	22.7	20.1	17.5	14.7	11.2
Q_{2R}	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	43.4	39.9	36.4	33.1	30.4	27.7	25.2	22.7	20.1	17.5	14.7	11.2
$Remanuf.Out$	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3	71.4	65.7	59.9	54.9	51.9	48.5	44.7	40.5	35.8	30.5	24.2	16.1
π_T	19392	17472	15652	13932	12312	10792	9372	8052	6856	5773	4774	3848	2982	2202	1510	905	392	-22	-319	-449
$\pi_T + \pi_{2R}$	21509	19519	17629	15839	14149	12559	11069	9679	8438	7322	6273	5291	4384	3554	2798	2115	1504	966	504	130
VRR	0.52	0.55	0.58	0.61	0.65	0.69	0.74	0.79	0.82	0.82	0.82	0.83	0.85	0.87	0.89	0.89	0.89	0.87	0.82	0.72
P_1	98.0	103.0	108.0	113.0	118.0	123.0	128.0	133.0	138.0	143.0	148.0	152.7	156.6	160.4	164.0	167.6	171.2	174.9	179.0	184.0
P_{2M}	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	64.3	67.2	70.0	72.5	74.1	75.8	77.6	79.7	82.1	84.8	87.9	92.0
P_{2R}	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	64.3	67.2	70.0	72.5	74.1	75.8	77.6	79.7	82.1	84.8	87.9	92.0

Table F.3: Outcome of duopolistic model with assumption of monopoly for increasing γ

γ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0
Q_{2M}	6.4	12.9	19.3	25.8	32.2	38.6	45.1	45.1	45.1	45.1	45.1	45.1	45.1	48.4	51.6	54.8	58.0	61.2	64.5	67.7
Q_{2R}	64.5	61.2	58.0	54.8	51.6	48.4	45.1	45.1	45.1	45.1	45.1	45.1	45.1	38.6	32.2	25.8	19.3	12.9	6.4	0.0
$Remanuf.Out$	58.3	61.0	63.6	66.3	68.9	71.6	74.2	74.3	74.3	74.3	74.3	74.3	74.2	71.6	68.9	66.3	63.6	61.0	58.3	55.7
π_T	13944	14199	14441	14668	14881	15080	15265	15394	15523	15652	15781	15910	16039	16270	16508	16752	17004	17263	17529	17802
$\pi_T + \pi_{2R}$	17798	17788	17770	17746	17714	17675	17630	17629	17629	17629	17629	17629	17630	17675	17714	17746	17770	17788	17798	17802
VRR	0.45	0.47	0.49	0.51	0.54	0.56	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.56	0.54	0.51	0.49	0.47	0.45	0.43
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0
P_{2M}	70.8	69.5	68.2	66.9	65.5	64.2	62.9	62.9	62.9	62.9	62.9	62.9	62.9	64.2	65.5	66.9	68.2	69.5	70.8	72.2
P_{2R}	70.8	69.5	68.2	66.9	65.5	64.2	62.9	62.9	62.9	62.9	62.9	62.9	62.9	64.2	65.5	66.9	68.2	69.5	70.8	0.0

Table F.4: Outcome of duopolistic model with assumption of monopoly for increasing τ

τ	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
e	0	0.605	0.785	0.872	0.894	0.88	0.847	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823
Q_1	171.0	174.2	177.0	178.5	178.9	178.7	178.1	178.0	179.0	180.0	181.0	182.0	183.0	184.0	185.0	186.0	187.0	188.0	189.0	190.0
Q_{2M}	0.0	8.7	13.3	17.9	22.4	26.8	31.2	35.6	40.3	45.0	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1
Q_{2R}	0.0	8.7	13.3	17.9	22.4	26.8	31.2	35.6	40.3	45.0	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1	45.1
$Remanuf.Out$	0.0	10.5	20.8	31.1	40.0	47.2	52.8	58.6	66.3	74.1	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3	74.3
π_T	14535	14469	14534	14656	14811	14969	15115	15226	15285	15314	15397	15482	15567	15652	15737	15822	15907	15992	16077	16162
$\pi_T + \pi_{2R}$	14621	14863	15259	15659	16011	16299	16530	16712	16838	16905	17082	17263	17446	17629	17814	17999	18186	18373	18562	18751
VRR	0.00	0.61	0.79	0.87	0.89	0.88	0.85	0.82	0.82	0.82	0.75	0.68	0.62	0.58	0.54	0.50	0.47	0.44	0.41	0.39
P_1	114.5	112.9	111.5	110.7	110.5	110.7	111.0	111.0	110.5	110.0	109.5	109.0	108.5	108.0	107.5	107.0	106.5	106.0	105.5	105.0
P_{2M}	0.0	94.7	89.6	84.4	80.0	76.4	73.6	70.7	66.9	63.0	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9
P_{2R}	0.0	94.7	89.6	84.4	80.0	76.4	73.6	70.7	66.9	63.0	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9	62.9

Table F.5: Outcome of duopolistic model with assumption of monopoly for increasing k

k	**0**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1400**	**1600**	**1800**	**2000**	**2200**	**2400**	**2600**	**2800**	**3000**	**3200**	**3400**	**3600**	**3800**
e	1	1	1	1	0.897	0.823	0.766	0.72	0.682	0.649	0.62	0.595	0.573	0.552	0.533	0.516	0	0	0	0
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0
Q_{2M}	40.0	40.0	40.0	40.0	42.9	45.1	46.9	48.4	49.5	50.5	51.3	52.0	52.5	52.9	53.1	53.3	0.0	0.0	0.0	0.0
Q_{2R}	40.0	40.0	40.0	40.0	42.9	45.1	46.9	48.4	49.5	50.5	51.3	52.0	52.5	52.9	53.1	53.3	0.0	0.0	0.0	0.0
$Remanuf.Out$	80.0	80.0	80.0	80.0	76.9	74.3	71.9	69.6	67.6	65.6	63.7	61.8	60.1	58.4	56.6	55.0	0.0	0.0	0.0	0.0
π_T	16440	16240	16040	15840	15736	15652	15581	15520	15466	15419	15378	15339	15304	15273	15245	15219	15640	15640	15640	15640
$\pi_T + \pi_{2R}$	18528	18328	18128	17928	17764	17629	17515	17414	17325	17245	17172	17105	17044	16987	16934	16885	16928	16928	16928	16928
VRR	0.62	0.62	0.62	0.62	0.60	0.58	0.56	0.54	0.52	0.51	0.49	0.48	0.47	0.45	0.44	0.43	0.00	0.00	0.00	0.00
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0
P_{2M}	60.0	60.0	60.0	60.0	61.5	62.9	64.1	65.2	66.2	67.2	68.2	69.1	69.9	70.8	71.7	72.5	0.0	0.0	0.0	0.0
P_{2R}	60.0	60.0	60.0	60.0	61.5	62.9	64.1	65.2	66.2	67.2	68.2	69.1	69.9	70.8	71.7	72.5	0.0	0.0	0.0	0.0

Table F.6: Outcome of duopolistic model with assumption of monopoly for increasing a_{2R}

a_{2R}	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**	**190**	**200**
e	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823	0.823
Q_1	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0
Q_{2M}	64.4	64.4	64.4	64.4	64.4	64.4	64.4	61.3	53.2	45.1	37.0	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5
Q_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.7	28.9	45.1	61.3	64.4	64.4	64.4	64.4	64.4	64.4	64.4	64.4	64.4
$Remanuf.Out$	53.0	53.0	53.0	53.0	53.0	53.0	53.0	60.9	67.6	74.3	80.9	82.2	82.2	82.2	82.2	82.2	82.2	82.2	82.2	82.2
π_T	16510	16510	16510	16510	16510	16510	16510	16236	15922	15652	15427	15389	15389	15389	15389	15389	15389	15389	15389	15389
$\pi_T + \pi_{2R}$	17798	17798	17798	17798	17798	17798	17798	17579	17493	17629	17988	18511	19041	19571	20101	20631	21161	21691	22221	22751
VRR	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.47	0.52	0.58	0.63	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
P_1	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0	108.0
P_{2M}	73.5	73.5	73.5	73.5	73.5	73.5	73.5	69.5	66.2	62.9	59.5	58.9	58.9	58.9	58.9	58.9	58.9	58.9	58.9	58.9
P_{2R}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	49.5	56.2	62.9	69.5	78.9	88.9	98.9	108.9	118.9	128.9	138.9	148.9	158.9