Review of The Higgs Mechanism in The Standard Model

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Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another University for the award of any other degree.

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Summary

This project presents a review of the Higgs mechanism in the SM. It shows theoretical basic Higgs-releated topics. Research into the Higgs sector has been done recently at the LHC. One of the dominant processes is gluon-gluon fusion for Higgs production. Moreover Higgs is observed via its decay into two photons. For this reason, this masters project makes a calculation of the cross-section of gluon fusion and that of the decay rate of Higgs to diphoton in the SM. The observed cross-section is larger almost by a factor of two. Thus, it may be a sign of new physics. For this reason the effects of new physics on $gg \to \gamma\gamma$ have been studied in this project.

Preface

In my project chapter 1 is revision of the SM. This chapter relies on the work of others provided by textbooks. Chapter 2 is partially my work. When calculated the SM cross section and decay rate I took some master integrals from puplished articles and dissertations. Moreover I took some complex mathematical processes from worked examples on Feynmann loop diagrams. In chapter 3 also contains reviewed works.

Acknowledgements

I would like to express my sincere gratitude to Dr Xavier Calmet for giving me much instruction in particle physics. In my undergraduate degree I did not take particle physics courses at my university. For this reason my background in particle physics was at an elementary level when I got my bachelors degree. However, I now believe that I have gained much knowledge in particle physics thanks to Dr Calmet's advice, help and encouragement. Without his help this project would not have materialised.

I would also like to thank my family. They have always believed in me and encourage me in whatever I decide to do. Many thanks for being in my life.

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Introduction

Elementary particles and their interactions are described by the best-known theory, the standard model (SM). The elementary particles are fermions, which are divided into quarks and leptons; the types of quark are up, down, charm, strange, top and bottom carrying both electrical and colour charges. Moreover, there are 6 leptons. However, only electrons, muons and taus have an electrical charge, while electron neutrinos, muon neutrinos and tau neutrinos do not have an electrical charge. Leptons carry a weak hypercharge interacting with the force carrier of the weak interaction, the W and Z bosons. Electrons, muons and taus also have an electrical charge so they have electromagnetic interaction as well. Quarks can make strong and electromagnetic interactions because they have colour and electrical charge. The SM explains three fundamental interactions: electromagnetic interaction, weak interaction and strong interaction. Force carrier particles are gluons and vector bosons such as γ, $W^{\pm}$ and Z.

Gauge theories are important field theories describing the interaction of fundamental particles. In this project, gauge theory and the meaning of gauge invariance will be described. Quantum electrodynamics is an Abelian gauge theory with the symmetry group $U(1)$. The standard model is also a gauge theory with the symmetry group $U(1) \times SU(2) \times SU(3)$ because the interaction of matter fields can be shown by gauge symmetries. We will describe invariance of some Lagrangians under some symmetry transformation groups.

The standard model depends on the process of spontaneous symmetry breaking to create mass to the elementary particle. Without it, the elementary particle would not have mass. Spontaneous symmetry breaking gives rise to the creation of a scalar particle called the Higgs boson [9]. The Higgs boson is now being found by ATLAS and CMS experiments at the LHC. In a high energy collider, Higgs is dominantly created by a fusion of two gluons and is seen via its decay into two photons [10]. The recent CMS announcement of a Higgs-like particle with a mass is 125.3Gev [32] and the recent Atlas announcement of this is 126Gev [17]. In addition, a different diphoton rate has been observed from the standard model prediction in Atlas and CMS experiments. The signal with two photons in the final state seems to be larger than expected within the standard model almost by a factor of two CMS announced $\sigma/\sigma_{SM} = 1.6\pm0.4$ [32] and Atlas announced $\sigma/\sigma_{SM} = 1.4\pm0.3$ [17]. To conclude, this CERN result may be a sign of new physics beyond the standard model.

Since the SM cross-section is different from the LHC cross-section, I will recalculate the SM cross-section for a gluon fusion loop diagram and recalculate the decay rate of Higgs to two photons by using dimensional regularisation techniques to remove divergence.

This masters project is organised in the following way. In section 2 we will review the Higgs mechanism in the SM. Then in section 3 we will show a full calculation of the gluon fusion cross-section in the SM and that of the decay rate of Higgs to two photons in the SM. Lastly, we will discuss the new physics in $gg \to H \to \gamma\gamma$.

Chapter 1

Review of Gauge Theory

1.1 Symmetries and Conservation Laws

There are many kinds of invariance principle and associated symmetry transformations that have a significant impact on physics. They guide the formulation of theories.

If the equations of motion are produced from a variational principle, a general and systematic procedure becomes ready to construct conservation theorems and constants of the motion as a result of invariance properties. Hence, conservation laws and section rules found in nature may be interrupted as Lagrangian symmetries, limiting its form. The general model for this program is obtained by Noether's theorem, which makes a connection between conservation law and every continuous symmetry transformation under invariant Lagrangian form. Two examples of this theorem are space-time, or geometrical, invariance and other internal symmetry.

First, an example of geometrical transformation of the space-time variables is described, translations of the form

$$x_\mu \to x'_\mu = x_\mu + a_\mu \tag{1.1}$$

Where the infinitesimal displacement a_μ does not depend on the coordinate x_μ, a Lagrangian, which is invariant under this kind of transformation, will shift by an amount

$$\delta\mathcal{L} = \mathcal{L}[x'] - \mathcal{L}[x] = a^\mu d\mathcal{L}/dx^\mu \tag{1.2}$$

Using a Lagrangian, which is obviously independent of coordinates, we might calculate equivalently the changes as

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi), \tag{1.3}$$

where

$$\delta\phi = \phi(x') - \phi(x) = a^\mu\partial_\mu\phi(x) \tag{1.4}$$

and

$$\delta(\partial_\mu\phi) = \partial_\mu\phi(x') - \partial_\mu\phi(x) = a^\nu\partial_\nu\partial_\mu\phi(x). \tag{1.5}$$

As a result, using Euler-Lagrange equations to remove $\partial\mathcal{L}/\partial\phi$, we yield

$$\begin{aligned}\delta\mathcal{L} &= \left[\partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}\right]a^\mu\partial_\mu\partial_\nu\phi \\ &= \partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}a^\mu\partial_\mu\phi.\end{aligned} \tag{1.6}$$

When two equations are equalised for δ we find

$$a_\mu\partial_\nu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}\partial^\mu\phi - g^{\mu\nu}\mathcal{L}\right] = 0, \tag{1.7}$$

which is convenient for arbitrary infinitesimal displacements a_μ. Consequently, the stress-energy-momentum progresses with tensor

$$\Theta^{\mu\nu} \equiv \frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}\partial^\mu\phi - g^{\mu\nu}\mathcal{L} \tag{1.8}$$

is comfortable with the conservation law

$$\partial_\mu\Theta^{\mu\nu} = 0. \tag{1.9}$$

To get invariant Lagrangian under such a transformation, we need

$$\delta\mathcal{L} = 0 \tag{1.10}$$

Specific computation gives

$$\begin{aligned}\delta\mathcal{L} &= \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) + \frac{\partial\mathcal{L}}{\partial_\mu\bar{\psi}}\delta(\partial_\mu\bar{\psi}) \\ &= \left[\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right]\frac{i}{2}\alpha.\tau\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\frac{i}{2}\alpha.\tau\psi(\partial_\mu\psi) \\ &= \partial_\mu\alpha.\left[\frac{i}{2}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\tau\psi\right],\end{aligned} \tag{1.11}$$

where the second line comes after the equations of motion. The term in square brackets may be interpreted as a conserved current (density),

$$J^\mu = \frac{i}{2}\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\tau\psi, \tag{1.12}$$

which is suitable for the continuity equation

$$\partial_\mu J^\mu = 0. \tag{1.13}$$

For special reasons related to the free nucleon Lagrangian, the exact form of the conserved current is

$$J^\mu = \bar{\psi}\gamma^\mu \frac{\tau}{2}\psi \tag{1.14}$$

which is known as the isospin current, in analogy with the familiar electromagnetic current for Dirac particles [1].

1.2 Gauge Revolution

In 1971 G. 't Hooft made a breathtaking discovery. 't Hooft explained that Yang-Mills gauge theory was renormalisable while its symmetry group was spontaneously broken. Thanks to this significant breakthrough, it is possible to express renormalisable theories of weak interactions, where W bosons are symbolised as gauge fields [2].

An earlier theory of Weinberg and Salam about weak interactions is a gauge theory built on the symmetry group $SU(2)\times U(1)$. However, since gauge theories were recognised to be renormalisable, real numerical predictions could be produced from various gauge theories and then investigated whether they regenerated with the experimental data or not. If the predictions of the gauge theory did not reach an agreement with the experimental data, they would have to be cancelled. Gauge theorists understood that the final judge of any theory depends on the experiment.

Within several years, the agreement between experiment and Weinberg-Salam theory was destroyed. The weak interactions vanished when a state of theoretical confusion changed to one of relative clarity within a brief period of time. Existence of gauge bosons $W^\pm$ and Z was predicted by Weinberg and Salam in 1983 by experiment. Thus, the experiment vindicated the theory [2].

The Weinberg-Salam model showed leptons using a simple method. The left-handed leptons could be based on $SU(2)$ doublets in three separate generations:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}; \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}; \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{1.15}$$

The intermediate vector bosons mediated between the interactions of these leptons vector bosons: $W^{\pm}_{\mu}$, Z_μ

The research into strong interactions also progressed quickly. The gauge revolution constructed Quantum Chromodynamics (QCD), which is a strong candidate for a theory about strong interactions. By requiring a new colour $SU(3)$ symmetry, the Yang-Mills theory now supplied a glue by which the quarks could be kept together.

The quarks in QCD are shown by:

$$\begin{pmatrix} u^1 & u^2 & u^3 \\ d^1 & d^2 & d^3 \\ s^1 & s^2 & s^3 \\ c^1 & c^2 & c^3 \\ \vdots & \vdots & \vdots \end{pmatrix} \tag{1.16}$$

where the $1, 2, 3$ index labels the colour symmetry. Experimental absence of quarks was explained plausibly by QCD. At low energy the effective $SU(3)$ colour coupling constant increased, and thus restricted the quarks permanently into the known hadrons [2].

At large energies the $SU(3)$ colour coupling constant decreased. This was known as asymptotic freedom which was found by Gross, Wilczek, Politzer and t' Hooft. At high energies, it could define the curious fact that the quarks behaved as if they were explained by a free theory. This occurred since the effective coupling constant became small in size with rising energy, giving the appearance of a free theory. As a result, the quark model worked much better than it is assumed to.

Soon, both the electroweak and QCD models were connected together to produce the standard model based on the gauge group $SU(3) \times SU(2) \times U(1)$. To have anomalies that threatened renormalisability, the leptons in the Weinberg-Salam model were described. Fortunately, these potentially destructive anomalies exactly cancelled against anomalies belonging to quarks. In other words, the lepton and quark sectors of the standard model compensated for each other's faults, which was a pleasurable theoretical success for the standard model. Consequently, because of this and other theoretical and experimental successes, the standard model was quickly accepted to be a first-order approximation to the final theory of particle interactions.

The spectrum of the standard model for the left-handed fermions is schematically shown here; they have neutrino ν, the electron e and the up and down quarks, which come in three colours labelled by the index, i. This model was used for the three generations [2]

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} u^i \\ d^i \end{pmatrix}; \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} c^i \\ s^i \end{pmatrix}; \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \begin{pmatrix} t^i \\ b^i \end{pmatrix} \tag{1.17}$$

In the standard model, the massive vector mesons mediate the forces between leptons and quarks, and the massless gluons mediate the forces between quarks.
Massive vector mesons: $W^{\pm}$, Z
Massless gluons: A_μ^a

1.3 Gauge Field Theories

1.3.1 Abelian Gauge Field Theories

Field theories involving massless spin 1 particles have been studied in this section due to the non-Abelian case (Yang-Mills theories) that renormalisability of the field theory needs a massless vector field [3].

Massless vector field is explained first by Quantum Electrodynamics (QED). Lagrangian density is for the interaction of the Dirac field with the electromagnetic field,

$$\mathcal{L}_1 = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma^\mu \psi A_\mu \tag{1.18}$$

If we use the gauge invariance principle we can maintain this form of Lagrangian. Firstly we examine the Lagrangian for the free Dirac field

$$\mathcal{L}_1 = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \tag{1.19}$$

$\mathcal{L}_1$ is invariant under the phase transformation.

$$\Psi(x) \to e^{-iq\lambda}\Psi(x) \tag{1.20}$$

where λ is an arbitrary real number. This global symmetry of Lagrangian might be transformed to a local symmetry, if invariance under the transformation could be arranged [3].

$$\psi(x) \to e^{-iq\lambda(x)}\Psi(x) \tag{1.21}$$

The Lagrangian of $\mathcal{L}_1$ is not invariant under this local symmetry. Its new form under transformation $\psi(x)$ is

$$\mathcal{L}_1 \rightarrow \mathcal{L}_1 + q\bar{\psi}\gamma^\mu\psi\partial_\mu\lambda. \tag{1.22}$$

If we desire gauge invariance we need extra terms for the Lagrangian. The next step might be carried out to achieve this. The derivative in $\mathcal{L}_1$(1.19) is replaced by a covariant derivative D_μ defined by

$$D_\mu\Psi = (\partial_\mu + iqA_\mu)\psi \tag{1.23}$$

The covariant derivative involves the vector field referred to as the 'gauge field' A_μ which transforms as

$$A_\mu \rightarrow A_\mu + \partial_\mu\lambda \tag{1.24}$$

under gauge transformation which acts on ψ. Then $D_\mu\psi$ follows the same way as ψ

$$D_\mu\psi \rightarrow e^{-iq\lambda}D_\mu\psi \tag{1.25}$$

As a result,the Lagrangian

$$\mathcal{L}_2 = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{1.26}$$

is gauge invariant.

We need to add gauge invariant terms for the vector field A_μ in $\mathcal{L}_2$. Then field strength tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.27}$$

is invariant under the gauge transformation (1.24). Thus, the final gauge invariant Lagrangian may be written as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1.28}$$

A massless vector field is explained by this Lagrangian density. A mass is not given the vector field by providing gauge invariance because the mass term $A_\mu A^\mu$ is not invariant under the gauge transformation of (1.24). As a result, a gauge invariant Lagrangian, which is an example of the interaction of a vector field with a spinor field, is a massless vector field [3].

1.3.2 Non-Abelian Gauge Field Theories

A theory about a number of Dirac spinor fields ψ_i, $i = 1, \cdots, p$ interacting with a number of vector fields A_a^μ, $a = 1, \cdots r$ is constructed. To achieve a gauge invariant theory, we might give each of the Dirac fields a 'charge' to couple to each vector field A_a^μ; this is then called a theory of r Abelian gauge fields [4].

It might be wondered whether generalisations of the principle of gauge invariance that disagree with having r distinct Abelian gauge field theories exist or not. To understand this presumption, first gauge transformation of (1.20) is generalised to

$$\psi(x) \to e^{-igT_a\lambda_a(x)}\psi(x) \tag{1.29}$$

Where T_a are p matrices which act on column vector $\psi(x)$, and the $\lambda_a(x)$ are arbitrary functions of x, g is going to be a coupling constant. Thus, we write

$$\psi(x) \to e^{-igt\lambda(x)}\psi(x) \tag{1.30}$$

where

$$T.\lambda(x) = T_a\lambda_a(x) \tag{1.31}$$

By correspondence with (1.23) we write

$$D^\mu\psi = (\partial^\mu + igT.A^\mu)\psi \tag{1.32}$$

An infinitesimal gauge transformation makes the development easier

$$\psi(x) \to (I - igT.\lambda)\psi(x) \tag{1.33}$$

Under this infinitesimal transformation

$$\partial^\mu\psi \to (I - igT.\lambda)\partial^\mu\psi - ig(T.\partial\lambda)\psi \tag{1.34}$$

We need to a gauge transformation property for the gauge fields

$$A_a^\mu \to A_a^\mu + \partial^\mu\lambda_a + gf_{abc}\lambda_b A_c^\mu \tag{1.35}$$

Where f_{abc} are constants. This is similar to (1.24) except for the last term. The last term, (1.34), has been defined to provide the gauge fields with an opportunity to achieve the role of cancelling out the unwanted terms in (1.34). The covariant derivative of ψ should be transformed in the same way as ψ [4].

$$D^\mu\psi \to (I - igT.\lambda)D^\mu\psi \tag{1.36}$$

This will occur provided

$$[T.\lambda, T.A^\mu] = if_{abc}T_a\lambda_b A^\mu_c. \tag{1.37}$$

As a result,

$$[T_a, T_b] = if_{abc}T_a \tag{1.38}$$

It is assumed that the coefficients f_{abc} are antisymmetric in all indices, in which case this may be written as

$$[T_a, T_b] = if_{bca}T_a \tag{1.39}$$

Consequently, the matrix T_a causes a representation of the Lie algebra with structure constants f_{abc}. If a gauge transformation with constant $\lambda_a(x)$ in (1.35) is taken. It is obvious that the gauge fields transform as the adjoint representation of the Lie group [4].

The finite gauge transformation of (1.30) is an appropriate use.

$$\psi(x) \to U(x)\psi(x) \tag{1.40}$$

where

$$U(x) = e^{-igT.\lambda(x)} \tag{1.41}$$

The infinitesimal transformation of (1.35) may cause the identical finite gauge transformation of the gauge fields. First the p matrix is introduced

$$A^\mu \to A^\mu + T.\partial^\mu\lambda - ig[T.\lambda, A^\mu] \tag{1.42}$$

This resembles finite transformation (1.42)

A$^\mu(x) \to U(x)(A^\mu - ig^{-1}\partial^\mu)U^{-1}(x)$(1.42)taken to linear order in λ_a.

Next a gauge invariant Lagrangian is formulated for the gauge fields themselves. To do this, we will require an object $F_a^{\mu\nu}$ with two Lorentz indices which transforms in a covariant way under the gauge group. This may be formulated directly from the covariant derivative of (1.32) by introducing

$$F^{\mu\nu} = F_a^{\mu\nu} T_a = -ig^{-1}[D^\mu, D^\nu] \tag{1.43}$$

Thus

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu] \tag{1.44}$$

Where we have cancelled a total derivative, or equivalently

$$F_a^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g f_{abc} A_b^\mu A_c^\nu \tag{1.45}$$

Where we have used (1.38) it is noticed that (1.47) is not dependent on the fermion representation chosen in (1.42).

We derive the transformation property of $F^{\mu\nu}$ under the gauge group from (1.44).

$$F^{\mu\nu}(x) \to U(x) F^{\mu\nu}(x) U^{-1}(x) \tag{1.46}$$

Now a gauge invariant Lagrangian L_{YM} for the gauge (or Yang-Mills) fields may be introduced. The generator t_a is usually used for the significant representation of the gauge group in (1.25). Then, it is correctly normalised as

$$\mathcal{L}_{YM} = -\frac{1}{2} Tr(F_a^{\mu\nu} F_{\mu\nu}^a) \tag{1.47}$$

or equivalently

$$\mathcal{L}_{YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a \tag{1.48}$$

The equivalence of these two forms comes after the conventional normalisation of generators of the gauge group, which gives the fundamental representation

$$Tr(t_a t_b) = \frac{1}{2}\delta_{ab} \tag{1.49}$$

To conclude, we may define gauge invariant Lagrangians, for Dirac spinor fields interacting with vector fields, of the form

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2} Tr(F_a^{\mu\nu} F_{\mu\nu}^a) \tag{1.50}$$

or equivalently

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a \tag{1.51}$$

Here the covariant derivative D_μ comes from (1.32) and covariant curl $F_{\mu\nu}$ (or $F^a_{\mu\nu}$) from (1.46) and (1.42) with T_a replaced by t_a, the generator of the fundamental representation (1.45). The gauge field turns into adjoint representation of the Lie group, and the spinor fields transform as a representation of the gauge group with the matrix generators T_a [4].

If the gauge group is an uncomplicated Lie group, there is a single gauge coupling constant, g. However, if the gauge group is a semi-simple one, which can be written as a product of simple factors ($e.g. SU(2) \times SU(2)$) then it leads to independent gauge coupling constants for the various simple factors.

1.4 Spontaneous Symmetry Breaking

Gauge invariance is important in weak interactions; for this reason some methods of generating gauge vector boson masses have to be provided without destroying the renormalisability of gauge theory. The gauge symmetry is broken by any such mass terms and spontaneous symmetry breaking is the only recognised method of carrying out a renormalisable procedure [5]. This is called spontaneous symmetry breaking, although the symmetry is not broken so much as secret or hidden.

1.4.1 Spontaneous Breaking of a Discrete Symmetry

Lagrangian has to respect the symmetry, but in a vacuum state, Lagrangian is not invariant under the symmetry transformation. If we had a spinor or vector field, then the vacuum would be discriminated against by a non-zero angular momentum $J(=\frac{1}{2}$ or $1)$ and the rotational invariance would be broken. Scalar fields break internal symmetry which we are interested in because scalar fields have non-zero value in vacuum [6].

This scalar field is called the Higgs field. To break the internal symmetry, its existence is postulated. Non-zero value in vacuum shows the existence of a non-zero classical field in vacua. Absence of any source means presence of a scalar field operator $\hat{\varphi}(x)$ having a non-zero expectation value (VEV).

$$< 0\,|\hat{\varphi}(x)|\,0 >= \varphi_c(x) \neq 0 \tag{1.52}$$

Where $\varphi_c(x)$ is the field measured in the vacuum, we need $\varphi_c(x)$ to be independent of x:

$$\varphi_c(x) = \varphi_c \tag{1.53}$$

The VEV of $\hat{\varphi}$ is zero in every order of perturbation theory, at least in $\lambda\varphi^4$ theory considered here; for this reason, spontaneous symmetry breaking must be a non-perturbative effect. The effective potential is shown by the potential $V(\varphi)$ by neglecting quantum effects for a while [6]. Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - V(\varphi) \tag{1.54}$$

with

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4!}\lambda\varphi^4 \tag{1.55}$$

The only symmetry of this simple model is the invariance under the discrete transformation

$$\varphi(x) \to \varphi\prime(x) \equiv -\varphi(x) \tag{1.56}$$

Obviously V will only have an absolute minimum if

$$\lambda \geq 0 \tag{1.57}$$

and in any case, we need this to be sure of the convergence of the functional integral. When μ^2 is positive and minimum value V is seen only at $\varphi = 0$ and μ is the mass of the field φ. V also has a minimum non-zero value of φ in a condition

$$\mu^2 < 0 \tag{1.58}$$

and then

$$\varphi_c = \pm(-6\frac{\mu^2}{\lambda})^{\frac{1}{2}} \tag{1.59}$$

This does not stabilise the sign of φ_c which is chosen by the system due to its symmetry. However, if symmetry is broken it does not depend on the sign of φ_c. Now a new field with zero VEV is introduced [6]

$$\tilde{\varphi} = \hat{\varphi} - \varphi_c \tag{1.60}$$

so that using $(1.35, 1.36)$

$$< 0\,|\tilde{\varphi}|\,0 >= 0 \tag{1.61}$$

L includes a function of $\tilde{\varphi}$, but it will not reflect symmetry $\tilde{\varphi} \rightarrow -\tilde{\varphi}$, since fluctuations about the asymmetric point $\varphi = \varphi_c$ are measured by $\tilde{\varphi}$. We maintain

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \tilde{\varphi})(\partial^\mu \tilde{\varphi}) + 2\mu^2 \tilde{\varphi}^2\right] - \frac{\lambda}{4!}(\tilde{\varphi}^4 + 4\tilde{\varphi}^3 \varphi_c) - \frac{1}{4}\mu^2 \tilde{\varphi}_c^2 \quad (1.62)$$

The cubic term $\tilde{\varphi}^3$ is a sign of spontaneous symmetry breaking, although this is the same Lagrangian as the symmetric (1.37). We can understand why the symmetry is defined as 'secret' by some. It is secret because only the special coefficient of the $\tilde{\varphi}^3$ term shown in (1.45) can be reconstructed in a symmetric form [6]. The spontaneous symmetry breaking is indeed non-perturbative because φ_c is proportional to $\lambda^{-\frac{1}{2}}$. Moreover, the mass squared of field $\tilde{\varphi}$ is clearly $-2\mu^2$

$$\frac{d^2V}{d\varphi^2}\,|_{\varphi=\varphi_c} = -2\mu^2 \quad (1.63)$$

1.4.2 Spontaneous breaking of a continuous global symmery

In the previous section we studied the real scalar field theory (1.54) which has only discrete symmetry (1.56). However, we will now determine a continuous gauge symmetry. The spontaneous breaking of a continuous symmetry has novel features, which are not seen in discrete cases. For this reason complex scalar field theory is introduced [7]. The Lagrangian

$$\mathcal{L} = (\partial^\mu \varphi)(\partial^\mu \varphi^*) - V(\varphi, \varphi^*) \quad (1.64)$$

is invariant under a global $U(1)$ gauge transformation

$$\varphi(x) \rightarrow \varphi\prime(x) \equiv e^{-iq\lambda}\varphi(x) \quad (1.65)$$

$$\varphi(x)^* \rightarrow \varphi\prime(x)^* = e^{iq\lambda}\varphi(x)^* \quad (1.66)$$

with (q, λ real and constant) provided

$$V(\varphi, \varphi*) = V(\varphi\varphi*) \quad (1.67)$$

If we focus on only renormalisable theories, then (1.67) implies that V has the form

$$V(\varphi, \varphi*) = \mu^2 \varphi\varphi * + \frac{1}{4}\lambda(\varphi\varphi*)^2 \quad (1.68)$$

If λ must be positive, μ^2 is positive and V takes the absolute minimum at $\varphi = 0$. If μ^2 is negative V obtains a minimum at a non-zero value φ_c of φ which satisfies

$$(\varphi_c^2) = -2\frac{\mu^2}{\lambda} \quad (1.69)$$

Any particular choice of φ_c breaks the symmetry spontaneously because under a gauge transformation (1.65, 1.66), the ground state $|\varphi_c >$ is transformed into a different state $\left|e^{-iq\lambda}\varphi_c >\right.$. If a new field having zero VEV is introduced when we break continuous symmetry. Let the phase of φ_c be δ, so that

$$\varphi_c = \frac{1}{\sqrt{2}} \upsilon e^{i\delta} \tag{1.70}$$

with

$$V = +(-4\mu^2/\lambda)^{\frac{1}{2}} \tag{1.71}$$

Similarly for $\hat{\varphi}$.

Then new φ may be expressed in terms of two real fields φ_1, φ_2 by

$$\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)e^{i\delta} \tag{1.72}$$

Since

$$< 0\,|\hat{\varphi}_i|\,0 >= \varphi_c \tag{1.73}$$

It follows from (1.70), (1.71) and (1.72) that only has a non-zero VEV:

$$< 0|\hat{\varphi}_i|0 >= \upsilon\delta_{i1} \qquad (i = 1, 2) \tag{1.74}$$

Thus new fields $\tilde{\varphi}_i$ having zero VEV are introduced

$$\tilde{\varphi}_i \equiv \hat{\varphi}_i - \upsilon\delta_{i1} \qquad (i = 1, 2) \tag{1.75}$$

The quadratic terms of the Lagrangian (1.68) are diagonalised by φ_1 φ_2 variables and we get

$$\mathcal{L} = \tfrac{1}{2}\left[(\partial_\mu\tilde{\varphi}_1)(\partial^\mu\tilde{\varphi}_1) + (\partial_\mu\tilde{\varphi}_2)(\partial^\mu\tilde{\varphi}_2) + 2\mu^2(\tilde{\varphi}_1)^2\right] \tag{1.76}$$

$$-\tfrac{1}{16}\lambda\left[(\tilde{\varphi}_1)^2 + (\tilde{\varphi}_2)^2\right]^2 - \tfrac{1}{4}\lambda\upsilon\tilde{\varphi}_1\left[(\tilde{\varphi}_1)^2 + (\tilde{\varphi}_2)^2\right] - \tfrac{1}{4}\mu^2\upsilon^2 \tag{1.77}$$

Undoubtedly the symmetry is spontaneously broken, as expected, and $\tilde{\varphi}_1$ has a positive mass squared of $-2\mu^2$

$$\frac{\partial^2 V}{\partial\varphi_1^2}|_{(\varphi_1,\varphi_2)=(\upsilon,0)} = -2\mu^2 \tag{1.78}$$

The novel feature is that the field $\tilde{\varphi}_2$ has not got mass [7].

$$\frac{\partial^2 V}{\partial\varphi_2^2}|_{(\varphi_1,\varphi_2)=(\upsilon,0)} = 0 \tag{1.79}$$

The spontaneous breaking of continuous global symmetry leads to Goldstone bosons. We understand this when we look at non-Abelian gauge symmetry G, defined in (1.30) and some scalar fields transforming as some representation of G without leaving from generality these may be defined in terms of n (say) real scalar fields.

$$\varphi(\mathsf{x}) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \vdots \\ \varphi_n(x) \end{pmatrix} \tag{1.80}$$

Under an infinitesimal global gauge transformation

$$\varphi(x) \to \varphi(x)\prime = \varphi(x) + \delta\varphi(x) \tag{1.81}$$

with

$$\delta\varphi(x) = -igT^a\lambda^a\varphi(x) \tag{1.82}$$

Where g, λ^a are real, and $T^a = (a = 1, \ldots, N)$ are the n matrices satisfying the Lie algebra (1.38); since iT^a is real and T^a is Hermitian, T^a must have antisymmetric properties. Since L is invariant under gauge transformation (1.81), (1.82) leads to conserved Noether current

$$J_a^\mu = \Pi_\mu^T(x)iT^a\varphi(x) \qquad (a = 1, \ldots, N) \tag{1.83}$$

where

$$\Pi_{\mu i} \equiv \frac{\partial L}{\partial(\partial^\mu\varphi_i)} \qquad (i = 1, \ldots, n) \tag{1.84}$$

The Euler-Lagrange equations give

$$\partial^\mu\Pi_\mu = \frac{\partial L}{\partial\varphi} \tag{1.85}$$

and current conservation then implies that

$$\left(\frac{\partial L}{\partial\varphi}\right)^T iT^a\varphi + \Pi_\mu iT^a\partial^\mu\varphi = 0 \tag{1.86}$$

In the field theories with which we are concerned, the Lagrangian has the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu)^T(\partial_\mu) - V(\varphi) \tag{1.87}$$

so

$$\Pi_\mu = \partial_\mu \varphi \tag{1.88}$$

$$\frac{\partial L}{\partial \varphi} = -\frac{\partial V}{\partial \varphi} \tag{1.89}$$

It follows from (1.88) that the second term of (1.86) vanishes, since T^a is antisymmetric and we presume that V satisfies

$$\frac{\partial V^T}{\partial \varphi} T^a \varphi = 0 \tag{1.90}$$

for all φ as a consequence of the symmetry.

The masses of various modes are determined by the behaviour of V in the region of its minimum. Since we are considering a spontaneous broken symmetry, V is the minimum at some value of φ which establishes the VEV of the field operators [7]. Thus,

$$< 0|\hat{\varphi}(x)|0 >= \upsilon \tag{1.91}$$

where

$$\frac{\partial V}{\partial \varphi}|_{\varphi=\upsilon} = 0 \tag{1.92}$$

Moreover, the ground state illustrated by V is not in general invariant under a gauge transformation, which means that

$$(1 - ig\lambda^a T^a)\upsilon \neq \upsilon \tag{1.93}$$

for all choices of infinitesimals λ^a. So, for at least one a,

$$iT^a \upsilon \neq 0 \tag{1.94}$$

We now illustrate fields

$$\tilde{\varphi} \equiv \hat{\varphi} - \upsilon \tag{1.95}$$

having zero VEV

$$< 0|\tilde{\varphi}|0 > \tag{1.96}$$

and express L in terms of $\tilde{\varphi}$. Then using (1.87) and (1.92)

$$\mathcal{L} = \frac{1}{2}\left((\partial_\mu \tilde{\varphi}_i)(\partial^\mu \tilde{\varphi}_i) - \tilde{\varphi}_i \tilde{\varphi}_j \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j}|_{\varphi=\upsilon} \right) - V(\upsilon) + O(\tilde{\varphi}^3). \tag{1.97}$$

Obviously the masses of the fields $\tilde{\varphi}$ are the eigenvalues of mass matrix

$$(\mu^2)_{ij} \equiv \frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}|_{\varphi=\upsilon} \tag{1.98}$$

Differentiating (1.90) with respect to φ and evaluating at $\varphi = \upsilon$, we find

$$(\mu^2)_i T^a \upsilon = 0 \qquad (a = 1, \ldots, N) \tag{1.99}$$

using (1.92). It comes after (1.94) that μ^2 has at least one eigenvector with zero eigenvalue, and consequently the linear combination $\tilde{\varphi} i T^a \upsilon$ is known as a Goldstone boson. Now assume that the ground state $|V>$ is left invariant under gauge transformations corresponding to some (maximal) subgroup S and G. Then we may choose generators $T^a(a = 1, \ldots, N)$ of G such that $T^a(a = 1, \ldots, M)$ accomplish S, since $|V>$ is invariant under the transformation corresponding to S,

$$T^a \upsilon = 0 \qquad (a = 1, \ldots, M) \tag{1.100}$$

but

$$T^a \upsilon \neq 0 \qquad (a = M + 1, \ldots, N) \tag{1.101}$$

The $N - M$ vectors $T^a \upsilon(a = M + 1, \ldots, N)$ are obviously independent, and it shows the existence of $N - M$ Goldstone bosons [7].

1.5 The Higgs Mechanism

In this section we have studied how global invariance of a field theory might be broken (or 'hidden') by the ground state (vacuum) spontaneously choosing one of the degenerate minima of the potential. This shows that we examine the implementation of breaking a local gauge invariance spontaneously, in the hope that the breaking will give rise to boson masses, whereas the renormalisability will be protected by the (hidden) symmetry [8].

The mechanism now called the Higgs mechanism is interpreted by using it on the locally gauge invariant version of the model. This model experiences the Lagrangian of 'scalar electrodynamics', but when it is spontaneously broken it is called the 'Higgs model'. Thus we begin with

$$\mathcal{L} = (D_\mu \varphi)(D^\mu \varphi^*) - \mu^2 \varphi * \varphi - \frac{1}{4}\lambda(\varphi\varphi^*)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1.102}$$

where

$$D_\mu \varphi \equiv (\partial_\mu + iqA_\mu)\varphi \tag{1.103}$$

$$D_\mu \varphi^* \equiv (\partial_\mu + iqA_\mu)\varphi^* \tag{1.104}$$

are the $U(1)$ gauge covariant derivatives and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.105}$$

is the gauge invariant field tensor. If μ^2 is positive the $U(1)$ invariance is not broken and obviously a scalar particle of mass μ and charge q interacting with a massless electromagnetic field is illustrated by (1.102); thus the name scalar electrodynamics [8].

We are interested in the case when $\mu^2 < 0$, so the symmetry is broken spontaneously and $\hat{\varphi}$ obtains a VEV

$$< 0|\hat{\varphi}(x)|0 >= \frac{1}{\sqrt{2}} v e^{i\delta} \tag{1.106}$$

Where v is shown in (1.71) and δ is arbitrary. As before, variables are changed and the fields, defined in (1.72) and (1.75) are used, which possess zero VEVs. In terms of these variables the covariant derivative is written:

$$D_\mu \hat{\varphi} = \frac{e^{i\delta}}{\sqrt{2}}[\partial_\mu \tilde{\varphi}_1 + i(\partial_\mu \tilde{\varphi}_2 + qvA_\mu) + iqA_\mu(\tilde{\varphi}_1 + i\tilde{\varphi}_2)]. \tag{1.107}$$

It is interesting that the former Goldstone boson $\tilde{\varphi}_2$ is unavoidably connected to the hitherto massless gauge field A_μ. Indeed, apart from interaction terms, $\tilde{\varphi}_2$ and A_μ goes into the Lagrangian only in the equation

$$A'_\mu \equiv A_\mu + \frac{1}{qv}\partial_\mu \tilde{\varphi}_2 \tag{1.108}$$

Put differently, due to the spontaneous symmetry breaking, the gauge field is combined with the Goldstone mode $\tilde{\varphi}_2$, which contributes a longitudinal degree of freedom in momentum space. As a result, this recommends that the field A'_μ is created, if A_μ is removed in favour of A'_μ in (1.85),

$$m(A') = qv \tag{1.109}$$

and the mass demands both the spontaneous symmetry breaking ($v \neq 0$) and coupling of the gauge field to the scalar field ($q \neq 0$). The exact form of L as a function of A'_μ, $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ shall not be presented, because $\tilde{\varphi}_2$ can be removed from Lagrangian. This can be seen by using the gauge invariance of L. When (1.108) is compared with (1.24) it is remarked that A'_μ may be acquired by a particular transformation, namely one with

$$\lambda(x) = \frac{1}{q\upsilon}\tilde{\varphi}_2(x) \tag{1.110}$$

This recommends that the whole of dependence of L upon $\tilde{\varphi}_2$ might transform into a (different) gauge transformation from (1.70) and (1.72) we have that

$$\varphi = \frac{1}{\sqrt{2}}(\upsilon + \tilde{\varphi}_1 + i\tilde{\varphi}_2)e^{i\delta} \tag{1.111}$$

Under a gauge transformation

$$\varphi \to \varphi' = e^{-iq\lambda}\varphi \tag{1.112}$$

$$\equiv \tfrac{1}{\sqrt{2}}(\upsilon + \tilde{\varphi}'_1 + i\tilde{\varphi}'_2)e^{i\delta} \tag{1.113}$$

So by choosing

$$q\lambda = \arctan\frac{\tilde{\varphi}_2}{\upsilon + \tilde{\varphi}_1} \tag{1.114}$$

we can set up that

$$\tilde{\varphi}_2 = 0 \tag{1.115}$$

In this gauge $\tilde{\varphi}_1$ is indicated by H, so that

$$\varphi(x) \to \varphi'(x) = \frac{1}{\sqrt{2}}[\upsilon + H(x)] \tag{1.116}$$

and using (1.107) this gives

$$D_\mu\varphi \to (D_\mu)'\varphi = \frac{e^{i\delta}}{\sqrt{2}}(\partial_\mu)H + iq\upsilon A'_\mu + iqA'_\mu H \tag{1.117}$$

where now A'_μ is the field A_μ gauge transformed using (1.114). If Lagrangian is gauge invariant it may be assessed in any gauge; in this gauge we acquire from (1.102), ((1.103), (1.104) and (1.105) by using (1.116) and (1.117)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}q^2 A'_\mu A'^\mu(\upsilon + H)^2 \tag{1.118}$$

$$-\frac{1}{2}\mu^2(\upsilon + H)^2 - \frac{\lambda}{16}\lambda(\upsilon + H)^4 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} \tag{1.119}$$

where

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \tag{1.120}$$

(118) and (119) may be made easy with the fact, shown by (1.71), that $\frac{\upsilon}{\sqrt{2}}$ minimises the potential, and lastly we get

$$\mathcal{L} = \frac{1}{2}[\partial_\mu H)(\partial^\mu H) + 2\mu^2 H^2] \tag{1.121}$$

$$-\frac{1}{4}\mu^2\upsilon^2 - \frac{\lambda}{16}(H^4 + 4\upsilon H^3) \tag{1.122}$$

$$-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}q^2 A'_\mu A'^\mu(\upsilon^2 + 2\upsilon H + H^2). \tag{1.123}$$

The gauge-transformed boson A'_μ having mass $q\upsilon$ has completely eaten the Goldstone mode. We have one real scalar field which is the Higgs field, H. Its mass is $(-2\mu^2)^{\frac{1}{2}}$. As a consequence, the total number (four) of degrees of freedom is unchanged. We have (transverse) modes, plus a complex field φ component of two real fields rather than a massless gauge boson. Now we possess a massive vector field A'_μ with three modes (two transverse and one longitudinal), and with one real scalar field, H. Undoubtedly, the gauge invariance is entirely broken, because A'_μ is massive and it is real. However, it is not clear whether the renormalisability of the theory has been protected [8].

To prove renormalisability, a different gauge from that specified in (1.114) is considered. The gauge shown in (1.114) is named 'unitary' gauge, since it explains that the Goldstone boson may be removed, (1.115) whereas the surviving fields H, A'_μ are remarkably normal fields possessing the normal propagators to massive scalar and vector particles. In other words, the only poles appearing in Green functions and Feynman diagrams are those obtaining from real particles. In all other gauges, in particular in the R_ξ gauges which shall be shortly determined, spurious vector and scalar poles which must cancel from S-matrix elements exist since they are not seen in the unitary gauge. In other words, the R_ξ gauges are not clearly unitary, but they are not obviously renormalisable; the ultraviolet divergences met are no worse than those appearing in QED. If a condition in the gauge field is imposed, the R_ξ gauge is specified. The addition of a gauge-fixing term to the Lagrangian

$$L_{GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 \tag{1.124}$$

makes sure that the gauge field A^μ may be made for satisfaction of the Lorentz condition

$$\partial_\mu A^\mu = 0 \tag{1.125}$$

In the present context it is useful to use a different gauge-fixing Lagrangian, first suggested by 't Hooft:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu - \upsilon\tilde{\varphi}_2)^2. \tag{1.126}$$

This makes sure that A_μ can be selected so that

$$\partial_\mu A^\mu = \xi q\upsilon\tilde{\varphi}_2 \tag{1.127}$$

supplied $q\upsilon$ is non-zero; this decreases to (1.115) in the limit $\xi \to \infty$. Consequently, it shall be anticipated that the unitary gauge becomes a limiting case of the R_ξ gauge. The bilinear mixing of A_μ and φ_2 is eliminated thanks to the t' Hooft gauge fixing. It is remembered that this derives from the $D_\mu D^\mu \varphi^*$ term of L and from (1.107). It is seen that this holds a quadratic term

$$\frac{1}{2}[(\partial_\mu\tilde{\varphi}_2)(\partial^\mu\tilde{\varphi}_2) + 2q\upsilon A^\mu(\partial_\mu\tilde{\varphi}_2) + (q\upsilon)^2 A_\mu A^\mu] \tag{1.128}$$

$$\frac{1}{2}[(\partial_\mu\tilde{\varphi}_2)(\partial^\mu\tilde{\varphi}_2) - 2q\upsilon A^\mu(\partial_\mu\tilde{\varphi}_2) + (q\upsilon)^2 A_\mu A^\mu] + q\upsilon\partial_\mu(A^\mu\tilde{\varphi}_2). \tag{1.129}$$

Since the total divergence does not affect the action, it may be dropped, and the cross-term now cancels this in (1.126) exactly. The full Lagrangian of the Higgs model in the R_ξ gauge is therefore $L + L_{GF}$ which defines

$$\begin{aligned}\mathcal{L}_{Higgs} = \frac{1}{2}[(\partial_\mu\tilde{\varphi}_1)(\partial^\mu\tilde{\varphi}_1) + 2\mu^2\tilde{\varphi}_1^2] - \frac{1}{4}\mu^2\upsilon^2 \\ +\frac{1}{2}[(\partial_\mu\tilde{\varphi}_2)(\partial^\mu\tilde{\varphi}_2) - \xi m_A^2\tilde{\varphi}_2^2] \\ +\frac{1}{2}[(1-\xi^{-1})(\partial_\mu A_\mu)^2 - (\partial_\mu A_\upsilon)(\partial^\mu A^\upsilon) + m_A^2 A_\mu A^\mu] \\ -\frac{\lambda}{16}[4\upsilon\tilde{\varphi}_1(\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2) + (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2)^2] \\ +qA^\mu\tilde{\varphi}_1 \overleftrightarrow{\partial_\mu}\tilde{\varphi}_2 + \frac{1}{2}q^2 A_\mu A^\mu\tilde{\varphi}_1^2 + q_\mu^2 A^\mu\tilde{\varphi}_1\end{aligned} \tag{1.130}$$

where

$$m_A = q\upsilon. \tag{1.131}$$

Consequently, the $\tilde{\varphi}_1$ field defines its mass squared as a $\mu^2 + 3\lambda\upsilon^2 = -2\mu^2$ and the former Goldstone boson mode $\tilde{\varphi}_2$ now defines its mass squared as a ξm_A^2. The propagator of the vector field may be written

$$i\tilde{\Delta}_{F\rho\sigma}(p) = -i\frac{g_{\rho\sigma} + (\xi-1)p_\rho p_\sigma(p^2 - \xi m_A^2)^{-1}}{p^2 - m_A^2 + i\epsilon} \tag{1.132}$$

This gives rise to the ordinary propagator of a massive vector boson in unitary limit $\xi \to \infty$ and is connected with ultraviolet behaviour. However for all finite values of ξ the manner of the (Euclidean) momentum $\bar{p} \to \infty$ is

$$\tilde{\Delta}_{F\rho\sigma}(p) \approx |\bar{p}|^{-2} \tag{1.133}$$

For this reason the R_ξ gauge is 'manifestly renormalisable'. The theory is actually logical. S-matrix elements are cancelled by the poles at $p^2 = \xi m_A^2$. This was done by 't Hooft [8].

Chapter 2

Production of the Higgs boson at the LHC

There are several ways to observe Higgs boson at the Large Hadron Collider (LHC). In the 2 photons, 2 b quarks, 2 vector bosons(WW^*/ZZ^*) and 2 tau leptons decay channels can search a light Higgs (mass) $< 150Gev/c^2$) [9]. A fusion of 2 gluons dominantly produces Higgs and Higgs is seen via its decays into 2 photons [10]. In this project $gg \rightarrow H$ Feynman diagram cross-section will be calculated and $H \rightarrow \gamma\gamma$ Feynman diagram decay rate will be calculated.

2.1 Gluon fusion cross Section

One of the significant ways to produce Higgs at the LHC is gluon fusion. To make the Higgs particle, two smashing protons radiate from two gluons. ($gg \rightarrow H$) Gluons and Higgs particles do not interact directly with each other. Thus, gluons and Higgs particles interact strongly with top quarks and anti-quarks [11]. The top quark dominates totally in the loop because of the strong Higgs coupling to the heavy top quarks.

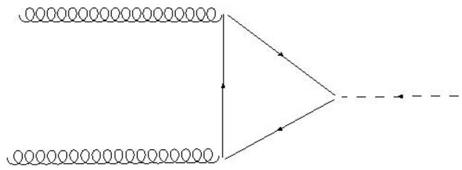

Figure 2.1: Gluon Fusion

In this masters project, we are going to study leading order (LO) gluon fusion. A Feynman diagram cross-section for the gluon to Higgs ($gg \to H$) is going to be calculated.

2.1.1 Feynman rules

There are two Feynman diagrams with a quark-loop and the same diagram with the incoming gluons crossed. These diagrams have the same contribution. We need to use Feynman rules to write down the matrix element of the two Feynman diagrams. To construct the matrix element without polarisation, we require the vertices of the top-quark with the gluon and the top-quark with the Higgs particle which are given in the standard model and the propagator of the top quark.

$$M_1^{\alpha\beta} = \int_{\infty}^{\infty} \frac{d^dk}{(2\pi)^d} \Big(\frac{-i}{\sqrt{2}} y_t \delta^{lj}\Big)\Big(\frac{(\not p_1 + \not k + m_t)}{(p_1+k)^2 - m_t^2}\Big)(-ig_s\gamma^\alpha T^a_{ji}). \tag{2.1}$$

$$\Big(\frac{i(\not k + m_t)}{k^2 - m_t^2}\Big)(-ig_s\gamma^\beta T^b_{il})\Big(\frac{i(\not k - \not p_2 + m_t)}{(k-p_2)^2 - m_t^2}\Big), \tag{2.2}$$

Where d is the number of dimensions. The trace is taken over closed fermion lines [12]

$$\Big(\frac{y_t g_s^2}{\sqrt{2}}\Big) T^a_{ji} T^b_{il} \delta^{lj} \frac{Tr[(\not p_1 + \not k + m_t)\gamma^\alpha(\not k + m_t)\gamma^\beta(\not k - \not p_2 + m_t)]}{[(p_1+k)^2 - m_t^2][k^2 - m_t^2][(k-p_2)^2 - m_t^2]} \tag{2.3}$$

Now for the generators of $SU(3)$, the identity $Tr[T^aT^b] = \frac{1}{2}\delta^{ab}$ is used. The matrix element takes then the form

$$M_1^{\alpha\beta} = \Big(\frac{y_t g_s^2}{2\sqrt{2}}\Big)\delta^{ab} \int_{\infty}^{\infty} \frac{d^dk}{(2\pi)^d} \frac{m_t Tr[\ldots]}{[(p_1+k)^2 - m_t^2][k^2 - m_t^2][(k-p_2)^2 - m_t^2]} \tag{2.4}$$

Expanding the trace and keeping in mind that odd products of γ matrices vanish, we only maintain odd powers of m_t inside the trace and can remove a factor m_t

$$\begin{aligned} Tr[\ldots] &= Tr[\underbrace{\not{p}_1\gamma^\alpha \not{k}\gamma^\beta + \not{p}_1\gamma^\alpha\gamma^\beta \not{k}}_{\not{p}_1\gamma^\alpha\gamma^\beta, k}] - Tr[\not{p}_1\gamma^\alpha\gamma^\beta \not{p}_2] + \\ &+ \underbrace{Tr[\not{k}\gamma^\alpha \not{k}\gamma^\beta + \gamma^\alpha \not{k}\gamma^\beta \not{k}]}_{Tr[2\not{k}\gamma^\alpha\not{k}\gamma^\beta]} + Tr[\not{k}\gamma^\alpha\gamma^\beta \not{k}] - \\ - & \quad Tr[\not{k}\gamma^\alpha\gamma^\beta \not{p}_2 + \gamma^\alpha \not{k}\gamma^\beta \not{p}_2] + m_t^2 Tr[\gamma^\alpha\gamma^\beta] \end{aligned} \tag{2.5}$$

Now we use the following properties for γ matrices:

- $\{\gamma^\alpha, \gamma^\beta\} = 2g^{\alpha\beta}$
- $Tr[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$
- $Tr[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$

The final result for the trace is

$$Tr[\ldots] = 4[2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta + g^{\alpha\beta}(-p_1p_2 - k^2 + m_t^2) - p_1^\alpha p_2^\beta + p_1\beta p_2\alpha + 4k^\alpha k^\beta] \tag{2.6}$$

When we look at figure (2), only incoming gluons are exchanged, the following exchanges have to be in the matrix element [12].

- $\alpha \leftrightarrow \beta$
- $p_1 \leftrightarrow p_2$
- $a \leftrightarrow b$

The matrix element for the Feynman diagram (figure 2) with a quark-loop and incoming gluons crossed is then

$$M_2^{\alpha\beta} \approx \int_\infty^\infty \frac{d^dk}{(2\pi)^d} \frac{4(2p_2^\beta k^\alpha - 2k^\beta p_1^\alpha + g^{\alpha\beta}(-p_1p_2 - k^2 + m_t^2) - p_2^\beta p_1^\alpha + p_2\alpha p_1^\beta + 4k^\beta k^\alpha)}{[(p_1+k)^2 - m_t^2][k^2 - m_t^2][(k-p_2)^2 - m_t^2]} \tag{2.7}$$

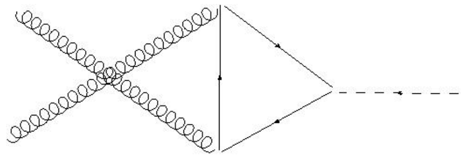

Figure 2.2: Second diagram for gluon fusion

This integral is the same as the integral for the first diagram. If we substitute $k \to -k$, the volume element d^4k does not change and we get exactly the same contribution as for the first diagram. For the whole matrix element we therefore have to take a factor of 2. The matrix element, introduced without polarisation, reads [12]

$$M^{\alpha\beta} = M_1^{\alpha\beta} + M_2^{\alpha\beta} = \Big(4m_t \frac{y_t g_s^2}{\sqrt{2}}\Big)\delta^{ab} \int_{\infty}^{\infty} \frac{d^d k}{(2\pi)^d} f^{\alpha\beta}(k), \tag{2.8}$$

where

$$f^{\alpha\beta}(k) = \frac{2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta g^{\alpha\beta}(-p_1p_2 - k^2 + m_t^2) - p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha + 4k^\alpha k^\beta}{[(p_1+k)^2 - m_t^2][k^2 - m_t^2][(k-p_2)^2 - m_t^2]} \tag{2.9}$$

2.1.2 Tensor reduction

We study the numerator of $f^{\alpha\beta}(k)$ and we want to get rid of the Dirac indices in k. We will use tensor reduction; as a result we narrow the terms in the numerator that depend on k, i.e. [12]

$$2p_1^\alpha k^\beta - 2k^\alpha p_2^\beta + 4k^\alpha k^\beta \tag{2.10}$$

If we make a general prediction for a numerator without Dirac indices in k it is [12]

$$A(k)g^{\alpha\beta} + B(k)p_1^\alpha p_1^\beta + C(k)p_2^\alpha p_2^\beta + D(k)p_2^\alpha p_1^\beta + E(k)p_2^\alpha p_2^\beta, \tag{2.11}$$

where $A(k), B(k), C(k), D(k), E(k)$ are unknown functions of k. To solve these, five different tensors are contracted to get five equations. Remember that p_1 and p_2 are the momenta of the gluons and because they are massless they fulfil $p_i^2 = m^2 = 0$.

- Contracting with $g^{\alpha\beta}$ gives

$$2(p_1k) - 2(p_2k) + 4k^2 = A(k).d + C(k)(p_1.P_2) + D(k)(P_1p_2)$$

- Contracting with $p_1^\alpha p_1^\beta$ gives

$$-2(p_1p_2)(kp_1) + 4(kp_1)^2 = E(k)(p_1p_2)^2$$

- Contracting with $p_1^\alpha p_1^\beta$ gives,

$$4(k.p_1)(kp_2) = A(k)(p_1.p_2) + D(k)(p_1p_2)^2$$

- Contracting with $p_2^\alpha p_1^\beta$ gives

$$2(p_1p_2)(kp_1) - 2(p_1p_2)(kp_2) + 4(kp_1)(kp_2) = A(k)(p_1p_2) + C(k)(p_1p_2)^2$$

- Contracting with $p_2^\alpha p_2^\beta$ gives

$$2(p_1p_2)(kp_2) + 4(kp_2)^2 = B(k)(p_1p_2)^2$$

Solving this system of equations we obtain the five unknown parameters and the numerator of $f^{\alpha\beta}(k)$ can then be explained without Dirac indices in k

$$\begin{aligned} &g^{\alpha\beta}\Big[\frac{4}{d-2}\Big(k^2 - \frac{2(kp_1)(kp_2)}{(p_1p_2)}\Big) + (m_t^2 - k^2 - (p_1p_2))\Big] \\ &+ p_1^\alpha p_1^\beta\Big[\frac{4(kp_2)^2}{(p_1p_2)^2} + \frac{2(kp_2)}{(p_1p_2)}\Big] \\ &+ p_1^\alpha p_2^\beta\Big[\frac{1}{d-2}\Big(4d\frac{(k.p_1)(kp_2)}{(p_1p_2)^2} + \frac{2k(p_1-p_2-2k)}{(p_1p_2)}\Big) - 1\Big] \\ &+ p_2^\alpha p_1^\beta\Big[\frac{4}{d-2}\frac{(kp_1)(kp_2}{(p_1p_2)^2} - \frac{k^2}{p_1p_2}\Big) + 1\Big] \\ &+ p_2^\alpha p_2^\beta\Big[\frac{4(kp_1)^2}{(p_1p_2)^2} - \frac{2(kp_1)}{(p_1p_2)}\Big]. \end{aligned} \tag{2.12}$$

2.1.3 Ward identity

At some point we need the polarisation vectors to contract our matrix element [12].

$$\epsilon_{1,\alpha}\epsilon_{2,\beta}M^{\alpha\beta}. \tag{2.13}$$

According to Ward identity, the amplitude M vanishes when the polarisation vector ϵ_μ is replaced by the momentum k_μ.

$$\epsilon_\mu M^\mu(k) = k_\mu M^\mu(k) = 0 \tag{2.14}$$

Since we have two longitudinally polarised massless gluons, the Ward identity offers us the condition [12]

$$p_{1,\alpha}p_{2,\beta}\Big(A(k)g^{\alpha\beta} + B(k)p_1^\alpha p_1^\beta + C(k)p_2^\alpha p_2^\beta + D(k)p_2^\alpha p_1^\beta + E(k)p_2^\alpha p_2^\beta\Big) = 0 \tag{2.15}$$

Note that terms like $\epsilon_{1,\alpha}p_1^\alpha = (\epsilon_1.p_1)$ vanish. We get

$$p_{1,\alpha}p_{2,\beta}\Big(A(k)g^{\alpha\beta} + D(k)p_2^\alpha p_1^\beta\Big) = 0 \;\Rightarrow\; A(k) = -D(k)(p_1p_2). \tag{2.16}$$

We can put together the above eq.(2.12) and get

$$M^{\alpha\beta} = \Big(4m_t\frac{y_t g_s^2}{\sqrt{2}}\delta^{ab}\Big)A(m_t)\Big(g^{\alpha\beta} - \frac{p_2^\alpha p_1^\beta}{(p_1.p_2)}, \tag{2.17}$$

where

$$A(m_t) \equiv \int_\infty^\infty \frac{d^dk}{(2\pi)^d}\frac{A(k)}{[(p_1+k)^2 - m_t^2][k^2 - m_t^2][(k-p_2)^2 - m_t^2]}. \tag{2.18}$$

Solving the integral in $A(m_t)$ is quite an over-long and technical calculation. It can be found in appendix A. The result we are going to obtain from further calculation is

$$A(m_t) = \frac{i}{(4\pi)^2}(1 + (1-\tau)f(\tau)], \tag{2.19}$$

where

$$f(\tau) = \begin{cases} \arcsin^2\frac{1}{\sqrt{\tau}}, & \tau \geq 1, \\ \frac{-1}{4}\left(\log[\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}] - i\pi\right)^2, & \tau > 1, \end{cases} \tag{2.20}$$

$$\tau = 4\left(\frac{m_t}{m_h}\right)^2 \tag{2.21}$$

2.1.4 Average Over Polarizations

We have to express eq.(2.17) again with the polarisation vectors of the gluons and sum of all polarisations. We have to divide eq.(2.17) by the number of polarisations N_p and N_g

$$|M|^2 = \sum_{pol}\left|\epsilon_{1,\alpha}^*(\lambda_1,p_1)\epsilon_{2,\beta}^*(\lambda_2,p_2)M^{\alpha\beta}\frac{1}{N_pN_g}\right|^2 \tag{2.22}$$

where $\sum_{pol}$ is $\lambda_1 = 1,2$ and $\lambda_2 = 1,2$. This is quite a long calculation. However the result is [12]

$$\sum_{pol}\left|\epsilon_{1,\alpha}^*(\lambda_1,p_1)\epsilon_{2,\beta}(\lambda_2,p_2)M^{\alpha\beta}\right|^2 = (d-2)\left(4m_t.\frac{y_tg_s^2}{\sqrt{2}}\delta^{ab}\right)^2|A(m_t)|^2. \tag{2.23}$$

The amplitude is

$$|M|^2 = \frac{\delta^{aa}(d-2)}{(N_pN_g)^2}\left(.\frac{4y_tg_s^2m_t}{\sqrt{2}}\right)^2 A(m_t).A^*(m_t) \tag{2.24}$$

where $y_t = \frac{m_t}{\nu}$ is the Yukawa coupling, with $\nu = 174$Gev. The constants are $d = 4$, $N_g = 8$, $N_p = 2$ and we plug in $g_s^2 = 4\pi\alpha_s(\mu)$. In addition $\delta^{aa} = Tr(I_{8\times 8}) = 8$ due to the existence of 8 different gluons. We get [12]

$$|M|^2 = \frac{1}{2}\frac{m_t^4}{\nu^2}(4\pi)^2\alpha_s^2(\mu)A(m_t).A^*(m_t) \tag{2.25}$$

We maintain a final result by using eq.(2.19)

$$|M|^2 = \frac{1}{2}\left(\frac{1}{4\pi}\right)^2\frac{m_t^4}{\nu^2}\alpha_s^2(\mu)\left|1 + (1-\tau)f(\tau)\right|^2 \tag{2.26}$$

2.1.5 Cross Section

The cross-section will be defined for incoming and outgoing particles. We have two incoming particles with mass and momentum m_1, p_1 and m_2, p_2 and one outgoing particle with mass and momentum m_h,q by using the matrix element [34]

$$d\sigma_{gg\to h} = \frac{1}{4\sqrt{(p_1.p_2)^2 - m_1^2m_2^2}}(2\pi)^4\delta^4(p_1 + p_2 - q)\left|M\right|^2\frac{d^{3q}}{(2\pi)^3}\frac{1}{2q_0} \tag{2.27}$$

After the above equation is integrated, the gluon gluon to Higgs cross-section is maintained

$$\sigma_{gg\to h} = \int d\sigma_{gg\to h} = \frac{\pi}{m_h^2}\left|M\right|^2\delta(2p_1.p_2 - m_h^2). \tag{2.28}$$

This LO gluon fusion result is standart textbook result which can be available in [33].

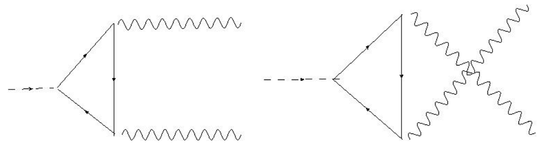

Figure 2.3: Higgs decay to photon pairs via fermion loop

2.2 Higgs Decay to photons via fermion loop

In this section we will calculate the decay with two photons. We have fermion-loop diagrams (figure 2.3). Photons do not couple directly to the Higgs field because they are massless. As a result, we have two Feynman diagrams with a fermion loop and the same diagram with the outgoing photons crossed. The matrix element for the fermion-loop diagram yields:

$$\begin{aligned} M^{ab\lambda\lambda'} = -i\frac{m_f}{v}(-1)N_c(f)Q_f^2 \int \frac{d^4l}{2\pi^4} tr\left[\frac{i(\not{l}-\not{k}+m_f)}{(l-k)^2-m_f^2+i\epsilon}(ie\gamma^\nu)\right. \\ \times \frac{i(\not{l}+m_f)}{l^2-m_f^2+i\epsilon}(ie\gamma^\mu)\frac{i(\not{l}+\not{p}+m_f)}{(l+p)^2-m_f^2+i\epsilon} tr[t^a t^b]\epsilon_\nu^{*\lambda'}(k)\epsilon_\mu^{*\lambda'}(p) \end{aligned} \quad (2.29)$$

To write down the matrix element we need to propagator of fermion, vertices of fermion with the photons, vertices of fermion with the Higgs particle and external photons. The sign in front of integral is related to the fermion loop and traces are taken of all Dirac and colour matrices. The definition of v in M is $v = \frac{e}{\sin\theta_w m_w}$. It is obvious that this diagram is potentially divergent because of the loop. The denominator is written again by using Feynman parametrisation: [14]

$$\begin{aligned} \frac{1}{((l-k)^2-m_f^2+i\epsilon)((l^2-m_f^2+i\epsilon)((l+p)^2-m_f^2+i\epsilon)} &= \\ \int dxdydz \frac{2\delta(x+y+z-1)}{(l^2-2l.(xk-zp)+xk^2+zp^2-m_f^2+i\epsilon)^3} &= \\ \int_0^1 dx \int_0^{1-x} dz \frac{2}{(l^2-2l.(xk-zp)-m_f^2+i\epsilon)^3} &= \\ \int_0^1 dx \int_0^{1-x} dz \frac{2}{(l'^2-\Delta)^3} & \end{aligned} \quad (2.30)$$

In the above equation Q_f is charge of fermion and N_c is colour factor.

In the last line the square was completed using this calculation:

$$\begin{aligned} l^2-2l.(xk-zp) = \quad & (l-(xk-zp))^2-(xk-zp)^2 \\ & = l'^2-(x^2k^2+z^2p^2-2xzk.p) \\ & = l'^2+2xzk.p = l'^2+xzm_h^2 \end{aligned} \quad (2.31)$$

and the definitions:

$$l' = l - (xk - zp) \tag{2.32}$$

$$\Delta = m_f^2 - xzm_f^2 - i\epsilon \tag{2.33}$$

We use trace techniques and the anticommutator identity $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ for the numerator:

$$\begin{aligned}
tr[\ldots] = [(\not{l} - \not{k}) + m_f)\gamma^\nu(\not{l} + m_f)\gamma^\mu(\not{l} + \not{p} + m_f)] \\
&= tr[(\not{l} - \not{k})\gamma^\nu l\gamma^\mu m_f + (\not{l} - \not{k})\gamma^\nu\gamma^\mu m_f(\not{l} + \not{p}) \\
&+ m_f\gamma^\nu \not{l}\gamma^\mu(\not{l} + \not{p}) + m_f\gamma^\nu\gamma^\mu m_f] \\
&= m_f tr[\not{l}\gamma^\nu\{\not{l}, \gamma^\mu\} - \not{k}\gamma^\nu\{\not{l}, \gamma^\mu\} - \not{p}\not{k}\gamma^\nu\gamma^\mu + \gamma^\nu \not{l}\gamma^\mu \not{l} + \{\not{l}, \gamma^\nu\}\gamma^\mu \not{p}] + 4m_f^3 g^{\mu\nu} \\
&= m_f(2l^\mu tr[l\gamma^\nu] - 2l^\mu tr[k\gamma^\nu] - 4p_\alpha k_\beta(g^{\alpha\beta} g\nu\mu - g^{\alpha\nu}g^{\beta\mu} + g^{\alpha\mu}g^{\nu\beta}) \\
&+ 4l_\alpha l_\beta(g^{\nu\alpha}g^{\mu\beta} - (g^{\nu\mu}g^{\alpha\beta} + (g^{\nu\beta}g^{\alpha\mu})2l^\nu tr[\gamma^\mu p] + 4m_f^2 g^{\mu\nu}) \\
&= 4m_f(2l^\nu l^\mu - 2l^\nu k\nu - p.kg^{\nu\mu} - p^\mu k^\nu + k\mu p^\nu + 2l^\mu l^\nu - g^{\nu\mu}l^2 + 2l^\nu p^\mu + m_f^2 g^{\nu\mu}) \\
&= 4m_f((4l^\mu l^\nu - l^2 g^{\nu\mu}) + (m_2^f - p.k)g^{\nu\mu} - (p^\mu k^\nu - k^\mu p^\nu) + 2(p^\mu l^\nu - l^\mu k^\nu)) \quad (2.34)
\end{aligned}$$

All factors proportional to p^μ and $k\nu$ vanish because of identity $p^\mu\epsilon_\mu(p) = 0$ (Ward identity). The factors proportional to $g^{\nu\mu}$ and $k^\mu p^\nu$ have to be kept.

We must write again the factors containing loop momenta l in terms of l' which is a new momentum one integrates over. Terms proportional to l' will be taken out since they vanish after integration:

$$\begin{aligned}
&4l^\mu l^\nu - l^2 g\nu\mu + 2(p^\mu l^\nu - l^\mu k^\nu) \\
&= 4(l' + (xk - zp))^\mu + (l' + (xk - zp))^\nu - (l' + (xk - zp))^2 g^{\nu\mu} \\
&+2(p^\mu(l' + (xk - zp))^\nu - (l' + (xk - zp))^\mu k^\nu) \\
&= 4l'^\mu l'^\nu + 4z^2 p^\mu p^\nu + 4xz(p^\mu k^\nu + k^\mu p^\nu) - (l'^2 - 2xzk.p)g^{\nu\mu} \\
&+2xp^\mu k^\nu - 2zp^\mu k^\nu - 2xk^\mu k^\nu + 2zp^\mu k^\nu \\
&= 4l'^\mu - l'^2 g^{\nu\mu} + (4z^2 - 2z)p^\mu p^\nu + (4x^2 - 2x)k^\mu k^\nu - 4xzk^\mu p^\nu \\
&+(2x + 2z - 4xz)p^\mu k^\nu + xzm_h^2 g^{\nu\mu}
\end{aligned}$$

Putting this together with all constant terms in equation (2.34) and changing the dimension from $d = 4$ to $d = 4 - 2\epsilon$ the numerator yields:

$$\begin{aligned}
tr[\ldots] = 4m_f\left(\left(\frac{4}{d} - 1\right)\right)l'^2 g^{\mu\nu} + \left(xzm_h^2 + m_f^2 - \frac{m_h^2}{2}\right)g^{\mu\nu} \\
+(4z^2 - 2z)p^\mu p^\nu + (4x^2 - 2x)k^\mu k^\nu + (1 - 4xz)k^\mu p^\nu + (2x + 2z - 1 - 4xz)p^\mu k^\nu
\end{aligned} \tag{2.35}$$

Now we come across two different terms: terms proportional to l'^2 are potentially divergent; all others are convergent. By using master formula of dimensional regularisation, the first term is integrated to obtain:

$$\int \frac{d^d l'}{(2\pi)^d}\left(\frac{4}{d}-1\right) l' \frac{2}{(l'^2-\Delta)^3} = \frac{(-1)^2 i}{(4\pi)^{\frac{d}{2}}}\frac{d}{2}\left(\frac{4}{d}-1\right)\frac{2\Gamma(3-\frac{d}{2}-1}{\Gamma(3)}\left(\frac{1}{\Delta}\right)^{(}3-\frac{d}{2}-1)$$
$$\frac{i}{(4\pi)^2}\epsilon\Gamma(\epsilon)\left(\frac{(4\pi)}{\Delta}\right)^{\epsilon} = \frac{i}{(4\pi)^2}\epsilon\left(\frac{1}{\epsilon}-\gamma+O(\epsilon)\right)(1+O(\epsilon))$$
$$= \frac{i}{(4\pi)^2}(1+O(\epsilon)) \overset{\epsilon\to 0}{\to} \frac{i}{(4\pi)^2} = -\frac{i}{16\pi^2}\frac{xzm_h^2-m_f^2}{\Delta} \quad (2.36)$$

This is convergent because the term $\frac{d}{2}\left(\frac{4}{d}-1\right)$ is proportional to ϵ. This removes the divergence of $\Gamma(x)$ at $x=0$. The other integral of the constant factor provides:

$$\int \frac{d^4 l'}{(2\pi)^4}\frac{2}{(l'^2-\Delta)^3} = \frac{(-1)^3 i}{(4\pi)^2}\frac{\Gamma(3-2)}{\Gamma(3)}\frac{2}{\Delta} = -\frac{i}{16\pi^2}\frac{1}{\Delta} \quad (2.37)$$

Taking all factors proportional to $g^{\mu\nu}$, $k^\mu p^\nu$ and neglecting other tensorial structures which vanish on multiplying with the polarisation vectors reads:

$$\int \frac{d^4 l'}{(2\pi)^4}\frac{2tr[\ldots]}{(l'^2-\Delta)^3} = -\frac{4m_f i}{16\pi^2}\frac{(xzm_h^2-m_f^2)g^{\mu\nu}+(xzm_h^2+m_f^2-\frac{m_h^2}{2})g^{\nu\mu}+(1-4xz)k^\mu p^\nu}{\Delta}$$
$$= \frac{m_f}{(4\pi)^2}\frac{1}{\Delta}\left(\left(\frac{m_h^2}{2}-2xzm_h^2\right)g^{\mu\nu}-(1-4xz)k^\mu p^\nu\right) = \frac{m_f i}{4\pi^2}\left(\frac{m_h^2}{2}g^{\nu\mu}-k^\mu p^\nu\right)\frac{1-4xz}{\Delta}$$
$$= \frac{i}{4m_f\pi^2}\left(\frac{m_h^2}{2}g^{\nu\mu}-k^\mu p^\nu\right)\frac{1-4xz}{1-xz\frac{m_h^2}{m_f^2}}$$

An integral over the remaining Feynman parameter x and z will be solved in appendix B.

$$I\left(\frac{m_h^2}{m_f^2}\right) = \int_0^1 dx \int_0^{1-x} dz \frac{1-4xz}{1-xz\frac{m_h^2}{m_f^2}} \quad (2.38)$$

After we multiply with polarisation vectors the expression for the numerator is invariant under $(p,\mu,\lambda) \leftrightarrow (k,\nu,\lambda')$. As a result, we get total amplitude when we multiply our amplitude by a factor 2 since contribution of both Feynman diagrams is the same.

$$2M^{ab\lambda\lambda'} = (-1)^2 i^6 N_c(f) Q_f^2 e^2 \frac{m_f}{\upsilon} tr[t^a t^b]\frac{i}{4m_f\pi^2}\left(\frac{m_h^2}{2}g^{\mu\nu}-k^\mu p^\nu\right) I\left(\frac{m_h^2}{m_f^2}\right)\epsilon_\nu^{*\lambda'}(k)\epsilon_\mu^{*\lambda'}(p)$$
$$= -\frac{iN_c(f)Q_f^2 e^2}{2\pi^2\upsilon} tr[t^a t^b] I\left(\frac{m_h^2}{m_f^2}\right)\left(\frac{m_h^2}{2}g^{\mu\nu}-k^\mu p^\nu\right)\epsilon_\nu^{*\lambda'}(k)\epsilon_\mu^{*\lambda'}(p)$$
$$= -\frac{iN_c(f)Q_f^2 e^3}{4\pi^2 \sin\theta_w m_w}\frac{1}{2}\delta^{ab} I\left(\frac{m_h^2}{m_f^2}\right)\epsilon_\nu^{*\lambda'}(k)\epsilon_\mu^{*\lambda'}(p) \quad (2.39)$$

If we first calculate the squared expression of the tensorial structures with the polarisation vectors, we can maintain easily the squared result [15]:

$$
\begin{aligned}
&\sum_{\lambda,\lambda'} |(A.g^{\mu\nu} + B.k^{\mu}p^{\nu})\epsilon_{\nu}^{*\lambda'}(p)\epsilon_{\mu}^{*\lambda'}(k)|^2 \\
&= (A^2.g^{\mu_1\nu_1}g^{\mu_2\nu_2} + B^2.k^{\mu_1}p^{\nu_1}k^{\mu_2}p^{\nu_2} + AB.g^{\mu_1\nu_1}g^{\mu_2\nu_2} + BA.k^{\mu_1}p^{\nu_1}g^{\mu_2\nu_2}) \\
&\times \left(\sum_{\lambda} \epsilon_{\mu 1}^{*\lambda'}(p)\epsilon_{\mu 2}^{\lambda'}(p)\right)\left(\sum_{\lambda'} \epsilon_{\nu 1(k)}^{*\lambda'}\epsilon_{\nu 2}^{\lambda'}(k)\right) \\
&= 4A^2 + B32(k^2p^2) + 2AB(k.p) = 4A^2 + 2AB(k.p)
\end{aligned}
\tag{2.40}
$$

The squared amplitude summed over all polarisation states and photon states lastly yields:

$$
\begin{aligned}
&|M|^2 = \sum_{\lambda,\lambda'}\sum_{a,b}\left|\sum_f 2M_q^{a,b,\lambda,\lambda'}\right|^2 = \\
&\frac{e^6Q_f^4N_c^2(f)}{16\pi^4m_w^2\sin^2\theta_w}\Big|\sum_f I\frac{m_h^2}{m_f^2}\Big|^2\frac{1}{4}\sum_{a,b}|\delta^{ab}|^2\sum_{\lambda,\lambda'}\Big|\left(\frac{m_h^2}{2}g^{\mu\nu}k^{\mu}p^{\nu}\right)\epsilon_{\mu(p)}^{\lambda *}\epsilon_{\nu(k)}^{*\lambda'} \\
&= \frac{e^6Q_f^4N_c^2(f)}{64\pi^4m_w^2\sin^2\theta_w}\left|\sum_f I\frac{m_h^2}{m_f^2}\right|^2 .8.\left(4\frac{m_h^4}{4} - 2\frac{m_h^2}{2}\frac{m_h^2}{2}\right) \\
&= \frac{e^6Q_f^4N_c^2(f)}{16\pi^4m_w^2\sin^2\theta_w}\left|\sum_f I\frac{m_h^2}{m_f^2}\right|^2
\end{aligned}
\tag{2.41}
$$

Therefore decay rate is

$$
\Gamma = \frac{1}{2m_h}\frac{e^6Q_f^4N_c^2(f)m_h^4}{16\pi^4m_w^2\sin^2\theta_w}\left|\sum_f I\frac{m_h^2}{m_f^2}\right|^2\frac{1}{8\pi} \tag{2.42}
$$

Chapter 3

New Physics in $gg \to H \to \gamma\gamma$

The effects of the recent discovery of 125 Gev Higgs SM-like particle at the $\sqrt{s} = 7$ and 8 Tev LHC and Tevatron will be studied. The signal with two photons in the final state may be larger than expected within the SM almost by a factor of two. The announced cross-section of ATLAS and CMS is $\sigma/\sigma_{SM} = 1.4 \pm 0.3$ and $\sigma/\sigma_{SM} = 1.6 \pm 0.4$ respectively [17, 32]. A larger branching ratio for Higgs to two photons is found by Tevatron, Atlas and CMS.

Investigation into the existence of new physics (NP) has been continued at the Tev scale. Acording to a hierarchy problem, Higgs must couple to these new states. Such coupling can impact on both its production and decay properties [16]. New physics can affect the properties of Higgs boson and contribute to extensions of the SM. For instance, it can be a signal of such light exotics at the LHC. This possibility is amazing in light of the recent results from LHC Higgs search.

3.1 Higgs Portal to Exotic Scalars

Interactions between the Higgs and new exotic particles S impact on modifications of loop-level Higgs production and decay process like $gg \to h$ and $h \to \gamma\gamma$. Moreover a new process $h \to SS$ can be opened in the case of a comparatively heavy Higgs. Thus, the specific quantum numbers of the new exotic states lead to many forms of interaction between the Higgs and exotics. A class of interactions, which are generic enough and universal form, are so-called Higgs portal interaction. The combining H^+H can be organised with an operator O_{NP}, which is a gauge and Lorentz invariant operator built out of exotic new fields. The Higgs portal interactions are parametrised by operators [16].

$$\mathcal{L} \supset \lambda H^{+} H O_{NP} \quad (3.1)$$

Additionally, $H^{+}H$ has two dimensions, so the Higgs portal interaction is typically of a low dimension and may be renormalisable if new exotic scalar exists. The operators $hG^{a}_{\mu\nu}G^{\mu\nu}_{a}$ and $hF_{\mu\nu}F^{\mu\nu}$ are produced. If we integrate new exotic states charged under colour or electromagnetism. The Higgs portal also couples to additional scalars S of the term

$$\mathcal{L} > -\lambda(H^{+}H)(S^{+}S) \quad (3.2)$$

and based on a set of possible $SU(3)_C \times SU(2)_L \times U(1)_Y$ representations of S. Fermions F and vector bosons V coupling via the Higgs portal may affect the phenomenology of the Higgs boson. For fermions and vector bosons the most generic Higgs portal couplings $(H^{+}H)(\bar{F}F)$ and $H^{+}HV^{\mu\nu}V_{\mu\nu}$ are not renormalised [19]. Thus, a minimal operator with scalar in eq.(3.7) should be maintained to get sizeable coupling and restrict new exotic scalars S [16].

To conclude, if deviations from an SM-like Higgs are maintained, encouraging and experimentally testable results will be produced by new light exotic matter coupling via Higgs portal, as such states can be seen precisely in the future at the LHC [16].

3.2 Doubly Charged Scalars Enhance Ratio Of $H \rightarrow \gamma\gamma$ In The Higgs Triplet Model

The Higgs triplet model (HTM) is a model of neutrino mass generation which suggests the existence of a doubly charged Higgs boson ($H^{\pm\pm}$) and singly charged Higgs boson ($H^{\pm}$). Such a particle could increase the branching ratio of a neutral Higgs boson decaying to two photons.

The lightest CP-even scalar (H_1) maintains the same couplings to the fermions and vector bosons as the Higgs boson of the SM [20]. As a result, the running searches for the SM Higgs boson also use H_1 of the HTM with little change. The loop-induced decay $H_1 \rightarrow \gamma\gamma$, which gets contribution from virtual $H^{\pm\pm}$ and $H^{\pm}$, is not often seen. Its branching ratio can be very different to that of the SM Higgs boson. As studied in [21], the running limits on BR($H_1 \rightarrow \gamma\gamma$) have a negative impact on the parameter space of $[m_{H^{\pm\pm}}, \lambda_1]$ where λ_1 represents a quartic coupling in the scalar potential. The case $\lambda_1 < 0$ can enhance the branching ratio of $H_1 \rightarrow \gamma\gamma$ [22].

3.2.1 Higgs Triplet Model

If HTM [23], a $Y = 2$ complex $SU(2)_L$ isospin triplet of scalar fields $T = (T_1, T_2, T_3)$ is available in the SM Lagrangian. The gauge invariant Yukawa interaction is

$$\mathcal{L} = h_{\ell\ell'} L_\ell^T C_{iT_2} \Delta L_{\ell'} + h.c \tag{3.3}$$

$h_{\ell\ell'}(\ell, \ell' = e, \mu, \tau)$ is a complex and symmetric coupling, C represents the Dirac charge conjugation operator, $T_i(i = 1-3)$ are the Paulimatrices, $L_\ell = (\upsilon_{\ell L}, \ell_L)^T$ left-handed lepton doublet, and Δ is 2×2 representation of the $Y = 2$ complex triplet fields:

$$\Delta = \mathbf{T}.\tau = T_1\tau_1 + T_2\tau_2 + T_3\tau_3 = \begin{pmatrix} \Delta^{+/\sqrt{2}} & \Delta^{++} \\ \Delta^{\circ} & -\Delta^{+}/\sqrt{2} \end{pmatrix} \tag{3.4}$$

where $T_1 = (\Delta^{++} + \Delta^{\circ})/2$, $T_2 = i(\Delta^{++} - \Delta^{\circ})/2$ and $T_3 = \Delta^{+}/\sqrt{2}$. A non-zero triplet vev $< \Delta^{\circ} >$ leads to the mass matrix for neutrinos:

$$m_{\ell\ell'} = 2h_{\ell\ell'} < \Delta^{\circ} >= \sqrt{2} h_{\ell\ell'} \upsilon_{\Delta} \tag{3.5}$$

Invariant Higgs potential [24, 25] which is shown (with $H = (\Phi^{+}, \Phi^{\circ})^T$):

$$\begin{aligned} V(H, \Delta) = & -m_h^2 H^{+} H + \frac{\lambda}{4}(H^{+}H)^2 + M_{\Delta}^2 Tr\Delta^{+}\Delta + (\mu H^T i\tau_2 \Delta^{+} H + h.c.) \\ & + \lambda_1 (H^{+}H) Tr\Delta^{+}\Delta + \lambda_2 (Tr\Delta^T \Delta)^2 + \lambda_3 Tr(\Delta^{+}\Delta)^2 + \lambda_4 H^{+}\Delta\Delta^{+} H. \end{aligned} \tag{3.6}$$

Here $m^2 < 0$ to make sure non-zero $< \Phi^{\circ} >= \upsilon/\sqrt{2}$ which spontaneously breaks $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$ while M_{Δ}^2.

In the HTM the scalar eigenstates are

1. the charged scalars $H^{\pm\pm}$ and $H^{\pm}$
2. the CP-even neutral scalars H_1 and H_2
3. a CP-odd neutral scalar A°

$H^{\pm}, H_2, A^{\circ}$ are a mixture of the triplet field, H_1 is a mixture of the doublet field.

The squared masses of H_1 and H_2 are:

$$m_{H_1}^2 = \frac{\lambda}{2}\upsilon^2 \tag{3.7}$$

$$m_{H_2}^2 = M_{\Delta}^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{2})\upsilon^2 + 3(\lambda_2 + \lambda_3)\upsilon_{\Delta}^2 \tag{3.8}$$

The squared mass of the CP-odd A° is [22]

$$m_{A^{\circ}}^2 = M_{\Delta}^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{2})\upsilon^2 + (\lambda_2 + \lambda_3)\upsilon_{\Delta}^2 \quad (3.9)$$

The squared mass of the $H^{\pm}$ is [22]

$$m_{H^{\pm}}^2 = M_{\Delta}^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{2})\upsilon^2 + (\lambda_2 + \sqrt{2}\lambda_3)\upsilon_{\Delta}^2 \quad (3.10)$$

The squared mass of the doubly-charged scalar is ($H^{\pm\mp} = \delta^{\pm\pm}$) is [22]

$$m_{H^{\pm\pm}}^2 = M_{\Delta}^2 + \frac{\lambda_1}{2}\upsilon^2 + \lambda_2\upsilon_{\Delta}^2 \quad (3.11)$$

3.3 The Fourth Generation of Standard Model impacts on Higgs Searches

The amazing possibility of a four-generation Standard Model(SM4) has been investigated [26]. The search for production of fourth generation quarks and leptons at colliders impacts on the electroweak parameters [27, 28] and on the Higgs boson production and decay partial widths [29, 30]. In brief, a fourth generation alters the Higgs branching fractions. Particularly, the coupling to gluons and photons is influenced at the loop level. Moreover they have a positive impact on the presence of heavy new particles. Existence of a fourth generation can be restrained by the precise measurements of the Higgs production rate and branching ratios [31].

There are three components to make an such exclusion possible. First, the gluon fusion production rate of a light Higgs boson by a factor of $O(10)$ would be increased by the fourth generation top and bottom quarks. Second, the partial decay width to diphotons can be prevented by as much as a factor of $O(100)$. Last, partial decay widths to final states which are controlled by tree-level amplitudes, such as $b\bar{b}$ and ZZ^*, get smaller corrections to the standard model prediction. As a result, substantial increase is seen in gluon fusion produced channels, but the diphoton channel shows an important decrease [31].

In the SM, the top induced one-loop contribution controls the gluon fusion. The SM4 presents two new heavy quarks into the loop, for which the leading-order (LO) contribution is nearly independent of the real masses. Therefore, the gluon fusion rate is enlarged by a factor of 9 at LO [31].

The fourth generation top and bottom changes the LO contributions to the Higgs, partial widths to diphotons and digluons as well. The $h \to gg$ is enhanced by a factor of 9. However, $h \to \gamma\gamma$, which is controlled by W-boson loop, is prevented since the extra fermions destructivly interrupt the W-boson contribution. At LO this shows that the diphoton width is decreasing by a factor of nearly 5 than the SM. In addition it does not depend on fermion masses. Finally, the other leading partial widths at tree-level stay without changing at LO [31].

The gluon fusion production and diphoton decay rate are especially sensitive to the existence of extra sequantial quarks and leptons. Thus an excellent opportunity is supplied by measurements of these rates to remind about the limits of the SM4. The discovered excess shows a cross-section that is quite a lot larger than the SM prediction. It cannot be interpreted as a clue for new physics, whereas it can be interpreted to put strong constraints on $SM4$ [31].

Conclusion

To understand Higgs mechanism we have studied Abelian and non-Abelian gauge field theories. Thanks to this theory we have discussed gauge invariant Lagrangian and spontaneous symmetry breaking which gives rise to Goldstone bosons. The Higgs mechanism has been defined to generate massive vector bosons in a gauge invariant theory. We defined the new form of Lagrangian (1.102) by using covariant derivatives instead of normal derivatives and adding free gauge fields A_μ. Moreover, we have seen how this Lagrangian gives mass to vector boson field A and scalar fields.

As we know the SM-like Higgs boson is now being discovered at the LHC. Higgs boson is observed by fusion of two gluons which is one of the dominant channels, but Higgs decays to photons easily. Thus, I have studied the SM cross-section of gluon fusion Higgs production loop diagram. All steps of the calculation have been done in detail. I have also calculated the decay of Higgs to diphoton loop diagram in the SM by using dimension regularisation.

Finally we have discussed the implications of the discovery at LHC of the observation of a Higgs-like particle with a mass nearly 125Gev in terms of new physics beyond the standard model. We have considered the effects of new exotic matter interacting through Higgs portal on SM Higgs boson searches. We have realised Higgs portal couplings could modify the Higgs production and decay patterns. Another effect of new physics has been studied in the Higgs triplet models. Doubly charged scalars $H^{\pm\pm}$ and singly charged scalars $H^{\pm}$ have been seen in HTM. $H^{\pm\pm}$ charged scalar can alter the branching ratio of $H_1 \to \gamma\gamma$. In addition we have studied the implications of a fourth generation of standard model on Higgs searches. A fourth generation would impact powerfully on the Higgs couplings to gluons and photons.

To conclude, the particle discovered at CERN could be Higgs boson or not. If the Higgs diphoton rate continues to be increased above the SM prediction in 2012 it means we could be close to new physics.

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Appendix A

Calculation for the gluon fusion

In this section the integral $A(m_t)$ from equation (2.18) is going to be computed.

A.0.1 Master Integrals

First integrals are defined with master integrals. All Dirac indices of k have been eliminated; now we require to make the numerators of $A(m_t)$ independent of k. There are 6 different types of numerator: k^2, $k.p_1$, $k.p_2$, $(k.p_1)(k.p_2)$, $(k.p_1)^2$ and $(k.p_2)^2$. For each of these we have to develop a different method [12].

- $\mathbf{k}^2$

$$\begin{aligned} &\frac{k^2+m_t^2-m_t^2}{[(2k.p_1)+k^2-m_t^2][k^2+m_t^2][k^2-2(k.p_2)-m_t^2]} \\ &=\frac{m_t^2}{(2k.p_1)+k^2-m_t^2][k^2-m_t^2][k^2-2(k.p_2)-m_t^2]}+ \\ &+\frac{1}{(2k.p_1)+k^2-m_t^2][k^2-2(k.p_2)-m_t^2]} \end{aligned} \tag{A.1}$$

- $\mathbf{k.p}_1$

$$\begin{aligned} &\frac{k.p_1+\frac{1}{2}(k^2-k^2+m_t^2-m_t^2)}{[(2p_1.k)+k^2-m_t^2][k^2-m_t^2][k^2-2(k.p_2)-m_t^2]} \\ &=\frac{\frac{1}{2}}{[k^2-m_t^2][k^2-2(k.p_2)-m_t^2]}- \\ &-\frac{\frac{1}{2}}{[(2k.p_1)+k^2-m_t^2][k^2-2(k.p_2)-m_t^2][k^2-2(k.p_2)-m_t^2]} \end{aligned} \tag{A.2}$$

- **k.p$_2$**

$$\frac{k.p_2 + \frac{1}{2}(k^2 - k^2 + m_t^2 - m_t^2)}{[(2p_1.k) + k^2 - m_t^2][k^2 - m_t^2][k^2 - 2(k.p_2) - m_t^2]}$$
$$\frac{\frac{-1}{2}}{[k^2 - 2(k.p_1) - m_t^2][k^2 - m_t^2]} + \tag{A.3}$$
$$+ \frac{\frac{1}{2}}{[(2k.p_1) + k^2 - m_t^2][k^2 - 2(k.p_2) - m_t^2][k^2 - 2(k.p_2) - m_t^2]}$$

- $(k.p_1)(k.p_2)$ Here we use the results of the $k.p_1$ case.

$$\underbrace{\frac{1}{2}\int_{\infty}^{\infty} d^dk \frac{p_2.k}{[k^2 - m_t^2][(k-p_2)^2 - m_t^2]}}_{\equiv I_1} +$$
$$\underbrace{\frac{1}{2}\int_{\infty}^{\infty} d^dk \frac{-p_2.k}{[(k+\frac{p_1}{2})^2 - m_t^2][(k-p_2)^2 - m_t^2]}}_{\equiv I_2} = \frac{1}{2}(I_1 + I_2) \tag{A.4}$$

forI$_1$ substitute $k \to k + \frac{p_2}{2}$

$$I_1 = \underbrace{\int_{\infty}^{\infty} d^dk \frac{p_2.k}{[(k+\frac{p_2}{2})^2 - m_t^2][(k-\frac{p_2}{2})^2 - m_t^2]}}_{\equiv 0} + \underbrace{p_2^2}_{0}(\ldots) = 0 \tag{A.5}$$

The first term in the sum is an odd function because it vanishes. For I_2 we substitute $k \to k - \frac{p_1}{2} + \frac{p_2}{2}$

$$I_2 = \underbrace{\int_{\infty}^{\infty} d^dk \frac{-p_2.k}{[(k+\frac{p_1}{2}+\frac{p_2}{2})^2 - m_t^2][(k-\frac{p_1}{2}-\frac{p_2}{2})^2 - m_t^2]}}_{\equiv 0} +$$
$$\frac{1}{2}\int_{\infty}^{\infty} d^dk \frac{p_1.p_2}{[(k+\frac{p_1}{2}+\frac{p_2}{2})^2 - m_t^2][(k-\frac{p_1}{2}-\frac{p_2}{2})^2 - m_t^2]} \tag{A.6}$$
$$\Rightarrow \frac{1}{2}(I_1 + I_2) = \frac{1}{4}\int_{\infty}^{\infty} d^dk \frac{p_1.p_2}{[(k+\frac{p_1}{2}+\frac{p_2}{2})^2 - m_t^2][(k-\frac{p_1}{2}-\frac{p_2}{2})^2 - m_t^2]} \tag{A.7}$$

- $(\mathbf{k.p_1})^2$

$$\frac{1}{4}\int_{\infty}^{\infty} d^dk \frac{p_1.p_2}{[(k+\frac{p_1}{2})^2 - m_t^2][(k-\frac{p_2}{2})^2 - m_t^2]} -$$
$$\frac{-1}{4}\int_{\infty}^{\infty} d^dk \frac{p_1.p_2}{[(k+\frac{p_1}{2}+\frac{p_2}{2})^2 - m_t^2][(k-\frac{p_1}{2}-\frac{p_2}{2})^2 - m_t^2]}. \tag{A.8}$$

- $(k.p_2)^2$ This case is analogous to before and we obtain precisely the same result as for $(k.p_1)^2$ [12].

We can now bring to all integrals two basic forms, which we call the master integrals for LO gluon fusion

$$J(a,b) \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^2 - m_t^2][(k+a)^2 - m_t^2]} \tag{A.9}$$

and

$$I(a,b) \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^2-m_t^2][(k+a)^2-m_t^2][(k+b)^2-m_t^2]}, \quad a,b \in \Re \tag{A.10}$$

$A(m_t)$ can now be defined with these integrals [12]

$$A(m_t) = \underbrace{I(p_1,-p_2)\Big(\frac{4}{d-2}m_t^2 - p_1.p_2\Big)}_{\equiv A_I} + \underbrace{J(p_1+p_2)\frac{4-d}{d-2}}_{\equiv A_j} \tag{A.11}$$

A.0.2 Feynman parameters and dimensional regularization

Feynman-parameters are required to solve the two master integrals $J(a)$and $I(a,b)$. We can find these integrals in [13] page 190.

The Integral J(a): The following Feynman parameter is used because we have two factors in the denominator.

$$\frac{1}{P.Q} = \int_0^1 dxdy \frac{\delta(x+y-1)}{(xP+yQ)^2} = \int_0^1 dx \frac{1}{(xP+(1-x)Q)^2} \tag{A.12}$$

In the last step we have assessed the $\delta - function$. Applied to $J(a)$ this is

$$J(a) = \int \frac{d^dk}{(2\pi)^d} \int_0^1 dx \frac{1}{[(1-x)(k^2-m_t^2)+x((k+a)^2-m_t^2)]^2} \tag{A.13}$$

If we make the substitution $k-ax$, we get

$$J(a) = \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^2+a^2x(1-x)-m_t^2]^2} \tag{A.14}$$

Now we can fix the following formula found in [13] page 250

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-\triangle)^n} = \frac{(-1)^n i\Gamma(n-\frac{d}{2})}{(4\pi)^{\frac{d}{2}}\Gamma(n)} \Big(\frac{1}{\triangle}\Big)^{n-\frac{d}{2}} \tag{A.15}$$

where Γ is the Gamma-function, defined by $\int_0^\infty x^{y-1}e^{-x}dx$, especially $\Gamma(n+1) = n!$ for $n \in N^0$. This gives us

$$J(a) = \int_0^1 dx \frac{i\Gamma(2-\frac{d}{2})}{(4\pi)^{\frac{d}{2}} \underbrace{\Gamma(2)}_{=1}} \big(-x^{a2}+m^2+a^2x^2\big)^{\frac{d}{2}-n} \tag{A.16}$$

The following step is to use dimensional regularisation, i.e. we substitute $d = 4-2\epsilon$, where $\epsilon > 0$

$$J(a) = \int_0^1 dx \frac{i\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}} (m^2+a^2x^2-xa^2)^{-\epsilon} \tag{A.17}$$

In the end we can use approximation $\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$, where γ 0.5772 is the Euler-Mascheroni constant. Using

$$\frac{x^{-\epsilon}}{\epsilon} = \frac{e^{-\epsilon \ln x}}{\epsilon} \approx \frac{1-\epsilon \ln x}{\epsilon} = \frac{1}{\epsilon} - \ln x \tag{A.18}$$

We maintain the final result, expanded in powers of ϵ

$$J(a) = \frac{1}{\epsilon}\frac{i}{(4\pi)^2} - \frac{i\gamma}{(4\pi)^2} - \int_0^1 dx \frac{i\gamma}{(4\pi)^2} \ln{[m^2 + a^2x^2 - xa^2]}. \tag{A.19}$$

We now insert $J(a)$ into eq.$A.11$. Looking at the second term in the sum, A_j with $d = 4 - 2\epsilon$, we get

$$\begin{aligned} A_J = J(p_1 + p_2)\frac{4-d}{d-2} &\approx \frac{i}{(4\pi)^2}\left[1 - \epsilon\left(1 + \int_0^1 dx \ln{[m_t^2 + x(p_1+p_2)^2(x-2)]}\right)\right] \\ &\approx \frac{i}{(4\pi)^2} + O(\epsilon) \end{aligned} \tag{A.20}$$

We understand that it is definitely essential to keep the number of dimensions open. At the end we can take the limit $\epsilon \to 0$ and thus understand that the contribution from A_J implies 1.

The integral $I(a,b)$: Here there are three factors in the denominator, so we use the Feynman parameter

$$\begin{aligned} \frac{1}{P.Q.R} &= \int_0^1 dxdydz \frac{2\delta(x+y+z-1)}{(Px+Qy+Rz)^3} = \\ &= \int_0^1 dxdy \frac{2}{(Px+Qy+R(1-x-y))^3}, \end{aligned} \tag{A.21}$$

to get

$$I(a,b) = 2\int_0^1 dxdy \int \frac{d^dk}{(2\pi)^d} \frac{1}{[x((k+a)^2 - m_t^2) + y((k+b)^2 - m_t^2) + (1-x-y)(k^2 - m_t^2)]^3} \tag{A.22}$$

As before we want to carry the denominator of the integrand to the form $(k^2 - \triangle)^n$. We substitute $k \to k - ax - by$ and get

$$I(a,b) = 2\int_0^1 dxdy \int \frac{d^dk}{(2\pi)^d} \frac{1}{[a^2x(x-1) + b^2y(y-1) + 2abxy + m_t^2]^3} \tag{A.23}$$

As before we apply $(A.15)$ with $d = 4$, which gives us

$$2\int_0^1 dxdy \frac{-i\overbrace{\tau(1)}^{=1}}{(4\pi)^{\frac{d}{2}}\underbrace{\tau(3)}_{=2}} \frac{1}{a^2x(x-1) + b^2y(y-1) + 2abxy + m_t^2} \tag{A.24}$$

A.0.3 Expressing A_J with dilogarithms

We introduce our result for $I(a,b)$ into Eq$(A.11)$ and look at the first term in the sum, A_I

$$\begin{aligned} A_I &= (2m_t^2 - p_1.p_2)I(p_1, -p_2) \\ &= \frac{-i}{(4\pi)^2}\int_0^1 dxdy \frac{2m_t^2 - p_1.p_2}{\underbrace{p_1^2}_{=0} x(x-1) + \underbrace{p_2^2}_{=0} y(y-1) - 2(p_1p_2)xy + m_t^2} \end{aligned} \tag{A.25}$$

In the denominator of the integrand a small imaginary part is added

$$A_I = \frac{i}{(4\pi)^2} \frac{2m_t^2 - p_1.p_2}{(2p_1p_2)} \underbrace{\int_0^1 dy \int_0^{1-y} dx \left(\frac{1}{xy - \frac{m_t^2}{2(p_1p_2)} + i\epsilon} \right)}_{\equiv Int} \tag{A.26}$$

For on-shell Higgs Boson we have that $q^2 = m_h^2$ and we can see in the Feynman graph $(figure1) q = p_1 + p_2$. Therefore $2(p_1p_2)^2 = m_h^2$. We substitute $R \equiv (\frac{m_t}{m_h})^2$. Eq.$A.11$ now yields

$$A(m_t) = \frac{i}{(4\pi)^2} \left[\left(2R - \frac{1}{2} \right) Int + 1 \right] \tag{A.27}$$

Polylogarithms: So for our related quantity A, the only thing that is left to calculate is Int. First we want to define this in terms of dilogarithms. The polylogarithm is a function $Li_s(z)$, for all complex numbers s and $|z| < 1$, defined by [12]

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s} \tag{A.28}$$

We can understand directly that the polylogarithm for $s = 1$ is $Li_s(z) = -\log(1-z)$. Another way is to express the polylogarithms recursively

$$Li_{s+1}(z) = \int_0^z dt \frac{Li_s(t)}{t} \tag{A.29}$$

The dilogarithm is the polylogarithm for $s = 2$. Using the above we can understand quite easily that the dilogarithm can be defined as

$$Li_2(z) = \int_0^z dt \frac{-\log(1-t)}{t} \tag{A.30}$$

With this information we can go on [12]

$$Int = \int_0^1 \frac{dy}{y} \int_0^{1-y} \frac{1}{x - \frac{R}{y} + i\epsilon} dx = \int_0^1 \frac{dy}{y} \log \left(\frac{1}{R} (y^2 + -y + R + i\epsilon) \right). \tag{A.31}$$

We factorise

$$(y^2 - y + R + i\epsilon) = \left(y - \underbrace{\frac{1}{2} - \sqrt{\frac{1}{4} - R}}_{\equiv \lambda_1} + i\epsilon \right) \left(y - \underbrace{\frac{1}{2} - \sqrt{\frac{1}{4} - R}}_{\equiv \lambda_2} - i\epsilon \right) \tag{A.32}$$

Now a case study is required.

Case a) For $R > \frac{1}{4} \Rightarrow \sqrt{\frac{1}{4} - R}$ is imaginary. So we don't want to $i\epsilon$. Now we can use $\log(ab) = +$ if $sgn(\varsigma a) = -sgn(\varsigma b)$.

$$\begin{aligned} Int &= \int_0^1 \frac{dy}{y} \left[\log \left(\frac{1}{\sqrt{R}} (y - \lambda_1) \right) + \log \left(\frac{1}{\sqrt{R}} (y - \lambda_2) \right] \right) \\ &= \int_0^1 \frac{dy}{y} \left[\underbrace{\log(\frac{\lambda_1}{\sqrt{R}}) + \log(\frac{\lambda_2}{\sqrt{R}})}_{=0} + \log(\frac{y}{\lambda_1} - 1) + \log(\frac{y}{\lambda_2} - 1) \right]. \end{aligned} \tag{A.33}$$

Using the dilogarithms above, this can be written as [12]

$$Int = -Li_2\left(\frac{\lambda_2}{R}\right) - Li_2\left(\frac{\lambda_1}{R}\right), \tag{A.34}$$

where we have used $\frac{1}{\lambda_1} = \frac{\lambda_2}{R}$ and $\frac{1}{\lambda_2} = \frac{\lambda_1}{R}$.

Case b). For $R < \frac{1}{4} \Rightarrow \sqrt{\frac{1}{4} - R}$ is real. We want to keep the $i\epsilon$. So we have

$$Int = \int_0^1 \frac{dy}{y} \log\left(\frac{1}{R}(y - \lambda_1 + i\epsilon)(y - \lambda_2 - i\epsilon)\right) \tag{A.35}$$

$$= -\left\{Li_2\left(\frac{\lambda_1}{R} + i\epsilon\right) + Li_2\left(\frac{\lambda_2}{R} - i\epsilon\right)\right\}. \tag{A.36}$$

Claim: a)

$$f(\tau) = \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) = \frac{1}{2}\left\{Li_2\left(\frac{\lambda_1}{R}\right) Li_2\left(\frac{\lambda_2}{R}\right)\right\}, \quad \tau \geq 1, \tag{A.37}$$

where $\tau \equiv 4R$

Proof: a). The next formulas are useful:

1. $\arcsin x = -i \log\left(ix + \sqrt{1 - x^2}\right)$
2. $\frac{-1}{2} \log^2(x) = Li_2(1 - x) + Li_2\left(1 - \frac{1}{x}\right)$
3. $Li_2(x) = -\int_0^1 \frac{dy}{y} \log(1 - x.y)$

With these the proof is above board [12].

$$2f(\tau) = 2\arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) \tag{A.38}$$

$$= -\frac{1}{2} \log^2\left[\left(\frac{i}{\sqrt{\tau}} + \sqrt{1 - \frac{1}{\tau}}\right)^2\right] \tag{A.39}$$

$$= Li_2\left[2\left(\frac{1 - i\sqrt{\tau - 1}}{\tau}\right)^2\right] + Li_2\left[1 - \left(\frac{\sqrt{\tau}}{i + \sqrt{\tau - 1}}\right)^2\right] \tag{A.40}$$

$$= Li_2\left[2\left(\frac{1 - i\sqrt{\tau - 1}}{\tau}\right)\right] + Li - 2\left[2\left(\frac{1 + i\sqrt{\tau - 1}}{\tau}\right)^2\right] \tag{A.41}$$

$$= \frac{1}{2}\left\{Li_2\left[\frac{1}{R}\left(\frac{-1}{2} - \sqrt{\frac{1}{4}}\right)\right] + Li_2\left[\frac{1}{R}\left(\frac{-1}{2} + \sqrt{\frac{1}{4}}\right)\right]\right\} \tag{A.42}$$

Claim: b)

$$\frac{1}{2}\left\{Li_2\left(\frac{\lambda_1}{R} + i\epsilon\right) + Li_2\left(\frac{\lambda_2}{R} - i\epsilon\right)\right\} = -\frac{1}{4}\left(\log\left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}}\right] - i\pi\right)^2 \tag{A.43}$$

Proof: b) Using 'proof: a) item 2' with

$$x = \frac{\tau - 2 - 2\sqrt{1 - \tau}}{\tau} - i\epsilon, \tag{A.44}$$

We get

$$\left\{Li_2\left(\frac{\lambda_1}{R}+i\epsilon\right)+Li_2\left(\frac{\lambda_2}{R}-i\epsilon\right)\right\} = -\frac{1}{2}\left(\log\left[\frac{\tau-2-2\sqrt{1-\tau}}{\tau}-i\epsilon\right]\right)^2$$
$$= -\frac{1}{2}\left(\log|\frac{\tau-2-2\sqrt{1-\tau}}{\tau}|-i\pi\right)^2 \tag{A.45}$$
$$= -\frac{1}{2}\left(\log\left[\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right]-i\pi\right)^2$$

Bringing everything together the final result is [12]

$$A = \frac{1}{(4\pi)^4}|1+(1-\tau)f(\tau)|, \tag{A.46}$$

$$f(\tau) = \begin{cases} \arcsin^2\frac{1}{\sqrt{\tau}}, & \tau \geq 1, \\ -\frac{1}{4}\left(\log\left[\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right]-i\pi\right)^2, & \tau < 1, \end{cases} \tag{A.47}$$

$$\tau = 4\left(\frac{m_t}{m_h}\right)^2 \tag{A.48}$$

Appendix B

Integration Of Form Factors

In this section we calculate form factor $I\left(\frac{m_h^2}{m_f^2}\right)$ in detail for Higgs decay to photons [15].

$$
\begin{aligned}
I\left(\frac{m_h^2}{m_f^2}\right) &= \int_0^1 dx \int_0^{1-x} dz \frac{1-4xz}{1-xz\frac{m_h^2}{m_f^2}} \\
&= \int_0^1 dx \left[\frac{1}{-x\frac{m_h^2}{m_f^2}} \ln(1-xz\frac{m_h^2}{m_f^2}) - 4x\left(\frac{z}{-x\frac{m_h^2}{m_f^2}} - \frac{1}{(-x\frac{m_h^2}{m_f^2})^2}\ln(1-xz\frac{m_h^2}{m_f^2})\right)\right]_{z=0}^{z=1-x} \\
&= \int_0^1 dx \left[-\frac{m_f^2}{xm_h^2}\ln(1-x(1-x)\frac{m_h^2}{m_f^2}) + \frac{4x(1-x)m_f^2}{xm_h^2} + \frac{4xm_f^4}{x^2m_h^4}\ln(1-x(1-x)\frac{m_h^2}{m_f^2})\right] \\
&= \int_0^1 dx \left[\left(\frac{4m_f^4}{m_h^4} - \frac{m_f^2}{m_h^2}\right)\frac{\ln\left(1-x(1-x)\frac{m_h^2}{m_f^2}\right)}{x} + \frac{4(1-x)m_f^2}{m_h^2}\right] \\
&= \left[\frac{4(x-\frac{x^2}{2})m_f^2}{m_h^2}\right]_{x=0}^{x=1} + \frac{m_f^2}{m_h^2}(4\frac{m_f^2}{m_h^2}-1)\int_0^1 dx \frac{\ln(1-x(1-x)\frac{m_f^2}{m_h^2})}{x} \\
&= 2\frac{m_f^2}{m_h^2} + \frac{m_f^2}{m_h^2}(4\frac{m_f^2}{m_h^2}-1)\int_0^1 dx \frac{\ln(1-x(1-x)\frac{m_f^2}{m_h^2})}{x}
\end{aligned}
\tag{B.1}
$$